AN ANALYSIS OF A PART OF A CONTINUOUS CONCRETE BRIDGE

Thesis for the Degree of B. S.

MICHIGAN STATE COLLEGE

Takashi Nakamura

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An Analysis of a Part of a Continuous Concrete Bridge

A Thesis Submitted to
The Faculty of
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of
AGRICULTURE AND APPLIED SCIENCE
by

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Candidate for the Degree of
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PREFACE

The purpose of this thesis is to put into practice some of the theory learned by the author in a course in indeterminate structures. The method used here is neither the best nor the easiest, but it makes possible the application of the fundamental principles necessary for the design of a continuous concrete bridge.

The drawing of influence line diagrams makes up the major part of this thesis. These diagrams can be drawn with the aid of a handbook of frame constants or the Portland Cement Association publication entitled "Continuous Concrete Bridges." The author used the method of moment distribution chiefly because of his lack of foundation to thoroughly understand and appreciate the other shortcut methods available.

The author would like to take this opportunity to express his deep appreciation to Professor William A. Bradley, of the C.E. Dept. of Mich State College whose kind and patient assistance made this thesis possible. Thanks also go to Mr. Rutter of the Mich. SHD who generously gave the author the plans for the bridge; Mr Murray, also of the Mich SHD, who made possible the comparison of this analysis with the actual design; and last but not least to Professor C L Allen, head of the C.E. Dept of Mich. State College, whose constructive criticism helped in the organization of this thesis.

T N

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LOADING SPECIFICATIONS
(A.A.S.H.O.-44)

3.2.1 - Loads.

Structures shall be proportioned for the following loads where they exist:

(a) dead load.

(b) live load.

(c) impact or dynamic effect of live load.

(d) wind loads.

(e) other forces, when they exist.

3.2.2 - Dead Load.

The dead load shall consist of the weight of the structure complete, including the roadway, sidewalks, and - - -

3.2.3. - Live Load.

The live load shall consist of the weight of the applied moving loads of vehicles, cars and pedestrians.

3.2.4 - Overload Provision.

The following provision for overload shall apply to all loadings except the H 20 and H 20-S16 loadings.

- -

3.2.5 - Highway Loadings.

(see loading diagram for loading used)

3.2.8 - (a) - Loading for Maximum Stresses.

The type of loading, whether lane loading or truck loading, to be used, and whether the spans be simple or continuous, shall be the loading which produces the maximum stress.

- -

For continuous spans, the lane loading shall be continuous or discontinuous, as may be necessary to produce maximum stresses, and the concentrated load or loads as specified in paragraph c shall be placed in such position as to produce maximum stresses.

(paragraph c applies to H loading only)

3.2.9 - Reduction in Load Intensity.

Where maximum stresses are produced in any member by loading any number of traffic lanes simultaneously, the following percentages of the resultant live load stresses shall be used in view of improbable coincident maximum loading:

- -

Four lanes or more - - - - - - - - - - - - 75 %

3.2.11.-(a) - Sidewalk Loading.

Sidewalk floors, stringers, and their immediate supports, shall be designed for a live load of 85 pounds per square foot of sidewalk area. - - -

3.2.12. - Impact.

$I = \frac{50}{L+125}$ (maximum 30 percent)

I = impact fraction (to be mult. by total L.L. stress, and this increment added to the total stress.)

L = length of the shortest span.

3.2.13. - Longitudinal Forces.

(not considered in design by Mich State Highway Dept.)

3.2.14. - Wind Loads.

(not considered in design by Mich State Highway Dept.)

DISTRIBUTION OF LOADS

3.3.2 - Distribution of Loads and Design of Concrete Slabs.

(a) Bending Moment.

Case C - Main reinforcement Parallel to Traffic

Span over 12 feet

Wheel loads: $E = \frac{10N + W}{2N}$

E = width of slab over which a wheel load is dist.

N = max. number of lanes of traffic permissible.

W = width of roadway between curbs on bridge.

Brief Outline of Superstructure Slab Analysis

I- Loading to obtain maximum effects.

A- Influence line diagrams.

1- These diagrams show the effect upon a particular section of the slab as a unit load is passed across the structure.

2- The method of moment distribution is used to obtain the effect of the unit load at the various sections.

a- In obtaining the carry over factors, a foot width of slab is used. The moment of inertia is calculated by using the cross section dimensions of the foot width of slab.

3- The reactions upon the slab as the load crosses the structure are found, and the various points of the influence line diagram (moment or shear) are obtained by using these reactions.

B- Loading the influence line diagrams

1- Having drawn the influence line diagrams for the various critical sections, the max. moment or shear for each section is found by loading the diagram to obtain the max effect. Both truck train and lane loading are used, and whichever gives the max. is taken as the controlling moment or shear.

a- The max effect by truck train method is obtained by moving the truck train over the diagram until the max condition is obtained (following AASHO specs).

II - Analysis of slab design

A - Maximum total moment diagram

1 - After the max moment on each critical section is found and [illegible] (see page [illegible]), a max. total moment diagram is drawn, from which the steel cut-off can be determined. (see pages 24 & [illegible]).

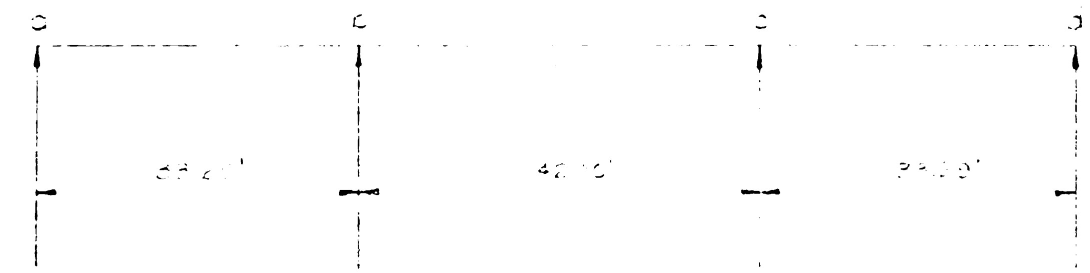
B - Checking slab at critical sections

1 - The slab is checked for [illegible] and negative moments (see pages [illegible]).

2 - Shearing and bond stresses [illegible] at the critical sections (see page [illegible]).

MOMENT DISTRIBUTION FOR INFLUENCE LINE DIAGRAM

(ANALYSIS OF SLAB DESIGN)



[illegible]

$I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(17)^3$

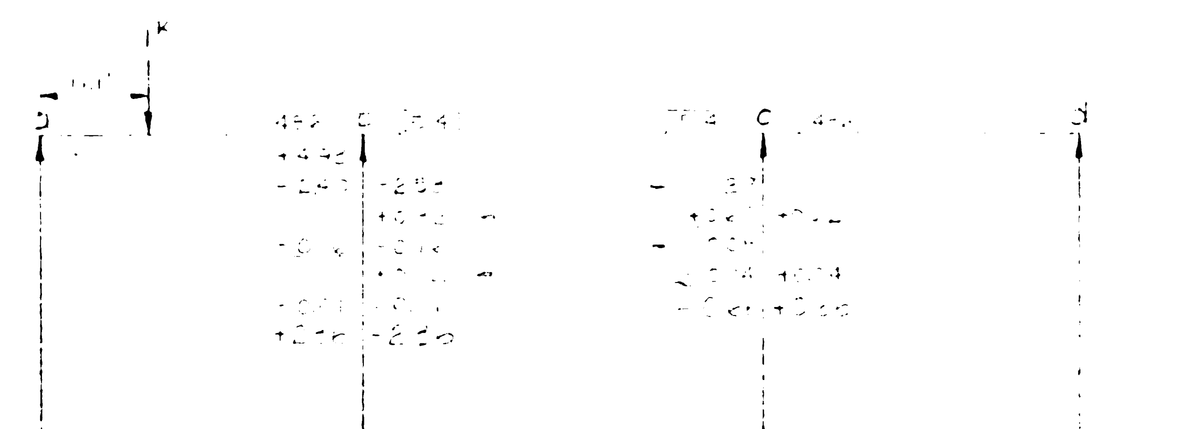
$= 4913 \text{ in}^4$

$CK_{ba} = \frac{EI}{L} = \frac{E\,4913}{[illegible]} = 442$; $\Sigma CK = 9[illegible]$; $r_{ba} = .482$

$CK_{bc} = \frac{EI}{L} = \frac{4913}{[illegible]} = 475$; $r_{bc} = .518$

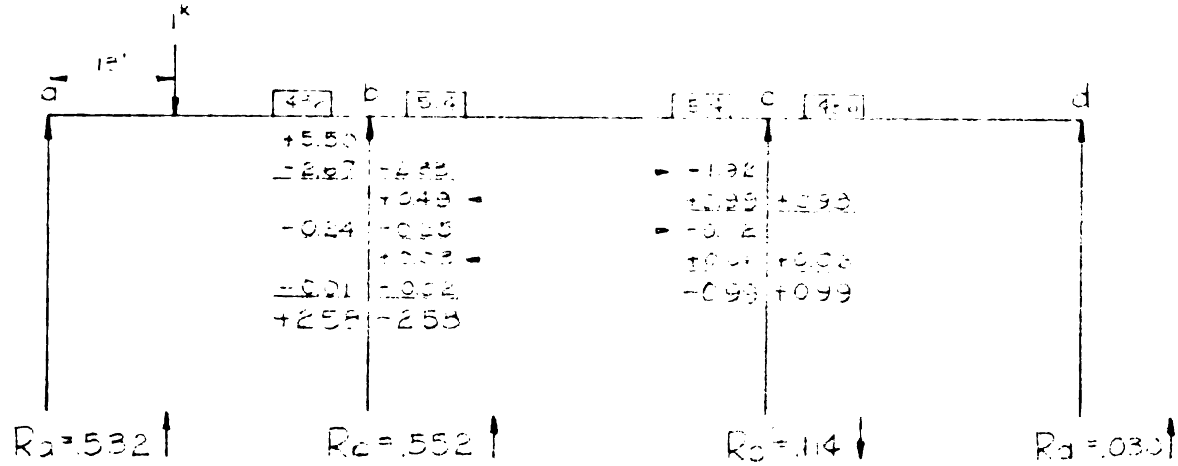
$CK_{cb} = 475$; $r_{cb} = .514$

$CK_{cd} = 442$; $r_{cd} = .486$

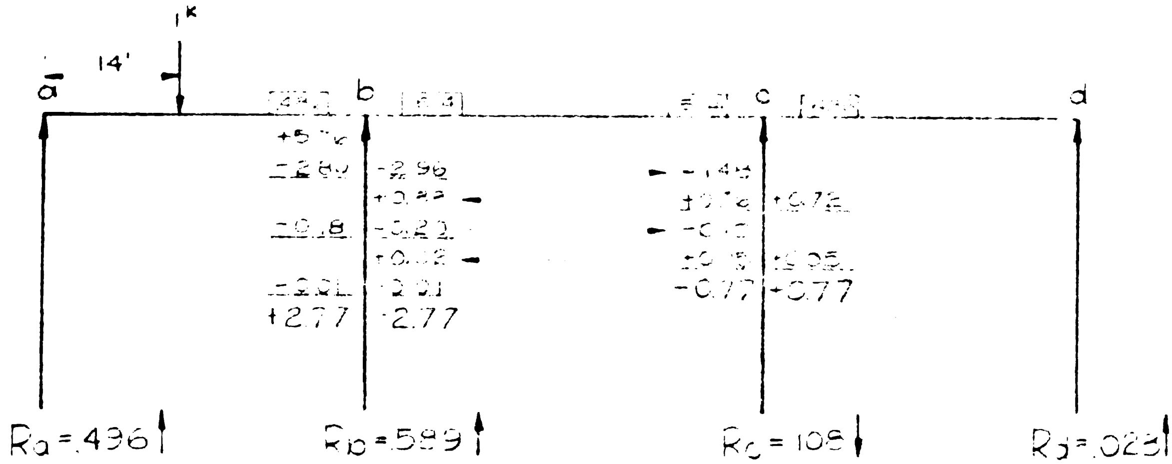


$R_a = .596$ $R_b = .475$ $R_c = [illegible]$ $R_d = .020$

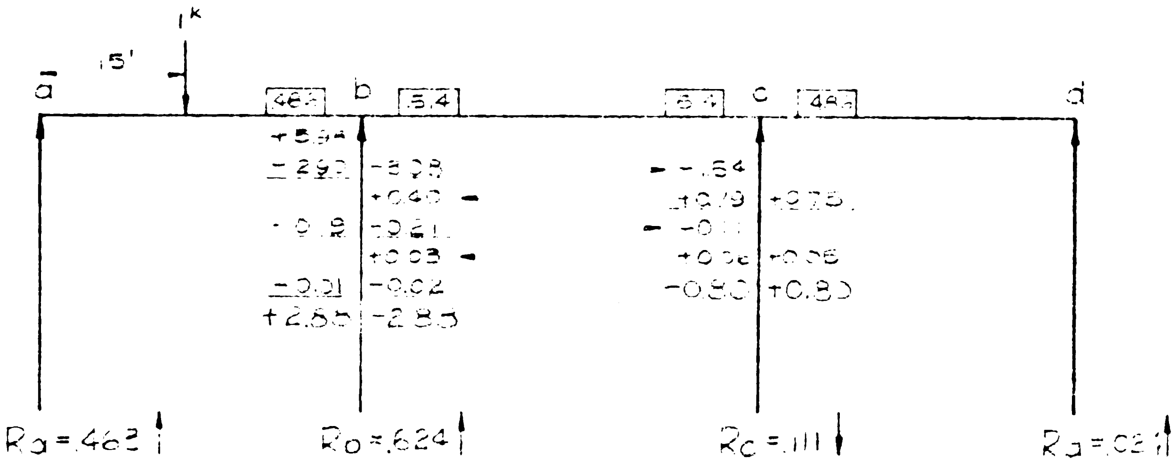
$M_{ba} = \frac{Pab}{L^2}(a+L) = [illegible] = 4.3 \text{ K-ft}$



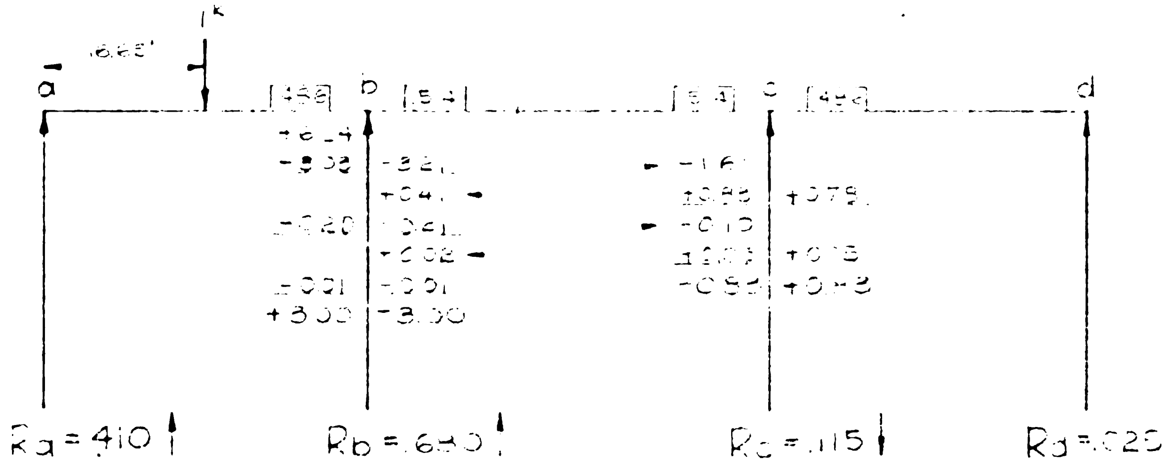
$$M_{ba} = \frac{(13)(20.3)}{2(33.3)^2}(13+33.3) = 5.50^{k'}$$



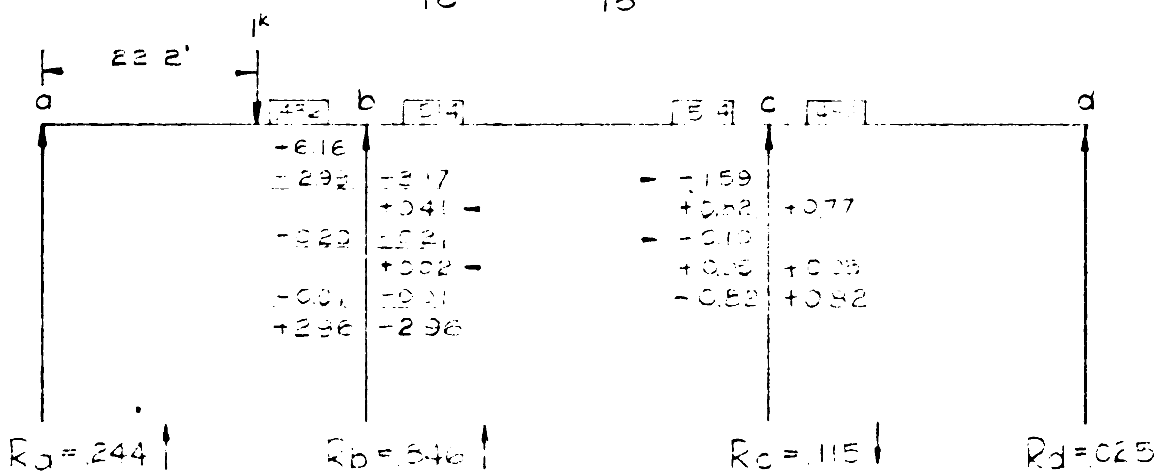
$$M_{ba} = \frac{(14)(19.3)}{2(33.3)^2}(14+33.3) = 5.76^{k'}$$



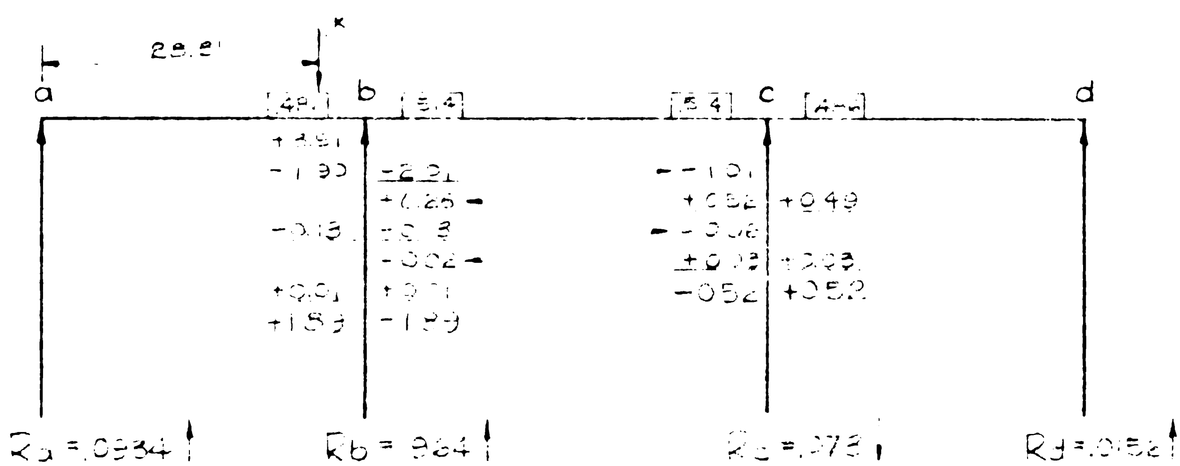
$$M_{ba} = \frac{(15)(18.3)}{2(33.3)^2}(15+33.3) = 5.98^{k'}$$



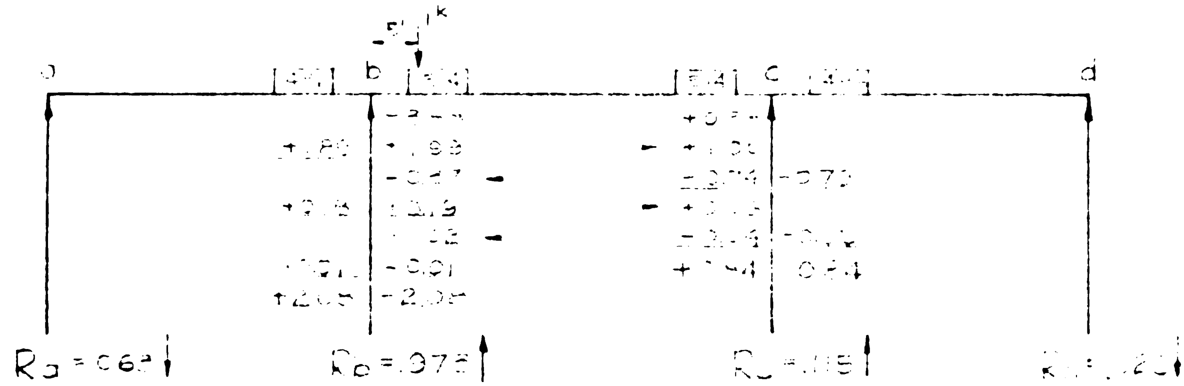
$$M_{bc} = \frac{3PL}{16} = \frac{(3)(33.3)}{16} = 6.24^{k\#}$$



$$M_{bc} = \frac{(22.2)(11.1)}{2(33.3)^2}(22.2 + 33.3) = 6.16^{k\#}$$

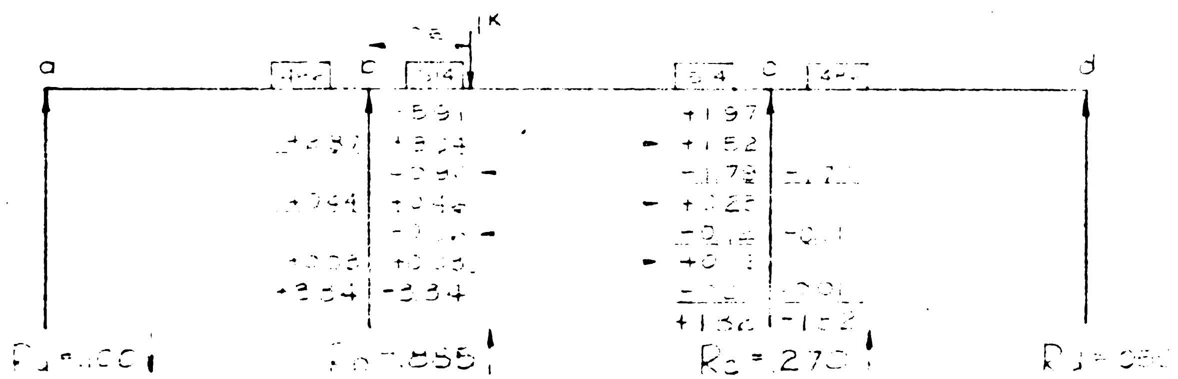


$$M_{ba} = \frac{(28.3)(5)}{2(33.3)^2}(28.3 + 33.3) = 3.91^{k\#}$$



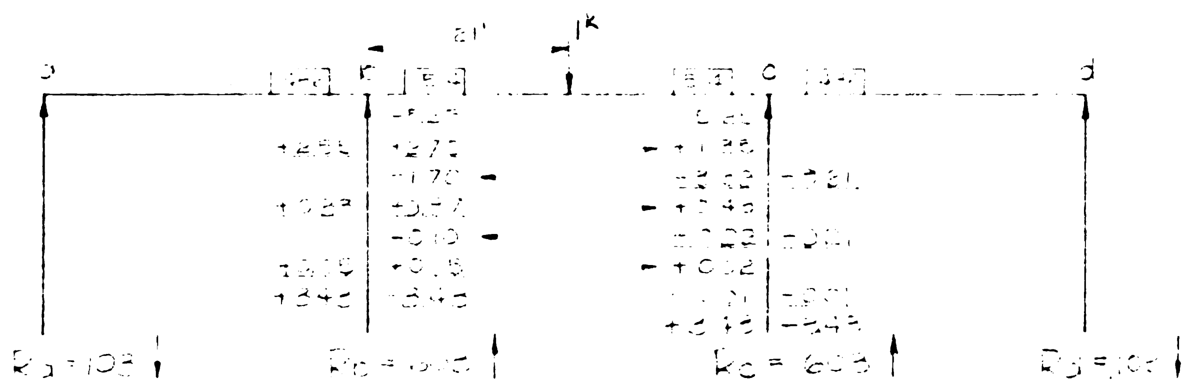
$$M_{bc} = \frac{Pab^2}{L^2} = \frac{(5)(37)^2}{(42)^2} = -3.88^{k'}$$

$$M_{cb} = \frac{(5)^2(37)}{(42)^2} = +.520^{k'}$$



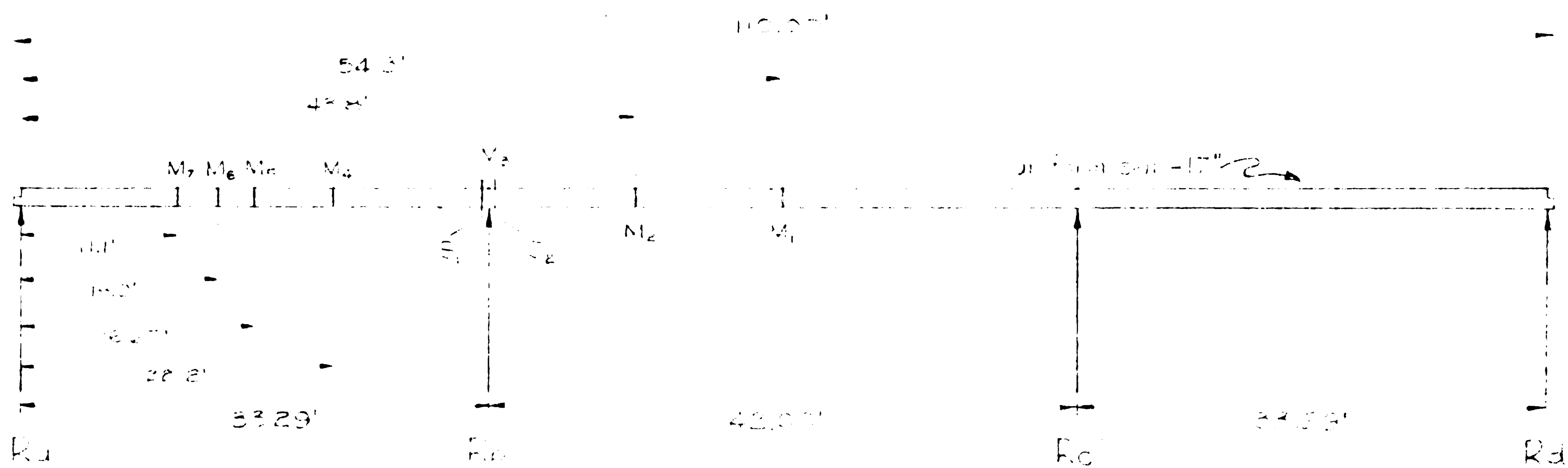
$$M_{bc} = \frac{(10.5)(31.5)^2}{(42)^2} = 5.91^{k'}$$

$$M_{cb} = \frac{(10.5)^2(31.5)}{(42)^2} = 1.97^{k'}$$



$$M_{bc} = M_{cb} = \frac{PL}{8} = \frac{42}{8} = 5.25^{k'}$$

INFLUENCE LINE DIAGRAMS FOR SLAB ANALYSIS

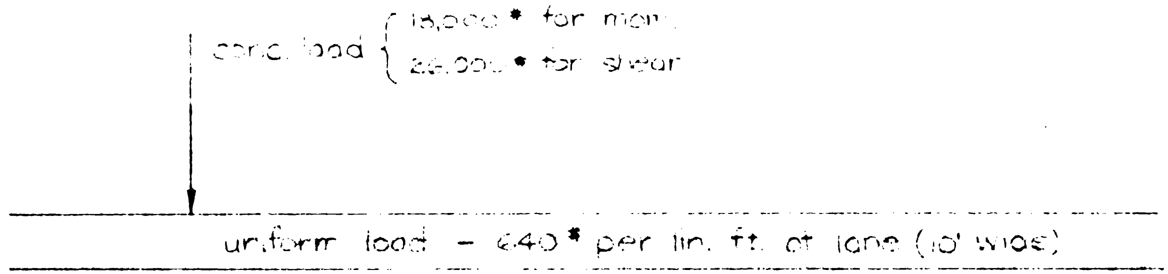


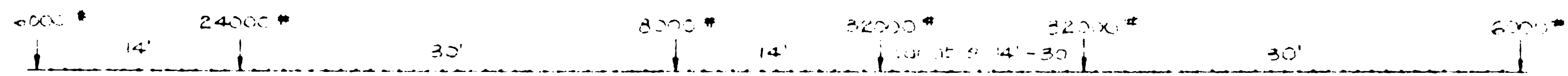
Sections for which influence lines are drawn

Scale: 1" = 12'-0"

H 20 - S 16 - 44

Maximum stress due to either lane or truck loading controls.





