WIDTH OF TWO-DIMENSIONAL ANOMALY SOURCES FROM GRAVITY AND MAGNETIC SECOND VERTICAL DERIVATIVES

Thesis for the Degree of M. S.

MICHIGAN STATE UNIVERSITY

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1971

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By

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A theoretical study of second vertical derivatives of gravity and magnetic anomalies shows that the width of the derivatives at zero value is useful in determining the width of two-dimensional tabular anomaly sources. The width of the zero values of the second vertical derivative increases with respect to the source width as the depth and depth extent increases and the dip decreases.

Widths of vertical two-dimensional tabular sources can be estimated if the depth and depth extent of the source are known using general families of curves relating true widths to observed widths as determined from the zero values of second vertical derivatives of gravity and vertical magnetic anomalies. In the gravity case if the width of the vertical tabular source is greater than approximately 4.5 times the depth, the zero values of second vertical derivatives are in error by less than 10 percent for any depth extent in predicting the width of the source. In the case of vertical magnetic anomalies, if the width exceeds the depth to the source by 2.0 or

more, the error in estimating width from second vertical derivatives is less than 10 percent for any depth extent of the source.

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A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

MASTER OF SCIENCE

Department of Geology

1971

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CHAPTER I

INTRODUCTION

Second vertical derivatives of gravity and magnetic fields have been used extensively for many years to enhance anomalies characterized by high frequency components. Another important property of second vertical derivatives is their relationship to geologic boundaries of anomalous sources. Vacquier and others (1951) have pointed out that the zero curvature or second vertical derivative contour of the total magnetic intensity closely approximates the outline of the vertical prismatic magnetic anomaly sources extending to great depths. Leney (1966) has used this principle in estimating the width of a nearly vertical, tabular magnetic ore body. Rudman and Blakely (1965) have used the second vertical derivative of both gravity and magnetic fields to outline the subsurface configuration of a balsaltic plug intruded into a granitic basement in Indiana. The delineation of the outline of any anomaly source is invaluable in the geophysical mapping of geologic units and is useful as a starting point in defining the anomaly source in the indirect method of interpreting gravity and magnetic anomalies.

Despite this important property of second vertical derivatives little study has been devoted to analysis of the method's applicability and accuracy. In this investigation, the relationship of the zero curvature values of gravity and vertical magnetic intensity anomalies to the width of a tabular, two-dimensional, dike-like source is determined analytically over a wide range of source parameters. The tabular, two-dimensional, dike-like source is considered because of its similarity to many geological features and its relatively well-known anomaly field. The study of the second vertical derivative of the vertical magnetic intensity has been restricted to induced magnetization in a vertical geomagnetic field, or the total magnetic intensity in this special case.

In order to calculate theoretical values, it was necessary to derive the equations for the second vertical derivative of gravity and vertical magnetic intensity of two-dimensional vertical rectangular prisms. In addition, equations were derived for second vertical derivatives of two-dimensional polygonal-shaped prisms which can be used to approximate the source of any two-dimensional gravity and magnetic feature. Furthermore, a method based on the least squares technique is described for calculating second vertical derivatives from field data derived from two-dimensional sources.

CHAPTER II

THE SECOND VERTICAL DERIVATIVE METHOD

Methods of approximating the second vertical derivative of gravity and magnetic fields were introduced into the geophysical industry in the early nineteen thirties to aid in isolating economically interesting anomalies. However, the first account of the method was not published until 1949 (Peters, 1949 and Henderson and Zietz, 1949). Subsequently, other approaches to the calculation of second vertical derivatives have been proposed (e.g.; Elkins, 1951; Rosenbach, 1953; and Henderson, 1960) and their relative merits discussed. Fuller (1967) has calculated the frequency response of several of the more common second vertical derivative calculation methods utilizing the Fourier transform.

In general, the second vertical derivative is determined from orthogonal second horizontal derivatives of the anomaly field (A) using LePlace's equation:

$$\frac{\partial^2 A}{\partial Z^2} = - \left(\frac{\partial^2 A}{\partial X^2} + \frac{\partial^2 A}{\partial Y^2}\right) \tag{1}$$

where X and Y are orthogonal coordinates on a horizontal surface and Z is the vertical component. The second horizontal derivative can be calculated directly from a contour map of the anomaly field. Most methods utilize a numerical method employing anomaly values at set radial distances (r) from the point of calculation. The second vertical derivative is calculated using an equation of the general form:

$$\frac{\partial^2 A}{\partial Z^2} = \frac{K}{r^2} (W_0 C_0 + W_1 C_1 + W_2 C_2 + . . . + W_n C_n) \qquad (2)$$

where C_0 is the value of the anomaly field at the point of calculation; C_1, C_2, . . . C_n are average anomaly values of data points on rings 1, 2, . . , n; W_0, W_1, W_2, . . , W_n, are weighting factors determined by the particular method either theoretically or empirically; K is a numerical coefficient; and r is the distance represented by the unit grid spacing of the anomaly values (Fig. 1).

The choice of weighting factors, number of rings, and the number of data points on each ring are a result of extensive comparative tests on theoretical and field data of various degrees of precision, frequencies, and data intervals. The final product, a second vertical derivative map, is only an approximation to the true second vertical derivative.

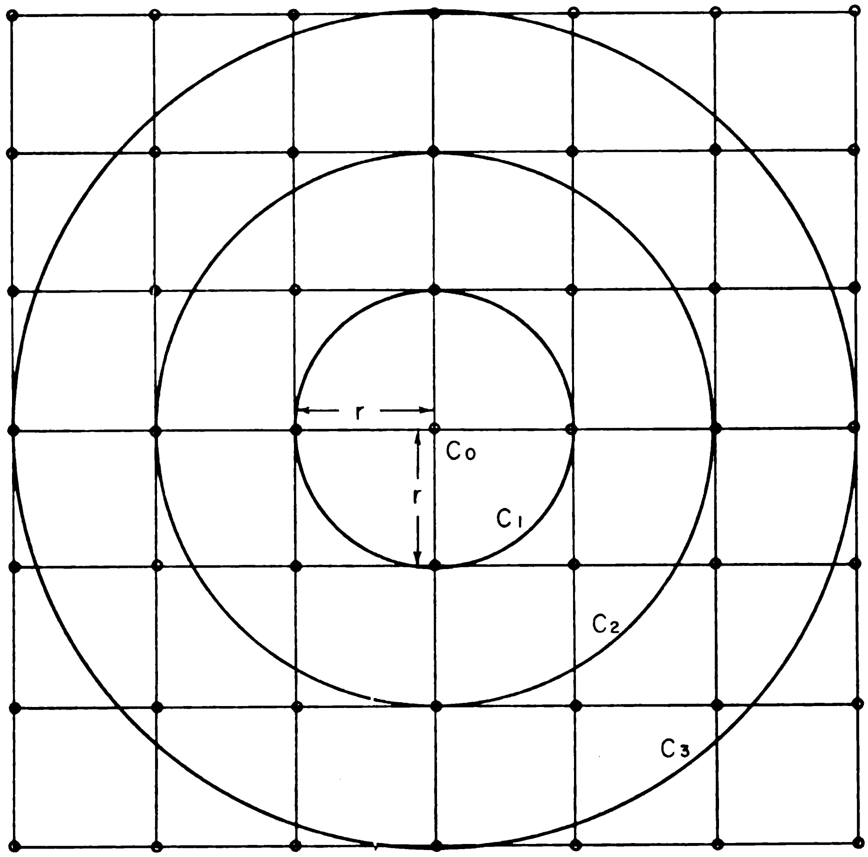
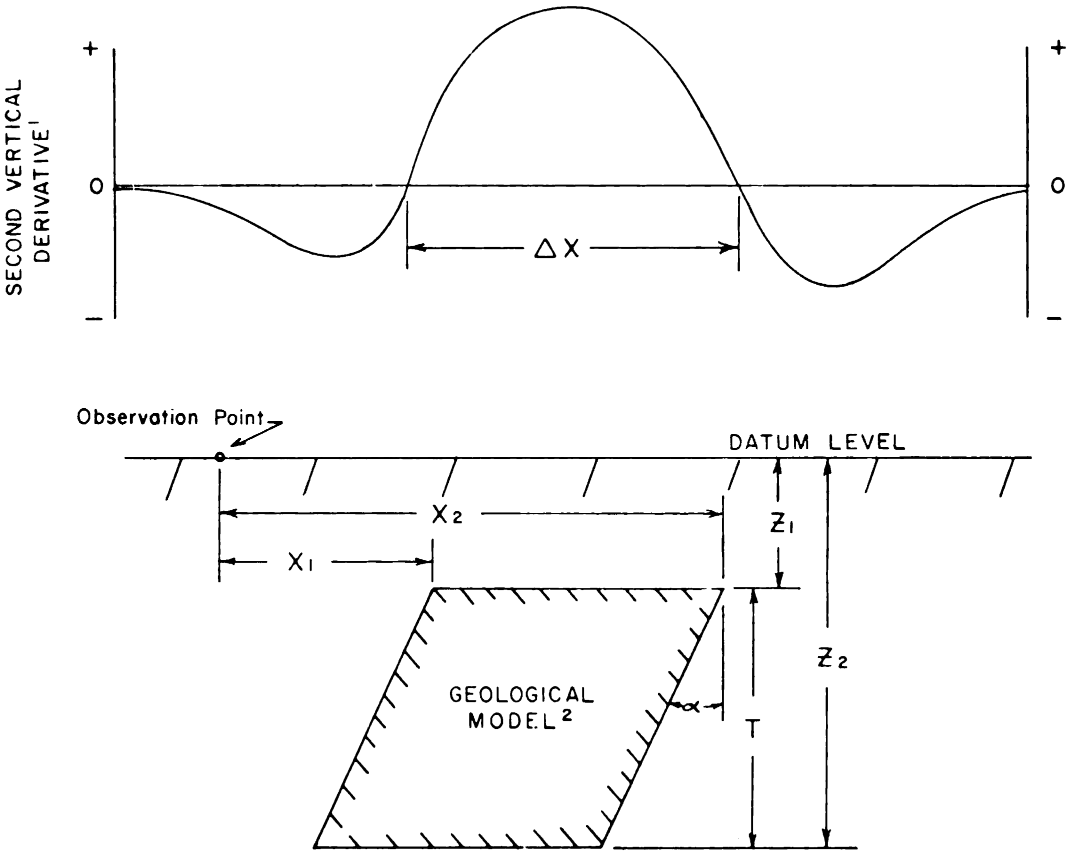


FIGURE 1.--Example of template used in calculating second vertical derivatives from gridded data. (Dots represent data points).

The second vertical derivative of gravity and magnetic anomalies has been used primarily to increase the perceptibility of steep-gradient anomalies derived from shallow sources. Romberg (1958) has pointed out that second vertical derivatives decrease two powers faster with depth than the original anomaly field. As a result second vertical derivatives attenuate rapidly as the depth to the source increases, thus increasing the perceptibility of shallow source anomalies. An additional use of second vertical derivatives is to aid in defining the boundaries of the anomaly source. As noted perviously, the zero second vertical derivative contour approaches the outline of the upper surface of the anomaly source. This is illustrated in the principal profile over a two-dimensional anomalous source (Fig. 2). The relationship between the distance between the zero cross-over points of the second vertical derivative profile (ΔX) and the parameters of tabular magnetic and gravity anomaly sources is the subject of this investigation.



X_1=Distance from the observation point to the near side of the geological model.
X_2=Distance from the observation point to the far side of the geological model.
Z_1=Depth to the top of the model.
Z_2=Depth to the bottom of the model.
T =Thickness of the model.
α =Dip of the model.
ΔX=Distance between cross-over points. The cross-over point is defined as the point where the second vertical derivative changes sign.

FIGURE 2.--Definition of parameters used for geological models and second vertical derivative calculations.

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[1]All second vertical derivative values given in the text are in units of 10^{-10}.

[2]For all gravitational sources a density of 1.0 gr./cc. was used, and for all magnetic sources a susceptibility of 0.001 emu/cc and induced field of 0.58 oersteds was used.

CHAPTER III

THE GRAVITY CASE

The equation for the gravitational anomaly due to a rectangular-shaped two-dimensional prism of density contrast $\Delta\rho$ as given by Heiland (1940) is

$$\Delta g = 2\gamma\Delta\rho[X_2 \ln\left(\frac{X_2^2+Z_2^2}{X_2^2+Z_1^2}\right)^{\frac{1}{2}} - X_1 \ln\left(\frac{X_1^2+Z_2^2}{X_1^2+Z_1^2}\right)^{\frac{1}{2}}$$

$$+Z_2\left(\tan^{-1}\frac{X_2}{Z_2} - \tan^{-1}\frac{X_1}{Z_2}\right) - Z_1\left(\tan^{-1}\frac{X_2}{Z_1} - \tan^{-1}\frac{X_1}{Z_1}\right)] \qquad (3)$$

where Δg is the calculated gravity anomaly, γ is the universal gravitational constant, and the other parameters are as defined in Figure 2. The first vertical derivative of gravity as determined by taking the derivative of equation 3 with respect to Z holding X constant is

$$\frac{\partial \Delta g}{\partial Z} = 2\gamma\Delta\rho[\tan^{-1}\frac{X_2}{Z_2} - \tan^{-1}\frac{X_1}{Z_2} - \tan^{-1}\frac{X_2}{Z_1}$$

$$+ \tan^{-1}\frac{X_1}{Z_1}]. \qquad (4)$$

Taking the partial derivative of equation 4 with respect to Z we obtain the second vertical derivative of gravity of a two-dimensional rectangular-shaped prism,

$$\frac{\partial^2 \Delta g}{\partial Z^2} = 2\gamma\Delta\rho\left[\frac{X_1}{X_1^2+Z_1^2} + \frac{X_2}{X_2^2+Z_2^2} - \frac{X_1}{X_1^2+Z_2^2} - \frac{X_2}{X_2^2+Z_1^2}\right]. \tag{5}$$

In order to obtain a more general relationship and specifically to calculate the second vertical derivative of dipping, two-dimensional, tabular bodies, we can consider the mathematical expression for the gravity anomaly of a two-dimensional prism of arbitrary cross-section (Grant and West, 1965):

$$\Delta g = 2\gamma\Delta\rho \sum_{K=1}^{N} \frac{b_K}{1+a_K^2}\left[\frac{1}{2}\ln\left(\frac{X_{K+1}^2+Z_{K+1}^2}{X_K^2+Z_K^2}\right) + a_K \left(\tan^{-1}\frac{X_{K+1}}{Z_{K+1}} - \tan^{-1}\frac{X_K}{Z_K}\right)\right] \tag{6}$$

Where:

$$a_K = \frac{X_{K+1}-X_K}{Z_{K+1}-Z_K}$$

$$b_K = \frac{X_K Z_{K+1} - X_{K+1} Z_K}{Z_{K+1}-Z_K}$$

Δg is the calculated gravity anomaly, γ is the universal gravitational constant, $\Delta\rho$ is the density differential between the anomalous source and the country rock,

X_K (K=1,2,3,4,. . .,N), X_{K+1} (K=1,2,3,4,. . .,N), Z_K (K=1,2,3,4,. . .,N), and Z_{K+1} (K=1,2,3,4,. . .,N) are horizontal and vertical distances respectively to the intersection points of the straight lines defining the shape of the source (see Fig. 3). The first vertical derivative of gravity as determined by taking the derivative of equation 6 with respect to Z holding X constant is:

$$\frac{\partial \Delta g}{\partial Z} = 2\gamma\Delta\rho\{[\frac{1}{2}\ ln(\frac{X_{K+1}^2+Z_{K+1}^2}{X_K^2+Z_K^2}) + a_K$$

$$(\tan^{-1}\frac{X_{K+1}}{Z_{K+1}} - \tan^{-1}\frac{X_K}{Z_K})][(\frac{1}{1+a_K^2})$$

$$(\frac{X_K-X_{K+1}}{Z_{K+1}+Z_K})]\} + \{[\frac{b_K}{1+a_K^2}] \tag{7}$$

$$[(\frac{(X_K^2+Z_K^2)Z_{K+1}-(X_{K+1}^2+Z_{K+1}^2)Z_K}{(X_K^2+Z_K^2)(X_{K+1}^2+Z_{K+1}^2)}) + a_K$$

$$(\frac{X_K}{X_K^2+Z_K^2} - \frac{X_{K+1}}{X_{K+1}^2+Z_{K+1}^2})]\}.$$

Taking the partial derivative of equation 7 with respect to Z we obtain the second vertical derivative of gravity,

