WORD PROBLEM STRUCTURE AND ITS EFFECT ON THE TRANSFER OF LEARNING TO SOLVE ALGEBRA WORD PROBLEMS

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ABSTRACT

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A problem in learning to solve mathematics word problems students have been facing is to transfer the learned problem-solving knowledge from one story context to another story context. Some studies have provided evidence showing that structure facilitates transfer of learning to solve word problems. However, it is still under development for what algebra word problem structures students can recognize and what kinds of structures are effective for what kinds of algebra word problems regarding transfer of learning. In this dissertation, I explored the following three questions: (1) "What are the structures that students can recognize in the domain of algebra word problem?" (2) "What are the difficulties students will encounter when trying to find structures of algebra word problems?" and (3) "Are particular structures helpful in teaching for transfer of learning to solve algebra word problems?" Sixty-one college students participated in a 2-hour controlled experiment and 10-minute one-to-one interview. The results showed several word problem structures students recognized or created, and multiple levels of difficulties students encountered when trying to structure algebra word problems. The results also showed that students who received structure-based instruction had better performance in some types of transfer of learning to solve algebra word problems.

DEDICATION

"Then you will know the truth, and the truth will set you free"
John 8:32
~This dissertation is dedicated to the ***Lord***, my lovely wife ***Ya-Wen Lin***~
and my two children ***Ian*** and ***Esther***

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Introduction

A vision of school mathematics education is to help students understand and be able to use mathematics in everyday life and in the workplace (NCTM, 2000). Word problems play an essential role for achieving the vision to help students appreciate mathematics in daily life and learn to solve real-life problems using mathematics.

However, it has been reported that students have various difficulties in learning to solve word problems. The difficulties include comprehending word problems (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980; Clement, 1982; Cummins, Kintsch, Reusser, & Weimer, 1988; Kintsch & Greeno, 1985; Herscovics & Kieran, 1980, Lewis & Mayer, 1987; MacGregor & Stacey, 1996; Nathan, Kintsch, & Young, 1992), translating word problems into equations (Stacey & MacGregor, 1999; Roth, 1996), and the transfer of solving word problems (Bassok & Holyoak, 1989; Fuchs et al., 2003, 2004; Gick & Holyoak, 1983; Hayes & Simon, 1977; Holyoak & Koh, 1987; Nickerson, Perkins, & Smith, 1985; Reed, Ernst, & Banerji, 1974; Reed, 1999).

The focus of this dissertation is to investigate students' difficulty in the transfer of learning to solve algebra word problems. The transfer issue means using previously learned knowledge and problem-solving skills to solve word problems whose contexts are different from those word problems solved before, or whose embedded equations are different from the equations embedded in the word problems solved before. The word problems that are different from word problems solved before, in this study, are called *transfer word problems*.

Why is the transfer of learning to solve word problems important? First, transfer has been claimed "the fundamental goal of education" (Marini & Genereux, 1995) or "the ultimate aim of teaching” (Macaulay, 2000). Marini & Genereux remarked that education would have failed if students could not apply (or transfer) what they learned in class in settings outside of school. Indeed, the vision of mathematics education is to enable students to apply mathematics learned in classrooms to real life like making purchasing decisions or choosing insurance or health plans (NCTM, 2000). Second, word problems have played the central role helping students apply formal mathematics knowledge and skills to real-life-like situations. If students cannot learn to solve word problems that are slightly different from what they solved before, then mathematics education for real-life will have failed. Third, unfortunately, the transfer of learning to solve word problems has been reported difficult to achieve (Catrambone & Holyoak, 1989; Fuchs, 2003; Gick & Holyoak, 1983; Holyoak & Koh, 1987; Reed, 1999; Ree, Dempster and Ettinger, 1985). Some studies (Catrambone & Holyoak, 1989; Reed, Dempster and Ettinger, 1985) showed that students even couldn’t apply problem-solving skills that they just learned to solve similar or analogical (word) problems.

Structure has been reported helpful in the transfer of learning to solve word problems (Bassok & Holyoak, 1989; Catrambone & Holyoak, 1989; Cooper & Sweller, 1987; Gick & Holyoak, 1983; Holyoak and Koh, 1987; Kaminski, Sloutsky and Heckler, 2008). Some additive or multiplicative structures have been reported helpful in solving elementary word problems (Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004; Jitendra, A. K., Griffin, C., Deatline-Buchman, A., & Sczesniak, E., 2007; Xin, 2008). Particularly, structure can help subjects organize and discriminate information of word problems and problem-solving skills, and help subjects recall types of problems and their associated solution methods when the subjects encounter similar/novel problems (Bassok & Holyoak, 1989; Blessing & Ross, 1996; Fuchs et al., 2004; Gick & Holyoak, 1987).

It is still not clear what kinds of structures are effective for what kinds of algebra word problems regarding transfer of learning. Although there is increasing evidence showing that certain structures are effective in learning to solve arithmetic word problems in the domain of elementary mathematics, still few studies concern algebra word problem structures and the structures' effect in transfer of learning. It is worth our attention that these few studies were mainly conducted by psychologists with emphasis on mental process and representation (e.g., what process induces good quality of schema or what quality of schema enhances transfer). Still, little research has been done by mathematics education researchers concerning what classifications of algebra word problem structures could be that would be helpful in teaching for transfer.

In addition, it has been reported that, for students, algebra word problem structures are difficult to recognize (Reed, 1989; 1999). Algebra word problems are more complex and require domain-specific knowledge than general word problems that had been studied in the psychology field (Reed, 1989). It requires substantial expertise and training to recognize the structures (Chi et al., 1982; Schoenfeld & Herrmann, 1982).

Hence, the purpose of the study is to answer the following three questions:

1. What are the structures of algebra word problems students can recognize?
2. What are the difficulties that students encounter in finding structures of algebra word problems?
3. Are particular structures helpful in teaching for transfer of learning to solve algebra word problems?

This study defines structure of a word problem as the particular components (e.g., objects, events) and the relationships between the components (Fuchs et al., 2004; Mayer, 1981). It is inherent in the word problem. For example, the following are two word problems.

Problem 1:

Two cars run away from each other on a straight road. One car is 25 miles per hour faster than the other car. After 3 hours, they are 225 miles apart. Find their speeds.

Problem 2:

Two pumps are used to fill a water storage tank at a resort. One pump can fill the tank 25 gallons per hour faster than the other one. When both pumps are operating together, the tank can be filled in 3 hours. The capacity of the tank is 225 gallons. Find the filling rate of each pump.

The two word problems have the same structure. The components are two events (i.e., two running cars or two running pumps), and the relationships between the components are: (1) the rate of one event is faster than the rate of the other event (i.e., 25 miles per hour faster or 25 gallons per hour faster), and (2) the two events end up with a certain time-rate relationship (i.e., after 3 hours, the two cars are 225 miles apart or the tank is filled of 225 gallons of water).

Problem type of word problems is the structure of word problems constrained by real-life contexts. For example, for the above problem 1 and 2, they have the same structure. However, they belong to different problem types. If we constrained the structure (the two components and the two relationships) by Distance-Rate-Time context (i.e., the events are constrained by car running and the rate is constrained by speed and time), then the structure with the associated context becomes the problem type of Distance-Rate-Time (like problem 1). Similarly, if we constrained the structure by filling-with-pump context (i.e., the events are constrained by pump filling and the rate is constrained by gallons per minute), then the structure with the associated context becomes the problem type of filling-with-pump (like problem 2). To sum up, problem type of word problems is the word problem structure constrained by a specific context.

Schema is the mental representation of structure. Since it is "mental representation", different subjects may have different mental representations for the same structure, for example, a diagram, a list of components, or images of objects, events, and situations (Armbruster, 1996; Dansereau, 1995). To illustrate, for the word problem of problem 1, one may mentally conceptualize the structure of problem 1 and represent it mentally as the diagram as follows (See Figure 1):

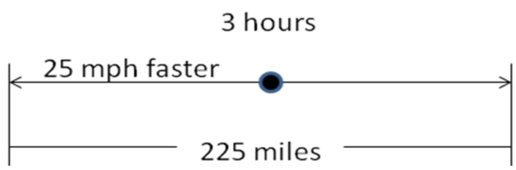

Figure 1: Mental Representation of Problem 1, Version 1

One may also use a diagram, but a different diagram, to conceptualize problem 1 mentally (See Figure 2)

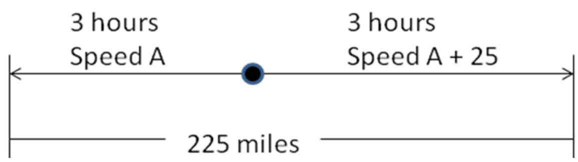

Figure 2: Mental Representation of Problem 1, Version 2

Another may mentally conceptualize the structure of problem 1 using a list of algebraic expressions:

$$A + B = 225$$

A is 25mph faster than B

A and B: the events of car running

In brief, schema is individual dependent. It depends on how individuals conceptualize structure. The same word problem structure could have various schemas created and exist in the mind of different individuals.

In summary, a word problem structure is more fundamental (or abstract) than a problem type because it is not constrained by any specific real-life context. Regarding schema, individuals may conceptualize a word problem structure mentally in various ways (e.g., a diagram or different diagrams or a component list) It is worth our attention that the three terms “structure”, “problem type” and “schema” are not well distinguished in the literature, and sometimes they are used as interchangeable. You will find the phenomenon in the next chapter, the review of the literature.

The one-variable linear equation word problem, as one kind of algebra word problems, was chosen for this study for the following three reasons. First, one-variable linear equations represent an important initial topic in algebra. Mastery of this topic plays a critical role in almost all subsequent mathematical courses and topics. Second, international assessments show that U.S. students have been facing difficulties and don’t do well in algebra, which includes solving algebra word problems (Blume & Heckman, 1997; Schmidt, McKnight, Cogan, Jakwerth, & Houang, 1999). Third, a literature search (e.g., ERIC), shows that little research has been conducted on the transfer of learning to solve linear equation word problems, although there are many studies about the comprehension and translation of linear equation word problems, for example, how to translate word statements to algebraic expressions and how to translate them correctly (Clement, 1982; Herscovics & Kieran, 1980; MacGregor & Stacey, 1996).

Since it is difficult and requires substantial training for students to recognize algebra word problem structures, and there are different qualities of word problem structures as identified in the literature, this study hypothesized that students would recognize different qualities of structures, and encounter different levels of difficulties with respect to the different qualities of structures, when asked to structure algebra word problems.

Hypothesis One: There will be various levels of sophistication in the algebra word problem structures that students recognize.

Hypothesis Two: There will be various levels of difficulties students encounter in structuring algebra word problems.

According to the cognitive psychology literature, structure or schema have been proven facilitating transfer in the general word problem domain. Teaching structures have been proved helpful in facilitating transfer in the domain of arithmetic. However, it is still unclear whether structure can also facilitate transfer in the domain of algebra word problems. Therefore, this study hypothesized teaching students algebra word problem structures could facilitate transfer.

Hypothesis Three: Students who receive structure-based instruction will have better performance in solving transfer algebra word problems, compared to students who receive non-structure-based instruction.

According to the literature, different qualities of structure have different effects in transfer. Is this also true in the domain of algebra word problems? This study hypothesized students who would recognize different qualities of structure have different performance in solving transfer word problems.

Hypothesis Four: Students who recognize different qualities of structure have different performance in solving transfer problems in the domain of algebra word problems.

Chapter 2

Review of Literature

Six elements that shape this study will be discussed in this literature review section. First, many studies have proved that transfer of learning is difficult to achieve. Some methods for improving transfer of learning to solve word problems which seem promising but in fact do not facilitate transfer will be discussed. Second, many studies also found that structure could facilitate transfer, especially for those good-quality structures. Views on the quality of structure will be discussed. Third, some studies have reported that it is difficult to recognize word problem structures. Factors that make it difficult to recognize will be discussed. Fourth, many studies have proved that teaching students schemas (or structures), rather than inducing structures solely by students themselves, can facilitate transfer. The definition of schema and some schema-based instruction methods will be discussed. Fifth, what algebra word problem structures can we teach to students to improve transfer? Some algebra word problem structures will be discussed. Sixth, how do we assess degree of transfer? Some measures of transfer will be discussed.

It Is Difficult to Solve Transfer Word Problems

This section is to present claims about the difficulty of achieving transfer in the domain of word problems, and to discuss several unsuccessful methods for improving transfer although they seem promising, which are reading and summarizing similar problems, studying the solution of a similar problem, and referring to the solution of a similar problem.

Transfer of learning has been claimed difficult to achieve (Cooper & Sweller, 1987; Fuchs, 2003; Catrambone & Holyoak, 1989; Gick & Holyoak, 1983; Holyoak & Koh, 1987; Detterman, 1993; Reed, 1999; Reed, Ernst, and Banerji, 1974; Reed, Dempster, and Ettinger, 1985; Macaulay, 2000; Marini & Genereux, 1995). Detterman (1993) reviewed several reviews concerning transfer (Baldwin and Ford, 1988; Singley and Anderson, 1989) and concluded "There have been several recent reviews concerning transfer….Reviewers are in almost total agreement that little transfer occurs" (p. 8). Marini and Genereux (1995) in their book chapter "The Challenge of Teaching for Transfer" noted "Unfortunately, achieving significant transfer of learning has proven to be a difficult chore" (p. 1). Reed (1999) in his book "Word Problems" described "Ideally, teachers would like students to transfer a learned solution to other problems that are solved in the same way. Unfortunately, this doesn't always occur because two problems that have identical solutions may be described so differently that it is not obvious that they share a common solution." (p. 103) ADD

Catrambone and Holyhoak (1989) found that examining similarities between problems did not facilitate transfer when students did not know how the to-be-solved transfer problems were related to the problems solved previously. They asked 66 college students to read and summarize two story analogs. After that, 35 students (the comparison group) were asked to write descriptions of similarities between the two stories, and the other 31 students summarized the two stories again. With a 30-minute delay (working on another task for the purpose of changing context), both groups of students were asked to solve an analogical problem. The results showed no significant difference in solving the (analogical transfer) problem between the two groups when students were not told that those stories they read previously were related to the analogical problem.

Reed, Dempster and Ettinger (1985) found that studying the solution of a similar algebra word problem did not help transfer when subjects were informed the studied solution could help solving to-be-solved transfer problems. They randomly assigned 48 college students who were taking college algebra course into two groups. Both of the two group students studied solutions of three practice problems (the distance, mixture, and work problem type) where each of the three practice problems was followed by two problems (equivalent or similar) for students to solve (3 problems in a set, 9 problems in total). The differences between the two groups were that, first, the practice problems were related (The related group) or unrelated (the unrelated group) to their followed problems. For example, the related group studied a mixture type practice problem and solved the same type of two other problems, but the unrelated group studied an area type problem and solved the other two problems with different type (e.g., the mixture type). Second, the related group students were told that the studied solutions could help them solve their followed problems (students were not allowed to refer to the solutions), but the unrelated group students were not. The equivalent problem meant the same solution procedure, that is, the same equation and unknown. The similar problem meant the solution procedure was similar but not identical. The results showed no significant difference between the two groups in solving the equivalent and similar problems.

Reed, Dempster and Ettinger (1985) also found that studying the solution of a similar algebra word problem did not help transfer when subjects were informed the studied solution could help solving to-be-solved transfer problems and they could refer back to the solution. Reed, Dempster and Ettinger (1985) randomly assigned 48 college students who were taking college algebra course into two groups. All 48 students were told in the beginning that each of the three solutions they were going to study should help them solve its followed two test problems (9 problems in total). The difference between the two groups of students was that one group could refer to the solutions when solving test problems and the other group could not. The results showed that there was no significant difference in solving the similar problems (See above for definition) between the two groups (z=0.32). However, there was a significant difference in solving the equivalent problems (See above for definition) between the two groups (z=3.78, $p<.01$).

To sum up, the literature has suggested that transfer of learning is difficult to achieve. Particularly, studies have shown that examining similarities of similar word problems or studying the solution of a similar problem does not facilitate in solving similar problems.

Structure Can Facilitate Transfer

This section is to present claims that structure/schema can facilitate transfer, and discuss the idea of quality of structure/schema by means of two studies. Particularly, two studies show that good-quality structures/schemas facilitate transfer, but poor-quality structures/schemas do not.

Structure or schema has been claimed helpful for improving transfer of learning. Gick and Holyoak (1987) proposed structure as one of the determinants of transfer. They noted "it is important to consider the structure of the task to be initially learned and its relationship to the transfer task…transfer will often depend on the acquisition of rules that characterize a category of tasks" (p. 20-21) Catrambone and Holyoak (1989) claimed that training on the induction of abstract rules or schemas can facilitate transfer. They noted "A number of studies have shown that inter-domain transfer can be facilitated by Manipulations designed to encourage the formation of generalized rules or schemas. The most direct manipulation of this sort involves explicit instruction in abstract rules, coupled with examples, in domains such as statistics and algebra. Such abstract training can produce substantial rates of spontaneous transfer (e.g., Bassok & Holyoak, 1989; Fong, Krantz, & Nisbett, 1986)." Dansereau (1995) also mentioned the recognition of structural features can enhance transfer. He noted "…giving students explicit instructions to compare analogs structurally further enhances transfer to superficially dissimilar situations (Brown, Kane, & Echols, 1986; Catrambone & Holyoak, 1989).” (p. 105).

Gick and Holyoak (1983) showed that the quality of schema had significant influence on analogical transfer. 98 subjects were asked to read two stories and write brief summary of each, and then they were asked to write ways in which the stories were similar, and then students were asked to solve an analogical transfer problem. Descriptions of similarities were categorized into three levels: good, intermediate, and poor. The three levels of schema (or similarity) quality were determined by comparing to a target schema, which was the convergence schema. The following is the convergence schema (p. 7):

Initial state

- Goal: Use force to overcome a central target.
- Resources: Sufficiently great force.
- Constraint: Unable to apply full force along one path.

Solution plan: Apply weak forces along multiple paths simultaneously

Outcome: Central target overcome by force.

A good schema was defined as “the basic idea of having forces converge from different directions had to be present” (p. 23). An intermediate schema was defined as “schema contained only one of these major features; e.g., “in both cases many small forces were used.”” (p. 23). A poor schema was defined as “contained none of the basic aspects of the convergence principle.” (p. 23). The results showed that the better quality the schema, the more successful the transfer performance ($G^2(4)=15.8$, $p<.005$)

Holyoak and Koh (1987) investigated the influence of surface and structural dissimilarity on analogical transfer and found that only structural dissimilarity had significant influence on analogical transfer. They asked 63 undergraduates to do three tasks (40 minutes in total). Task 1asked students to read and summarize a story. Task 2 was an “unrelated task to reduce demand characteristics that might cause subjects to relate the story task to the subsequent problems.” (p. 336). Task 3 asked students to solve a (radiation) problem and write as many solutions as possible. Students were randomly divided into four groups. Group 1, 2, and 3each had 16 students, and group 4 had 15 students. In task 1, group 1 had the story that had similar surface and similar structural features compared to the test problem in task 3. Group 2 had the story with similar surface and different structural features, group 3 had the story with different surface and similar structural features, and group 4 had the story with different surface and different structural features, compared to the test problem in task 3. The authors defined surface and structural features in terms of causal relationships involved in problem situations rather than purely syntactic criteria. Surface features were those that did not change causal relationships in a problem, and structural features were the causal relationships in a problem. Two problems may have different story contexts but share similar causal relationships involved in problem situations (surface dissimilarity). The authors gave the following example:

> “…for example, in the convergence analogies investigated by Gick and Holyoak (1980), there are many differences between the fortress attacked by the general and the tumor attacked by the doctor; but because the only causally relevant aspect is that each is a centrally located target, these differences are structure preserving.” (p. 334)

In this example, the two stories (fortress attacked by the general and the tumor attacked by the doctor) have different contexts (military war vs. doctor treatment), but they share the same causal relationship that a central located target (fortress or tumor) is attacked. The authors gave another example to illustrate two problems having different structural features (structural dissimilarity):

> "…for example, that the radiation problem stated that only one X-ray source was available. This difference would block use of multiple converging forces, and hence would violate the structure of the solution plan paralleling that which succeeded in the military story" (p. 334)

In this example, the two stories (doctor treatment and military war) share different causal relationship because in the military story there are multiple forces attacking a fortress. However, there is only one force (X-ray source) attacking a tumor. The results showed that structural dissimilarity impaired total transfer (78% correct on average for the two groups having similar structural features vs. 54% correct on average for the two groups having different structural features) [$G^2(1) = 4.31$, $p < .05$]. In contrast, surface dissimilarity had no effect on total transfer (68 % correct on average for the two groups having similar surface features vs. 64% correct on average for the two groups having different surface features) [$G^2(1) < 1$].

