A FORMALIZATION OF A THEORY OF DIFFERENTIATION IN ORGANIZATIONS

Thesis for the Degree of M. A.

MICHIGAN STATE UNIVERSITY
WILLIAM EARL VREDEVOOGD

1972

ABSTRACT

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by

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Peter M. Blau's theory of differentiation in work organizations is translated into the language of functional notation. As a result the theory is rendered in more general terms and can be seen as having broader empirical applications. In addition, the formalization uncovers what appear to be serious inconsistencies in the logical development of the theory in that certain of the lower order premises will not derive, and certain others are redundant.

A FORMALIZATION OF A THEORY OF DIFFERENTIATION IN ORGANIZATIONS

BY

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A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

MASTER OF ARTS

Department of Sociology

1972

I wish to respectfully dedicate this paper to the memory of the late James Riddle Hundley, Jr., Associate Professor of Sociology, Michigan State University.

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INTRODUCTORY REMARKS

Formalization, that is, the translation of a theory into symbolic representation,[1] is gaining acceptance as a tool, useful for the construction and analysis of social theories. This paper is an attempt to render, in formal terms, a theory by Peter M. Blau which was developed to explain differentiation in work organizations.[2] As a by-product of the formalization, an analysis of the logical structure of the theory will also be developed.

1. I am accepting the definition of formalization put forward by Berger, Cohen, Snell and Zelditch in Types of Formalization in Small Group Research (Houghton Mifflin & Co.; Boston; 1962) pg. 3 "By 'formalization' we refer to the general process of making explicit the logical structure of a set of assertions. We include in this process the activity of translating a set of statements about empirical phenomena into a particular formal language. Formal languages include the better known quantitative systems such as the calculus and, in addition, non-quantitative systems such as symbolic logic."

2. Blau, Peter M.; "A Formal Theory of Differentiation in Organizations"; American Sociological Review; vol. 35, April 1970, pp.201-218; Because of the brevity of Blau's paper, further references to specific page numbers will be omitted from the discussion as unnecessary.

Blau's theory seemed ideally suited for translation into a formal, or symbolic, language. Because it was created by its author to be a wholly deductive logical unit that would answer to Braithwaite's strict definition of theory,[3] the preliminary ordering of the component premises was already done. The remaining task should have been merely to choose an appropriate language, in this case the language of functional notation, and translate the theory.

The usual purpose of formalizing a theory is to render its contents in more general terms to see if any new and unforeseen applications for the hypotheses can be discovered. Such was the original goal of this paper. In the process of translating Blau's work, however, some additional questions occurred concerning the logical structure and some light is thrown on the problems and drawbacks of reasoning in ordinary language as opposed to reasoning in formal terms.

THE FORMALIZATION

The physical structure of the original theory is quite straight forward. Blau's theory consists of two general propositions and a set of lower level propositions

3. Braithwaite, Richard B.; Scientific Explanation; (Cambridge University Press; New York;1952) pg 22

which are purported to follow logically from the generalizations. The first of these general statements asserts that: "increasing size generates structural differentiation in organizations along various dimensions at decelerating rates." It should be noted that the generalization meets the requirements of being both universal and declarative.

For the sake of clarity, the definitions of several key terms in the statement will be noted. First, the organizations which Blau's theory encompasses are "work organizations," that is, those which are "deliberately established for explicit purposes and composed of employees." It is a limiting definition which serves to exclude all other types of organizations from the discussion. Its use is legitimate, although it is not the usual meaning given to the term, organizations.

In any case, the limiting of the hypothesis to work organizations is not as central to the formalizing of the theory as the meanings given to two other terms, size and differentiation. In Blau's work, size is operationally defined to be the total number of employees in an organization. This sum is apparently meant to include all employees, both managerial and clerical as well as the lower echelon workers normally connoted by the word, employee. Structural differentiation, on the other hand, is designated to mean the total number of component departments and divisions within the organization, specified in terms of any one criterion. Whether the chosen criterion partitions the organization along vertical or horizontal lines appears to be of no consequence. The interest lies only in the total number of distinct divisions.

