MATHEMATICAL ANALYSIS OF DEEP BED DRYING

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ABSTRACT

MATHEMATICAL ANALYSIS OF DEEP BED DRYING

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Characteristic K- and λ- values are defined to describe drying in deep beds. The K- value is the ratio of the proportion of temperature drop of the drying air from the i-th layer to the (i+1)-th layer, to the temperature of drying air in the same position in the same period of time. The λ-value is the ratio of the proportion of temperature gain of the drying air from the i-th layer to the (i+1)-th layer after the increase in vapor pressure of the grain due to heating, to the proportion of temperature drop of the drying air in the same position in the same period of time.

In the analysis of the drying process, the K- and λ-values are functions of position i and time j for a specific product. Various feasible relationships have been examined, for which mathematical equations are established. Also, these relationships are presented graphically. Because K- and λ-functions are defined for a given product, the drying process can be predicted by using the mathematical analysis presented.

Approved by Carl W. Hall

Jan 1964

MATHEMATICAL ANALYSIS OF DEEP BED DRYING

by

Hung C. Wu

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TABLE OF CONTENTS

LIST OF SYMBOLS

L = Length of the deep bin.

T_{oo} = Initial temperature before entering the grain.

i = Dummy index associated with layer position.

j = Dummy index associated with time.

P_{ij} = Proportion of heat lost of the drying air from i-th layer to (i + 1)-th layer at j-th instant.

R_{ij} = Proportion of temperature gain of the drying air from i-th layer to (i + 1)-th layer at j-th instant after vapor pressure of the grain increased.

T_{ij} = Temperature of the drying air in i-th layer at j-th instant.

$K_{ij} = \dfrac{P_{ij}}{T_{ij}}$

$\lambda_{ij} = \dfrac{R_{ij}}{P_{ij}}$

a = Constant and equals to $K(1-\lambda)$ if K and λ are constant.

U = Diagonal matrix of $m \times n$, which is a function of K_{ij}.

V = Diagonal matrix of $m \times n$, which is a function of λ_{ij}.

LIST OF FIGURES

INTRODUCTION

Inclement weather often prevents timely completion of harvest. When this happens, the farmer has to be very careful in operating a combine or picker. The harvesting loss of corn in October is 5 per cent as compared to that in November of 8.4 per cent and in December 18.4 per cent. The estimated loss is $6.00 per acre in October, $10.00 in November, and $22.00 in December.

Hall (1958) estimated that the losses between harvest and consumption of grain and hay amounts to 10 and 28 per cent, respectively, of the total production, and this loss in fruits and vegetables is as high as 35 per cent. Both harvesting and storage losses can be greatly minimized by adaptation and proper management of sound drying practices.

Recent studies have shown that crops harvested at a higher moisture content will have minimum field losses. For corn this moisture content is from 24 to 30 per cent; on the other hand, for safe storage of corn the moisture content should be below 13 per cent, wet basis. Hall (1957) also concluded that a saving of 75 per cent of the total losses which occur during harvesting and storage could be realized by combination of early harvest and subsequent drying. This leads to the importance of artificial drying.

Research about drying has been extensive. Drying of single kernel depth layers has been reasonably well described by mathematical equations. However, deep layer drying has been described by empirical equations, but not by using the single depth layers making up the bed. Hence, attempts to set up a general mathematical equation to describe the deep bed drying process have been of limited value. Much work has been done on the relationship between moisture content of the grain and drying time, but no work has been done in the field of the temperature distribution of the drying air along the deep bed. This is a new field of study. If we can explore this further and gather data, we can set up a temperature distribution equation in order to understand better the mechanism of the drying in the deep beds.

The objective of this study was to develop an equation and, furthermore, open the field of study to make up a series of charts and graphs for permanent usage and design to describe the drying process in deep layer drying.

REVIEW OF LITERATURE

The theory of drying for a single layer has been developed using Newton's law of heating or cooling, under the assumption that the surrounding air is at constant temperature and that the temperature difference between the drying air and grain is small. This equation is written as follows:

$$\frac{dT}{d\theta} = -K\ (T-T_e)$$

In this equation K is the heating or cooling constant, T is temperature in deg. F, θ is time in hours and T_e is external temperature in deg. F. Hall (1957) used Newton's equation to describe the falling rate of drying. The equation has been very satisfactory for the falling rate of drying. By substitution of the appropriate moisture contents (dry basis) for the temperatures in the above equation, the equation is changed to moisture content ratio:

$$\frac{M-M_e}{M_o-M_e} = e^{-K\theta}$$

Where M = moisture content of grain, wet basis, per cent.

M_e = equilibrium moisture content of grain, wet basis, per cent.

M_o = initial moisture content of grain, wet basis, per cent.

K = heating or cooling constant.

Bailey (1958) derived an equation for thin layer drying of hops in kilns and showed that drying time is inversely proportional to the difference between pressure of water at the temperature of drying and inlet and vapor pressure of water already present in the atmosphere. The drying time is directly proportional to the loss of water per square foot area per minute and was explained by the same equation in terms of air speed in feet per minute leaving the hops. He analysed the vapor pressure driving force in terms of pressure difference related to water at the inlet air temperature.

Simmonds (1953 b) proposed a method for predicting the rate of drying of wheat grain in beds 2 inches to 12 inches deep for air velocities of 12 to 130 feet per minute and an average temperature of 70 to 170 deg. F. with accuracies of $\pm$ 10 per cent. In this method, drying took place entirely in the decreasing rate period. The decreasing drying rate period of wheat was described in terms of its average moisture content. He believed this relationship would be more valuable if the drying rate constant could be simply correlated with air conditions.

He proposed that the mean temperature for predicting the performance of adiabatic dryers be the logarithmic mean temperature between the air temperature and its adiabatic saturation temperature. He has shown from a moisture balance equation that the point of maximum rate of drying is proportional to the bed depth, and the rate of drying depends on mass velocity of air. He was also convinced that a correlation of the rate constant with temperature would be sufficient for most practical purposes and considered that at any given temperature a certain fraction of molecules present in a system are capable of undergoing chemical reactions, diffusion, viscous flow, etc. He also developed an equation for the variation of the rate constant with air temperature and worked out a method for predicting drying performance based on the drying constant and logarithmic mean temperature of drying air and wet bulb temperatures. He suggested that the drying zone could be expressed in terms of drying percentage.

