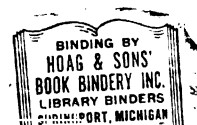


SOIL TEMPERATURE REGIME AND ITS
INTERACTION WITH SUBMERGED TILE

Thesis for the Degree of M. S.
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THESIS





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ABSTRACT

SOIL TEMPERATURE REGIME AND ITS INTERACTION WITH SUBMERGED TILE

By

Sirous Haji-Djafari

The purpose of this study was to investigate the effect of thermal environment on submerged drain tile and the conditions causing the cracking of clay tile due to the freezing of part of water within the tile. The location of interest for the investigation was near Saginaw, Michigan. A new technique was used to obtain soil temperatures from air temperatures.

In order to ascertain the behavior of a crack resulting from freezing the water within a drain tile, an experiment was conducted in the laboratory. The result of this experiment led to the determination of realistic conditions for solving the mathematical model of the freezing. In order to investigate the effect of submerged drain tile on the thermal environment, a computer model was developed and utilized to study four, six and eight inch submerged drain tile buried at 36, 42 and 48 inches depth.

The investigation led to following results: the average soil temperature varies as a function of depth but the average soil temperature is always greater than the average air temperature. When the soil temperature surrounding a submerged tile is reduced to less than 32°F for a period of time, the water inside the tile will freeze. Cracking will occur when about 1/3 of the water has frozen and the location will be at the interface of the ice water phase and the tile wall.

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SOIL TEMPERATURE REGIME AND ITS INTERACTION
WITH SUBMERGED TILE

By

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TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	ii
LIST OF TABLES	v
LIST OF FIGURES	vii
INTRODUCTION	1
 Chapter	
I. REVIEW OF BASIC POINTS	4
1.1 Introduction	4
1.2 Source and Amount of Heat	7
1.2.1 Latitude and Slope of Land	7
1.2.2 Vegetative Cover	8
1.3 Thermal Properties of Soils	10
1.3.1 Heat Capacity of Soils	10
1.3.2 Thermal Conductivity and Soil Factors Affecting It	15
1.4 General Differential Equation for the Temperature Field	19
1.5 Daily and Seasonal Changes in Soil Temperature	22
1.5.1 Damping Depth	25
II. DEFINITION OF THE PROBLEM	28
2.1 Background	28
2.1.1 Tile for Over-size Trenches	28
2.1.2 Clay Tile and Frost Action	29
2.2 Experimental Set-up	31
2.2.1 Procedure	31
2.2.2 Observations	32
2.3 Mathematical Model	35
2.3.1 General Assumptions	35
2.3.2 Specific Assumptions	37
2.3.3 Determination of Initial and Boundary Conditions	41
2.4 Solution of the Model	57
2.4.1 Formula and Method of Solution	59
2.4.2 Numerical Solution	59
2.4.3 Applying the Boundary and Initial Conditions in Solution of Related Formulas	62

Chapter	Page
III.	66
3.1 Results	66
3.1.1 Results of Experiments	66
3.1.2 Results of Soil Temperature Study	67
3.1.3 Results of Computer Modeling	70
3.2 Discussion of Results	77
IV. CONCLUSIONS	79
V. RECOMMENDATIONS FOR FURTHER STUDIES	81
LIST OF REFERENCES	83

Appendices

I. FOURIER SERIES MODELING OF PERIODIC PHENOMENA	85
II. COMPUTER MODELING FOR STUDY OF SOIL TEMPERATURE WITH SUBMERGED DRAIN TILE	92
III. CONVERSION FACTORS	108

LIST OF TABLES

Table		Page
1-1	Specific heat values c , densities ρ , and specific heat per unit volume C of several soil minerals and rock materials	12
1-2	Specific heat values c , densities ρ , and specific heat per unit volume C of several organic soils and soil materials	13
1-3	Specific heat values c , densities ρ , and specific heat per unit volume C of several soils after Kersten (1949)	14
1-4	Thermal properties of some typical soils and of the common soil constituents (at 10°C) .	20
1-5	Comparison of thermal properties of frozen and unfrozen soil	21
1-6	Damping depths of temperature fluctuation having various periods for several values of thermal diffusivity	27
2-1	Recorded daily soil temperature (°F.) at different depths in East Lansing Horticulture Farm in 1963	42
2-2	Annual average of soil temperature (°F.) for East Lansing Horticulture Farm Station	43
2-3	Annual average soil temperature (°F.) at the University of Minnesota, St. Paul Station. Soil is under sod cover	43
2-4	Effect of soil coverage on yearly average soil temperature (°F.) at different depths	46
2-5	Difference between air temperature and soil temperature at different depths in °F. . .	48
3-1	Effect of soil cover on soil temperature at different depths in the cold season . .	69

Table		Page
3-2	Effect of soil cover on soil temperature at different depths in the warm season	69
3-3	Time and required duration of submergance for cracking to take place in clay drain tile	76
I-1	The calculated fourier coefficients for 8 inches depths of soil	88
I-2	The calculated fourier coefficients for 20 inches depths of soil	89
I-3	The calculated fourier coefficients for 40 inches depths of soil	90
I-4	The calculated fourier coefficients for 80 inches depths of soil	91

LIST OF FIGURES

Figure		Page
1-1	Extrinsic factors which influence frost action	5
1-2	Intrinsic factors which influence frost action	6
1-3	Frost penetration in bare and grass-covered soils.	9
1-4	Heat conductivity and diffusivity in coarse quartz sand	18
1-5	Linear heat conduction	19
2-1a	Location of thermocouples in the experimental setup	33
2-1b	A picture of the cracked tile showing the general appearance. Note the thermocouple wires entering from the right . . .	33
2-2	Temperature variation for each thermocouple	34
2-3	The shape of ice causing cracking of a drain tile. The location of the crack is shown in the picture at the lower right of the tile	36
2-4	Grid system	39
2-5	Effect of soil coverage on soil temperature.	45
2-6	Variation of monthly average air and soil temperature by depth	47
2-7	The average monthly air temperature for East Lansing, Saginaw and Evart in 1968. .	50
2-8	Air temperature of East Lansing and Saginaw for the year 1963	51

Figure		Page
2-9	Calculated soil temperature for Saginaw, 1963	54
2-10	Recorded soil temperature for East Lansing, Horticulture Farm Station, 1963	55
2-11	Recorded soil temperature for East Lansing, Horticulture Farm Station, 1968	56
2-12	Approximation of drain tiles in grid system for computer model	58
3-1	Frost penetration in bare and sod surface soil	68
3-2	Temperature history in the soil around a four inch submerged drain tile burried at 36, 42, and 48 inch depths	71
3-3	Temperature history in the soil around 6 inch drain tile burried at 36, 42, and 48 inch depths	72
3-4	Temperature history in the soil around eight inch submerged drain tile burried at 36, 42, and 48 inch depths	73
3-5	The effect of water within the four-inch drain tile on soil temperature. (Iso- thermal lines are shown in the figure) . .	74
3-6	A photograph of an 8 inch clay tile removed from a field located near Saginaw, Michigan.. . . .	78

INTRODUCTION

Drainage plays an important role in agricultural production, both in arid regions as well as in regions where the primary role of drainage is in the removal of excess ground water from the plant root zone.

A well-designed drainage system requires high quality material which, for a plastic conduit system is synonymous with a material capable of supporting the over-burden load. For a rigid conduit system a crack-free tile is required which is capable of supporting the loading which may occur due to trench and/or over-burden, since a cracked tile, if disturbed, may collapse, thus causing blockage of the tile line and failure of the system.

Although progress has been made toward providing safe design criteria, a source of damage to buried tile lines remains uninvestigated, i.e., the cracking of tile due to water freezing within the conduit. Such a condition may occur in areas where good tile outlets are not available and pump drainage is utilized. Under these conditions, tile lines are often installed with minimum cover and submerged outlets are used. A large portion of the drainage system may be submerged during the cold season, and penetration of severe cold weather can cause freezing

of a portion of the water within the lines and subsequent cracking of the buried clay conduit. A knowledge of the condition conducive to freezing damage would lead to the establishment of design criteria which would ultimately increase the reliability of the drainage installed under these conditions.

In this thesis, a study has been made to investigate the effect of thermal environment on submerged drain tile and the conditions causing cracking of clay tile due to the freezing of part of the water within the tile. In this study the location of interest is near Saginaw, Michigan, where extensive areas of soil had been laid bare during the cold months. These areas are in large part pump drained and the drainage system pumps may not be operative during the winter season.

For this study soil temperatures were calculated from air temperatures based on a new technique which has been presented in the body of this thesis. The data required to support the hypothesis leading to the technique was obtained from United States Weather Bureau records from stations in Michigan, Minnesota, and Wisconsin.

It is felt that if it is demonstrated experimentally that cracking does occur under certain temperature conditions, and if a similar set of conditions can be predicted for a field location through application of the selective modeling technique, then one could expect that

tile drainage due to cracking can be anticipated for the location of interest. An experiment was conducted in the laboratory. The result of this experiment led to a determination of realistic conditions for solving the mathematical model of tile freezing. In order to investigate the effect of submerged drain tile on thermal environment, a computer model was developed and utilized in the theoretical analysis.

CHAPTER I

REVIEW OF BASIC POINTS

1.1 Introduction

In studying soil temperature and frost action around drain tile, many influencing factors must be considered. The factors which influence the soil temperature are divided into two groups: extrinsic and intrinsic factors (Kavianpour 1971).

Extrinsic factors are those which determine the ambient conditions. The extrinsic factors are summarized in the block diagram of Figure 1-1.

Intrinsic factors are those inherent to the soil material and include the thermal properties of soil, such as; the thermal conductivity, k , the specific heat, c , and the thermal diffusivity, α . The intrinsic factors are given in Figure 1-2.

In order to develop the basic concepts relating to thermal properties of soil, it is necessary to review some of the work which has been carried out with respect to the major factors and their influence on the overall thermal diffusivity of the soil. This is the objective of Chapter I.

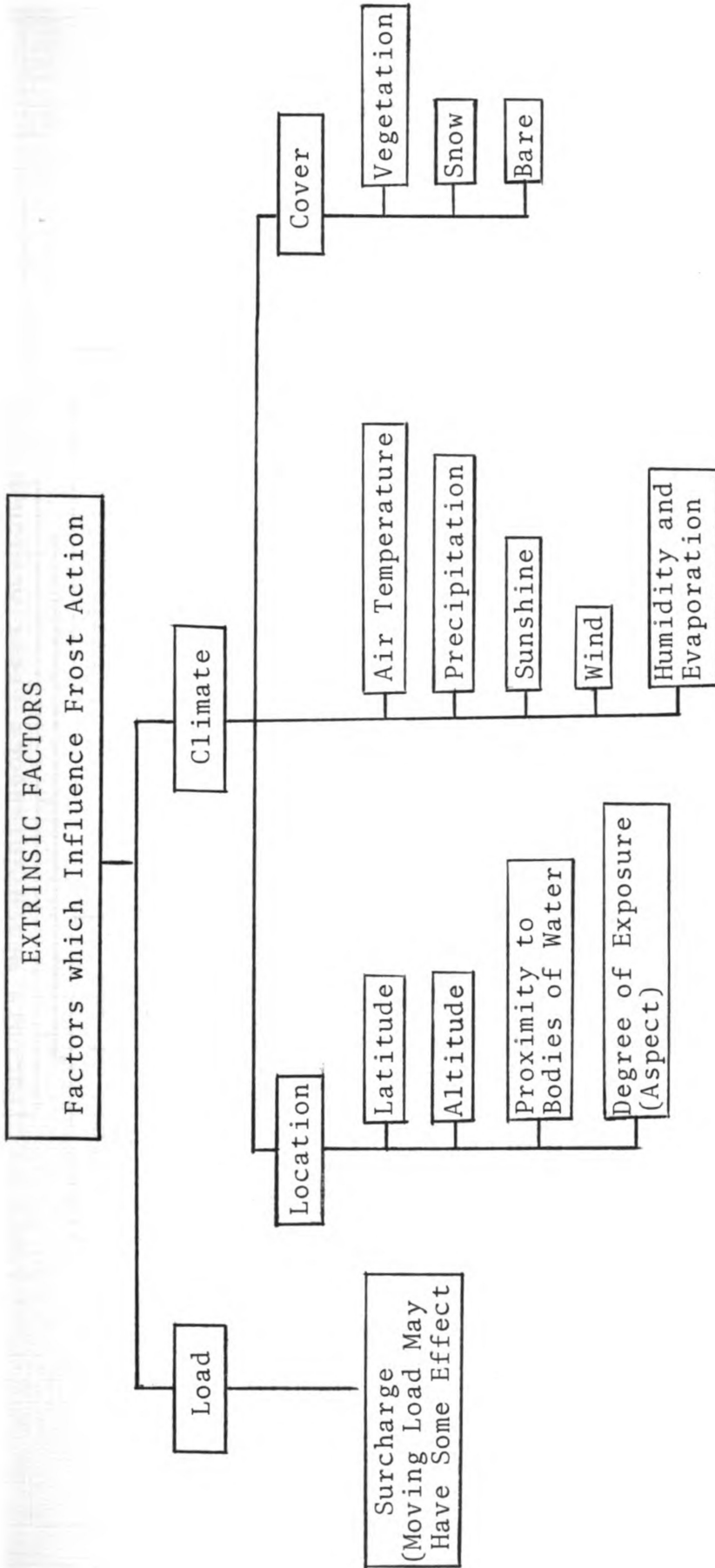


Figure 1-1. Extrinsic factors which influence frost action. [Adapted from Kavianpour, 1971, page 8.]

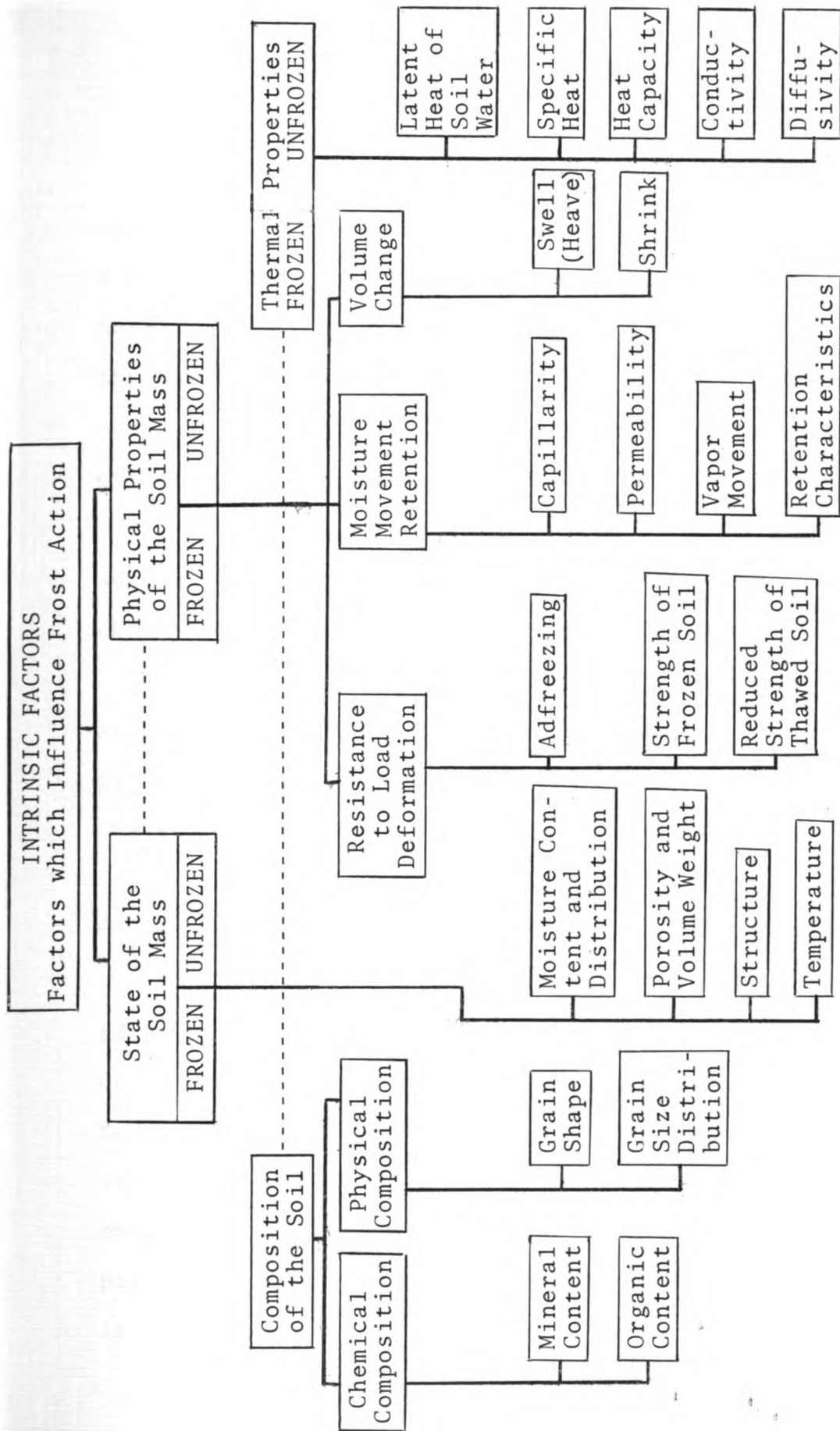


Figure 1-2. Intrinsic factors which influence frost action. [Adapted from Karianpour, 1971, page 7.]

1.2 Source and Amount of Heat

According to Baver, 1965, the temperature of the soil is primarily dependent upon the amount of radiant energy that is received from the sun. The quantities of heat reaching the earth's surface by conduction from within the earth, or resulting from chemical and biological processes, are so small that such sources have a negligible effect upon soil temperature.

The amount of radiation reaching the earth from the sun depends upon the amount emitted by the sun and the absorption of the radiation by the atmosphere.

In addition to the atmospheric conditions, there are several characteristics of the earth's surface that greatly affect the amount of radiation that is retained. (Baver 1965). These may be grouped as follows:

1.2.1 Latitude and slope of land

The angle at which the sun's rays meet the earth greatly influences the amount of radiation received per unit area. Radiation reaching the earth at an angle is scattered over a wider area than the same radiation striking the earth's surface perpendicularly. Consequently, in the former case the amount of heat received per unit area is decreased in proportion to the increase in area covered. The amount of radiation reaching the

earth per unit area is proportional to the cosine of the angle made between the perpendicular to the surface and the direction of the incoming radiation. Therefore, the radiation received per unit area decreases with an increase in this angle.

1.2.2 Vegetative cover

The major effect of vegetation on soil temperature is the insulating quality of plant cover on temperature fluctuations. Bare soil is unprotected from the direct rays of the sun and becomes very warm during the hottest part of the day. When cold seasons arrive, unprotected soil rapidly loses its heat to the atmosphere. In winter the vegetation acts as an insulating blanket to reduce the rate of heat loss from the soil. Consequently, a protected soil is cooler in summer and warmer in winter than one that is bare.

The investigation of Petit (see Baver, 1965, pp. 365-366) has shown that frost penetrates deeper and disappears slower under bare conditions than under grass or surface mulches. This is illustrated in Figure 1-3. It may be seen that the sod cover decreased the rate and depth of penetration of frost as compared with bare soil. When thawing occurred, frost disappeared from the protected soils sooner than from the bare soil owing to the fact that the latter soil was frozen to a greater depth.

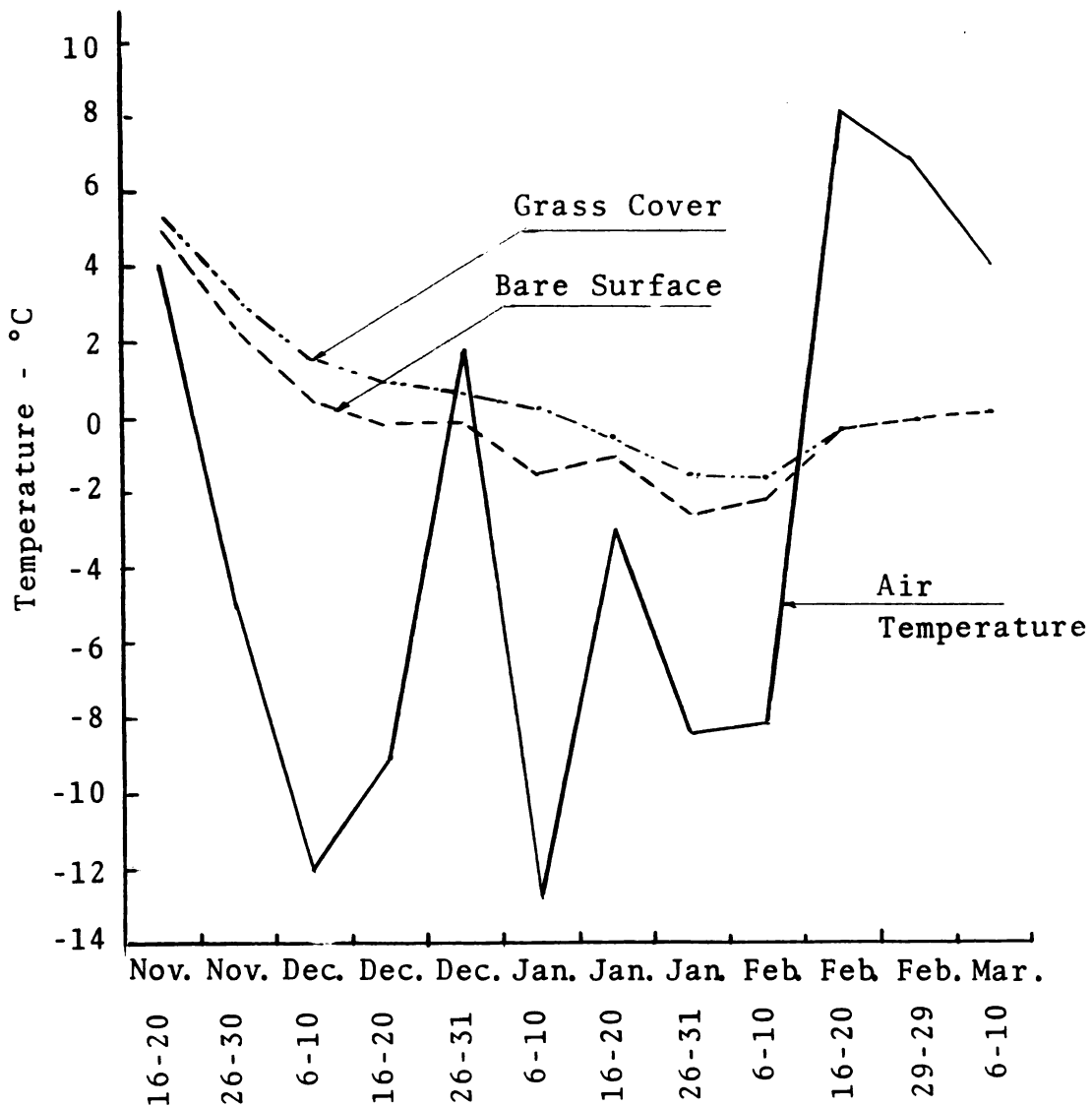


Figure 1-3. Frost penetration in bare and grass-covered soils [Petit (1893)]. Redrawn from Bayer 1965 p. 366.

