DEFINING PARAMETERS IN GRAVITY EXPLORATION FOR GROUNDWATER

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ABSTRACT

DEFINING PARAMETERS IN GRAVITY EXPLORATION
FOR GROUNDWATER

By

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Gravity exploration has been shown to be a useful assist in locating groundwater, particularly in areas such as the upper Mid-West which have a glacial overburden of up to several hundred feet on the bedrock. It is yet unknown, although, how the various parameters involved affect the applicability of the gravity method. It is intended, here, to define the physical characteristics of buried bedrock channels, which are reflected in gravity anomalies and to discuss accuracies in field measurements which effect interpretation.

Bedrock channels which produce anomalies as small as 0.14 milligals can be detected accurately with the present sensitivity of 0.01 milligals for gravity meters. Because of random errors in meter readings and elevation measurements, smaller anomalies would be obscurred. A standard deviation of 0.1 feet for elevation measurements is needed in detecting these small anomalies. More accurate elevation measurements can be determined, but this would be unnecessary

because of the overwhelming effect of the random errors in gravity meter readings.

Corrections to gravity data due to earth tides is, in most cases, accomplished more accurately when employing theoretical equations than when using the conventional method of reoccupying base stations a few times. It is likely that severe misinterpretations of the empirical diurnal variation can occur out of the presence of random errors in meter readings. Theoretically produced tidal curves have proven reliable by comparing them with detailed, instrumentally measured curves.

Gravity exploration is a useful method available for modest-budget exploration. Its accuracy, low cost and non-disruptiveness of the environment make it attractive.

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CHAPTER I

INTRODUCTION

Scope and Purpose of Study

The number of areas where consumable water can easily be obtained is rapidly decreasing. Many communities face the urgent and important problem of locating suitable water supplies for human consumption as well as irrigational purposes. This study is intended to investigate and develop the feasibility and applicability of gravity exploration for groundwater, particularly in glaciated areas. This is quantitative research of how various factors affect the applicability of the gravity method.

In recent years the problem of water pollution has become apparent. The dumping and spillage of sewage and harmful chemicals into bodies of surface water has greatly endangered this prime source for many of its important uses. This, along with the fact that many communities are not located near sufficiently large volumns of surface water, has increased the desire to investigate groundwater sources, although groundwater has long been the principle contributor to water needs in the upper mid-West. Groundwater is usually devoid of pollution associated with surface supplies and if found in suitable quantities can aid as a supplement or be used as a complete substitute for human uses. This need of additional groundwater sources encourages the development of new prospecting methods and refinement of old ones.

The southern boundary of Pleistocene, continental glaciation in North America forms a line which extends parallel to the coast of Southwestern Canada, running through Central Montana to the juncture of the Ohio and Mississippi rivers. From here it roughly follows the Ohio river and through the northwest corner of Pennsylvania and east through New York harbor. In glaciated areas, well sorted glacial sediments within buried bedrock channels often provide good reservoirs for water. These channels were formed by stream erosion and glacial abrasion and subsequently masked with a covering of glacial till. The channel deposits, being sand and gravel, are often very porous and permeable. Bedrock channels are favorable for groundwater because: 1) they contain thick, well sorted sediments; 2) they are usually the loci of buried outwash from previous glaciation; and 3) the outwash deposits on adjacent bedrock highs are more likely to be eroded (Horberg, 1945). These channel fill sediments are most commonly of lower density than the underlying bedrock and where topographically low bedrock channels occur, a negative density anomaly also exists. This lateral density contrast, from channel sands and gravels to consolidated bedrock, is the basis of the gravity method in groundwater prospecting.

Gravity exploration is certainly not new, but when applied to the problem of locating and evaluating groundwater aquifers, its inception was rather recent (Hall and Hajnel, 1962; Mcginnis, Kempton, and Heigold, 1963). This is due, in part, to the improvement in methods of analysis. Gravity can be used to indicate such features as: 1) configuration of bedrock drainage patterns; 2) location, size, and extent of glacial fill channels in the bedrock; and 3) depth to and slope of the bedrock surface.

The mapping of bedrock from gravity measurements has been extensively used in the recent past (Ibrahim, 1970; Lennox and Carlson, 1967; Rankin and Lavin, 1970). These studies dealt with surveys and theory. This investigation is keyed toward local surveys in glaciated areas wherein specific methods are refined and others developed. From this one can get an idea of what size feature is detectable with a given accuracy of measurements (i.e. meter reading and elevation measurements). An objective of this study is to define the geologic conditions which will result in observable gravity anomalies. It is intended, here, to answer such questions as:

1) Given a certain density contrast, what size of bedrock channel can be detected at a given depth for the available gravity meter sensitivity?

2) How accurate must elevation measurements be in order to minimize the effects of systematic errors?

3) What geologic conditions are reflected in observed gravity anomalies?

4) Can corrections for tidal variation be determined accurately from empirical measurements?

These and other objectives will be achieved using theoretical gravity anomalies, produced from realistic subsurface features. To illustrate the procedures, analysis of field surveys will be described.

Overall, the emphasis of this study is placed on defining the geologic conditions and the data analysis restrictions which make the gravity method useful. Obviously though, there are other conditions that allow gravity exploration to be useful. Economically, the method has the advantage of being quick, cheap and relatively easy to apply. In addition, gravity surveying is environmentally non-disruptive. It can provide a useful assist to other prospecting techniques such as electrical resistivity.

Theory

The basic formula for the force of gravity in the cgs system (centimeters-grams-seconds) is:

$$G = \gamma mm'/r^2 \qquad cm/sec^2 \qquad (1)$$

where G is the gravitational force of attraction, γ is the gravitational constant, m and m' are two masses and r is the distance between their centers. When measuring the force field of the earth on a unit test mass m', only the earth's mass is retained and the equation becomes:

$$G = \gamma m/r^2 \qquad cm/sec^2 \tag{2}$$

where r is the radius to the center of the earth.

In conducting a gravity survey, one is measuring the effects of a subsurface anomalous mass. Equation (2) can be manipulated to calculate such anomalous effects. The equation for a sphere becomes:

$$G = \gamma \Delta\rho V/r^2 \qquad cm/sec^2 \tag{3}$$

where $\Delta\rho V = \Delta m$ is substituted as a mass anomaly. $\Delta\rho$ is the density contrast (gm/cm^3) between the glacial sands and gravels and the adjacent bedrock, in our case, V (cm^3) is the volume of the subsurface sphere, and r is the distance to its center from the point of measurement.

Equation (3) can be used to determine the force of gravity at positions on the surface caused by various subsurface features. Buried bedrock channels can be approximated by a stack of infinitely long horizontal strips, each strip increasing in width up the stack (figure 1). Gravity anomalies over this type of buried structure are very similar to those measured over buried bedrock channels.

The formula for the force of gravity at positions relative to a buried infinite horizontal element can be written as:

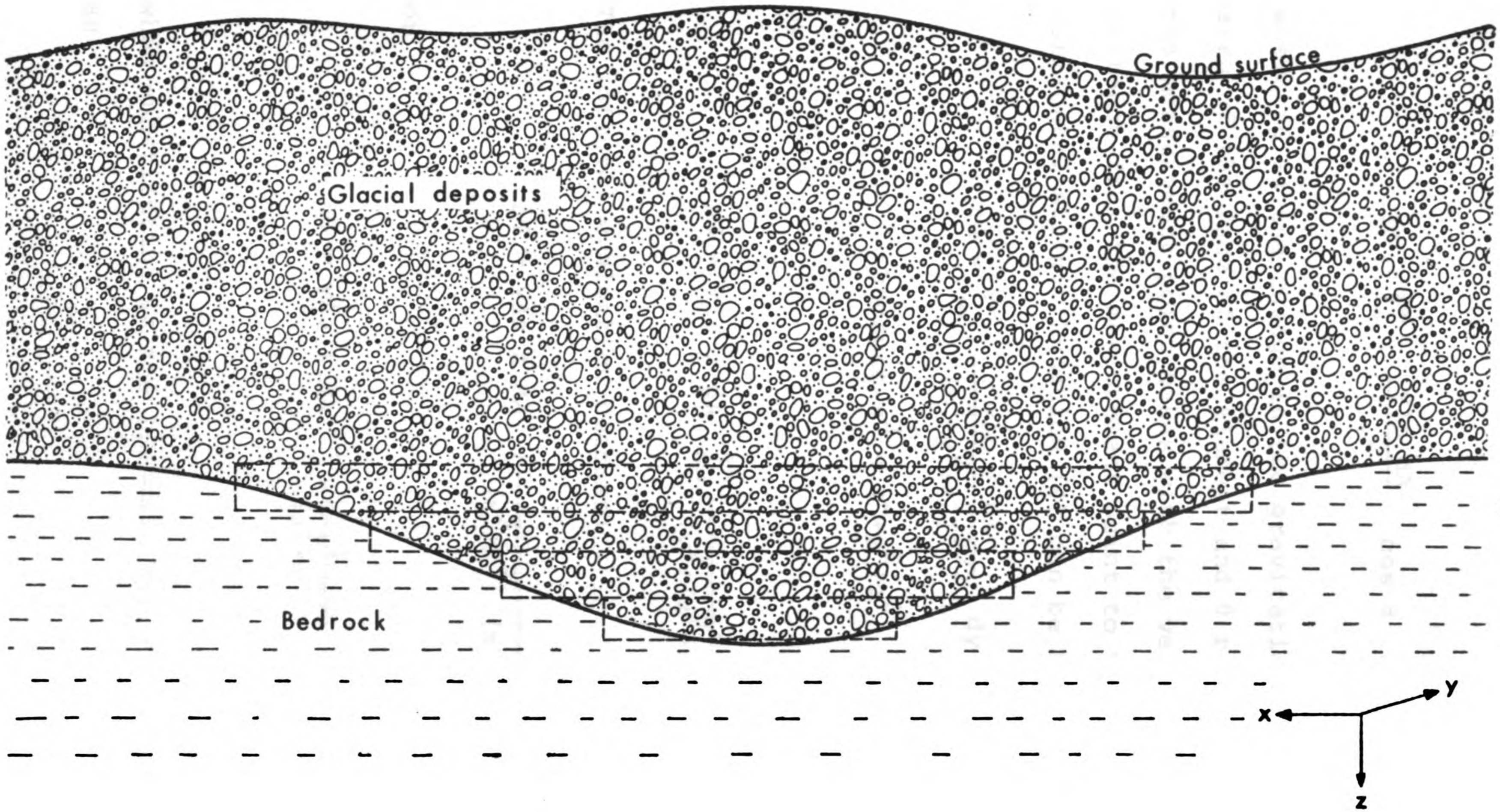

Fig. 1. Buried bedrock valley approximated by a stack of infinite horizontal strips.

$$\Delta G = \int_{-\infty}^{+\infty} \frac{\gamma \Delta\rho \; dA}{r^2} \cos\theta \; dy \qquad cm/sec^2, \qquad (4)$$

where ΔG is the change in the gravitational force, A is the area of the face of the element and θ represents the angle the measurement point makes with the vertical. r is the distance from the measurement point to the center of the horizontal strip. Equation (4) can be rewritten as:

$$\Delta G = \int_{-\infty}^{+\infty} \frac{\gamma \Delta\rho \; z \; dA}{(x^2+y^2+z^2)^{3/2}} \; dy \qquad cm/sec^2$$

because $r^2 = x^2+y^2+z^2$

or

$$\Delta G = \gamma \Delta\rho \; z \; dA \int_{-\infty}^{+\infty} \frac{1}{[y^2+(x^2+z^2)]^{3/2}} \; dy \quad cm/sec^2$$

The integral evaluation becomes:

$$\Delta G = \gamma \Delta\rho \; z \; dA \left[\frac{y}{(x^2+z^2) \; (x^2+y^2+z^2)^{1/2}} \right]_{-\infty}^{+\infty} cm/sec^2$$

or

$$\Delta G = \gamma \Delta\rho \; z \; dA \left[\frac{1}{(x^2+z^2)} (1) - \frac{1}{(x^2+z^2)} (-1) \right] \qquad cm/sec^2$$

when substituting in the limits. But, this can be rewritten as:

$$\Delta G = \int_{-\infty}^{+\infty} \frac{\gamma \Delta\rho \ dA}{r^2} \cos\theta \ dy \qquad cm/sec^2, \qquad (4)$$

where ΔG is the change in the gravitational force, A is the area of the face of the element and θ represents the angle the measurement point makes with the vertical. r is the distance from the measurement point to the center of the horizontal strip. Equation (4) can be rewritten as:

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or

$$\Delta G = \gamma \Delta\rho \ z \ dA \ \left[\frac{1}{(x^2+z^2)} (1) - \frac{1}{(x^2+z^2)} (-1)\right] \quad cm/sec^2$$

when substituting in the limits. But, this can be rewritten as:

$$\Delta G = 2\gamma\Delta\rho \; dA \; z \cdot \frac{1}{x^2+z^2} \quad cm/sec^2 \qquad (5)$$

Applying this form to the calculations for an infinite horizontal strip (figure 2), equation (5) can be written as:

$$\Delta G = 2\gamma\Delta\rho \; t \; z \cdot \int_{x_o-\alpha}^{x_o+\alpha} \frac{dx}{x^2+z^2} \quad cm/sec^2$$

where t is the thickness of the strip and $t \int_{x_o-\alpha}^{x_o+\alpha} dx = dA$ is substituted. Evaluation at the limits, after taking the integral, the above equation becomes:

$$\Delta G = 2\gamma\Delta\rho \; t \; [\theta_2-\theta_1] \quad cm/sec^2 \qquad (6)$$

(Hubbert-1948) (see figure 2).

Equation (6) will be used in a later chapter, to create a theoretical gravity anomaly from a stack of horizontal strips (figure 1). This stack is a good approximation to a buried bedrock valley.

In later chapters the force of gravity will be labeled as either gals or milligals. A gal equals 1 centimeter per second per second and a milligal is equal to one thousandth of a gal.

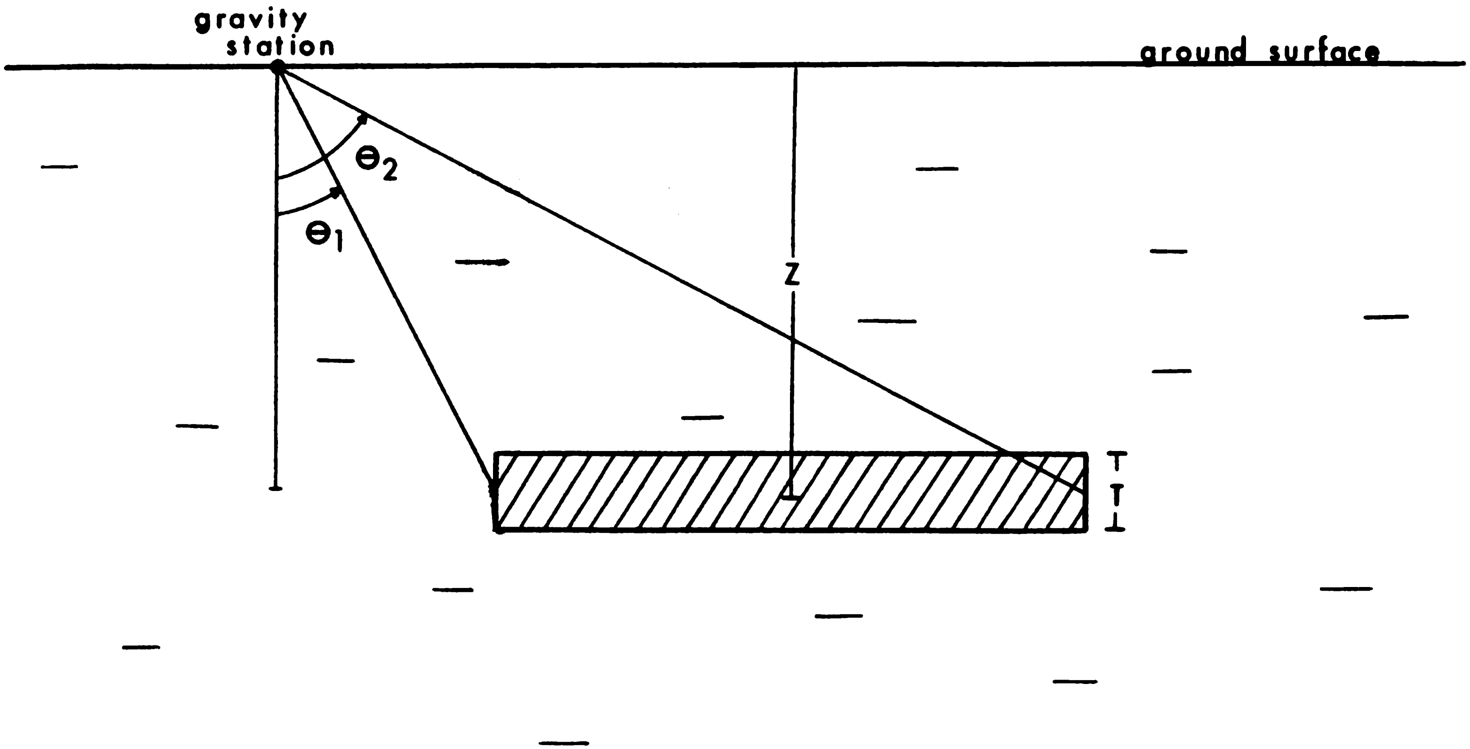

Fig. 2. Geometry of buried infinite horizontal strip.

CHAPTER II

BEDROCK CHANNELS

Introduction

This investigation deals with a method of detecting buried bedrock channels and the possible presence of suitable sediments for the production of water within these buried valleys. Well sorted sections of sand and gravel of appreciable thickness are desired in order to have best production.

Bedrock channels can be defined as troughs created into consolidated formations by means such as stream erosion and glacial abrasion. The covering of unconsolidated material upon these troughs establishes the term buried bedrock channel. The topography of buried bedrock channels are often completely masked by the glacial deposits above them. Furthermore, ancient drainage patterns can be quite different from the overlying present drainage patterns, not only in shape, but in direction of flow.

With this in mind, a brief discussion of bedrock topography and its relation to the presence of overlying glacial sediments as producers of groundwater is necessary.

Formation of Buried Bedrock Channels

Buried bedrock channels are remnants of ancient drainage patterns eroded into consolidated materials. They were formed by the action of surface water, by glacial abrasion and by glacial meltwater. These bedrock channels have since been covered by glacial sediments and in many cases have been completely obscured.

There are three general categories in which bedrock channels can be placed, depending on the method of creation: (1) Preglacial, (2) Periglacial, (3) Post-glacial (Wayne-1953). Preglacial channels, obviously, were formed as drainage paths prior to the Pleistocene glaciation. As in other geologic situations, the rivers and streams followed the least resistive sections of the rock surface in finding their way to an outlet. The most resistive areas served as drainage divides. These preglacial surfaces are analogous to present expressions in non-glaciated areas.

Periglacial bedrock channels are formed by means such as glacial abrasion, and erosion by ice marginal streams, proglacial streams, and in a minor way subglacial streams. Initially, the preglacial bedrock surface has in influence in guiding the glacial ice as it advances, the broad valleys having more of an effect than the smaller scale bedrock features. Valleys normal to the direction of ice movement were overridden, modified, and often completely destroyed. Channels parallel to the ice movement were more likely to be gouged, widened, and deepened into a U-shaped form (Flint-1971).

Glaciation was responsible for many drainage modifications of the world. Meltwaters from the ice caps usually flowed away as proglacial streams, cutting new channels or filling in old ones. Some meltwater streams, though, were largely ice-marginal. As the ice front receded, old routes were abandon for new ones and in some cases a network of drainage patterns developed. Many of these former drainage lines have little or no connection with existing stream courses. They may even cross present drainage divides through notches or grooves. Some of these abandon stream courses may have been cut by subglacial streams, but more often they represent successive drainage lines roughly paralleling a receding ice front (Thornbury-1969).