Hukill's (1947) approach for analysis of deep layer drying is as follows: the computed relationship between drying time, grain moisture, and grain depth units have been generalized to make them applicable to the drying problem. In the equation of drying rate he makes use of the drying time unit which is half response time, moisture content ratio, and depth factor.

Henderson (1955) suggested a method for calculating moisture content of the deep bed at any given time. The solution is based on stepwise integration from one layer to the other. He divided the depth into several thin layers and made an assumption of uniform drying of each layer to carry out the procedure. He illustrated a table of calculations which gives the moisture content at any depth with various intervals of time.

Ives (1953) showed some temperature-time-moisture relationships and process for drying grain, and in 1960 with co-authorship with Hukill and Black, employed dimensional analysis in planning the experiments and analyzing the data. With the help of the Buckingham Pi theorem, they developed a general empirical prediction equation for kernel drying times in counterflow grain drying:

$$T_k = (1.08 \ln \frac{M_i}{14}) \left(\frac{4400}{M_e^{1.41}} - 30\right) \left(\frac{0.608}{[(P_w - P_v)/P_b]^{.74}}\right) \left(\frac{0.311}{(P_v/P_b)^{.247}}\right)$$

Here (1) T_k is the kernel travel or drying time in minutes; (2) M_i is the grain moisture content <u>in</u>; (3) M_e is the grain moisture content <u>out</u>, per cent dry basis; (4) P_w is the saturation vapor pressure corresponding to the entering air temperature; (5) P_v is the partial vapor pressure of the entering air; (6) P_b is the barometric pressure, in lbs/sq.in.

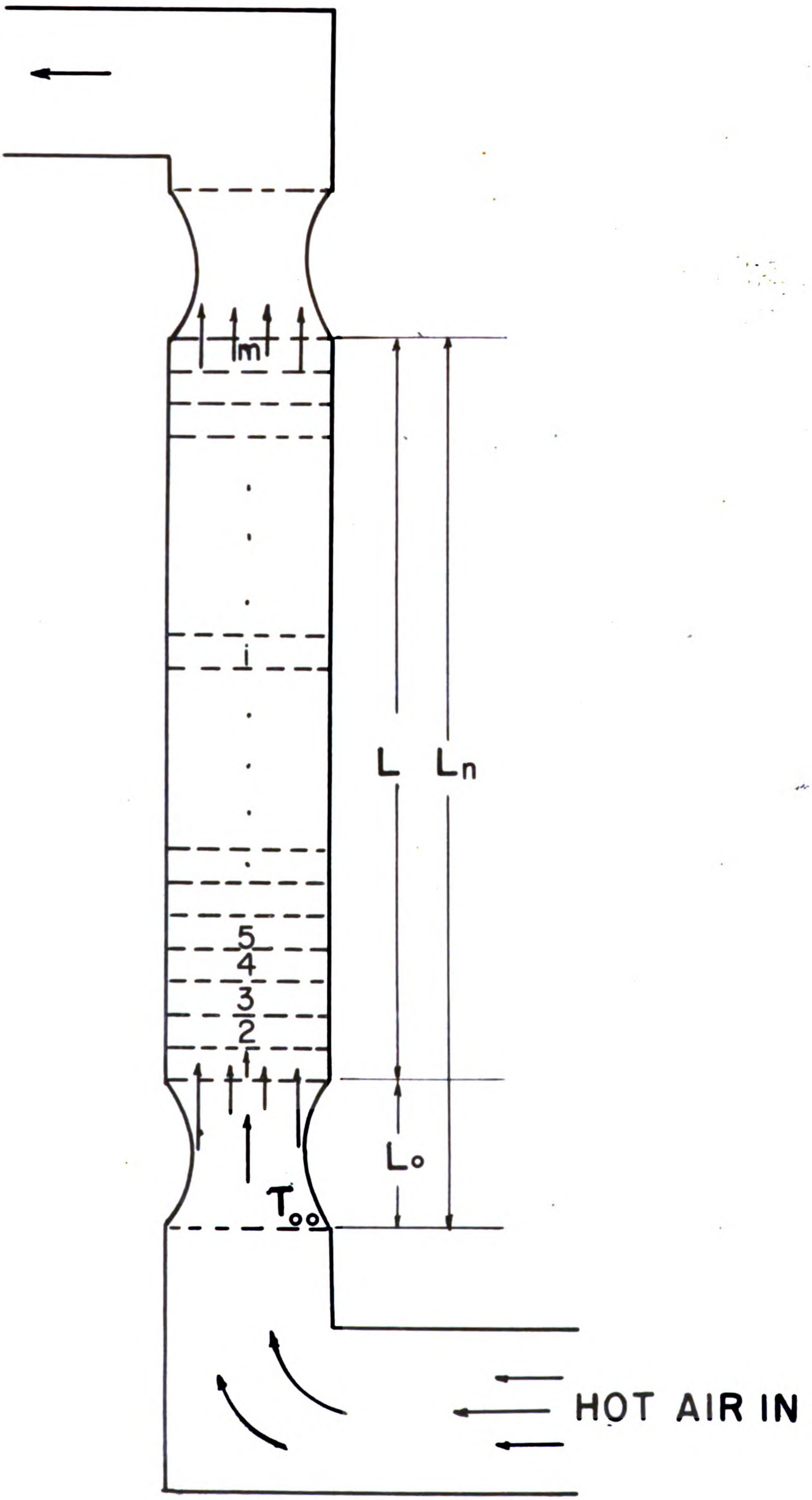

Fig. 1. Sketch of deep bin drying.

Defini

Defin

ANALYSIS AND RESULTS

Definition:

i-th index: associated with position (i-th strip)

j-th index: associated with time (j-th instant)

P_{ij}: proportion of temperature drop of the drying air from i-th layer to (i + 1)-th layer, at i-th instant. j

R_{ij}: proportion of temperature gain of the drying air from i-th layer to (i + 1)-th layer after vapor pressure of the grain increased, at j-th instant.

