

A SOLUTION OF A NUMBER
OF TYPICAL INDETERMINATE
STRUCTURAL PROBLEMS

Thesis for the Degree of M. S.
MICHIGAN STATE COLLEGE

John Ernest Meyer
1942



A SOLUTION OF A NUMBER OF
TYPICAL INDETERMINATE STRUCTURAL
PROBLEMS

by

John Ernest Meyer

A THESIS

Submitted to the Graduate School of Michigan
State College of Agriculture and Applied
Science in partial fulfilment of the
requirements for the degree of

MASTER OF SCIENCE

Department of Civil Engineering
1942

THESIS

Acknowledgement

I am indebted to Professor C.I.Allen for suggesting the subject matter of this thesis. Both Professor Allen and Professor C.I.Cade have given me much timely criticism and advice in the preparation of this manuscript.

Table of Contents

Title Sheet	Page 1
Acknowledgement	Page 2
Table of Contents	Page 3
Introduction	Page 4
Solutions	
Framed Structures in Space	Pages 5-18
Deflections of Structures	Pages 19-51
Classical Methods of Analysis	Pages 51-102
Continuous and Long Span Bridges	Pages 103-121
Modern Methods for Rigid Frames	Pages 122-152
Arches and Closed Rings	Pages 153-171
Explanation of Symbols	Pages 172-173
Bibliography	Page 174
Numbers in lower right corner are bibliography references.	
Notes on Contents	Pages 175-181
Discussion of the Theory of Three Moments and Area Moment Proposition	Pages 182-183.

Introduction

The presentation of the subject matter concerned with the calculation of the stresses in framed structures has been the principal concern of many authorities in the field of Civil Engineering. In many instances these men have been limited to the application of particular theories in the solution of complicated structural problems. They all recognized that the proper way to attack many indeterminate structural problems is to build a model, and then to actually weigh the particular stresses. Enough work has been done in this direction to allow the development and application of the theories of structural stresses to the solution of particular problems. However there is still a large amount of correlation to be accomplished in the matter of the application of the classical theories to the solution of the so-called, practical indeterminate structural problems.

Dean L. F. Grinter, of the Illinois Institute of Technology, is an acknowledged authority in the analysis of structural problems. In his book "Stresses in Framed Structures" Volume 2, he has presented a number of practical problems. Grinter has shown, that in many instances, we can apply the classical methods of analysis to the solution of these problems. In the present manuscript presented herewith these suggestions have in many cases, been carried out.

The application of some of the simpler classical methods to the solution of problems concerned with restrained beams and frames results in such an involved procedure that a more unique solution of such problems seemed worth while. This has been done in this thesis in such cases where it was felt that this method of attack simplified the solution of the problem.

Particular attention is called to the application of the Theorem of Three Moments and to the proposition of Area Moments to cases where the ordinary classical method of attack would not yield enough equations to render the problem determinate. These theorems, in combination with the bending moment diagram by parts, have made possible unique solutions of otherwise long and complicated problems. In the "Notes on Contents" in this thesis mention is made of this method of attack for the several classes of problems outlined in the "Table of Contents".

The field of Civil Engineering is old enough at the present time so that little remains in the way of new theorems to be developed for the solution of structural problems. It is felt, however, that there yet remains much work to be done in the intercorrelation of the classical theorems concerned with framed structures, and the applications of these intercorrelations of these classical theorems to the solution of practical indeterminate problems. The purpose of this thesis has been to present a phase of these intercorrelations.

Proble
De
examin

re

12

2

In

B

m

h

q

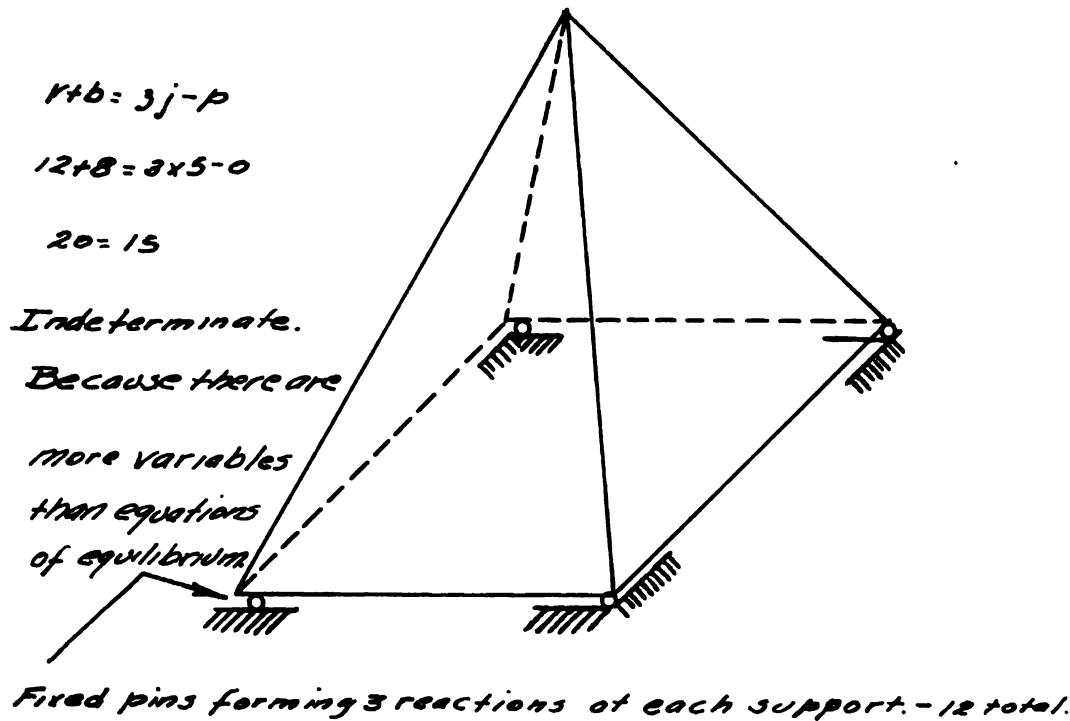
F

I,

h

Problem 1, Page 16

Determine whether the structure shown is statically determinate, statically indeterminate or unstable.



A test for static determinateness is as follows.

Since each joint furnishes 3 equations of statics, the following equation must be satisfied.

$$(r+b = 3j-p)$$

Now if $(r+b) > (3j-p)$, the figure is indeterminate,
If $(r+b) < (3j-p)$, the figure is unstable, and if $(r+b) = (3j-p)$,
the figure is determinate.

r = no. of reaction components.

b = " " bars.

j = " " joints

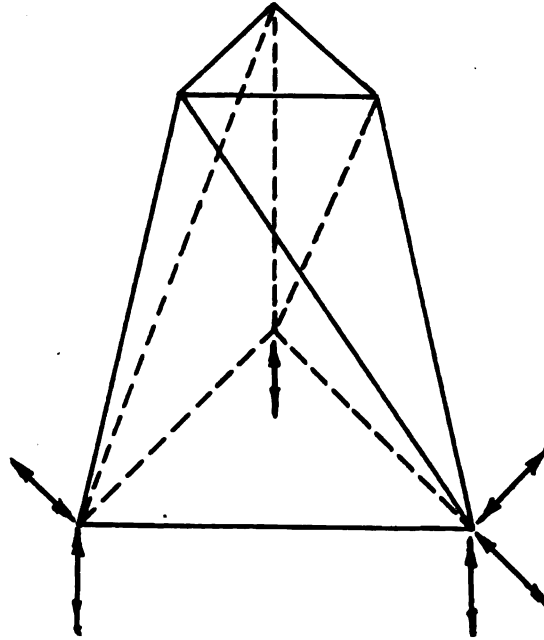
p = " " " all members coplanar.

8, 15, 16, 18.

Proble
Det
emina

Problem 2, Page 16

Determine whether the structure shown is statically determinate, statically indeterminate or unstable.



$$r + b = 3j - p$$

$$6 + 12 = 3 \times 6 - 0$$

$$18 = 18$$

This structure is determinate.

Problem
Dete
termina

Diage

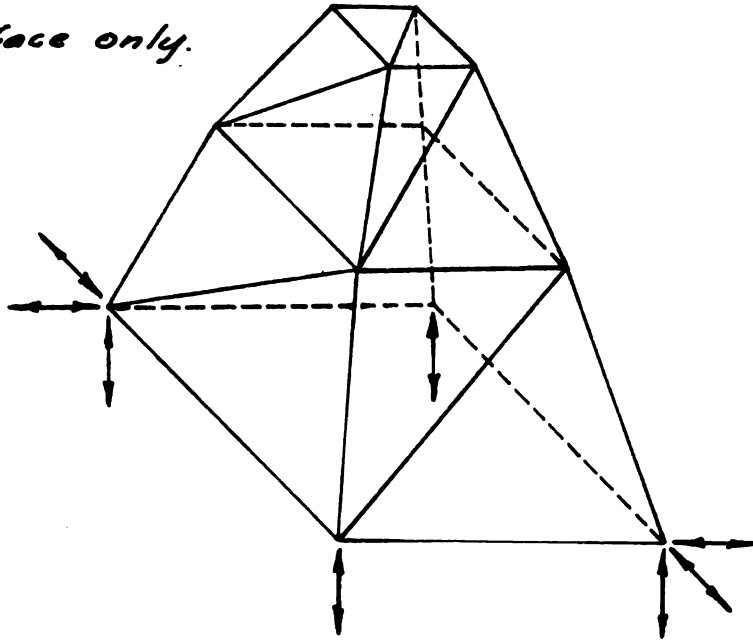
front

7/

Problem 3, Page 16.

Determine whether the structure shown is statically determinate, statically indeterminate or unstable.

*Diagonals shown in
front face only.*



$$r + b = 3j - p$$

$$8 + 25 = 3 \times 12 - 0$$

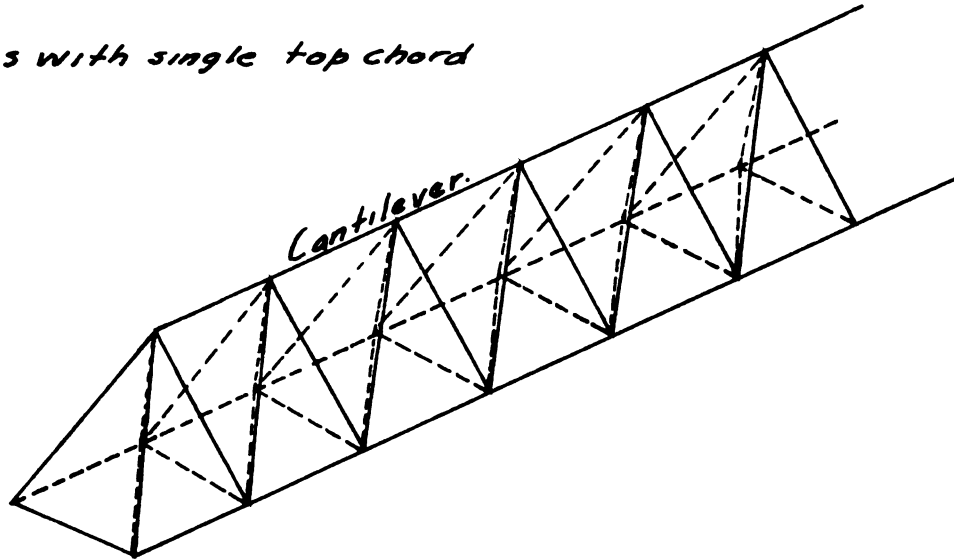
$$33 = 36$$

This structure is unstable.

Problem 4, Page 16.

Determine whether the structure shown is statically determinate, statically indeterminate or unstable.

Truss with single top chord



$$r + b = 3j - p$$

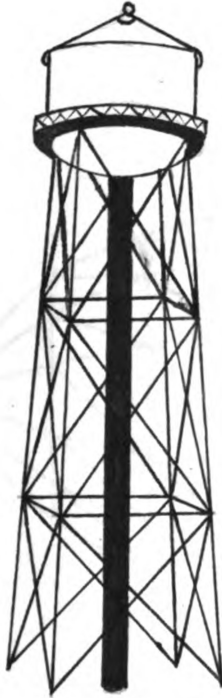
$$6 + 59 = 3 \times 20 - 0$$

$$65 = 60$$

This structure is indeterminate.

Problem 5, Page 16.

Study the layout of a typical four-legged tower and decide how it may be braced and made statically determinate. A three legged tower with battered legs is suggested.



$$r + b = 3j - p$$

$$12 + 48 = 3 \times 16 - 0$$

$$60 = 48$$

This structure is indeterminate.

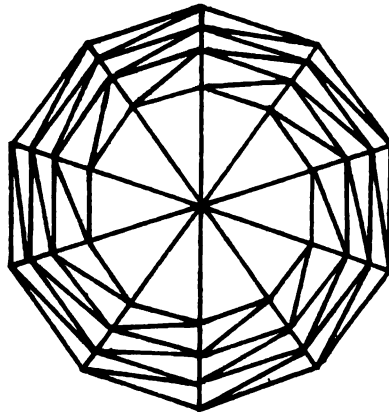
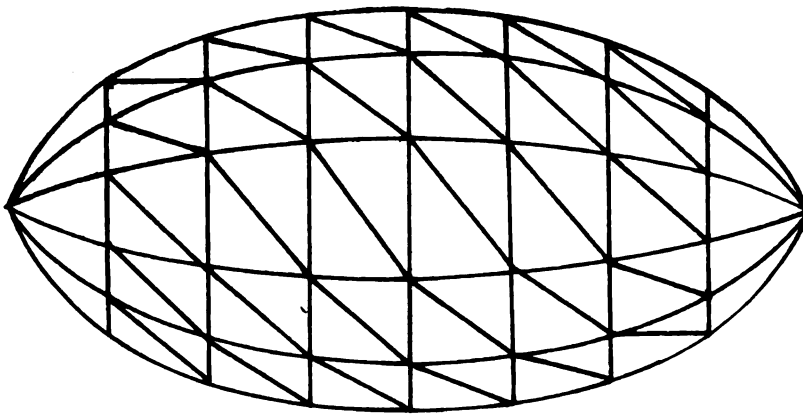
If 12 tie rods could be removed, it would then become determinate.

PROB
THE
ALLOU
ES CO



Problem 6, Page 16.

The dirigible frame shown is made up of decagonal rings. Allow for six reaction components and show that it's stresses could be calculated by the equations of statics.



$$r + b = 3j - p$$

$$6 + 210 = 3 \times 72 - 0$$

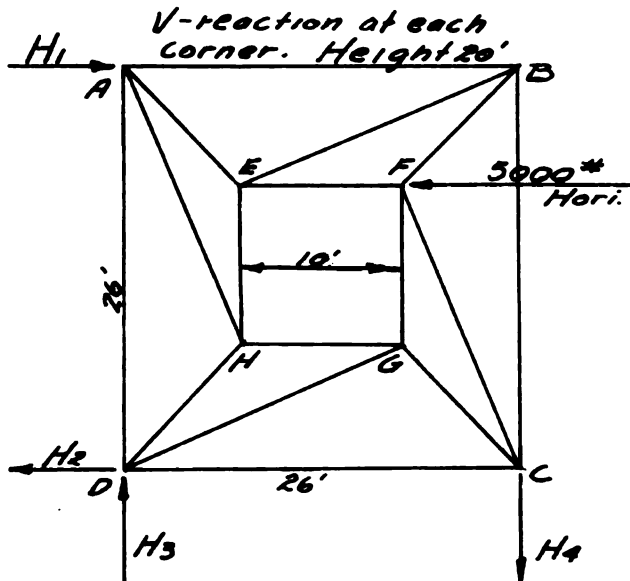
$$216 = 216$$

This structure is determinate.

8, 15, 16, 18

Problem 7, Page 30.

Analyze for the stresses in the framed pedestal shown. Note that although there are more than six reactions, you can start the analysis quite simply by observing that at each upper chord joint all members except one, lie in a plane.



$$r+b = 3j-p$$

$$8+16 = 3 \times 8 - 0$$

$$24 = 24$$

This structure is determinate.

Write $\sum F_x = 0$, $\sum F_y = 0$

$\sum F_z = 0$ at joints E, F

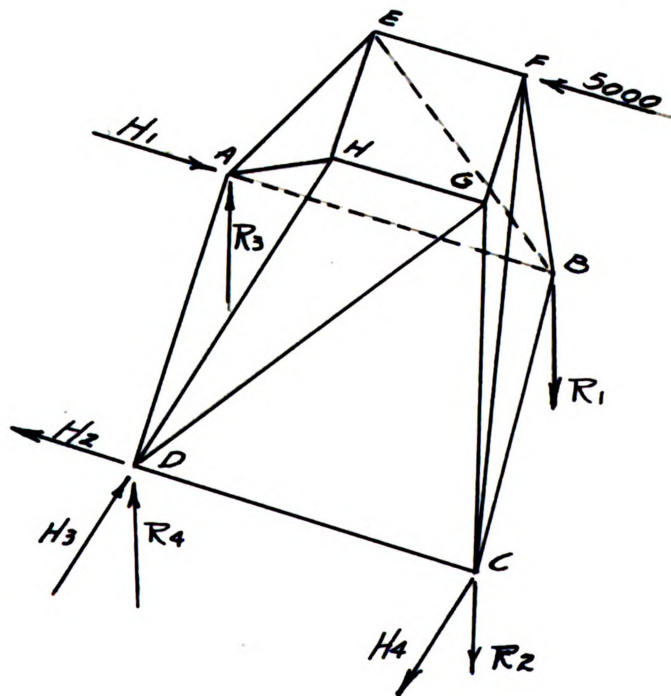
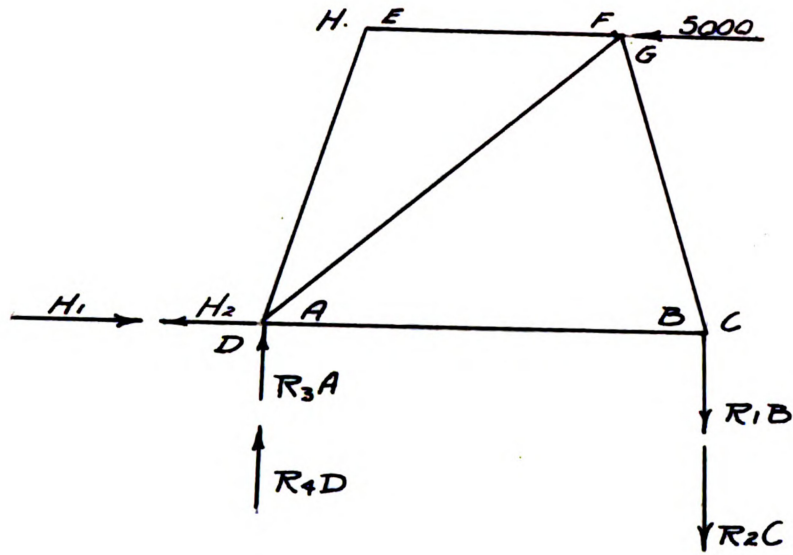
G & H.

- 1 $\sum F_x = 0$ at E $\frac{16}{28} EB + \frac{9}{22.9} EA - EF = 0$
- 2 $\sum F_y = 0$ at " $\frac{20}{22.9} EA - \frac{20}{28} EB = 0$
- 3 $\sum F_z = 0$ at " $\frac{9}{22.9} EA - \frac{9}{28} EB - EH = 0$
- 4 $\sum F_x = 0$ at F $EF - \frac{9}{22.9} FB + \frac{9}{28} FC - 5000 = 0$
- 5 $\sum F_y = 0$ at " $\frac{20}{22.9} FB - \frac{20}{28} FC = 0$
- 6 $\sum F_z = 0$ at " $-FG + \frac{9}{22.9} FB + \frac{16}{28} FC = 0$
- 7 $\sum F_x = 0$ at G $+HG - \frac{16}{28} DG - \frac{9}{22.9} CG = 0$
- 8 $\sum F_y = 0$ at " $\frac{20}{22.9} GL - \frac{20}{28} DG = 0$
- 9 $\sum F_z = 0$ at " $FG + \frac{9}{28} DG - \frac{9}{22.9} CG = 0$
- 10 $\sum F_x = 0$ at H $-HG - \frac{9}{28} HA + \frac{9}{22.9} HD = 0$
- 11 $\sum F_y = 0$ at " $\frac{20}{22.9} HD - \frac{20}{28} HA = 0$
- 12 $\sum F_z = 0$ at " $EH - \frac{9}{22.9} HD - \frac{16}{28} HA = 0$

8, 9, 12

Next Page

Prob 7 "cont."



Prob 7 'Cont':

From eq. 19 $EF = 5000 C$

$$Eq. 1 \quad -\frac{8}{22.9} AE + \frac{18}{28} EB = EF$$

$$Eq. 2 \quad -\frac{29}{22.9} AE - \frac{29}{28} EB = 0$$

$$Eq. 1 \times 2.5 \quad -\frac{20}{22.9} AE + \frac{45}{28} EB = 2.5 EF = 12500$$

$$\frac{65}{28} EB = 12500$$

$$EB = 5400 T$$

$$Eq. 2 \quad \frac{29}{22.9} AE = \frac{29}{28} \times 5400 \quad AE = 4420 C$$

$$Eq. 3 \quad \frac{8}{22.9} \times 4420 - 1540 = EH \quad EH = 0$$

$$Eq. 5 \quad \frac{29}{22.9} FB = \frac{29}{28} FC$$

Summary

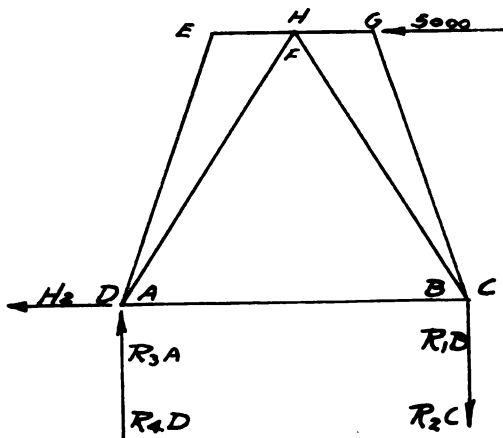
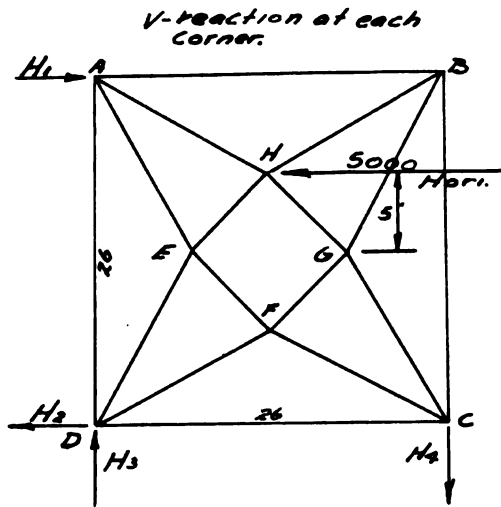
Member	Stress	Member	Stress
AE	4420C	AD	1540T
EB	5400T	H ₁	5000
EH	0	H ₂	0
FB	0	H ₁ & H ₂	1540
FC	0	DC	0
GC	0	AB	3460C
FG	0	EF	5000C
GH	0	AD & DH	0
BC	1540C	GD	0
R ₁ D	3860	R ₂ D	0
R ₁ A	3860	R ₂ C	0

Problem 8, Page 30

Explain in detail, why the analysis of this pedestal shown would be more difficult than the one of Problem 7. Can you write enough simultaneous equations from the equilibrium of joints E, F, G and H to obtain the value of any bar stress?

Can any stress be determined from less than 12 simultaneous equations for any combination of joints?

(Note) These questions form a written discussion by Prof. Grinter in Proceedings A.S.C.E. for September 1934, pages 1086-1091.



The solution of this problem would be more difficult than the preceding one, because, in Prob 7, at each upper chord joint, all members except 1 lie in a common plane. In

Prob. 8, at any joint there are only 2 members that lie in a common plane.

Enough simultaneous equations (12) could be written for the joints

E, F, G, H and a solution made, because there are only 12 unknowns. We must write 12 equations (3 for each joint)

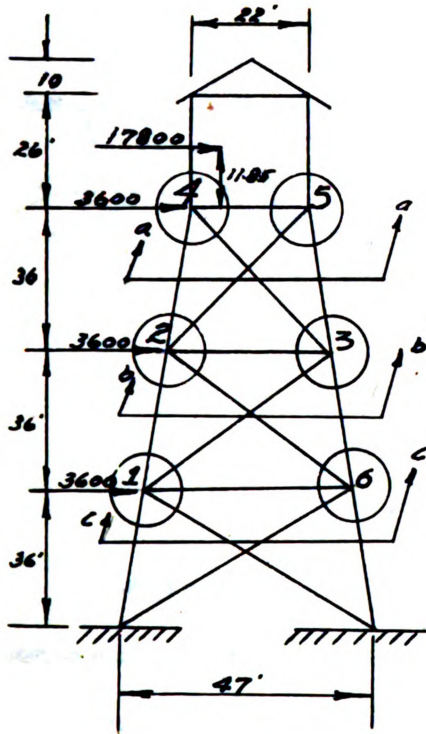
$\sum F_x = 0$, $\sum F_y = 0$, $\sum F_z = 0$, and no fewer will do for a solution.

See Proceedings A.S.C.E. Sept 1934, pages 1086-1091.

—

Problem 9, Page 30.

Analyze for the wind stresses in the 4 post water tower shown. This is a standard tower with tank of 100,000 gal. capacity. Wind pressure is obtained from 20 pounds per square foot on the dimetral plane of the tank and 200 pounds per vertical foot on the tower. Loads are given for the entire structure. Select rods to serve as diagonals at a working stress of 18000 pounds per square inch.



$$DC: 4830 T$$

$$CE: 14000 T$$

$$BC: 16250 T$$

$$CF: 10495 C$$

$$BA: 26100 T$$

$$DE: 1245 C$$

$$BF: 13500 T$$

$$EF: 60200 T$$

$$BG: 11720 C$$

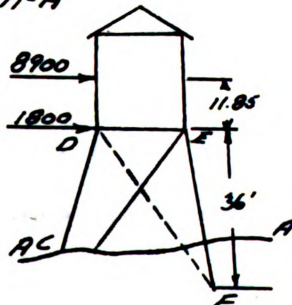
$$FG: 50600 T$$

$$AG: 14150 T$$

$$GH: 40800 T$$

$$\sum M_E = 0$$

Sec A-A



$$\sum M_F = 0$$

$$7800 \times 36 + 8900 \times 47.85 = 30 \frac{1}{2} BC_V$$

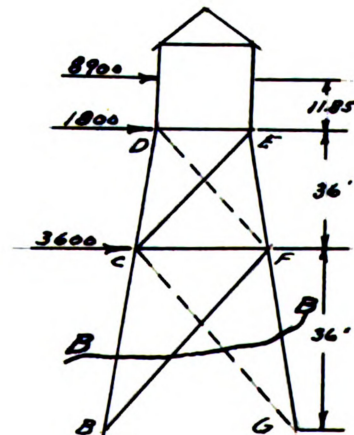
$$BC_V = 16175 \quad BC_H = 1910 \quad BC = 16250 T$$

$$8900 \times 11.85 = 22 DC_V$$

$$DC_V = 4800 \quad DC_H = 555$$

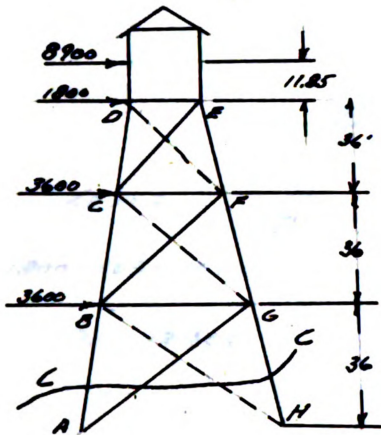
$$DC = 4830 T$$

Sec B-B



$$8, 9, 12$$

Prob 9 Cont

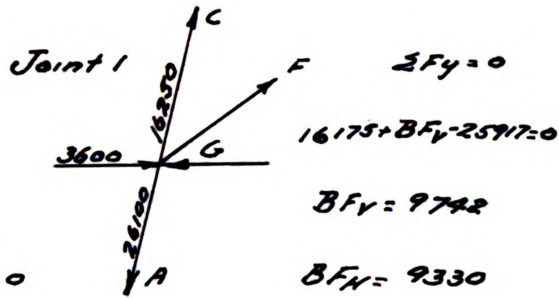


Sec. C-C

$$\sum M_G = 0$$

$$8900 \times 83.85 + 1800 \times 72 + 3600 \times 36 = 38 \frac{2}{3} B_{AV}$$

$$B_{AV} = 25917, B_{AH} = 3120 \quad B_A = 26100 T$$



$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$16175 + B_{FV} - 25917 = 0$$

$$B_{FV} = 9742$$

$$B_{FH} = 9330$$

$$B_G = 3600 + 1910 + 9330 - 3120 \quad B_F = 13600 T$$

$$B_G = 11720 C$$

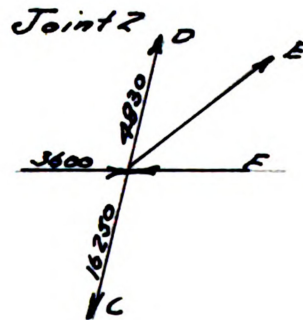
$$\sum F_y = 0$$

$$4830 + C_{EV} - 16175 = 0$$

$$C_{EV} = 11345$$

$$C_{EH} = 8250$$

$$C_E = 14000 T$$



$$\sum F_y = 0$$

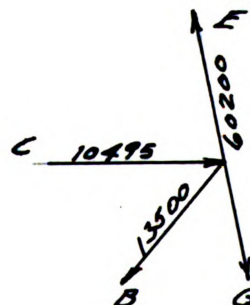
$$60000 = F_{Gy} + 9742$$

$$F_{Gy} = 50258$$

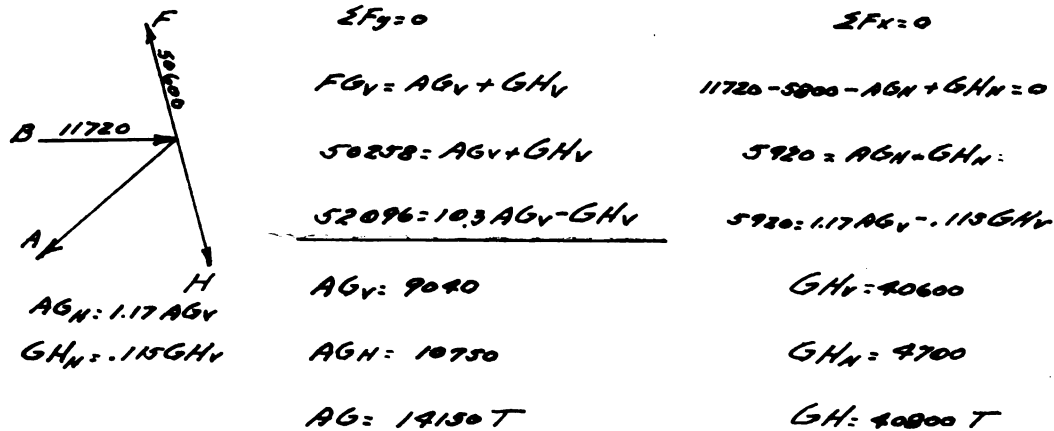
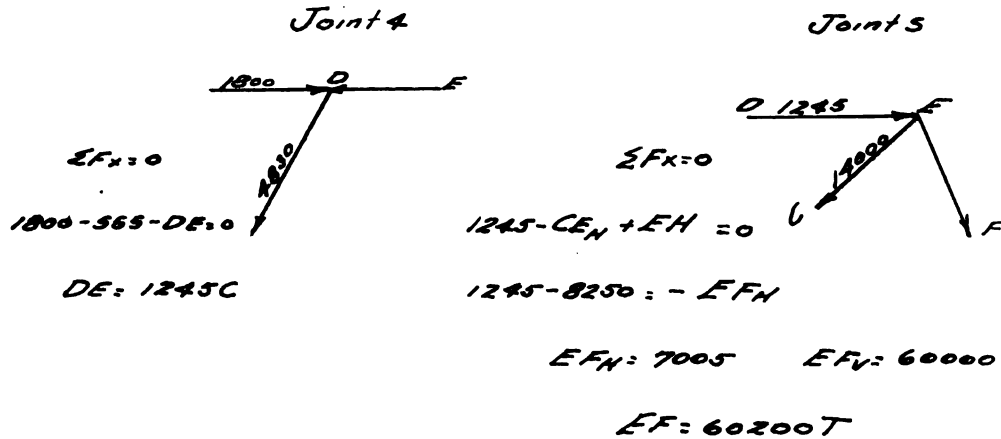
$$F_{GH} = 5800$$

$$F_G = 50600 T$$

Joint 3



Prob 9 Cont.



$AG = \frac{14130}{18000} = .7854''$ - Area 1" rod = .7854 - use 1" rod for member AG with upset end.

$BF = \frac{13500}{18000} = .75''$ use same size and kind as for AG

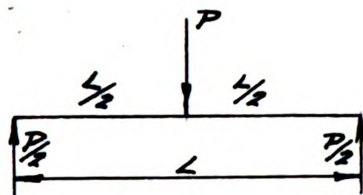
$CE = \frac{14000}{18000} = .778''$ use same size and kind as for AG & BF

1

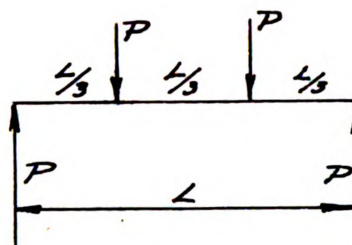
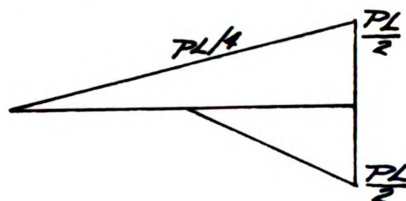
Problem 12, Page 34.

Obtain an expression for the deflection under a concentrated load placed at the center of this simple span beam.

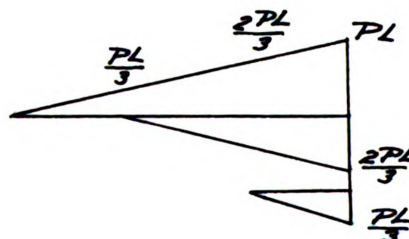
Repeat for deflection under one concentrated load for a simple beam carrying concentrated loads at the one third points.



Case 1



Case 2



$$EI \Delta = \frac{P}{2} \times \frac{L}{2} \times \frac{L}{2} \times \frac{1}{2} \times \frac{L}{2} \times \frac{2}{3}$$

$$\Delta = \frac{PL^3}{48EI}$$

$$EI \Delta_1 = \frac{PL}{2} \times \frac{L}{2} \times \frac{L}{2} \times \frac{2}{3} \times \frac{L}{2} - \frac{PL}{6} \times \frac{L}{6} \times \frac{L}{2} \times \frac{4}{9}$$

$$EI \Delta_1 = \frac{PL^3}{24} - \frac{PL^3}{162}$$

$$\Delta_1 = \frac{PL^3}{24EI} - \frac{PL^3}{162EI}$$

$$y = \Delta_1 - \Delta_2$$

$$= \frac{PL^3}{24EI} - \frac{PL^3}{162EI} - \frac{PL^3}{216EI}$$

$$y = \frac{5PL^3}{162EI}$$

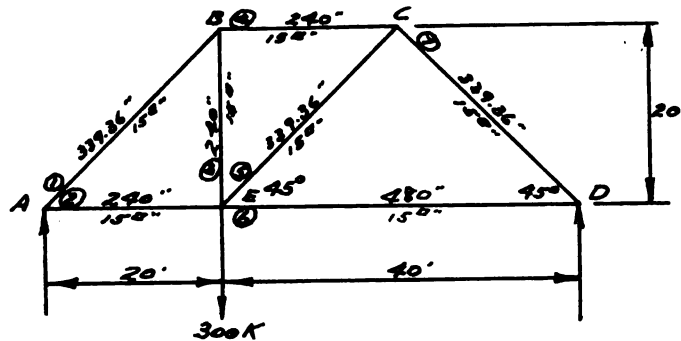
$$EI \Delta_2 = \frac{PL}{3} \times \frac{L}{6} \times \frac{L}{6} \times \frac{1}{2} + \frac{PL}{6} \times \frac{L}{2} \times \frac{L}{6} \times \frac{2}{3 \times 6} - \frac{PL}{6} \times \frac{L}{2} \times \frac{L}{6} \times \frac{2}{3} \times \frac{L}{6}$$

$$\Delta_2 = \frac{PL^3}{216EI}$$

4

Problem 13, Page 34.

Compute the deflection under the 300,000 pound load for the truss illustrated. Each member has an area of 15 square inches; $E=30,000,000$



Member	Length	Area	P	P_1	$\frac{PL}{AE}$	$\frac{PP_1L}{AE}$
1	339.36	15	-282800	- .943	-.213	+.20
2	240	"	+200000	+.667	+.107	+.071
3	"	"	"	"	"	+.071
4	"	"	-200000	-.667	-.107	+.071
5	339.36	"	+141400	+.471	+.107	+.050
6	480	"	+100000	+.333	"	+.035
7	339.36	"	-141400	-.471	-.107	+.050

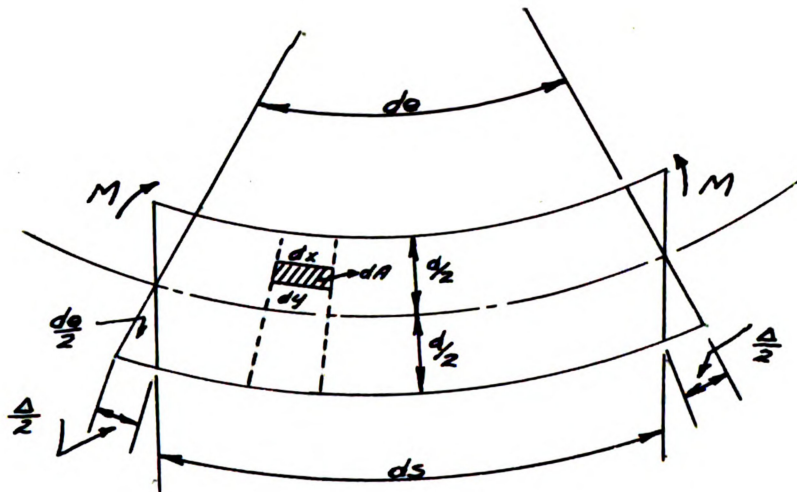
$$\Delta = .548 \text{ inches.}$$

8, 15, 18, 19

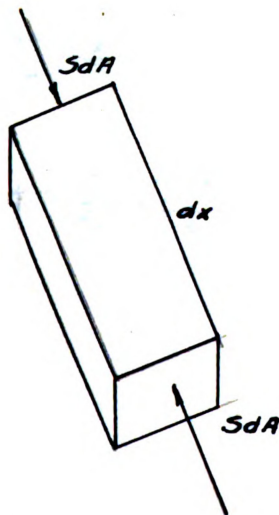
4

Problem 14, Page 34.

Derive an expression for the internal work in a beam. Make use of the sketch that represents a differential length of beam over which the moment may be considered to be constant. (hint) First determine the value of $\frac{d\theta}{2}$ with the aid of the flexure formula.



Take a differential area dA , a distance dy from the neutral axis of the beam, and assume it to act as a column.



$$\text{Load } SdA = P$$

$$S = \frac{My}{I} \quad \ell = dx$$

$$P = \frac{MydA}{I}$$

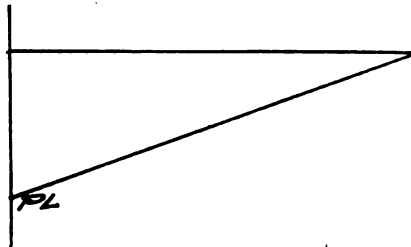
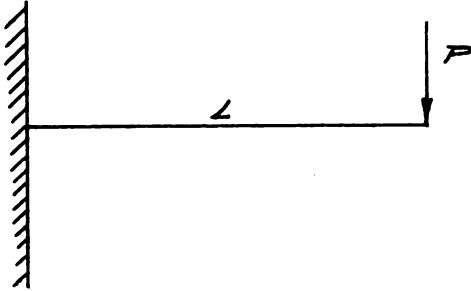
$$U = \frac{P^2 \ell}{2AE} = \frac{M^2 y^2 dA^2 dx}{2AE}$$

$$dU = \frac{M^2 y^2 dA^2 dx}{2I^2 dAE} \quad I = \int y^2 dA$$

$$U = \int dU = \iint \frac{M^2 y^2 dA dx}{2EI^2} = \frac{1}{2} \int \frac{M^2 dx}{EI} \quad 8, 15, 18, 19$$

Problem 15, Page 34

Prove that the internal work in a beam of any type is equal to $1/E.I.$ times the statical moment of the moment diagram about it's base. The loads are gradually applied.



Statical moment about
base of Moment Curve.

$$= PL \times \frac{1}{2} \times \frac{PL}{3} = \frac{P^2 L^3}{6}$$

$$W = \int_0^L \frac{M^2 dx}{2EI} = \int_0^L \frac{P^2 x^2 dx}{2EI} = \frac{P^2}{2EI} \int_0^L x^2 dx$$

$$W = \frac{P^2}{2EI} \left(\frac{x^3}{3} \right)_0^L = \frac{P^2 L^3}{6EI}$$

Work = $\frac{1}{EI} \left(\frac{P^2 L^3}{6} \right)$, which is $\frac{1}{EI}$ times the statical
moment.

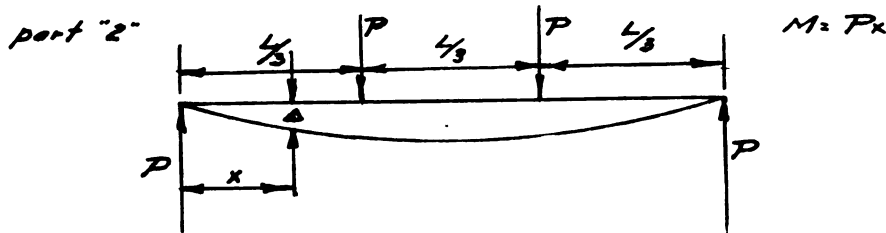
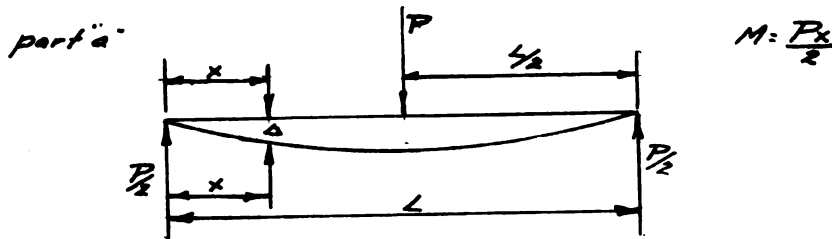
11

Problem 16, Page 35.

Work Problem 12 by use of Castigliano's theorem.

Problem 12 is stated as follows.

Obtain an expression for the deflection under a concentrated load placed at the center of a simple beam. Repeat the discussion for the deflection under one concentrated load for a simple beam carrying concentrated loads at the $1/3$ points.



$$a. U = \frac{P\Delta}{2} = 2 \int_0^{\frac{L}{2}} \frac{M^2 dx}{2EI} = 2 \int_0^{\frac{L}{2}} \frac{P^2 x^2 dx}{4 \times 2EI}$$

$$\frac{P\Delta}{2} = 2 \left(\frac{P^2 x^3}{3 \times 4 \times 2EI} \right)_0^{\frac{L}{2}} = \frac{2P^2 L^3}{8 \times 3 \times 4EI} = \frac{P^2 L^3}{8 \times 3 \times 4EI}$$

$$\Delta = \frac{PL^3}{48EI}$$

$$2. U = 2 \frac{P\Delta}{2} = 2 \int_0^{\frac{L}{3}} \frac{M^2 dx}{2EI} + \int_{\frac{L}{3}}^{\frac{2L}{3}} \frac{M^2 dx}{2EI}$$

$$= 2 \int_0^{\frac{L}{3}} \frac{P^2 x^2 dx}{2EI} + \left(\frac{PL}{3} \right)^2 \int_{\frac{L}{3}}^{\frac{2L}{3}} \frac{dx}{2EI}$$

$$= 2 \left(\frac{P^2 x^3}{6EI} \right)_0^{\frac{L}{3}} + \left(\frac{PL}{3} \right)^2 \left[\frac{x}{2EI} \right]_{\frac{L}{3}}^{\frac{2L}{3}} = 2 \left(\frac{P^2 L^3}{162EI} \right) + \frac{P^2 L^2}{9} \left(\frac{\frac{2L}{3} - \frac{L}{3}}{2EI} \right)$$

$$\frac{2P\Delta}{2} = \frac{2P^2 L^3}{162EI} + \frac{P^2 L^3}{54EI}$$

$$\Delta = \frac{5PL^3}{162EI}$$

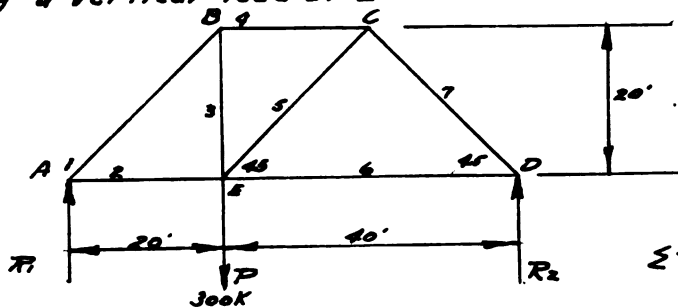
Problem 17, Page 35.

Work Problem 13 by use of Castiglian's theorem. When setting up the expression for internal work, let P equal the load.

Problem 13 is as follows.

Compute the deflection under the 300,000 pound load for the truss illustrated. Each member has an area of 15 square inches.

Apply a vertical load at E



$$\Sigma U = \frac{\Sigma P^2 L}{\Sigma A E}$$

$$R_1 = (300000 + P) \frac{2}{3}$$

$$\text{Total Load} = 300000 + P$$

$$R_2 = (300000 + P) \frac{1}{3}$$

$$S_5 = (300000 + P) \cdot 473$$

$$S_1 = (300000 + P) \cdot 943$$

$$S_6 = \quad \quad \quad .333$$

$$S_2 = \quad \quad \quad .666$$

$$S_7 = \quad \quad \quad .473$$

$$S_3 = \quad \quad \quad .666$$

$$S_4 = \quad \quad \quad .666$$

$$\Sigma U = \frac{[(300000 + P) \cdot 943^2] 339.36}{30 \times 30 \times 10^6} + 3 \frac{[(300000 + P) \cdot 666^2] \times 240}{30 \times 30^6} + 2 \frac{[(300000 + P) \cdot 473^2] 339.36}{30 \times 30^6}$$

$$+ \frac{[(300000 + P) \cdot 333^2] 480}{30 \times 30^6}$$

$$\frac{\partial U}{\partial P} = \delta = \frac{2(300000 + P) \cdot 89 \times 339.36}{30 \times 30^6} + \frac{2 \times 3(300000 + P) \cdot 443 \times 240}{30 \times 30^6} + \frac{2 \times 2(300000 + P) \cdot 22 \times 339.36}{30 \times 30^6} + \frac{2(300000 + P) \cdot 111 \times 480}{30 \times 30^6}$$

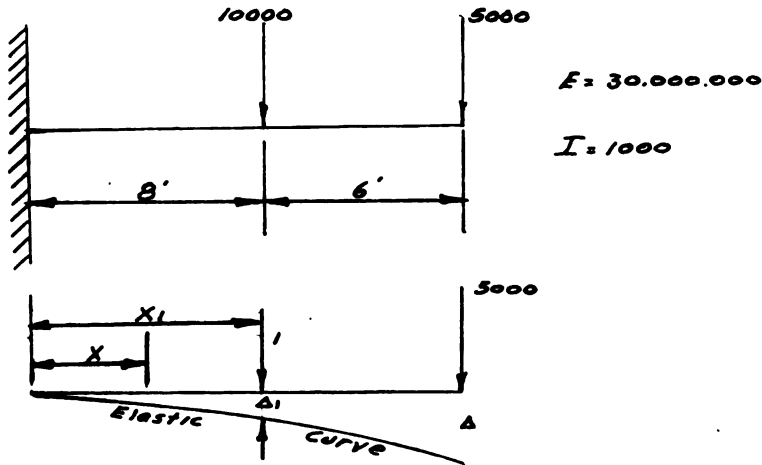
Let $P = 0$

$$\delta = .20144 + .21264 + .09954 + .03552 = .549''$$

8.18

Problem 18, Page 35.

Find the deflection under the 10,000 pound load for the cantilever beam illustrated. Let P equal the load when setting up the expression for internal work.



$$\begin{aligned}\delta_1 &= \int_0^{x_1} \frac{M M_1 dx}{EI} \\ EI \delta_1 &= \int_0^{x_1} P(l-x)(x_1-x) dx + \int_{x_1}^l M x_1 dx \\ &= P \int_0^{x_1} [l x_1 - l x - x x_1 + x^2] dx \\ &= P \left(l x_1^2 - \frac{l x_1^2}{2} - \frac{x_1^3}{2} + \frac{x_1^3}{3} \right) \quad x_1 = x \\ &= \frac{P}{6} (3l x_1^2 - x_1^3) = \frac{P x_1}{6} (3l - x_1)\end{aligned}$$

$$\delta_1 = \frac{P x_1^2}{6EI} (3l - x_1)$$

δ_1 for 5000 #

$$\delta_1 = \frac{5000 \times 96^2}{6 \times 1000 \times 30 \times 10^6} (3 \times 168 - 96) = .104448''$$

δ_1 for 10000 #

$$\delta_1 = \frac{10000 \times 96^2}{6000 \times 30 \times 10^6} (3 \times 96 - 96) = .0985$$

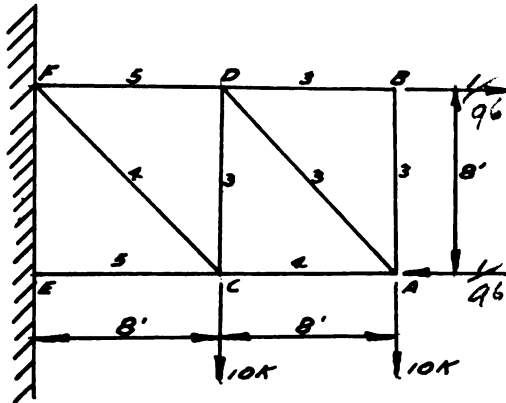
$$\delta_1 \text{ for } 5000 + \delta_1 \text{ for } 10000 = \delta$$

$$.1044 + .0985 = .2029''$$

8.18.19

Problem 20, Page 40.

Compute the angular rotation of the bar AB of the truss of Problem 19. (Problem 19 on previous page)



Bar	Kip	$\frac{1}{16}$ " at B $\rightarrow$ 4 $\frac{1}{16}$ " at A $\leftarrow$	L"	A	$\frac{S_u L}{A}$
AB	0	0	96	3	0
AC	-10	$-\frac{1}{16}$	"	4	2.5
AD	+14.14	0	135.5	3	0
BD	0	$+\frac{1}{16}$	96	"	0
DC	-10	0	"	"	0
DF	+10	$+\frac{1}{16}$	"	5	2
CE	-30	$-\frac{1}{16}$	"	5	6
FC	+28.28	0	135.5	4	0
$\sum \frac{S_u L}{A} =$					10.5

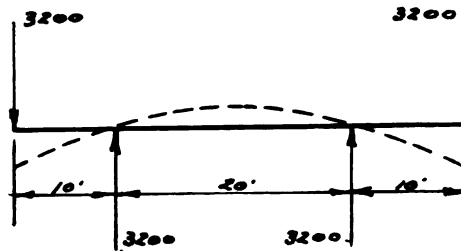
$$\sum \frac{S_u L}{AE} = \frac{10.5 \times 1000}{30 \times 10^6}$$

$$1 \times \theta = \frac{10.5 \times 1000 \times 57.3 \times 3600}{30 \times 10^6} = 72.3 \text{ sec.}$$

8, 15, 18, 19

Problem 21, Page 40.

Compute the maximum upward deflection of the 12 inch, 31.8 pound I-beam supported and loaded as shown.



12"-31.8" beam

$I = 215.8$

$E = 30 \times 10^6$

$G = 12 \times 10^6$

$M = \text{constant at } 3200 \times 120$

$$\Delta = \frac{\int Mm'dx}{EI} + \frac{\int VV'dx}{AG} \quad \frac{\int VV'dx}{AG} = 0$$

$$\Delta = 2 \int_0^{120} (3200 \times 120) \frac{x}{2} dx$$

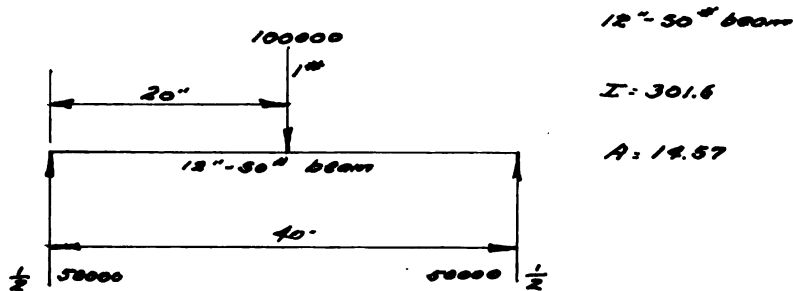
$$EI \Delta = \frac{2}{2} \left[3200 \times 120 \left(\frac{x^2}{2} \right) \right]$$

$$\Delta = \frac{3200 \times 120 \times 120 \times \frac{120}{2}}{215.8 \times 30 \times 10^6}$$

$$\Delta = \frac{3200 \times 120 \times 120 \times 60}{215.8 \times 30 \times 10^6} = .427''$$

Problem 22, Page 40.

Compute the bending and the shearing deflection at the center of a 12 inch 50 pound I-beam acting as a simple span of 40 inches. The beam carries a concentrated center load of 100,000 pounds. Assume that the beam is restrained against lateral buckling and that the web carries all the shear.



Bending
$$\Delta = \int_0^L \frac{M m dx}{EI} = 2 \int_0^{20} \frac{50000 x^2 dx}{301.6 \times 30 \times 10^6 \times 2}$$

$$= \frac{50000 \times 20^3}{3 \times 301.6 \times 30 \times 10^6} = .0147''$$

Web area : $.687 \times 12 = 8.24$

Shear :
$$2 \int_0^L \frac{V v ds}{AG} = 2 \int_0^{20} \frac{50000 dx}{AG \times 2}$$

$$= \frac{50000 \times 20}{8.24 \times 12 \times 10^6} = .0101''$$

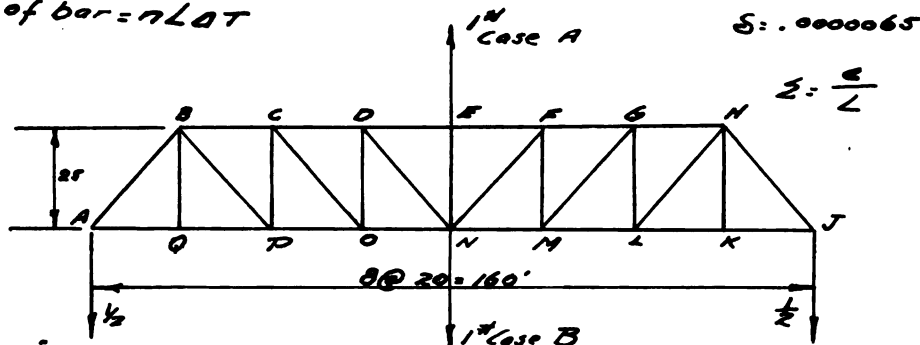
1.2.8.16.19

Problem 23, Page 41.