To sum up, the two studies concluded that good-quality schemas or structural features facilitated transfer, but poor-quality schemas or surface features did not.

It Is Difficult to Recognize Structure

This section is to show that it is difficult for students to recognize structures in the domain of mathematical word problems although structure can facilitate transfer. Three factors preventing subjects from recognizing structures will be discussed, which are (1) It is hard to find a super-ordinate concept to constrain the similarities between mathematical word problems (2) It requires substantial training to recognize quality structures, and (3) Detailed comparison of isomorphic word problems doesn't facilitate the formation of structure.

Reed (1999) claimed that it may be difficult for students to find algebra word problem structures, especially in creating "appropriate" or "useful" structures for grouping problems. Reed (1999) remarked that the creation of concepts for classifying problems require that the concepts are super-ordinate to the problems but cannot be too general to sufficiently constrain the matches between problems. Regarding the super-ordinate concepts, he noted that "abstraction requires creating concepts that are super-ordinate to the quantities in the Isomorphic problems but are not so general that they do not sufficiently constraint the solution. A constraint on creating an abstract solution is that it may be difficult to find such concepts." (p. 110). To illustrate, two problems may be grouped together by the concept "travel". However, the concept "travel" may be too general to capture important relations described in and between the problems. For example, the concept "travel" doesn't capture the spatial relation "the direction of traveling" if one problem is about two cars traveling toward each other with different speeds/rates and the other problem is about one car traveling successively after the other car with different speeds/rates. Therefore, it may be difficult to find a structure grouping problems appropriately.

Schoenfeld and Herrmann (1982) investigated the relationship between problem perception and expertise and found that novice mathematical problem solvers perceived problems on the basis of "surface structure", and students who received one-month mathematical problem-solving training perceived problems more like the experts who perceived much more "deep structures". They recruited 19 undergraduates in an experiment with 11 students in the experimental group and 8 students in the control group. All of the 19 students performed a card sorting task (about 20 minutes) and a mathematics test (5 problems) in the beginning. The card sorting task was to sort 32 problems into groups based on whether they were similar mathematically in their solution methods. After that, the experimental group students enrolled the “Techniques of Problem Solving” class (taught by Schoenfeld) focusing on general mathematical problem-solving strategies called “heuristics”. “The class met for 2.5 hours per day for 18 days with daily homework assignments that averaged 4-5 hours in length” (p. 486). In the mean time, the control group students enrolled the “Structured Programming” class focusing on nonmathematical problem solving using the computer. A month later, both groups of students performed the card sorting task and took another mathematics test again. The 32 mathematical problems (including word problems) were all accessible to students with a high school background in mathematics. Each of the 32 problems “was assigned an a priori mathematical “deep structure” and a mathematical “surface structure” characterization.” (p. 486). The authors defined “deep structure” as “the mathematical principles necessary for solution”, for example, the contradiction method (or principle) for proving an uniqueness argument (e.g., a function has only one inverse or two nonparallel lines has only one intersection point). The authors defined “surface structure” as “a naïve characterization of a problem, based on the most prominent mathematical objects that appear in it (polynomials, functions, whole numbers) or the general subject area it comes from (plane or solid geometry, limits).” (p. 486), for example, words or objects described in the problem statement. The results indicated “a strong change towards deep structure perceptions on the part of the experimental group and little or no change on the part of the control group” (p. 490) regarding the card sorting task.

Reed (1989) conducted three related experiments and found that a detailed comparison of isomorphic word problems did not result in the abstraction of schema. In experiment 1, 91 students who were in college algebra classes were given three mixture problems and three distances problems. For each of the three mixture and three distance problems, students were asked to construct an equation for the first problem (Trial 1), and then match concepts (or constructed variations) of the first and the second problem and attempt to construct an equation for the second problem (Trial 2), and then attempt to construct an equation for the third problem (Trial 3). The third problem was isomorphic to the previous two problems. Students could refer to the solution of the previous problems. The 91 students were randomly distributed into three groups. Thirty-two students were in the equivalent group where the first two of the mixture and the distance problems were equivalent (share the same story context and solution procedure). Thirty students were in the isomorph group where the first two of the mixture and the distance problems were isomorphic (have different story contexts but share a common solution procedure). Twenty-nine students were in the control group who received the same two problems as students in the isomorph group but were not given the concept-matching task. The results showed that the concept-matching task (Trial 2) did not facilitate schema abstraction, neither in the equivalent group nor in the isomorph group. The equivalent and isomorph groups matched concepts better than the control group did, but the two groups did not do better (52% and 48% correct equations) than the control group (55% correct equations) in solving the third problem (Trial 3). The author conducted experiment 2 to eliminate the factor that "it is possible that providing a specific solution may have discouraged students from using an abstract solution schema" (p. 536). The materials and procedure were the same as those in experiment 1 except that students were not allowed to examine the preceding solution when solving the third problem. They recruited 85 students who were in college algebra classes, where 31, 29, and 25 students were in the equivalent, isomorph, and control group respectively. The results showed that students who matched concepts in the isomorph group (26% correct equations) did not do better than students in the control group (48% correct equations) in solving the third problem. The author remarked “The results did not support predictions made on the basis of schema abstraction” (p. 536). According to the results of experiment 2, the authors hypothesized that students failed to create schema might due to the following two reasons: (1) students did not know the correct mapping when performing concept-matching task (2) students did not create schema because the concept-matching task was not helpful in the formation of schema. The author conducted another experiment (experiment 3) giving one group of students the correct answers of the concept-matching task and telling the students the common principle for the first two problems of the mixture, and the distance problems. The author recruited 107 students who were in college algebra classes, and the students were randomly distributed into four groups. There were 27 students in the mapping group, 26 students in the principles group, 28 students in the mapping/principles group, and 26 students in the control group. The procedure for the control group was the same as experiment 1, and for each of the other three groups was the same as the isomroph group in experiment 1. However, the concept-matching task was replace with instruction material indicating how the concepts of the first two problems were matched (for the mapping, and the mapping/principle groups) or what the common principle for the first two problems were (for the principle, and the mapping/principle groups). The results indicated that “having information about how the first two problems were related did not help students solve either the second or third problems in the series” (p. 538).

To sum up, to recognize structures between isomorphic word problems is hard to achieve. To recognize deep structures between mathematical (word) problems requires substantial training.

Schema-Based Instruction

Structure, as we discussed in the above “structure can facilitate transfer” section, has been proven facilitating transfer (e.g., Catrambone & Holyoak, 1987; Cooper & Sweller, 1987; Gick & Holyoak, 1983). However, it is also suggested that structure is hard to recognize (Cooper & Sweller, 1987; Mayer et al., 1999; Reed, 1999) as discussed in the above section. On the other hand, there is growing evidence regarding the benefits of explicit schema training, instead of having subjects induce schemas/structures on their own, on learning to solve elementary mathematics word problems (Fuchs et al., 2004, 2008; Fuson & Willis, 1989; Griffin & Jitendra, 2008; Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007; Jitendra et al., 2007; Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley, 1998; Jitendra & Hoff, 1996; Jitendra, Hoff, & Beck, 1999; Lewis, 1989; Willis & Fuson, 1988; Xin, 2007, 2008; Xin et al., 2005; Zawaiza & Gerber, 1993). In this section, schema theory and several studies on schema-based instruction which show some promises on transfer of learning will be discussed.

Schema Theory

Schema theory arose from psychology, particularly on the study of mental representation in cognition and learning. The representation can be an image (e.g., face, building, chair), a pattern (e.g., walk, ski, swim), an abstract structure (e.g., multiplication, equation, slope), or a list of components (the definition of nation or restaurant). Dansereau (1995) summarized schema theories and noted “In general, schema theories contain both structure and process considerations. The structural aspects are focused on how the information is represented and organized in memory (human or computer), whereas the processing aspects are focused on how these structures are developed, updated, and used” (p. 99). Briefly speaking, schema theory is to study the structure of schemas, the development/acquisition of schemas, and the use of existing schemas for future understanding in learning.

Armbruster (1996) characterized schema as “knowledge is represented in memory in general mental structures called schemas (or schemata). Schemas are prototypical or generic characterizations of objects, events, and situations.” (p. 253-254) For example, she illustrates a “restaurant schema”, which was originally given by Anderson, Spiro, & Anderson (1978), as “generic knowledge about checking in with a host or hostess, being ushered to a table, studying a menu that has food in typical categories, and placing an order with a waiter or waitress.” Dansereau (1995) defined schema as “A set of relatively abstract categories or placeholders and their interrelations. These schemas typically are formed by repeated experiences with the object, action, or event.” (p. 97). Some of the schemas, he described, are consciously derived or labeled such as the underlying structural components of a set of instances (e.g. homeostatic schema or certain text and story grammars), but other schemas are not (e.g., face schema, building schema, car schema).

Schema-Based Instruction

Jitendra et al. (2002) found that schema-based instruction was effective in students’ learning to solve multiplication and division word problems. Four 8th grade students with learning disabilities (scored at least 1 SD or more below the mean on a standardized mathematics achievement test) were given a pretreatment generalization test, intervention, and a post-treatment generalization test. Both of the generalization tests included multi-step problems, and all of the problems in the tests were novel. The intervention contained two phases. The first phase was the problem schemata identification training where the teacher demonstrated the problem schemata analysis (one Vary schema and one Multiplicative Comparison schema) using several examples to help students recognize and understand the key features and relations in the schemata. To illustrate, a Vary schema was composed of two identical diagrams where each of the diagram had one rectangle, one ellipse, and one arrow that connected the two shapes. The schema was illustrated by the problem statement “A car travels 25 miles on a gallon of gas. It can travel 75 miles on 3 gallon of gas”. The features and relations of the schema were expressed in terms of the shapes and the information associated with the shapes. Particularly, in the first diagram, the rectangle was labeled “1” and associated with “If gallons of gas”, the ellipse was labeled “25” and associated with “miles”, and the arrow was associated with “goes”. In the second diagram, the rectangle was labeled “3” and associated with “Then”, and the ellipse was labeled “75”. The first diagram expressed “If 1 gallon of gas goes 25 miles” and the second diagram expressed “Then 3 gallons of gas go 75 miles”. Therefore, the schema was illustrated by means of the running car example. This was a teacher-led demonstration along with frequent student exchanges. The second phase was a teacher-led demonstration and a facilitative questioning procedure in the context of word problems to allow students to identify and map critical elements of the specific problem onto the schemata. The intervention occurred for 35-40 minutes per session for a total of about 18 sessions (range = 16 to 20 sessions).The results showed that students’ problem-solving performance was substantially increased not only on one-step multiplication and division word problems, but also on the novel/transfer one-step and multi-step (not directly taught in the study) word problems. Particularly, the four students’ pretreatment generalization test scores were low (44%, 39%, 44%, and 28%), but each of the four students’ post-treatment generalization test scores was 100% (rate of correctness).

Fuchs et al. (2004) examined the effect of two types of schema-based instruction in learning to solve elementary mathematics word problems. They found that students who received the two types of instruction performed significantly better than students who received non-schema-based instruction. In the study, 24 teachers were randomly assigned to control, schema-based transfer instruction (SBTI), or Expanded SBTI conditions with 122, 110 and 119 third grade students (students in the classes of the 24 teachers) in each condition respectively (total 351). SBTI was a teacher-directed instruction for schema induction. Amount of 16-week math instruction was comparable across all three conditions. In the study, teachers taught students four types of word problems. For the control condition, teachers' instruction focused on the concepts underlying each type of problems with more practice in applying problem-solving rules and more emphasis on computation. The instruction was explicitly relying on worked examples, guided group practice, independent work with checking, and homework. However, the SBTI and Expanded SBTI conditions focused on teacher-guided schema-inducing instruction method. The teachers taught students the meaning of transfer using several examples. Particularly, the teachers taught "transfer means to move: Just as we transfer (move) to a different school, we can transfer (move) skills we learn to new situations." The following was one of the examples: "children learn to drink from a toddler cup, then "move" this skill to a real cup, then "move" this skill to a glass, then "move" this skill to a soda bottle" (p. 429). After teaching the meaning of transfer, the teachers taught each of the four types of problems and altered each problem's features to make some other problems unfamiliar to students without modifying the problem type or the required solution method to help student acquire the schema of the problem. In the mean time, students worked in pairs to apply the solution method to problems with varying transfer features, and then homework was assigned. Students were also asked to search novel problems for familiar problem types and apply the solution methods they knew. The following table (Table 1) shows types of problem feature altered by each of the three conditions.

Table 1: Instructional Focus by Condition

Instructional Focus	Control	SBTI	Expanded SBTI
Problem-solution rules	X	X	X
Transfer – Different format		X	X
Transfer – Different vocabulary		X	X
Transfer – Different question		X	X
Transfer – Irrelevant information			X
Transfer – Combining problem types			X
Transfer – Mixing superficial features			X

There were four measures of problem solving in the study: Transfer-1, Transfer-2, Transfer-3, and Transfer-4. These measures contained novel problems students had never seen before (or never used for instruction). Transfer-1 problems had the same structure with the problems used in the instruction. Transfer-2 problems were the varied problems from problems used in the instruction in terms of varying vocabulary. Transfer-3 problems were the varied problems from problems used in the instruction in terms of adding irrelevant information, combining two problem types, or mixing two transfer features (appearance/vocabulary and question). Transfer-4 problems were the far transfer problems including the combination of all four problem types and multiple pieces of numeric and narrative irrelevant information. The results showed that the SBTI and Expanded SBTI conditions performed significantly better than the control condition did on each of the four transfer measures. Particularly, Both of the two experimental groups exhibited more improvement than the control group on Transfer-1($F(2, 21) = 126.71$, $p < .001$, SBTI: ES=3.69, Expanded SBTI: ES=3.72) and Transfer 2 ($F(2, 21) = 69.37$, $p < .001$, SBTI: ES=1.95, Expanded SBIT: ES=2.10). For the transfer-3 measure, the improvement of both the SBTI and Expanded SBTI groups exceeded that of the control group ($F(2, 21) = 31.09$, $p < .001$, SBTI: ES=1.98, Expanded SBTI: ES=2.71), and the expanded SBTI group exhibited more improvement than the SBTI group (ES = 0.72). For the transfer-4 measure, the improvement of both the SBTI and Expanded SBTI groups again exceeded that of the control group ($F(2, 21) = 16.69$, $p < .001$, SBTI: ES=0.85, Expanded SBTI: ES=1.91), and the expanded SBTI condition improved more than the SBTI condition (ES=1.06).

In summary, schema-based instruction provides a method to help students acquire schemas / structures despite the fact that it is hard to recognize structures as suggested by the literature. Besides, several studies in the domain of mathematics word problems have showed that schema-based instruction is effective in transfer of learning. The discussed studies above also provide practical and detailed procedure for this study to design a teacher-directed instruction for schema induction.

Algebra Word Problem Structures

What structures can we teach in the domain of algebra word problems? In this section, some algebra word problem structures suggested in the literature will be discussed.

An equation consists of three fundamental components: multiplicative relationship, additive relationship, and equal relationship. A multiplicative relationship refers to those multiplicative terms in an equation, for example, the terms 2x and 3x in the linear equation $2x+1=3x+2$. An additive relationship refers to those algebraic terms that are added together. For example, the algebraic expression $2x+3x$ shows an additive relationship between the two algebraic terms 2x and 3x An equal relationship refers to an equal sign that relates two algebraic expressions or terms together. For example, the equal sign in the equation 2x+1=3x+2 equates the two algebraic expressions 2x+1 and 3x+2 to each other.

Yeshurun (1979) suggested three structures for explaining the multiplicative relationship in a linear equation. The first structure he proposed was Rate. The meaning of “ax=b” can be a quantity “a” times a rate “x” equals another quantity “b”. For example, 20 gallons (a=20) times 4 dollars per gallon (x= $4 per gallon) equals 80 dollars (b=80). The multiplicative term “ax” is a Rate structure. The second structure he proposed was Part/Whole. “ax=b” can be understood as a quantity “a” times a “ratio/percent” equals another quantity “b”. For example, one fourth (x=1/4) of 80 dollars (a=80) is 20 dollars (b=20). The multiplicative term “ax” is a Part/Whole structure. The third structure he proposed was Geometrical Relation. The meaning of “ax=b” can be geometrical relations like Length * Width = Area or Base Area * Height = Volume. The multiplicative term “ax” is a geometrical structure. Chang, Floden and Smith (2009) proposed another structure in explaining the multiplicative relationship, which was Multiple. To illustrate, “ax=b” can be understood as b is x times as much as a, which is an example of “multiple”. The four structures can be used to categorize or understand one-variable linear equation word problems, or recognizing the connections between algebra word problems regarding the multiplicative relationships in the problems.

Reed (1999) suggested three structures for classifying addition (or subtraction) word problems, which can be used as structures for explaining the additive relationship in a linear equation. The first structure he suggested was Change. The meaning of the equation “a+x=b” can be a quantity “a” and a changed quantity “x” that is taken from or added to “a”, and the resulted quantity “b” (e.g., “x” is the changed temperature from degree “a” to degree “b”). The additive term “a+x” is a Change structure. The second structure he suggested was Combine. The equation “a+x=b” can refer to a situation in which the sum of two quantities “a” and “x” is “b”. The additive term “a+x” is a Combine structure. The third structure he suggested was Compare. The equation “a+x=b” can mean one quantity “a” is “x” (quantity) less than or more than another quantity “b”. The additive term “a+x” is a Compare structure. The three structures can be used to categorize or understand one-variable linear equation word problems, or recognizing the connections between algebra word problems regarding the additive relationships in the problems.

In summary, four multiplicative relationships and three additive relationships that can be embedded in an algebra word problem were suggested in the literature. The four multiplicative relationships are Rate, Part/Whole, Multiple, and Geometrical relationships. The three additive relationships are Change, Combine, and Compare.

Measures of Transfer

In this section, the definition of transfer and measures of transfer will be discussed, which is necessary for the purpose of assessing the effect degree of transfer for any transfer-performance-related study. Particularly, “direction of transfer”, “distance / degree of transfer”, and transfer types in the domain of algebra word problems will be discussed.

Marini and Genereux (1995) defined transfer, in a broad way, as “prior learning affecting new learning or performance.” Particularly, the prior learned task (e.g., learning materials or practice problems) under a specific learning context (e.g., physical or social setting at home or school) affect the learning or performance of new tasks under another learning context.

The direction of transfer can be positive or negative, and the extent of transfer can be near-to-far, specific-to-general, or within-domain to cross-domain. Marini and Genereux (1995) mentioned:

> "extent of transfer can conceivably range from near, specific, one-dimensional transfer (involving one task or context only slightly different from original learning), to the opposite extreme of far, general, multidimensional transfer (encompassing a wide variety of tasks and contexts very different from original learning)" (p. 6)

Transfer does not have to be positive. Macaulay (2000) characterized transfer as positive if "what is learned during the first phase enhances learning during the second phase"; she characterized transfer as negative if "what is learned during the first phase is in some way detrimental to learning during the second phase" (p. 3).

About the distance (or degree) of transfer, Perkins and Salomon (1992) defined *near transfer* as "transfer between very similar contexts…as for instance…when a garage mechanic repairs an engine in a new model of car, but with a design much the same as in prior models". They defined *far transfer* as "transfer between contexts that, on appearance, seem remote and alien to one another. For instance, a chess player might apply basic strategic principles such as 'take control of the center' to investment practices, politics, or military campaigns". Gick and Holyoak (1987) referred "the effect of a task performance on the subsequent performance of the same task" as "self-transfer", because "each repetition in its own time and context" (p. 10) They described "self-transfer" as "those that are mere repetitions, "near transfer" as "those that are highly similar", and "far transfer" as "those that are very different". There is little agreement regarding the classification of near or far transfer, and within-domain or cross-domain transfer (Marini and Genereux, 1995, p. 5; Perkins and Salomon, 1992). One researcher may identify a transfer between two tasks as near and within-domain transfer, while another may describe the transfer between the two tasks as far and cross-domain transfer.

