

THE UTILITY OF FOURIER ESTIMATES OF GRAIN
SHAPE IN SEDIMENTOLOGICAL STUDIES

Thesis for the Degree of M. S.

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BRIAN REDMOND

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ABSTRACT

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By

Brian Redmond

Although grain shape variation has always been considered an important sedimentological variable it has not been successfully used in the solution of geologic problems. Results of recent research (Ehrlich & Weinberg, 1969) have indicated that use of a novel mathematical model of shape should permit shape to be utilized successfully. This paper concerns an evaluation of the Ehrlich-Weinberg shape variable in terms of its functional behavior and in a study of local provenance.

The results of the investigation show that there is no detectable correlation between size and shape in quartz as measured by the Fourier series for a size range of .1 to .3 mm mean radius. Standard deviation is not independent of the shape variable, the harmonic amplitude. High correlations between certain harmonic, especially the sixth and ninth harmonics, indicate possible structural control of quartz shape. The movement of sediments from tributary creeks to the Red Cedar River in central Michigan can be detected through roughness coefficient patterns. Thus the present results indicate that the Fourier series technique gives more promise for successful provenance work than any heretofore suggested.

THE UTILITY OF FOURIER ESTIMATES OF GRAIN SHAPE IN
SEDIMENTOLOGICAL STUDIES

By

Thesis
Brian Redmond

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INTRODUCTION

It is paradoxical that, although shape is one of the most prominent characteristics of matter in nature, the study of shape has received so little development in the geologic sciences. Such lack of rigor is not typical of the other sciences. For instance, thousands of years ago Euclid achieved fame when he founded a branch of mathematics based on shape. Modern-day chemists receive Nobel prizes for defining the shape of a molecule. Physicists and astronomers are able to so precisely compute projectile orbits that men have traveled to the moon's surface and back. Can it be that shape holds no significance for the geologist? Is the shape of a small particle devoid of meaning? Or is it that the shape of this small particle is not adequately described?

It is not for lack of appreciation of the utility of shape in solving geologic problems that geologists have failed to make better use of the shape parameter. Numerous geologic papers have been published dealing with this parameter. Unfortunately, the potential of shape to serve as a useful variable has remained unrealized.

Until recently, the shape of small particles has been described in terms of sphericity and roundness. Sphericity is a function of the ratio of particle surface area to volume. However, surface area, especially of a small and rugged sand grain, is quite difficult to measure. Such difficulty has given rise to various operational definitions of sphericity. These definitions usually involve the easier to measure circumscribed and inscribed circles of the grain projection (Riley, 1941), (Wadell, 1935), (Tickell, 1931), (Pentland, 1927), (Cox, 1927) or circumscribed spherical volume and actual volume (Krumbein &

Sloss, 1963), (Krumbein, 1941), (Wadell, 1933). Roundness is defined as being a measure of the change in the radius of curvature of segments of the particle's projection perimeter (Flemming, 1965). Some people feel that, together, sphericity and roundness might adequately describe grain shape but such is not the case.

The use of sphericity or any other regular polyhedron as a reference form in a shape approximation assumes that an irregular shape can be adequately approximated to that regular shape. To be sure, the sphere, rectangle, or any other regular shape is aesthetically pleasing and much easier to work with. The sphere can be quickly described with a radius, the rectangle by two orthogonal axes. However, few particles, unless strongly controlled by cleavage, approach or maintain the shape of a regular polyhedron. An adequate grain shape description should enable one to reconstruct the shape of the grain and this, one cannot uniquely do with just sphericity and roundness. A frankfurter, for instance, could be reconstructed from sphericity and roundness indices but the frankfurter would not be a unique solution for these indices. The final proof of the inadequacy of sphericity and roundness is in the singular lack of success in using such shape approximations to aid in the solution of geologic problems. Roundness and sphericity were differentiated by Robertus Anglicus in 1271 and little progress has been made since (Krynine, 1956).

Recently, a new approach to the problem of shape description was advanced using a Fourier series approximation (Ehrlich & Weinberg, 1969). Such an approximation, unlike previous shape statistics, does not reference the shape of a grain to a regular polyhedron. Moreover and most important, using the grain shape equation, the grain shape can be reproduced to within any precision. Grain shape so described has been used in the solution of a geologic problem such as the identification of different soil horizons within an Alaskan soil profile through an analysis of the shapes of the quartz grains in each horizon (Ehrlich & Weinberg, 1969).

It is the purpose of this paper to use the above method to investigate the nature of the shapes of quartz grains, to evaluate the utility of the method in a study of sediment provenance, and to examine empirical relationships among variables generated by the Fourier series and between these variables and grain size. Quartz was chosen because it is quite resistant to both chemical and mechanical weathering, because it has poorly expressed cleavage (if any at all), and because it is ubiquitous.

METHOD

The method used is the Fourier series shape approximation. For a detailed discussion of the method the reader is referred to Ehrlich & Weinberg (1969). The following description is largely taken from that source.

In general, grain shape is estimated by an expansion of the periphery radius as a function of angle about the grain's center of gravity by a Fourier series. The radius is therefore given by:

$$R(\theta) = R_0 + \sum_{n=1}^{\infty} R_n \cos(n\theta - \phi_n)$$

where θ is the polar angle measured from an arbitrary reference line. The first term in the series, R_0 , is equivalent to the average radius of the grain in the plane of interest. For the remainder of the terms, n is the harmonic order, R_n is the harmonic amplitude, and ϕ_n is the phase angle. The n 'th harmonic contributes to the shape of the particle a figure with n 'bumps'. That is, the 'zeroth' harmonic is a centered circle with an area equal to the total area, the first harmonic is an offset circle, the second is a figure eight, the third a trefoil, and the fourth a four-leaf clover. The center of gravity of the shape is chosen as the origin of the radius expansion to simplify interpretation of the generated series.

Coordinates of points on the periphery of the grain are necessary for expansion. At least twice the number of points must be known as the number of the highest harmonic. The initial origin of the periphery may be arbitrary because later in the analysis a transformation will place the origin at the grain center.

SAMPLING DESIGN

The sampling area is located in northeastern Ingham county just east of Lansing, Michigan. The object of the sampling was the upper Red Cedar River and one of its tributaries, Coon Creek (Figure 1). All samples were taken from the river and creek sediment. Sampling was conducted on the Red Cedar covering about 17 miles of river and on its tributary, Coon Creek from its headwaters to its mouth, a distance of at most five miles. Even within this small area, there is a great variety of glacial-fluvial materials laid down in the Pleistocene and soils developed from them. This study is planned to test whether that variation, which is slight geologically, is expressed by concomittant shape variation of quartz grains. The samples presumably constitute sediment in transport.

Two Red Cedar samples, one upstream and one downstream of the point where Coon Creek enters the Red Cedar, give an indication of the immediate effect of Coon Creeks contribution to the Red Cedar. At a point near the headwaters of Coon Creek, samples were taken from both the finer sediment located on the creek floodplain and from the coarser sand in the river scour. Being from the same location, these two samples can be compared to see if there is any relationship between grain shape and grain size.

A more general comparison can be made between the grains from all of the samples. Inter-relationships between harmonics can be examined. If there is a relationship between two harmonics, it would indicate that detrital quartz has a preferred shape. The number of grains for each sample required to attain a given precision for a given harmonic can also be computed.

PROCESSING

As a first step in processing, the samples are poured into a large mouthed plastic container and elutriated and decanted repeatedly with warm water in order to remove as much organic matter, silt, and clay as possible. The residue is transferred to a small glass beaker whereupon dilute hydrochloric acid with trace amounts of SnCl_2 is added. The beaker is then gently heated. The HCl will act to remove iron stains as well as any carbonate from the sample without altering the quartz grain shape. The presence of SnCl_2 will increase the rate of removal of any iron stain on the quartz. The process is repeated until the yellow stain in the solution indicating dissolved ferric ion no longer appears. The sample is then washed and dried and mounted on standard glass slides.

The projection area of the grains is obtained by means of a microprojector which linearly magnifies the image two-hundred times. Quartz is easily distinguished from other minerals by the use of polarized light in the microprojector. The image is traced onto a piece of paper and then taken to a digitizer which determines the coordinates of peripheral points and automatically punches these points onto Hollerith cards. The average number of peripheral points per grain is 70 points. The Hollerith cards are then submitted to either a CDC 3600 or CDC 6500 computer with the appropriate program.

PRECISION

Precision concerns the closeness of repeated measurements of the same parameter of the same sample. In this study there are two things which can be considered repeated measurements, the measurement of the shape of each individual grain and the composite shape of a multi-grain sample. Also of interest is the precision of two samples from the same location.

Defining the level of precision at each level, variation within a single grain, within a multi-grain sample, and between samples, is essential for determining the amount of real variation which can be detected. The greater the precision, the greater the resolving power of the analysis. Since precision is a limiting factor on the amount of visible information, if any, determination of precision at all levels will isolate the level with least precision so that efforts may be made toward increasing that precision and thereby the visible information at all levels. The number of grains per sample required to attain a given precision can also be calculated.

Up until this time, little data on the precision of the Fourier series shape approximation method was available except for a few measurements using Mahalanobis D by Ehrlich & Weinberg (1969). The analysis of precision in this paper will make up part of this lack and will give an indication of the applicability of the method.

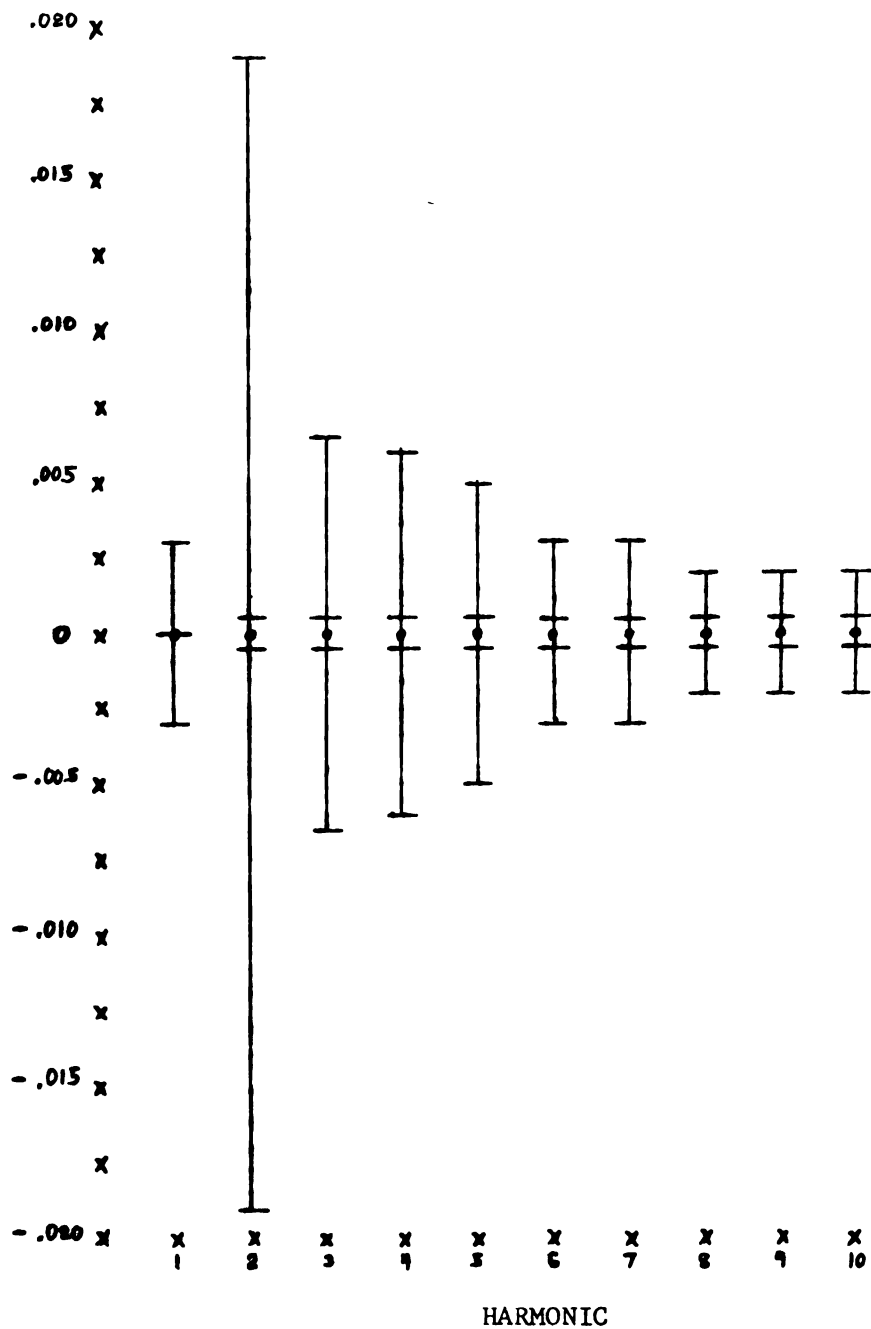
INDIVIDUAL GRAIN VARIATION VS. MULTI-GRAIN SAMPLE VARIATION

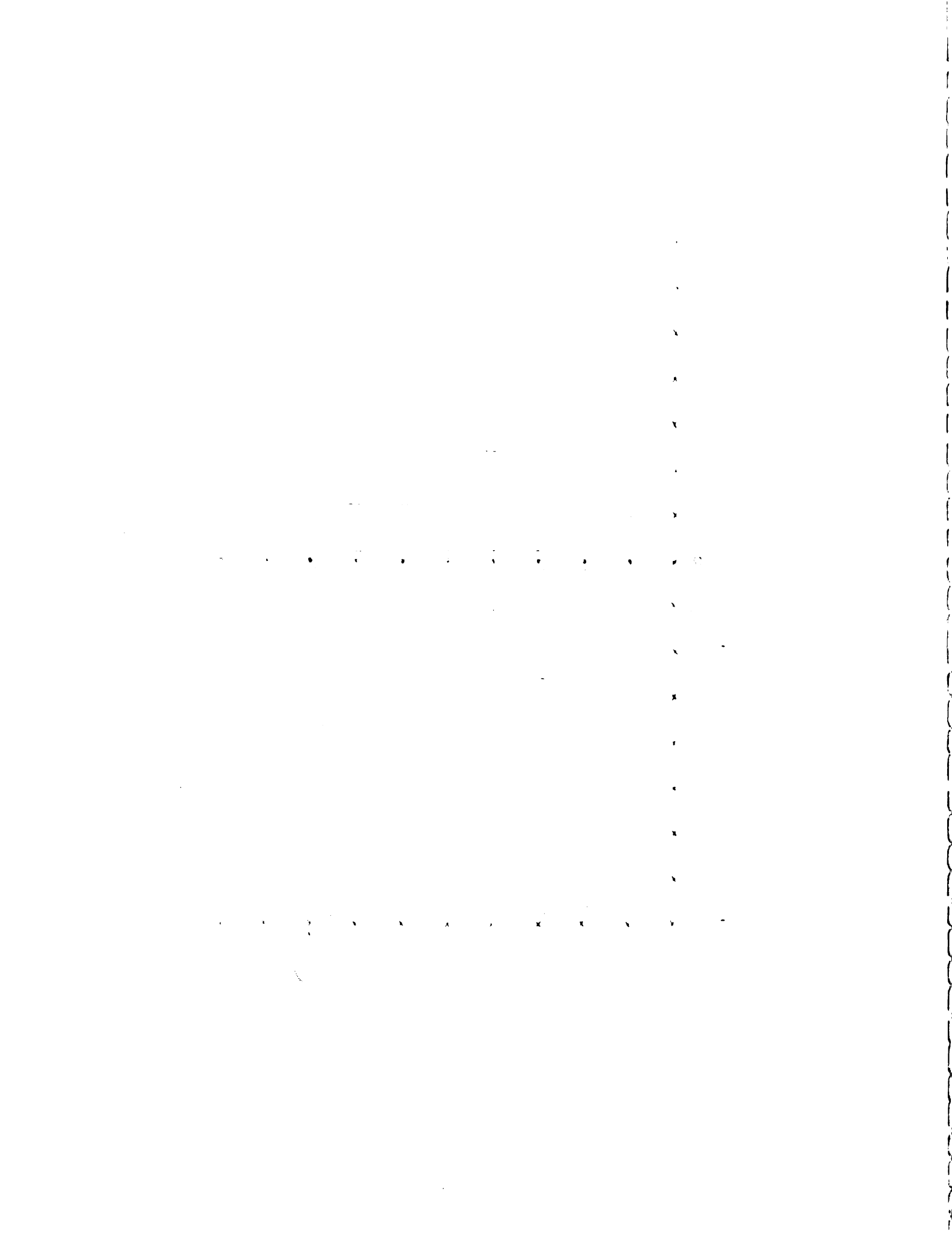
One of the 33 grains from sample B8 was chosen at random and duplicated or redigitized eleven times in order to determine the amount of induced variation in that single grain caused by measurement imprecision. Histograms of the spread of the harmonic amplitudes for the first ten harmonics were calculated (Appendix A). From the standard deviations, confidence intervals of .95 were calculated for each harmonic amplitude. This means that under repeated sampling with .95 confidence intervals constructed for each sample, about 95 out of 100 intervals will include the parametric value.

Histograms of the harmonic amplitudes were also made for the sample as a whole (Appendix B). Confidence intervals of .95 for the entire sample were calculated. The precision of the single grain can now be compared against the precision of the sample (Figure 2). As expected, the precision of the single grain is always greater than the precision of the sample since the sample variance contains as a component the within grain variation. It will be noticed that as the harmonic number increases, the within grain variation component ranges from negligible at the first harmonic to one fourth the variance of the multi-grain sample at the tenth harmonic. For the levels of precision of the individual grain and of the whole sample, it would seem that, up to the tenth harmonic, the imprecision of the sample mean is the limiting factor. An increase in the sample precision would result in an overall increase in the amount of visible information.

The sample size needed for a given precision can be determined. The estimated standard deviation is based on the amount of variation seen in the harmonic amplitude values for all samples and all harmonics from the eight Red Cedar River samples (Table 1). The amount of significant variation visible is

HARMONIC
AMPLITUDE
CONFIDENCE
INTERVAL

[illegible]



RED CEDAR RIVER
TABLE OF VALUES
TABLE 1

SAMPLE → PARAMETER ↓	B1	B2	B3	B4	B5	B6	B7	B8	AVERAGE
DISTANCE IN MILES	0	5.3	8.2	11.4	11.8	12.6	13.8	17.2	
1 SD CI.95	.011	.009	.007	.015	.009	.009	.009	.011	
1 MA CI.90	.014	.013	.010	.018	.012	.011	.012	.012	.0186
2 SD CI.95	.091	.079	.065	.083	.067	.077	.086	.064	.019
2 MA CI.90	.156	.183	.140	.171	.143	.150	.148	.127	.1523
3 SD CI.95	.041	.045	.041	.051	.034	.052	.036	.011	.013
3 MA CI.90	.085	.075	.059	.089	.079	.080	.070	.071	.0760
4 SD CI.95	.022	.028	.027	.028	.024	.028	.021	.020	
4 MA CI.90	.046	.047	.044	.048	.046	.043	.041	.018	.0454
5 SD CI.95	.018	.020	.020	.020	.014	.018	.016	.016	
5 MA CI.90	.034	.037	.032	.036	.029	.031	.029	.030	.0323
6 SD CI.95	.014	.013	.008	.013	.011	.015	.010	.011	
6 MA CI.90	.024	.024	.021	.024	.021	.023	.021	.025	.0229
7 SD CI.95	.009	.010	.012	.012	.007	.012	.007	.010	
7 MA CI.90	.017	.017	.017	.018	.017	.016	.016	.017	.0169
8 SD CI.95	.007	.009	.009	.007	.006	.009	.006	.006	
8 MA CI.90	.013	.016	.013	.014	.012	.015	.012	.014	.0136
9 SD CI.95	.006	.004	.007	.007	.006	.007	.005	.007	
9 MA CI.90	.010	.010	.012	.011	.012	.011	.009	.010	.0106
10 SD CI.95	.004	.005	.004	.004	.004	.006	.004	.006	
10 MA CI.90	.009	.009	.008	.008	.008	.010	.009	.009	.0088
N	31	37	26	37	39	30	32	33	33.1

HARMONIC NUMBER

SD = STANDARD DEVIATION MA = MEAN AMPLITUDE OF HARMONIC
CI = CONFIDENCE INTERVAL AT EITHER 90 OR 95% N = NUMBER OF GRAINS IN A SAMPLE

limited by intrasample precision. That is, if the standard deviation in harmonic amplitude among grains is .010, one could not expect to see a significant intersample variation of similar magnitude - it would be hidden in the intrasample variation. The intersample precision is related to the square root of the sample size - if the precision is to be doubled, the sample size will have to be increased fourfold.

Taking the eight Red Cedar River samples averaging 33 grains per sample and averaging the harmonic amplitudes between the samples for each harmonic will make possible the calculation of the amount of significant intersample variation which can be seen for that sample size for a designated size of type 1 and type 2 error. Estimates involving the type 2 error can only be made when prior estimates of the population standard deviations are available. One of the purposes of this paper is to provide such estimates. A probability of a type 2 error of .5 means that if a real difference exists, it will be manifest as a statistically significant difference about half of the time. A type 1 error set at .95 signifies that with the given standard deviation and sample size; the means of two samples drawn from identical populations will be interpreted as differing significantly five times in a hundred. Utilizing, as an example, the average size of the present sample set; and the empirical estimates of the standard deviation for data concerning type 2 error; and arbitrarily setting the type 1 error at .95; the smallest statistically discernable difference between two samples can be determined. Most of the requisite calculations can be performed beforehand and a graph can be constructed from which sample size or mean difference can be determined. Such a graph (Owen, 1962, p.25) is used for determining some of the values in Table 2. It can be seen in Table 2 that the total range of the mean of harmonic amplitude is of the same order of magnitude as the significant two sample difference. Since in this case we are

TABLE 2

Detectable Two-Sample Differences and Sample Size Calculations

Harmonic #	Average Mean	Observed Range	*2-Sample Difference	Desired 2-Sample Difference	*N For Desired Difference
1	.0126	.008	.006	.002	160
2	.1523	.056	.069	.020	190
3	.0760	.040	.035	.010	190
4	.0454	.007	.021	.006	190
5	.0323	.008	.015	.004	220
6	.0229	.004	.010	.003	190
7	.0169	.002	.008	.002	220
8	.0136	.004	.006	.002	170
9	.0106	.003	.005	.002	140
10	.0088	.002	.004	.001	240

N = Sample Size

*(Owen, 1962, p. 25)

dealing with many more than two samples, a total sample range exceeding the two sample range is to be expected by chance alone. Thus, those instances in this study where the total range exceeds the two-sample range in Table 2 must be treated circumspectly. The limited observation from Table 2 that the sample range is of the same order of magnitude as the two-sample difference means that in most cases we are at the extreme limit of our precision. If differences exist in those cases, greater sample sizes will be needed to detect them.

The only harmonic that appears to be at all robust at the present precision is the second harmonic. For this reason, only the second harmonic will be considered in a study of provenance in a succeeding section.

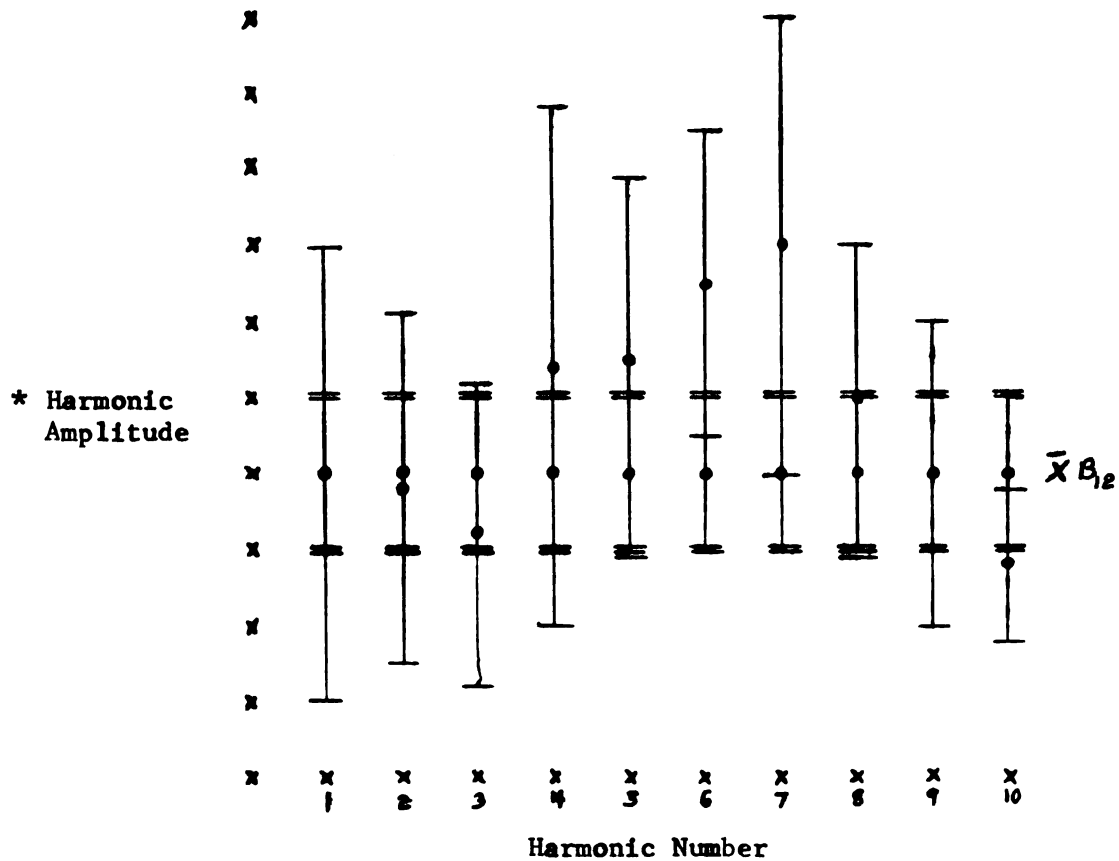
An estimate was made concerning sample sizes needed to render the other harmonics meaningful using Owen's graph. These sizes (Column 6, Table 2) are approximately 200 grains per sample.

One additional manipulatable variable not heretofore discussed is the number of peripheral points per grain. It is possible that a substantial increase in the number of points per grain might allow downward revision of the sample size estimate.

PRECISION OF TWO SAMPLES FROM THE SAME LOCATION

Two samples, B11 and B12, were taken from the same location, the headwaters of Coon Creek. In this case, it was decided to see what the maximum difference between the two samples would be; for, although from the same location, B11 was taken from the finer sediment in the floodplain of the creek and B12 from the coarser sediment in the river scour. Confidence intervals of .95 were constructed for each harmonic amplitude and the samples compared in Figure 3. Since B12 has 150 grains to the 42 grains of B11, the confidence intervals of B12 are smaller. A T-test was done on the difference in means to see if the difference was significant (Table 3). According to the test, there is no discernable significant difference between the two samples. This is graphically illustrated in Figure 3. In effect, one can justifiably assume that the two samples were drawn from the same population.

FIGURE 3
Comparison of the precisions
of two samples from the same location



single bar = .95 confidence limit of B11
double bar = .95 confidence limit of B12

* the confidence intervals and means
for the various harmonics of B12
were adjusted so that they would
all be equal

TABLE 3

T - test of the difference between the means of samples

B11 & B12

Harmonic #	(Standard Dev.) _{B11}	(Standard Dev.) _{B12}	(X ₁₂ - X ₁₁)	T
1	.010	.008	.000	.000
2	.094	.078	.003	.145
3	.040	.037	.004	.435
4	.039	.024	.004	.500
5	.018	.017	.003	.715
6	.014	.012	.005	.780
7	.011	.009	.003	1.250
8	.008	.007	.001	.556
9	.009	.006	.000	.000
10	.004	.006	.001	.910

FOR T₉₅

if T is less than 1.65 then there is
no significant difference

SIZE VS. HARMONIC AMPLITUDE

In the section of the precision of the two samples from the same location it was found that there was no discernable significant difference between the two samples and that the samples could have been drawn from the same population. The startling aspect of that finding is that although there was no shape difference detected between the two samples, there is an easily measureable size difference. The implication is that there is little or no correlation between size and shape.

To further investigate this idea that there is little or no size-shape relationship, scatter plots of the size, as measured by mean radius, versus shape, as measured by harmonic amplitude, were made for all harmonics for all Red Cedar River and Coon Creek samples (Appendix C). Correlation coefficients were calculated. The finding is that there is no significant correlation between size and shape for any sample. The conclusion is, therefore, that, at least for the area of the study and for the size range studied, .1 to .3mm mean radius, there is no discernable relationship between size and shape. Such an observation considerably simplifies shape study as no size effect need be calculated and removed.

STANDARD DEVIATION VS. HARMONIC AMPLITUDE

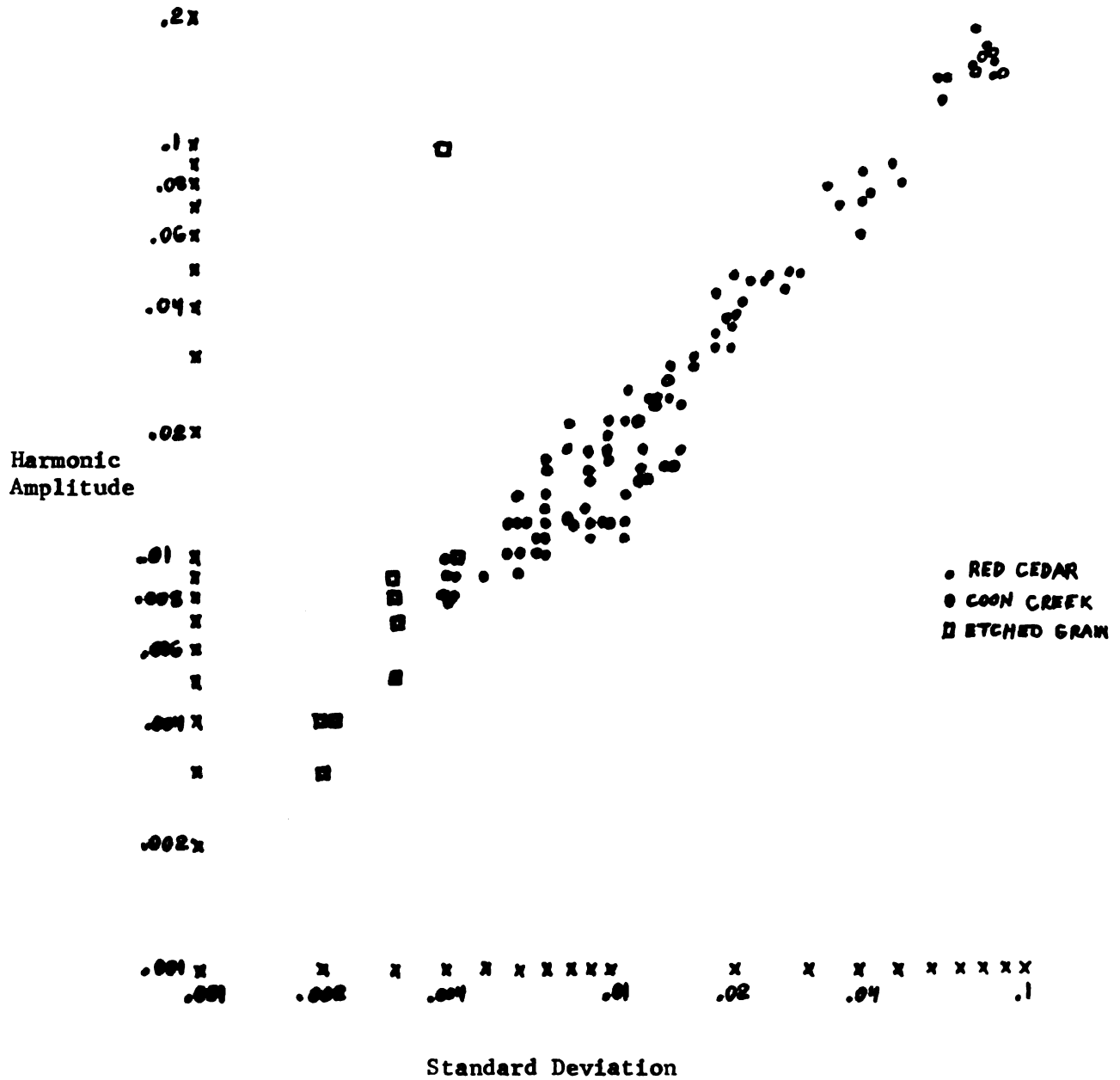
One of the necessary assumptions concerning data to be analysed by 'analysis of variance' technique is that the value of the mean is independent of that of the standard deviation. Cursory examination of the standard deviations and mean harmonic amplitudes in Table 1 suggest an exponential relationship. The two variables are therefore compared on a log-log graph (Figure 4). The various harmonic amplitudes were drawn from three sets, the Red Cedar River samples, the Coon Creek samples, and an artificially etched grain.