By substituting these definitions back into the original generalization, it can be rewritten as "the total number of divisions or parts in an organization, formally distinguished in terms of any one criterion, increases as the total number of employees in the whole organization increases, but at a declining rate." In this form, the statement is more open to translation into formal terms. Since the concern here is with "numbers of divisions" and "numbers of employees," the use of a mathematical language will not be inappropriate. Moreover, if one were describing the relationship expressed in the generalization, it might be said that the number of parts is a function of size. Together, these two characteristics suggest the actual language to be employed in this formalization, namely functional notation.

Obtaining a formal counterpart to the generalization is fairly straight forward. One merely replaces the defined entities with symbols and then finds a way to specify the relationships between them. In this case, the size of the organization, the number of employees, can be represented by X and the number of component parts or divisions can be designated as N. The relationship to be exhibited is that N increases as X increases and there are a number of different equations that, when graphed, produce the desired acclivous curves to reflect the correlation between X and N.

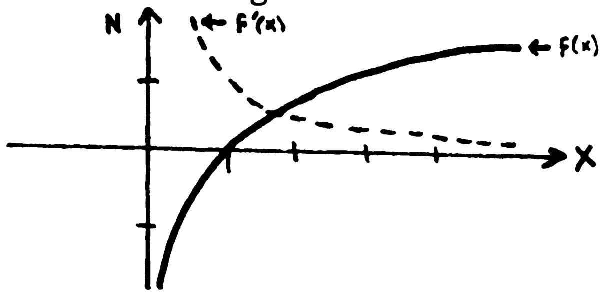
1) $N = aX + B$	(a linear function)
2) $N = X^a$	(an exponential function)
3) $N = \log X$	(a logrithmic function)
4) $N = \sin X$	(a periodic function)

However, the generalization also stipulates that the rate at which differentiation increases must decline and thus eliminates all but one of the curves from consideration.

In the calculus of functions, the rate of change is specified by the derivative of the function.[4] Since, of

4. Cedar,Jack G. and Outcalt,David L.; A Short Course in Calculus; (Worth Inc.; New York; 1968) chapter 6. For the most part, the mathematics involved in this paper will be trivial to the reader with a mathematical bent. For the uninitiated, this source provides an easily comprehensible check on the logic of the mathematical relationships. A second text employed by this author was Louis Leithold's, The Calculus with Analytic Geometry; (Harper and Row; New York; 1968) especially chapter 12.

the four equations noted above, only the logrithmic function has a declining derivative, it alone reflects the relationship expressed in the generalization.



The formalized version of Blau's initial statement then is $N = \log X$, or $N = a \log X$ where a is some positively valued constant. The latter expression will not be employed, however, since the issue central to this discussion will be the logical structure of the theory, and not the possible parameters of the relationships.

Consideration can now be given to the logical implications of the model and to the correctness of the individual derivations.[5] Based on the above generalization, Blau states a number of "derived" propositions which are purported to follow logically from it. The first of these reads as follows: "as the size of organizations increases, its marginal influence on differentiation decreases." In effect, what this says is that the change in the number of parts in

5. Since the words "derived, derivation and derivative" all occur in this discussion, care should be taken not to confuse their meanings. A logical derivation and a mathematical derivative are very different things.

an organization caused by a given increase in size is smaller for large groups than for small ones. In terms of the formalization, the difference in N caused by adding a given ΔX to X depends on the initial value of X. If X is small the corresponding change will be greater than if X is large. Although the proof will here be omitted, the most common measure of the effect that a given change in X will have on N is the derivative of the function. From the above graph of the derivative, $\frac{1}{X}$, it is apparent that the "marginal influence," or effect, of an increase in size gets smaller as size gets larger. The formalization has substantiated the proposition and consequently it can be said to follow logically from the higher level generalization.

The derivation of the second "lower level" proposition is not so straight forward. It states: "the larger an organization is, the larger the average size of its structural components of all kinds." This introduces a new term to the formalization, the size of the component part, which can be represented by the letter C. Since the whole is equal to the sum of its parts, the total size of the organization is the sum of all its components of any one kind. (The qualification "of one kind" is added merely to indicate that the divisions must be such that no double counting of employees takes place.) Representationally, then,...