Max. moments:

By lane loading:

$+(122.5 \times .64 + 7.07 \times 18) = +205.6^{k\#}$

$-(25.3 \times .64 \times 2 + 1.07 \times 18) = -51.7^{k\#}$

By truck-train loading:
(axial)

$\left.\begin{matrix} \underline{+29.1}^{k\#} \\ \underline{-11.1}^{k\#} \end{matrix}\right\}$ control

Design moments:

D.L./ft width of slab $= 150(\frac{17}{12})(1)$

$= .212^{k}/\text{ft of length}$

D.L. moment $= (122.5 - 50.6)\,.212$

$= \underline{+15.2}^{k\#}/\text{ft width-slab}$

* L.L. moment:

$+\frac{29.1^{k\#}}{(2)(5.25)} = +27.7^{k\#}/\text{ft width-slab}$

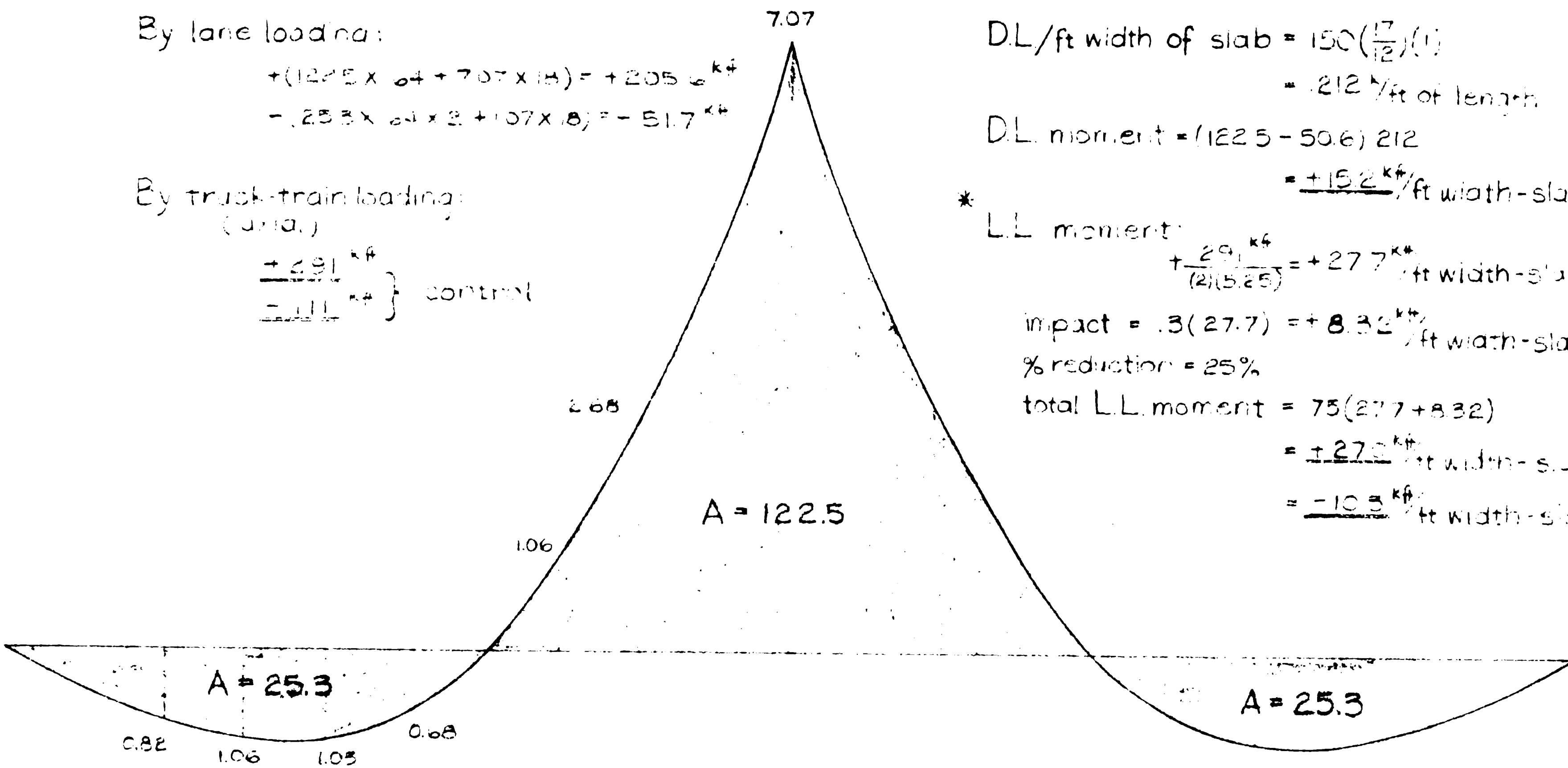
impact $= .3(27.7) = +8.32^{k\#}/\text{ft width-slab}$

% reduction = 25%

total L.L. moment $= 75(27.7 + 8.32)$

$= \underline{+27.0}^{k\#}/\text{ft width-slab}$

$= \underline{-10.5}^{k\#}/\text{ft width-slab}$



M_1

Scales:
Horizontal – 1" = 10'-0"
Vertical – 1" = 2.0

* see computation sheet for wheel load distribution (page 13)

Max moment:

By truck-train loading:

$\left.\begin{array}{l} \underline{+196^{kft}} \\ \underline{-110.4^{kft}} \end{array}\right\}$ control

Design moments/ft width - slab

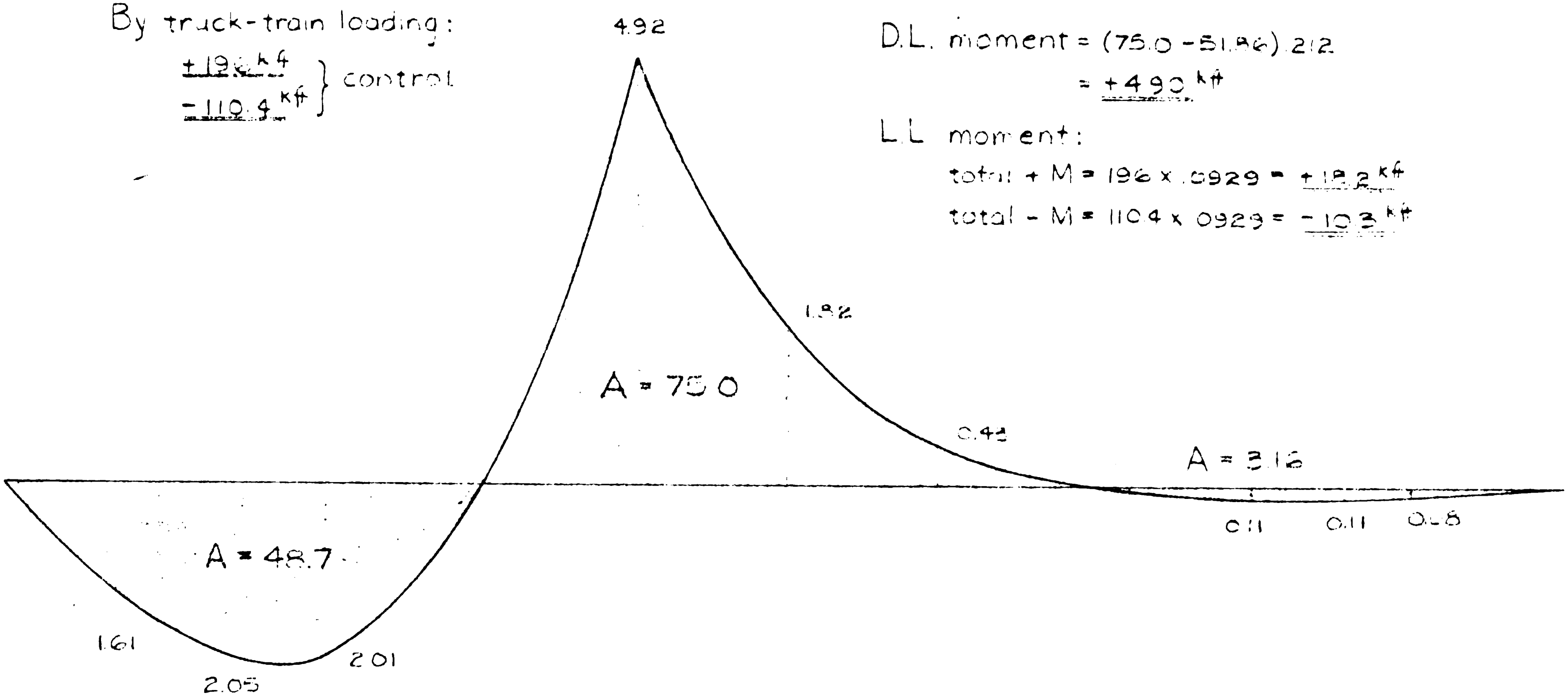
D.L. moment = (75.0 − 51.86) 2.12

= $\underline{+49.0^{kft}}$

L.L moment:

total + M = 196 x .0929 = $\underline{+18.2^{kft}}$

total − M = 110.4 x .0929 = $\underline{-10.3^{kft}}$



M_2

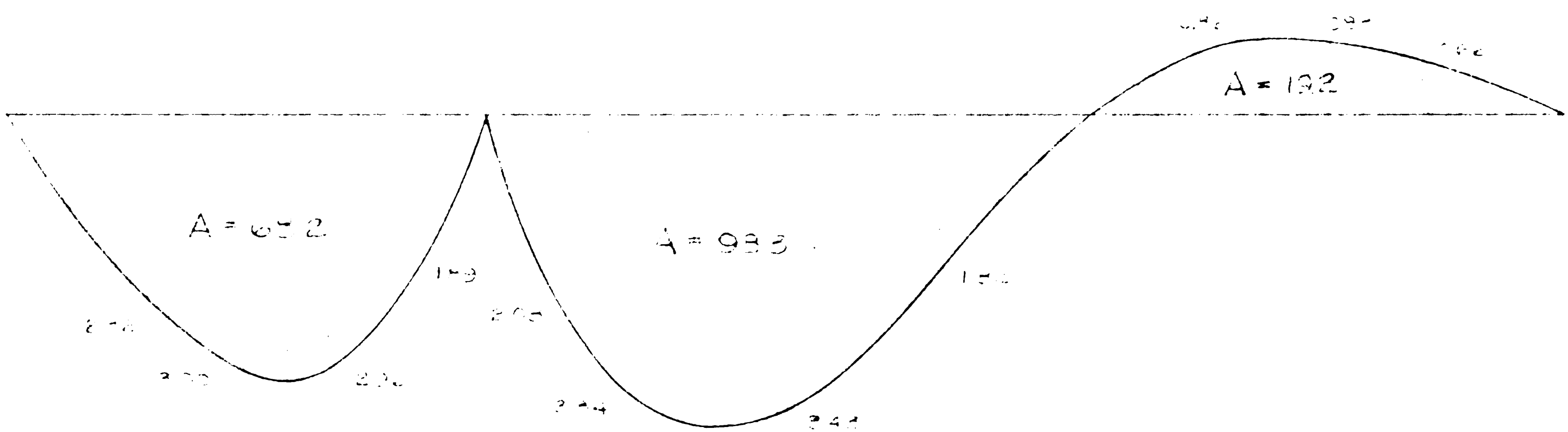
scale:
horizontal − 1" = 12'-0"
vertical − 1" = 2.0

Max. moments

Design moments

D.L. moment = −(... − 192, 2.8 = ...

L.L. moment:



M_3

Scale:
Horizontal – 1" = 8'-0"
Vertical

Max moment:

By truck-train loading:

$\left.\begin{array}{l} \underline{+252.1^{k\#}} \\ \underline{-13.8^{k\#}} \end{array}\right\}$ control

Design moments/ft width - slab

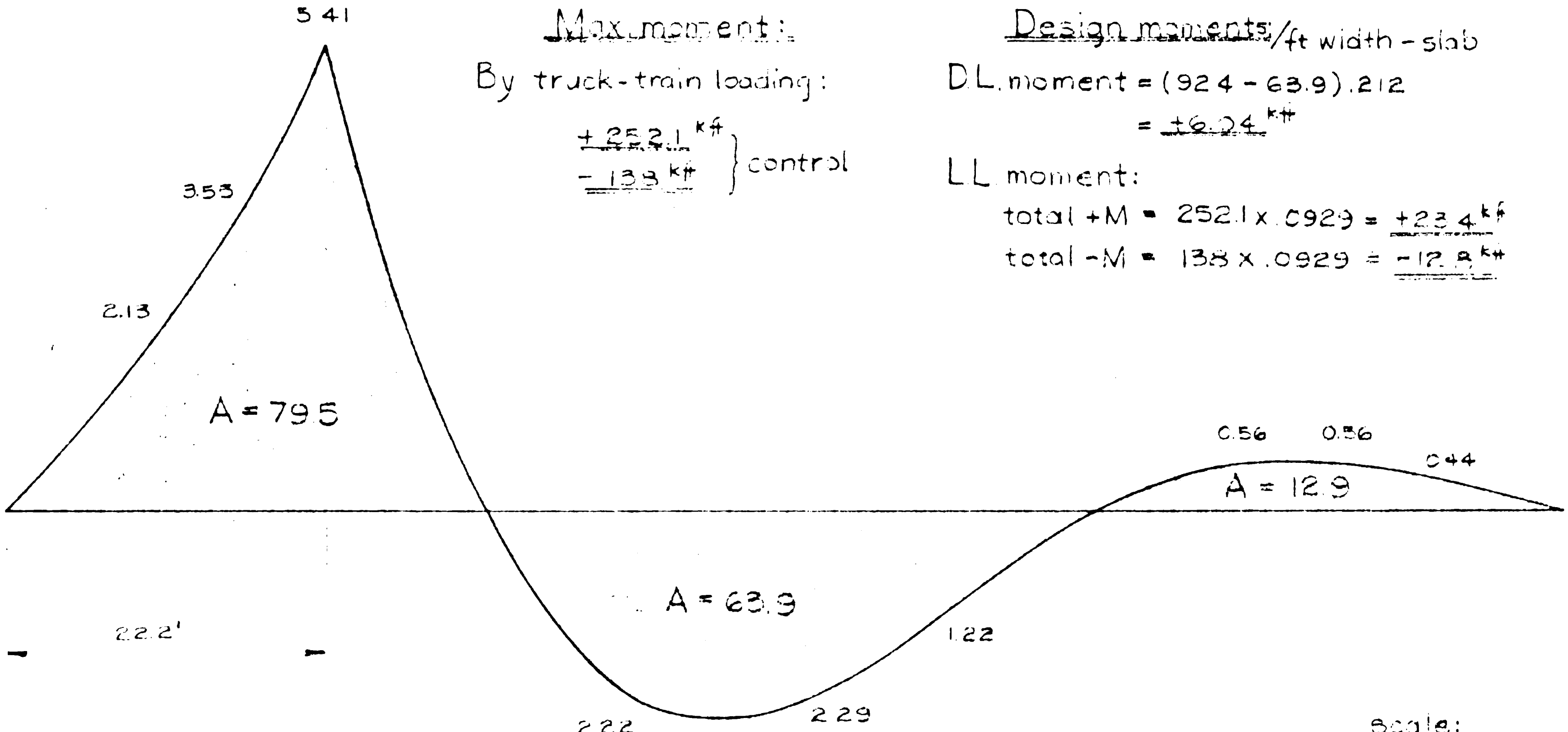
D.L. moment = (92.4 - 63.9) .212

= $\underline{+6.04^{k\#}}$

L.L moment:

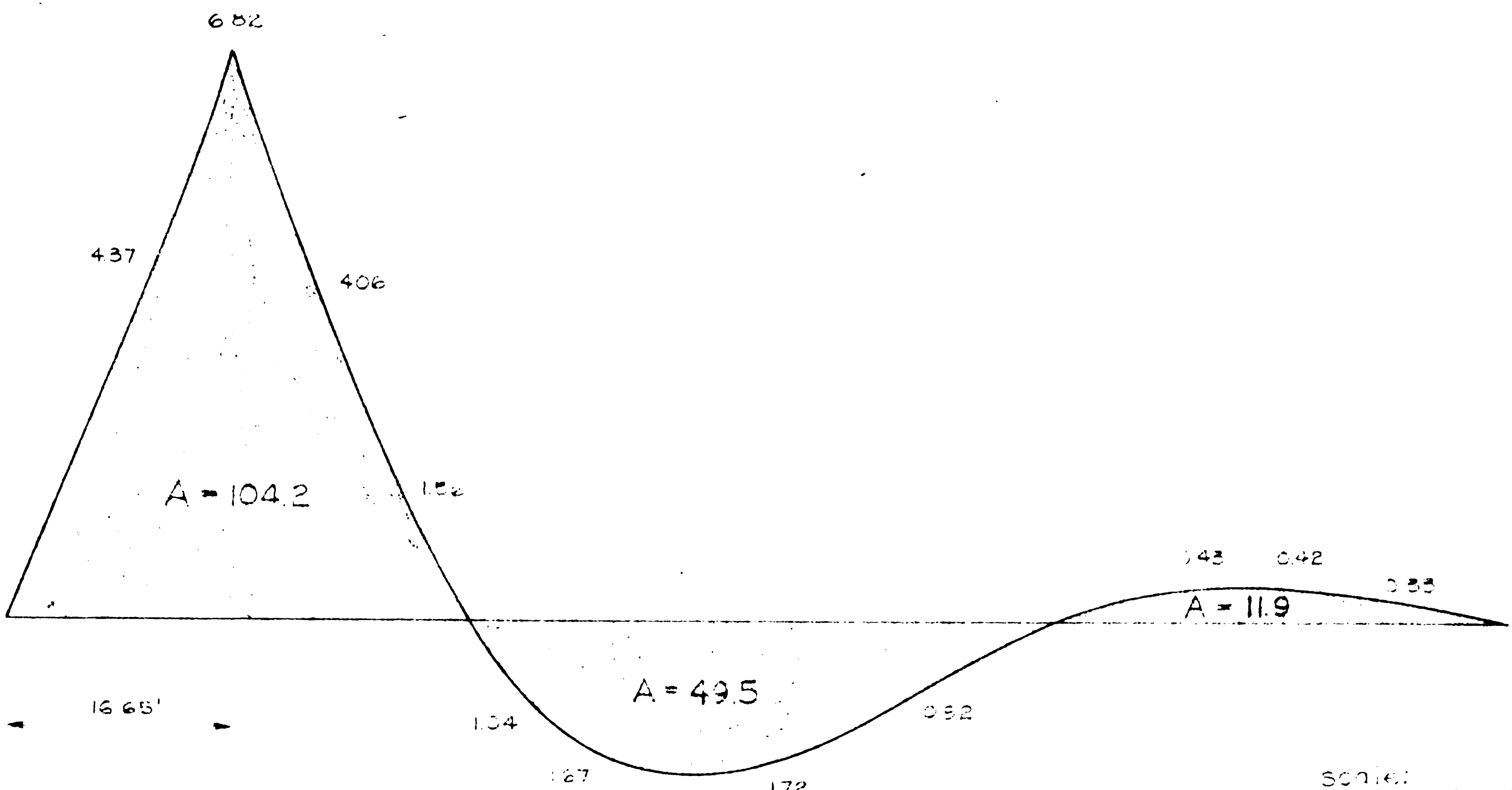
total +M = 252.1 x .0929 = $\underline{+23.4^{k\#}}$

total -M = 138 x .0929 = $\underline{-12.8^{k\#}}$



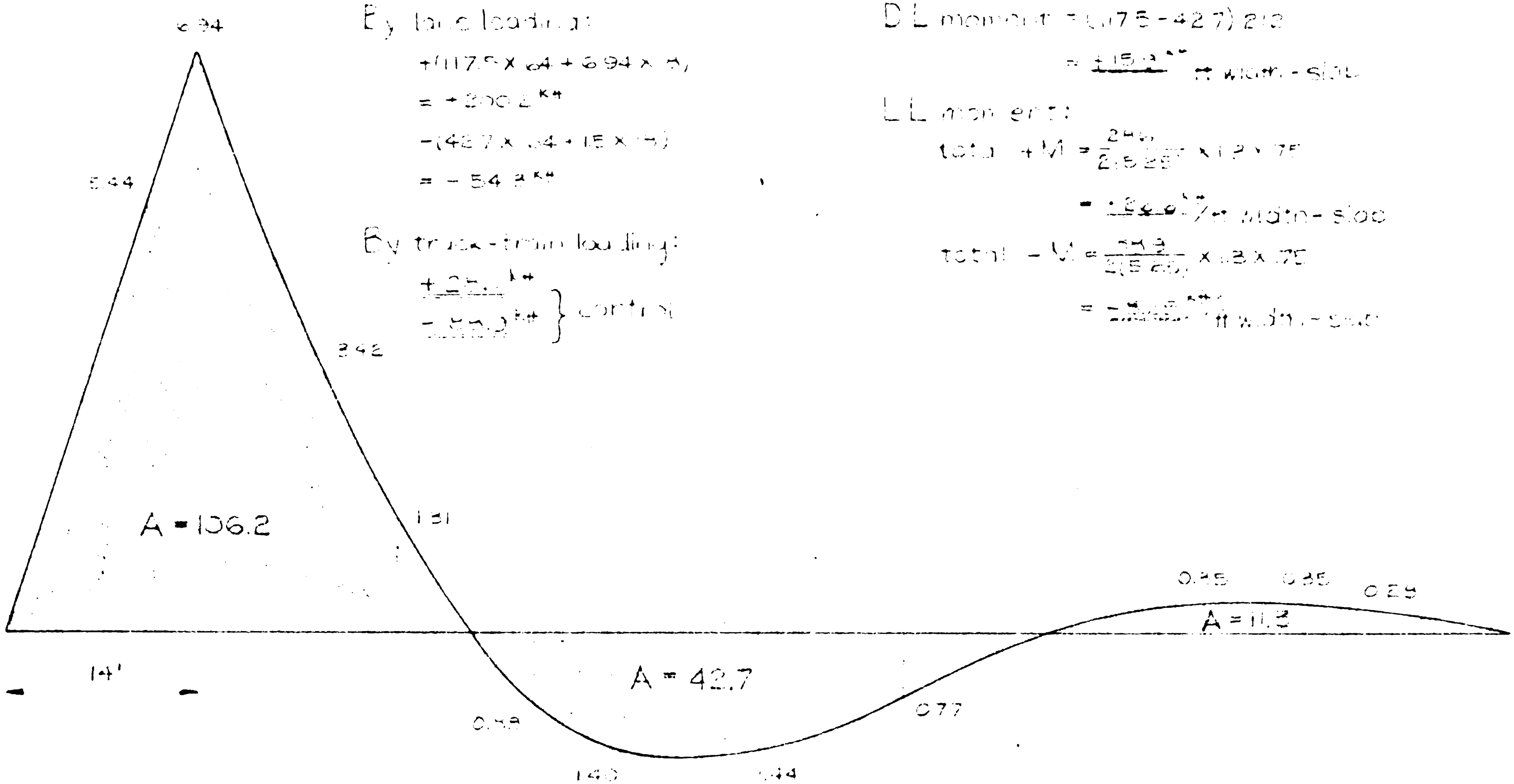
M_4

scale:
horizontal - 1" = 12'-0"
vertical - 1" = 2.0



M_5

scale:
horizontal – 1" = 12'-0"
vertical – 1" = 2.0



M_6

Scale:
horizontal – 1" = 12'-0"
vertical – 1" = 20

<u>Max moments</u>

By lane loading:

