

**PARTICIPATION AND COGNITIVE DEMAND: LINKING THE ENACTED  
CURRICULUM AND STUDENT LEARNING IN MIDDLE SCHOOL ALGEBRA**

By

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## ABSTRACT

### **PARTICIPATION AND COGNITIVE DEMAND: LINKING THE ENACTED CURRICULUM AND STUDENT LEARNING IN MIDDLE SCHOOL ALGEBRA**

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Many current policy initiatives focus on teacher qualifications and high-stakes assessments for students as a means to improve mathematics education in the United States, but this approach ignores the actual practice of teaching through which students have opportunities to learn mathematics. The present study is an effort to answer scholars' calls to focus on the enacted curriculum—what actually happens in the classroom—and the relationship between enacted curriculum and student learning in mathematics. In particular, within 12 middle school algebra classrooms from two states, each lesson focused on the introduction of variables as a means of representing changing quantities was analyzed for the levels of cognitive demand and the nature of student participation present throughout the phases of mathematical task enactments. Students' verbal participation is conceptualized through a new construct termed *participatory demand*, which attends to both the amount of student talk as well as the semantic nature of that talk. The results of the analyses of the enacted curriculum were then compared with class gain scores between a pre- and post-test related to the focal learning objective. Although cognitive demand in the enacted curriculum was not found to be significantly correlated with class gain scores, the nature of student participation was predictive of higher class gain scores.

Additionally, two narrative cases of teacher's task enactments are presented to illuminate issues of cognitive demand and participatory demand at a finer level of detail than the correlational analysis. Ms. Wyncott's task enactments constitute a case of declining cognitive demand and participatory demand that is not of a high mathematical nature, even though Ms.

Wyncott explicitly directed students to talk with one another in small groups about their mathematical work. Ms. Albert's task enactments constitute a case of cognitive demand that is maintained at a high level and participatory demand that is of a high mathematical nature both as students work and as they look-back on their work together as a whole class. These cases include particular attention to the look-back phase of enactment because this phase is a new addition to the Mathematical Tasks Framework (Stein, Grover, & Henningsen, 1996) that was used to structure analysis.

The final chapter of this thesis summarizes the ways in which the study contributes to an understanding of the relationship between cognitive demand and participatory demand within the enacted mathematics curriculum and to the body of research on the relationship between enacted curriculum and student learning. Implications for teacher education are discussed and potential avenues of future research are identified.

Dedicated to my wife, Chelsea, for her love and support.

## ACKNOWLEDGMENTS

I view this study as a culmination of my preparation as a secondary mathematics teacher, my endeavors in the realm of pure mathematics, and my experiences in the graduate program in mathematics education. I therefore extend thanks to the people who have supported me in all of those phases of life, especially Dave Coffey, John Golden, Rebecca Walker, and Dave Futer. A heartfelt thanks also goes to Filiz Doğru who was the first scholar to treat me as a colleague, even before I deserved it, and whose encouragement, love, and support has been invaluable. Furthermore, I am appreciative of my graduate school instructors, including Joan Ferrini-Mundy, Sharon Senk, Jack Smith, and Mike Shaughnessy, for introducing me to important ideas in the field and for pushing and probing my own thinking.

Of course, I owe the most for the work on this study to my dissertation committee—Beth Herbel-Eisenmann, Ralph Putnam, Kristen Bieda, and Mike Steele. The contributions this study has made to my own learning, and the potential contributions that it may make to the field, are largely due to them, though the weaknesses and flaws that remain are completely my own. I also thank Lorraine Males, Aaron Mosier, Aaron Brakoniecki, and Aladar Horvath for their input during the conceptualization of the study, and Kim Cervello Rogers for her help in its execution. I would be remiss, however, if I did not allude to the many ways in which Beth has supported me above and beyond her role as a dissertation advisor. I count myself lucky to have had the opportunity to work closely with her for many years and hope that the collaboration will continue into the future. She has helped me grow as a writer, a scholar, a colleague, and a person. I can only strive to impact my own future graduate students in the profound ways that she has impacted me.

I extend a special thank you to the teachers, students, and staff involved with *Project 2061*, without whom this study would not have been possible.

Finally, I express deep appreciation to my family. To my parents, Mary Anne and John, who have continually promoted a value in education, both in the classroom and in the larger world. To my brothers, Luke, Isiah, and Andrew, who encouraged me to pursue graduate studies while also constantly being ready with necessary diversions from the daily grind. And most of all, to my wife, Chelsea, for being a loving partner and passionate educator, and to our son, John Bryan, for providing the motivation to actually finish this dissertation (and for his patience in waiting until after my oral defense to make his appearance). Thank you.

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## LIST OF ABBREVIATIONS AND SYMBOLS FOR TRANSCRIPTION

AY:	Academic Year
ELL:	English Language Learners
LEP:	Limited English Proficiency
MTF:	Mathematical Tasks Framework
NCTM:	National Council of Teachers of Mathematics
RQ:	Research Question
TIMSS:	Trends in International Mathematics and Science Study
$\rho$ :	Spearman's rank correlation coefficient
[word]:	Replaced text such as a referent for its reference
[[ <i>action</i> ]]:	Description of classroom events within a transcript
...:	Omitted text or speech
__:	Verbal fill-in-the-blank
--,:	Self-interruptive speech
--.:	Speech interrupted by another speaker

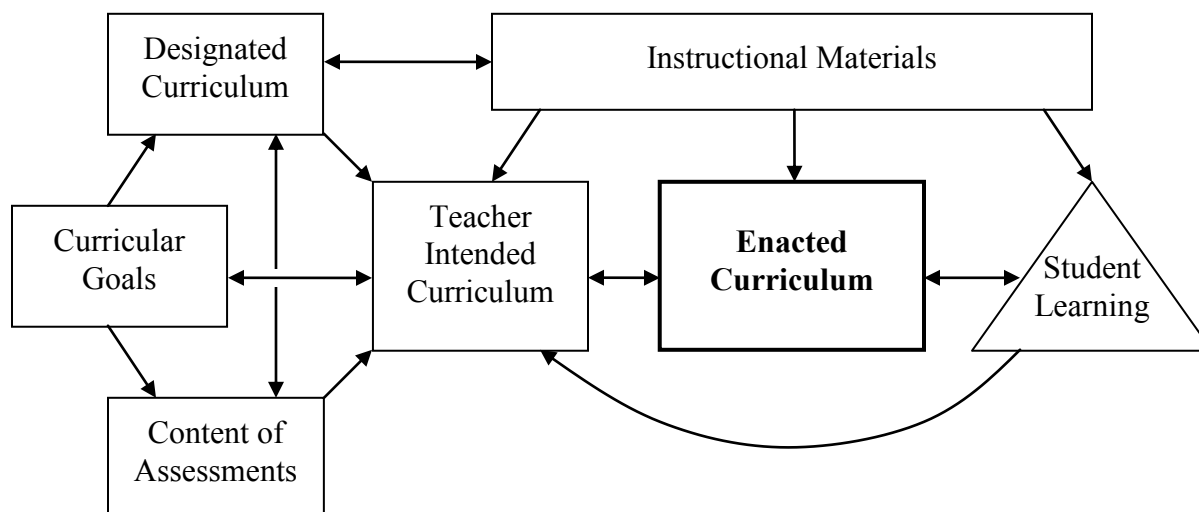
# CHAPTER 1

## INTRODUCTION

For decades the mathematics education system in the United States has been perceived as failing to produce students who are robust problem solvers or conceptual thinkers, potentially placing the United States at a disadvantage in the global marketplace. Several national reports have drawn attention to these issues—for example, *A Nation at Risk* (1983), *Everybody Counts* (1989), and *Foundations for Success* (National Mathematics Advisory Panel, 2008)—and various empirical studies have raised concerns about the quality of mathematics education in the United States as compared to other nations (e.g., Ma, 1999; Stigler & Hiebert, 1999). The resulting policy initiatives have focused primarily on two aspects of the mathematics education system: teachers and high-stakes assessment of students. Prior to this, the National Science Foundation funded the development of innovative curriculum materials in an effort to improve the quality of school mathematics in the United States. This focus on the personal characteristics of teachers and students and on curriculum materials, however, does not give explicit attention to the instructional practices and interactions that actually take place within mathematics classrooms, where the triad of teachers, students, and curriculum materials interact (Lampert, 2001). Moreover, the classroom is the site where students come into direct contact with other facets of the educational system and so the activities of the classroom have strong implications for student learning (Stein, Remillard, & Smith, 2007).

In contrast to governmental policy initiatives, recommendations originating from the mathematics education community itself have tended to focus on teaching practices. Instead of attending to teacher certification requirements or the frequency of standardized assessments, for example, the National Council of Teachers of Mathematics ([NCTM], 1989, 1991, 2000)

articulated a vision of what the activities within school mathematics classrooms might look like. Morris and Hiebert (2011) have recently renewed the call for reform efforts in mathematics education to be directed at improving *teaching* rather than *teachers*. Teaching is the process through which other factors such as the teacher’s knowledge and background, district resources, written curriculum materials, technological tools, and subject matter come together to shape what students actually learn (see Figure 1). This suggests that the enacted curriculum, defined as “what actually takes place in the classroom” (Remillard, 2005, p. 213), is a key leverage point for affecting change in school mathematics (Stein et al., 2007).



*Figure 1.* A depiction of how the various components of the educational system influence one another and ultimately culminate in student learning (adapted from Enacted Curriculum Framework Committee, 2010).

This is not to say that other components of the educational system are unimportant. Elements of the official curriculum, located on the left side of Figure 1, such as the *Common Core State Standards for Mathematics* (National Governors Association & Council of Chief State School Officers, 2010) or the content of high-stakes assessments certainly influence students’ mathematical experiences as they shape what is taught and how. The written

curriculum, represented across the top of Figure 1 as instructional materials such as textbooks, are also known to shape what it is that teachers do in the classroom (Chávez-López, 2003), both directly (through the physical use of materials during lessons) and indirectly (through the teacher's intended curriculum or lesson plan). All of these factors, however, are mediated through the enacted curriculum—they influence student learning primarily by influencing what goes on in the classroom, that is, the enacted curriculum.

This study examines the enacted curriculum in the context of middle school mathematics as well as the relationship between the enacted curriculum and student learning. With respect to Figure 1, then, this study concerns the enacted curriculum box as well as the left-to-right portion of the arrow between the enacted curriculum and student learning. The middle school setting is especially appropriate for this work because the enacted curriculum shapes not only the content that students learn but also their conceptions of what it means to do and learn mathematics. Middle school is an important transition phase wherein students often experience changes in motivation toward mathematics and may position themselves with respect to mathematics in ways that have positive or negative effects on students' future mathematical experiences and opportunities (Anderman & Midgley, 1997; Eccles et al., 1993). Although this study does not address such issues directly, they serve as an important backdrop for the aspects of enacted curriculum and students' learning that are examined.

### **Bounding the Study of Enacted Curriculum**

The enacted curriculum in school mathematics is complex and nuanced because it is not only a space where human beings interact, which is inherently multifaceted and multilayered, but a space that involves teachers and students interacting with each other and with mathematical content. To make sense of such complex phenomena, it is helpful to (a) use a framework that

parses the enacted curriculum in meaningful ways and (b) identify key areas of focus because of the untenable nature of attending to all aspects of the enacted curriculum. In this section, I describe how the present study was bounded in both of these ways.

In many ways, mathematical tasks serve as the key building blocks of the classroom lessons that comprise the enacted curriculum, with approximately 80% of lesson time in grade 8 mathematics classrooms spent around mathematical tasks (Hiebert et al., 2003). By mathematical tasks, I refer to problems or sets of related problems designed to focus on a particular mathematical concept, idea, or skill. This may be a single question that requires multiple steps to be taken to find an answer or a set of smaller items that are linked around a common problem context and mathematical topic. Stein and colleagues (Stein, Grover, & Henningsen, 1996), building on past curriculum research (e.g., Doyle & Carter, 1984), developed the Mathematical Tasks Framework (MTF) that has subsequently been used widely in both research (e.g., Boston & Smith, 2009; Charalambous, 2010; Henningsen & Stein, 1997; Stylianides & Stylianides, 2008) and teacher education (e.g., Arbaugh & Brown, 2005; Ely & Strowbridge Cohen, 2010; Stein & Smith, 1998; Stein, Smith, Henningsen, & Silver, 2000, 2009; Sullivan, Clarke, & Clarke, 2009). The MTF represents the progression of a mathematical task when used in practice, ultimately leading to student learning from the task (see Figure 2). The first phase of the MTF is the task as *written*, which shapes (though does not determine) the task enactment. This relationship corresponds to the fact that, in Figure 1, instructional materials and a teacher's intended curriculum influence the enacted curriculum. Then, within the enacted curriculum, the MTF identifies two phases—the set-up of the task and the implementation of the task. The *set-up* phase usually includes, but is not limited to, the teacher describing or clarifying the written task and communicating to the students what they are expected to do. The set-up phase is influenced

by such things as the teacher’s goals for the task, the teacher’s knowledge of the content, and the teacher’s knowledge of the students. In the *implementation* phase, students actually work to solve or complete the task. Influences on task implementation include classroom norms, task conditions, and student learning dispositions. In this study, I use a modified version<sup>1</sup> of the MTF as a structure for viewing the enacted curriculum allowing me to attend to various foci across time and organize them in meaningful ways. The MTF also bounds the scope of the study by restricting analysis to mathematical tasks at the exclusion of other classroom activities such as homework review, non-task-based explication of topics (e.g., lecture), or classroom business.

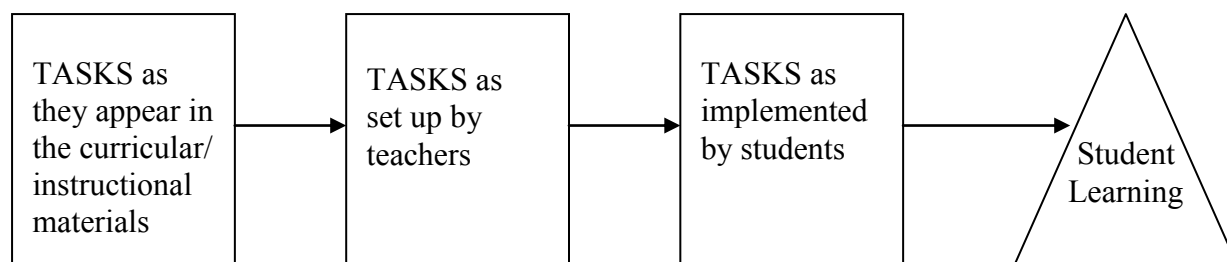


Figure 2. The Mathematical Tasks Framework (Stein & Smith, 1998).

The particular foci of this study are twofold—the cognitive demands placed on students within the enacted curriculum and the forms of student participation during mathematical task enactments. This pair of foci stems from the fact that learning in school mathematics involves both individual and collective processes. Students are held individually accountable for learning material and developing mathematical skills. Educational researchers and teachers have long recognized the importance of students’ individual thinking and understanding. Yet, at the same time, the students of a mathematics class are not learning in isolation from one another. They

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<sup>1</sup> The specific modification to the MTF is described later in this chapter.

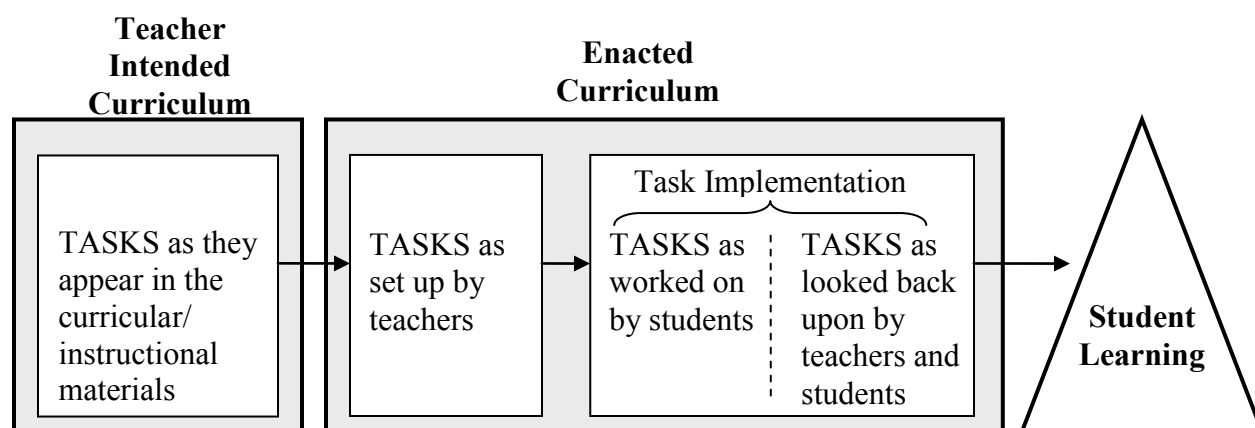


constitute a learning community, developing shared understandings and interacting with one another's as well as the teacher's thinking. It is important to bear in mind that the enacted curriculum does not act upon students; rather, students and their contributions during class are an active and integral part of the enacted curriculum itself.

The coexistence of individual and collective processes in the enacted curriculum warrants attention to both, which, in this study, is attained through the notions of cognitive demand and student participation, respectively. From a theoretical perspective, this also entails a balance between differing views on learning. Cognitive demand draws on the notion that students' thinking and mental actions have implications for their learning whereas my view of student participation stems from a sociocultural perspective that learning is not the accumulation of facts or knowledge in an individual's mind but a collaborative process by which a learner comes to participate in particular discourse communities (Lave & Wenger, 1991; Wenger, 1998). On the surface, these perspectives may seem to compete, but, as I describe in Chapter 2, they can also be seen as complementary.