$$C_1 + C_2 + C_3 + \ldots + C_N = X$$

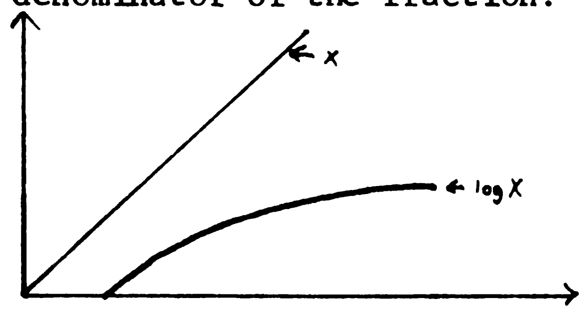
Now, the average size of the parts of a whole, that is the mathematical average, is found by dividing the total size by the number of parts, which is X/N. Thus, $\bar{C} = X/N$ and what the proposition contends is that $\bar{C}$ gets larger as X gets larger. For this to be true, one of the following conditions must exist: (a) N must be decreasing, (b) N must be holding constant, -- both of which are ruled out by the stipulation of the initial generalization that N is increasing as the log of X -- or (c) X must be increasing at a much faster rate than N. For values larger than X = 1, the logrithmic function does indeed indicate that X will increase at a much faster rate than the log of X, which is N. This being true, the proposition is deduceable from the generalization and is a correctly "derived" statement.

A word of caution goes with this apparent confirmation, however. Between the values of 0 and 1 on the X-axis, the log function rises almost vertically from the negative infinity. In this section of the graph the proposition would not hold true since N would be increasing by infinity while X was increasing by only 1. At first glance this may not appear to be an issue, for an organization with less than one employee does not carry any empirical meaning. Let it be remembered, however, that the parameters of the relationship, omitted from the discussion for the sake of convenience, have yet to be established and that therefore the placement of the graph within the quadrant has not been fixed. In the end, the vertical tail of the curve may find empirical relevance and the proposition would then be true only for organizations with sufficient size to be placed on the appropriate part of the graph. There is nothing in the generalization that says this will happen, but likewise there is nothing that says it cannot. The evaluation of the logical necessity of the statement must be qualified.

Along with this, a point that went unnoticed in the verbal presentation is that the rate at which the average size increases is itself growing along with size. Humman also takes notice of this fact in his discussion of Blau's work,[6] labelling the discovery as a new proposition. Blau has replied to this criticism in an unconvincing way, arguing that this cannot be since the average size of the

6. Humman, Norman P.; "A Mathematical Theory of Differentiation in Organizations" ASR 1971 vol 36 (April) pg. 300

parts must increase at a rate which is less than the rate of the whole.[7] In the formalization employed in this paper, which is somewhat less abstract than Humman's, it is clear that this condition of Blau's is not violated by the contention of increasing rate. The fraction is X/N, which is $X/\log X$, the derivative of which is $\log X - 1/\log X^2 > 0$. Since the derivative is positive the function is increasing. The rate of increase, is found in the second derivative, $[(\log X)^2/X - 2(\log X)^2 - 2(\log X)] / (\log X)^4$, which is also positive, indicating an increasing rate. The following graph demonstrates more simply the relationship between the numerator and denominator of the fraction.



As the denominator is increasing at a declining rate while the numerator is growing at a constant rate the distance between them grows at an increasing rate. The value of the fraction, therefore, is increasing at a slightly increasing rate.

7. Blau, Peter M., "Comments on Two Mathematical Formulations of the Theory of Differentiation in Organizations" ASR 1971, Vol 36, pg. 305

Further discussion of the second derived proposition will be bypassed, and consideration will now be given to the third. The third "lower level" theorem asserts that: "the proportionate size of the average structural component, as distinguished from its absolute size, decreases with increases in organizational size." Since the size of the average component has already been shown to be X/N, it only remains to find a comparable representation for the "proportionate size." Most commonly in mathematics the "proportion" is taken to be the ratio of the whole to the part, that is, the percentage of the total made up by any given part. This can be found by dividing the part by the whole.

$$\frac{\bar{C}}{X} = \frac{X/N}{X} = \frac{X}{N} + \frac{X}{1} = \frac{X}{N} \cdot \frac{1}{X} = \frac{1}{N}$$

Since the initial generalization stipulates that N must increase in correspondance to an increase in X, then the inverse, $\frac{1}{N}$, must decrease. The proportionate size of the average component must, therefore, decrease with an increase in organizational size. The formalized version shows the proposition to be a logical consequence of the generalization. To this can be added the observation that the rate of decrease itself declines with size, a point omitted in the original formulation.

