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AN EXPONENTIAL RELATIONSHIP
BETWEEN STRESS AND STRAIN WITH
APPLICATION TO PLASTIC BENDING

Thesis for the Degree of M. S.

MICHIGAN STATE COLLEGE

Nelson David Wolf

1955

This is to certify that the
thesis entitled
AN EXPONENTIAL RELATIONSHIP BETWEEN STRESS AND STRAIN
WITH APPLICATION TO PLASTIC BENDING

presented by

NELSON D. WOLF

has been accepted towards fulfillment
of the requirements for
MASTER OF SCIENCE degree in APPLIED MECHANICS

Charles O. Harris
Major professor

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AN EXPONENTIAL RELATIONSHIP BETWEEN STRESS AND
STRAIN WITH APPLICATION TO PLASTIC BENDING

by

Nelson David Wolf

A THESIS

Submitted to the School of Graduate Studies of Michigan
State College of Agriculture and Applied Science
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AN ABSTRACT

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Year 1955

Approved Charles O. Harris

The objectives of this study were to find a suitable mathematical relation between stress and strain; to find a relation between stress and bending moment where the stress-strain law is non-linear; and to check this relation by experiments.

The properties desired in a stress-strain relation are listed, and a formula between stress and strain was found. From this relation, the bending moment for a given cross-section was found as a function of the strain at the outer surface. Using the stress-strain curve, a plot of $\frac{Mc}{I}$ versus the maximum stress on the section was drawn.

From this study it was concluded that the proposed mathematical relation is suitable and can represent stress-strain data for certain types of magnesium and aluminum alloys. It was also concluded that the stress at the outer surface can be fairly well estimated by the use of this relation.

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STATEMENT OF THE PROBLEM

The objectives of the problem are: to find a suitable mathematical relation between stress and strain; to find a relation between stress and bending moment when the stress-strain law is non-linear; and to check this relation by suitable experiments.

CHAPTER I

MATHEMATICAL RELATIONSHIPS BETWEEN STRESS AND STRAIN WITH APPLICATION TO MAGNESIUM AND ALUMINUM ALLOYS

Introduction

The problem of representing stress-strain data by an analytical expression is not a new one. Many studies have been made on the subject with no one answer being the result. Below the proportional limit of the material Hooke's law seems to be universally accepted, but beyond this point no one formula is in general use.

Properties Desired in a Formula

In seeking formulas to represent stress-strain data, it is possible to state certain properties that a formula should have. This statement makes it possible to eliminate quickly certain types of formulas that otherwise would seem adequate at the first glance. The following is a list of such properties.¹

1. The equation should be simple.

¹ "Stress-Strain Formulas," W. R. Osgood, Journal of Aero. Science, Vol. 13, Jan. 1946, pp. 45-48.

2. It should be able to represent a large number of engineering materials.
3. It should go through the origin.
4. There should be at least two parameters in the formula, and they should be easy to calculate.
5. The slope at the origin should be equal to the modulus of elasticity.
6. The equation should be easy to integrate when substituted in integrals which arise in developing the theory of plastic bending or column analysis.

Each formula should have the properties listed above if it is to be useful in the general case.

Previously Proposed General Formulas

The following² is a list of formulas taken from the literature. The notation used is:

ϵ is unit strain,

$$\sigma = \frac{S}{E},$$

S is unit stress,

E is the slope of the stress-strain curve at the origin,

e is the base for the natural log, and

a, b, c, d, k, or α , β , γ are parameters.

² Ibid.

1. Hooke's law, the universal stress-strain formula,

$$\epsilon = \sigma,$$

usually written as,

$$S = E\epsilon,$$

needs no comment.

2. In 1729 Bulffingeri suggested,

$$\epsilon = k\sigma^n,$$

and it has been used in spite of the fact that the slope at the origin is not E.

3. Riccati proposed two formulas,

$$\sigma = k\epsilon^{-1/\epsilon},$$

and

$$\sigma = \frac{1}{a} \left(e^{ae/(1+\epsilon)} - 1 \right),$$

but they have only one parameter each, and the slope at the origin is not equal to E.

4. Gerstner's formula,

$$\sigma = \epsilon + b\epsilon^2,$$

also has only one parameter and therefore cannot be general.

5. In general, there is no objection to Poncelet's formula except that the parameters are hard to find. He suggested,

$$\epsilon = \sigma \left[1 + \beta (\epsilon^{\alpha\sigma} - 1) \right]$$

6. In Wertheim's formula,

$$\epsilon^2 = \alpha \sigma + \beta \sigma^2,$$

the slope at the origin is zero.

7. Hodgkinson's formula,

$$\sigma = \epsilon + b\epsilon^2 + c\epsilon^3 + d\epsilon^4,$$

is no more than a power series in ϵ ,
and the parameters are hard to find.

8. Cox suggested three formulas,

$$\epsilon = \sigma / (1 + \alpha \sigma)$$

$$\epsilon = \sigma + \beta \sigma^2 + \gamma \sigma^3$$

$$\sigma = \epsilon + b\epsilon^3 + c\epsilon^3$$

The first has only one parameter and the other two
are limited to only a few materials.

9. Inbert's formula,

$$\epsilon = \left(\frac{1}{\alpha} \right) (\epsilon^{\alpha \sigma} - 1),$$

has only one parameter.

10. Hartiz suggested two equations,

$$\sigma = \left(\frac{1}{\alpha} \right) (\epsilon^{\alpha \epsilon} - 1)$$

$$\sigma = \left[\epsilon / (1 - \epsilon) \right] \epsilon^{\alpha \epsilon}$$

but they both have only one parameter which is not
enough to represent accurately wide ranges of stress-
strain data.

11. In Schiile's formula,

$$\sigma = a\epsilon^n + b\epsilon^2,$$

the slope at the origin is not E.

12. Prager's formula,

$$\sigma = a\epsilon + b \tanh[(1-a)/b]\epsilon,$$

is good for many materials with a wide range of strain values, but the parameters are hard to find, and in problems in plastic bending the integrals do not integrate in closed form.

13. Holmquist and Nadai suggested the use of two formulas,

$$\epsilon = \sigma$$

$$\epsilon = \sigma + k(\sigma - \sigma_p)^n \quad \sigma \geq \sigma_p$$

where σ_p is the proportional limit stress.

The two form a powerful combination but the parameters are hard to find in the second equation.

14. Osgood and Ramberg suggested,

$$\epsilon = \sigma + k\sigma^n,$$

which represents a wide range of materials and the parameters are easy to find.

15. Rao and Leggett suggested,

$$\epsilon = \sigma + \beta(\cosh \alpha\sigma - 1)$$

but the parameters are hard to find for the best fit.

The Proposed Formula

After making an intensive study of the subject, Osgood and Ramberg suggested their formula,

$$\epsilon = \sigma + k\sigma^n$$

which is a special case of Holmquist's and Nadai's equation with σ_p equal to zero. They thought that this relationship was a good approximation to the stress-strain data and possessed the other properties desired in a formula.

The relationship suggested here is similar to that of Osgood and Ramberg with an interchange in the variables. The equation now takes the form,

$$\sigma = \epsilon + k\epsilon^n.$$

This is similar to the relationship suggested by Gerstner except that there are two parameters, whereas his formula replaces n by the numerical value of two.

The relationship does possess the properties discussed previously. First, the equation is certainly simple. Second, as is shown later, it can represent the stress-strain data of certain types of aluminum and magnesium alloys, and can therefore represent curves of similar shape for other materials. It best represents materials with a slight break in the lower range of the stress-strain curve. Third, the curve goes through the origin, and fourth, the parameters are easy to calculate. Fifth, the slope at the origin is equal to E , and sixth, the equation can be integrated in closed form when substituted in the integrals of plastic bending.