+(117.5 x .64 + 6.94 x 5)

= +200.2 k#

−(42.7 x .64 + 15 x 5)

= −54.3 k#

By truck-train loading:

+ 2[illegible] k# } control
− 55.2 k#

<u>Design moments.</u>

DL moment = (.175 − .427) 212

= ±15.2 k# / ft width-slab

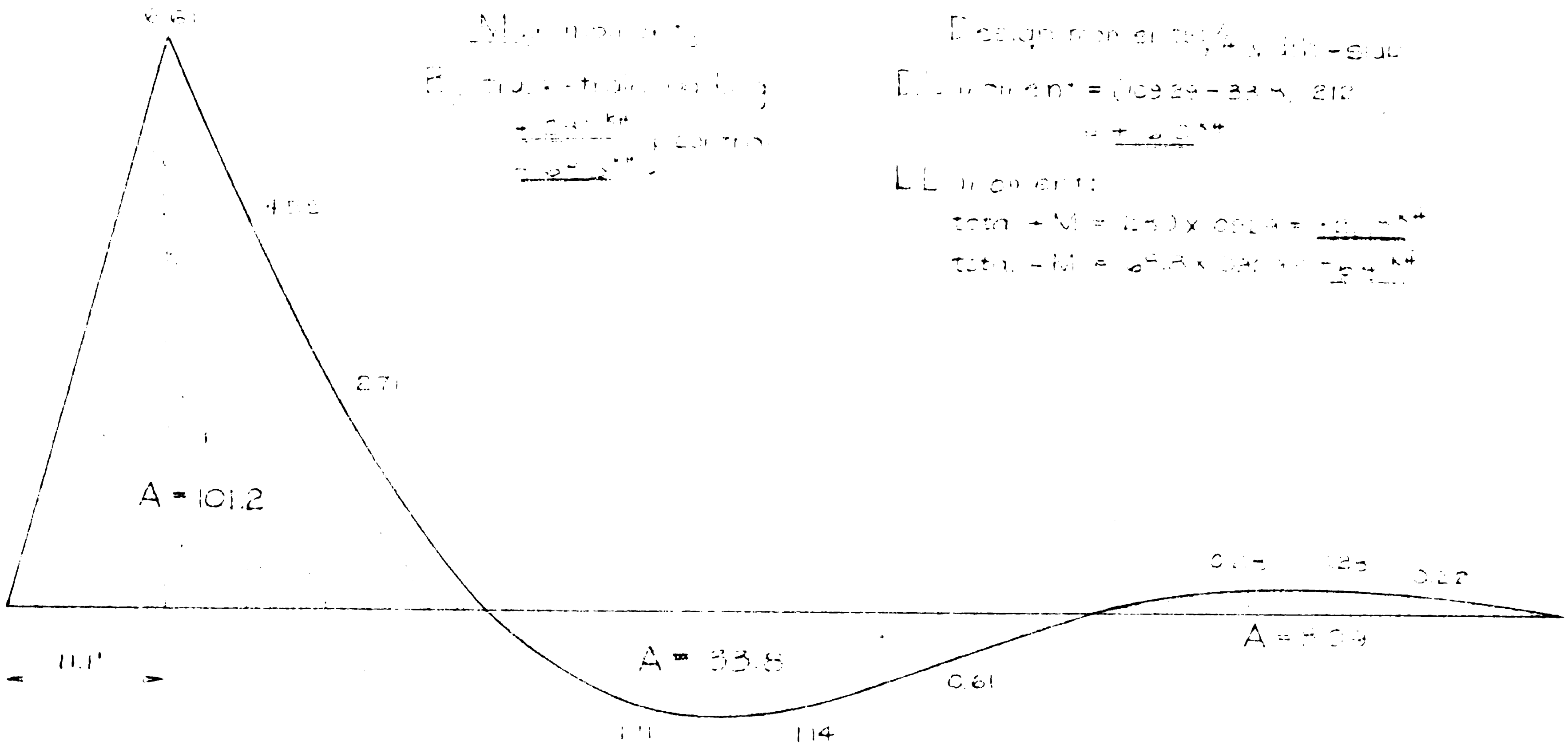
LL moment:

total +M = $\frac{2[illegible]}{2(5.25)}$ x 1.3 x .75

= [illegible] k# / ft width-slab

total −M = $\frac{55.2}{2(5.25)}$ x 1.3 x .75

= [illegible] k# / ft width-slab



M [illegible] moment [illegible]

B [illegible] from [illegible] ([illegible])

[illegible]

Design moment at [illegible] th - span

DL moment = (109.29 - 33.8) 212

= + [illegible]

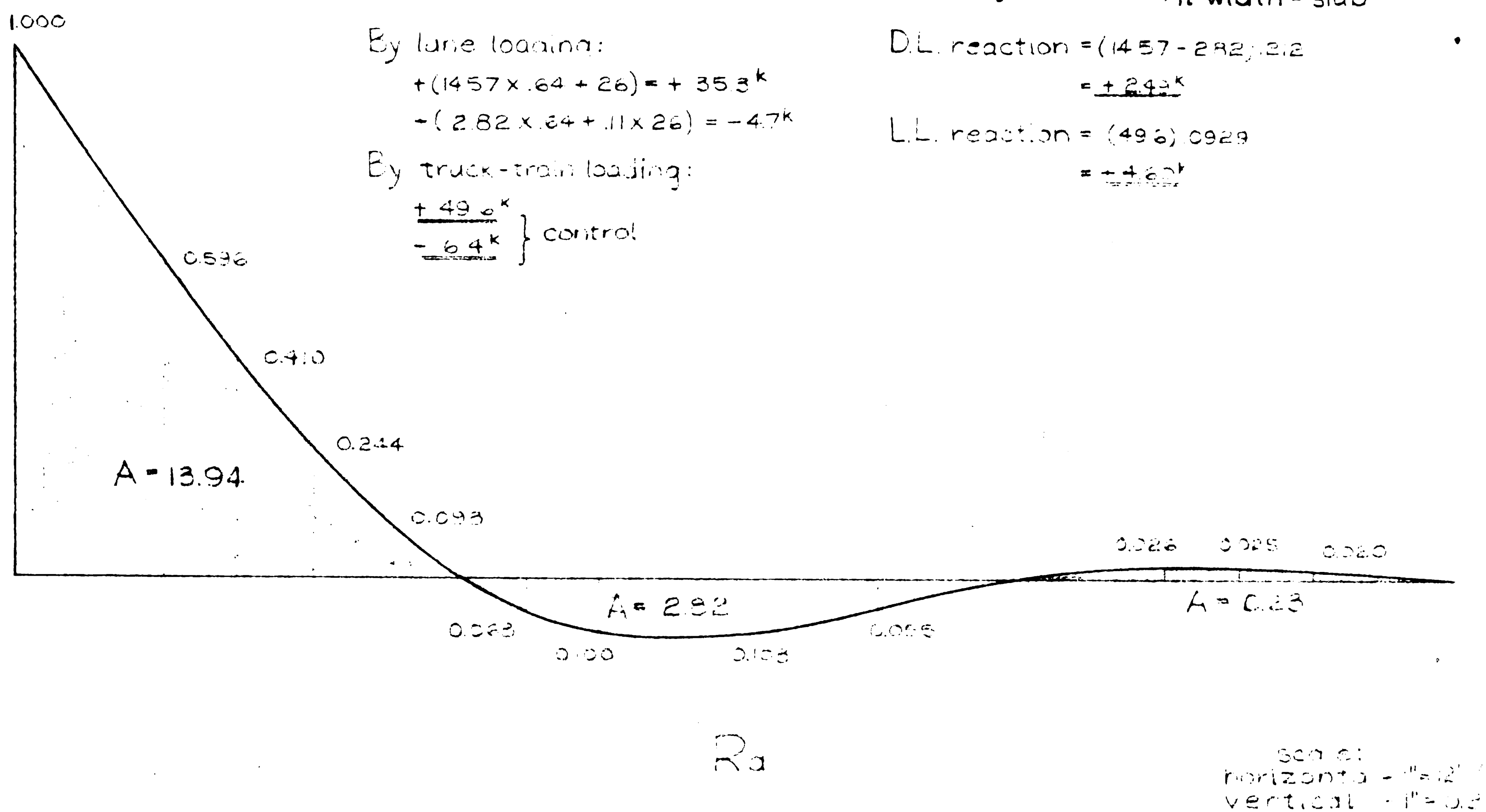
LL moment:

total + M = [illegible] x [illegible] = [illegible]

total - M = [illegible] x [illegible] = [illegible]

M_7

scale:
horizontal - 1" = 12'-0"
vertical - 1" = 2.0



__Max reaction:__

By lane loading:

$+(14.57 \times .64 + 26) = +35.3^k$

$-(2.82 \times .64 + .11 \times 26) = -4.7^k$

By truck-train loading:

$\left.\begin{array}{l} \underline{+49.6^k} \\ \underline{-6.4^k} \end{array}\right\}$ control

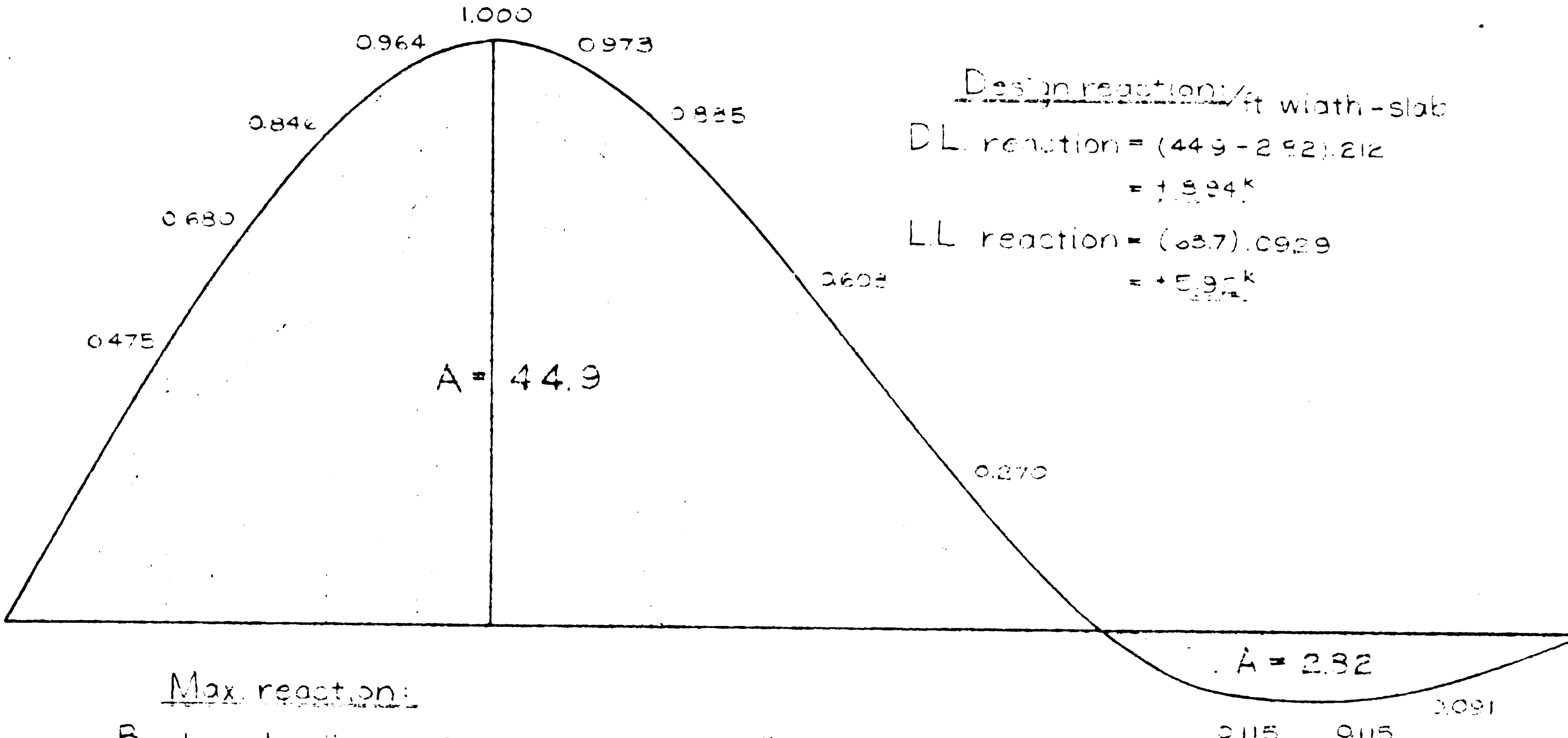
__Design reaction/ft width - slab__

D.L. reaction $= (14.57 - 2.82)\,2.12$

$= \underline{+24.9^k}$

L.L. reaction $= (49.6)\,.0929$

$= \underline{+4.61^k}$



Design reaction/ft width-slab

D.L. reaction = (44.9 − 2.82).212

= + 8.94^k

L.L. reaction = (63.7).0929

= + 5.9[illegible]^k

Max. reaction:

By lane loading: + (44.9 x .64 + 26) = + 54.8^k

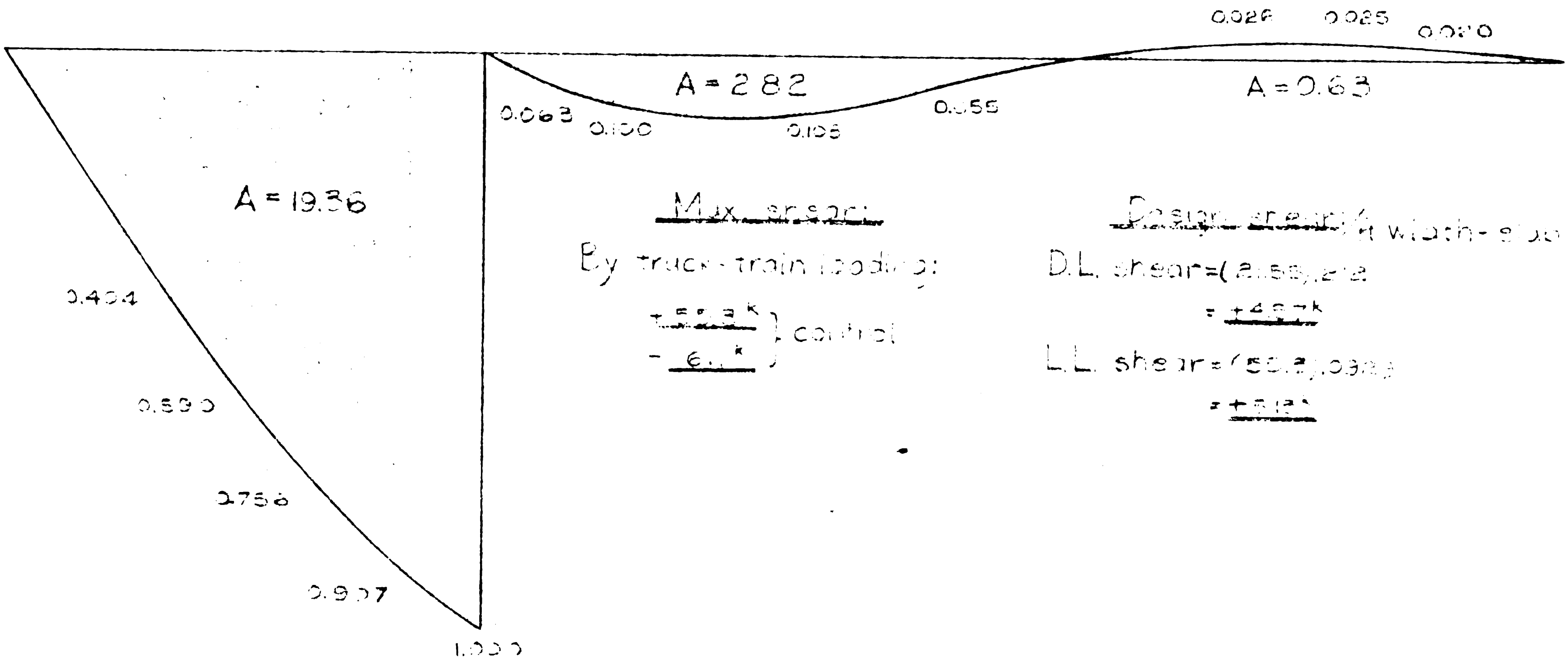
− (2.82 x .64 + .116 x 26) = − 4.8^k

By truck-train loading: + 63.7^k } control

− 6.4^k

R_b

scale:
horizontal − 1" = 12'-0"
vertical − 1" = 0.3



S_1

Scale
horizontal - 1" = 12'-0"
vertical - 1" = 0.3

Max shear:

By truck-trac loading:

$\left.\begin{array}{l} \underline{+56.4^k} \\ \underline{-4.3^k} \end{array}\right\}$ control

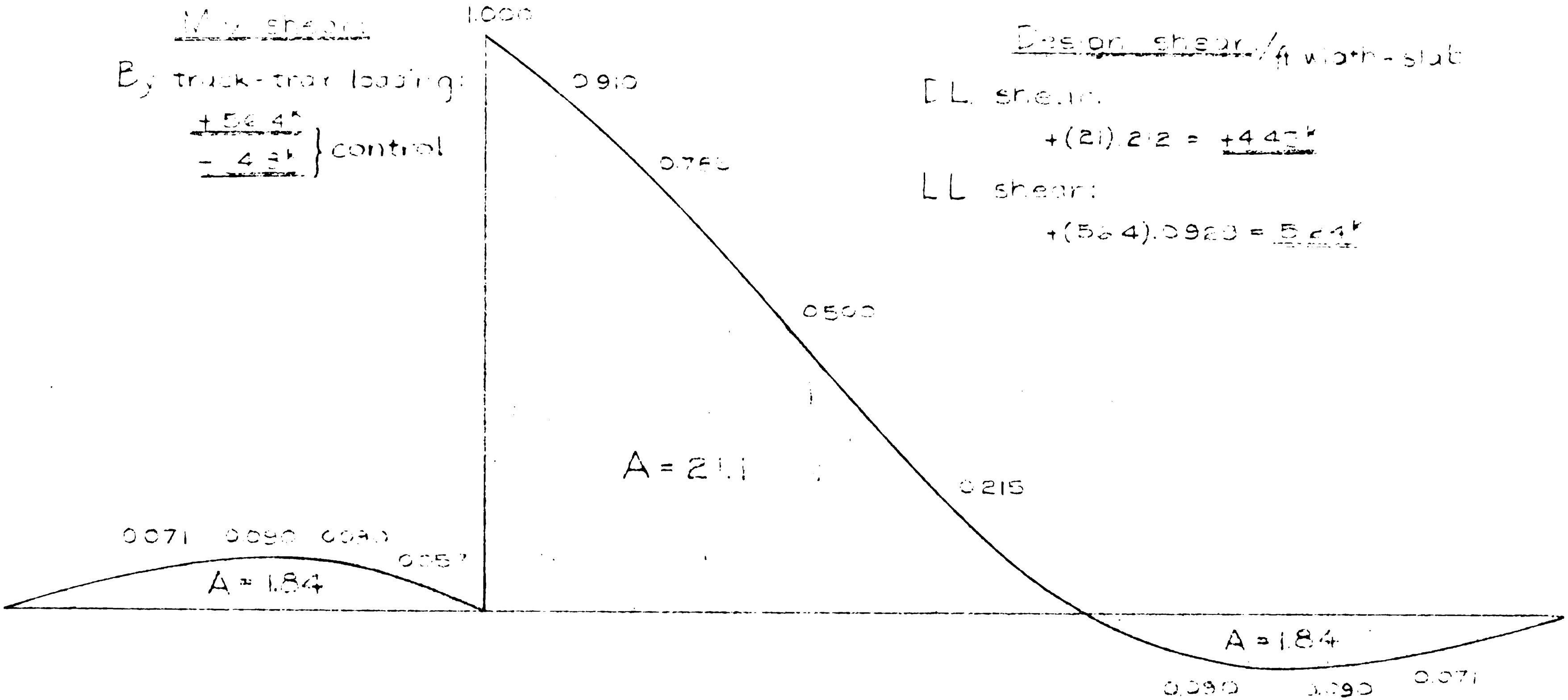
Design shear/ft width-slab

DL shear:

$+(2.1)\,2.2 = \underline{+4.62^k}$

LL shear:

$+(56.4).0923 = \underline{5.21^k}$



S2

Scale:
horizontal - 1" = 6.0"
vertical - 1" = 0.2

ANALYSIS OF SLAB DESIGN

SPAN:

Clear: 31'-11½" — 39'-9" — 31'-11½"

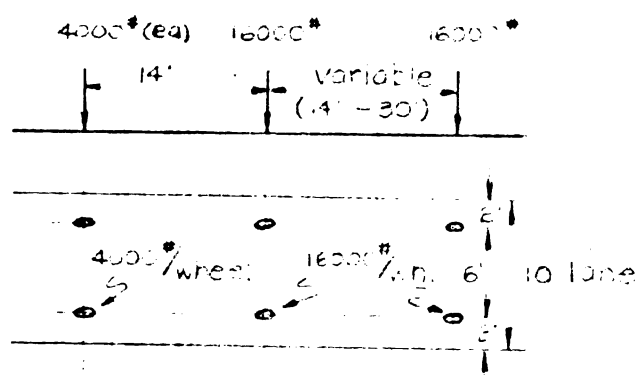
C/C : 3329' — 4200' — 3329'

WIDTH:

Two - 44' clear roadways

LOADING:

H20 - S16 - 44



IMPACT:

$$I = \frac{50}{L+125} \leq .30$$

$$I = \frac{50}{333+125} = .316 \quad \therefore \underline{\text{use } .30}$$

STRESSES:

$f'_c = 3000$ $\qquad$ $f_s = 18000$

$f_c = 1000$ $\qquad$ $n = 10$

v = 60 - no web reinforcement or special anchorage
= 90 - " " " with " "
= 140 - with " " no " "
= 180 - " " " and " "

u = 100 - with no special anchorage
= 150 - with special anchorage

WHEEL LOAD DISTRIBUTION:

$$E = \frac{10N + W}{4N}$$

$$E = \frac{10 \times 8 + 88}{4 \times 8} = \frac{168}{32} = 5.25'$$

∴ moment & shear/ft width of slab due to live load only =

$$\frac{\text{moment (or shear) due to axial load}}{2 \times 5.25}$$

MAXIMUM MOMENTS @ SECTIONS ALONG SLAB:

