

METHODOLOGICAL PROCEDURES AND
APPLICATIONS FOR
INCORPORATING ECONOMIC
CONSIDERATIONS INTO FERTILIZER
RECOMMENDATIONS

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METHODOLOGICAL PROCEDURES AND APPLICATIONS FOR
INCORPORATING ECONOMIC CONSIDERATIONS
INTO FERTILIZER RECOMMENDATIONS

By

Jack L. Knetsch

AN ABSTRACT

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ABSTRACT

Current fertilizer recommendations generally reflect inadequate attention to economic considerations. Although much research has been carried on in the past to promote the efficient use of fertilizer, this neglect of economic considerations remains. Many of the past efforts have been directed toward the maximization of yields, which is usually inconsistent with the more important concern of maximizing profits. Profits are increased only so long as the cost of adding fertilizer inputs is less than the added return derived from their use.

As the data needed to determine more accurately the optimum rate and combinations of fertilizer nutrients to use have generally been lacking, a project designed to produce such data was sponsored jointly by the Departments of Agricultural Economics and Soil Science of the Michigan State Experiment Station. The crops studied are corn, oats, wheat and alfalfa-brome in rotation. The variable nutrients studied are nitrogen, phosphorus and potassium. The data produced by these experiments permit more adequate analysis of fertilization rates, ratios and ultimately of crop sequences and fertility residuals. Only the corn data obtained from the first year of experimentation are analyzed in this thesis.

The analysis of these data are based on the concept of a continuous mathematical production function. According to this concept, yield responses to different fertilizer nutrients may be described by a continuous mathematical function which shows yield to be dependent upon the levels of the variable fertility nutrients. The optimum

application occurs where the value of a decreasing marginal product (first derivative of yield with respect to an input) is equal to the cost (price, under perfect competition) of adding another unit of input.

Four three-variable functions were fitted to the experimental data. After applying various statistical tests and less objective criteria, it was decided that the best fit was obtained by using the Carter-Halter equation of the form:

$$Y = a N^{b_1} c_1^N P^{b_2} c_2^P K^{b_3} c_3^K$$

where Y is yield and N , P and K are N , P_2O_5 and K_2O respectively. Only nitrogen was found to influence yield with the effects of phosphorus and potassium being statistically insignificant.

The equation was refitted using only nitrogen as an independent variable. The predicted yields given by the use of this equation were found to agree favorably with the averages of the observed yields at the various rates of nitrogen application.

The solution for the optimal quantity of nitrogen to apply was shown to be dependent upon the prices of both corn and nitrogen. For 1955, it was found that the recommended fertilization practices were far from those maximizing profits. The data, as analyzed, point out the need for information about the probability distribution of returns over time as well as for specific recommendations.

An empirical production function on which to base fertilizer recommendations should be based on data for a period of years. A production function fitted to such data would "average out" between

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the fifteenth is the fact that the

the sixteenth is the fact that the
the seventeenth is the fact that the
the eighteenth is the fact that the

year variations. Such data would also make it possible to estimate the probability of deviations from expected returns as well as expected deviations from the recommended amounts of fertilizer to use. This information would permit farmers to adjust their fertilization program to their particular capital positions in view of the risk and uncertainty involved.

Major problems encountered in this analysis are present regardless of the analytical method used but are made more evident by the added refinement of the functional analysis. The large variances in yields not associated with the independent variables cause difficulties and need to be overcome. Another problem is that of representativeness and applicability of the results obtained. Each experimental field is unique to itself and the results from such a field are applied to conditions known to differ.

A promising alternative is to incorporate within the experimental field, or fields, more of the variations in different farm fields. The experiments would then "average out" a range of differences more comparable to the range existing on farm fields. This would give the experimental production function a wider range of application. The practicability of these considerations, in view of the increased variances which would probably be encountered, is in need of empirical investigation.

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The author assumes responsibility for any errors still present in this manuscript.

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CHAPTER I

THE PROBLEM DEFINED

Introduction

Though the physical benefits of using commercial fertilizer in crop production are widely recognized, its economical use has not been adequately studied. As the productive contribution of fertilizer depends on the amounts and ratios of nutrients used, the yields resulting from different amounts and ratios must be known before accurate profit maximizing decisions can be made concerning its use. Efforts to make the use of fertilizer more profitable have been made for many years, with the problem becoming more important as the use of this resource has increased. The purpose of this study is to describe and apply research techniques of value in incorporating economic considerations into fertilizer recommendations and to point out several problems involved in such research and made evident by their application.

Contrary to an all too common belief, maximization of yields is not necessarily desirable. For the commercial farmer, a major concern is profit maximization. Maximum yields are seldom associated with maximum profits. Economically, higher yields are desirable to the extent they can be secured at an added cost less than their added value.¹

¹ This basic economic principle occurs throughout the literature of economics. A discussion of its application to agronomic problems is contained in: Earl O. Heady and W. D. Shrader, "The Interrelationships of Agronomy and Economics in Research and Recommendations to Farmers," Agronomy Journal, VI (October, 1953) pp. 496-502.

When an entire industry maximizes profits from the use of all inputs it employs, the difference between the total value of products produced by the industry and the value of items used by that industry is maximized within restrictions imposed by certain fixed elements in the situation, such as the distribution of fixed assets, technology, the institutional set up, the asset ownership pattern, etc. When profits are maximized, within these restrictions, the industry can be said to be operating at maximum efficiency.

The Basis for Current Fertilization Recommendations

The Logical Framework Behind Current Recommendations -- In the past and to a major extent at present not enough systematic effort has been made to incorporate economic considerations into the fertilizer recommendations of agriculturists. This circumstance was brought on by several factors; first, a primary concern of agriculturists has been to motivate farmers to use fertilizer in any quantity with only minor concern given to economic considerations; second, agronomists have had a primary interest in variance type studies for investigating responses to discrete treatments and the relation of such responses to soil characteristics rather than deriving response estimates to which economic interpretations might be attached; and third, until recently, fertilization experiments have been designed and conducted primarily by agronomists with economists taking little interest. Consequently, the economic aspects of fertilizer use have not been emphasized and techniques for determining the optimum application have been applied slowly.

Customarily, fertilizer recommendations to farmers have been based upon the results of field trials. Use of these trials has the limitation that recommendations are derived from experiments performed on only a partial list of the many soil types that may exist in a state. In Michigan there are three hundred such types recognized and it is a physical impossibility to test crops on all soils.² Therefore, recommendations made for all soil types must be generalized from experiments on only a few types. Fortunately, field trials have not been the only basis for recommendations. Other factors such as differences in cropping practices, past fertilization, the practicability and availability of the fertilizer recommended are commonly taken into account. Because the most profitable amount of fertilizer is not always used as a result of risk and uncertainty,³ such considerations also condition recommendations made to farmers. These and other factors less known to agriculturists⁴ make it necessary for recommendations to be based on general experience and judgment as well as experimental evidence.

² Department of Soil Science and Horticulture, Cooperative Extension Service, Fertilizer Recommendations for Michigan Crops, Extension Bulletin 159, Michigan State University, East Lansing, Michigan, June, 1953.

³ A recent example of an empirical verification of this commonly believed notion can be found in: Myron E. Wirth, "Production Responses to Agriculture Controls in Four Michigan Farming Areas in 1954," unpublished M. S. thesis, Department of Agricultural Economics, Michigan State University, 1956, pp. 46.

⁴ The term agriculturist will be used in this study to refer to the contributors to the solution of the problem made by both the agronomist and economist. It is used because it expresses the interdependence of the two disciplines in the solutions arrived at.

A quarter of a century ago VanSlyke⁵ suggested that "In the very nature of the case, the question of quantities and proportions of plant food to be used must always remain more or less a matter of individual experience and observation." He further states that each farmer should regard each fertilization as an experiment which should be repeated continuously until the most economical use of plant food is reached. This general trial and error suggestion has changed little through the years. In 1955 Collings⁶ wrote, "The kinds of fertilizer a farmer should apply, and the most profitable amounts for him to use are always somewhat of a guess, although his practice may be based on results of fertilizer test plots, field observations and use of improved soil testing techniques and leaf analysis methods." While such hit-or-miss recommendations have undoubtedly resulted in increased profits for those following them, they will seldom result in the maximum economic returns possible.

The Empirical Basis -- Present and past recommendations are often based on experiments which vary one variable nutrient or nutrient ratio at a time. Other variables are held constant at some level well above that at which deficiencies occur. This procedure usually either assumes that the response to the experimental variable is the same regardless of the level of the nonexperimental variables or, if this is not so, that farmers are interested only in the response

⁵Lucius L. VanSlyke, Fertilizer and Crop Production, (New York: Orange Judd Publishing Co., Inc., 1932), pp. 349.

⁶Gilbearth H. Collings, Commercial Fertilizers, (5th ed., New York: McGraw-Hill Book Co., Inc., 1955), pp. 492.

relationship associated with the fixed levels of the other variables. Thus, interaction of responses to the different nutrients as factors of production are somewhat underemphasized though many experiments show the importance of this phenomenon.

Analysis of experiments involving different rates of a single nutrient or single nutrient ratio usually consists of the determining of the mean yield for each discrete treatment and the statistical significance of that mean. One of these treatments is then designated as the "most profitable" or optimum application rate. The statistical significance of differences between treatment means is found either by variance analysis or by inspection of the data. Least significant differences may be computed. Such computations involve a comparison of the variance around treatment means with the variation between treatment means. The computed LSD then serves as an aid in locating the treatment rate beyond which no significant response occurs.

The location of the "best treatment" by inspection of the data usually predominates in the analysis of results from those more complex experiments that may involve more than one independent variable. Odland and Allbrighten illustrated these procedures by analyzing the results of an experiment involving nitrogen, phosphate and potash in the production of silage corn.⁷ Four levels of nitrogen and three levels each of phosphate and potash were used in twenty-four of the thirty-six possible combinations. Analysis of variance indicated

⁷T. E. Odland and H. B. Allbrighten, "The Effect of Various Amounts of Nitrogen Phosphoric Acid, and Potash on the Yields of Silage Corn," Proceedings of the Soil Science Society of America, XIV (1949), pp. 221-223.

that there were significant differences in yields due to the effect of the various treatments. The average effect of each nutrient at the various levels, disregarding the levels of the other two nutrients, were then set in tabular form. Inspection of this table produced the level of each nutrient that "seems to promise maximum economic returns."⁸

This type of experimental evidence, together with the experience and judgment of the individual making the recommendation, serves as the bases on which recommendations to farmers have been made. The question of opportunity costs and of price considerations has usually been omitted when recommendations have been made.

The salvation of using the present methods lies in the fact that profits are realized by using the recommendations so determined. While this is doubtless of value, the high profit application of fertilizer is not determined nor do such procedures show how the most profitable combination varies with prices of the product and of the fertilizer nutrients. As competition under cost-price squeezes intensifies, information which leads to the highest profit use of fertilizer will become increasingly important to the farm operator.

The Problem Stated

The **problem** of incorporating economic considerations into recommendations is twofold. The first is to secure data which permit an appropriate economic analysis. This portion of the problem requires experiments specifically designed for this purpose. The

⁸
Ibid.

second part of the problem is that of analyzing the results of field experiments so as to be able to make recommendations as to the most economic amounts of fertilizer to apply.

In the past, field experiments have not been designed to yield data which permit estimation of economically optimum applications of fertilizer. Data from past experiments are usually inadequate for a number of reasons. First, in most cases, only one nutrient, or one ratio of nutrients, has been permitted to vary, while holding all other conditions constant. While control over unstudied variables is a scientific necessity, it is desirable to vary more of the fertilizer nutrients under investigation in order to determine their interaction effects as well as their "primary" effects on yield. Secondly, many designs used do not include enough rates of application, nor combinations of nutrients at high enough levels to reach the point of maximum yield.

Experimental results are needed to provide data over a range of input levels wide enough to permit characterization of the economically relevant portion of the input-output relationship. This necessitates enough rates of application to provide sufficient information about the relationships to permit identification of the point of maximum profit under varying price conditions.

A further reason for inadequate economic interpretation of fertilizer data has been the failure to employ appropriate economic concepts and principles in analysis. The methods of analysis described in the previous section are inadequate to accomplish this.

The immediate need is for adequate input-output data necessary for the estimation of the functional relationship that exists between fertilizer and yield. After such a relationship is derived the optimal application for the appropriate price situation may be found by using appropriate economic concepts and principles.

The General Procedure to be Followed

In 1955 a project was initiated to secure the necessary data for an economic analysis of fertilizer use. This project was sponsored jointly by the Department of Agricultural Economics and Soil Science of Michigan State University. It includes a field experiment designed primarily to furnish data amenable to economic analysis as well as data of more agronomical interest. Members of both departments took active part in nearly all phases of the project including the design of the field experiments.

After the data were secured, estimates of the relationship between nutrients applied and yield were constructed. This estimation procedure was an integral part of the analysis. From the estimated input-output relationship, expressed in a mathematical form, the solutions of the most profitable rate and ratio of nutrients are found simultaneously.

While the method of deriving estimates of input-output relationships and determining optimum fertilizer applications from them is the most promising method for incorporating economic considerations into fertilizer recommendations, it is not without problems. Some of these problems are unique to this method of analysis but most are

present regardless of the methods used. The degree of refinement characteristic of this method makes obvious some shortcomings of field experimentation which have previously been overlooked.

Organization of Thesis

The concept of a production function and its economic implications will be presented in Chapter II. This discussion will serve as a basis for the analysis of the empirical data to be used in the analytical phase of this study.

The sources of empirical data will be discussed in Chapter III. Chapter IV will deal primarily with a description of the statistical analysis performed on the data. An appropriate mathematical function will be derived and the economic applications considered.

Chapter V will deal further with the interpretations of the results obtained in Chapter IV while Chapter VI will deal with the problems encountered in this type of research.

The summary and conclusions of the study will be presented in Chapter VII.

CHAPTER II

THE CONCEPT AND VALUE OF MATHEMATICAL PRODUCTION FUNCTIONS

The methods employed in this thesis for determining high profit combinations and amounts of fertilizers require the use of continuous mathematically expressed production functions. This chapter will clarify the concept of a production function and indicate its value in determining high profit combinations and amounts of fertilizer.