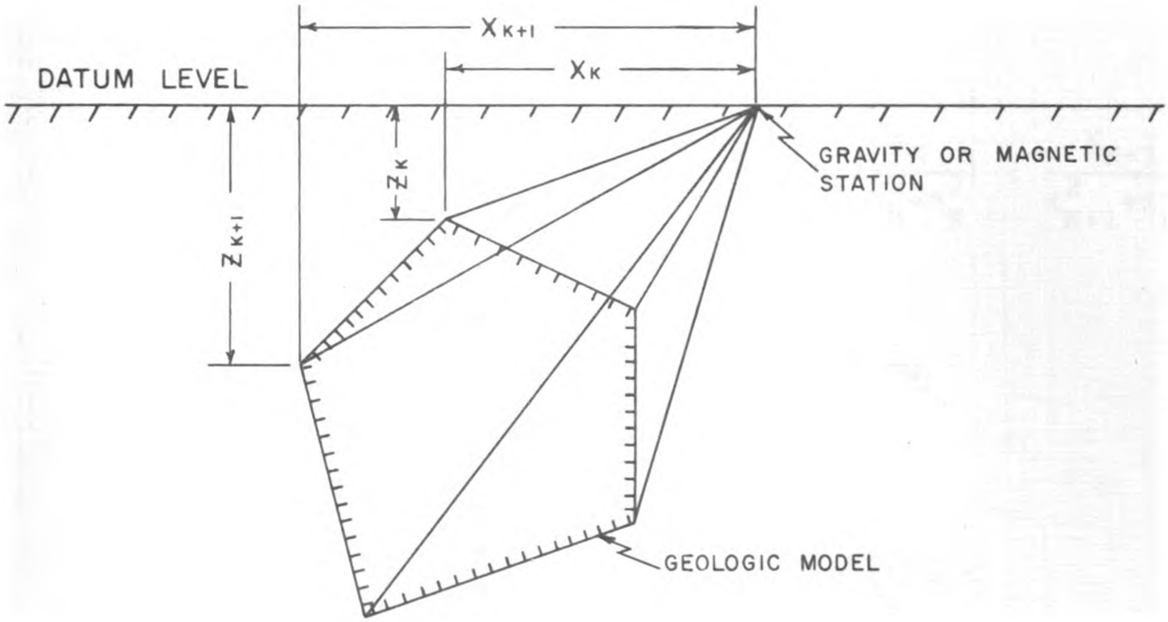

FIGURE 3.--Cross-section of a two-dimensional geological model represented by an irregular-shaped polygon.

$$\frac{\partial^2 \Delta g}{\partial z^2} = 2\gamma\Delta\rho\{\{[(\frac{1}{1+a_K^2})(\frac{x_K-x_{K+1}}{z_{K+1}-z_K})]$$

$$[(\frac{(x_K^2+z_K^2)z_{K+1}-(x_{K+1}^2+z_{K+1}^2)z_K}{(x_K^2+z_K^2)(x_{K+1}^2+z_{K+1}^2)})+a_K(\frac{x_K}{x_K^2+z_K^2}) - \frac{x_{K+1}}{x_{K+1}^2+z_{K+1}^2}]]$$

$$\}+\{[(\frac{(x_K^2+z_K^2)z_{K+1}-(x_{K+1}^2+z_{K+1}^2)z_K}{(x_K^2+z_K^2)(x_{K+1}^2+z_{K+1}^2)}) + a_K$$

$$(\frac{x_K}{x_K^2+z_K^2} - \frac{x_{K+1}}{x_{K+1}^2-z_{K+1}^2})][(\frac{1}{1+a_K^2})(\frac{x_K-x_{K+1}}{z_{K+1}-z_K})]\} +$$

$$\{[(\frac{b_K}{1+a_K^2})] \tag{8}$$

$$[(\frac{\{[(x_K^2+z_K^2)(x_{K+1}^2+z_{K+1}^2)][(x_K^2+z_K^2)+(x_{K+1}^2+z_{K+1}^2)]\}}{[(x_K^2+z_K^2)(x_{K+1}^2+z_{K+1}^2)]^2}$$

$$\frac{-\{[(x_K^2+z_K^2)z_{K+1}-(x_{K+1}^2+z_{K+1}^2)z_K][2(x_K^2+z_K^2)z_{K+1}+}{}$$

$$\frac{2(x_{K+1}^2+z_{K+1}^2)z_K]\}}{}) + a_K\ (\frac{2x_{K+1}z_{K+1}}{(x_{K+1}^2+z_{K+1}^2)^2} - \frac{2x_K z_K}{(x_K^2+z_K^2)^2}]]\}\}.$$

The computer programs for calculating the second vertical derivative of gravity according to equations 5 and 8 are given in Appendices A and B respectively.

CHAPTER IV

RESULTS OF GRAVITY MODELING

The horizontal distance between the zero second vertical derivative values of gravity (ΔX of Fig. 2) were calculated for a series of two-dimensional, vertical, tabular bodies from equation 5. The direction of the calculated profiles is perpendicular to the Y-axis along which the source is assumed to extend infinitely. A simple interation scheme was used to calculate the horizontal position of the zero value of the second vertical derivative from which ΔX can be computed. This interation appears as a subroutine to the computer programs in the appendices. The horizontal positions were calculated to a precision of 0.2 percent or less of the minimum distance parameter of the geologic models.

The ΔX values were computed for models varying from 500 to 10,000 units for depth to the top, width and thickness, except for the special case of $T/Z_1=100$ where T was set equal to 100000 units. The results are presented in Figure 4 with $\Delta X/Z_1$ plotted against W/Z_1 where W is the true width of the model and Z_1 is the depth to the top of the source from the observation surface. The values

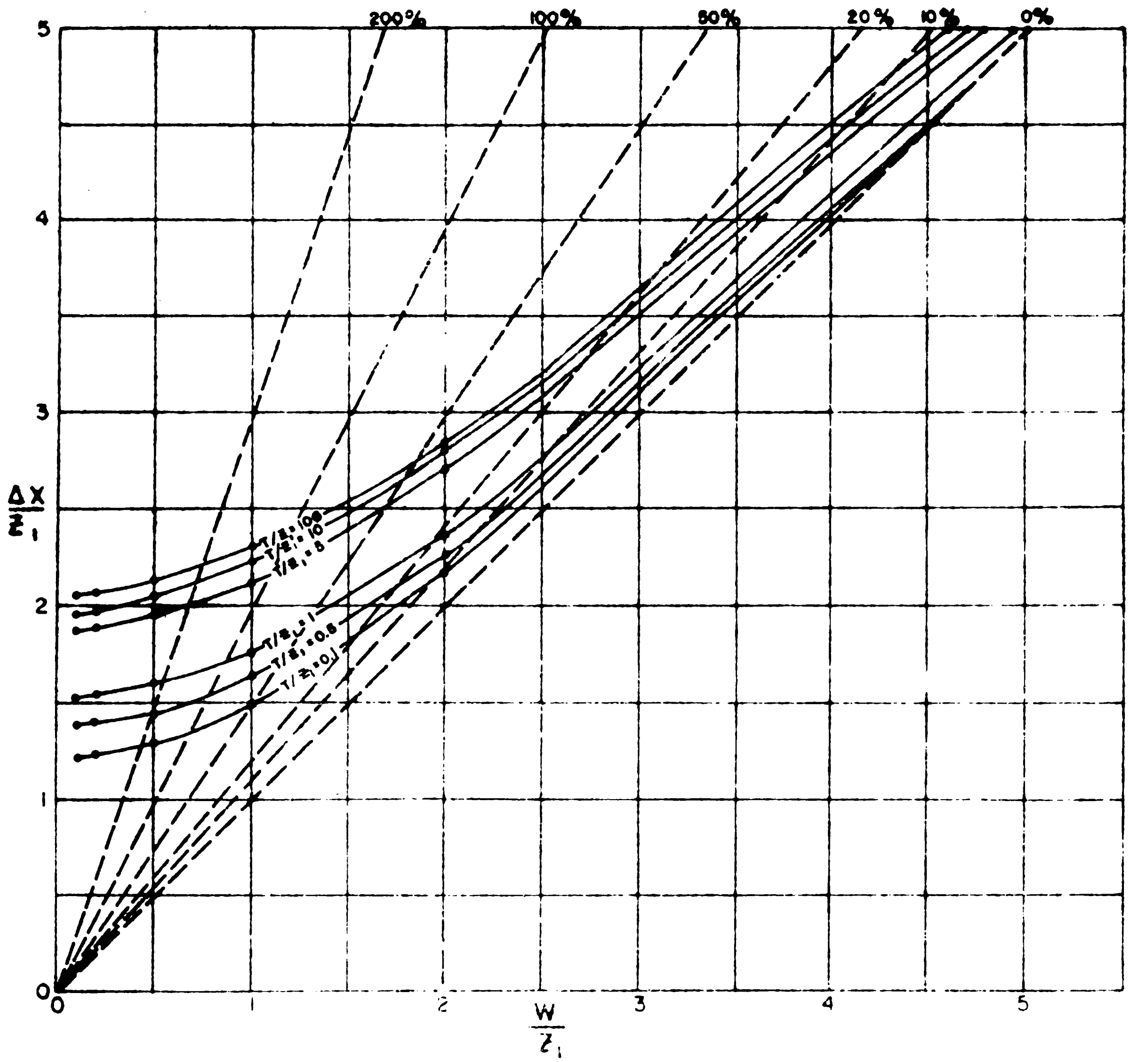

FIGURE 4.--Relationship between W, T, Z_1, and ΔX for a vertical tabular source of gravity anomalies.

of ΔX and W are divided by Z_1 to generalize the results. A family of curves is presented for values of T/Z_1 from 0.1 to 100. Error curves also have been plotted on Figure 4 which show the percent error according to the equation $((\Delta X-W)/W) \times 100$.

A number of very important relationships between the width, depth, and thickness of gravity anomaly sources and ΔX can be observed from Figure 4. First, for any given values of T, Z_1, and W, ΔX is always greater than W, except for the specific case of $Z_1=0$ where ΔX equals W. Secondly, as the ratio of W/Z_1 decreases, as, for example, when the depth increases while the width remains constant, the ratio of $\Delta X/Z_1$ decreases, but the percent error rapidly increases for all values of T/Z_1. Thirdly, as the width increases for a constant depth, the ratio of $\Delta X/Z_1$ increases and the error decreases for all values of T/Z_1. The curves asymptotically approach zero error as W/Z_1 increases toward large values. Fourthly, with width and depth constant, the error increases directly with the thickness of the source, but at a diminishing rate as thickness increases. The error difference between ratios of T/Z_1 of 10 and 100 are relatively small.

Utilizing the general relationships shown in Figure 4, if ΔX of a vertical two-dimensional tabular body is determined from a second vertical derivative of gravity map or profile and Z_1 is known from geological or geophysical data, a range of widths of the causative body can be determined or the true width determined if the thickness of the source is known. For example, if ΔX is found to be 2700 feet and Z_1 and T are estimated to be 1000 and 5000 feet respectively, the width of the body is 2000 feet. As a general statement, if W/Z_1 is approximately 4.5 or greater, ΔX will be in error by less than 10 percent, for any ratio of T/Z_1.

Plotted in Figure 4 is a curve of $T/Z_1=100$. In this case, the thickness of the source can be considered essentially infinite. At great depths the mass of this source has little influence on the observed gravitational field, and therefore little effect on the second vertical derivative and ΔX. Thus, any value of T can be considered essentially infinite when $T/Z_1>10$. This conclusion can be verified by the following example. Assume the model parameters are T=1000 feet, $Z_1=500$ feet ($T/Z_1=20$), and W=500 feet. Using the $T/Z_1=10$ curve of Figure 4, $\Delta X/Z_1=2.20$, while for the $T/Z_1=100$ curve, $\Delta X/Z_1=2.23$. The difference introduced in calculating ΔX by this slight variation in $\Delta X/Z_1$ is less than 1.5 percent. The actual value of $\Delta X/Z_1$, for $T/Z_1=20$ is 2.22 (not shown in Fig. 4).

The effect of dip of tabular bodies on the ΔX of gravitational fields is illustrated for a few special cases in Figure 5. In this figure the difference in the ΔX of dipping tabular bodies (ΔX_1) and vertical tabular bodies (ΔX_2) is plotted against T for specific cases of

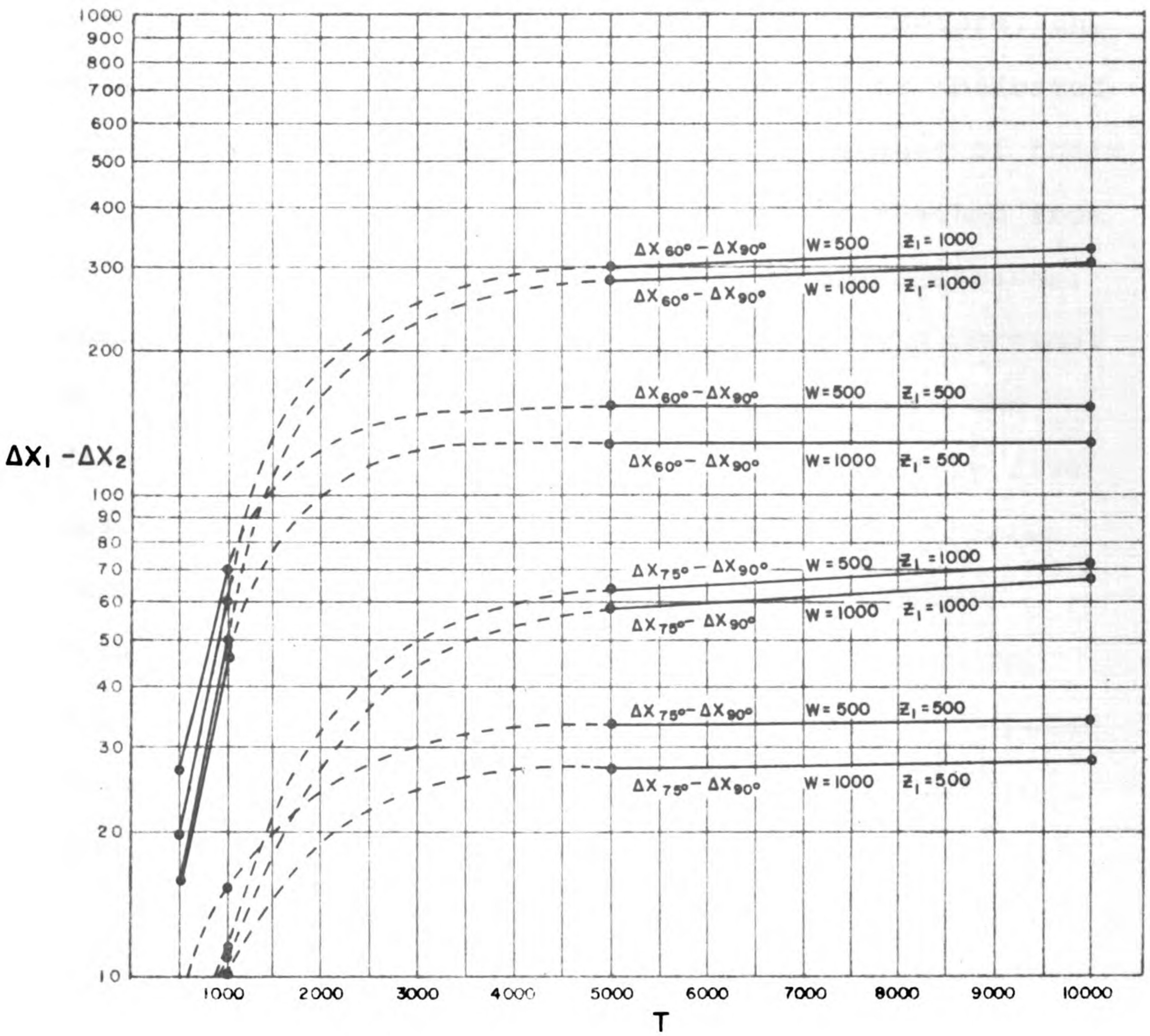

FIGURE 5.--The effect of dip of tabular sources on ΔX of gravitational fields.

anomaly sources dipping at 60° and 75°. Only a relatively few data points have been calculated and, therefore, the positions of the curves between data points as indicated by dashed lines may be somewhat in error because of their high curvature. Several conclusions can be reached from this figure. First, ΔX increases as the dip decreases. Second, the difference in the ΔX of dipping and vertical tabular bodies increases rapidly as thickness of the bodies increase up to thicknesses of approximately five times the depth to the top of the source. For greater thicknesses the difference in ΔX remains relatively constant and for one case starts to decrease. Third, the difference in the ΔX of dipping and vertical tabular bodies increases with increasing depth to the top and decreasing width of the source.