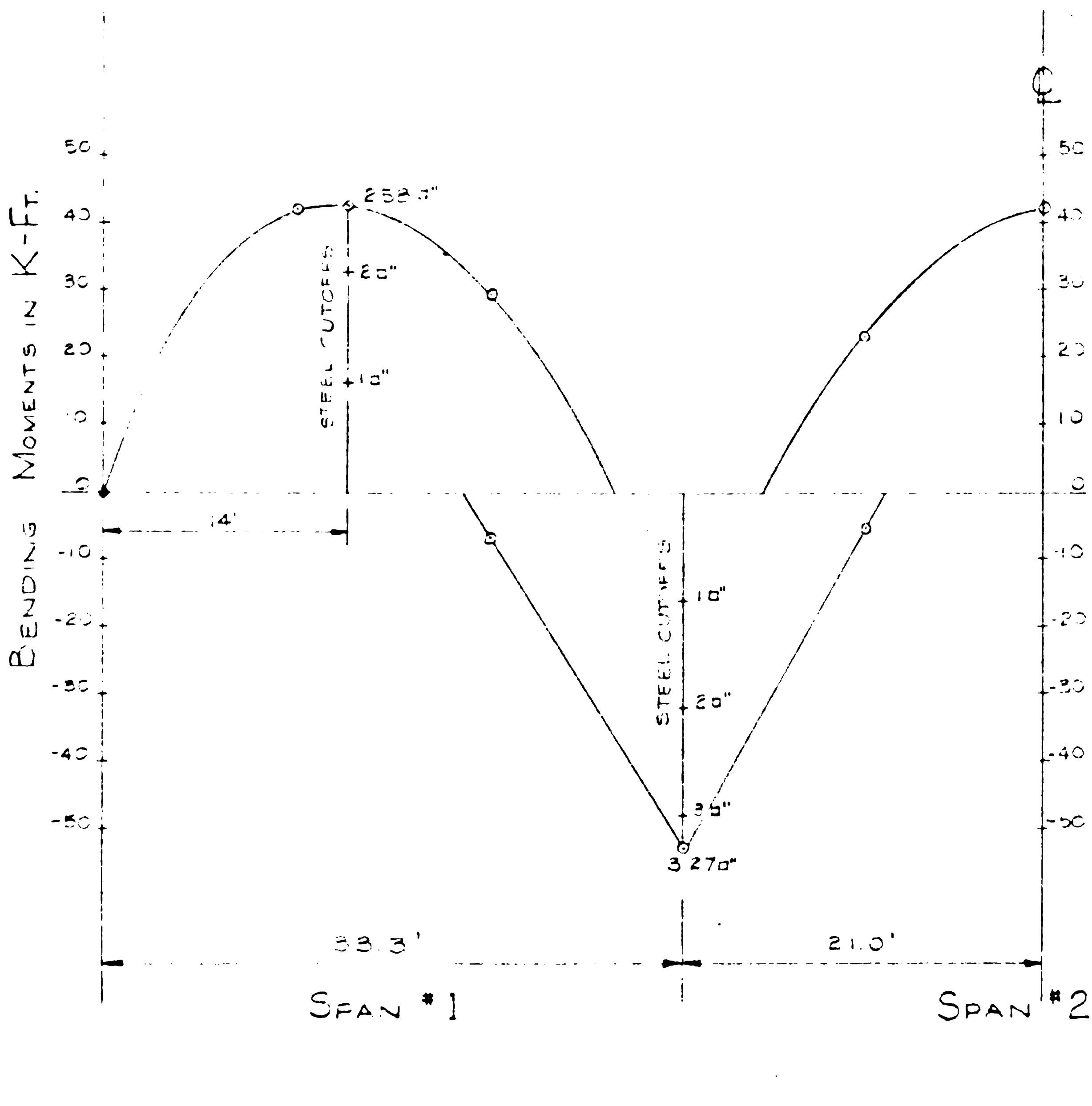
SPAN	SECTION	DL MOMENT (K FT)	LL + IMPACT MOM (K FT)	TOTAL (K FT)
1 & 3	0	0	0	0
	11.1'	+16.0	+26.0	+42.00
	11.1'	+16.0	-6.4	+ 9.60
	14.0'	+15.9	+26.6	+42.50
	14.0'	+15.9	-8.25	+ 7.65
	22.2'	+6.04	+23.4	+29.44
	22.2'	+6.04	-12.8	-6.76
2	33.3'	-31.2	-21.7	-52.90
	33.3'	-31.2	+4.36	-26.84
	43.8'	+4.90	+18.2	+23.10
	43.8'	+4.90	-10.3	- 5.40
	54.3'	+15.2	+27.0	+42.20
	54.3'	+15.2	-10.3	+ 4.90

MAXIMUM SHEAR:

At $R_a = +7.1^k$

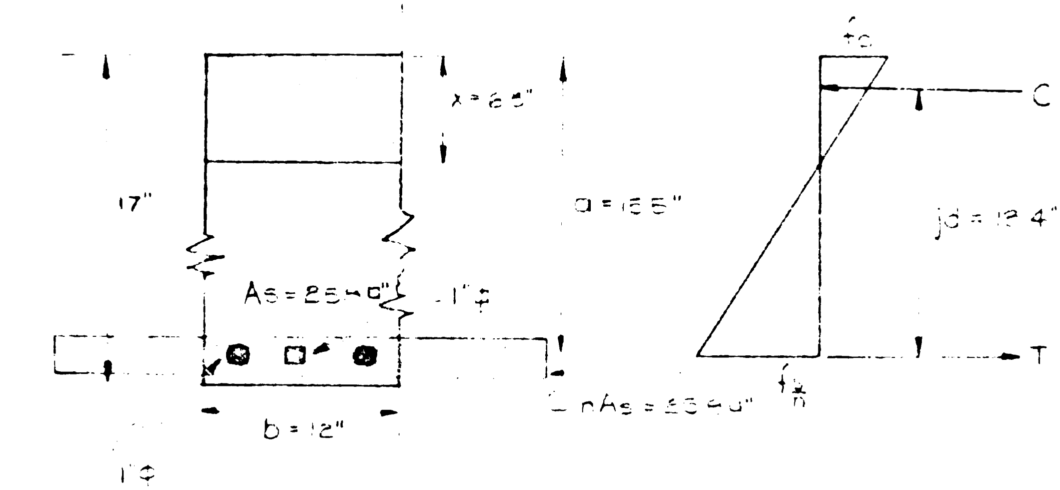
At $R_a = +9.7^k$

MAXIMUM TOTAL MOMENTS



SCALE: 1" = 10'-0"

CHECKING SLAB AT POINT OF MAX MOM. IN SPAN #2:



$$12x\left(\frac{x}{2}\right) = 25.4(15.5 - x)$$

$$6x^2 = 400 - 25.4x$$

$$x^2 + 4.2x - 66.7 = 0$$

$$x = \frac{-4.2 + \sqrt{(4.2)^2 + 4(66.7)}}{2} = \frac{-4.2 + 16.8}{2} = 6.3"$$

$$Cjd = B.M. = 42.5^{k'}$$

$$C = \frac{12 \times 6.3}{2} f_c = \frac{42500 \times 12}{12.4}$$

f_c = 1010 #/□" allowable = 1000 #/□"

$$A_s f_s = T = C = \frac{42500 \times 12}{12.4}$$

$$f_s = \frac{42500 \times 12}{2.54 \times 12.4} = \text{[illegible]} \ \#/□"$$

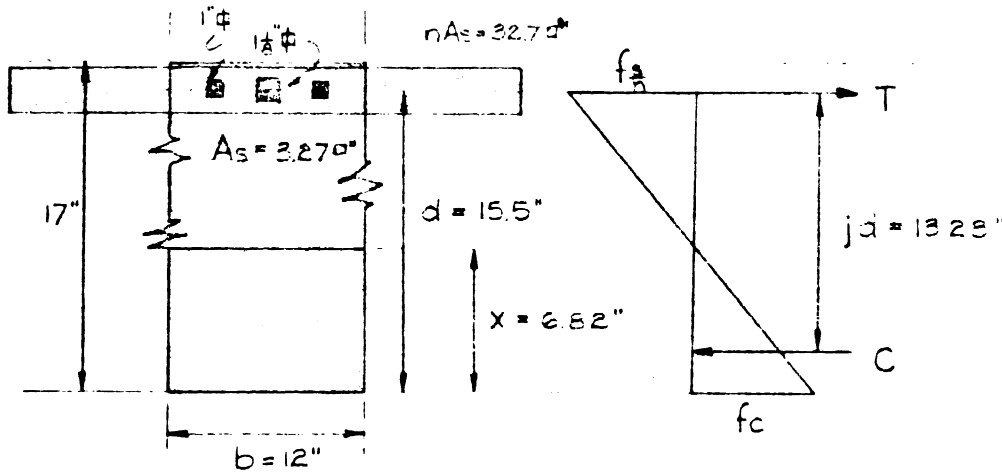
allowable = 18000 #/□"

Allowables by Mich State Highway Specs.

f_c = 1350 #/□"

f_s = 18000 #/□"

CHECKING SLAB AT POINT OF MAX. NEGATIVE MOM:



$$12x\left(\frac{x}{2}\right) = 32.7(15.5 - x)$$

$$6x^2 = 507 - 32.7x$$

$$x^2 + 5.55x - 84.5 = 0$$

$$x = \frac{-5.55 + \sqrt{(5.55)^2 + 4(84.5)}}{2} = \frac{-5.55 + 19.2}{2} = 6.82''$$

$$Cjd = B.M. = -52.9^{kft}$$

$$C = \frac{52900 \times 12}{13.28} = 48{,}000\#$$

$$C = \frac{12 \times 6.82}{2} f_c = 48{,}000$$

$$f_c = \underline{1170}\ \#/\square'' \quad \text{allowable} = 1000\ \#/\square''$$

$$T = A_s f_s = \frac{48000}{3.27} = \underline{14700}\ \#/\square''$$

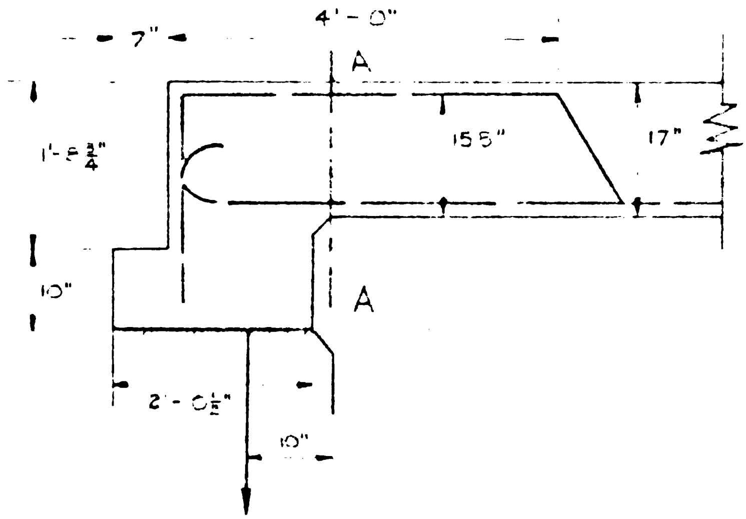
allowable = 18000 #/□"

allowables by Mich State Highway Dept.

$$f_c = 1350\ \#/\square''$$

$$f_s = 18000\ \#/\square''$$

CHECKING NEGATIVE MOMENT STEEL @ ABUTMENTS:



Assume slab fixed at section A-A.

From influence line diagram max reaction at end

$= (2.49 + 4.60) = 7.09 \ ^{k}/_{ft}$

$M = 7.09 \times 10 = 70.9 \ ^{k\,in}/_{ft}$ width of slab.

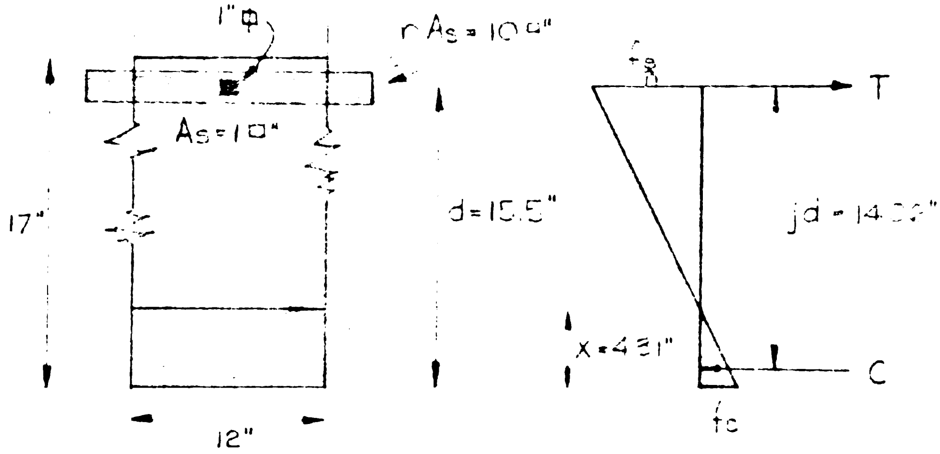
Using Mich State Highway Dept specs.,

$f'_c = 3000 \ ^{\#}/_{\square''}$

$f_c = 1350 \ ^{\#}/_{\square''}$

$f_s = 18000 \ ^{\#}/_{\square''}$

$n = 10$



$$12x\left(\frac{x}{2}\right) = 10(15.5 - x)$$

$$6x^2 = 155 - 10x$$

$$x^2 + 1.667x - 25.8 = 0$$

$$x = \frac{-1.667 + \sqrt{(1.667)^2 + 4(25.8)}}{2} = \frac{-1.667 + 10.3}{2} = 4.31''$$

$$Cjd = 70{,}900 \text{ in}\#$$

$$C = \frac{70{,}900}{14.06} = 5040\#$$

$$\frac{12(4.31)}{2} f_c = 5040$$

$f_c = \underline{195} \ \#/\square''$ allowable = 1250 $\#/\square''$

$$A_s f_s = 5040$$

$f_s = \underline{5040} \ \#/\square''$ allowable = 18000 $\#/\square''$

CHECKING STEEL CUTOFFS:

(see maximum total moment diagram)

1. At points 6' on each side of point of max negative moment steel required = 1.17 □" per foot width of span.

   steel provided = 2.27 □"

2. At points 10' on each side of point of max negative moment steel required = 0.28 □" per foot width of span

   steel provided = 1.27 □"

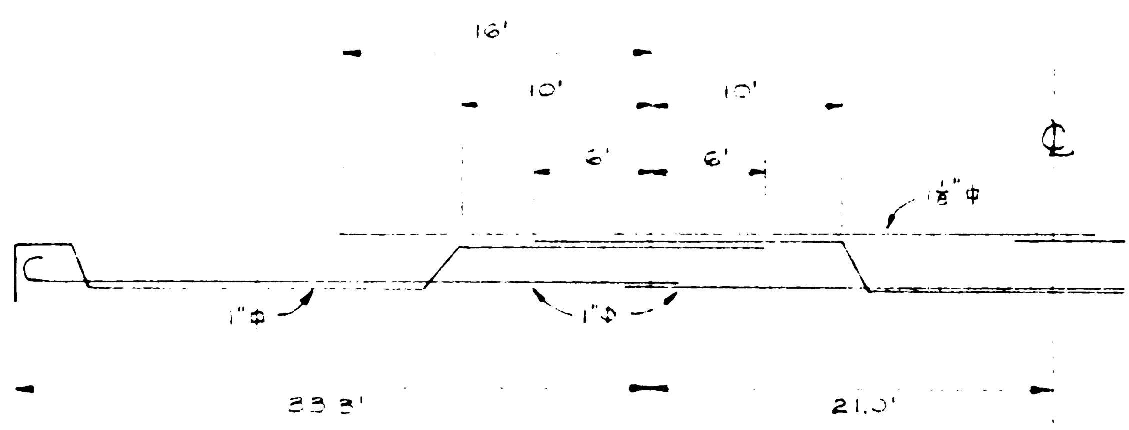
3. No negative moment steel required beyond point 13' on each side (+15 diameters = 13'+15"). Steel totally cut off in first span beyond 16' from pier.

4. For positive moment in spans one and two, steel required at points 10' from pier = 1.48 □"

   steel provided = 1.59 □"

5. For negative moment at abutment ends, Mich State Highway Dept. design arbitrarily provides a safe amount of steel.

   (see computation sheet for moment analysis)



CHECKING SHEAR & BOND:

$$v = \frac{V}{b_j d}$$

$V_{max.}$ @ $R_b = 97^k$

$v = \frac{9700}{12(13.4)} = \underline{\underline{60.2}}\ \#/\square''$ allowable $= 60\ \#/\square''$

$$u = \frac{V}{\Sigma o j d}$$

@ point of max negative moment $\Sigma o = 12.5''$

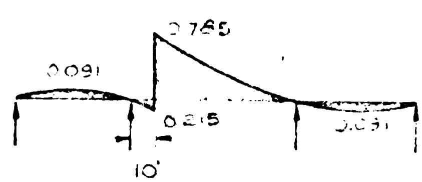
$u = \frac{9700}{12.5(13.4)} = \underline{\underline{58.0}}\ \#/\square''$ allowable $= 100\ \#/\square''$

@ reaction R_a, $\Sigma o = 6.3''$; $V = 71^k$

$u = \frac{7100}{6.3(12.23)} = \underline{\underline{92.2}}\ \#/\square''$ allowable $= 150\ \#/\square''$

@ points 10' from point of max neg mom in span 2 $\Sigma o = 8.0''$

$V = 3570\ \#$ (revising influence line diag S_2, & loading to get max. effect)



$u = \frac{3570}{8(12.4)} = \underline{\underline{36.2}}\ \#/\square''$ allowable $= 100\ \#/\square''$

CHECKING SLAB CROSS STEEL:

A.A.S.H.O. Specs. – 3.3.2 (c)

Percentage of main reinforcement to be placed at bottom of slab normal to the main steel =

$$\frac{100}{\sqrt{S}} \leq 50\%$$

S = effective span of slab

$$\frac{100}{\sqrt{32}} = 17.7\%$$

17.7% (2.58) = .46 □"/ft length of slab – required

.51 □"/ft provided ($\frac{3}{4}$" ϕ)

By Mich State Highway Dept. Specs.

For spans over 20':

middle half – 25% of main reinforcement

other sections – 15% " " "

.25(2.58) = .645 □"/ft length of slab

.15(2.58) = .387 □"/ft " " " } required

.62 □" for middle half

.38 □" for remaining section } provided

Temperature steel to be placed at top of slab normal to main reinforcement =

$\frac{1}{8}$ □"/ft width of surface (.125 □") required

.233 □" provided

BRIEF OUTLINE OF CROSSBEAM ANALYSIS

I - Loading to obtain maximum effect

A - Influence line diagrams

1 - See slab analysis outline.

B - Loading the influence line diagrams

1 - The live load to be distributed upon the beam to obtain the maximum effect is determined from the influence line diagram Rb (see page 57).

a - Maximum dead load and live load reactions per foot width of slab obtained from Rb gives the maximum dead load and live load upon the crossbeam per linear foot of beam.

2 - Dead load and live load effect of sidewalk is also included (see page 57).

II - Analysis of beam design

A - Checking maximum moments on beam

1 - The max. positive and negative moments on the beam are determined by properly loading the beam; and the reinforcing steel is checked (see pages 58, 61 & 62).

B - Checking shearing and bond stresses.

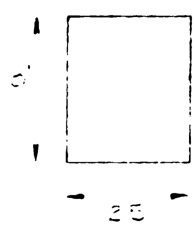
1 - See page 63

C - Checking stirrups

1 - See pages 64 & 65.

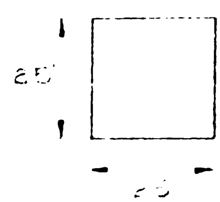
MOMENT DISTRIBUTION FOR INFLUENCE LINE DIAGRAM
(ANALYSIS OF PIER DESIGN)

Beam:

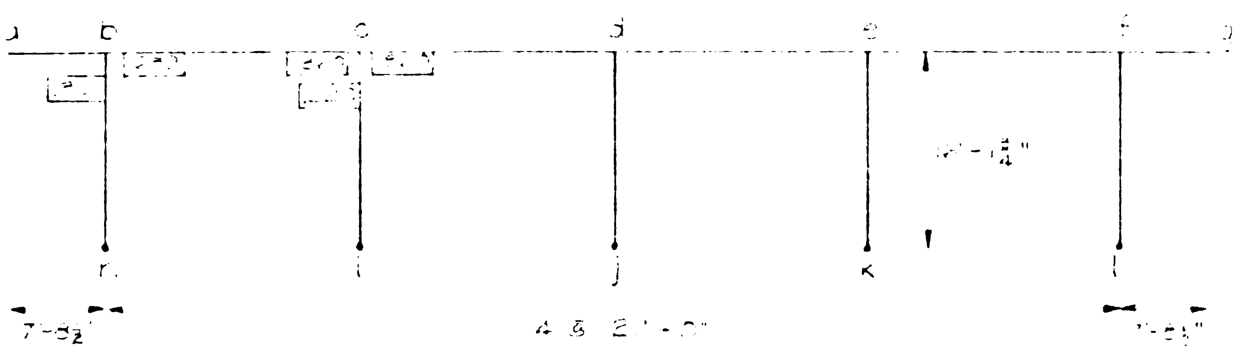


$I_b = \frac{1}{12}bh^3 = \frac{1}{12}(25)(\ldots)^3 = $ [illegible] in^4

Column:



$I_c = \frac{1}{12}b^4 = \frac{1}{12}(\ldots)^4 = $ [illegible] in^4



$$CK_{bh} = \frac{[illegible]}{[illegible]} = .205$$

$$CK_{bc} = \frac{4 \times [illegible]}{[illegible]} = .272$$

$$\Sigma CK = 1.677 \quad r_{bh} = [illegible] \quad r_{bc} = [illegible]$$

$$CK_{cb} = \frac{4 \times [illegible]}{[illegible]} = .272$$

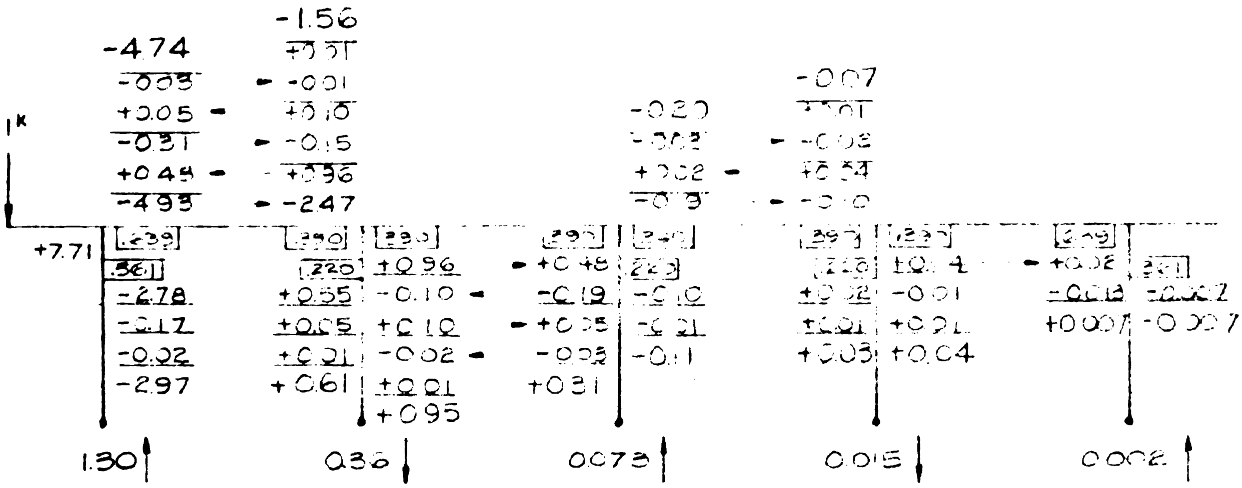
$$CK_{ci} = \frac{[illegible]}{[illegible]} = [illegible]$$

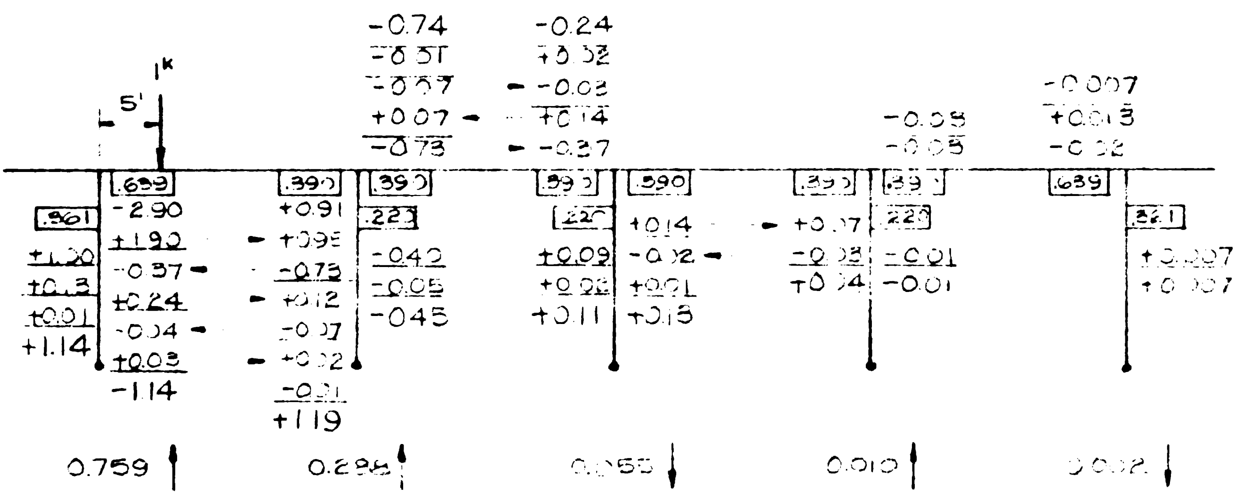
$$CK_{cd} = \frac{[illegible]}{[illegible]} = [illegible]$$

$$\Sigma CK = 2.749 \quad r_{cb} = [illegible] \quad r_{ci} = [illegible] \quad r_{cd} = [illegible]$$

Note:

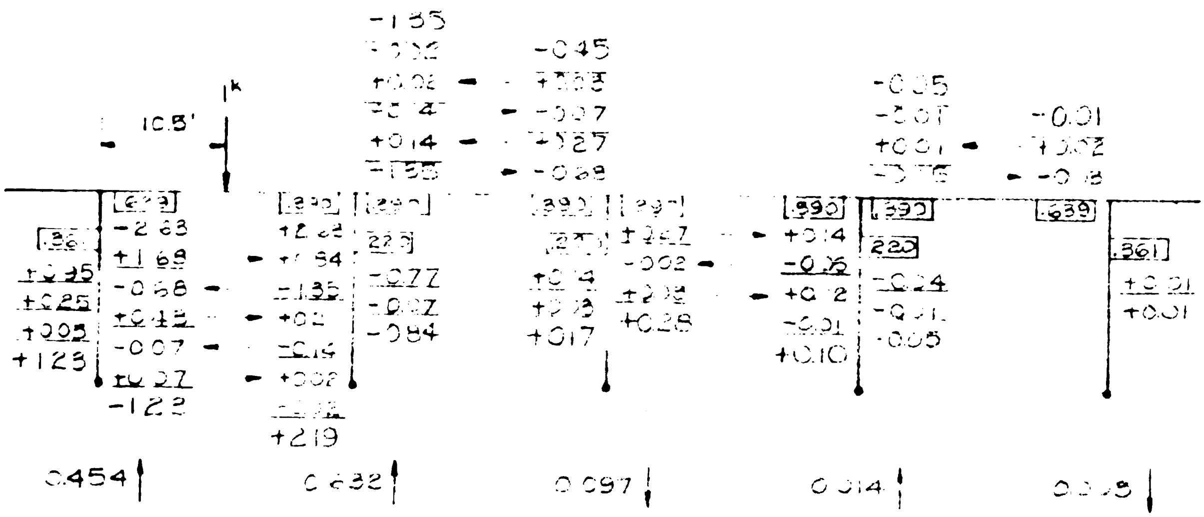
Because of the heavy uniform dead load [illegible] sidesway was considered in the following analysis.