Define $$K_{ij} = \frac{P_{ij}}{T_{ij}} \qquad \begin{array}{l}(i = o, 1, \ldots, m)\\(j = o, 1, \ldots, n)\end{array}$$

Since $P_{ij} < T_{ij}$; hence, $K_{ij} < 1$ and $P_{ij} = K_{ij} T_{ij}$ ... (1)

Furthermore, we define the fraction of R_{ij} on P_{ij} to be λ_{ij}; ie.

$$\lambda_{ij} = \frac{R_{ij}}{P_{ij}}$$

Since $R_{ij} < P_{ij}$; hence $\lambda_{ij} < 1$ and $R_{ij} = \lambda_{ij} P_{ij}$ ... (2)

By (1) and (2) we have:

$$R_{ij} = \lambda_{ij} P_{ij} = (\lambda_{ij} K_{ij}) T_{ij} \text{ and } K_{ij} \lambda_{ij} < 1$$

At the j-th instant, the temperature distribution of the deep bin could be determined as follows:

(i) In the first layer at the j-th instant:

$$T_{1j} = T_{oj} - P_{oj} + R_{oj}$$

$$= T_{oj} - K_{oj}T_{oj} + \lambda_{oj} K_{oj}T_{oj}$$

$$= T_{oj} (1 - K_{oj} + \lambda_{oj}K_{oj})$$

$$= T_{oj} [1 - K_{oj}(1-\lambda_{oj})] \qquad \ldots \qquad (3)$$

(ii) In the second layer at the j-th instant:

$$T_{2j} = T_{1j} - P_{1j} + R_{1j}$$

and $P_{1j} = K_{1j}T_{1j}$

$$R_{1j} = \lambda_{1j}P_{1j} = \lambda_{1j}K_{1j}T_{1j}$$

hence

$$T_{2j} = T_{1j} [1 - K_{1j} (1-\lambda_{1j})] \qquad \ldots \qquad (4)$$

By (3) and (4) therefore

$$T_{2j} = T_{oj} [1-K_{1j} (1-\lambda_{1j})][1 - K_{oj} (1-\lambda_{oj})]$$

. . . .

By induction we have for the m-th layer (the top layer) at the j-th instant:

$$T_{mj} = T_{oj} [1-K_{oj}(1-\lambda_{oj})][1-K_{1j} (1-\lambda_{1j})] \ldots [1-K_{ij} (1-\lambda_{ij})] \ldots [1-K_{m-1,j}(1-\lambda_{m-1,j})]$$

$$= T_{oj} \prod_{i=0}^{m-1} [1 - K_{ij} (1-\lambda_{ij})]$$

By a similar argument we can get a general equation for the temperature distribution of the drying air, with some change of index range from zero to (m-1) and (n-1) to one to m and n.

$$T_{mn} = T_{oo} \prod_{\substack{i=1 \\ j=1}}^{\substack{i=m \\ j=n}} [1-K_{ij}(1-\lambda_{ij})] \qquad \ldots \qquad (5)$$

(i=1, 2, ..., m.)
(j=1, 2, ..., n.)

GENERAL DISCUSSION OF EQUATION (5)

There are only four exclusive relationships between K_{ij}'s and λ_{ij}'s:

(i) $K_{ij} = K$ and $\lambda_{ij} = \lambda$ (both are constant)

(ii) The K_{ij}'s are unequal but $\lambda_{ij} = \lambda$

This could be subgrouped as follows:

(a) K_{ij}'s are monotonically increasing, with depth of bin, which is impossible in the process of drying.

(b) K_{ij}'s are monotonically decreasing:

(1) K_{ij}'s satisfy a linear equation.

(2) K_{ij}'s satisfy a nonlinear equation.

(iii) The K_{ij}'s are constant, and the λ_{ij}'s are unequal:

(a) λ_{ij}'s are monotonically increasing which is impossible in our case.

(b) λ_{ij}'s are monotonically decreasing:

(1) λ_{ij}'s satisfy a linear equation.

(2) λ_{ij}'s satisfy a nonlinear equation.

(iv) The K_{ij}'s are unequal and the λ_{ij}'s are also unequal.

(a) No monotonically increasing is possible in the process of drying.

(b) Function is linear in K_{ij}'s and λ_{ij}'s.

(c) Function is nonlinear in K_{ij}'s and λ_{ij}'s.

CASE I

(i) For $K_{ij} = K$ and $\lambda_{ij} = \lambda$

Then equation (5) becomes:

$$T_{mn} = T_{oo} \prod_{\substack{i=1 \\ j=1}}^{\substack{i=m \\ j=n}} [1-K(1-\lambda)] \quad \text{or}$$

$$T_{mn} = T_{oo} [1-K(1-\lambda)]^{mn}$$

Let $a = K(1-\lambda)$

Since $o \leq K < 1$ and $o \leq \lambda < 1$

Consequently $o \leq a < 1$

and

$$T_{mn} = T_{oo} (1-a)^{mn} \qquad \ldots \qquad (6)$$

By the binominal expansion[1], we obtain:

$$(1-a)^n = 1 - \frac{an}{1} + \frac{n(n-1)}{2!} a^2 - \frac{n(n-1)(n-2)}{3!} a^3 + \ldots$$

Where $o \leq a < 1$ and n is a positive integer.

[1]See appendix for further detail.

and since

$$e^{-an} = 1-an + \frac{n^2a^2}{2!} - \frac{n^3a^3}{3!} + \quad \dots$$

When n is large

$$n^2 \doteq n(n-1)$$

$$n^3 \doteq n(n-1)(n-2)$$

— {for $n = 100$, 3% difference
$n = 10$, 28% diff}

$\vdots$

Hence $(1-a)^n \doteq e^{-an}$ approximately, when n is large.

Therefore, equation (6) could be represented as

$$T_{mm} = T_{oo}\, e^{-amm} \qquad \dots \qquad (7)$$

Where $a = K(1-\lambda)$.

Equation (7) is the general formula for the temperature distribution in the deep bin.

So far we confined ourselves to the case where $K_{ij} = K$, $\lambda_{ij} = \lambda$, and hence a is a constant.