Periglacial streams, flowing away from glacial fronts, most commonly deposit well sorted sediments. This occurs continuously as the glacier retreats thus laying down a base of well sorted sands or gravels. Periglacial streams may also flow into the glacial front forming ice-contact lakes. These lakes may overrun, thus diverting previous patterns. This damming of drainage may also cause a complete reversal of flow and subsequently deepen the preglacial channel (Wayne-1953).

Post-glacial bedrock channels, as may be expected, are carved into the bedrock after the retreat of and not associated with the glacier. An example of this would be a stream which has eroded through the glacial debris and into the bedrock. A change to a depositional environment could subsequently cover the channel permitting the term buried bedrock channel to apply. Post-glacial channels are not known to exist extensively and are relatively unimportant.

Channel Deposits as Groundwater Aquifers

Depending upon the type of bedrock channel and the environment during the occurrence of glacial filling, the presence of groundwater could be quite substantial. Well sorted deposits of sand and gravel have high porosity and permeability, lending to the occurrence of large quantities of water.

The amount of sorting depends largely on the type of stream in which it was deposited. Streams that flowed toward the ice front became ponded and deposition of unsorted fine grained materials often resulted. These types of deposits serve as poor aquifers due to their low porosity. Peri-glacial streams, which flow away from the glacier front, on the other hand, tend to deposit well sorted sediments. Likewise, deposits which were laid down by streams running parallel to the ice front make good aquifers due to their degree of sorting, although not as good as proglacial streams.

Thick sequences of well sorted channel deposits, of course, are most desirable and depend in part on the activity of the ice front. An ice front which retreats rapidly causes the spreading of its contained sediments over a large area. A stagnent ice front, though, could fill a section of stream channel with relatively thick deposits of sorted alluvium (Wayne-1953).

A less important measurement of the quality of an aquifer, perhaps, is the age of the sediments. Sediments of an early glacial stage are generally more compact and more likely to be cemented. This constitutes a reduction in porosity.

CHAPTER III

DATA REDUCTIONS

Introduction

Observational gravity readings do not represent absolute values of the earth's gravitational force, but are representatives on a relative scale. These observed readings are not true indications of the picture of the subsurface structure. Corrections must be applied to station readings which are characterized by various elevations, latitudes, and terrains. By accounting for station corrections, all the values are reduced to a chosen datum plane, which may or may not be mean sea level. For local surveys it is most convenient to use the lowest station as the datum. Corrections include those due to (1) latitude, (2) elevation differences, (3) difference in local masses, (4) earth tides, and (5) instrument fatigue. After these have been applied, the resulting value is termed Bouguer Gravity Value. Reduced to a relative scale, the value becomes the Bouguer Gravity Anomaly.

The Bouguer Gravity Value can easily be calculated with a digital computer using the following equation:

$$G_{bgv} = g_o - g_l + g_e - g_m + g_t$$

where:

G_{bgv} = Bouguer Gravity Value

g_o = observed gravity

g_l = latitude correction

g_e = free air correction

g_m = Bouguer mass correction

g_t = terrain correction

Note: signs in the equation are for stations above datum elevation and poleward of a reference latitude.

Latitude Corrections

The earth, due to rotation, experiences an equatorial bulge. This implies that the force of gravity is somewhat less at the equator than at the poles, and increases gradually toward the poles. All stations must be corrected for latitude variations. The "International Gravity Formula" for the variation of normal gravity along the geoid with the latitude θ is:

$$G = 978.049\ (1+0.0052884\ \sin^2\theta - 0.0000059\ \sin^2 2\theta)\ \text{gals.}$$

The geoid is the surface within or around the earth that is everywhere normal to the direction of gravity. In localized surveys, an arbitrary reference latitude is generally chosen to which all readings are corrected. Stations within a degree of this arbitrary latitude can be set to a gradient and may be corrected by finding the product of this gradient and the north-south distance to the chosen reference latitude. The formula for determining the gradient to be applied is:

$$\text{gradient} = 1.307 \sin 2\phi \text{ milligals per mile,}$$

and is simply a differentiation of the International Gravity Formula. At 45° latitude the variation is about 0.1 milligals for each 400 feet (122 meters) of displacement in the north-south direction. Latitude corrections are to be subtracted for stations north of the reference latitude and added for stations south of the reference latitude.

Free Air Corrections

The free air correction accounts for the difference in elevation of the station in question to the chosen datum, to which all the data is reduced. Since the gravitational attraction of the earth's mass varies according to the inverse square law, as described in Chapter I, the attraction of the earth at a height h will be:

$$G = [R^2 / (R + h)^2]\, g_0 \text{ milligals}$$

where g_0 is the value at the datum level in milligals and R is the radius of the earth. From this, the gravity difference between the datum and the observations at elevations h can be shown to be approximately $(2g_0h) / R$, since $h \ll R$.

Substitution into this results in 0.094 milligals per foot. This multiplied by the height above the datum is the free air correction.

Bouguer Corrections

As just stated, the force of gravity varies as the reciprocal of the radius squared and due to added elevations, points above the datum must be reduced by additions of a correction. The free air correction, of course, ignores the added attraction of the mass of material between the station elevation and the datum. This Bouguer correction is based on the assumption that the surface of the earth is everywhere horizontal. So effectively, the correction deals with an infinate slab of material of thickness h. Hills projecting above this slab and valleys extending below violate the assumption of a slab, but this is taken care of by terrain corrections. The Bouguer correction, due to the added attraction of this infinate slab, where ρ is the rock density, is:

$$g_m = 2\pi\gamma\rho h. \quad \text{milligals}$$

Gamma (γ) is again the universal gravity constant. If ρ is given the value 2.00 g/cm^3 (possible density for bedrock channel deposits), then the Bouguer correction would be 0.025 milligals per foot (0.082 mgals/meter). This is subtracted if the station is above the datum because, in effect, a mass is being removed.

Since free air corrections and Bouguer corrections are proportional to the elevation above or below the datum, the two may be combined into a single correction. This correction would be:

$$\text{Elev. Corr.} = (0.09406 - 0.01276\rho)\ h. \text{ milligals}$$

where h is in feet.

Terrain Corrections

Bouguer corrections assume that the topography adjacent to the station is perfectly flat. This is seldom true, of course, and hills and valleys must be taken into account when reducing observed values to a datum. Hills rising above the elevation of the station constitutes a mass which is in opposition to the mass of the earth because the force due to the hill is upward. Corrections for high topographic features are added to the observed gravity readings to compensate for or off-set the upward component. Similarly, the absence of mass in the valleys must be added to the observed value in order to restore what was subtracted in the Bouguer correction, the terrain corrections are always added regardless of whether the feature is a hill or a valley.

In most gravity surveys, stations are chosen so that the surrounding area is relatively flat. This being the case, terrain corrections may be omitted. In the upper mid-west where glacial deposits prevail, the terrain is relatively flat except for a few isolated areas. In almost all cases in this area, correction for terrain may be omitted if stations are placed discriminately. A method for terrain correction was described by Hammer (1939).

Earth Tide Corrections

Due to the movements of the earth with respect to the sun and moon, diurnal variations in gravity readings occur on the earth. The effect known as tides is caused by the gravitational attraction of these celestial bodies on the earth. Although the magnitude of tides on the solid portions of the earth is much less than on the bodies of water, it is still of importance when undertaking a high-precision gravity survey. This bulging of the earth, however, causes small but measureable changes in the force of gravity as the distance to the center of the earth is altered. An upward force on the mechanism of the gravity meter itself, is also applied. The magnitude of the change varies with latitude, time of month, and time of year, but the complete tidal cycle is related to a gravity change of 0.2 to 0.3 milligals. Magnitudes such as this indicate that tidal corrections can be very important when dealing with small gravity anomalies.

There are essentially two methods for correcting the tidal effect. The method usually used is to return to the base stations often enough that these changes will be incorporated into the instrumental drift curve, which will be

discussed in the next section. The second method makes use of the formula derived by Heiland (1940), which gives the vertical component of the tidal force (Δg) caused by the sun and the moon on any point of the rigid earth:

$$\Delta g = (3\gamma r\ M_m/2\ D_m^3)(\cos 2\alpha_m + 1/3) - (3\gamma r\ M_s/2\ D_s^3)(\cos 2\alpha_s + 1/3) \text{ milligals}$$

where γ is the Universal Gravity Constant, r is the radius of the earth in centimeters, M_m and M_s are the masses of the moon and sun in grams, D_m and D_s are the distances to the moon and sun in centimeters, and α_m and α_s are the geocentric angles in degrees the respective celestial bodies make with the station.

The importance of using theoretical calculations for the tidal force, rather than returning to a base station throughout a survey, will be discussed in the next chapter.

Drift Adjustment

It is important to recognize that instrument fatigue alone may be attributable to the fact that a station may exhibit different readings at different times. It has been standard procedure in the past to return to the base station every couple of hours for additional readings. Since re-occupation of base station is no longer necessary when using theoretical corrections for earth tides, reoccupation for drift should either be eliminated also, or reduced. Instrument drift is linear, due to a constant relaxing, and can be

approximated by a constant for a particular instrument. This drift constant is rather large when the instrument is new and decreases with its age. The Lacoste-Romberg gravity meter owned by the Department of Geology at Michigan State University, for example, presently has a drift of about 0.003 milligals per hour. By multiplying this value by the time of day and subtracting this from the instrument reading, this corrects for drift.

When taking measurements for instrument drift, it must be remembered that these readings also include the effect of tides. Simply subtract the theoretical tides corrections corresponding to the time of measurements. The difference between the results should be the linear drift.

Choosing a Density

In interpreting gravity anomalies over bedrock channels, it is necessary to estimate the density of the glacial overburden before one can postulate on the structure of the bedrock. This estimate, as already known, is needed in the calculation of the Bouguer corrections.

Glacial sediments can exhibit a wide range of densities, both laterally and vertically. Variations in physical properties such as, porosity, composition, mineralogy and degree of saturation and cementation are all responsible for density deviations. Till, which can be poorly sorted and inhomogenous, usually varies within a rather wide range of densities as compared to well sorted, homogeneous outwash deposits, which may vary insignificantly.

Published densities for various consolidated and unconsolidated material are useful in that they indicate a range from which to make choices. Grant and West (1965) indicate that the density of soil and alluvium ranges from 1.6 to 2.2; sandstones from 2.2 to 2.7; shales from 1.9 to 2.8; and limestones from 2.2 to 2.8 gm/cm^2. The density range for till is 1.9 to 2.4 gm/cm^2, which was compiled from a number of authors: Hall and Hajnal (1962); Lennox and Carlson (1967); Rankin and Lavin (1970); Eaton and Watkins (1967); McGinnis, Kempton and Heigold (1963).

A method for determining the density of the material which constitutes the difference in elevation between stations along a traverse was suggested by Nettleton (1939). The procedure consists of taking gravity readings along a traverse, where the relief is significant, if possible. The corrections are then applied, including Bouguer corrections where various possible densities are used for the material comprising the topographic feature. The gravity and topographic profiles of a significant feature in the vicinity of the survey are compared for the various chosen densities and the density corresponding to the profiles which correlate the least, is chosen. When making use of a digital computer, the labor involved in this method is minimal and only requires judgement of the correlation between profiles.

Seigert (1941) suggested a method for making corrections due to elevation without choosing a density. This method eliminates personal judgement and shortens computation time considerably. If, however, a digital computer is used the time consumed is small. With the technique, one must assume that the observed gravity values along short segments of the profile vary linearly with distance. Gravity and elevation values of N stations in a straight line are plotted against distance. Points representing stations 1 and 3, in both profiles, are connected by straight lines. The interpolated values of gravity and elevation are taken to be the intersection of the straight lines with station 2. The difference between the true values and the interpolated values are termed g_2 and h_2. The same procedure is repeated to obtain g_3 and h_3, g_4 and h_4, etc. g_i and h_i, then, should satisfy the equation:

$$g_i = -kh_i$$

at each station, where k is the combined free air and Bouguer correction factor. By adding kh to the observed values of gravity, elevation corrections are applied and a density need not be determined. The combined value of k is determined with the following equation:

$$k = -\sum_{i=2}^{N-1} h_i g_i \Big/ \sum_{i=2}^{N-1} h_i^{2}.$$

By adding kh to the observed values of gravity, elevation corrections are applied and a density need not be chosen. The density may be determined, however, by substituting it into the equation for the elevation correction.

These two described methods have been tested extensively in this study and both have proven to be useful. Nettleton's method, in most cases, can be more accurate, but without use of a digital computer is obviously time consuming. In areas where surface elevations change only a few tens of feet, an error of up to 0.10 gm/cm^3 for a density estimation would result in a gravity anomaly error which is negligible. Seigert's method can be very useful, especially in smaller surveys where necessary assumptions are on a smaller scale. Where a digital computer is not available this method may be given extra consideration.

Consistent densities of the bedrock and overlying glacial sediments are assumed for surveys of the areal extent under consideration in this study. Interpretations using variable density methods are not applicable with the amount of information available to these local surveys. Thus, although substantial density variations are unlikely to occur over the range of a couple miles, changes that do occur represent a hazard of the gravity method.

All corrections applied in this study were accomplished with the use of the 6500 CDC digital computer at the Michigan State University Computer Center.

CHAPTER IV

THEORETICAL TIDE CORRECTIONS

Introduction

It has been common practice to make tidal corrections of gravity data by taking repeated readings at a base station and from this constructing an appropriate diurnal curve. The accuracy of this curve relies on the constructor's knowledge of tidal forces and its aperiodicity. Regardless of this ability, it is the writer's contention that a curve using a few repeated readings can be quite inaccurate as compared to the actual diurnal variation.

In order to avoid making conjectures in producing a tidal curve from measurements, one would have to reoccupy the base station at least every two hours in most cases. If a surveyor, for example, were taking gravity readings over an eight hour period, he would have to return to base station four times. This is often an inconvenience, especially when working in inaccessable areas. The time involved, alone, can be considerable. If one of these reoccupations involved a false reading, which does occur occasionally, a total misinterpretation of the tidal curve is very likely to happen. The statistical error in measurement, alone, can result in misinterpretation.

Theoretical Considerations

Heiland's formula, introduced in Chapter III, is for the calculation of the tidal force, due to the sun and moon, on a rigid earth. Wolf (1941) has demonstrated that while the tidal effect observed differs from the tidal force thus computed for a perfectly rigid earth, due to the effect of the distortion of the earth itself by the tidal force, the mean square residuals of the computed force as compared with the tidal effect will be a minimum if the computed force is increased by 20%. This may be most simply accomplished by rewriting Heiland's formula as:

$$C = -\frac{3.6\gamma\ rM_m}{2D_m^{\ 3}}(\cos 2\alpha_m + 1/3) - \frac{3.6\gamma\ rM_s}{2D_s^{\ 3}}(\cos 2\alpha_s + 1/3). \qquad (7)$$

where C is the vertical component of the tidal force for a non-rigid earth. Even though this formula is simple, the tidal force is given as a function of the geocentric angle between the heavenly body and the point of observation, and a method must be developed for computing this angle from the data available to the computer; namely, the latitude and longitude of the station and declination and hour-angle of the heavenly body.

Adler (1942) has derived equations for the computation of the tidal effect, using available information. His formulas for the corrections for the moon and sun, respectively, are:

$$C_m = K_m \, [\cos\delta_m \cos\phi \cos(\theta_m - \lambda) + \sin\delta_m \sin\phi]^2$$

$$C_s = K_s \, [\cos\delta_s \cos\phi \cos(\theta_s - \lambda) + \sin\delta_s \sin\phi]^2$$

if we let,

K_m = 0.1978 mgals (combination of constants from equation 7)

K_s = 0.0902 mgals (combination of constants from equation 7)

δ = declination of heavenly body (angular distance north or south of the celestial equator)

ϕ = latitude of observation station

θ = Greenwich hour angle of heavenly body (the time lapse converted to degrees since the sun was directly over the Greenwich meridan)

λ = longitude of observation station.

The correction for the moon and sun can be combined to form:

$$C = 1/3 \, (K_m + K_s) - (C_m + C_s) \text{ mgals.}$$

Adler (1942) has constructed charts from which these corrections may be computed using values taken from the annual publication of the American Ephemeris and Nautical Almanac.

A computer program has been developed for the purpose of calculating tidal corrections, which substantially reduces time and labor. Greenwhich hour angle is not an available statistic for use in the above equations and it may be calculated by converting Greenwich time to sidereal time from which the right ascention is subtracted. Conversion for sidereal time and right ascention are found also in the American Ephemeris and Nautical Almanac. Examples of tidal variations computed as above, for Lansing, Michigan and an area in Florida with the same longitude, are shown in figure 3. Variation in longitude can be shown to produce only minor differences in the tidal force.

Reliability of Theoretical Variation

At East Lansing, Michigan, measurements of diurnal variation for certain days have been made using a Lacoste Romberg gravity meter (sensitivity-0.01 milligals). Readings were taken at least three times per hour to establish an idea of the true tidal variation for the various periods. The linear instrument drift was removed from each reading and a computor derived least squares fit to the points for each period was plotted. Theoretical variations were also computed for these days and are plotted together with the measured curves (figures 4, 5, and 6). The theoretical and measured curves agree exceedingly well. The slight discrepancies that do exist, though, have been explained to be due to the water loading of the Great Lakes. It can be concluded, therefore, that theoretical calculations for the tidal variation approximate the true tidal variations extremely well.

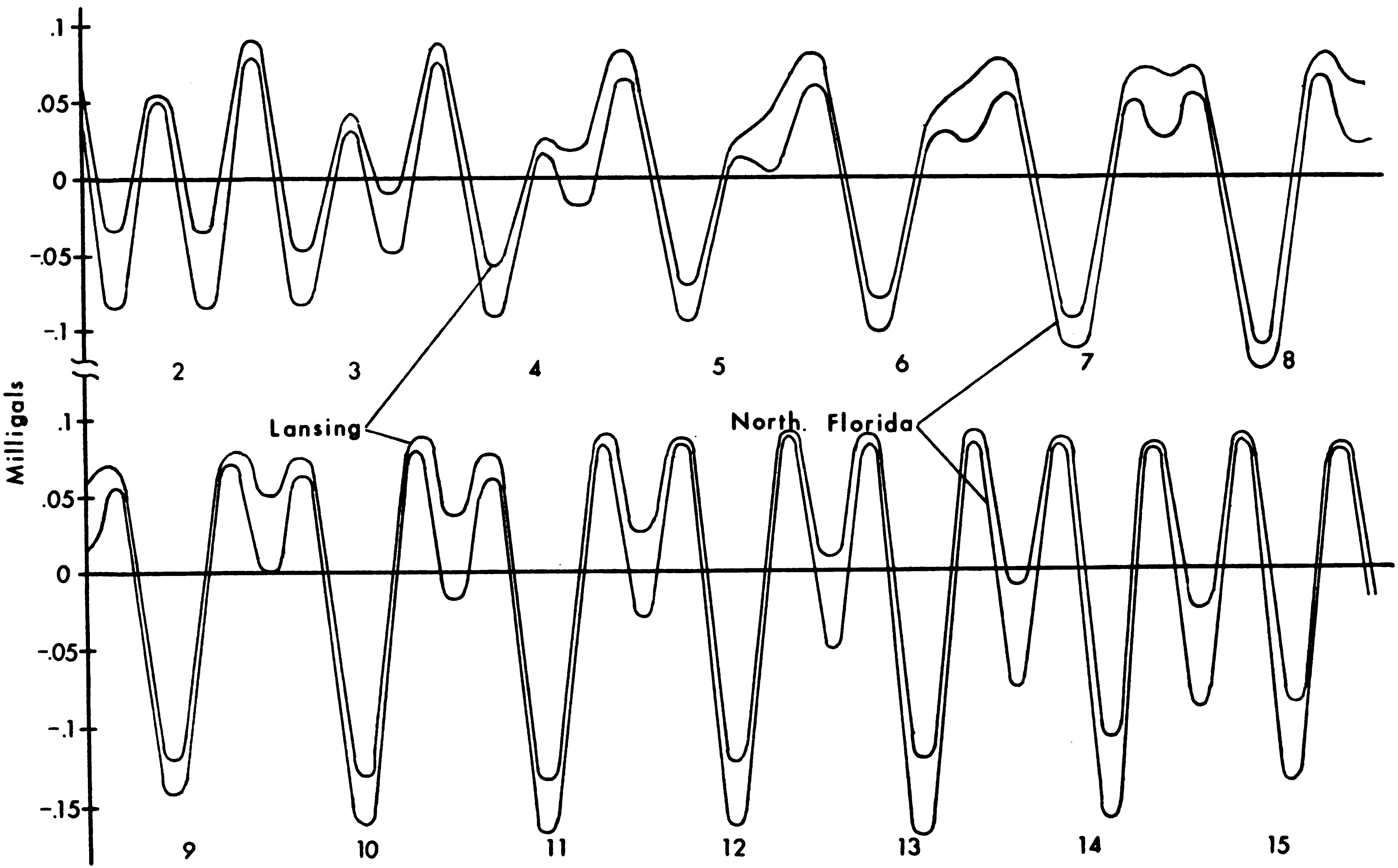

Fig. 3. Theoretical tidal variations for 14 days of August 1973.