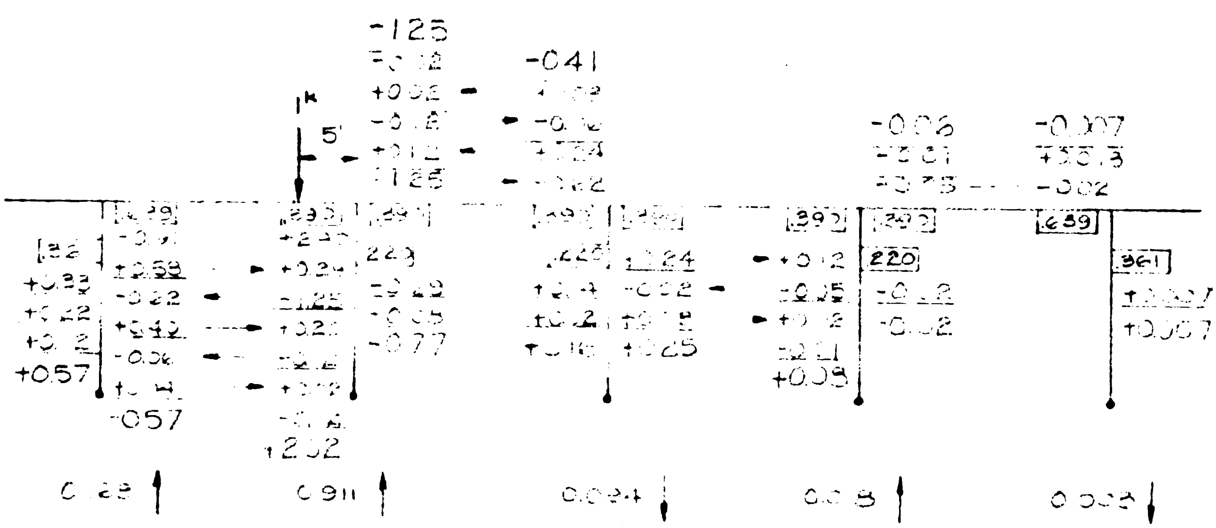


$$M_{bc} = \frac{Pab^2}{L^2} = \frac{(5)(16)^2}{(21)^2} = 2.90^{\,k\text{-}ft}$$

$$M_{cd} = \frac{(16)(5)^2}{(21)^2} = 0.907^{\,k\text{-}ft}$$

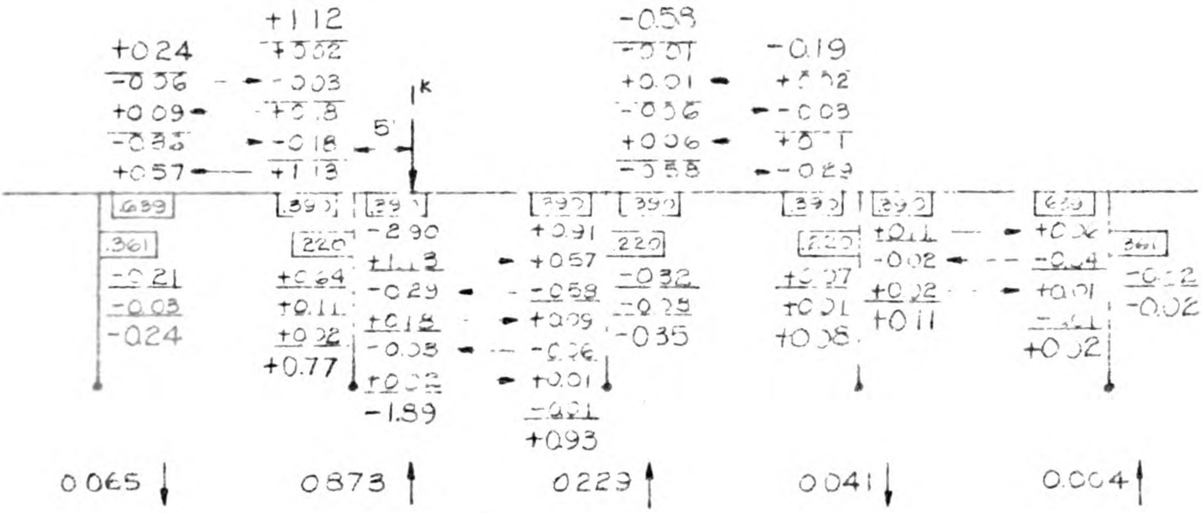


$$M_{bc} = M_{cd} = \frac{PL}{8} = \frac{21}{8} = 2.62^{k'}$$



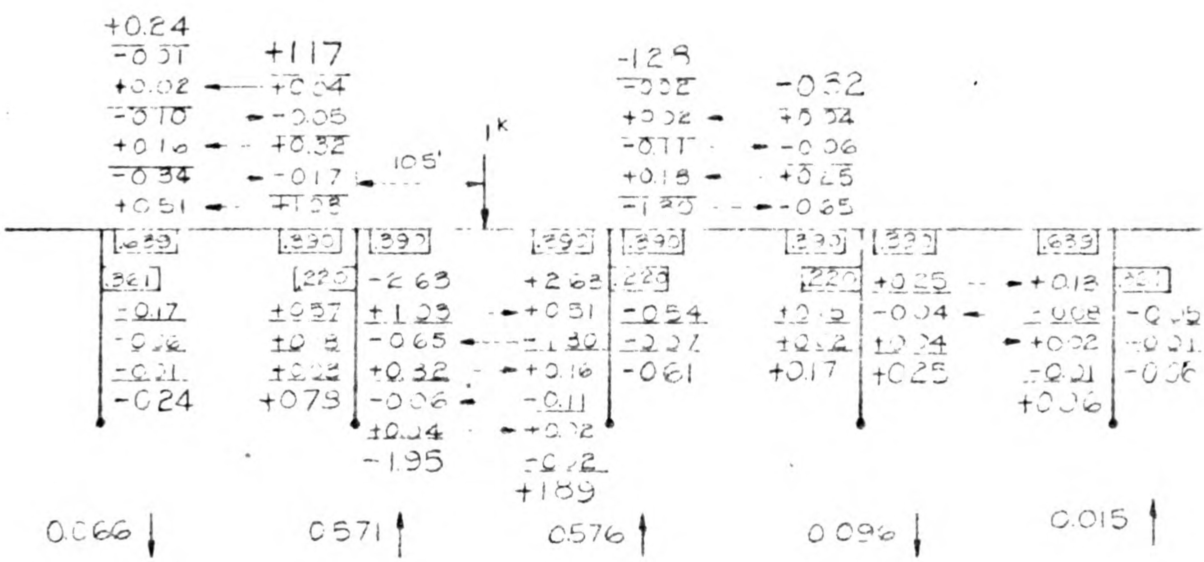
$$M_{bc} = \frac{Pab^2}{L^2} = \frac{(16)(5)^2}{(21)^2} = 0.907^{k'}$$

$$M_{cd} = \frac{(5)(16)^2}{(21)^2} = 2.90^{k'}$$

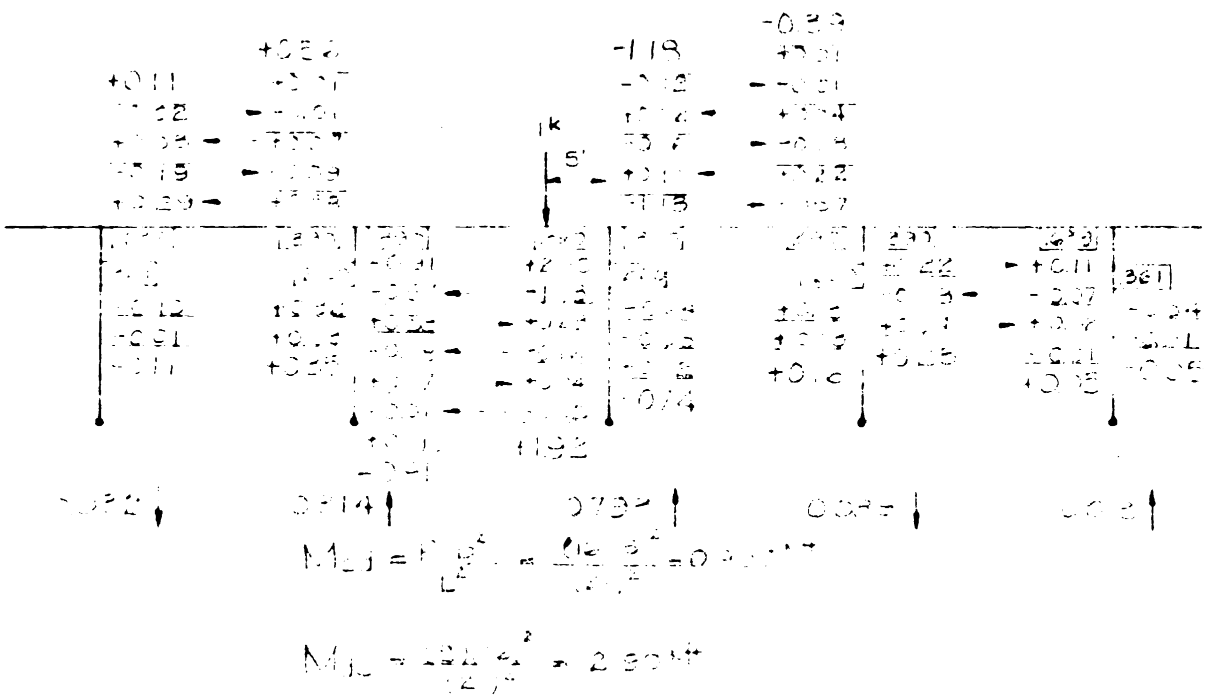


$$M_{cd} = \frac{Pab^2}{L^2} = \frac{(5)(16)^2}{(21)^2} = 2.90^{kft}$$

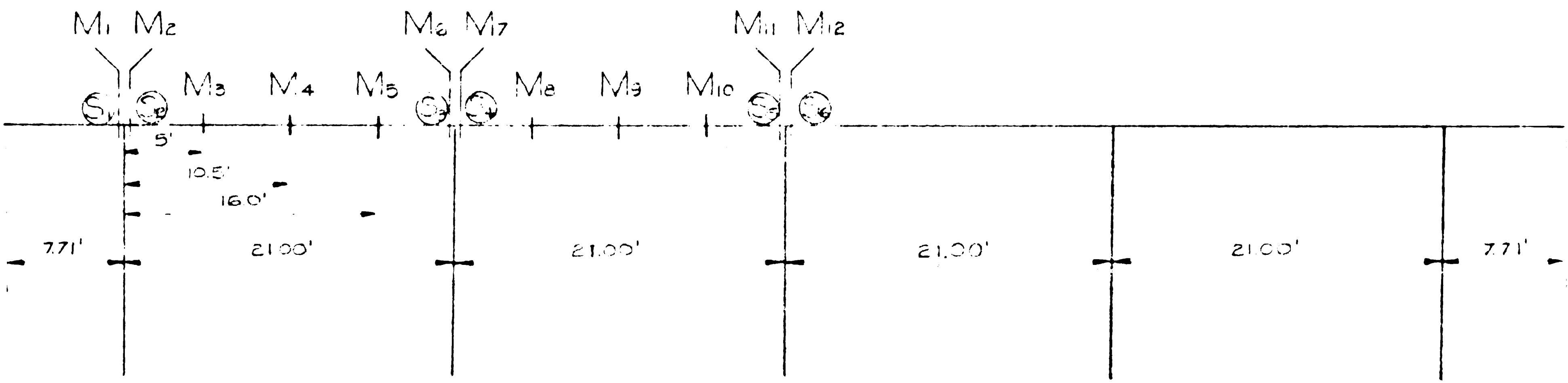
$$M_{dc} = \frac{(16)(5)^2}{(21)^2} = 0.907^{kft}$$



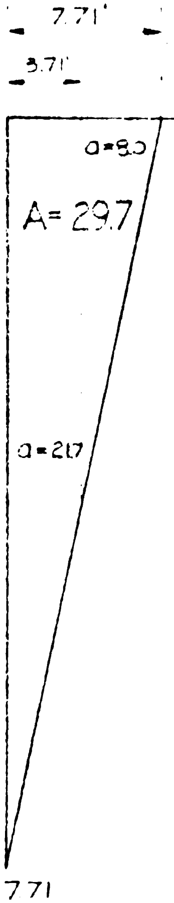
$$M_{cd} = M_{dc} = \frac{PL}{8} = \frac{21}{8} = 2.63^{kft}$$



INFLUENCE LINE DIAGRAMS FOR CROSSBEAM ANALYSIS:



Sections for which influence line diagrams are drawn.



Design moment at M_1

DL of beam = 150#/ft³ (2.5ft)(3.0ft) = 1.125 k/ft length of beam

* D.L. moment:

DL of beam = 29.7(1.125) = -33.4 k-ft

DL of sidewalk = 21.7(5.51) = -119.5

DL of slab = 29.7(8.84) = [illegible]

total = -479 k-ft

* L.L. moment:

LL on sidewalk = 21.7(2.82) = -63.0

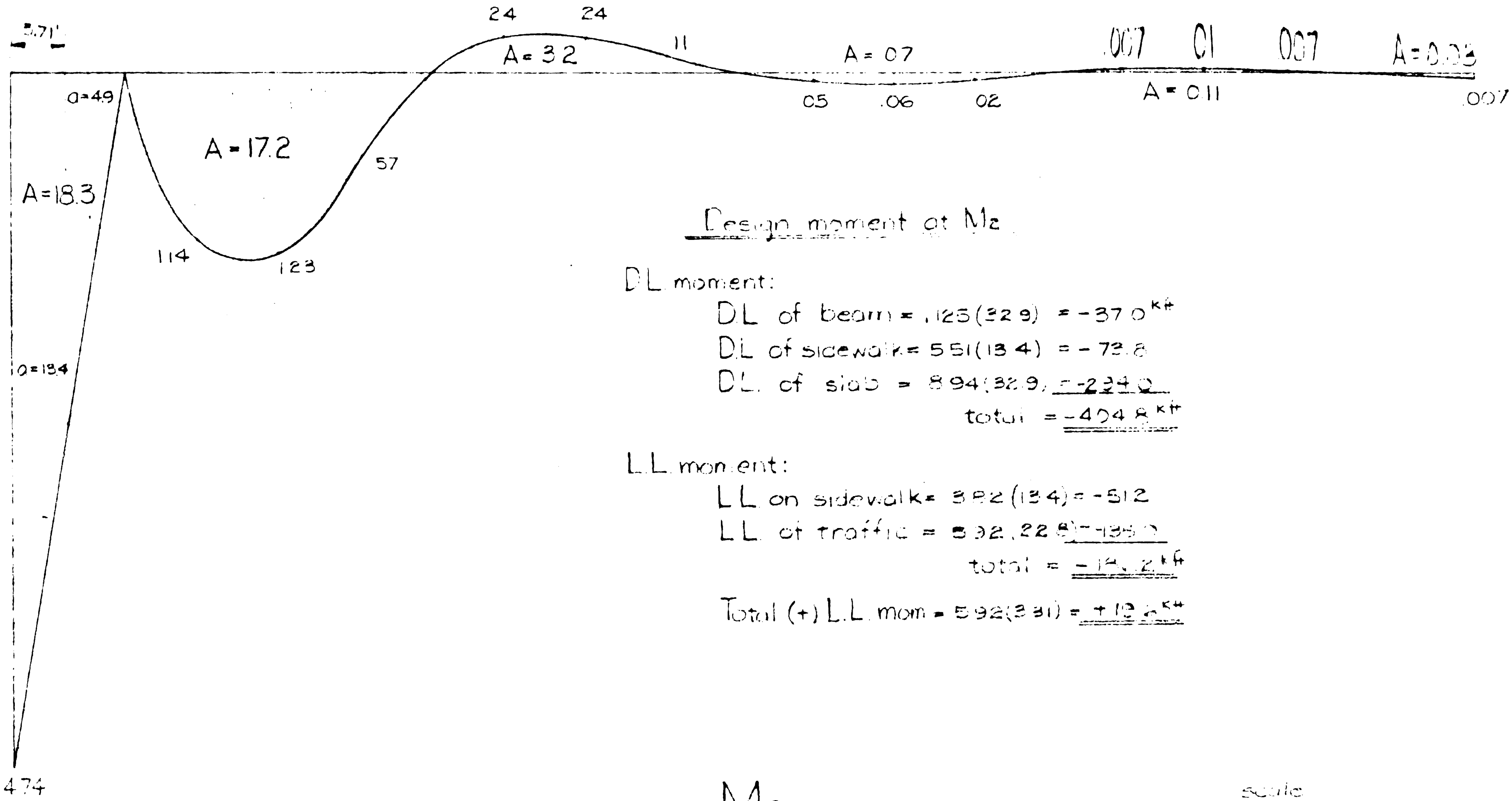
LL of traffic = 8.0(5.92) = -47.4

total = -110.4 k-ft

* see computation sheet on cross beam analysis for load distribution (see page 57)

M_1

scale:
horizontal - 1" = 10'-0"
vertical - 1" = 20 k-ft



Design moment at M_2

D.L. moment:

D.L. of beam = .125(32.9) = -37.0^{kft}

D.L. of sidewalk = 5.51(13.4) = −73.8

D.L. of slab = 8.94(32.9) = −294.0

total = -404.8^{kft}

L.L. moment:

L.L. on sidewalk = 3.82(13.4) = −51.2

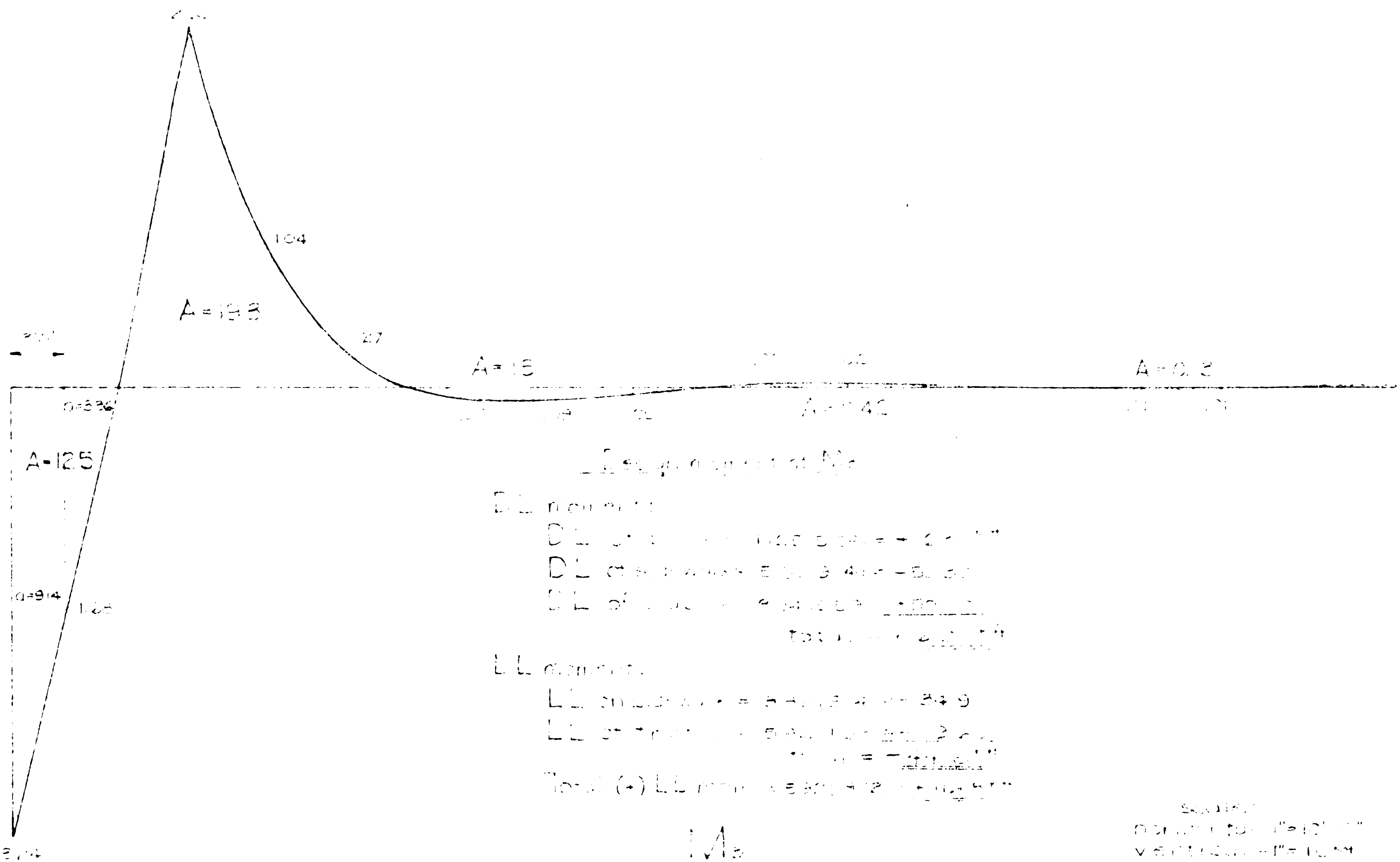
L.L. of traffic = 5.92(22.8) = −135.0

total = $-1[illegible].2^{kft}$

Total (+) L.L. mom = 5.92(3.31) = $+19.[illegible]^{kft}$

M_2

scale
horizontal − 1" = 10'−0"
vertical − 1" = 10^{k ft}



Influence line for M_B [illegible]

D.L. moment:

D.L. [illegible] = [illegible]

D.L. [illegible] = −5[illegible]

D.L. [illegible]

Total [illegible]

L.L. moment:

L.L. [illegible] = [illegible] 24.9

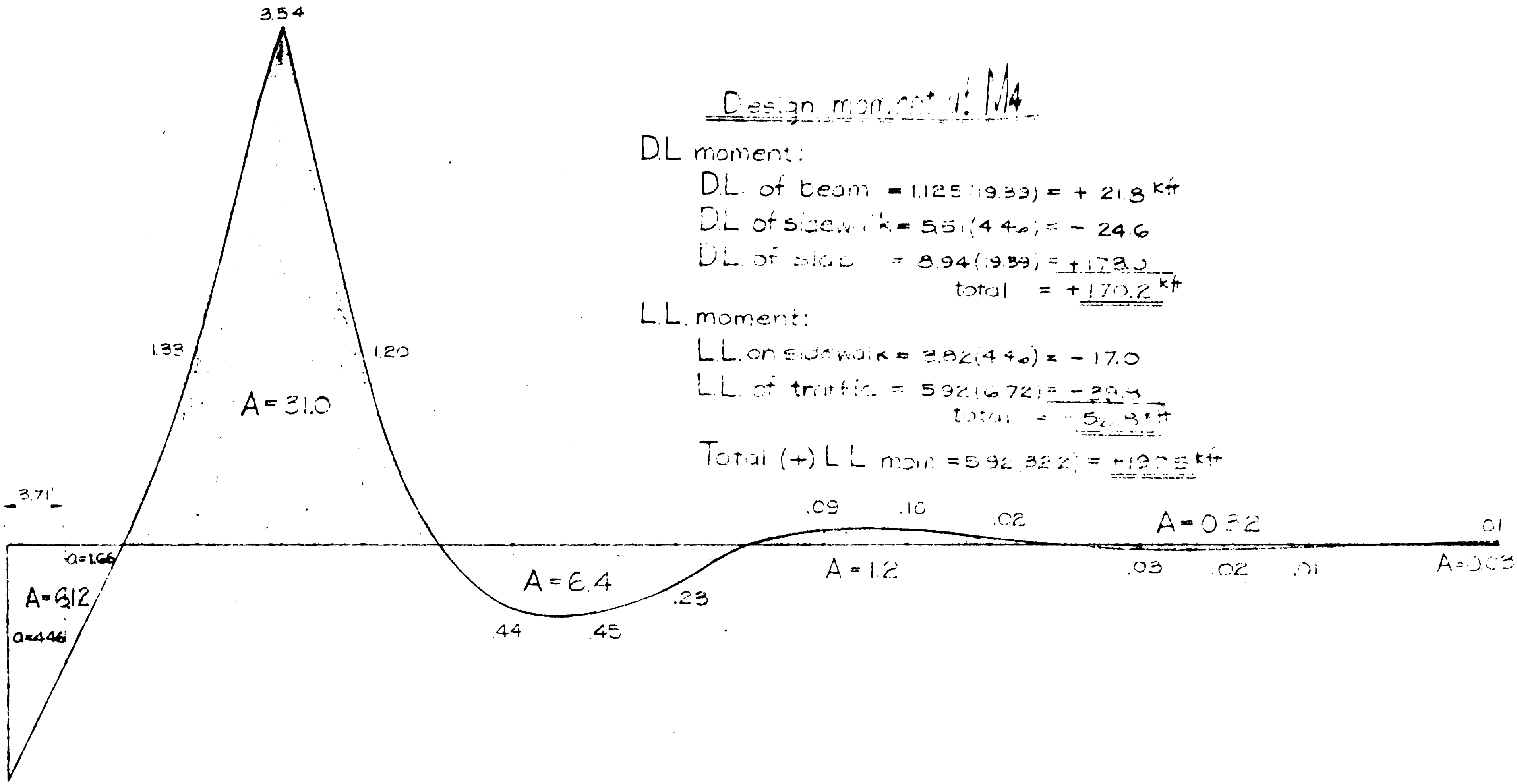
L.L. [illegible]

