INFLUENCE OF FOOD PRODUCT PROPERTIES ON THE FREEZING TIME

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ABSTRACT

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By

Rong-Ching Hsieh

The complicated factors associated with prediction of the freezing time of a food product are due to the continuous changing of food product properties and the lack of accuracy in measurement of the freezing characteristics of the product. Although Plank's equation has been widely used to predict the freezing time of a food product, the assumptions used in formulating the equation are too risky to be accepted for a transient heat transfer problem. Several attempts have been made to overcome these difficulties by utilizing a finite difference model and digital computers. For the most successful simulation model as developed by Gorby (1974), thermal property data are required only for unfrozen products. In addition, the temperature function curves for product properties and the freezing history curves can be generated besides the predicted freezing time.

The objective of this investigation was to utilize the computer simulation techniques to predict the freezing times and temperature history curves for different food products. Furthermore, the influences of some product properties of an actual food product on the freezing time can be explained by investigating the independent effect of the product properties.

The computer simulation program utilized in this investigation requires minimum

input information. The input information consists of the properties of a food product for temperatures above the initial freezing point, the initial product temperature and freezing conditions. The criterion for the freezing time was established by freezing the product to the instant when the mass average enthalpy of the product is equivalent to the value at $-25^{\circ}C$ which is the assumed storage temperature. The freezing time is a function of freezing conditions which include freezing medium temperature, surface heat transfer coefficient and product diameter. Further reduction of the freezing medium temperature below $-200^{\circ}C$ has only a small effect on reducing the freezing time. The same result can be predicted with further increase of the surface heat transfer coefficient above 200 $W/m^{\circ}K$. The predicted freezing time is directly proportional to the product diameter.

Under given freezing conditions, the freezing time is a function of initial freezing point, initial water content, product density and thermal conductivity of a food product. Influences of these parameters on the freezing time were discussed by using hypothetical combinations of product properties. A food product with a lower initial freezing point, a higher initial water content and a higher initial product density will have a longer freezing time. The deviations in the predicted freezing time among actual food products and hypothetical test products were due to the combined and complex influences of the product properties. It has been found that the prediction of freezing time is most sensitive to the accuracy of the measurement of the initial product density and the initial freezing point if the freezing point of a food product is above $-0.5^{\circ}C$. The influence of the accuracy of initial thermal conductivity data on freezing time is not important in the range of 0.45 to 0.55 $W/m^{\circ}K$ investigated. The combined effect of the inaccuracy in measuring these product properties will be significant even if the influence of an individual product property is small in predicting the actual freezing time and other freezing characteristics. However, the influences of product properties on the freezing time for actual food products are consistent with the trend predicted by the independent influence analysis.

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To My Parents

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a	Constant used in equation (9)
b	Volume fraction of the discontinuous phase used in equation (9)
C_{pa}	Apparent specific heat of food products, J/g°K (CPA)*
C_{pi}	Specific heat of ice, J/g °K(CPI)
C_{pw}	Specific heat of water, J/g °K (CPW)
dm	Differential mass, g
EMA	Mass average enthalpy of food products, J/g-product
$\bar{G}_A$	Molar Gibbs free energy of solvent, J/g-mole
$\bar{H}_A$	Heat content of the solvent, J/g-mole
HT	Total heat content of food products, J/g-product
h	Averaged surface heat transfer coefficient, $W/m^{2\circ}K$ (H)
IK	Initial thermal conductivity of food products, W/m°K
IPD	Initial product density, g/cm^3
IWC	Initial water content of food products, g-water/g-product
ICP	Initial specific heat of food products, J/g° K
k	Thermal conductivity of food products, W/m°K (K)
k_c	Thermal conductivity of continuous phase, W/m° K (KC)
k_d	Thermal conductivity of the discontinuous phase, W/m°K(KD)
L	Latent heat of fusion of pure solvent , J/g-solvent (HS)
M	Cubic root of the volume fraction of solids or discontinuous phase

M_A	Molecular weight of pure solvent, g/g-mole
M_B	Effective molecular weight of solute, g/g-mole
m_A	Mass of pure solvent, g
m_B	Mass of solute, g
R	Gas constant, 8.314 J/g-mole °K
S	Constant used in equation (B–13)
T	Absolute temperature, ° K
TF	Initial freezing point of food products, °C or °K
T	Freezing point of pure solvent, °K (TO)
T_{kr}	Cryoscopic temperature, °C or °K
T_n^t	Temperature of node n at time t, °C
V	Volume, cm^3
$\bar{V}$	Specific volume, cm^3/g (SV)
WC	Total water content, g-water/g-product
X_A	Mole fraction of solvent in solution
α	Thermal diffusivity, m^2/hr
ρ	Density, g/cm^3
ρ_i	Density of ice, g/cm^3 (DI)
ρ_w	Density of water, g/cm^3 (DW)
ρ_s	Density of product solids, g/cm^3 (DS)
θ	Time, minute or hour (TIME)
$\Delta\theta$	Computing time increment, hr (DTHETA)

* All symbols in the parenthesis are used in the computer program.

I. INTRODUCTION

The determination of the unsteady state heat conduction temperature distribution of frozen food products differs from that associated with ordinary cooling. Food products contain heat sources (latent heat evolution) during ice formation and the food product properties vary with temperature. Cryoscopic temperatures of the product are discontinuous and nondifferential.

The most critical factor limiting accuracy in predicting the freezing time is the lack of accurate method to describe the freezing characteristics of food products. Product properties data for food products are either inaccurate and/or difficult to be measured experimentally. To solve the complicated features of the continuous changing of product properties of frozen food products, numerical techniques and digital computers were utilized by Lescano and Heldman (1973), Heldman and Gorby (1974–B) and Gorby (1974) in recent years. These approaches provide an accurate and efficient way of predicting the freezing time, the refrigeration requirements and the product properties below the initial freezing point. This prediction model can be based on the available product properties data above the initial freezing point and the assumption of an ideal solution being present in food product system. In addition, simple mathematical caculations of thermal properties based on phase theories of physical chemistry and some empirical equations, such as Kopelman equation, Dickerson's equation to evaluate the specific heat of ice, the freezing point depression equation, and the basic equations to describe the transient heat conduction problems, are assumed to be reasonably correct.

The major difficulty involved in analyzing the influences of product properties of food products is the combined effect of these properties. The investigation of the independent influence of several product properties on freezing time is becoming essential in development of accurate methods.

The objectives of this investigation were:

1. To utilize the computer techniques which predict the product properties at below the initial freezing point and to apply the assembled prediction models developed by Gorby (1974) in predicting freezing time and temperature history for isotropic products.
2. To analyze the computer output data and compare freezing times, temperature history curves and product properties during freezing processes for several different food products.
3. To investigate the independent effect of different product properties on freezing time.
4. To explain the differences between freezing time and temperature history curves for different food products.

After the relationships between product properties and freezing time have been established, the freezing time of other food products with similar shapes and freezing conditions can be predicted. Furthermore, the relative significance of the accuracy in experimental determination of product properties will be more evident.

II. LITERATURE REVIEW

It has been pointed out by Barlett (1944), Ede (1949), Riedel (1949-A, 1949-B, 1956, 1957), Long (1955) and Hill (1966) that the water in a food product crystallizes over a range of temperatures. This fact was interpreted as the continuous depression of freezing point and variation in ice-water-solids composition by Staph and Woolrich (1951). Detailed explanations of the properties of water and ice during food freezing was presented by Fennama and Powrie (1964), Luyet (1969), Dickerson (1969) and Bakal and Hayakawa (1972).

The unfreezable water content may have a significant influence on refrigeration requirement for freezing. Energy can be saved by just reducing the food product to the temperature at which all the freezable water has been frozen. Duckworth (1971) proposed the techniques of differential thermal analysis (D.T.A.) to measure the unfreezable water content in various food products by utilizing powdered CO_2 or liquid nitrogen as a coolant.

Heldman (1974-B) predicted the relationship between unfrozen water fraction and temperature during food freezing by utilizing freezing point depression. It was established that accurate prediction of the relationship of unfrozen water percentage and temperature for products with considerable unfreezable water requires knowledge of the percent unfreezable water or improved knowledge of freezing characteristics of the product.

Several attempts had been made by Dickerson (1969) and others (Smith (1952), Long (1955), Lentz (1961), Hill (1967), etc.) to measure thermal properties of food products experimentally. The results were tabulated by Woodams and Nowrey (1968), Reidy(1968), Dickerson (1969) and Morley (1972). Long (1955) predicted the thermal conductivity of frozen codfish using a model of sodium chloride solution to simulate the cod muscle as a mixture of protein with a certain concentration of salt solution.

The prediction of refrigeration requirements during freezing was demonstrated by Heldman (1966) and verified experimentally for ice cream by Heldman and Hedrick (1970) using calorimetric determinations. The most widely used and straight forward empirical equation to predict the freezing rate in food products is the Plank (1913) equation. This equation was modified by Nagaoka et al.(1955) to take into account the initial water content and the sensible heat removed. Eddie and Pearson (1958) also modified Plank's equation for infinite slab products and pointed out that experimental measurements of thermal properties of food products are frequently not possible or may be inaccuate due to the inherent instrument errors. Carslaw and Jaeger (1959) presented the Neumann solution to the transient heat conduction problem. This solution was applied to the food freezing problem by Charm and Slavin (1962). A detailed analysis of several expressions for predicting freezing rates conducted by Bakal and Hayakawa (1972) indicates that all approaches have limitations of not taking into account the variable products properties during freezing.

To overcome the above limitations for analytical and empirical solutions, the numerical approach to solving the transient heat conduction by computer simulation with changing product properties was developed by Albasiny (1956), Hohner and Heldman(1970), Charm (1971), Cordell and Webb (1972), Bonacina and Comina (1973), Fleming (1973), Cullwick and Earle (1973), Lescano and Heldman (1973), Comina and Bonacina (1974), Heldman and Gorby (1974-B), Gorby (1974), and Tarnawski (1976). Several different computer simulations were compared by Heldman (1974–A).

An analysis of the output information for sweet cherries has been thoroughly discussed by Gorby (1974). The predicted values of unfrozen water content, thermal conductivity, product density, enthalpy and apparent specific heat for sweet cherries has been presented. In addition, the influence of surface heat transfer coefficient, product size and initial product temperature on freezing time was investigated. It has been shown that the initial product temperature (under normal handling conditions) has a little effect on the freezing time compared to the other factors. Heldman and Gorby (1975) demonstrated that low freezing medium temperature (−195°C or −320°F) in an individual-quick-freezing (IQF) process resulted in the heat content of the product being reduced to the desired level. Therefore, the refrigerant can be used more efficiently based on the heat content level.

III. THEORETICAL CONSIDERATIONS

Freezing time of food products change not only with the freezing conditions but also with the food product properties. The freezing conditions imposed on the food products during freezing are the freezing medium temperature, the surface heat transfer coefficient between the surface of food product and the freezing medium, the initial product temperature and the radius or other geometric factors of the product.

Changes in product properties during freezing are due to the continuous depression of the freezing point and therefore the continuous changing of the unfrozen water content. All the product properties are assumed to be constant above the initial freezing point.

3–1 Unfrozen water content

From the phase equilibrium described in the physical chemistry, the relationship between the mole fraction of solvent in a solution and the freezing point of the solution can be obtained by introducing the chemical potential of a pure solvent in the solid state and the chemical potential of the pure solvent in the liquid state. Under equilibrium conditions:

$$\frac{\overline{G}_A\,(\text{Solid}) - \overline{G}_A\,(\text{Liquid})}{RT} = \ln X_A \qquad (1)$$

where $\overline{G}_A$ = molar Gibbs Free energy

R = gas constant (8.314/g-mole°K)

T = absolute temperature (°K)

X_A = mole fraction of solvent in the solution

From the Gibbs-Helmholtz equation:

$$\frac{\bar{G}_A/T}{T} = -\frac{\bar{H}_A}{T^2} \qquad (2)$$

and differentiation of equation (1) with respect to T, the following equation is obtained:

$$\frac{\bar{H}_A\,(\text{liquid}) - \bar{H}_A\,(\text{solid})}{RT^2} = \frac{LM_A}{RT^2} = \frac{d\ln X_A}{dT} \qquad (3)$$

Integration of equation (3) with respect to T from the freezing point of pure solvent to any temperature below the freezing point results in the following equation:

$$\frac{LM_A}{R}\left[\frac{1}{T_o} - \frac{1}{T}\right] = \ln X_A \qquad (4)$$

where:

$$X_A = \frac{m_A/M_A}{m_A/M_A + m_B/M_B} = \text{mole fraction of solvent} \qquad (5)$$

and:

L = latent heat of fusion of pure solvent (J/g-solvent)

T_o = freezing point of pure solvent (^oK)

T = freezing point of the solution (^oK)

m_A = mass of pure solvent (g)

m_B = mass of solute (g)

M_A = molecular weight of pure solvent (g/mole)

M_B = effective molecular weight of solute (g/mole)

$T - T_o$ = freezing point depression (^oK)

In the case of food freezing, it is assumed that the liquid water is the solvent and ice together with food solids (including soluble solids) are the solute.

The unfrozen water content at a given temperature below the initial freezing can be obtained from the mole fraction of the unfrozen water. From equations (4) and (5):

$$X_A = \exp\left\{\frac{LM_A}{R}\left[\frac{1}{T_o} - \frac{1}{T}\right]\right\} \quad (6)$$

and:

$$m_A = \frac{M_B X_A M_A}{M_B(1 - X_A)} \quad (7)$$

where m_A is the effective mass of water and M_B is the apparent molecular weight of product solids. M_B in equation (7) can be determined by utilizing equation (6) with initial freezing point of the food product as T and solving equation (7) for M_B using the initial water content of the food product.

A small amount of water may remain unfrozen even at very low temperature in some food products. This portion of water is considered "inactive" in the dynamic freezing point depression. The above method was thoroughly discussed by Lescano (1973) and Heldman (1974-B).

The modified (or effective) water content used in equation (7) to determine the apparent molecular weight of product solids, is:

$$M_A = (\text{Initial water content}) - (\text{Unfreezable water content})$$

Hence, the total water content in the system at any temperature below the initial freezing point will be:

$$WC = M_A + (\text{Unfreezable water content}) \quad (8)$$

The unfrozen water content (WC) obtained from the above equation can be utilized in predicting the thermal properties of food products at temperature below the initial freezing point by determining the water, ice and solid fractions at any freezing instant. This method was proposed by Heldman (1974-B) and was shown to give good agreement with experimental data.

3–2 Thermal Conductivity

Long (1955) and Lentz (1961) have described the dependence of thermal conductivity of frozen food on the extent of product freezing. The results indicated that the thermal conductivities of a frozen food could be predicted by using an appropriate mathematical model and the frozen water fraction in the product as a function of temperature.

The mathematical model used by Long (1955), Lentz (1961) and Lescano (1973) was the Maxwell (1904) equation as first adapted to food products by Eucken (1940). The Maxwell equation is:

$$k = k_c \left[\frac{1 - (1 - a k_d / k_c)\, b}{1 + (a - 1)\, b} \right] \tag{9}$$

where:

k_c = thermal conductivity of the continuous phase

k_d = thermal conductivity of the discontinuous phase

$$a = \frac{3k_c}{2k_c + k_d}$$

$$b = \frac{V_d}{V_c + V_d}$$ = volume fraction of the discontinuous phase

A set of mathematical expressions was proposed by Kopelman (1967) to predict the thermal conductivity in a food system for both isotropic and anisotropic systems. In an anisotropic system with the thermal conductivity parallel to the fiber, the values are approximately 15 to 20% higher than that perpendicular to the fibers were proposed.

In this investigation, the Kopelman (1967) isotropic model was utilized. This model has been used successfully to predict thermal conductivities of frozen sweet cherries by Gorby (1974). For a two-component system:

$$k = k_c \left(\frac{1 - Q}{1 - Q\,(1 - M)} \right) \tag{10}$$

where:

$$Q = M^2 \left(1 - \frac{k_d}{k_c} \right)$$

M^3 = volume fraction of solids or discontinuous phase

Since the frozen food product consists of water, ice and product solids, a modification of the Kopelman model has been introduced by Heldman and Gorby (1974-A). It was found that the best agreement with experimental data is obtained by first considering two of the three product phases and reducing the system to a binary solution followed by reducing the system to a another binary system for a second use of the same expression. Water was considered a continuous phase for the water-ice system during the first use of the Kopelman equation, then the water-ice mixture was treated as a continuous phase and the product solids was taken as discontinuous phase for the second use of the equation.

The value of the thermal conductivity of product solids was found by using the Kopelman model and the experimental (or initial) thermal conductivity of the unfrozen product.