Find the maximum vertical deflection of the Pratt truss shown as produced by a 20 degree increase in temperature of the top chord and end posts. Take the coefficient of expansion to be .0000065. Is the deflection up or down?

Explain why the cross sectional areas of the members are not needed. What would be the maximum deflection if all the members of the lower chord were fabricated $\frac{1}{4}$ inch too long?

e : total change in length
of bar = $n \Delta L \alpha T$



Case A

Bar	S in #5	$u - 1$	L	$S u L$	Total $e = \sum \Delta L$
AB	$\frac{.0305 \times EA}{388.8}$	+1.646	388.8	.03262 EA	.0505
BC	$\frac{.0312 \times EA}{240}$	+1.8	240	.02496 EA	.0312
CD	"	+1.2	"	.03744 EA	"
DE	"	+1.6	"	.04992 EA	"
EF	"	+1.6	"	"	"
FG	"	+1.2	"	.03744 EA	"
GH	"	+1.8	"	.02496 EA	"
HJ	$\frac{.0505 \times EA}{388.8}$	+1.646	388.8	.03262 EA	.0515
$\Delta V = \sum \frac{S u L}{AE} = \frac{.28988 \times EA}{AE} = .28988"$					.28988

Deflection will be up. We do not need the value of the Area of cross section as it cancels out in the final solution.

8, 16, 18, 19

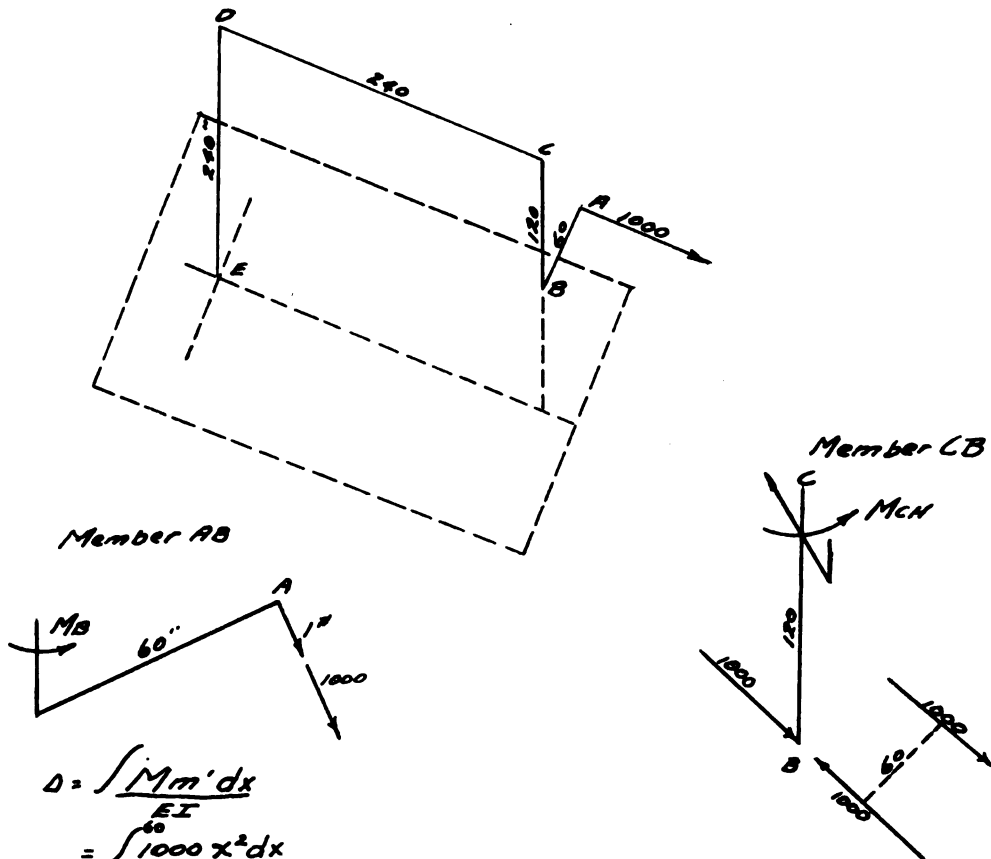
Prob 23 "Cont"

Case "B"

Bar	Size $\frac{1 \times 8 \text{ A}}{4 \times 240}$	Area in ²	L	SUL $\frac{1 \times 8 \text{ A} \times 4 \times 240}{4 \times 240}$	SUL/AE $\frac{1 \times 8 \text{ A} \times 4 \times 240}{4 \times 240 \times 8 \text{ A}} = .1$
AQ	"	-.4	240"	"	" = .1
QP	"	"	"	"	" = .1
PO	"	-.8	"	$\frac{1 \times 8 \text{ A} \times 8 \times 240}{4 \times 240}$	$\frac{1 \times 8 \text{ A} \times 8 \times 240}{4 \times 240 \times 8 \text{ A}} = .2$
ON	"	-1.2	"	$\frac{1 \times 8 \text{ A} \times 12 \times 240}{4 \times 240}$	$\frac{1 \times 8 \text{ A} \times 12 \times 240}{4 \times 240 \times 8 \text{ A}} = .3$
NM	"	"	"	"	" = .3
ML	"	-.8	"	$\frac{1 \times 8 \text{ A} \times 8 \times 240}{4 \times 240}$	$\frac{1 \times 8 \text{ A} \times 8 \times 240}{4 \times 240 \times 8 \text{ A}} = .2$
LK	"	-.4	"	$\frac{1 \times 8 \text{ A} \times 4 \times 240}{4 \times 240}$	$\frac{1 \times 8 \text{ A} \times 4 \times 240}{4 \times 240 \times 8 \text{ A}} = .1$
KJ	"	"	"	"	" = .1
				$\Sigma \frac{\text{SUL}}{\text{AE}}$	= 1.4"

Problem 24, Page 41.

A steel pipe is bent as shown to provide for expansion. The pipe has a polar moment of inertia of 292 and a moment of inertia about a diameter of 146. Compute the deflection of the point A in the direction of the 1000 pound load by taking into account flexure and torsion. Note that the member CD is bent in a diagonal plane.



$$\Delta = \int \frac{Mm'dx}{EI}$$

$$= \int_0^{60} \frac{1000x^2 dx}{EI}$$

$$= \frac{1000x^3}{3EI} \Big|_0^{60}$$

$$= \frac{1000 \times 60^3}{3 \times 146 \times 30 \times 10^6} = .0164''$$

$$BMA = \int \frac{Mm'dx}{EI} = \int_0^{120} \frac{1000x^2 dx}{EI}$$

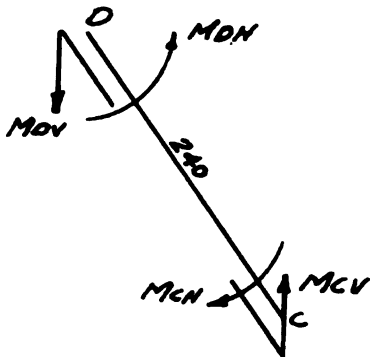
$$= \frac{1000 \times 120^3}{3 \times 146 \times 30 \times 10^6} = .131''$$

$$\text{Torsion } \Delta = \int \frac{Tt ds}{GJ} = \int_0^{120} \frac{60000 \times 60 dx}{GJ}$$

$$\Delta = \frac{60000 \times 60 \times 120}{12 \times 10^6 \times 292} = .123''$$

8, 13, 15, 16, 18, 19

Prob 24 Cont



$MCN = \text{constant}$

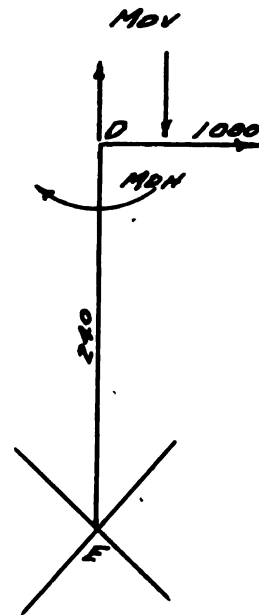
$$BMA = \int_0^{240} \frac{(60000)60dx}{EI}$$

$$\Delta = \frac{60000 \times 60 \times 240}{146 \times 30 \times 10^6} = .197''$$

$MCV = \text{constant}$

$$\Delta = \int_0^{240} \frac{Mmds}{EI}$$

$$= \frac{120000 \times 120 \times 240}{146 \times 30 \times 10^6} = .79''$$



$$MDV \Delta = \int_0^{240} \frac{1000 \times 240}{EI}$$

$$\Delta = \frac{1000 \times 240^3}{3 \times 146 \times 30 \times 10^6} = 1.05''$$

$$MDH \Delta = \int_0^{240} \frac{(60000)60dx}{GJ}$$

$$\Delta = \frac{60000 \times 60 \times 240}{12 \times 10^6 \times 292} = .245$$

Total Deflection in direction of A

.0164 - AB - bending mom.

.1310 - CB - " "

.1230 - CB - Torsion

1.05 - DE - bending mom

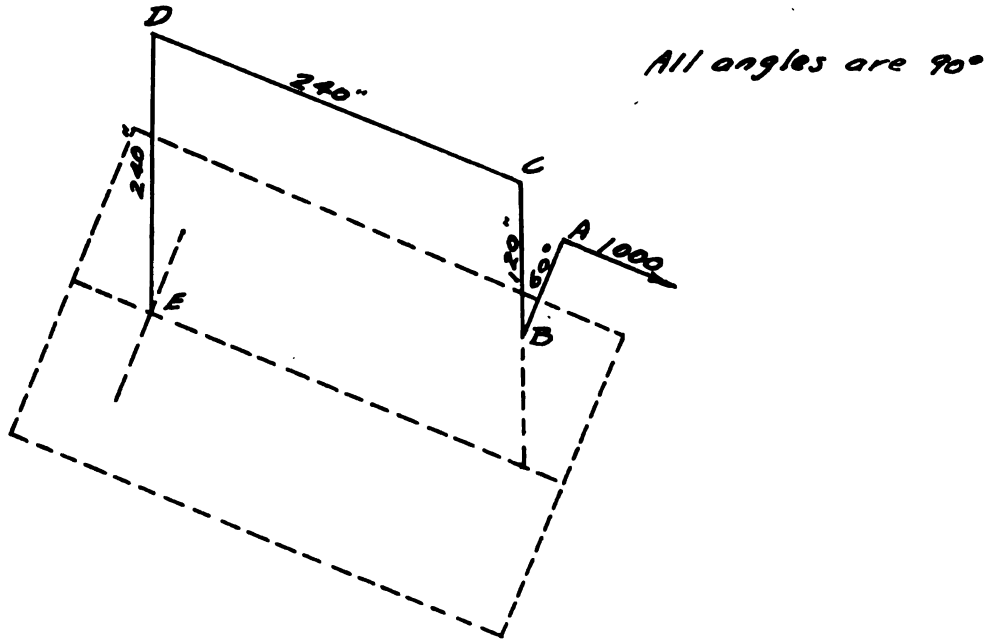
.245 - DE - Torsion

.197 - DC - bending mom

Total - 1.762''

Problem 25, Page 41.

Find the absolute deflection in space of point A of the pipe considered in the previous problem (Problem 24).



	Mem.	M	m'	T	t'	$\frac{Mm'}{EI}$	$\frac{Tt}{GJ}$	Total Δ
Z direction	AB		0		0	0	0	$\Delta z = 1.391$
	CB	0			0	0	0	
	CD	60x1000	X	0		.406	0	
	DE	0		60x1000	240		.985	
Y direction	AB	0		0		0	0	$\Delta y = .790$
	BC	0	0	0	0	0	0	
	CD	120x1000	X	0		0	.790	
	DE	0	0	0	0	0	0	

Δx from Prob 24 = 1.76

$$\Delta_T = \sqrt{1.76^2 + 1.391^2 + .79^2} = 2.38''$$

8, 13, 15, 16, 18, 19

Prob
Co
inch,
load
inch

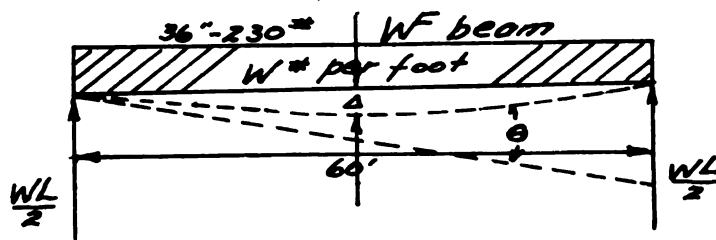
2

6

4

Problem 26, Page 47.

Compute the end slope and the maximum deflection of a 36 inch, 230 pound W.F. beam for a simple span of 60 feet, when loaded to it's allowable stress of 18000 pounds per square inch by a uniform load. Do not develop a formula.



$$I = 14988.4$$

$$S = \frac{Mc}{I}$$

$$M = \frac{WL^2}{8}$$

$$\frac{WL^2}{8} = \frac{SI}{C}$$

$$a \quad \theta = \int_0^{360} \frac{M dx}{EI} = \int_0^{360} \frac{\left(\frac{WLx}{2} - \frac{Wx^2}{2}\right) dx}{EI}$$

$$= \frac{1}{EI} \left[\int_0^{360} \frac{WLx^2}{4} - \frac{Wx^3}{6} \right] = \frac{1}{EI} \left[\frac{230 \times 720 \times 360^2}{4} - \frac{230 \times 360^3}{6} \right]$$

$$\theta = .00797 \text{ radians} \quad \text{or} \quad .456681^\circ$$

$$.456681 \times 60 = 0^\circ 27' 24''$$

$$b \quad \Delta = \frac{1}{EI} \int_0^{360} M x dx = \frac{1}{EI} \int_0^{360} \left(\frac{WLx^2}{2} - \frac{Wx^3}{3} \right) dx$$

$$= \frac{1}{EI} \left[\int_0^{360} \frac{WLx^3}{6} - \frac{Wx^4}{8} \right]$$

$$= \frac{1}{EI} \left[\frac{230 \times 720 \times 360^3}{6} - \frac{230 \times 360^4}{8} \right]$$

$$\Delta = 1.83''$$

Prot
A
fect
at 10
and t
effect

14

$\theta =$

$\bullet =$

$\theta =$

$\theta =$

$\Delta =$

$\Delta =$

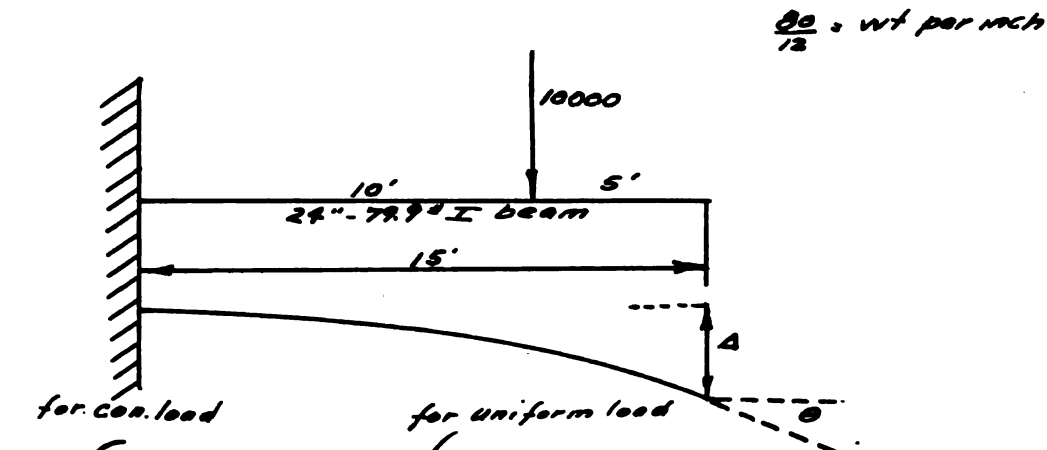
$M =$

$\theta =$

$S = \frac{M}{I}$

Problem 27, Page 47.

A 24 inch, 79.9 pound I-beam extends as a cantilever 15 feet long and carries a concentrated load of 10000 pounds at 10 feet from the fixed end. Find the maximum fiber stress and the slope and deflection at the free end. Include the effect of the dead load.



$$\theta = \int \frac{M dx}{EI} + \int \frac{M dx}{EI}$$

$$\theta = \int_0^{120} \frac{10000 x dx}{EI} + \int_0^{180} \frac{\frac{80}{12} x \cdot \frac{1}{2} dx}{EI}$$

$$\theta = \frac{10000 \times 120^2}{2 \times 30 \times 10^6 \times 2087.2} + \frac{\frac{80}{12} \times 180^3}{6 \times 30 \times 10^6 \times 2087.2} = .00113 + .00014$$

$$\theta = .00129 \times 57.3 \times 60 = 0^\circ - 4' - 25.8''$$

conc. load wt. of beam.

$$\Delta = \int \frac{M m dx}{EI} + \int \frac{M m dx}{EI}$$

$$\Delta = \int_0^{120} \frac{10000 x 120^3}{3 \times 30 \times 10^6 \times 2087.2} + \frac{\frac{80}{12} x 180^4}{8 \times 30 \times 10^6 \times 2087.2} = .175''$$

$$M = 10000 \times 120 = 1200000 \text{ "K}$$

$$80 \times 15 \times 90 = \frac{108000}{1.308.000 \text{ "K}}$$

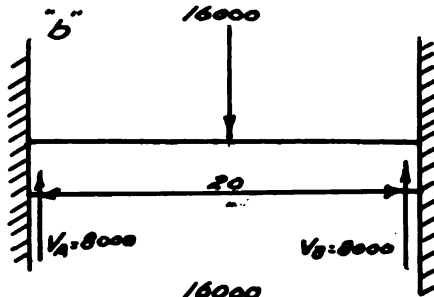
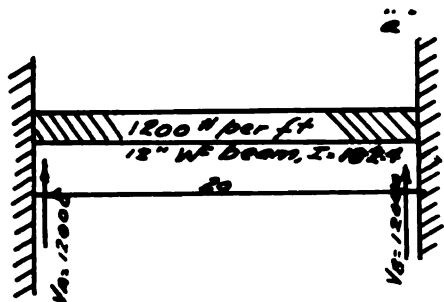
$$S = \frac{Mc}{I} = \frac{1.308.000 \times 12}{2087.2} = 7500 \text{ #/in}$$

Problem 28, Page 47.

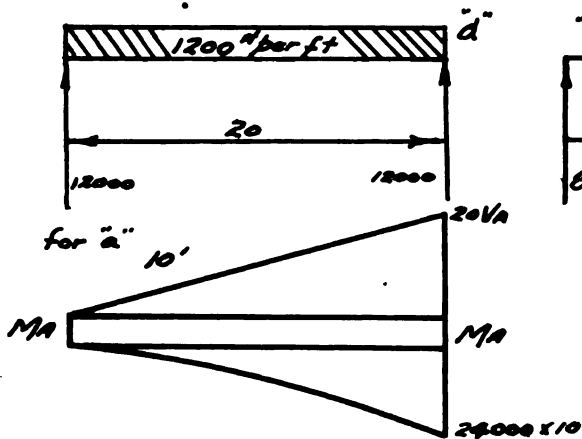
A beam fixed at both ends carries a total uniform load of 1200 pounds per foot on a span of 20 feet. Compute the maximum deflection if the section is a 12 inch W.F. beam having moment of inertia of 182.4. Compare with the maximum deflection caused by a center load of 16,000 pounds, which produces the same fiber stress. Make a further comparison with the maximum deflections of simple beams loaded in the same manner to this fiber stress and having the same section.

All beams have
Same section

Restrained beam



Simple Beam



Investigate to see if the
1200 lb/ft load and 16000 lb
concentrated load give the

Since the load is symmetrical,
the tangent at the center parallel beam.
to tangent at end.

Same unit stress on same

Area of M diagram between these 2 points = 0

$$0 = 12000 \times 10 \times 5 + 10 M_A - \frac{1}{3} \times 10 \times 12000 \times 5$$

$$M_A = -40000$$

$$\Delta = \frac{12000 \times 120 \times 60 \times 80 - 40000 \times 12 \times 120 \times 60 - 12000 \times 60 \times 40 \times 90}{30000000 \times 182.4}$$

$$\Delta = .16''$$

6.8.15

Pro

14

per

part

part

Prob 28 Cont

$$120000 \times 120 - 40000 \times 12 - 1200 \times 10 \times 60 = 240000 \text{ "}\cdot\text{"}$$

$$\text{Max Neg Mom at Support} = 480000 \text{ "}\cdot\text{"}$$

$$S = \frac{Mc}{I} = \frac{480000 \times 6}{182.4} = 15900 \text{ "}\cdot\text{"}$$

part "b" $\text{Max Mom} = \frac{5I}{8} = 480000 \text{ "}\cdot\text{"}$

$$\text{from table } \frac{WL}{8} = \frac{16000 \times 240}{8} = 480000$$

$$\Delta_{\text{max}} = \frac{WL^3}{192EI} = \frac{16000 \times 240^3}{192 \times 182.4 \times 30 \times 10^6} = .21 \text{ "}$$

part "c" Simple beam - conc. load at center - corresponding load.

$$\text{Using same stress, same length, } M = \frac{FL}{4}$$

$$M = 480000 \text{ from part "a"}$$

$$480000 = \frac{FL}{4} \quad F = 8000$$

$$\Delta = \frac{FL^3}{48EI} = \frac{8000 \times 240^3}{48 \times 182.4 \times 30 \times 10^6} = .42 \text{ "}$$

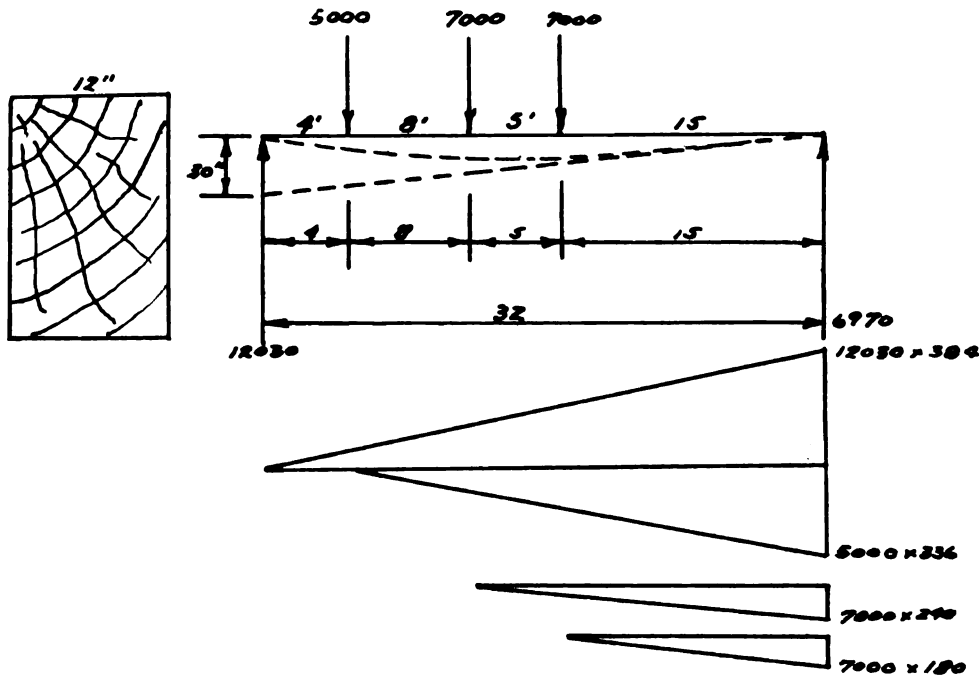
part "d" Simple beam - Same stress - same length - corresponding load $M = 480000 \text{ "}\cdot\text{"}$

$$M = \frac{WL}{8} = \frac{480000 \times 8}{240} = 16000 \text{ "}\cdot\text{" or } 800 \text{ "foot}$$

$$\Delta = \frac{5WL^3}{384EI} = \frac{5 \times 16000 \times 240^3}{384 \times 182.4 \times 30 \times 10^6} = .53$$

Problem 29, Page 47.

Find the maximum deflection of the beam shown due to the concentrated loads alone. The beam is of wood, 12 inches wide and 30 inches deep. Take the modulus of elasticity at 600,000 pounds per square inch.



$$\Delta_1 = \frac{12030 \times 384 \times 192 \times 256 - 5000 \times 336 \times 168 \times 272 - 7000 \times 290 \times 120 \times 304}{600000 \times 27000} - \frac{7000 \times 180 \times 90 \times 324}{60000 \times 27000}$$

$$\Delta_1 = 3.23'' \quad \theta = \frac{3.23}{384} = .00842 \text{ radians}$$

$$600000 \times 27000 \times .00842 = 12030 \frac{m^2}{2} - 5000 \left(\frac{m-48}{2} \right)^2 - 7000 \left(\frac{m-144}{2} \right)^2 - 7000 \left(\frac{m-204}{2} \right)^2$$

$$1360000000 = 6015m^2 - 2500m^2 + 240000m - 5760000 - 3500m^2 + 1008000$$

$$-72,576,000 - 3500m^2 + 142800m - 145,656,000$$

$$m^2 = 767.8m + 103300$$

$$m = 174.05'' \quad \text{use } 174''$$

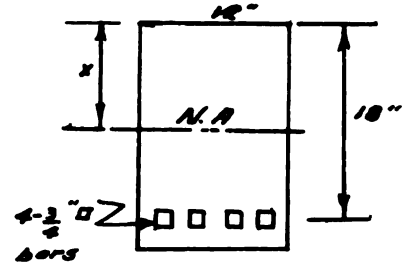
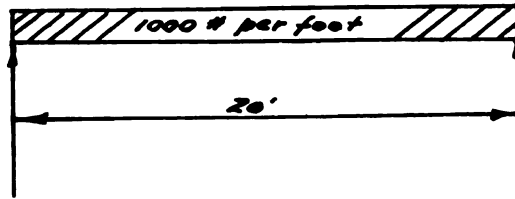
$$EI\Delta = \left[12030 \times 174 \times \frac{174}{2} \times \frac{2}{3} \times 174 \right] - \left[5000 \times 126 \times \frac{126}{2} \times \frac{1}{3} \right] - \left[7000 \times 30 \times \frac{30}{2} \times \frac{1}{3} \right]$$

$$\Delta = .94''$$

1.2, 8, 13

Problem 30, Page 47.

Since flexure formula for re-inforced concrete beams, $f_c = 2M/(k_j b d^2)$, must be equivalent to $f = Mc/I$, and since $c = kd$, we can readily show that $I = j k b d^3 / 2$. Show the derivation of this formula and then determine the theoretical deflection of a concrete beam 12 inches wide and 18 inches deep to the steel, which carries 4-3/4 inch square bars, if the simple span is 20 feet and the total load is 1000 pounds per foot. Let $E = 30,000,000$



$$f_c = \frac{2M}{k_j b d^2} = \frac{Mc}{I} \quad c = kd$$

$$I = \frac{j k^2 b d^3}{2} \quad n = 15$$

$$2M = n A_s (18 - x) = A_c \frac{x}{2} \quad \text{about N.A.}$$

$$15 \times 2.25 (18 - x) = \frac{12 x^2}{2}$$

$$6x^2 + 33.75x - 33.75 \times 18 = 0 \quad x = 7.64''$$

$$x = kd \quad jd = d - \frac{kd}{3} = 18 - \frac{7.64}{3} = 15.45$$

$$I = \frac{j d \times k d^2 \times b}{2} = \frac{15.45 \times 7.64^2 \times 12}{2} = 5170$$

$$M = \frac{w \ell^2}{8} = \frac{1}{8} \times \frac{1000}{12} \times 20 \times 20 \times 144 = 600,000$$

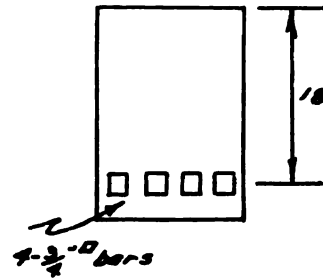
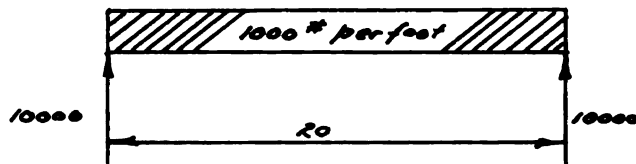
$$\frac{M}{EI} = \frac{600,000}{2 \times 10^6 \times 5170} = 5.8 \times 10^{-5} = m$$

$$\Delta = 120m \times \frac{2}{3} \times 120 - 120 \times m \times \frac{2}{3} \times 120 \times \frac{1}{8}$$

$$\Delta = 80 \times 5.8 \times 10^{-5} \times 120 \times \frac{5}{8} = .34''$$

Problem 31, Page 48.

Check the deflection of Problem 30 by computing a moment of inertia from the formula $f_s = M/pjbd^2$. Explain why the actual deflection of such a concrete beam could not be expected to be as great as the theoretical deflection.



$$p = \frac{As}{bd} = \frac{2}{4} \times \frac{1}{bd}$$

$$f_s = \frac{Mc}{I}$$

$$c = (d - kd)$$

$$I = pjbd^2(d - kd) = \frac{2}{4} \times \frac{1}{bd} jbd^2(d - kd)$$

$$= \frac{2}{4} jd(d - kd)$$

$$jd = 15.45$$

$$kd = 7.64$$

$$I = \frac{2}{4} (15.45)(18 - 7.64) = 360$$

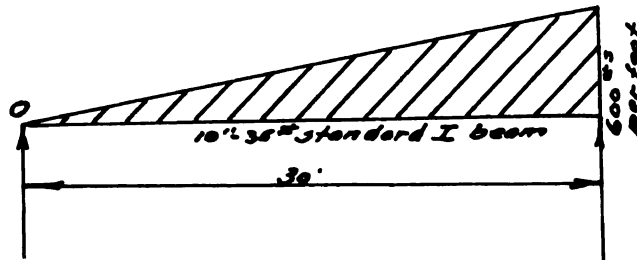
$$\frac{M}{EI} = \frac{600000}{30 \times 10^6 \times 360} = 5.56 \times 10^{-5}$$

$$\Delta = \frac{5}{8} \times 120 \times \frac{2}{3} \times 120 \times 5.56 \times 10^{-5} = .335$$

The beam will not actually deflect as much as computed, probably, because the stress capacity of the concrete between the steel and the Neutral Axis is utilized to some extent in carrying stress from steel to concrete in compression.

Problem 32, Page 48.

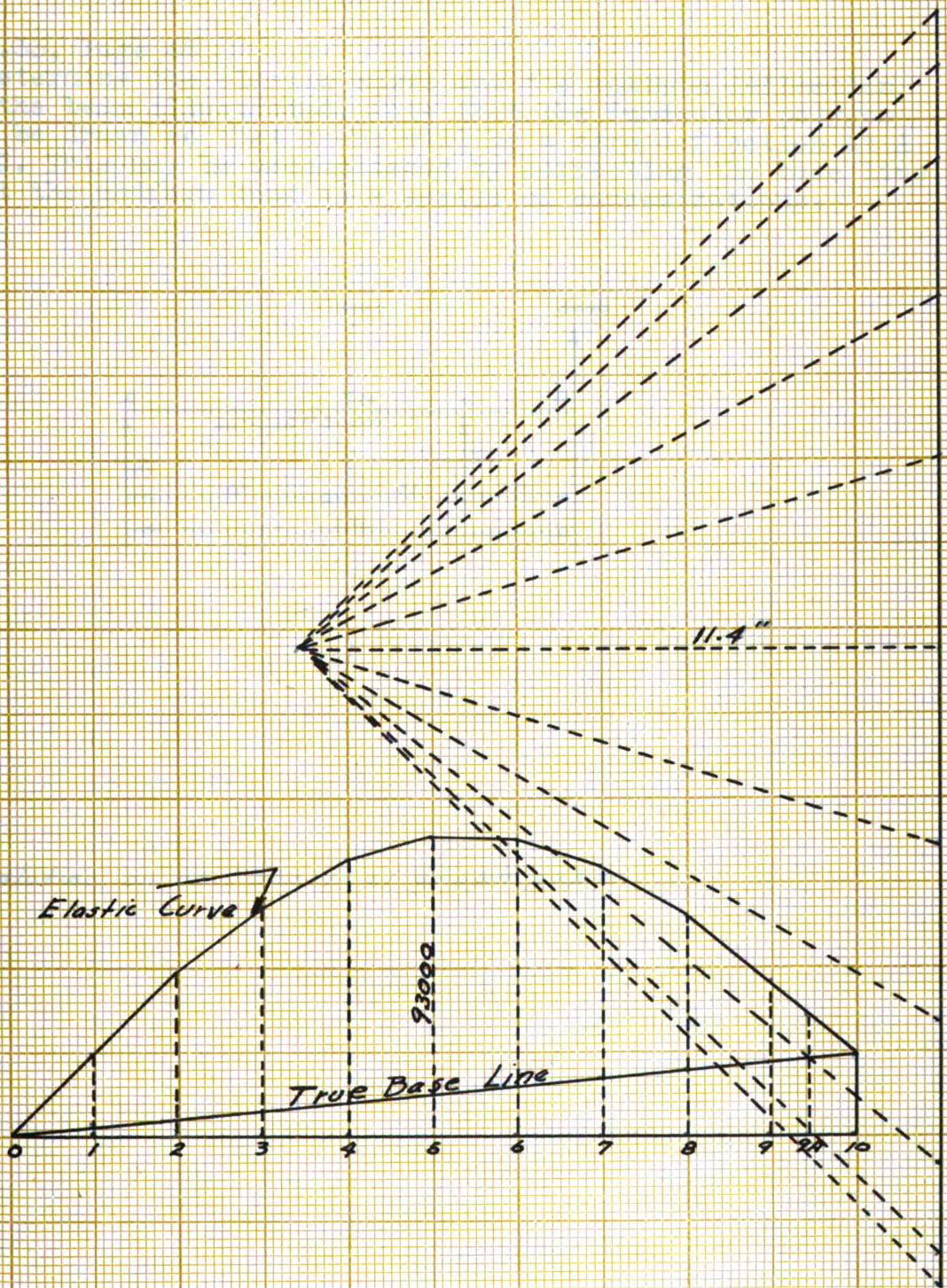
Graphically determine the elastic curve and the maximum deflection for a simply supported beam of 30 foot span loaded by a uniformly increasing load from zero at one end to 600 pounds per foot at the other end. The beam is a 10 inch 35 pound standard I-beam. Neglect the weight of the beam and compare the deflection obtained with the maximum allowable deflection of $1/360$ times the span.



X	$\frac{X^2}{3}$	M
3	3	8910
6	12	17300
9	27	24750
12	48	30200
15	75	38750
18	108	34550
21	147	32100
24	192	25900
27	243	15400
29	280	5800
30	300	0

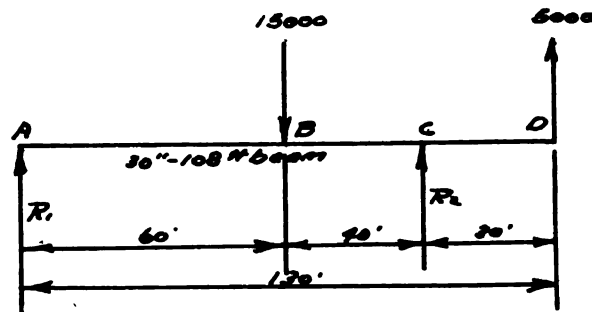
$$M = 3000X - \frac{10X^3}{3} = 10X(300 - \frac{X^2}{3})$$

$$\Delta = \frac{93000 \times 114 \times 3 \times 1728}{30 \times 10^6 \times 146} = .1265''$$

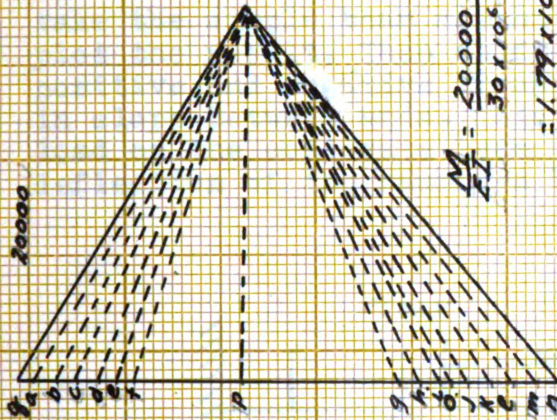


Prob.33, Page 48

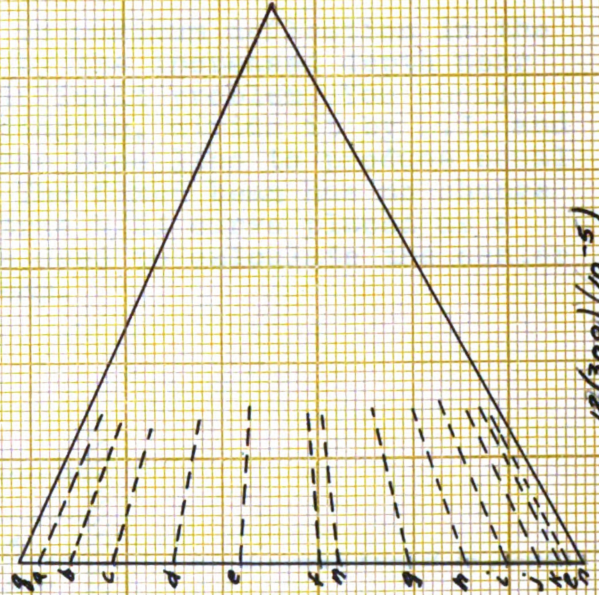
Determine the elastic curve and the maximum deflection for the beam illustrated. Note that the funicular polygon must be drawn so that there will be a zero deflection at A and C. This may be accomplished by correcting the base line. Treat the uniform load as a series of concentrations.



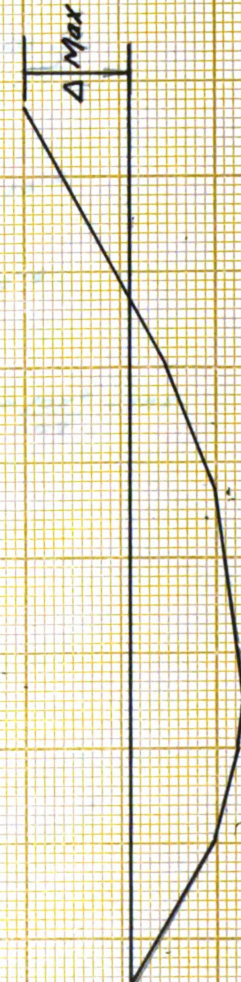
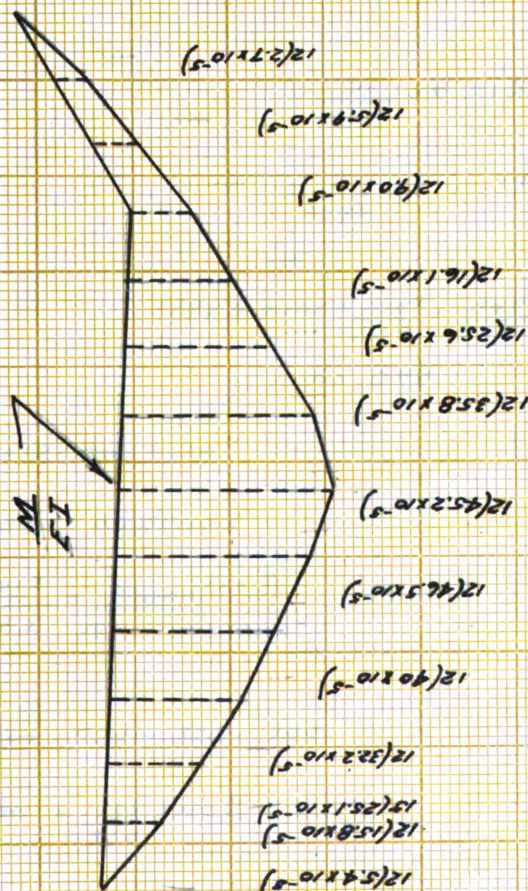
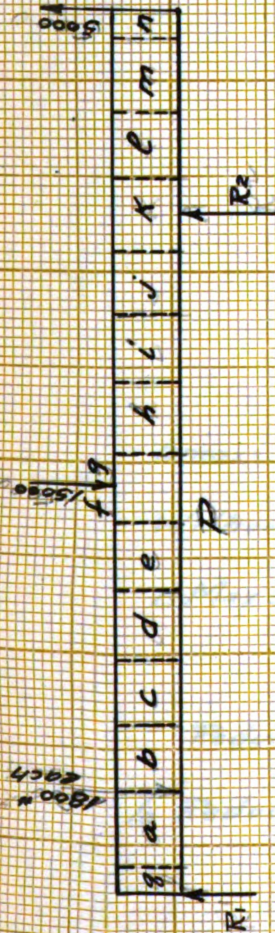
Solution on next sheet of graph paper.



$$\frac{M}{EI} = \frac{20000 \times 12 \times 10^3}{30 \times 10^6 \times 12} = 1.79 \times 10^{-6} \text{ (12)}$$



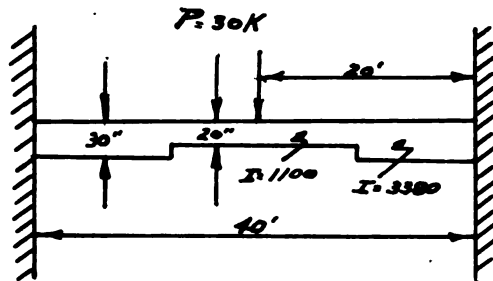
$$12(300)/(10^{-5}) = 0.035$$



$$\Delta_{Max} = 17 \times 12 \times 0.036 = 7.34''$$

Prob. 34, Page 48

Determine the % reduction in maximum deflection in the beam illustrated as produced by the increased depth near the ends. This is a rather difficult problem. You should take the simple beam moment diagram, divide it by E.I. and plot the result. Then shift the base line to produce equal positive and negative areas. The fixed beam has no change in slope between ends. This true M/EI curve placed as a load on the conjugate beam is used to calculate the center deflection of the haunched beam.



l = measured in feet.

$$\frac{M}{EI} = \frac{l(12)(25000)}{30 \times 10^6, I} = .01 \frac{l}{I}$$

$$= \frac{.01}{1100} \times 6 = .545 \times 10^{-4}$$

$$= \frac{.01}{3380} \times 6 = .178 \times 10^{-4}$$

$$.178 \times 5 + 10x + \frac{10x^2}{1.09} = \frac{.545 \times 10}{2} + \frac{10x^2}{1.09} - 10x$$

$$x = .092$$

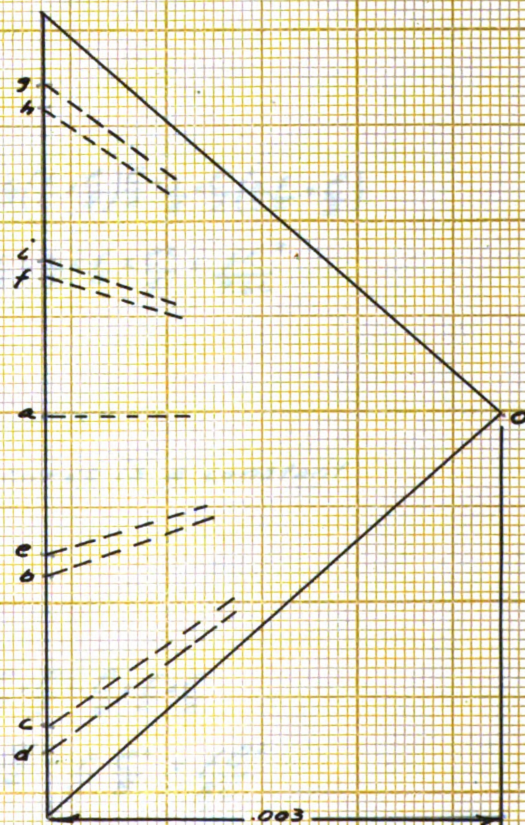
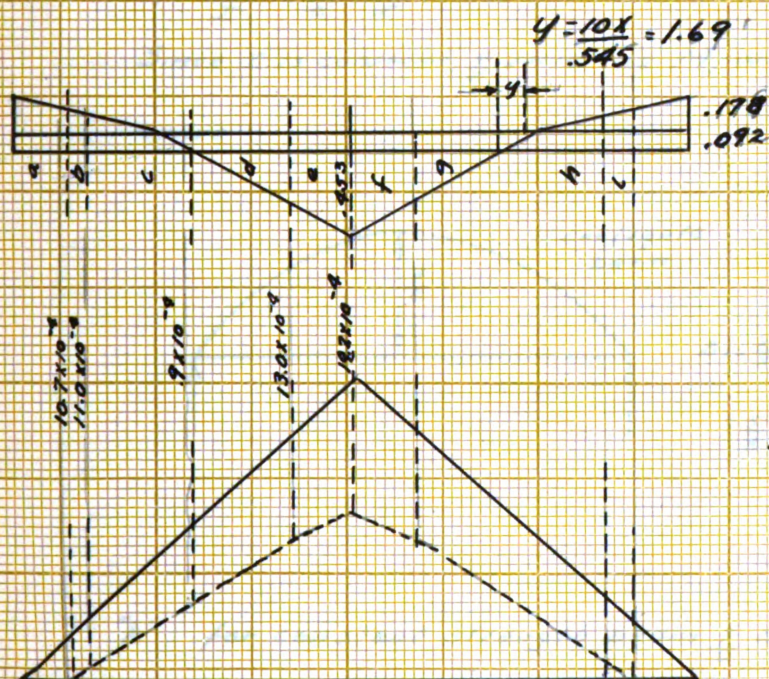
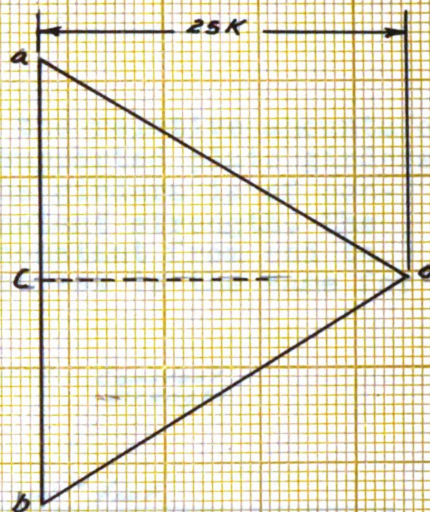
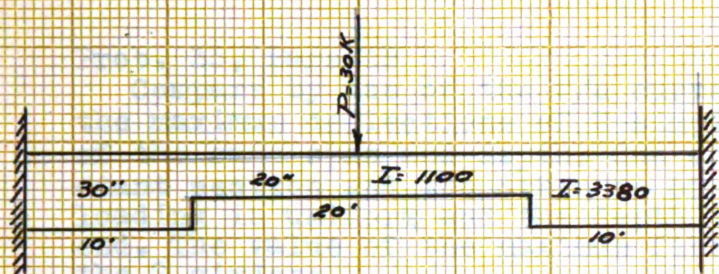
$$\Delta_{max} = 10 \times 12 \times .003 = .36''$$

Without ends widened

$$\Delta_{Max} = \frac{PL^3}{192EI} = \frac{30000(480^3)}{192 \times 1100 \times 30 \times 10^6} = .524''$$

$$\text{Reduction } .524 - .360 = .164$$

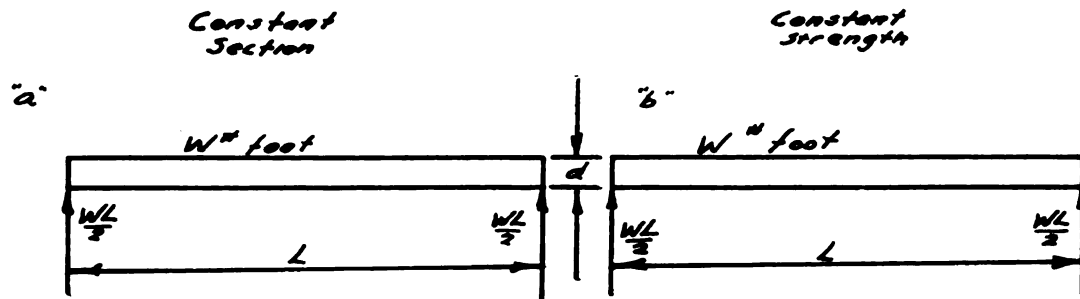
$$\% \text{ Reduction} = \frac{.164}{.524} \times 100\% = 31.3\%$$



Prob. 35, Page 48

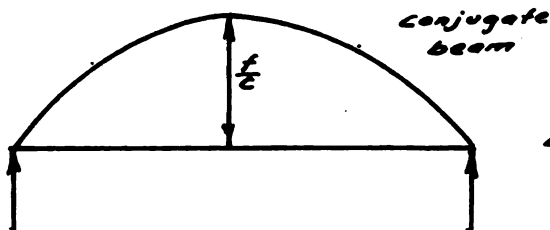
Compare by use of the f/c curves, the relative amounts of the maximum deflections of 2 simply supported plate girders of the same span and depth. One of the girders is of constant section and the other is of constant strength. The loading is uniform and the depth is constant from end to end. The value of the maximum fiber stress is the same for the 2 girders.

2 Girders



"a" $f, \frac{Mc}{I}, \frac{f}{c} = \frac{M}{I}$

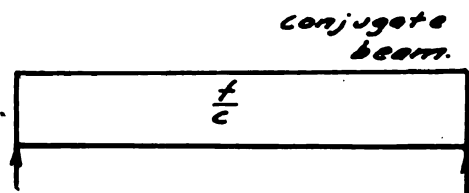
Since I is constant, $\frac{f}{c}$ varies as M , which gives a parabolic curve



$$\Delta_1 = \frac{2}{3} \left(\frac{f}{c} \times \frac{c}{2} \right) \frac{c}{2} - \frac{2}{3} \times \frac{c}{2} \left(\frac{2}{3} \frac{f}{c} \times \frac{c}{2} \right)$$

$$= \frac{5}{8} \times \frac{2}{3} \times \frac{f}{c} \times \frac{c^2}{4} = \frac{f c^2}{9.6 c}$$

"b" for constant strength beam $\frac{f}{c}$ curve is a constant



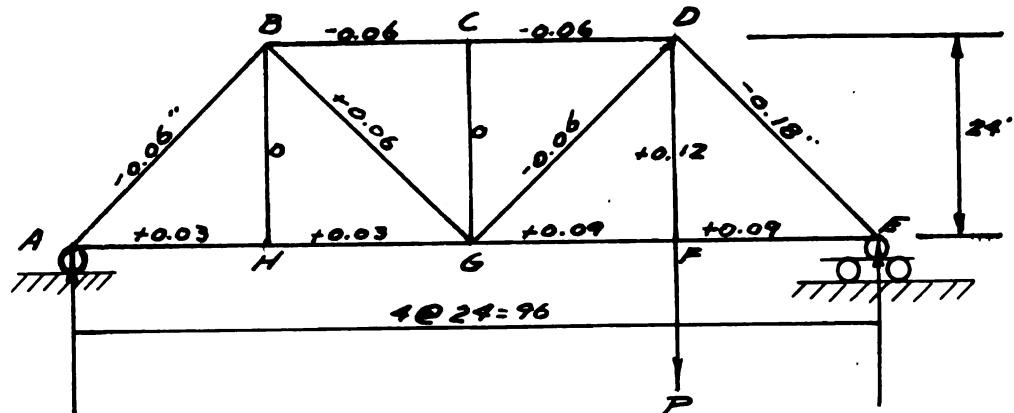
$$\Delta_2 = \frac{f}{c} \times \frac{c}{2} \times \frac{c}{2} - \frac{c}{4} \times \frac{f}{c} \times \frac{c}{2}$$

$$= \frac{f}{c} \times \frac{c^2}{4} - \frac{f}{c} \times \frac{c^2}{8} = \frac{f c^2}{8 c}$$

$$\frac{\Delta_1}{\Delta_2} = \frac{\frac{f c^2}{9.6 c}}{\frac{f c^2}{8 c}} = \frac{5}{6}$$

Prob. 36, Page 51

The actual changes in the lengths of the various truss members as caused by the load P are shown on the drawing of the truss. Obtain the shape of the deflected lower chord by the method of angle weights. $E=30,000,000$



Angle	Mem.	ΔL	n	$\frac{\Delta L}{n}$	Ratio	0 Load at P	Total
	Bb	0	24x12	0	1	0	
	Ba	+0.03	"	.000104	1	.000104	
AB	Ab	-.06	17x12	.000299	1	.000299	.000208
	Bc	+0.03	24x12	.000104	1	.000104	
	Cb	+.06	17x12	.000299	-1	-.000299	
	Bc	+0.03	24x12	.000104	0	0	
2x	bc	+.06	17x12	.000299	1	.000299	
	bc	"	24x12	"	1	"	
AC	Cc	0	✓	0	~	0	0
	cd	-.06	24x12	.000299	1	.000206	
2x	Cd	"	17x12	"	1	.000299	
	Dd	+.12	24x12	.000416	1	.000416	
	CD	+.09	"	.000312	0	0	
	CD	+.09	24x12	.000312	1	.000312	
	cD	-.06	17x12	.000299	1	.000299	
2x	Dd	+.12	24x12	.000416	2	.000832	.00242
	dE	+.18	17x12	.000882	1	.000882	
	DE	+.09	24x12	.000312	1	.000312	

$$\sum M_A = 0 \quad .000208 \times 24 + .00242 \times 72 - 96E = 0$$

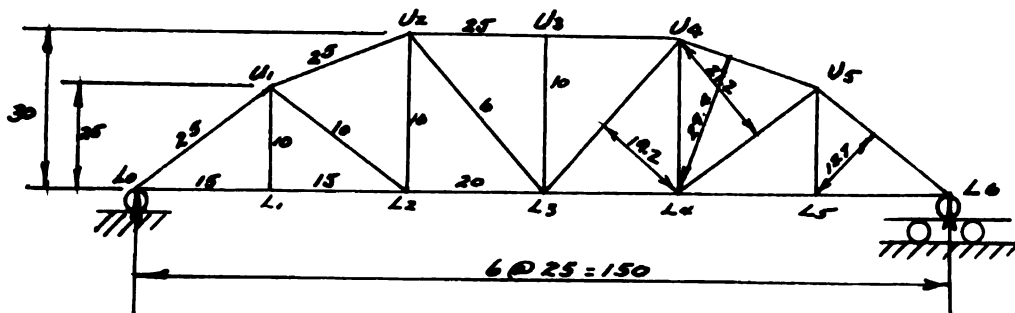
$$E = \frac{.00489 + .01731}{96} = .0021$$

$$y_D = M_D = .0021 \times 12 \times 24 = .58"$$

8.15.10

Prob. 37, Page 51

Determine the deflection diagram for the truss shown as caused by a dead load of 2000# per foot of the truss. The dead load stresses are given in Fig. 126 of Vol. 1. Areas are written on the members. Check the center deflection, obtained graphically, by a single calculation of virtual work.
 $E=30,000,000$

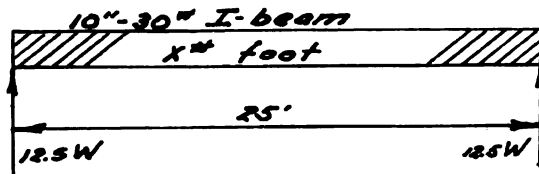


Mem	Stress S	$\frac{SL}{AE}$	r	$\frac{DL}{r}$	Ratio L1-L2-L3	DL Load @ L1	DL Load @ L2	DL Load @ L3
L0L1	+31.2	.0208	300	.0000693	1 - -	.0000693	-	-
L1L2	"	"	"	"	1 0 -	"	-	-
L2L3	+41.7	"	360	.0000578	- 1 0	-	.0000578	-
L3L4	"	"	300	.0000693	- - 0	-	-	-
L0U1	-44.1	.0250	212	.0001180	1 - -	.000118	-	-
U1U2	-42.5	.0173	352	.0000491	- 1 -	-	.0000491	-
U2U3	-46.8	.0189	360	.000052	- - 1	-	-	.000052
U3U4	"	"	"	"	- - 1	-	-	"
U1L1	+12.5	.0125	300 212	.0000417 .0000701	2 - 1 - 5	.0000834	.0000417 .0000701	-
U1L2	+14.7	.0208	254	.0000818	1 $\frac{4.2}{35.2}$ -	.0000991	.0000329	-
U2L2	+2.1	.0025	300	.0000083	$\frac{5}{30}$ 1 1	-	.000091	.0000083
U2L3	+8.1	.0211	230	.0000918	- 1 1	-	.0000918	.0000918
U3L3	0	0	0	-	-	-	-	-
U4L3	+8.1	.0211	230	.0000918	- - 1	-	-	.0000918
U4L4	+2.1	.0025	300	.0000083	- - 1	-	-	.0000083

B. 15.18

Prob. 42, Page 59

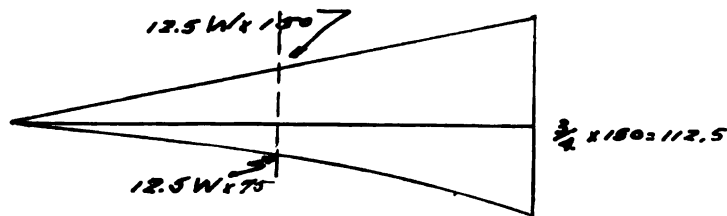
A 10", 30# I beam acts as a simple span 25 feet long. Determine the maximum uniform load and the maximum fiber stress if the limit on deflection is $1/360$ times the span.