For the transfer of learning to solve algebra word problems, Reed (1999) described similar ideas to *near* and *far transfer* using different terms and categories. He used the ideas of context and equation to create three types of transfer in the field of algebra word problems. Particularly, his classification was "based on whether the example problem and test problem share a common story context and a common solution procedure (equation)" (p. 101). He identified three kinds of transfer which were *equivalent*, *similar*, and *isomorphic*. He defined *equivalent* as the original problem and the transfer problem "share both common story context and common solution procedure" (p. 101). For example, suppose the following is the original problem:

"A small pipe can fill an oil tank in 12 hours and a large one can fill it in 8 hours. How long will it take to fill the tank if both pipes are used at the same time?" (p. 102) with the translated equation $(x/12) + (x/8) = 1$.

An problem *equivalent* to the original problem will be:

> "A small hose can fill a swimming pool in 6 hours and a large one can fill it in 3 hours. How long will it take to fill the pool if both hoses are used at the same time?" (p. 102)
>
> (The translated equation is $(x/6) + (x/3) = 1$)

He stated that the original problem and the transfer problem are *isomorphic* when they "have different story contexts but share a common solution" (p. 101). The following is a problem *isomorphic* to the original one:

> "Tom can drive to Bill's house in 4 hours and Bill can drive to Tom's house in 3 hours. How long will it take them to meet if they both leave their houses at the same time and drive toward each other?" (p. 102)

(The translated equation is $(x/4) + (x/3) = 1$)

He stated that the original problem and the transfer problem are *similar* if they “share a story context but require modifying the solution of the example to solve the test problem” (p. 101-102). The following is a problem *similar* to the original one:

> “A small pipe can fill a water tank in 20 hours and a large pipe can fill it in 15 hours. Water is used at a rate that would empty a full tank in 40 hours. How long will it take to fill the tank when both pipes are used at the same time, assuming that water is being used as the tank is filled?” (p. 102)
>
> (The translated equation is $(x/20) + (x/15) – (x/40) = 1$)

In summary, the three types of transfer (self, near, or far) are well-known measures for assessing the effect degree of transfer in the literature. However, there is little agreement in the classification of the three types of transfer. In the domain of algebra word problems, Reed (1999) defined three types of transfer (equivalent, similar, and isomorphic) using the ideas of story context, and equation that is embedded in story context. This domain-specific definition of transfer for algebra word problems is particular useful for this study, and will be discussed again in the next chapter.

Chapter 3

The framework

This chapter will discuss the framework for the experiment design and analysis of the study, which includes a design of structure-based instruction and traditional instruction, a method for analyzing algebra word problem structures regarding the structures students can recognize when asked to structure word problems, and a design for assessing the degree of transfer.

The Structure-Based Instruction and Traditional Instruction

In this section, the selection of "rate" structure (as the to-be-taught structure) and two rate types of word problems (distance and mixture), and the construction of the principle and procedure for the structure-based and traditional instruction will be discussed.

The Selection of "rate" Structure

This study chose the "rate" structure (e.g., miles per hour, dollars per item) as the to-be-taught structure for the structure-based instruction for two reasons. First, "Rate", the ratio relationship between two quantities with different units, has been the central focus of many studies about the transfer of algebra word problems (Bassok & Holyoak, 1989; Blessing & Ross, 1996; Reed, 1989; Reed, Dempster, and Ettinger, 1985). Second, teaching "rate" structure has been reported successful in facilitating transfer in the domain of elementary mathematics word problems (Fuchs et al., 2004; Jitendra et al., 2000).

Distance and mixture are the two rate types of word problems that have been adopted by many research studies about the transfer of algebra word problems. To illustrate, Reed, Dempster, and Ettinger (1985) studied three rate types of algebra word problems to evaluate the usefulness of analogous solutions for solving algebra word problems (discussed in the above section). The three rate-types were distance (Distance = Rate x Time), mixture (mixing liquids with different rates of solution), and work problem (working in different rates trying to complete a shared task). Speed is a kind of rate in the distance problem and liquid solution is also a kind of rate in mixing liquids with different rates of solution to get a certain rate of solution. Reed (1989) studied two rate types of algebra word problems, distance and mixture, to evaluate the effect of comparing isomorphic algebra word problems for the abstraction of schema. For the mixture type problems, he used problems about the mixture of liquids with different solutions, the mixture of peanuts with different prices, and interest problems. Each of the solution, price and interest is a kind of rate. For the distance type problems, he used convergence problems (e.g., two subjects moving toward each other), succession problems (e.g., one subject runs after the other subject), and divergence problems (e.g., two subjects moving in opposite directions). Blessing & Ross (1996) also used several rate types of word problems examining the relations between problem categorization and problem solving such as interest problems, mixture problems, motion problems (distance, rate, time), and work problems (work in different rates trying to complete a shared task).

Teaching rate structure has been reported successful in facilitating transfer in the domain of elementary mathematics by two studies. Jitendra et al. (2002) taught “vary” schema (rate relationship with a diagram saying “a car travels 25 miles on a gallon of gas; it can travel 75 miles on 3 gallons of gas” (p. 29), and students’ problem-solving performance was substantially increased in solving elementary multiplication and division word problems. Fuchs and his colleagues (2004) taught two rate types of word problems, the shopping list problem (price x number of items = cost) and the “buying bag” problem (a certain number of items for a bag, how many bags for a certain number of item), to help students induce schemas, and the results showed that students who received the schema-based instruction significantly outperformed students who received the non-schema-based instruction. The two studies were also discussed in chapter 2.

The Principle and Procedure of the Structure-Based Instruction

This study summarized several schema-based instruction methods used in different research studies (e.g., Fuchs et al., 2004; Jitendra et al., 2002; Xin, 2005), and proposed the following one principle and three steps of structure-based instruction for this study:

- Principle
  - Teacher-directed instruction principle
- Procedure
  - Teaching the definition of the structure
  - Helping students generate the schema
  - Application of the schema

Particularly, the principle was the core of the structure-base instruction where teachers guided students to induce schemas instead of students inducing schemas on their own. The first step of the procedure “teaching the definition of the structure” came from Jitendra and his colleagues’ study (2002) that the teacher demonstrated the problem schemata analysis (one Vary schema and one Multiplicative Comparison schema) using several examples to help students recognize and understand the key features and relations in the schemata. The second step “helping students generate the schema” came from Fuchs and colleagues’ study (2004) where the teacher taught a problem and altered the problem’s features to make some other problems students are not familiar with without modifying the problem type or the required solution methods to help students acquire the schema of the problem. The third step "Application of the schema" came from both of the studies by Jitendra et al. (2002) and Fuchs et al. (2004) where Jitendra's study asked participants to map problems to the learned schemata, and Fuchs' study asked students to search novel problems for familiar problem types and apply the solutions they knew.

The Traditional Instruction

The traditional instruction for learning to solve algebra word problems was derived from textbooks (e.g., Bellman et al., 2004), which was based on that several studies of evaluating the effectiveness of schema-based instruction designed their control group treatment by following the teaching plan (or script) on textbooks (e.g., Fuchs et al., 2004; Xin, 2005). Indeed, K-12 mathematics instruction relies on textbooks (McCrory, Francis, & Young, 2008). In addition, written curriculum is one of the conventional resources that enable or constrain teachers' instruction, and thus impact on students' learning and achievement (Cohen, Raudenbush, & Ball, 2003).

The traditional instruction consisted of the following four elements:

- Identify quantities
- Identify relationships between quantities
- Identify the unknown quantity and its relationships with other quantities
- Connect piecewise relationships together and translate it into an equation.

Identify quantities is to identify numbers with units, for example, 10 gallons, 5 dollars per meal, or 70 miles per hour. Identify relationships between quantities is to identify quantities that are related and find their relationships. For example, a car runs 60 miles per hour and the car has run 3 hours. The two quantities imply a distance (relationship), which is 60 x 3=180 miles. Identity the unknown quantity is to find the quantity that is unknown, for example, a question asked by the word problem (e.g., “what is the total hours the car run”?). Connect piecewise relationships together means to connect and integrate the found relationships (by the previous two steps) together. For example, the integrated relationship of cell phone monthly payment is to connect the two relationships together (charged minutes x 0.75 dollars per minute, monthly program fee) and get “Cell phone monthly payment = charged minutes x 0.75 dollars per minute + monthly program fee”.

In summary, the structure-based instruction design for this study was to teach rate structure following the teacher-directed instruction principle and the three steps of “teaching the definition of the structure”, “helping students to generate the schema”, and “application of the schema”. The traditional instruction for this study contained the following four elements: (1) identify quantities (2) identify relationships between quantities (3) identify the unknown quantity and its relationships with other quantities, and (4) connect piecewise relationships together and translate it into an equation.

The Framework for Analyzing Algebra Word Problem Structures

Recall that the definition of word problem structure, in this study, is the particular components and the relationships of objects, events, and situations in a word problem. It is inherent in word problems. Individuals’ recognition of word problem structures could be in the surface level such as superficial description about the content of word problems, or it could be in the deeper level about relationships in word problems. Therefore, a framework was constructed to capture the structures with different qualities (of recognition).

The framework defined two levels of sophistication of algebra word problem structures, the *surface structure* and the *deep structure*. The surface structure referred to words or objects that were part of the problem statement, or non-relational words summarizing word problems (e.g. money, time, interest). The deep structure referred to relationships in word problems, for example, “two cars running in opposite directions” (a relationship between two cars). The deep structure was divided into two sub-categories, partial and complete. The partial deep-structure meant the relationship in word problems was a part of the translated equations. For example, the rate relationship “miles per hour” is usually a part of an equation, especially for the multiplicative terms. The complete deep-structure meant the “equal” relationship in word problems. “Equal” can be understood as the equal sign of an equation. For example, the summary about several word problems “These problems are about two distances equaling each other” expresses an equal relationship between the two distances.

The definition of the *surface structure* was based on Schoenfeld & Herrmann’s (1982) “surface structure” (e.g., “a naïve characterization of a problem, based on the most prominent mathematical objects that appear in it”, p. 486) and Holyoak & Koh’s (1987) “surface features” (e.g., not violating the “causal relationship” in problems), which constituted the idea of “words or objects that were part of the problem statement” and the idea of “non-relational words” respectively in the definition. The definition of the *deep structure* was based on Holyoak & Koh’s (1987) “structural features” (e.g., the “causal relationship”) and Gick & Holyoak’s (1983) “good schema” (e.g., completeness of a structure), which constituted the ideas of “relationships in word problems” and “partial/complete” respectively in the definition.

In summary, the algebra word problem structures students recognized were analyzed by the quality of recognition, which resulted in the three structures, the surface structure, the deep-complete structure, and the deep-partial structure.

The Measures of Transfer

Three transfer measures were constructed to measure the effect degree of transfer of the structure-based instruction, which were SS (similar context and similar equation), SD (similar context and different equation), and DS (different contexts and similar equation), based on Reed's (1999) definitions of *equivalent*, *similar*, and *isomorphic* respectively. The following four examples illustrate the three transfer types (SS, SD, and DS) in terms of an original word problem and three transfer word problems:

The original word problem:

> Suppose you are helping to prepare a large banquet. You can peel 2 carrots per minute. You need 60 peeled carrots. How long will it take you to peel 60 carrots? (Modified from *Algebra 1*, p. 84 (Bellman et al., 2004))
>
> Equation translated from the word problem: $2x = 60$

1. The SS type (similar story context and similar translated equation):

> Suppose you are helping to prepare a large banquet. You can set up plates for 3 tables per minute. You need to set up 60 tables. How long will it take you to set up 60 tables?
>
> Equation translated from the word problem: $3x = 60$

2. The SD type (similar story context and different translated equations)

> Suppose you are helping to prepare a large banquet. You can set up plates on 3 tables per minute. You need to set up 60 tables. How long will it take you to finish if you have already set up 24 tables?
>
> Equation translated from the word problem: $3x = 60 – 24$

3. The DS type: (different story contexts and similar translated equation)

Suppose you are attending a marathon. You can run 3 miles per hour. You need to run 15 miles in the marathon. How long will it take you to finish the 15-mile marathon?

Equation translated from the word problem: $3x = 15$

The SS type problem has similar context (preparing materials for a banquet) compared to the context of the original problem. It also has similar equation structure compared to the equation of the original problem which is one variable term on the left side of the equation and one constant on the right side of the equation. Therefore, it is a similar context and similar equation (SS) type problem regarding the original problem. The SD type problem has similar context (preparing materials for a banquet) compared to the context of the original problem. However, it has different equation structure compared to the equation of the original problem, which is one more constant on the right side of the equation compared to the equation of the original equation. Therefore, it is a similar context and different equation (SD) type problem regarding the original problem. The DS type problem has different context (speed and distance) compared to the context of the original problem (preparing materials for a banquet). However, it has similar equation compared to the equation of the original problem which is one variable term on the left side of the equation and one constant on the right side of the equation. Therefore, it is a different context and similar equation (DS) type problem regarding the original problem.

According to Perkins and Salomon (1992), if only context is taken account regardless of equation, the SS and SD types of transfer can be categorized as *near transfer* because of holding similar context, and the DS type transfer can be categorized as far transfer because of holding different context. According to some psychologists like Gick and Holyoak (1983, 1987), Catrambone and Holyoak (1989), or Reed, Dempster, and Ettinger (1985), if only context is taken account regardless of equation (i.e., the difference/change of equation does not affect the classification of “near” or “far”), the SS type transfer can be categorized as analogous transfer or near transfer, and the DS type transfer can be categorized as isomorphic transfer or far transfer.

Chapter 4

Methodology

This dissertation study tried to answer the following four research questions:

1. What structures can students recognize when asked to structure algebra word problems, and what are the levels of sophistication of the structures?
2. What are the difficulties students will encounter when asked to structure algebra word problems, and what are the levels of the difficulties?
3. Is there performance difference between students who recognize different qualities of algebra word problem structures in solving transfer algebra word problems?
4. What is the effect of structure-based instruction on learning to solve transfer algebra word problems?

An experiment was designed to answer research questions 3 and 4. The experiment was a true experiment design, a pretest-posttest control group design, where participants were randomly assigned to two groups. The two groups were the experimental group, which received structure-based instruction, and the control group, which received traditional instruction. The experiment provided quantitative data for evaluating the effect of quality of algebra word problem structure on solving transfer algebra word problems, and for evaluating the effect of structure-based instruction and the effect of different structure qualities on solving transfer algebra word problems.

An interview was designed to answer research questions 1 and 2. One-to-one interviews were conducted immediately after the experiment to explore word problem structures students could recognize or create and the levels of sophistication of the structures (by asking students to classify all of the word problems they just solved in the posttest and tell why they grouped certain problems together), and to explore the difficulties and levels of the difficulties they encountered when classifying the word problems.

Participants

Sixty-one undergraduates (Male: 18, Female: 43) at Michigan State University (MSU) who were taking intermediate algebra (MTH 1825) or college algebra (MTH 103) in fall, 2009 participated in the study. The two courses were the most basic mathematics courses offered by the university. Students who were at lowest level of the MSU Mathematics Placement Service Exam (e.g., the score is 9 or below) were required to take intermediate algebra (MTH 1825). The intermediate algebra course was for students who were deficient in skills and knowledge of high school algebra, and its goal was to prepare students for college level mathematics courses, for example, college algebra (MTH 103). MTH 103 was for students whose scores were at the second lower level of the MSU Mathematics Placement Service Exam or who were not ready for pre-calculus courses. The participants were recruited through flyers (paper or email version) and through in-class announcements (by the instructor). Each student received $20 after finishing a 2-hour experiment including a one-to-one interview. Light refreshment was provided in the experiment.

The 61 students were randomly assigned to two groups, the experimental (or structure-based instruction) group and the control (or traditional instruction) group. The students were not randomly assigned to each of the two groups all at once. Students enrolled the study in different times during the Fall semester (September to December), 2009. In order to randomly assign students who expressed their interest in participating in the study through emails (in different times) to the two groups, a list of random numbers (“0” for control and “1” for experimental group) was pre-established. Students were placed on the list based on the order of the emails the author received. Hence, students were randomly assigned to either the control or the experimental group according to the numbers they were assigned (0 or 1).

Three instructors (two females, one male), who were doctoral students in mathematics education, were recruited to implement the teaching and interview tasks in the experiment. Each of the three instructors had experience in teaching college mathematics or mathematics education courses. The three instructors were assigned to teach a total of 12 experimental treatment sessions, and each instructor was required to teach sessions for the control group and for the experimental group. The teaching assignments were based on the instructor’s available times. Instructor One taught 15 control group students and 14 experimental group students. Instructor Two taught 11 control group students and 15 experimental group students. Instructor Three stopped her teaching commitment after teaching 3 control group students due to personal health reasons.

Procedure

Prior to the experiment, the three instructors were given five sessions of professional development. Each professional development session lasted about one hour. The first professional development session was to help the instructors understand the purpose of the study, the hypotheses, the research questions, the methods, and the experiment procedure of the study. The second included an introduction and practice of the traditional instruction treatment for the control group. The third included an introduction and practice of the structure-based instruction treatment for the experimental group. The fourth contained an introduction and practice of the interview protocol. The last professional development session was a discussion about possible scenarios that could happen during the experiment, for example, the time for instruction with subjects was delayed or no participants showed up. The discussion also included some logistical issues such as audio recording, students' paper work management, and real-time error correction strategy while the instructor was teaching using the experimental or control approach (e.g., the instructor was not following the teaching script or the instructor taught longer than the schedule indicated).

Each student was only in one treatment session, using either the structure-based or traditional approach. These treatment sessions were held 12 times within 7 weeks (6 sessions for the control group and 6 sessions for the experimental group). There were about 5 students on average in each session. A total of 61 students participated within the 7 weeks.

Each treatment session lasted about 2 hours containing a pretest (20 minutes), a treatment (1 hour), a posttest (20 minutes), and an interview (10-20 minutes). In the pretest and posttest, students were given 12 word problems to solve. The treatment contained a 15-minute lecture and a 45-minute problem-solving exercise. For the lecture, the experimental group students received structure-based instruction and the control group students received traditional instruction. For the problem-solving exercise, both groups of students solved 6 word problems, drawn from the pretest. For each problem, students were asked to solve it. Then the instructor explained the solution method to the students. Then students were asked to solve a problem that was the same as the problem just explained, but with different numerical quantities. For example, the quantities of 15 MPH, 3 hours, and 225 miles in the problem "Jane and Peter leave their home traveling in opposite directions on a straight road. Peter drives 15 miles per hour faster than Jane. After 3 hours, they are 225 miles apart. Find Jane's speed." were changed to 25 MPH, 4 hours, and 500 miles for students to solve. This pattern continued for each of the six exercise word problems. One-to-one interviews were conducted immediately after the posttest. Each student was asked three interview questions and students' oral responses were audio-recorded.

To ensure high fidelity of implementation, the researcher (the author) attended each experiment and evaluated the instructor's teaching based on instructor's teaching script, and helped the instructor follow time schedule of the experiment. The researcher gave real-time feedback and corrections whenever the instructor was not following teaching script, time schedule, or any part of the experimental procedure (e.g., worksheets should be collected before students take the posttest).

Materials

The Pretest and Posttest

The pretest consisted of 12 word problems. (See Appendix A) Within the 12 problems, 7 problems (1, 4, 5, 7, 9, 10, and11) were about travel and 5 problems (2, 3, 6, 8, and 12) were about mixture. The travel type problems contained speed, time and distance information where individuals might travel in opposite directions, the same direction, or make a round trip. The following three problems (from the pretest) illustrate the three travel scenarios.

Travel in the same direction:

Ellen and Kate raced on their bicycles to the library after school. They both left school at 3:00pm and bicycled along the same path. Ellen rode at a speed of 9 miles per hour and Kate rode at 12 miles per hour. Ellen got to the library 15 minutes later than Kate. How long did it take Ellen to get to the library?

Travel in opposite directions:

Two families meet at a park for a picnic. At the end of the day, one family travels east at an average speed of 42 miles per hour and the other travels west at an average speed of 50 miles per hour. Both families have 160 miles to travel. Find the time that will have elapsed when they are 100 miles apart.

Round trip travel:

Nora took her sister to the city airport. Because of traffic conditions, she drove 3.75 hours, but she drove only 1.5 hours from the airport to her home. On her way from the city airport to home, she averaged 40 miles per hour faster compared to her average speed from home to the airport. What is Nora's average speed from the airport to her home?

The Mixture type problems have things (e.g., liquids, nuts, or investments) mixed together for producing another product. The followings are two mixture type problems from the pretest.