The Concept of a Production Function

Behind the present attempt to improve the methods of making fertilizer recommendations is the concept of a production function. According to this concept, yield responses to fertilizer applications may be characterized by a continuous mathematical function. Output or production as the dependent variable is regarded as a function of inputs. In agronomic work such a function is commonly referred to as the response curve if only one nutrient is involved. If the assumptions of continuous functions are met, it is possible to describe such relationships mathematically. This in turn makes it possible to dismiss graphic forms which are limited in usefulness and work with their logical but much more versatile equivalent--a continuous mathematical function.¹ If such continuous mathematical functions can be derived, well known mathematical operations can be carried out

¹ Geometric, as contrasted to algebraic methods of analysis, are useful in investigating relationships prior to selecting and fitting mathematical functions.

to locate various optima. Thus, in the analytical work necessary for determining economic optima, it is convenient, but not essential, for fertilizer yield relationships to be expressed as a continuous mathematical function.

Crop production is a complex process involving many variables other than fertilizer nutrients. The production function describing production of a given crop is a sub-function of a more general function involving all products and all inputs or resources. This over-all function is far too complex and extensive to be dealt with and must be reduced to manageable sub-functions. The sub-function involved in producing one crop may be written in the form:

$$\text{Yield} = f(\text{plant nutrients, air, moisture, soil properties, heat, } X_1, \dots, X_n)$$

where $(X_1 \dots X_n)$ represents all other growth factors. The complexity of even this function exceeds our present mental and computational capacities. Therefore, in most studies it is necessary to specify a still more detailed and simpler sub-function such as:

$$\text{Yield of corn} = f(N, P_2O_5, \text{ and } K_2O/\text{air, moisture, } X_1 \dots X_n) + u$$

This more specific function states that the yield of corn is dependent upon N , P_2O_5 and K_2O which are studied variables, with air, moisture and all other inputs $(X_1 \dots X_n)$ fixed at specific conditions or levels. The "u" in the equation represents an influence on yields of the uncontrolled and unstudied variables present. The causes of the u's and hence the u's, themselves, are assumed to be randomly and

independently distributed with respect to the studied variables. If these assumptions are met, the effect of these uncontrolled and unstudied variables can be "averaged out" with statistical procedures.

Once the relationship between fertilizer and yield is expressed mathematically, incremental responses to the use of fertilizer can be determined. Mathematical functions representing physical input-output relationships can be converted to "budget" or profit functions if input and output prices are known. Well known mathematical procedures are available for locating the high profit point on such functions.² Such points vary with price changes and can be easily relocated for any new set of given prices. As such they provide a basis for specific profit maximizing fertilizer recommendations in contrast to less accurate recommendations characteristically resulting from the experience and analysis of field experiments previously discussed in Chapter I.

Problems in Choosing Functions

There are an infinite number of mathematical functions which might express the functional relationship between yields and fertilizer applications. Thus, the problem becomes one of choosing a function that is in some way best or better than others for the purpose of

² Glenn L. Johnson, "Interdisciplinary Considerations in Designing Experiments to Study the Profitability of Fertilizer Use," a paper presented at a Tennessee Valley Authority sponsored fertilizer economics symposium held in Knoxville, Tennessee, June 14-16, 1955. The proceedings of the symposium are in book form: Methodological Procedures in the Economic Analysis of Fertilizer Use Data, ed. E. L. Baum, Earl O. Heady and John Blackmore (Ames, Iowa; Iowa State College Press, 1956).

characterizing the data produced in a fertilizer experiment.³ After a function is selected, the next problem becomes one of estimating the necessary parameters. The method of least squares, which minimizes the sum of the squared deviations in the yield dimension, is usually employed in estimating the parameters of the production function and is generally accepted as being an efficient method. Ibach and Mendum suggest that graphical methods or the method of selected points offer alternatives by which these estimates may be obtained for certain functions.⁴ However, the method of least squares is more commonly employed.

Choosing one specific function from the array of possibilities is a major problem. The "true relationships" are complicated by a multiplicity of factors of a chemical, physical and biological nature whose influence on production are as yet very poorly understood. Presently such choices are made largely on a trial and error basis with experience, judgment, insight and familiarity with other results leading to decisions as to the most appropriate function. Little of the necessary work has been done on this problem by statistical theorists.

³ This discussion is limited, generally to the continuous function analysis which at this time appears to be the most promising. An approach involving experimentation at discrete points with use of linear programming techniques may be of eventual value. A serious limitation of this method appears to be the inability to determine accurately the yield at these discrete points. Cf. Clifford Hildreth, Economic Implications of Some Cotton Fertilizer Experiments, Cowles Commission Papers, new Series, no. 93, university of Chicago, (Chicago: Cowles Commission For Research In Economics, 1955).

⁴ D. B. Ibach and S. W. Mendum, "Determining Profitable Use of Fertilizer," U.S. Department of Agriculture, F. M. 105 (Washington: U. S. Government Printing Office, 1953).

Early Use of Functions -- Historically, attempts to relate fertilizer inputs to yield responses were in the direction of the formulation of "natural laws." An early concept of a production function was proposed by Justice von Liebig's "law of the minimum." This proposition stated that the yield of a crop is governed by the quantity of the most limiting factor and that as increments of this limiting factor are added, yields increase in direct proportion to the additions of this factor until another factor becomes limiting. This concept holds that yields increase as a linear function of the limiting factor and that factors of production are perfect complements. The economics of such a situation are inconsequential; if it pays to add any of the minimum factor, it pays to add that factor to the level at which it is no longer limiting. The concept of the law of the minimum has an influence on agriculturists even now as witnessed by various illustrations such as a water barrel with staves representing the different factors of profitable production.

Liebig's idea of simple proportional relationships has been refuted primarily on the grounds that it does not conform to empirical evidence. Instead of the assumed linear relationships, there has evolved the law of diminishing returns based on observations of curvilinear relationships. The law of diminishing returns holds that the addition of a variable input to fixed inputs results in total returns which first increase at an increasing rate, then increase at a decreasing rate and finally decrease. In most agronomic experimentation, the portion of the law stating that total returns increase at an increasing rate is not relevant due to the presence of nutrients

in the soil. In some extreme cases additions of an input may initially decrease total returns. This law has resulted from empirical observations which have shown it to be a nearly universal condition of production. It is essentially a modification of Liebig's original formulation made necessary by empirical observation.

The similarity of yield curves established by experimentation prompted Mitscherlich to suggest a single mathematical expression to quantify this relationship. This expression, which he called the "law of diminishing soil yield," assumes the existence of some maximum yield to occur when all conditions of growth are optimum and yield deficiency, short of the maximum, is brought on by shortages of essential growth factors. The function suggested by Mitscherlich states that the yield increase which occurs from additions of this factor is proportional to the original shortage of the factor.⁵ The essentials of Mitscherlich's formulation is given by $\frac{dY}{dX} = c(A - Y)$, where A is the maximum possible yield and c is the effect factor. Wilcox, a proponent of Mitscherlich's findings, has maintained that the slope of the response curve is the same regardless of soil conditions, as given by this equation.

In the original formulation by Mitscherlich, the yield increases brought about by additions of one factor were not modified by the levels of the other growth factor. Baule, a German mathematician

⁵ Eilhard Mitscherlich, "Das Gesetz des Minimums und das Gesetz des abnehmenden Bodenertrages." translation unknown, Landw Jahrb XXXVIII (1909) p. 537-552. This paper has been distributed by V. Sauchelli.

collaborating with Mitscherlich, recognized this shortcoming and modified the equation to include an interaction effect. This was done by making yield a product of the effect of all growth factors working together. The modified yield equation is

$$Y = A(1 - 10^{c_1 X_1}) (1 - 10^{c_2 X_2}) \dots (1 - 10^{c_n X_n})$$

with A the maximum yield, the c_i the effect factors and the X_i the variable growth factors. The reasoning behind this formulation brings to light the basis for the general emphasis on balanced fertility programs and the high degree of complementarity which appears to exist among nutrients.

At the time Mitscherlich worked on his expression, Spillman developed a similar equation for formulating the fertilizer-yield relationship. For a one independent variable function, the two can be written:

$$(\text{Spillman}): Y = A(1 - R^X); (\text{Mitscherlich}): Y = A(1 - e^{kX}).$$

In both equations A is the maximum yield attainable and e^k can be shown to be equal to R which make the equations equivalent.⁶

An important difference between the two formulations is that the proportionality factor (R) in Spillman's formula is assumed to be dependent upon the conditions encountered in the experimentation from which the observations were obtained while the proportionality

⁶ O. W. Wilcox "Evaluation of a Multiple Fertilizer Test," unpublished paper distributed by V. Sauchelli.

factor (e^k) of the Mitscherlich equation is considered to be constant and independent of such conditions. While the Mitscherlich equation has received wide attention, it has few followers owing primarily to a common rejection of the universality proposition of the effect factor. The Spillman function, on the other hand, is regarded as a valuable equation for expressing certain types of relationships. This function approaches asymptotically the maximum possible yield, thus eliminating the possibility of it representing diminishing yields. It also imposes limitations on changes in the elasticity of production as the ratio of subsequent increments in output is constant over all ranges of output. These characteristics must be kept in mind in using this equation.

Although most earlier attempts to quantify a functional relationship between fertilizer applications and yield have been refuted, the work has served as an important benchmark for further formulations. The use of the mathematical expression to characterize the fertilizer yield relationship has its roots in these earlier works as does the recognition of the law of diminishing yields.

Later Use of Functions -- More recent attempts to express the fertilizer-yield relationships mathematically have involved separate functions for the different relationships found in specific situations. The universal requirements imposed on these functions are that there be a definite orderliness in the relationship that can be described by smooth curves conforming, generally, to some of the second order conditions specific to the law of diminishing returns.⁷ No longer is

⁷Robert F. Mutton, An Appraisal of Research on the Economics of Fertilizer Use, Report No. T 55-1, Agricultural Economics Branch,

a specific law of growth held to exist for all conditions.

Since the number of functions more or less meeting the conditions of the law of diminishing returns is infinite, a major problem is to find a function that is in some way best for a specific set of data at hand. Among the functions most often used in recent years are the Cobb-Douglas, Spillman and various polynomials.

The Cobb-Douglas or power function of the form $y = aX_1^{b_1}X_2^{b_2} \dots X_n^{b_n}$ is linear when transformed to logarithmic form. This linearity characteristic simplifies the estimation of parameters by least squares. The function displays continuously increasing yield and constant elasticities with respect to all input variables. For functions displaying constant elasticity, a given percentage increase in an input brings about the same percentage increase in output regardless of the level of input and output. This function has the disadvantages of (1) taking on a value of zero whenever any input is zero, and (2) an inability to describe more than one of the following: increasing positive, decreasing positive or negative incremental returns to incremental inputs. With modification these disadvantages can be overcome.⁸

The Spillman function expressed in the general case as

$$Y = M(1-R_1^{x_1}) (1-R_2^{x_2}) \dots (1-R_n^{x_n})$$

Div. of Agriculture Relations, Tennessee Valley Authority (Knoxville, Tennessee: T.V.A. March, 1955), p. 13.

⁸H. O. Carter, "Modifications of the Cobb-Douglas Function to Destroy Constant Elasticity and Symmetry," unpublished M. S. Thesis, Department of Agriculture Economics, Michigan State University, 1955.

is still used at present, partly as a holdover from past studies but also because of its usefulness for describing relationships present in certain data. The Spillman function has limitations when more than one fertilizer variable is incorporated into the design as it allows for only a constant rate of interaction to occur. It has the additional disadvantages, for some purposes, of (1) being difficult to fit, (2) taking on a value of zero when any of the independent variables is zero, (3) having certain rigidities of elasticity of output with respect to inputs, (4) being unable to reflect decreasing positive incremental returns to incremental inputs, and (5) approaching a maximum output asymptotically.⁹

Another general type or family of functions consists of the various polynomials. The number of possibilities within this family is infinite though those involving equations of higher than the third degree have not been used as they appear to be inconsistent with biological logic as reflected in the law of diminishing returns.¹⁰ An example of a one variable form is $Y = a + b_1 X + b_2 X^2$. This function is easily fitted by the method of least squares and is flexible to the extent that terms may be added or subtracted to change the characteristics of the function. There are few premises to serve as

⁹For a fuller description of the Spillman function see: W. J. Spillman Use of the Exponential Yield Curve in Fertilizer Experiments, U. S. Department of Agriculture Technical Bulletin No. 348 (Washington: U. S. Government Printing Office, April 1933). Also Ibach and Mendum, op. cit.

¹⁰There is little agreement as to what determines the validity of biologic logic. The third degree equation, however, has been considered to incur this violation more so than an equation of the second order.

a guide for choosing the terms of the equation and in this way the choosing of a specific polynomial presents a problem much like that of choosing a general type of function.

Heady, Pesek and Brown¹¹ used two types of polynomials:

$$Y = a + b_1 \sqrt{N} + b_2 \sqrt{P} + b_3 N + b_4 P + b_5 \sqrt{NP}$$

$$Y = a + b_1 N + b_2 P + b_3 N^2 + b_4 P^2 + b_5 NP$$

The first of these the so-called square-root form was considered more satisfactory and used as the prediction equation for corn.

While application of the various forms of functions has been limited, it is now evident that no one type of function is superior to all other types. Nearly every study of the fertilizer-yield relationship to date has made use of a different function. Johnson working with alternative functions to describe a series of data from a nitrogen-corn experiment concluded that an equation of the form:

$$Y = a + b_1 X + b_2 X^2$$

gave the most satisfactory results.¹² In the previously described corn experiment by Heady, et al., 35 single variable functions were computed for the completed rows, columns and diagonals of their design. Each of the following five equations appear to be a "best fit" for at least one particular set of data:¹³

¹¹Earl O. Heady, John T. Pesek and William G. Brown, op. cit.

¹²Paul R. Johnson, "Alternative Functions for Analyzing a Fertilizer-Yield Relationship," Journal of Farm Economics, XXXV (November, 1953) pp. 519-529.

¹³Heady, et al., op. cit., pp. 303.

$$Y = a + b_1 X + b_2 \sqrt{X}$$

$$Y = M - A^X$$

$$Y' = a X^b$$

$$Y = a + b_1 X + b_2 X^2$$

$$Y = a + b_1 X + b_2 X^2 + b_3 \sqrt{X}$$

where: Y is total yield, Y' is yield above checks and X refers to quantity of the variable nutrient applied.