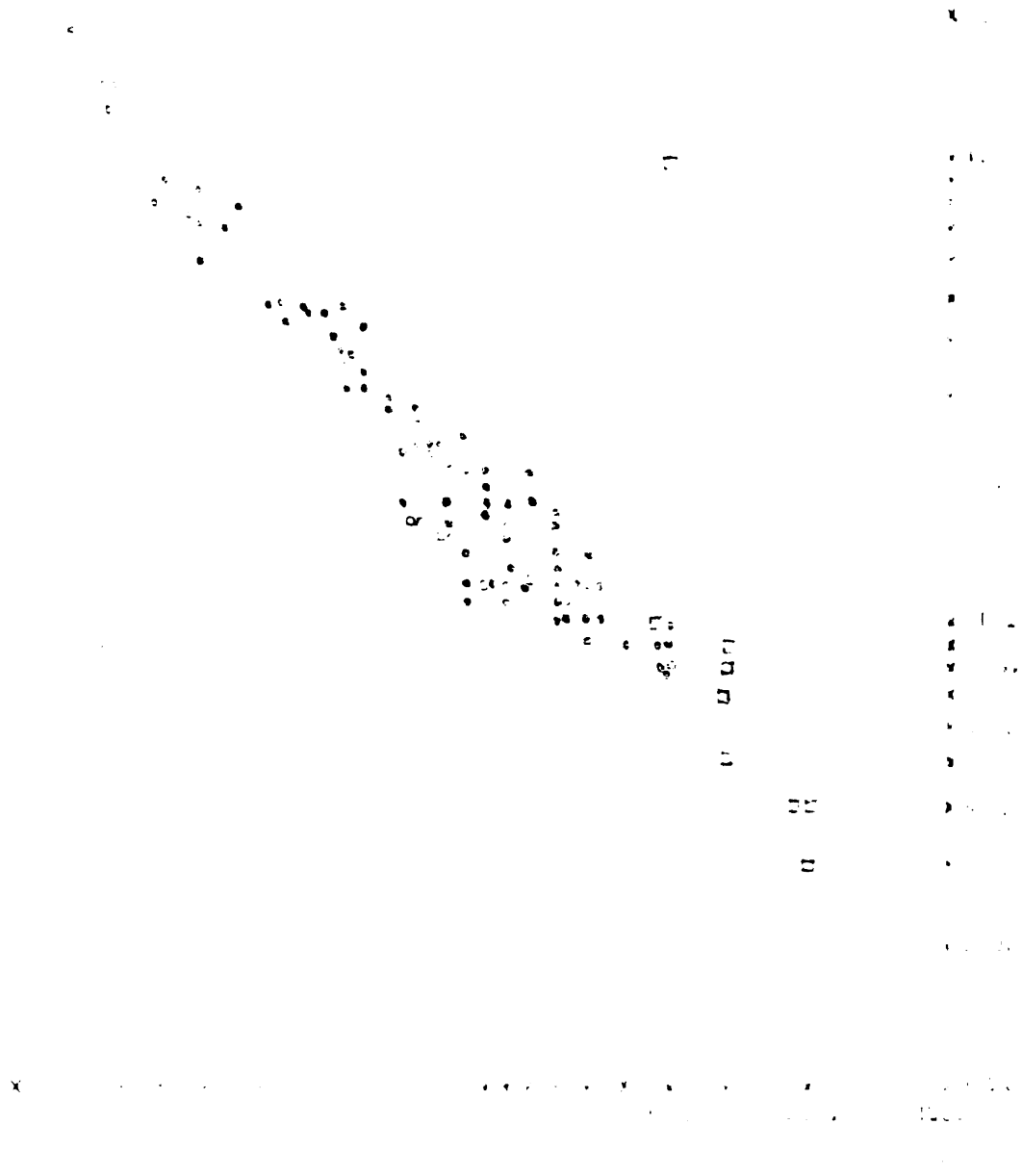
Examination of Figure 4, using the data from the Red Cedar and Coon Creek indicates a strong linear relationship between $\log$ (Standard deviation) and $\log$ (Mean harmonic amplitude). In addition, it can be seen that the linear relationship is approximately the same regardless of harmonic number. However, further inspection of the figure indicates a slightly more diffuse data array below mean amplitude values of approximately .012 and perhaps a slight lessening of slope. The mathematical and physical explanation cannot here be given with any confidence for this. However, the standard deviation can be thought to be composed of two components; one arising from intragrain variation and the other from intergrain variation.

Inspections of Tables 2 & 5 indicate that the size of the harmonic amplitudes at the higher harmonic numbers are estimated very imprecisely for their range of mean separation, many with only one significant figure. Variance achieves the same order of magnitude as the mean. This result raises the possibility that at higher harmonic numbers intragrain variation may be the principal component in the standard deviation whereas at low harmonic numbers intergrain variation is more important. This might explain the dual nature of the scatter in Figure 4.

FIGURE 4

a log-log comparison of standard deviation
versus harmonic amplitude





To test this hypothesis a single grain (the artificially etched grain used in the succeeding section) was replicated 32 times and analyzed using the Fourier shape technique. All values but one fall approximately in the lower trend. The exception, the third harmonic, has too low of a standard deviation for its harmonic amplitude, that is, that value is lower by a factor of ten than what the trend would predict. The third harmonic, however, has approximately the same standard deviation as the other harmonics of that grain. One might postulate that the distance from the third harmonic to the trend might represent the contribution of intergrain variation.

The variance can be freed from its dependence on the mean by suitable data transformation such as the arcsin square root transformation. If such a transformation is not performed, there seems little use in using the patterns of variations of both the mean and the standard deviation in an investigation.

INTERHARMONIC CORRELATIONS

Information carried by one harmonic might be duplicated by the information carried by another harmonic. To check for such possible duplication, linear correlation coefficients were calculated between harmonics (Table 4 & Appendix D). With 150 grains, any correlation coefficient exceeding .159 is considered significant at the .95 level. Ten of the 35 coefficients can be considered to arise from a nonrandom array (Table 4). However, most indicate that the linear relationship accounts for less than 5% of the total variation of the dependent variable. The linear relationship between the sixth and ninth harmonics is the highest of all the correlations, accounting for some twelve percent of the variation. Lesser correlations of from nine to twelve percent exist between: the tenth and fourth, the tenth and seventh, the third and ninth, and between the third and sixth. The presence of significant correlations between harmonics suggests that quartz possesses preferred shape modes - probably crystallographically or genetically controlled. As a preliminary test of the former hypothesis shape analysis was performed on the shape of a deeply etched quartz sphere.

Because of its structure quartz, when it dissolves does so at different rates in different crystallographic directions. A sphere of quartz subjected to some silica solvent will not become a successively smaller sphere (Fronde11, 1962). Rather the quartz will form a non-spherical solid which reflects the internal symmetry of quartz (Figure 5). The harmonic amplitude spectrum of the shape of a grain of quartz, originally spherical but subjected to prolonged etching with HF, was determined. The resulting spectrum was compared with that of the Red Cedar River samples (Table 5) and with the Coon Creek samples (Table 6). The shape of the etched grain is quite simple compared to that of natural detrital grains be-

TABLE 4

Interharmonic Correlation Coefficients

* Harmonic #	2	3	4	5	6	7	8
10	.174	.131	-.011	.200 ↗	.157	.215 ↗	.150
9	.093	.238 ↗	.097	.337 ↗	.340 ↗	.291 ↗	.132
8	.143	.092	.199 ↗	.112	.115	-.029	XXX
7	.159	.134	.033	.202 ↗	.172	XXX	XXX
6	.094	.192 ↗	-.013	.149	XXX	XXX	XXX
5	.155	.238 ↗	.080	XXX	XXX	XXX	XXX
4	.123	-.138	XXX	XXX	XXX	XXX	XXX
3	-.021	XXX	XXX	XXX	XXX	XXX	XXX

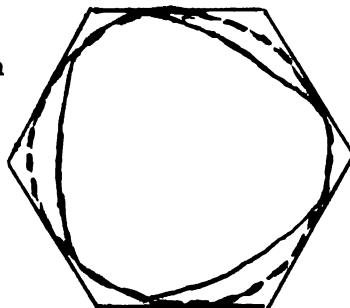
* not all possible harmonic combinations
have been calculated

significant correlations ↗

FIGURE 5

Etched Quartz Grain
(Projection Shape)

Rates of solution on
a sphere of left
quartz



The dotted line shows
the outline of the original
sphere; the inner lines the
outline of the solution-body
remaining after long continued
attack by HF.

(after Frondel, 1962)

TABLE 5

Comparisons of the Harmonic Amplitudes
of the eight Red Cedar River samples and the etched grain

Harmonic #	Etched gr.	B1	B2	B3	B4	B5	B6	B7	B8
1	.001	.014	.013	.010	.018	.012	.011	.012	.012
2	.010	.156	.183	.140	.171	.143	.150	.148	.127
3	.097	.085	.075	.059	.089	.079	.080	.070	.071
4	.007	.046	.047	.044	.048	.046	.043	.041	.048
5	.005	.034	.037	.032	.036	.029	.031	.029	.030
6	.009	.024	.024	.021	.024	.021	.023	.021	.025
7	.004	.017	.017	.017	.018	.017	.016	.016	.017
8	.004	.013	.016	.013	.014	.012	.015	.012	.014
9	.008	.010	.010	.012	.011	.012	.011	.009	.010
10	.003	.009	.009	.008	.008	.008	.010	.009	.009

TABLE 6

**Comparisons of the Harmonic Amplitudes
of the four Coon Creek samples and the etched grain**

Harmonic #	Etched gr.	B9	B10	B11	B12
1	.001	.015	.016	.012	.012
2	.010	.167	.166	.147	.150
3	.097	.075	.080	.073	.077
4	.007	.049	.050	.051	.047
5	.005	.033	.029	.034	.031
6	.009	.019	.023	.027	.022
7	.004	.017	.017	.019	.016
8	.004	.010	.016	.014	.013
9	.008	.009	.011	.010	.010
10	.003	.008	.010	.008	.009

cause its symmetry is exclusively controlled by internal structure. Therefore it is not surprising that the amplitude spectrum of the etched grain differs in two respects from natural quartz grains. The maximum amplitude of the etched grain is associated with the third harmonic rather than the second. The sixth and ninth harmonic amplitudes of the etched grain do not show the expected decrease of amplitude with increasing harmonic number as can be seen in all Red Cedar River and Coon Creek samples (Tables 5 & 6). Such a pattern is not unexpected in a trigonal mineral such as quartz.

It is interesting to note that in the preceeding section the high positive correlation between the sixth and ninth harmonics indicating, perhaps, a tendency towards crystallographic control of shape. Such control is reflected in the shape of the etched grain with its high sixth and ninth harmonic amplitudes. The slightly lower but still significant correlation between the fifth and ninth harmonics is at this time inexplicable.

PROVENANCE

The application of the data to the problem of provenance in the Red Cedar River proves more difficult than the formulation of general principles because of the lack of precision. It is only in the second harmonic with its high harmonic amplitudes that hope of detecting significant differences can blossom. Nonuniformity in the second harmonic amplitude pattern appears to consist of a set of samples (B2 & B4) with values of .17 and .18 and another set (all other Red Cedar samples) comprising 'background' level of approximately .15 (Figure 6). The higher values are from samples each immediately downstream from a major tributary suggesting that they represent a local tributary contribution which is diluted upon further transport. Corroborating evidence for the possibility lies in the value of the second harmonic amplitude of sediment taken near the mouth of Coon Creek (.17). The Red Cedar sample immediately downstream, B4, has a value of .17. Sample B2 (second harmonic amplitude of .18) lies downstream from Sloan Creek and again represents a major deviation from the general background of about .15. No rise is seen between B7 and B6 where Deer Creek enters the Red Cedar possibly because it has a much larger floodplain than either Coon Creek or Sloan Creek and because it has a lesser gradient. The lower end of Deer Creek becomes a slough when the Red Cedar is in flood, when the maximum sediment transport occurs. The Deer Creek sediment would then be deposited, not in the Red Cedar, but in the lower section of its own floodplain.

The picture can be made clearer by a look at the roughness coefficients of the samples. The Weinberg roughness coefficient is the sum of the squares of all the harmonic amplitudes (Ehrlich & Weinberg, 1969). This represents the average of the squared deviations of the grain perimeter from a circle of equal area. The

FIGURE 6
MEAN SECOND HARMONIC
RED CEDAR AND COON CREEK

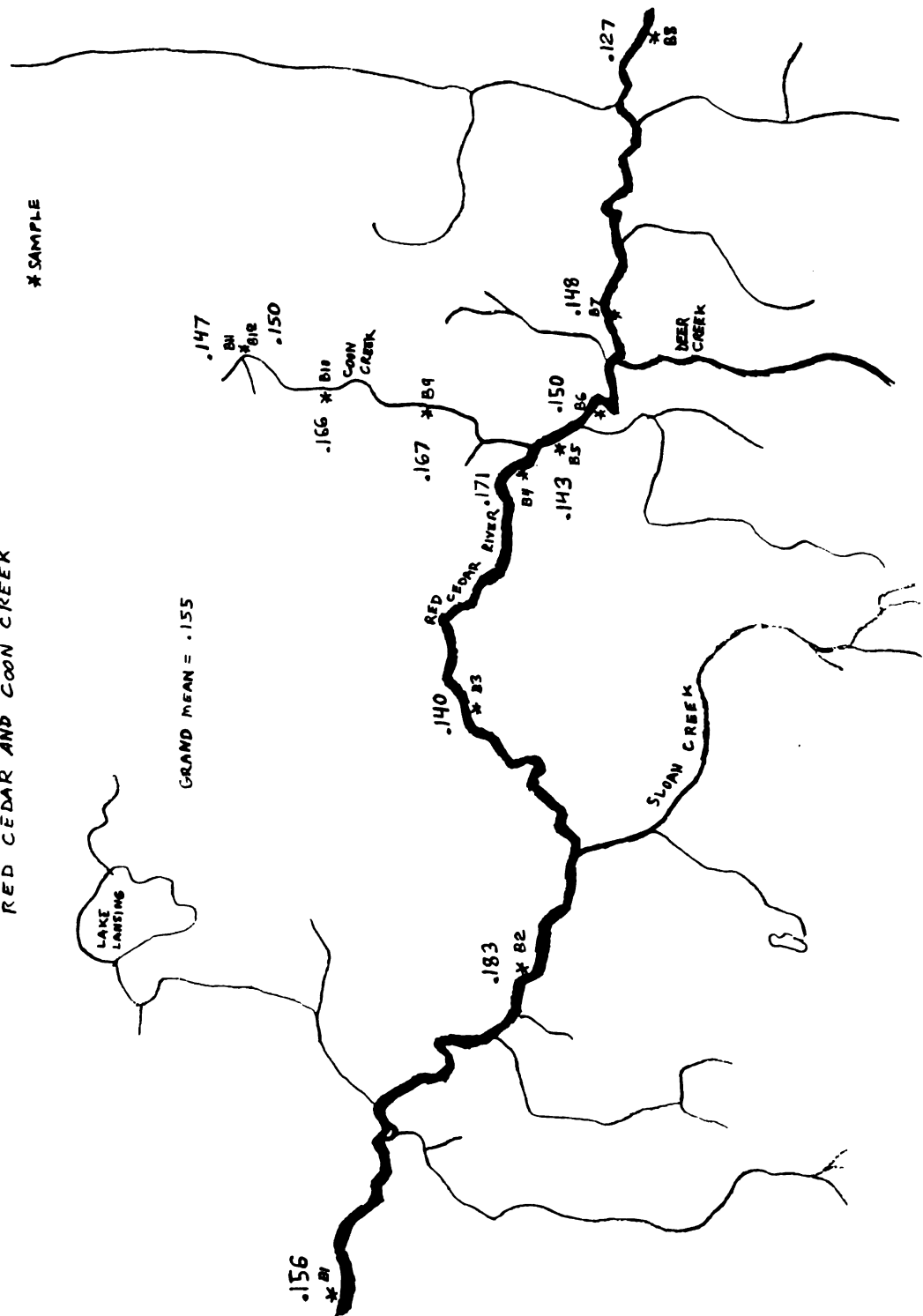
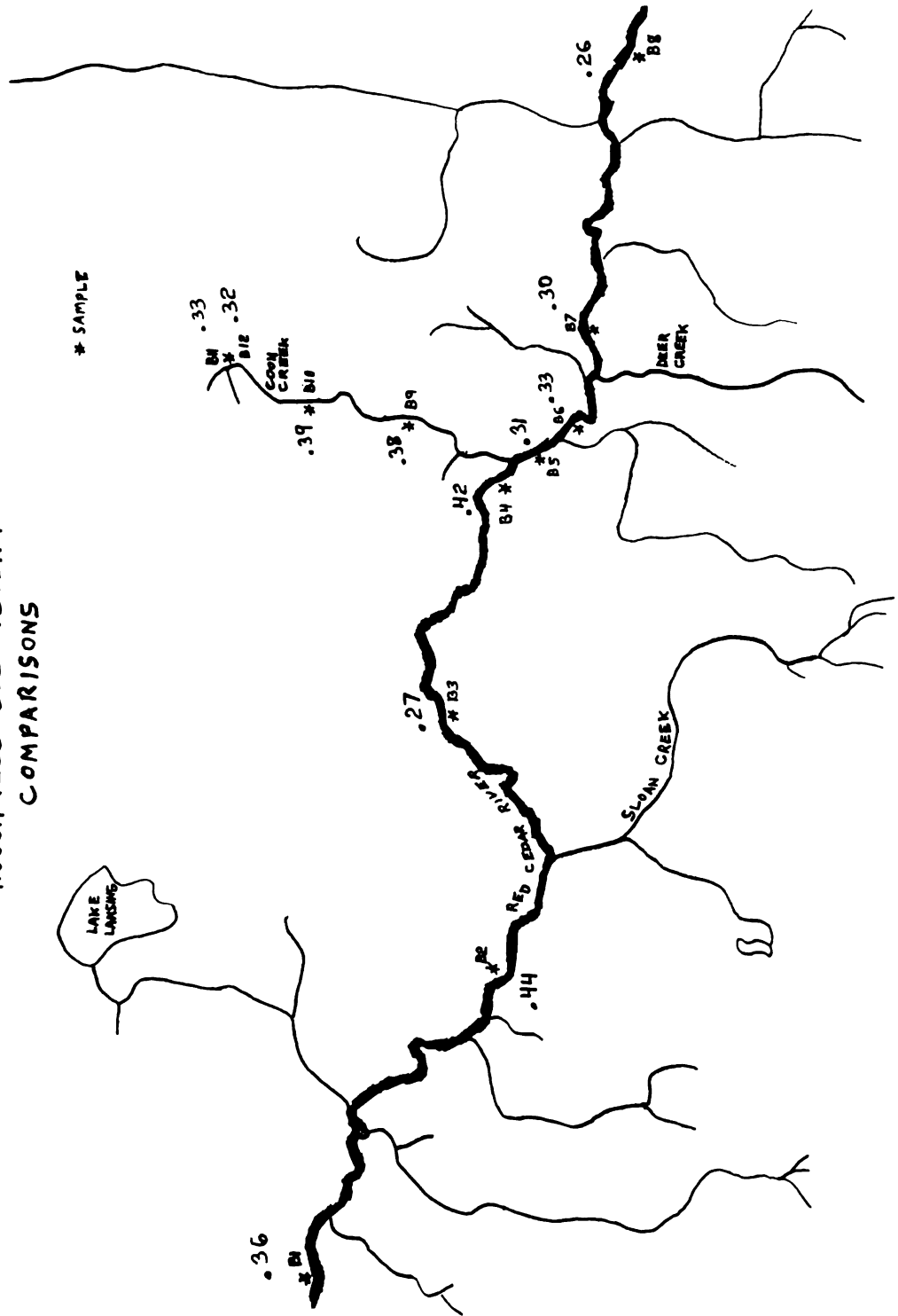
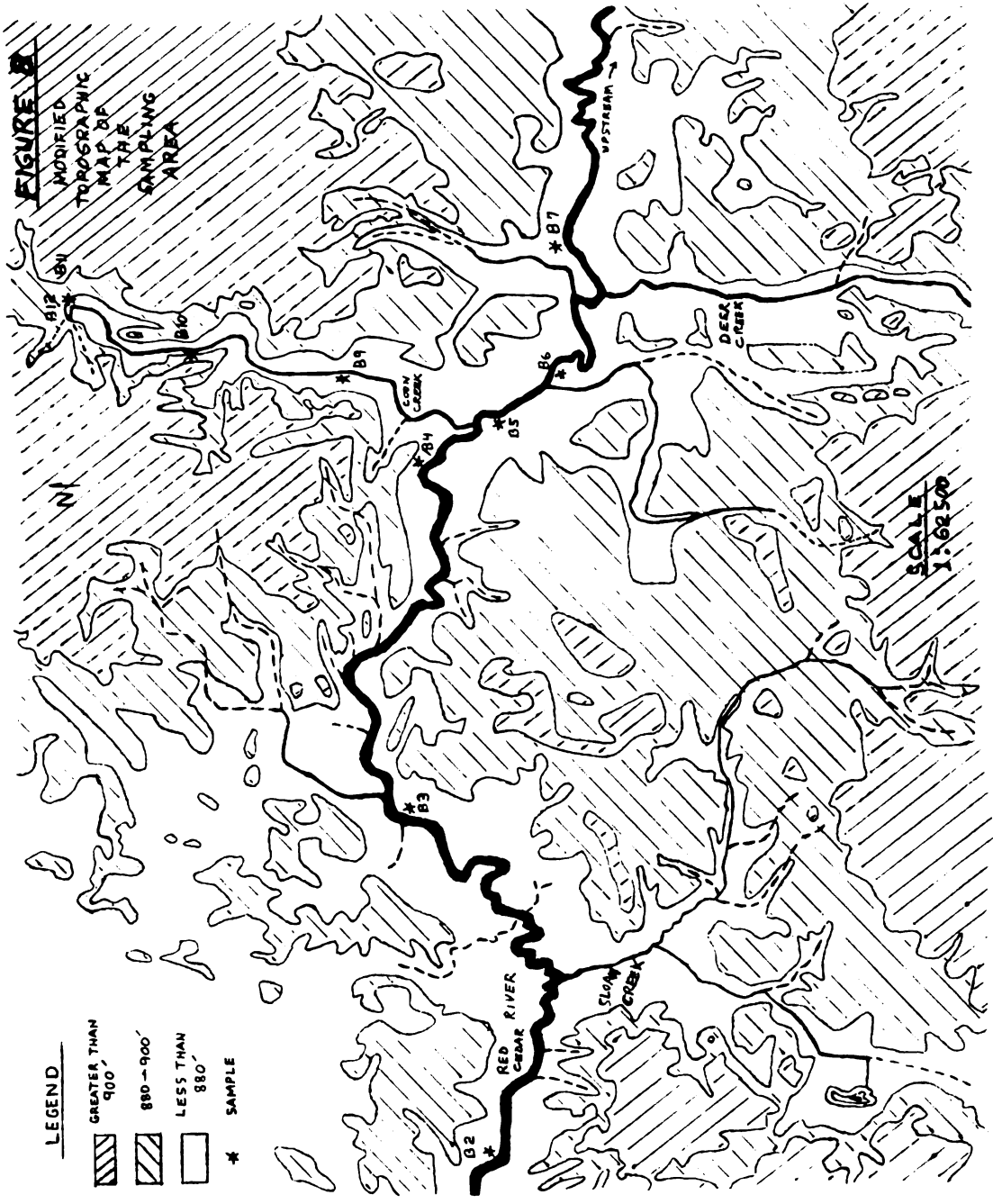


FIGURE 7
ROUGHNESS COEFFICIENT
COMPARISONS





higher the coefficient, the more rugged the grain shape. From Figure 7 it can be seen that the roughness coefficient values are distributed similarly to the second harmonic amplitudes of Figure 6 which is not too surprising as the second harmonic provides the greatest contribution to the roughness coefficient in the Red Cedar and Coon Creek samples. It is now seen that the tributary sediments, as illustrated by Coon Creek, consist of more rugged grains than those found in the Red Cedar. These more rugged grains are probably coming from glacial till composing the uplands through which Coon Creek and Sloan Creek are flowing (Figure 8).

With a more detailed study and greater precision it should be possible to quantify the amount of sediment entering the Red Cedar from the tributaries by an analysis of the change in the various harmonic amplitudes. It has been shown that a difference does exist between tributary and Red Cedar River sediment and that this difference is great enough to detect even under poor precision.

SUMMARY AND CONCLUSIONS

The relative precision of the harmonic values associated with the Fourier series approximations is inversely proportional to the harmonic number. Thus the number of points per grain is controlled more by consideration of precision than by the Nyquist Sampling Theorem which states that the number of points must be at least twice the order of the highest harmonic estimated. In addition, for studies such as this, more grains than thirty per sample will probably be necessary. Two-hundred grains per sample would reduce the standard error of the mean to approximately one-third the values in this study. Since no multivariate tests were performed in this study such as discriminate functions or multivariate analysis of variance, it is not known whether discrimination between samples could be improved in this way.

Empirical relationships between variables generated by the Fourier analysis should aid in further utilization of this technique. These include: There is no detectable correlation between size and shape as measured by the Fourier series approximation within the size range of the study, .1 to .3 mm mean radius. Therefore, no size effect need be calculated and removed from the shape equation. The standard deviation of the shape variable is not independent of the harmonic amplitude which means that certain statistical tests such as the T-test may be used on the data. Inspection of the correlation matrix of harmonic amplitudes indicates possible structural control of quartz shape. Results of this study demonstrate that provenance studies using this method are feasible but that the information will not come cheaply.

The movement of sediment from tributaries to the Red Cedar River can be detected through roughness coefficient patterns. The tributaries such as Coon Creek are carrying sediment with a rougher shape than that generally found in the Red Cedar. Tributary contribution to the Red Cedar can be detected by a high roughness coefficient in the Red Cedar sediment downstream from the mouth of a tributary. Sample sizes five or six times greater than those used here are probably necessary for a more detailed analysis of sediment provenance in this system. However, the present results indicate that the Fourier series technique gives more promise for successful provenance work than any heretofore suggested.

LIST OF REFERENCES

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References Cited

- Cox, E.P., 1927, "A Method of Assigning Numerical and Percentage Values to the Degree of Roundness of Sand Grains", J. of Paleontology, V-1, pp. 179 - 183.
- Ehrlich, R. & Weinberg, B., 1969, "An Exact Method for Characterization of Grain Shape", J. of Sed. Pet., (in press).
- Flemming, N.C., 1965, "Form and Function of Sedimentary Particles", J. of Sed. Pet., V-35, pp. 381 - 390.
- Fronzel, Clifford, 1962, The System of Mineralogy, John Wiley & Sons, Inc., New York, p. 162.
- Krumbein, W.C. & Sloss, L.L., 1963, Stratigraphy and Sedimentation, W.H. Freeman and Co., New York.
- Krynine, P.D., 1956, "Sphericity and Roundness Through the Ages", Bull. G.S.A., V-67, pp. 1661 - 1662.
- Owen, D.B., 1962, Handbook of Statistical Tables, Addison-Wesley Publ. Co., Inc., Reading, Mass., p. 39.
- Pentland, A., 1927, "A Method of Measuring the Angularity of Sands", Royal Soc. Canada, Proc. & Trans., V-21, Appendix C, Titles and Abstracts, p. xciii.
- Riley, N.A., 1941, "Projection Sphericity", J. of Sed. Pet., V-11, pp. 94 - 97.
- Tickell, F.G., 1931, The Examination of Fragmental Rocks, Stanford U. Press.
- Wadell, Hakon, 1933, "Sphericity and Roundness of Rock Particles", J. of Geology, V-41, pp. 310 - 331.
- , 1935, "Volume, Shape, and Roundness of Quartz Particles", J. of Geology, V-43, pp. 250 - 280.

GENERAL REFERENCES

- Aschenbrenner, B.C., 1955, "A Photogrammetric Method for the Tridimensional Measurement of Sand Grains", *Photogrammetric Eng.*, V-21, pp. 376 - 382.
- , 1956, "A New Method of Expressing Particle Sphericity", *J. of Sed. Pet.*, V-26, pp. 15 - 31.
- Bayly, B., 1968, Introduction to Petrology, Prentice-Hall Inc., p. 370.
- Beal, M.A. & Shepard, F.P., 1956, "A Use of Roundness to Determine Depositional Environments", *J. of Sed. Pet.*, V-26, pp. 49 - 60.
- Catacosinos, P.A., 1965, "Tables for the Determination of Sphericity and Shape of Rock Particles", *J. of Sed. Pet.*, V-35, pp. 354 - 365.
- Clifford, W.K., 1955, The Common Sense of the Exact Sciences, Dover Publ., Inc., New York.
- Gill, M.A., 1969, "Flatness Index of Non-Spherical Sedimentary Particles", *J. of Geology*, V-77, pp. 120 - 124.
- Glezen, W.H. & Ludwick, J.C., 1963, "An Automated Grain-Shape Classifier", *J. of Sed. Pet.*, V-33, pp. 23 - 40.
- Griffiths, J.C., 1967, Scientific Method in Analysis of Sediments, McGraw-Hill Inc., New York, p. 508.
- Hald, A., 1952, Statistical Tables and Formulas, John Wiley & Sons, New York, p. 39.
- Huitt, J.L., 1954, "Three-dimensional Measurement of Sand Grains", *Am. Assoc. Petroleum Geologists Bull.*, V-38, pp. 159 - 160.
- Krueger, W.C. Jr., 1963, Mineralogical Composition and Textural Properties of River Sediments from British Honduras, Rice University, Ph.D. thesis.
- Krumbein, W.C. & Pettijohn, F.J., 1938, Manual of Sedimentary Petrography, Appleton-Century, New York, p. 549.
- Krumbein, W.C., 1941, "Measurement and Geological Significance of Shape and Roundness of Sedimentary Particles", *J. of Sed. Pet.*, V-11, pp. 64 - 72.
- Krynine, P.D., 1940, "Petrology and Genesis of the Third Bradford Sand", *Pa. State College Bull. No. 29*, p. 81.
- Lees, G., 1964, "A New Method for Determining the Angularity of Particles", *Sedimentology*, V-3, pp. 2 - 21.

- Ore, Oystein, 1963, Graphs and Their Uses, Random House, Yale University.
- Ostle, Bernard, 1958, Statistics in Research, Iowa State College Press, Ames, Iowa, p. 459.
- Pettijohn, F.J. & Lundahl, A.C., 1943, "Shape and Roundness of Lake Erie Beach Sands", J. of Sed. Pet., V-13, pp. 69 - 78.
- Pettijohn, F.J., 1957, Sedimentary Rocks, Harper & Bros., New York, pp. 54 - 68.
- Powers, M.C., 1953, "A New Roundness Scale for Sedimentary Particles", J. of Sed. Pet., V-23, pp. 117 - 119.
- Pye, W.D. & Pye, M.H., 1943, "Sphericity Determinations of Pebbles and Sand Grains", J. of Sed. Pet., V-13, pp. 28 - 34.
- Rittenhouse, Gordon, 1943, "A Visual Method of Estimating Two-dimensional Sphericity", J. of Sed. Pet., V-13, pp. 79 - 81.
- Russell, R.D. & Taylor, R.E., 1937, "Roundness and Shape of Mississippi River Sands", J. of Geology, V-45, pp. 225 - 267.
- Russell, R.D., 1939, "Effects of Transportation of Sedimentary Particles", Recent Marine Sediments, Tulsa, pp. 32 - 47.
- Spock, L.E., 1953, Guide to the Study of Rocks, Harper and Bros., New York, p. 298.
- Wadell, Hakon, 1932, "Volume, Shape, and Roundness of Rock Particles", J. of Geology, V-40, pp. 443 - 451.
- Wentworth, C.K., 1922, "A Field Study of the Shapes of River Pebbles", Bull. U.S. Geol. Survey, 730-C, p. 114.
- , 1936, "An Analysis of the Shapes of Glacial Cobbles", J. of Sed. Pet., V-6, p. 89.
- Zingg, Th., 1935, "Beitrag zur Scholleranalyse", Schweiz. Mineralog. Petrog. Mitt., Bd. 15, pp. 39 - 140.

