RIGID FRAME BRIDGE
FOR FARM LANE

Thesis for the Degree of B. S.
Vincent Vanderburg
1937

Bridges
Tuttle

Civil engineering - Bridges + roofs

Rigid Frame Bridge

for Farm Lane

A Thesis Submitted to

The Faculty of

MICHIGAN STATE COLLEGE

of

AGRICULTURE AND APPLIED SCIENCE

by

Vincent Vanderburg

Candidate for the Degree of

Bachelor of Science

June 1937

THESIS

INTRODUCTION

Farm Lane is a public road bisecting the property of Michigan State College at East Lansing. The college campus and farms owned or leased comprise a total of more than nineteen hundred acres. When the first of this property was acquired there were few traveled highways and it was probably not intended that the lane through the center of the farm should ever be used as such. It was maintained as a typical farm lane for the purpose of moving cattle, farm machinery, etc., from one field to another, and from the farm buildings to and from the various fields and pastures. A simple Howe Truss bridge that had been previously used elsewhere was purchased and placed in position across the Red Cedar River.

With the expansion of college property south of Mt. Hope Road the gates across Farm Lane were eliminated. This opened it to public use and it became the most direct route between East Lansing and a prosperous farming section. It has been used for many years by the general public as a public highway.

In 1936 the W. P. A. through a cooperative project placed a hard surface on the roadway which has tended to increase the amount of traffic.

The present bridge was first condemned as unsafe for highway purposes by the Engineering Department in 1895. Its use becomes increasingly precarious with the passing of the years not only due to the deterioration of time but also due to the high speed with which heavy trucks traverse it.

The State Board of Agriculture has repeatedly requested the State Highway Department to build a substantial bridge at this point. In 1929 the legislature passed an enabling act authorizing the Highway Department to take over the construction and maintenance of roads and drives at the various State institutions. The Highway Department has not as yet agreed to undertake this project.

The college has placed warning signs on the bridge warning all traffic that it uses the bridge at its own risk. But, in spite of these precautions there is a moral responsibility on the college and the State to provide a safe bridge. The time is not far off when a new and adequate bridge must replace the present one.

The college campus now extends to the river at this point. The area east of Farm Lane is used for the Water Carnival and other similar purposes.

The new bridge in order to be in keeping with the fine appearance of the college buildings and the beautiful landscaped campus must be of an attractive design, and should combine the maximum beauty with the necessary utility.

With this in mind I am suggesting a Rigid Frame Concrete Bridge for this location resembling in general style the beautiful Rigid Frame Concrete Bridge in Krape Park, Freeport, Illinois.

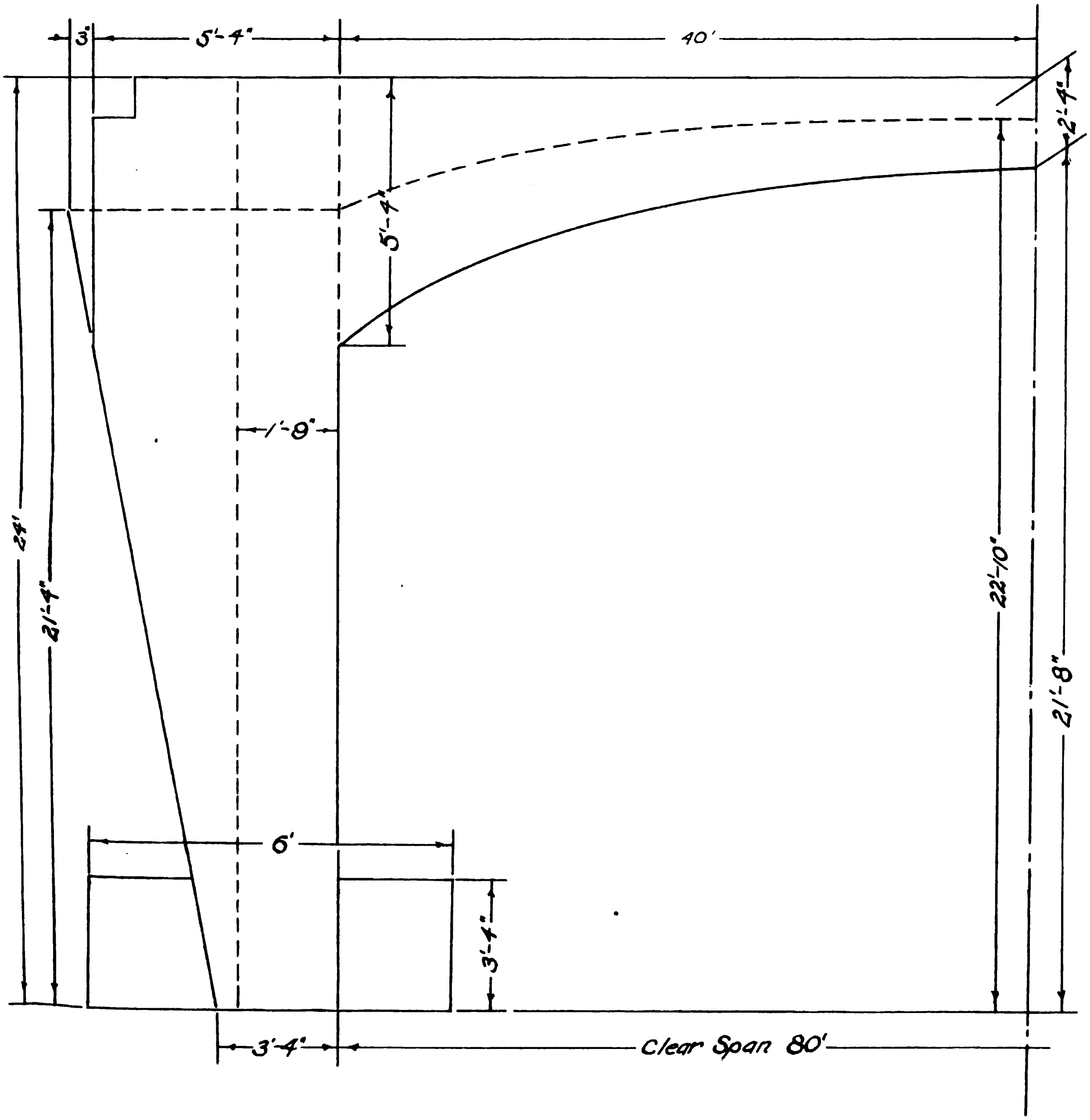
This type of bridge is economical to build, easy to maintain, sturdy and rigid, and can be made graceful and artistic in appearance. It represents a newer type of construction that should be of interest and value to the engineering department of the college for instructional purposes.

These specifications describe a bridge eighty feet long clear span, eighteen foot roadway with a single eight foot sidewalk with the clearance above normal water level of fifteen feet.

An adequate number of borings have been taken to demonstrate that no difficulty will be encountered in the construction of the foundation.

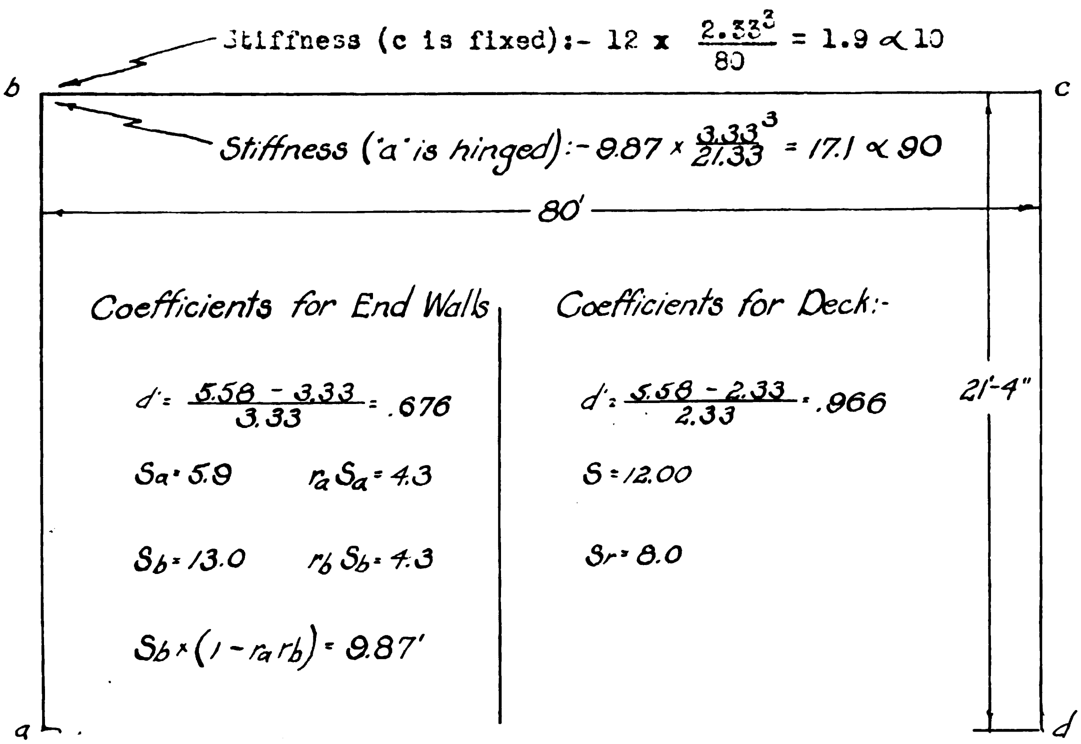
With this type of construction it is of the utmost importance that careful laboratory checks be made of the concrete poured in the monolithic structure.

