MATHEMATICAL SYSTEMS MODELING IN URBAN-REGIONAL PLANNING AS APPLIED TO A NATURAL WATER SYSTEM

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ABSTRACT

MATHEMATICAL SYSTEMS MODELING
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By

David L. Walker

Decision making is a product of logical and intuitive mental processes. It is judgment based upon systematic procedures starting from analyses of information about behavioral phenomena and ending with value considerations of appropriate alternatives. The quality of planning decision making is not just dependent upon basic data but also upon the preciseness and rationality of the techniques of analysis.

Planning is a maturing discipline and profession dependent upon quality decision making. It is searching for better analytical methodologies and theories for meaningful problem solving and guidance. The systems theory utilized in this thesis is emerging as a technique for fruitful analysis of socio-economic phenomena. This thesis is intended to provide a building-block in this approach to urban planning decision making.

The particular modeling theory applied here has developed in the engineering analysis of physical systems.

It is a technique for identifying and conceptualizing observable system phenomena and expressing their relationships mathematically. System components are expressed in terms of across (x) and through (y) variables. Component interconnections are related in a precise, systematic process by linear graph theory.

System theory is applied here to the natural water system (not the total hydrologic cycle) for a ten township area which includes Lansing, Michigan. Components of the system are conceptualized as ground-water elevation, surface water elevation, rivers and lakes elevation, rivers acquifer recharge, runoff to rivers, precipitation, groundwater inflow to the region, human water pumpage, infiltration and evapo-transpiration. The result is a dynamic, discrete state model structured from difference and algebraic equations relating the stated across and through variables. Numerical solutions relating behavioral phenomena may be readily obtained on digital computers.

Although the results (numerical) obtained from this initial modeling endeavor are not highly refined, they do indicate general trends and features which provide a guide to further research in developing a more operational model.

This type of model is both a predictive and simulative device, which provides a guide to data collection and measurement. In addition, it yields a methodology for relating exogenous elements, such as policy and value considerations, to planning problem solving. These features are highly desirable for guiding creative and design capabilities in the planner's search for more adequate decisions.

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David L. Walker

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INTRODUCTION

The primary emphasis of this thesis is to demonstrate how mathematical systems modeling can facilitate meaningful and consistent decision making in urban planning. There are various theories and techniques of both systems analysis and mathematical modeling, according to the demands of various fields of study. One of these has been selected to be explored in detail. This approach has emanated from the physical sciences, namely engineering, and is emerging as a means of identifying, mathematically describing, and analyzing physical and socio-economical phenomena as systems. The system components are characterized in terms of across and through variables and are interrelated by linear graph theory. The capacities of the selected systems analysis process are foreseen as a significant aid and advancement to contemporary planning.

A natural urban-regional water system has been selected as a model to illustrate the application of this methodology to planning problems. Water system phenomena (a physical system) lends itself to detailed quantification. There also are data available representing such features. These characteristics considerably lessen the task of mathematical systems modeling.

The organization of the thesis is as follows: Chapter I is a brief description of contemporary trends in methodologies and techniques being employed in the discipline of urban planning. It also includes a review of various models utilized in planning studies, their respective advantages and limitations, and their current uses.

Chapter II is a description of the selected modeling theory to be employed in this thesis. Most of the concepts have been developed for analytical study of physical phenomena, primarily from network theory and systems analysis utilized in electrical engineering.

Chapter III utilizes the theory presented in the previous chapter to structure a system model of water elevations and flows for a ten township region surrounding and including Lansing, Michigan. Data (partial) for making the water model operational are already available. Limitations and advantages of this particular modeling endeavor are also discussed, along with features for refining the model.

Chapter IV includes a summary of the methodology utilized and the results obtained in the course of the model development. A brief description of other possible applications for the selected modeling technique in planning is presented.

In the development of this thesis, mathematical systems modeling is used for analyzing a particular system, a natural water system for an urbanizing region. However, it is not limited to such detail and should be viewed broadly as a discipline for guiding planners and future planning activities in a systematic and coordinated process of data collection, analysis, synthesis, control and implementation. This is only a building-block to further application of this technique in public planning.

CHAPTER I

TRENDS IN CONTEMPORARY URBAN PLANNING METHODOLOGIES

Planning is a maturing profession that is striving to become established as a sophisticated institution for effectively solving urban problems through the selective process of designing and controlling urban phenomena. The evolution of this profession is similar to that of engineering and other professions. Initial efforts were largely intuitive. Next came the era of refined practice, based on empirical methods and the evolution of principles and standards. Finally, more complex analytical methodologies of applied sciences as devices for problem solving were established.

Planning is in the midst of an exciting change which is exemplified here by an abbreviated quote (Hirsch, 1964):

> "The urban government administrator may soon receive much greater help from science, which can provide him with informational and rational bases from which to strive for excellence. Urban interaction analysis, end-product-oriented program budgeting, effective analysis, and computerized information flow systems for planning and operating purposes hold promise for better decisions...."

Planning is at the threshold of becoming an applied social science. Many efforts now are oriented to the application of scientific methodologies to solving social and economic problems. This may be a long and arduous task, for the complexities of social phenomena are far more involved than those normally encountered in the physical sciences, mature disciplines of long standing. However, this condition can probably be surmounted, for man has progressed through the ages via his ability and intellect to increase the usefulness of his artifacts and his social institutions for a more desirable way of life.

Basic to planning and achieving these ends is accurate decision making. Because of the scope of this thesis, only a limited number of the most salient concepts contributing to these ends will be discussed. Recently, much has been written that points up the necessity for establishing the values, goals and objectives that form the restraints within which planners' decisions must be made. Also, policy planning is emerging as a process for stabilizing decision making within these value restraints.

General systems analysis (theory) of various forms is another discipline that is lending support to the establishment of planning as a scientific profession. This technique of analysis and others, such as cost-benefit analysis, tend toward quantification and mathematical representation. To carry through these techniques, it is necessary for the analyst to fully understand the system under study (total internal and external function) and to model it accurately. This demands great reflection, which in itself is an educational process basic to planning activities.

Mathematical modeling has been enhanced in recent years, as it has become readily structured to digital computers having advanced data storage and retrieval capabilities (Hertz, 1964). Advancements in modeling may be corollary to the advancement in computer utility. The latter has been a definite corollary to advancement in planning techniques. For now, it is possible to collect, store and analyze enormously large volumes of data in short, effective times that only a few years ago would have been economically not feasible to do.

Computers have advanced the usefulness of mathematical modeling and simulation in practical application. Models and strategies can be formulated that characterize urban environmental systems within particular constraints, as established empirically or theoretically. With the combined utilization of models and computers, various conditions, parameters and criteria may be simulated, and behavior immediately observed and evaluated. In this manner, real world conditions can be simulated without actually waiting to witness it develop or without structuring a total model environment, which is economically unfeasible. Coordinated decisions should emerge, for future developments can be predicted prior to final decision making, and corrective and control measures may be implemented where most effective.

Modeling is also useful as a guide to data collection. This can be of great application to urban planning, for presently many data collection endeavors are arbitrary and misguided. There exist today enormous volumes of data that were collected without adequate thought as to its utility and availability.

Desirably, only specified data of the proper form needs to be collected. The designing of models prior to data collection would streamline this process both functionally and economically and would provide the planner with more time to do constructive planning. The modeling process provides this framework for once a system has been formulated as a prototype, only pertinent data needed to make it function has to be collected.

Review of Mathematical Models

Planning is basically oriented to the future, particularly to the urban community's desired functional and spatial relationships. Planners are increasingly influencing urban development through advancement in guidance, controls, value considerations, and application of scientific techniques. They are striving for better plans through better analytical techniques. Advancement in modeling is a contributing factor to this end, for various alternatives may be readily evaluated in terms of desired policies and public actions. And action decisions may be implemented where impetus creates optimization.

Although public planning is very complex, owing to the interactions of numerous public authorities, the scale of application and private enterprise (an uncontrolable variable), urban functions are becoming better understood. Advancing analytical techniques, however, require the planning researcher to reflect thoroughly into urban phenomena for relating both exogenous and endogenous urban system relationships. It is an educational process for sophisticating and augmenting procedures for better urban understanding (Harris, 1965). Modeling is also a structure for operationally relating urban phenomena and for simulating (imitating) phenomena for analytical and predictive purposes. Some modeling techniques can accommodate comprehensive and systematic applications, as others can only be utilized in lesser domains.

Following is a discussion of models which includes a selected classification, scope of application, advantages, limitations and areas of current utilization. This is only a limited review of salient features for placing the systems modeling technique used here in its proper operational perspective.

Models Categorized

There are various categories in which models may be grouped. For instance, Lowry categorizes models as descriptive, predictive, and planning (Lowry, 1965). Crane likewise groups them broadly as logic and trend type models (Crane, 1964). For purposes of this discussion, Crane's broad grouping will be adequate.

Trend Models

Trend models are historically-based models, utilized extensively in planning because of the ease of formulation, reliability, and the fact that they are readily made operational on the basis of types of data currently collected and available. These are descriptive models that replicate existing phenomena in aggregate form, not as individual, behavioral components. These models are formulated upon the "how" phenomena develops, which implies an inevitable characteristic. Interrelationships among components are not explicitly expressed, but are absorbed in the aggregate data accounting. This is a serious limitation in distributing land uses at an adequate scale for effective comprehensive urban analysis.

Basic "through" or "across" variables (flows or stocks) individually are results of these models, such as population and/or commercial base projections. System interactions are not easily or explicitly considered, therefore, comprehensive and systematic analysis and simulation is not feasible. Growth model applications are developed upon a number of component submodels with little consideration made of the model interrelationships.

The basic operational assumptions have to be made too rigid for effective planning application and exogenous influences, such as policy decisions, can not be effectively simulated. These negative remarks are not to say that trend modeling is not useful for they do provide researchers with the best solutions at this time. It is a building block to more sophisticated and representative prototypes in the future.

Logic Models

Logic models are behavioral models that are structured of system components and respective component interactions as individual, identifiable elements. Resultant outputs are dependent on casual relationships which are indicative of real world phenomena and desirable for meaningful simulation. These can be very complex and highly sophisticated, but provides a great deal of promise for future analytical endeavors. A high level of knowledge of system assemblage is necessary at this stage to explicitly and quantifiably describe each system. It is apparent that this demands adequately qualified personnel and highly specifical data to properly construct these models. Presently, results are risky, owing to the necessity of making assumptions about adequate parameters, data, variables and variable domains, when appropriate data describing such features are not available.

These models can have great utility in planning, for cause and effect (dynamic results) relationships may be incorporated for a comprehensively-scaled urban system. Optimization and simulation features are readily accommodated, especially in particular logic models. Both exogenous (includes value considerations) and endogenous variables may be utilized in providing for real world relationships. This is a desirable feature in most planning endeavors.

It should also be pointed out that parameters relating variables, variable domains of influence, and conceptualization of components and component relationships are difficult to establish. Also, it is necessary to manipulate massive amounts of data in most endeavors to identify the proper data for analytical purposes. This can be most effectively accomplished through various statistical means (factor analysis, correlation-regression, etc.), for many times a system phenomena may be almost totally defined by a few pertinent characteristics. This reduces data manipulation to workable scales. These are some of the reasons that make logic modeling risky at the present time.

Lowry's planning model types are similar to this group. However, these are not widely utilized in planning, as yet, owing to difficulty of quantifying socio-economic phenomena. Although logic models have been widely utilized in the social sciences having only analytical solutions, the focus here is on precise formulations having numerical solutions.

Trend and Logic Modeling Summary Remarks

Presently trend models are the most operational models. They are simple constructs that may be formulated and manipulated by not highly trained personnel. Recent refinements have created a great deal of confidence in their usage, they are no longer great economic risks. Through widespread use, desired results are now being obtained in universal applications, i.e., land use allocations, population forecasts, etc.

However, trend models do have limitations. Large amounts of data are required for highly disaggregated and reliable models, components are aggregate phenomena, results are forecasts of future states but don't indicate the behavioral pattern for obtaining such states, and trends connote inevitability, a state of non-control.

Logic models, on the other hand, are complicated devices that require highly trained personnel for formulation and manipulation. Potentially, these types of models seem to have broad future application. Behavior phenomena can be predicted upon little historical data. However, there is considerable risk in making these models operational. At this time, few agencies have finances for making such changes. Much of real world phenomena are non-linear relationships which contributes greatly to the complexities of these models. However, it is at this very stage that oversimplification is lessened, and results may become more meaningful. Other important advantages of these models are their accommodations to component interactions, the external policy (value) considerations, and fruitful simulation procedures.

Current Application of Models

Mathematical modeling in public planning is a relatively new, but developing approach. Presently there are few models that are operational. However, through increased use and refinement, interest in models will probably increase at an increasing rate. The objective here is to describe only principal models that have been or are being implemented in planning today.

Transportation studies have been foremost in the advancement of model application for urban planning, although now land use and renewal models are also being employed in various studies. This can be attributed to the level of economic resources of local political units and to federal interest in comprehensive planning (Voorhees, 1965).

The Chicago Area Transportation Study, in the late 1950's, utilized a set of trend models (linear equations) in projecting population, economic activity, and various land uses, and transportation requirements to produce desired output for plan design.

Baltimore has employed a regression trend-type model for its land use study. Various land uses are related by linear regression submodels. The mean results are established within rigid assumptions.

Boston's activity allocation model is a trend device that distributes land use activities as functions of selected socio-economic clustering elements, e.g., white and blue collar workers, water and sewers, employment and residential centers and others.

The Southeastern Wisconsin regional planning commission's regional activity model is a logic model utilized extensively as a simulation device. Economic specifications and relationships (behavioral decisions) form the bases of analysis calculated upon the constraints of an economic input-output matrix.

The Penn-Jersey transportation study land use forecast model has utilized a linear programing model in making predictions related to spatial distribution of housing features, industrial and commercial activities, and land uses. Numerous complex inputs are characterized by linear equations. The model has yet to become operational, but the study has indicated the non-linearity of real-world phenomena in furthering land use modeling.

The Pittsburgh area transportation study (PATS), the Upper New York State transportation study (UNYTS), and the Cleveland-Seven County transportation - land use study (SCOTS), are utilizing opportunity (trend) models for distributing land use activities. This is capable of distributing population, land use activities (trip ends) and trips in one operation. This opportunity model is conceptualized on the probability that activities of a zone will be distributed as a portion of the opportunities of the remaining zones.

Particular theorems of the physical sciences are being employed by Koenig and colleagues in modeling socio-economic phenomena. A simulation state model is the result of relating algebraic, difference or differential equations (representing component stock and flow features) through linear graph theory. This is a logic model, either dynamic or static. This application has great utility, for it generates a state model and respective outputs that can be further used for other mathematical application; control techniques, linear and dynamic programming, sensitivity, simulation, optimization, and others. A dynamic state model representing church monetary support has been prepared by Koenig. A static state model of recreation travel has likewise been prepared (Ellis, 1964). Koenig, Marquis and Goodnuff are exploring applications to university, inter-undustry, and traffic control system models, respectively.

The remainder of this thesis will explore the application of Koenig's technique to an urban-regional natural water system as it relates to urbanization and natural phenomena. This application was chosen because of the obvious system characteristics of natural water flows and elevations and the availability of considerable data. Many recent efforts suggest the importance of water quantity and quality to urban areas, justifying these initial modeling steps as critical to urban-regional planning.

CHAPTER II

A SYSTEMS THEORY

The particular technique of mathematical systems modeling used as the basis for this thesis is a discipline that may contribute significantly to the advancement of urban planning. This is a process of modeling system components in precise, systematic, quantitative forms in formulating dynamic and static system models which can simulate system behavior (Koenig, 1963). Once this has been established, deliberate and structured methodologies are available for making rational, functional, and consistent decisions about system control and design within the framework of established value considerations. This is desirable for establishing and maintaining lasting policies and coordinated planning action decisions.

In recent years, engineering disciplines have utilized mathematical system modeling extensively for analyzing physical systems. It should be understood that although systems analysis and mathematical modeling are two distinct processes, they can be used in combinations. The methodology is now being introduced into many socioeconomic disciplines as an analytical tool and may evolve as a significant comprehensive approach to decision making.

Efforts at modeling non-physical phenomena are fragmented in various fields. However, these initial efforts are building blocks for furthering the study of social systems and for developing an interdisciplinary cohesiveness among various social sciences. Aggregation of such efforts could create an interdisciplinary communication and information flow link that is apparently lacking at the present time. Principal features of this basic systems approach are presented in this chapter, while the actual structuring of the water system model is presented in the subsequent chapter and Appendix A.

The material to be presented here has been abstracted from the works of Herman Koenig (lectures, books, and papers), William Grecco (doctoral thesis, 1962), and Jack Ellis (lectures and papers, 1964). This systems theory is outlined here as a framework for discussion to follow.

