EXPERIMENTAL ANALYSIS OF THE VIBRATION OF VARIABLE THICKNESS PLATES

Thesis for the Degree of M. S.

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Myrl W. Thompson

1962

This is to certify that the

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EXPERIMENTAL ANALYSIS OF THE
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Myrl W. Thompson

has been accepted towards fulfillment of the requirements for

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C. A. Tatro
Major professor

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by

Myrl W. Thompson

AN ABSTRACT

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ABSTRACT

Previous work in the Applied Mechanics Department at Michigan State University has resulted in the theoretical analysis of the various mode shapes of vibrating, variable thickness plates. The objectives of this thesis are to develop an experimental method of measuring the relative amplitudes of vibration of plates and to make readings of these amplitudes for variable thickness plates such that a comparison can be made between the experimental and theoretical results that are available.

The development of equipment proved to be a major part of the work that was done in this investigation. The capacitive transducer that was used as a measuring device was a variable-gap, parallel plate capacitor having a shielded probe as one of its plates and the vibrating plate itself as its second parallel capacitor plate. Readings were taken at each of 225 data points by the probe as it was carried over the plate by a traverse mechanism that was developed for the purpose.

The experimental results that were obtained indicated that further refinements of the capacitive probe were needed to insure accurate measurement of the more complicated mode shapes. The fair to good agreement between the theoretical and experimental results for simpler mode shapes and the reliability exhibited by the capacitive probe in static tests warrant its further development.

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The author wishes to take this opportunity to acknowledge the guidance and assistance given by Dr. Clement A. Tatro in this investigation. The method of displacement measurement was, in fact, the result of a suggestion made by Dr. Tatro. The author also wishes to express gratitude to Dr. William A. Bradley for his assistance in the work and to Mr. Carroll Lenning of Oldsmobile Division of General Motors Corporation for the financial assistance that was made available.

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I. Introduction

Background

Several individuals in the past generations of master's and doctoral candidates have worked on variable thickness plates in the Applied Mechanics Department at Michigan State University. The experimental work that has been done has resulted in equipment that is being used in a continuing investigation of the behavior of plates in vibration and under static loading.

Purpose

The purpose of this investigation is to experimentally verify the finite difference approximation of mode shapes for vibrating plates made by someone else. These calculations predict the mode shapes of a variable thickness, aluminum plate and provide approximations of the relative amplitudes of vibration to be expected.

An important part of the work done in fulfilling the objectives of this investigation has been the development and proving of measuring instruments and equipment.

Order of Reporting

The presentation of the work done in this investigation will be given according to the following order of reporting.

1) Objectives and Procedure of Experimental Analysis
2) Experimental Equipment
3) Presentation of the Results

II. Objectives of Experimental Analysis

The objectives of this thesis stem from a recommendation made by a previous investigator of variable thickness plates. This recommendation...

> "...(to develop)... a technique to measure the amplitudes in the vibrating plate."

came about after numerical calculations had been made to approximate the relative amplitudes of the various modes of vibration.[1] The following sections present a summary of the work previously done with these plates and outlines the work done in this thesis to extend the investigation.

Summary of Previous Work

In as much as the purpose of this thesis is to experimentally verify the results of finite difference calculations, it is appropriate at this time to present a brief summary of the numerical method and the results that it produced.

The Finite Difference Method The theoretical analysis of plate deflections, either under static loading or in vibration, requires the solution of differential equations which describe the behavior of the plate. The finite difference method offers a means of approximating the solution of these differential equations whose exact solution is very difficult or perhaps even impossible in some cases. In applying the finite difference method to plates, the plate

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[1] Raju, B. Basava, Bending and Vibration in Plates of Variable Thickness. Michigan State University Ph.D. Thesis, Applied Mechanics, 1961.

is first divided with a grid such as that shown in Figure 1 page 13 . Note in the Figure the designation of the intersection points of the grid. It is at these points that the differential equation is approximated. The approximations used in a plate analysis may be stated as...

$$\frac{\partial f_o}{\partial x} = \frac{f_1 - f_2}{2h} \qquad (1)$$

$$\frac{\partial^2 f_o}{\partial x^2} = \frac{f_1 - 2f_o + f_2}{h^2} \qquad (2)$$

where in the case of plate deflection, f is a function of the deflections, where x is the coordinate in the direction of the deflection, and where h is the size of the grid over which the approximation is made. Note that in the equations (1) and (2) above that the subscripts 1 and 2 refer to points just to either side of some point o at which the differential equation is being approximated. In actual practice, the subscripts are replaced with the subscripts e,w,n, and s to indicate the function (f) evaluated at the points that are respectively east, west, north, or south of the point o. Forpurposes of this thesis it is sufficient to say that this method requires the application of boundary conditions such that a series of simultaneous algebraic equations are obtained which may be solved. The solution of these simultaneous equations using a matrix method produces the desired approximate solution of the differential equations of the plate.

This then is the basic method that was used in the theoretical analysis of variable thickness plates. The method, although rather simple in concept, is tedious in application to an involved problem such as that presented by the variable thickness plates. The actual results that are to be experimentally verified in this thesis were obtained using the digital computer which greatly reduced the effort and time involved in solving the differential equations that were encountered.

Predicted Mode Shapes The results that were obtained by a previous investigator using the finite difference method described above are approximations of the relative amplitudes of vibration of variable thickness plates. The results for the one plate that was experimentally measured in this thesis are presented on the following pages 5 through 10 in Tables 1 through 6. This one plate, that designated plate as plate "A" in the reference cited at the bottom of page 2, was chosen because the clamped edge promised less boundary condition problems than the other harder-to-obtain simply supported edges. It was felt that in as much as the technique of measurement to be used was essentially untried, it would be best to not introduce unnecessary complications.

The finite difference results are presented for a grid which is one-half that used in the experimental work. The theoretical mode shapes are shown in pictorial views in a later chapter for each of the mode shapes in Figures 7 through 12 on pages 41, 43, 45, 47, 49, and 51, respectively.

	1	2	3	4	5	6	7
1	.018	.064	.110	.129	.110	.064	.018
2	.064	.217	.370	.433	.370	.217	.064
3	.110	.370	.665	.794	.665	.370	.110
4	.129	.433	.794	1.00	.794	.433	.129
5	.110	.370	.665	.794	.665	.370	.110
6	.064	.217	.370	.433	.370	.217	.064
7	.018	.064	.110	.129	.110	.064	.018

The data shown on this page for the first mode and on the following pages for modes 2 through 6 is that obtained by B. Basava Raju in his Bending and Vibration in Plates of Variable Thickness. (See reference at bottom of page 2).

Table 1 First Mode-Finite Difference Results

	1	2	3	4	5	6	7
1	.067	.166	.150	.000	-.150	-.166	-.067
2	.200	.506	.468	.000	-.468	-.506	-.200
3	.323	.826	.841	.000	-.841	-.826	-.323
4	.372	.958	1.00	.000	-1.00	-.958	-.372
5	.323	.826	.841	.000	-.841	-.826	-.323
6	.200	.506	.468	.000	-.468	-.506	-.200
7	.067	.166	.150	.000	-.150	-.166	-.067

Table 2 Second Mode-Finite Difference Results

1	.191	.421	.365	.000	-.365	-.421	-.191
2	.421	1.00	.896	.000	-.896	-1.00	-.421
3	.365	.896	.929	.000	-.929	-.896	-.365
4	.000	.000	.000	.000	.000	.000	.000
5	-.365	-.896	-.929	.000	.929	.896	.365
6	-.421	-1.00	-.896	.000	.896	1.00	.421
7	-.191	-.421	-.365	.000	.365	.421	.191
	1	2	3	4	5	6	7

Table 3 Third Mode-Finite Difference Results

	1	2	3	4	5	6	7
1	.000	.099	.365	.497	.365	.099	.000
2	-.099	.000	.626	1.00	.626	.000	-.099
3	-.356	-.626	.000	.589	.000	-.626	-.356
4	-.497	-1.00	-.589	.000	-.589	-1.00	-.497
5	-.356	-.626	.000	.589	.000	-.626	-.356
6	-.099	.000	.626	1.00	.626	.000	-.099
7	.000	.099	.356	.497	.356	.099	.000

Table 4 Fourth Mode-Finite Difference Results

	1	2	3	4	5	6	7
1	.081	.173	.173	.146	.173	.173	.081
2	.173	.363	.282	.163	.282	.363	.173
3	.173	.282	-.053	-.387	-.053	.282	.173
4	.146	.163	-.387	-1.00	-.387	.163	.146
5	.173	.282	-.053	-.387	-.053	.282	.173
6	.173	.363	.282	.163	.282	.363	.173
7	.081	.173	.173	.146	.173	.173	.081

Table 5 Fifth Mode-Finite Difference Results

1	.255	.490	.375	.000	-.375	-.490	-.255
2	.380	.774	.554	.000	-.554	-.774	-.380
3	.065	-.008	-.243	.000	.243	.008	-.065
4	-.186	-.647	-1.00	.000	1.00	.647	.186
5	.065	-.008	-.243	.000	.243	.008	-.065
6	.380	.774	.554	.000	-.554	-.774	-.380
7	.255	.490	.375	.000	-.375	-.490	-.255
	1	2	3	4	5	6	7

Table 6 Sixth Mode-Finite Difference Results

Qualitative Measurements Work has been done that has qualitatively verified the mode shapes predicted by the finite difference method. This previous work amounted to using fine sand or glass beads on the vibrating plate to outline the nodes of vibration such that each mode shape could be observed and photographed.[2] The results of the qualitative work that was done show that good clear mode shapes were obtainable by the previous investigator. The symmetry exhibited by these photographs of the glass beads outlining the nodal lines appears to be good. This same method of using glass beads on the vibrating plate was used in the present work to adjust the frequency and amplitude of the vibration such that the clearest and finest mode shapes could be obtained. It should also be noted at this point that although good outlines of the modes were obtained in this investigation, the degree of symmetry in the second, third, and fourth modes was not as sharply defined as those presented in the reference above. Instead, it was found that certain nodal lines tended to blur as if there was not a good node existing. Once the mode shape had been obtained using the glass beads, the quantitative measurements to be presented later were taken with the equipment that will be described later in this thesis.

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[2] Raju, B. Basava, Bending and Vibration in Plates of Variable Thickness. Michigan State University Ph.D. Thesis, Applied Mechanics, 1961. See Figure 30, page 59.

<u>Experimental Analysis</u>

The experimental work that was done made use of a plate that was used in the qualitative measurements taken by a previous investigator. This plate was the same as one used in the theoretical analysis described in the <u>Summary of Previous Work</u> section previously presented.

<u>The Plate</u> The plate used in this work was a square aluminum plate of variable thickness. This plate was held rigidly in a heavy frame such that its edges were effectively clamped. The variable thickness of the plate was obtained by machining a .125 of an inch thick plate such that the plate was symmetrical about 4 axes. The thickness variation was from .125 of an inch at its edges to .0625 of an inch at its center as shown in Figure 1 , page 13.

To facilitate the measurement of the displacement at each point of the plate, the flat surface was marked with a grid. This grid was drawn such that it was approximately twice as fine as that used in the theoretical analysis using the finite difference method. The intersection points of the grid were then taken as the measuring points during the tests. The 15 space by 15 space grid resulted in 225 points of measurement for each of the six mode shapes investigated.

<u>Quantitative Measurements</u> In experimentally verifying the results of the finite difference analysis, there are

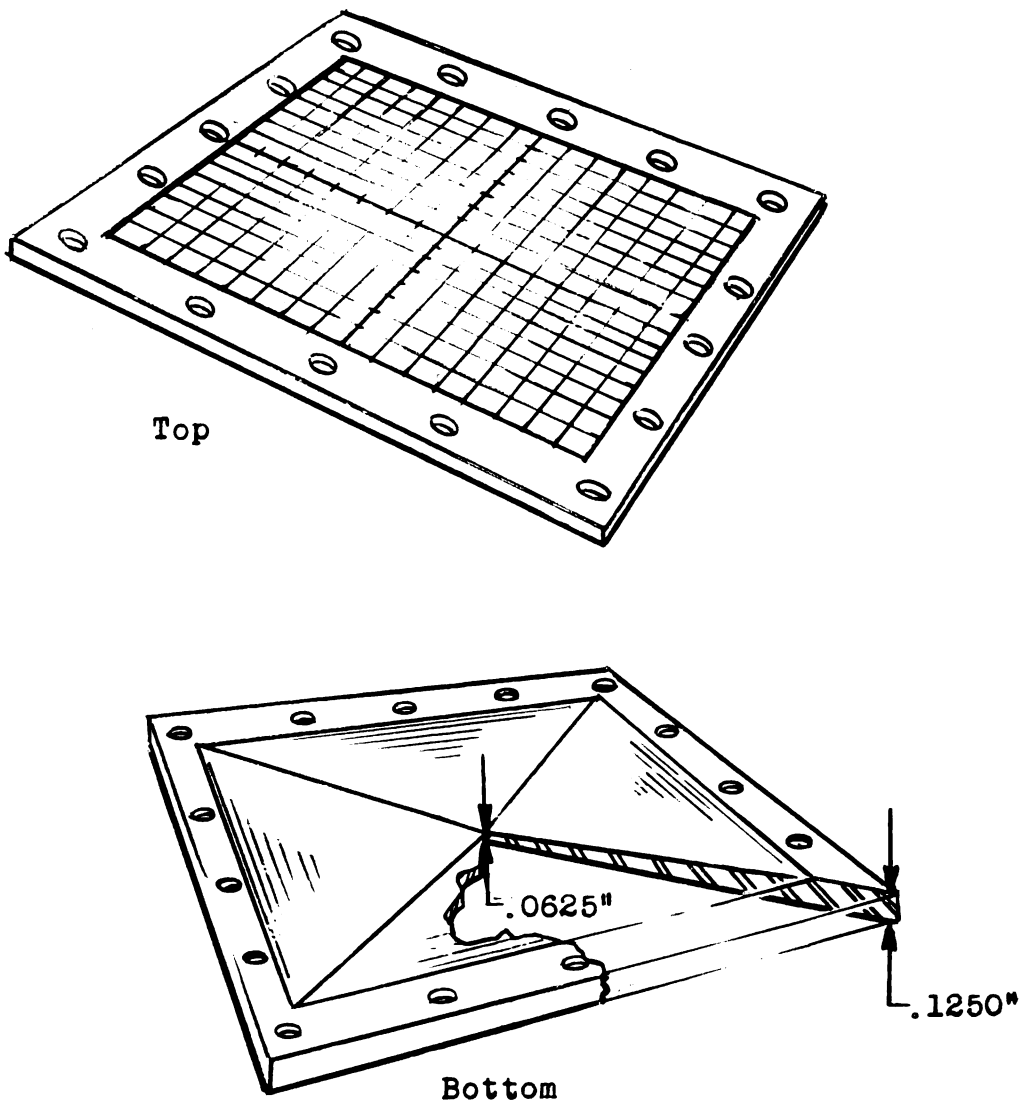

Figure 1 The Variable Thickness Plate

certain specific areas of interest. These areas are listed below...