1.3 Thermal Properties of Soils

The temperature of a soil depends not only on the quantity of energy transported to it, but also on its ability to absorb this energy (Geiger 1965). The basic factors which control soil temperature are the thermal properties of the soil. Other factors such as intensity of absorption of energy (related to soil color) also affect this ability.

Two independent thermal properties enter into a quantitative description of the heat transfer by conduction, viz., the thermal conductivity, k , and the heat capacity per unit volume, $C = \rho c$. In many equations the ratio of these two quantities appears. The ratio is called the thermal diffusivity and is denoted by the symbol

$$\alpha = \frac{k}{C} = \frac{k}{\rho c}.$$

DeVries (van Wijk, et al. 1963), Geiger (1965), and Baver (1965) present a detailed discussion relating to thermal properties of soils.

1.3.1 Heat capacity of soils

The specific heat of any substance is defined as the number of calories of heat required to raise one gram of the substance one degree on the centigrade scale (Baver 1965). The heat capacity per unit volume of a given material C as defined above is equal to its specific heat, c , times its

mass density, ρ . The specific heat of water is 1.00 calorie per gram per $^{\circ}\text{C}$ at 16°C (one BTU/lb_m per $^{\circ}\text{F}$ at 60°F). All other constituents of soils have much lower specific heats.

It may be seen from the data in Tables 1-1 and 1-2 that quartz has the lowest specific heat of the major soil constituents and humus the highest, excepting water. The aluminosilicate kaolin has a slightly higher specific heat than quartz. Since the major constituents in most soils are quartz, aluminosilicates, water and humus, it is evident that humus and water will affect the heat capacity considerably.

The heat capacity per unit volume of soil can be found by adding the heat capacities of the different soil constituents in one cm^3 (DeVries, see van Wijk, et al., 1963). Thus, if x_s , x_w , and x_a denote the volume fractions of solid material, water (or ice) and air, respectively, one has

$$C = x_s C_s + x_w C_w + x_a C_a$$

The third term in the right-hand side can usually be neglected. The value of $C_w = 1.00 \text{ cal cm}^{-3} ^{\circ}\text{C}^{-1}$ in the case of water, $0.45 \text{ cal cm}^{-3} ^{\circ}\text{C}^{-1}$ in case of ice at 0°C and $.43 \text{ cal cm}^{-3} ^{\circ}\text{C}^{-1}$ for ice at -20°C . The specific heats of twelve different mineral soils and material were measured by Kersten (van Wijk, et al., 1963) (see Table 1-3). He found that the specific heat of most soil minerals varied

Table 1-1. Specific heat values c , densities ρ , and specific heat per unit volume C of several soil minerals and rock materials.*

Material	c (cal g ⁻¹ °C ⁻¹)	Mean temp (°C)	ρ (g cm ⁻³)	Reference	C (cal cm ⁻³ °C ⁻¹)
Quartz	0.191	~60	2.65	Ulrich (1894)	.506
Kaolin	0.224	~60	2.6	"	.582
CaCO ₃	0.208	~60	2.71	"	.563
MgCO ₃	0.246	~60	3.04	"	.747
CaSO ₄	0.195	~60	2.45	"	.477
Fe ₂ O ₃	0.165	~60	5.24	"	.864
Al ₂ O ₃	0.217	--	3.7	Lang (1878)	.802
Fe(OH) ₃	0.226	~60	3.6	Ulrich (1894)	.813
Orthoclase	0.194	~60	2.56	"	.496
Oligoclase	0.205	~60	2.64	"	.541
Potash mica	0.208	~60	2.9	"	.603
Magnesia mica	0.206	~60	2.9	"	.597
Hornblende	0.195	~60	3.2	"	.624
Apatite	0.183	~60	3.2	"	.585
Dolomite	0.222	~60	2.9	"	.643
Talc	0.209	~60	2.7	"	.564
Granite	0.192	~56	2.6	Landolt-Börnstein (1952)	.499
Syenite	0.199	~59	2.7	"	.537
Diorite	0.194	~65	2.9	"	.562
Andesite	0.199	~59	2.4	"	.477
Basalt	0.213	~55	3.0	"	.639

(From van Wijk, et al., 1963 page 213)

*See Appendix III for conversion factors from metric to British units.

Table 1-2. Specific heat values c, densities ρ , and specific heat per unit volume C of several organic soils and soil materials.*

Material	c (cal g ⁻¹ °C ⁻¹)	Mean temp (°C)	ρ (g cm ⁻³)	Reference	C (cal cm ⁻³ °C ⁻¹)
Humus	0.477	--	1.26	Lang (1878)	.601
	0.443	~60		Ulrich (1894)	
"Verwitterungserde"	0.416			Bracht (1949)	
"Bolster"	0.46	28	1.36	DeVries and De Wit (1954)	.625

(From van Wijk, et al., 1963 page 212)

*See Appendix III for conversion factors from metric to British units.

Table 1-3. Specific heat values c , densities ρ , and specific heat per unit volume C of several soils after Kersten (1949).*

Soil	c_1 (cal g ⁻¹ °C ⁻¹)	Mean temp. (°C)	ρ_1 (g cm ⁻³)	C (cal cm ⁻³ °C ⁻¹)
Northway fine sand	0.197	61.0	2.76	.543
Northway sand	0.171	- 6.7	2.74	.468
	0.185	18.8		--
	0.191	60.2		--
Northway silt loam	0.168	-10.4	2.70	.453
	0.176	20.4		--
	0.193	60.4		--
Chena river gravel	0.194	61.0	2.70	.523
	0.196	60.4		--
Fairbanks silt loam	0.164	- 8.4	2.70	.442
	0.183	18.8		--
	0.194	61.3		--
Graded Ottawa sand	0.157	- 9.5	2.65	.416
	0.164	18.5		--
	0.176	37.7		--
	0.189	60.3		--
20-30 Ottawa sand	0.183	37.8	2.65	.48
	0.189	59.9		--
Lowell sand	0.159	- 9.5	2.67	.420
	0.188	19.7		--
	0.188	60.9		--
Crushed quartz	0.190	60.9	2.65	.503
Crushed trap rock	0.193	59.9	2.97	.573
Crushed feldspar	0.190	59.3	2.56	.486
Crushed granite	0.161	-13.3	2.67	.429
	0.174	19.4		--
	0.189	60.9		--

(Adopted from van Wijk, et al., 1963 page 212)

*See Appendix III for conversion factors from metric to British units.

linearly from $.16 \pm .01 \text{ cal cm}^{-3} \text{ }^{\circ}\text{C}^{-1}$ at -18°C to $.19 \pm .01 \text{ cal cm}^{-3} \text{ }^{\circ}\text{C}^{-1}$ at 60°C . Since the specific mass of these minerals is about 2.7 g cm^{-3} an average value of C_s of about .46 holds for a mineral soil at 10°C .

The specific heat of soil organic matter was determined by several authors (see van Wijk, et al., 1963), the most probable value being $.46 \text{ cal g}^{-1} \text{ }^{\circ}\text{C}^{-1}$ (extremes are $.42$ and $.48 \text{ cal g}^{-1} \text{ }^{\circ}\text{C}^{-1}$). An average value of specific mass of the organic materials in soil is 1.3 g cm^{-3} and therefore, $C_s = .60$ is a good average value in the case of organic soils.

DeVries concludes that if the volume fractions of soil minerals and of organic matter are denoted by x_m and x_o respectively, the heat capacity per unit volume, equals

$$C = .46 x_m + .60 x_o + x_w \text{ cal cm}^{-3} \text{ }^{\circ}\text{C}^{-1}$$

1.3.2 Thermal conductivity and soil factors affecting it

Geiger (1965) defines the thermal conductivity k as the amount of heat in calories that will flow through a one cm. cube of substance in one second, when the temperature difference between opposite faces is one degree centigrade, and there are no variations in temperature.

In natural soils, k varies not only from place to place, but also in one place as the water content of the soil changes (van Wijk 1965).

1.3.2.1 Soil composition and porosity

Von Schwarz and others (Baver 1965) have shown that the heat conductivity of different soils follows the order: sand > loam > clay > peat.

The degree of packing and porosity of the soil seems to be the major factor determining the thermal transfer. Smith and Byers (1938) arrived at the following approximate expression for the thermal conductivity of a dry soil,

$$k = k_2 p + k_1 (1-p)$$

where k_2 and k_1 are the conductivities of dry air and of the soil material respectively, and p is porosity. This expression shows that the conductivity of the soil decreases as the porosity increases. Skaggs and Smith (1968) conclude that the thermal conductivity of different soils can be calculated in terms of soil porosity.

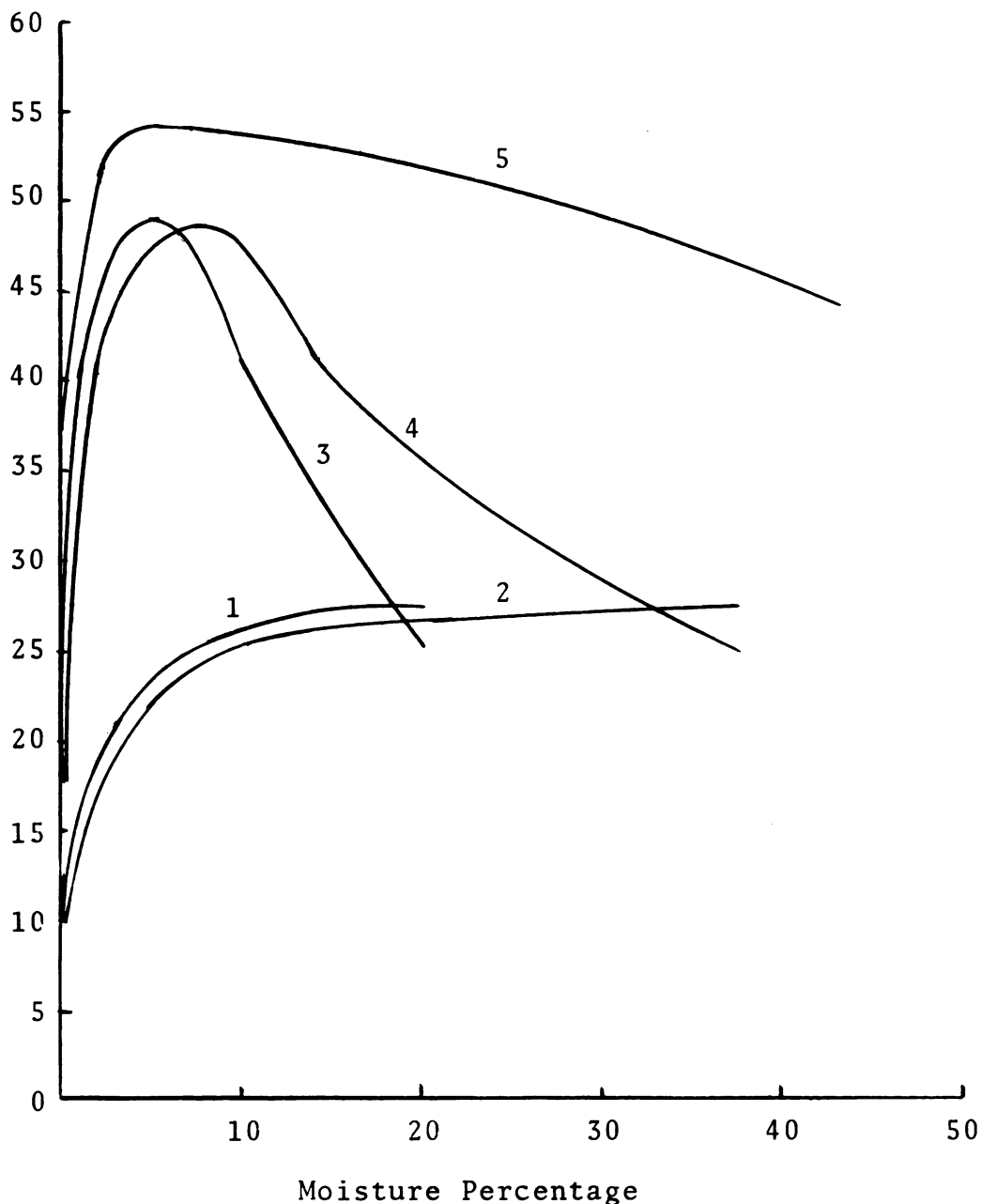
1.3.2.2 The influence of soil moisture in thermal conductivity

Earlier investigations have shown that the heat conductivity of soils and soil materials increases with the moisture content. Patten (see Baver, 1963, pp. 376-379) studied the conductivity of various soils at different moisture contents. He found that the conductivity of dry quartz particles, as well as of soil, was only about

one-half to one-third that of water (.005 calorie per sq. cm. per second per degree change in temperature gradient) and about one-fifteenth to one-twentieth that of a solid quartz block. Thus, the conductivity of quartz, for example, is greatly decreased when it is divided into particles. The reduction in conductivity is due to the small amount of surface contact between particles through which heat will readily flow. The presence of water between the particles increases the conductivity to values higher than that of pure water. The presence of a water film at the points of contact of the particles replaces the air which is a poor heat conductor and thus improves the thermal contact between the particles.

A typical example of Patten's curves is given in Figure 1-4. It is seen that heat conductivity increases with moisture content. It is also to be noted that the greatest rate of increase in conductivity takes place at the lower moisture contents. On the other hand, with fine quartz powder and fine-textured soils, Patten's original curves indicate that the greatest rate of change of heat conductivity apparently occurred at the higher moisture contents.

Also, it can be seen from Figure 1-4 that the diffusivity $\alpha = \frac{k}{\rho c}$ increases rapidly at first with increasing moisture to a maximum, and then decreases. This is due primarily to a greater rise in conductivity at the



- 1 = Heat conductivity, K , c.g.s. $\times 10^4$ plotted against H_2O in per cent by wet weight
- 2 = Same as 1 but with H_2O expressed in per cent by volume
- 3 = Diffusivity, $K/C \times 10^2$, plotted against H_2O in per cent by wet weight
- 4 = Same as 3 but with H_2O expressed in per cent by volume
- 5 = Total porosity in per cent by volume

Figure 1-4. Heat conductivity and diffusivity in coarse quartz sand. (Redrawn from Bevar, 1965, p. 377.)

lower moisture contents, as compared with the increase in heat capacity. As the moisture content becomes larger, however, the value of c becomes greater and diffusivity decreases. Thermal properties of some typical soils and the common soil constituents are given in Tables 1-4 and 1-5.

1.4 General Differential Equation for the Temperature Field

The basic law which quantitatively defines heat conduction is generally attributed to the French mathematician Jean Fourier (1768-1830). With reference to Figure 1-5

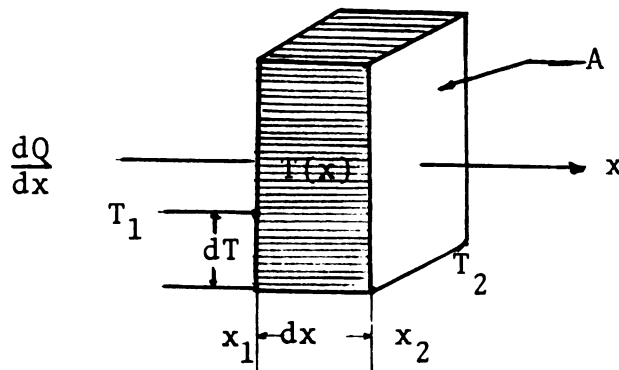


Figure 1-5. Linear heat conduction.

the one dimensional form of the Fourier law states that the quantity of heat dQ conducted in the x -direction of a homogeneous solid in time dt is a product of the conducting

Table 1-4. Thermal properties of some typical soils and of the common soil constituents (at 10°C).*

Substance	x_w Volume fraction of water	x_s Solid matter	K cal cm sec (°C)	C cal cm ³ (°C)	α $\frac{\text{cm}^2}{\text{sec}} \times 10^{-3}$	α $\frac{\text{ft}^2}{\text{hr}} \times 10^{-3}$	D cm damping depth
Quartz		21	10.48	43.75	175.0		
Clay Minerals		7	.48	14.58	58.33		
Organic matter		.6	.6	1	4		
Water		1.37	1.00	1.37	5.48		
Air		.06	.0003	200	800		
Ice (0°C)		5.2	.45	11.56	46.24		
Sand	.0	.6	.7	.3	2.33	9.33	8.0
	.2	.6	4.2	.5	8.4	33.6	15.2
	.4	.6	5.4	.7	7.7	30.86	14.6
Clay	.0	.6	.6	.3	2	8	7.4
	.2	.6	2.8	.5	5.6	22.4	12.4
	.4	.6	3.8	.7	5.43	21.71	12.2
Peat	.0	.2	.14	.12	1.17	4.67	5.7
	.4	.2	.7	.52	1.35	5.38	6.1
	.8	.2	1.2	.92	1.30	5.22	6.0

(From van Wijk, 1965)

*See Appendix III for conversion factors from metric to British units.

Table 1-5. Comparison of thermal properties of frozen and unfrozen soil.*

Parameter**	Silt (17-1/2 percent)		Clay (65 percent)		Water/ice	
	Frozen	Thawed	Frozen	Thawed	Frozen	Thawed
Thermal conductivity (mcal per cm sec °C)	5.0	3.4	4.0	2.5	5.4	1.2
Volumetric specific heat (cal per cm ³ °C)	.42	.57	.52	.83	.45	1.00
Thermal diffusivity (cm ² per sec)	.012	.006	.008	.003	.012	.0012
Latent heat per unit volume (cal per cm ³)		21.6		50.4		72
Moisture content (per- cent wet weight)		15		39.5		100
Moisture content (per- cent dry weight)		17.5		65		∞
Moisture content (per- cent by volume)		28.5		63		100
Wet density (gm per cm ³)	1.8	1.8	1.6	1.6	.9	1.0
Wet density (lb per cu ft)	112	112	100	100	56.2	62.4

(From Geological survey circular 632, p. 6).

*See Appendix III for conversion factors from metric to British units.

**Parameters: Thermal conductivity, volumetric specific heat, moisture content, and wet density are independently chosen parameters; the others are derived.

area A normal to the flow path x , the temperature gradient $\frac{dT}{dx}$ along the path, the thermal conductivity k .

Expressed analytically:

$$\frac{dQ}{dt} = - kA \frac{dT}{dx} \quad [1.1]$$

The Equation [1.1] defines the transient heat flow in a linear conductor, and from this is derived the general partial differential equation satisfied by the transient temperature field in a three-dimensional volume.

Schneider (1955) derives the equation for three-dimensional and obtains:

$$\frac{d}{dx} (k_T \frac{dT}{dx}) + \frac{d}{dy} (k_T \frac{dT}{dy}) + \frac{d}{dz} (k_T \frac{dT}{dz}) + \dot{Q} = c\rho \frac{dT}{dt} \quad [1.2]$$

where $\dot{Q}$ = rate of generation or utilization of energy per unit volume.

The most general partial-differential equation for heat conduction is the same form, but with k_T , c , ρ , and $\dot{Q}$ replaced by $k(x,y,z,T)$, $c(x,y,z,T)$, $\rho(x,y,z,T)$ and $\dot{Q}(x,y,z,T,t)$ to include spatial and temporal as well as temperature dependence.

1.5 Daily and Seasonal Changes in Soil Temperature

The movement of heat into or out of the soil depends upon the difference in temperature between the surface of the soil and lower layer of the soil, i.e., the soil

temperature gradient. Much information regarding the temperature regime of the soil can be obtained by solving the differential equation of heat flow for the soil.

For a homogenous and isotropic soil, assuming no significant generation or utilization of heat occurs in the soil, equation [1.2] can be written (Merva 1970)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad [1.3]$$

where:

T = temperature

x, y, z = Cartesian coordinates, z increasing downward

α = thermal diffusivity = $k/\rho c$

t = time

With the further assumption that no gradient of temperature exists in the x, y directions, equation [1.3] reduces to

$$\frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad [1.4]$$

The solution of equation [1.4] will describe the behavior of the temperature in the soil mass.

Merva (1970) solves this equation with the following initial and boundary conditions:

$T(z, t)$ = some quasi-steady state condition (a)

$T(0, t)$ = a known periodic function of time (b)

$T(\infty, t)$ = a finite value (c)

and obtains:

$$T(z,t) = A_0 + \sum_{n=1}^{\infty} e^{-\sqrt{\frac{n\pi}{\alpha\tau}} z} \left[a_n \cos\left(\frac{2n\pi t}{\tau} - \sqrt{\frac{n\pi}{\alpha\tau}} z\right) + b_n \sin\left(\frac{2n\pi t}{\tau} - \sqrt{\frac{n\pi}{\alpha\tau}} z\right) \right] \quad [1.5]$$

where:

τ is the maximum period of time for which the solution is valid. For a diurnal temperature variation τ is 24 hr. or 86,400 sec. while for the annual temperature variation, $\tau = 365$ days.

z = depth of soil.

A_0 , a_n , and b_n are Fourier series coefficients that can be obtained from

$$A_0 = \frac{1}{\tau} \int_0^{\tau} f(t) dt \quad [1.6a]$$

$$a_n = \frac{2}{\tau} \int_0^{\tau} f(t) \cos \frac{2n\pi t}{\tau} dt \quad [1.6b]$$

$$b_n = \frac{2}{\tau} \int_0^{\tau} f(t) \sin \frac{2n\pi t}{\tau} dt \quad [1.6c]$$

where $f(t)$ is the surface temperature. In many cases the surface temperature data is available from ESSA, Weather Bureau.

For any soil A_0 is the average temperature of the soil surface for a period of time equal to one complete cycle of the longest periodic component of the Fourier Series solution, and in practice A_0 would be determined by averaging temperatures over a length of time equal to the length of record used to obtain for the Fourier series representation. The interpretation of Equation [1.5] as Merva states is:

The exponential multiplier therefore tells us to what depth a fluctuation with a frequency of $2n\pi/\tau$ will penetrate. It is immediately apparent that the lowest frequency components of the Fourier Series expansion influence temperature at greater depths than do high frequency components. Thus, one would expect the yearly temperature fluctuation to be detectable at far greater depths than the daily fluctuations.

Also from the Equation [1.5] we note that the argument of the sinusoidal fluctuation contains an angular frequency which is a function of time, and a phase shift which is a function of depth. The result of the phase shift is that the time of occurrence of the maximum or minimum temperature is shifted with depth.

