ERRORS IN MEAN RAINFALL DETERMINATIONS ON SMALL WATERSHEDS

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THESIS

ABSTRACT

ERRORS IN MEAN RAINFALL DETERMINATIONS
ON SMALL WATERSHEDS

by Hugh Albert Curry

Data from 22 recording rain gages were analyzed to determine the error in estimating mean rainfall covering a small area. The gages were located on a 35.5 square mile area in south-central Michigan. Six years of records were available for the study.

The data were checked for reliability and consistency. Results indicated a need for further study on rain gage stack heights as they affect precipitation catch. Need for consideration of micro-topography near rain gage stations was disclosed.

The hourly rainfall data were reduced to 296 storm events. Storms were then divided into groups according to rainfall amount. The average error for various gage densities in each group was determined and an equation relating this error, mean rainfall amount, and gage density was developed. A regression analysis utilizing a digital computer provided a least squares fit to the data. A graphical plot of the equation and the conditions under which the results are valid is presented.

Approved E H Kidder
Major Professor

Approved Carl W Hall
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ERRORS IN MEAN RAINFALL DETERMINATIONS
ON SMALL WATERSHEDS

By

Hugh Albert Curry

A THESIS

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INTRODUCTION

The design of agricultural drainage facilities, water utilization projects, and other hydrologic engineering endeavors requires a knowledge of precipitation over an area. Accurate determination of this precipitation is fundamental to providing adequate and economical designs for these projects. Most rainfall data have been secured from large areas utilizing very few rain gages. Available precipitation data on small areas with dense rain gage networks are limited. In order to obtain this and other hydrologic data from small agricultural areas, the Michigan Water Resources Commission; Surface Water Branch, Michigan District Office, United States Geological Survey; United States Weather Bureau office at East Lansing; and the Agricultural Engineering Department, Michigan State University, East Lansing, cooperatively initiated a research project (Ash *et al*., 1958). Adjacent watersheds selected for this study were the Sloan and Deer Creek areas located in south-central Michigan. These watersheds, as shown in Figure 1, form an area approximately 12 miles long and 3 miles wide. The topography is flat to gently undulating.

Since 1958, twenty-two recording rain gages located in the Deer-Sloan area have provided a dense network from which errors in mean rainfall could be studied.

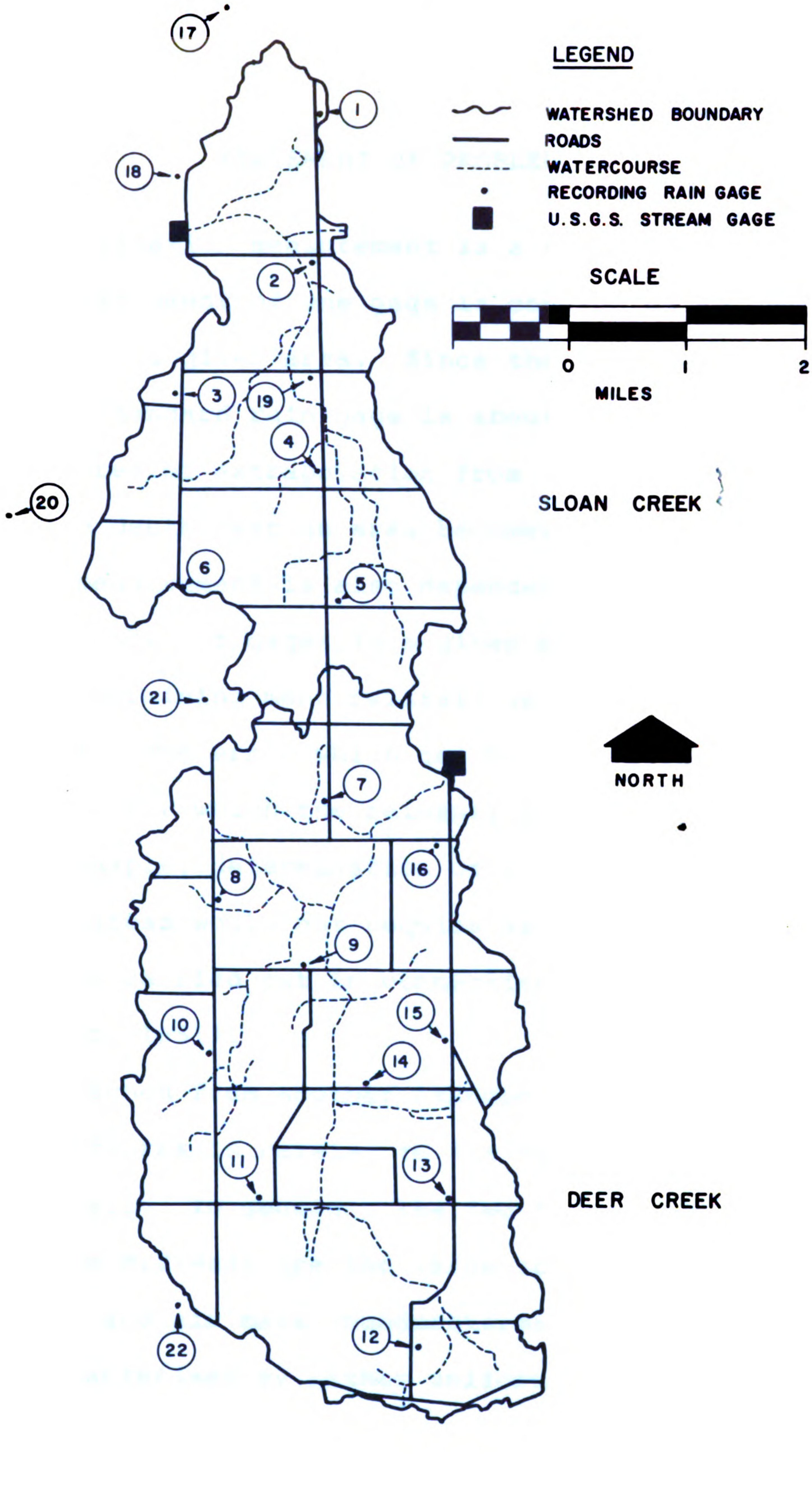


FIG. I. RECORDING RAINGAGE DISTRIBUTION
SLOAN-DEER CREEK BASIN

STATEMENT OF PROBLEM

Precipitation measurement is a sampling process in that the catchment of one gage is considered to be representative for a given area. Since the catchment area of an 8 inch standard rain gage is about 1/80,000,000 square mile, the degree of extrapolation from gage catch to computed average depth over an area becomes evident. Accuracy of rainfall measurement is also dependent on the gage density or the number of gages in a given area. Errors are induced in determining mean rainfall as the gages are placed further apart. The error which can be tolerated depends on the purpose for which the rainfall information is to be used. For example, determination of long term averages on fairly level areas would not require as dense a network as would projects carried out in connection with thunderstorm studies (Light, 1951).

Approached from another standpoint, the adequacy of various networks is related to the type of storm producing the rainfall. In general, the two types of storms prevailing in the Mid-West are the large scale frontal or cyclonic storms and air mass thunderstorms. The warm frontal storm is characterized by rather uniform rainfall intensity, a narrow distribution range, and coverage over a large area. To measure the rainfall produced by this type of storm, a few well placed rain gages are entirely adequate. Thunderstorms produce rainfall characterized by widely varying intensities, spotty distribution, and coverage over small areas (Jens, 1951). Owing to these differences a network of rain gages adequate for sampling large-scale cyclonic storms will generally fail to give the correct pattern of rainfall resulting from scattered thunderstorms (ASCE, 1949). Several studies have been carried out in an attempt to determine the error resulting from the use of various rain gage densities. The prerequisite to such a study is a dense rain gage network like that found in the Deer-Sloan area. Few of these networks exist and little analytical work has been possible (Linsley et al., 1951).

Data collected from the Deer-Sloan network permitted rainfall studies to be initiated. For this problem, errors in mean rainfall determinations were considered. The specific objectives were (1) to compare methods of determining mean rainfall over an area; (2) to determine the error in calculating mean rainfall over small areas when utilizing various gage densities; (3) to relate this error to precipitation amount and gage density; and (4) to present this error relationship in usable form.

REVIEW OF LITERATURE

The achievement of accurate measurement of mean precipitation is the basis for all the studies on gage density. Horton (1923) found one of the earliest solutions to the problem with his reference to the following statement by Sir John Benton from a 1920 irrigation manual.