1) To establish the accuracy and reliability of the measuring technique.

2) To obtain the relative amplitudes of vibration and to check their agreement with predicted results.

3) To analyze the actual symmetry of the data obtained in the light of the symmetry expected with the plate that was used.

The procedure used and the equipment involved in the investigation of the above areas of interest are presented in the following chapter.

III. Experimental Equipment

A great deal of the apparatus and equipment that was used in this investigation was available as the result of previous work. The work that was required to adapt the existing equipment to this investigation proved to be, however, a large part of the work that was done. Specifically, the development and calibration of the capacitive transducer required extensive work to establish its accuracy and reliability in the measurement of the deflection of plates. This transducer as well as the circuitry it required, and the two degree of movement traverse mechanism that was designed are presented in the following section.

Transducer Design and Application

The choice of the transducer to measure the dynamic displacement of a vibrating plate was based on the following criteria:

1) The transducer must have a sensitivity that will enable it to read changes in deflection of .0001 inch.
2) The transducer must not change the behavior of the plate in its application...i.e. it must not touch the plate or restrain its deflection.
3) It must have sufficient stability to enable a set of readings to be taken over a period of time.

On the basis of these criteria, the capacitive transducer was chosen for the deflection measurements. Not only did it theoretically satisfy each of the requirements, but it also offered an ease of application which further prompted its choice.[3]

<u>Principle of Operation</u> The principle stating the relationship between the capacitance of a two plate capacitor and its dimensions is...

$$C = .225 \frac{\epsilon_0 A}{d} \qquad (1)$$

where ϵ_0 is the dielectric constant of the material between the two plates of the capacitor, A is the overlapping area of the two plates, and d is the distance between the two plates. Note in equation (1) above the inverse (hyperbolic) relationship of capacitance (C) to the distance between the plates (d). It is this hyperbolic relationship, referred to later as the Calibration Curve, upon which the conversion of data from relative capacitance to amplitude of deflection will be made. A typical calibration curve is shown in Figure 2 , page 17.

By using the plate itself as one-half of a variable capacitor, it is possible to construct a circuit in which

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[3] To avoid confusion it should be noted at this point that the capacitive transducer is often referred to as the capacitive probe or pickup in the remainder of this text. This nomenclature is the result of common usage.

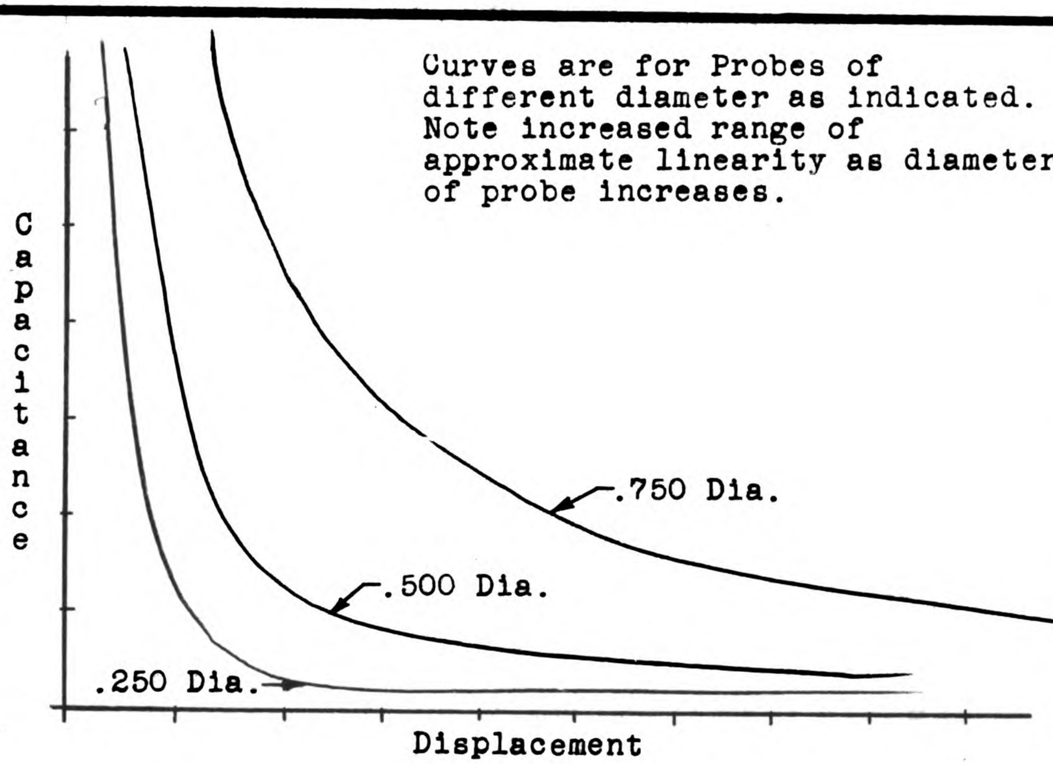

Figure 2 Capacitance vs. Displacement Curves

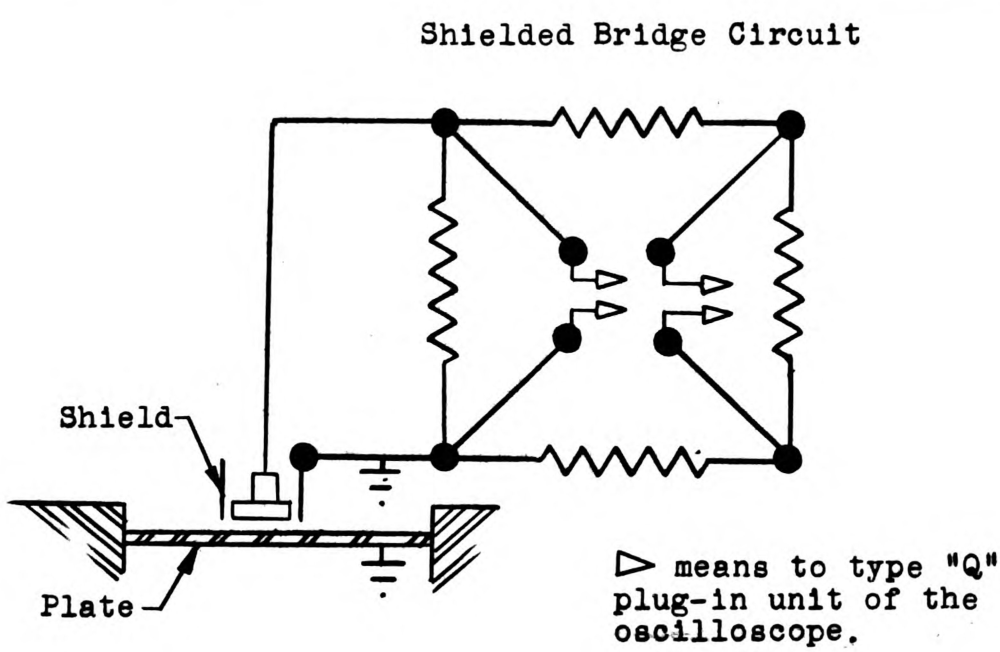

Figure 3 Schematic Diagram of Circuit

the capacitance varies with the amplitude of displacement of the plate. The variable capacitor and the circuit that it requires are shown schematically in Figure 3 , page 17. Note the amplifying bridge circuit in Figure 3 . This amplification was necessary so that the signal could be observed on the Tektronix Oscilloscope.

<u>Sensitivity</u> Because of the hyperbolic relationship between capacitance and displacement, the capacitive transducer has a theoretically infinite sensitivity. In other words, as the distance between the plates of the capacitor becomes smaller, the change in capacitance per unit change in displacement becomes greater. In actual practice, there is a limit to the sensitivity caused by the voltage breakdown limit of the gap between the plates of the capacitor.[4]

The sensitivity of the capacitive probe can be brought to within .0001 of an inch without having the probe touch the plate by choosing the proper reference distance between the plates of the capacitor and the corresponding probe diameter. The probe finally chosen for measurement of the plate deflections was a .750 of an inch diameter probe with an initial gap of .0300 of an inch between the plates of the capacitor. These dimensions were the result of a compromise between sensitivity, amplitude of oscilloscope response, and of the range of reasonable capacitance vs. displacement linearity.

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[4] <u>Instrumentation in Scientific Research</u>, Kurt S. Lion, McGraw Hill Book Company, Inc., page 67.

Plate Restraint The second criterion that the transducer have no effect on the behavior of the plate is well satisfied by the capacitive probe. Not only is there no physical contact between the probe and the plate, but the forces exerted by the plates of the capacitor upon each other are found to be small. The principle expressing this force is...

$$F = \frac{8.85 \times 10^{-12}}{2} \frac{E^2 A}{d^2} \qquad (2)$$

where E is the voltage applied across the plates of the capacitor, A is the area of the overlapping plates, and d is the distance between the plates. If we use the dimensions of the capacitive probe used in this work in the above equation (2) and make the calculation, we find that the forces involved are of the order of a few dynes...where 10^5 dynes = 1 pound of force.

The vibrating plate will also displace air from between the capacitive probe and the plate, and this too could very well be a source of force exerted on the plate. Considerations given this problem in this work are presented in the Appendix of this Thesis on page 67.

Stability The criterion of stability is the most difficult to satisfy in the case of the capacitive probe. Not only is capacitance a function of temperature, pressure, and humidity, but it also is affected by stray signals, noise in the highly amplifying circuitry, and the surrounding background vibration. This problem is best approached with extensive shielding of all leads and the capacitive probe itself, and by such precautions as taping all leads down to prevent movement and noise in the circuit, by using rubber insulation where possible, and by using very heavy supports for the plate and probe carriers so that surrounding vibration from running machinery and operator movement could be damped out. As a result of these precautions, all problems of stability were sufficiently reduced except for an annoying internal drift in capacitance caused by extreme drafts or temperature changes.[5]

The setup in its final form is shown in Figure 4, page 21. The cross section shown is a detailed drawing of the capacitive transducer shown in the photograph.

<u>Static Calibration</u>

This part of the development of the capacitive probe was necessary to establish the accuracy and reliability of the capacitive pick-up. The preliminary calibration attempts amounted to a comparison of the deflection measured with a dial indicator with those measured with the probe. In the final calibration tests, the same plate that was to be used in the dynamic vibrations tests was uniformly loaded for the static tests. Measurements of this deflected plate were then taken again with both the dial indicator and the capacitive probe that by this time had been fully developed.

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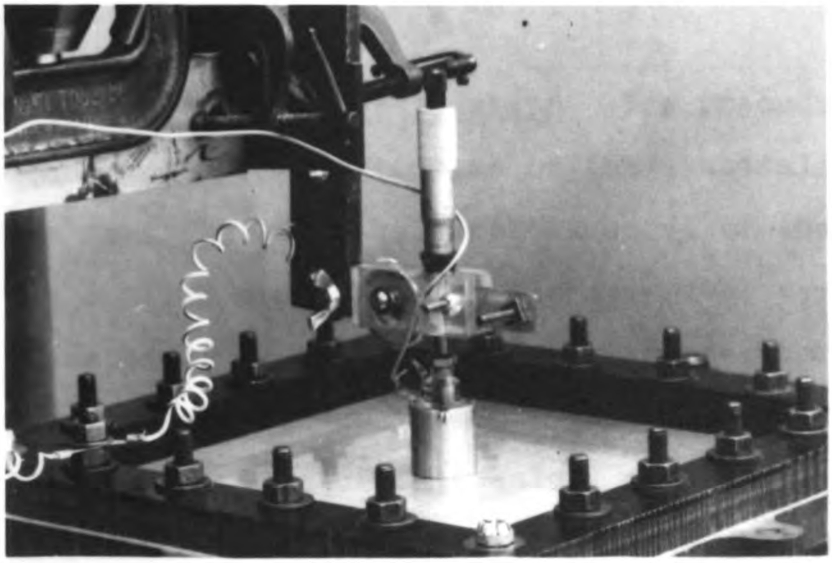
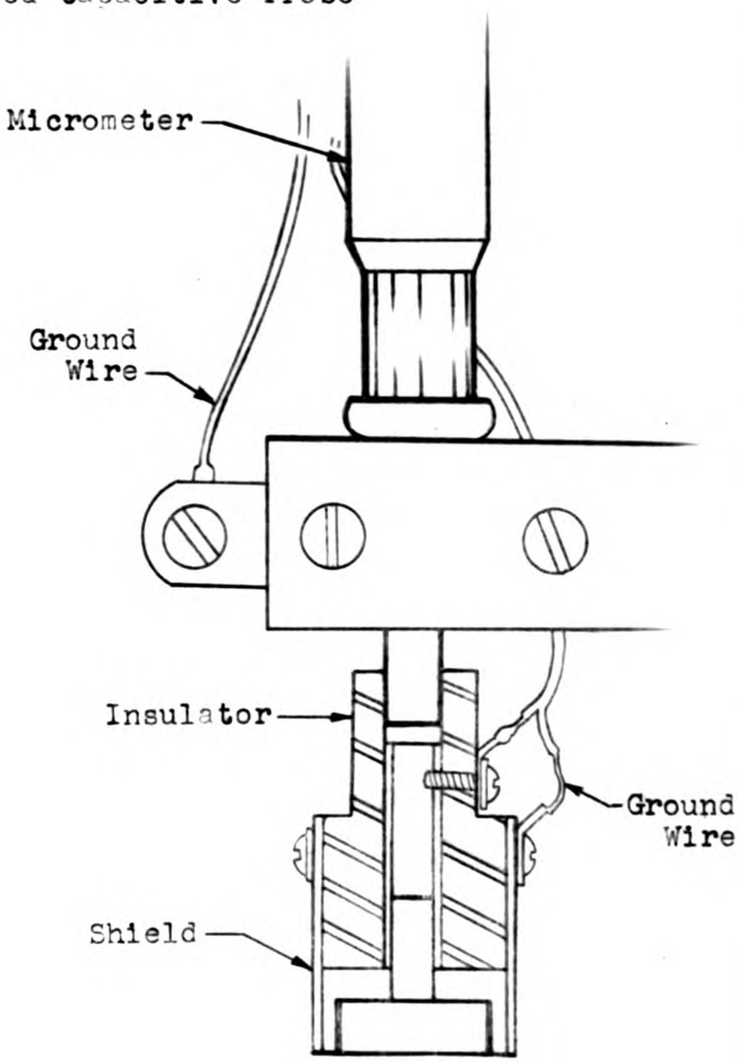
[5] This problem was resolved by eliminating the source of the drafts and temperature changes. Data was taken late at night when traffic through the area was nonexistent.

