A GROWTH MODEL FOR ASPARAGUS

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ABSTRACT

A GROWTH MODEL FOR ASPARAGUS

By

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The object of this study is to develop a model that will accurately describe the growth characteristics of asparagus spears in the field for application as a prediction tool for selecting time of harvest. An equation for spear height was derived by solving a differential equation for growth rate. The constants of the equation were found by least squares regression analysis and by direct calculation from the field data. The least squares regression also indicated that growth rate is higher, by a factor of two, for daytime growth than for night growth.

A relatively large variability was found to exist between, and to some extent within, spears. To account for the variability constants were selected randomly from a cumulative probability distribution for use in height calculations.

To verify the ability of the model to predict spear heights, spear heights were calculated using the same growth conditions that the measured spears underwent.

Because of the variability, single spears could not be compared directly, so distributions were used for direct comparison of measured and calculated heights. The calculated and measured values were compared in two ways. First, all spears from one measurement group were "grown," under similar conditions, to the next measurement point. Second, spears with initial heights within a particular range were "grown" for 7 and 24 hours. In almost all instances the means of the measured and calculated height distributions differed by less than two centimeters, but the calculated distributions were more compact than the measured value distributions.

After verification, two examples are offered as to how the model might be used as a prediction tool. One uses the results of the verification calculations and the other takes a hypothetical situation and describes how the farmer might go about making a time of harvest decision.

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A GROWTH MODEL FOR ASPARAGUS

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INTRODUCTION

As with most fields, technology is advancing very rapidly in the area of vegetable harvesting machinery. The past two decades have seen the development of machinery which has allowed a farmer to cultivate large acreages of vegetables. Previously, these same vegetables were grown in smaller, hand harvested acreages. Examples of such crops are tomatoes, lettuce, cucumbers and asparagus.

As far as asparagus is concerned, most attempts to develop a selective harvester have proven to be impractical or uneconomical, as stated in Stout (1967). The alternative to a selective harvester is one that cuts all spears, i.e., nonselective. A recent development is the sled harvester. Although this harvester is nonselective, the high harvest rate and low initial cost enable it to operate economically. The sled operates at speeds up to 15 mph compared to the 1 to 3 mph speeds of a selective harvester (Carpenter, 1967).

The primary problem of increased volume, aside from physical handling, is that the time of harvest becomes very critical. In Michigan, processors accept only 10 per cent of the spears over 7.5 inches long before docking the grower because taller spears have more fibrous chunks than short spears. Therefore, considering the height of the cutter bar, the grower wants to harvest when less than ten per cent of the spears are more than ten inches (25 centimeters) high. During high temperature conditions asparagus has been observed to grow as much as three inches in four hours. A harvesting error of even a couple of hours could be quite costly because the spears might grow through the acceptable range before the harvesting is completed. The optimum situation would allow selecting the harvest time at the point where the duration of harvest coincides with the period that the field grows through the desired height range.

The objective of this study is to develop a growth model for asparagus which can be used to aid the farmer in predicting the optimum time to harvest asparagus. The model must be able to predict the height of a spear at time $T + \Delta t$, given the height distribution at time t and the average temperature during the period Δt.

REVIEW OF LITERATURE

To date only limited work has been done toward modeling the growth characteristics of asparagus spears; however, several people have investigated the elongation of asparagus spears. Although the objectives of their studies differed, the parameters found to affect the growth rate did not. The primary factors influencing growth were temperature and height. The fact that height is involved eliminates the possibility of a simple temperature model since growth rate increases as the spear gets longer.

Asparagus, like many other plants, is divergent during its early growth; that is, taller spears grow faster than short ones, leaving a field less mature with time. Divergence is accentuated by the fact that new spears emerge continually during the entire growth period, thus increasing nonuniformity. Growth rate is also sensitive to changes in temperature resulting in even greater nonuniformity at higher temperatures. The effect of temperature, as well as height, can be seen in the two typical growth curves shown in Figure 1.

One of the earliest studies was by Culpepper and Moon (1939), in which they measured the elongation of asparagus stems over a height range of 0 to 250 centimeters

