SIMULATION OF COUNTER-FLOW DRYING

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ABSTRACT

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by

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A theoretical model of counter-flow drying of biological products was developed. The model was solved by invariant imbedding, making certain estimations. A new mathematical technique was developed (invariant programming), which allowed direct solution of the model without trial and error. The theoretical results compared well with the experimental data of Ives (1967).

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SIMULATION OF COUNTER-FLOW DRYING

by

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NOMENCLATURE

A	cross-sectional area of the dryer, ft^2
D	Chu's diffusion coefficient, ft^2/hr
G_a	air flow rate, lbm dry air/hr ft^2
G_p	product flow rate, lbm dry product/hr ft^2
H	air humidity ratio, lbm water vapor/lbm dry air
$\overline{M}$	average product moisture content, lbm water/lbm dry product
M_1, M_2, M_3	local product moisture contents, lbm water/lbm dry product
P_{atm}	atmospheric pressure, psi
P_{sat}	saturated vapor pressure, psi
R	feasible set of outlet product properties
S	feasible set of outlet air properties
T	air temperature, F
XINF	equilibrium moisture content, lbm water/lbm dry product
W	width of the dryer, ft
a	length of the dryer, ft
a'	specific surface area of the product, ft^2/ft^3
c	specific heat of liquid water, Btu/lbm F
c_a	specific heat of dry air, Btu/lbm F
c_p	specific heat of dry product, Btu/lbm F
c_v	specific heat of water vapor, Btu/lbm F
h	convective heat transfer coefficient, Btu/hr ft^2 F

h_D	convective mass transfer coefficient, ft/hr
h_{fg}	latent heat of evaporation, Btu/lbm water
rh	relative humidity, decimal
x	coordinate distance from product inlet, ft
x'	arbitrary position within the dryer, ft
z	coordinate from the surface of the kernel, ft
ε	porosity, ft^3 product/ft^3 total
ρ	density of particles, lbm dry product/ft^3
θ	product temperature, F

as subscripts:

i	inlet
o	outlet

as superscripts:

*	minimizes the dimensionless norm
'	desired value

INTRODUCTION

Heated-air grain dryers are used on many farms and elevators in the Midwest. These dryers have been typically of the deep bed type. Due to the seasonal nature of grain drying most farmers and elevator owners are not capable of handling the magnitude of grain to be dried during the harvest season for economic reasons. Attempts have been made to speed up the drying process by increasing the inlet air temperature. This process usually lowers the quality of the dried grain. The deep bed grain dryer requires time for dumping, cleaning and refilling. Since continuous-flow dryers do not require a shut-down time, their use will speed up the total drying process.

Another problem of the stationary deep bed grain dryer is that the moisture content of the grain is not uniform throughout the bed depth. The product is overdried at the air's inlet, but is underdried at the air exit. This is also true of the continuous-flow cross-flow grain dryer. However, the concurrent and the counter-flow dryers exit the grain at a uniform moisture content.

It was observed by Thompson (1967) that counter flow dryers remove more moisture per foot of dryer depth than the other continuous-flow dryers. It should also be noted that the counter-flow dryer makes less efficient use of the internal energy of the inlet air, since a larger quantity of the air's energy is used to heat the grain and thus, less energy is available for the evaporation of moisture.

In order to optimize a given drying system, a concise model of the dryer is essential. Deep bed grain dryers have been successfully modeled for a wide range of inlet and initial condition by Thompson (1967) and Bakker-Arkema et al. (1969). Thompson's model utilizes an empirical drying rate equation. Bakker-Arkema considers the diffusion of moisture within the kernel using a variable diffusion coefficient for corn. Thompson has had moderate success in modeling of concurrent and cross-flow grain dryers. The Bakker-Arkema model should work well when extended to these two systems.

It is more difficult to model the counter-flow dryer than either the cross-flow or the concurrent-flow grain dryers. In a counter-flow dryer the product enters at one end of the dryer, while the air enters the system at the other end. Therefore, both the air and the product properties are not known at any one location. Thompson solved this problem by assuming that the air temperature is equal to the product temperature for all points within the system. This assumption will yield accurate results only if the thermal capacity of the product flow is approximately equal to the thermal capacity of the air flow.

Ives (1967) considered a special family of counter flow dryers, in which the air flow rate is much greater than the product flow rate. He applied thermodynamic relationships to determine the equilibrium values for the outlet air temperature and humidity ratio. The usefulness of his research is limited to dryers in which the outlet air and product properties have come to equilibrium.

OBJECTIVES

The objectives of this research are the following:

1) To study the theoretical mechanisms of counter-flow drying of biological products.

2) To develop a mathematical model of this system.

3) To develop a concise and direct solution of the model.

MODEL OF SYSTEM

Mathematical modeling of counter-flow drying consists of formulating balances of the conserved properties (mass and energy) of the system. The equations resulting from these balances will represent the system if the correct assumptions are made. The assumptions made in this model are the following:

a) Temperature and moisture ratios are the controlling potentials.

b) The temperature gradients within the particles are negligible.

c) The temperature and moisture gradients in the y and z directions are zero.

d) The conduction by particle to particle contact is negligible.

e) The system is in steady state.

In the following balance six dependent variables are considered: product temperature (θ), three local product moisture contents (M_1, M_2 and M_3), air humidity ratio (H) and air temperature (T). All of these variables are functions of the position within the dryer (x) only.

In Figure 1, an elemental volume of the system, $A\Delta x$, is isolated for consideration. Since it is assumed that the system is in steady state only the input and output air and product properties are essential for the evaluation of these balances.

Within the differential volume there are two modes of energy transfer; changes in internal energy of the product and the air and

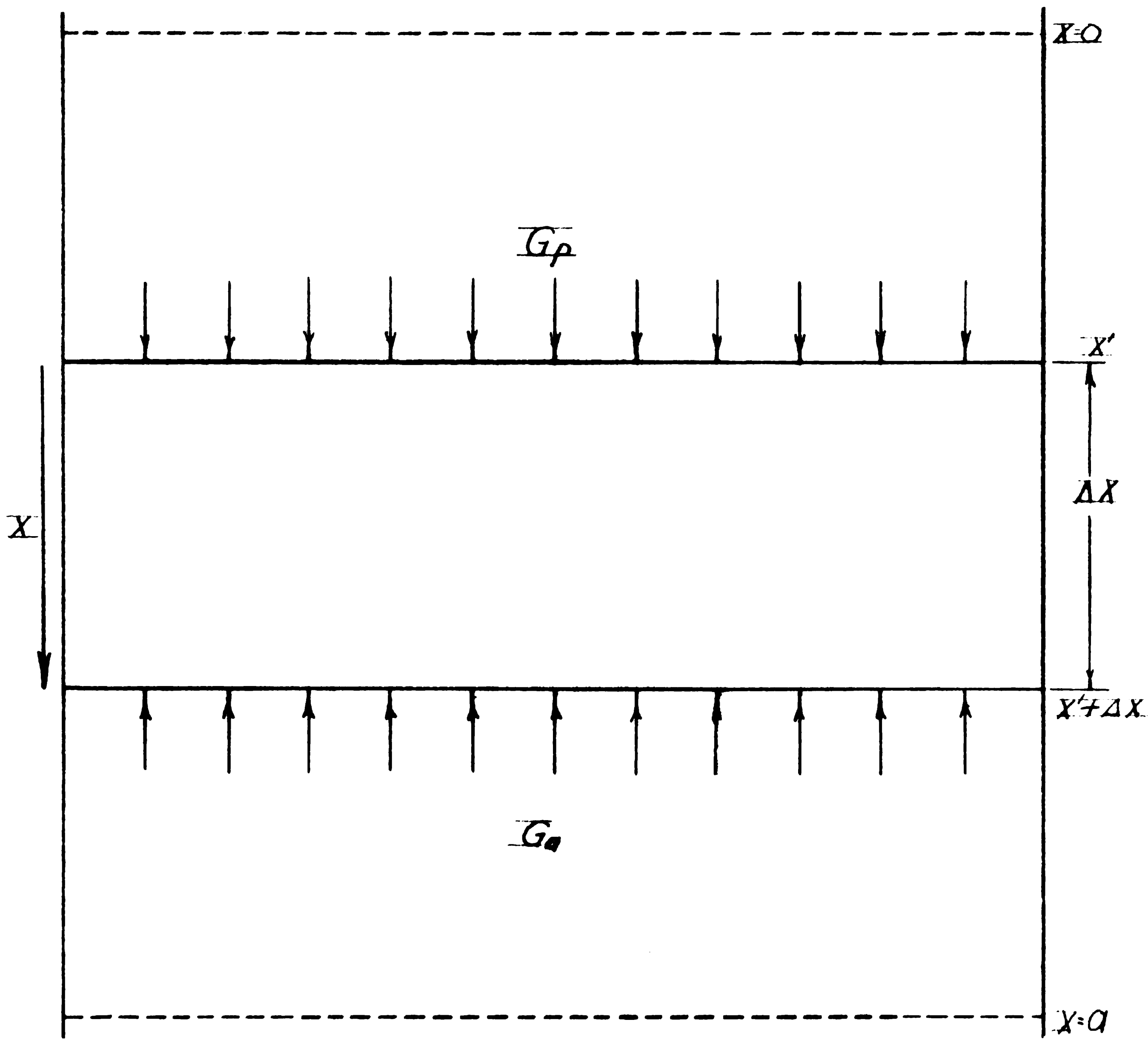

Figure 1. An Elemental Dryer Volume

latent energy of evaporation. The incoming air at $x' + \Delta x$ has an internal energy which can be expressed as:

$$(c_a G_a + c_v H G_a) A T_{x' + \Delta x} \tag{1}$$

When the air exist the elemental volume at x', its internal can be represented by:

$$(c_a G_a + c_v H G_a) A T_{x'} \tag{2}$$

The entering product at x' has an internal energy of:

$$(c_p G_p + c \bar{M} G_p) A \theta_{x'} \tag{3}$$

The internal energy of the outlet product can be written as:

$$(c_p G_p + c \bar{M} G_p) A \theta_{x' + \Delta x} \tag{4}$$

The latent energy of the entering air can be expressed as:

$$A G_a h_{fg} H_{x' + \Delta x} \tag{5}$$

The air exiting the control volume has a latent energy equal to:

$$A G_a h_{fg} H_{x'} \tag{6}$$

Since it was assumed that the system is in steady state, the inputs to the elemental volume must equal the outputs. Therefore, equation (1) plus equation (3) plus equation (5) is equal to equation (2) plus equation (4) plus equation (6). Combining similar terms yields:

$$(c_a G_a + c_v H G_a)(T_{x' + \Delta x} - T_{x'}) = (c_p G_p + c \bar{M} G_p)$$

$$(\theta_{x' + \Delta x} - \theta_{x'}) - G_a h_{fg} (H_{x' + \Delta x} - H_{x'}) \qquad (7)$$

Dividing equation(7) by Δx and allowing Δx to approach zero, results in equation (8):

$$\frac{dT}{dx} = \frac{c_p G_p + c \bar{M} G_p}{c_a G_a + c_v H G_a} \frac{d\theta}{dx} - \frac{G_a h_{fg}}{c_a G_a + c_v H G_a} \frac{dH}{dx} \qquad (8)$$

Consider an energy balance for a single particle of grain within the elemental volume. There is an inflow of heat to the product due to convection at the particle surface:

$$a' h \; (T_{x' + \frac{1}{2}\Delta x} - \theta_{x' + \frac{1}{2}\Delta x}) \; \Delta x \; A \qquad (9)$$

The product's internal energy at x' and the product's internal energy at $x' + \Delta x$ have been previously expressed by equations (3) and (4), respectively. The change in internal energy of the product in passing from x' to $x' + \Delta x$ is equal to the energy gained by convection. Therefore, equation (4) minus equation (3) equals equation (9):

$$(c_a G_a + c \bar{M} G_p)(\theta_{x' + \Delta x} - \theta_{x'}) = a'h (T_{x' + \frac{1}{2}\Delta x} - \theta_{x' + \frac{1}{2}\Delta x}) \Delta x \quad (10)$$

Dividing equation (10) by Δx and allowing Δx to approach zero, yields:

$$\frac{d\theta}{dx} = \frac{a'h}{c_p G_p + c \bar{M} G_p} (T - \theta) \quad (11)$$

Due to the asumption of steady state, the mass lost by the product in the elemental volume must equal the mass gained by the air. The moisture held by the product at x' is equal to:

$$G_p A \bar{M}_{x'} \quad (12)$$

The mass of water contained by the product at $x' + \Delta x$ is:

$$G_p A \bar{M}_{x' + \Delta x} \quad (13)$$

The total water in the form of vapor held by the air at $x' + \Delta x$ can be expressed as:

$$G_a A H_{x' + \Delta x} \quad (14)$$

The mass of water vapor held by the air exiting the elemental volume can be represented by:

$$G_a A H_{x'} \quad (15)$$

Performing a mass balance the inlet mass equals the outlet mass. Therefore, equation (12) plus equation (14) must equal equation (13) plus equation (15). Combining similar terms yields:

$$G_a (H_{x' + \Delta x} - H_{x'}) = G_p (\bar{M}_{x' + \Delta x} - \bar{M}_{x'}) \quad (16)$$

Again, dividing equation (16) by Δx and allowing Δx to approach zero, results in the following equation:

$$\frac{dH}{dx} = \frac{G_p}{G_a} \frac{d\bar{M}}{dx} \tag{17}$$

The corn kernel was modeled as an infinite flat plate. In Figure 2 an elemental volume $4\Delta z \Delta x W$ is isolated for consideration. Three node positions were chosen within the flat plate: the surface, the midway point between the surface and the center of the flat plate, and the center.

The surface node represents a volume of $\frac{1}{2}\Delta z \Delta x W$. There are inputs of moisture to this elemental volume: the moisture of the product at x' and the diffusion of water from the interior portions of the kernel. The moisture of the product x' can be represented by:

$$G_p \; (\Delta z \;\; W/2\varepsilon) \; (M_1)_{x'} \tag{18}$$

The diffusion of moisture from the interior portions of the kernel can be expressed as:

$$(\rho \; D\Delta x \; W) \; [(M_2)_{x' + \frac{1}{2}\Delta x} - (M_1)_{x' + \frac{1}{2}\Delta x}] \; / \; \Delta z \tag{19}$$

There are also outputs of moisture from the elemental volume: the moisture of the product at $x' + \Delta x$ and the convection at the surface of the kernel. The moisture of the product at $x' + \Delta x$ can be represented by:

$$G_p \; (\Delta z \; W/2\varepsilon) \; (M_1)_{x' + \Delta x} \tag{20}$$

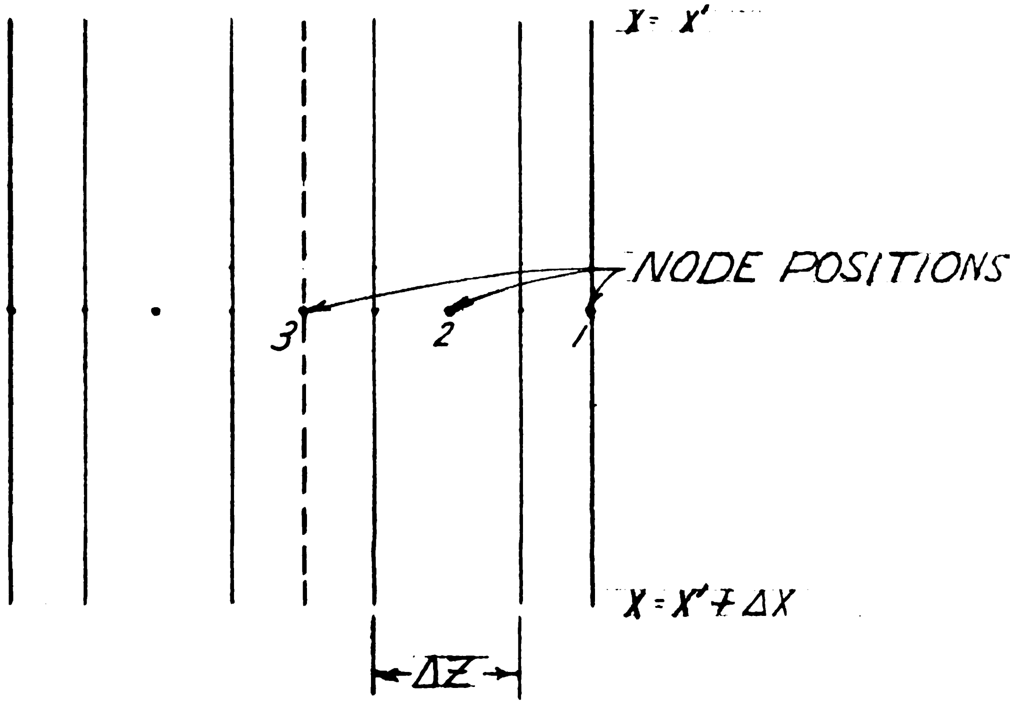

Figure 2. Grain Kernel

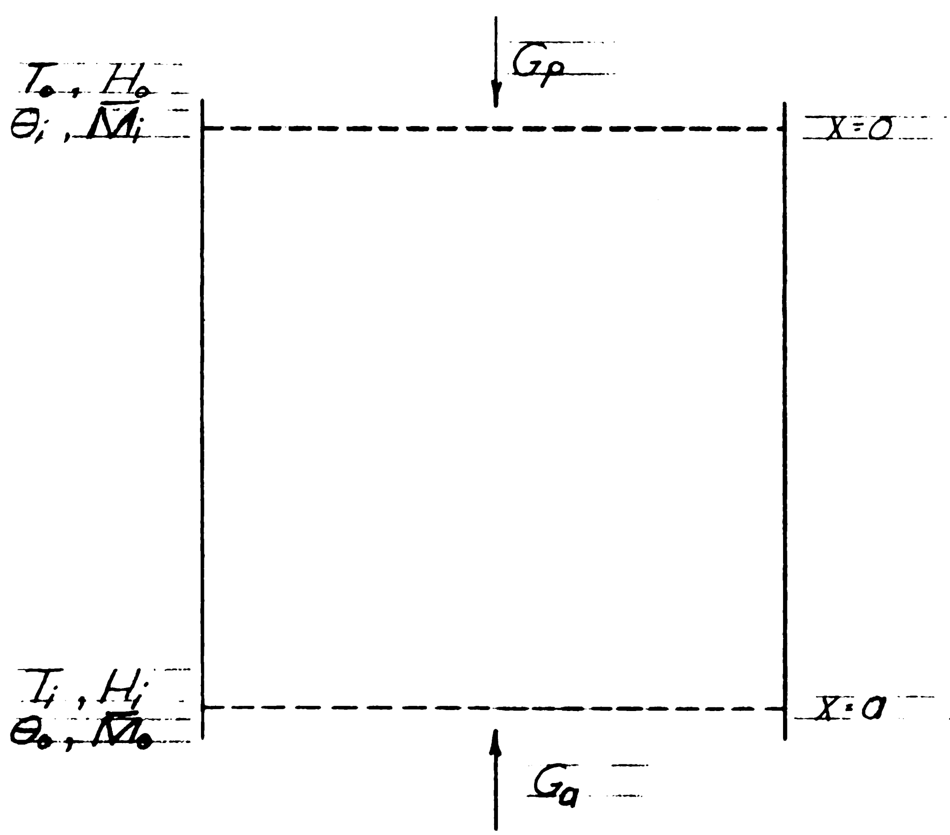

Figure 3. Counter-Flow Dryer

The convection of water at the surface of the kernel can be modeled as:

$$(\rho\ h_D\ \Delta x\ W)\ [(M_1)_{x' + \frac{1}{2}\Delta x} - XINF_{x' + \frac{1}{2}\Delta x}] \tag{21}$$

Performing a mass balance the inputs must equal the outputs. Therefore, equation (18) plus equation (19) equals equation (20) plus equation (21):

$$G_p(\Delta z/2\varepsilon)\ [(M_1)_{x' + \Delta x} - (M_1)_{x'}] = (\rho D \Delta x)\ [(M_2)_{x' + \frac{1}{2}\Delta x} - (M_1)_{x' + \frac{1}{2}\Delta x}]/\Delta z + (\rho h_D \Delta x)\ [XINF_{x' + \frac{1}{2}\Delta x} - (M_1)_{x' + \frac{1}{2}\Delta x}] \tag{22}$$

Dividing equation (22) by Δx and allowing Δx to approach zero, yields:

$$\frac{dM_1}{dx} = \frac{2\rho D\varepsilon}{(\Delta z)^2 G_p}(M_2 - M_1) + \frac{2\rho h_D \varepsilon}{\Delta z G_p}(XINF - M_1) \tag{23}$$

XINF, which was introduced in equation (21) is calculated from the empirical moisture equilibrium isotherm equation developed by Thompson (1968) from the data by Rodriquez - Arias (1963):

$$XINF = 0.01\ \{-\ln\ (1-rh)/[3.82 \times 10^{-5}\ (\theta + 50)]\}^{\frac{1}{2}} \tag{24}$$

Where rh is the relative humidity:

$$rh = HP_{atm}/[P_{sat}\ (H + .622)\ ] \tag{25}$$

D represents Chu's (1968) diffusion coefficient for corn which is a function of the product temperature and the average moisture content of the corn kernel:

$$D = 0.001629\ EXP\ [\ (0.00045\ \emptyset - 0.05485)\ 100.0\ \bar{M} - 2513.0/\emptyset\ ] \tag{26}$$

where $\emptyset$ is θ expressed in degrees Kelvin. It should be noted here that the Chu's diffusion coefficient was determined from data between the product temperatures 120 F and 160 F. It has been shown by earlier research (Bakker-Arkema et al., 1969) that this coefficient fits thin layer drying data down to 100 F, but one cannot assert that Chu's diffusion coefficient would also represent data above 160 F.