It can be seen that the relationship possesses the desired properties, but the equation does have limitations. In

attempting to apply the equation to a number of different actual stress-strain curves for magnesium and aluminum alloys, as well as a number of hypothetical curves, the following observations were made.

The first and biggest limitation found was the fact that the equation holds good over only a portion of the entire stress-strain curve. For the cases tried, and for the best fit, it was found that the equation holds for values of strain up to the yield strength, at 0.2 percent offset, of the material in every case, and in some cases up to one and one-half to two times the yield strength strain. Therefore it can be seen that the equation cannot be used over the entire stress-strain curve, that is up to rupture of the material, and the point up to which it holds is best found by applying the relation to the particular data.

The second limitation found was the fact that for the cases tried, usually, a perfect fit could not be obtained. The closeness of the fit depended upon the range to which the equation was being applied, as well as the material itself. The lower part of the mathematical curve is not a straight line like that found in many materials, but when plotted and drawn to a reasonable scale it cannot be distinguished from it.

Method of Application of the Formula

The general equation can be written in the following form.

$$\sigma = \epsilon + k \epsilon^n \quad (1)$$

$$\sigma - \epsilon = k \epsilon^n \quad (2)$$

$$\log (\sigma - \epsilon) = \log k + n \log \epsilon \quad (3)$$

The parameters to be found are k and n . Plotting values of $(\sigma - \epsilon)$ versus ϵ , obtained from actual tension and compressive tests, on log-log paper should give a straight line if the data is to fit this type of curve. If the result is a straight line, or nearly so, the constant k is the intercept, and n the actual slope of the line on log-log paper.

Before going further, the question of algebraic signs should be investigated. Let M be any point on the stress-strain curve past the proportional limit OP . See Fig. 1. The coordinates of this point are (ϵ_1, S_1) . Let E be the slope of the straight part of the curve. PN is a continuation of the straight line OP and has slope E . From the Fig.,

$$\epsilon_2 = \frac{S_1}{E} ,$$

and by definition,

$$\frac{S_1}{E} = \sigma_1 .$$

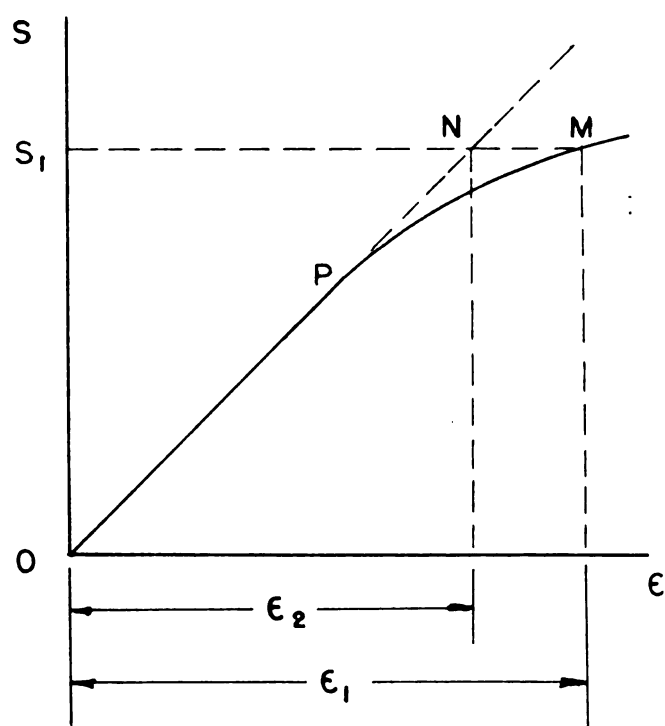


FIG. 1

Again from the Fig.,

$$\epsilon_1 > \epsilon_2 \quad ,$$

or
$$\epsilon_1 > \sigma_1 \quad .$$

In general

$$\epsilon \geq \sigma$$

and $(\sigma - \epsilon)$ is a negative quantity if $\epsilon = \sigma$ is not considered.³

From equation (2) it is seen that k must be a negative number. Therefore the absolute values of the quantities must be considered in equation (3).

$$\log |\sigma - \epsilon| = \log |k| + n \log \epsilon \quad (4)$$

The general procedure in calculating k and n is as follows.

1. Pick a number of points from the stress-strain curve that is to be fitted.
2. For each point (ϵ, S) , calculate σ .
3. Calculate $|\sigma - \epsilon|$.
4. Plot $|\sigma - \epsilon|$ versus ϵ on log-log paper.
5. Draw a straight line through the points.
6. Find the actual slope of this line which is n .⁴

³ For $\epsilon = \sigma$, the curve is a straight line. This is very often the case for small values of ϵ . When ϵ is very small, the term $k\epsilon^n$ is much smaller than ϵ , and when plotting the curve the term ϵ cannot be distinguished from $\epsilon - k\epsilon^n$.

⁴ It may be found that the curve is not a perfect straight line. In this case a compromise must be made in drawing the line and determining the slope n .

7. Substitute a typical point in equation (4) and calculate k .⁵
8. Plot the curve just obtained and compare with the original stress-strain data.

Application to Magnesium and Aluminum Alloys

The aluminum alloy used was ordinary stock, Alcoa 24S-T4.⁶ Six tensile specimens and six compression specimens were tested. They were cut from two different bars, each twelve feet long; one being of square cross-section one inch on a side; the other of circular cross-section one inch in diameter. One of each type of specimen was cut from each end, and one of each type from the middle of each bar.⁷

The magnesium alloy was a laboratory produced experimental material of a type designated ZK60A-B by the Dow Chemical Company. The bar was 30 inches long and had a circular cross-section one inch in diameter. One tension specimen, and one compression specimen was cut from each end.

⁵ Actually k could be read directly from the graph, but due to the physical size of the graph paper and numerical size of numbers involved, it is best found in this manner.

⁶ For properties and heat treat see, Alcoa Structural Handbook, Aluminum Company of America, 1950, pp. 14, 15, 22.

⁷ The reason for this procedure will be seen in Chapter III.

The tensile specimens were standard⁸ 0.505 inches in diameter. The compression specimens were three inches long and one inch in diameter.

The stress-strain tests were performed with a 60,000 pound, universal testing machine, with automatic stress-strain recorder.

The procedure followed was that normally used when running standard tensile and compressive tests.

The tests results for the aluminum alloy, square section, are shown in Fig. 2, for the circular section, Fig. 3. The tests results for the magnesium alloy are shown in Fig. 4.

The following example shows the procedure used in determining the stress-strain equation for the aluminum alloy. Notice how steps one, two and three have been tabulated in Table I for easier calculation. Fig. 5 shows steps four, five and six, where the graph has been plotted, the line drawn, and the slope obtained from it. Step seven is to pick a point from the graph and calculate the value of k .

Thus, for a point at which

$$\epsilon = 0.007$$

$$|\sigma - \epsilon| \neq 0.0021$$

and

$$n = 3$$

$$\log_{10} |.0021| = \log_{10} |k| + 3 \log_{10} (.007)$$

⁸ For standard tensile specimens see ASTM Standards, 1952, Part I, pp. 506, 507.