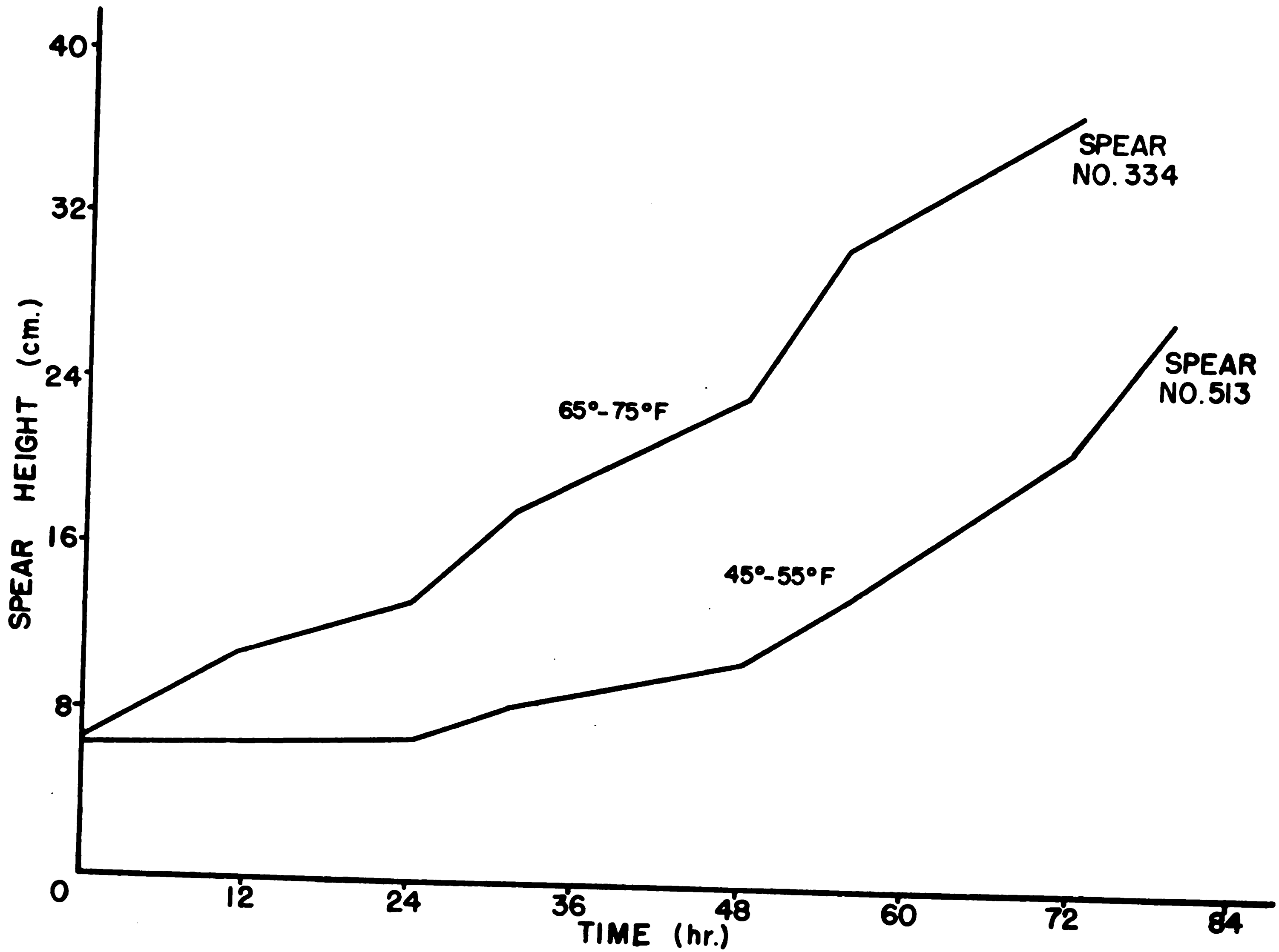

FIGURE I. GROWTH CURVES FOR TWO TYPICAL SPEARS with temperature ranging from 45° to 95°F. They found considerable variability in elongation rates and ultimate heights reached. They concluded that growth rate continues to increase up to a height of about 60 to 70 centimeters (25 inches) which is well past the 25 centimeter (10 inch) height of harvested spears. They also found that within the given temperature range, growth rate increased with increased temperature; however, little growth took place below 40°F.

Another early study by Tiedjens (1924) recognized the effect of increasing temperature on growth rate. A more recent study of stem elongation by Downs (1962) confirms the fact that taller spears grow more rapidly than shorter spears during the same time period. He found that length could be correlated with initial height in a linear function for initial heights ranging from two to nine inches.

Only one person has attempted to propose a model for spear growth, Blumenfield (1961). In this study, temperature and height were assumed to be the primary factors influencing growth. Field measurements were taken over a range of temperatures and heights and then analyzed using multiple regression techniques. The proposed model for growth rate was

$$GR = a + b\,SH + c\,T + d\,(SH)^2 + e\,SHT + f\,(T)^2 \qquad [1]$$

where

T = average air temperature

SH = spear height in centimeters

and

a, b, c, d, e, f are constants.

After the regression analysis, the squared and cross product terms were eliminated because they only accounted for four per cent of the variation while the linear terms accounted for 80 per cent. The remaining 16 per cent was attributed to variables such as soil moisture, fertility, temperature or individual spear variation. The final equation for the growth rate was

$$GR = -15.25 + 0.3163\ (SH) + 0.3544\ (T) \qquad [2]$$

This equation indicates that an increase in height of one centimeter will cause the growth rate to increase 0.3163 cm/day and that an increase of 1°F will increase the growth rate by 0.3544 cm/day.

The Blumenfield equation can be represented by a plane in three dimensional space with axes of height, temperature and growth rate as seen in Figure 2. Of particular interest in this plane are the temperatures at which growth stops. These are represented by the line of intersection between the equation's plane with the

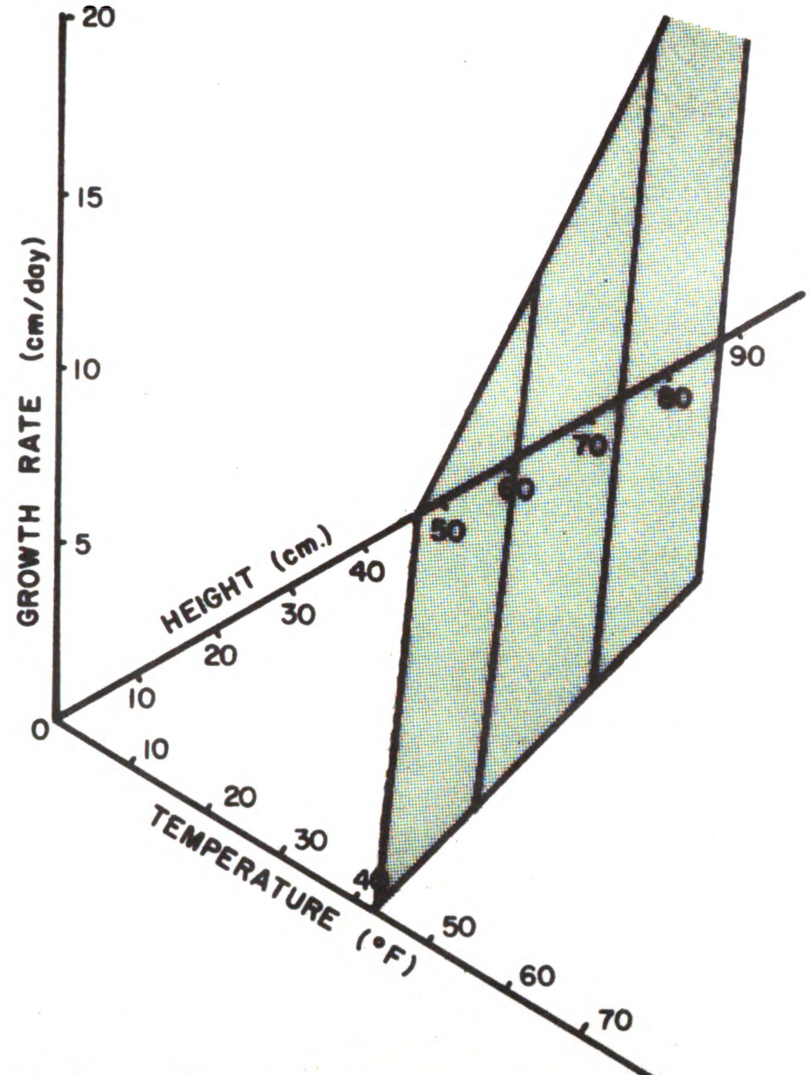

FIGURE 11. THREE DIMENSIONAL REPRESENTATION OF GROWTH RATE EQUATION

height-temperature plane. The line of intersection shows that shorter spears require higher temperatures to sustain growth. The plot represents the equation rather than actual conditions thus explaining why unlikely situations appear in Figure 2.

