AN ATTEMPT TO APPLY ELEMENTARY VIBRATION THEORY TO A STACK OF PACKAGES

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ABSTRACT

AN ATTEMPT TO APPLY ELEMENTARY VIBRATION THEORY TO A STACK OF PACKAGES

By

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This thesis is a determination of the degree of applicability of elementary vibration theory to a stack of packages. If such an application could be used to explain the behavior of a stack of packages, a simple and very powerful tool would be available to better design a functional package.

From the research it can be concluded that there is little correlation of elementary vibration theory to the laboratory data. The difference between the laboratory value and a corresponding theoretical value, increases as the number of containers in the stack increases.

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Thomas Joseph Kusza

A THESIS

Submitted to
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To Nita Jo, my wife

ACKNOWLEDGMENTS

I would like to thank Dr. James W. Goff for his help during my program. Without his guidance and support this thesis could not have been written. I would also like to thank Dr. William Sharpe, Jr. whose patience and direction were tremendous aids in getting me to the level of understanding which was necessary to perform the research in this paper.

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INTRODUCTION

In transit packages are stacked in various multiples to efficiently use space. Vibrations are transmitted through the floor of the truck or rail car and are experienced by the units because they are conveyed through the stack. Each container's response to vibration input will differ due to the influence of the other stack members. Since the stacked configuration has a marked influence, it is important to study its effects and determine if they are predictable.

Little effort has been spent in trying to apply elementary vibration theory to stacked packages. The application, if successfully made, would provide the designer an invaluable tool.

The purpose of this paper is to determine if a simple linear undamped model could be used to describe stacked packages. A more complicated model with damping and nonlinear springs would be expected to be more accurate, but the object is to see how useful a simple model could be.

There are two reasons why simple vibration theory might not work. First, the package stack might be too complicated to treat as a discrete mass system with only a few elements and second, the typical packaging material may not behave as a linear undamped spring. This investigation will attempt to isolate the second premise and avoid the first by using a simple model made up of steel plates to simulate package mass.

Dow Ethafoam* 220 and 275 (mullen test in lbs) c-flute corrugated fiberboard were the materials proposed for the simple model. They were tested individually, while bonded together, and under various loadings, to simulate real conditions. The high and low loadings for the polyethylene cushion were taken from a static-stress-peak-deceleration curve[1] and a resonant frequency curve[2] respectively. The high loading gives optimal cushion performance and the low gives a measure of predictability to the natural frequency.

The bulk of the research involves the determination of the error introduced by treating the materials as linear undamped systems. Linearity allows ready prediction of performance and this allows for great savings in time and money.

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*Registered Trademark, the Dow Chemical Company.

The body of the thesis is broken up into six parts. The first discusses single mass vibration theory. The second is composed of three sections: the first section deals with measuring static-spring constants; the second contains an investigation of single mass resonant frequency as a function of static loading; the third section has a study of resonant frequency as a function of input acceleration. The third part is a discussion of the results of the second. The fourth part is an analysis of a stack of two packages. The fifth is an analysis of a stack of four packages. The last contains the conclusions.

ELEMENTARY SINGLE MASS VIBRATION THEORY

> The subject of vibration deals with the oscillatory motion of dynamic systems. A dynamic system is a combination of matter which possess mass and whose parts are capable of relative motion. All bodies possessing mass and elasticity are capable of vibration. The mass is inherent in the body, and the elasticity is due to the relative motion of the parts of the body.
>
> The objective of the designer is to control or minimize the vibration when it is objectionable and to utilize and enhance the vibration when it is desirable.[3]

The elements that constitute a vibratory system are illustrated in Figure 1. The mass is assumed to be a rigid body. The spring element is elastic and is assumed to be of negligible mass. A linear spring is one that obeys Hooke's law, that is, the spring force is proportional to the spring deformation. The constant of proportionality, measured in force per unit deformation, is called the spring constant (K). The damping element has neither mass nor elasticity. Damping force is proportional to the relative velocity between the two ends of the damper.

The work energy or input into a damper is non-conservative. Energy enters a system through the application of an excitation force to the system.