The node located at the midpoint between the surface and the center in Figure 2 has a volume of $\Delta x \Delta z W$. There is an input of moisture to this elemental volume at x'. This input is the inflow of moisture carried by the inlet product, which can be represented as:

$$(G_p \Delta z W/\epsilon)\ (M_2)_{x'} \tag{27}$$

There is also an inflow of mass due to diffusion from the innermost node. This diffusion of mass can be expressed as:

$$(\rho D \Delta x W)\ [\ (M_3)_{x' + \frac{1}{2}\Delta x} - (M_2)_{x' + \frac{1}{2}\Delta x}\ ]\ /\ \Delta z \tag{28}$$

The diffusion of moisture from the node being analyzed to the surface node has been previously expressed by equation (19). The outlet product at $x' + \Delta x$ contains the quantity of moisture expressed by the following equation:

$$(G_p \Delta z W/\epsilon)\ (M_2)_{x' + \Delta x} \tag{29}$$

Formulating a mass balance, equation (27) plus equation (28) equals equation (19) plus equation (29). Combining similar terms yields:

$$(G_p \Delta z/\epsilon)\ [(M_2)_{x' + \Delta x} - (M_2)_{x'}] = (\rho D \Delta x)\ [(M_3)_{x' + \frac{1}{2}\Delta x}$$

$$-2(M_2)_{x' + \frac{1}{2}\Delta x} + (M_3)_{x' + \frac{1}{2}\Delta x}]\ /\ \Delta z \tag{30}$$

Again, dividing equation (30) by Δx and allowing Δx to approach zero, yields:

$$\frac{dM_2}{dx} = \frac{\rho D \epsilon}{G_p(\Delta z)^2} (M_1 - 2M_2 + M_3) \tag{31}$$

The center node also has a volume of $\Delta x \Delta z W$. The only input of moisture to this elemental volume is the inlet moisture at x'. This inlet product moisture can be expressed as:

$$(G_p \Delta z W/\epsilon) \; (M_3)_{x'} \tag{32}$$

The moisture held by the product at $x' + \Delta x$ can be written as:

$$(G_p \Delta z W/\epsilon) \; (M_3)_{x' + \Delta x} \tag{33}$$

There is diffusion of moisture to the neighboring nodes on both sides of the center node. This transfer of moisture is equal to twice equation (28). From a mass balance, equation (32) must equal two times equation (28) plus equation (33):

$$(G_p \Delta z/\epsilon) \; [(M_3)_{x' + \Delta x} - (M_3)_{x'}] = 2\rho D \Delta x \; [(M_2)_{x' + \frac{1}{2}\Delta x} - (M_3)_{x' + \frac{1}{2}\Delta x}] \; / \; \Delta z \tag{34}$$

Dividing equation (34) by Δx and allowing Δx to approach zero, yields:

$$\frac{dM_3}{dx} = \frac{2\rho D\epsilon}{G_p(\Delta z)^2} (M_2 - M_3) \tag{35}$$

Noting in Figure 2 that the node midway between the surface and the center of the flat plate has twice the volume of the other two nodes, the average moisture content can be represented by:

$$\bar{M} = (M_1 + 2M_2 + M_3) \;/4 \tag{36}$$

Similarly, the total derivative of the average moisture content with respect to x can be expressed as:

$$\frac{d\bar{M}}{dx} = \left(\frac{dM_1}{dx} + 2\frac{dM_2}{dx} + \frac{dM_3}{dx} \right) /4 \tag{37}$$

A typical counter-flow dryer is shown in Figure 3. The product enters at the x equal zero end of the dryer with a known moisture content and temperature:

$$M_1(0) = M_2(0) = M_3(0) = \bar{M}_i \tag{38}$$

$$\theta\ (0) = \theta_i \tag{39}$$

The air enters at the x equal a end of the dryer with a known humidity ratio and temperature:

$$H(a) = H_i \tag{40}$$

$$T(a) = T_i \tag{41}$$

The nonlinear differential equations (8), (11), (17), (23), (31), and (35) plus the boundary conditions (38) through (41) represent a two point boundary value problem. In its present form, this problem can be solved only by trial and error. In the following sections these equations will be cast into a form which allows direct solution.

INVARIANT IMBEDDING

It is a common practice to formulate a system equation as a function of time or position. If sufficient initial and boundary conditions are known this equation can be solved analytically or numerically for the state variable as a function of time or position.

Considering this system equation, invariant imbedding expresses the state variable at a given position or time as a function of the length of a continuous mechanism or the time of an event and the boundary conditions of the system equation. The initial condition of an invariant imbedding problem is the system's response when the time of an event or the length of a continuous mechanism is zero. In most cases, there is no response at the initial condition. The boundary conditions of an invariant imbedding problem usually are known system responses to specific boundary conditions of the original system equation.

The system equations for a counter-flow dryer are equations (8), (11), (17), (23), (31), and (35) and the independent variable is the position within the bed (x). The boundary conditions to the system are represented by equations (38) through (41). In the invariant imbedding formulation it is sought to express the state variables (T, H, θ, M_1, M_2, and M_3) as functions of the dryer length a and the boundary conditions of the system equations.

Consider a basic dryer as shown in Figure 3. The inlet product properties are θ_i and $\overline{M}_i$ at x = 0 and the inlet air properties at x = a are T_i and H_i. It is desired to express the outlet properties of the product (θ_o and $\bar{M}_o$) and the outlet properties of the air (H_o and T_o) as functions of the dryer length and the inlet (air or product) properties. Looking at the x equal a end of the dryer, the outlet product temperature (θ_o) can be represented as a function of the inlet air temperature, the inlet air humidity ratio and the length of the dryer:

$$\theta_o = \theta_o(T_i, H_i, a) \tag{42}$$

The total derivative of equation (42) with respect to the independent variables T_i, H_i and a can be written as:

$$d\theta_o = \frac{\partial \theta_o}{\partial T_i}\bigg|_{H_i, a} dT_i + \frac{\partial \theta_o}{\partial H_i}\bigg|_{T_i, a} dH_i + \frac{\partial \theta_o}{\partial a}\bigg|_{T_i, H_i} da \tag{43}$$

The partial derivatives in equation (43) can be thought of as sensitivity coefficients depicting the rate at which the state variable (θ_o) changes as a function of the inlet conditions (T_i and H_i) and the dryer length a. Dividing equation (43) by the incremental distance dx yields:

$$\frac{d\theta_o}{dx} = \frac{\partial \theta_o}{\partial T_i}\bigg|_{H_i, a} \frac{dT_i}{dx} + \frac{\partial \theta_o}{\partial H_i}\bigg|_{T_i, a} \frac{dH_i}{dx} + \frac{\partial \theta_o}{\partial a}\bigg|_{T_i, H_i} \frac{da}{dx} \tag{44}$$

Since equation (42) was written for the location where x equals a, the total derivative of a with respect to x must equal 1. Thus, equation (44) can be rewritten as:

$$\frac{d\theta_o}{dx} = \frac{\partial \theta_o}{\partial T_i}\bigg|_{H_i, a} \frac{dT_i}{dx} + \frac{\partial \theta_o}{\partial H_i}\bigg|_{T_i, a} \frac{dH_i}{dx} + \frac{\partial \theta_o}{\partial a}\bigg|_{T_i, H_i} \tag{45}$$

The total derivatives with respect to x can be calculated from equations (8), (11) and (17) using the properties at the x equal a end of the dryer.

For a dryer of length zero the outlet product temperature must equal the inlet product temperature:

$$\theta_o\ (T_i, H_i, 0) = \theta_i \tag{46}$$

The boundary conditions for equation (45) will be discussed in a later section.

The three local moisture contents (M_1, M_2 and M_3) at x equal a can also be expressed as functions of the inlet air temperature, the inlet air humidity ratio, and the dryer length.

$$(M_j)_o = [M_j(T_i, H_i, a)]_o \qquad j = 1, 2, 3 \tag{47}$$

Following the same development as presented in equations (43) through (45), results in:

$$\frac{d(M_j)_o}{dx} = \left.\frac{\partial (M_j)_o}{\partial T_i}\right|_{H_i,a} \frac{dT_i}{dx} + \left.\frac{\partial (M_j)_o}{\partial H_i}\right|_{T_i,a} \frac{dH_i}{dx} + \left.\frac{\partial (M_j)_o}{\partial a}\right|_{T_i,H_i} \qquad j = 1, 2, 3 \tag{48}$$

The total derivatives with respect to x can be calculated by equations (8), (17), (23), (31), and (35) using the properties at the air's inlet.

For a dryer of length zero the outlet product moisture content is equal to the inlet product moisture content:

$$[M_j\ (T_i, H_i, 0)]_o = \overline{M}_i \qquad j = 1, 2, 3 \tag{49}$$

The two boundary conditions will be discussed in a later section.

It is also desirable to know the outlet air temperature and the outlet air humidity ratio as a function of the inlet air temperature, the inlet air humidity ratio and the length of the dryer. The relationship of the outlet air temperature as a function of these independent variables can be written as:

$$T_o = T_o(T_i, H_i, a) \tag{50}$$

The total derivative of the outlet air temperature with respect to the independent variables can be expressed as:

$$dT_o = \frac{\partial T_o}{\partial T_i}\Big|_{H_i, a} dT_i + \frac{\partial T_o}{\partial H_i}\Big|_{T_i, a} dH_i + \frac{\partial T_o}{\partial a}\Big|_{T_i, H_i} da \tag{51}$$

With a given value of T_i, H_i and a at x equal a there exists a unique value of the outlet air temperature. If the air humidity ratio and the air temperature at x equal a are held equal to H_i and T_i, respectively, while the dryer length is increased to a + da, the outlet air temperature will remain unchanged. Therefore, if the appropriate values of dT_i and dH_i are chosen in equation (51), dT_o can be assumed to be equal zero. Selecting dT_i equal to $\frac{dT_i}{dx}$ da and dH_i equal to $\frac{dH_i}{dx}$ da will satisfy this criterion:

$$\frac{\partial T_o}{\partial T_i}\Big|_{H_i, a} \frac{dT_i}{dx} da + \frac{\partial T_o}{\partial H_i}\Big|_{T_i, a} \frac{dH_i}{dx} da + \frac{\partial T_o}{\partial a}\Big|_{T_i, H_i} da = 0 \tag{52}$$

Dividing equation (52) by da, yields the following equation:

$$\frac{\partial T_o}{\partial T_i}\bigg|_{H_i,a} \frac{dT_i}{dx} + \frac{\partial T_o}{\partial H_i}\bigg|_{T_i,a} \frac{dH_i}{dx} + \frac{\partial T_o}{\partial a}\bigg|_{T_i,H_i} = 0 \qquad (53)$$

The total derivatives with respect to x can be calculated by equations (8) and (17). For a dryer of length equal zero the inlet air temperature is equal to the outlet air temperature:

$$T_o\ (T_i,\ H_i,\ 0) = T_i \qquad (54)$$

The boundary conditions to equation (53) will be discussed in a later section.

The outlet air humidity ratio can also be expressed as a function of the inlet air temperature, the inlet air humidity ratio and the dryer length. Following the procedure outlined by equations (50) through (53), equation (55) results:

$$\frac{\partial H_o}{\partial H_i}\bigg|_{T_i,a} \frac{dH_i}{dx} + \frac{\partial H_o}{\partial T_i}\bigg|_{H_i,a} \frac{dT_i}{dx} + \frac{\partial H_o}{\partial a}\bigg|_{T_i,H_i} = 0 \qquad (55)$$

Again, the total derivatives with respect to x can be calculated by equations (8) and (17). The initial condition to equation (55) is expressed as:

$$H_o\ (T_i,\ H_i,\ 0) = H_i \qquad (56)$$

Reversing the dryer such that the air enters at the x equal zero end and the product inlet is at the x equal a end of the dryer, the outlet air and product properties can be expressed as functions of the inlet product temperature, the inlet product moisture content and the dryer length. This relationship for the outlet air temperature can be written as:

$$T_o = T_o(\theta_i, \bar{M}_i, a) \tag{57}$$

Following the procedure used to derive equation (45) yields:

$$\frac{dT_o}{dx} = \frac{\partial T_o}{\partial \theta_i}\Big|_{\bar{M}_i, a} \frac{d\theta_i}{dx} + \frac{\partial T_o}{\partial \bar{M}_i}\Big|_{\theta_i, a} \frac{d\bar{M}_i}{dx} + \frac{\partial T_o}{\partial a}\Big|_{\theta_i, \bar{M}_i} \tag{58}$$

Similarly, the outlet air humidity ratio can be expressed as a function of the inlet product temperature, the inlet product moisture content and the dryer length. For this property the following relationship can be written:

$$\frac{dH_o}{dx} = \frac{\partial H_o}{\partial \theta_i}\Big|_{\bar{M}_i, a} \frac{d\theta_i}{dx} + \frac{\partial H_o}{\partial \bar{M}_i}\Big|_{\theta_i, a} \frac{dM_i}{dx} + \frac{\partial H_o}{\partial a}\Big|_{\theta_i, \bar{M}_i} \tag{59}$$

The total derivatives with respect to x can be calculated by equations (11) and (36) using the properties at the product's inlet.

The outlet product properties can also be represented as functions of the inlet product properties and the dryer length. Following the procedure outlined by equations (50) through (53) yields equations (60) and (61):

$$\frac{\partial \theta_o}{\partial \theta_i}\Big|_{\bar{M}_i, a} \frac{d\theta_i}{dx} + \frac{\partial \theta_o}{\partial \bar{M}_i}\Big|_{\theta_i, a} \frac{d\bar{M}_i}{dx} + \frac{\partial \theta_o}{\partial a}\Big|_{\theta_i, \bar{M}_i} = 0 \tag{60}$$

$$\frac{\partial (M_j)_o}{\partial \theta_i}\Big|_{\bar{M}_i, a} \frac{d\theta_i}{dx} + \frac{\partial (M_j)_o}{\partial M_i}\Big|_{\theta_i, a} \frac{d\bar{M}_i}{dx} + \frac{\partial (M_j)_o}{\partial a}\Big|_{\theta_i, \bar{M}_i} = 0 \tag{61}$$

$$j = 1, 2, 3$$

Once the appropriate boundary conditions are chosen equations (45), (48), (53), and (55) can be solved numerically for the outlet air and product properties as functions of the inlet air properties and the dryer length. Similarly equations (58), (59), (60) and (61) could be solved for the outlet properties as functions of the inlet product properties and the dryer length.

EQUILIBRIUM STATES

A great deal of insight into the mechanisms of counter-flow drying may be gained by an investigation of equilibrium states. Equilibrium states refer to values of the outlet air properties (T_o,H_o) and the outlet product properties (θ_o, $\overline{M}_o$) such that these properties remain constant while the dryer is increased in length. It should be noted again that the dryer was assumed to be in steady state.

Equilibrium State 1

In Figure 4 the inlet product temperature (θ_i) is equal to the outlet air temperature (T_o) and the outlet air humidity ratio (H_o) is in equilibrium with the inlet product moisture content ($\overline{M}_i$). Since the outlet air is in equilibrium with the inlet product with respect to both heat and mass transfer, it is not possible for the product to increase its outlet temperature or to decrease its outlet moisture content.

Equilibrium State 2

In Figure 5 the outlet product temperature (θ_o) is equal to the inlet air temperature (T_i) and the outlet product moisture content ($\overline{M}_o$) is in equilibrium with the inlet air humidity ratio (H_i). It is physically impossible for the outlet product to absorb any more thermal energy from the air or transfer additional moisture to the air. Therefore, the air cannot decrease its internal energy or increase its humidity ratio by heat and mass transfer.