> The least number of rainfall stations inside the boundaries of a catchment area which will afford a reasonably safe estimate of the rainfall may be assumed to be as follows:

	Area in Square Miles					
From	0	50	100	200	350	500
To	50	100	200	350	500	750
Sta. Needed	1	2	3	4	5	6

This information probably served as a guide and did not carry with it a full analysis on how these values were determined. It does show an early recognition and concern for the problem.

Horton (1923) studied the departures of recorded average rainfall from the true rainfall for annual precipitation values and developed a formula of the form

$$\delta = K \frac{R}{\sqrt{N}} \tag{1}$$

where

δ = departure in inches

K = a constant

R = greatest difference in inches of the mean annual rainfall at any two points within the area

N = number of stations

This formula was used and the constant determined when 13 years (1900 through 1913) of data from the 48.87 square mile Derwent River Basin in England was analyzed. A total of 42 rain gages were located in the area. Groups ranging from 1 to 20 gages were selected at random with the only restriction being that two stations in a group could not be adjacent. All 42 gages were used to determine the true average rainfall. Both the arithmetic average and the Thiessen method were used. Comparison of these methods led Horton (1923) to conclude that little advantage was gained by using the Thiessen method with the number of stations used in this study. The study was somewhat unusual in that only random sampling was used to select the groups of gages and only annual precipitation values were considered.

The following four studies have been carried out in the Middle West. All are concerned with errors connected with various densities of rain gage networks. Probably the biggest variable was the size of area over which the rain gages were placed. All interpret true rainfall over an area to be that rainfall amount computed when all gages located in the area are used in the computation. It must be assumed that the true rainfall is an accurate measure of the mean rainfall over the area and is the basis from which all errors are determined. These errors represent the difference between the true rainfall and the mean rainfall determined by a gage network less dense than the one from which the true rainfall was calculated.

Light (1947) utilized data from an 8000 square mile area for the period 1937 through 1941. Thirty-eight relatively intense storms were selected for analysis. The number of gages varied from 250 to 500.

Light (1947) presented a complete statistical theory from which his conclusions were drawn. A comparison was also made between random selection of gage groups and selection of gage groups representing uniform spacing over an area. The terms "random-gage errors" and "uniform gage errors" were applied respectively to the two methods of selection. Results showed that the uniform gage errors were less than the random gage errors for comparable storms. This conformed to the theory advanced by Light (1947) and appealed to the author's logical thinking on this matter. A graph relating percent standard error and gage density for a given size area was presented. Light (1947) did not consider the effect of rainfall amount in this study. In addition, the areas (up to 8000 square miles) were larger than those used in the following studies.

Linsley and Kohler (1951) used data collected in 1947 and 1948. Analysis of 68 storms producing precipitation over a 220 square mile area on which 55 rain gages were distributed was carried out. Gages were placed in various groups to provide different gage densities. The stations for each group were selected on as nearly a uniform grid pattern as the network permitted. Examination of the data from this study indicated an equation of the form

$$E = K P^{n} N^{m} \qquad (2)$$

would give a good fit. In this equation

E = average error in inches

P = storm precipitation in inches

N = number of gages

K, n, and m = constants

The constants varied when the area under consideration was changed. Linsley and Kohler (1951) stated that area should be introduced as an additional parameter and suggested that the constant K might be equal to kA^{c} where A is the area and k and c constants. However, the sample in their study was too limited to develop such a relationship.

Huff and Neill (1957) did include area as a variable. Several watersheds located in Illinois with areas of 25, 50, 100, 200, and 400 square miles were considered. An equation was developed for each of the above areas expressing the average error, E, as a function of the sample mean rainfall, P, and the gage density, G. These equations were of the form

$$\text{Log } E = a + b P^{0.5} + c \text{ Log } G \qquad (3)$$

E and P are expressed in inches and G in square miles per gage. A regression technique was used to fit the data to the above equation and determine the constants a, b, and c. To include area as a variable, a combined equation of the following form was developed.

$$\text{Log } E = -2.642 + 0.794\, A^{-0.07}\, P^{0.5} + 0.966\, A^{-0.12}\, \text{Log } G \qquad (4)$$

Area in square miles is represented by A while the other variables remain the same.

Sampling techniques used in arriving at gage groupings for various gage densities were investigated. Three sampling plans referred to as (1) random start, (2) combined, and (3) best-centered were carried out on the 100 square mile network. Although none of these plans would be considered completely random, the first two tended to be more random than the best-centered plan which gave a fairly uniform grid pattern. In analyzing the results, Huff and Neill (1957) came to the following conclusion:

> The best-centered sampling plan approaches the practical situation which hydrologists must contend with in using the results of sampling error analysis. In addition, this plan ranked well in the comparison with other sampling plans which were considered.

A graph expressing the range of mean rainfall in terms of the mean rainfall for a given gage density was presented. This was developed by determining the standard deviation of the errors and again determining regression equations. Then 95 percent confidence limits were obtained by taking two standard deviations and adding and subtracting from the mean rainfall to obtain the range of mean rainfall which could be expected.

McGuinness (1963) carried out a study on a 7.16 square mile watershed. The errors were determined in the same way as in previous works and a multiple regression technique used to develop the relationship

$$E = 0.03\ P^{0.54}\ G^{0.24} \qquad (5)$$

where

E = the absolute difference in rainfall catch in inches between the 'true' and one of the less dense networks

P = rainfall in inches for the entire network

G = gage density in square miles per gage

Subwatersheds delineated within the 7.16 square mile watershed provided study areas of 0.46, 1.44, 2.37, and 4.01 square miles. There was no indication that the coefficients in the equation varied as the area being considered changed.

McGuinness (1963) combined his work with that of Huff and Neill (1957) and Light (1947) to develop a nomogram for estimating the average error of mean watershed rainfall. Combining these studies allowed both large and small gage densities to be worked into the nomogram. The nomogram relates error in inches to the mean watershed rainfall in inches, the gage density in square miles per gage, and geographic location as specified by 5-year frequency values of 24-hour duration point rainfall taken from Hershfield's atlas (1961). The area over which the nomogram was considered valid was the north-central part of the United States with Missouri and Kentucky serving as the southwestern and southeastern corners.

DESIGN OF ANALYSIS

The Review of Literature dealt primarily with four studies. This was not meant to infer they are the only ones available on the subject. Work by such investigators as Wilkinson (1955) and Wilm, Nelson, and Storey (1939) was not covered since their studies were carried out in mountainous terrain. The results would not be applicable or comparable to the work presented in this thesis. Studies made in the southwestern part of the United States are not presented due to acute geographic and climatic variations of these areas from that of the Middle West. Radically different types of analyses were not apparent in these studies.

Several errors are inherent in the collection of precipitation data. These might result from instrument deficiencies, improper gage exposure, orographic effects, and variations in (1) rainfall intensity, (2) rainfall amount, (3) storm duration, and (4) storm direction. No attempt was made to relate error to variations in rainfall intensity or storm duration. The north-south orientation of the Deer-Sloan area makes study of storm direction difficult. Nearly all storms in south-central Michigan travel in an easterly direction and are over the study area for only a short time.

Discussion of Data

Data and other information came from three main sources: (1) U.S. Weather Bureau, East Lansing; (2) Agricultural Engineering Department, Michigan State University, East Lansing; and (3) personal observation of the Deer-Sloan area. Hourly rainfall data were obtained from recording rain gage charts which were collected every four or five days. Agricultural Engineering personnel had the responsibility of changing the charts and helping to maintain all field equipment in working order. Weather Bureau personnel tabulated the rainfall data and calibrated the instruments.

As shown in Figure 1, gages were distributed in a fairly uniform pattern throughout the watershed area. Sixteen gages were installed in April, 1956, and six additional ones in April, 1958. In placing a gage, consideration was given to exposure, accessibility, and gage security. The last mentioned of these considerations was carried out by positioning gages within sight of existing farm building sites. All the gages were weighing type recording rain gages equipped with 8 inch diameter funnels.

Only that data obtained since April, 1958, was used for this study. It was at this time that the network was expanded to 22 gages. This analysis was not concerned with snowfall and data from the four winter months of the year, November, December, January, and February were not used. The rainfall data for the months May through October, 1958, and the months March through October, 1959, through 1963 were utilized. This provided a cumulative total of 46 months of warm weather rainfall records.