CASE II

If the K_{ij}'s are unequal and $\lambda_{ij} = \lambda$, then the general formula (5) becomes:

$$T_{mm} = T_{oo}\,[1-K_{11}(1-\lambda)][1-K_{12}(1-\lambda)] \dots [1-K_{ij}(1-\lambda)] \dots [1-K_{mm}(1-\lambda)]$$

$$= T_{oo} \prod_{\substack{i=1\\j=1}}^{\substack{i=m\\j=m}} [1-K_{ij}(1-\lambda)]$$

$$= T_{ee} \begin{vmatrix} [1-K_{1j}(1-\lambda)], & 0, & \dots, & 0 \\ 0, & [1-K_{2j}(1-\lambda)], & \dots, & 0 \\ \cdot & & \dots, & 0 \\ \cdot & & \dots, & 0 \\ \cdot & & \dots, & 0 \\ 0, \dots & & \dots, & 0 \\ 0, \dots & & , & [1-K_{nj}(1-\lambda)] \end{vmatrix} \quad j = 1, \dots, n$$

If we define matrix B_j such that

$$B_{j(mxn)} = \begin{vmatrix} [1-K_{1j}(1-\lambda)], & 0, & \dots, & 0 \\ 0, & [1-K_{2j}(1-\lambda)], & \dots, & 0 \\ \cdot & & \cdot & 0 \\ \cdot & & \cdot & 0 \\ \cdot & & \cdot & 0 \\ 0, \dots & & \cdot & 0 \\ 0, \dots & & , & [1-K_{nj}(1-\lambda)] \end{vmatrix} \quad j = 1, \dots, n$$

Then

$$T_{mn} = T_{ee} \prod_{j=1}^{n} B_{j(mxn)} \qquad \dots \qquad (7a)$$

Furthermore, if we define the matrix U such that

$$U_{mxn} = \begin{vmatrix} B_1, & 0, \dots\dots & , 0 \\ 0, & B_2, \dots & , 0 \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ 0, & & , B_n \end{vmatrix}$$

Then general temperature distribution equation will be very much simplified and

$$T_{mn} = T_{ee}\, U \dots \qquad \dots \qquad (8)$$

In the developing of this equation, we assume that the K_{ij}'s satisfy a linear relation and are monotonically decreasing. Let K be the first layer at the beginning of the drying process, then $K_{11} = K$ and K_{mm} is the value of K_{ij} at the last layer at the last moment of the drying process.

Furthermore, if we assume the length of the deep bin is L, since $L_n - L_e = L$ = constant, then

$$\frac{K_{11} - K_{mm}}{L_n - L_e} = M \quad \text{Where } M \text{ is the slope of the line;}$$

$$\text{i.e.}\quad K_{11} - K_{mm} = (L_n - L_e)\,M$$
$$= ML$$

Since K_{11}, K_{mm} and L are all measurable, hence M is entirely determined. Once M is determined, we can go back to the equation (7a) to find out each of the K_{ij}'s and thus $[1-K_{ij}(1-\lambda)]$ and thus $B_{j(mmm)}$ and thus U_{mmm}, and finally T_{mm}, because

$$T_{mm} = T_{ee}\,U$$

In case (ii)(b)(2), since K_{ij}'s are nonlinear, under the condition of monotonically decreasing, we can limit the function of K_{ij}'s to a limited number of functions with respect to X's (depth of bin). For example, hyperbolic function, $f(x) = e^x$, etc., are some examples.

CASE III

K_{ij}'s are constant and λ_{ij}'s are unequal and λ_{ij}'s are monotonically decreasing.

From general equation (5), since K_{ij}'s are constant, hence:

$$T_{mn} = T_{oo} \prod_{\substack{i=1 \\ j=1}}^{\substack{i=m \\ j=n}} \left[1 - K\,(1-\lambda_{ij})\right]$$

$$= T_{oo} \left[1-K\,(1-\lambda_{11})\right] \left[1-K\,(1-\lambda_{12})\right] \ldots \left[1-K\,(1-\lambda_{ij})\right] \ldots \left[1-K(1-\lambda_{mn})\right]$$

$$= T_{oo} \begin{vmatrix} [1-K(1-\lambda_{1j})], & 0, & \ldots, & 0 \\ 0, & [1-K(1-\lambda_{2j})], & , & 0 \\ \cdot & & , & 0 \\ \cdot & & , & 0 \\ \cdot & & , & 0 \\ 0, \ldots & & , & [1-K(1-\lambda_{mj})] \end{vmatrix} \quad j = 1, \ldots, n$$

Define A_j such that

$$A_j = \begin{vmatrix} [1-K(1-\lambda_{1j})], & 0, & \ldots, & 0 \\ 0, & [1-K(1-\lambda_{2j})], & , & 0 \\ \cdot & & , & 0 \\ \cdot & & , & 0 \\ \cdot & & , & 0 \\ 0, \ldots & & , & [1-K(1-\lambda_{mj})] \end{vmatrix} \quad j = 1, \ldots, n$$

Then

$$T_{mn} = T_{oo} \prod_{j=1}^{n} A_{j(m \times m)} \qquad \ldots \qquad (9)$$

Suppose we define a matrix V_{mxm} such that

$$V_{mxm} = \left| \begin{array}{llll} A_1, & 0, & \ldots, & 0 \\ 0 & A_2, & \ldots, & 0 \\ 0 & & & \\ \ldots & & , & A_n \end{array} \right|$$

The general temperature distribution equation of (5) will be in the following form:

$$T_{mn} = T_{oo} \, V \qquad \ldots \qquad (9a)$$

Where

$$V_{mxm} = \left| \begin{array}{llll} A_1, & 0, & \ldots, & 0 \\ 0 & A_2, & \ldots, & 0 \\ 0 & & & \\ \ldots & & , & A_n \end{array} \right|$$

and A_i's were defined previously.

CASE IV

(i) The K_{ij}'s and λ_{ij}'s are linear function of i and j.

