THE EFFECT OF THE COLUMN TO PARTICLE DIAMETER RATIO ON THE CORRELATION OF THE PRESSURE DROP VERSUS FLOW RATE OF FLUIDS THROUGH PACKED BEDS

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ABSTRACT

THE EFFECT OF THE COLUMN TO PARTICLE DIAMETER RATIO ON THE CORRELATION OF THE PRESSURE DROP VERSUS FLOW RATE OF FLUIDS THROUGH PACKED BEDS

by Devendra Mehta

Ergun has established a correlation between the pressure drop in a packed column and the flow rate of a fluid through it. This correlation is considered valid only when the diameter of the column is much larger than the diameter of the packed particles; the Ergun correlation does not include the wall effect on the pressure drop versus flow rate correlation.

In this project, the Ergun correlation was modified to include the wall effect which eliminated the assumption that the diameter of the column should be much larger than that of the packed particles.

Experiments to obtain data for the pressure drop in packed beds were performed for cases where the wall effect was important. The investigations were performed with an one-half inch diameter column packed with spherical glass beads. Water was used as a fluid flowing through packed beds.

Pressure drop - flow rate data were plotted on a log-log graph in the form of the packed friction factor, $(\frac{\Delta p g_c \rho}{G^2})(\frac{D_p}{L}) \times (\frac{\epsilon^3}{1-\epsilon})$ versus Reynold's number, $(\frac{D_p G}{\mu(1-\epsilon)})$. It was observed that, when the wall effect was significant, the plot of the above groups deviated from the plot of the Ergun correlation. However;

after the friction factor and Reynold's number, $(\frac{\Delta p g_c \rho}{G^2})(\frac{D_p}{L}) \times (\frac{\epsilon^3}{1-\epsilon})(\frac{1}{\frac{4D_p}{6D_c(1-\epsilon)} + 1})$ and $(\frac{D_p G}{\mu(1-\epsilon)})(\frac{1}{\frac{4D_p}{6D_c(1-\epsilon)} + 1})$ respectively, were modified to include the wall effect as a parameter, a log-log plot of the above groups coincided with the Ergun correlation.

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TO

MY PARENTS AND BROTHERS

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INTRODUCTION

Ergun (5) has established a correlation between the pressure drop in a packed column and the flow rate of a fluid through it. The Ergun equation when stated in dimensionless groups is:

$$\left(\frac{\Delta p g_c \rho}{G^2}\right)\left(\frac{D_p}{L}\right)\left(\frac{\epsilon^3}{1-\epsilon}\right) = \left(\frac{150\mu(1-\epsilon)}{D_p G}\right) + 1.75$$

where Δp = pressure drop through a column having length L,

g_c = gravitational constant,

ρ = density of the fluid flowing through the column,

G = mass flow rate per unit area,

D_p = diameter of the particles,

L = length of the packed column,

ϵ = void fraction,

μ = viscosity of the fluid.

Many experiments have been performed by various investigators (2, 3, 4, 6, 7, 8, 9, 10), and it was found that the results from those experiments fit the Ergun equation well. However, Ergun's correlation has the limitation that the effect of the wall on the pressure drop through the bed is neglected. The deviations from the Ergun correlation are negligible if the ratio of the diameter of the column to the diameter of the packing particles is larger than 50:1. This assumes that the wall effect on the hydraulic radius -- the characteristic length dimension in the Reynold's number -- of the packed bed is negligible. Several investigators (3, 4, 5) have stated that the wall effect may be important for column to particle diameter ratios in the range of 8:1 to 20:1. In fact, it is safe to state that, if the ratio of the column to particle diameter is less than 50:1, the wall effect on the hydraulic radius is significant. Investigators failed to establish the magnitude of the wall effect.

The purpose of this investigation was to examine the effect of the wall on the flow characteristics of fluids in packed beds and to determine the validity of the modified Ergun equation which includes the wall effect on the pressure drop through a packed column. Experiments were performed at low flow rates with packed bed Reynold's numbers, ranging from 0.1 to 10.0.

The apparatus was a column packed with spherical glass beads. Nitrogen pressure was used to maintain constant flow in the bed. The pressure drop between two positions in the bed was measured at various liquid flow rates through the packed column.

The column diameter was kept constant, whereas the particle size was varied such that the column to particle diameter ratios ranged from 92:1 to 8:1. The glass beads used in packing the column were uniform and spherical. Water was used as the fluid.

Pressure drop and flow rate data were obtained at various diameter ratios. Values of $\left(\frac{\Delta p g_c \rho}{G^2}\right)\left(\frac{D_p}{L}\right)\left(\frac{\epsilon^3}{1-\epsilon}\right)$ and $\frac{D_p G}{\mu(1-\epsilon)}$ were calculated and a log-log plot of these groups was made in order to compare it with the Ergun plot. It was found that at the smaller diameter ratios, the wall effect on the hydraulic radius or on the flow characteristics of fluids in packed beds cannot be neglected. This was observed from the deviation of the plot from the Ergun equation for the smaller ratio of column to particle diameter.

If the corrected Ergun equation is used, there is no deviation observed when the data for smaller diameter ratios, as well as for larger diameter ratios, are replotted. Therefore, if corrections are made by including the wall effect in the Ergun equation, it is not necessary to assume that the diameter of the column should be much larger than that of the particles. This correction involves including the effect of the tube wall on the hydraulic radius of the packed bed. Details of this correction are explained in the theory section.

The pressure drop due to the flow of fluids through packed columns has been the subject of experimental investigation by many workers (2, 3, 4, 6, 7, 8, 9, 10) to determine the correlation between the pressure drop and the flow rate of fluid through packed columns. Those correlations differ in many respects; some are to be used for low flow rates, while others are to be used at high flow rates. Previous workers (2, 3, 4, 6, 9) derived relations using different assumptions and correlated the particular experimental data obtained with or without some of the data published earlier. They agreed that the expressions relating the pressure drop along the length of bed and the flow of fluid through the bed contain the following factors:

1. Pressure drop along the length of bed and the flow rate of the fluid,
2. Density and viscosity of fluid,
3. Void fraction of the bed,
4. Shape and the surface of the particles.

It was Reynolds (5), who first formulated the relation between the pressure drop and the flow rate. He stated that the resistance offered by friction to the motion of the fluid was the sum of two terms. He proposed that the first term was proportional to the first power of the velocity of the fluid and the second term was proportional to the product of the density of the fluid and the second power of the velocity.

$$\frac{\Delta p}{L} = a\ v_o + b\ \rho\ v_o^2 \qquad (2)$$

Here, Δp is the pressure drop along the bed of length L, v_o is the linear velocity of fluid, ρ is the density of the fluid, and a and b are constants. This relation was tested by Ergun and Orning (6) as well as by Morcom (9). They plotted the values of $\frac{\Delta p}{Lv_o}$ against $\rho\ v_o$ which were obtained from their investigation, and straight lines were obtained as expected from equation 2. They noted that the values of a and b were different depending on other conditions such as the viscosity of the fluid, the closeness of particles, etc.

Ergun and Orning (6) tried to develop relationships for the constants a and b in order to predict their experimental values. They were partially successful in deriving the mathematical model for those constants for the general case. This will be discussed in detail later.

Morcom (9) used gases such as air, carbon dioxide, etc., as fluids and used different granular materials with various types of packing. He mentioned that pressure drop is also a function of closeness of packing. He showed that the pressure drop is inversely proportional to the cube of the void factor. This is true, but it will be shown later that pressure drop is also proportional to $(1-\epsilon)^2$ for low flows and $(1-\epsilon)$ for high flow rates, where ϵ is the void fraction.

It can be seen from equation 2 that the velocity approaches zero, the quantity $\frac{\Delta p}{Lv_o}$ approaches a constant value, a. This is the condition for viscous flow, and it can be seen that the above equation is similar to the Poiseulle equation (1, 4, 5);

$$\frac{\Delta p}{Lv_o} = \text{constant}$$

and to Darcy's law (4);

$$v_o = \frac{k\Delta p}{L}, \quad \text{where } k \text{ is a constant.}$$

If the velocity of the fluid is high, then in comparison with the term $b \rho v_o$, the constant "a" is negligible. In other words, it is the condition for turbulent flow where the resistance to the flow is constituted by kinetic energy losses. So the resistance to the flow is the sum of the two factors - loss of viscous energy and loss of kinetic energy. The loss of viscous energy is due to the friction between two layers of the fluid, and the magnitude of viscous energy depends on local velocity gradient of the layers. The loss of kinetic energy is due to the motion of the fluid as a bulk and the magnitude of it depends on the bulk average velocity.

It can be seen that, if the constant "a" is replaced by $a'\mu$ where a' is a constant, a' depends only on the characteristics of the bed.

Kozeny (1, 4, 5) developed the correlation between the pressure drop and the flow rate as:

$$\frac{\Delta p g_c}{L} = k_1 \frac{(1-\epsilon)^2 \mu v_o}{\epsilon^3 D_p^{\ 2}} \qquad (3)$$

where k_1 is a constant. Comparing equations 2 and 3, it can be seen that equation 3 is similar to equation 2 for low flow rates. Then the constant "a" is equal to $k_1 \frac{(1-\epsilon)^2\mu}{\epsilon^3 D_p^2}$.

So the Kozeny (4) equation is in partial agreement with equation 2. Carman (4), Lea (7), Nurse (7), and others have verified experimentally that constant "a" in the Reynold's equation is equal to:

$$k_1 \frac{(1-\epsilon)^2\mu}{\epsilon^3 D_p^2}$$

However, most of their experiments were performed at low flow rates; so they failed to see the effects at high flow rates. Carman (4) did work in the high flow regions and found deviations in his results from those expected from Kozeny equation; however, he neglected it as an experimental error.

Carman (4) did extensive work on the flow of fluids through granular beds. He developed the correlation between pressure drop and flow rate and arrived at the same equation that was mentioned by Kozeny. He used Darcy's law and included the void fraction as a parameter to describe packed bed data;

$$v_o = \frac{K\Delta p}{L}, \tag{4}$$

K is a constant. Furthermore, he also used the two dimensionless groups, friction factor and Reynold's number for packed beds as: $\frac{\Delta p g_c \epsilon^3}{L\rho v_o^2 s}$ and $\frac{\rho v_o}{\mu s}$ where s is the surface area of packed bed per unit volume of bed. He added further that there is a linear relation between the flow rate and the pressure drop through the bed. From this relation, he concluded that the dimensionless group $\frac{\Delta p g_c D_p^2 \epsilon^3}{36 \mu L v_o (1-\epsilon)}$ be designated as a constant j. The above group is valid for a bed made up of spherical particles. For other shapes of particles, the dimensionless group should be modified. Carman tried to calculate the values of j from the results of the other investigations on the flow of fluids through packed beds and showed that the values of j for different sets of conditions varied. He argued that the variation was due to the expansion of the fluid when it passed through the bed and the expansion of the fluid resulted in a change of viscosity of the fluid. In addition, the values of ϵ may also change if the bed is under high pressure. The above variations were the important factors which caused the variation of j. Carman (4) reported in his papers that wall effect is also a factor in the variation of j. He used Coulson's modification which applies to Kozeny equation and which includes the wall effect as a parameter. However, the corrected values of j still varied.