3–3 Product Density

Constant densities for water, ice and solids below the initial freezing point have been assumed in predicting the product density. In an ideal case, the product density change is only due to the proportion changes of the mixture.

The specific volume (or volume per unit mass) was found by adding the volume per unit mass of product fractions. Thus:

$$\overline{V} = 1/\rho = m_i/\rho_i + m_w/\rho_w + m_s/\rho_s \qquad (11)$$

with the density of product solids (ρ_s) found by utilizing equation (11) with the initial product density and solving for the density.

3–4 Enthalpy

The total heat content of a frozen food product as proposed by Lescano and Heldman (1973) was expressed by considering zero enthalpy at −40°C.

$$HT = m_s C_{ps}\,(T + 40) + m_w L + m_w C_{pw}\,(T + 40) + m_i C_{pi}(T + 40) - m_w\,(\text{at } -40^\circ C)\, L \qquad (12)$$

At −40°C, where enthalpy is zero, the total water content (m_w) is equal to the water content at −40°C. This portion of the unfrozen water at −40°C was considered unfreezable water content and used as an additional input parameter in the prediction of unfrozen water contents for cod fish and lean beef by Heldman (1974-B).

The solids content (m_s) can be obtained from the initial water content. The unfrozen water content (m_w) can be predicted by equations (6), (7), and (8). The unfreezable water content (m_w at −40°C) is known from experimental data. The specific heat of solids (C_{ps}) is determined from the initial product specific heat and the initial water content by the following equation:

$$ICP = IWC \cdot C_{pw} + m_s \cdot C_{ps} \qquad (13)$$

In general, m_s, C_{ps}, C_{pw} and L are not functions of temperature, while m_w, m_i and C_{pi} are predicted. The specific heat of ice (C_{pi}) as a function of temperature was obtained by Dickerson (1969):

$$C_{pi} = A + B \cdot T \qquad (14)$$

where:

A = 1.9507941

B = 0.00206153

T = degree K.

Heldman and Gorby (1974-B) predicted the refrigeration requirements using equation (12).

3–5 Apparent Specific Heat

The apparent specific heat of a food product is defined as the differential change in enthalpy if the enthalpy-temperature function is continuous:

$$C_{pa} = \Delta H/\Delta T \qquad (15)$$

if ΔT is chosen as 0.028°C (0.05°F) and ΔH can be obtained from equation (12), the apparent specific heat can be predicted.

3–6 Heat Transfer

Several attempts has been made to solve the heat transfer problem and predict the freezing time of food products. The exact solution from Plank's (1913) equation is not applicable due to the assumptions required when applied to food products.

Some modifications of Plank's equation have been proposed. Yet, the major problems imposed by an unsteady state heat transfer have not been solved. The techniques of a finite difference model to solve the transient heat transfer problem has been proved accurate and effective.

For the transient heat conduction problem of a spherical product with temperature dependent thermal property values, the appropriate differential equations of energy balance in terms of finite difference model can be generated from the following equation:

$$q_n^{in} - q_n^{out} = q_n^{stored} \qquad (16)$$

where q_n is the heat transfer rate at node n. The expanded differential forms of the above equation for the center, interior and surface nodes are presented in Appendix B.

There are several ways to solve the energy balance differential equation by the finite difference methods. The most commonly used methods are the forward difference (explicit) and the backward difference (implicit) for the time derivative. The simplest finite difference representation of equation (16) utilizes the central difference formula for the spatial derivative and a forward difference for the time derivative. This method is presented in Appendix B.

The following assumptions have been made in formulating the finite difference model:

(1) Energy transport occurs only in the radial direction.

(2) The initial product composition and temperature are uniform and the food products are homogeneous and isotropic.

(3) Mass transfer within the product or on the products surface is neglected.

(4) The specific heats of solids and water, density of solids, water and ice, and the latent heat of fusion for water are constant.

(5) All product properties are constant above the initial freezing point.

(6) At a discontinuous point of product properties there exists a separation limit phase, at which temperature is T_{kr} (cryoscopic temperature). This divides the examined food product into unfrozen and frozen parts.

(7) The freezing medium temperature and surface heat transfer coefficient are assumed to be constant throughout the freezing processes.

The above assumptions have been proposed by Gorby (1974) and Tarnawski (1976). The solution of equation (16) can be put in various forms for different approaches. A set of solutions for the food product was derived by Gorby (1974) for the surface, interior and center nodes from both explicit and implicit methods. The implicit method was used in this investigation.

The implicit method, or backward difference method, has the advantage that any time increment can be used (Kreith, 1973). The explicit method, or forward difference method, has the problem of stability. Choosing a computing time increment which is small enough to stabilize the calculation for a given value of shell thickness in the computer is essential upon using the explicit method. The method is described in Appendix B.

3–7 Criterion for Freezing Time

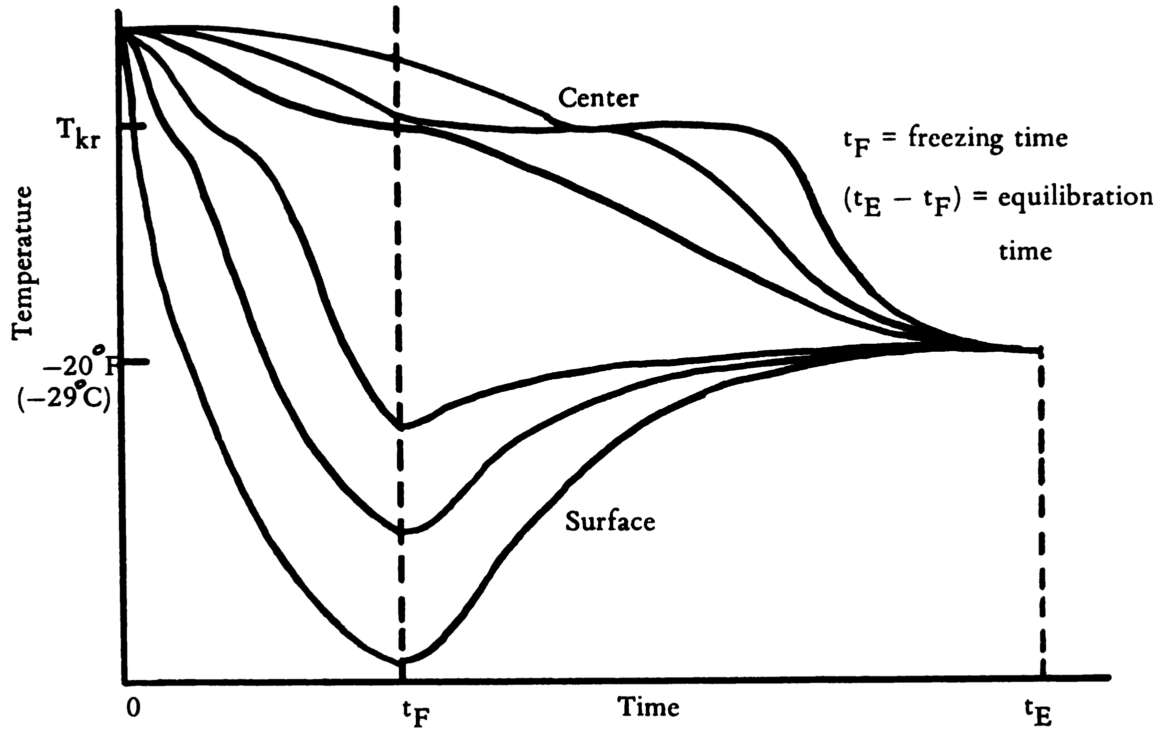
The freezing time in a quick freezing process is small compared to conventional freezing. Because of the fast freezing rate, the temperature distribution within the product at the end of freezing may span a wide temperature range. Therefore, the commonly used method which applied thermal arrest time or center temperature criterion can not be utilized for the quick freezing processes. Two types of temperature history diagrams shown in Figure 1 were found by Gorby (1974). Comparison of these two figures shows that the lower freezing medium temperature [case (a)]has a more dramatic influence since a large temperature gradient exists between the freezing medium and the product surface. Beyond the end point of the freezing process (t_F) the product is exposed to environmental parameters with a surface heat transfer coefficient of 1 BTU/hrft^2°F (5.7 W/m^2°K) and a storage temperature of –20°F (–29°C) until the product temperature is uniform at point t_E. The time period between t_F and t_E is the equilibration time. Shorter freezing time and longer equilibrating time were predicted by using lower freezing medium temperature.

It has been proposed by Gorby (1974) that the total heat content be a criterion for food freezing processes. The freezing process stops at the instant when the total enthalpy from equation (12) reaches the equivalent product enthalpy at the desired storage temperature.

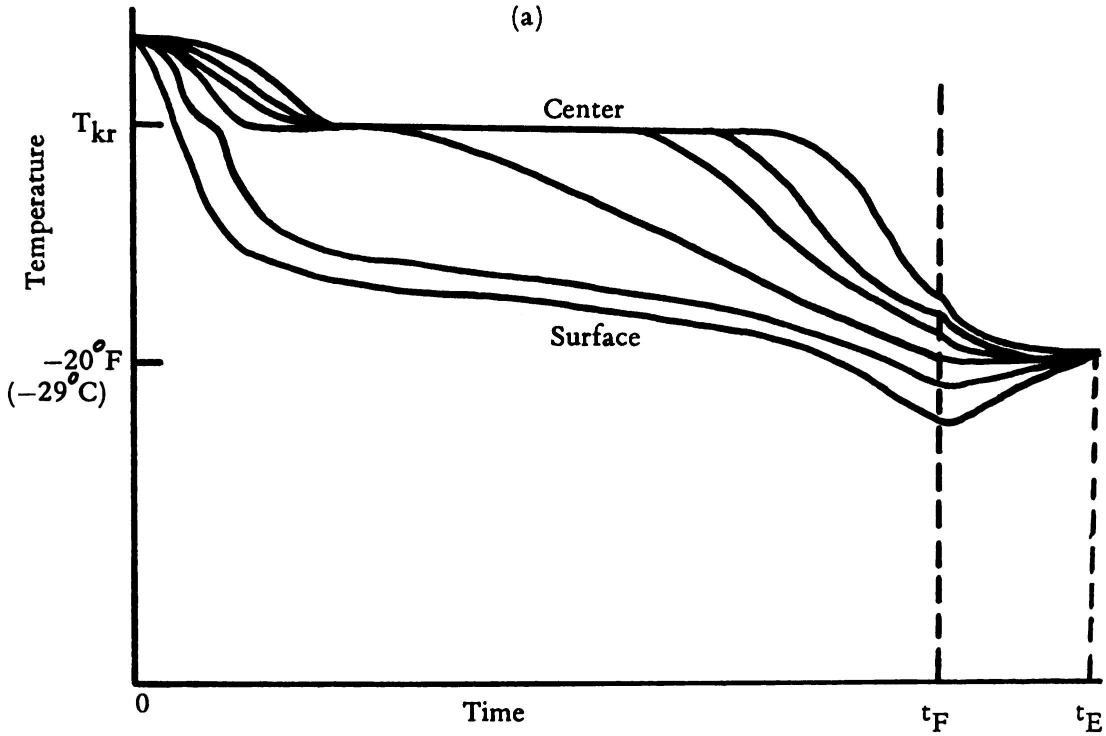
The mass average enthalpy (EMA, J/g-product) is defined mathematically as:

$$EMA = \frac{1}{m} \int_m H dm \tag{17}$$

The above equation can be approximated by the same divisions in the finite difference



(a)



(b)

Figure 1. Types of temperature history profile as a function of freezing medium temperature when (a) the freezing medium temperature is very low and (b) the freezing medium temperature is higher.

model as:

$$EMA = \frac{1}{V} \sum_{I=1}^{M} H(I)V(I) \qquad (18)$$

where M is the number of nodes and V is the total volume of the individual product.

The uneven temperature distribution at the end of freezing process will redistribute heat within the product until it reaches equilibrium. This provides the lowest possible residence time in the freezing medium and also the lowest refrigerant requirement.

IV. EXPERIMENTAL CONSIDERATIONS

Asparagus, carrots, cherries, peas, plums and strawberries were chosen to compare the freezing time. The reason for selection of these products were as follow:

1. They are commercially available in frozen form and the quick freezing process is the best and most efficient way to preserve these products and maintain product quality.
2. The product properties of these food products cover the range typical of most fruit and vegetable products encountered in food freezing preservation.
3. These products are isotropic or can be described by isotropic conditions.
4. These products are of comparable size or can be cut into comparable sizes for use in comparing the effects of product properties on the freezing time.

A computer program based on the mathematical model presented in the preceding chapter was developed by Gorby (1974, see Appendix E). The program can generate the predicted product properties as a function of temperature and the predicted temperature history under an input set of initial conditions.

The following input information is necessary to generate the program:

1. Properties of food product
   a) Initial product temperature (TI), degree C.
   b) Initial freezing point (TF), degree C.
   c) Initial water content (IWC), decimal.
   d) Unfreezable water content (UFWC), g-water/g-product.

e) Initial product density (IPD), g/cm^3

f) Initial specific heat (ICP), J/g°K

g) Initial thermal conductivity (IK), W/m°K

2. Freezing medium temperature (TINF), degree K

3. Convective heat transfer coefficient (H), W/m^2°K

4. Computing time increment (DTHETA), hour.

5. Print time increment (TPRINT), minute.

6. Product diameter (DIA), cm

7. Equivalent enthalpy temperature for the program to stop (STEMP), degree C

8. Number of shells into which product will be divided (LAYERS), integer

A STEMP value of −25°C was utilized and 10 LAYERS was assumed throughout the investigation unless otherwise specified.

Additional input data were required:

1. Conversion to degrees absolute (A) = 273.15

2. Gas constant (R) = 8.314J/g-mole°K

3. Heat of fusion for water (HS) =334.72 J/g

4. Density of water (DW) = 1.00 g/cm^3

5. Density of ice (DI) = 0.917 g/cm^3 (at 0 °C)

6. Freezing point of pure water (TO) = 0 °C

7. Thermal conductivity of ice (KI) = 2.21 W/m°K (at 0 °C)

8. Thermal conductivity of water (KW) = 0.57 W/m°K

9. Specific heat of water (CPW) = 4.2 J/g°K

The following parameters were chosen to evaluate:

1. Product diameters of 1cm for asparagus and peas, 2cm for carrots, cherries, plums, and strawberries.

2. Initial product temperature: 10°C

3. Surface heat transfer coefficients: 25, 70, 170, and 340 W/m^{2}°K

4. Freezing medium temperatures: −35, −75, −130, and −195°C

Times for the products to reach an equivalent heat content at −25°C were predicted. Table A-1 shows the property data for these products. These data were obtained or estimated by Riedel (1951), Reidy (1968), Dickerson (1969) or Bedford (1976).

A set of test runs was conducted using carrots to generate the relationships between the freezing time and each separate product property by fixing all the other parameters except the one which was investigated. The variations of these parameters were in the range of the property values of these products.

A diameter of 2cm was assumed for asparagus and peas in order to compare the freezing rates of the same product size with the rest of the products.

The output information includes:

1. Predicted apparent specific heat, thermal conductivity, product density, unfrozen water content and enthalpy of food products as a function of temperature.
2. Predicted temperature history for the center and surface nodes.
3. Predicted freezing time for the whole product to reach the equivalent enthalpy temperature of −25°C.

V. RESULTS AND DISCUSSIONS

Four input values for the freezing medium temperature (−35°C, −75°C, −130°C, and −195°C) and surface heat transfer coefficient (25, 70, 170 and 340 $W/m^{2o}K$) were selected in order to generate reasonable curves for their influences on the freezing time.

It is safe to assume that all the individual products or cut pieces are homogeneous. The pits in cherries and plums have thermal properties similar to those of the fruit but have slightly lower water contents. Since the freezing time is very short, the effect of the cylindrical or cubical shapes of carrots and asparagus pieces was assumed to be very small since the food product shapes investigated are spherical.

The temperature influence on product properties was investigated in order to demonstrate the influence of these properties on the freezing time. Figure 2 illustrates the predicted values of unfrozen water content as a function of temperature for the various products. The unfrozen water contents remains constant until the initial freezing point is reached and is reduced rapidly due to the crystallization of water in the food product while the product temperature remains constant. The freezing point of these food porducts decreases due to the higher ratio of solids to water content. The unfrozen water fraction remains almost constant at the temperatures below −40°C.