The specifications used in working these necessary problems were obtained from the Michigan State Highway Department. Much valuable information was obtained from the Portland Cement Association through Mr. J. O. Granum. Grateful acknowledgment is also made to Professor Allen of the M. S. C. Civil Engineering Department, and to J. G. Martin of the Portland Cement Association for their many helpful and valuable suggestions.


3"
5'-4"
40'
2'-4"
5'-4"
1'-9"
24'
21'-4"
22'-10"
21'-8"
6'
3'-4"
3'-4"
Clear Span 80'

Problem 1.

Frame Dimensions. Ax/s and Coefficients.



Deck coefficients:-

$$d^1 = \frac{5.58 - 2.33}{2.33} = \overset{1.14}{\cancel{.966}}$$

From Chart II

S = 12.0

rS = 8.0

$S \times \frac{I}{L}$, or proportional to $12.00 \times \frac{(2.33)^3}{80} = 1.9$

Carry over factor, r, equals

$$\frac{rS}{S} = \frac{8}{12} = .666$$

$d^1 = \frac{5.58 - 3.33}{3.33} = .676$ Sa = 5.9 Sb = 13.0 ra Sa = 4.3 rbSb = 4.3

Re

Imma

0	
1.9351	
0	
.90	
0	
0	
0	
.6920	
:	
:	
.9039	
0	

Comput

1

$$S_b \times \frac{I}{L}(1 - r_a r_b) = 13. \times \left(1 - \frac{(4.3)^2}{13 \times 5.9}\right) \times \frac{I}{L} = 9.87 \times \frac{I}{L} = 9.87$$

$$\times \frac{3.33^3}{21.33} = 17.06$$

The relative stiffness in per cent at "b" is then

$$\frac{1.9}{1.9 + 17.06} \times 100 = 10. \text{ for the deck}$$

$$\frac{17.06}{1.9 + 17.06} \times 100 = 90. \text{ for the wall}$$

Problem 2.

Distribution of Fixed End Moment

b (90)	10	.66	10	c (90)
0.	+100.00	(1) Fixed end mom.	0.	0.
90.	- 10.	Distributed mom.	0.	0.
0.	0.	(2) Carry-over mom.	-6.6	0.
0.	0.	Distributed "	+ .66	+5.94
0.	+ .4356	(3) Carry-over mom.	0.	0.
-.3920	- .04356	Distributed "	0.	0.
0.	0.	(4) Carry-over mom.	-0.0287496	0.
0.	0.	Distributed "	+ .002874	+.025874
-90.39	+90.39	Total Moments	-5.965874	+5.965874
a				d

Computations -

1st Cycle - 100 x .10 = - 10 in bc

- 100 x .90 = -90 in ba

total moments at end of 1st cycle

- 90 + 90 at "b" zero at "c" zero

3rd Cy

4th Cy

I

the en

releas

-

+

1

-

+

(

by eli

2nd Cycle - Moment carried from b to c equals

- 10 x .66 = - 6.6

Distributed moments at "c" are

+ 6.6 x .1 = + .66 in cb

+ 6.6 x .9 = + 5.94 in cd

total moment at end 2nd cycle

- 90 & 90 at b,

- 5.94 & + 5.94 at c

3rd Cycle + .66 x .66 = .4356

- .4356 x .1 = .04356 in bc

- .4356 x .9 = .39204 in ba

total moment at end of 3rd cycle

- 90.39204 & + 90.39204 at b

- 5.94 & + 5.94 at c

4th Cycle

- .04356 x .66 = -.0287496

+ .0287496 x .1 = +.00287496 in cb

+ .0287496 x .9 = + .025874 in cd

If a Fixed End Moment - 100.00 is applied in "ba" at "b" (i.e., in the end wall immediately below the corner joint) and the joints are then released, the final corner moments become:-

- 100.00	+ 10.00
+ 90.	- .4356
+ .39204	+ .04356
- 9.60796 in "ba" at "b"	+ 9.60796 in "bc" at "b"
+ 5.965874 and	- 5.965874 at "C"

(These relative moment values will be helpful for subsequent analysis by eliminating repetition of computations.)

Problem 5.

The

ings and

Ele

nce of 2

plus a

Fixed

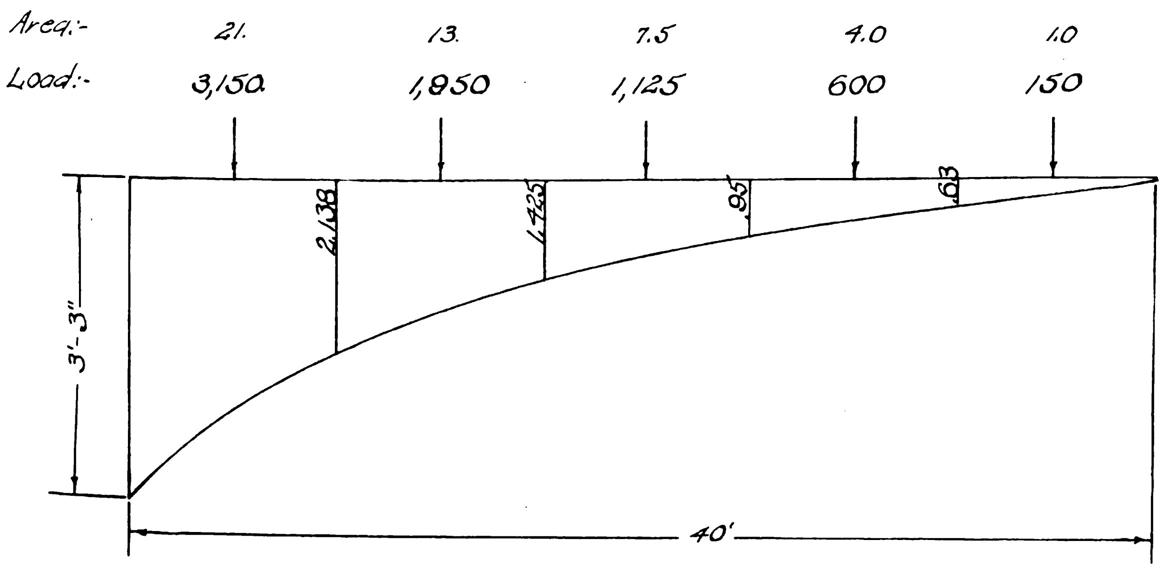
U

Problem 5.

Dead Load

The weight at the end walls is carried directly down to the footings and creates no moments.

Michigan State Highway Department Specifications call for an allowance of 20#/sq. ft. of roadway for additional separate wearing surface; plus a "½" additional thickness to provide monolithic wearing surface.



(Concrete weighs approximately 150# 1 cu. ft.)

Fixed End Moment per foot of width:-

Uniform load:-

$$2.37\ (150) + 20 = 356 + 20 = 376\#/\text{sq. ft.}$$

$$376 \times 80^2 \times .102 = 245,452 \text{ ft. lbs.}$$

value

stati

Equivalent concentrated loads

$$
\begin{aligned}
3,150 \times 80 \times .05 &= 12,600 \\
1,950 \times 80 \times .125 &= 19,500 \\
1,125 \times 80 \times .18 &= 16,200 \\
600 \times 80 \times .20 &= 9,600 \\
150 \times 80 \times .187 &= 2,240 \\
150 \times 80 \times .14 &= 1,680 \\
600 \times 80 \times .1 &= 4,800 \\
1,125 \times 80 \times .04 &= 3,600 \\
1,950 \times 80 \times .012 &= 1,870 \\
3,150 \times 80 \times .002 &= \underline{\quad 500}
\end{aligned}
$$

318,042 ft. lbs.

Using values determined in Problem 2, pages 6 and 7, the numerical values of the corner moments at "b" are:-

318,042 x .9039 due to Fixed End Mom. at "b"

318,042 x .05965 " " " " " " "c"

Total moment when deck is straight is:-

$318,042 \times (.9039 + .05965) = 306,000$ ft. lbs.

and produces tension in the outside corner.

Correcting this moment for curvature of deck:-

(Raise of deck axis is 1.62')

$$306,000 \times \frac{21.33 + .5 \times 1.62}{21.33 + 1.62} = 306,000 \times \frac{22.14}{22.95} =$$

$306,000 \times .965 = 295,000$ ft. lbs.