Blake (2) approached the problem of the fluid flow in a packed bed by comparing it with the fluid flow in a circular pipe. It has been established for the flow in pipes that a unique plot is obtained if the dimensionless groups, $(\rho v_e D_e/\mu)$ and $R/\rho v_e^2$, are plotted against one another. Here, R = frictional force per unit area, D_e = pipe diameter, and v_o = actual velocity in the channel.

Blake showed that the above groups can be modified for a packed bed if dimensional homogenity is used in comparing the flow in a packed bed with that in a circular pipe. He substituted D_e by the hydraulic radius R_h where R_h = cross-sectional area per perimeter presented to fluid. For granular beds, $R_h = \epsilon/s$; s = surface area of particle / unit volume of bed. Again, R is defined in terms of Δp as:

$$R = \frac{\Delta p g_c \epsilon}{Ls}$$

So the dimensionless groups for packed beds are modified as:

$$\left(\frac{\Delta p g_c \epsilon^3}{L\rho\ v_o^2 s}\right) \quad \text{and} \quad \left(\frac{\rho\ v_o}{\mu s}\right)$$

These groups are known as the Blake dimensionless groups (4, 5, 6), since he was first to recognize the importance of them. These groups can be plotted on log-log graphs, and a unique graph is obtained.

Burke and Plummer (3) proposed that the pressure drop was due to the loss of kinetic energy. That is to say that Δp is proportional to v^2. They assumed that the granular bed was equivalent to a group of parallel channels; they regarded the bed to be made up of the sum of the separate resistances of the individual particles in it - as measured from the rate of free fall in the fluid. The force, F, acting on an isolated sphere suspended in a fluid stream is equal to $\frac{3\pi\mu D_p v}{g_c \epsilon}$. The number of particles per unit of packing volume is $6(1-\epsilon)/\pi D_p^3$. Burke and Plummer stated that the rate of work W done due to the flow of the fluid is:

$$W = \left(\frac{3\pi\mu D_p v_o}{g_c \epsilon}\right)\left(\frac{v_o}{\epsilon}\right)\left(\frac{6(1-\epsilon)}{\pi D_p^3}\right).$$

But W in terms of Δp is also equal to $\frac{\Delta p}{\rho L}$. Combining the above equations, they proposed the following proportionality for the flow of fluids through packed beds:

$$\frac{\Delta p g_c}{L} \quad \alpha \quad \frac{(1-\epsilon)\rho\, v_o^2}{\epsilon^3 D_p} \tag{5}$$

This proportionality has been verified for high flow rates; however, it fails in low flow regions. This is due to the assumption that the pressure drop is due to only kinetic energy term.

If the above equation is compared with equation 2, it can be shown that if the flow rates are high, equation 2 is similar to equation 5, as the constant a in equation 2 becomes negligible at high flow rates. The constant b is equal to: $k\,\frac{(1-\epsilon)}{\epsilon^3 D_p}$. Using equation 2 and using the values of the constants a and b, the following equation is obtained:

$$\frac{\Delta p g_c}{L} = k_1 \frac{(1-\epsilon)^2 \mu\, v_o}{\epsilon^3 D_p^2} + k_2 \frac{(1-\epsilon)\rho\, v_o^2}{\epsilon^3 D_p} \tag{6}$$

This equation is known as the Ergun equation (1,5). The above equation can be rearranged in terms of dimensionless groups:

$$\left(\frac{\Delta p g_c}{\rho\ v_o^2}\right)\left(\frac{D_p}{L}\right)\left(\frac{\epsilon^3}{1-\epsilon}\right) = k_1 \frac{D_p \rho\ v_o}{\mu(1-\epsilon)} + k_2 \qquad (6a)$$

This equation shows how the Blake dimensionless groups fit with the Ergun equation (5).

Many workers (3, 4, 5) have stated that the wall effect is an important factor when considering the flow of fluids through packed beds. The magnitude of the wall effect has not been determined by past investigators. They assumed that the wall effect is negligible if the column to particle diameter ratio is very large. It will be proved in the later section that the Ergun equation can be modified to include the frictional effect due to the wall.

SCOPE OF THE PROBLEM

Ergun has developed an empirical relationship between the pressure drop and the flow rate in a packed bed. This relationship is represented in the form of a friction factor and Reynold's number. For a packed bed, the friction factor and the Reynold's number are defined as:

$$\text{Friction factor} = \left(\frac{\Delta p g_c \rho}{G^2}\right)\left(\frac{6\ R_h \epsilon^2}{L}\right)$$

$$\text{and Reynold's number} = \frac{6GR_h}{\mu\epsilon}\ .$$

Here, R_h is the hydraulic radius which is defined as the ratio of the cross-section available for flow to the wetted perimeter. For packed beds, Ergun used the expression for the hydraulic radius:

$$R_h = \frac{\epsilon D_p}{6(1-\epsilon)} \qquad (7)$$

This expression does not include the wall effect. Hence, it shows no dependency on the column diameter; since, during the development of equation 7, it was assumed that the packed particle diameter was much smaller in comparison with that of the column diameter. In this project, the diameter of the column is not considered too large when compared with the particle diameter. This results in modification of the Ergun equation. The object of this research is to examine the validity of the modified Ergun equation which includes the wall effect on the hydraulic radius. The following expression is obtained for the hydraulic radius - when the wall effect is included:

$$R_h = \frac{\epsilon}{\frac{6(1-\epsilon)}{D_p} + 4/D_c} \tag{8}$$

Thus, friction factor $= \left(\frac{\Delta p g_c \rho}{G^2}\right)\left(\frac{6\,\epsilon^3}{L}\right)\left(\frac{1}{\frac{6(1-\epsilon)}{D_p} + 4/D_c}\right)$

and Reynold's number $= \frac{6\ G}{\mu\{\frac{6(1-\epsilon)}{D_p} + 4/D_c\}}$

where D_c = column diameter, D_p = particle diameter, and ϵ = void fraction.

The validity of the modified friction factor and Reynold's number was determined by making a log-log plot of the above groups calculated from experimental data and comparing that graph with a similar plot of the Ergun equation.

Many chemical processes are carried out in packed beds where the wall effect is important. As the design of large scale equipment is based on small laboratory models, it is important to understand the flow characteristics so that large scale equipment can be reliably designed. With the help of the modified Ergun equation, laboratory data which are obtained under conditions where the wall effect is important can be successfully scaled to size where the wall effect is not important.

THEORY

First, the derivation of the Ergun equation which is a correlation relating the pressure drop to the flow rate in a packed bed will be made with the assumption that the diameter of the packed particles is very small in comparison with that of a column. Then, this equation will be modified to take into account the effect of the column wall on the hydraulic radius. The problem is confined to spherical particles in a cylindrical tube having a constant cross-section.

First, consider the flow of an incompressible fluid with density ρ through a pipe.

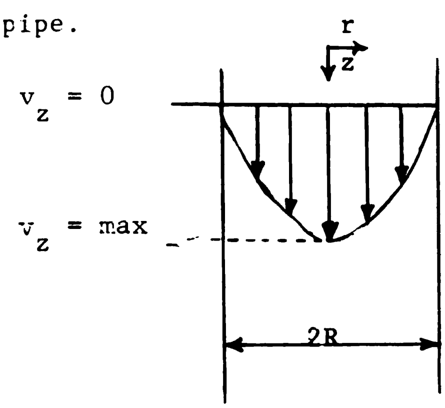

Figure 1: Velocity distribution for the flow in a cylindrical tube.

Making a force balance on a shell of thickness Δr, the following differential equation is obtained:

$$\therefore \quad \frac{d\left(-r\mu \frac{dv_z}{dr}\right)}{dr} = \frac{\Delta p g_c r}{L} \qquad (9)$$

Solving the above equation, and using the following boundary conditions: 1. at r = R, the velocity v_z is maximum; and

2. at $r = 0$, the velocity v_z is zero,

the velocity distribution in z direction is defined as:

$$v_z = \frac{\Delta p g_c R^2}{4\mu L} \{1 - \frac{r^2}{R^2}\} \qquad (10)$$

The average velocity $<v_z>$ is calculated by summing up all velocities over the cross-section and then dividing by the cross-sectional area:

$$<v> = \frac{\int_o^{2\pi} \int_o^{R} v_z r dr d\theta}{\int_o^{2\pi} \int_o^{R} r dr \, d\theta} \qquad (11)$$

$$\therefore \quad <v> = \frac{\Delta p R^2 g_c}{8\mu L} \qquad (12)$$

This is the well known Poiseulle equation. Developing the above equation, it was assumed that

1. the fluid was newtonian,
2. the end effects were neglected,
3. the temperature was constant,
4. the steady state was established when considering the force balance.

This is the case for the column without any packing. All flow systems do not have same shape or same cross-section available for the flow of fluid. It will be assumed that the same equations which describe flow in a pipe describe flow in a packed bed. It is further assumed that the hydraulic radius is the characteristic length parameter in the Reynold's number. Now consider the steady flow of fluid through a cylindrical tube filled with spherical beads. Assume that the packed bed is a tube made up of very complicated cross-section with a hydraulic radius R_h. It can be shown that if void fraction is one, four times the hydraulic radius is equal to the diameter of the tube.

Equation 12 is transformed in terms of the hydraulic radius R_h by replacing R with $2R_h$, so that dimensional homogenity can be used to compare the flow properties in packed column with that in an empty cylinder.

$$\therefore \quad \langle v \rangle = \frac{\Delta p R_h^2 g_c}{2\mu L} \tag{13}$$

Now,
$$R_h = \frac{\text{Cross-section available for flow}}{\text{Wetted perimeter}}$$
$$= \frac{\text{Volume available for flow}}{\text{Total wetted surface}}$$
$$= \frac{\text{Volume of voids / Volume of bed}}{\text{Wetted surface / Volume of bed}} \; .$$

If the diameter of the column is considered too large when it is compared with that of the particle, the resistance offered by the wetted surface of the column is negligible when it is compared with the wetted surface of the packed bed.

Now,
$$\frac{\text{Volume of voids}}{\text{Volume of bed}}$$
$$= \frac{\epsilon \text{ X Volume of bed}}{\text{Volume of bed}}$$

where ϵ = void fraction.

