STRUCTURAL SUPPORTS FOR REFINERY VESSELS

Thesis for the Degree of M. S.

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Zigurds Janis Michelsons

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This is to certify that the

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STRUCTURAL SUPPORTS

FOR REFINERY VESSELS

by

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A THESIS

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INTRODUCTORY STATEMENT

The subject matter of this thesis was chosen in order to become acquainted with the applications of structural engineering to the petroleum industry and to determine some of the special problems encountered by a structural engineer in the design of a petroleum refinery.

While the theory used in the design of structural parts of an oil refinery is not basically different from structural design in general, there are design requirements and loading conditions that have created some distinct types of structures and details best adapted to the needs of the chemical and petroleum industry.

As there are no books published on the subject of structural supports for refinery vessels, all the information had to be obtained from the oil industry and from articles published in periodicals. Only a limited number of articles can be found in periodicals of the petroleum industry, obtainable from specialized libraries; because the subject is too specialized for discussion in general civil engineering magazines and also is not the direct concern of chemical engineers.

Thus, a major part of the effort in writing this discussion had to be spent in finding possible sources of information. After obtaining and evaluating the information, some insight in the requirements and problems of the oil industry has been gained, conditioned, of course, by the fact that the writer has no actual experience in the industry.

The first part of the thesis is a general discussion of some aspects of the design of structural supports for vessels of a refinery. The second part is a moment analysis of a simple frame that supports a vertical vessel.

ACKNOWLEDGEMENTS

The assistance by numerous organizations in providing informative materials for this thesis is hereby gratefully acknowledged. Names of these companies, who have helped by sending pamphlets, reprints of articles, illustrative plans and suggestions, are listed alphabetically on the following page.

A special acknowledgement is given to Dr. Richard H. J. Pian of Michigan State College under whose supervision this thesis has been prepared, and to Dr. F. E. Wolosewick, an authority on structural design in the oil industry, whose design charts have been the starting point for the mathematical analysis, using the Moment Distribution method, in the second part of this thesis.

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Union Oil Company

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PART ONE

GENERAL DISCUSSION

Supports of vessels in refineries range from simple pedestals with a footing to huge open-type frameworks of skyscraper proportions, supporting numerous vessels, equipment and a maze of piping, as well as operating and inspection platforms, stairways and ladders.

LOADS

The most important part in the design of a structural support is the determination and evaluation of loads and loading combinations to be supported. The following loads and forces must be considered: dead loads, live loads, impact, vibration, thermal forces, test loads, erection loads, wind and earthquake.

Dead loads include the weight of the structure, empty equipment and piping, and its fireproofing and insulation. For piping under one foot in diameter the structure is often designed by assuming a specified uniformly distributed loading, in order to simplify calculations. Points of support for larger pipes must be predetermined and considered as concentrated loads including the weight of the pipe, the fittings, valves, insulation and the weight of the fluid.

Live loads consist of movable loads, including personnel, portable machinery and equipment, and gravity forces caused by the fluid in the equipment and piping under normal operation. But what forces are created by the liquid, solid or viscous materials in the equipment must oftentimes be determined from observations of the performance of similar equipment in the past.

Because provisions must be made for mechanical handling of materials, equipment and parts, under operating and replacement conditions, the structural framing must support, in many cases, also elevators, trolleys, beams, and hoists in convenient locations. The frame must be designed to resist not only the weight of these handling devices and the weight carried by them, but also the vertical and horizontal impact forces due to their operation.

The disturbing forces produced by equipment having a tendency to vibrate, and by surging fluids must be considered in the design of the supports. Possible vibration caused by nearby vibrating equipment must also be investigated.

Because many of the vessels operate at high temperatures, large thermal expansion forces are set up by the vessel and the piping, that must be absorbed by the structural frame. The temperature differential is

minimized by proper insulation of the vessels and piping or by facilitating heat transfer to the supports by continuous welds or heating coils in the supports. Nevertheless, thermal expansion cannot be avoided and must be taken care of by arranging the supports to permit clearances for expansion of the vessels. To minimize the thrust of the expanding vessels and piping on the supports, sliding bearings and expansion joints are provided.

Before putting the completed structure into operation, it is usually tested by filling the vessel and the piping with water. In most cases water is heavier than the operating fluid, therefore this is considered as a separate loading condition.

Temporary loads and forces may be caused under erection conditions and must be considered in the design of the structural supports.

Wind forces are computed assuming a uniformly distributed load on the vertical projection of the vessel, piping and framing. The magnitude of this uniform load is computed by considering the design wind velocity and the shape and height of the exposed surfaces. Commonly used wind design velocity is 100 M.P.H., giving a wind pressure of about 30 lbs. per square foot.

Where necessary, an earthquake force is considered. The most commonly assumed seismic force is 0.1 or 0.2 of the weight of the mass acting horizontally at the center of gravity.

LOADING COMBINATIONS

After determination and evaluation of all these loads and forces, the structure has to be analyzed for the critical loading conditions. Ordinarily the loads to be considered in each loading condition are specified in standards of the organization designing the structure[33]. In general, the structure is investigated for the erection, testing and operating condition.

The erection condition gives the most critical case for stability of the structure due to the overturning moment of wind or earthquake, with the least stabilizing moment of the supports and the empty equipment. Under erection condition the specified minimum stability factor is ordinarily less than under the operating condition. Also the allowable stresses may be increased by one third when uncluding wind or earthquake forces in the design; this, however, does not necessarily permit increase of the safe soil bearing values.

The testing condition includes forces due to the weight of the completed structure and equipment plus the weight of the test fluid filling the vessel and pipes. No wind or earthquake force is included in this case or, at least, the forces are reduced in magnitude.

In analyzing the structure for the normal operating condition, full wind or earthquake force, whichever is greater, is considered, in addition to the weight of the completed structure and equipment, the operating weight of fluid, applicable live loads from platforms and walkways, thermal forces, vibration and impact. Only live loads from permanent storage are included when checking the stability against horizontal overturning forces or when designing anchor bolts.

VESSELS

The vessels to be supported are of cylindrical form with elliptical, hemispherical or conical heads. The rounded outline of the vessels is necessitated by the fact that many of the vessels operate under high pressure, and the sphere and the cylinder, of course, are best suited as containers of pressure. In addition to the internal pressure, the cylindrical shell of the vessel has to resist external forces at points where the vessel is attached to its supports and it has to act as a hollow beam in resisting the bending moment due to wind or other horizontal forces. In most cases, ladders, platforms, cranes or other equipment are supported from the vessel shell, as well as internal brackets and beams supporting internal parts of the vessel. Complex stresses of highly indeterminate nature are caused in the vessel shell at these points of support.[16] The stress analysis of the vessel shell is made more difficult by the high temperatures at which many of the vessels have to operate, requiring the vessel to be built of special material, becoming a problem partly in metallurgy. Investigations aimed at determining more exactly the stresses in the vessel shell due to the various factors and finding a practical design method are very much welcomed by the oil industry.

The design and construction of the vessels is largely governed by the "API-ASME Code for the Design, Construction, Inspection, and Repair of Unfired Pressure Vessels for Petroleum Liquids and Gases" that specifies the allowable stresses for different conditions. Different strength theories have been used for computing the required thickness of the shell. The Maximum Shear theory seems to be the most widely used; others are Maximum Stress theory, Maximum Strain theory, and the Modified Strain-Energy theory[23].

The Code specifies a minimum permissible shell thickness as a function of the diameter of the vessel. The required shell thickness is computed to resist the internal pressure, the weight of the vessel, its

contents and superimposed loads, and bending moments from wind or other forces. The bending stresses are computed by the flexure formula, considering the vessel as a thin-walled hollow beam with a section modulus $\frac{\pi D^2 t}{4}$[2]. The height of thin-walled tall towers may be limited by its strength against buckling, the maximum height being a function of the allowable compressive stress in the shell[10]. To the computed required plate thickness a corrosion allowance (usually 1/8") is added, to insure longer life. The inner parts of the vessel, being exposed to chemical erosion, may require the protection of wear plates in addition to a corrosion allowance.[22]

Most vessels are of welded construction because high temperatures and pressure preclude the use of riveted construction, also a smooth inside is required.[29] Welded construction permits highest efficiency of joints and effects savings in materials and weight.[18]

When considering their supports, cylindrical vessels may be divided into vertical and horizontal vessels. Vertical pressure vessels vary in height and diameter. Also the ratio of height to diameter varies from a drum with proportions of a barrel to slender fractionating columns in excess of 150 feet in height and only about 10' diameter.

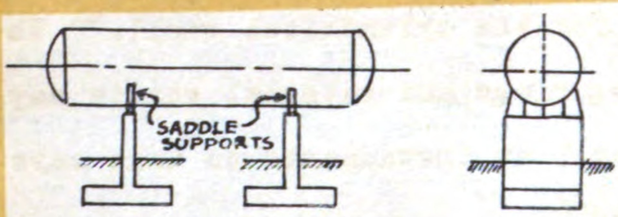


FIG.1

HORIZONTAL VESSEL SUPPORTS

Horizontal vessels usually rest on saddle supports (Fig. 1). Welded saddle supports insure a more uniform distribution of bearing stresses in the vessel shell than other types of supports. The supports should be set at such a distance from the ends of the vessel, so that the stresses in the shell are the same at the center of the span as over the supports, or else the supports should be located near the ends where the head acts as a stiffener for the cylindrical shell.[21] When necessary, stiffening rings and internal struts may be installed in the vessel at the supports to take care of the bending stresses.[34]

Usually only two piers are used for supporting horizontal vessels. Using separate footings for more than two piers, differential settlement may distribute the bearing stresses at the supports unequally. Also the vessel may be lifted off the middle support due to a possible greater expansion at the top of the vessel which is more exposed to sun's heat and less cooled by the liquids contained in the vessel.[21]

Generally, a slide plate is provided at the free end of the vessel to take care of thermal expansion. Rollers may be required for expansion of horizontal vessels operating under high temperatures. In that

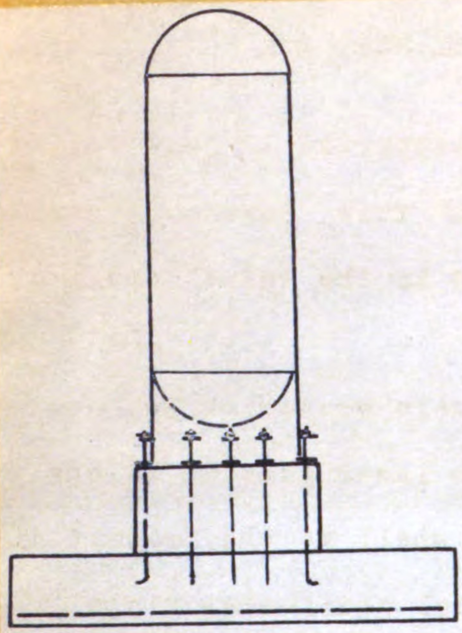


FIG. 4

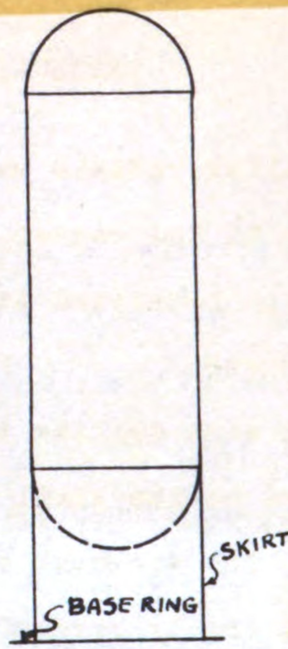


FIG. 2

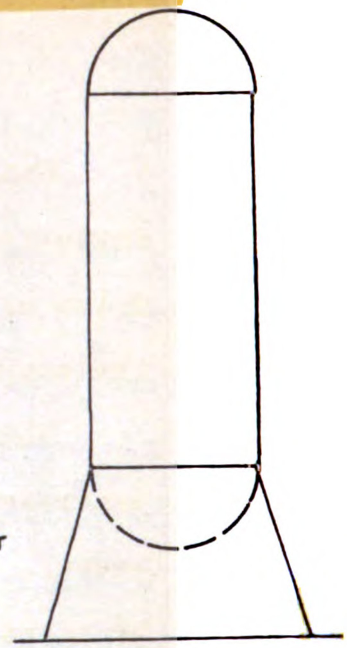


FIG. 3

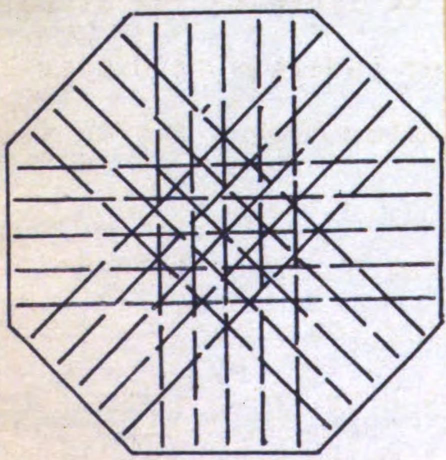


FIG. 5

case the fixed-end pier may have to be designed to resist the total seismic force parallel to the longitudinal axis of the vessel.

VERTICAL VESSEL SUPPORTS

Smaller vessels may be supported by legs welded directly to the vessel shell. This, however, introduces high localized stresses in the vessel shell at the supports.

The same applies to vessels supported by lugs bracketed to the shell at the lower portion of the vessel. The strength of the shell at the support may be increased by circumferential stiffening rings.

Most large vessels rest on an extension skirt of steel with a base ring (Fig. 2). Tapered skirts(Fig. 3) are used where it is required to increase the diameter of the base ring to give larger leverage to the anchor bolts in resisting the overturning moment due to wind or other forces.