The concept of a storm event was developed to reduce the rainfall data to a usable form. A storm event, as perceived in this study, is a period of rainfall when greater than 0.01 inches was recorded in a 1 hour period. In addition, one or both of the following conditions have to be met:

1. One gage in the network records 0.20 inches or more rainfall in a period of 1 hour.
2. Greater than one-half of the gages in the network record 0.10 inches or more rainfall in a period of 1 hour.

These criteria are a basis for Table 1 which presents a breakdown of the number of storm events recorded for the 6 years of study.

Table 1. Number of storm events determined from the Deer-Sloan data

Year	No. of Storm Events	Collection Period
1958	37	May through October
1959	54	March through October
1960	49	
1961	60	
1962	58	
1963	38	
Total	296	

DISCUSSION OF RESULTS

Reliability of Data

According to Horton (1919), precipitation measurements are subject to various errors, most being individually small. It is important to consider the conditions and limitations under which engineering data are gathered and tabulated.

As brought out in the preceding section, an effort was made to locate gages in areas which were considered "to be of good exposure," but still maintaining accessibility and gage security. Frequent servicing and adequate maintenance procedures tended to reduce errors due to functional disorders of the equipment. Where there was an obvious error in tracings on the charts, Weather Bureau personnel estimated the rainfall amounts and time of occurrence for the missing gages by using surrounding gages as references. It was necessary to estimate only time of rainfall as totals were usually available (Eichmeier and Wheaton, 1960). Since this is true, very little error should be induced due to those estimates.

Table 2 shows that three of the gages were moved during the period in which data were collected.

A check on the consistency of record from these three gages was made to indicate whether the move produced

Table 2. Gage relocation data

Gage Number	Month Moved	Approximate Distance Moved
4	May, 1959	400 ft.
16	May, 1959	1/2 mi.
19	June, 1960	300 ft.

any relative change in the precipitation catch. This check was made by applying a double-mass analysis which tests the consistency of the record at a station by comparing its accumulated annual or seasonal precipitation with the concurrent accumulated values of mean precipitation for a group of surrounding stations. Using this method, a change due to meteorological causes would not cause a change in slope as all base stations would be similarly affected (Linsley _et al_., 1958).

Records from May, 1958, through December, 1963, were utilized for the double-mass analysis. Accumulated monthly precipitation for each of the stations indicated in Table 2 was compared with the accumulated values of mean precipitation for all the gages in the network.

Figures 2, 3, and 4 show the results of the double-mass analysis as applied to gages 4, 16, and 19, respectively. On each of the graphs, the slope of the mean line connecting the points does not change for the 6 year period being considered. This is interpreted to mean that the relocation

