FUNDAMENTALS OF ANTENNA RADIATION

By

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PREFACE

The purpose of this thesis is to present in a logical and straightforward manner the fundamentals of antenna radiation and related concepts which are not clearly explained in some texts. The author wishes to express his thanks to Dr. R. D. Spence of the Department of Physics at M.S.C., whose criticism made the author self-critical of his own ideas, to Mr. Stephen S. Atwood of the Department of Electrical Engineering at the U. of M., whose help and encouragement were invaluable when even the dilemmas had dilemmas, and to Dr. J. A. Strelzoff of the Department of Electrical Engineering at M. S. C. for his help in the development of this thesis, and for his patience in reading the manuscript.

H. A. Myers

INTRODUCTION

It is a natural tendency to classify the fundamentals of antenna radiation along with the fundamentals of D.C. circuit analysis, operation of simple motors, etc., all of which is pretty cut-and-dried material. However, the answer to the question, "How does an antenna radiate?", is quite difficult, and many authors and teachers who include the study of antennas in their course material show a lack of clarity on this point. It is one thing to arrive at the equations for the radiation field and describe the radiation from there, but quite another to describe how this radiation is generated by the antenna. The crux of the dilemma is this: The tangential component of the electric field intensity is necessarily practically zero at the surface of the conductor because it is approximately zero inside, and the tangential component is continuous across the boundary. How then can there be any power radiated? Poynting's vector ralation is $\bar{P} = \bar{E}x\bar{H}$, and if $\bar{E} = 0$, then $\bar{P} = 0$, because $\bar{H}$ is certainly finite. Bronwell & Beam[1] present the conventional radiation-from-the-gap solution to this problem. Their book is chosen as a specific target because it is used in our senior courses; however, they are not alone in

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[1] Bronwell, A. B., & Beam, R. E. Theory and Application of Microwaves 1947 New York: McGraw-Hill

their lack of clarity on this point. Section 20.05 of Bronwell & Beam* is quoted in its entirety so that the reader may better grasp the problem of this thesis and so that the author will not be accused of quoting out of context.

"20.05. VALIDITY OF THE INDUCED-EMF- METHOD. - Although the induced emf method of evaluating antenna impedances yields results which agree favorably with measured values, this method embodies certain inconsistencies which lead us to question its validity. If we assume that the antenna is a perfect conductor, then the tangential intensity must be zero in order to satisfy the boundary conditions. This presents an embarrassing situation, since if E_z is zero at the condusting surface, then the normal component of Poynting's vector is likewise zero and there can be no power flow normal to the surface of the antenna. Does this mean that a perfectly conducting antenna could not radiate power? Experimental evidence indicates that the radiating properties of an antenna are actually improved as the conductivity of the antenna conductor increases. Intuitively we would expect that a perfectly conducting antenna would radiate just as effectively as an antenna with finite conductivity. If the radiated power does not leave the surface of the conductor, then where does it leave the antenna?

Before attempting an explanation of the anomaly, let us straighten out the matter of current distribuition in the antenna. It is evident that, for a perfectly conducting antenna, the assumption of a sinusoidal current distribution is erroneous, since it yields a tangential intensity at the surface of the antenna which we know cannot exist. However, the value of E_z computed on the basis of an assumed sinusoidal current distribution is quite small and, for thin antennas, only a slight modification of the current distribution is necessary in order to cause E_z to vanish at the surface of the conductor, thereby satisfying the boundary conditions. Consequently, the current distribution along a thin, perfectly conducting antenna would differ slightly from a sinusoidal distribution, the discrepancy being most pronounced at the current nodes. As the conductor diameter approaches zero, the current distribution approaches a sinusoidal distribution.

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* Ibid 1

"In order to answer the question "where does the radiated power leave the antenna," we must search for a surface over which the normal component of Poynting's vector is not zero. Could this be the extreme end surfaces of the antenna? In order to have a normal component of Poynting's vector, it would be necessary to have a tangential component of electric intensity at the end surfaces, but this is again ruled out by our assumption of a perfectly conducting antenna.

"Let us now consider the antenna shown in Fig. 4, in which it is assumed that there is a finite separation distance between conductors at the feed point ab. An electric field exists in the region ab such that the line integral of electric intensity over this interval is equal to the impressed emf. A magnetic field also exists in this region owing to the current flow. Consequently we would expect to find a normal component of Poynting's vector over any surface enclosing the region ab. This leads us to suspected that the radiated power might depart from the antenna in the region between a and b. As the distance ab is decreased, the electric intensity increases in such a way that the line integral of electric intensity from a to b always equals the applied emf. In the foregoing derivations, a point generator was assumed, implying that the distance ab approaches zero. If this were true, however, the electric intensity would approach infinite value over the infinitesimal distance ab.

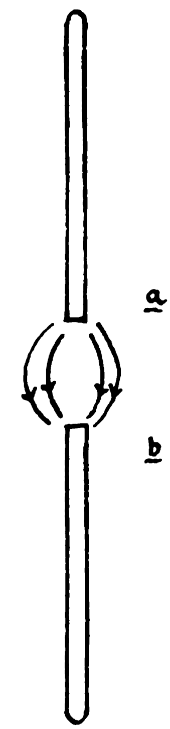

Fig. 4. - Field in the vicinity of the feed point of a dipole antenna.

"Since the induced-emf method apparantly yields reasonably correct results, we conclude that the complex power leaving a thin, perfectly conducting antenna may be determined by either of two methods.

1. Assume a sinusoidal current distribution and a point source feeding the antenna. Ignoring boundary conditions, evaluate E_z by the methods outlined above and insert this in either Eq. (20.04-1 or 2) to obtain the complex power flow.

2. With the true current distribution, E_z is zero at the surface of the antenna, but it is not zero in the region ab.

We may therefore obtain the complex power flow by integrating Poynting's vector over a surface enclosing the region ab.

"It is not clear why the two methods give essentially the same result. Apparantly the slight modification in antenna current, which is necessary to satisfy the boundary conditions, causes a shift from a situation of distributed power flow from the conductor surface to one of concentrated power flow emanating from the region ab, without appreciably altering the value of the power or the complex impedance computed from it. Since all practical antennas have finite conductivity, the value of E_z is small but need not be zero."

For many reasons which will be brought out in the text, the theory that an antenna radiates from the gap is unsatisfactory. Also, Stratton[2] uses the e.m.f. method with no such qualms and obtains thereby the correct value for the radiation resistance. The following alternative explanation will be presented.

For batteries, transmitting antennas - in general, for generators of all kinds, as contrasted with passive elements - it is necessary and logical to separate the net electric field into two components: one is the charge-separating or applied field, which is the electrical equivalent of whatever force (mechanical, chemical, etc.) it is that actually separates the charges, and the other is the induced field, which is the usual electric field between the charges. From this it will be demonstrated that it is logical that a tangential component of the induced field exists at the surfaces of the generator. Then it will be shown that the

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[2]Stratton, J. A. Electromagnetic Theory 1941
pp.457 - 460. New York: McGraw-Hill

Poynting vector theorem applies only to the induced component of the net electric field, and from this it follows that the power can be and is radiated from the elements of an antenna.

The thesis will consist essentially of three parts. First, in the QUALITATIVE IDEAS section, it will be explained in non-mathematical terms how as antenna actually radiates. Second, in the ANALYSIS section, the mathematical analysis will be presented. Finally, in EXPERIMENTS, the experimental evidence will be presented that confirms the mathematical and physical arguments given in the previous sections.

QUALITATIVE IDEAS

There are many different ways to analyze antenna behavior. One is the theory of Schelkunoff[3] which is based fundamentally on the concept that the antenna and the space surrounding it are two wave guides. Maxwell's equations are rigorously applied to a conical antenna, and then the usual thin cylindrical antenna encountered in practice is approximated by letting the angle of the conical apices approach zero. However, analyses such as this are best appreciated by those already familiar with antenna theory; the viewpoint to be presented is that of Hertz[4], because this viewpoint, at least in a qualitative sense, is more simple and straightforward.

In 1889 Heinrich Hertz*, after several years of research on the nature and effects of electric oscillations and radiation showed how the theory of Maxwell explained the mechanism of radiation. Hertz applied Maxwell's equations to an oscillating doublet and plotted the lines of electric field intensity for various values of time. These curves are shown in Figures 1 to 8 on the following page. The Mathematics involved is not very lengthy, but the actual plotting

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3 Schelkunoff, S. A. Electromagnetic Waves pp. 441-479 1943 New York: D. Van Nostrand Co.

4 Hertz, Heinrich (translated by D. C. Jones) Electric Waves 1900 New York Macmillan & Co.

* Ibid 4 pp. 137-150

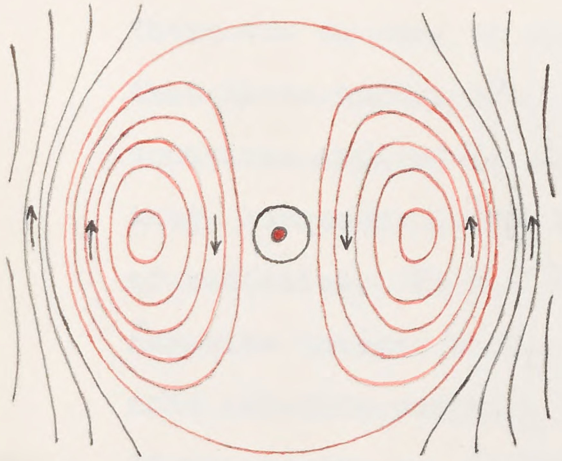


Fig. 1

t = 0

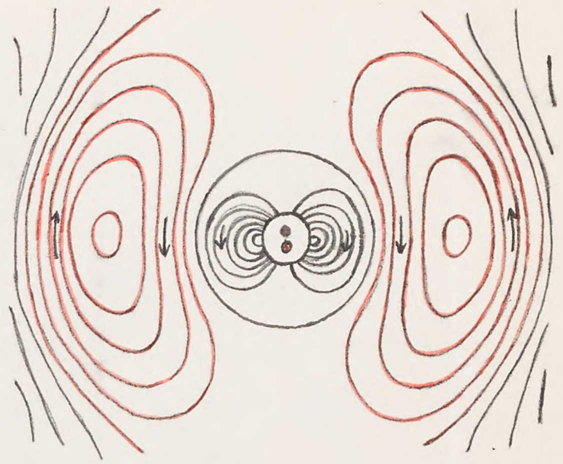
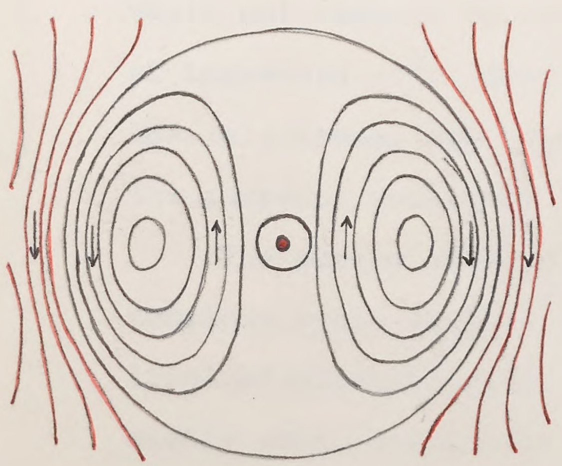