Steps:

1. Initially, identify the system to be solved.
2. Identify system components which can be mathematically characterized by across and through variables.
3. Measure components as functions of across and through variables, in common dimensions, or obtain available data to describe the same.
4. Establish terminal equations of components in terms of across and through variables.
5. Utilize the linear graph theory, a system graph and tree (T).
6. Establish equations representing component interconnections as established in the system graph.
7. Formulate the static or dynamic state model through a precise application of the systems theory.

8. Finally, obtain numerical and graphical solutions. Behavioral characteristics of the model may be determined from the transition matrix as an initial step to numerical solutions.

These steps are grouped under more general headings to provide a structure for an integrated and meaningful discussion.

Establish the System, System Components, and Component Terminal Equations

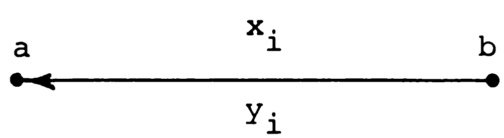
A system is an aggregation of distinct, individual components interrelating together to achieve a common purpose. Systems theory is predicated by the identification and description of such components and respective component interconnections in quantifiable terms, i.e., to be mathematically expressed and measured. These are the conceptual and developmental stages where the researcher must reflect extensively to establish appropriate system components. Intuitive and design demands are at a premium at this point, for all components and their respective interconnections must be identified and established accurately or the model will not function as the intended system prototype.

Each component is modeled individually, expressed diagrammatically as a terminal graph (a line segment called an edge), and modeled by an equation in terms of across (x) and through (y) variables (Figure 1a) as illustrated by the manner of measurement across or through terminal points. An example will assist basic understanding at this point: current flow (through, or y variable) in electric transmission lines is created by voltage differential (across, or x variable). The terminal graph represents the component as a two-terminal line segment. The orientation of such a line is in the direction of the flow from the point of origin to the point of destination. It is important to reiterate that each component has to be identified, conceptualized and measured in terms of across and through variables that are related in common dimensions.

Interconnections Characterized

The system graph, the aggregate of terminal graphs, diagrams all components of the system with respective component interconnections commonly called interfaces, (Figure 1b). The interfaces are indicated by vertices and are the common terminal points of respective components. This methodology also may be utilized with multiterminal components. However, this intricacy will not be used in the model development to follow.

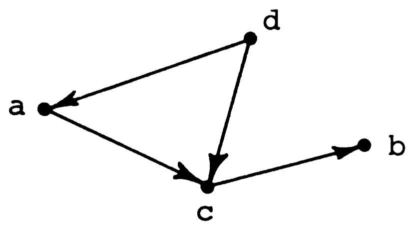
Terminal Graph



(a)
Figure 1

Each system must meet definite, established, linear graph criteria. Namely, across (x's) variables must sum zero about a closed circuit. This is called the circuit postulate and is illustrated by Figure 1b, as the algebraic sum of edges a-c, d-c, and d-a must equal zero (0). Through (y's) variables relating to a vertex must sum zero (0). This is illustrated by Figure 1b, as the algebraic sum of edges a-c, d-c, and c-b must equal zero at vertex c. These theorems interrelate models of the individual components and will be referred to as the constraint equations.

System Graph



(b)
Figure 1

<u>System Characterized</u>

The ultimate end of the modeling process is to structure system state models, dynamic or static, in an orderly manner for ease of programming for numerical and graphical solutions by a digital computer. When utilizing difference equations as terminal equations, the dynamic state model takes the following form:

$$Z_{p1}(n+1) = PZ_{p1}(n) + QZ_{s1}(n)$$
$$Z_{p2}(n) = MZ_{s2}(n) + NZ_{s2}(n+1) + F\ (n)$$

The vectors $Z_p = (Z_{p1}, Z_{p2})$ and $Z_s = (Z_{s1}, Z_{s2})$ represent the primary and secondary variables from the system graph tree (T) and co-tree (T'). A tree is an open end path through the system graph that connects all vertices. The co-tree is made up of the remaining edges. P, Q, M, and N are matrices of coefficients, and F(n) is a vector of known inputs. Through substitution and manipulation of variables, the secondary variables are expressed in terms of the primary variables and take the following form:

$$Z(n+1) = PZ(n) + QZ(n) + F(n)$$

Thus, a system state model is formulated that can be numerically solved.

Closing Remarks

As can be observed from this chapter, this particular system theory is an orderly and precise methodology for modeling distinct system components, relating the interconnections among such components, and obtaining behavioral, graphical, and numerical system solutions. However, the process is not as parsimonious as might be assumed at first observation. A paramount problem in relating this theory to practical application is in the conceptualization phase, that of identifying an adequate system, correct variables for representing components, functional relationships between across and through variables (parameters) and the interconnections of the system components.

Owing to such difficulties, it is often necessary to experiment with various components, terminal equations, parameters (transition matricies) and trees of the linear graph until adequate elements can be adequately established. A researcher having great awareness of the system phenomena and having intuitively creative abilities can be empirically effective at this stage in establishing a desirable system prototype.

Through variables are normally readily observable, for researchers have routinely manipulated these elements in various planning activities. However, there are few planning activities where across variables have been explicitly established. Consequently, they are not as readily identifiable as flow variables and may have to be empirically established from simulated solutions of an assumed state model, as discussed above. This may require numerous feedback and revision sequences. However, considerable effort has to be expended at this phase, for it is the basis on which the model is developed.

Granted, generally, socio-economic systems are more complex and respective components and variables are more difficult to identify and establish their physical system; however, if there is to be further development of this approach in urban-regional planning, a great deal of effort by socio-economic researchers will be necessary in achieving proficiency in its use.

Also, as in many planning endeavors, this methodology is limited by the availability of appropriate information measurement. If components can not be characterized quantifiably because of this inadequacy, then it is necessary to make new measurements. This is a real limitation in applying this technique; also, this may require great amounts of additional data. In addition, qualified researchers are necessary at this stage of development for streamlining data collection procedures.

CHAPTER III

MODEL DEVELOPMENT

The objective of this chapter is to relate the modeling technique described in Chapter II to water elevations and flows in the Lansing, Michigan region. A study of this type has application to planning, for it is a methodology for relating resources to the demands of urbanization.

Initially, it was assumed that urbanizing features would play a major component role in the water system model. However, it was discovered that because of the complexities of an urban system it would be necessary to start the modeling process by characterizing the natural water system (not the total hydrologic cycle) separately. In conceptualizing the components it became necessary to describe human water usage (pumpage) as one of a number of flows between the components. Pumpage can be described as an individual component representing human use of water, a vector of elements which might include industrial, residential, commercial, institutional and other uses.

Study Area Described

The study area is a ten township region, approximately 230,000 acres, with Lansing, Michigan at its geographic center. The townships forming the region are Oneida, Delta, Watertown, Dewitt, Bath, Meridian, Alaiedon, Delhi, Windsor and Lansing. The primary water user is the Lansing Metropolitan complex which pumps approximately 25 million gallons of water per day. Grand Ledge and numerous private supplies demand a significant amount of water, approximately 5 million gallons per day. Detailed information was obtained from the Battelle Memorial Institute Report to the Tri-County Regional Planning Commission (Swager, 1963).

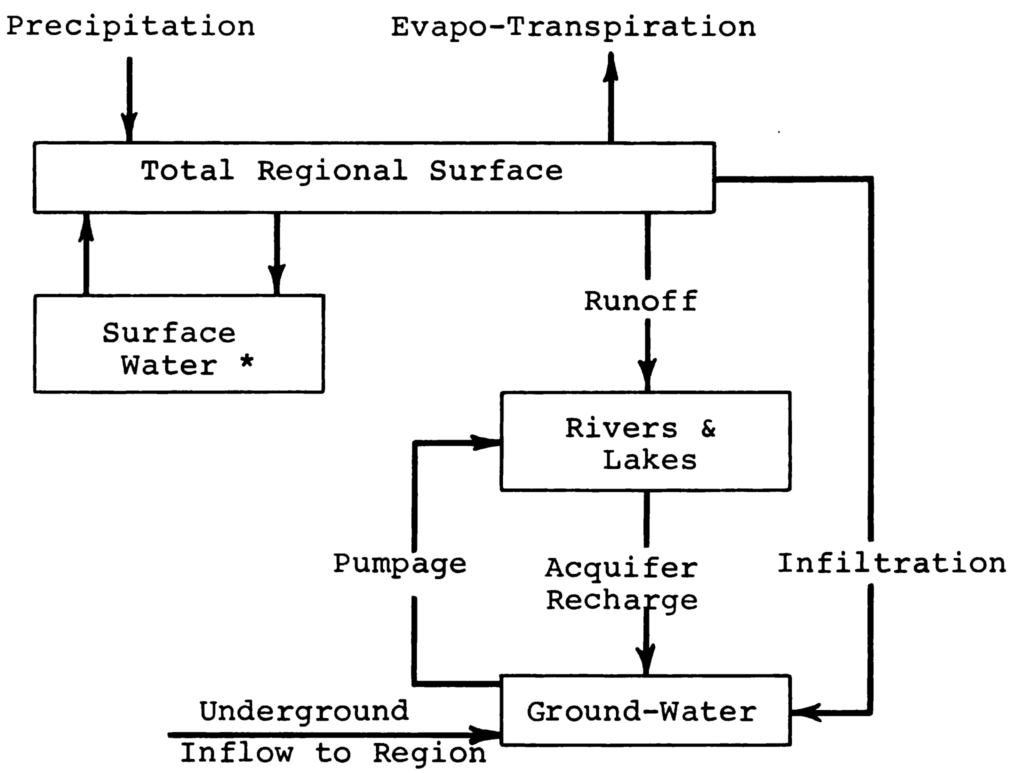
System Conceptualized

The most difficult task in applying the model theory presented in Chapter II is system conceptualization, selecting the components and characterizing them mathematically. As in most modeling endeavors utilizing this approach, it is difficult to determine the across variable. Across variables are not generally identified during research; consequently, planners are not familiar with their functions. In some planning endeavors, flows are routinely measured and readily identified. Even after numerous empirical efforts in trying to identify which is the appropriate across variable of the water system (stocks, elevation or possibly others), this has not been reliably established. This has led to the need for assuming some of the component terminal equations. This is further discussed in a later section on model improvements.

Initially, endeavors to establish suitable, distinct system components revolved about such possible components as wells, human water usage, ground water cone of depression, integrated surface supply (river, lakes, and land), deep and shallow water supplies, runoff, infiltration, stream acquifer recharge, precipitation, evaporation,

transpiration, water supply accretion and leakage, and interception. After extensive research, however, the following components were selected: ground water elevation, rivers and lakes elevation, surface water elevation, runoff, infiltration, inflow to region, precipitation, evapotranspiration, river acquifer recharge, and human usage (well pumpage). Following is the schematic diagram of the selected natural water system and respective components (Wisler, 1949):

Schematic Natural Water System



*other than Rivers & Lakes

Figure 2

Terminal Equations Described

The following equations express each component in terms of across (x) and through (y) variables. Units in measurement for these variables are elevation in feet (relative to a zero (0) surface level) and gallons of water flow per year, respectively.

Ground (9), streams and lakes (10), and surface (11) (portions of the earth's crust that have water containing capabilities) water reserves are storage components characterized by difference equations of the form $x(n+1) = px(n) + qy(n)$ (see Appendix A). Elevation (x) at a future time is a function of the present elevation and "build-up" flows between the present and that future time. The assumption is that these water storage reservoirs have rectangularly-shaped cross sections so that elevation and volume measurements are linearly related. The respective parameters (g, p, q) are discussed further in a following section in this Chapter.

Precipitation (1), evapo-transpiration (2), river acquifer recharge (6), and human water usage pumpage (7) are assumed known flows and are described as functions of time, $y(n) = f(n)$ (see Appendix A). These components are assumed as aggregate flows which are not dependent on x variables. A system can readily become over-specified by incorporating too many known functions, for the solution is trivial and a state model does not exist. This was an initial problem in conceptualizing this system, since much of the data are inappropriately measured for detailed variable representation.

Runoff (3), infiltration (4), and inflow of groundwater into the region (8) are characterized as algebraic equations of the form y(n) = gx(n) (see Appendix A). It is assumed that flows are dependent on elevation differentials (if no elevation differential exists, there will be no flow) and are related by parameters (g). As discussed further in the following section, these components have also been measured in aggregate by water specialists. However, to eliminate over-specification of the system, it was assumed these components tend toward this type of characterization. This assumption may not be completely correct, as is discussed further in the section on future model improvements.

A well component (5) had been considered a part of the system until the time of finalizing the system model, but was then eliminated. This is the reason number 5 is not included in the numbering of components in the final presentation.

System Graph

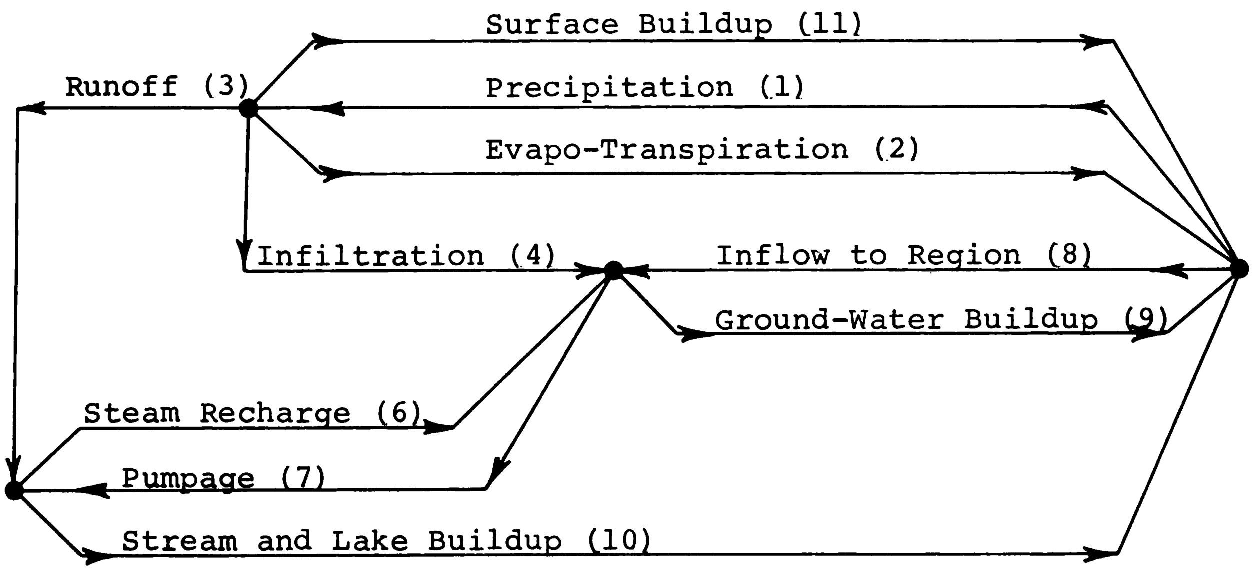


Figure 3

The appropriate tree (T) includes edges 9, 10, and 11 as branches on the linear graph. Constraint equations are structured on this basis (see Appendix A).

Model Structured

Applying the precise modeling theory (see Appendix A), the resultant discrete state model is:

$$\begin{bmatrix} x_9(n+1) \\ x_{10}(n+1) \\ x_{11}(n+1) \end{bmatrix} = \begin{bmatrix} p_9-(q_9g_4-q_9g_8) & & q_9g_4 \\ 0 & p_{10}-q_{10}g_3 & q_{10}g_3 \\ q_{11}g_4 & q_{11}g_3 & p_{11}-(q_{11}g_3+q_{11}g_4) \end{bmatrix} \begin{bmatrix} x_9(n) \\ x_{10}(n) \\ x_{11}(n) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & q_9 & -q_9 \\ 0 & 0 & -q_{10} & q_{10} \\ q_{11} & -q_{11} & 0 & 0 \end{bmatrix} \begin{bmatrix} f_1(n) \\ f_2(n) \\ f_6(n) \\ f_7(n) \end{bmatrix}$$

Quantitative solutions of the discrete state model were obtained from the Michigan State University digital computer, CDC 3600, utilizing a previously structured program (Ellsworth, March, 1965). Both discrete time response (numerical data) and respective plots may be obtained by simply introducing an appropriate transition matrix and an initial condition state vector into this program.

The program considers the model to be of the general, homogeneous form $Z(n+1) = QZ(n)$. The matrix Z is a state vector and the matrix Q is a square transition matrix of the order n. Since the water model is of the non-homogeneous form $Z(n+1) = PZ(n) + QE(n)$, it is necessary to augment the non-homogeneous model into the equivalent, homogeneous form for solution. This form is also shown in Appendix A.

Simulations Discussed

Simulations are stepwise at one year time increments iterated 10 times to illustrate the dynamics of a 10 year period. Input quantities are varied for each discussion case to indicate behavioral implications of the system. Numerous iterations were made on the computer with various combinations of parameters (altered transition matrices) and initial conditions and inputs, before an appropriate system response was consumated and selected for presentation in the following sections.