Blau's fourth derived theorem posits that, "the larger the organization is, the wider the supervisory span of control." This introduces a new expression, "supervisory span of control," which is defined to be, in operational terms, the number of subordinates per manager. The path of reasoning leading to this hypothesis is circuitous. On the basis of the third proposition, Blau argues that "if...the proportionate size of any[8] organizational component declines with increasing size, and if this applies to the proportion of managers, it follows that the number of subordinates per manager...must expand with size." It appears to be faulty reasoning.

The terminology employed in the propositions has apparently caused some confusion. The third derived theorem, the one dealing with proportionate size, does not refer to just any component, as is here implied, but rather it has to do with the proportionate size of the average component, $\overline{C}$. Now, the average component doesn't actually exist. It's a mathematical construct whose size is dictated by the total size of the organization, X, and the number of parts or divisions, N. There is no possible way to predict the size of any given part, C_k, simply by knowing the average size

8. emphasis mine

of the various parts. Nor is there anything in the initial generalization or in the other two propositions from which the relationship can be deduced. If the fourth theorem is true, and there are reasons for believing it is not, it certainly cannot be construed to be a "derived" proposition, since it is not a logical consequence of anything that has preceded it. This, in itself, is enough to deny Blau's formulation the distinction of being called a theory. By the definition he himself advocates, such an entity must be a logically deductive system.

Not only is this fourth proposition not logically derived, but there are two very good reasons for also doubting its accuracy. First, one needs to question whether the theorem is internally consistent in its rationale. If the main contention,--which holds that the proportionate size of any part decreases as the size of the whole increases,--can be seen as applying to the management component, can it not be legitimately argued that it should apply to the non-management component as well? Blau's treatment of the premise suggest such a dichotomy of parts by assuming that a decrease in the proportionate size of the management component will result in a corresponding increase in the non-management element. This would be true because their sum must always equal 100%. Since a decrease in both of these parts is a mathematical impossibility, and since the hypothesis cannot apply to the managers without also applying to the non-managers, the argument is self contradictory.

The second, and perhaps the more important, reason for questioning the accuracy of the hypothesis is that it is a simple matter to construct an example in which the behavior runs counter to what is predicted by postulate four and yet agrees with everything that precedes it. Let's start with an organization of some given size, say X = 190. Increasing that size by a fixed amount, say $\Delta X = 50$, should decrease the proportionate size of the management component; if the theorem is correct. Suppose such an organization exists and is divided according to the levels in the management hierarchy. To agree with the formalized generalization there should be 5 such levels or parts whose sizes can be set as follows:

levels	C_1	C_2	C_3	C_4	C_6
workers	1	4	10	15	160
division	Managers				Non-managers

Within this organization, C_1 is the highest level in the hierarchy and C_5 is the lowest or non-management component. The sizes of the various parts are arbitrary, since nothing in the theory to this point has predicted or in any way concretely established their size. However, they were chosen to be the kind of distribution one might easily be able to find in real organizations.

Now, if the size of the whole organization is increased by 50 the generalization, predicts a corresponding increase in the number of parts. In this case, a new level must be added to the organizations hierarchy. The expanded distribution might then be:

C_1	C_2	C_{3a}	C_{3b}	C_4	C_5
1	4	5	10	20	200

In this new organization the third level of management has subdivided into two parts, creating the 6th component to comply with the generalization. The average size of the parts, which was 40 before the increase, is now 41. This is consistent with proposition 2. The proportionate size of the average component has decreased from 20% to 16.5%. which is the change predicted by proposition 3. The proportionate size of the management component, however, has not gone down, as was predicted by the fourth theorem, but rather it has remained at 20%. This serves to abrogate the universality of the hypothesis. The example constructed ahered to the relationships in the earlier propositions, and yet failed to exhibit the behavior predicted by theorem 4. What is called into question by this is not whether the relationship expressed by the theorem can be observed to exist, but rather the question is whether it can be logically derived, and so the theory is no longer a deductive system.

Both of these questions concerning the fourth proposition may be due, in part, to an inadequate definition of what constitutes the management component. One thing is clear, however, from what Blau has provided to this point, the theorem will not derive.