1.5.1 Damping depth

The depth to which fluctuation will penetrate to soil mass before being reduced by a factor of e^{-1} is called the damping depth and can be calculated (van Wijk et al., 1963) from:

$$D = \sqrt{\frac{\alpha t}{n\pi}} \quad [1.6]$$

The physical explanation of the damping and retardation of the temperature variation with depth lies in the fact that a certain amount of heat is stored or released in a layer when the temperature in that layer increases or decreases respectively.

The damping depth depends on the period of temperature variation. It is $\sqrt{365} \approx 19$ times larger for the annual variation than for the diurnal variation in a given soil. Table 1-6 presents damping depths for several values of thermal diffusivities.

Table 1-6. Damping depths of temperature fluctuation having various periods for several values of thermal diffusivity.*

Thermal Diffusivity in cm^2/sec	Damping Depth D (cm)		
	Period of Fluctuation		
	hourly	daily	yearly
.001	1.07	5.24	100.19
.002	1.51	7.42	141.69
.003	1.85	9.08	173.54
.004	2.14	10.49	200.38
.005	2.39	11.73	224.03
.006	2.62	12.85	245.42
.007	2.83	13.87	265.08
.008	3.03	14.83	283.38
.009	3.21	15.73	300.57
.010	3.39	16.58	316.83
.011	3.55	17.39	332.30
.012	3.71	18.17	347.07
.013	3.86	18.91	361.24
.014	4.01	19.62	374.88
.015	4.15	20.31	388.04
.016	4.28	20.98	400.76
.017	4.41	21.62	413.10
.018	4.54	22.25	425.07
.019	4.67	22.86	436.72
.020	4.79	23.45	448.07

*See Appendix III for conversion factors from metric to British units.

CHAPTER II

DEFINITION OF THE PROBLEM

2.1 Background

Drain tiles in common use are clay, concrete or plastic. A well-designed drainage system requires high-quality material as well as proper installation to insure a long-lasting drain for the more severe exposure conditions that are likely to be encountered in ordinary farm drainage.

2.1.1 Tile for over-size trenches

Many tests have been made (Manson, in Luthin, 1957) to determine experimentally and theoretically the loads to which drain tile are subjected in service. These tests look to design of tile systems which will avoid cracking of the tile from overloading after installation. The effect of numerous factors have been studied, such as soil type, depth of cover, width of trench, and bedding conditions of the tile as laid. Based on these studies, tables (for example see Luthin, p. 316 or Standards for Drainage, p. 34) have been prepared to show safe allowable depths of trench for drain tile of different diameters.

2.1.2 Clay tile and frost action

The frost resistance of clay products is largely dependent on the quality and handling of the raw clay or shale previous to, and during burning.

In order to obtain factual information regarding the durability of clay tile under actual service conditions in a cold region, Miller and Manson (Luthin, 1957, pp. 321-322) dug up some tile which had been installed for an average of 33 years and concluded that:

1. The shale tile examined were all in first-class condition even where the depth of cover averaged but 1.7 feet.
2. Many of the surface clay tile were in poor condition where the depth of cover was 2.00 feet and less.

Frost penetration is an important factor in tile line freezing. There are a number of factors other than depth of penetration that influence the effects of frost action on drain tile. Manson summarizes them as:

1. Quantity and source of water carried by the tile during cold weather.
2. Vegetative cover.
3. Physical condition of the soil cover--fall plowed, etc.
4. Depth and duration of snow cover.
5. Frequency and duration of winter temperatures.

Considerable attention should be paid to factor one, especially in areas where good tile outlets are not available and pump drainage is utilized, because tile lines are often installed with minimum cover and submerged outlets are used. Thus a large portion of the drainage system may

be submerged and close enough to the surface that severe cold weather can cause freezing of the water within the lines, with subsequent cracking of the buried conduct if the pump is not operated during the winter period.

In this thesis, a study has been made to investigate the temperature history around submerged drain tile to determine the reasonable depth at which cold weather will not result in freezing of the water inside the drain tile. It is noted that the presence of water within the tile is expected to inhibit the cooling action and thus the freezing of the water. To determine if freezing will occur, therefore, it is necessary to ascertain the penetration and duration of the 32° front in soil at the desired location. It is felt that if it can be demonstrated experimentally that cracking does occur under certain temperature conditions, and, if a similar set of conditions can be predicted for a field location through selective modeling techniques, then one could expect that tile damage due to cracking can be anticipated for the selective location. In the present study the location of interest is near Saginaw, Michigan, where extensive areas of soil lay bare during the cold months. These areas are in large part pump drained and the drainage system pumps may not be operative during the winter season.

2.2 Experimental Set-up

In order to ascertain the behavior of a crack resulting from freezing of the water inside a drain tile, an experiment was conducted in the Physical Properties Laboratory at Michigan State University. The result of this experiment led to a determination of realistic conditions for solving the mathematical model of tile freezing.

2.2.1 Procedure

Two four-inch clay drain tiles were selected for the experimental phase of the study. The ends were blocked with wood sealed to the tile with silicon rubber sealer. Four thermocouples located midway between the ends of the tile were placed one inch apart on a diameter inside one of the tiles, while a second experiment was performed using only three thermocouples. When the silicon rubber seal was dry, the tiles were soaked in water for 24 hours and were then filled with 40°F. water.

A No. 40 Constantan wire was wrapped around the second tile and the ends of wire were fixed with a nail to the wood used to block the tile ends. The wire was attached to a strain gauge amplifier indicator in order to determine the exact time of cracking so that when the tile cracked, the wire broke and the meter on the amplifier went to full scale.

Both tiles were embedded in a box of dry soil and additional thermocouples were located on each of four sides of the tiles as shown in Figure 2-1. The whole system was placed in a freezer with the average ambient temperature maintained at 0°F. The thermocouples were connected to a self-balancing potentiometer recorder. In addition the temperatures of the thermocouples were checked with a Millivolt potentiometer.

The instruments used were:

1. Strain gage amplifier from Daytronic, model 300 D, with type 90 strain gage input model.
2. Self-balancing potentiometer from Texas Instrument, point Multiriter.
3. Millivalt potentiometer from Leeds & Northrup Company
Catalog No: 8686.

2.2.2 Observations

The temperature readings of thermocouples corresponding to the points 1,2,3,4 are given in Figure 2-2. The other experiment showed a similar behavior.

Since the top surface of tile was exposed to ambient air with average temperature zero degrees F., points 1,2,3 cooled in this order respectively. Point 4 cooled faster than other points because the thermal diffusivity of the soil was greater than that of water.

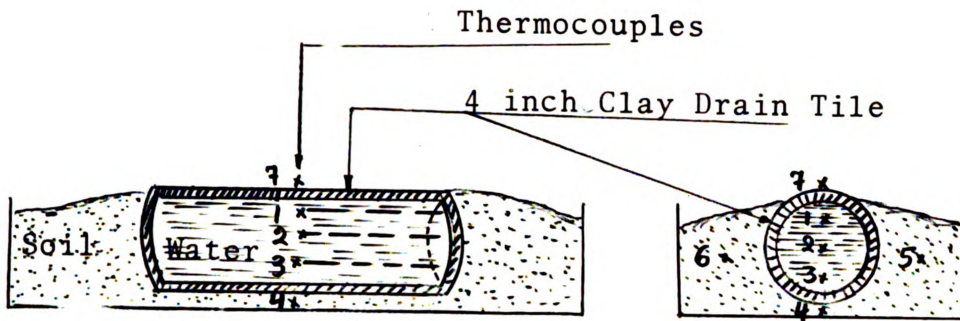


Figure 2-1a. Location of thermocouples in the experimental setup.



Figure 2-1b. A picture of the cracked tile showing the general appearance. Note the thermocouple wires entering from the right.

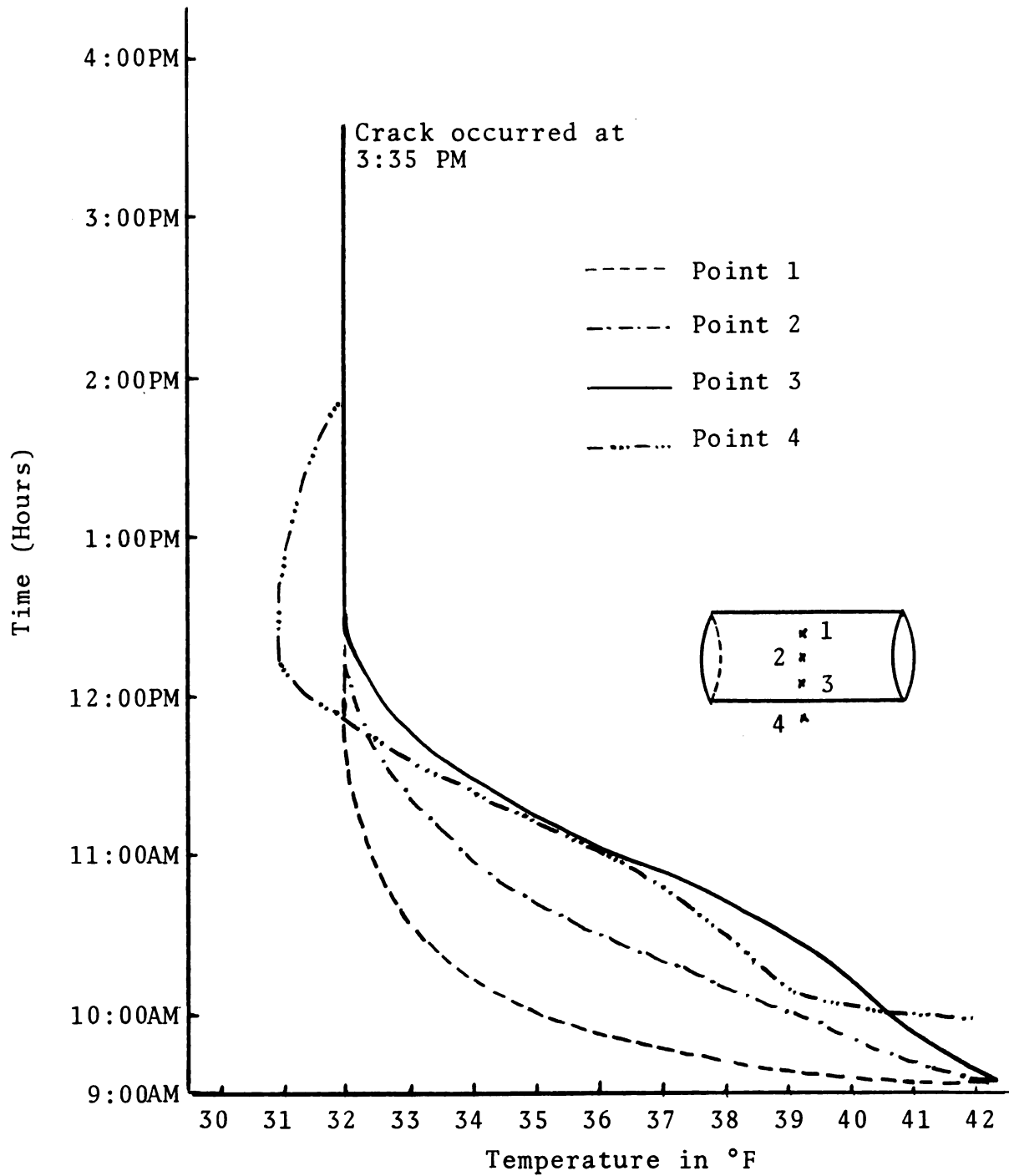


Figure 2-2. Temperature variation for each thermocouple.

When the temperature of point one reached 32°F. it remained constant at this value. The temperature of points 2,3 decreased respectively until they reached 32°F. and then they also remained constant. The temperature of point 4 dropped below 32°F. When the temperature of all of the water inside the drain tile became 32°F., the formation of ice started. Ice was formed in the upper portion of the tile. The formation was crest-shaped and occupied about one-third of the volume inside the tile at the time of cracking. The shape of the ice and location of the crack are shown in Figure 2-3. Thus, it has been demonstrated that cracking of a clay tile due to freezing of water within the tile is possible provided the tile system remains at 32°F. for a sufficiently long duration of time. It is probable that the condition indicated will occur if the temperatures surrounding the tile drop to less than 32°F. on all sides.

2.3 Mathematical Model

2.3.1 General assumptions

Heat is transported in the soil mainly by conduction, i.e., the transfer of thermal energy on a molecular scale. To facilitate the mathematical treatment the following general assumptions are made:

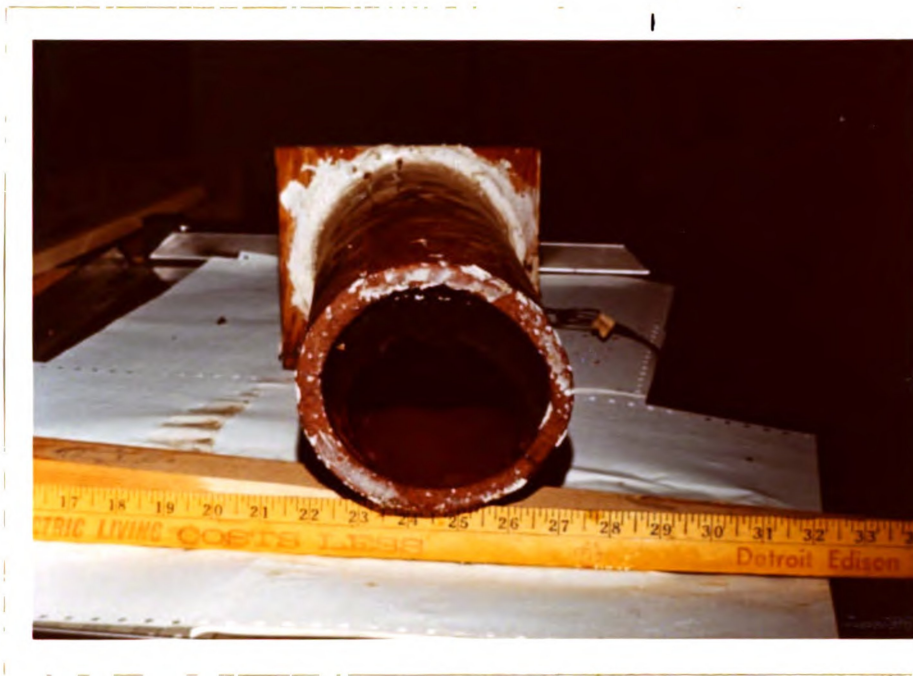


Figure 2-3. The shape of ice causing cracking of a drain tile. The location of the crack is shown in the picture at the lower right of the tile.

1. The soil is almost saturated in the winter, therefore, the thermal conductivity is uniform throughout the soil mass. This assumption has a deficit in that the thermal properties of frozen soil are different from those of unfrozen soil. Consequently, when the soil starts freezing there will be a moving boundary within the soil mass across which the thermal properties differ with time and temperature. Since we are interested only in a critical temperature around a drain tile for simplification we assume that the soil is homogenous and isotropic and therefore the

thermal properties are independent of space and time coordinates. The thermal diffusivity of frozen soil is used in the calculations since frozen soil has the higher value. The resulting temperatures will thus be conservative.

2. The latent heat of fusion of water can be ignored in the model during the phase change.

3. The medium is at rest, i.e., that no motion of matter on a macroscopic scale occurs.

4. The heat flow is in the vertical and horizontal directions only.

5. No heat is generated or utilized within the soil medium.

2.3.2 Specific assumptions

The surface of the soil is subjected to a seasonal variation in temperature. The temperature fluctuation is sinusoidal and its daily and yearly fluctuation decreases with depth. At the damping depth the temperature is almost constant for a period of one cycle (see Sec. 1.5). The damping depth is a function of the thermal diffusivity and differs for different soils. The damping depth for daily temperature fluctuations for most soils normally is less than 12 inches (see Table 1-6).

The upper coordinate boundary for the present solution of the mathematical model is located at 13 inches below the soil surface so that daily temperature

fluctuations can be ignored (see Figure 2-4). For a soil having a different thermal diffusivity, this depth should be calculated and the upper coordinate boundary chosen accordingly.

It is assumed for the purposes of the model that $T(x,z,t)$ represents the temperature of the soil at depth z and distance x from the center of the tile at time t . In the solution, for the upper boundary it is assumed that one inch above the first x coordinate line the temperature is constant at some value G . The assumption is necessary to facilitate solving of the tridiagonal matrix associated with the solution. In the computer solution it is possible to change the value of G at every time increment, thus, a slow temperature variation of the upper boundary is possible.

Since the soil is a semi-infinite body, it is assumed that at infinite depths soil temperature remains constant, i.e., $T(x,m,t) = \text{constant}$ for each period of time where m represents the maximum grid point, which in the present problem is 68 inches. Since the soil temperature at the yearly damping depth usually is nearly constant, it is desirable that the value of m be equal or greater than the yearly damping depth.

It is assumed that at the right hand side of the soil system there exists a boundary beyond which the temperature of the water in the tile will not effect the soil temperature. This point is taken 68 inches from the center

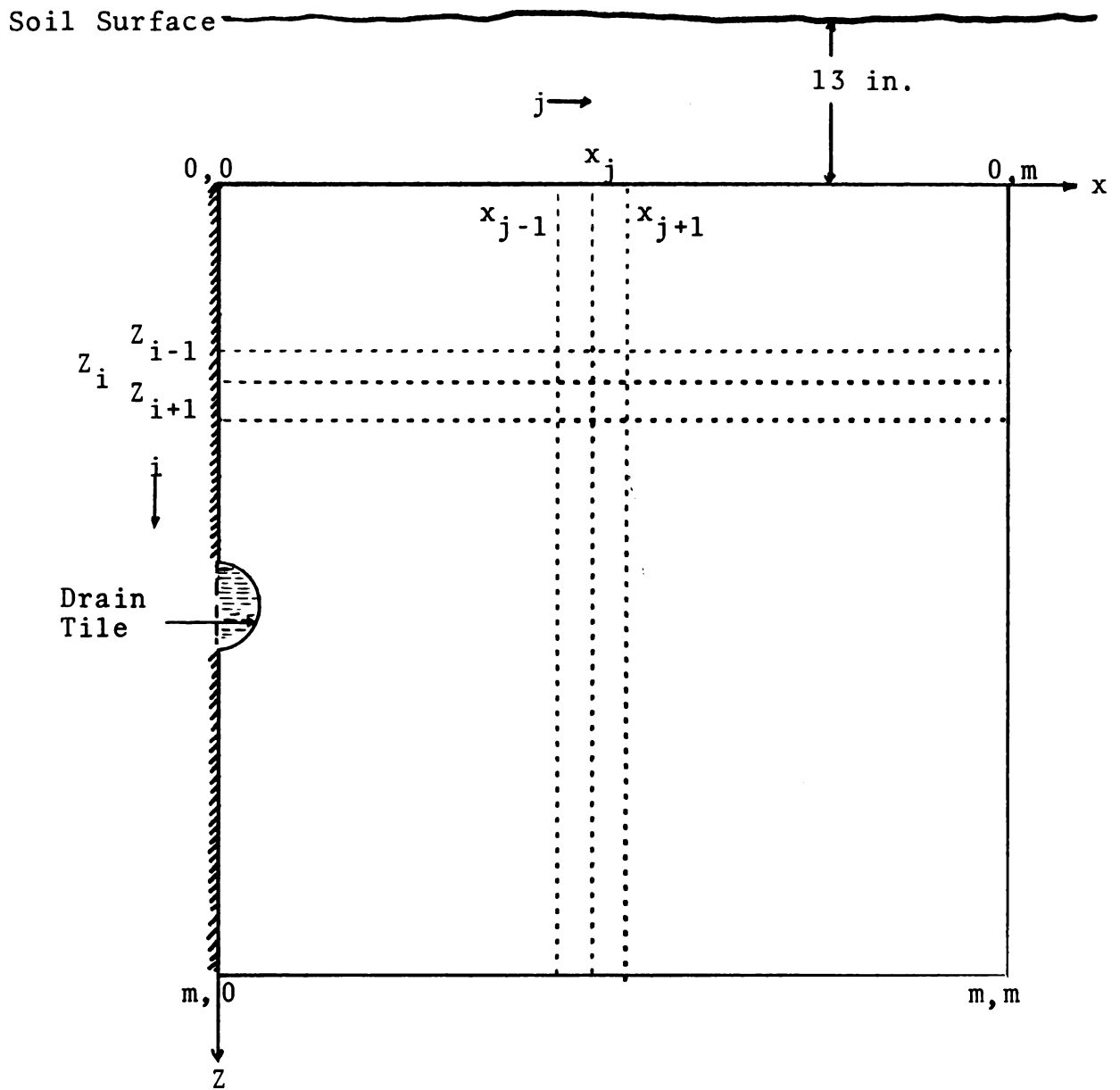


Figure 2-4. Grid system.

of the drain tile. Mathematically, the assumption can be stated as $T(m,z,t) = T(z,t)$, a known temperature at every depth and time.

Due to symmetry, the vertical line passing through the center of the tile is an insulated boundary, i.e., $\partial T / \partial x = 0$ for $T(0,z,t)$.

Finally it was assumed that the tile line was submerged by 32°F. water, and that the temperature in the tile remains constant at 32°F. until a point one inch below the tile line reaches 32°F. (see Sec. 2.2). Formation of ice will begin from the coldest point and will extend downward in the tile as was demonstrated by experiment. Because of the complexity of solution for a two phase, moving boundary, ice-water problem, the entire system is considered as a single phase system. It is assumed the cracking will occur when the temperature one inch below the bottom of the tile reaches 32°F. The assumption is justifiable since experimental evidence shows only about one-third of the water must turn to ice to initiate cracking of the tile. It is thought that the time required to attain 32°F. at one inch below the bottom of the tile is more than adequate to allow for conversion of one-third of the water to ice.

2.3.3 Determination of initial and boundary conditions

2.3.3.1 Determination of soil temperature

The soil temperature readings to a depth of 36 inches in Miami fine sandy loam soil under fescue grass at the East Lansing, Horticulture Farm were obtained from the U.S. Weather Bureau Climatological records for 1963, the coldest year of record at this station for the past 15 years.

The temperature of the soil for eleven days (from February 21 until March 4, 1963) was chosen (Table 2-1) and by interpolation the temperature for every inch of soil depth was estimated and used as the daily temperature data.

Because the annual average of soil temperature and soil cover and soil characteristics differs from location to location, the temperature reading of the East Lansing Station can not be assumed to be representative of the soil temperature for other locations. Since the recorded temperature for an arbitrary location is not available, the soil temperatures were estimated using a Fourier series technique as follows.

The annual average soil temperature A_0 is not equal throughout the soil profile, it varies with depth. Tables 2-2 and 2-3 give annual average values of soil temperatures

Table 2-1. Recorded daily soil temperature (°F.) at different depths in East Lansing Horticulture Farm in 1963.