The crown moment for straight deck centerline can now be found by statics. The total positive moment assuming a simply supported deck is:-

Th

created

is the

Co

(tensio

Cr

use of

Concen

Unifor

$$3{,}150 \times 80 \times .05 = 12{,}600$$
$$1{,}950 \times 80 \times .15 = 23{,}400$$
$$1{,}125 \times 80 \times .25 = 22{,}500$$
$$600 \times 80 \times .35 = 16{,}800$$
$$150 \times 80 \times .45 = 5{,}400$$
$$1/8 \times 376 \times 80^2 = \underline{300{,}800}$$
$$381{,}500 \text{ ft. lbs.}$$

The difference between this moment and the negative corner moment created by the same loading:-

$$381{,}500 - 306{,}000 = 75{,}500 \text{ ft. lbs.}$$

is the moment at the crown with straight deck.

Correct for curvature of deck and determine the final crown moment (tension in bottom of deck).

$$75{,}500 \times \frac{21.33 + .5 \times 1.62}{21.33 + 1.62} = 75{,}500 \times .965 = 72{,}800 \text{ ft. lbs.}$$

Checking on the final corner and crown moments will now be made by use of influence lines (Chart I).

$$\frac{M}{S} = \frac{21.33}{80} = .266 \text{ (Interpolating between .54 and .18 on Chart I)}$$

Concentrated Loads

$$3{,}150 \times 80 \times .048 = 12{,}100$$
$$1{,}950 \times 80 \times .113 = 17{,}620$$
$$1{,}125 \times 80 \times .182 = 16{,}400$$
$$600 \times 80 \times .198 = 9{,}500$$
$$150 \times 80 \times .185 = 2{,}220$$
$$150 \times 80 \times .138 = 1{,}655$$
$$600 \times 80 \times .089 = 4{,}270$$
$$1{,}125 \times 80 \times .048 = 4{,}320$$
$$1{,}950 \times 80 \times .02 = 3{,}120$$
$$3{,}150 \times 80 \times .004 = 1{,}010$$

Uniform Load $376 \times 80^2 \times .102 = \underline{245{,}452}$

$$317{,}667 \text{ ft. lbs.}$$

Total

and

Corr

ment is

Tot

The

created

is the

Co

(tensio

F

F

Total moment when the deck is straight is:-

$$317,667 \times (.9039 - .05965) =$$

$$317,667 \times .96355 = 305,000 \text{ ft. lbs.}$$

and produces tension in outside of the corner.

Correcting this moment for curvature of the deck, the final corner moment is:-

$$305,000 \times \frac{21.33 + .5 \times 1.62}{21.33 + 1.62} = 305,000 \times .965 = 294,000 \text{ ft. lbs.}$$

Total positive moment has been previously computed:-

(page) 10 - 381,500 ft. lbs.

The difference between this moment and the negative corner moment created by the same loading:-

$$381,500 - 305,000 = 76,500 \text{ ft. lbs.}$$

is the moment at the crown of the frame with straight deck.

Correct for curvature of deck and determine the final crown moment (tension in bottom of deck):-

$$76,500 \times \frac{21.33 + .5 \times 1.62}{21.33 + 1.62} = 76,500 \times .965 = 73,800 \text{ fl. lbs.}$$

	Corner	Crown
From Chart I	- 294,000	+ 73,800
From Moment Distribution	- 295,000	+ 72,800

Total dead load of the frame, one foot wide, is:-

Wearing surface:- 20* x 80 =	1,600
Deck:- 2.37 x 80 x 150 =	27,640
Deck:- .33 x 3.25 x 80 x 150 =	13,000
Corners:- 3.25 x 5.58 x 2 x 150 =	5,442
Walls:- .5 x (5.58 + 3.33) x 18.54 x 2 x 150 =	24,800
Footings:- (6.0 - 3.33) x 3.33 x 2 x 150 =	2,670
	75,152

The vertical reaction on each footing is:-

0.5 x 75,152 = 37,576 lb. say 37,500 lb.

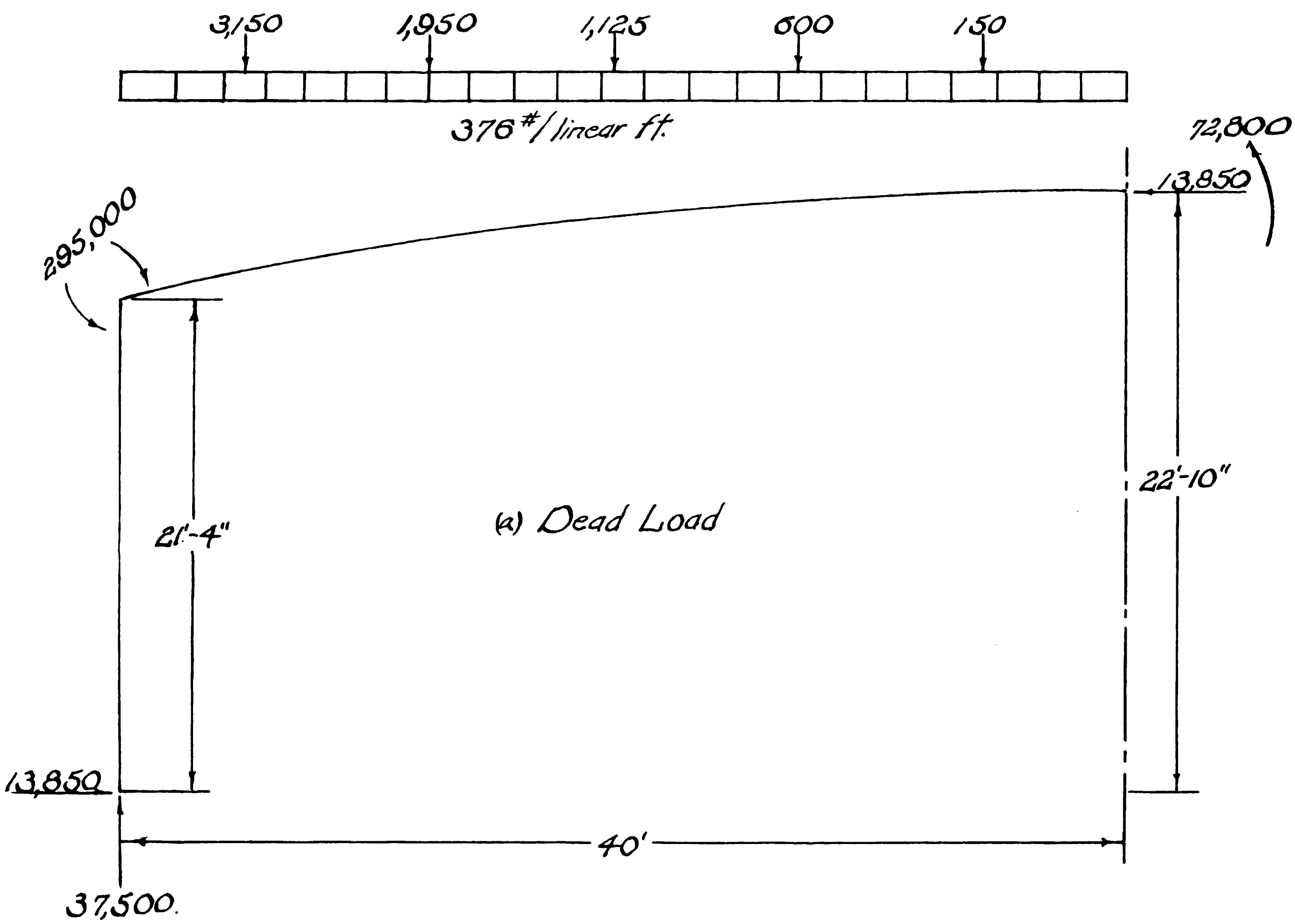
The horizontal thrust at the footing, when the deck is curved, is:-

$$\frac{295,000}{21.33} = 13,850 \text{ lb.}$$

The crown thrust also equals 13,850 lb., since all the loads are gravity loads.

*Future Wearing Surface - Michigan State Highway Department Specifications.

Problem 4.



Live loads taken from:- "Theory of Modern Steel Structures" - By Grinter - Page 107 - Article 113.

These loads are considered by the American Association of State Highway Officials as the loadings designed for the various types of bridges.

We have selected according to the specifications of the A. A. S. H. O. the alternate loading or equivalent loading to take the place of the truck train for long spans. The loads are for a lane 9 feet wide.

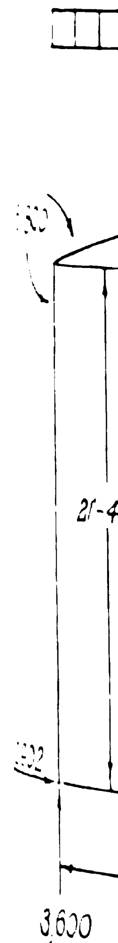
H 15 Alternate loading or equivalent loading is used.

Without Impact:
- Concentrated load = 13,500 lbs. for moment; 20,500 lbs. for shear
- Uniform loading = 480 lb. per linear foot.