[illegible] = −[illegible]

Total (+) L.L. [illegible]

M_B

Scales:
Horizontal [illegible]
Vertical [illegible]



Design moment at M_4

D.L. moment:

- D.L. of beam = 1.125(19.39) = + 21.8 k-ft
- D.L. of sidewalk = 5.51(4.46) = − 24.6
- D.L. of slab = 8.94(19.39) = + 173.0
- total = + 170.2 k-ft

L.L. moment:

- L.L. on sidewalk = 3.82(4.46) = − 17.0
- L.L. of traffic = 5.92(6.72) = − 39.8
- total = − 56.8 k-ft

Total (+) L.L. mom = 5.92(32.2) = + 190.5 k-ft

M_4

scale:
horizontal − 1" = 10'-0"
vertical − 1" = 1.0 k-ft

Design moment at M_5

D.L moment:

D.L. of beam = $1.125(2.38) = +2.68^{kft}$

D.L. of sidewalk = $5.51(0.06) = +0.33$

D.L. of slab = $8.94(2.38) = +21.30$

total = $\underline{\underline{+24.31^{kft}}}$

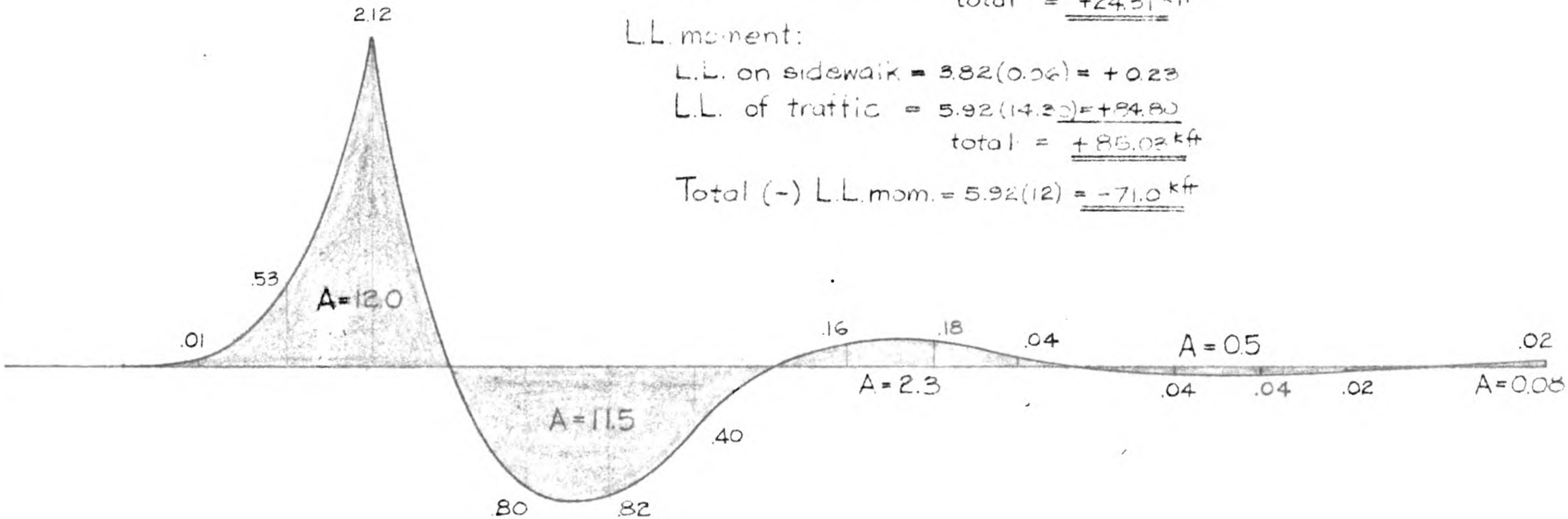
L.L. moment:

L.L. on sidewalk = $3.82(0.06) = +0.23$

L.L. of traffic = $5.92(14.30) = +84.80$

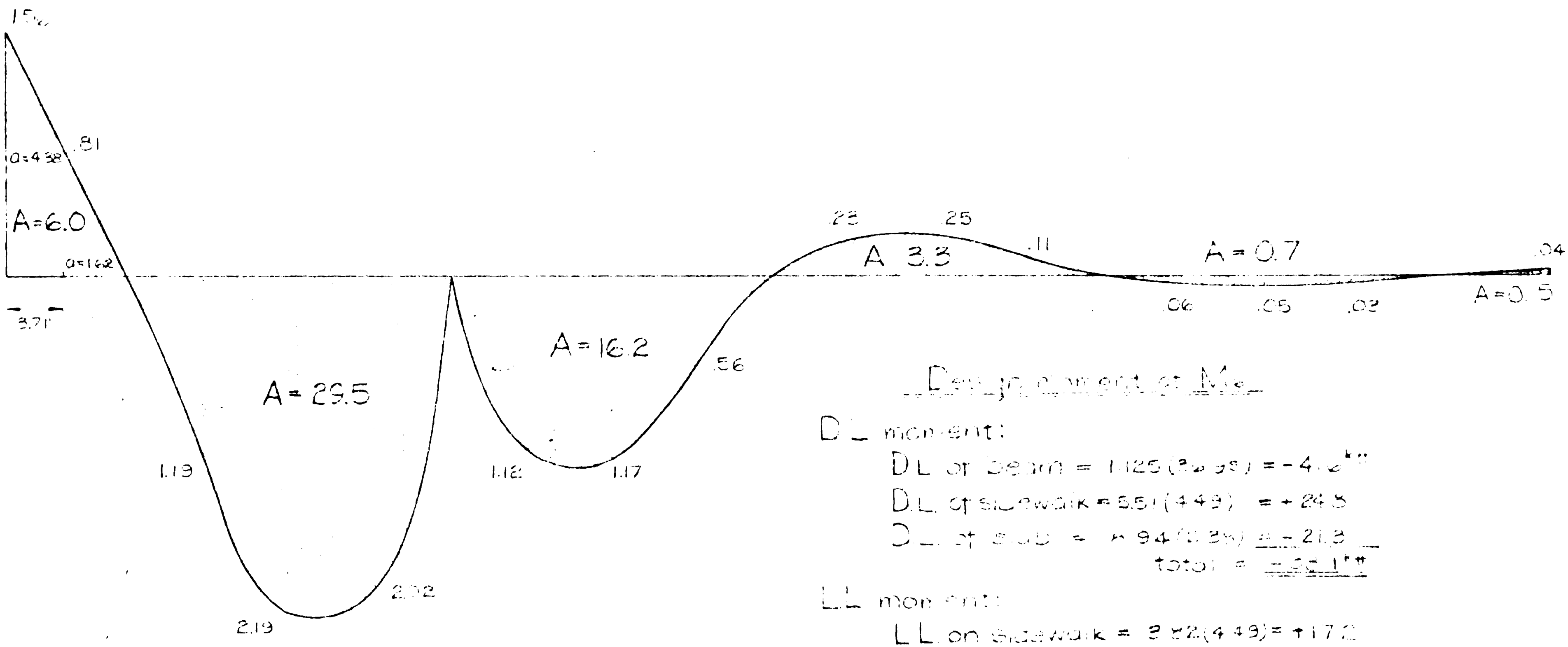
total = $\underline{\underline{+85.03^{kft}}}$

Total (−) L.L. mom. = $5.92(12) = \underline{\underline{-71.0^{kft}}}$



M_5

scale:
horizontal − 1" = 10'-0"
vertical − 1" = 1.0^{kft}



Design moment at M_6

DL moment:

DL of beam = 1.125([illegible]) = $-4.[illegible]^{k'}$

D.L. of sidewalk = 5.51(4.49) = +24.5

DL of slab = [illegible].94([illegible]) = −21.8

total = $-[illegible].1^{k'}$

LL moment:

LL on sidewalk = 3.82(4.49) = +17.2

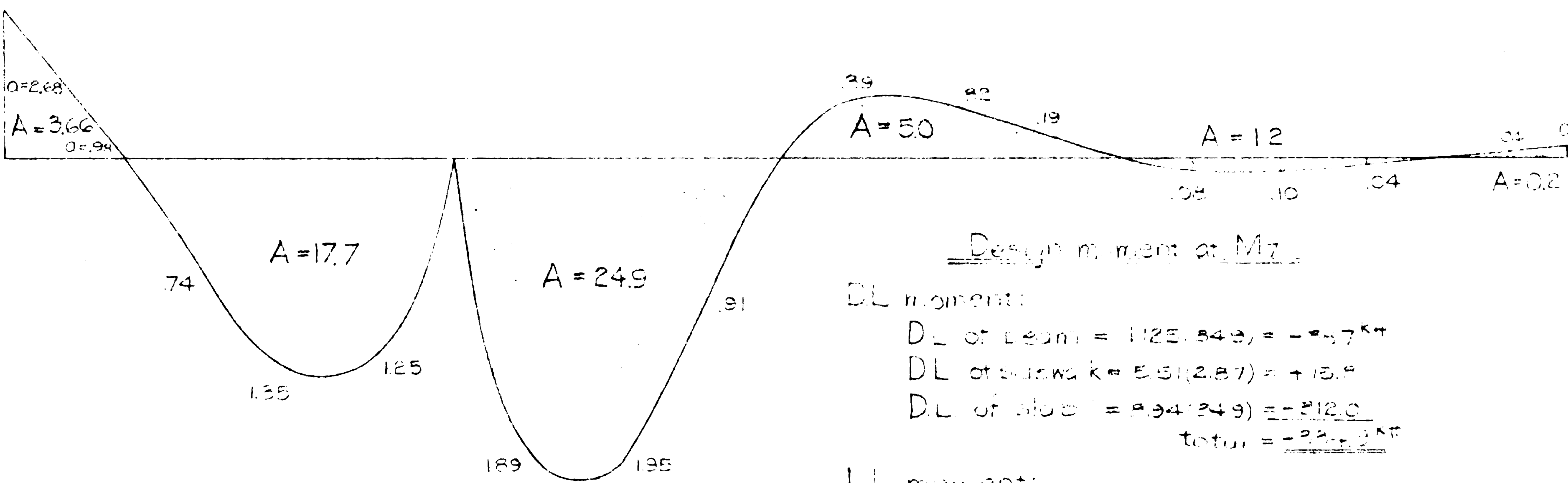
LL of traffic = 5.[illegible](3.30) = +19.5

total = $+36.7^{k'}$

Total (−) LL mom. = 5.32([illegible]) = $-2[illegible]^{k'}$

M_6

scale:
horizontal - 1" = 10'-0"
vertical - 1" = 10k'



Design moment at M_7

D.L moment:

D.L of beam = 1.125(84.9) = −[illegible]$^{k'}$

D.L of sidewalk = 5.51(2.87) = +15.8

D.L of slab = 8.94(24.9) = −[illegible]

total = −[illegible]$^{k'}$

L.L moment:

L.L of sidewalk = 3.82(2.87) = +11.0

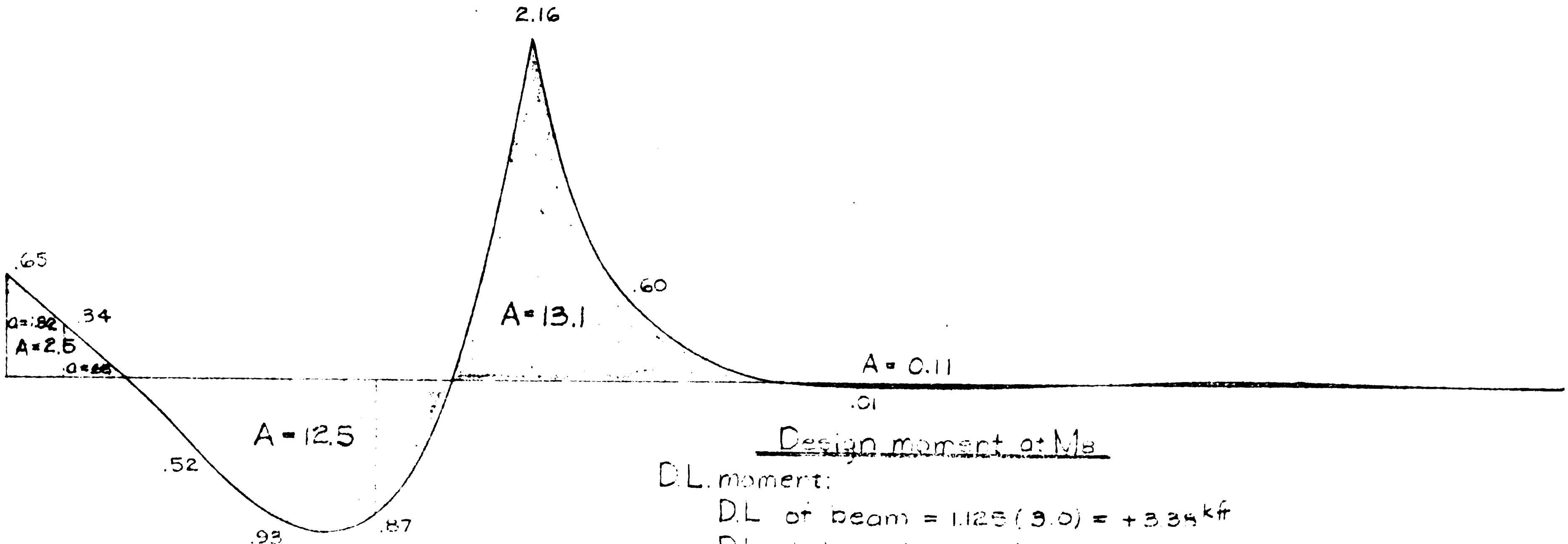
L.L of traffic = 5.92(5.0) = +29.6

+40.6$^{k'}$

Total (−) L.L mom = 5.92(43.8) = −259.3$^{k'}$

M_7

scale:
horizontal − 1" = 10'−0"
vertical − 1" = 1.0$^{k'}$



Design moment at M_B

D.L. moment:

D.L. of beam = 1.125 (3.0) = + 3.38 kft

D.L. of sidewalk = 5.51 (1.82) = + 10.02

D.L. of slab = 8.84 (3.0) = + 26.50

total = + 40.2 kft

L.L. moment:

L.L. on sidewalk = 3.82 (1.82) = + 6.95

L.L. of traffic = 5.92 (13.1) = + 77.50

total = + 84.5 kft

Total (–) L.L. mom. = 5.92 (12.6) = – 74.6 kft

M_B

scale:
horizontal – 1" = 10'-0"
vertical – 1" = 1.0 kft

Design moment at M_3

D.L. moment:

D.L. of beam = 1.25 (7.74) = + [illegible]$^{k'}$

D.L. of sidewalk = .5 (.41) = + [illegible]

D.L. of slab = 8.94 (7.74) = + [illegible]

total = + [illegible]

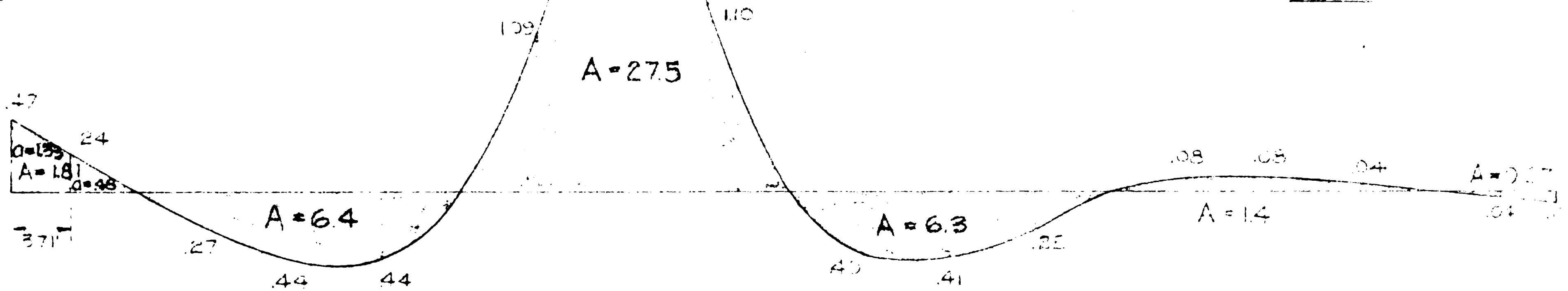
L.L. moment:

L.L. on sidewalk = [illegible] (.4) = + [illegible]

L.L. of traffic = [illegible] = + [illegible]

total = + [illegible]$^{k'}$

Total (−) L.L. mom. = 5.92 (27) = [illegible]$^{k'}$



M_3

scale:
horizontal — 1" = 10'-0"
vertical — 1" = 1.0$^{k'}$

Design moment at M_{10}

DL moment:

DL of beam = [illegible] = + [illegible] k'

DL of sidewalk = .55 (.41) = − .2 [illegible]

DL of slab = .94 (.58) = + [illegible]

total = + [illegible] k'

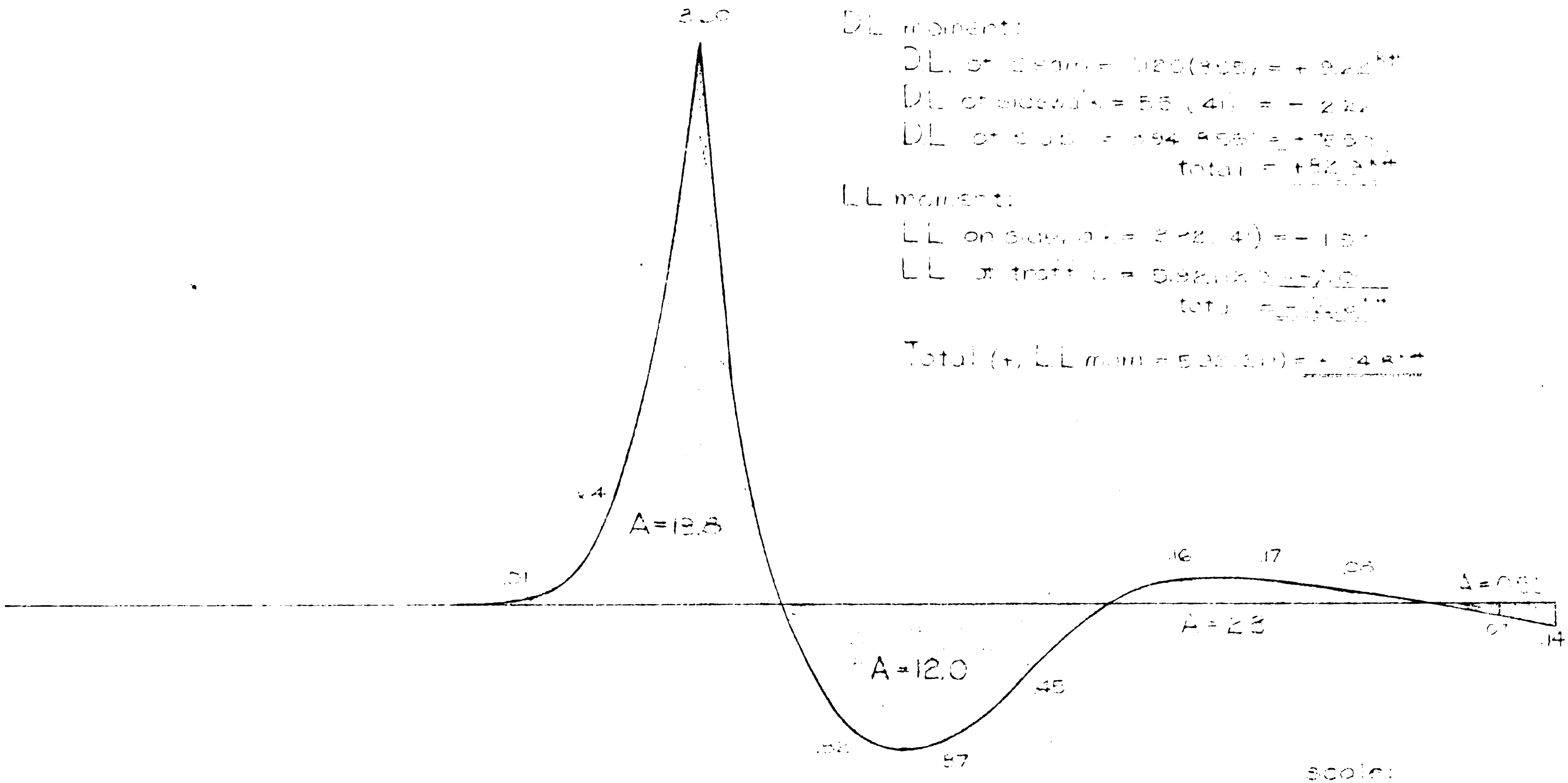
LL moment:

LL on sidewalk = [illegible] (.4) = − .1 [illegible]

LL of traffic = 5.92 [illegible] = + [illegible]

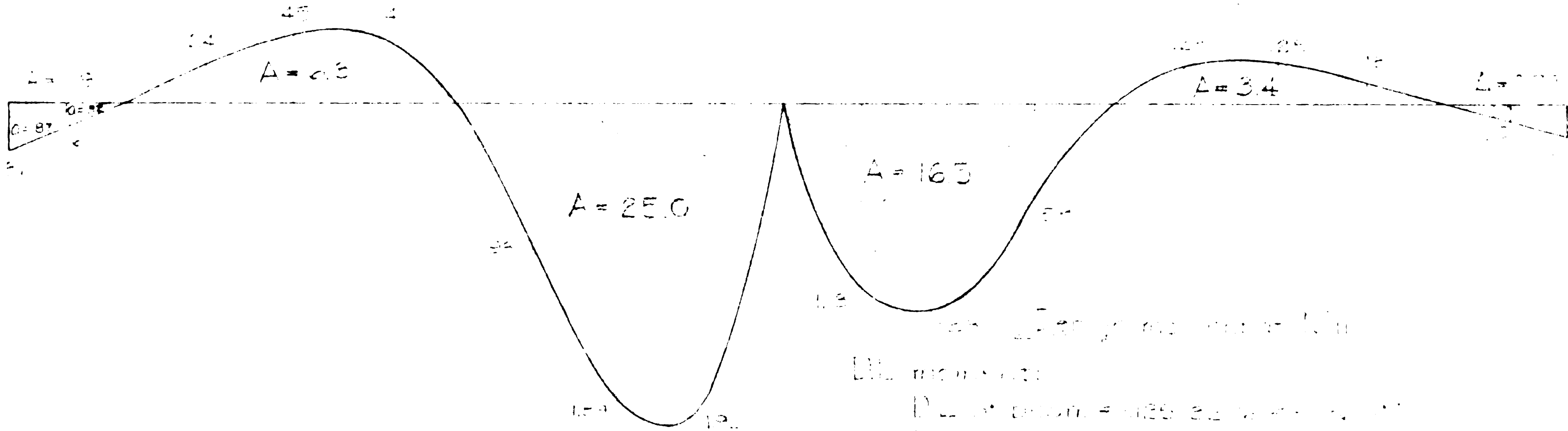
total = + [illegible] k'

Total (+ LL mom = 5.92 [illegible]) = + [illegible] k'



M_{10}

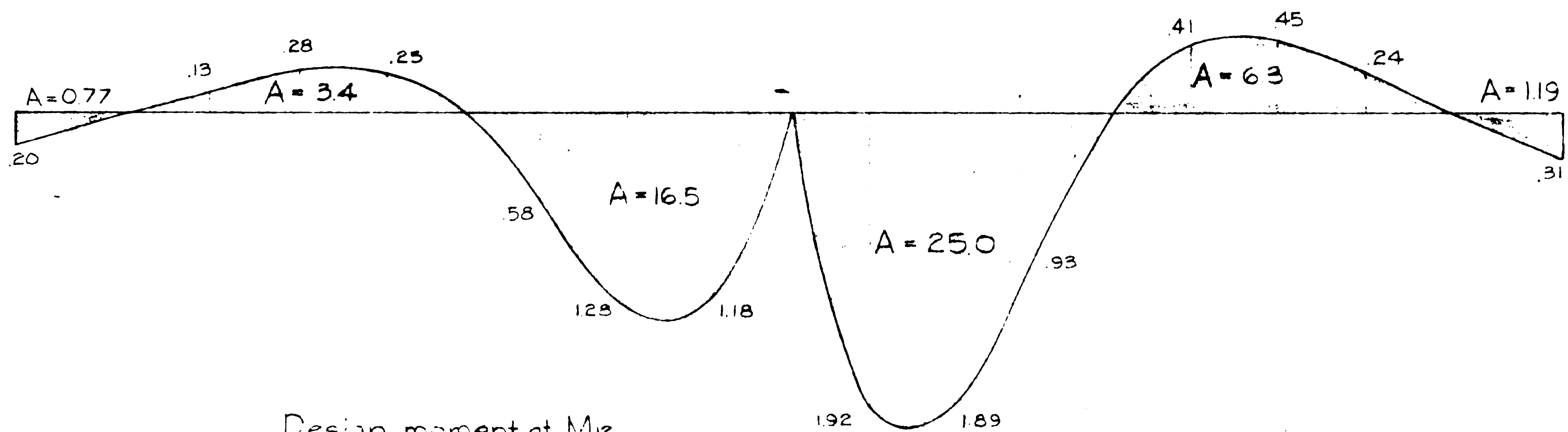
scale:
horizontal − 1" = 10'-0"
vertical − 1" = 1.0k



