# DESIGN OF A REINFORCED CONCRETE BRIDGE ON M-39 NEAR GRAND LEDGE

Thesis for the Degree of B. S.
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# Design of a Reinforced Concrete Bridge on M-39 Near Grand Ledge

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by

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Introduction

This thesis covers the complete drawings and design for a bridge on M 39 near Grand Ledge.

At the present time there is at the site a 25 foot by 18 foot reinforced concrete bridge. It is at the bottom of two comparatively steep slopes and is too narrow for the present volume of traffic on the highway.

It is my object in this thesis to design a bridge with improved approach grades and wide enough to carry the present and future volume of traffic.

Keeping in mind future road work the grade of the bridge was established as shown on drawing number 1. No attempt has been made to show the future road work but when the highway is improved it can be tied in very readily with the designed bridge approach.

Both slab and girder bridges were investigated and it was desided that the proper bridge for the existing conditions is a slab bridge. The span of the bridge was controled by the existing conditions and was established as 25 feet clear.

"Specifications for The Design of Highway Bridges" adopted by the Michigan State Highway Department, November, 1936, were used throughout.atAt times it was deemed advisable to vary a small amount from these specifications. This variation is noted in the design.

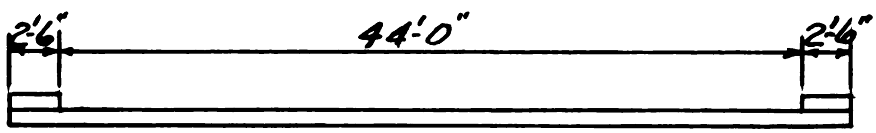
I wish to take this opportunity to thank Mr. Martin of the Portland Cement Association and Mr. Shuttleworth of the Michigan State Highway Department for their valuable aid and advice in the preparation of this thesis.

# II

## Design of Slab

Width of Roadway -

For four lane highways, the Michigan State Highway Department specifications suggest 44 ft. curb to curb, measured at right angles to the longitudinal centerline of the bridge.




2 ft. 6 in. from the curb to the edge of the bridge allows 1 ft. 6 in. clear distance from the curb to the railing , 1 ft. assumed for the railing.

Effective Width for the Distribution of Concentrated Wheel Loads on the Slab -

The specifications, Article 41, states:

$$B = 0.7S + 2$$

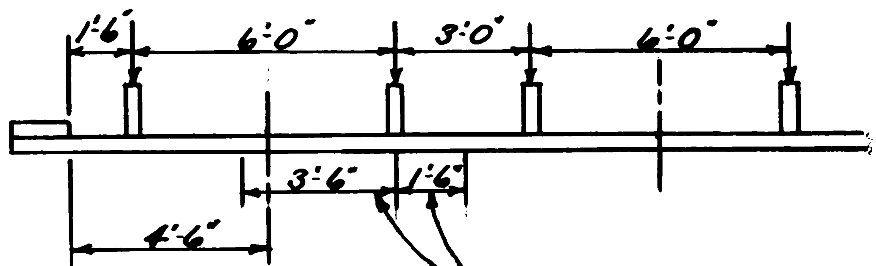
where $B$ = Effective width of slab in feet.
$S$ = Span of slab, center to center of supports.

assume 12 in. abutment wall.
$S = 25 + 1 = 26$ ft.

$B = 0.7 \times 26 + 2 = 18.2 + 2 = 20.2$ ft.

max. $B = 7$ ft (specifications, Article 41)

Article 31 of the specifications assumes that the traffic lanes will not occupy a position in which the center of the lane is nearer than 4 ft. 6 in. to the roadway face of the curb.




assumed effective width of
wheel load B

With traffic lanes as shown, two wheels can come within 3 ft. of each other, and their effective widths overlap and a new distribution must be calculated. The specifications, Article 41 also gives :

$$\text{effective width} = \frac{B + C}{2}$$

where C = distance center to center of loads.

$$\text{effective width} = \frac{7 + 3}{2} = 5 \text{ ft.}$$

distance from curb to edge of distributed wheel load B = 4.5 + 3.0 - 3.5 = 4 ft.

assume d = $13\frac{1}{4}$ in.

depth of slab = $13\frac{1}{4} + 2 + \frac{1}{2}$ = 15 3/4 in.

Article 59 calls for a protective coverage of $1\frac{1}{2}$ diams. (min. of $1\frac{1}{2}$ in.) from the surface of the concrete to the center of the nearest bar, but after investigation it was decided that a protective coverage of 2 in. clear would be better for this bridge deck.

## Dead Load

$\text{slab} - \frac{15.75}{12} \times 1 \times 150 = 196.75$

future wearing surface (Specs. Art. 30) $= 20.00$

$\frac{1}{2}$ in. wearing surface (Specs. Art. 26) $\frac{0.5}{12} \times 1 \times 150 = \underline{6.25}$

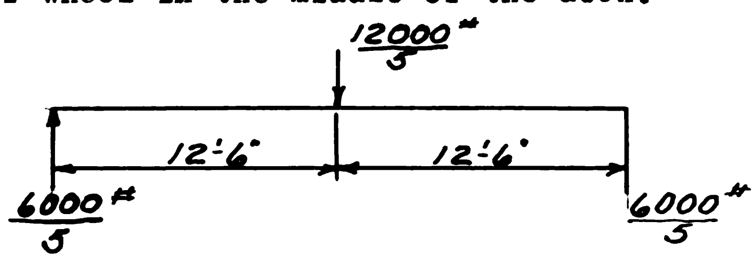
$223.00$ lbs. per sq. in.

$$\text{dead load moment} = \frac{wl^2}{8}$$

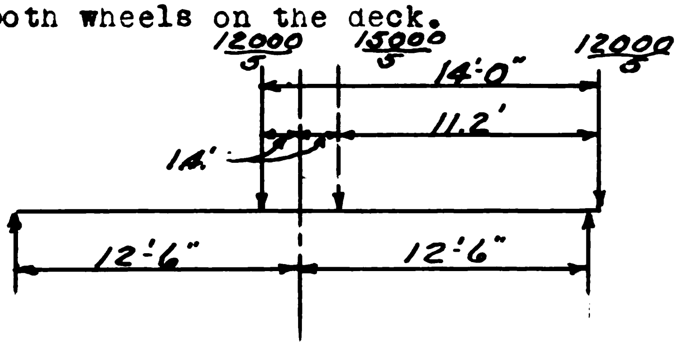
$$= 1/8 \times 223 \times \overline{25}^2 \times 12 = 209{,}000 \text{ in. lbs.}$$

## Live Load

Rear wheel in the middle of the deck.




Try both wheels on the deck.




Maximum moment occurs when the 12,000 lb. wheel is at the center of the span, and is:

Live load moment $= \frac{6,000}{5} \times 12.5 \times 12.0 = 180,000$ in. lbs.

Impact

Article 37 of the specifications states that the impact coefficient shall be:

$$\text{Impact coefficient} = I = \frac{L + 20}{6L + 20}$$

L = Span length

$$I = \frac{25 + 20}{6 \times 25 + 20} = \frac{45}{170} = 0.265$$

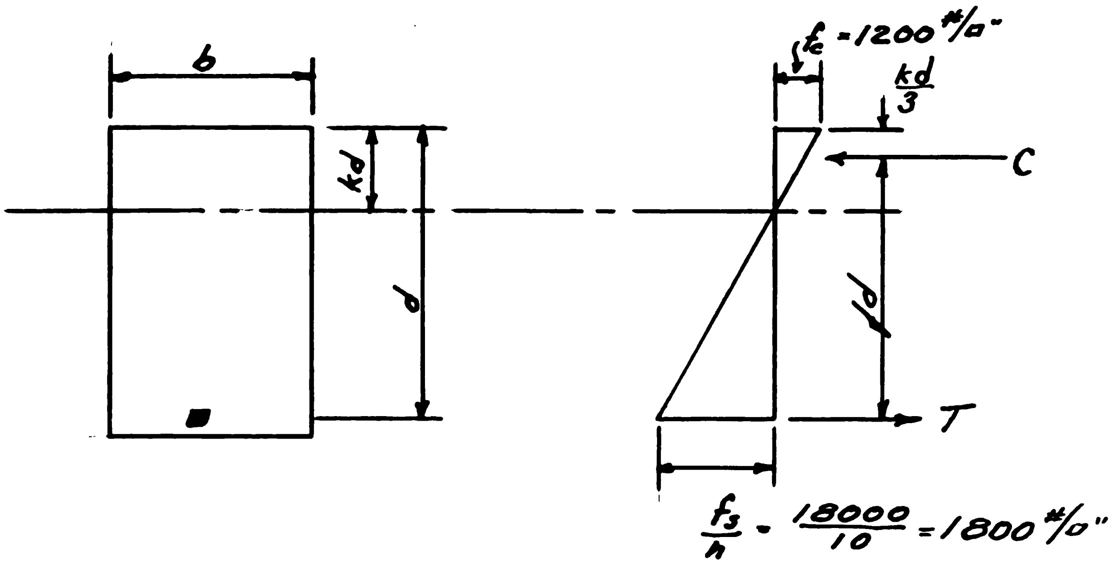
Impact moment = 0.265 x 180,000 = 47,700 in. lbs.