To summarize, this study of the enacted curriculum and its relation to student learning in middle-school mathematics uses the MTF as a structure for parsing the enacted curriculum, allowing for a meaningful focus on cognitive demand and student participation during mathematical task enactments. Because of the attention to student participation, however, my past work (Otten, 2010a) suggests that a slight modification to the MTF is appropriate. In particular, I found that the form of student participation tended to not only shift between the set-up and implementation phases (e.g., from teacher monologue to student small-group work) but also within implementation, from silent student work or small-group work back to teacher-led whole-class interactions. This shift can also be interpreted with respect to the life cycle of a

mathematical task as represented in the original MTF, that is, the shift within the implementation phase typically involves students ceasing their active work to solve the task and beginning to look back on that work. The process of looking back can serve many different functions, such as sharing answers, summarizing, generalizing, formalizing, or connecting ideas stemming from the task (Otten, 2010b). In addition to the shifting participation format, these activities may be more or less cognitively demanding than the original work students were doing. Thus, for this study I employ the modified version of the MTF in which the implementation phase is sub-divided into a *working* phase and a *look-back* phase, allowing for a more refined analysis of the cognitive demand and student participation of mathematical task enactments. Figure 3, which is similar to the framework used by Jackson and colleagues (2011, April), depicts this modified MTF in relation to some of the components from the curriculum framework of Figure 1.



*Figure 3.* The Mathematical Tasks Framework situated within the context of various forms of curriculum and with sub-phases of the implementation phase identified.

### **A Final Note of Introduction**

In the preceding paragraphs, I attempted to set the intellectual scene for this work as a study of the enacted curriculum, particularly cognitive demand and student participation, and its relation to student learning in the context of middle-school mathematics. The basis for this study,

however, is not solely intellectual—there is also a personal component. During the student teaching year of my secondary teaching certification program I had the opportunity to teach algebra and geometry courses at the middle-school level. I strove to create a classroom community of active mathematical thinkers and believe that I had a great deal of success in achieving this goal. I problematized content from the exposition of the course textbooks so that students could think through new pieces of mathematical knowledge before it was presented to them as fact. I tried to set up rich tasks that had multiple points of entry so that all the students could feel as though the mathematical ideas were approachable. I established a culture of classroom interaction that was intended to mirror mathematical practices—we began investigations by clarifying the situation and the assumptions, we collectively explored and generated conjectures that were collected on the public “conjecture wall,” we expected mathematical justifications from one another and used the process of justifying to disprove, refine, or prove the conjectures and then add them to the public record, often taking a proven claim as the inspiration for the next round of exploration. The students also knew that when I walked over to my desk and rolled my chair to the center of the classroom it was time to stop and have a discussion about what it was that we were doing as mathematical thinkers.

There were several pieces of evidence that this approach was producing positive results and the sources of this evidence varied from myself to those in authority over me (e.g., my mentor teacher, my field instructor) to the actual students. From my own perspective, I loved doing mathematics and I also loved coming in to school every day to work with these students because, although the content was different, the processes that we were engaging in together were the same processes that characterize the disciplinary practices of mathematicians. I took this to be a sign that things were going well. From the perspectives of my mentor teacher, my

principal, and my field instructors, I received compliments about the attitudes and mathematical performance of the students. The most powerful feedback however, was the students saying that they enjoyed coming to math class, not because it was with their friends or because I was a lenient teacher but actually because they were enjoying the work that we were doing. There were students who would work on unassigned problems at home and come in wanting to add a conjecture to the wall. There were even multiple students who at the beginning of the school year did not know that mathematicians existed, but by the winter break were citing mathematician as their career goal.

Following this experience I entered graduate school in mathematics education and I would often connect research I was reading to my activities with those students. I began formulating questions as well as hypotheses as I reflected back upon that time. Certainly the students deserved a great deal of credit. I felt as though I was lucky to be working with such wonderful students who had such rich mathematical ideas. But I also realized that it could not entirely be explained by the students because the other mathematics teachers in the same middle school did not have nearly the positive results that I was achieving.<sup>2</sup> I also found, in reading work such as the Cognitively Guided Instruction research (e.g., Carpenter, Hiebert, & Moser, 1981) and other studies, that many if not all students are capable of the types of insightful mathematical thinking that I was seeing in my students but, unfortunately, they do not necessarily have the opportunity to do so. This led me to reflect more on the practices that I

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<sup>2</sup> A hypothesis I have regarding the more teacher-centered practices of the other mathematics teachers is that it is related to the fact that they all majored in a subject other than mathematics, but exploring this hypothesis will have to wait until another study.

utilized. Gradually, my thoughts centered on two dimensions of practice—cognitive demand and student participation. With respect to cognitive demand, I felt that I actually increased the cognitive demand of the tasks in the written curriculum materials by enacting them within a classroom of mathematical processes and practices. Such increases between written and enacted tasks, however, are known to be rare (Henningsen & Stein, 1997; National Center for Education Statistics, 2003)—usually one needs to start with a high-level task and work to maintain it. Moreover, I felt that it was through the promotion of active student participation that I was able to maintain the high levels of cognitive demand. I could not have kept the high level on my own. Instead, I depended on students to share mathematical ideas, supply justifications, engage in intellectual debates, and come to consensus around a new piece of mathematical knowledge, with me acting as a guide. This participatory demand, as I call it in the next chapter, was not only a means of maintaining high levels of cognitive demand but was also an explicit end in and of itself. I wanted my students to see themselves as mathematical beings and as active participants in their own learning. As I have come to more fully enter the community of mathematics educators, I find that I am certainly not alone in these beliefs about the power of student participation in mathematics classroom, both as a means and an end. The present study, then, can be viewed as an endeavor to examine these beliefs empirically, with the potential to confirm or disconfirm some of my hypotheses about those experiences.

A final point to be taken from this personal story is the significance of it taking place in a middle-school setting. I felt as though middle-school students were at an important turning point, advanced enough in their thinking to engage with some sophisticated mathematical topics but not yet to a point where they had completely written off mathematics as a subject that is not for them. Unfortunately, many high-school students already find themselves in a place where their

mathematical trajectory and their attitudes toward the subject are permanently fixed, and not necessarily in a productive direction. If students' mathematical participation in the enacted curriculum at the middle-school level has the potential to shape students' learning and disposition in productive ways, then it may also ripple through their own personal mathematical storylines for years to come.

### **Overview of Forthcoming Chapters**

In the next chapter, I present background literature regarding the link between the enacted curriculum and student learning as well as research on aspects of the enacted curriculum that relate to directly the present study. Chapter 2 also contains an explication of the theoretical perspectives that serve as a foundation for this work. In particular, Chapter 2 presents the case that the thinking that students engage in and the ways in which students participate are not only integral to the enacted curriculum but influence what students learn. Chapter 3 begins with the specific research questions that guided this study and then proceeds with descriptions of the classroom data, the measures of student learning, and the specific procedures of analysis that constitute the method of this study. I describe how student thinking and student participation are captured in this study and how those aspects of the enacted curriculum are investigated in relation to student learning. In Chapter 4 I present results of the cognitive demand and participatory demand analysis conducted across multiple classrooms as well as the results of the correlational analysis between these dimensions of the enacted curriculum and student learning. These results suggest that the dimensions of cognitive demand and participatory demand are both worthy of attention with regard to mathematics learning. Chapter 5, then, contains narrative cases of two classrooms, illuminating salient issues with regard to the enacted curriculum specifically in the look-back phase of task enactments. This chapter also depicts some of the potential

interplay between cognitive demand and participatory demand as lessons play out. Finally, in Chapter 6, I conclude with a discussion of potential answers to the guiding research questions, key implications that I see stemming from this study, connections to past research, and directions for future research.

## **CHAPTER 2**

### **BACKGROUND LITERATURE AND THEORY**

The goal of this chapter is to situate the present study within the work of other scholars. I begin by surveying past research interrogating the link between the enacted mathematics curriculum and student learning in order to identify and situate components of the enacted mathematics curriculum. This review includes work from the process-product paradigm as well as studies of cognitive demand. I then transition to the importance of student participation and cite work dealing specifically with that aspect of the enacted curriculum, giving special attention to research related to what I am calling the look-back phase of mathematical task enactments. The chapter proceeds with a description of the theoretical perspectives I use to clarify how I am attempting to balance the individual and collective aspects of the enacted curriculum. This theoretical perspective also leads to my conceptualization of student participation in mathematics discourse in general and the construct of participatory demand in particular, and allows for the overarching question of this study to be expressed at the conclusion of the chapter.

#### **Research on the Enacted Curriculum**

##### **The Process-Product Paradigm**

In the 1970s and early 1980s, there was a broad movement in mathematics education research to identify those teaching behaviors that were linked to student achievement scores (Brophy & Good, 1986). This movement was known as the process-product paradigm, with the name stemming from a model developed by Dunkin and Biddle (1974) to represent the several categories of variables at play in relation to classroom teaching (see Figure 4). In particular, process variables were those present in actual classrooms, such as teacher behaviors and pupil behaviors, whereas product variables were essentially outcomes such as immediate pupil growth



and long-term pupil effects. Process-product research, therefore, involved identifying links between observable behaviors in the classroom and pupil outcomes, typically achievement test scores. Teacher behaviors that correlated with high achievement were considered effective practices. Examples of effective practices were covering the material on the test (Brophy & Good, 1986), spending relatively little time going over homework assignments (Good & Grouws, 1977), and having a well-organized classroom with academically-focused lessons (Evertson, Anderson, Anderson, & Brophy, 1980). In addition to the many correlational studies, there were also experimental studies (e.g., Gage, 1978, cited in Brophy & Good, 1986), but the goal was the same—linking observable or “low inference” process variables and student achievement scores.

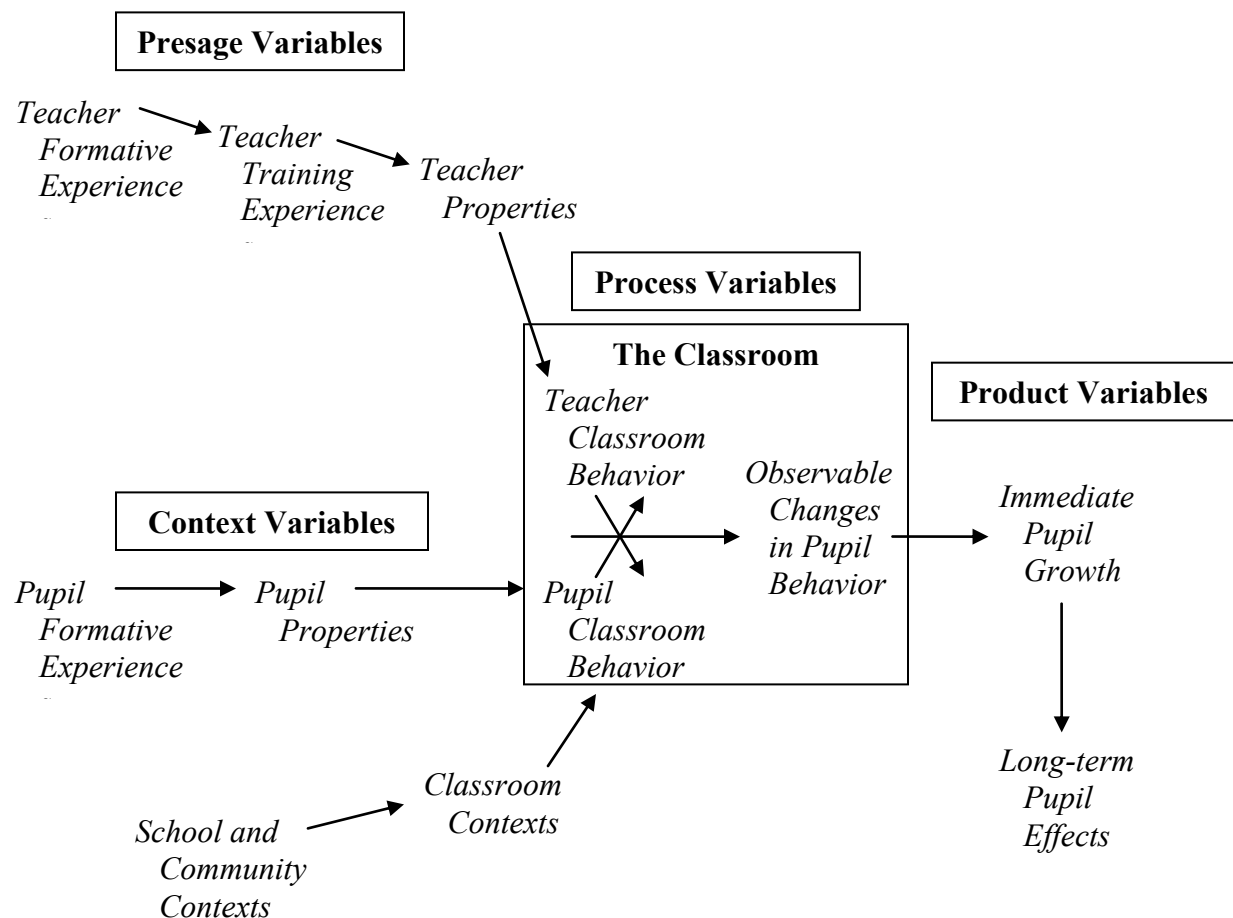


Figure 4. A model for the study of classroom teaching used within the process-product paradigm, adapted from Dunkin and Biddle (1974) as printed in Shulman (1986).

Although this line of research had several strengths, such as its appeal to practitioners and policymakers as well as its reliance on actual (rather than laboratory) classrooms, it began to draw criticism from various scholars. For example, Peterson and Swing (1982) pointed out the lack of attention given to students' roles in shaping what happens in classrooms as well as the absence of a consideration of how students think about and make sense of what happens. Winne and Marx (1982) argued for an attention to both students' and teachers' thought processes in the classroom. Shulman (1986) synthesized the critiques that were growing with regard to process-product research, emphasizing the fact that the paradigm was largely subject free and theory free. Although some studies (e.g., Good & Grouws, 1977) involved mathematics classrooms in particular, the program of research overall was aimed at identifying general teaching practices and thus was not equipped to incorporate the nuances of what it meant to teach particular subjects. With respect to theory, Shulman argued that by focusing primarily on what works, the paradigm was not able to answer the question of why it works, which should be an essential characteristic of any scientific or intellectual endeavor.

This history of process-product research is relevant to the present study because of the connections between process and product variables and the more contemporary notions of enacted curriculum and student learning (from Figure 1), respectively. There are important distinctions to draw, however, when considering the present study. First, the enacted curriculum is conceived in a way that incorporates multiple dimensions of classrooms rather than only readily observable behaviors. In particular, this study attends to the dimensions of students' potential cognitive processes as well as their social interactions in the enacted curriculum, which echoes Shulman's (1986) point that there are social and cognitive mediators between teaching and learning that are not adequately addressed in process-product research. I agree with

Shulman's description of the "simultaneity of the two processes in the learner" (p. 17) and have attempted with this study to consider the simultaneity in ways that address the critique of process-product research.

Second, because of the conceptualization of enacted curriculum within a broader curriculum theory, the subject matter is central to the notion of enacted curriculum in ways that it is not in the process variables. As described in the next section, the construct of cognitive demand that I use to address students' cognitive processes in the enacted curriculum was developed specifically for the subject of mathematics. Similarly, the construct of participatory demand that I develop below to address students' collective processes deals specifically with mathematics discourse.<sup>3</sup>

A third distinction between the process-product paradigm and the curriculum framework underpinning this study concerns student outcomes. In process-product research, student achievement scores were the prevalent product measure but, as Shulman (1986) noted, this fairly limited operationalization of student learning started to become the goal of education. In other words, by framing "effective teaching" as those practices associated with high achievement scores, forms of teaching that perhaps yielded other important forms of student outcomes (e.g., problem solving abilities, collaborative skills, productive attitudes or dispositions) were decreasing in value. The notion of student learning within the curriculum framework, on the

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<sup>3</sup> Although cognitive demand and participatory demand were developed specifically in the realm of mathematics education, there are clear adaptations that could be made to each, rendering them useful in other subject areas as well.

other hand, is construed broadly to cover knowledge, skills, beliefs, affect, and so forth. A variety of research techniques are required to capture such a variety of student outcomes.<sup>4</sup>

### **More Recent Work on the Enacted Curriculum**

In the decades since the peak of the process-product paradigm, different approaches have been taken, particularly within mathematics education, in looking at the enacted curriculum and student learning (Stein et al., 2007). Some studies have investigated the enacted curriculum as a mediator between instructional materials and student learning. For example, Tarr and colleagues (2008) found that what they called a *Standards*-based learning environment (referring to the *Standards* documents of NCTM, 1989, 1991, 2000) acted as a mediator between the impact of *Standards*-based curriculum materials and student achievement. Characteristics of a *Standards*-based learning environment included opportunities to conjecture, emphasis on conceptual understanding and multiple solution strategies, and teachers building on students' mathematical thinking. One of the key findings of this study was that classrooms using *Standards*-based curriculum materials and incorporating a moderate or high level of *Standards*-based learning environment had higher scores on one of the outcome measures than did other classrooms; *Standards*-based curriculum materials alone did not exhibit this relationship. Similarly, Senk and Thompson (2003) collected research on *Standards*-based curriculum materials and how they

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<sup>4</sup> Although the curriculum framework underlying the present study involves a broad conception of student learning, this study itself relies only on a written student assessment. Even this, however, is different from a typical process-product achievement test in that it was designed around a particular mathematical learning goal and includes a variety of types of items (see Chapter 3).

relate to students' mathematical learning. An overarching theme of this work was that students using *Standards*-based materials performed comparably with students using other materials, such as publisher-developed textbooks, on assessments of students' procedural and factual competencies but there is some evidence that the former population outperformed the latter on more complex problem-solving assessments. This potential difference may stem from the differences in enacted curriculum and thus differences in students' actual experiences in mathematics class.

Lloyd (2008) examined a teacher's decisions as a mediator between curriculum materials and the classroom discourse that occurred in the enacted curriculum. In particular, she focused on a high-school teacher as he implemented a new curriculum and shifted from a predominantly small-group format of discourse to whole-class discussions that involved extended student contributions. Lloyd's analysis was not only based on the behaviors exhibited in the enacted curriculum but also the teacher's and students' expectations, comforts, and discomforts participating in that space. Herbel-Eisenmann has also investigated the role that teacher's discourse choices, whether conscious or unconscious, have in shaping the classroom discourse within the enacted curriculum (Herbel-Eisenmann & Wagner, 2010; Herbel-Eisenmann, Wagner, & Cortes, 2010; Wagner & Herbel-Eisenmann, 2008). Additionally, Choppin (2011) analyzed teacher's attention to student thinking as it relates to the ways in which the teacher enacted challenging mathematical tasks.