Figure 4

The Shielded Capacitive Transducer
(See Next Page)



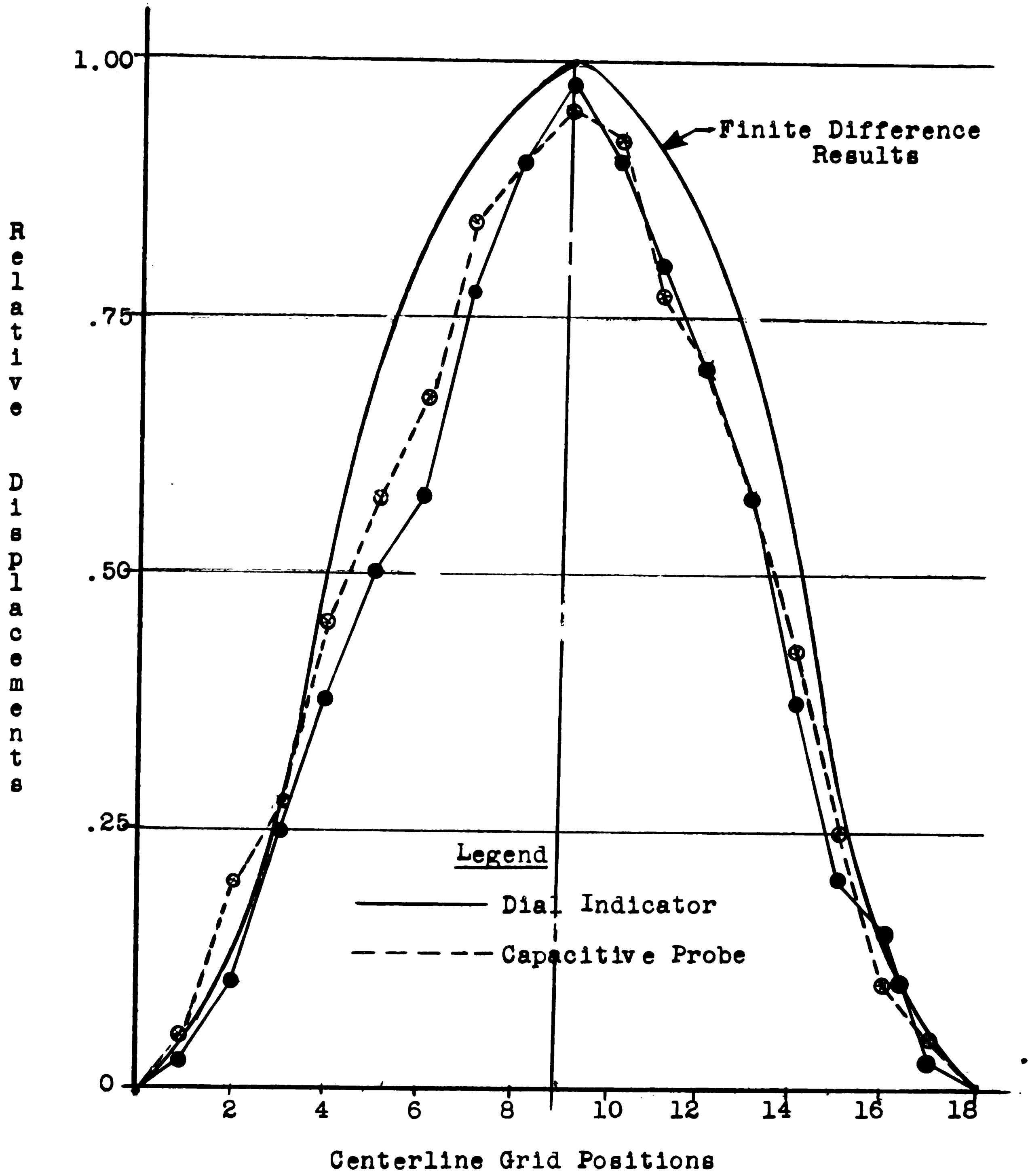
The Shielded Capacitive Probe



Procedure of Calibration The procedure used in determining the deflection of the statically loaded plate was to first run a calibration curve of the probe. This was nothing more than just an accurate, very detailed capacitance vs. distance-between-plates curve. This curve was then used to convert the relative voltage changes read from the oscilloscope to actual amplitudes of deflection of the uniformly loaded static plate. This conversion not only resulted in quantitative amplitudes of deflection, but also corrected for the slight non-linearity of the capacitance vs. displacement relationship that existed in the range in which the probe was used.

Once the calibration curve was determined, the probe was carefully gapped at .0300 of an inch from the center of the plate at zero load. From this point a complete set of readings was taken of the 225 grid points of the plate in its undeflected condition. This gave a picture of the plate surface before loading. This step was necessary since the plate was not perfectly flat. After taking readings of the loaded plate, the total deflection of the plate from unloaded to loaded condition then was the difference between initial and final readings at each point of the plate. Readings were taken at each point of the plate by moving the probe with the traverse mechanism that had been designed for the purpose.

The measurements that were obtained using the probe and the dial indicator were then compared to the results of



Graph 1 Comparison of Capacitive Probe, Dial Indicator, and Finite Difference Results- Static Test.

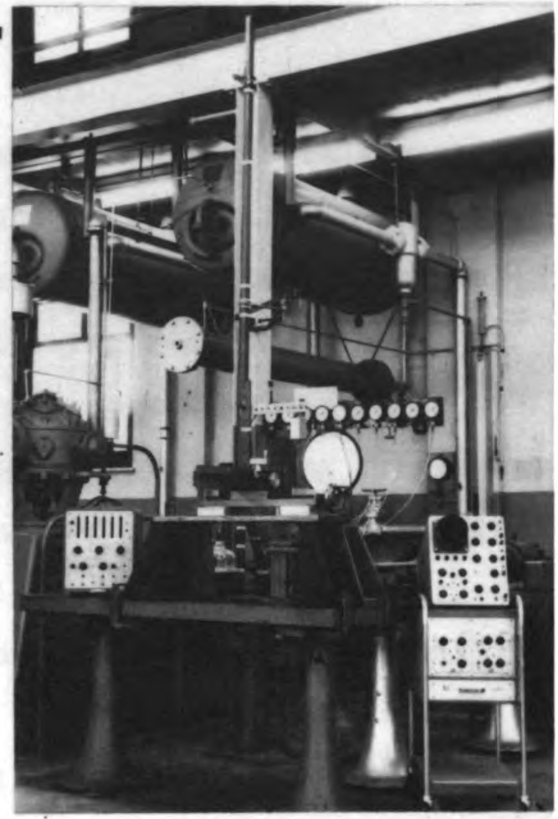
a finite difference analysis which were available. The results of this comparison are shown in Graph 1 , page 24. From this graph it can be seen that there is good agreement between the three sets of data plotted. A capacitive probe will, then, give an accurate representation of a deflected plate under static loading. There remains now only the application of this probe to the dynamic vibration of the plate.

Equipment of Calibration The equipment used in the calibrations tests was the same as that used later in the vibrations work, except for the method of loading. In the calibration work, the load was applied with a head of water through a water bag placed in a closed chamber below the plate. The water pressure standpipe and the apparatus used for calibration are shown in Figure 5 , page 27 . The water sac was made of plastic material bonded together with Goodyear Pliobond Cement.

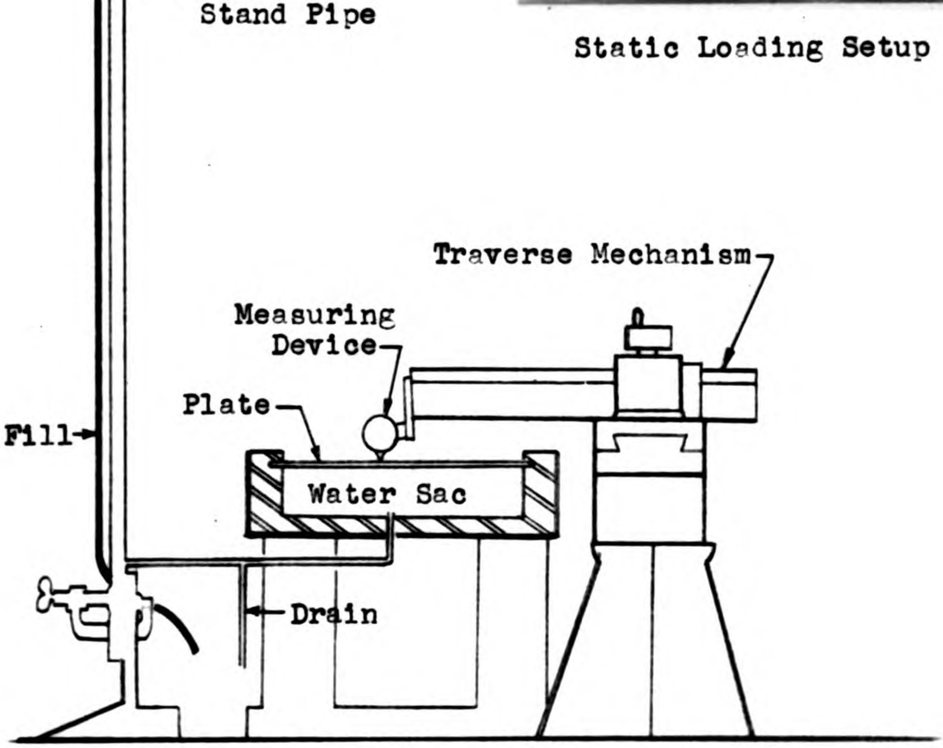
In obtaining the calibration curve, which was required only for the static tests.....a large diameter plate was mounted on a micrometer head which enabled data to be taken for the plotting of the calibration curve. The data for the curve was taken as the deflection of the trace on the oscilloscope screen. The oscilloscope was rebalanced after each full scale deflection of the trace. The data taken showed excellent repeatibilitv. It should be mentioned, however, that any geometric change of the probe or its components requires that a new calibration curve be run.

Figure 5

Static Calibration Apparatus
(See Next Page)



Static Loading Setup



Dynamic Measurements

The actual taking of dynamic data represented the final realization of the objective of the investigation. The equipment used in these measurements and the procedure that was followed are presented in the following section.

Equipment In the experimental work that was done, it was necessary to vibrate the plate at both varying frequency and amplitude. This required range of vibration was obtained with a siren and a high volume air compressor that provided a pulsating air blast which was used to vibrate the plate. Measurements of the amplitude of vibration of the plate were taken at each of the 225 points on the plate by moving the capacitive probe across the plate with the traverse mechanism. In as much as the compressor and siren were available for use from previous work[6], only a detailed description of the traverse mechanism will be presented here.

Measurement of the amplitude of vibration of the plate required that the traverse mechanism have two degrees of motion that could maintain an accurate vertical position of the capacitive probe. For this reason, the traverse was built heavy enough so that there would be no problem in keeping the probe in position with respect to the plate. The traverse mechanism, shown in Figure 6 , page 30.

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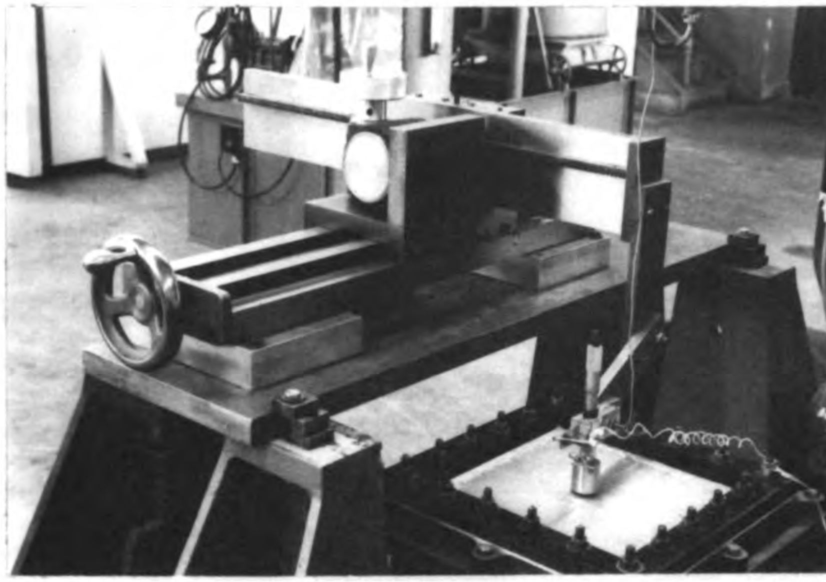
[6] For detailed description of the equipment and the compressor, see Raju, B. Basava, Bending and Vibration in Plates of Variable Thickness. Michigan State University Ph.D. Thesis, Applied Mechanics, 1961.

Figure 6

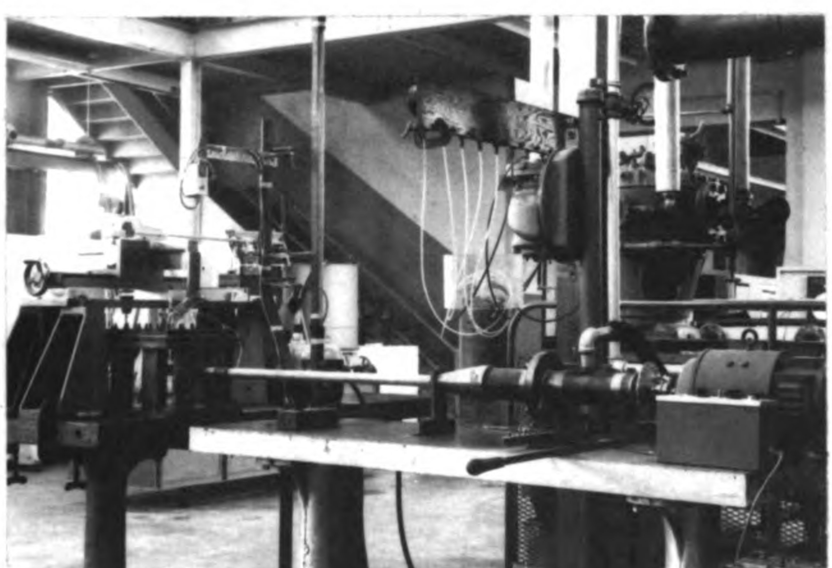
The Traverse Mechanism
(Next Page)

Figure 7

The Setup in Final Form
(Next Page)



THE TRAVERSE MECHANISM



THE SETUP IN FINAL FORM

consists primarily of two high quality dovetails. As shown in Figure 6, the larger screw driven slide supports the small dovetail and the rack and pinion driven slide. It is this smaller cross slide that supports the capacitive probe. The final setup is shown in Figure 7, page 30.