Fig. 2

t = T/8

● - Positive Charge

● - Negative Charge



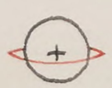


Fig. 5

t = T/2

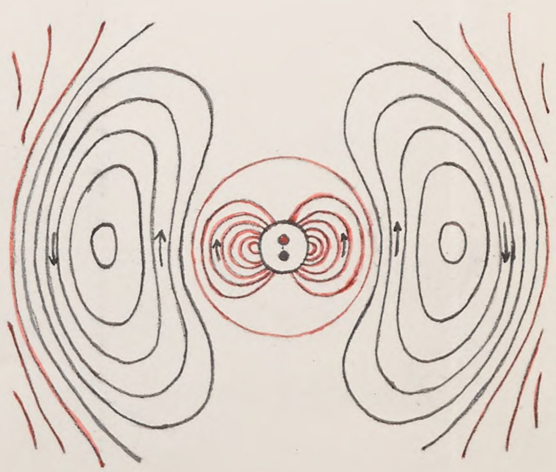
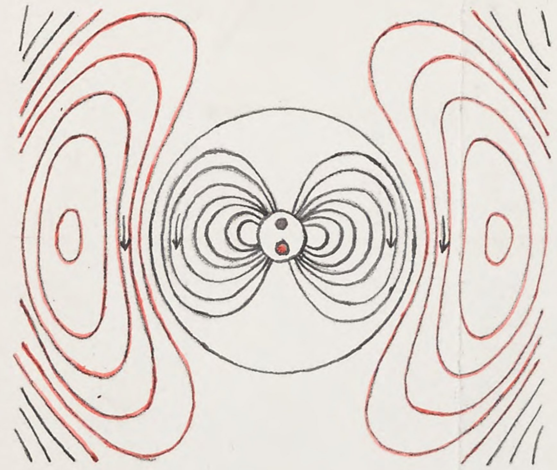


Fig. 6

t = 5T/8



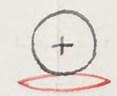


Fig. 3

t = T/4

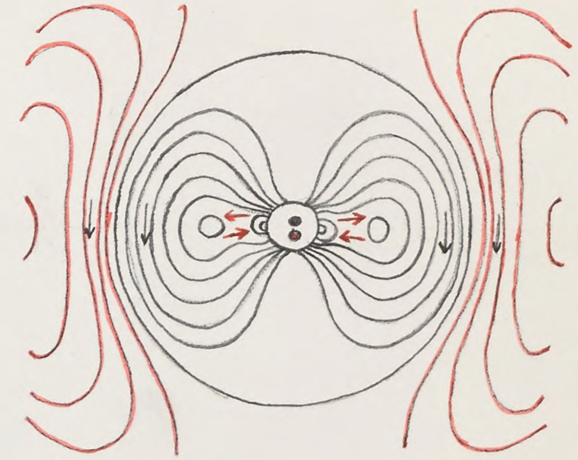
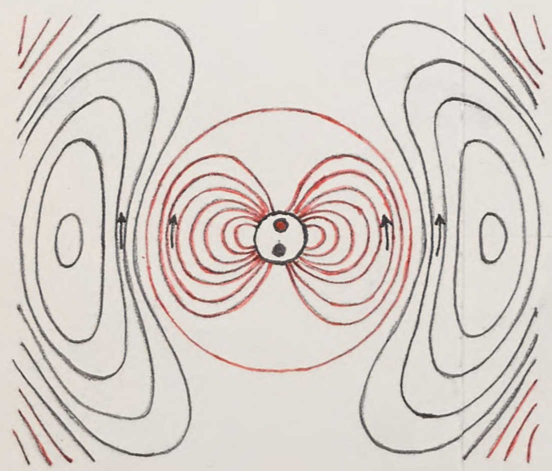


Fig. 4

t = 3T/8

T - Period of complete oscillation



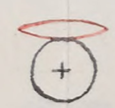
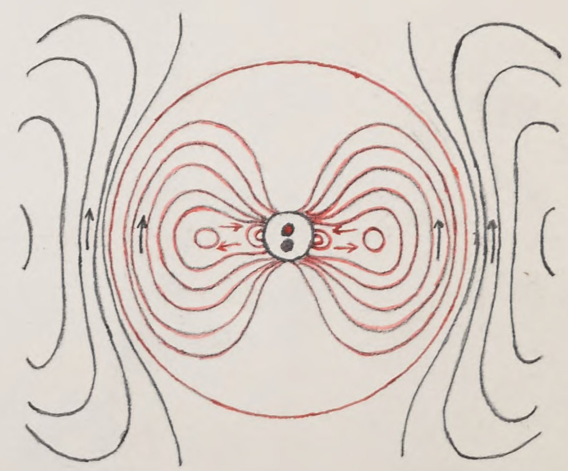


Fig. 7

t = 3T/4



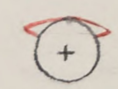


Fig. 8

t = 7T/8

of the curves is obviously a time-consuming job, because there are so many of them. However, the important point is that these curves are plotted from the results of Maxwell's equations applied to an oscillating dipole, and therefore they represent a true mathematical picture of the mechanism of radiation. In the center of each figure the positive and negative charges of the dipole are depicted in their approximate relative position at the instant of time for which the figure is drawn. As Hertz mentions, the lines of force are not continued right up to this picture, for the formulae assume that the oscillator is infinitely short, and therefore become inadequate in the neighborhood of a finite oscillator. However, the infinitesimal oscillator that exists in practice in the conductor of an antenna is the nucleus of the atom and the associated free electron (or electrons). Or, in poor conductors, the elementary oscillator may be thought of as an atom with oscillating polarization. Poor conductors, then, would not radiate as well as good conductors not only because of increased I^2R losses, but also because the excursion of the electrons, and therefore the moment of the dipole (or the current) would not be as great.

The curves of Figures 1 to 8 are drawn in color for a complete cycle so that the mode of generation (of the black lines of electric field intensity, for example) can be most easily seen. The arrows indicate the direction of the field intensity. Figure 1 shows the condition when the positive and negative charges of the dipole are superimposed. In Figure 2, one eighth of a cycle later, the electric field lines begin to build up as shown, pushing fields which were built up previously away from the source; this process continues in Figure 3, where the separation of the charges is a maximum and the current is zero. In figure 4, as the charges begin to come together, the important part of the radiation process begins to occur. The outer black field lines are propagating outward and expanding with the velocity of light and are meanwhile in the process of being cut off at their source. The red arrows indicate the direction of the lines of force near the source. As the charges come together the lines of force must also, and since they are equal and oppositely directed (as the red arrows show), they cancel in this region. The net electric field becomes zero here, leaving the closed curves of field intensity shown in Figure 5, where the current is now a maximum and oppositely directed to that in Figure 1. As the charges begin to separate again in Figure 6 a new field builds up, oppositely directed to that in Figure 2, and pushes the black field lines away from the source. Thus, radiation is effected.

The crude, symbolic pictures of atoms in various states of polarization (or, atomic nuclei and the associated free electrons) associated with each figure is the author's attempt to show how the atoms of a radiating element actually serve as oscillating dipoles. The red rings represent the electrons, and the black spheres represent the nucleus and all the shells except the outer one. Thus, by comparing the state of the atom with the position of the dipole charges in each figure, it is easy to see how an atom can radiate electromagnetic energy. Hertz writes:

"... This loss of energy corresponds to the radiation into space. In consequence of this the oscillation would of necessity soon come to rest unless impressed forces restored the lost energy at the origin. In treating the oscillation as undamped, we have tacitly assumed the presence of such forces."

It remains now only to show how the atoms in the elements of an antenna can be forced to oscillate in the manner illustrated in Figs. 1 to 8, so that radiation will result.

Consider the half wave dipole antenna shown in Figure 9. At a particular instant of time the transmission line feeding the antenna imposes positive charges on the gap end of one element, while negative charges are imposed on

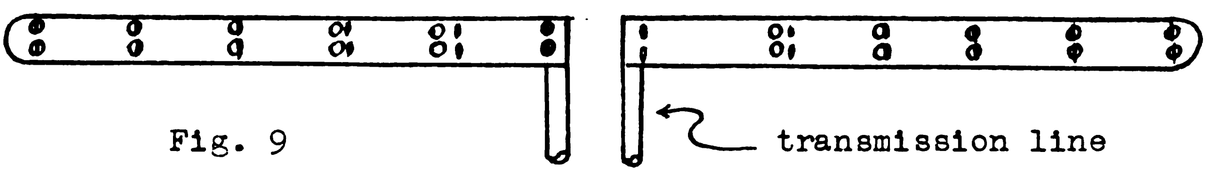


Fig. 9

the gap end of the other element. As a result, the atoms are polarized (or, the free electrons are attracted or repelled) as shown. Near the center the positive charges attract the electrons quite strongly, as shown. Then adjacent nuclei attract the electrons of atoms further from the center less strongly, and so on. This attraction (repulsion) effect actually travels along the element with approximately the velocity of light, and the result is that a standing wave of current is established which is approximately sinusoidal, with the maximum at the center and the nodes at the ends of the antenna. The charges at the end of transmission line oscillate, of course, and thus force the atoms in the elements to oscillate, and therefore radiate. The radiation of the antenna is the summation of the radiation of its atoms. The energy required to sustain the oscillations is supplied by the transmission line, and as far as the transmission line is concerned, that energy is dissipated in a "load". The load is represented as a resistance for computation purposes, and this is, of course, the radiation resistance of the antenna.