EQUILIBRIUM STATES

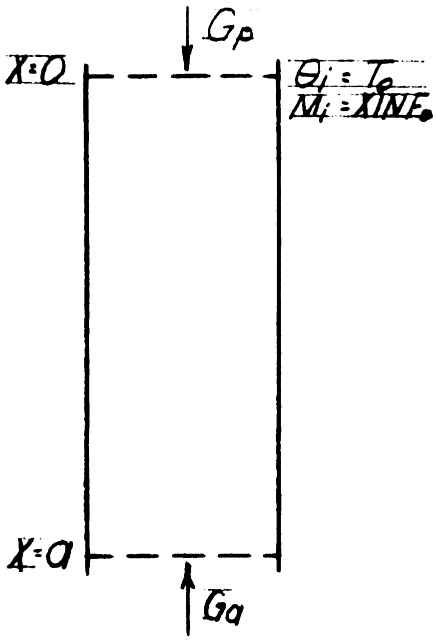

Figure 4.
Equilibrium State 1

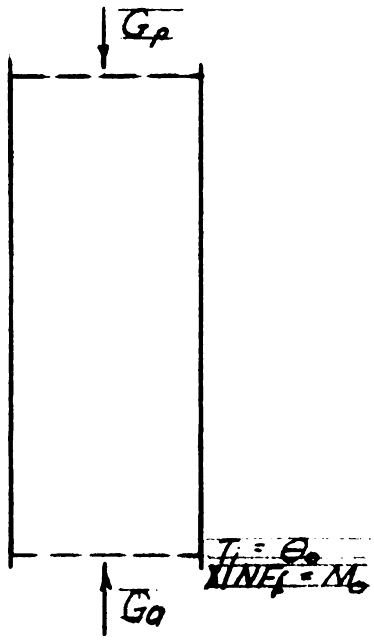

Figure 5.
Equilibrium State 2

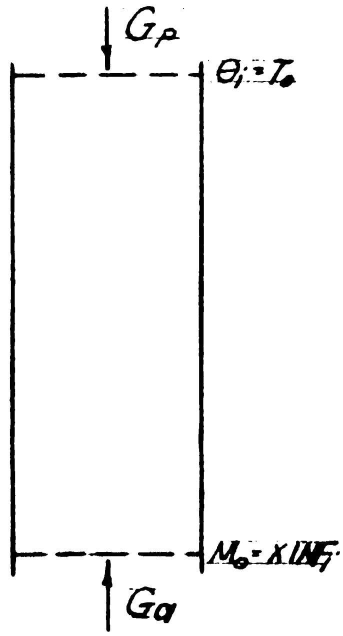

Figure 6.
Equilibrium State 3

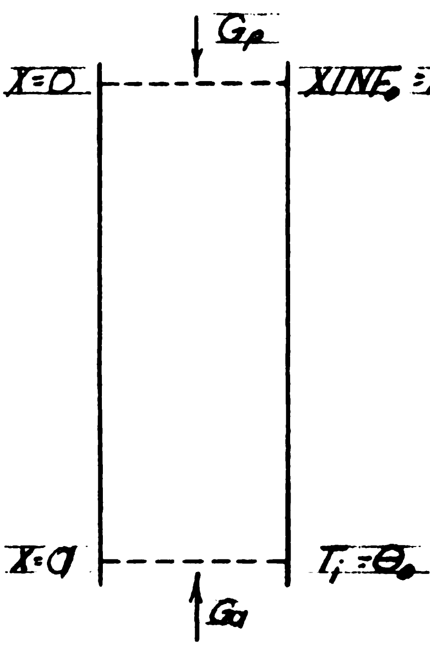

Figure 7.
Equilibrium State 4

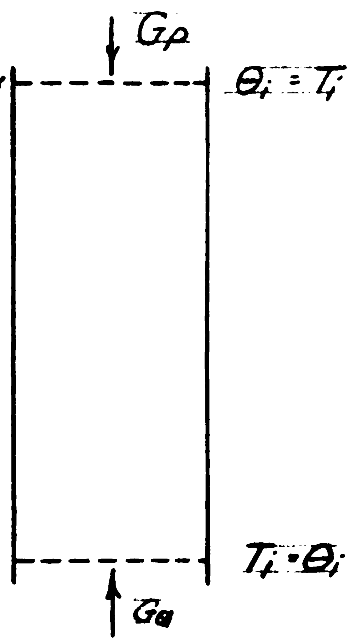

Figure 8.
Equilibrium State 5

Equilibrium State 3

In the case shown in Figure 6 the outlet air temperature (T_o) is equal to the inlet product temperature (θ_i) and the outlet product moisture content ($\overline{M}_o$) is the equilibrium value with respect to the inlet air properties (T_i and H_i) and the outlet product temperature (θ_o). It is obvious that the air cannot transfer anymore thermal energy. Since it requires thermal energy to evaporate moisture from the product, which is already in moisture equilibrium at its outlet, no further moisture can be exchanged between the air and the product. Similarly the product cannot increase its internal energy for the air cannot transfer anymore thermal energy.

Equilibrium State 4

In Figure 4 the outlet product temperature (θ_o) is equal to the inlet air temperature (T_i) and the outlet air humidity ratio (H_o) is in equilibrium with the inlet product moisture content ($\overline{M}_i$). The product cannot increase its internal energy by heat transfer from the air. Since the air is already in moisture equilibrium with the product, it is not possible for the air to use its thermal energy for evaporation.

Before considering the fifth equilibrium state two assumptions must be made. Again, it should be noted that the system's response is assumed to be steady state. The additional assumptions made are the following:

a) The outlet air temperature must asymptotically approach the inlet product temperature as the dryer length is increased. Similarly, the outlet product temperature must asymptotically approach the inlet air temperature as the dryer length is increased.

b) Energy transfer controls over mass transfer. In other words, if there is no energy available for evaporation, there will be no evaporation.

Equilibrium State 5

In Figure 8 the inlet product temperature (θ_i) is equal to the inlet product temperature. From assumption (a) the outlet product temperature must equal the inlet product temperature as the dryer length is increased. The outlet air temperature is equal to the inlet air temperature for a dryer of length zero. Thus, the outlet air temperature must equal the inlet air temperature regardless of the dryer length. Thermal energy is required to evaporate moisture from the product, therefore, there can be no mass transfer.

BOUNDARY CONDITIONS

If the appropriate boundary conditions are determined, the invariant imbedding equations (45), (48), (53) and (55) along with the initial conditions (46), (49), (54) and (56) can be solved numerically for the outlet properties (air and product) as functions of the inlet air humidity ratio, the inlet air temperature and the dryer length. From equilibrium state 5, the outlet properties (θ_o, $\overline{M}_o$, T_o, H_o) are equal to the inlet properties (θ_i, $\overline{M}_i$, T_i, H_i) if the inlet air temperature (T_i) is equal to the inlet product temperature. Therefore, the following set of boundary conditions can be written:

$$\theta_o\ (\theta_i, H_i, a) = \theta_i \tag{62}$$

$$[M_j\ (\theta_i, H_i, a)]_o = \overline{M}_i \qquad j = 1,2,3 \tag{63}$$

$$T_o\ (\theta_i, H_i, a) = T_i \tag{64}$$

$$H_o\ (\theta_i, H_i, a) = H_i \tag{65}$$

Given another set of boundary conditions at a specific inlet air humidity ratio, the invariant imbedding equations can be solved numerically.

Consider the equilibrium value of the air humidity ratio H_{eq} for an air temperature θ_i, a product temperature θ_i and a product moisture content $\overline{M}_i$. If the inlet air humidity ratio is equal to H_{eq}, it was assumed that the outlet air and the outlet product properties are independent of the inlet air humidity ratio. Therefore, when the inlet air humidity ratio is equal to H_{eq}, it was assumed that the second term on the right-hand side of equations (45) and (48) and the second term on the left-hand side of equations (53) and (55) are zero. Rewriting these equations yields:

$$\frac{d\theta_o}{dx} = \frac{\partial\theta_o}{\partial T_i}\bigg|_{H_{eq},a} \frac{dT_i}{dx} + \frac{\partial\theta_o}{\partial a}\bigg|_{H_{eq},T_i} \tag{66}$$

$$\frac{d(M_j)_o}{dx} = \frac{\partial(M_j)_o}{\partial T_i}\bigg|_{H_{eq},a} \frac{dT_i}{dx} + \frac{\partial(M_j)_o}{\partial a}\bigg|_{H_{eq},T_i} \quad j = 1,2,3 \tag{67}$$

$$\frac{\partial T_o}{\partial T_i}\bigg|_{H_{eq},a} \frac{dT_i}{dx} + \frac{\partial T_o}{\partial a}\bigg|_{H_{eq},T_i} = 0 \tag{68}$$

$$\frac{\partial H_o}{\partial T_i}\bigg|_{H_{eq},a} \frac{dT_i}{dx} + \frac{\partial H_o}{\partial a}\bigg|_{H_{eq},T_i} = 0 \tag{69}$$

With an inlet air humidity ratio H_{eq}, equations (66) through (69) along with the initial conditions (46), (49), (54) and (56) and the boundary conditions (62) through (65) can be solved numerically for the outlet properties as functions of the inlet air temperature (T_i) and the dryer length (a). The solutions to these equations will generate the necessary set of boundary conditions to solve equations (45), (48), (53) and (55).

The assumptions made in deriving equations (66) through (69) should be considered in more detail. It was assumed that the second term on the right-hand side of equations (45) and (48) and the second term on the left-hand side of equations (53) and (55) were neglible when compared with the other terms of these equations. For equation (45) this term can be assumed small, while for equation (48) the assumption is reasonable only for low outlet relative humidities. The assumption applied to equations (53) and (55) is also valid if the outlet relative humidities are low; however, for high relative humidities this assumption is very poor.

As the dryer length a is increased a counter-flow dryer can be said to be approaching one of the equilibrium states (1 through 4). In other words, the system is approaching a mass equilibrium at one end of the dryer and a thermal equilibrium at an outlet of the system. One end of the dryer is limiting the degree of mass transfer, while the same or the other end is controlling the heat transfer. For example, the outlet product temperature may be very close to the inlet air temperature, while the outlet air temperature may be much greater than the inlet product temperature. Therefore, as a is increased the quantity of heat transfer will be determined by the heat transfer at the air inlet. Thus in this case, the rate at which the product absorbs energy from the air dictates the rate at which energy is transferred in the system.

Equations (66) through (69) assume that both heat and mass transfer are controlled by the x equal a end of the dryer. Therefore, the system must be approaching equilibrium state 2. Since the outlet air temperature (T_o) is usually lower than the inlet air temperature (T_i), the x equal zero end of the dryer limits mass transfer in most cases. This means that equations (66) through (69) alone cannot predict the outlet properties with the inlet humidity ratio equal H_{eq}.

Equations (58), (59), (60) and (61) express the outlet properties (T_o, θ_o, $\overline{M}_o$, H_o) as a function of the inlet product temperature, the inlet product moisture content and the dryer length. The total derivatives with respect to x in these equations are evaluated at the x equal zero end. Therefore, these equations assume that the x equal zero end of the dryer controls both heat and mass transfer. In other words, the system is approaching equilibrium state 1. Again, it cannot be assumed that equations (58) through (61) alone will accurately predict the outlet properties at H_i equal H_{eq}. A combination of equations (58) through (61) and equations (66) through (69) can be used to calculate the outlet properties.

Both equations (69) and (59) can be evaluated for the outlet air humidity ratio. Equation (69) assumes that the x equal a end of the dryer controls the mass transfer, while equation (59) assumes that the other end of the dryer limits the mass transfer. The maximum feasible mass transfer is depicted by the minimum of the H_o's calculated by equations (59) and (69). This value is the correct H_o for a dryer of length a with the inlet properties θ_i, $\overline{M}_i$, H_{eq} and T_i. The appropriate values of $(M_j)_o$ j = 1,2,3 can be calculated by the same procedure using equations (61) and (67). Since the degree of mass transfer is almost always controlled by the x equal zero end of the dryer the errors induced by the assumption prior to equation (66) can be ignored.

Prior to the evaluation of θ_o and T_o, a system energy balance must be derived. Consider the system as a black box with inputs and outputs of energy at both ends. There is an input of thermal energy by the product at x equal zero of:

$$A\ (c_pG_p + c\bar{M}_iG_p)\theta_i \tag{70}$$

At the output the thermal energy of the product can be expressed as:

$$A\ (c_pG_p + c\bar{M}_oG_p)\theta_o \tag{71}$$

The inlet thermal energy of the air at x equal a can be written as:

$$A\ (c_aG_a + c_vH_iG_p)T_i \tag{72}$$

The outlet air has a thermal energy equal to:

$$A\ (c_aG_a + c_vH_oG_p)T_o \tag{73}$$

There is also energy transferred within the system in the form of latent heat of evaporation. The latent energy of the air at its inlet is:

$$Ah_{fg}G_aH_i \tag{74}$$

The latent energy of the air a x equal zero can be expressed as:

$$Ah_{fg}G_aH_o \tag{75}$$

Performing an energy balance equation (70) plus equations (72) and (74) equals equation (71) plus equations (73) and (75):

$$(c_aG_a + c_vH_iG_a)(T_o - T_i) = -(c_pG_p + c\bar{M}_oG_p)(\theta_o - \theta_i)$$

$$+ G_ah_{fg}(H_o - H_i) - c_vG_a(H_o - H_i)T_o + cG_a(H_o - H_i)\theta_i \tag{76}$$

θ_o can be calculated by equation (66) assuming that heat transfer is controlled at the air inlet. Since H_o, $\bar{M}_o$, θ_o, T_i and $H_i(H_{eq})$ are known, equation (76) can be solved for T_o. It seems possible to evaluate equation (68) for T_o; however, it has previously been noted that this equation cannot be trusted at high outlet relative humidities. Assuming that heat transfer is controlled by the x equal zero end of the dryer, T_o can be evaluated by equation (58). This value of T_o is compared with the T_o calculated by equation (76) and the largest value is considered to be the appropriate value.

The correct value of θ_o can be obtained by selecting the smallest θ_o generated by equations (66) and (60).

In order to evaluate the partial derivatives of the outlet properties with respect to θ_i and $\overline{M}_i$ in equations (58) and (61) it was necessary to calculate the outlet properties for the inlet conditions $(\theta_i, \overline{M}_i)$, $(\theta_i + d\theta_i, \overline{M}_i)$ and $(\theta_i, \overline{M}_i + d\overline{M}_i)$. The partials in equation (60) can be approximated by:

$$\left.\frac{\partial \theta_o}{\partial \theta_i}\right|_{\overline{M}_i, a} = \frac{(\theta_o)_{\theta_i + d\theta_i, \overline{M}_i} - (\theta_o)_{\theta_i, \overline{M}_i}}{d\theta_i} \tag{77}$$

$$\left.\frac{\partial \theta_o}{\partial \overline{M}_i}\right|_{\theta_i, a} = \frac{(\theta_o)_{\theta_i, \overline{M}_i + d\overline{M}_i} - (\theta_o)_{\theta_i, \overline{M}_i}}{d\overline{M}_i} \tag{78}$$

Similar expressions can be written to calculate the partials of the other outlet properties with respect to the inlet product properties.

Using the procedure outlined in determining the outlet properties (θ_o, $\overline{M}_o$, T_o and H_o) at a specific inlet air humidity ratio (H_{eq}) plus the initial conditions [equations (46), (49), (54) and (56)] and the boundary conditions at a specific inlet air temperature (θ_i) [equations

(62) through (65)], the invariant imbedding equations (45), (48), (53) and (55) can be solved numerically for the outlet properties as functions of the inlet air properties and the dryer length. It should be noted here that the technique used to calculate the outlet properties at a specific inlet air humidity ratio (H_{eq}) is by no means unique. Undoubtedly, there exist better methods.

If the partial derivatives of the outlet properties with respect to the inlet air humidity ratio in equations (45), (48), (53) and (55) were known at a specific inlet air humidity ratio (H_{eq} in this case), the invariant imbedding equations could be solved directly for the outlet properties at this boundary. Unfortunately, this is not the case.

NUMERICAL SOLUTION

Equations (45), (48), (53), and (55) were backward differenced in the (a) direction and forward differenced in the (T_i) and (H_i) directions. The finite-difference approximation of equation (45) can be written as:

$$\theta_o \Big|_{\substack{a+\Delta a \\ T_i+\Delta T_i \\ H_i+\Delta H_i}} = \left(\theta_o \Big|_{\substack{a \\ T_i+\Delta T_i \\ H_i+\Delta H_i}} + \frac{dT_i}{dx}\Big|_{\substack{a \\ T_i+\Delta T_i \\ H_i+\Delta H_i}} \frac{\Delta a}{\Delta T_i}\, \theta_o \Big|_{\substack{a+\Delta a \\ T_i \\ H_i+\Delta H_i}} + \right.$$

$$\left. \frac{dH_i}{dx}\Big|_{\substack{a \\ H_i+\Delta H_i \\ T_i+\Delta T_i}} \frac{\Delta a}{\Delta H_i}\, \theta_o \Big|_{\substack{a+\Delta a \\ H_i+\Delta H_i \\ T_i+\Delta T_i}} \Delta a \right) \Big/ \left(1.0 + \right.$$

$$\left. \frac{dT_i}{dx}\Big|_{\substack{a \\ T_i+\Delta T_i \\ H_i+\Delta H_i}} \frac{\Delta a}{\Delta T_i} + \frac{dH_i}{dx}\Big|_{\substack{a \\ T_i+\Delta T_i \\ H_i+\Delta H_i}} \frac{\Delta a}{\Delta H_i} \right) \tag{79}$$

It should be noted that the total derivatives with respect to x are evaluated at (a). Similar expressions can be written for the remaining equations (48), (53) and (55). These finite difference equations are stable for large values of Δa(.1 ft), ΔT_i(10 F) and ΔH_i (.01 lbm water/lbm dry air), if T_i and H_i are chosen such that:

$$\frac{dT_i}{dx}\Big|_{\substack{a \\ T_i+\Delta T_i \\ H_i+\Delta H_i}} \frac{\Delta a}{\Delta T_i} \quad \text{and} \quad \frac{dH_i}{dx}\Big|_{\substack{a \\ T_i+\Delta T_i \\ H_i+\Delta H_i}} \frac{\Delta a}{\Delta T_i} \quad \text{are positive.}$$

Since the total derivatives with respect to x at the x equal a end of the dryer rarely change signs when (a) is increased, it is easy to choose T_i and H_i approximately. This is not true for the x equal zero end of the dryer. The total derivative of the inlet moisture content [equations (58) through (61)] with respect to x changes sign when condensation occurs. Therefore, it is difficult to maintain stability when these equations are used. Since these equations are employed in the estimation of the outlet properties on the H_i equal H_{eq} boundary, the solution at this boundary is not stable when condensation occurs within the dryer.

The finite difference approximations of equations (45), (48), (53) and (55) with their initial conditions and the boundary conditions can be solved numerically for the outlet properties. Using these outlet properties, it is possible to solve equations (8), (11), (17), (23), (31) and (35) for the air temperature, the product temperature, the air humidity ratio and the local product moisture contents as functions of x. Solutions to these can be obtained by any numerical integration routine. Stability problems may occur if the appropriate initial conditions are not chosen. Consider equations (8) and (11) written in state variable form:

$$\frac{d}{dx}\begin{bmatrix} T \\ \theta \end{bmatrix} = \begin{bmatrix} \dfrac{a'h}{G_a c_a + G_a H c_v} & \dfrac{-a'h}{G_a c_a + G_a H c_v} \\ \dfrac{a'h}{G_p c_p + G_p \overline{M} c} & \dfrac{-a'h}{G_p c_p + G_p \overline{M} c} \end{bmatrix} \begin{bmatrix} T \\ \theta \end{bmatrix} - \frac{G_a h_{fg}}{G_a c_a + G_a H c_v} \begin{bmatrix} \dfrac{dH}{dx} \\ 0 \end{bmatrix} \tag{80}$$

If the elements of the square matrix are asumed constant and the total derivative of the air humidity ratio with respect to x is assumed zero, the eigenvalues of this square matrix must be less than or equal to zero. Therefore, if the system of equations is stable the following criterion must be satisfied:

$$\frac{G_p c_p + G_p \overline{M} c}{G_a c_a + G_a H c_v} \leq 1 \qquad (81)$$

Since the total derivative of the humidity ratio with respect to x is negative, the criterion for stability is more conservative than equation (81). The stability criterion (81) is for the case when the initial properties are chosen at the product inlet. If one chooses to solve equations (8), (11), (17), (23), (31) and (35) using the known properties at x equal a, the following criterion must be satisfied:

$$\frac{G_a c_a + G_a H c_v}{G_p c_p + G_p \overline{M} c} \leq 1 \qquad (82)$$

In this case, the derivative of the air humidity ratio with respect to x is positive. Thus, the actual criterion is less conservative than equation (82). Therefore, a region exists where $G_a c_a + G_a H c_v$ is slightly greater than $G_p c_p + G_p \overline{M} c$ in which one must guess which initial conditions to use in the solution of the system model.

If condensation occurs within the dryer, the system must be modeled using the initial properties at the air inlet. Since condensation usually occurs when the air flow is small with respect to the product flow, this is not a severe limitation on the system model. It should be noted that there are special cases [equation (81) is satisfied and condensation occurs within the dryer] for which it is not possible to solve equations (8), (11), (17), (23), (31) and (35) for the air and product properties as functions of x.

INVARIANT PROGRAMMING

The motivations for using the invariant imbedding concept for the solution of the counter-flow dryer are the following:

1) This formulation yields the outlet air and product properties directly and in a known number of steps.

2) No estimates of the unknown air temperature, air humidity ratio, product temperature and product moisture content are needed.

3) In generating the desired results other pertinent information for system design and system use is calculated; e.g. the outlet properties as functions of the dryer length and the inlet air or product properties.