With the present knowledge of the fertilizer-yield relationship, the usual method of choosing a function is to use one out of a limited number of different alternatives which appear to best fit the experimental observations. Judgments as to which is best are not made by highly objective rules or statistical tests. Such judgments rest, instead, primarily upon the researcher's experience and familiarity with the data. The most that statistical measures can do, as they do not provide a direct objective test, is contribute inconclusive information as to "goodness of fit." Specific statistics commonly of help in determining the best fit are the standard error of estimate, coefficient of determination, and standard errors of the regression coefficients. The standard errors of estimate indicate the closeness of the observed values to the predicted values. The coefficient of determination, multiplied by 100, measures the percentage of variance of the dependent variable "explained by" or associated with the independent variables. The standard errors of the regression coefficients measure the accuracy of the estimated regression coefficients.

Other aids in choosing a function are (1) plotting the functions or sub-portions thereof on a graph along with a scatter diagram of the observations, and (2) a study of expected biological relationships. Mathematical knowledge of help in choosing functions includes: knowledge of the shape various functions take on with different values of the parameters and the knowledge that the expected shape of a response function, through most of the relevant range, is convex from above.

Economic Optima

The goal in developing an appropriate function is to be able to make more efficient recommendations for the economic use of fertilizer. With the production relationship expressed mathematically the rates of fertilizer application which maximize profits are easily determined. Problems involved in projecting these rates into recommendations for field conditions will be taken up in Chapter V.

To determine an optimum application, the price per unit of the fertilizer, the price per unit of the crop grown and the marginal product of fertilizer in production of the crop (i.e., the partial derivative of yield with respect to the fertilizer nutrient involved) are needed.

For a single nutrient or single combination of nutrients, the optimum rate is attained and profits are maximized when the cost of adding another unit of the nutrient or nutrient combination is just equal to the return derived from its use. This can be defined as the application for which the marginal value product of the input $(\frac{\partial Y}{\partial X} P_Y)$

equals the cost of using another unit of the input (usually the input price), provided the marginal value product is diminishing (i.e., that the second derivative of the production function be negative at this point). The optimal point can also be stated as being the output where a rising marginal cost per unit of product equals marginal revenue (product price).¹⁴

The optimal application then is the rate of application where the marginal product, multiplied by the price, is equal to the cost of adding more nitrogen or

$$P_Y (MPP_N) = P_N$$

Dividing both sides of this expression by P_Y we have:

$$MPP_N = \frac{P_N}{P_Y}.$$

Since the marginal physical product is determined from the derived yield equation, as the derivative, the optimal condition can be written as:

$$\frac{dY}{dN} = \frac{P_N}{P_Y}$$

Substituting appropriate prices for nitrogen and corn and solving for N determines the most profitable amount of the nutrient to apply.

¹⁴

The appropriate marginal cost will depend on price only as a lower limit and upon opportunity costs at all but this limit. Opportunity cost would take into account possibilities of making a greater profit from the additional expenditure at the margin in some other use of the factor.

The economizing principle can be illustrated by a simple hypothetical case of a production function assumed to be of a simple form such as:

$$Y = a + bX + cX^2.$$

Then:

$$\frac{dY}{dX} = b + 2cX$$

The optimal application of X, in perfect competition, is then given by

$$b + 2cX = \frac{P_X}{P_Y}$$

Substituting hypothetical values into this expression we may have:

$$Y = 25.0 + 6.0 X - .5X^2$$

$$\frac{dY}{dX} = 6.0 - X$$

Equating this derivative to a price ratio, to determine the economic optimum of X to apply, when Y is priced at \$1.20 and the price per unit of X is \$3.60:

$$6.0 - X = \frac{3.60}{1.20}$$

$$X = 3.$$

Under these conditions, three units of X maximizes profit. This would result in a yield of

$$Y = 25.0 + 6.0(3) - .5(3)^2 = 47.5$$

It may be pointed out that maximum yields would result where the MPP is equal to zero or at 6 units. Use of these six units would result in a maximum yield of 79 units, but even though this yield is greater by 31.5 units it is less profitable than the lower yield.

Solution For Multi-Variable Case

In the previous analysis, principles of profit maximization were used to determine the quantity of one fertilizer nutrient applied for profit maximization. To determine the optimal fertilization program when more than one input is being used involves the simultaneous determination of the most profitable combination of all nutrients and the most profitable amounts of this combination to use. These conditions are determined when the ratio of the marginal value product of each nutrient to its cost is the same for all nutrients and equal to one, under perfect competition and an unlimited capital condition. Under capital restrictions these ratios are based to an opportunity cost, which is the return that could be made on the investment if used elsewhere in the business.

For a three nutrient case these relationships can be made operational by setting the marginal products, or partial derivatives, for each nutrient equal to their price ratios and solving the three equations simultaneously for the quantities of the three nutrients to apply for maximum profits.¹⁵

¹⁵ This can be written mathematically for the general case of three variable inputs producing Y as $Y = f(X_1, X_2, X_3)$

When π = profit, the profit equation is:

$$\pi = P_Y Y - P_{X_1} X_1 - P_{X_2} X_2 - P_{X_3} X_3$$

Partial differentiation with respect to the three variable yields:

$$\frac{\partial \pi}{\partial X_1} = \frac{\partial Y}{\partial X_1} P_Y - P_{X_1}$$

$$\frac{\partial \pi}{\partial X_2} = \frac{\partial Y}{\partial X_2} P_Y - P_{X_2}$$

$$\frac{\partial \pi}{\partial X_3} = \frac{\partial Y}{\partial X_3} P_Y - P_{X_3}$$

Setting the three partial derivatives equal to zero expresses the mathematical condition for maximum profit, assuming perfect competition, perfect knowledge and the law of diminishing returns which specifies the second order conditions necessary for a maximum.

CHAPTER III

SOURCE OF EMPIRICAL DATA

The preceding chapter provides the basic concepts and procedures needed for determining the incremental yield responses to applications of one or more fertilizer nutrients. It also showed how the incremented response data can be incorporated into an analysis for determining the most profitable amounts and combinations of fertilizers to apply. The source of the data to be used in the analysis presented in this thesis will be discussed in the present chapter.

In recent years, field experiments have been designed to produce data to which these newer analytical techniques can be applied. The project which produced the data on which this thesis is based was initiated jointly by the Departments of Soil Science and Agricultural Economics at the Michigan Agricultural Experiment Station.¹ The field experiments were designed specifically for the purpose of producing data to be used in estimating the fertilizer production function for the crop studied.

General Characteristics of Appropriate Experimental Designs

Experimental designs which yield adequate data suitable for economic analysis are necessarily more extensive than many past designs.

¹

This project is supported in part from funds received from the Midwest Soil Improvement Association, Davison Chemical Company and the National Plant Food Institute.

More information is needed to fit mathematical functions which represent larger portions of the fertilizer-yield surface. For functional analysis it is important that the extreme combinations and rates of fertilizer be included in the design along with the observations close to the expected "practical" range.

When only one factor of production is being investigated, the problem is relatively simple. An experiment can easily be designed to cover the entire range of response including enough rates, properly distributed, to permit estimation of a continuous function.

Investigation of two or more factors of production increases the problem as provisions for measuring interaction must be made. Before interaction can be measured, yields must be observed for many more combinations and rates of fertilization than have generally been included in past fertilization experiments. Multi-variable, incomplete factorial layouts involving several rates of application of each variable are used. It is usually desirable to increase the number of rates without using all of the possible combinations of the rates rather than using a complete factorial design with fewer rates. This design technique produces information needed for measuring interaction terms while keeping the size of the experiment within reason. This procedure becomes even more important as the number of variables is increased. Though this procedure yields information about many more points on the function, less reliability can be attached to yield estimates for a specific point than would be produced by extensive replication of the fertilizer treatment represented by that point.

A field of high uniformity is generally selected. Generally speaking, the more uniform a field is, the more unique it is and the smaller the range of farm conditions represented by it. There is a real conflict between the need for uniformity and the need for representativeness, which will be discussed in Chapter VI, after the analysis of the present data has been made and discussed. In addition to controls imposed by the soil characteristics, management practices are used that are both desirable for, and attainable by, the farm operators expected to use the results. Unless moisture level is included as a separate variable, it is usually held constant for all treatments either through irrigation or more commonly by prevailing weather conditions. Other factors over which controls can be extended are usually controlled. If controls are impossible but measures can be made, increasing attempts are being made to incorporate them in the analysis. Such incorporation often requires extensive use of agronomic principles and concepts.

Other factors over which neither controls nor measurements can be made are allowed to vary and are assumed or made to behave randomly. If this assumption is met and/or randomization attempts are successful, the effects of such variables can be averaged out by statistical procedures.

Experimental Design Used

The crops being studied in this joint project are corn, oats, wheat and alfalfa, grown in four year rotation. Each crop is to be grown each year on one of the four experimental fields. Three variable

nutrients, nitrogen, phosphorus and potassium were used on each crop. Only the results of the corn experiment for the 1955 crop year will be analyzed in this thesis.

The design used for this experiment includes six rates, including the zero application, for each of the three variable nutrients: nitrogen, phosphorus and potassium. These rates of application are given in Table I. Extrapolation of yield equations beyond the range of the observations used for the fitting of the functions cannot be made with confidence, which makes it imperative that the experiment be so designed as to allow reliable estimates to be derived for all relevant yields.

Observations were obtained from an incomplete factorial experimental design of the nature indicated in Table I. Ninety-one points on the surface out of a possible 216 were sampled with 39 of these points being replicated twice. Eleven replications of the check treatment (no fertilizer applied) were obtained to establish more accurately the origin of the fitted function. Two replications were made of the second, fourth, and sixth level of treatment for each nutrient and for all points on the "main diagonal" of the design. Thus a total of 130 observations are made for each crop.

The field size necessary to accommodate this design is slightly over 3.7 acres including alley-ways between rows of plots. Non-uniform factors that were neither controlled nor measured but which affect production were assumed to be randomized. This was accomplished by a complete randomization of the treatments over the total plot area.

Although the design is not perfect from many viewpoints it is a fairly efficient design for obtaining the necessary data. It represents a necessary compromise between perfection and the cost of perfection. This design, with randomized plots, provides sufficient observations over the relevant range of the input-output relationship for estimation of the three variable production function.

The experimental work was initiated in the spring of 1955 on Kalamazoo sandy loam soil at two sites in southwestern Michigan.² This soil is not excessively droughty by Michigan standards but lack of moisture-holding capacity together with a natural low fertility limits crop yields. While the experimental fields are located on private farms, all field work was done by Experiment Station personnel.

Nitrogen was applied in the form of 33.5% ammonium nitrate, phosphorus as 45% superphosphate and potassium as 50% muriate of potash. The fertilizer was broadcast prior to plowing and then plowed down. No supplemental fertilizer was added.

The hybrid variety used was Michigan 250. The yields resulting from the various treatments fall into a general range of from twenty to fifty bushels per acre. The check plot yields averaged 27.4 bushels, having a high of 45.5 and a low of 19.3 bushels per acre. The yields for all plots and treatments are given in the Appendix.

The experimental plots measured 14' by 50' in size with a harvested area of 7' by 50' or approximately $\frac{1}{125}$ of an acre taken from the

² The experiments are located in Kalamazoo and Calhoun County, with two experimental fields located at each site. The corn experiment reported in this thesis was at the Ewald Fich farm in Calhoun County.

TABLE I

DESIGN OF EXPERIMENT FROM WHICH YIELD DATA WERE OBTAINED
Each "X" Represents an Experimental Plot

Pounds of N Per A	Pounds of K ₂ O Per A	Pounds of P ₂ O ₅ Per A					
		0	40	80	160	320	480
0	0	11*			X		
	20		X				
	40						
	80	X			X		X
	160						
	240				X		X
20	0		X				
	20	X	XX		XX		XX
	40			X		X	
	80		XX		XX		XX
	160			X		X	
	240		XX		XX		XX
40	0						
	20			X		X	
	40		X	XX	X		X
	80			X		X	
	160		X		X		X
	240			X		X	

*

Eleven plots received no fertilizer.

TABLE I - Continued

Pounds of N Per A	Pounds of K ₂ O Per A	Pounds of P ₂ O ₅ Per A					
		0	40	80	160	320	480
80	0	X			X		X
	20		XX		XX		XX
	40			X		X	
	80	X	XX		XX		XX
	160			X		X	
	240	X	XX		XX	X	XX
160	0						
	20			X		X	
	40		X	X	X		X
	80			X		X	X
	160		X		X	XX	X
	240			X	X	X	X
240	0					X	X
	20		XX		XX		XX
	40					X	
	80	X	XX		XX		XX
	160			X		X	X
	240	X	XX		XX	X	XX

center of each plot. Seeding rate; variety; planting, cultivation and harvesting times and practices; weed control measures; soil characteristics within the bounds that a field of the size used are uniform; past management practices and past cropping history were all held constant for all of the experimental plots. The corn population was thinned to 12,500 plants per acre, which was considered optimum for the soil type. All other factors of production such as soil variations, insect and disease infestation, as well as errors of fertilizer application and yield measurement were considered to vary randomly, except for the three variable nutrients and the varying amounts of labor and machinery needed for the application of the fertilizer and harvesting of the corn. Past history of the experimental field included a corn crop in 1954 and wheat in 1953.

The 1955 growing season was unfavorable for the growth of corn. The expected average yields of corn, with good management is 65 bushels per acre,³ which is far above that obtained from the experimental plots for this particular year. This reduction in expected yield was brought on by a drought period extending from mid-July through mid-September.

Soil samples were taken and tested for phosphorus and potassium before application of the fertilizer and again prior to growing the succeeding crop. Although the results of these tests will not be used in the analysis presented in this thesis, they will be used in further attempts to more appropriately characterize the existing relationship between added fertilizers and yields of crops.

³James Porter, Stanley Alfred, Eugene Whitside and Robert Lucas, Get the Most from Your Farmland, Michigan State University Cooperative Extension Service, East Lansing, Michigan, pp. 17.