The simplest foundation for a vertical vessel is a pedestal, resting on a footing, to which the skirt base ring is fastened by bolts (Fig. 4). A commonly used shape of the pedestal and the footing is octagonal. The most customary arrangement of reinforcing steel in the footing is shown in Fig. 5. By necessity, the steel must be placed in four planes, requiring a

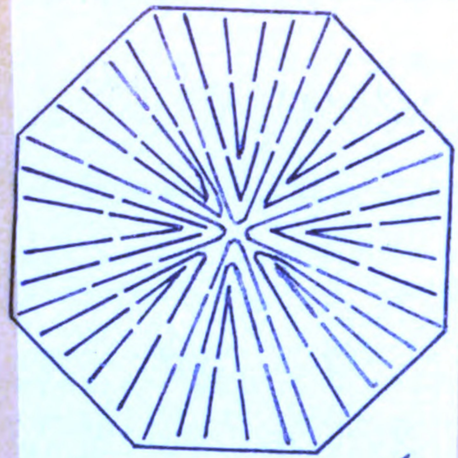


FIG. 6

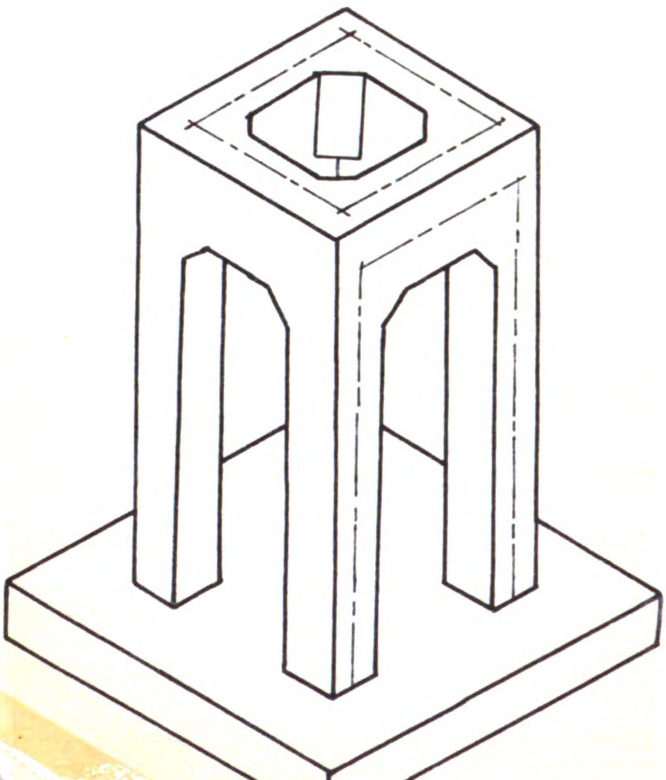


FIG. 7

greater footing depth. Another disadvantage of this steel arrangement is the unnecessarily close grid underneath the pedestal, making placing of concrete difficult. An alternate arrangement is obtained using bent bars radiating from the center (Fig. 6). These bars can be placed in one plane, thus providing a greater effective footing depth.[11]

A too large footing for high towers may be avoided by maintaining stability by means of guy wires, fastened to the vessel, instead of by a self supporting foundation.

Ordinarily, the vessel has to be held a certain distance above grade to accomodate piping below the vessel. The length of the skirt is increased accordingly. Holes have to be provided in the skirt for piping and additional openings may be necessary for servicing and ventilation.

A structural steel or reinforced concrete framework is used if it is required to support the vessel at a higher elevation with more space available underneath the vessel. Such a frame in its simplest form, a table top support, is shown in Fig. 7. This frame has been chosen for a Moment Distribution analysis in the second part of this thesis. This support is economical for various sizes of vessel diameters and heights of support up to 30 feet. For supporting larger vessels, additional columns and horizontal bracing may have to be added to avoid a bulky appearance of the support. Bending in the columns due to wind shears may be decreased by struts to adjacent structures or by guying the vessels.

When additional equipment and platforms are necessary that cannot be supported by a simple frame or from the vessel itself, the vessel is enclosed by a multy-level framework on which the vessel, the servicing and inspection platforms, auxiliary equipment, and piping are supported. Due to the high fire hazard in the oil industry, all main structural members are enclosed by concrete. Gunite is commonly used for fireproofing structural steel members.

The most outstanding example of a refinery structure, because of its large size and complicated design, is the intricate framework of a catalytic cracker, supporting numerous large vessels, piping and other equipment. The carefully calculated lay-out, determined by process requirements, results in a compact structure that permits replacements with minimum loss of production time. The required structural framework is an irregular structure with many strength considerations subordinated to the needs of the chemical process. The structure has to be analyzed separately for various loading conditions.

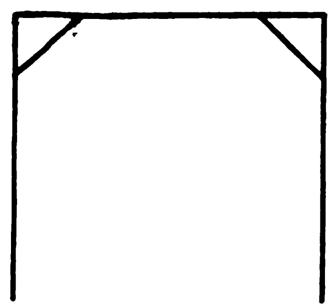


FIG. 8

The catalytic cracking structure may be about 20 stories high, narrow and with its heaviest load - the regenerator close to the top. This presents special structural design problems. The structural frame being an open type structure, it has to withstand wind pressures without the aid of masonry walls or floor slabs. Cross or "K" bracing cannot always be used to resist wind moments because it interferes with the piping, expansion loops and the operating platforms. In such cases knee braces (Fig. 8) are designed[8] to give the needed stiffening to the framework to resist the effects of sidethrust due to pipe stresses, wind, and equipment thermal expansion or contraction forces. Sometimes also special wind trusses may be required underneath the girders. Moment resistant beam-to-column connections[28] may be undesirable because requiring large columns to resist bending, when no advantage can be taken of continuity due to irregular framing of the beams. This discontinuity of the framework is caused by the many large openings required for equipment and piping, often demanding the use of diagonal beams.

Great care has to be taken in leveling the main girders supporting the large vessels, in order to insure a uniform bearing surface for the skirt base ring. Special bearing plates may be fitted to the tops of

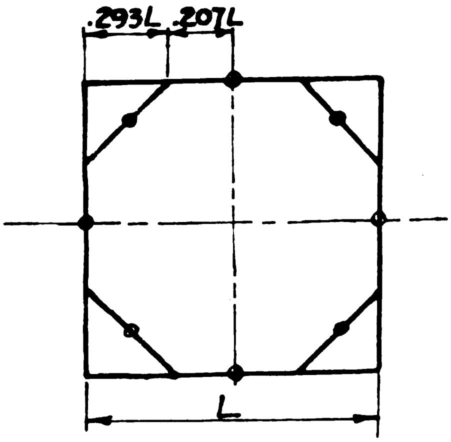


FIG. 9

girders and planed level for skirt bearing to avoid unequal bearing stresses. In designing the girders, the vessel load is often assumed to be uniformly distributed over the length of the girder. This, however, is not in accordance with the actual conditions because when the vessel is set onto the supporting girders, it usually rests on shims placed between the vessel skirt plate and the girder. When later grout of the unexpanding type is poured underneath the skirt base ring between the shimming points, this still does not produce a continuous bearing on the girder because of shrinkage of the concrete. To insure equal reactions at all points of support, the supporting girders must be designed for equal deflections at the loading points. A common type of girder framing used to support a vertical vessel at 8 points with opening for the head of vessel is shown in Fig. 9. The short diagonal beams should have the same stiffness as the main girders, otherwise each point of support will not take its proportionate share of the total load.

On the other hand, in case of a reinforced concrete support where the whole top of the support is poured monolitically, the assumption of concentrated loads may have less meaning and a uniform load distribution may be the best assumption.

In case of hot vessels resting on the supporting frame directly through lugs or base ring angles that are integral with the hot shell, radial expansion forces from the vessel are transferred to the supporting horizontal bent. Before the base of the hot tower is able to expand, it has to overcome the frictional resistance of the support. In case of friction between the base plate of the tower and a smooth concrete surface of the support, the radial expansion force, transferred to the supporting frame, will be about 20% of the supported weight at each point of support. When using fitted sole plates, the coefficient of friction may be reduced to 13-15%. Still the thermal expansion forces may be high, causing ring tension, in case of an integrally poured concrete top of the support. In critical cases special low friction bearing plates are sandwiched in between, permitting radial expansion of the vessel on these sliding bearings. A bearing plate made of bronze with trepanned holes filled with graphite sticks and both faces greased with a graphite lubricant reduces the coefficient of friction to 5-7%.[5] In case of a large vessel, whose stability is not overcome by wind or other horizontal forces, rollers in boxes anchored to the concrete foundation ring have been used to permit radial expansion but not side slippage of the vessel.[22]

In general, there are two methods of reducing the destructive force of the thermal expansion and contraction, either by designing rigid piping that displaces

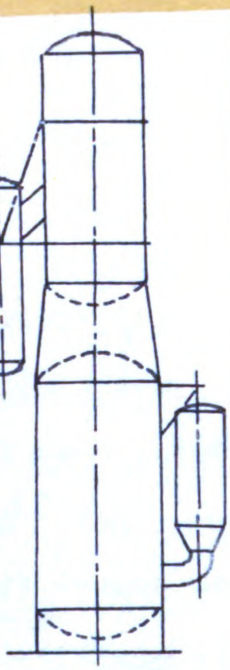


FIG. 10

the major equipment (on sliding bearings) through the full amount of the expansion, or by keeping the vessels stationary and absorbing the expansions by bends or expansion joints in the interconnecting piping.[13] Differential thermal expansion may also be decreased by connecting various process equipment together (Fig.10). Designing expansion connections for a refinery is a large field of enterprise for research investigations.

ANCHOR BOLTS

The providing of adequate bolts at the base of the tower, anchored to the foundation to give stability to the vessel against overturning moments, is an important part of the structural design.

The anchor bolts are located on a bolt circle in the base plate outside the skirt of the vessel, providing at least 2½-3½" clearance from bolt circle to skirt. Ordinarily, at least 8 bolts or more are used for larger vessels to give a more even distribution of stresses along the circumference of the vessel and to minimize the danger in case of a loose bolt. The anchor bolts should be developed to their full tensile strength by embedment into the foundation. This requirement limits the practical size of the bolts to 3" diameter, usually, though, below 2½" diameter. The bolt stresses are transferred to the tower by lugs

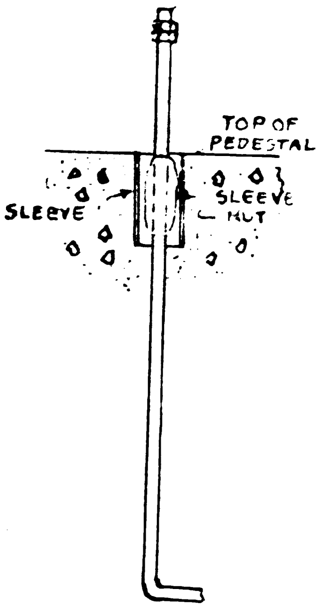


FIG. 11

welded to the skirt of the vessel. The length of the lugs should be long enough to allow space for sufficient weld of the required strength. It is safer to design the lugs for only one weld for every lug because the welds may be difficult to make on the inner faces of the lugs.[6] For larger bolts two nuts are used. Also an initial tension is introduced into the bolts by tightening them after erection of the tower. The ends of the anchor bolts, protruding from the foundation, may be bent or their threads damaged when setting the tower in place, therefore a sleeve nut is sometimes attached to the end of the bolt below the top of the concrete (Fig. 11), into which a stud bolt is inserted from the top through the lugs of the tower.[2]

The force resisted by each bolt is proportional to its distance from the neutral axis of the base circle. The neutral axis is, usually, assumed to be located at the center of the bolt circle and bolt stresses computed by the flexure formula, using the section modulus of the whole bolt group. Using another procedure, the stresses per foot of circumference of the skirt are computed and the bolts designed to take care of tensile forces within their segment of the base ring.[2] Although the assumption that the rotation occurs about the center line of the base ring is not exactly correct, it is convenient from design standpoint and it is practical to design the anchor bolts for most towers by this

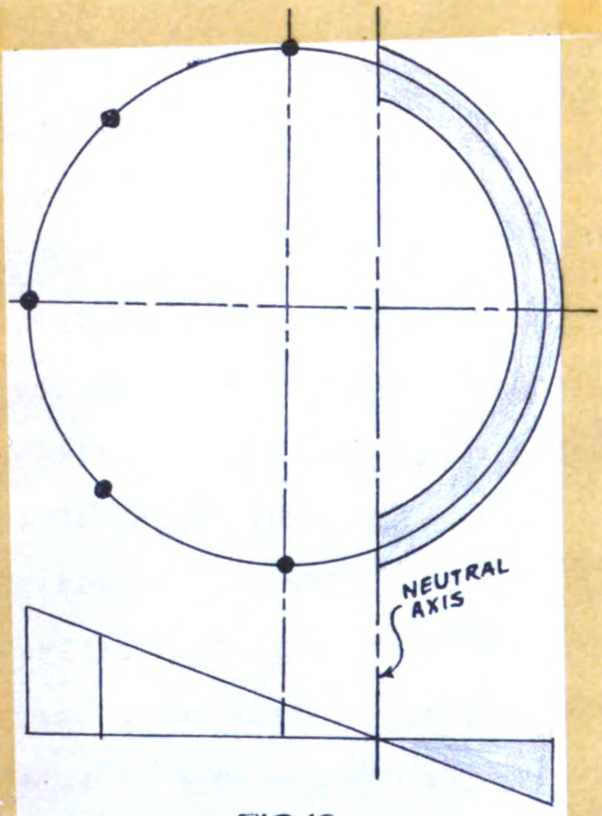


FIG. 12

method with a minimum of effort.