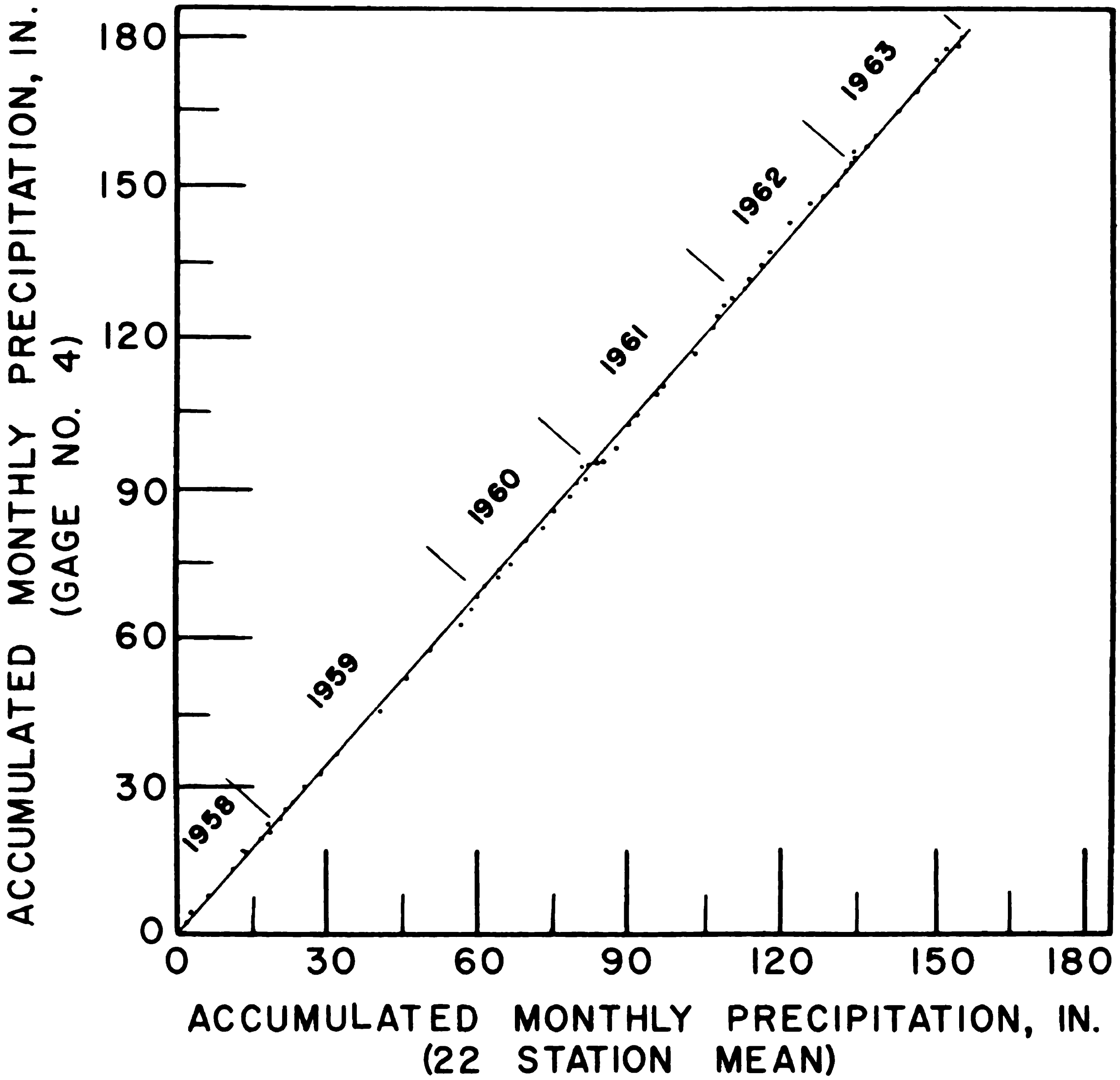


FIGURE 2. DOUBLE-MASS ANALYSIS
GAGE NUMBER 4

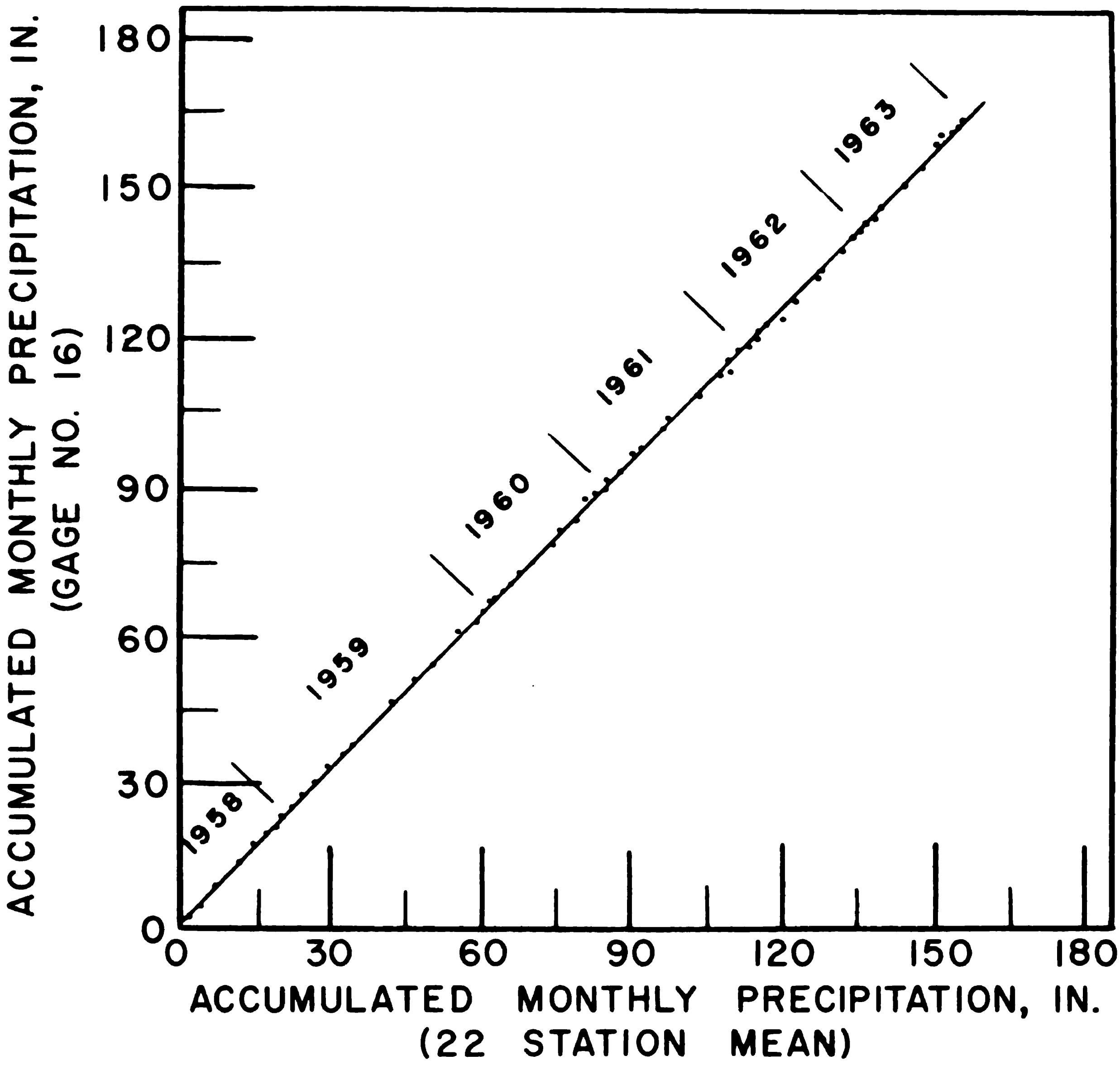


FIGURE 3. DOUBLE-MASS ANALYSIS
GAGE NUMBER 16

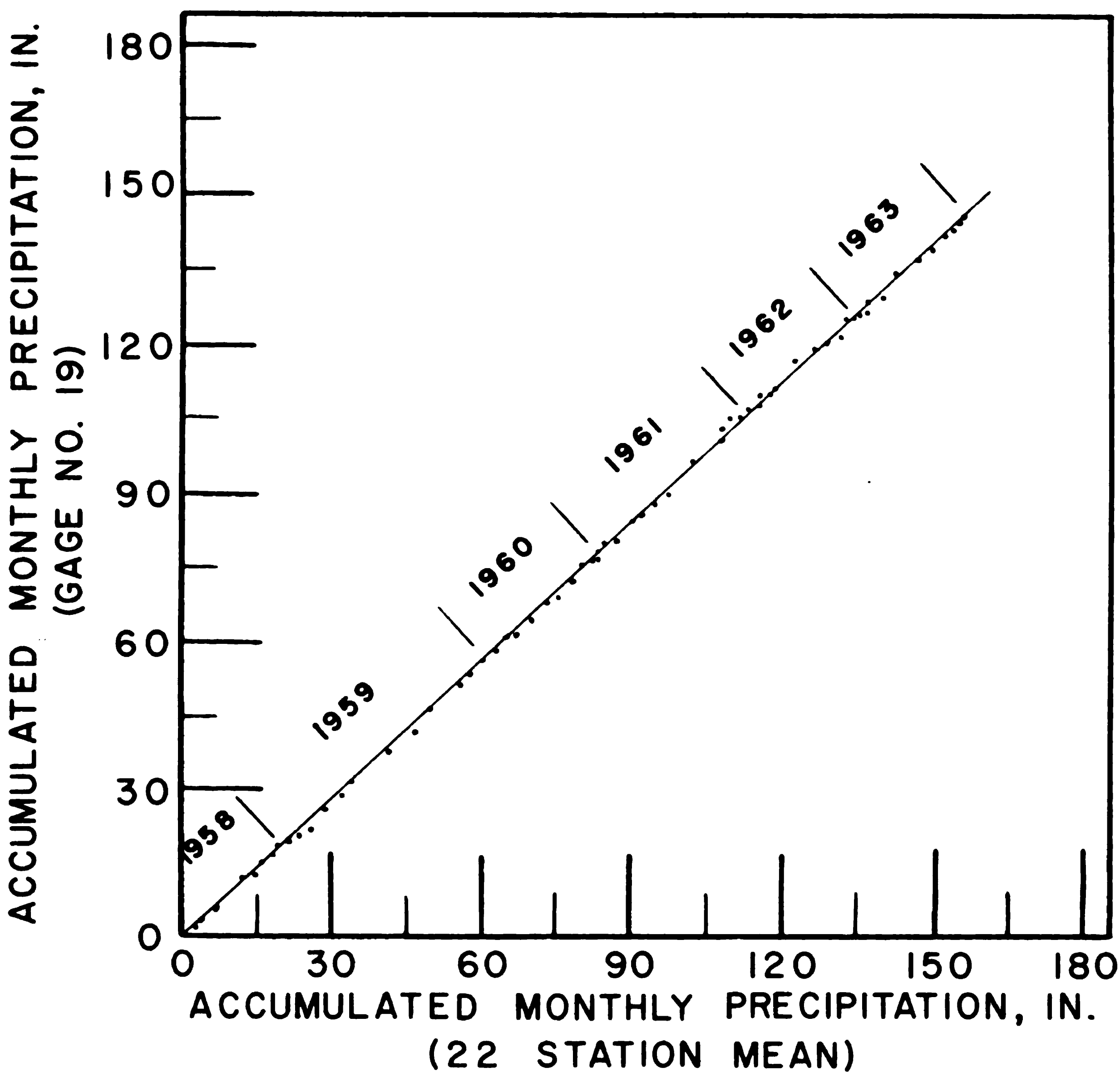


FIGURE 4. DOUBLE-MASS ANALYSIS
GAGE NUMBER 19

of these gages from their original position had no significant effect on precipitation catch.

Another check on the data was to determine if any persistent areal pattern was detectable. Figure 5 shows the Thiessen network for the Deer-Sloan area. Development of this network is presented in the following section. The network is shown again in Figure 6 and total precipitation values for the period May, 1958, through December, 1963, indicated. From this figure, no inconsistencies in areal pattern involving several gages can be found.

Table 3 shows each gage along with the total precipitation catch for the period indicated above. The last column ranks the gages according to total precipitation catch. Examination discloses that gage 4 collected the greatest amount of precipitation and gage 20 the least, the difference being 34.95 inches. However, the difference between gage 16, the second ranking gage, and gage 20 was 18.97 inches. This would indicate that gage 4 was collecting a considerably greater amount of precipitation than the other gages. Further evidence of this is found in Figure 2 which shows the double-mass analysis for gage 4. If this gage was collecting the same amount of precipitation as the other gages in the network, a 1 to 1 slope would result. Instead, the slope of the line in Figure 2 is 1.00 horizontal to 1.15 vertical. In Figures 3 and 4, the slopes are 1.00 to 1.03 and 1.00 to 0.94 respectively.