Rather than considering one factor of the educational system (see Figure 1) as a mediator of another, some studies have illuminated important issues with regard to the enacted curriculum by comparing the enacted curriculum from different countries. For instance, Clarke and colleagues (Clarke, Emanuelsson, Jablonka, & Mok, 2006; Shimizu, Kaur, Huang, & Clarke,

2010) have produced books that draw comparisons and contrasts between the classroom practices and routines within classrooms around the world. Stigler and Hiebert (1999), drawing on the TIMSS video study (Martin & Kelly, 1998), described patterns in how U.S. mathematics teachers organized the enacted curriculum and how those patterns differed from other countries such as Germany or Japan. Mullis and colleagues (2000), also drawing on TIMSS video data, characterized the enacted curriculum by identifying the types of activities that were taking place and the durations of those activities. For example, across countries, teacher lecture accounted for 23% of class time and teacher-led student practice, which may be a whole-class format for the working phase of the MTF if the practice was centered on a mathematical task, accounted for 22% of class time. Student independent practice, which would include individual or small-group format working phases, had an international average of 15% of class time. The United States, in particular, was found to be slightly below the international averages in lecture and guided practice (20% and 18%, resp.) and slightly above average in independent practice (17%). Because Mullis and colleagues did not interpret the data through the MTF, however, it was not clear into what categories Mullis and colleagues placed the set up of a mathematical task or the process of looking back.

### **Mathematical Task Enactments and Cognitive Demand**

The research around mathematical tasks and cognitive demand has taken a close look at the enacted curriculum to investigate the nature of the activity taking place in mathematics classrooms. This focus is based on the premise that the enactment of mathematical tasks has implications not only for the content that students' learn but also for students' conceptions of what it means to do and learn mathematics (Doyle, 1983, 1988), which is a learning outcome in its own right and also influences students' subsequent learning. The MTF, which was introduced

in Chapter 1 (see Figure 2), has been used in studies of the enacted curriculum together with key constructs such as task features and cognitive demand, which can be traced through the phases of the MTF. Task features are aspects of mathematical tasks that are known to relate to student reasoning, multiple solution paths, multiple representations, and justification (Stein et al., 1996). Stein and colleagues found that these features typically remained consistent from the written task through to the set-up phase of enactment. However, they found that the other construct of interest—cognitive demand—tended to decline during enactment.

Cognitive demand of a mathematical task refers to “the kind of thinking processes entailed in solving the task” (Stein et al., 1996, p. 461). These processes can range from recalling facts to the rote execution of procedures to a meaningful use of procedures or authentic problem solving process.<sup>5</sup> Although different instructional goals may call for different types of cognitive demand (Stein et al., 2000), mathematics education literature (Hiebert & Lefevre, 1986; Kilpatrick, Swafford, & Findell, 2001) and policy documents (NCTM, 1989, 2000) call for a regular presence of mathematical tasks at high levels of cognitive demand for students. Stein and Lane (1996), in a study of several hundred task enactments from four middle school sites, also found that student performance on a cognitively demanding assessment was positively associated with the use of cognitively demanding tasks in instruction. Moreover, the strongest gains occurred when highly demanding written tasks were not only used but the cognitive demand was maintained at a high level through the set-up phase and implementation phase.

Because these findings suggest the importance of considering cognitive demand in particular, Henningsen and Stein (1997) explored the factors that seem to relate to the

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<sup>5</sup> Chapter 3 contains more details about each of the levels of cognitive demand.

maintenance or decline of cognitive demand during enactment. They found that factors such as building on students' prior knowledge, scaffolding, giving an appropriate amount of time, modeling high-level performance, and pressing for explanation and meaning were frequently involved in the maintenance of high cognitive demand. On the other hand, giving too much or too little time, having a lack of accountability, shifting focus from the process to the solution, and removing the intellectual burden on students were some of the factors commonly associated with declines in cognitive demand during enactment.

The present study, which builds on this past work involving cognitive demand, has the potential to make several contributions. First, by using the modified MTF that separates task implementation into the working phase and the look-back phase (see Figure 3), there is the potential to distinguish between tasks that shift in cognitive demand between the set-up and working phases versus those that shift between the working and look-back phases. With the original MTF, both cases would be considered a shift in cognitive demand from set-up to implementation. For example, consider a task enactment in which a high-level task declines during implementation due to a focus on correct answers. Using the modified MTF, one can illuminate whether this decline occurred as the students were working or whether they worked at a cognitively high level but then, in sharing out their work during the look-back phase, focused exclusively on their answers instead of their solution strategies or sense-making. Because it is possible for substantial amounts of time to be spent both working and looking back at that work and the emphases during these times can be different (Otten, 2010b), it is valuable to be able to parse these phases of enactment. Unfortunately, for reasons discussed in Chapter 3, detailed analysis of the working phases was not possible in this study, though the potential of the modified framework under different circumstances remains as a contribution.



A second contribution of this study, beyond past work, has to do with units of analysis. Stein and Lane's work (1996) is particularly relevant to the current study because it examined links between the enacted curriculum and students' mathematical learning. In Stein and Lane's study, however, the unit of analysis was the school site, essentially yielding a sample size of four. This limited sample size effectively illustrated a connection between cognitive demand and student performance gains because Site A and Site D contrasted noticeably in their task enactments and also in their gain scores. It is difficult to know, however, the extent to which this illustration is generalizable beyond those particular sites. In the present study, I shift the unit of analysis from school site to individual classrooms and am able to use statistical methods to look for relationships between enactment and learning, rather than relying only on inspection. I also draw data from a wider variety of school settings. The generalizability still remains modest, however, because this study does not have a large sample size, is correlational in nature, and is restricted to a single content topic (see Chapter 3).

Third, by focusing on the thinking processes entailed in mathematical tasks, which is admittedly an important facet of intellectual work in mathematics and thus worthy of attention, past studies of cognitive demand move to the background other important considerations such as the nature of student participation in mathematics classroom discourse. For example, Stein, Grover, and Henningsen (1996) reported overall patterns of participation (e.g., 69% of tasks allowed for students to use one another as resources, 30% of tasks involved students working alone or contributing to whole-class interactions as individuals) and paid attention to student explanations but did not fully consider patterns of participation in relation to cognitive demand or learning. In the next section, I outline why such a focus might be worthwhile.

## **Student Participation in Mathematics Classrooms**

### **A Sociocultural Perspective**

As mentioned in Chapter 1, sociocultural perspectives in education view learning not as the accumulation of facts or knowledge in an individual's mind but as a collaborative process by which a learner comes to participate in particular discourse communities (Lave & Wenger, 1991; Wenger, 1998). Lemke (1990) reflected this perspective when he wrote that "learning science means learning to *talk* science. It also means learning to use this specialized conceptual language in reading and writing, in reasoning and problem solving, and in guiding practical action in...daily life" (p. 1; emphasis in original). Of course, the same perspective can be applied to the learning of mathematics. Learning and "knowing" mathematics means being able to engage in mathematical pursuits as a legitimate participant of a community and being able to converse mathematically in the sense of effectively using mathematical language, both formal and informal, and making meaning from the language use of others. Recognizing the collective aspects of learning brings with it several complex dimensions to consider. Systemic Functional Linguistics (Halliday & Matthiessen, 2003), for example, draws attention to the fact that any use of language involves not only the content being communicated but also relationships being negotiated and text being organized. All of these phenomena, not just the first, are at play in the enacted mathematics curriculum and are worthy of examination.

As I explain in the next section of this chapter, I take the sociocultural perspective to be valuable for its attention to collective and communal aspects of student learning, not as a replacement of cognitive considerations. But first, I review some literature that has informed this study with regard to student participation.

## Research Attending to Student Participation

As stated previously, the enacted curriculum is extraordinarily complex. Scholars in mathematics education have found ample phenomena of interest to investigate, from students' developing identities (Cobb, Gresalfi, & Hodge, 2009) to equity issues in groupwork (Esmonde, 2009) to teachers' development of a community of inquiry in their classrooms (Goos, 2004). The nature of student participation has been found to be of utmost importance in mathematics classrooms. For example, the roles that students take on in mathematics classrooms at various grade levels has implications not only for the content that they learn but also the status of the individuals in the classroom and the equity of the learning opportunities (Boaler & Staples, 2008; Featherstone et al., 2011). There are other studies that have examined student participation during what I call the working phase of mathematical task enactments (e.g., Fuchs, Fuchs, Hamlett, & Karns, 1998), but of particular interest here are studies of the whole-class discussions that often look back upon the processes and ideas that came up during the working.

With specific regard to student participation in middle-school mathematics classrooms, Wagner (2007) has spoken directly with secondary students about their perceptions of mathematics discourse with a particular focus on the issue of human agency, which is often absent from formal mathematical language and can give the impression that mathematics is devoid of human actors. Jansen (2006, 2008) studied students' motivations for participation in mathematics classrooms. Her work demonstrated that students in mathematics classrooms are navigating social relationships and their learning of mathematics and that there are many factors related to students' participation in whole-class discussions. For example, Jansen (2008) found through interviews with 15 students from classrooms using the *Connected Mathematics* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998a) textbook series that students who felt somewhat

threatened by whole-class discussions tended to contribute in procedural ways and students who believed that certain behaviors were inappropriate were less likely to critique their classmates' solutions or explanations. Although the present study does not delve into student beliefs or motivations in this way, I agree with Jansen's premise that student participation is a central component of the enacted curriculum, worthy of close inspection.

Several other scholars have focused on the features of productive whole-class discussions during what I call the look-back phase of enactment.<sup>6</sup> Staples and Colonis (2007) have written about the value of *collaborative discussions* over *sharing discussions*, with the former characterized by the collective development of mathematical understandings and the latter characterized by the mere reporting out of solutions. They also described characteristics of pedagogy that were associated with collaborative discussions, such as positioning students to attend to one another's ideas, managing wrong answers, and linking across ideas (see also Staples, 2007). Wagner and Herbel-Eisenmann (2008) illuminated the notion of opening up and closing down classroom discourse, which is related to collaborative versus sharing discussions, though from a more linguistic perspective. In particular, teachers and written curriculum materials (and students) make linguistic choices that may function to restrict or expand the possibilities that students have for engaging in the classroom discourse, and certain acts of closing down the discourse may lead to sharing discussions whereas acts of opening up the discourse may help to cultivate collaborative discussions. Herbel-Eisenmann, Wagner, and

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<sup>6</sup> Shimizu (2006) studied the conclusions of mathematics lessons, but his consideration of teacher's summary or wrap-up procedures was more restrictive than what I capture in the look-back phase.

Cortes (2010) also made the point that coming to participate in mathematics classroom discourse is distinct from, though related to, participation in the formal discourse of the discipline of mathematics. This point underlies the present study as I examine participatory demand in the context of school classrooms, not communities of practicing mathematicians.

The scholars involved with the original MTF have also taken up this issue of facilitating mathematically productive discussions at the conclusion of mathematical tasks by focusing on what teachers can do in preparation and execution of such discussions. In particular, they have identified five practices—anticipating, monitoring, selecting, sequencing, and connecting—that can be used by teachers to strategically work toward a discussion that achieves their learning goals for the lesson (Stein, Engle, Smith, & Hughes, 2008; Stein & Smith, 2011). In much of this past work, students are taken to be active participants in the discourse of the whole-class discussions, working to collectively make connections and build meaning rather than only receiving meaning or summaries from the teacher. The focus, however, was on the teacher's role in orchestrating productive whole-class discussions (or their actions that seem to work against such discussions). In the present study, I focus not on the teacher's discourse during enactment but on the students', which has been tied to learning in preliminary reports of research on mathematics discourse (Chapin & O'Connor, 2004).

Additionally, the attention in the five practices work was primarily on the mathematical ideas being developed and the mathematical goals of the lesson. There are several other purposes of the look-back phase of enactment that might also be considered. For example, the look-back phase may be used to engage in collective reflection (Cobb, Boufi, McClain, & Whitenack, 1997) in which the processes of mathematical activity themselves are made an explicit object of reflection. A classroom community may also establish taken-as-shared understandings during the

look-back phase, establishing a sort of public record of ideas, and, as highlighted by Systemic Functional Linguistics, they are always simultaneously negotiating various relationships.

A conclusion of much of this work is that teachers have a difficult balance to strike during whole-class discussions because they are attempting to value and build upon students' idiosyncratic constructions of knowledge while also moving the class toward conventional mathematical language and practices (Ball, 1993; Lampert, 2001; Wood, Cobb, & Yackel, 1993). Just as teachers must navigate delicate balances during task enactments, one of my goals in this study is to balance the attention to cognitive demand that has been fruitful in past research with an attention to the nature of student participation during task enactments.

### **A Blended Approach**

One of the central critiques of process-product research, which could also apply to research between the enacted curriculum and student learning, was that it was atheoretical in nature (Shulman, 1986). Here I articulate my theoretical stance, which draws both on cognition at an individual level and student participation at a collective level, to ground this study in ways that allow for hypotheses to be formed (see Chapter 3) and for results (see Chapters 4 and 5) to be interpreted not simply as “what works” but in terms of their relations to these theoretical ideas.

Although I ascribe to the notion that individual cognition is a useful construct for considering student learning in mathematics, there are several critiques of this perspective to be aware of. One is a critique of the “representational perspective” of learning, which Cobb, Yackel, and Wood (1992) characterize as the process by which learners come to understand and internalize concepts that exist in external representations. From the representational perspective, a number line, for example, contains within it the notions of continuity, the ordering of the real numbers, and the density of the rational numbers. The goal for students, then, is to understand

and internalize these mathematical ideas from the representation. One of the problems that Cobb, Yackel, and Wood point out with respect to this perspective is that students are said to come to comprehend a new concept embedded in an external representation, but they could only do so if they already had the mental ability to understand this concept. Another problem is that there are countless interpretations of an external representation that are possible, yet many students come to understand precisely the interpretation that is already held by individuals who know the subject. Cobb, Yackel, and Wood also argue from a more practical standpoint that the representational perspective on learning leads to instruction that is misaligned with the goals of the reform movement in mathematics education.

This critique of the representational perspective is not meant to discredit the individual processes involved in coming to learn a subject such as mathematics. In fact, Cobb, Yackel, and Wood (1992) are clear in identifying both individual and collective aspects of learning:

[M]athematics teachers and students [are] active constructors of their ways of knowing and participants in social practices rather than mirrors of a world independent of experience, history, and culture. Knowing would then be seen as a matter of being able to participate in mathematical practices in the course of which one can appropriately explain and justifying one's actions... we suggest that it is potentially more fruitful for our purposes as mathematics educators to view students as actively constructing mathematical ways of knowing that make it possible for them to participate increasingly in taken-as-shared mathematical practices. (pp. 15–16)

One can see in this quote references to individual constructive, cognitive processes as well as communal, participatory processes involved in the mathematics learning.

Similarly, when thinking about learners, teachers, and researchers, Sfard (1998) argued for the necessity of including the individual level, which can be viewed through the metaphor of acquisition of knowledge, as well as the social level, which can be viewed through the metaphor of participation in discourse communities. In an attempt to achieve this balance, Sfard (2008, 2012) has proposed a framework that purports to unify thinking and social interactions through the notion of communication, in particular, that thinking is a form of communicating with oneself. In this study, I attempt to strike a balance in a different way. As described in the next section, I build upon the existing cognitive demand framework by adding a complementary framework that focuses on the nature of student participation in the classroom discourse.

In summary, several scholars have called for a balance between cognitive and sociocultural perspectives on learning, which are themselves complementary. One must be careful, however, that cognitivism and socioculturalism are not “thrown together with little thought for the differences in their purposes, assumptions, or perspectives” (Shulman, 1986, p. 33). Rather, a disciplined approach must be taken whereby analytic decisions are guided by a careful consideration of what each theoretical perspective affords. Such an approach is the goal of the present study. The enacted curriculum within mathematics classrooms is conceptualized in terms of students’ individual thought processes as well as interactions between individuals and participation in the classroom community. To ground these aspects of the enacted curriculum, I use the existing construct of cognitive demand and the new construct of participatory demand.

### **Cognitive Demand and Participatory Demand**

The cognitive demand construct, defined above and explicated in the next chapter, is designed to promote the tracing of the thinking processes entailed in solving a mathematical task throughout the phases of enactment. In a sense, cognitive demand can be used to answer the



following question: What mental (and mathematical) actions are students expected to engage in to solve this task, and what do they actually engage in as they solve it? Recalling memorized facts and justifying a general claim, then, are substantially different answers to these questions, as are enacting a rote procedure and explaining why a procedure produces a correct result. These considerations are an important aspect of the enacted curriculum because I believe that all students are individually capable of rich mathematical activity and that providing opportunities for students to experience authentic mathematical practices such as problem solving and reasoning-and-proving is an end in and of itself.

To complement cognitive demand, I introduce the construct of participatory demand, which is defined as the extent and nature of the interactions expected of students during the enactment of a mathematical task.<sup>7</sup> The details of how I operationalized this construct for the present study are described in Chapter 3. Here I simply mention that including both words “extent” and “nature” in the definition was done intentionally. Although the amount of student participation during task enactments has obvious implications with regard to students coming to be legitimate participants in the mathematics classroom community, it is insufficient to attend

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<sup>7</sup> As with cognitive demand, participatory demand is defined as an expectation in the written and set-up phases but as the actual participation in the working and look-back phases. Moreover, the label of “participatory demand” was specifically chosen to parallel the label of cognitive demand. One reason for this choice was that cognitive demand is well-known in various communities of mathematics educators to the extent that it is nearly a “demand” brand. By using participatory demand, I am explicitly linking to that brand. Another reason is that, by using parallel phrasing, I hope to promote the two constructs being viewed as a complementary pair.

only to the sheer quantity. I have observed several mathematics classrooms in which the student participation was extremely high during the points in a class period when the teacher shifted conversation away from mathematics. For instance, some teachers elicit much more participation when they begin talking about extra-curricular activities or popular culture than when they ask for mathematical explanations or justifications, and in some task enactments the discussion of the “real world” problem context brings forth more student participation than the whole-class discussion at the conclusion of the task. For this reason, it is important to also attend to the *nature* of the student participation, which I capture through a consideration of the thematics of the student discourse.