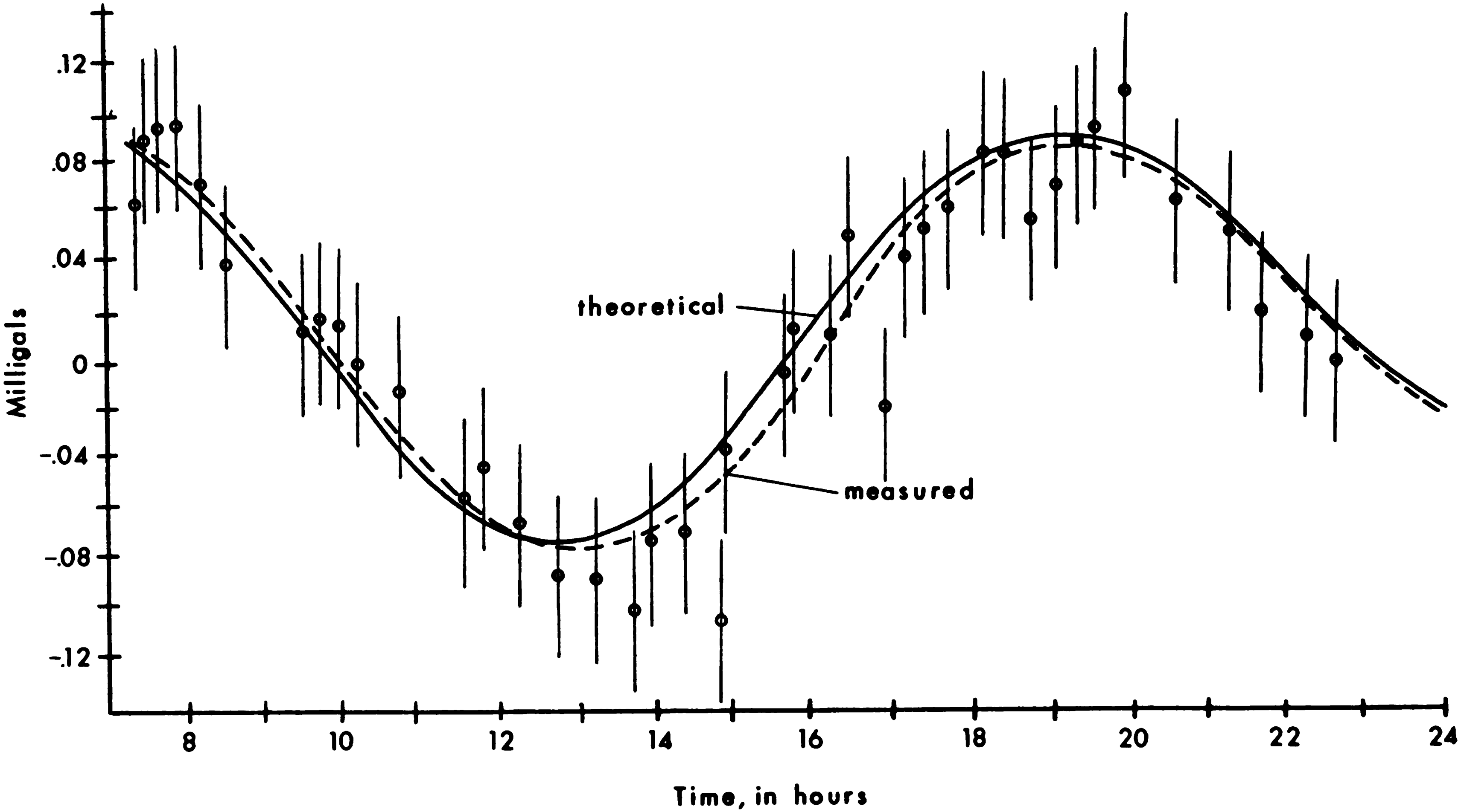

Fig. 4. Comparison of empirical and theoretical tidal approximations, March 9, 1974 Empirical is least-squares-fitted.

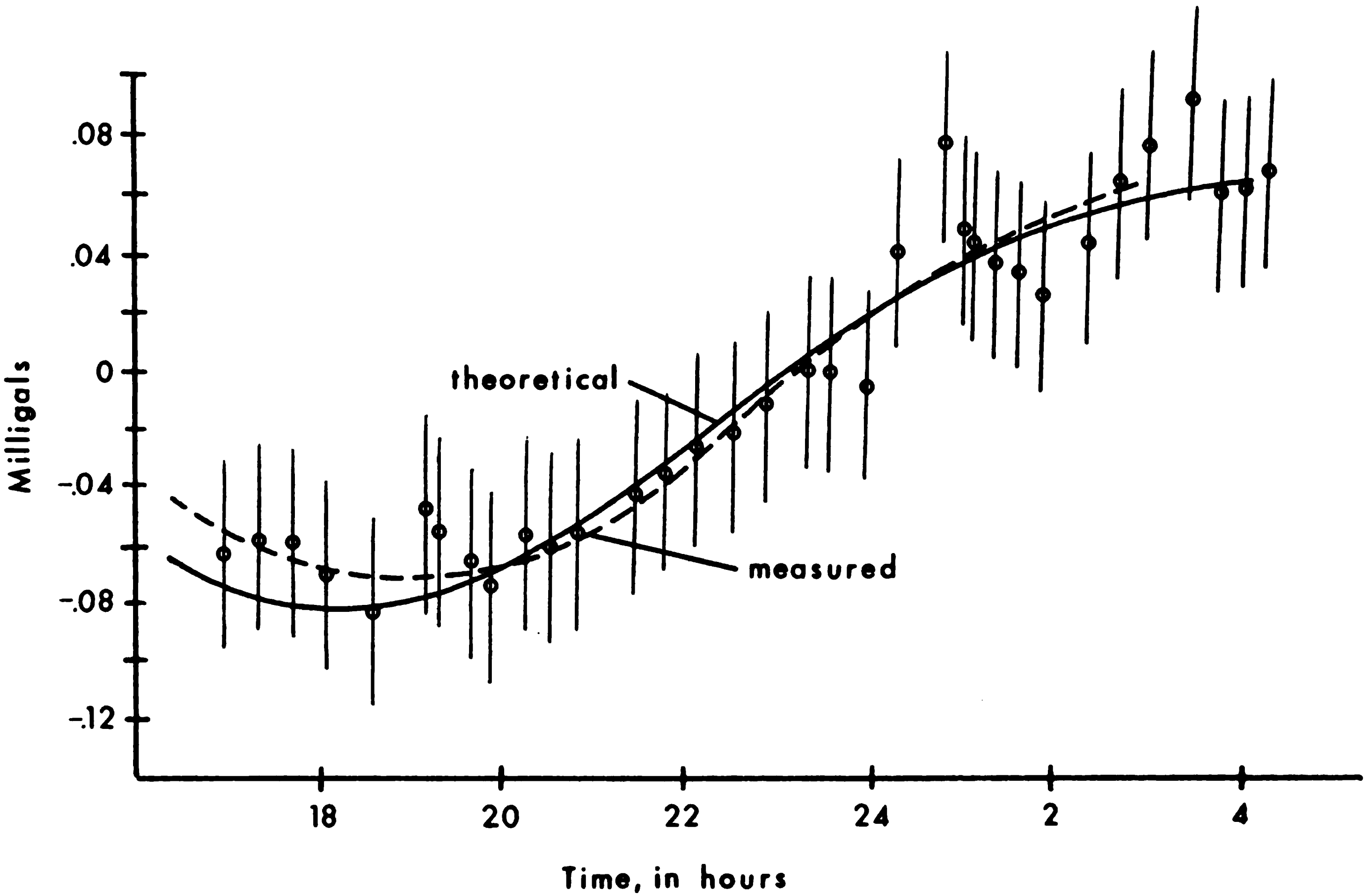

Fig. 5. Comparison of empirical and theoretical tidal approximations, March 1-2, 1974, using least-squares fit.

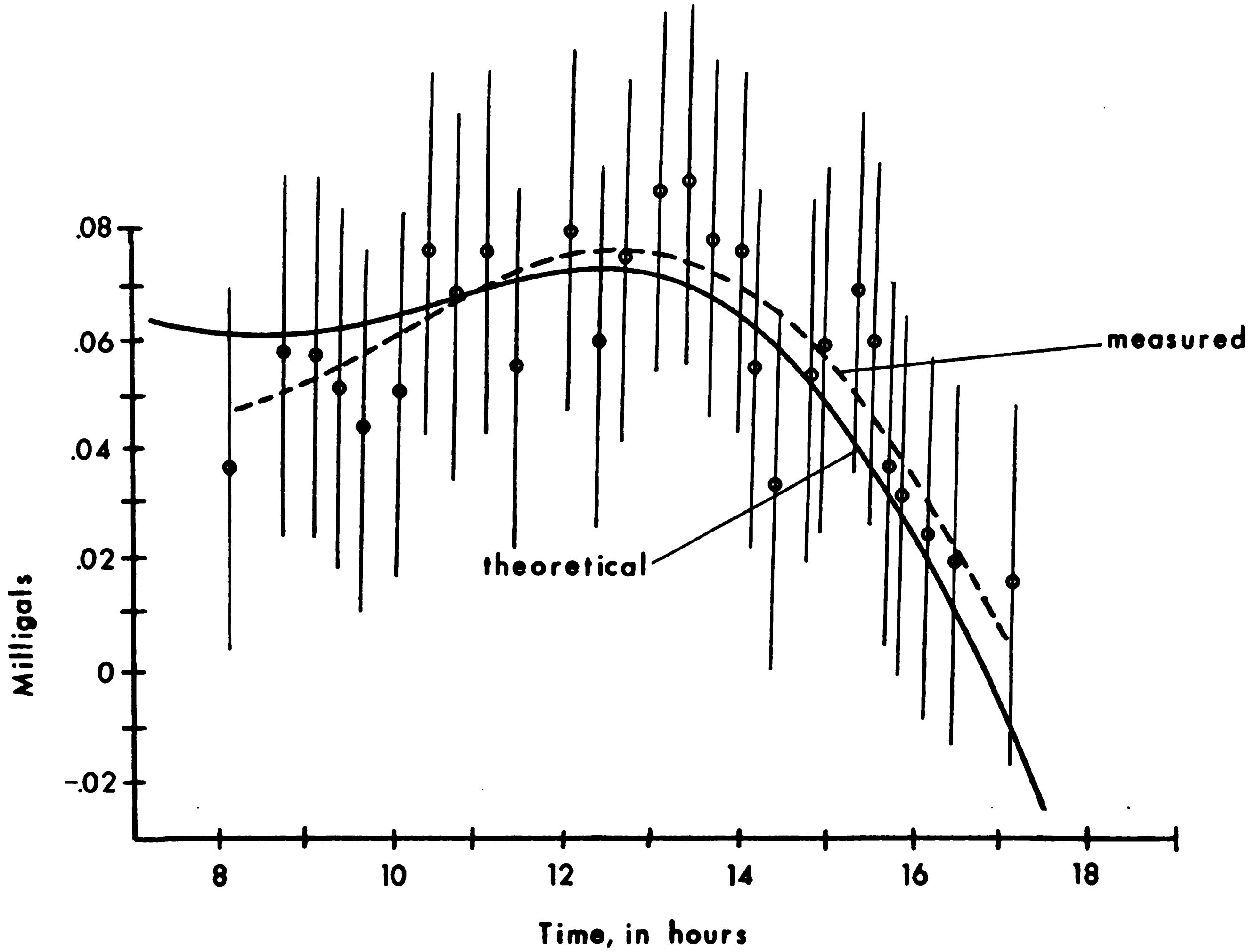

Fig. 6. Comparison of empirical and theoretical tidal approximations, February 2, 1974, using least-squares fit.

Comparison of Results using Theoretical and Empirical Variations

To test the theoretical and empirical tidal correction methods, a hypothetical gravity profile has been devised. By comparing the results after applying theoretical corrections to those upon applying the empirical method, one could make conclusions as to the reliability of the latter. This is acceptable, of course, because theoretical corrections have been shown here to be reliable.

Figure 7 indicates the tidal drift curves used to modify a hypothetical gravity profile. The empirical curve was produced using measurements from an actual gravity survey of August 7, 1973. It is evident that this curve is substantially different from the theoretically produced curve of the same day. It is difficult to establish definite reasons for this, but perhaps slight measurement errors were involved in creating the empirical variation. The theoretical curve, for August 1, 1973, has also been entered to help indicate that the tidal variation is not identical from day to day, and that it can change substantially within days.

To illustrate the effects of using different methods in correcting for earth tides, figure 8 was produced. The theoretical method applied to the data is assumed to yield a realistic, totally reduced gravity anomaly produced by a buried bedrock channel (solid line, figure 8). Using the diurnal variations of figure 7, the "raw data" (not shown)

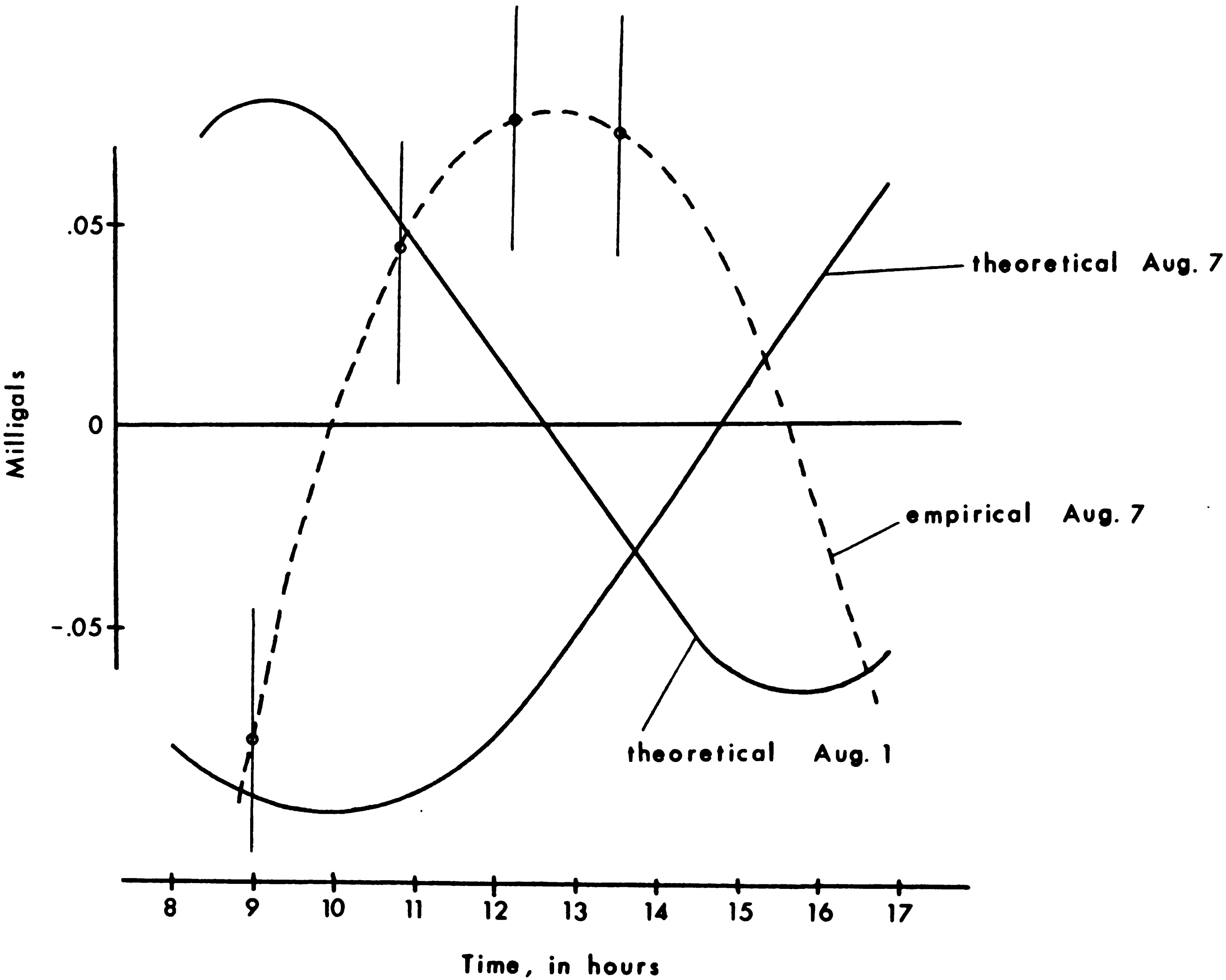

Fig. 7. Tidal force curves used in a survey.

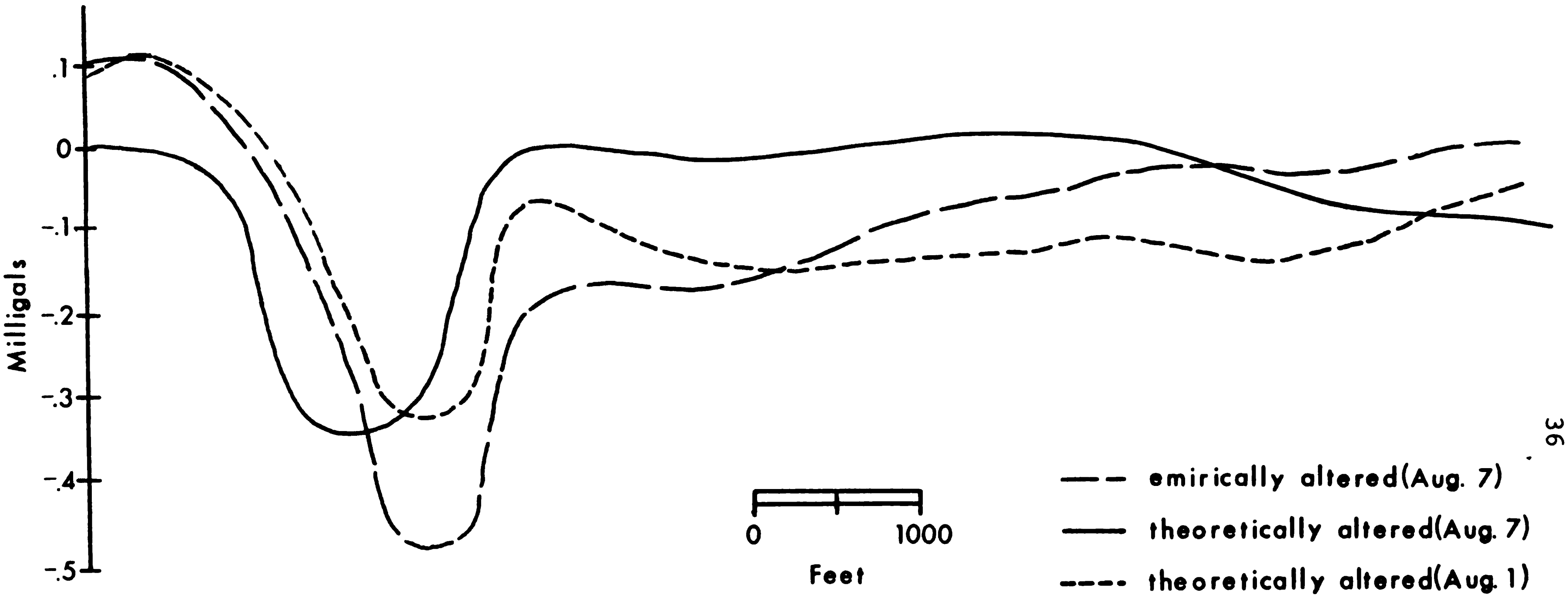

Fig. 8. Gravity profiles reduced theoretically and empirically.

to be expected is computed by adding the theoretical tidal correction to this realistic anomaly. An empirical representation, as would be computed using an empirical diurnal variation, was then produced by subtracting the latter from "raw data". The original hypothetical data represents a survey where the gravity readings were taken at equal time in intervals over a six hour period and the stations were also equally spaced.