CHAPTER V

THE MAGNETIC CASE

The equation for the vertical magnetic intensity due to a rectangular-shaped two-dimensional prism of susceptibility contrast Δk as given by Cook (1950) is:

$$\Delta V = 2\Delta kH\left(\tan^{-1}\frac{X_1}{Z_1} - \tan^{-1}\frac{X_1}{Z_2} - \tan^{-1}\frac{X_2}{Z_1} + \tan^{-1}\frac{X_2}{Z_2}\right) \qquad (9)$$

where ΔV is the calculated vertical magnetic intensity, Δk is the susceptibility contrast, H is the Earth's magnetic field, and the other parameters are as defined in Figure 2. The first vertical derivative of the vertical magnetic intensity as determined by taking the derivative of equation 9 with respect to Z holding X constant is:

$$\frac{\partial \Delta V}{\partial Z} = 2\Delta kH\left(\frac{X_1}{X_1^2+Z_1^2} - \frac{X_1}{X_1^2+Z_2^2} - \frac{X_2}{X_2^2+Z_1^2} + \frac{X_2}{X_2^2+Z_2^2}\right). \qquad (10)$$

Taking the partial derivative of equation 10 with respect to Z we obtain the second vertical derivative of magnetics of a two-dimensional rectangular-shaped prism,

$$\frac{\partial^2 \Delta V}{\partial Z^2} = 4\Delta k H\left[\frac{X_1 Z_1}{(X_1^2+Z_1^2)^2} - \frac{X_1 Z_2}{(X_1^2+Z_2^2)^2} - \frac{X_2 Z_1}{(X_2^2+Z_1^2)^2} + \frac{X_2 Z_2}{(X_2^2+Z_2^2)^2}\right]. \quad (11)$$

In order to obtain a more general relationship and specifically to calculate the second vertical derivative of dipping, two-dimensional, tabular bodies, we can consider the mathematical expression for the vertical magnetic intensity of a two-dimensional prism of arbitrary cross-section (Grant and West, 1965):

$$\Delta V = 2\Delta k H \sqrt{1-\cos^2\lambda\cos^2 I}\ \sum_{K=1}^{N} \left(\frac{1}{1+a^2}\right)$$

$$\{[(a_K \sin\beta + \cos\beta) X_K -$$

$$[\ln \sqrt{\frac{(1+a_K^2) Z_{K+1}^2 + 2a_K b_K Z_{K+1} + b_K^2}{(1+a^2 k) Z_K^2 + 2a_K b_K Z_K + b_K^2}} -$$

$$(a_K \cos\beta - \sin\beta) X_K][\tan^{-1}\left(\frac{(1+a_K^2) Z_{K+1}}{b_K} + a_K\right)$$

$$-\tan^{-1}\left(\frac{(1+a_K^2) Z_K}{b_K} + a_K\right)]\} \quad (12)$$

where:

$$a_K = \frac{X_{K+1}-X_K}{Z_{K+1}-Z_K}, \quad b_K = \frac{X_K Z_{K+1}-X_{K+1} Z_K}{Z_{K+1}-Z_K}$$

$$\beta = \tan^{-1}\left(\frac{\tan I}{\sin\lambda}\right)$$

where ΔV is the vertical magnetic intensity, Δk is the susceptibility contrast between the anomalous source and the country rock, H is the Earth's magnetic field, λ is the strike of the body, I is the inclination of the Earth's magnetic field, and X_K (K=1,2,3,. . ,N), X_{K+1} (K=1,2,3, . . , N), Z_K (K=1,2,3,. . ,N), Z_{K+1} (K=1,2,3, . . ,N) are horizontal and vertical distances respectively to the intersection points of the straight lines defining the shape of the source (See Fig. 3). The first vertical derivative of the vertical magnetic intensity as determined by taking the derivative of equation 12 with respect to Z holding X constant is:

$$\frac{\partial \Delta V}{\partial Z} = 2\Delta kH \sqrt{1-\cos^2\lambda\cos^2 I} \sum_{K=1}^{N} e_K$$

$$\left[\frac{j_K v_K - i_K m_K}{j_K v_K}\right] - g_K\{[(\frac{1}{1+s_K})$$

$$(\frac{b_K u_K - c_K u_K Z_{K+1}}{b_K^2})] - [(\frac{1}{1+t_K}) \qquad (13)$$

$$(\frac{b_K u_K - c_K u_K Z_K}{b_K^2})]\}$$

where:

$$c_K = \frac{X_K - X_{K+1}}{Z_{K+1} - Z_K}$$

$$e_K = a_K \sin\beta + \cos\beta$$

$$g_K = a_K \cos\beta - \sin\beta$$

$$i_K = u_K Z_{K+1}^2 + 2a_K b_K Z_{K+1} + b_K^2$$

$$j_K = u_K Z_K^2 + 2a_K b_K Z_K + b_K^2$$

$$m_K = 2(u_K Z_K + a_K b_K + a_K c_K Z_K + b_K c_K)$$

$$s_K = 1 + \left(\frac{u_K Z_{K+1}}{b_K} + a_K\right)^2$$

$$t_K = 1 + \left(\frac{u_K Z_K}{b_K} + a_K\right)^2$$

$$u_K = (1 + a_K^2)$$

$$v_K = 2(u_K A_{K+1} + a_K b_K + a_K c_K Z_{K+1} + b_K c_K).$$

Taking the partial derivative of equation 13 with respect to Z we obtain the second vertical derivative of the vertical magnetic intensity,

$$\frac{\partial^2 \Delta V}{\partial Z^2} = 2\Delta kH \sqrt{1-\cos^2\lambda\cos^2 I} \sum_{K=1}^{N} \{e_K$$

$$\left[\frac{[j_K v_K (j_K n_K + m_K v_K - i_K n_K + m_K v_K)]}{(1_K j_K)^2} + \right. \tag{14}$$

$$\left.\frac{[(j_K v_K - i_K m_K)(m_K v_K + j_K n_K)]}{}\right]\} +$$

$$\{g_K[\frac{j_K v_K - i_K m_K}{v_K j_K}]\} \quad - r_K\{[(\frac{1}{1+s_K})$$

$$(\frac{b_K\ _K - c_K\ _K Z_{K+1}}{b_K^2})] \ - \ [(\frac{1}{1+t_K})$$

$$(\frac{b_K u_K - c_K u_K Z_K}{b_K^2})]\} \ - \ g_K\{[(\frac{1}{1+p_K})$$

$$(\frac{b_K u_K - c_K u_K Z_{K+1}}{b_K^2})] \ + \ [(\frac{1}{1+s_K})$$

$$(\frac{b_K^2(c_K u_K) - (c_K u_K) + (b_K u_K - c_K u_K Z_{K+1})}{b_K^4}$$

$$\frac{(2b_K c_K)}{\quad})] \ - \ [(\frac{1}{1+w_K})(\frac{b_K u_K - c_K u_K Z_K}{b_K^2})]$$

$$+[(\frac{1}{1+t_K})(\frac{b_K^2(c_K u_K) - (c_K u_K) +}{b_K^4}$$

$$\frac{(b_K u_K - c_K u_K Z_k)(2b_K c_K)}{\quad})]\}$$

where:

$$n_K = 2[u_K + 2a_K c_K + c_K^2]$$

$$p_K = \frac{a_K b_K + u_K Z_{K+1} + u_K}{b_K}$$

$$r_K = -a_1 \sin\beta + \cos\beta$$

$$w_K = \frac{a_K b_K + u_K Z_K + u_K}{b_K} .$$

The computer programs for calculating the second vertical derivative of the vertical magnetic intensity according to equations 11 and 14 are given in Appendices C and D respectively. An inclination of 90° was used for the models, therefore, the vertical magnetic intensity is equal to the total magnetic intensity.

CHAPTER VI

RESULTS OF MAGNETIC MODELING

The horizontal distance between the zero second vertical derivative of vertical magnetic intensity (ΔX of Fig. 2) was calculated for the same series of two-dimensional, vertical tabular bodies that was considered in the gravity case. Vertical magnetic polarization of the anomaly sources was assumed. The results of these calculations are presented in Figure 6 in the same manner as the results of the gravity calculations are shown in Figure 4. The single exception is that the $T/Z_1=10$ curve is not plotted for the magnetic case because it is nearly the same as the $T/Z_1=5$ curve.

Figure 6 may be used in magnetic interpretation as Figure 4 can be used in gravity interpretation and in general the conclusions reached from Figure 4 are applicable to the magnetic case considered here. However, the measured horizontal distance between the zero second vertical derivatives of vertical magnetic intensity is more nearly equivalent to the true width of the anomaly source than ΔX in the gravity case. As a general statement, if W/Z_1 is 2.0 or greater, ΔX will be in error by

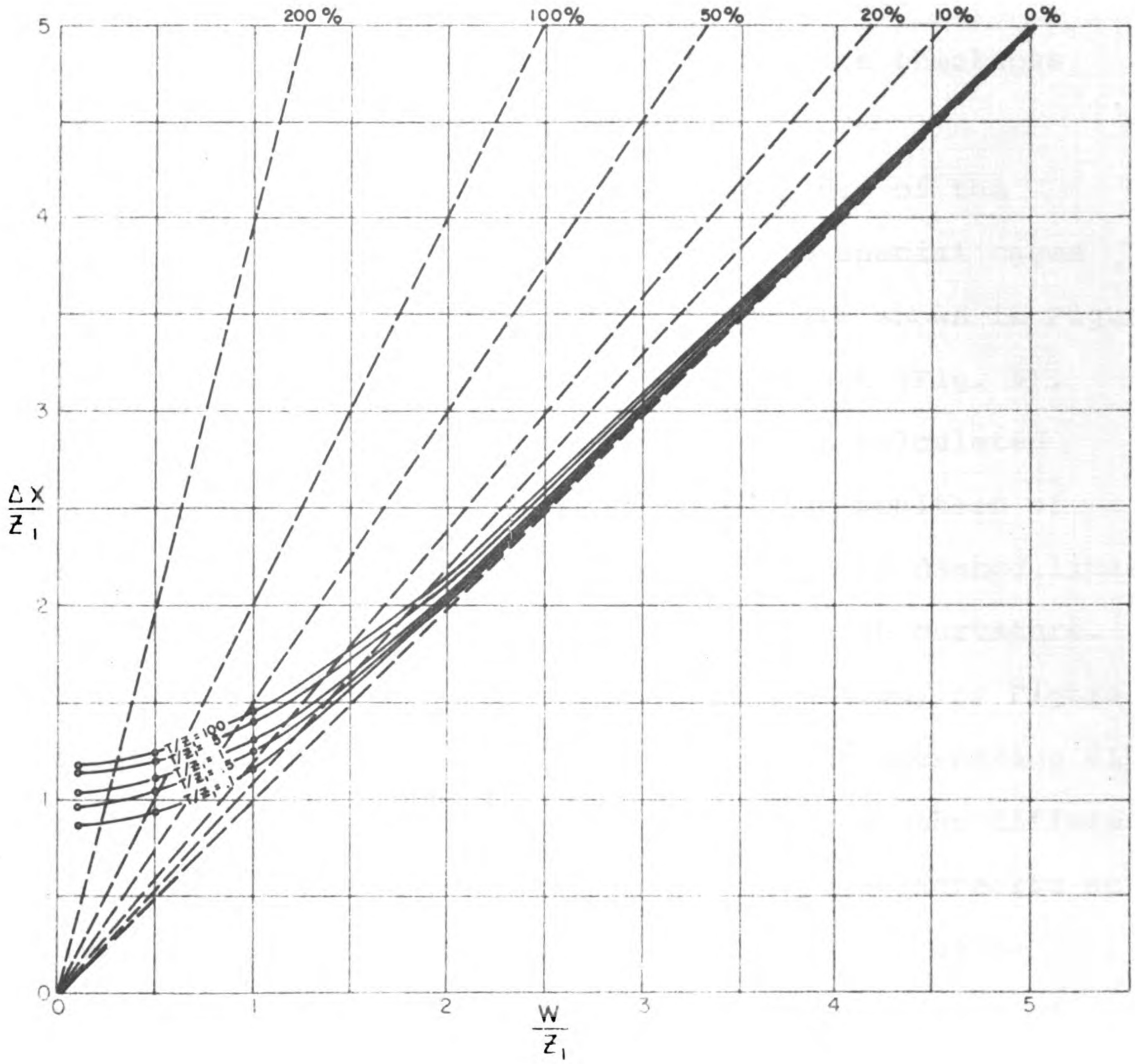

FIGURE 6.--Relationship between W, T, Z_1, and ΔX for a vertical tabular source of vertical magnetic intensity anomalies in a vertical magnetic field.

less than 10 percent for any value of T/Z_1. Also the effect of varying T is less in the magnetic case than in gravity. For all practical purposes in this regard, the source can be assumed to have an infinite thickness if $T/Z_1>5$.

As in gravity case, the effect of dip of the anomaly source on ΔX was studied for a few special cases in magnetics. The results of this study are shown in Figure 7 in an equivalent manner to the gravity case (Fig. 5). Only a relatively few data points have been calculated and, therefore, as in the gravity case, the position of the curves between data points as indicated by dashed lines, may be somewhat in error because of their high curvature. The significant conclusions reached from a study of Figure 7 are the following. First, ΔX increases with decreasing dip for any given set of body dimensions. Second, the difference in ΔX of dipping and vertical tabular sources increases as the depth increases and the width decreases all other dimensions being equal. Third, the difference in ΔX of dipping and vertical tabular bodies increases rapidly as the thickness (T) of the source increases. The difference in ΔX reaches a maximum and then gradually decreases. The value of T at which the maximum occurs increases as the depth to the top of the source increases.