Parameters

The paramount difficulty in quantifying across and through variables was in the establishment of parameters for relating these variables in common dimension. This is the area in which much research and appropriate data measurement is still needed. It is sufficient to say here that many of the parameters are assumed and are rough approximations at best. How the parameters were approximated and how reliable they may be follows. Specific magnitudes and references are listed in Appendix B.

It is assumed that parameters p_9, p_{10}, and p_{11} each equal one (1.0). This implies that all water accretions and leakages for each component sum zero and that all storage at a given point in time is carried over without loss or gain to the next period except losses due to evapo-transpiration which is included as a component.

Parameters q_9, q_{10}, and q_{11} are approximations relating respective build-up flows to differences in elevation. Thus, for each gallon per year of flow there will be q additional feet of elevation build-up over a rectangularly shaped area for surface, rivers and lakes, and ground water storages. Parameter q_9 assumes that the total ground water build-up is uniform over the total region. This is to say that all underground formations are horizontal; this would only happen in an ideal situation. Parameter q_{10} assumes that a tenth of the study area is in rivers and lakes (for ease of computation) and that river waters through the region change four times daily. Parameter q_{11} assumes that a tenth of the surface (other than lakes and rivers) area has water storage capabilities. That is, owing to soil types (clay, muck), climate, and depressions in the surface crust, water is contained on the surface. This is reduced only by evapo-transpiration.

In the algebraic equations, parameters g_3, g_4, and g_8 are approximations relating elevation differentials to flows. Thus, a one foot difference in elevation will mean g gallons per year of water flow. Parameter g_3 relates the estimated amount of average infiltration per year (15% of precipitation) and the difference between surface crust and ground water table elevations as a constant function. This differential is assumed as 100 feet. Parameter g_4 likewise is an estimation of the total amount of runoff per year (23% of precipitation) and the difference between surface crust and river elevations; about 25 feet. This is a constant function.

Parameter g_8 relates the amount of ground-water inflow to the region for each foot of elevation change. This is assumed on the basis of a transmissibility rate of about 23,000 and the perimeter of the region which is about 72 miles (Wisler, 1949).

Inputs

Models of the general nature structured in this chapter are very useful analytical devices in that simulation may be readily accommodated. This, of course, provides a researcher with additional assistance in forecasting future phenomena and for better understanding past system behavior.

This section will deal with inputs to the simulation model. Not only is this an exercise in simulation, but a stage of model calibration, refinement and validation. The sensitivity of input and initial condition changes may be readily analyzed and revisions initiated. Likewise, guidance in data collection may become more stringent and useful, for appropriate substance may be measured in the appropriate format.

Discussions of twelve selected cases, those having salient features, are presented here with specific numerical and graphical solutions displayed in Appendix B. Each case is presented in Appendix B as a separate printout continuing numerical and graphical solutions and general description of the state vector. For all cases, the parameters will remain identical. Consequently, the transition matrix remains constant (see Appendix B). Other parameters have been explored in previous iterations; however, the prototype solutions displayed here are the most representative of the natural water system being studied.

Input and initial conditions for each case are described in the respective discussions. Specific inquiry of each case can be made by referring to Appendix B.

Case 1: Generally, this is the prototype of the existing natural water system, a basis for comparing and analyzing the following eleven (11) cases. The transition matrix remains the same for all cases, while inputs and initial conditions are varied to determine empirically their degree of sensitivity to the system. Inputs to the system are average annual flows (Y's); initial conditions (X's) are generally assumed elevations.

Generally, with these conditions being constant for ten (10) iterations, representing a ten year time period, ground water buildup is rapid for the first five (5) years and then only gradual increase prevails. Surface water decreases for the first three (3) years and then increases, almost linearly, for the remaining study time. This latter rate of increase is greater than ground-water build-up, creating a diversion in slope curves. Rivers and lakes build-up at a low and steady rate.

Case 2: Inputs and initial conditions are the same as for Case 1, except precipitation (Y_1) is doubled. Generally, a tremendous increase in surface water buildup develops as the capacities for infiltration and runoff are exceeded; ground-water elevations and rivers and lakes elevations likewise increase substantially. Intuitively, these phenomena should develop; however, refinement of the parameters (g's and q's) should make these solutions more realistic. This will be discussed later in this chapter.

Case 3: Inputs and initial conditions are the same as for Case 1, except evapo-transpiration is increased slightly. Generally the behavior is similar to Case 1; however, ground-water build-up is slightly less as is rivers and lakes buildup. Surface water reacts profoundly; initially, it decreases to greater depths and then increases at a lesser rate than in Case 1. The divergence from ground-water curve is less, and the numerical solution at the end of the study time is much reduced.

Case 4: Inputs and initial conditions are the same as for Case 1, except river recharge is doubled. The results are the same as for Case 1. Therefore, this slight input increase is insignificant to change the behavior of the system.

Case 5: Inputs and initial conditions are the same as for Case 1, except that river recharge is increased substantially, being almost squared. Contrary to Case 4, this increase does affect the system solution significantly; ground water and surface water levels increase, and rivers and lakes decrease substantially. Large amounts of these waters are recharging the acquifer (ground water supply). This is intuitively obvious and a phenomenon that could be implemented by adding storage dams to the water system.

Case 6: Inputs and initial conditions are the same as for Case 1, except human usage, well pumpage, is doubled. Generally the behavioral results are nearly the same as for Case 1, with slight differences. However, these slight increments indicate reduced ground and surface water levels and an increase in rivers and lakes. Intuitively these phenomena should develop.

Case 7: Inputs and initial conditions are the same as for Case 1, except that evapo-transpiration is reduced 60%. This reduction creates a profound increase to surface water level, and to a lesser degree ground water level. Rivers and lakes levels are also increased. Intuitively these phenomena should develop.

Case 8: Inputs and initial conditions are the same as for Case 1, except that precipitation is reduced to nearly total drought conditions. General behavior of the system is similar to that one would intuitively conceive; all water levels decrease, surface water decreasing profoundly. However, ground water levels generally increase. This behavior is characteristic in all cases discussed and perhaps is much too prevalent for detailed analytical study. Additional research into component interactions or conceptualization or parameter measurement is necessary to check this unrealistic feature. More detailed information on this follows in the next section.

Case 9: Inputs are the same as for Case 1. However, initial conditions have been altered to show the influence of elevation changes on the overall system function. Increment values between the respective elevations remain the same, -90 feet and -10 feet. General behavior is the same; but all buildups increase at greater rates, especially surface water. This indicates that significant relationships exist between the equation-parameters and elevations.

Case 10: Initial conditions are the same as for Case 1 except that the inputs have been changed to simulate spring conditions iterated for ten (10) years period. Gross water supply is reduced somewhat from Case 1, and human usage is considerably reduced. Generally behaviors are similar; however, ground water level is less and surface water and rivers and lakes are significantly less.

Intuitively this is agreeable.

Case 11: Inputs are the same as for Case 10, but the elevations are changed. This is to indicate the sensitivity of elevation changes on the overall system function. Generally behaviors are only slightly similar, for the rate of change of each buildup is much more severe than in Case 10. This can be attributed to increased elevation differentials. Elevation changes have definite and significant affects on model solutions.

Case 12: Inputs are the same as for Case 10, but elevations are changed; also, the elevation differentials are reduced from those in Case 11. General behavior is similar to both Cases 10 and 11, but the rate of change is much reduced from them both. This probably indicates the significance of the depth of ground-water elevation on the water system.

Model Improvements

As in many research efforts, initial results are building-blocks to further study. However, through experimental and theoretical refinements, more meaningful results may be obtained. Results obtained in this study presently remain in the conceptualization stage, ready to be utilized fundamentally as a building-block to further refinement. If adequate data and knowledge for conceptualizing and modeling the components had been available, the model solutions could have been more meaningful and refined. However, the simple model structured here with these limitations does illustrate the principal features of this modeling technique.

Following is discussion regarding features for refining the water system model:

1) Many of the parameters utilized in the terminal equations were assumed, as discussed previously. Accurate parameters for relating across and through variables (g_3, q_4, g_8, p_9, p_{10}, p_{11}, q_9, q_{10}, and q_{11}) have to be measured before establishing reliable state model numerical solutions. This is another necessary refinement as indicated in step 3 of the overall modeling process, Chapter II.

2) It is possible that some of the phenomena may not best be characterized with linear equations, but with non-linear and differential equations. Bounds to these equations should be established, for linear approximations only exist for specific domains.

Examples of non-linear phenomena are:

a) Transmissibility, soil permeability and storage capacities (Y's in Figure 4) may change in relationship to elevation of water table, as differential equations.

Flows vs. Elevations

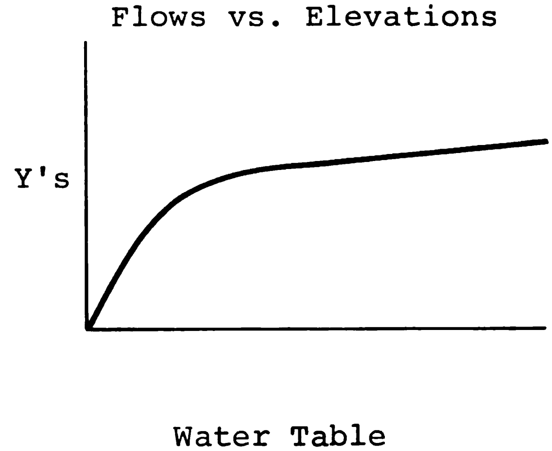


Figure 4

b) Both evaporation (Y_1) and runoff (Y_3) could be represented by a family of curves (Figure 5). Number 1 indicates that as precipitation increases, runoff and evaporation increase correspondingly when the total regional surface is either completely covered by water, frozen or paved. Numbers 2 and 3 describe similar elements, however, they are of lesser extremes.

Flows vs. Soil Conditions

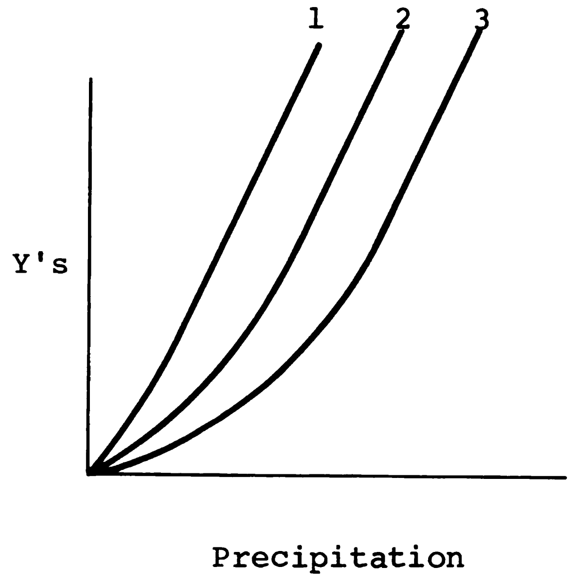


Figure 5

c) Evaporation and runoff may best be characterized as functions of storm frequency, storm duration, storm magnitude, and piezometric surface elevation, (assuming shorter time periods).

3) Precipitation, transpiration, evaporation and human usage (pumpage) are features that could be more meaningfully modeled as time-variant (changing throughout the year and over a duration of years) rather than constant inputs. Thus, a model might be developed with seasonal (quarterly) or monthly time periods with changing inputs.

4) Population is increasing at an increasing rate, likewise per capita water usage is increasing. This is characterized by a non-linear curve similar to Figure 6. This affects the influence of the human usage component significantly.

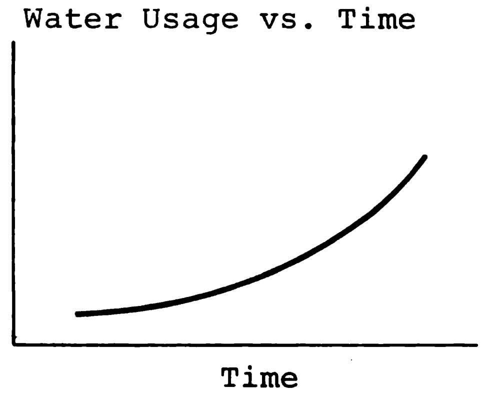


Figure 6

5) Snow is a peculiar element for there is a significant time-lag between precipitation and infiltration. This may be affected by the soil conditions, temperature and winds.

6) Plant wilting conditions may indicate something about the water potentials that are contained in the soil between the land surface and the water table. This was not considered at all in the state model, however, this may be where precipitation carry-over is implemented (Hariri, 1960).

7) Storm frequency and distribution; condition of surface (forest litter, pavement, etc.) and many other elements can be expressed in more refined models.

8) Wind conditions, temperature, and humidity also influence the capabilities of water, and could be explicitly represented. These were considered as aggregate features affecting evapo-transpiration, not as distinct components.

9) A significant alteration is to shorten the time period from an annual to either a seasonal, monthly, or a daily basis. The dynamics of infiltration, weather and soil conditions, time-lags, draw-down and other factors could be meaningfully solved for this time scale. This would be very fruitful for short-run decision-making and to reduce the chance for over-specification of the system, (a problem in conceptualizing this system). Conceivably, it eliminates all chance of mixing short-run and long-run elements which might have created some initial problems in conceptualizing the water system.

10) It is important at this time to point out that the conceptualized through and across variables might not be the correct system variables. Also, the system phenomena might not be conceptualized adequately. Exploration with alternate variables and phenomena would assist in possibly improving and validating the model.

With the findings presented in this chapter as a common basis for further discussion among modeling analysts and water specialists, model refinements could be very fruitful. Results from coordinated theoretical and empirical research at this level of knowing can be quickly evaluated because the basic model is computerized at this stage. Sensitivity of alternate components, component interconnections, parameters, and inputs to the system model can be evaluated and improvements made expeditiously.

CHAPTER IV

CONCLUSION

The modeling technique explored in this thesis may lead to extensive application in various socio-economic disciplines and thus, to urban planning. Although the water model developed here is not highly sophisticated, it does point up a systematic analytical methodology. It is more than comprehensively systematic, it also relates a precise methodology for modeling the internal features of a system. Therefore, it has extensive analytical application, both externally and internally.

Another feature that adds to its potential is that various sophisticated mathematical applications can be implemented in conjunction with this methodology, owing to the manner in which the resultant model is quantitatively derived. Some such features are control techniques, optimization, linear and dynamic programming, synthesis and design, sensitivity tests and others. This modeling technique, along with its associated ones, provides comprehensive and fruitful processes for simulating real world phenomena prior to their existence. These could provide action decisions which are very useful tools for guiding and controlling future development.

This thesis has shown, as Crane has previously expressed, that logic modeling contains a high degree of risk. If relationships between across and flow variables are not known, or if there is insufficient and/or inappropriate data, various relationships and data have to be assumed. It becomes obvious for obtaining reliable results that measurement criteria and devices for securing necessary basic data will have to become more refined and directive. Until personnel and appropriate data are available for generating these types of logical modeling operations, unreliability will prevail.

It seems at this point it should be stressed that emerging planning concepts should be developed about more reliable and consistent criteria than the traditional intuitive processes. Many planners have propagated the concept that socio-economic phenomena can not be quantified and analyzed in detail as is required in an application such as the one utilized in this thesis. It is my belief that statements of this nature are generated and perpetuated because few planners have observed the real insights that applied science may hold for planning. Granted, most socio-economic phenomena are more complex than those of the physical sciences and engineering. However, the axioms and theorems of the more mature professions may provide the structure for more meaningful study in the social sciences. The facility of digital computers and other electronic data processing techniques will assist in the planner's application of theorems of mature disciplines to the maturing planning profession.

Further, the very process of identifying and establishing system phenomena is of utmost importance. The reflective depth one is required to achieve in conceptualizing behavioral systems phenomena assists in understanding and establishing individual sub-systems in proper relationship to more comprehensive and complex systems, a hierarchy of systems derived from the most elemental component systems. It is conceivable that this arrangement of systems could be the component sub-systems of a community, and the community system an aggregate of such systems (Marquis, 1963). Here the planner's role becomes very significant, for his intuitive, creative, and design talents are directed to establishing these system models and the criteria for making simulation or plan design modeling (Logic) meaningful. This furthers the planner's role in providing needed foresight.

It is also conceivable that the modeling theory utilized in this thesis may provide a structure for analyzing the community as a mechanism of functioning, individual systems. Stock and flow characteristics, or their aggregates, of one system may provide the driving impetus for other systems. Each system structure exerts behavioral constraints upon the individual components, consequently each variable functions within an established domain, in a particular manner and with an established relationship to other variables. These characteristics, parameters, may be observed in input-output matrices for variable relationships may be directly observed and computed. Systems modeling then provides the vehicle for further analysis and understanding.

At this point examples of this application may be helpful. The simplified natural water system, structured in Chapter III, a physical system which is influenced by socio-economic considerations, provides insights into this process. Many of the components could be meaningfully broken down into sub-components or sub-systems. Precipitation could be characterized as a vector made up of hail, snow, and rain. Infiltration may be derived of industrial pumpage into ground, acquifers being directly exposed to precipitation, and seepage from swamps, could likewise be detailed and represented by a vector of elements.