In figure 8, the troughs of the theoretically and empirically produced profiles of August 7th have been shifted approximately 500 feet relative to each other. This lateral shift would seemingly cause a well driller to miss the peak production area by 500 feet, resulting in a substantial decrease in production of water or perhaps even missing the channel sands altogether. Undulations also begin to appear in the empirical curve, leading perhaps, to misinterpretations. The third profile of figure 8 helps to indicate the magnitude of changes that can occur by using different diurnal variation curves in altering the anomaly.

Figure 9 represents the gravity profiles created as in figure 8, but using an alternate sequence of station readings. Here, the troughs did not change perhaps enough, to cause a misinterpretation of the location of the channel sands, but merely a false indication of the size and depth. The noticeable undulation indicative of the right hand portion of the theoretically altered profile is completely

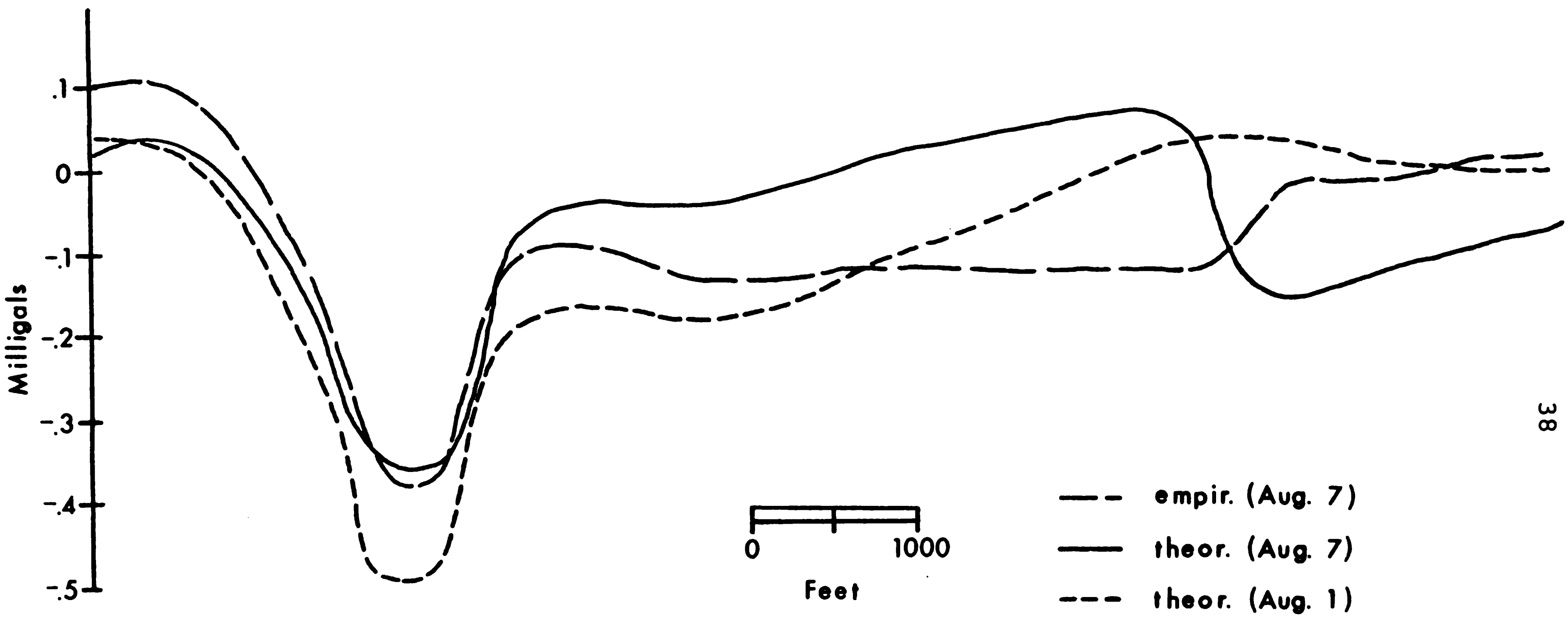

Fig. 9. Gravity profiles reduced theoretically and empirically by using alternate station sequences.

masked in the empirical profile. An anomaly of this size can certainly not be disregarded. Again, in figure 9, a gravity profile is produced using a tidal variation of a different day, but using the same sequence of station occupations. The magnitude of change, as can be seen, is substantial.

Additional indications of the changes that occur due to the use of different diurnal variations in altering a gravity anomaly are shown in figures 10 and 11. The dashed profile was created by subtracting out the tidal correction from the original hypothetical data for August 7th and then adding the corrections, for the same times and station sequence, of August 1st. In effect, this would be creating new raw data. But, by comparing the original raw data to the newly created raw data, one can get an idea of the effect that tidal corrections can have on the shape of a gravity profile and the misinterpretations which can result. In figure 10, a lateral shifting of the trough occurs. Figure 11, which represents a different station occupation sequence from that of figure 10, indicates an anomaly that one profile depicts while the other does not.

Conventionally, empirical corrections due to earth tides seem to be the simplest and easiest method, but a closer investigation has shown that this method not only can be unnecessarily inconvenient and time consuming in field procedure, but can lead to unfortunate interpretations. Theoretical calculation and reduction for the tidal force serves

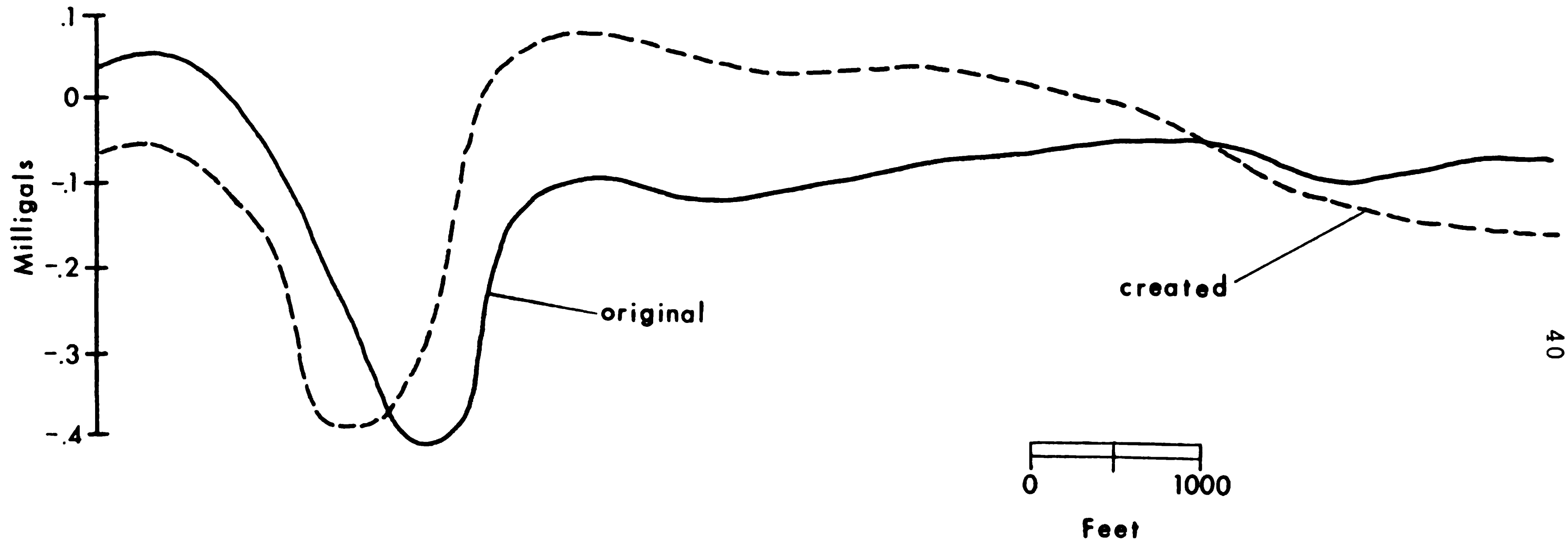

Fig. 10. Effects of using incorrect diurnal variation.

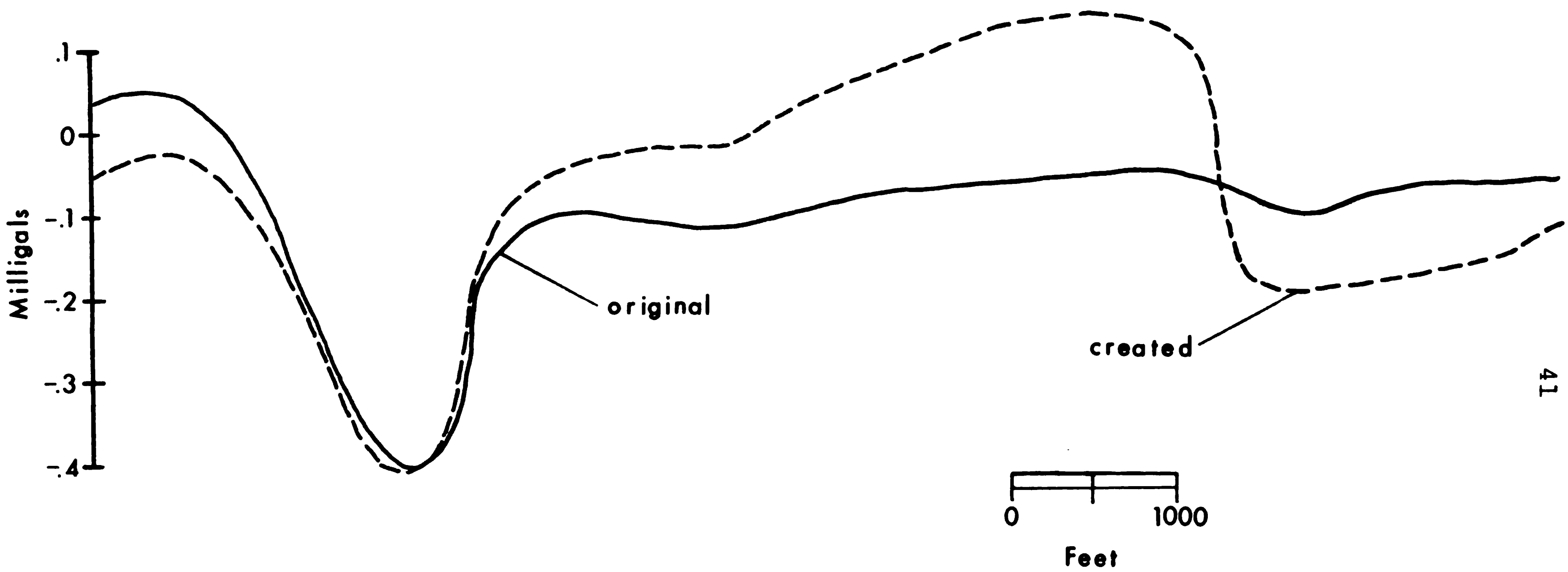

Fig. 11. Effects of using incorrect diurnal variation - alternate station sequence.

as an alternate method. Tidal variation curves produced theoretically have shown to be a good approximation of the true tidal force for inland areas. This was accomplished by comparing them with detailed measurements from a gravity meter. By actually applying the two methods to the same set of data it was determined that the empirical technique could create substantial differences in the outcome as compared to the theoretical method. This is because tidal force interpretations, resulting from the few values taken over the period of a day and used in the empirical method, can contain misinterpretations from errors arising, due to the limited data set. These differences in outcomes, from using the different methods, could lead to false interpretations of the subsurface.

If the theoretical method is as reliable as a detailed empirical curve, then it has the advantages of being quicker, less expensive, and void of errors. It is suggested, therefore, that the theoretical method of applying tidal corrections be used whenever possible.

CHAPTER V

ISOLATION OF RESIDUALS

Introduction

When corrected gravity values are plotted on a map or profile the result is seldom useful in depicting subsurface geology or more particularly in our case, bedrock topography. These values incorporate forces not only due to the near-surface features of interest, but also deep seated regional features. In order to isolate the near-surface objectives from the deeper components, one must apply a method of interpretation. Several interpretation methods have been developed for this purpose, of which each has its particular advantages and disadvantages.

The terms "Regional" and "Residual" need be defined in understanding the separation of shallow and deep gravitational components. The "regional" is defined as the force caused by deep-seated structural features producing variations in gravity at the surface which are much larger in areal extent than the structures ordinarily of interest in prospecting. The forces produced by the structures of interest alone are termed "residuals." These two components are complementary and the residual is determined by subtracting the regional from the Bouguer Gravity anomaly. Methods for approximating the regional are described here.

Conventional Methods

There are two general categories of methods for the removal of the regional from observed gravity. They are graphical and analytical. With the graphical techniques the regional trends must be estimated from the contours or profiles of the observed data. The regional lines are superimposed over the existing lines where a certain amount of arbitrary interpolation is involved. A great deal of judgement and experience is needed to derive the proper regional from the Bouguer anomaly. Graphical methods have the advantage that all geologic information available can be used to determine the regional, but in the same instance much experience is required for good results.

The method first used for separating the regional effect from the observed values was by visual smoothing. The procedure consists of connecting the undisturbed portions of the observed profiles or contours with straight lines or smooth curves. It is assumed with this method that the regional is broad and smooth and otherwise non-complex. The smoothing approach can not be applied with any accuracy to areas which exhibit complex regionals, but with small local surveys, where the area covered, in most cases, is much smaller than the major structural feature which governs the regional trends, the method can be used effectively (Nettleton, 1954).

A similar version of the smoothing method is cross profiling. A series of profiles are drawn on a contour map as a network of intersecting lines. There is an additional control in the fact that the profiles must be adjusted so that values of the crossing profiles are identical at points of intersection. Like smoothing, though, cross profiling can lead to erroneous results when performed on a complex area. Where smooth, broad regional trends are characterized the profiling method can prove quite accurate and useful.

These graphical methods of contour and profile smoothing depend on the judgement and experience of the interpreter. With analytical methods of determining the residual, mathematical operations of the observed data make it possible to isolate anomalies without depending upon a great amount of judgement. An unfortunate consequence of analytical techniques is that they often become too mechanical and known geologic factors may be completely ignored when interpreting an area.

This disadvantage is exemplified by the grid system of calculating the residual (Griffin, 1949). By averaging the gravity values at equally spread stations along the periphery of a circle, the regional is determined for the station which lies at the center. The residual is simply the observed value at the center of the circle minus the average of the periphery values. This method was designed to eliminate judgement by the interpretor, but it has been pointed out that choice of a radius is important and somewhat arbitrary (Dobrin, 1960). The circle must lie completely outside the anomaly, but within other disturbances.

Other analytical methods include vertical second derivative and downward continuation. They are very similar in theory and calculation. The two methods have been examined and explained by Elkins (1951) and Trejo (1954) respectively. While these techniques have proven valuable in amplifying the high frequency anomalies with respect to the low, total elimination of the regional does not exist. With these methods, hidden local anomalies become noticeable, but quantitative interpretation is not possible. Since the regional trends are still present, in effect, judgements on the physical properties of the subsurface can be in error. If mere location of a subsurface feature is desired, this does not present a disadvantage. The vertical derivative and downward continuation methods require the use of charts and grids. Weighting coefficients also have to be selected depending on the type of problem. It is evident, then, that vertical derivatives and downward continuation require a certain amount of experience and judgement. If quantitative analysis of buried bedrock channels is needed, these two methods are not applicable.

Digital filtering can also be employed using the Fourier integral. This method involves the elimination of a certain range of frequencies from a given "gravity signal."

Since the regional is assumed to be the broad trends, the long wavelengths of the signal are filtered out. A problem exists from the question of what size frequencies should be eliminated and what size should be permitted to pass. Usually a specific filter is needed for each specific problem. Digital filtering, along with the previously described analytical methods, does not produce true residuals (Skeels, 1967). An overall regional is not approximated and subtracted out. Therefore the only available analytical method in which a true regional is approximated is the least-square technique (Skeels, 1967).

Least-squares Polynomial Method

In evaluating and interpreting gravity data it is desirable to incorporate the most accurate and reliable methods available. Speed and simplicity are also of concern when choosing the best technique. The above discussion demonstrates that gravity residuals obtained by applying many of the previously described methods can be quite subjective. Others have shown to be non-quantitative. Erroneous conclusions can be obtained in using these methods. This is particularly important when conducting surveys for locating buried bedrock channels, where very accurate procedures are necessary. The least-squares method has the advantage that it is analytical, but also known geologic information can be applied for its use. The subjectivity involved in its procedure is minimal.

The least-squares polynomial approximation is a widely used and popular method. It involves the fitting of a line or a surface to a set of gravity data. The degree of the polynomial which is to be fitted depends upon the trends of the regional. If the regional happens to slope with a continuous gradient in one direction, for example, then the polynomial should be of the first degree. To obtain the residual the fitted polynomial is subtracted from the corrected gravity data.

A least-squares approximation is defined as a line or surface of which the sum of the squares of the vertical distances from the polynomial fit to each of the original points is a minimum. A one dimensional approximation is of the form: $Y = a + bx + cx^2 + dx^3 = . . .$, where x is a Cartesian coordinate and a,b,c,d, etc. are the coefficients determined by the least-square procedure. The degree of the fit is related to the number of terms in the equation. A two dimensional approximation is of the form: $Z = a + bx + cy + dxy + . . .$

The suggested means of performing a least-squares interpretation on a set of gravity data assumes that the residuals occur randomly and that positive and negative undulations have an equal probability of occurrence (Ibrahim, 1970). With this assumption, first degree polynomial approximations along with higher order approximations are calculated. The polynomial equation which produces the smallest sum of the squares is chosen as the regional approximation. This is an analytical method, thus leaving no judgement to the interpreter. In opposition to applying least-squares in this means, the polynomial with the smallest sum of the squares is not the best regional approximation. Generally, as the order of the polynomial increases, the sum of the squares approaches a minimum. It is conceivable that a very high degree equation would approach the actual Bouguer anomaly and would incorporate local objectives. Subtraction of a regional such as this would eliminate all anomalies.