W includes wt of beam

$$\Delta = \frac{1}{360} \text{ span}$$

$$\frac{1}{360} \times 300 = .833''$$



$$.833 = \frac{12.5W \times 150 \times 75 \times 100 - \frac{1}{3} \times 12.5W \times 75 \times 150 \times 112.5}{30 \times 10^6 \times 133.5}$$

$$.833 \times 8000 \times 133.5 = 2343.75 W$$

$$W = \frac{8000 \times 133.5 \times .833}{2343.75} = 380 \text{ #/foot. Includes wt. of beam}$$

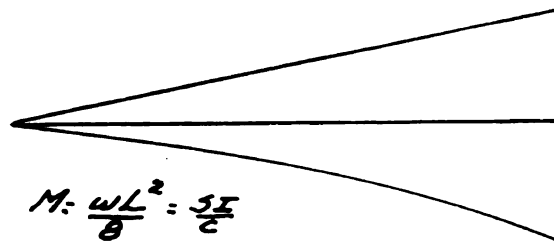
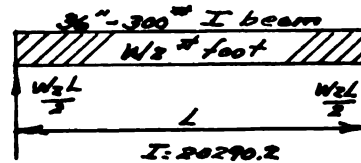
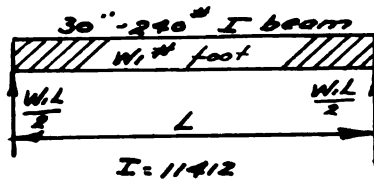
$$\frac{380 \times 25}{2} = 4750 \text{ # } R_1 = R_2$$

$$S = \frac{Mc}{I} = \frac{(4750 \times 150 - 4750 \times 75)5}{133.5} = 13400 \text{ #/in}$$

Prob. 43, Page 59

A 30", 240# beam and a 36", 300# beam carry uniform loads over the same spans and have equal maximum deflections. Determine the fiber stress in the 30" beam if the 36" beam is stressed to 18000# per square inch. Explain

Equal Deflections



$$M = \frac{S I}{C}$$

$$S = \frac{M C}{I}$$

$$M = \frac{W L^2}{8} = \frac{S I}{C}$$

$$\Delta_2 = \frac{5 W_2 L^4}{384 E I}$$

$$= \frac{18000 \times 20290.2}{18} = 20290200$$

$$W L^2 = 162321600$$

$$W = \frac{162321600}{L^2}$$

$$\Delta_2 = \frac{5 \times 162321600 L^2}{384 \times 30 \times 10^6 \times 20290.2} = \frac{5 \times 5.41 L^2}{384 \times 20290.2}$$

$$\Delta_1 = \Delta_2$$

$$\Delta_1 = \frac{5 W_1 L^4}{384 E I} = \frac{5 \times 5.41 L^2}{384 \times 20290.2}$$

$$W_1 L^2 = \frac{5 \times 5.41 \times 384 \times 30 \times 10^6 \times 11412}{384 \times 20290.2} = 5.41 \times 30 \times 10^6 \times 563$$

$$M = \frac{W L^2}{8}$$

$$S = \frac{M C}{I}$$

$$S = \frac{W L^2 C}{8 I} = \frac{W L^2 \times 15}{8 \times 11412}$$

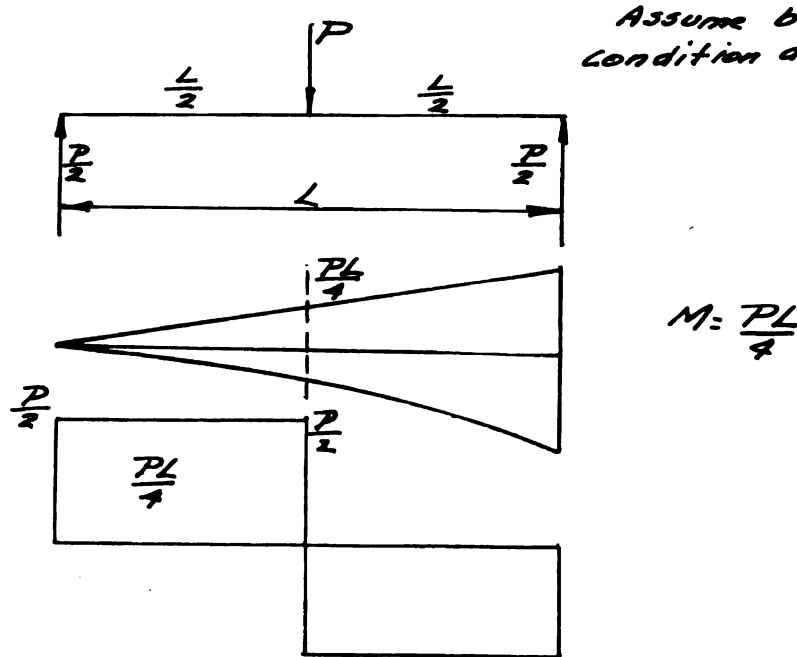
$$W_1 = \frac{5.41 \times 30 \times 10^6 \times 563}{L^2}$$

$$S = \frac{(5.41 \times 30 \times 10^6 \times 563) \times L^2 \times 15.37}{L^2 \times 8 \times 11412} = 15300 \text{ # stress in } 30\text{-}240\text{ # I beam.}$$

8, 13, 16, 18, 19

Prob. 44, Page 59

An I- beam has a fixed deflection. A particular beam is found to be 50% overstressed when it is deflected the required amount. What reduction in depth will produce the required stress. Why does the moment of inertia of the beam not enter into this problem.



$$E I \Delta = \frac{P}{2} \times \frac{L}{2} \times \frac{L}{2} \times \frac{2}{3} \times \frac{L}{2} = \frac{PL^3}{48EI} \quad \text{Fixed Mom.}$$

Use S_1 as $\frac{3}{2}$ allowable

" C_1 " depth of wider beam

" S " allowable stress in second beam

" C_2 " depth of narrow beam

$$\frac{3}{2} S = \frac{PLC_1}{4I}$$

$$S = \frac{PLC_2}{4I}$$

$$S = \frac{2PLC_1}{12I}$$

$$\frac{2PLC_1}{12I} = \frac{PLC_2}{4I}$$

I cancels out and has no effect on problem

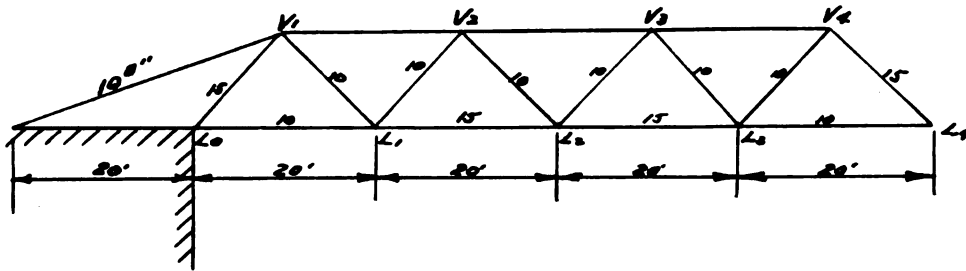
$$C_1 = \frac{6C_2}{4} = \frac{3}{2} C_2$$

$\therefore C_2$ must be $\frac{2}{3}$ of C_1 or a reduction

of $\frac{1}{3}$ or $33\frac{1}{3}\%$

Prob. 45, Page 59

The truss shown is to be erected by the cantilever method. If the dead load is 500# per foot of truss and if there is a concentrated load of 4500# at L_1 , produced by the erection equipment, find the proper length of the erection tieback in order for L_4 to land on a seat at the same level as L_0 . Areas of all members are shown on the figure. If the lower chord members have excessive L/r values, how would you stiffen them to permit erection.



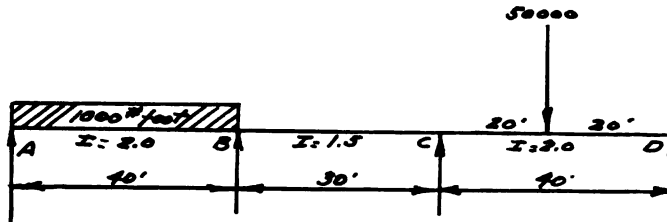
Mem	ES	L	A	u	$\frac{SL}{AE}$	$\frac{SUL}{AE}$
L_0V_1	90.55	14.1	15	-7.1	-.0028	.02
L_0L_1	137.5	20	10	-7.0	-.0091	.064
L_1L_2	78.5	"	15	-5.0	-.0035	.018
V_1V_2	108.0	"	20	+6.0	+.0039	.023
V_1L_1	51.9	14.1	10	+1.4	+.0024	.003
L_1V_2	37.7	"	"	-1.4	-.0018	.0025
V_2L_2	38.6	"	"	+1.4	+.00177	.0025
V_2V_3	49.0	20	20	+4.0	+.00163	.0065
V_2L_3	23.6	14.1	10	+1.4	+.00111	.0016
L_2L_3	29.5	20	15	-3.0	-.00131	.0039
V_3V_4	10.5	"	20	+2.0	+.00033	.0007
V_3L_4	7.1	14.1	15	+1.4	+.002	.0003
L_3L_4	5.0	20	10	-1.0	-.00033	.0003
L_3V_4	7.1	14.1	"	-1.4	-.00033	.0004
L_3V_3	23.6	"	"	+1.4	-.00111	.0015
T	292.5	31.623	"	12.65	-.031	.392

$.5403 \times 12 = 6.5''$ total deformation
 $\frac{6.5}{80} = \frac{x}{6.33}$ $x = .51''$ change in length
 Length of Backstay = 31.632' or 31'-7.49"
 $31'-7.49'' - .51'' = 31'-6.98''$ or $31'-6\frac{15}{16}''$

Prob. 46, Page 64

Draw the final moment diagram for the two examples given in Paragraph 47

2" Settlement of support.



1st. Draw B.M diagram for continuous beam with supports all on same level

2nd Draw B.M diagram for continuous beam with settlement of 1 support.

Use values for BMas found in illustrated problem. It is also necessary to compute shears and draw shear diagram.

$$M_B = M_A + 40 V_{AR} - 800000$$

$$M_C = M_B + 30 V_{BR}$$

$$-56600 = 0 - 800000 + 40 V_{AR}$$

$$-173300 = -56600 + 30 V_{BR}$$

$$V_{AR} = 18585$$

$$V_{BR} = -3890$$

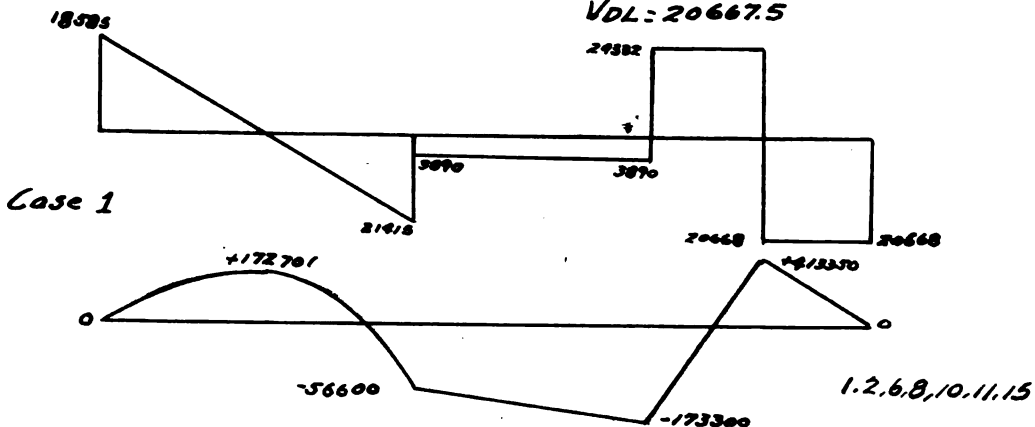
$$V_{BL} = 21415$$

$$V_{CL} = +3890$$

$$M_D = M_C + 40 V_{CR} - 1000000$$

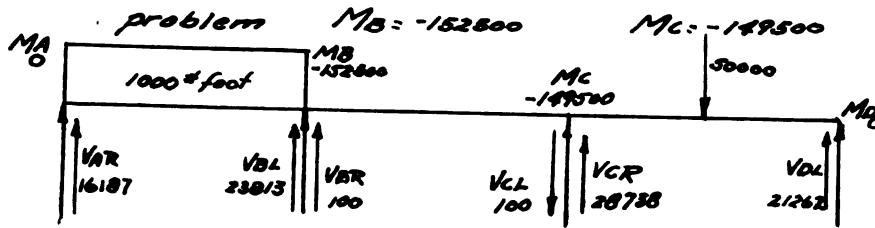
$$V_{CR} = 29332.5$$

$$V_{DL} = 20667.5$$



Prob 46 "cont"

Case "2" Settlement Condition. BMs taken from illustrated



$$M_B = M_A + 90 VAR - 800000$$

$$M_C = M_B + 30 V_{BR}$$

$$-152500 = 0 - 800000 + 90 VAR$$

$$-149500 = -152500 + 30 V_{BR}$$

$$VAR = 16187$$

$$V_{BR} = 100$$

$$V_{BL} = 23813$$

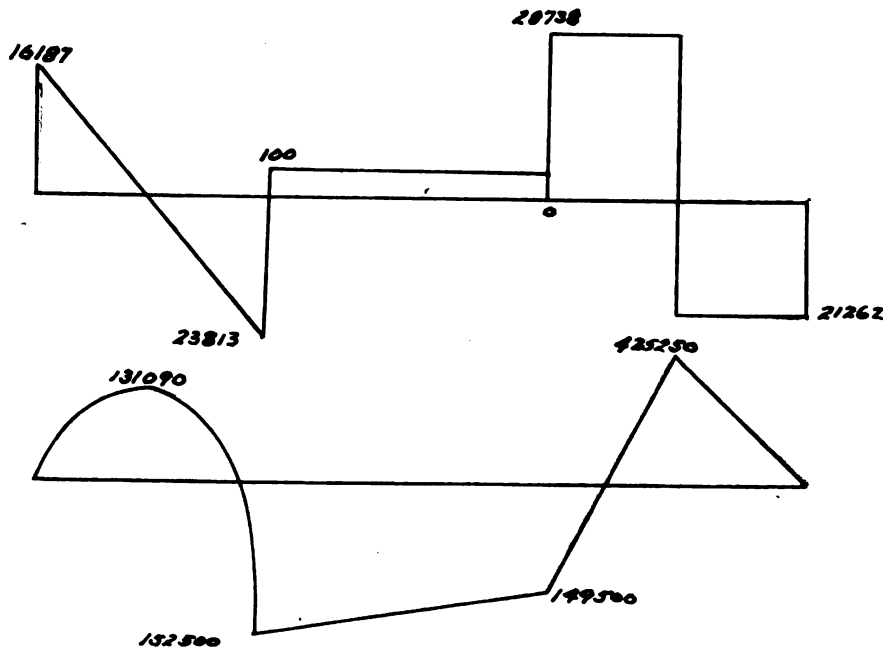
$$V_{CL} = -100$$

$$M_D = M_C + 40 V_{CR} - 1000000$$

$$0 = -149500 + 40 V_{CR}$$

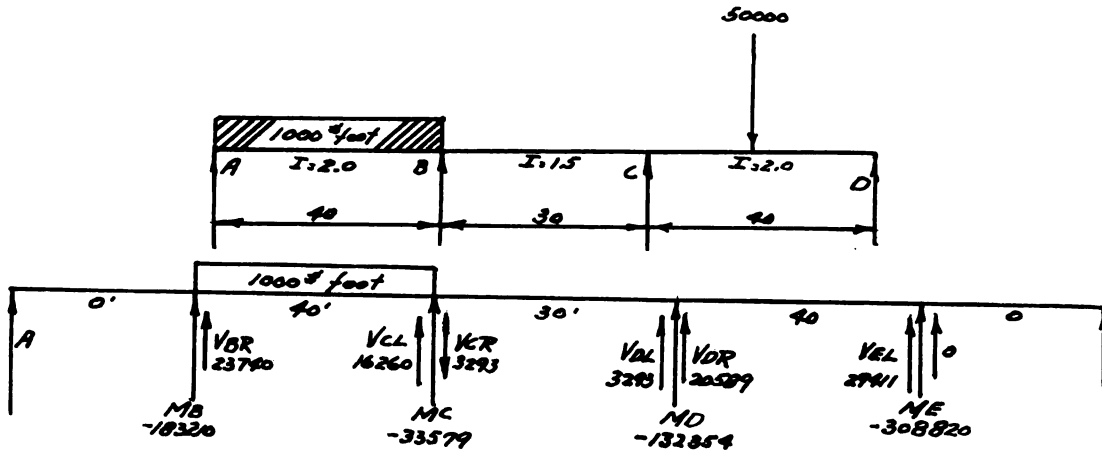
$$V_{CR} = 28738$$

$$V_{DL} = 21262$$



Problem 47, Page 64

Repeat the analysis of the example of Paragraph 47 without settlement where the ends of the beam are fixed against rotation.



Work as a continuous beam by theory of 3 moments. —

replacing fixed ends by beams of 0 length, 0 load

$$1 \quad M_A L_0 + 80 M_B + 40 M_C = -16000.000$$

$$2 \quad 40 M_B + 140 M_C + 30 M_D = -16.000.000$$

$$3 \quad 30 M_C + 140 M_D + 40 M_E = -30.000.000$$

$$4 \quad 40 M_D + 80 M_E + M_F L_0 = -30.000.000$$

Solving equations 1, 2, 3, 4, we arrive at the following

values. $M_B = -183210$ $M_C = -33579$ $M_D = -132354$

$$M_E = -308820$$

$$M_C = M_B + 40 V_{BR} - 800000$$

$$M_E = M_D + 40 V_{DR} - 1000000$$

$$-33579 = -183210 - 800000 + 40 V_{BR}$$

$$-308820 = -132354 + 40 V_{DR}$$

$$V_{BR} = +23740$$

$$V_{CL} = +16260$$

$$V_{DR} = 20589$$

$$V_{EL} = 2941$$

$$M_D = M_C + 30 V_{ER}$$

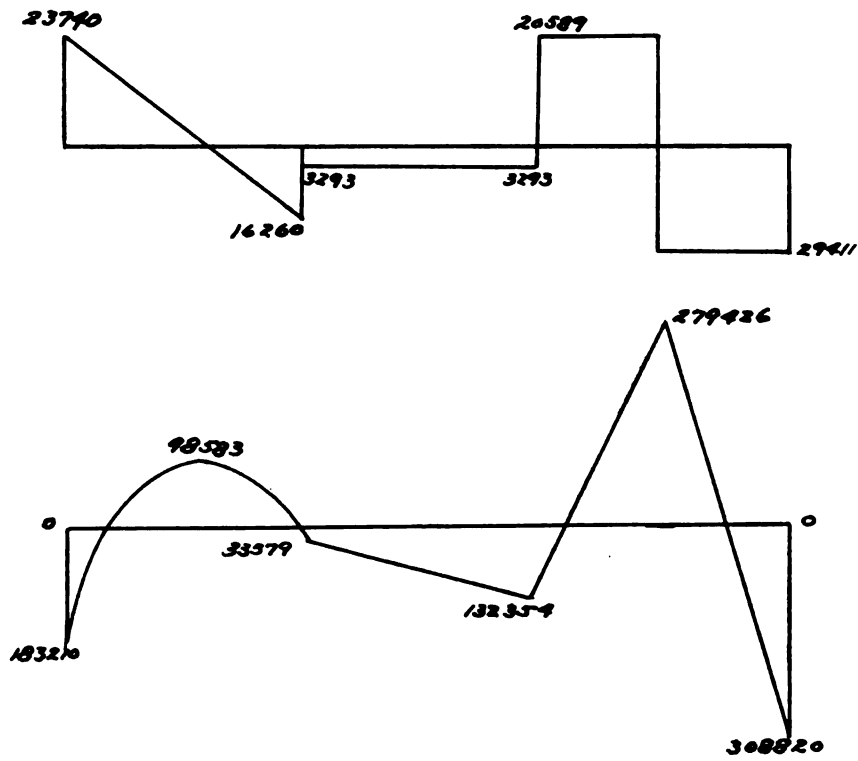
$$-132354 = -33580 + 30 V_{ER}$$

$$V_{ER} = -3293$$

$$V_{DL} = +3293$$

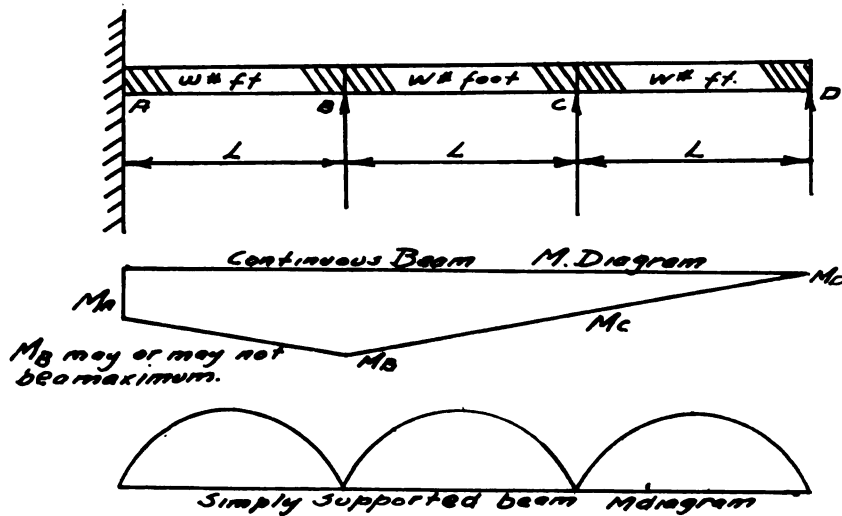
1, 2, 6, 8, 10, 11, 15

Prob 47 "Cont"



Problem 48, Page 64

Obtain the moment diagram for a continuous three span beam fixed at one extreme end and simply supported at the other, where the spans are of equal lengths and the loading is W # per foot over the entire length. Such beams commonly are designed for maximum moments of $WL^2/12$ for interior spans and $WL^2/10$ for simply supported end spans.



There are 3 equations to be written. Instead of a restrained beam, I assumed a 0 length span and load.

$$M_0 L + 2M_A(L+L) + M_B L = -\frac{W_0 L^3}{4} - \frac{WL^3}{4}$$

$$\textcircled{1} \quad 2M_A L + M_B L = -\frac{WL^3}{4} \quad M_A = -\frac{WL^3}{4L^2} - \frac{M_B}{2}$$

$$M_A L + 2M_B(2L) + M_C L = -\frac{WL^3}{4} - \frac{WL^3}{4} = -\frac{WL^3}{2}$$

$$M_A L + 4M_B L + M_C L = -\frac{WL^3}{2}$$

$$\text{Substituting for } M_A, \quad -\frac{WL^3}{8} - \frac{M_B L}{2} + 4M_B L + M_C L = -\frac{WL^3}{2}$$

$$-WL^3 - 4M_B L + 32M_B L + 8M_C L = -4WL^3$$

$$\textcircled{2} \quad 28M_B L + 8M_C L = -3WL^3$$

$$M_B L + 2M_C(2L) + M_0 L = -\frac{WL^3}{2}$$

$$\textcircled{3} \quad M_B L + 4M_C L = -\frac{WL^3}{2}$$

$$\textcircled{2} \quad 28M_B L + 8M_C L = -3WL^3$$

$$\textcircled{3} \times 2 \quad 2M_B L + 8M_C L = -WL^3$$

$$26M_B L = -2WL^3$$

$$M_B = -\frac{WL^2}{13}$$

$$M_A = -\frac{12}{204} WL^2$$

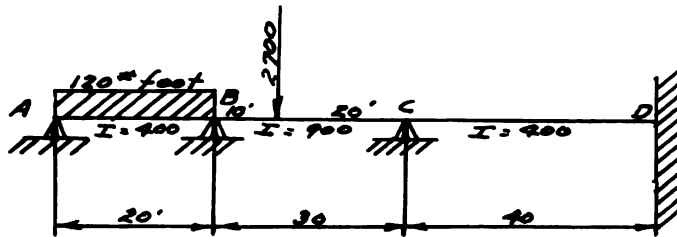
$$M_D = 0$$

$$M_C = \frac{1}{9.5} WL^2$$

1, 2, 6, 8, 10,
11, 15, 16

Problem 49, Page 70

Use least work method to obtain the moment diagram for the beam shown.



Find moment over support.

$$M_{AB} = \frac{WL^2}{12}$$

$$= \frac{120 \times 400}{12} = 4000'$$

$$M_{BC} = \frac{2700 \times 10 \times 400}{900} = 12000'$$

$$M_{CB} = \frac{2700 \times 20 \times 100}{900} = 6000'$$

$$M_{CD} = 0$$

$$K = \frac{I}{L}$$

$$K_B = \frac{400}{20 \times 12} = 1.67$$

$$K_C = \frac{900}{12 \times 30} = 2.5$$

$$K_D = \frac{400}{40 \times 12} = .833$$

0	100	40	60	75	25	Fixed End
+90	-40	+120	60	0	0	0
-16	-32	-48	-24			
-24	-12	+32	+65	+21		+105
-4	-8	-12	-6			
+4	+2	+2.3	+6.5	+1.5		+1.8
-8	-17	-2.6	-7.3			
+8	+4	+4	+10	+3		.1
0	-3	-5	-22.8	-22.8		11.4
	91.6	91.6				

$$M_A = 0, M_B = 9160, M_C = 2280, M_D = 1140$$

The way in which the moments divide where the K values are unequal is computed as follows

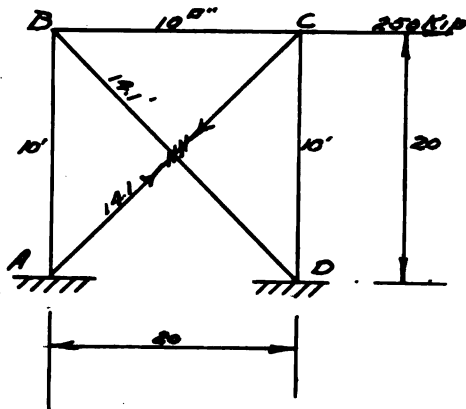
$$\% \text{ for BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{1.67}{1.67 + 2.50} = 40\%$$

$$\frac{15.16}{8.18, 14}$$

8, 15, 16

Problem 51, Page 70

Analyze the pin-connected bent for the stresses caused by the 250 kip load.



Assume member AC is the redundant member, cut it and put the force X in place. Using 1st joint C, then joint B, the horizontal section thru the truss, we find the stress in all members.

Bar	L/A	Total Stress	$\frac{\partial S}{\partial X}$	$\frac{SL}{A} \frac{\partial S}{\partial X}$
AB	2	$250 - .707X$	$-.707$	$-178 + .5X$
BC	2	"	$-.707$	$-178 + .5X$
CD	2	$.707X$	$-.707$	$.5X$
BD	2	$354 - X$	-1	$-354 + X$
AC	2	$-X$	-1	$-X$
				$-710 + 3.5X = 0$

$$AC = X = \frac{710}{3.5} = 203K$$

$$BC = 250 - .707 \times 203 = 107K$$

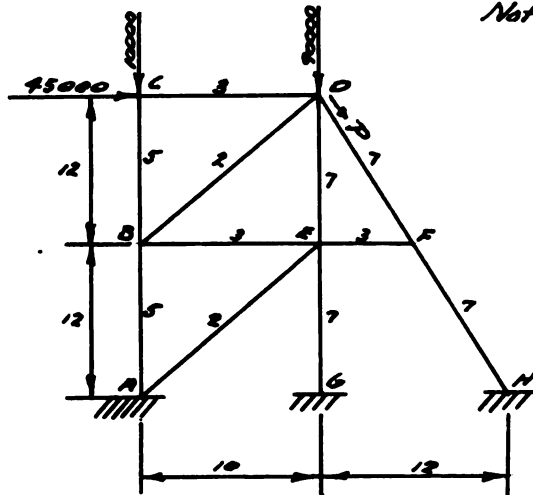
$$CD = .707 \times 203 = 143K$$

$$AB = BC = 107K$$

$$BD = 354 - 203 = 151K$$

Problem 52, Page 70

Analyze this mine head frame for the stresses caused by the forces which are produced by the cable pull. Areas of members are given on the figure. The joints are pin-connected. Take the stress in the sloping back brace as the redundant stress S .



Note - No stress in EF

A more convenient way to compute the stresses is to compute S for all loads except P and then for P alone. This is of course the same way we compute U .

M&m	stress	L/A	$\frac{\partial S}{\partial P}$	$\frac{SL}{A} \frac{\partial S}{\partial P}$
CD	45K	3.33	0	0
CB	10K	2.40	0	0
DB	70.2 - .69P	7.80	-.69	-381 + 3.77P
DE	144 - 1.44P	1.72	-1.44	-352 + 3.50P
BA	-38.1 + .446P	2.40	+.446	-38 + .46P
BE	59.1 - .585P	3.33	-.585	115 + 1.14P
AE	70.2 - .695P	7.80	-.69	-381 + 3.77P
EG	198 - 1.98P	1.72	-1.98	-652 + 3.77P
EF	0	2.00	0	0
DH	P	3.85	+1	3.85P

$$P = \frac{1919}{23.04} = 83.3 \text{ Kip}$$

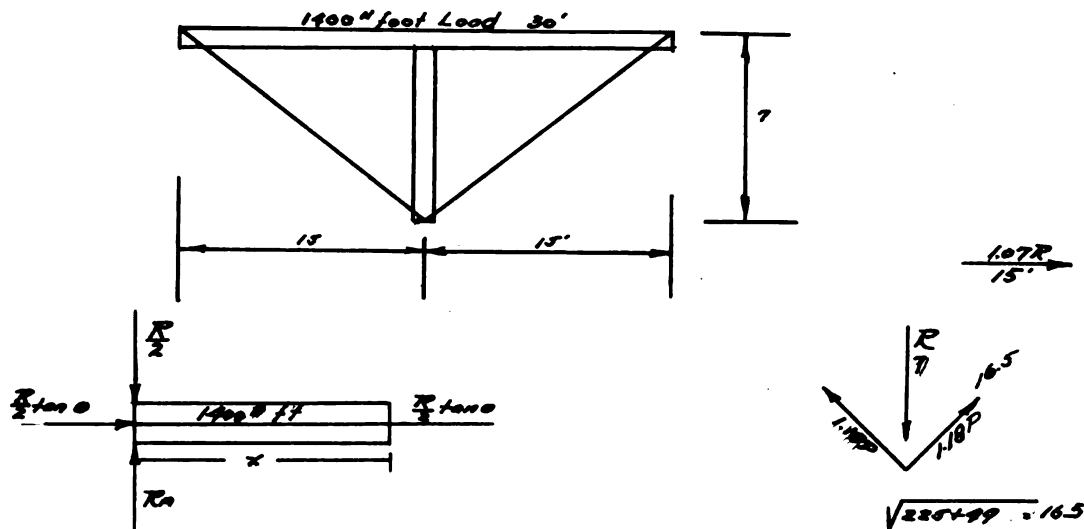
$$1919 + 23.04P = 0$$

8.15.18.19

11

Problem 53, Page 70

Analyze the trussed beam for the stresses caused by a uniform load over the entire length of 1400# per foot. How could the design be improved if the maximum fiber stress is limited to 10,000# per sq.inch.



$$M_x = \frac{wLx}{2} - \frac{1}{2}Rx - \frac{wL^2}{2} = \frac{1400 \times 80 \times 15}{2} - 7.5R - \frac{1400 \times 80^2}{2} = 36000 - 7.5R - 157500$$

$MX: 157500 - 7.5R$

$$W = \sum_0^n \frac{S^2 L}{2EA} + 2 \int_0^{\frac{L}{2}} M_x^2 dx$$

$$2 \int_0^{\frac{1}{2}} \frac{(157500 - 7.5R)^2 dx}{2EI} = \frac{(157500 - 7.5R)^2 \times 15 \times 144^2}{EI}$$

Bending in Beam

$$\angle \frac{S^2 L}{2AE} = \frac{1.07 R^2 \times 30 \times 129}{2 \times 5 \times 14.69}$$

Compression in Beam.

R² x 7 x 144
2 x 5 x 6.14

Compression in Strut

$$2 \left[\frac{1.18 R^2 \times 16.5 \times 144}{2 \times 3} \right]$$

Tension in Rods

$$W_{TOTAL} = \frac{(157500 - 7.5R)^2 \times 15 \times 1.94}{E \times 287} - \frac{1.07R^2 \times 130 \times 1.94}{2 \times E \times 19.64} - \frac{R^2 \times 78 \times 1.94}{2 \times E \times 6.4} - 2 \left[\frac{140R^2 \times 16.5 \times 1.94}{2 \times 3 \times E} \right]$$

$$\frac{\partial W}{\partial R} = 0$$

8,15,18
next Page

5.

Prob 53 "Cont"

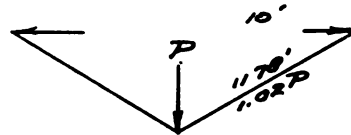
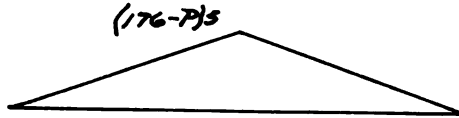
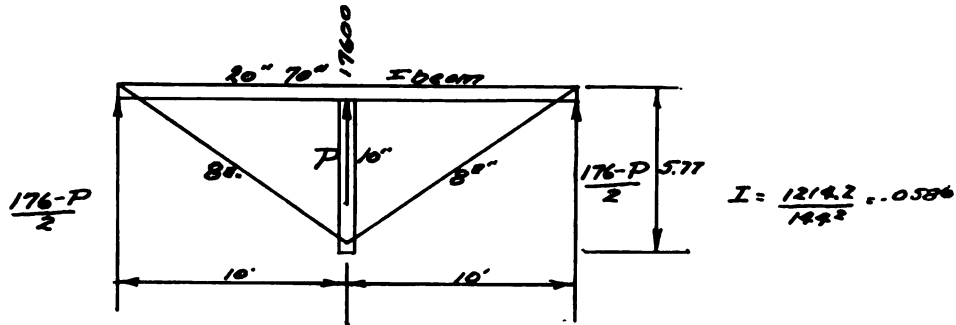
$$\frac{dW}{dR} = 29,806,250.000 \times 1080 - 2,362,500 \times 1080 R + 60750 R^2 - 170 R^3 - 83 R^4 - 1110 R^5 = 0$$

$$-2,551,500,000 + 5938712 R = 0$$

$$R = 21.5 \text{ Kips.}$$

Problem 54, Page 70

Calculate the maximum fiber stress in the trussed beam.
 $E=30,000,000\#$ per sq.inch. Take S_x in the vertical strut as redundant.



$$W = \frac{\int M^2 dx}{2AE} = \sum \frac{S^2 L}{2AE}$$

$$W_B = \frac{1}{.0586} \left(\left[\frac{(176-P)5}{2} \times \frac{20}{2} \right] \left[\frac{(176-P)5}{2} \right]^{\frac{1}{3}} \right)$$

$$= (176^2 - 2 \times 176P + P^2) 1420$$

$$W_B = \frac{(.863P)^2 \times 20}{2 \times 141} = 52.8P^2$$

$$W_T = \frac{2 \left(\left[\frac{(1.02P)^2 \times 11.70}{2} \right] \times 144 \right)}{2 \times 8} = 220P^2$$

$$W_P = \frac{P^2 \times 5 \times 144}{2 \times 10} = 36P^2$$

$$\frac{\partial W}{\partial P} = -2 \times 176 \times 1420 + 2 \times 1420 \times P$$

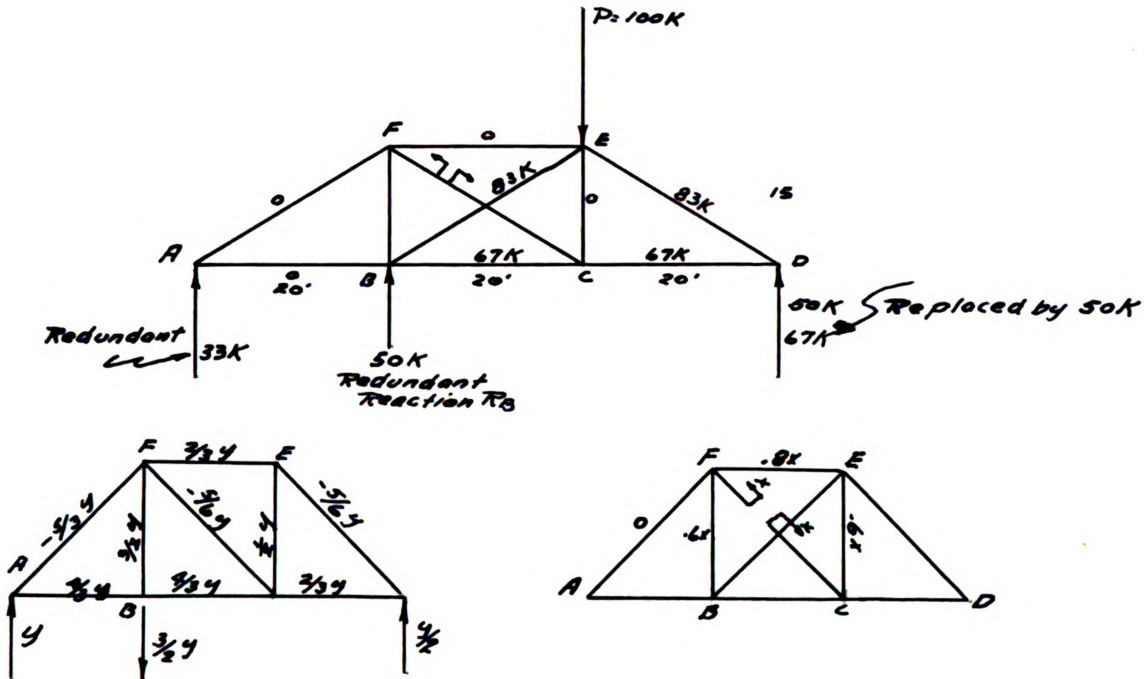
$$\frac{\partial W}{\partial P} = 618P$$

$$2828P + 618P = 500000$$

$$P = 145.5 \text{ Kips.}$$

Problem 55, Page 70

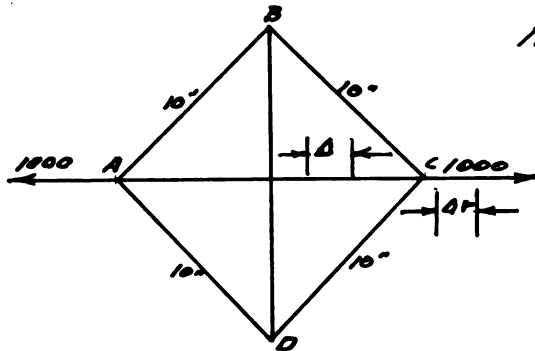
Check the results obtained in Fig. 62, page 68, by another analysis using the left hand reaction and the stress in the diagonal EC as the redundant forces.



Mem	$\frac{L}{A}$	Total Stress	$\frac{\partial S}{\partial y}$	$\frac{\partial S}{\partial x}$	$\frac{SL}{A} \frac{\partial S}{\partial y}$	$\frac{SL}{A} \frac{\partial S}{\partial x}$
AB	2.0	$+33y$	$\frac{4}{3}$	0	$3.56y$	0
BC	"	$67K + 33y + .8x$	"	.8	$1787 + 3.56y + 2.135x$	$107 + 2.13y + 1.28x$
CD	"	$67K + 33y$	$\frac{2}{3}$	0	$89.3 + .892y$	0
DE	2.5	$-83K - \frac{5}{6}y$	$-\frac{5}{6}$	0	$173 + 1.735y$	0
EF	2.0	$-\frac{3}{2}y + .8x$	$-\frac{3}{2}$	+.8	$-1.888y - 1.066x$	$-1.066y + 1.28x$
FA	2.5	$-\frac{5}{3}y$	$-\frac{5}{3}$	0	$6.95y$	0
BF	3.0	$+\frac{3}{2}y + .6x$	$+\frac{3}{2}$	+.6	$6.75y + 2.7x$	$2.7y + 1.08x$
BE	5.0	$-83K - x$		-1	0	$415 + 5x$
CE	3.0	$+\frac{1}{2}y + .6x$	$+\frac{1}{2}$	+.6	$.75y + .9x$	$.9y + 1.08x$
CF	5.0	$-\frac{5}{6}y - x$	$-\frac{5}{6}$	-1	$3.47y + 4.16x$	$4.16y + 5x$
					$441 + 28.56y + 8.73x$	$522 + 8.22y + 19.72x$
					$8.82y + 19.72x = -522$	
					$28.56y + 8.73x = -441$	
					$y = -5.65K$	
					$x = -31.8K$	

Problem 56, Page 74

The frame illustrated has all members of the same area. Use a direct application of virtual work to obtain the stress in the member in line with the loads.



Let AC be the redundant member.

Notes in this case, we get the equation

$$S\Delta = P\delta \text{ or}$$

$$S \times 1 = \frac{P\delta}{\Delta}$$

$$\delta = S\Delta$$

$$\Delta = u^2 L$$

Member	S due to P=1	u due to P=1	$S\Delta$ = δ	$u^2 L$ = Δ	L
AB	.707	.707	5	5	10
BC	"	"	"	"	"
CD	"	"	"	"	"
DA	"	"	"	"	"
AC	0	1.00	0	14.14	14.14
BD	1	1.00	14.14	14.14	14.14
			34.14	48.28	

$$S \times 48.28 = 1000 \times 34.14$$

$$S = \frac{1000 \times 34.14}{48.28}$$

$$S = 707.2$$

8.16.18.19

11

Problem 61, Page 76.

Analyze for the stresses in the frame of Problem 56 by use of the general method of indeterminate structures.

$$X = \frac{\Delta X^P}{\Delta X^{x=1}}$$

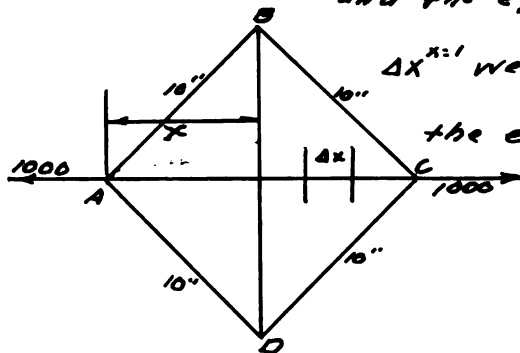
To determine ΔX^P , we use least work

and the equation $\Delta X^P = \sum S u L$. To find

$\Delta X^{x=1}$ we again use least work, but

the equation is $\Delta X^P = \sum u^2 L$.

S = stress due to 1000^{lb} force.



L	Mem	S	u	$\sum u L$	$\sum u^2 L$
10	AB	707	.707	500	5
"	BC	"	"	"	"
"	CD	"	"	"	"
"	DA	"	"	"	"
14.14	AC	0	1.00	—	14.14
"	BD	1000	1.00	14.14	14.14

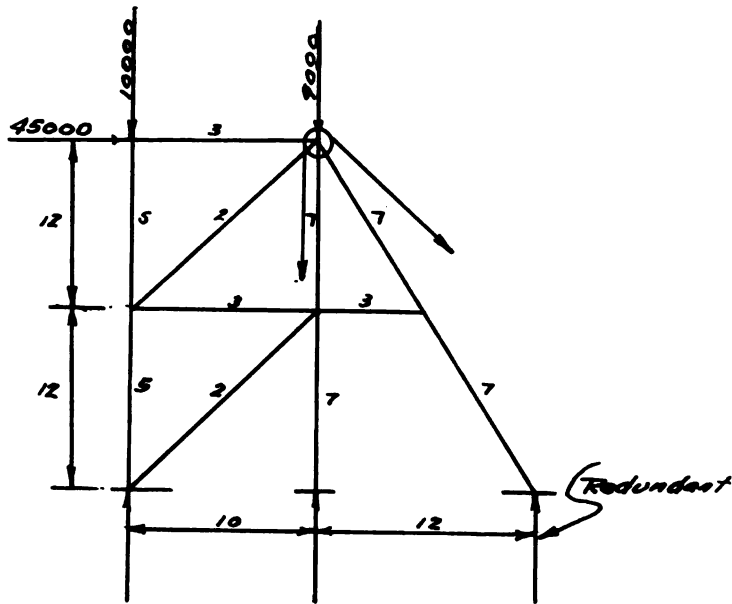
$$\Delta X^P = 3414$$

$$48.28 = \Delta X^{x=1}$$

$$X = \frac{3414}{48.28} = 707^*$$

Problem 62, Page 76.

Analyze for the stresses in the frame of Problem 52 by the general method of indeterminate structures.



By a close inspection of the table in Prob 52, we arrive at the following conclusions.

1- That the stress caused by the real load is the 1st part of the term under stress.

2- That $u = \frac{\delta S}{\delta P}$ from text page 41

3- That the term $\frac{\delta L}{\delta A} \frac{\delta S}{\delta P}$ is in reality $\frac{\delta u L}{\delta A} + \frac{u^2 L}{\delta A} (P)$

Therefore further computations are unnecessary as the values we want can be taken from table of Prob 52

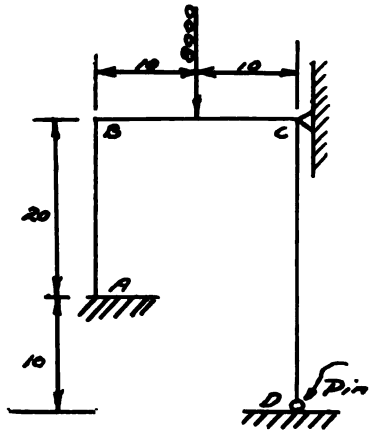
$$\therefore \Delta x^P = 1919 \quad \Delta x^{x=1} = 23.04 X(1)$$

Note~ in this case X is the redundant which is P in Prob 52

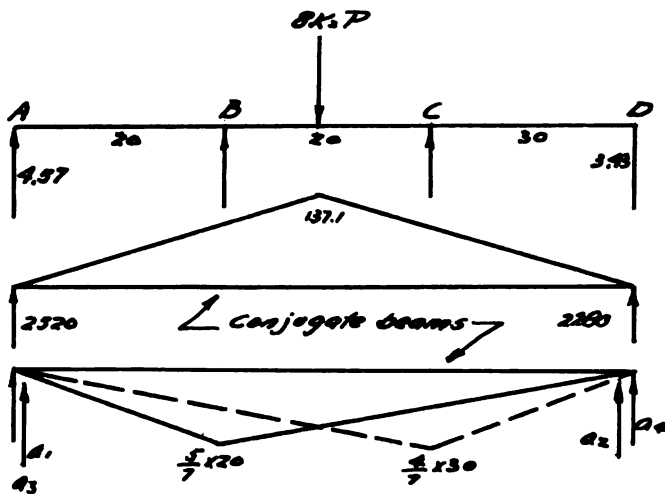
$$\text{Then } X = \frac{\Delta x^P}{\Delta x^{x=1}} = \frac{1919}{23.04} = 83.3K$$

Problem 63, Page 76.

Check the moments given in Problem 50. Use the general method of indeterminate structures.



To apply this method, we will straighten the bent out into a continuous beam over 4 supports, compute the redundant reactions, then the end reactions and finally the moment over the interior supports which equal the moment at the corner of the bent.



By Moments

$$\Delta x^P = 44320$$

$$\Delta y^P = 53000$$

$$A_1 = 285 \quad A_2 = 218$$

$$A_3 = 285 \quad A_4 = 315$$

$$\Delta x^{x=1} = 285 \times 20 - \frac{5}{7} \times 20 \times \frac{1}{3} \times 20 \times \frac{30}{2} = 4746$$

$$\Delta y^{y=1} = 315 \times 30 - \frac{5}{7} \times 30 \times \frac{3}{2} \times \frac{30}{2} \times \frac{1}{3} \times 30 = 6890$$

$$\Delta x^{y=1} = \Delta y^{x=1} = 218 \times 30 - \frac{5}{7} \times 20 \times \frac{3}{2} \times \frac{30}{2} \times \frac{1}{3} \times 30 = 5165$$

Substitute in equation of General Moments

$$44320 + x \cdot 4746 + y \cdot 5165 = 0$$

Prob 63 "cont"

$$53000 + y \cdot 6890 + x \cdot 5165 = 0$$

$$\sum M_{R_2} = 0 = 30 \times 4.1 - 40 \times 8 + 50 \times 9.8 - 70 R_1 = 0$$

$$\sum M_{R_1} = 0 = 20 \times 4.8 + 40 \times 2.1 - 30 \times 9 + 70 R_2 = 0$$

$$M_B = .615 \times 20 = -12300 \text{ 'k}$$

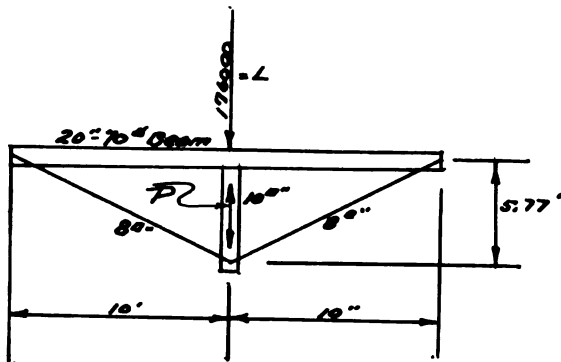
$$y = 4.1 \quad x = 4.8$$

$$M_C = .312 \times 30 = -9360 \text{ 'k}$$

$$R_1 = .615 \quad R_2 = .312$$

Problem 64, Page 76.

Use the general method of indeterminate structures to obtain the stress in the redundant strut of Problem 54.



$$P = \frac{\Delta P^L}{\Delta P^{P=1}}$$

$$\Delta P^L = \sum \frac{S u L}{A E} + \int \frac{M m' ds}{E I}$$

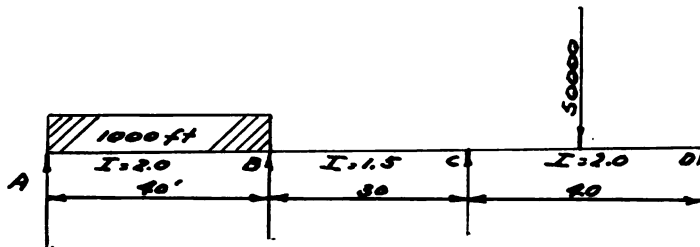
$$\Delta P^{P=1} = \sum \frac{u'^2 L}{A E} + \int \frac{m'^2 ds}{E I}$$

Mem	$\sum u M$	u or m'	$\frac{S u L}{A E} + \frac{M m' ds}{E I}$	$\frac{u'^2 L}{A E} + \frac{m'^2 ds}{E I}$
AB	$\frac{176}{2} \times 10$	$\frac{1}{2} \times 10$	500 000	2830
AC	"	.86	0	112
AD	"	1.02	0	219
CD	"	1.02	0	219
BD	"	1	0	82
		$\Delta P^L =$	500 000	$3461 = \Delta P^{P=1}$

$$S = \frac{500000}{3461} = 144.5 \text{ Kips.}$$

Problem 65, Page 76.