When designing tall towers, the use of a method, based on the actual location of the neutral axis, may be justified by resulting in a base design with smaller anchor bolts, lugs, and base plate. A theoretical approach to the problem is to consider the section between the base ring and the concrete pedestal as a hollow cylindrical reinforced concrete cantilever beam of balanced design, with an axial load equal to the weight of the tower and an external bending moment equal to the overturning moment.[24] After finding the neutral axis of the base section by the transformed area method (Fig. 12), the tensile stresses in the bolts and the concrete bearing stress may be calculated in accordance with the theory of flexure. This, of course, requires the base plate to be stiffened, in order that it may be considered rigid, with negligible deflection. In this analysis the usual initial tension in the bolts may be neglected. To find the most economical arrangement of anchor bolts, it may be necessary to repeat these calculations several times, trying out different number and sizes of anchor bolts and some variation of the bolt circle diameter. For the purpose of finding the minimum anchor bolt area that is consistent with a given base ring area and a given working stress in steel, nomograms have been devised that produce the solution without the need of numerous cumbersome trials.[15]

CONCLUSION

This summary of applications of structural engineering to the refinery industry should give a general survey to an engineer outside the industry, about some of the problems encountered in the structural design of a refinery. A more detailed information on this subject may be obtained from articles listed as references at the end of this thesis. This list is believed ~~to~~ to include the major part of articles on the structural design of refineries, published in American periodicals within the last ten years preceding 1952.

PART TWO

MOMENT ANALYSIS

OF A SIMPLE SUPPORT.

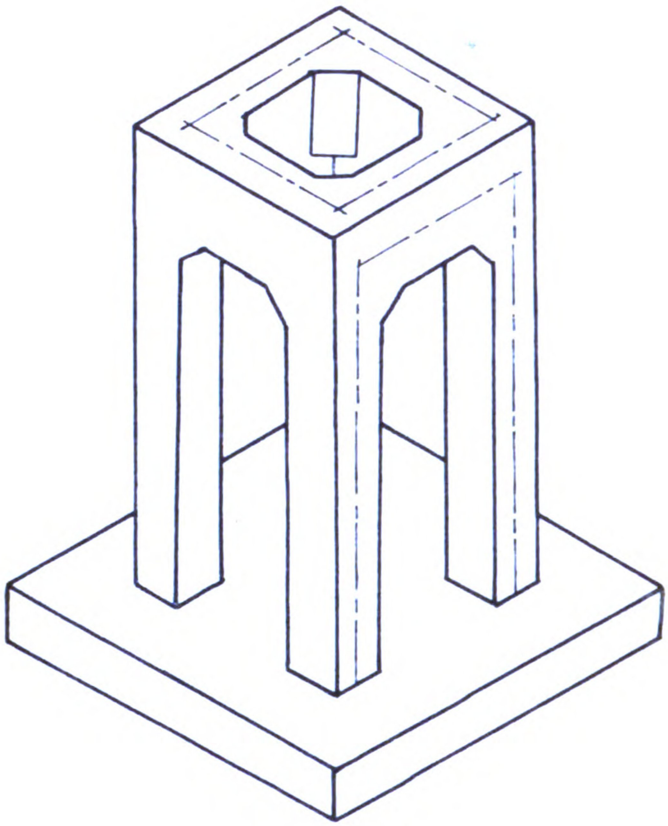


FIG.7

INTRODUCTION

Dr. F. E. Wolosewick has prepared charts for the functions of bending moments in the beams, columns, and foundation girders, as well as vertical reactions, end shears in girders, and unit soil pressures for the simple frame of the type shown in Fig. 7, and has found the functions to have a straight-line variation.[17]

Following his example, end-moments of the members of such a frame are analyzed here by moment distribution, extending the study by taking into consideration the effects of varying amount of end-restraint of the columns, varying loading conditions, haunching of the members and side-sway of the frame.

First, expressions are derived for the end moments in the members, then, values for the end-moments are computed for different ratios of beam stiffness to column stiffness and various loading conditions, and, finally, the functions are plotted in charts, for easier interpretation.

NOTATIONS USED

A, B, C, D, E, F	Corner points on the centerline of the members of the bent
c	Carry-over factor for the beam
c_c	Carry-over factor for the column from the column-beam joint to the column end at the footing
$d = \frac{r}{r+1}$	Moment distribution factor for the beam at the joint with a column, assumed fixed at the footing
$d_m = \frac{r}{r+0.75}$	Moment distribution factor for the beam at the joint with a column, assuming columns pin-connected
F	Fixed end moment
H	Horizontal thrust, applied at top of bent
h	Height of bent, from top of footing to centerline of beam
$k_B = r$	Relative stiffness factor of the beam
$k_c = 1$	Relative stiffness factor of the column
L	Length of beam span, center to center of columns
M	Applied moment at top of frame due to the horizontal force W
M_{AB}, M_{BA}, etc.	End moments in members of a bent caused by vertical loads without sidesway
P	Concentrated vertical load on beam
Q	Concentric Weight supported by frame
$r = \frac{k_B}{k_c}$	Ratio of beam stiffness factor to column stiffness factor; also relative stiffness factor of the beam, with k_c equal to one

$$S_1 = 1-(1-c)d-(c-c^2)d^2-(c^2-c^3)-\dots-(c^n-c^{n+1})d^{n+1}-\dots$$

$$S_2 = 1-d+c^2d^2-c^2d^3+c^4d^4-c^4d^5+\dots+c^{2n}d^{2n}-c^{2n}d^{2n+1}\dots$$

$$S_3 = cd-cd^2+c^3d^3-c^3d^4+\dots+c^{2n+1}d^{2n+1}-c^{2n+1}d^{2n+2}\dots$$

$$S_4 = \frac{d}{2} - \left(\frac{d}{2}\right)^2 + \left(\frac{d}{2}\right)^3 - \left(\frac{d}{2}\right)^4 + \left(\frac{d}{2}\right)^5 - \dots - (-1)^n\left(\frac{d}{2}\right)^n\dots$$

W Horizontal force applied on vessel with moment arm a, causing a moment M about the top of frame

X Coefficient for the fixed end moments in the beam, caused by the loads due to Q

Y Coefficient for the fixed end moment at the left end of the beam, caused by the loads due to M.

Z Coefficient for the fixed end moment at the right end of the beam, caused by the loads due to M.

MOMENT SIGN CONVENTION

The Moment Distribution sign convention is used in the computations, adapting the following criterion in determining the algebraic signs of the resisting moments in the ends of the structural members acting on the joint:

End moments of the member are positive when they act in a clockwise direction on the joint, and negative when they act in the opposite direction.

This sign convention gives positive fixed end moments at the left end of a beam with downward loading and negative moments at the right end.

Sidesway causes positive fixed end moments when the line joining the ends of the member rotates in a clockwise direction during relative lateral displacement and negative - otherwise.

GENERAL CONSIDERATIONS

The table-top supports may be either of reinforced concrete or structural steel. When making the frame of structural steel, it is also encased in concrete, for fireproofing purposes, developing end-restraints at the joints. The structure will be treated as a rigid frame, using centerline distances of the members for the Moment Distribution analysis.

Only frames with equal length of beams, giving a square outline of the frame in top view, will be considered in this analysis, because these table-top supports will be designed to carry only one vessel at the center. Because the vessel load will be assumed to act symmetrically about its center, and the support will be designed for horizontal forces from either direction, all vertical ~~and symmetrical~~ bents of the frame will be alike and symmetrical about their vertical centerlines.

The frame is analyzed as a plane structure. This is much simpler than a three-dimensional analysis; besides, analysis of a rectangular frame as a three-dimensional structure gives values for end-moments that differ only by a few per cent,in most cases, from those obtained by plane analysis.[35] This is because a member, framing into a joint perpendicular to the plane of the bending moment, does not add much to the stiffness of the joint because the torsional stiffness

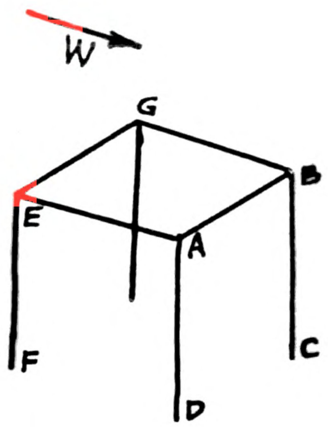


FIG. 19

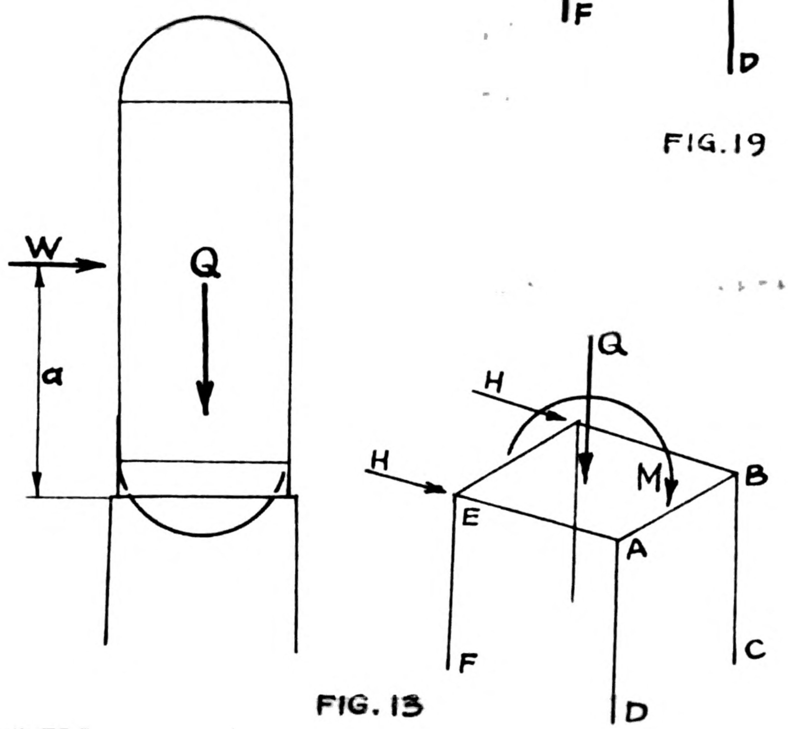


FIG. 13

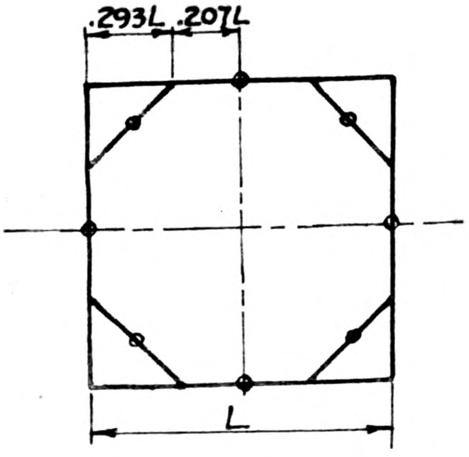


FIG. 9

of structural members is much smaller than their bending stiffness. This small difference in results justifies the analysis of the frame as divided into two-dimensional bents.

Only the vertical bents DABC and FEAD (Fig. 19) are considered, the thermal or other forces acting on the horizontal bent ABGE at the top of the support being neglected for purposes of this investigation.

The frame is acted upon by forces cue to the weight of the vessel Q, and due to a horizontal force W from wind, earthquake, or other causes, that creates a bending moment M at the top of the support (Fig. 13).

The bent DABC supports vertical loads due to Q and M that may either be considered as concentrated at several points or as uniformly distributed along the length of the beam AB. The bent FEAD resists a horizontal thrust H=W/2, in addition to vertical forces.

A typical bent DABC on the leeward side of the frame, supporting the largest downward loads, will be analyzed for end moments due to vertical loads without sidesway.

Two kinds of vertical loading on the horizontal beam AB will be considered: a uniformly distributed load along the entire length L of the beam, or concentrated loads from bolts at 8 points, with support top framing as shown in Fig. 9. For computing loads on the beams due to M, the simplified assumption of rotation about the centerline of the vessel will be made.

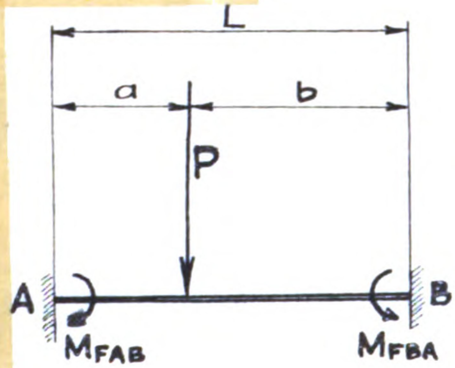


FIG. 14

FIXED END MOMENTS

The familiar formulas:

$$MFAB = \frac{Pab^2}{L}$$

$$MFBA = -\frac{Pba^2}{L}$$ for a concentrated load, and

$$MFAB = -MFBA = \frac{wL^2}{12}$$ for uniform load (Fig. 14)

are used to compute the fixed end moments for beams of uniform moment of inertia along their entire length. The computations give a result that may be expressed in general terms as $MF = XPL$ where the value of X is determined by the location of the load P.

As the vertical loads, in the present analysis, are a combination of effects from Q and M, the fixed end moments will be expressed in the form $MFAB = XLQ + YM$, where X and Y are functions both of the location of the load P and of its expression in terms of Q or M. The weight of the tower Q is assumed to be distributed equally to all points of loading which are symmetrically located, and therefore gives equal fixed end moments in the beams at both ends. A general expression for the fixed end moment at the right side of the beam is $MFBA = -(XLQ + ZM)$.