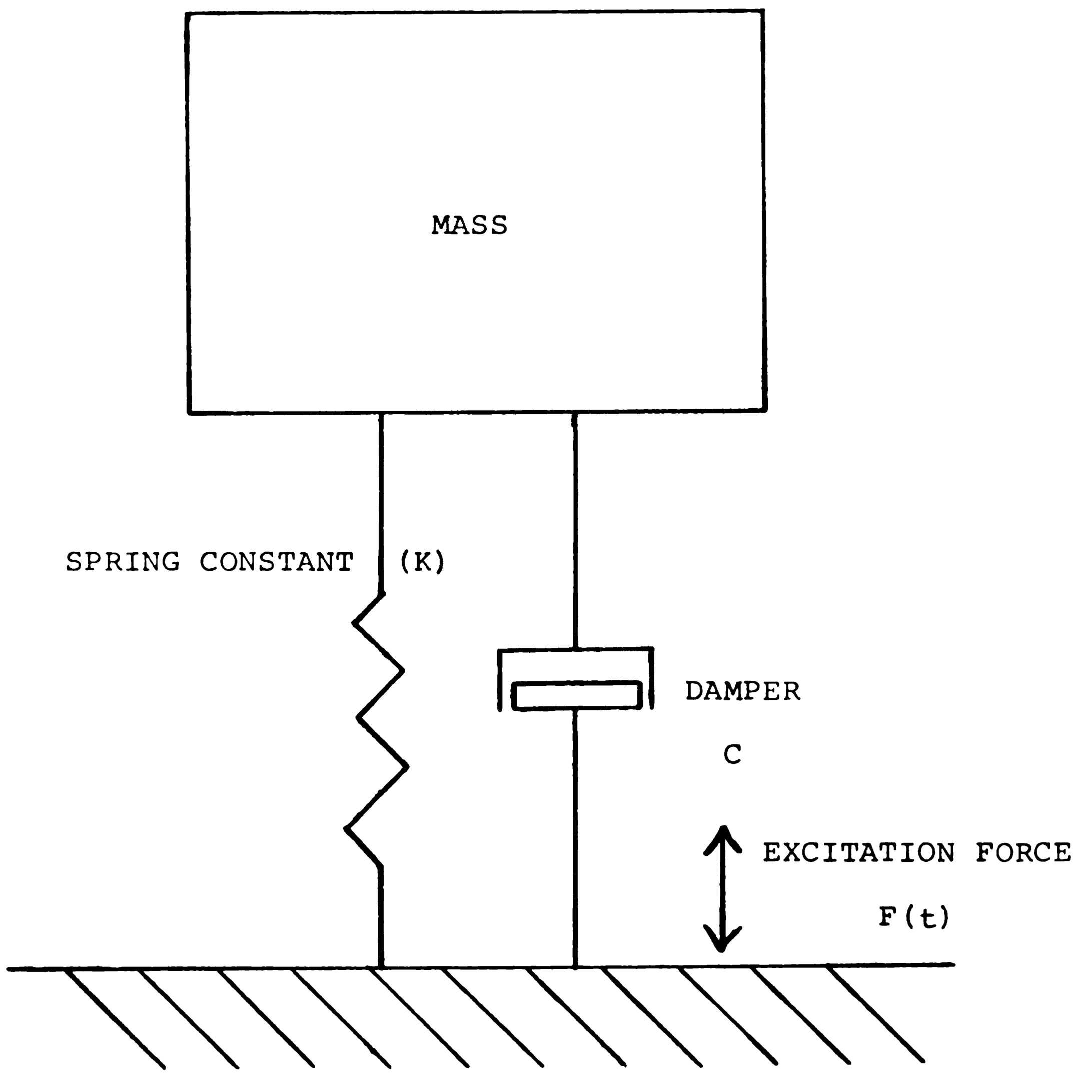


Figure 1. Single degree of freedom system.

The focal values are the mass and the spring constant. The excitation force is discussed later and the damping factor is neglected completely. Damping does exist in all physical systems to some extent, but the purpose is to compare linear undamped systems with actual stacks of packages.

Natural frequency is a property of a dynamic system. The natural frequency is determined by the mass and spring constant. This value is calculated from the following equation:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{K}{m\,(\text{mass})}}$$

The natural frequency will be in cycles per second (H_z). The mass is determined by dividing the weight in pounds by 386, the acceleration of gravity in inches/second. The spring constant is determined by taking the slope of the static load deflection curve for that material. If the graph is curvilinear, then the spring constant is not a constant and varies as a function of the load on the cushion. The system is no longer simple and prediction from the simple theory is impossible.

"When the frequency of the exciting force coincides with one of the natural frequencies of the system, a condition of resonance is encountered, and dangerously large amplitudes may result."[4]

If the natural frequency of the system is known, the resonance condition maybe avoided through design. This is why natural frequency is so important. Tremendous savings could be realized if the simple equation, previously mentioned, could be used in place of tedious laboratory testing.

EXPERIMENTAL INVESTIGATION OF CUSHIONING MATERIALS

Static Spring Constants

In the preceding chapter the formula for natural frequency is presented. The spring constant (one of the independent variables, the other being the mass) is important since each product's mass is virtually fixed. By treating the mass as a constant, the natural frequency varies as a function of the spring constant. This section is an experimental determination of the spring constant, with an application of the formula to demonstrate its effects on natural frequency.

The specimens were cut to proper dimensions, 1.5" x 1.5" x 1" and 3" x 3" x 1", to yield 2.0 and 0.5 psi loading respectively for an 18 pound mass. The corrugated fiberboard was only tested at the 0.5 psi loading.

An Instron laboratory testing maching, model TT-B, was used to test the samples. The test procedure used is similar to ASTM D 1225-66 (flat crush of corrugated board). The corrugated fiberboard was tested two sheets at a time with the flutes running perpendicular to simulate their arrangement in the flaps of a box.

The testing procedure produces load deflection plots such as the one in Figure 2. Since the masses weighed 18 pounds and 4 cushions would be used for each mass, each cushion would bear 4.5 pounds. From a tangent drawn at this point, the spring constant can be calculated by taking the slope of the tangent.

With the natural frequency equation, a theoretical value may be determined by substituting for the mass and spring constant. The results of the tests and calculations are presented in Table 1.