Mixture - nuts:

A grocer mixes two kinds of nuts that cost \$2 per pound and \$3 per pound respectively, where the total weight of the two kinds of nuts is 10 pounds. The new product's (mixed nuts) cost has been calculated at \$2.40 per pound so that the value of the new product is the same as the value of the two kinds of nuts combined. How much of each kind of nut is put into the mixture?

Mixture - investment:

Dana invested in two funds paying 9.5% and 11% simple interest. She invested \$2000 more in the fund with 11% rate compared to the amount invested in the fund with 9.5% rate. During one year, she earned \$1040 from the two funds in total. How much did Dana invest in each fund?

The 12 posttest word problems were generated based on four types of equation combined with the two context types(travel and mixture), which were (1) ax+b(cx+d)=2 (2) ax+bx=c (3) ax=b(cx+d), and (4) ax+b(c-dx)=e*f . Three problems were generated for each equation type. The problem numbers (1-12) for each of the four types of problems were randomly assigned to avoid the situation that problems appeared in a particular order with respect to equation type (See Table 2).

Table 2: Design of the Pretest Problems

Equation Type	Context	Problem Number
Type 1 ax+b(cx+d)=2	1.1 Travel - Two-car opposite	5
	1.2 Mixture - investment	12
	1.3 Travel - One-car one-way different speeds	7
Type 2 ax+bx=c	2.1 Travel - Two-car toward each other	11
	2.2 Travel - Two-car toward each other with time unknown	4
	2.3 Mixture - investment	8
Type 3 ax=b(cx+d)	3.1 Travel - Two-car follow-up	9
	3.2 Travel - Two-car end different	1
	3.3 Travel - One-car two-way different speeds	10
Type 4 ax+b(c-dx)=e*f	4.1 Mixture - nuts	6
	4.2 Mixture - liquids	12
	4.3 Mixture - investment	3

The posttest also consisted of 12 word problems, which were different from the pretest word problems. (See Appendix B) To allow for assessment of transfer, each posttest problem was matched with a pretest problem in one of the following three ways, Similar context and Similar equation (SS), Similar context and Different equation (SD), or Different context and Similar Equation (DS) (See Table 3 and Appendix C)

Table 3: Design of the Posttest Problems

Pretest Design	Problem Number	Posttest Design	Problem Number (randomly assigned)
1.1	5	SS	4
1.2	12	SD	5
1.3	7	DS	2
2.1	11	SS	8
2.2	4	SD	1
2.3	8	DS	10
3.1	9	SS	12
3.2	1	SD	9
3.3	10	DS	3
4.1	6	SS	7
4.2	12	SD	11
4.3	3	DS	6

Within the 12 posttest word problems, four were **SS (Similar context, Similar equation),** four were **SD (Similar context, Different equation)**, and four were **DS (Different context,**

Similar equation). For example, the following problem was a pretest problem with the story context of two cars running in opposite direction and with the translated equation 3x+3(x+15)=225.

Context: Two-car opposite; Equation: 3x+3(x+15)=225

Jane and Peter leave their home traveling in opposite directions on a straight road. Peter drives 15 miles per hour faster than Jane. After 3 hours, they are 225 miles apart. Find Jane's speed.

The following SS problem was generated for the posttest which had similar context (two bicycles driving in opposite direction) and similar equation (two variable terms on one side of an equation) regarding the above pretest problem.

Context: Two-bicycle opposite; Equation: 2x+2(x+5)=70

Two bicyclists ride in opposite directions from the same place. The speed of the first bicyclist is 5 miles per hour faster than the second. After 2 hours they are 70 miles apart. Find their speeds.

The following problem was a pretest problem with investment story context and with the equation 0.095x+0.11(x+2000)=1040.

Context: Mixture - Investment; Equation: 0.095x+0.11(x+2000)=1040

Dana invested in two funds paying 9.5% and 11% simple interest. She invested $2000 more in the fund with 11% rate compared to the amount invested in the fund with 9.5% rate. During one year, she earned $1040 from the two funds in total. How much did Dana invest in each fund?

The following SD problem was generated for the posttest which had similar investment context but different equation (subtraction between two variable terms instead of addition) regarding the above pretest problem.

Context: Mixture - Investment; Equation: $0.04(x+4000)-0.025x=500$

Jose deposited money into two bank accounts paying 2.5% and 4% simple interest. He put $4000 more in the account with the 4% rate compared to the amount put in the account with the 2.5% rate. During one year, he earned $500 more from the 4% account compared to the 2.5% account. How much did Jose deposit in each account?

The following problem was a pretest problem with the story context of one car driving one way with different speeds and with the translated equation $58x+52(x+2.75)=418$.

Context: One-car one-way different speeds; Equation: $58x+52(x+2.75)=418$

On the first part of a 418-mile trip, a salesman averaged 58 miles per hour. He averaged only 52 miles per hour on the last part of the trip because of an increased volume of traffic. Find the amount of time at each of the speeds if the last part of the trip took 2.75 hours more compared to the first part of the trip.

The following DS problem was generated for the posttest which had different story context (Two pumps filling a tank) and similar equation (Two variable terms on one side of an equation) regarding the above pretest problem.

Context: Two pumps filling a tank; Equation: $10x+10(x+5)=250$

Two pumps are used to fill a water storage tank at a resort. One pump can fill the tank 5 gallons per minute faster than the other one. When both pumps are operating together, the tank can be filled in 10 minutes. The capacity of the tank is 250 gallons. Find the filling rate of each pump.

The 12 posttest word problems were also categorized into three real-life event categories which were Travel, Mixture, and Other. Similar to the pretest, the travel type problems were about speed, time and distance. The Mixture type problems were about things combined for producing a product. The Other category consisted of problems that did not belong to the travel or mixture type. The following table (Table 4) shows the three real-life problem types and the posttest problems under each type.

Table 4: Three Real-life Problem Types and the Posttest Problems

Real-life Type	Travel	Mixture	Other
Problems	1, 4, 8, 9, 12	5, 6, 7, 11	2, 3, 10

The Instructional & Exercise Materials

For the one-hour treatment in the treatment session, the experimental group received 15 minutes structure-based instruction and 45 minutes structure-based exercise session. The control group received 15 minutes traditional instruction and 45 minutes traditional exercise session.

The structure-based instruction included the instruction of Rate structure (e.g., miles per hour, dollars per item) and Two-Car model (See Appendix D, E). For the instruction of the rate structure, the teacher first asked students what rate was, and showed students examples of rate in daily life and in equations. Second, the teacher taught students the definition of rate (a ratio relationship between two quantities with different units). Third, the teacher helped students form and consolidate the rate structure by giving students several rates and asking students what real-life contexts were related to these rates. For example, given the rate “pages per minute” and “dollars per month”, what are the related contexts? (Example answers: “copy machine” and “salary” respectively). Fourth, the teacher helped students apply the rate structure by (1) asking students to translate several real-life rate relationships to symbolic expressions (e.g., “If you have the information *x* miles per hour and 3 hours, what does 3*x* mean?”), and (2) asking students translating several monomials to real-life situations (e.g., “Can you find a real-life story for the expression “60x”? Try to identify a rate first, for example, can 60 be a rate?”). Finally, the teacher summarized the instruction by asking students the following two questions (1) “When you read a story problem, can you recognize the rate relationship(s)?”, and (2) “When you read an equation, can you generate a real-life rate example for the multiplicative term(s)?” The teacher discussed students’ answers if their answers were “No” or others. For the instruction of the two-car model, the teacher taught students three scenarios of two cars for comprehending travel type problems. The two-car model consisted of three scenarios that were (1) Two cars travel in opposite directions (2) Two cars travel toward each other, and (3) Two cars travel in the same direction.

In the 45-minute structure-based problem-solving exercise of the experimental group, students spent time practicing six exercise problems (See Appendix H), and then the instructor explained solution methods of the exercise problems to students using structure-based instruction method. Four elements were included in the structure-based instruction explaining the solution methods: 1. Ask students to use the two-car model to comprehend the word problem they were asked to solve if the model is applicable2. Ask students to look for rates in the word problem 3. Ask students to look for rate relationships in the word problem 4. Connect the observed relationships. The following is the teaching script for the solution method of problem 1 in the exercise session of the experimental group (Also see Appendix I).

Problem 1:

Jane and Peter leave their home traveling in opposite directions on a straight road. Peter drives 15 miles per hour faster than Jane. After 3 hours, they are 225 miles apart. Find Jane's speed.

Teaching Script:

1. "What scenario is this problem about according to the Two-Car Model?"

(The teacher discusses with students about possible scenarios, and gives the correct answer)

2. Looking for rates

"Can you find any rate in this problem?" (The teacher waits for students' answers, and then proceeds)

"3 hours, 225 miles apart, 15 miles per hour faster, Jane's speed, Peter's speed (Jane's speed and peter's speed are rates, although they have no numbers associated.)

3. Looking for rate relationships (other quantities that associated with rates):

"Are there quantities that can be connected to the rates you find?" (The teacher waits for students' answers, and then proceeds)

"Jane's speed is a rate, and Jane drives 3 hours. Therefore, the distance Jane drives is Jane's speed time 3. *If we don't have a number regarding a quantity, we just suppose it with a symbol to help us make inferences and get more information.* Suppose Jane's speed is x, then the distance Jane drives is 3x. Since Peter drives 15 miles faster than Jane, Peter's speed is x+15. Peter's speed is a rate, and Peter also drives 3 hours. Therefore, the distance Peter drives is 3(x+15)."

4. Connecting the relationships we have:

"Since after three hours they are 225 miles apart, the sum of the distances (two mileages) Jane and Peter drove is equal to 225. Therefore, the translated equation is 3x+3(x+15) = 225."

The traditional instruction consisted of the four-step strategy and an application of the strategy (See Appendix F, G). The teacher taught the first step "Identify quantities" by first describing the definition of quantity (a number associated with a unit), and illustrated with examples (e.g., 3 persons, 10 gallons, 70 miles per hour). The teacher taught the second step "Identify relationships between quantities" by illustrating with examples how two quantities could be related together. For example, "We know the speed of a car is 70 miles per hour, and we know the car has run 3 hours. The two quantities imply a distance relationship, e.g., 70 MPH x 3 hours = 240 miles." The teacher taught the third step "Identify the unknown quantities and find relationships related to the quantities" by illustrating that some quantities might be implicit, which meant there were no numbers associated, and illustrating with examples how an unknown quantity could be related to other quantities in a word problems (similar to step 2). The teacher taught the last step "Find the integrated relationship" by illustrating with examples how to connect piecewise relationships found in the previous three steps to make an integrated relationship. For example, the integrated relationship "Cell phone monthly payment = charged minutes x 0.75 dollars per minute + monthly program fee" is composed of the two relationships that are "charged minutes x 0.75 dollars per minute is part of the total charge" and "monthly program fee is part of the total charge". After teaching the four-step strategy, the teacher explained with an example how to apply the strategy. The following is the application of the strategy in the lecture:

The MacNeills rented a moving truck for \$50 plus \$.50 per mile. The total cost was \$90. Find the number of miles the truck was driven.

- Identify quantities:
  - \$50, \$.50 per mile, total cost \$90, and miles the truck was driven.
- Identify relationships between quantities:
  - The total cost includes \$50
  - Part of the cost is the rate .50 times the number of miles the car run.
- Identify the unknown quantities and find relationships related to the quantities:
  - Suppose the truck run x miles, then part of the cost is $0.5x$
- Figure out the integrated relationship and translate it:
  - Total cost = \$50 + 0.5x. The translated equation is $90=0.5x+50$ or $50+0.5x=90$

In the 45-minute traditional problem-solving exercise of the control group, students spent time practicing six exercise problems (See Appendix H), and then the instructor explained solution methods of the exercise problems to students using traditional instruction method. Four elements were included in the traditional instruction explaining the solution methods: 1. Ask students to find quantities in the word problem 2. Ask students to find relationships between the quantities 3. Ask students to identify the unknown quantities and relationships related to the quantities 4. Ask students to find the integrated relationship and translate it into a symbolic expression. The following is the teaching script for the solution method of problem 1 in the exercise session of the control group (Also see Appendix J).

Teaching Script:

"Let's draw a picture to figure out what the problem is about." (The teacher discusses with students what the picture would be, and draw the picture on the blackboard)

"First, can we find some quantities from the problem…..? (The teacher waits for students' answers, and then proceeds) The quantities we can find are 15 miles per hour faster, 3 hours, and 225 miles apart, Jane's speed, Peter's speed. Some quantities may be implicit, which means there are no numbers associated, but you know they are quantities."

"Second, can we find any relationship between the quantities…? (The teacher waits for students' answers, and then proceeds) The relationships we can find are: 1. Peter drives 15 miles per hour faster than Jane 2. Each of them drives 3 hours, which means each of their speeds times 3 is the distance each of them drives. 3. They are 225 miles apart after 3-hour driving, which means the sum of the distances they drive is 225."

"Third, for a quantity with no number associated, just suppose it with a symbol to help us make inferences and figure out other relationships. Suppose x is Jane's speed, then (x+15) is Peter's speed, and the distance Jane drives is 3x and the distance Peter drives is 3(x+15)."

"Fourth, therefore, the integrated relationship is: the mileage Peter drives + the mileage Jane drives in 3 hours = 225 miles. The translated equation is $3(x+15) + 3x = 225$"

The Interview Protocol

In the interview, students were asked three questions: (1) Can you classify the 12 word problems (the posttest problems) into five groups based on similarities between the problems (problems can be reused)? (2) Can you tell me why you grouped the problems together for each group? (3) What were the difficulties you encountered when you tried to classify the problems? The first two questions were intended to explore algebra word problem structures from students' point of view (See Appendix K).

Chapter 5

Analysis and Results

In this chapter, four results will be presented with respect to the four research questions. To answer the first and second research questions, a qualitative analysis on students' interview transcripts was conducted to identify and analyze structures students recognized, and to identify and categorize difficulties students encountered when asked to structure algebra word problems. To answer the third research question, a mixed method of qualitative and quantitative analysis was conducted. Particularly, students were categorized to groups with different qualities of structure based on the analysis of the structures students recognized (or the result for the first research question). Then the posttest scores between the groups were compared. To answer the fourth research question, several statistical comparisons of the posttest scores of the structure-based instruction group and the traditional instruction group were conducted.

The Structures of Algebra Word Problems

The first result, the structures of algebra word problems students recognized and the levels of sophistication of the structures, is to answer the first research question: What structures students can recognize when asked to structure algebra word problems, and what are the levels of sophistication of the structures? To answer the research question, this study conducted one-to-one interviews immediately after the posttest to explore word problem structures students could recognize or create and the levels of sophistication of the structures by asking students to classify all 12 word problems they just solved in the posttest based on problems' similarities and asking students to tell why they grouped certain problems together.

To attain the structures of algebra word problems students recognized or created, 61 interview data regarding the two interview questions (1) “Can you classify the 12 word problems into groups based on similarities between the problems?, Can you create five groups, if possible?”, and (2) “Can you tell me why you grouped the problems of each group together?” were transcribed by 5 mathematics education doctoral students, and proofread by the 5 doctoral students and one elementary school teacher who holds a master degree in education. Each interview data was transcribed and proofread by different persons.

The framework defined two levels of sophistication of algebra word problem structures, the *surface structure* and the *deep structure*. The deep structure was divided into two sub-categories, complete and partial. The complete (deep-structure) means the “equal” relationship in word problems. The partial (deep-structure) means the relationship in word problems is a part of the translated equations. A coding scheme was constructed based on the framework to help categorize structures from students’ responses to the two interview questions. The coding scheme consisted of the following four categories: (1) complete relationship (2) partial relationship (3) Real-life event, and (4) Not related. Students’ responses were characterized as the complete relationship when the “equal” relationship in a word problem was mentioned. For example, the cost of the final product is “equal” to the costs of two components combined. The relationship could be mentioned in terms of symbolic expressions (e.g., x+y=z) or real-life events (e.g., miles per hour). Students’ responses were characterized as the partial relationship when multiplicative or additive relationship, but not the equal relationship, was mentioned. For example, “miles per hour” or “dollars per item” is a rate and can form a multiplicative relationship with a quantity. Similarly, the relationship could be mentioned by symbolic expressions (e.g., 3x where x stands for miles per hour) or by real-life events (e.g., miles per hour). Students' responses were characterized as "real-life event" if there was no relationship mentioned at all but only single real-life terms such as "time", "travel, or "money". Students' responses were characterized as "not-related" if they were not about the event or structure of the problem. For example, some students might say "the problem is hard". Descriptions of the four categories and coding examples can be found in the following table (Table 5).

Table 5: Coding Scheme for Categorizing Word Problem Structures

Category		Description	Example
Complete Relationship	symbolic expression	"Equal" relationship is mentioned in terms of symbolic expressions.	"These problems were grouped together because they have variables on both sides"
	event expression	"Equal" relationship is mentioned in terms of real-life event.	"These problems were grouped together because they 1) have the same/equal distance, or 2) make a mixture from two things"
Partial Relationship	symbolic expression	Additive or multiplicative relationship is mentioned in terms of symbolic forms.	"These problems were grouped together because they 1) have a multiplied x, or 2) plus and divide something"

Table 5: (continued).

	event expression	Additive or multiplicative relationship is mentioned in terms of real-life event.	"These problems were grouped together because they 1) have rates, or 2) have two things combined"
Real-life event		No relationship is mentioned but only single real-life terms.	"These problems were grouped together because they were about (1) time, or (2) travel, or (3) money"
Not related		Not about the event or structure of the problem	"These problems were grouped together because they were hard to solve"

The study was designed to include four word problem structures with different qualities. These qualities range from deep to surface level based on as indicated in the table above: "complete relationship", "partial relationship", "general structure", and the "real-life event". (Actually, "general structure" was not in the original design, but interviews indicated that some students used schemas that did not fit with the original three problem structures, so this was added as a way to classify these additional schemas.) All students' schemas can be classified as fitting within one of these four problem structures. Further sub-classification can be done within some structures. Four sub-types fall under the "complete relationship" structure. Two sub-types fall under the "partial relationship" structure; similarly, two fall under the "general structure".

All of the interview excerpts in this section are students' responses to the interview question "Can you tell me why you grouped the problems together?"

The Complete Relationship

Four sub-types of schema under the "complete relationship" structure were identified, which indicate the equal relationship in word problems (See Table 6).

Table 6: Four Types of Schema under the Complete Relationship Structure

Structure	Item	Schema Type
Complete Relationship	1	Objects of Equal Relationship
	2	Action of Equal Relationship
	3	Meeting point of Equal Relationship
	4	Symbols of Equal Relationship

The first schema type is called "Objects of Equal Relationship", which means the equaled objects in an equal relationship are specified. The equaled objects can be distance, rate, or amount of money. Each of the following two interview excerpts indicates an equal relationship between the posttest word problems 3 and 12, and the equaled objects in each of the two equal relationships are specified as distance or money.

> "OK, um, group 3: 3, 6, and 12. Let's see, 3, 6, and 12. Um, I think I did this one because it was, like, all equal. Like, when does this, when does this distance equal this one? Mm hmm And when does this free parking equal the free parking of that one." (Student ID: 7)
>
> "then group 5 was 12 and 3 *(mm hmm)* and I thought they were similar because you're finding where they're the same, sort of like here where it overtook the first

one you have to find where they're at the same distance. Well you're finding the time but you have to find like where they would be the same distance and this one you have to find when the prices would be the same, or how much you would be charged, at what number of hours, the charge would be the same." (Student ID: 39)

The second schema type is called "Action of Equal Relationship", which means the action of making things equal is specified. Each of the following two interview excerpts indicates an equal relationship between the posttest word problems 2, 7 and 11, and the action that makes things equal is specified as mixing.

> "(*Then in your second group you put together problems 2, 7, and 11. So why was that?)* It was about two objects mixing together to make like a combined object." (Student ID: 52)
>
> "I grouped numbers 2, 7 and 11 together um because they're similar in that you have to find how much of um two groups will combine together to make a third group" (Student ID: 21)
>
> *Note: The italic words mean they were said by the interviewer*

The third schema type is called "Meet-Point of Equal Relationship", which means the meeting point of an equal relationship is specified. The following two interview excerpts indicate an equal relationship between the word problems 1, 4, 12 (Student Id:41) and 3, 9 (Student ID: 14), and the meeting point is specified as a place.