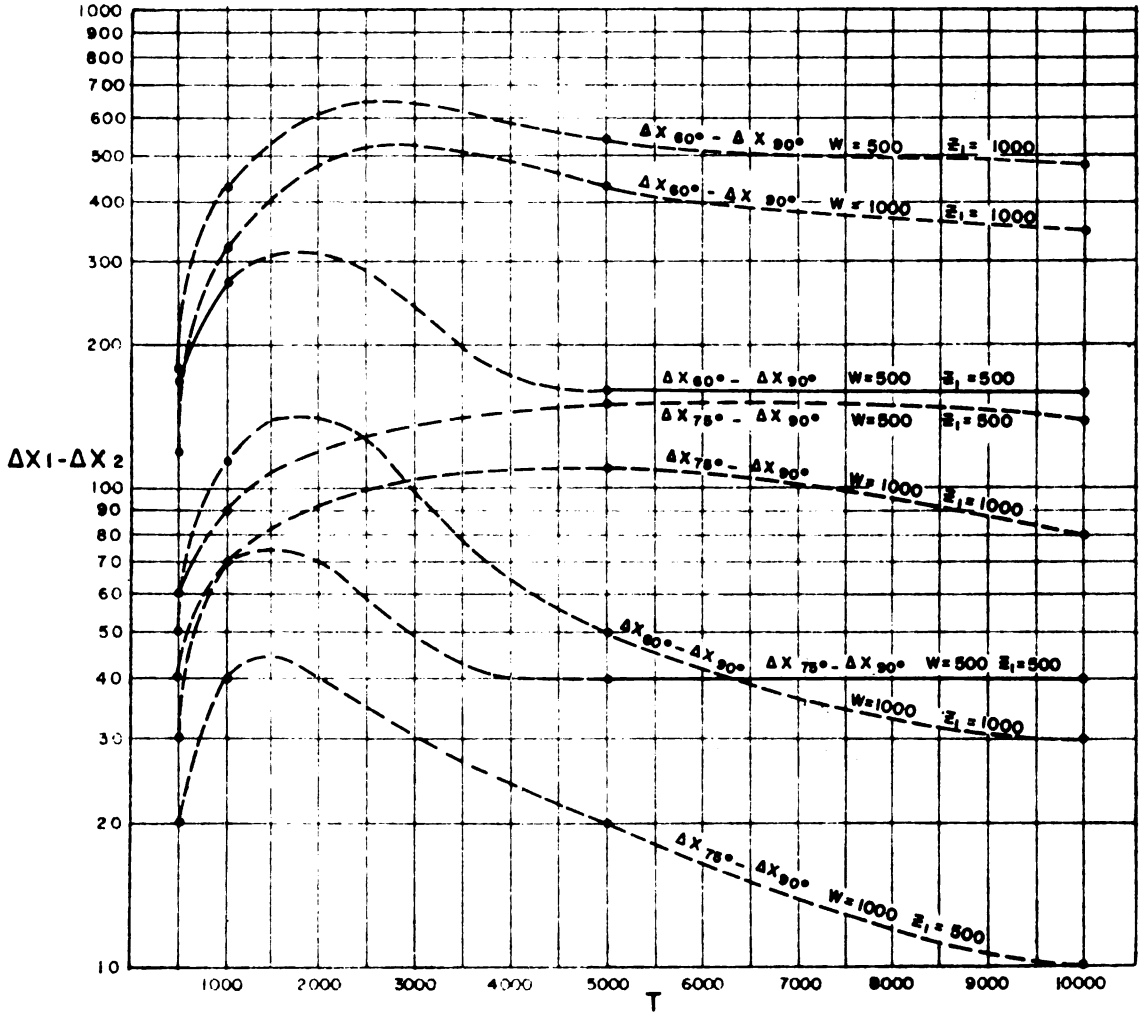

FIGURE 7.--The effect of dip of tabular sources on ΔX of vertical magnetic intensity anomalies.

CHAPTER VII

A LEAST SQUARES SECOND VERTICAL DERIVATIVE APPROXIMATION METHOD

The relationships previously determined in this study are based on theoretical second vertical derivatives. However, in actual practice second vertical derivatives calculated from observed anomalies are only approximations to the true or theoretical second vertical derivatives. Therefore, conclusions based on the results previously presented will be in error by an amount determined by the accuracy of the method used to calculate the approximate second vertical derivative.

Elkins (1951) and Swartz (1954) have outlined a second vertical derivative procedure which is applicable to two-dimensional gravity and magnetic anomalies. This method, which is based on LaPlace's equation, utilizes an extrapolation of the curve of "r^2" versus the average anomaly at a distance "r" from the calculation point to the zero value of r. The curve can be approximated by the method of least squares to determine the second vertical derivative.

The theory is developed in the following manner assuming that the field data collected from a gravity or magnetic survey obeys LaPlace's equation.

LaPlace's equation is,

$$\frac{\partial^2 A}{\partial Z^2} = -\left(\frac{\partial^2 A}{\partial X^2} + \frac{\partial^2 A}{\partial Y^2}\right) \tag{15}$$

where A is the observed anomaly. Equation 15 can be simplified to

$$\frac{\partial^2 A}{\partial Z^2} = -\frac{\partial^2 A}{\partial X^2} \tag{16}$$

if the anomaly is two-dimensional in the Y direction, i.e., $\partial^2 A/\partial Y^2$ is equal to zero. Referring to Figure 8 where c is the anomaly value at the point of calculation of the second vertical derivative, and b_1 and b_2 are the anomaly values along the X direction at a distance "r" from c, the second horizontal derivative parallel to the X axis is:

$$\frac{\partial^2 A}{\partial X^2} \simeq \frac{1}{r}\left[\frac{b_1 - c}{r} - \frac{c - b_3}{r}\right] = \frac{b_1 + b_3 - 2c}{r^2} \tag{17}$$

substituting $-\partial^2 A/\partial Z^2$ for $\partial^2 A/\partial X^2$ the approximate relationship in equation 17 becomes exact as the distance r approaches zero, and thus:

$$\frac{\partial^2 A}{\partial Z^2} = \lim_{r \to 0} \frac{2c - b_1 - b_3}{r^2}\ . \tag{18}$$

Utilizing a series of values at different distances ("r") from c, a least-squares curve method can be readily

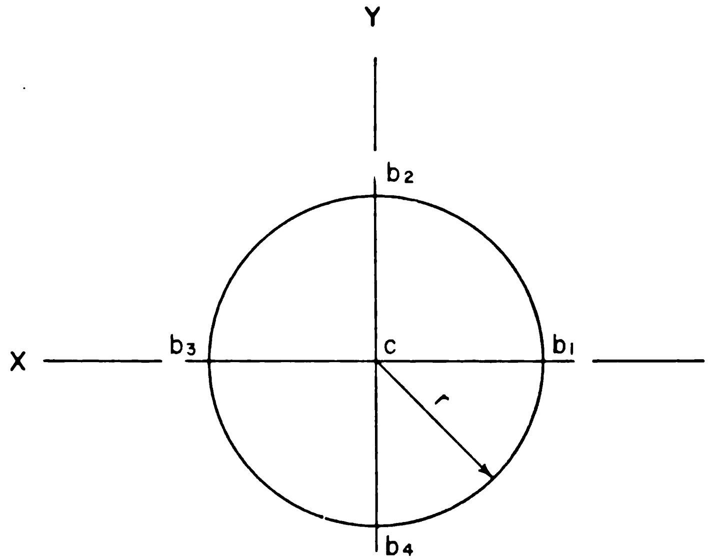

FIGURE 8.--Grid for calculating a three dimensional second vertical derivative.

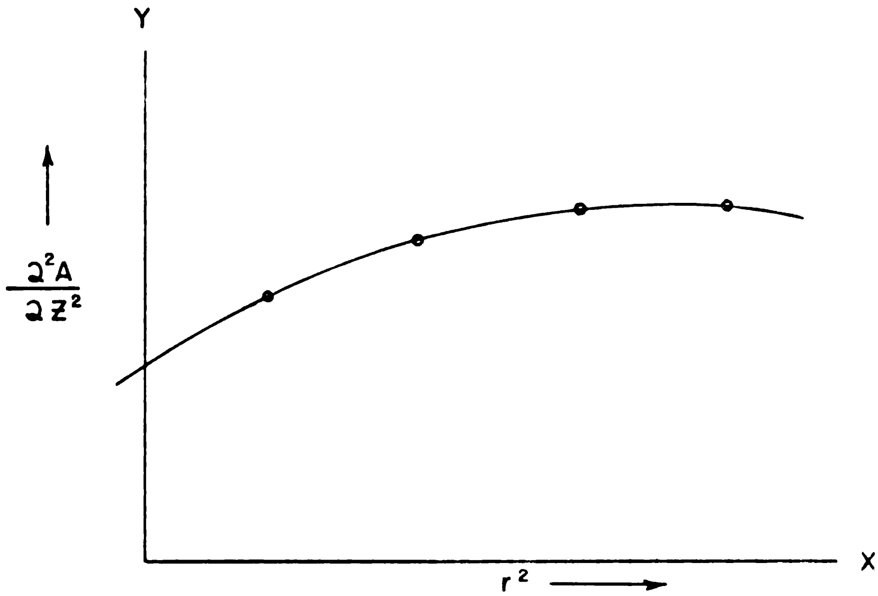

FIGURE 9.--Approximate second vertical derivative plotted as a function of r^2, and shown fitted with a least squares curve of degree two.

applied to calculate $\partial^2 A/\partial Z^2$ using equation 18. The procedure consists of calculating the approximate second vertical derivative values from profile data at various values of r. An example is illustrated in Figure 9 where $\partial^2 A/\partial Z^2$ is plotted against the appropriate "r^2" values. Next a polynomial equation of degree N is fitted to the data points using the least squares method. After solving for the coefficients of the equation, "r" is set equal to zero and the resulting equation is solved for $\partial^2 A/\partial Z^2$ according to equation 18. The data points represent a smooth function, therefore, a second degree equation (N=2) was chosen for the analysis. A listing of the computer programs used for the gravity and magnetic cases are given in Appendices E and F respectively.

A theoretical gravity anomaly was calculated for a vertical tabular source and used to test the least squares method of finding second vertical derivatives. The theoretical second vertical derivative was calculated for the vertical tabular source using the same set of parameters (Z_1=1000 feet, W=1000 feet, and Z_2=100,000 feet).

The theoretical gravity and theoretical second vertical derivative of gravity are presented in Table 1. A varying number of data points were used in calculating the approximate second vertical derivative. The "five point method" implies that the center point and two adjacent points, two on either side, were used in the calculation. Columns 4 through 7 show the difference between the approximate and theoretical second vertical derivative for various "point methods".

TABLE 1.--Difference between theoretical and approximate second vertical derivatives of gravity of a vertical tabular source using various approximation methods ($r = 1/2\ Z_1$).

(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	12.112	- 1.570	0.000	+0.000	+0.000	+0.000
2	12.523	- 1.886	0.000	+0.000	+0.000	+0.001
3	12.977	- 2.297	0.000	+0.000	+0.000	+0.002
4	13.485	- 2.837	0.000	+0.001	+0.002	+0.008
5	14.059	- 3.536	+0.002	+0.002	+0.007	+0.022
6	14.716	- 4.381	-0.011	+0.003	+0.011	-0.005
7	15.474	- 5.113	-0.038	-0.002	-0.053	-0.203
8	16.348	- 4.382	-0.043	-0.060	-0.159	-0.177
9	17.313	+ 2.689	-0.316	-0.099	-0.382	-0.871
10	18.188	+21.880	-0.005	-0.054	-0.144	-0.172
11	18.567	+35.011	+0.055	+0.194	+0.671	+1.384
Absolute mean error =			0.043	0.038	0.130	0.254

Column (1) Station number (interval = 500 feet).

Column (2) Observed gravity anomaly (mgals) over a vertical tabular source. Station 11 is over the center of the source.

Column (3) Theoretical second vertical derivative of gravity in mgals/cm^2 x 10^{-10}.

Column (4) Difference between the theoretical and approximate second vertical derivative using the five point method.

Column (5) Same as (4) for the seven point method.

Column (6) Same as (4) for the nine point method.

Column (7) Same as (4) for the eleven point method.

TABLE 2.--Difference between theoretical and approximate second vertical derivatives of gravity of a vertical tabular using various data intervals for the five point method.

(1)	(2)	(3)	(4)	(5)	(6)
1	12.112	- 1.570	+0.000	+0.000	
2	12.523	- 1.886	+0.000	+0.000	+0.001
3	12.977	- 2.297	+0.000	+0.000	
4	13.485	- 2.837	+0.001	+0.000	-0.013
5	14.059	- 3.536	+0.001	+0.002	
6	14.716	- 4.381	-0.001	-0.011	-0.239
7	15.474	- 5.113	-0.002	-0.038	
8	16.348	- 4.382	-0.004	-0.043	+0.486
9	17.313	+ 2.689	-0.021	-0.316	
10	18.188	+21.880	+0.005	-0.005	+0.238
11	18.567	+35.011	+0.041	+0.055	

Source Parameters: Z_1=1000 units; W=1000 units; and Z_2=100,000 units.

Column (1) Station number (interval = 500 feet).

Column (2) Observed gravity anomaly (mgals) over a vertical tabular source. Station 11 is over the center of the source.

Column (3) Theoretical second vertical derivative of gravity in mgals/cm^2 x 10^{-10}.

Column (4) Difference between the theoretical and approximate second vertical derivative for r=1/4 Z_1.

Column (5) Same as (4) for r=1/2 Z_1.

Column (6) Same as (4) for r=Z_1.

The average mean error was calculated to aid in evaluating the various "point methods".

In the example given the "Seven point method" gave the best overall results. However, in other cases depending on (1) the station spacing, (2) the noise within the data, and (3) the frequency spectra of the anomalies; one of the other methods may be more suitable. Comparative tests on the set of data being studied would be necessary in most cases to determine which method gave the best results. This could be easily programmed as a simple input parameter.

A test was also made to determine the effect of the data interval on the accuracy in calculating the second vertical derivative of gravity of the same model used in evaluating the various "point methods". Various data intervals ("r") were chosen as a function of Z_1. A value of r equal to 1/4 Z_1 gave the best results for the theoretical case (see Table 2). However, field data may contain high frequency noise, therefore, the closest data interval chosen would not necessarily give the best results. A compromise between theoretical accuracy and the influence of the noise within the data would have to be made in choosing the optimum data interval.

The same procedure and anomaly source used to evaluate the second vertical derivative of gravity was applied to the magnetic case assuming vertical magnetic polarization. Table 3 for magnetics is equivalent to Table 1 for gravity. The same general conclusions can be drawn for

TABLE 3.--Difference between theoretical and approximate second vertical derivatives of magnetics of a vertical tabular source ($r=1/2\ Z_1$).

(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	3.35	- 10.824	+ 0.008	- 0.001	- 0.004	- 0.017
2	4.36	- 16.095	+ 0.016	- 0.002	- 0.011	- 0.038
3	5.76	- 24.900	+ 0.035	- 0.055	- 0.021	- 0.058
4	7.75	- 40.354	+ 0.079	- 0.007	- 0.011	- 0.177
5	10.70	- 68.990	+ 0.158	+ 0.021	+ 0.326	+ 2.094
6	15.30	- 124.859	+ 0.073	+ 0.337	+ 2.179	+ 4.399
7	22.91	- 235.855	- 2.267	+ 1.127	- 0.669	- 3.156
8	36.16	- 424.515	-11.032	- 7.167	-22.936	- 35.535
9	59.06	- 444.462	+26.095	+ 7.070	+32.650	+ 76.145
10	89.95	+ 624.256	-10.871	- 6.852	-20.795	- 30.023
11	106.41	+1598.162	+48.049	+16.406	+64.549	+129.997
Absolute mean error =			8.97	4.20	13.11	25.60

Column (1) Station number (interval = 500 feet).

Column (2) Observed vertical magnetic intensity (gammas) over a vertical tabular source in a vertical magnetic field. Station 11 is over the center of the source.

Column (3) Theoretical second vertical derivative of magnetics in gammas/$cm^2 \times 10^{-10}$.

Column (4) Difference between the theoretical and approximate second vertical derivative for the five point method.

Column (5) Same as (4) for the seven point method.

Column (6) Same as (4) for the nine point method.

Column (7) Same as (4) for the elven point method.