From equation (5) we have

$$T_{mn} = T_{oo} \prod_{j=1}^{i=m} \left[1-K_{ij} \, (1-\lambda_{ij})\right]$$

In the previous equation K_{ij} and λ_{ij} are the only two unknowns. Since K_{ij}'s and λ_{ij}'s are in a linear relation with respect to time and position, K_{ij} or λ_{ij} surface is a plane in three dimension which could well be set up by experiment, once the surface is described, one can look at the surface and determine the value of K_{ij}'s and λ_{ij}'s and insert into the general equation, and T_{ij} could be then determined. This procedure will be demonstrated in the illustration section.

(ii) The K_{ij}'s and λ_{ij}'s are nonlinear function of i and j. Using the same general equation, we encounter the same problems as (i). However, since K_{ij}'s and λ_{ij}'s are in nonlinear relationship with respect to time and position, K_{ij} and λ_{ij} surfaces have different shapes; some of the shapes are suggested in the Figures, 4,5 and 6 which could be utilized in calculating the temperature distribution of the drying air along the deep bed.

ILLUSTRATION

For the sake of simplicity, consider the first case when $K_{ij} = K$ and $\lambda_{ij} = \lambda$. Since K's are always less than one, for engineering purposes, we can assume its value to be 0.05, 0.10, 0.15, ..., 0.95, and since proportion of the heat gain of the drying air in the i-th layer to (i + 1)-th layer after vapor pressure of the grain increased is almost negligible; hence the assumption of $\lambda = 0.1$ will be a good approximation, and $a = K(1-\lambda)$ could be determined by Fig. 2.

By Fig. 2, if $\lambda = 0.1$ and $K = 0.78$ then $a = 0.7$.

By equation (7) we have

$$T_{mn} = T_{oo}e^{-0.7mn}$$

or

$$\ln \left(\frac{T_{mn}}{T_{oo}}\right) = -0.7mn$$

$$\text{i.e. } \frac{\ln T_{oo} - \ln T_{mn}}{0.7m} = n \qquad \ldots \qquad (10)$$

In equation (10) T_{oo} is given (could be easily measured) and m is determined arbitrarily, which is shown in Fig. 3. When $T_{oo} = 120°F$.