Impact:-

New and

With Imp



Maxim

is placed a

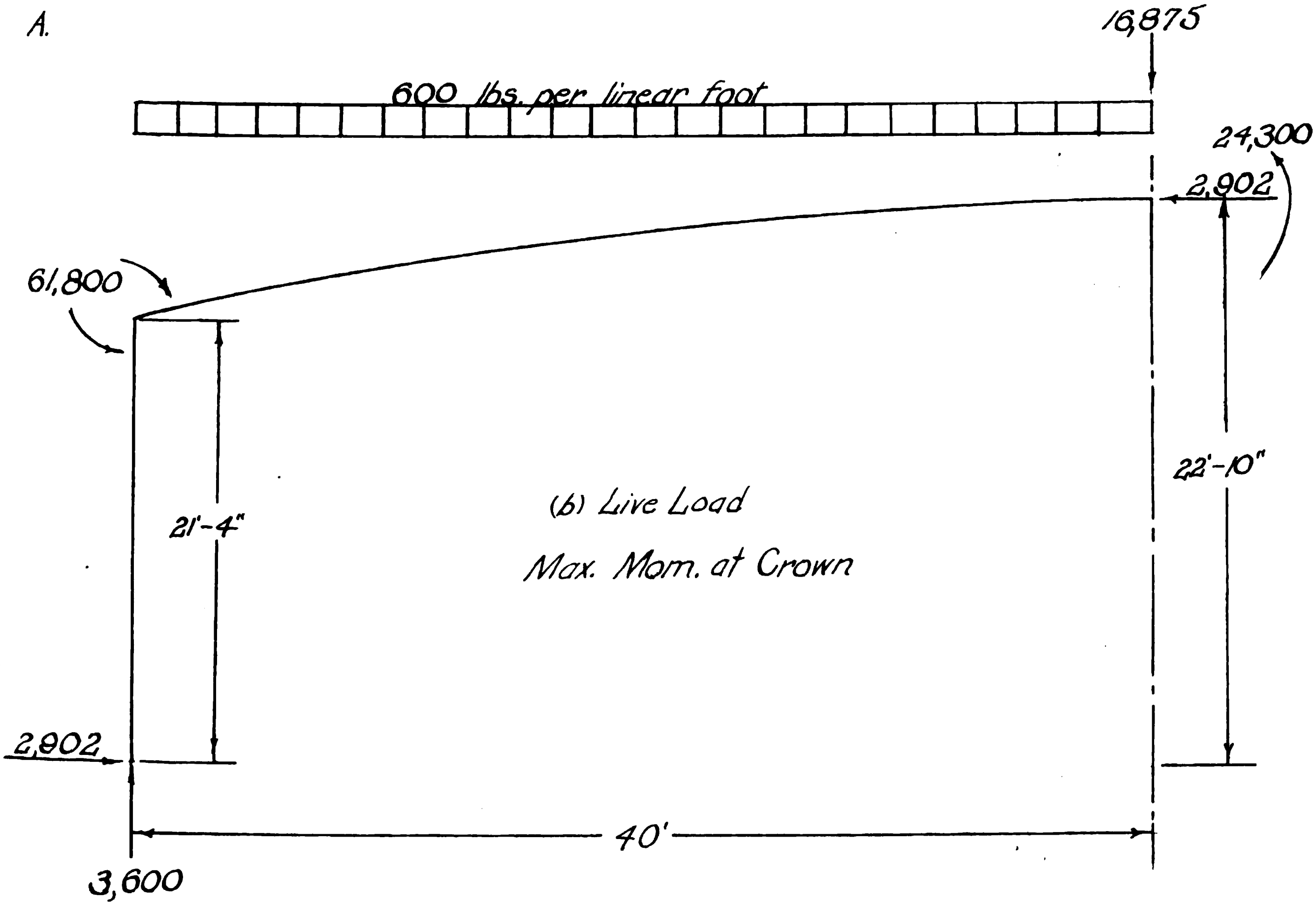
The first g

efficients

Impact:- $I = \frac{50}{L + 125} = \frac{50}{205} = 24.3$ or 25%

New and used values for:

(16,875 lbs. for moment
(Concentrated load = (
With Impact ((25,625 lbs. for shear
(
(Uniform load = 600 lbs. per linear foot



Maximum moment is produced at the crown when the concentrated load is placed at the midpoint and the uniform load covers the entire span. The first step is the analysis[13] to determine the fixed end moment coefficients by entering Chart II with $d^1 = .966$.

Fixed End Moment:- (9 foot lane)

Uniform load:- $600 \times 80^2 \times .101 = \dfrac{387,840}{9} = 43,000$

Concentrated load:- $16,875 \times 80 \times .168 = \dfrac{227,000}{9} = \underline{25,200}$

68,200 ft. lbs.

By using the values from Problem 2, the corner moment is found to be:- $68,200 \times (.90592 + .059658) = 68,200 \times .965578 = 65,600$ ft. lbs.

(Straight Deck)

The total positive crown moment assuming a simply supported deck is:- $600 \times 40 \times .5 \times 40 = \dfrac{480,000}{9} = 53,300$

$.5 \times 16,875 \times 40 = \dfrac{337,500}{9} = \underline{37,500}$

90,800 ft. lbs.

The difference between this moment and the negative corner moment created by the same loading.

$90,800 - 65,600 = 25,200$ ft. lbs.

is the moment at the crown of the frame with straight deck.

Correct for curvature of deck and determine the final crown moment (tension in bottom of deck):

$$25,200 \times \frac{21.33 + .5 \times 1.62}{21.33 + 1.62} = 25,200 \times .965 = 24,300 \text{ ft. lb.}$$

A check on the final crown moment can be obtained by use of the influence lines in Chart I.

$600 \times 80^2 \times .021 = \dfrac{80,640}{9} = 8,950$

$16,875 \times 80 \times .078 = \dfrac{105,000}{9} = \underline{11,680}$

20,630 ft. lb.

The moments, thrusts and shears for the position of the live load that gives the maximum moment at the crown are shown.

The corner moment when the deck is curved is:-

$$65{,}600 \times \frac{21.53}{22.83} = 61{,}800 \text{ ft. lb.}$$

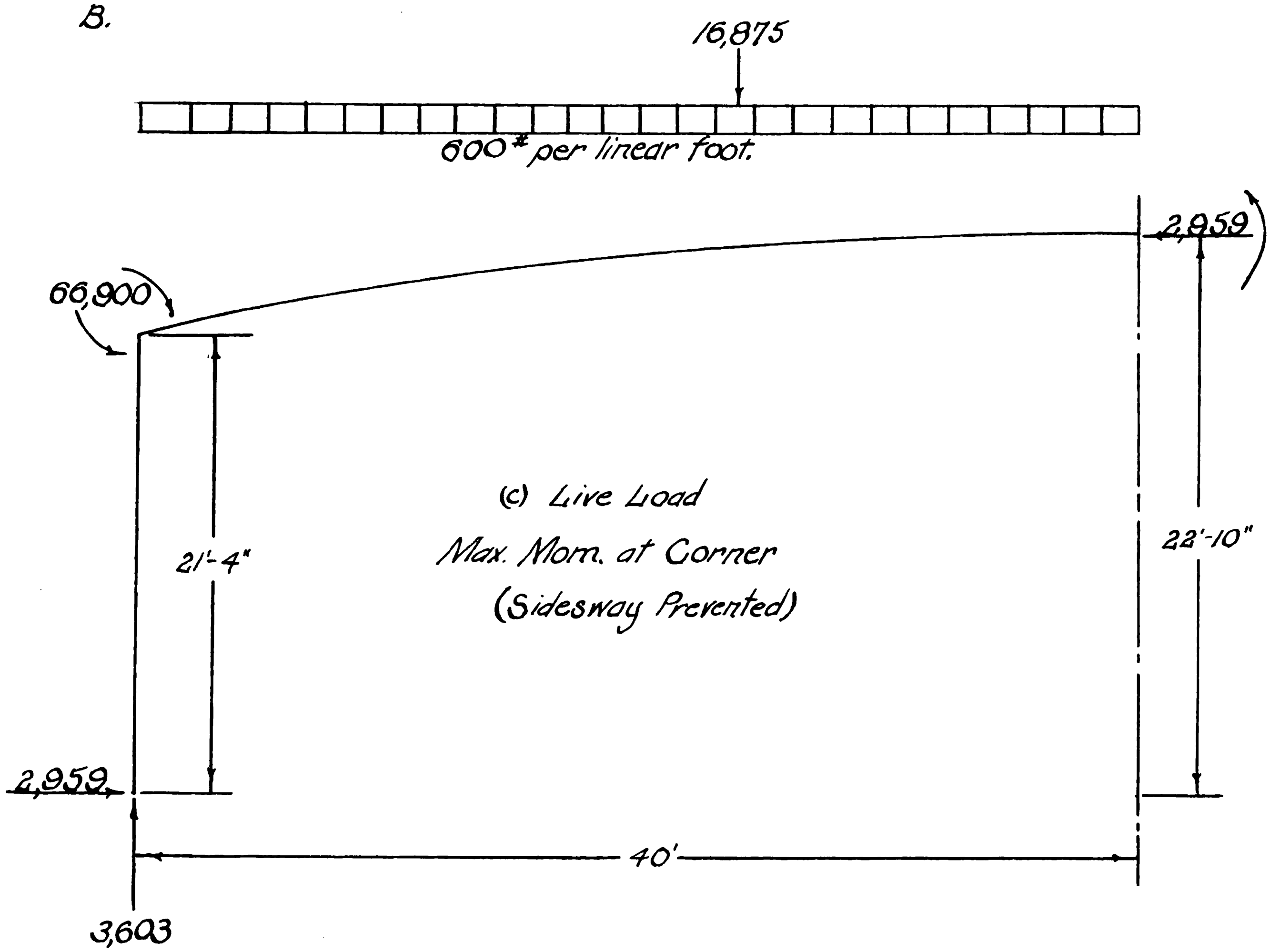
The corresponding horizontal thrust is:-

$$\frac{61{,}800}{21.33} = 2{,}902 \text{ lb.}$$

The vertical reaction on each footing is:-

$$600 \times .5 \times 80 + .5 \times 16{,}875 = \frac{32{,}437}{9} = 3{,}600 \text{ lb.}$$

B.