MATHEMATICAL MODEL

As stated in the introduction, the object of this study is to develop a growth model for asparagus that will assist the farmer in selecting the optimum time to harvest his crop. Blumenfield's regression analysis, using height and temperature as the significant parameters, can be modified to yield a differential equation by defining growth rate as

$$GR = \frac{dH}{dt}$$

This definition allows Blumenfield's equation to be written as

$$GR = \frac{dH}{dt} = a + bH + cT \tag{3}$$

or

$$\frac{dH}{dt} - bH = a + cT \tag{4}$$

where the constants have the units

a = cm/day

b = 1/day

c = cm/day °F.

The solution of [4] consists of a complementary solution and a particular solution. The complementary solution

$$H = C_o e^{bt} \qquad [5]$$

where C_o is an undetermined constant, is the solution to the homogeneous equation

$$\frac{dH}{dt} - bH = 0$$

The particular solution can be obtained by using the method of undetermined coefficients once the temperature is known as a function of time. For the moment this will be met by representing the temperature by the Fourier Series

$$T = A_o + \sum^{n} (A_n \cos\omega nt + B_n \sin\omega nt) \qquad [6]$$

thus giving a general type solution of the form

$$\frac{dH}{dt} - bH = a + c(T_{ave} + \sum^{n} (A_n \cos\omega nt + B_n \sin\omega nt)) \qquad [7]$$

At this point two observations can be made about the constants a, b, and c that will make handling the particular solutions less involved. These observations are based on both Blumenfield's regression analysis and least squares regression of a limited amount of growth data from the 1971 season. The first is that constants b anc c are equal in magnitude but dimensionally different. The second is that the ratio a/c represents a zero growth temperature for an initially emerging spear; i.e., at zero height. This relationship may be obtained by setting the height (H) and the growth rate (GR) equal to zero in equation [3] and solving for a in terms of c and T_o, yielding

$$a = -cT_o \qquad [8]$$

The effect of these assumptions is to reduce the numerical values in the growth equation but not the dimensional units. In further development the factors may not appear consistent in units but, in fact, they are because the units remain after cancellation of magnitudes.

With the above two observations in mind the particular solution of equation [7] may be rewritten as

$$\frac{dH}{dt} - bH = b(T_{ave} - T_o) + b \sum^{n} (A_n \cos\omega nt + B_n \sin\omega nt) \qquad [9]$$

The particular solution for the constant term is

$$H_{P1} = \frac{bT_{ave} - bT_o}{b} = T_{ave} - T_o = \Delta T \qquad [10]$$

and the form of the particular solution for the Fourier series term is

$$H_{P2} = \sum^{n} (C_n \cos\omega nt + D_n \sin\omega nt) \qquad [11]$$

Where the values of C_n and D_n are obtained using the method of undetermined coefficients. This procedure gives the following values

$$C_n = \frac{-b^2 A_n + \omega nbB_n}{b^2 + (n\omega)^2}$$

and

$$D_n = \frac{\omega nbA_n - b^2 B_n}{b^2 + (n\omega)^2}$$

If the complementary solution [5] and the two particular solutions [10] and [11] are combined the result is the total solution for the height

$$H = C_o e^{bt} - \Delta T + \sum^{n} (C_n \cos\omega nt + D_n \sin\omega nt) \qquad [12]$$

Using the boundary conditions $H = H_i$ at $t = 0$, the constant C_o may be determined as

$$C_o = H_i + \Delta T - \sum^{n} C_n \qquad [13]$$

Combining [12] and [13] gives the general equation for spear height after time t as

$$H = (H_i + \Delta T - \sum^{n} C_n)e^{bt} - \Delta T + \sum^{n} (C_n \cos\omega nt + D_n \sin\omega nt) \quad [14]$$

When a digital computer is available, the Fourier solution does not appear too unwieldly; however, to enable practical application a simpler solution is preferable. The most obvious simplification is to represent temperature by a constant; the average temperature over the growth period. This procedure also has merit because it represents the type of information available to growers. For the case of constant temperature all of the summation terms can be neglected and [14] reduces to

$$H = (H_o + \Delta T)e^{bt} - \Delta T \quad [15]$$

where the ΔT terms contain the factor $b/c = 1$ cm/°F to make the units consistant with height. The growth rate equation [3] from which [15] was derived can be described in three dimensional space just as Blumenfield's equation was in Figure 2. In comparing the general shape of Figure 2 with what [15] represents it becomes apparent that the ΔT term is affected by spear height. The temperature at which growth stops becomes lower as the spear becomes taller. The relationship between the zero growth temperature and height may be determined by resolving [3] using the conditions $H = H_i$ and $T = T_o$ and $GR = 0$. The result is that

$$T_o = -\frac{a}{c} - \frac{b}{c} H_i \qquad [16]$$

Using this result produces

$$\Delta T = T_{ave} - T_o = (T_{ave} - \frac{a}{c}) + \frac{b}{c} H_i \qquad [17]$$

If this equation is substituted into [15] the resulting equation for height is

$$H = [(1 + \frac{b}{c}) H_i + T_{ave} + \frac{a}{c}] \, e^{bt} - [\frac{b}{c} H_i + T_{ave} + \frac{a}{c}] \qquad [18]$$

With the establishment of the usable model the task remains to define the constants and verify that the model will calculate spear heights within an acceptable range of accuracy.