Figure 3 illustrates the predicted thermal conductivity changes during the freezing process. The significant increase at temperatures below the initial freezing point is related to the presence of ice which has a higher (about 4 or 5 times higher) thermal conductivity than that of water. The values of thermal conductivity in the frozen region are functions of unfrozen water content and temperature. At temperatures below about −50°C the increase in the values of thermal conductivity is very small and values remain fairly constant at very low temperature. It is also shown that the food product with a higher initial thermal conductivity does not necessarily have a higher thermal conductivity in the frozen region because other factors such as the product density and water content of the food products influence the thermal conductivity at temperatures below the initial freezing point. This is demonstrated by Kopelman's equation (Equation 10) where M^3, volume fraction of solids or discontinuous phase, is a function of the product density and unfrozen water content at that instant. In addition, the temperature functions of product density and unfrozen water content are unique for each food product and their combined influences on the evaluation of thermal conductivity below the initial freezing point are complicated.

The temperature influence on the product density is presented in Figure 4. The shape of the curves is similar to those for unfrozen water content (Figure 2) since the determination of product density in the frozen region is directly related to the amount of unfrozen water present in the food product (Equation 12). By comparing the values of product properties at initial temperature and at temperatures far below the initial freezing point, it is evident that the value of thermal conductivity increases almost 4 times the initial value, whereas the product density decreases about 10%.

The enthalpy values for the food products at temperatures below the initial freezing point are presented in Figure 5. Removal of latent heat contributed most to the reduction in heat content at temperatures just below the initial freezing point where the temperature decrease is very small. The large amount of enthalpy reduction in that region results in very high values of apparent specific heat as illustrated in Figure 6. Curves in Figure 6 are the temperature derivatives of the corresponding curves in Figure 5.

In order to investigate the independent influence of product properties on the freezing time, a series of tests were conducted using carrots as the test product. The results are

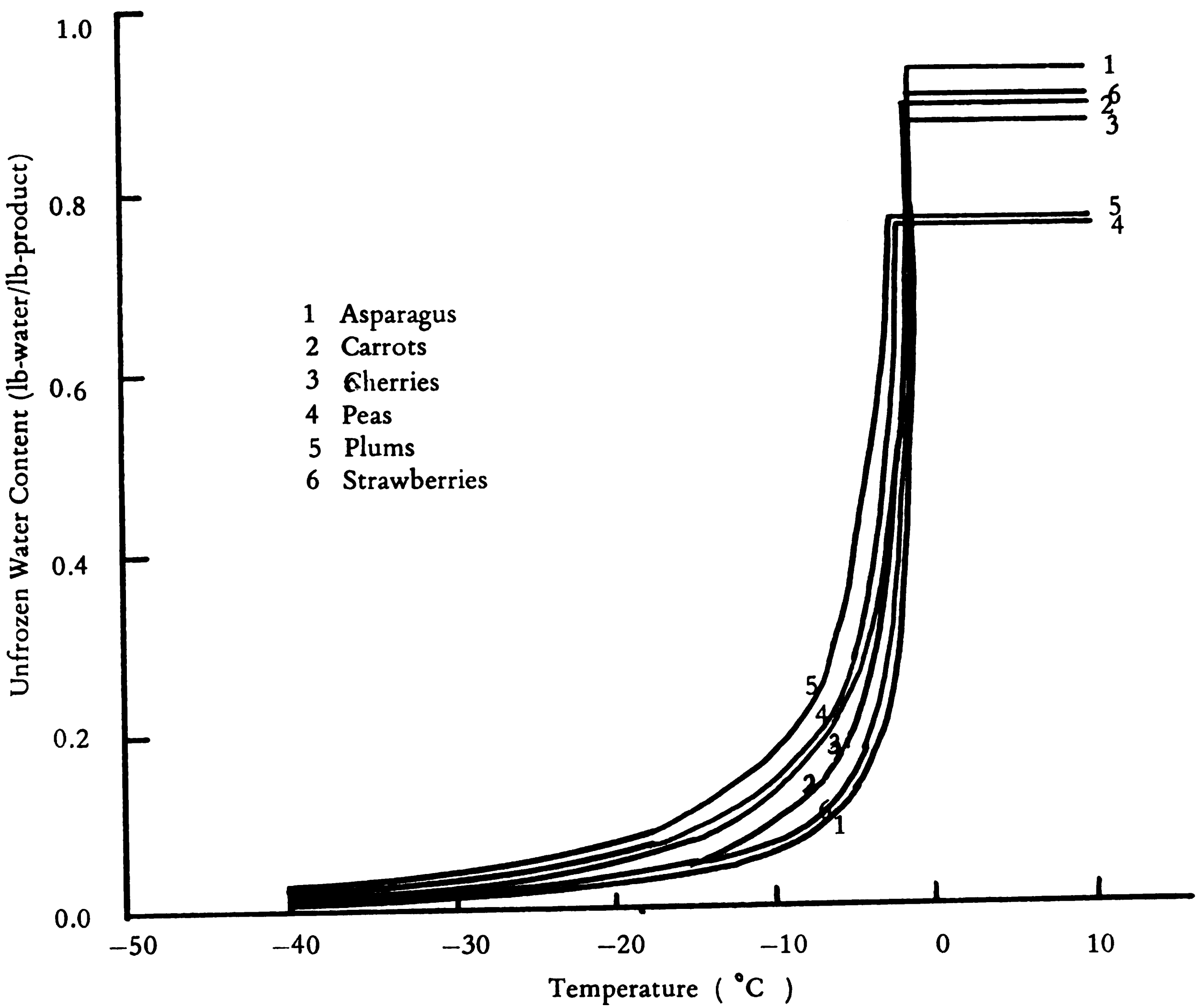


Figure 2. Predicted unfrozen water content for different food products.

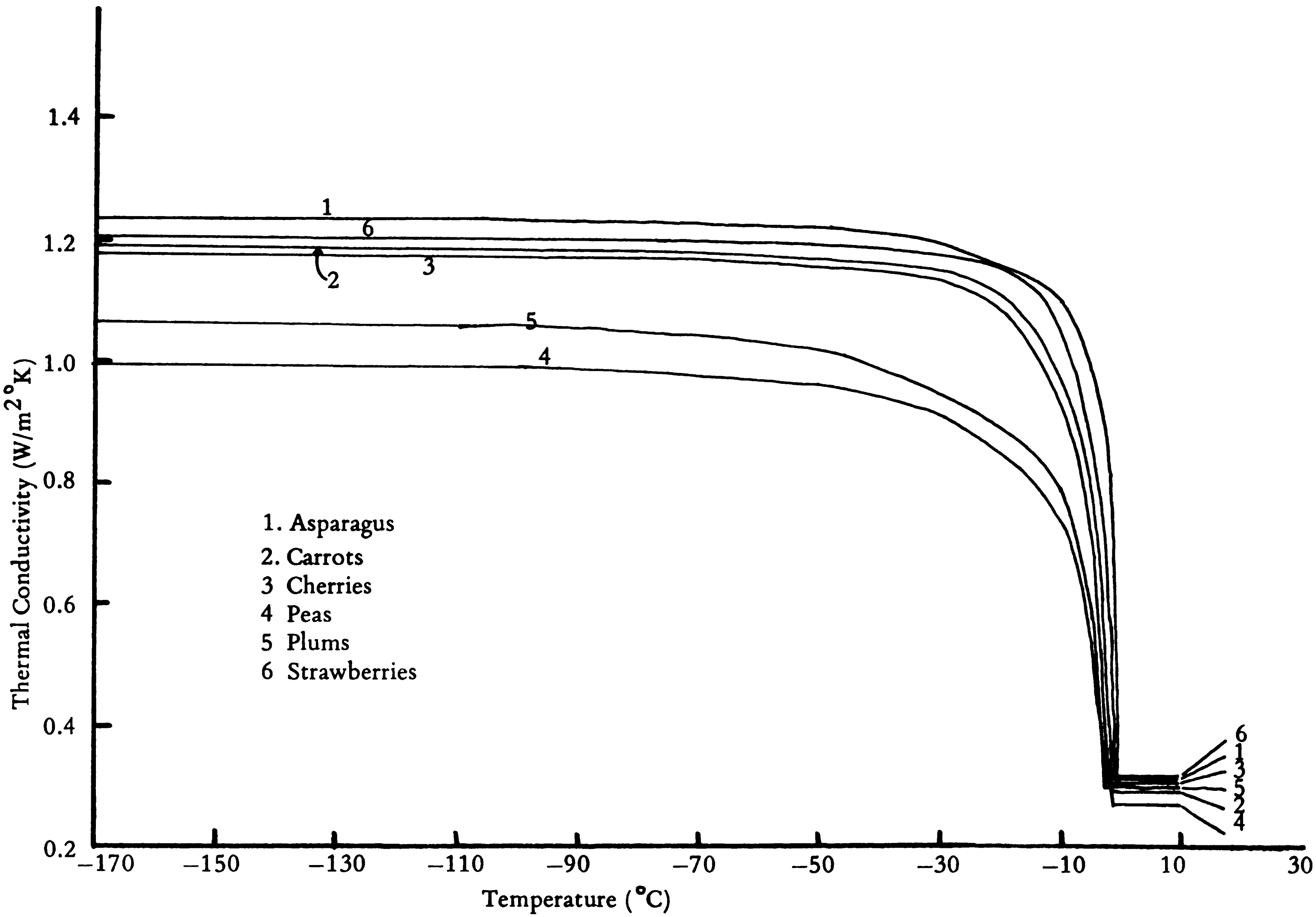


Figure 3. Predicted thermal conductivity for different food products.

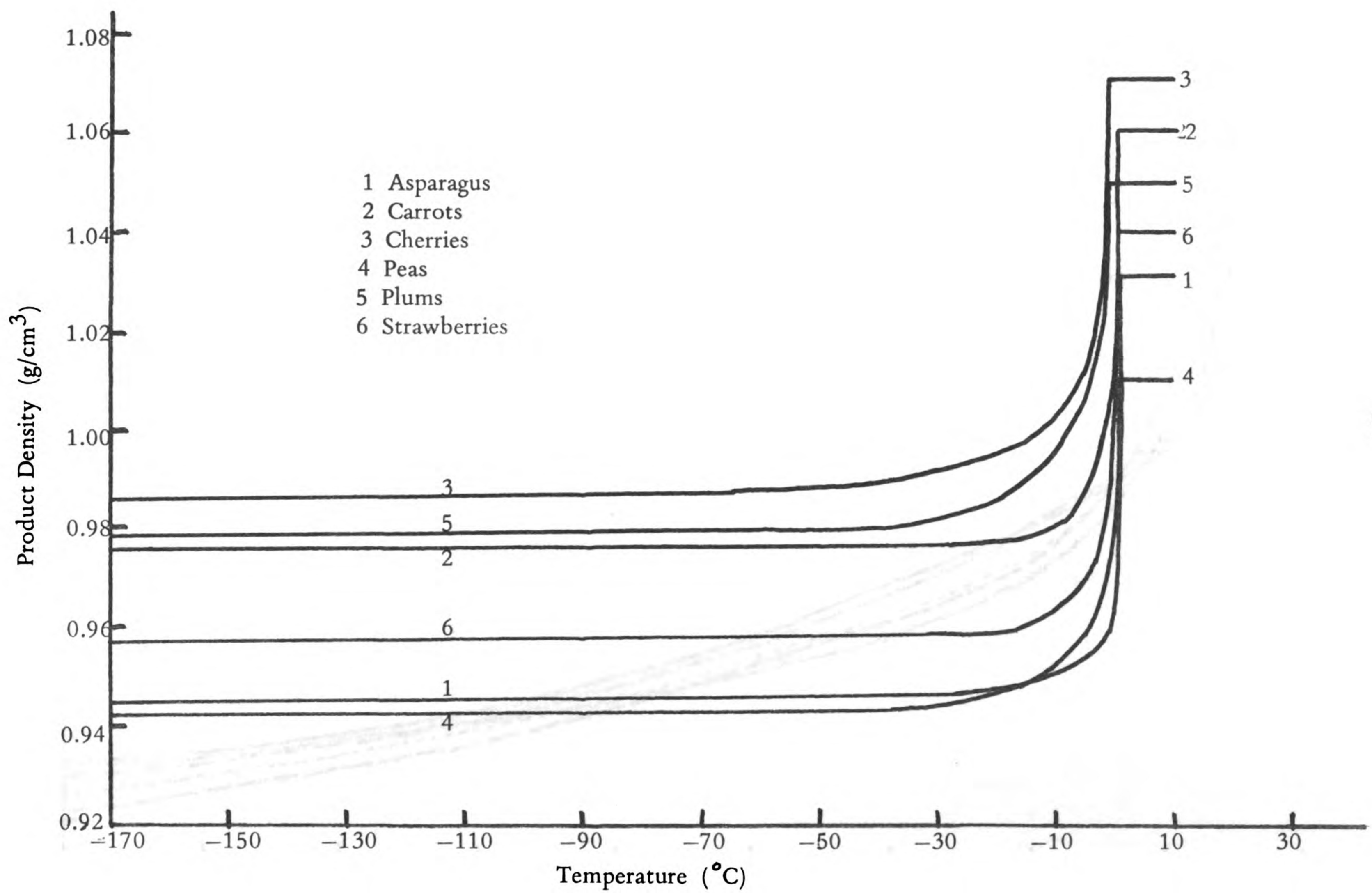


Figure 4. Predicted product density for different food products.

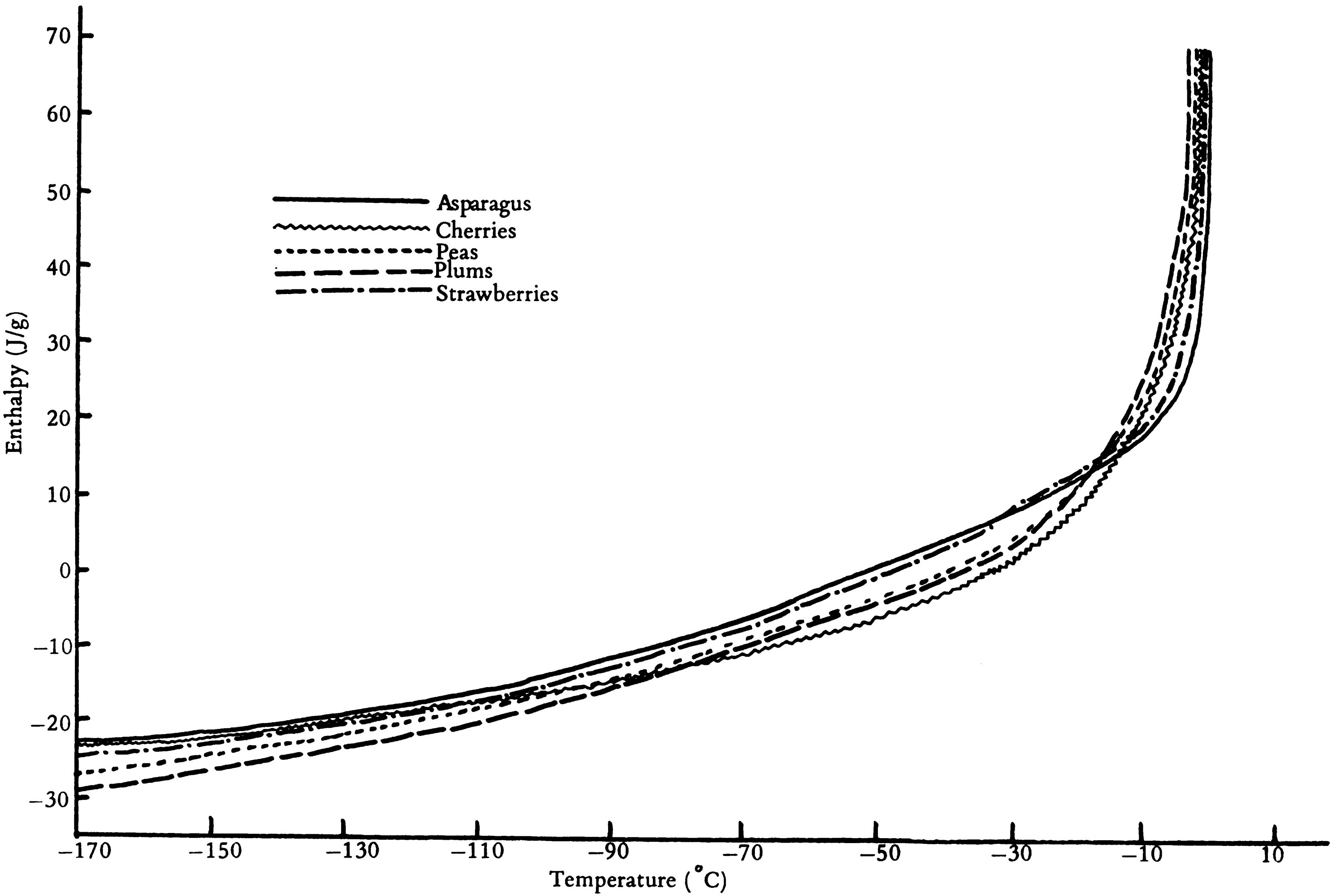


Figure 5. Predicted enthalpy for different food products.