TENSION AND COMPRESSION TESTS

For Alcoa 24S-T4 Al

SQUARE SECTION

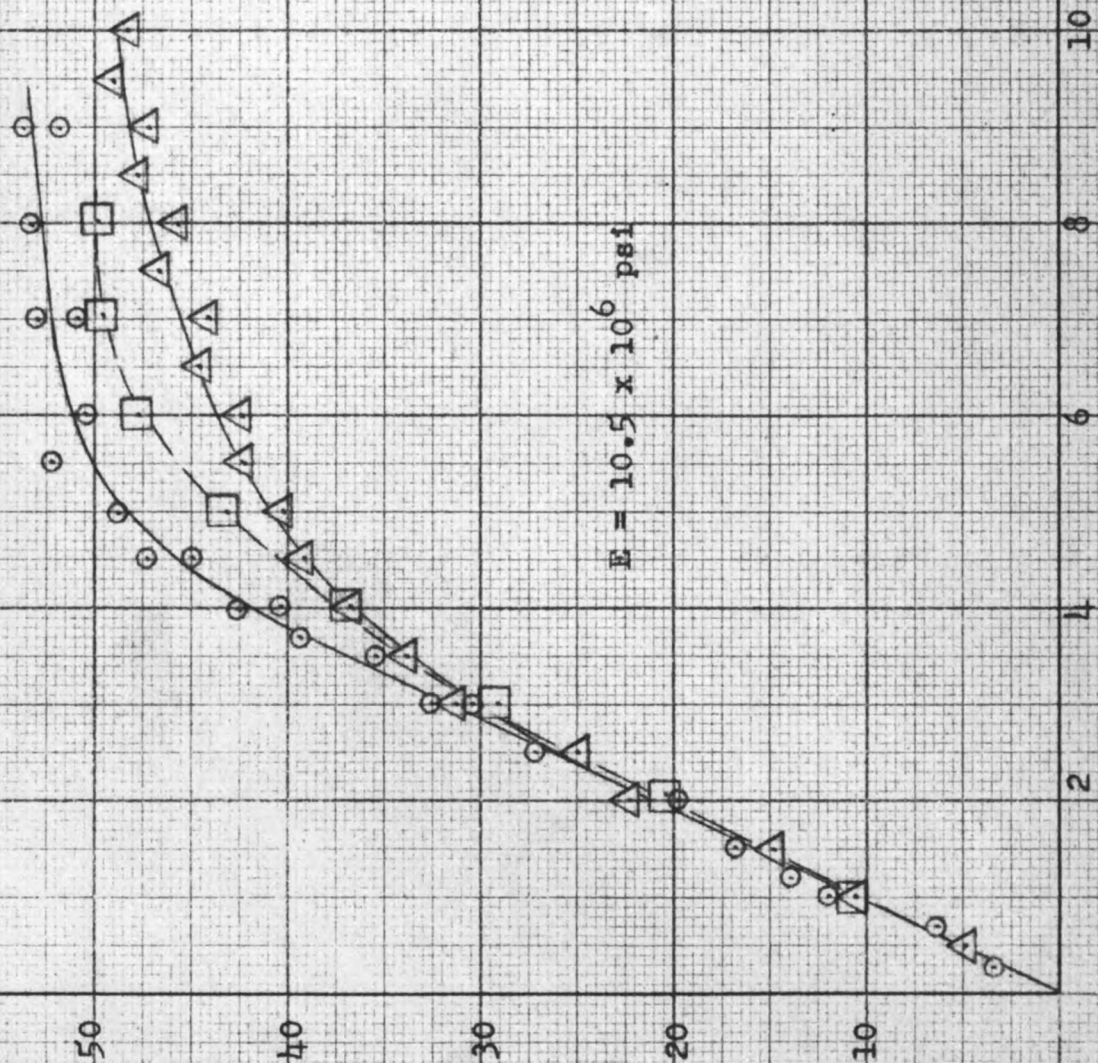
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Fig. 2

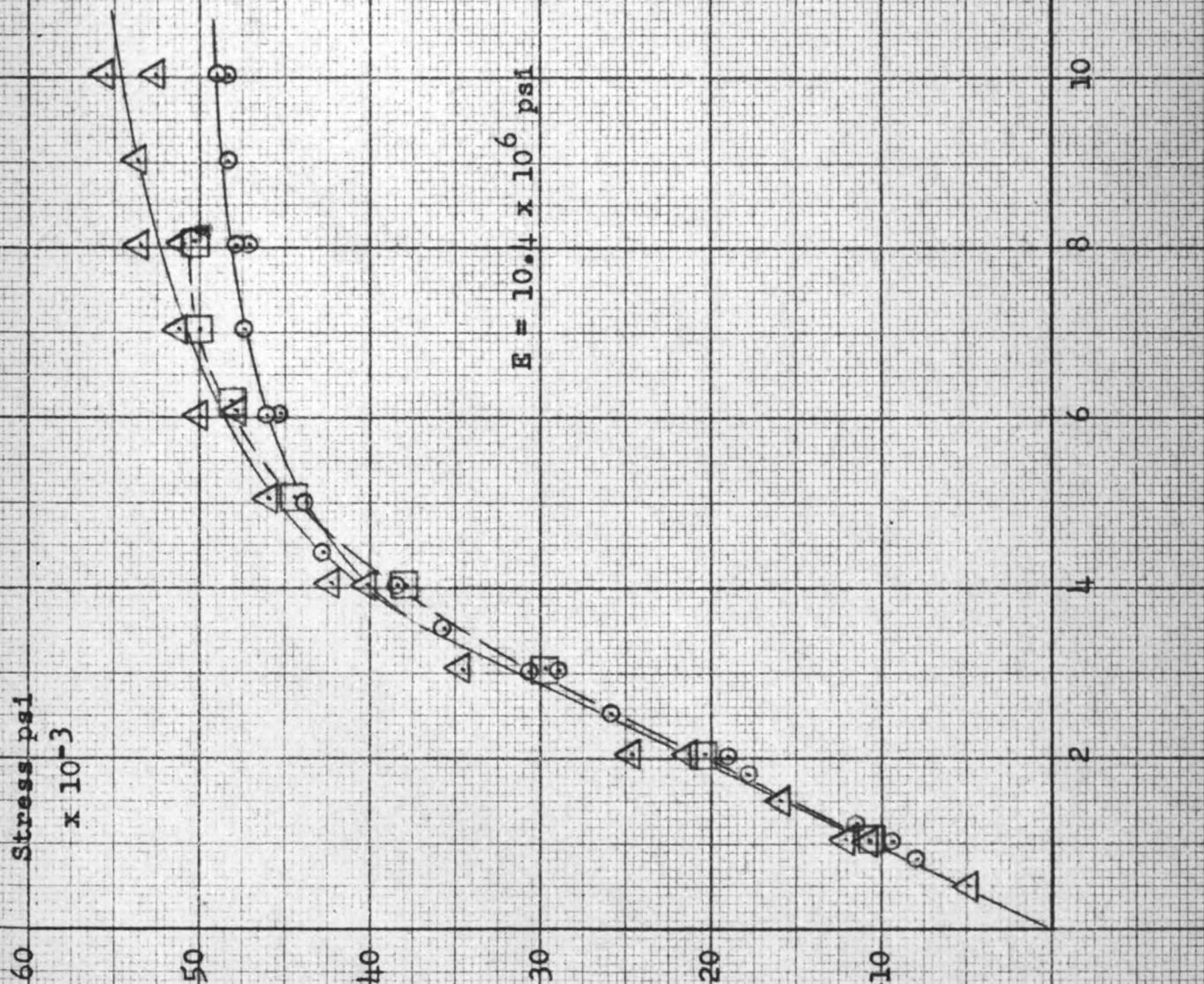
Stress, psi
 $\times 10^3$ $E = 10.5 \times 10^6 \text{ psi}$

○ — Tension
 △ — Compression
 □ — $\sigma = \epsilon = 6100 \epsilon^3$

Strain in./in.
 $\times 10^3$ 

TENSION AND COMPRESSION TESTS
For Alcoa 24S-T4 Al.
CIRCULAR SECTION
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Fig. 3



TENSION AND COMPRESSION TESTS
For Mg, Type ZK60A-B By Dow Co.
CIRCULAR SECTION
Nelson Wolf 12-3-54