In this model human pumpage is an aggregate of extensive sub-systems as one component. This component could be developed from individual systems as industrial, residential, commercial, agricultural, recreational and others. Activity allocations and individual consumption models of each could be developed and utilized by resource people individually or in combination. This model could be a component in a more comprehensive physical growth model.

Other examples may also illustrate the utility of this modeling technique. A population model may be established that indicates population growth is not the sole result of natural increase but also of migration. Employment may be the flow driver for establishing migration; and migration, a possible flow driver which creates population (stock) changes in the model, may be characterized in relation to heads of households and family characteristics. The population model may become the across driver (creates flows) to a commercial spending potential model which is dependent on population composition and related spending. This model then obviously may become a flow driver to change commercial floor space as shown in Figure 7. By gradually perfecting models of individual urban systems, a complex urban system model may be established using these sub-system models as edges for a comprehensive,community system graph. This step has been generally formulated by Marquis, and here with a systematic methodology of application, it may be implemented. Public revenues and expenditure capabilities, taxes and others may also be afforded their respective influences within the total system.

Schematic Community System

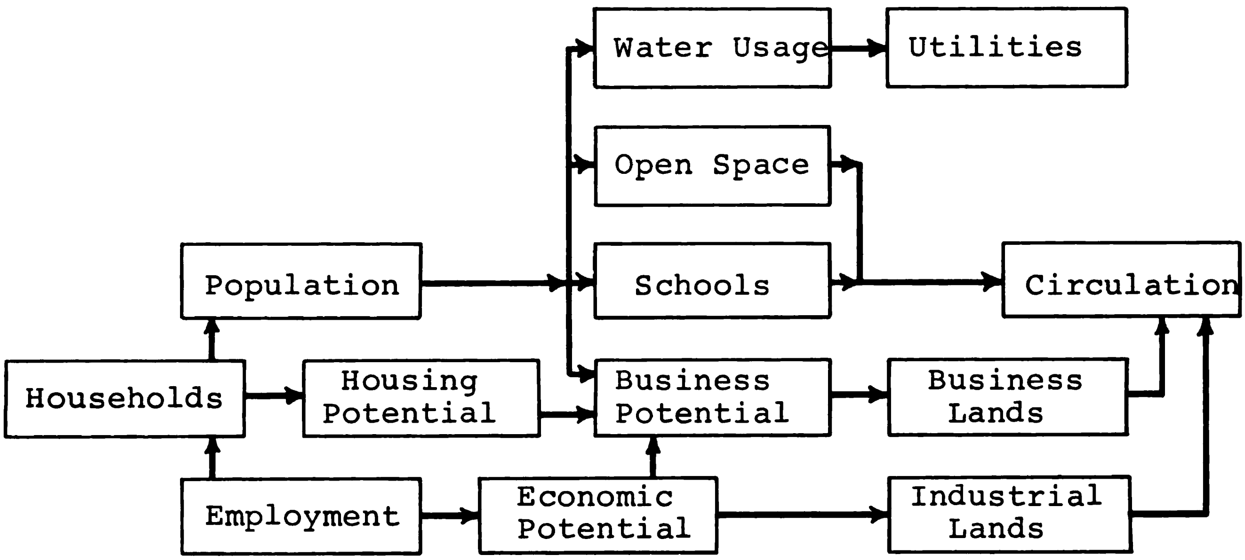


Figure 7

Another possible modeling and computer application of this technique may be related to control theory and to the development of desired communities. Presently, zoning and other regulations used for "guiding" desirable community development. Performance standards are being emphasized for guiding future development in our expansive, developing urban complexes. It is conceivable that a desired community structure could be programmed on computers, along with existing community structure. Policies, objectives, design standards and performance characteristics, desired land use relations, could likewise be established and similarly programmed into the computer. Decisions on future development, that which is to be the difference between desired and existing conditions, can be made from alternatives obtained from the computer which compared needed types of development to proposed development in regard to established performance standards (intensity, use type, others). Control theory and other modeling methodologies may provide useful means for analyzing these problems. It could prove to be a desirable substitute to the rigid and sometimes unrealistic application of zoning. Performance guidance may become a directed, operational notion.

Preparation of accurate component equations and parameters requires creative design abilities. Intuitive and creative talents of planners can be utilized to the utmost at this level, for this requires the talents of disciplined specialists and planners having an understanding of the urban community. Once the system model is established, alternative decision strategies may be evaluated in respect to previously established goals and objectives. This should assist in providing for more meaningful planning efforts in the future.

APPENDIX A

Water System Model Structured

1. <u>Terminal Equations</u>

$y_1(n) = f_1(n)$

$y_2(n) = f_2(n)$

$y_3(n) = g_3x_3(n)$

$y_4(n) = g_4x_4(n)$

$y_6(n) = f_6(n)$

$y_7(n) = f_7(n)$

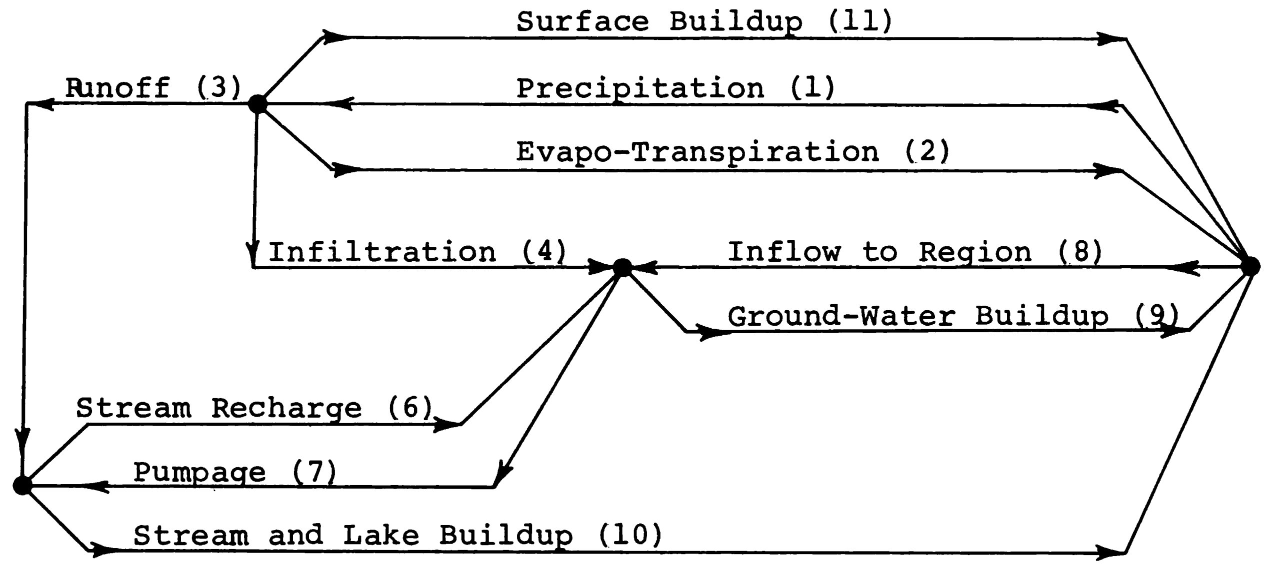
$y_8(n) = g_8x_8(n)$

$x_9(n+1) = p_9x_9(n) + q_9y_9(n)$

$x_{10}(n+1) = p_{10}x_{10}(n) + q_{10}y_{10}(n)$

$x_{11}(n+1) = p_{11}x_{11}(n) + q_{11}y_{11}(n)$

2. <u>Water System Graph</u>



3. Rearrangement of difference and algebraic equations into matrix form

$$\begin{bmatrix} x_9(n+1) \\ x_{10}(n+1) \\ x_{11}(n+1) \end{bmatrix} = \begin{bmatrix} p_9 & 0 & 0 \\ 0 & p_{10} & 0 \\ 0 & 0 & p_{11} \end{bmatrix} \begin{bmatrix} x_9(n) \\ x_{10}(n) \\ x_{11}(n) \end{bmatrix} + \begin{bmatrix} q_9 & 0 & 0 \\ 0 & q_{10} & 0 \\ 0 & 0 & q_{11} \end{bmatrix} \begin{bmatrix} y_9(n) \\ y_{10}(n) \\ y_{11}(n) \end{bmatrix}$$

$$\begin{bmatrix} y_3(n) \\ y_4(n) \\ y_8(n) \end{bmatrix} = \begin{bmatrix} g_3 & 0 & 0 \\ 0 & g_4 & 0 \\ 0 & 0 & g_8 \end{bmatrix} \begin{bmatrix} x_3(n) \\ x_4(n) \\ x_8(n) \end{bmatrix} \qquad \begin{bmatrix} y_1(n) \\ y_2(n) \\ y_6(n) \\ y_7(n) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_1(n) \\ f_2(n) \\ f_6(n) \\ f_7(n) \end{bmatrix}$$

4. Utilizing linear graph theory, the appropriate tree includes edges 9, 10 and 11

Cut-set Equations

$$\begin{bmatrix} y_9(n) \\ y_{10}(n) \\ y_{11}(n) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ 1 & -1 & -1 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1(n) \\ y_2(n) \\ y_3(n) \\ y_4(n) \\ y_6(n) \\ y_7(n) \\ y_8(n) \end{bmatrix}$$

Circuit Equations

$$\begin{bmatrix} x_1(n) \\ x_2(n) \\ x_3(n) \\ x_4(n) \\ x_6(n) \\ x_7(n) \\ x_8(n) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_9(n) \\ x_{10}(n) \\ x_{11}(n) \end{bmatrix}$$

5. Substitution of cut-set and circuit equations (step 4) into terminal equations (step 3) and, subsequently, rearranging and substituting all equations into terms of primary variables

$$\begin{bmatrix} x_9(n+1) \\ x_{10}(n+1) \\ x_{11}(n+1) \end{bmatrix} = \begin{bmatrix} p_9 & 0 & 0 \\ 0 & p_{10} & 0 \\ 0 & 0 & p_{11} \end{bmatrix} \begin{bmatrix} x_9(n) \\ x_{10}(n) \\ x_{11}(n) \end{bmatrix} + \begin{bmatrix} -(q_9g_4+q_9g_8) & 0 & q_9g_4 \\ 0 & -q_{10}g_3 & q_{10}g_3 \\ q_{11}g_4 & q_{11}g_3 & -(q_{11}g_3+q_{11}g_4) \end{bmatrix} \begin{bmatrix} x_9(n) \\ x_{10}(n) \\ x_{11}(n) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & q_9 & -q_9 \\ 0 & 0 & q_{10} & q_{10} \\ q_{11} & -q_{11} & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1(n) \\ y_2(n) \\ y_6(n) \\ y_7(n) \end{bmatrix}$$

6. The water system discrete state model (general form is $Z(n+1) = PZ(n) + QE(n)$

$$\begin{bmatrix} x_9(n+1) \\ x_{10}(n+1) \\ x_{11}(n+1) \end{bmatrix} = \begin{bmatrix} p_9-(q_9g_4-q_9g_8) & 0 & q_9g_4 \\ 0 & p_{10}-q_{10}g_3 & q_{10}g_3 \\ q_{11}g_4 & q_{11}g_3 & p-(q_{11}g_3+q_{11}g_4) \end{bmatrix} \begin{bmatrix} x_9(n) \\ x_{10}(n) \\ x_{11}(n) \end{bmatrix} + \begin{bmatrix} 0 & 0 & q_9 & -q_9 \\ 0 & 0 & -q_{10} & q_{10} \\ q_{11} & -q_{11} & 0 & 0 \end{bmatrix} \begin{bmatrix} f_1(n) \\ f_2(n) \\ f_6(n) \\ f_7(n) \end{bmatrix}$$

7. The general form of the model is non-homogeneous. To be computerized, it is necessary to augment the transition matrix into the general homogeneous form $Z(n+1) = PZ(n)$. This model contains a vector of constant inputs $QE(n)$, therefore the appropriate homogeneous structure is as follows:

$$X(n+1) = \begin{bmatrix} p_9-(q_9g_4+q_9g_8) & 0 & q_9g_4 & 0 & 0 & q_9 & -q_9 \\ 0 & p_{10}-q_{10}g_3 & q_{10}g_3 & 0 & 0 & -q_{10} & q_{10} \\ q_{11}g_4 & q_{11}g_3 & p_{11}-(q_{11}g_3+q_{11}g_4) & q_{11} & -q_{11} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_9(n) \\ x_{10}(n) \\ x_{11}(n) \\ f_1(n) \\ f_2(n) \\ f_6(n) \\ f_7(n) \end{bmatrix}$$

APPENDIX B

Basic Data, Parameters, Transition Matrices, Initial Conditions, and Numerical Solutions

Basic Data* — Mean Annual

Study Area

	Mean Annual
Precipitation	32 inches (2.0×10^{11} gallons)
Evaporation	17 inches
Transpiration	4 inches
Runoff	7 inches
Infiltration	4 inches
Human Usage	3.0×10^{7} gals. per day or approximately 1.0×10^{11} gals. per year

1 acre-foot = 3.26×10^{6} gallons
1 cubic foot = 7.5 gallons

*Basic References; Swager, 1963, and U.S. Weather Bureau, 1965.

Parameters

$p_9 = 1$, $p_{10} = 1$, $p_{11} = 1$

$q_9 = 1.3 \times 10^{-11}$, $q_{10} = 1.0 \times 10^{-11}$, $q_{11} = 1.3 \times 10^{-10}$

$g_3 = 3.0 \times 10^{9}$, $g_4 = 1.5 \times 10^{9}$, $g_8 = 3.0 \times 10^{10}$

Following are the transition matrix, initial conditions and inputs, numerical and graphical solutions (10-1 year increments) for each salient simulation (Case 1 to 12) on separate sheets of print-out.

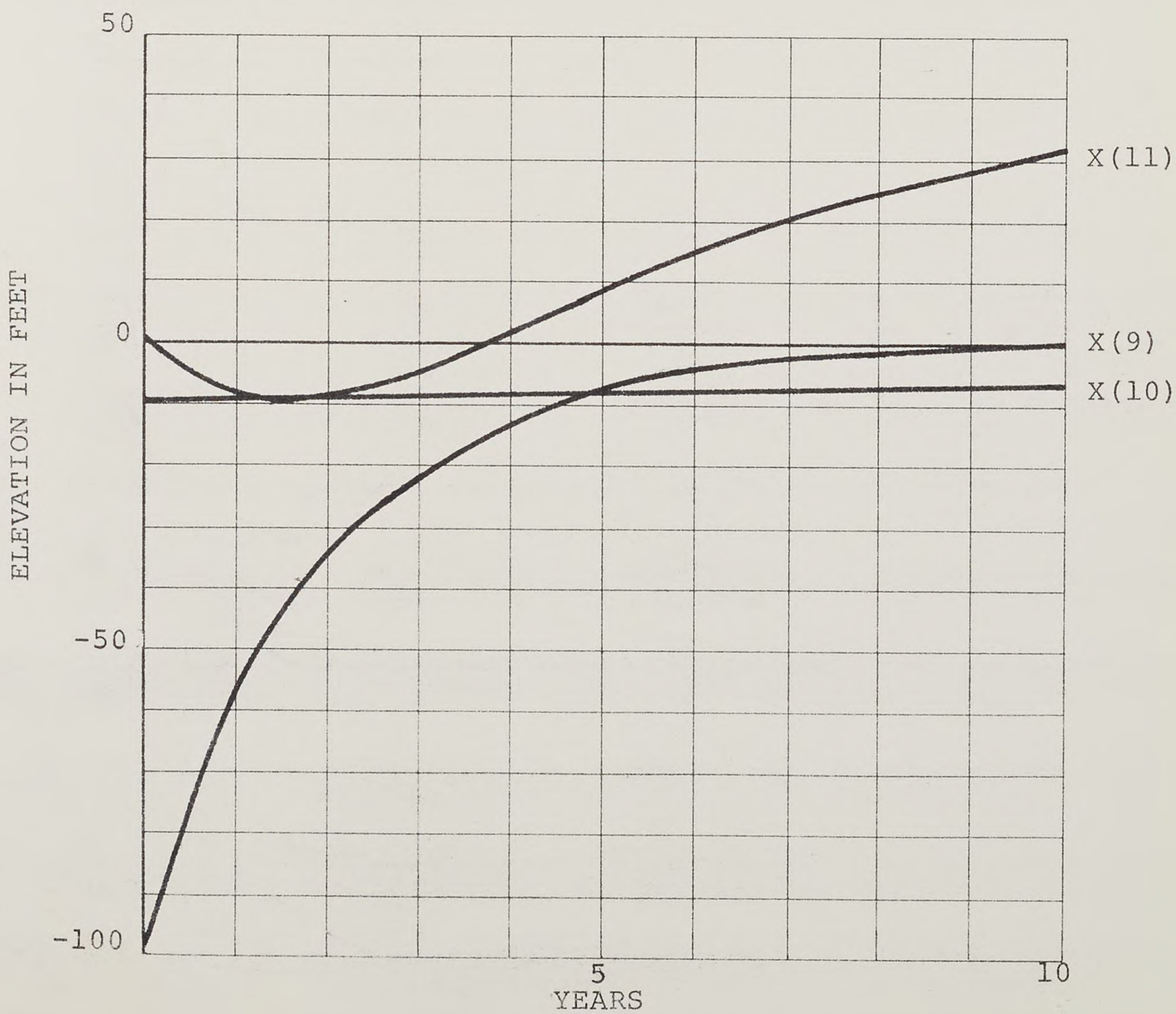
CASE 1

TRANSITION MATRIX

6.0000-001	-0.0000+000	2.0000-002	-0.0000+000	-0.0000+000	1.3000-011	-1.3000-011
-0.0000+000	9.9000-001	3.0000-003	-0.0000+000	-0.0000+000	-1.0000-011	1.0000-011
2.0000-001	4.0000-002	7.6000-001	1.3000-010	-1.3000-010	0.0000+000	0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000