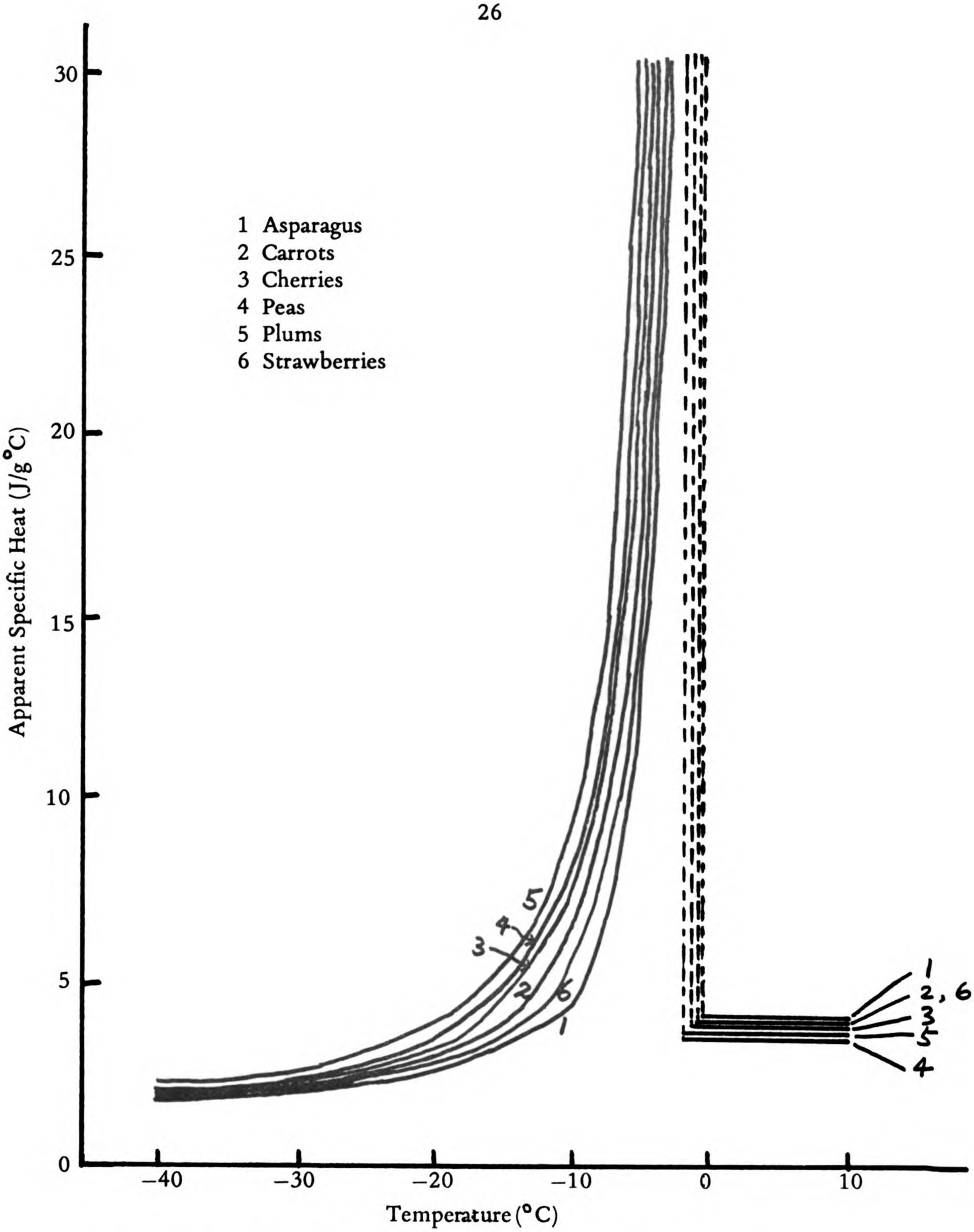


Figure 6. Predicted apparent specific heat of different food products.

presented in Figures 7, 8, 9, 10 and 11. These test runs were set up in the range of the product properties typical for the products. The results shown in Figure 7 illustrate that a food product with a higher initial freezing point has a shorter freezing time. It is fairly safe to assume that the temperature function curves of unfrozen water content and enthalpy for the products with different initial freezing points will have similar shape of curves. Besides, these curves do not cross each other. The only difference between these curves is the position of the discontinuous point and the whole curve have been shifted. The above reasoning is based on the fact that all the parameters except the initial freezing point have been kept constant. Therefore, the product with a higher initial freezing point will possess lower unfrozen water content and enthalpy values than the other product with a lower initial freezing point at the same freezing temperature, and a shorter freezing time for the product with a higher initial freezing point is expected.

Figure 8 shows that a food product with a higher initial water content has a longer freezing time. It is apparent that the product with a higher initial water content has a higher total heat content due to the extra contribution of the latent heat from higher initial water content.

The influence of the unfreezable water content on the freezing time is significant for fibrous products. A higher value of unfreezable water content results in less freezable water while keeping the total water content constant. Less freezable water present in a food product will require shorter time to freeze. This is consistent with the results presented in Figure 9.

Figure 10 predicts the influence of initial product density on the freezong time. From the heat transfer equation presented in Appendix B, it is evident that a product with a higher density will have a slower heat transfer rate. Thus, a longer freezing time can be expected for the product with a higher density.

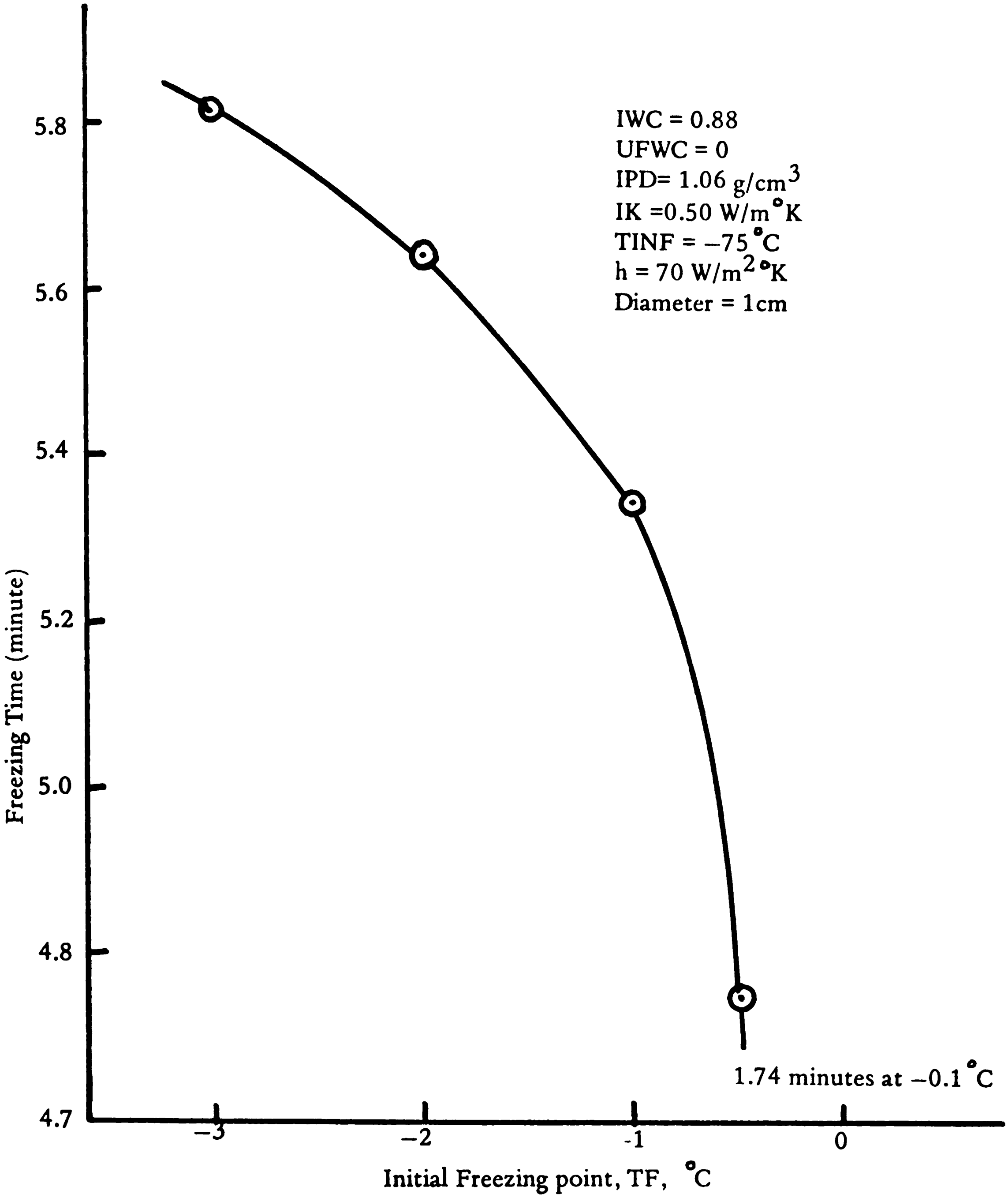


Figure 7. Predicted influence of initial freezing point on the freezing time.

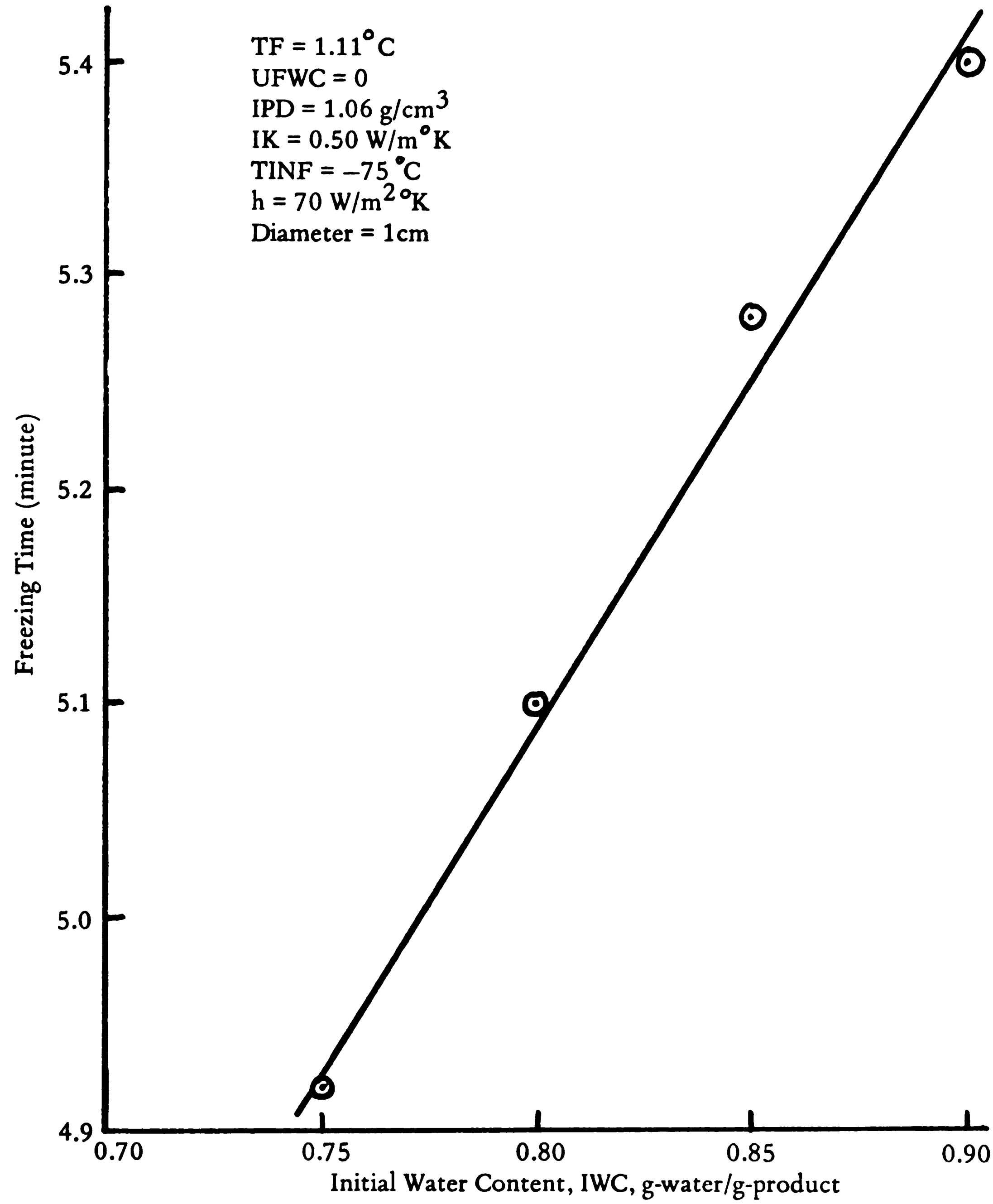


Figure 8. Influence of initial water content on the freezing time.

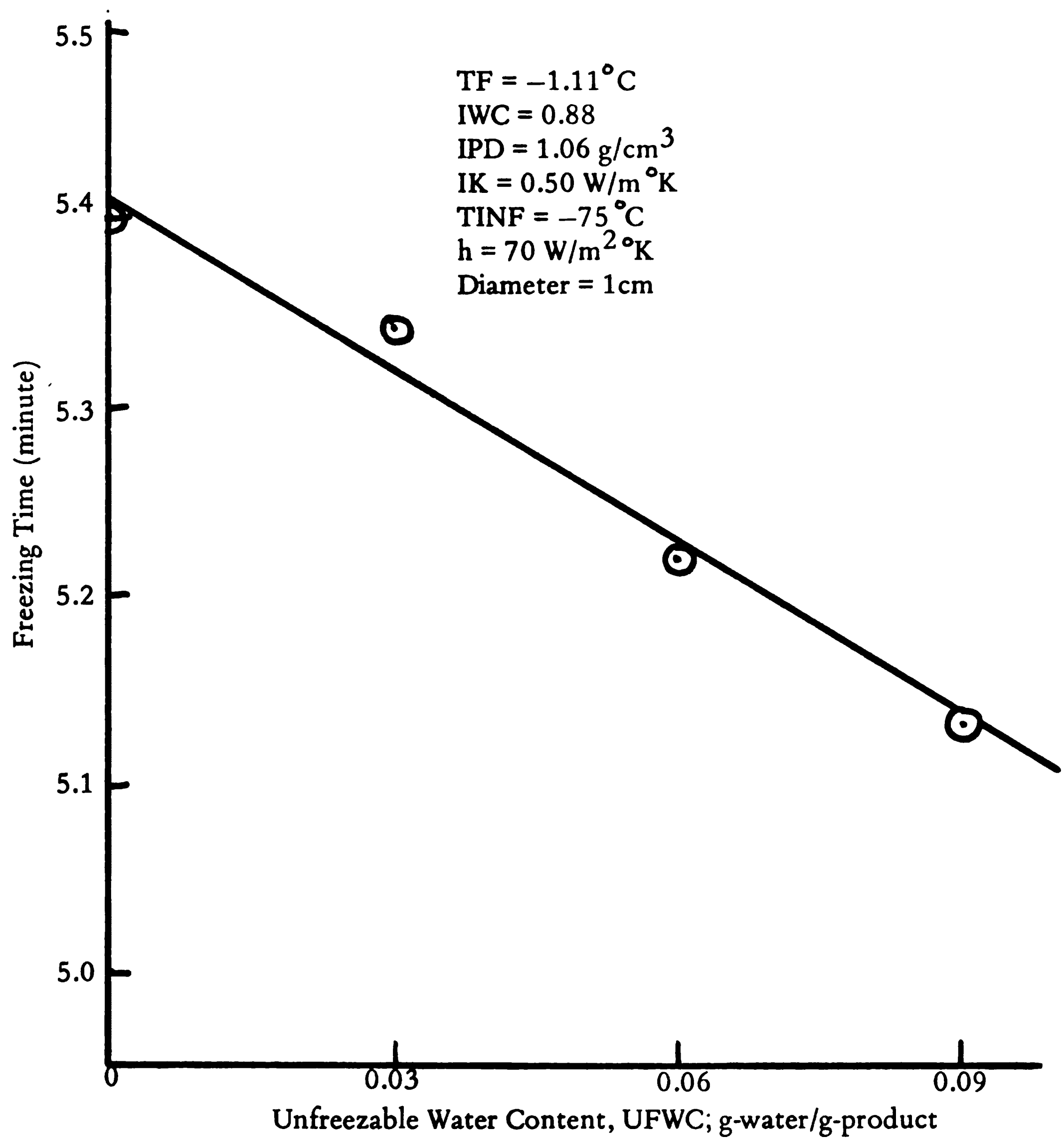


Figure 9. Influence of unfreezable water content on the freezing time.