This is, then, the physical picture of the way in which an antenna radiates. It is only qualitative, but certainly any quantitative analysis should check the physical facts. Specifically, an analysis which reveals that the radiation can come only from the gap should be questioned, because all physical reasoning and experience indicate that it is the element that does the radiating.

G. H. Livens[5] derives the equations for, and presents the radiation curves of the Hertzian oscillator, and he also examines the case and plots the curves taking into account the damping which is really existent. In the opinion

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[5] Livens, G. H. The Teory of Electricity pp. 292 - 313
1926 Cambridge University Press (2nd. ed.)

of the author, these curves,if not the mathematics, should be presented and explained early in the study of electro-magnetic fields, so that radiation will be better understood.

ANALYSIS

The expression for the retarded vector potential $\bar{A}$ is given as Eq. 16 in Appendix I as:

$$A = \frac{\mu}{4\pi}\iiint_V \frac{\bar{i}(t-r/c)dv}{r} \qquad (16)$$

An example of the use of the retarded vector potential $\bar{A}$ is the derivation of the field of an incremental current element (or antenna). The incremental antenna is shown in Fig. 10, having a length dz and carrying a uniform current Ie^{jwt}.

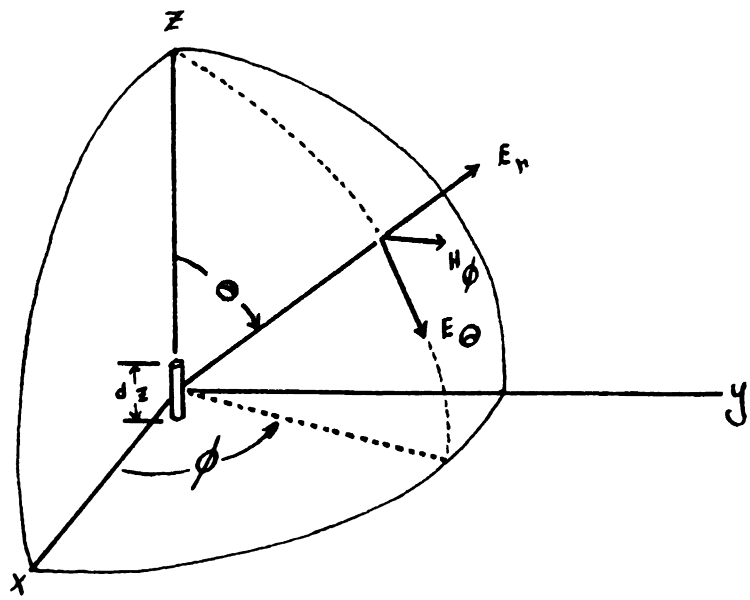

Fig. 10 - Coordinates for the incremental antenna

Applying Eq. (16) to a differential current element, replacing $\bar{i}dv$ by $\bar{k}Idz$, noticing that $\bar{A}$ is in the same direction as the current and is therefore $\bar{k}A_z$, the vector potential at a point distant r from the antenna becomes:

$$A_z = \frac{\mu Idz}{4\pi r}e^{jw(t-r/c)} \qquad (22)$$

Since $\beta = \omega/c$, Eq. (22) can also be written

$$A_z = \frac{\mu Idz}{4\pi r}e^{j(\omega t-\beta r)}$$

Expressing the vector potential in spherical coordinates and dropping the time function $e^{j\omega t}$, we obtain

$$A_r = A_z\cos\theta = \frac{\mu Idz}{4\pi r}\cos\theta e^{-j\beta r} \qquad (24)$$

$$A_\theta = A_z\sin\theta = -\frac{\mu Idz}{4\pi r}\sin\theta e^{-j\beta r} \qquad (25)$$

$$A_\phi = 0$$

The magnetic intensity is obtained by inserting A_r and A_θ into $\overline{H} = (1/\mu)\mathrm{curl}\overline{A}$ (in spherical coordinates). With the additional substitution of $\beta = 2\pi/\lambda$ and remembering that $\partial/\partial\phi = 0$, we obtain $H_r = H_\theta = 0$

$$H_\phi = \left(\frac{j2\pi}{\lambda r} + \frac{1}{r^2}\right)\frac{Idz}{4\pi}\sin\theta e^{-j\beta r} \qquad (26)$$

To obtain the electric intensity, insert H_ϕ from Eq. (26) into $\mathrm{Curl}\overline{H} = j\omega\mu\overline{E}$ (Maxwell's second equation), giving

$$E_r = \eta\left(\frac{1}{r^2} - \frac{j\lambda}{2\pi r^3}\right)\frac{Idz}{2\pi}\cos\theta e^{-j\beta r} \qquad (27)$$

$$E_\theta = \eta\left(\frac{j2}{\lambda r} + \frac{1}{r^2} - \frac{j\lambda}{2\pi r^3}\right)\frac{Idz}{4\pi}\sin\theta e^{-j\beta r} \qquad (28)$$

The above derivation was taken from Bronwell & Beam*. The following discussion is quoted from them.

"The electric and magnetic intensities contain terms varying as $1/r$, $1/r^2$, and $1/r^3$. The components containing $1/r^2$ and $1/r^3$ predominate in the immediate vicinity of the antenna and are known as the <u>induction</u> <u>field</u> of the antenna. The induction field represents reactive energy which is stored in the field during one portion of the cycle and returned to the source during a later portion of the cycle.The induction field terms become vasishingly small at remote distances from the antenna and hence do not contribute to the radiation of power from the antenna.

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* Ibid 1, pp. 402 - 405

"The terms varying as $1/r$ in the intensity expressions comprise the <u>radiation field</u> of the antenna. The radiation field is comprised of electromagnetic waves traveling radially outward with a propagation factor $e^{j(\omega t-\beta r)} = e^{j\omega(t-r/c)}$ and with intensities which vary inversely as the first power of the distance from the source. Equation (26) shows that the induction and radiation field components of magnetic intensity are equal at a distance of $r = \lambda/2\pi$, or approximately a sixth of a wavelength from the antenna.

It is interesting to observe that if we had assumed that the field builds up instantaneously throughout space, <u>i.e.</u>, if we had used $e^{j\omega t}$ instead of $e^{j\omega(t-r/c)}$ in Eq.(23), the radiation-field terms would not have been present in the resulting intensity equations. Radiation is therefore dependent upon the fact that the field has a finite velocity of propagation. In conventional circuit analysis it is customary to ignore the finite velocity of propagation of the field. This approximation is valid if most of the field is confined to a region which is very small in comparison with the wavelength. It leads to what is known as the <u>quasi-stationary analysis</u>.

"The radiation-field terms, taken alone, comprise a spherical TEM wave propagating radially outward from the source with a wave impedance equal to the intrinsic impedance of free space. Discarding the j factor in Eqs. (26) and (28), we obtain the radiation field intensities,

$$H_\phi = \frac{I dz \sin\theta}{2\lambda r} e^{-j\beta r} \qquad (29)$$

$$E_\theta = \frac{\eta I dz \sin\theta e^{-j\beta r}}{2\lambda r} \qquad (30)$$

The ratio of electric to magnetic intensity is equal to the intrinsic impedance of the medium, thus

$$E_\theta / H_\phi = \eta$$

"The power radiated by the incremental antenna is found by integrating the normal component of Poynting's vector over the surface of an imaginary sphere having the antenna at its center. For convenience we choose a sphere which is large enough so that the induction-field terms are negligible. Inserting E_θ from Eq. (30) into the expression for power

$$P_{ave} = \frac{|E|^2}{2\eta}$$

and dropping the phase-shift term $e^{-j\beta r}$, we obtain for the time-average power density,

$$p = \frac{|E_\theta|^2}{2\eta} = \frac{\eta (I dz)^2 \sin^2\theta}{8 r^2 \lambda^2} \qquad (32)$$

The total radiated power is therefore

$$P = \int_0^{\pi} p 2\pi r^2 \sin\theta$$

$$= \frac{\eta (Idz)^2}{8r^2\lambda^2}\int_0 2\pi r^2 \sin^3\theta d\theta$$

$$= \frac{\pi\eta}{3\lambda^2} (Idz)^2 \qquad (33)$$

The radiated power is independent of the radius of the sphere over which the power density is integrated. This is a consequence of assuming a lossless transmission medium."

Schelkunoff* arrives at equations identical to Eqs. (27) and (28) by slightly different means and carries the process one step farther by finding an expression for E_z on the axis of the current element. Referring to Fig. 11: If the value of E_z is to be found on the axis, then $\theta = 0$. Therefore E_θ is equal to zero ($r \neq 0$), and E_z equals E_r evaluated at $\theta = 0$.

$$E_z = \frac{\eta Idz}{2\pi r^2}(1 + 1/j\beta r)e^{-j\beta r} \qquad (34)$$

Expanding $e^{-j\beta r}$ into its equivalent series, multiplying through and colleting like terms gives,

$$E_z = \frac{\eta Idz}{2\pi}\left(\frac{1}{j\beta r^3} - \frac{j\beta}{2r} - \frac{\beta^2}{3} + \ldots\right) \qquad (35)$$

Schelkunoff then writes:

"The first two terms are in quadrature with I and on the average do no work; but the third term is 180° out of phase with I and work is done against the field by the <u>impressed</u> electromotive force. The in-phase component of this force is then

$$Re(V^1) = - dzRe(E_z) = \frac{\eta\beta}{6\pi}{}^2dz^2I = \frac{2\pi\eta dz^2I}{3\lambda^2} \qquad (36)$$

The work done by this force per second is seen to be equal to <u>P</u> in Eq. (33)."

....