Depth	Days													
	2/21	2/22	2/23	2/24	2/25	2/26	2/27	2/28	3/1	3/2	3/3	3/4		
air max.	11	10	24	22	22	12	17	25	25	37	36	36		
temp. min.	-1	5	8	12	4	-5	0	16	-1	2	28	28		
3 inches	25	24	27	30	28	26	25	28	27	28	30	32		
6 inches	27	25	27	30	28	27	26	28	27	28	30	32		
12 inches	32	27	28	31	29	28	27	28	28	29	30	31		
24 inches	33	33	32	33	32	32	31	31	31	31	31	32		
36 inches	35	34	34	35	34	34	33	33	33	33	33	33		
80 inches	37	36	36	36	36	36	36	35	35	35	35	35		
Extrapolation														

Miami fine sandy loam under fescue grass. Degree of slope 0.

Data are taken from U.S. Weather Bureau, Climatological Data, Michigan.

Table 2-2. Annual average of soil temperature (°F.) for East Lansing Horticulture Farm Station.

Depth	1967	1968	1969	1970
Air	46.7	47.9	47.4	48.1
2 in.	53.3	53.2	53.2	52.4
4 in.	51.8	51.5	51.3	51.0
8 in.	50.1	49.6	49.9	50.1
20 in.	49.3	49.1	49.5	49.5
40 in.	49.3	49.2	49.4	49.5
80 in.	49.1	49.0	49.4	49.1

Data from U.S. Weather Bureau, Climatological Data for Michigan.

Table 2-3. Annual average soil temperature (°F.) at University of Minnesota, St. Paul Station. Soil is under sod cover.

Depth	1963	1964	1965
Average air temp.	45.9	46.9	43.6
1 cm.	51.2	51.7	47.9
5 cm.	50.6	51.0	47.2
10 cm.	49.8	50.2	46.4
20 cm.	48.2	48.7	45.4
40 cm.	47.9	48.0	45.0
80 cm.	47.2	48.2	45.1
120 cm.	47.5	48.2	45.6
160 cm.	47.7	--	46.4
320 cm.	47.3	--	46.9
480 cm.	47.5	--	47.1
640 cm.	48.0	--	47.9
800 cm.	48.7	--	47.7
960 cm.	48.0	48.2	47.8
1120 cm.	47.8	48.1	47.7
1280 cm.	48.0	47.9	47.8

Data from U.S. Weather Bureau, Climatological Data for Minnesota.

for the East Lansing, Horticulture Farm Station from 1967-1970 and annual average soil temperatures for different depths and related annual air temperatures for the University of Minnesota, St. Paul Station, respectively. The recorded soil temperatures reveal that the soil surface has the highest yearly average temperature.

Annual average soil temperature rapidly decreases with increasing depth to a certain depth. Beyond this depth gradually A_0 increases as shown in Table 2-3. At any depth the yearly average soil temperature is greater than annual average air temperature for a given location.

Soil temperature is also highly affected by surface cover. A_0 has a greater value for bare soil than for grass or sod-covered soil as is shown in Figure 2-5 and Table 2-4.

As discussed above, since each depth of soil has a different annual average temperature, the annual air temperatures can not be used in a Fourier series to obtain the boundary conditions to be substituted for the soil surface temperatures to obtain a solution using Equation [1.5]. Indeed, if one analyzes data for the soil temperature, the air temperature and the temperature one inch above the ground, the results show that the annual average temperature at one inch above the ground is greater than the annual air temperature. This fact is illustrated in Figure 2-6.

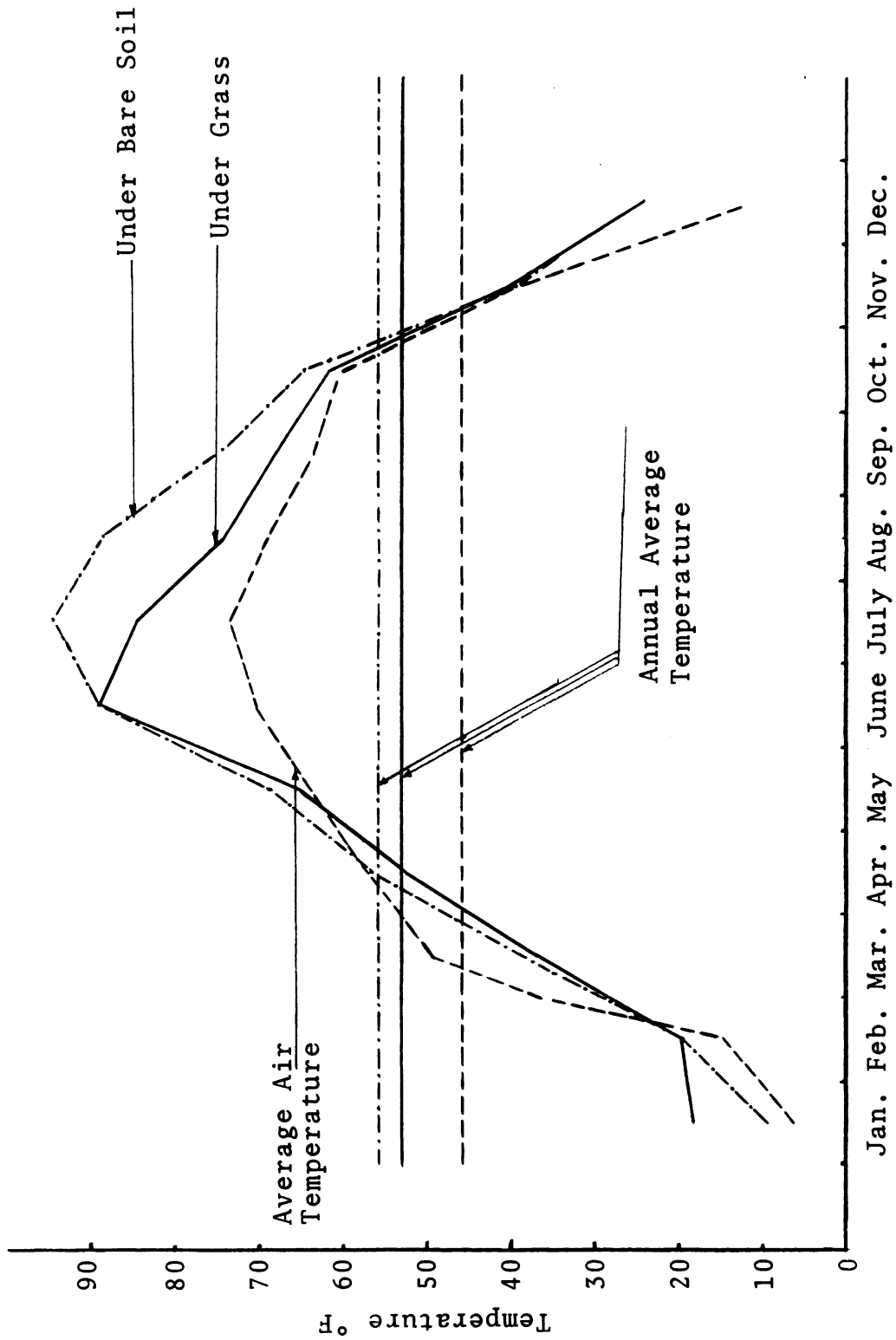


Figure 2-5. Effect of soil coverage on soil temperature. [Data taken from U.S. Weather Bureau (1963). Climatological data, University of Minnesota, St. Paul Station.]

Table 2-4. Effect of soil coverage on yearly average soil temperature (°F.) at different depths.

Depth	under sod	under bare soil	under soybean covered soil
Air 45.9°F.			
1 centimeter	51.2	55.8	53.1
5 centimeters	50.6	55.8	51.6
10 centimeters	49.8	53.0	50.8
20 centimeters	48.2	49.9	48.9
40 centimeters	47.9	48.1	47.2
80 centimeters	47.5	47.5	47.1

Data from U.S. Weather Bureau, 1963. Climatological Data, University of Minnesota, St. Paul Station, Minnesota.

In order to calculate the soil temperature at different depths, a Fourier series was fit to the soil temperatures at each depth as recorded at the East Lansing Horticulture Farm. Such a technique gives a function which describes the temperature fluctuation at each depth in the soil for the above mentioned station. In order to obtain the soil temperatures for another location in Michigan it was postulated that the same general nature of temperature fluctuation would be present; however, the mean soil temperatures A_0 for each depth would be different depending upon whether the corresponding average annual air temperatures were greater or less than the average annual air temperature at the East Lansing location. Thus, the

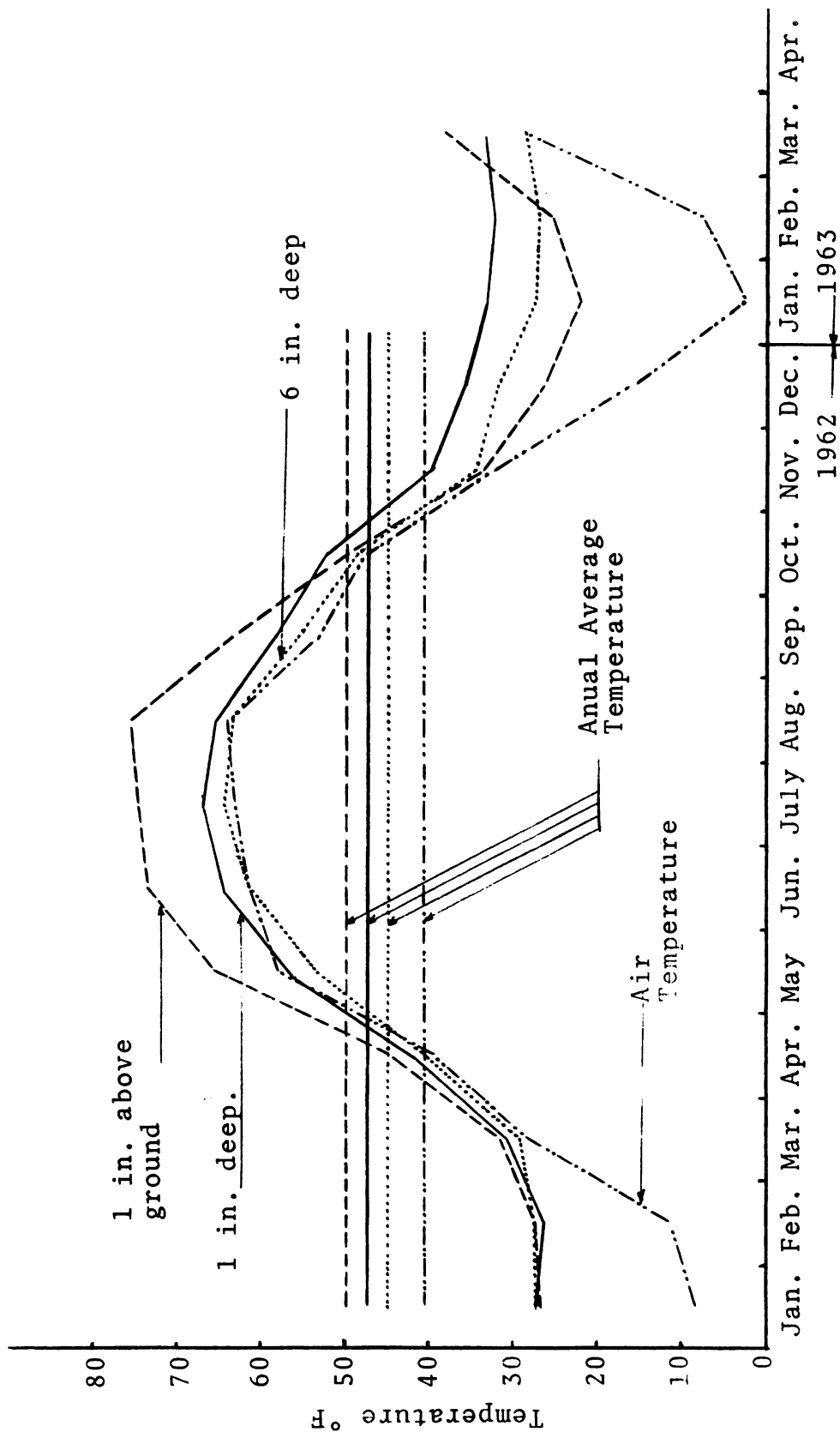


Figure 2-6. Variation of monthly average air and soil temperatures by depth.

correct value of A_0 must be determined. At the present time, calculating A_0 is difficult, because soil temperature is recorded in only a few locations in every state, and the relation between soil temperature and air temperature has not been determined. Therefore, to get an estimate of annual average soil temperature at different depths, the following procedure was applied.

The difference between the annual average air and soil temperatures at selected depths was calculated from Tables 2-2 and 2-3 and the results are given in Table 2-5 for the Minnesota and Michigan data.

Table 2-5. Difference between air temperature and soil temperature at different depths in °F.

Depth	University of Minnesota St. Paul				East Lansing Horticulture Farm			
	1963	1964	1965	average	1968	1969	1970	average
1 cm.	6.3	5.8	4.3	5.5	--	--	--	--
5 cm.	4.7	4.1	3.6	4.1	5.3	5.8	4.3	5.1
10 cm.	3.9	3.3	2.8	3.3	3.6	3.8	2.9	3.4
20 cm.	2.3	1.8	1.8	2.0	1.9	2.5	2.0	2.1
40 cm.	2.0	1.1	1.4	1.5	--	--	--	--
50 cm.	--	--	--	--	1.2	2.1	1.4	1.6
80 cm.	1.3	1.3	1.5	1.4	--	--	--	--
100 cm.	--	--	--	--	1.3	2.0	1.4	1.6
120 cm.	1.6	1.3	2.0	1.6	--	--	--	--
200 cm.	--	--	--	--	1.8	2.0	1.0	1.6

As can be seen from Table 2-5, the annual average soil temperature at 8 inches is about two degrees F. higher than air temperature for any given year, and between 20-80 inches depth the difference reduces to 1.5 degrees F. It is worthwhile to note that this difference depends on soil coverage as well as kind of soil, and soil moisture content. For example, for bare soil the variation is greater than sod-covered soil (Table 2-4).

Based upon the above reasoning it is felt that adding two F. degrees to the annual average air temperature, will with good accuracy yield an estimate of the annual average soil temperature at 8 inches for a soil that has a sod cover. Similarly, adding 1.5F. degrees to the annual average air temperature will give an estimate of the average annual soil temperature between 20-80 inches depth.

The temperature fluctuations for every soil depth are dependent upon weather conditions, surface coverage, type of soil and soil moisture content. It is assumed in the present study that if the daily air temperature variation of a region has a similar pattern in comparison with another region, then the soil temperature fluctuation for the two regions with the same kind of soil will also have a similar shape.

The daily air temperature variations for East Lansing, Saginaw and Evart (Central Lower Michigan) for 1968 are given in Figure 2-7. In Figure 2-8 a similar

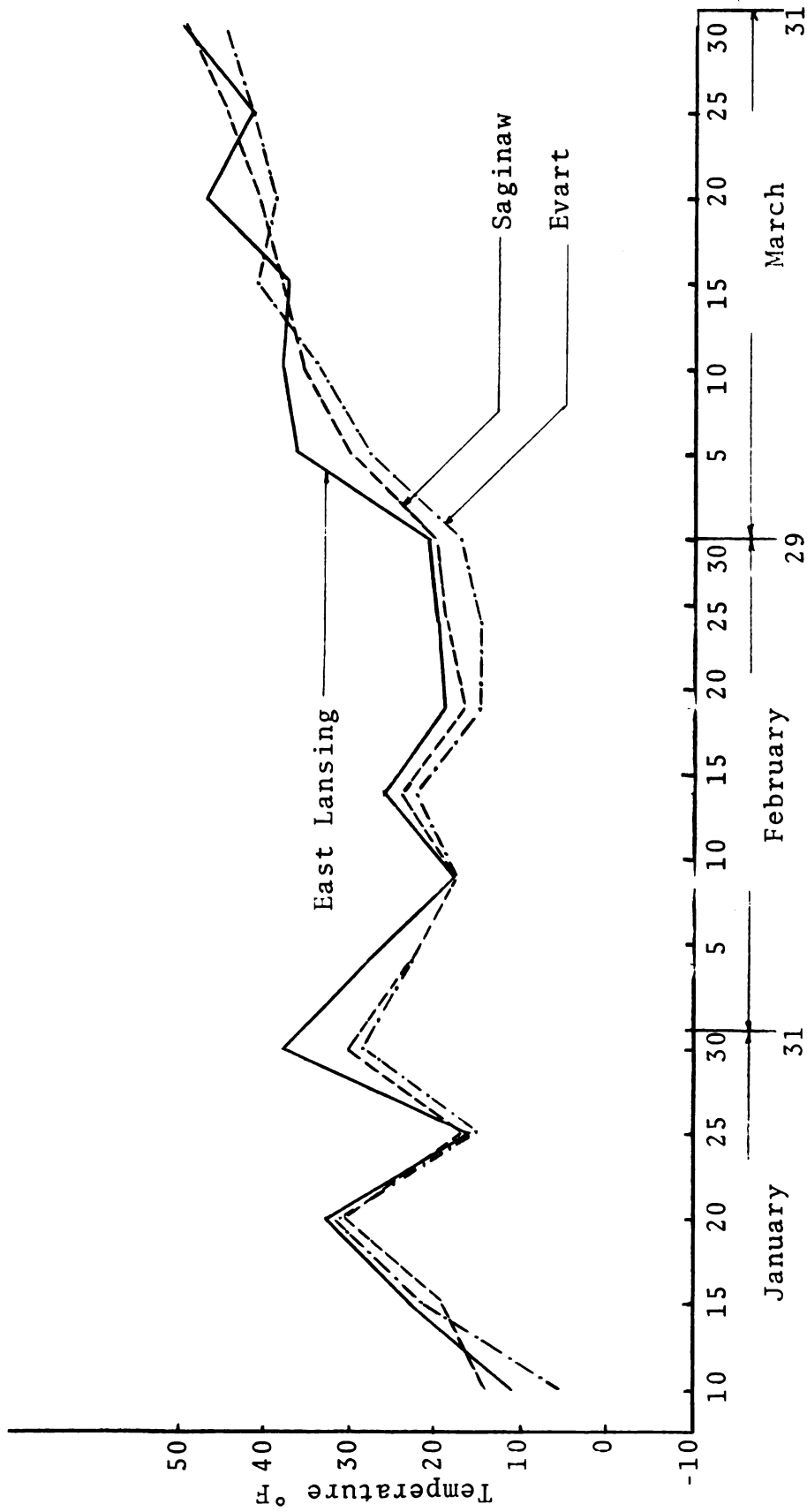


Figure 2-7. The average monthly air temperature for East Lansing, Saginaw, and Ewart in 1968. [Data taken from U.S. Weather Bureau climatological data, Michigan.]

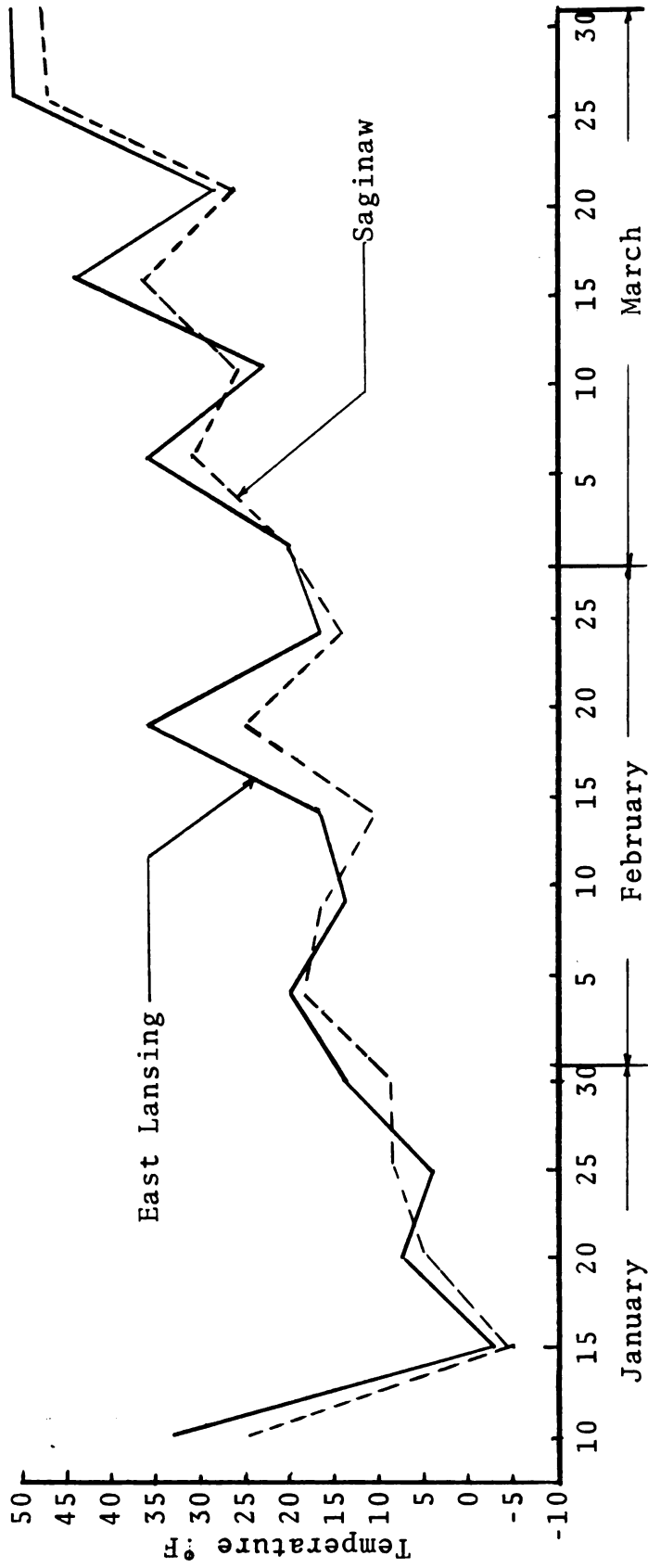


Figure 2-8. Air temperature of East Lansing and Saginaw for the year 1963.

graph is given for the air temperature of East Lansing and Saginaw for 1963. It can be seen from Figures 2-7 and 2-8 that air temperature of Saginaw has almost the same shape as for East Lansing, except for a few days in February 1963 (from 14-25), which were much cooler.

Accordingly, the reasoning discussed above was applied to calculate the soil temperature for Saginaw. To obtain the temperature fluctuations, the Fourier coefficients were calculated for each depth at which the temperature was recorded and available. Since the recorded daily soil temperature in East Lansing is not complete for 1963, the recorded soil temperature at depths 8, 20, 40 and 80 inches of 1968 were used to calculate the Fourier coefficients. For accuracy the first 73 coefficients for 8, 20, 40 and 80 inches were calculated and the results are given in Appendix I.

In the present study the temperature distribution for Saginaw for 1963 at 8, 20, 40 and 80 inches depths were estimated, based on average values of soil temperature as calculated above, and soil temperature fluctuations which are characteristic of Central Lower Michigan as determined from the East Lansing data.