Take reactions as redundants and solve for the moments at B and C in the continuous beam of Fig. 56, page 63. Neglect settlement. (Slope Deflection Method)



$$C_{AB} = -\frac{1000 \times 40^2}{12} = -133.3$$

$$C_{BA} = +133.3$$

$$C_{CD} = -C_{DC} = -\frac{5000 \times 40}{8} = -250$$

$$C_{BC} = C_{CB} = 0$$

$$M_{AB} = 2EX \cdot 0.5(2\theta_A + \theta_B) - 133.3$$

$$K = \frac{2}{40} = .05 \quad K' = K''$$

$$M_{BA} = 2EX \cdot 0.5(2\theta_B + \theta_A) + 133.3$$

$$M_{BC} = 2EX \cdot 0.5(2\theta_B + \theta_C)$$

$$M_{AB} = 0 = M_{BC}$$

$$M_{CB} = 2EX \cdot 0.5(2\theta_C + \theta_B)$$

$$M_{BA} + M_{BC} = 0$$

$$M_{CD} = 2EX \cdot 0.5(2\theta_C + \theta_D) - 250$$

$$M_{CB} + M_{CD} = 0$$

$$M_{DC} = 2EX \cdot 0.5(2\theta_D + \theta_C) + 250$$

Substitute in the above equations & simplify

$$\textcircled{a} 2\theta_A + \theta_B - 133.3 = 0$$

$$\textcircled{b} \theta_A + 4\theta_B + \theta_C + 133.3 = 0$$

~Note~

$$\theta_B + \theta_C + 2\theta_D + 250 = 0$$

These are true normal

Solve

equations & could be

$$\theta_D = -1920 \quad \theta_B = -760$$

solved by least squares.

$$\theta_C = 1340 \quad \theta_A = 1100$$

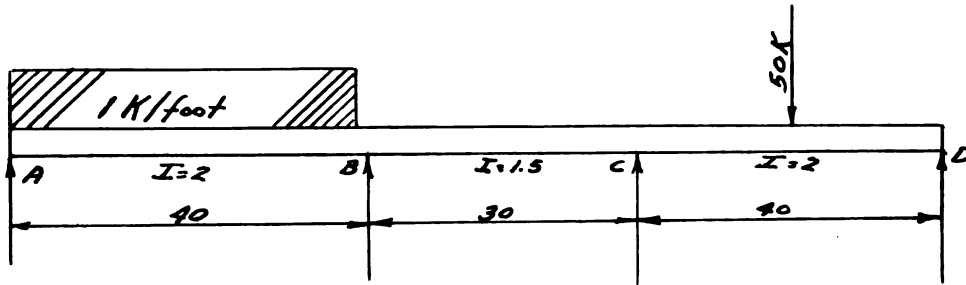
$$M_{BC} = 2X \cdot 0.5(-760 + 1340) = 58KF = 58000 \text{ 'Ks}$$

$$M_{CD} = 2X \cdot 0.5(2 \times 1340 + -1920) - 250 = 174000 \text{ 'Ks}$$

8, 11, 13, 16

Problem 65, Page 76.

Take reactions as redundants and solve for the moments at B and C in the continuous beam of Fig. 56, page 63. Neglect settlement. (Balancing Angles Method)

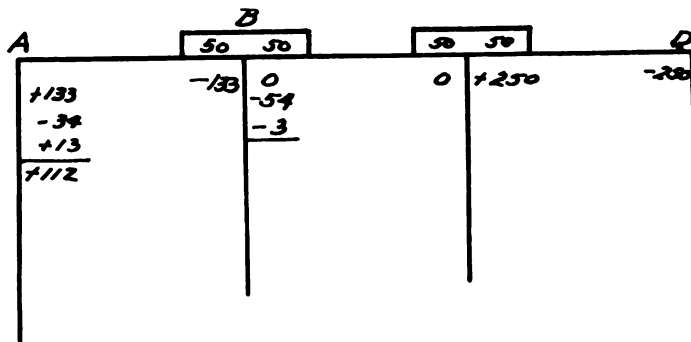


$$\theta_A = -\theta_B = \frac{1000 \times 40^3}{24 \times 2 \times E} = 1,333,000 \div E \quad K: \text{constant} = .05$$

$$\theta_B = -\theta_C = 0$$

$$\theta_C = \theta_D = \frac{50000 \times 40^2}{16 \times 2 \times E} = \frac{2,500,000}{E}$$

-67	+17	
	+125	
<u>-27</u>	<u>-6</u>	↘ Change of joint
-94	+136	



$$M_{AB} = .1(2,240,000 - 940,000) - 133,000 = 0$$

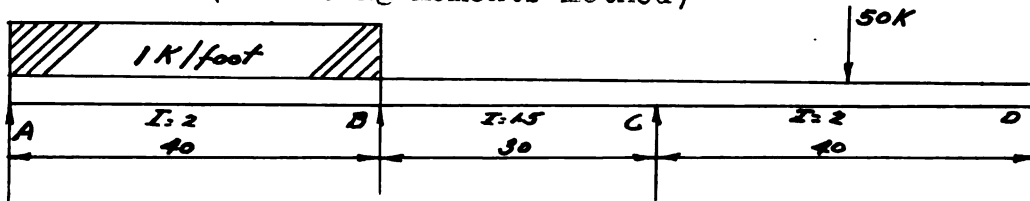
$$M_{BA} = .1[2(-940,000) + (1,120,000)] + 133,000$$

$$= -76,000 + 133,000 = 57,000$$

$$M_{CB} = .1[2(1,360,000) + (-940,000)] = 178,000$$

Problem 65, Page 76.

Take reactions as redundants and solve for the moments at B and C in the continuous beam of Fig. 56, page 63. Neglect settlement. (Balancing Moments Method)



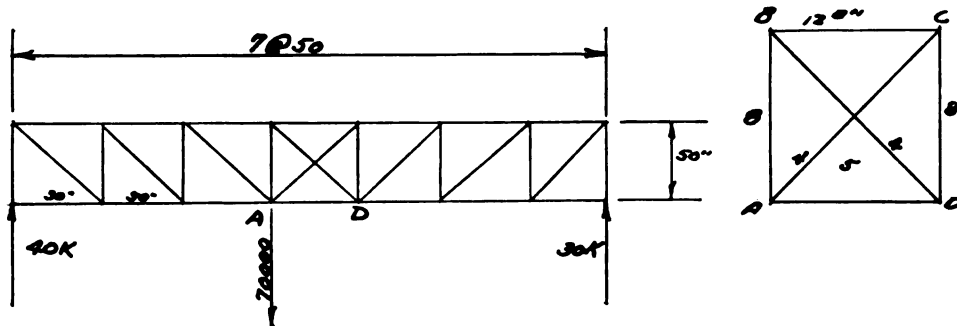
See prob 65 (by slope deflection) for complete details of problem

A	B	C	D	
+133	-133	0	-250	$K = \text{constant}$
-133	-67	0	+250	
<u>+50</u>	<u>+100</u>	<u>+50</u>	<u>-106</u>	$C_{AB} = 133$
-50	-25	-212	+106	
<u>+32</u>	<u>+65</u>	<u>+32</u>	<u>-21</u>	$C_{CB} = 0$
-32	-16	-42	+21	
<u>+9</u>	<u>+18</u>	<u>+9</u>	<u>-5</u>	$C_{CD} = 250$
-9	-5	-9	+5	
<u>+5</u>	<u>+5</u>	<u>+2.5</u>	<u>-2.5</u>	
0	-50	-172	0	
	+58	+172		

* Note~ When coming to a balance, it is always necessary to finish up with a pair of balancing values, in this case +5, -5; but when desired accuracy is obtained you omit the carry over factors. Thus each joint will balance on each side.

Problem 66, Page 76.

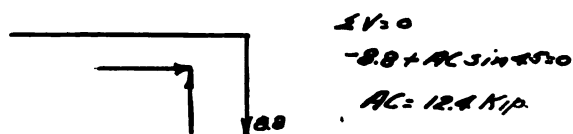
Find the stresses in the center diagonals of the truss as illustrated. The single load is 70,000 pounds.



Note - The effect of the redundant stress occurs only in the members making up that panel and we only need to compute the L.L. stresses in this panel. BD is redundant.

Mem	L	A	S	u	$\frac{u^2 L}{A}$	$\frac{SuL}{A}$
AB	50	8	30T	.707C	3.13	-132
BC	"	12	90C	"	2.08	265
CD	"	8	0	"	3.13	0
DA	"	5	120T	"	5.00	-850
BD	70.7	4	42.5C	1T	17.65	-751
AC	"	4	0	"	17.65	0
$\Delta X^{x=1}$					48.64	-1468

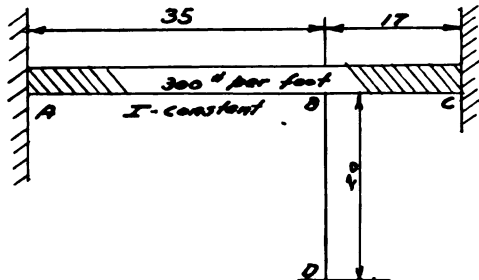
$$S = \frac{1468}{48.64} = 30.1 \text{ Kip Tension}$$



8.15.16.18

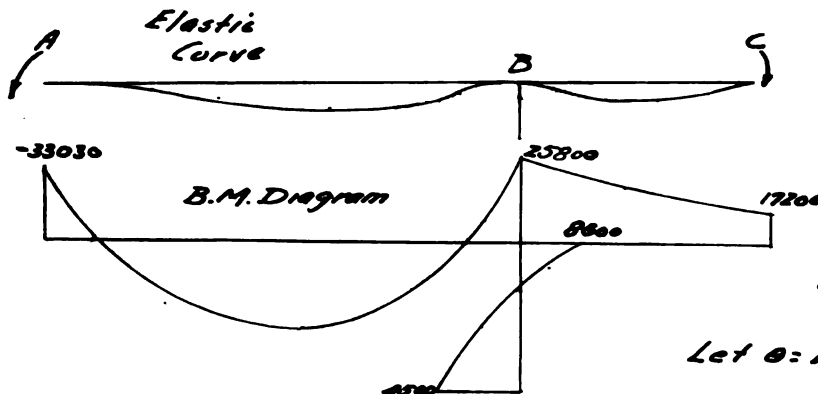
Problem 67, Page 81.

Obtain the moment diagrams for the loaded structure shown. In this case check the results by statics and by drawing a deflected structure to see that it is reasonable.



$$-C_{AB} = \frac{wL^2}{12} = \frac{300 \times 35^2}{12} = 30600$$

$$C_{BC} = \frac{wL^2}{12} = \frac{300 \times 17^2}{12} = 7230$$



$$K = \frac{I}{L} = \frac{100}{L}$$

Where I is

constant and equal to 100.

Let $\theta = E\theta$, since it is eliminated ultimately

$$K = \frac{100}{35} \quad K' = 5.89 \quad K'' = 5.00$$

$$M_{AB} = 2EK\theta_B - C_{AB}$$

$$M_{BA} = 2EK\theta_B + C_{AB}$$

$$M_{BC} = 2EK'(2\theta_B) - C_{BC}$$

$$M_{BD} = 2EK'(2\theta_B) + 0$$

$$M_{BA} + M_{BC} + M_{BD} = 0$$

$$2 \times 2.86 \times 2\theta_B + 30600 + 2 \times 5.89 \times 2\theta_B - 7230 + 2 \times 5 \times 2\theta_B = 0$$

$$45\theta_B = -23370 \quad \theta_B = -520$$

~ Note ~

Mom Diagram is plotted with positive moment on Tension side of beam.

T.B. 10

Prob 67 "cont"

$$M_{AB} = 2 \times 2.86 \times (-425) - 30600 = 33030$$

$$M_{BA} = 2 \times 2.86 \times (2 \times -425) + 30600 = 25800$$

$$M_{BC} = 2 \times 5.89 \times (2 \times -425) - 7230 = -17200$$

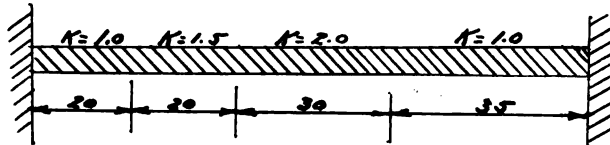
$$M_{BD} = 2 \times 5.00 \times (2 \times -425) = -8500$$

$$M_{DB} = 2 \times 5.00 \times (-425) = -4500$$

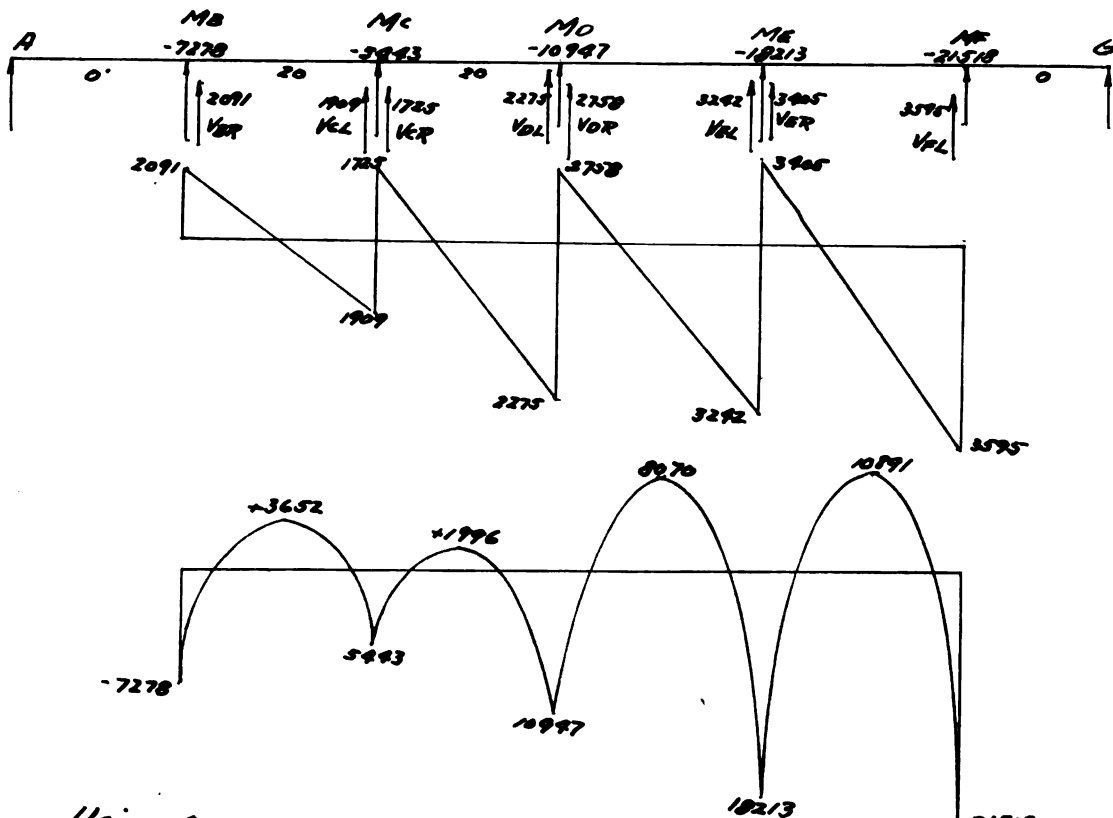
Problem 68, Page 81.

Obtain the moment diagrams for the loaded structure shown. In this case check the results by statics and by drawing a deflected structure to see that it is reasonable.

Assume a continuous beam condition by removing restrained ends and adding 2 extra end beams of 0 length.



Load 200^{lb}/foot over all spans



Using 3 Moment Equations with end beams of 0 length and 0 load.

$$1- M_A L_0 + 2 M_B (20 \times 10) + 20 M_C = - 800000$$

$$2- 20 M_B + 80 M_C + 20 M_D = - 800000$$

$$3- 20 M_C + 100 M_D + 30 M_E = - 1,750,000$$

$$4- 35 M_E + 70 M_F = - 2143750$$

1, 2, 6, 8, 13, 15

Prob 68 Cont

$$5 = 30M_D + 130M_E + 35M_F = -3493750$$

A solution of equations 1, 2, 3, 4, 5, will give the following values

$$M_D = -7278$$

$$M_E = -18213$$

$$M_C = -5443$$

$$M_F = -21518$$

$$M_D = -10947$$

Computations of shears

$$M_F = M_E + 35V_{ER} - 7000 \times 17.5$$

$$M_C = M_D + 20V_{DR} - 40000$$

$$-21518 = -18213 - 122500 + 35V_{ER}$$

$$-5443 = -7278 - 40000 + 20V_{DR}$$

$$V_{ER} = 3405$$

$$V_{DR} = 2091$$

$$V_{EL} = 7000 - 3405$$

$$V_{CL} = 4000 - 2091 = 1909$$

$$V_{FL} = 3595$$

$$M_D = M_C + 20V_{CR} - 40000$$

$$M_E = M_D + 30V_{DR} - 90000$$

$$-10947 = -5443 - 40000 + 20V_{CR}$$

$$-18213 = -10947 - 90000 + 30V_{DR}$$

$$V_{CR} = 1725$$

$$V_{DR} = 2758$$

$$V_{DL} = 4000 - 1725$$

$$V_{EL} = 6000 - 2758$$

$$V_{DL} = 2275$$

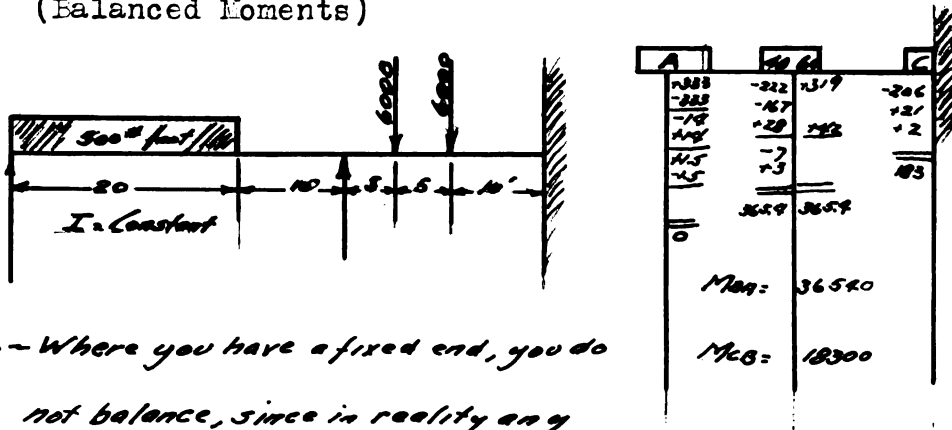
$$V_{EL} = 3242$$

~Note~

These values were all used to draw the shear diagrams and hence the Moment Diagrams.

Problem 69, Page 81.

Obtain moment diagrams for the loaded structure shown. In this case check the results by statics and by drawing a deflected structure to see that it is reasonable (Balanced Moments)



Note - Where you have a fixed end, you do not balance, since in reality any

moment going into the joint remains there.

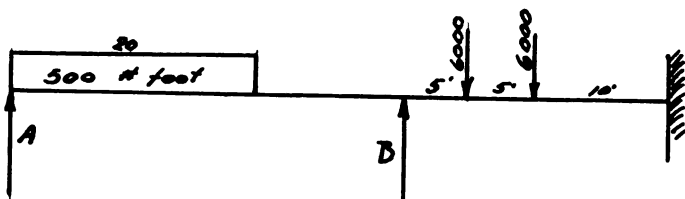
The physical conception of balancing moments is to hold each joint fixed, and release one at a time. Then let it rotate. Repeat on the next joint, and so on until every joint has been released and then clamped again.

This process is repeated until every joint has come to static equilibrium and the imaginary clamps can be removed. This obtains when the moments on each side of a joint are equal and opposite.

The following values were obtained from Prob 69 (Method of Slope Deflection) and are listed on following page.

$$\begin{aligned}
 C_{AB} &= 333 & C_{CB} &= 206 & I &= \text{constant } 100 \\
 C_{BA} &= 222 & K_{AB} &= 3.33 \\
 C_{BC} &= 319 & K_{BC} &= 5
 \end{aligned}$$

Prob 69- "Cont" "Balancing Mom" & "Slope Deflection"



$$K = \frac{1}{30} \quad P = 0$$

$$K = \frac{1}{20} \quad \theta_c = 0$$

$$C_{AB} = \frac{500 \times 20^3}{12 \times 30^2} (3 \times 20^2 - 8 \times 20 \times 30 + (6 \times 30^2)) = -33300$$

$$C_{BA} = \frac{500 \times 20^3}{12 \times 30^2} (4 \times 30 - 3 \times 20) = 22200$$

$$C_{BC} = -\frac{6000 \times 5 \times 15^2}{400} - \frac{6000 \times 20}{8} = 31900$$

$$C_{CB} = \frac{6000 \times 15 \times 5^2}{400} + \frac{6000 \times 20}{8} = 20630$$

$$M_{AB} = 2EK(2\theta_A + \theta_B) - 33300 = 0 \quad M_{AB} = 0$$

$$M_{BA} = 2EK(2\theta_B + \theta_A) + 22200 \quad M_{BA} + M_{BC} = 0$$

$$M_{BC} = 2EK(2\theta_B + \theta_C) - 31900$$

$$M_{CB} = 2EK(2\theta_C + \theta_B) + 20630$$

$$\textcircled{a} \quad 2 \times \frac{1}{30} \times (2\theta_A + \theta_B) - 33300 = 0$$

$$\textcircled{b} \quad 2 \times \frac{1}{30} \times (2\theta_B + \theta_A + 22200) + 2 \times \frac{1}{20} (2\theta_B) - 31900 = 0$$

$$\textcircled{a} \quad .1335 \theta_A + .0668 \theta_B - 33300 = 0$$

$$\textcircled{b} \quad .0668 \theta_A + .3335 \theta_B - 9700 = 0$$

$$\textcircled{a} \quad .1335 \theta_A + .0668 \theta_B - 33300 = 0$$

$$\textcircled{b} \times 2 \quad .1335 \theta_A + .667 \theta_B - 19400 = 0$$

$$-.6 \theta_B - 13900 = 0$$

$$\theta_B = -23200$$

$$\theta_A = \frac{.0668(-23200) - 33300}{.1335} = 26100$$

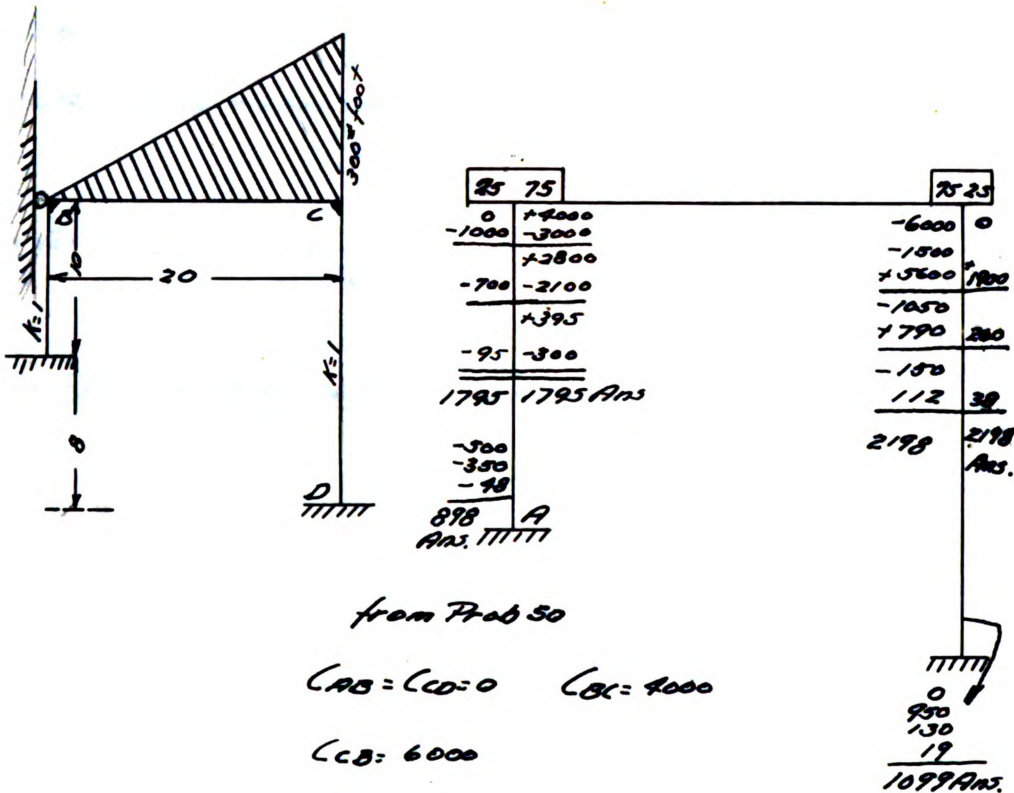
$$M_{CB} = 2 \times \frac{1}{20} (-23200) + 20630 = 18300$$

$$M_{BC} = 2 \times \frac{1}{20} [2(-23200)] - 31900 = 36540$$

Problem 70, Page 81.

Obtain moment diagrams for the loaded structure shown. In this case check the results by statics and by drawing a deflected structure to see that it is reasonable.

(Balancing Moments)



No side sway because of restraint

$$\left. \begin{aligned} \textcircled{a} \quad M_{BA} + M_{BC} &= 0 \\ \textcircled{b} \quad M_{CB} + M_{CD} &= 0 \end{aligned} \right\} \text{because of corner support } p=0$$

Because of fixed ends $\theta_A + \theta_D$ are 0

$$M_{AB} = 2 \times E \times I \times (\theta_B) \quad M_{BA} = 2 \times E \times I \times (2\theta_B)$$

$$M_{BC} = 2 \times E \times I \times (2\theta_B + \theta_C) - 4000 \quad M_{CB} = 2 \times E \times I \times (2\theta_C + \theta_B) + 6000$$

$$M_{CD} = 2 \times E \times I \times (2\theta_C) \quad M_{DC} = 2 \times E \times I \times (\theta_C)$$

$$\textcircled{a} \quad 2(2\theta_B) + 2 \times 3(2\theta_B + \theta_C) - 4000 = 0$$

$$\textcircled{b} \quad 2 \times 3(2\theta_C + \theta_B) + 6000 + 2 \times 2\theta_C = 0$$

Prob 70 Cont"

$$\textcircled{a} \quad 16\theta_B + 6\theta_C - 4000 = 0$$

$$\textcircled{b} \quad 16\theta_C + 6\theta_B + 6000 = 0$$

$$\text{from } \textcircled{a} \quad \theta_C = \frac{4000 - 16\theta_B}{6}$$

$$\text{from } \textcircled{b} \quad \frac{16 \times 4000 - 16\theta_B}{6} + 6\theta_B + 6000 = 0$$

$$10670 - 42.6\theta_B + 6\theta_B + 6000 = 0 \quad \theta_B = 455$$

$$BC = \frac{4000 - 7280}{6} = 546$$

$$M_{AB} = 2\theta_B = 910$$

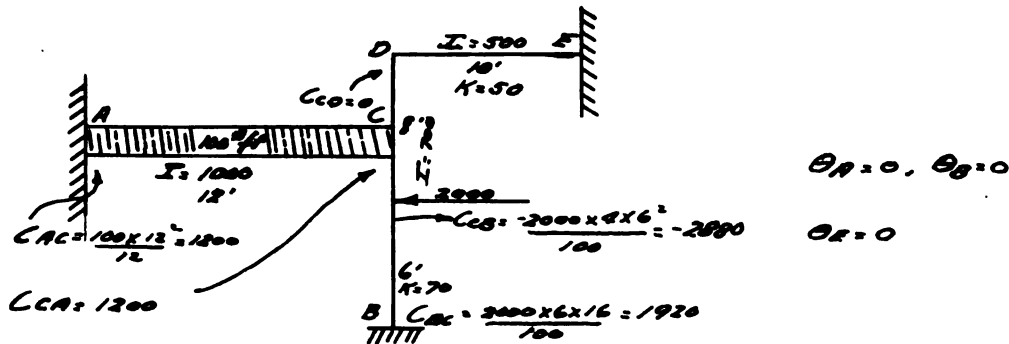
$$M_{BA} = 4\theta_B = 1820$$

$$M_{CD} = 4\theta_C = 2185$$

$$M_{DC} = 2\theta_C = 1092$$

Problem 71, Page 81.

Obtain moment diagrams for the loaded structure shown. In this case check the results by statics and by drawing a deflected structure to see that it is reasonable.



$$M_{AC} = 2E83.3(2\theta_A + \theta_C) - 1200 = 166.6E\theta_C - 1200$$

$$M_{CA} = 2E83.3(2\theta_C + \theta_A) + 1200 = 333.2E\theta_C + 1200$$

$$M_{CB} = 2E70(2\theta_C + \theta_B) - 2880 = 280E\theta_C - 2880$$

$$M_{BC} = 2E70(2\theta_B + \theta_C) + 1920 = 140E\theta_C + 1920$$

$$M_{CD} = 2E175(2\theta_C + \theta_D) = 700E\theta_C + 350E\theta_D$$

$$M_{DC} = 2E175(2\theta_D + \theta_C) = 700E\theta_D + 350E\theta_C$$

$$M_{DE} = 2E50(2\theta_D + \theta_E) = 200E\theta_D \quad M_{CA} + M_{CD} + M_{CB} = 0$$

$$M_{ED} = 2E50(2\theta_E + \theta_D) = 100E\theta_D \quad M_{DC} + M_{DE} = 0$$

$$1213.2E\theta_C + 350E\theta_D - 1680 = 0$$

$$900E\theta_D + 350E\theta_C = 0$$

$$E\theta_C = 1.561$$

$$E\theta_D = -60.8$$

$$M_{AC} = -940$$

$$M_{CA} = +1564$$

$$M_{CB} = -2442$$

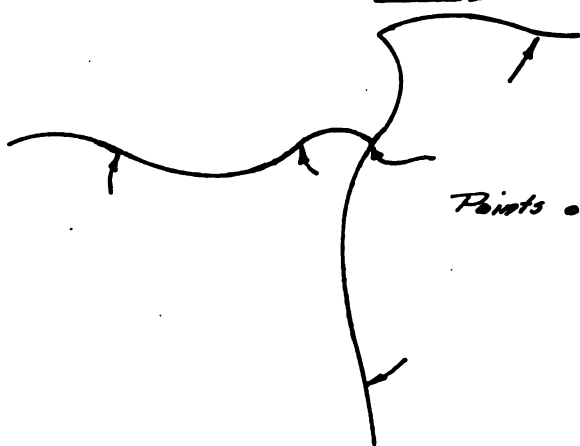
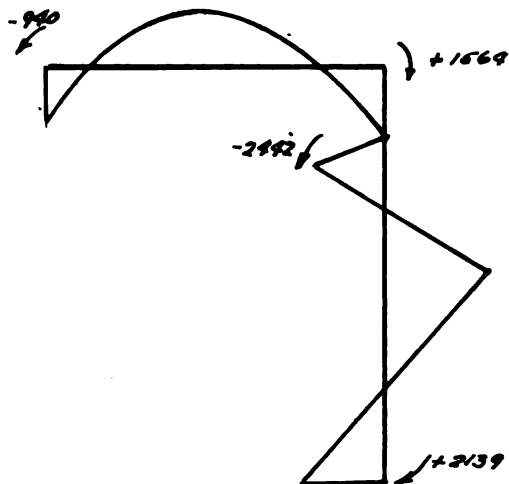
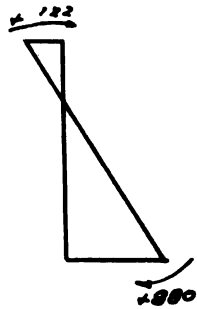
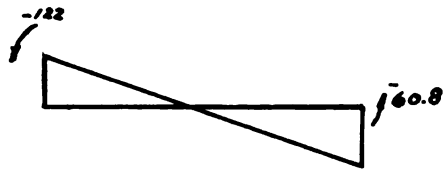
$$M_{CD} = +880$$

$$M_{DC} = +122$$

$$M_{DE} = -122$$

$$M_{BC} = +2139$$

$$M_{ED} = -60.8$$



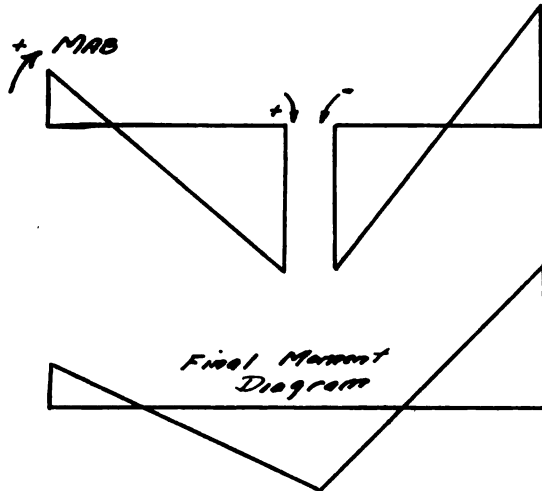
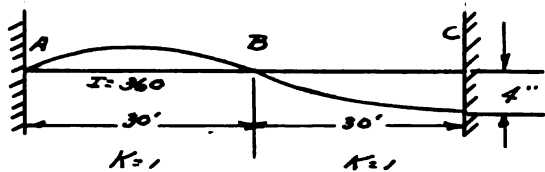
Points of contraflexure.

Problem 72, Page 81.

Obtain the moment diagram for the beam caused by the settlement of the support C. $E = 30,000,000$

$$\theta_A = 0$$

$$\theta_C = 0$$



$$M_{AB} = 2E(2\theta_A + \theta_B) = 2E\theta_B$$

$$M_{BA} = 2E(2\theta_B + \theta_A) = 4E\theta_B$$

$$M_{BC} = 2E(2\theta_B + \theta_C - 3\frac{\delta}{360}) = 4E\theta_B - \frac{F}{15}$$

$$M_{CB} = 2E(2\theta_C + \theta_B - \frac{\delta}{30}) = 2E\theta_B - \frac{F}{15}$$

$$M_{BA} + M_{BC} = 0$$

$$8E\theta_B = \frac{F}{15}$$

$$\theta_B = \frac{1}{120}$$

$$M_{BC} = 30,000,000 \left(\frac{4}{120} - \frac{1}{15} \right) = -1,000,000 \text{ "ft-lb"}$$

$$M_{BA} = 30,000,000 \left(\frac{4}{120} \right) = +1,000,000 \text{ "ft-lb"}$$

$$M_{AB} = 500,000 \text{ "ft-lb"}$$

$$M_{CB} = \frac{30,000,000}{60} - 2,000,000 = -1,500,000 \text{ "ft-lb"}$$

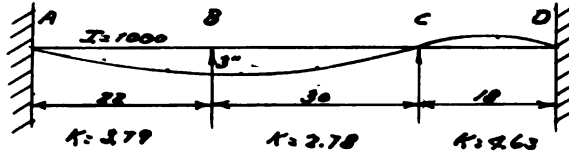
8, 11, 18

Problem 73, Page 81

Obtain the maximum moment for this beam caused by the settlement at B. E is 30,000,000

$$\theta_A = 0$$

$$\theta_D = 0$$



$$M_{AB} = 2E(2.79)(\theta_B - 3 \frac{3}{22 \cdot 12}) = 7.58 E \theta_B - .259 E$$

$$M_{BA} = 2E(2.78)(2\theta_B - .0341) = 11.12 E \theta_B - .1898 E$$

$$M_{BC} = 2E(2.78)(2\theta_B + \theta_C + 3 \frac{3}{30 \cdot 12}) = 11.12 E \theta_B + 5.56 E \theta_C + .139 E$$

$$M_{CB} = 2E(2.78)(2\theta_C + \theta_B + 3 \frac{3}{30 \cdot 12}) = 11.12 E \theta_C + 5.56 E \theta_B + .139 E$$

$$M_{CD} = 2E(4.63)(2\theta_C) = 18.52 E \theta_C$$

$$M_{DC} = 2E(4.63)(\theta_C) = 9.26 E \theta_C$$

$$M_{BA} + M_{BC} = 0, 11.12 E \theta_B - .1898 E + 11.12 E \theta_B + 5.56 E \theta_C + .139 E = 0$$

$$M_{CB} + M_{CD} = 0, 11.12 E \theta_C + 5.56 E \theta_B + .139 E + 18.52 E \theta_C = 0$$

$$22.24 E \theta_B + 5.56 E \theta_C - .0508 E = 0$$

$$5.56 E \theta_B + 29.64 E \theta_C + .139 E = 0$$

$$-62.91 \theta_C = -.3377$$

$$\theta_C = .00537$$

$$\theta_B = \frac{5.56(.00537) + .0508}{22.24} = .00362$$

$$M_{AB} = 7.58(30,000,000)(.00362) - .259(30,000,000)$$

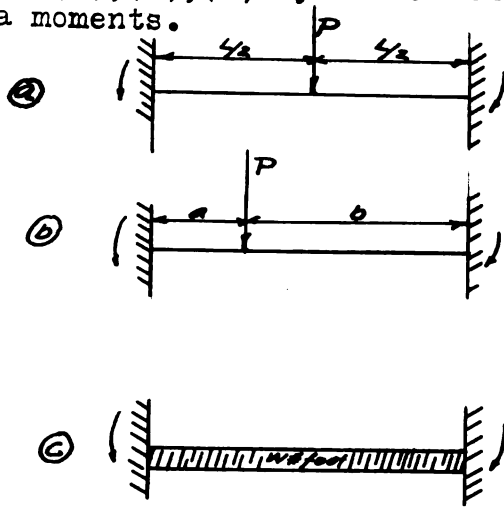
$$= .2315(30 \times 10^6) = 6,940,000 \text{ #ft}$$

$$\text{or } 577,000 \text{ #ft}$$

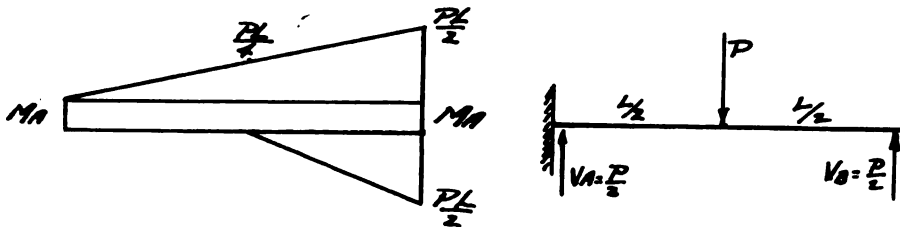
8.11.16.18

Problem 74, Page 81.

Check the fixed end moments as given in Figure 69 for the beams (a), (b), (c) by use of the first two propositions of area moments.



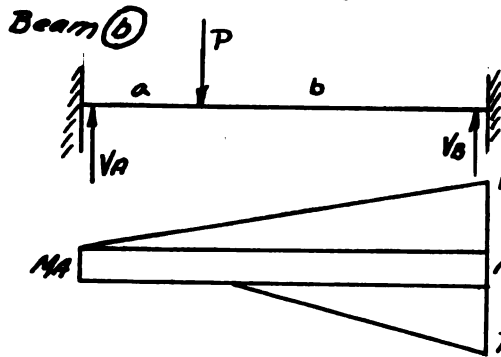
Beam (a) Because tangent at center is $\parallel$ to tangent at end, this area of M diagram between 2 points = 0



$$\frac{PL}{8} \times \frac{L}{2} \times \frac{L}{2} + M_A \frac{L}{2} = 0$$

$$\frac{PL^2}{16} = -\frac{M_A L}{2}$$

$$M_A = -\frac{PL}{8}$$



$$\frac{V_A L^2}{2} + M_A L = \frac{P b^2}{2}$$

$$V_A L \text{ ① } V_A L^2 + 2 M_A L = P b^2$$

$$\text{①} \times \text{②} = 3 V_A L^2 + 6 M_A L^2 = 3 P b^2 L$$

$$\frac{V_A L^2}{2} \times \frac{L}{3} + \frac{M_A L^2}{2} = \frac{P b^2}{2} \times \frac{L}{3}$$

$$\frac{V_A L^3}{6} + \frac{M_A L^2}{2} = \frac{P b^3}{6}$$

$$\text{②} \times \text{③} = V_A L^3 + 3 M_A L^2 = P b^3$$

$$\text{③} \times \text{②} = 2 V_A L^3 + 6 M_A L^2 = 2 P b^3$$

8, 15, 16, 18

Prob 74 Cont

$$VAL^3 = 3Pb^2L - 2Pb^3$$

$$VA = \frac{Pb^2(3L-2b)}{L^3}$$

Using $\frac{VAL^2}{2} + MAL = \frac{Pb^2}{2}$

$$\left(\frac{Pb^2(3L-2b)}{L^3} \right) \frac{L^2}{2} + MAL = \frac{Pb^2}{2}$$

$$Pb^2(3L-2b) + 2MAL^2 = Pb^2L$$

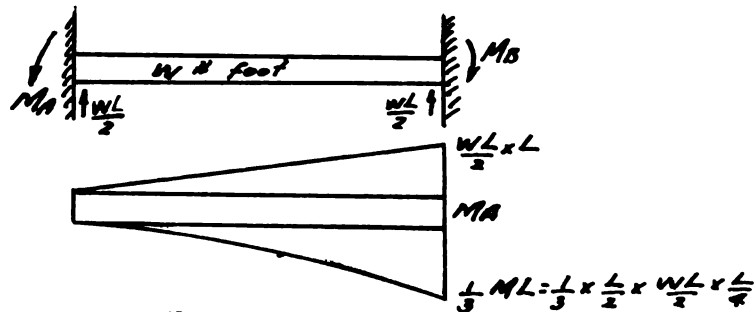
$$3Pb^2L - 2Pb^3 + 2MAL^2 = 2Pb^2L \quad (L-b) = a$$

$$2Pb^2L - 2Pb^3 = -2MAL^2$$

$$\frac{2Pb^2L - 2Pb^3}{2L^2} = -MA$$

$$MA = -\frac{Pab^2}{L^2}$$

beam ©



$$\frac{WL}{2} \times \frac{L}{2} \times \frac{L}{4} + \frac{MAL}{2} = \frac{1}{3} \times \frac{L}{2} \times \frac{WL^2}{8}$$

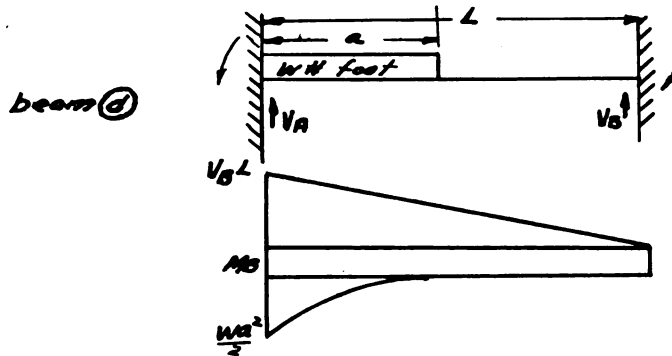
$$\frac{WL^3}{16} + \frac{MAL}{2} = \frac{WL^3}{48}$$

$$3WL^3 + 24MAL = WL^3$$

$$MAL = -\frac{2WL^3}{24L} = -\frac{WL^2}{12}$$

Problem 75, Page 81.

Develop the expressions for fixed end moments for any beam from (d) to (h) as given in Figure 69. Use any method discussed in this chapter or use the area moment proposition.



$$\frac{V_B L^2}{2} + M_B L = \frac{1}{3} \frac{w a^3}{2}$$

$$3 V_B L^2 + 6 M_B L = w a^3$$

$$3 V_B L^2 = w a^3 - 6 M_B L$$

$$V_B = \frac{w a^3}{3 L^2} - \frac{2 M_B}{L}$$

$$\frac{V_B L^2}{2} \times \frac{L}{3} + \frac{M_B L^2}{2} = \frac{1}{3} \frac{w a^3}{2} \times \frac{a}{4}$$

$$\frac{V_B L^2}{6} + \frac{M_B L^2}{2} = \frac{w a^4}{12 \times 2}$$

$$4 V_B L^2 + 12 M_B L^2 = w a^4$$

$$4 \left(\frac{w a^3}{3 L^2} - \frac{2 M_B}{L} \right) + 12 M_B L^2 = w a^4$$

$$12 M_B L^2 = 3 w a^4 - 8 w a^3 L$$

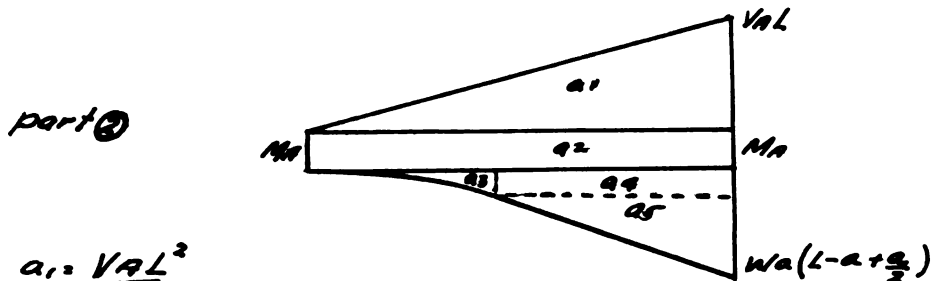
$$\frac{4 w a^3 L^3}{3 L^2} - \frac{8 M_B L^3}{L} + 12 M_B L^2 = w a^4$$

$$M_B = \frac{w a^3}{12 L^2} (3a - 4L)$$

$$\frac{4 w a^3 L}{3} - 8 M_B L^2 + 12 M_B L^2 = w a^4$$

$$\frac{4 w a^3 L}{3} + 4 M_B L^2 = w a^4$$

$$4 w a^3 L + 12 M_B L^2 = 3 w a^4$$



$$a_1 = \frac{V_A L^2}{2}$$

$$a_2 = M_A L$$

$$a_3 = \frac{1}{3} \frac{w a^3}{2} \cdot a$$

$$a_4 = \frac{w a^2}{2} (L - a)$$

$$a_5 = \left[w a \left(L - \frac{a}{2} \right) - \frac{w a^2}{2} \right] \left(\frac{L - a}{2} \right)$$

Prob 75 "Cont"

sheet 2

$$\frac{V_R L^2}{2} + M_R L = \frac{W a^2}{2} (L-a) + \frac{W a^3}{6} + \left[W a \left(L - \frac{a}{2} \right) - \frac{W a^2}{2} \right] \left(\frac{L-a}{2} \right)$$

$$\frac{V_R L^2}{2} + M_R L = \frac{W a^2 L}{2} - \frac{W a^3}{2} + \frac{W a^3}{6} + \frac{W a L^2}{2} - W a^2 L + \frac{W a^3}{2}$$

$$3 V_R L^2 + 6 M_R L = 3 W a^2 L - 3 W a^3 + W a^3 + 3 W a L^2 - 6 W a^2 L + 3 W a^3$$

$$3 V_R L^2 - 3 W a^2 L + W a^3 + 3 W a L^2 - 6 M_R L$$

$$V_R = \frac{W a^3}{3 L^2} - \frac{W a^2}{L} + W a - \frac{2 M_R}{L}$$

$$\begin{aligned} \frac{V_R L^2}{2} \cdot \frac{L}{2} + \frac{M_R L}{2} &= \frac{W a^2}{2} \left(\frac{L-a}{2} \right)^2 + \left(\frac{1}{2} \frac{W a^2}{2} \times a \right) \left(L-a + \frac{a}{2} \right) + \left[W a \left(L - \frac{a}{2} \right) - \frac{W a^2}{2} \right] \times \left(\frac{L-a}{2} \right)^2 \\ &= \frac{W a^2 L^2}{4} - \frac{2 W a^3 L}{4} + \frac{W a^4}{4} + \frac{W a^3}{6} - \frac{W a^4}{8} + \frac{W a L^3}{6} - \frac{3 W a^2 L^2}{6} + \frac{3 W a^3 L}{6} - \frac{W a^4}{6} \end{aligned}$$

$$\frac{V_R L^3}{6} = -\frac{W^2 L^2}{4} + \frac{W a^3 L}{6} - \frac{W a^4}{24} + \frac{W a L^3}{6} - \frac{M_R L^2}{2}$$

$$4 V_R L^3 = -6 W^2 L^2 + 4 W a^3 L - W a^4 + 4 W a L^3 - 12 M_R L$$

$$V_R = \frac{-3 W^2}{2 L} + \frac{W a^3}{L^2} - \frac{W a^4}{2 L^3} + W a - \frac{3 M_R}{L}$$

$$V_R = V_R$$

$$\frac{W a^3}{3 L^2} - \frac{W a^2}{L} + W a - \frac{2 M_R}{L} = \frac{W a^3}{L^2} - \frac{3 W^2}{2 L} - \frac{W a^4}{2 L^3} + W a - \frac{3 M_R}{L}$$

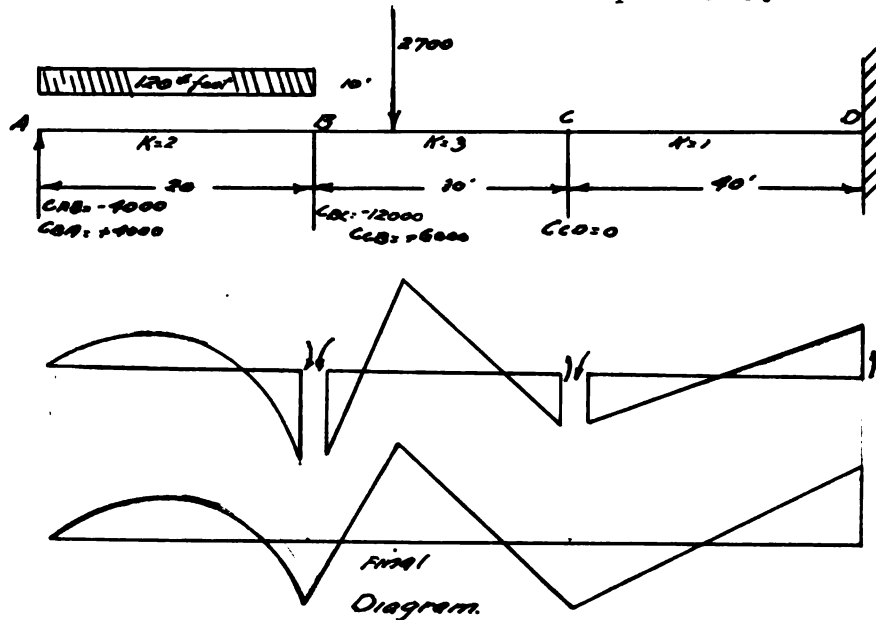
$$4 W a^3 L - 12 W a^2 L^2 + 12 W a L^3 - 3 a M_R L^2 = 12 W a^3 L - 18 W^2 L^2 - 3 W a^4 + 12 W a L^3 - 36 M_R L^2$$

$$12 M_R L^2 = 8 W a^3 L - 6 W a^2 L^2 - 3 W a^4$$

$$M_R = \frac{-W a^2}{12 L^2} (3 a^2 - 8 a L + 6 L^2)$$

Problem 76, Page 84.

Re-analyze for the moments in the continuous beam of Figure 70, using only two simultaneous equations.



Since $M_{AB} = 0$ $\rho = 0$

$$M_{BA} = 3EK(\theta_B) + H_{BA} \quad \text{Where } H_{BA} = C_{BA} + \frac{C_{AB}}{2}$$

$$= 6E\theta_B + 9000 + 2000 = 6E\theta_B + 6000$$

$$M_{BC} = 2EK(2\theta_B + \theta_C) - 12000$$

$$= 12E\theta_B + 6E\theta_C - 12000$$

$$M_{CB} = 2EK(\theta_B + 2\theta_C) + 6000$$

$$= 6E\theta_B + 12E\theta_C + 6000$$

$$M_{CD} = 2EK(2\theta_C + \theta_D) + 0 = 4E\theta_C$$

Then $M_{BA} + M_{BC} = 0$

$$6E\theta_B + 6000 + 12E\theta_B + 6E\theta_C - 12000 = 18E\theta_B + 6E\theta_C - 6000 = 0$$

$$M_{CB} + M_{CD} = 0$$

$$6E\theta_B + 12E\theta_C + 6000 + 4E\theta_C = 0, \quad 6E\theta_B + 16E\theta_C + 6000 = 0$$

$$M_{BA} = 6(524) + 6000 = 9144$$

$$42E\theta_C = -24000$$

$$E\theta_C = -571$$

$$E\theta_B = +524$$

$$8, 13.15$$

Prob 76 "Cont"

$$M_{AC} = 12(524) + 6(-571) - 12000 = -9144$$

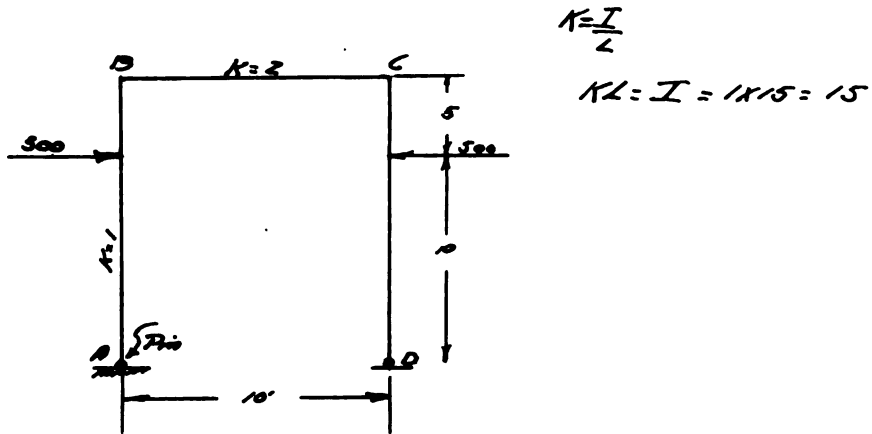
$$M_{CB} = (-571 \times 12) + 6(524) + 6000 = 2290$$

$$M_{CD} = 4E\theta_c = -2284$$

$$M_{DC} = 2(-571) = -1142$$

Problem 77, Page 84.

Analyze for the moments at B and C of the bent shown. Use H-values.



$$K = \frac{I}{L}$$

$$K_L = I = 1 \times 15 = 15$$

$$K = \frac{I_{AB}}{L}$$

$$n = \frac{K_L h}{I_{AB}} = \frac{2 \times 15}{15}$$

$$n = 2$$

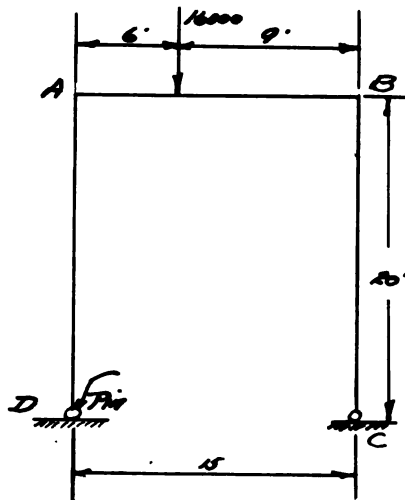
$$M_{AB} = M_{AC} = - \left[\frac{2n}{2n+3} \right] \frac{P_0 b}{2h^2} (h+b)$$

$$= \frac{-4 \times 500 \times 5 \times 10 \times 25}{7 \times 2 \times 225} = \frac{50000}{63} = 793 \text{ 'ft}$$

Equation derived in previous graduate study on
Rigid Frames

Problem 78, Page 84.