Fig. 4

Stress psi
 $\times 10^{-3}$

$E = 6.3 \times 10^6$ psi

○ --- Tension
△ --- Compression
□ --- $\sigma = \epsilon - 425 \epsilon^{2.6}$

Strain-in/in.
 $\times 10^3$

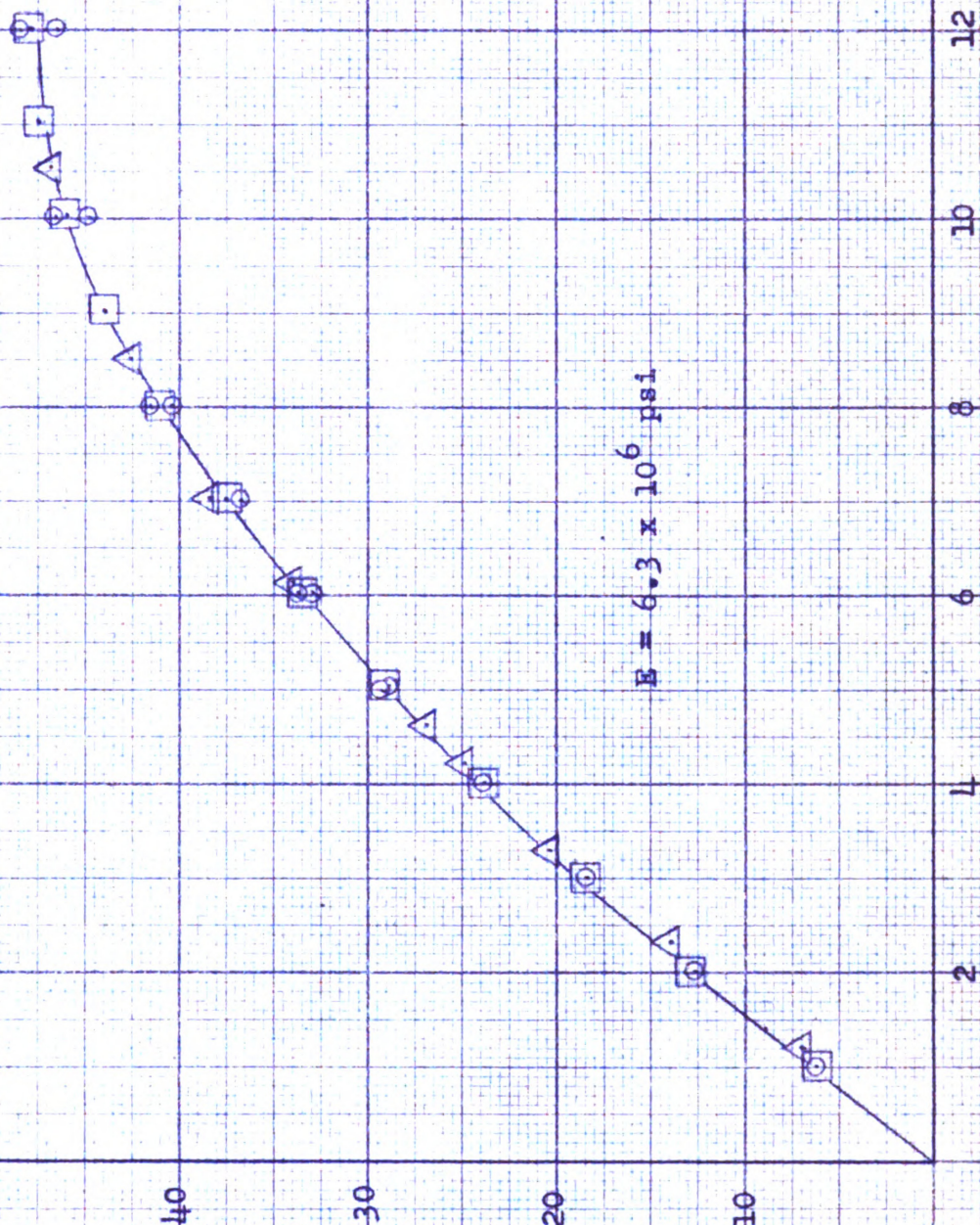


TABLE I

s	ϵ	σ	$ \sigma - \epsilon $
38,500	.004	.0037	.0003
40,500	.004	.0039	.0001
43,000	.0044	.0041	.0003
44,000	.005	.0042	.0008
46,300	.005	.0045	.0005
46,000	.006	.0044	.0016
48,000	.006	.0046	.0014
50,500	.006	.0049	.0011
47,250	.007	.0045	.0025
51,500	.007	.0050	.0020
48,000	.008	.0046	.0034
51,500	.008	.0050	.0030
54,000	.008	.0052	.0028
48,500	.009	.0047	.0043
54,000	.009	.0052	.0038
49,000	.010	.0048	.0052
53,000	.010	.0051	.0049
56,000	.010	.0054	.0046

$$7.3222 - 10 = \log_{10} |k| + 3(7.8451 - 10)$$

$$\log_{10} |k| = 3.7869$$

$$|k| = 6120$$

$$k \approx -6100$$

$$\sigma = \epsilon - 6100\epsilon^3$$

Step eight is carried out with the use of the tabulation shown in Table II. The curve is plotted in Fig. 2 and the results observed.

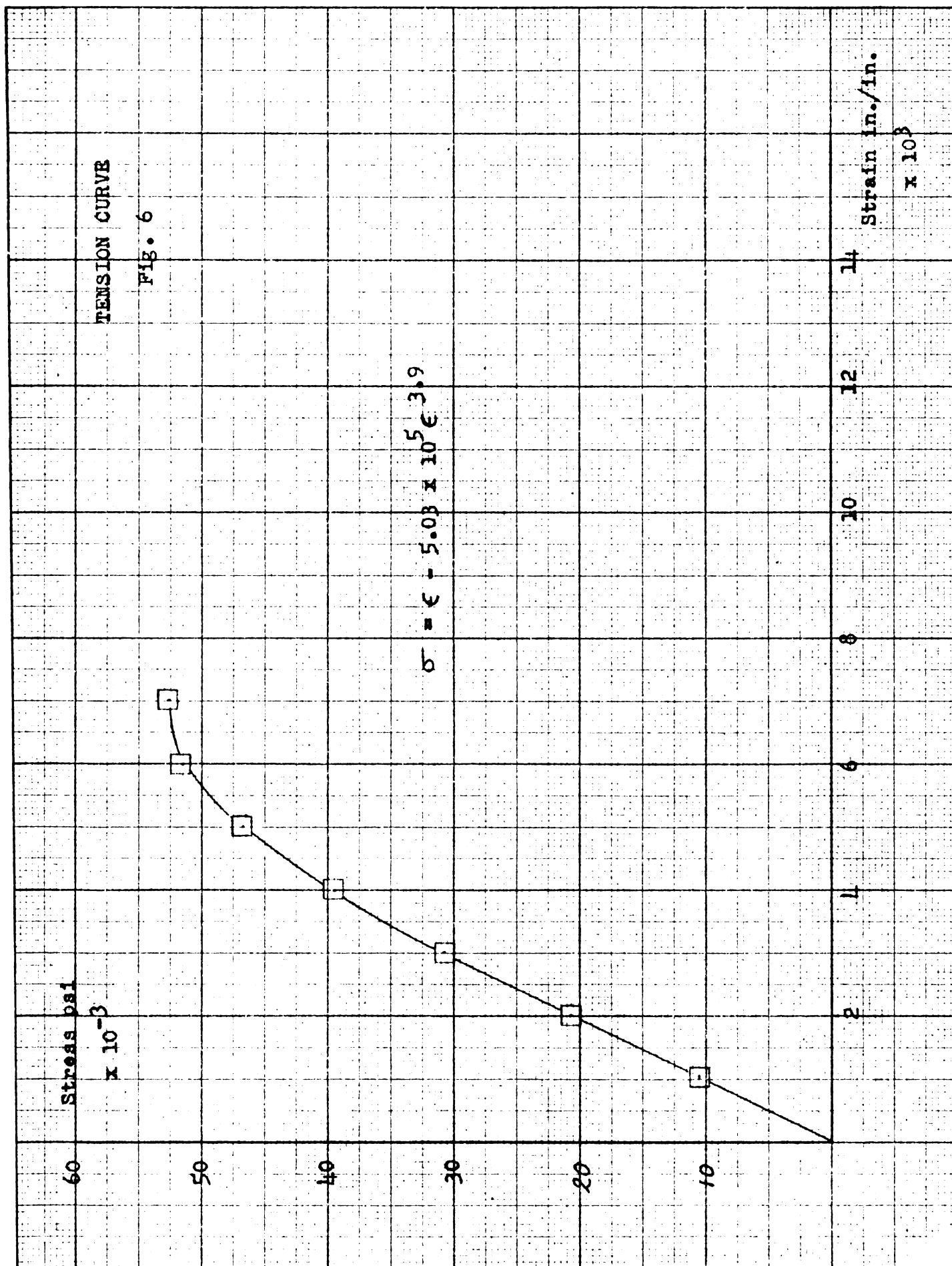
The equation fits well for values of strain up to 0.008 in./in. In Fig. 2, this is somewhat past the yield point strain of 0.007 in ./in. in tension and 0.006 in./in. in compression at 0.2 percent offset. The equation as found here is an average curve between the tension and compressive tests; the reason for this procedure will be explained later. If so desired, a curve could be found that would fit either the tension or compressive curve more closely. For the closer approximations, see Figs. 6 and 7. Notice the equation of the curve is given in each figure.