Single Mass Natural Frequency as a Function of Static Loading

This part deals with the laboratory determination of the natural frequency of a system. The product, while mounted on the cushions, is subjected to a vibration input through a range of 0-50 H_z. The acceleration of the mass is monitored to locate the resonance point, which is defined to be the point at which the frequency of the vibrating table is equal to natural frequency of the package.

The cushions were tested at four loadings. Along with the polyethylene cushion and corrugated fiberboard, a combination of the two was tested to simulate actual package conditions.

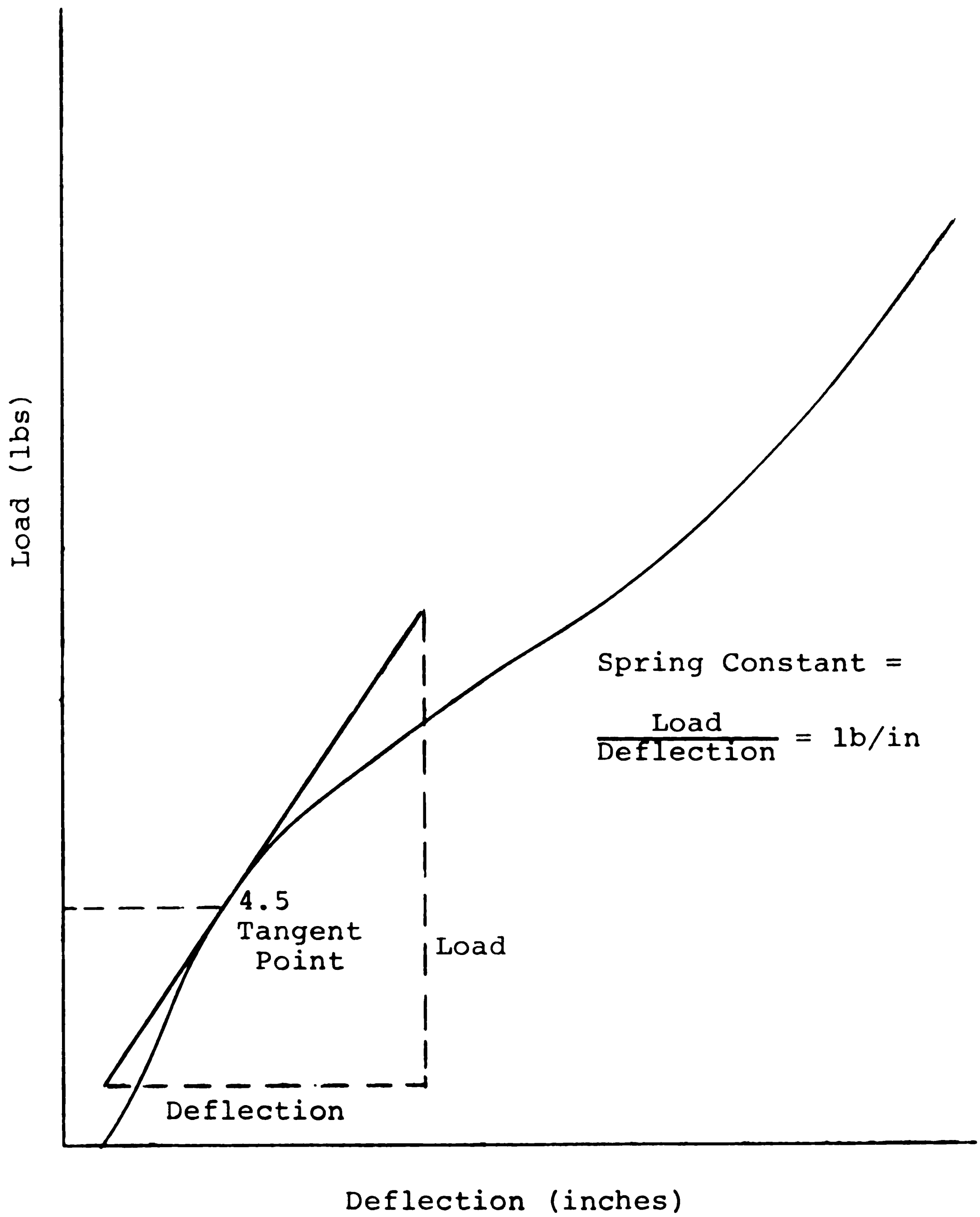


Figure 2. Sample of a Static Load Deflection Curve. Material: Ethafoam[a] 220; Size: 1.5" x 1.5" x 1".

[a]Registered Trademark, The Dow Company.

TABLE 1.--Laboratory spring constants and theoretical natural frequencies.