CHAPTER IV

FUNCTIONAL ANALYSIS OF EXPERIMENTAL RESULTS

A conceptual presentation of appropriate methods for economic analysis of fertilization data was made in Chapter II. The source of the empirical data to be used in this thesis was described in Chapter III. In this chapter analysis of these data will be carried out, beginning with fitting an appropriate mathematical production function and ending with determination of the most profitable applications of fertilizer for varying price situations.

Functions Fitted to Data

As indicated in the preceding chapter, the number of possible mathematical functions which might be used to describe a fertilizer-yield relationship is infinite. The task of choosing among these alternatives, while sometimes difficult, can be done by an agriculturist able to draw upon past experience in this and related kinds of work. While a great deal of judgment and subjectivity is involved, experience and familiarity with the data at hand enables reasonably good choices to be made from a number of alternative functions.

The primary requirement of the mathematical function to be chosen is that it describe the technical production relationships. Its usefulness is particularly determined by its performance in the economic range of response. As attempts are made to add detail to the description of the observed relationship, it becomes apparent

that such detail almost invariably has as its price loss of computational ease. Although, it may be possible to gain both computational expediency and detail, in some cases, by and large one can be increased only at the expense of the other. This is true not only of fertilizer yield relationships but in most research. To value the payment for detail too highly may defeat the objective of the analysis which is finding useable answers to the problem faced. Conversely, to value it too lightly may result in misuse of research resources and failure to produce results. The less variance exhibited in the experiment data, the less difficult the choice between functions becomes and the more variance present the less important the choice. In the present study, as in all such studies, better fits could probably be obtained by using a function which has not been considered. But, considering the variance of the data and the possible rewards for accuracy, a wider search for a better function was felt to be unwarranted.

Four different functions were originally selected and fitted to the data obtained from the 130 plot, corn experiment.

These functions were:

(1) Cross-product polynomial¹

$$Y = a + b_1 N + b_2 P + b_3 K + b_4 N^2 + b_5 P^2 + b_6 K^2 + b_7 NP + b_8 NK + b_9 PK$$

(2) Square root polynomial¹

$$Y = a + b_1 N + b_2 P + b_3 K + b_4 \sqrt{N} + b_5 \sqrt{P} + b_6 \sqrt{K} + b_7 \sqrt{NP} + b_8 \sqrt{NK} + b_9 \sqrt{PK}.$$

¹These functions have been given names in order to simplify reference in the text, with no particular relevance other than describing a particular characteristic of each equation.

(3) Cobb-Douglas

$$Y = a N^{b_1} P^{b_2} K^{b_3}$$

(4) Carter-Halter

$$Y = a N^{b_1} c_1 N^{b_2} c_2 P^{b_3} c_3 K$$

The polynomials used are only two of many possible forms. These particular forms have been used previously with some success. As the data indicate no range of increasing marginal returns, the polynomial equations contain no terms of higher than the second degree. Being polynomials involving first and second or one-half degree terms, they are capable of showing both the diminishing marginal yields and diminishing total yields evident in scatter diagrams of the data. Although no complete interaction term is present in either of the equations, it was felt that interactions are measured by the last three terms of each equation about as accurately as the variance in the data permit. While the two equations are similar in many respects, they were both used because of differing past experience with each equation. Though the cross-product form has been more commonly used, Heady found the square root form to be of value, particularly in cases where the marginal products are large at low inputs and small at higher rates of inputs.²

² Earl O. Heady, "Technical Considerations for Estimating Production Functions in Studies of Farm Resource Use," a paper presented at North Central Farm Management Research Conference on Farm Scale and Resource Productivity, October, 1954. The proceeding of this conference are in book form: Resource Productivity, Returns to Scale, and Farm Size, ed. Earl O. Heady, Glenn L. Johnson and Lowell S. Hardin (Ames, Iowa: Iowa State College Press, 1956).

A Cobb-Douglas, or power function, was also fitted to the data. Since this function is incapable of describing the diminishing total yields apparent in the data, observations involving the highest nutrient applications were omitted.³ Functions which allow use of all experimental observations are more efficient under many conditions than those necessitating exclusion of a part of the data. Though this may be so, it is also true that the main interest is in obtaining an accurate description of the relationship in the range of economic relevance i.e. the range for which yields are increasing and in which the partial derivatives of yield with respect to the nutrient are positive and decreasing.

As the Cobb-Douglas function has the further property of taking on zero values when any of the inputs are zero, a transformation of the data was used to overcome this difficulty. This was accomplished by the addition of one-tenth of a unit⁴ to all amounts of fertilizer nutrients applied which introduces some slight biases at the lower ranges of observations. This bias occurs because the function must have a value of zero at the origin, but is forced close to the check plot yields at input values of one-tenth unit. Thus some upward bias comes about in estimating yields for the next higher increments of yields in much the same way that a piece of springsteel bulges up

³ That is, plots receiving: N at 240 pounds, P_2O_5 at 480 pounds or K_2O at 240 pounds per acre were omitted from the regression analysis.

⁴ Inputs were measured in twenty pound units as a computational expedient. The transformation thus added two pounds which may be considered negligible.

when bent sharply at one end. This bias was judged to be less serious than that resulting from other possible modifications or from letting the function pass through the origin in the usual way.

The fourth expression fitted to the data was the Carter-Halter exponential function which also requires a logarithmic transformation before fitting. This function, which is a modification of the Cobb-Douglas, allows more flexibility in the shape of the function without foregoing certain important properties of expediency and simplicity of parameter estimation.⁵ The important advantage of this function is its flexibility which is greater than that of the unmodified Cobb-Douglas. Offsetting this advantage is the necessity of estimating two parameters for each variable included which is more than required for the Cobb-Douglas but less than involved for any polynomial that has commonly been used. Another disadvantage of the Carter-Halter function is the greater complexity encountered in locating various economic optima.

For the Carter-Halter function to conform to that portion of the law of diminishing returns usually encountered in fertilizer studies, the b_i 's and c_i 's should be positive and less than one. The function will then indicate a total product increasing at a decreasing rate and eventually decreasing. Though this function has been fitted to

⁵ The original work describing this modification of the Cobb-Douglas equations appears in H. O. Carter's M. S. thesis, op. cit. A further description with more comprehensive treatment will appear in a paper by H. O. Carter and A. Halter to be submitted to the Journal of Farm Economics. The name of Carter-Halter will be used for reference to this function in the text.

few sets of empirical data, preliminary results show it to be capable of describing all three stages of production simultaneously.

Results of Function Fitting

When these four functions were fitted to the common set of experimental data, the parameters took the values indicated below. The N, P and K refer to twenty pound units of N, P₂O₅ and K₂O respectively. Yield is measured in bushels per acre.

Cross-product polynomial:

$$Y = 30.70 + 3.59557 N + .54670 P - .11615 K - .23719 N^2 - .00112 P^2 + .03696 K^2 - .02300 NP - .01932 NK - .02921 PK$$

Square-root polynomial:

$$Y = 27.29 - 3.42776 N + .54818 P + .19308 K + 15.98638 \sqrt{N} - 1.17115 \sqrt{P} - .71484 \sqrt{K} - .02928 \sqrt{NP} - .17406 \sqrt{NK} + .07762 \sqrt{PK}$$

Cobb-Douglas:

$$Y = 35.12 (N + 0.1)^{.15558} (P + 0.1)^{-.00804} (K + 0.1)^{-.02732}$$

Carter-Halter:

$$Y = 38.54 (N + 0.1)^{.19272} 0.960^{(N + 0.1)} (P + 0.1)^{-.02135} 1.006^{(P + 0.1)} (K + 0.1)^{-.00163} 1.001^{(K + 0.1)}$$

All of the three variable functions fitted indicate a strong response to nitrogen applications. All show insignificant responses to phosphorus and potassium except the cross-product polynomial which indicates some response to phosphorus. However, none of the coefficients of the cross-product polynomial involving phosphorus are significant at the ten percent level indicating that there may in fact be insufficient reason to suspect that a response to phosphorus exists.

The Cobb-Douglas function shows negative responses to both phosphorus and potassium; however, due to the small size of the coefficients and their relatively large standard errors,⁶ it can be presumed that little if any change in yields results from changing the application levels of these nutrients. The response due to nitrogen is sizeable as given by the coefficient of the N term and a low standard error indicates a high degree of statistical significance.

The Carter-Halter fit indicates the same relationships--very significant responses to nitrogen but little response to phosphorus and potash. The coefficients for the N terms are highly significant, ($P > .99$) while the coefficients of the P and K terms lack a dependable level of significance as given by the "t" test ($P < .90$). Further indications of the lack of response to phosphorus and potash are given by (1) the small size of the coefficients and (2) signs for the coefficients contrary to logical expectations. As the coefficients are both small and insignificant, the probability of contrary signs is greater than if the coefficients were large and significant. An analysis of variance indicates still further that there are no responses to phosphorus or potash but very significant ($P > .99$) responses to nitrogen.

✓ Selection of the "Best" Function

After fitting a number of equations to a single set of data, the choice of function that most accurately represents the existing relationship between added nutrients and yield had to be made.

⁶ For "t" values of the coefficients see Table 2.

As previously noted, there are no purely "objective" criteria for making this selection. Reliance must, instead, be put on rather indirect and subjective measures of "goodness of fit." Among these are such statistics as the coefficient of multiple correlation, magnitudes of residual variance quantities not associated with the regression, plottings of the alternative functions along with a scatter diagram of the experimental observations, logical expectations, and knowledge of the technical relationships inherent in the data. Such measures usually help isolate one function as being "more reasonable" than the others.

The basic statistics relating to the four, three-variable functions fitted in this study are given in Table II. The large variances inherent in the data are evidenced by the low coefficients of multiple determination (R^2) which denote the proportions of the total variance "explained by" or associated with the changes in the dependent variables.

As a further test for determining the "best fit," F ratios were computed on the amount of the total sum of the squared deviations from the means explained by regression and that independent of regression. The computational form given in Table III was used. In this table n is the number of observations, m is the number of variants and S_y^2 is the sum of the squared deviations from the mean.⁷ The computed F ratios are given in Table IV.

⁷ George W. Snedecor, Statistical Methods, (4th ed., Ames, Iowa: State College Press, 1953), pp. 348.

TABLE II

STATISTICS FOR THREE-VARIABLE EQUATIONS FITTED TO CORN DATA,
KALINZOO SANDY LOAN SOIL, SOUTHWESTERN MICHIGAN IN 1955

Equation	Value of R	Value of R ²	Value of the constant term "a"	Value of "t" for coefficients, together with terms involved	
Cross-product polynomial	.5408	.29	30.70	N: 4.945** K: .157 P: .004 NP: .810 PK: .957	P: 1.073 N ² : 4.440** K ² : .688 NK: .529
Square-root polynomial	.6377	.41	27.29	N: 5.788** K: .336 $\sqrt{P}$: .789 $\sqrt{NP}$: .058 $\sqrt{PK}$: .157	P: 1.231 $\sqrt{N}$: 6.112** $\sqrt{K}$: .278 $\sqrt{NK}$: .308
Cobb-Douglas	.7145	.51	35.12	(N + 0.1) : 5.695** (P + 0.1) : .382 (K + 0.1) : 1.021	
Carter-Halter	.7010	.49	38.54	(N + 0.1) : 8.145** 5.628** (P + 0.1) : 1.117* 1.921* (K + 0.1) : .069 .127	

**Significant at 1%; *Significant at 5-10%. All other coefficients are not significant at 10%.

Two "t" values are given for each fertilizer nutrient for the Carter-Halter equation because each nutrient occurs twice in the equation.

TABLE III
FORM OF COMPUTATIONS USED FOR COMPUTING ANALYSIS
OF VARIANCE IN REGRESSION

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F Ratio
Total	$n-1$	S_y^2	$\frac{S_y^2}{n-1}$	
Due to regression	$m-1$	$R^2 S_y^2$	$\frac{R^2 S_y^2}{m-1}$	$\frac{R^2 S_y^2}{m-1}$
Independent of regression	$n-m$	$(1-R^2) S_y^2$	$\frac{(1-R^2) S_y^2}{n-m}$	$\frac{(1-R^2) S_y^2}{n-m}$

TABLE IV
F RATIOS FOR ANALYSIS OF VARIANCE IN REGRESSION FOR THREE-VARIABLE
FUNCTIONS FITTED TO CORN DATA. KALAMAZOO SANDY LOAM SOIL,
SOUTHWESTERN MICHIGAN IN 1955

Function	Total D F	F Ratio*
Cross-product polynomial	129	6.31
Square-root polynomial	129	10.45
Cobb-Douglas	64	32.88
Carter-Halter	129	24.16

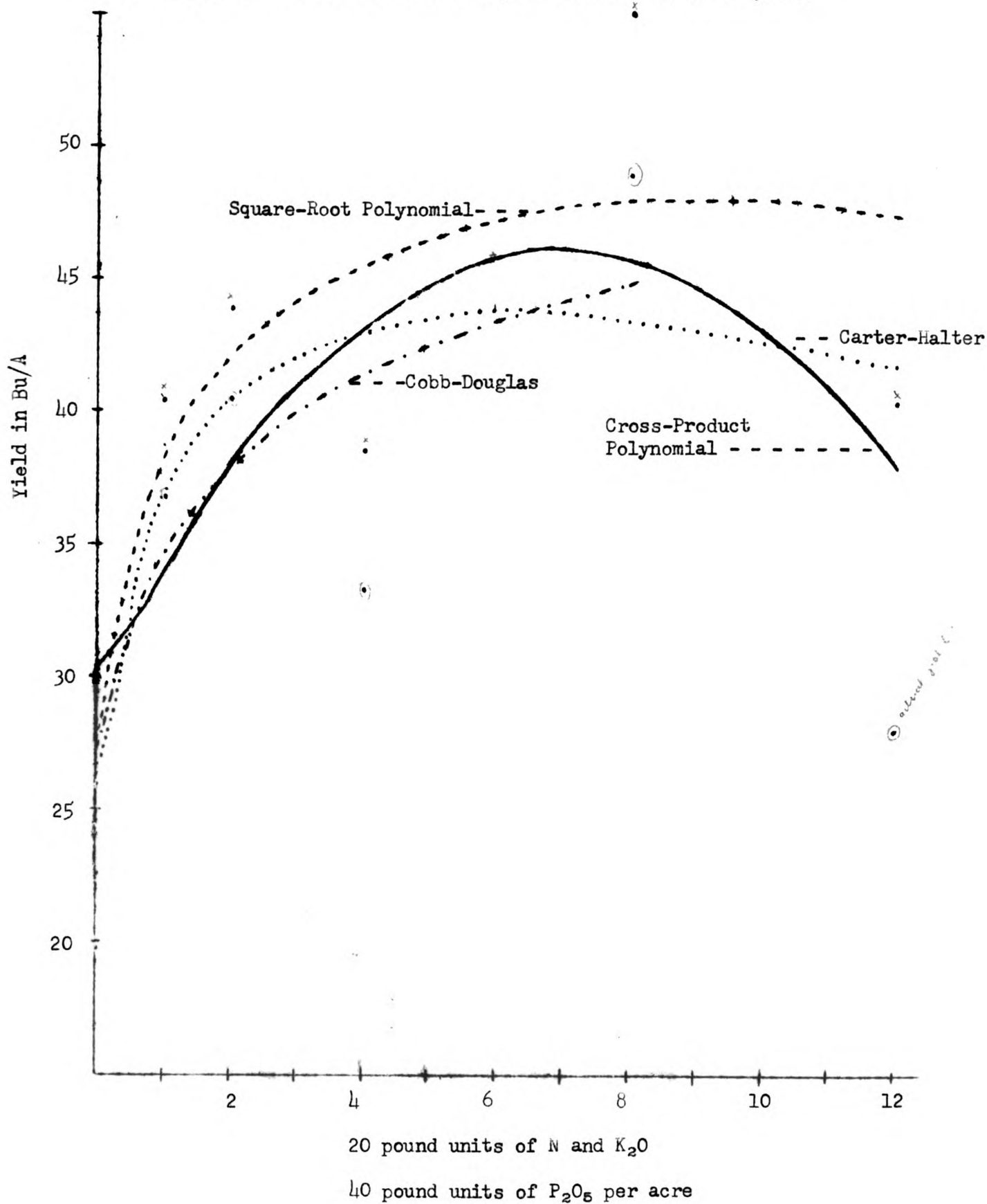
*

All of these F ratios are significant at 1%.