In this study, I trace both cognitive demand and participatory demand through the phases of the modified MTF (see Figure 3). Within the theoretical perspective of this study, it is important to note the student learning triangle to the right of the enacted curriculum in this figure. The arrow emanating from the enacted curriculum directed toward student learning should not be interpreted to mean that the enacted curriculum determines what students learn or that the relationship between enactment and learning is unidirectional. Rather, this arrow represents the fact that the enacted curriculum *influences* student learning in various ways. Some maintain that task implementation is an “especially” important influence on learning (Stein et al., 2000, p. 4), but there are many other influences as well. Also, in the opposite direction, student learning has a profound impact on the enacted curriculum in that students’ existing knowledge and what they are thinking and learning moment to moment has implications for their interactions and for the

decisions that teachers make during enactment.<sup>8</sup> Also in light of the discussion above, student learning is multifaceted and should not be viewed merely as students' performance on written assessments (although it may be operationalized that way in some studies, the present study included). From a sociocultural perspective, students' participation in the enacted curriculum is, to an extent, the learning itself—not separate from it.

### **The Scope of the Present Study**

Any study of the enacted mathematics classroom necessarily foregrounds certain aspects and backgrounds others. I fully recognize that this study, by focusing on cognitive demand and participatory demand of mathematical task enactments, fails to attend to many issues that are of the utmost important to students' experiences in mathematics classrooms. For example, I do not give explicit attention to the role of the textbook during enactment, the teachers' decisions, students' beliefs or emotions, and countless other domains. Furthermore, I do not look outside the classroom in space or time. Students' out-of-school experiences certainly make an impact inside the classroom, and Radford (2006) argued that it is not sufficient to examine what takes

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<sup>8</sup> Another clarification about the modified MTF diagram is that the arrows are meant to show influence, not determination, and the phases are neither chronological nor strictly linear. The written task influences but does not determine the way that it is set-up and implemented. Similarly, the set-up phase influences but does not determine the working phase and the working phase certainly does not determine the look-back phase as a teacher may make many different choices about what to emphasize or not while looking back. Also, a single task enactment might move back and forth between setting up the task and working and between working and looking back on that work.

place in a classroom even if one's goal is to make sense of what is taking place in that classroom. In particular, he points to historical dimensions at play in the enacted curriculum: "the meanings circulating in the classroom cannot be confined to the interactive dimension that takes place in the class itself; rather they have to be conceptualized according to the context of the historical-cultural dimension" (p. 23). These omissions should not be construed as an indication that I devalue these issues but rather an inherent and necessary limitation of doing such work.

What I do focus on in this study, however, is the unfolding of mathematical tasks in several different middle school mathematics classrooms. I examine the written tasks and trace the cognitive demand and participatory demand throughout the phases of the task enactment. I then investigate the potential relationships between these features of the enacted curriculum and students' mathematical learning as exhibited on a written assessment. The overarching goal is twofold: (a) to gain insight into the nature of the enacted curriculum in middle school mathematics classrooms, particularly with respect to the new construct of participatory demand and the new identification of the look-back phase within the MTF, and (b) to gain insight into the association between the enacted curriculum and student learning (in the forward direction). Just as this study represents an attempt to balance individual and collective perspectives on learning, I also pursue these goals with both quantitative and qualitative methods. In the next chapter, I present the specific research questions that guide this study and then describe the mixed method procedures of analysis that I employed.

## **CHAPTER 3**

### **METHOD**

This study is an examination of the enacted curriculum in middle school mathematics classrooms and its relation to students' mathematical learning. The data come from a multi-year professional development project in which classroom video recordings were made and student pre- and post-tests were administered. In the sections that follow, I first describe the research questions before giving background information about the professional development project overall. Then I describe the specific participants, settings, data, and analytic procedures of the study.

#### **Research Questions**

The following questions guided this study:

1. How does the enacted curriculum—specifically, the levels of cognitive demand and the forms of student participation during the phases of mathematical task enactments—relate to student learning? (RQ1)
2. In what ways do the levels of cognitive demand and forms of student participation interact during the look-back phase of mathematical task enactments? (RQ2)

At its broadest level, RQ1 situates this study as an interrogation of the link between the enacted curriculum and student learning, as described in Chapter 1. The relationship between the enacted curriculum and student learning, however, is bidirectional and extraordinarily complex. For example, what students have learned in the past and are learning in the moment influence the ways in which classroom interactions play out. There are also countless layers of phenomena that one could attend to in the enacted curriculum, from modes of presentation to affective responses to physical arrangements and movements. In the present study, RQ1 specifies that cognitive

demand and student participation, within the structure of the Mathematical Tasks Framework (MTF), are the particular areas of focus. Attending to cognitive demand allows for this study to build directly on past work (e.g., Stein & Lane, 1996) in a way that is a modified replication, while simultaneous attention to student participation adds a new dimension to the analysis.

As described below, the general methodological approach to answering RQ1 is through qualitative coding leading to correlational analysis. In addressing RQ2, I take a complementary approach by conducting a qualitative analysis of two classrooms. The intent of RQ2 is to closely examine the enacted curriculum in and of itself and, specifically, to consider cases of interactions between cognitive demand and student participation. The site of these examinations is specified as the look-back phase of mathematical task enactments. The rationale for narrowing in on this particular phase is twofold. First, extensive research already exists with respect to other phases of task enactments such as the set-up phase (Crespo, 2003; Jackson et al., 2011) and the implementation phase overall (Henningsen & Stein, 1997; Stein et al., 1996). As a new phase in the MTF, however, the look-back phase has not yet received the same consideration.<sup>9</sup> Second, the look-back phase has practical implications because it is commonly the phase of a task enactment where the key mathematical outcomes of the task are publicly shared and established, which has implications for cognitive demand, and where the students have opportunities to

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<sup>9</sup> Whole-class discussion, which is a common occurrence during the look-back phase of the MTF, have been the focus of past research (e.g., Stein et al., 2008; Wood et al., 1993; Zolkower & Shreyar, 2007), but this work was not situated as a phase of mathematical task enactments in the way that it is in the present study.

participate with their teacher and all of their classmates. Thus, the look-back phase appears to be ripe for an investigation of interactions between cognitive demand and student participation.

### **Evolution of the Research Questions**

It should be noted that the research questions, presented here in their final forms, were not the research questions that initially guided the development of the study proposal. At first, this study was conceived as an examination of mathematical processes—for example, problem solving, reasoning and proof, or communication (NCTM, 2000)—as they were exhibited in middle school mathematics classrooms during the look-back phases of task enactments. Eventually, the language of these questions shifted to adopt the standards of mathematical practice as enumerated in the *Common Core State Standards for Mathematics* (2010), such as making sense of problems and persevering in solving them, reasoning abstractly and quantitatively, attending to precision, looking for and making use of structure, and so forth. Through the process of developing the study, however, I realized that it is difficult (and perhaps not especially useful) to examine mathematical practices without also attending to other features of the task enactments. Cognitive demand (Doyle, 1983; Stein et al., 1996), in particular, seemed important to consider because it is well established in the field of mathematics education and incorporates a notion of “doing mathematics” that relates directly to such mathematical practices as problem solving, conjecturing, and justifying. It also became increasingly clear that my interest in mathematical practices within the enacted curriculum stems from a belief that students deserve the opportunity to participate in mathematical activity that is “intellectually honest” (Bruner, 1960; Lampert, 1990) or true to the spirit of the discipline. To understand such participation, however, requires more than a focus on key practices; it necessitates a framework

for thinking about student participation more broadly.<sup>10</sup> After clarifying my theoretical stance with regard to students' coming to enter the mathematical discourse of the classroom, I realized that what was needed was a framework, like that for cognitive demand, which can be used to trace students' mathematical participation throughout the phases of the MTF.

This thought process led to RQ1 in its present state, but RQ2 was phrased around the mathematical practices in the look-back phase of mathematical task enactments. Although I still find importance in examining students' opportunities to engage in mathematical practices during the enacted curriculum, initial analysis of the classroom data revealed the need to dig deeper into the issues of cognitive demand and student participation and their interrelations as a follow-up to RQ1, leading to a final revision of RQ2. Ironically, the original motivation for the study—the mathematical practices—is no longer present in the research questions, but the new focus of the study lays a foundation for eventually understanding how the practices fit into students cognitive and collective activities in middle school mathematics classrooms.

### **Additional Questions that this Study Addresses**

As a final comment on the research questions of this study, I would like to point out that there are underlying questions that, although not rising in prominence to the level of a guiding research questions, are nevertheless answered to an extent by this work. One such question

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<sup>10</sup> This is not to say that the resulting framework for student participation used in this study captures all of the important facets of student participation in classrooms—far from it. My point here is simply that participation in the key mathematical practices during the look-back phase was too narrow and, as a course correction, the subsequent attention to students' engagement in mathematical discourse throughout the phases of task enactments was more appropriate.



relates to the phases of task enactments as depicted in the original MTF and the modified MTF used here: What empirical value is there in identifying two sub-phases of the implementation phase of enactment? If there is no apparent value in the modification to the MTF, then researchers would be wise to adhere to the original, which has been empirically established (e.g., Stein & Lane, 1996). As will be shown in Chapter 4, however, and further articulated in Chapter 5, important shifts in the life of the task took place between the working phase and the look-back phase, implying value in drawing the distinction.

Another question that is important to consider with respect to this study is the following: Are cognitive demand and participatory demand theoretically and empirically distinguishable dimensions? This question is important to consider because, although on the surface cognitive demand and participatory demand seem to be distinct, it is not guaranteed a priori that they are separable. For instance, it may be that student participation is linked to cognitive demand in that students interact more richly when engaged in rich tasks and interact sparingly when engaged in low-level tasks. If this were the case, cognitive demand and participatory demand would be two aspects of the same phenomenon rather than conceptualizations of distinct phenomena. I make the case later in this chapter that the two dimensions are theoretically separable, and show through the results of the study that they are independent in practice as well.

Next, I describe the source of the data for this study, the nature of the subset of data used in this specific study, and the procedures of analysis.

### ***Project 2061 Professional Development and Research Program***

This study used data collected as part of a three-year voluntary professional development project within a five-year research study that was a joint initiative of the American Association for the Advancement of Science and the U.S. Department of Education. This project, entitled

*Project 2061*, was conducted jointly in Delaware and Texas and involved a total of 50 middle school mathematics teachers and nearly 2,500 students. The professional development was built around research on the teaching and learning of specific mathematical learning goals—one related to the use of variables to represent change, another related to the analysis of data, and a third related to number and operations.

During the first summer of the project, teachers met for 1–2 days to learn what the three focal learning goals were and identify whether these goals would be addressed in their forthcoming year of instruction. During the subsequent academic year, project staff video-recorded the lessons directed at the focal learning goals, but there was no intervention with the teachers at this point, so this served as a baseline year for instruction. Pre- and post-tests were administered by the teachers to their students before and after the focal lessons. The second summer of the professional development involved 4–5 sessions that centered on instructional strategies related to the focal learning goals, in particular, how teachers might use different mathematical representations (e.g., tables, diagrams, physical models) in their teaching. These instructional strategies were based on research from the mathematics education community. All lessons directed at the focal learning goals were again video-recorded and pre- and post-tests were again administered. The third summer again consisted of 4–5 sessions, in this case focusing on students' mathematical understanding as exhibited by the teachers' own students in videos and written assessments from the previous academic year. A third set of pre- and post-tests and classroom video-recordings were collected. It should be noted that not all teachers from the first year of the professional development continued throughout all three years, and some teachers joined the project in year two or year three. Furthermore, a few teachers shifted grade levels during the three-year project and so did not teach precisely the same content each year. These

considerations, however, are not central to the present study because my investigations are not longitudinal nor are they concerned with the interventions that may have influenced teachers' enactments of tasks. Rather, this study deals with what happens in the enacted curriculum (not why it happens) and how that relates to student learning of mathematics.

The research goals of *Project 2061* were different than mine. Their overall aim was to learn what “professional development and continuing support teachers need to improve student achievement” in the context of middle school mathematics (DeBoer et al., 2004, p. 1). The logic model for their investigation is depicted in Figure 5. Moving through the model, the *Project 2061* team designed their professional development experiences based on research literature in mathematics education and purposefully recruited within teacher populations using various textbook series, including some whose development was funded by the National Science Foundation to follow the recommendations contained in mathematics education reform documents (NCTM, 1989, 1991). Viewing teacher knowledge, skills, and attitudes as a mediator between the professional development experiences and actual teaching practices, *Project 2061* collected information about each teacher's college major and course-taking as well as their teaching experience and exposure to past professional development sessions. A majority of attention was then given to teacher behavior, or classroom practices, as well as student learning, as measured by the pre- and post-tests. Examples of specific empirical reports that were undertaken by the *Project 2061* team are an investigation of the link between teachers' guiding questions and students learning of middle school algebra content (Wilson & Roseman, under review) and the predictive strength of teaching practices such as encouraging student explanations or representing ideas effectively with respect to student learning (Wilson et al., 2009).

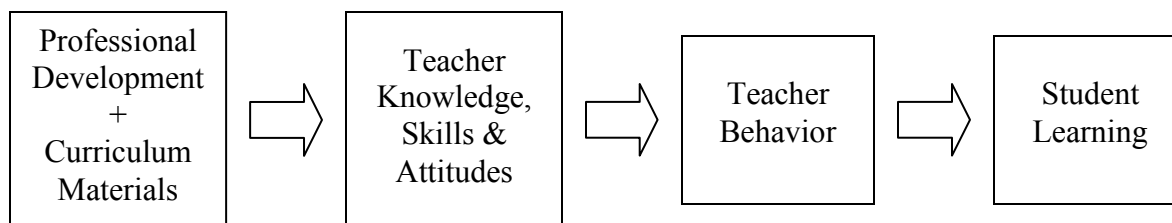


Figure 5. The logic model for the *Project 2061* professional development and research project.

Although *Project 2061* constitutes the research background of the present study, this dissertation was conducted independently and with different intentions. In particular, the professional development aims of *Project 2061* and questions about what influences teachers' practices were not directly tied to the present aim of examining the relationship between the enacted curriculum and student learning. The participants and data of the present study, however, were inherited from the broader project.

## Participants and Setting

### Teachers

From the larger project described above, I selected 12 middle-school mathematics teachers (8 from Delaware, 4 from Texas) as potential participants for the present study. Selection of these 12 teachers was based primarily on the mathematical content of their instruction. In particular, only those teachers who taught the early algebra learning goal (i.e., using variables to represent change) were considered; those teaching the number and operations or data analysis learning goals were excluded from consideration. I made other exclusions based on curriculum use. Specifically, I excluded all but two of the teachers using Glencoe's publisher-developed textbook series because the analytic framing of the current study depends upon mathematical task enactments rather than teacher exposition, and publisher-developed textbooks are typically not as task-centered as the NSF-funded curricular programs that were also present

in the *Project 2061* sample.<sup>11</sup> Also, I excluded one teacher using *Connected Mathematics* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998b) and one teacher using *Mathematics in Context* (National Center for Research in Mathematical Sciences Education & the Freudenthal Institute, 1998) because several other teachers were using these curriculum series. The specific teachers excluded in this latter case were based on conversations with *Project 2061* staff and their recommendations regarding the richness of the classroom practice, with those teachers identified as having ample student engagement sure to be included. This process left 12 teachers that I invited to participate in the current study. I was unable to gain consent from three of these teachers, however, and thus Table 1 contains information about the 9 participating teachers, where it can be seen that they vary in their background characteristics.

Table 1

*Teacher location and background characteristics*

Teacher	State	College Major	Years Teaching	Grade Level	Textbook Series
Ms. Albert	DE	Middle School*	7	8	<i>Mathematics in Context</i>
Ms. Cavillon	DE	Middle School	9	7	<i>Connected Mathematics</i>
Ms. Cesky	TX	Mathematics*	6	8	<i>Math Thematics</i>
Ms. DePalma	DE	Mathematics	8	8	<i>Mathematics in Context</i>
Ms. James	TX	Mathematics	8	8	<i>Glencoe</i> ‡
Mr. Johnson	DE	Elementary	6	7	<i>Connected Mathematics</i>
Ms. Mendoza	TX	Mathematics	13	8	<i>Math Thematics</i> ‡
Mr. Milson	DE	Elementary	5	7	<i>Connected Mathematics</i>
Ms. Wyncott	DE	Elementary†	4	8	<i>Mathematics in Context</i>

Notes: \* indicates a Master's degree. † indicates a specialization in Middle School education. ‡ indicates teachers who used self-supplied activities for the focal lessons.

<sup>11</sup> I was not able to view the videos to confirm this assumption but simply had to make the exclusion because of limited resources. This decision to exclude teachers using publisher-developed textbooks is a limitation to the study.

It should be noted that this set of participating teachers<sup>12</sup> are not necessarily representative of middle school mathematics teachers in general for the following reasons. First, these teachers all voluntarily became involved in the *Project 2061* activities, most of them for multiple years. This behavior is not characteristic of all middle school mathematics teachers. Second, I purposefully selected against some of those teachers who used publisher-developed textbooks, thus biasing my sample in favor of textbook series more commonly associated with student-centered instructional models. Third, because *Project 2061* utilized personnel from large research universities in Delaware and Texas, the participating teachers—indeed, all the teachers involved in *Project 2061* overall—taught in districts that were relatively close to these universities. Although it was not the case that the teachers came solely from districts populated by university families, it must be assumed that there is regional bias to some extent. Nevertheless, it can be safely assumed that there is variation in their teaching practices because each is a unique individual with a unique set of background experiences, they are located in different regions of the United States, they have different levels of experience, and, as described below, there are differences in their student populations as well as their curriculum materials. This variation in the enacted curriculum, situated around a single learning goal, is the key that allows for differences in classroom enactment to be explored with respect to student learning.

### **Students and School Settings**

The previous section described the 9 participating teachers in this study. As will be described below, however, the unit of analysis is a *set of lessons* around the focal learning goal

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<sup>12</sup> All teacher and student names are pseudonyms.

and, for two of the participating teachers, there were multiple sets of lessons for those teachers (one set from each for multiple academic years). Thus, there are 9 teachers but 12 sets of lessons and 12 classes of students involved in the present study. The 246 student participants are the 7th or 8th grade students of the 9 participating teachers who were present for both the pre- and post-test (see Table 2). Thus, actual class sizes may have been larger than indicated in Table 2.