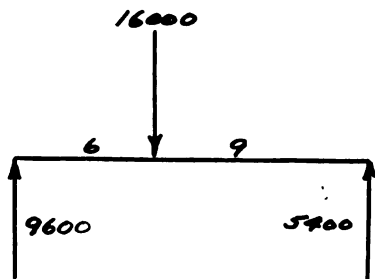
Analyze for the moments in the bent illustrated. The columns and girder are of the same section.



$$\begin{aligned}
 \text{From previous work, } M_{AD} &= -M_{BC} = \frac{3P_0b}{2L} \left(\frac{L}{2n+3} \right) \\
 &= \frac{3 \times 16000 \times 6 \times 9}{2 \times 15} \left(\frac{L}{9/5 + 3} \right) \\
 &= 15247.
 \end{aligned}$$

$$K = \frac{I_{AB}}{L} = \frac{I_{AB}}{15}$$

$$n = \frac{Kh}{I_{AD}} = \frac{I_{AB}h}{I_{AD} \cdot 15} = \frac{20}{15} = \frac{4}{3}$$



$$9600 \times 6 = 57600$$

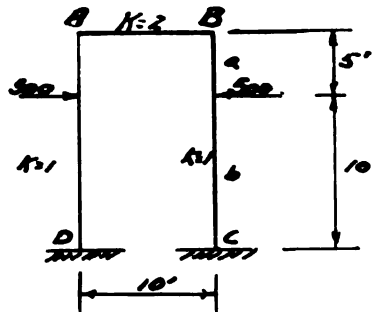
$$\frac{15247}{}$$

$$M_{AB} = 42.353 \text{ KF}$$

Problem 79, Page 84

Repeat the analysis of Problem 77 with the bent changed to the condition of fixed end columns.

$$K = \frac{I_{AB}}{L}$$



$$n = \frac{K_h}{I_{AD}} = \frac{K_h}{I_{BC}}$$

$$K=2 \quad K = \frac{I_{AB}}{L}$$

$$M_{AB} = M_{BC} = \frac{-n}{n+2} \left(\frac{Pab^2}{h^2} \right)$$

$$n = \frac{2 \times 15}{15} = 2$$

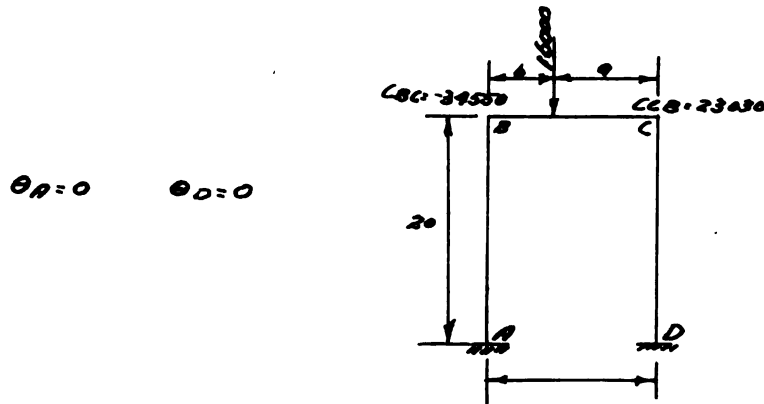
$$M_{CB} = M_{DA} = \frac{Pab}{h^2} \left(\frac{b}{n+2} + a \right)$$

$$M_{AB} = M_{BC} = \frac{2}{4} \times \frac{500 \times 5 \times 100}{225} = 555 \text{ ft-lb}$$

$$M_{CB} = M_{DA} = \frac{500 \times 5 \times 10}{225} \left(\frac{10}{4} + 5 \right) = 833 \text{ ft-lb}$$

Problem 80, Page 84

Repeat the analysis of Problem 78 with the bent changed to the condition of fixed end columns.



$$\theta_A = 0 \quad \theta_D = 0$$

$$M_{AB} = 2E(\theta_B - 3P) = 2E\theta_B - 6EP$$

$$M_{BA} = 2E(2\theta_B - 3P) = 4E\theta_B - 6EP$$

$$M_{BC} = 2E(2\theta_B + \theta_C) - 34550 = 4E\theta_B + 2E\theta_C - 34550$$

$$M_{CB} = 2E(2\theta_C + \theta_B) + 23030 = 4E\theta_C + 2E\theta_B + 23030$$

$$M_{CD} = 2E(2\theta_C - 3P) = 4E\theta_C - 6EP$$

$$M_{DC} = 2E(\theta_C - 3P) = 2E\theta_C - 6EP$$

$$M_{BA} + M_{BC} = 0 \quad 8E\theta_B + 2E\theta_C - 6EP - 34550 = 0 \quad (a)$$

$$M_{CB} + M_{CD} = 0 \quad 2E\theta_B + 8E\theta_C - 6EP + 23030 = 0 \quad (b)$$

$$\frac{M_{BA} + M_{AB}}{20} + \frac{M_{CD} + M_{DC}}{20} = 0 \quad 3E\theta_B - 12EP + 3E\theta_C = 0 \quad (c)$$

$$2a-c \quad 13E\theta_B + E\theta_C = 69100$$

$$2b-c \quad E\theta_B + 13E\theta_C = -46060$$

$$168E\theta_C = -668000$$

$$E\theta_C = -3980$$

$$168E\theta_B = 934000$$

$$E\theta_B = 5560$$

$$EP = \frac{1590}{4} = 395$$

7.8.10.16

Prob 80 "Cont"

$$M_{AB} = 2(5560) - 6(395) = 8750$$

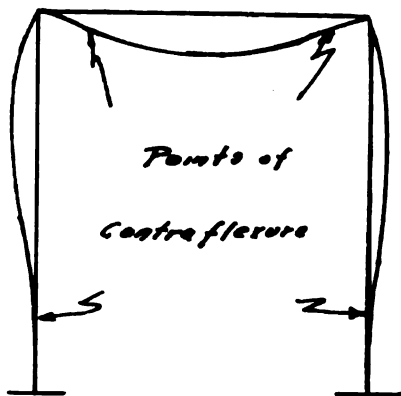
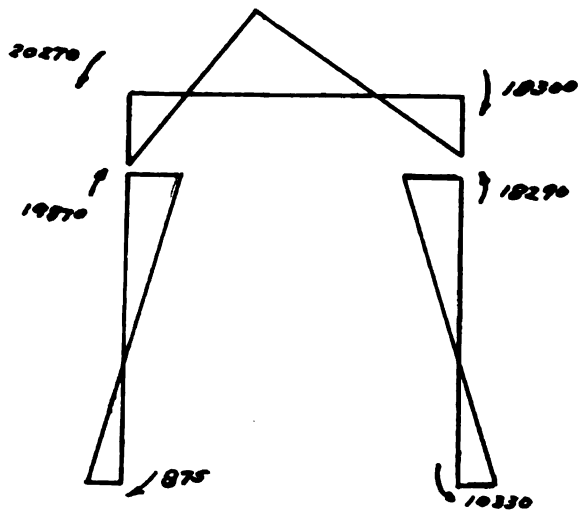
$$M_{BA} = 4(5560) - 6(395) = 19870$$

$$M_{BC} = 4(5560) + 2(-3980) - 34550 = -20270$$

$$M_{CB} = 4(-3980) + 2(5560) + 23030 = +18230$$

$$M_{CD} = 4(-3980) - 6(395) = -18290$$

$$M_{DC} = 2(-3980) - 6(395) = -10330$$

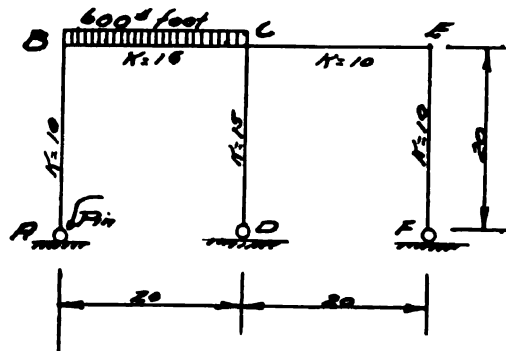


Problem 81, Page 84

Obtain a moment diagram for the two bay bent illustrated. Use equation (23) to simplify the work. Draw the deflected structure by use of the final moments, as a check on the reasonableness of the work.

$$C_{BC} = \frac{WL^2}{12} = -20000$$

$$C_{CB} = +20000$$



$$M_{BA} = 3E \times 10 (\theta_B - P) = 30E\theta_B - 30EP$$

$$M_{BC} = 2E \times 15 (2\theta_B + \theta_C) - 2000 = 60E\theta_B + 30E\theta_C - 20000$$

$$M_{CB} = 2E \times 15 (2\theta_C + \theta_B) + 20000 = 60E\theta_C + 30E\theta_B + 20000$$

$$M_{CD} = 3E \times 15 (\theta_C - P) = 45E\theta_C - 45EP$$

$$M_{CE} = 2E \times 10 (2\theta_C + \theta_E) = 40E\theta_C + 20E\theta_E \quad \sum M = 0$$

$$M_{EC} = 2E \times 10 (2\theta_E + \theta_C) = 40E\theta_E + 20E\theta_C \quad \sum \text{Shear} = 0$$

$$M_{EF} = 3E \times 10 (\theta_E - P) = 30E\theta_E - 30EP$$

$$\textcircled{1} \quad 90E\theta_B + 30E\theta_C - 30EP - 20000 = 0$$

$$\textcircled{2} \quad 30E\theta_B + 145E\theta_C + 20E\theta_E - 45EP + 20000 = 0$$

$$\textcircled{3} \quad 20E\theta_C + 40E\theta_E - 30EP = 0$$

$$\textcircled{4} \quad 30E\theta_B + 45E\theta_C + 30E\theta_E - 105EP = 0$$

$$(-1 + 3\textcircled{2}) \quad 405E\theta_C + 60E\theta_E - 105EP = -80000$$

$$\textcircled{5} \quad 20E\theta_C + 40E\theta_E - 30EP = 0$$

$$\textcircled{6} = \textcircled{2} - \textcircled{5} \quad 100E\theta_C - 10E\theta_E + 60EP = -80000$$

$$\textcircled{7} = 7\textcircled{6} + 6\textcircled{5} \quad 2715E\theta_C + 555EP = -560000$$

$$\textcircled{8} = \textcircled{3} + 7\textcircled{6} \quad 720E\theta_C + 390EP = -140000$$

7, 8, 13, 15, 16

Prob 81 "Cont"

$$4.89E\theta_c - EP = -1008$$

$$1.846E\theta_c + EP = -359$$

$$6.736E\theta_c = -1359$$

$$E\theta_c = -202$$

$$EP = -322$$

$$E\theta_E = +195.7$$

$$E\theta_B = +182.2$$

$$M_{CE} = -4172$$

$$M_{BA} = +15126$$

$$M_{CD} = -5400$$

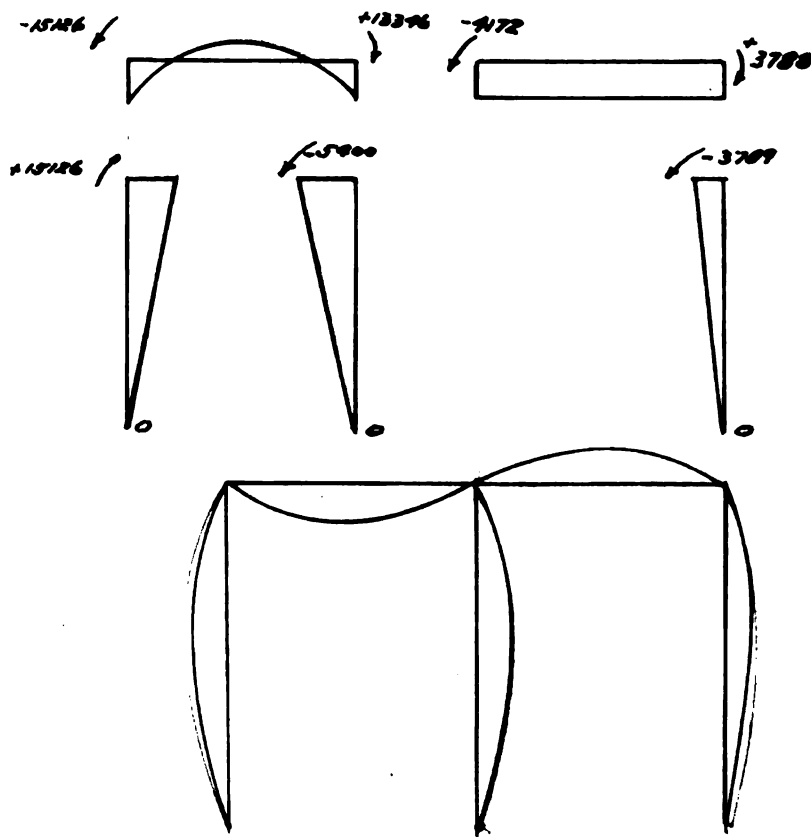
$$M_{EC} = +3788$$

$$M_{BC} = -15128$$

$$M_{AB} = M_{DC} = M_{FE} = 0$$

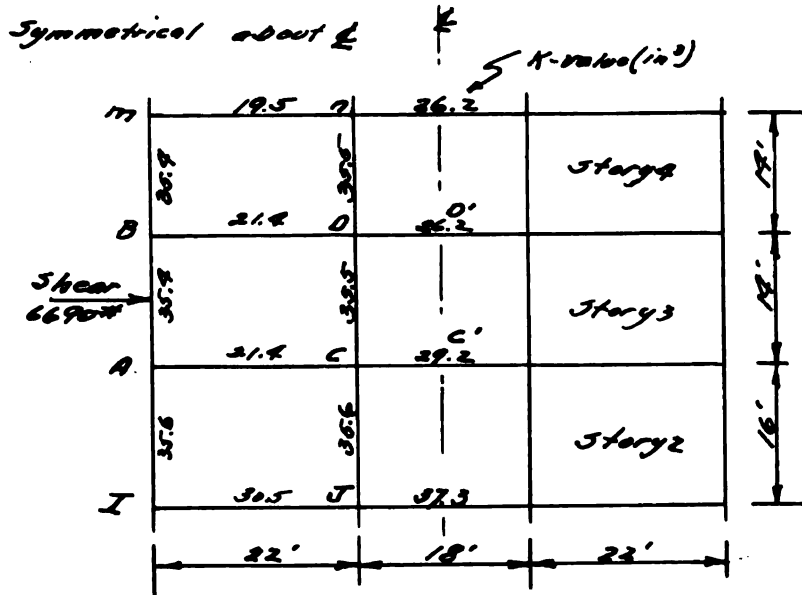
$$M_{EF} = -3789$$

$$M_{CB} = 13346$$



Problem 82, Page 87

The building frame shown represents the 2nd, 3rd, and 4th stories of the Wilson-Maney bent.* Compute the wind moments and check with these values: $M_{AB} = 90,700$; $M_{BA} = 100,300$; $M_{CD} = 182,000$; $M_{DC} = 187,500$ in-lb.



Assumptions made in work $\theta_A = \theta_C$ $\theta_B = \theta_M$ $p = \text{const.}$

$$\theta_C = \theta_j \quad \theta_n = \theta_D \quad \theta_D = \theta_{D'}$$

$$\theta_C = \theta_{C'}$$

General Equation are $\sum M = 0$ $\sum F_x = 0$ $C = 0$

$$M_{AB} = 2EK(2\theta_A + \theta_B - 3p) - C_{AB}$$

Applying these facts and omitting $2E$, we arrive at

$$\begin{aligned} \sum M_A = 0 \quad & 35.6(3\theta_A - 3p) + 21.4(2\theta_A + \theta_C) + 35.4(2\theta_A + \theta_B - 3p) = 0 \\ & = 220.4\theta_A + 35.4\theta_B + 21.4\theta_C - 213.0p = 0 \end{aligned}$$

$$\begin{aligned} \sum M_B = 0 \quad & 35.4(2\theta_B + \theta_A - 3p) + 35.4(3\theta_B - 3p) + 21.4(2\theta_B + \theta_D) = 0 \\ & = 219.8\theta_B + 35.4\theta_A + 21.4\theta_C - 212.4p = 0 \end{aligned}$$

$$\begin{aligned} \sum M_C = 0 \quad & 35.6(3\theta_C - 3p) + 35.5(2\theta_C + \theta_D - 3p) + 22.2(3\theta_C + 21.4[2\theta_C - \theta_A]) = 0 \\ & 21.4\theta_A + 308.2\theta_C + 35.5\theta_D - 213.3p = 0 \end{aligned}$$

$$\begin{aligned} \sum M_D = 0 \quad & 35.5(2\theta_D + \theta_C - 3p) + 35.5(3\theta_D - 3p) + 21.4(2\theta_D + \theta_B - 3p) + 26.2(3\theta_D) = 0 \\ & = 21.4\theta_B + 35.5\theta_C + 299\theta_D - 213p = 0 \end{aligned}$$

4, 7, 8, 10, 15, 16

Prob 82 "Cont"

$$\Sigma F_H = [2(35.4[3\theta_A + 3\theta_B - 6\rho] + 35.5[3\theta_C + 3\theta_D - 6\rho]) : 1412] + \frac{6690}{2} = 0$$

$$= 12.2\theta_A + 12.2\theta_B + 12.23\theta_C + 12.23\theta_D - 48.9\rho + 3345 = 0$$

To solve these equations simultaneously, we arrange them in a table and eliminate by equating coefficients and subtracting.

Eq.	Makeup	$E\theta_A$	$E\theta_B$	$E\theta_C$	$E\theta_D$	$E\rho$	Constant
1		220.4	35.4	21.4	0	-213.0	
2		35.4	219.8	0	21.4	-212.4	
3		21.4	0	302.2	35.5	-213.3	
4		0	21.4	35.5	299.0	-213.0	
5		12.2	12.2	12.23	12.23	-48.9	+3345
6	$1 - \frac{220.4}{35.4} (2)$	0	-1323.6	+21.4	-133.2	+1110.0	
7	$1 - \frac{220.4}{21.4} (3)$	0	+35.4	-3159	-366.0	+1972.0	
8	4	0	21.4	35.5	299.0	-213.3	
9	$1 - \frac{220.4}{12.2} (5)$	0	-185.0	-199.6	-221.0	+672	-60500
10	$9 + \frac{185}{39.4} (7)$		0	-16700	-2131	+10970	-60500
11	$9 + \frac{185}{21.4} (8)$		0	+109.4	+2364	-1172	"
12	$9 - \frac{185}{1323.6} (9)$		0	-202.6	-202.4	+517	"
13	$10 + \frac{16700}{109.4} (11)$			0	343200	-168000	-9,296,000
14	$12 - \frac{202.6}{16700} (10)$			0	-176.6	+384	-59770
15	$\frac{13}{100} + \frac{3439}{176.6} (14)$					580	-1,255,000
VAL		2070	949	1218	1332	2164	

Prob 82 "Cont"

$$MAB = 2 \times 35.6 (2 \times 2070 + 949 - 3 \times 2164) = -99500$$

$$MBA = 70.8 (2 \times 949 + 2070 - 6492) = -108000$$

$$MCD = 71.0 (2 \times 1218 + 133.2 - 6492) = -193300$$

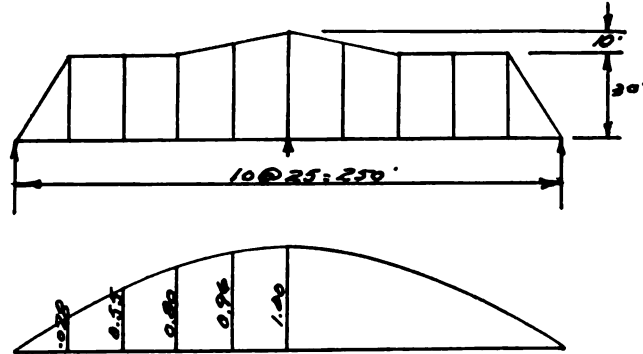
$$MDC = 71.0 (2 \times 1332 + 1218 - 6492) = -185000$$

Note - These values check book within 10%

Problem 90, Page 136

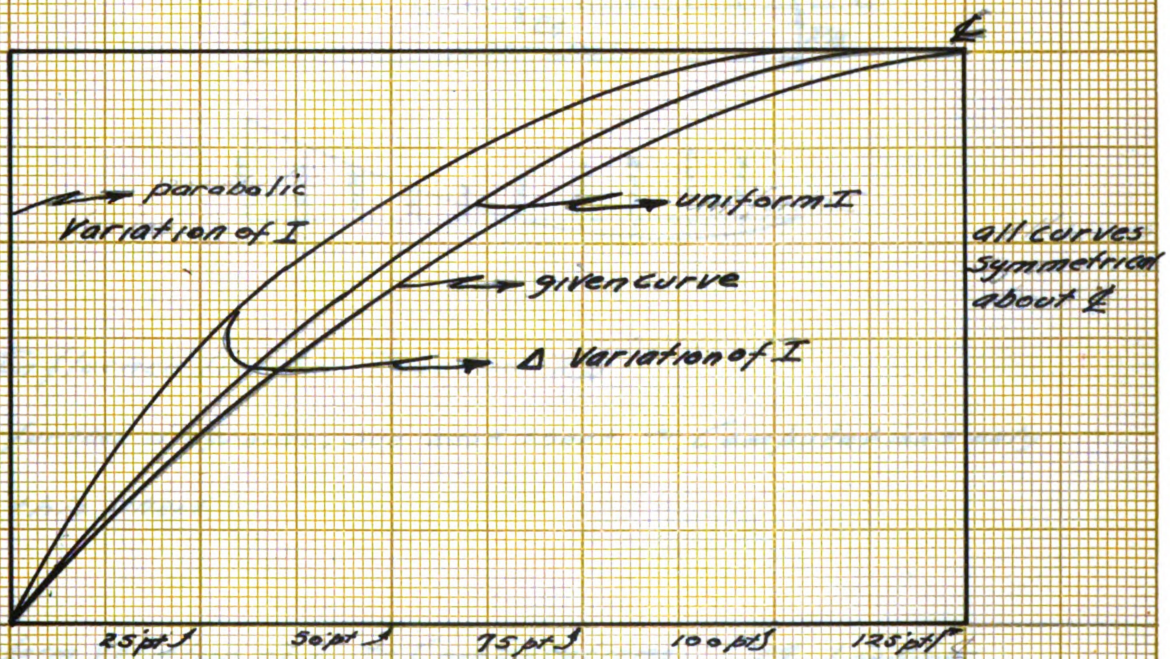
The influence for center reaction is given for the truss illustrated. Compare with the influence lines for a girder on the basis of 3 assumptions.

- (1) uniform I
- (2) triangular variation of I
- (3) parabolic variation of I

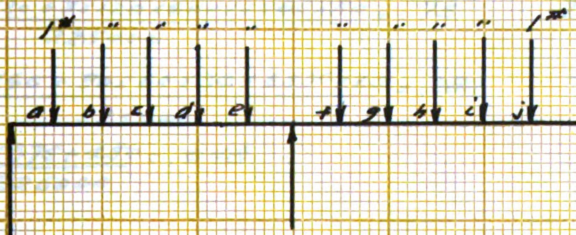


$\sum M_R = 0$ (with load at specified point)

- (a) $5 \times 28 - 9 \times 1 + 10 R_R = 0$ $R_R = .76$
- (b) $5 \times 55 - 8 \times 1 + 10 R_R = 0$ $R_R = .525$
- (c) $5 \times 85 - 7 \times 1 + 10 R_R = 0$ $R_R = .200$
- (d) $5 \times 96 - 6 \times 1 + 10 R_R = 0$ $R_R = .120$
- (e) $5 \times 1.00 - 5 \times 1 + 10 R_R = 0$ $R_R = .000$
- (f) $5 \times 96 - 4 \times 1 + 10 R_R = 0$ $R_R = .08$
- (g) $5 \times 80 - 3 \times 1 + 10 R_R = 0$ $R_R = .100$
- (h) $5 \times 55 - 2 \times 1 + 10 R_R = 0$ $R_R = .075$
- (i) $5 \times 28 - 1 \times 1 + 10 R_R = 0$ $R_R = .04$



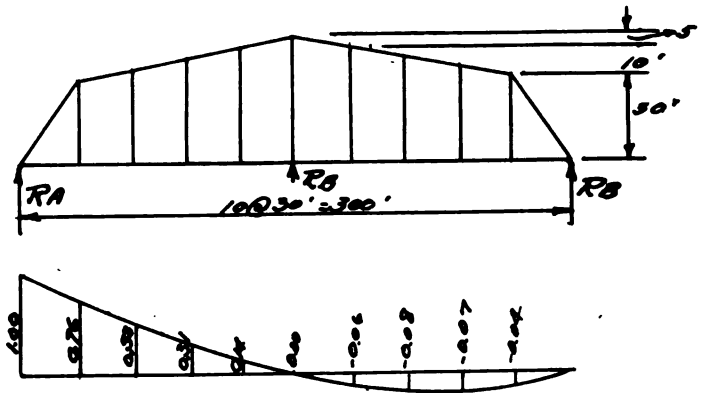
Influence Line for End Reaction.



Problem 91, Page 136

The influence for end reaction is given for the truss illustrated. Compare with the influence lines for a girder on the basis of 3 assumptions.

- (1) uniform I
- (2) triangular variation of I
- (3) parabolic variation of I



Influence line for Reaction R_A

For further diagrams, see next sheet. ~ Computations only on this sheet.

Uniform Variation of I ~ Computations. $\frac{M}{EI}$ Loading

$$15(150)(150) - 375(300) = 225000$$

$$\text{or } \frac{15 \times 150 \times 150 - 375 \times 300}{225000} = 1.00$$

$$\frac{750 \times 120 + 3 \times 120 \times 60 + 12 \times 60 \times 80}{225000} = .75$$

$$\frac{750 \times 80 + 60 \times 90 \times 45 + 9 \times 45 \times 60}{225000} = .517$$

$$\frac{750 \times 60 + 90 \times 60 \times 30 + 6 \times 30 \times 20}{225000} = .288$$

$$\frac{-120 \times 375 + 2280}{225000} = .072$$

$$\frac{-30 \times 375 + 450}{225000} = .048$$

$$\frac{50}{150} \times 15 \times 75 = R = 375$$

Prob 91 "Cont"

Triangular Variation of I $\frac{M}{EI}$ Loading

$$75 \times 300 - 300 \times 150 = 22500$$

$$\frac{90 \times 75 + \frac{90^2}{2}}{22500} = .48$$

$$\frac{60 \times 75 + \frac{60^2}{2}}{22500} = .28$$

$$\frac{30 \times 7.5 + \frac{30^2}{2}}{22500} = .12$$

$$\frac{75 \times 120 - \frac{120^2}{2}}{22500} = .08$$

$$\frac{75 \times 60 - \frac{60^2}{2}}{22500} = .12$$

$$\frac{75 \times 30 - \frac{30^2}{2}}{22500} = .08$$

$$\frac{150 \times 75}{150} = R = 75$$

Parabolic Variation of I

$$37.5(300) - 200 \times 150 = 18750$$

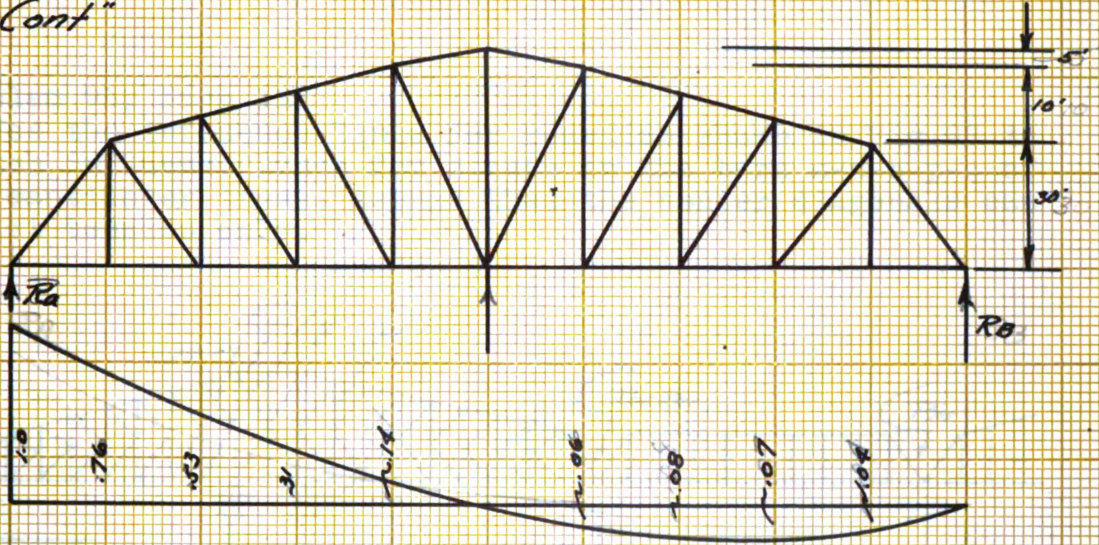
$$\frac{2}{3} \times 1 \times 150 = 100$$

$$\frac{62.5 \times 30 + 29.4 \times 15}{18750} = .123$$

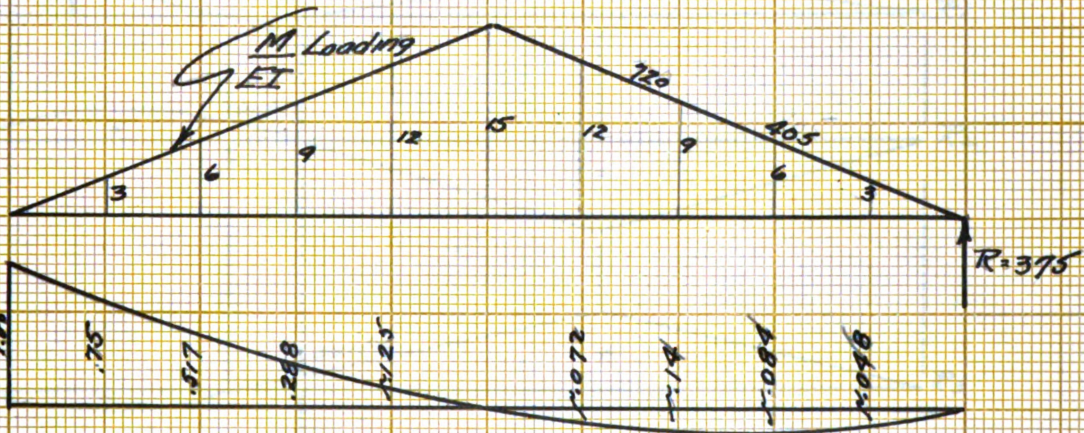
$$R = 100 \times \frac{3}{8} = 37.5$$

$$\frac{30 \times 37.5 - 7.8 \times 30 \times .35}{18750} = .0655$$

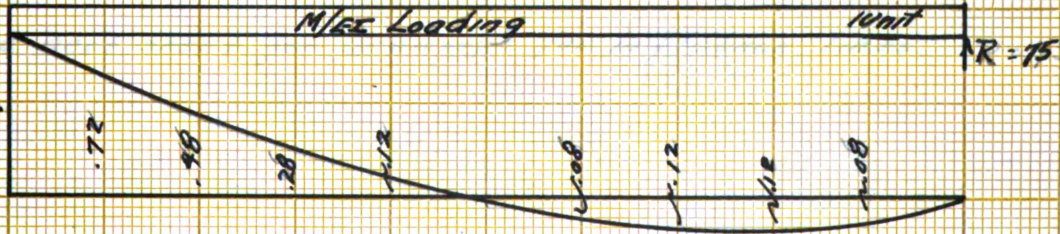
Prob 91 "Cont"



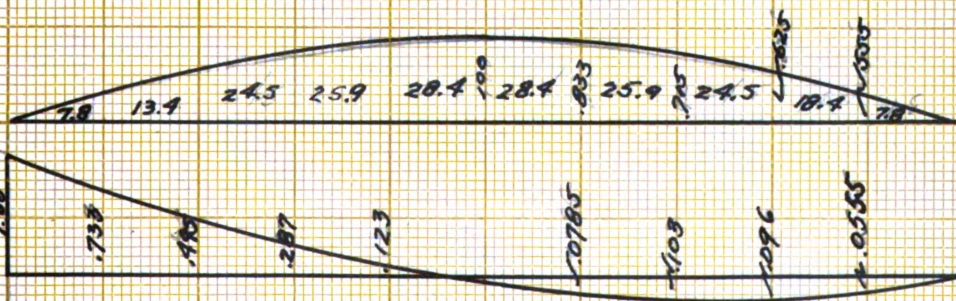
Uniform
Variation of I



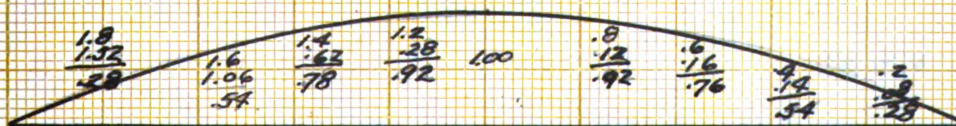
Triangular
Var. of I



Parabolic
Var. of I

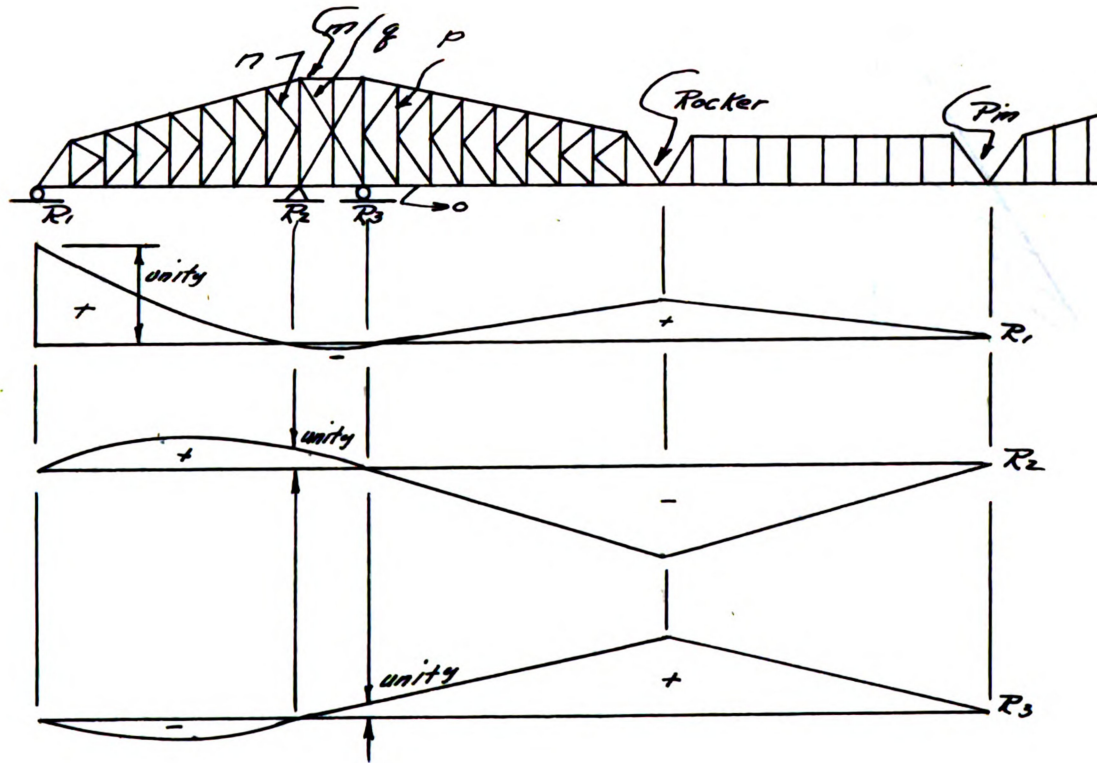


Infl. Line
For R_b



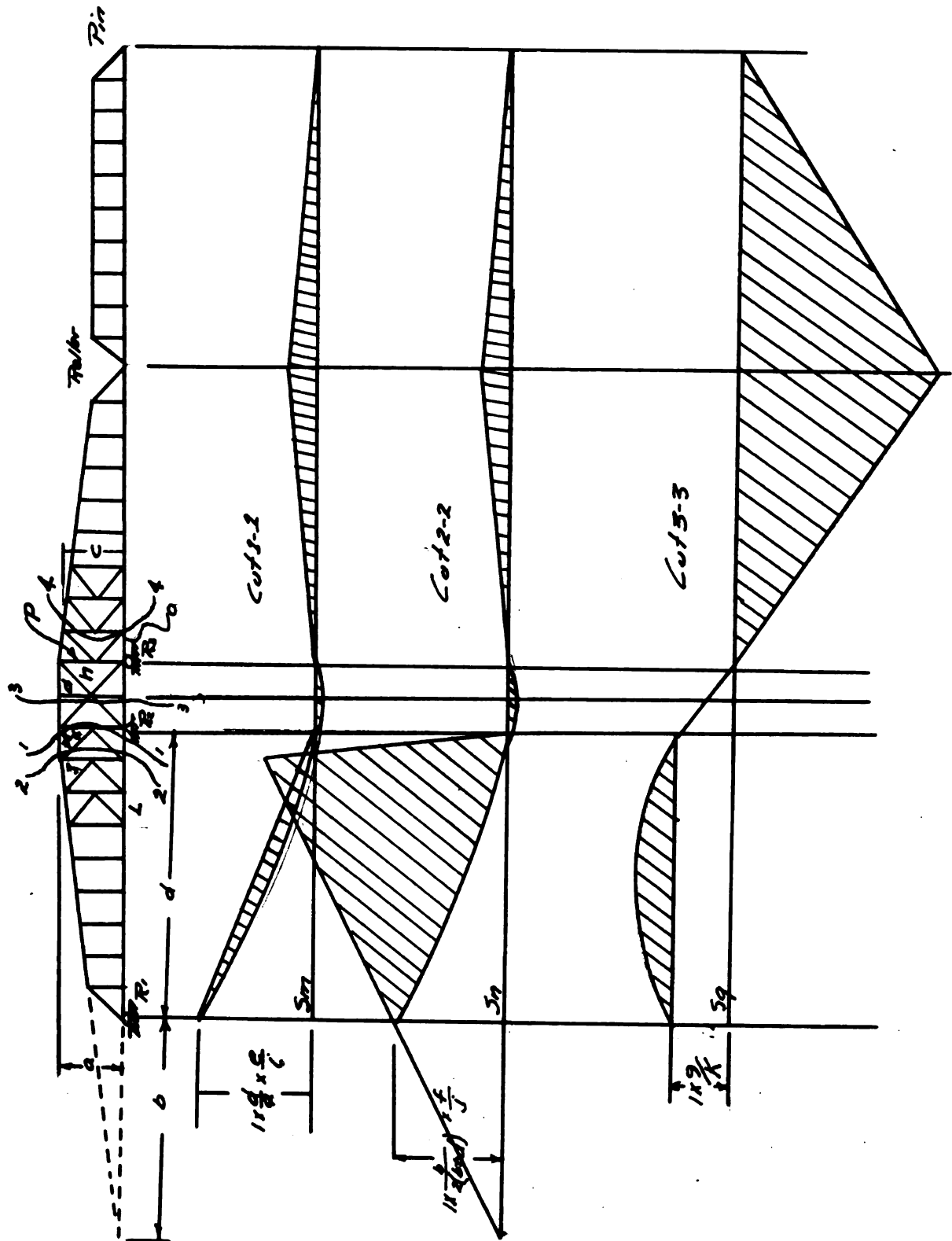
Problem 95, Page 137

Use the influence lines for reaction shown in Fig. 97 to sketch influence lines for stress in the bars m-n-o-p-q of the cantilever truss. Assume that the center diagonals can resist tension only.

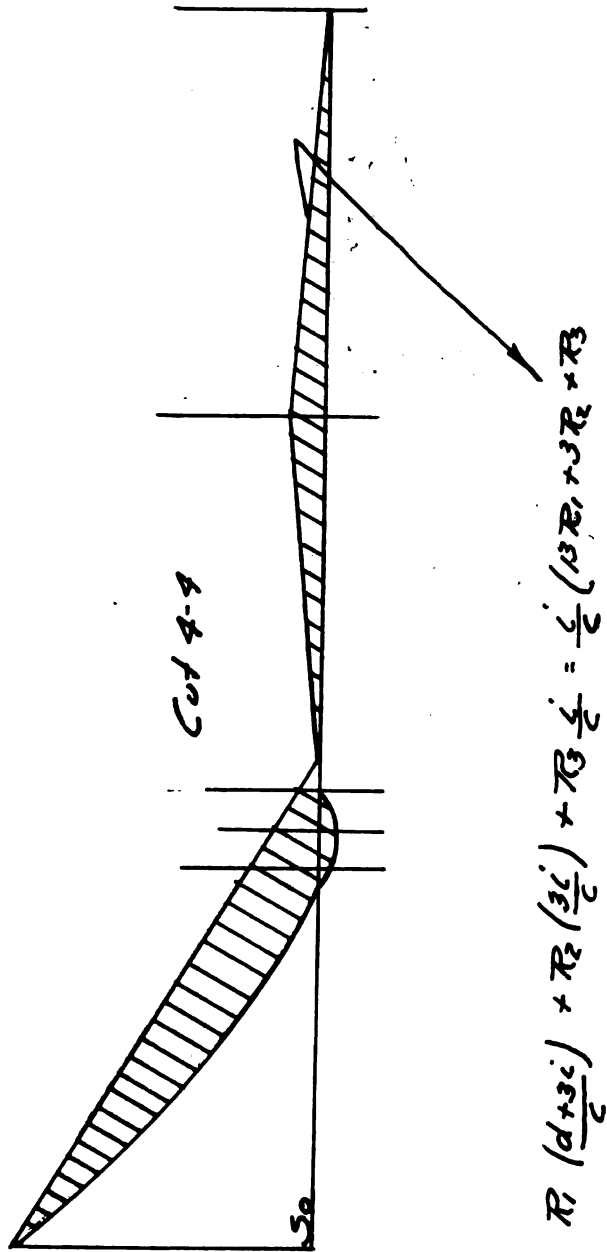


See next sheet for drawings for influence lines.

Prob 95 "Cont"

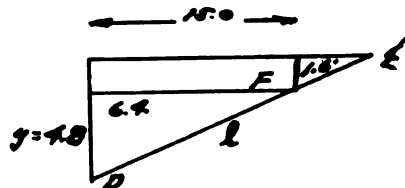
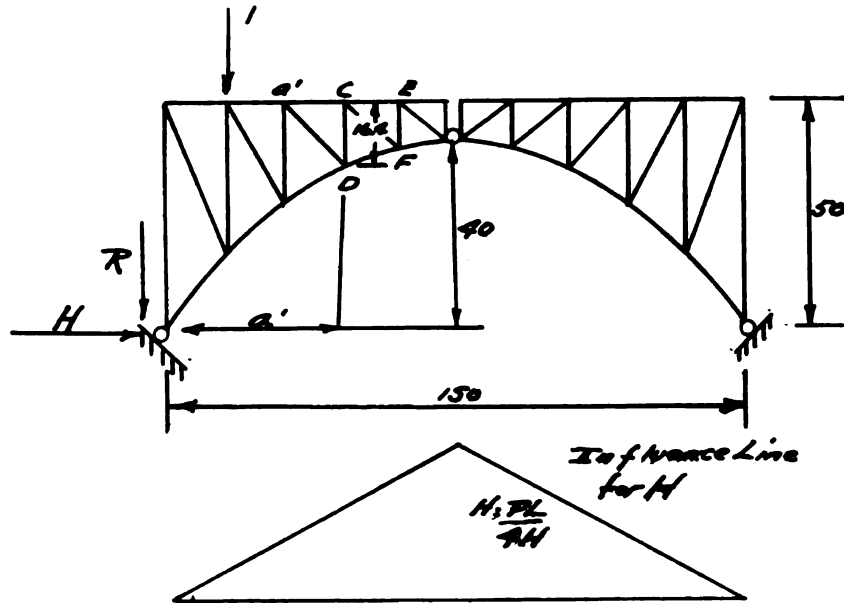


Prob 95 "Cont"



Problem 97, Page 137

The following dimensions apply to the three hinged parabolic arch of Fig. 99. $L=150$ feet; $h=40$ feet; $h_c=50$ feet obtain the maximum values of H - S_{DF} and S_{CF} for a uniform D.L. of 3000# per foot and a uniform load (L.L.) of 5000# per foot.



For a uniform load the AREA of the influence line gives the value of the function. $\therefore H_c = \frac{1 \times 150 \times 8000 \times 150}{4 \times 40} = 563200$

$$y = Kx^2 \quad 40 = Kx(75)^2 \quad K = .00712$$

$$\therefore y_F = .00712(15)^2 = 1.6' \quad y_D = .00712(30)^2 = 6.4'$$

$$C = \sqrt{15^2 + 4.8^2} \quad C = 15.75$$

$$\frac{y}{4.8} = \frac{x}{15} = \frac{C}{15.75} \quad C = \frac{x \cdot 15.75}{15} = 1.05x$$

Influence line for $DF = C = 1.05x$ Moment about C

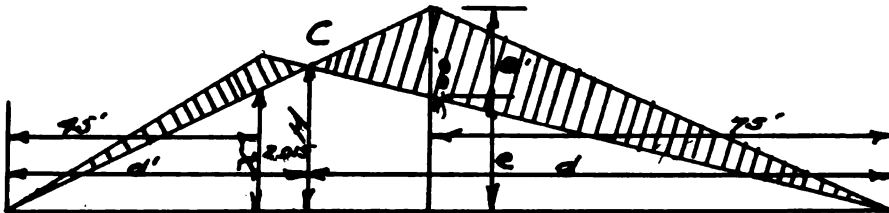
$$R \cdot a - a \cdot 1 - 0.8x = 0 \quad Rx45 - ax1 = 16.4x$$

$$\text{or } Rx45 - ax1 = \frac{Cx16}{1.05} = 15.6C$$

5.8.17

Prob 97 Cont

$C = \frac{R \times 45 - ax1}{15.6}$. Formula for Influence line DF. From this formula, we get the Δ with Max Ord = 2.015.



Subtracting the influence line for $H \times 4$, we get the influence line S_{DF} (cross hatched). S_{DF} for dead load = difference between Δ 's (Load all way across)

$$C, \left(\frac{2.015 \times 150}{2} - \frac{3 \times 150}{2} \right) 3000 = 220000^*$$

For max live load stress, we must find point C and put the load from there to right end. $\frac{2.015}{105} = \frac{H}{d}$ $\frac{3}{75} = \frac{H}{d}$
 $d + d' = 150$

$$(150 - d) \frac{2.015}{105} = \frac{3}{75} d'$$

$$2.88 - .0192d' = .04d'$$

$$d = 101.4$$

$$d' = \frac{2.88}{.0592} = 48.6$$

$$H = \frac{3 \times 48.6}{75} = 1.945$$

$$\frac{2.015}{105} \times 75 = C \quad C = 1.44$$

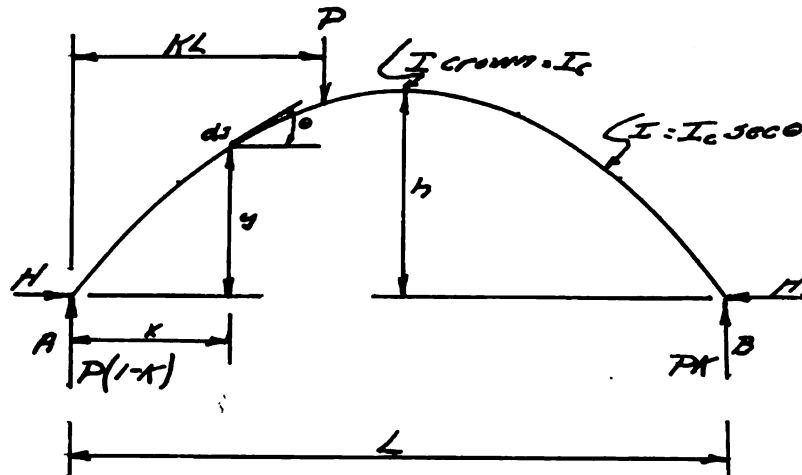
$$C' = 3.00 - 1.44 = 1.56$$

$$\text{Live Load} = \frac{1.56 \times 101.4 \times 5000}{2} = 396000^*$$

$$\text{Max } S_{DF} = 396000 + 220000 = 616000^*$$

Problem 98, Page 137

Derive equation (2) of Par. 90 by use of the general method of indeterminate structures.



Since the arch ring is a parabola, the moment when a unit horizontal load is applied is $xy = y$

$$\text{or } \delta^{H=1} = \int \frac{y^2 ds}{EI} = \int \frac{y^2 dx}{EI_c}$$

$$\text{But } \frac{ds}{I} = \frac{dx}{I_c} \quad \begin{cases} I = I_c \sec \theta \\ ds = dx \sec \theta \end{cases}$$

But this is $\frac{1}{2}$ the Statical Moment of the Parabola about the base: or $\delta^{H=1} = \frac{2}{EI_c} \left(\frac{2}{3} h \cdot h \right) \frac{3}{5} L = \frac{8}{15} \frac{Lh^2}{EI_c}$

$$\text{and } \therefore H^{\delta=1} = \frac{1}{\delta^{H=1}} = \frac{15}{8} \frac{EI_c}{Lh^2}$$

From figure

$$M_{0 \sim KL} = P(1-K)x + f(M)_{KL \sim L} = PK(L-x)$$

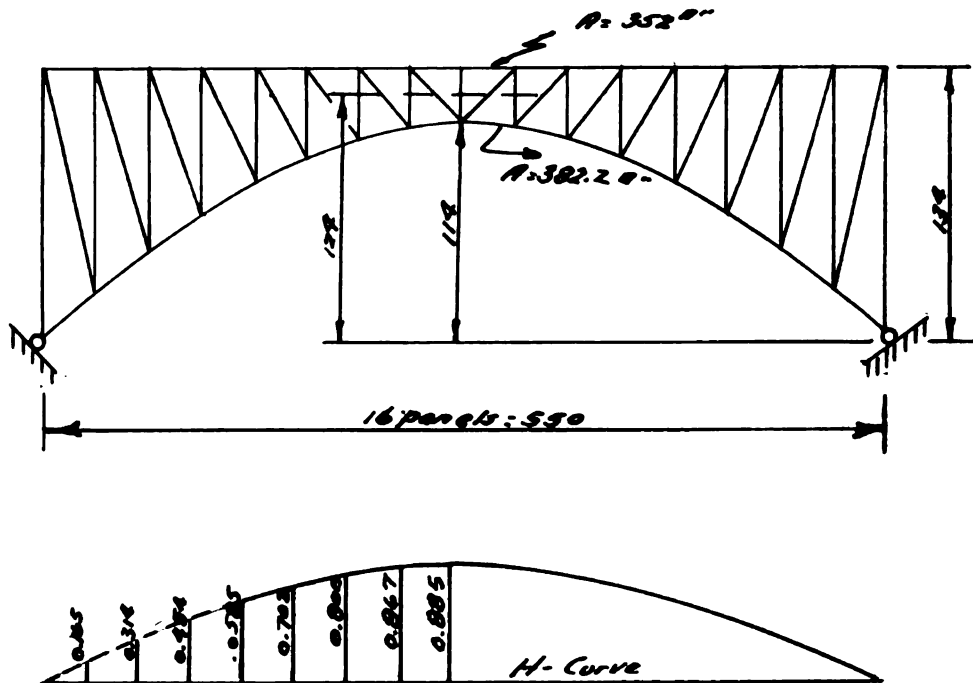
$$m = y = 4h \left(\frac{x}{L} - \frac{x^2}{L^2} \right)$$

$$\delta^P = \int_0^{KL} \frac{(P)(1-K)x \cdot 4h \left(\frac{x}{L} - \frac{x^2}{L^2} \right) dx}{EI_c} + \int_{KL}^L \frac{PK(L-x) \cdot 4h \left(\frac{x}{L} - \frac{x^2}{L^2} \right)}{EI_c}$$

$$= \frac{4Ph}{L^2 EI_c} \left[(1-K) \int_0^{KL} (Lx^2 - x^3) dx + K \int_{KL}^L (xL^2 - 2x^2L + x^3) dx \right] \quad \text{5.8.17}$$

Problem 99, Page 137

Compare the influence ordinates as given for the Niagara Gorge Arch with those given by equation (2) and with the semi-cubical parabola having a maximum ordinate $0.2L/h$. Compute the horizontal thrust produced by a 60° rise in temperature and the corresponding unit stresses in the members whose areas are given. Rise is 124 feet.



Influence ordinate at center N.G.A. = .841

Semi Cub. Parabola $A = .2 \frac{L}{h} = .2 \times \frac{550}{124} = .886$

Sec. Var. $A = .195 \frac{L}{h} = .195 \times \frac{550}{124} = .834$

Infl. Ord. at $\frac{1}{4}$ pt N.G.A. = .545

Semi Cub. Parabola $A = .129 \frac{L}{h} = .129 \times \frac{550}{124} = .55$

Sec. Var. $A = .139 \frac{L}{h} = .139 \times \frac{550}{124} = .594$

$$H_{\text{temp}} = \frac{15}{8} \frac{\delta t E I_c}{h^2} \quad \delta = .0000065 \quad t = 60^\circ$$

$$E = 30 \times 10^6 \quad h = 124$$

5.7.8, 16.17

Prob 99 "Cont"

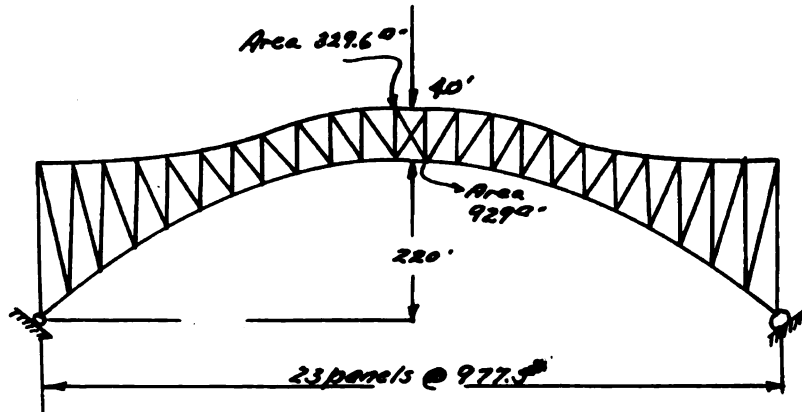
$$I_c = (352 + 382.2)(10^2) = 734.2 \times 100$$

$$H_T = \frac{15}{8} \times \frac{6.5 \times 10^{-6} \times 30 \times 10^6 \times 734.2 \times 10^2 \times 60}{124128}$$

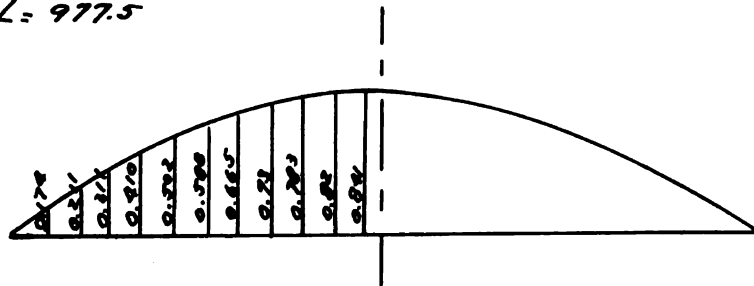
$$H_{Temp} = 104500^*$$

Problem 100, Page 137.