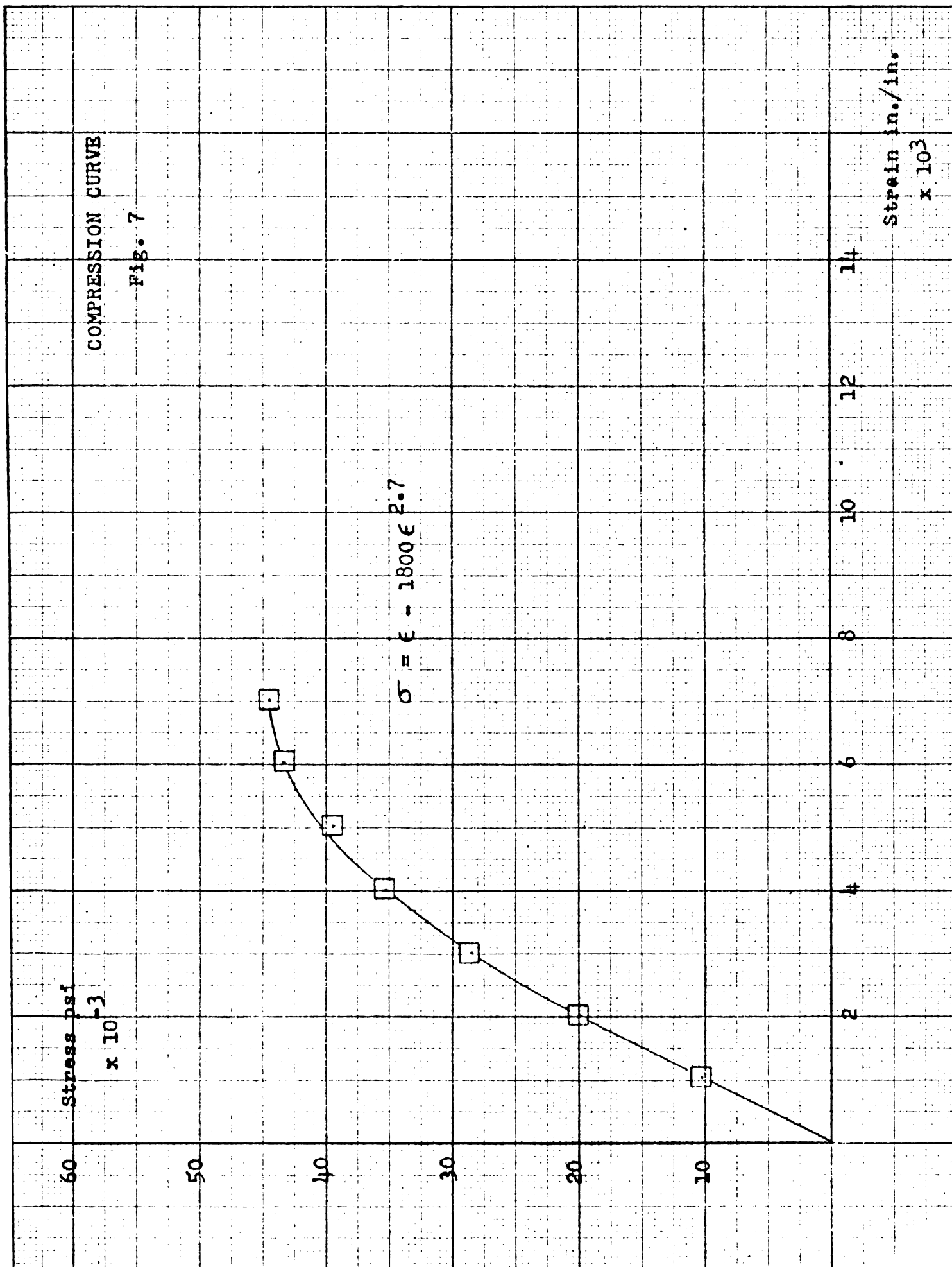
If the curves shown in Figs. 6 and 7 are compared with the actual stress-strain data, Fig. 2, very close agreement can be seen.

The stress-strain data, and the curve plotted for the equation is shown in Fig. 4 for the magnesium alloy. The fit is almost perfect as can be observed, and the equation holds for values of strain up to 0.012 in./in. Beyond this

TABLE II

ϵ	ϵ^3	$6100^3 \epsilon^3$	σ	s
0.001	1×10^{-9}	6.1×10^{-6}	0.00100	10,250
0.002	8×10^{-9}	48.8×10^{-6}	0.00195	20,000
0.003	27×10^{-9}	$165. \times 10^{-6}$	0.00284	29,100
0.004	64×10^{-9}	$390. \times 10^{-6}$	0.00361	37,000
0.005	125×10^{-9}	$762. \times 10^{-6}$	0.00424	43,500
0.006	216×10^{-9}	$1320. \times 10^{-6}$	0.00468	48,000
0.007	343×10^{-9}	$2090. \times 10^{-6}$	0.00491	50,200
0.008	512×10^{-9}	$3120. \times 10^{-6}$	0.00488	50,000
0.009	729×10^{-9}	$4440. \times 10^{-6}$	0.00456	46,700





point the curve falls off rapidly as is the case for the other equations. There is a slight break in the curve for low values of ϵ ; this leads to a better fit as well as a wider range of application.

CHAPTER II

APPLICATION OF THE STRESS-STRAIN FORMULA TO PLASTIC BENDING

Introduction

The flexure formula,

$$s = \frac{Mc}{I}$$

found in elementary books on strength of materials, uses Hooke's law in its derivation. Hooke's law,

$$s = E\epsilon,$$

can be used where the stress-strain data can be fitted to a straight line. For non-linear stress-strain curves, and for stresses past the proportional limit, the flexure formula cannot be used because Hooke's law does not apply. Therefore, when these cases exist, some other relationship between stress and bending moment is necessary.