DL moment:

LL moment:

M_u

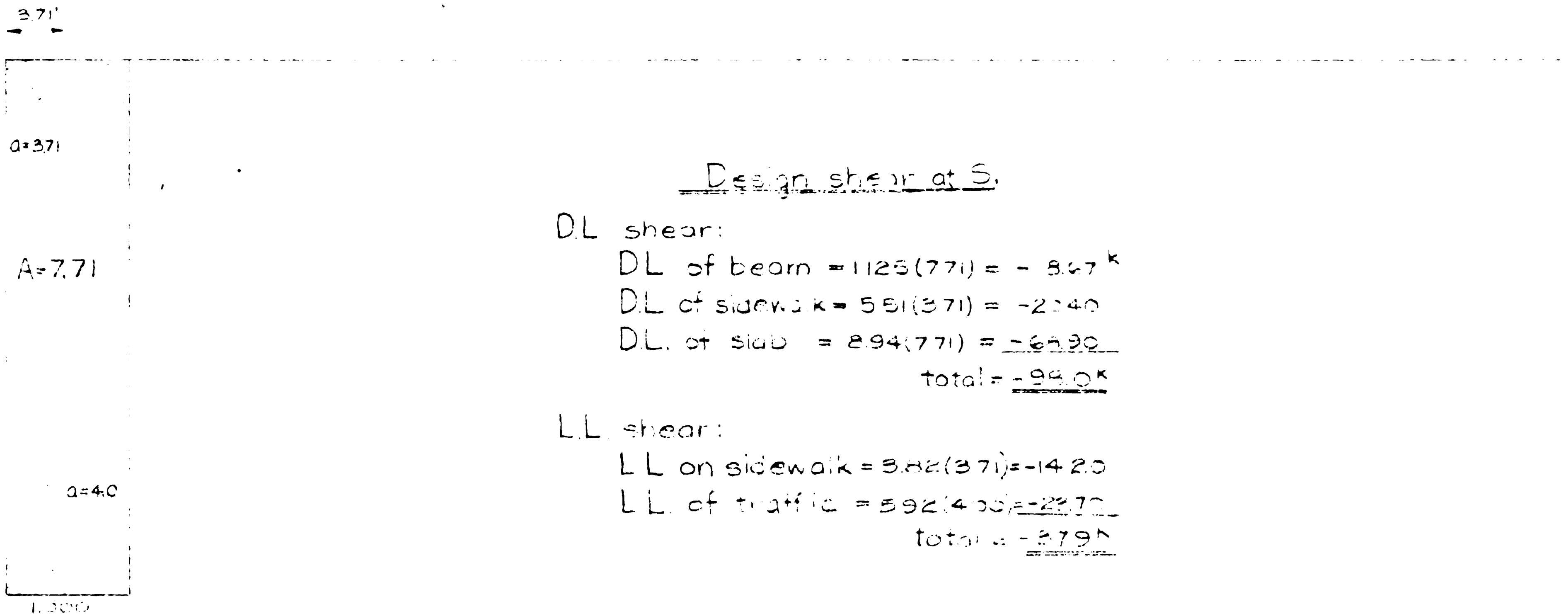


Design moment at M_{12}

See M_{11}

M_{12}

scale:
horizontal - 1" = 10'-0"
vertical - 1" = 1.0K#



Design shear at S_1

D.L shear:

DL of beam = 1125(771) = − 8.67^k

D.L of sidewalk = 551(371) = −2[illegible]40

D.L. of slab = 8.94(771) = −68.90

total = −99.0^k

L.L shear:

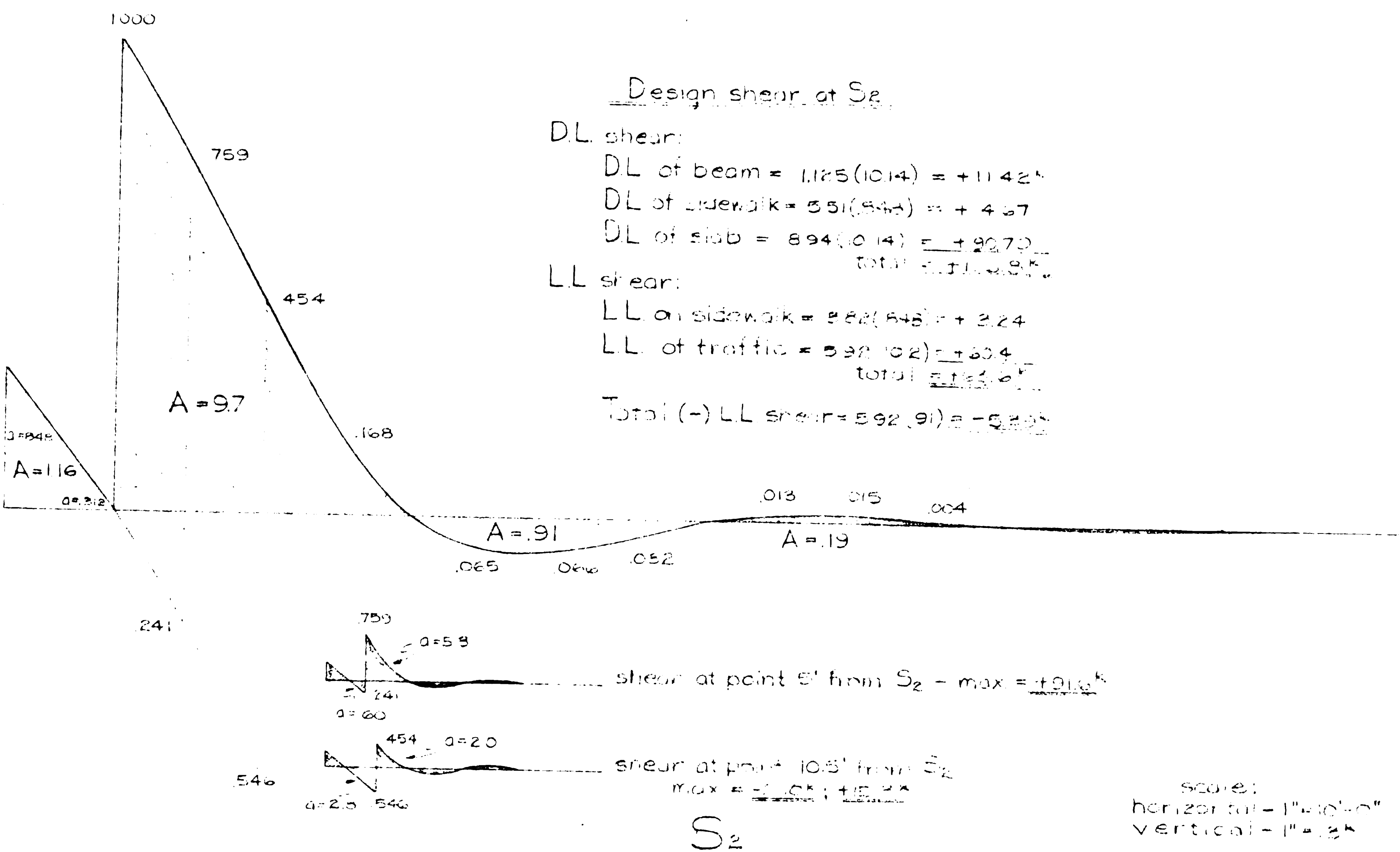
LL on sidewalk = 3.82(371) = −14.20

L.L. of traffic = 592(400) = −23.70

total = −37.9^k

S_1

scale:
horizontal − 1" = 10'-0"
vertical − 1" = 3^k



Design shear at S_2

D.L. shear:

D.L of beam = 1.125(10.14) = +11.42^k

D.L of sidewalk = .551(8.48) = +4.67

D.L of slab = .894(10.14) = +92.70

total = [illegible]^k

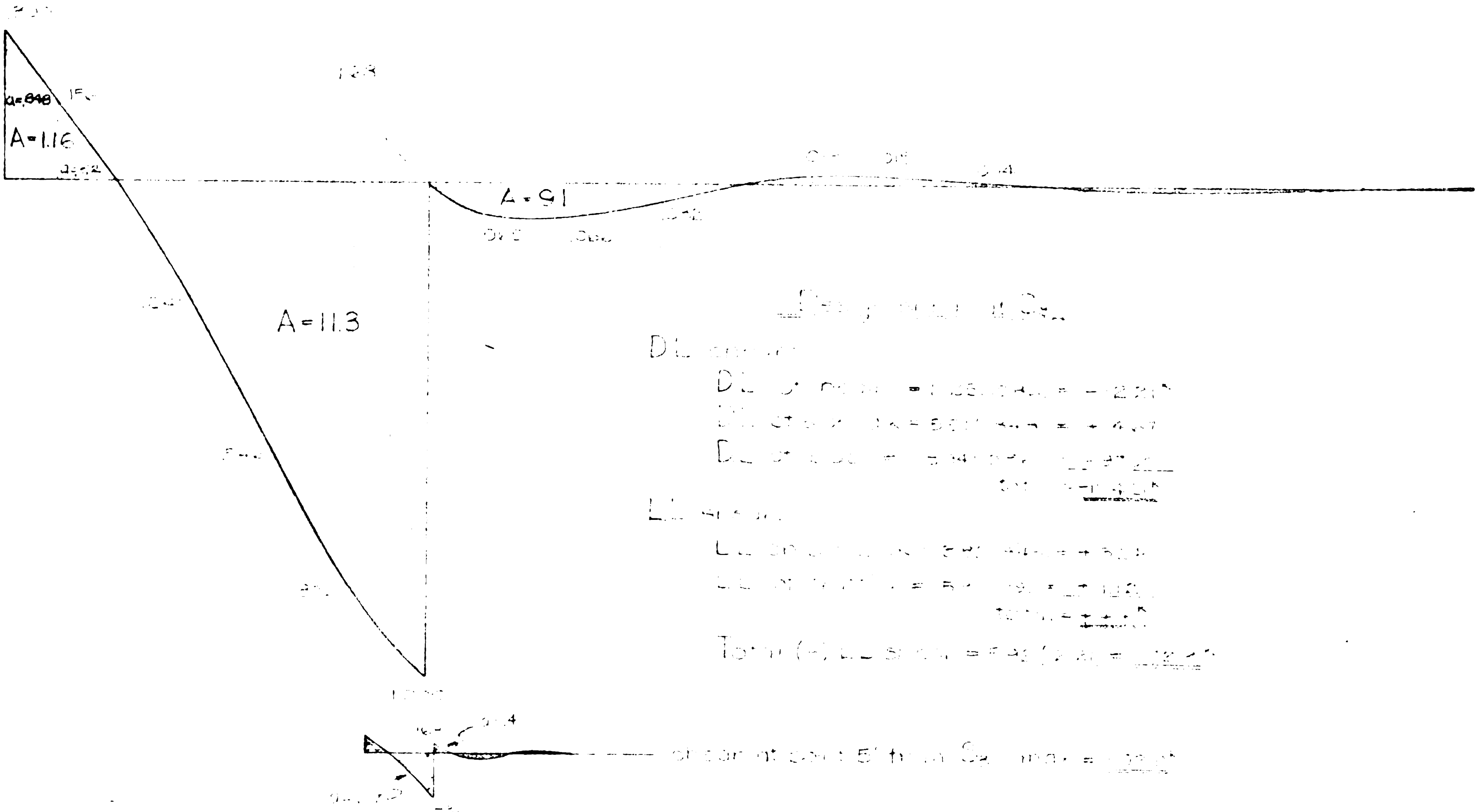
L.L shear:

L.L. on sidewalk = .882(.848) = +3.24

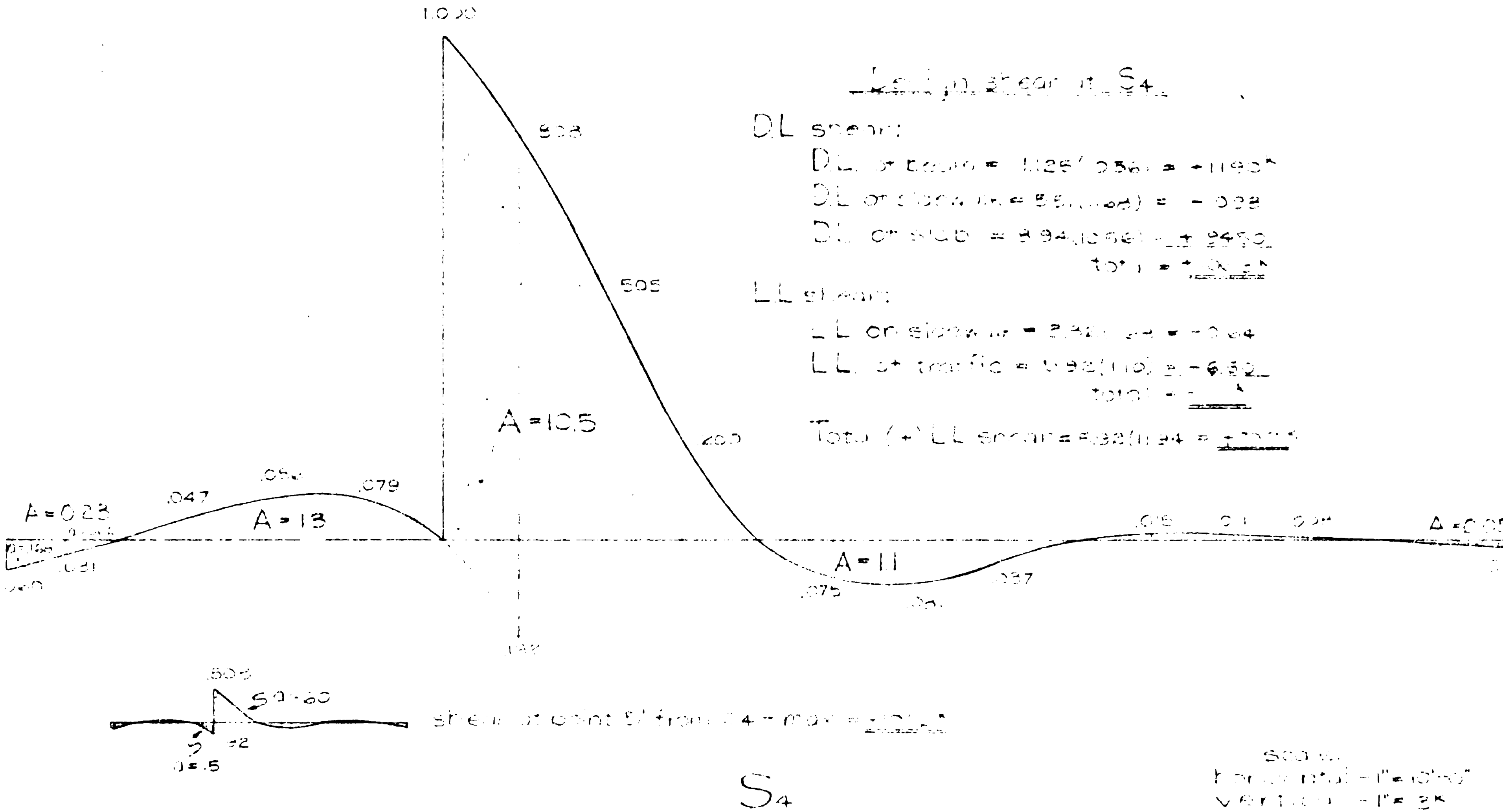
L.L. of traffic = .592(102) = +60.4

total = [illegible]^k

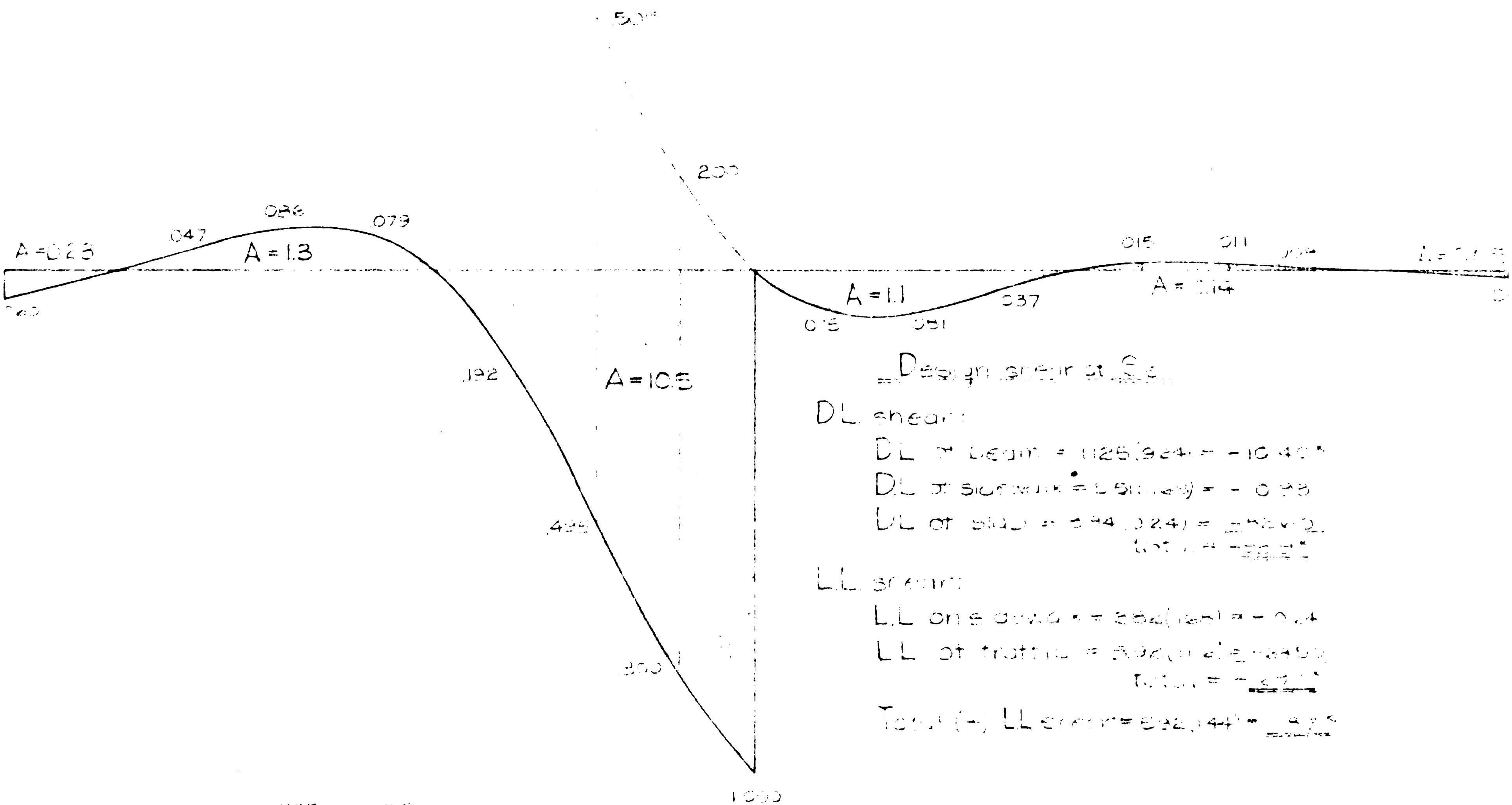
Total (–) L.L shear = .592(.91) = –[illegible]^k



S_8



1.000
.908
.505
.200
A=10.5
A=.023
.047
.079
A=.13
.031
A=.11
.075
.037
.018
A=0.05
DL shear:
LL shear:
.502
S4



Design shear at S_5

DL shear:

DL on beam = .1125(92.4) = −10.40[illegible]

DL of sidewalk = 1.5(...) = − 0.98

DL of slab = .544(.324) = [illegible]

total = [illegible]

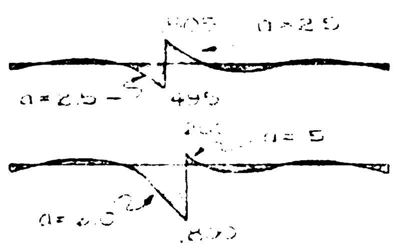
LL shear:

LL on sidewalk = .582(...) = − 0.[illegible]

LL of traffic = [illegible]

total = [illegible]

Total (+) LL shear = .582(.144) = [illegible]

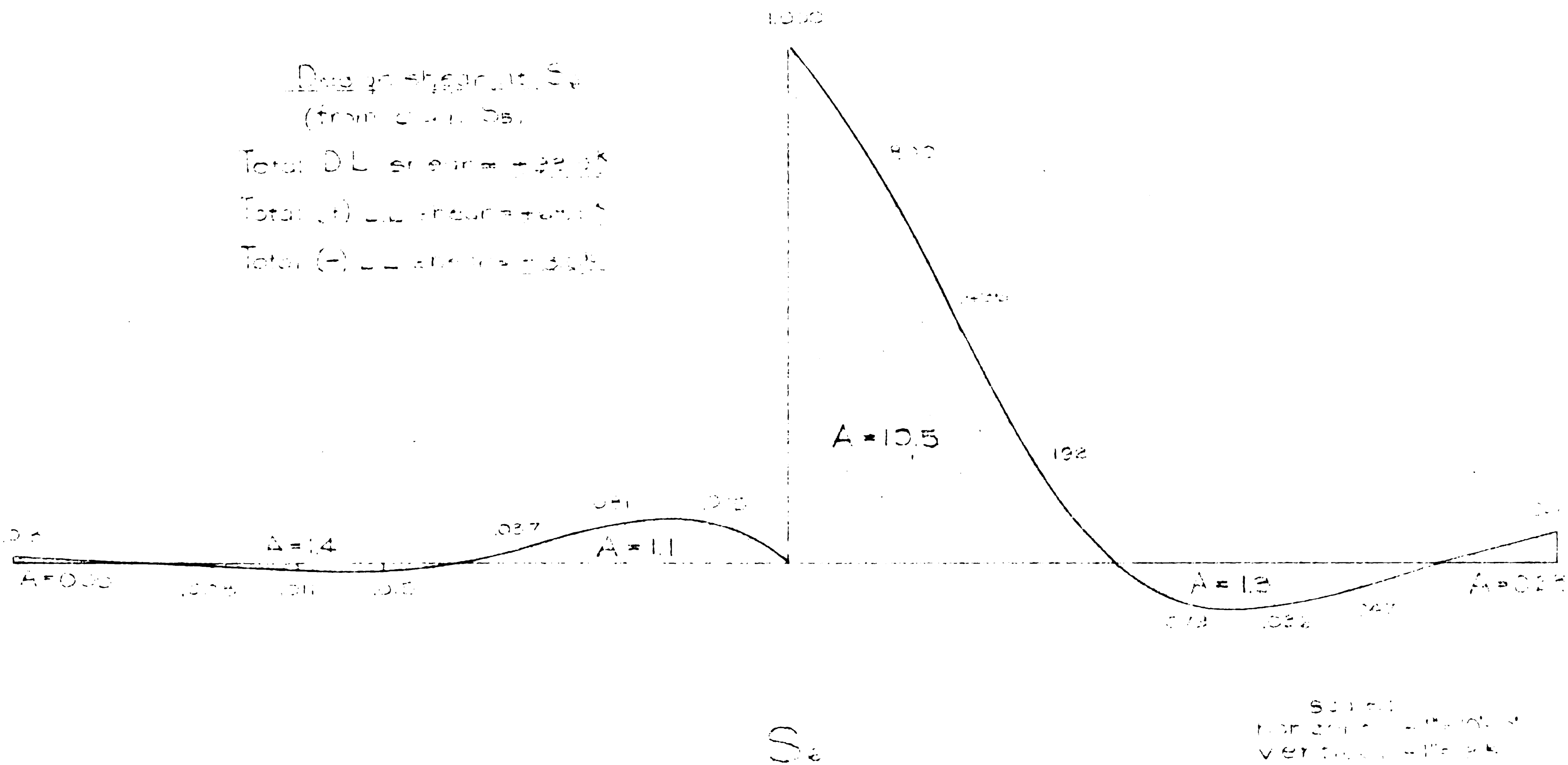


shear at point 10.5' from S_5 − max = +[illegible] : [illegible]

shear at point 5' from S_5 − max = −[illegible]