Also, Volume of bed

$$= \frac{\text{Volume of sphere}}{1 - \epsilon}$$

In addition, $\frac{\text{Wetted surface}}{\text{Volume of bed}}$

$$= \frac{\pi D_p^2}{\frac{\pi D_p^3}{6(1-\epsilon)}}$$

$$= \frac{6(1-\epsilon)}{D_p}$$

Hence, the hydraulic radius R_h can be defined as:

$$R_h = \frac{D_p \epsilon}{6(1-\epsilon)}$$

Substituting R_h in equation 13;

$$\langle v \rangle = \frac{\Delta p g_c \epsilon^2 D_p^2}{36(1-\epsilon)^2 . 2\mu L}$$

$$= \frac{\Delta p g_c \epsilon^2 D_p^2}{72(1-\epsilon)^2 \mu L} \quad (14)$$

However, $\langle v \rangle = \frac{v_o}{\epsilon}$ (15)

where v_o = velocity of fluid if there was no packing in the column.

$$\therefore \quad v_o = \frac{\Delta p g_c \epsilon^3 D_p^2}{72(1-\epsilon)^2 \mu L} \quad (16)$$

It is assumed that the path of liquid flowing through bed is

L ft. But, this is not true since - due to the bed - it makes a zigzag path which increases the effective length L. The experimental measurements indicate that the number 72 in the denominator be replaced by 150 (1, 5). Hence, the equation 16 changes as:

$$v_o = \frac{\Delta p g_c \epsilon^3 D_p^{\ 2}}{150(1-\epsilon)^2 \mu L} \tag{17}$$

This equation is known as the Blake - Kozeny (1, 4, 5) equation and is valid for low flow rates.

For highly turbulent flow, the friction factor is only a function of roughness when Reynold's number is high. The friction factor f is also called as a drag coefficent and it is a dimensionless quantity. It is approximately a constant at higher Reynold's number. Now, for the flow of a fluid through a bed of spheres, the pressure drop Δp is as follows:

$$\Delta p = {}^{F}/{}_{Ag_c} \tag{18}$$

where F = Force exerted on the solid surfaces

and A = cross-sectional area.

Consider the fluid flowing through an empty column. The fluid will exert force F on the solid surfaces which is equal to:

$$F = A'Kf \tag{19}$$

where A' = the surface area of column or wetted surface,

K = Kinetic energy / unit volume,

and f = friction factor.

So for circular tubes of radius R and length L;

$$K = \tfrac{1}{2}\rho <v>^2 \tag{20}$$

and $A' = 2\pi RL$

$$F = (2\pi RL)(\tfrac{1}{2}\rho <v>^2)f \tag{21}$$

But $\Delta p = F/g_c A$ (18)

$$= \frac{F}{g_c \pi R^2} \tag{22}$$

So substituting and rearranging the above equations, following equation is obtained:

$$f = \frac{R}{2L}\left\{\frac{\Delta p g_c}{\tfrac{1}{2}\rho <v>^2}\right\} \tag{23}$$

Again, the hydraulic radius

$$R_h = {}^{R}/2 \ .$$

So substituting the value of R in the terms of R_h in equation 23;

$$f = \frac{R_h}{L}\left(\frac{\Delta p g_c}{\tfrac{1}{2}\rho <v>^2}\right)$$

$$\therefore \frac{\Delta p g_c}{L} = \frac{1}{R_h}\frac{1}{2}\rho <v>^2 f \tag{24}$$

But $<v> = {}^{v}{}_{o}/\epsilon$ (15)

and $R_h = \dfrac{\epsilon D_p}{6(1-\epsilon)}$

$$\frac{\Delta p g_c}{L} = \frac{3(1-\epsilon)\rho\, v_o^2 f}{\epsilon^3 D_p} \tag{25}$$

Experimental data (1, 5) indicate that

$6f = 3.5$

Hence,

$$\frac{\Delta p g_c}{L} = \frac{1.75\rho\, v_o^2 (1-\epsilon)}{\epsilon^3\, D_p} \qquad (26)$$

Equation 26 is known as the Burke-Plummer equation (1, 3, 5). When equation 17 and equation 26 are combined, following equation results:

$$\left(\frac{\Delta p g_c}{L}\right) = \frac{150\mu v_o}{D_p^2}\left(\frac{(1-\epsilon)^2}{\epsilon^3}\right) + \frac{1.75\rho\, v_o^2(1-\epsilon)}{D_p\,\epsilon^3} \qquad (27)$$

This is known as the Ergun equation (1, 5). This can be also written in terms of G, the mass flow rate and in the dimensionless groups:

$$\left(\frac{\Delta p g_c \rho}{G^2}\right)\left(\frac{D_p}{L}\right)\left(\frac{\epsilon^3}{1-\epsilon}\right) = \frac{150\mu(1-\epsilon)}{D_p G} + 1.75 \qquad (28)$$

It can be seen that in the low flow regions where most of the experiments are performed, the plot of $\log\left(\frac{\Delta p g_c \rho D_p \epsilon^3}{G^2\, L(1-\epsilon)}\right)$ versus $\log \frac{D_p G}{\mu(1-\epsilon)}$ will be a straight line with the slope of -1.

At very low flow rates or in laminar flow, $\frac{(1-\epsilon)\mu}{D_p G}$ factor is dominant in the right side of the equation 28. At very high flow rates or in turbulent flow rates, $\frac{(1-\epsilon)\mu}{D_p G}$ becomes very negligible comparing to 1.75. So,

$$\left(\frac{\Delta p g_c \rho\, D_p \epsilon^3}{G^2\, L(1-\epsilon)}\right)$$

remains constant at 1.75.

The above derived relation does not include the wall effect. If the diameter of the column is very large compared to that of the packing particles, the pressure drop and the flow rates are unaffected by the friction due to the wall.

Assume now that the diameter of the column is not large when compared with that of the packing particles. Then there will be a correction required in the hydraulic radius relation in the final equation.

Again, the ratio of the wetted surface to the colume of the bed can be written as:

$$= \frac{\text{Wetted surface of spheres + Wetted surface of wall}}{\text{Volume of the bed}}$$

$$= \frac{\text{Wetted surface of spheres}}{\text{Volume of bed}} + \frac{\text{Wetted surface of wall}}{\text{volume of bed}}$$

$$= \frac{\pi D_p^2}{\pi D_p^3/6(1-\epsilon)} + \frac{\pi D_c L}{\pi D_c^2 L/4} \qquad (29)$$

where D_c = diameter of the column.

So,

$$\frac{\text{wetted surface}}{\text{volume of bed}}$$

$$= \frac{6(1-\epsilon)}{D_p} + \frac{4}{D_c} \qquad (30)$$

$$\therefore \quad R_h = \frac{\epsilon}{\frac{6(1-\epsilon)}{D_p} + 4/D_c} \qquad (31)$$

Substituting the values of R_h in equation 13 and using the relation of equation 15, the following equation is written for v_o:

$$v_o = \frac{\epsilon^3 \Delta p g_c}{2\mu L\left(\frac{6(1-\epsilon)}{D_p} + \frac{4}{D_c}\right)^2} \qquad (32)$$

$$\text{or} \quad v_o = \frac{\epsilon^3 \Delta p g_c D_p^2}{72\mu L(1-\epsilon)^2} \left(\frac{1}{{}^4/6 \frac{D_p}{D_c(1-\epsilon)} + 1} \right)^2 \tag{33}$$

$$\text{Let} \quad M = \frac{4D_p}{6D_c(1-\epsilon)} + 1 \tag{34}$$

and 72 in the denominator be replaced by 150 as stated previously.

$$\therefore \quad v_o = \frac{\epsilon^3 \Delta p g_c D_p^2}{150(1-\epsilon)^2 \mu L \ M^2} \tag{35}$$

This can be considered as modified Blake-Kozeny equation.

Similarly, for the turbulent flow:

$$\frac{\Delta p g_c}{L} = \frac{\rho}{2R_h} \langle v \rangle^2 f \tag{24}$$

Again, substituting R_h from equation 31 and using the relation of equation 15, the equation changes to the modified Burke-Plummer equation:

$$\frac{\Delta p g_c}{L} = \frac{1.75\rho \ v_o^2(1-\epsilon) \ M}{D_p \ \epsilon^3} \tag{36}$$

Hence, by adding these two modified relations, the correlation becomes:

$$\therefore \quad \frac{\Delta p g_c}{L} = \frac{150\mu \ v_o(1-\epsilon)^2 M^2}{D_p^2 \ \epsilon^3} + \frac{1.75\rho \ v_o^2(1-\epsilon)M}{D_p \ \epsilon^3} \tag{37}$$

or in terms of G, the mass flow rate and in the dimensionless groups, equation 37 is transferred to as:

$$\left(\frac{\Delta p g_c \rho}{G^2}\right)\left(\frac{D_p}{L}\right)\left(\frac{\epsilon^3}{1-\epsilon}\right)\left(\frac{1}{M}\right) = \frac{150\mu(1-\epsilon)M}{D_p G} + 1.75 \tag{38}$$

The above equation can be stated as the modified Ergun equation. When $D_p << D_c$, M = 1. Therefore, for $D_p << D_c$ equation 38 is same equation as equation 28.

Equation 38 includes as a parameter the effect of the wall. Therefore, data for the pressure drop versus flow rate for the column in which the wall effect is important should be represented, as well as data for which the wall effect is not important.

EXPERIMENTAL EQUIPMENT

The apparatus used for this experimentation consisted of an one-half inch glass column, a pressurized flow systems, packing particles for the column, U-tube manometer, and a graduated cylinder. The schematic diagram is shown in figure 2. Nitrogen tank A was used as a pump to pressurize the flow of water from the water tank through the packed column, whereas nitrogen tank B was used as a controlling device to maintain a constant flow rate. The U-tube manometer was connected to two taps of the column which were 1.5 ft. apart. Various flow rates were obtained by adjusting the valves at the top and bottom of the column.

GLASS BEADS

The bed was packed with glass beads which were spherical and uniform in size. Figure 11 is a microphotograph of the glass beads used in this experimentation. To obtain the various ratios of the column to packing particle diameter, various sizes of the beads were used. The diameter of the column was kept constant. The beads used in this project were of uniform size, and there was no mixing of different sizes. The diameter of beads was measured using either the micrometer or the microscope.