In general, when a bar is subjected to bending, the stress distribution curve may be thought of as a stress-strain curve, if the stress axis of the stress-strain curve is assumed to lie in the neutral surface of the beam. See Fig. 8. Section A-A is considered to be any cross-section. The stress-strain properties in tension and compression are assumed to be the same.

In applications this may not be the case, so the average of the stress-strain curves in tension and compression could be used for the stress distribution in the cross-section. This is the reason why the average curve was calculated for the aluminum alloy in Chapter I.

Plane sections are assumed to remain plane before and after bending, in the elastic range as well as the plastic range.

From this assumption and Fig. 9,

$$\epsilon = \frac{(R + y) d\theta - R d\theta}{R d\theta} = \frac{y}{R}$$

$$y = R \epsilon \quad (5)$$

Bending Moment for a Rectangular Cross-Section

From Fig. 10,

$$M = \int S y dA$$

$$dA = b_1 dy$$

$$M = 2b_1 \int_0^{h/2} S y dy,$$

and from equation (1)

$$S = E (\epsilon + k \epsilon^n).$$

$$M = 2 b_1 E \int_0^{h/2} (\epsilon + k \epsilon^n) y dy$$

From (5) $dy = R d\epsilon$

and

$$M = 2 b_1 E R^2 \int_0^{\epsilon_c} (\epsilon^2 + k \epsilon^{n+1}) d\epsilon$$

where ϵ_c is the strain at the outer surface.

Using (5)

$$M = \frac{b_1 E h^2}{2} \left(\frac{\epsilon_c}{3} + \frac{k \epsilon_c^n}{n+2} \right) \quad (6)$$

The values of k and n , as found in Chapter I, can be substituted in equation (6). This formula gives the relation between the bending moment and strain at the outer surface for a rectangular section.

$\frac{Mc}{I}$ can be calculated and plotted against the maximum stress on the cross-section by using the stress-strain curve. This graph gives a useful relation between a given bending moment and maximum stress on a rectangular section for a given material.

Bending Moment for a Circular Cross-Section

The same assumptions are made here as in the treatment of the rectangular section. From Fig. 11,

$$M = \int S y dA$$

$$dA = 2x dy$$

$$M = 4 \int_0^r S x y dy$$

Using equations (1) and (5), with the equation of the circle gives,

$$M = 4 R^3 E \int_0^{\epsilon_c} (\epsilon^2 + k \epsilon^n + 1) (\epsilon_c^2 - \epsilon^2)^{1/2} d\epsilon$$

Integration by parts, a sufficient number of times gives,

$$M = 4 R^3 E \left\{ \frac{\pi \epsilon_c^4}{16} + \frac{k}{n+2} \int_0^{\epsilon_c} \frac{\epsilon^{n+3}}{(\epsilon_c^2 - \epsilon^2)^{1/2}} d\epsilon \right\} \quad (8)$$

Making the substitution, $\epsilon = u \epsilon_c$, the integral becomes,

$$\epsilon_c^{n+3} \int_0^1 \frac{u^{n+3}}{(1-u^2)^{1/2}} du$$

From a table of definite integrals,

$$= \frac{1}{2} \epsilon_c^{n+3} B\left(\frac{n+4}{2}, \frac{1}{2}\right)$$

where B denotes Beta function. This can be written in terms of Gamma functions as follows,

$$= \frac{1}{2} \frac{\Gamma\left(\frac{n+4}{2}\right) \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{n+5}{2}\right)} \epsilon_c^{n+3}$$

Substituting this back in equation (8), using (5) and noting $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ gives,

$$M = \pi r^3 E \left\{ 0.250 \epsilon_c + \frac{2k}{\sqrt{\pi} (n+2)} \frac{\Gamma\left(\frac{n+4}{2}\right)}{\Gamma\left(\frac{n+5}{2}\right)} \epsilon_c^n \right\} \quad (9)$$

The values of k and n as found in Chapter I can be substituted in equation (9). This gives the relation between the bending moment and strain at the outer surface for a circular cross-section, when the stress-strain data for the material can be fitted to equation (1).

$$^9 \quad \Gamma(m) = \int_0^{\infty} x^{m-1} e^{-x} dx$$

By using the same procedure as before, a graph can be drawn that will relate $\frac{Mc}{I}$ to the maximum stress on the section. This will be for a circular cross-section and a given material.

CHAPTER III

THE BENDING TESTS

Introduction

The bending theory developed in the previous chapter can be checked by performing bending tests. These test results can be used to show how close the stress predicted by the theory approaches the actual stress developed at the outer surface, and also to illustrate the procedure in plotting the stress versus $\frac{Mc}{I}$ diagram.

Description of Experiment

Fig. 12 shows a diagram of the test setup. The load was applied by the use of dead weights. Two SR-4 type electric strain gages were used. One gage was located directly at the highest point of the section, while the other was located at the lowest point of the section. The strain at the outer surface was taken as the average of the two readings. This was done to account for the fact that the stress-strain properties in tension and compression were not the same as assumed in the theory.

The Brush amplifier, along with the Brush oscillograph, was used to record the strain values.

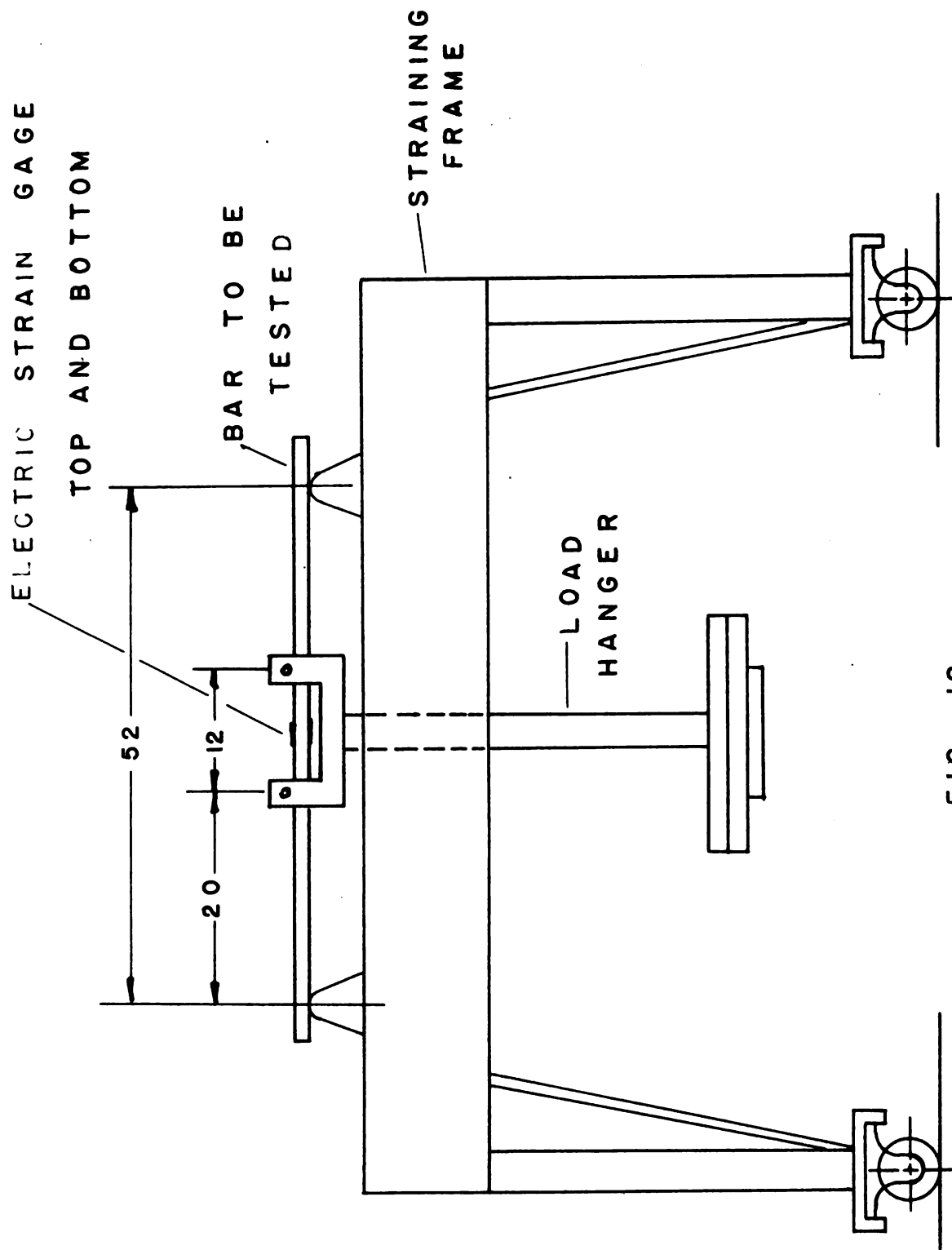


FIG. 12

The procedure was as follows:

1. Load increments of 100 lbs. were used.
2. Both values of strain were recorded for each load.

With this data, $\frac{Mc}{I}$ can be calculated, and the value of stress can be found from the stress-strain diagram corresponding to the average value of strain. These two values are then plotted.

The Aluminum Alloy

The tensile and compressive specimens were cut from the same bars that were used for the bending tests. In order to determine if the properties of the bar were the same throughout, the specimens were cut out of the bar as described in Chapter I.

From the average stress-strain curve of the aluminum alloy, Figs. 2 and 3,

$$k = -6100$$

$$n = 3.$$

For the square section,

$$b_1 = 1 \text{ in.}$$

$$n = 1 \text{ in.}$$

$$E = 10.5 \times 10^6 \text{ psi,}$$

and from equation (6),

$$M = 5.25 \times 10^6 \epsilon_c (0.330 - 1250 \epsilon_c^2). \quad (10)$$

For the circular section,

$$r = 0.50 \text{ in.}$$

$$E = 10.4 \times 10^6 \text{ psi}$$

and from equation (9),

$$M = 4.09 \epsilon_c (0.250 - 762 \epsilon_c^2) 10^6 \quad (11)$$

Calculating $\frac{Mc}{I}$, and substituting values of strain for ϵ_c in (10 and (11), and finding the corresponding stress value from the average curve, Figs. 2 and 3, the calculated curves in Figs. 13 and 14 were drawn. The actual test results are also shown for both the square and circular sections.

For the rectangular section there is good agreement between the theory and test data up to a stress value of about 30,000 psi, and again from about 45,000 psi to 50,000 psi. Between these points the stress values are low. One reason for the low values may be the fact that the average stress-strain curve, Fig. 2, is not as good an average in this range, as in the other parts of the curve.

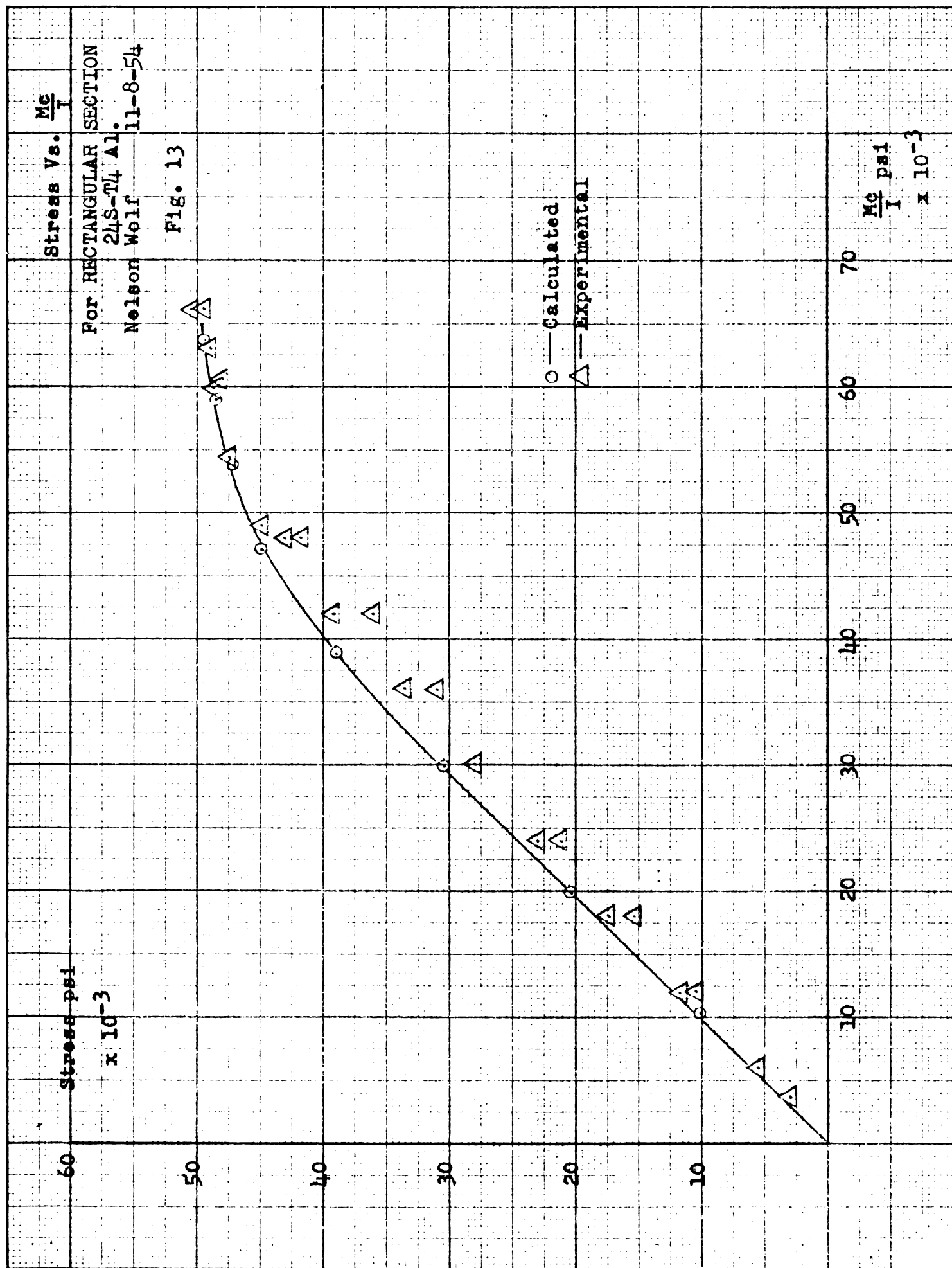
For the circular section good agreement exists over the entire range of the curve.