The correct way to apply the least-squares method involves the analytical advantages as well as the graphical advantages. Judgements can be made by knowing what type anomalies are produced by buried bedrock channels, and the accuracy of mathematical modeling can be included. By inspection of the Bouguer anomaly the general trends of the regional can be determined. Fluctuations as small as those produced by bedrock channels are either obscure or very small compared to the total picture. Since these, then, are distinguishable from the large scale trends, determinations as to the degree of the polynomial which best approximates the regional become straight forward. After determining the order of the equation, the calculations for the best fit is performed and subtracted from the original data to obtain the residual. In cases where the local anomalies are quite large the interpreter may choose to use only the data, which is not incorporated in these anomalies, to calculate the regional (Skeels, 1967). These local anomalies can distort the true regional when least-squares is performed. This procedure need only be used, though, when desiring very accurate determinations for large buried bedrock valleys.

Some examples of isolating the residuals using one dimensional least-squares are shown in figures 12 and 13. In part (a) of figure 12 it is evident that the overall trend is linear. An equation of the first degree would then be used to approximate the regional. Similarly, in figure 13a, the general trend of the profile is arcuate. A second order polynomial is, of course, used to approximate an arc shaped curve. In both figures the regional, fitted by least-squares, is indicated and subtracted from the Bouguer anomaly to produce the residual. In small scale surveys, which this study is directed toward, only simple, low-order regionals are likely to occur. Thus regional trends which can best be approximated by a fourth degree polynomial, or higher, are unlikely to prevail in surveys which have an areal dimension up to a few miles. Additional examples of regional fittings are included in Chapter VII where sample surveys are described. The smoothing method is analogous to the least-squares technique and in certain cases can be used accurately in profile surveys. This is particularly true when the survey is at a small scale.

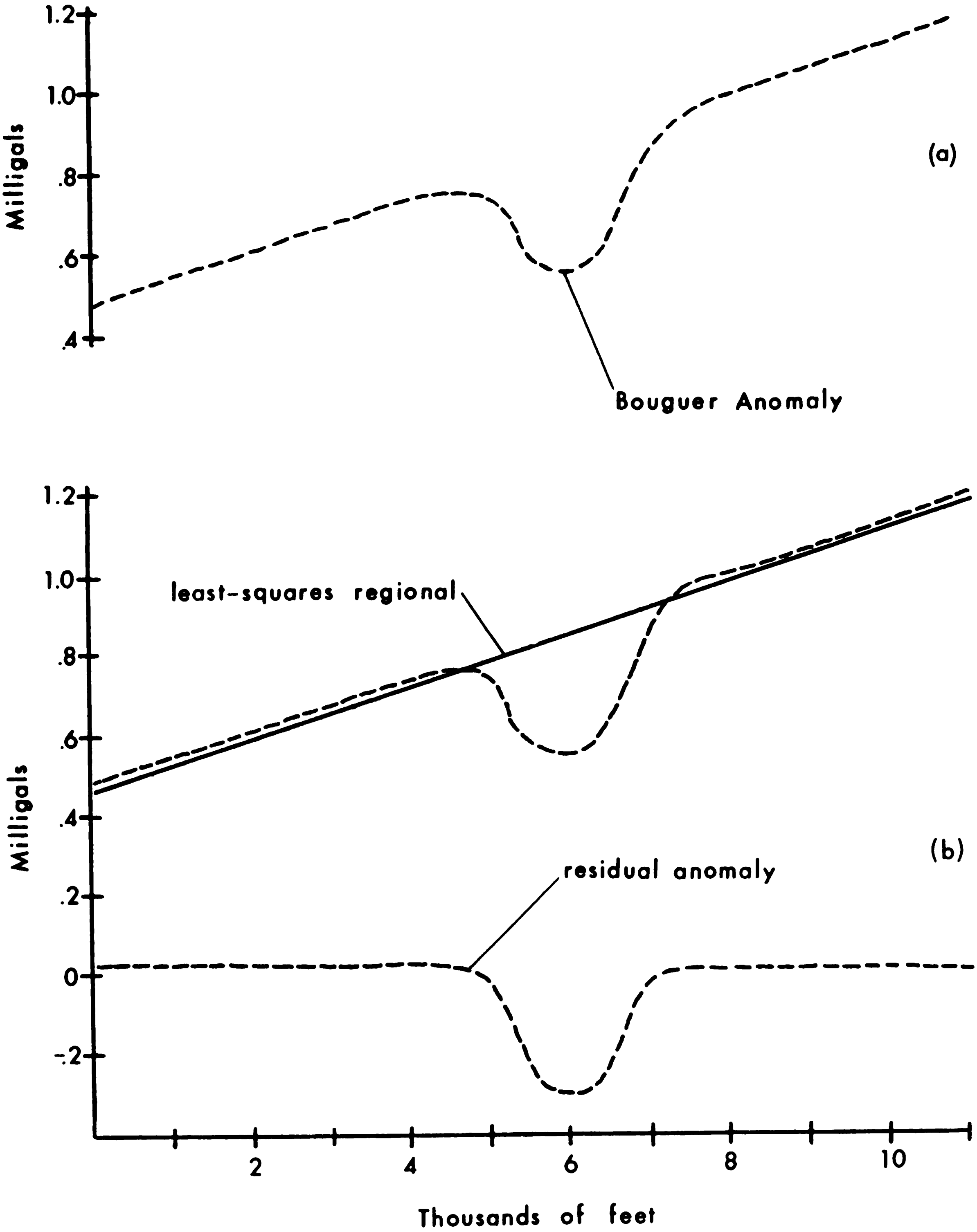

Fig. 12. 1st degree polynomial approximation of regional.

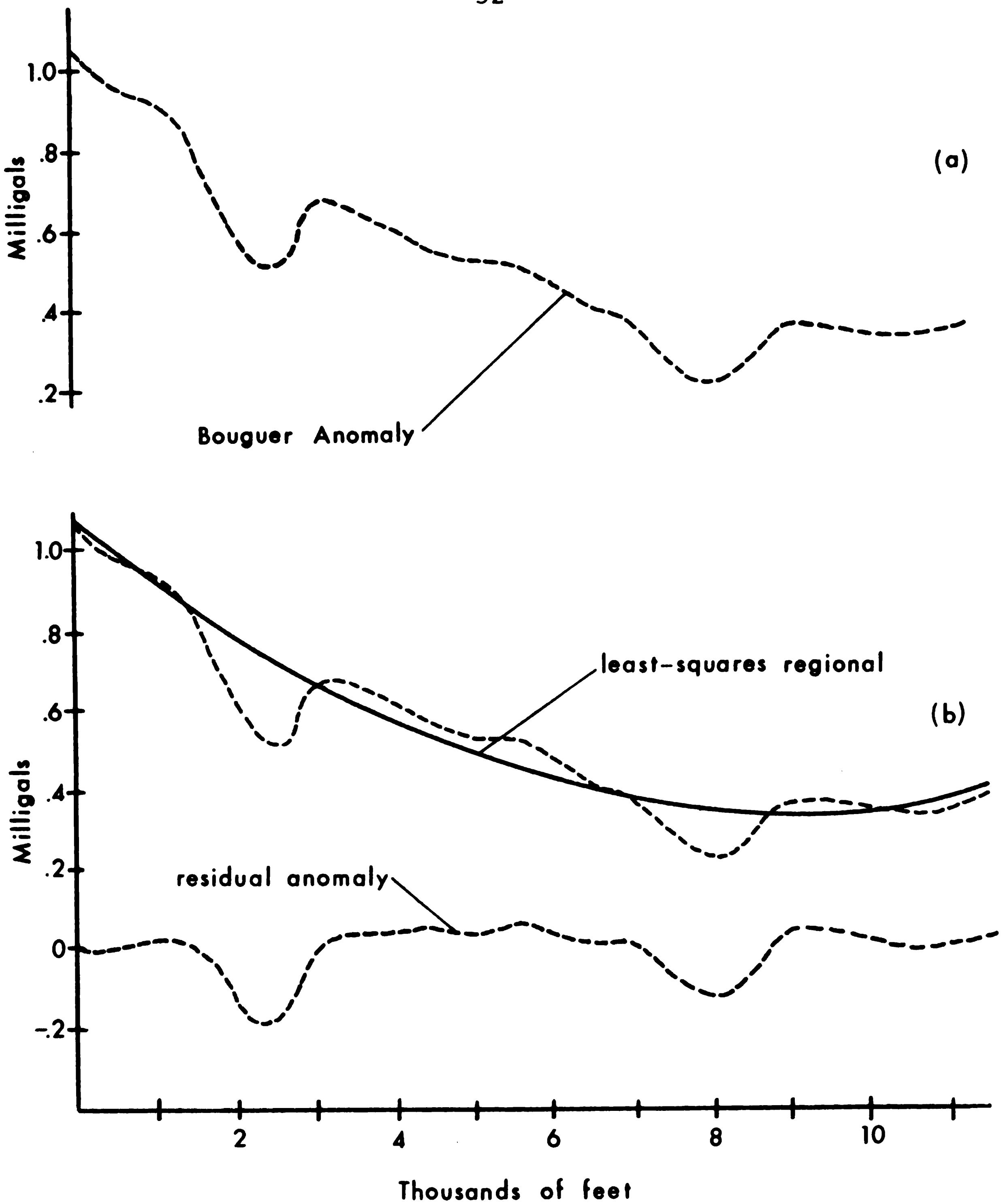

Fig. 13. 2nd degree polynomial approximation of regional

One problem inherent in this study is uncertainty as to the origin of the observable gravity anomalies. Hopefully the observable fluctuations are produced by buried bedrock channels, but other subsurface structures can also produce small wavelength anomalies. Anomalies which do not resemble those created by bedrock channels present no problem, but conceivably similar anomalies which are of a different origin can exist. There is no available method of separating anomolous curves due to bedrock valleys from those due to other sources when the curves are similar. A gridded survey, as compared to profiling, would certainly reduce the chances that an anomaly produced by a pinnacle reef, for example, would be mistaken for a buried bedrock channel.

CHAPTER VI

INTERPRETING THE RESULTS

Introduction

An important phase in conducting a survey is interpretation. In this case, the problem is to determine bedrock configurations from gravity anomalies. These anomalies should indicate such features as: configuration of bedrock drainage pattern; location, size and extent of glacial fill channels in the bedrock; depth to and slope of the bedrock surface. A basic understanding of the types of gravity anomalies, which various shaped bedrock channels produce, is very helpful.

After isolation of the shallow objectives from the deeper regional trends, the resulting gravity anomaly is not an aboslute representation of the bedrock surface. This curve is defined by points which incorporate certain errors in measurement, which include those due to: (1) instrument readings; (2) elevation measurements; (3) latitude measurements; (4) station distances, etc. When interpretating a set of reduced data, these errors must be kept in mind and a certain amount of smoothing is often necessary. Depending upon the accuracy in the measurements, which were used to make corrections upon raw data, various true anomalies can be obscurred, shifted lateraly, attenuated or amplified. Creation of false anomalies can also be a result. Although it is desirable that all measurements be made with high precision, the fact must be considered that increasing the precision usually increases the time and labor in more than direct proportion. It therefore becomes the duty of the surveyor to maintain a degree of precision as high (but no higher) as can be justified by the purpose of the survey. The accuracy in measurement when attempting to locate large subsurface features need not be as great as that needed in locating small features.

Modeling of Gravity Anomalies

The purpose here is to describe bedrock configurations given various gravity anomalies. To gain a knowledge of the shape of anomalies that certain buried bedrock valleys create, various theoretical anomalies have been produced by the method described in Chapter I. Four average sized bedrock valleys, the size in which this study is concentrated, were fitted with stacks of infinite horizontal strips (figure 1). Nine theoretical gravity profiles were calculated for each valley using equation 6 of Chapter I. The nine profiles for each individual channel reflects the various combinations of density contrasts and depths of burial. The gravity profiles, here, have been assumed perpindicular to the channel, but could be computed for any other angle.

For each of the 36 theoretical gravity profiles, the minimum point of the anomaly was plotted against its respective density contrast. Points representing equal burial depths (vertical distance from shoulder of bedrock channel to ground surface) for a given channel size were then connected (figure 14). This graph displays the effects that the various parameters have on the magnitude of the anomaly. It is evident, from figure 14, that depth of burial plays a minor role in producing the curve. For channel 1, for example, using a density contrast of 0.4 gm/cm^3 the minimum point for the 100 foot burial is 0.244 mgals, and only 0.255 and 0.266 for the 75 and 50 foot burials respectively. The modification, here, of 0.011 milligals for every additional 25 feet of burial is relatively minimal. The difference in slope between the lines representing the depths of burial is itself an indication of the relative insignificance. Figure 15 indicates the likeness between produced gravity anomalies where the only difference is in amount of overburden.

The values for thickness of overburden used in this study are characteristic of those for Northern North America, and of concern for economic groundwater exploration. Although changes within this range of several hundred feet have little effect on an anomaly, a much deeper bedrock would have a significant effect. Objectives at these greater depths would produce anomalies which are much broader, and thus harder to isolate from regional effects and other anomalies.

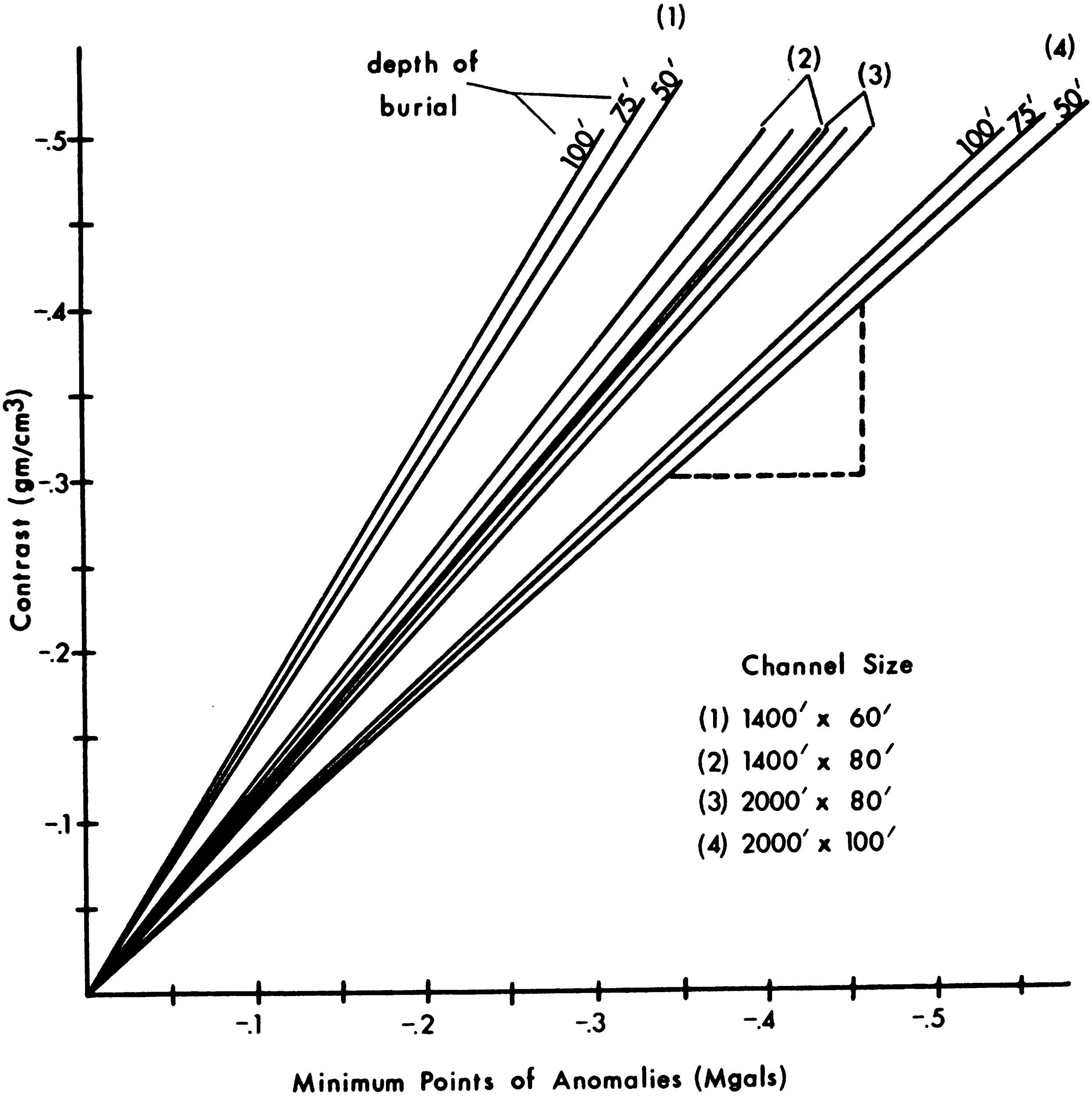

Fig. 14. Minimum point anomalies for various shaped bedrock channels with various densities.

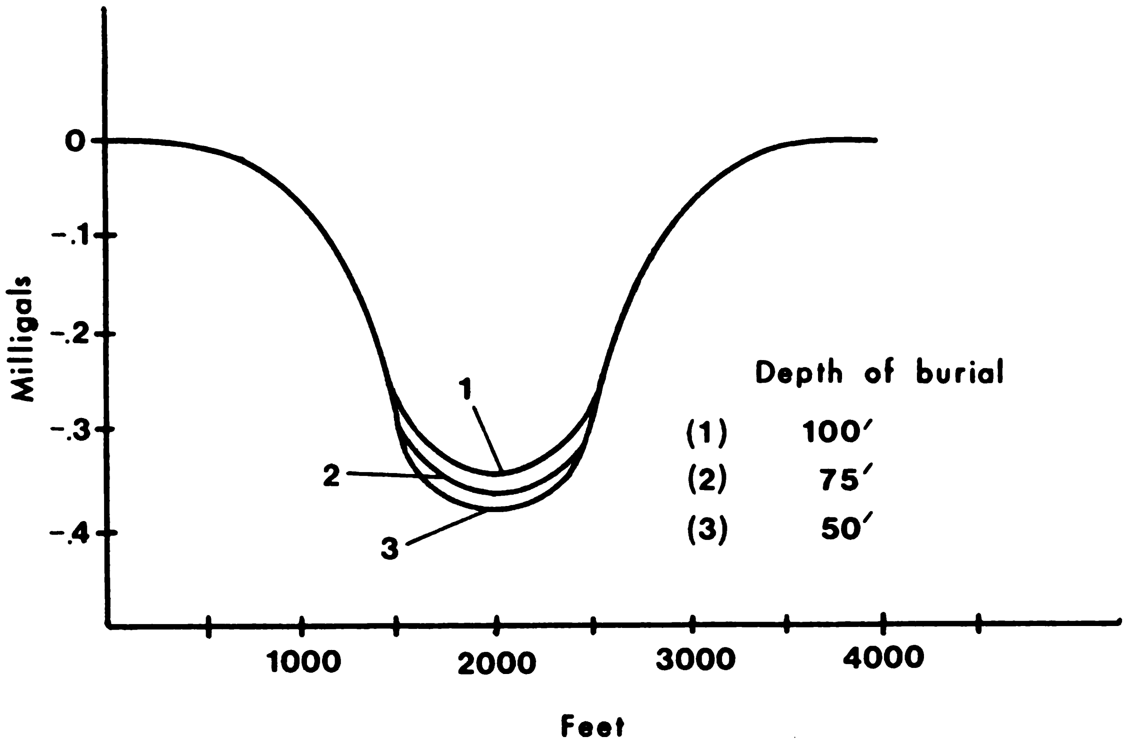

Fig. 15. Depth of burial effect on anomaly.