Maximum moment at the corner (point 1.0 in Chart II) is produced with uniform load over the entire span and when the concentrated load is placed at or near point .625. The following values are obtained with the load of 16,875# at point 0.625.

Fixed End Moment: (9 foot lane) $d^1 = .966$

$$\text{At point 1.0} \quad 16,875 \times 80 \times .2 = \frac{270,000}{9} = 30,000$$

$$\text{At point 0.0} \quad 16,875 \times 80 \times .1 = \frac{135,000}{9} = 15,000$$

Corner moment after distribution:-

at 1.0 $\quad 30,000 \times .9039 + 15,000 \times .0596 = 27,100 - 894 = 27,994$

at 0.0 $\quad 30,000 \times .0596 + 15,000 \times .9039 = 1,789 - 13,580 = 15,369$

Maximum corner moment (including uniform load) when the deck is straight.

$27,994 + 43,000 \times (.9039 + .0596) = 27,994 + 41,400 = \underline{69,394}$ ft. lbs.

Final corner moment allowing for curvature of deck.

$$69,394 \times \frac{21.33 + .5 \times 1.62}{21.33 + 1.62} = 69,394 \times .965 = \underline{\underline{66,900}} \text{ ft. lb.}$$

Check by Chart I with 16,875 lb., at point .625

$$600 \times 80^2 \times .095 = \frac{364,800}{9} = 40,600$$

$$16,875 \times 80 \times .187 = \frac{252,000}{9} = \underline{28,000}$$

$$68,600 \text{ ft. lb.}$$

The moments, thrusts and shears for the position of the live load that gives the maximum moment at the crown are shown:-

The corner moment when the deck is curved is:-

$$66,900 \times \frac{21.58}{22.83} = 63,000 \text{ ft. lbs.}$$

The corresponding horizontal thrust is:-

$$\frac{63,000}{21.35} = 2,959 \text{ lb.}$$

The vertical reaction on each footing:-

$$600 \times .5 \times 80 + .5 \times 16,875 = \frac{32,437}{9} = 3,603 \text{ lb.}$$

Problem 5.

Change in Length of Deck and Horizontal Displacement

A relative change in length of deck may be either a shortening (temperature drop, shrinkage, outward displacement of footings) or a lengthening (temperature rise, inward displacement of footings).

Assume that the frame in Problem 1 is subject to a deck shortening due to (a) temperature drop of 45° F., (b) shrinkage corresponding to a shrinkage factor of 0.0002, and (c) outward horizontal displacement of the footings equivalent to a contraction coefficient of .0002.

The shortening per unit of length is:-

$$45 \times 0.0000065 + .0002 + .0002 = .0006925$$

(M. S. H. D. Specifications)

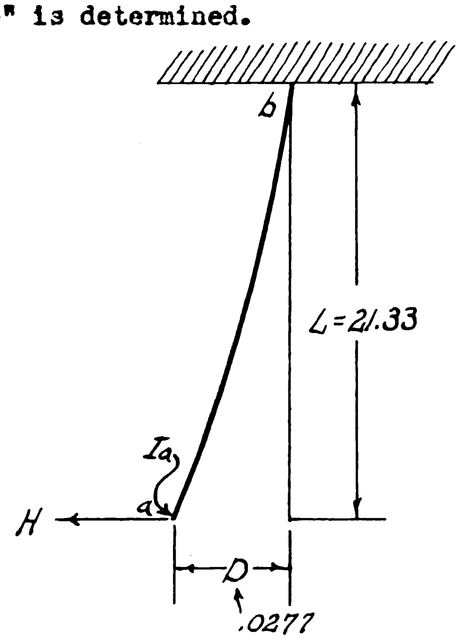
The total shortening in the span of 80 feet is:-

$$.0006925 \times 80 = .0554 \text{ ft.}$$

This is equivalent to an outward displacement of .0277' at "a" and "d".

When analyzing the frame by moment distribution, begin by locking the joints "b" and "c".

Figure below illustrates the static conditions from which the fixed end moment at "b" is determined.



It can be shown that:-

$$\text{F. E. M. at "b"} = \frac{E \times D}{L} \times S_b \times \frac{I_a}{L} \times (1 - r_a r_b) =$$

$$\frac{(3 \times 10^6 \times 12^2) \times 0.0277}{21.33} \times 13 \times \frac{(1/12) \times 3.33^3}{21.33} \times \left(1 - \frac{4.3^2}{13.0 \times 5.9}\right) = 786{,}000$$

F. E. M. at "b" = 786,000 ft. lb.

According to Problem 2, the formula for both corner and crown moment when deck is straight may be written as:-

$$\text{Moment} = 786{,}000 \times \underbrace{[(1.0000 - 0.900 - .392) - .0598]}_{.0174} = 13{,}700 \text{ ft.lb.}$$

Final Moments - allowing for curvature of deck - equal

$$\text{Corner:} \quad 13{,}700 \times \left(\frac{21.33}{21.33 + 1.62}\right)^2 = 11{,}820 \text{ ft. lb.}$$

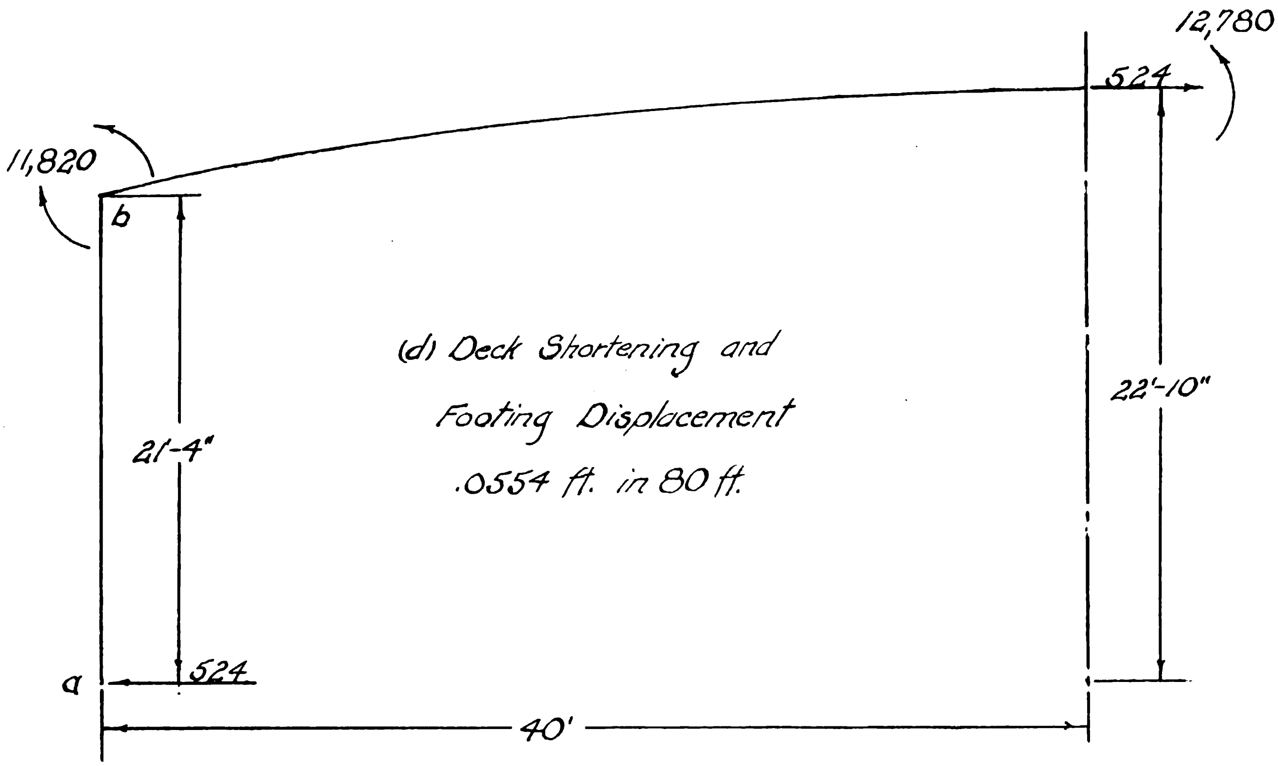
$$\text{Crown:} \quad 13{,}700 \times \left(\frac{21.33}{21.33 + 1.62}\right) = 12{,}780 \text{ ft. lb.}$$

The effect of changes in the thickness at the bottom of the wall is relatively insignificant.