The annual average air temperature at Saginaw in 1963 was 45.5 degrees F. The annual average of soil temperature at 8 inches was taken to be equal to 47.5 degrees F.

and at 20, 40 and 80 inches depths the calculated value of A_0 was found to be 47.0°F.

The calculated Fourier coefficients for 8, 20, 40 and 80 inches and the assumed annual average value of soil temperature estimated as above were used in the Fourier series model to estimate the daily temperatures for the above depths. The calculated temperatures for Saginaw are given in Figure 2-9 and the recorded temperatures for East Lansing in 1963 and 1968 at different depths are given in Figures 2-10 and 2-11.

By interpolation, the temperature for every inch of soil depth was estimated and used as the daily temperature data where such information was required as initial condition to the computer solution.

2.3.3.2 Thermal diffusivity

Since the temperature readings in East Lansing are obtained in fine sandy loam, a thermal diffusivity was chosen equal to $.006 \text{ cm}^2/\text{sec}$ and with interpolation $\alpha = .009 \text{ cm}^2/\text{sec}$ is used for frozen soil (see Table 1-5). In calculating the soil temperature for Saginaw, $\alpha = .009 \text{ cm}^2/\text{sec}$ was used assuming a frozen silt loam soil.

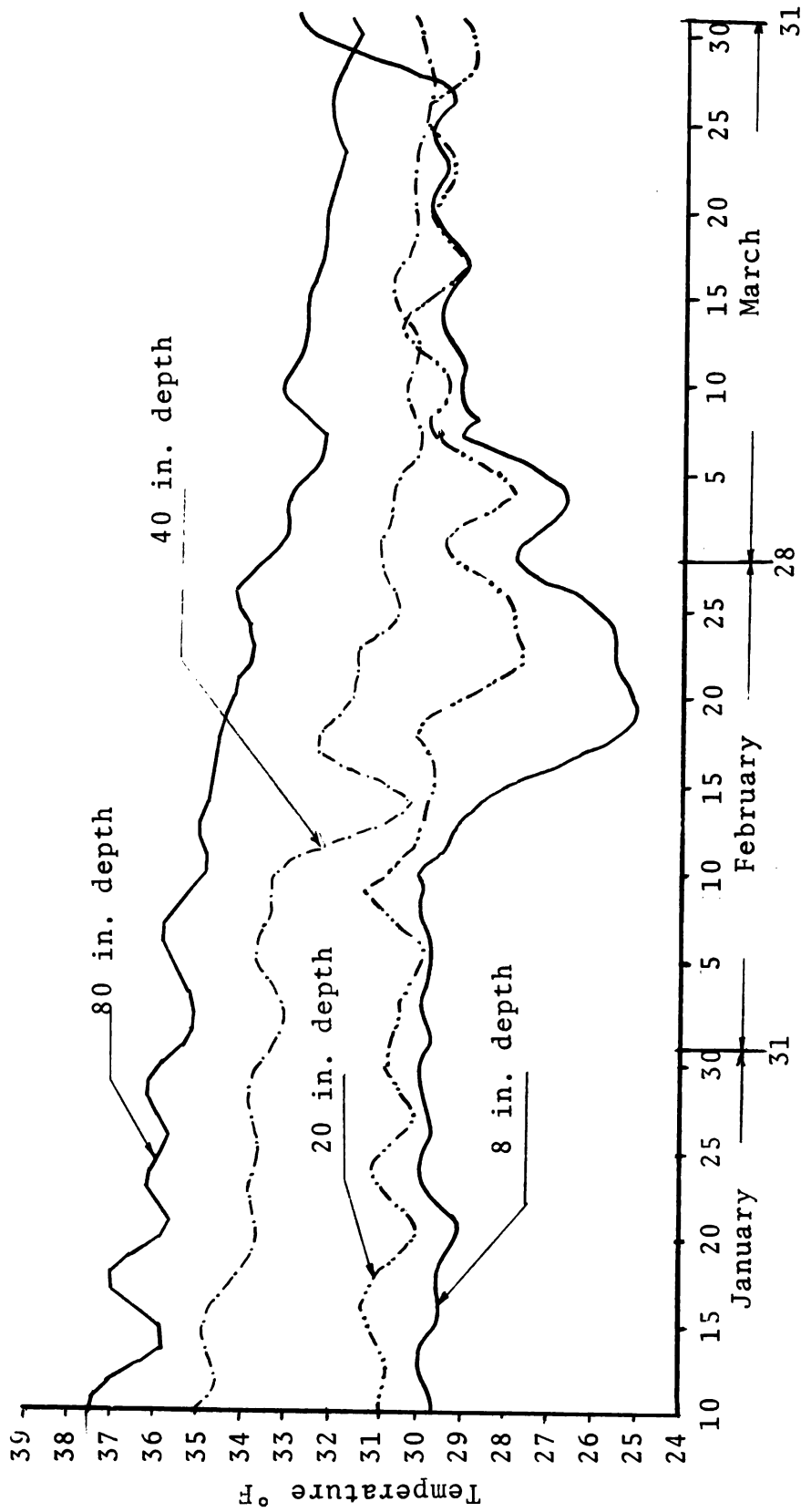


Figure 2-9. Calculated soil temperature for Saginaw, 1963.

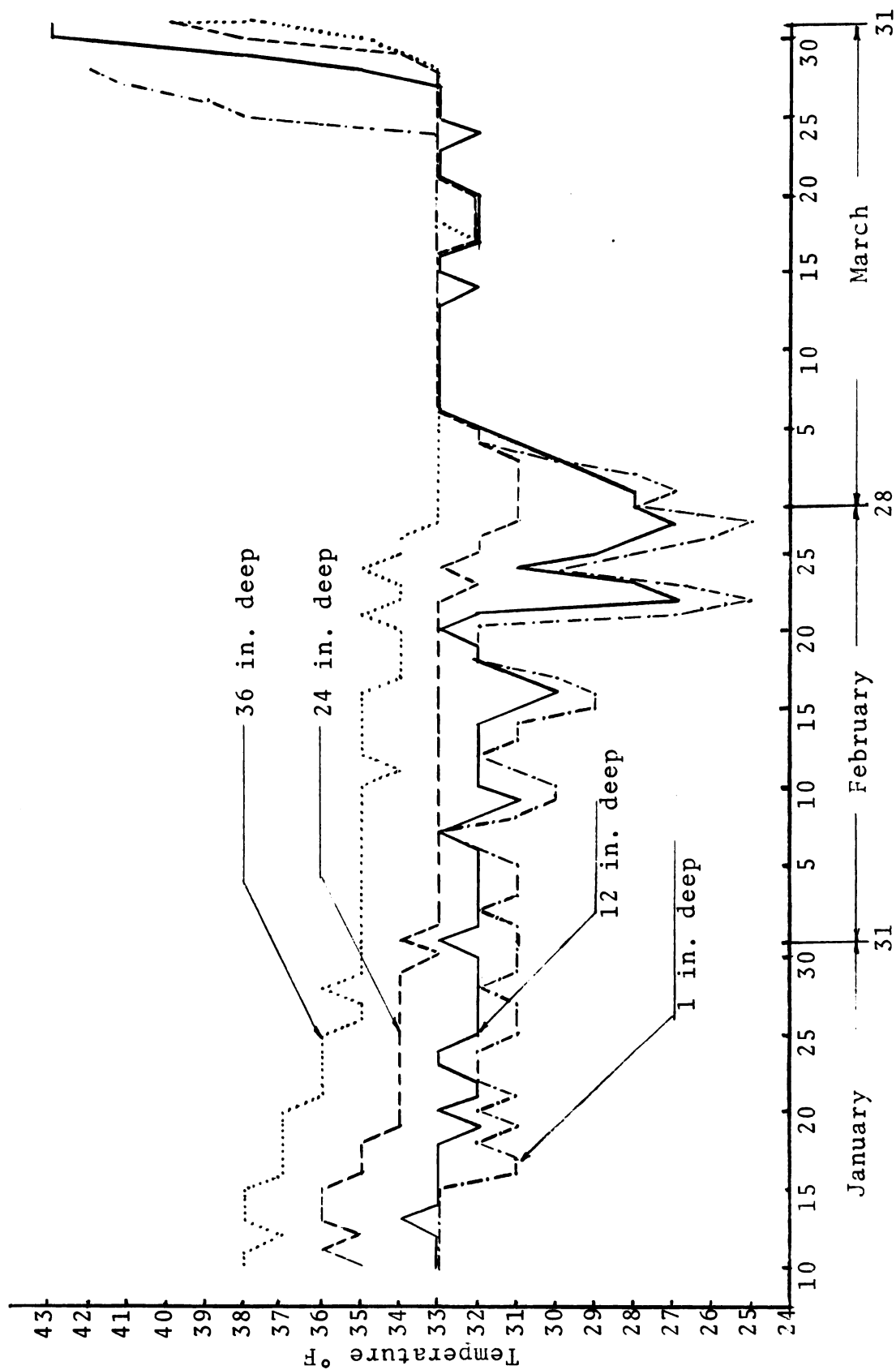


Figure 2-10. Recorded soil temperature for East Lansing, Horticulture Farm Station, 1963.

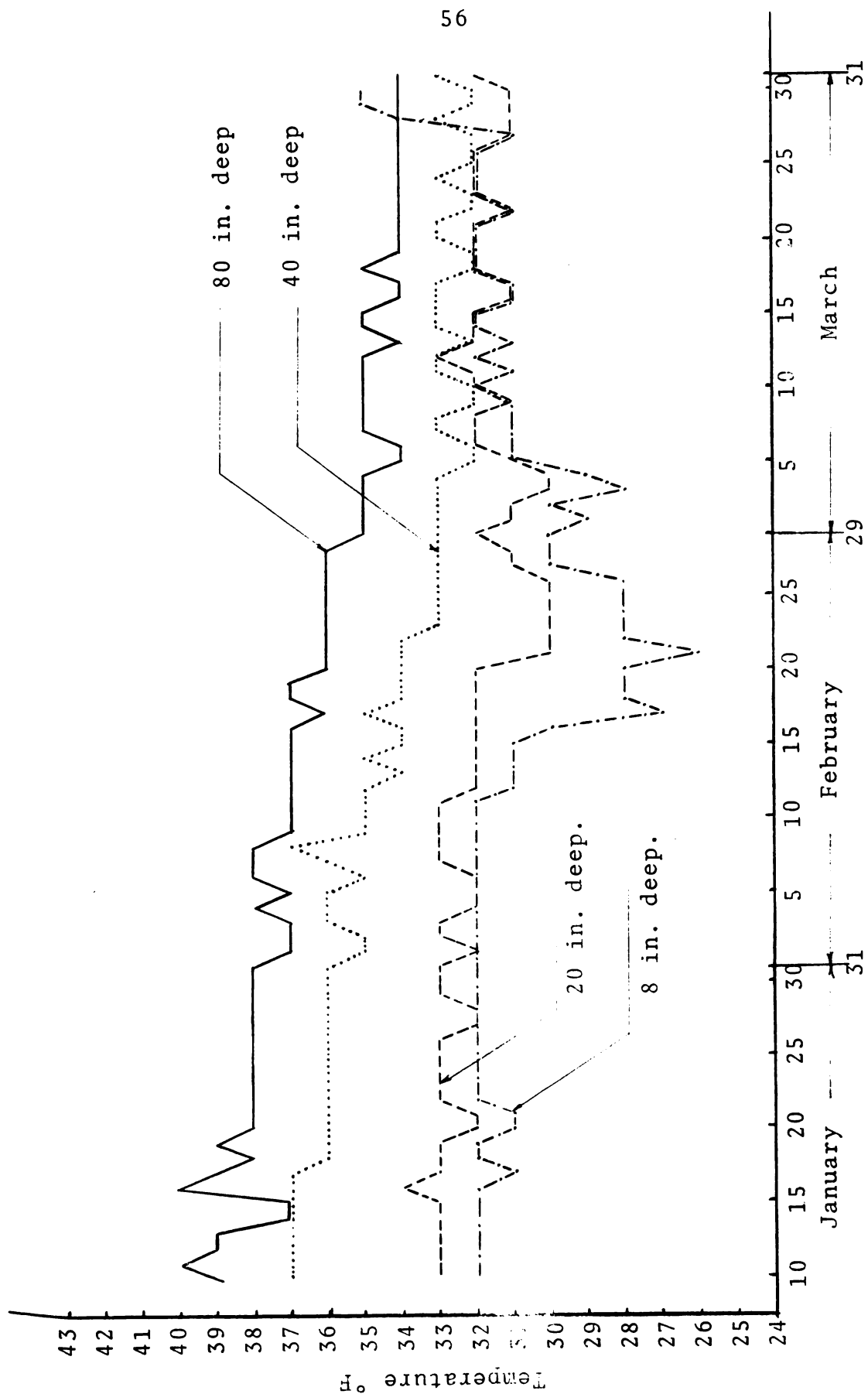


Figure 2-11. Recorded soil temperature for East Lansing, Horticulture Farm Station, 1968.

2.3.3.3 Depth and diameter of drain tile

The most often used drain tile in drainage systems are four, six, and eight inch tile. Depths vary depending upon the type of soil and drainage conditions. The most-recommended depth for drainage of Michigan soils is from 36 to 48 inches (Standards for Drainage of Michigan Soils, 1963). In the computer solution the tile boundaries are delineated by grid points as shown in Figure 2-12. Solutions obtained by applying the model using the boundary and initial conditions determined above were examined for four, six and eight inch tile for depth of 36 in., 42 in., and 48 in.

2.4 Solution of the Model

Based on the concepts advanced in preceeding sections, the temperature distribution around a tile and the cracking of a submerged tile due to freezing of the water inside the drain can be studied using the following approach.

The cross section of soil which is our problem space is covered with a grid as shown in Figure 2-4. The left boundary of this grid passes through the center of the tile. Since the solution is symmetrical about a vertical line passing through the center of the tile, only the right half of the solution need to be computed. The grid lines

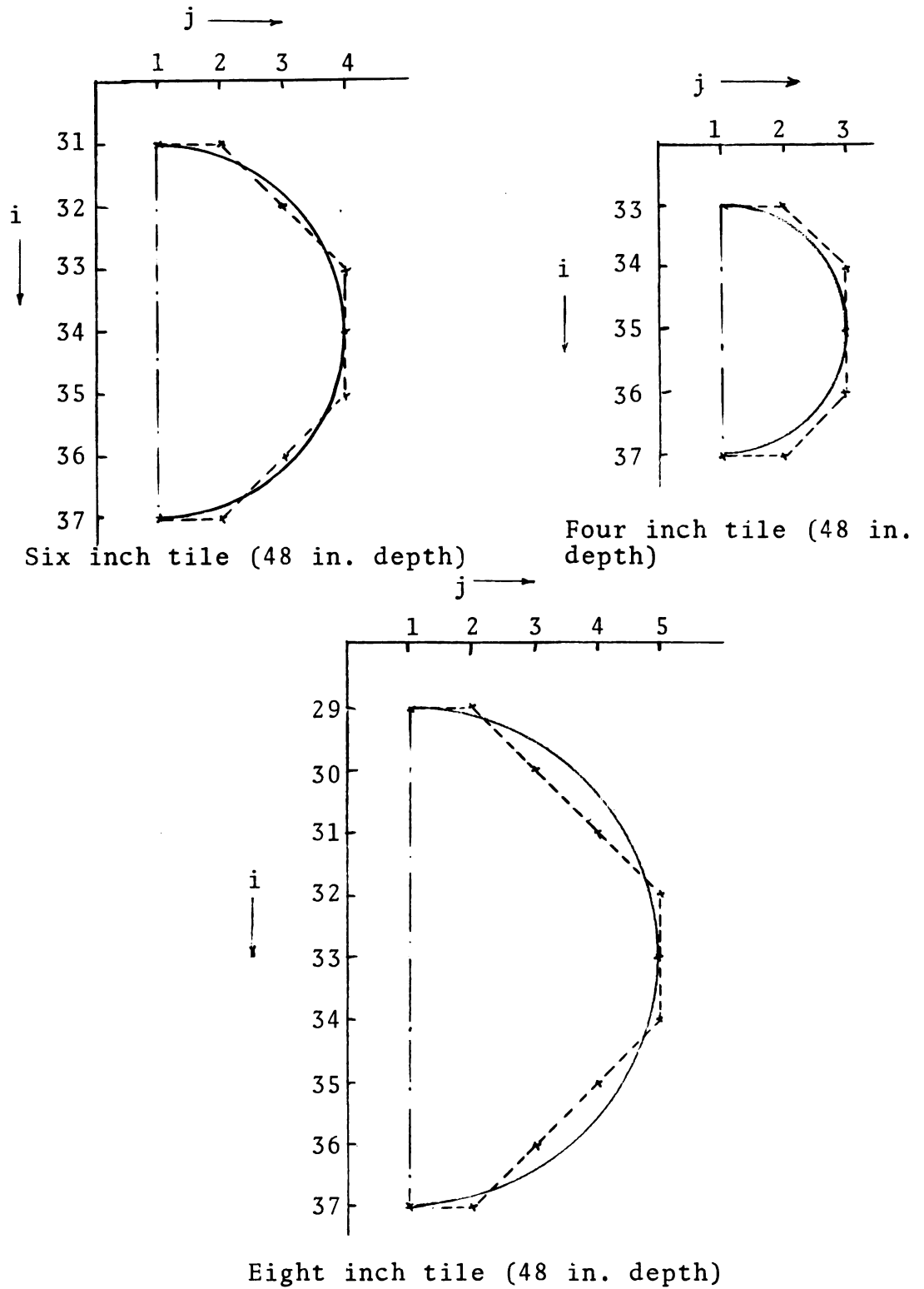


Figure 2-12. Approximation of drain tiles in grid system for computer model.

are spaced one inch apart and cover a square of 68-inch width and depth. Grid points corresponding to the tile boundary vary depending upon the diameter and depth of the tile.

2.4.1 Formula and method of solution

The general heat conduction equation is (see section 1.4):

$$\frac{\partial}{\partial x}(k_T \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k_T \frac{\partial T}{\partial y}) + \frac{\partial}{\partial z}(k_T \frac{\partial T}{\partial z}) = \rho c \frac{\partial T}{\partial t} \quad [2.1]$$

With the assumptions stated above, the equation can be written:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad [2.2]$$

where:

T = Temperature

z,x = Cartesian coordinates, z increasing downward,
and x increasing horizontally from the center
of the tile to the right

α = Thermal diffusivity of the soil

t = time

2.4.2 Numerical solution

The analytical solution of Equation [2.2] is complicated. A numerical approximation to the solution of

Equation [2.2] may be obtained by the step-wise solution of an associated difference equation. Three approaches are possible. They are:

a. The explicit difference method, which yields equations that are simple to solve, but require an uneconomically large number of time steps of limited size.

b. The implicit difference method, which yields equations that do not possess a singularity to limit the time step, but which require at each time step the solution by iteration of large sets of simultaneous equations.

c. The alternating-direction implicit method (Cranahan, 1969) has been presented by Peaceman and Rachford (1955), and Douglas (1955) and is used in this paper.

The alternating-direction implicit procedure requires the line-by-line solution of small sets of simultaneous equations that can be solved by a direct, non-iterative method.

Essentially, the principle is to employ two difference equations which are used in turn over successive time-steps, each of duration $\Delta t/2$. The first equation is implicit only in the x-direction and the second is implicit only in the y-direction. Let $T(i,j,n)$ represent $T(z,x,t)$ where $Z = i\Delta z$ and $X = j\Delta x$ (Figure 2-4). Thus if $\hat{T}_{i,j}^*$ is an intermediate value at the end of the first time-step (see Appendix II for detail), we have:

$$\begin{aligned}
-\bar{T}_{i-1,j}^* + 2\left(\frac{1}{\lambda} + 1\right) \bar{T}_{i,j}^* - \bar{T}_{i+1,j}^* &= T_{i,j-1,n} + 2\left(\frac{1}{\lambda} - 1\right) \\
&\quad T_{i,j,n} + T_{i,j+1,n} \quad [2.3]
\end{aligned}$$

followed by:

$$\begin{aligned}
-T_{i,j-1,n+1} + 2\left(\frac{1}{\lambda} + 1\right) T_{i,j,n+1} - T_{i,j+1,n+1} &= \\
\bar{T}_{i-1,j}^* + 2\left(\frac{1}{\lambda} - 1\right) \bar{T}_{i,j}^* + \bar{T}_{i+1,j}^* &\quad [2.4]
\end{aligned}$$

where:

$$\lambda = \frac{\alpha \Delta t}{\Delta x^2} = \frac{\alpha \Delta t}{\Delta z^2} \quad [2.5]$$

α = Thermal diffusivity;

Δt = Time increment;

Δx = Distance increment in the horizontal direction;

Δz = Depth increment in the vertical direction.

$$\text{Let } b = 2\left(\frac{1}{\lambda} + 1\right) \quad [2.6a]$$

$$\text{and } f = 2\left(\frac{1}{\lambda} - 1\right) \quad [2.6b]$$

Equations [2.3] and [2.4] can then be written in the form:

$$\begin{aligned}
-\bar{T}_{i-1,j}^* + b\bar{T}_{i,j}^* - \bar{T}_{i+1,j}^* &= T_{i,j-1,n} + fT_{i,j,n} \\
&\quad + T_{i,j+1,n} \quad [2.7]
\end{aligned}$$

and

$$-T_{i,j-1,n+1} + bT_{i,j,n+1} - T_{i,j+1,n+1} =$$

$$\bar{T}_{i-1,j} + f\bar{T}_{i,j} + \bar{T}_{i+1,j} \quad [2.8]$$

In general the equation [2.7] is solved for the intermediate values $\bar{T}$, which are then used in Equation [2.8], thus leading to the solution $T_{i,j,n+1}$ at the end of the whole time interval Δt . A tridiagonal coefficient matrix is used in the solution of Equations [2.7] and [2.8] (Carnahan, et al., 1969, pp. 441-446).

2.4.3 Applying the boundary and initial conditions in solution of related formulas

Based on the preceeding sections, the behavior of soil surrounding a submerged tile will be predicted by solving Equations [2.7] and [2.8]:

$$-\bar{T}_{i-1,j} + b\bar{T}_{i,j} - \bar{T}_{i+1,j} = T_{i,j-1,n} + fT_{i,j,n}$$

$$+ T_{i,j+1,n} \quad [2.7]$$

$$-T_{i,j-1,n+1} + bT_{i,j,n+1} - T_{i,j+1,n+1} =$$

$$\bar{T}_{i-1,j}^* + f\bar{T}_{i,j}^* + \bar{T}_{i+1,j}^* \quad [2.8]$$

where T and $\bar{T}$ refer to temperature at the beginning and end of half time step $\Delta t/2$; and with the following boundary and initial conditions:

Time	Temperature	Condition
at $t=0$ and at multiples of 24 hours	$T =$ known throughout the soil mass	
	$T =$ known	$z = -1 \quad 0 < x \leq m$
at $t > 0$	$T =$ known	$z = m \quad 0 < x \leq m$
	$T =$ constant	$x = m \quad 0 > z \geq m$
	$\frac{\partial T}{\partial x} = 0$ along the side $x=0$	$0 \leq z \leq m$

The temperature of water in the tile will be held constant at 32°F., for $t > 0$, until a grid point one inch below the tile line becomes 32°F.