STATE VECTOR

Ground Water X(9)	Rivers & Lakes X(10)	Surface Waters X(11)	Precip-itation Y(1)	Evapo-Trans. Y(2)	Acquifer Recharge Y(6)	Pumpage Y(7)	Years
-1.00+002	-1.00+001	0.00+000	2.00+011	1.20+011	4.00+006	1.00+010	0.00+000
-6.01+001	-9.80+000	-1.00+001	2.00+011	1.20+011	4.00+006	1.00+010	1.00+000
-3.64+001	-9.63+000	-9.62+000	2.00+011	1.20+011	4.00+006	1.00+010	2.00+000
-2.22+001	-9.46+000	-4.58+000	2.00+011	1.20+011	4.00+006	1.00+010	3.00+000
-1.35+001	-9.26+000	2.11+000	2.00+011	1.20+011	4.00+006	1.00+010	4.00+000
-8.20+000	-9.08+000	8.93+000	2.00+011	1.20+011	4.00+006	1.00+010	5.00+000
-4.87+000	-8.87+000	1.52+001	2.00+011	1.20+011	4.00+006	1.00+010	6.00+000
-2.75+000	-8.63+000	2.06+001	2.00+011	1.20+011	4.00+006	1.00+010	7.00+000
-1.37+000	-8.38+000	2.52+001	2.00+011	1.20+011	4.00+006	1.00+010	8.00+000
-4.47-001	-8.13+000	2.89+001	2.00+011	1.20+011	4.00+006	1.00+010	9.00+000
1.80-001	-7.86+000	3.20+001	2.00+011	1.20+011	4.00+006	1.00+010	1.00+001



DESCRIPTION:

Inputs (Y's) are annual average flows into the natural water system. The initial conditions (X's) are assumed elevations. The purpose for this case is to establish a basis for following case comparisons.

Headings for the remaining cases are the same as for this case. Therefore, they will not be repeated.

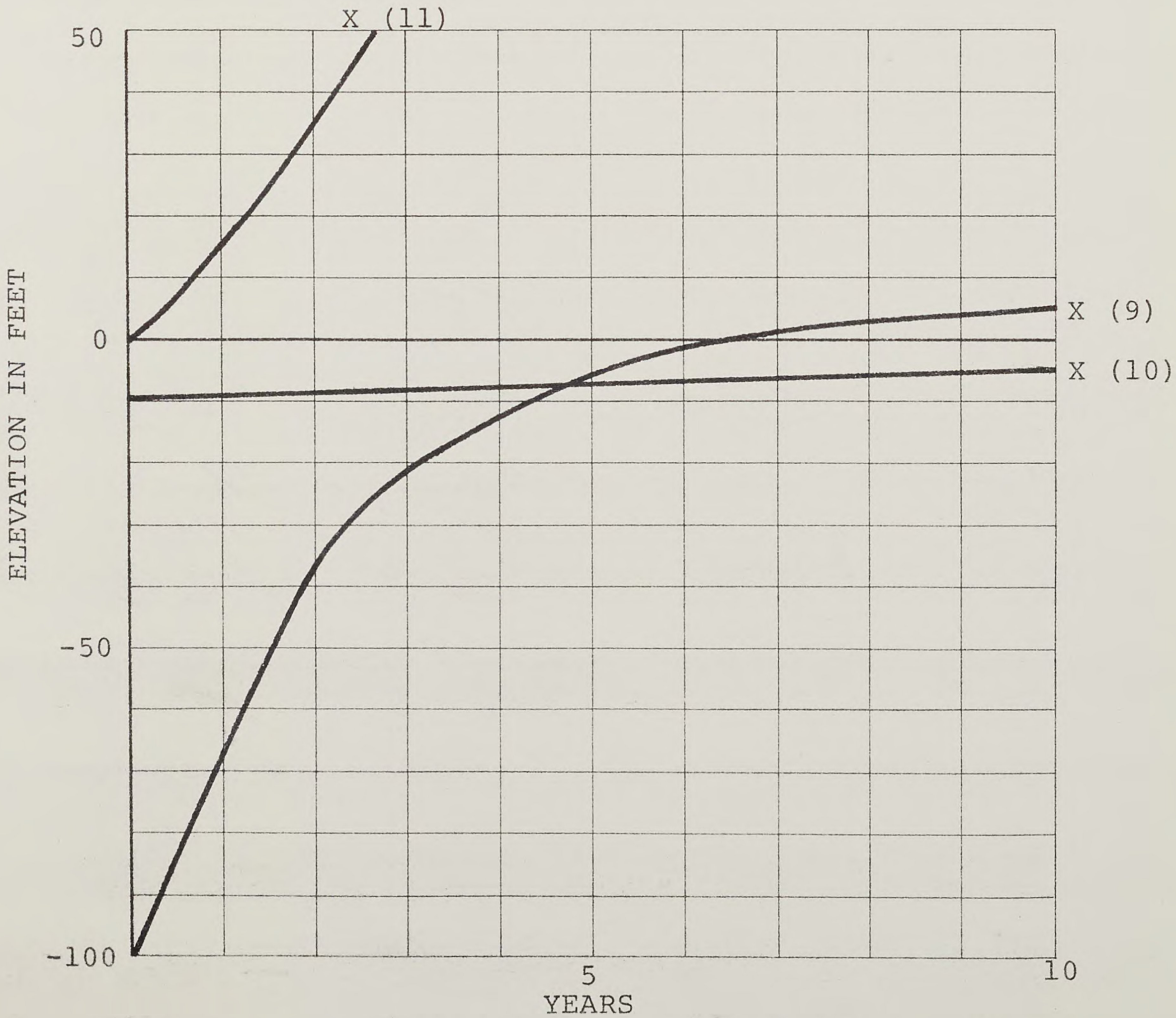
CASE 2

TRANSITION MATRIX

6.0000-001	-0.0000+000	2.0000-002	-0.0000+000	-0.0000+000	1.3000-011	-1.3000-011
-0.0000+000	9.9000-001	3.0000-003	-0.0000+000	-0.0000+000	-1.0000-011	1.0000-011
2.0000-001	4.0000-002	7.6000-001	1.3000-010	-1.3000-010	0.0000+000	0.0000+000
-0.0000+000	-0.0000+000	-1.0000+000	1.0000+000	-0.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-1.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-1.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-1.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000

STATE VECTOR

X(9)	X(10)	X(11)	Y(1)	Y(2)	Y(6)	Y(7)	Years
-1.00+002	-1.00+001	0.00+000	4.00+011	1.20+011	4.00+006	1.00+010	0.00+000
-6.01+001	-9.80+000	1.60+001	4.00+011	1.20+011	4.00+006	1.00+010	1.00+000
-3.59+001	-9.55+000	3.61+001	4.00+011	1.20+011	4.00+006	1.00+010	2.00+000
-2.09+001	-9.25+000	5.63+001	4.00+011	1.20+011	4.00+006	1.00+010	3.00+000
-1.16+001	-8.89+000	7.45+001	4.00+011	1.20+011	4.00+006	1.00+010	4.00+000
-5.58+000	-8.48+000	9.05+001	4.00+011	1.20+011	4.00+006	1.00+010	5.00+000
-1.67+000	-8.02+000	1.04+002	4.00+011	1.20+011	4.00+006	1.00+010	6.00+000
9.43-001	-7.53+000	1.15+002	4.00+011	1.20+011	4.00+006	1.00+010	7.00+000
2.73+000	-7.01+000	1.23+002	4.00+011	1.20+011	4.00+006	1.00+010	8.00+000
3.97+000	-6.47+000	1.30+002	4.00+011	1.20+011	4.00+006	1.00+010	9.00+000
4.86+000	-5.91+000	1.36+002	4.00+011	1.20+011	4.00+006	1.00+010	1.00+001



DESCRIPTION:

All conditions are identical to Case 1 except that precipitation is doubled in magnitude.

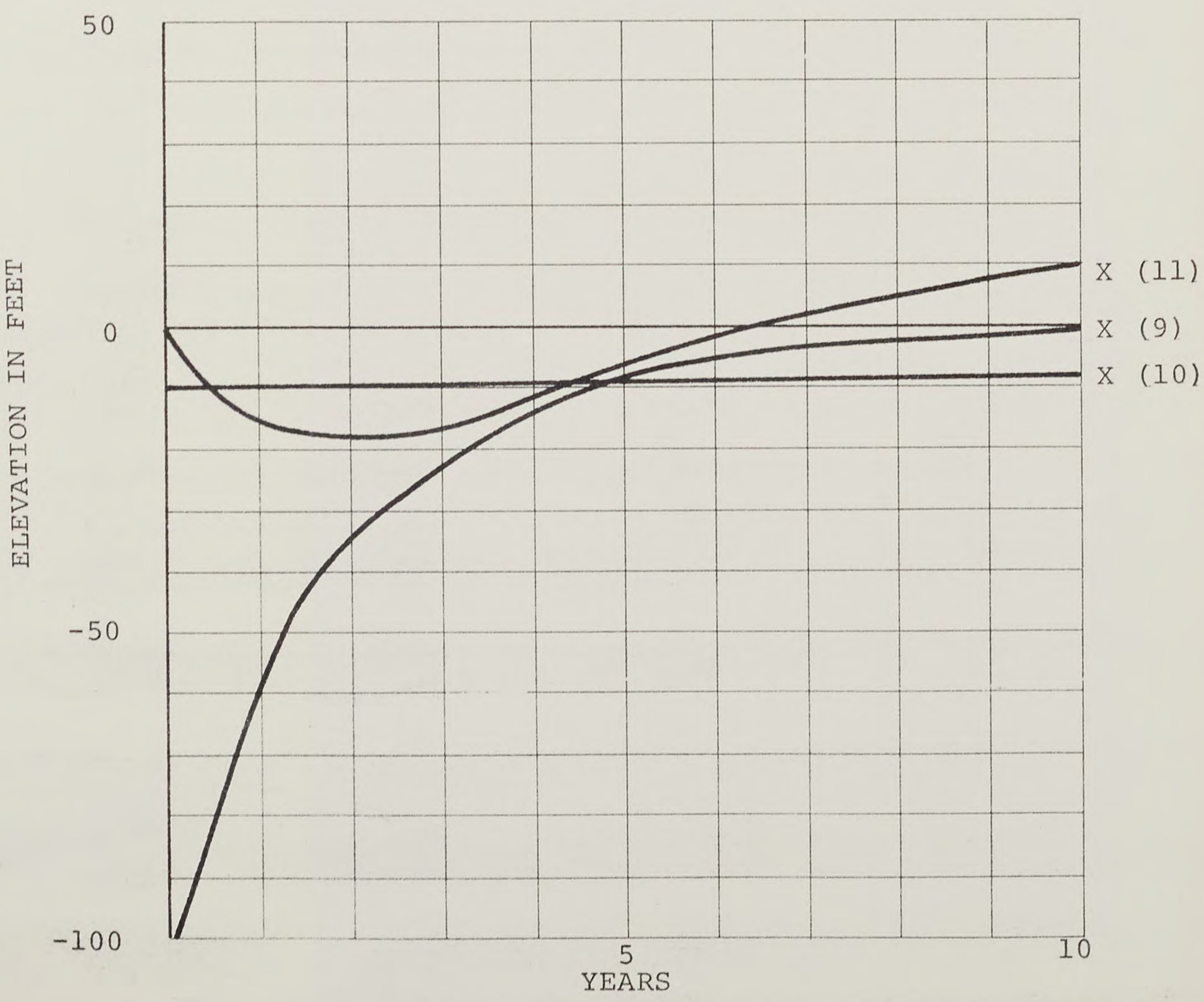
CASE 3

TRANSITION MATRIX

6.0000-001	-0.0000+000	2.0000-002	-0.0000+000	-0.0000+000	1.3000-011	-1.3000-011
-0.0000+000	9.9000-001	3.0000-003	-0.0000+000	-0.0000+000	-1.0000-011	1.0000-011
2.0000-001	4.0000-002	7.6000-001	1.3000-010	-1.3000-010	0.0000+000	0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000

STATE VECTOR

X(9)	X(10)	X(11)	Y(1)	Y(2)	Y(6)	Y(7)	Years
-1.00+002	-1.00+001	0.00+000	2.00+011	1.60+011	4.00+006	1.00+010	0.00+000
-6.01+001	-9.60+000	-1.52+001	2.00+011	1.60+011	4.00+006	1.00+010	1.00+000
-3.65+001	-9.65+000	-1.38+001	2.00+011	1.60+011	4.00+006	1.00+010	2.00+000
-2.24+001	-9.51+000	-1.63+001	2.00+011	1.60+011	4.00+006	1.00+010	3.00+000
-1.39+001	-9.36+000	-1.24+001	2.00+011	1.60+011	4.00+006	1.00+010	4.00+000
-8.73+000	-9.21+000	-7.38+000	2.00+011	1.60+011	4.00+006	1.00+010	5.00+000
-5.51+000	-9.04+000	-2.52+000	2.00+011	1.60+011	4.00+006	1.00+010	6.00+000
-3.49+000	-8.85+000	1.32+000	2.00+011	1.60+011	4.00+006	1.00+010	7.00+000
-2.19+000	-8.66+000	5.55+000	2.00+011	1.60+011	4.00+006	1.00+010	8.00+000
-1.33+000	-8.46+000	8.62+000	2.00+011	1.60+011	4.00+006	1.00+010	9.00+000
-7.56-001	-8.25+000	1.11+001	2.00+011	1.60+011	4.00+006	1.00+010	1.00+001



DESCRIPTION:

All conditions are identical to Case 1, except that evapotranspiration is increased 33% in magnitude.