When assuming a uniformly distributed load on the beam, the load per unit length will be taken as

$$w = \frac{Q}{4L} \pm \frac{4M}{\pi L^2} ,$$

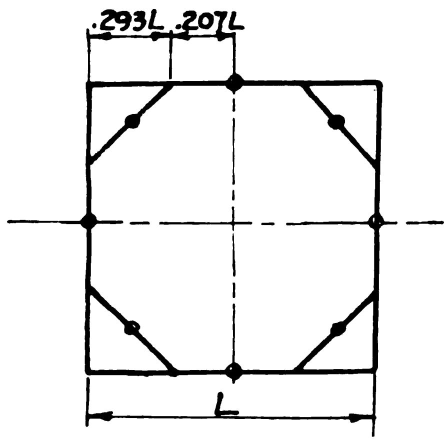


FIG. 9

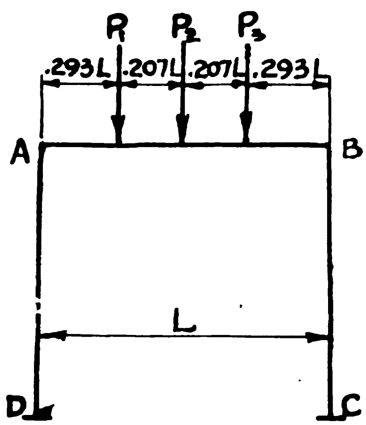


FIG. 15

where the second part of the expression is the maximum stress in a unit segment of the skirt shell, computed by the flexure formula using the approximate expression for a section modulus of a thin cylindrical shell $\frac{\pi D^2 t}{4}$, transferred to a unit length of the bolt circle with a diameter L.

Computed fixed end moments in the beam AB for uniformly distributed load with wind direction perpendicular to the beam:

$$MFAB = - MFBA = \frac{wL^2}{12} = \frac{1}{48}LQ + \frac{1}{3\pi} M$$

$$X = \frac{1}{48} \qquad Y = \frac{1}{3\pi}$$

For wind direction perpendicular to the beam AB, and framing arrangement of Fig. 9 with 8 bolts, the beam AB carries loads located as shown in Fig. 15. Computations by the flexure formula, using L^2 as moment of inertia for the group of 8 equally spaced bolts about the centerline of its bolt circle, gives the following values for the vertical loads:

$$P_2 = \frac{Q}{8} + \frac{M}{2L}$$

$$P_1 = P_3 = \frac{Q}{16} + \frac{0.17675M}{L}$$

P_1 and P_3 is half of the load carried by the short diagonal beams, transferred to the beam AB.

Superposition of the effects of the three loads gives the following expression for fixed end moments of

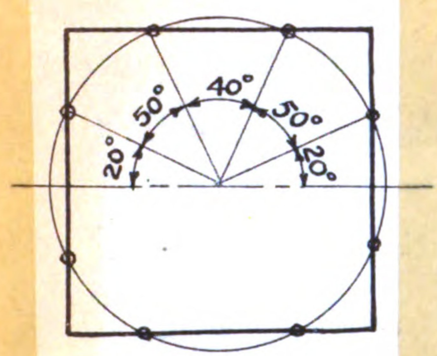


FIG. 16

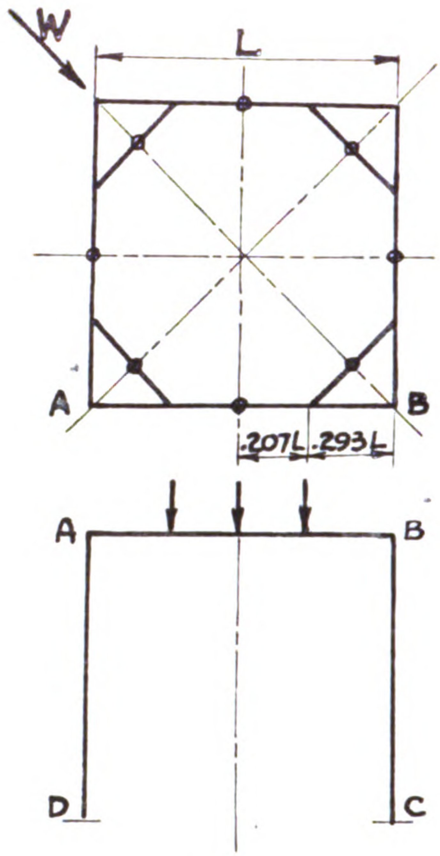


FIG. 17

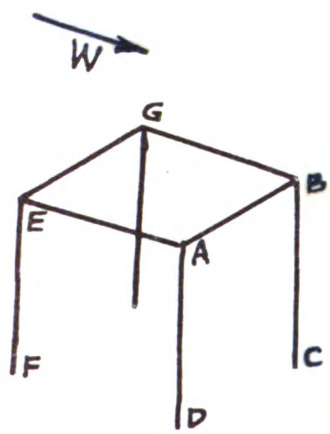


FIG. 19

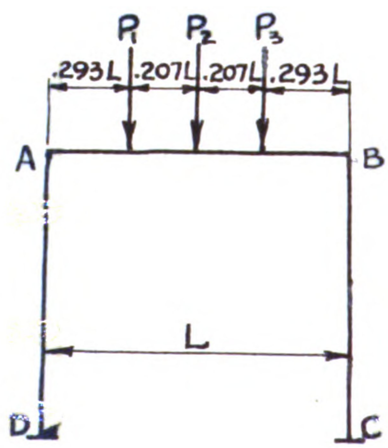


FIG. 15

the beam AB for 8-point loading on the frame at perpendicular wind:

$$MFAB = -MFBA = 0.00818198LQ + 0.02838263M$$

Fixed end moments due to another 8-point loading arrangement, used in the practice (Fig. 16), were investigated and found to be smaller than in the other 8-point loading case, therefore fixed-end moment values as above will be used as representing the more significant concentrated loading.

Assuming wind from a diagonal direction (Fig. 17), with rotation about the diagonal, the following magnitude of the concentrated loads (Fig. 15) has been computed:

$$P_1 = \frac{Q}{16} \qquad P_2 = \frac{Q}{8} + \frac{0.3535M}{L}$$

$$P_3 = \frac{Q}{16} + \frac{M}{4L}$$

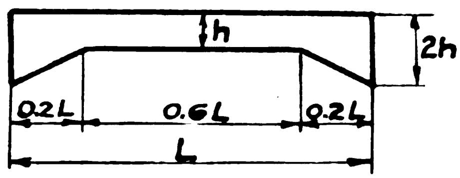
The fixed end moments in the beams caused by these loads due to 8-point loading on the frame at diagonal wind are as follows:

$$MFAB = 0.02857194LQ + 0.059362M$$

$$MFBA = -(0.02857194LQ + 0.080814M)$$

$$X = 0.02857194 \quad Y = 0.059362 \quad Z = 0.0808014$$

A wind parallel to the plane of the bent FEAD (Fig. 19) causes the following fixed end moments in the beams due to concentrated loads:



HAUNCHED BEAM

$k_{AB} = k_{BA} = 7.81$

$C_{AB} = C_{BA} = 0.659$

FIG. 18

$$MFEA = 0.02857194LQ - 0.01515817M$$

$$FFAE = -(0.02857194LQ + 0.01515817M)$$

For the haunched beam shown in Fig. 18, the fixed end moments due to a uniformly distributed load along its entire length, neglecting the effect of the haunch loads, are[35]:

$$MFAB = -MFBA = 0.0993wL^2 = 0.0993 \frac{4M}{\pi} + \frac{QL}{4} =$$

$$= 0.1264M + 0.024825LQ$$

Wind is assumed to be perpendicular to the beam.

MOMENT DISTRIBUTION OF FIXED END MOMENTS CAUSED BY VERTICAL LOADS ON THE BEAM WITHOUT SIDESWAY OF THE BENT

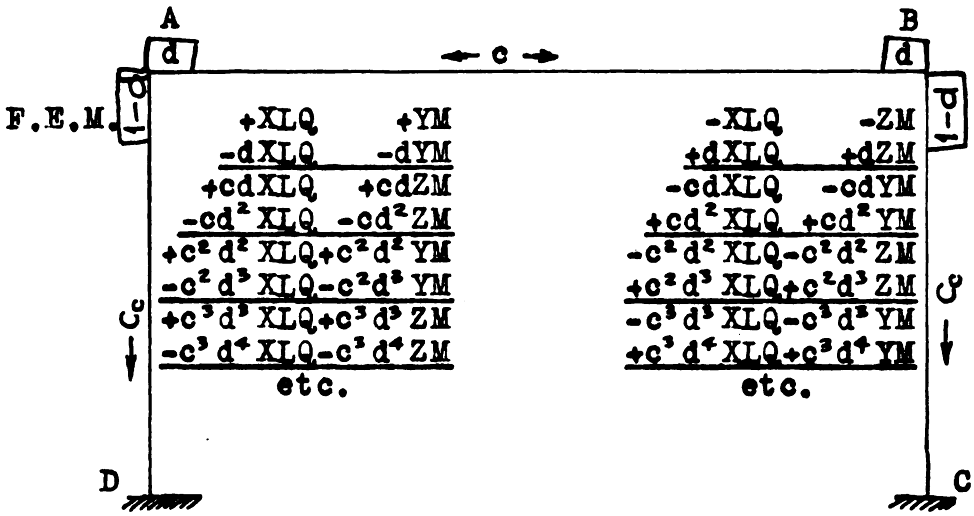
A general case of moment distribution has been carried out on the following sheet for a symmetrical beam. The beam is supported on columns, fixed at the footing, forming a bent symmetrical about the midspan of the beam.

"d" is the distribution factor for the beam at the beam-column joint. The distribution factor for the column is (1 - d). "d", in terms of the ratio "r" of the beam stiffness to column stiffness, is $d = \frac{r}{r+1}$; $r = k_B/k_C$

"c" is the carry-over factor for the beam. "c_C" is the carry-over factor for the column, from the beam-column joint to the base of the column.

Because no moments are carried over from the fixed end of the column at the footing to the beam-column joint, the moment distribution procedure is shown only for the beam end-moments. The final moment in the column at the beam-column joint is of the same magnitude as the end-moment in the beam at the same joint. Because there are no initial fixed end moments in the column, the final end-moment in the column at the footing is equal in magnitude to the moment at the upper end, multiplied by c_C.

Moment distribution of fixed end moments of a beam
caused by vertical loads on the beam
without sidesway of the bent,
carried out in general terms.



Moment distribution results
in the following end-moments:

$$M_{AB} = XLQ\left[1-(1-c)d-(c-c^2)d^2-(c^2-c^3)d^3-\ldots-(c^n-c^{n+1})d^{n+1}\ldots\right]+$$
$$+YM\left[1-d+c^2d^2-c^2d^3+c^4d^4-c^4d^5+\ldots+c^{2n}d^{2n}-c^{2n}d^{2n+1}+\ldots\right]+$$
$$+ZM\left[+cd-cd^2+c^3d^3-c^3d^4+\ldots+c^{2n+1}d^{2n+1}-c^{2n+1}d^{2n+2}+\ldots\right]$$

Designating the infinite series by S_1, S_2, and S_3, respectively, gives final end-moments in an abbreviated form:

$$M_{AB} = XLQ[S_1] + YM[S_2] + ZM[S_3]$$

$$M_{BA} = -\left\{XLQ[S_1] + YM[S_3] + ZM[S_2]\right\}$$

$$M_{AD} = -M_{AB} \qquad M_{BC} = -M_{BA}$$

$$M_{DA} = c_c M_{AD} = -c_c M_{AB}$$

$$M_{CB} = \quad -c_c M_{BA}$$

It can be seen that the moment distribution results in end-moment expressions that contain the infinite series S_1, S_2, and S_3.

If the loading is symmetrical, the fixed end moments in the beam are equal ($Y = Z$). Their moment distribution gives a simplified expression for the final moments:

$$M_{AB} = -M_{BA} = (XLQ + YM)S_1 \ ,$$

S_2 plus S_3 being equal to S_1.

For a beam with a uniform moment of inertial c is $\frac{1}{2}$ and the infinite series are simplified as follows:

$$S_1 = 1 - \frac{d}{2} - \frac{d^2}{2^2} - \frac{d^3}{2^3} - \frac{d^4}{2^4} - \dots - \frac{d^n}{2^n} - \dots$$

$$S_2 = 1 - d + \frac{d^2}{2^2} - \frac{d^3}{2^2} + \frac{d^4}{2^4} - \frac{d^5}{2^4} + \dots + \frac{d^{2n}}{2^{2n}} - \frac{d^{(2n+1)}}{2^{2n}} + \dots$$

$$S_3 = \frac{d}{2} - \frac{d^2}{2} + \frac{d^3}{2^3} - \frac{d^4}{2^3} + \frac{d^5}{2^5} - \frac{d^6}{2^5} + \dots + \frac{d^{(2n+1)}}{2^{(2n+1)}} - \frac{d^{(2n+2)}}{d^{(2n+1)}} + \dots$$

The same formulas can be used also to find final end-moments in a bent with columns hinged at the bottom, only d in the series S_1, S_2, and S_3 has to be substituted by a modified distribution factor d_m. For a bent with members having a uniform moment of inertia d_m is equal to $\frac{r}{r+0.75}$, the relative stiffness factor of the column being modified multiflying by 3/4. The end moments at the bottom of the column are, of course, equal to zero.

MOMENT DISTRIBUTION FOR SIDESWAY

If the bent is not restrained from swaying in a lateral direction, horizontal forces and unsymmetrical vertical loading cause lateral translation of the joints, inducing end-moments, in addition to those caused by the vertical loads.