Test No.	Crosshead Speed (in/min)	Ethafoam[a] 0.5 psi Spring Constant (lb/in)	Ethafoam[a] 0.5 psi Natural Frequency (H_z)	Ethafoam[a] 2.0 psi Spring Constant (lb/in)	Ethafoam[a] 2.0 psi Natural Frequency (H_z)
1	0.5	240.1	22.9	133.3	17
2	0.5	201.6	20.9	123.1	16.4
3	0.5	246.5	23.2	123.3	16.4
1	0.05	237.5	22.7	114.6	15.8
2	0.05	194.4	20.6	. .[b]	. .
3	0.05	250	23.3	. .[b]	. .
1	0.5	271	24.3	126.5	16.6
2	0.5	255	23.6	98	14.6
3	0.5	225	22.1	131.8	16.9
1	5	233	22.5	130.8	16.9
2	5	305	25.8	137	17.3
3	5	277	24.5	145	17.8

Test No.	Crosshead Speed (in/min)	Corrugated Fiberboard 0.5 psi Spring Constant (lb/in)	Corrugated Fiberboard 0.5 psi Natural Frequency (H_z)	Corrugated Fiberboard 0.5 psi Load Range (lbs)
1	0.1	176.5	19.6	200
1	0.1	205	21.1	100
1	0.1	152.5	18.2	50
1	0.1	196	20.6	50

[a]Registered Trademark, The Dow Company.

[b]Test was not repeated due to length.

A hydraulic shaker was used to vibrate the model through the frequency range. The cushions were placed under the corners of the mass as seen in Figure 3. A small amount of adhesive was used to prevent slippage and possible damage. The equipment was then calibrated and set up (see Appendix A-1).

Figure 4 is a plot which was used to calculate natural frequency (see Appendix A-1). The sample calculations show how the natural frequency was determined. Each loading for each material (polyethylene foam, corrugated fiberboard, combination of first two) was tested at least twice and the results are presented in Table 2.

Single Mass Natural Frequency as a Function of Input Acceleration

In trying to design a test that was repeatable, the possibility of keeping displacement constant was investigated. It was found that natural frequency varied from run to run. By keeping the input excitation force constant the results became very consistent.

To investigate what might actually happen if the input changed, the input was varied and its effects on natural frequency were observed. All three materials were tested but at only one loading, 0.5 psi. The results are presented in Table 3.

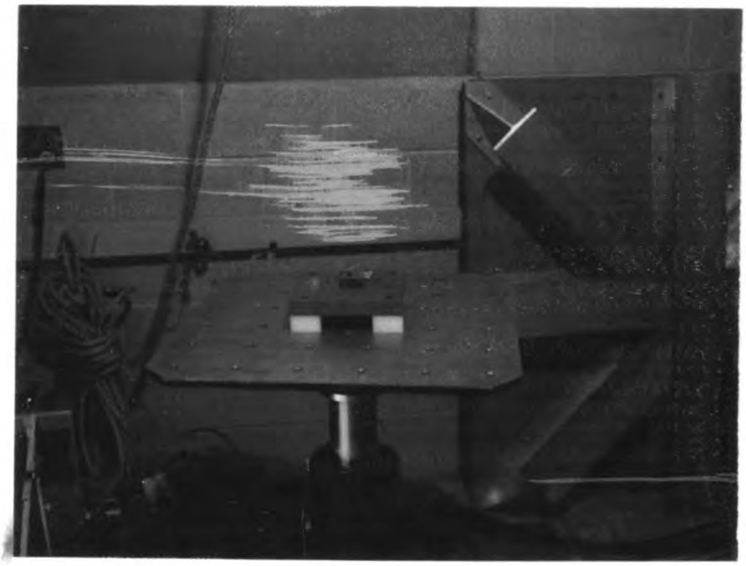
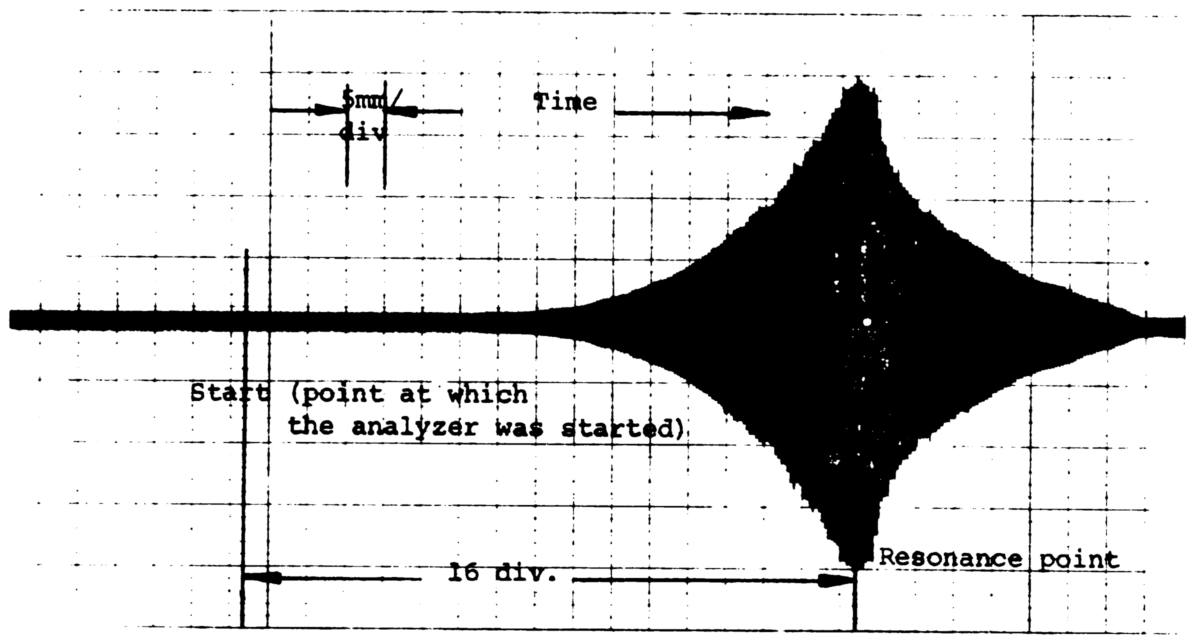


Figure 3. Plate illustrating the one mass model mounted on the table.