Equation [2.7] was applied to each point $i = 1, 2, \dots, m-1$ in the j column and the following tridiagonal system for the j column was obtained:

$$\begin{array}{rcl}
 i = 0 & b\bar{T}_{0,j}^* - \bar{T}_{1,j}^* & = d_0 \\
 i = 1 & -\bar{T}_{0,j}^* + b\bar{T}_{1,j}^* - \bar{T}_{2,j}^* & = d_1 \\
 i = 2 & -\bar{T}_{1,j}^* + b\bar{T}_{2,j}^* - \bar{T}_{3,j}^* & = d_2 \\
 & \text{-----} & \text{-----} \\
 i = i & -\bar{T}_{i-1,j}^* + b\bar{T}_{i,j}^* - \bar{T}_{i+1,j}^* & = d_i \\
 & \text{-----} & \text{-----} \\
 i = m-2 & -\bar{T}_{m-3,j}^* + b\bar{T}_{m-2,j}^* - \bar{T}_{m-1,j}^* & = d_{m-2} \\
 i = m-1 & -\bar{T}_{m-2,j}^* + b\bar{T}_{m-1,j}^* & = d_{m-1}
 \end{array} \quad \left. \vphantom{\begin{array}{rcl} i = 0 \\ i = 1 \\ i = 2 \\ i = i \\ i = m-2 \\ i = m-1 \end{array}} \right\} [2.7a]$$

with

$$\begin{array}{rcl}
 d_0 = 2T_{0,1} + fT_{0,0} + G & \text{for } i = 0 & \\
 d_i = 2T_{i,1} + fT_{i,0} & \text{for } i = 1, 2, \dots, m-1 & \\
 \left. \vphantom{\begin{array}{rcl} d_0 = 2T_{0,1} + fT_{0,0} + G \\ d_i = 2T_{i,1} + fT_{i,0} \end{array}} \right\} & & \text{for } j = 0 \\
 d_0 = T_{0,j-1} + fT_{0,j} + T_{0,j+1} + G & \text{for } i = 0 & \\
 d_i = T_{i,j-1} + fT_{i,j} + T_{i,j+1} & \text{for } i = 1, 2, 3, \dots, m-1 & \\
 \left. \vphantom{\begin{array}{rcl} d_0 = T_{0,j-1} + fT_{0,j} + T_{0,j+1} + G \\ d_i = T_{i,j-1} + fT_{i,j} + T_{i,j+1} \end{array}} \right\} & & \text{for } j = 1, 2, \dots, m-1 \\
 & & \\
 d_0 = 2T_{0,m-1} + fT_{0,m} + G & \text{for } i = 0 & \\
 d_i = 2T_{i,m-1} + fT_{i,m} & \text{for } i = 1, 2, \dots, m-1 & \\
 \left. \vphantom{\begin{array}{rcl} d_0 = 2T_{0,m-1} + fT_{0,m} + G \\ d_i = 2T_{i,m-1} + fT_{i,m} \end{array}} \right\} & & \text{for } j = m
 \end{array} \quad \left. \vphantom{\begin{array}{rcl} d_0 = 2T_{0,1} + fT_{0,0} + G \\ d_i = 2T_{i,1} + fT_{i,0} \\ d_0 = T_{0,j-1} + fT_{0,j} + T_{0,j+1} + G \\ d_i = T_{i,j-1} + fT_{i,j} + T_{i,j+1} \\ d_0 = 2T_{0,m-1} + fT_{0,m} + G \\ d_i = 2T_{i,m-1} + fT_{i,m} \end{array}} \right\} [2.7b]$$

Comparing the coefficients of Equation [2.7a] with coefficients of the tridiagonal matrix (see Carnahan, et al., 1969, pp. 441-446) it can be seen that $a = -1$, $b = b$, $c = -1$ except for the first equation where $a = 0$, and for the last equation where $c = 0$.

This procedure is valid except for points that have a constant temperature (for example water in the tile). At such locations the tridiagonal matrix must be modified. This has been done through consultation with Dr. J. V. Beck, Associate Professor of Mechanical Engineering, Michigan State University, as follows.

When one of the unknowns in the tridiagonal system becomes known and constant, the value of the b coefficient is given a magnitude of unity and the other coefficients are taken equal to zero in the equation. The known value was substituted instead of the value d (value of the right hand side of Equation [2.7a]).

The other coefficients in the column of the tridiagonal matrix related to this point will be zero. For detail see sample computer program in Appendix II.

The procedure is repeated for successive columns $j = 0, 1, \dots, n-1$ until all the $\hat{T}_{i,j}^*$ are found at the end of the first half time-step. The temperatures at the end of the second half time step are found similarly, by applying Equation [2.8] with related boundary and initial conditions, to each point in a row ($j = 0, 1, \dots, n-1$), for successive rows ($i = 0, 1, \dots, n-1$).

CHAPTER III

3.1 Results

The results of this research have been outlined in three headings:

3.1.1 Results of Experiments

An experiment was conducted in order to determine the realistic condition for solving the mathematical model of freezing water inside a drain tile. The results of this experiment can be summarized as follows:

1. In the process of cooling the water inside the drain tile there was a gradient between the water and the ambient temperature and it was greatest between the water and the upper surface of the tile.
2. Water started to freeze when the temperature throughout the water inside the tile reached 32° F., and the temperature surrounding the tile was less than 32° F.
3. The conversion of only a portion of the water to ice caused the cracking of the tile.

4. The location of the crack was at the water-ice interface on the interior tile surface in the lower third of the tile (see Figure 2-3).

3.1.2 Results of Soil Temperature Study

1. The annual average soil temperature is not equal throughout the soil profile; it varies with depth.
2. Annual average soil temperature rapidly decreases with increasing depth to a certain depth. Beyond this depth gradually its value decreases. At any depth the yearly average soil temperature is greater than annual average air temperature for a given location.
3. The highest annual average temperature has been observed at one inch above the ground of given location.
4. Annual average temperature of bare soil at certain depth is greater than sod-covered soil (see Figure 3-1) and the frost penetration is deeper in bare soil than sod covered soil.
5. In the winter the bare soil is much colder than sod or grass covered soil. In the summer the bare soil is warmer than soil under sod. This fact is illustrated in Tables 3-1, 3-2.

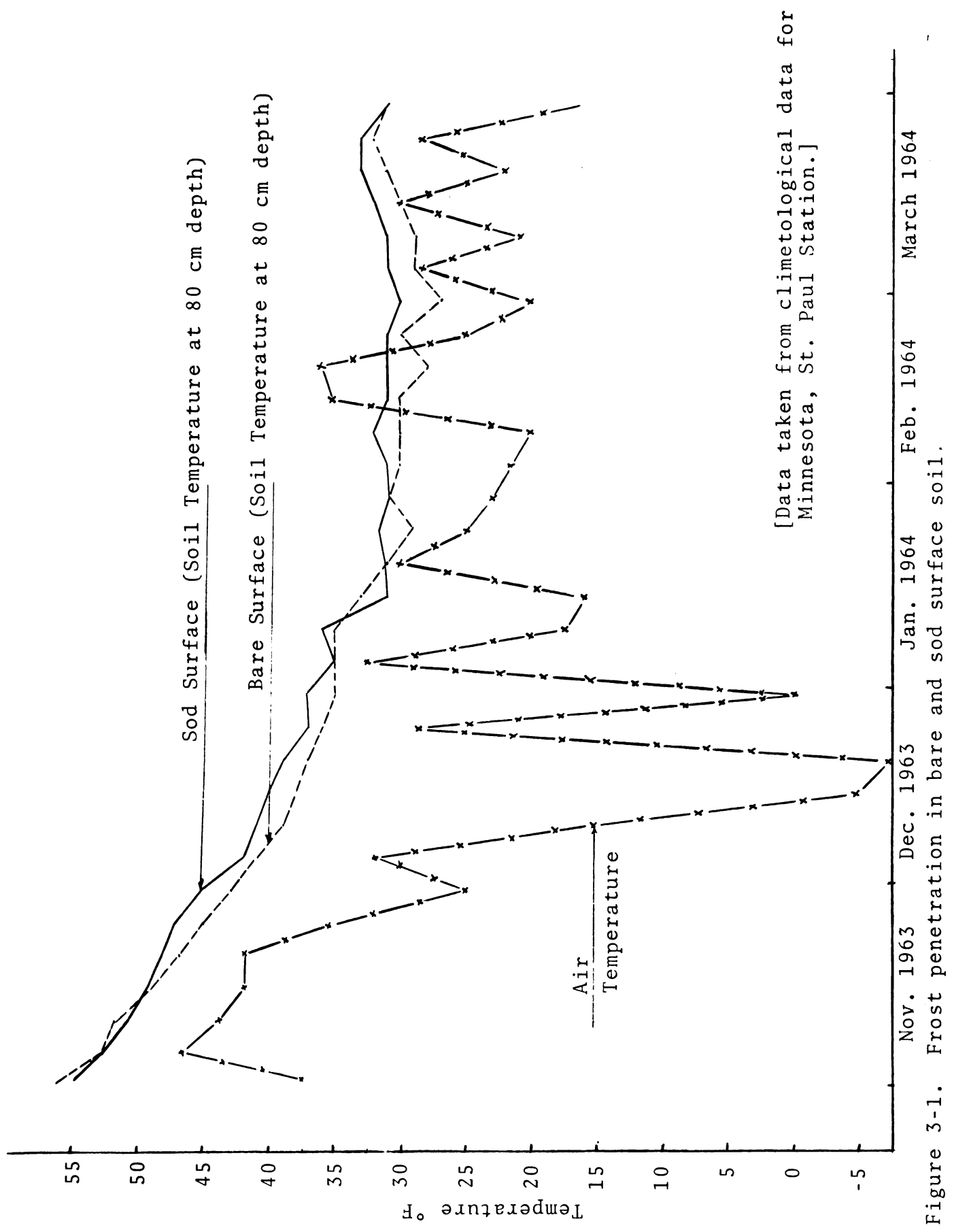


Figure 3-1. Frost penetration in bare and sod surface soil.

Table 3-1. Effect of soil cover on soil temperature at different depths in the cold season.

Average air Temp.		January			February		
		6.2°F			14.8°		
Average Soil temp. at		under sod	under bare soil	under soybean	under sod	under bare soil	under soybean
1	centimeter	24.5	9.9	18.2	22.0	19.7	19.8
5	centimeters	24.9	11.2	18.7	21.9	20.5	19.4
10	"	25.7	12.0	19.4	21.9	19.7	19.5
20	"	26.8	12.2	20.6	24.1	18.0	19.5
40	"	30.4	14.5	23.9	27.5	17.3	20.4
80	"	34.2	22.5	30.7	32.8	20.1	24.4

Source: Data are taken from Climatological Data, University of Minnesota, St. Paul, 1963.

Table 3-2. Effect of soil cover on soil temperature at different depths in the warm season.

Average air Temp.		June			July		
		70.4°F			73.5		
Average Soil temp. at		under sod	under bare soil	under soybean	under sod	under bare soil	under soybean
1	centimeter	73.5	89.	89.1	80.6	94.6	84.8
5	centimeters	72.5	87.1	82.7	79.3	93.1	79.9
10	"	70.5	81.5	79.4	76.8	87.8	78.6
20	"	66.5	74.0	73.3	72.6	82.0	75.8
40	"	62.3	68.5	66.2	68.2	76.9	71.8
80	"	57.5	61.8	59.8	63.6	71.5	67.0

Source: Data are taken from Climatological Data, University of Minnesota, St. Paul, 1963.

3.1.3 Results of Computer Modeling

In order to investigate the effect of thermal environment on submerged drain tile a computer model was developed. The calculated soil temperature for Saginaw, Michigan for 1963 was used in the model, and the boundary and initial conditions were based on the experimental results.

The results of the computer program modeling soil temperature around four, six and eight inch submerged drain tile buried at 36, 42 and 48 inch depths are given in Figures 3-2, 3-3, and 3-4.

Figure 3-2 represents the temperature history of the soil around four inch drain tile. It can be observed that on February 9, the soil temperature at the 31 inches depth was 32° F. and at 37 inches depth the temperature was 32.4° F. On this day for the purposes of the model, water was assumed to have entered the drain tile at a constant temperature equal to 32° F. The effect of water within the four-inch drain tile on soil temperature has been shown in Figure 3-5. The model results indicate that the presence of water inside the drain tile inhibited rapid cooling of the soil around the tile, and 7 days were required before soil one inch below the drain tile reached 32° F. or less. On this day the soil one foot from the drain tile at the 37 inch depth had a temperature equal to 31.54° F. Since the ambient soil temperature was less than 32° F. it is

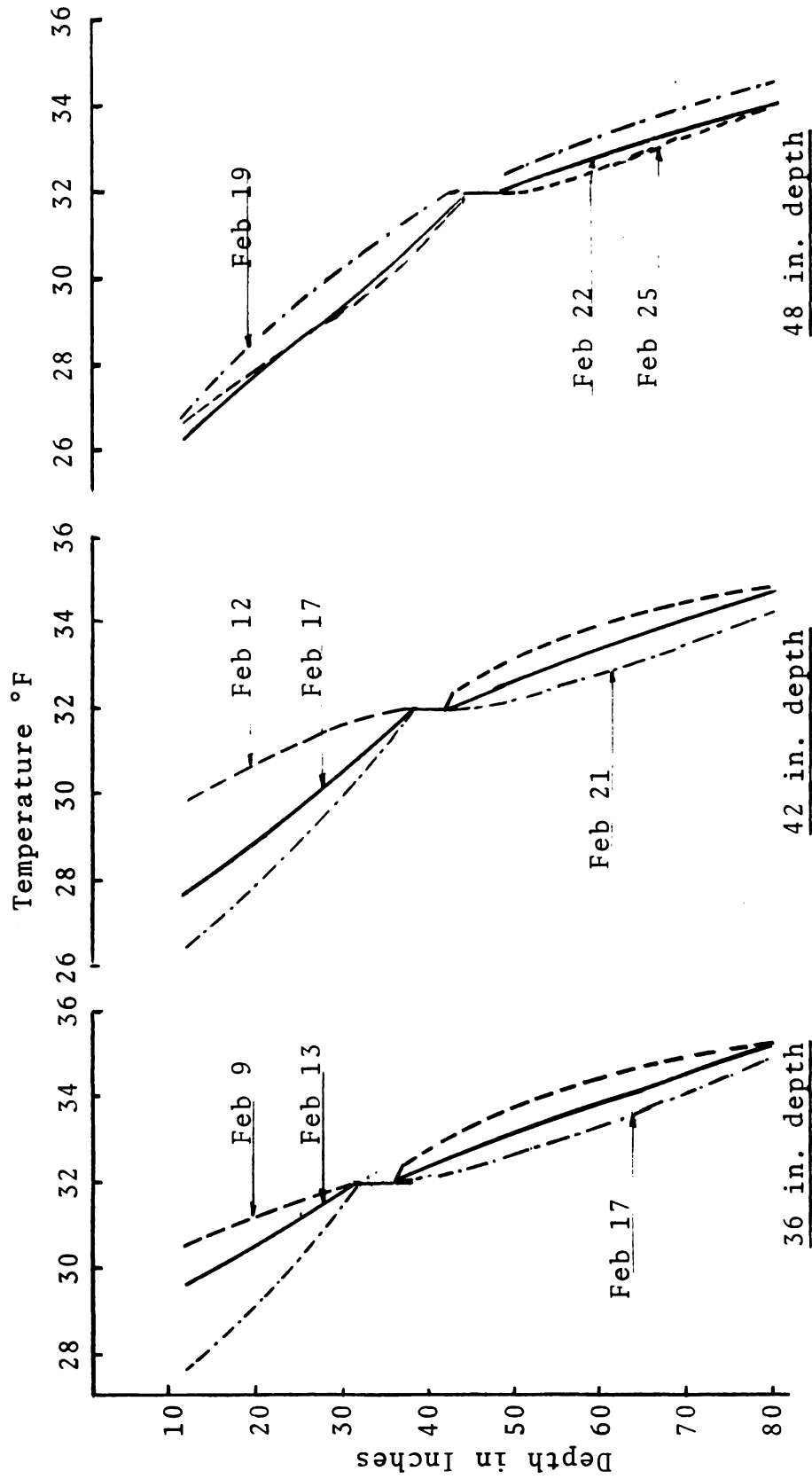


Figure 3-2. Temperature history in the soil around a four inch submerged drain tile burried at 36, 42, and 48 inch depths.

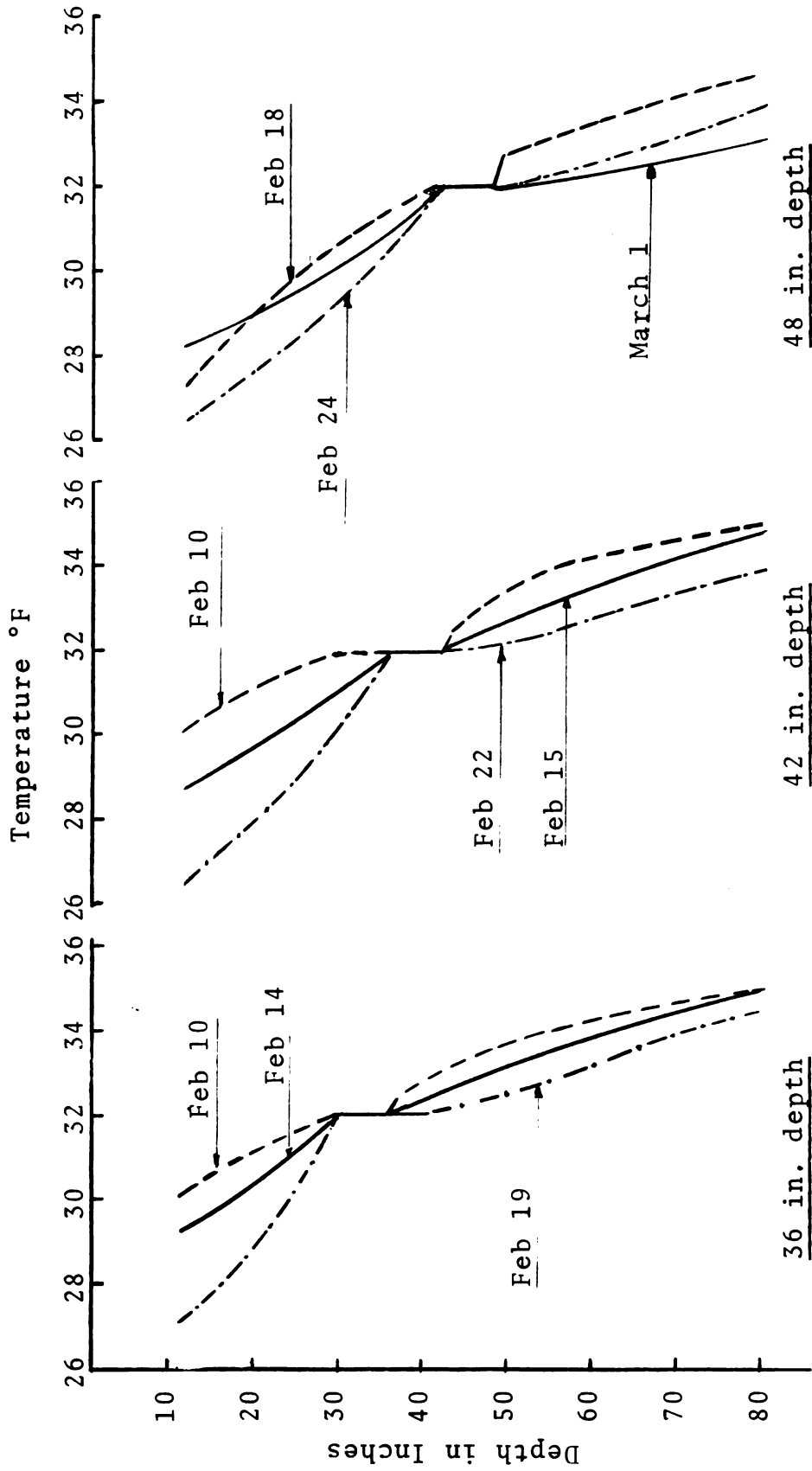


Figure 3-3. Temperature history in the soil around 6 inch drain tile buried at 36, 42, and 48 inch depths.

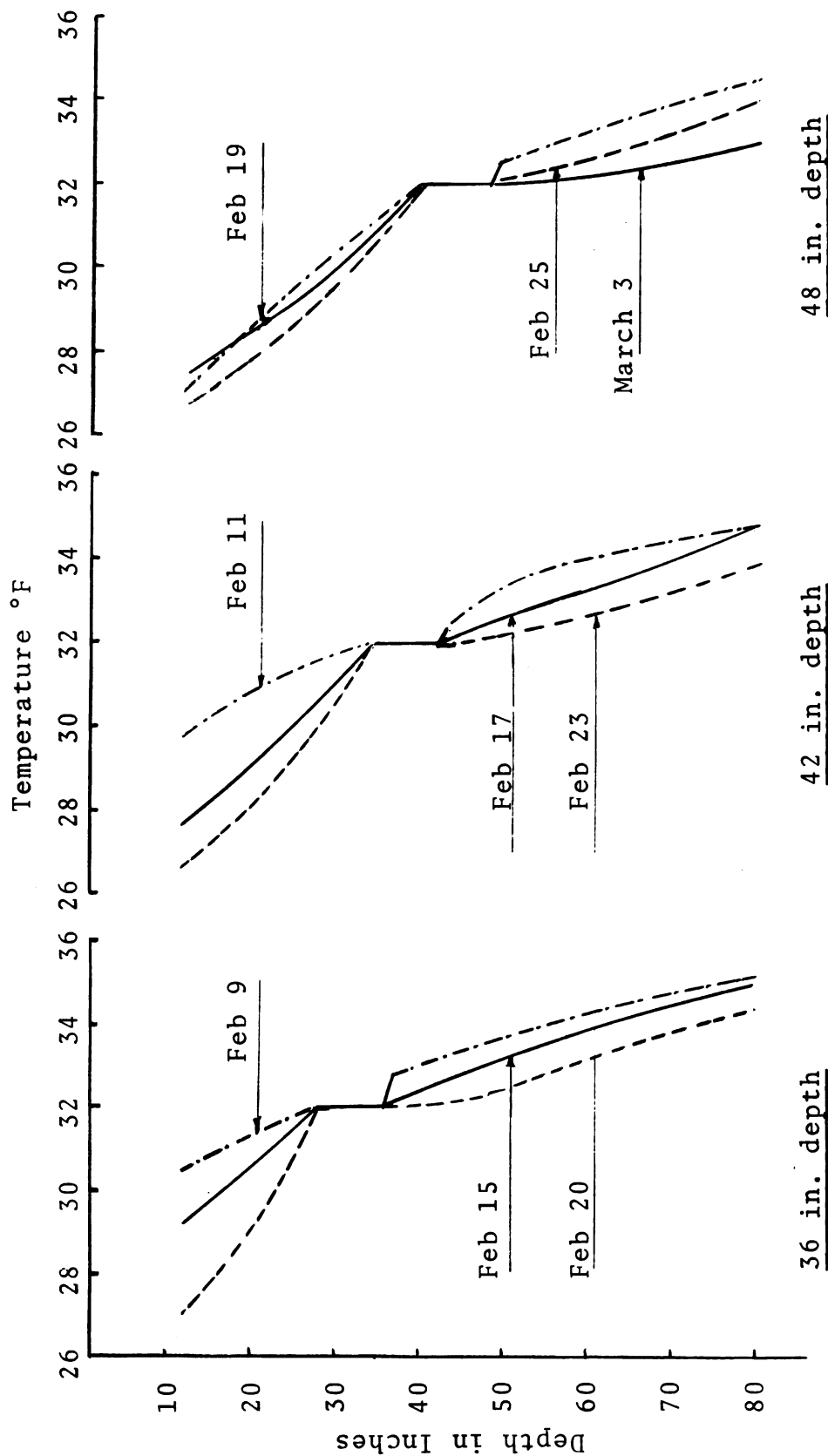


Figure 3-4. Temperature history in the soil around eight inch submerged drain tile buried at 36, 42, and 48 inch depths.