*Ibid 3, pp. 131 and 133-134

Thus,

$$P_{ave} = \frac{VI}{2} = \frac{\pi\mu}{3\lambda^2}(Idz)^2 \qquad (37)$$

which checks Eq. (33). The underlining of the word impressed in the above discussion is the author's. The important point is this: For each differential time-varying current element there is a component of its associated electric field $\bar{E}$ which acts in a direction opposite to the current direction. Thus, an impressed or applied field must act in the direction of the current to drive the current against the component of E_z that is 180° out of phase with I. Therefore, in a radiating conductor the net field is

$$\bar{E}_{net} = \bar{E}^i + \bar{E} = \bar{i}/\sigma \qquad (38)$$

Since the conductivity σ is usually very large, $\bar{E}_{net}$ may be practically zero, but this does not require that $\bar{E}$ be zero. In fact, the above derivations show that $\bar{E}$ will not be zero. E, as used above, is termed the induced field. The concept of the induced field is not an antenna phenomenon; it is used throughout the whole range of Electrical Engineering, from D.C. batteries on, as will be shown later. In fact, the point of this thesis is that many authors and teachers do not distinguish between the induced and the applied fields in antenna derivations, and as a result they run into the difficulties explained in the Introduction.

Poynting's theorem will now be examined critically. Maxwell's equations are:

$$\mathrm{curl}(\bar{E}) = -\partial\bar{B}/\partial t \qquad (I)$$

$$\mathrm{curl}(H) = \bar{i} + \partial\bar{D}/\partial t \qquad (II)$$

Form the scalar product of (I) by $\bar{H}$ and (II) by $\bar{E}$ and apply the identity

$$\mathrm{div}(\bar{E}\mathrm{x}\bar{H}) = \bar{H}\cdot\mathrm{curl}(\bar{E}) - \bar{E}\cdot\mathrm{curl}(\bar{H})$$

to arrive at

$$\mathrm{div}(\bar{E}\mathrm{x}\bar{H}) + \bar{E}\cdot\bar{i} = -\bar{E}\cdot(\partial\bar{D}/\partial t) - \bar{H}\cdot(\partial\bar{B}/\partial t) \qquad (45)$$

Finally, integrating over a volume V bounded by a surface S, and applying the divergence theorem to the first term in (45):

$$\int_S (\bar{E}\mathrm{x}\bar{H})\cdot\bar{n}da + \int_V \bar{E}\cdot\bar{i}dv = -\int_V (\bar{E}\cdot\partial\bar{D}/\partial t + \bar{H}\cdot\partial\bar{B}/\partial t)dv \qquad (46)$$

Stratton* is now quoted because his interpretation of the $\int_V \bar{E}\cdot\bar{i}dv$ term in Eq. (46) is the same as the author's. Some authors[6] write that "... the third term $(\bar{E}\cdot\bar{i})$ is the usual ohmic term and so represents energy dissipated in heat per unit time." This is incorrect, as will be seen from Stratton's explanation.

"This result (46) was first derived by Poynting in 1884, and again in the same year by Heaviside. Its customary interpretation is as follows. We assume that the formal expressions for densities of energy stored in the electromagnetic field are the same as in the stationary regime. Then the right-hand side of (46) represents the rate of decrease of electric and magnetic energy stored within the volume. The loss of available stored energy must be accounted for by the terms on the

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* Ibid 2, pp. 132, 133

6 Ramo, Simon, & Whinnery, J. R. Fields and Waves in Modern Radio pp. 427 - 428 New York 1944 Wiley & Sons

left-hand side of (46). Let σ be the conductivity of the medium and $\bar{E}'$ the intensity of impressed electromotive forces such as arise in a region of chemical activity - the interior of a battery, for example. Then

$$i = \sigma(\bar{E} + \bar{E}'), \qquad \bar{E} = \bar{i}/\sigma - \bar{E}' \tag{47}$$

and hence

$$\int_V \bar{E}\cdot\bar{i}dv = \int_V (\bar{i}^2/\sigma)dv - \int_V \bar{E}'\cdot\bar{i}dv \tag{48}$$

The first term on the right of (48) represents the power dissipated in Joule heat - an irreversible transformation. The second term expresses the power expended by the flow of charge against the impressed forces, the negative sign indicating that these impressed forces are doing work on the system, offseting in part the Joule loss and tending to increase the energy stored in the field. If, finally, all material bodies in the field are absolutely rigid, thereby excluding possible transformations of electromagnetic energy into elastic energy of a stressed medium, the balance can be maintained only by a flow of electromagnetic energy across the surface bounding V. This, according to Poynting, is the significance of the surface integral in (46). The diminution of electromagnetic energy stored in V is partly accounted for by the Joule heat loss, partly compensated by energy introduced through impressed forces; the remainder flows outward across the bounding surface S, representing a loss measured in joules per second, or watts, by the integral

$$\int_S \bar{P}\cdot\bar{n}da = \int_S (\bar{E}x\bar{H})\cdot\bar{n}da \tag{49}$$

The Poynting vector $\bar{P}$ defined by

$$\bar{P} = \bar{E}x\bar{H} \qquad \text{watts/meter}^2, \tag{50}$$

may be interpreted as the intensity of energy flow at a point in the field; i.e., the energy per second crossing a unit area whose normal is oriented in the direction of the vector $\bar{E}x\bar{H}$."

By comparing $\bar{E}$ in Eq. (50) to Stratton's definition of $\bar{E}$ in (47), it is seen that Poynting's vector theorem applies only to the induced electric field intensity, $\bar{E}$. This result follows from the fact that Maxwell's Equations themselves involve only the induced field E, as Strattons' development implies.

This will be illustrated by the specific case of a d.c. battery. See Fig. 11

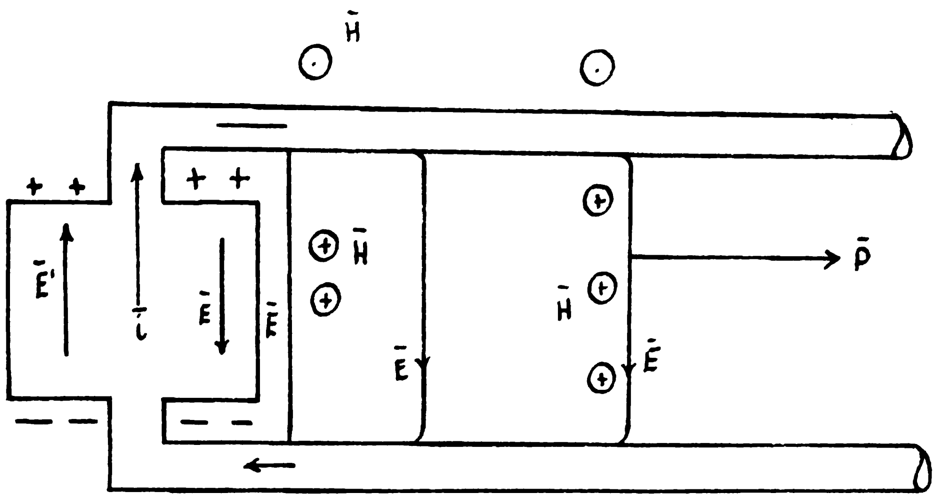

Fig. 11 Electric and magnetic fields of a d.c.battery.

It is common practice to consider only what happens outside the battery, because an engineer is interested primarily in circuit problems. However, it is rather widely known that inside the battery there must be an impressed or applied force which is in the direction of the current and which drives the current against the resistance of the battery and against the field due to the charges at the battery terminals. This force is termed the impressed field E', and it is simply the electrical equivalent of the chemical force that separates the charges. Now, according to the usual procedure, the power flow down the line is represented as shown in Fig. 11. But notice which component of the net electric field is used, It is the component that is directed downward from the positive to the negative charges, and this component is the <u>induced</u> field! Since <u>impressed</u> fields usually exist only in generators, the case of the d.c. battery illustrates the general fact that Poynting's theorem applies only to the <u>induced</u> field.

Note that $\bar{E}'$, the impressed field, is in general not strictly a field; it is usually only the electrical equivalent of some force which exists inside a generator and not outside. Thus, $\bar{E}$ generally can not satisfy the boundary conditions imposed by Maxwell's equations because it can have a tangential component on the generator side of the interface between the generator and the outside, but it is zero everywhere outside. Therefore, Maxwell's equations can not be applied to $\bar{E}'$ <u>in general</u> simply because sometimes it is not a field but a force due to chemical, mechanical, etc. action. On the other hand, the induced field $\bar{E}$ satisfies Maxwell's equations for both the static and the time-varying states. Therefore, since $\bar{E}$ is the only component of the net electric field intensity that can satisfy Maxwell's equations, we conclude that Maxwell's equations in general can be applied rigorously only to $\bar{E}$, the induced field. Since outside the generating source $\bar{E}'$ is usually zero, the net field is equal to $\bar{E}$, and here all authors are in agreement as to the propagation of electric waves, reflection phenomena, etc.

The reason for the qualifying "in general" and "usually" in the statements above is that in the special case of a transmitting antenna the field which is the driving force for the antenna currents is itself the induced field of the transmission line, and therefore Poynting's vector applies to it. However, this field is closely associated with the currents and exists only in the immediate vicinity of the currents. When we are considering an antenna as a generator we need not concern ourselves with how the antenna currents are sustained, any more than we need to know the chemical action by which the currents are sustained in a d.c.battery. We are primarily interested in the power emanating from the generator, whether this generator is a d. c. battery, transmitting antenna, or a 50,000 kw. alternator. To calculate this generated power at any point in the field we must apply Poynting's theorem to the induced electric field, the field of the charges and currents, and not to the field or force that drives the currents.

We are now in a position to examine how a differential time-varying current element can radiate power. Eq. (4) shows that the real component of the tangential induced field intensity associated with a differential cureent element is 180° out of phase with the current. It has been shown above that the generated power can be calculated by applying Poynting's vector to the induced component of the net electric field intensity. Therefore, Poynting's vector, $\overline{P} = \overline{E}x\overline{H}$, must be directed as shown in Fig. 12.