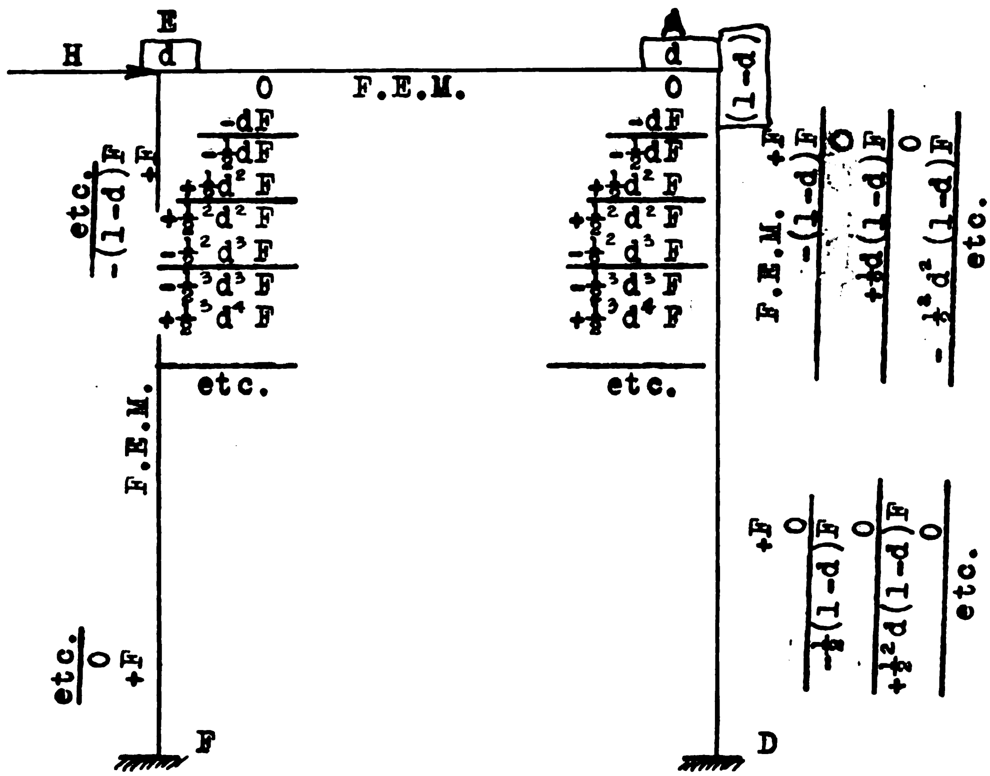
In case of a symmetrical bent with members having a uniform moment of inertia, the fixed end moments caused by the sidesway are equal in magnitude at all joints.

These fixed end moments have been denoted by F and moment distribution carried out on a following page, for a bent with fixed-end columns.

The moment distribution gives end moments expressed in a formula containing the infinite series S_4. The magnitude of F can be computed from an equation of equilibrium satisfying the condition that the sum of the sidesway end-moments resists the horizontal shears caused by the horizontal forces and the asymmetry of the vertical forces.

To find an algebraic expression for the end-moments in the members, with consideration of the effects of the sidesway, it would be necessary to express F in terms of the horizontal thrust H, and the height of the bent h. In order to keep the expressions for end moments in the same simpler terms as the previous end moment formulas, this has not been done. However, the formula for the sidesway moments can be used to add the effects of sidesway to numerical values of moments computed from the previously developed end-moment formulas or their graphical expression in form of charts, when applying equations of equilibrium to the bent.

Moment distribution for sidesway moments for a bent with members of uniform moment of inertia, carried out in a general form.



Moment distribution results in the following final end-moments:

$$M_{AE}=M_{EA}=F\left[-d-\tfrac{1}{2}d+\tfrac{1}{2}d^2+\tfrac{1}{2}^2d^2-\tfrac{1}{2}^2d^3-\ldots\right]=$$

$$=F\left[-3(d/2)+3(d/2)^2-3(d/2)^3+\ldots+(-1)^n3(d/2)^n-\ldots\right]=$$

$$=3F\left[-d/2+(d/2)^2-(d/2)^3+\ldots+(-1)^n(d/2)+\ldots\right]$$

$$M_{FE}=M_{DA}=F\left\{1-\tfrac{1}{2}(1-d)+d/2^2(1-d)-d^2/2^3(1-d)+d^3/2^4(1-d)-\ldots\right.$$

$$\left.\ldots-d^{2n}/2^{2n+1}(1-d)+d^{2n+1}/2^{2n+2}(1-d)-\ldots\right\}=$$

$$=F\left\{1+(1-d)(-\tfrac{1}{2}+d/2^2-d^2/2^3+d^3/2^4-\ldots-d^{2n}/2^{2n+1}+d^{2n+1}/2^{2n+2}-\ldots\right\}=$$

$$=F\left\{-\tfrac{1}{2}-3d/2\ -3d\ /2\ -3d\ /2\ -\ldots-3d\ /2\ -3d\ /2\ -\ldots\right\}=$$

$$=F\left\{+\tfrac{1}{2}+3/2\left[d/2-(d/2)^2+(d/2)^3-\ldots-(-1)^n(d/2)^n-\ldots\right]\right\}$$

In abbreviated form:

$$M_{AE}=M_{EA}=3F\left[-S_4\right]$$

$$M_{FE}=M_{DA}=F\left\{\tfrac{1}{2}-3/2\left[S_4\right]\right\}$$

VALUES FOR THE INFINITE SERIES

A systematic computation of approximate values of the four infinite series is given in the tables #1, #2, #3, #4, #5, for r values from 1 to 10. The computations have been carried out to the seventh decimal place and to the d^8 member of the series.

Computations of the series for bents with uniform moment of inertia beams with c equal to $\frac{1}{2}$ are given in tables #1, #2, #3, and #4. Table #5 contains values for S_1 for a haunched beam with c equal to 0.659.

In compiling values for S_1 with c equal to $\frac{1}{2}$, it was realized that the exact value of S_1 is $\frac{2}{r+2}$, for bents with fixed end columns. A proof of this is given on the next sheet by expanding the function $\frac{2}{r+2}$ into a Maclaurin's series, resulting in S_1.

$$d = \frac{r}{r+1} \qquad\qquad r = \frac{d}{1-d}$$

$$\frac{2}{r+2} = \frac{2-2d}{2-d}$$

$$f(d) = \frac{2-2d}{2-d} \qquad\qquad f(0) = 1$$

$$f'(d) = \frac{-2}{(2-d)^2} \qquad\qquad f'(0) = -\tfrac{1}{2}$$

$$f''(d) = \frac{-4}{(2-d)^3} \qquad\qquad f''(0) = -\tfrac{1}{2}$$

$$f'''(d) = \frac{-12}{(2-d)^4} \qquad\qquad f'''(0) = -3/4$$

$$f^{iv}(d) = \frac{-48}{(2-d)^5} \qquad\qquad f^{iv}(0) = -3/2$$

$$f^{v}(d) = \frac{-240}{(2-d)^6} \qquad\qquad f^{v}(0) = -\frac{15}{4}$$

$$f^{vi}(d) = \frac{-1440}{(2-d)^7} \qquad\qquad f^{vi}(0) = -\frac{95}{8}$$

$$f(d) = f(0) + \frac{f'(0)}{1!}d + \frac{f\ (0)}{2!}d^2 + \frac{f\ (0)}{3!}d^3 + \ldots + \frac{f^{(n-1)}(0)}{(n-1)!}d^{(n-1)} + \ldots$$

$$= 1 - \frac{d}{2} - \frac{d^2}{4} - \frac{d^3}{8} - \frac{d^4}{16} - \frac{d^5}{32} - \ldots$$

$$\therefore \quad \frac{2}{r-2} = s_1$$

TABLE #1

r		d	d^2	d^3
1	d	.5	.25	.125
	dm	.5714285	.3265305	.186
2	d	.6666667	.4444445	.296
	dm	.7272727	.5289256	.384
3	d	.75	.5625	.421
	dm	.8	.64	.512
4	d	.8	.64	.512
	dm	.8421052	.7091412	.597
5	dm	.8333333	.6944444	.578
	dm	.8695652	.7561436	.657
6	d	.8571428	.7346937	.629
	dm	.8888889	.7901235	.702
7	d	.8750000	.7656250	.669
	dm	.9032258	.8158168	.736
8	d	.8888889	.7901235	.7023
	dm	.9142857	.8359183	.7642
9	d	.9	.81	.729
	dm	.9230769	.8520710	.7865
10	d	.9090909	.8264463	.7513
	dm	.9302325	.8653325	.8049

TABLE #1

r		d	d^2	d^3	d^4	d^5	d^6	d^7	d^8
1	d	.5	.25	.125	.0625	.03125	.010625	.0053125	.0026563
	dm	.5714285	.3265305	.1865888	.1066222	.0609269	.0348154	.0198945	.0113683
2	d	.6666667	.4444445	.2962963	.1975309	.1316873	.0877915	.0585277	.0390185
	dm	.7272727	.5289256	.3846731	.2797622	.2034634	.1479734	.1076170	.0782691
3	d	.75	.5625	.421875	.3164063	.2373047	.1779785	.1334839	.1001129
	dm	.8	.64	.512	.4096	.32768	.262144	.2097152	.1677722
4	d	.8	.64	.512	.4096	.32768	.262144	.2097152	.1677722
	dm	.8421052	.7091412	.5971715	.5028812	.4234789	.3566138	.3003063	.2528895
5	dm	.8333333	.6944444	.5787037	.4822531	.4018776	.3348960	.2790816	.2325680
	dm	.8695652	.7561436	.6575162	.5717532	.4971767	.4323276	.3759370	.3269017
6	d	.8571428	.7346937	.6297374	.5397749	.4626642	.3965693	.3399165	.2913570
	dm	.8888889	.7901235	.7023320	.6242951	.5549290	.4932702	.4384624	.3897444
7	d	.8750000	.7656250	.6699219	.5861817	.5129090	.4487954	.4487954	.3926960
	dm	.9032258	.8158168	.7368668	.6655571	.6011483	.5429727	.4904270	.4429663
8	d	.8888889	.7901235	.7023320	.6242951	.5549290	.4932702	.4384624	.3897444
	dm	.9142857	.8359183	.7642681	.6987594	.6388657	.5841058	.5340396	.4876888
9	d	.9	.81	.729	.6561	.59049	.531441	.4782969	.4304672
	dm	.9230769	.8520710	.7865271	.7260250	.6701769	.6186248	.5710383	.527123
10	d	.9090909	.8264463	.7513148	.6830135	.6204214	.5644740	.5131582	.4665074
	dm	.9302325	.8653325	.8049604	.7488003	.6965584	.6479613	.6027547	.5607020

TABLE #2

COMPUTATIONS FOR S_1 & S_4

r		$\frac{d}{2}$	$\frac{d^2}{2^2}$	$\frac{d^3}{2^3}$	$\frac{d^4}{2^4}$	$\frac{d^5}{2^5}$	$\frac{d^6}{2^6}$	$\frac{d^7}{2^7}$	$\frac{d^8}{2^8}$
1	d	.25	.0625	.015625	.0039063	.0009766	.0001660	.0000415	.0000104
	dm	.2857143	.0816326	.0233236	.0066639	.0019040	.0005440	.0001554	.0000444
2	d	.3333333	.1111111	.0370370	.0123457	.0041152	.0013717	.0004572	.0001524
	dm	.3636364	.1322314	.0480841	.0174851	.0063582	.0023121	.0008408	.0003057
3	d	.375	.140625	.0527344	.0197754	.0074158	.0027809	.0010428	.0003911
	dm	.4	.16	.064	.0256	.01024	.004096	.0016384	.0006554
4	d	.4	.16	.064	.0256	.01024	.004096	.0016384	.0006554
	dm	.4210526	.1772853	.0746464	.0314301	.0132337	.0055721	.0023461	.0009878
5	d	.4166667	.1736111	.0723380	.0301408	.0125587	.0052328	.0021803	.0009085
	dm	.4347826	.1890359	.0821895	.0357345	.0155368	.0067551	.0029370	.0012770
6	d	.4285714	.1836734	.0787172	.0337360	.0144583	.0061964	.0026556	.0011381
	dm	.4444444	.1975309	.0877915	.0390184	.0173415	0077073	.0034255	.0015224
7	d	.4375000	.1914063	.0837402	.0366364	.0160284	.0070124	.0030679	.0013422
	dm	.4516129	.2039542	.0921084	.0415973	.0187859	.0084839	.0038315	.0017303
8	d	.4444444	.1975309	.0877915	.0390184	.0173415	.0077073	.0034255	.0015224
	dm	.4571429	.2089796	.0955335	.043725	.0199646	.0041722	.0041722	.0019050
9	d	.45	.2025	.091125	.0410063	.0184528	.0083038	.0037369	.0016815
	dm	.4615385	.2130178	.0983159	.0453766	.0209430	.0096660	.0044612	.0020590
10	d	.4545455	.2066116	.0939144	.0426883	.0194038	.0088199	.0040090	.0018223
	dm	.4651163	.2163331	.1006201	.0468000	.0217675	.0101244	.0047092	.0021902