TABLE 4.--Difference between theoretical and approximate second vertical derivatives of magnetics of a vertical tabular source using various data intervals for the five point method.

(1)	(2)	(3)	(4)	(5)	(6)
1	3.35	- 10.824	+0.000	+ 0.008	
2	4.36	- 16.095	+0.000	+ 0.016	+ [illegible].313
3	5.76	- 24.900	+0.000	+ 0.035	
4	7.75	- 40.354	+0.004	+ 0.079	+ 1.381
5	10.70	- 68.990	+0.008	+ 0.158	
6	15.30	- 124.859	+0.006	+ 0.073	- 9.002
7	22.91	- 235.855	-0.107	- 2.267	
8	36.16	- 424.515	-0.836	-11.032	-25.199
9	59.06	- 444.462	+1.480	+26.095	
10	89.95	+ 624.256	-0.802	-10.871	-32.426
11	106.41	+1598.162	+2.769	+48.049	

Source Parameters: Z_1=1000 units; W=1000 units; and Z_2=100,000 units.

Column (1) Station number (interval=500 feet).

Column (2) Observed vertical magnetic intensity (gammas) over a vertical tabular source in a vertical magnetic field. Station 11 is over the center of the source.

Column (3) Theoretical second vertical derivative of magnetics in gammas/cm^2 x 10^{-10}.

Column (4) Difference between the theoretical and approximate second vertical derivative for r-1/4 Z_1.

Column (5) Same as (4) for r=1/2 Z_1.

Column (6) Same as (4) for r=Z_1.

magnetics except that the magnitude of error is greater for magnetics. This can be explained since the equivalent magnetic anomaly is larger in magnitude and narrower in width, thus containing sharper gradients. Table 4 for magnetics is equivalent to Table 2 for gravity. The data interval r=1/4 Z_1 gave the best results as in the equivalent gravity case.

Observed magnetic data in general contains higher frequency components than gravity data from the same sources. Thus, the selection of the best "point method" is more critical for magnetics due to greater difficulty in separating noise from signal.

CHAPTER VIII

CONCLUSION

The distance between zero values of the second vertical derivative of gravity and magnetic anomalies is useful as a guide to the width of two-dimensional tabular sources. The accuracy of the zero value in defining the width decreases as the true width decreases, the depth extent increases, and the dip decreases for a given depth to the source. The effect of these variations is to increase the width as defined by the zero second vertical derivative values over the true width. The error is greater for gravity than for vertical magnetic intensity.

General family of curves relating true widths of vertical two-dimensional tabular bodies to observed widths as determined from the zero values of second vertical derivatives of gravity and vertical magnetic intensity can be used to estimate true widths providing the depth and depth extent of the source are known. In any event, in the case of gravity, if the width of the vertical tabular source is greater than approximately 4.5 times the depth, the zero values of second vertical derivatives are in error by less than 10 percent for any depth extent in predicting the width of the source. In the case of magnetics, if the width exceeds the depth to the source two-fold or more, the error in estimating the width from second vertical derivatives is less than 10 percent for any depth extent of the source.

The least squares method of approximating the second vertical derivatives of two-dimensional gravity and magnetic anomalies is a viable method of approximating the true second vertical derivative.

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APPENDIX A

COMPUTER PROGRAM FOR CALCULATING THE THEORETICAL SECOND VERTICAL DERIVATIVE OF GRAVITY OVER A VERTICAL TABULAR MODEL

```
*****************************************************************

PRØGRAM FØR CALCULATING THE THEØRETICAL SECØND VERTICAL
DERIVATIVE ØF GRAVITY ØF A VERTICAL TABULAR SØURCE
PRØGRAMMED IN FØRTRAN FØR A XDS SIGMA 5  (1/72)

*****************************************************************


*****************************************************************

DESCRIPTIØN ØF INPUT DATA:

CARD 1  (I5)
   LTØT=TØTAL NUMBER ØF PRØFILES
CARD 2  (I5)
   JTØT=TØTAL NUMBER ØF SØURCES PER PRØFILE
CARD 3  (2F10.2,2I5)
   DELX=STATIØN SPACING
   FØ=X-CØØRDINATE ØF THE FIRST STATIØN
   M=TØTAL NUMBER ØF STATIØNS PER PRØFILE
   N=NUMBER ØF CØØRDINATES PLUS ØNE FØR EACH SØURCE
CARDS 4-->N  (2F10.3)
   X(I)=X-CØØRDINATES ØF THE SØURCE
   Z(I)=Z-CØØRDINATES ØF THE SØURCE

   CARDS 4-->N ARE REPEATED JTØT NUMBER ØF TIMES
   CARDS 2 THRØUGH 4-->N ARE REPEATED LTØT NUMBER ØF TIMES

*****************************************************************
```

```
      COMMON SECDERIV(500),FX(500),M,X(50),Z(50),N
      READ (5,5) LTOT
    5 FORMAT (I5)
      DO 49 L=1,LTOT
      READ (5,7) JTOT
    7 FORMAT (I5)
      READ (5,9) DELX,FO,M,N
    9 FORMAT (2F10.2,2I5)
      DO 47 J=1,JTOT
      WRITE (6,11)
   11 FORMAT ('1')
      WRITE (6,13) L
   13 FORMAT (//,50X,'RESULTS FOR PROFILE NUMBER ',I5,///)
      DO 15 I=1,M
      RI=I
   15 FX(I)=FO-DELX+DELX*RI
      WRITE (6,17) J
   17 FORMAT (50X,'COORDINATES FOR BODY NUMBER ',I5,//)
      WRITE (6,19)
   19 FORMAT (50X,'X-COORDINATE',6X,'Z-COORDINATE',//)
      DO 25 I=1,N
      READ (5,21) X(I),Z(I)
   21 FORMAT (2F10.2)
      WRITE (6,23) X(I),Z(I)
   23 FORMAT (46X,2F15.3)
   25 CONTINUE
      Z1=Z(1)
      Z2=Z(3)
      IF (Z1) 28,27,28
   27 Z1=.01
   28 DO 30 I=1,M
      X1=FX(I)-X(2)
      X2=FX(I)-X(1)
      CALC3=(X2/(Z1**2+X2**2))-(X1/(Z1**2+X1**2))
      CALC4=(X1/(Z2**2+X1**2))-(X2/(Z2**2+X2**2))
      SECDERIV(I)=4.377E-06*(CALC3+CALC4)
   30 CONTINUE
      WRITE (6,35)
   35 FORMAT (///,30X,'I',8X,'FX(I)',8X,'SECOND DERIVATIVE GRAVITY (2D T
     1HICK DIKE CASE)',/)
      DO 39 I=1,M
      WRITE (6,37) I,FX(I),SECDERIV(I)
   37 FORMAT (27X,I4,F13.3,22X,E13.6)
   39 CONTINUE
      CALL XINTERCT (XINTERCP)
      WRITE (6,41)
   41 FORMAT (//,50X,'THE CROSS-OVER POINT IS LOCATED AT',/)
      WRITE (6,43) XINTERCP
   43 FORMAT (60X,'X=',F10.3)
   47 CONTINUE
```

```
49 CONTINUE
   STOP
   END
   SUBROUTINE XINTERCT (XINTERCP)
   COMMON DERIV(500),FX(500),NUMBER,X(50),Z(50),N
   XX1=X(1)
   XX2=X(2)
   Z1=Z(1)
   Z2=Z(3)
   FX1=FX(1)
   IF (DERIV(1).LE.0) ISET=0
   IF (DERIV(1).GT.0) ISET=1
   IF (Z1) 10,5,10
 5 Z1=.01
10 FX1=FX1+1.
   X1=FX1-XX2
   X2=FX1-XX1
   XX1=X(1)
   XX2=X(2)
   CALC3=(X2/(Z1**2+X2**2))-(X1/(Z1**2+X1**2))
   CALC4=(X1/(Z2**2+X1**2))-(X2/(Z2**2+X2**2))
   DERIV1=4.377E-06*(CALC3+CALC4)
   IF (ISET.EQ.0.AND.DERIV1.LT.0) GO TO 10
   IF (ISET.EQ.1.AND.DERIV1.GT.0) GO TO 10
   XINTERCP=FX1
   RETURN
   END
```

APPENDIX B

COMPUTER PROGRAM FOR CALCULATING THE THEORETICAL SECOND VERTICAL DERIVATIVE OF GRAVITY OVER A IRREGULAR-SHAPED POLYGON

```
*******************************************************************

PROGRAM FOR CALCULATING THE THEORETICAL SECOND VERTICAL
DERIVATIVE OF GRAVITY OF A PRISM-SHAPED SOURCE
PROGRAMMED IN FORTRAN FOR A XDS SIGMA 5  (1/72)

*******************************************************************


*******************************************************************

DESCRIPTION OF INPUT DATA:

CARD 1  (I5)
   LTOT=TOTAL NUMBER OF PROFILES
CARD 2  (I5)
   JTOT=TOTAL NUMBER OF SOURCES PER PROFILE
CARD 3  (2F10.2,2I5)
   M=TOTAL NUMBER OF STATIONS PER PROFILE
   DELX=STATION SPACING
   FO=X-COORDINATE OF THE FIRST STATION
   N=NUMBER OF COORDINATES PLUS ONE FOR EACH SOURCE
CARDS 4-->N  (2F10.3)
   X(I)=X-COORDINATES OF THE SOURCE
   Z(I)=Z-COORDINATES OF THE SOURCE

   CARDS 4-->N ARE REPEATED JTOT NUMBER OF TIMES
   CARDS 2 THROUGH 4-->N ARE REPEATED LTOT NUMBER OF TIMES

*******************************************************************
```

```
      COMMON SECDERIV(500),FX(500),X(50),Z(50),XINTERCP(50),M,N,LL
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      READ (5,5) LTOT
    5 FORMAT (I5)
      DO 49 L=1,LTOT
      READ (5,7) JTOT
    7 FORMAT (I5)
      READ (5,9) DELX,FO,M,N
    9 FORMAT (2F10.2,2I5)
      DO 47 J=1,JTOT
      WRITE (6,11)
   11 FORMAT ('1')
      WRITE (6,13) L
   13 FORMAT (//,50X,'RESULTS FOR PROFILE NUMBER ',I5,///)
      DO 15 I=1,M
      RI=I
   15 FX(I)=FO-DELX+DELX*RI
      WRITE (6,17) J
   17 FORMAT (50X,'COORDINATES FOR BODY NUMBER ',I5,//)
      WRITE (6,19)
   19 FORMAT (50X,'X-COORDINATE',6X,'Z-COORDINATE',//)
      DO 25 I=1,N
      READ (5,21) X(I),Z(I)
   21 FORMAT (2F10.2)
      WRITE (6,23) X(I),Z(I)
   23 FORMAT (46X,2F15.3)
   25 CONTINUE
      NN=N-1
      DO 33 I=1,M
      SECDER2=0
      DO 31 K=1,NN
      X1=X(K)-FX(I)
      X2=X(K+1)-FX(I)
      ZZ1=Z(K)
      Z1=Z(K)
      ZZ2=Z(K+1)
      Z2=Z(K+1)
      IF (ZZ1-ZZ2) 29,27,29
   27 ZZ2=ZZ2+.00001
   29 BB1=(X1*Z2-X2*Z1)/(ZZ2-ZZ1)
      A1=(X2-X1)/(ZZ2-ZZ1)
      B1=(1/(1+A1**2))*((X1-X2)/(ZZ2-ZZ1))
      B2=((X1**2+Z1**2)*Z2)-((X2**2+Z2**2)*Z1)
      B3=(X1**2+Z1**2)*(X2**2+Z2**2)
      B4=A1*((X1/(X1**2+Z1**2))-(X2/(X2**2+Z2**2)))
      C1=B1*((B2/B3)+B4)
      B5=((X1**2+Z1**2)*Z2)-((X2**2+Z2**2)*Z1)
      B6=(X1**2+Z1**2)*(X2**2+Z2**2)
      B7=A1*((X1/(X1**2+Z1**2))-(X2/(X2**2+Z2**2)))
      B8=(1/(1+A1**2))*((X1-X2)/(ZZ2-ZZ1))
```

```
      B9=BB1/(1+A1**2)
      B10=(X1**2+Z1**2)*(X2**2+Z2**2)
      B11=(X1**2+Z1**2)-(X2**2+Z2**2)
      B12=((X1**2+Z1**2)*Z2)-((X2**2+Z2**2)*Z1)
      B13=((X1**2+Z1**2)*2*Z2)+((X2**2+Z2**2)*2*Z1)
      B14=((X1**2+Z1**2)*(X2**2+Z2**2))**2
      B15=A1*(((2*X2*Z2)/((X2**2+Z2**2)**2))-((2*X1*Z1)/((X1**2+Z1**2)
     1**2)))
      C2=(((B5/B6)+B7)*B8)+(B9*((((B10*B11)-(B12*B13))/B14)+B15))
      SECDER1=C1+C2
      SECDER2=SECDER1+SECDER2
   31 CØNTINUE
      SECDERIV(I)=4.377E-06*SECDER2
   33 CØNTINUE
      WRITE (6,35)
   35 FØRMAT (///,30X,'I',8X,'FX(I)',8X,'SECØND DERIVATIVE GRAVITY (2D G
     1ENERAL CASE)',/)
      DØ 39 I=1,M
      WRITE (6,37) I,FX(I),SECDERIV(I)
   37 FØRMAT (27X,I4,F13.3,22X,E13.6)
   39 CØNTINUE
      CALL XINTERCT
      WRITE (6,41)
   41 FØRMAT (//,50X,'THE CRØSS-ØVER PØINTS ARE LØCATED AT',/)
      DØ 45 I=1,LL
      WRITE (6,43) XINTERCP(I)
   43 FØRMAT (60X,'X=',F10.3)
   45 CØNTINUE
   47 CØNTINUE
   49 CØNTINUE
      STØP
      END
      SUBRØUTINE XINTERCT
      CØMMØN DERIV(500),FX(500),X(50),Z(50),XINTERCP(50),NUMBER,N,LL
      IMPLICIT DØUBLE PRECISIØN (A-H,Ø-Z)
      LL=0
      NN=N-1
      JJ=1
    5 IF (DERIV(JJ)) 7,7,15
    7 DØ 13 I=1,NUMBER
      IF (NUMBER-JJ) 35,35,9
    9 JJ=JJ+1
      IF (DERIV(JJ)) 13,11,11
   11 FX1=FX(JJ-1)
      KK=1
      LL=LL+1
      GØ TØ 23
   13 CØNTINUE
   15 DØ 21 I=1,NUMBER
      IF (NUMBER-JJ) 35,35,17
```

```
17 JJ=JJ+1
   IF (DERIV(JJ)) 19,19,21
19 FX1=FX(JJ-1)
   KK=0
   LL=LL+1
   GO TO 23
21 CONTINUE
23 FX1=FX1+1.
   SECDER2=0
   DO 29 K=1,NN
   X1=X(K)-FX1
   X2=X(K+1)-FX1
   ZZ1=Z(K)
   Z1=Z(K)
   ZZ2=Z(K+1)
   Z2=Z(K+1)
   IF (ZZ1-ZZ2) 27,25,27
25 ZZ2=ZZ2+.00001
27 BB1=(X1*Z2-X2*Z1)/(ZZ2-ZZ1)
   A1=(X2-X1)/(ZZ2-ZZ1)
   B1=(1/(1+A1**2))*((X1-X2)/(ZZ2-ZZ1))
   B2=((X1**2+Z1**2)*Z2)-((X2**2+Z2**2)*Z1)
   B3=(X1**2+Z1**2)*(X2**2+Z2**2)
   B4=A1*((X1/(X1**2+Z1**2))-(X2/(X2**2+Z2**2)))
   C1=B1*((B2/B3)+B4)
   B5=((X1**2+Z1**2)*Z2)-((X2**2+Z2**2)*Z1)
   B6=(X1**2+Z1**2)*(X2**2+Z2**2)
   B7=A1*((X1/(X1**2+Z1**2))-(X2/(X2**2+Z2**2)))
   B8=(1/(1+A1**2))*((X1-X2)/(ZZ2-ZZ1))
   B9=BB1/(1+A1**2)
   B10=(X1**2+Z1**2)*(X2**2+Z2**2)
   B11=(X1**2+Z1**2)-(X2**2+Z2**2)
   B12=((X1**2+Z1**2)*Z2)-((X2**2+Z2**2)*Z1)
   B13=((X1**2+Z1**2)*2*Z2)+((X2**2+Z2**2)*2*Z1)
   B14=((X1**2+Z1**2)*(X2**2+Z2**2))**2
   B15=A1*(((2*X2*Z2)/((X2**2+Z2**2)**2))-((2*X1*Z1)/((X1**2+Z1**2)
  1**2)))
   C2=(((B5/B6)+B7)*B8)+(B9*((((B10*B11)-(B12*B13))/B14)+B15))
   SECDER1=C1+C2
   SECDER2=SECDER1+SECDER2
29 CONTINUE
   DERIV1=4.377E-06*SECDER2
   IF (KK) 31,33,31
31 IF (DERIV1) 23,34,34
33 IF (DERIV1) 34,34,23
34 XINTERCP(LL)=FX1
   GO TO 5
35 CONTINUE
   RETURN
   END
```