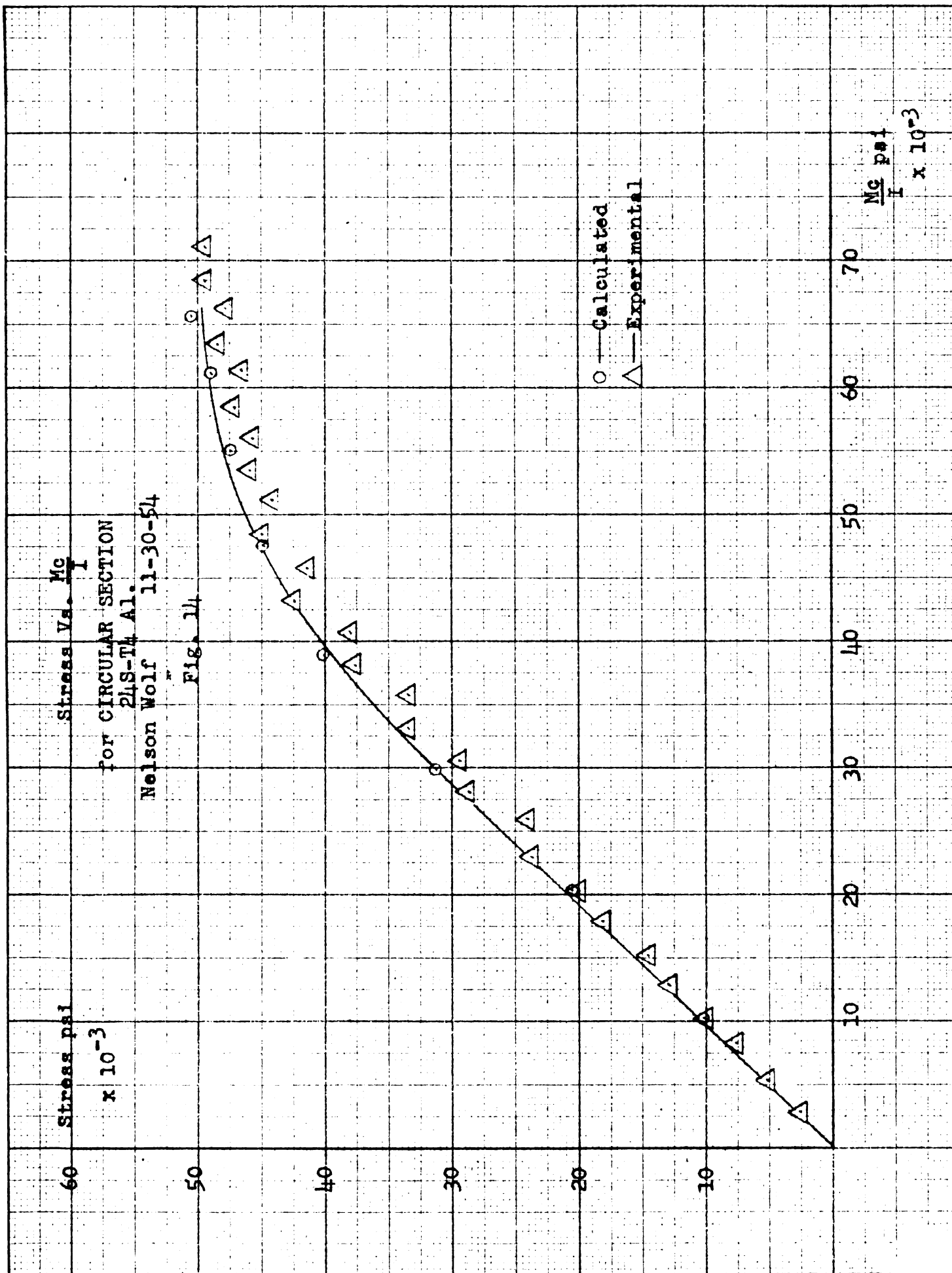
The Magnesium Alloy

The same procedure was applied to the magnesium alloy as was applied to the aluminum alloy. From Fig. 4, and from the data on the circular section,

$$n = 2.6$$

$$k = -425$$





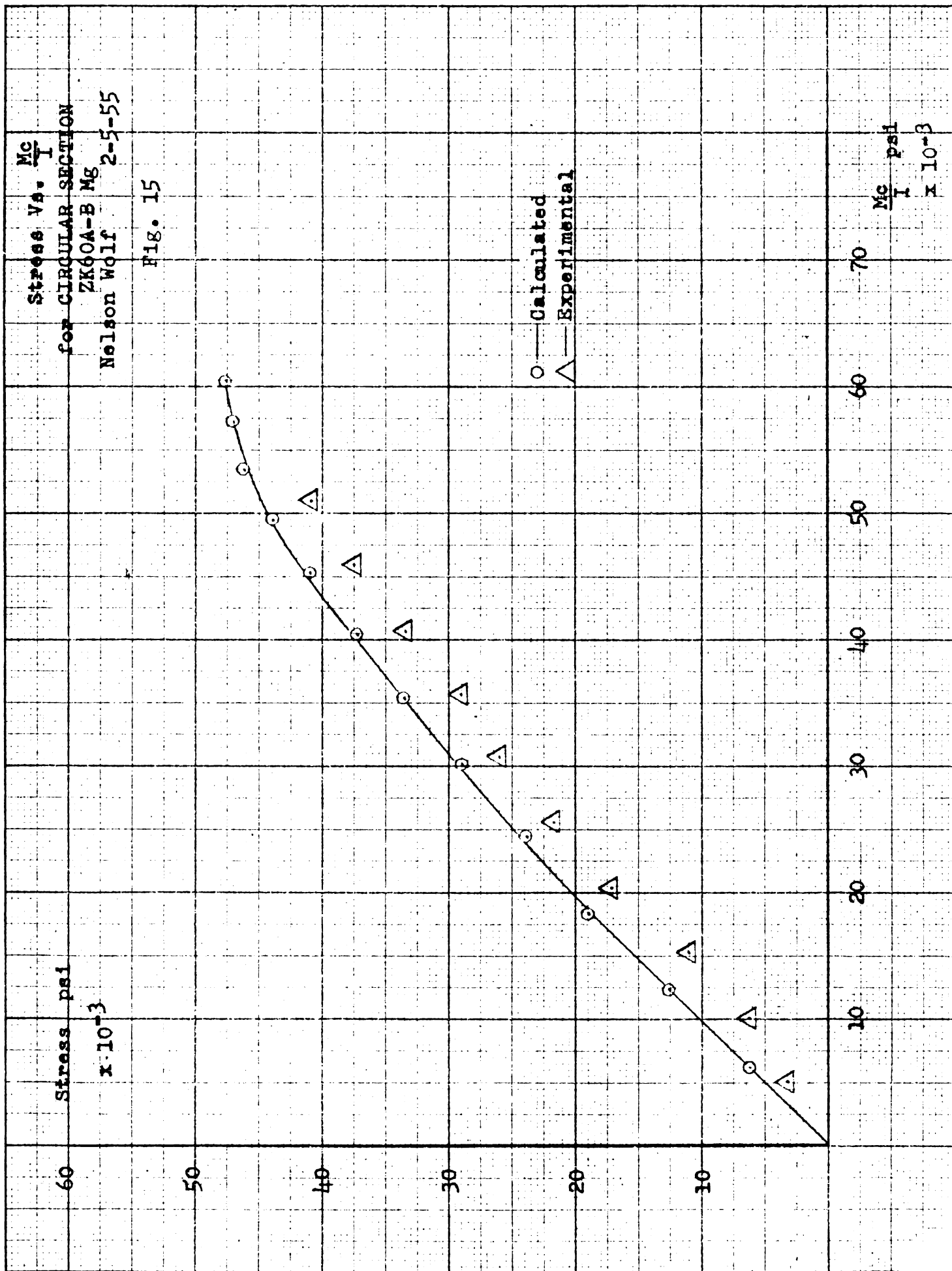
$$r = 0.50 \text{ in.}$$

$$E = 6.3 \times 10^6 \text{ psi}$$

and from equation (9),

$$M = 2.47 \epsilon_c (0.250 - 60 \epsilon_c^{1.6}) 10^6$$

Using the same procedure as before, the results are shown in Fig. 15. Better agreement was expected because the equation relating stress to strain agrees very well with the stress-strain data. The error was assumed to be due to experimental technique.



CHAPTER IV

CONCLUSIONS

From this study it may be concluded:

1. The proposed mathematical relationship is suitable and can represent stress-strain data for certain types of magnesium and aluminum alloys.
2. A relation between stress and bending moment was found when the stress-strain law is non-linear.
3. The maximum stress at the outer surface can be fairly well estimated by the use of this relation.

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