In addition to the statistical measures used, other less objective evidence was accumulated on which to choose the function which best describes the relationships in the data. Plotting of the alternative functions against actual observations and a familiarity with the data were useful in choosing among the functions. Although these "tests" are not independent of the statistics previously cited, they do provide an added basis for the final choice. An example of the plotting procedures used is given in Figure 1. For this example, the estimated yields for inputs added in 1-2-1 ratio (up the main diagonal of the design) are plotted along with the actual yields as they occurred in the field experiment. As a significant response occurred only to additions of nitrogen, emphasis was placed on plottings in the nitrogen-yield dimension.

FIGURE 1

ESTIMATED YIELDS OF CORN FOR NUTRIENTS ADDED IN SIMPLE PROPORTIONS
INDICATED BY MAIN DIAGONAL OF DESIGN USING THREE VARIABLE
FUNCTIONS, KALAMAZOO SANDY LOAM SOIL, SOUTHWESTERN MICHIGAN, 1955



The plottings of field observations show large responses up to 60 pound applications of nitrogen. Yield increases continue to occur until around the 100 pound level and then total yields slowly decrease at higher rates of application.

Results of Fitting the Cross-Product Polynomial -- This function gave the poorest fit as indicated by the statistics presented in Tables II and IV. Only 29 percent of the variance about the mean yield was associated with this fit.

The function originates at a yield slightly over 30 bushels, which is approximately three bushels greater than the average of the eleven check plots and over four bushels greater than the average of all plots receiving no application of nitrogen. Increases in yield occur on this function beyond the level evident in the data. The latter portion of the nitrogen-yield curve indicates yields to be severely depressed by the higher rate of application. While this portion of the curve is relatively unimportant for economic considerations, it gives an added indication of how well a function describes the data. This function appears to be the poorest description, among the functions fitted, of the fertilizer-yield data.

Results of Fitting Square-Root Polynomial -- This function fitted the data better than the cross-product polynomial according to all criteria of evaluation used. Specifically, this equation fitted the yield data better than the cross-product polynomial for low applications of nitrogen. Forty-one percent of the variance in the original yield data was associated with this regression. This function

originates at 27.29 bushels which is near the average yield for the check plots. The function follows the data very well at the lower observations but indicates yields that are well above those observed for nitrogen applications in excess of forty pounds. While this function had certain advantages over the other polynomial fitted, its failure to fit at high levels along with its poor showing with respect to objective statistical measures leaves much to be desired.

Results of Fitting the Cobb-Douglas Function -- Fifty-one percent of the variance is associated with this function, greater than any other function used. The F ratio computed for analysis of variance in regression is also larger than any other. As these statistics apply to the data in logarithmic form, they do not necessarily indicate that this function approximates the experimental data better than the polynomials. Plotting the function and data in natural numbers revealed certain undesirable characteristics which, in turn, provide the basis for rejecting it as the best representation of the data. This function, while originating at a satisfactory yield, indicates that yield increases occur throughout the entire range of observation. This characteristic is a property inherent in the function and is not itself a serious disadvantage if the marginal products are estimated satisfactorily in the relevant range. However, the marginal products on this function are smaller at the lowest rates of application and larger at the higher rates of application than appears to be the case.

Results of Fitting Carter-Halter -- The Carter-Halter equation appears to give the best description of the field observations among

the functions used. It indicates that yields originate at 26 bushels per acre and increase rapidly with additions of nitrogen until approximately 60 pounds is reached. At higher rates of applications, smaller yield increases occur until a maximum of 46 bushels is reached at an application of 120 pounds of nitrogen per acre. At still higher levels of application this function indicates that yield is reduced somewhat.

The statistical measures of the "goodness of fit" indicate that this function fits the data nearly as well as the Cobb-Douglas. Whether or not the fit is superior to the fits for the polynomials used is not entirely clear from the statistical measures as they apply to the data in logarithmic form in the case of the Carter-Halter and Cobb-Douglas functions. However, on the basis of all things considered--the statistics computed, plots of functions and data, and knowledge of the situation at hand--the Carter-Halter equation was accepted as giving the "best fit" among the functions considered. It should not be inferred from the acceptance, however, that this equation will fit best for all similar experiments, for all experiments on corn or even for an identical experiment conducted on the same soil in another year. However, there is good reason to believe that inherent properties of this equation will cause it to describe rather adequately many similar types of relationships. Successive years' results will have to be analyzed in a similar manner until an appropriate expression can be formulated for the relationship of fertilizer application to yield of corn on this type of soil in this climatic area. At such

time it should also be possible to estimate probability distributions of the coefficients.

Prediction of Yields

Using the Carter-Halter equation, yield estimates were made for various levels and combinations of the three nutrients. These estimates are presented in Table V. It can be seen from these figures that nitrogen exerts the predominant effect on yields. The signs of the P and K coefficients cause small additions of these nutrients to depress predicted yields somewhat with larger additions increasing yields at a slight, but increasing, rate--a situation which is not logically acceptable. The probability of these inappropriate signs occurring is large in view of the large standard errors of the coefficients involved. These large standard errors are in turn due to the lack of yield response to these nutrients and to the size of the unexplained variance.

With the parameters of the "best" three-variable function determined, yield predictions could be made for any combination of nutrients. This ability to predict yields for any point in the function rather than being limited only to points of actual field observation gives the continuous function technique an analytical advantage over other methods of analysis.

Statistical Reliability of Estimates

The individual coefficients in an estimating equation often have little economic meaning as the economic significance of the equation

TABLE V

PREDICTED YIELDS OF CORN FROM THREE-VARIABLE CARTER-HALTER EQUATION,
KALAMAZOO SANDY LOAM SOIL, SOUTHWESTERN MICHIGAN IN 1955

Pounds of N per A	Pounds of K ₂ O per A	Pounds of P ₂ O ₅ per A						
		0	80	160	240	320	400	480
0	0	26.0	24.4	24.8	25.2	25.5	26.2	26.8
	60	25.9	24.5	24.8	25.2	25.6	26.1	26.7
	120	25.9	24.6	25.8	25.2	25.7	26.2	26.7
	180	26.0	24.6	24.9	25.3	25.7	26.3	26.8
	240	26.1	24.7	24.9	25.3	25.8	26.3	26.9
20	0	39.0	37.0	37.3	37.9	38.6	39.4	40.2
	60	38.9	36.9	37.2	37.8	38.5	39.3	40.1
	120	39.0	36.9	37.3	37.9	38.6	39.4	40.2
	180	39.0	37.0	37.5	38.0	38.7	39.4	40.3
	240	39.2	37.1	37.5	38.0	38.8	39.5	40.4
40	0	42.8	40.6	41.0	41.6	42.4	43.2	44.1
	60	42.7	40.5	40.9	41.5	42.3	43.1	44.0
	120	42.8	40.5	40.9	41.6	42.4	43.2	44.1
	180	42.9	40.6	41.0	41.7	42.4	43.3	44.2
	240	43.0	40.7	41.1	41.8	42.5	43.4	44.3
80	0	45.2	42.7	43.2	43.9	44.7	45.6	46.5
	60	45.1	42.7	43.1	43.8	44.5	45.5	46.4
	120	45.1	42.7	43.1	43.8	44.7	45.5	46.5
	180	45.2	42.8	43.2	43.9	44.7	45.6	46.6
	240	45.3	42.9	43.3	44.0	44.9	45.7	46.7
120	0	45.0	42.6	43.1	43.8	44.6	45.5	46.4
	60	44.9	42.5	43.0	43.7	44.5	45.3	46.3
	120	45.0	42.6	43.0	43.7	44.5	45.4	46.4
	180	45.0	42.7	43.1	43.8	44.6	45.5	46.5
	240	45.2	42.8	43.2	43.9	44.7	45.6	46.6
160	0	43.9	41.5	42.0	42.7	43.4	44.3	45.2
	60	43.8	41.5	41.9	42.5	43.3	44.2	45.1
	120	43.9	41.4	41.9	42.6	43.4	44.3	45.2
	180	44.0	41.5	42.0	42.7	43.5	44.4	45.3
	240	44.1	41.6	42.1	42.8	43.6	44.5	45.4
200	0	42.3	40.0	40.4	41.1	41.8	42.7	43.6
	60	42.2	39.9	40.3	41.0	41.7	42.6	43.5
	120	42.2	40.0	40.4	41.0	41.8	42.6	43.5
	180	42.3	40.1	40.5	41.1	41.9	42.7	43.6
	240	42.4	40.2	40.6	41.2	42.0	42.8	43.7
240	0	40.4	38.2	38.6	39.2	40.0	40.8	41.6
	60	40.3	38.1	38.5	39.1	39.9	40.7	41.5
	120	40.4	38.2	38.6	39.2	39.9	40.7	41.6
	180	40.4	38.3	38.7	39.3	40.0	40.8	41.7
	240	40.5	38.4	38.7	39.4	40.1	40.9	41.8

depends on (1) the partial derivatives of yield with respect to each of the factors considered and (2) the predicting ability of the equation both of which often depend on several of its coefficients. To omit certain terms of an equation may reduce the meaningfulness of these derivatives in terms of biological premises. The omission becomes more disturbing and the probability of drawing unwarranted conclusions increases as the size of the coefficient of the omitted variable becomes larger. If the coefficient is of considerable size, it may be meaningful in an economic sense though a large standard error may indicate that it does not differ significantly from zero. Especially in connection with polynomials, it has been common practice to omit an independent variable if its coefficient proves to be insignificant at some arbitrary level and then refit the equation. Though in a strictly statistical sense this may appear to be an appropriate practice, economic considerations indicate the danger of erroneous or unwarranted conclusions.

An alternative approach to the problem of statistical reliability is to measure the reliability of the partial derivatives. In this way confidence levels for an economically meaningful portion of the expression could be stated. A derivative used in locating an economic optimum may be statistically questionable although some of the coefficients entering into the partial derivatives are significant. Conversely, statistically significant derivatives, based in part on statistically insignificant coefficients, would be useable. Measurement of the reliability of a derivative at a point is complicated by the fact that such measures are functions of the reliability of both

the input coefficients and of the output or yield at that point. Statistical methods for determining the standard error of partial derivatives for the equations commonly used in fertilizer response experiments are not yet developed.

Despite the lack of a statistical measure of reliability, the partial derivatives of the three-variable Carter-Halter function, with respect to both phosphorus and potassium, are regarded as insignificant. This conclusion is based on the statistical insignificance of all of the coefficients involving these variables as well as their illogical signs.