In as much as theorem 4 is not the last proposition to be evaluated, the final judgment of its merits and demerits will be put aside until later. Instead, a way must be devised to incorporate its content so that the logical derivations of the subsequent propositions can be considered. This will require some rather arbitrary maneuvering.

Because proposition 4 cannot be logically derived from the original generalization or from the subsequent propositions, the question to be answered is, from where did it come? It was noted that in moving from theorem 3 to theorem 4 Blau has made the assumption that any given component will have the same quality of decreasing proportionate size that the average component exhibits. Since this has already been shown to be a mathematically unwarranted assumption, it might better be regarded as a stipulation. The exact wording is unimportant, it could even take the form of another generalization, so long as it states in some manner that the managerial component will behave the same as the average of the components as size increases.

This brings up a side issue. It will be recalled, although it was not mentioned at the time, that there were no logical antecedents to the original generalization. In fact, no explanation is ever given for why that original relationship between size and differentiation should exist; that is, no explanation other than its being an observed phenomenon in the data is ever given. This approach is adumbrative of a glorified curve fitting that is wholly unsuited to strong logical development. Its recurrence at this point,--the decrease in the proportionate size of the management component is also observable in the data--gives emphasis to the weakness of the approach. A generalization is supposed to be the logical rock on which a theory can be built. It is not supposed to be a title you apply to underivable statements.

If the new generalization can be limited to apply to the management component, rather than to any and all components, then a rational argument might be made for its existence. Each division in an organization will have one or more individuals who function as managers, so therefore the actual number of supervisory personnel will depend importantly on the number of divisions and on the size of those divisions. The only indicator of the size of the parts is their mathematical average, which is $\bar{C}$. The size of the management component can then be formalized as a proportion of the average component, $a\bar{C}$ where $\underline{a}$ is less than 1 and greater than 0. This is essentially the same correction that Hummon was forced to make in developing his model of Blau's theory.[10] It is not clear, however, that Hummon was aware of the assumptive nature of the $a\bar{C}$ correction.

If, indeed, these assumptions can be made, then it is possible to derive the fifth of Blau's propositions. It states: "organizations exhibit an economy of scale in management." Although admittedly cryptic, this is intended

10. Hummon...pg 299

to mean that organizations will undergo a decline in the proportionate size of the managerial component. This is virtually the same thing that theorem 4 said and so the derivation is trivial. However, the formal version of the deduction is interesting. As before, the proportionate size of a part is given by the ratio of the part to the whole, C_m/X in this case. By substituting for C_m, the management component, one gets...

$$C_m/X = a\bar{C}/X = (a)\ \frac{X}{N}/X = \frac{aX}{N} \cdot \frac{1}{X} = a/N$$

Since the denominator of the fraction, a/N, will increase at a declining rate as X grows large, while the numerator is a fixed constant, the value of the whole fraction must decrease at a declining rate.

The wording is poor, but it becomes clearer in conjunction with the sixth derived proposition which says that "the (increase in the) economy of scale in administrative overhead itself declines with increasing organizational size." Here what Blau seems to be implying by "economy of scale" is the rate at which the proportionate size of the management part is decreasing. In the symbolic formulation, the rate of change would be the mathematical derivative of the function under consideration. In this case it is the derivative of a/N.

$$d(a/N)/dx = a\ d\ (1/N)/dx - a\ d(N^{-1})/dx = -a/N^2$$

Being negative, this fraction increases toward zero at a declining rate. Hence, the "economy of scale" increases at a rate comensurate with the rate at which the proportionate size of the management component is decreasing. When that rate declines, as it does when X gets large, then the economy of scale declines also.

The reader should keep in mind that these derivations would not be possible without the assumption that the management component behaves the same as the average of the parts, that is, that $C_m = a\bar{C}$. It cannot be emphasized too strongly that this is a completely arbitrary assumption. Its adoption is solely as a convenience device so that the logic of the latter deductions can be examined. While the propositions may appear to derive, it must not be forgotten that they make their foundation in a logical phantom and so, in reality, are without substance themselves. A possible approach might be to mentally preface each derivation with the phrase, "if indeed it could be shown that $C_m = a\bar{C}$, then ..."