Total moment = 209,000 + 180,000 + 47,700
= 436,700 in.lbs. per ft. width of slab.

Calculate depth of slab

Use: $f_c$ = 3,000 lbs. per sq. in. and n = 10

intermediate grade steel $f_s$ = 18,000 lbs. per sq. in.



$$\frac{kd}{1200} = \frac{d}{3,000}$$

$$kd = 0.4d$$

$$jd = d - \frac{kd}{3} = 0.867d$$

$$T = C = \frac{1200}{2} \times .4d \times 12 = 2880d$$

$$M = T \times jd$$

$$436,7000 = 2880d \times .867d$$

$$d^2 = \frac{436,700}{2880 \times .867} = 175$$

$$d = 13.22 \text{ in.}$$

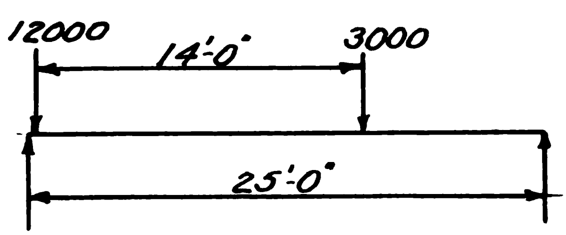
use $d = 13\frac{1}{4}$ in. as was assumed

$$T = A_s f_s$$

$$A_s = \frac{2,880 \times 13.25}{18,000} = 2.12 \text{ sq. in.}$$

use 1 in. square bars @ 5.5 in. $A = 2.18$ sq. in.

Check Shear




Live load shear $= V_L = 12,000 + \frac{11}{25}\ 3,000 = 13,320$ lbs.

Impact shear $= V_I = 0.265 \times 13,320 = 3,535$ lbs.

Dead load shear $= V_D = 223 \times 12.5 = 2790$ lbs.

Dead load unit shear $= v_D = \frac{V}{bjd}$

$$= \frac{2790}{12 \times 0.867 \times 13.25}$$

$$= 20.25 \text{ lbs. per sq. in.}$$

There have been many attempts to establish a formula for the effective width for shear distribution. Kirkham in "Highway Bridges", page 118, assumes that the end shear on simple slabs due to live load and impact may be distributed transversely over 2.5'

Caughey in his book "Reinforced Concrete", on page 149, gives the results of an experiment at the Iowa Engineering Experiment Station in which the formula was found to be as follows:

$$E = .75 + 1.2t + x$$

where $t$ = slab thickness in feet

$x$ = distance of load from unsupported edge of slab in feet, never to be assigned a greater value than $1 + 2.4t$

$$x = 1 + 2.4t = 1 + \frac{2.4 \times 13.25}{12} = 3.68 \text{ ft.}$$ which is less than the distance to the edge of the slab for the point at which we are figuring the shear.

E then becomes $E = .75 + 1.2 \times \frac{13.25}{12} + 3.68 = 5.76$ ft.

The specifications do not limit their effective width of slab formula, to figuring moment so it is assumed that it can be used for shear.

There are many other formulas advocated for use which compare favorably with the last two shown above. In view of all this it was decided to use the same distribution for shear that was used for moment, namely 5 feet

Live load & impact shear = 16,855 lbs. distributed over 5 ft. transversely

Live Load Impact unit shear $= V_L = \dfrac{V}{bjd}$

$$= \frac{16,855}{12 \times 5 \times 867 \times 13.25}$$

$= 24.4$ lbs. per sq. in.

total unit shear $= V = V_L + V_D$

$= 24.4 + 20.25$

$= 44.65$ lbs per sq. in

allowable $V = 60$ lbs per sq. in.

<u>Check Bond</u>

$$\mu = \frac{V}{\Sigma_o jd}$$

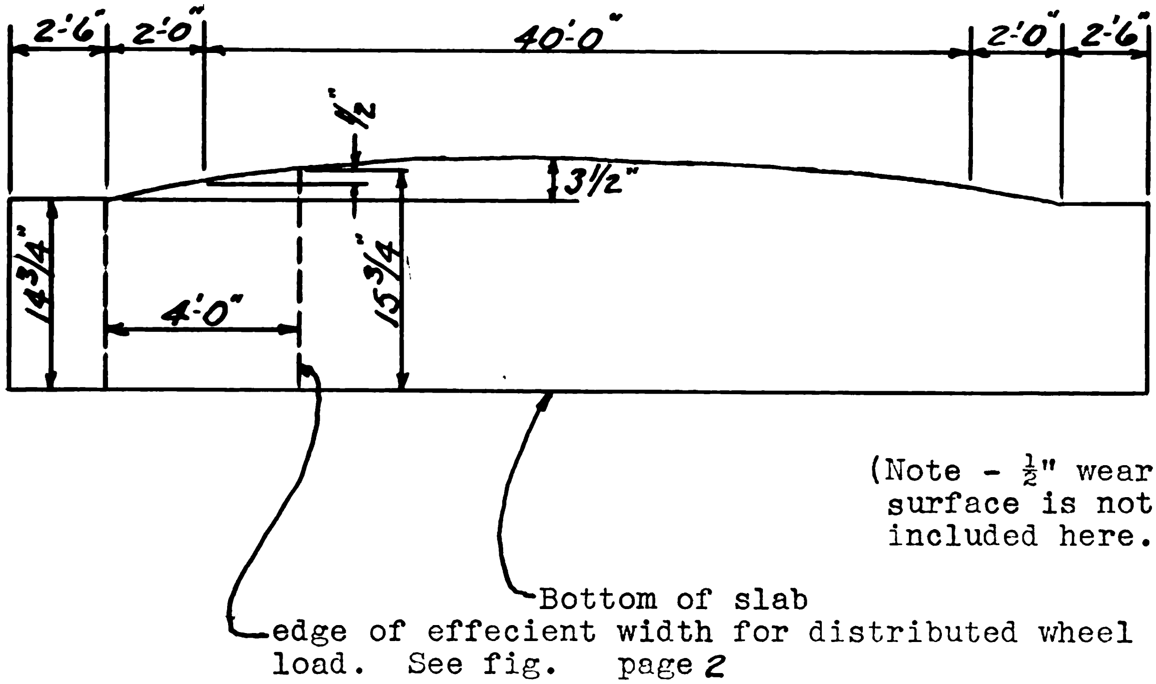
$$V = \frac{16855}{5} + 2790 = 6161 \text{ lbs.}$$

$$\mu = \frac{6161}{4 \times \frac{12}{3.5} \times 867 \times 13.25} = 61.4 \text{ lbs per sq in.}$$

allowable $\mu = 150$ lbs. per sq. in.

Special anchorage is not shown to be necessary but good practice will require a standard hook on the main longitudinal steel.

Crown of Roadway




(Note - ½" wearing surface is not included here.)

$C$ = $.00187R^2$ (for middle 40 ft. - Specs. Art. 24)

$C$ = Crown of roadway in inches

$R$ = Width of roadway in feet

$C$ =- $.00187 \times \overline{40}^2$ = 3 in.

The two feet outside of the middle 40 feet is tangent to the curve with a slope equivalent to 3 inch vertical in 10 foot horizontal.

$$\frac{3}{10} = \frac{x}{2}$$

$x$ = .6 use ½ inch

Determine crown 4 feet from the face of the curb.

$$40 - 4 = 36 \text{ ft.}$$
$$C = .00187 \times 36^2 = 2.42 \text{ in. say } 2.5 \text{ in.}$$
$$3 - 2.5 = 0.5 \text{ in.}$$

Temperature Reinforcement

"Temperature reinforcement shall be placed at exposed surfaces and shall provide not less than one eigth square inch of reinforcement per foot of width of surface." Specs. Art. 66.