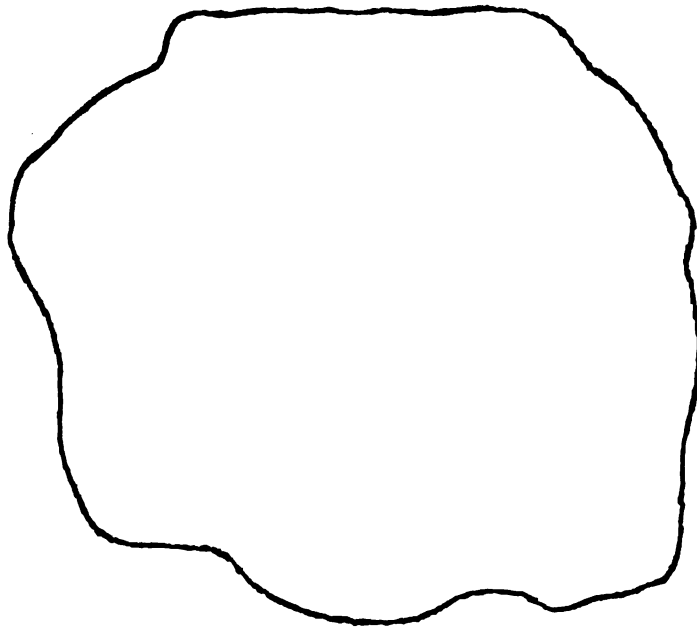
APPENDICES

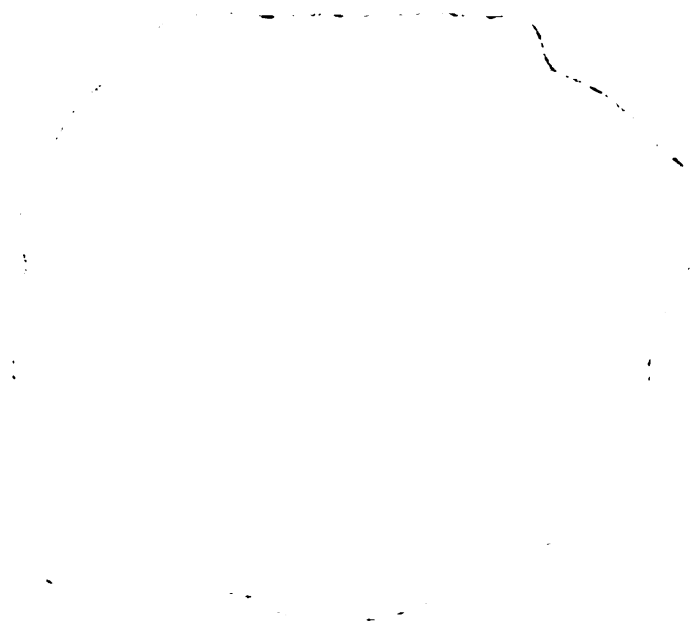
APPENDIX A

Histograms of the Harmonic Amplitudes of the
Test Grain
With a Listing of the Harmonic Amplitudes of Each Replication

The test grain was replicated eleven times. The first nine replications had the same orientation. The last two replications each had a different orientation. The harmonic values of the last two replications are encircled.

The Test Grain





SFT 1

VARIABLE 1

N = 11
← NUMBER OF REPLICATIONS

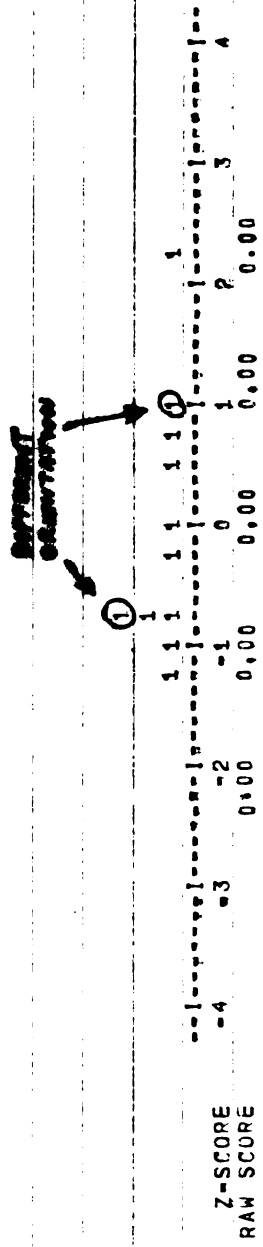
MEAN = 0.003
STD. DEV. = 0.000 (0.000)
VARIANCE = 0.000 (0.000)
EF. OF VAR. = 7.569 (7.938)
SKEWNESS = 0.868 (1.012)
KURTOSIS = 0.127 (1.045)

LOWER QUARTILE = 0.003
MEDIAN = 0.003
UPPER QUARTILE = 0.004

RANGE = 0.003 TO 0.004

HISTOGRAMS

GRAIN



SFT 1

VARIABLE 2

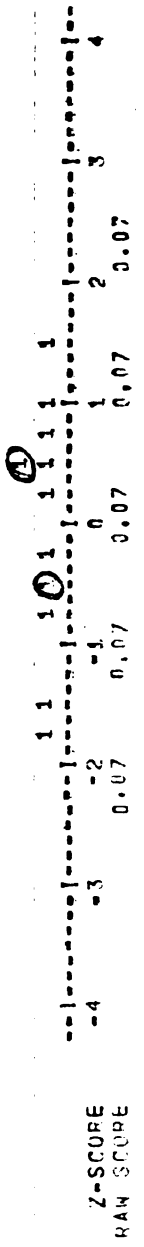
HARMONIC NUMBER

N = 11

MEAN = 0.068
STD. DEV. = 0.001 (0.001)
VARIANCE = 0.000 (0.000)
EF. OF VAR. = 1.407 (1.476)
SKEWNESS = -0.386 (-0.450)
KURTOSIS = -0.850 (-0.583)

LOWER QUARTILE = 0.067
MEDIAN = 0.068
UPPER QUARTILE = 0.069

RANGE = 0.066 TO 0.069



100

100

100

100

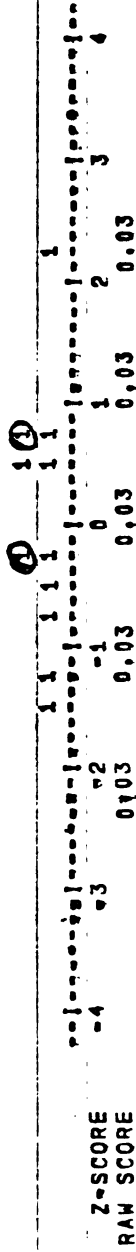
GRAIN

SET 1 VARIABLE 3

N = 11
 MEAN = 0.031
 STD. DEV. = 0.002
 VARIANCE = 0.000
 EF. OF VAR. = 5.131
 SKEWNESS = 0.440
 KURTOSIS = -0.349

LOWER QUARTILE = 0.030
 MEDIAN = 0.031
 UPPER QUARTILE = 0.033

RANGE = 0.029 TO 0.035



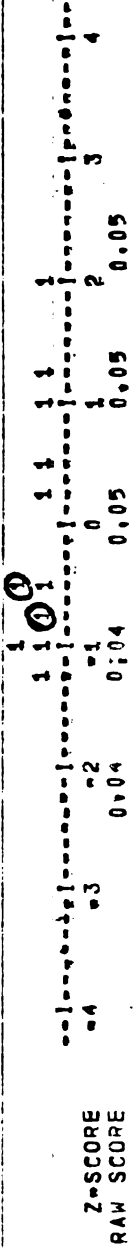
40

SET 1 VARIABLE 4

N = 11
 MEAN = 0.046
 STD. DEV. = 0.001
 VARIANCE = 0.000
 EF. OF VAR. = 3.136
 SKEWNESS = 0.487
 KURTOSIS = -1.065

LOWER QUARTILE = 0.045
 MEDIAN = 0.046
 UPPER QUARTILE = 0.048

RANGE = 0.044 TO 0.049

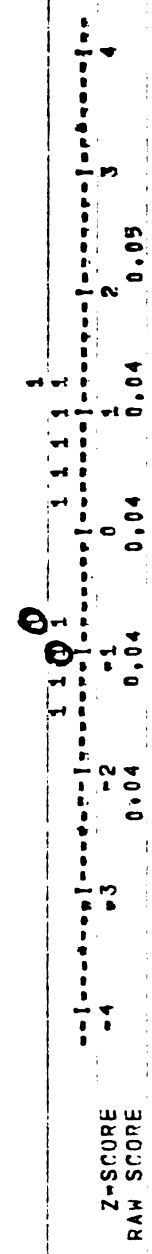


TEST
GRAIN

SET 1 VARIABLE 7

N = 11
 MEAN = 0.043
 STD. DEV. = 0.001 (0.001)
 VARIANCE = 0.000 (0.000)
 COEFF. OF VAR. = 1.931 (2.026)
 SKEWNESS = -0.067 (-0.078)
 KURTOSIS = -1.488 (-1.647)
 LOWER QUANTILE = 0.043
 MEDIAN = 0.044
 UPPER QUANTILE = 0.044

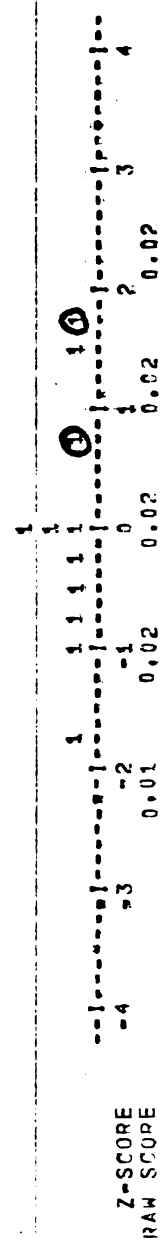
RANGE = 0.042 TO 0.045



SET 1 VARIABLE 8

N = 11
 MEAN = 0.016
 STD. DEV. = 0.001 (0.001)
 VARIANCE = 0.000 (0.000)
 COEFF. OF VAR. = 7.832 (8.214)
 SKEWNESS = 0.332 (0.387)
 KURTOSIS = -0.562 (-0.104)
 LOWER QUANTILE = 0.016
 MEDIAN = 0.016
 UPPER QUANTILE = 0.017

RANGE = 0.014 TO 0.019



1

1

100

1

10

1



1

1

100



1

APPENDIX B

Histograms of the Harmonic Amplitudes of
The Red Cedar River and Coon Creek Samples

Red Cedar River

Sample B1	-----	46
Sample B2	-----	51
Sample B3	-----	56
Sample B4	-----	61
Sample B5	-----	66
Sample B6	-----	71
Sample B7	-----	76
Sample B8	-----	81

Coon Creek

Sample B9	-----	86
Sample B10	-----	91
Sample B11	-----	96
Sample B12	-----	101

RED CEDAR

SET 1 VARIABLE 1

N = 31

MEAN = 0.014
 STD. DEV. = 0.011 (0.011)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 76.560 (77.826)
 SKEWNESS = 0.867 (0.911)
 KURTOSIS = -0.362 (-0.207)

LOWER QUANTILE = 0.007

MEDIAN = 0.011

UPPER QUANTILE = 0.023

RANGE = 0.001 TO 0.039



SET 1 VARIABLE 2

N = 31

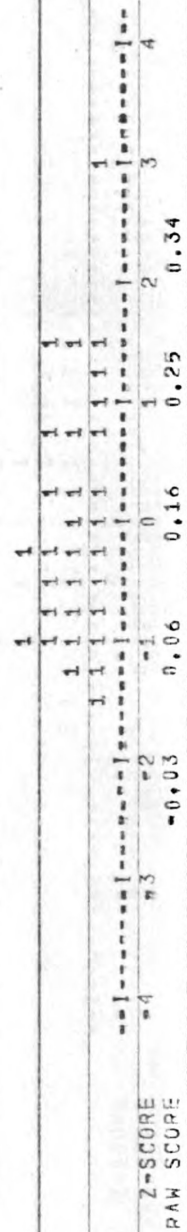
MEAN = 0.156
 STD. DEV. = 0.091 (0.093)
 VARIANCE = 0.008 (0.009)
 COEF. OF VAR. = 58.618 (59.587)
 SKEWNESS = 0.815 (0.857)
 KURTOSIS = 0.313 (0.592)

LOWER QUANTILE = 0.089

MEDIAN = 0.138

UPPER QUANTILE = 0.225

RANGE = 0.028 TO 0.420





SET	1	VARIABLE	5
N = 31			
MEAN =	0.034		
STD. DEV. =	0.018	(	0.018)
VARIANCE =	0.000	(	0.000)
COEF. OF VAR. =	53.563	(	54.448)
SKEWNESS =	0.927	(	0.869)
KURTOSIS =	0.851	(	1.228)
LOWER QUARTILE =	0.019		
MEDIAN =	0.030		
UPPER QUARTILE =	0.044		
RANGE =	0.004 TO	0.089	
Z-SCORE			
RAW SCORE			
-4			
-3			
-2			
-1			
0			
1			
2			
3			
4			

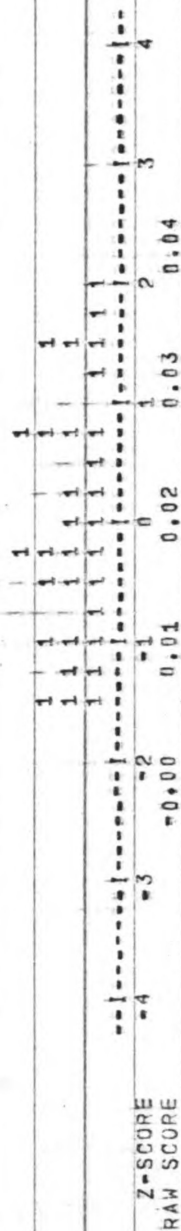
SET	1	VARIABLE	6
N = 31			
MEAN =	0.024	(	0.014)
STD. DEV. =	0.014	(	0.000)
VARIANCE =	0.000	(	0.000)
COEF. OF VAR. =	55.955	(	56.880)
SKEWNESS =	0.730	(	0.767)
KURTOSIS =	0.040	(	0.174)
LOWER QUARTILE =	0.011		
MEDIAN =	0.022		
UPPER QUARTILE =	0.032		
RANGE =	0.006 TO	0.057	
Z-SCORE			
RAW SCORE			
-4			
-3			
-2			
-1			
0			
1			
2			
3			
4			

SET 1 VARIABLE 7

N = 31
 MEAN = 0.017
 STD. DEV. = 0.009 (0.010)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 56.607 (57.543)
 SKEWNESS = 0.286 (0.301)
 KURTOSIS = -0.958 (-0.911)

LOWER QUARTILE = 0.008
 MEDIAN = 0.014
 UPPER QUARTILE = 0.024

RANGE = 0.001 TO 0.035



SFT 1 VARIABLE 8

N = 31
 MEAN = 0.013
 STD. DEV. = 0.007 (0.007)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 56.666 (57.603)
 SKEWNESS = 0.394 (0.414)
 KURTOSIS = -0.768 (-0.687)

LOWER QUARTILE = 0.006
 MEDIAN = 0.013
 UPPER QUARTILE = 0.018

RANGE = 0.002 TO 0.029

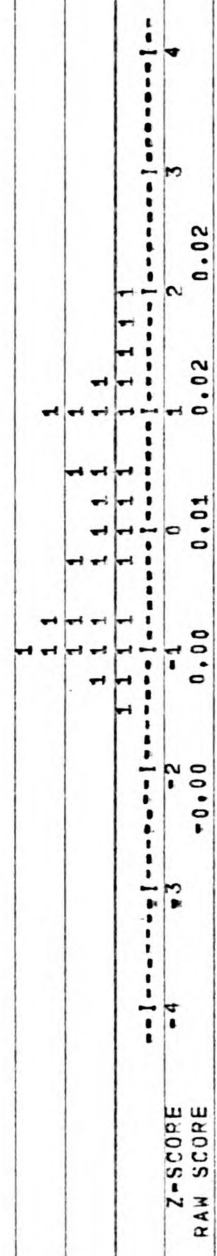


SET 1 VARIABLE 9

N = 31
MEAN = 0.010
STD. DEV. = 0.006 (0.006)
VARIANCE = 0.000 (0.000)
COEFF. OF VAR. = 60.648 (61.650)
SKEWNESS = 0.309 (0.325)
KURTOSIS = -1.141 (-1.127)

LOWER QUANTILE = 0.004
MEDIAN = 0.010
UPPER QUANTILE = 0.016

RANGE = 0.001 TO 0.023

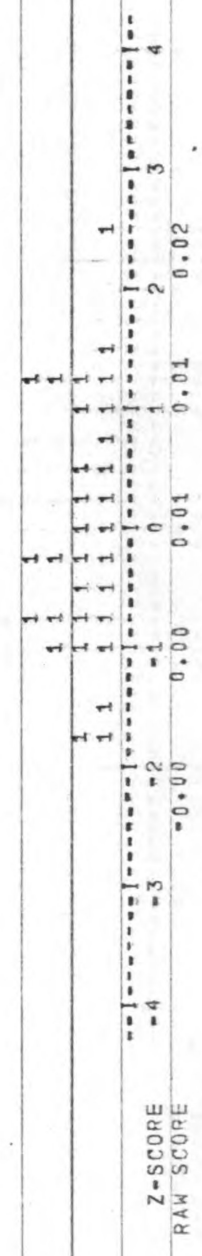


SET 1 VARIABLE 10

N = 31
MEAN = 0.009
STD. DEV. = 0.004 (0.005)
VARIANCE = 0.000 (0.000)
COEFF. OF VAR. = 51.017 (51.860)
SKEWNESS = 0.271 (0.285)
KURTOSIS = -0.507 (-0.378)

LOWER QUANTILE = 0.005
MEDIAN = 0.008
UPPER QUANTILE = 0.013

RANGE = 0.001 TO 0.019



SFT 1

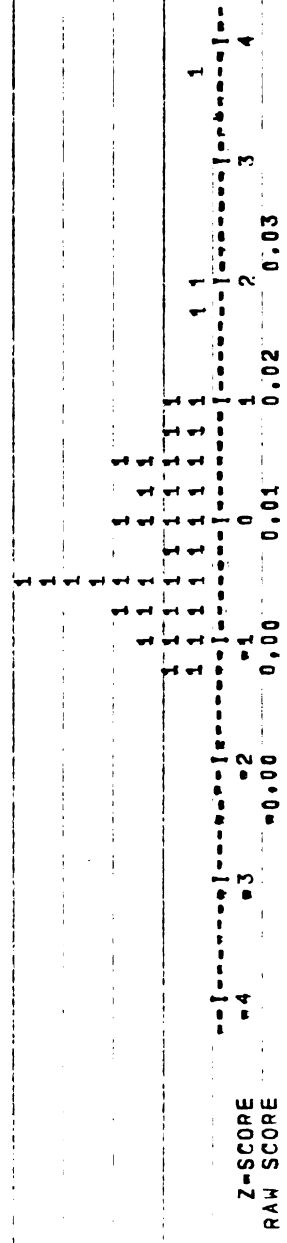
VARIABLE 1

N = 37

MEAN = 0.013
STD. DEV. = 0.009 (0.009)
VARIANCE = 0.000 (0.000)
COEF. OF VAR. = 67.692 (68.626)
SKEWNESS = 1.725 (1.799)
KURTOSIS = 4.072 (4.862)

LOWER QUANTILE = 0.008
MEDIAN = 0.012
UPPER QUANTILE = 0.017

RANGE = 0.001 TO 0.048



SFT 1

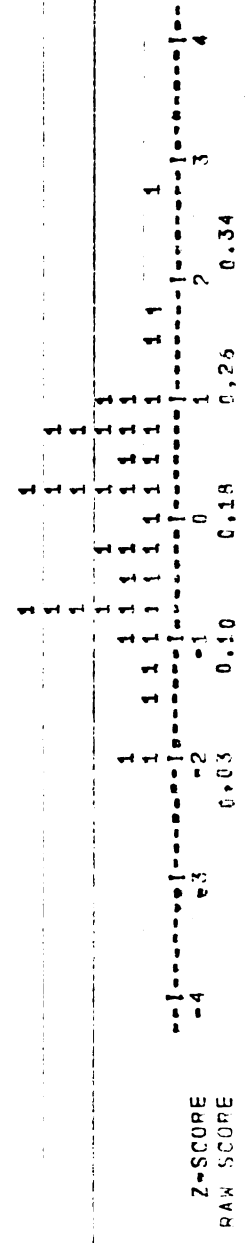
VARIABLE 2

N = 37

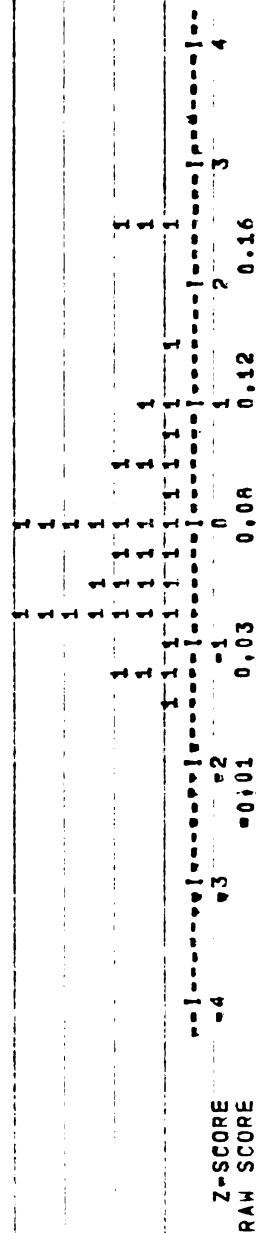
MEAN = 0.183
STD. DEV. = 0.079 (0.080)
VARIANCE = 0.006 (0.006)
COEF. OF VAR. = 43.137 (43.732)
SKEWNESS = 0.268 (0.279)
KURTOSIS = 0.464 (0.714)

LOWER QUANTILE = 0.123
MEDIAN = 0.194
UPPER QUANTILE = 0.236

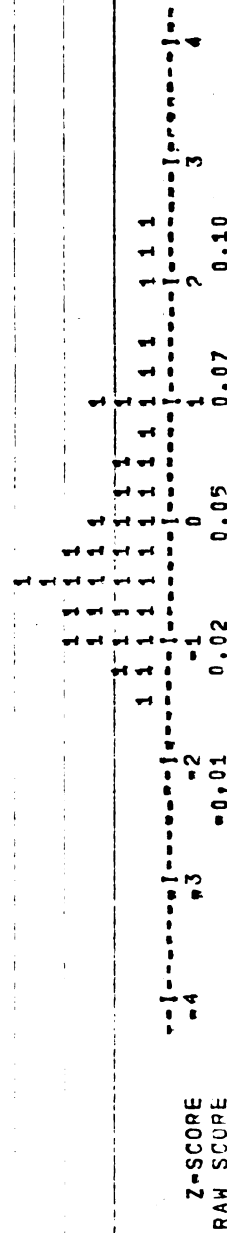
RANGE = 0.020 TO 0.410



RANGE = 0.014 TO 0.188



RANGE = 0.005 TO 0.113

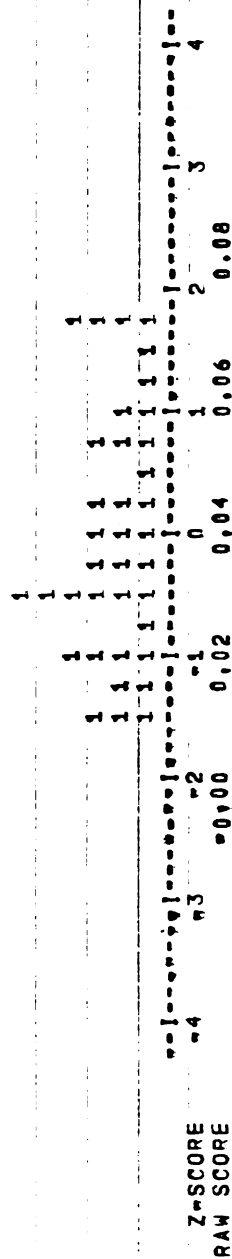


SPT 1 VARIABLE 5

N = 37
 MEAN = 0.037
 STD. DEV. = 0.020 (0.020)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 52.841 (53.570)
 SKEWNESS = 0.272 (0.284)
 KURTOSIS = -0.964 (-0.927)

LOWER QUANTILE = 0.022
 MEDIAN = 0.034
 UPPER QUANTILE = 0.052

RANGE = 0.006 TO 0.073

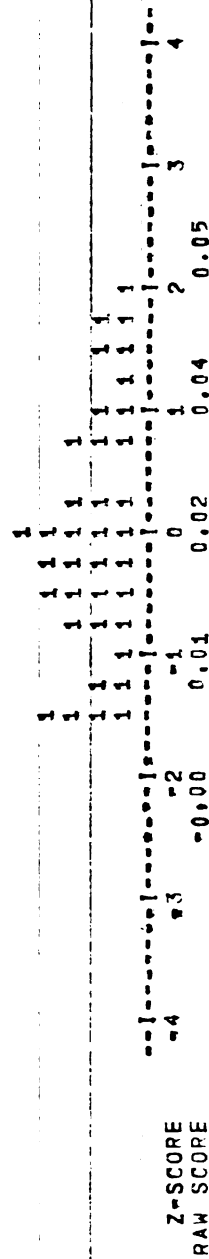


SPT 1 VARIABLE 6

N = 37
 MEAN = 0.024
 STD. DEV. = 0.013 (0.014)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 55.891 (56.662)
 SKEWNESS = 0.381 (0.398)
 KURTOSIS = -0.761 (-0.693)

LOWER QUANTILE = 0.015
 MEDIAN = 0.022
 UPPER QUANTILE = 0.034

RANGE = 0.003 TO 0.052

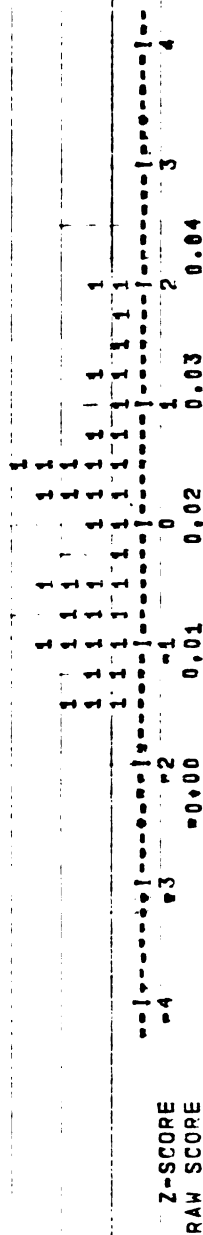


SFT 1 VARIABLE 7

N = 37
 MEAN = 0.017
 STD. DEV. = 0.010 (0.010)
 VARIANCE = 0.000 (0.000)
 CEF. OF VAR. = 57.899 (58.698)
 SKEWNESS = 0.320 (0.334)
 KURTOSIS = -0.817 (-0.758)

LOWER QUARTILE = 0.008
 MEDIAN = 0.019
 UPPER QUARTILE = 0.022

RANGE = 0.001 TO 0.037



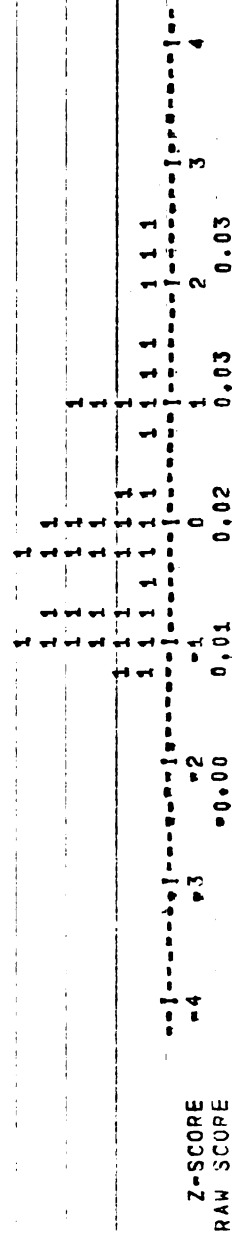
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SFT 1 VARIABLE 8

N = 37
 MEAN = 0.016
 STD. DEV. = 0.009 (0.010)
 VARIANCE = 0.000 (0.000)
 CEF. OF VAR. = 60.774 (61.613)
 SKEWNESS = 0.923 (0.962)
 KURTOSIS = -0.083 (0.087)

LOWER QUARTILE = 0.008
 MEDIAN = 0.012
 UPPER QUARTILE = 0.024

RANGE = 0.003 TO 0.040

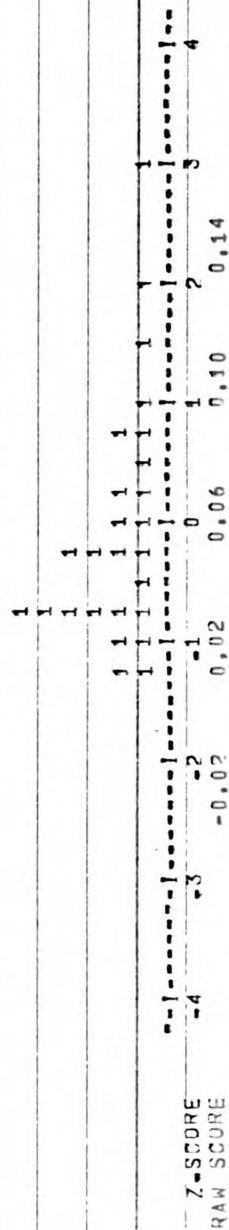


SET 1 VARIABLE 3

N = 25
 MEAN = 0.059
 STD. DEV. = 0.041 (0.042)
 VARIANCE = 0.002 (0.002)
 COEF. OF VAR. = 68.637 (69.996)
 SKEWNESS = 1.353 (1.437)
 KURTOSIS = 1.467 (2.090)

LOWER QUARTILE = 0.030
 MEDIAN = 0.048
 UPPER QUARTILE = 0.078

RANGE = 0.010 TO 0.183



SET 1 VARIABLE 4

N = 25
 MEAN = 0.044
 STD. DEV. = 0.027 (0.028)
 VARIANCE = 0.001 (0.001)
 COEF. OF VAR. = 62.458 (63.695)
 SKEWNESS = 0.770 (0.818)
 KURTOSIS = -0.017 (0.251)

LOWER QUARTILE = 0.024
 MEDIAN = 0.037
 UPPER QUARTILE = 0.060

RANGE = 0.010 TO 0.113



SFT 1 VARIABLE 5

N = 24
 MEAN = 0.932
 STD. DEV. = 0.029 (0.021)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 65.491 (64.752)
 SKEWNESS = 0.362 (0.384)
 KURTOSIS = -1.063 (-1.035)

LOWER QUANTILE = 0.915
 MEDIAN = 0.929
 UPPER QUANTILE = 0.954

RANGE = 0.003 TO 0.076

Z-SCORE	RAW SCORE
-4	-3
-3	-2
-2	-1
-1	0
0	0.01
0.01	0.03
0.03	0.05
0.05	0.07
0.07	

SFT 1 VARIABLE 6

N = 24
 MEAN = 0.021
 STD. DEV. = 0.004 (0.008)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 40.174 (40.124)
 SKEWNESS = 0.309 (0.328)
 KURTOSIS = -0.441 (-0.207)

LOWER QUANTILE = 0.014
 MEDIAN = 0.021
 UPPER QUANTILE = 0.027

RANGE = 0.007 TO 0.039

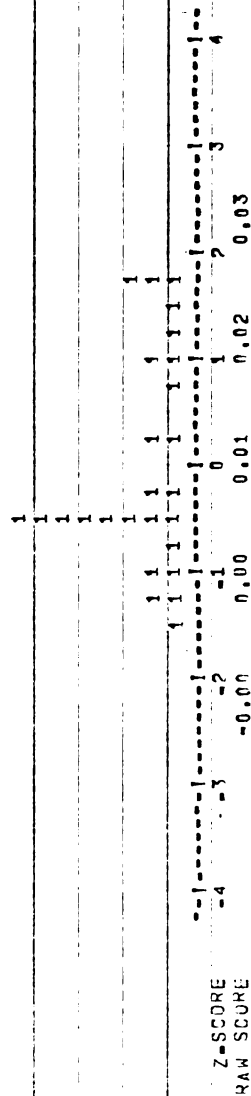
Z-SCORE	RAW SCORE
-4	-3
-3	-2
-2	-1
-1	0
0	0.01
0.01	0.02
0.02	0.03
0.03	0.04
0.04	