Inserting numerical values in the empirical formula gives:-

$$\text{Corner moment:-} \quad 4.35 \times 10^6 \times \underbrace{\frac{21.33 \times .0277}{(21.33 + 1.62)^2}}_{.00109} \times 2.33^2 = 24{,}604 \text{ ft.lb.}$$

$$\text{Crown moment:-} \quad 12{,}700 \times \underbrace{\frac{21.33 + 1.62}{21.33}}_{1.075} = 13{,}650 \text{ ft. lb.}$$



Corner moment when the deck is curved is:-

$$11{,}820 \times \frac{21.58}{22.83} = 11{,}180 \text{ ft. lb.}$$

Corresponding horizontal thrust is:-

$$\frac{11{,}180}{21.33} = 524 \text{ lb.}$$

Problem 6.

Earth Pressure

Rigid frames should be designed to withstand two groups of influences, (1) the forces characteristic of continuous structures; and (2) the dead and live loads, tractive forces and earth pressure.

The loads of group (2) are identical with those acting on ordinary simple-span bridges with the exception of the earth pressure on the end walls. Earth pressure on abutments for simple-span bridges is usually active pressure, produced by the backfill moving toward the abutment. In rigid frame bridges it is possible - at least theoretically - to develop some passive earth pressure by a movement of the end wall against the backfill. Tests are recorded which indicate that little passive earth pressure is developed; it may ordinarily be disregarded.

Problem 7.

Dissymmetry and Sidesway

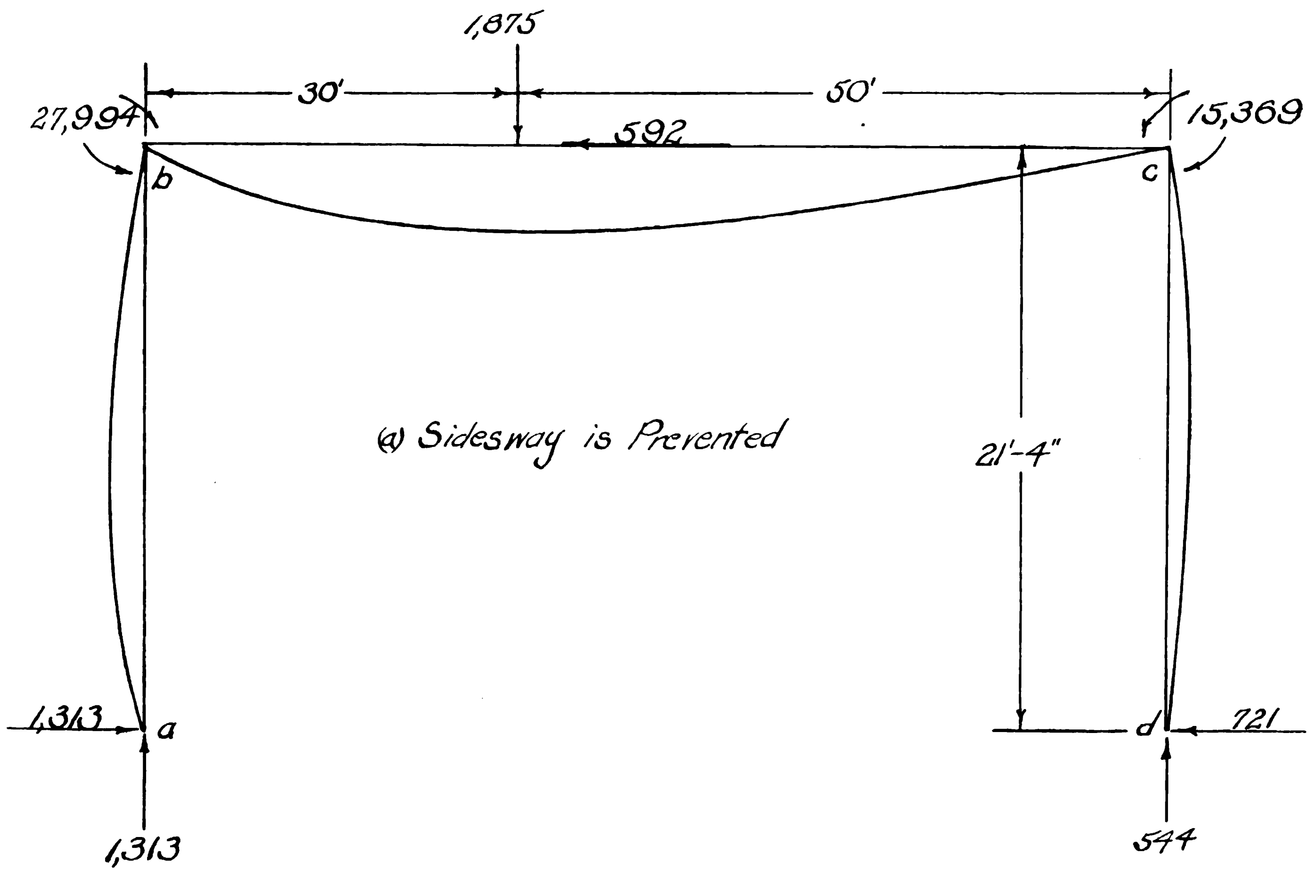
If the frame or loading is unsymmetrical, the moment distribution method as discussed and applied in the foregoing gives horizontal thrusts that apparently do not satisfy the statical requirements for equilibrium.

Take the frame on page 5 loaded in 16,875 lb. at point 0.625 with a lane 9 feet wide. The corner moments, determined in Problem 4 for straight deck are:-

at point 1.0: 27,994

at point 0.0: 15,869

The corresponding horizontal thrusts are:-



$$\frac{27,994}{21.33} = 1,313 \text{ lb. at "a"}$$

and

$$\frac{15,369}{21.33} = 721 \text{ lb. at "d"}$$

The algebraic sum of the horizontal forces is 1,313 - 721 = 592 lb., but it should be zero to satisfy the static requirement that the sum of the projections of all external forces on any line must be zero. This apparent discrepancy will be clarified by the discussion of sidesway which follows.

It is evident that the deck "bc" in figure shown above will tend to move sidewise relative to "a" and "d" whenever the frame or the loading is unsymmetrical, and also that a lateral displacement of "bc" will set up moments at the corners. Refer to Problems 2 and 4 and observe that no fixed end moment due to displacement of "bc" was included in the analysis.

The significance of this omission is that points "b" and "c" have been kept in their original position vertically above "a" and "d"; or, as it is called, sidesway of the frame has been prevented.

It is obvious that an external force must be added in the line "bc" when horizontal displacement of "bc" is to be prevented. The laws of equilibrium require that the force equal 592 lb. The loads, reactions and deflected axis for the frame in which sidesway is prevented are shown in the above figure. The force of 592 lb. in "bc" increases the vertical reaction at "a" (and decreases the vertical reaction at "d"), thereby make it equal to