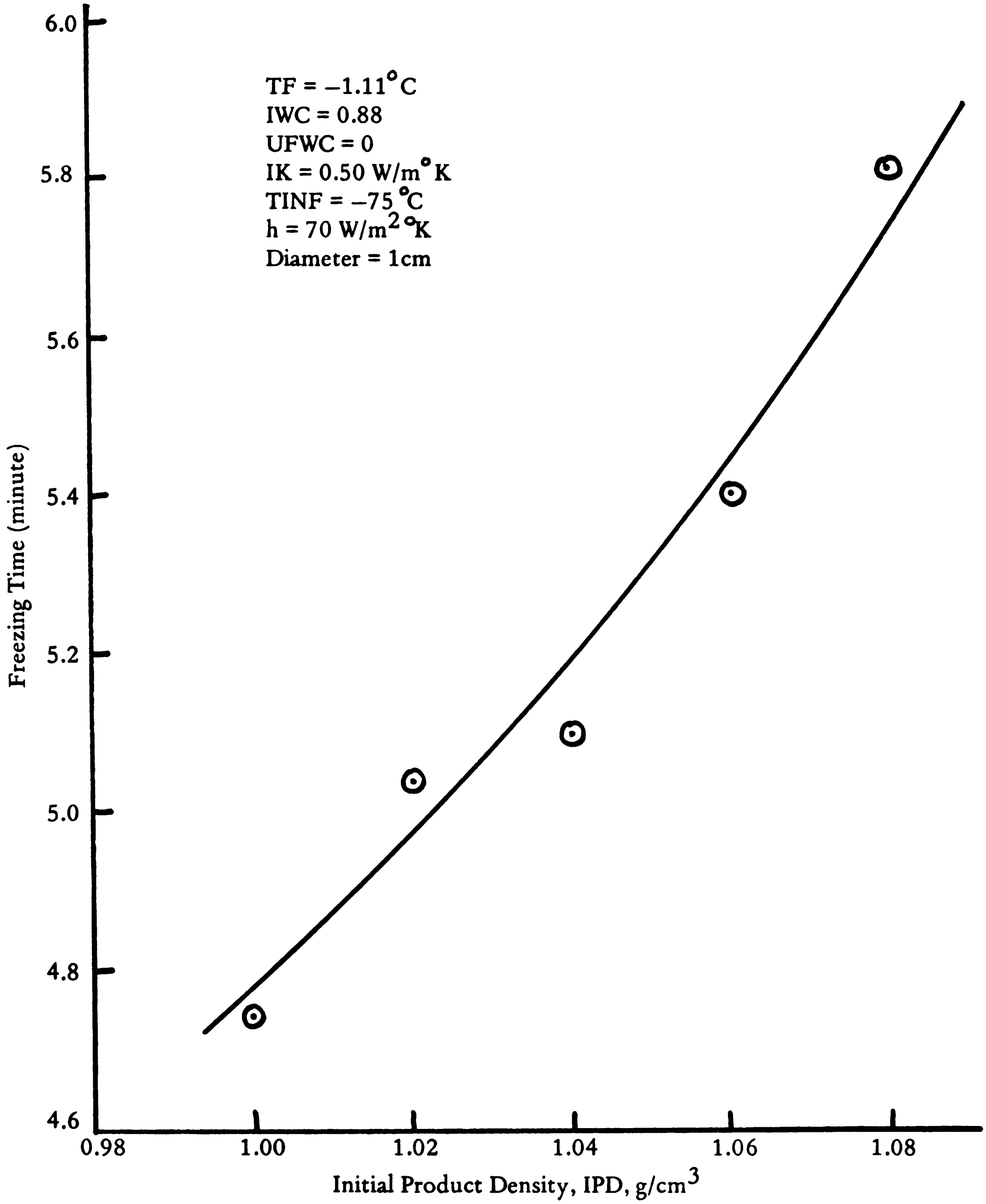


Figure 10. Influence of initial product density on the freezing time.

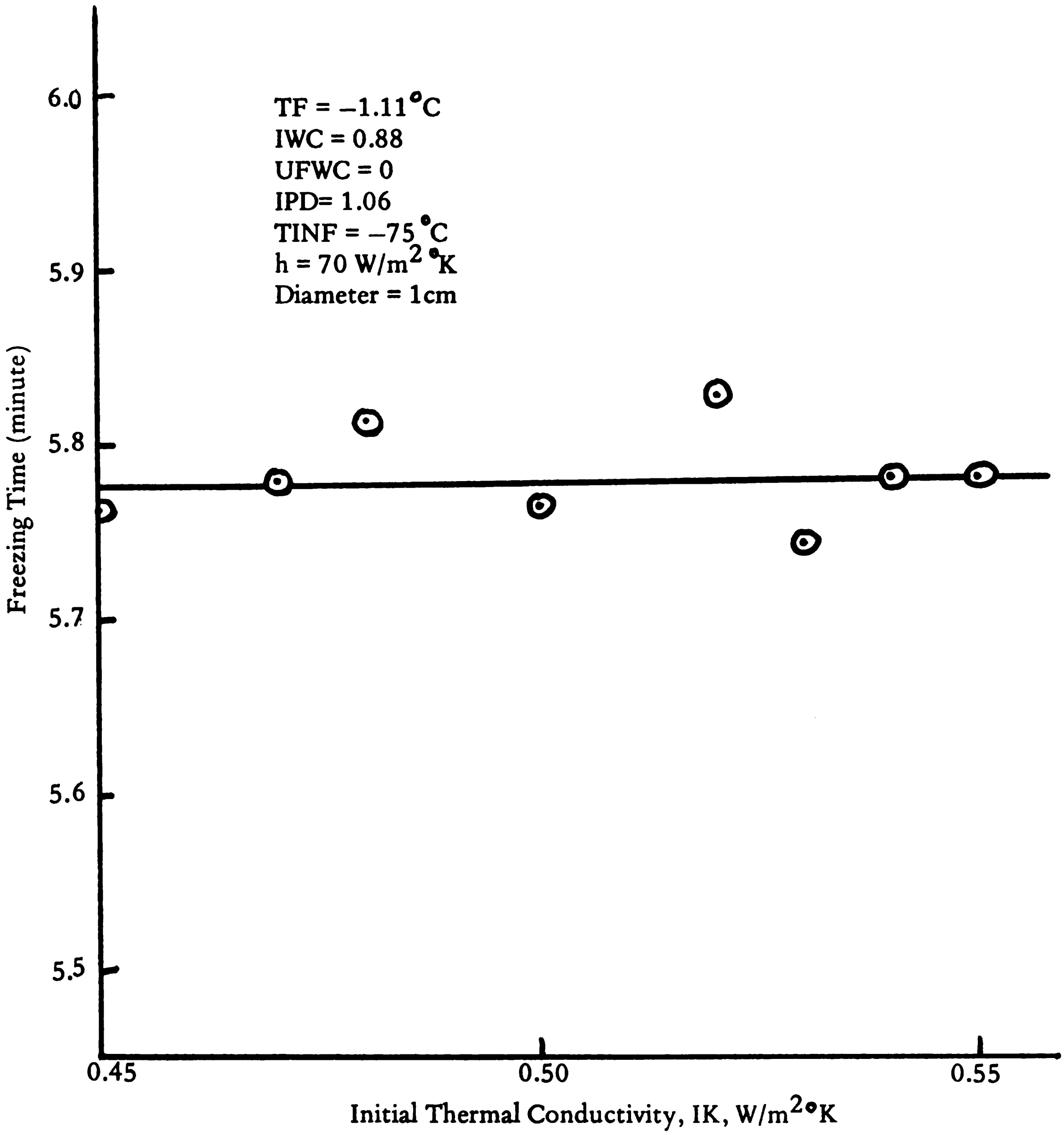


Figure 11. Influence of initial thermal conductivity on the freezing time.

The influence of the initial thermal conductivity shown in Figure 11 demonstrates that the values of the initial thermal conductivity have almost no influence on the predicted freezing time. The values of freezing time in Figure 11 were generated by dividing the product into 20 layers and utilizing a smaller computing time increment to get more accurate results. Similar shapes of curve were obtained by dividing the product into 10 layers which had been used throughout this investigation. Better results were obtained by dividing the food product into more layers and utilizing a smaller computing time increment, and accordingly, a longer computing time is required. The results for the freezing times of different initial thermal conductivities generated by dividing the product into 10 layers had even larger deviations and were unable to identify the influence. Therefore, a 20-layer division was utilized and the results appeared to be close to a horizontal straight line which means the initial thermal conductivity has little effect in predicting the freezing time.

The results in Table 1, which were generated from Figures 7 to 11, illustrate the maximum allowable deviations from the actual property values when measuring product properties with 1 to 4% variations in the prediction of freezing times. The results are based on arbitrarily chosen conditions: (a) initial product temperature of 10°C, (b) freezing medium temperature of -75°C, (c) surface heat transfer coefficient of 70 W/m$^{2\circ}$K and (d) product diameter of 2cm. Values of allowable deviations for product properties in freezing time prediction presented in Table 1 were calculated by dividing the freezing time variations by the average slope of curves in Figure 7 to 11 and multiplying the stated ranges of product properties. The product with a higher initial freezing point requires a more accurate measurement. A 0.1g-water/g-product inaccuracy in measuring the initial water content or unfreezable water content will cause 3% of deviation in predicting the freezing time.

The tolerance of deviations for the initial product density values appears to be the

Table 1 Maximum allowable deviations in measuring product properties from the actual values for the stated variations in the prediction of freezing time. Based on carrots with initial product temperature of 10°C, the freezing medium temperature of –75°C, the surface heat transfer coefficient of 70W/m^{2}°K and the product diameter of 2cm.

Product Properties	Range	Units	Variations of Freezing Time			
			±1%	±2%	±3%	±4%
Initial Freezing Point	–0.1 to –0.5	deg C	±0.013	±0.025	±0.038	±0.05
Initial Freezing Point	–0.5 to –1	deg C	±0.04	±0.08	±0.12	±0.16
Initial Freezing Point	–1 to –2	deg C	±0.02	±0.4	±0.6	±0.8
Initial Freezing Point	–2 to –3	deg C	±0.3	±0.6	±0.9	±1.2
Initial Water Content	0.75 to 0.90	g-water/g-product	±0.03	±0.06	±0.10	±0.13
Unfreezable Water Content	0 to 0.1	g-water/g-prodcut	±0.04	±0.07	±0.10	±0.14
Initial Product Density	1.0 to 1.1	g/cm^3	±0.009	±0.018	±0.027	±0.036
Initial Thermal Conductivity	0.45 to 0.55	W/m°K	–	–	–	–

smallest. A 0.036 g/cm^3 deviation in measuring the initial product density will cause 4% of variation in predicting the freezing time. The slope of the curve in Figure 11 is zero which shows that the influence of the inaccuracy in measuring the thermal conductivities is very small in the range of 0.45 to 0.55 W/m^oK.

Comparing the temperature history of freezing with various values of initial thermal c onductivity indicates that at any freezing instant, the product with a higher initial thermal conductivity has a higher surface temperature and a lower center temperature. In addition, the mass average enthalpy values at any instant are almost the same. The consistency of the values of mass average enthalpy explains the close freezing time. The mass average enthalpy is the criterion to terminate the freezing process. The freezing process will be stopped once the product reaches the mass average enthalpy of that equivalent to the temperature of -25^oC.

Figures 12 and 13 illustrate the influence of freezing conditions on the freezing ti me for the various products. At freezing medium temperatures below -200^oC, reduction in the freezing medium temperature has a small effect on the freezing time. However, at freezing medium temperatures above -100^oC, the appropriate selection of coolant with a lower boiling temperature will decrease the freezing time significantly. The influence of the surface heat transfer coefficient on the freezing time is very large at the values of surface heat transfer coefficient lower than 100 $W/m^{2o}K$. Above the value of 200 $W/m^{2o}K$ for the surface heat transfer coefficient, an increase in surface heat transfer coefficient has a very small influence on the freezing time. The results in Figure 14 demonstrate the increase in freezing time for product with larger diameters. The freezing time is almost doubled with a two-fold increase in the product diameter. Although the total heat content to be removed is proportional to the mass (or approximately the volume) of the product, the exposed surface area of the product to the freezing medium is also increased. The net

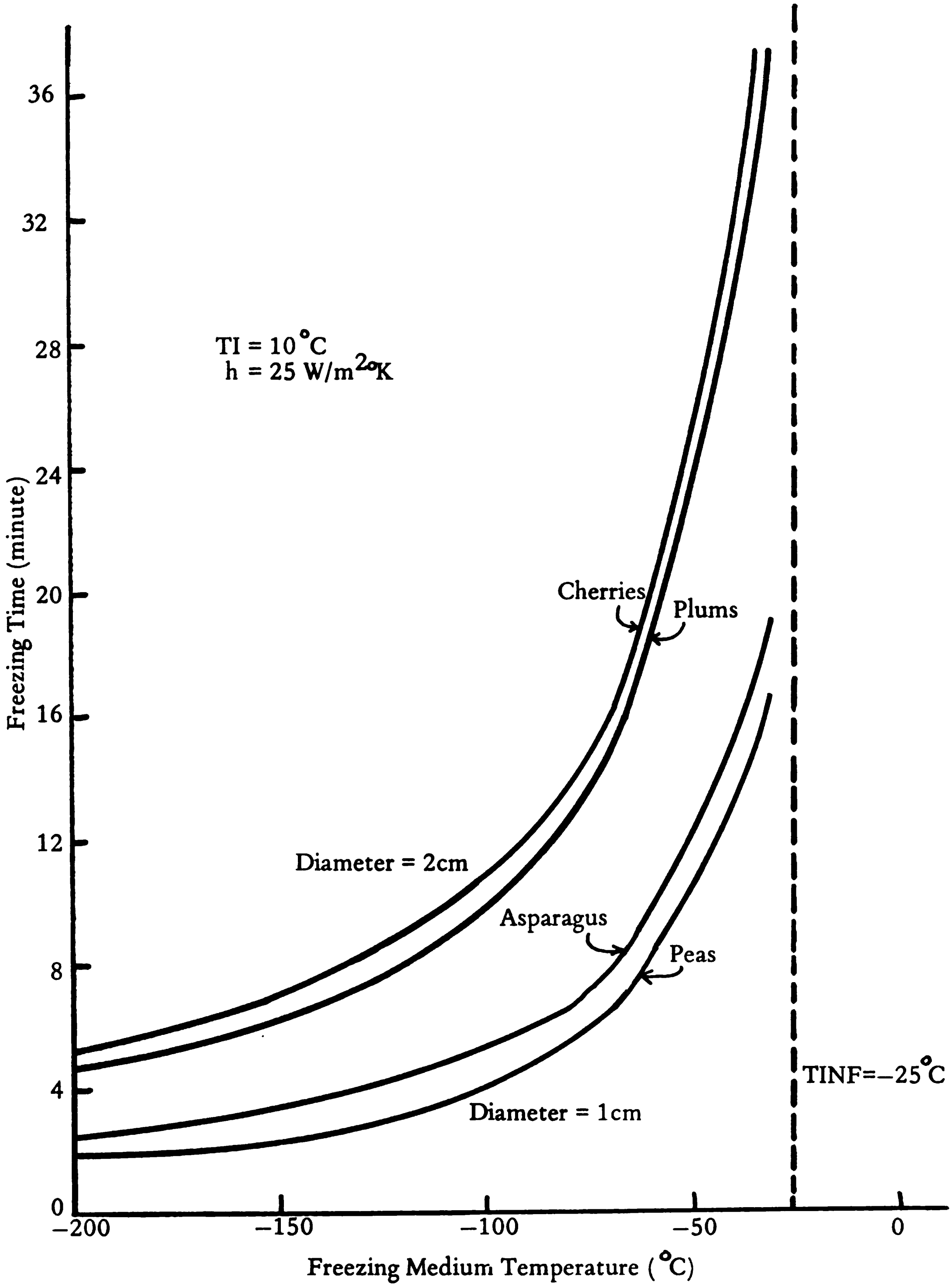


Figure 12. Predicted freezing time versus freezing medium temperature for various food products of different product diameters.