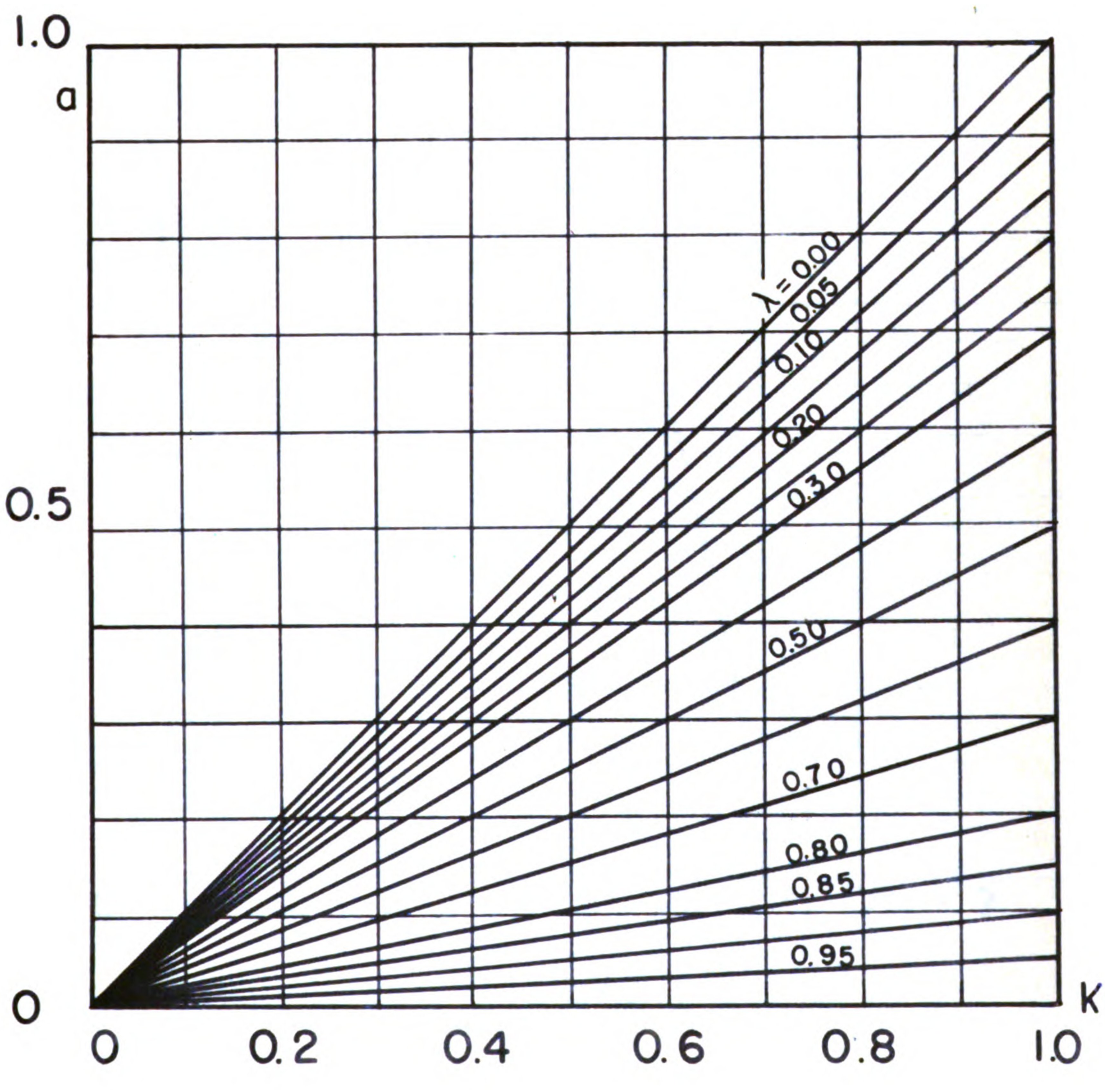

Fig. 2. $a = K(1-\lambda)$

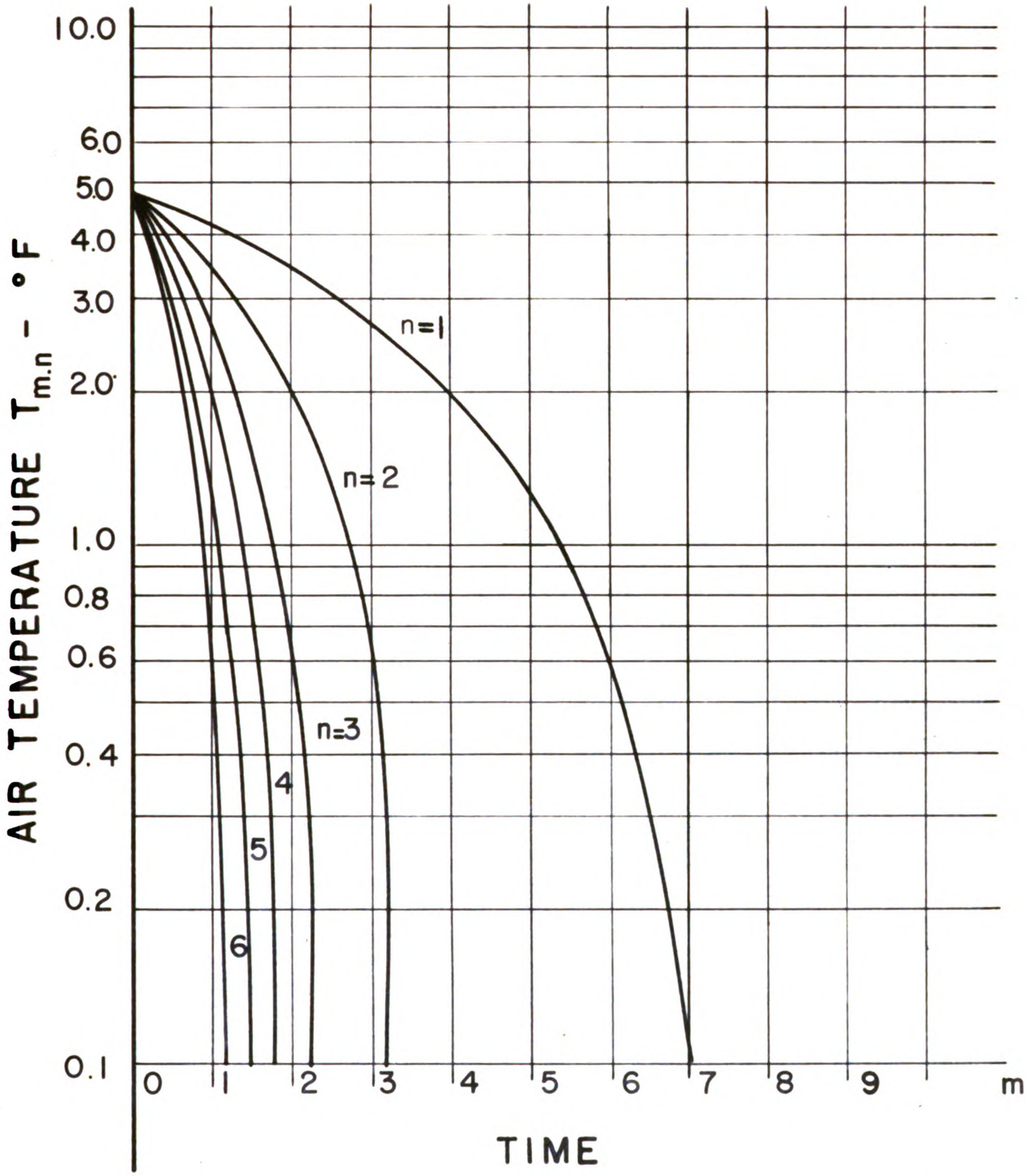

Fig. 3. $\ln T_{mn} = \ln T_{oo} - \alpha mn$ When $\alpha = 0.7$ $T_{oo} = 120^{\circ}$ F.

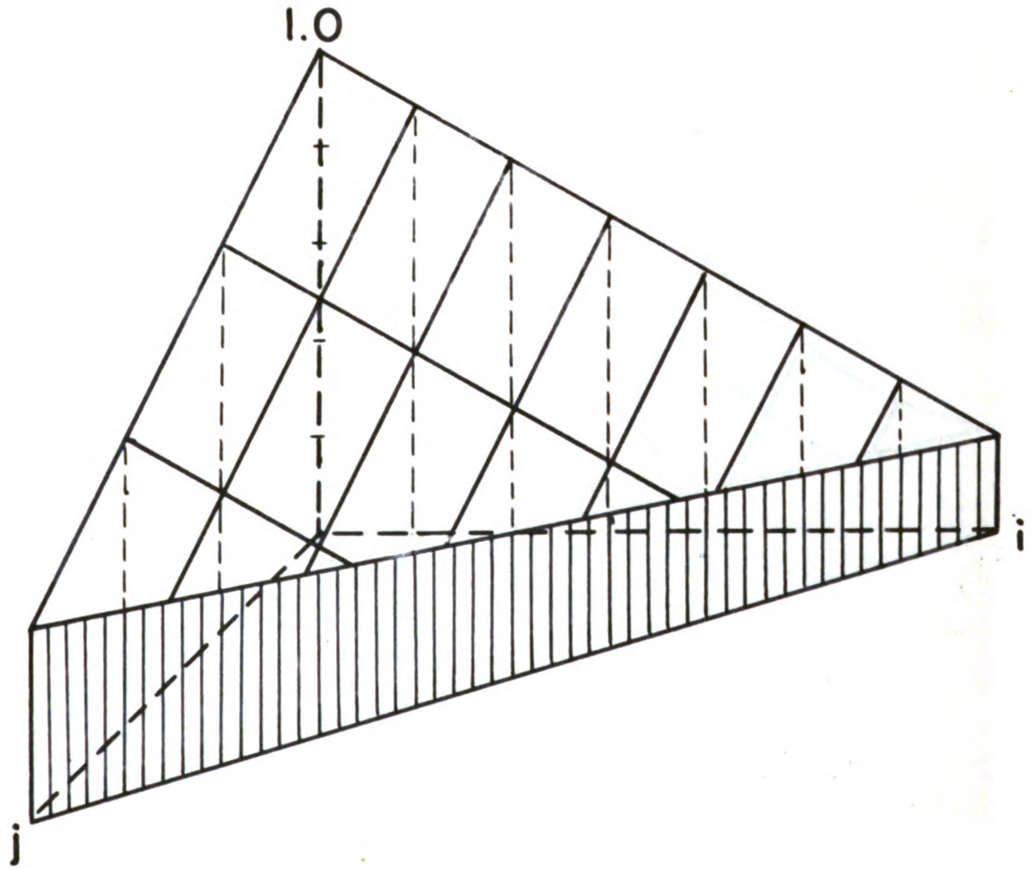

Fig. 4.