EXPERIMENTAL DATA

Field Procedure

Height and temperature data were taken at Michigan State University Horticulture Research Center. The plants used were five years old and of the California 711 variety. Height measurements were taken twice daily at 0900 and 1600 hours from May 8 through June 6, 1972. Numbered pieces of wood, held in the ground by a nail, were used for identification and as a base for measurement. Measurements were made on an individual spear until the spear exceeded 33 centimeters (13 inches), then it was cut to keep the field clean. A total of 3920 measurements were made on 519 spears. Hourly air temperature values were taken from a continuous recorder located at the research center. The hourly values were used to calculate the average temperature for each growth period.

Multiple Regression Analysis

The measurement data were divided into two groups. Sixty per cent of the data was used in the least squares regression analysis and for the determination of the growth constant. The remaining 40 per cent was used for comparison with height distributions calculated using the mathematical model. To distribute the data uniformly between the two groups the data were sorted using the last digit of the identification number. The 60 per cent consisted of all spears with the last digit 1, 3, 4, 6, 8, and 9; and the 40 per cent those with the last digits of 2, 5, 7, and 0.

The height and time data were converted to growth rate and a multiple regression analysis was performed using height and temperature as the important parameters influencing growth rate. The day and night growth periods were analyzed separately to determine any differences in the constants.

The first regression performed was a repeat of Blumenfield's original equation [1] which included the squared and cross product terms of temperature and height. The coefficients for the squared and cross product terms appeared considerably smaller than the linear terms in both the day and night periods. Another reason for neglecting the squared and cross product terms is that they did not contribute much to explaining the variability. The percentage increase in explaining variability due to the squared and cross product terms was four and one per cent for the day and night equations respectively. Included in the regression analysis was the determination of the significance of each coefficient. The analysis that included the square and cross product terms yielded several coefficients with significance at a level greater than 0.10, while the analysis with only the linear terms gave all the coefficients significant at the 0.0005 level. This fact was also used to make the decision to neglect the nonlinear terms. The results for both analysis can be seen in equations [19] through [22].

$$GR_D = -106.098 - 1.078H + 3.232T - 0.003H^2 - 0.023T^2$$
$$+0.026H\ T \qquad [19]$$

$$GR_N = -48.449 - 0.228H + 1.557T + 0.003H^2 - 0.012T$$
$$+ 0.006H\ T \qquad [20]$$

$$GR_D = -34.668 + 0.506H + 0.606T \qquad [21]$$

$$GR_N = -16.741 - 0.259H + 0.297T \qquad [22]$$

An advantage derived from neglecting the second order terms is the many-fold reduction in effort required to solve the differential equation.

Two important results are obtained from the above regression equations. First, the ratios of the constants obtained from the 1972 season appear consistant with both the previous season's and Blumenfield's values. Since it is the ratios a/c and b/c of the constants and not their magnitudes that are used in the final equation, the previous assumptions of their values are not critical. If these two ratios are compared between the day and night equations they are very close even though the individual magnitudes differ by a factor of two. Secondly, the difference in magnitude points out that the spears grow more slowly during the night. This fact agrees with another finding of Blumenfield's (1961) that 60 to 80 per cent of the growth took place between 7 AM and 7 PM. The difference in growth rates also means that the day and night growth periods must be handled separately in further analysis. Graphing height equation [21] or [22] would yield a plane in three-dimensional space similar to the plot in Figure 2.

Distribution of Growth Constants

The growth constant b was determined by first fixing the ratios b/c and a/c at 0.835 and -57.20 respectively, yielding as the equation for height

$$H = [(1.835)H_i + T_{ave} - 57.20]e^{bt} - (0.835)H_i + T_{ave} - 57.20$$

This equation was solved for b

$$b = \frac{\ln\left[\frac{H_n + (0.835H_{(n-1)} + T_{ave} - 57.20)}{1.835H_{(n-1)} + T_{ave} - 57.20}\right]}{t_n - t_{(n-1)}} \qquad [23]$$

The height, temperature and time data used in the regression analysis were used to calculate a value of b for each measurement interval.

After evaluating the results it was concluded that the values represented a truncated normal distribution with a mean of 0.1500 and a standard deviation of about 0.05. If the objective is to grow an average spear, then the average value of the growth constant could be used, but this would not give an indication of the distributions of spear height. In looking at the calculated constants, the values for a particular spear are somewhat arbitrary within the distribution with respect to the other spears. There is also some degree of randomness within a spear when compensation is made for the day-night differences. The constants for several spears selected randomly are shown in Table 1. The measurement groups referred to in Table 1, and again in Table 3, represent the interval between measurements. A group does not necessarily represent a particular height range because a range of heights were selected as initial measurement points. However, the higher the group number the higher the average height of the spears in the group. All initial measurements were made in the morning so that the first and all even numbered groups are always the daytime growth periods and the odd numbers are the night growth periods.

In order to simulate the dispersion of the growth constants a procedure was used in which values could be selected randomly from a particular distribution. The distribution of the calculated values of the growth

TABLE 1.--Growth Constants for Spears Selected Randomly from 1972 Data.

Measurement Group	Spear Number						
	9	12	125	208	349	359	451
2	.0362	.0623	.1614	.1007	.0649	.0790	.0975
3	.0393	.0294	.1423	.1177	.1598	.1383	.1941
4	.0380	.1312	.2016	.0901	.0926	.1281	.1421
5	.0407	.0874	.1614	.1802	.1762	.1735	.1774
6	.0494	.1069	.0912	.0585	.1171	.1383	.1492
7	.1069	.1154		.1986		.1369	.1703
8	.0640	.2213		.0958			
9	.1175	.1201					

constant, after dividing the day values by two, is given in Table 2 and shown graphically in Figure 3. The distribution used for selecting constants was the cumulative probability curve, also shown in Figure 3.

TABLE 2.--Distribution of Constant b for 1750 Values.