Since the calculation of the outlet properties at the specific inlet air humidity (H_{eq}) can be considered to be no more than an estimation, the procedure presented does not satisfy the second motivation. In the following pages, an alternative formulation (invariant programming) will be analyzed.

Consider a set S of discrete feasible outlet air humidity ratios, H_o and outlet air temperatures, T_o. In deriving equations (53) and (55) it was observed that for a given dryer length (a) there exists a unique ordered pair of inlet air properties (T_i,H_i) corresponding to each ordered pair of outlet air properties (T_o,H_o). Similarly for a given value of a the ordered pairs (T_o,H_o) map one to one into the set of ordered pairs (θ_o,$\overline{M}_o$).

It is desired to determine the outletproperties (θ_o, $\overline{M}_o$, T_o, H_o) for a given dryer length and given inlet conditions. Define T_i' and H_i' as the desired inlet air properties. One can search the ordered pairs (T_i,H_i) corresponding to the set S to find the discrete values (T_i^*,H_i^*) which minimize the dimensionless norm.

$$\left[\left(\frac{T_i' - T_i}{T_i'} \right)^2 + \left(\frac{H_i' - H_i}{H_i'} \right)^2 \right]^{\frac{1}{2}} \tag{83}$$

The air properties (T_o^*,H_o^*) are the ordered pair which mapped into these values of (T_i^*,H_i^*). Similarly the outlet properties $(\theta_o^*,\overline{M}_o^*)$ are the ordered pair corresponding to the outlet air properties (T_o^*,H_o^*).

Generally the dimensionless norm (83) is not equal to zero. Therefore, to obtain more accurate values of the outlet properties, it is essential to interpolate between the discrete values of (T_i,H_i).

For a given dryer length the inlet air properties are functions of the outlet air properties:

$$T_i = T_i(T_o,H_o) \tag{84}$$

$$H_i = H_i(T_o,H_o) \tag{85}$$

Expanding equations (84) and (85) by Taylor's Series about the point (T_o^*,H_o^*), yields:

$$T_i = T_i^* + \left.\frac{\partial T_i}{\partial T_o}\right|_{H_o^*} (T_o - T_o^*) + \left.\frac{\partial T_i}{\partial H_o}\right|_{T_o^*} (H_o - H_o^*) \tag{86}$$

$$H_i = H_i^* + \left.\frac{\partial H_i}{\partial T_o}\right|_{H_o^*} (T_o - T_o^*) + \left.\frac{\partial H_i}{\partial H_o}\right|_{T_o^*} (H_o - H_o^*) \tag{87}$$

Equations (86) and (87) can be evaluated for the desired outlet air properties (T_o', H_o') corresponding to the inlet air properties (T_i', H_i'). Thus:

$$H_o' = H_o^* + \left[\frac{\partial T_i}{\partial T_o} (H_i' - H_i^*) - \frac{\partial H_i}{\partial T_o} (T_i' - T_i^*) \right] \Big/ \left(\frac{\partial T_i}{\partial T_o} \frac{\partial H_i}{\partial H_o} - \frac{\partial H_i}{\partial T_o} \frac{\partial T_i}{\partial H_o} \right) \tag{88}$$

$$T_o' = T_o^* - \left[\frac{\partial T_i}{\partial H_o} (H_o' - H_o^*) - (T_i' - T_i^*) \right] \Big/ \frac{\partial T_i}{\partial T_o} \tag{89}$$

For a given dryer length the outlet product properties $(\theta_o, \overline{M}_o)$ can be expressed as functions of the outlet air properties (T_o, H_o).

$$\theta_o = \theta_o(H_o, T_o) \tag{90}$$

$$\overline{M}_o = \overline{M}_o(H_o, T_o) \tag{91}$$

Expanding equations (90) and (91) by the first two terms of Taylor Series about H_o^* and T_o^*, yields:

$$\theta_o' = \theta_o^* + \frac{\partial \theta_o}{\partial T_o} (T_o' - T_o^*) + \frac{\partial \theta_o}{\partial H_o} (H_o' - H_o^*) \tag{92}$$

$$\overline{M}_o' = \overline{M}_o^* + \frac{\partial \overline{M}_o}{\partial T_o} (T_o' - T_o^*) + \frac{\partial \overline{M}_o}{\partial T_o} (H_o' - H_o^*) \tag{93}$$

Where T_o' and H_o' are the values calculated in equations (88) and (89).

For a dryer of zero length, it is obvious that the outlet properties are equal to the inlet properties:

$$(\theta_o, \overline{M}_o) = (\theta_i, \overline{M}_i) \tag{94}$$

$$(T_i,H_i) = (T_o,H_o) \tag{95}$$

When the dryer length is increased by Δa the inlet air properties and the outlet product properties can be calculated by the following formulas:

$$(\theta_o,\overline{M}_o)\bigg|_{\Delta a} = \left(\theta_o + \frac{d\theta_o}{dx}\bigg|_0 \Delta a, \overline{M}_o + \frac{d\overline{M}_o}{dx}\bigg|_0 \Delta a \right) \tag{96}$$

$$(T_i,H_i)\bigg|_{\Delta a} = \left(T_i + \frac{dT_i}{dx}\bigg|_0 \Delta a, H_i + \frac{dH_i}{dx}\bigg|_0 \Delta a \right) \tag{97}$$

The total derivatives with respect to x can be calculated by equation (8), (11), (17) and (37). Equations (96) and (97) can be written for an arbitrary dryer length (a) as:

$$(\theta_o,\overline{M}_o)\bigg|_{a+\Delta a} = \left(\theta_o + \frac{d\theta_o}{dx}\bigg|_a \Delta a, \overline{M}_o + \frac{d\overline{M}_o}{dx}\bigg|_a \Delta a \right) \tag{98}$$

$$(T_i,H_i)\bigg|_{a+\Delta a} = \left(T_i + \frac{dT_i}{dx}\bigg|_a \Delta a, H_i + \frac{dH_i}{dx}\bigg|_a \Delta a \right) \tag{99}$$

Using the initial conditions (94) and (95) and equations (98) and (99) the values of $\theta_o,\overline{M}_o,T_i$ and H_i which yield the outlet air properties T_o and H_o for a dryer of length $a + \Delta a$ can be calculated.

The calculation of the outlet properties by invariant programming can be outlined as follows:

1) Determine a set S of feasible T_o and H_o values for the desired inlet air properties (T_i' and H_i'). A recommended set would include discrete outlet air temperatures between the inlet

product temperature and the maximum desired inlet air temperature. The range of outlet humidity ratios should include discrete values between the minimum desired inlet air humidity ratio and the humidity ratio at the wet bulb temperature of the maximum desired inlet air temperature and the maximum desired inlet air humidity ratio.

2) Initialize the outlet product properties and the inlet air properties by equations (94) and (95).

3) Step the dryer length and calculate the appropriate set of outlet product properties and inlet air properties by equations (98) and (99).

4) Repeat step three until computer output is desired.

5) For each pair of desired inlet air temperatures and inlet air humidity ratios, search the set of calculated inlet air properties to find the order pair (T_i^*, H_i^*) which minimizes the dimensionless norm (83).

6) Calculate the desired outlet properties by equations (88), (89), (92), and (93).

7) Return the step three and repeat the procedure until the maximum desired dryer length is reached.

To improve the accuracy of the solution choose more discrete ordered pairs (T_o, H_o) or use more terms in the Taylor Series for equations (86), (87), (92) and (93).

It should be noted that this procedure will only model the system if criterion (81) is satisfied and no condensation occurs within the dryer. If criterion (82) is satisfied, one must consider a set R of feasible outlet product properties $(\theta_o, \overline{M}_o)$. Following a similar procedure will yield the desired outlet properties.

Invariant programming requires no estimates and will give very accurate results if the number of ordered pairs in S or R is large. This is a definite advantage over invariant imbedding.

RESULTS AND DISCUSSION

Errors

Most numerical techniques yield approximations of the original mathematical models, therefore the numerical solutions are not identical to the exact solutions. Due to numerical approximations both the invariant imbedding and the invariant programming formulations of the counter-flow drying problem are subject to errors. The invariant imbedding solution is subject to errors due to the assumptions and estimations necessary to calculate the outlet properties along the boundaries. The errors induced in the invariant imbedding formulation can be summarized as the following:

1) Errors due to the assumptions (a and b, pages 24 and 25) made along the T_i equal θ_i boundary of the invariant imbedding problem.

2) Errors due to the estimation procedure (pages 26 through 32) used to calculate the outlet properties along the H_i equal H_{eq} boundary.

3) Truncation errors.

If the step sizes in T_i,H_i and a are chosen small enough the truncation errors are small with respect to the other errors.

In order to study the effects of the first two types of errors, the values of the outlet product moisture content calculated by the invariant imbedding solution were compared with the values of this property which satisfy the model. In Figures 9, 10 and 11, the outlet moisture contents are compared near the $T_i = \theta_i$ boundary, near the

$H_i = H_{eq}$ boundary and for a point in the center of the mesh. The inlet conditions and parameters for the dryer considered are the following:

G_a	200 lbm dry air/hr
G_p	35 lbm dry product/hr
θ_i	80 F
$\overline{M}_i$	.33

In Figure 9, the outlet product moisture contents calculated by the invariant imbedding and the invariant programming formulations are plotted. The values obtained from the invariant programming formulation were tested with the system model and proven to satisfy the model. It should be noted that the inlet air temperature is very close to the inlet product temperature. Also the inlet air humidity ratio is much less than H_{eq}. Therefore, it was assumed that the errors shown in this figure are due largely to errors of kind number one. Though the absolute magnitude of the errors are small, the relative errors near this boundary are quite large.

In Figure 10 the inlet air temperature is much greater than the inlet product temperature, while the inlet air humidity ratio is only slightly greater than H_{eq}. In this case, the errors were assumed to be of the second type. The estimation procedure appears to have over-estimated the magnitude of the mass transfer. The relative errors near this boundary are small, but the absolute errors are quite large.

In Figure 11, an inlet air temperature and air humidity ratio in the center of the mesh are chosen. Even though the errors along the boundaries are large, the errors for inlet air properties a distance from these boundaries are small. Therefore, if the invariant imbedding formulation is used the outlet properties calculated near these boundaries

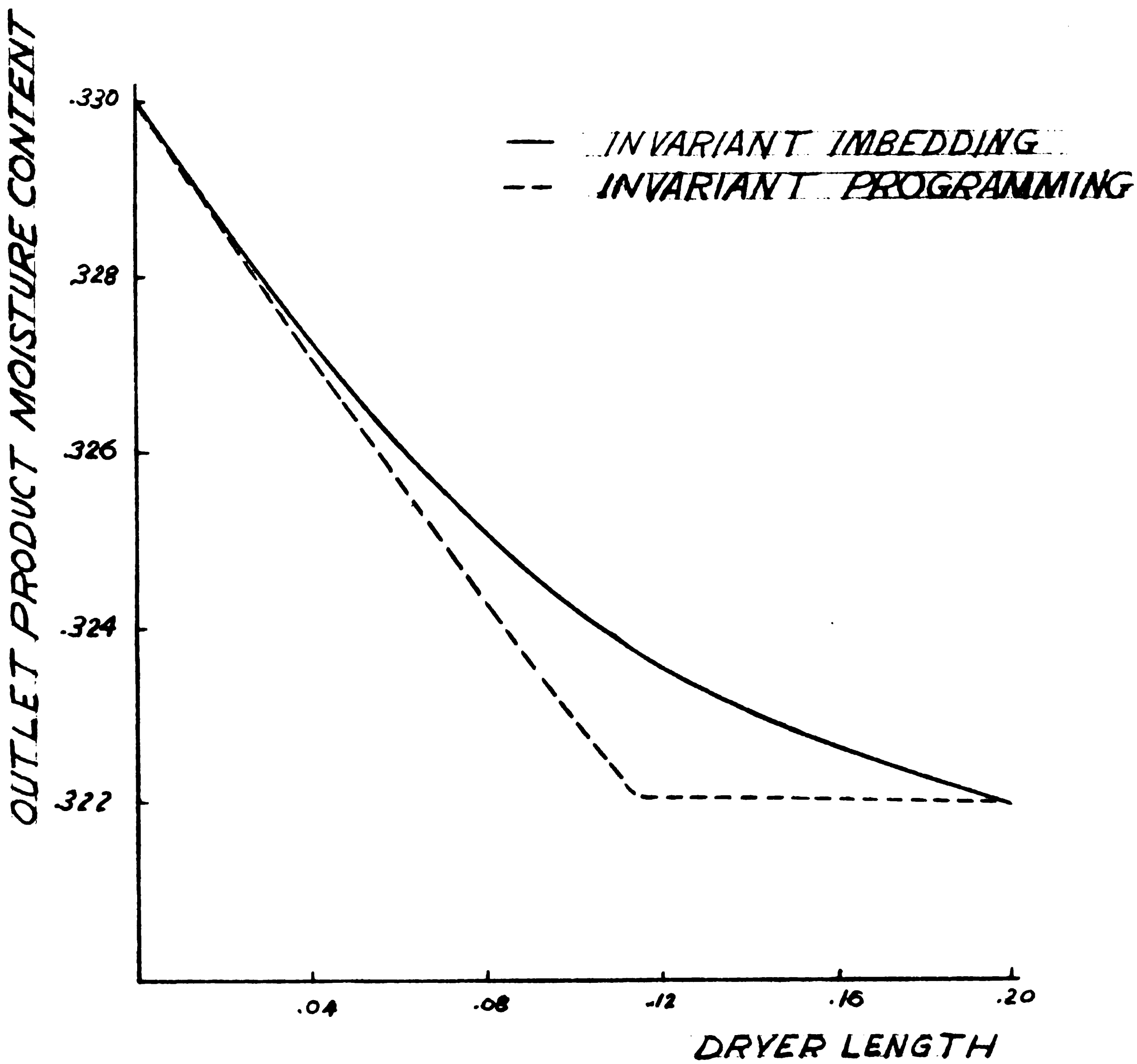

Figure 9. Errors in the Invariant Imbedding Solution near the $T_i = \theta_i$ **Boundary**

T_i = 90.0 F

H_i = .0117 lbm water vapor/lbm dry air

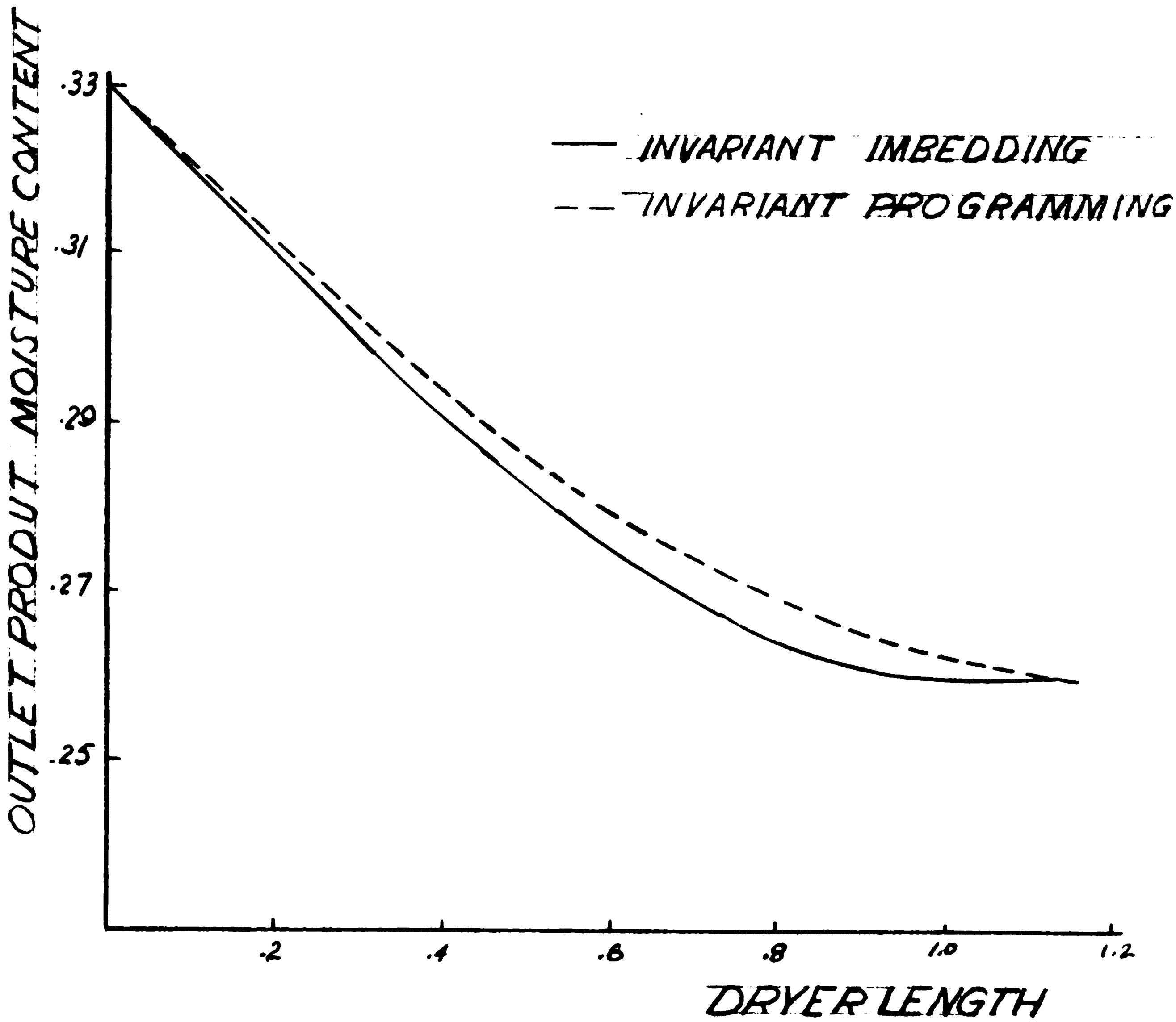

Figure 10. Errors in the Invariant Imbedding Solution near the $H_i = H_{eq}$ Boundary

$T_i = 180$ F

$H_i = .02003$ lbm water vapor/lbm dry air

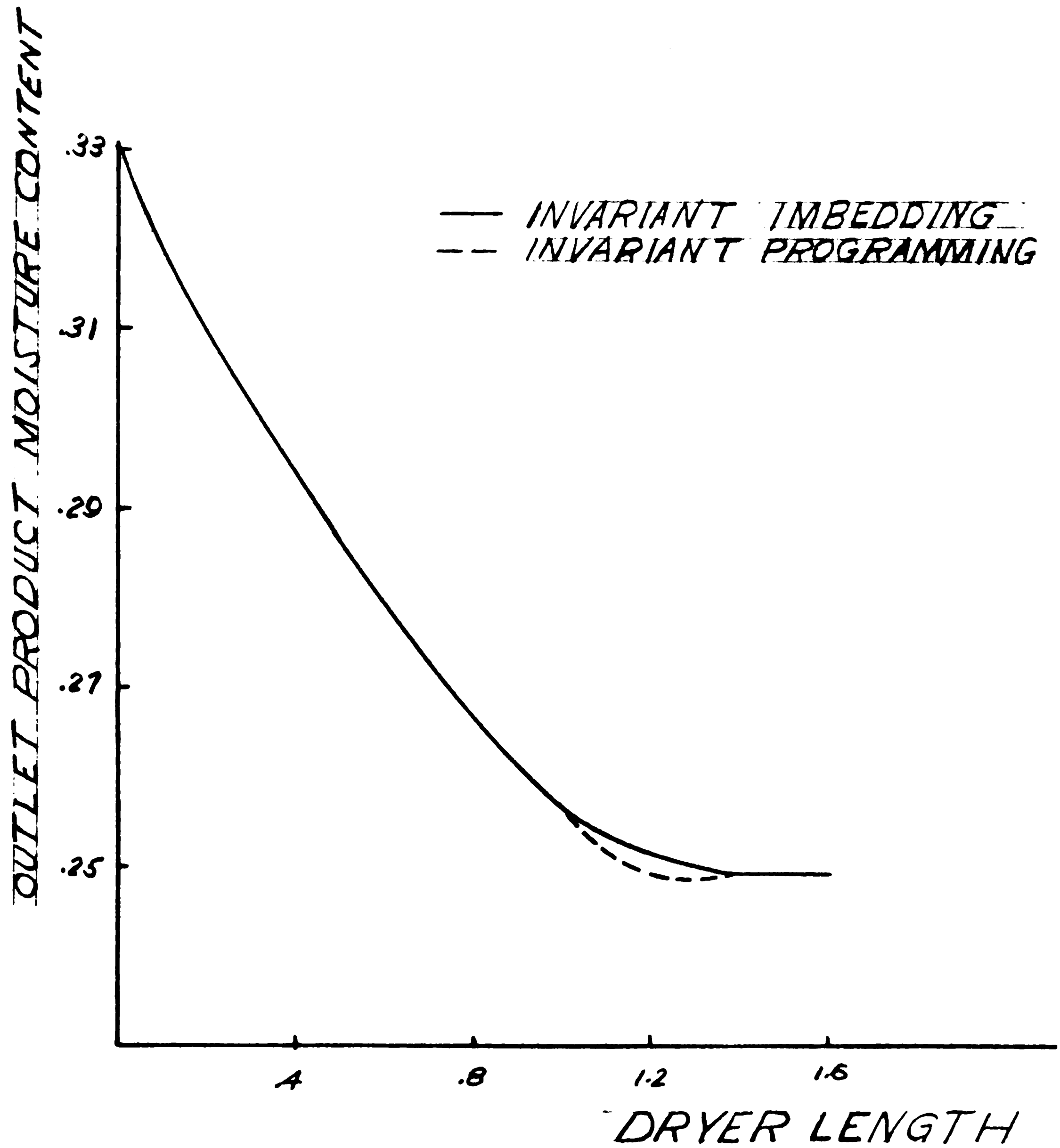

Figure 11. Errors in the Invariant Imbedding Solution for a Center Mesh Point

T_i = 180 F

H_i = .0117 lbm water vapor/lbm dry air

should not be trusted.