The effect of the lateral extent of the bedrock channel on the minimum point is also indicated in figure 14. Comparing the 100 foot burials between channels 2 and 3, where the only difference is the width of the valley, the minimum point for the 1400 foot wide channel using a density contrast of 0.5 gm/cm^3 is 0.400 mgals while the 2000 foot channel reflects a 0.438 mgal value. Thus, the change in width of a bedrock valley also plays a rather minor part in producing the minimum points of a gravity profile. Although there is an added mass in this situation, unlike the variation in thickness of overburden, the mass is added laterally and has a small effect on the force over the central portion of the channel. There is an overall significant difference, though, in the shape of the curves produced by these varying width channels. The shoulders of the gravity anomaly over the 1400 foot channel are much narrower than the anomaly produced by the wider channel. Therefore, the width of the shoulders of the curve are a good indication of the width of the underlying channel. In measuring the anomalies produced by approximating bedrock valleys with infinite horizontal strips, it was determined that the width of the anomalies at 80% of the height is a good estimate for the lateral extent of the subsurface feature (figure 16). The anomalies are produced from 2000 and 1400 foot channel widths as indicated. The width of the anomalies are also 2000 and 1400 feet at 80% of the height.

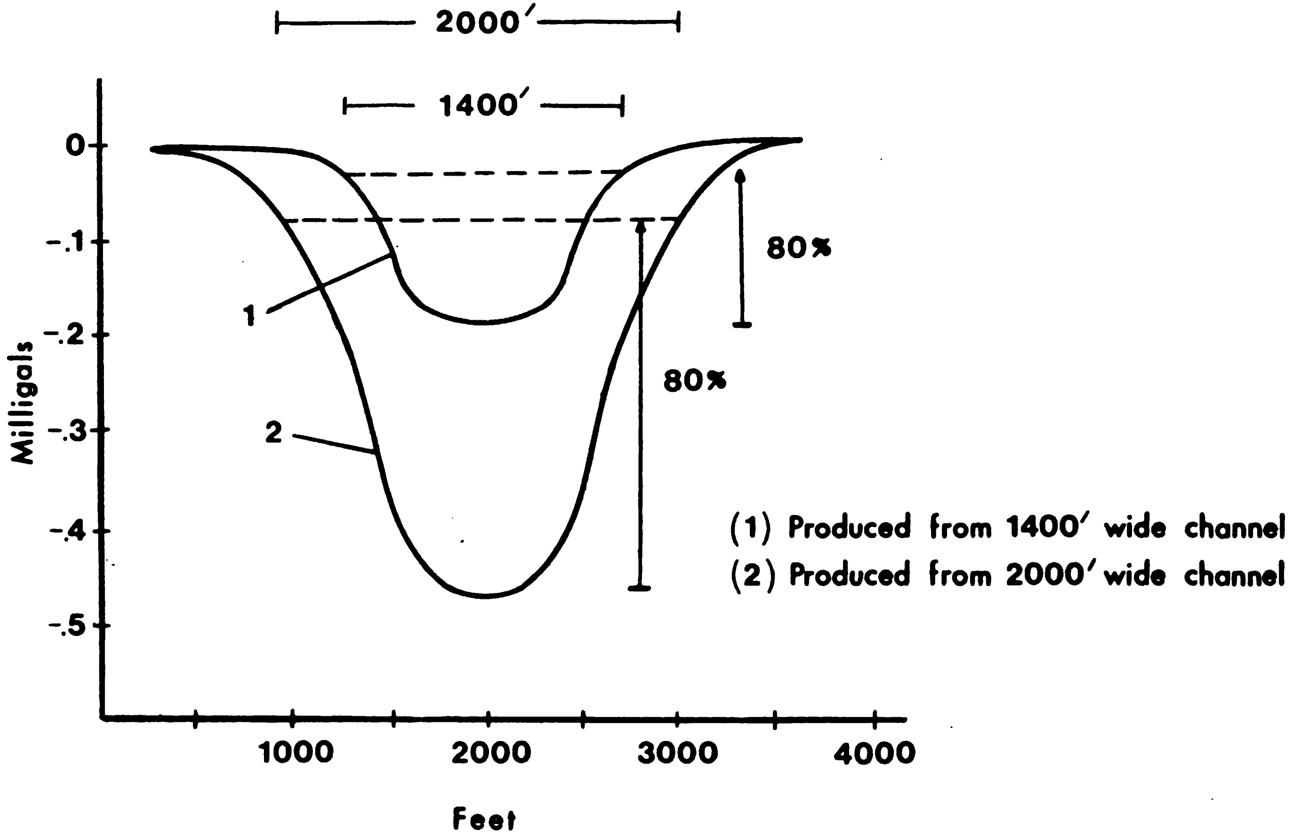

Fig. 16. Channel width estimation.

The depth of the bedrock channel (vertical distance from trough to shoulders) plays a more significant role in producing the minimum points of the anomaly. The added mass in this case is displayed in the minimum points. From figure 14, the difference between minimum points for the 75 foot burials of channels 3 and 4 is 0.064 mgal at a density contrast of -0.3 gm/cm^3. As indicated this difference increases with increasing density contrast where at -0.5 gm/cm^3 the additional force becomes 0.107 mgals. The wider angle between groups 3 and 4 exhibits the fact that depth of the channel is more significant in producing minimum points than width of the channel, as displayed by the angle between group 2 and 3. The angle between groups 1 and 2, where a difference in channel height is indicated, is comparable, of course, to the angle between 3 and 4.

The final parameter which effects the magnitude of the gravity anomaly is the density contrast between the channel sands and gravels and the consolidated bedrock deposits. Like the channel depth, this also is of more importance than the depth of burial and lateral extent of the bedrock valley. A change in density contrast of only 0.1 gm/cm^3, as indicated by the dashed vertical line of figure 14, corresponds to a 0.114 mgal change represented by the dashed horizontal line. This effect of density contrast is more evident, of course, in the larger channels, because of the greater anomalous mass that is involved. This is indicated by the slope of the lines of figure 14. The larger slopes of channel 1 correspond to a comparably small channel size and relate to a 0.067 mgal change per 0.1 gm/cm^3 change in density contrast.

Determination of Bedrock Topography

Even though certain estimates can be made of the bedrock surface from residual gravity anomalies, it is unreasonable to use direct visual judgement alone to predict buried topography. Various different subsurface situations, for example, can produce similar gravity anomalies at the ground surface (figure 17). Channels of different sizes with different density contrasts, here, are shown to create comparable force fields at the surface. It is virtually impossible to describe the subsurface from these anomaly curves. If methods to convert residual anomalies to bedrock elevation are used, visual judgement can be totally unnecessary.

One method, proposed in this study simply involves the interpolation technique. In the process of setting up an array or traverse for a gravity survey, one should include two or more drill hole sites as gravity stations. If drill holes, where depth to bedrock is established, are not within the area of interest, others may be chosen from outside, with the forfeiture of some accuracy. It may be wise, however, to choose an area that does have a certain degree of well control. The readings taken at these well sites are to be corrected using the normal procedures previously described. The residual

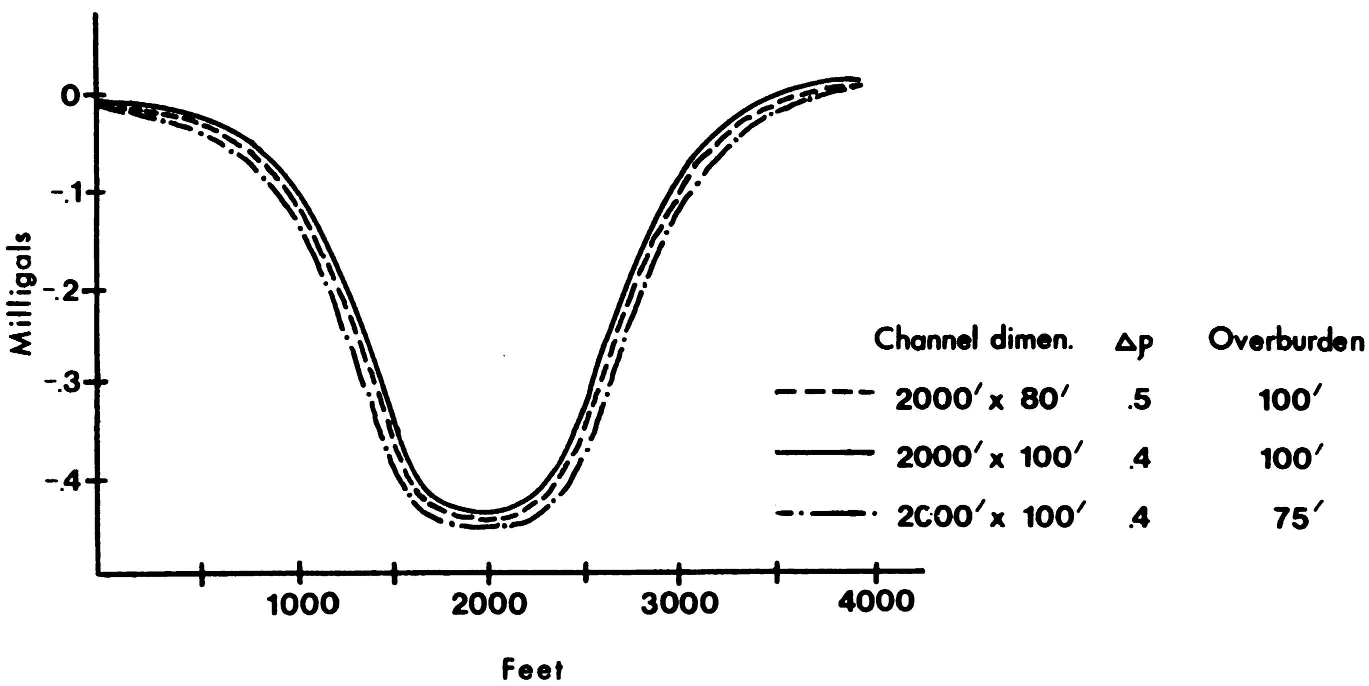

Fig. 17. Similarity of anomalies produced from different channels

values of the drill hole readings are then plotted against their known bedrock elevations and fitted with a straight line. From this line bedrock elevations can be determined for each station, which correspond to the various residual values. Refer to figure 25 of Chapter VII for an example. Ibrahim (1970) used a similar graph, but only to evaluate the accuracy of methods of residual isolation. The effectiveness of this method relies on the dependability of the drill hole data and the number of drill holes used.

Another method to convert residual anomalies to bedrock elevations was suggested by Ibrahim and Hinze (1972). This method requires the knowledge of the density contrast, and assumes the bedrock is in the form of an infinite horizontal slab. It makes use of the equation:

$$H_n = [gr_n \,/\, 2\pi\gamma\Delta\rho] + D$$

where H_n = elevation of the bedrock surface,

gr_n = residual gravity anomaly at the n^{th} observation sight,

γ = gravitational constant,

$\Delta\rho$ = density contrast, and

D = elevation of the bedrock surface at the drill hole.

The limitation of this method is that it requires an accurate density contrast. However, only one drill hole is needed while the first method requires at least two.

Accuracy of Measurements

The size of bedrock channel which can be detected depends chiefly on the accuracy in which the various measurements were taken, in order to apply corrections to the raw readings. It can be shown that the instrument reading and the surface elevation determinations are the only measurements which have to be taken into consideration when making error adjustments. Values for latitude and station distances can deviate substantially and still not create detectable changes. Errors due to the tidal approximation may also be neglected when using the theoretical method.

In order to see what effects the errors due to instrument reading and elevation measurements have on anomalies, a gravity profile was devised to which these errors were applied. The total anomaly was assembled by using two model anomalies produced from different channel sizes. The two individual anomalies were created theoretically, as described previously, and represent a relatively small and large magnitude within the range of anomalies under consideration. Instrument readings from actual field surveys and measurements for diurnal variation were used to establish a value of 0.005 mgals for the standard deviation of the error associated with the gravity meter. Standard deviations used for the error in elevation measurements are 0.05, 0.1, 0.2, and 0.3 feet. These errors correspond to various degrees of leveling (Davis and Kelly, 1967). Using the random number

function from the digital computer, instrumental reading errors and leveling errors were combined and applied to the values representing the devised anomaly. These errors were first set to a normal Gaussian distribution about zero using the stated standard deviations. Six trials were run on the devised data for each standard deviation representing elevation errors.

Combining random errors for the instrument reading and elevation measurement correspondong to the 0.05 feet standard deviation, and applying them to the theoretical gravity profile only produced slightly observable changes to the original anomalies. It is unlikely that different interpretations for the two anomalies could occur at this level of accuracy. Since ± 0.05 mgal represents the maximum combined error which could result, it is possible to apply errors in order to cause an alternate interpretation, but very improbable. Applying random errors to a set of data a number of times, such as this, has an indication of the probability that alternate interpretations could occur.

Application of a normal distribution of errors with a standard deviation of 0.1 feet poses a slightly different picture. Slight shifts of the smaller anomaly tend to occur. Also, small but noticeable undulations can appear where the profile was originally flat-lying. Figure 18a represents these occurrences, where the dashed profiles were produced by adding random errors to the solid profile. The anomaly

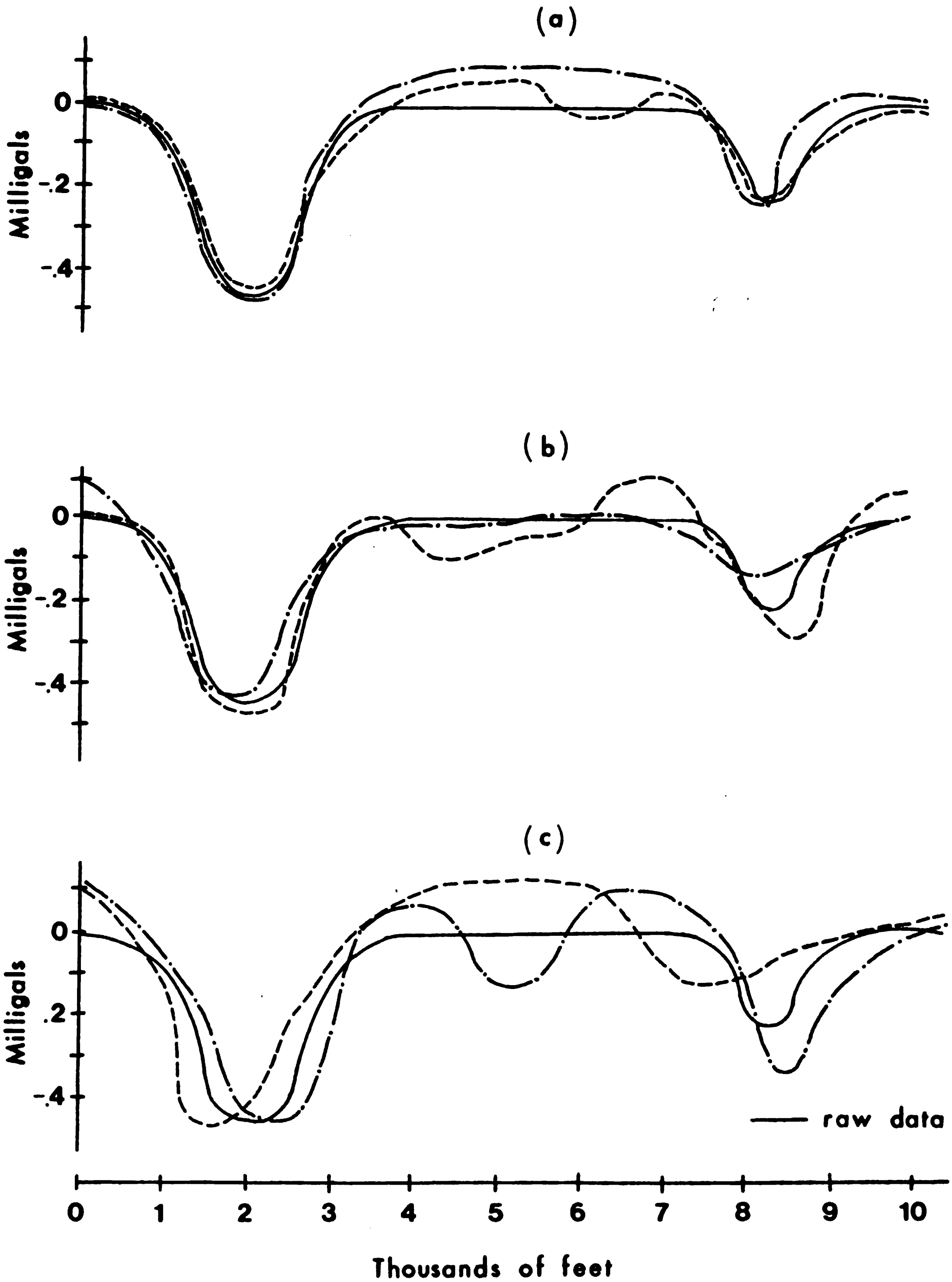

Fig. 18. Changes to a gravity profile due to the introduction of random errors. Normal distribution, with standard deviations of (a) 0.1 ft, (b) 0.2 ft., (c) 0.3 feet.

shift could cause one to miss the peak water production zone, while the added feature could possibly be interpreted as a small channel. It should be noticed, though, that the larger anomaly of the profile remains unchanged and that an alternate interpretation of this anomaly could not exist. The maximum possible error, here, would be ± 0.07 mgals.

The next level of accuracy in elevation measurements, of course, tends to create even more drastic changes. Samples of the six trials of randomly applied errors corresponding to a standard deviation of 0.2 feet are shown in figure 18b. It is evident that errors of this magnitude tend to (1) shift the larger anomaly slightly, (2) severely shift the small anomaly, (3) create false anomalies, and (4) attenuate and broaden the smaller anomaly.

Finally, the application of random errors associated with the 0.3 feet standard deviation for elevation determinations are indicated in figure 18c. Obviously, these errors produce the most severe alteration to the original data. The maximum possible error in this case is 0.15 mgals. These errors tend to (1) obscure the smaller anomaly, (2) substantially accentuate the smaller anomaly, (3) severely broaden the smaller anomaly, (4) attenuate the larger anomaly, and (5) substantially shift the smaller and larger anomaly. It is intuitive that quite opposing interpretations can exist between the original data and the data produced by errors of this magnitude.

With the accuracy used to produce the curve of figure 18a, it is reasonable to state that anomalies smaller than 0.20 milligals can be detected. From the maximum possible errors associated with levels of accuracy in which certain sized anomalies were radically altered, it is conjectured that anomalies as small as 0.14 milligals can be detected accurately. More accurate elevation measurements, than have been discussed here, could be determined, but anomalies smaller than 0.14 milligals would be overwhelmed by the error in the reading of the gravity meter. It is evident from figure 18 that anomalies larger than 0.4 milligals can be detected accurately with a standard deviation of 0.2 feet for elevation measurements, but smaller magnitude anomalies must be associated with more accurate elevation measurements to achieve the same certainty.

As we have determined previously, the two major parameters which effect the amplitude of anomalies are density contrast and depth of channel. To describe what size channels produce 0.14 mgal and larger anomalies, a graph of density contrast verses channel depth has been prepared (figure 19). This was produced using a constant depth of burial and channel width. An anomaly of -0.22 milligals, for example, could be contributed to a density contrast of -0.3 gm/cm^3 and a channel depth of 65 feet or a density contrast of -0.5 gm/cm^3 and a channel depth of 44 feet.