MANOMETER FLUID

Mercury and tri-chloro ethylene were used as a manometer fluid to measure the pressure drop in the column. For the high

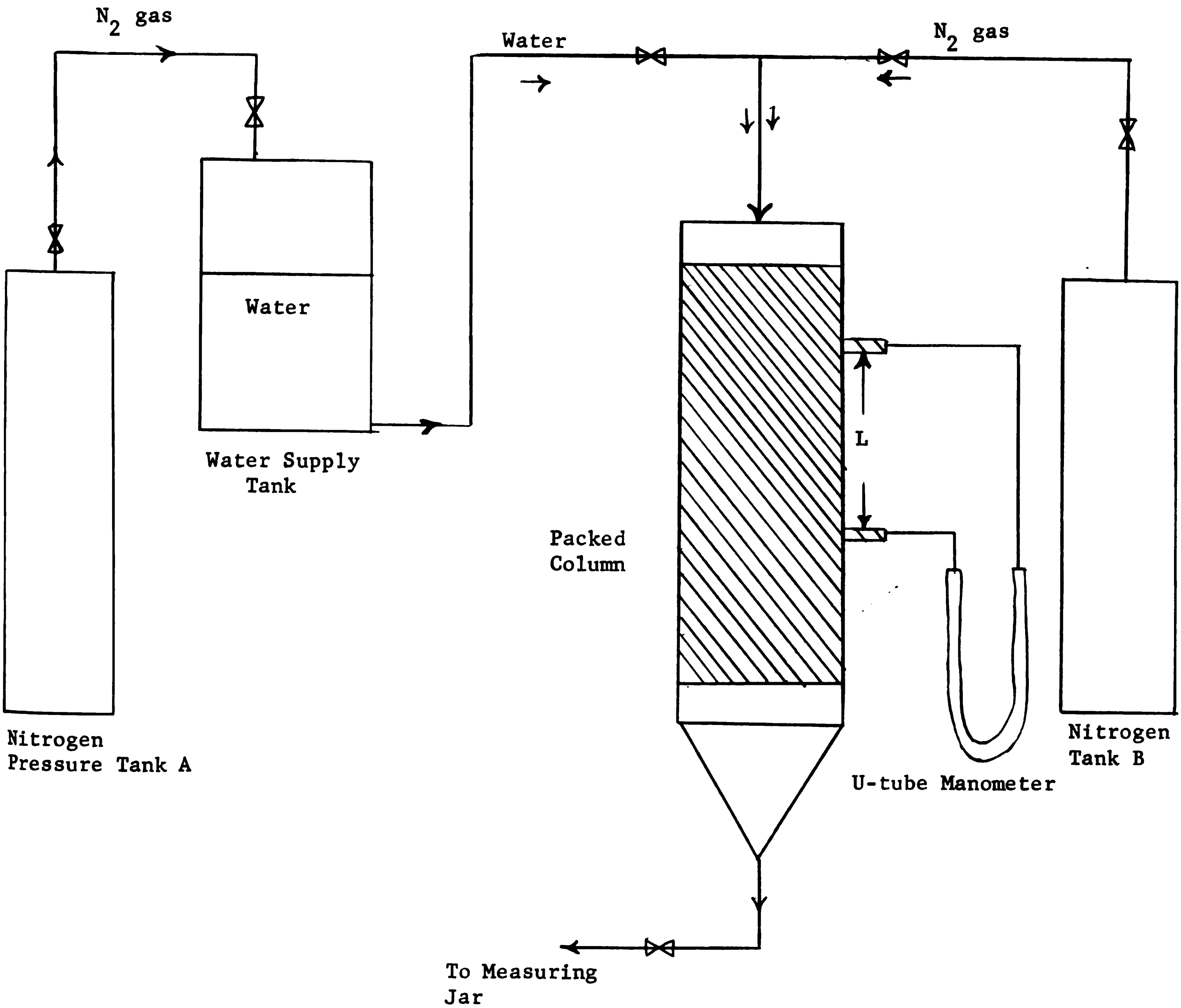

FIGURE 2

Schematic Diagram of the Equipment

pressure drop measurements, mercury was used; whereas for the low pressure drop measurements, tri-chloro ethylene was used as a manometer fluid. The difference between the liquid levels in the two arms of the manometer was measured, and from it the pressure drop in the packed column was calculated.

$$\Delta p = \frac{62.4gh(\rho' - 1)}{g_c}$$

where h = difference in the liquid levels in the manometer in feet.

g_c = gravitational constant, $\frac{\text{lbm-ft.}}{\text{lbf-sec}^2}$

g = gravitational acceleration, ft/sec^2

ρ = density of the fluid, lbm/ft^3

MEASUREMENT OF VOID FRACTION

The void factor ϵ was determined by the following procedure. First, the graduated cylinder was filled with water. The height of water level was x cms. Then, dry glass beads were poured in the cylinder. The cylinder was vibrated till the beads settled down uniformly. The water level in the cylinder rose because the beads were added. The height of the beads in the cylinder was z cms, whereas the new height of the water level was y cms. These heights are proportional to the corresponding volumes.

Apparent volume of bed = z cms.

Volume of beads = (y-x) cms.

Void fraction = $(1 - \frac{y-x}{z})$

As x, y, and z were known, ϵ was calculated accordingly.

PACKING PROCEDURE

Packing procedure was important as there was a possibility of air entrapment in the bed during packing. It was required that there be no air trapped in the column when the experiments were performed.

The glass spherical beads were soaked for a day or more in a beaker filled with water. The wet packing particles were transferred to the column which was filled with water, and care was taken to see that no air bubbles were in the system. Afterwards, the column was vibrated to let the packing particles be settled in the bed uniformly. After obtaining the data for one size beads, the beads were removed; and the column was thoroughly washed and dried. The same procedure was used for beads of different diameters.

RUN PROCEDURE

To obtain a steady flow, constant liquid level above the bed was required. Water was forced through the packed column using nitrogen pressure from the tank A as shown in figure 2. The constant liquid level above the bed was achieved by applying the gas pressure to the top of the liquid level in the column from the gas tank B. Constant flow rates, constant water level at the top of the column, and the constant manometer reading were indicators of the steady flow. The valve on gas tank B was adjusted until the above three observations remained constant. Sometimes, the valve connected at the bottom of the column was also adjusted.

After achieving the steady state conditions, the flow rate was measured by collecting the water which flowed through the column in a graduated cylinder for a specific time. Simultaneously, the corresponding pressure drop in the column was measured on the manometer. This was repeated for three or more runs, and the arithmetic average of the above readings was used to calculate the two dimensionless groups. The same procedure was applied for the different flow rates.

For each set of data, the temperature of the fluid was also noted. Corresponding to the temperature of the water, the viscosity of the water was obtained from Perry's (11) handbook. The temperature measurement of water was necessary, since the variation of the viscosity of water is significant if the small change in the temperature occurs. e.g., viscosity at $70^{o}F$ = 0.98 C.P. and viscosity at $75^{o}F$ = 0.922 C.P.

RESULTS

The values of flow rates and the corresponding values for pressure drops for various ratios of the column diameter to packed particle diameter are shown in the Appendix I. From these values of the pressure drop and the flow rate, the values of the two dimensionless groups: the friction factor, $\frac{\Delta p g_c \rho D_p \epsilon^3}{G^2 L(1-\epsilon)}$, and the Reynold's number for the packed bed, $\frac{D_p G}{\mu(1-\epsilon)}$ were calculated. These groups are designated as y and x respectively.

Ratios of the column to the particle diameter used for the experiments were

91:1, 50:1, 36:1, 25:1, 18:1, and 7.7:1.

The first three ratios are high; so D_p/D_c is almost zero, and the correction factor M becomes one. The plots of the log y versus log x should coincide with the Ergun plot. It can be seen from the Figures 3, 4, and 5 that these plots do coincide with the plot from the Ergun equation. Furthermore, it can be seen that the Ergun equation can be extended to the lower range of x = 0.1 to 1.0. Previous data did not extend into this lower range.

The last three ratios of the column to particle diameter are small. Hence, D_p/D_c is not negligible when it is compared with 1. So the factor M will be greater than 1. The modified dimensionless groups, $\frac{\Delta p g_c \rho D_p \epsilon^3}{G^2 L(1-\epsilon)M}$ and $\frac{D_p G}{\mu(1-\epsilon)M}$ will be different than the uncorrected values of y and x. Let the modified groups be designated as Y and X.

$$\therefore \quad Y = y/M \quad \text{and} \quad X = x/M.$$

It can be seen that, if the value of M is considered as 1 instead of the real value, the corresponding values of Y and X will be greater than the real values of Y and X. This will result in the shift of the graph of log Y versus log X to the right side of the plot from the Ergun equation (5). The plots are shown in Figures 6, 7, 8, and 9. It can be seen that these plots do not coincide with the plot from the Ergun equation. Furthermore, the magnitude of the deviation of the plots depends on the magnitude of the ratio D_p/D_c. The larger the ratio of D_p/D_c, the greater the deviation of the plot of log $\mathfrak{Y}$ versus log $\mathfrak{X}$ from the Ergun plot. In addition, it can be seen from the Figures 6, 7, 8, and 10 that, if the values of M are used to correct the data, plots of log Y versus log X coincide with the Ergun plot.

A plot of log Y versus log X in the set having ratio 18:1 does not exactly coincide with the Ergun plot. However, the above plot can be considered as coinciding with the Ergun plot within the experimental error. The deviation in that plot may be due to the small error in measuring the void fraction ϵ. Y contains the factor ϵ^3, a small error in the value of ϵ will change appreciably the value of Y. This may explain the deviation in the above plot.

Thus, the modified Ergun equation which includes the wall effect is valid and it does not require the assumption that the column diameter should be much larger than that of the particle diameter.

CONCLUSIONS

The equation which relates the pressure drop with the flow rate of fluids through packed beds is:

$$\left(\frac{\Delta p g_c \rho}{G^2}\right)\left(\frac{D_p}{L}\right)\left(\frac{\epsilon^3}{1-\epsilon}\right)\left(\frac{1}{\frac{4D_p}{6D_c(1-\epsilon)}+1}\right) = \frac{150(1-\epsilon)\mu}{D_p G}\left(\frac{4D_p}{6D_c(1-\epsilon)}+1\right) + 1.75$$

This equation includes the effect of the wall on the hydraulic radius as a parameter. Data for pressure drop versus flow rate through packed beds are correlated more accurately when the friction due to the wall surface area or the effect of the wall on the hydraulic radius is taken into account. This correction is particularly important when the column to particle diameter ratio is less than 50:1.

Furthermore, the range over which the above equation was verified was from a packed bed Reynold's number lower limit of 0.1 to a higher limit of 10.0.

In this project, water was used as a fluid for experimentation. To confirm that the above correlation is valid for other newtonian fluids, it is suggested that the further experiments be performed using different newtonian fluids.

In addition, it is recommended that the above equation be verified in the regions above higher limit 10.0 of packed bed Reynold's number. This will confirm that the above correlation is valid for very high flow rates.

APPENDIX A

SETS OF TABLES CONTAINING DATA

D_c/D_p	=	91:1
Diameter of the Beads	=	0.0055 inches
Diameter of the Column	=	½ inches
Void Fraction	=	0.36
Length of the Packed Column	=	1.5 ft.
Density of the Manometer Fluid	=	13.6 gms./c.cms
Temperature of Water	=	75^o F
Viscosity of Water	=	0.922 C.P.
Density of Water	=	62.3 lbm/c.ft.