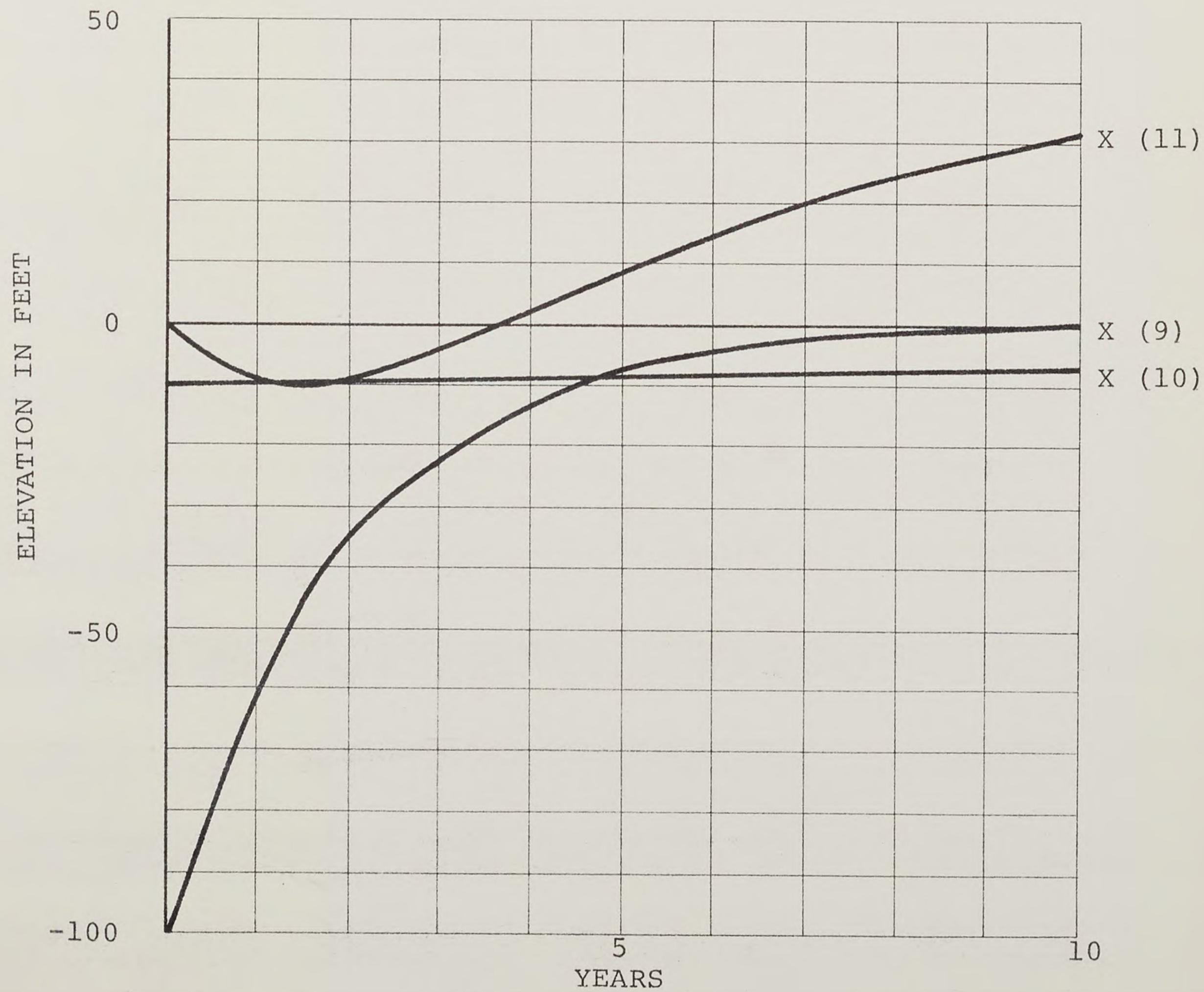
CASE 4

TRANSITION MATRIX

6.0000-001	-0.0000+000	2.0000-002	-0.0000+000	-0.0000+000	1.3000-011	-1.3000-011
-0.0000+000	9.9000-001	3.0000-003	-0.0000+000	-0.0000+000	-1.0000-011	1.0000-011
2.0000-001	4.0000-002	7.6000-001	1.3000-010	-1.3000-010	0.0000+000	0.0000+000
-0.0000+000	-0.0000+000	-1.0000+000	1.0000+000	-0.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-1.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-1.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-1.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000

STATE VECTOR

X(9)	X(10)	X(11)	Y(1)	Y(2)	Y(6)	Y(7)	Years
-1.00+002	-1.00+001	0.00+000	2.00+011	1.20+011	1.20+006	1.00+010	0.00+000
-6.01+001	-9.80+000	-1.00+001	2.00+011	1.20+011	1.20+006	1.00+010	1.00+000
-3.64+001	-9.63+000	-9.62+000	2.00+011	1.20+011	1.20+006	1.00+010	2.00+000
-2.22+001	-9.46+000	-4.58+000	2.00+011	1.20+011	1.20+006	1.00+010	3.00+000
-1.35+001	-9.28+000	2.11+000	2.00+011	1.20+011	1.20+006	1.00+010	4.00+000
-8.20+000	-9.08+000	8.95+000	2.00+011	1.20+011	1.20+006	1.00+010	5.00+000
-4.87+000	-8.87+000	1.52+001	2.00+011	1.20+011	1.20+006	1.00+010	6.00+000
-2.75+000	-8.63+000	2.06+001	2.00+011	1.20+011	1.20+006	1.00+010	7.00+000
-1.37+000	-8.38+000	2.52+001	2.00+011	1.20+011	1.20+006	1.00+010	8.00+000
-4.47-001	-8.13+000	2.89+001	2.00+011	1.20+011	1.20+006	1.00+010	9.00+000
1.80-001	-7.86+000	3.20+001	2.00+011	1.20+011	1.20+006	1.00+010	1.00+001



DESCRIPTION:

All conditions are identical to Case 1, except that river acquifer recharge is reduced 33% in magnitude.

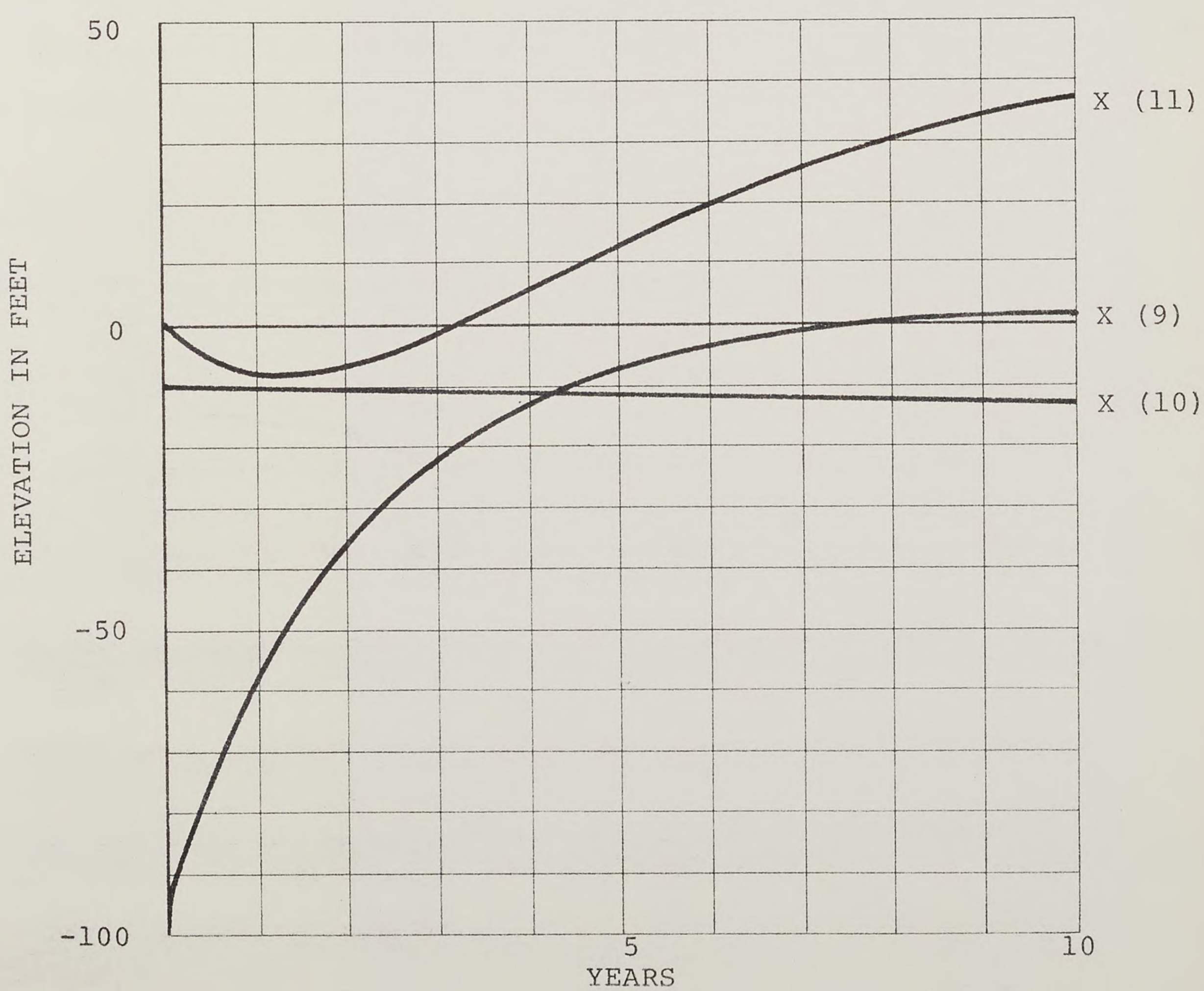
CASE 5

TRANSITION MATRIX

6.0000-001	-0.0000+000	2.0000-002	-0.0000+000	-0.0000+000	1.3000-011	-1.3000-011
-0.0000+000	9.9000-001	3.0000-003	-0.0000+000	-0.0000+000	-1.0000-011	1.0000-011
2.0000-001	4.0000-002	7.6000-001	1.3000-010	-1.3000-010	0.0000+000	0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000

STATE VECTOR

X(9)	X(10)	X(11)	Y(1)	Y(2)	Y(6)	Y(7)	Years
-1.00+002	-1.00+001	0.00+000	2.10+011	1.20+011	4.00+010	1.00+010	0.00+000
-5.96+001	-1.02+001	-3.70+000	2.10+011	1.20+011	4.00+010	1.00+010	1.00+000
-3.56+001	-1.04+001	-7.24+000	2.10+011	1.20+011	4.00+010	1.00+010	2.00+000
-2.11+001	-1.06+001	-1.33+000	2.10+011	1.20+011	4.00+010	1.00+010	3.00+000
-1.23+001	-1.08+001	6.02+000	2.10+011	1.20+011	4.00+010	1.00+010	4.00+000
-6.86+000	-1.10+001	1.34+001	2.10+011	1.20+011	4.00+010	1.00+010	5.00+000
-3.46+000	-1.12+001	2.01+001	2.10+011	1.20+011	4.00+010	1.00+010	6.00+000
-1.28+000	-1.13+001	2.58+001	2.10+011	1.20+011	4.00+010	1.00+010	7.00+000
1.36-001	-1.14+001	3.06+001	2.10+011	1.20+011	4.00+010	1.00+010	8.00+000
1.08+000	-1.15+001	3.45+001	2.10+011	1.20+011	4.00+010	1.00+010	9.00+000
1.73+000	-1.16+001	3.77+001	2.10+011	1.20+011	4.00+010	1.00+010	1.00+001



DESCRIPTION:

All conditions are identical to Case 1, except that river acquifer recharge is nearly squared in magnitude.

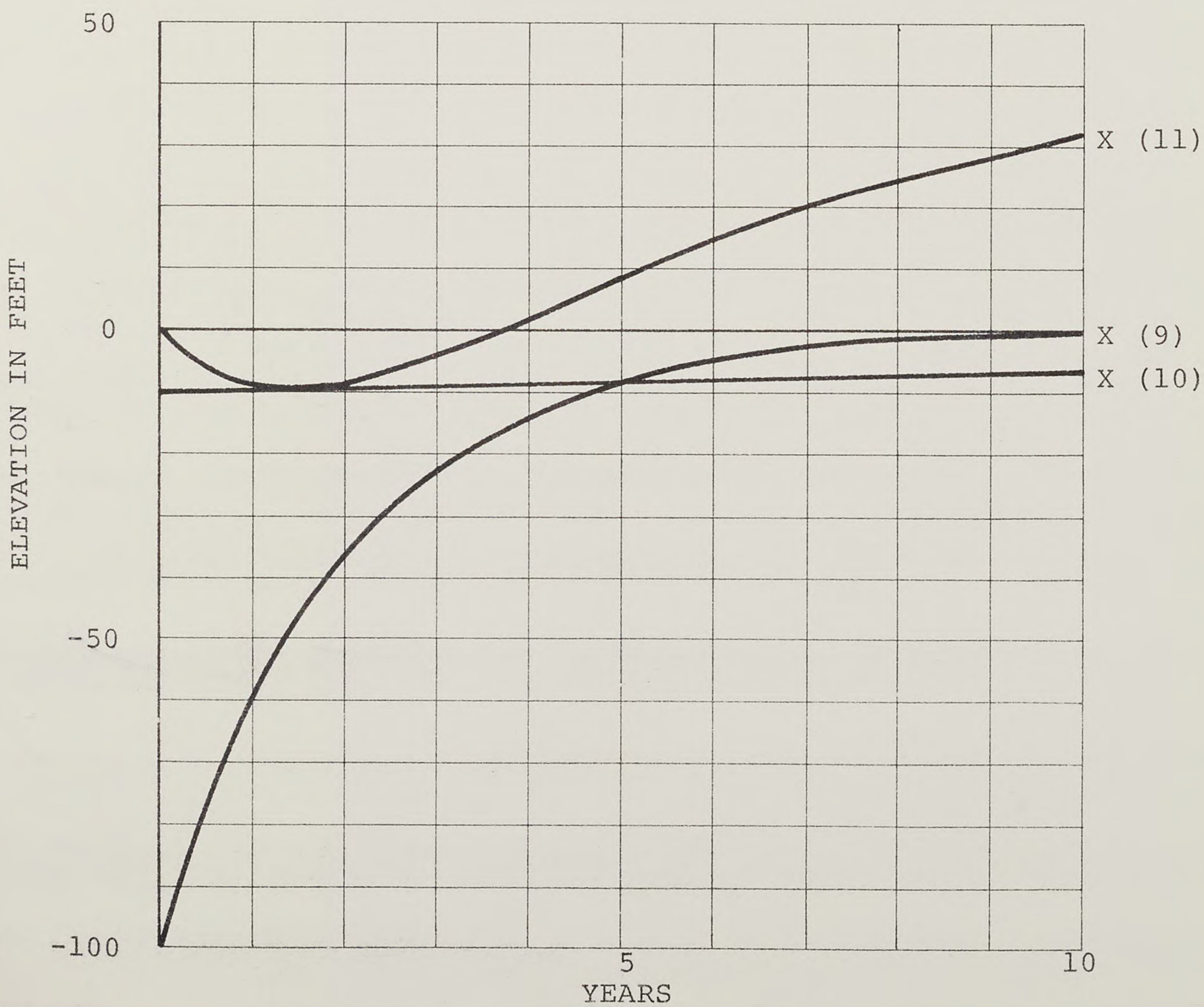
CASE 6

TRANSITION MATRIX

6.0000-001	-0.0000+000	2.0000-002	-0.0000+000	-0.0000+000	1.3000-011	-1.3000-011
-0.0000+000	9.9000-001	5.0000-003	-0.0000+000	-0.0000+000	-1.0000-011	1.0000-011
2.0000-001	4.0000-002	7.6000-001	1.3000-010	-1.3000-010	0.0000+000	0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000

STATE VECTOR

X(9)	X(10)	X(11)	Y(1)	Y(2)	Y(6)	Y(7)	Years
-1.00+002	-1.00+001	0.00+000	2.00+011	1.20+011	4.00+006	2.00+010	0.00+000
-6.03+001	-9.70+000	-1.00+001	2.00+011	1.20+011	4.00+006	2.00+010	1.00+000
-3.66+001	-9.43+000	-9.54+000	2.00+011	1.20+011	4.00+006	2.00+010	2.00+000
-2.24+001	-9.17+000	-4.65+000	2.00+011	1.20+011	4.00+006	2.00+010	3.00+000
-1.38+001	-8.89+000	2.05+000	2.00+011	1.20+011	4.00+006	2.00+010	4.00+000
-8.50+000	-8.59+000	8.85+000	2.00+011	1.20+011	4.00+006	2.00+010	5.00+000
-5.19+000	-8.26+000	1.51+001	2.00+011	1.20+011	4.00+006	2.00+010	6.00+000
-3.07+000	-7.95+000	2.05+001	2.00+011	1.20+011	4.00+006	2.00+010	7.00+000
-1.69+000	-7.61+000	2.50+001	2.00+011	1.20+011	4.00+006	2.00+010	8.00+000
-7.75-001	-7.26+000	2.86+001	2.00+011	1.20+011	4.00+006	2.00+010	9.00+000
-1.49-001	-6.90+000	3.16+001	2.00+011	1.20+011	4.00+006	2.00+010	1.00+001



DESCRIPTION:

All conditions are identical to Case 1, except that human usage is doubled in magnitude.