APPENDIX C

COMPUTER PROGRAM FOR CALCULATING THE THEORETICAL SECOND VERTICAL DERIVATIVE OF MAGNETICS OVER A VERTICAL TABULAR MODEL

```
********************************************************

PRØGRAM FØR CALCULATING THE THEØRETICAL SECØND VERTICAL
DERIVATIVE ØF MAGNETICS ØF A VERTICAL TABULAR SØURCE
PRØGRAMMED IN FØRTRAN FØR A XDS SIGMA 5  (1/72)

*******************************************************


********************************************************

DESCRIPTIØN ØF INPUT DATA:

CARD 1   (I5)
   LTØT=TØTAL NUMBER ØF PRØFILES
CARD 2   (I5)
   JTØT=TØTAL NUMBER ØF SØURCES PER PRØFILE
CARD 3 (2F10.2,2I5,3F10.2)
   DELX=STATIØN SPACING
   FØ=X-CØØRDINATE ØF THE FIRST STATIØN
   M=TØTAL NUMBER ØF STATIØNS PER PRØFILE
   N=NUMBER ØF CØØRDINATES PLUS ØNE FØR EACH SØURCE
   SUS=SUSCEPTIBILITY CØNTRAST  (EMU/CC)
   ZINDUCED=MAGNITUDE ØF THE EARTH'S REGIØNAL MAGNETIC
   FIELD (ØERSTEDS)
CARDS 4-->N  (2F10.3)
   X(I)=X-CØØRDINATES ØF THE SØURCE
   Z(I)=Z-CØØRDINATES ØF THE SØURCE

   CARDS 4-->N ARE REPEATED JTØT NUMBER ØF TIMES
   CARDS 2 THRØUGH 4-->N ARE REPEATED LTØT NUMBER ØF TIMES

********************************************************
```

```
      CØMMØN SECDERIV (500),FX(500),M,X(50),Z(50),SUS,INCL,
     1ZINDUCED,N,VERTMAG
      REAL INCL
      READ (5,5) LTØT
    5 FØRMAT (I5)
      DØ 70 L=1,LTØT
      READ (5,7) JTØT
    7 FØRMAT (I5)
      READ (5,9) DELX,FØ,M,N,SUS,ZINDUCED,INCL
    9 FØRMAT (2F10.2,2I5,3F10.2)
      DØ 10 I=1,M
   10 FX(I)=FØ-DELX+DELX*I
      DØ 60 J=1,JTØT
      WRITE (6,20)
   20 FØRMAT ('1')
      WRITE (6,22) L
   22 FØRMAT (///,50X,'RESULTS FØR PRØFILE NUMBER',I5,///)
      WRITE (6,24) SUS,ZINDUCED,INCL
   24 FØRMAT (/,'SUSCEPTIBILITY=',F7.4,10X,'INDUCED MAGNETIZATIØN=',
     1F10.2,1X,'ØERSTEDS',10X,'INCLINATIØN ØF THE MAGNETIC FIELD=',
     1F5.2,1X,'DEGREES',//)
      WRITE (6,30) J
   30 FØRMAT (50X,'CØØRDINATES FØR BØDY NUMBER',I5,//)
      WRITE (6,32)
   32 FØRMAT (50X,'X-CØØRDINATE',6X,'Z-CØØRDINATE',//)
      DØ 38 I=1,N
      READ (5,34) X(I),Z(I)
   34 FØRMAT (2F10.2)
      WRITE (6,36) X(I),Z(I)
   36 FØRMAT (46X,2F15.3)
   38 CØNTINUE
      RADANGLE=INCL*.01745329
      VERTMAG=ZINDUCED*SIN(RADANGLE)
      Z1=Z(1)
      Z2=Z(3)
      IF (Z1) 45,40,45
   40 Z1=.01
   45 DØ 50 I=1,M
      X1=FX(I)-X(2)
      X2=FX(I)-X(1)
      CALC1=(X1*Z2)/(X1**2+Z2**2)**2
      CALC2=(X1*Z1)/(X1**2+Z1**2)**2
      CALC3=(X2*Z1)/(X2**2+Z1**2)**2
      CALC4=(X2*Z2)/(X2**2+Z2**2)**2
      SECDERIV(I)=SUS*VERTMAG*4.0356E02*(CALC1-CALC2+CALC3-CALC4)
   50 CØNTINUE
      WRITE (6,52)
   52 FØRMAT (///,30X,'I',8X,'FX(I)',8X,'SECØND DERIVATIVE MAGNETICS (2D
     1 THICK DIKE CASE)',/)
      DØ 54 I=1,M
```

```
      WRITE (6,53) I,FX(I),SECDERIV(I)
   53 FORMAT (27X,I4,F13.3,22X,E13.6)
   54 CONTINUE
      CALL XINTERCT (XINTERCP)
      WRITE (6,56)
   56 FORMAT (//,50X,'THE CROSS-OVER POINT IS LOCATED AT',/)
      WRITE (6,57) XINTERCP
   57 FORMAT (60X,'X=',F10.3)
   60 CONTINUE
   70 CONTINUE
      STOP
      END
      SUBROUTINE XINTERCT (XINTERCP)
      COMMON SECDERIV (500),FX(500),M,X(50),Z(50),SUS,INCL,
     1ZINDUCED,N,VERTMAG
      XX1=X(1)
      XX2=X(2)
      Z1=Z(1)
      Z2=Z(3)
      FX1=FX(1)
      IF (SECDERIV(1).LE.0) ISET=0
      IF (SECDERIV(1).GT.0) ISET=1
      IF (Z1) 20,15,20
   15 Z1=.01
   20 FX1=FX1+1.
      X1=FX1-XX2
      X2=FX1-XX1
      CALC1=(X1*Z2)/(X1**2+Z2**2)**2
      CALC2=(X1*Z1)/(X1**2+Z1**2)**2
      CALC3=(X2*Z1)/(X2**2+Z1**2)**2
      CALC4=(X2*Z2)/(X2**2+Z2**2)**2
      DERIV1=SUS*VERTMAG*4.0356E02*(CALC1-CALC2+CALC3-CALC4)
      IF (ISET.EQ.0.AND.DERIV1.LT.0) GO TO 20
      IF (ISET.EQ.1.AND.DERIV1.GT.0) GO TO 20
      XINTERCP=FX1
      RETURN
      END
```

APPENDIX D

COMPUTER PROGRAM FOR CALCULATING THE THEORETICAL SECOND VERTICAL DERIVATIVE OF MAGNETICS OVER A IRREGULAR-SHAPED POLYGON

```
**************************************************************

PROGRAM FOR CALCULATING THE THEORETICAL SECOND VERTICAL
DERIVATIVE OF MAGNETICS OF A PRISM-SHAPED SOURCE
PROGRAMMED IN FORTRAN FOR A XDS SIGMA 5  (1/72)

**************************************************************


**************************************************************

DESCRIPTION OF INPUT DATA:

CARD 1   (I5)
   LTOT=TOTAL NUMBER OF PROFILES
CARD 2   (I5)
   JTOT=TOTAL NUMBER OF SOURCES PER PROFILE
CARD 3 (2F10.2,2I5,3F10.2)
   DELX=STATION SPACING
   FO=X-COORDINATE OF THE FIRST STATION
   M=TOTAL NUMBER OF STATIONS PER PROFILE
   N=NUMBER OF COORDINATES PLUS ONE FOR EACH SOURCE
   SUS=SUSCEPTIBILITY CONTRAST  (EMU/CC)
   ZINDUCED=MAGNITUDE OF THE EARTH'S REGIONAL MAGNETIC
   FIELD (OERSTEDS)
CARDS 4-->N   (2F10.3)
   X(I)=X-COORDINATES OF THE SOURCE
   Z(I)=Z-COORDINATES OF THE SOURCE

   CARDS 4-->N ARE REPEATED JTOT NUMBER OF TIMES
   CARDS 2 THROUGH 4-->N ARE REPEATED LTOT NUMBER OF TIMES

**************************************************************
```

```
      COMMON SECDERIV(500),FX(500),X(50),Z(50),SUS,STRIKE,
     1XINTERCP(50),BETA,BBB,ZINDUCED,INCL,M,N,LL
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      REAL INCL
      READ (5,5) LTOT
    5 FORMAT (I5)
      DO 70 L=1,LTOT
      READ (5,7) JTOT
    7 FORMAT (I5)
      READ (5,9) DELX,FO,M,N,SUS,ZINDUCED,INCL,STRIKE
    9 FORMAT (2F10.2,2I5,4F10.2)
      DO 10 I=1,M
   10 FX(I)=FO-DELX+DELX*I
      IF (STRIKE) 17,15,17
   15 STRIKE=.00001
   17 STRIKE=STRIKE*.01745329
      INCL=INCL*.01745329
      BETA=DATAN(DTAN(INCL)-DSIN(STRIKE))
      NN=N-1
      BBB=DSQRT(1-((DCOS(STRIKE))**2*(DCOS(INCL))**2))
      STRIKE=STRIKE/.01745329
      INCL=INCL/.01745329
      DO 60 J=1,JTOT
      WRITE (6,20)
   20 FORMAT ('1')
      WRITE (6,22) L
   22 FORMAT (///,50X,'RESULTS FOR PROFILE NUMBER',I5,///)
      WRITE (6,24) SUS,ZINDUCED,INCL
   24 FORMAT (/,'SUSCEPTIBILITY=',F7.4,10X,'INDUCED MAGNETIZATION=',
     1F10.2,1X,'OERSTEDS',10X,'INCLINATION OF THE MAGNETIC FIELD=',
     1F5.2,1X,'DEGREES',//)
      WRITE (6,26) STRIKE
   26 FORMAT (50X,'STRIKE OF THE BODY=N',F5.2,'E',//)
      WRITE (6,30) J
   30 FORMAT (50X,'COORDINATES FOR BODY NUMBER',I5,//)
      WRITE (6,32)
   32 FORMAT (50X,'X-COORDINATE',6X,'Z-COORDINATE',//)
      DO 38 I=1,N
      READ (5,34) X(I),Z(I)
   34 FORMAT (2F10.2)
      WRITE (6,36) X(I),Z(I)
   36 FORMAT (46X,2F15.3)
   38 CONTINUE
      DO 51 I=1,M
      DERIV2=0
      DO 50 K=1,NN
      X1=X(K)-FX(I)
      X2=X(K+1)-FX(I)
      ZZ1=Z(K)
      Z1=Z(K)
```

```
      ZZ2=Z(K+1)
      Z2=Z(K+1)
      IF (ZZ1-ZZ2) 42,40,42
   40 ZZ2=ZZ2+.01
   42 A1=(X2-X1)/(ZZ2-ZZ1)
      B1=(X1*Z2-X2*Z1)/(ZZ2-ZZ1)
      IF (B1) 46,44,46
   44 B1=.00001
   46 H2=1+A1**2
      AA=1/H2
      BB=A1*DSIN(BETA)+DCOS(BETA)
      DD=A1*DCOS(BETA)-DSIN(BETA)
      FF=((1+A1**2)*Z2**2)+(2*A1*B1*Z2)+B1**2
      GG=((1+A1**2)*Z1**2)+(2*A1*B1*Z1)+B1**2
      H1=(X1-X2)/(ZZ2-ZZ1)
      SN=((H2*Z2)/B1)+A1
      SP=((H2*Z1)/B1)+A1
      HH=(2*Z2*H2)+(2*A1*B1)+(2*A1*Z2*H1)+(2*B1*H1)
      SI=(2*Z1*H2)+(2*A1*B1)+(2*A1*Z1*H1)+(2*B1*H1)
      SJ=2*(H2+(2*A1*H1)+H1**2)
      SL=1+SN**2
      SM=1+SP**2
      QQ=(GG*FF)*(((GG*SJ)+(HH*SI))-((FF*SJ)+(SI*HH)))
      RR=((GG*HH)-(FF*SI))*((GG*HH)+(FF*SI))
      SS=.5*BB*(QQ-RR)/(GG*FF)**2
      UUU=B1-Z2*H1
      VVV=B1-Z1*H1
      UU=1/SL*(-2*H1*H2*UUU)/B1**3
      WW=1/SM*(-2*H1*H2*VVV)/B1**3
      VV=((H2*UUU)/B1**2)*(-2*SN*(H2*UUU)/B1**2)/SL**2
      XX=((H2*VVV)/B1**2)*(-2*SP*(H2*VVV)/B1**2)/SM**2
      YY=DD*((UU+VV)-(WW+XX))
      DERIV1=AA*(SS-YY)
      DERIV2=DERIV1+DERIV2
   50 CONTINUE
      SECDERIV(I)=2.0178E02*SUS*ZINDUCED*BBB*DERIV2
   51 CONTINUE
      WRITE (6,52)
   52 FORMAT (///,30X,'I',8X,'FX(I)',8X,'SECOND DERIVATIVE MAGNETICS (2D
     1 GENERAL CASE)',/)
      DO 54 I=1,M
      WRITE (6,53) I,FX(I),SECDERIV(I)
   53 FORMAT (27X,I4,F13.3,22X,E13.6)
   54 CONTINUE
      CALL XINTERCT
      WRITE (6,56)
   56 FORMAT (//,50X,'THE CROSS-OVER POINTS ARE LOCATED',/)
      DO 58 I=1,LL
      WRITE (6,57) XINTERCP(I)
   57 FORMAT (60X,'X=',F10.3)
```

```
58 CONTINUE
60 CONTINUE
70 CONTINUE
   STOP
   END
   SUBROUTINE XINTERCT
   COMMON DERIV(500),FX(500),X(50),Z(50),SUS,STRIKE,
  1XINTERCP(50),BETA,BBB,ZINDUCED,INCL,NUMBER,N,LL
   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
   REAL INCL
   LL=0
   NN=N-1
   JJ=1
 5 IF (DERIV(JJ)) 7,7,13
 7 DO 12 I=1,NUMBER
   IF (NUMBER-JJ) 39,39,9
 9 JJ=JJ+1
   IF (DERIV(JJ)) 12,11,11
11 FX1=FX(JJ-1)
   KK=1
   LL=LL+1
   GO TO 21
12 CONTINUE
13 DO 19 I=1,NUMBER
   IF (NUMBER-JJ) 39,39,15
15 JJ=JJ+1
   IF (DERIV(JJ)) 17,17,19
17 FX1=FX(JJ-1)
   KK=0
   LL=LL+1
   GO TO 21
19 CONTINUE
21 FX1=FX1+1.
   DERIV2=0
   DO 31 K=1,NN
   X1=X(K)-FX1
   X2=X(K+1)-FX1
   ZZ1=Z(K)
   Z1=Z(K)
   ZZ2=Z(K+1)
   Z2=Z(K+1)
   IF (ZZ1-ZZ2) 25,23,25
23 ZZ2=ZZ2+.01
25 A1=(X2-X1)/(ZZ2-ZZ1)
   B1=(X1*Z2-X2*Z1)/(ZZ2-ZZ1)
   IF (B1) 29,27,29
27 B1=.00001
29 H2=1+A1**2
   AA=1/H2
   BB=A1*DSIN(BETA)+DCOS(BETA)
```

```
      DD=A1*DCOS(BETA)*DSIN(BETA)
      FF=((1+A1**2)*Z2**2)+(2*A1*B1*Z2)+B1**2
      GG=((1+A1**2)*Z1**2)+(2*A1*B1*Z1)+B1**2
      H1=(X1-X2)/(ZZ2-ZZ1)
      SN=((H2*Z2)/B1)+A1
      SP=((H2*Z1)/B1)+A1
      HH=(2*Z2*H2)+(2*A1*B1)+(2*A1*Z2*H1)+(2*B1*H1)
      SI=(2*Z1*H2)+(2*A1*B1)+(2*A1*Z1*H1)+(2*B1*H1)
      SJ=2*(H2+(2*A1*H1)+H1**2)
      SL=1+SN**2
      SM=1+SP**2
      QQ=(GG*FF)*(((GG*SJ)+(HH*SI))-((FF*SJ)+(SI*HH)))
      RR=((GG*HH)-(FF*SI))*((GG*HH)+(FF*SI))
      SS=.5*BB*(QQ-RR)/(GG*FF)**2
      UUU=B1-Z2*H1
      VVV=B1-Z1*H1
      UU=1/SL*(-2*H1*H2*UUU)/B1**3
      WW=1/SM*(-2*H1*H2*VVV)/B1**3
      VV=((H2*UUU)/B1**2)*(-2*SN*(H2*UUU)/B1**2)/SL**2
      XX=((H2*VVV)/B1**2)*(-2*SP*(H2*VVV)/B1**2)/SM**2
      YY=DD*((UU+VV)-(WW+XX))
      DERIV1=AA*(SS-YY)
      DERIV2=DERIV1+DERIV2
   31 CONTINUE
      DERIV1=2.0178E02*SUS*ZINDUCED*BBB*DERIV2
      IF (KK) 33,35,33
   33 IF (DERIV1) 21,37,37
   35 IF (DERIV1) 37,37,21
   37 XINTERCP(LL)=FX1
      GO TO 5
   39 CONTINUE
      RETURN
      END
```