$$\frac{16,875}{9} \times \frac{50}{80} + 592 \times \frac{21.33}{80} =$$

$$1,875 \times .625 + 592 \times .268 = 1,331 \text{ lb.}$$

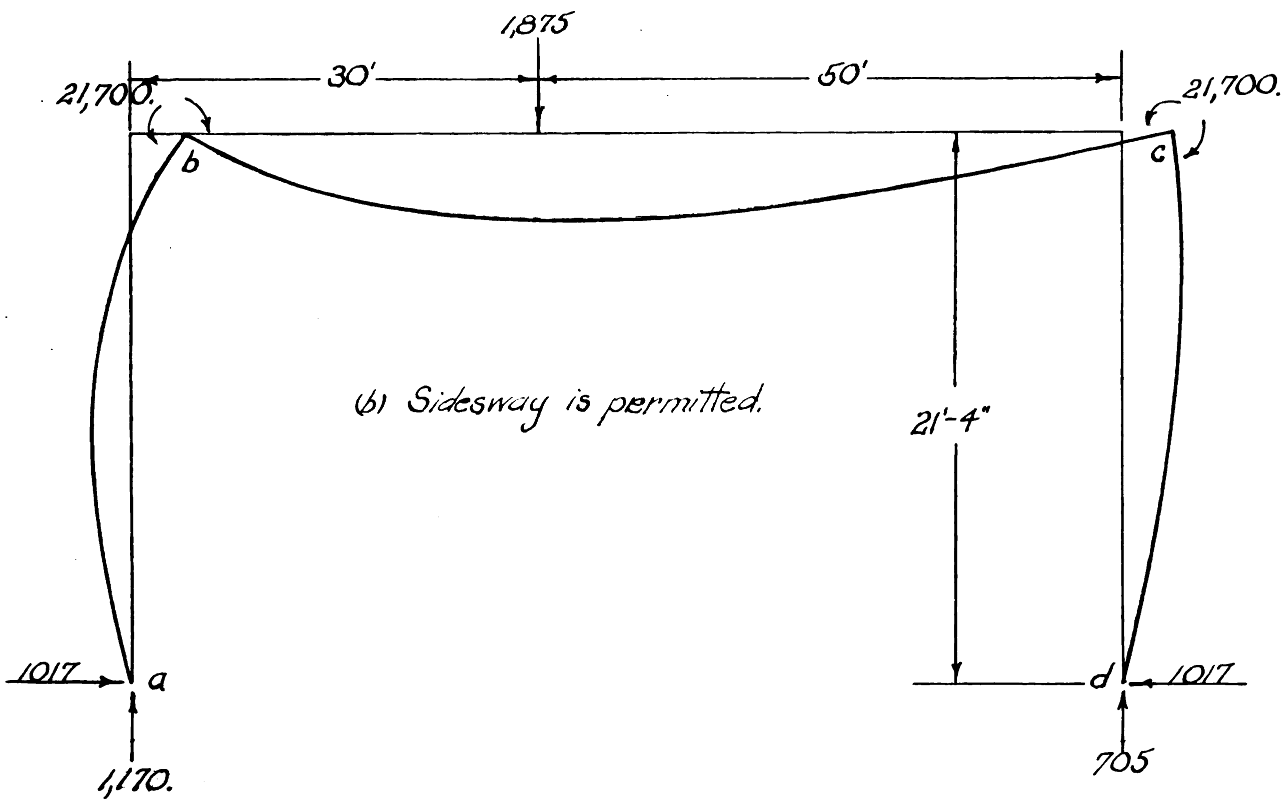
Of the two assway and no sidesway - the latter is obviously closer to the actual condition in the ordinary rigid frame for highway bridges. The assumption that no sway takes place is therefore preferable, especially since it gives the greater corner moment. It shall be illustrated, however, how readily results obtained by moment distribution may be adjusted to allow for the assumption that sidesway is permitted. Consider, for example, the frame in the figure below analyzed by moment distribution, in which a force of 592 lb. is required to prevent sidesway. Eliminate this force by adding another equal but opposite force in "bc", simultaneously displacing the deck horizontally in the direction from "b" to "a". Determine the F. E. M. and then by moment distribution - in a manner similar to that in Problem 5 - the final corner moments. This general procedure can often be simplified. In the frame in the figure below for example, the added force of 592 lb., obviously creates the same horizontal thrust at both footings when sidesway is permitted must therefore be:-

1,313 - 296 = 721 + 296 = 1017 lbs.,

and the corner moments at joints "b" and "c" are:-

1,017 x 21.33 = 21,691 ft. lb., say 21,700 ft. lb.

The corresponding maximum negative corner moment is 27,994 when sidesway is prevented.



Problem 8.

Shears

Loads used on page 14 are:-

Concentrated load = 20,500 lb.

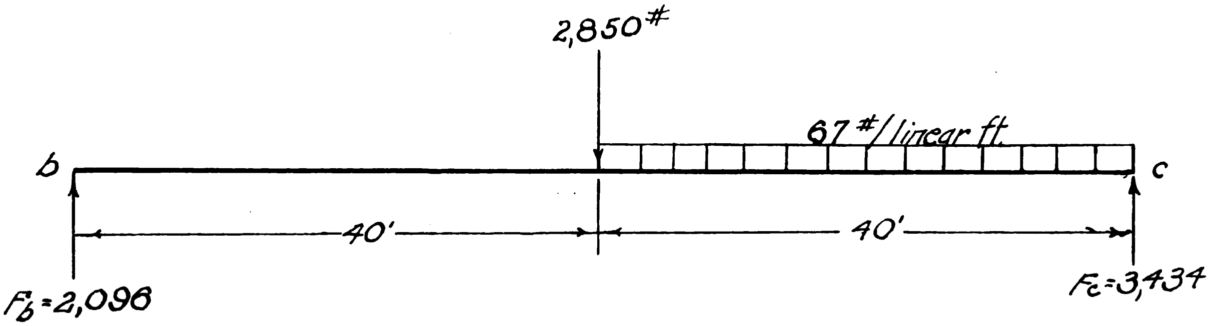
Uniform loading = 480#/ lin. ft.

these loads are for a 9 foot traffic lane, and the loads per foot of width for shear loads are:-

$$\text{Concentrated load} = \frac{20,500}{9} \times 1.25 = 2,850 \text{ lb.}$$

$$\text{Uniform load} = \frac{480}{9} \times 1.25 = 67\#/\text{sq. ft.}$$

Find the maximum total shear and unit shearing stresses at (a) the crown (b) the corner, and (c) the top of the footing. (a) Crown. The shear is zero due to dead load, deck shortening note previous problem. The shear calculations for live loads are simplified if sidesway is assumed to be permitted, since the shears in the deck then equal the shears in a simply supported beam with a span length of 80 ft. The maximum shear due to live loads equal 2,096 lb. and is produced by the loading arrangement shown in the figure below.



$$M_b = 2,850 \times 40 + (67 \times 40) \times 60 - F_c \times 80 =$$

$$= 114,000 + 160,800 - 80\ F_c = 274,800 - 80\ F_c$$

$$F_c = \frac{274,800}{80} = 3434$$

$$2,850 + 2,680 - 3,434 - F_b = 0$$

$$F_b = +\ 2096$$

The corresponding unit shearing stress is:-

$$\frac{2,096}{12 \times 7/8 \times 22} = 9.06 \text{ pounds per sq. in.}$$

Further investigation of the shear based upon the assumption that sidesway is prevented (see Problem 7) is unwarranted. (b) Corner. The total dead load from face to face of end walls is:-

Wearing surface:- 20 x 80 = 1,600

Deck:- 2.33 x 150 x 80 = 27,960

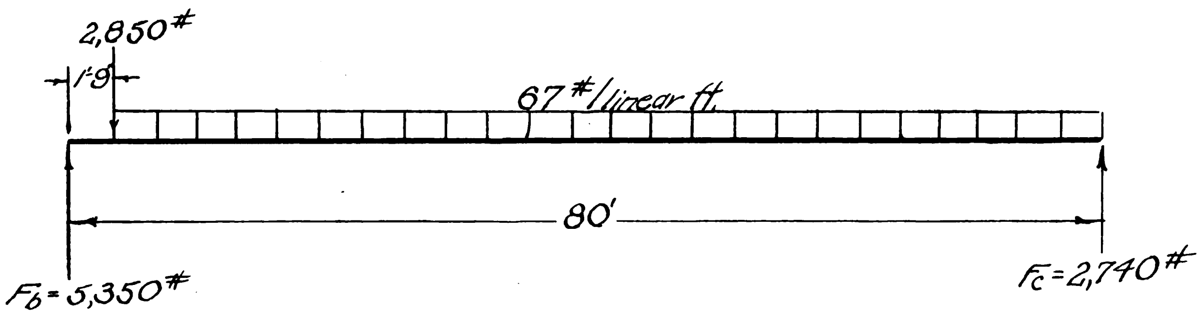
Deck:- .333 x 3 x 150 x 80 = 83,880

113,440 lb.

Maximum shear due to dead load is:-

$$\tfrac{1}{2} \times 113{,}440 = 56{,}720 \text{ lb.}$$

Deck shortening and symmetrical earth pressure produce no vertical shears. The maximum shear at the face of the end wall due to live load equals 5,350 lb., and is produced by the loading arrangement shown in the figure below.



$$M_b = 2{,}850 \times 1.75 + (67 \times 78.25) \times 40.875 - 80 \times Fa =$$

$$4{,}980 + 214{,}200 - 80\ Fc = 219{,}180 - 80\ Fc$$

$$Fc = \frac{219{,}180}{80} = 2{,}740 \text{ lb.}$$

$$2{,}850 + 5{,}240 - 2{,}740 - F_b =$$

$$F_b = 5{,}350 \text{ lb.}$$

The total dead and live load shear is:-

$$56{,}720 + 5{,}350 = 62{,}070 \text{ lb.}$$

The corresponding unit stress is:-

$$\frac{62{,}070}{12 \times 7/8 \times 62} = 95 \text{ pounds per sq. in.}$$

(c) Top of footing. The maximum shear equals the horizontal thrust at the support. The dead load thrust is:-

$$\frac{295,000}{21.33} = 13,850 \text{ lb.}$$

The maximum horizontal thrust due to live load is produced by the same load arrangement that causes maximum corner moment. The maximum corner moment, derived from the analysis in Problem 4 is:-

$$\frac{2,850}{1,875} \times 27,994 + 41,400 = 42,500 + 41,400 = 83,900 \text{ ft. lb.}$$

with straight deck; but allowing for curvature of deck it equals:-

$$83,900 \times \frac{21.33 + .5 \times 1.62}{21.33 + 1.62} = 83,900 \times .965 = 81,000 \text{ ft. lb.}$$

The horizontal thrust is:-

$$\frac{81,000}{21.33} = 3,800 \text{ lb.}$$

The horizontal thrusts produced by earth pressure and deck shortening counteract the thrusts due to dead and live load. It is therefore on the safe side to disregard earth pressure and deck shortening and to take the maximum shear as:-

$$3,800 + 13,850 = 17,650 \text{ lb.}$$

The corresponding unit shearing stress:-

$$\frac{17,650}{12 \times 7/8 \times 40} = \frac{17,650}{420} = 42 \text{ lb. per sq. in.}$$

Problem 9.