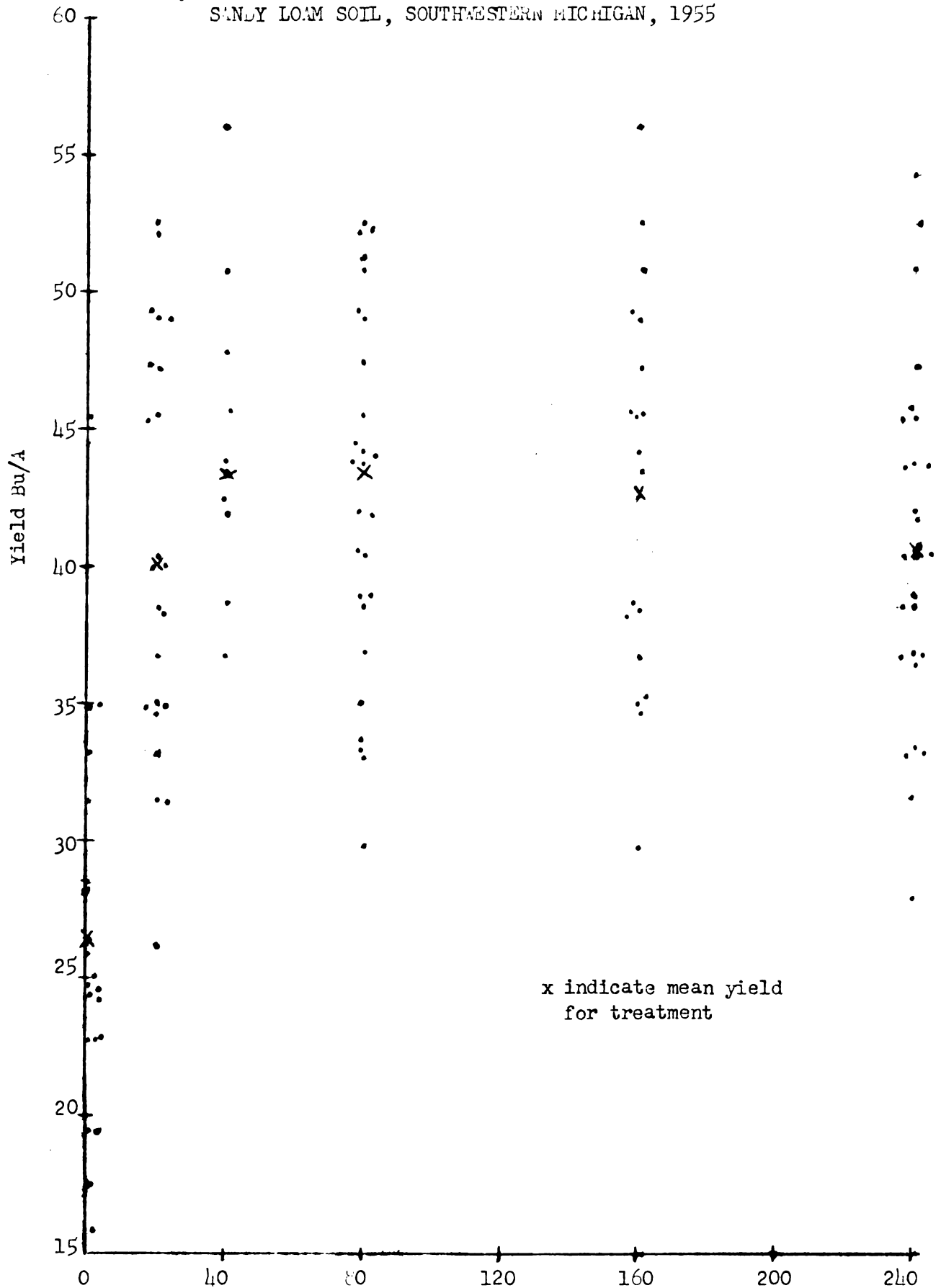
One Variable Function

Because of the lack of a significant responses to phosphorus and potash, the Carter-Halter function was refitted with the P and K terms omitted. The following results were obtained where N is measured in twenty pound units:

$$Y = 39.71 (N + 0.1)^{.18627} .96230^{(N + 0.1)}$$

The "b" value (.18627) is less than one indicating that the total physical product (yield) rises at a decreasing rate and the "c" value (.96230) is also less than one insuring that the total physical product will eventually decrease. This conforms to the relationship apparent in the data. Figure 2 shows the scatter diagram of yields as a function of nitrogen applications. The check yield denoted by this function is 25.8 bushels which compares favorably with the average yield of 26.3 bushel for plots receiving no nitrogen. The function shows the

OBSERVED YIELDS OF CORN RESULTING FROM VARYING LEVELS OF N, P_2O_5
AND K_2O VARIED EXPERIMENTALLY FOR EACH RATE OF N. KALAMAZOO
SANDY LOAM SOIL, SOUTHWESTERN MICHIGAN, 1955



marginal physical product to be large at the smaller application and equal to zero at 85 pounds of nitrogen with a yield of 44.2 bushels of corn.

The coefficient of multiple correlation (R) is .6884. Forty-seven percent of the variance is "explained by" the function (see Table I for comparisons with the three-variable functions). The "t" values for the coefficients are 9.76 and 5.7, both highly significant. The F ratio of variance in regression is 112.43, far in excess of any of the previous equations. On the basis of these statistics, plots of the function against the other functions and the observations, and the discrepancies noted among coefficients of the three-variable equations, this equation was accepted to be the most nearly characteristic of the observed relationships. Predicted yields and marginal products using this prediction equation are given in Table VI, utilizing rates similar to those of Table V.

Solution for Optimal Application

In solving for the optimal amount of nitrogen on the one-variable prediction equation previously developed, the problem is in principle identical to that discussed in Chapter II though the mathematics of the solution becomes somewhat more involved.

For illustrative purposes, a price of \$1.20 per bushel for corn and 12 cents per pound of nitrogen will be considered. The prices used are at this point quite immaterial. The appropriate price to be used for the product is the expected price at the time of harvest or

TABLE VI

OBSERVED AND PREDICTED YIELDS AND MARGINAL PRODUCTS FROM ONE VARIABLE
CARTER-HALTER EQUATION, CORN, KALAMAZOO SANDY LOAM SOIL,
SOUTHWESTERN MICHIGAN IN 1955

N per Acre	Average of Actual Yields	Number of Plots	Predicted Yield of Corn*	Predicted Marginal Product
<u>Pounds</u>	<u>Bu/A</u>	<u>Number</u>	<u>Bu/A</u>	<u>Bu/A</u>
0	26.3	18	25.8	27.10
20	40.6	24	38.2	5.65
40	43.5	14	41.8	2.29
80	43.4	29	44.1	.35
120	--	--	44.0	-.33
160	42.8	18	43.0	-.65
200	--	--	41.5	-.81
240	40.7	27	39.8	-.91

*This equation does not include P and K as variables, although these nutrients were varied experimentally for each rate of nitrogen applied.

when the product is to be sold. The price of the fertilizer should include not only its purchase price at the time of application but its cost of application as well.⁸ The input units in the prediction

⁸ Cost of application is, in a sense, a fixed cost as it does not increase appreciably with the rate or proportion of fertilizer changes. The choice is to apply or not to apply fertilizer. Only if the net revenue from use of fertilizer is greater than the cost of application at any point will it be profitable to make an application. The most profitable point is then given by the price ratio. If the application is on a custom basis, where price plus application is the same regardless of the rate, the price ratio varied on this aggregate price should be used.

equation are twenty pounds. Thus, the price per pound of N is multiplied by twenty giving a price ratio of $\frac{2.40}{1.20}$ or 2.0.

Starting from the equation as previously derived,

$$Y = 39.71 (N + 0.1)^{.18627} .96230^N$$

The marginal product of $(N + 0.1)$ is given by the derivative:

$$\frac{dY}{d(N + 0.1)} = 39.71 [(N + 0.1)^{.18627} .96230^N + 0.1 \ln .96230 + .96230^N + 0.1 (N + 0.1)^{-.81373} .18627]$$

Simplifying this derivative and setting it equal to the price ratio gives:

$$39.71 (.96230^N + 0.1 (N + 0.1)^{.18627}) (.03874 + (N + 0.1)^{-1} .18627) = 2.0$$

Solution of this equation, for the optimal $(N + 0.1)$, is accomplished by Newton's method of successive approximation.⁹ A first approximation of the desired root of the above equation is obtained graphically. After the first approximation, X_1 , successively closer approximations can be found to the desired accuracy from the following:

$$X_{i+1} = X_i - \frac{\frac{b}{X_i} + \ln c}{\left[\ln c + \frac{b}{X_i}\right]^2 - \frac{b}{X_i^2}} + \frac{m}{aX_i^b c^{X_i} \left[\left(\ln c + \frac{b}{X_i}\right)^2 - \frac{b}{X_i^2}\right]}$$

where m equals the price ratio.

The first approximation, found by using a graphic determination, of the solution of the equation, is 2.25 units. With a price ratio of 2.0 the second approximation determined by using the above

⁹M. E. Smith, Meyer Salhower and Howard K. Justice, Unified Calculus, (New York: John Wiley and Sons, Inc., 1947), pp. 186.

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expression is found to be 2.13 units. At 22 pounds of N $[(N + 0.1)20]$ per unit less $2 = 20(0.1)$ the optimum application is 44.86 pounds of nitrogen per acre. The yield at this optimal rate is found by substituting this value of N into the original equation. In this instance the optimal predicted yield is 42.13 bushels per acre.

The solution found in this way is a solution giving the greatest profits under the assumed price conditions. If these prices are not appropriate the solution is not appropriate. If the cost of nitrogen decreases as a cheaper source of nitrogen such as anhydrous ammonia is used, the optimal application becomes greater. This points out a shortcoming of recommendations which do not take into account prices and changes in prices. Table VII gives the application which would result in maximum profit and the resulting yields, under varying price situations. The importance of price considerations in determination of optimal fertilization is brought out by the wide range of the recommendations under different price conditions.

TABLE VII

MOST PROFITABLE APPLICATION AND PREDICTED YIELD OF CORN UNDER
VARIOUS PRICE CONDITIONS ESTIMATED FROM ONE VARIABLE
CARTER-HALTER EQUATION, KALAMAZOO SANDY LOAM,
SOUTHWESTERN MICHIGAN IN 1955

Price of Corn per bu.	Price of N per pound	Optimum application of N (lbs. per A)	Predicted yield (bu. per A)
\$2.00	\$.18	46.5	42.44
	.15	53.6	42.83
	.12	59.6	43.20
	.09	67.0	43.56
	.06	76.1	43.87
1.60	.18	42.3	41.88
	.15	47.1	42.32
	.12	53.5	42.82
	.09	61.4	43.29
	.06	71.3	44.28
1.20	.18	34.5	40.94
	.15	38.6	41.46
	.12	45.0	42.13
	.09	53.5	42.82
	.06	64.4	42.89
.80	.18	24.7	39.30
	.15	29.0	40.10
	.12	34.5	40.94
	.09	42.3	41.88
	.06	61.4	43.20
.40	.18	12.0	35.64
	.15	14.8	36.69
	.12	18.8	37.89
	.09	24.7	39.30
	.06	34.5	40.94

CHAPTER V

MEANING AND USE OF THE EXPERIMENTALLY DERIVED ESTIMATES

The data derived from the field experiment were analyzed in the previous chapter. An appropriate production function was derived and the most economic fertilizer applications were determined for a wide range of input and product price situations. In the present chapter the meaning and usefulness of the derived estimates will be investigated from the standpoints of the farm operator, the economist and the agronomist. Also the results obtained from this method of analysis will be discussed and compared to the results obtained by using a more conventional method as shown by Odland and Allbrighten and discussed in the first chapter of this thesis.

Discussion of Results Obtained

The field experiment from which the present data were secured was designed to determine the economic use of fertilizer for a corn, oats, wheat, alfalfa-brome rotation. The results analyzed are for only one of these crops, corn, and for the first year of the rotation. The weather experienced during the crop year was unfavorable for corn. Although this was not an extremely unusual occurrence, 1955 must be considered a unique corn year. This is probably true of all crop years. For this reason experimental results for one year are never an adequate basis for inferences about future crop years. Optimal fertilizer rates based on these data result in maximum profits for

only 1955 conditions. The problem encountered in securing production functions which have reliability over time will be taken up in the succeeding chapter.

Despite the uniqueness of one year's data, there are statements of significance that can be made from the first year results of the experiment.

For a year such as 1955, it was found that the corn responded significantly only to additions of nitrogen. This may have been due to the unfavorable growing conditions occurring during the late summer.

✓ Interpretation of Results Obtained for Farmers -- Though, in general, results obtained from this analysis cannot be used for future crop years they have important implications for farm operators.

The current fertilizer recommendation for corn made by the Extension Service for this soil type, in this climatic area, is 200 pounds of 3-12-12 fertilizer.¹ This recommendation is for row applications and should be approximately doubled for the broadcast applications used in the field experiment. Therefore, the recommended amounts are 12 pounds of N, 48 pounds of P_2O_5 and 48 pounds of K_2O .

If the recommendations had been followed for the experimental year, the investment made in phosphorus and potassium would have been lost as far as returns from the immediate crop were concerned. With P_2O_5 and K_2O costing ten and eleven cents per pound respectively, the residual P_2O_5 and K_2O left in the soil cost 10.08 dollars per acre.

¹ Porter, et al., op. cit.

The expenditure on nitrogen would have returned a profit on its cost from the immediate crop. Nitrogen earnings, however, would not have been large enough to cover the cost of all three nutrients. Problems of carry-over effects of fertilizer will be discussed further in the next chapter.

The twelve pounds of nitrogen would have brought about a yield increase of approximately 9.5 bushels of corn per acre. At \$1.20 a bushel the increased yield was worth \$11.40. With nitrogen valued at \$.15 per pound the net return, above nitrogen cost disregarding costs of application and of harvesting the added yield, would have been about \$9.60. However, if the optimum application of nitrogen had been used, the return would have been still greater. With corn valued at \$1.20 per bushel and nitrogen at \$.15 per pound the most profitable application was 39 pounds per acre at a cost of \$5.58. The resulting increased yield is estimated at 15.8 bushels with a value of \$18.96. The net return for the applied fertilizer is \$13.11 which is a \$3.51 greater profit per acre from use of the optimal amount over the recommended amount. This difference in profits occurred for what was considered an unfavorable growing season. With a more favorable growing season the difference would probably be even greater. If the recommended amounts of phosphorus and potash had been applied, the results would have been even less favorable for a part of the cost of the phosphorus and potash at least would have to be subtracted from the return obtained from nitrogen.

The importance of applying the optimal amounts of fertilizer can easily be seen from the standpoint of the farm operator. What level of fertilization represents the optimal level for the average year or for other kinds of years is not known at this time. Further experimentation is needed for making such determinations.

It must be kept in mind that the present results are based on broadcast applications and that different production relationship would be expected for other application methods.

This analysis, for the first year, also indicates that it will not be sufficient to know the optimal application for different price situations for the average growing season. It will also be necessary to have a probability distribution of returns secured from different years if the farm operator is to adjust his fertilization program to his particular capital position in view of the risks and uncertainties involved. Even if the Extension Service's recommended fertilization, given earlier, proves optimal for an average year under average price conditions, this recommendation did not result in maximum or near maximum profits for 1955. For this particular year, the recommended amounts of phosphorus and potash were too high and that of nitrogen too low. For another year this situation may change entirely and the amounts of all nutrients may be too low or too high. Farmers need to know something about the probability distributions of these functions over time. A farmer may choose to invest in alternative enterprises in which the certainty of the desired return is greater, or conversely, he may be in a position to "play the odds" for greater gains.