Same as Problem 99 except for the Hell Gate Arch as shown. Use a 72° rise in temperature and compute I_c about the center of gravity of the chord areas at the crown. Use the rise as 220 feet. For additional data on this arch refer to Transactions A.S.C.E. 1918, pp852-1137.



$$f = 220 \quad L = 977.5$$



Find I_c - Locate Neutral Axis

$$\sum M_{cg} = 0$$

$$\bar{y} = \frac{329.6 \times 40}{329.6 + 929} \quad \bar{y} = 10.5$$

$$I_c = 929(10.5)^2 + 329.6 \times 29.5^2 = 102300 + 287000 = 389300$$

$$H = \frac{15}{8} \left(\frac{\delta t E I_c L_T}{f^2 L} \right) \quad L_T = L$$

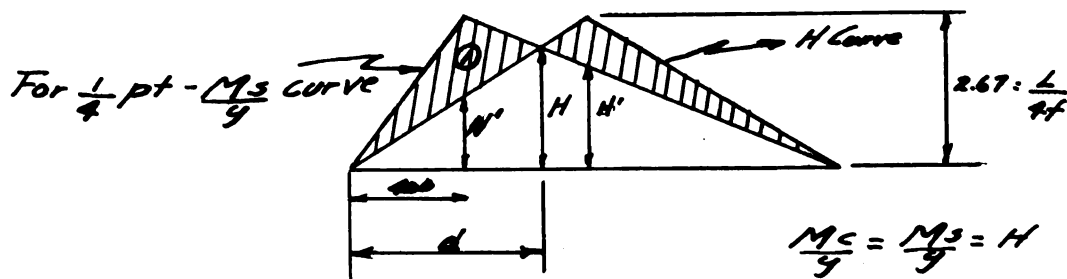
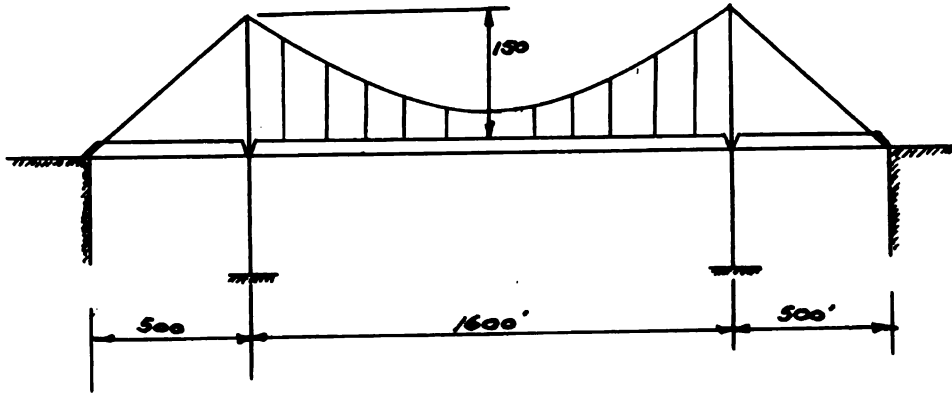
$$= \frac{15}{8} \left(\frac{6.5 \times 10^{-6} \times 72 \times 30 \times 10^6 \times 389300}{220 \times 220} \right) = 211800 \text{ lbs}$$

Note - in this equation if δ = ratio $t = ^\circ F$ $E = \text{psi}$

I in ft^2 & f is in feet, then H will be in pounds.

Problem 101, Page 137.

Assume that the suspension bridge shown for Problem 103 has a pin at the midpoint of the stiffening truss. Compute and plot the maximum moments in the stiffening truss for each 1/8th point of the span as produced by a uniform live load of 3000 pounds per foot, per truss.



$$\frac{2.67}{800} = \frac{H}{d}, \quad \frac{2.67}{1200} = \frac{H}{d-1600}$$

$$\frac{2.67d}{800} = \frac{2.67}{1200} (d-1600)$$

$$d = \frac{2}{3}(1600-d)$$

$$d + \frac{2}{3}d = \frac{2}{3} \times 1600 \quad d = 641'$$

$$1600 - d = 959$$

$$H = \frac{2.67}{800} \times 641 = 2.14$$

$$\frac{2.14}{641} = \frac{H'}{400} \quad H' = 1.335$$

$$2.67 - H' = 1.335$$

Δ_1 has the largest area $\therefore$ put live load from left end to there, then

$$S = 1.335 \times 641 \times 3000 = 1,285,000^*$$

$$y = Kx^2, \quad 150 = K(800)^2 \quad K = 2.343 \times 10^{-4}$$

$$y' = 150 - 2.343 \times 10^{-4} x^2 = 150 - 2.343 \times 10^{-4} \times 400^2$$

$$y = 112.5$$

$$5, 7, 9, 12$$

Prob 101 "Cont"

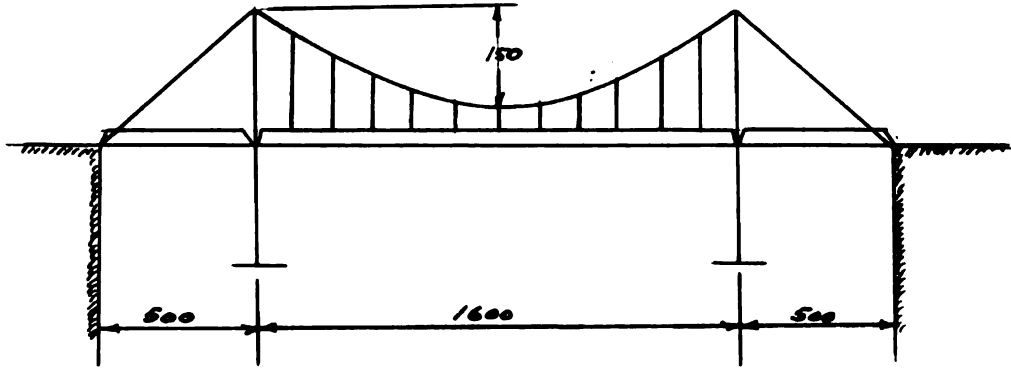
$$M_c = 1.285,000 \times 112.5$$

$$= 144,700.000 \text{ '}\#3$$

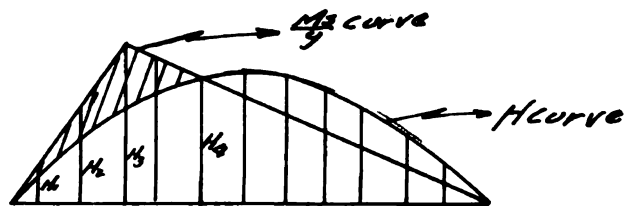
*Similarly the ordinates for the other points
can be computed and the curve sketched.*

Problem 102, Page 137.

Same as Problem 101 except to take the stiffening truss without a center hinge and of uniform moment of inertia. Let equation (9) page 129 represent the influence line for H.



Given equation for $H = .6 \frac{L}{f} (K - 2K^3 + K^4)$ where $K = \frac{x}{L}$



$$H_1 = .6 \left(\frac{1600}{150} \right) \left(\frac{1}{8} - 2 \times \frac{1}{8}^3 + \frac{1}{8}^4 \right) = .800$$

$$H_2 = .6 \left(\frac{1600}{150} \right) \left(\frac{1}{4} - 2 \times \frac{1}{4}^3 + \frac{1}{4}^4 \right) = 1.425$$

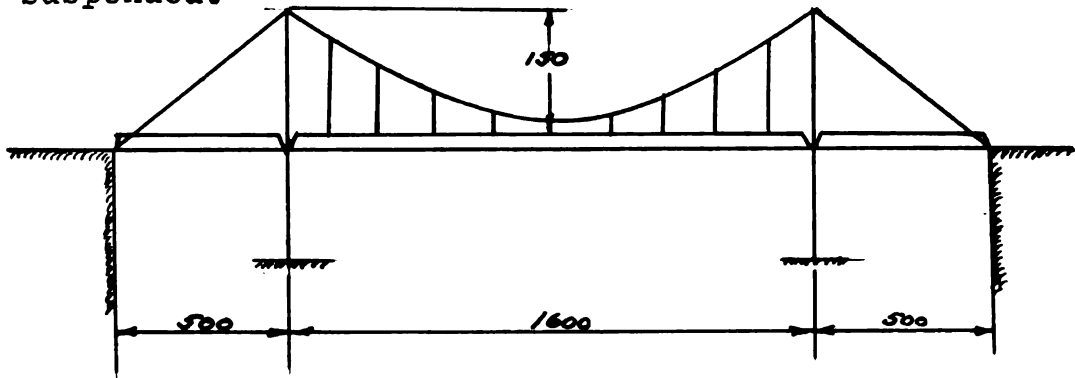
$$H_3 = .6 \left(\frac{1600}{150} \right) \left(.375 - 2 \times .375^3 + .375^4 \right) = 1.86$$

$$H_4 = .6 \left(\frac{1600}{150} \right) \left(\frac{1}{2} - 2 \times \frac{1}{2}^3 + \frac{1}{2}^4 \right) = 2.00$$

The procedure from here on is same as in Prob 101, except in this case, the H curve is not a standard Geometric figure and we find the area by a summation. We will place the uniform load up to point C and find area ① x w. which will give the influence ordinate.

Problem 104, Page 137.

Obtain the value of the horizontal cable pull caused by a temperature drop of 60°F . for the suspension bridge of Problem 103. Use equation (10) page 129, based upon an average value of I of $3,000,000$ inches⁴. Express the results in terms of stress in the chords of the stiffening truss if these chords have equal areas of 67 square inches, at the center spaced 25 feet apart. What would be the approximate percentage reduction of the cable pull if the side spans were suspended.



$$H = \frac{15}{8} \left(\frac{6.5 \times 10^{-6} \times 60 \times 36 \times 10^6 \times 3 \times 10^6 \times 2600}{150 \times 150 \times 1600 \times 144} \right)$$

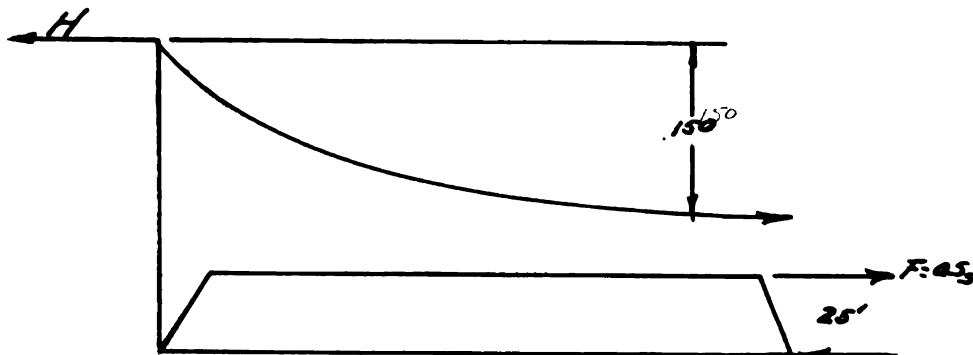
$H = 3.30 \times 10^4$ Value of cable pull caused by Temp. drop of 60°

BM caused by Temp. stress is $H \times y = 3.30 \times 10^4 \times 150 = 4.96 \times 10^6$

This BM will be resisted by the stiffening truss

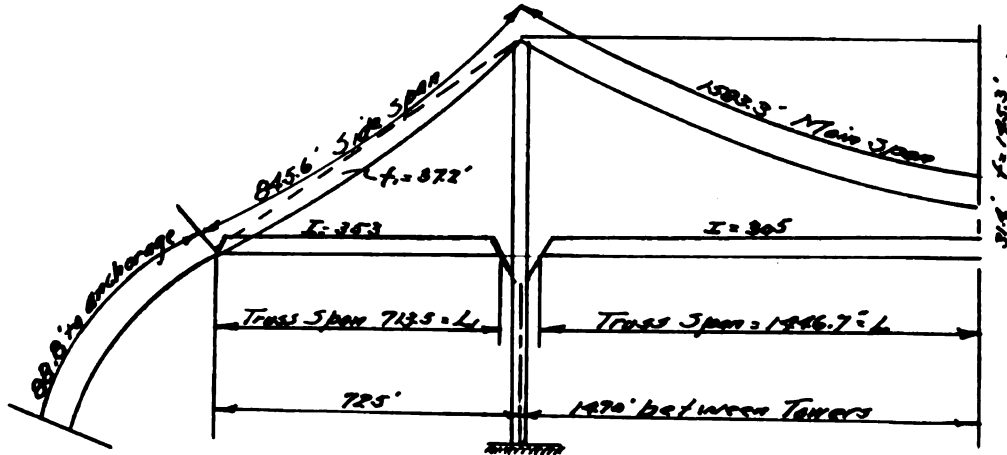
$$M_T = 25 \times 6.7 \times S_s = 4.96 \times 10^6$$

$$S_s = \frac{4.96 \times 10^6}{25 \times 6.7} = 2960 \text{ in}^2$$



Problem 105, Page 137.

The exact value of H for the Manhattan Suspension Bridge is 6,600,000 pounds, when the Live Load is placed on the center span. It is 347,000 pounds when the Live Load is placed on the side spans. Compute the maximum positive and negative deflections of the center span as suggested in Paragraph 99.



$$L = 1470 \quad E = 30 \times 10^6 \text{ psi} \quad I = 305$$

$$W = 4000 \quad H = 6.6 \times 10^6 \quad f = 145.3$$

$$\Delta f = \frac{5}{384} \frac{L^4}{EI} \left(W - \frac{8Hf}{L^2} \right)$$

$$= \frac{5}{384} \times \frac{1470^4}{144 \times 30 \times 10^6 \times 305} \left(4000 - \frac{8 \times 6.6 \times 10^6 \times 145.3}{1470^2} \right)$$

$$= .046 \times 460 = 21.2 \text{ " positive deflection}$$

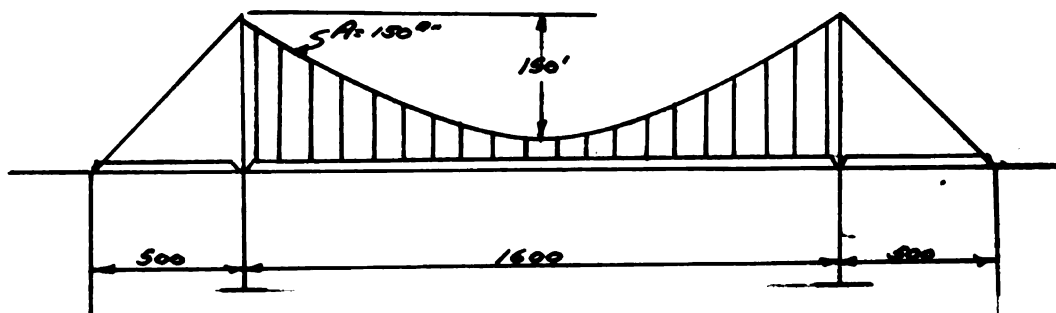
$$\Delta f' = \frac{5}{384} \frac{L^4}{EI} \left(\frac{8Hf}{L^2} \right)$$

$$= \frac{5}{384} \times \frac{1470^4}{144 \times 30 \times 10^6 \times 305} \left(\frac{8 \times 347000 \times 145.3}{1470 \times 1470} \right)$$

$$= .046 \times 187.5 = 8.6 \text{ " negative deflection.}$$

Problem 106, Page 138.

Assume that the suspension bridge shown in Problem 103 has a stiffening truss of constant section where I is 3,500,000 inches⁴. Each cable has an area of 150 square inch, and carries a Live Load of 3000 pounds per foot. Compute the approximate maximum deflections with and without suspended side spans. The cable extends 100 feet into the abutment to the point of anchorage. The sag in the suspended side span is found from the relation given in Paragraph 96.



$$I = 3,500,000 \text{ in}^4$$

$$\text{Extra length of cable} = 100'$$

$$L_T = 2800$$

$$W = 3000 \text{ #/ft}$$

$$\text{Cable } \Delta f_1 = \frac{3}{128} \frac{WL^3 L_T}{AE f^2} = \frac{3}{128} \frac{(3000)(1600)^3 (2800)}{(150)(30 \times 10^6)(150^3)} = 8.0'$$

$$\Delta f_2 \text{ suspended sides} = \frac{1}{40} \frac{WL^3 L_1 f_1^2}{EI_1 f^2}$$

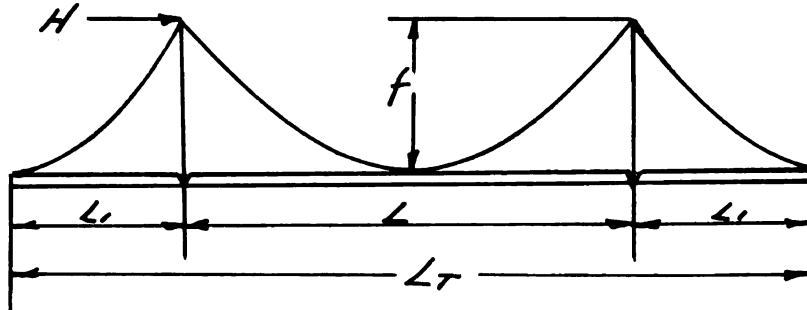
$$\begin{aligned} f_1 &= f \left(\frac{L_1}{L} \right)^2 \\ &= 150 \left(\frac{500}{1600} \right)^2 \\ &= 14.65' \end{aligned}$$

$$\Delta f_2 = \frac{1}{40} \frac{(3000)(1600^3)(500)(14.65^2)}{30 \times 10^6 \times 3,500,000 \times 150^2} = 2.0'$$

$$\Delta f_{\text{Total}} = 8.0 + 2.0 = 10.0'$$

Problem 107, Page 138.

Develop a general expression for temperature deflection of a two hinged suspension bridge with straight back-stays and use it to find deflection of the Manhattan Bridge when $t=60^{\circ}\text{F}$. The exposed length of the cable is 3320 feet.



L_c = Length of cable

We know that the change in horizontal length of the cable is $e f L_c$. Also that the horizontal pull on the cable due to a 1* load at center is $\frac{3}{16} \frac{L}{f}$

Then from the equation of virtual work we have

$$1 \cdot \Delta f = U \delta$$

$$\text{or } \Delta f^{\text{Temp}} = \frac{3}{16} \frac{L}{f} (e t L_c)$$

$$\text{Given } L = 1470' \quad e = .0000065 \quad L_c = 3320'$$

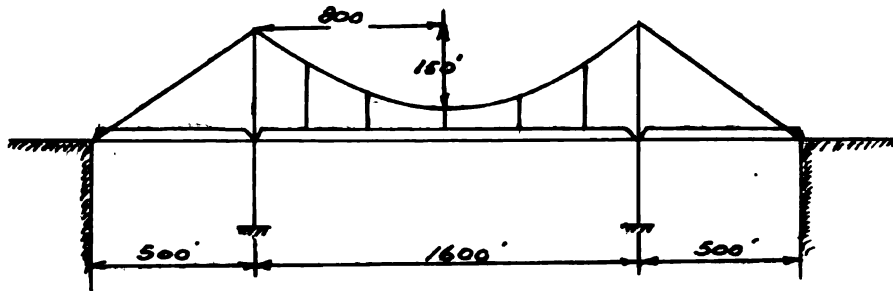
$$f = 145.3' \quad t = 60^{\circ}\text{F}$$

$$\text{then } \Delta f^{\text{Temp}} = \frac{3}{16} \cdot \frac{1470}{145.3} (.0000065)(60)(3320)$$

$$= 2.45'$$

Problem 108, Page 158.

Compute the approximate temperature deflection for the bridge of Problem 103, when $t: 65^{\circ}\text{F}$.

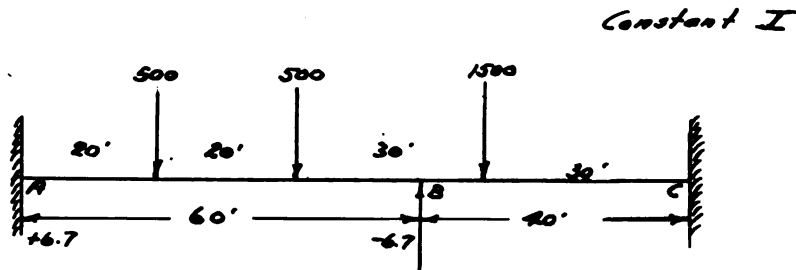


$$\Delta f^{Temp} = \frac{3}{16} \frac{L}{f} e t L_t$$

$$\Delta f^{Temp} = \frac{3}{16} \cdot \frac{1600}{150} (.0000065)(65)(2600) = 2.2'$$

Problem 110, Page 147.

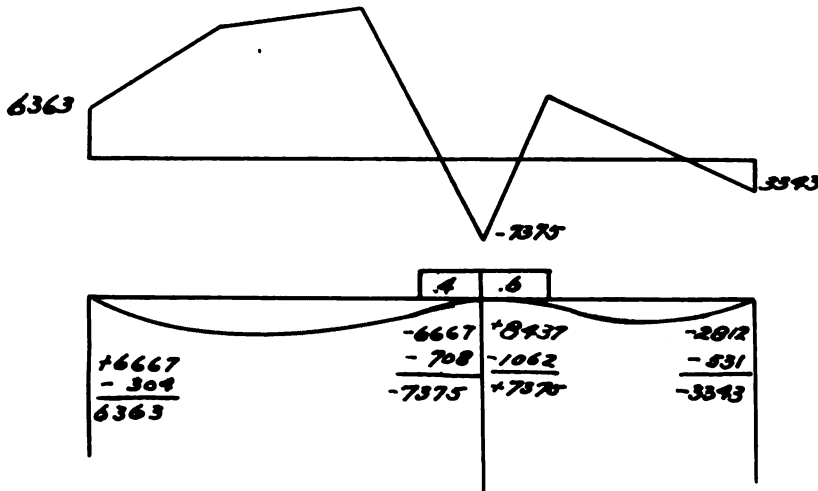
Analyze the continuous beam shown and draw the final moment diagrams. Calculate the moments under each concentrated load and determine the values of the maximum moments. Locate points of contraflexure and sketch the elastic curve for the beam.



$$K_1 = \frac{I}{L} = \frac{I}{60} \quad \frac{I_1}{60} + \frac{I_2}{40} \quad \frac{I_1}{60} / \frac{40I_1 + 60I_2}{2400}$$

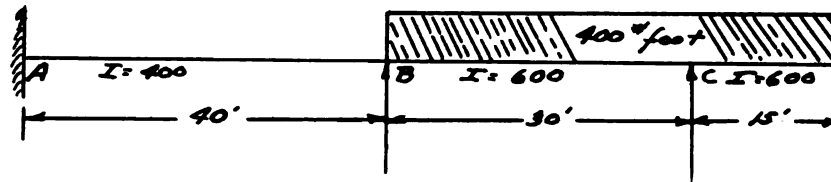
$$K_2 = \frac{I}{40}$$

$$\frac{I_1 \times 2400}{60(40I_1 + 60I_2)} \cdot \frac{4I_1}{4I_1 + 6I_2} = \frac{4}{10} = \frac{2}{5}$$



Problem 111, Page 147.

Analyze the continuous beam shown and draw the final moment diagrams. Calculate the moments under each concentrated load and determine the values of the maximum moments. Locate the points of contraflexure and sketch the elastic curve for the beam.

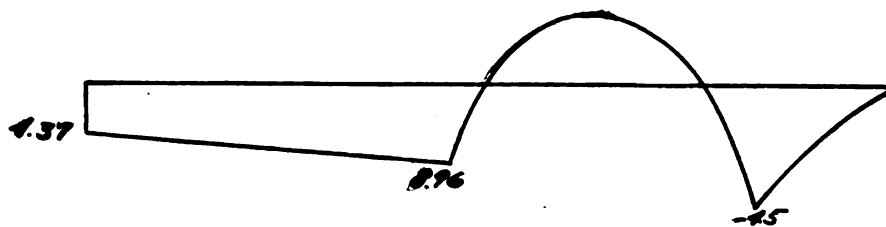


$$K_1 = \frac{I}{L} = 10$$

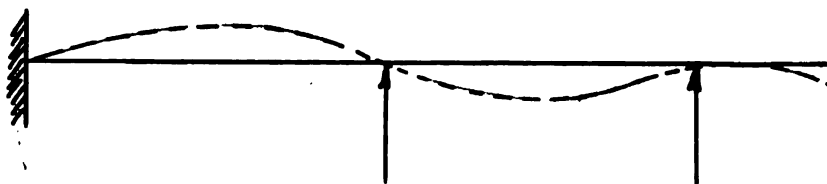
$$K_2 = \frac{600}{30} = 20$$

$$K_3 = \frac{600}{15} = 40$$

Moment Curve



A		B		C	
		33	67	100	0
		0	130	-30	+4.5
-3.75		-7.5	-15	-15	
		-7.5	-15	-7.5	
-6.2			13.75	+7.5	
		-1.25	-2.50	-1.25	
-9.37			-6.2	+1.25	
		-2.11	-4.1		
-9.37		-8.96	+8.96	-4.5	

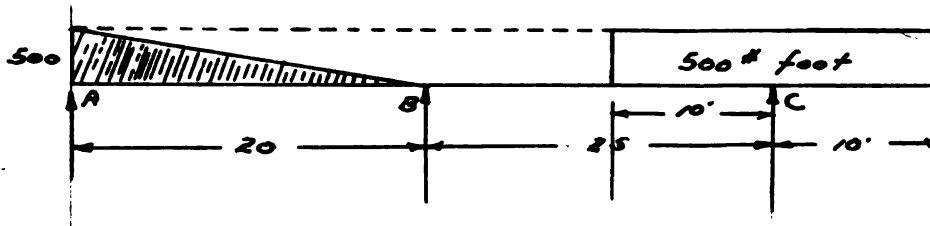


Elastic Curve

1, 2, 6B, 15, 16, 18

Problem 112, Page 147.

Analyze the continuous beam shown and draw the final moment diagrams. Calculate the moments under each concentrated load and determine the values of the maximum moments. Locate the points of contraflexure and sketch the elastic curve for the beam.



$$C_{AB} = \frac{WL^2}{20} = \frac{500 \times 20 \times 20}{20} = 10000$$

$$C_{BC} = \frac{500 \times 1000}{12 \times 25 \times 25} (4 \times 25 - 3 \times 10)$$

$$C_{BA} = \frac{WL^2}{20} = \frac{500 \times 20 \times 20}{30} = 6700$$

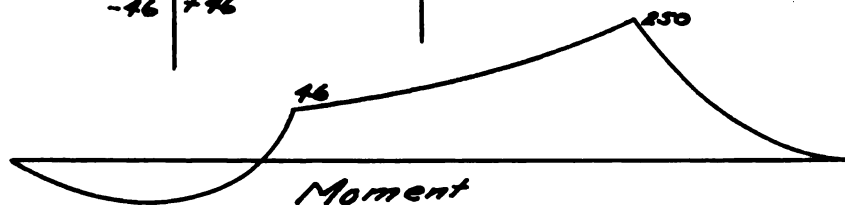
$$C_{BC} = 4600$$

$$C_{CB} = 500 \times 10 \times 5 = 25000$$

$$C_{CB} = \frac{500 \times 10 \times 10}{12 \times 25 \times 25} (3 \times 100 - 8 \times 10 \times 25 + 6 \times 25 \times 25) = 13600$$

Note - the moment at the overhanging end is figured in the usual manner, but since there is no restraint, the balanced moment is distributed 100% to the other side.

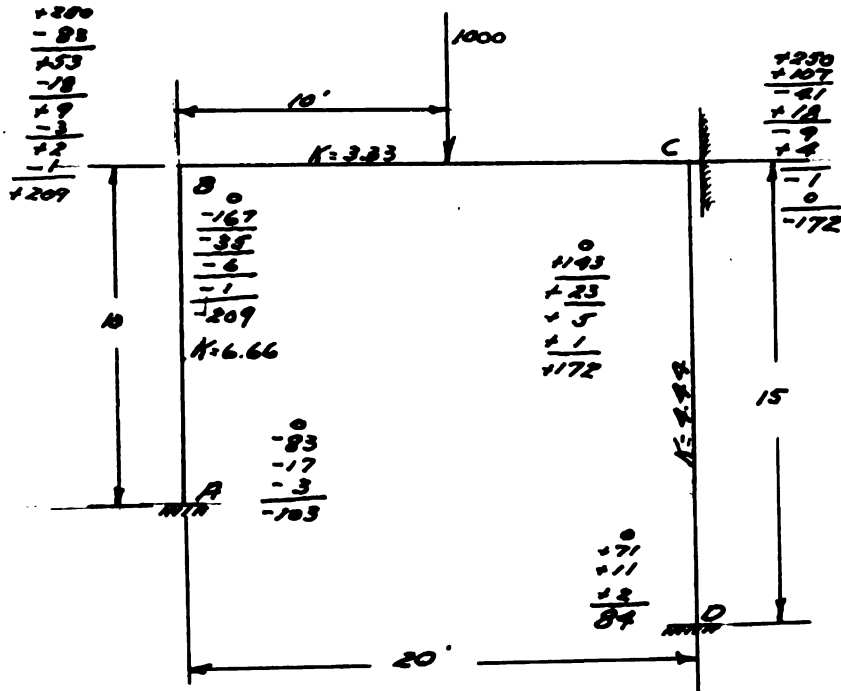
	5/9	4/9	100	0
1/100	-67	+96	-136	1200
-100	-50	-57	-119	
+36	+71	+57	28	
-36	-18	-18	-28	
0	+18	+18	-250	+250
	-46	+96		



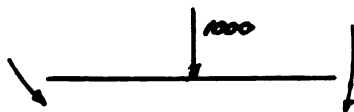
1.2.6.8, 15, 16, 18

Problem 113, Page 149.

Analyze for the joint moments and draw moment diagrams for the frame shown. There are no joint movements. Locate points of contraflexure and sketch the deflected structure.



$$F.E.M. \frac{PL}{8} = \frac{1000 \times 10}{8} = 1250$$

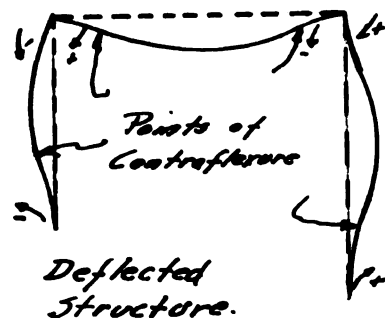
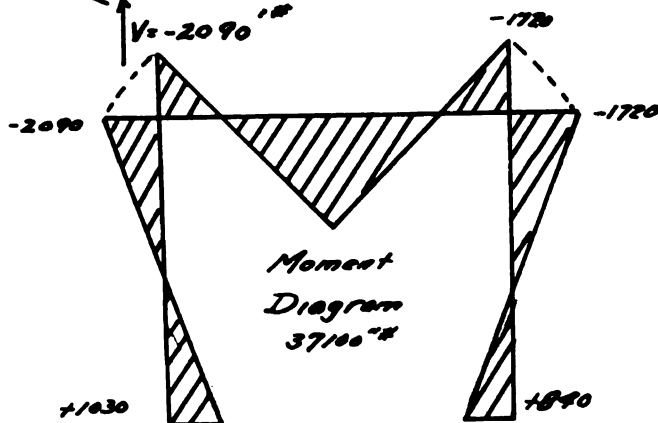


$$H = \frac{2090 + 1030}{10} = 312$$

$$V = \frac{1000 \times 10}{20} + \frac{312 \times 10}{20} - \frac{1780 + 1030}{20} = 518.5$$

$$\text{Moment of Load} = 1030 + 518.5 - 3120 = 3095$$

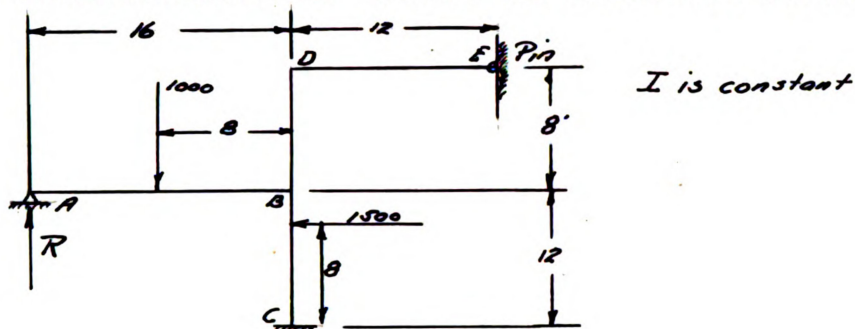
$$= 37180$$



7, 8, 10, 11, 14, 15, 16, 20.

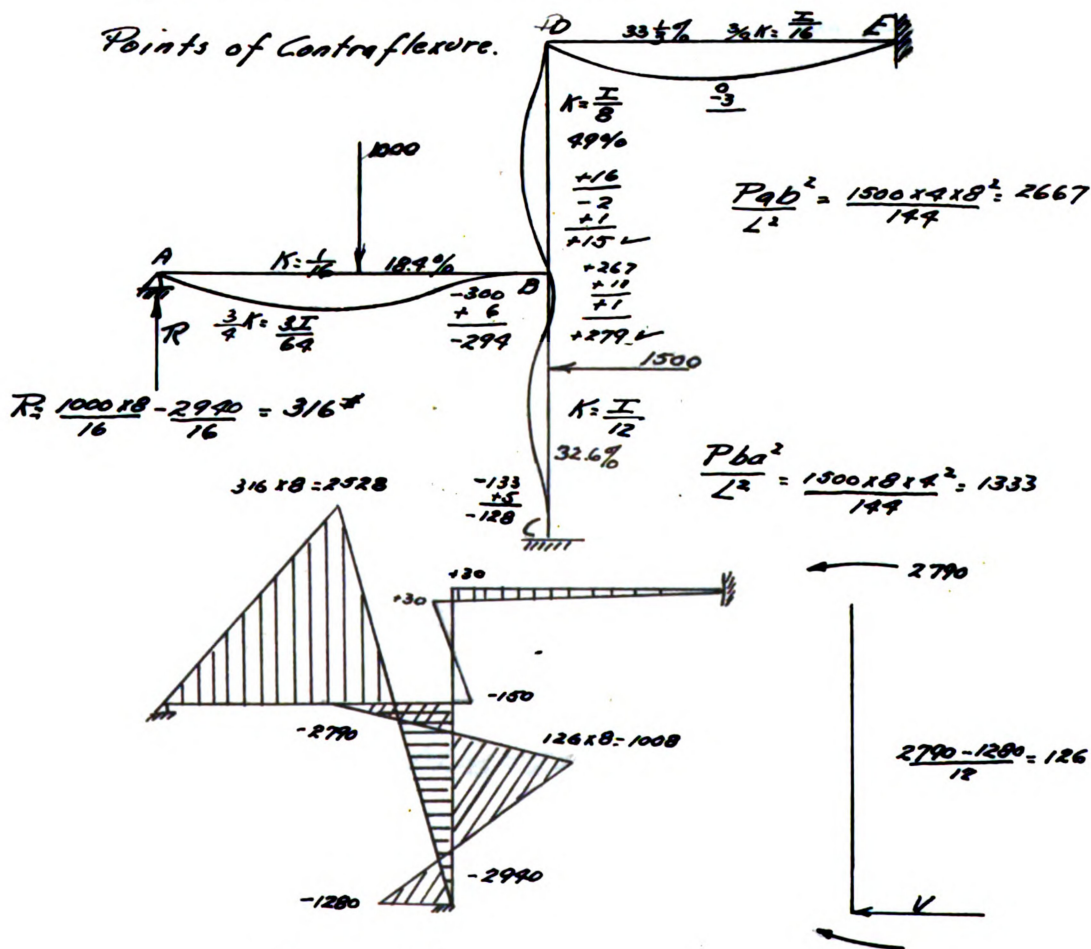
Problem 114, Page 149.

Analyze for the joint moments and draw moment diagrams for the frame shown. There are no joint movements. Locate points of contraflexure and sketch the deflected structure.



Deflected Structure showing

Points of Contraflexure.

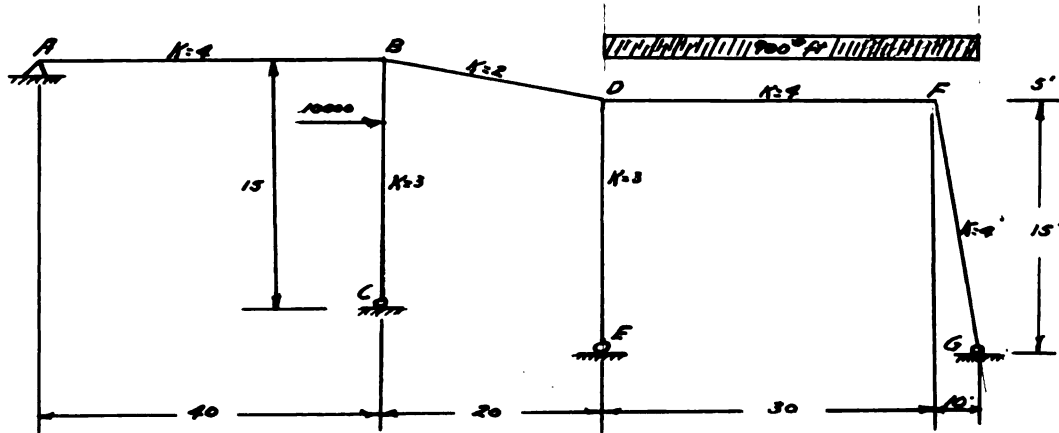


Moment Diagram.

7.8, 10, 11, 14, 15, 16, 21

Problem 116, Page 149.

Analyze for the joint moments and draw the moment diagram for the frame shown. There are no joint movements. Locate points of contraflexure and sketch the deflected structure.



We will use the simplified treatment of pin ends. This consists of reducing the K values for pin ended members by 25% or multiply by $\frac{3}{4}$ and increasing the fixed end moments of the opposite end by $\frac{1}{2}$ of the fixed end moment of the pin end ($FE = MFE + \frac{1}{2} MP$)

$$K_{AB} = \frac{3}{4} \times 4 = 3 = K_{FG}$$

$$K_{BC} = \frac{3}{4} \times 3 = \frac{9}{4}$$

$$K_{DE} = \frac{3}{4} \times 3 = \frac{9}{4}$$

$$M_{BC} = \frac{10 \times 8 \times 49}{225} + \frac{1}{2} \left(\frac{10 \times 7 \times 64}{225} \right) = 27.4$$

$$M_{DE} = M_{FD} = \frac{.9 \times 900}{12} = 67.5$$

$$M_{FG} = 7.5 + \frac{27.5}{2} = 10.9$$

Simple Mom in DF

Simple Mom. in FG

$$\frac{.9 \times 900}{8} = 101.5$$

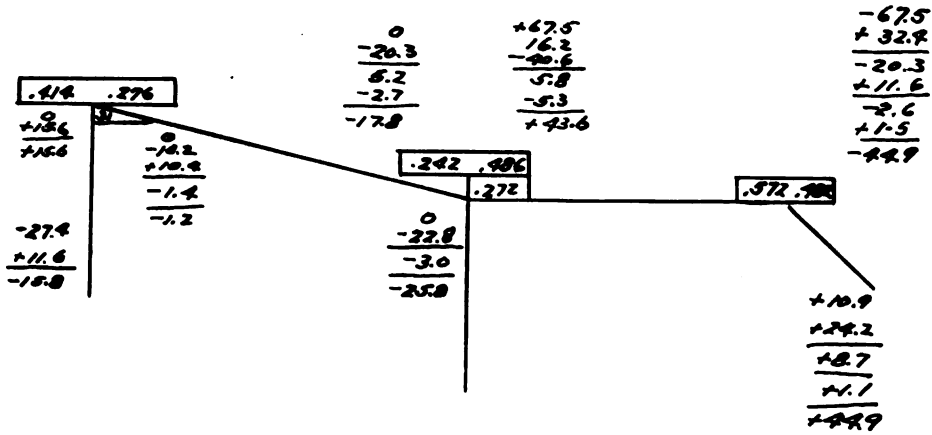
$$\frac{.9 \times 100}{8} = 11.2$$

$$\text{Max Mom} = 101.3 - \frac{(43.6 + 44.9)}{2}$$

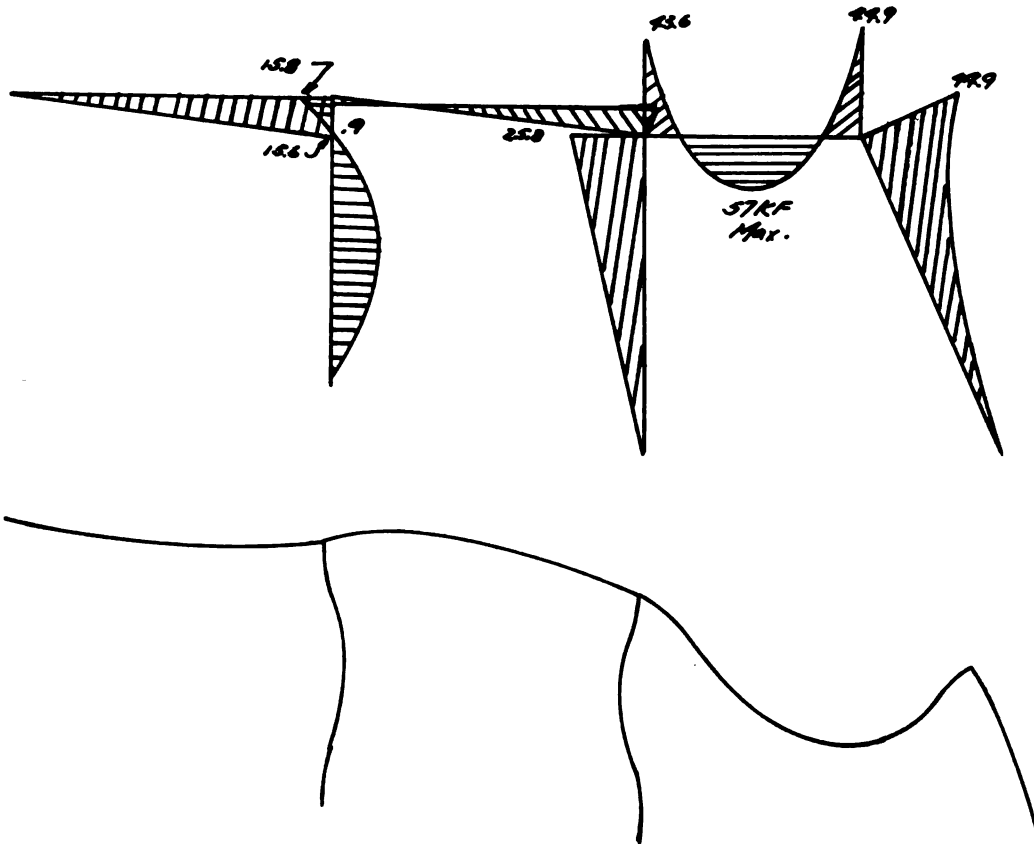
$$= 101.3 - 44.3 = 57 \text{ Kip ft.}$$

7.8, 10, 11, 14, 15, 20

Prob 116 "Cont"

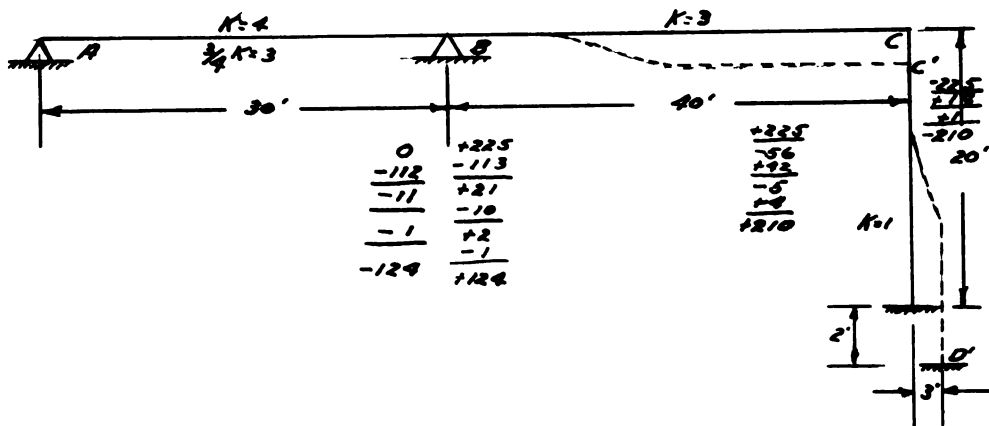


Moment Diagram & Deflected Structure



Problem 117, Page 152.

Assume that a landslide moves the joint D in Figure 122 a distance of 2 inches down and 3 inches to the right. Compute the maximum fiber stress in girder and column.



$$M = \frac{6EA\Delta}{L} = \frac{6(30,000,000)3 \times 2}{40 \times 12} = 2,250,000 \text{ in}^2$$

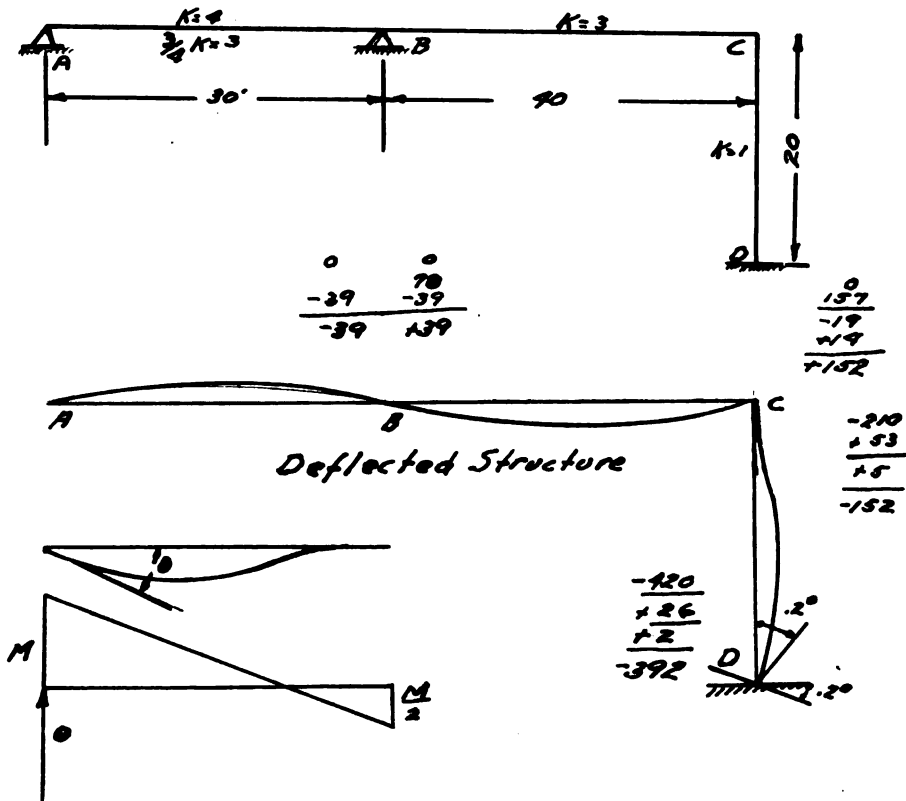
$$M = \frac{6(30,000,000)1 \times 3}{20 \times 12} = 2,225,000 \text{ in}^2$$

$$\text{Max. fiber stress in Girder} = \frac{2,100,000 \times 10}{1490} = 14,600 \text{ psi}$$

$$\text{Column} = \frac{2,100,000 \times 6}{240} = 54,500 \text{ psi}$$

Problem 118, Page 152

Determine the maximum fiber stresses in the frame of Fig. 122 caused by a clockwise rotation of 0.2 of the footing at D. Sketch the shape of the deflected structure.



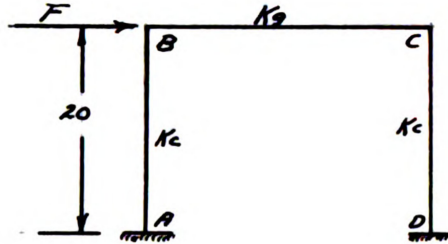
$$\theta = \frac{ML}{2EI} - \frac{ML}{4EI} = \frac{ML}{4EI}$$

$$M = \frac{4EI\theta}{L} = 4EK\theta = 4 \times 30 \times 10^6 \times 1 \times \frac{2\pi}{180} = 420000$$

$$\text{Max. Fiber Stress in Column} = \frac{392000 \times 6}{240} = 9800 \text{ psi}$$

Problem 119, Page 157

Locate the points of contraflexure in the columns of this simple bent for 3 cases of relative stiffness, (a) $K_g = 2K_c$, (b) $K_g = K_c$, (c) $K_g = K_c/2$. Sketch the general shape of the deflected structure.



If we let the structure have

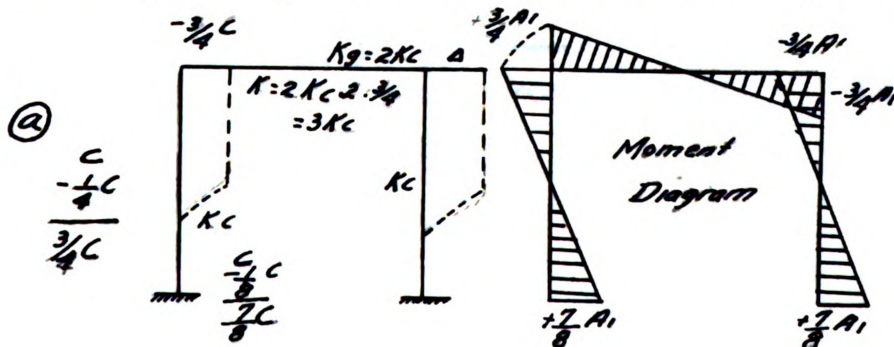
a side sway of Δ , then

$$\text{the F.E.M.} = M_{FE} = \frac{6EK\Delta}{L}$$

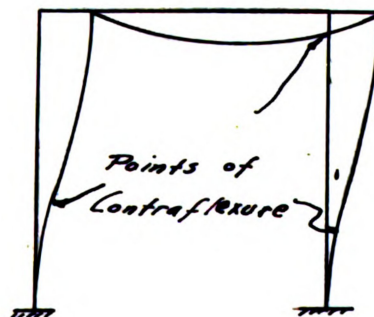
$$\text{or if we let } C = \frac{6EA}{L}$$

$M_{FE} = C$. If we consider

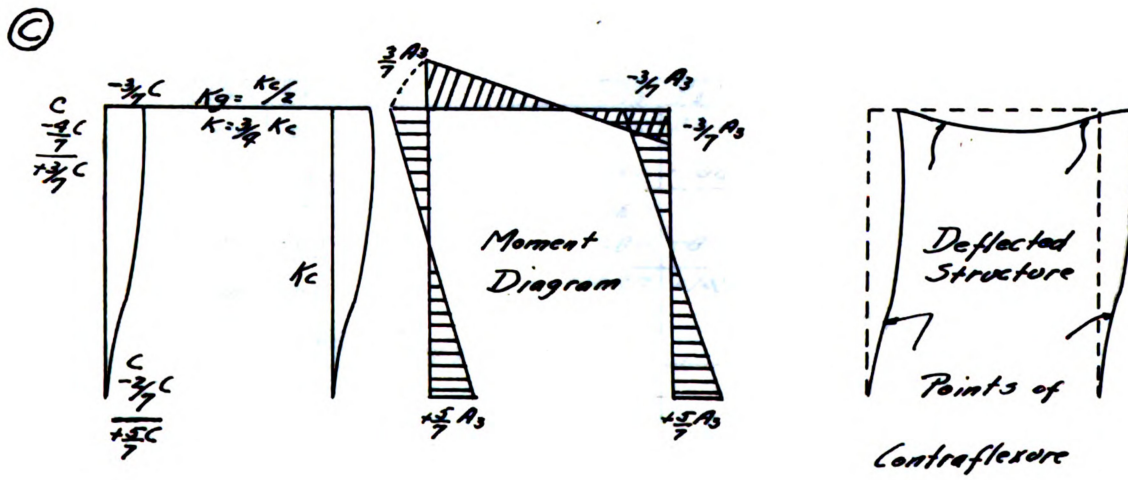
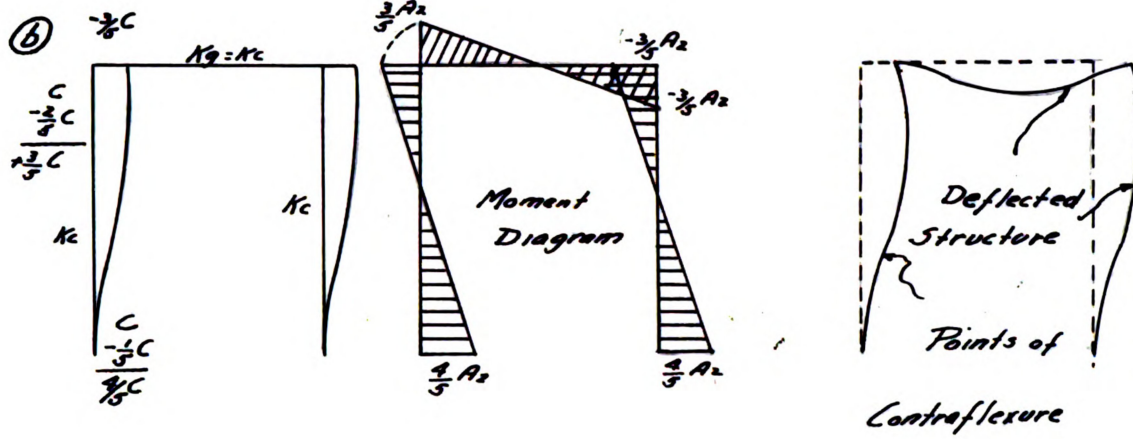
half the upper girder we must multiply the K value by 2, and since there is a pin at the center, we must also multiply by $3/4$. Since the results given need only be multiplied by a constant, we may call all the constants together A, A_2, A_3 etc.



Deflected Structure

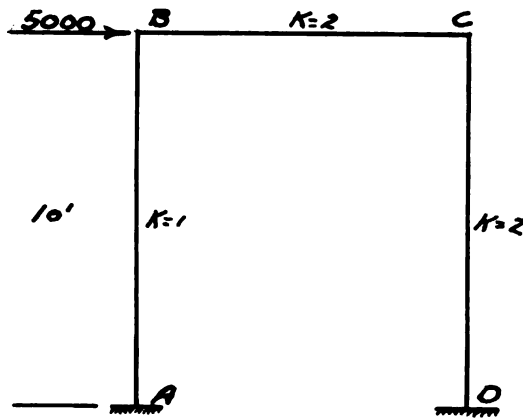


Prob 119 "Cont"

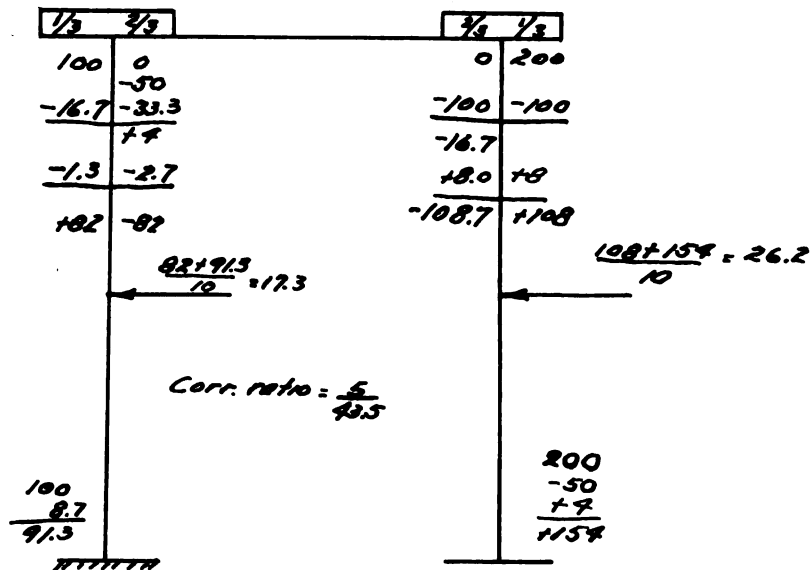


Problem 120, Page 157

Check the following values for the moments in the bent shown.