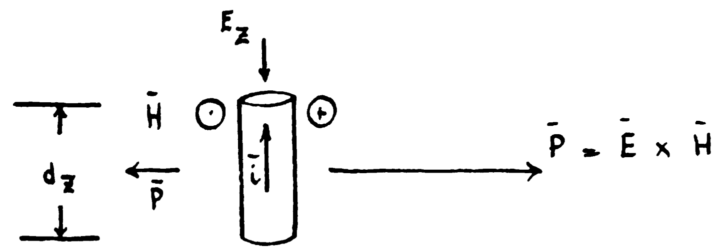

Fig. 12. Radiation of a differential

time-varying current

The current must be time-varying because Eq. (35) shows that $Re(E_z)$ is equal to a constant times the frequency squared. Thus, it is seen that a differential time-varying current element radiates power. Therefore, any conductor carrying a time-varying current will radiate, because its current is merely the summation of the differential current elements. The radiation of a twin lead transmission line is negligible only because the conductors are close together compared with a wavelength; and since they are carrying current in opposite directions, the radiation of one is cancelled by the radiation of the other.

Thus we are led to expect that the current-carrying elements of an antenna must radiate electromagnetic energy. If a good approximation to the actual current distribution is found that can be integrated to give the associated electric and magnetic field intensities, we should expect that Poynting's vector P will give the correct value for the power radiated by the antenna. King[7] shows that a sinusoidal distribution is good approximation to the actual current distribution, and when the induced field associated with this current is calculated it is found that is has a real component tangential to the conductor which is oppositely directed to the current. From the above analysis, this result should be expected. Furthermore, when the total generated power is calculated by integrating $\bar{P}$ over the surface of the conductors not only the correct value for the radiation resistance is found, but also the correct value for the antenna reactance is given!

In spite of these facts, almost every author except Stratton writes that E_z should be zero at the surface of the conductor, and therefore no power can be radiated by the conductors. The point they miss is that $\bar{E}_{net} = \bar{E}' + \bar{E} = \bar{i}/\sigma$ is certainly almost zero, but $\bar{E}$, the induced field from which the radiated power must be calculated is definitely not zero. These authors write that the E_z calculated from a sinusoidal distribution is not zero because the sinusoidal distribution is not sufficiently close to the actual current distribution. The truth is that no matter what kind of a distribution is assumed the real component of the tangential induced field will not equal zero and will be oppositely directed to the

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7 King, Mimno, Wing Transmission Lines, Antennas, and Wave Guides pp. 90-93 New York 1945 McGraw-Hill

current. Since the differential current elements have oppositely directed $Re(E_z)$s, if there is any current distribution at all it will have a $Re(E_z)$ 180° out of phase. Stratton* writes,

"The method of Poynting, which has thus far been employed to calculate the radiation resistance, is based entirely on a determination of the outward flow of energy at great distances from the center of the radiating system.

Now if energy is constantly lost from the system it must also be supplied by the source; consequently, work must be done on the antenna currents at a constant rate. There must be a component of the e.m.f. parallel to the wire at its surface which is 180° out of phase with the antenna current."

Serious difficulties are encountered as the result of insisting that E_z be zero at the surface of the conductor. It follows immediately, as Bronwell and Beam** show, that the radiation must come from the gap. This seems just barely plausible for a half-wave dipole. But then, why does a full-wave dipole radiate off at angles instead of perpendicular to the gap? How does a delta-fed half-wave dipole manage to radiate at all? It doesn't have a gap, as shown in Fig. 13.

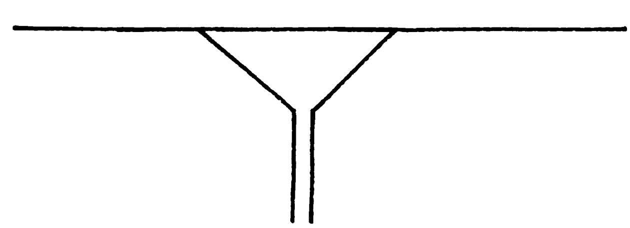

Fig. 13 Delta-fed half-wave dipole

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* Ibid 2 p. 454

** Ibid 1 pp. 432 - 433.

In fact, why have elements on the dipole at all, if it is the gap that does the radiating? Perhaps we need the current so that a magnetic field will exist in the gap. Then how

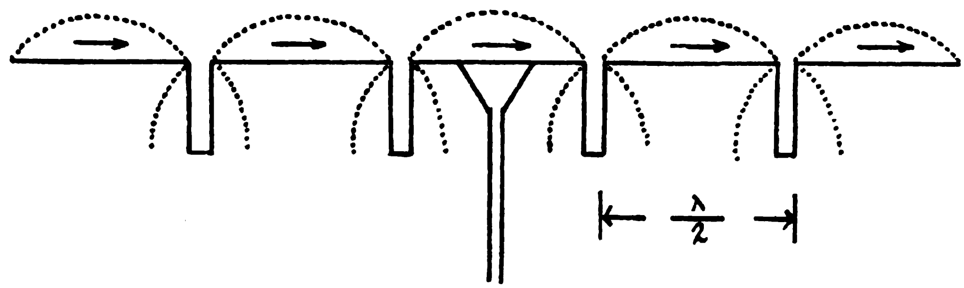

Fig. 14. Current distribution on a theoretical colinear array.

do the elements of the colinear array shown in Fig. 5 radiate? They have no gaps at the current maximums. Also, high frequency transmission lines have a small amount of radiation loss, and there are certainly no gaps in the transmission line wires. These questions alone, without the support of Stratton or the mathematics presented in this thesis, were sufficient to cause the author to be skeptical of the validity of the radiation-from-the-gap argument.

It is instructive to examine the lines of power flow (Poynting vector) surrounding an antenna and its transmission line feeder. See Fig. 15. In Fig. 15 (a) the usual representation[8] is given. This shows the power flowing down the line and being dissipated in the resistive load, which

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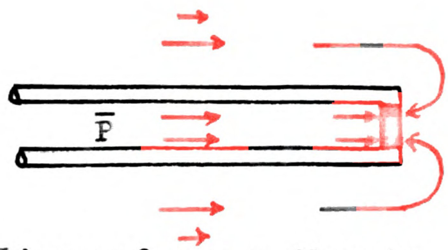
[8] See, for example, Attwood, S. S. Electric and Magnetic Fields 2nd. Ed. p. 123 1941 New York Wiley & Sons.

is represented by the red section at the end of the line. The resistance of the transmission line wires is neglected. In Fig. 15 (b) the power-flow lines are shown terminating on the elements of the antenna. These power lines would be calculated from the impressed field, E', which is the field of the transmission line. As shown previously, this impressed field drives the antenna currents against the induced field due to the currents.

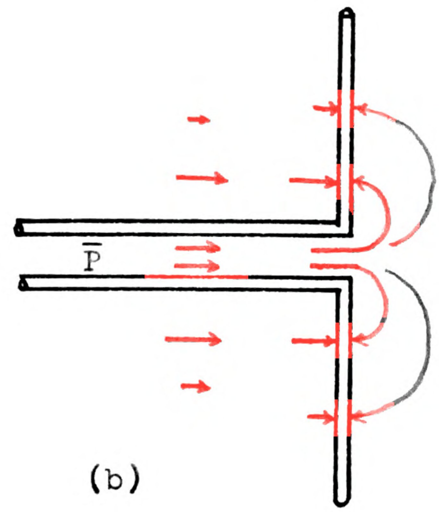
It cannot be overemphasised that when we are considering a transmitting antenna as a generator of electromagnetic radiation we must calculate the generated (radiated) power by applying Poynting's vector to the induced field, just as is done in the case of a battery or any other generator. Thus, the power radiated by the induced electric field is shown in Fig. 15 (c).

If there are no losses in the antenna, $\bar{E}'$ and $\bar{E}$ are equal in magnitude but oppositely directed, the net tangential electric field intensity is zero, and there is no net power flow from the antenna. But the radiation field still emanates from the elements because it is compensated for by the equal and opposite field of the transmission line. Note that Fig. (15) gives only a physical picture of the lines of power flow because Poynting's theorem can be applied rigorously only to closed surfaces. See Fig. (19) and the discussion given in the CONCLUSION section.



Lines of power flow in the vicinity of a lossless line and a load.

(a)



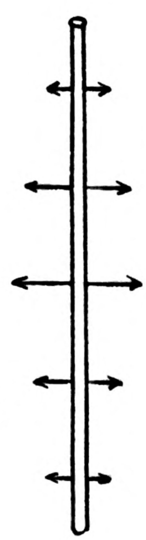
(b)

Fig. 15

Lines of power flow due to <u>impressed</u> electric field intensity ($\overline{E}'$) which drives the antenna currents against their reactive forces.

(c)

Lines of power flow due to <u>induced</u> field ($\overline{E}$) of antenna charges and currents.