✓ Significance of Results from an Economic Standpoint -- The results obtained from the present analysis have determined the optimal fertilizer rates and ratios for the 1955 crop year. These results constitute one annual observation of the total number to be obtained from which a probability distribution of annual yields and returns can be computed. For the time being, lack of returns to phosphorus and potassium can be considered due to the unfavorable weather experienced. However, if a similar lack of response shows up in a large percentage of the years, it will have very real and important economic consequences in terms of profit maximization.

The least-cost combination of nutrients was computed from the results of the field experiment. This combination is somewhat unique in that it contains only one of the three nutrients studied and remains the same for all yield levels. The stability of this relationship can be determined only after further investigations are made for future years.

The possibility of opportunity costs must also be considered from an economic standpoint before the recommendations can be used by farmers. Under the usual capital situations of most farmers it is possible to make expenditures in alternative enterprises, within the business, and realize varying rates of returns. Expenditures on fertilizer must return a rate which is at least as great as that for any other possible expenditure if profits for the whole farm unit are to be maximized. In Chapter II the optimum condition, without capital restrictions, was stated as the application where the marginal

product multiplied by the product price is equal to the cost of adding another unit of input. In this case, the cost was considered to be the price of a unit of input. When opportunity costs are being considered, the price of the input is its earning power in the highest alternative use.

As an example of the presence of an opportunity cost condition, a situation may be considered whereby a farmer is able to realize a return of \$1.10 on a dollar expenditure on some other enterprise, e.g. for protein supplement fed to beef cattle. The appropriate fertilizer cost then becomes its price per unit multiplied by 1.10, as a dollar would earn \$1.10 if spent for protein supplement. The optimal condition can then be written as:

$$P_Y (MPP_X) = 1.1 (P_X)$$

where X is the fertilizer input and Y the product. The marginal value product given in this expression includes not only the value derived from the increment to the crop, but also the residual value of the applied fertilizer as well. The problem encountered in placing a value on this residual will be discussed in the following chapter.

✓ Significance of Results from an Agronomic Standpoint -- The results obtained from the field experiment of primarily agronomic interest is the lack of response to phosphorus and potassium and the positive response to nitrogen that occurred. The unfavorable crop conditions prevailing during the late stages of the season are assumed to have limited the yield to the extent that phosphorus and potassium did not become limiting enough for responses to occur. The use made

of nitrogen indicated that the soil was unable to furnish adequate amounts of this nutrient for even the low yield attained.

A further agronomic consideration which is also of interest to economists but not reported in this study is the extent that the soil test results are associated with the responses not associated with the independent variables evident in the data.

The results of this study point out the need for more satisfactory soil test measures of plant nutrients, especially nitrogen. Also shown to be needed are improved measures of the various factors which contribute to the unexplained variations in yields. As yet the technical relationships that exist between yields and the various factors of production are relatively poorly understood. There is need for more abstract thinking, to develop more fully these relationships and the implications of them.

✓ Significance of Results from a Statistical Standpoint -- The outstanding statistical characteristic present in the plot data is the large unexplained variances. The problems presented by the large variances were evident in the difficulty experienced in a choice of mathematical function used to express the existing relationships. A further discussion of the problems presented by large variances and methods of overcoming their effects is given in the following chapter.

The coefficients derived for the nitrogen terms, when nitrogen was considered alone, were found to be significantly different from zero at the one percent level, for all of the functions used. The

terms involving the phosphorus and potassium coefficients lacked a dependable level of significance. The problem encountered of deciding whether or not an input variable effects output significantly was discussed in Chapter III. In this analysis the phosphorus and potassium inputs were considered to lack a dependable level of significance in explaining yield differences and were omitted from the analysis.

The discussion in Chapter III has pointed out the statistical need for a measure of confidence limits of the derivative of the derived production function. This confidence measure is needed to determine the reliability of the derived optima for use of both the economist and the farm operator using the results. A further statistical need is for a more objective means of choosing among alternative production functions.

Comparison of Results by Conventional Techniques

It is interesting to compare the analysis in this study with Odland's and Allbrighten's² analysis technique which was discussed in Chapter I. Their method, essentially, compares average yields of plots receiving increasing amounts of the variable nutrients. The optimal level of each nutrient is found by examination of data for the different nitrogen treatments without regard to the levels of the other variable nutrients. There are few precise economic criteria or sets of statements given for defining the optimal application.

² T. E. Odland and H. G. Allbrighten, op. cit.

the first of these is the fact that the *Journal* has been published for over 100 years, and has thus become a part of the history of the country.

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The average values are judged as to their relative profitability primarily by inspection and conjecture.

In applying their procedure to the data used in this study, the yields of corn are grouped in order of increasing amounts of application of nitrogen, phosphorus and potash applied in the fertilizer, together with the number of observations in each group in Table VIII.

TABLE VIII

AVERAGE YIELD OF CORN GROUPED ACCORDING TO INCREASING AMOUNTS OF N, P_2O_5 and K_2O APPLIED, WITH NUMBER OF OBSERVATIONS IN EACH GROUP
KALAMAZOO SANDY LOAM SOIL, SOUTHWESTERN MICHIGAN, 1955

Nutrient	Level lbs/A	number of Observations	Average Yield in Bu/A
N	0	18	26.3
	20	24	40.6
	40	14	43.5
	80	29	43.4
	160	18	42.8
	240	27	40.7
P_2O_5	0	18	32.2
	40	24	38.3
	80	14	43.1
	160	27	40.0
	320	17	41.3
	480	30	43.2
K_2O	0	18	31.4
	20	24	43.3
	40	14	39.9
	80	28	40.5
	160	16	39.5
	240	30	41.8

From the results shown in this tabulation it appears that responses occur to all three of the variable nutrients and, hence, all should be applied for a profitable return. From the evidence available,

it appears, using their methods, that 40 pounds of N, 80 pounds of P_2O_5 and 20 pounds of K_2O per acre seem to result in the most profitable yield.

Although this method of analysis has a great computational advantage over the function estimation technique, the results are in error. The lack of a response to phosphorus and potash is not revealed. Furthermore, the most profitable application is not readily defined and remains obscure.

CHAPTER VI

PROBLEMS INVOLVED

In Chapter IV, data from the field experiment were analyzed and economically optimum applications were derived for varying price conditions. The meaning and usefulness of these results were discussed from various standpoints in the preceding chapter. Various problems occurred when analyzing and drawing economic implications from the data produced from the field experiments. These problems will be considered specifically in the present chapter. Though some of these problems are unique to the type of analysis carried out in this thesis most of them are present regardless of the type of analysis used. However, the refinement in functional analysis brings to light problems often overlooked or ignored in other less precise methods of analysis.

Analysis Over Time

As previously stated, the results of the first year's experimentation, described and analyzed in the preceding sections, cannot be used indiscriminately for making predictions for other years. The derived production function is only a first approximation to a long run, average production relationship for the three variable nutrients and yield of corn under the specified conditions. If the conditions which generated the present data could be expected to hold for future crop years, the application found to be most profitable

for the past year would be expected to be the most profitable for the future crop years. As 1955, like all years, was somewhat unique, further experimental work will be needed before the average production relationships and their behavior over time will be known. When the relationships are studied in a time sequence such factors as weather differences, rotation effects and accumulation and depletion of soil productivities become important considerations.

Weather Variations Over Time -- As fertilizer-yield relationships are studied over a period of years, weather differences will cause variations in the results obtained. Yield variation due to weather factors will differ in intensity with different climatic areas, crops and soils. The long run average production function is not an adequate basis for making annual decisions. It is also desirable to have information on the probabilities of the various possible outcomes. The probability distribution of returns is of importance as an aid to the determination of the degree of risk and uncertainty involved in a fertilizer program. The variance about the average production function should be broken down into between years and within years components. With a quantitative knowledge of these distributions a farmer can make adjustments to counteract their effects--he will be in a better position to "play the odds."

As each year's data are obtained, additional production functions can be estimated. Each such function is an estimate of the true production relationship under the conditions generating the yield data. Subsequent reformulations of the "average" production function

based on successive accumulation of annual data will approach the long run average production relationship. In this way, each year's estimate of the production function will contribute to the final formulation of the relationship and to the probability distributions of annual functions about the long run average functions.

The average functions will exhibit characteristics through their general form, or type, and their estimated parameters that are directly applicable only to the average general conditions present in the experimental situation from which they were derived. The standard error of estimate for an observation for the average function will probably be greater than for an observation on one of the component relationships owing to the greater yield variability occurring over time. However, the deviations of the annual functions from the long run average function may not be large. Only empirical investigation will reveal how the economically optimum applications will vary from year to year.

Problems of Rotation Effects Over Time -- To analyze the role of fertilization in the total farm program, rotation effects must be considered in addition to annual fertilization of single crops. A farmer formulating a fertilization program for his farm should make decisions on the basis of all crops grown in the rotation not for just a single crop.

Also of importance to the farm operator are predictions of the interaction effects between fertilizer rates, ratios and crop sequences. Decisions as to the economically optimal rotation depend,

in part, upon the relationships between yields of various crops in the different rotations and the fertilization program.

Most field experiments designed to study the fertilizer-yield relationship have lacked provisions for studying rotation effects. The emphasis, instead, has been on the derivation of a production function for a single crop. Though the data necessary to study the interaction effects of fertilization and crop sequence have not been accumulated yet, the study being conducted in Michigan, of which the corn data analyzed herein are a part, is designed to obtain some measures of pertinent interaction effects for the particular rotation under study. Application of all of the rates and ratios, indicated by the design presented earlier, is being made to all crops in the rotation. The response of the different crops to the applied nutrients and to the residual nutrients, applied earlier in the rotations will be measured and analyzed subsequently.

If it is found, for instance, that corn responds greatly to nitrogen but not to phosphorus or potash while alfalfa-brome responds to phosphorus and potash but not nitrogen, important and profitable adjustments in fertilization patterns will be called for.

After sufficient annual data have been collected some of these interactions will become known and redesign of the rotation experiment may be called for.

Problems of Fertility Buildup and Depletion Over Time -- Closely akin to the problem of rotational considerations discussed in the previous section, are the problems brought about by fertility build-ups and depletions which occur as fertilization programs are carried on over time.

This is essentially an accounting problem. Values must be placed on beginning and ending inventories of fertilizer nutrients in the soil. Evaluation of such nutrients necessitates two types of measurements. First the nutrients must be measured or tested. As yet soil tests give poor indications of the fertility level present in soil. The results obtained from such tests are mainly used as indexes of the fertility levels, for the immediate present. The second measurement problem is that of measuring the value of the nutrients in a soil in terms of their productivity potentials for different crops. Current market prices of the nutrients may be meaningless for nutrients in the soil. Such nutrients are fixed. Their value in production should be measured, through experimentation, in terms of their ability to produce growth in subsequent crops.

The original fertility levels varied widely from plot to plot over the fields on which the current experiments were run. Steps will be taken to include these measurements in subsequent analyses of these data. The study also provides for soil tests after each crop in the rotation is harvested. Hence, some at least of the basic measurements for studying the problem of fertility accumulation and depletion have been made.

Variance Problems

Large between-plot variances not associated with the independent variable are present in the experimental data. These large variances are characteristic of all fertility research. In these experiments, these large variances are reflected in the low coefficients of multiple

correlation obtained for the function fittings and are evident on inspection of the experimental results given in the Appendix. These variances present problems (1) in selecting the appropriate mathematical function to fit and (2) of accuracy of optima located on the selected functions.¹

The variance present in experimental data are believed to be larger than those experienced on the farms to which the results are applied. If this were not the case, i.e., if variances on the experimental plots were not at least as large as those existing on farms, the experimental results obtained would have very limited usefulness. Thus, the problem is to reduce the variance in the experimental results to more closely conform to that of the farmer's fields enabling the economic optimal conditions to be stated more precisely.

Potential methods of solving this very troublesome problem include (1) averaging out the variance by replication and larger experimental plots, (2) measuring and studying the causes of variance, (3) selection of form free functions and (4) placing more stringent controls in experimental procedures on the causes of variance. At the present time it appears that a combination of these several means will ultimately prove more effective than any one of them used alone.

Averaging Out Variance -- The large variance can be averaged out in two ways: by replication and by increasing the size of the

¹ Glenn L. Johnson, "A Critical Evaluation of Fertilization Research," paper read before Western Farm Management Research Committee Meeting, Corvallis, Oregon, March, 1956.

individual plots. Basically these two methods are similar. When the causes of variance are randomly distributed throughout the experimental area the use of larger plots will be successful. To the extent the causes of variance are not randomly distributed but are correlated between adjacent small plots replication of plots becomes relatively more effective than larger plots in reducing variance. As the correlations between adjacent plots are not perfect and as there are some economies to plot size, a compromise will probably be found to be the most satisfactory means of reducing between plot variance in yield data. The success of using both larger plots and more replications depends in part on the amount of additional variance introduced in going to larger experimental fields to accommodate the larger plots or added replications. In view of their importance in economic analysis, it appears unwise to cut the number of treatments substantially below the number in the present experiments in order to avoid larger fields.

If the size of the harvested area were increased some of the added variance in larger fields would be averaged out within each plot and the standard error of estimate might be reduced. Farmers operating with large fields, it appears, must experience a much lower standard deviation of expected yield than found with small experimental plots. It may be possible to overcome the difficulties, presented by large between-plot variances, by the use of much larger plots in experimental work. By encompassing within the plots more causes of variance and averaging out their influences easier selection of appropriate functions may result. At the same time, the experimental

1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes that proper record-keeping is essential for transparency and accountability, particularly in financial matters. The text suggests that organizations should implement robust systems to track every detail, from procurement to sales, to ensure that all data is reliable and accessible.

2. In the second section, the focus shifts to the role of technology in modern business operations. It highlights how digital tools and software can streamline processes, reduce errors, and improve overall efficiency. The author argues that embracing technology is not just a luxury but a necessity for staying competitive in today's fast-paced market. Examples of various software solutions and their benefits are provided to illustrate this point.

3. The third part of the document addresses the challenges of managing a diverse workforce. It discusses the importance of effective communication and collaboration across different departments and cultures. The text offers practical advice on how to foster a positive work environment, encourage innovation, and resolve conflicts. It also touches upon the need for continuous training and development to keep the workforce up-to-date with the latest industry trends.