SET 1 VARIABLE 9

N = 24
 MEAN = 0.012
 STD. DEV. = 0.007 (0.007)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 50.864 (50.030)
 SKEWNESS = 0.537 (0.570)
 KURTOSIS = -0.907 (-0.935)

LOWER QUANTILE = 0.008
 MEDIAN = 0.009
 UPPER QUANTILE = 0.016

RANGE = 0.001 TO 0.024

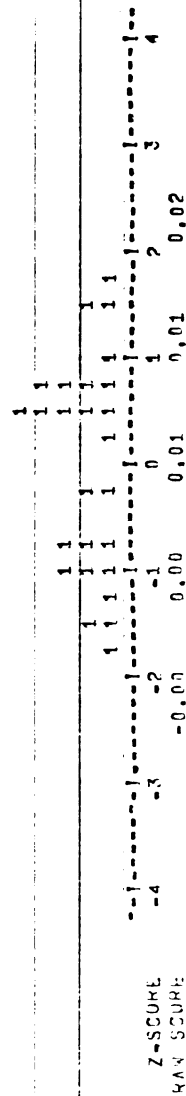


SET 1 VARIABLE 10

N = 24
 MEAN = 0.006
 STD. DEV. = 0.004 (0.004)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 51.248 (52.263)
 SKEWNESS = -0.247 (-0.156)
 KURTOSIS = -1.213 (-1.212)

LOWER QUANTILE = 0.005
 MEDIAN = 0.010
 UPPER QUANTILE = 0.011

RANGE = 0.001 TO 0.016



SET 1 VARIABLE 3

N = 37

MEAN = 0.089
 STD. DEV. = 0.051 (0.052)
 VARIANCE = 0.003 (0.003)
 COEF. OF VAR. = 57.134 (57.922)
 SKEWNESS = 0.548 (0.571)
 KURTOSIS = -0.427 (-0.309)

LOWER QUARTILE = 0.055
 MEDIAN = 0.079
 UPPER QUARTILE = 0.120

RANGE = 0.005 TO 0.207

Z-SCORE	RAW SCORE
-4	0.03
-3	-0.01
-2	0.04
-1	0.09
0	0.14
1	0.19
2	
3	
4	

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SET 1 VARIABLE 4

N = 37

MEAN = 0.048
 STD. DEV. = 0.028 (0.028)
 VARIANCE = 0.001 (0.001)
 COEF. OF VAR. = 58.539 (59.346)
 SKEWNESS = 0.699 (0.729)
 KURTOSIS = 0.054 (0.243)

LOWER QUARTILE = 0.025
 MEDIAN = 0.045
 UPPER QUARTILE = 0.062

RANGE = 0.005 TO 0.125

Z-SCORE	RAW SCORE
-4	-0.01
-3	0.02
-2	0.05
-1	0.08
0	0.10
1	
2	
3	
4	

SET 1 VARIABLE 5

N = 37

MEAN = 0.036
 STD. DEV. = 0.020 (0.020)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 54.495 (55.247)
 SKEWNESS = 0.840 (0.876)
 KURTOSIS = -0.256 (-0.112)

LOWER QUARTILE = 0.022
 MEDIAN = 0.029
 UPPER QUARTILE = 0.052

RANGE = 0.006 TO 0.084

Z-SCORE	RAW SCORE
-4	3
-3	2
-2	1
-1	1
0	1
0.02	1
0.04	1
0.06	1
0.07	1
0.08	1
0.09	1
0.10	1
0.11	1
0.12	1
0.13	1
0.14	1
0.15	1
0.16	1
0.17	1
0.18	1
0.19	1
0.20	1
0.21	1
0.22	1
0.23	1
0.24	1
0.25	1
0.26	1
0.27	1
0.28	1
0.29	1
0.30	1
0.31	1
0.32	1
0.33	1
0.34	1
0.35	1
0.36	1
0.37	1
0.38	1
0.39	1
0.40	1
0.41	1
0.42	1
0.43	1
0.44	1
0.45	1
0.46	1
0.47	1
0.48	1
0.49	1
0.50	1
0.51	1
0.52	1
0.53	1
0.54	1
0.55	1
0.56	1
0.57	1
0.58	1
0.59	1
0.60	1
0.61	1
0.62	1
0.63	1
0.64	1
0.65	1
0.66	1
0.67	1
0.68	1
0.69	1
0.70	1
0.71	1
0.72	1
0.73	1
0.74	1
0.75	1
0.76	1
0.77	1
0.78	1
0.79	1
0.80	1
0.81	1
0.82	1
0.83	1
0.84	1
0.85	1
0.86	1
0.87	1
0.88	1
0.89	1
0.90	1
0.91	1
0.92	1
0.93	1
0.94	1
0.95	1
0.96	1
0.97	1
0.98	1
0.99	1
1.00	1

SET 1 VARIABLE 6

N = 37

MEAN = 0.024
 STD. DEV. = 0.013 (0.013)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 53.643 (54.383)
 SKEWNESS = 0.923 (0.962)
 KURTOSIS = 0.390 (0.630)

LOWER QUARTILE = 0.014
 MEDIAN = 0.021
 UPPER QUARTILE = 0.035

RANGE = 0.007 TO 0.063

Z-SCORE	RAW SCORE
-4	3
-3	2
-2	1
-1	1
0	1
0.01	1
0.02	1
0.03	1
0.04	1
0.05	1
0.06	1
0.07	1
0.08	1
0.09	1
0.10	1
0.11	1
0.12	1
0.13	1
0.14	1
0.15	1
0.16	1
0.17	1
0.18	1
0.19	1
0.20	1
0.21	1
0.22	1
0.23	1
0.24	1
0.25	1
0.26	1
0.27	1
0.28	1
0.29	1
0.30	1
0.31	1
0.32	1
0.33	1
0.34	1
0.35	1
0.36	1
0.37	1
0.38	1
0.39	1
0.40	1
0.41	1
0.42	1
0.43	1
0.44	1
0.45	1
0.46	1
0.47	1
0.48	1
0.49	1
0.50	1
0.51	1
0.52	1
0.53	1
0.54	1
0.55	1
0.56	1
0.57	1
0.58	1
0.59	1
0.60	1
0.61	1
0.62	1
0.63	1
0.64	1
0.65	1
0.66	1
0.67	1
0.68	1
0.69	1
0.70	1
0.71	1
0.72	1
0.73	1
0.74	1
0.75	1
0.76	1
0.77	1
0.78	1
0.79	1
0.80	1
0.81	1
0.82	1
0.83	1
0.84	1
0.85	1
0.86	1
0.87	1
0.88	1
0.89	1
0.90	1
0.91	1
0.92	1
0.93	1
0.94	1
0.95	1
0.96	1
0.97	1
0.98	1
0.99	1
1.00	1

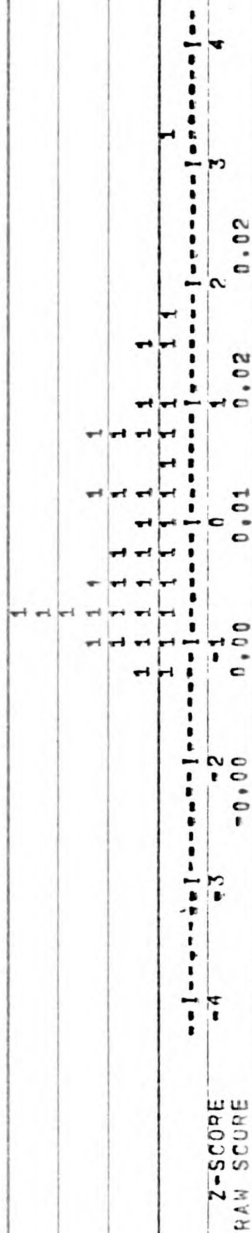
SET 1 VARIABLE 9

N = 37

MEAN = 0.011
 STD. DEV. = 0.007 (0.007)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 65.835 (66.743)
 SKEWNESS = 1.166 (1.216)
 KURTOSIS = 1.481 (1.884)

LOWER QUARTILE = 0.005
 MEDIAN = 0.009
 UPPER QUARTILE = 0.015

RANGE = 0.001 TO 0.034



SET 1 VARIABLE 10

N = 37

MEAN = 0.008
 STD. DEV. = 0.004 (0.004)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 54.709 (55.464)
 SKEWNESS = 0.601 (0.627)
 KURTOSIS = -0.266 (-0.124)

LOWER QUARTILE = 0.005
 MEDIAN = 0.006
 UPPER QUARTILE = 0.011

RANGE = 0.001 TO 0.019



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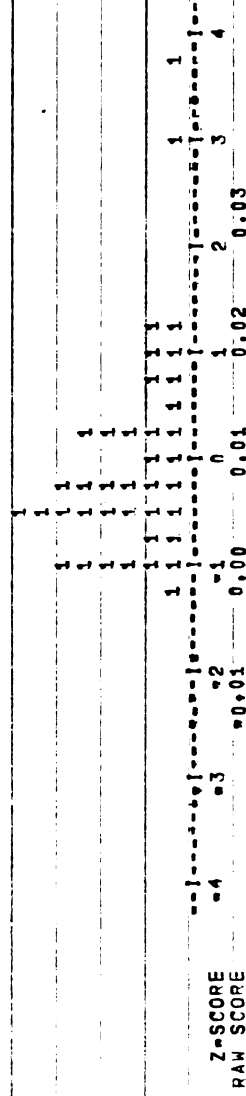
SPT 1 VARIABLE 1

N = 39

MEAN = 0.012
 STD. DEV. = 0.009 (0.009)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 74.107 (75.075)
 SKEWNESS = 1.815 (1.888)
 KURTOSIS = 3.830 (4.541)

LOWER QUANTILE = 0.007
 MEDIAN = 0.009
 UPPER QUANTILE = 0.015

RANGE = 0.002 TO 0.044



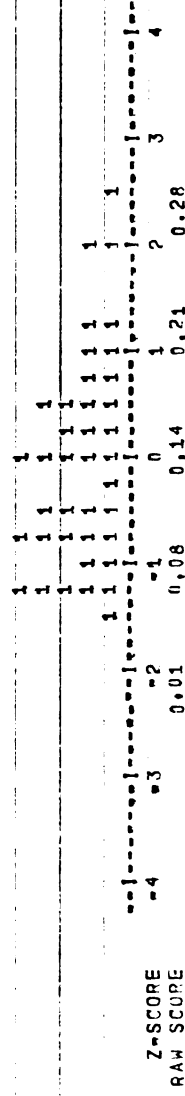
SPT 1 VARIABLE 2

N = 39

MEAN = 0.143
 STD. DEV. = 0.067 (0.068)
 VARIANCE = 0.004 (0.005)
 COEF. OF VAR. = 46.938 (47.552)
 SKEWNESS = 0.628 (0.654)
 KURTOSIS = 0.136 (0.016)

LOWER QUANTILE = 0.092
 MEDIAN = 0.139
 UPPER QUANTILE = 0.183

RANGE = 0.041 TO 0.316

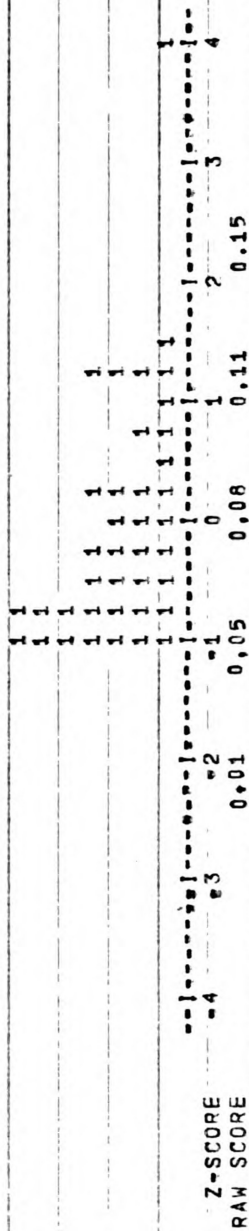


SET 1 VARIABLE 3

N = 39
 MEAN = 0.079
 STD. DEV. = 0.034 (0.034)
 VARIANCE = 0.001 (0.001)
 DEF. OF VAR. = 42.889 (43.449)
 SKEWNESS = 1.759 (1.830)
 KURTOSIS = 4.378 (5.167)

LOWER QUARTILE = 0.053
 MEDIAN = 0.072
 UPPER QUARTILE = 0.097

RANGE = 0.043 TO 0.215

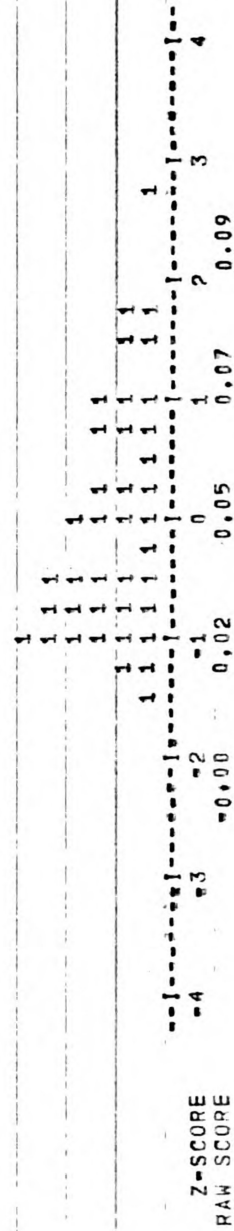


SET 1 VARIABLE 4

N = 39
 MEAN = 0.046
 STD. DEV. = 0.024 (0.024)
 VARIANCE = 0.001 (0.001)
 DEF. OF VAR. = 52.406 (53.091)
 SKEWNESS = 0.708 (0.737)
 KURTOSIS = -0.301 (-0.172)

LOWER QUARTILE = 0.027
 MEDIAN = 0.037
 UPPER QUARTILE = 0.066

RANGE = 0.012 TO 0.110



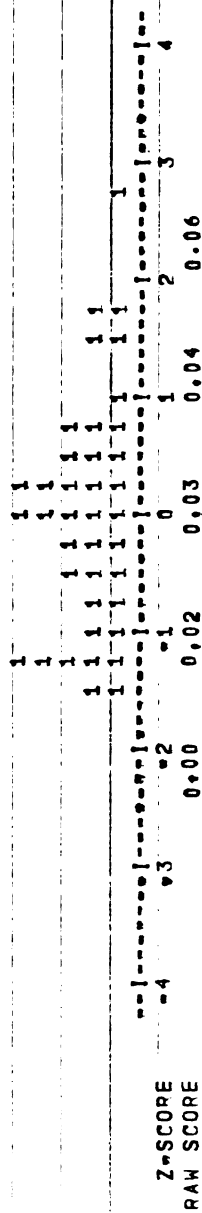
SET 1 VARIABLE 5

N = 39

MEAN = 0.029
 STD. DEV. = 0.014 (0.014)
 VARIANCE = 0.000 (0.000)
 DEF. OF VAR. = 48.375 (49.007)
 SKEWNESS = 0.507 (0.527)
 KURTOSIS = 0.076 (0.085)

LOWER QUANTILE = 0.019
 MEDIAN = 0.029
 UPPER QUANTILE = 0.038

RANGE = 0.007 TO 0.068



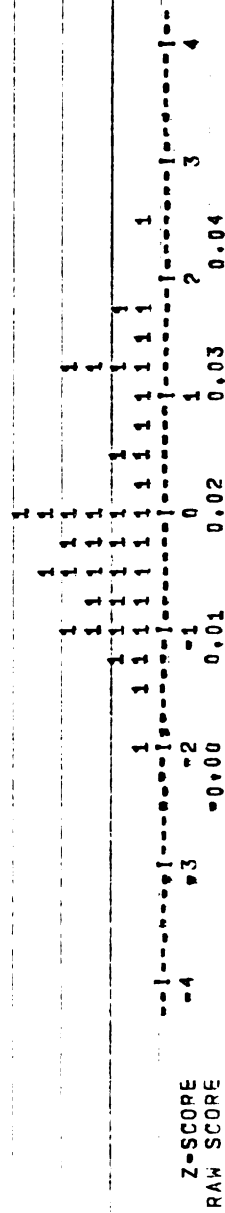
SET 1 VARIABLE 6

N = 39

MEAN = 0.021
 STD. DEV. = 0.011 (0.011)
 VARIANCE = 0.000 (0.000)
 DEF. OF VAR. = 50.032 (50.686)
 SKEWNESS = 0.539 (0.561)
 KURTOSIS = 0.205 (0.063)

LOWER QUANTILE = 0.014
 MEDIAN = 0.020
 UPPER QUANTILE = 0.028

RANGE = 0.001 TO 0.049

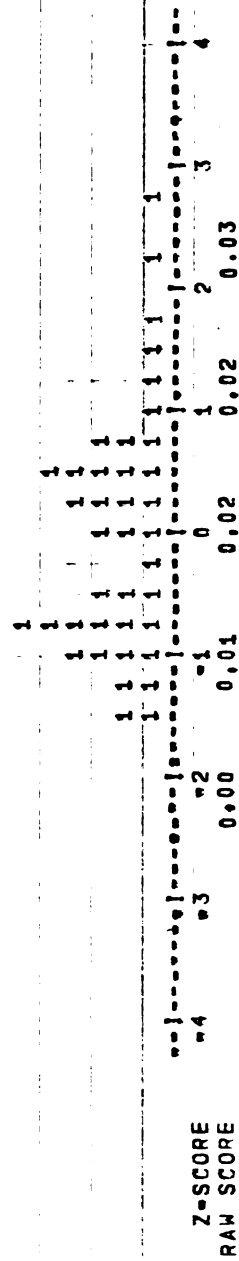


SFT 1 VARIABLE 7

N = 39
 MEAN = 0.017
 STD. DEV. = 0.007 (0.007)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 42.984 (43.546)
 SKEWNESS = 0.720 (0.749)
 KURTOSIS = 0.158 (0.352)

LOWER QUARTILE = 0.011
 MEDIAN = 0.017
 UPPER QUARTILE = 0.021

RANGE = 0.005 TO 0.038

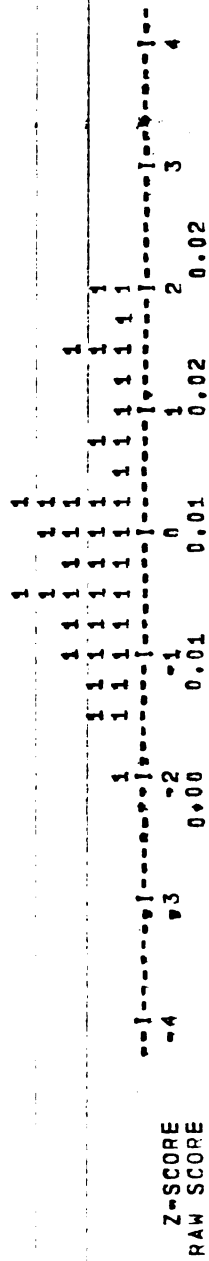


SFT 1 VARIABLE 8

N = 39
 MEAN = 0.012
 STD. DEV. = 0.006 (0.006)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 49.888 (50.540)
 SKEWNESS = 0.330 (0.343)
 KURTOSIS = -0.659 (-0.581)

LOWER QUARTILE = 0.008
 MEDIAN = 0.012
 UPPER QUARTILE = 0.017

RANGE = 0.000 TO 0.025



SET 1 VARIABLE 9

N = 39
 MEAN = 0.012
 STD. DEV. = 0.006 (0.006)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 51.126 (51.794)
 SKEWNESS = 0.444 (0.462)
 KURTOSIS = 0.489 (0.387)

LOWER QUARTILE = 0.007
 MEDIAN = 0.011
 UPPER QUARTILE = 0.017

RANGE = 0.001 TO 0.028



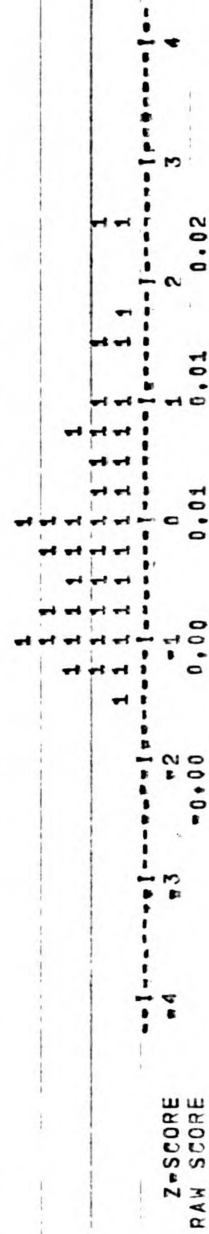
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SET 1 VARIABLE 10

N = 39
 MEAN = 0.008
 STD. DEV. = 0.004 (0.005)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 55.006 (55.726)
 SKEWNESS = 0.797 (0.829)
 KURTOSIS = 0.012 (0.185)

LOWER QUARTILE = 0.005
 MEDIAN = 0.007
 UPPER QUARTILE = 0.011

RANGE = 0.002 TO 0.020



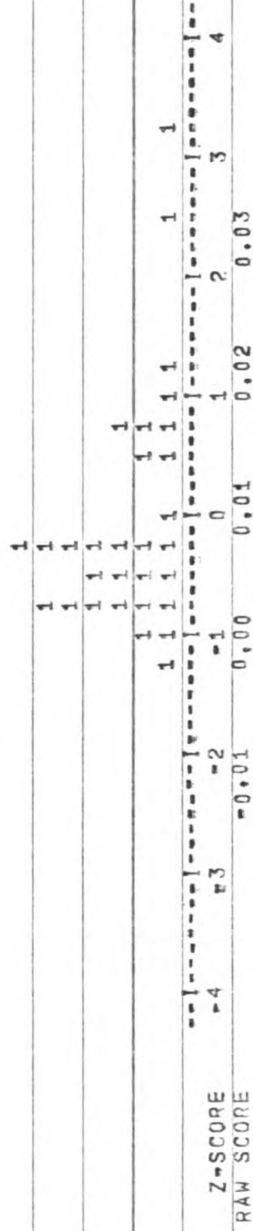
SET 1 VARIABLE 1

N = 30

MEAN = 0.011
STD. DEV. = 0.009 (0.009)
VARIANCE = 0.000 (0.000)
COEF. OF VAR. = 80.826 (82.207)
SKEWNESS = 1.643 (1.731)
KURTOSIS = 2.581 (3.299)

LOWER QUARTILE = 0.005
MEDIAN = 0.008
UPPER QUARTILE = 0.016

RANGE = 0.000 TO 0.041



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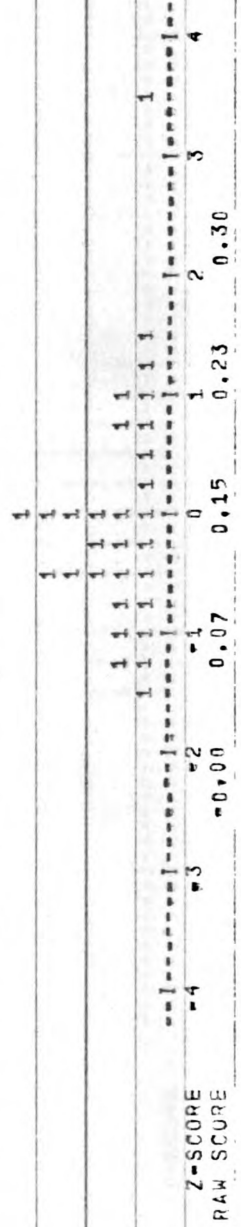
SET 1 VARIABLE 2

N = 30

MEAN = 0.150
STD. DEV. = 0.077 (0.078)
VARIANCE = 0.006 (0.006)
COEF. OF VAR. = 51.190 (52.065)
SKEWNESS = 1.502 (1.582)
KURTOSIS = 3.360 (4.226)

LOWER QUARTILE = 0.103
MEDIAN = 0.140
UPPER QUARTILE = 0.192

RANGE = 0.042 TO 0.427



SET 1 VARIABLE 3

N = 30

MEAN = 0.080
 STD. DEV. = 0.052 (0.052)
 VARIANCE = 0.003 (0.003)
 COEF. OF VAR. = 64.464 (65.566)
 SKEWNESS = 1.112 (1.171)
 KURTOSIS = 0.260 (0.539)

LOWER QUANTILE = 0.041
 MEDIAN = 0.061
 UPPER QUANTILE = 0.119

RANGE = 0.024 TO 0.213

Z-SCORE	RAW SCORE
-4	3
-3	2
-2	1
-1	0
0	0.03
1	0.08
2	0.13
3	0.18
4	

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SET 1 VARIABLE 4

N = 30

MEAN = 0.043
 STD. DEV. = 0.028 (0.028)
 VARIANCE = 0.001 (0.001)
 COEF. OF VAR. = 64.942 (66.052)
 SKEWNESS = 0.979 (1.032)
 KURTOSIS = 1.177 (1.629)

LOWER QUANTILE = 0.026
 MEDIAN = 0.042
 UPPER QUANTILE = 0.059

RANGE = 0.002 TO 0.123

Z-SCORE	RAW SCORE
-4	3
-3	2
-2	1
-1	0
0	0.02
1	0.04
2	0.07
3	0.10
4	

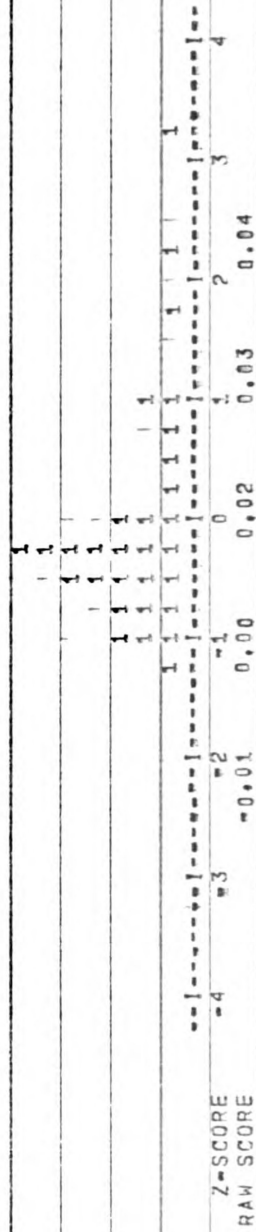
SET 1 VARIABLE 7

N = 30

MEAN = 0.016
 STD. DEV. = 0.012 (0.012)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 73.238 (74.490)
 SKEWNESS = 1.655 (1.743)
 KURTOSIS = 2.619 (3.345)

LOWER QUARTILE = 0.009
 MEDIAN = 0.013
 UPPER QUARTILE = 0.019

RANGE = 0.001 TO 0.057



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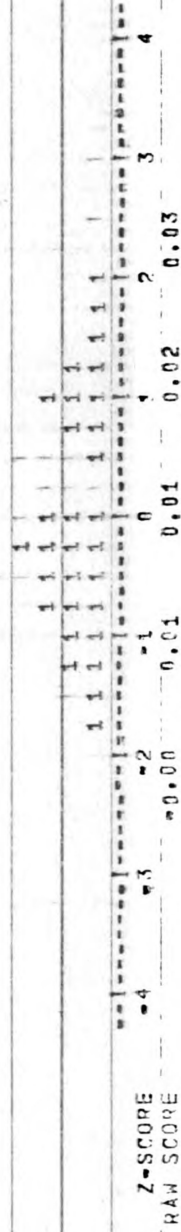
SET 1 VARIABLE 8

N = 30

MEAN = 0.015
 STD. DEV. = 0.009 (0.009)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 60.526 (61.560)
 SKEWNESS = 0.311 (0.328)
 KURTOSIS = -1.008 (-0.969)

LOWER QUARTILE = 0.009
 MEDIAN = 0.012
 UPPER QUARTILE = 0.023

RANGE = 0.000 TO 0.032



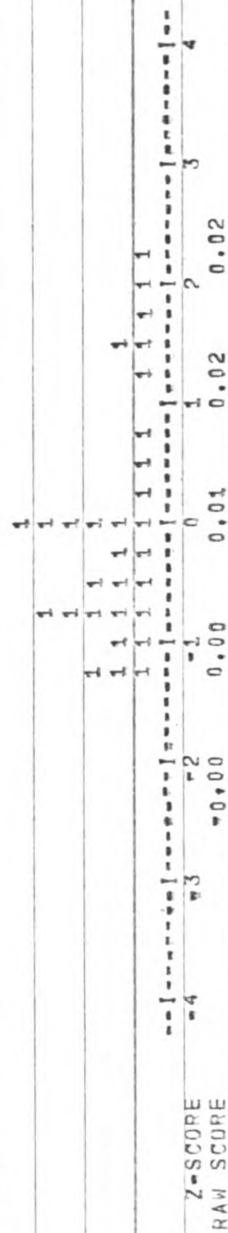
SET 1 VARIABLE 9

N = 30

MEAN = 0.011
 STD. DEV. = 0.007 (0.007)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 56.806 (57.777)
 SKEWNESS = 0.743 (0.783)
 KURTOSIS = -0.475 (-0.335)

LOWER QUARTILE = 0.007
 MEDIAN = 0.010
 UPPER QUARTILE = 0.015

RANGE = 0.003 TO 0.025



75

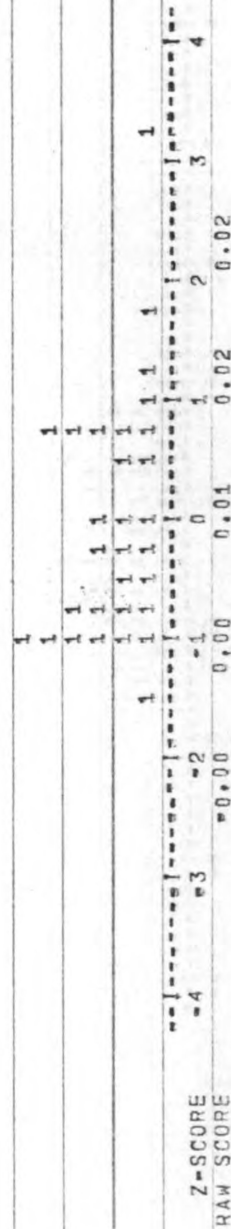
SET 1 VARIABLE 10

N = 30

MEAN = 0.010
 STD. DEV. = 0.006 (0.006)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 58.919 (59.927)
 SKEWNESS = 1.050 (1.106)
 KURTOSIS = 1.431 (1.932)

LOWER QUARTILE = 0.006
 MEDIAN = 0.009
 UPPER QUARTILE = 0.015

RANGE = 0.001 TO 0.029



155 1

VARIABLE 1

VARIABLE 1

35
"
2

MEAN = 0.912

600.00

ANCE = 0.000

WALC	=	0.000
VAL	=	73.435

YES. = 1-713
YES. = 1-713

1.713
3.353

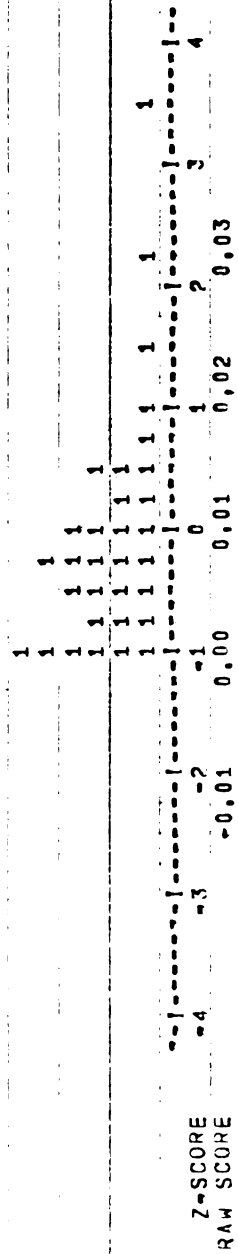
$$0.005 = 0.005$$

ME: JIAN = 0.010

[illegible]

RANGE = 0.002 TO 0.044

RANGE = 0.002 TO 0.044



76

SET 1

VARIABLE 2

VARIABLE 2

32
=

$$M_{\Sigma}^2 \Delta N = 0.148$$

0.006

Adjusted = 0.007

VA	50.093
VA	50.093

10. 093
10. 773

[illegible]