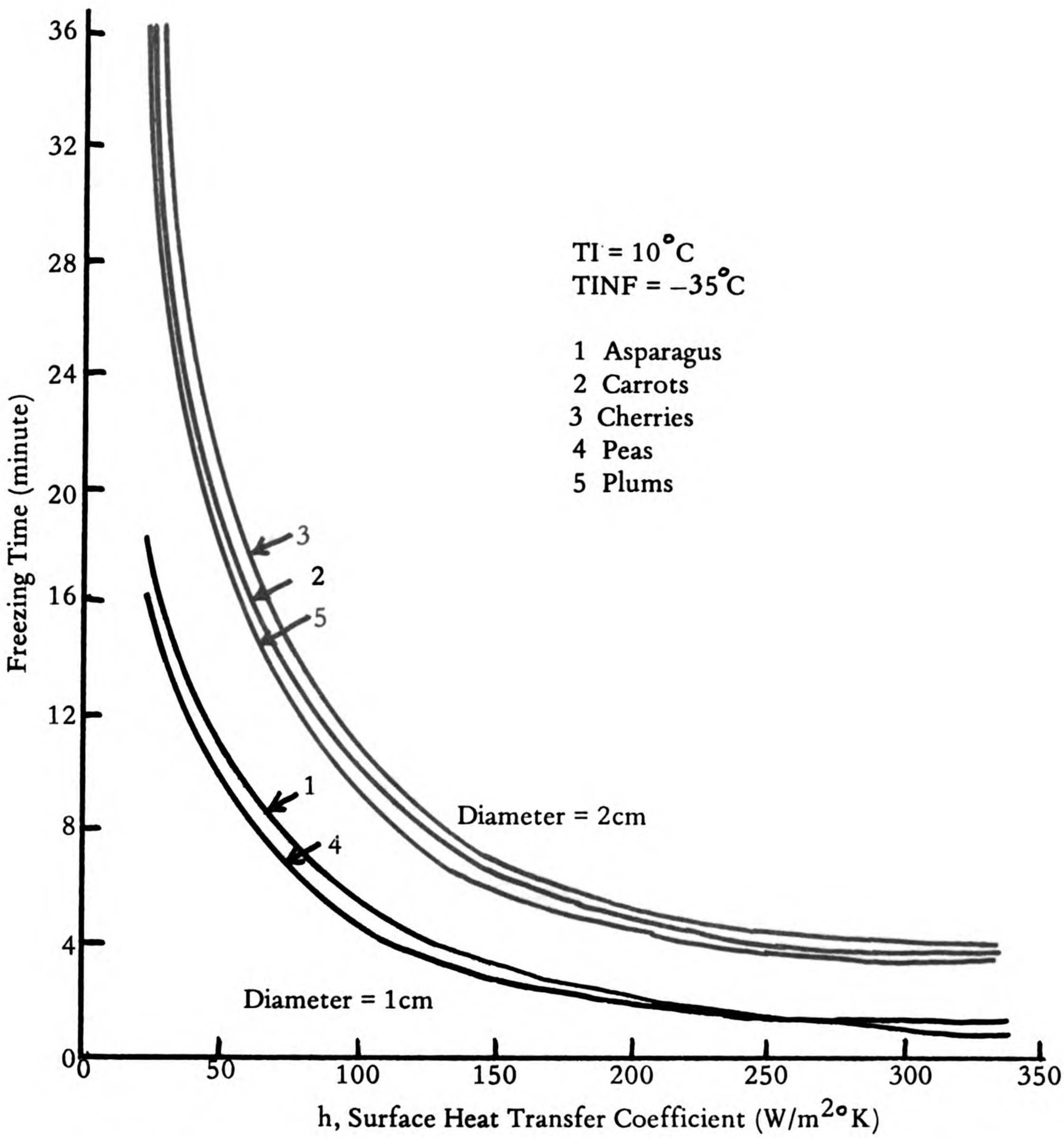


Figure 13. Predicted freezing time versus surface heat transfer coefficient for various food products with different product diameters.

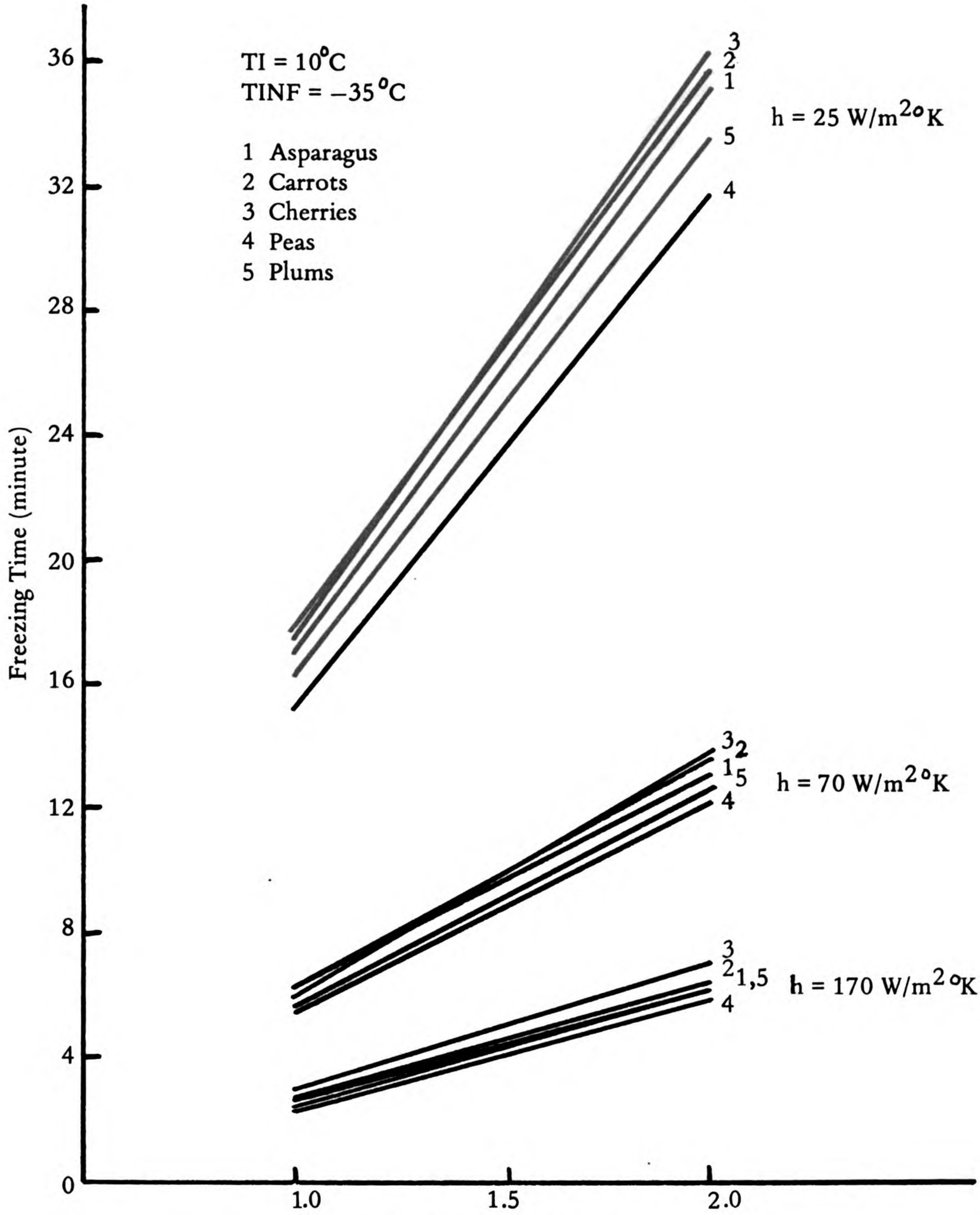


Figure 14. Predicted freezing time versus product diameter for various food products at different surface heat transfer coefficients.

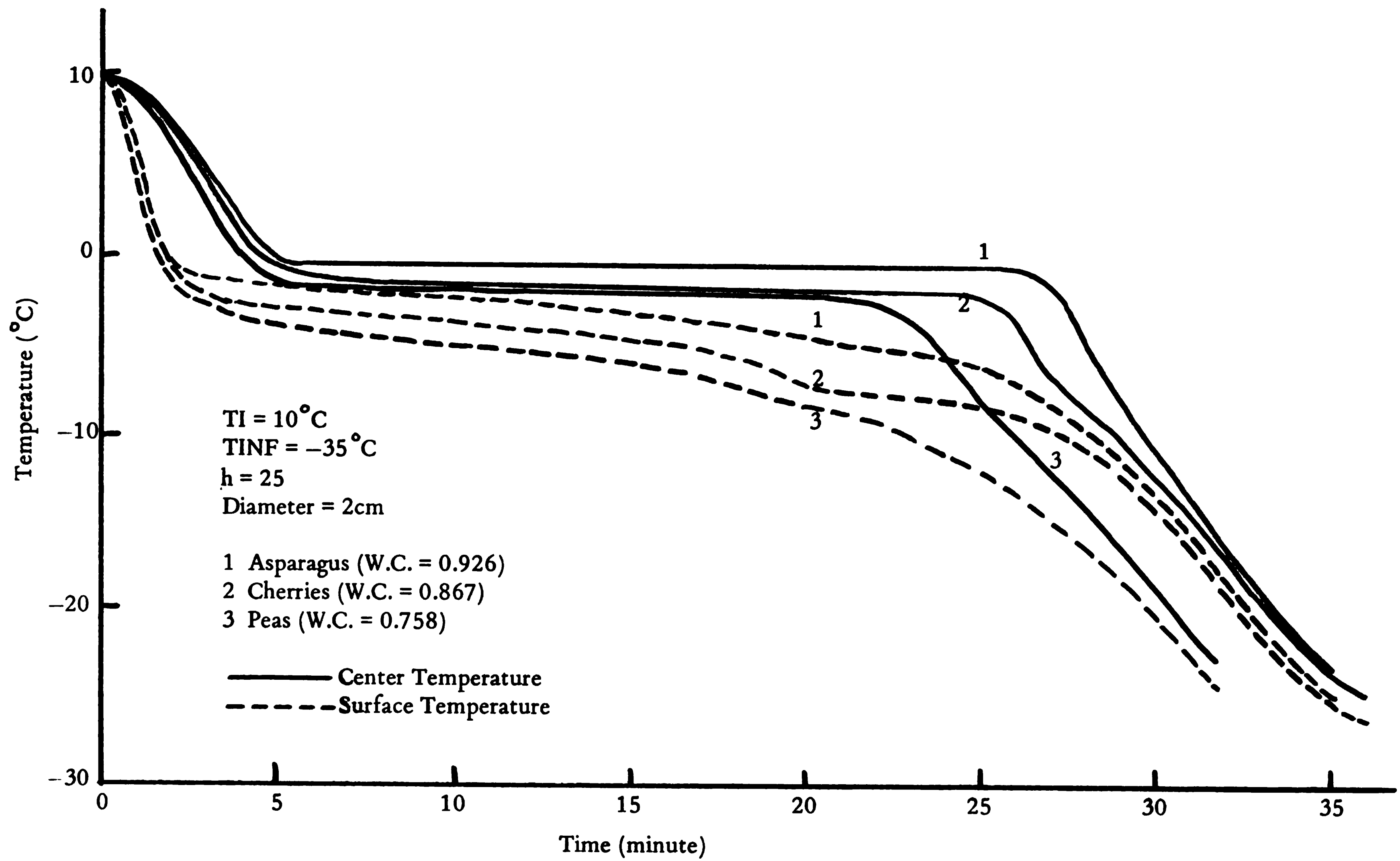


Figure 15. Predicted temperature history for different food products.

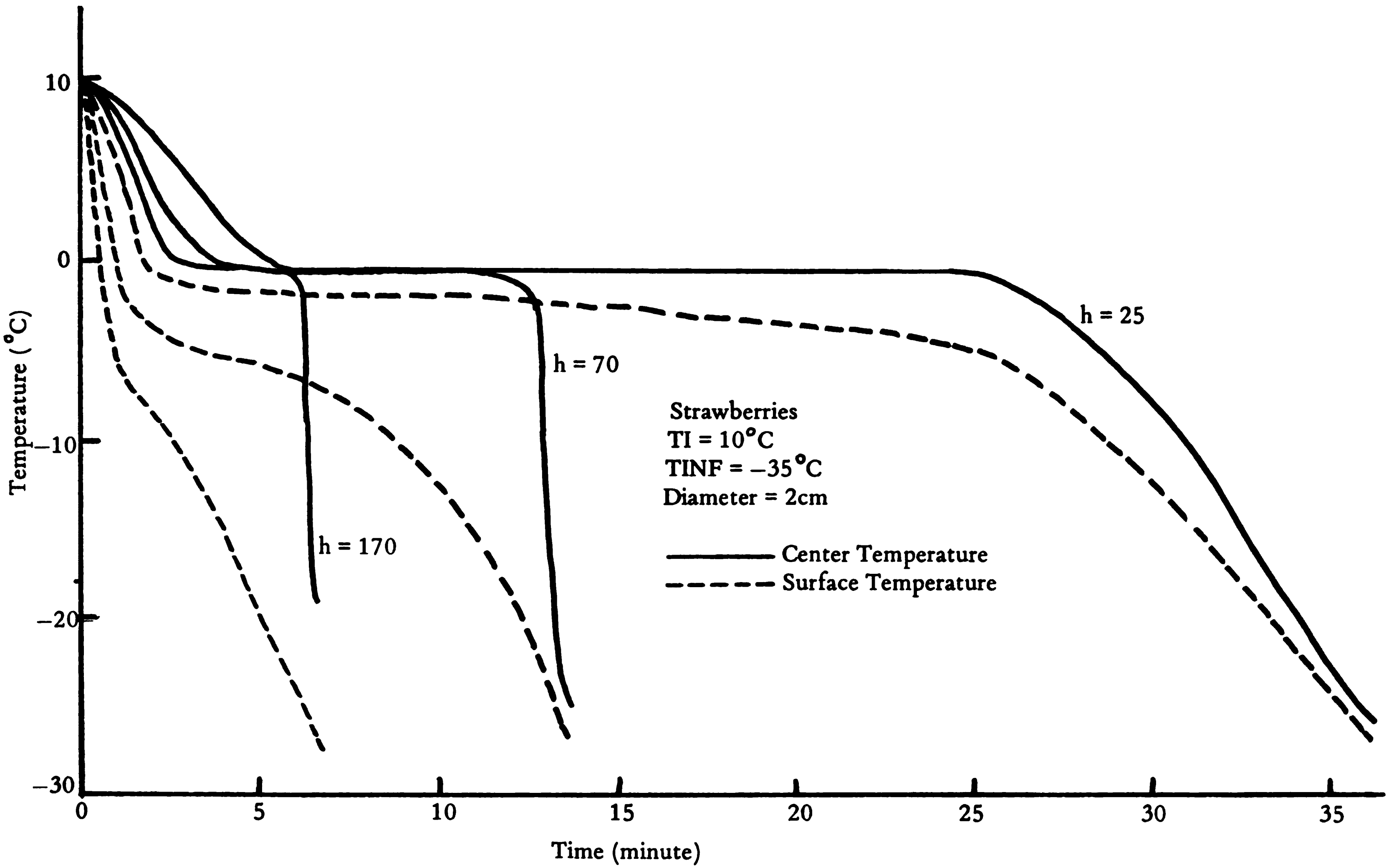


Figure 16. Predicted temperature history of strawberries freezing at different values of surface heat transfer coefficient.

effect shows that the freezing time is approximately proportional to the diameter of the product.

Two plots of freezing history were shown in Figures 15 and 16. Figure 15 compares the freezing history of asparagus, cherries and peas which have different initial water contents (asparagus = 0.926, cherries = 0.867 and peas = 0.758). The center temperature of asparagus stays in the horizontal region for a longer period compared to the other two products, and a longer freezing time was expected for asparagus. Figure 16 shows the influence of surface heat transfer coefficient on the temperature history of strawberries. At a lower value of the surface heat transfer coefficient, the product center temperature stayed at the cryoscopic temperature for a longer period. The time that the center temperature stayed at initial freezing temperature covers almost one half of the total freezing time. The horizontal portion of the curve was reduced as the temperature history curve was moving toward the center of the product and eventually approached a continuous curve when the surface heat transfer coefficient was very large. The influence of the latent heat removal on the surface temperature was not very clearly shown on the temperature history curve, yet the convex shape of surface temperature history curves was recognized as the retardation of the latent heat removal during water crystallization. This was illustrated by equation (4) such that at very low temperatures, there is still some water unfrozen. The removal of latent heat of unfrozen water retards the temperature drop.