Table 2

*Number of students present for pre- and post-tests in each set of lessons*

Teacher	AY	Grade	Participating students
Ms. Albert	2003-2004	8	23
Ms. Cavillon	2002-2003	7	22
Ms. Cavillon	2004-2005	7	19
Ms. Cesky	2003-2004	8	16
Ms. DePalma	2003-2004	8	28
Ms. James	2003-2004	8	20
Mr. Johnson	2002-2003	7	16
Ms. Mendoza	2002-2003	8	20
Ms. Mendoza	2003-2004	8	15
Ms. Mendoza	2004-2005	8	24
Mr. Milson	2002-2003	7	22
Ms. Wyncott	2003-2004	8	21

Specific background characteristics of the students participating in this study are not known because the student data was stripped of identifying information due to the fact that students could not be reached to give consent. To gain a general sense of the characteristics of students involved in this study, however, I gathered information at the district level. As Table 3 shows, the 9 participating teachers taught in 5 different districts, which I have labeled anonymously. The statistics contained in Table 3 and described below come from the websites of the Delaware Department of Education (<http://profiles.doe.k12.de.us>) and the Texas Education Agency (<http://www.tea.state.tx.us/districtinfo.aspx>). Although the statistics are for a more

recent academic year (unspecified for anonymity) than when the data used in this study was collected, they are meant to give a broad sense of the district populations.

Table 3

*Percents of various student characteristics by school district*

	Delaware			Texas	
	Alpha	Beta	Gamma	Delta	Epsilon
White	56%	44%	63%	29%	59%
African American	34%	33%	27%	23%	14%
Hispanic / Latino	5%	19%	6%	47%	18%
Other ethnicity	5%	4%	4%	1%	9%
ELL / LEP	1%	5%	1%	17%	6%
Low Income*	29%	58%	34%	71%	33%
Special Education	11%	11%	10%	7%	8%
Total enrollment	9,319	17,190	7,579	15,536	10,061
Teacher(s)	Ms. Albert Ms. Wyncott	Mr. Johnson Mr. Milson	Ms. Cavillon	Ms. Cesky Ms. Mendoza	Ms. James

Note: \* This label was used by both the Delaware Department of Education and the Texas Education Agency but I do not know if the criteria for inclusion were equivalent.

In Alpha school district, there is an average of 750 students in each of the three grade 6–8 middle schools. Middle school students in the Alpha district typically score higher than the state average on the Delaware Comprehensive Assessment in mathematics, with 72% having met the mathematics standards. In Beta school district, there is an average of 1000 students in each of the four grade 6–8 middle schools. Their students are below the state average in mathematics performance, with 48% having met the mathematics standards on the Delaware Comprehensive Assessment. In Gamma school district, there is an average of 740 students in each of the three grade 6–8 middle schools. In the Gamma district, 64% of middle school students met the mathematics standards, which is slightly above the state average.

Within the state of Texas, Delta school district has four middle schools (grades 6–8) with an average of 560 students per building. On the Texas Assessment of Knowledge and Skills,



72% of Delta middle school students met the mathematics standards. In Epsilon school district, there are two grade 7–8 middle schools with an average of 590 students per building. Their students performed above the state average on the Texas Assessment of Knowledge and Skills, with 90% having met the mathematics standards.

Another important consideration with regard to the students involved in this study is their algebraic background. In all cases, the mathematics class being studied constituted the students' first formal experiences with algebra in general and the use of variables in particular (Wilson et al., 2009), although, as noted above, this occurred in grade 7 for some students and grade 8 for others. The next section ("Mathematical Setting") provides more detail about the mathematical content that serves as the backdrop for this study of the enacted curriculum as well as the curriculum materials being used.

### **Mathematical Setting**

The mathematical content setting of the study is introductory algebra, specifically the learning goal of using symbolic equations to describe and represent quantitative relationships of change. This use of variables is typically students' first substantial and explicit departure from arithmetic into algebra and is significant because of the recent emphasis on algebra in policy documents (National Governors Association & Council of Chief State School Officers, 2010; National Mathematics Advisory Panel, 2008) and the importance of variable in students' subsequent mathematical experiences (Carragher & Schliemann, 2007; Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005). Additionally, there have been substantial increases in the number of middle-school students taking algebra. The way in which algebra is taught has implications for students' future experiences because algebra is not only a gateway for college and many careers, it is also a gateway to students' future mathematical experiences (Chazan, 1996). Yet

studies have shown that algebra, and particularly the notion of variable, can be difficult for students (Carraher & Schliemann, 2007; Knuth et al., 2005; Sfard & Linchevski, 1994). This suggests there is much to learn from an examination of the enacted curriculum in this mathematical context.

Table 1 above contains a column indicating the textbooks used by teachers in this study. Eight of the participating teachers used one of the curriculum series that were explicitly designed to adhere to NCTM's (1989, 1991) recommendations for school mathematics—the *Connected Mathematics Project* (Lappan et al., 1998a), *Mathematics in Context* (National Center for Research in Mathematical Sciences Education & the Freudenthal Institute, 1998), and *Mathematics* (Billstein et al., 1999). The other participating teacher used a publisher-developed textbook from *Glencoe* (Collins et al., 1998); however, she and one of the *Mathematics* teachers supplied their own activities for the set of lessons addressing the focal learning goal. This proportion of Standards-based textbooks (i.e., eight out of nine) is higher than the national proportion (Dossey, Halvorsen, & Soucy McCrone, 2008) but is the result of two factors: one is that the vast majority of teachers involved in *Project 2061* used Standards-based textbooks, and the second is that I favored these textbooks in my selection for the present study because they are more likely to be enacted in task-centered ways that align with the Mathematical Task Framework that I am employing in my analysis (Senk & Thompson, 2003), though this is certainly not a perfect correspondence. Methodologically, this choice means that I will be unable to draw conclusions about mathematics teaching practices in general (e.g., purely lecture-based instruction), but there is value in examining the variation amongst the enacted curriculum of several different teachers operating within the same general instructional scheme and its relation to student learning. A certain degree of focus is required for any productive research endeavor.

Within *Connected Mathematics*, the unit entitled *Variables and Patterns* (Lappan et al., 1998b) contained the lessons identified by the teachers and the *Project 2061* staff as relating directly to the focal learning goal—in particular, the last part of Investigation 3 (pp. 40–41) and the entirety of Investigation 4 (pp. 49–53).<sup>13</sup> These activities involve modeling various relationships within the context of a bicycle tour business. For example, there are costs and profits that vary with the number of customers served on the bike tour, and in Investigation 4 there are relationships between time and distance that depend on traveling speed. The written tasks involve tables, graphs, and the explicit development of variables as a means of encoding the relationships in mathematical equations.

Within *Mathematics in Context* (National Center for Research in Mathematical Sciences Education & the Freudenthal Institute, 1998), the focal lessons occur in the *Graphing Equations* unit, particularly section E entitled *Solving Equations* (pp. 31–34).<sup>14</sup> These activities are built

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<sup>13</sup> The three teachers using *Connected Mathematics* all taught grade 7, which is where the lessons focused on using variables to represent changing quantities were located in the curriculum materials. The other teachers taught grade 8 (see Table 1).

<sup>14</sup> *Project 2061* identified pages 31–39 as relevant to the focal learning goal. I, however, restricted attention to pages 31–34 for two reasons. First, even though the textbooks items are numbered consecutively from page 31 to page 39, implying a unified set of tasks, I believe that the latter portion of these pages focuses on solving linear equations rather than using variables to write equations. Tasks included from other textbooks do not emphasize equation solving in that

around the context of frogs jumping. Students are expected to model with diagrams the relationship between frogs' starting positions, the length of their jumps, and their subsequent positions. The variable  $x$  is used as a label for unknowns in the diagrams and the written tasks involve students writing equations and solving for  $x$ . These activities are similar to those in *Connected Mathematics* in that a non-mathematical context was used as the source of quantitative relationships and variables were used to represent those relationships in mathematical equations. *Mathematics in Context* is different, however, because all the equations were of the same form (i.e., frog's starting position plus the product of the jump length and the number of jumps, with the jump length unknown) and involved the same aspects of the problem context. In the *Connected Mathematics* activities, there were several different equations generated from the various relations embedded in the bicycle tour context.

Within *Math Thematics: Book 3* (Billstein et al., 1999), the unit entitled *Equations and Graphs* contained an exploration in which students were expected to use variables to write an equation capturing the relationship between a car's speed and the recommended safe following distance (pp. 179–181). Later in the same unit (pp. 189–192), there was an exploration where students work from a mystery story to write equations that relate the amount of time a missing person spent walking to the distance that person would have traveled. Similar to the previous textbooks, tables and graphs were explicitly included in both explorations. It should be noted, however, that Ms. Cesky was the only teacher who used the actual *Math Thematics* textbook in her classroom enactment. Ms. Mendoza, for whom there are three sets of lessons included in this

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way. Second, one of the teachers using *Mathematics in Context*, Ms. Albert, had a video error that made analysis of the last day of enactment impossible.

study, had access to the *Math Themes* textbooks but, for the focal lessons, used supplementary materials in the form of loose-leaf handouts of student activities. Her activities, of which there were three, followed a similar format of presenting a visual pattern (e.g., arrangements of dots, larger and larger buildings, larger and larger pinwheels) and asking the students to fill in a table and write an equation with variables that relates the sequence term to the number of objects in the pattern. Although she consistently used this format, the patterns themselves changed from year to year. The pattern of buildings was the only carry-over activity, which she used in AY 2003–2004 and AY 2004–2005.

Ms. James, a teacher with a classroom set of *Glencoe* (Collins et al., 1998) textbooks, also used loose-leaf student activities to teach the focal learning goal. Her activities, however, varied in type. The first activity involved students stacking cups, recording the height of the stack for different numbers of cups, and developing an equation to model this relationship. The second activity was calculator-based, with students comparing given equations to their graphs. The third activity involved the non-mathematical context of comparing cell phone plans, with students expected to generate tables, graphs, and equations to represent the cost of the plans and then compare and contrast those representations.

In summary, this study involves 9 middle school mathematics teachers, 246 middle school students, and 12 sets of lessons. The school settings vary, as do the curriculum materials being used, but there is consistency in the learning goal that is the focus of the sets of lessons, namely, using variables to represent relationships between changing quantities. There is also similarity with respect to the fact that the lessons involve tables and graphs, with the variables being employed to generate mathematical equations. In the next sections, I describe the specific

data used in this study and the processes of analysis that were undertaken to answer the research questions.

## **Data and Analysis**

### **Classroom Data**

The study draws on two general categories of data: classroom data of the enacted curriculum and student pre- and post-test data used as a measure of learning. The classroom data consist of video recordings from a single camera located near the back of each classroom. Additionally, I used textbooks and instructional artifacts (e.g., copies of handouts) as an aid for interpreting the classroom events from the video recordings. Classroom data were collected for every class period that the teacher, together with *Project 2061* staff, identified as being directly related to the focal learning goal. There are, therefore, different numbers of observations per teacher depending on how many lessons were devoted to the objective, with 2 class periods being the minimum and 9 class periods the maximum (see Table 4). Four sets of lessons were drawn from the baseline year (i.e., the first year of *Project 2061*, before any substantial PD intervention), six from the second year of the project, and two from the third year, totaling 12 sets of lessons and 52 class periods. Although the summer PD sessions may have led to changes in the teachers' practices during the second or third year—and there is evidence that they did (Wilson & Roseman, under review)—this possibility does not invalidate the current study, which focuses on enactment and student learning, not the influences on enactment or changes in enactment over time. In other words, this study is not concerned with the forces that may have shaped the enacted curriculum in various ways but rather the enacted curriculum as it is and how it might relate to student learning.

Table 4

*Number of video-recorded lessons used in this study for each participating teacher*

	Albert	Cavillon	Cesky	DePalma	James	Johnson	Mendoza	Milson	Wyncott
AY 2002–2003		6				7	4	7	
AY 2003–2004	3		3	3	3		3		2
AY 2004–2005		7					4		

### Student Data

In this study, pre- and post-tests of mathematical content knowledge serve as the measure of student learning in the effort to answer the research questions. The pre- and post-tests were administered to students by their teachers, in collaboration with *Project 2061* staff. The tests were designed to be equivalent to one another, exhibiting only changes of values within items that were intended to leave the difficulty of the items unchanged. The tests contained different item formats, specifically, seven multiple-choice items, eight constructed-response items, and one multi-part item designed to assess higher-level thinking in ways that are difficult through single-part items. The tests were designed by *Project 2061* staff to assess the focal learning goal of using variables to represent changing quantities. This focus does not imply, however, that all of the items directly involve using variables to encode a relationship between varying quantities. There are items addressing, for instance, students' facility with equations containing unknowns, their understanding of symbolic representations, and their knowledge of tables and graphs (see sample items in Figure 6; Appendix A contains an example of an entire test). The rationale behind this instrument design is that pieces of mathematical knowledge do not exist in isolation

from one another but are connected in a web of meaning; the tests are designed to assess not only the focal learning goal but also adjacent ideas and skills in the web of meaning.<sup>15</sup> With that being said, the tests do contain items that closely mirror the process entailed in all of the different classroom tasks, that is, modeling a nonmathematical relationship between quantities by assigning variables and generating an equation. On the tests, these came in the form of a phone plan where the cost depended on the number of minutes used and a patio pattern where the number of tiles depended on the size of the patio. *Project 2061* staff piloted and analyzed the alignment of the tests to the learning goal prior to implementation.

<p>Mary has some trading cards. Julie has 3 times as many trading cards as Mary. They have 36 trading cards in all.</p> <p>Which of these equations represents their trading card collection?</p> <p>A. <math>3x = 36</math>            B. <math>x + 3 = 36</math>            C. <math>x + 3x = 36</math>            D. <math>3x + 36 = x</math>.</p>	<p><math>a = b - 2</math> is a true statement when <math>a = 3</math> and <math>b = 5</math>.</p> <p>Find a different pair of values for <math>a</math> and <math>b</math> that also make this a true statement.</p> <p><math>a =</math> _____</p> <p><math>b =</math> _____</p>
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Figure 6. Sample items from the student pre- and post-test.

The pre-test was administered at the beginning of the academic year in which data were collected and the post-test was administered several weeks after the lessons related to the learning goal were complete. Specific timing varied depending on each teacher's schedule and

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<sup>15</sup> This notion of a web of meaning relates to the theoretical framing of this study in which the meaning construed through mathematics discourse also depends on webs of semantic relations rather than single terms having meaning in and of themselves.



preferences. *Project 2061* staff scored the multiple-choice items for correctness and used a 3-point rubric to score the constructed-response items and each part of the multi-part item. This process yielded a single numeric score less than or equal to 27 for each student. Inter-rater agreement, as calculated on the *Project 2061* data overall, was above 90% (Wilson & Roseman, under review).

In the present study, as mentioned above, the unit of analysis is not the student or the teacher but the set of lessons ( $n = 12$ ) around the focal learning goal. Thus, the standardized gain scores between the 12 class averages on the pre- and post-tests were used to represent student learning, rather than individual student scores. Additionally, by using the gain scores rather than simply the post-tests scores, there is some accounting for differences in prior mathematical knowledge that students may have had in the web of content around variables representing changing quantities. Also particular to this study, as described below, is the attention to cognitive demand and student participation. An analysis of the pre- and post-tests revealed that there were multiple items of both low and high cognitive demand, which raised the possibility of detecting relationships between student learning and differences in the cognitive demand of the enacted curriculum. Yet, with respect to student participation, which in this study is restricted to verbal participation, the pre- and post-test constitute a different medium of communication. This misalignment between what is observed in the enacted curriculum (i.e., verbal participation) and what is observed as an outcome (i.e., students' written work and solutions) means that I was unable to draw conclusions about how students' participation during the focal lessons impacted their ability to participate mathematically in the future, but this is not unexpected. The premise for the study, as outlined in Chapters 1 and 2, is that student participation is an important component of students' learning of mathematics overall, which consists of both individual

knowledge and skill development as well as community membership. The student data collected through the pre- and post-tests allows for the more indirect relationship between student participation and individual knowledge and skill development to be examined.

### **Analysis for RQ1**

The analysis of data was oriented around the research questions presented above. First, I describe my processes for examining the relationship between the enacted curriculum and student learning, through the lens of cognitive demand and student participation.

**Phases of task enactment.** Within the 52 study class periods, there were 37 enacted mathematical tasks (some lasting more than one class period). The beginning of a task enactment was identified by the teacher calling for a shift in attention to the written task, either by handing it out to students or asking the class to turn to the particular page in their textbooks. The end of the task was marked by a shift to another mathematical or academic task. A teacher may have returned to one of these mathematical tasks at a later point in time, for example, if ideas from the task came up in a subsequent discussion or if a student asked a question about the task during a review of the unit. Such cases were not captured in the data used for this study.

As noted in Chapter 1, a mathematical task has been defined as a problem or set of related problems designed as a cohesive whole, focusing on a particular mathematical concept, idea, or skill (adapted from Stein, Grover, & Henningsen, 1996). In this study, the mathematical tasks corresponded with single “investigations” or “explorations” in the textbooks or with individual hand-outs prepared by the teacher. After identifying the enacted mathematical tasks, I parsed them into the phases of the modified MTF (see Figure 3 in Chapter 1). The parsing of phases was based on the teacher’s directives, the apparent purpose of segments of the lesson, and the discourse cues documented in my previous work (Otten, 2010a). In general, the set-up phase

involved the teacher addressing the entire class with the intention of clarifying directions, orienting students to the content or context of the task, and, in some instances, the teacher leading preliminary work on the task as an example of how to begin. The working phase is characterized by actual progress being made (or expecting to be made) in solving the task. This phase varied in its talk formats from students working independently to students working in small groups to teachers leading as the whole class worked through the task together. The look-back phase—which is described in detail in Otten (2010a), where it is referred to as the conclusion phase<sup>16</sup>—is typically a whole-class interaction, like the set-up phase. It is defined by the fact that the teacher and students are no longer expected to progress on the task but instead are looking back at the work that they undertook during the working phase. It is important to

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<sup>16</sup> After several trial labels, I now call the final phase of task enactment the *look-back phase*, a phrasing that is meant to parallel to the set-up phase and is an attempt to be equally broad in scope because it does not define the purpose one has for looking back. In my initial work, I avoided the term *summary* because it connotes a specific purpose one might have for the final phase. I also considered calling it the *whole-class discussion* phase, but this phrase highlighted the talk format rather than the stage in the task's lifespan, and teachers might use a whole-class format for the other phases as well or may not truly have a discussion during the phase in question. In Otten (2010a) I used conclusion phase to convey that it was the last phase of enactment, but later I found it difficult to escape the connotation that conclusion means wrapping up or shutting down, even though this phase may involve opening many ideas up. After deciding that *collective reflection phase* was too bulky, I landed on the *look-back phase* which I am quite pleased with thus far.

note that work still takes place during this phase, but it is of a different kind as the teacher and students may, for example, reflect, draw connections, make generalizations, summarize, or formalize their thinking on the task. Alternatively, they may share and check answers. Just as the set-up phase entailed collectively looking ahead to the work that would be undertaken, the look-back phase entails collectively looking back at the work that was just undertaken.