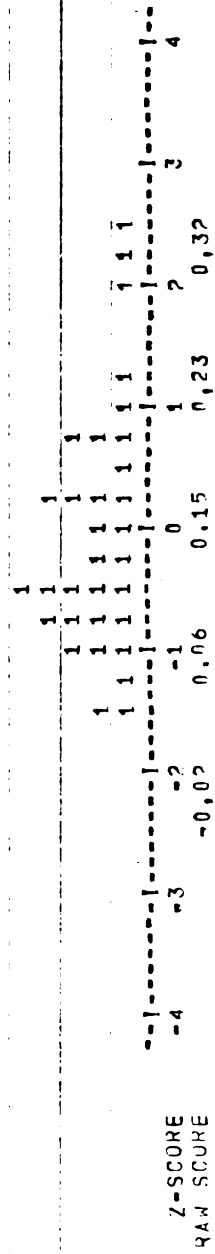
LOWFEE QUALITY = 0.084

COEFFICIENT =	0.1184
MEAN =	0.127

1953 011110 336d11
H-011110 336d11
0.127 0.202

RAVGE =	0.024 TO	0.359
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RAVGE =	0.024 TO	0.359
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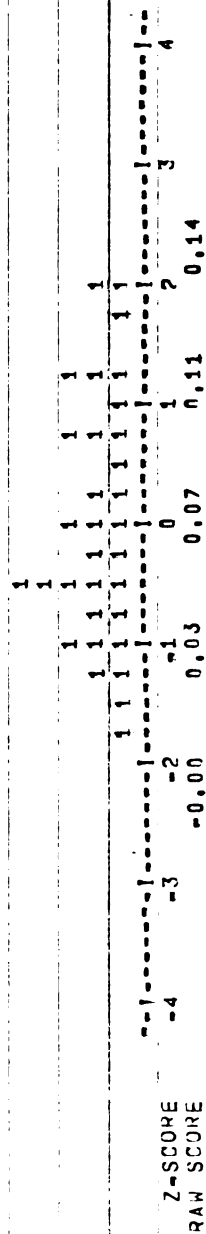


SET 1 VARIABLE 3

N = 32
 MEAN = 0.070
 STD. DEV. = 0.036 (0.036)
 VARIANCE = 0.001 (0.001)
 COEF. OF VAR. = 51.055 (51.872)
 SKEWNESS = 0.347 (0.364)
 KURTOSIS = -0.777 (-0.700)

LOWER QUARTILE = 0.040
 MEDIAN = 0.066
 UPPER QUARTILE = 0.096

RANGE = 0.007 TO 0.145



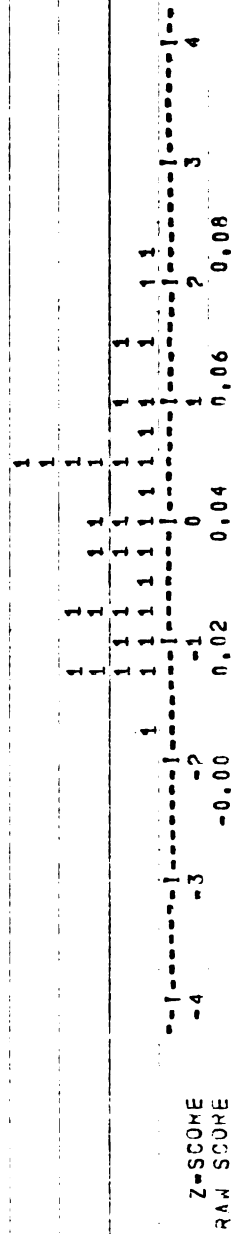
77

SET 1 VARIABLE 4

N = 32
 MEAN = 0.041
 STD. DEV. = 0.021 (0.022)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 51.653 (52.480)
 SKEWNESS = 0.376 (0.397)
 KURTOSIS = -0.552 (-0.435)

LOWER QUARTILE = 0.025
 MEDIAN = 0.040
 UPPER QUARTILE = 0.054

RANGE = 0.006 TO 0.090

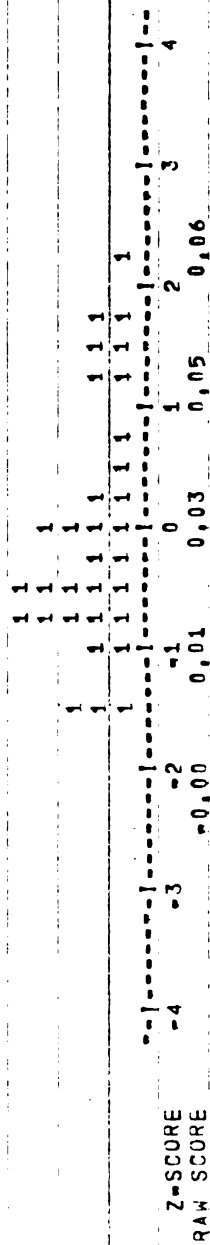


SET 1 VARIABLE 5

N = 32
 MEAN = 0.029
 STD. DEV. = 0.016 (0.016)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 55.001 (55.881)
 SKEWNESS = 0.560 (0.588)
 KURTOSIS = -0.600 (-0.597)

LOWER QUANTILE = 0.017
 1-QUANT = 0.027
 UPPER QUANTILE = 0.040

RANGE = 0.005 TO 0.064



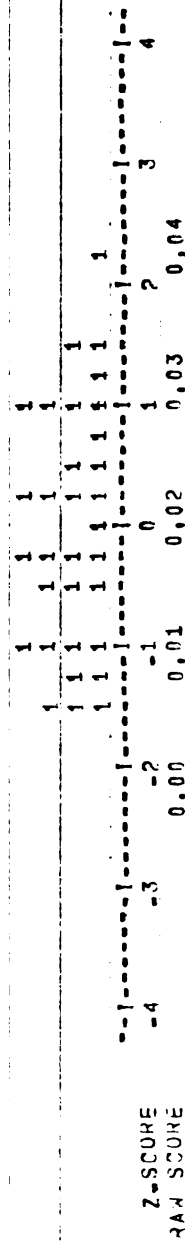
78

SET 1 VARIABLE 6

N = 32
 MEAN = 0.021
 STD. DEV. = 0.010 (0.010)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 47.493 (48.243)
 SKEWNESS = 0.234 (0.245)
 KURTOSIS = -0.756 (-0.675)

LOWER QUANTILE = 0.011
 1-QUANT = 0.020
 UPPER QUANTILE = 0.030

RANGE = 0.005 TO 0.044



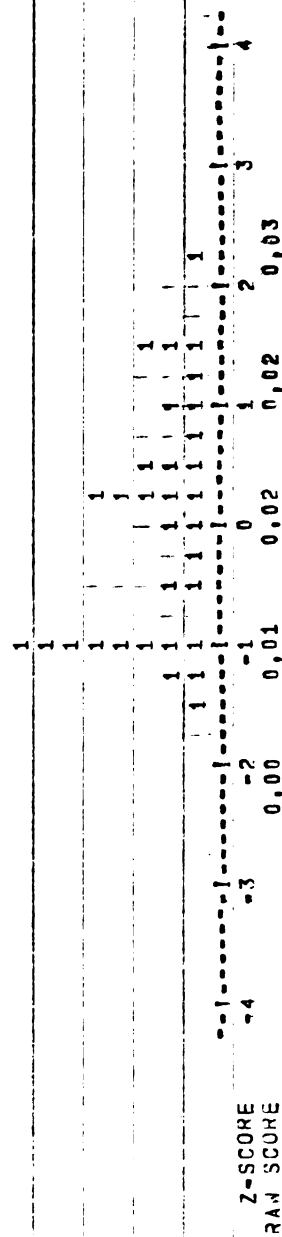
SET 1 VARIABLE 7

N = 32

MEAN = 0.016
 STD. DEV. = 0.007 (0.007)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 44.028 (44.752)
 SKEWNESS = 0.333 (0.349)
 KURTOSIS = -0.801 (-0.729)

LOWER QUANTILE = 0.009
 MEDIAN = 0.017
 UPPER QUANTILE = 0.020

RANGE = 0.005 TO 0.032



79

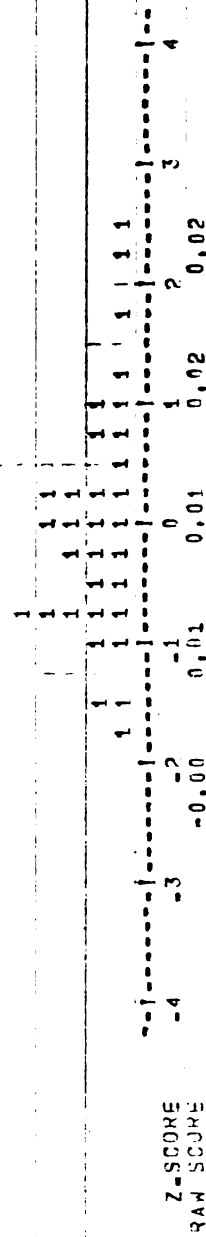
SET 1 VARIABLE 8

N = 32

MEAN = 0.012
 STD. DEV. = 0.006 (0.006)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 52.871 (53.717)
 SKEWNESS = 0.639 (0.671)
 KURTOSIS = -0.034 (0.173)

LOWER QUANTILE = 0.007
 MEDIAN = 0.011
 UPPER QUANTILE = 0.016

RANGE = 0.002 TO 0.028



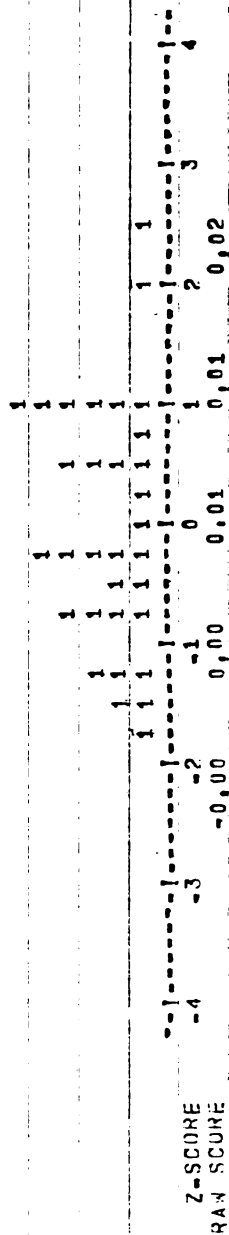
SET 1 VARIABLE 9

N = 32

MEAN = 0.009 (0.005)
 STD. DEV. = 0.000 (0.000)
 VARIANCE = 56.948 (0.000)
 COEF. OF VAR. = 0.295 (0.310)
 SKEWNESS = -0.513 (-0.390)
 KURTOSIS =

LOWER QUANTILE = 0.005
 MEDIAN = 0.008
 UPPER QUANTILE = 0.013

RANGE = 0.000 TO 0.021



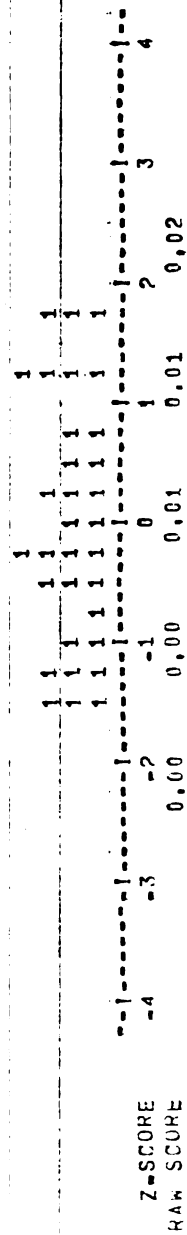
SET 1 VARIABLE 10

N = 32

MEAN = 0.009 (0.004)
 STD. DEV. = 0.004 (0.000)
 VARIANCE = 48.325 (0.000)
 COEF. OF VAR. = 0.157 (0.164)
 SKEWNESS = -1.046 (-1.016)
 KURTOSIS =

LOWER QUANTILE = 0.005
 MEDIAN = 0.008
 UPPER QUANTILE = 0.012

RANGE = 0.002 TO 0.016



HISTOGRAMS

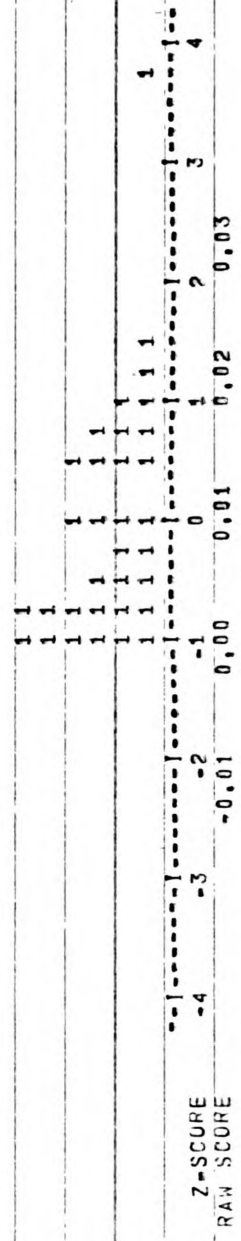
SET 1 VARIABLE 1

← SIZE OF SAMPLE

N = 37
 MEAN = 0.012
 STD. DEV. = 0.011 (0.011)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 9.567 (90.956)
 SKEWNESS = 1.634 (1.713)
 KURTOSIS = 3.559 (4.721)

LOWER QUANTILE = 0.004
 MEDIAN = 0.010
 UPPER QUANTILE = 0.018

RANGE = 0.011 TO 0.053



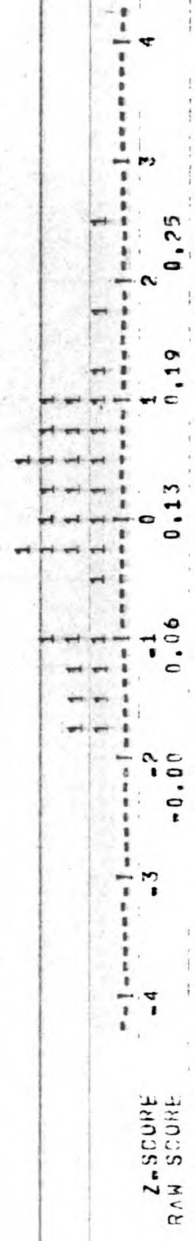
HARMONIC NUMBER

SET 2 VARIABLE 2

N = 37
 MEAN = 0.127
 STD. DEV. = 0.064 (0.065)
 VARIANCE = 0.004 (0.004)
 COEF. OF VAR. = 51.192 (51.192)
 SKEWNESS = 0.099 (0.104)
 KURTOSIS = -0.342 (-0.193)

LOWER QUANTILE = 0.065
 MEDIAN = 0.130
 UPPER QUANTILE = 0.169

RANGE = 0.013 TO 0.286



Z-SCORE
 RAW SCORE

100

100

[illegible]

1

11



11

1

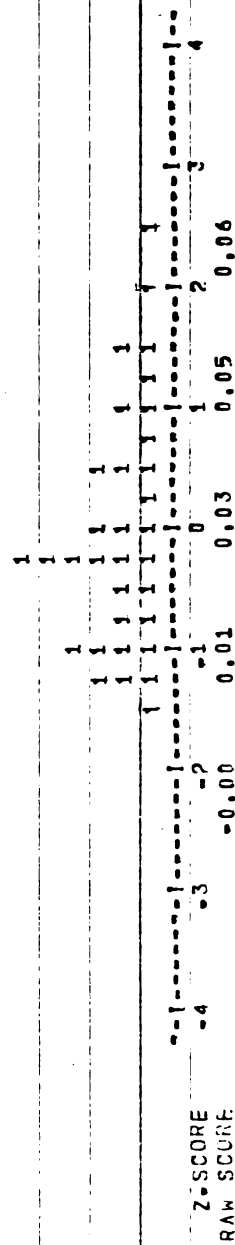


SET 1 VARIABLE 5

N = 33
 MEAN = 0.030
 STD. DEV. = 0.016 (0.016)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 52.850 (53.162)
 SKEWNESS = 0.667 (0.699)
 KURTOSIS = -0.278 (-0.072)

LOWER QUANTILE = 0.017
 MEDIAN = 0.027
 UPPER QUANTILE = 0.040

RANGE = 0.000 TO 0.072



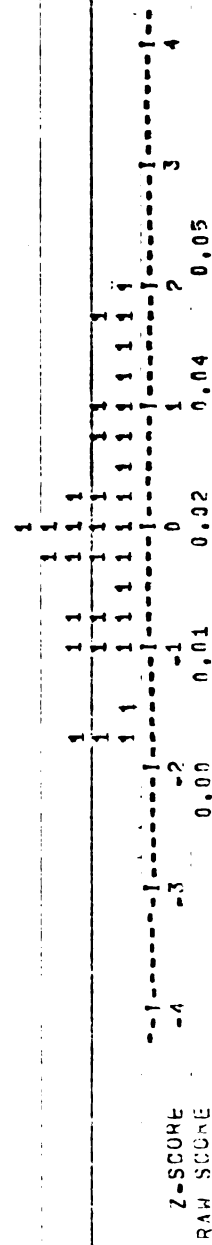
83

SET 1 VARIABLE 6

N = 33
 MEAN = 0.025
 STD. DEV. = 0.011 (0.011)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 44.824 (45.214)
 SKEWNESS = 0.136 (0.143)
 KURTOSIS = -0.671 (-0.579)

LOWER QUANTILE = 0.017
 MEDIAN = 0.026
 UPPER QUANTILE = 0.032

RANGE = 0.005 TO 0.047

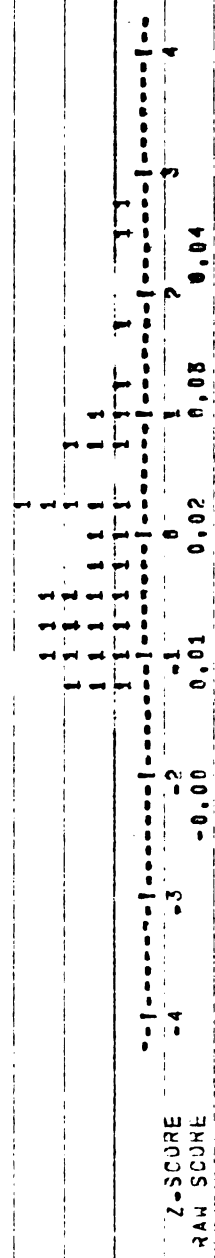


SET 1 VARIABLE 7

N = 33
MEAN = 0.017
STD. DEV. = 0.010 (0.011)
VARIANCE = 0.000 (0.000)
COEF. OF VAR. = 62.293 (63.256)
SKEWNESS = 1.025 (1.973)
KURTOSIS = 0.627 (0.940)

LOWER QUANTILE = 0.000
MEDIAN = 0.015
UPPER QUANTILE = 0.024

RANGE = 0.004 TO 0.045

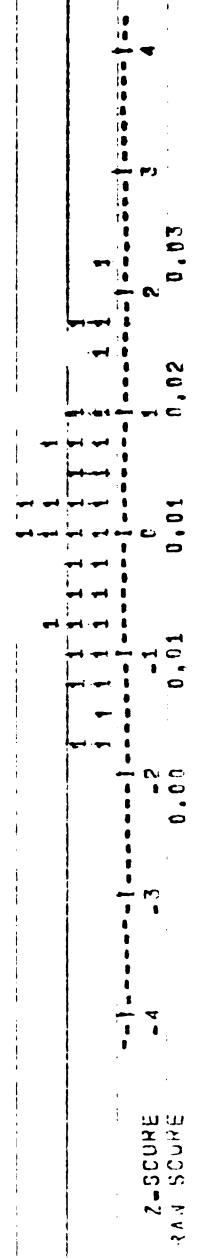


SET 1 VARIABLE 8

N = 33
MEAN = 0.014
STD. DEV. = 0.006 (0.006)
VARIANCE = 0.000 (0.000)
COEF. OF VAR. = 42.869 (43.655)
SKEWNESS = 0.227 (0.258)
KURTOSIS = -0.541 (-0.426)

LOWER QUANTILE = 0.000
MEDIAN = 0.014
UPPER QUANTILE = 0.018

RANGE = 0.004 TO 0.028



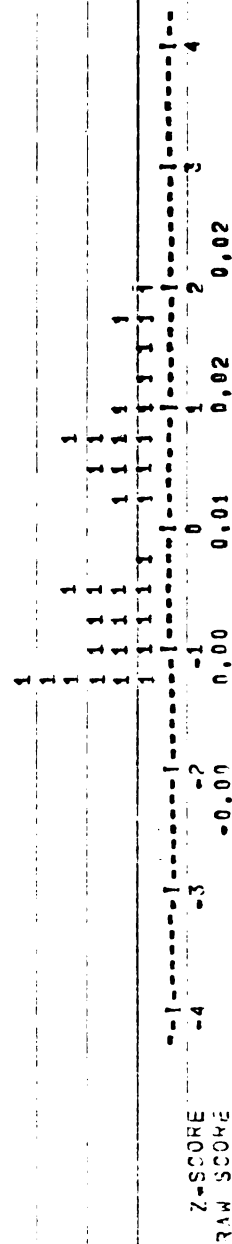
SET 1 VARIABLE 9

N = 33

MEAN = 0.010
 STD. DEV. = 0.007 (0.007)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 59.922 (71.007)
 SKEWNESS = 0.291 (0.305)
 KURTOSIS = -1.220 (-1.220)

LOWER QUANTILE = 0.003
 MEDIAN = 0.007
 UPPER QUANTILE = 0.015

RANGE = 0.000 TO 0.025



85

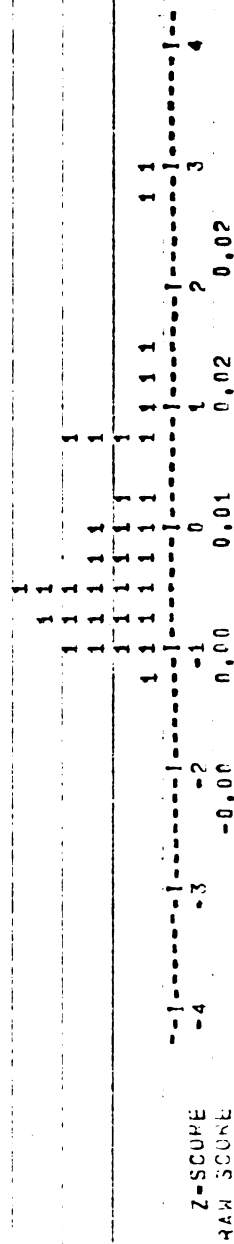
SET 1 VARIABLE 10

N = 33

MEAN = 0.009
 STD. DEV. = 0.006 (0.006)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 67.356 (66.370)
 SKEWNESS = 1.329 (1.393)
 KURTOSIS = 1.348 (1.784)

LOWER QUANTILE = 0.005
 MEDIAN = 0.007
 UPPER QUANTILE = 0.013

RANGE = 0.002 TO 0.027



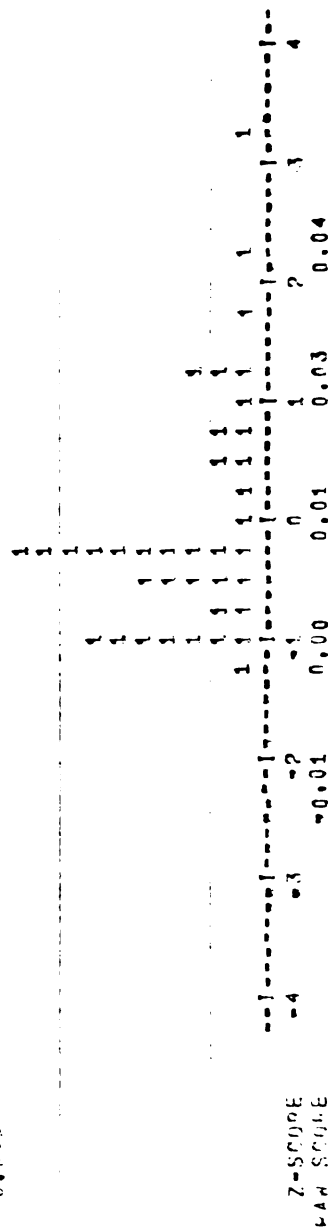
VARIABLE 1

N = 58

MEAN = 0.015
 STD. DEV. = 0.012 (0.012)
 VARIANCE = 0.000 (0.000)
 CORR. OF VAR. = 0.2382
 CORRELATION = 1.374 (1.389)
 AUTOCORR = 1.000 (2.121)

COEFF. OF VARIATION = 0.005
 MEAN = 0.012
 CORRELATION = 0.2382

MEAN = 0.011 TO 0.055



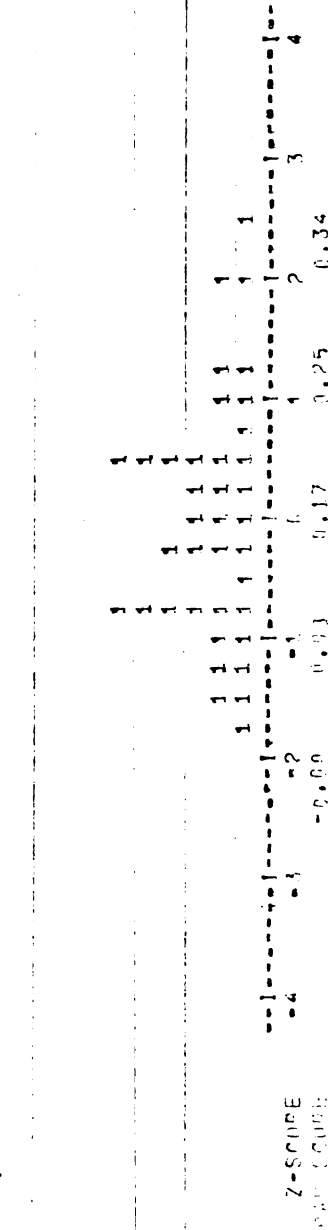
SET 1 VARIABLE 2

N = 58

MEAN = 0.007
 STD. DEV. = 0.007 (0.007)
 VARIANCE = 0.000 (0.000)
 CORR. OF VAR. = 0.176 (1.643)
 CORRELATION = 0.427 (0.445)
 AUTOCORR = 0.000 (-0.106)

COEFF. OF VARIATION = 0.007
 MEAN = 0.007
 CORRELATION = 0.176

MEAN = 0.012 TO 0.377

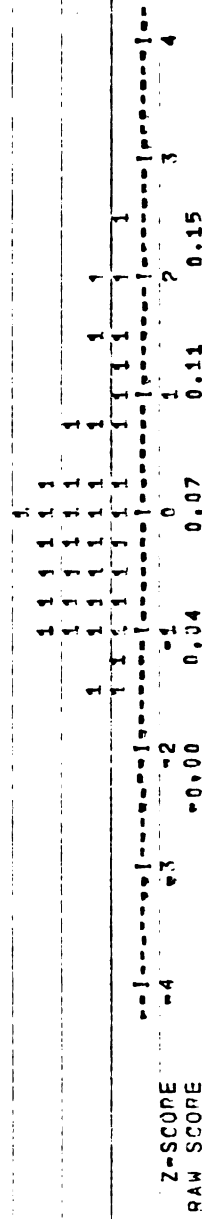


SFT 1 VARIABLE 3

N = 38
 MEAN = 0.075
 STD. DEV. = 0.038 (0.039)
 VARIANCE = 0.001 (0.001)
 COEF. OF VAR. = 50.511 (51.493)
 SKEWNESS = 0.721 (0.751)
 KURTOSIS = -0.103 (0.059)

LOWER QUANTILE = 0.044
 MEDIAN = 0.070
 UPPER QUANTILE = 0.099

RANGE = 0.016 TO 0.171

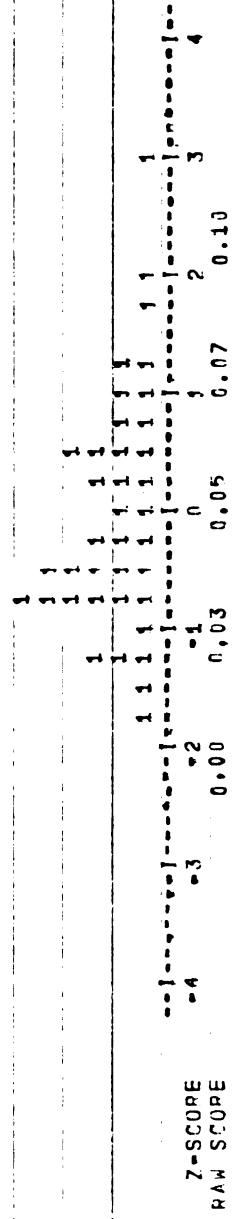


SFT 1 VARIABLE 4

N = 38
 MEAN = 0.049
 STD. DEV. = 0.023 (0.023)
 VARIANCE = 0.001 (0.001)
 COEF. OF VAR. = 47.272 (47.906)
 SKEWNESS = 0.748 (0.780)
 KURTOSIS = 0.424 (0.661)

LOWER QUANTILE = 0.032
 MEDIAN = 0.045
 UPPER QUANTILE = 0.063

RANGE = 0.010 TO 0.117



SET 1 VARIABLE 5

N = 38

MEAN = 0.033

STD. DEV. = 0.017 (0.017)

VARIANCE = 0.000 (0.000)

PER. OF VAR. = 51.314 (52.003)

SKEWNESS = 1.040 (1.004)

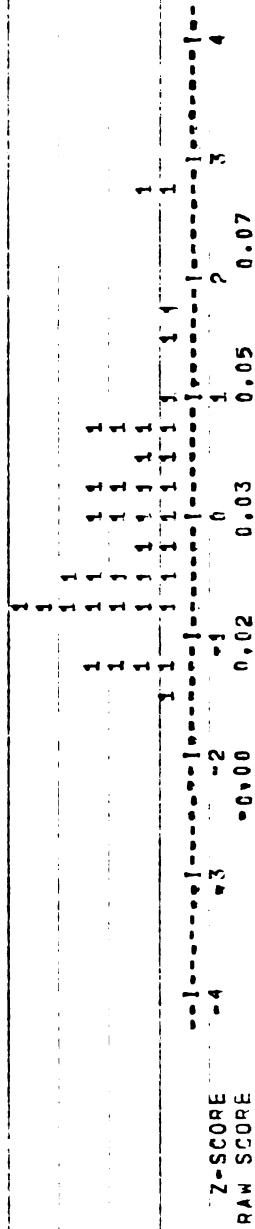
KURTOSIS = 0.901 (1.208)

LOWER QUARTILE = 0.021

MEDIAN = 0.031

UPPER QUARTILE = 0.044

RANGE = 0.009 TO 0.082



SET 1 VARIABLE 6

N = 32

MEAN = 0.019

STD. DEV. = 0.010 (0.010)

VARIANCE = 0.000 (0.000)

PER. OF VAR. = 51.077 (51.763)

SKEWNESS = 0.403 (0.513)

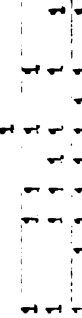
KURTOSIS = 0.341 (-0.214)

LOWER QUARTILE = 0.012

MEDIAN = 0.018

UPPER QUARTILE = 0.025

RANGE = 0.004 TO 0.044

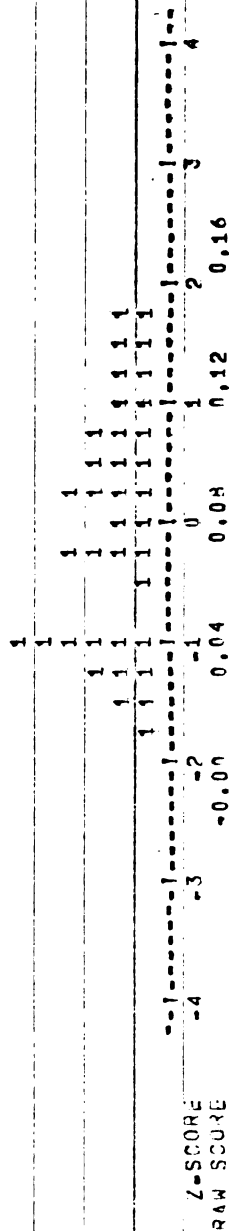


N = 37

MEAN = 0.080
 STD. DEV. = 0.041 (0.041)
 VARIANCE = 0.002 (0.002)
 COEF. OF VAR. = 50.006 (51.568)
 SKEWNESS = 0.002 (0.005)
 KURTOSIS = -1.173 (-1.167)