TABLE #3

COMPUTATIONS FOR S_2 & S_3

r	d	$\frac{d^2}{2}$	$\frac{d^3}{2^2}$	$\frac{d^4}{2^3}$	$\frac{d^5}{2^4}$	$\frac{d^6}{2^5}$	$\frac{d^7}{2^6}$	$\frac{d^8}{2^7}$
1	.5	.125	.03125	.0078125	.0019531	.0003320	.0000830	.0000208
2	.6666667	.2222222	.0740741	.0246914	.0082305	.0027435	.0009145	.0003048
3	.75	.28125	.1054688	.0395508	.0148315	.0055618	.0020857	.0007821
4	.8	.32	.128	.0512	.02048	.008192	.0032768	.0013107
5	.8333333	.3472222	.1446759	.0602816	.0251174	.0104656	.0043607	.0018169
6	.8571428	.3673469	.1574346	.0674719	.0289165	.0123928	.0053112	.0022762
7	.8750000	.3828125	.1674805	.0732727	.0320568	.0140249	.0061359	.0026844
8	.8888889	.3950618	.1755830	.0780369	.0346831	.0154147	.0068510	.0030449
9	.9	.405	.18225	.0820125	.0369056	.0166075	.0074734	.0033630
10	.9090909	.4132232	.1878287	.0853767	.0388076	.0176398	.0080181	.0036446

TABLE #4

r		FROM TABLE #2 (a) $\sum \frac{d^n}{2^n}$	COMPUTED 1-(a) $\underline{S_1}$	ACTUAL $\frac{2}{r+2}$ S_1	FROM TABLE #2 (b) $\sum \frac{d^{2n+1}}{2^{2n+1}}$	FROM TABLE #2 (c) $\sum \frac{d^{2n+2}}{2^{2n+2}}$	(b)-(c) $\underline{S_4}$	FROM TABLE #3 (d) $\sum \frac{d^{2n+1}}{2^{2n}}$	FROM TABLE #3 (e) $\sum \frac{d^{2n+2}}{2^{2n+1}}$	1+(c)-(d) $\underline{S_2}$	(b)-(e) $\underline{S_3}$
1	d	.3332258	.6667742	2/3 =.6666667	.2666431	.0665827	.2000604	.5332861	.1331653	.5332966	.1334778
	dm	.3999822	.6000178		.3110973	.0888849	.2222124	.			
2	d	.4999236	.5000764	1/2 =.5	.3749427	.1249809	.2499618	.7498858	.2499619	.3750951	.1249808
	dm	.5712538	.4287462		.4189195	.1523343	.2665852				
3	d	.5997654	.4002346	2/5 =.4	.4361930	.1635724	.2726206	.8723860	.3271447	.2911864	.1090483
	dm	.6662298	.3337702		.4758784	.1903514	.2855270				
4	d	.6662298	.3337702	1/3 =.3333333	.4758784	.1903514	.2855270	.9517568	.3807027	.2385946	.0951757
	dm	.7265541	.2733449		.5112788	.2152753	.2960035				
5	d	.7136369	.2863631	2/7 =.2857133	.5037437	.2098832	.2938505	1.0074873	.4197863	.2024059	.0839574
	dm	.7682484	.2317416		.5354459	.2328025	.3026434				
6	d	.7491464	.2508536	1/4 =.25	.5244025	.2247439	.2996586	1.0488051	.4494878	.1759388	.0749147
	dm	.7987819	.2012181		.5530029	.2457790	.3072239				
7	d	.7767338	.2232662	2/9 =.2222222	.5403365	.2363973	.3039392	1.0806732	.4727945	.1557241	.0675420
	dm	.8221044	.1778956		.5663387	.2557657	.3105730				
8	d	.7987819	.2012181	1/5 =.2	.5530029	.2457790	.3072239	1.1060060	.4915583	.1397730	.0614446
	dm	.8404970	.1595030		.5769132	.2636838	.3131294				
9	d	.8168063	.1831937	2/11=.1818182	.5633147	.2534916	.3098231	1.1266290	.5069830	.1268626	.0564317
	dm	.8553780	.1446220		.5852586	.2701194	.3151392	1.			
10	d	.8318148	.1681852	1/6 =.1666667	.5718727	.2599421	.3119306	1.1437453	.5198843	.1161968	.0519864
	dm	.8676608	.1323392		.5922131	.2754477	.3167654				

TABLE #5

COMPUTATIONS FOR HAUNCHED BEAM

c	c^2	c^3	c^4	c^5	c^6	c^7	c^8
.659	.4342810	.2861912	.1886000	.1242874	.0819054	.0539757	.0355700
$1-c$	$c-c^2$	c^2-c^3	c^3-c^4	c^4-c^5	c^5-c^6	c^6-c^7	c^7-c^8
.341	.2247190	.1480898	.0975912	.0643126	.0423820	.0279297	.0184057

r	$(1-c)$	$(c-c^2)d^2$	$(c^2-c^3)d^3$	$(c^3-c^4)d^4$	$(c^4-c^5)d^5$	$(c^5-c^6)d^6$	$(c^6-c^7)d^7$	$(c^7-c^8)d^8$		
1	.1705	.0561798	.0185112	.0060995	.0020098	.0004504	.0001484	.0000489	.2539480	.7460520
2	.2273333	.0998751	.0438785	.0192773	.0084692	.0037208	.0016347	.0007182	.4049071	.5950929
3	.25575	.1264044	.0624754	.0308785	.0152617	.0075431	.0037282	.0018426	.5038339	.4961161
4	.2728	.1438202	.0758220	.0399734	.0210740	.0111102	.0058573	.0030880	.5735451	.4264549
5	.2841667	.1560549	.0857001	.0470637	.0258458	.0141036	.0077947	.0042806	.6251001	.3748999
6	.2922857	.1650996	.0932577	.0526773	.0297551	.0168074	.0094938	.0053626	.6647392	.3352608
7	.298375	.1720505	.0992086	.0572062	.0329865	.0190208	.0109679	.0063244	.6961399	.3038601
8	.3031111	.1775558	.1040082	.0609257	.0356889	.0209058	.0122461	.0071735	.7216151	.2783849
9	.3069	.1820224	.1079575	.0640296	.0379759	.0225235	.0133587	.0079231	.7426907	.2573093
10	.3100000	.1857182	.1112621	.0666561	.0399331	.0239235	.0143324	.0085864	.7604118	.2395882

DISCUSSION OF CHARTS

The formulas for end-moments obtained by moment distribution have been represented graphically in charts #1, #2, #3, and #4. The appropriate constants X, Y, Z and values for S_1, S_2, S_3 from table #4 or #5 were substituted in the formulas and the end-moments M_{AB} and M_{BA} plotted as ordinates in terms of QL, with M/QL as abscissas, using r values from 1 to 10.

Chart #1 gives a comparison between end-moments of a fixed column bent and a hinged column bent for symmetrical loading. The darker lines in chart #1 represent the functions of the end moments of a beam in a bent with fixed-end columns, the lighter lines - for a bent with hinged columns. Lines representing functions for the same r value have been connected by arrows, indicating the possible range of variation of beam end moments in a given bent due to varying end restraint of columns.

It can be seen from the charts that the difference between the end moments in beams of fixed column bents and those of hinged column bents is larger for higher r values. As this difference is larger both absolutely and relatively, it indicates the greater necessity of determining the end condition of the columns in analyzing a bent with a relatively stiff girder.

The difference between the beam end-moments of a fixed-column bent and those of a hinged column bent is caused by the smaller end restraint given to the beam by the hinged columns. The smaller end restraint of the beams is taken care of in the moment distribution procedure by modifying the stiffness factor of the column, resulting into a modified distribution factor. "r" being a function of the stiffness factors, the modification from a fixed to a hinged column bent can be expressen by means of a modified r. The modified r for a hinged column bent is 4/3 of the r for a fixed column bent. Using this relationship, beam end moments for a hinged column bent can be obtained from the graphs of moment functions of a fixed column bent by using the modified r. Thus, e.g., the end moments for a bent with r equal to 6, having hinged-end columns, can be obtained from the graph for a fixed-end column bent with r equal to 8.

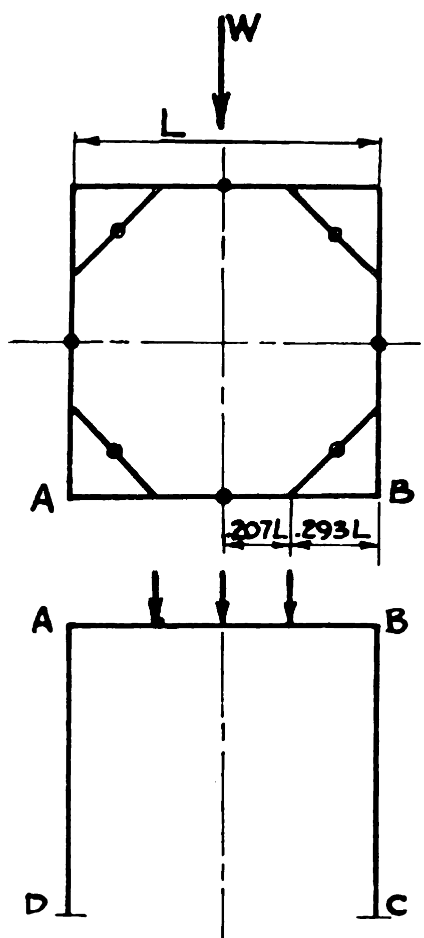
As can be expected, bents with a greater r ratio have relatively greater beam end moments due to a greater end restraint given to the beams by relatively stiffer columns. The column stiffness has an increasing influence on the beam end moments with a decreasing r ratio. This indicates the need for a closer evaluation of stiffness of the members of a bent with relatively stiff columns, when moments in the beam are computed.

Of course, the beam moments increase with increasing load. The intercept of the end-moment line with the vertical axis of the coordinate system (M/QL equal to zero) represents the magnitude of the beam end-moments when only a concentric load Q is applied to the frame.

The chart gives magnitude of the end moments in the beam AB on the side of the frame where Q and M both cause downward load and, consequently, end moments of the same sign. As the M/QL ratio increases, the influence of M on the end-moments increases. When M causes an end moment greater than that caused by Q (the ordinate for the particular M/QL being more than two times greater than the ordinate at the intercept with the vertical axis), it may be necessary to investigate end moments in the beam on the opposite side of the frame, where uplift loads are imposed on the bent with end-moments of an opposite sign in the beam.

The slopes of the functions with lower r are steeper, indicating a greater rate of increase of the end-moments with an increase in the loading for a bent with a relatively flexible beam.

<u>Chart #2</u> represents graphically the influence of uniformly distributed and concentrated loading on the end-moments of the beam. The darker lines are the functions of the end-moments when the frame is loaded at 8 points as shown in the figure. Because not less than 8 bolts are used for anchoring a large vertical vessel, and this particular 8-point loading causes the greatest end-moments, the area between lines of equal r values may be taken as representing the range of variation in the end-moments due to concentrated or uniform load assumption in practical cases.