Although some task enactments proceed linearly from set-up to working to look-back phases, there are many variations to this sequence. For example, a teacher may briefly go over the directions to a task and then have students work for several minutes. The teacher may then pull the class back together to clarify an important aspect of the task context and take questions about the directions before setting the student back to work. In this case, neither the set-up phase nor the working phases were contiguous in time. A similar thing can happen with the look-back phase if a teacher stops the class mid-way through the task to reflect on what they have done so far and then leads a whole-class discussion at the end of the task to formalize their results. Both of these events would be part of the look-back phase of the task enactment. Even though there can be non-linearity through the phases, these phases served as the analytic framework for the RQ1 coding of the enacted curriculum and so, as described below, multiple segments of a particular phase (e.g., a set-up phase that existed as two separate portions of a class period) were merged and considered as one entity of enactment.

**Cognitive demand.** The levels of cognitive demand, from high to low, are *doing mathematics*, *procedures with connections to meaning*, *procedures without connections to meaning*, and *memorization* (see Figure 7). There are other levels as well, such as *non-mathematical activity* or *unsystematic exploration*, which were not explicitly used in the present coding scheme because they do not represent cognitive demand of mathematical activity.

Moreover, because the data for this study did not allow a close inspection of each student or each small group as they worked, analysis of cognitive demand was restricted to those phases other than the working phase. That is, the written task and the set-up and look-back phases of enactment were each coded using the existing framework of cognitive demand. For example, a single task enactment may have been written at a level of doing mathematics, set up at a level of procedures with connections to meaning, and the look-back phase may have declined to procedures without connections to meaning. The characteristics of each level as articulated by Stein, Smith, Henningsen, and Silver (2000, p. 16) were used as the basis for the code determinations, with at least two characteristics from a particular level being required to warrant coding at that level. If characteristics were present from multiple levels simultaneously, the phase in question was assigned a code at the higher of the levels. The numbers 4, 3, 2, and 1 were used to mark each phase with a level (see Figure 7).

	Level	Description
High	(4) Doing mathematics	Exploring novel mathematical ideas and engaging in complex, mathematical thinking and reasoning strategies such as conjecturing, justifying representing, or looking for patterns. Tasks typically involve only a few problems.
	(3) Procedures with connections to meaning	Using formulas, algorithms, or well-defined procedures and understanding why they work, making sense of the procedures in relation to mathematical concepts. Tasks typically involve only a few problems.
Low	(2) Procedures without connections to meaning	Using formulas, algorithms, or well-defined procedures that have been previously learned or exemplified without providing conceptual explanations. Tasks can involve dozens of similar exercises.
	(1) Memorization	Committing facts, rules, or definitions to memory or reproducing previously learned facts, rules, or definitions without the use of a procedure. Tasks can involve dozens of similar exercises.

*Figure 7.* An analytic framework for cognitive demand of mathematical tasks, adapted from Stein, Grover, and Henningsen (1996) and Stein, Smith, Henningsen, and Silver (2000).

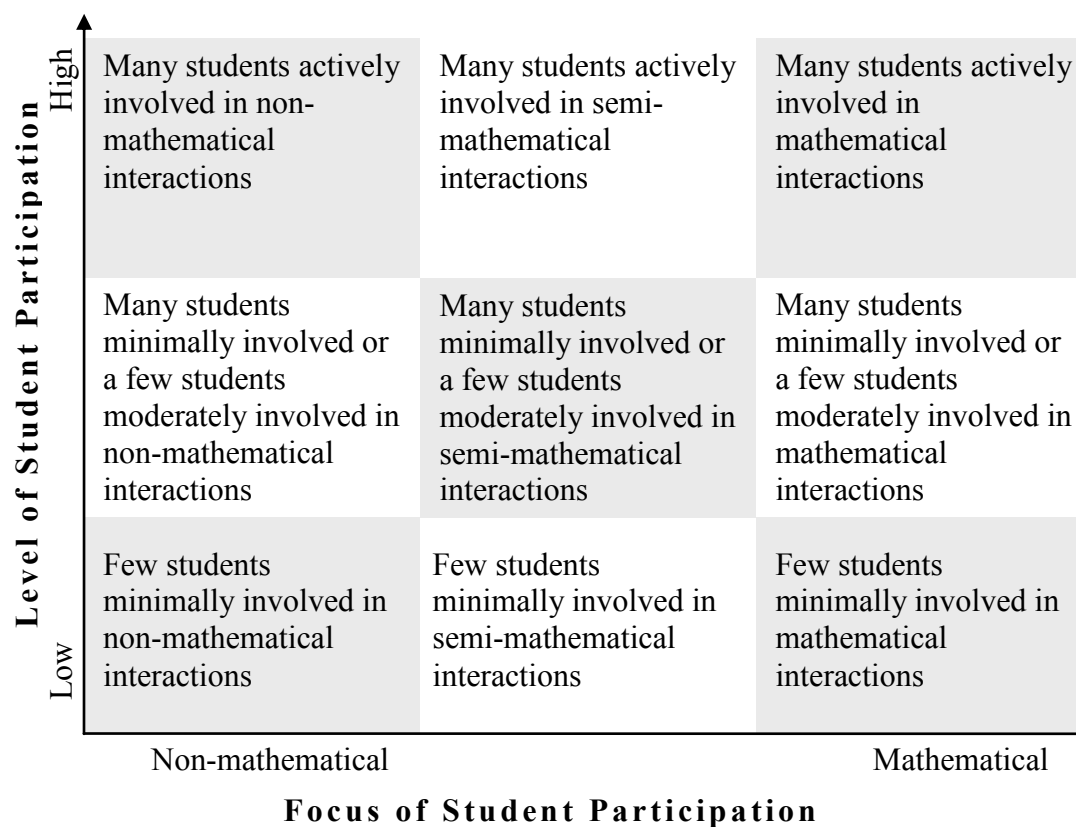
For written tasks and tasks as set up by the teacher, cognitive demand was coded according to what was expected of the students if they were to complete the task as indicated in the directions. For the look-back phase, coding was based on the public processes being undertaken in the class or encouraged by the teacher, looking at the classroom overall rather than at individual students. As Stein and colleagues (1996) stated, “When coding the cognitive demands of the task as implemented, coders were asked to make judgments about the kinds of cognitive processes in which the majority of the students appeared to be engaged” (p. 467). It is important to note that cognitive demand depends heavily upon the classroom context. For example, a doing mathematics task in a middle school algebra class might be at the procedures-without-connections level or the memorization level in a university calculus class.

A second mathematics education researcher analyzed 15% of the enacted mathematical tasks to verify reliability of the cognitive demand coding scheme used in this study. With respect to the four levels of cognitive demand, initial interrater agreement was less than 60%, which was deemed unacceptable. We realized upon inspection, however, that the majority of disagreements in the coding were within the high and low categories of cognitive demand (i.e., 4 vs. 3 or 2 vs. 1) rather than between the high and low categories. Inter-rater agreement was 83% when the levels of cognitive demand were collapsed to the binary classification of high or low. This collapse of codes maintains integrity of the construct because the authors of the framework themselves have stated that the distinction between high and low cognitive demand is more significant than the distinctions within those categories (Stein et al., 2000). Hence, for analysis, only the distinction between high and low cognitive demand was used. Discrepancies were resolved by myself based on notes left by the second coder and by rewatching the classroom video as necessary.

**Participatory demand.** Similar to the process used for cognitive demand, a single participatory demand code was given to the written task, the set-up phase, and the look-back phase for each of the 37 mathematical task enactments. Again, the working phase was omitted due to limitations in the data. Recall from Chapter 2 that participatory demand is defined as the extent and nature of the interactions expected of students during the enactment of a mathematical task. As with cognitive demand, there is a subtle shift in the meaning of this construct between the written and set-up phases and the working and look-back phases. In particular, the demand of a mathematical task before implementation is thought of based on what would be entailed if the task was enacted as intended. In other words, cognitive demand and participatory demand of written tasks and tasks as set up by teachers refer to the expectations of student thinking and interaction, respectively. During the working phase and the look-back phase, however, these constructs shift and now refer to the actual thinking (as inferred by teachers and researchers) and interactions students engage in at those times. It is no longer a future expectation but in fact a representation of what is believed to have occurred.

In the present study, the original intention for participatory demand coding was to look for explicit expectations of student participation embedded in the written tasks (with the understanding that these may often be absent) and to look for teachers explicitly outlining during the set-up phase their expectations for student participation on the task. During analysis, however, this approach had to be modified as the majority of teachers made no explicit mention of their expectations for student participation. (This absence is itself a finding that is mentioned in Chapter 4.) Instead, the set-up phases were coded for participatory demand based on actual student participation, as was done for the look-back phases.

Pilot analysis (Otten, 2010b) suggested that a framework for participatory demand could be constructed along two dimensions: the level of student participation (how involved are students in the classroom discourse?) and the focus of student participation (how mathematically-oriented is the content of students' contributions?). Figure 8 depicts this initial framework. Although these dimensions are related (e.g., students talking more often are more likely to say something mathematical), it is important to capture both aspects of participation because it is possible for students to be highly involved in a classroom discussion that is non-mathematical, such as a discussion of the context of a task.



*Figure 8.* An initial analytic framework for participatory demand of mathematical tasks.

Although the initial framework in Figure 8 gives a general sense of the categories of participatory demand, it is not specific enough to allow for reliable coding of the enacted



curriculum to take place. Thus, I make explicit the dimensions along which I operationalized this construct.<sup>17</sup> For the *level of student participation* dimension, I used the number of students speaking on task (i.e., off-task or non-endorsed spoken contributions were not considered) and the quantity of student speech. In particular, a phase of enactment in which only a few students speak and their utterances are brief responses, such as those in an Initiate-Respond-Evaluate interaction pattern (Mehan, 1979), constitutes a *low* (L) level of student participation. On the other hand, a *high* (H) level of student participation involves multiple students contributing paragraph-length utterances to the publicly-endorsed classroom discourse. A *medium* (D) level of student participation was defined with respect to low and high levels. Specifically, student participation that was not identifiable as low or high was given a medium code for level of student participation. Students reading aloud directly from the written task were not included in the consideration of level of participation.

The focus of student participation dimension was operationalized through thematic discourse analysis (Herbel-Eisenmann & Otten, 2011; Lemke, 1990). In particular, thematic discourse analysis involves the identification of thematic items, which are essentially mathematical terms or concepts such as variable, expression, equation, or multiplication, that are present in the discourse as well as the identification of semantic relationships between those

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<sup>17</sup> These operationalizations are certainly not the only ones possible. For example, level of student participation may be tracked on an individual rather than collective basis or focus of student participation may be tied more to particular mathematical ideas or strategies rather than general semantics. Also, participation overall may be conceived as including more than just verbal contributions, which is a justifiable position, though not the one taken in this study.

thematic items, such as subset/superset, process/goal, or part/whole. Interactions in which the thematic items of the students' discourse are predominantly non-mathematical, for example, they reference the context of a real-world problem rather than the mathematical concepts, were coded as *non-mathematical* (N) in focus. If mathematical thematic items appear in students' discourse but are almost always expressed in isolation from one another, with semantic relations between items rarely occurring in the student talk, then the *semi-mathematical* (S) code was used. In most instances of semi-mathematical participation, there were still mathematical relations present in the discourse but the teacher was verbalizing the relations while students were only supplying individual semantic items. Finally, if mathematical thematic items and semantic relationships were both expressed in students' discourse, this was coded as a *mathematical* (M) focus.

Examples of these codes are presented in Table 5 and examples of maps of semantic relations in Figure 9. (See Appendix B for notes on the transcriptions in this study.) There were also some auxiliary codes included for participatory demand. In particular, for written tasks, an *implicit* (I) code was used if there was no mention at all of expected student interactions and, in the case that there was no student verbalization during a phase of enactment, an L code was given for level of participation but an O code (resembling a 0) was given for focus of participation to represent the fact that there was no student discourse with which to have a focus.

Table 5

*Examples of participatory demand codes*

Source	Data Excerpt	Code
Ms. Cavillon 2/10/2003 Set-up phase	Ms. Cavillon: Alright, let's look at [Investigation] 4.2. Keep your [Investigation] 4.1 paper out, though. Keep your paper out and look at the bottom of page 51. [[Student reads aloud the introductory paragraphs of the task.]] How many times have you been on a trip, going somewhere, you're making good time but then you have to stop? And you get McDonald's, you have	Level: L  Only 6–7 students speak at all and 3 utterances are single words.

Table 5 (cont'd)

	<p>that urge for those French fries. Or nature's telling you you need to do something. Or you're thirsty, or whatever the case might be. Okay. You see a Cracker Barrel and you're determined to always stop at every Cracker Barrel you see on the way.</p> <p><i>Student:</i> Yeah.</p> <p><i>Ms. Cavillon:</i> Because they might have the new Beanie Baby. You know that's where they started, right?</p> <p><i>Student:</i> Yeah.</p> <p><i>Ms. Cavillon:</i> Did you know that?</p> <p><i>Sally:</i> What?</p> <p><i>Trenton:</i> I knew that.</p> <p><i>Ms. Cavillon:</i> Beanie Babies became big at the Cracker Barrels. <i>[[Ms. Cavillon briefly describes the spread of Beanie Babies to their region of the country and a student says that they know someone with over 100 Beanie Babies.]]</i> Anyway, going on, when you're traveling on the interstate, you might be doing a really good distance, er, speed. But then you stop. What does that do to your speed?</p> <p><i>John:</i> It makes the average go down.</p> <p><i>Ms. Cavillon:</i> It makes the average go down. Very good, John. <i>[[Ms. Cavillon relates a stopped car to having a 0 on a grade sheet, bringing the averages down.]]</i> So, what they want you to do for letter A is... <i>[[Ms. Cavillon goes over the task directions.]]</i></p>	<p>Focus: <b>N</b></p> <p>Looking only at the student turns, all but one of the turns are absent of mathematical thematic items. John does utter a mathematical relation, but this alone does not warrant an S or M code.</p>
<p>Mr. Milson 10/7/2002 Look-back</p>	<p><i>Mr. Milson:</i> ...Now, [part] C [of Investigation 4.1]. If the students continue driving at their steady rate of 55 miles per hour, how far would they go in 10 hours, 12-and-a-third hours, 15 hours? Ten hours, how far would they go, Gibson?</p> <p><i>Gibson:</i> Probably, 550.</p> <p><i>Mr. Milson:</i> 550 miles. Good job. What about 12 and a third hours? <i>[[pause]]</i> Allison, how far would they go in 12 and a third hours?</p> <p><i>Allison:</i> Um, I don't know.</p> <p><i>Mr. Milson:</i> Robert.</p> <p><i>Robert:</i> 678.</p> <p><i>Mr. Milson:</i> 678 miles. Good estimate. And finally, 15 hours. Javon.</p> <p><i>Javon:</i> Um, can I guess, like, 750?</p> <p><i>Mr. Milson:</i> A little low. Emily?</p> <p><i>Emily:</i> 825.</p> <p><i>Mr. Milson:</i> Good. 825. Okay, all good... <i>[[Mr. Milson continues on to parts D and E.]]</i> What would we use to write that [rule] as an equation, then? Brad?</p>	<p>Level: <b>D</b></p> <p>7 students speak during this excerpt, and others were involved at other times in the look-back phase, warranting a D code.</p> <p>Focus: <b>S</b></p> <p>The students verbalize mathematical thematic items but Mr. Milson</p>

Table 5 (cont'd)

	<p><i>Brad</i>: Um, times, well, multiplication.</p> <p><i>Mr. Milson</i>: Good. <i>[[Mr. Milson writes <math>55 \times h = d</math> on an overhead transparency.]]</i> That's fine, 55 times <math>h</math> equals <math>d</math>, which is distance. <i>[[Tony raises his hand.]]</i></p> <p>What is it, Tony?</p> <p><i>Tony</i>: <math>D</math> equals 55 <math>h</math>?</p> <p><i>Mr. Milson</i>: That's fine. Means the same thing... <i>[[Mr. Milson explains that the multiplication symbol can be omitted and that the choice of letters is arbitrary.]]</i></p>	tends to form the semantic relations.
<p>Ms. DePalma 4/7/2004 Look-back</p>	<p><i>Ms. DePalma</i>: ...Make sure you're watching [as students come to the overhead projector to present their work on problem 17].</p> <p><i>Mary</i>: <i>[[Mary places her diagram of a frog starting 5 units to the right and jumping an unknown distance to the right 4 times.]]</i> She starts at 5 and then she takes four jumps forward.</p> <p><i>Ms. DePalma</i>: Good. And she has an <math>x</math> over each jump. <i>[[Mary removes her transparency and sits back down.]]</i></p> <p>And [part] B, James?</p> <p><i>James</i>: <i>[[James places his incorrect diagram of a frog starting 12 units to the right and jumping an unknown distance to the left 6 times.]]</i> The starting point's 12 and he jumps back 6 times.</p> <p><i>Ms. DePalma</i>: Okay, and we need to fix something on James's. Does anyone know what we have to fix?</p> <p>Clark?</p> <p><i>Clark</i>: Well, it subtracts the six jumps, and those are going backward, so it's going to be <math>6x</math> equals 12.</p> <p><i>Ms. DePalma</i>: Um, <i>[[pause]]</i> close. Isaiah?</p> <p><i>Isaiah</i>: You have to put the negatives on the <math>x</math>.</p> <p><i>Ms. DePalma</i>: Yeah, put negatives in front of the <math>x</math>'s to notify that you're going backwards. And there's something else, too. Look at [James's] starting point.</p> <p><i>Student</i>: It's wrong.</p> <p><i>Ms. DePalma</i>: Pete, do you know?</p> <p><i>Pete</i>: I think [the starting point of the jumps] is supposed to be on the other end, like, the end of the <math>x</math>'s.</p> <p><i>Ms. DePalma</i>: Why?</p> <p><i>Pete</i>: Um, because they're jumping--, um, they're starting at the 12 and jumping backward 6 times.</p> <p><i>Ms. DePalma</i>: Right. Do you understand that, James? <i>[[James nods]]</i> You're 12 should be at the end of that diagram because... <i>[[Ms. DePalma corrects James's diagram and then continues with the look-back phase.]]</i></p>	<p>Level: <b>H</b></p> <p>Throughout the look-back phase, nearly 20 students contribute verbally to the classroom discourse. Also, as seen in this excerpt, many student turns are sentence or paragraph length.</p> <p>Focus: <b>M</b></p> <p>Semantic relations between mathematical terms are expressed in the students' talk (e.g., Clark's turn, Isaiah's turn, Pete's turns).</p>

Ms. Mendoza

11/18/2004

Set-up

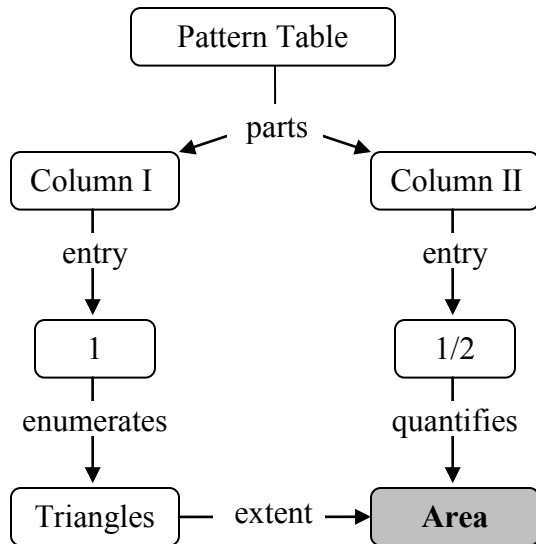
*Ms. Mendoza:* *[[Discussing a pinwheel pattern composed of a certain number of triangles, each with an area of one-half square units.]]* The 1 represents the number of triangles in our pattern. Where's the half coming from? It's the what?