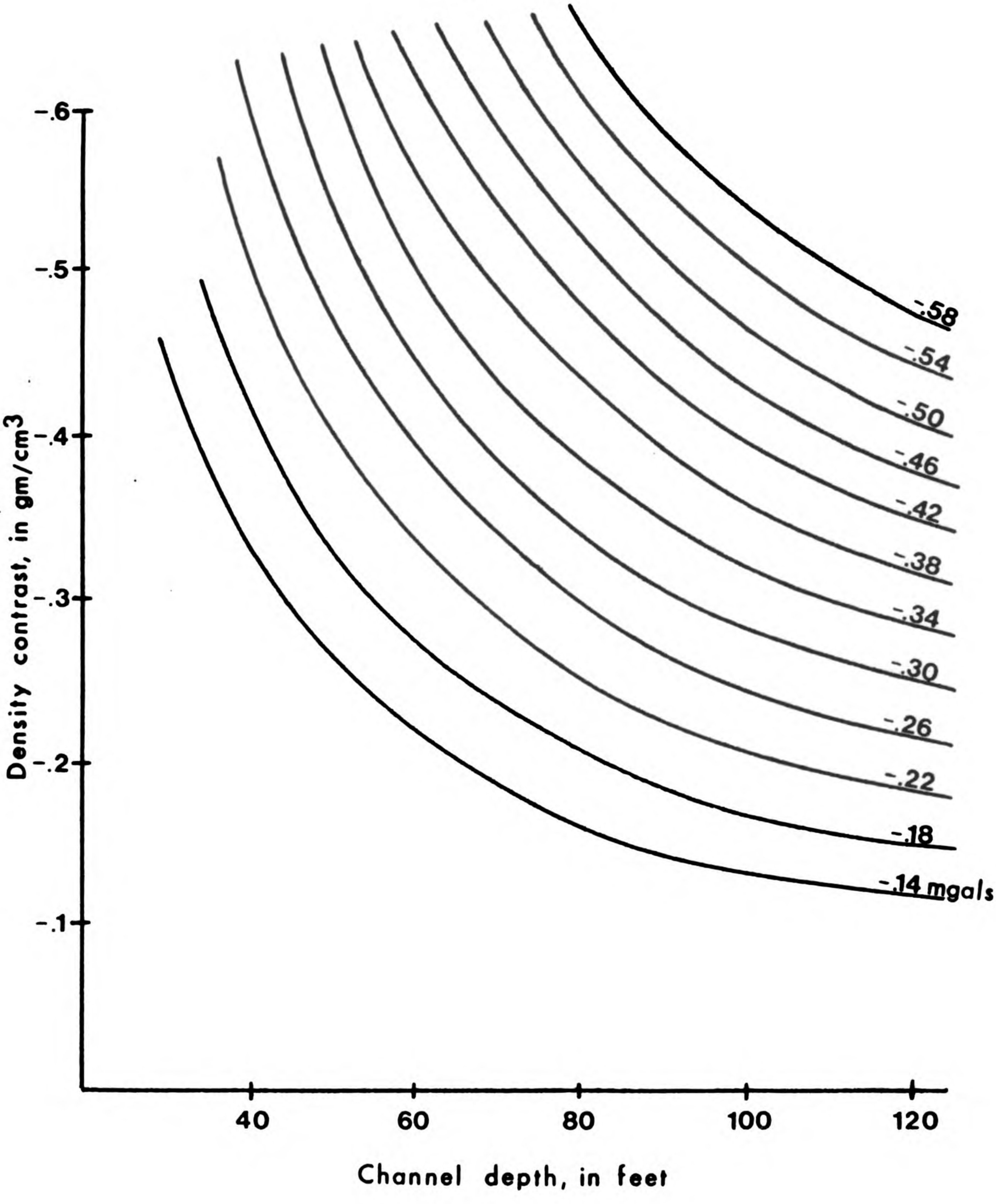

Fig. 19. Magnitude of anomalies from variable density contrasts and channel depths.

CHAPTER VII

SAMPLE SURVEYS

The purpose of this chapter is to describe the application of suggested corrections and interpretations to actual gravity surveys, and to indicate the outcomes of each phase of the case studies. Of the two surveys chosen, one is a gravity traverse, and was conducted near Durand, Michigan. The second was undertaken near Hartford City, Indiana as a gridded survey. By using the suggested techniques and applying them as described, the gravity method in exploration for groundwater can be used with a minimum of error by relatively inexperienced personnel.

Durand, Michigan Survey

This particular survey represents the first application of gravity in Michigan, as far as can be established, for the specific purpose of locating groundwater in a local channel in the bedrock. Durand has experienced a lack of sufficient amounts of suitable water for its community needs. Conventional methods such as resistivity and test drilling failed to uncover any prospects. The gravity method was finally chosen for a further attempt.

Twenty three gravity stations were set up by a local consulting firm along a straight traverse. Careful placement was utilized so as to avoid topographic features such as gullies and mounds. This was to allow terrain corrections to be omitted. The choice of location was partially determined by the presence of two water well, which were used as gravity stations. Station positions, elevations, and gravity readings were then determined. With these measurements, the corrections were applied using the methods described in previous chapters. Gravity profiles were plotted corresponding to various possible densities for the glacial overburden and compared to the surface topography. The method of performing Bouguer corrections suggested by Seigert (1941), which eliminates the choice of a density, was applied and the resulting profile happened to correlate least with the surface topography of all plotted profiles. Seigert's method was then used for elevation corrections. The next step was to reduce the values for tidal effects. The theoretical tidal effects for August 7, 1973, at Durand, Michigan are shown in figure 7 of Chapter IV. These forces were then subtracted from the previous values to produce the Bouguer values. Figure 20 represents a plot of the Bouguer values. From this, minor fluctuations are noticeable, but the occurance of a bedrock channel can hardly be determined.

From the trends of the Bouguer anomaly, it is evident that a relatively strong regional is present. By inspection,

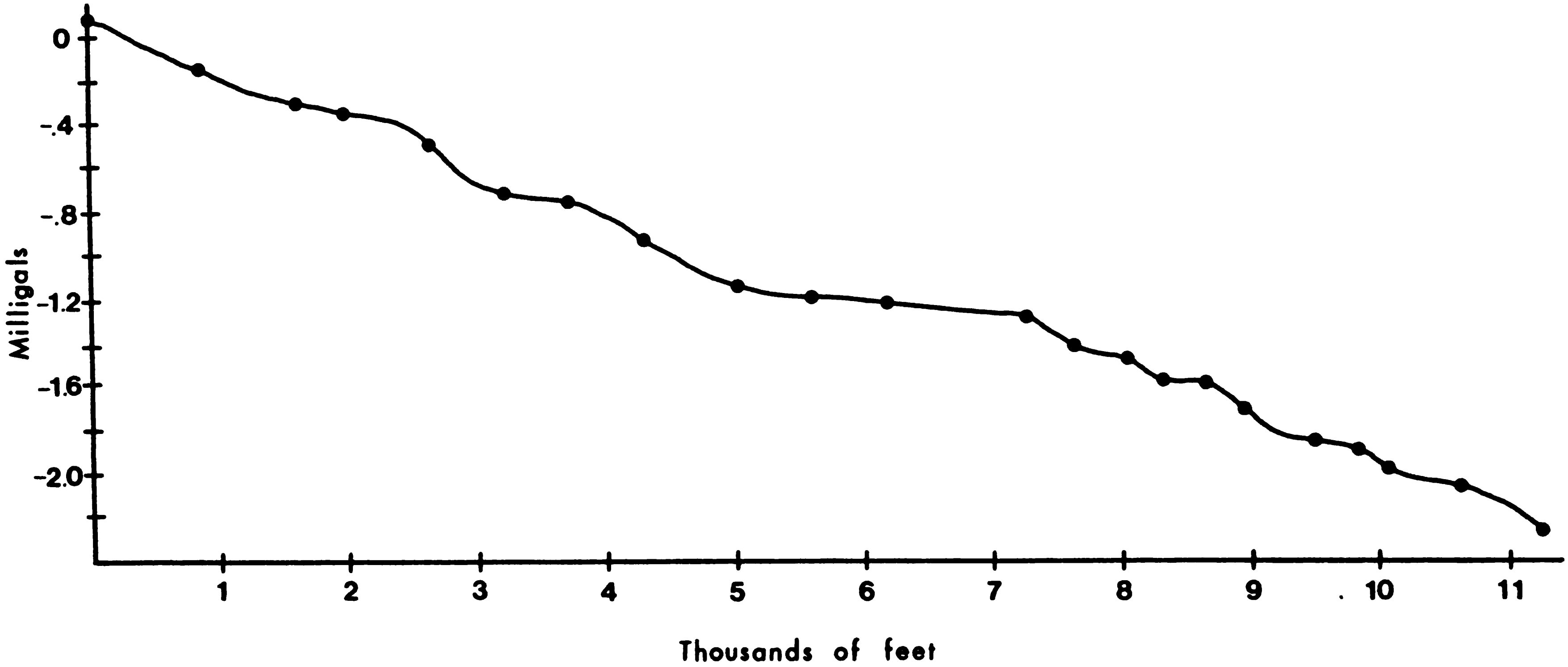

Fig. 20. Bouguer Anomaly — Durand, Michigan.

this regional can best be approximated by a straight line. A first order least-squares fit was performed on the data and subtracted to produce the residual (figure 21). The seemingly erratic nature in the string of data represents the random measurement error introduced. Error bars are used to help smooth the anomaly into a realistic curve. The importance of eliminating the regional from the Bouguer anomaly can be understood by comparing figures 20 and 21. This seemingly unanomolous profile is interpreted to indicate a 0.17 milligal depression.

Since the true density contrast between the bedrock and the overlying glacial deposits is not known, the method involving formula 7 of Chapter VI can not be used to determine bedrock elevations. Instead we use interpolation from station readings of known bedrock elevations to arrive at bedrock elevations for other station readings. This was explained in Chapter VI. A profile of the bedrock topography would be identical in shape to the residual anomaly, thus the elevations are also indicated in figure 21.

The detected anomaly certainly resembles the shapes produced by buried bedrock channels and one would have no choice but to assume it to have that origin. In most cases, further definition of the channel would be desired before suggestions as to a proposed well site would be made. A gridded survey across the appropriate area would, perhaps, lead to more favorable results.

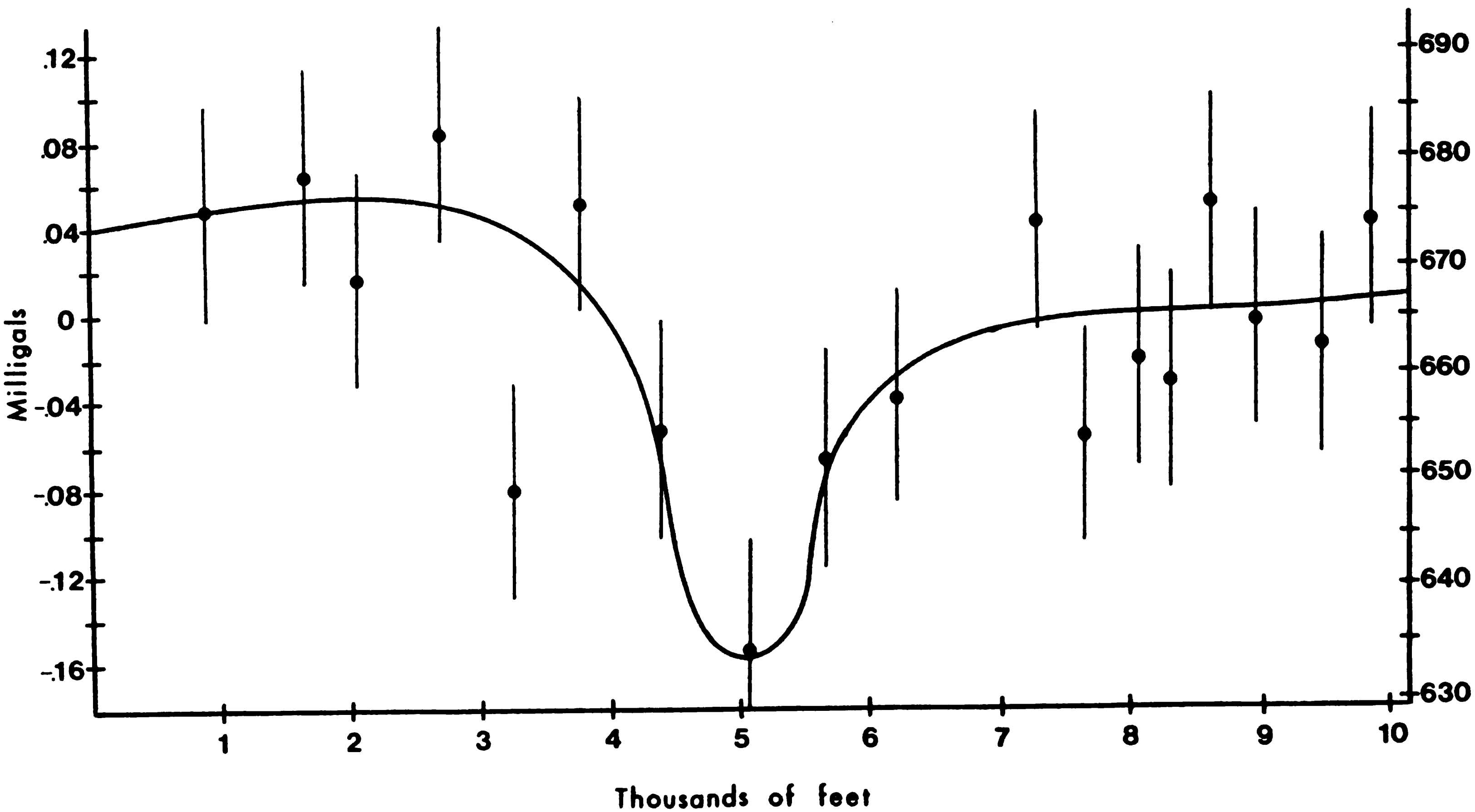

Fig. 21. Residual Anomaly — Durand, Michigan.

Because of labor problems the Durand Gravity Survey, by printing time, has not as yet been followed up by drilling. A bedrock study by Mencenberg (1963) though, indicates a topographic low in approximately the same area.

Hartford City Survey

This survey was undertaken for the purpose of locating additional water resources for the Hartford City, Indiana community. Thirty eight gravity stations were established in a grid on a tract of city-owned land. Three additional stations were set up at well locations of known bedrock depth to aid in the quantitative interpretation of the bedrock. A string of stations, which transcended the largest topographic feature in the area, was used to determine the correct density for the glacial deposits. The gravity profile corresponding to a specific overburden density, which correlated least with the surface topography, was choosen as representing the true density. This density of 2.05 gm/cm^3 was then used in performing the Bouguer corrections. Along with other corrections, the theoretical tidal forces were computed and applied to the raw data. A computer drawn diagram of the resulting Bouguer anomaly is shown in figure 22.

The Bouguer anomaly indicates a general trend dipping to the southeast. This regional trend can best be depicted by a plane surface. Thus, a two-dimensional least-squares fit of the first order was established and subtracted from

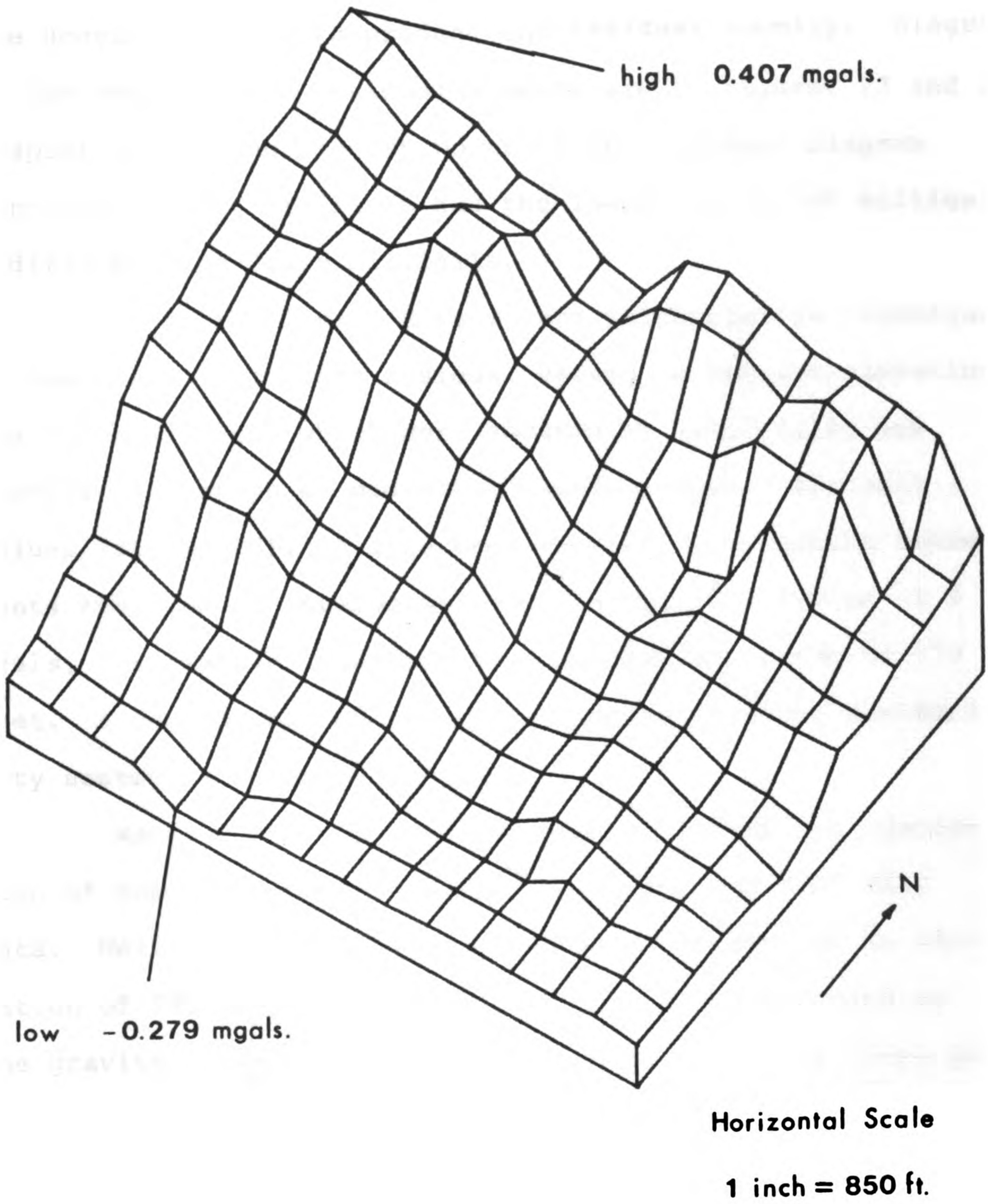

Fig. 22. Bouguer Anomaly - Hartford City, Indiana.

the Bouguer anomaly to produce the residual anomaly. Diagrams of the regional and residual anomaly are in figures 23 and 24, respectively. The highest point of the residual diagram represents 0.126 milligals and the lowest is -0.169 milligals, a difference of 0.295 milligals.

As in the Durand Survey, the interpolation technique is used here to convert residual values to bedrock elevations. The three stations which were located at drill holes are plotted on a graph of bedrock elevations versus residual values (figure 25). A line best fitting these points represents the line of interpolation. A residual value of -0.1 mgals, for example, converts to a bedrock elevation of 770 feet. A contour map of bedrock topography for the Hartford City survey is shown in figure 26.

Water wells have been drilled following the completion of the original interpretation of the Hartford City data. Well "A" in figure 26 encountered bedrock at an elevation of 775 feet, almost exactly what was determined by the gravity survey. It has been rated as a 1400 gallons per minute well. Drilling at location "B" established a bedrock elevation of 750 feet. Production for this well was set at 1200 gallons per minute. In both instances production has more than doubled the previous best rate in the area. These follow-ups serve to verify the gravity method as a useful tool in prospecting for groundwater.