1/8 sq. in. reinforcement per foot of width of surface =
$\frac{1}{2}$ in. round bars @ 19 in.

Use $\frac{1}{2}$ in. round @ 18 in. logitudinaly in the top of the slab.

There is no restraint of the slab transversely in the middle of the span so that except for tie bars, no temperatüre reinforcement is necessary. There is, however, restraint transversely near and at the abutmnets and transverse temperature reinforcement should be used.

Use $\frac{1}{2}$ in. round @ 18 in. transversely in the top of the slab for 5 feet next to the abutments. and

Use $\frac{1}{2}$ in. round @ 30 in. transversely in the top of the slab for the middle 15 feet.

Transverse Steel in Bottom of Slab

There should be steel in the bottom of the slab to take care of the moment perpendicular to the span of the slab. It may be argued that there have been no failures in slabs due to this moment, but this is not due to the fact that the m moment does not exist. Putting steel in a slab does not mean that all of the moment is in the direction of this steel. For a balanced design, steel should be added perpendicular to the main reinforcement.

For spans of this length the Michigan State Highway Department uses 15% of the main reinforcement in the outer $\frac{1}{4}$ of the span and 25% of the main reinforcement in the middle $\frac{1}{2}$ of the span.

1 in. sq. bars @ 5.5 in. $A_s = 2.18$ sq. in.

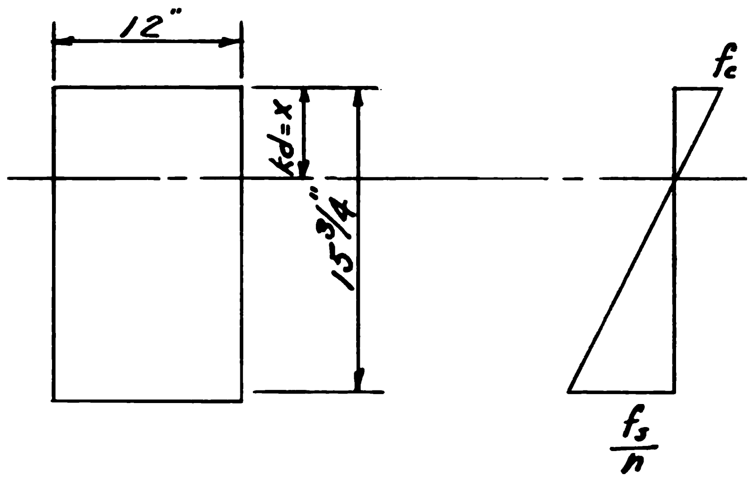
$2.18 \times .15 = .327$ sq. in.
$2.18 \times .25 = .545$ sq. in.

Use $\frac{1}{2}$ in sq. bars at 9 in. in the outer $\frac{1}{4}$ span.

Use $\frac{1}{2}$ in sq. bars at 5.5 in. in the middle $\frac{1}{2}$ of the span.

Check the Stresses at the Centerline

Depth of slab $= 15\ 3/4 + 2\frac{1}{2} = 18\frac{1}{4}$ in, $d = 15\ 3/4$ in.




$A_s = 2.18^{\circ}$ in.

Transformed area steel = n x $A_s$ = 10 x 2.18 = 21.8 sq. in.

Take moments about neutral axis.

$$12 \times X \times \frac{x}{2} = 21.8 \times (15.75 - x)$$

$$6x^2 = 344 - 21.8x$$

$$6x^2 + 21.8x - 344 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-21.8 \pm \sqrt{476 + 8260}}{12} = 5.98 \text{ in.}$$

$$jd = 15.75 - \frac{5.98}{3} = 13.76 \text{ in.}$$

Dead Load

| | |
|---|---|
| slab - $\frac{18.25}{12}$ x 1 x 150 | 228.00 |
| future wearing surface - Specs. Art. 30 | 20.00 |
| $\frac{1}{2}$ in. wearing surface | 6.25 |
| | 254.25 lbs. per sq. in. |

Dead load moment $= \dfrac{w\,l^2}{8}$

$= 1/8 \times 254.25 \times 25^2 \times 12 = 239{,}000$ in. lb.

Live load and Impact moment $= 180{,}000 + 47{,}700 = 227{,}700$ in. lb.

Total moment $= 227{,}700 + 239{,}000 = 466{,}700$ in. lb.

$$M = \frac{f_c}{2} \times k\,d \times b \times j\,d$$

$$466{,}700 = \frac{f_c}{2} \times 5.98 \times 12 \times 13.76$$

$f_c = 944$ lbs. per sq. in.
allowable $f_c = 1{,}200$ lbs. per sq. in.

$$f_s A_s = \frac{f_c}{2} \times kd \times b$$

$$f_s = \frac{944}{2} \times \frac{5.98 \times 12}{2.18}$$

$= 15{,}550$ lbs. per sq. in.
allowable $f_s = 18{,}000$ lbs. per sq. in.

$$V = \frac{16855}{5} + 254.25 \times 12.5$$

$= 3371 + 3180 = 6551$ lbs.

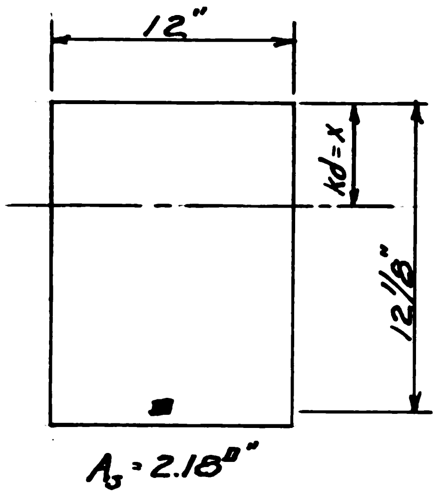
$$v = \frac{V}{bjd} = \frac{6551}{12 \times 13.76} = 39.6 \text{ lbs. per sq. in.}$$

allowable $v = 60$ lbs. per sq. in.

$$u = \frac{V}{\Sigma o\,jd} = \frac{6551}{4 \times \frac{12}{5.5} \times 13.76} = 54.6 \text{ lbs. per sq. in.}$$

allowable $u = 150$ lbs. per sq. in.

Check Stresses at Curb




Depth of slab = 15 3/4 - 1 1/8 in.
= 14 5/8 in.
d = 12 1/8 in.

Transformed area steel = $nA_s$ = 10 x 2.18 = 21.8 sq. in.

Take moments about neutral axis.

$$12 \times X \times \frac{X}{2} = 21.8 \ (12.125 - X)$$

$$6X^2 + 21.8X - 264 = 0$$

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2\,a}$$

$$= \frac{-\ 21.8 \pm \sqrt{476 + 6340}}{12}$$

$$= 5.05 \text{ in.}$$

$$jd = d - \frac{kd}{3} = 12.13 - \frac{5.05}{3} = 10.45 \text{ in.}$$

Dead load
slab - $\frac{14.63}{12}$ x 1 x 150 — 182.75
future wearing surface - specs. Art. 30 — 20.
½ in. wearing surface — 6.25

209.0 lbs. per sq. in.

$$\text{Dead load moment} = \frac{w\,l^2}{8}$$
$$= 1/8 \times 209 \times 25^2 \times 12 = 196{,}500 \text{ in. lbs.}$$

Effective width for concentrated load

Max. B is 7 ft., but the distance from a wheel at the curb to the edge of the slab is less than $\frac{B}{2}$ so the formula;

eff. width = $\frac{B}{2}$ + distance to edge of slab, must be used.