Pressure drop in h inches	Flow Rate V^{cc}/sec.	x	y	X	Y
22.8	0.333	0.620	228	0.615	226
17.9	0.246	0.458	330	0.453	326
23.8	0.342	0.636	226	0.628	224
11.9	0.167	0.310	478	0.306	472
7.2	0.100	0.186	800	0.184	792
9.0	0.130	0.242	590	0.239	584
14.6	0.217	0.402	345	0.398	341
15.9	0.233	0.434	324	0.428	321
38.2	0.567	1.060	132	1.050	131

TABLE 1: Data for D_{c/D_p} = 91:1

D_c/D_p	=	45:1
Diameter of the Beads	=	0.011 inches
Diameter of the Column	=	½ inches
Void Fraction	=	0.40
Length of the Packed Column	=	1.5 ft.
Density of the Manometer Fluid	=	13.6 gms./$_{c.cms.}$
Temperature of Water	=	76^{o} F
Viscosity of Water	=	0.918 C.P.
Density of Water	=	62.4 lbm/$_{c.ft.}$

Pressure drop in h inches	Flow Rate V^{cc}/sec.	x	y	X	Y
4.8	0.383	1.53	107.0	1.49	104.5
13.0	1.133	4.52	33.0	4.42	32.2
21.0	1.783	7.12	21.6	6.96	21.1
27.2	2.083	8.22	20.2	8.04	19.7
14.9	1.268	5.07	30.4	4.95	29.6
28.4	2.265	9.05	18.0	8.84	17.5
2.4	0.195	0.78	206.0	0.76	201.0
10.0	0.808	3.20	50.0	3.10	48.9

TABLE 2: Data for D_{c/D_p} = 45:1

D_c/D_p	=	36:1
Diameter of the Beads	=	0.014 inches
Diameter of the Column	=	0.5 inches
Void Fraction	=	0.36
Length of the Packed Column	=	1.5 ft.
Density of the Manometer Fluid	=	13.6 gms./c.cms.
Temperature of Water	=	76^{o} F
Viscosity of Water	=	0.918 C.P.
Density of Water	=	62.3 lbm/c.ft.

Pressure drop in h inches	Flow Rate V^{cc}/sec.	x	y	X	Y
10.5	1.008	4.970	29.4	4.820	28.5
16.3	1.458	7.180	21.8	6.980	20.2
1.3	0.117	0.575	270.0	0.560	262.0
2.8	0.250	1.230	127.0	1.200	123.0
12.4	1.108	5.450	28.6	5.280	27.8
8.7	0.783	3.850	40.2	3.730	39.0
6.9	0.608	2.990	53.1	2.900	51.5
5.2	0.467	2.300	69.0	2.230	67.0

TABLE 3: Data for D_c/D_p = 36:1

D_c/D_p	=	25:1
Diameter of the Beads	=	0.020 inches
Diameter of the Column	=	0.5 inches
Void Fraction	=	0.40
Length of the Packed Column	=	1.5 ft.
Density of the Manometer	=	13.6 gms./$_{c.cms.}$
Temperature of Water	=	70^{o} F
Viscosity of Water	=	0.98 C.P.
Density of Water	=	62.4 lbm./$_{c.ft.}$

Pressure drop in h inches	Flow Rate V^{cc}/sec.	x	y	X	Y
5.000	1.167	8.00	20.6	7.67	19.7
4.000	1.000	6.85	23.8	6.56	22.8
0.425	0.108	0.74	236.0	0.71	226.0
1.200	0.287	1.97	86.8	1.88	83.0
2.600	0.633	4.33	38.5	4.15	36.8
3.200	0.750	5.14	34.0	4.92	32.6
4.700	1.116	7.66	22.3	7.34	21.3
6.000	1.367	9.38	19.1	8.99	18.3

TABLE 4: Data for D_{c/D_p} = 25:1

D_c/D_p	=	18:1
Diameter of the Beads	=	0.028 inches
Diameter of the Column	=	0.5 inches
Void Fraction	=	0.39
Length of the Packed Column	=	1.5 ft.
Density of the Manometer Fluid	=	1.466 gms./$_{c.cms.}$
Temperature of Water	=	72^{o} F
Viscosity of Water	=	0.9579 C.P.
Density of Water	=	62.4 lbm./$_{c.ft.}$

Pressure drop in h inches	Flow Rate V^{cc}/sec.	x	y	X	Y
46.0	0.600	5.78	36.0	5.42	33.8
34.0	0.450	4.32	47.8	4.06	44.9
30.0	0.390	3.75	55.8	3.53	52.0
40.5	0.540	5.20	39.2	4.90	34.8
26.0	0.340	3.27	63.5	3.08	59.7
6.4	0.087	0.83	242.0	0.78	228.0
3.6	0.047	0.45	468.0	0.42	444.0
16.3	0.220	2.11	95.5	1.98	90.0
10.4	0.130	1.25	174.0	1.18	164.0

TABLE 5: Date for D_{c}/D_{p} = 18:1

D_c/D_p	=	7.7:1
Diameter of the Beads	=	0.065 inches
Diameter of the Column	=	0.5 inches
Void Fraction	=	0.415
Length of the Packed Column	=	1.5 ft.
Density of the Manometer Fluid	=	1.466 gms./$_{c.cms.}$
Temperature of Water	=	72^{o} F
Viscosity of Water	=	0.9579 C.P.
Density of Water	=	62.4 lbm./$_{c.ft.}$

Pressure drop in h inches	Flow Rate V^{cc}/sec.	x	y	X	Y
4.90	0.420	9.70	22.1	8.45	19.3
2.60	0.230	5.31	39.4	4.63	34.3
3.00	0.250	5.77	38.0	5.02	33.0
1.80	0.160	3.69	56.0	3.21	48.7
3.60	0.250	8.08	23.4	7.05	20.4
0.90	0.065	1.50	167.0	1.31	148.0
1.40	0.105	2.42	97.0	2.11	84.5
1.50	0.130	3.00	70.5	2.62	61.5
0.65	0.440	1.02	266.0	0.89	232.0

TABLE 6: Data for D_{c}/D_{p} = 7.7:1

APPENDIX B

PLOTS OF THE FRICTION FACTOR VERSUS REYNOLD'S NUMBER OF THE PACKED BED

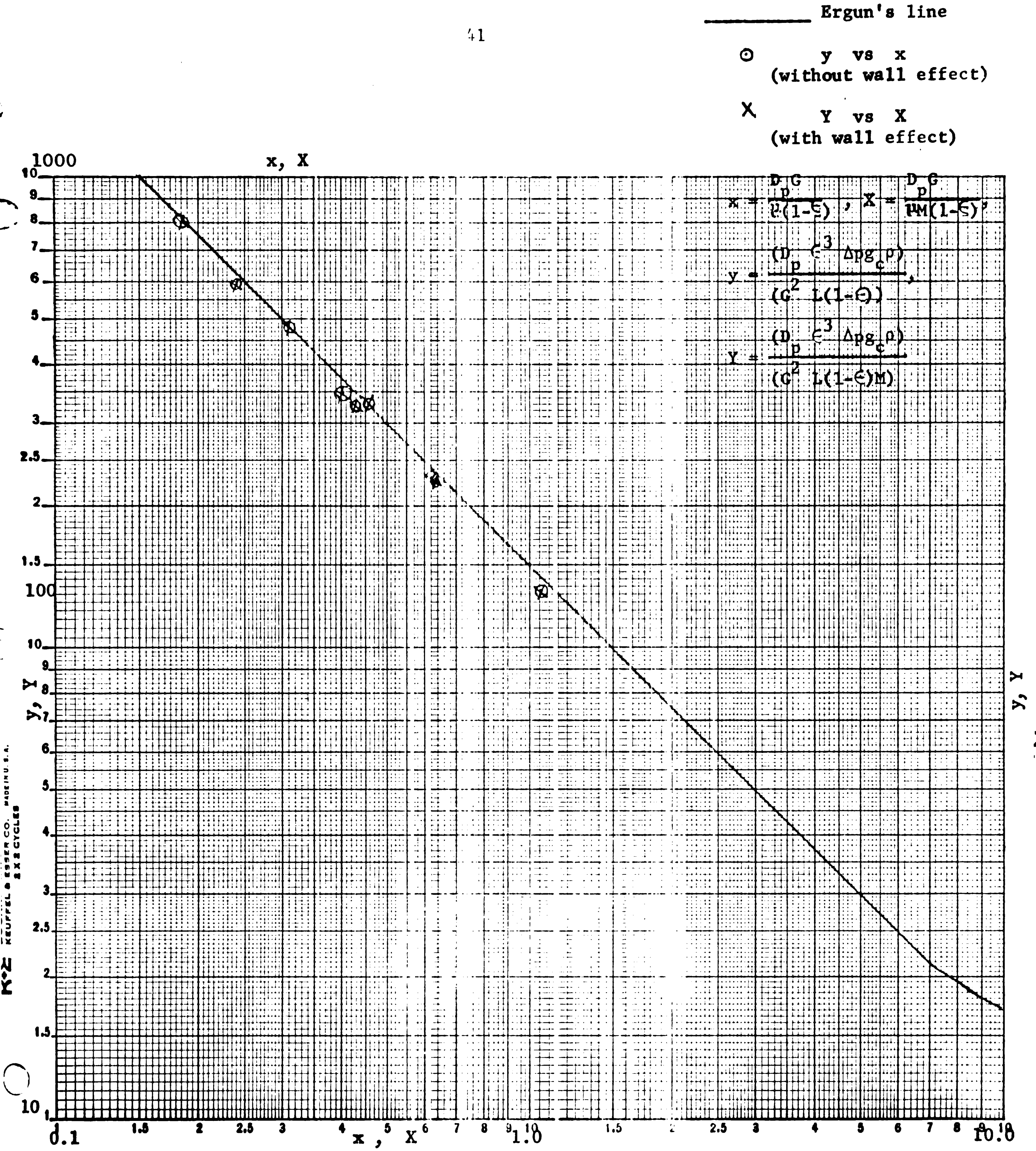

Figure 3: Plot of the friction factor versus Reynold's number of the packed bed for D_{c/D_p} = 91:1.