There exist two basic types of errors in the invariant programming solution. These are the following:

1) Errors due to the linear interpolation between the discrete ordered pairs (T_o,H_o). [equations (88), (89), (92) and (93)].

2) Truncation errors in the numerical solution of equations (98) and (99).

If the step size in a is small the truncation errors can be considered small with respect to the interpolation errors.

In Figure 12 a typical set S of feasible outlet air properties is shown. The inlet conditions and parameters for the dryer considered are the following:

G_a	261.3 lbm dry air/hr
G_p	18.7 lbm dry product/hr
θ_i	80.5 F
$\overline{M}_i$	.481

The desired inlet air properties are:

T_i'	180.0 F
H_i'	.0053 lbm water vapor/lbm dry air

The set S includes outlet air temperatures from 150 F to 80.5 F and outlet air humidity ratios from .03 to .0053 lbm water vapor/lbm dry air. Each dot in Figure 12 represents a discrete value of the ordered pair (T_o,H_o). It should be noted that the line in the lower right-hand

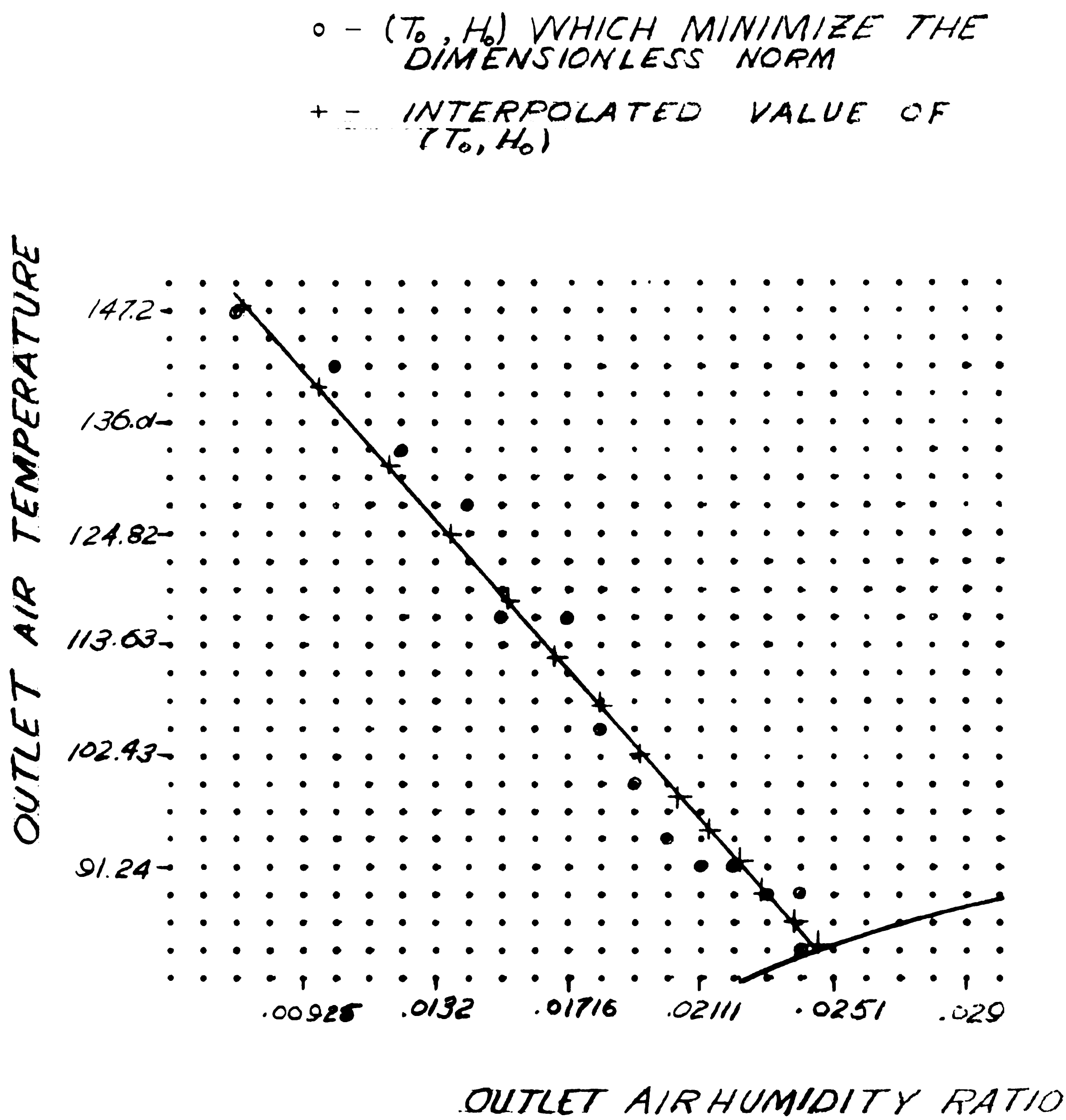

Figure 12. Schematic Diagram of the Solution of a Counter-Flow Drying Problem, Using Invariant Programming.

corner is the saturation line. For any point below and to the right of this line, the outlet relative humidity would be greater than 100%. Therefore, these points cannot be included in the set of feasible outlet air properties.

Starting at the upper left-hand corner of this figure, the circled dot is the ordered pair (T_o, H_o) which minimizes the dimensionless norm [equation (83)] for this dryer with a length of one inch. The corresponding cross (+) is the interpolated value of this order pair. As the dryer is increased in length, one inch at a time, the ordered pair which minimizes the dimensionless norm moves down and to the right. For each circle there is a corresponding interpolated value of the ordered pair (T_o, H_o). When the dryer length is equal to 14 inches the interpolated values of (T_o, H_o) are approximately on the saturation line. Since the air flow rate is much greater than the product flow rate, the outlet product temperature is equal to the inlet air temperature. Thus, the dryer has approached equilibrium state 4. Increasing the dryer length will not change the outlet properties $(T_o, H_o, \theta_o, \overline{M}_o)$.

It should be noted that the order pair (T_o, H_o) which minimized the dimensionless norm is not necessarily the nearest discrete ordered pair to the interpolated value of (T_o, H_o). This can be explained by the fact that the dimensionless norm is minimized for the other end of the dryer.

It can be seen that the ordered pair which minimizes the dimensionless norm may be quite distant from the interpolated value of the ordered pair. More accurate results may be obtained by considering more discrete values of (T_o,H_o), thus reducing this distance. This is especially true near the saturation line, where the inlet air properties (T_i,H_i) and the outlet product properties $(\theta_o,\overline{M}_o)$ are very sensitive to the outlet air properties.

Contrary to the invariant imbedding formulation the invariant programming can yield results with as much accuracy as desired if the step size in a is small the number of discrete ordered pairs (T_o,H_o) is large. A reasonable estimate of the interpolation error is one-half the distance between two neighboring values of the discrete ordered pairs (T_o,H_o). For the reaons presented above the invariant programming formulation should be used whenever possible.

Experimental Versus Theoretical Results

The theoretical model was compared with the experimental results of Ives (1967) (experiments III-56, III-58, and III-60). The inlet conditions, the flow rates and the parameters for the experiment considered are the following:

G_a	261.3 lbm dry air/hr
G_p	18.7 lbm dry product/hr
θ_i	80.5 F
$\overline{M}_i$	.481
h_D	2.12×10^{-3} ft/hr

In Table 1, the experimental and the theoretical outlet product moisture contents are compared for three dryer lengths.

Table 1. Experimental Versus Theoretical Results for Three Dryer Lengths

Dryer Length	Outlet Product Moisture Contents	
	Experimental	Theoretical
10 inches	.244	.256
14 inches	.224	.215
24 inches	.216	.215

In Figure 13 the theoretical product moisture content versus x is compared with the experimental product moisture content for a dryer of length 14 inches. In Figure 14, the theoretical and the experimental values of the air temperature are compared for the same dryer.

The agreement between the experimental and the theoretical results is quite good. From Table 1, it will be noted that for a 10 inch long dryer the theoretical value of the outlet product moisture content is slightly greater than the corresponding experimental value. It appears that the theoretical model under-estimates the quantity of mass transfer. It was noted earlier that the Chu's diffusion coefficient for corn was calculated from data between 120 F and 160 F. Therefore, one cannot be certain that the diffusion coefficient calculated by equation (26) at 180 F is the correct value. Also, little is known about convective mass transfer within a bed of particles. Therefore, the value of the convective mass transfer coefficient used is no more than a crude estimate.

Quasilinearization

For a dryer of a given length and with given inlet air and product properties a quasilinearization or a non-linear estimation pro-

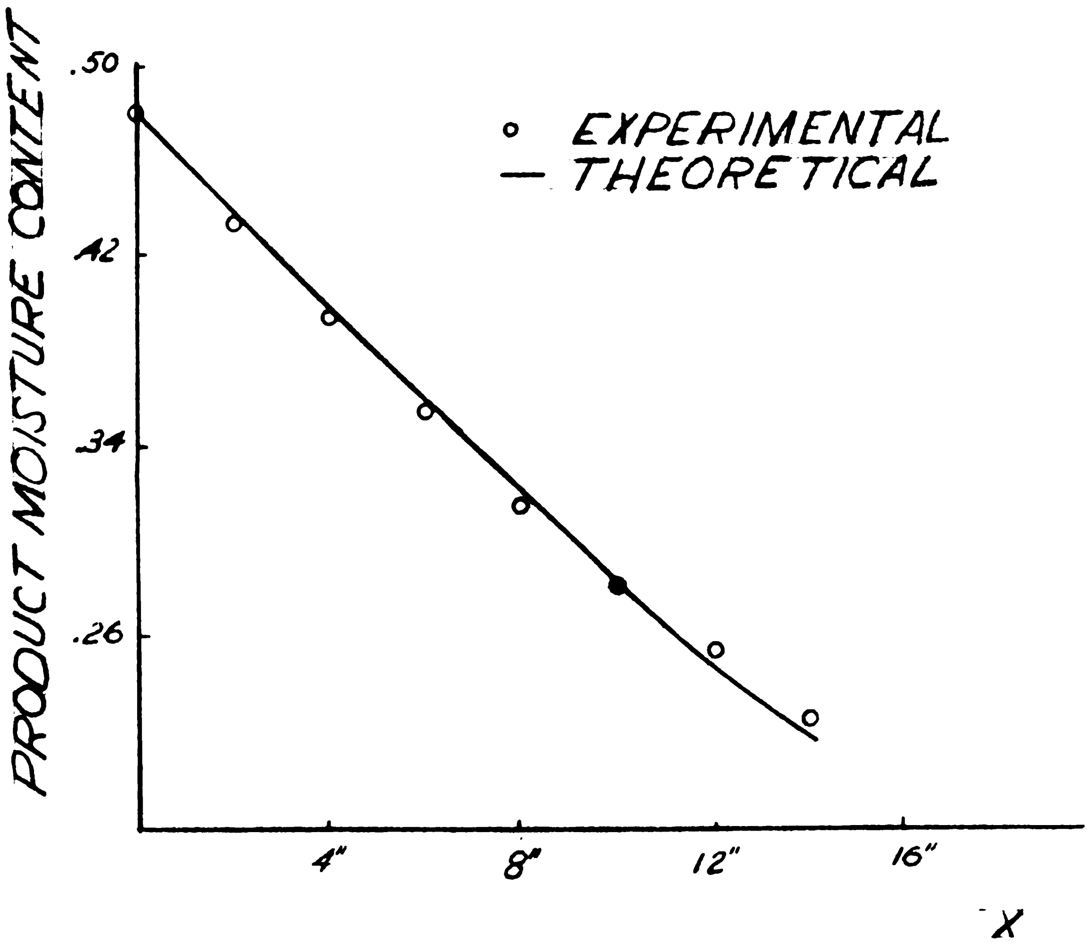

Figure 13. Comparison of Theoretical and Experimental Product Moisture Contents within the Dryer

T_i = 180.0 F

H_i = .0053 lbm water vapor/lbm dry air

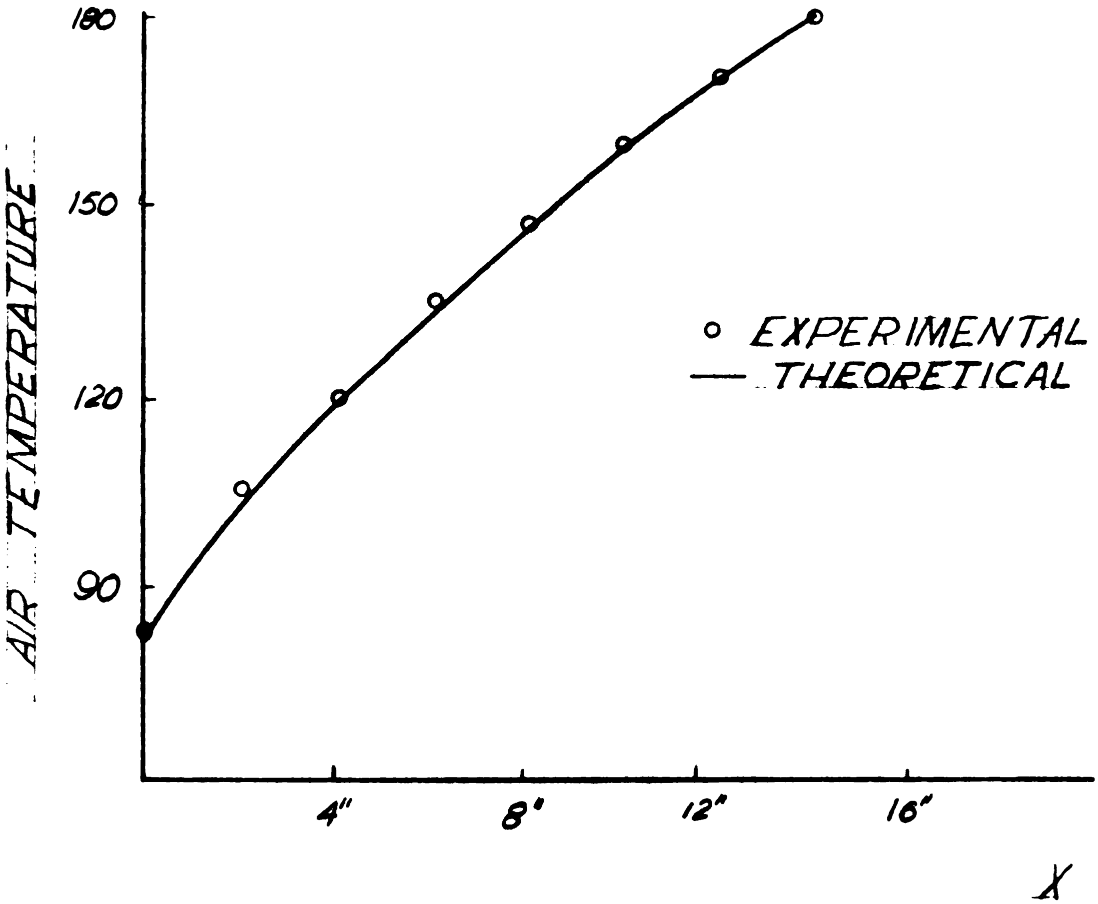

Figure 14. Comparison of Theoretical and Experimental Air Temperatures within the Dryer.

T_i = 180.0 F

H_i = .0053 lbm water vapor/lbm dry air

cedure could also be used to determine the outlet properties (θ_o, $\overline{M}_o$, T_o and H_o). The techniques presented in this paper have two major advantages over quasilinearization. First contrary to the invariant imbedding and the invariant programming formulations, the quasilinearization procedure requires a good initial estimate of the outlet properties or the system equations (8), (11), (17), (23), (31) and (35) will become unstable. Secondly, in generating the solutions the quasilinearization procedure yields no other practical information. The invariant imbedding and the invariant programming formulations generate the outlet properties as functions of the dryer length and the inlet air or product properties.