Assume the wheel at the curb.

$$\text{distance to edge of slab} = 2.5 + \frac{7.5}{12}$$
$$= 3.125$$

where $\frac{7.5}{12}$ = distance of center of tire from face of curb in feet.

$$\text{effective width} = \frac{7}{2} + 3.125 = 6.625 \text{ ft.}$$

$$\text{Live load moment} = \frac{6{,}000}{6.625} \times 12.5 \times 12 = 136{,}000 \text{ in. lbs.}$$

$$\text{Impact moment} = .265 \times 136{,}000 = 36{,}100 \text{ in lbs.}$$

$$\text{Total moment} = 196{,}500 + 136{,}000 + 36{,}100$$
$$= 368{,}600 \text{ in lbs.}$$

$$M = \frac{f_c}{2} \times kd \times b \times jd$$

$$368{,}600 = \frac{f_c}{2} \times 5.05 \times 12 \times 10.45$$

$$f_c = 1165 \text{ lbs per sq. in.}$$

$$f_s A_s = \frac{f_c}{2} \times kd \times b$$

$$f_s = \frac{\frac{1165}{2} \times 5.05 \times 12}{2.18} = 16{,}200 \text{ lbs. per sq. in.}$$

allowable = 18,000 per sq. in.

$$V = \frac{16{,}855}{6.625} + 209 \times 12.5 = 5{,}150 \text{ lbs.}$$

$$v = \frac{V}{bjd} = \frac{5{,}150}{12 \times 10.45} = 41 \text{ lbs. per sq. in.}$$

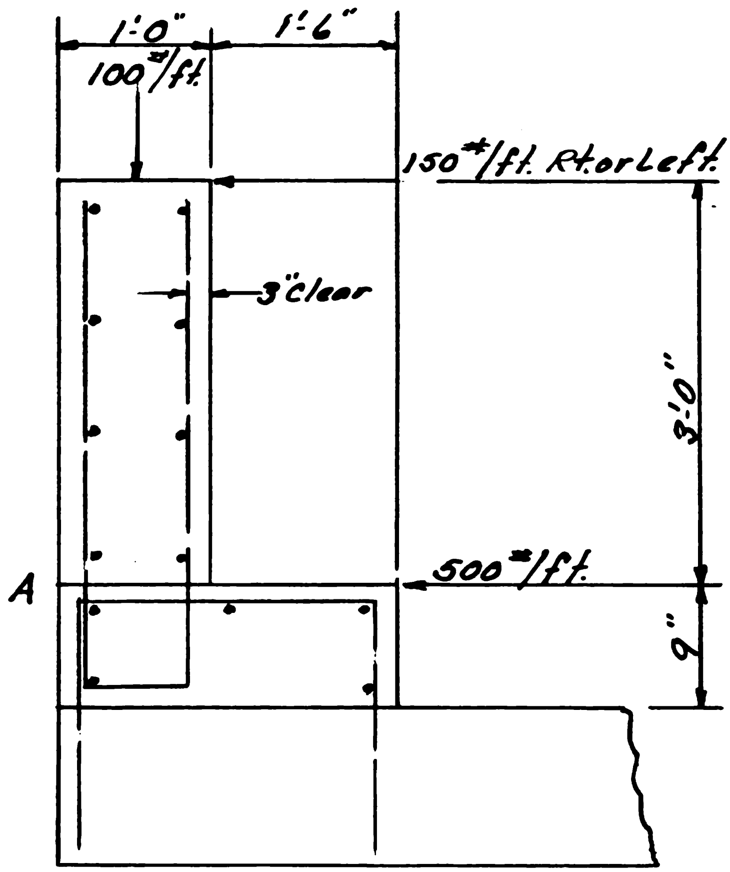
allowable $v = 60$ lbs. per sq. in.

$$\mu = \frac{V}{\Sigma_0 jd} = \frac{5{,}150}{4 \times \frac{12}{5.5} \times 10.45} = 56.5 \text{ lbs. per sq. in.}$$

allowable $\mu = 150$ lbs per sq. in.

III

Design of Railing




Assume a 12 in. railing with ½ in. round bars @ 16 in. vertically.

<u>Check Shear</u>

If only reasonable care is taken with the joint at the base of the rail it may be expected to develop at least 50% of the shearing resistance of monolithic concrete.

As the unit shear is only; $\frac{150}{144}$ = 1.05 lbs. per sq. in., the railing of course is safe in shear.

Check Railing for Over Turning About Point A

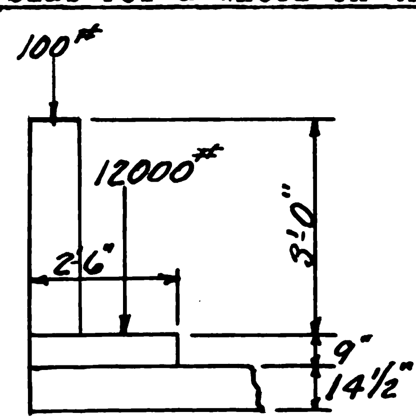
Moment about A = 150 x 3 - 100 x .5 - 150 x 3 x .5
= 450 - 50 - 225 = 175 ft. lbs.

Area of steel needed to withstand this moment per ft. of railing = $A_s$ = $\frac{175 \times 12}{18,000}$ = .117 sq. in.

$\frac{1}{2}$ in. round @ 16 in. have an area of .20 x $\frac{12}{16}$
= .15 sq. in. per ft. of rail

Use $\frac{1}{2}$ in round @ 16 in. vertically in each face. Use 4 - $\frac{1}{2}$ in. round horizontally for temperature steel in each face. The moment and shear are both less on the curb section and $\frac{1}{2}$ in. round @ 16 in. vertically may also be used. Use 5 - $\frac{1}{2}$ in. round horizontally as shown.

Check the Slab For a Wheel on the Curb



Dead load

| | | | |
|---|---|---|---|
| railing | $3 \times 1 \times 150$ | = | 450 |
| curb and slab | $\frac{9 + 14\frac{1}{2}}{12} \times 1 \times 150$ | = | 294 |
| load | 100 | = | 100 |
| | | | 844 lbs. per ft. of curb |

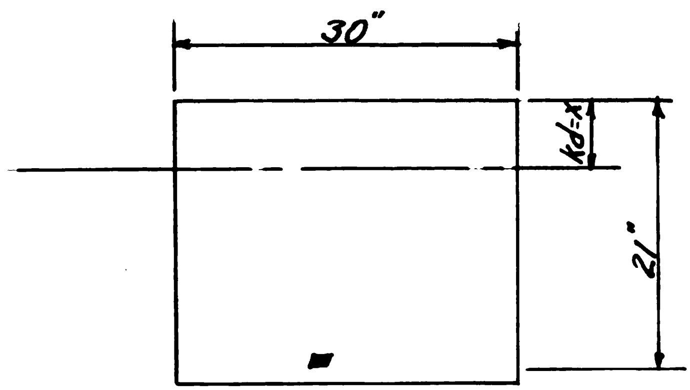
$$\text{Dead load moment} = \frac{w\,l^2}{8}$$

$$= 1/8 \times 844 \times 25^2 \times 12 = 791{,}000 \text{ in. lbs.}$$

$$\text{Live moment} = \frac{6{,}000}{2.5} \times 12.5 \times 12 = 360{,}000 \text{ in. lbs.}$$

$$\text{Impact moment} = .265 \times 360{,}000 = 95{,}400 \text{ in. lbs.}$$

$$\text{Total moment} = 791{,}000 + 360{,}000 + 95{,}400 = 1{,}246{,}400 \text{ in. lbs.}$$




$A_s = 2.18 \times 2.5 = 5.45\square''$

Transformed area steel $= n\,A_s = 54.5$ sq. in.

$$30 \times X \times \frac{x}{2} = 54.5\,(21 - x)$$

$$15x^2 + 54.5x - 1145 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-54.5 \pm \sqrt{2970 + 68700}}{30} = 7.1 \text{ in.}$$

$$jd = 21 - \frac{7.1}{3} = 18.6 \text{ in.}$$

$$T = C = \frac{f_c}{2} \times 30 \times 7.1$$

$$M = C \times jd = 1{,}246{,}400 = \frac{f_c}{2} \times 30 \times 7.1 \times 18.6$$

$f_c$ = 630 lbs. per sq. in.
allowable $f_c$ = 1,200 lbs per sq. in.

$$A_s f_s \times jd = M$$

$$f_s = \frac{1{,}246{,}400}{5.45 \times 18.6} = 12{,}300 \text{ lbs per sq. in.}$$

allowable $f_s$ = 18,000 lbs per sq. in.

Check Shear -

Dead load shear $V_d$ = 844 x 12.5 = 1,0550 lbs.

Live load shear $V_l = 12{,}000 + \frac{11}{25} 3{,}000 = 13{,}320$ lbs

$$v = \frac{V}{bjd} = \frac{23{,}870}{2.5 \times 867 \times 21} = 523 \text{ lbs per sq. in.}$$

The curb is not designed for a wheel load on it.

Dead load $v = \frac{10{,}550}{2.5 \times 867 \times 21} = 231$ lbs per sq. in.