If indeed it could be shown that $C_m = a\bar{C}$, then even proposition 4 would be deduceable. Recall that the theorem referred to the number of employees per manager. This can be found by dividing the former by the latter. Although the absolute size of either of these components is not known, together they make up the whole of the organization, being, one must assume, mutually exclusive and exhaustive categories. The number of non-management employees, represented by C_e, can be found by subtracting the sum of the managers from the whole: $X-C_m=C_e$. Substituting gives...

$$C_e/C_m = \frac{X-Cm}{Cm} = \frac{X-a(X/N)}{a\ (X/N)} = \frac{NX-aX}{N} \cdot \frac{N}{aX} = \frac{X(N-a)}{Xa} = \frac{N-a}{a}$$

In this case, it is the numerator of the fraction that is growing along with N so the value of the fraction will be increasing at a declining rate. The proposition would be a direct and logical consequence of the arbitrary assumption.

Time has been spent on this fourth proposition for two reasons: namely, it is the first of Blau's derived propositions that flatly cannot be derived; and secondly it is the central proposition by which the theory lives or dies. Meyer, in his comments on Blau's work,[11] gives evidence of the importance of the proposition by making it the "given" hypothesis or generalization and then showing that the propositions and statements which precede it, can actually be derived from it. In fact, Meyer goes so far as to say

11. Meyer, Marshall W.; "Some Constraints in Analyzing Data on Organizational Structures: A Comment on Blau's Paper" ASR Vol 36, April '71, pg. 295.

that the other theorems are so interrelated as to be indistinguishable-that they are merely parts of a whole. In creating his reformulation of Blau, Meyer assumes two things about organizations, unity of command and symmetry, for which he is criticized by Blau for working with a "special case."[12] Yet, the point to be emphasized is that Meyer recognized Blau's fourth theorem for what it was, the central theorem of the theory and one that could not be derived from previous propositions. In fact, in opposition to Blau, the belief that the fourth proposition implies the others, is more logically correct. It is because of the importance of the role of this theorem that the necessary assumptions were made to allow its derivation. Without it the latter parts of the theory had no logical base, at all.

Returning to the order of statements as they occur in Blau's paper, attention can now be turned to the postulate labeled as the "second generalization." It posits that "structural differentiation in organizations enlarges the administrative component." That is to say, an increase in the number of parts causes an increase in the number of managers. Here, reasoning in ordinary language has caused

12. Blau; "Comments..." ASR V. 36 p. 306. Blau argues, and rightly. that Meyer is off base when discussing the "interrelatedness" of the propositions. The propositions can, and must be, interrelated without being tautological.

the derivability of this statement to be overlooked. The relationship expressed is easily demonstrable on the basis of what has gone before. Mathematically, if $N = \log X$ then it is also true that $X = e^N$. What this implies is that if an increase occurs in the number of parts, there must be a coinciding increase in the size of the whole. It has already been established that $\bar{C} = X/N$, so any increase in size also increases the size of the average component. Now, again employing the infamous assumption, if $C_m = a\bar{C}$ then an increase in the number of parts must result in an increase in the number of managers. The statement is no higher level generalization, but rather the formalization shows it to be derivable from previous statements.

Following this generalization, there are three more "derived propositions." The first of these says that "the large size of an organization indirectly raises the ratio of administrative personnel through the structural differentiation it generates." The question here is what does Blau mean by "indirectly raises?" Both Hummon[13] and Meyer[14] have provided conceptualizations of this relationship which call into play "other factors" that effect differentiation. The

13. Hummon 301
14. Meyer 296

indirect effect, then, would be the effect of size on these other factors. This Blau acknowledges as precisely what he had intended.[15] But even so, the statement is merely meant to explain that something is acting upon the organization to make the rate, at which the proportionate size of the management component is decreasing, itself decline. In so much as this declining rate has been logically established through the development of proposition 5, the necessity of this new theorem is called into question. Its derivability is not open to question, since that has already been established earlier, but merely whether this is a legitimate proposition or a new wording of theorem 5.