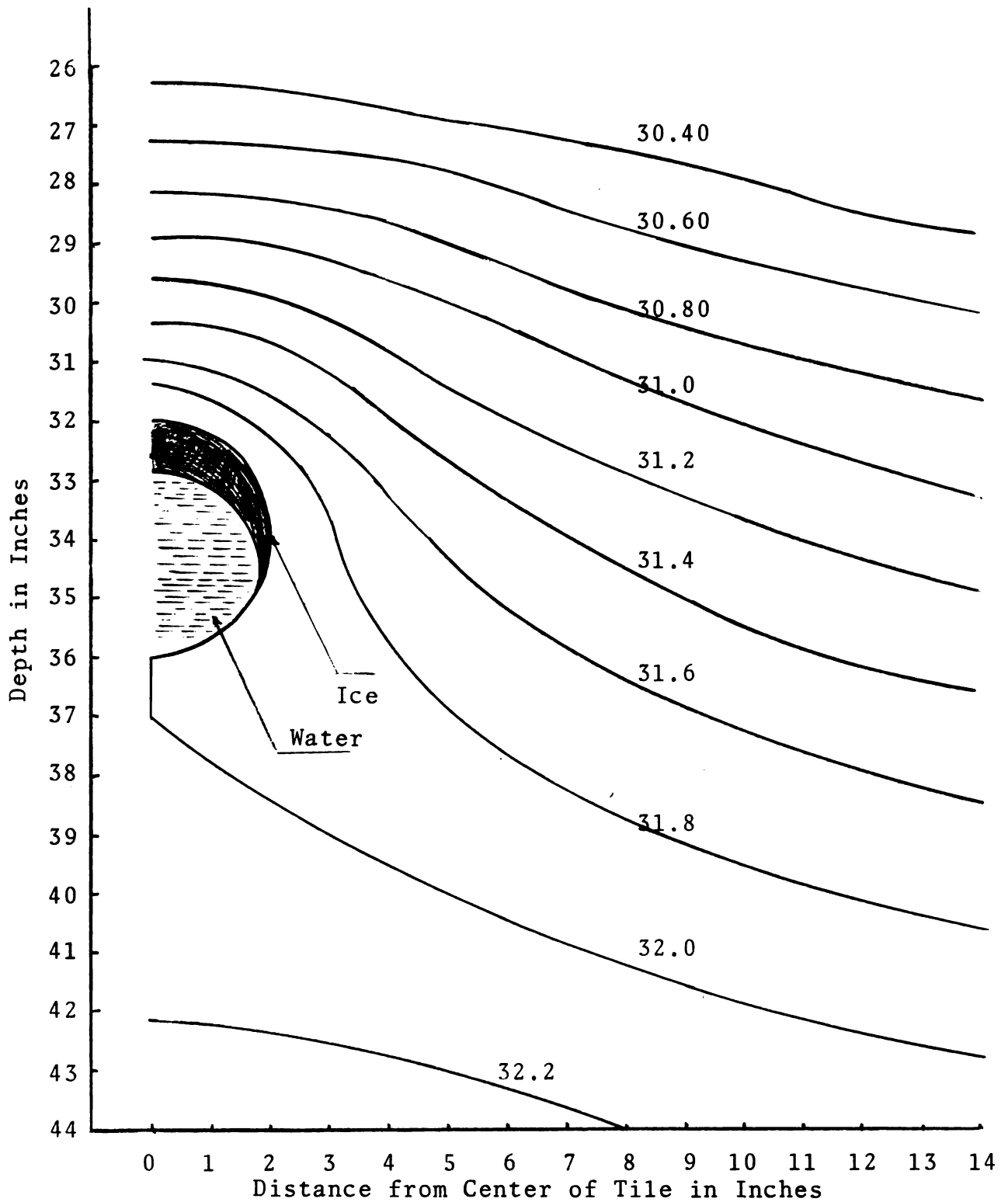


Figure 3-5. The effect of water within the four-inch drain tile on soil temperature. (Isothermal lines are shown in the figure.)

felt, in accordance with the experimental evidence, that the water inside the drain tile would begin to freeze. At a point one inch below the four inch drain tile buried at 42 inch and 48 inch depths the temperature reached 32° F. or less on February 21 and on February 25, respectively.

One inch above the six inch drain tile the temperature was 32° F. on February 10, while one inch below the tile the temperature was 32.22 degrees F. and on February 19 (after 9 days) one inch below the six inch tile the temperature attained a value of 32° F. or less. On February 22, and March 1, the temperature one inch below the six inch tile buried at 42 and 48 inch depths reached 32° F. or less, respectively.

Similar observations were noted for the eight inch drain tile buried at 36, 42, and 48 inch depths except more time was required to cool the eight inch tile. February 20 and February 23 and March 3 were the dates at which the temperature one inch below the drain tile became 32° F. or less respectively. Other results are summarized in Table 3-3 and can be stated as follows:

1. The tile which had been buried deeper, froze later in the season than shallow-buried tile.
2. Tile which had less diameter froze faster than big diameter tile.

Table 3-3. Time and required duration of submergence for cracking to take place in clay drain tile.

Diameter of Drain Tile (Inches)	Description*	36	Tile Depth (Inches) 42	48
4	A	2/10	2/11	2/19
	B	2/17	2/21	2/25
	C	7 days	10	6
6	A	2/10	2/11	2/19
	B	2/19	2/22	3/1
	C	9 days	11	10
8	A	2/10	2/11	2/19
	B	2/20	2/23	3/3
	C	10 days	12	12

*Description:

- A) Date at which soil above tile reached 32° F.
- B) Date at which ice was assumed to have formed in tile.
- C) Required duration of submerge for cracking to take place under the assumed conditions.

3.2 Discussion of Results

The results of this research give a useful tool for determining the behavior of water inside submerged drain tile as it may relate to the cracking of clay tile. This research investigates the variation of soil temperature and its relation with air temperature as a means of determining the soil temperature at every depth. As indicated previously, the results are conservative, because:

1. The recorded soil temperatures for Saginaw were not available, therefore the soil temperature for Saginaw for 1963, the coldest year on record, had to be calculated based on assumptions relating soil temperature and air temperature, and the general nature of soil temperature fluctuations.
2. The latent heat of fusion of water and variation of thermal diffusivity of frozen and unfrozen soil and also ice and water were not taken into account due to the complexity of the required model.

However, it is believed the results of the present study are logical and in agreement with field observations. Figure 3-6 is a photograph of an 8 inch clay tile removed from a field located near Saginaw. The tile had been buried at 48 inches. Note that the position of the

cracks are in agreement with the general location of cracks noted in the experimental investigation. Also, the fact that the model suggests that freezing can occur in such tile buried at 48 inches gives support to the hypothesis that cracking of this tile resulted from freezing of water since our model indicates that for 1963 at Saginaw, Michigan, especially for bare soil, the soil temperature up to 48 inch depth had reached 32° F. or less, and cracking due to freezing could result.



Figure 3-6. A photograph of an 8 inch clay tile removed from a field located near Saginaw, Michigan. Compare the location of cracks with that shown in Figure 2-3.

CHAPTER IV

CONCLUSIONS

The following conclusions are based on the investigation conducted on soil temperature and effect of thermal environment on submerged drainage conduits.

1. The annual average soil temperature is not equal throughout the soil profile; it varies with depth. Annual average soil temperature is always greater than annual average air temperature.
2. When the temperature of soil surrounding a submerged drain tile is reduced to less than 32° F. for some period of time, the water inside the drain tile will freeze and as a result of the phenomena when about 1/3 of the water has been changed to ice the clay drain tile will crack. The primary crack will occur at the water-ice interface on the interior tile surface, which usually is located in the lower half of the tile.

3. The tile which has been buried deeper will freeze later in the season than shallow-buried tile.
4. Tile of less diameter will freeze faster than big diameter tile.
5. The time which is required to freeze the water inside the drain tile in order to crack the clay tile depends upon the temperature gradient between water and ambient. When the temperature gradient increases the water will freeze faster than with a low temperature gradient.

CHAPTER V

RECOMMENDATIONS FOR FURTHER STUDIES

1. This study is the first step in evaluating the effect of environment temperature and frost penetration on submerged drain tile. Further research is needed in order to utilize the result of this study in developing a complete model for predicting soil temperature around drain tile.

2. Soil temperature reading should be extended for more locations in every state under different soil cover, such as bare soil, grass covered soil, and sod covered soil.

3. The process of data recording should be accompanied by the measurement of the solar radiation, which is a governing factor in increasing the soil surface temperature. Solar radiation data are required in studying the relationship between surface and air temperature.

4. The soil temperature measurement should be accompanied by the measurement of the moisture content of soil. Variation of soil moisture is required in the estimation of thermal properties of soil as well as the study of the migration of moisture in the soil.

5. Investigation should be continued to complete the mathematical model in order to determine the soil temperature at every depth with respect to soil surface temperature or air temperature. This study should include effect of the latent heat of fusion, and the variation of thermal diffusivity in moving media (i.e., frozen and unfrozen soil as well as water and ice) in determining the soil temperature, and water inside the drain tile in different seasons.

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APPENDICES

APPENDIX I

FOURIER SERIES MODELING OF PERIODIC PHENOMENA

APPENDIX I

FOURIER SERIES MODELING
OF PERIODIC PHENOMENA

In engineering analysis and design it is often necessary to work with environmental parameters which change with time such as the temperature of the earth's surface or the relative humidity within a plant canopy. The parameters may vary continuously and the function which describes the process, i.e., the mathematical model, may not be known. As a first step in the description of the process one has instead a sequence of data obtained over some interval of time, for example, the series of average daily temperatures for each day of the year. In many cases, much information could be obtained about a process from a continuous mathematical model. (Merva 1970).

For most environmental parameters (temperature, relative humidity, partial pressure of CO_2 , etc.) directly related to the plant environment, there exists a considerable amount of periodicity. The presence of periodicity suggests that some type of periodic function could be constructed to fit the observed data. Such a function would be useful in that it would serve to model the behavior of these parameters. Fourier Series will furnish such a function.

A Fourier series approximation of this function can be written:

$$f(t) = A_0 + \sum_{n=1}^k a_n \cos \omega_n t + \sum_{n=1}^k b_n \sin \omega_n t \quad (I-1)$$

where $f(t) = T$ = temperature at some point of interest;
 A_0 , a_n and b_n are all constant scalar coefficients.

It is necessary to determine the coefficients A_0 , a_n and b_n and the arguments $\omega_n t$ up to a value of k such that the function is sufficiently accurate. To determine the arguments $\omega_n t$ it is necessary only that as t varies from zero to τ where τ is the interval of time over which the model is desired, the argument must vary from zero to some integer multiple of 2π . The necessary conditions are fulfilled if $\omega_n = \frac{2n\pi}{\tau}$.

Equation (I-1) for a function being modeled over the interval τ can be written:

$$f(t) = A_0 + \sum_{n=1}^k \left(a_n \cos \frac{2n\pi}{\tau} t + b_n \sin \frac{2n\pi}{\tau} t \right) \quad (I-2)$$

A_0 , a_n and b_n can be obtained from

$$A_0 = \frac{1}{\tau} \int_0^{\tau} f(t) dt \quad (I-3)$$

$$a_n = \frac{2}{\tau} \int_0^{\tau} f(t) \cos \frac{2n\pi}{\tau} t dt \quad (I-4)$$

$$b_n = \frac{2}{\tau} \int_0^{\tau} f(t) \sin \frac{2n\pi}{\tau} t dt \quad (I-5)$$

The function $f(t)$ can now be approximated to the desired accuracy by performing the integrations indicated in equations (I-3), (I-4), and (I-5). In most cases only

A_0 , a_1 , a_2 and b_1 and b_2 must be found to obtain a reasonable approximation to the function $f(t)$. Determining more coefficients will usually give a better representation of $f(t)$ although not necessarily a better model for the fluctuations of the parameter being investigated. The accuracy will depend on what one uses for $f(t)$ in the integration.

The first 73 Fourier coefficients for 8, 20, 40, 80 inches depth of soil have been computed by equations I-3, I-4, and I-5 and the results are given in Tables (I-1), (I-2), (I-3), and (I-4). (See section 2.3.3.1).

TABLE I-1

THE CALCULATED FOURIER COEFFICIENTS FOR 8 INCHES DEPTHS OF SOIL

NO	A COEFF.	B COEFF.	NO	A COEFF.	B COEFF.
1	-18.552	-11.084	38	-.041	.004
2	.451	1.179	39	.074	.296
3	1.018	.759	40	.134	.086
4	.163	1.335	41	-.000	-.091
5	-.510	.278	42	.190	-.130
6	-.239	-.932	43	-.132	.101
7	.169	.224	44	.069	.240
8	-.767	.464	45	.257	-.420
9	.379	-.507	46	-.151	-.046
10	-.248	.420	47	-.479	.408
11	1.626	-.266	48	.568	.016
12	-.972	.299	49	.147	-.169
13	-.316	-.113	50	.061	-.001
14	.519	.375	51	-.148	-.104
15	-.098	-.418	52	.027	.022
16	-.024	-.460	53	-.017	-.094
17	-.397	.275	54	-.116	-.159
18	.013	-.004	55	-.128	.135
19	.261	.546	56	.201	.031
20	.357	-.517	57	-.121	.085
21	-.417	.020	58	.104	.009
22	-.345	-.011	59	-.005	-.081
23	.097	-.008	60	-.046	.052
24	-.042	.234	61	.019	.017
25	.454	-.122	62	.312	-.105
26	-.282	-.222	63	-.149	-.094
27	.096	.090	64	-.240	-.054
28	.199	.285	65	.060	.182
29	-.357	-.361	66	.154	-.033
30	.180	.002	67	-.108	.075
31	-.185	.003	68	.080	-.048
32	.115	.098	69	-.037	.005
33	.048	-.109	70	.149	-.187
34	-.068	.065	71	-.142	.037
35	-.201	-.172	72	-.248	-.217
36	-.108	.151	73	.039	.172
37	-.115	.119			

TABLE I-2

THE CALCULATED FOURIER COEFFICIENTS FOR 20 INCHES DEPTHS OF SOIL

NO	A COEFF.	B COEFF.	NO	A COEFF.	B COEFF.
1	-15.583	-11.904	38	.007	.038
2	.293	1.043	39	.017	.262
3	.560	.884	40	-.054	.115
4	-.212	.856	41	-.001	-.096
5	-.614	-.103	42	.179	-.022
6	.089	-.704	43	-.015	-.033
7	.181	.475	44	.042	.167
8	-.683	.299	45	.408	-.203
9	.387	-.257	46	-.010	-.054
10	-.255	.252	47	-.305	-.014
11	1.106	.230	48	.268	.171
12	-.737	.030	49	.015	-.103
13	-.076	-.187	50	.041	.070
14	.389	.264	51	-.021	-.100
15	.047	-.273	52	.089	.122
16	.081	-.249	53	-.052	-.060
17	-.375	.104	54	.019	-.209
18	.037	-.035	55	-.003	.007
19	-.017	.458	56	.037	.059
20	.320	-.218	57	-.099	-.040
21	-.265	-.066	58	.028	-.036
22	-.036	-.143	59	.075	-.047
23	.041	.088	60	-.074	-.124
24	-.281	.184	61	-.037	.039
25	.290	.136	62	.068	.153
26	-.076	-.189	63	-.014	-.028
27	.084	.106	64	.045	-.054
28	-.080	.203	65	.031	.002
29	-.149	-.232	66	.052	.018
30	.164	.120	67	-.048	.045
31	-.074	.065	68	.037	.093
32	-.064	.192	69	-.044	-.030
33	.051	.054	70	.114	-.038
34	.001	-.070	71	-.068	.046
35	-.043	-.175	72	.034	-.103
36	-.055	.123	73	.090	.011
37	-.092	.027			

TABLE I-3

THE CALCULATED FOURIER COEFFICIENTS FOR 40 INCHES DEPTHS OF SOIL

NO	A COEFF.	B COEFF.	NO	A COEFF.	B COEFF.
1	-11.044	-12.678	38	-.004	-.017
2	.399	.750	39	.020	.094
3	.289	.763	40	-.104	-.006
4	-.330	.736	41	.098	-.059
5	-.421	-.174	42	-.004	.035
6	.343	-.452	43	-.063	-.161
7	.077	.295	44	.034	.109
8	-.520	-.037	45	.158	-.044
9	.282	-.053	46	.001	.036
10	-.228	-.060	47	.027	-.147
11	.548	.416	48	-.059	.047
12	-.295	-.232	49	.015	-.022
13	-.052	-.093	50	-.045	.051
14	.030	.301	51	.085	-.070
15	-.028	.006	52	-.005	-.001
16	.082	.026	53	.060	.001
17	-.180	-.082	54	.018	-.026
18	.067	-.115	55	.005	.032
19	-.108	.029	56	-.084	.020
20	.122	.108	57	-.062	-.039
21	-.072	-.031	58	.025	-.013
22	.023	-.066	59	-.001	-.009
23	.019	.180	60	-.017	-.111
24	-.143	-.077	61	.036	-.043
25	-.014	.083	62	-.063	.027
26	.110	-.125	63	-.045	-.063
27	-.034	-.022	64	-.036	.073
28	-.097	-.081	65	-.040	.025
29	.114	.027	66	-.068	-.029
30	.010	.148	67	-.023	.008
31	-.092	.017	68	.010	-.017
32	-.045	.141	69	.004	.031
33	-.029	.017	70	.032	.024
34	.010	-.047	71	-.043	-.010
35	.062	-.086	72	.033	.046
36	-.003	-.053	73	-.044	-.063
37	.023	-.025			

TABLE I-4

THE CALCULATED FOURIER COEFFICIENTS FOR 80 INCHES DEPTHS OF SOIL

NO	A COEFF.	B COEFF.	NO	A COEFF.	B COEFF.
1	-7.894	-12.433	38	.066	.010
2	.017	.747	39	.020	-.036
3	-.186	.615	40	-.047	.000
4	-.424	.398	41	.048	-.004
5	-.426	-.269	42	-.045	.058
6	.323	-.221	43	-.030	-.025
7	-.026	.206	44	-.044	.066
8	-.310	-.014	45	-.025	.072
9	.221	.040	46	.003	.070
10	-.068	-.042	47	.067	-.039
11	.215	.306	48	-.037	.053
12	-.076	-.208	49	.054	.013
13	-.041	-.166	50	-.027	.027
14	-.122	.119	51	.016	-.002
15	.061	.067	52	.041	.023
16	-.036	.102	53	.075	.020
17	-.045	-.063	54	-.069	-.067
18	.048	.086	55	.037	-.084
19	-.058	-.017	56	-.070	-.043
20	.081	.073	57	-.023	-.018
21	-.118	-.026	58	.088	-.005
22	.015	.015	59	.042	-.058
23	-.063	.194	60	.062	-.054
24	-.021	-.125	61	.029	-.008
25	-.004	.045	62	-.007	-.129
26	.127	.044	63	-.017	-.061
27	-.043	-.004	64	.015	.052
28	-.097	-.086	65	-.004	.060
29	.012	.095	66	.103	.006
30	.045	.010	67	.000	-.027
31	.024	-.021	68	.061	-.058
32	.034	-.047	69	-.063	-.033
33	-.012	.007	70	-.029	-.030
34	-.071	-.100	71	-.039	.020
35	-.055	.012	72	-.018	.010
36	.050	-.071	73	-.047	-.037
37	-.017	-.064			

APPENDIX II

COMPUTER MODELING FOR STUDY OF
SOIL TEMPERATURE WITH
SUBMERGED DRAIN TILE

APPENDIX II

COMPUTER MODELING FOR STUDY OF SOIL TEMPERATURE WITH SUBMERGED DRAIN TILE

II.1 Differential Equation

Unsteady heat conduction equation in the soil is given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (\text{II.1})$$

Let $T(i,j,n)$ represent $T(z,x,t)$, where $Z = i\Delta z$ and $x = j\Delta x$ (Figure 2-4). The finite difference approximate derivative of partial differential equation by the Taylor's expansion is given

$$\frac{\partial T}{\partial t} = \frac{T_{i,j,n+1} - T_{i,j}}{\Delta t} + O(\Delta t) \quad (\text{II.2.a})$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i,j-1,n} - 2T_{i,j,n} + T_{i,j+1,n}}{(\Delta x)^2} + O[(\Delta x)^2] \quad (\text{II.2.b})$$

$$\frac{\partial^2 T}{\partial z^2} = \frac{T_{i-1,j,n} - 2T_{i,j,n} + T_{i+1,j,n}}{(\Delta z)^2} + O[(\Delta z)^2] \quad (\text{II.2.c})$$

Where $O()$ represents the discretization error.