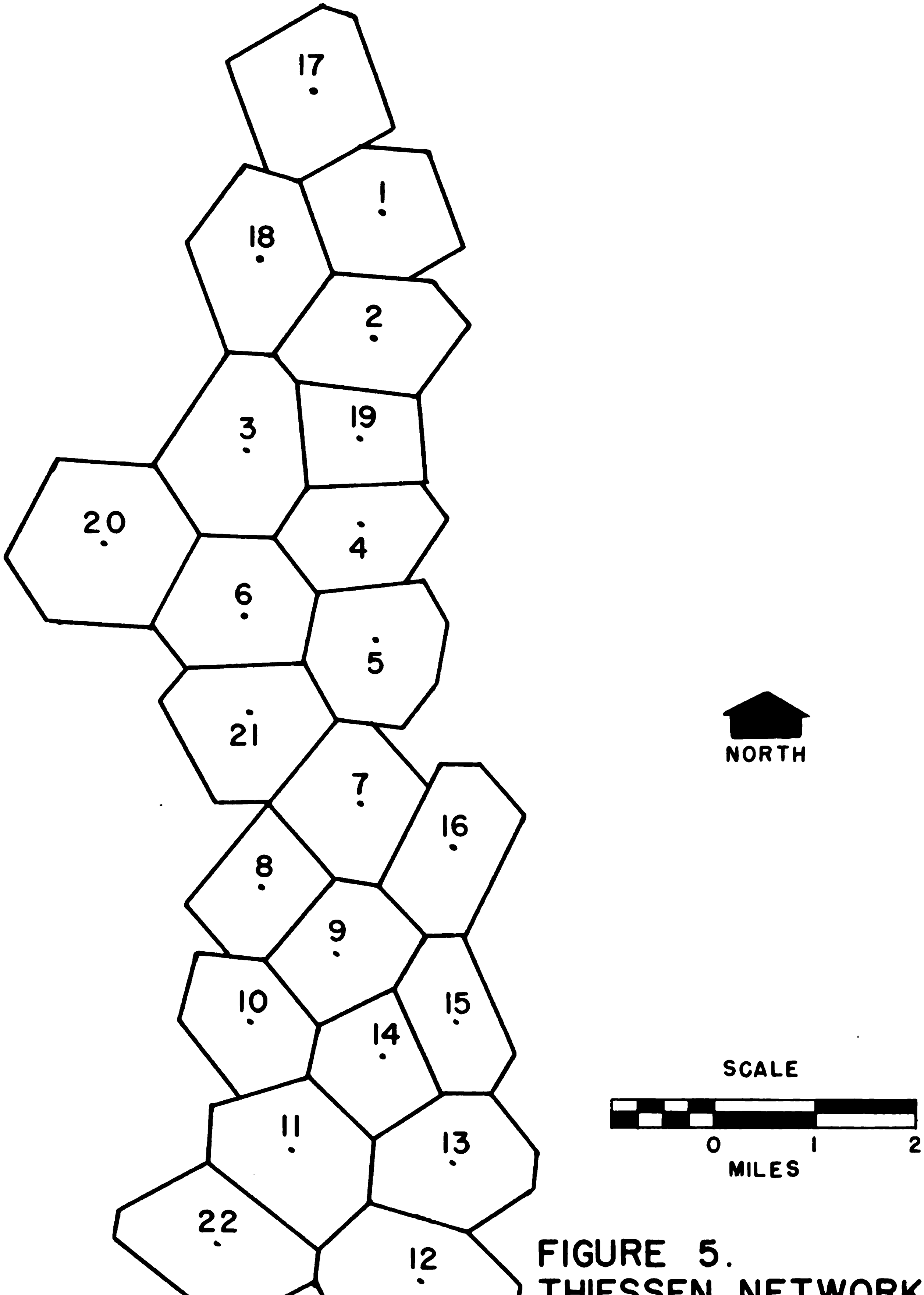
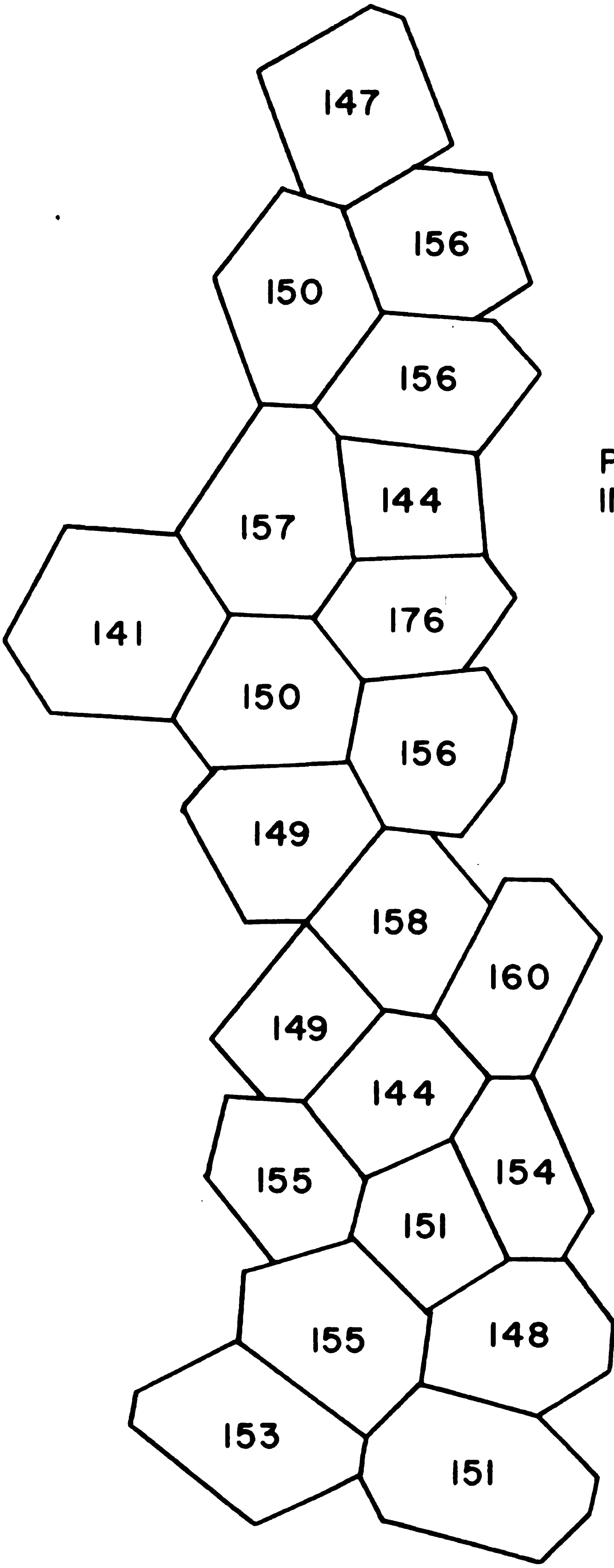


FIGURE 5.
THIESSEN NETWORK
DEER-SLOAN AREA



PRECIPITATION VALUES IN INCHES

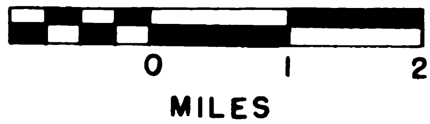


FIGURE 6.
TOTAL PRECIPITION
MAY, 1958 THROUGH
DECEMBER, 1963

Table 3. Total precipitation catch for each gage from May, 1958, through December, 1963

Gage Number	Total Precipitation Catch, Inches	Rank
1	156.25	5
2	156.00	6
3	157.10	4
4	176.18	1
5	155.52	7
6	149.99	14
7	157.89	3
8	148.89	16
9	144.03	21
10	154.79	8
11	154.62	9
12	151.41	12
13	148.12	18
14	150.89	13
15	154.23	10
16	160.20	2
17	146.82	19
18	149.53	15
19	144.35	20
20	141.23	22
21	148.57	17
22	152.81	11

Thus gages 9 and 16 produce mean lines nearer to the 1 to 1 slope and have catches which conform more to the network average. Gage 4, by exhibiting a steeper slope, indicates a catch greater than that of the remainder of the network.

Early studies on the Deer-Sloan rain gage network assumed all gages would record approximately the same amount of precipitation. Eichmeier, Wheaton, and Kidder (1959) stated:

> It is believed that a longer period of records will show that no one gage consistently receives the highest amount of precipitation.

Later investigations involving this network have not substantiated this statement. Meyers (1960), in discussing the use of gage 4, made the following observation:

> . . . this gage recorded more rainfall than the Thiessen average 13 of the 18 times; several times by over 0.5 inch and one of these being 43 percent larger. As the location and calibration of this gage met all standard specifications, why it recorded consistently high has been of great concern.

Records compiled by A. H. Eichmeier (1964) show that in 66 months of record, gage 4 has ranked first with regard to total monthly rainfall 26 times when compared to the other 21 gages in the network.

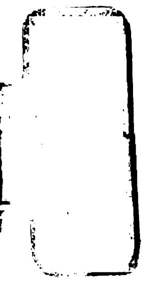
All of the factors mentioned above indicate gage 4 consistently collects more rainfall than the other gages. Since the location of this gage adheres to the requirements of a "good" gage exposure and is functioning properly, it is difficult to explain the deviation of its recordings.