16 div. x 5 mm/div. = 80 mm.
80 mm ÷ 2 mm/sec (recorder speed) = 40 secs.
40 sec x 0.1 H_z/sec = 4 H_z
The resonant point is at 4 H_z

If the analyzer sweeps
through a 50 H_z range
in 500 secs it sweeps
through 0.1 H_z/sec

Figure 4. Acceleration time plot from Brush Recorder.

TABLE 2.--Natural frequency of a single mass.

Loading	Material		
	Ethafoam[a]	Ethafoam[a] Corrugated Fiberboard	Corrugated Fiberboard
0.5 psi	39.4 H_z	42.2 H_z	19.4 H_z
0.5 psi	39.0 H_z	38.5 H_z	26.6 H_z
1.0 psi	32.3 H_z	29.8 H_z	25.6 H_z
1.0 psi	32.1 H_z	32.4 H_z	23.0 H_z
1.5 psi	30.1 H_z	29.9 H_z	24.9 H_z
1.5 psi	30.6 H_z	26.4 H_z	22.1 H_z
2.0 psi	25.1 H_z	20.8 H_z	20.4 H_z
2.0 psi	25.2 H_z	18.5 H_z	20.1 H_z

[a]Registered Trademark, The Dow Company.

TABLE 3.--Input acceleration vs. natural frequency.

Input (g's)	Ethafoam[a]	Corrugated Fiberboard	Combination
	Natural Frequency (H_z)	Natural Frequency (H_z)	Natural Frequency (H_z)
0.071	38.5	17.5	47.5
0.143	37.5	17.5	47.5
0.214	36.5	17.5	45
0.286	34	17.5	43.5
0.357	32.5	17.5	42.5
0.428	31	17.5	40
0.50	30	17.5	36
0.571	31	16	36

[a]Registered Trademark, The Dow Company.

DISCUSSION OF RESULTS FROM CHAPTER III

In comparing the results of the theoretical natural frequency calculated from the spring constants and the laboratory results for the polyethylene cushion loaded at 0.5 psi, there is no agreement. The lab value is almost twice that of the calculated natural frequency. When the cushion is loaded to 2.0 psi the natural frequency values are much closer. The difference is approximately 28 percent. This does not allow for a reasonable amount of predictability, but it does lean closer to the theory. It can be concluded that as you go to the higher loading, towards the flat part of the resonant frequency curves and away from the optimal range on the deceleration curves, the laboratory values agree more with the theoretical natural frequency.

On the other hand, the corrugated fiberboard material appeared to be very predictable at all loadings tested. Laboratory values when compared to the theoretical results, appeared to show little difference. The greatest difference was when the cushion was loaded to 0.5 psi. The corrugated fiberboard, for the loadings tested, appears to be a linear spring, which should allow for a great deal of predictability.

In analyzing the spring constants alone, the polyethylene foam loaded to 0.5 psi had the largest values. The corrugated pads followed next with the polyethelene cushions loaded at 2.0 psi being last. In looking at the effects of changing the crosshead speed, the differences were negligible.

The corrugated board acts like a linear spring making the natural frequency very predictable. For the loadings which were tested, the amount of loading on the corrugated pads has no effect on the natural frequency. For the foam alone, there is a shift of 15 H_z from the low loading with a high natural frequency of 39.0 H_z to the higher loading with the low natural frequency of 25.0 H_z. In the combination the polyethylene foam dominates at the lower loading, 0.5 and 1.0 psi, while the corrugated dominates at the 1.5 and 2.0 psi loadings. By "dominating" it is meant that the natural frequency of the combination is similar to the natural frequency of the material which dominates it.

The final area of discussion is that of input acceleration. Tests run on the corrugated material show the natural frequency was not affected by input acceleration. The polyethylene foam loaded to 0.5 psi (as was the corrugated fiberboard), showed that by changing input acceleration the natural frequency could be moved. By moving from a low of 0.071 g. to a high of 0.571 g. the natural frequency could be lowered by 7.5 H_z. Thus, the input acceleration is very critical in determining natural frequency for non-linear materials.

While this simple model is not adequate, obviously a model could be developed, but this work indicates that it would have to include damping, non-linear effects or other considerations that would make it very complicated, and it is this complexity which has caused restricted use.

TWO MASS SYSTEM

Elementary Theory

> The degrees of freedom of a system are equal to the number of independent coordinates necessary to describe the motion of the system. A system with n degrees of freedom will have n natural frequencies which characterize the behavior of the system.[5]

Any system which requires two independent coordinates to describe its motion is a two-degrees-of-freedom system, as illustrated in Figure 5 The spring constant (K) is the value determined in Chapter III, multiplied times 4 (4 cushions). The spring constant (K) will be half the value of spring constant (K_1) because each cushion is 2" thick. The double thickness results in a softer cushion. This configuration was used to simulate the arrangement of one package on top of another.