Under Blau's "second generalization" the next derived proposition states that "the direct effects of large organizational size lowering the administrative ratio exceed its indirect effects raising it owing to the stuctural differentiation it generates." The wording of this particular theorem, with the references to direct and indirect effects, is very imprecise and confusing. Moreover, when the implications of the statement are unravelled, it contains little other than a description of the behavior of the

15. Blau "Comments" 304

proportionate size of the managerial component. It simply says that the rate at which the proportionate size is decreasing is itself declining. In as much as this has already been demonstrated in the formalized version of the fifth proposition, above a restatement is unnecessary. A new derivation cannot be claimed through the paraphrasing of an earlier proposition.

The third derived theorem, and the last in Blau's theory, says that the "differentiation of large organizations into subunits stems the decline in the economy of scale in management with increasing size." This is in direct conflict with proposition 6, above. There, it was established that the economy of scale does not decrease; it increases. Again the wording of the statement is at fault. What Blau might have intended in place of "economy of scale" is the proportionate size of the management component, which does indeed decrease with an increase in size. This being the case, the theorem is merely a restatement of theorem 6 which says that the rate of decrease is itself declining. Alternatively the word "increase" could be substituted for "decline" in the proposition so that it read "the differentiation of large organizations into subunits stems the <u>increase</u> in the economy of scale..." This too would be concurrent with the earlier propositions. In either case, the original wording of the statement is unacceptable.

A SUMMARY WITH ADDITIONAL OBSERVATIONS

Consider the changes caused by the formalized treatment of rates of change as the mathematical derivatives of functions. In the first place, the formalization provided a convenient way of dealing with rates, a way of "seeing" changes in rates through the graphs of the derivatives. Secondly, it provided a uniform approach to rates of change free of linguistic ambiguities. In a number of instances--namely, in the propositions dealing with "economy of scale" in management and with the "indirect raises" in the percentage of managers--Blau was actually referring to rates of change and was apparently unaware of it. This was easily recognizable in the formal version, however, and served to clarify the meanings of the terms.

A second instance of the weakness of reasoning in ordinary language can be found in the manifest changes in meaning that the "average of the components" undergoes in connection with the fourth proposition. The term begins as the "average size" of the components, moves quickly into being just the "average component" and finally ends up being interpreted as "any component." This specialized use of the word, average, might have some basis in colloquial contexts--("Now you take your ordinary average student...")--but its presence in a scientific theory means trouble. On the other hand, in the formal interpretation the term begins as the mathematical average, X/N, and it remains exactly that even though in the end the proposition will not derive because of it.

The power of formal representations to furnish exact and vivid definitions of terms is important to the construction of good theory. Of equal importance is the ability to delineate clearly the essential character of the expressed relationships. This is an especially strong quality of mathematical languages because the logical basis of relationships in mathematics has already been elaborately worked out. When a given relationship, such as that expressed in the first generalization above, is translated into a formal mathematical language, N=logX, there are a number of benefits to be gained. In the first place, it is obviously more concise than the verbal expression. Secondly, a well established set of rules is provided governing what kinds of operations can be performed on the expression. One knows, for example, that both sides of the expression can be operated on by _e_, the base of the natural logrithms, to produce $e^N = X$. On the other hand, one also knows what operations cannot be performed. Because of this, it is possible to work out the logical consequences of a relationship or, as above, to check on the accuracy of derivations made in other ways. Finally, a symbolic representation simplifies, as well as clarifies, a relationship. Working with such a representation becomes a simple matter of manipulating the symbols. It therefore does away with the confusion of dealing with a bulky verbal description.

The second benefit of formalizing which follows from clearer definitions is that it contributed to the development of a better logical structure. It allowed us to find errors in Blau's logic that might otherwise go unnoticed. The formal translation uncovered quite a few such errors. It was found that proposition 3 held true only under special conditions, that proposition 4 could not be derived, from existing propositions, that if assumptions were made that would allow theorem 4 to stand then the second generalization was also deduceable and not a generalization at all, and finally that the last three propositions either contradicted what had gone before or were irrelevant descriptions of earlier relationships. Blau's theory has fallen short of his goal, which was to produce a strictly deductive explanation of the pheonemon. Moreover, the reader will recall that each of these errors was readily discernable in the formal translation of the theory, and hence could have been avoided.