The influences of several product properties on the freezing time for the actual food products is demonstrated in Figures 17, 18, 19 and 20. The combined effect of these properties on the freezing time is complicated. However, the trend of curves shown in Figures 17 through 20 were found to be consistent except for some cases where abnormally high values of the parameters were used. The unexpected longer freezing times of cherries and carrots shown in Figure 17 was due to the higher product density. A considerably

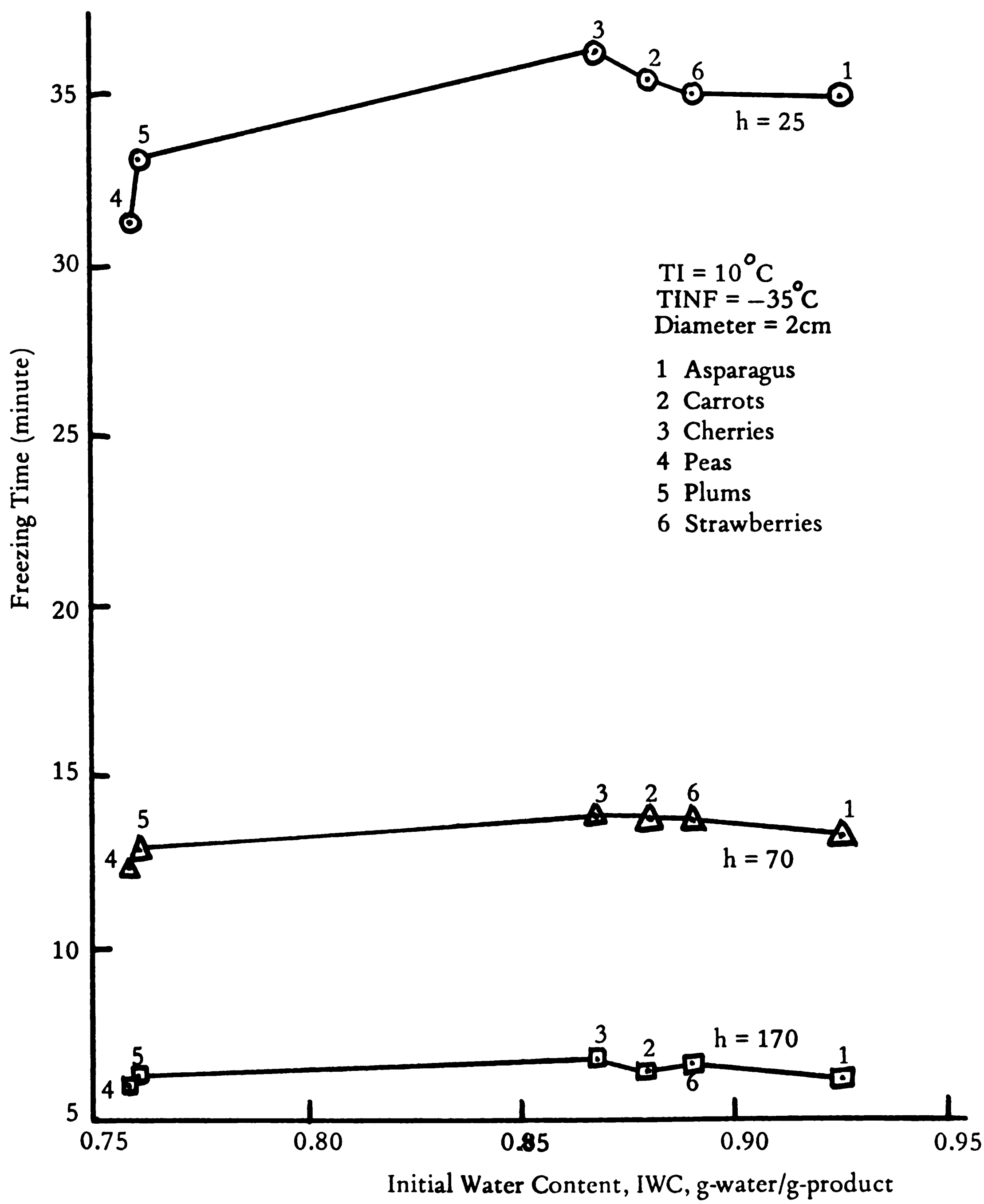


Figure 17. Predicted freezing time versus initial water content for different food products at different values of surface heat transfer coefficient.

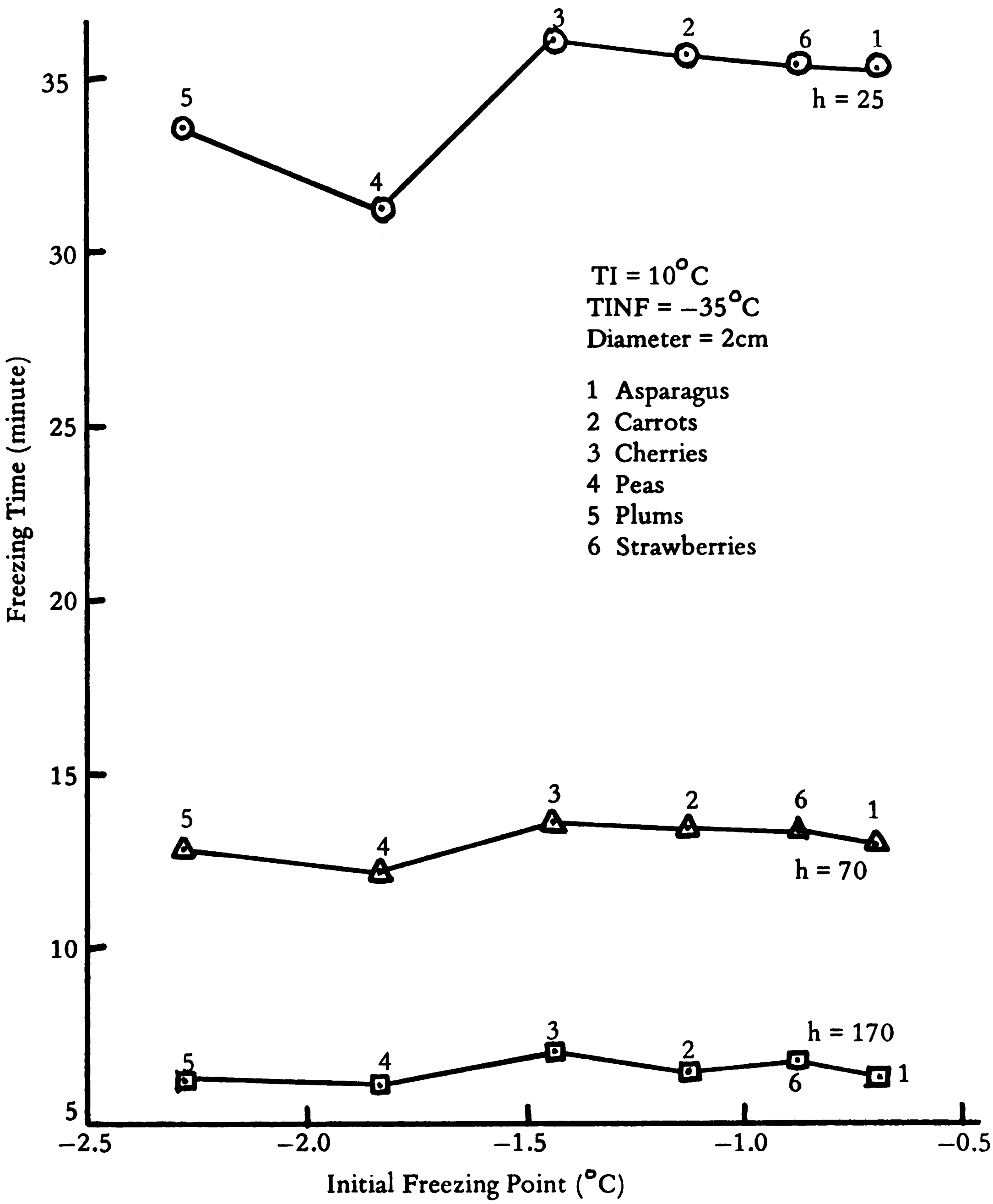


Figure 18. Predicted freezing time versus initial freezing point for different food products at different values of surface heat transfer coefficient.

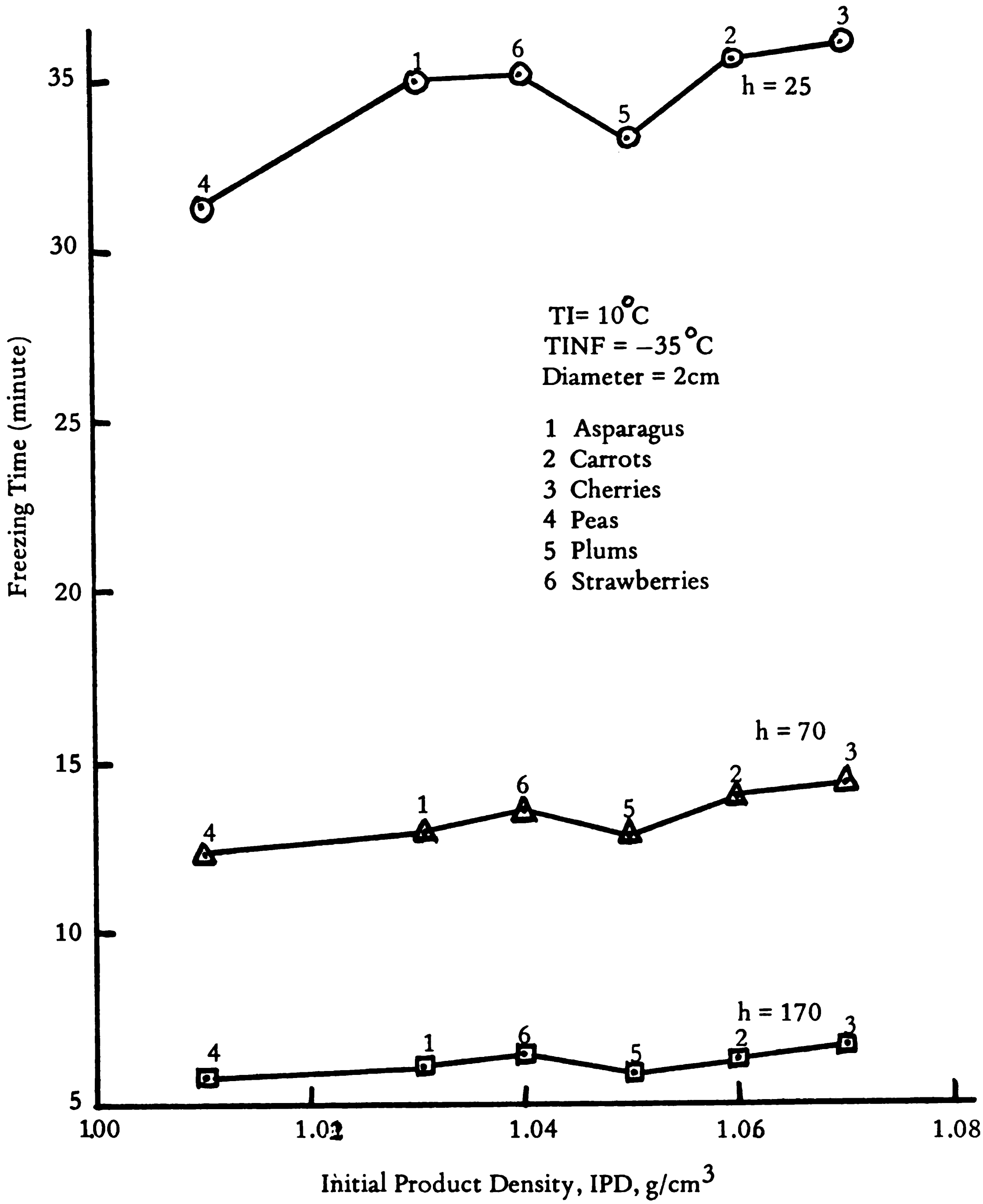


Figure 19. Predicted freezing time versus initial product density for different food products at different values of surface heat transfer coefficient.

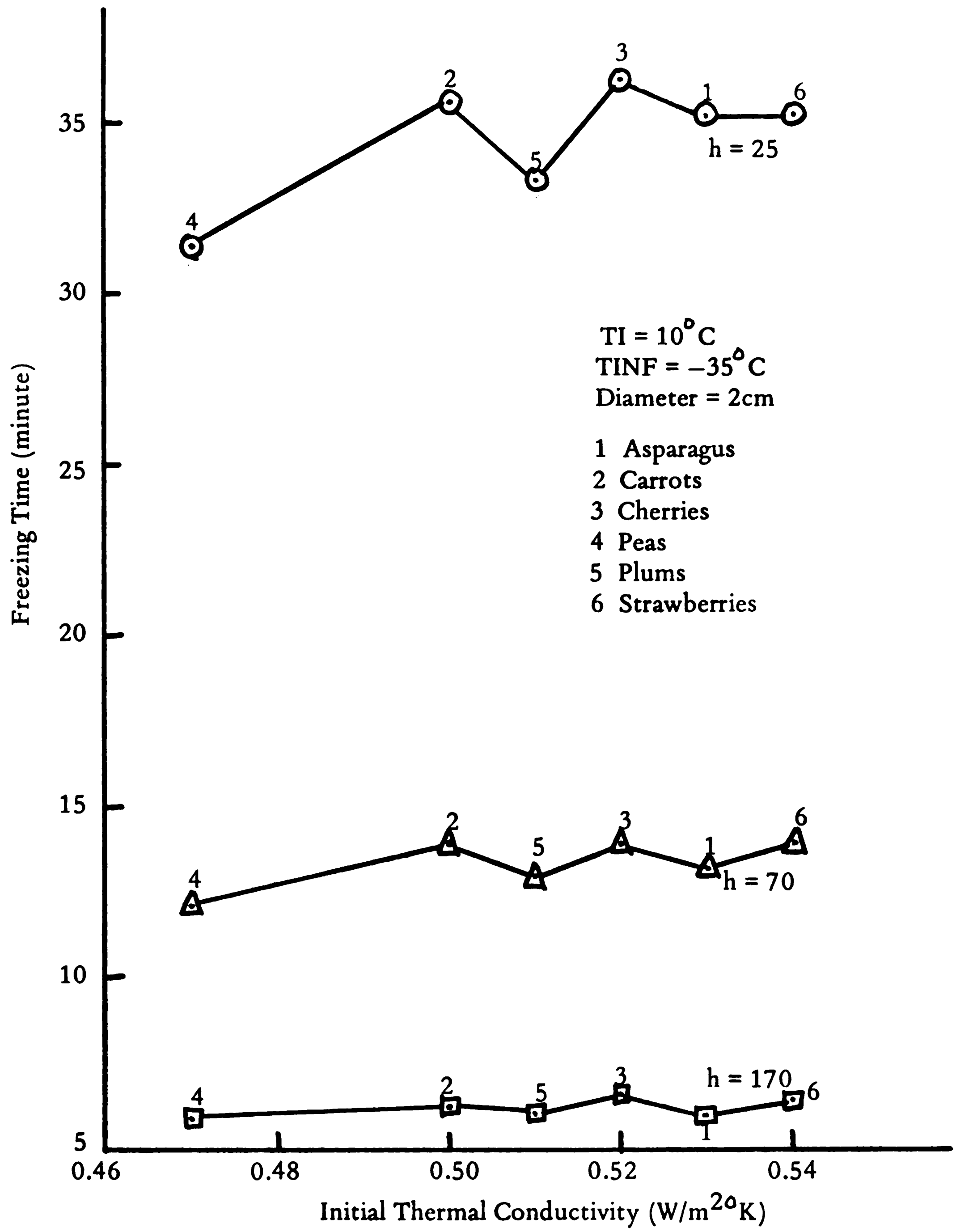


Figure 20. Predicted freezing time versus initial thermal conductivity for different food products at different values of surface heat transfer coefficient.

shorter freezing time for peas compared to that of plums is because of the lower product density for peas. Low initial water contents of plums and peas caused the shorter freezing times in Figures 18 and 19 in which the other points were consistent with the previous predictions. The small influence of initial thermal conductivity was demonstrated in Figure 20. The shorter freezing times of peas and plums were caused by the lower initial water content.

V. CONCLUSIONS

1. A computer prediction model which requires only the unfrozen product property data can be used to compare the freezing times of different food products.
1. A food product possesses a specific cryoscopic temperature where all the temperature function curves of product properties are discontinuous and nondifferential.
3. More accurate results can be obtained in predicting the freezing time by dividing the food product into more layers in the finite difference formulation. Accordingly, a longer computing time is required.
4. At freezing medium temperatures below $-200^{\circ}C$, reduction in the freezing medium temperature has a small effect on reducing the freezing time. Furthermore, above the value of 200 $W/m^{2\circ}K$ for the surface heat transfer coefficient, an increase in surface heat transfer coefficient has a very small influence on the freezing time.
5. The freezing time of a food product is a function of the initial freezing point, water content, product density and thermal conductivity. In addition, the independent influences of the above product properties on the freezing time can be predicted through the usage of hypothetical combinations of product properties.
6. A food product with a lower initial freezing point, a higher initial water content and a higher initial product density has a longer freezing time.
7. The freezing time is most sensitive to the accuracy of the measurement of the initial product density and the initial freezing point while the initial freezing point of the food product falls above $-0.5^{\circ}C$. Little influence of the accuracy of initial thermal

conductivity data is observed for fruit and vegetable products.