Division	Frequency	Cumulative Frequency	Cumulative Per Cent
0.0000-.0099	3	3	.0017
.0100-.0199	2	5	.0029
.0200-.0299	12	17	.0097
.0300-.0399	12	29	.0166
.0400-.0499	16	45	.0257
.0500-.0599	28	73	.0417
.0600-.0699	33	106	.0606
.0700-.0799	42	148	.0846
.0800-.0899	54	202	.1154
.0900-.0999	69	271	.1549
.1000-.1099	85	356	.2034
.1100-.1199	99	455	.2600
.1200-.1299	109	564	.3223
.1300-.1399	133	697	.3983
.1400-.1499	135	832	.4754
.1500-.1599	153	985	.5629
.1600-.1699	138	1121	.6406
.1700-.1799	109	1230	.7029
.1800-.1899	87	1317	.7526
.1900-.1999	69	1386	.7920
.2000-.2099	57	1443	.8246
.2100-.2199	55	1498	.8560
.2200-.2299	49	1547	.8840
.2300-.2399	47	1594	.9109
.2400-.2499	33	1627	.9297
.2500-.2599	30	1657	.9469
.2600-.2699	28	1685	.9629
.2700-.2700	28	1713	.9789
.2800-.2899	21	1734	.9909
.2900-.2999	10	1744	.9966
.3000-.3099	6	1750	1.0000

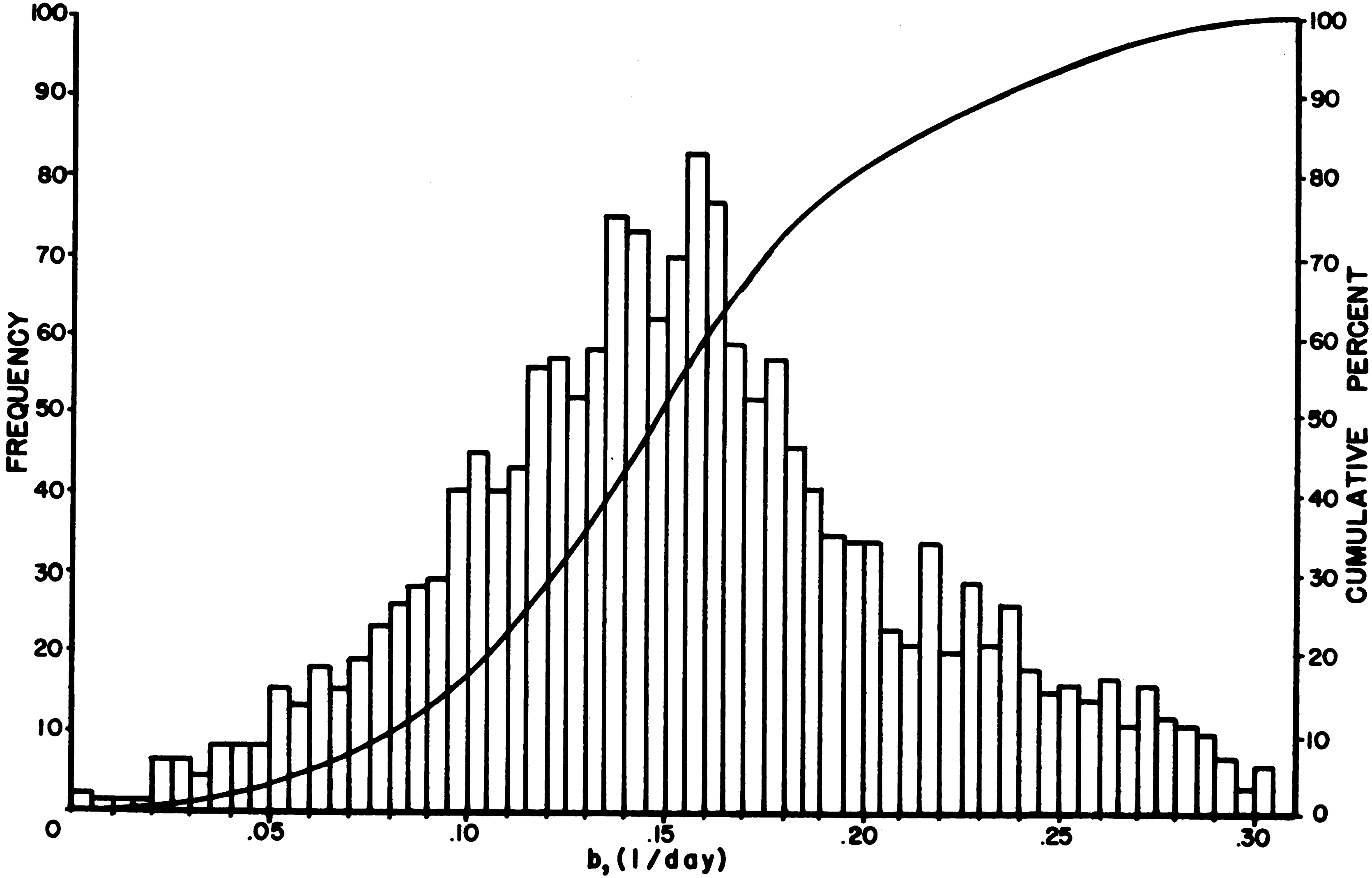

FIGURE III. FREQUENCY AND CUMULATIVE DISTRIBUTION OF GROWTH CONSTANT b FOR 1750 VALUES

RESULTS

Verification

The first approach to verification is to compare the height distributions of field data with a height distribution calculated using identical growing conditions. Equation [18] was employed to calculate the spear heights. The field data used was the 40 per cent of the measured data which was not used in the regression analysis.

To calculate heights under identical conditions required careful selection of values for the parameters involved; initial height, average temperature, growth time and a growth constant. The initial height for each growth period was defined as the measured height at the start of the growth period. The average temperature was taken to be the same as that determined for the measured growth period. The growth time was taken to coincide with each interval in the measured data. Therefore, for each calculated spear there is a real spear with the same initial height; growing at the same average temperature for the same length of time. The fourth parameter, the growth constant, was selected from the distribution generated in the previous chapter. Using existing computer programs a random number between zero and one was generated and a growth constant value was selected from the cumulative probability distribution shown in Figure 3. A new growth constant was generated for each growth period rather than retaining the same constant for the entire spear. The growth constant was multiplied by two for the day growth periods.

Both the measured and calculated heights were sorted and simple statistics were calculated. Figure 4 shows the mean values of the height distributions at the end of each measurement period. Table 3 gives the same information in numerical form. There was no difference between the means of groups five, seven, eight, and nine at the five per cent probability level when compared using the Student's-t test. Although no tolerance limits have been clearly defined, it is questionable whether the rejection limit of about one centimeter truly reflects field tolerances.