$\overline{P} = \overline{E} x \overline{H}$

EXPERIMENTAL RESULTS

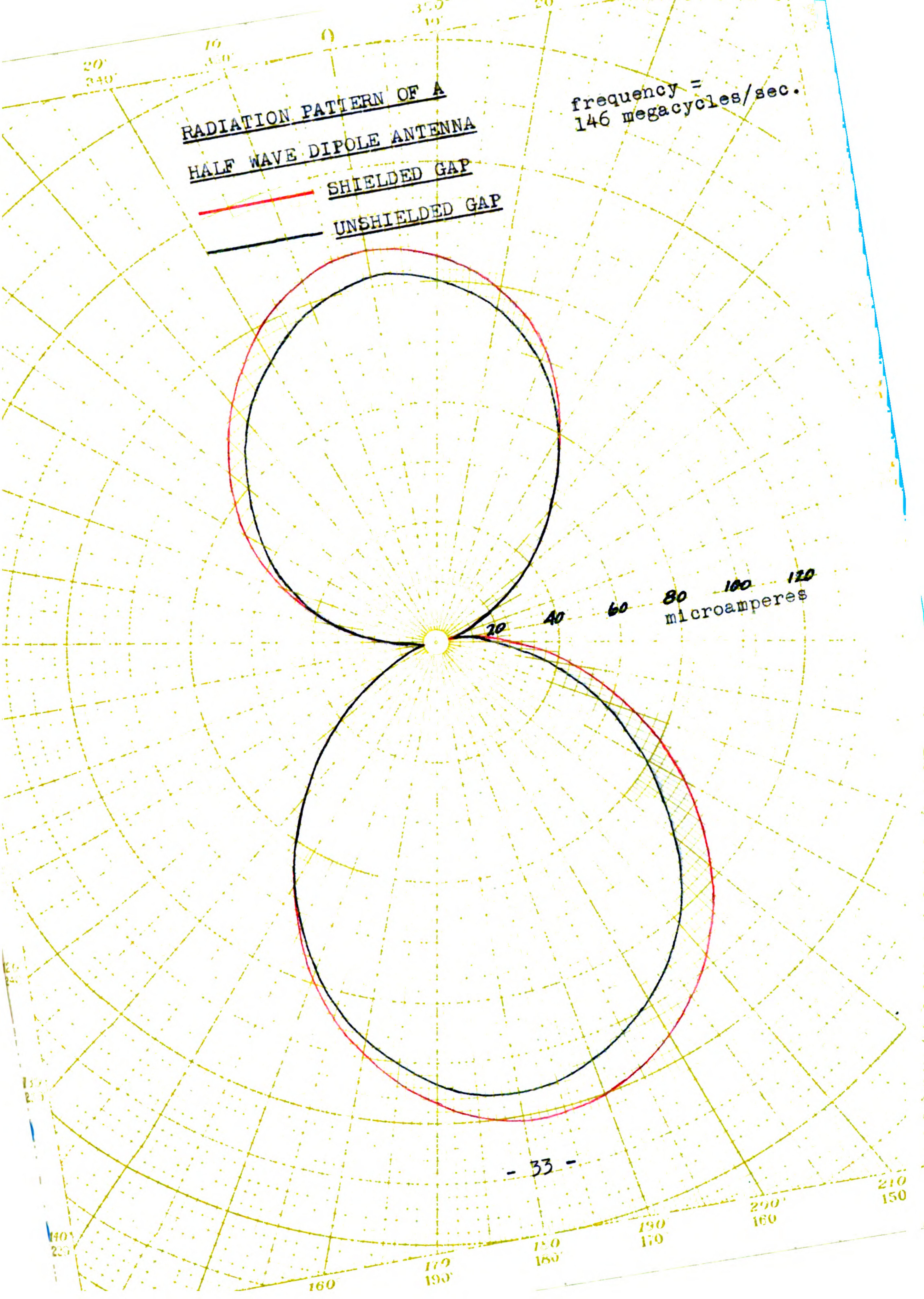
A simple qualitative experiment to determine whether most of the radiation from an antenna comes from the air gap was performed as follows: Two half-wave dipole antennas were constructed with elements of half-inch copper tubing fifty centimeters long. The supporting masts were constructed in the form of a Y so that the gaps were readily accesible. The resonant frequency of the antennas was found to be 146 megacycles, and a small oscillator was used to drive the transmitting antenna. See Figs. 16 and 17. When the receiving antenna was placed one wavelength (approximately 2 meters) away from the transmitting antenna, the maximum pickup on a crystal-microammeter combination was 160 microamperes, or about half scale on the 300 microampere range. 75 ohm twin lead transmission line was used to match the antennas. Since all walls were several wavelengths away, reflections were reduced to a negligible value.

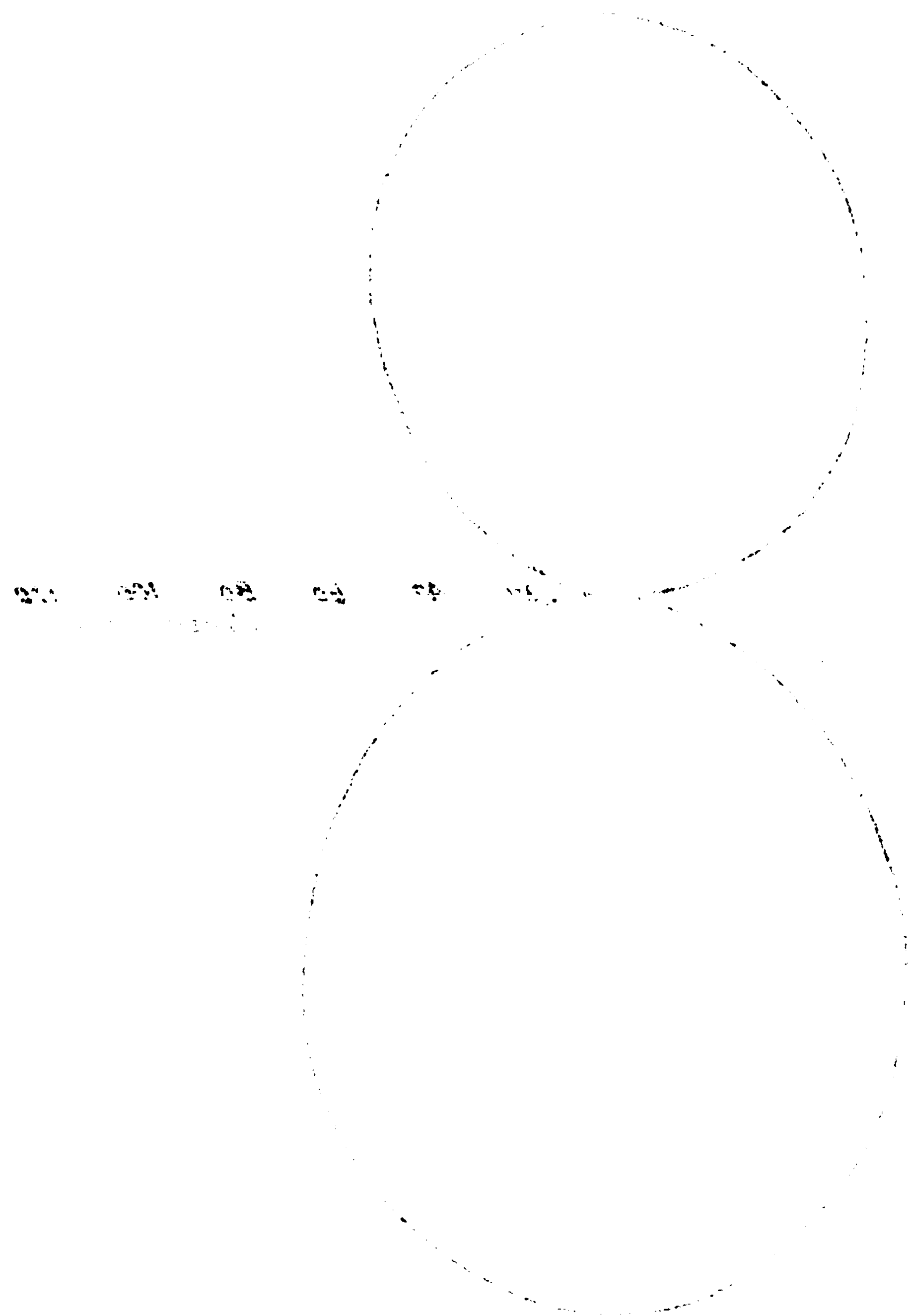
In the first run, the gap of the transmitting antenna was left unshielded, and the match at the oscillator was adjusted for maximum pickup. The transmitting antenna was then rotated and readings on the meter at the receiving antenna were taken for ten degree intervals. The results are plotted in Fig. 18. One half of the pattern is slightly smaller than the other due to the loss in the transmission line as it wrapped around the mast of the transmitting antenna. This difference is unavoidable in the simple set-up used, and it is of no great importance anyway. The radiation pattern is slightly fatter than it should be due to the nonlinearity of the crystal-microammeter combination in the lower range, but this is also unimportant.

In the second run, a coffee can ($3\frac{1}{2}$" high by $4\frac{1}{2}$" dia.) with the cover on tightly was used to shield the gap. 5/8 inch holes were cut in the top and bottom to admit the dipole elements, and the can was supported by polystyrene insulators between itself and the elements. A one-inch hole was cut in the side to admit the transmission line feeder. With the same match at the oscillator as used in the previous case, the run was repeated for the antenna with the shielded gap. The results are shown as the slightly larger pattern in Fig. 17. The transmitting antenna was jiggled a couple of degrees in the process of shielding the gap, and this shows up in the patterns. The effect of the shield can be seen because the match to the oscillator was even slightly better with the gap shielded than when the gap was unshielded. However, this difference is only on the order of 5%, and merely shows that the shield affects the match, as would be expected. The important point is this: <u>The radiation pattern of a dipole antenna with the gap shielded is essentially the same as the pattern with the gap unshielded</u>. The conclusion is obvious: The radiation from an antenna does not come from the gap. If it did, at least a large part of the radiation energy would be dissipated in I^2R losses in the shield, and the pattern

Fig. 16 Transmitting antenna with unshielded gap.

Fig. 17 Transmitting antenna with shielded gap.


RADIATION PATTERN OF A
HALF WAVE DIPOLE ANTENNA
SHIELDED GAP
UNSHIELDED GAP
frequency =
146 megacycles/sec.
20
40
60
80
100
120
microamperes



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would be drastically affected.

Now certainly a small amount of energy is radiated by the gap because there is a time-varying electric field in the gap, which induces a time-varying magnetic field in the vicinity, which induces another electric field in its vicinity, and so on. But the above experiments show that at most this is a relatively small amount of the total radiation of the antenna.

A second experiment immediately suggested itself.Try shielding the elements, since they are assumed to be the source of the radiation energy. Thus, if the elements are shielded, the radiation should decrease. Now this is not a good experiment, because a shield around the elements lowers the capacitance between them, alters the current distribution, and so on.

However, this experiment was performed in a qualitative manner. Two sections of a cylindrical wave guide were used as shields for the elements, and it was found, as expected, that as the shields covered more and more of the elements, the radiation became less and less. The amount of power decrease was roughly proportional to the length of element shielded. That is all that can and should be said about this particular experiment.

CONCLUSION

The experiments in themselves do not present conclusive evidence that most of the radiation of an antenna comes from the elements, because it can argued that the fields inside the shield will leak through the necessary loles and induce currents on the outside of the shield which would in turn radiate. However, it seems probable that such currents would introduce losses and/or distortions of the resultant field pattern, and this was not found to be the case.