> "so I put um 1, 4, and 12 and…they both were coming together equal in one spot" (Student ID: 41)
>
> "when are they going to be the same, and they're trying to get the same place too." (Student ID: 14)

The fourth schema type is called “Symbols of Equal Relationship”, which means the equal relationship specified is related to symbolic expressions. The following interview excerpt indicates an equal relationship between the posttest word problems 2, 3, and 7, and the equal relationship is specified as two equivalent equations.

> “um, group 2 I chose problem numbers 2, 3, and 7, and I chose numbers 2, 3, and 7 because…I guess setting equations equal to each other, um, you get two different equations and then set them equal to each other, I think that’s why I chose those” (Student ID: 37)

The following interview excerpt indicates an equal relationship between the word problems 3 and 10, and the equal relationship is specified as a variable on each side of an equation.

> “Group 3, uh that, those equations look like to me they were set up where they had a variable on each side of the equals sign, so I thought those were similar” (Student ID: 26)

The Partial Relationship

Two sub-types of schema under the “partial relationship” structure were identified, which indicate the partial relationship in word problems (See Table 7).

Table 7: Two Types of Schema under the Partial Relationship Structure

Structure	Item	Schema Type
Partial Relationship	5	Multiplicative Relationship
	6	Additive Relationship

The fifth schema type refers to multiplicative relationship in word problems. As mentioned in the theoretical framework, a multiplicative relationship can be rate, multiple, or part/whole relationship. The following two interview excerpts indicate a multiplicative relationship between the posttest problems 2 and 10 (Student ID: 32) and problems 1, 4, and 9 (Student ID 37), and the specified multiplicative relationship is rate.

> "group number 2, I did 2 and 10 because it had to do with um rates, and this one also had to do with rates" (Student ID: 32)
>
> "um in group 1 I had numbers 1, 4, and 9 and I put numbers 1, 4 and 9 in the same group because they all dealt with um time and finding like speed or how far apart, um, it takes to like reach the um maybe like half way or in the middle, um, just a lot of them involved time and like miles per hour" (Student ID: 37)

The sixth schema type refers to additive relationship in word problems. As mentioned in the theoretical framework, an additive relationship can be Combine, Change, or Compare relationship. The following two interview excerpts indicate an additive relationship between the posttest problems 5, 11 (Student ID: 15) and problems 3 and 7 (Student ID: 22).

> "And then for group 2, I put 5 and 11.mostly because I had to set up the problem kind of in the same way…two variable whereas I need to minus one from the other." (Student ID: 15)
>
> "*(and then you also had uh group of problem 7 and problem 3 what was those or what were those?)* uh they are the ones that had like trying to figure out … different combinations with their prices (ok) with each other" (Student ID: 22)

The General Relationship

Two sub-types of schema under the "general structure" were identified, which indicate some relationship in word problems. However, it is not clear whether the relationship is equal relationship, additive relationship, or multiplicative relationship (See Table 8).

Table 8: Two Types of Schema under the General Structure

Structure	Item	Schema Type
General Structure	7	Similar Context Relationship
	8	Isomorphic Context Relationship

The seventh schema type is called "Similar Context Relationship", which refers to a relationship where elements of the relationship are constrained by the content of specific word problems. The following three interview excerpts indicate some relationships, where they are neither equal relationship nor additive or multiplicative relationship, and the relationship is described in terms of and directly related to the problem statement of some travel type (with direction specified) word problems.

> "all the ones that had people going in opposite directions"
>
> (Student ID: 33, Grouped Problems: 1, 4, and 6)
>
> "group 1 was problem 1, 4, 8 and 9 *(right, so why do you group them together?)* those um those all had to do with uh finding distances or finding when someone would meet wherever you traveled either away from each other, towards each other or to the same spot"
>
> (Student ID: 44, Grouped Problems: 1, 4, 8, and 9)
>
> "grouped 1, 4, 8, 9, and 12 together. *OK. So why do you group them together?* I grouped these ones together because they all have to do with, like, travelling and

dist--, like, yeah. They all travel. *Mm hmm*. They're all travelling problems, like, finding something to do with two objects travelling at or away from each other, or in the same direction."

(Student ID: 6, Grouped Problems: 1, 4, 8, 9, and12)

The eighth schema type is called "Isomorphic Context Relationship", which refers to a relationship where elements of the relationship are not constrained by the content of specific word problems. The following interview excerpt indicate some relationship, where it is neither equal relationship nor additive or multiplicative relationship, and the relationship is not described directly through or directly related to the problem statement of the word problems (5, 6, 11). Problem 5 is about depositing money into two bank accounts, problem 6 is about taking two kinds of courses, and problem 11 is about mixing three kinds of juice. The relationship that constrains the three problems is described in a more general way.

"I put numbers 5, 6 and 11 together because they were all asking for a certain percentage in the sense that you had to find an amount after, like how much of each amount was used" (Student ID: 17)

The Real-Life Event

The real-life event structure means no relationships are mentioned but only single real-life terms such as "money", "percentage", "time". The following two interview excerpts indicate that problems 3, 7, and 5 were grouped because they were about money, and problems 1, 4, 8, 9, 12 were grouped because they were about distance.

"Number 3, 7, and 5 go together because they all have working with money."

(Student ID = 6)

> "I put problems 1, 4, 8, 9, and 12 together. I did that because they were all distance problems." (Student ID = 23)

The Other

Some ideas students used to classify word problems are not about the content of word problems. For example, some students grouped problems together because those problems were hard, or they didn't know how to solve the problems. The following excerpt illustrates the case.

> "Um, group four were the easy, number 7, 1, and 4. Thought those were easier problems. And group five was 2 and 9. Those were the harder problems" (Student ID = 9)

In summary, this study identified four structures with different qualities, which are, from deep to surface level, the complete relationship, partial relationship, general structure, and the real-life event. This study also found four sub-types of schema under the complete relationship structure which are the "objects of equal relationship", "action of equal relationship", "meeting point of equal relationship", and the "symbols of equal relationship", two sub-types of schema under the partial relationship structure which are the "multiplicative relationship" and "additive relationship", and two sub-types of schema under the general structure which are the "similar context relationship" and the "isomorphic context relationship".

The Difficulties in Structuring Algebra Word Problems

The second result, the difficulties in structuring algebra word problems, is to answer the second research question: "What are the difficulties students will encounter when asked to structure algebra word problems, and what are the levels of the difficulties?" To answer the research question, this study conducted one-to-one interviews immediately after the posttest by first asking students to classify all 12 word problems they just solved in the posttest based on their similarities and asking students to tell why they grouped certain problems together, and then students were asked if they encountered any difficulty in classifying the word problems.

To attain the difficulties students encountered, 61 interview data regarding the interview questions “What were the difficulties/struggles/hard parts you encountered when you tried to classify the problems?” were transcribed by 5 mathematics education doctoral students, and proofread by the 5 doctoral students and one elementary school teacher who holds a master degree in education. Each interview data was transcribed and proofread by different persons.

The difficulties students encountered (when structuring word problems) were categorized based on the following two principles: First, students may not know what or which features they can/have to attend to when classifying the problems. This difficulty is based on the fact that students classify word problems according to surface features or deep features of word problems, which was discussed in chapter 2. Second, students may not be able to find appropriate concepts or categories to describe the similar features they observe among word problems. This difficulty was discussed by Reed (Also see chapter 2).

The difficulties students encountered in structuring / grouping the word problems were categorized to the following five types: (1) the difficulty in comparing word problems. Students in this type had difficulty in finding features from the word problems and comparing them. (2) The difficulty in finding similarities between word problems. Students in this type expressed it was hard to find similar features between the word problems. (3) The difficulty in wording similarities. Students in this type had difficulty in finding an appropriate concept/category to constrain the similarity they found between some word problems. (4) The difficulty in differentiating similarities from differences. Students in this type had difficulty in determining which features to compare. Some features of a set of word problems can be used to group the set of word problems. However, some other features of the set of word problems may ungroup the set of word problems. (5) The difficulty in justifying the legitimacy of a group. All of the interview excerpts in this section were students' responses regarding the interview question "What were the difficulties/struggles/hard parts you encountered when you tried to classify the problems?**"**

The Difficulty in Comparing Problems

Some students had difficulty in comparing word problems. They did not know what to compare or how to compare, even though they were prompted to compare and group word problems by similarities between the problems. The following two interview excerpts illustrate the difficulty that students didn't know what to do when structuring problems with different contents or contexts.

> "what is the hard part of classifying them? um they're worded so different, they have different kinds of numbers, different units they're not exactly asking the same thing, so that's hard to," (Student ID = 39)
>
> "where words [3:50] such is like gallons as compared to like investments, it just differs in my mind, so I don't find them the same" (Student ID = 49)

The Difficulty in Finding Similarities between Problems

Some students were able to find similarities between word problems and grouped them together. However, they admitted it was hard to find similarities. The following two interview excerpts illustrate the difficulty in finding similarities when grouping problems.

"But the difficult part was, um, they used different terminology in each one. So trying to figure out, um, which, which problems, uh, were similar *mm hmm* and how they, how they corresponded to each other." (Student ID: 1)

"Trying to, like, I guess, find enough things that they have similar together" (Student ID: 6)

The Difficulty in Wording Similarities

Some students found similarities between word problems and grouped the problems together. However, they had difficulty in expressing the character of the group or naming the group. The following interview excerpt shows that a student had difficulty in giving a name to a group she created.

"It is difficult to figure out which word I want to use, I mean in my head I know why I grouped them together, but a certain word I wanna use, I can't think of it right now. But I mean, I don't know how I wanna word it," (Student ID = 11)

The Difficulty in Differentiating Similarities from Differences

Students had difficulty in differentiating or grouping word problems when there were both similarities and differences between the problems. The following two excerpts illustrate how differences between word problems prevented students from grouping the problems together when the students had seen similarities between the problems.

"those were kind of hard to group together because it has to do with these two people meeting each other, and uh it has to do with distance but it also has to do with time, and this one also has to do with different time but they're going the same distance when this one is not the same distance *(not the same distance)* yeah, it is different time, *(ok, ok, so you mean 8, and 9 right?)* yes, *(ok, so it's hard to* *group them because sometimes so you see some similarities)* some similarities but also different *(so it's hard to group them)* yeah." (Student ID: 32)

"it's confusing by like trying to like [4:15] connect all of them together (mm hmm) because like all of them like I said they all have something to do with rate like per per time, per second, per whatever, like when they meet (mm hmm), but they all like asking different things so I (ok) just got confused" (Student ID: 43)

Note: Words in the parentheses are from the interviewer

The Difficulty in Justifying the Legitimacy

Students may still feel uncertain when they can structure word problems into groups. The following interview excerpt illustrates that a student still felt uncertain to some groups she created even though these groups were created with plausible reasons.

"I wasn't sure what to what to group them into, for instance, I grouped um 3 and 6 because they both had variables on the each side, and each side and I grouped 5 and 11 because they um deal with percentages, but, um, but 1, 2, for group 1 I chose 1, 2, 4, 8, 9 and 12 and those are kind of to me they were kind of the same type of problem I use kind of the same method to figure them out, so I grouped them and to a certain group but um, yeah it was kind of difficult overall, I wasn't sure how you wanted me to group them. *(ok, so, um what is the hard part? So, um, so you you don't um so when when finding the similarities and saying that these are a group, these are a group, why it is um to you is um you are not so um confident or hesitate to say that's a)* um because to me I wasn't sure what um what ones went with, with different ones, and I kind of like different, different ideas of what I think, um, what I think are the similarities, but it's this difficult for me because I wasn't sure if that's what you were asking" (Student ID: 58)

Note: Words in the parentheses are from the interviewer

Some students did not have any difficulty in classifying word problems (about 5% of the participants). The followings are interview excerpts from such kind of students.

"I think it was easy to classify, it was easy for me to um point out just the way I set up the problems with the diagrams like, you know. it was easy for me to separate which ones looked similar." (Student ID = 45)

"I don't think it was anything hard about it, 'cause you just find similarities, so I don't think that's hard *(so that's not that hard)* no *(ok)*." (Student ID: 60)

Note: Words in the parentheses are from the interviewer

In summary, this study found five levels of difficulties students encountered when asked to structure algebra word problems, which are (1) The difficulty in comparing word problems (2) The difficulty in finding similarities between word problems (3) The difficulty in wording similarities (4) The difficulty in differentiating similarities from differences, and (5) The difficulty in justifying the legitimacy of a group.

The Effect of Structure Quality

The third result, the effect of structure quality, answers the third research question: Is there performance difference between students who recognize different qualities of algebra word problem structures in solving transfer algebra word problems? To answer this research question, two ways of quality differentiation were employed, which were the complete structure (CS) vs. the partial structure (PS), and the number of complete structures created by each student.

Descriptive and inferential statistical analyses were conducted to compare the performance of the CS and PS groups. Three linear regressions were conducted to compare the performance of students who recognized different numbers of complete structures.

The Effect of the Complete Structure and the Partial Structure

According to the first result, a student who recognized at least one complete relationship was placed in the complete structure (CS) group. (Each student recognized more than one structure because they were asked to generated 5 groups/structures when asked to classify the word problems) A student who did not recognize any complete relationship and s/he recognized more than 50% partial relationships among the structures/groups he/she recognized was placed in the partial structure (PS) group.

The structures each student recognized were coded as Complete Structure (CS), Partial Structure (PS), Real-Life event (RL), and Not-Related (NL) based on the coding scheme (See result 1). Particularly, an inter-rater reliability coding was conducted. Two coders (the author and one who transcribed the interview data) coded the 61 interview transcripts with respect to the first two interview questions ("Can you classify the 12 word problems into groups based on similarities between the problems? Can you create five groups, if possible?" and "Can you tell me why you grouped the problems of each group together?"). Ten percent of the transcripts were used for coding training. The two coders coded the ten percent separately, then compared their codes and discussed the discrepancies. Another ten percent of the transcripts were used for inter-rater reliability coding. The two coders achieved 89% inter-rater reliability. The author coded the rest 80% interview transcripts. The coding result showed that 33 students were placed in the CS group, 24 students were placed in the PS group, and four students were place in the RL group. This study excluded the 4 RL group students from the statistical analysis due to the small number.

The effect of the two kinds of structure (CS, PS) in solving word problems was evaluated in terms of the three transfer types (SS, SD, DS) and the three real-life problem types (travel, mixture, other).

According to the mean scores, students who recognized the complete structure (i.e., schema types 1, 2, 3, or 4) in structuring word problems (the CS group) performed better than students who only recognized partial structure (i.e., schema types 5, 6, 7, or 8) in structuring word problems (the PS group) on each of the three transfer types, SS, SD, and DS. Students' average ratio of correctness in solving SS, SD, and DS type transfer word problems in the posttest is illustrated in Table 9.

Table 9: Means of the CS and PS Groups by Transfer Type

Group		SS	SD	DS
CS	Mean	.4672	.1793	.3763
	N	33	33	33
	Std. Deviation	.33002	.22450	.27173
PS	Mean	.4479	.1736	.3646
	N	24	24	24
	Std. Deviation	.30477	.21691	.28320

According to the mean scores, the CS group also performed better than the PS group on each of the three real-life event types. Students' average ratio of correctness in solving Travel, Mixture, and Other type word problems in the posttest is illustrated in Table 10.

Table 10: Means of the CS and PS Groups by Problem Type

Group		Travel	Mixing	Other
CS	Mean	.4288	.1515	.4596
	N	33	33	33
	Std. Deviation	.32527	.24425	.33601
PS	Mean	.3896	.1389	.4028
	N	24	24	24
	Std. Deviation	.22300	.21656	.30660

A t-test was conducted to evaluate if the two groups (CS, PS) had different problem-solving performance (regarding the posttest). The result showed that there was no significant performance difference between the two groups (t=0.403, df=55, $p>0.1$; See Table 11).

Table 11: Statistics of the Posttest Scores of the CS and PS Groups

Group	N	Mean	Std. Deviation	Std. Error Mean
CS	33	.353571	.2016437	.0351017
PS	24	.332561	.1837487	.0375076

The Effect of the Complete Structure

Three linear regressions were conducted. The independent variable was the number of complete relationship structures students created, and the dependent variable was students' posttest scores. Particularly, the three regressions were conducted for the three different groups of subjects: the total 61 participants, the students (N=15) who enrolled intermediate algebra (MTH 1825), and the students (N=46) who enrolled college algebra (MTH 103).

Each of the three regressions showed that the number of complete relationship structures students recognized did not significantly predict students' posttest scores ($p > 0.1$).

The Effect of Structure-based Instruction in Solving Algebra Word Problems

The fourth result, the effect of structure-based instruction in solving algebra word problems, is to answer the fourth research question: what is the effect of structure-based instruction in learning to solve transfer algebra word problems? To answer this research question, the overall posttest scores, the posttest scores regarding the three types of transfer, and the posttest scores regarding the three real-life problem types of the two conditions (control/tradition instruction, experimental/structure-based instruction) were compared. First, several t-tests were conducted to evaluate if students from the structure-based instruction group (the SB group) and from the traditional-based instruction group (the TB group) had different performance in solving algebra word problems. Second, two MANOVA tests were conducted to evaluate if the two groups of students had different performance in the three types of transfer (SS, SD, and DS) and the three real-life problem types (travel, mixture, and other). Both of the two tests had the same independent variable, the instructional method. The dependent variables of the two tests were the posttest scores of the three types of transfer (SS, SD, DS) and the posttest scores of the three types of real-life problem (Travel, Mixture, Other) (See Table 12).

Table 12: Variables of the Two MANOVA Tests

	Independent Variable	Dependent Variable
MANOVA Test 1	Instruction Method • Group 1: Structure-based instruction • Group 2: Traditional instruction	Posttest scores of the three transfer types • SS • SD • DS
MANOVA Test 2	Instruction Method • Group 1: Structure-based instruction • Group 2: Traditional instruction	Posttest scores of the three real-life problem types • Travel • Mixture • Other

The Treatment Effect in General

To analyze if there was a significant performance difference between students who received structure-based instruction (the SB group) and students who received tradition-based instruction (the TB group), three statistical analyses were conducted. First, the means of the pretest scores of the two groups were compared to see if the two groups were equivalent. Second, the pretest and posttest scores were compared for each of the two groups to see if the treatment was effective. Third, the means of the gain scores of the two groups were compared.

The first analysis showed that the two groups were equivalent (t=0.538, p>0.1; See Table 13).

Table 13: Statistics of the Pretest Scores of the SB and TB Groups

Condition	N	Mean	Std. Deviation	Std. Error Mean
SB	29	.101635	.1450173	.0269290
TB	32	.085243	.0890356	.0157394

The second analysis showed that the treatment for the SB group was significantly effective ($t=6.392$, $df=28$, $p<0.001$; See Table 14), and the treatment for the TB group was also significantly effective ($t=6.952$, $df=31$, $p<0.001$; See Table 15).

Table 14: Paired Samples Statistics of the SB Group

Test	Mean	N	Std. Deviation	Std. Error Mean
Posttest	.373807	29	.2105664	.0391012
Pretest	.101635	29	.1450173	.0269290

Table 15: Paired Samples Statistics of the TB Group

Test	Mean	N	Std. Deviation	Std. Error Mean
Posttest	.307475	32	.1720435	.0304133
Pretest	.085243	32	.0890356	.0157394

The third analysis showed that students' performance of the two groups were not significantly different ($t=0.949$, $df=59$, $p>0.1$; See Table 16).

Table 16: Statistics of the Gain Scores of the SB and TB Groups

Condition	N	Mean	Std. Deviation	Std. Error Mean

Table 16 (continued).

SB	29	.272172	.2293141	.0425826
TB	32	.222232	.1808195	.0319647

This shows that the groups were initially equivalent, and the students in each group improved during the instruction, but the amount of improvement was not significantly different between these two groups.

The Treatment Effect on the Three Transfer Types

According to the mean scores, the structure-based (SB) instruction group performed better than the traditional-based (TB) instruction group on the SS type and SD type transfer problems, but not on the DS type transfer problems (See Table 17). The SB group had higher mean scores (0.06 and 0.11 higher) than the TB group had on the SS and SD type word problems. However, the SB group had lower mean score (0.01 lower) than the TB group had on the DS type word problems.