It is interesting to note that the means of the calculated values reflect an underestimate for the shorter spears with the best accuracy occurring near 28 centimeters. For application purposes it would be desirable to lower the crossover point to the region of interest. It appears that for taller spears the initial height terms become more significant. Also, close evaluation of the growth constant determination could yield greater accuracy in the region below 25 centimeters.

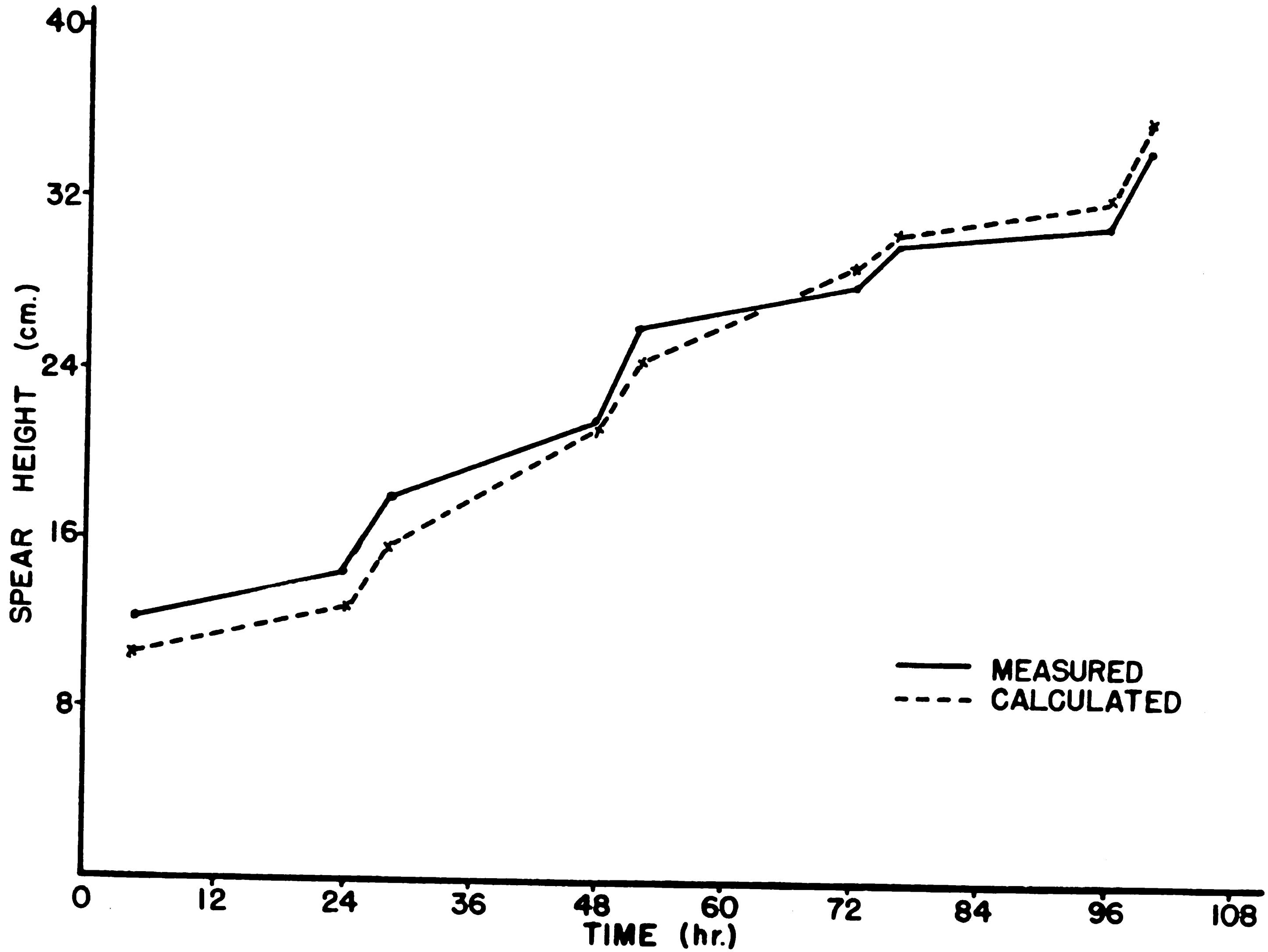

FIGURE IV. MEANS OF CALCULATED AND MEASURED HEIGHT DISTRIBUTIONS

TABLE 3.--Distribution Means for Measured and Calculated Distributions at Each Measurement Group.

Measurement Group	Measured		Calculated		Difference in Means	No. in Sample
	Mean	Std. Dev.	Mean	Std. Dev.		
2	12.22	3.14	10.38	3.96	-1.84	195
3	14.60	3.39	12.85	3.87	-1.75	195
4	18.08	4.63	15.69	4.01	-2.39	195
5	21.79	5.88	21.04	5.69	-0.75	194
6	26.03	6.88	24.55	6.00	-1.48	184
7	28.26	6.62	29.04	6.98	+0.78	141
8	30.27	6.32	30.60	6.78	+0.33	96
9	31.06	4.59	32.06	5.76	+1.00	48
10	34.71	5.46	36.14	6.43	+1.43	25

A second approach to verification was to compare height distributions of measured and calculated values that have initial values within particular ranges. The idea is to begin with a distribution that would be similar to values found in a field sample. The intervals selected were 14-17, 17-20, 20-23, 23-26 centimeters. They were chosen because they represent initial height groups that approach the 25 centimeter height value in increments of approximately one inch. The initial conditions were those of the field data and the growth constants were selected using the same procedure as described for the previous calculations. In the present situation the spears were only grown through two measurement intervals thus simulating an early morning, 0800 hours, initial observation and two subsequent observations, one at about 1600 hours and another 24 hours after the initial observation. The initial height for the second time period was the calculated result of the first interval rather than the measured distribution of the second group. The means of the distribution are given in Table 4. Figure 5 shows the frequency distributions for the 17-20 centimeter height group for both the calculated and measured data at 1600 hours and at 0900 hours the next day. All but one of the calculated means are within two centimeters of the means of the measured distributions. The distributions of the calculated values were more compact than the

TABLE 4.--Means for Height Distribution after 7 and 24 Hours.