The K_{ij} and λ_{ij} surface, when K_{ij} and λ_{ij} satisfy a linear relationship with respect to i, position, and j, time.

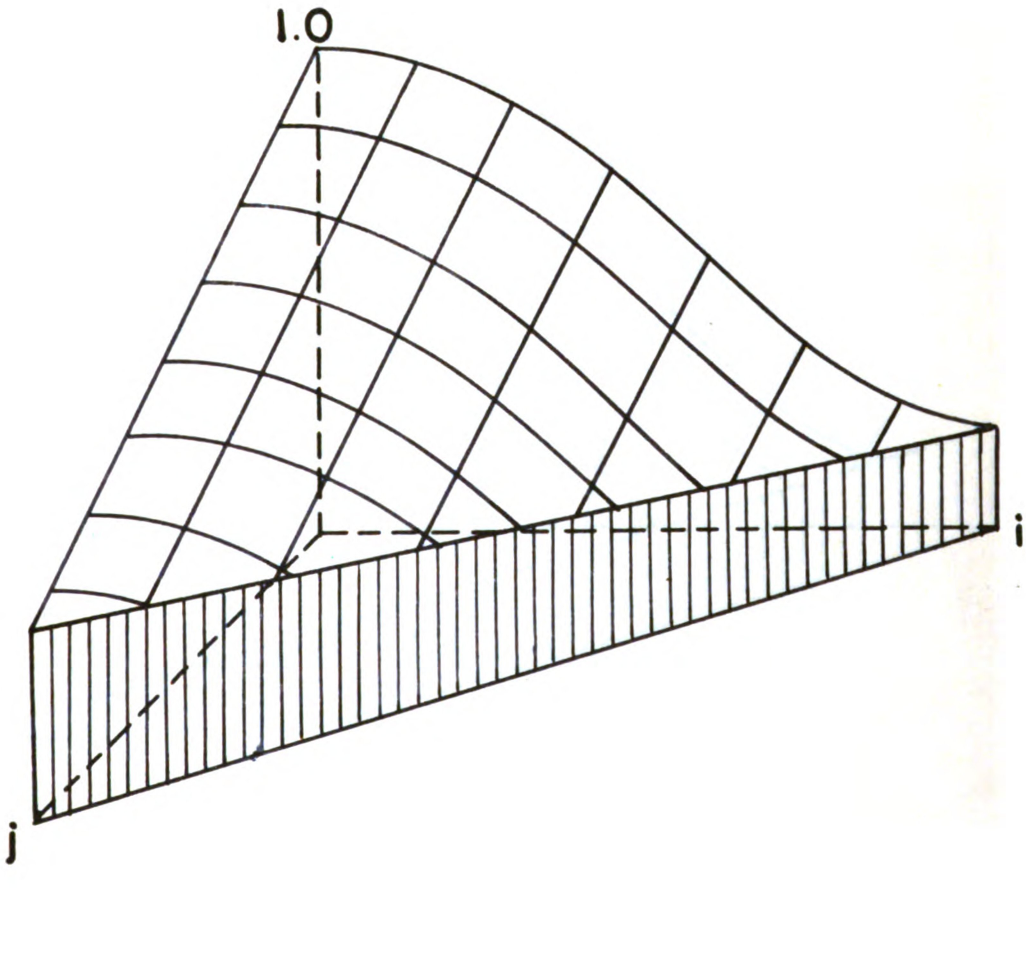

Fig. 5.

The K_{ij} and λ_{ij} surface, when K_{ij} or λ_{ij} is a nonlinear function of i, but a linear function of j.