<u>Procedure</u> Dynamic measurements of the amplitude of vibration of the vibrating plate were made in a manner similar to that used in the static calibration work. Readings of the change in capacitance were read from the oscilloscope as the peak to peak height of a signal displayed on the screen. This signal was that read from the circuit containing the variable capacitor consisting of the capacitive probe and the vibrating plate. Readings were taken in the dynamic case at the same points used in the static work...i.e. at the 225 intersection points of the grid drawn on the plate. In processing the data resulting from the dynamic readings, the capacitance vs. displacement relationship was considered to be linear. This assumption is permissible since the part of the hyperbolic curve along which readings were taken was small. A discussion of the errors introduced by this assumption is presented in the section <u>Sources of Error</u>. At this time it is sufficient to say that the assumption of linearity greatly reduced the problems of data taking and data processing without introducing appreciable error.

The problem of instability that is inherent in the capacitive probe made it advisable to incorporate some type of reference check in the data taking procedure. This was done by taking the data in a spiral pattern and by checking at a point previously read at every two or three rounds in the 45 to 60 minute data taking period.

Sources of Error

The sources of error can be broken into three general areas...

1) Errors in Capacitive Probe

2) Errors from Traverse Mechanism

3) Errors Resulting from Method of Handling Data

Each of these sources of error and the probable amount of error that they contribute are explored in detail in the following sections.

Errors from the Capacitive Probe As mentioned previously in the discussion of the stability of the capacitive probe, the capacitance is affected by the pressure, temperature, and humidity, as well as, it seems, by drafts and air movements. Examining first the effects of changes in pressure on the capacitive probe measurements, it is known that for air at 66°F, a change in pressure of from 1 atmosphere to 100 atmospheres causes a change in the dielectric constant of from 1.0006 to 1.0548. This represents a deviation of 5.42% in the dielectric constant with an extreme change of 100 atmospheres of pressure. In order to get some idea of the pressure variations existing between the vibrating plate and the capacitive probe, consider the most extreme case possible which would be if the varying capacitor is thought of as a closed cylinder of changing volume. In such a case it is found that the pressure changes would be of the order of 2 atmospheres. From this we can conclude that the errors caused by pressure changes can be neglected in the light of the 5.42% error with a change in pressure of 100 atmospheres.

The fringe effect or stray capacitance at the edges of the plate introduce some error in the simple expression for capacitance shown in equation (1) on page 16. According to Lion in <u>Instrumentation for Scientific Research</u>, the fringe effect introduces an error of about 6% if the ratio of the radius to distance between the plates is 50. In the case where this ratio can be kept over 200, the error is less than 1%. In the case of the probe used in this work with its .750 diamenter and .0300 of an inch gap, the ratio of radius to gap is 125. In the light of this ratio and since the probe used was shielded, the error can be considered to be well below 3%.

The errors caused by temperature and humidity changes may be considered to have negligible effect on the measurements that were made since these variables may be considered to be constant in the time that it took for a set of readings to be taken.

<u>Errors from Traverse Mechanism</u> The traverse mechanism supported the capacitive probe on a cantilever and could

conceivably contribute error to the readings that were taken. For this reason, the traverse mechanism was designed to have very rigid components. Checks made of the deflection of the cantilever showed that a very heavy load would be required to have an appreciable deflection at even the most extreme cantilever position. In the light of the ruggedly built mechanism, and the care that was used in moving the probe, this source of error can be neglected since the .75# load of the probe caused an insignificant deflection in the 1" X 4" steel cantilever even at its maximum extension of 18 inches.

Errors Resulting from Data Evaluation An important source of possible error is that of the assumptions made and the technique used in data handling and evaluation.

The one assumption made in converting the dynamic capacitive reading to relative amplitudes of vibration was that the capacitance vs. displacement relationship was linear. This assumption introduced little error as shown in Figure 8 page 35. As seen in the figure, the range in which the change in deflection occurred about the .0300 inch initial gap at the center of the plate amounted to a $\pm$20 units of oscilloscope deflection. This portion of the curve is a small and essentially linear portion of the total hyperbolic calibration curve. Even considering the fact that the plate was not flat, which would vary the .0300 inch initial gap, we find that the $\pm$ .0015 inch variation in the undeflected plate surface

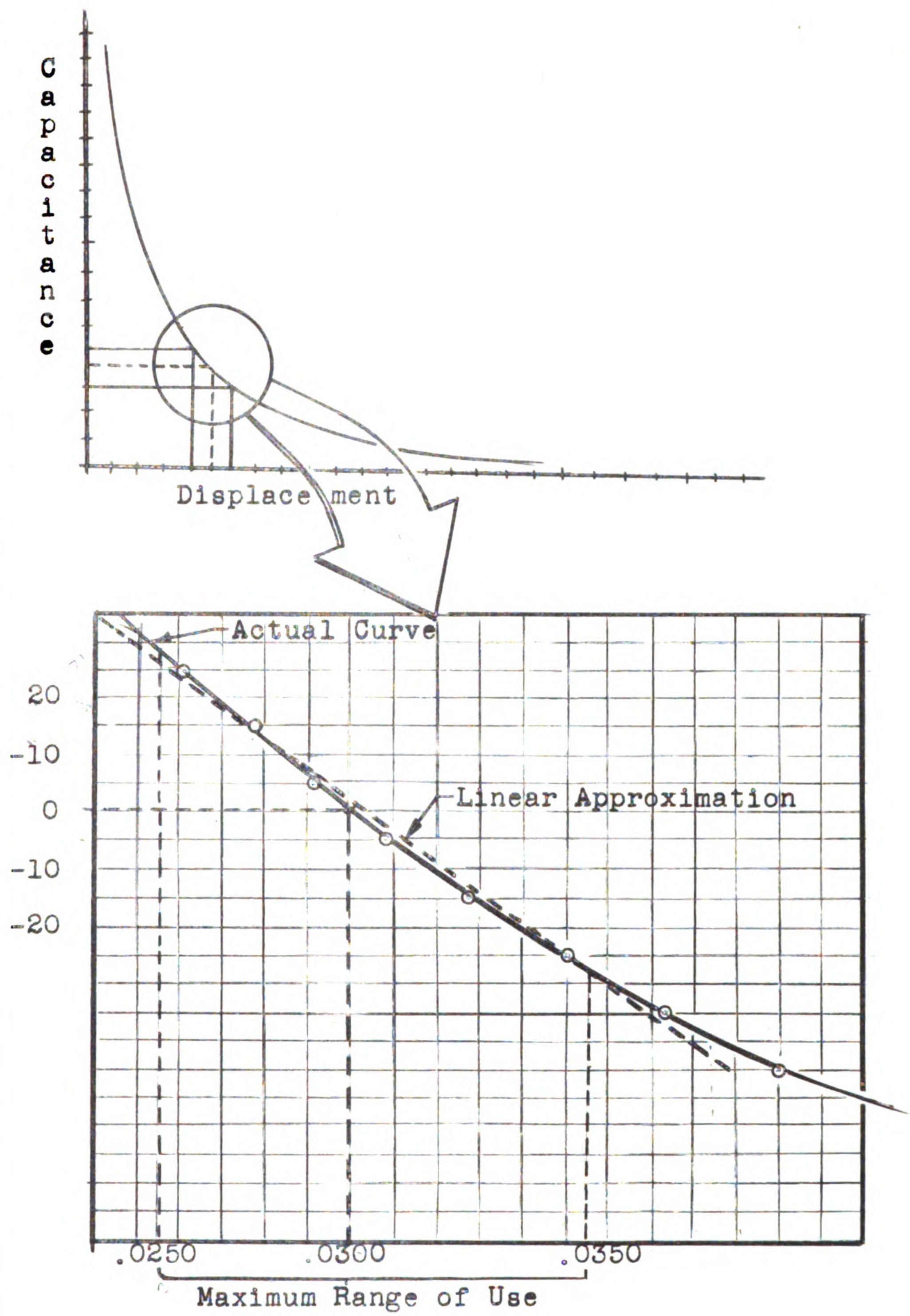


Figure 8 Capacitance vs Displacement Linearity

would expand the range of measurement to that indicated on the curve in Figure 8. The percent of error introduced by the nonlinearity of the calibration curve, then, is 3 %.

Another source of error resulted from readings of the scope trace deflection. The signal that resulted, especially in the higher modes, varied somewhat in the absolute peak-to-peak height. This variation was "averaged out" by setting the scope such that a wide band was displayed on the screen instead of the clear single sweep signal. The reading taken as data then was the height of the brightest part of the band. Stray fringes of the band were considered to be the variation of the signal. In as much as the bright band that was measured was stable, the primary source of error here would be the reading of the height. The screen was gridded with 40 units, and considering that readings could be estimated with an accuracy of $\pm$ 1 unit, this amounts to an error of approximately $\pm$ 2%.

<u>Summary of Errors</u> A summary of the sources of errors and the amount that each is considered to have contributed to the total error of the results that were obtained is presented in the table below.

Probe Errors	
Pressure Changes	Negligible
Fringe Effect	3.0%
Humidity, Temperature	Negligible
Traverse Mechanism	
Cantilever Deflection	Negligible
Data Evaluation	
Linearity Assumption	3%
Data Reading	$\pm$ 2.0%

IV. Presentation of Results

The quantity of experimental data to be presented and compared with the finite difference results presents a problem. This problem, one of keeping the data presentation compact and to the point while still presenting enough detail to enable the reader to get a feel for the results of the thesis work, was handled in the following manner. For each mode shape a pictorial view of the theoretical mode shape has been developed. The pictorial mode shapes are shown as absolute values, although positive and negative areas have been marked where their inclusion aids in visualizing the actual physical shape of the plate in vibration. Following the pictorial view of each mode shape, the experimental data is plotted on a set of graphs which present the relative deflection of the plate along each of the cross sections formed by the 15 by 15 space grid which was drawn on the plate. The sets of graphs are keyed to the data sheets in that the column of graphs marked N S is that data taken from the appropriate data sheet as the rows marked 1 through 15, while the column of graphs marked W E is that data appearing as columns 1 through 15 on the data sheets.

These same graphs have superimposed upon them the finite difference results in red which were available. The sets of graphs, then, not only indicate the degree of symmetry and the experimental mode shapes that were found to exist, but also provide an excellent comparison between experimental and finite difference data.

Mode Shapes

First Mode A pictorial view of the absolute value shape of the first mode shape is presented in Figure 9, page 41. The experimental data that was obtained for the first mode is shown plotted in the set of graphs shown in Graph 2, page 42. The agreement of the experimental data with the finite difference results for this simple mode is good. The degree of symmetry is also good. Note that the finite difference results in this case are neither consistently above nor below the actual experimental values obtained.

Second Mode A pictorial view of the absolute value shape of the second mode is shown in Figure 10, page 43. It should be emphasized here that the actual physical shape of the vibrating plate is that implied by the positive and negative signs shown in the pictorial drawing. The data plotted for each of the cross sections of the 15 by 15 space grid is shown in Graph 3, page 44. It can be seen that there is fair to good agreement between the experimental data and the finite difference results shown in red on these graphs. Symmetry is not as good as that obtained with the first mode, but is fair. The lack of symmetry may be attributed to the fact that the plate in its second mode of vibration must by necessary be driven from a point off center.

Third Mode The pictorial view of the third mode absolute value shape is presented in Figure 11, page 45. The mode shape is marked with positive and negative areas to indicate its actual physical shape. The plots of the data for each of the cross sections is shown in Graph 4, page 46 . There is a marked lack of symmetry in the experimental data, although the mode shape is apparent. Note also that for this more complex mode shape, the .750 diameter probe has been unable to detect the detailed shape of the mode. The lack of good agreement between the experimental and finite difference data might again be attributed to the fact that the plate had to be driven off center in this mode of vibration.

Fourth Mode The pictorial view of the fourth mode is presented in Figure 12, page 47 . The positive and negative signs are again shown to indicate the physical shape of the vibrating plate. The agreement between the finite difference results and the experimental data shown in Graph 5, page 48, is only fair at best. The large diamenter probe that was used was unable to pick out the fine detail of the complex intersection of the four surfaces of the fourth mode shape. The probe only read an average of the true surface in which much detail was lost. Symmetry in this case is not good. This is attributable to the fact that the plate was by necessity driven at one corner since in this mode of vibration a node passed thru the center.

<u>Fifth Mode</u> The pictorial view of the fifth mode shape is shown in Figure 13, page 49. The agreement of the experimental data to the finite difference results as shown in Graph 6, page 50, was fair to good. The mode shape is clearly evident in the graphs of the experimental data and the symmetry is seen to be fair to good. Note that in this case that the finite difference results were consistently less than the experimental data.

<u>Sixth Mode</u> The pictorial view of the sixth mode of vibration is shown in Figure 14, page 51. The mode shape is discernible from the experimental data shown plotted in Graph 7, page 52. The symmetry, however, is not good. As in the other complex mode shapes, namely the third and fourth modes, the capacitive probe has rounded off the data to such an extent that detail of the mode shape has been lost. Here, again, the plate was driven off center.

<u>General Comparison of Mode Shapes</u> In general, the mode shapes that were simpler in form and which could be obtained by driving the plate from the center provided good agreement between the finite difference and experimental results. The lack of good symmetry in some of the experimental data and the lack of good agreement between relative amplitudes of the experimental and finite difference methods may be attributed to the following....