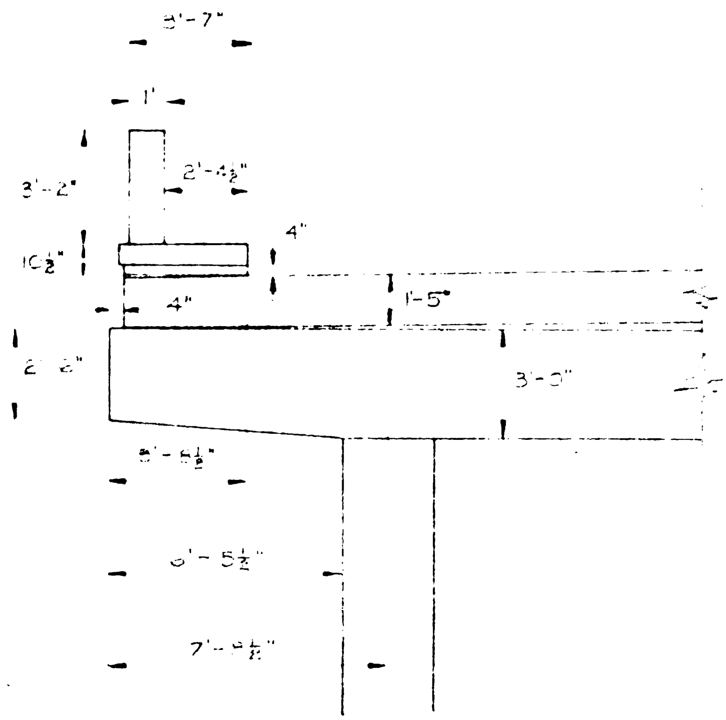
S_5

scale:
horizontal − 1" = 10'-0"
vertical − 1" = .3"


1.000
A = 10.5
192
.037
A = 1.1
A = 1.4
A = 1.3

ANALYSIS OF CROSS-BEAM

LOAD DISTRIBUTION



Maximum load on beam due to train & weight of slab:

* total DL = [illegible]^k/ft width of slab
total LL = 532^k/ft width of slab

Maximum load on beam due to sidewalk load:

DL = $150^{\#}/ft^3$ ([illegible] ft) = [illegible]/ft width [illegible]
LL = $85^{\#}/ft^2$ = .085^k/ft width of [illegible]

* total DL = (443)([illegible]) = [illegible]^k/ft width of sidewalk
total LL = (443)([illegible]) = [illegible]/ft width of sidewalk

* see diagram R8

MAXIMUM MOMENTS AT SECTIONS ALONG BEAM:

SPAN	SECTION	D L MOM (k#)	LL + IMP MOM (k#)	TOTAL (k#)
0	M_1 - 0'	-417.9	-130.4	-548.3
1 & 4	M_2 - 0'	-404.8	-186.2	-591.0
		-404.8	+19.6	-385.2
	M_3 - 5.0'	+6.0	-44.6	-38.6
		+6.0	+116.8	+122.8
	M_4 - 10.5'	+170.2	-56.8	+113.4
		+170.2	+190.5	+360.7
	M_5 - 16.0'	+24.8	+85.0	+109.8
		+24.8	-71.0	-46.7
	M_6 - 21.0'	-38.1	+36.7	-1.4
		-38.1	-275.0	-313.1
2 & 3	M_7 - 21.0'	-334.9	+40.6	-294.3
		-334.9	-259.0	-593.9
	M_8 - 26.0'	+40.2	+84.5	+124.7
		+40.2	-74.6	-34.4
	M_9 - 31.5'	+184.7	+174.8	+359.5
		+184.7	-75.2	+109.5
	M_{10} - 37.0'	+82.9	-72.6	+10.3
		+82.9	+124.8	+207.7
	M_{11} - 42.0'	-337.8	-261.5	-599.3
		-337.8	+57.4	-280.4

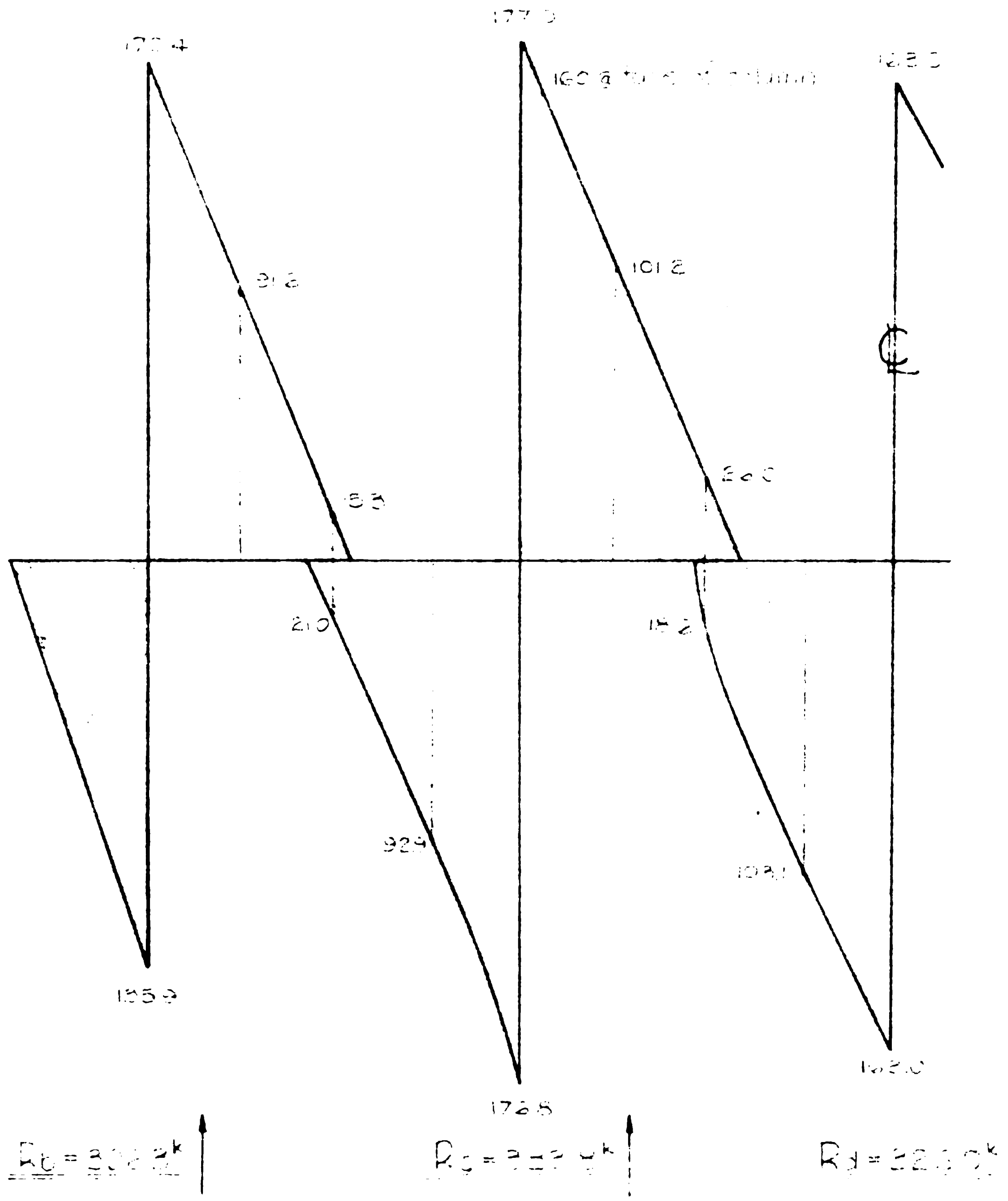
maximum positive moment = +360.7 k# @ M_4

maximum negative moment = -593.9 k# @ M_7

MAXIMUM SHEAR ALONG CROSS-BEAM:

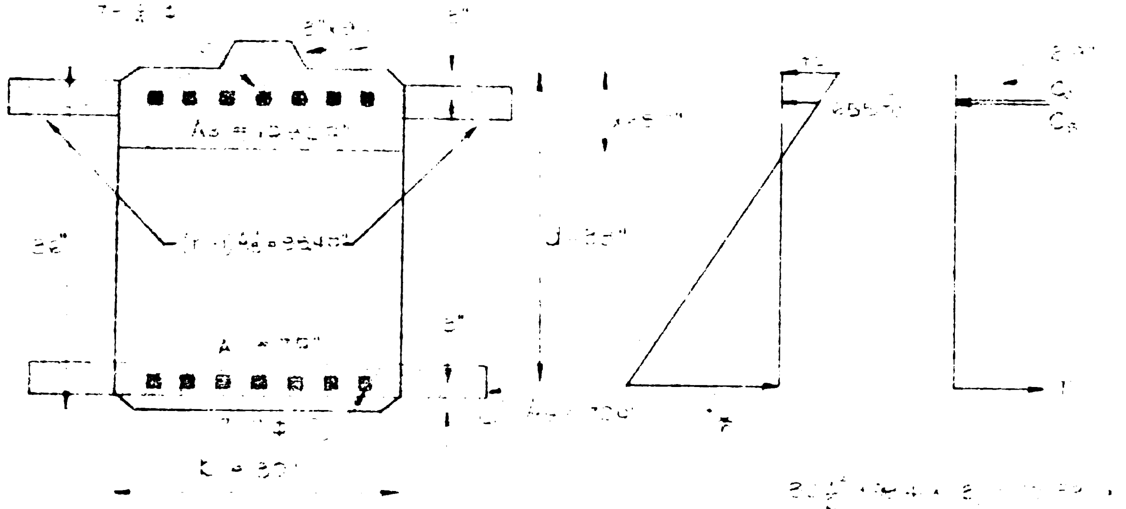
SPAN	SECTION	D.L. SHEAR (K)	L.L. SHEAR (K)	TOTAL
0	S_1 – 0	– 98.0	–37.9	–135.9
1 & 4	S_2 – 0	+106.8	+63.6	+170.4
	5.0'			+ 3.6
	10.5'			+ 15.3
	10.5'			– 21.0
	16.0'			– 92.9
	S_3 – 21.0'	–104.3	–72.3	–176.5
2 & 3	S_4 – 0	+106.3	+70.7	+177.0
	5.0'			+10.2
	10.5'			+20.0
	10.5'			– 18.6
	16.0'			–109.1
	S_5 – 21.0'	–93.9	–69.1	–163.0
3	S_6 – 0	+93.9	+69.1	+162.0

MAXIMUM TOTAL SHEAR DIAGRAM:



Scale:
horizontal
vertical

CHECKING BEAM AT POINT OF MAX POSITIVE MOM.



Compression [illegible]

C_s = [illegible]

[illegible]

[illegible]

[illegible] $^{\#}$

* [illegible] $^{\#}$

[illegible] $^{\#}$

C_c = [illegible] $^{\#}$

[illegible]

* [illegible] $^{\#}$

[illegible] $^{\#}$

[illegible] $^{\#}$

[illegible]

[illegible]

[illegible]

[illegible]

x = [illegible]

x = [illegible] / 2

x = 8.7"

* note:

Mich. State Highway Dept. [illegible]

[illegible]

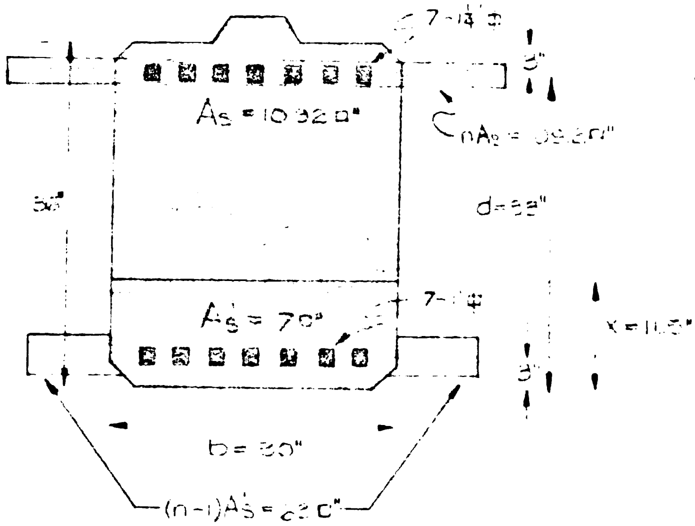
[illegible]

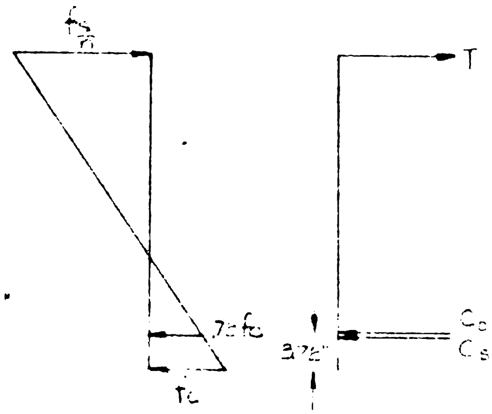
[illegible] $^{\#}$

[illegible]

[illegible] $^{\#}$

CHECKING BEAM AT POINT OF MAX NEGATIVE MOM.





$$30\frac{x^2}{2} + 63(x-3) = 109.2(33-x)$$

$$15x^2 + 63x - 189 = 3600 - 109.2x$$

$$15x^2 + 172.2x - 3789 = 0$$

$$x^2 + 11.5x - 253 = 0$$

$$x = \frac{-11.5 \pm \sqrt{132.2 + 1012}}{2}$$

$$x = \frac{-11.5 + 33.8}{2}$$

$$x = 11.15''$$

Compression ... C ... [illegible]

$C_s = (.73f_c)(63) = 46f_c \times 30'' = 1380f_c$

$C_c = (\frac{1}{2}f_c)(30 \times 11.5) = 167f_c \times 29.25'' = 4890f_c$

$213f_c \qquad 6270f_c$

$jd = 29.4''$

$6270f_c = 593{,}900 \times 12$ in-#

* $f_c = 1132\ \#/\square''$

all. value $= 1850\ \#/\square''$

$C = T = 213f_c = 242{,}000\#$

$T = A_sf_s = 242{,}000\#$

* $f_s = 22{,}120\ \#/\square''$ allowable $= 18{,}000\ \#/\square''$

$\frac{f_s'}{n} = .73f_c = 830$; $f_s' =$ [illegible] $\#/\square''$

Since the entire beam was reinforced as illustrated, it was not necessary to check any other section for moment.

*note:

By Mich State Highway Dept Method;

$M = (-354.3 -$ [illegible]$) = -$ [illegible] in-#

$f_c = 1132\ \#/\square''$

$f_s = 22{,}1$[illegible] $\#/\square''$

CHECKING BOND:

$$u = \frac{V}{\Sigma o j d}$$

At point of max negative moment; $\Sigma o = 35''$

$V_{max.} = 160,000^{\#}$ at face of column

$$u = \frac{160,000}{35 \times 29.4} = \underline{\underline{155}}\ ^{\#}/_{\square''}$$

allowable = 150 $^{\#}/_{\square''}$ with special anchorage

Without impact, $V = 143,000^{\#}$

$$u = \underline{\underline{139}}\ ^{\#}/_{\square''}$$

At point of inflection; $\Sigma o = 29''$

From max. total moment table, point of inflection $\cong 6'$ from support.

From max. total shear diagram;

$V \cong 108,000^{\#}$

$$u = \frac{108,000}{28 \times 30.05} = \underline{\underline{122}}\ ^{\#}/_{\square''}$$ allowable = 100 $^{\#}/_{\square''}$ without S.A.

Without impact, $V \cong 90,000^{\#}$

$$u = \underline{\underline{107}}\ ^{\#}/_{\square''}$$

CHECKING SHEAR:

$$v = \frac{V}{bjd}$$

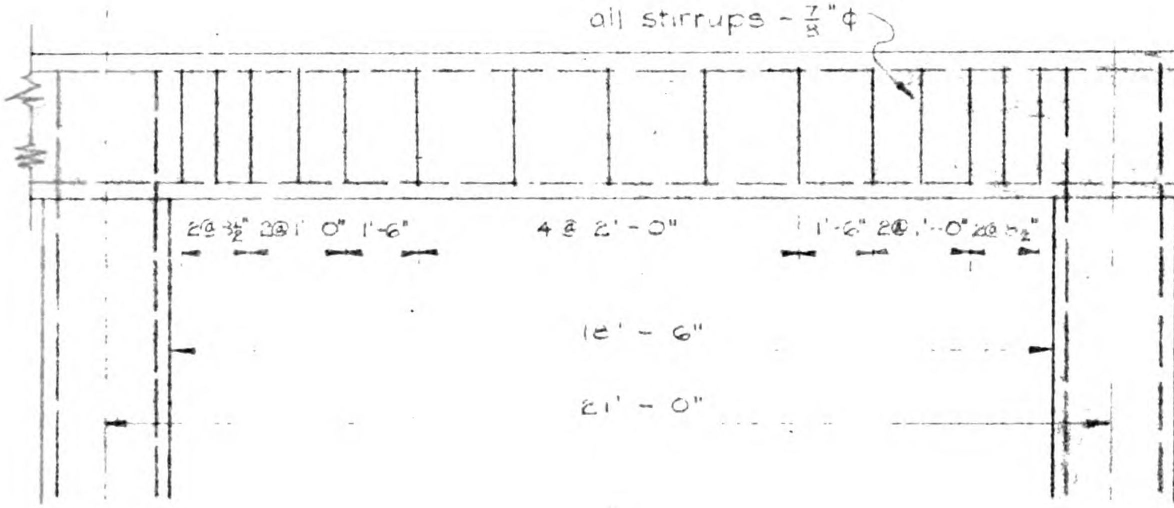
V_{max} without impact = 143,000$^{\#}$

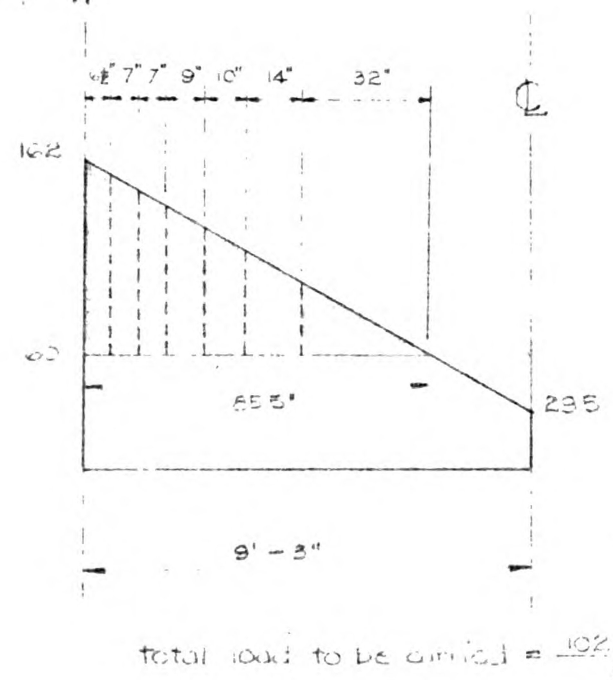
$$v = \frac{143,000}{30 \times 29.4} = \underline{\underline{162}}\ ^{\#}/_{\square''}$$

allowable = 180 $^{\#}/_{\square''}$ with web reinforcement & S.A.

CHECKING VERTICAL STIRRUPS:

Since the stirrup spacing is symmetrical for each span; and also symmetrical about the centerline of the span, only one half of the span is analyzed for max. shear





$V_{max.} = 162\ \#/\square''$

$V_{min.} = \frac{26000}{30 \times 29.4} = 29.5\ \#/\square''$

(from total shear diag.)

with 7/8"φ each stirrup can take:

$(1.20\square'')(16,000\ \#/\square'') = 19,200\ \#$

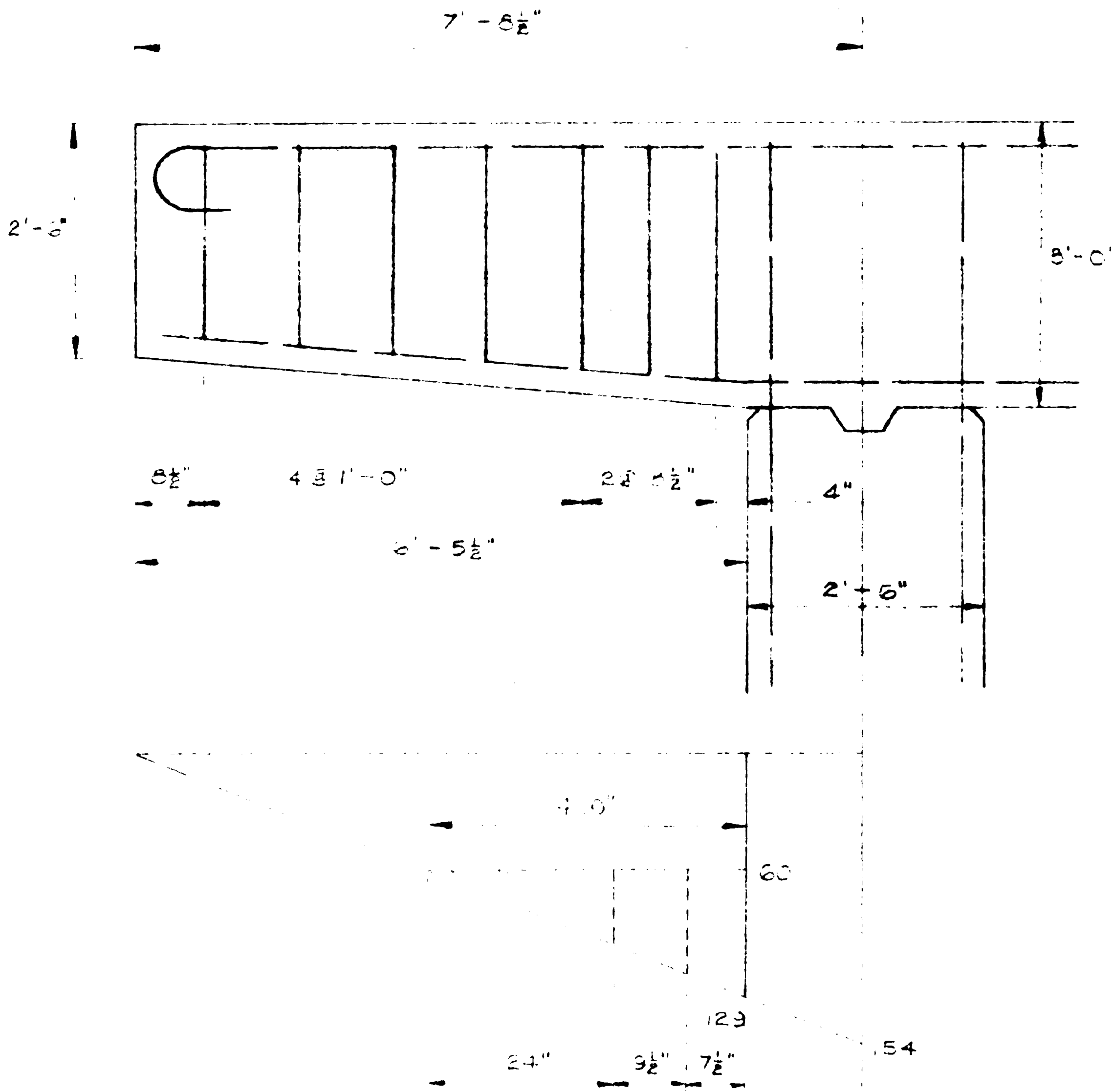
total load to be carried $= \frac{102 \times 85.5}{2}(30) = 131,000\ \#$

number of stirrups required $= \frac{131}{19.2} = 6.8$, say 7

number provided = 7.

theoretical spacing shows that stirrups should be closer together.

CHECKING CANTILEVER SECTION OF CROSS-BEAM



$$v = \frac{V}{b_j d} = \frac{125{,}900}{30 \times 29.4} = 154\ \#/\square''$$

each stirrup takes 19,200 #

total load to be carried = $\frac{4\ \ \ \ }{2}$ (60) = 42,400 #

number of stirrups required = $\frac{42.4}{19.2}$ = 2.2 say 3

number provided = 7

Theoretical spacing shows that stirrups are properly placed.

CONCLUSION

Due to the limited time allowed for the writing of this thesis, the author was not able to go through the complete analysis. For anyone wishing to complete this work a suggestive outline is:

I - Analysis of pier.

  A - Check column.

  B - Check footing.

II - Analysis of abutment *

  A - Check the four cases specified by Mich. S.H.D.

III - Analysis of railing.

The completed analysis of the slab showed that the concrete and steel stresses are within allowable limits. Upon a comparison of this work with the design by the Mich S.H.D., it was found that the design was based on the H20 loading while the H20 - S16 loading was used for this analysis. This difference was somewhat compensated by the fact that an additional dead load for future wearing surface was included in the design but not in this analysis.

The crossbeam analysis showed that the steel is over-stressed by 2000#/□" for negative moment (without impact). This can be due to the different method used for design and this analysis. The highway dept. used the approximate method of loading the various spans and using the maximum effect thus obtained; whereas influence line diagrams were drawn for this analysis, and maximum effect was obtained by properly loading the diagrams. The difference in loading can

* see thesis by Robert C McLarvy for this analysis.

Bond and shearing stresses for both the slab and beam are within allowable limits.

The object of this thesis - that of putting into application some of the theory learned in a course in indeterminate structures — was fulfilled.

BIBLIOGRAPHY:

The American Association of State Highway Officials, Standard Specs. for Highway Bridge, Fourth Edition.

L. C. Maugh, Statically Indeterminate Structures, John Wiley and Sons.

Portland Cement Association, Continuous Concrete Bridges, Second Edition.

Hale Sutherland and Raymond C. Reese, Introduction to Reinforced Concrete Design, John Wiley and Sons.

American Concrete Institute, Reinforced Concrete Design Handbook, First Edition.