LOADING CONDITION
FOR CHART No 1

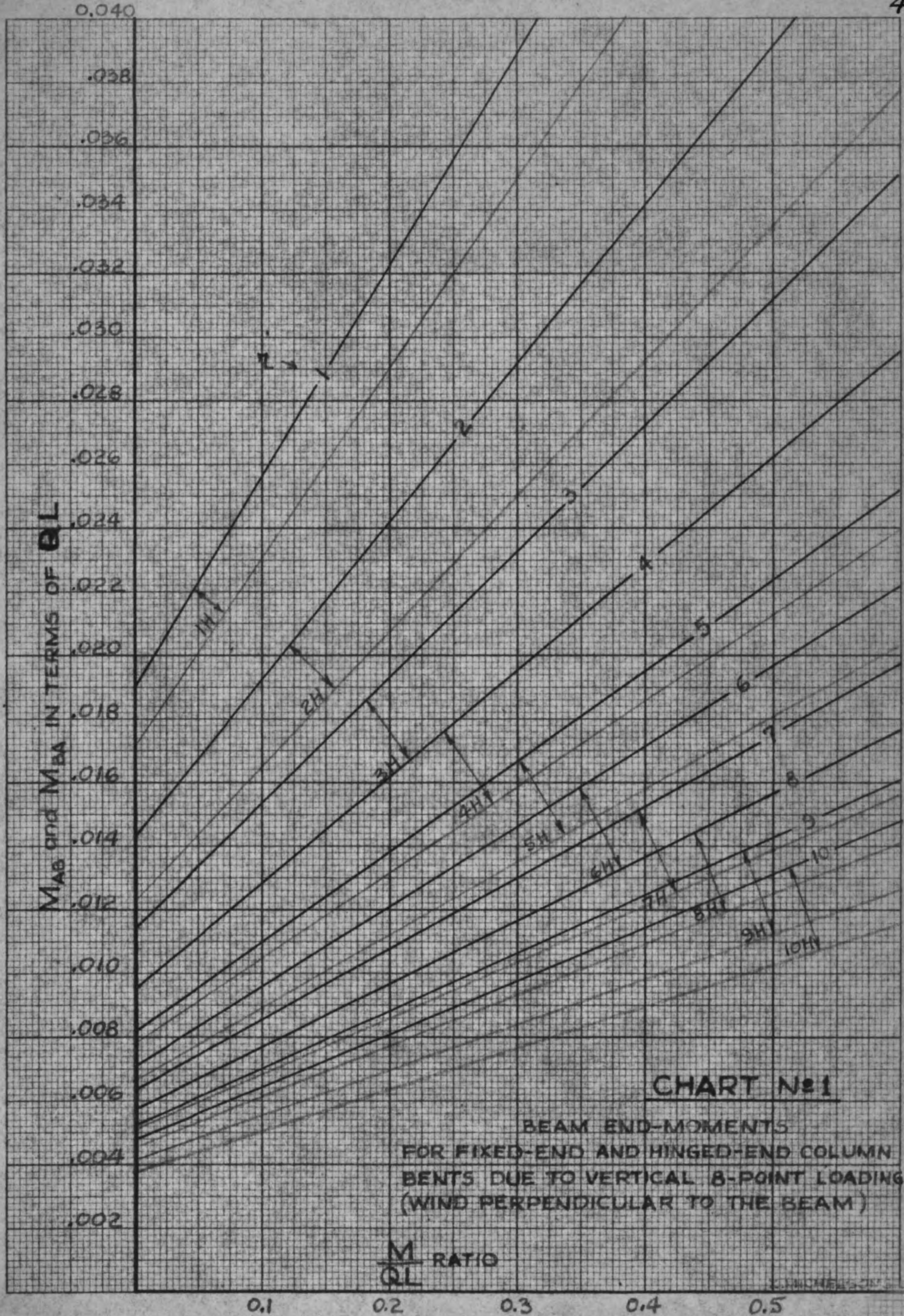
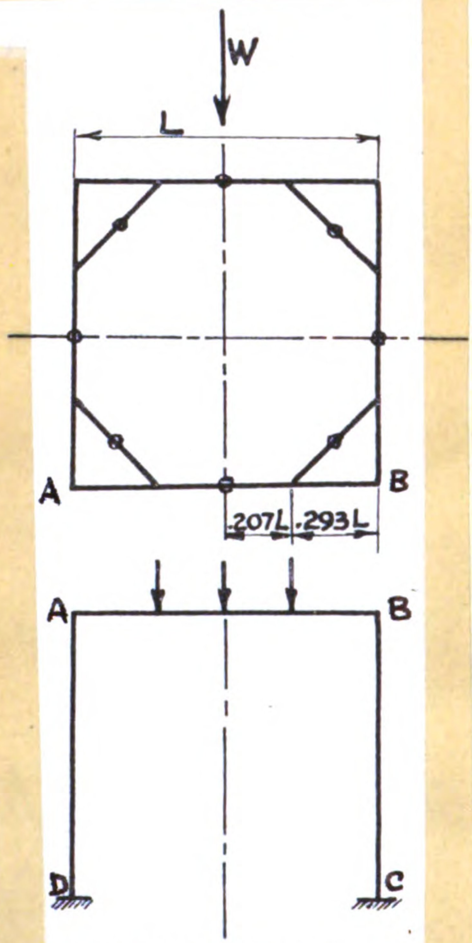
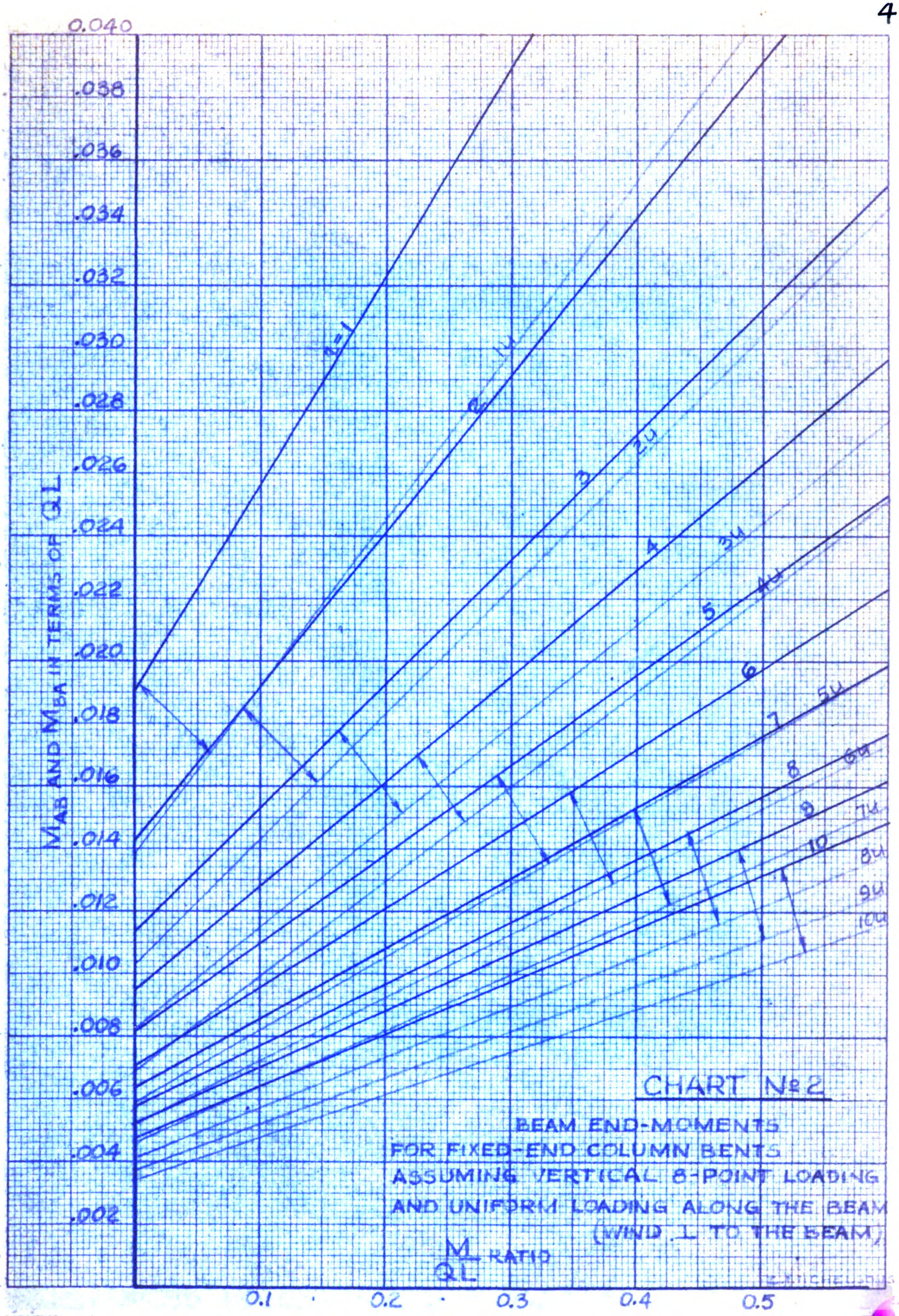


CHART No 1
BEAM END-MOMENTS
FOR FIXED-END AND HINGED-END COLUMN
BENTS DUE TO VERTICAL 8-POINT LOADING
(WIND PERPENDICULAR TO THE BEAM)
M_{AB} and M_{BA} IN TERMS OF QL
$\frac{M}{QL}$ RATIO
0.040
.038
.036
.034
.032
.030
.028
.026
.024
.022
.020
.018
.016
.014
.012
.010
.008
.006
.004
.002
0.1
0.2
0.3
0.4
0.5
1
2
3
4
5
6
7
8
9
10
1H
2H
3H
4H
5H
6H
7H
8H
9H
10H



LOADING CONDITION
FOR CHART № 2
(8-POINT LOADING)


0.040
.038
.036
.034
.032
.030
.028
.026
.024
.022
.020
.018
.016
.014
.012
.010
.008
.006
.004
.002
0.1
0.2
0.3
0.4
0.5
M_{AB} AND M_{BA} IN TERMS OF QL
$\frac{M}{QL}$ RATIO
8=1
1U
2
2U
3
3U
4
4U
5
5U
6
6U
7
7U
8
8U
9
9U
10
10U
CHART № 2
BEAM END-MOMENTS
FOR FIXED-END COLUMN BENTS
ASSUMING VERTICAL 8-POINT LOADING
AND UNIFORM LOADING ALONG THE BEAM
(WIND ⊥ TO THE BEAM)

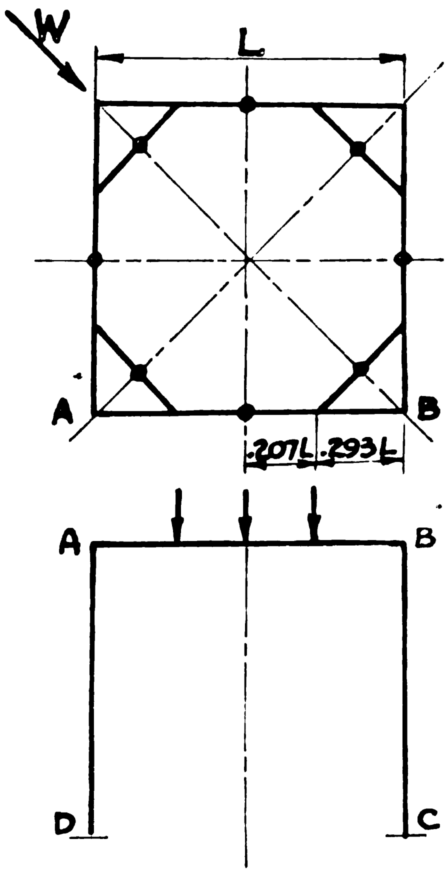


FIG. 17
LOADING CONDITION
FOR CHART No 3

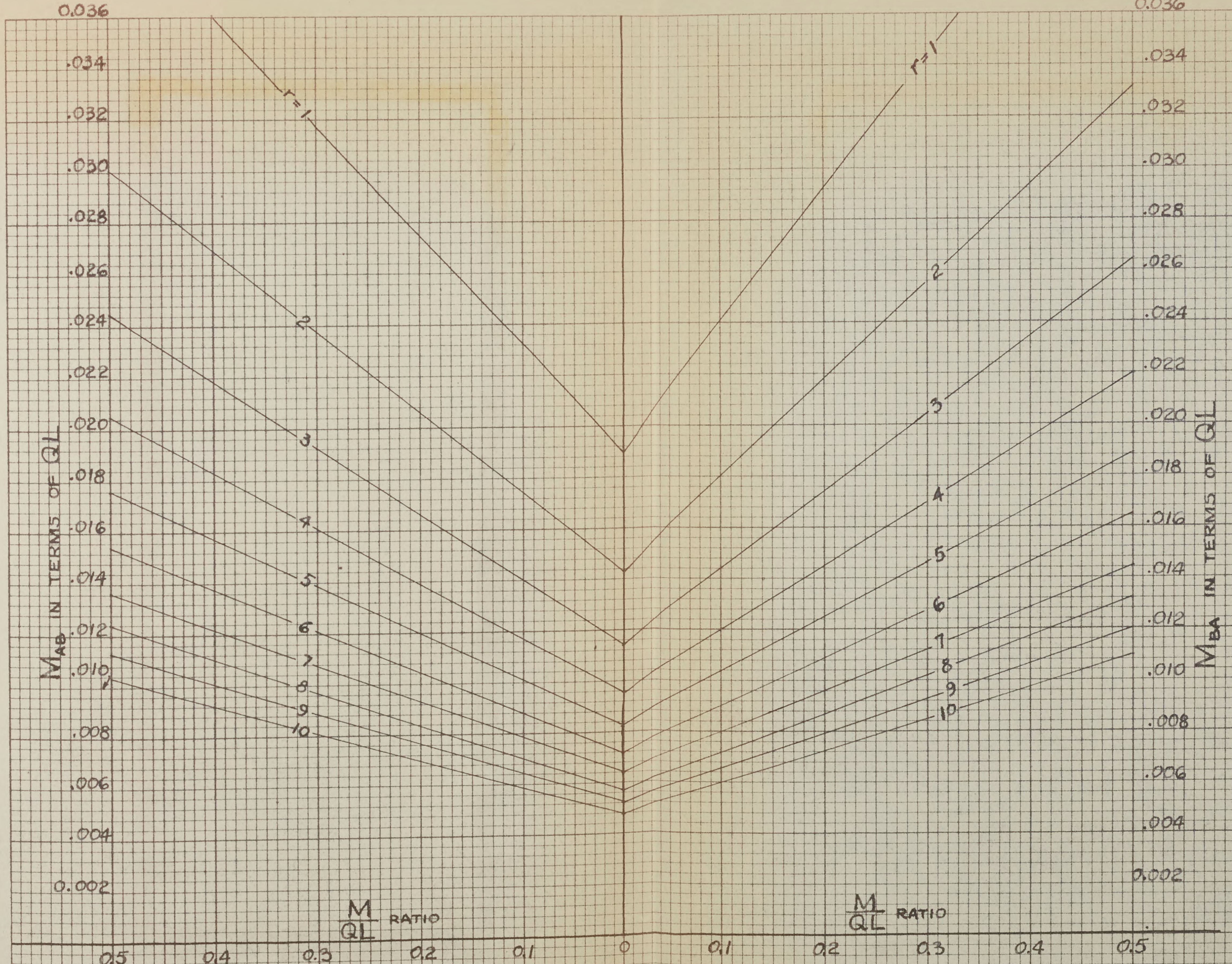
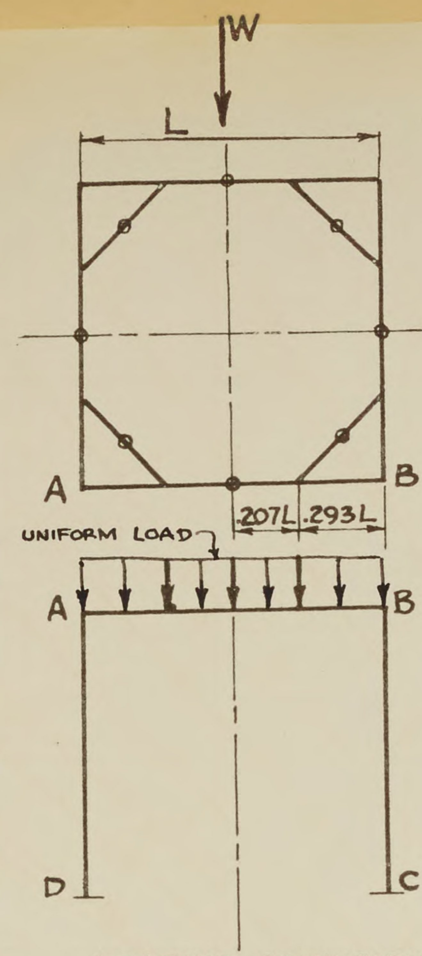