Parameter Study

In Figures 15, 16 and 17 the outlet product moisture content is shown as a function of the dryer length and the inlet air properties for a family of dryers with the following inlet properties and flow rates:

G_a	200 lbm dry air/hr
G_p	35 lbm dry product/hr
θ_i	80.0 F
$\overline{M}_i$	.33

In Figure 15 the outlet product moisture content is plotted for various inlet air temperatures as a function of dryer length for a given inlet air humidity ratio. With the inlet air temperatures equal to 100 F and 120 F, the dryer approaches a condition where the inlet product temperature is equal to the outlet air temperature and the inlet air temperature is equal to the outlet product temperature. These dryers reach this state at .3 and .6 feet, respectively. With inlet

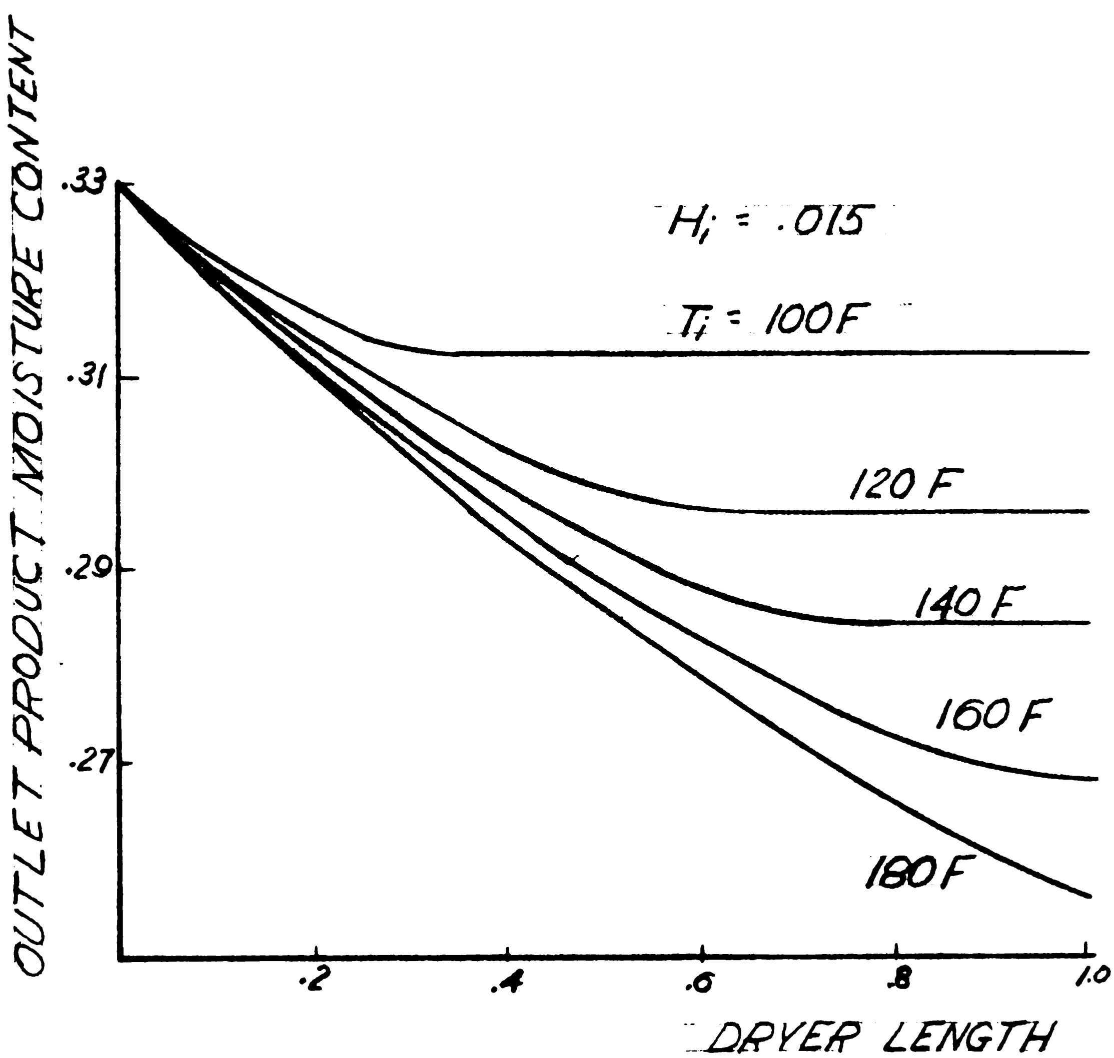

Figure 15. The Outlet Product Moisture Content for Various Inlet Air Temperatures Versus Dryer Length for a Given Inlet Air Humidity Ratio

air temperatures equal to 140 F, 160 F and 180 F the dryer approaches equilibrium state 4. For inlet air temperatures of 140 F and 160 F the dryer reaches equilibrium state 4 for a dryer length of .8 and .95 feet, respectively.

The basic shape of these curves is typical for counter-flow grain dryers. The dryer decreases its outlet product moisture content rapidly as the dryer length is increased until an equilibrium state is reached. As the dryer length is increased the outlet air temperature is lowered due to the energy required for evaporation of the moisture and the energy used to increase the outlet product temperature. The outlet air humidity ratio is increased as moisture is transferred from the product. Therefore, for the dryer which approaches equilibrium state 4, the system rapidly approaches a condition where the outlet air is saturated.

In Figure 16 the outlet product moisture content for various inlet air humidities is plotted as a function of dryer length for an inlet air temperature equal to 180 F. All of these dryers approach equilibrium state 4. For the inlet air humidity ratios .04 and .03 lbm water vapor/lbm dry air the system reaches equilibrium state 4 when the dryer length is increased to .8 feet and .9 feet, respectively.

In Figure 17, the outlet product moisture content for various inlet air humidity ratios is shown as a function of the inlet air temperature for a given dryer length. For the inlet air humidity ratios .02, .03 and .04 lbm water vapor/lbm dry air all of the dryers have reached equilibrium. For the inlet air humidity ratio .01 lbm water vapor /lbm dry air the dryer has reached equilibrium if the inlet air temperature is

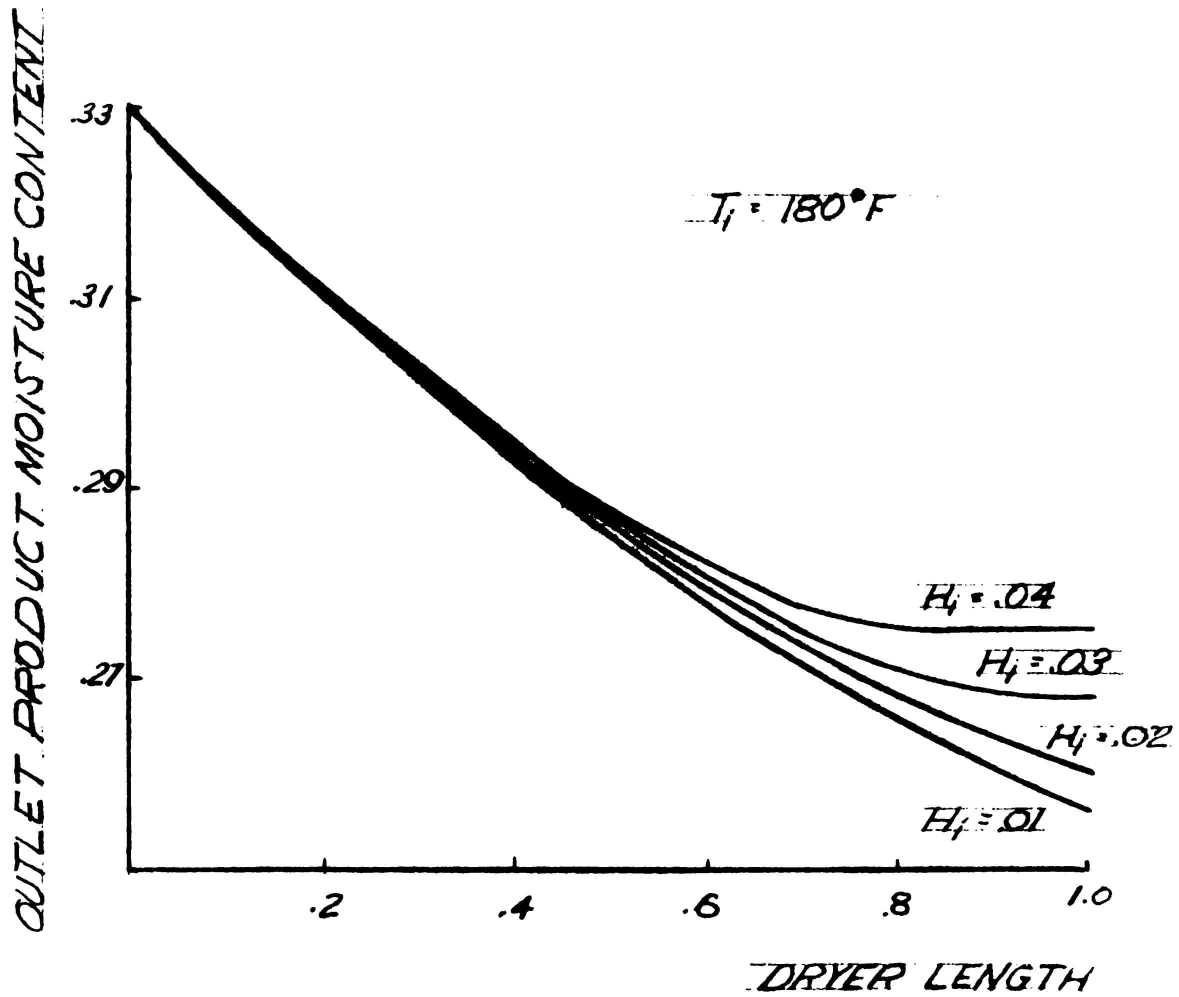

Figure 16. The Outlet Product Moisture Content for Various Inlet Air Humidity Ratios Versus Dryer Length for a Given Inlet Air Temperature.

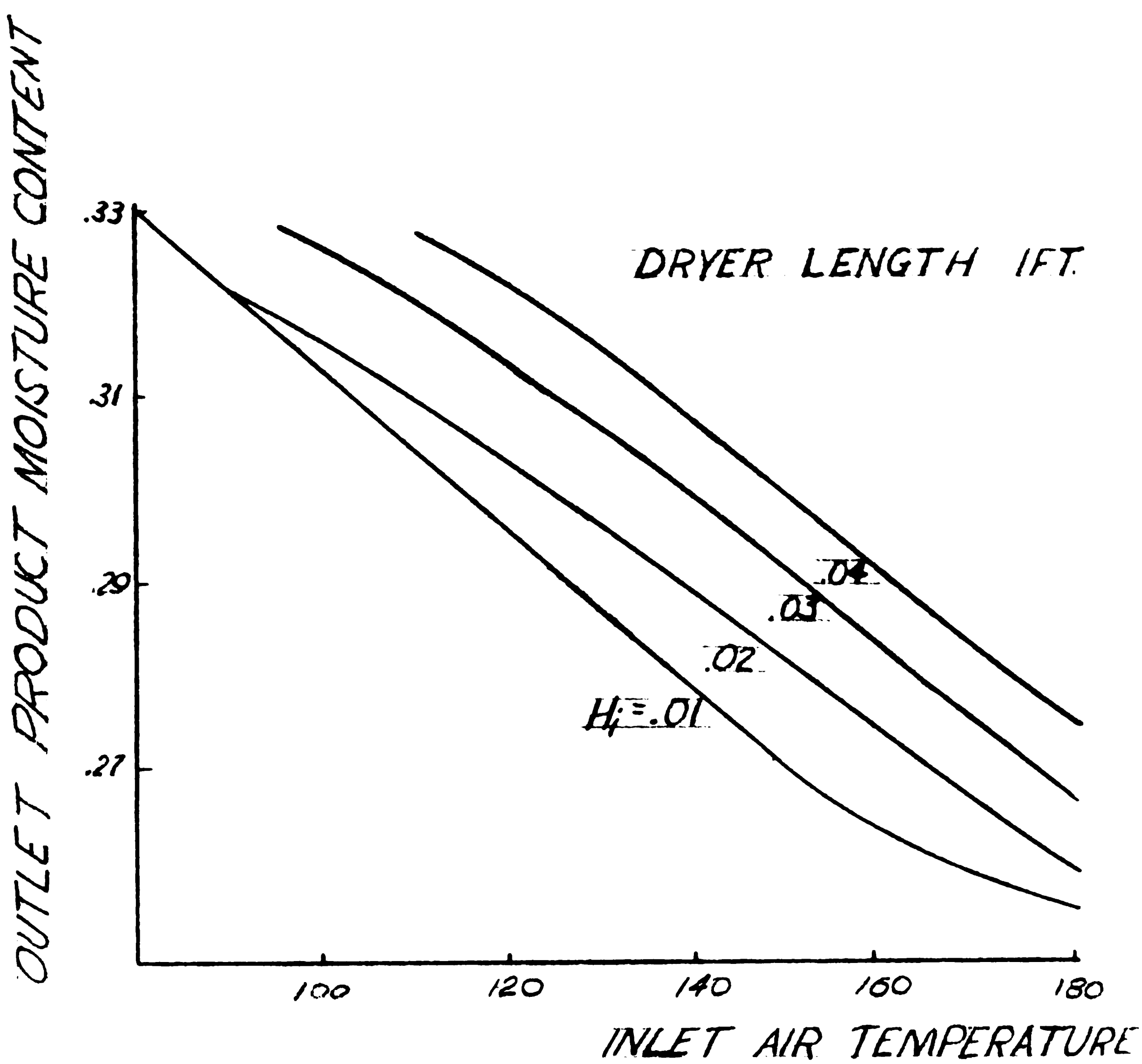

Figure 17. The Outlet Product Moisture Content for Various Inlet Air Humidity Ratios Versus Inlet Air Temperature for a Given Dryer Length.

less than or equal to 150 F. When the inlet air humidity ratio is equal to .03 or .04, all of the dryers have reached equilibrium state 4. If the inlet air humidity ratio is equal to .02 the system approaches equilibrium state 4 if the inlet air temperature is greater than 90 F. Similarly, if the inlet air humidity ratio is .01 the system approaches equilibrium state 4 if the inlet air temperature is greater than 150 F. If the inlet air humidity ratio is equal to .01 and the inlet air temperature is less than or equal to 150 F or if the inlet air humidity ratio is equal to .02 and the inlet air temperature is less than or equal to 90 F, the system will approach a condition in which the inlet air temperature is equal to the outlet product temperature and the inlet product temperature is equal to the outlet air temperature.

SUMMARY AND CONCLUSIONS

Invariant Imbedding Versus Invariant Programming

Both invariant imbedding and invariant programming are ideally suited to solve first order two-point boundary value problems. In this research the first order equations are equations (8), (11), (17), (23), (31) and (35) and the two-point boundary conditions are equations (38) through (41). In general n of these boundary conditions will be at x equal zero. The remaining m boundary conditions will be at x equal a. For the model presented in this paper n equals 4 and m equals 2.

Invariant imbedding equations can be written for the m unknown properties at x equal zero and the n unknown properties at x equal a. There exist two basic forms of these invariant imbedding equations.

1) For given values of the n known properties at x equal zero, the n + m unknown properties can be expressed as functions of the known properties at x equal a and the dryer length. The outlet air and product properties are expressed as functions of the two known properties at x equal a and the dryer length in equations (45), (48), (53) and (55).

2) For given values of the m known properties at x equal a, the n + m unknown properties can be expressed as functions of the known properties at x equal zero and the dryer length. The outlet properties are expressed as functions of the four known properties at x equal zero and the dryer length in equations (58) and (61).

For both types of invariant imbedding equations initial conditions are known values of the n + m unknown properties at a specific dryer length. Boundary conditions for the first type of invariant imbedding euqations are known values of the n + m unknown properties at specific values of each of the m known properties at x equal a. Therefore, to solve for the n + m unknown properties using the first set of invariant imbedding equations it is essential that n + m initial conditions and m(n+m) boundary conditions be known.

Boundary conditions for the second set of invariant imbedding equations consist of known values of the n + m unknown properties at specific values of each of the n known properties at x equal zero. In this case n + m initial conditions n(n+m) boundary conditions are necessary to solve the invariant imbedding equations.

In some cases not all n + m initial conditions and m(n+m) boundary conditions are known for the invariant imbedding equations of the first type. Similarly there exist cases where the n + m initial conditions and n(n+m) boundary conditions are not all known for the invariant imbedding equations of the second type. In these cases it is difficult to solve the invariant imbedding equations. Therefore, invariant programming should be used. For the model studied in this paper, there were insufficient boundary conditions. Thus, it is essential to use invariant programming.

Model Limitations

It was stated on page 35 that there exist special cases [equation (81) is satisfied and condensation occurs within the dryer) for which it is not possible to solve the system equations (8), (11), (17), (23), (31) and (35). In the design of a counter-flow dryer one is interested in maximizing the degree of drying. Therefore, a system in which condensation occurs is of no practical interest.

Since invariant programming utilizes the model directly, this method can be solved for the outlet properties if the model can be evaluated.

Suggestions for Further Study

The model presented would be capable of representing the system for a wider range of inlet conditions, if the diffusion coefficient for corn was known for higher product temperatures. More research is needed in the area of convective mass transfer within packed beds of particles.

With further study the model could be written in terms of more representative driving potentials (free energies and chemical potentials). With this modification the equilibrium states should be redefined and experimentally verified.

On page 41 it was noted that if criterion (82) was satisfied, one must consider a set R of feasible outlet product properties. Theoretical work is yet to be done for this special class of dryers.

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APPENDIX

a) Computer Program which solves for the outlet properties using invariant imbedding.

b) Computer Program which solves the system model.