**Students:** The area.

*Ms. Mendoza:* The area of that one triangle. That shape. So the two things that we're looking at is, one, the number of triangles that we're using, and the other thing is the \_\_\_.

**Students:** The area.

*Ms. Mendoza:* The area of the shape that we've made.



Ms. Albert

4/1/2004

Look-back

**Clark:** *[[Discussing distances composed of a starting distance and some number of equal jumps.]]* **I found out that, if each jump was between two and six decimeters, Fred will finish between twenty-four and thirty-six decimeters because three times six decimeters equals eighteen because Fred jumps three times at six decimeters, equals eighteen, and eighteen plus eighteen is thirty-six, because Fred starts at eighteen. And two times three because Fred jumps two times and three decimeters, it equals six, plus eighteen equals twenty-four.**

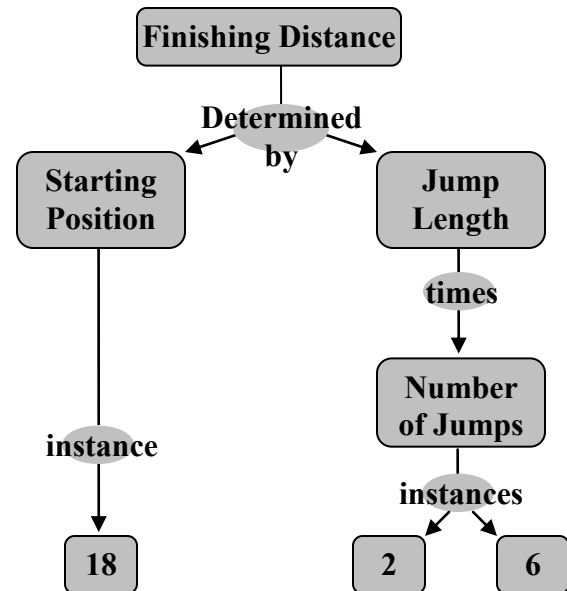


Figure 9. A thematic map in which students supply only a mathematical term (S) and one in which a student supplies a full semantic relation (M). Student contributions are shaded.

As with cognitive demand, the most generous code within reason was given. For instance, if 2 minutes of a 6-minute whole-class discussion involved students expressing semantic relationships between multiple mathematical thematic items, and the other 4 minutes involved students only expressing mathematical thematic items in isolation, this phase would be given an M code because that is the most generous code applicable. If, on the other hand, only 30 seconds of a 10 minute whole-class discussion involved students expressing semantic relationships, this does not reasonably warrant an M code.

Based on the second coder's analysis of the 15% subset of mathematical task enactments, the inter-rater agreement for participatory demand was 94% on level of student participation and 83% on focus of student participation. This agreement was considered sufficient to allow for conclusions to be drawn about participatory demand in this study. In fact, the agreement is quite high when one considers the inherent messiness of the enacted curriculum in schools. Discrepancies were resolved by myself based on notes left by the second coder and by rewatching the classroom video as necessary.

In this operationalization, participatory demand comprises only students' verbal contributions in class. This restriction is based on practical factors, such as observability and the limited time and resources of the present study, and should not be construed as a philosophical privileging of verbal participation over other forms of participation. I fully recognize that a great deal of student participation, possibly even the vast majority, involves listening and thinking about the mathematical ideas at play in a given lesson. My colleagues and I have, elsewhere, considered the role of listening in classroom discourse (Otten, Herbel-Eisenmann, Cirillo, Steele, & Bosman, 2011), and in this study, aspects of student thinking are captured by the cognitive demand construct. Reading is another important form of student participation and the study

lessons do frequently involve students reading aloud from their textbooks. As mentioned above, however, these verbalizations were not considered with respect to participatory demand because they are different than when students speak up to verbalize their own thoughts or ideas.

Furthermore, as articulated in Chapter 2, there is a rationale for focusing on verbal discourse because coming to do and learn mathematics is inseparable from coming to engage in mathematics discourse—mathematical thoughts are meant to be communicated and placed into dialogue with the mathematical thoughts of others.

**Distinguishing cognitive demand and participatory demand.** Because participatory demand is a new construct intended to complement the well-established framework of cognitive demand of mathematical tasks, it is important to stop and consider whether the two are theoretically separable. One might suppose, for example, that a high level of mathematically-focused student participation occurs precisely when students are engaged with a cognitively demanding task. I contend, however, that the two constructs are theoretically separable.<sup>18</sup> For example, consider a task enactment in which many students talk about the procedures they carried out to solve the task. Such interactions were described as calculational discourse by Thompson, Philipp, Thompson, and Boyd (1994). Because students in this case are highly involved in the classroom discourse and may be articulating semantic relations as they describe their procedures (even if they are not necessarily conceptual relations), this would be an H / M category of participatory demand. Because the work and discussion of the task are procedurally focused, however, without a press for understanding why the procedures are valid, this would be a low level of cognitive demand. Conversely, consider a task enactment in which high levels of

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<sup>18</sup> Chapter 4 contains evidence that the constructs are also empirically separable.

cognitive demand are maintained throughout the task phases, but students work individually and in complete silence. This situation would constitute low participatory demand. These thought experiments suggest that the two constructs are capturing different aspects of the enacted curriculum.

Another distinction to note between cognitive demand and participatory demand is that the former is ordinal whereas the latter is categorical. In other words, there is a meaningful order to the original four levels of cognitive demand as well as to the collapsed codes of high and low cognitive demand. For participatory demand, however, there is no reason to believe that, for instance, an H / S code is inherently “higher” than a D / M code or even an L / N code. When ascribing value to participatory demand, it is necessary to consider many other factors, such as the teacher’s intention and the needs of the students, and to recognize that different types of participatory demand may be appropriate at different points in the task enactment. For example, a low level of student participation may be appropriate if the teacher is trying to give directions during the set-up phase and a non-mathematical focus may be appropriate if students are attempting to connect what they have found mathematically to the non-mathematical context of the task. In the next section, I use H / M as a benchmark for considering participatory demand in relation to student learning, but not because H / M is necessarily the “best” form of student participation—this category is singled out simply because it stands at the upper-right extreme of the framework.

**Relating enactment to student learning.** Following the analyses described above, each written, set-up, and look-back phase of each mathematical task enactment has attached to it codes for cognitive demand and the two aspects of participatory demand (i.e., level of student participation and focus of student participation). To compare these features of the enacted



curriculum with student learning, Spearman's rank correlation coefficient (Myers & Well, 2003) was used. This statistic, because it is nonparametric in nature, does not require an assumption of normality in the variables' distributions and is appropriate for small sample sizes (Ott & Longnecker, 2001), such as that in the present study. To calculate Spearman's rank correlation coefficient, the 12 sets of classroom lessons were ranked separately according to cognitive demand, participatory demand, and standardized gain scores on the pre- and post-test. With respect to cognitive demand, I calculated for each of the 12 sets of lessons the percent of enacted tasks that had at least one enactment phase (set-up or look-back) at a high level. These percents were used to rank the sets from highest to lowest. Ties were broken first by the prevalence of high level phases during enactment (e.g., set-up and look-back phases at a high level were ranked above only the look-back phase at a high level) and next by the percent of high-level written tasks. In this way, the cognitive demand of both the written task and the enacted task were considered, but enactment was given priority. Because participatory demand is a two-dimensional variable, a simple ranking was not possible. Instead, I ranked the sets of task enactments from high to low in terms of the percent of enacted tasks that had at least one phase with H / M participation. Ties were broken based on the prevalence of other phases with a mathematical (M) focus of participation.

Spearman's rank correlation coefficient was calculated between the cognitive demand ranking and the participatory demand ranking to confirm that the constructs were distinct (i.e., they were not correlated). Then, Spearman's rank correlation coefficient was calculated between cognitive demand and the standardized gain scores as well as between participatory demand and the standardized gain scores. The resulting statistics, like other estimates of correlation, describe the strength of relationship between the given variables. For example,  $\rho = 0.85$  for cognitive

demand and gain scores would indicate a fairly strong positive relationship between those variables, thus confirming past research (Stein & Lane, 1996). If  $\rho = 0.15$  for participatory demand and class gain scores, then this would fail to provide evidence in support of a strong positive relationship between those variables. Each calculation of  $\rho$  was accompanied by a Student's  $t$ -test of significance to determine the probability that observed differences in ranking arose by chance.

This study's investigation of the relationship between cognitive demand of mathematical task enactments and students' mathematical learning is a loose replication of past work that identified a positive relationship (e.g., Stein & Lane, 1996). Similarities include the notion of cognitive demand itself as a potential predictor of student learning, the situating of the study at the middle school level, and the observation of the enacted curriculum paired with written assessments of students. There are, however, important differences between the studies. In particular, Stein and Lane (1996) employed greater resources and investigated many more classrooms than in the present study, allowing for a more refined (and powerful) statistical approach. Stein and Lane also considered task features along with cognitive demand, used the original MTF without distinguishing between the working and look-back phases, and traced the trajectory of cognitive demand through the phases of enactment rather than simply looking for the presence of high cognitive demand. They also needed to refine their outcome measures in important ways in order to be able to detect the positive differences in student learning that were correlated with the maintenance of high cognitive demand. Because of these and other distinctions in method, this study cannot be described as a replication of Stein and Lane (1996) or other work from that program (e.g., Stein, Grover, & Henningsen, 1996). Moreover, whereas

Stein and Lane (1996) used the school as their unit of analysis, with a sample size of 2, the present study uses sets of lessons within a classroom as the unit of analysis.

Even with the differences in mind, however, it is reasonable to hypothesize based on this past work that the students in the present study who are exposed to a greater degree of high cognitive demand will exhibit higher gain scores. One might also hypothesize, based on the importance ascribed to student discourse in mathematics classrooms (Chapin, O'Connor, & Anderson, 2009; Chapman, 2003; Herbel-Eisenmann & Cirillo, 2009; Ryve, 2011) and my own theoretical perspective on learning, that active student participation in mathematically-focused discourse will also be predictive of higher gain scores.<sup>19</sup> Note that these hypotheses are correlational in nature and do not refer to cognitive demand or participatory demand *causing* higher gain scores.

**Limitations.** The approaches just described for answering RQ1 have their limitations. Some of these limitations have already been alluded to, such as the fact that the sample size of 12 sets of lessons is not large (though not small either), traditional teacher-centered modes of instruction have largely been excluded, and only one area of content is being examined. All of these limitations place bounds on the kinds of claims that can be made from this study. Furthermore, because the statistical analysis is correlational, causal claims cannot be made and any findings between variables must come with appropriate caveats. Another consideration, though not necessarily a limitation because it stems from a necessary part of the empirical process, is the fact that the variables of cognitive demand and participatory demand have been

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<sup>19</sup> Because of this hypothesized directionality of the relationships, I used one-tailed *t*-tests of significance in the analysis of RQ1.

conceived of in a particular way, whereas others might operationalize them differently. Hence, for example, findings with respect to participatory demand only apply to student participation as defined here, not other forms of student participation. The analysis also focuses on the typically whole-class phases of set-up and look-back, without analysis of cognitive demand and participatory demand during the working phase.

Other limitations include the fact that important facets of the enacted curriculum are not being attended to because of finite resources and the need to select foregrounded factors. For example, the teacher's discourse moves in and of themselves are being essentially ignored (though any results these moves may have in the student discourse is attended to) and the time spent addressing the focal learning goal is not being accounted for in relation to student learning, even though it varied from 3 class periods to 7 class periods. Also missing is an attention to various types of important instructional decisions, such as how to represent a specific mathematical idea (Brenner et al., 1997) or the selecting and sequencing of solutions during the look-back phase (Stein & Smith, 2011). Most of these limitations are to be expected, however, because, although they are an important part of the enacted curriculum, they do not directly fall in the purview of the research questions guiding this study. Other limitations are due to the fact that RQ1 is being addressed from the standpoint of an overview across several different classrooms. To partially remedy this situation, I use a different methodological approach with respect to RQ2.

## **Analysis for RQ2**

My second guiding research question deals with the interaction of cognitive demand and participatory demand specifically within the look-back phase of task enactments. The analysis of this question is purely qualitative and takes as its focus only two of the 12 sets of lessons

described above (which themselves were a subset of the larger set of classroom data collected by *Project 2061*). Taken together, this study approaches the type of mixed methods approach encouraged by methodological scholars (e.g., Creswell, 2009).

As noted previously, the look-back phase especially deserves detailed attention because it is a newly demarcated portion of the MTF (Otten, 2010a). Moreover, the look-back phase can be the time when teachers and students make connections and draw meaning out of their work in prior phases, which has implications for cognitive demand, and also typically involves a whole-class public discourse that students may be involved in to varying extents, which has implications for participatory demand. Thus, there is a rationale for examining the interaction between cognitive demand and participatory demand during this phase specifically.

Whereas the answers to RQ1 come in the form of the results and interpretations of statistical tests between the variables of interest, the answers to RQ2 come in a much different form, namely, narrative cases.<sup>20</sup> By narrative case I mean a written account of an event—in this case, classroom interactions—that is not merely an anecdote or reporting of objective observations but a story that is situated within an intellectual framework that allows the narrative case to be seen as a case of something (Shulman, 1992). I felt that the use of narratives cases in addressing RQ2 was the most appropriate course of action because, as Bruner (1986) and others (e.g., Sinclair, Healy, & Reis Sales, 2009) have pointed out, we understand our world not only in the form of propositional knowledge but also in the form of narratives. By seeking an underlying

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<sup>20</sup> Note that this is not a *case study* research method, which would require a more intimate knowledge of the teachers, students, classrooms, and communities than I have available to me in this study and would serve a different purpose.

story of what is taking place in a particular classroom with respect to cognitive demand and participatory demand during look-back phases of enactment, and constructing from that a narrative case, new understandings can be achieved. This understanding is reliable to the extent that it rings true as an authentic and believable narrative case and it is generalizable to the extent that others resonate with its themes and conclusions. Looking forward, the narrative cases can serve as a foundation for further research (e.g., investigating the prevalence in other classrooms of certain characteristics of the narrative case, testing hypotheses inspired by the themes of the narrative case) and can also serve a practical function as a source of reflection for educators (Steele, 2008). Moreover, they need not be limited to the interpretations or framings that I personally bring in this thesis to the narrative cases. Part of the power of narrative cases in education “is their potential for reinterpretation and multiple representation” (Shulman, 1992, p. 17).

**Developing the narrative cases.** To begin, it was necessary to reduce the number of sets of lessons to serve as the basis for the narrative cases because attending to all 12 was unfeasible for this study. Based on the analysis of RQ1, in which I viewed and developed analytic memos on all 12 sets of lessons, I identified four candidates for the development of a narrative case based on particularly interesting activities that occurred in their look-back phases. One of these candidates was identified primarily because of its prevalence of one of the standards for mathematical practice from the *Common Core State Standards for Mathematics* (2010). When these practices were removed from the guiding research questions, as described above, this candidate was removed from consideration for analysis with respect to RQ2. I also identified interesting phenomena with respect to the relationship between cognitive demand and participatory demand that might be ripe for a detailed analysis. For example, discrepancies

between the two constructs (e.g., high cognitive demand but low student participation, low cognitive demand but high mathematical student participation) seemed intellectually intriguing, as did shifts in either the cognitive demand or the participatory demand during the look-back phase. Because two of the three candidates involved teachers using the same curriculum materials, I decided it was wise to focus on those two sets of lessons with respect to RQ2 because this would provide consistency in the specific tasks with which the students across the narrative cases were engaging.

The two selected sets of lessons are Ms. Wyncott's from AY 2003–2004 and Ms. Albert's, also from AY 2003–2004. Both sets of lessons came from the second year of the *Project 2061* PD sequence and both teachers' written tasks came from *Mathematics in Context* (1998). They both taught in the Alpha school district, although Ms. Albert had been teaching longer (7 years) than Ms. Wyncott (4 years). The analysis for RQ1 revealed important differences in the enacted curriculum in their respective classrooms, as well as differences in student learning as measured on the pre- and post-test. In particular, Ms. Albert had consistently high marks for cognitive demand throughout the phases of enacted tasks whereas Ms. Wyncott did not necessarily maintain cognitively demanding written tasks at a high level. There were also differences in participatory demand as Ms. Wyncott was consistently coded as having medium and semi-mathematical participatory demand. Ms. Albert, on the other hand, tended to have limited student participation during the set-up phase but active and mathematically-focused participatory demand during the look-back phase. All of these differences in the enacted curriculum become especially intriguing when one learns that Ms. Albert's students gained approximately 0.5 standard deviations from the pre-test to the post-test whereas Ms. Wyncott's students were the only class to decline from the pre-test to the post-test. For these reasons,

narrative cases constructed around these two sets of lessons seemed promising for the issues that might arise both within the individual cases as well as between the cases.