8. The functional curves of the influences of product properties on the freezing time for actual food products were found to be consistent with the trend predicted by the usage of independent influence analysis. Unexpected longer or shorter freezing times can be explained by the abnormally high or low values of the product properties except the one under investigation.
9. The combined inaccuracy of measuring the product property values may have a very significant effect on prediction of the exact freezing time.

VI. SUGGESTIONS FOR FURTHER STUDY

1. To experimentally verify or modify the model used in this paper to predict thermal properties and freezing time.
2. To utilize the anisotropic model for thermal conductivity in a simulation for fibrous products.
3. To modify the model to take into account the changes in product density (about 10%) and product diameter which occur during freezing.
4. To modify the model to predict thermal properties and freezing time for a slab, a cube or a cylindrical product.
5. To utilize the present model or modified model to investigate the influence of product properties on freezing rate for other food products to check the consistency.
6. To utilize this model to predict the product properties below the initial freezing point for food products of which the thermal properties above the freezing point have been experimentally measured.

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APPENDIX A

Table A–1 Values of physical properties of several food products utilized in the simulation.

PROPERTIES	UNITS	ASPARAGUS	CARROTS	CHERRIES	PEAS	PLUMS	STRAWBERRIES
Initial Freezing Point (TF)	°C	–0.67	–1.11	–1.44	–1.83	–2.28	–0.89
Initial Water Content (IWC)	g-H_2O/g-product	0.926	0.88	0.867	0.758	0.76	0.893
Unfreezable Water Content(UFWC)	g-H_2O/g-product	0	0	0	0	0	0
Initial Product Density (IPD)	g/cm^3	1.03	1.06	1.07	1.01	1.05	1.04
Initial Specific Heat (ICP)	J/g °K	3.97	3.89	3.79	3.56	3.64	3.93
Initial Thermal Conductivity (IK)	W/m °K	0.53	0.50	0.52	0.47	0.51	0.54

Table A–2 Predicted freezing times for different food products with different freezing conditions and product sizes.

TINF	$\bar{h}$	Freezing Time (Minutes)					
Freezing Medium Temperature	Surface Heat Transfer Coefficient	Product Diameter =1cm		Product Diameter = 2cm			
°C	$W/m^2\,°K$	Asparagus	Peas	Carrots	Cherries	Plums	Strawberries
–35	25	17.11	15.35	35.56	36.08	33.44	35.25
–35	70	6.45	5.80	13.69	13.72	12.90	13.62
–35	170	2.65	2.53	6.29	6.69	6.13	6.58
–35	340	1.13	1.39	3.78	3.99	4.04	3.53
–75	25	7.15	6.06	14.76	14.58	13.48	14.72
–75	70	2.44	2.23	5.40	5.15	4.79	5.02
–75	170	1.18	1.02	2.37	2.23	2.20	2.29
–75	340	0.52	0.553	1.36	1.40	1.25	0.95
–130	25	4.06	3.33	7.99	8.26	7.43	7.91
–130	70	1.51	1.07	2.85	2.57	2.36	2.78
–130	170	0.62	0.533	0.89	1.18	1.16	0.91
–130	340	0.22	0.286	0.652	0.584	0.762	0.611
–195	25	2.45	2.24	5.27	5.28	4.71	4.97
–195	70	0.87	0.788	1.51	1.62	1.65	1.72
–195	170	0.385	0.297	0.755	0.698	0.745	0.606
–195	340	0.117	0.141	0.413	0.393	0.45	0.43

Table A–3 Predicted freezing times of several food products with the product diameter of 1cm.

TINF ($^\circ C$)	h ($W/m^2\,^\circ K$)	Freezing Time (Minutes)					
		Asparagus	Carrots	Cherries	Peas	Plums	Strawberries
−35	25	17.11	17.64	17.50	15.35	16.20	16.92
−35	70	6.45	6.34	6.30	5.80	6.09	6.65
−35	170	2.65	2.90	3.00	2.53	2.78	2.79

Table A–4 Predicted freezing times of several food products with the product diameter of 2cm.

TINF ($^\circ C$)	h ($W/m^2\,^\circ K$)	Freezing Time (Minutes)					
		Asparagus	Carrots	Cherries	Peas	Plums	Strawberries
−35	25	35.12	35.56	36.08	31.50	33.44	35.25
−35	70	12.98	13.69	13.72	12.22	12.90	13.62
−35	170	6.19	6.29	6.69	5.90	6.13	6.58

Table A–5 Predicted freezing times for different food products of diameter 2 centimeter at different surface heat transfer coefficients with initial product temperature 10 °C and freezing medium temperature –35 °C. (Refer to Fig. 17, 18, 19 and 20)

Products	Density	Thermal Conductivity	Initial Freezing Point	Initial Water Content	Freezing Time (Minutes)		
					h = 25	h = 70	h = 170
Asparagus	1.03	0.53	–0.67	0.926	35.12	12.98	6.19
Carrots	1.06	0.50	–1.11	0.88	35.56	13.69	6.29
Sweet Cherries	1.07	0.52	–1.44	0.867	36.08	13.72	6.69
Tall Peas	1.01	0,47	–1.83	0.758	31.50	12.22	5.90
Small Plums	1.05	0.51	–2.28	0.76	33.44	12.90	6.13
Strawberries	1.04	0.54	–0.89	0.893	35.25	12.62	6.58

APPENDIX B

EXPLICIT NUMERICAL SOLUTIONS

AND STABILITY

The analysis of a spherical transient heat conduction problem can be simplified by cosidering heat transport being only in the radial direction. In the case of spherical food freezing, the direction of positive heat flow is defined to be pointed radially outward. Then, the energy equilibrium equation at any node n of the sphere is:

$$q_n^{in} - q_n^{out} = q_n^{stored} \qquad \text{(B1)}$$

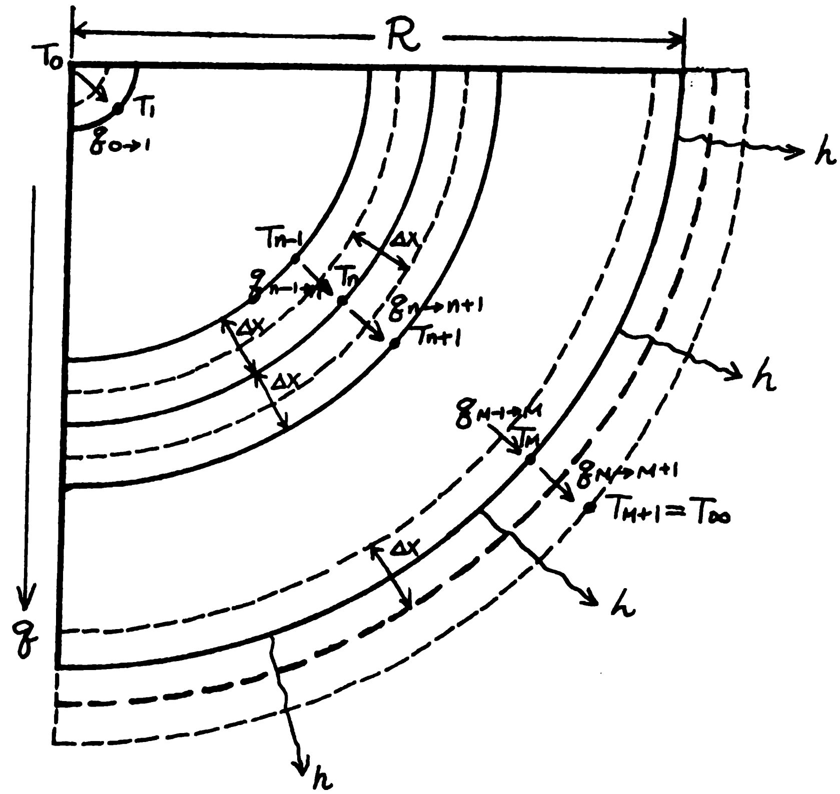

Figure B–1 Schematic representation of the spherical nodal heat transfer system of a finite difference method.

For the general interior nodes (except the center node), the above equation can be written as:

$$\left(\frac{k_{n-1} + k_n}{2} \right) \frac{A_{n1}}{\Delta r} \left(T_{n-1} - T_n \right) - \left(\frac{k_n + k_{n-1}}{2} \right) \frac{A_{n2}}{\Delta r} \left(T_n - T_{n+1} \right) = \rho_n C_n V_n \frac{dT}{d\theta} \qquad \text{(B2)}$$

where:
- T_n = nodal tempetature
- k_n = thermal conductivity of the product at node n and temperature T_n
- Δr = thickness of the shell
- A_{n1} = area of shell closer to the center
- A_{n2} = area of shell closer to the surface
- V_n = volume of the shell n
- P_n = density of the product at temperature T_n
- C_n = apparent specific heat of the product at temperature T_n
- θ = time

Since $\frac{dT}{d\theta} = \frac{T_n^{\theta+1} - T_n^{\theta}}{\Delta\theta}$, equation (B2) can be solved by the explicit method as follows

$$\frac{T_n^{\theta+1} - T_n^{\theta}}{\Delta\theta} = \frac{(k_{n-1} + k_n)A_{n1}}{2\rho_n C_n V_n} \left(\frac{T_{n-1}^{\theta} - T_n^{\theta}}{\Delta r} \right) - \frac{(k_n + k_{n+1})A_{n2}}{2\rho_n C_n V_n} \left(\frac{T_n^{\theta} - T_{n+1}^{\theta}}{\Delta r} \right) \qquad \text{(B3)}$$

Let $W_1 = \frac{(k_{n-1} + k_n)A_{n1}}{2\rho_n C_n V_n \Delta r}$, $W_2 = \frac{(k_n + k_{n+1})A_{n2}}{2\rho_n C_n V_n \Delta r}$

Equation (B3) may be simplified as:

$$\begin{aligned} T_n^{\theta+1} &= W_1\Delta\theta(T_{n-1}^{\theta} - T_n^{\theta}) - W_2\Delta\theta(T_n^{\theta} - T_{n+1}^{\theta}) + T_n^{\theta} \\ &= (1 - W_1\Delta\theta - W_2\Delta\theta)\, T_n + W_1\Delta\theta T_{n-1} + W_2\Delta\theta T_{n+1} \end{aligned} \qquad \text{(B4)}$$

To stabilize the above equation, the coefficient of T_n^{θ} should not be negative, thus

$$\Delta\theta \le \frac{1}{W_1 + W_2} \qquad \text{(B5)}$$

Since $k_{n-1} < k_n < k_{n+1}$ because of $T_{n-1} > T_n > T_{n+1}$ and the thermal conductivity of food products increases with decreasing temperature (refer to Figure 3). Besides,

$$A_{n1} < A_{n2}$$

$$V_n / A_{n2} \approx \Delta r$$

In order to minimize $\Delta\theta$, equation (B5) can be expressed as:

$$\Delta\theta \le \frac{\rho_n C_n \Delta r^2}{2k_{n+1}} , \quad n = 2, 3, ..., M \qquad \text{(B6)}$$

For the center node:

$$q^{in} = 0$$

$$q^{out} = \left(\frac{k_1 + k_2}{2}\right) \frac{A_{12}}{\Delta r} (T_1 - T_2)$$

$$q^{stored} = \rho_1 C_1 V_1 \frac{dT}{dt}$$

Taking $Y_2 = \frac{(k_1 + k_2) A_{12}}{2\rho_1 C_1 V_1 \Delta r}$, and $\frac{dT}{d\theta} = \frac{T_1^{\theta+1} - T_1^{\theta}}{\Delta\theta}$, equation (B1) for the center node can be solved by the explicit method as:

$$T_1^{\theta+1} = (1 - Y_2\Delta\theta)\, T_1^{\theta} + Y_2 \Delta\theta\, T_2^{\theta} \qquad \text{(B7)}$$

To stabilize the above calculation, it requires:

$$\Delta\theta \le \frac{1}{Y_2} = \frac{2\rho_1 C_1 V_1 \Delta r}{(k_1 + k_2) A_{12}}$$

Since

$$V_1 / A_{12} = \Delta r / 2$$

$$k_1 < k_2$$

Thus

$$\Delta\theta \leq \frac{\rho_1 C_1 \Delta r^2}{2k_2} \tag{B8}$$

Equations (B6) and (B8) can be combined as:

$$\Delta\theta \leq \frac{\rho_n C_n \Delta r^2}{2k_{n+1}} \quad , \quad n = 1, 2, 3, \ldots, n \tag{B9}$$

for the center and interior nodes.

For the surface node M,

$$q_M^{in} = \left(\frac{k_{M-1} + k_M}{2}\right) \frac{A_{M1}}{\Delta r} \left(T_{M-1} - T_M \right)$$

$$q_M^{out} = h\, A_{M2} \left(T_M - T_\infty \right)$$

$$q_M^{stored} = \rho_M C_M V_M \frac{dT}{d\theta}$$

where

h = surface heat transfer coefficient

T_∞= freezing medium temperature

Let $X_1 = \dfrac{(k_{M-1} + k_M)\, A_{M1}}{2\rho_M C_M V_M \Delta r}$, $X_2 = \dfrac{h\, A_{M2}}{\rho_M C_M V_M}$ and $\dfrac{dT}{d\theta} = \dfrac{T_M^{+1} - T_M}{\Delta\theta}$

equation (B1) for the surface node can be solved by the explicit method as

$$T_M^{\theta+1} = (1 - W_1 \Delta\theta - W_2 \Delta\theta)\, T_M^{\theta} + W_1 \Delta\theta T_{M-1}^{\theta} + W_2 \Delta\theta\, T_\infty \tag{B10}$$

To stabilize the above equation, it requires that

$$\Delta\theta \leq \frac{1}{W_1 + W_2} \tag{B11}$$

Since

$$k_{M-1} < k_M$$

$$A_{M1} < A_{M2}$$

$$V_M / A_{M2} \approx \Delta r / 2$$

Equation (B11) can be written as

$$\Delta\theta \leq \frac{\rho_M C_M \Delta r^2}{2(k_M + h\Delta r)} \qquad \text{(B12)}$$

in order to minimize the right hand term of equation (B11).

To obtain the minimum value of $\Delta\theta$, it is necessary to consider the temperature function of ρ, C and k, and take the minimum value of ρ and C, the maximum value of k. From figures 3, 4 and 6,

$$k_{max} \approx 4\, k_o$$

$$\rho_{min} \approx 0.9 \rho_o$$

$$C_{min} \approx C_o$$

where k_o, ρ_o, C_o are the product property values above the initial freezing point.

Comparing equations (B9) and (B12), $\Delta\theta$ in equation (B12) is smaller than that in equation (B9) because of $h\Delta r$ is positive and $k_M \geq k_{n+1}$, $C_M < C_n$, whereas the influence of density is very small.

This concludes our search for the minimum value of the computing time increment ($\Delta\theta$) expressed as the initial property value to stabilize the explicit finite difference solution throughout the freezing processes, and

$$\Delta\theta \leq \frac{1}{S}\left[\frac{\rho_o C_o \Delta r^2}{2\,(k_o + h\Delta r)}\right] \qquad \text{(B13)}$$

where $S \geq 5$.

APPENDIX C

UNITS CONVERSION

Temperature	$C = \frac{5}{9}(F+32)$
	$K = C + 273.15$
	$K = \frac{5}{9}(F + 459.67)$
	$K = \frac{5}{9}R$
Length	1m = ~~0.3048~~ 3.28 ft
Mass	$1Kg = 2.2046 lb_m$
Density	$1Kg/m^3 = 1 \;\; 10^{-3}\, g/cm^3 = 0.0624 \frac{lb_m}{ft^3}$
Specific Heat	$1\ J/g^{\circ}K = 0.239\ BTU/lb_m{}^{\circ}F$
Thermal Conductivity	$1\ W/m^{\circ}K = 0.578\ BTU/hrft^{\circ}F$
Surface Heat Transfer Coefficient	$1\ W/m^{2\,\circ}K = 0.176\ BTU/hrft^{2\,\circ}F$
Energy	$1\ J = 9.48 \times 10^{-4}\ BTU$
Heat Transfer Rate	$1\ W = 3.414\ BTU/hr$