Stresses at Crown and Corner

The frame in Problem 1 is subject to dead load, live load and change in length of deck. A summary of these loads and the moments and thrusts they create is given in the figure on pages 13, 14, 16 and 20. Choose tensile and compressive reinforcement and ascertain that the corresponding unit stresses do not exceed the allowable working stresses, which will e chosen c

lbs./sq. in

Moment

Dead

Live

Chan

Ecce

The

less tha

A

2.00 s

still

pounds

axial

spaced

say

will

be chosen comparatively as Fc = 1,000 lbs./sq. in., and Fs = 18,000 lbs./sq. in.

Moments and thrusts at the midpoint of the deck are:-

	Moment	Axial Thrust
Dead Load	+ 73,800	+ 13,850
Live Load	+ 24,300	+ 2,902
Change in Deck Length	+ 12,780	- 540
	+ 110,880	+ 16,212

Eccentricity with respect to the centerline:- $\frac{110,880}{16,212} = 6.8$ ft.,

say $\underline{82"}$

The tensile steel area for this moment and thrust must be somewhat less than that required when the axial thrust is disregarded; namely:-

$$\frac{110,880 \times 12}{18,000 \times 7/8 \times 22} = 3.84 \text{ sq. in.}$$

A tensile reinforcement of 1 - in. square bars spaced 6 in. (A^1 = 2.00 sq. in.) will be chosen. With this reinforcement - and axial thrust still disregarded - the extreme fiber stress in concrete is less than 900 pounds per square inch. This stress will be raised by the addition of axial thrust, and compressive reinforcement equal to 1 in. square bars spaced 12 in. will be chosen.

The depth to the neutral axis in the concrete section equals:-

$$d\left(\sqrt{2\,p\,m + (pn)^2} - p\,n\right) =$$

d = 22", $A^1 = 2.00^{\square}$ in., n = 12, and E is infinity

$$22\left(\sqrt{2 \times .01 \times 12 + (.01 \times 12)^2} - .01 \times 12\right) = 22(.50 - .12) =$$

8.36 in.

Adding axial thrust will tend to increase the effective depth to, say 9 in. The section coefficients with the estimated value of z = 9 will be:-

Estimated Section Coefficients		Correction	Corrected Value
$A = 12 \times 9 + (12 - 1) \times 1.00 + 12 \times 2.00 =$	143	-13.44	129.6
$Q = \frac{1}{2} \times 12 \times 9^2 + 11 \times 2 + 24 \times 22$	$= 1{,}036$	-13.44×8.44	922.5
$I = 1/3 \times 12 \times 9^3 + 11 \times 2^2 + 24 \times 22^2$	$= 14{,}576$	-13.44×8.44^2	13,620

$E = 82 - .5 \times 24 = 70$

If Z has been correctly chosen it should satisfy the equation:-

$$Z = \frac{14{,}576 + 70 \times 1{,}036}{1{,}036 + 70 \times 143} = \frac{87{,}096}{11{,}046} = 7.88 \text{ in.}$$

Using the second value of Z = 7.88, correct A, Q, and I used above for the discrepancy in effective concrete area which equals

$$12 \times (9.00 - 7.88) = 13.44 \text{ sq. in.,}$$

the center of which is at a distance of:-

$$7.88 + \tfrac{1}{2} \times (1.12) = 8.44 \text{ in.}$$

below extreme concrete fiber.

Determine the final value of Z as:-

$$Z = \frac{13{,}620 + 70 \times 923}{923 + 70 \times 130} = \frac{78{,}230}{10{,}023} = 7.8 \text{ in.}$$

Now compute:-

$$g = \frac{923}{130} = 7.1, \quad c = 70 + 7.1 = 77$$

$$Ig = 13{,}620 - \frac{(923)^2}{130} = 13{,}620 - 6{,}550 = 7{,}070$$

and determine stresses:-

$$f_c = \frac{16{,}212 \times 77}{7{,}070} \times 7.8 = 1{,}375 \text{ ft. lb.}$$

f_c = 3010

Fro

it is ne

In the a

out the

From the previous problem worked problem(9) I have found out that it is necessary in that the Rigid Frame Bridge be equipped with ribbing. In the analysis of Rigid Frames that I have studied it does not carry out the application of ribbing.

BIBLIOGRAPHY

1. "Analysis and Tests of Rigidly Connected Reinforced Concrete Frames" M. Abe, Bulletin 107, Univ. of Ill. Eng. Exp. Sta., 1918.

2. "Structural Laboratory Investigations in Reinforced Concrete Made by Concrete Ship Section, Emergency Fleet Corporation", W. A. Slater, Proc. A. C. I., 1919, pp. 24-62.

3. "An Accurate Mechanical Solution of Statically Indeterminate Structures by Use of Paper Models and Special Gages", G. E. Beggs, Proc. A. C. I., 1922, pp. 58-82; Engineering and Contracting, May 24, 1922, pp. 497-501; American Architect, July 5, 1922, pp. 52-56; American Architect, July 19, 1922, pp. 69-74; Proc. A. C. I., 1923, pp. 55-66.

4. "Thrust of Skew Barrel Arch Measured on Laboratory Model", Clyde T. Morris, Engineering News-Record, Volume 88, 1922, pp. 638-40.

5. "Continuous Frame Design Used for Concrete Highway Bridges", A. G. Hayden, Engineering New-Record, January 11, 1923, pp. 73-75.

6. "Tests of Knees for Continuous Frame Concrete Bridges", A. G. Hayden, Engineering News-Record, January 18, 1923, pp. 108-110.

7. "Analysis of the Stresses in the Ring of a Concrete Skew Arch", J. C. Rathbun, Trans. A. S. C. E., 1924, pp. 611-680.

8. "Researches in Concrete", W. K. Hatt, Bulletin 24, Purdue University.

9. "Rigid Frames in Concrete Bridge Construction", A. G. Hayden, Engineering News-Record, April 29, 1926, pp. 686-689; Engineering News-Record, August 12, 1926, p. 275.

10.

11.

12.

13.

14.

15.

16.

17

18

19

10. "The Essentials of Rigid Frame Design", E. H. Harder, Concrete, October, 1927, pp. 13-15; Concrete, November, 1927, pp. 43-45; Concrete, December, 1927, pp. 37-42.

11. "Rigid-Frame Solid-Section Design Applied to Skew Bridges", A. G. Hayden, Engineering News-Record, December 1, 1927, pp. 867-869.

12. "Rigid Frame Bridge Construction by the Westchester County Park Commission", A. G. Hayden, The Municipal Eng. Jl., Sec. Quarterly Issue, 1928.

13. "Tests of the Effect of Brackets in Reinforced Concrete Rigid Frames", F. E. Richart, Bur. of Stds. Jl. of Research, August, 1928, pp. 189-253.

14. "Continuity as a Factor in Reinforced Concrete Design", Hardy Cross, Proc. A. C. I., 1929, pp. 669-711.

15. "Simplified Rigid Frame Design", Hardy Cross, Jl. A. C. I., December, 1929, pp. 170-183.

16. "Cost Economies in Concrete Bridges", C. B. McCullough, Proc. 10th Annual Meeting of the Highway Research Board, 1930, pp. 281-323.

17. "Design of a Reinforced Concrete Skew Arch, " B. L. Weiner, Proc. A. S. C. E., January, 1931, pp. 3-112.

18. "Rigid Frame Bridge of 101-ft. Span at San Antonio", J. W. Beretta, Engineering News-Record, June 25, 1931, pp. 1048-1049.

19. "Long Rigid Frame Bridge Erected by Cantilever Method", R. Schjodt, Engineering News-Record, August 6, 1931, pp. 208-209.

20. "The Rigid-Frame Bridge", A. G. Hayden, published by John Wiley & Sons, New York, 1931.

21.

22.

23.

24.

25.

26.

27.

28.

29.

21. "Analysis of Continuous Frames by Distributing Fixed End Moments", Hardy Cross, Trans. A. S. C. E., Vol. 96, pp. 1-156.

22. "Unique Three-Span Rigid Frame Bridge", J. W. Beretta, Civil Engineering May, 1932, pp. 309-311.

23. "Continuous Frames of Reinforced Concrete", Cross and Morgan, published by John Wiley & Sons, 1932.

24. "Rigid Frames and Continuous Concrete Spans", J. W. Beretta, Civil Engineering, September, 1932, pp. 557-562.

25. "Rigid Frame Bridges", Hardy Cross, Bulletin 353, Amer. Rwy. Eng. Assn., pp. 580-588.

26. "The Modified Slope-Deflection Equations", L. T. Evans, Proc. A. C. I., 1932, pp. 109-130.

27. "Design of Continuous Frames Having Variable Moment of Inertia", Thor Germundsson, Civil Engineering, October, 1932, pp. 647-648.

28. "Ten Years of Achievement with Rigid-Frame Bridges", Engineering News-Record, April 27, 1933, pp. 531-533.

29. "New Developments in Grade Separation Structures", Charles P. Disney, Jl. Western Society Engineers, April, 1934, pp. 17-25.

[redacted]