The third advantage of using a formal approach is also the original purpose for doing a formalization. It allowed us to create a more general theory than Blau's and uncover new and unforeseen applications for it. In his original formulations Blau was concerned only with the changes in differentiation caused by increasing size in an organization. The symbolic interpretation, however, can serve to predict the effects of changes in either direction. From the graph of the equation, $N = \log X$, it is clear that the relationship holds true for decreases in X as well as for increases. The concern of the theory should more properly be directed at effect upon differentiation of any changes in size, rather than only the effects of increases. Moreover, the formalization also indicated that if the first equation is true then it is also true that $X = e^N$, that is, that size can be predicted by knowing the number of parts. This would seem to indicate that size and differentiation are mutually dependent upon each other. If this were the case the theory would apply to a wider set of events as it would then encompass those organizations and situations where an increase in the number of parts has led to an increase in size.

The dynamic nature of the relationships as they are expressed in this formalization, raises one last question concerning the original formulation. Although, until now, no mention has been made of it, the supporting evidence reported by Blau seems ill suited to his theory. Upon examination it can be noted that all of the propositions and generalizations in the theory carry an implicit notion of time, that is, they are processually articulated. They refer, without exception, to changes which occur as the size of an organization increases over time.[16] The empirical evidence, however, does not report the effects or differences caused as the size of an organization increases. Instead, it reflects the observed differences between large organizations and small organizations, and the assumption must be made that these differences represent some kind of an on-going process through which small organizations must pass on their way to becoming large. The necessity for making

16. Hummon...p. 302 Hummon also noted that, in mathematical form, the theory is ideally suited to a dynamic interpretation-and, in fact, develops just such a model with size and differentiation as functions of time.

such an assumption could have been avoided. Had the functional notation been employed, the interest of the theory would have been focused on the effects of changes in size (not just increases in size) in an organization. Since the graph of the relationship between differentiation and size took the form of a continuous function, it would have been clear that the data should also be continuous. This being so, a time study of organizations could have been made to measure changes, in either direction, of the size and related variables. Such data would have been a more accurate measure of the relationships expressed in the theory.

There is one other bothersome aspect to Blau's approach to this theory and it has nothing to do, at least directly, with the formalization.

From reading Blau's paper one gets the impression that the data reported as evidence was in existence prior to the theory. This can be inferred from the lack of logical antecedents to the first generalization, from the order in which the arguments occur, from the emphasis on the management component to the exclusion of others, and from the nature of the data itself which does not fit a time oriented theory. It is as if Blau had tried to create a theory to explain the data at hand, and then used that same data to test the theory's explanatory powers.[17] Perhaps this is due to the pervading influence of the inductive method that has held dominance in social research for so long.[18] Whatever the reason, it is an approach that greatly increases the chances of making a logical error. It is a very difficult, and seldom accomplished, task to make deductive arguments out of the "creative leaps" in thought so central to the inductive approach. It is doubly difficult when the evidence employed to test the deduction is the very data from which it was inductively spawned. Moreover, this leads to a situation where theory is being guided by research rather than the other way around; it carries the mark of a kind of cloaked empiricism.

FINAL REFLECTIONS

While the possibility may exist, in practice it is highly improbable that a scientific theory can be achieved through reasoning in non-formal languages. This is true because the inherent ambiguity and imprecision of the

17. Blau, "Comments" p. 304. Blau alludes to this approach in his opening comments on Hummons work, giving the suggestion that this is indeed the method he employed.

18. This method has recently been under increasing attack by the scientific philosophers. See Popper...pg. 46

English language are themselves the source of most errors in reasoning. As was shown by the formalization of Blau's theory, many errors may exist which can only be found by employing a symbolic representation. Formalizations, then, are an invaluable tool in the construction of scientific social theories.

On the other hand, the development of such scientific, by which is meant deductive, theories is imperative if progress is to continue in the field of sociology. The field has long been critized, from within and without, for its lack of a cumulative body of knowledge. Yet, the nature of deductive structure is such that it holds the promise of producing theories in an integrateable form. One can only speculate at what the state of the field might be today if all those who heeded Merton's call for theories of the middle range had known the value of deductive structures and been able to employ formal languages. But it is not speculation to say that the sum total of sociological knowledge need no longer be represented by an accumulation of uncorrelated facts. The development of scientific social theories can at last bring an end to that period in the field's development. And formal languages can play an important role in speeding the process along.

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