LOWER QUANTILE = 0.039
 MEDIAN = 0.082
 UPPER QUANTILE = 0.113

RANGE = 0.041 TO 0.153



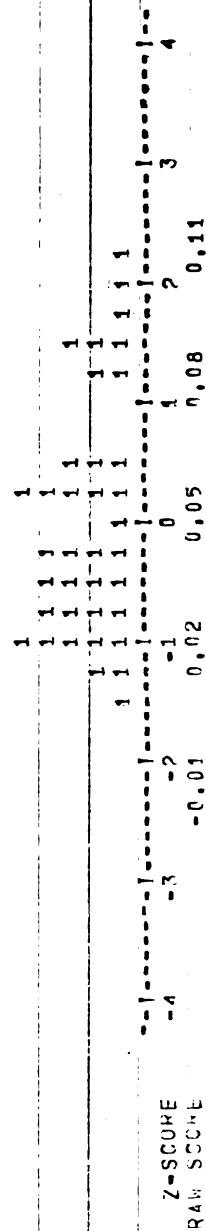
SET 1 VARIABLE 4

N = 37

MEAN = 0.050
 STD. DEV. = 0.071 (0.031)
 VARIANCE = 0.001 (0.001)
 COEF. OF VAR. = 41.560 (62.429)
 SKEWNESS = 0.577 (0.602)
 KURTOSIS = -0.669 (-0.587)

LOWER QUANTILE = 0.028
 MEDIAN = 0.042
 UPPER QUANTILE = 0.065

RANGE = 0.004 TO 0.119

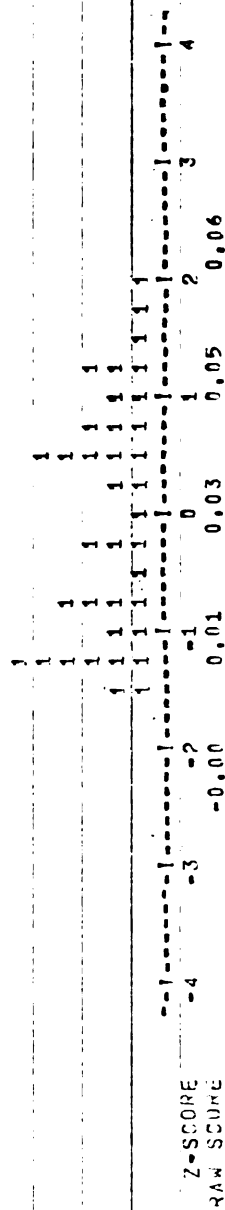


SET 1 VARIABLE 5

N = 37
 MEAN = 0.029
 STD. DEV. = 0.017 (0.017)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 57.779 (56.576)
 SKEWNESS = 0.150 (0.150)
 KURTOSIS = -1.196 (-1.196)

LOWER QUANTILE = 0.013
 MEDIAN = 0.031
 UPPER QUANTILE = 0.043

RANGE = 0.033 TO 0.064



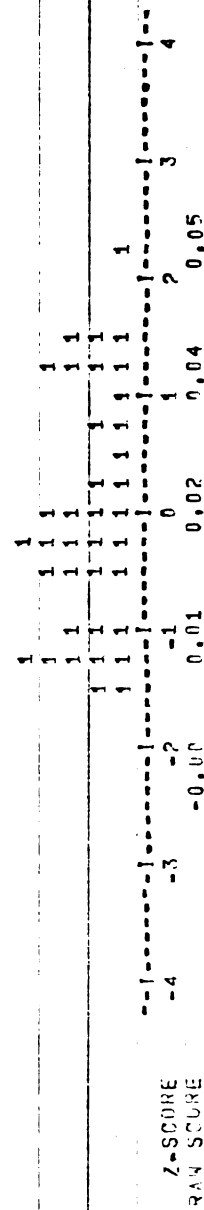
93

SET 1 VARIABLE 6

N = 37
 MEAN = 0.023
 STD. DEV. = 0.013 (0.013)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 55.821 (52.641)
 SKEWNESS = 0.303 (0.469)
 KURTOSIS = -0.777 (-0.712)

LOWER QUANTILE = 0.011
 MEDIAN = 0.021
 UPPER QUANTILE = 0.034

RANGE = 0.014 TO 0.053



N	=	37
MEAN	=	0.017
DEV	=	0.009
ANGLE	=	0.000
VAR	=	1.769
SE	=	0.917
ST	=	0.200

LOWER QUANTILE #	0.011
MEAN #	0.016
UPPER QUANTILE #	0.029

$$\text{Average} = \frac{0.0470}{6.040}$$

RAW SCORE	Z-SCORE
-4	-1.34
-3	-0.67
-2	0.00
-1	0.67
0	1.34
1	2.01
2	2.68
3	3.35
4	4.02

SET 1
VARIABLE 9

N	=	37
MEAN	=	0.016
DEV	=	0.019
AGE	=	0.010
VAR	=	54.914
STDEV	=	0.715
COVS	=	-0.129

LOWER QUANTILE =	0.109
MEDIAN =	0.018
UPPER QUANTILE =	0.023

RAYLE =	0.074 17	0.037
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Z-SCORE	RAW SCORE
-4	1
-3	1
-2	1
-1	1
0	1
1	1
2	1
3	1
4	1

SFT 1 VARIABLE 1

N = 42

MEAN = 0.012
 STD. DEV. = 0.010 (0.010)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 87.930 (88.996)
 SKEWNESS = 2.760 (2.863)
 KURTOSIS = 9.889 (11.333)

LOWER QUARTILE = 0.005
 MEDIAN = 0.010
 UPPER QUARTILE = 0.014

RANGE = 0.002 TO 0.061

Z-SCORE	RAW SCORE
-4	3
-3	0
-2	0
-1	0
0	0
1	0
2	0
3	0
4	0

96

SFT 1 VARIABLE 2

N = 42

MEAN = 0.147
 STD. DEV. = 0.094 (0.095)
 VARIANCE = 0.009 (0.009)
 COEF. OF VAR. = 63.798 (64.572)
 SKEWNESS = 1.706 (1.769)
 KURTOSIS = 3.254 (3.835)

LOWER QUARTILE = 0.087
 MEDIAN = 0.119
 UPPER QUARTILE = 0.179

RANGE = 0.029 TO 0.496

Z-SCORE	RAW SCORE
-4	3
-3	0
-2	0
-1	0
0	0
1	0
2	0
3	0
4	0

MEAN	=	0.073
STD. DEV.	=	0.040
VARIANCE	=	0.002
COEF. OF VAR.	=	54.978
SKEWNESS	=	0.397
KURTOSIS	=	0.828
		(0.041)
		(0.002)
		(55.645)
		(0.412)
		(-0.778)

LOWER QUANTILE =	0.039
MEDIAN =	0.071
UPPER QUANTILE =	0.107

RANGE = 0.005 TO 0.162

Z-SCORE	RAW SCORE
-4	1
-3	1
-2	1
-1	1
0	1
1	1
2	1
3	1
4	1

SET 1 VARIABLE 4

	MEAN =	0.051	
STD. DEV.	=	0.039	
VARIANCE	=	0.001	
COEF. OF VAR.	=	75.874	
SKENESS	=	1.626	
KURTOSIS	=	3.308	

LOWER QUARTILE =	0.024
MEDIAN =	0.038
UPPER QUARTILE =	0.068

RANGE = 0.007 TO 0.200

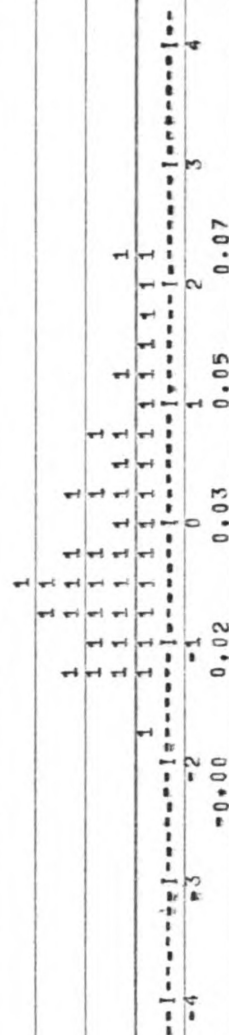
[illegible]

SET 1 VARIABLE 5

N = 42
 MEAN = 0.034
 STD. DEV. = 0.018 (0.018)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 51.585 (52.210)
 SKEWNESS = 0.627 (0.650)
 KURTOSIS = -0.458 (-0.360)

LOWER QUARTILE = 0.021
 MEDIAN = 0.030
 UPPER QUARTILE = 0.047

RANGE = 0.006 TO 0.075



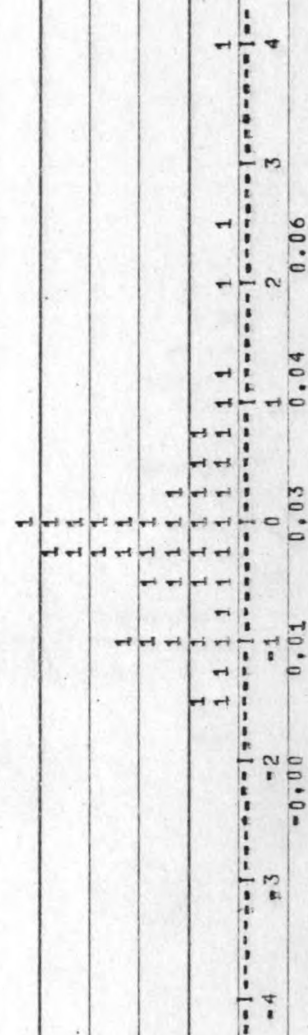
98

SET 1 VARIABLE 6

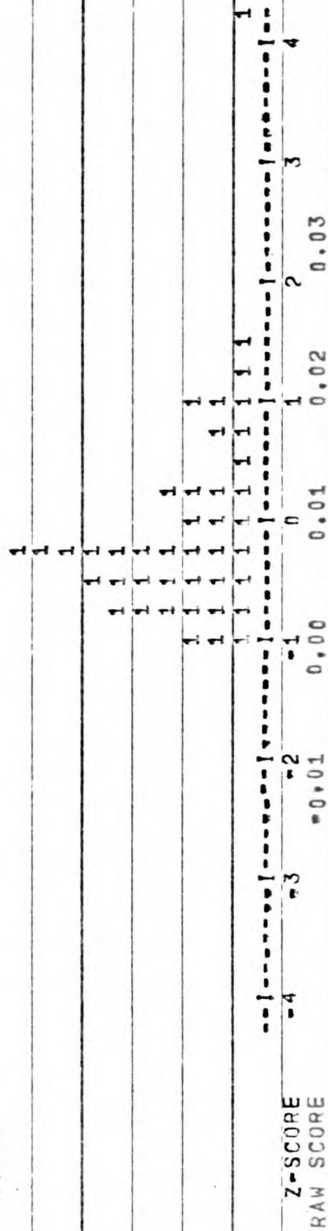
N = 42
 MEAN = 0.027
 STD. DEV. = 0.014 (0.015)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 53.582 (54.232)
 SKEWNESS = 1.689 (1.752)
 KURTOSIS = 4.130 (4.825)

LOWER QUARTILE = 0.019
 MEDIAN = 0.025
 UPPER QUARTILE = 0.030

RANGE = 0.005 TO 0.083

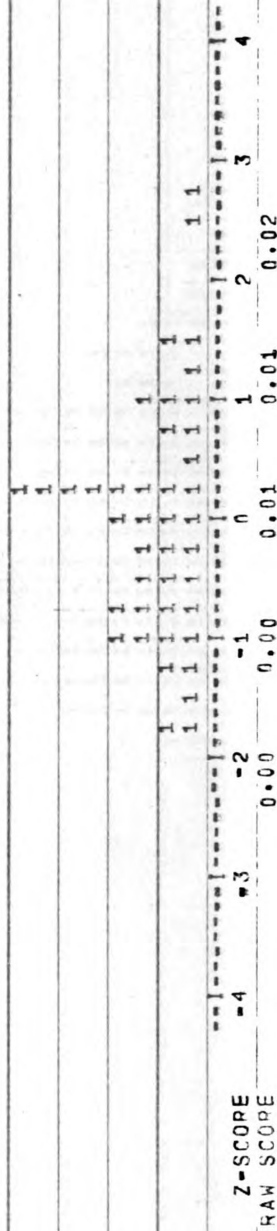


RANGE = 0.002 TO 0.052



SFT 1

RANGE = 0.002 TO 0.019

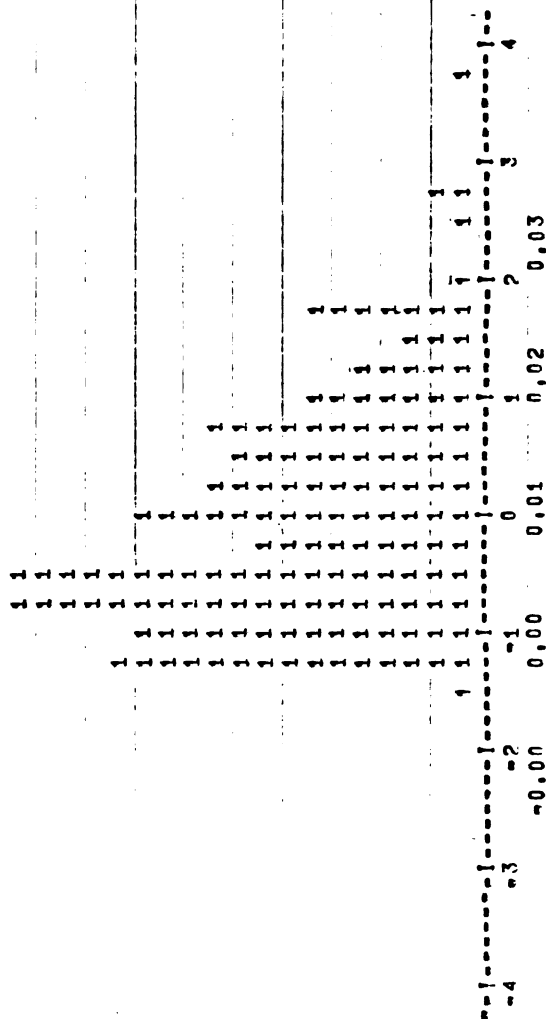


DATA SET 1

VARIABLE 1

N = 157
 MEAN = 0.012
 STD. DEV. = 0.008
 VARIANCE = 0.000
 COEF. OF VAR. = 67.334
 SKEWNESS = 0.885
 KURTOSIS = 0.644

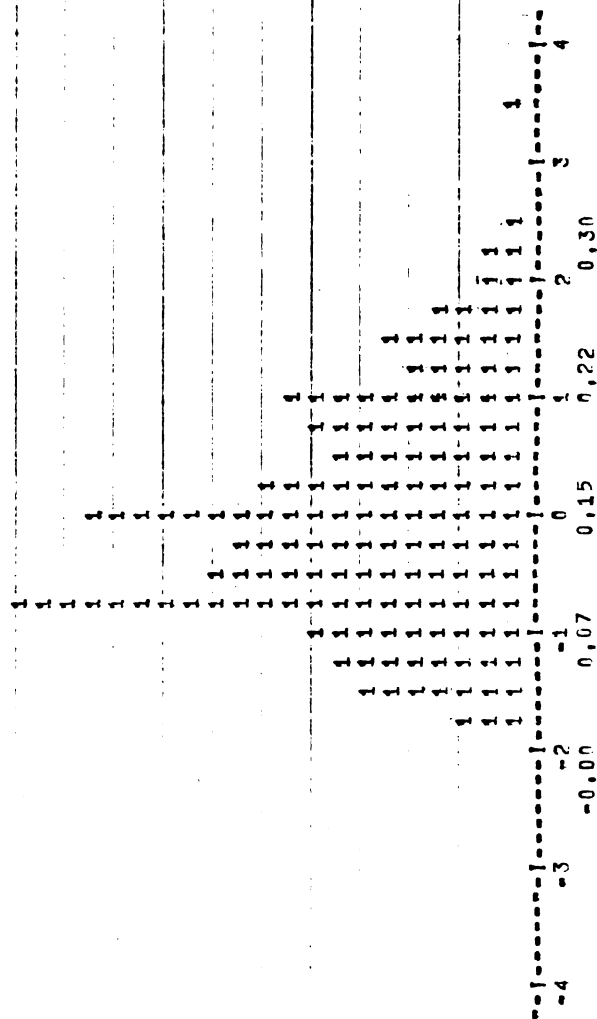
LOWER QUANTILE = 0.006
 MEDIAN = 0.011
 UPPER QUANTILE = 0.018
 RANGE = 0.001 TO 0.044



SET 1 VARIABLE 2

N = 157
 MEAN = 0.148
 STD. DEV. = 0.076
 VARIANCE = 0.006
 COEF. OF VAR. = 51.384
 SKEWNESS = 0.622
 KURTOSIS = 0.164

LOWER QUANTILE = 0.091
 MEDIAN = 0.140
 UPPER QUANTILE = 0.200
 RANGE = 0.005 TO 0.418

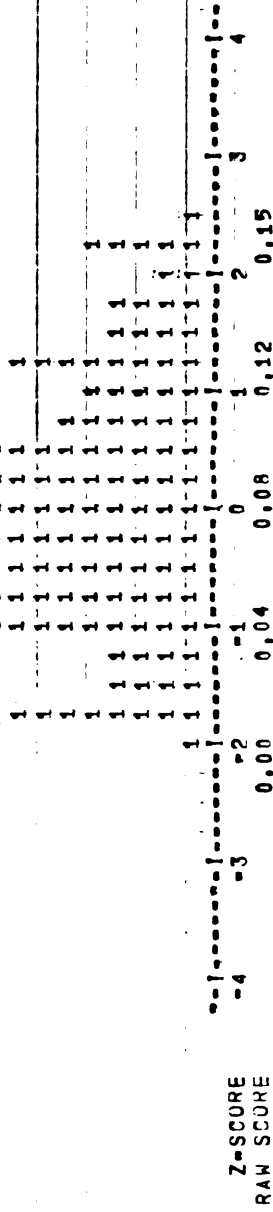


SET 1 VARIABLE 3

N = 150
 MEAN = 0.078
 STD. DEV. = 0.038
 VARIANCE = 0.001
 COEF. OF VAR. = 48.382
 SKEWNESS = 0.379
 KURTOSIS = -0.308

LOWER QUARTILE = 0.053
 MEDIAN = 0.077
 UPPER QUARTILE = 0.100

RANGE = 0.000 TO 0.168

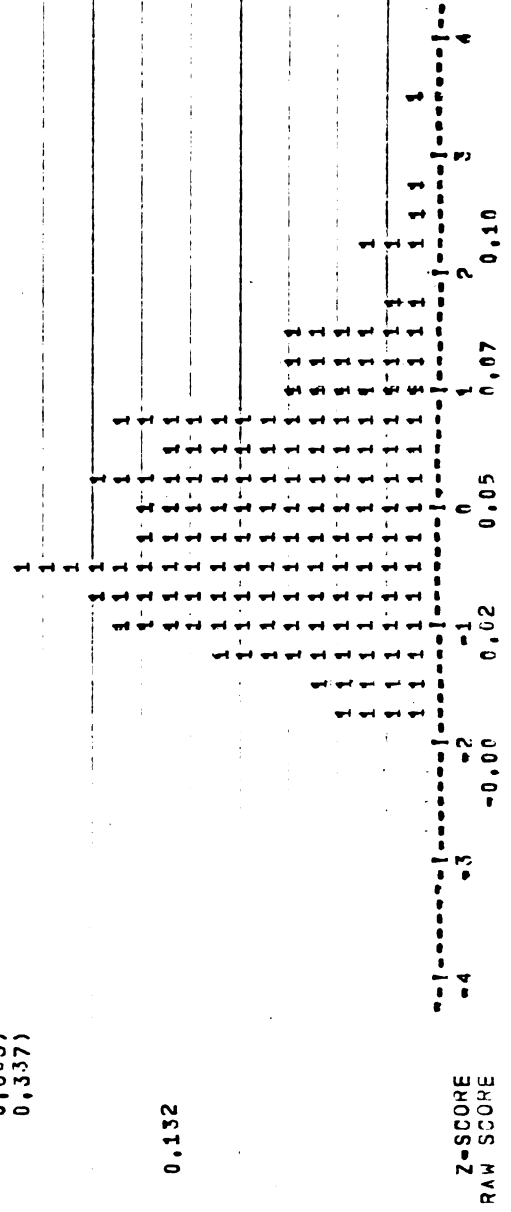


SET 1 VARIABLE 4

N = 150
 MEAN = 0.047
 STD. DEV. = 0.024
 VARIANCE = 0.001
 COEF. OF VAR. = 51.304
 SKEWNESS = 0.597
 KURTOSIS = 0.286

LOWER QUARTILE = 0.030
 MEDIAN = 0.044
 UPPER QUARTILE = 0.064

RANGE = 0.002 TO 0.132

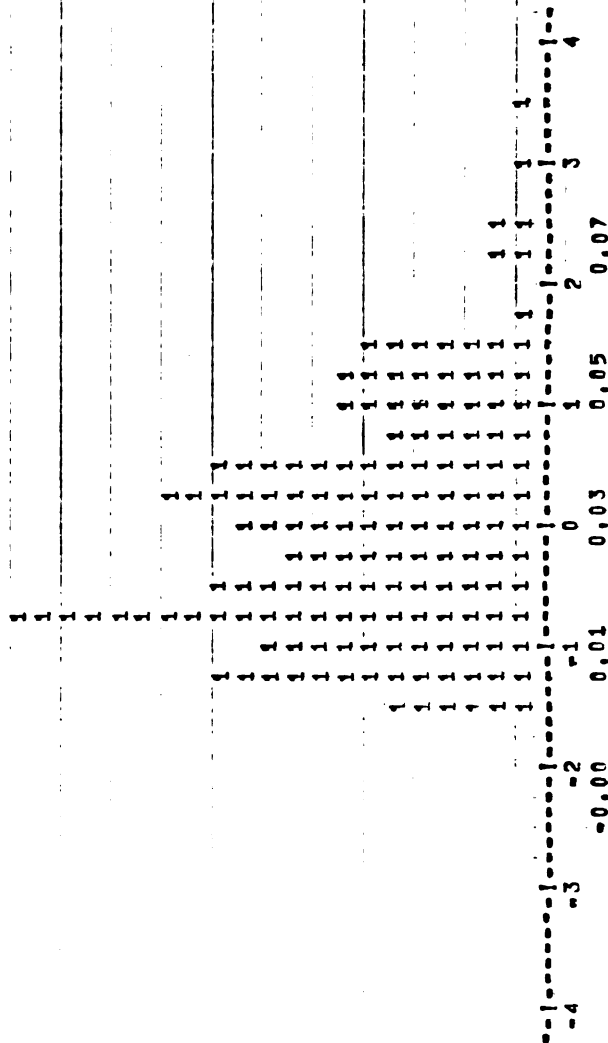


SET 1 VARIABLE 5

N = 151
 MEAN = 0.031
 STD. DEV. = 0.017 (0.017)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 56.295 (56.483)
 SKEWNESS = 0.731 (0.739)
 KURTOSIS = 0.398 (0.453)

LOWER QUANTILE = 0.018
 MEDIAN = 0.029
 UPPER QUANTILE = 0.041

RANGE = 0.003 TO 0.091

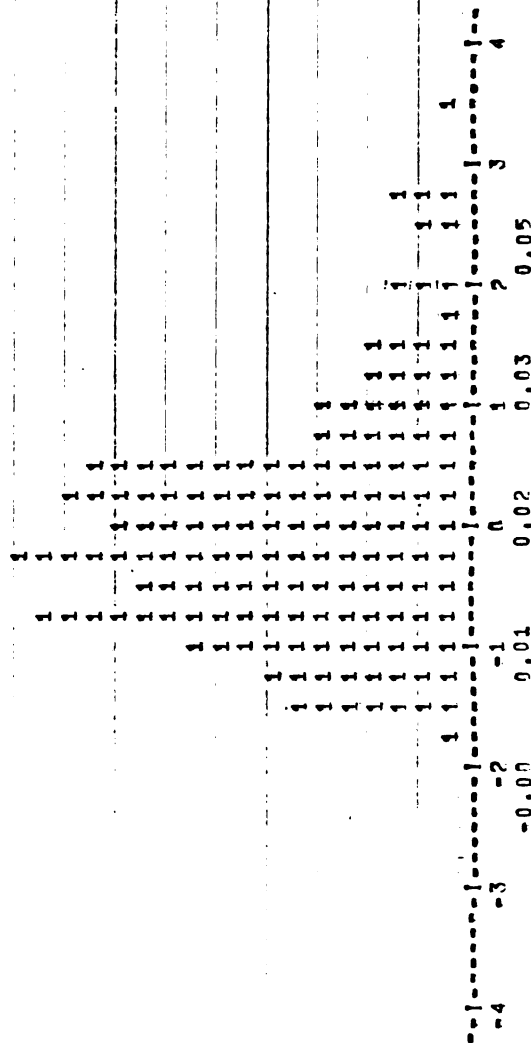


SET 1 VARIABLE 6

N = 151
 MEAN = 0.022
 STD. DEV. = 0.012 (0.012)
 VARIANCE = 0.000 (0.000)
 COEF. OF VAR. = 55.204 (55.389)
 SKEWNESS = 0.938 (0.948)
 KURTOSIS = 1.002 (1.078)

LOWER QUANTILE = 0.013
 MEDIAN = 0.020
 UPPER QUANTILE = 0.028

RANGE = 0.002 TO 0.063

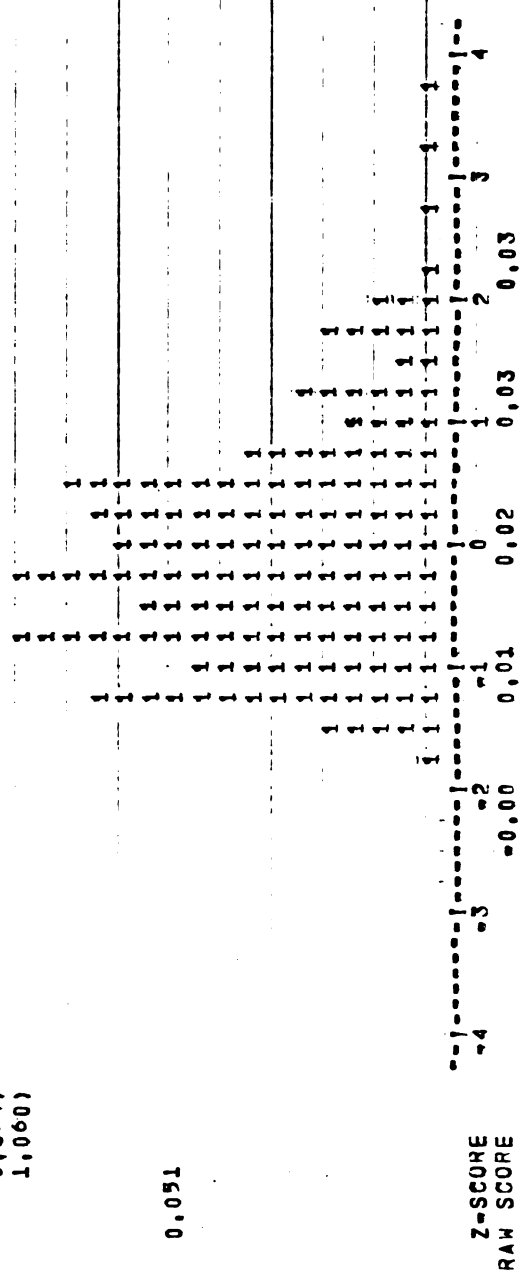


SET 1 VARIABLE 7

N = 150
MEAN = 0.016 (0.009)
STD. DEV. = 0.009 (0.000)
VARIANCE = 0.000 (55.816)
DEF. OF VAR. = 55.630 (0.874)
SKEWNESS = 0.865 (1.060)
KURTOSIS = 0.985 (

LOWER QUARTILE = 0.010
MEDIAN = 0.015
UPPER QUANTILE = 0.021

RANGE = 0.002 TO 0.051

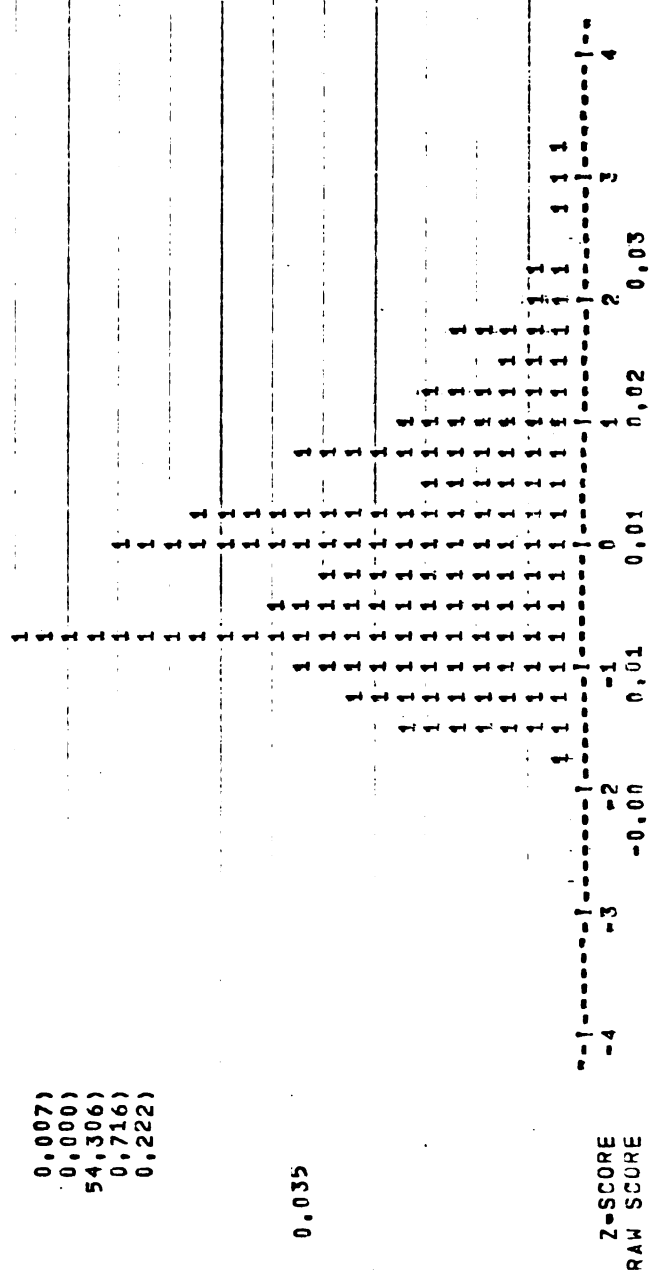


SET 1 VARIABLE 8

N = 150
MEAN = 0.013 (0.007)
STD. DEV. = 0.007 (0.000)
VARIANCE = 0.000 (54.306)
DEF. OF VAR. = 54.125 (0.716)
SKEWNESS = 0.709 (0.222)
KURTOSIS = 0.175 (

LOWER QUARTILE = 0.007
MEDIAN = 0.012
UPPER QUANTILE = 0.018

RANGE = 0.001 TO 0.035

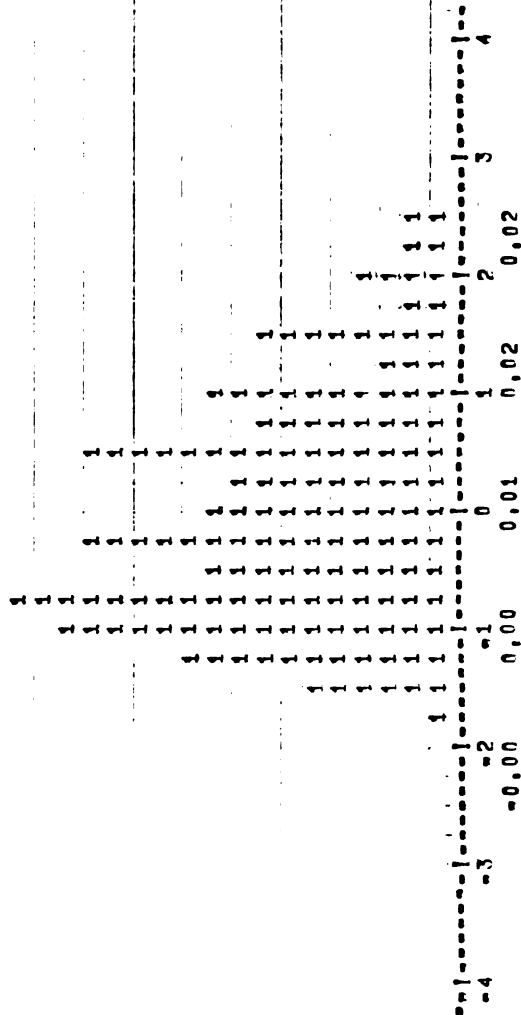


SET 1 VARIABLE 9

N = 155
 MEAN = 0.010 (0.006)
 STD. DEV. = 0.006 (0.000)
 VARIANCE = 0.000 (54.666)
 C. EF. OF VAR. = 54.504 (0.497)
 SKEWNESS = 0.492 (-0.532)
 KURTOSIS = -0.554 (