Referring again to Table 3, it is evident that gages

17 through 22 were located in the lower portion of the table based on precipitation catch. This is significant since these six gages were added to the network approximately 2 years after the first 16 gages were put in place. The gages were interspersed with those already existing in the network, so differences due to gage distribution were ruled out. In an attempt to explain why these gages rank low in total precipitation catch, a determination was made to see if they differed significantly from the others. Investigation showed that the only physical difference was in the height of the 8 inch diameter stack or funnel which extends above the conical top of the recording rain gage. On some of the gages, there was a stack 6 inches in height while on others, the stack was only 3 inches high. In order to relate this observation to the 22 gages, Table 4 was prepared. This table distributes the gages shown in Table 3 in order of their rank. In general, gages with 3 inch stacks are clustered toward the lower end of the column. This indicated these gages collected less precipitation than gages with 6 inch stacks. Verification of this fact is shown by totaling the 6 years of precipitation catch for each type of gage and determining the average catch per gage. For the 10 gages with 3 inch stacks, the average catch was 150.52 inches. The 12 gages which had 6 inch stacks received an average of 154.52 inches for the 6 year period.

Table 4. Distribution of gages according to total precipitation catch

Rank	Gage Number	Gages with 3 Inch Stacks Marked X Blanks Indicate Gages with 6 Inch Stacks
1	4	
2	16	
3	7	X
4	3	
5	1	
6	2	
7	5	
8	10	X
9	11	
10	15	
11	22	X
12	12	X
13	14	X
14	6	
15	18	X
16	8	
17	21	X
18	13	X
19	17	X
20	19	X
21	9	
22	20	



Data from the Deer-Sloan area disclose the relationship indicated above. Additional study will be needed to further confirm these results.

True Rainfall, Gage Density, and Errors in Mean Rainfall

To adequately fulfill the objectives, the terms "true rainfall," "gage density," and "errors in mean rainfall" become important.

True rainfall is used to indicate the best possible measure of rainfall which covers an area. It is calculated by utilizing all gages available in a given network. This interpretation of true rainfall concurs with that used in the studies of Light (1947), Linsley and Kohler (1951), Huff and Neill (1957), and McGuinness (1963) which were cited earlier.

To determine the mean rainfall on an area over which several gages are distributed, three methods are generally accepted. The first method simply determines the arithmetic average of the recorded rainfall amounts from all gages. This is known as the arithmetic mean method. The Thiessen method utilizes a weighing factor based on the area of influence for each gage. The relative size of these areas varies with the gage distribution. The areas are arrived at by plotting the rain gage stations on a map, connecting adjacent stations by straight lines, and constructing the perpendicular bisectors of these lines.

These perpendicular bisectors form the boundaries of polygons which define the area over which a given gage is considered representative. The size of each area is determined and expressed as a percentage of the total area to determine the weighing factor. This factor is often called the Thiessen coefficient. The third method, known as the isohyetal method, requires that contours of equal rainfall be drawn on a map of the area under consideration. These contours are known as isohyets. Determination of area between isohyets allows for the computation of total rainfall over an area. This method is especially adapted to areas where orographic effects need to be taken into account.

The Deer-Sloan area is nearly level and orographic effects are practically nonexistent. Since this is true, the isohyetal method was not used. The decision was made to use both the Thiessen and the arithmetic mean method so the results could be compared. The Thiessen method served the additional function of establishing the total area over which the rain gages could be considered representative. Figure 5 shows this area as formed by the Thiessen network. In the cases where outside boundaries were not defined by the perpendicular bisectors, mirror images of the inner boundaries around that particular gage were used. For example, the outside boundaries of gage 1 were produced by drawing lines parallel to the boundaries between gage 1 and gages 17, 18, and 2. These lines were

placed at the same distance from the gage as were the inner boundaries. With the Thiessen network completed, the total area included within the boundaries was 35.54 square miles. This is about 10 square miles larger than the area contained within the watershed boundaries as shown in Figure 1. The larger area is used since the Deer and Sloan watershed boundaries have no meaning for rainfall determinations. Table 5 shows the gages and the area over which each gage was considered representative. The Thiessen coefficient is shown as a percentage. Since gage 16 was moved a considerable distance on May 5, 1959, it was necessary to correct the Thiessen coefficients of gages 7, 8, 9, 15, and 16. The total area and coefficients for the remainder of the gages were not changed appreciably. Gages 4 and 19 were also relocated, but the distance which they were moved was short and the coefficients were unaffected. Two sets of Thiessen coefficients are shown in Table 5: (1) those to be used in evaluating all storms occurring before May 5, 1959, and (2) those to be used for storms occurring after May 5, 1959.

With these coefficients and the rainfall data discussed earlier, the true rainfall over the total 35.54 square mile area could be determined. Figure 7 shows a sample computation on the form developed to determine rainfall amounts. The true rainfall was determined by both the Thiessen and the arithmetic mean methods. Table 6 compares the difference in these values for the 296 storm events.

THESIS

Table 5. Thiessen coefficients for the Deer-Sloan area

Gage No.	Area, Sq. Mi.		Thiessen Coefficient, %	
	Before May 5, 1959	After May 5, 1959	Before May 5, 1959	After May 5, 1959
1	1.56	*	4.4	*
2	1.61		4.5	
3	1.90		5.3	
4	1.44		4.1	
5	1.74		4.9	
6	1.72		4.8	
7	1.76	1.60	5.0	4.5
8	1.26	1.27	3.5	3.6
9	1.27	1.35	3.6	3.8
10	1.37		3.9	
11	1.86		5.2	
12	2.15		6.1	
13	1.65		4.6	
14	1.23		3.5	
15	1.08	1.17	3.0	3.3
16	1.06	1.57	4.5	4.4
17	1.74		4.9	
18	1.87		5.3	
19	1.03		2.9	
20	2.19		6.2	
21	1.58		4.4	
22	1.93		5.4	
Total	35.54		100.0	

*Note: Those left blank did not change.

DATE OF STORM APRIL 15, 1960
10 A.M.-1 P.M.

GAGE NUMBER	THIESSEN COEFF., %	OBSERVED PREC., IN.	WEIGHTED PREC., IN.
1	4.4	0.29	1.3×10^{-2}
2	4.5	0.30	1.4
3	5.3	0.32	1.7
4	4.1	0.36	1.5
5	4.9	0.40	2.0
6	4.8	0.41	2.0
7	4.5	0.30	1.4
8	3.6	0.35	1.3
9	3.8	0.35	1.3
10	3.9	0.27	1.1
11	5.2	0.43	2.2
12	6.1	0.40	2.4
13	4.6	0.51	2.3
14	3.5	0.35	1.2
15	3.3	0.31	1.0
16	4.4	0.25	1.1
17	4.9	0.32	1.6
18	5.3	0.34	1.8
19	2.9	0.33	1.0
20	6.2	0.34	2.1
21	4.4	0.40	1.8
22	5.4	0.57	3.1
TOTAL	100.0	7.90	0.36
AVERAGE		0.36	

FIGURE 7. SAMPLE COMPUTATION OF RAINFALL AMOUNTS

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Table 6. Comparison of Thiessen and arithmetic mean methods of determining true rainfall

Difference Between Thiessen & Arith. Ave. Values of True Rainfall, Inches	Number of Storms	Percent of Total
0.00	181	61.2
0.01	95	32.1
0.02	13	4.4
0.03	6	2.0
0.04	1	0.3
Total	296	100.0

This table reveals little difference in the arithmetic mean method and the Thiessen method of determining true rainfall values for this dense network. The Thiessen value was used in this study. Table 7 shows the distribution of storms based on the Thiessen value of true rainfall.

The term, gage density, was used to designate the number of rain gage stations in an area. The boundaries of the area and the distribution of gages within the area must also be known to carry out an analysis. It is expressed as a quantity having the units of square miles per gage.

True rainfall is based on the most dense gage distribution available. This occurs when data from all 22 gages located in the 35.54 square mile area are utilized and result in a gage density of 1.6 square miles per gage.