Table 17: Means of the SB and TB Groups by Transfer Type

Condition		SS	SD	DS
SB	Mean	.49	.22	.36
	N	29	29	29
	Std. Deviation	.36	.23	.29
TB	Mean	.43	.11	.37
	N	32	32	32
	Std. Deviation	.28	.19	.26

The first MANOVA test was conducted comparing the mean scores of the structure-based (SB) instruction group (N=29) and the traditional-based (TB) instruction group (N=32) regarding the three transfer types, SS, SD and DS. The test showed the two groups performed significantly different on SD type word problems (F=4.3, p=0.042). The effect size (Partial Eta Squared = 0.68) of the difference is medium (Cohen, 1992). The structure-based instruction group performed significantly better than the traditional-based instruction group on SD type word problems. However, there was no difference between the two groups on SS and DS type word problems. Statistics are showed in Table 18.

Table 18: Statistics of the First MANOVA Test

Source	Dependent Variable	df	F	Sig.	Partial Eta Squared
Corrected Model	SS	1	.666	.418	.011
	SD	1	4.308	.042	.068
	DS	1	.022	.884	.000

The Treatment Effect on the Three Real-Life Problem Types

According to the mean scores, students in the structure-based (SB) instruction group performed better than students in the traditional (TB) instruction group on the Travel and Mixture type word problems, but not on the Other type problems (See Table 19). The SB group had higher mean scores (0.14 and 0.03 higher) than the TB group had on the Travel and Mixture type word problems. However, the SB group had lower mean score (0.01 lower) than the TB group had on the Other type word problems.

Table 19: Means of the SB and TB groups by Problem Type

Condition		Travel	Mixture	Other
SB	Mean	.4770	.1638	.4253
	N	29	29	29
	Std. Deviation	.29481	.26209	.33802
TB	Mean	.3344	.1380	.4323
	N	32	32	32
	Std. Deviation	.25593	.20148	.30190

The second MANOVA test was conducted comparing the mean scores of the structure-based (SB) instruction group (N=29) and the traditional-based (TB) instruction group (N=32) regarding the three real-life problem types, Travel, Mixture and Other. The test showed the two groups performed significantly different on Travel type word problems (F=4.1, p=0.048). The effect size (Partial Eta Squared = 0.065) of the difference is medium (Cohen, 1992).The structure-based instruction group performed significantly better than the traditional instruction group on Travel type word problems. However, there was no performance difference between the two groups on the Mixing and Other type word problems. Statistics are showed in Table 20.

Table 20: Statistics of the Second MANOVA Test

Source	Dependent Variable	df	F	Sig.	Partial Eta Squared
Corrected Model	Travel	1	4.091	.048	.065
	Mixing	1	.187	.667	.003
	Other	1	.007	.932	.000

Chapter 6

Discussion and Implication

Students have been facing difficulty in transferring prior learned problem-solving skills to novel problems, and structure has been proposed helpful in facilitating transfer. Examining the research literature, only a few studies explored algebra word problem structures and how well the structures were helpful in facilitating transfer. Most of the studies focused on mental process and representation (e.g., what process induces good quality of schema or what quality of schema enhances transfer). Little research has been done concerning what classifications of algebra word problem structures could be that would be helpful in teaching for transfer. On the other hand, algebra word problem structures have been reported difficult to recognize or it requires substantial training to recognize the structures.

Therefore, the purpose of the study is to explore (1) the structures of algebra word problems students can recognize (2) the difficulties students encounter in recognizing structures of algebra word problems, and (3) if particular structures are helpful in teaching for transfer of learning to solve algebra word problems. The first and second results of the study respond to the first two purposes respectively. The third and fourth results of the study respond to the third purpose.

For the first result, four structures with different qualities (from surface to deep structure) were identified. Different schemas or conceptions under the structures were also found. For example, the complete relationship structure has four different ways of conception or four types of schema recognized by students regarding the equal relationship in a word problem.

For the second result, five levels of difficulties students encountered when asked to structuring algebra word problems were identified. Difficulty levels range from the surface level of finding features of word problems for comparison, recognizing the similarities among problem features, wording similarities, to the deeper level of differentiating similarities from differences.

For the third result, although the descriptive statistical analysis showed that students who recognized good-quality algebra word problem structures had higher performance compared to students who recognized partial algebra word problem structures, the inferential statistical analyses (the t-test and the three linear regressions) showed that the quality of word problem structures did not significantly affect students' performance of solving algebra word problems.

For the fourth result, the inferential statistical analyses showed that students who received the structure-based instruction (the SB group) performed significantly better than students who received the traditional instruction (the TB group) in solving the SD transfer type of word problems (but not the SS and DS type transfer problems) and the travel type word problems (but not the mixture and other type word problems). However, the overall performances regarding the posttest scores of the two groups (SB, TB) were not significantly different. Although the structure-based instruction was effective in improving students' performance in solving algebra word problems, the traditional-based instruction was as effective as the structure-based instruction in improving students' performance.

The Structures of Algebra Word Problems

Four word problem structures were identified in this study. The levels of sophistication of the four structures, from deep/good to surface/poor (according to Gick & Holyoak (1983), Holyoak & Koh (1987), and Schoenfeld & Herrmann (1982)), are the complete relationship, the partial relationship, the general structure, and the real-life event. The general structure is the only one that was created during the analysis, that is, not in the coding scheme. Some students recognized good quality structures (e.g., the equal relationship). Some students recognized intermediate quality structures (e.g., the additive or multiplicative relationship). Some students recognized poor quality structures which indicate no relationships, for example, single real-life terms (e.g., travel, money, interest).

Gick and Holyoak (1983) proposed three types of schema quality, good, intermediate, and poor, based on how many core components a schema captures (Also see chapter 2). Similarly, the quality of the four structures suggests a 2x2 quality table which differentiates the quality into four types (See Table 21). A structure with better quality captures more relationships embedded in a story and in an equation of a word problem. In other words, more story content and more equation components are captured. On the other hand, a structure with poor quality captures less story content and less equation components. Specifically, schema types 1-4 of the complete relationship structure which specify the equal relationship can be regarded as good quality (Cell A1) because they capture the story on each side of an equation (the equaled things) and capture the complete equation component (the equal relationship). Schema types 5 and 6 of the partial relationship structure which specify additive or multiplicative relationship can be regarded as intermediate quality (Cell A2). They capture less story content (partial relationship). Schema types 7 and 8 of the general structure which specify some relationship (not operational relationships such as equal, multiplicative or additive relationships) can be regarded as intermediate quality (Cell B1). They capture less equation components (no equal, additive or multiplicative relationships specified). The real-life event that specifies no relationship can be regarded as poor quality (Cell B2) because it captures much less of story content and equation components compared to the other three qualities.

Table 21: Four Qualities of the Four Structures

		Story Content	
		More	Less
Equation Component	More	Good Quality (A1)	Intermediate Quality (A2)
	Less	Intermediate Quality (B1)	Poor Quality (B2)

The quality table provides researchers an empirical-based framework for further exploration on which quality of algebra word problem structures would be effective in solving which type of transfer word problems (i.e., SS, SD, DS). For example, will students perform differently between those who recognize A1structure and those who recognize B1 structure with respect to the three types of transfer (SS, SD, and DS)?

Conceptual understanding of symbolic expressions has been the central goal of mathematics education reform, especially in school algebra (NCTM, 2000). Teachers may use the eight types of schema of the structures as a framework, which combines story structure and equation structure together, to conceptually connecting the equal, additive and multiplicative relationships in an equation to real-life situations. For example, the equal sign in an equation can be explained/understood as two objects (e.g., two distances) that are equal to each other (i.e., schema type 1) or the meeting point of two quantities (e.g., cost) that are equal to each other (i.e., schema type 3). The multiplicative term in an equation can be explained/understood as a rate structure such as “miles per hour times hours” or “dollars per minute times minutes” (i.e., schema type 5). In other words, the equation-and-story-combined character of the eight types of schema provides teachers ways of relating symbolic expressions to real-life situations for teachers to conceptually elaborate and make sense of algebraic operations and equations.

Another aspect of the eight types of schema that is worth our attention is the different perspectives or approaches in describing similar structures. For example, the study found students used four different perspectives (object, action, meeting point, and symbol) describing the structure of equal relationship, two perspectives (addition and multiplication) describing operational relationships, and different single terms characterizing word problems (e.g., money, travel, and mixture).

The different perspectives informed researchers that students' discourses of describing word problems and their relationships need to be analyzed more carefully because students' languages may vary and ideas may also vary when describing the same algebra word problem structure. In other words, researchers need to be aware of different ways of expression (languages or ideas) when examining whether a student holds a word problem structures.

The different perspectives (e.g., object, action, meeting point, or symbol in describing the equal relationship in an equation) in describing word problems inform teachers that students may have different comprehensions or ways of understanding word problems. For example, there are different ways of explaining or comprehending "equal sign" in the following problem: Two cars run in opposite direction with different speeds (60 mph and 70 mph) at 2pm. When were the two car 260 miles away from each other? If the variable is time traveled, then the equal sign can be explained as two distances equaling each other (schema type 1), or can be explained as combining two cars to make 260 miles traveled in total (schema type 2), or can be explained as two variable terms combined equaling a constant (schema type 4). Teachers may want to know the different ways of expression / understanding of the equal sign in an equation / word problem when teaching students the equal relationship or listening to students about their understanding of the equal relationship in an equation or word problem.

The Difficulty in Structuring Word Problems

The difficulties students encountered (when structuring algebra word problems) are categorized to five types or levels. In the study, some students had difficulty in the surface level of finding features from word problems and comparing them, or finding similarities between word problems. When similarities were found, some students had difficulty in wording the similarities, or, had difficulty in finding an appropriate concept/category to constrain the similarities. For the deeper level, some students had the difficulty in differentiating similarities from differences. Some features (similarities) of a set of word problems can be used to group the set of word problems. However, some other features (differences) of the set of word problems may ungroup the set of word problems.

These levels of difficulty indicate a progression of structuring algebra word problems (See Table 22). The lower/surface levels (e.g., 1, 2, and 3) refer to the beginning stages and the higher/deep levels (e.g., 4 and 5) refer to the closing stages in structuring word problems.

Table 22: Five Levels of Structuring Algebra Word Problems

Level	Ability
1	Compare word problems
2	Find similarities between word problems
3	Word similarities
4	Differentiate similarities from differences
5	Justify the legitimacy of a group

The levels of progression of structuring algebra word problems extend our understanding of the induction process of algebra word problem structures. Examining the literature, almost all of the studies about the induction of word problem structures emphasized on induction methods, for example, summarizing story analogs and/or describing similarities between stories (Catrambone & Holyoak, 1989; Cummins, 1992; Gick & Holyoak, 1983), solving analogical problems (Cooper & Sweller, 1987), or directly instructing schemas (Fuchs et al., 2003, 2004; Jitendra et al., 2002; Xin, 2005). Few studies addressed difficulties in the induction process.

The 5 levels inform researchers of four fundamental issues underlying schema induction methods that have been widely discussed in the psychology literature, which are (1) how to compare problems or solutions (2) how to find similarities between problems (3) how to differentiate similarities from differences, and (4) how to categorize problems.

The 5 levels inform teachers that it may not be intuitive to students to differentiate word problems in terms of similarity or difference. In other words, students may have difficulty in seeing similarities or differences between word problems. The progression of the 5 levels of seeing word problem structures may help teachers diagnose students' difficulties in seeing similar / different things between word problems (e.g., similar on which or different on which).

The Effect of Algebra Word Problem Structures

This study showed that the quality of word problem structures does not affect students' performance of solving algebra word problems, and the structure-based instruction does not make significant difference in helping students solve algebra word problems compared to the traditional-based instruction. However, this study showed that teaching rate structure and two-car model (the structure-based instruction group) helped students achieve significant immediate transfer when contexts were similar and equations embedded in problems were altered, or when problems were travel type problems (e.g., distance, speed, and time). Teaching structures did not make significant difference on the transfer types of SS and DS, and on the mixture or other problem types, compared to the traditional instruction. The results suggest that (1) rate structure and two-car model are effective in facilitating SD type of transfer (similar context but different equation) and travel type of word problems, or more generally, (2) structure-based instruction method is effective in teaching for transfer with respect to the SD type of transfer.

There are two possible explanations for the failure of the structure-based instruction on the SS and DS types of transfer. First, traditional instruction group students could easily recall what they just learned (the stories and their solution methods) when encountered problems with similar context and similar equation (SS type). Second, since the treatment lasted only one hour, the dosage of treatment for the structure-based instruction group might not be enough to result in significant transfer on DS type (different contexts and similar equation) problems, compared to the traditional instruction group. DS type of transfer, compared to the other two types of transfer, is far transfer (or far from what students had just learned), which typically takes more practice time and still is difficult to achieve.

Teaching rate structure and two-car model also helped students achieve significant immediate transfer when problems were about travel (speed, time, and distance). However, teaching the structure and model did not make significant transfer difference on mixture and other types of problems compared to the traditional instruction. There are two possible explanations. First, the two-car model was directly related to the travel type problems. It could help students comprehend travel type stories. However, the model was not directly related to the other types of word problems, although isomorphic relationship could exist (e.g., two pumps with different rates filling a tank can be understood as two cars running toward each other), and therefore the two-car model might not help students comprehend the other types of word problems. Second, since the treatment lasted only one hour, the dosage of treatment for the structure-based instruction group might not be enough to result in significant transfer in solving problems with different contexts/stories, that is, different from the two-car model.

Limitations

The mathematics domain of this study was restricted to one-variable linear equation word problems. The results (e.g., the structures students recognized, difficulties in structuring word problems, or the effect of structure-based instruction) may not be applied to other algebra topics such as quadratic equations or trigonometric functions. For example, structures students could see in quadratic equation word problems or difficulties students could encounter in structuring quadratic equation word problems may be different from the findings of the study.

The structure adopted by the structure-based instruction was restricted to rate structure and the two-car model. It is still unknown whether the effectiveness of structure-based instruction on SD type of transfer proved in this study is true for other kinds of structure (e.g., multiplicative structures like multiple or part/whole, or additive structures like change or compare). More generally, it is unknown whether teaching other kinds of structure will have different effect on the three types of transfer (SS, SD, and DS). Therefore, future explorations to test the effectiveness of other kinds of structure by means of structure-based instruction are needed for the two purposes (1) to generalize the effectiveness of structure-based instruction in the transfer of learning to solve algebra word problems, and (2) to explore the effective relationships in facilitating transfer between kinds of structure and types of word problems (e.g., two-car model for travel type).

The intervention time of this study was restricted to one hour due to limited budget. It would be too early to conclude that structure-based instruction is not effective in solving SS and DS types of transfer word problems. Studies with longer intervention time may provide more insights on the effect of structure-based instruction on the SD and DS types of transfer.

Conclusion

This study has several contributions to the research of word problems, and to the research and teaching of algebra word problems. Specifically, this study addresses two research issues of word problems, which are the influence of schema and the induction process of word problem structures. This study addresses two research issues of algebra word problems, which are the effect of schema-based instruction in the transfer of solving algebra word problems and student's discourse analysis regarding algebra word problems. This study also addresses three issues of teaching algebra word problems, which are the conceptual understanding of algebraic expressions, students' ways of comprehending word problems, and students' difficulty in comparing word problems.

Regarding the research on word problems, this study examines the influence of the quality of schema. Some studies (e.g., Gick & Holyoak, 1983; Holyoak & Koh, 1987) show that subjects who hold good quality of schema have better performance than subjects who hold poor quality of schema in solving some types of non-mathematics word problems (e.g., finding a way to destroy tumor using rays with the constrain that high-intensity rays from one direction is not possible; Possible solution: apply weak forces along multiple paths simultaneously). However, this study found that the quality of schema did not affect students' performance in the domain of algebra word problems. Specifically, there was no difference between the performance of students who recognize more good-quality schemas and students who recognize less good-quality structures or who recognize poor-quality schemas in solving one-variable linear equation word problems. Second, this study examines the difficulty in the induction process (e.g., comparing word problems) of word problem structures/schemas and finds five levels of difficulty. It is worth for currently structure/schema-induction method researchers to consider the five levels of difficulty as helping students overcome the difficulties of comparing word problems may affect the induction outcome.

Regarding the research of algebra word problems, this study examines the effect of schema-based instruction. Several studies (e.g., Fuchs et al., 2004; Jitendra et al., 2002) show that teaching certain types of multiplication or division schema (e.g., the division schema of "buying bag", for example, "Jose needs 32 party hats for his party. Party hats come in bags of 4. How many bags of party hats does Jose need?") improve near transfer (e.g., similar context, similar solution) and far transfer (e.g., different contexts, similar solution). This study found that the structure-based instruction was effective in improving students' performance of solving algebra word problems. However, the study also found that the traditional-based instruction was as effective as the schema-based instruction in improving students' performance. Particularly, there was no significant difference between the two instruction methods in solving near transfer problems (i.e., similar context, different equation structures) and far transfer problems (i.e., different contexts, similar equation structure), although the structure-based instruction group performed significantly better than the traditional group in solving problems with similar context and different equation structures. Second, this study provides insights to students' discourse analysis of algebra word problems. Specifically, this study informs researchers that students' discourses of describing word problems and relationships in the problems need to be analyzed more carefully because students' languages may vary and concepts may also vary when describing the same algebra word problem structure (e.g., the four schemas of the complete relationship structure). In addition, this study proposes a framework (See Table 21) for future research on the effect of structure in the transfer of learning to solve algebra word problems.

Regarding the teaching of algebra word problems, this study provides insights to teachers about students' ways of comprehending word problems. Specifically, students may have different conceptions regarding a word problem (structure), for example, the four conceptions / schemas toward the complete relationship structure. This study also provides insights to teachers about student's difficulty in comparing word problems. Specifically, this study informs teachers that it may not be intuitive to students to differentiate word problems in terms of similarity or difference. The progression of the 5 levels of seeing word problem structures can help teachers diagnose students' difficulties in seeing similar / different things between word problems.

In summary, algebra word problems are difficult to students in many ways. This study is just the beginning episode of finding solutions to help students and teachers deal with the complexity of solving algebra word problems. More research is needed to find effective ways to help students transfer their problem-solving knowledge and skills to problems situated in relatively novel contexts. Similarly, more research is needed to provide teachers effective strategies to teach algebra word problems, and to provide teachers the knowledge of algebra word problems and the skill of analyzing algebra word problems so that they may help their students in solving algebra word problems.

APPENDICES

Appendix A
Pretest

Please solve the following 12 word problems and show all your work

1. Ellen and Kate raced on their bicycles to the library after school. They both left school at 3:00pm and bicycled along the same path. Ellen rode at a speed of 9 miles per hour and Kate rode at 12 miles per hour. Ellen got to the library 15 minutes later than Kate. How long did it take Ellen to get to the library?

2. Suppose you work in a restaurant. You want to make 10 liters of punch with 15% alcohol solution for your friend. You decide to mix 10% solution with 30% solution, to make your own 15% solution. How many liters of 10% solution and 30% solution should you use?

3. A store has $30,000 of inventory in 12-in and 19-inch color televisions. The profit on a 12-inch set is 22% and the profit on a 19-inch set is 40%. The profit for the entire stock is 35%. How much was invested in each type of television?

4. Two families meet at a park for a picnic. At the end of the day, one family travels east at an average speed of 42 miles per hour and the other travels west at an average speed of 50 miles per hour. Both families have 160 miles to travel. Find the time that will have elapsed when they are 100 miles apart.

5. Jane and Peter leave their home traveling in opposite directions on a straight road. Peter drives 15 miles per hour faster than Jane. After 3 hours, they are 225 miles apart. Find Jane’s speed.

6. A grocer mixes two kinds of nuts that cost $2 per pound and $3 per pound respectively, where the total weight of the two kinds of nuts is 10 pounds. The new product’s (mixed nuts) cost has been calculated at $2.40 per pound so that the value of the new product is the same as the value of the two kinds of nuts combined. How much of each kind of nut is put into the mixture?

7. On the first part of a 418-mile trip, a salesman averaged 58 miles per hour. He averaged only 52 miles per hour on the last part of the trip because of an increased volume of traffic. Find the amount of time at each of the speeds if the last part of the trip took 2.75 hours more compared to the first part of the trip.

8. Lena invested the same amount of money into two different bank accounts with 8% and 11% simple interests. During one year, she earned $560. How much did she invest in each account?

9. A train leaves a train station at 1pm. It travels at an average rate of 60 miles per hour. A high-speed train leaves the same station an hour later. It travels at an average rate of 90 miles per hour. The second train follows the same route as the first train on a track parallel to the first. In how many hours will the second train catch up with the first train?

10. Nora took her sister to the city airport. Because of traffic conditions, she drove 3.75 hours, but she drove only 1.5 hours from the airport to her home. On her way from the city airport to

home, she averaged 40 miles per hour faster compared to her average speed from home to the airport. What is Nora's average speed from the airport to her home?