I have chosen the fixed ends moments arbitrarily large, but in proportion to the K values.



$$M_A = \frac{5}{43.5} \times 91.3 = 10.5$$

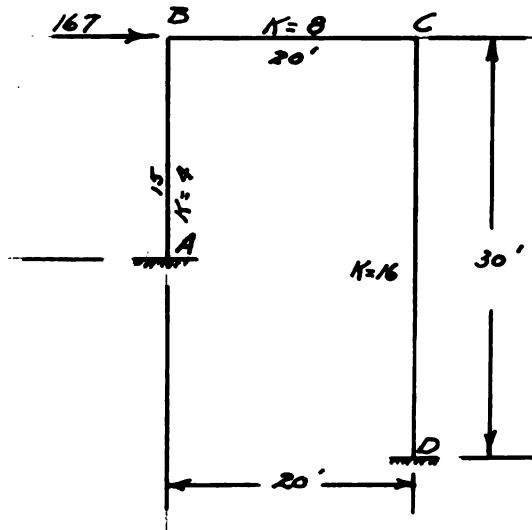
$$M_B = \frac{5}{43.5} \times 82 = 9.45$$

$$M_C = \frac{5}{43.5} \times 108 = 12.4$$

$$M_D = \frac{5}{43.5} \times 154 = 17.7$$

Problem 121, Page 157

Check the following values for the moments in the bent shown.



Select moments proportional to $\frac{K}{L}$ values

$$\frac{4/15}{16/30} = \frac{1}{2}$$

Say 400 & 800

$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
400	0	0	800
	-135	-270	-530
-88	-177	-88	
	+15	29	59
-5	-10	-5	
		2	-3
+307	-307	-332	+326
	400		
	-48		
	-2		
	+354		

Shear in AB = $\frac{354 + 307}{15} = 44$

" " CD = $\frac{326 + 565}{30} = 29.7$

$$\begin{array}{r} 800 \\ -265 \\ +30 \\ \hline +565 \end{array}$$

Correct ratio = $\frac{167}{73.7} = 2.27$

$M_{AB} = 805 \text{ 'K}$

$M_{BC} = 700 \text{ 'K}$

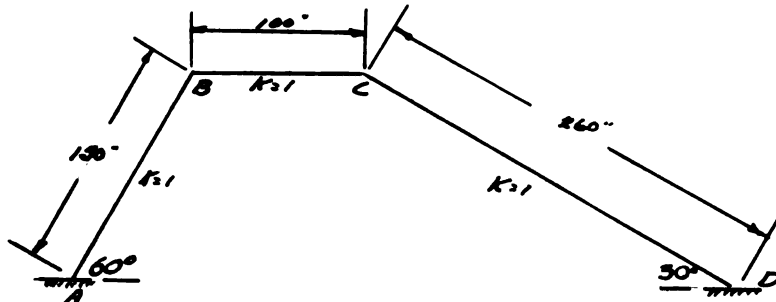
$M_{CB} = 755 \text{ 'K}$

$M_{DC} = 1280 \text{ 'K}$

7.8, 10.16

Problem 132, Page 157

Check the following values for the moments in the bent with sloping legs as caused by a 1 inch horizontal side lurch where $E=30,000,000$ per sq. in. Draw a moment diagram and sketch the deflected structure.



Assuming a 1" lateral deflection, I compute the values of C . Using the formula $C = \frac{6EK\Delta}{L}$. If the lateral movement is 1", the deflection at C is $\Delta \cos 60^\circ = 1$, at B it is

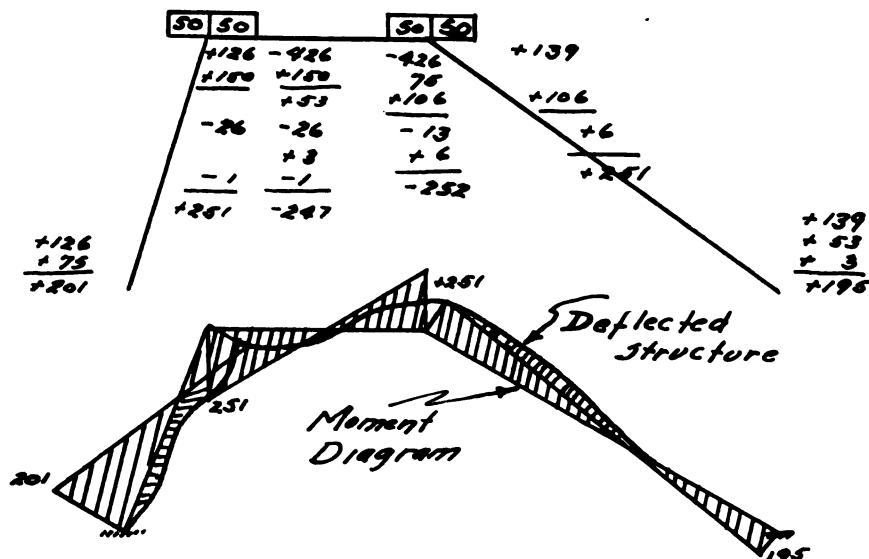
$$\Delta \cos 30^\circ = 1 \quad \Delta C = \frac{1}{\cos 30^\circ} = 2 \quad \Delta B = 1.15$$

$$C_{BA} = \frac{6 \times 30 \times 10^6 \times 1.15 \times 1}{150} = 1.26 \times 10^6$$

$$C_{CD} = \frac{6 \times 30 \times 10^6 \times 1 \times 20}{260} = 1.39 \times 10^6$$

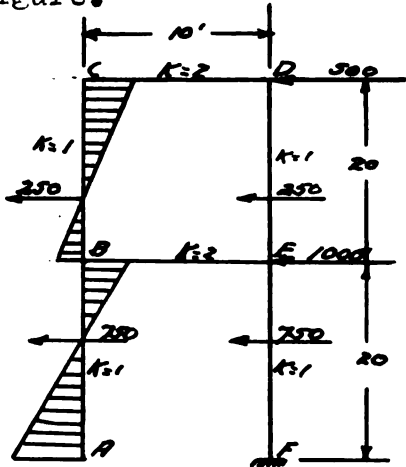
The vertical deflection is $2 \sin 60^\circ + 1.15 \sin 30^\circ = 1.732 + .575 = 2.307$

$$C_{BC} = C_{CB} = \frac{6 \times 30 \times 10^6 \times 2.307}{100} = 4.26 \times 10^6$$



Problem 124, Page 157

Obtain joint moments and sketch the shape of the deflected structure for the 2 story bent shown. The members are all of the same cross-section, i.e., K values of girders are twice the K values of columns. Final column moments are given on the figure.

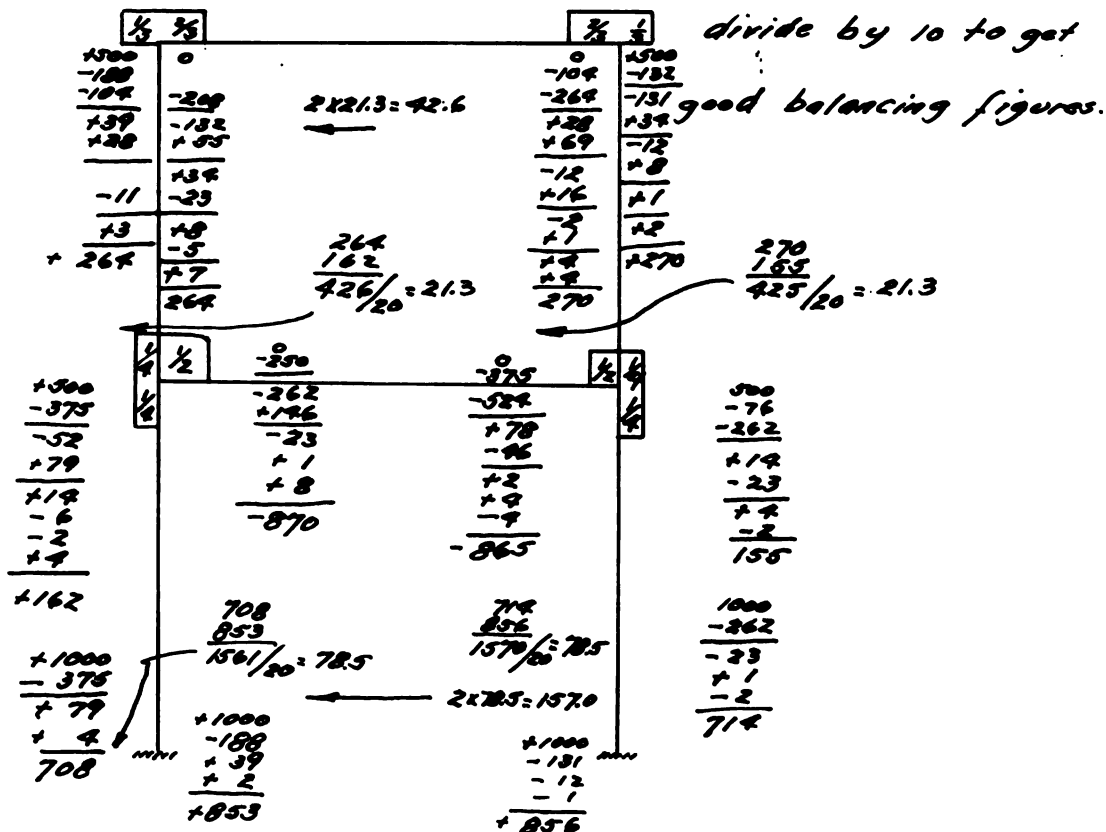


$$\frac{C_B + C_C}{20} = 250 = \frac{C_D + C_E}{20}$$

$$C_B = C_C = 20000$$

$$\frac{C_A + C_B}{20} = 750 \quad C_A = C_B = 7500$$

To get a better approximation,
we increase the lower story to
10000 & the upper to 5000, then



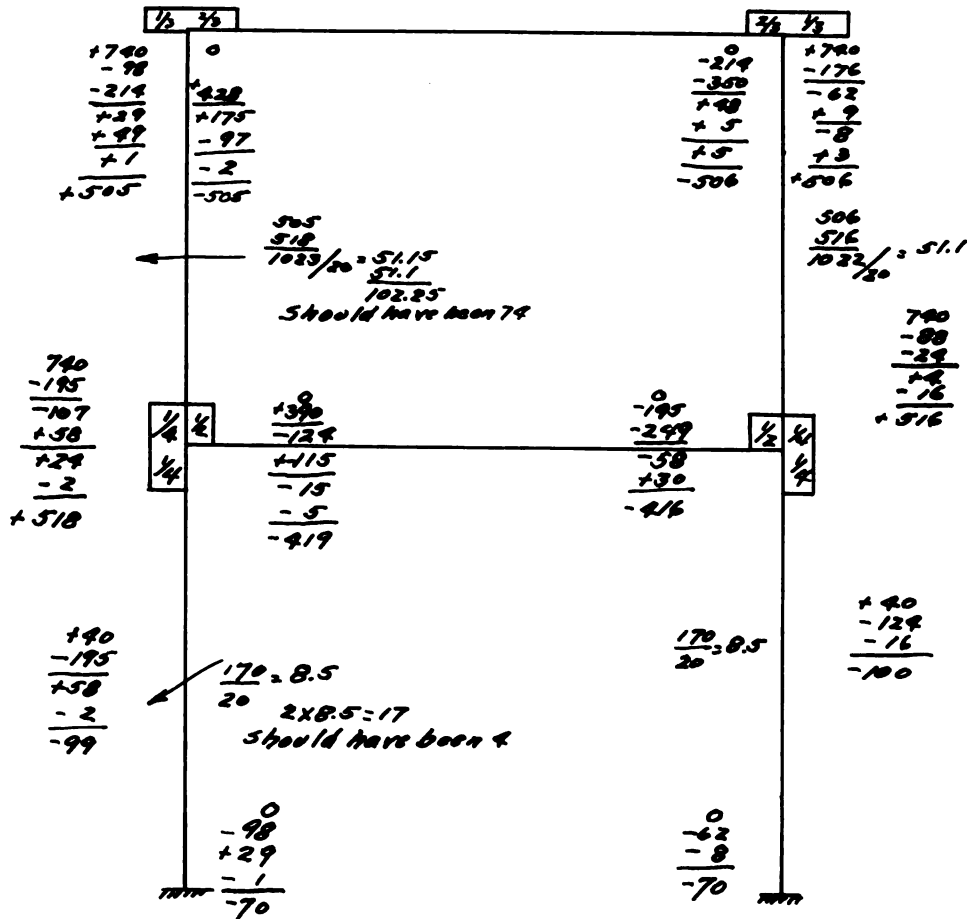
Note: we must multiply by 10 to get the actual values.

7, 8, 10, 16

Prob 124 'Cont' Page 2

The shears as a result of the balancing process are 426* to left in the upper story and 1570* in the lower story. We have restraining forces of 74* left & right in upper & lower story. The shears should have been 500 and 1500 respectively, thus permitting the restraining forces mentioned. Reverse the restraining forces and compute the moments

$$\frac{C_B + C_C}{20} = \frac{74}{2} = \frac{C_D + C_E}{20} \quad C_B = C_E = 740 \quad \frac{C_A + C_B}{20} = \frac{4}{2}, C_A = C_B = 40$$



Prob 129 'Cont' Page 3

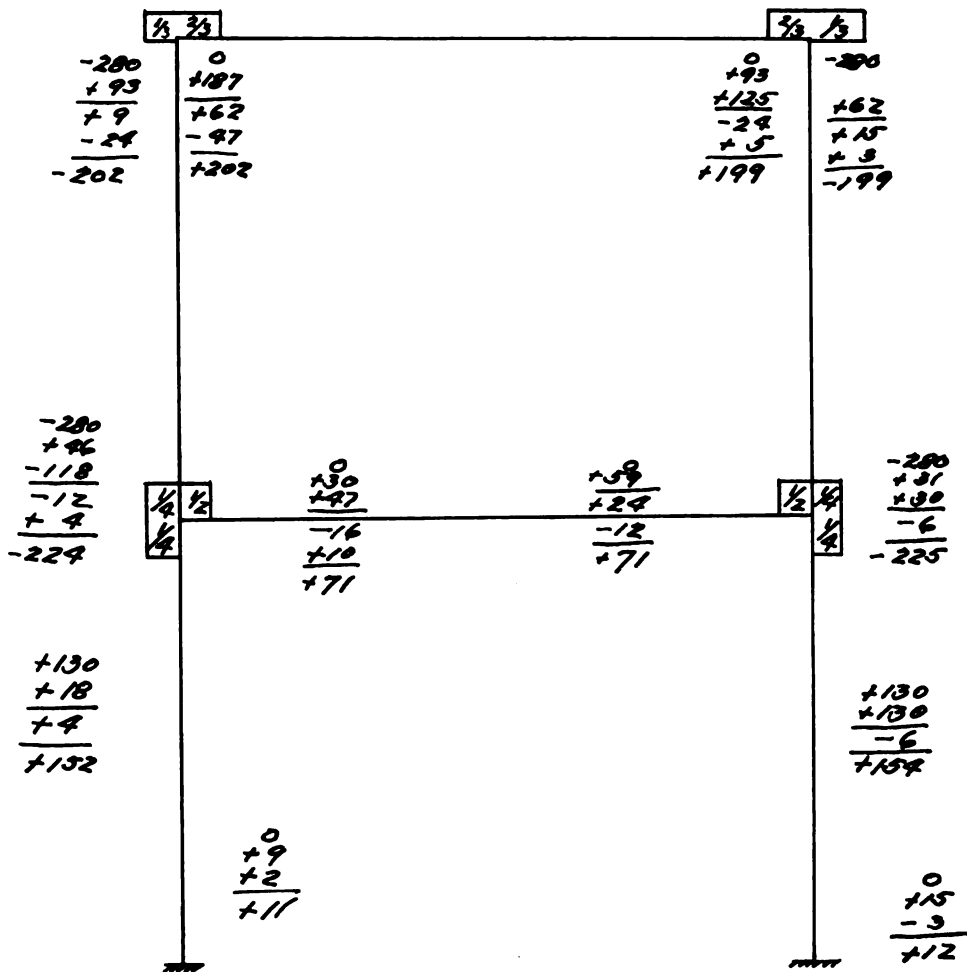
Proceeding exactly as before, we have

$$\frac{74}{102} = \frac{28}{20} \text{ - joint restraint on upper story}$$

$$\frac{170}{130} = \frac{40}{130} \text{ - joint restraint on lower story}$$

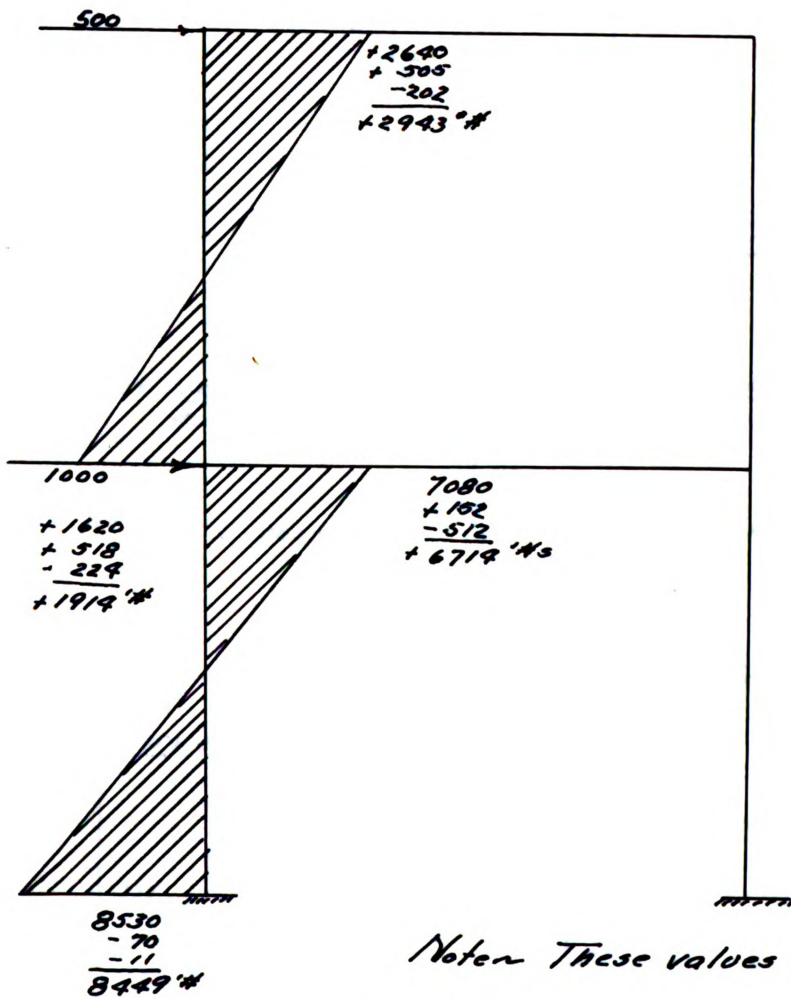
$$\frac{C_B + C_C}{20} = \frac{28}{2}, C_B = C_C = -280$$

$$\frac{C_A + C_D}{20} = \frac{13}{2}, C_A = C_D = 130, \text{ increase to } +150$$



Prob 124 'Cont' Page 4

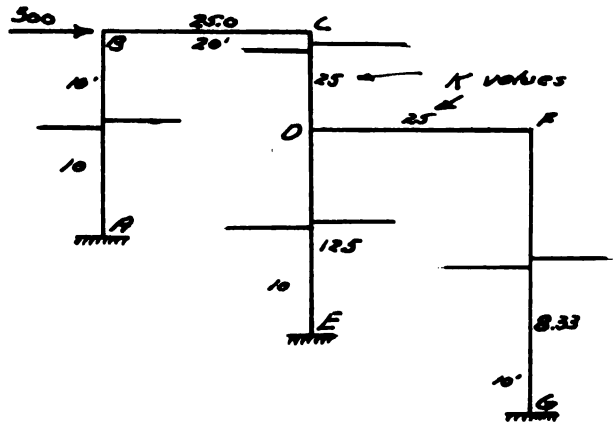
We will now sum up the moments, assuming that we have approximated the actual values. Plotting on tension side.



Note~ These values are approximately within 10% since only 3 approximations were made, and moments were not balanced quite exactly.

Problem 126, Page 162

Check the distribution of shears to the fixed end columns as given in Fig. 131 (b) and (f).



$$\textcircled{b} \quad K = \frac{EI}{L}$$

$$\text{Stiff: } \frac{EI}{L^3} = \frac{K}{L^2}$$

$$CD = \frac{25}{100} = .25 = 3.0$$

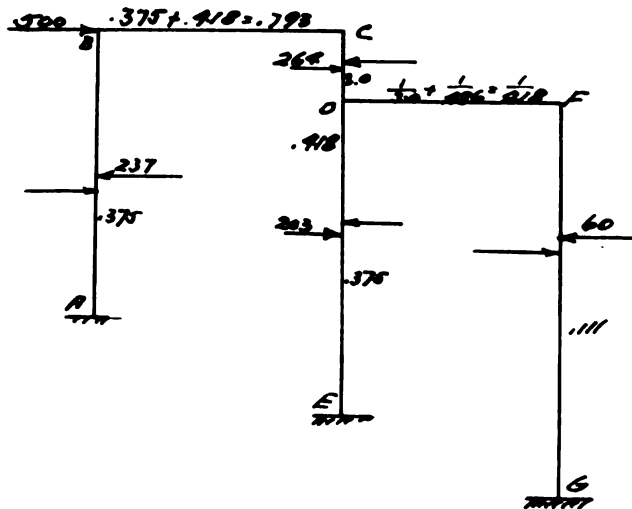
$$AB = \frac{12.5}{400} = .0313 = .375$$

$$DE = \frac{12.5}{400} = .0313 = .375$$

$$BC = \frac{25}{400} = .0625 = .75$$

$$DF = \frac{25}{400} = .0625 = .75$$

$$FG = \frac{0.33}{900} = .00925 = .111$$



$$AB = \frac{.375 \times 300}{.793} = 237$$

$$CD = \frac{.918 \times 300}{.793} = 264$$

$$DE = \frac{.375 \times 264}{.406} = 203$$

$$FG = \frac{.111 \times 264}{.406} = 60$$

Prob 126 "Cont"

Given a joint restraint of 172*

All we have to do is substitute 172 in place of 500 in part b and multiply shears by $\frac{172}{500}$

$$AB: 237 \times \frac{172}{500} = 81$$

$$DE: 203 \times \frac{172}{500} = 70$$

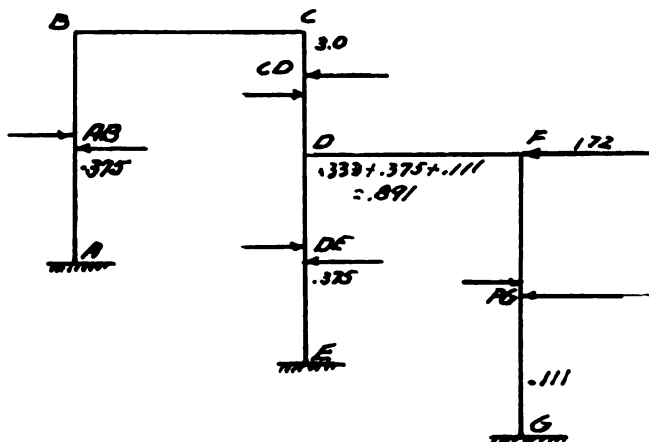
$$CD: 263 \times \frac{172}{500} = 91$$

$$FG: 60 \times \frac{172}{500} = 21$$

Note - Apply force at C - horizontally to right.

Problem 127, Page 102

Check the distribution of shears to the fixed end columns as given in Fig. 131 (g).



$$FG: \frac{172}{819} \times .111 = .23$$

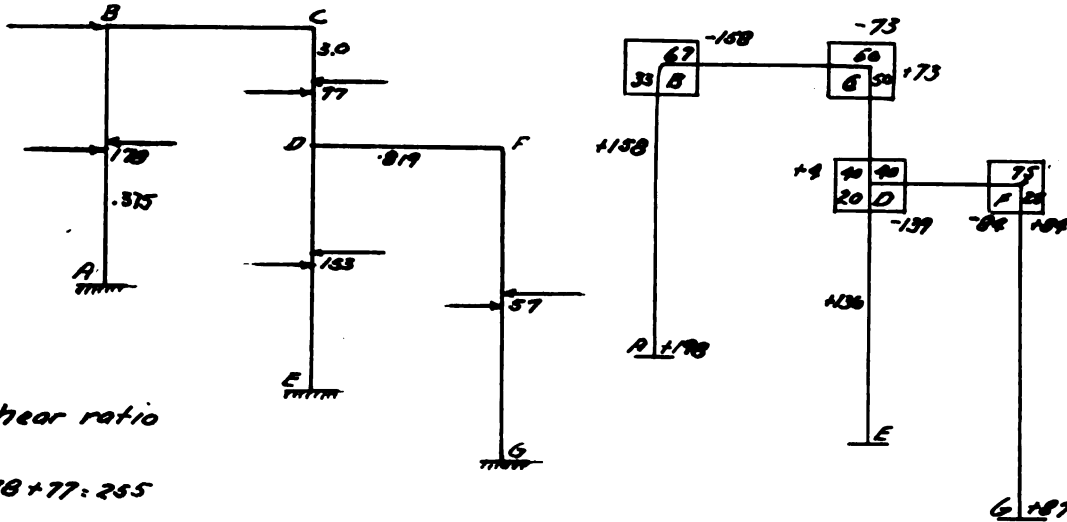
$$DE = \frac{170}{819} \times 375 = 79$$

$$CD = \frac{170}{.819} \times .333 = 70$$

$$AB = -CD = -70$$

Problem 128, Page 162

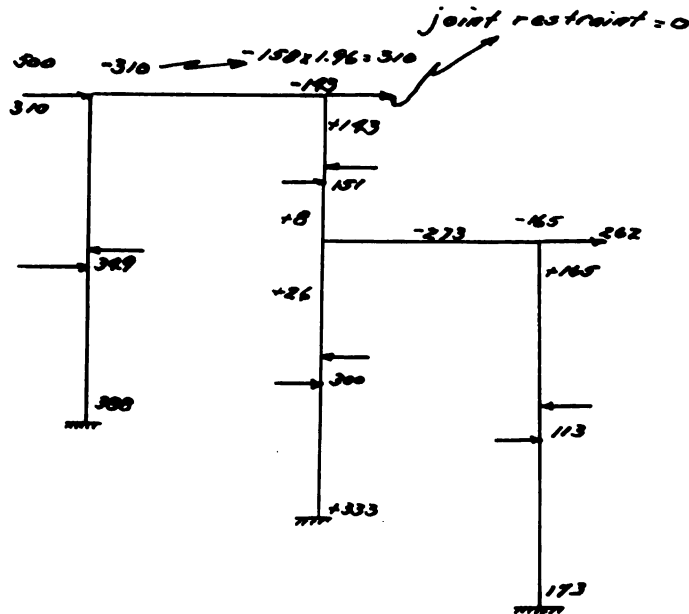
Repeat the analysis of the example of Fig. 131 by adjusting shears in (d) by use of the upper correction ratio, 1.96. It will be found that this procedure simplifies the solution. Explain the discrepancy in final moments.



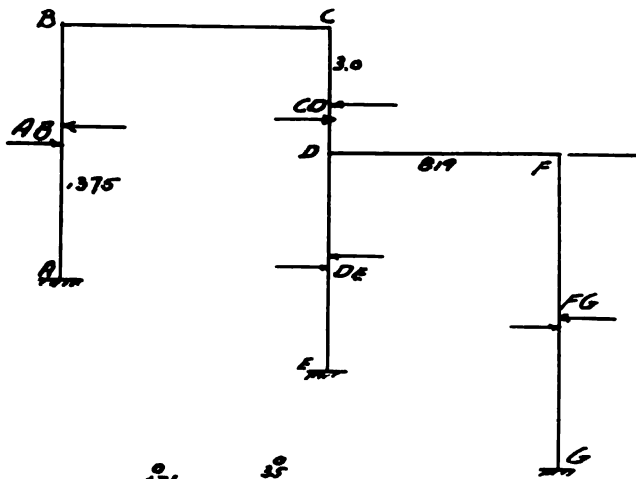
Shear ratio

$$178 + 77 = 255$$

$$\text{Corr Ratio } \frac{500}{255} = 1.96$$



Prob 128 "Cont" page 2

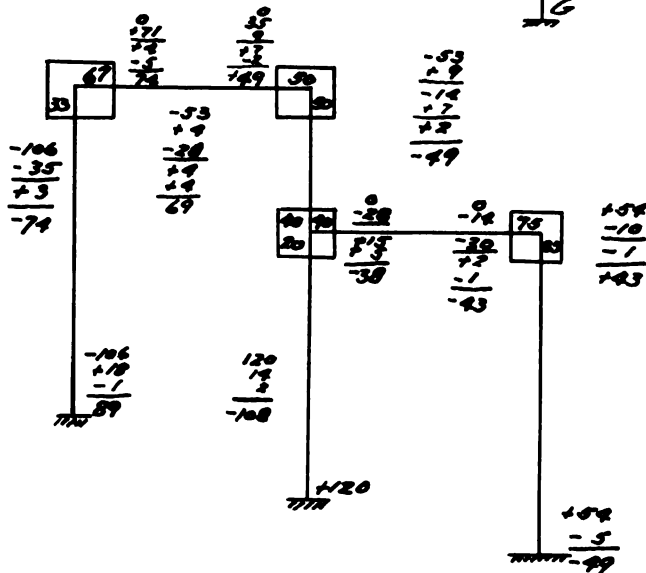


$$FG = \frac{262}{8.19} \times 1.111 = 36$$

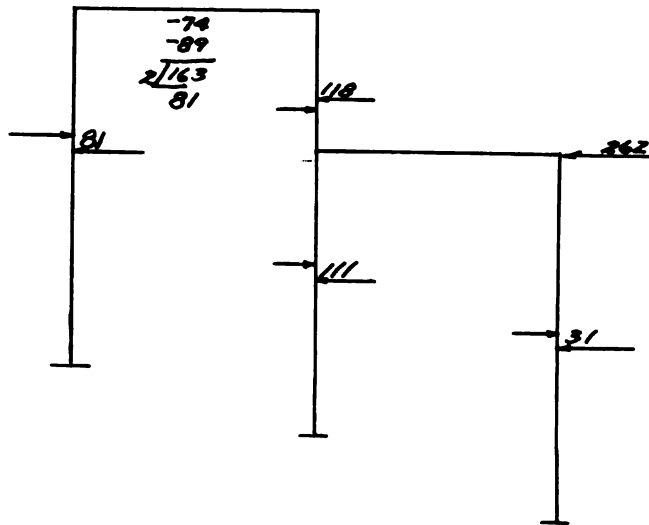
$$DE = \frac{262}{8.19} \times 1.375 = 120$$

$$CD = \frac{262}{8.19} \times 1.333 = 106$$

$$AB = -CD = -106$$



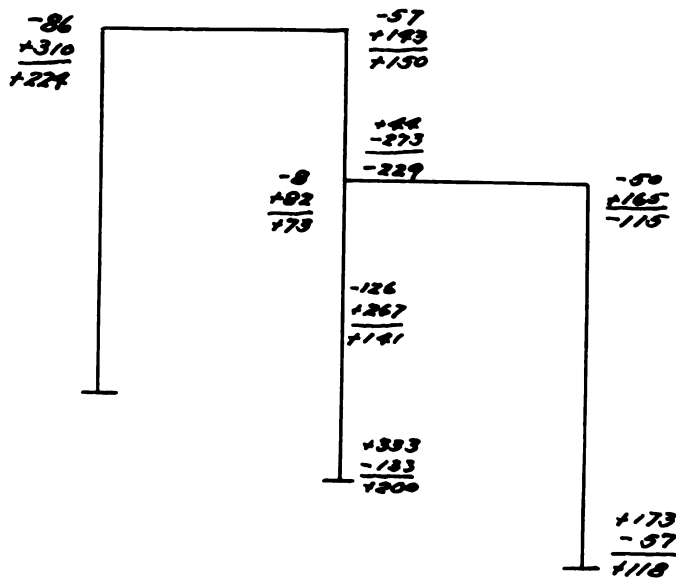
Prob 128 Cont Page 3



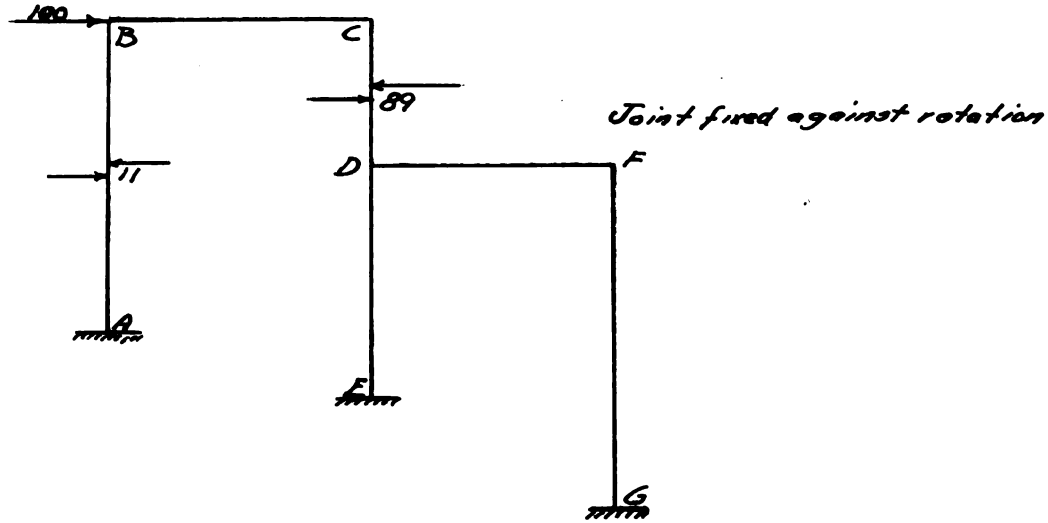
Shear 223

$$\text{Ratio } \frac{262}{223} = 1.17$$

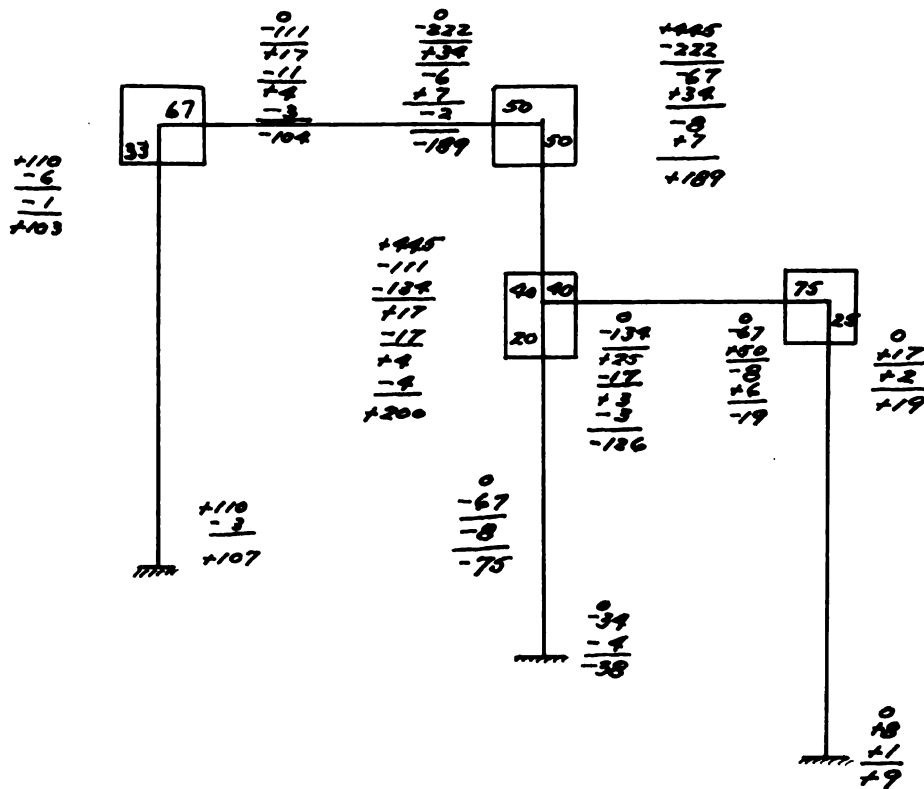
Final Moments



Problem 128, Page 162
Special method.

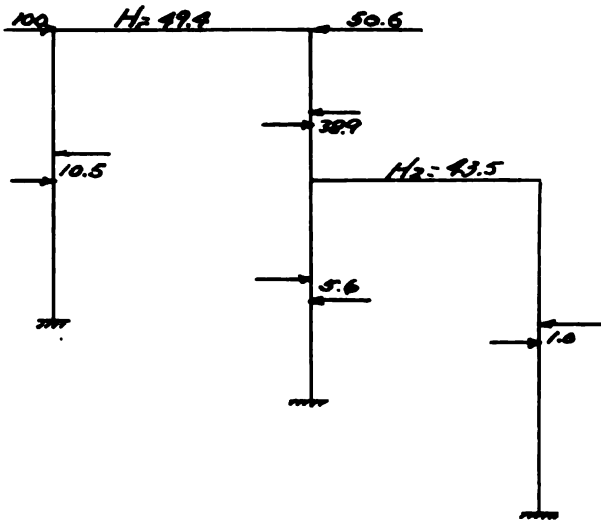


② Effect of joint force at B

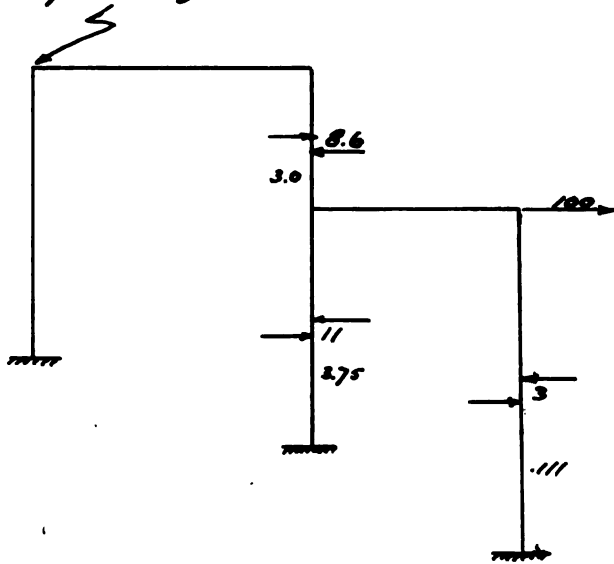


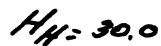
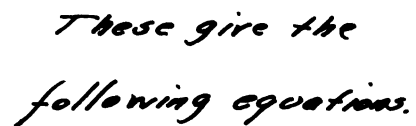
Prob 128 Special "Cont" page 2

②



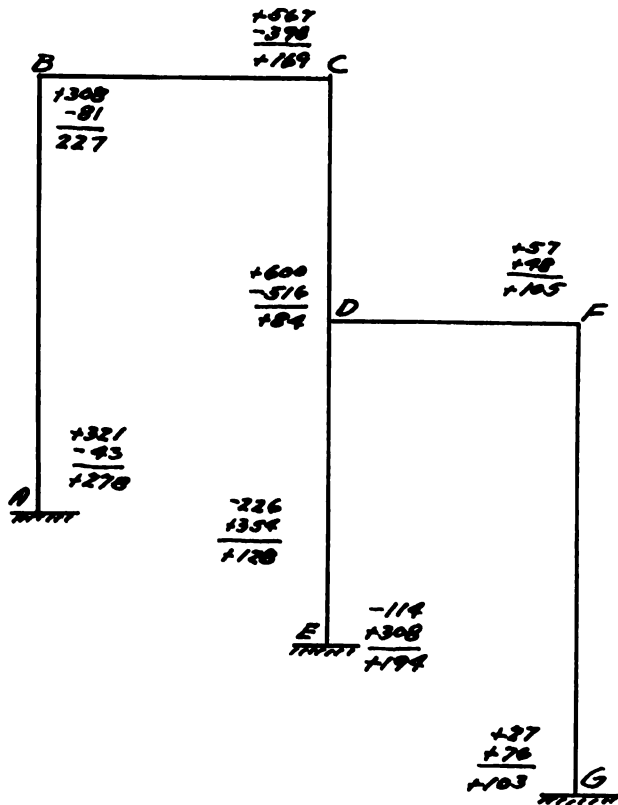
⑤ Joint fixed against rotation.





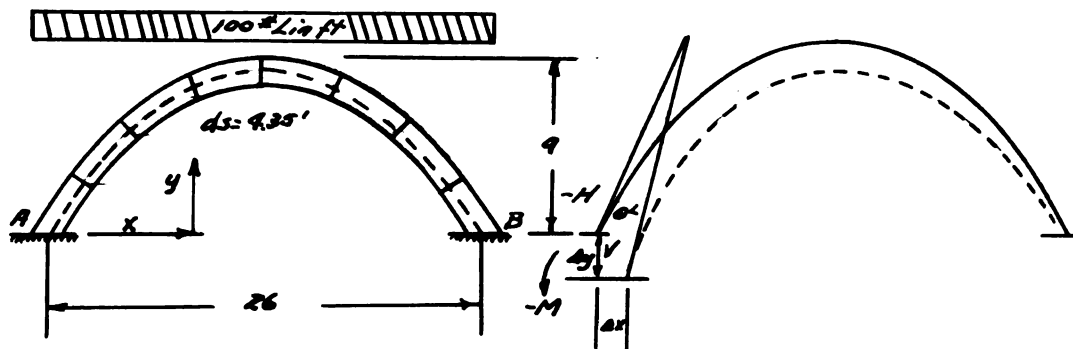
Prob 128 special Cont" Page 4

$$\text{Final Moments} = M_{HL} \times 2.3 + M_{HH} \times 3.0$$



Problem 146, Page 204

Accurately determine the springing moment for the arch of Fig. 156 as caused by a uniform vertical load of 100 $\frac{lb}{ft}$ per ft. over the entire span.



Point	Assume $\frac{ds}{dx}$	X	Y	x^2	y^2	xy	M_s	M_{sx}	M_{sy}
1	1	0.4	2.2	.2	4.8	.9	10	4	22
2	1	2.9	5.7	8.4	32.5	16.5	420	1218	2394
3	1	6.6	7.8	43.6	60.9	51.5	2180	14388	17004
4	1	10.8	8.8	116.8	77.4	95.2	5840	63072	51392
5	1	15.2	8.8	231.5	77.4	134.0	11575	175980	101860
6	1	19.4	7.8	376.0	60.9	151.6	18800	369720	146640
7	1	23.1	5.7	533.0	32.5	131.7	26650	615615	151905
8	1	25.6	2.2	655.0	4.8	56.3	32750	838900	72050
Σ	8	109.0	49.0	1964.5	351.2	637.7	98225	2078357	543267

$$\Theta^L \cdot \Sigma M_s \frac{ds}{EI} = -98225$$

$$\delta \Theta^{N=1} = \delta X^{M=1} = \Sigma y \frac{ds}{EI} = -49$$

$$\Delta y^L = \Sigma M_s x \frac{ds}{EI} = -2078357$$

$$\delta y^{N=1} = \delta x^2 \frac{ds}{EI} = +1964.5$$

$$\Delta x^L = \Sigma M_s y \frac{ds}{EI} = +543267$$

$$\delta x^{N=1} = \delta y^2 \frac{ds}{EI} = +351.2$$

$$\delta \Theta^{M=1} = \Sigma \frac{ds}{EI} = +8$$

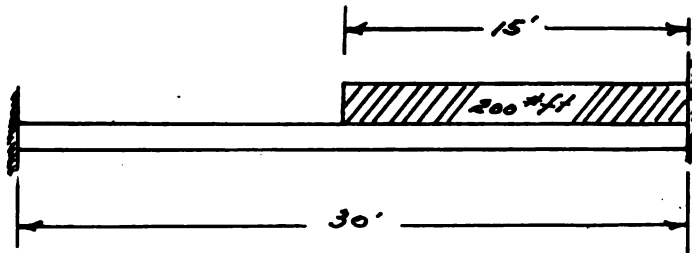
$$\delta y^{M=1} = \delta x^{N=1} = \Sigma xy \frac{ds}{EI} = -637.7$$

$$\delta \Theta^{V=1} = \delta y^{M=1} = \Sigma x \frac{ds}{EI} = +109$$

5, 8, 11, 17

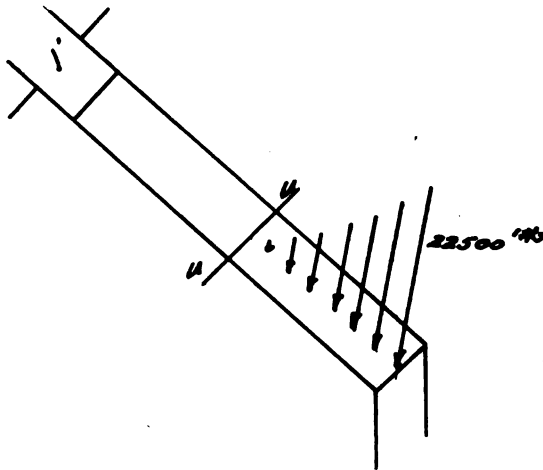
Problem 160, Page 229

Determine the end moments for the fixed end beam of 30 foot span loaded with a uniform load of 200#/ft over the left half of the span.



No change in cross-section

$$\frac{1}{EI} = 1$$



$$A = 30 \quad W = \frac{1}{2} \times 15 \times 200 = 112500$$

$$I_{yy} = \frac{1}{12} \times 12 \times 30^3 = 2250$$

$$M_{yy} = W \times 15 \times \frac{3}{4} = 1,270,000$$

$$f = M_{iA} = \frac{W}{A} \pm \frac{M_{yy} \bar{y}}{I_{yy}}$$

$$= \frac{112500}{30} + \frac{1,270,000 \times 15}{2250} = 12200$$

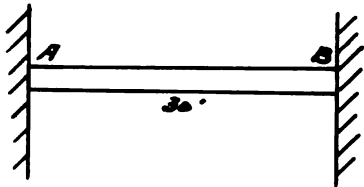
$$M_{iB} = 3750 - 8450 = 4700$$

$$M_A = 22500 - 12200 = 10300$$

$$M_B = 4700$$

Prob 160 Cont

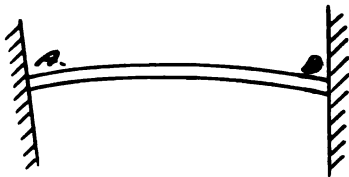
With an angle change



$$\theta_A = .1^\circ$$

$$E = 30 \times 10^6$$

$$I = 1000$$



$$\Delta = \frac{d\delta}{dx} = \frac{30 \times 12}{EI} = \frac{12}{30 \times 10^6 \times 1000} = \frac{12}{10^9}$$

$$W = \theta \cdot \frac{1}{57.7} = .00173 \text{ rad.}$$

$$x = 15$$

$$M_{yy} = .00173 \times 15 \times 12$$

$$I_{yy} = \frac{1}{12} \times \frac{1}{30 \times 10^9} (30 \times 12)^3$$

$$M_A = \frac{.00173 \times 10^9}{12} + (15 \times 12) \left(\frac{.00173 \times 15 \times 12 \times 10^9}{900 \times 144} \right)$$

$$= 142100 + 432500 = 574600 \text{ "K} \text{ or } 42000 \text{ "K}$$

$$M_B = 142100 - 432500 = -290400 \text{ "K} \text{ or } -22000 \text{ "K}$$

These answers are in correct ratio.

Problem 161, Page 229

Compute the corner moments at B and D for the bent of Fig. 171 and complete the moment diagram. Sketch the shape of the deflected structure.

From Example

$$A = 30^\circ \quad W = 5000$$

$$I_x = 333 \quad I_y = 583$$

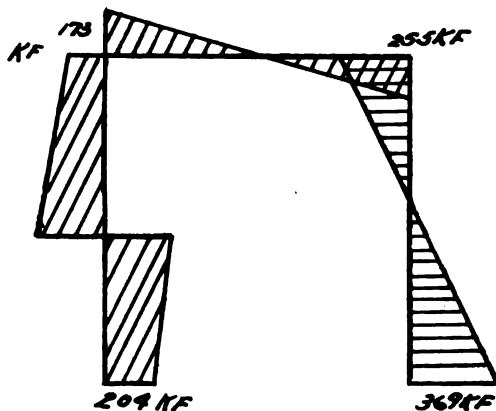
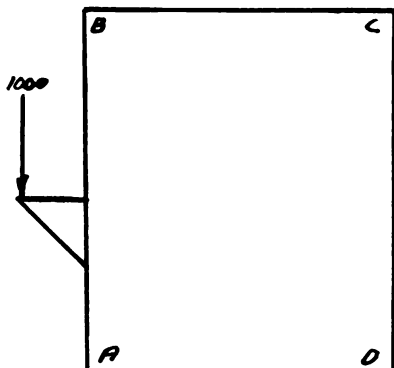
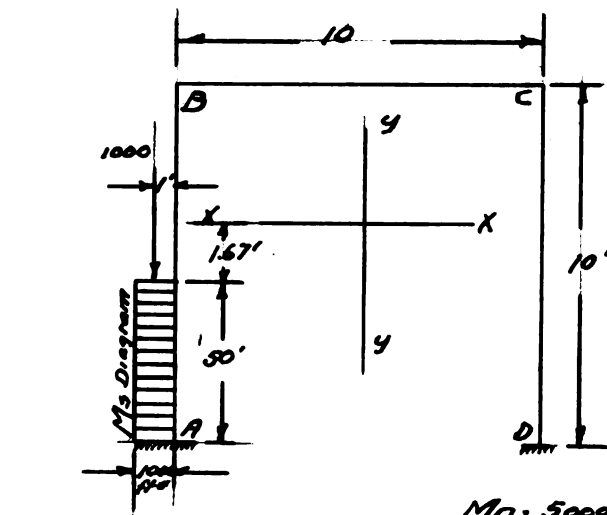
$$M_x = -20800 \quad M_y = -25000$$

$$M_B = \frac{5000}{30} + \frac{25000 \times 5}{583} - \frac{20800 \times 3.33}{333} = 173 \text{ KF}$$

$$M_D = \frac{5000}{3} - \frac{25000 \times 5}{583} + \frac{20800 \times 6.67}{333} = 369 \text{ KF}$$

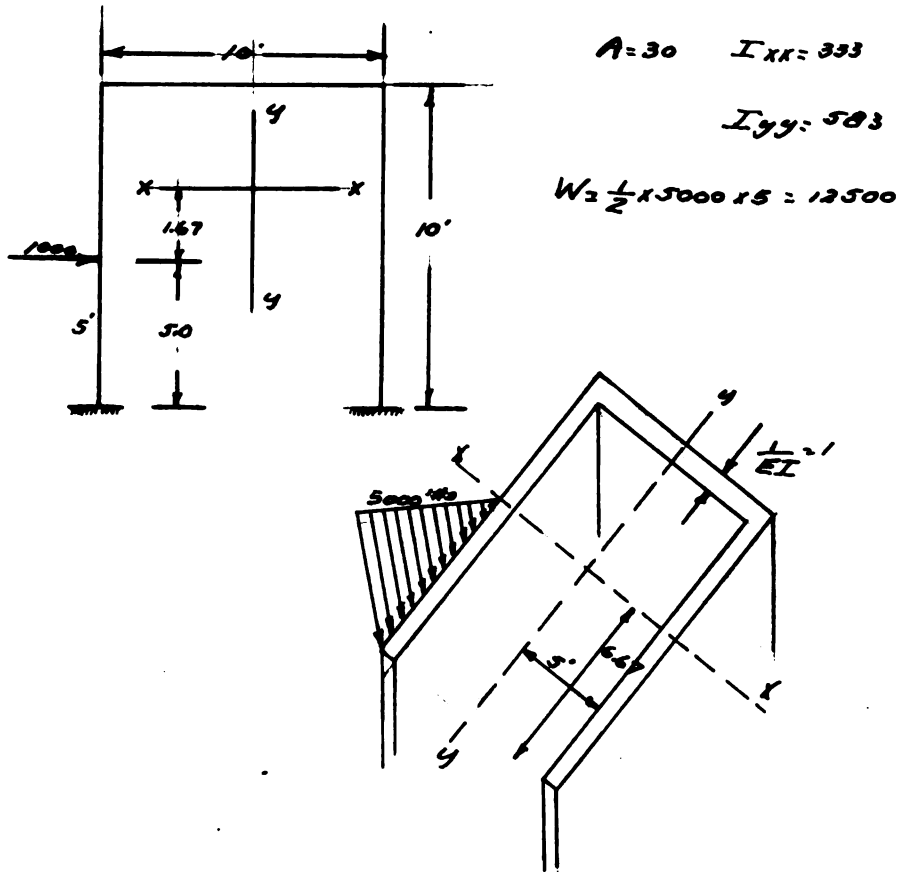
$$M_A = 204 \text{ KF}$$

$$M_C = 255 \text{ KF}$$



Problem 162, Page 229

Obtain the moment diagram for the bent of Fig. 171 as caused by a horizontal load of 1000# at the mid-height of the left leg. Sketch the shape of the deflected structure.



$$M_{xx} = (1.67 + \frac{2}{3} \times 5) \times 12500 = 62500$$

$$M_{yy} = 12500 \times 5.0 = 62500$$

$$M_A = 5000 - \left(\frac{12500}{30} + 6.67 \frac{62500}{333} + \frac{62500 \times 5}{583} \right)$$

$$M_B = \frac{12500}{30} - \frac{62500 \times 3.33}{333} - \frac{62500 \times 5}{583}$$

$$M_C = \frac{12500}{30} - \frac{62500 \times 3.33}{333} - \frac{62500 \times 5}{583}$$

$$M_D = \frac{12500}{30} + \frac{62500 \times 6.67}{333} - \frac{62500 \times 5}{583}$$

$$M_A = 5000 - (413.3 + 1250 + 536) = -289$$

$$M_B = -(413 - 625 + 536) = -322$$

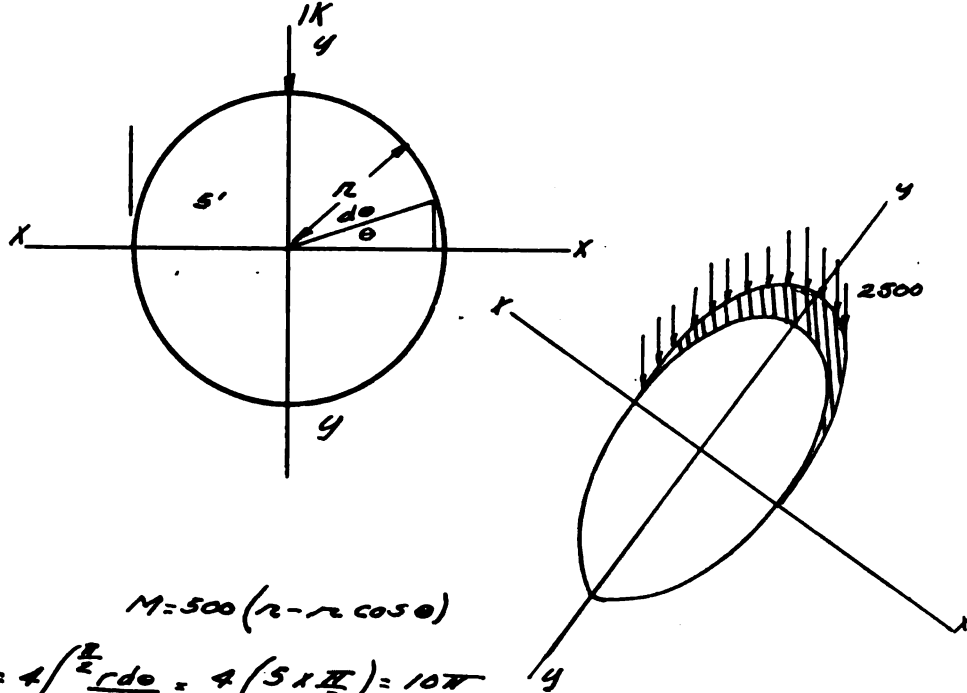
$$M_C = -(413 - 625 - 536) = -748$$

$$M_D = -(413 + 1250 - 536) = -1127$$

8.11.15

Problem 163, Page 229

Obtain the moments in a circular ring of 10 inch diameter supported on a point reaction and carrying a concentrated load at the top of 1000#. If the ring is made up of a circular rod, find the diameter (required) of the rod to work at a fiber stress of 16000# per sq.inch.



$$ds = r d\theta \quad M = 500(r - r \cos \theta)$$

$$A = \int \frac{ds}{EI} = 4 \int_0^{\frac{\pi}{2}} \frac{r d\theta}{EI} = 4 \left(5 \times \frac{\pi}{2} \right) = 10\pi$$

$$W = 4 \times 500 \times 25 \int_0^{\frac{\pi}{2}} (1 - \cos \theta) d\theta$$

$$= 50000 \left[\theta - \sin \theta \right]_0^{\frac{\pi}{2}} = 50000 \left[\frac{\pi}{2} - 1 \right]$$

$$= 50000 \times 2.57$$

$$M_1 = \frac{W}{A} = \frac{50000 \times 2.57}{10 \times 3.142} = 910$$

$$M_A = 2500 - 910 = 1590$$

$$S = \frac{Mc}{I} = 16000$$

$$r^2 = \frac{1590}{.7854 \times 16000} = .125$$

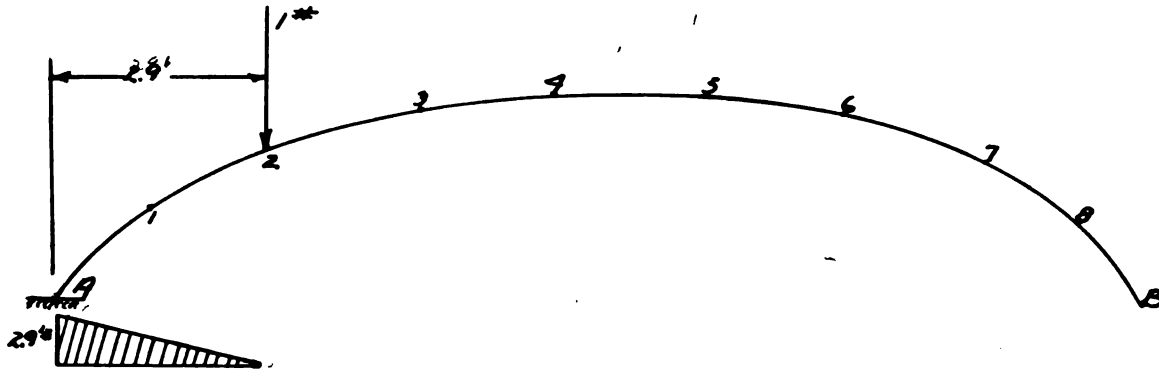
$$r = .5" \quad d = 1.0"$$

✓ Because the $\frac{Mc}{I}$ terms cancel due to symmetry

Problem 164, Page 229

Check the moments obtained in Art. 163 for the elliptical arch by using a cantilever from the right end as a static structure.