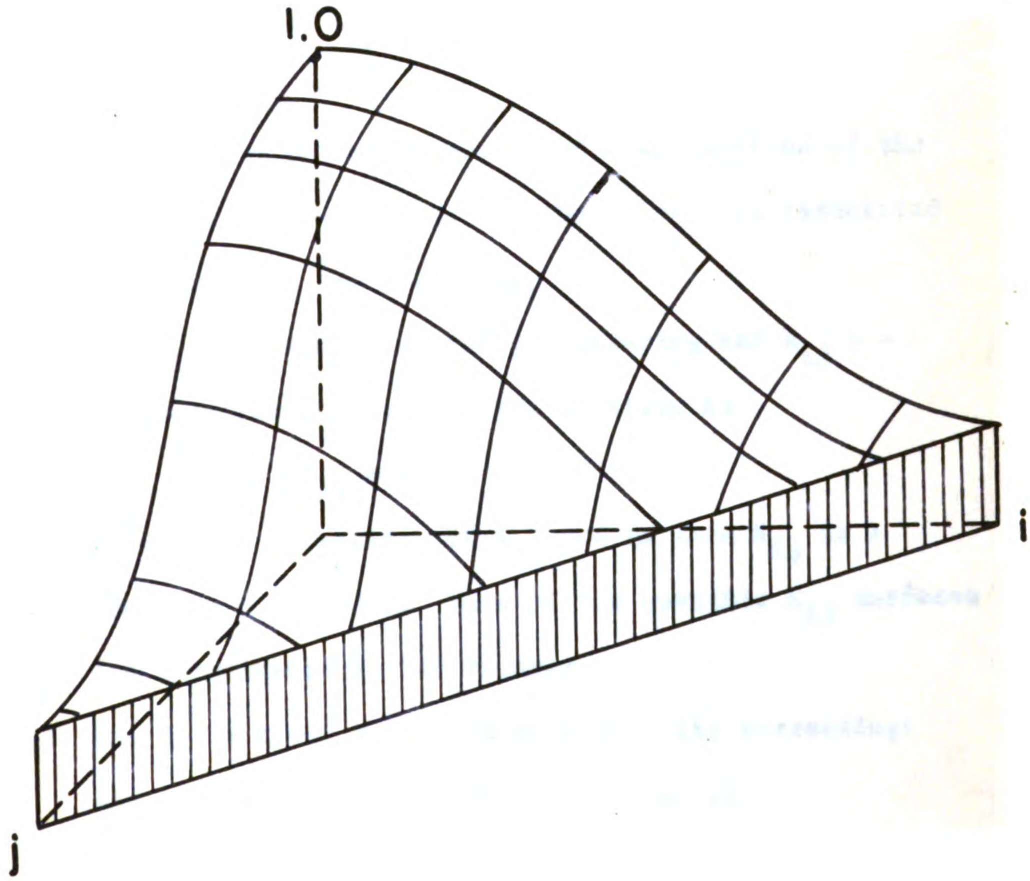

Fig. 6.

The K_{ij} and λ_{ij} surface, when K_{ij} or λ_{ij} is both a nonlinear function of i and j.

CONCLUSION

By defining K_{ij} and λ_{ij}, according to the previous analysis and discussion, the temperature distribution equations of the drying air along a deep bin could be calculated as follows:

(i) If $K_{ij} = K$ and $\lambda_{ij} = \lambda$

Then $T_{mn} = T_{oo}e^{-amn}$

Where the m index is associated with position of the grain as the m-th strip and the n index is associated with time, as the n-th time.

(ii) The K_{ij}'s are monotonically decreasing and $\lambda_{ij} = \lambda$

The temperature distribution equation is

$$T_{mn} = T_{oo}U$$

Where U is a function of K_{ij} and in turn K_{ij} is a function of i and j. Some of the possible K_{ij} surfaces are illustrated in the figures.

(iii) The $K_{ij} = K$ and λ_{ij}'s are monotonically decreasing: the temperature distribution equation is

$$T_{mn} = T_{oo}V$$

Where V is a function of λ_{ij}; furthermore, since λ_{ij} is a function of i and j, hence V is a function of i and j. Some of the possible λ_{ij} surfaces are illustrated in the figures.

(iv) The K_{ij}'s and λ_{ij}'s are monotonically decreasing and

(a) a linear function of i and j;

(b) a nonlinear function of i and j.

Then the temperature distribution function of the drying air is:

$$T_{mn} = T_{oo} \prod_{\substack{i=1 \\ j=1}}^{\substack{i=m \\ j=n}} \left[1-K_{ij}(1-\lambda_{ij})\right] \qquad \ldots \qquad (5)$$

Some K_{ij} and λ_{ij} surfaces are illustrated in the figures.

SUMMARY

In the process of this analysis, the concept of K's and λ's was used in a mathematical equation to represent the drying pattern. It is appropriate to define such values, since during drying each agricultural product has a particular set of K's and λ's. These values could be determined experimentally. In this paper, although a particular and practical set of λ's and K's are not included, expected values of K's and λ's have been shown. It is also shown how to utilize these procedures.

The drying process of agricultural products could be described by a simple equation using simple values of K and λ, such as:

$$T_{mn} = T_{oo} e^{-amn}$$

Where $a = K(1-\lambda)$

m = numbers of strips in the bin

n = numbers of unit time required for drying.

When K and λ are constant, as shown in the beginning of the analysis, each K- and λ- value is between zero and one. Even when the K's and λ's are not constant, the range of variation is not too wide. Consequently, a good approximation can be made even when K's and λ's are not constant by the simple equation $T_{mn} = T_{oo} e^{-amn}$. If more precise results are wanted, the K_{ij} and λ_{ij} surfaces can be

set up first and those values inserted into equations depending on different drying conditions. Therefore, if the K's and λ's are available, the temperature at different positions in the bin at different times can be used with the figures to find the required time for the K-values to approach one, which is the required time for drying the product.

The general temperature distribution function of the drying air is:

$$T_{mn} = T_{oo} \prod_{\substack{i=1 \\ j=1}}^{\substack{i=m \\ j=n}} \left[1-K_{ij}(1-\lambda_{ij})\right]$$

RECOMMENDATIONS

The temperature distribution along the depth of a bin is described by the K and λ values, which are functions of position i and time j. Before the theory can be used in practical applications, several experiments should be run to determine the values of K and λ.

Therefore, it is recommended that a project be set up to obtain the data needed to set up K and λ figures or surfaces for different drying conditions.

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APPENDIX

Show that if $0 < a < 1$ that for n large

$$(1-a)^n \doteq e^{-an}$$

$$(1-a)^n = 1-n(1)^{n-1}a + \binom{n}{2}(1)^{n-2}a^2 - \binom{n}{3}(1)^{n-3}a^3 + \binom{n}{4}(1)^{n-4}a^4 \ldots$$

$$= 1-\binom{n}{1}a + \binom{n}{2}a^2 - \binom{n}{3}a^3 + \binom{n}{4}a^4 - \ldots + (-1)^k\binom{n}{k}a^k$$

$$e^n = 1 + n + \frac{n^2}{2!} + \frac{n^3}{3!} + \ldots + \frac{n^k}{k!} + \ldots$$

$$e^{-an} = 1 - an + \frac{a^2n^2}{2!} - \frac{a^3n^3}{3!} + \frac{a^4n^4}{4!} - \ldots + (-1)^k \frac{a^kn^k}{k!} + \ldots$$

If n is large, is $\binom{n}{k} \doteq \frac{n^k}{k!}$?

$$\frac{n!}{\cancel{k!}\,(n-k)!} \doteq \frac{n^k}{\cancel{k!}}$$

i.e. $n! \doteq n^k\,(n-k)!$

$k = 1 \quad n! \doteq n(n-1)! = n!$ exact

$k = 2 \quad n! \doteq n^2(n-2)! = n(\frac{n}{n-1})\,(n-2)!\,(n-1) = n!\,(\frac{n}{n-1})$

$k = 3 \quad n! \doteq n^3(n-3)! = n!\,(\frac{n}{n-1})(\frac{n}{n-2})$

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Therefore $(1-a)^n \doteq e^{-an}$ when n is large.