The allowable unit shear with special anchorage and web reinforcement = 270 lbs per sq. in.

$v$ = 231 lbs per sq. in.
$v_c$ = 90 lbs per sq. in.

$v_s = v - v_c$ = 141 lbs per sq. in.

$$v_s = \frac{a_s f_s}{b\ s \sin\alpha}$$

½ in. round bars $a_s$ = .20

$$s = \frac{.20 \times 18{,}000}{2.5 \times 1 \times 141} = 10 + \text{ inches}$$

Use 9 inch spacing. Try shear at $\frac{1}{4}$ point.

$$V = 844 \times 12.5 - 844 \times 6.25$$

$$= 5{,}280 \text{ lbs.}$$

$$v = \frac{5{,}280}{2.5 \times .867 \times 21} = 115 \text{ lbs per sq. in.}$$

$$v_s = 115 - 90 = 25 \text{ lbs per sq. in.}$$

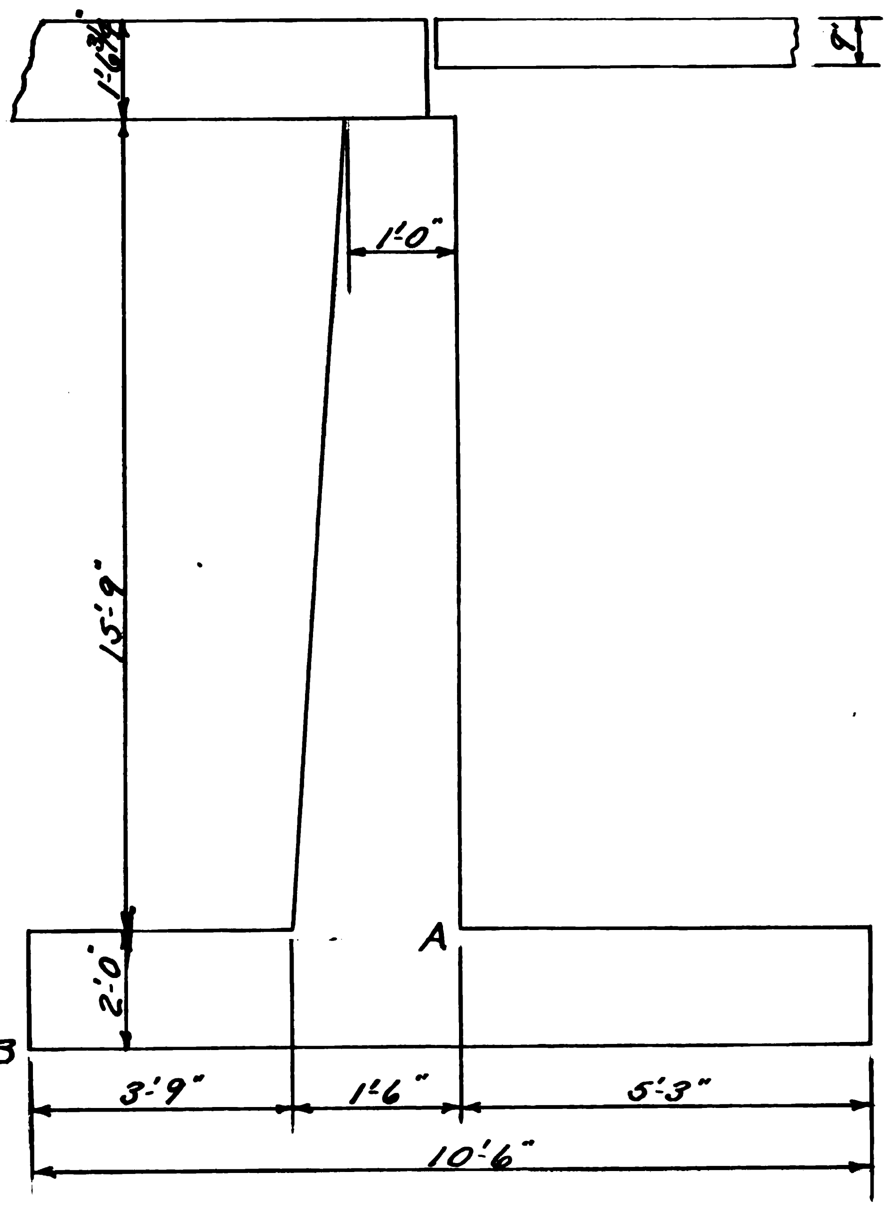
$$s = \frac{.20 \times 18{,}000}{2.5 \times 1 \times 25} = 50 \text{ in.}$$

$$\text{max. } s = \frac{45d}{\alpha + 10} = \frac{45 \times \frac{1}{2}}{100} = 22.5 \text{ in.}$$

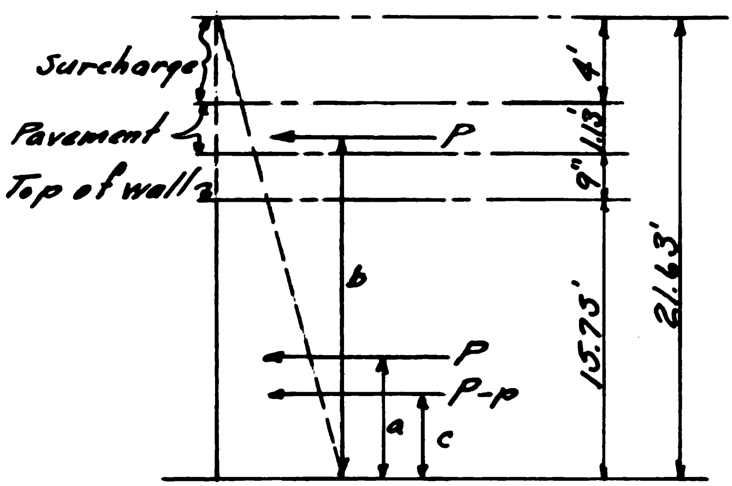
Use $\frac{1}{2}$ in. round stirrups at 9 in. in outside $\frac{1}{4}$ of span and $\frac{1}{2}$ in. round stirrups at 18 in. in middle half of span.

IV

Design of Abutment Walls



Design of stem




*Max pressure on stem.*

"Retaining walls, abutments and structures built to retain fills shall be designed to resist pressures determined in accordance with "Rankine" theory of pressure distribution in non cohesive granular material provided that no structure shall be designed for an equivalent fluid pressure of less than 30 lbs per sq. ft.", Specifications, Article 39.

An equivalent earth surcharge of four feet is used as the live load.

Values of $C_e$ = .33 and W = 100 lbs per cu. ft. are used by the Michigan State Highway Department for soil conditions similar to those on this job.

$$.75 \text{ ft. of concrete} = \frac{.75 \times 150}{100} = 1.13 \text{ ft. of earth}$$

$$P = C_e \frac{w\,h^2}{2} = .33 \quad \frac{100 \times \overline{21.63}^2}{2} = 7{,}730 \text{ lbs.}$$

$$a = \frac{21.63}{3} = 7.21 \text{ ft.}$$

$$p = .33 \quad \frac{100 \times \overline{5.88}^2}{2} = 570 \text{ lbs.}$$

$$b. = \frac{5.88}{3} + 15.75 = 17.71 \text{ ft.}$$

$$P\text{-}p = 7{,}730 - 570 = 7{,}160 \text{ lbs.}$$

Take moments about A

$$C = \frac{7{,}730 \times 7.21 - 570 \times 17.71}{7{,}160} = 6.38 \text{ ft.}$$

Moment at base = M = 7160 x 6.38 x 12 = 548,000 in lbs.

$$d = \sqrt{\frac{M}{Kb}}$$

for $f_c$ = 1,200 lbs per sq. in.

and $f_s$ = 18,000 lbs per sq. in.

$$K = 208$$

$$d = \sqrt{\frac{548{,}000}{208 \times 12}} = 14.82 \text{ say } 15 \text{ in.}$$

Depth of slab = 15 + 3 = 18 in.

$$M = f_s A_s jd$$

$$k = \frac{1}{1 + \frac{t_s}{nf_c}} = \frac{1}{1 + \frac{18{,}000}{10 \times 1{,}200}} = 0.4$$

$$J = 1 - \frac{k}{3} = .867$$

$$A_s = \frac{548{,}000}{18{,}000 \times .867 \times 15} = 2.34 \text{ sq. in.}$$

Use 1 in. round @ 4 in. $A_s$ = 2.36 sq. in.