11. The distance along a shipping route between San Francisco and Honolulu is 2,100 miles. If one ship leaves San Francisco at the same time another leaves Honolulu, and if the former travels at 15 miles per hour and the latter at 20 miles per hour, how long will it take the two ships to meet each other?

12. Dana invested in two funds paying 9.5% and 11% simple interest. She invested $2000 more in the fund with 11% rate compared to the amount invested in the fund with 9.5% rate. During one year, she earned $1040 from the two funds in total. How much did Dana invest in each fund?

Appendix B
Posttest

Please solve the following 12 word problems and show all your work.
1. Ming and Janet's families camped at a national park. At the end of the day, Ming's family traveled east at an average speed of 65 miles per hour. Half an hour later, Janet's family traveled west at an average speed of 70 miles per hour. Find the time Ming's family traveled when they were 235 miles apart.

2. Two pumps are used to fill a water storage tank at a resort. One pump can fill the tank 5 gallons per minute faster than the other one. When both pumps are operating together, the tank can be filled in 10 minutes. The capacity of the tank is 250 gallons. Find the filling rate of each pump.

3. In an airport, the short term parking rate is $3 per hour. The long term parking rate is $8 per hour, but customers get two hours parking for free. When will short term and long term parking charges be the same?

4. Two bicyclists ride in opposite directions from the same place. The speed of the first bicyclist is 5 miles per hour faster than the second. After 2 hours they are 70 miles apart. Find their speeds.

5. Jose deposited money into two bank accounts paying 2.5% and 4% simple interest. He put $4000 more in the account with the 4% rate compared to the amount put in the account with the 2.5% rate. During one year, he earned $500 more from the 4% account compared to the 2.5% account. How much did Jose deposit in each account?

6. Jason has 7 courses to take this semester including some selective and some core courses. After some investigations, he discovers that the probability of getting a grade of 3.0 from the core courses is 60%, and getting a grade of 3.0 from the selective courses is 74%. If he wants to get a grade of 3.0 with 70% probability, how many core and selective courses should he take?

7. A grocer mixes two kinds of fruits that cost $1.19 per pound and $ 1.69 per pound respectively, where the total weight of the two kinds of fruits is 10 pounds. The new product (a mixed fruit basket) cost has been calculated $1.29 per pound so that the value of the new product is the same as the value of the two kinds of fruits combined. How much of each kind of fruit is put into the mixture?

8. Kenisha and Sarah walk toward each other from the chemistry lab and the library respectively. Kenisha walks with an average speed of 3 miles per hour and Sarah walks with an average speed of 5 miles per hour. The distance between the lab and the library is 4/10 of a mile. How long will it take for the two persons to meet each other?

9. Allen and Susan both left from their homes for a conference at 1pm. They both took the same eastbound highway. Allen averaged 60 miles per hour and Susan averaged 70 miles per hour. Susan lives 5 miles west of Allen's house. Allen arrived at the conference 15 minutes later than Susan. How long did it take Allen to get to the conference?

10. An old piece of equipment can print, stuff, and label 38 letters per minute. A newer model can handle 82 per minute. How long will it take for both pieces of equipment to prepare a mailing of 6,000 letters?

11. A company produces bottles of tropical juice. Each bottle is 64 fl oz with 80% pure juice. Three kinds of juice, mango, pineapple, apple, are mixed together with 95%, 85%, and 70% pure juice respectively to make a bottle of 80% tropical juice. The company uses the same amount of mango and pineapple juice in the mixture. How many ounces of mango, pineapple, and apple juice are used for a bottle of tropical juice?

12. A jet leaves the Charlotte, North Carolina, airport traveling at an average rate of 564 km/h. Another jet leaves the airport one half hour later traveling at 744 km/h in the same direction. How long will it take for the second jet take to overtake the first?

Appendix C
Design of the Pretest and Posttest

The Design of the Pretest

Type 1

1.1 Two-car opposite
1.2 Mixture - investment
1.3 One-car one-way different speeds

Type 2

2.1 Two-car toward each other
2.2 Two-car toward each other with time unknown
2.3 Mixture - investment

Type 3

3.1 Two-car follow-up
3.2 Two-car end different
3.3 One-car two-way different speeds

Type 4

4.1 Mixture - nuts
4.2 Mixture - liquids
4.3 Mixture - investment

Type 1

1.1 Two-car opposite: (College Algebra), #5 in the pretest

Jane and Peter leave their home traveling in opposite directions on a straight road. Peter drives 15 miles per hour faster than Jane. After 3 hours, they are 225 miles apart. Find Jane's speed.
3x+3(x+15)=225

1.2 Mixture - Investment: (College Algebra, Larson, p. 173, modified), #12 in the pretest

Dana invested in two funds paying 9.5% and 11% simple interest. She invested $2000 more in the fund with 11% rate compared to the amount invested in the fund with 9.5% rate. During one year, she earned $1040 from the two funds in total. How much did Dana invest in each fund?
0.095x+0.11(x+2000)=1040

1.3 One-car one-way different speeds: (Algebra & Tri., p. 165, Q43, modified), #7 in the pretest

On the first part of a 418-mile trip, a salesman averaged 58 miles per hour. He averaged only 52 miles per hour on the last part of the trip because of an increased volume of traffic. Find the amount of time at each of the speeds if the last part of the trip took 2.75 hours more compared to the first part of the trip.
58x+52(x+2.75)=418

Type 2

2.1 Two-car toward each other: (College Algebra), #11 in the pretest

The distance along a shipping route between San Francisco and Honolulu is 2,100 miles. If one ship leaves San Francisco at the same time another leaves Honolulu, and if the former travels at 15 miles per hour and the latter at 20 miles per hour, how long will it take the two ships to meet each other?

15x+20x=2100

2.2 Two-car toward each other with time unknown: (Algebra & Trig. P. 165, Q44), #4 in the pretest

Two families meet at a park for a picnic. At the end of the day, one family travels east at an average speed of 42 miles per hour and the other travels west at an average speed of 50 miles per hour. Both families have 160 miles to travel. Find the time that will have elapsed when they are 100 miles apart.

42x+50x=100

2.3 Mixture - Investment, #8 in the pretest

Lena invested the same amount of money into two different bank accounts with 8% and 11% simple interests. During one year, she earned $560. How much did she invest in each account?

0.08x+0.11x=560

Type 3

3.1 Two-car follow-up: (Algebra 1, p. 105), #9 in the pretest

A train leaves a train station at 1pm. It travels at an average rate of 60 miles per hour. A high-speed train leaves the same station an hour later. It travels at an average rate of 90 miles per hour. The second train follows the same route as the first train on a track parallel to the first. In how many hours will the second train catch up with the first train?

60x=90(x-1)

3.2 Two-car end different: (Algebra 1, p. 109, Q22), #1 in the pretest

Ellen and Kate raced on their bicycles to the library after school. They both left school at 3:00pm and bicycled along the same path. Ellen rode at a speed of 9 miles per hour and Kate rode at 12 miles per hour. Ellen got to the library 15 minutes later than Kate. How long did it take Ellen to get to the library?

9x=12(x-0.25)

3.3 One-car two-way different speeds: (Algebra 1, p. 108, Q12, modified), #10 in the pretest

Nora took her sister to the city airport. Because of traffic conditions, she drove 3.75 hours, but she drove only 1.5 hours from the airport to her home. On her way from the city airport to home, she averaged 40 miles per hour faster compared to her average speed from home to the airport. What is Nora's average speed from the airport to her home?

1.5x=3.75(x-40)

Type 4

4.1 Mixture - nuts: (Algebra & Trig., p. 166, Q56), #6 in the pretest

A grocer mixes two kinds of nuts that cost $2 per pound and $3 per pound respectively, where the total weight of the two kinds of nuts is 10 pounds. The new product's (mixed nuts) cost has been calculated at $2.40 per pound so that the value of the new product is the same as the value of the two kinds of nuts combined. How much of each kind of nut is put into the mixture?

2x+3(10-x)=10(2.4)

4.2 Mixture - liquid: (Internet), #2 in the pretest

Suppose you work in a restaurant. You want to make 10 liters of punch with 15% alcohol solution for your friend. You decide to mix 10% solution with 30% solution, to make your own 15% solution. How many liters of 10% solution and 30% solution should you use?
$0.1x+0.3(10-x)=10(0.15)$

4.3 Mixture - Investment: (College Algebra, Larson, p. 173), #3 in the pretest
A store has $30,000 of inventory in 12-in and 19-inch color televisions. The profit on a 12-inch set is 22% and the profit on a 19-inch set is 40%. The profit for the entire stock is 35%. How much was invested in each type of television?
$0.22x + 0.4(30000-x)=0.35(30000)$

The Design of the Posttest

Type 1

- 1.1 Two-bicycle opposite
- 1.2 Mixture - investment
- 1.3 Other - two pumps filling a tank

Type 2

- 2.1 Two-person toward each other
- 2.2 Two-car opposite
- 2.3 Other - two equipments working together

Type 3

- 3.1 Two-flight the same direction
- 3.2 Two-car the same direction
- 3.3 Other - parking fee

Type 4

- 4.1 Mixture - fruits
- 4.2 Mixture - juice
- 4.3 Mixture - course taking

Type 1

1.1 (SS, Algebra 1, p. 108, Q15), #4 in the posttest
Two bicyclists ride in opposite directions from the same place. The speed of the first bicyclist is 5 miles per hour faster than the second. After 2 hours they are 70 miles apart. Find their speeds.
$2x+2(x+5)=70$

1.2 (SD), #5 in the posttest
Jose deposited money into two bank accounts paying 2.5% and 4% simple interest. He put $4000 more in the account with the 4% rate compared to the amount put in the account with the 2.5% rate. During one year, he earned $500 more from the 4% account compared to the 2.5% account. How much did Jose deposit in each account?
$0.04(x+4000)-0.025x=500$

1.3 (DS, College Algebra, Barnett, p. 84), #2 in the posttest
Two pumps are used to fill a water storage tank at a resort. One pump can fill the tank 5 gallons per minute faster than the other one. When both pumps are operating together, the tank can be filled in 10 minutes. The capacity of the tank is 250 gallons. Find the filling rate of each pump.

$10x+10(x+5)=250$

Type 2
2.1 (SS), #8 in the posttest
Kenisha and Sarah walk toward each other from the chemistry lab and the library respectively. Kenisha walks with an average speed of 3 miles per hour and Sarah walks with an average speed of 5 miles per hour. The distance between the lab and the library is 4/10 of a mile. How long will it take for the two persons to meet each other?
$3x+5x=0.25$

2.2 (SD, Algebra & Trig., p.165, Q44, modified), #1 in the posttest
Ming and Janet's families camped at a national park. At the end of the day, Ming's family traveled east at an average speed of 65 miles per hour. Half an hour later, Janet's family traveled west at an average speed of 70 miles per hour. Find the time Ming's family traveled when they were 235 miles apart.
$65x+70(x-0.5)=235$

2.3 (DS, College Algebra, Barnett, p. 82), #10 in the posttest
An old piece of equipment can print, stuff, and label 38 letters per minute. A newer model can handle 82 per minute. How long will it take for both pieces of equipment to prepare a mailing of 6,000 letters?
$38x+82x=6000$

Type 3
3.1 (SS, Algebra 1, p. 108, Q11), #12 in the posttest
A jet leaves the Charlotte, North Carolina, airport traveling at an average rate of 564 km/h. Another jet leaves the airport one half hour later traveling at 744 km/h in the same direction. How long will it take for the second jet take to overtake the first?
$564x=744(x-0.5)$

3.2 (SD), #9 in the posttest
Allen and Susan both left from their homes for a conference at 1pm. They both took the same eastbound highway. Allen averaged 60 miles per hour and Susan averaged 70 miles per hour. Susan lives 5 miles west of Allen's house. Allen arrived at the conference 15 minutes later than Susan. How long did it take Allen to get to the conference?
$60x+5=70(x-0.25)$

3.3 (DS), #3 in the posttest
In an airport, the short term parking rate is $3 per hour. The long term parking rate is $8 per hour, but customers get two hours parking for free. When will short term and long term parking charges be the same?
$3x=8(x-2)$

Type 4
4.1 (SS), #7 in the posttest

A grocer mixes two kinds of fruits that cost \$1.19 per pound and \$ 1.69 per pound respectively, where the total weight of the two kinds of fruits is 10 pounds. The new product (a mixed fruit basket) cost has been calculated \$1.29 per pound so that the value of the new product is the same as the value of the two kinds of fruits combined. How much of each kind of fruit is put into the mixture?

$1.19x + 1.69(10-x) = 10(1.29)$

4.2 (SD), #11 in the posttest

A company produces bottles of tropical juice. Each bottle is 64 fl oz with 80% pure juice. Three kinds of juice, mango, pineapple, apple, are mixed together with 95%, 85%, and 70% pure juice respectively to make a bottle of 80% tropical juice. The company uses the same amount of mango and pineapple juice in the mixture. How many ounces of mango, pineapple, and apple juice are used for a bottle of tropical juice?

$0.95x+0.85x+0.7(64-2x)=0.8(64)$

4.3 (DS), #6 in the posttest

Jason has 7 courses to take this semester including some selective and some core courses. After some investigations, he discovers that the probability of getting a grade of 3.0 from the core courses is 60%, and getting a grade of 3.0 from the selective courses is 74%. If he wants to get a grade of 3.0 with 70% probability, how many core and selective courses should he take?

$0.74x+0.6(7-x)=0.7x7$

Appendix D
Teaching Script for the Lecture of the Experimental Group

1. Word problems with Rate relationships

- **In a word problem:** There are many word problems that are about rate. What is your understanding of Rate?
- **In an equation:** In an equation, you may interpret the multiplicative term with Rate. For example, in the equation $60x+20=140$, $60x$ can be interpreted as a rate story, which is 60 miles per hour times x hours.

2. The definition of Rate:

- Rate is the comparison of two quantities with different units. When I say quantity, I mean a number associated with a unit, for example, 10 persons, 2 minutes, 30 miles.
- Here are some examples of rate: 80 miles per hour (the two quantities are 80 miles and per hour), 10 dollars per item (the two quantities are 10 dollars and per item), 72 heartbeats per minute, or 4 dollars per pound.
- Sometime a rate doesn't appear with the format something per something. However, we may transform it. For example, the rate of driving 240 miles for 3 hours is 240/30=80 miles per hour. The rate of 50 dollars for 10 persons is 50/10 or 5 dollars per person"

3. Understanding Rate:

- "Can you give a real-life example for the rate: pages per minute? (copy machine)"
- "Can you give a real-life example for the rate: dollars per month? (salary)"
- "Can you give a real-life example for the rate: persons per day? (museum or zoo)"
- "Can you generate real-life rate examples based on the following units? (e.g., gallon, cup, piece, box, person, dollars, minute, hour, day, month) Please choose two units to create a rate"

4. Explain the translation from real-life cases to symbols and ask students questions: "Now let's see the translation from real-life rate examples to symbols."

- "If we have the information x miles per hour and 3 hours, then we know $3x$ means….miles in three hours."
- "If we have the information 3 miles per hour and x hours, then we know $3x$ means…miles in x hour. Therefore, 3 can be the rate, or not the rate."
- "If we have the information 8 cents per page and x pages, then the total charge for x pages is…($0.08x$)"
- "If we have the information x dollars per minute and 20 minutes, then the total charge for 20 minutes is…($20x$)"

5. Explain the transformation from symbols to real-life cases and ask students questions: "Now let's see the translation from symbols to real-life rate examples, or, you can call it the interpretation of symbols."

- "60x can mean…suppose **60 is the rate** like miles per hour, and x is the hours"
- "10x can mean…suppose **x is the rate** like miles per hour and 10 is the hours"
- "Can you give a real-life rate story for 10000x? Which one do you suppose is the rate? What is the rate? What does 10000x mean?"
- "Can you give a real-life story for "5x"? Which one do you suppose is the rate? What is the rate? What does 5x mean?"

6. Conclusion:

- When you read a story problem, can you recognize a rate relationship?

- When you read an equation, can you generate a real-life rate example for the multiplicative term(s)?

7. The Two-Car model for understanding

Here are the three possible two-car relationships/scenarios in traveling: (1) Traveling in opposite way (2) Traveling toward each other (3) Traveling the same path (one follows the other). Many word problems can be understood or interpreted using this model. Of course, there are other scenarios for the two cars, for example, one traveling east and one traveling north.

Appendix E
Student Worksheet for the Lecture of the Experimental Group

1. Word problems with Rate relationships

- **In a word problem**
- **In an equation**

2. The definition of Rate:

- **Relationship between two quantities**
  - **A quantity is a number associated with a unit**
- **80 miles per hour, 10 dollars per item, 72 heartbeats per minute**
- **240 miles for 3 hours →**
- **50 dollars for 10 persons →**

3. Understanding Rate in real life:

- **Pages per minute**
- **Dollars per month**
- **Persons per day**
- **Generate rate example: gallon, cup, piece, box, person, dollars, minute, hour, day, month**

4. Translation from real-life cases to symbols:

- **A car with the speed x miles per hour, travels 3 hours, what is the traveled distance?**
- **A car with the speed 3 miles per hour, travels x hours, what is the traveled distance?**
- **A copy machine charges 8 cents per page, how much for making x copies?**
- **A game charges x dollars per minute, how much for 20 minutes?**

5. Translation from symbols to real-life cases:

- **60x can mean… (suppose 60 as a rate, e.g., miles per hour)**
- **10x can mean… (suppose x as a rate, e.g., dollars per minute)**
- **10000x can mean…**
- **5x can mean…**

6. Conclusion:

- **When you read a story problem, can you recognize a rate relationship?**
- **When you read an equation, can you generate a real-life rate example for the multiplicative term(s)?**

7. The Two-Car Model for understanding word problems

Appendix F
Teaching Script for the Lecture of the Control Group

Teaching Scripts:
"Now I will teach you a method for solving word problems. You may find it useful for the mathematics courses you are going to take."

1. State the character of solving a word problem:
"Usually our main task to solve a word problem is to translate the story of the problem into an equation.

2. The four steps for translating a word problem:

- Identify quantities:
  - A quantity means a number associated with a unit
  - For example: 3 persons, 10 gallons, 5 dollars per meal, 70 miles per hour, total charge, total travel distance
- Identify relationships between quantities:
  - We know the speed of a car is 70 miles per hour, and we know the car drive 3 hours. The two quantities imply a distance relationship, e.g., 70 MPH x 3 hours = 240 miles.
  - We know the rate for renting a car is 0.25 dollars per mile, and we know the traveled mileage is 40 miles. The two quantities imply a cost relationship, e.g., 0.25x40=10 dollars.
- Identify the unknown quantities and find relationships related to the quantities:
  - Some quantities may be implicit, which means there are no numbers associated, but you know they are quantities. You can suppose an implicit quantity with a symbol (e.g., x) to help you figure out relationships between this implicit quantity and other quantities implicit.
  - For example, the speed (miles per hour) of a car is unknown but the hours the car run is 4. Therefore, we can inference that the car has run the distance of $4x$ miles
- Figure out the integrated relationship and translate it:
  - Organizing pieces of relationships to get the whole picture, or, the integrated relationship
  - For example: Total car rental fee = traveled mileage x 0.25 dollars per mile + equipment fee. Therefore, $45=0.25x+ 25$
  - Another example: Cell phone monthly payment = charged minutes x 0.75 dollars per minute + monthly program fee. Therefore, $35=0.75x+15$

3. Apply the four steps
The MacNeills rented a moving truck for $50 plus $.50 per mile. The total cost was $90. Find the number of miles the truck was driven.