It must be emphasised that the comparison of a Hertzian dipole to an atomic nucleus and its associated free electron(s) in the presence of an impressed time-varying electromagnetic field is not a rigorous one, because Maxwell's equation can not be rigorously applied at atomic dimensions. Nevertheless, in the opinion of the author, this comparison gives one a good picture of the physical mechanism of radiation, and so it is presented here.

The important point of this thesis, however, is that the generated power of a transmitting antenna must be calculated by applying Poynting's theorem to the induced electric field due to the currents and charges of the antenna. This concept resolves the dilemma quoted from Bronwell & Beam in the INTRODUCTION. Now Poynting's theorem can be rigorously applied only to closed surfaces*, and this is shown in Fig. 19.

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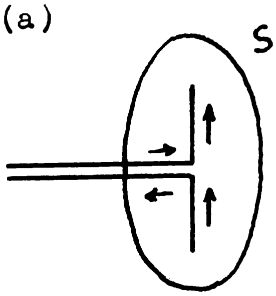
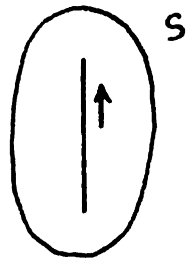
* ... and only to electric and magnetic fields that are functionally related, in the opinion of the author. However, this is a subject for a thesis in itself.

Fig. 19(a)



$$P_{net} = \iint_S (\bar{E}_{net} x \bar{H}) \cdot \bar{n} ds = 0$$

Fig. 19(b)



$$P_{radiated} = \iint_S (\bar{E} x \bar{H}) \cdot \bar{n} ds$$

In 19(a) the <u>net</u> power integrated over the closed surface <u>S</u> is zero (neglecting losses), because the power flowing into <u>S</u> due to the transmission line just equals the power radiated by the antenna. The <u>radiated</u> power must be calculated from the <u>induced</u> field $\bar{E}$, as in 19(b). Since $\bar{E}$ and $\bar{H}$ are distributed over the total length of the antenna, it is reasonable that the <u>whole</u> antenna radiates. The vector $\bar{P}$ is often interpreted as the intensity of power flow per unit area, and Fig. 15 illustrates these flow lines. Also, Friis[9] uses $\bar{P}$ as power flow per unit area in deriving a transmission formula for antennas which has proven valid during years of intensive use. Thus, since it has been shown that the physical theory, Poynting's theorem, and the experimental evidence are in agreement, it seems reasonable that <u>most of the radiation of an antenna emanates from the elements and not from the air gap</u>.

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[9] Friis, H. T. "A Note on a Simple Transmission Formula" <u>Proc. Inst. Radio Eng.</u>, Vol. 34, pp. 254 - 256 May, 1946.

APPENDIX I

ELEMENTARY DERIVATION OF RETARDED VECTOR POTENTIAL

As a result of his experiments and derivation from Maxwell's equations on the propagation and reflection of electromagnetic effects, Hertz* proved that these "electric actions" were waves which propagated with a high but finite velocity, this velocity being, in point of fact, the velocity of light. The curves in the QUALITATIVE IDEAS section show how this is accomplished. Modern Radar makes spectacular use of this finite velocity of propagation (which will be designated by the letter c, after common usage). The fact that this is a finite velocity should appear logical even to the layman, because the only alternative is that it be an infinite velocity. But Modern Physics teaches that all electromagnetic waves have a mass associated with them, and an infinite velocity would require an infinite amount of energy to initiate it, which is a physical impossibility.

At low frequencies the wavelengths are so long and the velocity of propagation so high that all circuits are electrically short, and the effect of the finite velocity is negligible; but nevertheless, it exists. In general, from the considerations of the previous paragraph, it can be said that all electromagnetic action is propagated with a finite

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* Ibid 4, pp. 107 - 136.

velocity. Thus, if the effect of a current (or charge) located at the origin of a coordinate system is being measured at a point P, which is at a distance r from the origin, P at any specific time t can measure only the effect of a current which existed at some earlier time (t-r/c). The reason that P cannot measure the current at time t is that the effect of this current has not reached P. Therefore all of the standard low frequency equations would be more accurate if their currents or charges were written as functions of (t-r/c) instead of merely as functions of **t**. This accuracy is, of course, necessary at the higher frequencies.

Consider, for example, Ampere's law, which is illustrated in Fig. 20. All currents will henceforth be written as I(t-r/c) to emphasise the fact of the finite velocity of propagation.

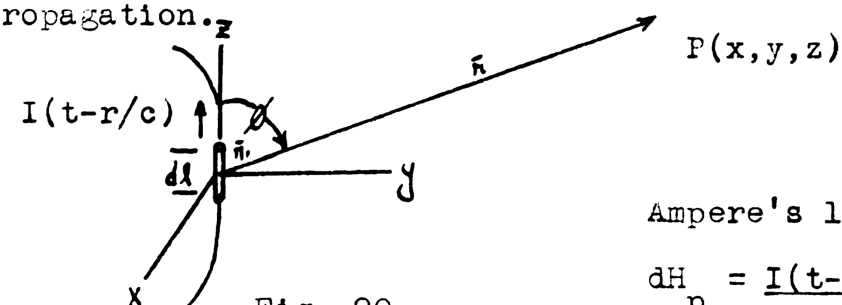

Fig. 20

Ampere's law:

$$dH_p = \frac{I(t-r/c)dl}{r^2}\sin\phi \quad (1)$$

But (1) is only an equation of magnitudes. The vector form of Ampere's law is:

$$d\bar{H} = \frac{I(t-r/c)\overline{dl}x\bar{r}_1}{r^2} \quad (2)$$

$\bar{r}_1$ is the unit vector in the $\bar{r}$ direction

Or, (2) can be written

$$d\bar{H} = \frac{I(t-r/c)d\,\bar{l}x\bar{r}}{r^3} \quad (3)$$

Integrating:

$$\bar{H} = \int \frac{I(t-r/c)d\bar{l}x\bar{r}}{r^3} \quad (4)$$

Consider now the portion of the integrand, $\frac{d\bar{l}x\bar{r}}{r^3}$.

Expanding grad (1/r) in cylindrical coordinates gives

$$\text{grad}(1/r) = \frac{-\bar{r}}{r^3}$$

Therefore, $\frac{d\bar{l}x\bar{r}}{r^3} = \text{grad}(1/r)xd\bar{l}$ (5)

Using the vector identity* $\bar{F}x\text{grad}V = V\text{curl}\bar{F} - \text{curl}V\bar{F}$, simple algebra gives $\text{grad}(1/r)xd\bar{l} = \text{curl}\frac{d\bar{l}}{r} - \frac{\text{curl}d\bar{l}}{r}$ (6)

But since $d\bar{l}$ is only a directed line segment and can be expressed in general as (idx + jdy + kdz), simple expansion of the curl gives $\text{curl}(d\bar{l}) = 0$. (7)

Therefore, $\frac{d\bar{l}x\bar{r}}{r^3} = \text{curl}(d\bar{l}/r)$ (8)

Thus Eq. (4) reduces to $\bar{H} = \int I(t-r/c)\text{curl}(d\bar{l}/r)$. (9)

But I(t-r/c) is a scalar, and since we are dealing with continuous functions, the order of differentiation and integration can be interchanged. Eq. (9) then becomes:

$$\bar{H} = \text{curl}\left[\int \frac{I(t-r/c)d\bar{l}}{r}\right] \quad (10)$$

But, by definition,

$$\bar{B} = \mu\bar{H} = \text{curl}\bar{A} \quad (11)$$

Therefore, we define $\bar{A} = \mu \int \frac{I(t-r/c)d\bar{l}}{r}$ (12)

But $I(t-r/c) = \bar{i}(t-r/c)\cdot d\bar{a}$ (13)

where da is the differential cross-section area through which the current density $\bar{i}(t-r/c)$ flows.

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* Ibid 3. p. 13, Eq. 8 - 4.

And $\left[\bar{i}(t-r/c)\cdot d\bar{a} \quad d\bar{l}\right] = \bar{i}(t-r/c)dv,$ (14)

where dv is the differential element of volume, because $\bar{i}(t-r/c)$ is in the same direction as $d\bar{l}$.

Therefore $$\bar{A} = \mu \iiint_V \frac{\bar{i}(t-r/c)dv}{r} \qquad (15)$$

The above derivation is patterned after that presented in Ramo & Whinnery*, in which the unrationalized m.k.s. system of units is used. When written in the rationalized m.k.s. system of units, the expression for $\bar{A}$ becomes

$$\bar{A} = \frac{\mu}{4\pi}\iiint_V \frac{\bar{i}(t-r/c)dv}{r} \qquad (16)$$

$\bar{A}$ is termed the retarded vector potential. The current density i is actually a function of x, y, z, <u>and</u> t - r/c. Abraham & Becker[10] write the expression for the current as i (x,y,z,t-r/c), but this is usually considered too cumbersome, and so Eq. (16) is the formula usually found in the literature. Most derivations, such as the one given in King[11], are too complicated for presentation to seniors. On the other hand, Brainerd, Koehler, Reich, & Woodruff[12] get around the problem by not defining $\bar{A}$ explicitly in the antenna derivations. However, the retarded vector potential $\bar{A}$ is an important concept, and so its derivation is presented here.

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* Ibid 6 pp. 73 - 74 and 162 - 164

10 Abraham, M., and Becker, T. <u>Classical Electricity and Magnetism</u>, pp. 220-222, 1932, New York, G. E. Stechert

11 King, R. W. P. <u>Electromagnetic Engineering</u> pp. 224-232 1945 New York McGraw-Hill

12 Brainerd, J. G., Koehler, G., Reich, J.J., & Woodruff, L.F. <u>Ultra-High-Frequency Techniques</u> pp. 392-394 1942 New York D. Van Nostrand Co.

RETARDED SCALAR POTENTIAL

The derivation given here is similar to the one presented by Skilling[13]. The potential due to a point charge distance $\bar{r}$ away (taking into account the finite velocity of propagation) is:

$$V = \frac{q(t-r/c)}{4\pi r} \qquad (17)$$

If the potential is due to many charges located at various points in space, the potential is merely the sum of the various potentials

$$V = \frac{1}{4\pi\epsilon}\sum_{i=1}^{n}\frac{q_i(t-r/c)}{r_i} \qquad (18)$$

Finally, if the number of charges q_i increases indefinitely in a given space so that a continuous volume distribution of charge is approached, then

$$V = \frac{1}{4\pi\epsilon}\iiint_{Vol.}\frac{\rho(t-r/c)dv}{r} \qquad (19)$$

ρ is the charge density perunit volume and is a continuous function. V is termed the retarded scalar potential.