1) The capacitive probe that was developed and proven in static test was too large to read the

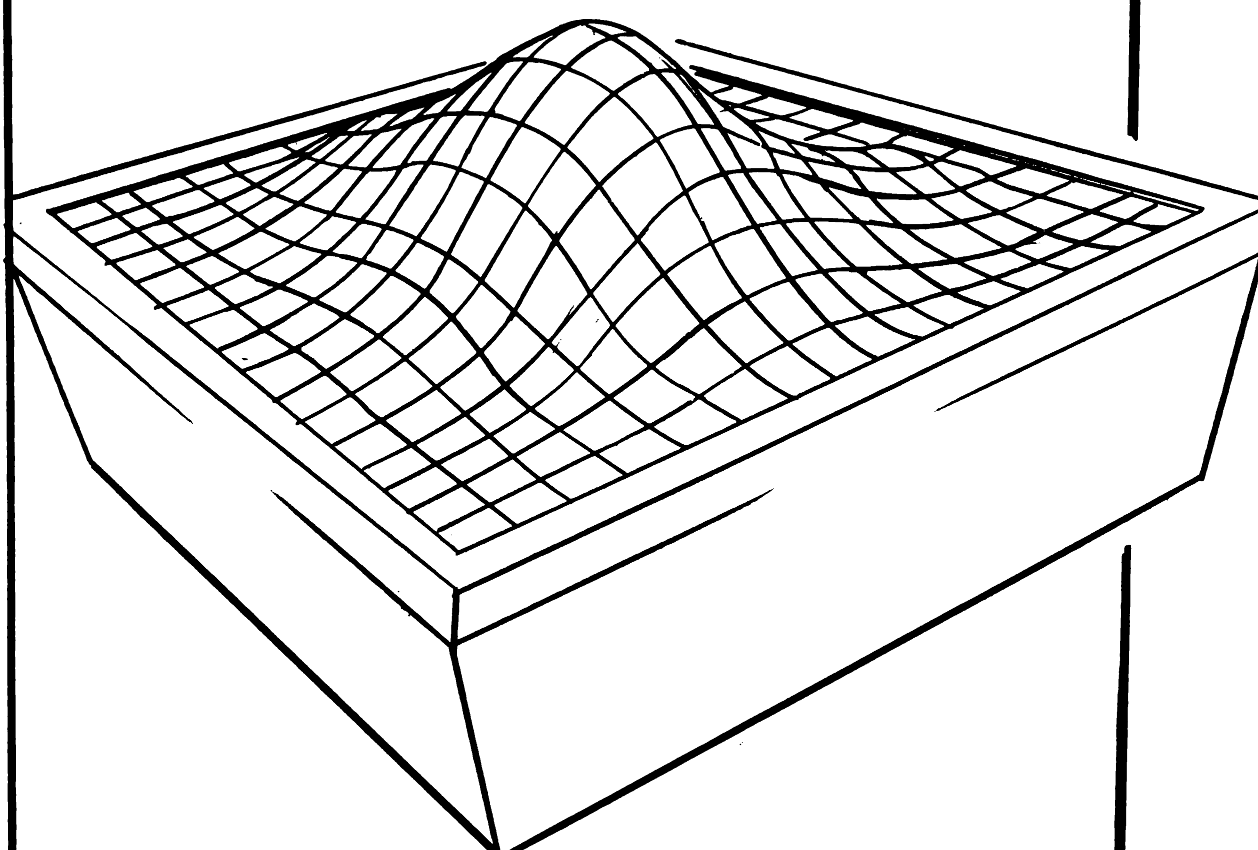
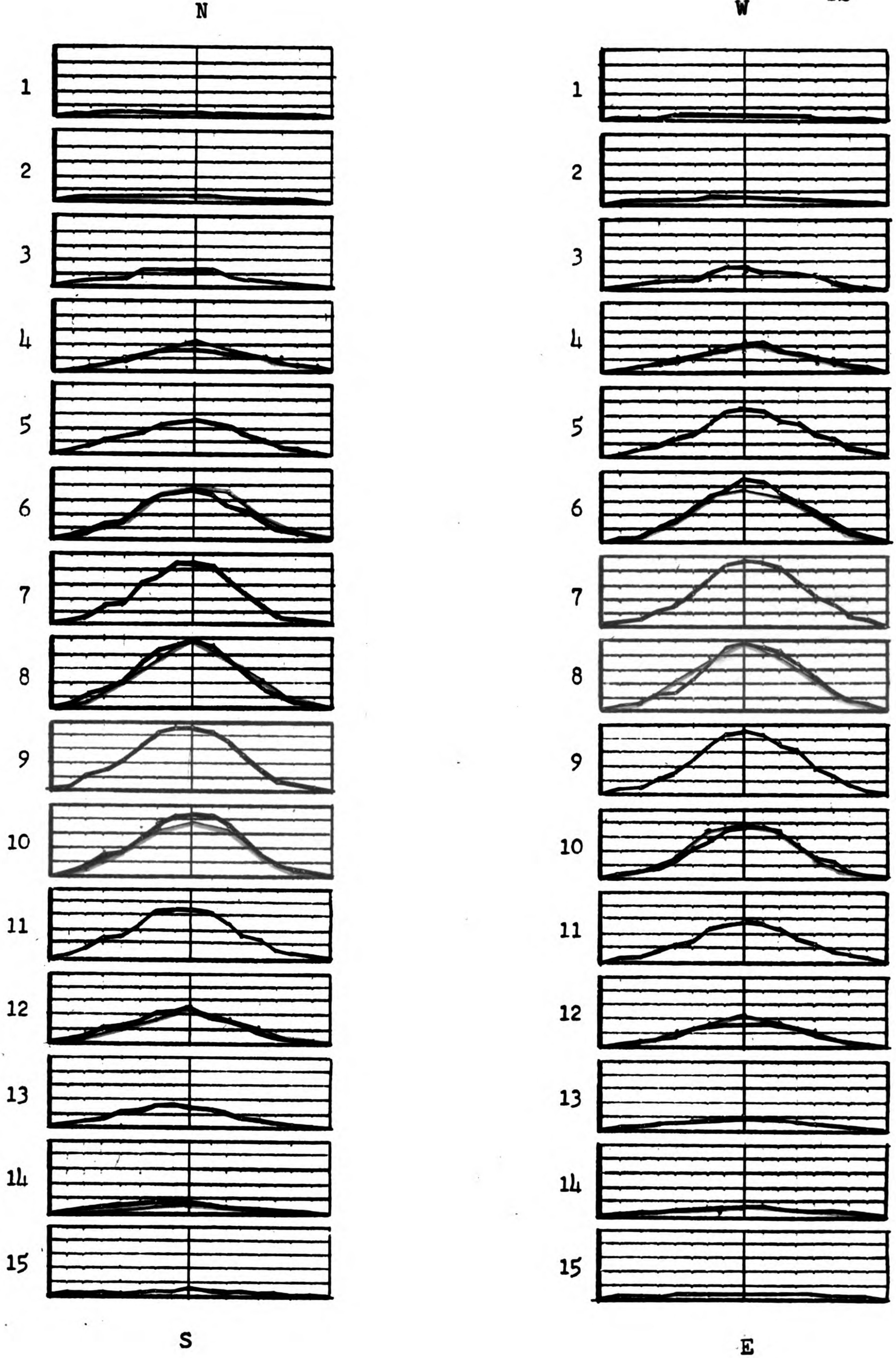


Figure 9-Pictorial View of First Mode Shape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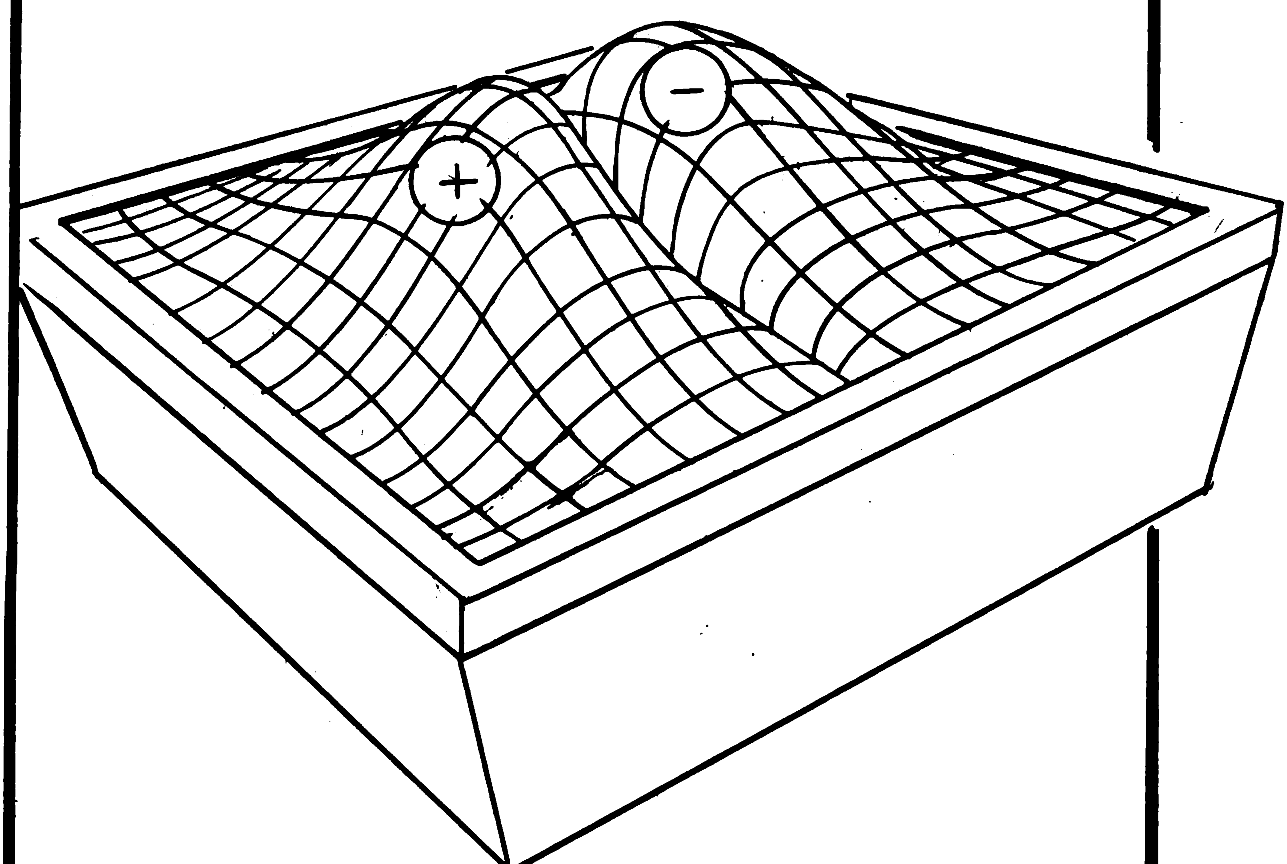
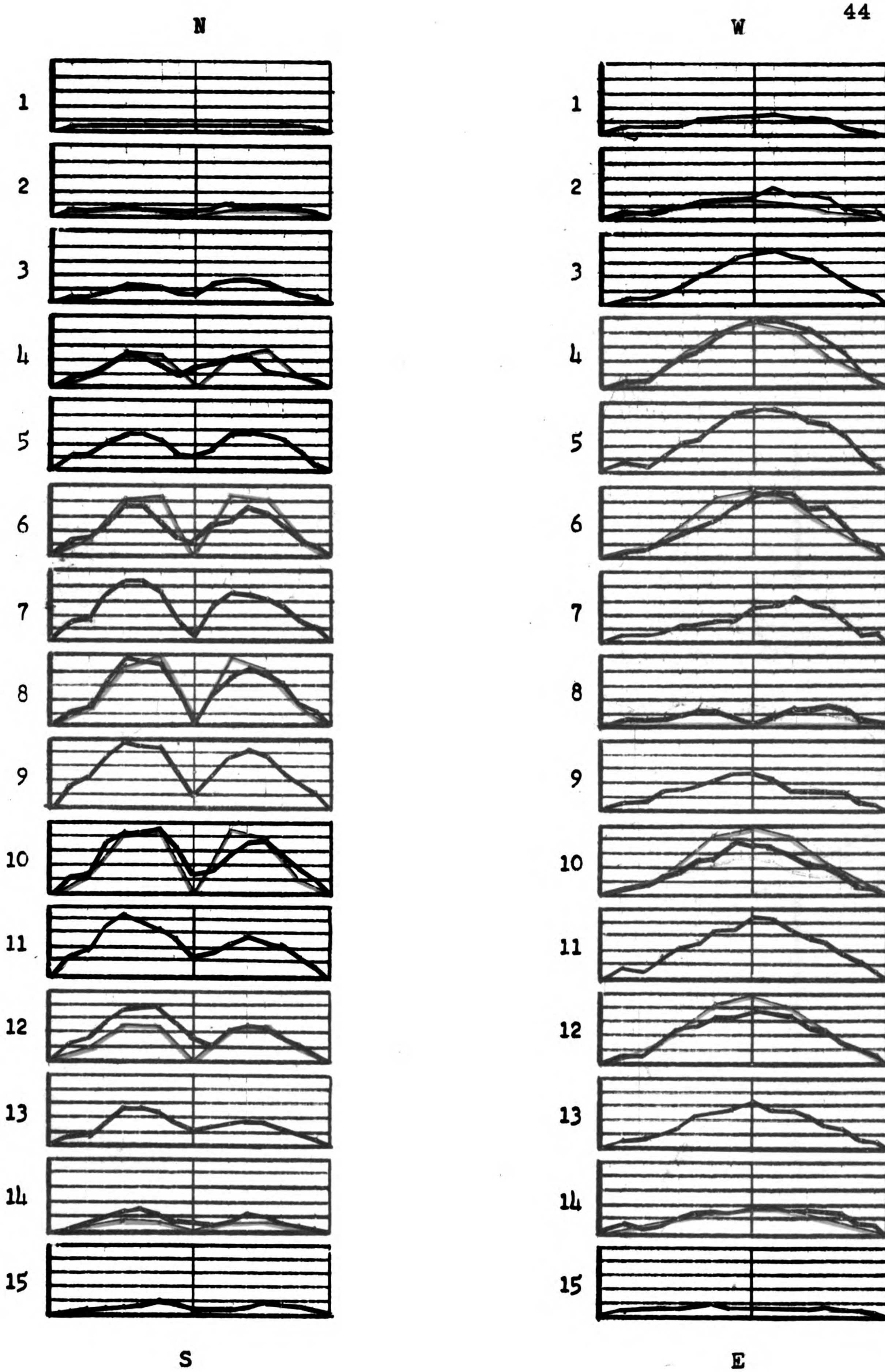
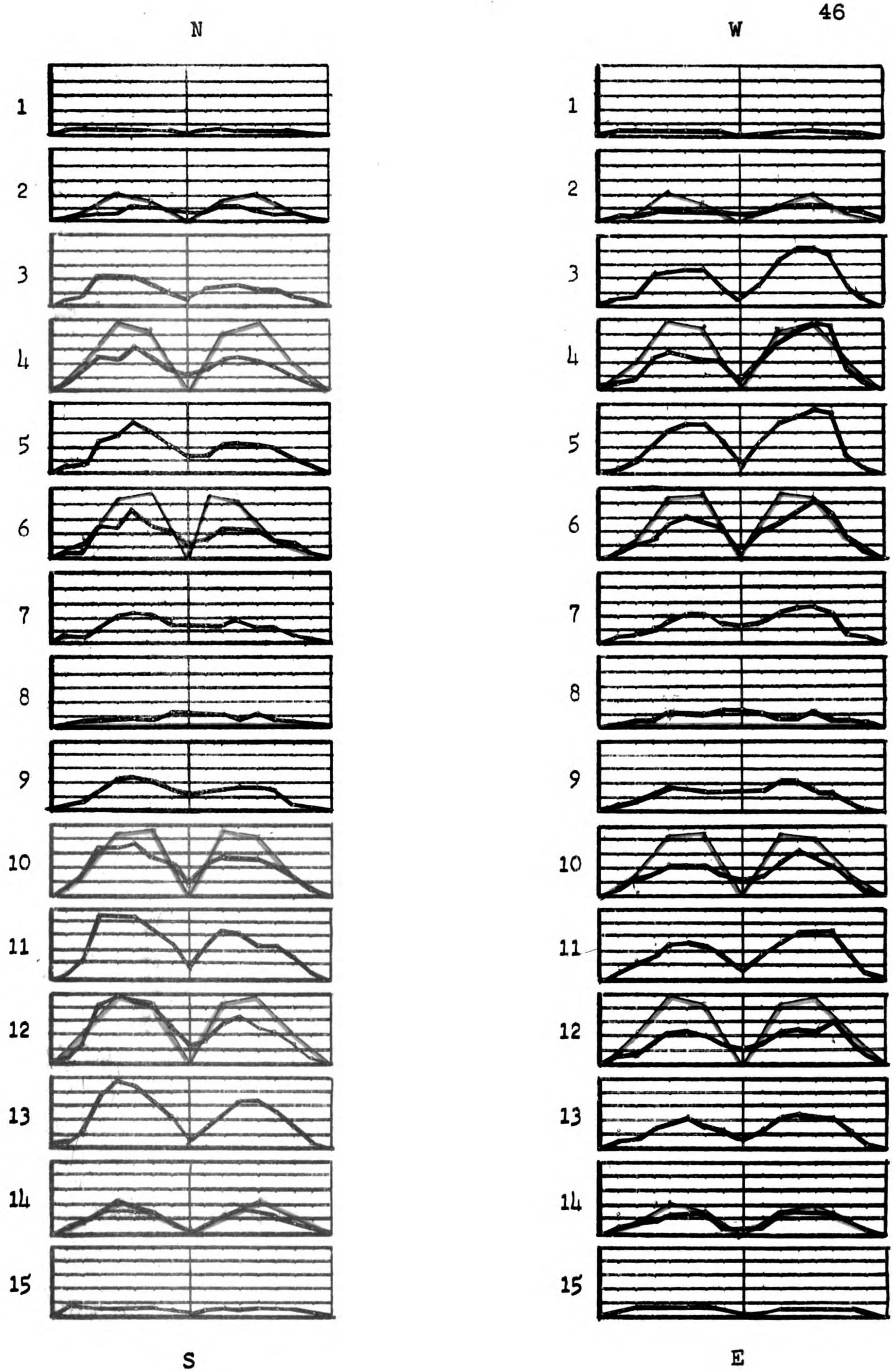