APPENDIX E

COMPUTER PROGRAM FOR CALCULATING THE SECOND VERTICAL DERIVATIVE OF THEORETICAL GRAVITY USING THE LEAST SQUARES METHOD

```
*************************************************************

PROGRAM FOR CALCULATING THE APPROXIMATE SECOND VERTICAL
DERIVATIVE OF THEORETICAL GRAVITY USING THE LEAST SQUARES
METHOD
PROGRAMMED IN FORTRAN FOR A CDC 3600 COMPUTER (12/66)

*************************************************************


*************************************************************

DESCRIPTION OF INPUT DATA:

CARD 1   (I5)
   LTOT=TOTAL NUMBER OF PROFILES
CARD 2   (2I10,2F10.2)
   JTOT=TOTAL NUMBER OF SOURCES PER PROFILE
   M=TOTAL NUMBER OF STATIONS PER PROFILE
   DELX=STATION SPACING
   FO=X-COORDINATE OF THE FIRST STATION
CARD 3   (I10,F10.2)
   N=NUMBER OF COORDINATES PLUS ONE FOR EACH SOURCE
   RHO=DENSITY CONTRAST  (GR/CC)
CARDS 4-->N   (2F10.3)
   X(I)=X-COORDINATES OF THE SOURCE
   Z(I)=Z-COORDINATES OF THE SOURCE

   CARDS 4-->N ARE REPEATED JTOT NUMBER OF TIMES
   CARDS 2 THROUGH 4-->N ARE REPEATED LTOT NUMBER OF TIMES

*************************************************************
```

```
      PROGRAM DGDZ
      COMMON/1/GRAV(500),FX(500),M,APPRDER(500),X(50),Z(50),N,DELX,FO,
     1RHO,THEODER(500),NUMBODY,MM,NN
      COMMON/2/ DUMMY(2011)
      MM=1  $  NN=2
      READ3,LTOT
    3 FORMAT(I5)
      DO 20 J=1,LTOT
      PRINT 7
    7 FORMAT (1H1)
      PRINT 4,J
    4 FORMAT (///50X,*RESULTS FOR PROFILE NUMBER*,I5///)
      CALL APPROX
      PRINT 11
   11 FORMAT(1H1)
      PRINT 12
   12 FORMAT (5X,*I*,10X,*STATION*,10X,*GRAVITY ANOMALY*,10X,*APPROXIMAT
     1E SECOND DERIVATIVE*,10X,*THEORETICAL SECOND DERIVATIVE*//)
      DO 13 I=1,M
   13 THEODER(I)=THEODER(I)*1E10
      PRINT 15,(I,FX(I),GRAV(I),APPRDER(I),THEODER(I),I=1,M)
   15 FORMAT (I6,8X,F10.3,9X,F10.3,23X,E13.6,26X,E13.6)
      PUNCH 510,(GRAV(I),APPRDER(I),THEODER(I),I=1,M)
  510 FORMAT (6E13,6)
   20 CONTINUE
      END
```