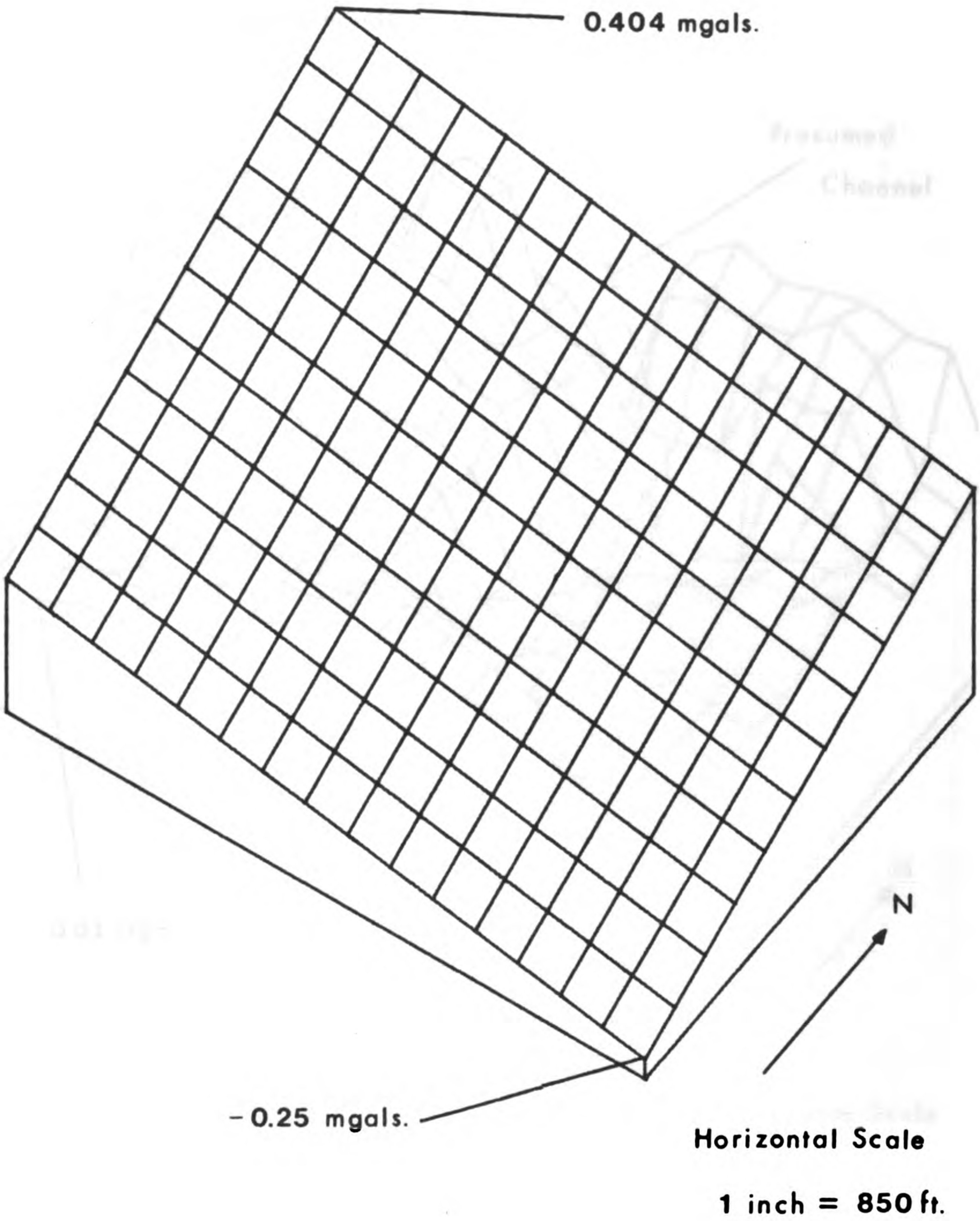

Fig. 23. Regional approximation - Hartford City.

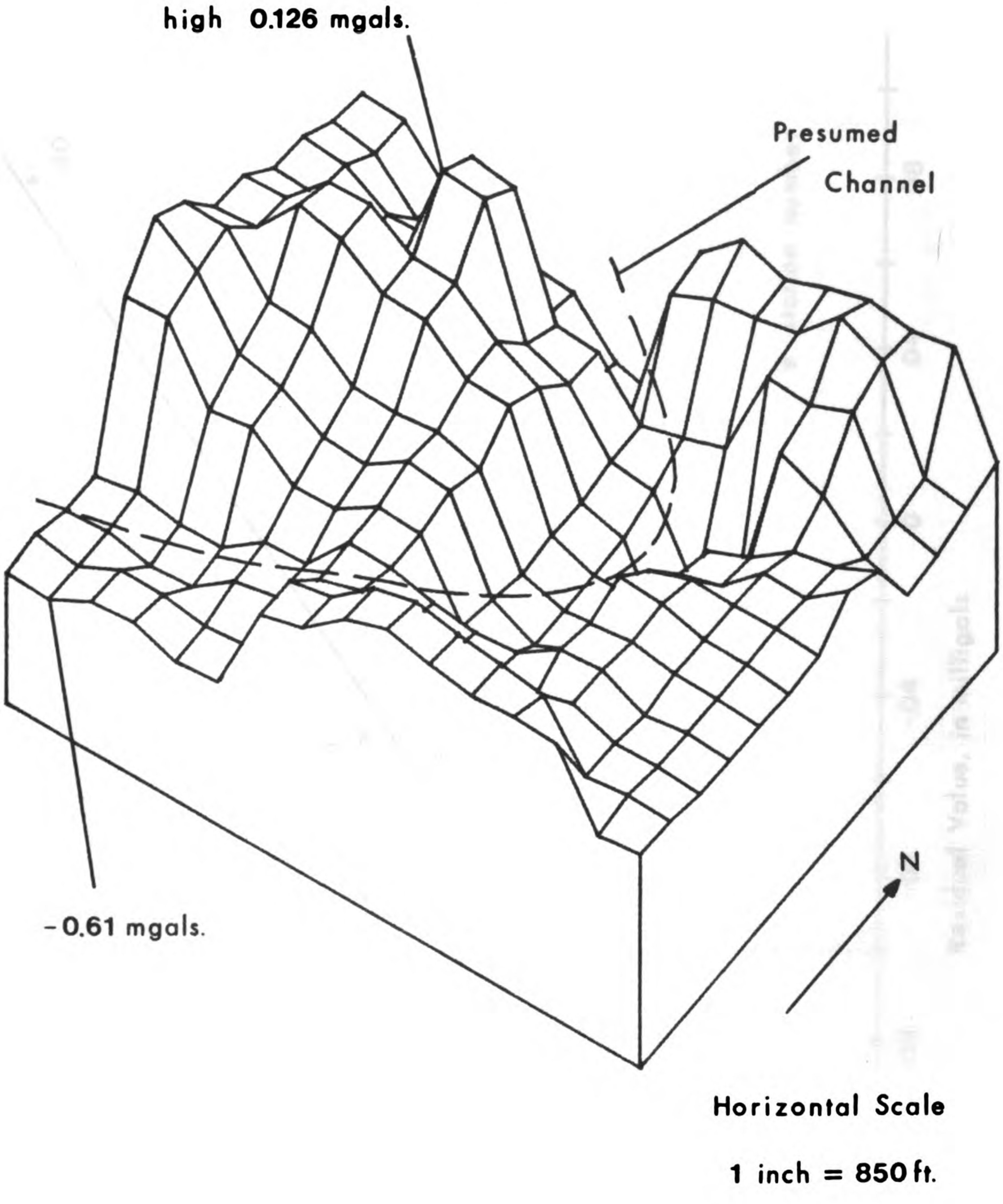

Fig. 24. Residual Anomaly - Hartford City.

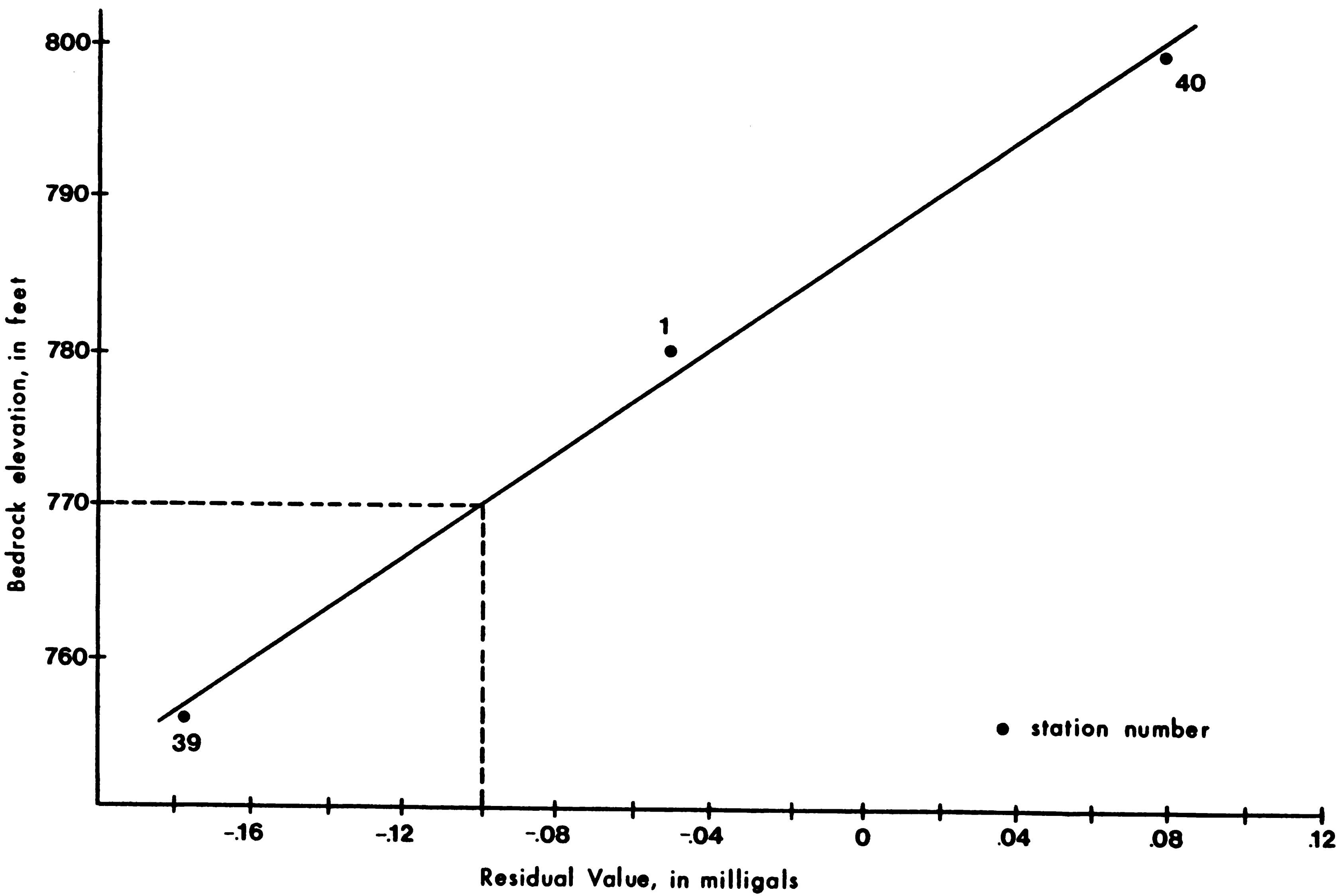

Fig. 25. Interpolation of bedrock elevation.

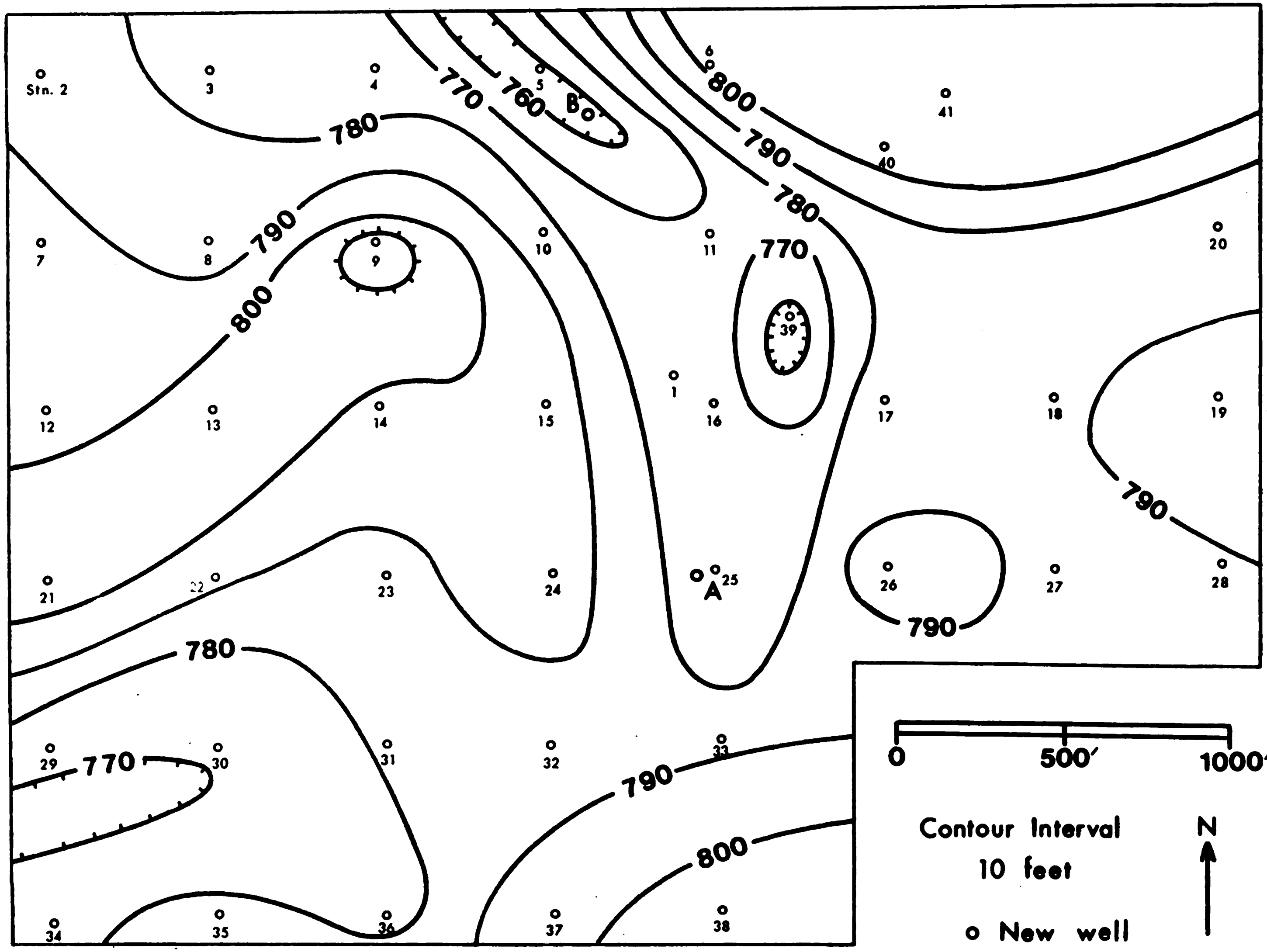

Fig. 26. Bedrock topography - Hartford City, determined from gravity survey.

CHAPTER VIII

CONCLUSION

It has been the intent of this study to develop the feasibility and applicability of gravity exploration in groundwater prospecting, and in the process to suggest and refine methods for correcting and interpretating the gravity data. Buried bedrock channels are prime sources for groundwater and can create anomalies as small as can be measured by gravity meters. Thus, accuracy in methods and measurements are of vital importance.

Various conclusions of this study can be summarized.

1. Theoretical tidal corrections can play a useful role in reducing the raw data in high precision surveys. Conventional methods can be very inaccurate and have been shown to lead to erroneous interpretations.

2. The least-squares method of isolating the residuals is useful when locating buried bedrock channels with gravity. A certain amount of judgement is required while still retaining the accuracy of mathematical approximation. Also, polynomial fitting allows the quantitative interpretation of anomalies.

3. Use of accurate methods in leveling enable bedrock channels which produce anomalies as small as 0.14 milligals, to be detected. Although even smaller anomalies are theoretically measureable with the present sensitivity in gravity meters (0.01 milligals), they tend to be substantially obscured by random errors in their readings.

4. The gravity method can be successfully used in locating bedrock valleys and indicating their physical propertites.

There are reasons in addition to accuracy that make the gravity method feasible in exploration for groundwater.

1. The method is non-disruptive of the environment. Gravity involves measurement of an existing potential field with hand carried equipment. Reflection seismology, in contrast, can rely on the releasing of energy, with attendant dangers and disruptions, and limitations in urban and built-up areas.

2. A gravity survey is relatively quick and inexpensive, and is thus attractive for the typical economics of groundwater exploration. Perhaps the most time consuming phase is in setting up gravity stations and obtaining elevations. When digital computing is used, as in this study, the time is substantially reduced. The field procedures require a maximum of two people. The second person is needed only for measurements of distances and elevations.

The preceeding statements indicate the value of gravity exploration for groundwater resources. The method should continue to gain interest, particularly for use in glaciated areas.

APPENDIX A

Computer Programs

APPENDIX A

Computer Programs

Various computer programs were used in this study, to aid in mathematical calculations, some of which were written by the author. Brief comments will be made of each.

Gravity Data Reductions

This computer program was used to perform the corrections described in Chapter III to the original gravity readings. This includes latitude corrections, free air corrections, Bouguer corrections and instrumental drift corrections.

Earth Tide Calculations

This computer program involves the theoretical calculation of tidal forces due to the attractions of the moon and sun on the earth. The formulas involved are indicated in Chapter IV.

One Dimensional Polynomial Fitting

"Best-fits" to diurnal gravity measurements were calculated by the use of a one dimensional least-squares program. This computer program is also used for approximating the regional trends of gravity traverses from Bouguer Anomaly values.

Two Dimensional Polynomial Fitting

Polynomial fitting of a gridded set of gravity data, for approximating the regional trend, was accomplished using this program.

Linear Fitting

This program was used for fitting a straight line to a set of data. This was more efficient than using the one dimensional polynomial fitting program which computes various degree fits.

Channel Modeling

This program was used to calculate the gravitational effects at the surface produced by hypothetical buried bed-rock channels.

Random Error Introduction

Random errors due to gravity meter readings and elevation measurements were introduced to a hypothetical gravity anomaly with this program.

APPENDIX B

Data for Hartford City, Indiana Survey

APPENDIX B

Data for Hartford City, Indiana Survey

November 26-27, 1973

Station #	Reading	Time	Elevation
B1	3697.42	3:31PM	866.53 ft.
16	3697.45	3:44	865.85
15	3697.29	3:51	871.11
14	3696.52	4:03	884.23
13	3697.10	4:11	876.88
8	3696.99	4:18	876.39
12	3696.53	4:30	885.62
7	3696.16	4:36	890.81
2	3696.41	4:44	886.44
3	3696.50	4:52	883.31
4	3696.41	5:00	883.55
5	3696.01	5:07	887.73
9	3696.25	5:24	890.05
10	3695.63	5:31	896.23
11	3696.38	5:41	883.04
B1	3697.49	5:48	866.53
B1	3697.49	8:34AM	866.53
39	3697.21	8:43	867.77

Station #	Reading	Time	Elevation
40	3696.40	8:48AM	883.86 Ft.
41	3696.19	8:52	887.48
6	3695.41	9:00	900.23
22	3697.09	9:16	876.00
21	3696.33	9:32	890.04
29	3697.65	9:42	866.52
34	3697.83	9:58	864.03
30	3697.62	10:07	866.20
23	3697.46	10:16	868.82
24	3697.62	10:25	866.21
B1	3697.52	10:34	866.53
B25	3697.33	10:44	868.10
32	3696.94	10:56	876.61
31	3697.62	11:03	866.04
35	3697.68	11:12	866.11
36	3697.18	11:20	872.58
37	3696.23	11:26	888.37
38	3696.73	11:32	880.58
33	3696.52	11:41	881.80
B25	3697.35	11:51	868.10
17	3697.36	12:07PM	867.45
26	3696.94	12:15	875.13
28	3696.35	12:26	882.41
27	3696.61	12:33	879.00

Station #	Reading	Time	Elevation
19	3696.77	12:40 PM	876.34 Ft.
18	3697.37	12:46	867.04
20	3696.89	12:53	874.72
B25	3697.35	1:00	868.10

Latitude - 40.46° North Longitude - 84.35° West

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