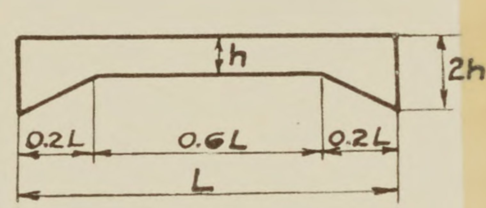
CHART № 3

BEAM END-MOMENTS FOR FIXED-END COLUMN BENTS
DUE TO VERTICAL 8-POINT LOADING (WIND DIAGONALLY)

Z. MICHELSONS



LOADING CONDITION
FOR CHART No 4



HAUNCHED BEAM

$k_{AB} = k_{BA} = 7.81$

$C_{AB} = C_{BA} = 0.659$

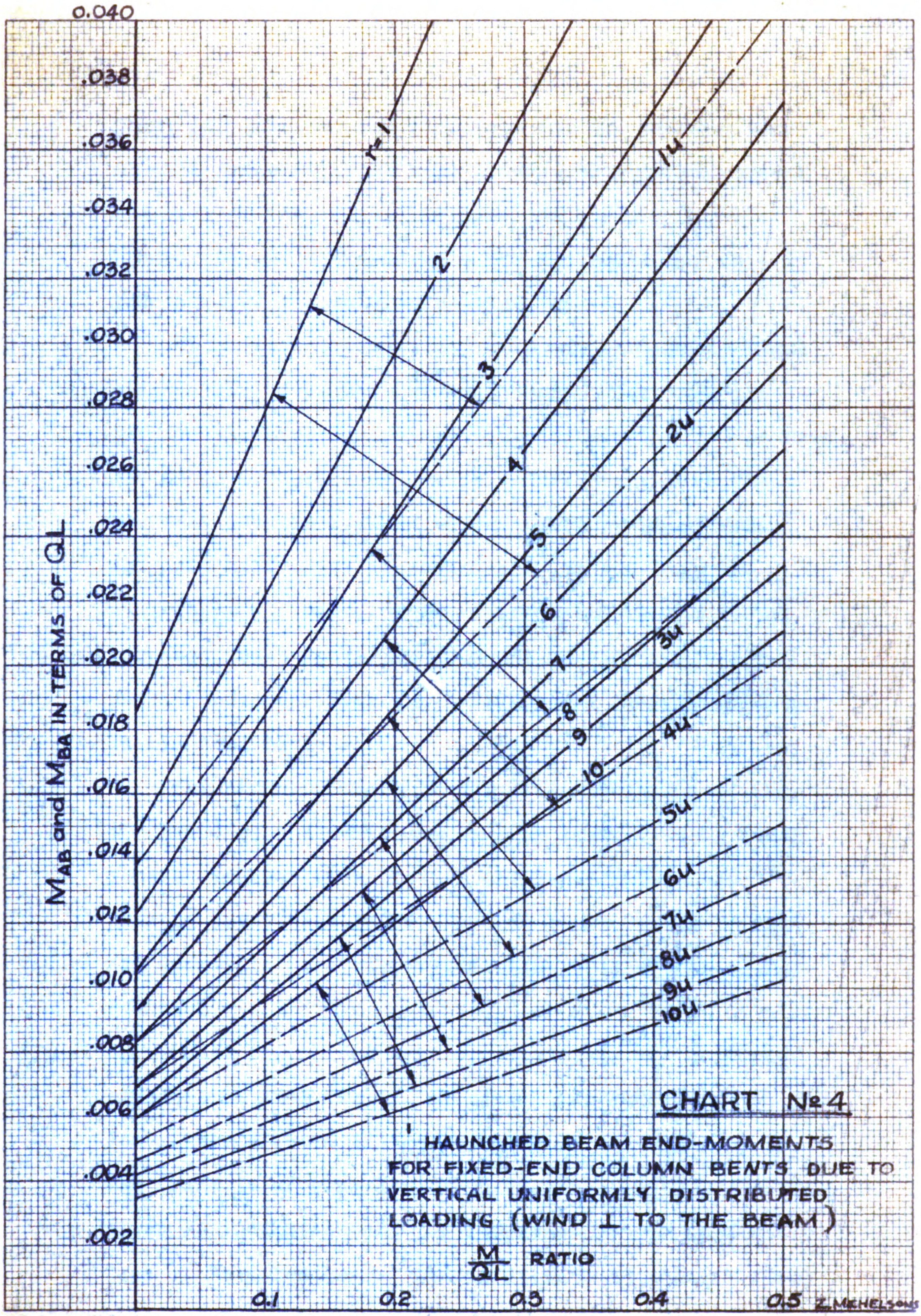


CHART № 4

HAUNCHED BEAM END-MOMENTS FOR FIXED-END COLUMN BENTS DUE TO VERTICAL UNIFORMLY DISTRIBUTED LOADING (WIND ⊥ TO THE BEAM)

Chart #3 gives moment functions for the leeward beam AB of a frame with the wind blowing along a horizontal diagonal of the frame. Because the loading is not symmetrical, the end-moments are not equal and have been plotted separately for each end.

This loading condition, with the horizontal force diagonally, gives smaller end-moments than when the horizontal force is applied perpendicularily to the beam AB, as in the previous charts. This is because the heaviest load is applied to the short diagonal beam and only half of it is transferred to the beam AB.

Chart #4 was plotted to determine the influence of haunched beams on the end-moments of the bent. The solid lines represent the functions of the end-moments in haunched beams.

The haunched beams have greater end-moments than the beams with a uniform moment of inertia along their length, because of a greater fixed end moment and greater carry-over factor for the haunched beam. This difference is more significant for bents with a higher r ratio.

Most reinforced concrete table-top supports have haunches at ends ~~fxx~~ of the beams, therefore the evaluation of the effects of the haunches on the stresses in the frame is of practical significance. Although chart #4 indicates only the influence of one particular type of haunched beams, the difference in results seems to justify the consideration of the haunches in frame analysis.

The practical significance of the error arising from neglecting the haunches at the joints is, however, minimized by the fact that the moments at the midspan of the beam are considerably greater than the end-moments and thus the midspan moments are likely to govern the design of the beam. Neglecting the effects of the haunches, would result in a safer design of the beam for greater midspan moments, while the haunches at the joints would take care of the increased end-moments.

The practical importance of the moment distribution with consideration of the haunches is also decreased by the difficulty of obtaining actual stiffness factor values fer reinforced concrete members with varying reinforcement and thus varying moment of inertia along the length of the member. Because the moment distribution constants cannot be determined exactly for the members, the consideration of the haunches not always is justified, because the application of the more convenient uniform member moment distribution procedure is easier.

Due to the need of considering additional variables, when analizing the frame for sidesway moments, no charts have been plotted that would include the effect of the sidesway on the end moments. An investigation, using numerical examples, however, indicates that the influence of the sidesway is of major importance, even to the extent of reversing stresses in the members due to change of sign of the end-moments. In cases where sidesway can occur, its effects should be investigated when designing the frame.

NUMERICAL EXAMPLES

Given: Design weight of vessel, contents and auxiliary equipment and piping supported from the vessel: $Q = 10000$ lbs. Exterior bending moment about the top of support due to wind earthquake or other horizontal forces acting on the vessel: $M = 20000$ ft. lbs. Beam span (determined by vessel diameter): $L = 10$ ft. Ratio of beam stiffness to column stiffness: $r = 6$.

Finding fixed-end moments in structural members of a table-top support. The M/QL ratio is computed from the above design data to be 0.2 . Assuming the columns to be fixed at the footing and wind perpendicular to the beam under consideration, the end-moments in the beam can be obtained from chart #1 by reading the ordinate on the moment line for $r = 6$, corresponding to the abscissa 0.2. Result: $M_{AB} = M_{BA} = 0.0121QL = 1210$ ft.lb. The end-moment in the columns at the footing is $\frac{1}{2}$ of this value.

For hinged-end column support the beam end moment is obtained from the 6H line (which happens to coincide with the moment line of a fixed-column bent having $r = 8$). $M_{AB} = M_{BA} = 1170$ ft.lb. When a uniform distribution of the vessel load along the length of the beam is assumed, the end moment is obtained from the 6u line in chart #2 as being $M_{AB} = M_{BA} = 920$ ft.lb.

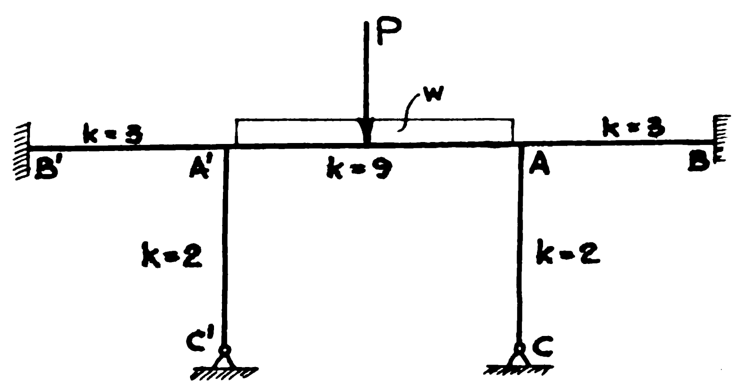


FIG.20

APPLICATION OF SERIES S_1.

The algebraic expression $\frac{2}{r+2}$ for S_1 can be used to find end-moments in symmetrical frames with members of uniform moment of inertia, with the central member loaded symmetrically and the other members being fixed or hinged at one end and rigidly connected to the central member at the other end.

The end moments of the loaded member can be found directly by the formula

$$M_{AA'} = M_{A'A} = \frac{2}{r+2}(F),$$

where "r" is the ratio of the stiffness factor of the loaded member to the sum of the stiffness factors of the other members at the joint. F is the fixed end moment in the loaded member due to the load.

Example. Find the end moments in the frame shown in Fig. 20. The computed stiffness factors of the members are indicated on the frame. The fixed end moment in the loaded beam AB is computed to be F = 120 ft.lb.

First, the stiffness factor of the column AC is modified by 3/4 to account for its hinged end: $k_{AC} = (3/4)2 = 1.5$. Then r is found: $r = \frac{9}{1.5+3} = 2$. The end moment in A'A is:

$$M_{AA'} = \frac{2}{r+2}(F) = \frac{2}{2+2}(120) = 60 \text{ ft.lb.}$$

This moment is counteracted by the members AC and AB at the joint A in the ratio $1.5/3 = \frac{1}{2}$. Thus, $M_{AC} =$

$(1/3)60 = 20$ ft.lb. and $M_{AB} = (2/3)\ 60 = 40$ ft.lb. M_{BA} is $\frac{1}{2} M_{AB} = 20$ ft.lb. $M_{CA} = 0$. The other end - moments are symmetrical about the center of the span A'A.

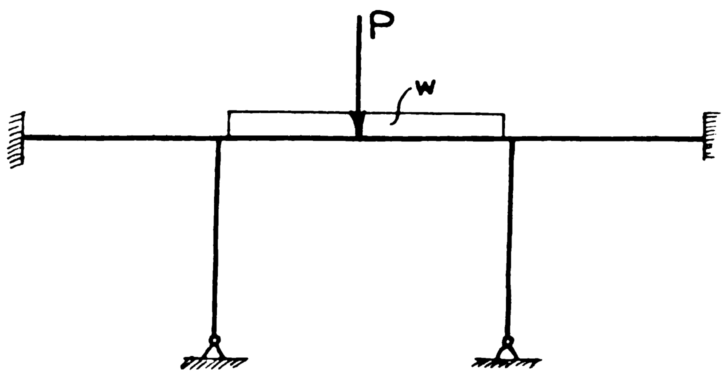


FIG. 20

CONCLUSION

This moment distribution analysis is helpful in recognizing the factors affecting the end moments of members in simple symmetrical bents.

Application of infinite series is shown to be a possible means of conveniently solving moment distribution problems. The series S_1 was shown to have a value that can be expressed in a simple algebraic equation. By use of this relationship, values of end-moments for symmetrical bents having members of uniform moment of inertia caused by symmetrical vertical loading can be obtained directly, without the need of a moment distribution, by substituting the appropriate values into the equation:

$$M_{AB} = XPL\left(\frac{2}{r+2}\right),$$

when the vertical loading is expressed in terms of a load P.

This relationship is not confined solely to the simple bents considered in this thesis, but is applicable to all symmetrical plane frames with members of a uniform moment of inertia and a symmetrical loading on the central member, with the other members having fixed or hinged ends. An example of such a frame is given in Fig. 20. The r value for such a frame is the ratio of the stiffness factor of the loaded member to the sum of the stiffness factors of the other members at the joint

modified to take care of the hinged-end members.

It may be possible to find a simple solution also for end moments in frames of a different structure and loading condition when simple expressions, exact solutions or convenient approximations can be found to express the moment distribution relationships.

The graphs of the end-moments can be used for design purposes, by computing the M/QL ratio from known design data and entering the chart with this value to find the end-moment in terms of QL. For practical purposes, it would be necessary to supplement the charts for end moments with charts of maximum positive moments and other necessary design function graphs.

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33) Standard Oil Development Company's engineering standard: design loads; open type structures. Structures; wind and earthquake design requirements

34) Standard Oil Company of California design guide: design of stiffening rings and internal struts at supports of horizontal cylindrical vessels.

35) Dean Peabody, Jr.
The Design of Reinforced Concrete Structures
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36) Portland Cement Association
Handbook of Frame Constants