4. The fourth section explores the impact of external factors on business performance. It examines how economic conditions, market fluctuations, and regulatory changes can affect an organization's bottom line. The author provides strategies for risk management and contingency planning to help businesses navigate these uncertainties. It stresses the importance of staying informed about the external environment and being prepared to adapt quickly to any changes.

5. Finally, the document concludes with a call to action, urging organizations to adopt a proactive and forward-thinking approach to their operations. It encourages the implementation of best practices and the pursuit of excellence in all aspects of the business. The author believes that by following these guidelines, organizations can achieve long-term success and sustainable growth.

conditions could, with care, be made more like those on typical farms in the sense that more of the variability common to all of the farms to which results are applied would be present in the experimental plots and in the sense that the causes of variance would have about the same average for both farm and experimental conditions. Empirical evidence is needed to investigate the relative value of these procedures.

Measuring and Studying Causes of Variance -- This means of handling experimental variance is a very useful one provided such measures can be devised and incorporated in the analysis thereby reducing unexplained variations. This method requires improved means of measuring the factors of production which introduce the unexplained variance in yield results. Also called for is a more definite understanding of the technical relationships that exist between yield and the various factors of production.

Control of the Causes of Variance -- This method of reducing variance has been emphasized by all scientists including agronomists. It consists of placing experimental controls on the causes of variance so as to isolate the influence of the independent variables being studied. In carrying out this method, serious questions of applicability can result because the experimental situation may be made so unique by the controls that it will not correspond to farm situations.

Fitting Form-Free Functions -- This method consists generally of estimating discrete points of the response function with little regard to the shape of the surface except that the function is

The first of these is the fact that the
theoretical model of the system is
based on a number of assumptions which
are not always valid in practice.
For example, the model assumes that
the system is in a steady state, and
that the input and output signals are
linear and time-invariant. In practice,
the system may be non-linear and
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generally concave. This method seems to avoid the problem of selecting appropriate functions in the presence of large variances rather than of reducing those variances.

Representativeness and Applicability of Derived Functions

A production function derived from a field experiment is representative of the experimental field from which the results were obtained. Any experimental field, regardless of how well chosen, has associated with it certain fixed factors more or less unique to it. Because of this, the problem arises of applying the results obtained from unique experimental conditions to farmers' fields.

The problem is further intensified by choosing experimental fields in such a way as to minimize within field variability. This minimization is desirable if the factors whose variability is minimized average near the average for a significant number of farms. But this is seldom the case. For example, soil type which is so rigorously controlled in the experimental area is not and cannot be controlled on all farms using the results of the experiment or even on a single farm field. If larger experimental fields and/or more than one field were used, more of the variability known or assumed to exist under a given soil type, or types, to which the results are to apply would be encompassed within the experiment. This would tend to insure that the average level of the uncontrolled factors would correspond in experimental and farm fields. The experiments

² Hildreth, op. cit.

would then "average out" a range of differences that exist in different soils and different fields in such a way as to be more comparable to conditions existing on the farm fields permitting the derived production functions to have a wider range of application.

Extending the experimentation in this manner, while increasing the usefulness of results, will also increase the amounts of variance not associated with the independent variables. Therefore, it may be difficult or impossible to obtain accurate or reliable estimates of the production function even with increased numbers of observations. Larger plots and/or replications, or the methods previously discussed may overcome this problem. The practicability of these considerations is in need of empirical investigation.

In some ways increased usefulness has been accomplished by defining the applicability of the derived function, thereby allowing the proper choice of production equation to be made by the farmer. This is done by defining the crop grown, soil types, geographic area to which the particular equation is relevant, and the general cultural practices adhered to. If these conditions are beyond the limits of adjustment for the farmer, he must (1) use another function better suited to his conditions, or (2) adjust his conditions to fit the given function. But there are a great number of factors over which the farmer has no control and which are fixed at differing levels than are present in any of the experimental fields. In this case the farmer cannot adjust his conditions to come under a particular function or choose another function altogether, for his conditions

are different than those used to derive the various defined experimental functions. Thus by over-specification of experimental conditions the derived functions are applicable to only a few farm conditions and the farms not meeting these conditions are unable to use the results.

Other means of making the results obtained from unique experiments applicable to farm conditions are (1) by synthesizing the farm conditions from results of different experimental conditions and (2) by interpolation between the results of two or more experimentally defined sets of conditions. The use of either of these methods involves a larger degree of subjectivity than other methods but may for the time being at least be the most practical.

The problem of applicability of results is present regardless of the analysis of results used and hence is not unique to a regression type, designed to study the economic implications of the experiment.

Problems of Selection of Specific Functions

The selection of a specific mathematical function to describe the fertilizer-yield relationship is often difficult, especially when large unexplained variances are present in the data. The discussion of this problem has been taken up in Chapter II, and procedures for making a choice of function are demonstrated in Chapter III.

CHAPTER VII

SUMMARY AND CONCLUSIONS

Current fertilizer recommendations generally reflect inadequate attention to economic considerations. Although much research has been carried on in the past to promote efficient use of fertilizer, this neglect of economic considerations still remains. Some past efforts have been directed towards methods and rates of application which maximize yields, an aim which is often inconsistent with profit maximization. Profits are increased only so long as the cost of adding fertilizer inputs is less than the added return.

Inadequacies of past empirical work on the determination of optimum amounts of fertilizers include: (1) studying only one nutrient or ratio of nutrients at a time, (2) using rates of application too low to locate the point of maximum profit which may range up to the point beyond which total product does not increase as more fertilizer is applied, and (3) the emphasis on studies for investigating responses to discrete treatments and relating these treatments to soil characteristics, rather than on continuous response surfaces.

As the data needed to determine more accurately the optimum rates and combinations of fertilizer nutrients to use have generally been lacking, a project was sponsored jointly between the Department of Agricultural Economics and Soil Science of the Michigan State Experiment Station. Members of both departments took active part in nearly all

phases of the project, including the design of the field experiments. The need for considering fertilization of a crop in view of the interactions within rotations and changing fertility levels is known but economic study of these considerations has been often overlooked. The data produced by these experiments permit more adequate analysis of fertilization rates, ratios, and ultimately, of crop sequences and fertility residuals. The variable nutrients studied are N, P_2O_5 and K_2O with all other factors of production either fixed or allowed to vary randomly. The crops involved are corn, oats, wheat and an alfalfa-brome mixture. Only the corn data are analyzed in this thesis.

The analysis of these data is based on the concept of a continuous mathematical production function. According to this concept, yield responses to different fertilizer nutrients may be described by a continuous mathematical function which treats yield as a dependent variable and the different fertility nutrients as independent variables. After such a mathematical function is derived, various economic maxima can be found by well-known mathematical operations. The law of diminishing returns, which is empirical in origin, insures the necessary second order conditions for the existence of the economically optimum points. The optimum applications occur where the value of a decreasing marginal product (first partial derivative of yield with respect to an input) is equal to the cost (price, under perfect competition) of adding another unit of input. A principle advantage of using continuous mathematical functions is that estimates can be made for all conceivable levels of input within the experimental ranges rather than for only a few discrete points.

The problem of choosing a particular mathematical function to fit involves some objective tests or criteria but rests importantly on the experience and judgment of the researcher. This weakness in functional analysis points up the need for theoretical statistical work. Other mathematical work is needed on appropriate functions to describe the experimentally derived production relationships. Also, statistical work is needed on the problem of placing confidence limits on (1) the derivatives of these functions and (2) the economic optima determined with derivatives.

Four three-variable functions were fitted to the experimental data. After applying various statistical tests and less objective criteria, it was decided that the best fit was obtained by using the Carter-Halter equation of the form:

$$Y = a N^{b_1} c_1^N P^{b_2} c_2^P K^{b_3} c_3^K$$

where Y is yield and N , P_2O_6 and K_2O are represented respectively by N , P and K . Coefficients of the N terms were found to be significantly different from zero at the one percent level. Coefficients for the P and K terms, however, lacked a dependable level of significance. Primarily on this basis, but also because of inconsistencies in the signs of these coefficients, it was decided that the additions of phosphorus and potash had no significant effect on yield. The least cost combination of nutrients was thus found to contain only nitrogen, with no applications of phosphorus and potash called for.

The equation was refitted using only nitrogen as an independent variable. The coefficients of this equation were all found to be

significant at the one percent level and the relationships expressed in the function were also evident in the plottings of the observed yields. The derived coefficients for the one-variable equation were very similar to the nitrogen coefficients of the three-variable equations.

The solution for the optimal quantity of nitrogen to apply was shown to be dependent upon the prices of both nitrogen and corn. For the profit maximizing application, the partial derivative of yield with respect to nitrogen is equal to the price ratio of the nitrogen and corn. Solutions for such optima, presented for a wide range of price variations, were found to vary from a low of 12 pounds to a high of 76 pounds per acre.

An empirical production relation on which to base fertilizer recommendations should be based on experimental data for a period of years. Such data would permit an average production function to be derived which would "average out" between year variations. Such data would also make it possible to estimate the probability of deviations from expected returns as well as expected deviations from the recommended amounts of fertilizer to use. This information would permit farmers to adjust their fertilization program to their particular capital positions in view of the risk and uncertainty involved.

For 1955 it was found that the recommended fertilization applications were far from the applications which would maximize profits if the values of residual fertility left in the soil as a result of the fertilizer applications were not considered. Even if these

the first of these is the fact that the system is not a simple one, but a complex one, in which the various parts are interrelated and interdependent. The second is that the system is not a static one, but a dynamic one, in which the parts are constantly changing and evolving. The third is that the system is not a closed one, but an open one, in which the parts are constantly interacting with the environment. The fourth is that the system is not a linear one, but a non-linear one, in which the parts are constantly interacting with each other in a non-linear fashion. The fifth is that the system is not a deterministic one, but a probabilistic one, in which the parts are constantly interacting with each other in a probabilistic fashion. The sixth is that the system is not a simple one, but a complex one, in which the parts are interrelated and interdependent. The seventh is that the system is not a static one, but a dynamic one, in which the parts are constantly changing and evolving. The eighth is that the system is not a closed one, but an open one, in which the parts are constantly interacting with the environment. The ninth is that the system is not a linear one, but a non-linear one, in which the parts are constantly interacting with each other in a non-linear fashion. The tenth is that the system is not a deterministic one, but a probabilistic one, in which the parts are constantly interacting with each other in a probabilistic fashion.

residual values were considered, the recommended rates would not be optimum.

Large variances in yields not associated with the independent variables were experienced. Statistical and experimental procedures are needed to overcome the effects of large variances. Among the procedures holding some promise are (1) use of more replications or of larger experimental plots to average out the causes of variance, (2) measuring and studying the causes of variance and incorporating this information into the analysis, (3) selection of form free functions as a means of avoiding the difficulties brought on by large variances, and (4) placing more stringent controls in experimental procedures on the causes of variance. It appears that a combination of these several means will ultimately prove to be more effective than any of them used alone. Soil tests and their place in the study of unexplained variance are very important aspects of the problem. Also needed, in this connection, is a better understanding of the technical relationships that exist between various factors of production.

Another problem in research underway in fertility studies is representativeness of experimental fields in terms of applicability of the results obtained. This problem is closely related to the variance problem. Each experimental field is more or less unique to itself and each farm to which the results are applied has various characteristics that differ from those of the experimental field. It appears that if larger experimental fields and/or more than one field were

used, more of the variability known or assumed to exist within the farm fields, to which the results are to apply, would be encompassed within the experiment. The experiments would then "average out" a range of differences more comparable to the range existing on the farm fields permitting the derived production function to have a wider range of application. The practicability of these considerations, in view of the increased variances which would probably be encountered, is in need of empirical investigation.

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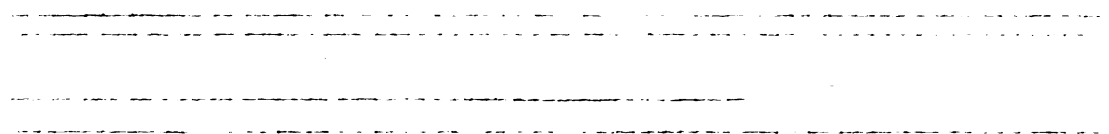
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APPENDIX

EXPERIMENTAL YIELDS OF CORN FOR VARYING LEVELS OF APPLICATION
OF NITROGEN, PHOSPHORUS AND POTASSIUM ON
KALAMAZOO SANDY LOAM SOIL IN 1955
(YIELDS ARE IN BUSHELS PER ACRE)

Pounds of N Per A	Pounds of K ₂ O Per A	Pounds of P ₂ O ₅ Per A					
		0	40	80	160	320	480
0	0	*			17.5		
	20		24.8				
	40						
	80				15.8		24.5
	160	22.8					
	240				31.5		35.0
20	0		35.0				
	20	52.5	36.8 40.3		40.3 47.3		35.0 49.0
	40			45.5		31.5	
	80		35.0 38.5		38.5 40.3		49.0 45.5
	160			26.2		31.5	
	240		33.3 35.0		38.5 52.5		47.3 49.0
40	0						
	20			42.0		56.0	
	40		38.5	40.3 43.8	40.3		45.5
	80			47.3		43.8	
	160		40.3		36.8		42.0
	240			50.8		42.0	

* The eleven check plot yields were: 24.5, 26.3, 35.0, 26.3, 22.8, 33.3, 24.5, 19.3, 24.5, 19.3, 45.5.



Pounds of N Per A	Pounds of K_2O Per A	Pounds of P_2O_5 Per A					
		0	40	80	160	320	480
80	0	36.8			52.5		43.8
	20		42.0 40.3		50.8 47.3		42.0 64.8
	40			52.5		33.3	
	80	43.8	50.8 33.3		33.3 38.5		45.5 35.0
	160			38.5		29.8	52.5
	240	49.0	43.8 49.0		38.5 40.3	42.0	43.8 43.8
160	0						
	20			49.0		35.0	
	40		38.5	35.0	43.8		36.8
	80			52.5		50.8	45.5
	160		29.8		35.0	49.0 56.0	45.5
	240			38.5	47.3	43.8	38.5
240	0					33.3	43.8
	20		36.8 36.8		36.8 43.8		43.8 45.5
	40					33.3	
	80	42.0	40.3 36.8		33.3 54.3		50.8 47.3
	160			42.0		38.5	38.5
	240	31.5	45.5 38.5		45.5 40.3	52.5	40.3 28.0

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