c) Computer Program which solves for the outlet properties using invariant programming.

```
      PROGRAM COUNTER
      DIMENSION R1(31,26,3),R2(31,26,3),R11(10,5 ),R22(10,5 ),CX1(10,5 )
     1 ,CX2(10,5 ),TX1(100),HX1(100),TPX1(100),XMX1(100),XD1(100),R3(31,
     226,3),R4(31,26,3),TO(31,26,3),HO(31,26,3),TOUT(10,5 ),RHO(10,5 )
     3,ISP(31,26,3),XMC(3),XMP10(3),XMCINL(3),THETAL(3),XIN(3),XJB7(3)
      READ(60,1)  GAIR,CAIR,GPROD,CPROD,HFG,AP,H,HD
      PRINT 11
      WRITE(61,2) GAIR,CAIR,GPROD,CPROD,HFG,AP,H,HD
    1 FORMAT(8F10.0)
    2 FORMAT(* *,8E15.4)
   11 FORMAT(* *,*MASS FLOW OF AIR*,7X,*CAIR*,3X,*MASS FLOW OF PROD*,6X,
     1*CPROD*,5X,*LATENT HEAT*,10X,*A*,7X,*CONV COEF*,3X,*MASS CONV COEF
     2*)
      READ(60,3) A1,B1,C1,D1,E1
      PRINT 12
      WRITE(61,2) A1,B1,C1,D1,E1
    3 FORMAT(5E15.5)
   12 FORMAT(* *,7X,*A1*,13X,*B1*,13X,*C1*,13X,*D1*,13X,*E1*)
      READ (60,4) NA,NC1,NC2,A,TAIR,TPROD,HAIR
      PRINT 13
      WRITE(61,5) NA,NC1,NC2,A,TAIR,TPROD,HAIR
    4  FORMAT(3I10,4F12.0)
    5 FORMAT(* *,3I10,4E15.5)
   13 FORMAT(* *,2X,*NODES A*,3X,*NODES C1*,2X,*NODES C2*,1X,*LENGTH OF
     1DRYER *,6X,*TEMP*,11X,*THETA*,11X,*HSP*)
      READ(60,1) XMCIN,EPSLON,RHOP,DELZ,DELP,DXMCIN,DTHETA
      PRINT 14
      WRITE(61,2)XMCIN,EPSLON,RHOP,DELZ,DELP,DXMCIN,DTHETA
   14 FORMAT(* *,5X,*IN MCDB*,8X,*POROSITY*,7X,*DENSITY PROD*,8X,*DELZ*,
     18X,*DELP*,8X,*DXMCIN*,8X,*DTHETA*)
      READ(60,6) NPC1,NPC2,NPR1,NPR2,NS,NH
      PRINT 18
    6 FORMAT(8I10)
      WRITE(61,7) NPC1,NPC2,NPR1,NPR2,NS,NH
    7 FORMAT(* *,8I10)
   18 FORMAT(* *,7X,*NPC1*,7X,*NPC2*,7X,*NPR1*,7X,*NPR2*,8X,*NS*,8X,*NH*
```

```
1)
 THETA=TPROD
 TEMP=TAIR
 HSP=HAIR
 HBLIN=A1+B1*THETA+C1*THETA**2+D1*THETA**3+E1*THETA**4
 RHI=1.0-EXP(-0.0000382*(THETA+50.0)*(XMCIN*100.0)**2)
 HBLIN=HBLIN*RHI
 HBLIN=0.622*(HBLIN/(14.7-HBLIN))
 C22=HSP-HBLIN
 C11=TEMP-THETA
 XNC1=NC1-1
 XNC2=NC2-NH
 XNA=NA
 XNS=NS
 DC1=C11/XNC1
 DC2=C22/XNC2
 DC6=DC1-DTHETA
 DC5=DC1
 XMCINL(1)=XMCIN
 XMCINL(2)=XMCIN
 XMCINL(3)=XMCIN+DXMCIN
 THETAL(1)=THETA
 THETAL(2)=THETA+DTHETA
 THETAL(3)=THETA
 NH1=NH+1
 XNH=NH-1
 DC3=DC2
 DC4=DC2/(XNH+1.0)
 DA=-A/XNA
 XNPC1=NPC1
 XNPC2=NPC2
 INPC1=NPC1+1
 INPC2=NPC2+1
 N1=XNC1/XNPC1
 N2=(XNC2+XNH)/XNPC2
 NC11=(NC1-1)/NPC1
```

```
      NC22=(NC2-1)/NPC2
      DC11=DC1*XNPC1
      DC22=DC2*XNPC2
      DO 10I=1,NC1
      DO20J=1,NC2
      CM1=I-1
      CM2=J-1
      IF(J.LE.NH1) DC2=DC4
      IF(J.GT.NH1) DC2=DC3
      IF(J.GT.NH1) CM2=J-NH
      DO 22 K=1,3
      R1(I,J,K)=0.0
      R2(I,J,K)=0.0
      R3(I,J,K)=0.0
      R4(I,J,K)=0.0
      H0(I,J,K)=CM2*DC2+HBLIN
      T0(I,J,K)=CM1*DC1+THETA
      ISP(I,J,K)=5
   22 CONTINUE
   20 CONTINUE
   10 CONTINUE
      A=0.0
      IA=0
      LA=0
      DO 1100 I=1,100
      TX1(I)=0.0
      HX1(I)=0.0
      TPX1(I)=0.0
      XMX1(I)=0.0
      XD1(I)=0.0
 1100 CONTINUE
      THETM=0.0
      NPF=0
      XMP3=-HD*RHOP/(DELZ*GPROD)*EPSLON
      XMP5=GPROD/GAIR
      BA1=CAIR*GAIR
```

```
      BA2=CPROD*GPROD
      BA3=H*AP
      BA4=-RHOP*EPSLON/(GPROD*DELZ**2)
      BA5=0.446*GAIR
      BA9=GAIR*(1094.0-0.57*THETA)
      XIN(1)=0.01*(1.0/(0.0000382*(THETA+50.0)))**0.5
      XIN(2)=0.01*(1.0/(0.0000382*(THETAL(2)+50.0)))**0.5
      XIN(3)=XIN(1)
      XMP11=XMP3*DELP/XNS*2.0
      TAB=(THETA-32.0)*0.55555+273.2
      TABT=(THETAL(2)-32.0)*0.55555+273.2
      D=0.001629*EXP((0.00045*TAB-0.05485)*XMCIN*100.0-2513.0/TAB)
      DT=.001629*EXP((0.00045*TABT-.05485)*XMCIN*100.0-2513.0/TABT)
      DM=.001629*EXP((0.00045*TAB-0.05485)*XMCINL(3)*100.0-2513.0/TAB)
      XMP10(1)=BA4*D*DELP/XNS*4.0
      XMP10(2)=BA4*DT*DELP/XNS*4.0
      XMP10(3)=BA4*DM*DELP/XNS*4.0
      DO 23 I=1,3
      XJB7(I)=BA3/(BA2+GPROD*XMCINL(I))
   23 CONTINUE
   99 A=A-DA
      IA=IA+1
      LA=LA+1
C CALCULATIONS AT C2=0.0
      DC2=DC4
      DO 30 I=2,NC1
      HGRADT=HO(I,1,2)-HO(I,1,1)
      HGRADM=HO(I,1,3)-HO(I,1,1)
      TGRADT=TO(I,1,2)-TO(I,1,1)
      TGRADM=TO(I,1,3)-TO(I,1,1)
      C12=I-1
      C12=C12*DC1
      C2=0.0
      HUMRO=C2+HBLIN
      XJB2=BA1+BA5*HUMRO
      XJB5=1.0/XJB2
```

```
      DO 31 L=1,3
      IF(ISP(I,1,1).EQ.0) GO TO 904
      IF(L.EQ.2.AND.I.EQ.2) DC1=DC6
      THETAC=1.001*THETAL(L)
      XMCINC=.999*XMCINL(L)
      TEM=R1(I,1,L)+THETAL(L)
      THET=THETA+C12
      THETC=.990*THET
      IF( THET.GE.212.0) GO TO 807
      HDPC1=A1+B1*THET+C1*THET**2+D1*THET**3+E1*THET**4
      RHI=HUMRO*14.7/(HDPC1*(0.622+HUMRO))
      IF(RHI.GE.1.0) GO TO 707
      XINF=0.01*((-ALOG(1.0-RHI))/((0.0000382)*(TEM+50.0)))**0.5
      GO TO 708
  807 XINF=0.0
      GO TO 708
  707 XINF=1.0
  708 CONTINUE
      XINFC=1.001*XINF
      XMCAVE=(R2(I,1,L)+2.0*R3(I,1,L)+R4(I,1,L))/4.0+XMCINL(L)
      IF(TEM.GT.THETC.AND.XINFC.GT.XMCAVE.OR.TO(I,1,L).LT.THETAC) ISP(I,
     11,L)=0
      IF(ISP(I,1,L).EQ.0) GO TO 904
      TAB=(TEM-32.0)*.555555+273.2
      DIFF=0.001629*EXP((0.00045*TAB-0.05485)*XMCAVE*100.0-2513.0/TAB)
      XJB1=BA2+GPROD*XMCAVE
      XMP1=XJB1/XJB2
      HFG=1094.0-0.57*TEM
      XMP2=HFG*GAIR/XJB2
      XMP4=BA3/XJB1
      XMP6=BA4*DIFF
      G1=-XMP4*(C12-R1(I,1,L)-THETAL(L)+THETA)
      G2=XMP3*(XINF-XMCINL(L)-R2(I,1,L))+XMP6*(R3(I,1,L)-R2(I,1,L))
      G2=2.0*G2
      G3=XMP6*(R4(I,1,L)-2.0*R3(I,1,L)+R2(I,1,L))
      G4=2.0*XMP6*(R3(I,1,L)-R4(I,1,L))
```

```
      F2=XMP5*(G2+2.0*G3+G4)/4.0
      F1=XMP1*G1-XMP2*F2
      IF(TO(I,1,L).GE.212.0) GO TO 801
      HDPO=A1+B1*TO(I,1,L)+C1*TO(I,1,L)**2+D1*TO(I,1,L)**3+E1*TO(I,1,L)*
     1*4
      RH=HO(I,1,L)*14.7/(HDPO*(0.622+HO(I,1,L)))
      IF(RH.GE.1.0) GO TO 701
      XINF=XIN(L)*(-ALOG(1.0-RH))**0.5
      GO TO 702
  801 XINF=0.0
      GO TO 702
  701 XINF=1.0
  702 CONTINUE
      IF(XMCINC.LT.XINF.AND.TEM.GT.THETC) ISP(I,1,L)=0
      IF(ISP(I,1,L).EQ.0) GO TO 904
      TE1=F1*DA/DC1
      TE3=G1*DA
      TE4=G2*DA
      TE5=G3*DA
      TE6=G4*DA
      DR2=R1(I,1,L)
      R1(I,1,L)=(R1(I,1,L)+TE1*R1(I-1,1,L)+TE3)/(1.0+TE1)
      DR1=R1(I,1,L)-DR2
      R2(I,1,L)=(R2(I,1,L)+TE1*R2(I-1,1,L)+TE4)/(1.0+TE1)
      R3(I,1,L)=(R3(I,1,L)+TE1*R3(I-1,1,L)+TE5)/(1.0+TE1)
      R4(I,1,L)=(R4(I,1,L)+TE1*R4(I-1,1,L)+TE6)/(1.0+TE1)
      XMCAVE=(R2(I,1,L)+2.0*R3(I,1,L)+R4(I,1,L))/4.0+XMCINL(L)
      HOUT= XMP5*(XMCINL(L)-XMCAVE)+HUMRO
      THET=TO(I,1,L)-XMP1*DR1-XMP2*(HOUT-HO(I,1,L))
      CB1=BA1+BA5*HO(I,1,L)
      CB1=1.0/CB1
      XMP8=BA3*CB1
      XMP9=BA9*CB1
      XMC1=XMCINL(L)
      XMC2=XMCINL(L)
      XMC3=XMCINL(L)
```

```
      XMC4=XMCINL(L)
      XMC5=XMCINL(L)
      DO 1001 K=1,NS
      XMC6=XMC1-(XMP11*(XINF-XMC1)  +XMP10(L)*(XMC2-XMC1))*2.0
      XMC7=XMC2-XMP10(L)*(XMC1-2.0*XMC2+XMC3)
      XMC8=XMC3-XMP10(L)*(XMC2-2.0*XMC3+XMC4)
      XMC9=XMC4-XMP10(L)*(XMC3-2.0*XMC4+XMC5)
      XMC10=XMC5-2.0*XMP10(L)*(XMC4-XMC5)
      XMC1=XMC6
      XMC2=XMC7
      XMC3=XMC8
      XMC4=XMC9
      XMC5=XMC10
 1001 CONTINUE
      XMA=(XMC1+2.0*XMC2+2.0*XMC3+2.0*XMC4+XMC5)/8.0
      U2=(XMCINL(L)-XMA)/DELP
      V1=XMP5*U2
      U1=XJB7(L)*(THETAL(L)-TO(I,1,L))
      V2=XMP8*(THETAL(L)-TO(I,1,L))-XMP9*V1
      TE7=-V1*DA
      TE8=-V2*DA
      TE9=U1*DA/DTHETA
      TE10=U2*DA/DXMCIN
      DTOUT=TO(I,1,L)
      DHOUT1=HO(I,1,L)
      TO(I,1,L)=TO(I,1,L)+TGRADT*TE9+TGRADM*TE10+TE8
      HO(I,1,L)=HO(I,1,L)+HGRADT*TE9+HGRADM*TE10+TE7
      DHOUT=HO(I,1,L)-DHOUT1
      XMP8=XMP8/XJB7(L)
      IF(L.EQ.1.AND.A.LE.0.2) GO TO 910
      GO TO (909,902,911,909), ISP(I,1,1)
  910 IF(ISP(I,1,1).EQ.4.OR.ISP(1,1,1).EQ.1) GO TO 909
      IF(V1.GT.F2) GO TO 903
  909 XMCAVE1=XMCINL(L)+(HUMRO-HO(I,1,L))/XMP5
      THET=DTOUT-XMP1*DR1-XMP2*DHOUT
      E2=4.0*R2(I,1,L)*(XMCAVE1-XMCAVE)/(R2(I,1,L)+2.0*R3(I,1,L)+R4(I,1,
```

```
     1L))
      E3=E2*R3(I,1,L)/R2(I.1.L)
      E4=E2*R4(I,1,L)/R2(I.1,L)
      R2(I.1,L)=R2(I,1,L)+E2
      R3(I.1.L)=R3(I,1,L)+E3
      R4(I.1.L)=R4(I,1,L)+E4
      IF(L.EQ.1.AND.A.LE.0.2) GO TO 912
      GO TO (913,902,911,901),ISP(I.1,1)
  912 IF(G1.GT.U1) GO TO 901
  913 ISP(I,1,L)=1
      R1(I.1.L)=DR2-((TO(I,1.L)-DTOUT)+XMP9*DHOUT)/XMP8
      GO TO 904
  901 ISP(I,1,L)=4
      TO(I,1,L)=THET
      GO TO 904
  903 IF(G1.GT.U1) GO TO 902
  911 ISP(I.1.L)=3
      R1(I.1.L)=DR2-((TO(I,1.L)-DTOUT)+XMP9*(HOUT-DHOUT1))/XMP8
      HO(I.1.L)=HOUT
      GO TO 904
  902 ISP(I,1,L)=2
      HO(I.1.L)=HOUT
      TO(I.1.L)=THET
  904 CONTINUE
      DC1=DC5
   31 CONTINUE
   30 CONTINUE
C CALCULATIONS OF ENTIRE MESH
      L=1
      DO 50I=2,NC1
      CM1=I-1
      CM1=CM1*DC1
      DO 60J=2,NC2
      IF(J.GT.NH1) DC2=DC3
      IF(J.LE.NH1) DC2=DC4
      CM2=J-1
```

```
      IF(J.GT.NH1) CM2=J-NH
      CM2=CM2*DC2
      HUMRO=CM2+HBLIN
      XJB2=BA1+BA5*HUMRO
      THETAC=1.001*THETAL(L)
      XMCINC=0.999*XMCINL(L)
      IF(ISP(I,J,L).EQ.0) GO TO 908
      RM1=R1(I,J,L)
      TEM=RM1+THETAL(L)
      THET=CM1+THETA
      THETC=0.999*THET
      IF(THET.GT.212.0)GO TO 815
      HDPM=A1+B1*THET+C1*THET**2+D1*THET**3+E1*THET**4
      RHI=HUMRO*14.7/(HDPM*(.622+HUMRO))
      IF(RHI.GE.1.0) GO TO 715
      XINF=0.01*((-ALOG(1.0-RHI))/((0.0000382)*(TEM+50.0)))**0.5
      GO TO 716
  815 XINF=0.0
      GO TO 716
  715 XINF=1.0
  716 CONTINUE
      XINFC=1.001*XINF
      XMCAVE=(R2(I,J,L)+2.0*R3(I,J,L)+R4(I,J,L))/4.0+XMCINL(L)
      IF(TEM.GT.THETC.AND.XINFC.GT.XMCAVE.OR.TO(I,J,L).LT.THETAC) ISP(I,
     1J,L)=0
      IF(ISP(I,J,L).EQ.0) GO TO 908
      TAB=(TEM-32.0)*.555555+273.2
      DIFF=0.001629*EXP((0.00045*TAB-0.05485)*XMCAVE*100.0-2513.0/TAB)
      XJB1=BA2+GPROD*XMCAVE
      XMP1=XJB1/XJB2
      HFG=1094.0-0.57*TEM
      XMP2=HFG*GAIR/XJB2
      XMP4=BA3/XJB1
      XMP6=BA4*DIFF
      G1=-XMP4*(CM1-RM1-THETAL(L)+THETA)
      G2=XMP3*(XINF-XMCINL(L)-R2(I,J,L))+XMP6*(R3(I,J,L)-R2(I,J,L))
```

```
      G2=2.0*G2
      G3=XMP6*(R4(I,J,L)-2.0*R3(I,J,L)+R2(I,J,L))
      G4=2.0*XMP6*(R3(I,J,L)-R4(I,J,L))
      F2=XMP5*(G2+2.0*G3+G4)/4.0
      F1=XMP1*G1-XMP2*F2
      IF(TO(I,J,L).GE.212.0) GO TO 806
      HDPO=A1+B1*TO(I,J,L)+C1*TO(I,J,L)**2+D1*TO(I,J,L)**3+E1*TO(I,J,L)*
     1*4
      RH=HO(I,J,L)*(14.7-HDPO)/(0.622*HDPO)
      IF(RH.GE.1.0) GO TO 706
      XINF=XIN(L)*(-ALOG(1.0-RH))**0.5
      GO TO 704
  806 XINF=0.0
      GO TO 704
  706 XINF=1.0
  704 CONTINUE
      IF(XMCINC.LT.XINF.AND.TEM.GT.THETC) ISP(I,J,L)=0
      IF(ISP(I,J,L).EQ.0) GO TO 908
      TE1=F1*DA/DC1
      TE2=F2*DA/DC2
      TE3=G1*DA
      TE4=G2*DA
      TE5=G3*DA
      TE6=G4*DA
      R1(I,J,L)=(R1(I,J,L)+TE1*R1(I-1,J,L)+TE2*R1(I,J-1,L)+TE3)/(1.0+TE1
     1+TE2)
      R2(I,J,L)=(R2(I,J,L)+TE1*R2(I-1,J,L)+TE2*R2(I,J-1,L)+TE4)/(1.0+TE1
     1+TE2)
      R3(I,J,L)=(R3(I,J,L)+TE1*R3(I-1,J,L)+TE2*R3(I,J-1,L)+TE5)/(1.0+TE1
     1+TE2)
      R4(I,J,L)=(R4(I,J,L)+TE1*R4(I-1,J,L)+TE2*R4(I,J-1,L)+TE6)/(1.0+TE1
     1+TE2)
      THET=(TO(I,J,L)+TE1*TO(I-1,J,L)+TE2*TO(I,J-1,L))/(1.0+TE1+TE2)
      HOUT=(HO(I,J,L)+TE1*HO(I-1,J,L)+TE2*HO(I,J-1,L))/(1.0+TE1+TE2)
      HO(I,J,L)=HOUT
      TO(I,J,L)=THET
```

```
908 CONTINUE
    DC1=DC5
 60 CONTINUE
 50 CONTINUE
    DO 15 I=INPC2,NC2,NPC2
    DO 25 J=INPC1,NC1,NPC1
    NI=(I-1)/NPC2
    NJ=(J-1)/NPC1
    XNI=NI
    XNJ=NJ
    RHO(NJ,NI)=HO(J.I,1)
    TOUT(NJ,NI)=TO(J,I,1)
    R11(NJ,NI)=R1(J,I,1)+THETA
    R22(NJ,NI)=(R2(J,I,1)+2.0*R3(J,I,1)+R4(J,I,1))/4.0+XMCIN
    CX1(NJ,NI)=XNJ*DC11+THETA
    CX2(NJ,NI)=XNI*DC22+HBLIN-DC2*XNH
 25 CONTINUE
 15 CONTINUE
 35 IF(LA.NE.NPR1) GO TO 45
    PRINT 16,A
 16 FORMAT(*1*,*DATA WITH DRYER LENGTH EQUAL*,F6.3,2X,*FEET*)
    PRINT 17
 17 FORMAT(* *,4X,*IN TEMP*,7X,*OUT THETA*,8X,*IN HSP*,8X,*OUT MCDB*,8
   1X,*OUT HSP*,8X,*OUT TEMP*)
    WRITE(61,8) ((CX1(I,J),R11(I,J),CX2(I,J),R22(I,J),RHO(I,J),TOUT(I,
   1J),J=1,N2),I=1,N1)
  8 FORMAT(* *,6E15.4)
    LA=0
 45 IF(IA.LT.NA) GO TO 99
    IF(NPR2.NE.1) GO TO 69
    NPF=1
    CALL       MODEL(GPROD,A1,B1,C1,D1,E1,A,TO(NC1,NC2,1),THETA,HO(NC1