This system will have two natural frequencies and when it oscillates at one of them it is said to be vibrating at a principal mode.

The system does not really appear to be much more involved but its complexity can be viewed in the equations. The following equation is used to find the angular natural frequencies (K_2 for this system is zero).

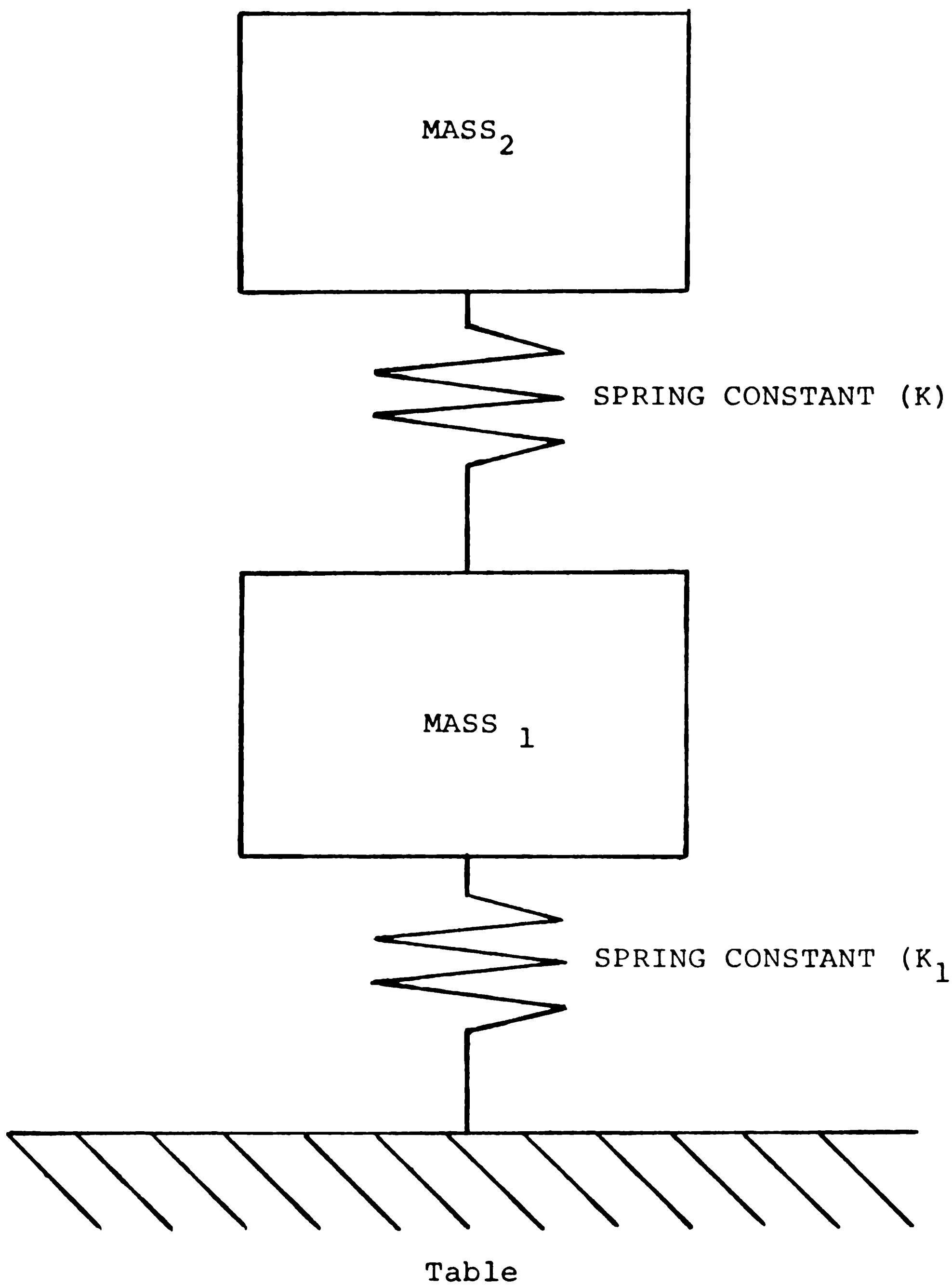


Figure 5. Two degree of freedom system.

$$\omega^4 - \frac{K_1 + K}{m_1} \omega^2 + \frac{K_1K_2 + K_1K + K_2K}{m_1m_2} = 0$$

Experimental Procedure

Another accelerometer was used so that both masses could be monitored. Previous procedures were followed with the additional equipment (see Appendix A-1).

Results

The phasing and amplitude ratios were determined from pictures (see Appendix A-2). The theoretical phasing and amplitude ratios were determined from a computer program (see Appendix B). The data along with its theoretical counterpart is presented in Table 4.

Discussion

There is very little agreement between laboratory data and the theoretical values. In comparing the natural frequencies, the lower frequencies agree with their theoretical counterparts while only the second frequency of the polyethylene cushion loaded to 0.5 psi agrees with its theoretical value. The first natural frequency could be predicted for all three cases and the generalization that all first natural frequencies may be calculated for all loadings of the foam and corrugated fiberboard could be made.