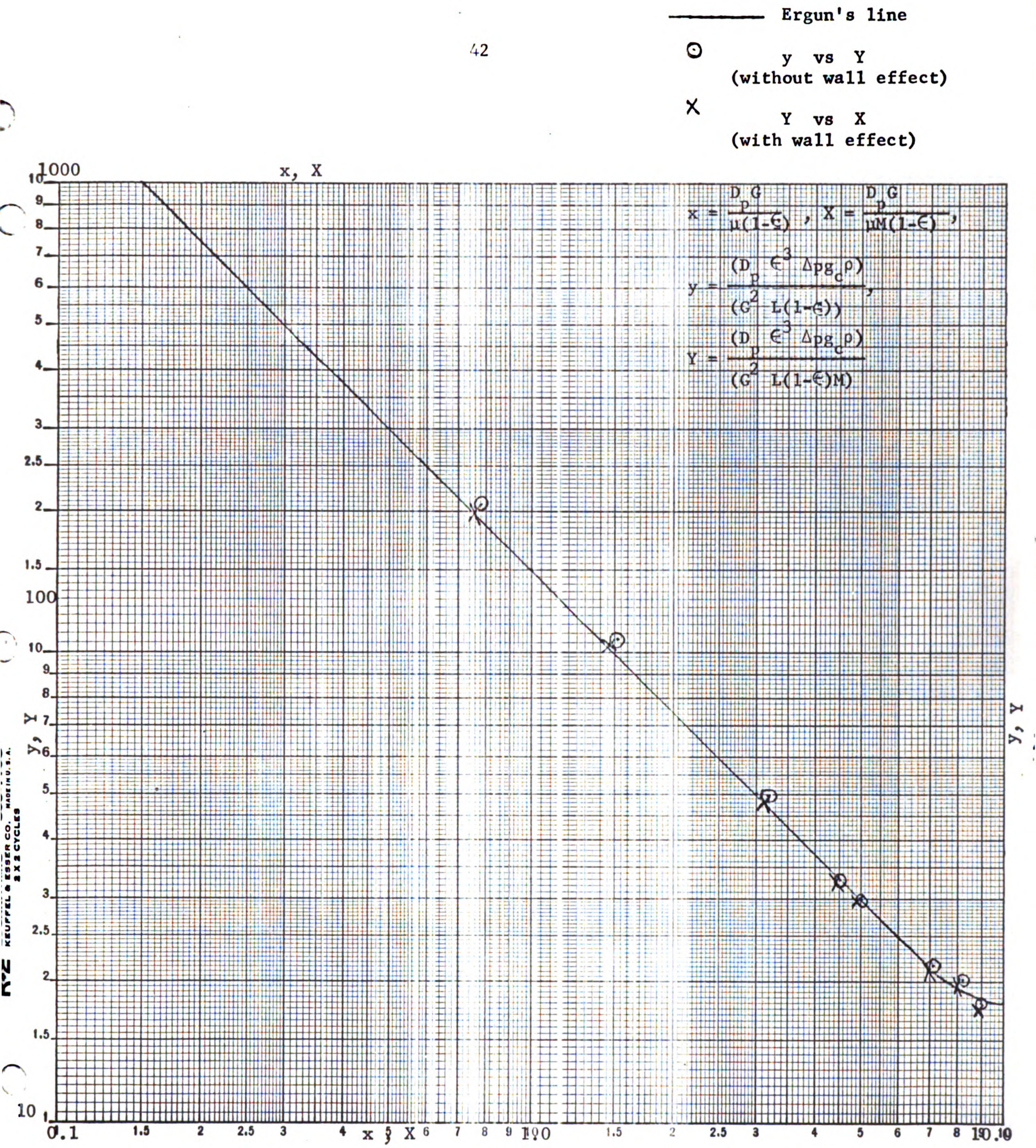

Figure 4: Plot of the friction factor versus Reynold's number of the packed bed for D_c/D_p = 45:1.

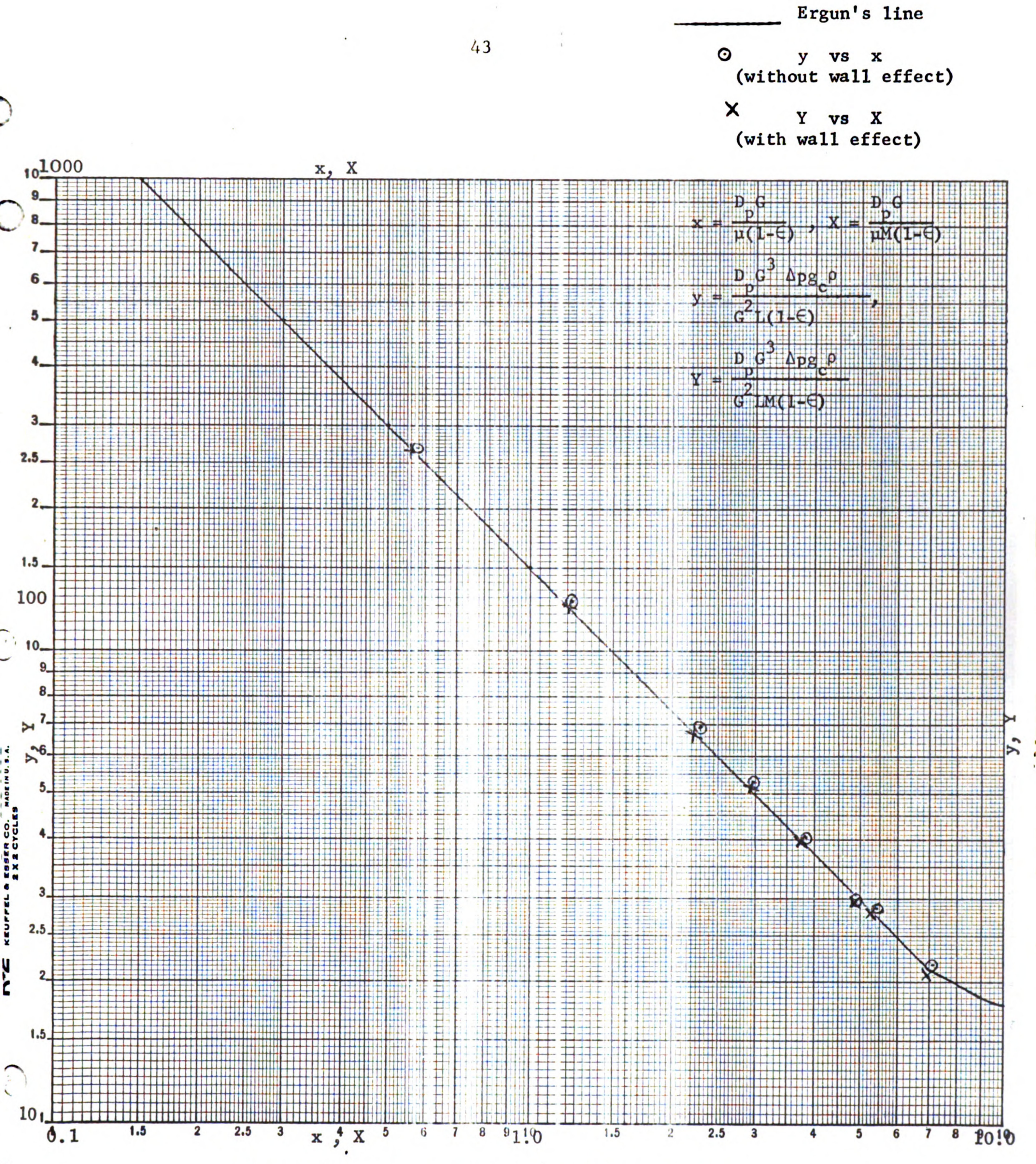

Figure 5: Plot of the friction factor versus Reynold's number of the packed bed for D_c/D_p = 36:1

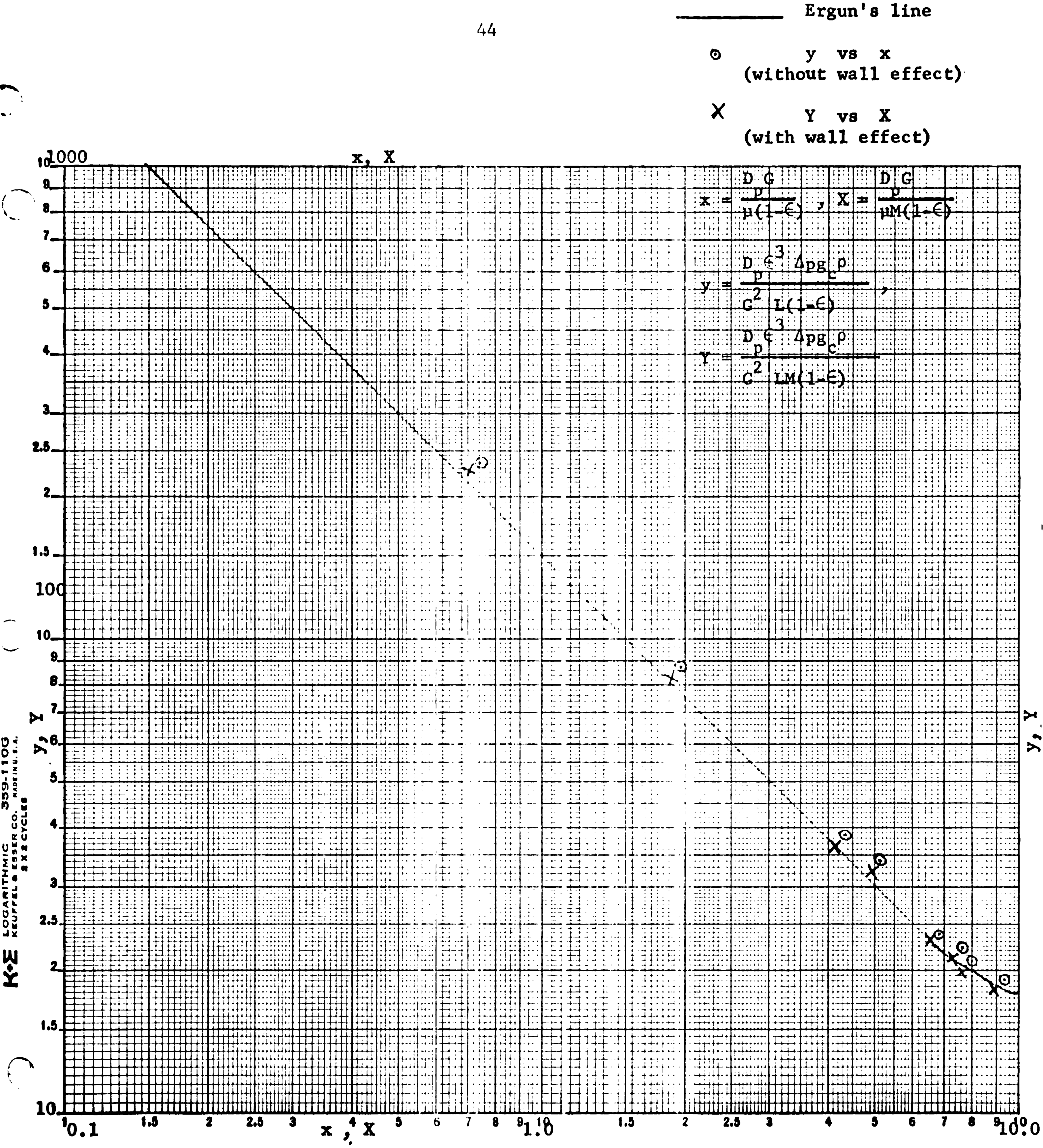

Figure 6: Plot of the friction factor versus Reynold's number of the packed bed for D_{c}/D_{p} = 25:1