   1,NC2,1),XMCIN,XMC,THET,DA,BA1,BA2,BA3,BA4,BA5,XMP3,XMP5,NPF,GAIR,T
   1EMP,HSP)
 69 END
```

```
      SUBROUTINE MODEL(GPROD,A1,B1,C1,D1,E1,A,TEMM,THETM,HSM,XMCIM,XMC
     1 ,THET,DA,BA1,BA2,BA3,BA4,BA5,XMP3,XMP5,NPF,GAIR,TEMP1,HSP1)
      DIMENSION XMC(3),TX1(100),HX1(100),TPX1(100),XMX1(100),XD1(100)
      KA=0
      X1=0.0
      DX1=-DA
      X=0.0
      TEMP=TEMM
      THETA=THETM
      HSP=HSM
      XMCIN=XMCIM
      DO10I=1,3
      XMC(I)=XMCIN
   10 CONTINUE
   77 XJB2=BA1+BA5*HSP
      XMCIN=(XMC(1)+2.0*XMC(2)+XMC(3))/4.0
      XJB1=BA2+GPROD*XMCIN
      HFG=1094.0-0.57*THETA
      XMP1=XJB1/XJB2
      XMP2=HFG*GAIR/XJB2
      XMP4=BA3/XJB1
      TAB=(THETA-32.0)*.555555+273.2
      DIFF=0.001629*EXP((0.00045*TAB-0.05485)*XMCIN*100.0-2513.0/TAB)
      XMP6=BA4*DIFF
      IF(TEMP.GT.212.0) GO TO 851
      HDPM=A1+B1*TEMP+C1*TEMP**2+D1*TEMP**3+E1*TEMP**4
      RHI=HSP*14.7/(HDPM*(0.622+HSP))
      IF(RHI.GE.1.0) GO TO 717
      XINF=0.01*((-ALOG(1.0-RHI))/((0.0000382)*(THETA+50.0)))**0.5
      GO TO 718
  851 XINF=0.0
      GO TO 718
  717 XINF=1.0
  718 CONTINUE
      G1=XMP3*(XINF-XMC(1))+XMP6*(XMC(2)-XMC(1))
      G1=2.0*G1
```

```
      G2=XMP6*(XMC(3)-2.0*XMC(2)+XMC(1))
      G3=XMP6*2.0*(XMC(2)-XMC(3))
      DHSP=XMP5*(G1+2.0*G2+G3)/4.0
      DTHETA=-XMP4*(TEMP-THETA)
      DTEMP=XMP1*DTHETA-XMP2*DHSP
      DXH=0.00025/DHSP
      DXH=ABS(DXH)
      DXTH=2.0/DTHETA
      DXTH=ABS(DXTH)
      DXT=2.0/DTEMP
      DXT=ABS(DXT)
      DX=AMIN1(DXT,DXTH,DXH,DX1)
      DX=-DX
      X=X-DX
      IF(X.GT.A) DX=X+DX-A
      X1=X1-DX
      XMC(1)=XMC(1)+G1*DX
      XMC(2)=XMC(2)+G2*DX
      XMC(3)=XMC(3)+G3*DX
      HSP=HSP+DHSP*DX
      THETA=THETA+DTHETA*DX
      TEMP=TEMP+DTEMP*DX
      IF(NPF.EQ.0) GO TO 78
      IF(X1.LT.0.02) GO TO 78
      KA=KA+1
      TX1(KA)=TEMP
      HX1(KA)=HSP
      TPX1(KA)=THETA
      XMX1(KA)=(XMC(1)+2.0*XMC(2)+XMC(3))/4.0
      XD1(KA)=X
      XKA=KA
      X1=X-XKA*0.02
   78 IF(X.LT.A) GO TO 77
      IF(NPF.EQ.0) GO TO 69
      PRINT 19
   19 FORMAT(*1*,7X,*X*,13X,*TEMP*,11X,*THETA*,11X,*HSP*,11X,*MCDB*)
```

```
      WRITE(61,8)(XD1(I),TX1(I),TPX1(I),HX1(I),XMX1(I),I=1,KA)
    8 FORMAT(* *,5E15.4)
   69 END
```

```
      PROGRAM COUNT
C   INVARIANT PROGRAMMING
C   DICTIONARY
C   GAIR = AIR FLOW RATE , LBM D.A./HR
C   CAIR = SPECIFIC HEAT DRY AIR , BTU/LBM F
C   GPROD = PRODUCT FLOW RATE , LBM DRY PRODUCT/HR
C   CPROD = SPECIFIC HEAT DRY PRODUCT , BTU/LBM F
C  HFG =  LATENT HEAT OF EVAPORATION , BTU/LBM WATER
C   AP = SPECIFIC SURFACE AREA OF PROD , FT**2/FT**3
C   H = CONVECTIVE HEAT TRANSFER COEFFICIENT , BTU/FT**2 HR F
C   HD = CONVECTIVE MASS TRANSFER COEFFICIENT ,  1.0/HR
C   A1,B1,C1,D1,E1 = COEFFICIENTS OF POWER SERIES EXPANSION OF THE
C  SATURATION PRESSURE LINE
C   NA = NUMBER OF STEPS IN DRYER LENGTH
C   NC1 = NUMBER OF DISCRETE OUTLET AIR TEMPS IN FEASIBLE SET S
C   NC2 = NUMBER OF DISCRETE OUTLET AIR HUMIDITIES IN FEASIBLE SET S
C   A = TOTAL DRYER LENGTH , FT
C   TMAX = MAXIMUM FEASIBLE OUTLET AIR TEMPERATURE , F
C   TMIN = MINIMUM FEASIBLE OUTLET AIR TEMPERATURE , F
C   HMAX = MAXIMUM FEASIBLE OUTLET AIR HUMIDITY , LBM WATER / LBM DA
C   HMIN = MINIMUM FEASIBLE OUTLET AIR HUMIDITY , LBM WATER/LBM DA
C  XMCIN = INLET PRODUCT MOISTURE RATIO , DRY BASIS
C   EPSLON = POROSITY , FT**3 PRODUCT/ FT**3 TOTAL
C   RHOP = DENSITY OF PRODUCT , LBM/FT**3
C   THETA = INLET PRODUCT TEMPERATURE , F
C   DELZ = STEP SIZE WITHIN THE PRODUCT, FT
C   NPC1 =  NUMBER OF DESIRED INLET AIR TEMPERATURES
C   NPC2 = NUMBER OF DESIRED INLET AIR HUMIDITY RATIOS , LBM WATER/LBM DA
C   NPR1 = NUMBER OF STEPS IN DRYER LENGTH BETWEEN OUTPUT
C   TCHECK = MAXIMUM ALLOWABLEINLET AIR TEMPERATURE , SHOULD BE GREATER
C  THAN THE MAXIMUM DESIRED INLET AIR TEMPERATURE
C   HCHECK = MIMIMUM ALLOWABLE INLET AIR HUMDITY RATIO , SHOULD BE POSITIVE
C   AND LESS THAN THE MINIMUM DESIRED INLET AIR HUMIDITY RATIO
C   HDS= DESIRED INLET AIR  HUMIDITY RATIOS
C   TDS = DESIRED INLET AIR TEMPERATURES
C   TIN = INLET AIR TEMPERATURES
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C   HIN = INLET AIR HUMIDITY RATIOS
C   THO = OUTLET PRODUCT TEMPERATURES
C   XMO = OUTLET LOCAL MOISTURE RATIOS
      DIMENSION TIN(51,26),HIN(51,26),THO(51,26),XMO(51,26,3),HOUT(10,10
     1),TOUT(10,10),XMOUT(10,10),TDS(10),HDS(10),NF1(51),THOUT(10,10)
      READ(60,1)  GAIR,CAIR,GPROD,CPROD,HFG,AP,H,HD
      PRINT 11
      WRITE(61,2) GAIR,CAIR,GPROD,CPROD,HFG,AP,H,HD
    1 FORMAT(8F10.0)
    2 FORMAT(* *,8E15.4)
   11 FORMAT(* *,*MASS FLOW OF AIR*,7X,*CAIR*,3X,*MASS FLOW OF PROD*,6X,
     1*CPROD*,5X,*LATENT HEAT*,10X,*A*,7X,*CONV COEF*,3X,*MASS CONV COEF
     2*)
      READ(60,3) A1,B1,C1,D1,E1
      PRINT 12
      WRITE(61,2) A1,B1,C1,D1,E1
    3 FORMAT(5E15.5)
   12 FORMAT(* *,7X,*A1*,13X,*B1*,13X,*C1*,13X,*D1*,13X,*E1*)
      READ (60,4) NA,NC1,NC2,A,TMAX,TMIN,HMAX
      PRINT 13
      WRITE(61,5) NA,NC1,NC2,A,TMAX,TMIN,HMAX
    4  FORMAT(3I10,4F12.0)
    5 FORMAT(* *,3I10,4E15.5)
   13 FORMAT(* *,2X,*NODES A*,3X,*NODES C1*,2X,*NODES C2*,1X,*LENGTH OF
     1DPYER *,6X,*TMAX*,11X,*TMIN*,11X,*HMAX*)
      READ(60,1) XMCIN,EPSLON,RHOP,HMIN,THETA,DELZ
      PRINT 14
      WRITE(61,2)XMCIN,EPSLON,RHOP,HMIN,THETA,DELZ
   14 FORMAT(* *,5X,*IN MCDB*,8X,*POROSITY*,7X,*DENSITY PROD*,8X,*HMIN*,
     18X,*THETA*,8X,*DELZ*)
      READ(60,6) NPC1,NPC2,NPR1
      PRINT 18
      WRITE(61,7)NPC1,NPC2,NPR1
    6 FORMAT(8I10)
    7 FORMAT(* *,8I10)
   18 FORMAT(* *,7X,*NPC1*,7X,*NPC2*,7X,*NPR1*)
```

```
C   READ DESIRED TEMPS IN ASCENDING ORDER
C   READ DESIRED HUMIDITIES IN DESCENDING ORDER
      READ(60,1) TCHECK,HCHECK,(TDS(I),I=1,NPC1),(HDS(J),J=1,NPC2)
      PRINT 19
   19 FORMAT(* *,5X,*TCHECK*,10X,*HCHECK*)
      WRITE(61,2) TCHECK,HCHECK
      PRINT 21
   21 FORMAT(* *,10X,*DESIRED INLET AIR TEMPS*)
      WRITE(61,2) (TDS(I),I=1,NPC1)
      PRINT 22
   22 FORMAT(* *,10X,*DESIRED INLET HUMIDITIES*)
      WRITE(61,2) (HDS(I),I=1,NPC2)
      THETAC=1.001*THETA
      C11=TMAX-TMIN
      C22=HMAX-HMIN
      XNC1=NC1-1
      XNC2=NC2-1
      DC1=C11/XNC1
      DC2=C22/XNC2
      XNA=NA
      DA=A/XNA
      NF2=NC1
      NF3=NC2
C   INITIALIZE FEASIBLE SOLUTIONS
      DO 10 I=1,NF2
      DO 20 J=1,NF3
      CM1=I-1
      CM2=J-1
      HIN(I,J)=HMAX-CM2*DC2
      TIN(I,J)=TMIN+CM1*DC1
      DO 30 K=1,3
      XMO(I,J,K)=XMCIN
   30 CONTINUE
      THO(I,J)=THETA
   20 CONTINUE
   10 CONTINUE
```

```
      IA=0
      IK=0
      XMP3=-HD*RHOP/(DELZ*GPROD)*EPSLON
      XMP5=GPROD/GAIR
      BA1=CAIR*GAIR
      BA2=CPROD*GPROD
      BA3=H*AP
      BA4=-RHOP*EPSLON/(GPROD*DELZ**2)
      BA5=0.446*GAIR
      DO 40 I=1,NF2
      NF1(I)=1
   40 CONTINUE
      DO 45I=1,NPC1
      DO 65J=1,NPC2
      TOUT(I,J)=TDS(I)
      HOUT(I,J)=HDS(J)
      XMOUT(I,J)=XMCIN
      THOUT(I,J)=THETA
   65 CONTINUE
   45 CONTINUE
      A=0.0
C   STEP DRYER LENGTH
C CALCULATE NEW SET OF  INLET AIR PROPERTIES AND OUTLET PRODUCT PROPERTIES
   99 IA=IA+1
      IK=IK+1
      A=A+DA
      DO 50 I=1,NF2
      NF=NF1(I)
      DO 60 J=NF,NF3
      THET=TIN(I,J)
      HUMRO=HIN(I,J)
      TEM=THO(I,J)
      IF(HUMRO.LE. HCHECK) GO TO 60
      IF(THET.GE.TCHECK) GO TO 60
      IF(THET.GT.212.0)GO TO 815
      HDPM=A1+B1*THET+C1*THET**2+D1*THET**3+E1*THET**4
```

```
      RHI=HUMRO*(14.7-HDPM)/(.622*HDPM)
      IF(RHI.GE.1.0) GO TO 715
      XINF=0.01*((-ALOG(1.0-RHI))/((0.0000382)*(TEM+50.0)))**0.5
      GO TO 716
  815 XINF=0.0
      GO TO 716
  715 XINF=1.0
  716 CONTINUE
      IF(XINF.GE.XMCIN) NF1(I)=J+1
      XMCAVE=(XMO(I,J,1)+2.0*XMO(I,J,2)+XMO(I,J,3))/4.0
      TAB=(TEM-32.0)*.555555+273.2
      DIFF=0.001629*EXP((0.00045*TAB-0.05485)*XMCAVE*100.0-2513.0/TAB)
      XJB2=BA1+BA5*HUMRO
      XJB1=BA2+GPROD*XMCAVE
      XMP1=XJB1/XJB2
      HFG=1094.0-0.57*TEM
      XMP2=HFG*GAIR/XJB2
      XMP4=BA3/XJB1
      XMP6=BA4*DIFF
      XM1=XMO(I,J,1)
      XM2=XMO(I,J,2)
      XM3=XMO(I,J,3)
      G1=XMP4*(THET-TEM)
      G2=-2.0*(XMP3*(XINF-XM1)+XMP6*(XM2-XM1))
      G3=-XMP6*(XM1-2.0*XM2+XM3)
      G4=-2.0*XMP6*(XM2-XM3)
      F2=XMP5*(G2+2.0*G3+G4)/4.0
      F1=XMP1*G1-XMP2*F2
      IXH=ABS(F2*DA/.0005)+1.0
      IXT=ABS(G1*DA/2.00)+1.0
      XXH=IXH
      XXT=IXT
      DXH=DA/XXH
      DXT=DA/XXT
      DX=AMIN1(DXH,DXT)
      IXS=AMAX0(IXH,IXT)
```

```
      DO 55 K=1,IXS
      THO(I,J)=THO(I,J)+G1*DX
      XMO(I,J,1)=XMO(I,J,1)+G2*DX
      XMO(I,J,2)=XMO(I,J,2)+G3*DX
      XMO(I,J,3)=XMO(I,J,3)+G4*DX
      TIN(I,J)=TIN(I,J)+F1*DX
      HIN(I,J)=HIN(I,J)+F2*DX
      IF(K.EQ.IXS) GO TO 55
      THET=TIN(I,J)
      HUMRO=HIN(I,J)
      TEM=THO(I,J)
      IF(HUMRO.LE. HCHECK) GO TO 60
      IF(THET.GE.TCHECK) GO TO 60
      IF(THET.GT.212.0)GO TO 915
      HDPM=A1+B1*THET+C1*THET**2+D1*THET**3+E1*THET**4
      RHI=HUMRO*(14.7-HDPM)/(.622*HDPM)
      IF(RHI.GE.1.0) GO TO 615
      XINF=0.01*((-ALOG(1.0-RHI))/((0.0000382)*(TEM+50.0)))**0.5
      GO TO 616
  915 XINF=0.0
      GO TO 616
  615 XINF=1.0
  616 CONTINUE
      IF(XINF.GE.XMCIN) NF1(I)=J+1
      XMCAVE=(XMO(I,J,1)+2.0*XMO(I,J,2)+XMO(I,J,3))/4.0
      TAB=(TEM-32.0)*.555555+273.2
      DIFF=0.001629*EXP((0.00045*TAB-0.05485)*XMCAVE*100.0-2513.0/TAB)
      XJB2=BA1+BA5*HUMRO
      XJB1=BA2+GPROD*XMCAVE
      XMP1=XJB1/XJB2
      HFG=1094.0-0.57*TEM
      XMP2=HFG*GAIR/XJB2
      XMP4=BA3/XJB1
      XMP6=BA4*DIFF
      XM1=XMO(I,J,1)
      XM2=XMO(I,J,2)
```

```
      DTO=-DC1
      IF(IS.EQ.1) DTO=-DTO
      IF(JS.EQ.NF1(IS)) DHO=-DHO
      PTITO=(TIN(IS1,JS)-TIN(IS,JS))/DTO
      PTIHO=(TIN(IS,JS1)-TIN(IS,JS))/DHO
      PHITO=(HIN(IS1,JS)-HIN(IS,JS))/DTO
      PHIHO=(HIN(IS,JS1)-HIN(IS,JS))/DHO
      PTHOTO=(THO(IS1,JS)-THO(IS,JS))/DTO
      PTHOHO=(THO(IS,JS1)-THO(IS,JS))/DHO
      PXMOTO=(XMO(IS1,JS,1)+2.0*XMO(IS1,JS,2)+XMO(IS1,JS,3)-XMO(IS,JS,1)
     1-2.0*XMO(IS,JS,2)-XMO(IS,JS,3))/DTO
      PXMOHO=(XMO(IS,JS1,1)+2.0*XMO(IS,JS1,2)+XMO(IS,JS1,3)-XMO(IS,JS,1)
     1-2.0*XMO(IS,JS,2)-XMO(IS,JS,3))/DHO
C   CALCULATE THE CORRESPONDING OUTLET PROPERTIES
      HOUT(K,L)=HMAX-(JS-1)*DC2+(PTITO*(HDS(L)-HIN(IS,JS))-PHITO*(TDS(K)
     1-TIN(IS,JS)))/(PTITO*PHIHO-PHITO*PTIHO)
      TOUT(K,L)=TMIN+(IS-1)*DC1-(PTIHO*(HOUT(K,L)-HMAX+(JS-1)*DC2)-(TDS(
     1K)-TIN(IS,JS)))/PTITO
      THOUT(K,L)=THO(IS,JS)+PTHOTO*(TOUT(K,L)-TMIN-(IS-1)*DC1)+PTHOHO*(H
     1OUT(K,L)-HMAX+(JS-1)*DC2)
      XMOUT(K,L)=XMCIN-(HOUT(K,L)-HDS(L))/XMP5
      IF(TOUT(K,L).LT.THETA) TOUT(K,L)=THETA
   80 CONTINUE
   70 CONTINUE
      PRINT 16,A
   16 FORMAT(*1*,*DATA WITH DRYER LENGTH EQUAL*,F6.3,2X,*FEET*)
      PRINT 17
   17 FORMAT(* *,4X,*IN TEMP*,7X,*OUT THETA*,8X,*IN HSP*,8X,*OUT MCDB*,8
     1X,*OUT HSP*,8X,*OUT TEMP*)
    8 FORMAT(* *,6E15.4)
      WRITE(61,8)((TDS(I),THOUT(I,J),HDS(J),XMOUT(I,J),HOUT(I,J),TOUT(I,
     1J),J=1,NPC2),I=1,NPC1)
      IK=0
      IF(IA.LT.NA) GO TO 99
   69 END
```

```
      XM3=XMO(I,J,3)
      G1=XMP4*(THET-TEM)
      G2=-2.0*(XMP3*(XINF-XM1)+XMP6*(XM2-XM1))
      G3=-XMP6*(XM1-2.0*XM2+XM3)
      G4=-2.0*XMP6*(XM2-XM3)
      F2=XMP5*(G2+2.0*G3+G4)/4.0
      F1=XMP1*G1-XMP2*F2
   55 CONTINUE
   60 CONTINUE
   50 CONTINUE
C     SEARCH FOR ORDERED PAIR(T,H) WHICH MINIMIMIZES THE NORMALIZED NORM
      IF(IK.NE.NPR1) GO TO 99
      DO 70 K=1,NPC1
      DO 80 L=1,NPC2
      IF(TOUT(K,L).LE.THETAC) GO TO 80
      DO 90 I=1,NF2
      NF=NF1(I)
      DO 100 J=NF,NF3
      IF(TIN(I,J).GE.TCHECK.OR.HIN(I,J).LE.HCHECK) GO TO 100
      XH=((HDS(L)-HIN(I,J))/HDS(L))**2
      XT=((TDS(K)-TIN(I,J))/TDS(K))**2
      XTH=XH+XT
      IF(I.EQ.1.AND.J.EQ.NF1(I)) GO TO 25
      IF(XTH.GE.XTHS) GO TO 100
   25 JS=J
      IS=I
      XTHS=XTH
  100 CONTINUE
   90 CONTINUE
C     INTERPOLATE TO ACQUIRE MORE ACCURATE SOLUTION
      IF(HDS(L).GT.HIN(IS,NF1(IS))) GO TO 80
      IS1=IS-1
      IF(IS.EQ.1) IS1=IS+1
      JS1=JS-1
      IF(JS.EQ.NF1(IS)) JS1=JS+1
      DHO=DC2
```