Initial Height Group	After 7 Hours			After 24 Hours		
	Measured	Calculated	Diff.	Measured	Calculated	Diff.
14-17	18.39	16.87	-1.52	21.99	19.69	-2.30
17-20	22.45	20.79	-1.66	26.79	25.06	-1.73
20-23	25.76	24.77	-0.99	30.90	30.83	-0.07
23-26	29.49	29.03	-0.46	35.25	36.51	+1.26

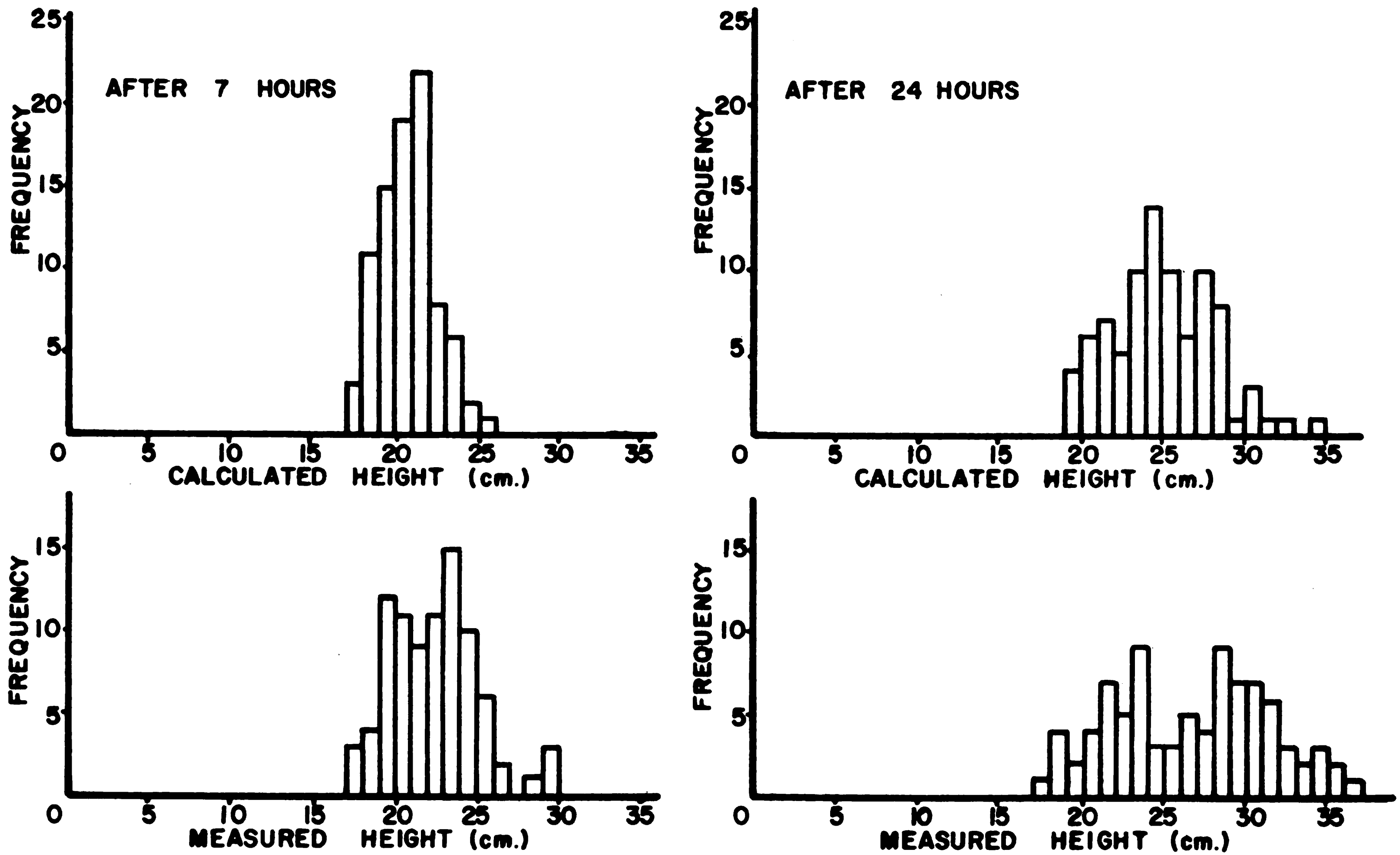

FIGURE V. HEIGHT DISTRIBUTION FOR SPEARS GROWN 7 AND 24 HOURS FROM INITIAL HEIGHTS BETWEEN 17-20 CM.

measured values even though the means appeared to be quite close. This situation means that in the present state, with limited data, the model does not handle the distribution tails as well as it should. One possible explanation is that constants for short spear heights were included in the distribution from which the growth constants were selected. This may have made the distribution less effective for taller spears. Closer evaluation of the growth constant distribution could possibly correct the deviation.

APPLICATION OF THE MODEL

How can this model be used to predict optimum time of harvest? The values in Table 4 offer an example. If the majority of taller spears in the field are in the 14-17 centimeter range it can be predicted that the field would not be ready for harvesting because most of the spears are around 20 centimeters after 24 hours. When the 17-20 centimeter group is considered, the field should be harvested before the next morning because more than half of the spears will be over the desired height at that time. Growth of the 20-23 centimeter spears indicates that harvesting should probably be completed by the end of the same day. Although the above example is not based on the same conditions for each spear, it gives some idea of how the model can be used.

Another example illustrating how the model can be used is to consider the hypothetical situation of a grower who samples his field at 0900 and finds that his maximum spear height is around eight inches (20 cm). He also knows that the day's average temperature will be about 70°F. The farmer is faced with a decision whether to harvest today or wait until the following morning.

Information about the height of the eight inch spears at some future time can be obtained by growing this spear a large number of times. Table 5 shows the height distributions, at two hour intervals, for 600 spears grown at 70°F from an initial height of 20 centimeters (8 inches). After four hours there are two per cent of the spears over 25 centimeters, but after six hours 26.8 per cent of the spears are over 25 centimeters. If the crop is left to grow ten hours, to 1900 hours, 75.5 per cent of the spears will be over 25 centimeters. It is obvious that the farmer will have to harvest today and he should start sometime after lunch, i.e., after four hours.