Hint. Slide rule accuracy will not be satisfactory here because of the difference of large quantities involved in the term $Mx - x_0$.



PT	$\frac{ds}{EI}$	x	y	x_0	y_0	x_0^2	y_0^2	M_s	$M_s x_0$	$M_s y_0$
1	1	.4	2.2	-12.6	-3.9	158.7	15.2	2.5	-31.5	-8.5
2	1	2.9	5.7	-10.1	-.4	102.0	.2			
3	1	6.6	7.8	-6.4	+1.7	41.0	2.9			
4	1	10.8	8.8	-2.2	+2.7	4.8	7.3			
5	1	15.2	8.8	+2.2	+2.7	4.8	7.3			
6	1	19.4	7.8	+6.4	-1.4	41.0	2.9			
7	1	21.3	5.7	+10.1	-.4	102.0	.2			
8	1	25.6	2.2	+12.6	-3.9	158.7	15.2			
Σ		104	49.0			613	51.2	2.5	-31.5	-8.5

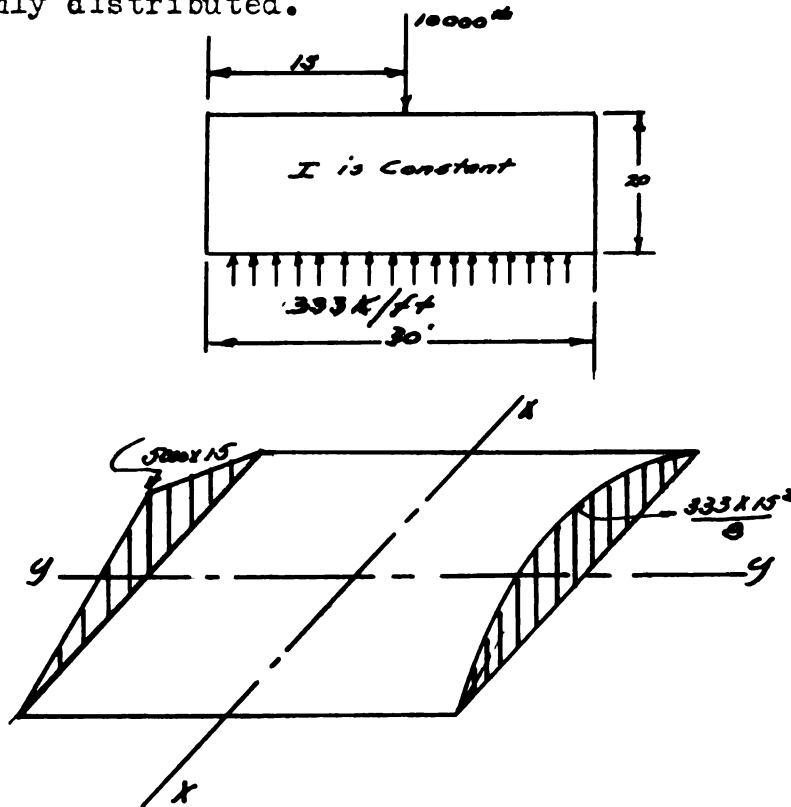
$$M_A = -2.9 + \frac{2.5}{8} + \frac{31.5}{613} \times 13 + \frac{8.5}{51.2} \times 6.12 = -.90$$

$$M_B = \frac{2.5}{8} - \frac{31.5 \times 13}{613} + \frac{8.5 \times 6.12}{51.2} = +.66$$

Note: A small error in M_s is greatly magnified.

Problem 166, Page 229

Determine the maximum moment in the box culvert illustrated when the 10,000# wheel load is placed at the center of the span. The reaction under the box is assumed to be uniformly distributed.



We will place pins at all 4 corners. This is still statically determinate for symmetrical loading.

$$A = 2 \times 30 + 2 \times 20 = 100$$

$$I_{yy} = 2 \left(\frac{20^3}{12} + 20 \times 15^2 \right) = 13500$$

$$I_{xx} = 2 \left(\frac{30^3}{12} + 30 \times 10^2 \right) = 7330$$

$$W = \frac{3}{8} (30 \times 37500) + \frac{75000 + 30}{2} = 1,870,000$$

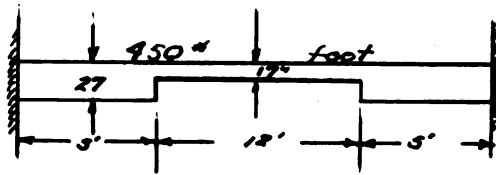
$$M_{xx} = 10 (1,120,000 - 750,000) = 10 (370,000) = 3,700,000$$

$$M_p = 75000 - \left(\frac{1,870,000}{100} + \frac{3,700,000 \times 10}{7330} \right)$$

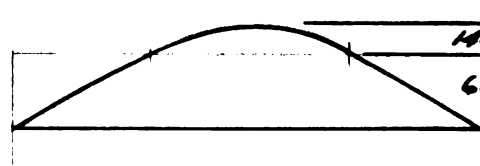
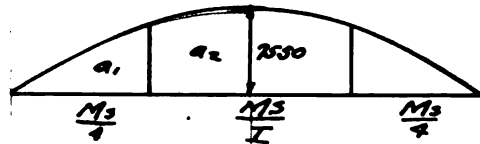
$$M_p = 75000 - 23200 = 52800 \quad \text{**}$$

Problem 167, Page 229

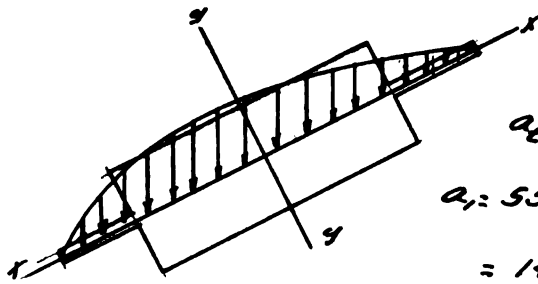
The symmetrically deepened beam illustrated carries a uniform vertical load of 450# per ft. Obtain the moment diagram.



Make statically determinate by placing pins at the ends. Because of symmetry, only the $\frac{W}{A}$ term is present



$$\frac{1}{EI_1} = 1, \frac{1}{EI_2} = \frac{1}{4} = .25$$



$$Q_2 = 7500 \times \frac{2}{3} \times 11 = 55500$$

$$Q_1 = 55500 - (53000 \times 6) - \frac{2}{3} \times 2250 \times 6 = 14700$$

$$Q_2 = 53000 \times 6 + \frac{2}{3} \times 2250 \times 6 = 40800$$

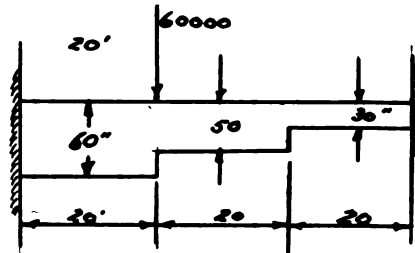
$$A = 2(25 \times 5) + 12 \times 1 = 14.5$$

$$W = \left(\frac{14700}{4}\right)2 + 2(40800) = 88950$$

$$M_A = M_B = \frac{88950}{14.5} = 6125$$

Problem 168, Page 230

Analyze the unsymmetrically deepened beam illustrated for the effect of the concentrated load. Obtain the moment diagram to scale. Choose a cantilever extending from the left hand end as the static structure.



Proportional widths

1, 1.73, 8

$$\frac{1}{EI} = \frac{1}{63} + \frac{1}{216} + \frac{1}{125} + \frac{1}{27}$$

$$\bar{X} = \frac{20 \times 10 + 1.73 \times 20 \times 30 + 8 \times 20 \times 50}{20 + 34.6 + 160} = \frac{9238}{214.6} = 43.1' \text{ from left end}$$

$$I_{yy} = 160 \times 6.9^2 + \frac{1}{12} \times 8 \times 20^3 = 12490$$

$$= 34.6 (13.1)^2 + \frac{1}{12} \times 1.73 \times 20^3 = 7090$$

$$= 20 (33.1)^2 + \frac{1}{12} \times 1 \times 20^3 = 22500$$

$$42600$$

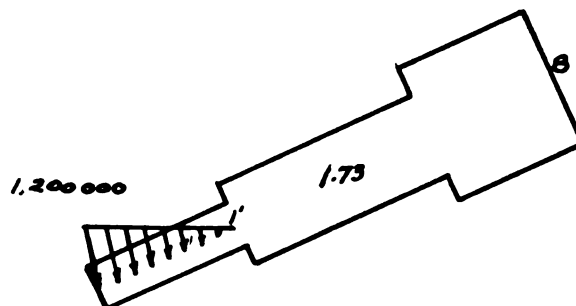
$$W = \frac{1,200,000 \times 20}{2} = 12,000,000$$

$$M_{yy} = 12 \times 10^6 \times 36.44 = 437 \times 10^6$$

$$A = 20 + 34.6 + 160 = 214.6$$

$$M_A = -120 - \frac{437 \times 10^6}{1000 \times 426000} \times 43.1 - \frac{12 \times 10^6}{214.6 \times 1000} = -120 - 499 = -701 \text{ K.F.}$$

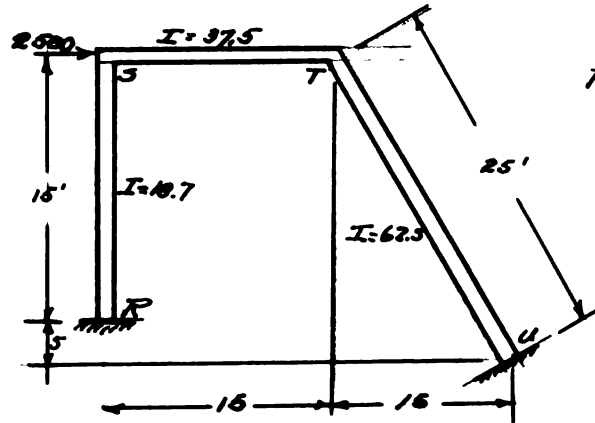
$$M_B = -\frac{437 \times 10^6}{426000} \times 16.9 + \frac{12 \times 10^6}{214.6 \times 1000} = -176 + 59 = -117 \text{ K.F.}$$



58.17

Problem 169, Page 237

Obtain a final moment diagram for the bent of Fig. 176 and sketch the shape of the deflected structure.



from fig 176

$$A = 1.6 \quad W = 150$$

$$M_{xx} = -56.2$$

$$M_{yy} = -112.5$$

$$I_{xx} = 50.8$$

$$I_{yy} = 150$$

$$I_{xy} = -25$$

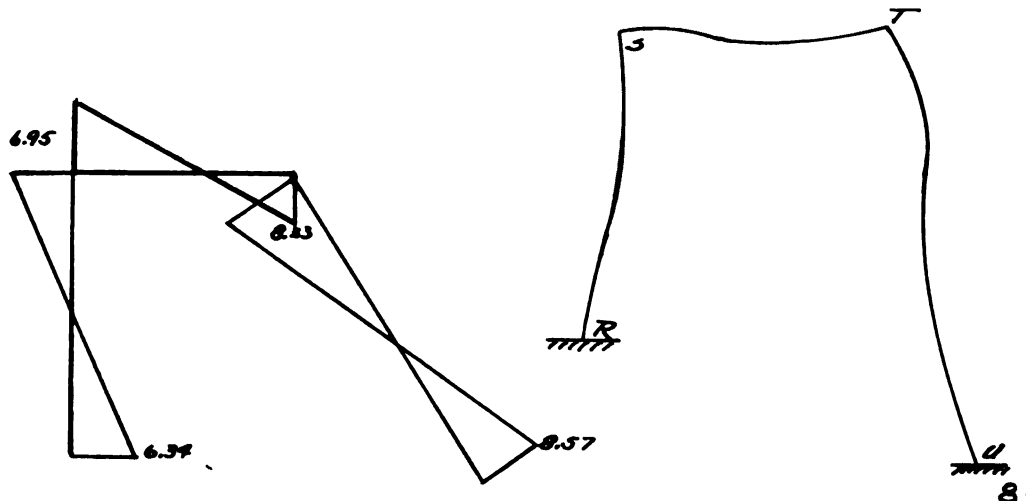
$$M_R = 6.34 \text{ KF}$$

$$M_S = 0 - \left[\frac{150}{1.6} + 7.5 \left(\frac{-112.5 + 50.8 - 56.2 \times 25}{50.8 \times 150.9 - (25)^2} \right) - 6.25 \left(\frac{-56.2 \times 150.9 - 112.5 \times 25}{50.8 \times 150.9 - 625} \right) \right]$$

$$= -[+9.37 + 7.65 - 10.05] = -6.95 \text{ KF}$$

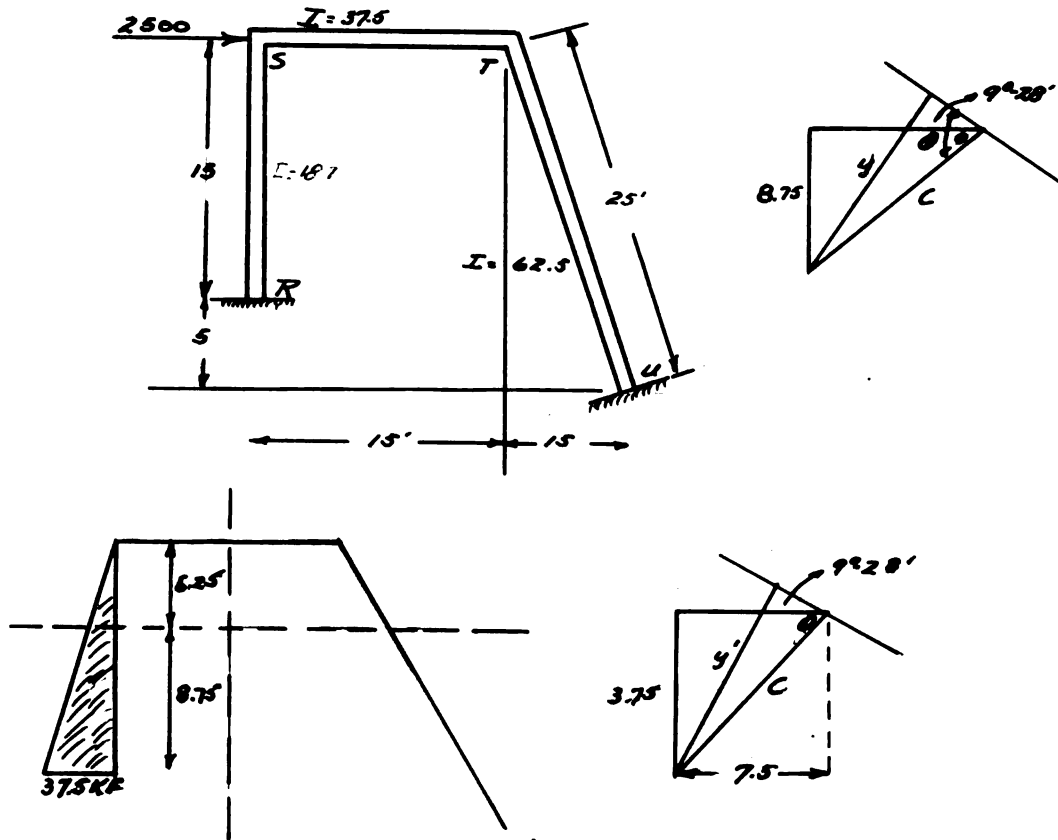
$$M_T = [9.37 - 7.5(1.02) - 6.25(1.61)] = -[9.37 - 7.65 - 10.05] = 8.33 \text{ KF}$$

$$M_U = [9.37 - 22.5(1.02) + 13.75(1.61)] = -[9.37 - 23 + 22.2] = -8.57 \text{ KF}$$



Special Problem.

Analysis of an unsymmetrical bent by the conjugate beam axis.



From preliminary work in the book $I_x = 50.8$, $I_y = 150$
 $I_{xy} = 25$ $A = 1.60$. Select $y-y$ axis as 1 conjugate axis.

$$\tan \theta = \frac{I_{xy}}{I_{yy}} = \frac{-25}{150} = -.1668 \quad \theta = 180^\circ - (9^\circ 28')$$

$$I_{x'} = \left(I_{xx} - \frac{I_{xy}^2}{I_{yy}} \right) \cos^2 \theta = \left(50.8 - \frac{625}{150} \right) .972 = 45.4$$

$$M_{yy} = Wx_0 = 112.5$$

$$\tan \theta = \frac{3.75}{7.5}$$

$$\theta = 26.6^\circ$$

$$9.46^\circ$$

$$\phi = 36.06^\circ$$

$$y' = 8.375 \sin \phi = 4.93$$

$$C = 8.375$$

$$M_{x'x'} = Wy' = 15 \times 4.93 = -74$$

Special Problem "Cont"

$$C = 11.5$$

$$\theta = 40.4^\circ$$

$$y = 11.5 \sin 57.9^\circ = 9.85$$

$$\phi = \frac{9.5^\circ}{57.9}$$

$$M_R = 37.5 - \left(\frac{M}{I} + \frac{My_y}{I_{yy}} + \frac{Mx'x'y'}{I_{xx'}} \right)$$

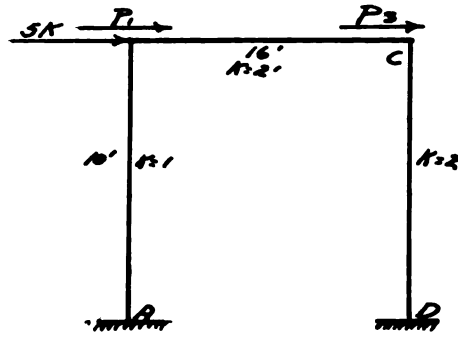
$$M_R = 37.5 - \left(\frac{45}{16} + \frac{-74}{45.4} \times -9.85 + \frac{-112.5}{150} \times -7.5 \right)$$

$$= 37.5 - (9.36 + 16.1 + 5.60) = 6.44 \text{ K.F.}$$

Problem 170, Page 237

Make use of the column analogy and the general column flexure equation to check the moments as given for the bent of Problem 120. Place pins at B and C to produce M_s .

Hint. Look for a possible negative column load.



$$\frac{L}{A} = \frac{\int \frac{ds}{EI}}{A} = \frac{1}{k_1} = 1$$

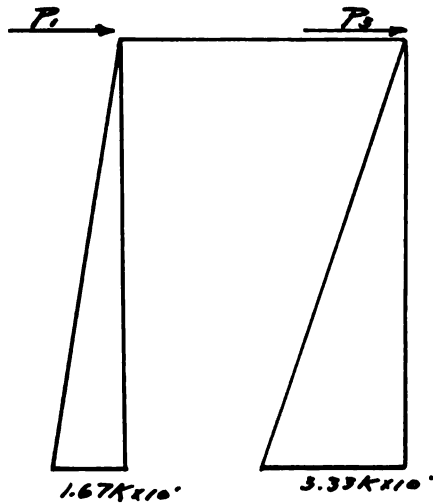
$$\frac{1}{k_2} = \frac{1}{2}, \quad \frac{1}{k_3} = \frac{1}{2}$$

$$\frac{P_1}{1} = \frac{P_2}{2}$$

$$P_2 = 2P_1$$

$$P_1 = 1.67K$$

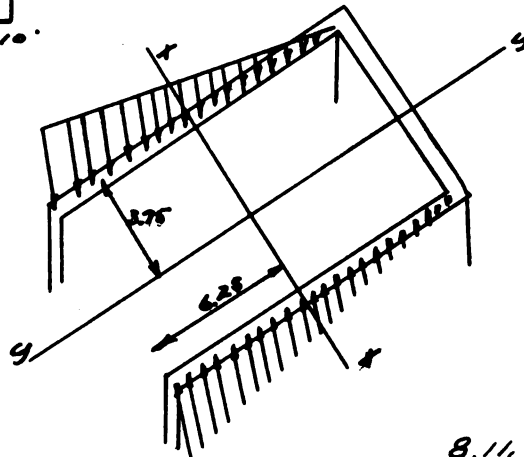
$$P_2 = 3.33K$$



$$P_1 + P_2 = 5$$

$$3P_1 = 5$$

$$P_1 = 1.67K$$



8, 11, 14, 20, 21

Prob 170 "Cont"

Mem	L	T	A	x	A _x	y	A _y	X _o	Y _o	W	M _{x1}	M _{xy}	I _{xx}	I _{yy}	I _{xy}
AD	10	10	1	0	0	5	5	-3.75	-1.25	8.33	-104	-31.2	990	14.05	14.77
BC	10	20	$\frac{1}{2}$	5	2.5	10	5	+1.25	+3.75	0	0	0	7.05	4.95	12.3
CD	10	20	$\frac{1}{2}$	10	5	5	2.5	6.25	-1.25	-8.33	+104	-52	4.95	19.60	-3.9
Σ			2		7.5		12.5			0	0	-83.2	21.90	38.50	13.11

$$X_o = \frac{7.5}{2} = 3.75$$

$$Y_o = \frac{12.5}{2} = 6.25$$

$$M_D = 33.3 - \left[0 + (6.25) \left(\frac{-83.2 \times 21.9 - 0}{834.3} \right) + \left(\frac{-6.25}{834.3} \right) (-83.2 \times 3.11 + 310.5) \right] = 17.7' \#$$

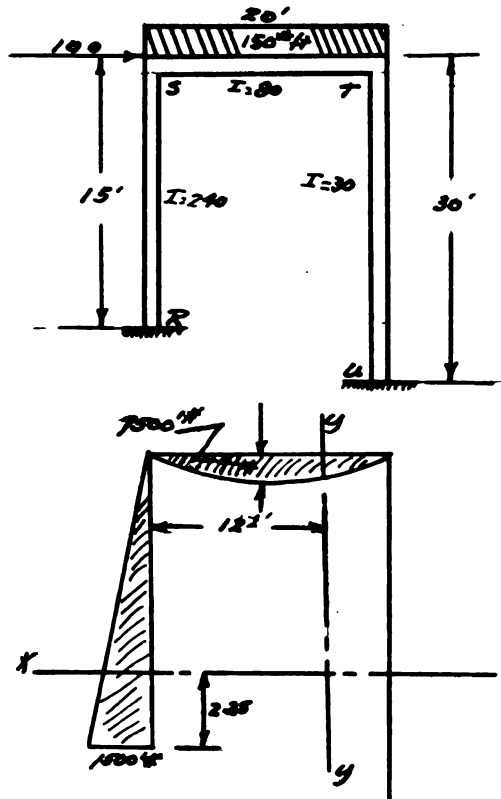
$$M_k = 0 - 13.65 + (3.75 \times 3.105) = 12.5' \#$$

$$M_B = (-2.185) (-3.75) + 1.2 = 9.4' \#$$

$$M_A = -16.7 + 8.2 - 1.9 = 10.4' \#$$

Problem 172, Page 237

Repeat the calculations of Fig. 177 for a bent of identical dimensions but where the moments of inertia of the vertical legs are interchanged. Check the results by use of principal axes.



Mem	L	I	A	x	Ax	y	Ay	x _o	y _o	W	M _{xx}	M _{yy}	I _{xx}	I _{yy}	I _{xy}
RS	15	240	.063	0	0	7.5	.47	-12	+5.25	.7	1.93	-8.4	12.9	9.0	-3.93
ST	20	80	.25	10	2.5	15.0	9.75	-2	12.75	25	31.0	+50	40.4	9.25	-6.37
TU	30	30	1	20	20	0	0	+8	-2.25	0	0	0	80.3	64.0	-16.00
Σ			1.88		22.5		4.22			24.3	31.7	41.6	133.6	82.25	-28.3

$$X = \frac{22.5}{1.88} = 12 \quad Y = \frac{4.22}{1.88} = 2.25$$

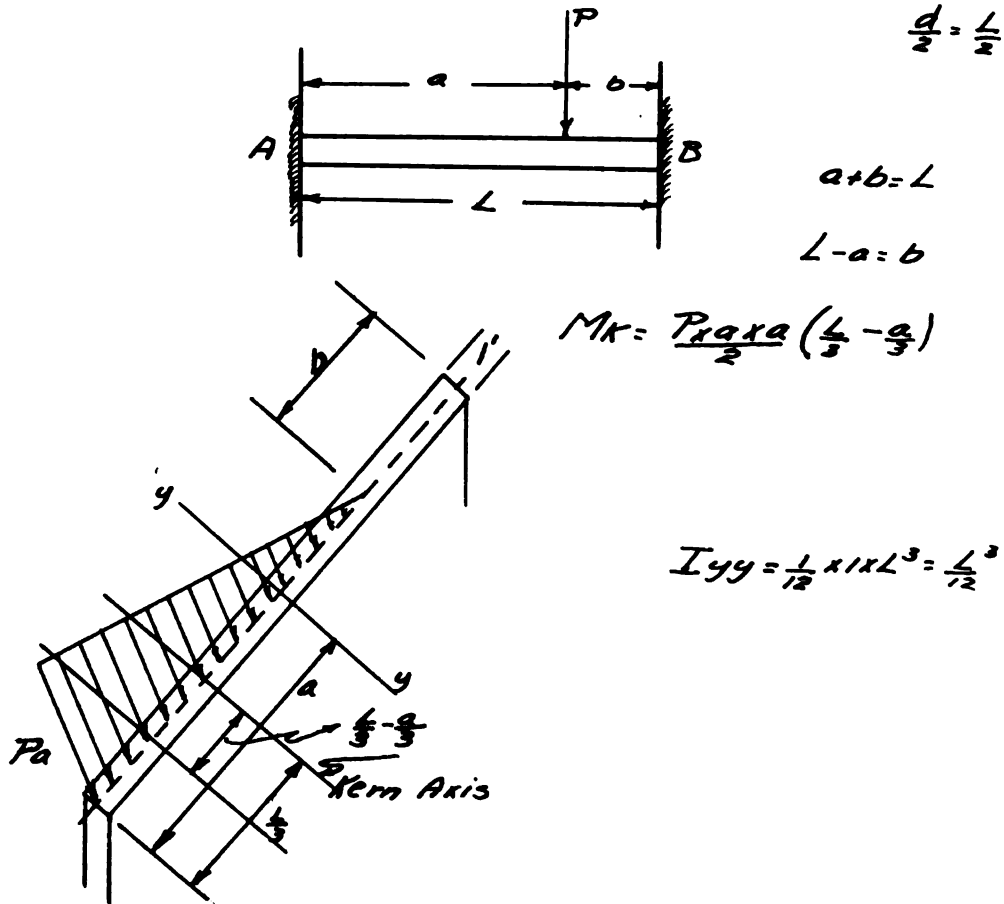
$$M_R = 1.5 - \left[\frac{-24.3}{1.88} + (-12) \times \frac{41.6 \times 133.6 - (-31.7 \times -28.3)}{133.6 \times 82.25 - (28.3)^2} + (-2.25) \frac{(-31.7 \times 82.25) - 41.6(-28.3)}{94.70} \right]$$

$$= 1.5 - [-12.9 + 4.4 + 6.0] = 1.5 - (-2.5) = 4.0 \text{ K.F.}$$

8.11.14.20.21

Problem 173, Page 238

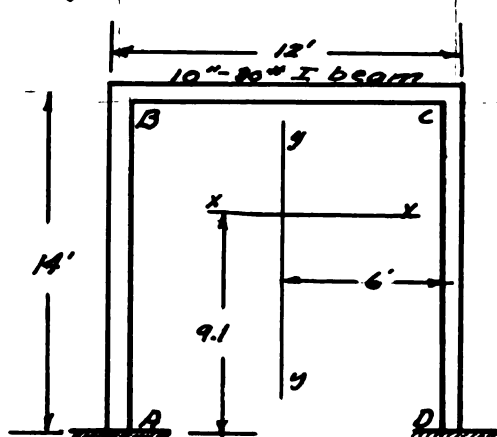
Make use of the kern relationship in the column analogy to determine a general expression for the end moment in a fixed end beam carrying a concentrated load P at distances a and b from the two ends.



$$f = \frac{\frac{P a^2}{2} \times \frac{1}{3} (L - a) \frac{L}{2}}{\frac{1}{12} \times 1 \times L^3} = \frac{P a^2 b}{L^2}$$

Problem 175, Page 242

A fixed end bent 14 ft. high and 12 ft. wide is made up of 10 in. 30# I beams with their webs placed in the plane of the bent. Find the maximum fiber stress produced by each of three movements of the left footings, namely $x=1$ in., $y=1$ in. and $\theta=0.1^\circ$. Use the dimensions given as center line dimensions.

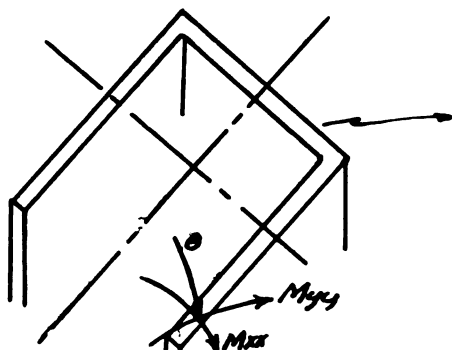


$$E = 30 \times 10^6 \quad I = 133.5$$

$$I/c = 26.7$$

②

Mem	L	I	A	A'	x	A _x	y	A _y
AB	14'	133.5	14x12 ÷ EI	7	0	0	7	49
BC	12'	"	12x12 ÷ EI	6	6	36	14	84
CD	14	"	14x12 ÷ EI	7	12	84	7	49
Σ			12x10 ⁻⁸	20		120		182



$$M_D = f = (9.1 \times 12) \frac{1 \times 10^{-4}}{3 \times 72} = 293000 \text{ #}$$

$$S = \frac{293000}{26.7} = 11000 \text{ #/in}^2$$

$$\bar{x} = 6 \quad \bar{y} = \frac{182}{20} = 9.1$$

$$\frac{1}{EI} = \frac{1}{30 \times 10^6 \times 133.5} = \frac{2.495}{10^{10}}$$

$$I_{xx} AB = 1.240 \times 10^{-4}$$

$$I_{xx} BC = 1.240 \times 10^{-4}$$

$$I_{xx} CD = 1.240 \times 10^{-4}$$

$$I_{xT} = 3.72 \times 10^{-4}$$

8.11.14.20.21

Prob 175 Cont

⑥ $A_y = 1''$ $M_{yy} = 1$

$$I_{yy} = 2 \left(\frac{2.495}{10^{10}} \times 14 \times 12 \times \left[\frac{2.495}{10^{10}} \right]^2 + \left(\frac{2.495}{10^{10}} \times 14 \times 12 \times 72^2 \right) + \left(\frac{1}{12} \times 12^3 \times \frac{2.495}{10^{10}} \times [12 \times 12]^2 \right) \right)$$

$$= \left(\frac{4.18}{10^8} \times 5180 \right) 2 + \frac{6210}{10^8} = 4.965 \times 10^{-4}$$

$f = 72$ $\frac{1 \times 10^4}{4.965} = 145000''$

$S = \frac{145000}{26.7} = 5.440''$

⑦ $\theta = .1^\circ = .00175 \text{ rad}$

$M_{yy} = .00175 \times 6 \times 12$

$M_{xx} = .00175 \times 9.1 \times 12$

$A = 12 \times 10^{-8}$

$I_{yy} = 4.965 \times 10^{-4}$

$I_{xx} = 3.72 \times 10^{-4}$

$$f_c = \left(\frac{.00175 \times 10^8}{12} + (4.9 \times 12) \left(\frac{.00175 \times 72 \times 10^4}{4.965} \right) + 72 \left(\frac{.00175 \times 9.1 \times 12 \times 10^8}{3.72} \right) \right)$$

$= 24580 + 24920 + 37000 = 88000''$

$S = \frac{88000}{26.7} = 3330''$

Explanation of Symbols

Δ (delta)	deflection, usually caused by the main loads.
$\Delta A, \Delta B, \Delta C$	changes of angles of a triangle caused by stress.
Δf	change of the sag of a suspended cable.
ΔL	change in length of a member.
δ (delta)	deflection, usually caused by a unit load.
δL	change in length of a small fiber.
$\delta \theta$	small change in .
θ (theta)	end slope of a member of a rigid frame.
κ (kappa)	factor defining change in stiffness due to end restraint.
ρ (rho)	rotation of a member from joint translation.
Σ (sigma)	summation sign.
ϕ (phi)	end angle change of a pin end member
a	short length, usually a part of a span.
A	cross-sectional area, particularly of the analogous column.
A_s, A_c	areas of the S and C diagrams.
b	numbers of bars in a space frame.
c	extreme fiber distance for cross-section resisting bending.
C	carry over factor used in balancing angles.
C_{AB}	fixed end moment at A of the beam AB.
d	depth of a beam or of any cross-section.
e	eccentricity of a load; coefficient of temperature change.
E	modulus of elasticity
f	unit stress; sag of a cable in a suspension bridge
G	shearing modulus of elasticity.
h	height of a truss or story; rise of an arch.
H_o, V_o, M_o	neutral point forces; thrust, shear, moment.
H_{AB}	C (slope deflection)
I_x, I_y, I_{xy}	moments of inertia and product of inertia.
j	number of joints in a space frame.
J	polar moment of inertia.
k	defined by the relation $a = kL$
K	stiffness or I/I ; subscripts c and g refer to column and girder.
L	length as of a truss bar or of an entire span.
m or m'	virtual moment at any point of a structure.
M_{AB}	moment at end A of member AB (slope deflection); moment about AB.
M_{fs}	fixed end moment, see C for slope deflection usage.
M_K	kern moment.
M_s, M_i	static and indeterminate moments.
M_t	total moment in a story, wind shear times story height.
M_{x-x}, M_{y-y}	moment of forces about x-x and y-y axes.
M_{xy}, M_{xz}	moment of forces in the xy or xz plane.
O	designation of the neutral point, used as a subscript.

Explanation of Symbols
Continued

p	number of space joints where all bars lie in one plane.
Q	statical moment of the cross-section of the analogous column.
r	radius of a moment center.
r-ratio	stiffness of columns to girders.
S	total stress in a bar.
t	change in temperature in degrees F.
t'	torque moment caused by the virtual loading.
v-value	restraint factor in moment distribution.
v'	total shear caused by the virtual loading.
w	unit load.
W	total load; Pds/El load; total work.
x_o, y_o	coordinates from the neutral point.
$\bar{x}, \bar{y}, \bar{z}$	distances referred to a centroid.
X, Y, Z	redundant stresses or reactions.

Bibliography

1. American Institute of Steel Construction-Steel Construction, 173-300, 358-371, American Institute of Steel Construction, New York, 1941.
2. Abbott H.H. and Doland J.J.-Water Supply Engineering, 466, McGraw-Hill, New York, 1929.
3. Bethlehem Steel Company-Catalogue S-30, 284-304, Bethlehem Steel Company, Bethlehem Pa., 1929.
4. Bishop C.T.-Structural Design, 195-222, John Wiley & Son, New York, 1938.
5. Jaughay R.A.-Reinforced Concrete, 163-193, D. Van Nostrand, New York, 1936.
6. Brinker I.E.-Design of Modern Steel Structures, 385-395, MacMillan, New York, 1941.
7. Brinker I.E.-Theory of Modern Steel Structures Vol.1, 77-100, 188-196, 226-284, MacMillan, New York, 1936.
8. Brinker I.E.-Theory of Modern Steel Structures Vol.2, MacMillan, New York, 1937.
9. Hool G.A. and Hinne W.S.-Steel and Timber Structures, 402-438, McGraw-Hill, New York, 1924.
10. Hool G.A. and Hinne W.S.-Stresses in Framed Structures, 476-445, 448-466, McGraw-Hill, New York, 1923.
11. Laurson F.C. and Cox W.J.-Mechanics of Materials, 101-206, 211-230, 270-300, 301-315, John Wiley & Son, New York 1938.
12. Seeley F.B. and Ensign K.T.-Analytical Mechanics for Engineers, 93-99, 99-101, John Wiley & Son, New York, 1933.
13. Seeley F.B.-Resistance of Materials, 88-63, 266-276, 298-306, 307-317, John Wiley & Son, New York, 1935.
14. Shedd T.C.-Structural Design in Steel, 244-255, 293-296, John Wiley & Son, New York, 1934.
15. Shedd T.C. and Vawter J.-Theory of Simple Structures, 153-156, 340-413, 440-465, John Wiley & Son, New York, 1941.
16. Sutherland H. and Bowman F.L.-Structural Design, 4-24, 41-67, 141-157, John Wiley & Son, New York, 1938.
17. Sutherland H. and Clifford W.W.-Reinforced Concrete Design, 289-325, John Wiley & Son, New York, 1926.
18. Timoshenko S. and MacCullough G.H.-Elements of Strength of Materials, 190-224, 292-303, D. Van Nostrand, New York, 1936..
19. Urquhart I.C. and O'Rourke C.I.-Elementary Structural Engineering, 192-216, McGraw-Hill, New York, 1941.
20. Urquhart I.C. and O'Rourke C.I.-Design of Steel Structures. 24-26, McGraw-Hill, New York, 1930.
21. Young C.R.-Structural Problems, 191-211, John Wiley & Son, New York, 1935.

Notes on Contents

Problem 7, page 11.

Because this structure is determinate, the stresses in the members can be found very easily by writing the equations of equilibrium for non-coplanar, concurrent forces at each joint. Page 13 shows the method of tabulating these equations to expedite their solution as simultaneous equations.

Problem 9, page 16.

This is intended to be considered only as a planar structure, and the ordinary methods of Free Body Diagrams and the equations of equilibrium for concurrent forces can be applied.

Problem 11, page 19.

This could be solved by several methods, but the Area Moment Proposition is the most readily understood and applied.

Problem 13, page 20.

An engineering theorem which follows from the conservation of energy, states that the internal work of deformation equals the external work of the applied loads; accordingly if there is but one concentrated load on the structure, the deflection of it's point of application in the direction of the load can be determined from the equation of real work.

Problem 16, page 23.

Castigliano also deals with internal work as produced by a gradually applied load. Castigliano's law states "The derivative of the total internal work with respect to any one load is equal to the deflection of the load point in the direction of the load."

Problem 19, page 26.

The method of virtual work as used in this problem is widely accepted as the most satisfactory method of determining the deflection of any point on a body, in a direction caused by all possible kinds of internal distortions.

Problem 24, page 32.

Virtual work methods are very easily applied to this type of problem because the steel pipe has a combination of bending and torsion in several different planes.

Problem 28, page 39.

The Area Moment Proposition method is the most expedient method for this type of problem.

Problem 32, page 41.

Since deflection may be computed as the bending moment in the conjugate beam, produced by it's M/EI loading, it is

clear that the elastic curve of the real beam is identical with the moment diagram for the conjugate beam. Hence we can draw a force diagram and a funicular polygon for the M/EI loads; so that the funicular polygon will represent the moment diagram for the conjugate beam or the elastic curve of the real beam.

Problem 30, page 51.

The application of the Area Moment Proposition can be extended to the field of truss deflections, where it is known as the method of elastic weights or angle weights.

Problem 46, page 52.

This type of problem is best worked by the Theorem of Three Moments. It is a so called "Classical" method of analysis. The formulas can be developed by double integration, combined with the Area Moment Proposition. It is possible to work this by other methods, but they are less practical.

Problem 42, page 48-Problem 43, page 49-Problem 44, page 50.

The Area Moment Proposition is the most practical solution for these types of problems.

Problem 47, page 54-Problem 48, page 56.

The Theorem of Three Moments, ordinarily applied to a continuous beam, is here applied to a combination of continuous and restrained beams. We can apply it to a restrained beam by considering the restraint to be removed and replaced with a beam of zero length and zero load.

Problem 49, page 57.

This problem is worked by the Moment Distribution method. The moments at the supports are found in the ordinary way and then are distributed to each beam in the ratio of their K values.-see bottom page 57.

Problem 51, page 59.

This truss is internally indeterminate to the first degree; hence, we remove bar to obtain the static structure. The stress in each bar is expressed in terms of the applied load, and the redundant stress X . A redundant bar is a bar that is designed to take only one kind of stress and is very often subjected to the opposite kind. Under these conditions it does not act and hence is called inactive or redundant.

Problem 55, page 61.

This truss is both internally and externally indeterminate to the first degree. Hence, we remove one reaction and cut one bar to obtain the static structure. The stress in each bar is expressed in terms of the applied load,

the redundant stress X and the redundant reaction Y .

Problem 68, page 71.

The use of the slope deflection method for the analysis of continuous beams and frames illustrates a different procedure from any previous problem. Each of the previous problems has made use of certain stress functions, such as bar stresses, joint moments or reactions as redundant. It is clear that the joint rotations and deflections of a continuous beam or frame would serve equally well as unknowns. Thus, there is developed a slope deflection method of analysis. This method, while useful, is by no means advantageous, as an unusual sign convention has initiated against its acceptance.

Problem 65, page 73.

The Balancing Moments Method was originated by Hardy Cross and is often referred to as the Hardy Cross Method. It is applicable directly to any frame where the joints rotate, but do not translate. The method depends upon a single physical conception, the idea that joints of the unloaded structure actually might be locked against rotation, and then, when the loads were applied, that these joints might be successively released and relocked, one by one, into new positions, as many times as necessary, until finally each joint would have rotated into its proper position.

Problem 82, page 100.

The application of the full slope deflection method to the analysis of a multi-storied building is too cumbersome to be useful. The analysis of a ten story, unsymmetrical frame, three bays wide would require the solution of fifty simultaneous equations. The method here used is an approximate one and is very useful. Any story is isolated from the rest of the building, and it is assumed that the side lurch or ρ value of the story above and below are the same as for the story under consideration. It is further assumed that the θ values for joints I and II are the same as for joints B and D, and that the values for joints I and J are the same as for A and C. Hence for any one story, there are only five unknowns to be determined, $\theta_A, \theta_B, \rho, \theta_D$ and ρ .

Problem 97, page 179.

The shape of the influence line is commonly used in the selection of an equivalent uniform live load. This explains why the analysis of the three hinged arch for live load will be studied by means of the influence line. The horizontal thrust in a three hinged arch is statically determinate; the influence line for horizontal thrust H is a triangle with maximum ordinate at the center hinge. Since the moment at the center hinge must be zero, we may equate the simple beam moment that would exist there to the moment produced by H .

Problem 101, page 116.

This structure, after adding the pin at the midpoint of the stiffening truss, is statically indeterminate to the first degree. The redundant member usually is taken as the horizontal component of the stress in the cable. The influence line for H can be obtained as the deflected load line, when the cable is cut at it's lowest point and is shortened by a unit amount.

Problem 105, page 120.

This is a two hinged span, and the simplest method of calculating the maximum deflection of a two hinged span, is to compute the deflection of the stiffening truss in the center span as a simple beam, loaded with the difference between the uniform live load, and the uniform hanger pull. If the exact value of H for live loading on the center span is known, then the hanger pull per foot, p , is $8Hf/L^2$, and the effective uniform load, which deflects the stiffening truss downward becomes $W - 8Hf/L^2$.

Problem 106, page 124.

The Balancing Moments method was first referred to in Problem 65 page 177 of this appendix, but a little more discussion probably could be carried on in this article. The following steps should be followed in Balancing moments.

- 1-Compute fixed end moments for each loaded span and record them with correct signs at their proper locations on a diagram of the structure.
- 2-Compute distribution factors at the joints from the K values; record these values inside a box at each joint.
- 3-Select any joint which has a large unbalanced moment, and compute the balancing moment from the distribution factors. Record each balancing moment underneath the corresponding fixed end moment. Balancing moments are of opposite sign to the unbalanced joint moment. Draw a horizontal line under each balancing moment, to show that the moments above this line are balanced and need not be balanced again.
- 4-Record the carry over moments at the far ends of the members. Place the carry over moments in a column under-

the fixed end moments or the balancing moment existing there. Each carry over moment is of the same sign as the corresponding balancing moment, but of one half its value. They must be considered as additional fixed end moments at the joints where they are recorded.

5-Successively balance each other joint of the structure, taking care to record balancing moments and carry over factors with proper sign. Note that the carry over moments unbalance, previously balanced joints, and that such joints must be rebalanced. Only those moments recorded below the last horizontal lines are considered during the process of rebalancing.

6-The process of balancing joints is carried on in any desired sequence, until the carry over moments become negligibly small, which shows that the joints are balanced. This result will be reached after each joint has been balanced about three times. The time required should not exceed ten minutes per span for a continuous beam, even when the analysis is made by an inexperienced operator.

Problem 113, page 128.

This is the application of the Balancing Moment method to a frame, instead of a continuous beam. The method is exactly the same as explained in the previous article.

Problem 117, page 132.

This is a simple case of joint translation, which may occur by accident or design. However, the regular method of balancing moments, coupled with a simple application of the Area Moments can be used.

Problem 114, page 139.

One cannot fully understand this method of successive corrections, unless he can form a mental picture of the physical behavior of the structure, that corresponds to each step of the process. It is desirable, to consider that joint rotation and joint translation as separate movements to be permitted alternately. The structure is allowed to move with joints fixed against rotation. Then, the joints are allowed to rotate, but the structure is restrained against translation. This is permitted as many times as possible, until the structure reaches its true deflected position, with all imaginary restraints removed.

Since each joint is to be balanced only once before additional fixed end moments are introduced into the column, it is known ahead of time, that such correction moments will be positive. The following rules should be helpful in this kind of solution.

1.-The structure is allowed to deflect under the loads while the joints are fixed against rotation and the fixed end moments are calculated.

- 2.-The fixed end moments are balanced around the joints until each joint is in equilibrium; that is the summation of moments at each are equal to zero. During this process, no translations of the joints are allowed. This step is the same as the Cross method of balancing fixed end moments.
- 3.-From the balanced moments of step 2, the column shears are determined, and from the differences between the calculated story shears and the true story shears, the artificial restraining forces at the joints are found.
- 4.-The artificial joint restraints found in step 3 are eliminated by applying equal and opposite joint forces.
- 5.-The structure is allowed to deflect with fixed joints under the action of this new force system, fixed end moments are calculated, joint moments are balanced, and a second set of joint restraints is determined. It will be found that this second set of joint restraints is smaller and of less importance than the first set.
- 6.-The process is repeated until a final set of joint restraints is determined that is small enough to neglect entirely.
- 7.-The final moments are obtained by adding algebraically, the several sets of balanced moments obtained from the original loads, and the several sets of forces that balance the artificial joint restraints.

Problem 137, page 145.

A bent such as this one is best analyzed for wind moments by the general method of successive corrections. As a starting point, one must be able to distribute the shears produced by the wind forces, among the columns when the joints are held against rotation. This distribution is made according to the stiffness ratio.

Problem 146, page 153.

The length of the arch ring is divided by trial with a pair of dividers into eight sections, which scale approximately 4.35 feet each. The centers of the divisions are then located, the coordinates x and y from the left hand springing, of each center point being scaled. The arch is cut away at the left abutment; thus it becomes a cantilever from the right hand end. The value of $ds/2I$, which is constant for each division, will be taken as unity. Since it appears in each term of the simultaneous equation that will be written, $ds/2I$ may be given a unit value, whenever it is constant.

Problem 160, page 154.

This problem is solved by the "Column Analogy" method, combined with what is commonly called the "Neutral

Point Conception". This Neutral Point Conception allows us to represent certain Arch properties as zero quantities, if the origin of coordinates, or the point of application of the redundant reactions $(1, V, 1)$ is changed to the centroid of the elastic weights and where either the x-x or y-y axis is an axis of symmetry. Therefore, for a standard symmetric arch, the N.P. is on the vertical axis of symmetry.

In the development of the column analogy from the Neutral Point method, the following equation is arrived at; $M_i = M_o + \left[\frac{W}{A} + \frac{M_o}{I_x} y + \frac{M_o}{I_y} x \right]$

This equation, which represents the indeterminate moment M_i caused by the neutral point reaction, corresponds in form to the common equation for determining the stress in an eccentrically loaded column, namely

$$S = P/A + M_x/I_x + M_y/I_y$$

In the conjugate beam method, one visualized the beam as loaded with the M/EI diagram, hence there should be no difficulty in visualizing it here, as a load acting upon an analogous column. If such a column is visualized, A must be it's cross sectional area, I_x -x, I_y -y the moments of inertia about the x-x and y-y axis through the neutral point. This analogous column is loaded with an imaginary distributed load W , which, because of it's eccentricity produces tilting moments I_x -x and I_y -y about the centroidal axis. Then the indeterminate moment M_i is found at any point in the ring as the stress existing there in this eccentrically loaded column.

A Short Discussion of the Theory of Three Moments
and Area Moment Proposition, using the Bending
Moment Diagram by Parts Application.

The originator of the Theory of Three Moments, Emile Clapeyron (1799-1864), was the consultant engineer on some very important research work carried on in Russia in the year 1824. He tested the strength of Iron bridges, that were encountered while on a military mission for the French Government.

In 1883, he set forth a theory concerning the equality of internal work of stresses, to the external work of the loads. This theory is the basic principal of our modern theory of indeterminate structures.

About 1857, he presented the Theory of Three Moments for the case of a uniformly loaded span of uniform cross-section. Due to Professor Clapeyron's retiring nature, he never published this theory. The importance and wide application of the method has since been realized, and it is now given mention as one of the important theories regarding the solution of beams and frames.

The method involves the considering of any three successive spans of any continuous span beam, starting from the left end, as one unit. Equations are then formed in terms of the bending moment over each support, length of each of the three spans, and the statical moment of the bending moment diagram. This bending moment diagram is a result of whatever system of loading is present on the span.

The usual application of this method has been to continuous beams, simply supported at their extreme ends. The beams can be of any number of spans of the same or different lengths, and of the same or different cross-sections. The loads can be of any kind, concentrated, uniform, or any any combination of the two.

While it is not mentioned in text books, it can also be applied very advantageously to other structures, such as four sided frames and restrained beams. This is quite a unique application of the Theory of Three Moments, and on numerous occasions in this Thesis, it has been used as the principal method of solution. The application to restrained beams offers a much more simple solution than any of the standard methods.

To apply it to a restrained beam (one embedded in masonry at both ends), we must remove the restraint from each end, and replace the same with a span of zero length, and zero load. We then have the equivalent of three spans, loaded as desired. Our equations, as mentioned at the beginning of this article, can then be applied.

This can still be a very complicated procedure, if we draw the ordinary bending moment diagrams. However, by using the Area Moment Proposition method, with the Bending Moment Diagram by parts, instead of a composite bending moment diagram, we can simplify the taking of the statical moments.

The bending moment diagram by parts, as used in these

solutions, in the first place, to the problem of the Area Moment Proposition. It means, that we plot the bending moment diagram for each load separately, instead of drawing the composite diagram. This makes it much easier to find the distance from a moment center to the centroid of the bending moment diagram.

These statical moments are then incorporated into the proposition that The statical moment of the area of the Bending Moment diagram between any two points A and B, divided by the product of E , the modulus of elasticity, and I , the moment of inertia, is equal to the deflection of one of the points above or below the other. For instance, if we take the statical moment of the area about the point B, then we would find the deflection of B above or below A.

ROOM USE ONLY

May 19'44

8701 81 40

Jun 29'50

21 21 32

ROOM USE ONLY

MICHIGAN STATE UNIVERSITY LIBRARIES



3 1293 03177 4312