N5.3B 12/08/66

```
      SUBROUTINE APPROX
      COMMON/1/GRAV(500),DUMMY1(500),M,APPRDER(500),DUMMY2(101),DELX,
     1DUMMY3(505)
      COMMON/2/DUMMY4(931),VAL1(30),VAL2(30),DUMMY5(1020)
      CALL GRAV2D
      PRINT 5
    5 FORMAT (1H1)
      MAX=M-2
      DO 15 I=3,MAX
      RAD =0
      DO 10 K=1,2
      RAD=RAD+DELX
      VAL1(K)=(RAD*30.48)**2
      VAL2(K)=(2*GRAV(I)-GRAV(I-K)-GRAV(I+K))/VAL1(K)
   10 VAL2(K)=VAL2(K)*1E10
      CALL LSTSQPOL (DERIV)
      APPRDER(I)=DERIV
   15 CONTINUE
      END
```

```
      SUBROUTINE GRAV2D
      COMMON/1/GRAV(500),FX(500),M,APPRDER(500),X(50),Z(50),N,DELX,FO,
     1RHO,THEODER(500),JTOT,DUMMY1(2)
      COMMON/2/ DUMMY2(2011)
C
C     N IS THE NUMBER OF BODY POINTS PLUS ONE
C     L IS THE INDEX FOR PROFILES
C     J IS THE INDEX FOR BODIES/PROFILE
C     M IS THE NUMBER OF STATIONS
C     ELE IS THE ELEVATION OF THE PROFILE
C     FO IS THE X COORDINATE OF 1ST STATION
C     BODY COORDINATES, STATION SPACING AND FO MUST BE CONVERTED TO KFT
C
      READ 12,JTOT,M,DELX,FO
   12 FORMAT (2I10,2F10.2)
      DO 15 I=1,M
      GRAV(I)=0.0  $  FX(I)=(FO-DELX)+DELX*I  $  APPRDER(I)=0.0
   15 THEODER(I)=0.0
      DO 13 J=1,JTOT
      READ 16,N,RHO  $  ELE=0.0
   16 FORMAT (I10,F10.2)
      READ 20,(X(I),Z(I),I=1,N)
   20 FORMAT(2F10.3)
      PUNCH 500,N,M,RHO,FO,DELX
  500 FORMAT (2I10,3F10.2)
      PUNCH 505,(X(I),Z(I),I=1,N)
  505 FORMAT (10F8.0)
      CALL CALCDER
      PRINT 8,J
    8 FORMAT(50X,*COORDINATES FOR BODY NUMBER*,I5//)
      PRINT 9
    9 FORMAT(50X,*X-COORDINATE*,6X,*Z-COORDINATE*//)
      PRINT 14,(X(I),Z(I),I=1,N)
   14 FORMAT (49X,F15.3,2X,F15.3)
      DO420 K=1,M
      SDEL7=0.0
      I=1
  205 EXXX=X(I)-FX(K)
      ZEEE=Z(I)-ELE
      RR=EXXX*EXXX+ZEEE*ZEEE
      IF(EXXX)210,240,280
  210 IF(ZEEE)220,230,230
  220 THETR=ATANF(ZEEE/EXXX)-3.1415927
      GO TO300
  230 THETR=ATANF(ZEEE/EXXX)+3.1415927
      GO TO 300
  240 IF(ZEEE)250,260,270
  250 THETR=-1.5707963
      GO TO 300
  260 THETR=0.0
      GO TO 300
  270 THETR=1.5707963
      GO TO 300
  280 THETR=ATANF(ZEEE/EXXX)
  300 IF(I-1)3001,3002,3001
 3002 EXX=EXXX
```

```
      ZEE=ZEEE
      R=RR
      THETA=THETB
      IF(I-1)205,200,205
  200 I=2
      GO TO 205
 3001 CHECK=EXX*ZEEE-ZEE*EXXX
      IF(CHECK)320,310,320
  310 DELZ=0.0
  320 OMEGA=THETA-THETB
      IF(OMEGA)3201,3202,3202
 3202 IF(OMEGA-3.1415927)330,330,340
 3201 IF(OMEGA+3.1415927)340,330,330
  330 DTHET=OMEGA
      GO TO 370
  340 IF(OMEGA)350,360,360
  350 DTHET=OMEGA+6.2831853
      GO TO 370
  360 DTHET=OMEGA-6.2831853
  370 AAA=(EXXX-EXX)*(EXXX-EXX)
      AA=(ZEEE-ZEE)*(ZEEE-ZEE)
      A=CHECK/(AAA+AA)
      B=(EXXX-EXX)*DTHET
      IF(RR)371,371,403
  371 C=0.0
      GO TO 402
  403 IF(R)371,371,401
  401 CCC=RR/R
      CC=LOGF(CCC)
      C=.5*(ZEEE-ZEE)*CC
  402 DELZ=A*(B+C)
  400 SDELZ=SDELZ+DELZ
      IF(I-N)3003,3005,3005
 3003 I=I+1
      GO TO 3002
 3005 SUM   =.004066032*RHO*SDELZ
      GRAV(K)=GRAV(K)+SUM
  420 CONTINUE
   13 CONTINUE
      END
```

```
      SUBROUTINE CALCDER
      COMMON/1/GRAV(500),FX(500),M,DUMMY1(500),X(50),Z(50),N,DELX,FO,RHO
     1,THEODER(500),DUMMY2(3)
      COMMON/2/ DUMMY3(2011)
      NN=N-1
      DO 15 I=1,M
      SEC DER2=0
      DO 14 K=1,NN
      X1=X(K)-FX(I)  $  X2=X(K+1)-FX(I)  $  Z1=ZZ1=Z(K) $ Z2=ZZ2=Z(K+1)
      IF(Z1-Z2)16,10,16
   10 Z2=Z2+.01
   16 BB1=(X1*ZZ2-X2*ZZ1)/(Z2-Z1)
      A1=(X2-X1)/(Z2-Z1)
      B1=(1/(1+A1**2))*((X1-X2)/(Z2-Z1))
      B2=((X1**2+Z1**2)*Z2)-((X2**2+Z2**2)*Z1)
      B3=(X1**2+Z1**2)*(X2**2+Z2**2)
      B4=A1*((X1/(X1**2+Z1**2))-(X2/(X2**2+Z2**2)))
      C1=B1*((B2/B3)+B4)
      B5=((X1**2+Z1**2)*Z2)-((X2**2+Z2**2)*Z1)
      B6=(X1**2+Z1**2)*(X2**2+Z2**2)
      B7=A1*((X1/(X1**2+Z1**2))-(X2/(X2**2+Z2**2)))
      B8=(1/(1+A1**2))*((X1-X2)/(Z2-Z1))
      B9=BB1/(1+A1**2)
      B10=(X1**2+Z1**2)*(X2**2+Z2**2)
      B11=(X1**2+Z1**2)-(X2**2+Z2**2)
      B12=((X1**2+Z1**2)*Z2)-((X2**2+Z2**2)*Z1)
      B13=((X1**2+Z1**2)*2*Z2)+((X2**2+Z2**2)*2*Z1)
      B14=((X1**2+Z1**2)*(X2**2+Z2**2))**2
      B15=A1*(((2*X2*Z2)/((X2**2+Z2**2)**2))-((2*X1*Z1)/((X1**2+Z1**2)**
     12)))
      C2=(((B5/B6)+B7)*B8)+(B9*((((B10*B11)-(B12*B13))/B14)+B15))
      SECDER1=C1+C2
      SECDER2=SECDER1+SECDER2
   14 CONTINUE
      SECDERIV    =4.377E-06*SECDER2*RHO
      THEODER(I)=THEODER(I)+SECDERIV
   15 CONTINUE
      END
```

```
      SUBROUTINE LSTSQPOL(DERIV)
      COMMON/1/DUMMY1(2106),M,N
      COMMON/2/A(30,30),B(30),MP1,X(30),Y(30),C(30,30),DUMMY2(120)
      DO 30 I=1,N
   30 C(I,1)=1.0
      MP1=M+1
      DO 35 J=2,MP1
      DO 35 I=1,N
   35 C(I,J)=C(I,J-1)*X(I)
      DO 40 I=1,MP1
      DO 40 J=1,MP1
      A(I,J)=0.0
      DO 40 K=1,N
   40 A(I,J)=A(I,J)+C(K,I)*C(K,J)
      DO 45 I=1,MP1
      B(I)=0.0
      DO 45 K=1,N
   45 B(I)=B(I)+C(K,I)*Y(K)
      CALL MATRIX
      PRINT 600,N,M
  600 FORMAT (* NUMBER OF GIVEN DATA POINTS=*,I2,10X,*DEGREE POLYNOMIAL=
     1*,I2)
      DO 50 I=1,MP1
      II=I-1
   50 PRINT 800,II,B(I)
  800 FORMAT (5X,I2,* DEGREE COEFFICIENT=*,E13.6)
      DERIV=B(1)
      END
```

```
      SUBROUTINE MATRIX
      COMMON/1/ DUMMY1(2108)
      COMMON/2/A(30,30),B(30),N  ,DUMMY2(960),IPVOT(30),INDEX(30,2),
     1PIVOT(30)
      EQUIVALENCE(IROW,JROW),(ICOL,JCOL)
      DET=1.0
      DO 17 J=1,N
   17 IPVOT(J)=0
      DO 135 I=1,N   $   T=0
      DO 9 J=1,N
      IF(IPVOT(J)-1)13,9,13
   13 DO 23 K=1,N
      IF(IPVOT(K)-1)43,23,81
   43 IF(ABSF(T)-ABSF(A(J,K)))83,23,23
   83 IROW=J   $   ICOL=K   $   T=A(J,K)
   23 CONTINUE
    9 CONTINUE
      IPVOT(ICOL)=IPVOT(ICOL)+1
      IF(IROW-ICOL)73,109,73
   73 DET=-DET
      DO 12 L=1,N
      T=A(IROW,L)
      A(IROW,L)=A(ICOL,L)
   12 A(ICOL,L)=T
      T=B(IROW)   $   B(IROW)=B(ICOL)   $   B(ICOL)=T
  109 INDEX(I,1)=IROW   $   INDEX(I,2)=ICOL
      PIVOT(I)=A(ICOL,ICOL)   $   DET=DET*PIVOT(I)   $   A(ICOL,ICOL)=1.0
      DO 205 L=1,N
  205 A(ICOL,L)=A(ICOL,L)/PIVOT(I)
      B(ICOL)=B(ICOL)/PIVOT(I)
      DO 135 LI=1,N
      IF(LI-ICOL) 21,135,21
   21 T=A(LI,ICOL)
      A(LI,ICOL)=0.0
      DO 89 L=1,N
   89 A(LI,L)=A(LI,L)-A(ICOL,L)*T
      B(LI)=B(LI)-B(ICOL)*T
  135 CONTINUE
  222 DO 3 I=1,N
      L=N-I+1
      IF(INDEX(L,1)-INDEX(L,2))19,3,19
   19 JROW=INDEX(L,1)
      JCOL=INDEX(L,2)
      DO 549 K=1,N
      T=A(K,JROW)   $   A(K,JROW)=A(K,JCOL)   $   A(K,JCOL)=T
  549 CONTINUE
    3 CONTINUE
   81 CONTINUE
      END
,1.59,4500,7,   ,

CUTION STARTED AT    2224 -31
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APPENDIX F

COMPUTER PROGRAM FOR CALCULATING THE SECOND VERTICAL DERIVATIVE OF THEORETICAL MAGNETICS USING THE LEAST SQUARES METHOD

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**************************************************************

PROGRAM FOR CALCULATING THE APPROXIMATE SECOND VERTICAL
DERIVATIVE OF THEORETICAL MAGNETICS USING THE LEAST SQUARES
METHOD
PROGRAMMED IN FORTRAN FOR A CDC 3600 COMPUTER  (1/67)

**************************************************************


**************************************************************

DESCRIPTION OF INPUT DATA:

CARD 1  (I5)
   LTOT=TOTAL NUMBER OF PROFILES
CARD 2  (2I10,2F10.2)
   JTOT=TOTAL NUMBER OF SOURCES PER PROFILE
   M=TOTAL NUMBER OF STATIONS PER PROFILE
   DELX=STATION SPACING
   FO=X-COORDINATE OF THE FIRST STATION
CARD 3  (I10,4F10.2)
   N=NUMBER OF COORDINATES PLUS ONE FOR EACH SOURCE
   SUS=SUSCEPTIBILITY CONTRAST  (EMU/CC)
   STRIKE=STRIKE OF THE SOURCE  (POSITIVE CLOCKWISE
   (DEGREES) FROM THE MAGNETIC NORTH POLE)
   DIP=INCLINATION OF THE EARTH'S REGIONAL MAGNETIC FIELD
   (POSITIVE DOWNWARD (DEGREES))
   ZINDUCED=MAGNITUDE OF THE EARTH'S REGIONAL MAGNETIC
   FIELD (OERSTEDS)
   INCL =INCLINATION OF THE EARTH'S REGIONAL MAGNETIC FIELD
   (POSITIVE DOWNWARD (DEGREES))
CARDS 4-->N  (2F10.3)
   X(I)=X-COORDINATES OF THE SOURCE
   Z(I)=Z-COORDINATES OF THE SOURCE

   CARDS 4-->N ARE REPEATED JTOT NUMBER OF TIMES
   CARDS 2 THROUGH 4-->N ARE REPEATED LTOT NUMBER OF TIMES

**************************************************************
```

```
      PROGRAM FGDZ
      COMMON/1/ZMAG(500),FX(500),M,APPRDER(500),X(50),Z(50),N,DELX,FO,
     1SUS,THEODER(500),NUMBODY,MM,NN,STRIKE,DIP,ZINDUCED
      COMMON/2/ DUMMY(2011)
      MM=2   $   NN=3
      READ3,LTOT
    3 FORMAT(I5)
      DO 20 J=1,LTOT
      PRINT 7
    7 FORMAT (1H1)
      PRINT 4,J
    4 FORMAT (///50X,*RESULTS FOR PROFILE NUMBER*,I5///)
      CALL APPROX
      PRINT 11
   11 FORMAT(1H1)
      PRINT 12
   12 FORMAT (5X,*I*,10X,*STATION*,10X,*MAGNETIC ANOMALY*,10X,*APPROXIMA
     1TE SECOND DERIVATIVE*,10X,*THEORETICAL SECOND DERIVATIVE*//)
      DO 13 I=1,M
   13 THEODER(I)=THEODER(I)*1E10
      PRINT 15,(I,FX(I),ZMAG(I),APPRDER(I),THEODER(I),I=1,M)
   15 FORMAT (I6,8X,F10.3,9X,F10.3,23X,E13.6,26X,E13.6)
      PUNCH 510,(ZMAG(I),APPRDER(I),THEODER(I),I=1,M)
  510 FORMAT (6E13.6)
   20 CONTINUE
      END
```

```
      SUBROUTINE APPROX
      COMMON/1/ZMAG(500),DUMMY1(500),M,APPRDER(500),DUMMY2(101),DELX,
     1DUMMY3(508)
      COMMON/2/DUMMY4(931),VAL1(30),VAL2(30),DUMMY5(1020)
      CALL MAG2D
      PRINT 5
    5 FORMAT (1H1)
      MAX=M-3
      DO 15 I=4,MAX
      RAD =0
      DO 10 K=1,3
      RAD=RAD+DELX
      VAL1(K)=(RAD*30.48)**2
      VAL2(K)=(2*ZMAG(I)-ZMAG(I-K)-ZMAG(I+K))/VAL1(K)
   10 VAL2(K)=VAL2(K)*1E10
      CALL LSTSQPOL (DERIV)
      APPRDER(I)=DERIV
   15 CONTINUE
      END
```

```
      SUBROUTINE MAG2D
      COMMON/1/ZMAG(500),FX(500),M,APPRDER(500),X(50),Z(50),N,DELX,FO,
     1SUS,THEOIER(500),JTOT,DUMMY1(2),STRIKE,DIP,ZINDUCED
      COMMON/2/ DUMMY2(2011)
      READ 12,JTOT,M,DELX,FO
   12 FORMAT (2I10,2F10.2)
      DO 6 I=1,M
      APPRDER(I)=0.0
    6 FX(I)=FO-DELX+DELX*I
      DO 13 J=1,JTOT
      READ 16,N,SUS,STRIKE,DIP,ZINDUCED  $  FLE=0.0
   16 FORMAT (I10,4F10.2)
      READ 20,(X(I),Z(I),I=1,N)
   20 FORMAT(2F10.3)
      PUNCH 500,N,M,FO,DELX,SUS,STRIKE,DIP,ZINDUCED
  500 FORMAT (2I5,6F10.4)
      PUNCH 505,(X(I),Z(I),I=1,N)
  505 FORMAT (10F8.0)
      PRINT 8,J
    8 FORMAT(50X,*COORDINATES FOR BODY NUMBER*,I5//)
      PRINT 9
    9 FORMAT(50X,*X-COORDINATE*,6X,*Z-COORDINATE*//)
      PRINT 14,(X(I),Z(I),I=1,N)
   14 FORMAT (49X,F15.3,2X,F15.3)
      PRINT 5,SUS,ZINDUCED,DIP
    5 FORMAT (* SUSCEPTIBILITY=*,F7.4,10X,*INDUCED MAGNETIZATION=*,F10.2
     1,10X,*INCLINATION OF THE MAGNETIC FIELD=*,F5.2,1X,*DEGREES*//)
      PRINT 15,STRIKE
   15 FORMAT (* STRIKE OF THE BODY=N*,F5.2,*E*//)
      IF(STRIKE)10,19
   19 STRIKE=.00001
   10 STRIKE=STRIKE*.01745329  $  DIP=DIP*.01745329
      CALL CALCDER
      BETA=ATANF(TANF(DIP)/SINF(STRIKE))
      NN=N-1  $  BBB=SQRTF(1-((COSF(STRIKE))**2*(COSF(DIP))**2))
      DO 15 I=1,M
      ZINTEN2=0
      DO 14 K=1,NN
      X1=X(K)-FX(I)  $  X2=X(K+1)-FX(I)  $  ZZ1=Z1=Z(K)  $ ZZ2=Z2=Z(K+1)
      IF(ZZ1-ZZ2)16,17,16
   17 ZZ2=ZZ2+.01
   16 A1=(X2-X1)/(ZZ2-ZZ1)
      B1=(X1*Z2-X2*Z1)/(ZZ2-ZZ1)
      IF(B1)38,36
   36 B1=.001
   38 H2=1+A1**2  $  AA=1/H2
      BB=A1*SINF(BETA)+COSF(BETA)
      DD=A1*COSF(BETA)-SINF(BETA)
      FF=((1+A1**2)*Z2**2)+(2*A1*B1*Z2)+B1**2
      GG=((1+A1**2)*Z1**2)+(2*A1*B1*Z1)+B1**2
      HH=SQRTF(FF/GG)
      IF(HH)40,40,42
   40 HH=1
   42 CC=LOGF(HH)
      SN=((H2*Z2)/B1)+A1  $  SP=((H2*Z1)/B1)+A1
      EE=ATANF(SN)-ATANF(SP)
```

```
      ZINTEN1=AA*((BB*CC)-(DD*EE))
      ZINTEN2=ZINTEN1+ZINTEN2
   14 CONTINUE
      ZMAG(I)   =2E5*SUS*ZINDUCED*BBB*ZINTEN2
   15 CONTINUE
   13 CONTINUE
      END
```

```
      SUBROUTINE CALCDER
      COMMON/1/ZMAG(500),FX(500),M,DUMMY1(500),X(50),Z(50),N,DELX,FO,SUS
     1,THEODER(500),DUMMY2(3),STRIKE,DIP,ZINDUCED
      COMMON/2/ DUMMY3(2011)
      BETA=ATANF(TANF(DIP)/SINF(STRIKE))
      NN=N-1  $  BBB=SQRTF(1-((COSF(STRIKE))**2*(COSF(DIP))**2))
      DO 15 I=1,M
      DERIV2=0
      DO 14 K=1,NN
      X1=X(K)-FX(I)  $  X2=X(K+1)-FX(I)  $  ZZ1=Z1=Z(K)  $ ZZ2=Z2=Z(K+1)
      IF(ZZ1-ZZ2)16,17,16
   17 ZZ2=ZZ2+.01
   16 A1=(X2-X1)/(ZZ2-ZZ1)
      B1=(X1*Z2-X2*Z1)/(ZZ2-ZZ1)
      IF  (B1)22,21
   21 B1=.01
   22 H2=1+A1**2  $  AA=1/H2
      BB=A1*SINF(BETA)+COSF(BETA)
      DD=A1*COSF(BETA)-SINF(BETA)
      FF=((1+A1**2)*Z2**2)+(2*A1*B1*Z2)+B1**2
      GG=((1+A1**2)*Z1**2)+(2*A1*B1*Z1)+B1**2
      H1=(X1-X2)/(ZZ2-ZZ1)
      SN=((H2*Z2)/B1)+A1  $  SP=((H2*Z1)/B1)+A1
      HH=(2*Z2*H2)+(2*A1*B1)+(2*A1*Z2*H1)+(2*B1*H1)
      SI=(2*Z1*H2)+(2*A1*B1)+(2*A1*Z1*H1)+(2*B1*H1)
      SJ=2*(H2+(2*A1*H1)+H1**2)
      SL=1+SN**2  $  SM=1+SP**2
      QQ=(GG*FF)*(((GG*SJ)+(HH*SI))-((FF*SJ)+(SI*HH)))
      RR=((GG*HH)-(FF*SI))*((GG*HH)+(FF*SI))
      SS=.5*BB*(QQ-RR)/(GG*FF)**2
      UUU=-1-Z2*H1  $  VVV=B1-Z1*H1
      UU=1/SL*(-2*H1*H2*UUU)/B1**3
      WW=1/SM*(-2*H1*H2*VVV)/B1**3
      VV=((H2*UUU)/B1**2)*(-2*SN*(H2*UUU)/B1**2)/SL**2
      XX=((H2*VVV)/B1**2)*(-2*SP*(H2*VVV)/B1**2)/SM**2
      YY=DD*((UU+VV)-(WW+XX))
      DERIV1=AA*(SS-YY)
      DERIV2=DERIV1+DERIV2
   14 CONTINUE
      THEODER(I)=2.1527E2*SUS*ZINDUCED*BBB*DERIV2
   15 CONTINUE
      END
```

N5.3B 01/06/67

```
      SUBROUTINE LSTSQPOL(DERIV)
      COMMON/1/DUMMY1(2106),M,N,DUMMY2(3)
      COMMON/2/A(30,30),B(30),MP1,X(30),Y(30),C(30,30),DUMMY3(120)
      DO 30 I=1,N
   30 C(I,1)=1.0
      MP1=M+1
      DO 35 J=2,MP1
      DO 35 I=1,N
   35 C(I,J)=C(I,J-1)*X(I)
      DO 40 I=1,MP1
      DO 40 J=1,MP1
      A(I,J)=0.0
      DO 40 K=1,N
   40 A(I,J)=A(I,J)+C(K,I)*C(K,J)
      DO 45 I=1,MP1
      B(I)=0.0
      DO 45 K=1,N
   45 B(I)=B(I)+C(K,I)*Y(K)
      CALL MATRIX
      PRINT 600,N,M
  600 FORMAT (* NUMBER OF GIVEN DATA POINTS=*,I2,10X,*DEGREE POLYNOMIAL=
     1*,I2)
      DO 50 I=1,MP1
      II=I-1
   50 PRINT 800,II,B(I)
  800 FORMAT (5X,I2,* DEGREE COEFFICIENT=*,E13.6)
      DERIV=B(2)
      END
```

```
      SUBROUTINE MATRIX
      COMMON/1/ DUMMY1(2111)
      COMMON/2/A(30,30),B(30),N   ,DUMMY2(960),IPVOT(30),INDEX(30,2),
     1PIVOT(30)
      EQUIVALENCE(IROW,JROW),(ICOL,JCOL)
      DET=1.0
      DO 17 J=1,N
   17 IPVOT(J)=0
      DO 135 I=1,N   $   T=0
      DO 9 J=1,N
      IF(IPVOT(J)-1)13,9,13
   13 DO 23 K=1,N
      IF(IPVOT(K)-1)43,23,81
   43 IF(ABSF(T)-ABSF(A(J,K)))83,23,23
   83 IROW=J   $   ICOL=K   $   T=A(J,K)
   23 CONTINUE
    9 CONTINUE
      IPVOT(ICOL)=IPVOT(ICOL)+1
      IF(IROW-ICOL)73,109,73
   73 DET=-DET
      DO 12 L=1,N
      T=A(IROW,L)
      A(IROW,L)=A(ICOL,L)
   12 A(ICOL,L)=T
      T=B(IROW)   $   B(IROW)=B(ICOL)   $   B(ICOL)=T
  109 INDEX(I,1)=IROW   $   INDEX(I,2)=ICOL
      PIVOT(I)=A(ICOL,ICOL)   $   DET=DET*PIVOT(I)   $   A(ICOL,ICOL)=1.0
      DO 205 L=1,N
  205 A(ICOL,L)=A(ICOL,L)/PIVOT(I)
      B(ICOL)=B(ICOL)/PIVOT(I)
      DO 135 LI=1,N
      IF(LI-ICOL) 21,135,21
   21 T=A(LI,ICOL)
      A(LI,ICOL)=0.0
      DO 89 L=1,N
   89 A(LI,L)=A(LI,L)-A(ICOL,L)*T
      B(LI)=B(LI)-B(ICOL)*T
  135 CONTINUE
  222 DO 3 I=1,N
      L=N-I+1
      IF(INDEX(L,1)-INDEX(L,2))19,3,19
   19 JROW=INDEX(L,1)
      JCOL=INDEX(L,2)
      DO 549 K=1,N
      T=A(K,JROW)   $   A(K,JROW)=A(K,JCOL)   $   A(K,JCOL)=T
  549 CONTINUE
    3 CONTINUE
   81 CONTINUE
      END
N,1.59,6500,7,   ,

ECUTION STARTED AT    0322 -56
```