$V = (P - p) = 7{,}160$ lbs.

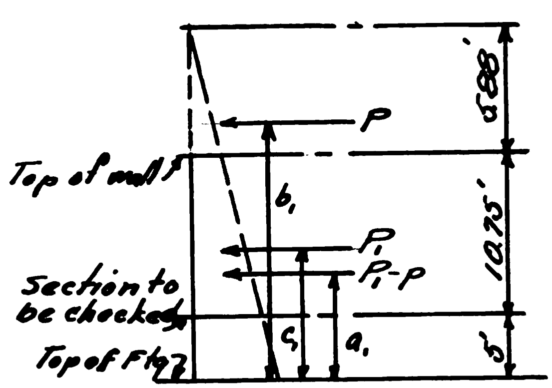
$$\text{max. } v = \frac{V}{bjd} = \frac{7{,}160}{12 \times .867 \times 15} = 45.5 \text{ lbs per sq. in.}$$

allowable 60 lbs per sq. in.

$$\mu = \frac{V}{\Sigma_o jd} = \frac{7{,}160}{3.14 \times \frac{12}{3} \times .867 \times 15} = 45.3 \text{ lbs per sq. in.}$$

allowable $\mu$ = 150 lbs per sq. in

Check Stem 5 ft. Above the Footing




$$P_1 = .33 \frac{100 \times \overline{16.63}^2}{2} = 4{,}560 \text{ lbs.}$$

$$a_1 = \frac{16.63}{3} = 5.54 \text{ ft.}$$

$p = 570$ lbs. $b_1 = \frac{5.88}{3} + 10.75 = 12.71$ ft.

$$P_1 - p = 4{,}560 - 570 = 3{,}990 \text{ lbs.}$$

$$C_1 = \frac{4{,}560 \times 5.54 - 570 \times 12.71}{3{,}990} = 4.50 \text{ ft.}$$

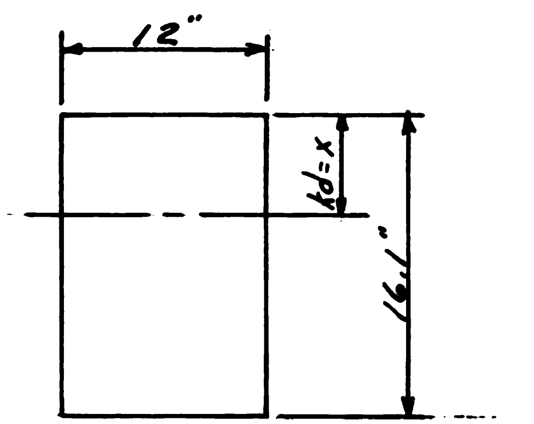
$$M = 3{,}990 \times 4.50 \times 12 = 215{,}500 \text{ in lbs.}$$

$$d = \sqrt{\frac{215{,}500}{208 \times 12}} = 9.25 \text{ in.}$$

Depth of stem needed = $9.25 + 3 = 12.25$ in.

Depth of stem available = $12 + \dfrac{6 \times 10.75}{15.75} = 16.1$ in.
$d = 13.1$ in.

Assume that ½ the steel will be terminated 5 ft. above the footing.




$$12 \times X \times \frac{X}{2} = 1.18 \times 10(13.1 - X)$$

$$6X^2 + 11.8\,X - 154.5 = 0$$

$$X = \frac{-11.8 \pm \sqrt{139.4 + 3{,}700}}{12}$$

$$kd = X = 4.17 \text{ in.}$$

$jd = 13.1 - \frac{4.17}{3} = 11.71$ in.

$215,500 = \frac{f_c}{2} \times 4.17 \times 12 \times 11.71$

$f_c = 735$ lbs. per sq. in.

allowable $f_c = 1,200$ lbs per sq. in.

$f_s \times 1.18 = \frac{735}{2} \times 4.17 \times 12$

$f_s = 15,600$ lbs per sq. in.

allowable $f_s = 18,000$ lbs per sq. in.

Check for anchorage

allowable tension in a round rod $= \frac{\pi a^2}{4} \times 18,000$

Bond developed in a bar $= \pi \times a \times l \times 150$

$\frac{\pi a^2}{4} \times 18,000 = \pi al \times 150 \quad l = 30a$

Anchor ½ of the bars 30 diameters or 30 in. beyond 5 ft. above the footing.

Temperature Steel in the Stem

The specifications call for 1/8 sq. in. of temperature steel per foot of surface.

Spacing for ½ in. round bars.

$\frac{.125}{12} = \frac{.20}{s} \qquad s = 19$ in. ≠

Use ½ in. round @ 18 in. horizontal and vertical in the front face and use ½ in. round @ 18 in. horizontal at 18 in. in the back face.

Determine Dimention for the Abutment

The cross-section will be analysed using three methods of loading.

Cas I - No superstructure load
No live load

Case II - Superstructure dead load
All live load surcharges

Case III - Superstructure dead load
No surcharges

Case I

for dimension see sketch

Wt. of stem - $15.75 \times 1 \times 1 \times 150 + \frac{.5 \times 1 \times 150 \times 1.575}{2}$

| | |
|---|---|
| = 2,360 + 591 | = 2,951 lbs. |
| Wt. of base = 10.5 x 1 x 2 x 150 | = 3,150 " |
| Wt. of earth = 5.25 x 15.75 x 1 x 100 | = 8,275 " |
| Total weight | 14,376 lbs. |

Distance of resultant of vertical forces from point

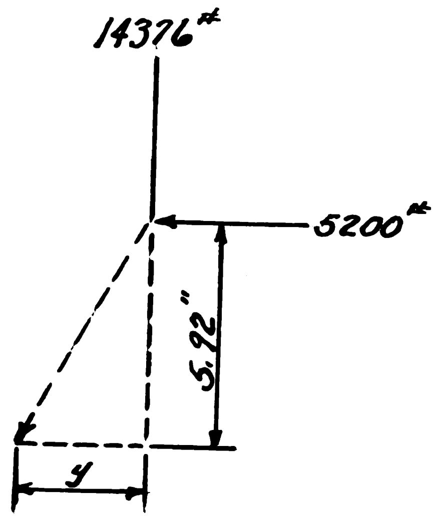
B = X

$$X = \frac{2.360 \times 4.75 + 591 \times 4.08 + 3,150 \times 5.25 + 8,275 \times 7.875}{14376}$$

$= 6.63$

Earth pressure = $P_2$ = $.33 \; \frac{100 \times \overline{17.75}^2}{2}$ = 5,200 lbs.

$a_2 = \frac{17.75}{3} = 5.92$ in.




$$\frac{y}{5.91} = \frac{5,200}{14,376}$$

$$y = 2.14 \text{ ft.}$$

Toe distance to resultant = 6.63 - 2.14
= 4.49 ft.

Eccentricity = e = 5.25 - 4.49 = .76 ft.

$\frac{10.5}{6}$ = 1.75 ft. Resultant passes through the middle third of the base.

Sliding factor of safety = f = $\frac{14,376 \times .4}{5,200}$ = 1.105

Overturning factor of safety = $\frac{14,376 \times 6.63}{5,200 \times 5.92}$ = 3.10

Earth pressures = p =- $\frac{P_2}{b} \quad (1 \pm \frac{6\,e}{b}$

$$= \frac{14,376}{10.5} \; ( 1 \pm \frac{6 \times .76}{10.5}$$

= 1965 lbs per sq. ft.

= 775 lbs per sq. ft.

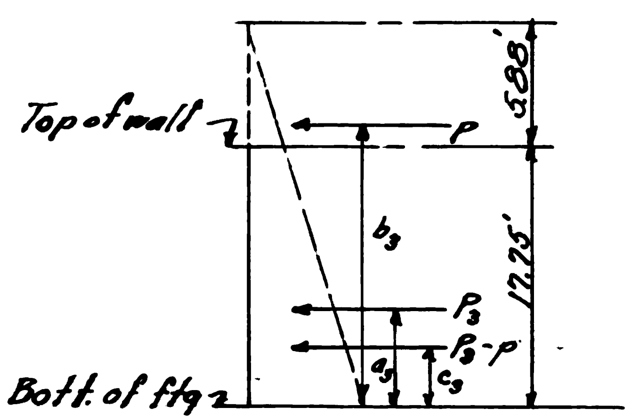
Case II

| | |
|---|---|
| Wt. of deck = 1.5 x 1 x 13.5 x 150 | = 3,040 lbs. |
| Wt. of surcharge on deck = 4 x 1 x 13.5 x 100 | = 5,400 " |
| Wt. of stem = 2,360 + 591 | = 2,951 " |
| Wt. of base | = 3,150 " |
| Wt. of earth and surcharge = 21.63 x 5.25 x 100 | = 11,350 |
| Total weight | 25,891 lbs. |

Distance of resultant of vertical forces from point B = X

$$X = \frac{(3,040 + 5,400 + 2,360)\ 4.75 + 591 \times 4.08 + 3.50 \times 11350 \times 7.875}{25,891}$$

= 6.17 ft.




$$P_3 = .33\ \frac{100 \times \overline{23.63}^2}{2}$$

$= 9{,}210$ lbs. $\qquad a_3 = \frac{23.63}{3} = 7.88$ ft.