LOWER QUANTILE = 0.006
 MEDIAN = 0.010
 UPPER QUANTILE = 0.014

RANGE = 0.001 TO 0.025

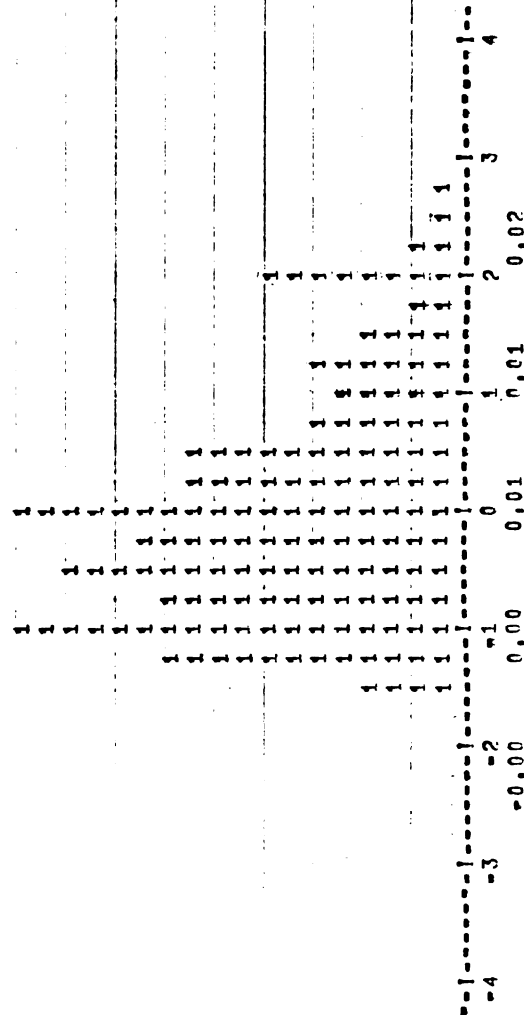


SET 1 VARIABLE 10

N = 155
 MEAN = 0.009 (0.005)
 STD. DEV. = 0.005 (0.000)
 VARIANCE = 0.000 (61.277)
 C. EF. OF VAR. = 61.072 (0.671)
 SKEWNESS = 0.671 (-0.307)
 KURTOSIS = -0.307 (-0.276)

LOWER QUANTILE = 0.004
 MEDIAN = 0.008
 UPPER QUANTILE = 0.012

RANGE = 0.000 TO 0.024



APPENDIX C

**Scatter Plots of the Mean Radius versus Harmonic Amplitude
for
Two Representative Samples from Coon Creek and the Red Cedar River**

Red Cedar River

Sample B5 -----107

Sample B8 -----112

Coon Creek

Sample B9 -----117

Sample B12 -----122

	DATA SET	:	Y Y I
			Y 3-
N ■	39		Y I I
			MAR MONIC
			ANNALS

VARIABLE 11	VARIABLE 1	2
(Y)	(X)	
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6
7	7	7
8	8	8
9	9	9
10	10	10
11	11	11
12	12	12
13	13	13
14	14	14
15	15	15
16	16	16
17	17	17
18	18	18
19	19	19
20	20	20
21	21	21
22	22	22
23	23	23
24	24	24
25	25	25
26	26	26
27	27	27
28	28	28
29	29	29
30	30	30
31	31	31
32	32	32
33	33	33
34	34	34
35	35	35
36	36	36
37	37	37
38	38	38
39	39	39
40	40	40
41	41	41
42	42	42
43	43	43
44	44	44
45	45	45
46	46	46
47	47	47
48	48	48
49	49	49
50	50	50
51	51	51
52	52	52
53	53	53
54	54	54
55	55	55
56	56	56
57	57	57
58	58	58
59	59	59
60	60	60
61	61	61
62	62	62
63	63	63
64	64	64
65	65	65
66	66	66
67	67	67
68	68	68
69	69	69
70	70	70
71	71	71
72	72	72
73	73	73
74	74	74
75	75	75
76	76	76
77	77	77
78	78	78
79	79	79
80	80	80
81	81	81
82	82	82
83	83	83
84	84	84
85	85	85
86	86	86
87	87	87
88	88	88
89	89	89
90	90	90
91	91	91
92	92	92
93	93	93
94	94	94
95	95	95
96	96	96
97	97	97
98	98	98
99	99	99
100	100	100

MEAN	247,894	0,012	1
STD. DEV.	50,401	0,009	1
VARIANCE	2540,213	0,000	1

COVARIANCE = -0.133

Q = 0.299297

ETA(Y,X) = 0.74991 ETA(X,Y) = 0.49063

REFR. COEFS. I

A(Y,X)	A(X,Y)	-2-
268,25	0.02	

$$P(Y, X) = 0.0001$$

3

[illegible]

107

	DATA SET	Y Y I
1	SAMPLE	Y Y 3-
2	STATE	Y Y I

39

MEAN	VARIABLE 11 (Y)	VARIABLE 2 (Y)	2"
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9
10	10	10	10
11	11	11	11
12	12	12	12
13	13	13	13
14	14	14	14
15	15	15	15
16	16	16	16
17	17	17	17
18	18	18	18
19	19	19	19
20	20	20	20
21	21	21	21
22	22	22	22
23	23	23	23
24	24	24	24
25	25	25	25
26	26	26	26
27	27	27	27
28	28	28	28
29	29	29	29
30	30	30	30
31	31	31	31
32	32	32	32
33	33	33	33
34	34	34	34
35	35	35	35
36	36	36	36
37	37	37	37
38	38	38	38
39	39	39	39
40	40	40	40
41	41	41	41
42	42	42	42
43	43	43	43
44	44	44	44
45	45	45	45
46	46	46	46
47	47	47	47
48	48	48	48
49	49	49	49
50	50	50	50
51	51	51	51
52	52	52	52
53	53	53	53
54	54	54	54
55	55	55	55
56	56	56	56
57	57	57	57
58	58	58	58
59	59	59	59
60	60	60	60
61	61	61	61
62	62	62	62
63	63	63	63
64	64	64	64
65	65	65	65
66	66	66	66
67	67	67	67
68	68	68	68
69	69	69	69
70	70	70	70
71	71	71	71
72	72	72	72
73	73	73	73
74	74	74	74
75	75	75	75
76	76	76	76
77	77	77	77
78	78	78	78
79	79	79	79
80	80	80	80
81	81	81	81
82	82	82	82
83	83	83	83
84	84	84	84
85	85	85	85
86	86	86	86
87	87	87	87
88	88	88	88
89	89	89	89
90	90	90	90
91	91	91	91
92	92	92	92
93	93	93	93
94	94	94	94
95	95	95	95
96	96	96	96
97	97	97	97
98	98	98	98
99	99	99	99
100	100	100	100

MEAN	247.894	0.143
STD. DEV.	50.401	0.067
VARIANCE	2540.213	0.004
		ABSOLUTE

COVARIANCE = 0.895

$$R = -0.265595$$
$$\text{ETA}(Y,X) = 0.53807 \quad \text{ETA}(X,Y) = 0.53440$$

REGR. COEFS. !

$$A(Y, X) = 276.41 \quad A(X, Y) = 0.23$$
$$B(Y,X) = -200.0838 \quad B(X,Y) = -0.0004$$

5

HARMONIC AMPL.

1.1

1.2

1.3

1.4

1.5

1.6

— I — Y — Y

	3	2	1	0	1	2	3
X	X						X
X							
Y							

DATA SET ; Y Y Y Y

39 - N

VARIABLE 11	VARIABLE 5	2
(Y)	(X)	

MFAN : 247,894

STD. DEV. 59.40%

VARIANCE = 2540,213

COVARIANCE = .076

R - 0.106693

ETA(Y,X) = 0.54146 ETA(X,Y) = 0.62864

REFR. COES.

$$A(Y, X) = 236.78 \quad A(X, Y) = 0.02$$
$$P(Y, X) = 379.5200 \quad P(X, Y) = 0.0000$$

3

DATA SET 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840,

39 ■ 2

VARIABLE 11	VARIABLE 6	27
(Y)	(X)	
1	1	1
2	2	4
3	3	9
4	4	16
5	5	25
6	6	36
7	7	49
8	8	64
9	9	81
10	10	100
11	11	121
12	12	144
13	13	169
14	14	196
15	15	225
16	16	256
17	17	289
18	18	324
19	19	361
20	20	400
21	21	441
22	22	484
23	23	529
24	24	576
25	25	625
26	26	676
27	27	729
28	28	784
29	29	841
30	30	900
31	31	961
32	32	1024
33	33	1089
34	34	1156
35	35	1225
36	36	1296
37	37	1369
38	38	1444
39	39	1521
40	40	1600
41	41	1681
42	42	1764
43	43	1849
44	44	1936
45	45	2025
46	46	2116
47	47	2209
48	48	2304
49	49	2401
50	50	2500
51	51	2601
52	52	2704
53	53	2809
54	54	2916
55	55	3025
56	56	3136
57	57	3249
58	58	3364
59	59	3481
60	60	3600
61	61	3721
62	62	3844
63	63	3969
64	64	4096
65	65	4225
66	66	4356
67	67	4489
68	68	4624
69	69	4761
70	70	4900
71	71	5041
72	72	5184
73	73	5329
74	74	5476
75	75	5625
76	76	5776
77	77	5929
78	78	6084
79	79	6241
80	80	6400
81	81	6561
82	82	6724
83	83	6889
84	84	7056
85	85	7225
86	86	7396
87	87	7569
88	88	7744
89	89	7921
90	90	8100
91	91	8281
92	92	8464
93	93	8649
94	94	8836
95	95	9025
96	96	9216
97	97	9409
98	98	9604
99	99	9801
100	100	10000

$$MEAN = 247.894$$

SYN. DEVS. = 50.401

VARIANCE = 2540.213

COVARIANCE = -0.013

R = -0.024183

ETA(Y,X) = 0.68203 ETA(Y,Y) = 0.52845

REGA, COEF,

$$A(Y, X) = 250,33 \quad A(X, Y) = 0,02$$
$$z(y, x) = -115.4513 \quad r(x, y) = -0.0000$$

5

DATA SET 1

VARIABLE	11	VARIABLE	1	2
(Y)	(Y)	(Y)		
MEAN =	752.547	0.012		
STD. DEV. =	74.863	0.011		
VARIANCE =	5604.664	0.009		
COVARIANCE =	-0.237			
R =	-0.295729			
ETA(V,X) = 0.71255	ETA(V,Y) = 0.66063			
REGR. COEFS.				
A(V,X) =	377.26	A(X,Y) =	0.03	
B(V,X) = *****	B(X,Y) =	-0.000		

112

DATA SET 1

VARIABLE	11	VARIABLE	2	3
(Y)	(Y)	(Y)		
MEAN =	752.547	0.127		
STD. DEV. =	74.863	0.064		
VARIANCE =	5604.664	0.004		
COVARIANCE =	-0.135			
R =	-0.029265			
ETA(V,X) = 0.50159	ETA(V,Y) = 0.47541			
REGR. COEFS.				
A(V,X) =	306.74	A(X,Y) =	0.14	
B(V,X) = -33.1194	B(X,Y) =	-0.000		

112

	x	y
1	-3	-2
2	-1	-1
3	0	1

DATA SET 1

VARIABLE 11 (Y)	VARIABLE 9 (X)
MEAN = 352.547	0.010
STD. DEV. = 74.863	0.007
VARIANCE = 5604.464	0.000
COVARIANCE = 3.135	
R = 1.261905	
ETA(Y,X) = 0.63334	ETA(X,Y) = 0.64149
REG. COEFS.	
A(Y,X) = 324.51	A(X,Y) = 0.00
B(Y,X) = 2451.0045	B(X,Y) = 0.0000

116

DATA SET 1

VARIABLE 11 (Y)	VARIABLE 10 (X)
MEAN = 352.547	0.009
STD. DEV. = 74.863	0.006
VARIANCE = 5604.464	0.000
COVARIANCE = -0.015	
R = -5.033353	
ETA(Y,X) = 0.58137	ETA(X,Y) = 0.76334
REG. COEFS.	
A(Y,X) = 356.37	A(X,Y) = 0.01
B(Y,X) = -414.9317	B(X,Y) = -0.0000

DATA SET 1

N = 30

VARIABLE 11 (Y)	VARIABLE 3 (X)	2
MEAN = 323.597	0.075	1
STD. DEV. = 87.934	0.038	1
VARIANCE = 7732.346	0.001	1

COVARIANCE = 0.555

R = 0.165936

ETA(Y,X) = 0.45552 ETA(Y,Y) = 0.64268

REGR. COEFS.

A(Y,X) = 294.80 A(X,Y) = 0.05

B(Y,X) = 383.7094 B(X,Y) = 0.0001

-3-

118

DATA SET 1

N = 38

VARIABLE 11 (Y)	VARIABLE 4 (X)	2
MEAN = 323.597	0.049	1
STD. DEV. = 87.934	0.023	1
VARIANCE = 7732.346	0.001	1

COVARIANCE = 0.049

R = -0.020425

ETA(Y,X) = 0.72561 ETA(X,Y) = 0.62806

REGR. COEFS.

A(Y,X) = 327.40 A(X,Y) = 0.05

B(Y,X) = 77.7152 B(X,Y) = -0.0000

-3-

-3	-2	-1	0	1	2	3
						X X
						X X
						X X

35-Y

35-Y

35-Y

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VARIABLE 11	VARIABLE 5
(Y)	(X)
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97	97
98	98
99	99
100	100

MEAN	=	323.597	=	0.033
STD.DEV.	=	87.934	=	0.017
VARIANCE	=	7732.346	=	0.000

COVARIANCE = 0.061

$$R = 0.040530$$

ETA(Y,X) = 0.42563 ETA(X,Y) = 0.59267

• ১৫৩৮ •

$$A(Y, X) = 316.65 \quad A(X, Y) = 0.03$$
$$u(Y, X) = 209.1055 \quad p(X, Y) = 0.0000$$

3

$\frac{1}{x}$

DATA SET 1-1

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32

VARIABLE 11	VARIABLE 6
(Y)	(X)
1	1
2	2
3	3
4	4
5	5
6	6
7	7
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10	10
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98	98
99	99
100	100

MEAN =	323.597	0.019
STD. DEV. =	87.934	0.010
STANDARD	7732.346	0.000

COVARIANCE = = 0.292

R = -0.337315

$$\text{ETA}(Y, X) = 0.62254 \quad \text{ETA}(X, Y) = 0.66578$$

REFR. UNES.

$$A(Y, X) = 380,9 \text{ H} \quad A(X, Y) = 0,03$$
$$E(Y, X) = \text{*****} \quad F(X, Y) = -0.0000$$

5

[illegible]

DATA SET 1

VARIABLE	11	VARIABLE	9
(Y)	(X)		
MEAN =	323.597	0.009	
STD.DEV. =	87.934	0.005	
VARIANCE =	7732.346	0.000	
COVARIANCE =	0.266		
R =	0.42837		
ETA(Y,X) = 0.62262	ETA(X,Y) = 0.77763		
REGR. COEFS.			
A(Y,X) =	242.22	A(X,Y) =	0.00
B(Y,X) =	8744.7619	B(X,Y) =	0.0000

-3 -2 -1 0 1 2 3
 /-----|-----|-----|-----|-----|-----|-----|
 X X
 X X
 X X

121

DATA SET 1

VARIABLE	11	VARIABLE	10
(Y)	(X)		
MEAN =	323.597	0.008	
STD.DEV. =	87.934	0.004	
VARIANCE =	7732.346	0.000	
COVARIANCE =	0.617		
R =	0.047508		
ETA(Y,X) = 0.64011	ETA(X,Y) = 0.75730		
REGR. COEFS.			
A(Y,X) =	315.96	A(X,Y) =	0.01
B(Y,X) =	1011.5502	B(X,Y) =	0.0000

-3 -2 -1 0 1 2 3
 /-----|-----|-----|-----|-----|-----|-----|
 X X
 X X
 X X

SAMPLE B12 COON CREEK

DATA SET 1

N = 150

VARIABLE 11 VARIABLE 1
(Y) (X)

MEAN = 383.572 0.012
STD. DEV. = 104.330 0.008
VARIANCE = 10884.574 0.009

COVARIANCE = -0.055

R = -0.063316

ETA(Y,X) = 0.31511 ETA(X,Y) = 0.26009

REGR. COEFS.

A(Y,X) = 393.38 A(X,Y) = 0.01

B(Y,X) = -793.5487 R(X,Y) = -0.0000



122

DATA SET 1

N = 150

VARIABLE 11 VARIABLE 2
(Y) (X)

MEAN = 383.572 0.148
STD. DEV. = 104.330 0.076
VARIANCE = 10884.574 0.006

COVARIANCE = -0.011

R = -0.001394

ETA(Y,X) = 0.34945 ETA(X,Y) = 0.26559

REGR. COEFS.

A(Y,X) = 383.46 A(X,Y) = 0.15

B(Y,X) = -1.9113 R(X,Y) = -0.0000



DATA SET 1

N = 150

VARIABLE 11 VARIABLE 3
(Y) (X)

MEAN = 383.372 0.078
 TD.DEV. = 104.330 0.038
 VARIANCE = 10884.574 0.001

COVARIANCE = 0.162

R = 0.041430

TA(Y,X) = 0.27450 ETA(X,Y) = 0.32391

RFGP, COEFS:

A(Y,X) = 374.64 A(X,Y) = 0.07

B(Y,X) = 115.0675 B(X,Y) = 0.0000

123

DATA SET 1

N = 150

VARIABLE 11 VARIABLE 4
(Y) (X)

MEAN = 383.372 0.047
 TD.DEV. = 104.330 0.024
 VARIANCE = 10884.574 0.001

COVARIANCE = -0.134

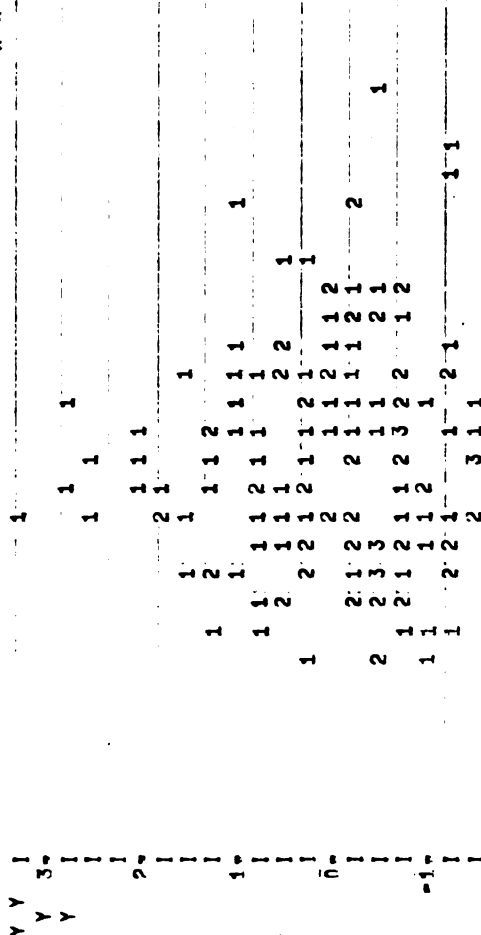
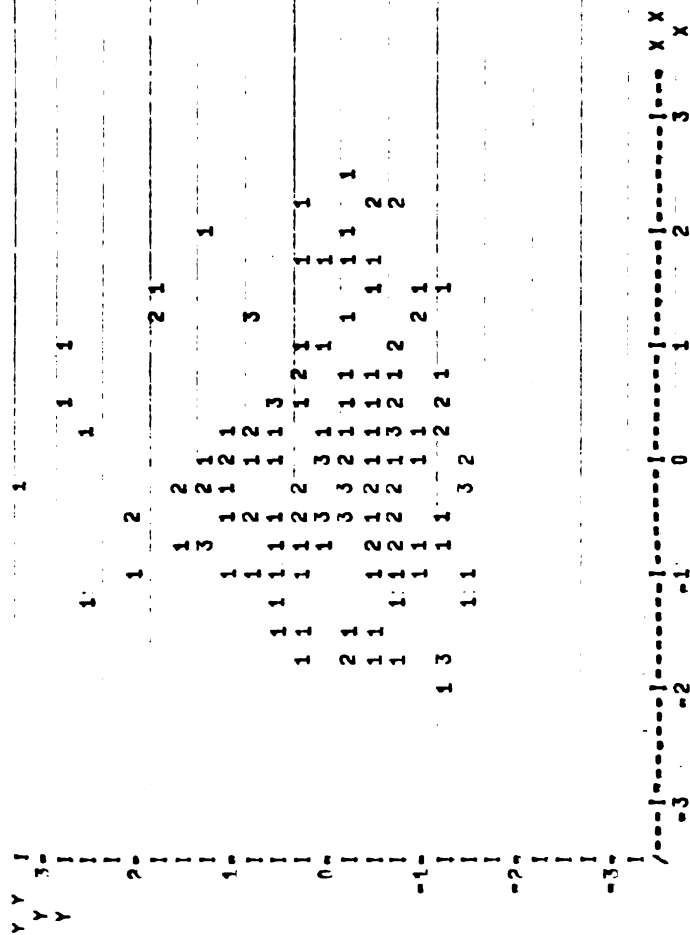
R = -0.053284

TA(Y,X) = 0.32009 ETA(X,Y) = 0.33421

RFGP, COEFS:

A(Y,X) = 394.41 A(X,Y) = 0.05

B(Y,X) = -229.0601 B(X,Y) = -0.0000



DATA SET 1

N = 150

VARIABLE 11 VARIABLE 5
(Y) (X)

MEAN = 383.372 0.031
STD. DEV. = 104.330 0.017
VARIANCE = 10884.574 0.000

COVARIANCE = -0.177

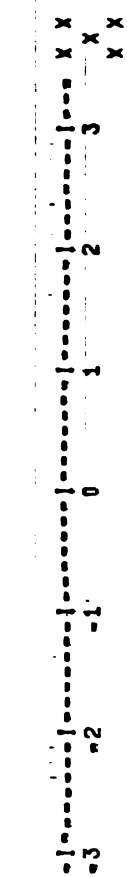
R = -0.097783

TA(Y,X) = 0.33802 ETA(X,Y) = 0.25524

REGR. COEFS.

A(Y,X) = 401.69 A(X,Y) = 0.04

B(Y,X) = -586.9604 R(X,Y) = -0.0000



DATA SET 1

N = 150

VARIABLE 11 VARIABLE 6
(Y) (X)

MEAN = 383.372 0.022
STD. DEV. = 104.330 0.012
VARIANCE = 10884.574 0.000

COVARIANCE = -0.238

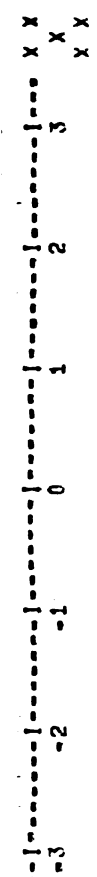
R = -0.191541

TA(Y,X) = 0.36859 ETA(X,Y) = 0.42237

REGR. COEFS.

A(Y,X) = 419.77 A(X,Y) = 0.03

B(Y,X) = ***** R(X,Y) = -0.0000



$$\begin{aligned} A(Y', X) &= 391,74 & A(X, Y) &= 0,01 \\ B(Y', X) &= -634,1291 & B(X, Y) &= -0,0000 \end{aligned}$$

$$g(y', x) = \dots \quad R(x, y) = -0.0000$$

APPENDIX D

**Scatter Plots of Interharmonic Correlations in
Sample B12 of Coon Creek**

DATA SET 1

HARMONIC NUMBER

N = 149

VARIABLE 2 VARIABLE 3
(Y) (X)

MEAN = 0.142 0.077
STD. DEV. = 0.076 0.037
VARIANCE = 0.006 0.001

COVARIANCE = -0.000 CORRELATION
R = -0.021276 COEFFICIENT

TA(Y,X) = 0.32011 STA(Y,Y) = 0.30796

REGRESS. COEFS.

A(Y,X) = 0.15 A(X,Y) = 0.08
B(Y,X) = -0.0432 B(X,Y) = -0.0105

DATA SET 1

SAMPLE SIZE

N = 149

VARIABLE 2 VARIABLE 4
(Y) (X)

MEAN = 0.149 0.047
STD. DEV. = 0.076 0.024
VARIANCE = 0.006 0.001

HARMONIC AMPLITUDE

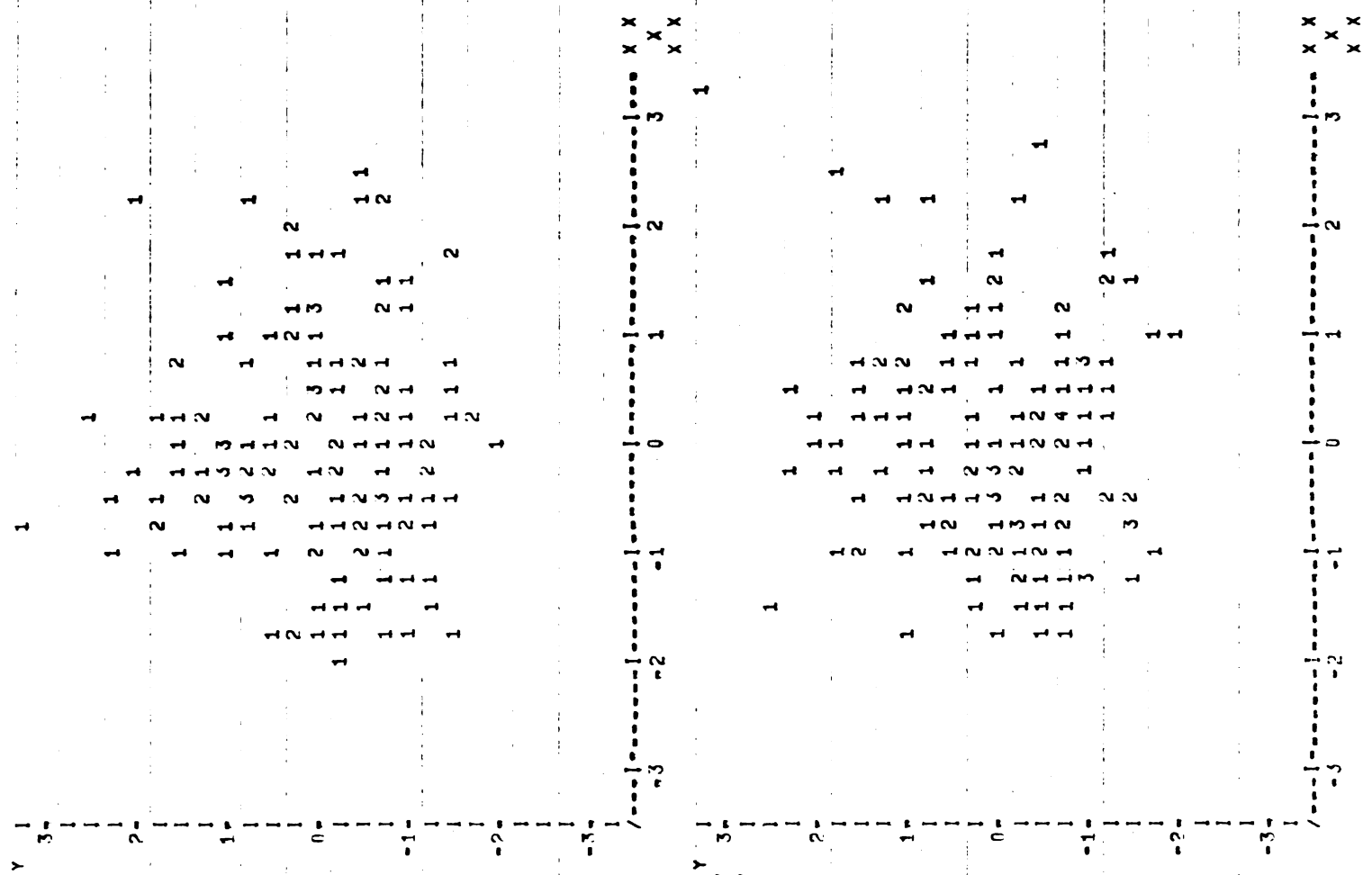
0.000

R = 0.122879

TA(Y,X) = 0.45742 STA(Y,Y) = 0.42419

REGRESS. COEFS.

A(Y,X) = 0.13 A(X,Y) = 0.04
B(Y,X) = 0.0861 B(X,Y) = 0.0391



1

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DATA SET 1

N = 149

VARIABLE 2 VARIABLE 3
(Y) (X)

MEAN = 0.149 0.031
 STD. DEV. = 0.076 0.017
 VARIANCE = 0.006 0.000

COVARIANCE = 0.000

R = 0.95278

ETA(Y,X) = 0.41637 ETA(Y,Y) = 0.35841

REG. COEFS.