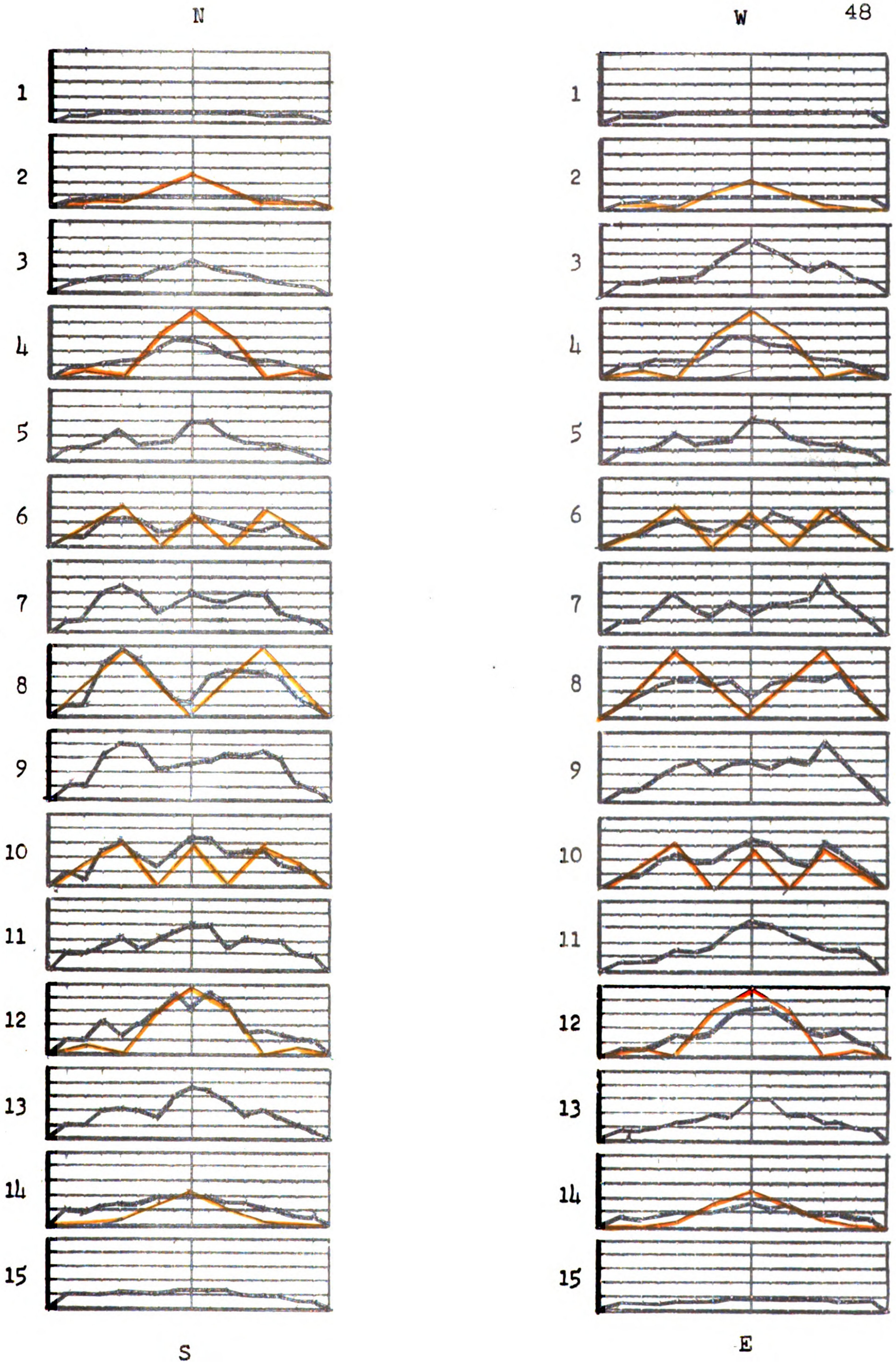
Figure 10 Pictorial View of Second Mode Shape



Graph 3. Second Mode Experimental Data



Graph 4. Third Mode Experimental Data



Graph 5. Fourth Mode Experimental Data

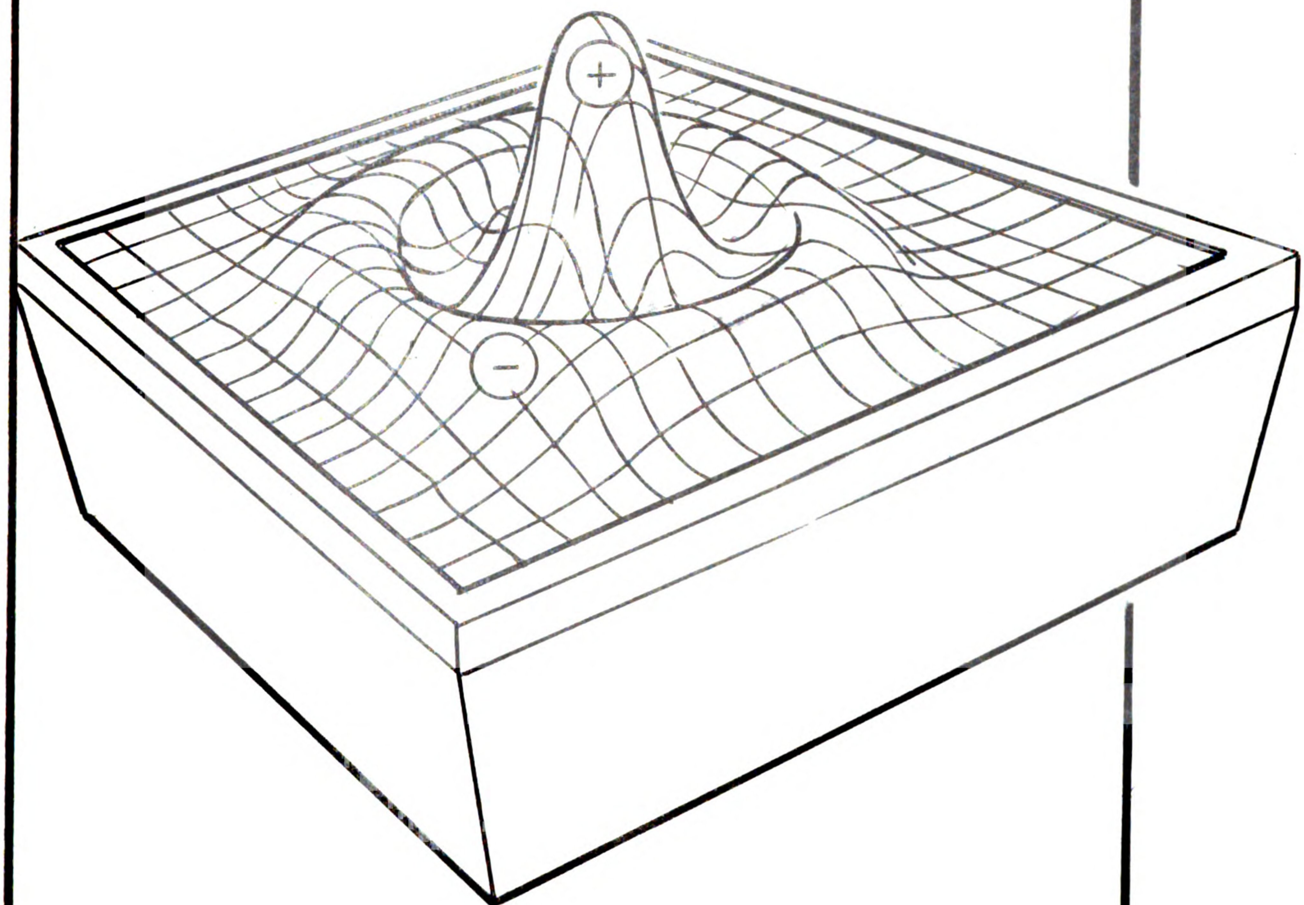
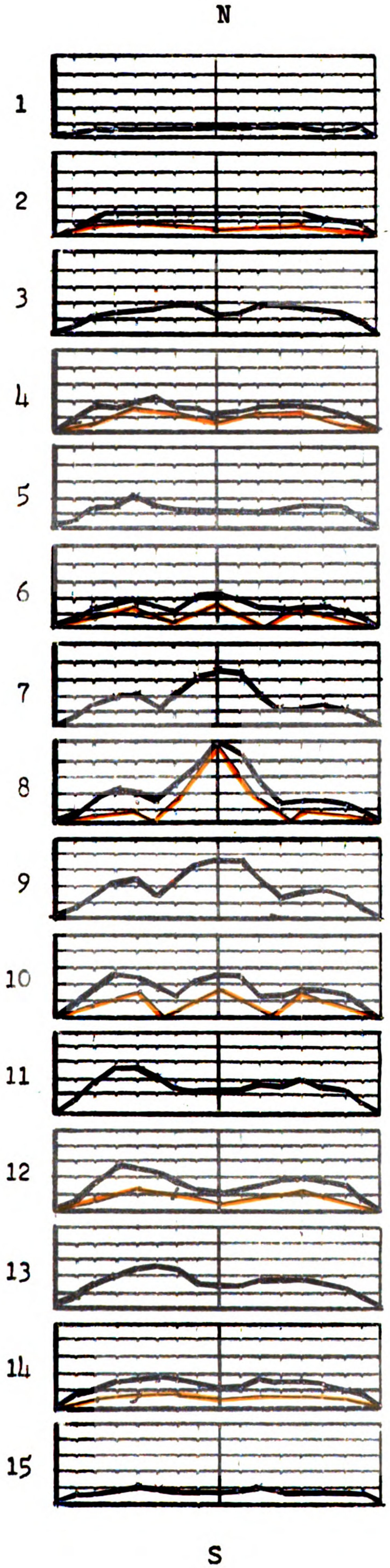