$p = 570$ lbs. $\qquad b_3 = \frac{5.88}{3} + 17.75 = 19.71$ ft.

$P_3 - p = 9{,}210 - 570 = 8640$ lbs.

$$C_3 = \frac{9{,}210 \times 7.88 - 570 \times 19.71}{8{,}640}$$

$= 7.10$ ft.

$$\frac{y}{7.10} = \frac{8{,}640}{25{,}891}$$

$y = 2.37$ ft.

Toe distance to resultant $= 6.17 - 2.37 = 3.80$ ft.

$C = 5.25 - 3.80 = 1.45$ ft.

Sliding factor of safety $= \frac{25891 \times .4}{8640} = 1.20$

Overturning factor of safety $= \frac{25{,}891 \times 6.17}{8{,}640 \times 7.10} = 2.61$

Earth pressures $= \frac{2{,}5891}{10.5} \left(1 \pm \frac{6 \times 1.45}{10.5}\right)$

$= 4{,}500$ lbs. per sq. ft.

$= 427$ lbs per sq. ft.

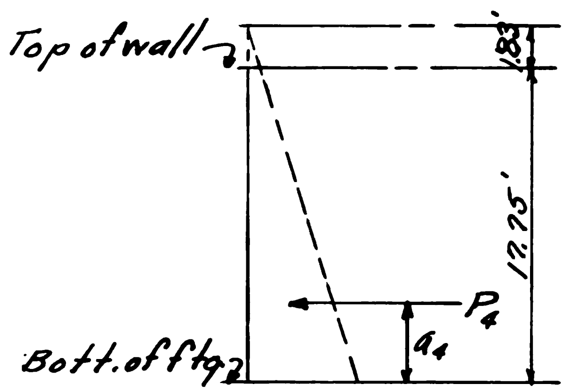
Case III

| | |
|---|---|
| Wt. of deck | = 3,090 lbs. |
| Wt. of stem = 2,360 + 591 | = 2,951 " |
| Wt. of base | = 3,150 " |
| Wt. of earth = 17.63 x 5.25 x 1 x 100 | = 9,250 " |
| Total weight | 18,391 lbs. |

Distance of resultant of vertical forces from point B = X

$$X = \frac{(3040 + 2360)\ 4.75 + 591 \times 4.08 + 3150 \times 5.25 + 9250 \times 7.875}{18391}$$

$= 6.39$ ft.




$P_4 = .33\ \dfrac{100 \times 19.63^2}{2} = 6{,}370$ lbs. $\quad a_4 = \dfrac{19.63}{3} = 6.54$

$\dfrac{y}{6.54} = \dfrac{6370}{18391} \qquad y = 2.27$ ft.

Toe distance to resultant = 6.39 - 2.27 = 4.12 ft.

e = 5.25 - 4.12 = 1.13 ft.

Sliding factoreof safety $= \dfrac{18391 \times .4}{6370} = 1.155$

Overturning factor of safety $= \dfrac{18391 \times 6.39}{6370 \times 6.54} = 2.82$

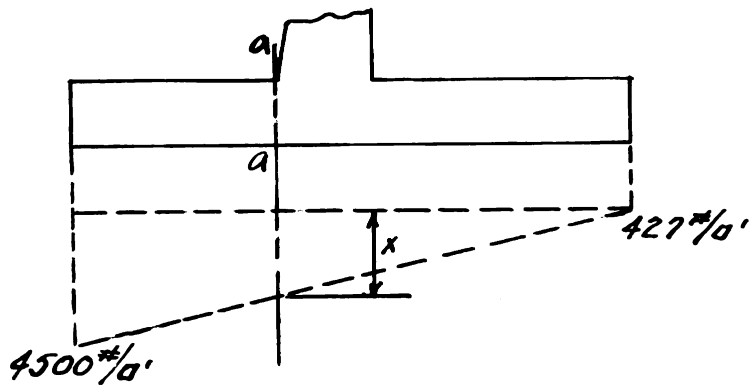
Earth pressures = p = $\dfrac{18391}{10.5}\left(1 \pm \dfrac{6 \times 1.13}{10.5}\right)$

= 2,940 lbs. per sq. ft.

= 564 lbs. per sq, ft.

## Design of Toe Slab

Maximum moment occurs under Case II




$4500 - 427 = 4073$ lbs per sq. ft.

$$\frac{x}{4073} = \frac{6.75}{10.5} = 2620 \text{ lbs. per sq. ft.}$$

$4500 - 2620 - 427 = 1453$ lbs. per sq. ft.

Wt. of concrete = 300 lbs. per sq. ft.

Moment about a - a = M

$$M = 3047 \times 3.75 \times \frac{3.75}{2} + \frac{1453 \times 375}{2} \times \frac{2 \times 3.75}{3}$$

$$- 300 \times 3.75 \times \frac{3.75}{2} = 26{,}150 \text{ ft. lbs.}$$

$$d = \sqrt{\frac{26{,}150 \times 12}{208 \times 12}} = 11.2 \text{ in. say } 11.5 \text{ in.}$$

Depth of toe slab = $11.5 + 3.5 = 15$ in.

Check shear -

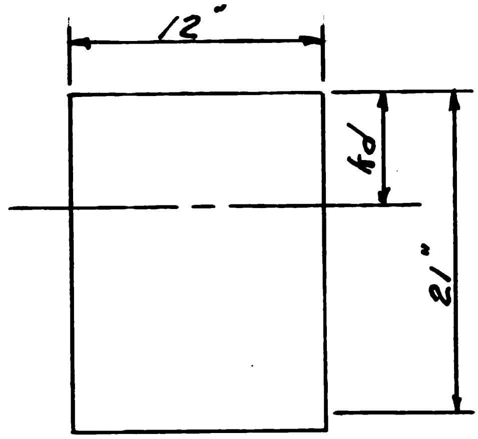
$$V = 3047 \times 3.75 + \frac{1453 \times 3.75}{2} - 300 \times 3.75 = 13015 \text{ lbs.}$$

$$v = \frac{V}{bjd} = \frac{13015}{12 \times .867 \times 11.5} = 109 \text{ lbs. per sq. in.}$$

$$d \text{ needed} = \frac{109}{60} \times 11.5 = 20.9 \text{ in. say } 21 \text{ in.}$$

Depth of toe slab = 24.5 in.

The 24 in. assumed checks.




$$C = \frac{1200}{2} \times 12 \times kd$$

$$M = 26{,}150 \times 12 = \frac{1200}{2} \times 12 \times kd \left(21 - \frac{kd}{3}\right)$$

$$2400\,(kd)^2 - 151{,}200\,kd + 313{,}800 = 0$$

$$kd = \frac{+151.2 \pm \sqrt{2{,}286{,}144 - 301{,}248}}{48}$$

$$= \frac{1512 - 1409}{48} = 2.15 \text{ in.}$$

$$jd = 20.283$$

$$T = C = A_s f_s \qquad j = .97$$

$$A_s = \frac{1200}{2} \times \frac{12 \times 2.15}{18{,}000} = .86 \text{ sq. in.}$$

Use 3/4 in. round bars @ 6 in.

$A_s$ = .88 sq. in.

Check Bond

$$\mu = \frac{13015}{4.71 \times .97 \times 21} = 135 \text{ lbs. per sq. in.}$$

allowable $\mu$ = 150 lbs. per sq. in.