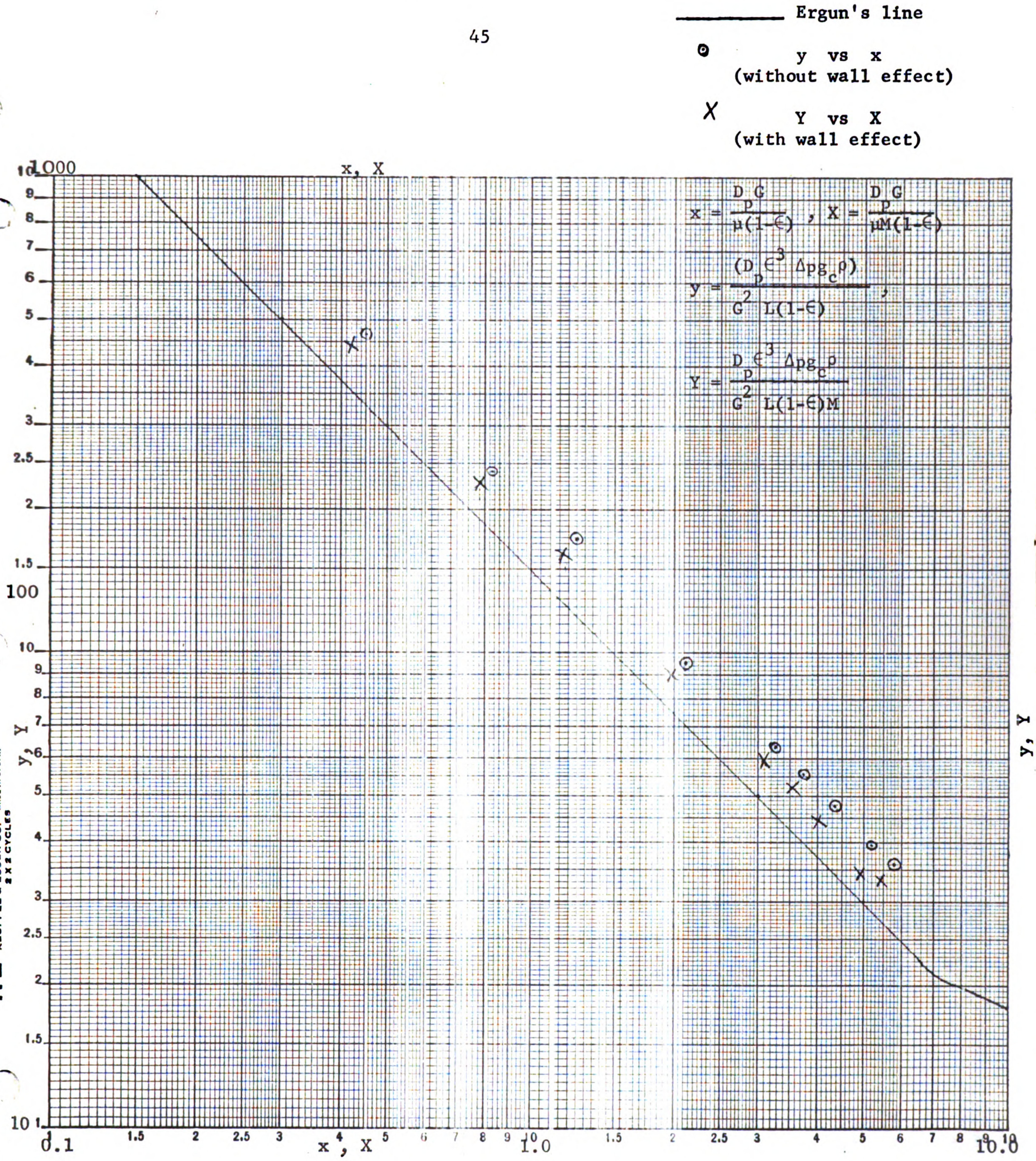

Figure 7: Plot of the friction factor versus Reynold's number of the packed bed for $D_c/D_p = 18:1$

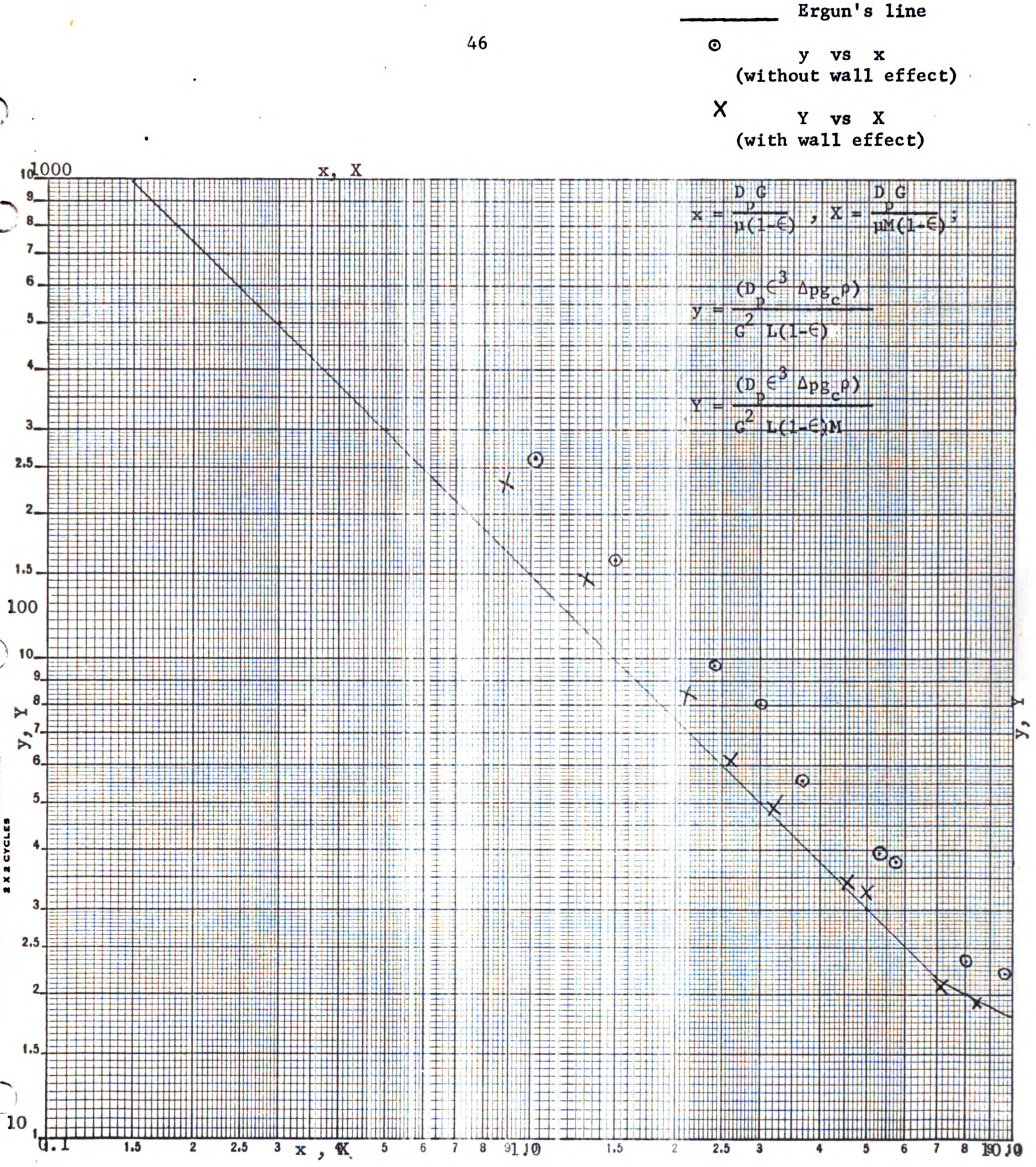

Figure 8: Plot of the friction factor versus Reynold's number of the packed bed for $D_c/D_p = 7.7:1$.

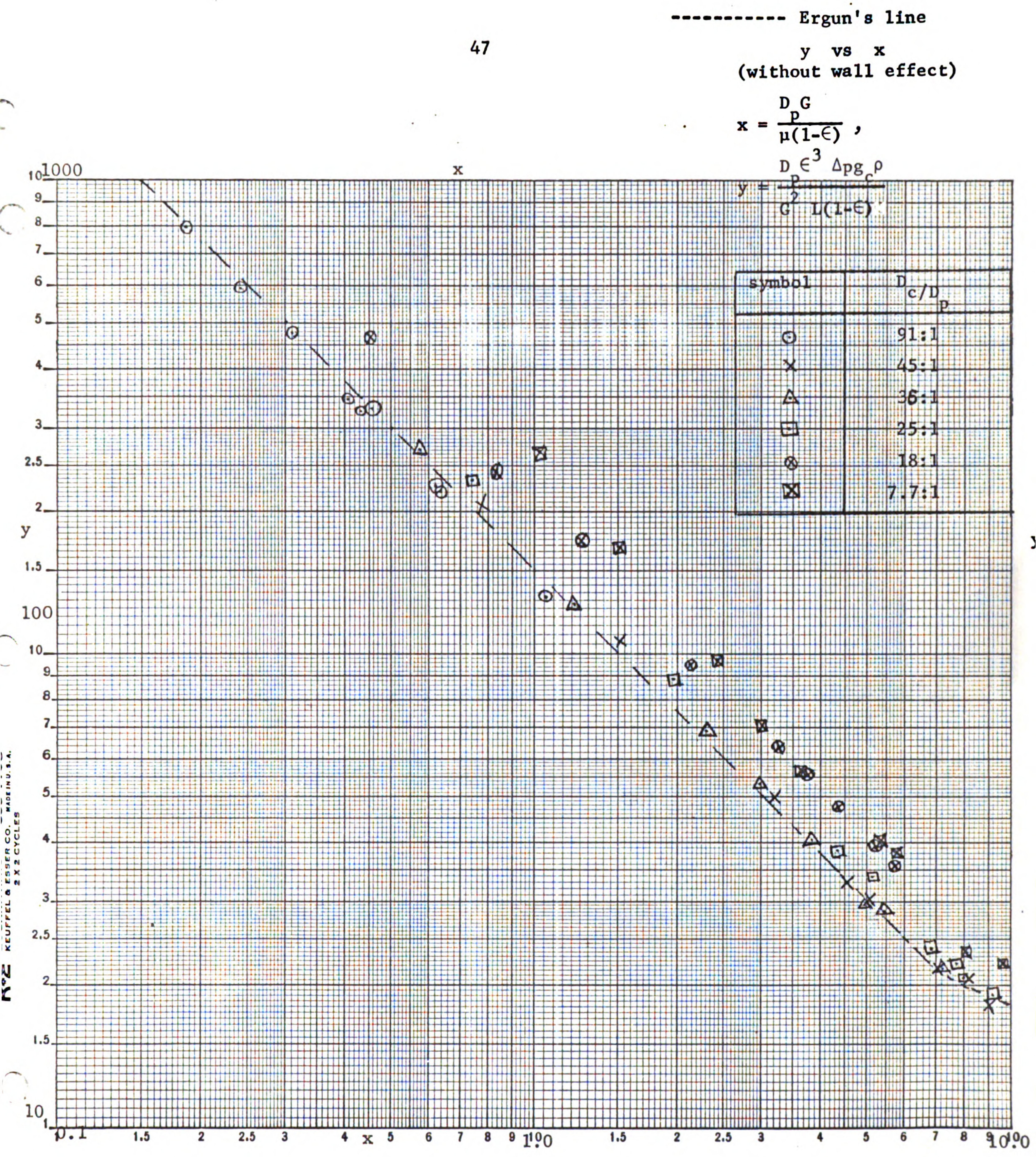

Figure 9: Plot of the friction factor versus Reynold's number of the packed bed for various D_{c}/D_{p}, wall effect neglected.