The growth of 600 spears, 20 centimeters high, starting at 1600 hours and growing at 60°F is also given in Table 5. After 16 hours the distribution of heights appear equivalent to about seven hours or less than half of the day's growth for the same period. The difference is caused both by the difference in the day and night growth constants and by the lower growth temperature. If the farmer had the above situation, he could let the spears grow over night.

TABLE 5.--Height Distribution at Two Hour Intervals for 600 Spears Grown Under Same Conditions.

Height in Groups		Hours of Growth					
		Day*					Night**
In.	Cm.	4	6	8	10	12	16
8.0	20.00-20.99	3.5	2.0	1.0	0.4	0.3	1.3
	21.00-21.99	20.0	6.3	3.4	1.8	1.6	4.0
9.0	22.00-22.99	43.0	15.6	7.8	3.9	2.2	13.5
	23.00-23.99	22.7	27.5	11.3	7.9	4.5	21.3
	24.00-24.99	8.8	21.7	19.0	10.5	4.3	27.0
10.0	25.00-25.99	2.0	13.7	21.8	14.0	8.5	15.5
	26.00-26.99	0.0	7.5	15.2	17.5	10.5	9.3
11.0	27.00-27.99	0.0	5.3	6.6	14.5	13.7	5.1
	28.00-28.99	0.0	0.3	8.0	9.8	13.1	3.0
	29.00-29.99		0.0	3.0	5.6	11.5	0.0
12.0	30.00-30.99		0.0	2.3	6.3	7.5	0.0
	31.00-31.99		0.0	0.5	3.5	5.7	0.0
13.0	32.00-32.99			0.0	2.0	5.2	
	33.00-33.99			0.0	2.1	4.3	
	34.00-34.99			0.0	0.2	2.4	
14.0	35.00-35.99				0.0	2.5	
	36.00-36.99				0.0	1.4	
15.0	37.00-37.99				0.0	0.8	
	38.00-38.99					0.0	
	39.00-39.99					0.0	
16.0	40.00-40.99					0.0	
Mean		22.70	24.11	25.56	27.06	28.60	24.46
Standard Dev.		.968	1.494	2.047	2.630	3.244	1.643

*Tave = 70°F
H_o = 20.0 cm.

**Tave = 60°F
H_o = 20.0 cm.

SOURCES OF ERROR

When dealing with natural systems there are always a number of factors that can influence the results. Such parameters as soil moisture and fertility, soil temperature, temperature gradients, and inherent plant variation may contribute substantially to asparagus growth. Blumenfield (1961) attributed 16 per cent of the variation in the growth rate to the above factors, but chose to neglect them and focus on height and temperature.

It has been hypothesized, but not proven, that a particular distribution of growth constants, b, is particular to the location and variety. This fact would indicate that it may be possible to compensate for location dependent parameters such as soil fertility and temperature gradients by using the random procedure. The same thought applies to inherent plant variations. This type of compensation would necessitate calibrating a field by taking limited data and generating the distribution for the growth constants.

Another possible source of error was the use of air temperature rather than the temperature at the growing region which, according to Culpepper and Moon (1939), is primarily at the spear tip. Depending on the absorptive and reflective factors of the soil surface, the temperature in the first ten centimeters could be $\pm 8°F$ from the average air temperature, according to Geiger (1965). However, this difference is less pronounced above 15 centimeters where the interest of the model lies. It was also thought that average air temperature is a parameter easily obtained by the growers.

Physical limitations in measurement might also be responsible for some error introduction. Heights were measured to the nearest 0.1 centimeter, and in the case of low temperatures, daily growth was only several tenths of a centimeter which could result in large percentage errors in the growth rate. Measurement was also hindered by the fact that many spears tended to grow curved or at an angle from vertical, making consistent referencing difficult.

SUMMARY OF RESULTS

The objective of this study was to obtain a model that could accurately describe the growth characteristics of asparagus for use in predicting time of harvest.

Field Data

During the 1972 season more than 500 spears were measured twice a day until they exceeded 30 centimeters thus giving from 5 to 12 measurements per spear and 3920 measurements over all. Temperature was recorded continually for the entire measurement period. These data were divided; 60 per cent being used for model development and 40 per cent for verification.

Regression Analysis

Sixty per cent of the field data was analyzed using a least squares regression analysis. The linear, squared, and cross product terms of air temperature and spear height were the dependent variables and growth rate the independent variable. Preliminary analysis showed that the square and cross product terms could be neglected and the model based on the linear terms. Values for constants and their ratios were established. A difference was recognized between the day and night values. Even though the difference was a factor of two it did not affect the ratios.

Model Development

By defining growth rate as the change in height over the change in time the regression equation was written as a differential equation. Solution of the differential equation yielded an equation for spear height at a time Δt as a function of initial height, average temperature and a characteristic growth constant. It was recognized, from the regression analysis, that differences existed between day and night growth constants. It was further determined that a single value for the growth constant could not simulate accurately the spread in heights of spears which were initially the same height.

Growth Constant Distribution

To handle the problem of variability between spears existing computer programs were utilized to generate a random number between zero and one which in turn was used to select a growth constant value from a cumulative probability distribution. The cumulative distribution for the growth constant b was generated by calculating the value using equation [23], for each measurement period. Variability within a spear was handled by selecting a new constant for each growth period rather than retaining a single value throughout the spear.

Model Verification

Using the above procedure, and the remaining 40 per cent of the field data, heights were calculated using equation [18]. The resulting height distributions were compared. The first approach took all the spears from each measurement group and calculated the height at the next measurement point. The second approach selected spears within a particular height group and calculated heights for the next two measurement points; 7 and 24 hours later. In both comparison distributions the means of the calculated data, around the area of interest, differed by less than two centimeters from the means of the measured height distributions. Although the means were close, the calculated values were more compact than the measured values. The model does not adequately handle the distribution tails as well as it should for use as a prediction tool. The model; however, is realistically accurate when considering all of the factors that might influence the growth of asparagus.

FUTURE RESEARCH

A closer evaluation of the distribution of the growth constants is needed. Possibilities would be to evaluate temperature and height effects to see if a refinement of the distribution would produce better predictions in the tail regions of the height calculations. The model needs to be field tested by selecting a sample of spears, calculating the heights based on forecasted temperatures, and compare the calculated results with what really exists in the field. Further research could also focus on the development of a series of tables or graphs that, after field calibration, could be used by a grower.

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