A(Y,X) = 0.13 A(X,Y) = 0.03

B(Y,X) = 0.703 B(X,Y) = 0.0355

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1
-3 -2 -1 0 1 2 3
X X
X X
X X

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DATA SET 1

N = 149

VARIABLE 2 VARIABLE 6
(Y) (X)

MEAN = 0.149 0.022
 STD. DEV. = 0.076 0.012
 VARIANCE = 0.006 0.000

COVARIANCE = 0.000

R = 0.94011

ETA(Y,X) = 0.37696 ETA(Y,Y) = 0.48397

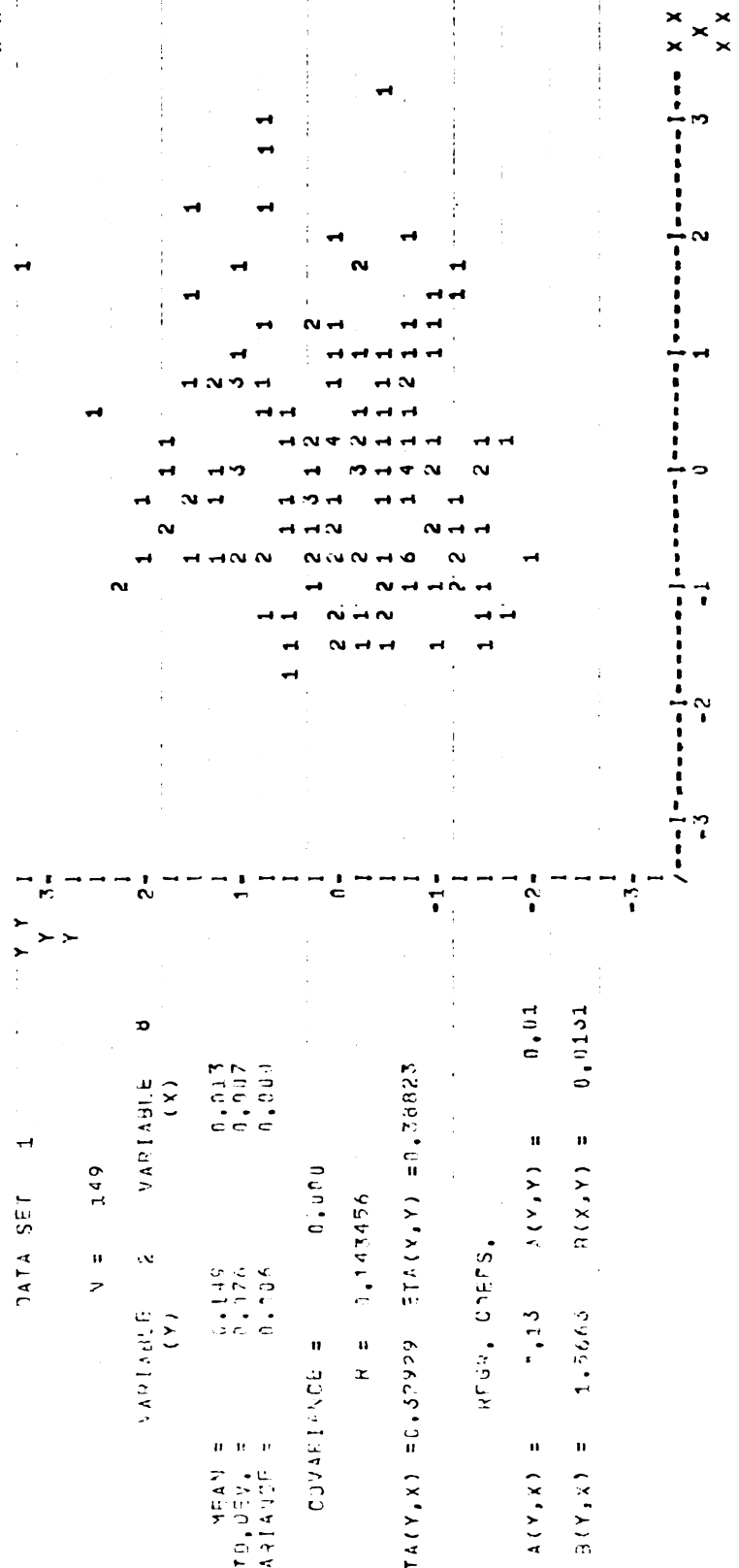
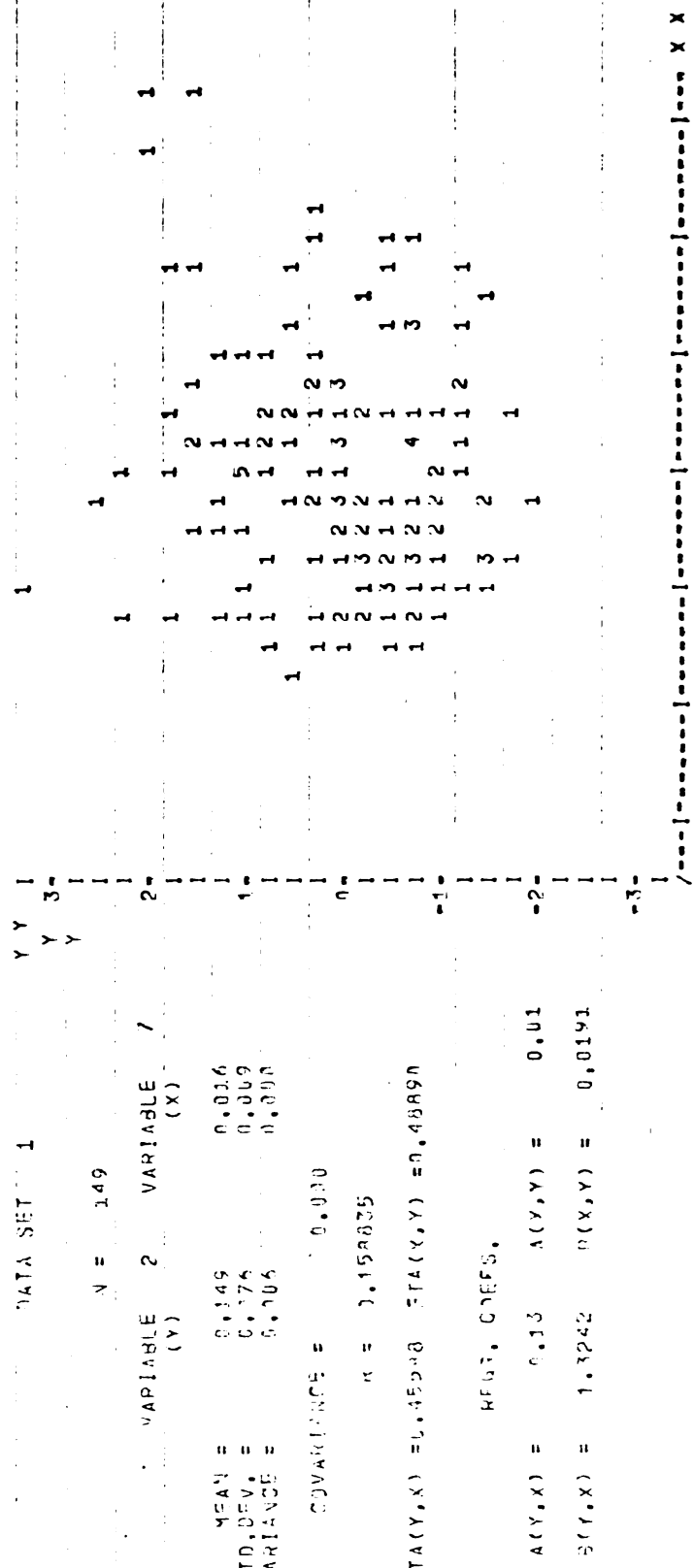
REG. COEFS.

A(Y,X) = 0.14 A(X,Y) = 0.02

B(Y,X) = 0.6027 B(X,Y) = 0.0147

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1
-3 -2 -1 0 1 2 3
X X
X X
X X



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DATA SET 1

      J = 149

      VARIABLE 2  VARIABLE 9
      (Y)      (Y)

      MEAN = 0.149 0.010
      STD. DEV. = 0.076 0.006
      VARIANCE = 0.006 0.000

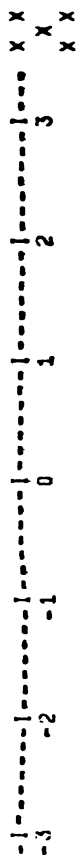
      COVARIANCE = 0.000

      R = 0.023423

      FTA(Y,X) = 0.3061 ETA(X,Y) = 0.3689

      FOUR CORRES.

      A(Y,X) = 0.14 A(X,Y) = 0.01
      B(Y,X) = 1.2463 B(X,Y) = 0.0070
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DATA SET 1

N = 149

STEP 1: 2 VARIABLE 10
(Y) (X)

MEAN = .149 0.000
STD. DEV. = .076 1.005
VARIANCE = .005 1.000

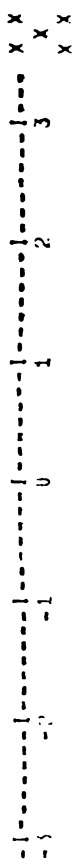
COVARIANCE = 0.000

R = 0.173571

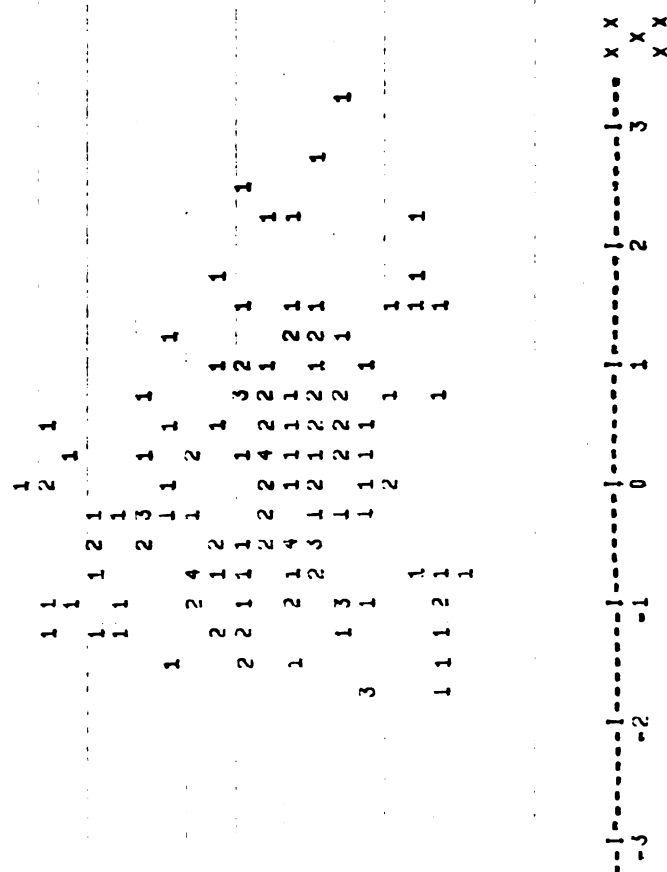
ETA(Y,X) = 0.33749 ETA(X,Y) = 0.15923

REGR. COEFFS.

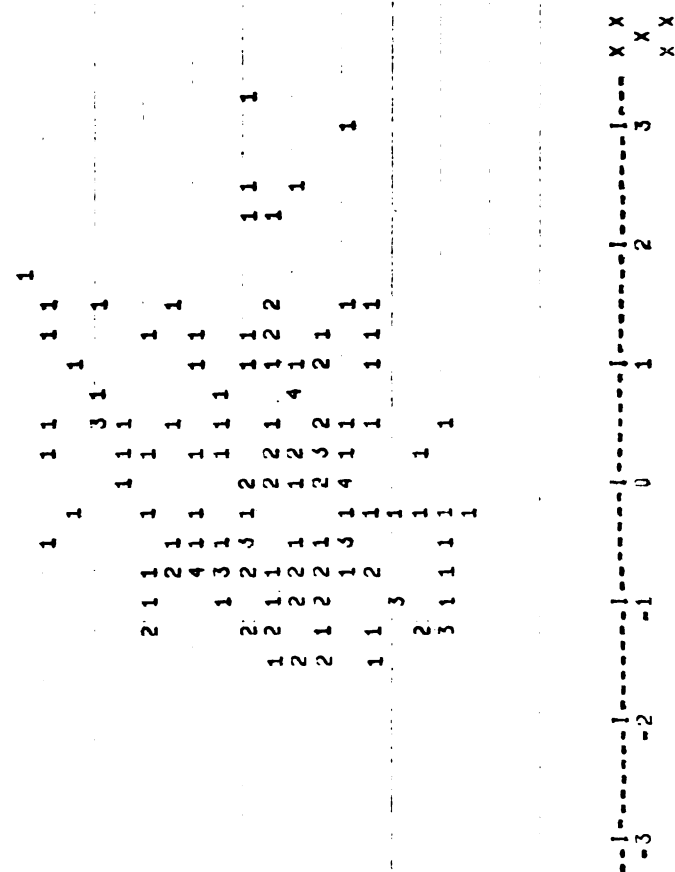
A(Y,X) = .036 A(X,Y) = 0.01
B(Y,X) = 2.4320 B(X,Y) = 0.0128
```



DATA SET 1
Y Y 1
Y 3
Y
N = 149
VARIABLE 3 VARIABLE 4
(V) (X)
MEAN = 0.077 0.047
STD. DEV. = 0.037 0.024
VARIANCE = 0.001 0.001
COVARIANCE = -0.000
R = -1.137666
TA(Y,X) = 0.38150 ETA(Y,Y) = 0.42090
REG. COEFS.
A(Y,X) = 0.39 A(X,Y) = 0.05
B(Y,X) = -0.2129 B(X,Y) = -0.0820
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DATA SET 1
Y Y 1
Y 3
Y
N = 149
VARIABLE 3 VARIABLE 4
(Y) (X)
MEAN = 0.077 0.031
STD. DEV. = 0.037 0.017
VARIANCE = 0.001 0.000
COVARIANCE = 0.000
R = 0.238160
TA(Y,X) = 0.38377 ETA(Y,Y) = 0.38503
REG. COEFS.
A(Y,X) = 0.06 A(X,Y) = 0.02
B(Y,X) = 0.5120 B(X,Y) = 0.1108
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DATA SET 1

N = 149

VARIABLE 3 VARIABLE 10
(Y) (X)

MEAN = 0.027 0.009
STO. DEV. = 0.037 0.005
VARIANCE = 0.001 0.000

COVARIANCE = 0.000

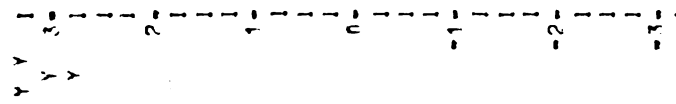
R = 0.131343

ETA(Y,X) = 0.00037 ETA(X,Y) = 0.33811

REG. COEFS.

A(Y,X) = 0.07 A(X,Y) = 0.01

B(Y,X) = 0.0071 B(X,Y) = 0.0191



DATA SET 1

N = 149

VARIABLE 4 VARIABLE 5
(Y) (Y)

MEAN = 0.047 0.031
STO. DEV. = 0.024 0.017
VARIANCE = 0.001 0.000

COVARIANCE = 0.000

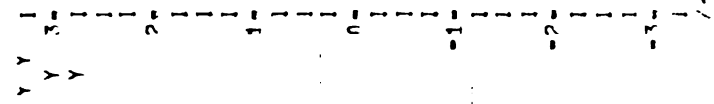
R = 0.070858

ETA(Y,X) = 0.28540 ETA(X,Y) = 0.31891

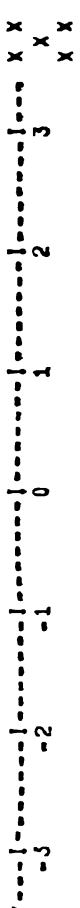
REG. COEFS.

A(Y,X) = 0.04 A(X,Y) = 0.03

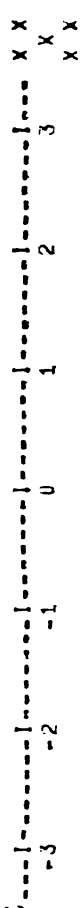
B(Y,X) = 0.1110 B(X,Y) = 0.0574



DATA SET 1
Y Y
Y 3
Y
N = 149
VARIABLE 4 VARIABLE 5
(Y) (X)
MEAN = 0.047 0.013
STD. DEV. = 0.024 0.007
VARIANCE = 0.001 0.000
COVARIANCE = 0.000
R = 0.199350
ETA(Y,X) = 0.37245 ETA(Y,Y) = 0.41657
REG. COEFS.
A(Y,X) = 0.04 I(X,Y) = 0.01
B(Y,X) = 0.0020 R(Y,Y) = 0.0574
/



DATA SET 1
Y Y
Y 3
Y
N = 149
VARIABLE 4 VARIABLE 5
(Y) (X)
MEAN = 0.047 0.010
STD. DEV. = 0.024 0.006
VARIANCE = 0.001 0.000
COVARIANCE = 0.000
R = 0.097497
ETA(Y,X) = 0.33461 ETA(Y,Y) = 0.36779
REG. COEFS.
A(Y,X) = 0.04 I(X,Y) = 0.01
B(Y,X) = 0.0137 R(Y,Y) = 0.0230
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100

DATA SET 1

N = 149

VARIABLE 6 VARIABLE 7
(Y) (X)

MEAN = 0.022 0.016
STD. DEV. = 0.012 0.009
VARIANCE = 0.000 0.000

COVARIANCE = 0.000

R = .374915

ETA(Y,X) = 0.43254 ETA(Y,Y) = 0.40528

REG. COEFFS.

A(Y,X) = 0.02 A(X,Y) = 0.01

B(Y,X) = 0.220 B(X,Y) = 0.1322



DATA SET 1

N = 149

VARIABLE 6 VARIABLE 6
(Y) (X)

MEAN = 0.022 0.013
STD. DEV. = 0.012 0.007
VARIANCE = 0.000 0.000

COVARIANCE = 0.000

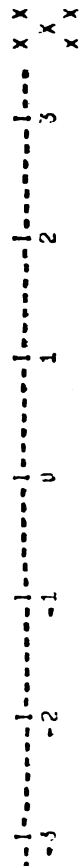
R = .315902

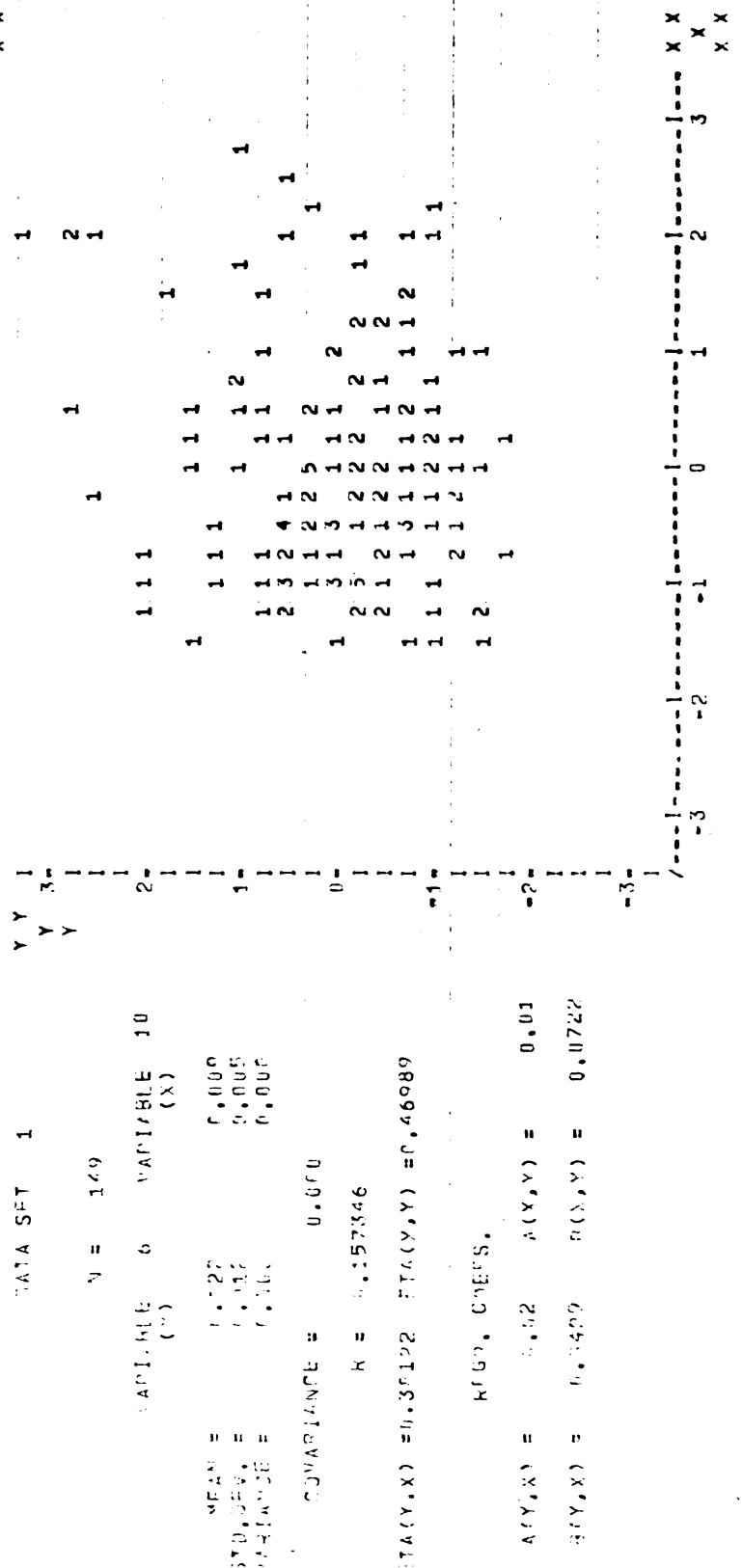
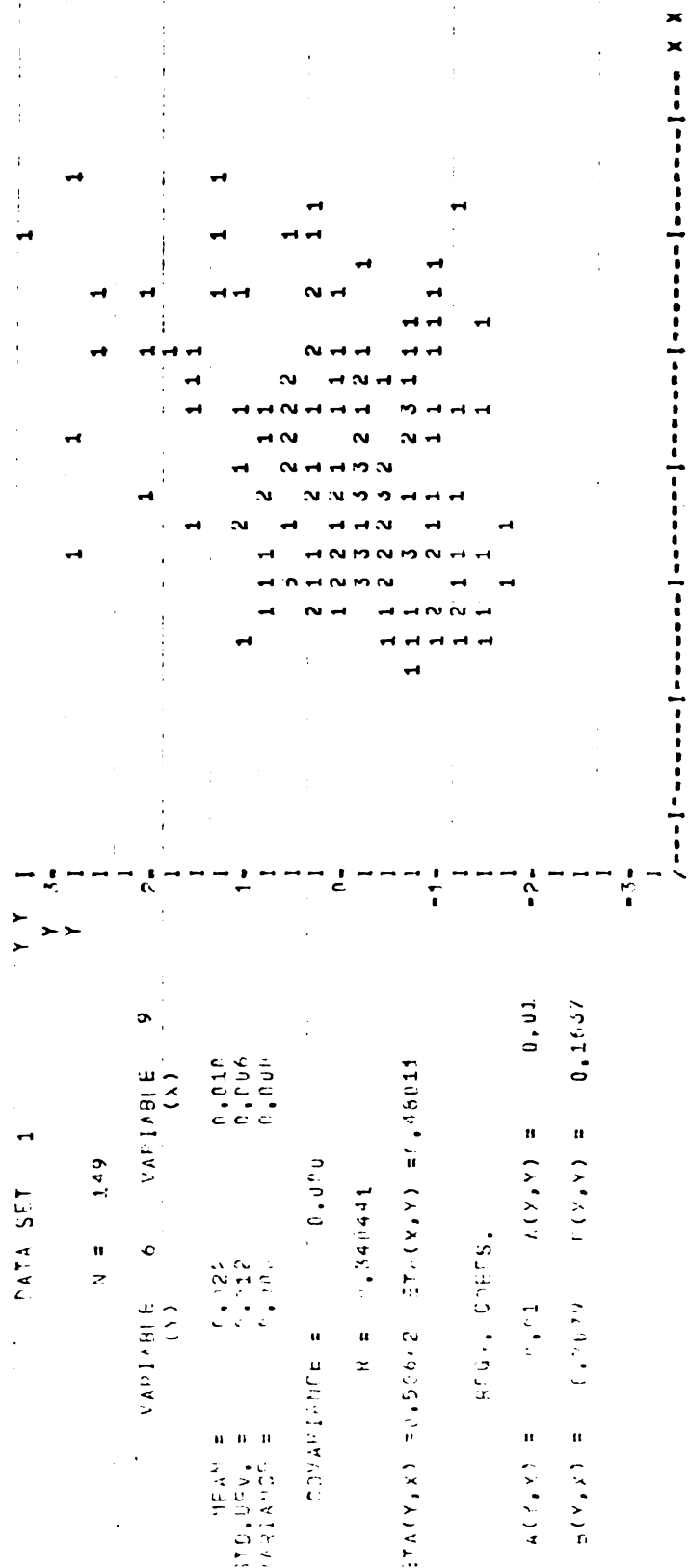
ETA(Y,X) = 0.36376 ETA(Y,Y) = 0.49129

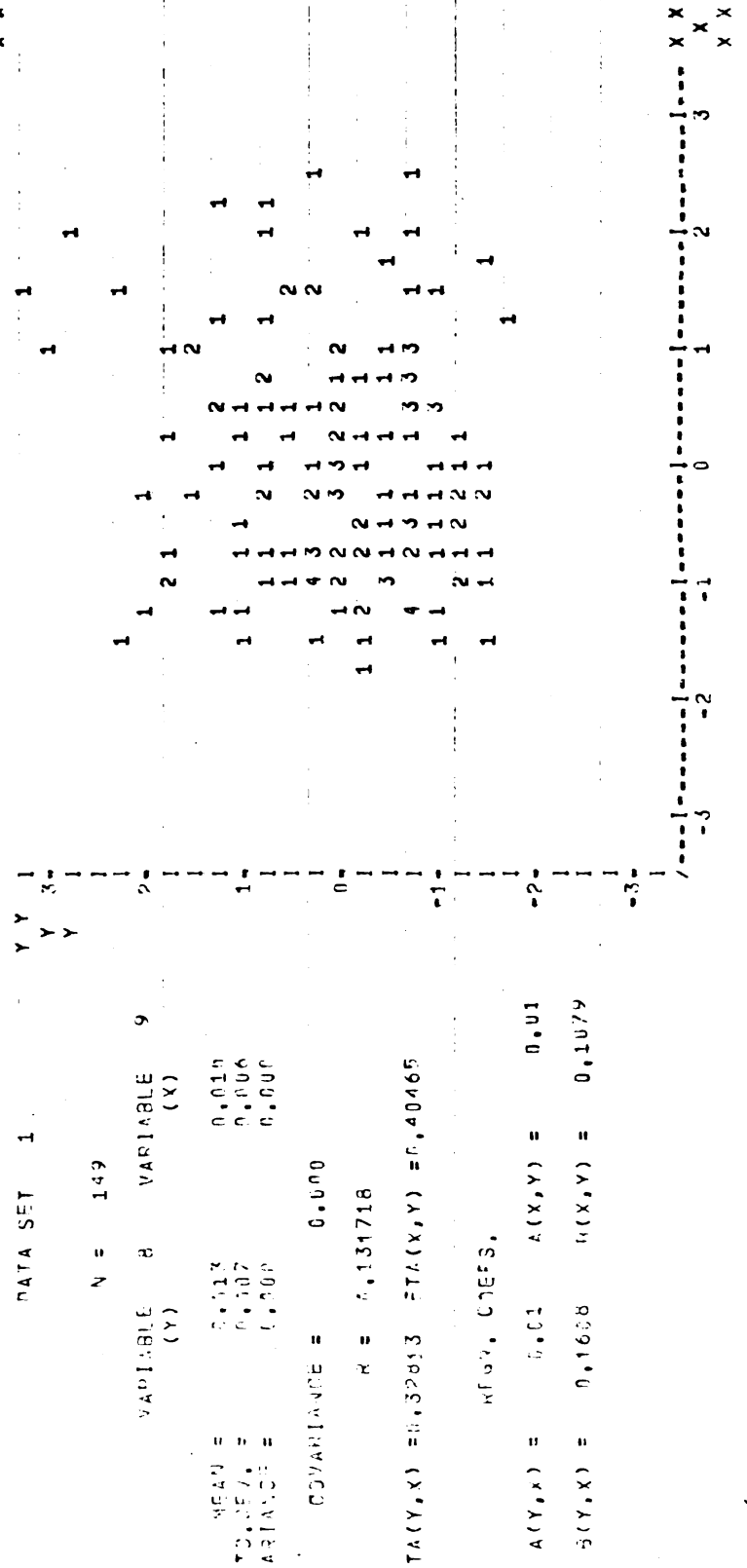
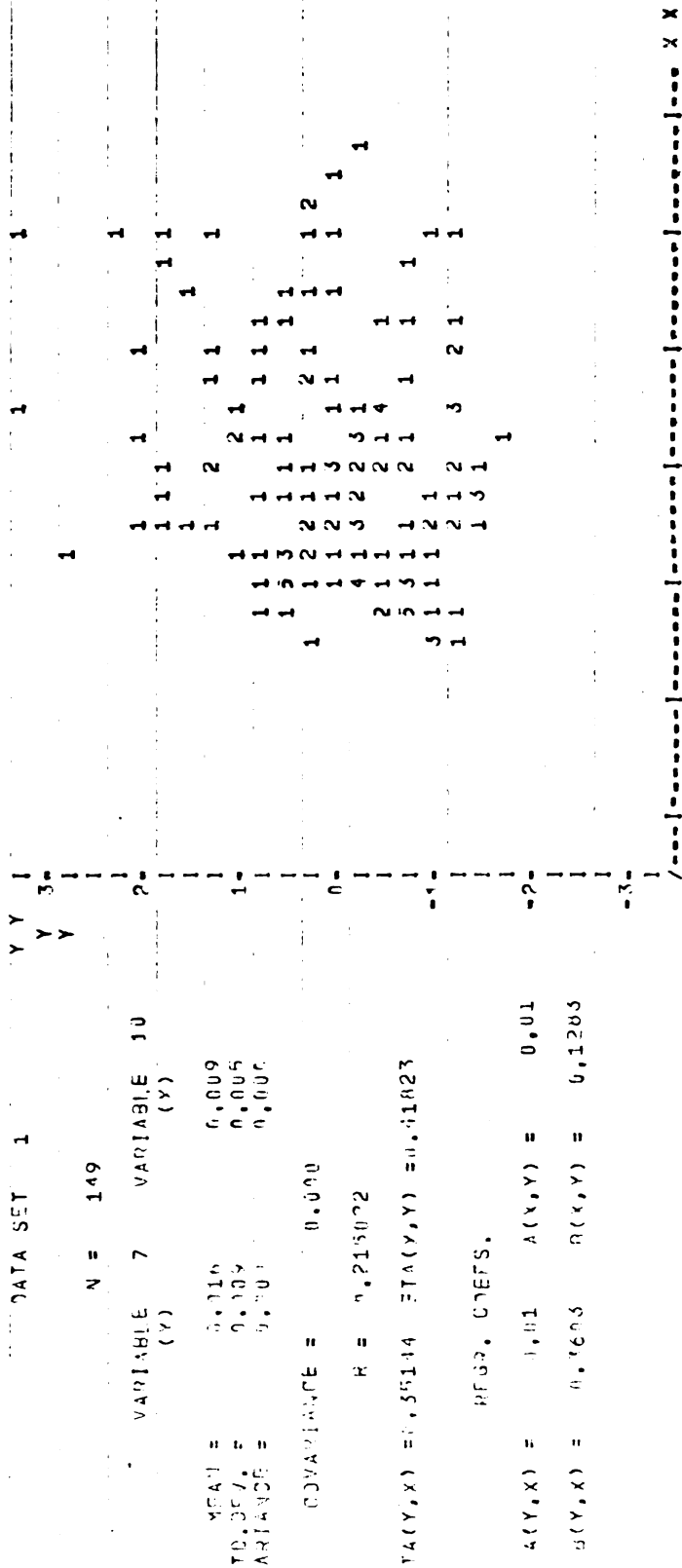
REG. COEFFS.

A(Y,X) = 0.02 A(X,Y) = 0.01

B(Y,X) = 0.1974 B(X,Y) = 0.0580







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DATA SET 1 Y Y 1

N = 149

VARIABLE 6 VARIABLE 10
(Y) (Y)

MEAN = 0.213 0.009
STD. DEV. = 0.207 0.005
VARIANCE = 0.000 0.000

COVARIANCE = 0.000

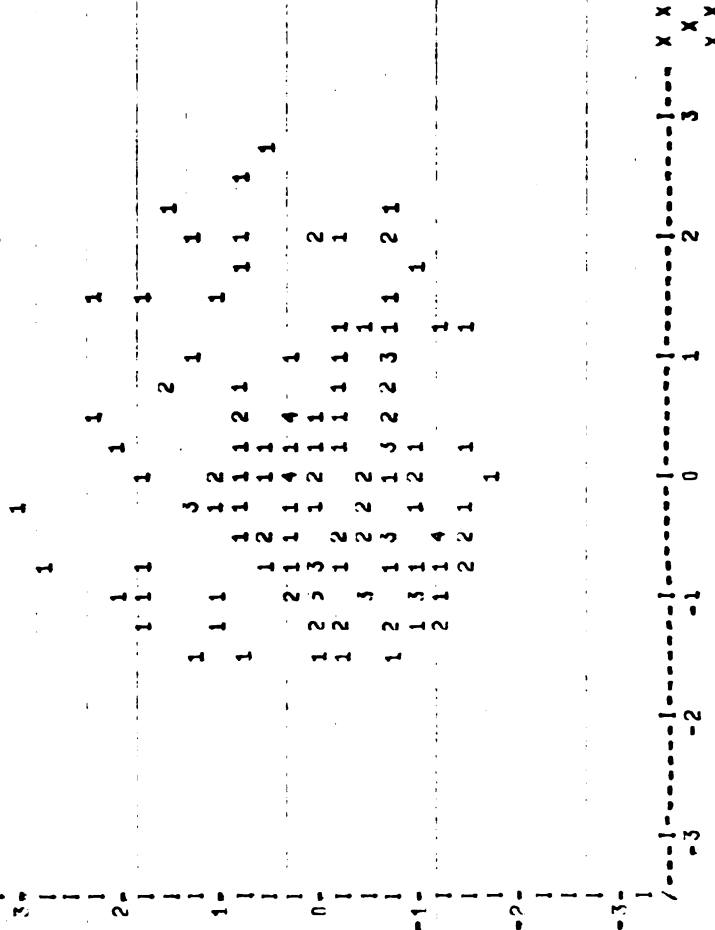
R = 0.140086

ETA(X,Y) = 0.37850 ETA(X,Y) = 0.42509

RESID. CORRS.

A(Y,X) = 0.01 A(Y,Y) = 0.01

B(Y,X) = 0.1912 B(Y,Y) = 0.1111



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