The amplitude ratios show no agreement.

TABLE 4.--Data from the two mass system.

	Natural Frequency 1		Natural Frequency 2		Amplitude Ratio 1		Amplitude Ratio 2		Phasing at Natural Frequency 1		Phasing at Natural Frequency 2	
	Lab Results	Theory Results	L.R.[a]	T.R.[b]	L.R.	T.R.	L.R.	T.R.	L.R.	T.R.	L.R.	T.R.
3"x3"x1" Ethafoam 0.5 psi	11 H_z	12.5 H_z	28 H_z	30.1 H_z	.625	.414	.500	-2.475	IN	IN	IN	OUT
1 1/2" x 1 1/2" x 1" Ethafoam 2.0 psi	10 H_z	9.1 H_z	30 H_z	21.9 H_z	.699	.414	-1.274	-2.416	IN	IN	OUT	OUT
3"x3"x1" Corrugated 0.5 psi	10 H_z	10.8 H_z	20 H_z	26 H_z	.923	.414	.856	-2.418	IN	IN	IN	OUT

[a] L.R. = Laboratory results.

[b] T.R. = Theoretical results.

In checking the phasing for the first natural frequency, there is total agreement. For the second natural frequency, only the polyethylene foam loaded to 0.5 psi agrees.

FOUR MASS SYSTEM

Elementary Theory

There is no real way to sufficiently present vibration theory for multiple-degree of freedom systems in a few lines. The step from one mass to two masses is complicated, but the step from two masses to four masses is infinitely more so.

Figure 6 is a four mass system. Two masses and springs have been added on the same manner as in Chapter V. The values of K_2 - K_4 are the same as the spring constant in Chapter V. The solution for this system involves a series of matrices, which are solved though a computer program. This program is on file at Michigan State University (see Appendix B).

Like the two mass system, there are three areas of comparison: natural frequency, amplitude ratio, and phasing.

Experimental Procedure

The equipment used was the same as in Chapter V. Since only two accelerometers were used, their positions were shifted to get the reaction of all four masses.