Table 7. Distribution of storms according to true rainfall

Range, Inches		Number of Storms	Percent of Total
0.00 - 0.09		37	12.5
0.10 - 0.19		103	34.8
0.20 - 0.29		46	15.5
0.30 - 0.39		34	11.4
0.40 - 0.49		25	8.4
0.50 - 0.59		9	3.4
0.60 - 0.69		4	1.3
0.70 - 0.79		13	4.4
0.80 - 0.89		2	0.7
0.90 - 0.99		7	2.3
1.00 - 1.09		2	0.7
1.10 - 1.19		4	1.3
1.20 - 1.29		1	0.3
1.30 - 1.39		2	0.7
1.40 - 1.49		2	0.7
1.50 - 1.59		1	0.3
1.60 - 1.69		1	0.3
1.70 - 1.79		1	0.3
1.80 - 1.89		2	0.7
	Total	296	100.0

Values of mean rainfall for less dense networks are obtained by considering data from fewer than the total of 22 gages. Groups of gages were selected to represent the various gage densities. Selection was made to provide the most uniform pattern of gage distribution possible. This has the advantage of approaching the situation which generally appears in the field and is a method which can be reproduced. It was pointed out in the Review of Literature that this method is better than any other when compared with the so-called "random sampling" methods.

For the Deer-Sloan area, utilization of only one gage to define the network results in a gage density of 35.5 square miles per gage. Therefore, by selecting various numbers of gages, gage densities between 1.6 and 35.5 square miles per gage were obtained. For this analysis, networks consisting of 1, 2, 3, 4, 5, 7, and 12 gages were used. Since the Deer-Sloan area is long and narrow, consideration for the uniformity of gage spacing was made along the north-south axis only. Orientation of the gages in an east-west direction across the narrow portion of the watershed was not taken into account. This procedure is justified by the shape of the area and the fact that most storms in this area follow a west to east course. All gages placed on an east-west line will lie in the path of these storms.

The procedure outlined above resulted in gage networks as shown in Table 8. It is with these networks that the analysis was carried out.

Table 8. Gage networks used in the analysis of data

No. of Gages in Network	Gage Density, Sq. Mi./Gage	Gages Used in Each Network
1	35.5	21
2	17.8	2,10
3	11.8	1,11,21
4	8.9	1,3,13,16
5	7.1	9,17,19,21,22
7	5.1	2,3,7,12,14, 18,21
12	3.0	1,3,6,9,12,13, 14,16,17,18, 19,21
22	1.6	1 through 22

These networks form the most nearly uniform pattern possible based on spacing in a north-south direction. The only exception to this statement is that gage 4 was not considered in any network having less than the maximum of 22 gages. Elimination of this gage from the less dense networks is justified by the bias exhibited in the data. This is discussed under the section entitled Reliability of Data.

The third term mentioned in the opening paragraph of this section is error in mean rainfall. This error represents the absolute difference between true rainfall and mean rainfall as determined by one of the less dense networks.

The first part of this section explained that true rainfall is determined using the Thiessen method and all 22 gages in the network. Mean rainfall is the average rainfall over an area for one of the less dense networks. It is determined by the arithmetic mean method since this method produces nearly the same answer as does the Thiessen method (see Table 6). It was felt that the increased time necessary to use the Thiessen method could not be justified.

Data to calculate the mean rainfall was obtained from forms like those shown in Figure 7. Table 9 serves as an example of how the error for one storm at various gage densities is determined. These errors were determined for each of the 296 storm events used in the study.

Table 9. Example of error determination for a single storm

Date of Storm - October 29, 1963.
True Rainfall Amount - 0.23 inches.

Gage Density Sq. Mi./Gage	Mean Rainfall, Inches	Error, Inches*
35.5	0.10	0.13
17.8	0.26	0.03
11.8	0.16	0.07
8.9	0.27	0.04
7.1	0.21	0.02
5.1	0.23	0.00
3.0	0.25	0.02

*Absolute difference between true rainfall and mean rainfall.

Analysis of Data

This section explains how the data were utilized to carry out an analysis. The final result is an expression of the error inherent in calculating mean rainfall over an area.

Error in determining mean rainfall is related to the precipitation amount and gage density. References also indicate that this error is a function of the size of area under consideration. However, this study deals with only one area and this factor could not be included as a variable. In general, it was evident that the error became larger as the mean rainfall increased and also as the gages became less dense. Preliminary plots of the data showed that an equation of the form

$$\text{Log } E = a + b\,P + c\,G \qquad (6)$$

gave a good fit. E is error in mean rainfall in inches, P the mean rainfall in inches, G the gage density in square miles per gage, and a, b, and c are constants.

To determine the constants in the above equation, all the data were subjected to a multiple regression analysis. The digital computer at Michigan State University was utilized to carry this out. The "CORE routine," a program written by Agricultural Experiment Station personnel at Michigan State University, and the data were submitted to the computer to obtain a least squares fit. Since little additional work is necessary to fit the data to more than

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one regression equation, equations of the form used by Linsley and Kohler (1951) and Huff and Neill (1957) were included. These equations are found in the Review of Literature and are of the following form:

$$E = a\ P^m\ G^n \tag{7}$$

$$\text{Log } E = a + b\ P^{0.5} + c \text{ Log } G \tag{3}$$

In the equation, a, b, c, P, and G represent the same values as in equation (6); m and n are constants.

Results from the computer produced the following constants:

$$\text{Log } E = -1.936 + 0.352\ P + 0.015\ G \tag{6,a}$$

$$E = 0.013\ P^{0.31}\ G^{0.47} \tag{7,a}$$

$$\text{Log } E = -2.346 + 0.499\ P^{0.5} + 0.472 \text{ Log } G \tag{3,a}$$

Multiple correlation coefficients were 0.49, 0.47, and 0.49 for equations (6,a), (7,a), and (3,a) respectively. Although this magnitude of correlation is significant, it was desired that a greater degree of accuracy be obtained. In addition, plotting the equations showed considerable variation in the value obtained for E, especially at high values of mean rainfall, P, and high values of gage density, G.

The first consideration given to the question of greater accuracy was development of a different form for the equation. Examination of the data showed that the scatter of absolute error values was so great that no one equation would provide a better fit than those used. A method similar to that described by Linsley and Kohler (1951) was

then employed. The storm events were divided into groups based on the true rainfall value for each storm. Table 10 shows the range and number of storm events in each group. The midpoint of each range is the mean rainfall value considered representative of the range.

Table 10. Number of storm events in various precipitation ranges

Precipitation Range, Inches	Mean Rainfall, Inches		Number of Storm Events in Group
0.00 - 0.09	0.05		37
0.10 - 0.19	0.15		103
0.20 - 0.29	0.25		46
0.30 - 0.39	0.35		34
0.40 - 0.49	0.45		25
0.50 - 1.00	0.75		35
1.00 - 1.82	1.41		16
		Total	296

The average errors for each group were calculated in two steps: (1) errors in mean rainfall for each gage density were totaled for all storm events in the group, (2) this total was divided by the number of storm events in the group. This resulted in seven average errors for each group which correspond to the seven gage densities shown in Table 8. These errors and gage densities, along with the precipitation values, are tabulated in the Appendix.

The data involving average error were fitted to equations of the same form used previously. The regression coefficients arrived at by submitting the data and program to the computer are as follows:

$$\text{Log } E = -1.966 + 0.485\ P + 0.022\ G \qquad (6,b)$$

$$E = 0.013\ P^{0.44}\ G^{0.66} \qquad (7,b)$$

$$\text{Log } E = -2.554 + 0.711\ P^{0.5} + 0.663\ \text{Log } G \qquad (3,b)$$

Multiple correlation coefficients for equations (6,b), (7,b), and (3,b) were 0.93, 0.88, and 0.92 respectively. Graphical plots of these equations showed equation (3,b) provided a better fit at high values of mean rainfall amounts. Figure 8 presents a graph of equation (3,b) relating average error and mean rainfall for gage densities of 5, 10, 20, and 30 square miles per gage. These values would be valid for mean rainfall amounts covering areas of approximately 35 square miles. Since the error found is an average error, values of absolute error will deviate around this average. A few of these deviations are quite large.