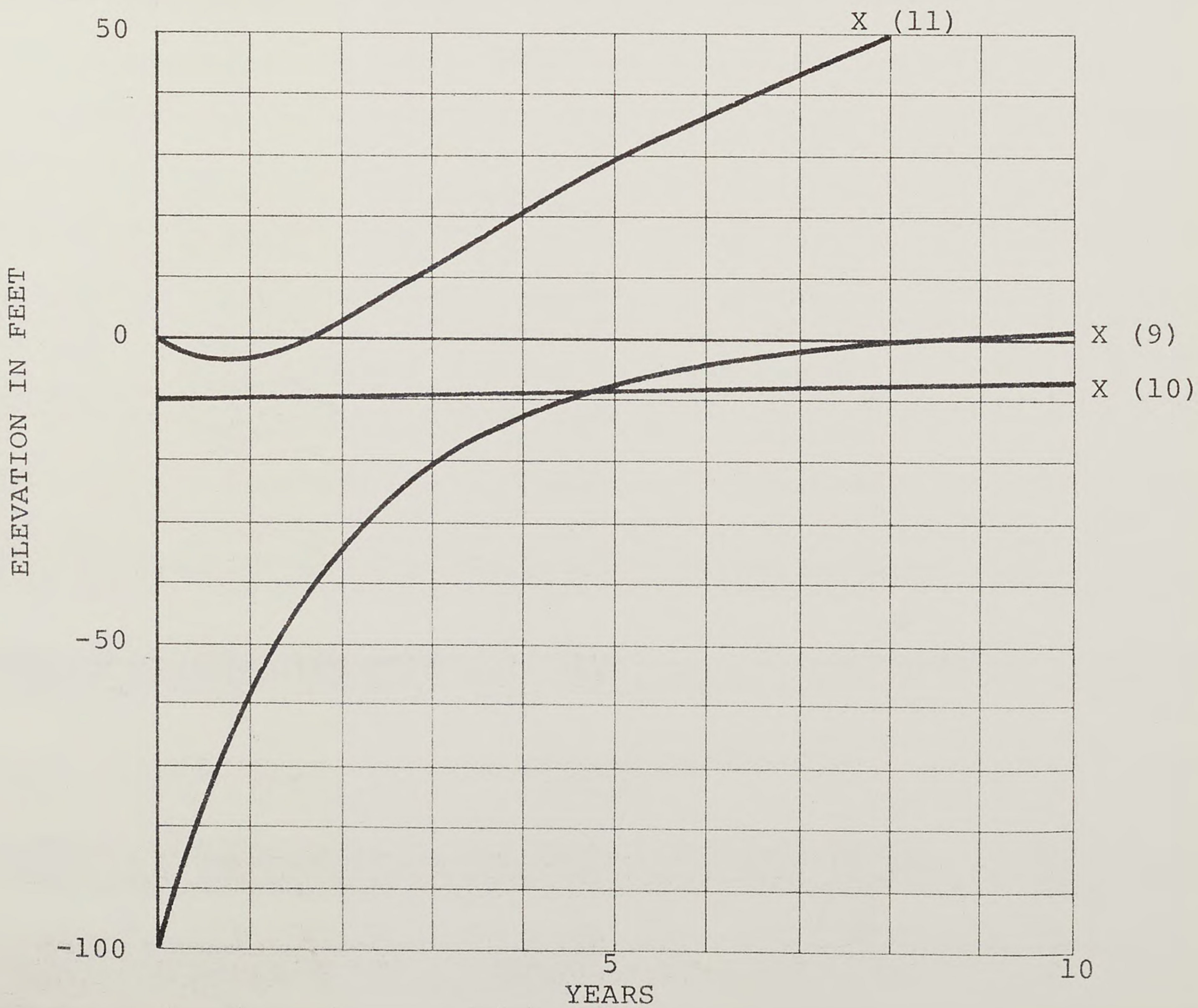
CASE 7

TRANSITION MATRIX

6.0000-001	-0.0000+000	2.0000-002	-0.0000+000	-0.0000+000	1.3000-011	-1.3000-011
-0.0000+000	9.9000-001	3.0000-003	-0.0000+000	-0.0000+000	-1.0000-011	1.0000-011
2.0000-001	4.0000-002	7.6000-001	1.3000-010	-1.3000-010	0.0000+000	0.0000+000
-0.0000+000	-0.0000+000	-1.0000+000	1.0000+000	-0.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.1000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000

STATE VECTOR

X(9)	X(10)	X(11)	Y(1)	Y(2)	Y(6)	Y(7)	Years
-1.00+002	-1.00+001	0.00+000	2.00+011	7.00+010	4.00+006	1.00+010	0.00+000
-6.01+001	-9.80+000	-3.60+000	2.00+011	7.00+010	4.00+006	1.00+010	1.00+000
-3.63+001	-9.61+000	1.82+000	2.00+011	7.00+010	4.00+006	1.00+010	2.00+000
-2.19+001	-9.41+000	1.16+001	2.00+011	7.00+010	4.00+006	1.00+010	3.00+000
-1.30+001	-9.19+000	2.02+001	2.00+011	7.00+010	4.00+006	1.00+010	4.00+000
-7.55+000	-8.93+000	2.93+001	2.00+011	7.00+010	4.00+006	1.00+010	5.00+000
-4.07+000	-8.66+000	3.73+001	2.00+011	7.00+010	4.00+006	1.00+010	6.00+000
-1.83+000	-8.36+000	4.41+001	2.00+011	7.00+010	4.00+006	1.00+010	7.00+000
-3.44-001	-8.04+000	4.97+001	2.00+011	7.00+010	4.00+006	1.00+010	8.00+000
6.58-001	-7.71+000	5.43+001	2.00+011	7.00+010	4.00+006	1.00+010	9.00+000
1.35+000	-7.37+000	5.80+001	2.00+011	7.00+010	4.00+006	1.00+010	1.00+001



DESCRIPTION:

All conditions are identical to Case 1, except that evapotranspiration is reduced 60% in magnitude.

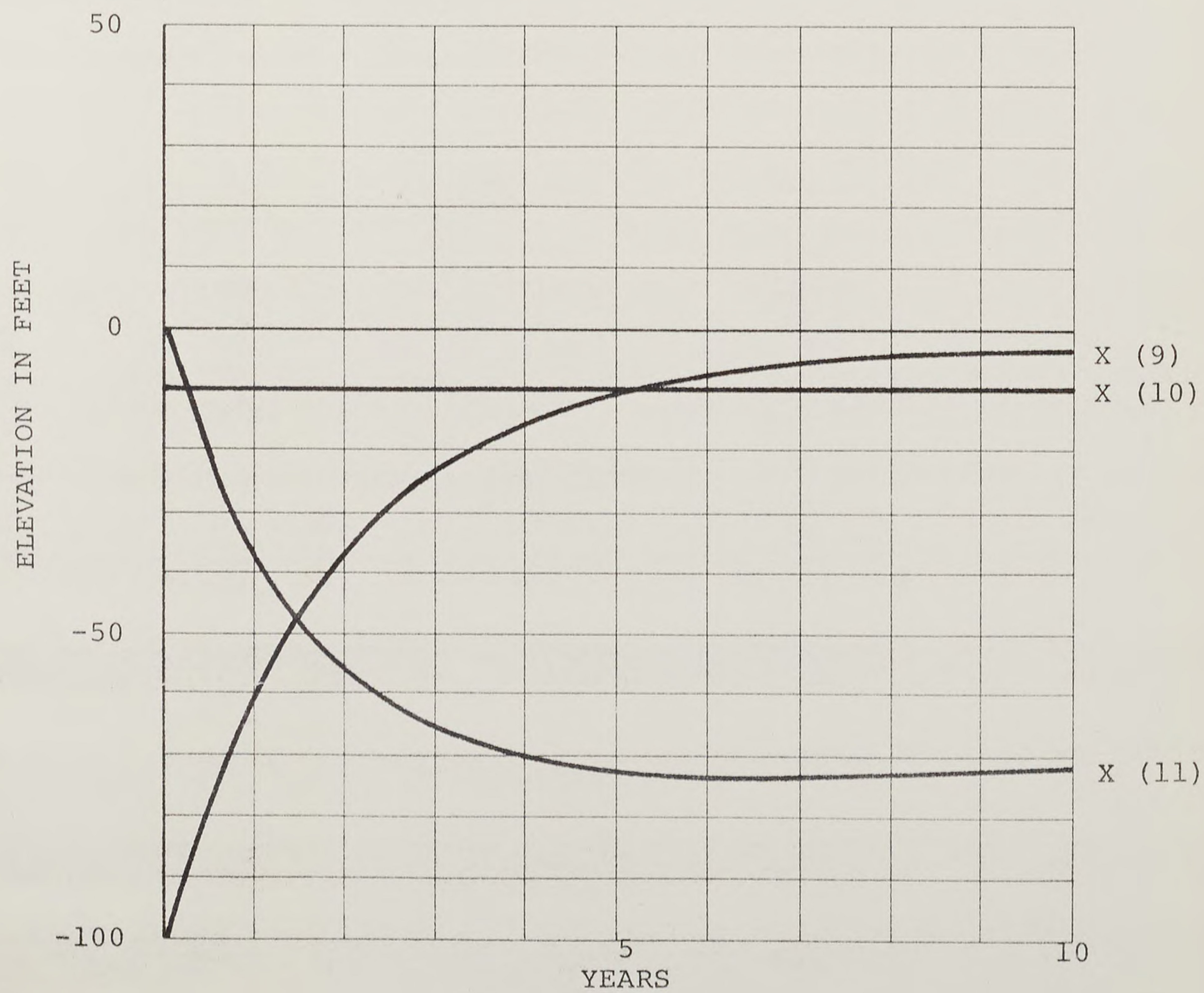
CASE 8

TRANSITION MATRIX

6.0000-001	-0.0000+000	2.0000-002	-0.0000+000	-0.0000+000	1.3000-011	-1.3000-011
-0.0000+000	9.9000-001	3.0000-003	-0.0000+000	-0.0000+000	-1.0000-011	1.0000-011
2.0000-001	4.0000-002	7.6000-001	1.3000-010	-1.3000-010	0.0000+000	0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000

STATE VECTOR

X(9)	X(10)	X(11)	Y(1)	Y(2)	Y(6)	Y(7)	Years
-1.00+002	-1.00+001	0.00+000	2.00+004	1.20+011	4.00+006	1.00+010	0.00+000
-6.01+001	-9.80+000	-3.60+001	2.00+004	1.20+011	4.00+006	1.00+010	1.00+000
-3.69+001	-9.71+000	-5.54+001	2.00+004	1.20+011	4.00+006	1.00+010	2.00+000
-2.34+001	-9.68+000	-6.55+001	2.00+004	1.20+011	4.00+006	1.00+010	3.00+000
-1.55+001	-9.68+000	-7.04+001	2.00+004	1.20+011	4.00+006	1.00+010	4.00+000
-1.08+001	-9.69+000	-7.26+001	2.00+004	1.20+011	4.00+006	1.00+010	5.00+000
-8.08+000	-9.71+000	-7.33+001	2.00+004	1.20+011	4.00+006	1.00+010	6.00+000
-6.44+000	-9.74+000	-7.36+001	2.00+004	1.20+011	4.00+006	1.00+010	7.00+000
-5.46+000	-9.76+000	-7.30+001	2.00+004	1.20+011	4.00+006	1.00+010	8.00+000
-4.87+000	-9.78+000	-7.26+001	2.00+004	1.20+011	4.00+006	1.00+010	9.00+000
-4.50+000	-9.80+000	-7.21+001	2.00+004	1.20+011	4.00+006	1.00+010	1.00+001



DESCRIPTION:

All conditions are identical to Case 1, except that precipitation is reduced to a nearly total drought situation.

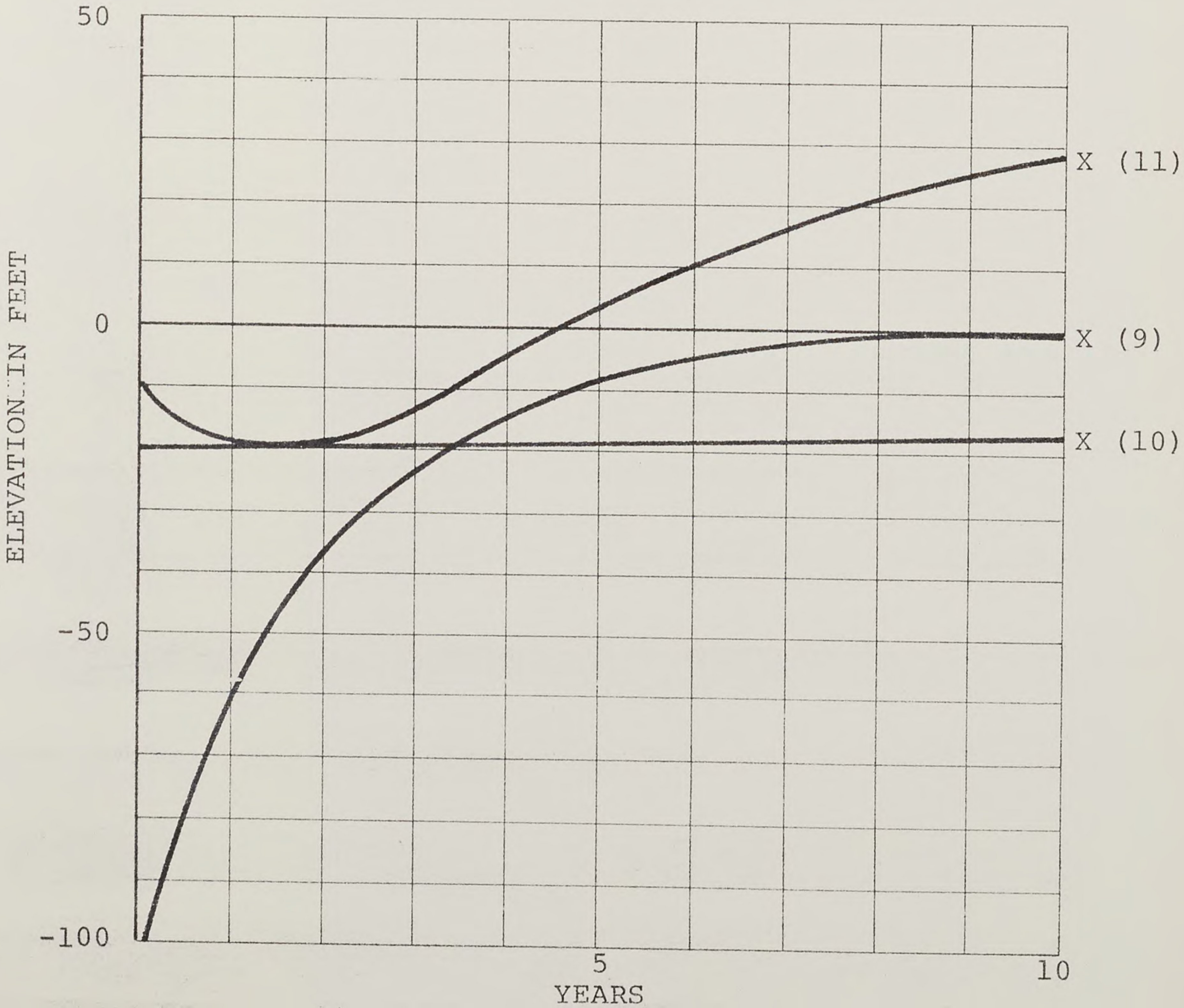
CASE 9

TRANSITION MATRIX

6.0000-001	-0.0000+000	2.0000-002	-0.0000+000	-0.0000+000	1.3000-011	-1.3000-011
-0.0000+000	9.9000-001	5.0000-003	-0.0000+000	-0.0000+000	-1.0000-011	1.0000-011
2.0000-001	4.0000-002	7.6000-001	1.3000-010	-1.3000-010	0.0000+000	0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000

STATE VECTOR

X(9)	X(10)	X(11)	Y(1)	Y(2)	Y(6)	Y(7)	Years
-1.10+002	-2.00+001	-1.00+001	2.00+011	1.20+011	4.00+006	1.00+010	0.00+000
-6.63+001	-1.97+001	-2.00+001	2.00+011	1.20+011	4.00+006	1.00+010	1.00+000
-4.03+001	-1.95+001	-1.89+001	2.00+011	1.20+011	4.00+006	1.00+010	2.00+000
-2.47+001	-1.93+001	-1.28+001	2.00+011	1.20+011	4.00+006	1.00+010	3.00+000
-1.52+001	-1.90+001	-5.02+000	2.00+011	1.20+011	4.00+006	1.00+010	4.00+000
-9.35+000	-1.87+001	2.76+000	2.00+011	1.20+011	4.00+006	1.00+010	5.00+000
-5.69+000	-1.84+001	9.90+000	2.00+011	1.20+011	4.00+006	1.00+010	6.00+000
-3.34+000	-1.81+001	1.60+001	2.00+011	1.20+011	4.00+006	1.00+010	7.00+000
-1.82+000	-1.78+001	2.12+001	2.00+011	1.20+011	4.00+006	1.00+010	8.00+000
-7.95-001	-1.74+001	2.54+001	2.00+011	1.20+011	4.00+006	1.00+010	9.00+000
-9.84-002	-1.71+001	2.89+001	2.00+011	1.20+011	4.00+006	1.00+010	1.00+001



DESCRIPTION:

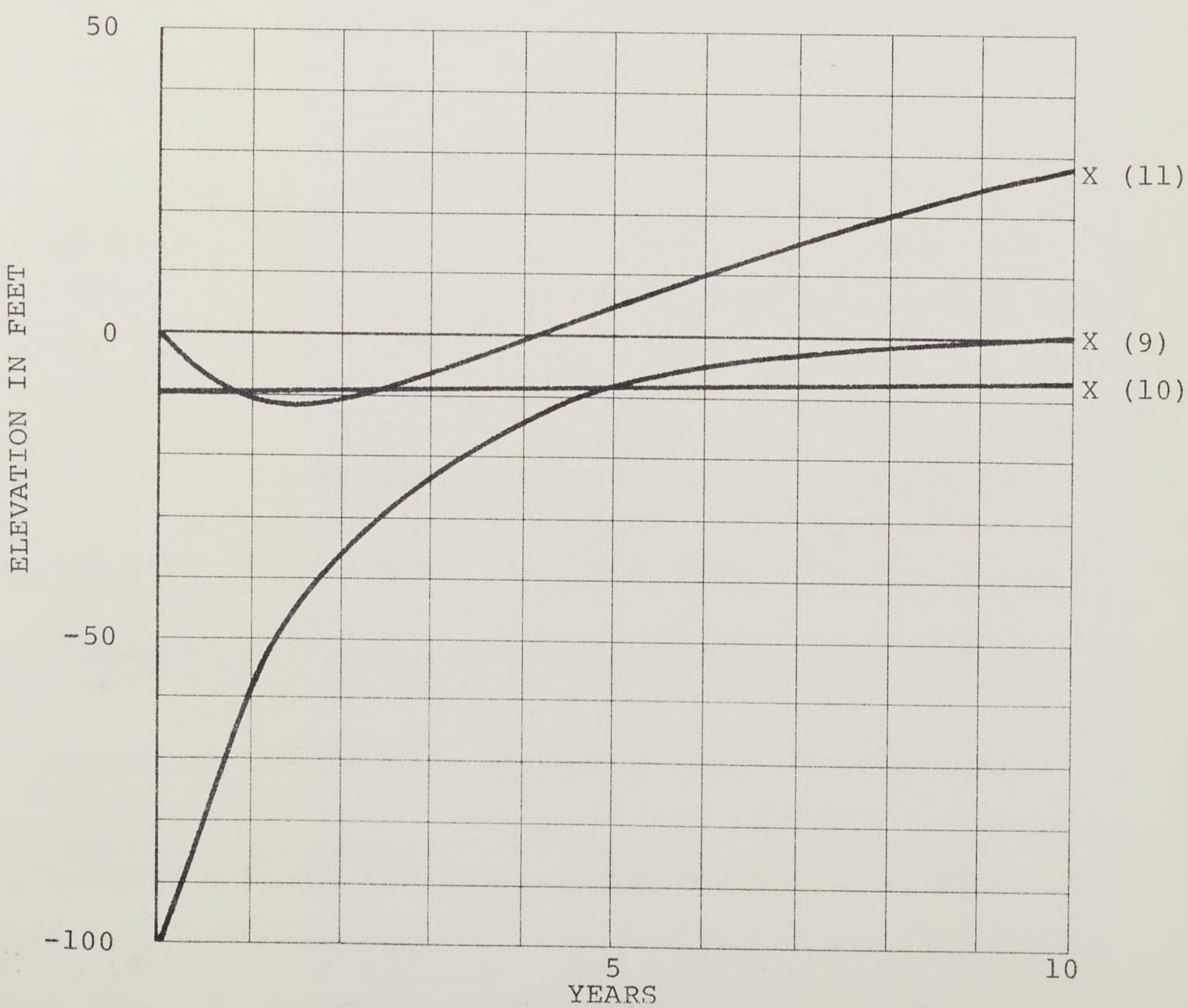
Inputs (Y's) are identical to Case 1, however, initial conditions (X's) are altered to show the influence of elevations on system solutions.

CASE 10

TRANSITION MATRIX

TRANSITION MATRIX	6.0000-001	-0.0000+000	2.0000-002	-0.0000+000	-0.0000+000	1.3000-011	-1.3000-011
	-0.0000+000	9.9000-001	3.0000-003	-0.0000+000	-0.0000+000	-1.0000-011	1.0000-011
	2.0000-001	4.0000-002	7.5000-001	1.3000-010	-1.3000-010	0.0000+000	0.0000+000
	-0.0000+000	-0.0000+000	-1.0000+000	1.0000+000	-0.0000+000	-0.0000+000	-0.0000+000
	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000
	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000
	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000

	X(9)	X(10)	X(11)	Y(1)	Y(2)	Y(6)	Y(7)	Years
STATE VECTOR	-1.00+002	-1.00+001	0.00+000	3.00+011	2.30+011	5.00+006	8.00+009	.00+000
	-6.01+001	-9.82+000	-1.13+001	3.00+011	2.30+011	5.00+006	8.00+009	1.00+000
	-3.64+001	-9.68+000	-1.19+001	3.00+011	2.30+011	5.00+006	8.00+009	2.00+000
	-2.22+001	-9.55+000	-7.51+000	3.00+011	2.30+011	5.00+006	8.00+009	3.00+000
	-1.36+001	-9.36+000	-1.90+000	3.00+011	2.30+011	5.00+006	8.00+009	4.00+000
	-8.27+000	-9.21+000	4.87+000	3.00+011	2.30+011	5.00+006	8.00+009	5.00+000
	-4.97+000	-9.03+000	1.08+001	3.00+011	2.30+011	5.00+006	8.00+009	6.00+000
	-2.87+000	-8.82+000	1.59+001	3.00+011	2.30+011	5.00+006	8.00+009	7.00+000
	-1.51+000	-8.61+000	2.13+001	3.00+011	2.30+011	5.00+006	8.00+009	8.00+000
	-6.03-001	-8.38+000	2.39+001	3.00+011	2.30+011	5.00+006	8.00+009	9.00+000
	1.20-002	-8.15+000	2.68+001	3.00+011	2.30+011	5.00+006	8.00+009	1.00+001



DESCRIPTION:

Initial conditions are identical to Case 1, however the inputs represent spring type conditions. Precipitation is increased 50%, evapo-transpiration is increased 50%, river acquifer recharge is increased 20% and pumpage is decreased substantially.

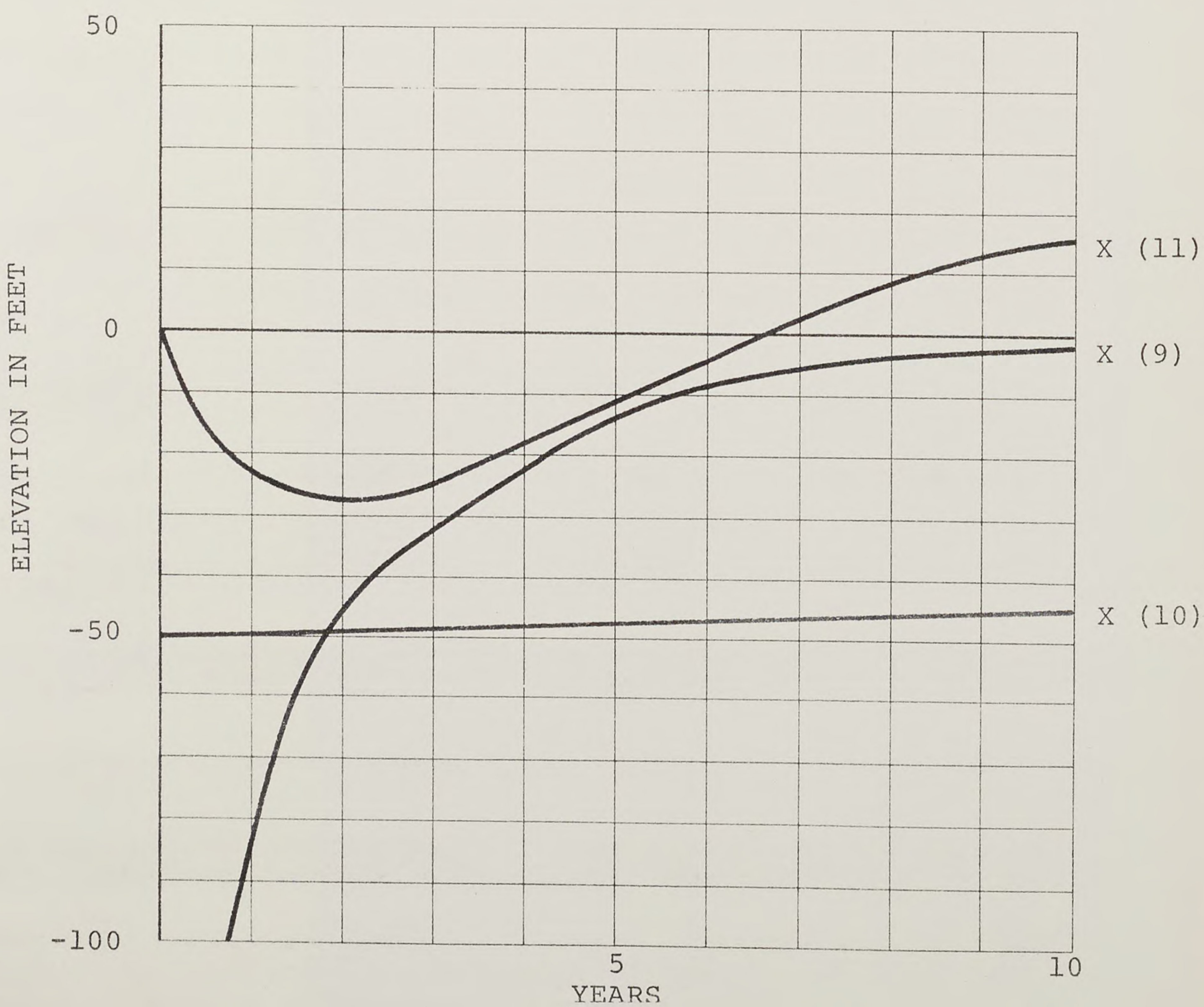
CASE 11

TRANSITION MATRIX

6.0000-001	-0.0000+000	2.0000-002	-0.0000+000	-0.0000+000	1.3000-011	-1.3000-011
-0.0000+000	9.9000-001	3.0000-003	-0.0000+000	-0.0000+000	-1.0000-011	1.0000-011
2.0000-001	4.0000-002	7.6000-001	1.3000-010	-1.3000-010	0.0000+000	0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000

STATE VECTOR

X(9)	X(10)	X(11)	Y(1)	Y(2)	Y(6)	Y(7)	Years
-1.50+002	-5.00+001	0.00+000	3.00+011	2.30+011	5.00+006	8.00+009	0.00+000
-9.01+001	-4.94+001	-2.29+001	3.00+011	2.30+011	5.00+006	8.00+009	1.00+000
-5.46+001	-4.89+001	-2.83+001	3.00+011	2.30+011	5.00+006	8.00+009	2.00+000
-3.34+001	-4.84+001	-2.53+001	3.00+011	2.30+011	5.00+006	8.00+009	3.00+000
-2.07+001	-4.79+001	-1.87+001	3.00+011	2.30+011	5.00+006	8.00+009	4.00+000
-1.29+001	-4.74+001	-1.12+001	3.00+011	2.30+011	5.00+006	8.00+009	5.00+000
-8.06+000	-4.69+001	-3.39+000	3.00+011	2.30+011	5.00+006	8.00+009	6.00+000
-5.02+000	-4.64+001	2.66+000	3.00+011	2.30+011	5.00+006	8.00+009	7.00+000
-3.06+000	-4.58+001	3.25+000	3.00+011	2.30+011	5.00+006	8.00+009	8.00+000
-1.78+000	-4.53+001	1.29+001	3.00+011	2.30+011	5.00+006	8.00+009	9.00+000
-9.10-001	-4.47+001	1.68+001	3.00+011	2.30+011	5.00+006	8.00+009	1.00+001



DESCRIPTION:

Inputs are identical to Case 10, except that initial conditions are altered considerably to show the influence of elevations on the overall system function.

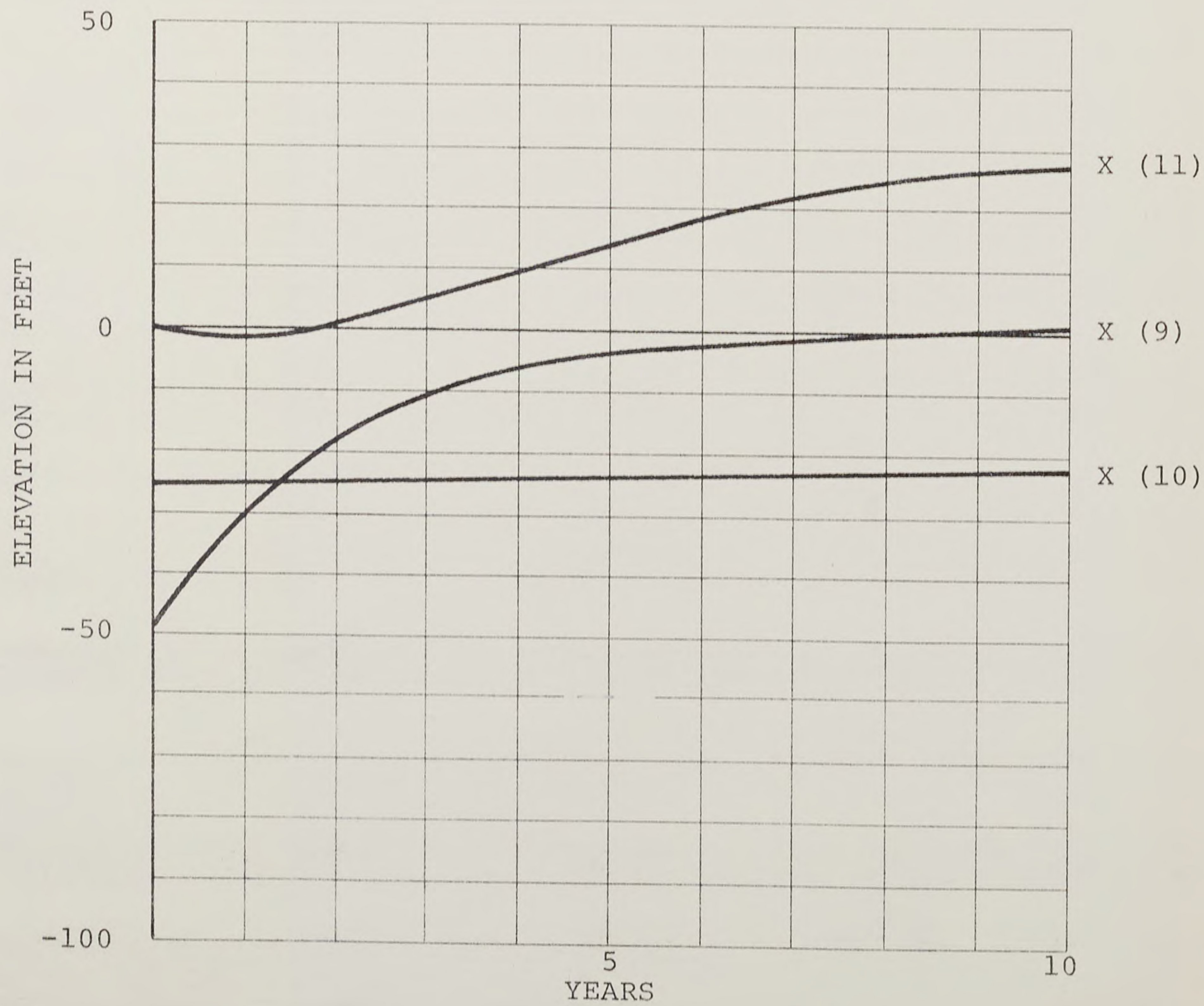
CASE 12

TRANSITION MATRIX

6.0000-001	-0.0000+000	2.0000-002	-0.0000+000	-0.0000+000	1.3000-011	-1.3000-011
-0.0000+000	9.9010-001	3.0000-003	-0.0000+000	-0.0000+000	-1.0000-011	1.0000-011
2.0000-001	4.0000-002	7.6000-001	1.3000-010	-1.3000-010	0.0000+000	0.0000+000
-0.0000+000	-0.0000+000	-1.0000+000	1.0000+000	-0.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-1.0000+000	-0.0000+000	1.0000+000	-0.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-1.0000+000	-0.0000+000	-0.0000+000	1.0000+000	-0.0000+000
-0.0000+000	-0.0000+000	-1.0000+000	-0.0000+000	-0.0000+000	-0.0000+000	1.0000+000

STATE VECTOR

X(9)	X(10)	X(11)	Y(1)	Y(2)	Y(6)	Y(7)	Years
-5.00+001	-2.50+001	1.00+000	3.00+011	2.30+011	5.00+006	8.00+009	0.00+000
-3.01+001	-2.47+001	-1.90+000	3.00+011	2.30+011	5.00+006	8.00+009	1.00+000
-1.82+001	-2.43+001	6.46-001	3.00+011	2.30+011	5.00+006	8.00+009	2.00+000
-1.10+001	-2.40+001	4.96+000	3.00+011	2.30+011	5.00+006	8.00+009	3.00+000
-6.61+000	-2.37+001	9.72+000	3.00+011	2.30+011	5.00+006	8.00+009	4.00+000
-3.88+000	-2.33+001	1.42+001	3.00+011	2.30+011	5.00+006	8.00+009	5.00+000
-2.15+000	-2.30+001	1.82+001	3.00+011	2.30+011	5.00+006	8.00+009	6.00+000
-1.03+000	-2.26+001	2.16+001	3.00+011	2.30+011	5.00+006	8.00+009	7.00+000
-2.89-001	-2.23+001	2.44+001	3.00+011	2.30+011	5.00+006	8.00+009	8.00+000
2.11-001	-2.19+001	2.67+001	3.00+011	2.30+011	5.00+006	8.00+009	9.00+000
5.56-001	-2.15+001	2.86+001	3.00+011	2.30+011	5.00+006	8.00+009	1.00+001



DESCRIPTION:

Inputs are identical to Case 10, except that initial conditions are altered considerably to show the influence of elevations on the overall system function.

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