## Design of Heel Slab

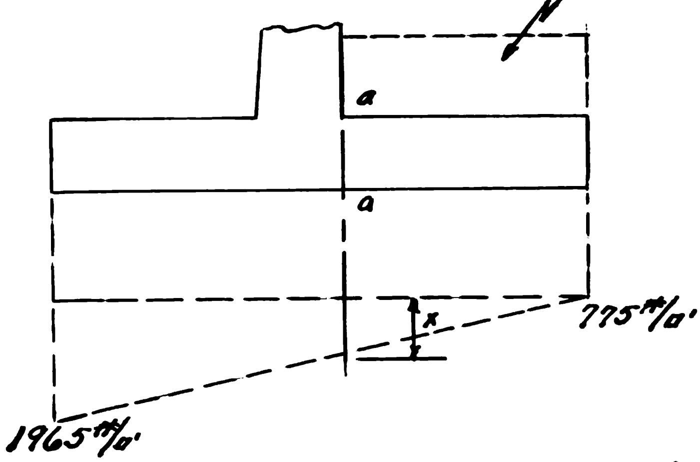
Determination of maximum moment.

Case I

15.75 x 1 x 1 = 1575 lbs. per sq. ft.

300

1875 lbs. per sq. ft.




1965 - 775 = 1190 lbs. per sq. ft.

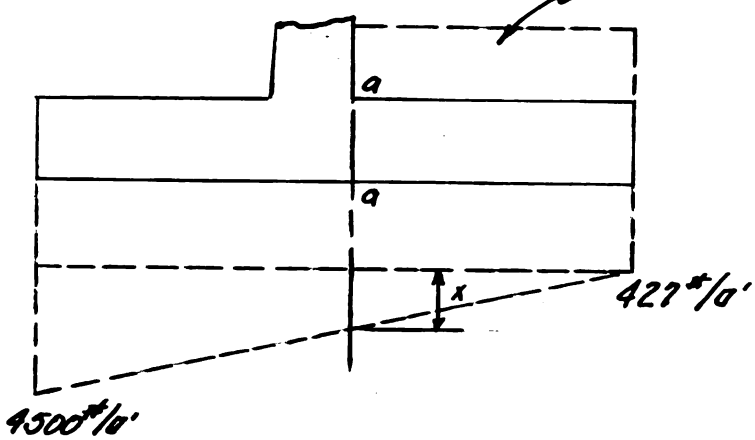
$$x = \frac{1190}{10.5} \times 5.25 = 595 \text{ lbs. per sq. ft.}$$

Moment at a - a = M

$$M = 1875 \times \frac{5.25^2}{2} - \frac{775 \times 5.25^2}{2} - \frac{595 \times 5.25}{2} \times \frac{5.25}{3}$$

$$= 12{,}410 \text{ ft. lbs.}$$

Case II




21.63 x 1 x 100 =
2163 lbs. per sq. ft
300
2463 lbs. per sq. ft.

4500 - 427 = 4073 lbs. per sq. ft.

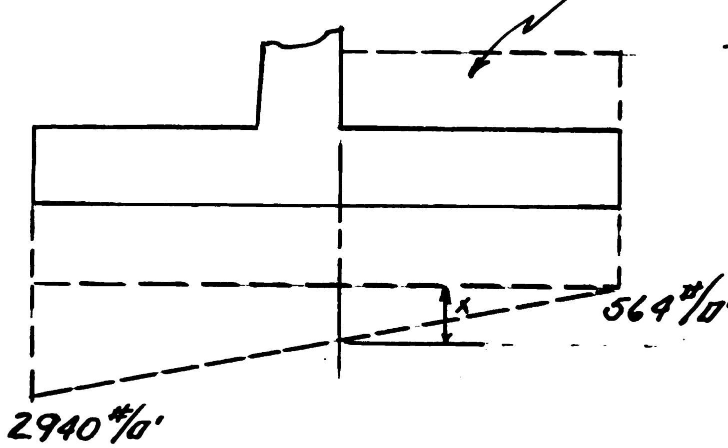
$$x = \frac{4073}{10.5} \times 5.25 = 2036 \text{ lbs. per sq. ft.}$$

Moment at a-a = M

$$M = \frac{2463 \times 5.25^2}{2} - \frac{427 \times 5.25^2}{2} - \frac{2036 \times 5.25}{2} \times \frac{5.25}{3}$$

M = 18,760 ft. lbs.

Case III




17.63 x 1 x 100 =
1763 lbs. per sq. ft.
300
2063 lbs. per sq. ft.

2940 - 564 = 2376 lbs. per sq. ft.

$$x = \frac{2376}{10.5} \times 5.25 = 1188 \text{ lbs. per sq. ft.}$$

Moment at a-a = M

$$M = \frac{2063 \times \overline{5.25}^2}{2} - \frac{564 \times \overline{5.25}^2}{2} - \frac{1188 \times 5.25}{2} \times \frac{5.25}{3}$$

$$= 15{,}170 \text{ ft. lbs.}$$

Maximum moment = 18,760 ft. lbs.

$$d = \sqrt{\frac{18760 \times 12}{208 \times 12}} = 9.5 \text{ in.}$$

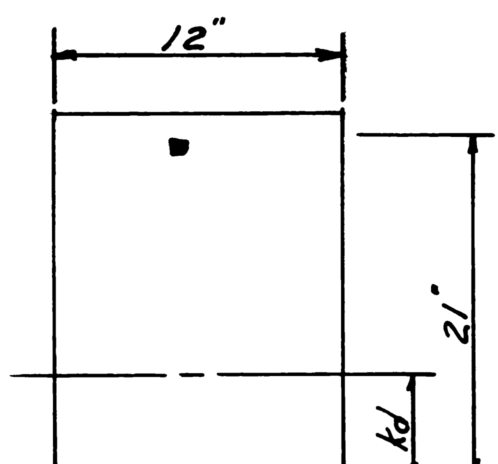
Depth of heel slab = 9.5 + 3.5 = 13 in.

Check Shear

$$V = 2463 \times 5.25 - 427 \times 5.25 - \frac{2036 \times 5.25}{2} = 5370 \text{ lbs.}$$

$$v = \frac{5370}{12 \times .867 \times 9.5} = 54.4 \text{ lbs. per sq. in.}$$

Use the same effective depth for the heel slab as was used for the toe slab - 21 in.




$$C = \frac{1200}{2} \times 12 \times kd$$

$$M = 18760 \times 12 = \frac{1200}{2} \times 12 \times kd\ (21 - \frac{kd}{3})$$

$$2400(kd)^2 - 151200\ kd + 225120 = 0$$

$$kd = \frac{15120 \pm \sqrt{228614400 - 21,611,520}}{480} = 1.53 \text{ in.}$$

$$T = C = A_s\ f_s$$

$$A_s = \frac{1200}{2} \times \frac{12 \times 1.53}{18000} = .613 \text{ sq. in.}$$

Use 5/8 round @ 5½ in. $A_s$ = .67 sq. in.

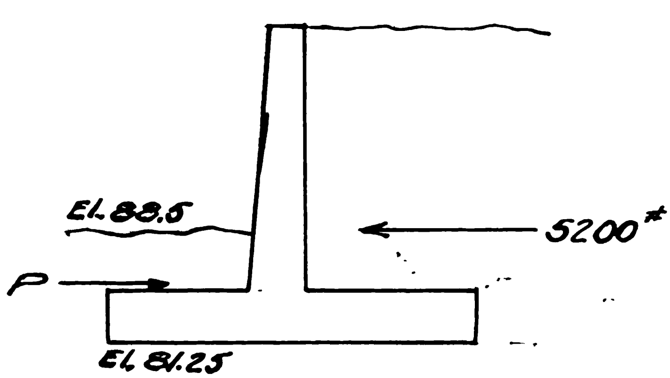
$jd = 20.49$
$j = .97$

Check Bond

$$\mu = \frac{5370}{1.96 \times \frac{12}{5.5} \times .97 \times 21} = 61.7 \text{ lbs. per sq. in.}$$

allowable $\mu$ = 150 lbs. per sq. in.

Check factor of safety against sliding




Factor of safety against sliding is smallest in Case I

f = 1.105.

passive $C_e = \frac{1}{.33} = 3$ to be safe use $C_e = 2$

$$p = 2 \times \frac{100 \times \overline{7.25}^2}{2} = 5260 \text{ lbs.}$$

$$Wt. = 8270 + 2951 + 5.75 \times 5.25 \times 100 + 3075 = 16266 \text{ lbs.}$$

$$f = \frac{16266 \times .4 + 5260}{5200} = 2.26$$

II

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