Note that V is directly obtainable from the fundamental Coulomb's law:

$$\bar{F} = \frac{q_1 q_2}{4\pi\epsilon r^2}\bar{r}_1 \qquad (20)$$

$\bar{F}$ is the force between the two charges. The electric field intensity $\bar{E}$ is defined as the force on a unit positive

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13 Skilling, H. H. Fundamentals of Electric Waves pp. 127-128 1942 New York John Wiley & Sons

charge, so that $\bar{E}$ (taking into account the finite velocity of propagation) is given by

$$\bar{E} = \frac{q(t-r/c)\bar{r}_1}{4\pi\epsilon r^2} \qquad \text{volts/meter} \qquad (21)$$

$\bar{E}$ is then integrated from infinity to <u>r</u> to give Eq. (17).

Thus, the expressions for the retarded vector and scalar potentials are easily derived from three fundamental facts about electricity: (1) Electromagnetic effects are propagated with a finite velocity: (2) Ampere's law: and (3) Coulomb's law.

APPENDIX II

TANGENTIAL INDUCED ELECTRIC FIELD OF DIPOLE ANTENNA

A sinusoidal distribution is assumed for two reasons: (1) For thin, cylindrical antennas, the sinusoidal distribution is a good approximation to the actual distribution. See footnote 7, page 25. (2) The sine distribution is the only one that can be integrated with any degree of simplicity.

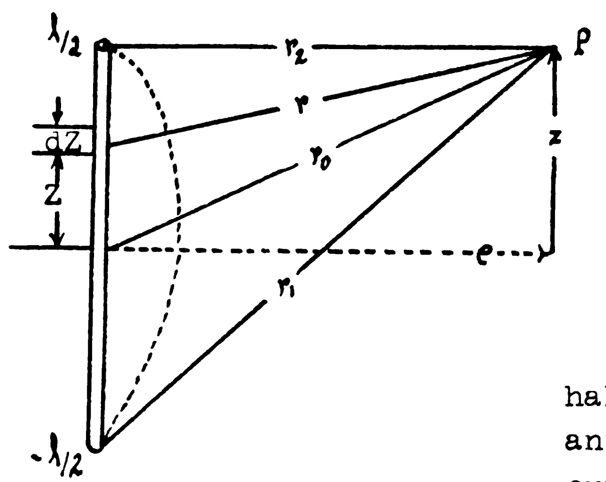

Fig. 21 half-wave dipole antenna with sinusoidal current distribution

The various parameters are illustrated in Fig. 21. The current is given by

$$I = I_o \sin\beta(\ell/2 - Z) \qquad (51)$$

where the time function $e^{j\omega t}$ is assumed.

The charge distribution may be obtained by inserting the current from Eq. (51) into the equation of continuity

$$\mathrm{div}(\bar{i}) = -\partial q_v/\partial t$$

$q_v \equiv \rho$, the charge per unit volume. In one dimension

$$\partial I/\partial Z = -\partial q_\ell/\partial t = -j\omega q_\ell$$

q_ℓ is charge per unit length. Differentiating (51) and substituting into the above equation gives

$$I_o \cos\beta(\ell/2 - Z) = -j\omega q$$

$$q_\ell = -jI_o\left(\frac{2\pi/\lambda}{2\pi f}\right)\cos\beta(\ell/2 - Z) = -\frac{jI_o}{\lambda f}\cos\beta(\ell/2 - Z)$$

$$q_\ell = -\frac{jI_o}{c}\cos\beta(\ell/2 - Z) \qquad (52)$$

The retarded vector and scalar potentials are obtained by inserting current and charge from Eqs. (51) and (52) into (16) and (19).

$$\bar{A} = \frac{\mu}{4\pi}\iiint_V \frac{\bar{i}(t-r/c)dv}{r} \qquad (16)$$

$$V = \frac{1}{4\pi e}\iiint_{Vol.} \frac{q_v(t-r/c)dv}{r} \qquad (19)$$

For time functions of the form $e^{j\omega(t-r/c)}$,*

$$e^{j\omega(t-r/c)} = e^{j\omega t}e^{-j\omega r/f} = e^{j\omega t}e^{-j\beta r}$$

Since $q_v dv = q_\ell dZ$ and $\bar{i}dv = Id\bar{Z}$, (16) and (19) become ($e^{j\omega t}$ assumed)

$$V = \frac{1}{4\pi e}\int_{-\ell/2}^{+\ell/2} -\frac{jI_o\cos\beta(\ell/2 - Z)e^{-j\beta r}dZ}{r}$$

after algebra

$$V = \frac{-jI_o M}{4\pi}\int_{-\ell/2}^{+\ell/2} \frac{\cos\beta(\ell/2 - Z)}{r} e^{j\beta r} dZ \qquad (53)$$

$$A_Z = \frac{I_o}{4\pi}\int_{-\ell/2}^{+\ell/2} \frac{\sin\beta(\ell/2 - Z)}{r} e^{j\beta r} dZ \qquad (54)$$

The field intensities may be evaluated in terms of the potentials by using Eqs. (11) and (43).

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* Stratton, Ibid 2 p. 455, uses $e^{-j\omega(t-r/c)}$ and this is the only difference in our answers.

$$\bar{H} = (1/\mu)\,\mathrm{curl}(\bar{A}) \qquad (11)$$

$$\bar{E} = -\,\mathrm{grad}V - \partial\bar{A}/\partial t \qquad (43)$$

when expressed in cylindrical coordinates for this particular problem, these become

$$E_z = -\partial V/\partial z - j\omega A_z; \quad E_\rho = -\partial V/\partial \rho; \quad E_\phi = 0 \qquad (55)$$

$$H_\phi = -(1/\mu)\partial A_z/\partial \rho \qquad (56)$$

The intensity component E_z at ρ resulting from the charge and current in the antenna is obtained by inserting Eqs. (53) and (54) into the first of (55). Letting $f(r) = \frac{e^{-j\beta r}}{r}$, we have:

$$E_z = \frac{JI}{4\pi}\left\{\mu\int_{-l/2}^{+l/2}\frac{\partial}{\partial z}\left[f(r)\cos\beta(l/2-Z)\right]dZ - \omega\mu\int_{-l/2}^{+l/2} f(r)\sin\beta(l/2-Z)dZ\right\} \qquad (57)$$

Referring to Fig. 17, we obtain

$$r^2 = \rho^2 + (z - Z)^2 \qquad (58)$$

$$\partial r/\partial z = \not{2}\frac{(z-Z)}{r}(1); \quad \partial r/\partial Z = \not{2}\frac{(z-Z)}{r}(-1)$$

Therefore $\partial r/\partial z = -\partial r/\partial Z$

$$\partial f(r)/\partial z = (\partial f(r)/\partial r)\partial r/\partial z; \quad \partial f(r)/\partial Z = (\partial f(r)/\partial r)\partial r/\partial Z$$

Therefore $\partial f(r)/\partial z = -\partial f(r)/\partial Z$

This result is now substituted into the first integral in Eq. (57) and the integration is then completed by parts.

$$E_{z(\text{first integral})} = -\mu\int_{-l/2}^{+l/2}\partial f(r)/\partial Z\cos\beta(l/2 - Z)dZ$$

Set $u = \cos\beta(l/2 - Z)$ $\qquad$ $v = f(r)$

$du = +\beta\sin\beta(l/2 - Z)dZ$ $\qquad$ $dv = (\partial f(r)/\partial Z)dZ$ (59)

$$E_z(\text{st.}\int) = -\mu f(r)\cos\beta(\ell/2 - Z)\Big|_{-\ell/2}^{+\ell/2} + \omega\mu\int_{-\ell/2}^{+\ell/2} f(r)\sin\beta(\ell/2 - Z)\,dZ$$

$$\text{note that } \mu\beta = \sqrt{\frac{\mu}{\epsilon}}\cdot\frac{\omega}{c} = \sqrt{\frac{\mu}{\epsilon}}\cdot\omega\sqrt{\mu\epsilon} = \omega\mu$$

When Eq. (59) is substituted into (57), the two integral terms cancel, leaving

$$E_z = -\frac{jI_o\mu e^{-j\beta r}}{4\pi r}\cos(\ell/2 - Z)\Big|_{-\ell/2}^{+\ell/2}$$

$$E_z = -\frac{jI_o}{4\pi}\mu\left[e^{-j\beta r_2}/r_2 - (e^{-j\beta r_1}/r_1)\cos\beta\ell\right] \qquad (60)$$

For a $\frac{1}{2}$ wave dipole $\beta\ell = (2\pi/\lambda)\lambda/2 = \pi$; $\cos\pi = -1$

$$E_z = -j30I_o\left(\frac{e^{-j\beta r_2}}{r_2} + \frac{e^{-j\beta r_1}}{r_1}\right) \qquad (61)$$

It must be noted that r can not be equal to zero in the integrals used in this derivation, because then the integrals would be improper and would have to be evaluated by the methods of Advanced Calculus. Also, r_1 and r_2 can not be zero at the ends of antenna, because then E_z would be infinite. All these conditions are automatically taken care of in practice because the elements of the antenna have a finite radius.

Eq. (35) shows that, for a differential current element, it is the real component of E_z that is oppositely directed to the current. Thus, we would expect that the real component of E_z in Eq. (61) would be oppositely directed to the current, since the current in any wire is merely the summation or integral of its differential currents. Expanding Eq. (61) gives

$$E_z = -j30I_o \left[\frac{1}{r_2} (\cos\beta r_2 - j\sin\beta r_2) + \frac{1}{r_1} (\cos\beta r_1 - j\sin\beta r_1) \right]$$

Taking the real part:

$$Re(E_z) = -30I_o \left(\frac{\sin\beta r_2}{r_2} + \frac{\sin\beta r_1}{r_1} \right) \qquad (62)$$

Thus we see that the real component of $\bar{E}$ is oppositely directed to the current as expected.

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