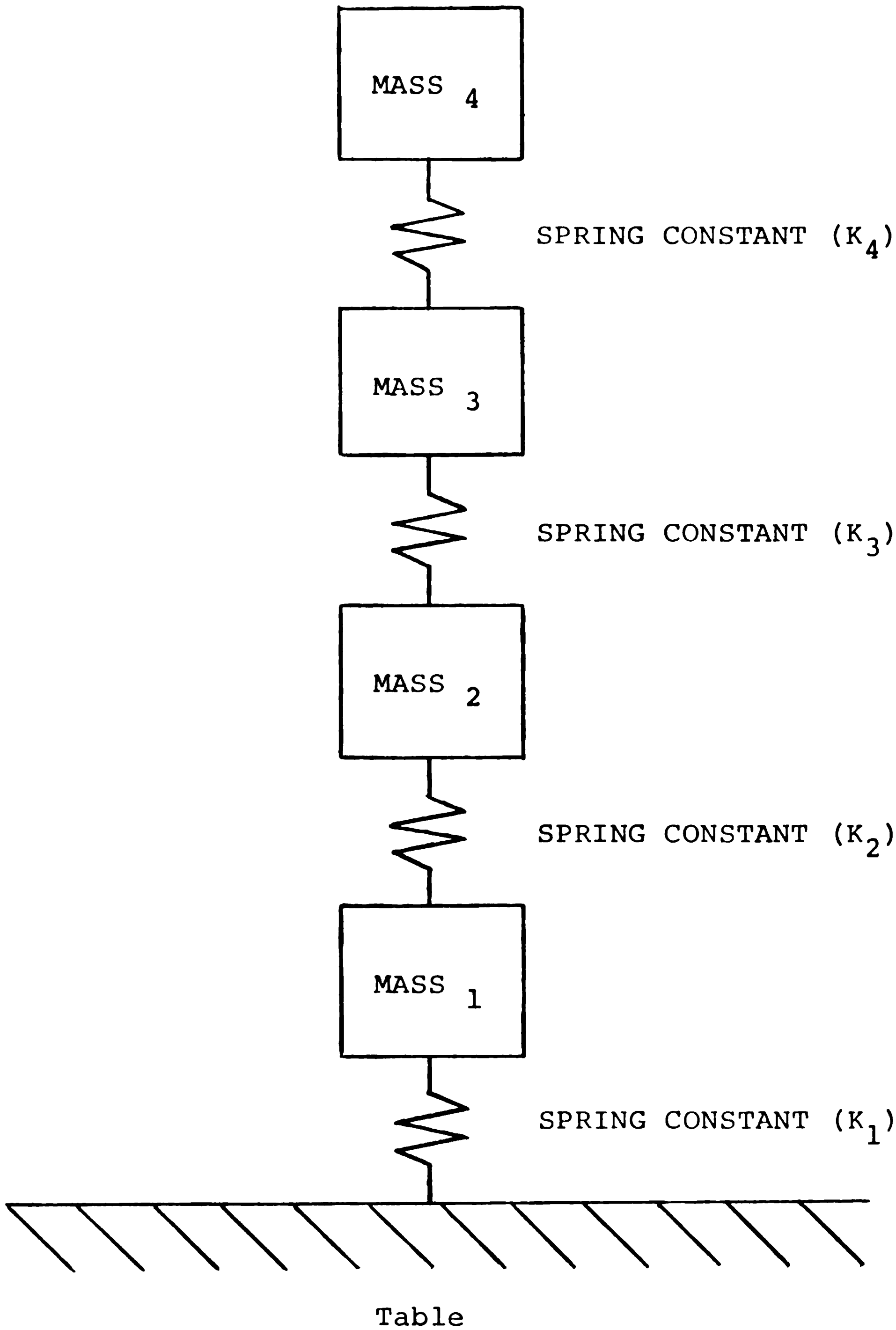


Figure 6. Four degree of freedom system.

Results

The data is collected in the same manner as was previously stated. The results of the tests along with its theoretical counterpart is presented in Table 5.

Discussion

There were four natural frequencies found, which shows some agreement with theory, but the agreement ends there. The frequencies do increase as do the theoretical ones, but actual values show no agreement.

The amplitude ratios show no agreement.

The phasing shows occasional agreement but there is not enough consistency to allow for any predictions.

TABLE 5.--Data from the four mass system.

3"x3"x1" Ethafoam[a] 0.5 psi							
Natural Frequency 1		Natural Frequency 2		Natural Frequency 3		Natural Frequency 4	
L.R.[b]	T.R.[c]	L.R.	T.R.	L.R.	T.R.	L.R.	T.R.
5.6 H_z	16.6 H_z	14.4 H_z	19.6 H_z	24 H_z	29.4 H_z	48 H_z	83.6 H_z

Amplitude Ratios

	Mass 1		Mass 2		Mass 3		Mass 4	
	L.R.	T.R.	L.R.	T.R.	L.R.	T.R.	L.R.	T.R.
Mass 1	. .	. .	3.86	1.766	.274	1.18	1.50	.199
Mass 2	.599	.351	. .	. .	.336	5.029	2.58	.234
Mass 3	.599	.234	1.39	5.029	. .	. .	1.28	.351
Mass 4	.399	.199	.819	1.180	.150	1.766	. .	. .

Phasing

	Mass 1		Mass 2		Mass 3		Mass 4	
	L.R.	T.R.	L.R.	T.R.	L.R.	T.R.	L.R.	T.R.
Mass 1	. .	. .	IN	IN	OUT	OUT	OUT	OUT
Mass 2	OUT	IN	. .	. .	IN	OUT	IN	IN
Mass 3	OUT	IN	OUT	IN	. .	. .	IN	OUT
Mass 4	OUT	IN	OUT	OUT	OUT	OUT	. .	. .

[a]Registered Trademark, The Dow Company.

[b]L.R. = Laboratory results.

[c]T.R. = Theoretical results.

CONCLUSIONS

The data in general does not allow for any predicability from the theoretical calculations. The differences are wide and too frequent to put complete faith in the theory. Yet, the few instances of agreement are not chance occurrences. They are real and give a certain amount of credibility to the analysis. It was found that as the investigation became more complex, the disparity between values increased till they possessed no similarity.

Although there is little proof, the theoretical approach is still a valid plan of attack. What has been shown is that both materials defy analysis through elementary theory. If the effective spring constant is treated as a non-linear function and damping is also introduced, there could be an aligning of the laboratory and theoretical values.

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APPENDICES

APPENDIX A-1--A-2

APPENDIX A-1

An Endevco accelerometer no. 2265-20 was mounted to the steel plate to measure the acceleration of the mass. The accelerometer output was fed into an Endevco signal conditioner, no. 4470 and calibrated to 625 mv/g. A Krohn-Hite filter no. 3750 was used to filter the output of the signal conditioner on a low pass of 60 H_z. The output of the filter was fed onto a Gould 280 brush recorder which was set at a speed of 2 mm/sec and a sensitivity of 50 mv/line. These charts are then used to calculate natural frequency (see Figure 4).

To keep the input constant, a Kistler accelerometer no. 818, calibrated for 10.4 mv/g., was mounted on the table. The accelerometer was monitored on a Bruel and Kjaer RMS meter, no. 2417 and kept at a constant input of 0.0015 v RMS (0.21 g.). The input was kept constant to allow the easy comparison of results.

To aid in the accurate location of the natural frequency a Quan-Tech Wave Analyzer no. 304T was used to sweep through the frequency range of 0-50 H_z in either 50 or 500 seconds (both time intervals produced comparable results). By starting the analyzer and simultaneously marking the start on the brush recorder, the point of resonance was very easily determined.

APPENDIX A-2

Along with the additional equipment for the two mass tests, a Tektronix no. 502 oscilloscope and camera were used to determine phasing and amplitude ratios. Pictures were used to provide permanent records for future use.

APPENDIX B

APPENDIX B

The mass and stiffness matrices were computed for the two and four mass models, through a computer program. The program name is EIGEN, NROOT and is on file at Michigan State University. This program calculated the theoretical values for the natural frequencies, mode shapes (phasing), and amplitude ratios.