Figure 13 Pictorial View of Fifth Mode Shape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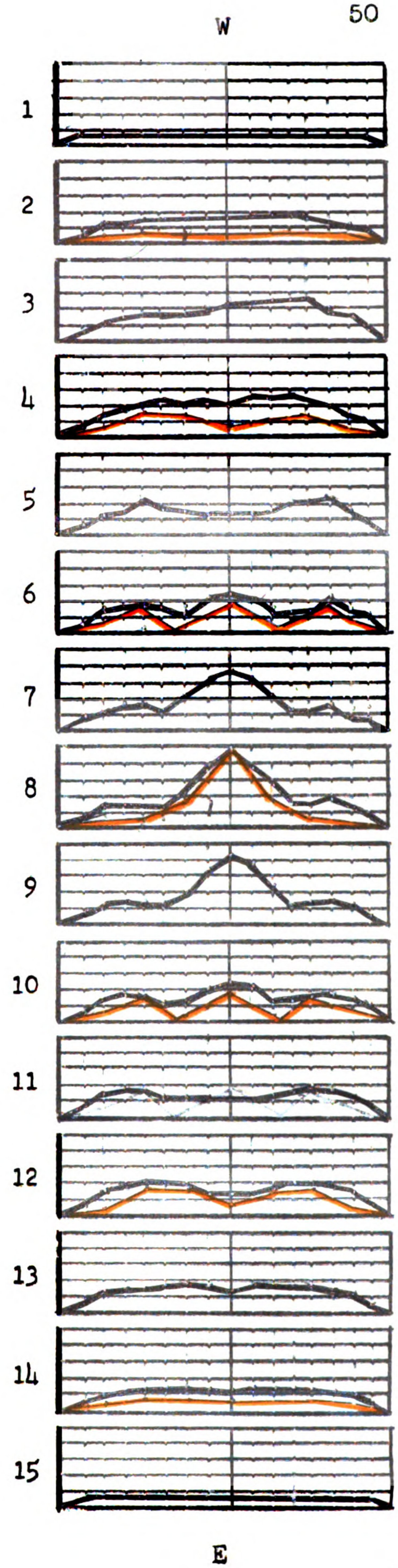
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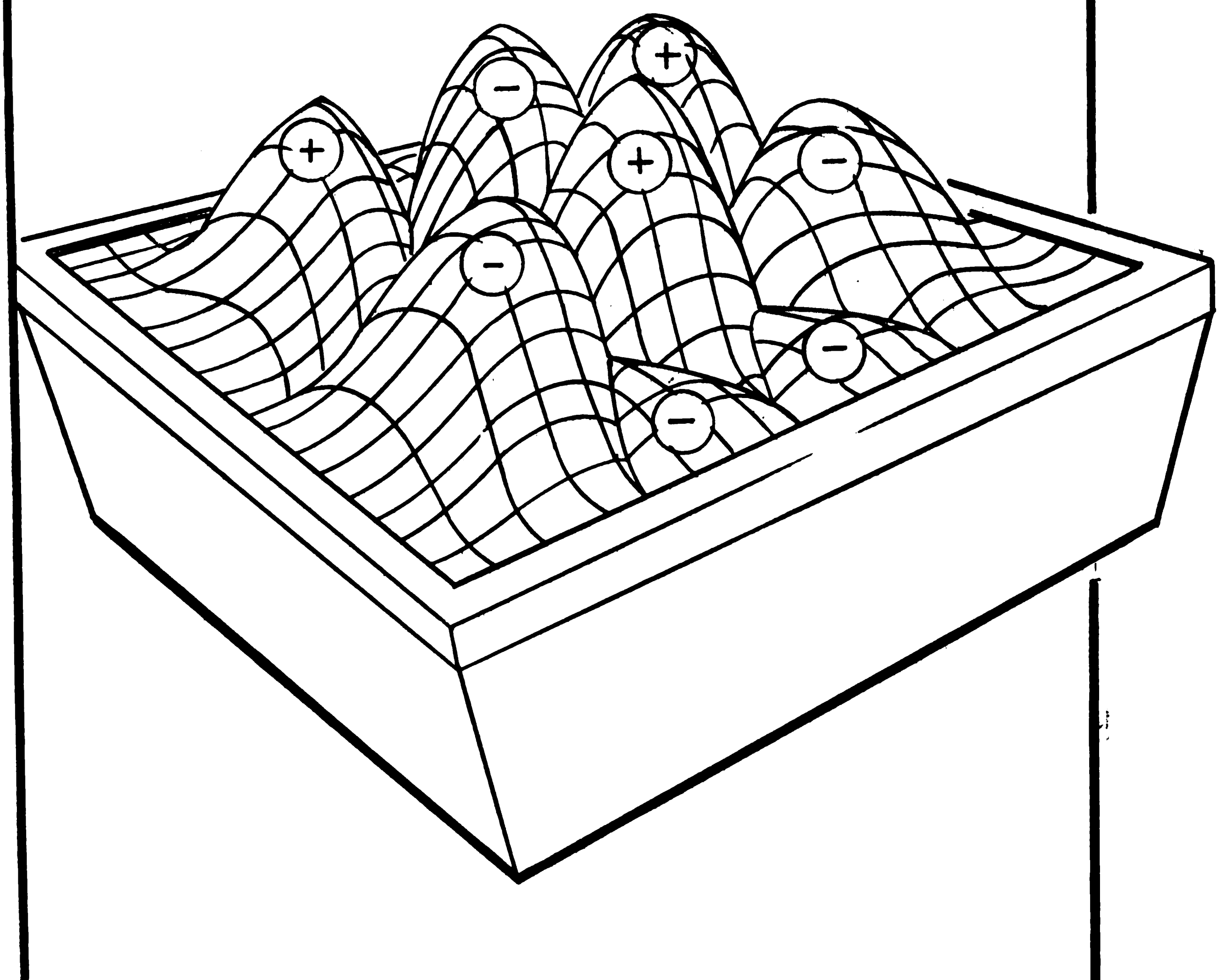
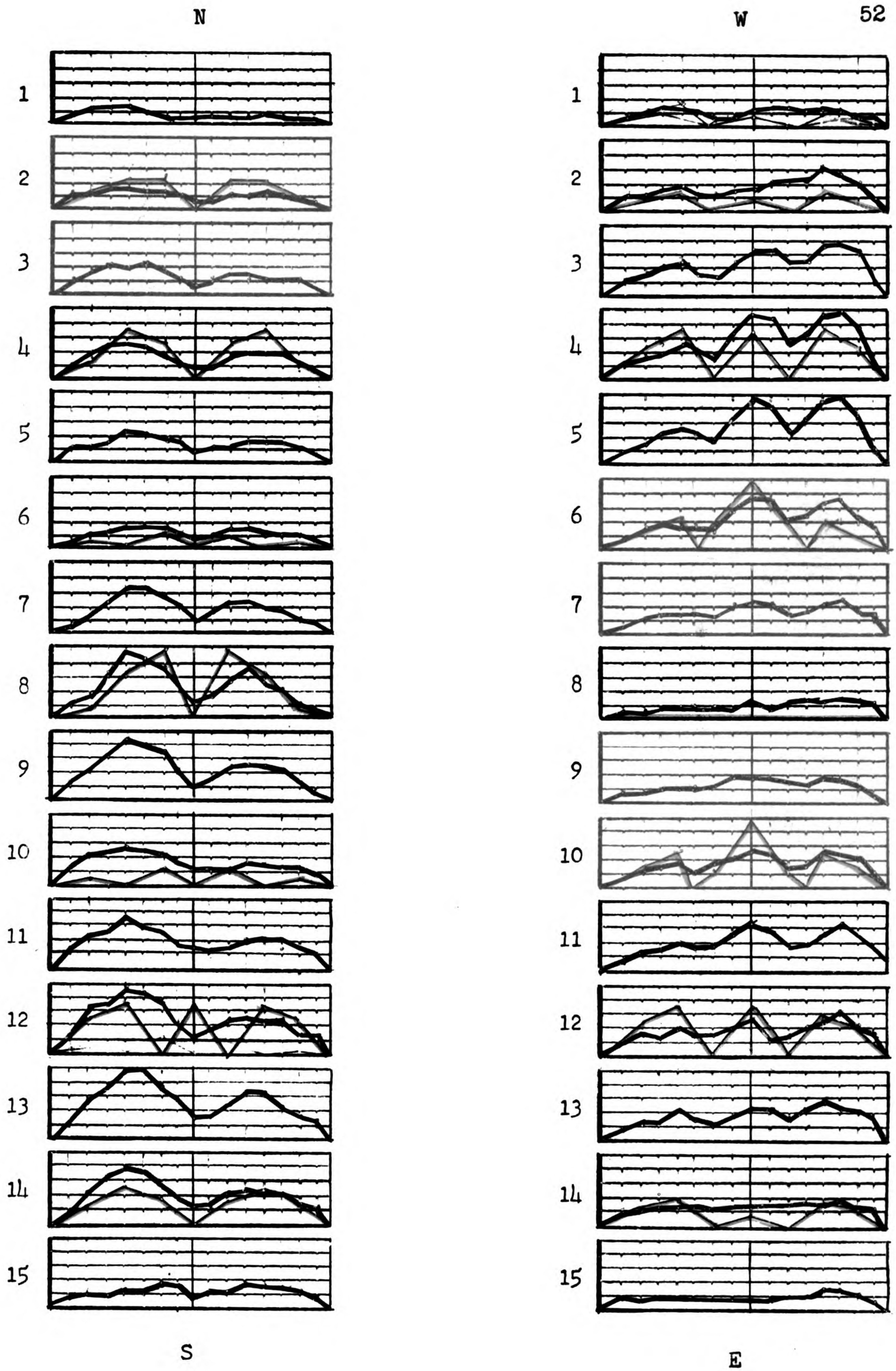


Figure 14. Pictorial View of Sixth Mode Shape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detail of complicated intersection of some of the surfaces of the vibrating plate.

2) The plate was by necessity driven off center for certain modes. This caused a marked lack of symmetry as the plate was forced to vibrate at larger amplitudes at the points of driving.

3) The probe reads only absolute values and loses detail in the area of the nodes.

4) The finite difference method does not produce exact results. Better results would require a finer grid, and considerations which would make the theoretical model more representative of the real structure.

Capacitive Probe Output Signal

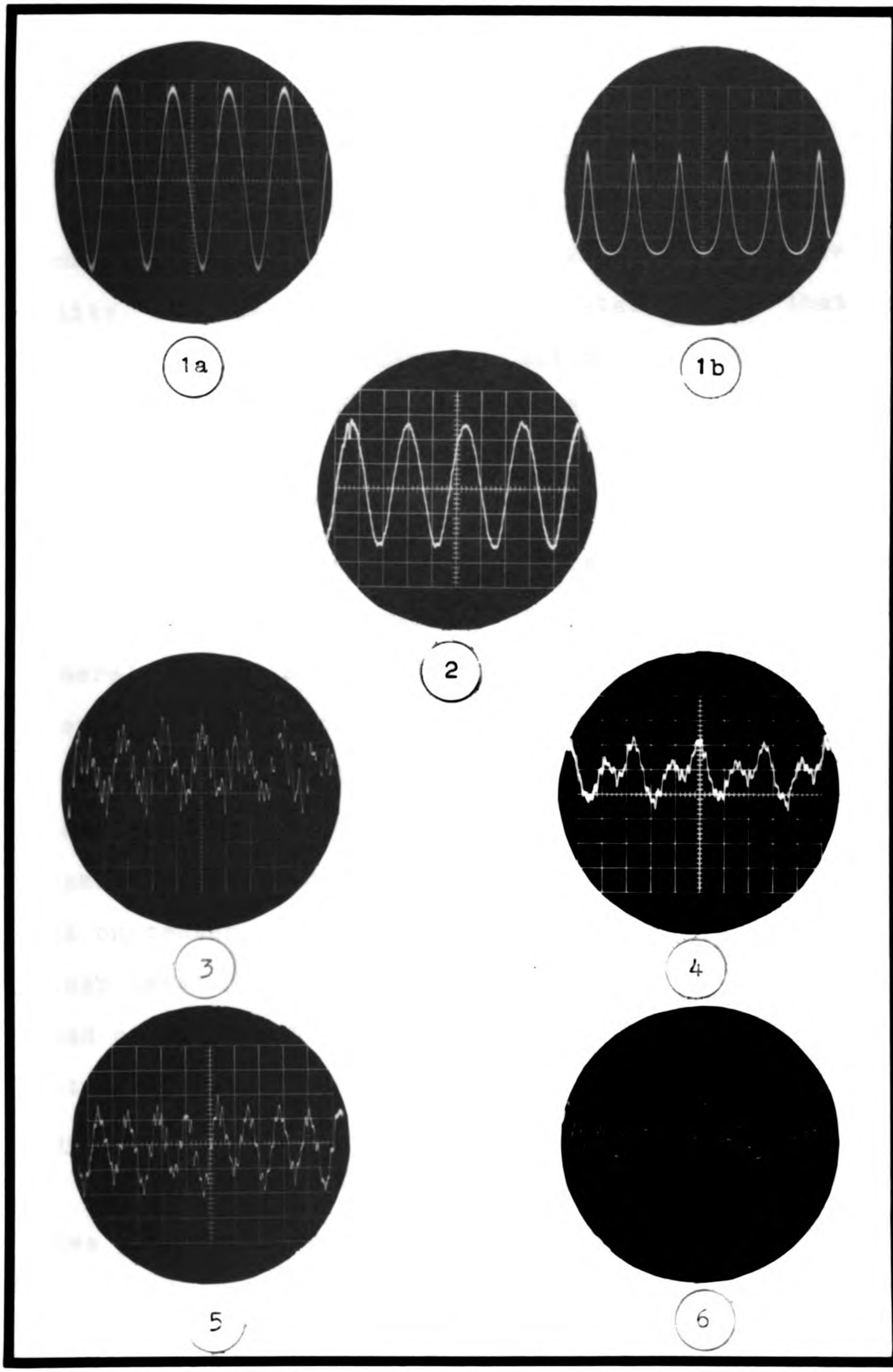
The output from the capacitive probe was presented on an oscilloscope. The absolute value of the signal, being proportional to the deflection of the vibrating plate, was read as the data upon which the previously presented graphs are based. For each mode shape a single signal was displayed on the screen of the oscilloscope and photographed. These photographs are shown in Figure 15, page 56. Note that for the first mode shape both large and small deflection signals are presented. The small deflection signals for the first mode is essentially a sine wave, while the large deflection signal is distorted as the plate is drive beyond its natural amplitude of vibration. In all but the first mode, a great deal of noise was found to be superimposed upon the signal obtained. The source of this noise, it is felt, is a combination of a non-linear behavior of the plate at more complicated mode shapes and of a non-sinusoidal signal produced by the siren used to vibrate the plate.

It was found during the investigation that the output signal from the capacitive probe was stable in form only for the more simple mode of vibration such as the first, fifth, and to some extent the second. The more complicated mode shapes, the third, fourth, and sixth, were less repeatable. The photographs of these more complicated mode shapes shown in parts 3, 4, and 6 of Figure 15, page 56, should be considered only to be representative of the type of signal obtained, rather than the actual signal that could be obtained at a given run.

Figure 15

The Signals Read from the Probe
(Next Page)

1a	First Mode	(Small Deflections)
1b	First Mode	(Large Deflections)
2	Second Mode	(Small Deflections)
3	Third Mode	(Small Deflections)
4	Fourth Mode	(Small Deflections)
5	Fifth Mode	(Small Deflections)
6	Sixth Mode	(Small Deflections)



Signals from Capacitive Probe

V. Conclusions

Summary

The capacitive probe that has been developed has been shown to have satisfactory sensitivity, accuracy, repeatibility, and ease in application in static tests that were made of the measuring device. Actual measurements of a statically deflected plate produced experimental results that agreed with a dial indicator measurement to within a 10% deviation.