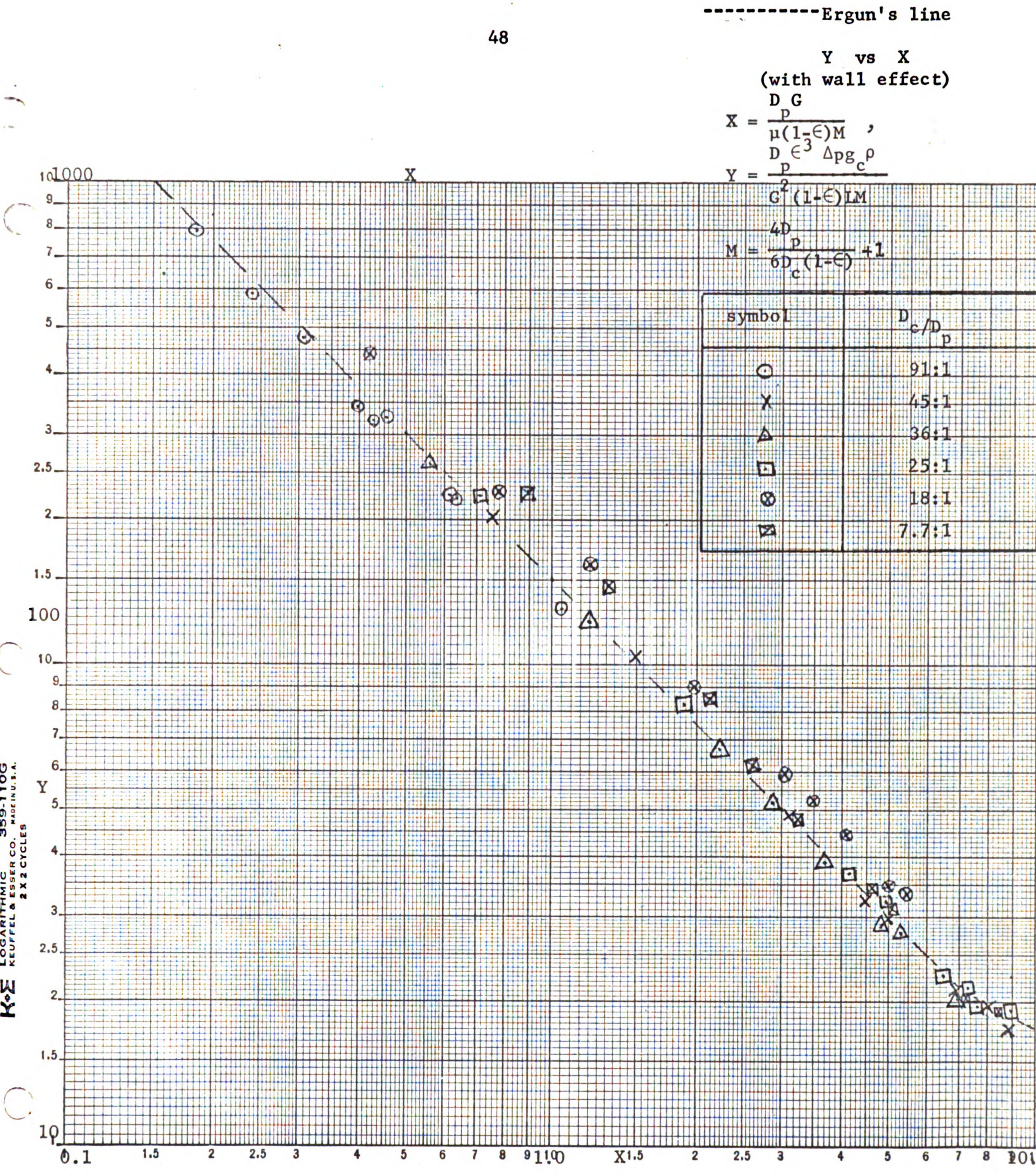

Figure 10: Plot of the friction factor versus Reynold's number of the packed bed for various D_{c/D_p}, wall effect included.

APPENDIX C

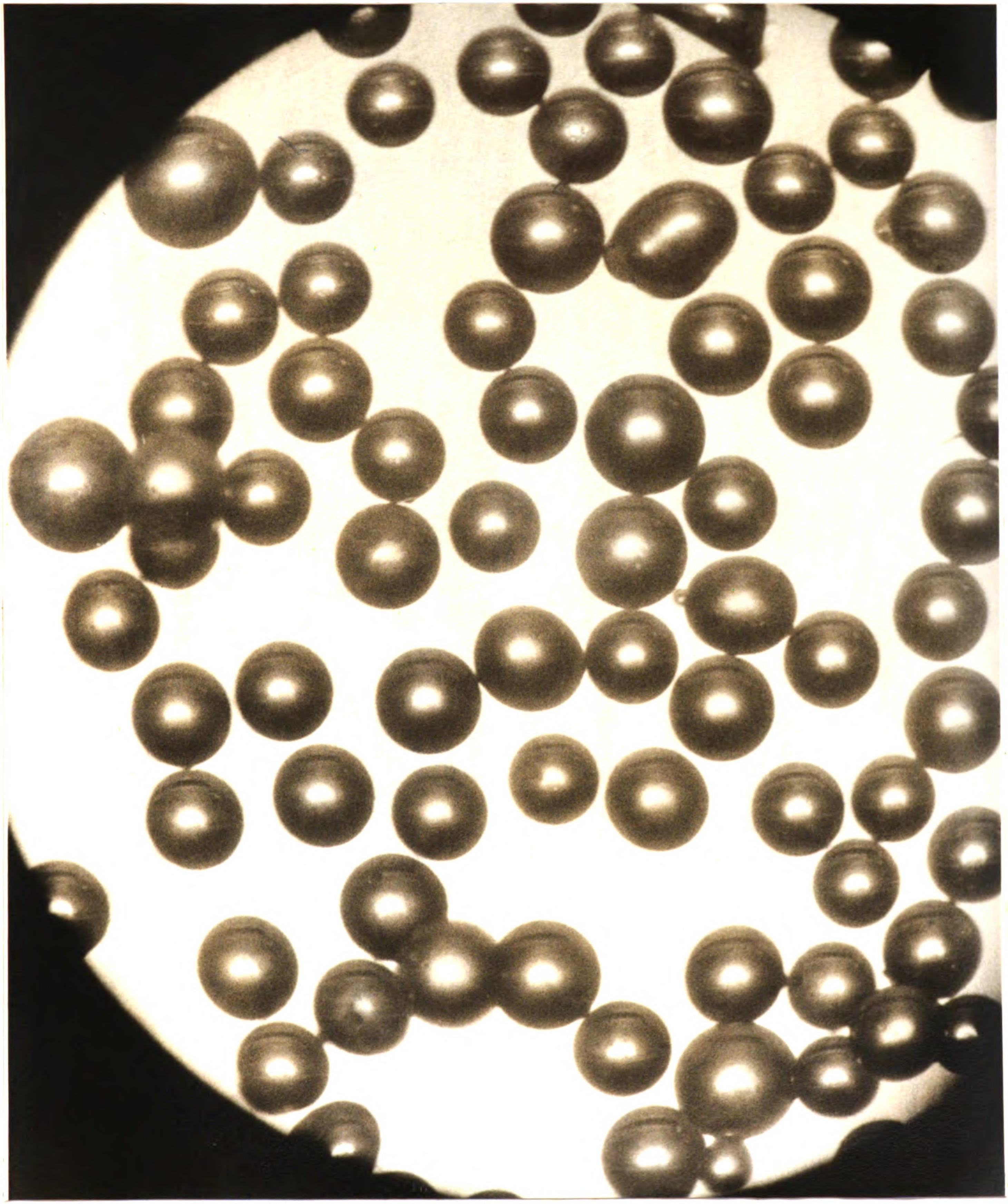

FIGURE 11: MICROPHOTOGRAPH OF THE GLASS BEADS

NOMENCLATURE

A	=	Cross-sectional area of the column.
A'	=	Surface area.
a	=	Constant in Reynold's equation.
b	=	Constant in Reynold's equation.
D_c	=	Diameter of the column.
D_e	=	Diameter of the container.
D_p	=	Diameter of the glass beads.
f	=	Friction factor.
F	=	Force exerted by the fluid.
G	=	Mass flow rate.
g	=	Gravity acceleration.
g_c	=	Gravitational constant.
h	=	Difference in the liquid levels in the manometer.
j,k	=	Proportionality constants.
L	=	Length of the bed.
M	=	Correction factor.
R	=	Radius of the cylinder.
R_h	=	Hydraulic radius.
s	=	Surface area of bed / Unit volume of bed.
$\langle v \rangle$	=	Average velocity of the fluid in a cylinder.
v_e	=	Velocity of the fluid in channels.
v_o	=	Apparent flow rate of the fluid.
v_z	=	Velocity of the fluid in z direction.
x	=	Modified Reynold's number, wall effect neglected.
X	=	Modified Reynold's number, wall effect included.

y	=	Modified friction factor, wall effect neglected.
Y	=	Modified friction factor, wall effect included.
Δp	=	Pressure drop.
ϵ	=	Void fraction.
μ	=	Viscosity of the fluid.
ρ	=	Density of the fluid.
ρ'	=	Density of the manometer fluid.

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