- Identify quantities:
  - $50, $.50 per mile, total cost $90, and miles the truck was driven.
- Identify relationships between quantities:
  - The total cost includes $50
  - Part of the cost is the rate .50 times the number of miles the car run.
- Identify the unknown quantities and find relationships related to the quantities:
  - Suppose the truck run x miles, then part of the cost is $0.5x$
- Figure out the integrated relationship and translate it:
  - Total cost = $50 + 0.5x. The translated equation is $90=0.5x+50$ or $50+0.5x=90$

Appendix G
Student Worksheet for the Lecture of the Control Group

1. The main task of solving a word problem
2. The four steps for translating a word problem:

- Identify quantities:
  - A quantity means a number associated with a unit
  - For example: 3 persons, 10 gallons, 5 dollars per meal, 70 miles per hour, total charge, total travel distance
- Identify relationships between quantities:
  - We know the speed of a car is 70 miles per hour, and we know the car drive 3 hours. The two quantities imply a distance relationship, e.g., 70 MPH x 3 hours = 240 miles.
  - We know the rate for renting a car is 0.25 dollars per mile, and we know the traveled mileage is 40 miles. The two quantities imply a cost relationship, e.g., 0.25x40=10 dollars.
- Identify the unknown quantities and find relationships related to the quantities:
  - Some quantities may be implicit, which means there are no numbers associated, but you know they are quantities. You can suppose an implicit quantity with a symbol (e.g., x) to help you figure out relationships between this implicit quantity and other quantities.
  - For example, the speed (miles per hour) of a car is unknown but the hours the car run is 4. Therefore, we can inference that the can has run the distance of $4x$ miles
- Figure out the integrated relationship and translate it:
  - Organizing pieces of relationships to get the whole picture, or, the integrated relationship
  - For example: Total car rental fee = traveled mileage x 0.25 dollars per mile + equipment fee. Therefore, $45=0.25x+ 25$
  - Another example: Cell phone monthly payment = charged minutes x 0.75 dollars per minute + monthly program fee. Therefore, $35=0.75x+15$

3. Apply the four steps
The MacNeills rented a moving truck for $50 plus $.50 per mile. The total cost was $90. Find the number of miles the truck was driven.

- Identify quantities:
  - $50, $.50 per mile, total cost $90, and miles the truck was driven.
- Identify relationships between quantities:
  - The total cost includes $50
  - Part of the cost is the rate .50 times the number of miles the car run.
- Identify the unknown quantities and find relationships related to the quantities:
  - Suppose the truck run x miles, then part of the cost is $0.5x$
- Figure out the integrated relationship and translate it:
  - Total cost = $50 + 0.5x. The translated equation is $90=0.5x+50$ or $50+0.5x=90$

Appendix H
Exercise Worksheet

Problem A:
Jane and Peter leave their home traveling in opposite directions on a straight road. Peter drives 15 miles per hour faster than Jane. After 3 hours, they are 225 miles apart. Find Jane's speed.

Problem B:
The distance along a shipping route between San Francisco and Honolulu is 2,100 miles. If one ship leaves San Francisco at the same time another leaves Honolulu, and if the former travels at 15 miles per hour and the latter at 20 miles per hour, how long will it take the two ships to see each other?

Problem C:
A train leaves a train station at 1pm. It travels at an average rate of 60 miles per hour. A high-speed train leaves the same station an hour later. It travels at an average rate of 90 miles per hour. The second train follows the same route as the first train on a track parallel to the first. In how many hours will the second train catch up with the first train?

Problem D:
A grocer mixes two kinds of nuts that cost $2 per pound and $3 per pound respectively, where the total weight of the two kinds of nuts is 10 pounds. The new product (mixed nuts) cost has been calculated $2.4 per pound so that the value of the new product is the same as the value of the two kinds of nuts combined. How much of each kind of nuts is put into the mixture?

Problem E:
Dana invested in two funds paying 9.5% and 11% simple interest. She invested $2000 more in the fund with 11% rate compared to the amount invested to the fund with 9.5% rate. During one year, she earned $1040 from the two funds in total. How much did Dana invest in each fund?

Problem F:
Ellen and Kate raced on their bicycles to the library after school. They both left school at 3:00pm and bicycled along the same path. Ellen rode at a speed of 9 miles per hour and Kate rode at 12 miles per hour. Ellen got to the library 15 minutes later than Kate. How long did it take Ellen to get to the library?

Appendix I
Selected Teaching Script for the Exercise of the Experimental Group

Problem A:
Jane and Peter leave their home traveling in opposite directions on a straight road. Peter drives 15 miles per hour faster than Jane. After 3 hours, they are 225 miles apart. Find Jane's speed.
Exercise: 25 miles faster, after 4 hours, 500 miles apart
Teaching Script:
1. "What scenario is this problem about according to the Two-Car Model?"
2. Looking for rate
"Can you find any rate in this problem?"
"3 hours, 225 miles apart, 15 miles per hour faster, Jane's speed, Peter's speed (Jane's speed and peter's speed are rate, although they have no numbers associated.)
3. Looking for rate relationships (other quantities that associated with rates):
"Are there quantities that can be connected to the rates you find?"
"Jane's speed is a rate, and Jane drives 3 hours. Therefore, the distance Jane drives is Jane's speed time 3. *If we don't have a number regarding a quantity, we just suppose it with a symbol to help us make inferences and get more information.* Suppose Jane's speed is x, then the distance Jane drives is 3r. Since Peter drives 15 miles faster than Jane, Peter's speed is x+15. Peter's speed is a rate, and Peter also drives 3 hours. Therefore, the distance Peter drives is 3(x+15)."
4. Connecting the relationships we have:
"Since after three hours they are 225 miles apart, the sum of the distances (two mileages) Jane and Peter drove is equal to 225. Therefore, the translated equation is 3x+3(x+15) = 225."

Problem B:
The distance along a shipping route between San Francisco and Honolulu is 2,100 miles. If one ship leaves San Francisco at the same time another leaves Honolulu, and if the former travels at 15 miles per hour and the latter at 20 miles per hour, how long will it take the two ships to see each other?
Exercise: 1800 miles, former 20 MPH, latter 25MPH

Problem C:
A train leaves a train station at 1pm. It travels at an average rate of 60 miles per hour. A high-speed train leaves the same station an hour later. It travels at an average rate of 90 miles per hour. The second train follows the same route as the first train on a track parallel to the first. In how many hours will the second train catch up with the first train?
Exercise: First 70MPH, Second 80MPH
Teaching Script:
1. "What scenario is this problem according to the Two-Car Model?"
2. Looking for rate:
"Can you find any rate in this problem?"
"60 miles per hour is a rate and 96 miles per hour is another rate.
3. Looking for rate relationships: (other quantities that associated with rates)
"Are there quantities that can be connected to the rates you find?"

"60 miles per hour is a rate. So we know that if the first train runs a couple of hours, then the distance the train run is 60 times a couple of hours."
"*If we don't have a number regarding a quantity, we just suppose it with a symbol to help us make inferences and get more information.* We suppose the first train runs x hours. The distance the first train runs is 60x miles."
"Since the high-speed train leaves an hour later, the high-speed train travels x-1 hours."
"Since the high-speed train travels 96 miles per hour and it travels x-1 hours. So the mileage the high-speed train travels is 96(x-1)."
4. Connecting the relationships and translate:
"We know that at the catch-up point both trains travel the same mileage. At the catch-up point, the train travels x hours and the high-speed train travel x-1 hours. Therefore, 60x = 96(x-1)"

Problem D:
A grocer mixes two kinds of nuts that cost $2 per pound and $3 per pound respectively, where the total weight of the two kinds of nuts is 10 pounds. The new product (mixed nuts) cost has been calculated $2.4 per pound so that the value of the new product is the same as the value of the two kinds of nuts combined. How much of each kind of nuts is put into the mixture?
Exercise: One kind costs $2.5 per pound, the other kind costs $5 per pound, the total weight is 20 pounds, the mixed nuts are $4 per pound
Teaching Script:
"Let's draw a picture to figure out what the problem is about."
1. Looking for rate:
"$2 per pound, $3 per pound, and $2.4 per pound."
2. Looking for rate relationships:
"Are there quantities that can be connected to the rates you find?"
"2 dollars per pound is a rate. If the weight of type 1 nuts is x pound, then the value of type 1 nuts is 2x. *If we don't have a number regarding a quantity, we just suppose it with a symbol to help us make inferences and get more information.* Since the weight of the two types of nuts combined is 10, the weight of type 2 is 10-x. Since type 2 nuts are $3 per pound, the value of type 2 nuts is 3(10-x). Another rate is $2.4 per pound for nuts of after mixing. Since we know the weight of the mixed nuts is 10, the value of the mixed nuts is 10x2.4=24.
3. Connecting the relationships and translate:
Since the value of the nuts before and after mixing should be the same, 2x+3(10-x)=24.

Problem E:
Dana invested in two funds paying 9.5% and 11% simple interest. She invested $2000 more in the fund with 11% rate compared to the amount invested to the fund with 9.5% rate. During one year, she earned $1040 from the two funds in total. How much did Dana invest in each fund?
Exercise: 8.5%, 10%, 4000 more in the fund with 10%, earned 1510

Problem F:
Ellen and Kate raced on their bicycles to the library after school. They both left school at 3:00pm and bicycled along the same path. Ellen rode at a speed of 9 miles per hour and Kate rode at 12 miles per hour. Ellen got to the library 15 minutes later than Kate. How long did it take Ellen to get to the library?
Exercise: Ellen rode 10 mph, Kate rode 14 mph, and Ellen arrived half an hour later than Kate

Appendix J
Selected Teaching Script for the Exercise of the Control Group

Problem A:
Jane and Peter leave their home traveling in opposite directions on a straight road. Peter drives 15 miles per hour faster than Jane. After 3 hours, they are 225 miles apart. Find Jane's speed.
Exercise: 25 miles faster, after 4 hours, 500 miles apart

Teaching Script:
"Let's draw a picture to figure out what the problem is about." (For the purpose of understanding the problem)
"First, can we find some quantities from the problem…..? The quantities we can find are 15 miles per hour faster, 3 hours, and 225 miles apart, Jane's speed, Peter's speed. Some quantities may be implicit, which means there are no numbers associated, but you know they are quantities."
"Second, can we find any relationship between the quantities…? The relationships we can find are: 1. Peter drives 15 miles per hour faster than Jane 2. Each of them drives 3 hours, which means each of their speeds times 3 is the distance each of them drives. 3. They are 225 miles apart after 3-hour driving, which means the sum of the distances they drive is 225."
"Third, for a quantity with no number associated, just suppose it with a symbol to help us make inferences and figure out other relationships. Suppose x is Jane's speed, then (x+15) is Peter's speed, and the distance Jane drives is 3x and the distance Peter drives is 3(x+15)."
"Fourth, therefore, the integrated relationship is: the mileage Peter drives + the mileage Jane drives in 3 hours = 225 miles. The translated equation is $3(x+15) + 3x = 225$"

Problem B:
The distance along a shipping route between San Francisco and Honolulu is 2,100 miles. If one ship leaves San Francisco at the same time another leaves Honolulu, and if the former travels at 15 miles per hour and the latter at 20 miles per hour, how long will it take the two ships to meet each other?
Exercise: 1800 miles, former 20 MPH, latter 25MPH

Problem C:
A train leaves a train station at 1pm. It travels at an average rate of 60 miles per hour. A high-speed train leaves the same station an hour later. It travels at an average rate of 90 miles per hour. The second train follows the same route as the first train on a track parallel to the first. In how many hours will the second train catch up with the first train?
Exercise: First 70MPH, Second 80MPH

Teaching Script:
"Let's draw a picture to figure out what the problem is about."
"First, can we find some quantities from the problem…..? The quantities we can find are 60 miles per hour (the first train), an hour later, and 96 miles per hour (the second train), the hours the second train travel. Some quantities may be implicit, which means there are no numbers associated, but you know they are quantities."

“Second, can we find any relationship between the quantities…? The relationships we can find are: (1) The mileage the first train travels is the same as that of the second train (2) The time the high speed train travels is an hour less than the time the train travels;

“Third, for a quantity with no number associated, just suppose it with a symbol to help us make inferences and figure out other relationships. Suppose the first train travels x hour, then the second train travels x-1 hour. The distance the first train travels is 60x, and the second train travels 96(x-1)”

“Fourth, therefore, we have the integrated relationship: speed (first train) x time = speed (second train) x time (an hour less).The translated equation will be 60t=96(t-1)”

Problem D:

A grocer mixes two kinds of nuts that cost \$2 per pound and \$3 per pound respectively, where the total weight of the two kinds of nuts is 10 pounds. The new product (mixed nuts) cost has been calculated \$2.4 per pound so that the value of the new product is the same as the value of the two kinds of nuts combined. How much of each kind of nuts is put into the mixture?

Teaching Script:

“First, can we find some quantities from the problem…..? The quantities we can find are 2.49 dollars per pound (type 1 nuts), 3.89 dollars per pound (type 2 nuts), 100 pounds (mix), and 3.19 dollars per pound (mixed nuts), and the weight of each kind of nuts. Some quantities may be implicit, which means there are no numbers associated, but you know they are quantities.”

“Second, can we find any relationship between the quantities…? The relationships we can find are: (1) the weight of type 1 nuts x 2.49 = the total value of type 1 nuts (2) the weight of type 2 nuts x 3.89 = the total value of type 2 nuts (3) The total weight of the two kinds of nuts is 100 pounds (4) The values of before and after mixing are equal (5) The value after mixing is equal to 3.19 per pound x 100”

“Third, for a quantity with no number associated, just suppose it with a symbol to help us make inferences and figure out other relationships. Suppose the weight of type 1 nuts is x pounds, then the type 2 nuts is 100-x pounds.

“Fourth, therefore, we have the integrated relationship: 3.19 dollars per pound x 100 = the sum of the values of the two kinds of nuts / the weight (pound). The translated equation will be (2.49x+3.89(100-x)) = 3.19x100”

Problem E:

Dana invested in two funds paying 9.5% and 11% simple interest. She invested \$2000 more in the fund with 11% rate compared to the amount invested to the fund with 9.5% rate. During one year, she earned \$1040 from the two funds in total. How much did Dana invest in each fund?

Exercise: 8.5%, 10%, 4000 more in the fund with 10%, earned 1510

Problem F:

Ellen and Kate raced on their bicycles to the library after school. They both left school at 3:00pm and bicycled along the same path. Ellen rode at a speed of 9 miles per hour and Kate rode at 12 miles per hour. Ellen got to the library 15 minutes later than Kate. How long did it take Ellen to get to the library?

Exercise: Ellen rode 10 mph, Kate rode 14 mph, and Ellen arrived half an hour later than Kate

Appendix K
Interview Protocol

Part I – Interview Questions:

Do not turn on the recorder. Give your interviewee some time to work on the first interview question.

1. **Can you classify the 12 word problems into groups based on similarities between the problems?**
   - e.g., you may think 2, 4, 5 or 2, 5, 7, 10 can be grouped
   - Each group may have 1, 2, 3,…, or 10 problems
   - You may create as many groups as you want?
   - You may use the similarities you observe among the problems to classify the problems.

Note: Ask the interviewee to generate 5 groups if possible, write the five groups on his/her posttest paper (e.g., group1, group2, group3 group4 and group5).

Turn on the recorder

Push the REC button

Speak: Today is _______ (Date). This is ______ (Dan/Jia/Sasha/Leo) interviewing _____ (Can you say your name?)

Have your interviewee say aloud what problems they group together (e.g., 1, 2, 5, 9 for group 1).

2. **Can you tell me why you group the problems together?**
   - Why do you group these problems together?
   - Can you tell me more about the similarities you use for grouping these problems?
   - These problems were grouped together because…

Part II – Interview Question

3. **What were the difficulties/struggles/hard parts you encountered when you tried to classify the problems?**
   - What were the difficulties you encountered when you tried to find clues for classifying these problems?
   - Were there difficult parts when you tried to find similarities between the problems?
   - Can you say more about "this difficulty"?

Appendix L
Student Consent Form

Dear Student,

You are invited to participate in a research project (Research title: Exploring the Effect of Ability to Classify Problems and Abstract Schemas for Classifying Problems on Solving Transfer Algebra Word Problems). Researchers are required to provide a consent form to inform you about the study, to convey that participation is voluntary, to explain risks and benefits of participation, and to empower you to make an informed decision. You should feel free to ask the researchers any questions you may have.

From this study, the researchers hope to learn whether abstract schemas and the ability to classify word problems have positive effects on students' performance of solving one-variable linear equation (e.g., $2x+4=8$) word problems, such that new problem-solving strategies may be developed to enhance teachers' teaching and students' learning for solving linear equation word problems. The entire study will take you about two hours including a pretest (20 minutes), one session of lecture (15 minutes), one session of exercise (40 minutes), a posttest (20 minutes), and an interview (15 minutes). You may be asked to take a retention test (20 minutes) two weeks after you finish the 2-hour study. You will receive the compensation of $20 from the researchers within one week after you complete the entire 2-hour research study (not including the retention test). You will receive another compensation of $5 from the researchers within one week after you complete the retention test. Food (e.g., pizza, chips, or beverage) will be provided in the experiment.

In the study, your written work (e.g., pretest, posttest, exercise worksheet, interview task etc.) will be collected, and your activities and conversations with the instructor, researchers, or with other students in the lecture, exercise and interview may be video-taped and/or audio-recorded. The researchers may take notes during the lecture, exercise, and interview. The potential benefit for you to participate in this study is that you will receive a session of lecture and a session of exercise to help you learn to solve word problems, which is part of the course contents of MTH 1825, MTH 103, MTH 110, MTH 112, MTH 114, and MTH 116 you are taking or you may take in the future.

There are minimal risks involved in participating. You may feel tired in the experiment (solving word problems) or feel uncomfortable when the researchers are video and/or audio recording your conversations. All your privacy will be protected. The data collected will be coded such that your name and personal information will not show up in or linked to any reports of the research project. Information about you will be kept confidential to the maximum extent allowable by law. The data will be stored and locked in a steel cabin in the office of the researchers in the College of Education at the Michigan State University. Only the researchers of the project or the Institutional Review Board (IRB) of the Michigan State University can access to the information about you and the data collected from you. All data (not including audiotapes that have been transcribed) must be retained for a minimum of 3 years following closure of project.

Your participation in this project is completely voluntary. You are free to withdraw your participation at any time and for any reason, and the data collected from you will be destroyed immediately after your withdrawal. However, you will get the compensation of $20 if and only if you finish the entire 2-hour study, and you will get the compensation of $5 if and only if you finish the entire 20-minute retention test. A copy of the research results after this project is completed will be given to you at your request.

If you have concerns or questions about this study, such as scientific issues, how to do any part of it, or to report an injury, please contact the researcher: Dr. Robert Floden, 116M Erickson Hall, Michigan State University, Email: floden@msu.edu, Phone: (517) 355-3486. If you have questions or concerns about your role and rights as a research participant, would like to obtain information or offer input, or would like to register a complaint about this study, you may contact, anonymously if you wish, the Michigan State University's Human Research Protection Program at 517-355-2180, Fax 517-432-4503, or e-mail irb@msu.edu or regular mail at 202 Olds Hall, MSU, East Lansing, MI 48824.

PLEASE INITIAL EACH LINE YOU APPROVE

_______ I agree to participate in this research study

_______ I Do Not want to be audio recorded

_______ I Do Not want to be video recorded

Your Name (Print): ________________________________

Your Signature: ___________________________________ Date: _____________________

Appendix M
Student Contact Information Form

First Name: ________________________ Last Name: __________________________

Gender: M / F (please circle one)

Email Address: __________________________

Phone Number (optional): ________________________

Major: __________________________

Years in college (optional):
(Check the box that applies to you)

- ☐ First Year
- ☐ Second Year
- ☐ Third Year
- ☐ Beyond Third Year

Mathematics course(s) you are required to take:
(Check box(es) that applies to you)

- ☐ MTH 1825 – Intermediate Algebra
- ☐ MTH 103 – College Algebra
- ☐ MTH 110 – Finite Mathematics & Elements of College Algebra
- ☐ MTH 112 – Finite Mathematics: Applications of College Algebra
- ☐ MTH 114 - Trigonometry
- ☐ MTH 116 – College Algebra & Trigonometry

Mathematics course(s) you are taking:
(Check box(es) that applies to you)

- ☐ MTH 1825 – Intermediate Algebra
- ☐ MTH 103 – College Algebra
- ☐ MTH 110 – Finite Mathematics & Elements of College Algebra
- ☐ MTH 112 – Finite Mathematics: Applications of College Algebra
- ☐ MTH 114 - Trigonometry
- ☐ MTH 116 – College Algebra & Trigonometry

Mathematics course(s) you will take:
(Check box(es) that applies to you)

- ☐ MTH 1825 – Intermediate Algebra
- ☐ MTH 103 – College Algebra
- ☐ MTH 110 – Finite Mathematics & Elements of College Algebra
- ☐ MTH 112 – Finite Mathematics: Applications of College Algebra
- ☐ MTH 114 - Trigonometry
- ☐ MTH 116 – College Algebra & Trigonometry

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