For convenience let define

$$\delta_x^2 T_{i,j,n} = \frac{T_{i,j-1,n} - 2T_{i,j,n} + T_{i,j+1,n}}{(\Delta x)^2} \quad (\text{II.3.a})$$

and

$$\delta_z^2 T_{i,j,n} = \frac{T_{i-1,j,n} - 2T_{i,j,n} + T_{i+1,j,n}}{(\Delta z)^2} \quad (\text{II.3.b})$$

Essentially, the principle of the implicit alternating-direction method (Cranahan, 1969) has been used in this paper to employ two difference equations which are used in turn over successive time-steps, each of duration $\Delta t/2$. The first equation is implicit only in the x-direction and the second is implicit only in the y-direction. Thus, if $T_{i,j}^*$ is an intermediate value at the end of the first time-step, the Equation (II-1) can be written

$$\frac{T_{i,j}^* - T_{i,j,n}}{\Delta t/2} = \delta_x^2 T_{i,j}^* + \delta_z^2 T_{i,j,n} \quad (\text{II.4.a})$$

followed by:

$$\frac{T_{i,j,n+1} - T_{i,j}^*}{\Delta t/2} = \delta_x^2 T_{i,j}^* + \delta_z^2 T_{i,j,n+1} \quad (\text{II.4.b})$$

Written in full and rearranged, with $x = z$ for simplicity, the equations (2-3) and (2-4) can be obtained.

II.2 Expansion of the Formula and Applying Related Boundary and Initial Conditions

The Equations (2-7) and (2-8) with the related boundary and initial conditions are used in computer modeling. The expansion of the Equation (2-7) is given in section (1.2.4.3). The expansion of the Equation (2-8) in the tridiagonal system for the i rows is as following.

$$\begin{array}{rcl}
 j = 0 & bT_{i,0} - 2T_{i,1} & = d_0 \\
 j = 1 & -T_{i,0} + bT_{i,1} - T_{i,2} & = d_1 \\
 j = 3 & -T_{i,1} + bT_{i,2} - T_{i,3} & = d_2 \\
 & \dots & \\
 j = j & -T_{i,j-1} + bT_{i,j} - T_{i,j+1} & = d_j \\
 & \dots & \\
 j = m-2 & -T_{i,m-3} + bT_{i,m-2} - T_{i,m-1} & = d_{m-2} \\
 j = m-1 & -T_{i,m-1} + bT_{i,m-1} & = d_{m-1}
 \end{array} \quad \left. \vphantom{\begin{array}{rcl} j = 0 \\ j = 1 \\ j = 3 \\ j = j \\ j = m-2 \\ j = m-1 \end{array}} \right\} \text{(II.5.a)}$$

with

$$\begin{array}{rcl}
 d_0 & = \hat{T}_{1,j} + f\hat{T}(0,j) + G \\
 d_j & = \hat{T}_{i-1,j} + f\hat{T}_{i,j} + \hat{T}_{i+1,j} \\
 d_{m-1} & = \hat{T}_{i-1,j} + f\hat{T}_{i,j} + \hat{T}_{i+1,j} + \hat{T}_{i,m}
 \end{array} \quad \left. \vphantom{\begin{array}{rcl} d_0 \\ d_j \\ d_{m-1} \end{array}} \right\} \text{(II.5.b)}$$

The coefficients of the tridiagonal matrix (see Cranahan, et al., pp. 441-446) are:

$$\begin{array}{rcl}
 a_1 = 0 & \text{and} & a_k = -1 \quad k = 2, 3, \dots, m \\
 & & b_k = b \quad k = 1, 3, \dots, m \\
 c_m = 0 & \text{and} & c_k = -1 \quad k = 1, \dots, m-1
 \end{array} \quad \text{(II.6)}$$

where $b = 2 \left(\frac{1}{\lambda} + 1 \right)$ (See equation 2-6a).

When one or two or more points attain a constant temperature the tridiagonal matrix should be modified. This condition happens when the temperature inside the drain tile remains constant for a period of time.

For simplicity let us assume that only one point, for example point (9.5), has a known constant temperature, say T_c .

In tridiagonal system for $j = 5$ column the equations are

$$i = 8 \quad -\overset{*}{T}_{7,5} + b\overset{*}{T}_{8,5} - \overset{*}{T}_{9,5} = d'_8 \quad (\text{II.7.a})$$

$$i = 9 \quad -\overset{*}{T}_{8,5} + b\overset{*}{T}_{9,5} - \overset{*}{T}_{10,5} = d'_9 \quad (\text{II.7.b})$$

$$i = 10 \quad -\overset{*}{T}_{9,5} + b\overset{*}{T}_{10,5} - \overset{*}{T}_{11,5} = d'_{10} \quad (\text{II.7.c})$$

Since the $\overset{*}{T}_{9,5}$ is constant, the condition of (II-6) no longer can be used in determining the coefficients of Equations (II-7).

Equation (II.7.a) can be written

$$-\overset{*}{T}_{7,5} + b\overset{*}{T}_{8,5} = d'_8 + \overset{*}{T}_{9,5} = d_8$$

hence $c_8 = 0$; $d_8 = T_c + d'_8 = T_c + (T_{8,4} + fT_{8,5} + T_{8,6})$

Similarly in Equation (II.7.c) $b\overset{*}{T}_{10,5} - \overset{*}{T}_{11,5} = d'_{10} + \overset{*}{T}_{9,5} = d_{10}$

hence $a_{10} = 0$; $d_{10} = T_c + d'_{10} = T_c + (T_{10,4} + fT_{10,5} + T_{10,6})$

and finally the Equation (II-7.b) can be modified

$$a_9 = 0, \quad b_9 = 1, \quad c_9 = 0, \quad d_9 = T_c$$

The equations of tridiagonal system for $i = 9$ row are

$$j = 4 \quad -T_{9,3} + bT_{9,4} - T_{9,5} = d_4' \quad (\text{II.8.a})$$

$$j = 5 \quad -T_{9,4} + bT_{9,5} - T_{9,6} = d_5' \quad (\text{II.8.b})$$

$$j = 6 \quad -T_{9,5} + bT_{9,6} - T_{9,7} = d_7' \quad (\text{II.8.c})$$

A similar procedure should apply to Equation (II-8).

For two or more constant temperatures (see Figure 2-11) more equations are involved (see computer program).

II.3 List of Principal Variables

<u>Program Symbol</u>	<u>Definition</u>
A,B,C,D	coefficient vectors of tridiagonal matrix
DTAU	time step Δt
DX	space increment, Δx (one inch)
F	$f = 2(1/\lambda - 1)$
I,J	row and column subscripts, i, j
ICOUNT	counter on the number of time-steps
IFREQ	number of time-steps between successive printings of temperatures
G	constant temperature at one inch above x coordinate

M	number of space increment (68)
RATIO	$\lambda = \alpha \Delta t / (\Delta x)^2$
T	matrix of temperatures T at each grid point
TAU	Time, t.
TMAX	Maximum time for successive runs (24 hr)
TPRIME	Vector for temporary storage of temperatures computed by TRIDG. These values T' are then placed in the appropriate row of T or column $\bar{T}^*$
TRIDAG	Subroutine for solving a tridiagonal system of simultaneous equations (see Carnahan, <u>et al.</u> , 1969)
TSTAR	Matrix of temperatures $\bar{T}^*$ at the end of the first half time-step
ZERO	Constant temperature (32° F.)

Because of FORTRAN limitations all subscripts are advanced by one, when they appear in the program; e.g., $T_{1,0}$ $T_{m,m}$ became T(2,1) and T(M + 1, M + 1), respectively.


```

A(I) = -1.
33 C(I) = -1.
HRS = 0.
666 CONTINUE
ICOUNT = 0
TAU = 0.
34 TAU = TAU + DTAU
HRS = HRS + DTAU
C(I) = -1.
ICOUNT = ICOUNT + 1

C
C .....
C ..... COMPUTE TEMPERATURES AT END OF HALF .....
C ..... TIME INCREMENT (IMPLICIT HY COLUMNS) .....
C
DO 36 J = 1,M
DO 37 I = 1,M
IF(I,HRS,1) GO TO 36
D(I) = 2. * T(I,2) + F * T(I,1)
GO TO 37
36 D(I) = T(I,J-1) + F * T(I,J) + T(I,J+1)
37 CONTINUE
D(I) = D(I) + G
D(I) = D(I) + T(MP,J)
CALL IRIIAG (1,M,A,M,C,D,TPRIME)
DO 38 I = 1,M
38 TSTAR(I,J) = TPRIME(I)

C
C .....
C ..... COMPUTE TEMPERATURES AT END OF WHOLE .....
C ..... TIME INCREMENT (IMPLICIT HY ROWS) .....
C .....
C(I) = -2.
DO 62 I = 1,M
DO 61 J = 1,N
IF(I,HRS,1) GO TO 60
D(I) = TSTAR(2,J) + F * TSTAR(1,J) + G
GO TO 61
60 D(I) = TSTAR(I-1,J) + F * TSTAR(I,J) + TSTAR(I+1,J)
61 CONTINUE
D(I) = D(I) + TSTAR(I,MP)
CALL IRIIAG(1,M,A,M,C,D,TPRIME)
DO 62 J = 1,M
62 T(I,J) = TPRIME(J)

C
C .....

```

```

C      .....PRINT TEMPERATURES THROUGHOUT THE QUADRANT .....
C*****
      IF (ICOUNT.NE. IFRF) 60 TO 45
      ICOUNT = 0
      IF (TAU - TMAX) 34,31,31
      31 CONTINUE
      TIME = 0.05
      WRITE (61,61.3) TIME
      613 FORMAT(43,10X, 'HOURS TEMPERATURES IN QUADRANT
      IF (J.EQ.1) J=10 ARE*)
C*****
C      ...FLOW-THROUGH DRAIN TILE LINE HAS BEEN HURIED AT 48 INCHES DEPTH .....
C*****
C
      DO 5 I=29,37
      DO 5 J=1,2
      T(I,J) = ZERO
      5 ISTAR(I,J) = ZERO
      DO 6 I=31,35
      DO 6 J=3,6
      T(I,J) = ZERO
      6 ISTAR(I,J) = ZERO
      T(36,3) = ZERO      % T(32,5) = ZERO
      T(34,5) = ZERO      % T(34,5) = ZERO
      ISTAR(30,3) = ZERO  % ISTAR(36,5) = ZERO
      ISTAR(33,5) = ZERO  % ISTAR(34,5) = ZERO
C*****
C*****
C*****
      250 FORMAT(43,10X, 'THE INITIAL TEMPERATURES FROM J=1 TO J=16 ARE*)
      DO 258 I=1,16
      258 WRITE (61,2020) I, (T(I,J), J=1,16)
C      .....
      ICOUNT = 0
      15 IF (TAU - TMAX) 19,4,4
      4 CONTINUE
      TAU = 0.
      GOLD = 6
      READ (60,1020) DTAU,K
      IF (N.GT.40) GO TO 1
      READ (60,100) ZERO, TMAX, IFRF
      WRITE (61,551) K
      READ (60,101) (P(I), I=1,16)
      WRITE (61,203) (P(I), I=1,16)
      RATIO = (DTAU*ALFA)/(OX*DX)

```

```

      WRITE (61.200)DTAU,DX,RATIO,IMAX,M,IFREQ ,6
      IF (6 - 6*LD)GOTO 815,816,817
      815 CONTINUE
      DO 820 I=1,M
      DO 820 J=1,M
      IF (P(I) - T(I,1))820,821,821
      820 T(I,J) = P(I)
      821 CONTINUE
      GO TO 816
      817 CONTINUE
      DO 822 I=1,M
      DO 822 J=1,M
      IF (T(I,1) - P(1))822,823,823
      822 T(I,J) = P(1)
      823 CONTINUE
      816 CONTINUE
      CAT = 1./RATIO
      H(1) = 2.*CAT + 1.
      F = 2.* (CAT -1.)
      WRITE (61.200)CAT,H(1) , F
      DO 852 I=1,M
      T(I,M) = P(1)
      552 TSTAB(I,30) = P(1)
      GO 853 J=1,M
      T(40,J) = P(40)
      553 TSTAB(40,J) = P(40)
      19 IAT = IAU + DTADU
      ICOUNT = ICOUNT + 1
      *****
C
C
C      ..... COMPUTE TEMPERATURES AT END OF HALF
C      TIME INCREMENT (IMPLICIT BY COLUMNS).....
C
C*****
      DO 3 I = 2,M
      A(I) = -1.
      H(I) = H(1)
      3 C(I) = -1.
      C(I) = -1.
      *****
      J= 5
      A(32)=0.          % H(32)=1.          % C(32)=0.
      A(33)=0.          % H(33)=1.          % C(33)=0.
      A(34)=0.          % H(34)=1.          % C(34)=0.
      DO 325 I = 1,M

```

```

325 D(1) = T(1,J-1) + F*T(1,J) + T(1,J+1)
D(1) = D(1) + G
D(32)=ZERO $ D(34)=ZERO $ D(34)=ZERO
D(8) = D(1) + I(MP,J)
CALL T10A6 (1,M,A,R,C,D,TPRIME)
DO 326 I = 1,M
325 TSTAR(I,J) = TPRIME(I)
C
J = 4
A(31)=0. $ H(31)=1. $ C(31)=0.
A(32)=0. $ H(32)=1. $ C(32)=0.
A(33)=0. $ H(33)=1. $ C(33)=0.
A(34)=0. $ H(34)=1. $ C(34)=0.
A(35)=0. $ H(35)=1. $ C(35)=0.
DO 320 I = 1,M
320 D(1) = T(1,J-1) + F*T(1,J) + T(1,J+1)
D(1) = D(1) + G
D(32)=ZERO $ D(34)=ZERO $ D(34)=ZERO
D(31)=ZERO $ D(35)=ZERO
D(8) = D(1) + I(MP,J)
CALL T10A6 (1,M,A,R,C,D,TPRIME)
DO 321 I = 1,M
321 TSTAR(I,J) = TPRIME(I)
C
C
J = 3
A(30)=0. $ H(30)=1. $ C(30)=0.
A(31)=0. $ H(31)=1. $ C(31)=0.
A(32)=0. $ H(32)=1. $ C(32)=0.
A(33)=0. $ H(33)=1. $ C(33)=0.
A(34)=0. $ H(34)=1. $ C(34)=0.
A(35)=0. $ H(35)=1. $ C(35)=0.
A(36)=0. $ H(36)=1. $ C(36)=0.
DO 315 I=1,M
315 D(1) = T(1,J-1) + F*T(1,J) + T(1,J+1)
D(1) = D(1) + G
D(32)=ZERO $ D(34)=ZERO $ D(34)=ZERO
D(31)=ZERO $ D(35)=ZERO
D(30)=ZERO $ D(36)=ZERO
D(8) = D(1) + I(MP,J)
CALL T10A6 (1,M,A,R,C,D,TPRIME)
DO 316 I=1,M
316 TSTAR(I,J) = TPRIME(I)
C
C
J = 2

```

```

A(29)=0.
A(30)=0.
A(31)=0.
A(32)=0.
A(33)=0.
A(34)=0.
A(35)=0.
A(36)=0.
A(37)=0.
DO 725 I=1,4
725 B(I) = T(I,J-1) + F*T(I,J) + T(I,J+1)
D(1) = D(1) + G
D(29)=ZERO
D(30)=ZERO
D(31)=ZERO
D(32)=ZERO
D(33)=ZERO
D(34)=ZERO
D(35)=ZERO
D(36)=ZERO
D(37)=ZERO
CALL TPRIME(1,M,A,B,C,D,TPRIME)
DO 726 I=1,4
726 TSTAR(I,J)=TPRIME(I)
A(29)=0.
A(30)=0.
A(31)=0.
A(32)=0.
A(33)=0.
A(34)=0.
A(35)=0.
A(36)=0.
A(37)=0.
DO 320 I=1,4
320 B(I) = F*T(I,2) + F*T(I,1)
D(1) = D(1) + G
D(32)=ZERO
D(33)=ZERO
D(34)=ZERO
D(35)=ZERO
D(36)=ZERO
D(37)=ZERO
D(38) = D(4) + T(38,1)
CALL TPRIME(1,M,A,B,C,D,TPRIME)
DO 311 I=1,4
311 TSTAR(I,1) = TPRIME(I)
C
.....
DO 72 I = 2,4
A(I) = -1.
B(I) = -1.
72 C(I) = -1.

```

```

DO 4 J=1,M
DO 7 I=1,M
D(I) = T(I,J-1)+F*I(I,J)+T(I,J+1)
7 CONTINUE
D(1) = D(1) + G
D(M) = D(M) + T(IP,J)
CALL TRIDAB (1,M,A,M,C,D,IPRIME)
DO 4 I=1,M
4 TSTAR(I,J) = IPRIME(I)
*****
C ..... COMPUTE TEMPERATURES AT END OF WHOLE
C ..... TIME INCREMENT (IMPLICIT BY ROWS).....
*****
C(1) = -2.
DO 12 I=1,2N
DO 11 J=1,M
IF (I.NE.1) GO TO 10
D(J) = TSTAR(2,J) + F*TSTAR(1,J) + G
GO TO 11
10 D(J) = TSTAR(1-1,J) + F*TSTAR(1,J) + TSTAR(1+1,J)
11 CONTINUE
D(1) = D(1) + TSTAR(1,MP)
CALL TRIDAB (1,M,A,M,C,D,IPRIME)
DO 12 J=1,M
12 T(1,J) = TPRIME(J)
+2 CONTINUE
C .....
A(1) = 1.      % C(1) = 0.
A(2) = 0.      % H(2) = 1.      % C(2) = 0.
DO 332 I=29,37,M
DO 330 J = 1,M
330 D(J) = TSTAR(I-1,J) + F*TSTAR(1,J) + TSTAR(I+1,J)
D(1) = Z(M)
D(2) = Z(M)
D(3) = D(3) + TSTAR(1,MP)
CALL TRIDAB (1,M,A,M,C,D,IPRIME)
DO 331 J = 1,M
331 T(1,J) = IPRIME(J)
332 CONTINUE
C .....
C(1) = 0.      % H(1) = 1.
A(2) = 0.      % H(2) = 1.      % C(2) = 0.
A(3) = 0.      % H(3) = 1.      % C(3) = 0.
DO 336 I=30,36,M
DO 337 J = 1,M

```

```

337 D(J) = TSTAR(I-1,J) + F*TSTAR(I,J) + TSTAR(I+1,J)
D(1) = ZERO
D(2) = ZERO
D(3) = ZERO
D(4) = D(4) + TSTAR(I,MP)
CALL TPRIME(1,M,A,H,C,D,TPRIME)
DO 335 J = 1,M
335 T(I,J) = TPRIME(J)
336 CONTINUE
C
.....
C(1) = 0.      % H(1) = 1.
C(2) = 0.      % H(2) = 1.      % C(2) = 0.
C(3) = 0.      % H(3) = 1.      % C(3) = 0.
C(4) = 0.      % H(4) = 1.      % C(4) = 0.
DO 340 I = 31,35,4
DO 341 J = 1,M
341 D(J) = TSTAR(I-1,J) + F*TSTAR(I,J) + TSTAR(I+1,J)
D(1) = ZERO
D(2) = ZERO
D(3) = ZERO
D(4) = D(4) + TSTAR(I,MP)
CALL TPRIME(1,M,A,H,C,D,TPRIME)
DO 342 J = 1,M
342 T(I,J) = TPRIME(J)
343 CONTINUE
C*****
C(1) = 0.      % H(1) = 1.
C(2) = 0.      % H(2) = 1.      % C(2) = 0.
C(3) = 0.      % H(3) = 1.      % C(3) = 0.
C(4) = 0.      % H(4) = 1.      % C(4) = 0.
C(5) = 0.      % H(5) = 1.      % C(5) = 0.
DO 740 I = 32,34
DO 741 J = 1,M
741 D(J) = TSTAR(I-1,J) + F*TSTAR(I,J) + TSTAR(I+1,J)
D(1) = ZERO
D(2) = ZERO
D(3) = ZERO
D(4) = ZERO
D(5) = D(5) + TSTAR(I,MP)
CALL TPRIME(1,M,A,H,C,D,TPRIME)
DO 742 J = 1,M
742 T(I,J) = TPRIME(J)
743 CONTINUE

```



```

551 FOR4AT(*1*.10X,*1THE NEW TEMPERATURE AT*.15.* TH DAY OF YEAR ARE*)
IF(1(34.1).LT.32.)GO TO 1
GO TO 15
1 CONTINUE
C .....
WRITE(61.223)K
223 FOR4AT(*0*.14X,*AT*.15.*TH DAY THE TEMPERATURES IN SOIL QUADRANT
IF(04 J=17 TO J=32 ARE*)
DO 445 I=1.44P
445 WRITE(61.220)I.(1(1.J),J=17.32)
WRITE(61.225)K
225 FOR4AT(*0*.14X,*AT*.15.*TH DAY THE TEMPERATURES IN SOIL QUADRANT
IF(04 J=33 TO J=52 ARE*)
DO 447 I=1.44P
447 WRITE(61.220)I.(1(1.J),J=33.52)
WRITE(61.224)K
224 FOR4AT(*0*.14X,*AT*.15.*TH DAY THE TEMPERATURES IN SOIL QUADRANT
2 FROM J=53 TO J=69 ARE*)
DO 446 I=1.44P
446 WRITE(61.220)I.(1(1.J),J=53.69)
END

```

FOR 48

```

C *****
SUBROUTINE FETDAB (IF,L,ASH,C,D,V)
C *****
DIR=DSIGN(A(1),C(1),D(1),V(1),HETA(101),GAMMA(101)
HETA(1F) = B(1F)
GAMMA(1F) = D(1F)/HETA(1F)
IF(1 = IF + 1
DO 1 I = IFP1,L
HETA(I) = B(I) - A(I)*C(I-1)/HETA(I-1)
1 GAMMA(I) = D(I) - A(I) * GAMMA(I-1))/ HETA(I)
V(I) = GAMMA(I)
LAST = L - IF
DO 2 K=1,LAST
I = L - K
2 V(I) = GAMMA(I) - C(I) *V(I+1)/HETA(I)
RETURN
END

```

APPENDIX III

CONVERSION FACTORS

APPENDIX III

CONVERSION FACTORS

Length:

$$12 \text{ in.} = 1 \text{ ft} = 30.48 \text{ cm}$$

Mass:

$$1 \text{ kg} = 1000 \text{ g} = 2.205 \text{ lb}_m$$

$$454 \text{ g} = 1 \text{ lb}_m$$

$$1 \text{ lb}_m = .45359 \text{ kg}$$

Energy:

$$1 \text{ Btu} = 252 \text{ cal}$$

$$1 \text{ cal} = 3.9657 \times 10^{-3} \text{ Btu}$$

$$1 \text{ hp-hr} = 2545 \text{ Btu}$$

Specific heat:

$$1 \text{ cal/g}^\circ\text{C} = .999346 \text{ Btu/lb}^\circ\text{F}$$

Thermal conductivity:

$$1 \text{ cal/sec-cm-}^\circ\text{C} = 242 \text{ Btu/hr-ft-}^\circ\text{F} = 5806 \text{ Btu/day-ft-}^\circ\text{F}$$

Thermal diffusivity:

$$1 \text{ cm}^2/\text{sec} = 3.875 \text{ ft}^2/\text{hr}$$

Volumetric heat capacity:

$$1 \text{ cal/cm}^3\text{-}^\circ\text{C} = 62.48 \text{ Btu/ft}^3\text{-}^\circ\text{F}$$

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