The results of applying the capacitive probe to dynamic measurements of the amplitude of vibration of a plate in general are not as good as those obtained in static test runs. There is good or only just fair agreement between the experimental and finite difference data for only the simpler mode shapes. The more complicated mode shapes lack symmetry because of the necessity of driving the plate at a point not on center. The capacitive probe was unable to read the finer detail of the more complicated mode shapes since it read only absolute values and could not pick out surface intersections.

Recommendations

Even though it has been shown that the specific capacitive probe that was developed and used in this investigation was unable to provide good readings of all mode shapes, it should not be interpreted that this technique is not a good one. On the basis of the good qualities that the capacitive probe technique of dynamic measurement offers, and in the light of the results of static calibration tests, it is felt that this method warrants further work. Specifically,recommendations are....

1) Develope a probe of smaller diameter so that increased detail can be read by the probe. This will involve an increase in the non-linearity of the probe to displacement, a decrease in the range of measurement that may be covered, and perhaps will require increased amplification of the signal which will further amplify the problems of stability and shielding that will be encountered.
2) Investigate the large deflection of statically loaded variable thickness plates using the equipment that is now available from the calibration work that was done with the capacitive probe. A dial indicator was found to produce satisfactory results for static measurements, although the capacitive probe provides a more sensitive, although harder to use method of measurement.
3) Investigate the forces existing between the probe and the plate.
4) Investigate higher modes of vibration of plates.

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3. Jaeger, J.C., Elasticity, Fracture, and Flow, John Wiley and Sons, New York, N.Y.

APPENDIX

1	.03	.03	.03	.06	.06	.06	.06	.08	.06	.06	.03	.03	.03	.03	.03
2	.03	.06	.08	.08	.11	.14	.14	.14	.14	.11	.08	.08	.06	.06	.03
3	.03	.08	.11	.14	.20	.22	.22	.22	.22	.20	.14	.11	.08	.06	.03
4	.03	.11	.17	.20	.25	.31	.33	.33	.31	.25	.22	.14	.11	.08	.06
5	.03	.14	.22	.28	.39	.47	.50	.53	.47	.42	.33	.22	.14	.08	.06
6	.06	.17	.28	.33	.50	.64	.70	.75	.67	.56	.45	.28	.17	.11	.06
7	.06	.20	.33	.39	.62	.78	.89	.89	.83	.70	.50	.33	.20	.11	.06
8	.08	.22	.36	.45	.67	.89	.95	1.0	.92	.75	.58	.33	.22	.14	.06
9	.08	.22	.33	.45	.64	.83	.92	.95	.86	.72	.53	.33	.20	.14	.06
10	.06	.20	.31	.39	.58	.75	.81	.83	.78	.67	.50	.31	.20	.11	.06
11	.06	.17	.28	.33	.50	.62	.64	.67	.62	.50	.39	.28	.14	.11	.06
12	.06	.14	.22	.25	.33	.42	.42	.53	.39	.33	.28	.20	.11	.08	.03
13	.03	.11	.17	.22	.28	.33	.36	.33	.28	.22	.19	.14	.08	.06	.03
14	.03	.06	.08	.14	.17	.19	.19	.19	.17	.14	.11	.08	.06	.06	.03
15	.03	.03	.06	.06	.08	.08	.08	.11	.08	.08	.08	.06	.06	.03	.03
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Table 7 First Mode-Experimental Data

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	.14	.14	.14	.14	.17	.14	.14	.14	.14	.14	.17	.14	.14	.14	.11
2	.14	.09	.14	.17	.14	.14	.11	.11	.14	.20	.14	.17	.17	.11	.09
3	.14	.14	.20	.29	.29	.23	.17	.17	.29	.34	.34	.31	.20	.14	.11
4	.17	.20	.31	.46	.43	.34	.23	.20	.37	.40	.43	.43	.29	.20	.14
5	.23	.29	.46	.57	.57	.43	.23	.23	.40	.57	.54	.48	.43	.29	.17
6	.23	.29	.57	.71	.71	.57	.31	.23	.43	.57	.74	.63	.46	.29	.20
7	.29	.31	.68	.83	.86	.74	.37	.17	.57	.77	.71	.63	.57	.34	.23
8	.29	.37	.77	1.0	.94	.91	.51	.09	.57	.77	.83	.80	.63	.37	.26
9	.34	.46	.80	1.0	.97	.97	.57	.20	.48	.74	.83	.77	.54	.37	.23
10	.29	.34	.71	.91	.91	.94	.66	.29	.37	.60	.71	.71	.57	.34	.20
11	.29	.34	.68	.86	.80	.71	.57	.29	.31	.48	.60	.54	.46	.29	.17
12	.23	.31	.57	.74	.77	.80	.54	.34	.29	.43	.51	.48	.34	.26	.14
13	.14	.17	.37	.57	.57	.48	.37	.26	.29	.31	.40	.34	.29	.20	.11
14	.09	.11	.20	.23	.31	.23	.17	.17	.14	.17	.23	.20	.17	.14	.09
15	.06	.11	.11	.14	.17	.20	.17	.14	.14	.14	.14	.17	.11	.11	.09

Table 8 Second Mode-Experimental Data

1	.11	.11	.11	.11	.13	.11	.11	.08	.11	.13	.13	.11	.11	.11	.08
2	.11	.13	.16	.16	.25	.22	.19	.16	.19	.22	.25	.19	.16	.16	.13
3	.11	.19	.42	.44	.42	.33	.22	.16	.28	.33	.36	.33	.28	.19	.13
4	.13	.16	.50	.47	.61	.55	.36	.22	.39	.44	.47	.42	.39	.25	.13
5	.11	.19	.55	.58	.75	.61	.42	.25	.36	.44	.47	.44	.42	.25	.16
6	.11	.16	.55	.55	.72	.55	.42	.22	.28	.44	.44	.42	.33	.28	.16
7	.11	.13	.28	.42	.47	.42	.28	.28	.28	.33	.39	.33	.28	.16	.13
8	.08	.11	.13	.19	.19	.16	.25	.28	.28	.25	.19	.25	.16	.13	.08
9	.08	.13	.33	.47	.47	.42	.33	.22	.28	.33	.39	.36	.30	.13	.08
10	.11	.28	.69	.69	.78	.58	.42	.19	.42	.55	.55	.53	.42	.28	.11
11	.13	.28	.83	.83	.83	.69	.55	.19	.42	.64	.61	.55	.55	.30	.11
12	.11	.28	.83	1.0	.94	.86	.55	.25	.33	.58	.64	.55	.53	.30	.11
13	.11	.25	.78	.97	.89	.64	.42	.13	.36	.55	.67	.64	.47	.28	.11
14	.11	.19	.33	.30	.28	.28	.16	.11	.16	.22	.39	.33	.25	.16	.11
15	.13	.13	.11	.13	.13	.13	.11	.08	.11	.11	.13	.13	.11	.11	.08
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Table 9 Third Mode-Experimental Data

1	.16	.16	.20	.20	.20	.20	.20	.20	.20	.20	.20	.16	.16	.12	.12
2	.16	.16	.20	.20	.20	.20	.20	.20	.20	.20	.20	.16	.16	.12	.12
3	.16	.20	.24	.24	.28	.40	.40	.48	.40	.36	.24	.20	.20	.16	.12
4	.20	.20	.24	.28	.32	.44	.60	.60	.56	.48	.32	.32	.28	.20	.16
5	.20	.20	.32	.44	.24	.36	.36	.60	.60	.40	.32	.28	.32	.20	.16
6	.20	.20	.42	.42	.44	.28	.36	.56	.42	.40	.36	.36	.40	.20	.16
7	.20	.20	.60	.76	.60	.36	.44	.60	.56	.56	.60	.60	.36	.28	.20
8	.20	.20	.80	1.0	.88	.56	.36	.36	.60	.68	.68	.68	.60	.32	.20
9	.20	.20	.64	.80	.80	.44	.44	.56	.56	.64	.64	.68	.60	.24	.20
10	.20	.16	.56	.64	.44	.28	.44	.64	.64	.42	.56	.56	.36	.28	.20
11	.20	.20	.36	.44	.28	.44	.56	.60	.60	.36	.44	.40	.40	.20	.20
12	.20	.20	.44	.28	.40	.60	.84	.60	.88	.64	.36	.36	.32	.20	.20
13	.20	.20	.40	.40	.40	.36	.60	.68	.64	.56	.36	.40	.32	.20	.16
14	.20	.20	.24	.28	.32	.40	.40	.40	.40	.36	.36	.24	.20	.16	.16
15	.20	.20	.20	.20	.20	.20	.24	.24	.24	.24	.20	.20	.20	.16	.16
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Table 10 Fourth Mode-Experimental Data

1	.17	.17	.17	.17	.17	.17	.20	.17	.14	.14	.17	.17	.14	.14	.17
2	.17	.26	.26	.29	.26	.26	.26	.26	.26	.26	.26	.26	.23	.20	.17
3	.17	.26	.32	.34	.34	.37	.34	.29	.29	.34	.34	.32	.29	.23	.17
4	.17	.32	.34	.40	.43	.37	.34	.29	.29	.32	.34	.37	.32	.26	.17
5	.17	.32	.37	.43	.37	.29	.29	.29	.29	.26	.29	.34	.34	.29	.17
6	.17	.32	.37	.40	.32	.29	.43	.46	.40	.29	.26	.32	.32	.29	.17
7	.17	.32	.40	.43	.29	.43	.63	.77	.66	.43	.26	.29	.32	.26	.17
8	.17	.32	.43	.40	.29	.49	.77	1.0	.86	.49	.23	.29	.29	.26	.17
9	.17	.34	.43	.46	.29	.43	.66	.72	.72	.43	.23	.29	.34	.29	.17
10	.17	.34	.49	.49	.37	.29	.43	.54	.49	.29	.26	.34	.34	.29	.17
11	.17	.37	.52	.54	.43	.34	.26	.29	.23	.29	.34	.40	.34	.29	.17
12	.17	.34	.52	.49	.43	.34	.26	.26	.29	.34	.40	.40	.34	.29	.17
13	.17	.29	.40	.43	.46	.43	.34	.37	.37	.37	.34	.37	.32	.29	.17
14	.17	.23	.34	.32	.34	.32	.29	.26	.29	.34	.29	.29	.26	.20	.17
15	.17	.14	.17	.23	.20	.17	.17	.17	.14	.20	.17	.17	.17	.17	.17
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

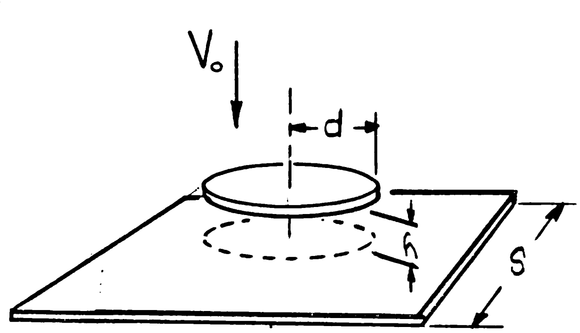
Table 11   Fifth Mode-Experimental Data

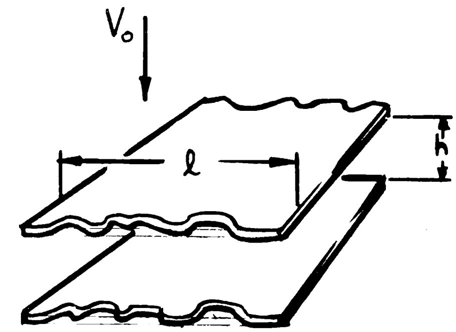
1	.18	.23	.23	.25	.20	.18	.13	.13	.18	.18	.18	.20	.18	.18	.15
2	.23	.23	.37	.37	.30	.30	.26	.15	.18	.25	.25	.30	.25	.20	.13
3	.28	.37	.45	.40	.43	.37	.25	.18	.20	.28	.30	.25	.25	.23	.15
4	.25	.40	.50	.52	.50	.37	.25	.20	.20	.35	.40	.40	.40	.23	.18
5	.25	.25	.33	.45	.43	.38	.28	.20	.25	.25	.33	.30	.30	.25	.15
6	.18	.25	.30	.35	.37	.35	.25	.20	.25	.35	.35	.30	.25	.23	.15
7	.18	.30	.50	.68	.68	.58	.43	.20	.35	.45	.50	.40	.38	.25	.15
8	.25	.37	.68	.95	.90	.78	.50	.23	.38	.58	.63	.50	.43	.25	.18
9	.28	.43	.65	.88	.80	.73	.43	.20	.35	.50	.55	.50	.43	.28	.18
10	.30	.43	.50	.58	.50	.43	.33	.25	.28	.33	.35	.33	.30	.30	.20
11	.23	.45	.50	.68	.58	.50	.38	.30	.25	.33	.40	.43	.43	.30	.20
12	.25	.63	.68	.95	.90	.68	.45	.25	.38	.50	.53	.50	.50	.30	.30
13	.23	.58	.75	1.0	1.0	.75	.55	.30	.33	.50	.63	.63	.43	.37	.30
14	.18	.43	.63	.75	.70	.50	.38	.25	.25	.43	.50	.45	.43	.30	.25
15	.13	.20	.20	.25	.25	.35	.33	.15	.13	.25	.37	.37	.30	.25	.18
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Table 12 Sixth Mode-Experimental Data

Air Displacement Force Consideration

The complete analysis of the problem dealing with the force acting on the plate that is caused by the flow of air from between the plate and the probe is quite involved. The problem, one of forcing air out from between a disk and a large plate, is schematically presented in the drawing below. Although a complete and detailed analysis is beyond the scope of this investigation, the consideration that was made is presented below.





The solution of a similar problem is available in a reference[7] and through this it is possible to at least obtain some idea of the order of magnitude of the force of the air on the plate. The case for which the solution is known is shown schematically in the drawing above. From the reference in the footnote on this page, it is found that the force (P) is expressed in the form of ....

$$P = 2\mu V_o\, l^3/h^3$$

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7 Jaeger, J.C., Elasticity, Fracture, and Flow, John Wiley and Sons, New York, N. Y. (See page 140 - 142)

where (V_0) is the velocity of the plates with respect to one another, (l) is the width of the strips, and (h) is the distance between the strips. η is viscosity of fluid between plates.

Making the calculation indicated in equation (3), using l = .750 inches, h = .0300 inches, V_0=15 inches per second, we find that the force is small, less than .01#, but of a magnitude that could have an effect. Since the case for which the solution is known is definitely a worse condition than that encountered in this investigation, we can say that the effect is small, although not necessarily negligible. There definitely is need for further work in this area, as indicated in the recommendations at the end of this thesis.