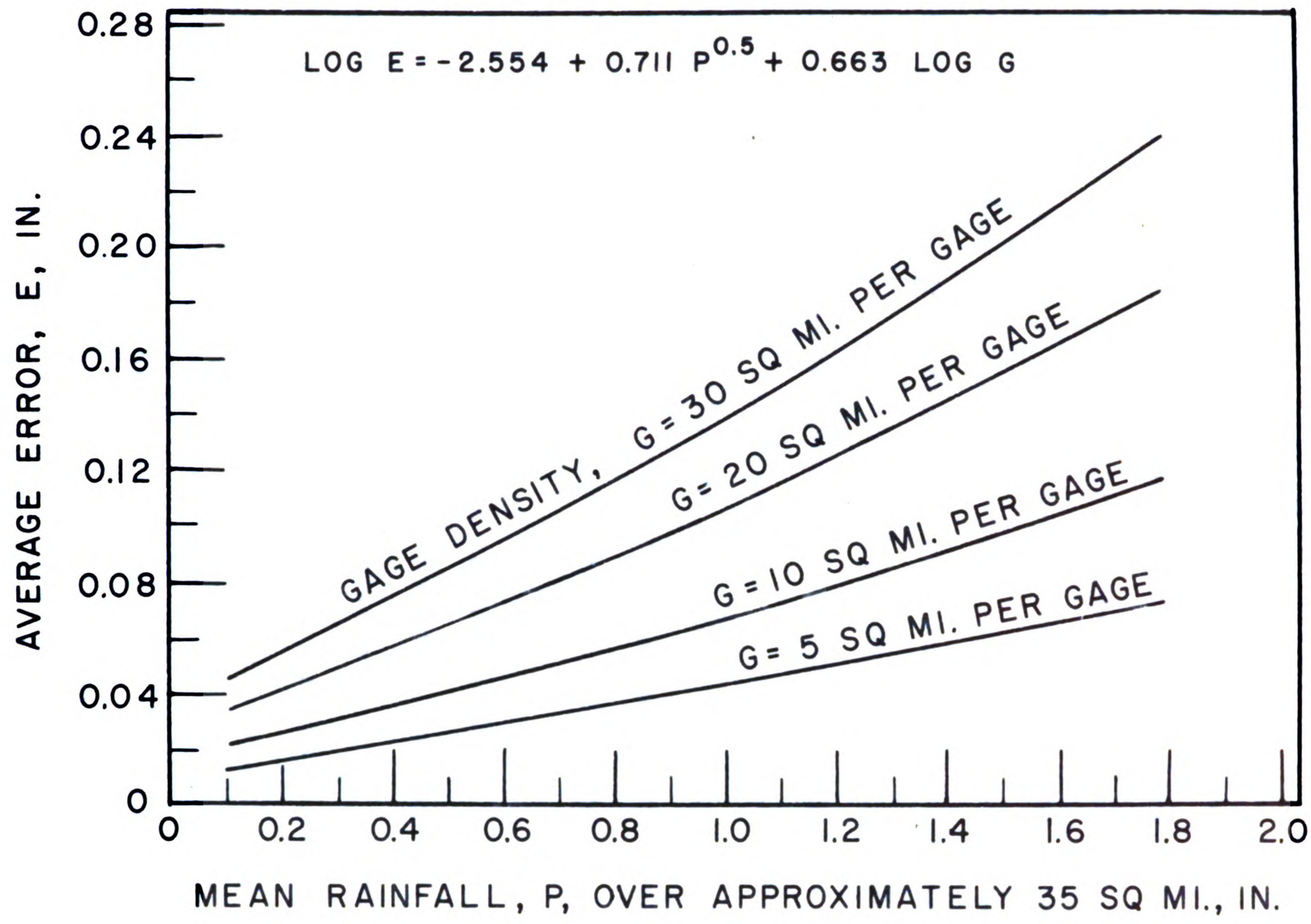

FIGURE 8. AVERAGE ERROR IN MEAN RAINFALL

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SUMMARY

A dense recording rain gage network established on Deer and Sloan Creek Watersheds in south-central Michigan provided data for rainfall analysis. The network consisted of 22 recording rain gages covering an area of approximately 35.5 square miles. The rain gages were installed and calibrated by U.S. Weather Bureau Personnel. The Agricultural Engineering Department at Michigan State University helps to maintain the equipment and change the charts.

Six years of rainfall records obtained from 1958 through 1963 were available for analysis. Hourly rainfall data tabulated by the Weather Bureau were used to select 296 storm events. Snowfall data were not utilized in the study.

Checks were made to test the reliability of data. For each of three gages that were moved, a double-mass analysis was carried out. Observations were made of the distribution of total precipitation catch for the 6 year period. Results showed one gage in the network was recording a significantly greater amount of precipitation than the other gages. Also, a group of gages in the network tended to record lesser amounts of precipitation than the remaining gages.

The Thiessen network was developed to define the

area over which each gage was considered representative. True rainfall, or the mean rainfall as determined by all 22 gages, was calculated by both the arithmetic mean method and the Thiessen method. The results from these two methods were compared. Errors in mean rainfall were calculated by obtaining the absolute difference between true rainfall and mean rainfall as determined by a gage network utilizing less than 22 gages. Seven gage densities ranging from 3.0 to 35.5 square miles per gage were used in the analysis. Errors were computed for all 296 storm events.

The average error in mean rainfall estimates was related to gage density and mean rainfall amount. A regression analysis utilizing the digital computer was used with the Deer-Sloan data. This analysis furnishes a least squares fit to the data and provides constants to be used in an equation relating the variables.

CONCLUSIONS

The conclusions resulting from this investigation were:

1. Rainfall amounts from a single storm vary widely, even over small areas.

2. The Thiessen method and the arithmetic mean method of determining mean rainfall will result in nearly the same value for dense rain gage networks.

3. In south-central Michigan, nearly 75 percent of the storm events, as defined by this thesis, resulted in rainfall amounts of less than 0.40 inches. About one-third fell in the 0.10 to 0.19 inch range.

4. Values of absolute error are too scattered to develop an equation relating this error to mean rainfall and gage density.

5. A relationship between average error, mean rainfall, and gage density was developed. This relationship is valid for areas approximately the same size as that from which the data were obtained. Infrequent deviations of rather large magnitude must be permissible if average error data are used.

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RECOMMENDATIONS FOR FUTURE STUDY

1. A means of comparing true rainfall values as derived from different studies needs to be developed.

2. The effect of rain gage stack height on precipitation catch needs to be investigated.

3. The effect of micro-topography near rain gage stations needs further study.

4. A determination of other factors besides gage density and rainfall amount which affect estimates of mean rainfall is needed.

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APPENDIX

AVERAGE ERROR DATA

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Appendix. Average error data for the Deer-Sloan area

Mean Rainfall, Inches	Gage Density, Sq. Mi./Gage	Average Error, Inches
0.05	35.5	0.05
0.15	35.5	0.06
0.25	35.5	0.08
0.35	35.5	0.11
0.45	35.5	0.14
0.75	35.5	0.14
1.41	35.5	0.23
0.05	17.8	0.04
0.15	17.8	0.02
0.25	17.8	0.04
0.35	17.8	0.04
0.45	17.8	0.05
0.75	17.8	0.06
1.41	17.8	0.14
0.05	11.8	0.03
0.15	11.8	0.02
0.25	11.8	0.03
0.35	11.8	0.03
0.45	11.8	0.05
0.75	11.8	0.06
1.41	11.8	0.07
0.05	8.9	0.02
0.15	8.9	0.01

Appendix. (Continued)

Mean Rainfall, Inches	Gage Density, Sq. Mi./Gage	Average Error, Inches
0.25	8.9	0.02
0.35	8.9	0.03
0.45	8.9	0.06
0.75	8.9	0.04
1.41	8.9	0.09
0.05	7.1	0.02
0.15	7.1	0.01
0.25	7.1	0.02
0.35	7.1	0.02
0.45	7.1	0.03
0.75	7.1	0.04
1.41	7.1	0.05
0.05	5.1	0.02
0.15	5.1	0.01
0.25	5.1	0.02
0.35	5.1	0.02
0.45	5.1	0.03
0.75	5.1	0.03
1.41	5.1	0.07
0.05	3.0	0.01
0.15	3.0	0.01
0.25	3.0	0.01
0.35	3.0	0.02

Appendix. (Continued)

Mean Rainfall, Inches	Gage Density, Sq. Mi./Gage	Average Error, Inches
0.45	3.0	0.03
0.75	3.0	0.03
1.41	3.0	0.06

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