Having selected the two sets of lessons that would constitute the data of the narrative cases, I went about the following process to formulate the narratives. I reexamined the written tasks from the *Mathematics in Context* curriculum and reviewed the set-up and working phases of enactment to establish a background for the look-back phases, which would receive primary attention. Then, I watched the look-back phase videos without a particular agenda in mind and took field notes on any emergent themes. This viewing, however, was not meant to be nor could it be a tabula rasa analysis because I had previously analyzed these same clips for RQ1. Nevertheless, it did allow me to take in the broad strokes of the look-back phase such as the mathematical ideas being shared or the instructional purpose apparently being served by the look-back phase. Next, I went through the look-back phases again to trace cognitive demand and participatory demand at a fine-grained level. In the analysis for RQ1, a single cognitive demand code and a single pair of participatory demand codes were assigned to the entire look-back phase, but there were often moments or single exchanges within the phase that have implications for cognitive demand and participatory demand. In constructing the narrative cases, I looked for these moments and interactions that seemed important with regard to the overall story of the task enactment. This process also allowed me to hone in on potential interactions between cognitive demand and participatory demand. For example, was there a specific teacher question that led not only to an increase in cognitive demand but also resulted in several students vocalizing mathematical relations? Were there particular aspects of the written task that, when brought out in the look-back phase, seemed to alter the complexion of the cognitive demand, participatory demand, or both? As the narratives of the two sets of look-back phases began to take shape, I



also revisited the classroom video to consider alternative interpretations or alternative storylines. These alternatives were not only meant as a check on the trustworthiness of my original interpretations but, in some cases, contributed to the narrative in that I could include the possibility of an alternative interpretive in the written case directly. At the conclusion of this process, I then wrote narratives for Ms. Wyncott and Ms. Albert in a way that, rather than being a true-to-life description of events (which is impossible), brought to the surface some of the salient issues regarding cognitive demand and participatory demand in the look-back phase while still being a fair representation of what occurred in the classrooms. Thus, it becomes necessary to note some of the limitations of this narrative method.

**Limitations.** With respect to RQ2, there are limitations associated to my particular choices as well as limitations tied to the narrative case method itself. The first and perhaps most important choice that I made was in the selection of the two teachers for this analysis. Had I chosen different teachers, different themes and a different overall story would have resulted. For example, Ms. Cesky is discussed in Chapter 4 as a teacher who had low participatory demand in her task enactments but her students achieved relatively high gain scores. Her story would likely be different from Ms. Albert's, for example, though the fact that Ms. Albert is more aligned with the general trends observed in this study than Ms. Cesky is a justification for making the choice that I did. Another key choice, as with RQ1, was in my operationalization of cognitive demand and, particularly, participatory demand. Because of how I conceived these constructs, I attended to certain aspects of the cases and not others.

With respect to the narrative case method overall, it is important to bear in mind that, in essence, I am the research instrument. The things I see, the interpretations I make, and the conclusions I draw all occur through the filter of my own unique background, beliefs,

assumptions, and values, which means that consumers of these narrative cases should banish the thought that they are reading the true or definitive account of the task enactments but should always be mindful that they are reading *my* account of the cognitive demand and participatory demand of the task enactments. With this in mind, such an account can nevertheless have value and provide insight into issues of mathematics education.

### **Summary**

At the broadest level, this is a study of the enacted curriculum in middle school mathematics classrooms and of the relationship between two particular features of the enacted curriculum—cognitive demand and participatory demand—relate to student learning of algebra. The classroom data consisted of 12 sets of lessons from 9 teachers in 5 school districts in 2 states, whereas student learning was measured by gain scores between a pre- and post-test of algebra content. In addressing RQ1, cognitive demand was coded using an existing framework (Stein, Grover, & Henningsen, 1996; Stein & Lane, 1996) and participatory demand was coded based on the amount of students' verbal contributions and the thematic content (Herbel-Eisenmann & Otten, 2011) of those contributions. Correlational analysis was then conducted between the ranks of cognitive demand and participatory demand and the ranks of students' standardized gain scores. For RQ2, narrative cases were written around the look-back phases of Ms. Wyncott's and Ms. Albert's mathematical task enactments. These narrative cases complement the analysis for RQ1 by exploring in greater detail some of the potential relationships between cognitive demand and participatory demand and exemplifying aspects of task enactments that may spur reflection or future research.

In the two following chapters, results of these analyses are presented, organized by research question. In particular, Chapter 4 focuses on RQ1 and the correlational analysis across

all 12 sets of lessons, demonstrating the independence of cognitive demand and participatory demand in the enacted curriculum yet their joint importance with respect to student learning across the classrooms. Chapter 5 contains the two narrative cases addressing RQ2, illustrating ways in which cognitive demand and participatory demand can play out and interact with one another.

## **CHAPTER 4**

### **ENACTMENT AND LEARNING ACROSS THE CLASSROOMS**

In this chapter, I present an overview of the range of phenomena observed in the enacted curriculum, beginning with the various ways in which the phases of mathematical task played out in the teachers' classrooms. This preliminary section is meant to provide a general sense of the range of practices I found in the classroom data in order to provide a sense of the various classrooms and teachers' practices. Next, I describe the nature of cognitive demand and participatory demand that was observed before presenting the standardized class gain scores for the 12 classes involved in the study. Finally, I share the results of the correlational analysis with respect to cognitive demand and the class gain scores as well as participatory demand and the class gain scores, providing evidence of the relations these have to student learning.

#### **Phases of Mathematical Task Enactments**

Although the 12 sets of lessons in this study all focused on the same learning goal in middle school algebra, there were many variations in the ways the teachers enacted their mathematical tasks aimed at this learning goal. The variations may stem from the facts that the teachers had different backgrounds, were located in different districts and in different states, were using different curriculum materials, had different teaching styles, and had students who are all individually unique. Regardless of the sources, this section describes the variations as well as the patterns in the mathematical task enactments across the 12 sets of classroom lessons, focusing particularly on the phases of enactment from the modified MTF (see Figure 3). The working phase is included in this overview section, though it was not treated in the analysis of cognitive demand and participatory demand, whose results are presented subsequently, for reasons explained in Chapter 3.

## Written Tasks

As mentioned in Chapter 3, tasks from three different textbook series were used by teachers in this study—*Connected Mathematics* (Lappan et al., 1998b), *Mathematics in Context* (National Center for Research in Mathematical Sciences Education & the Freudenthal Institute, 1998), and *Mathematics* (Billstein et al., 1999). Additionally, there were two teachers who employed loose-leaf hand-outs to address the focal learning objective of using variables to representing relationships between changing quantities. Although these teachers did have access to textbook series—namely, *Mathematics* and *Glencoe* (Collins et al., 1998)—they used teacher-supplied hand-outs exclusively in teaching the focal learning objective. Table 1 in Chapter 3 depicts which teacher used which type of curricular and supplemental materials.

There were similarities and differences in the written tasks from these various sources. One important commonality shared by all of the written tasks was the presence of relationships represented by tables and graphs that then, after assigning variables to the quantities involved in the relationship, were translated into equations. For example, in Investigation 4 from *Connected Mathematics* (Lappan et al., 1998b), students are asked to complete a table for the time and distance traveled by a tour group driving 55 miles per hour and are later expected to generate an equation such as  $d = 55t$  to represent the rule from the table. Another similarity was that none of the written tasks used a naked numerical relationship as the basis of this representational work. The tasks, differed, however, in the type of context that was provided for the relationships. For example, *Connected Mathematics* developed a bicycle rental company context that allowed for the development of various relationships between customers and income as well as between speed, distance, and time. *Mathematics in Context*, in a slightly less true-to-life fashion, posited hopping frogs to represent linear expressions of the form  $a + bx$ , with  $a$  being the starting point

of the frog and  $b$  being the number of hops of length  $x$ . Whereas the tasks in *Connected Mathematics* included a more robust and realistic context, *Mathematics in Context* allowed for and explicitly encouraged the image of hopping frogs (see Figure 10) to be used as a general representation of linear expressions and linear equations, including those beyond the written tasks themselves. In *Math Thematics*, like a portion of *Connected Mathematics*, speed, distance, and time were the related quantities. Initially, the context is automobiles and safe following distances and later it shifts to walking speeds. The teacher-supplied tasks, however, commonly shifted context from one task to the next.

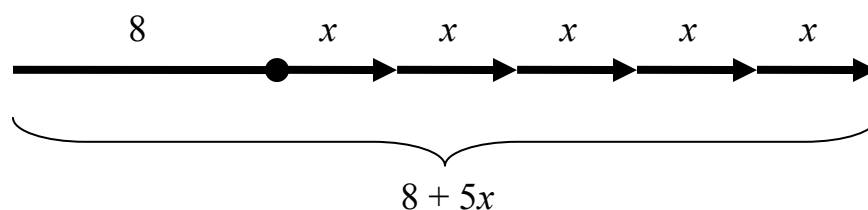


Figure 10. An example of the diagrams contained in *Mathematics in Context* (1998) tasks.

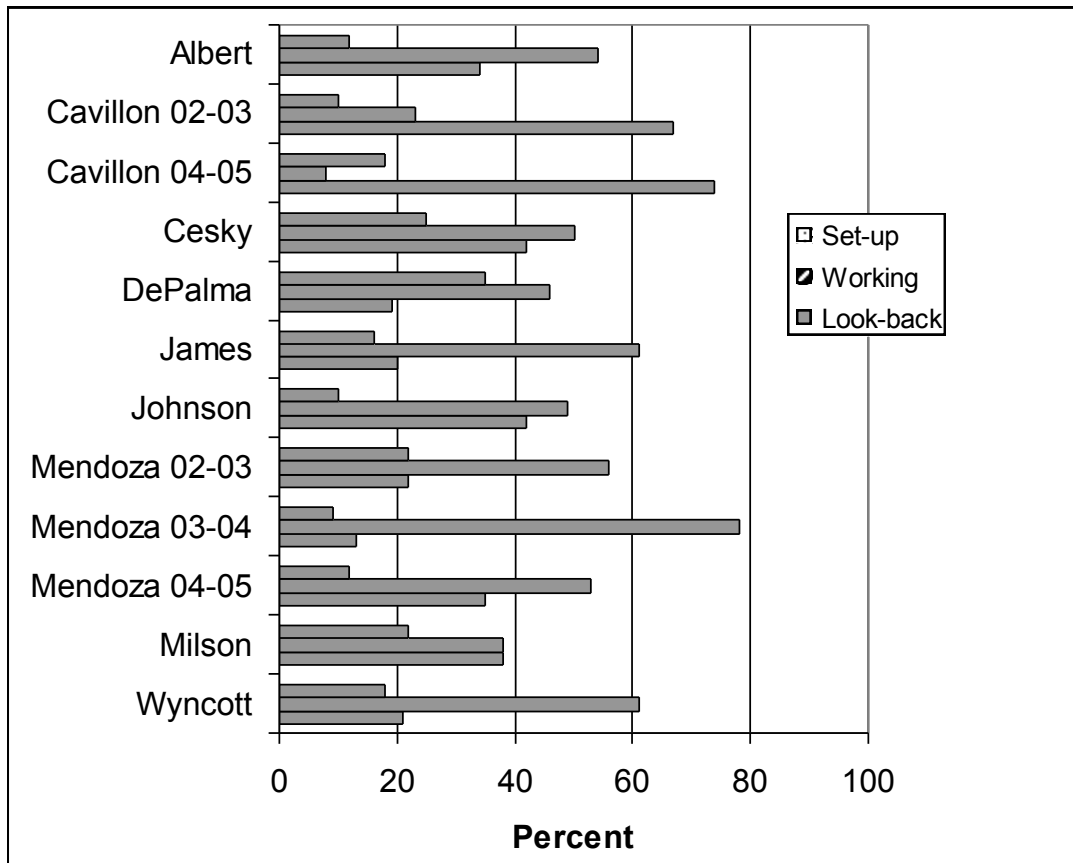
Another difference between the textbook tasks and the teacher-supplied tasks was that, in all of the textbook tasks, the relationships were between varying measurable quantities, whether magnitude (e.g., distance, time, income) or multitude (e.g., number of customers, number of hops). Some of the teacher-supplied tasks, on the other hand, involved relationships between a quantity and an abstract numeration or ordinal number. For example, Ms. Mendoza, in AY 2003–2004 and AY 2004–2005, relied exclusively on growing patterns where the relationship of change was between the number of items in the pattern and the sequence number.

### Set-Up Phase

There were some consistent features across the set-up phases of all 12 sets of lessons. In particular, teachers during the set-up phase directed students' attention to the written task and, at

some point, paraphrased or read verbatim the directions to at least the first portion of the task. Half of the sets of lessons—Ms. Albert, Ms. Cesky, Ms. DePalma, Ms. Mendoza (AY 2003–2004), Mr. Milson, and Ms. Wyncott—included students reading aloud from the written task, usually the orienting paragraph(s) that describe the context of the task. A few of the remaining teachers instead read aloud themselves. Some teachers also used the set-up phase to organize materials and resources (e.g., graph paper, calculators) to be used on the task.

The portion of time spent in the set-up phase varied. The topmost bars in Figure 11 depict, for each of the 12 sets of lesson, the percent of time spent in the set-up phase with respect to the total time spent enacting the mathematical tasks. Ms. DePalma spent the most time in the set-up phase, at 35%, whereas Ms. Cavillon (AY 2002–2003), Mr. Johnson, and Ms. Mendoza (AY 2003–2004) spent 10% or less. The average percentage across the 12 sets was 17.4%. With regard to set up, the teachers varied in the way that they divided this time. Sixteen of the 37 mathematical task enactments contained a single segment of set-up at the beginning, whereas the remaining 21 enactments involved a segment of set-up at the beginning as well as later segments of set-up. As an example of the latter technique, Mr. Johnson, in his enactment of Investigation 3.4 from *Connected Mathematics*, spent 6 minutes setting up part A of the task, gave his students 16 minutes to work on part A, and then spent another minute setting up parts B through D specifically.



*Figure 11.* Percent of time spent in each phase of mathematical task enactments.

Other observations indicating the range of activities in the set-up phases included Ms. Mendoza, in AY 2002–2003 and AY 2004–2005, solving the first problem of her self-supplied task as an example for students of how to proceed and Ms. Wyncott, along with other teachers, explicitly telling students how long they will have to work in each segment of the working phase. Additionally, in one instance, Ms. Albert had the written task projected on the front board so that students began working for a few minutes without any explicit set up. Then, she called the whole class together to explain how the work on the first few problems related to the work that would take place on the rest of the task, thus giving the directions and articulating the goals for the task overall.



## **Working Phase**

The working phase is characterized by students being expected to work and progress through the written task. Their work took place in several different formats, with small-group or partner work being the most prevalent. Some teachers, such as Ms. Cesky and Mr. Milson, used a combination of individual work time and small-group work, whereas Ms. DePalma, Ms. Mendoza, and Ms. Wyncott tended to use a combination of individual and whole-class working of the task. Ms. Cavillon, on occasion, assigned the work to be done as homework, so that only the set-up and look-back phases took place during class time. A few other teachers gave work time during class but, if students were unfinished, asked them to continue working at home.

The working phase was commonly divided into discontinuous segments. In some cases, as mentioned above, work would cease while the teacher set up the next part of the task, and in other cases the teacher would lead students in looking back at their work thus far before allowing them to continue working. The middle bars in Figure 11 depict the percent of enactment time spent in the working phase. Ms. Cavillon has the smallest percents because of her use of homework. The majority of teachers fall between the upper 40s and the lower 60s, with the average across the 12 sets of lessons being 48.1%. The average of those who had students work predominantly during class (i.e., excluding Ms. Cavillon) is 54.6%.

## **Look-Back Phase**

The look-back phase was perhaps the most varied of the phases of enactment. Simply noting the various percents of time spent looking back, represented in Figure 11 by the lowermost bars, reveals differences between teachers (and even within a single teacher in the case of Ms. Mendoza). Ms. Cavillon used the largest percentage of in-class time in the look-back phase, but this was primarily because her working phases were minimal due to her use of

homework. The next highest look-back percentages were Mr. Johnson and Ms. Cesky at 42%. Ms. DePalma and Ms. Mendoza (AY 2003–2004), on the other hand, spent less than 20% of their task enactment time in the look-back phase. The average percentage was 35.6%.

All of the look-back phases within this study took place in the whole-class format. Although it is conceivable that teachers might have small groups of students reflect on their work on the task or have individual students write a summary of the solution approaches, I did not observe such occurrences. I observed several discourse structures during the look-back phase, some of which are addressed in greater detail in the section below on participatory demand. The most prevalent structure of the look-back phase was for the teacher to lead the classroom discourse with the IRE interaction pattern (Mehan, 1979). In this way, the teacher could solicit for public scrutiny the solutions to the task and could make remarks about those solutions. As a representative example of this IRE pattern, Ms. Cavillon had students report out the values they found for a table by asking questions such as, “What was a hundred fifty-five times two customers?” This served as an initiation of an interaction, and a student responded, “Three ten.” Ms. Cavillon revoiced the response as “Three hundred and ten dollars” with a downward inflection and she wrote the answer in the appropriate cell of the table projected on the front board, which one might interpret as evaluating the response as the correct answers. Similarly, Ms. Mendoza (AY 2003–2004) began a look-back phase by asking the question, “If I wanted an eighteen-story building, how many cubes do I need?” This initiation yielded several student responses. Ms. Mendoza said, “Some of you said eighteen, some said thirty-six.” She then repeated her question, “If I wanted an eighteen-story building, how many cubes would I need?” This re-initiation led to a student responding with “thirty-six” again. Ms. Mendoza said, “thirty-six, right?” She then moved on to ask about a twenty-six story building, indicating that the tag

question (i.e., "...right?") was marking the fact that thirty-six was the correct answer. In contrast to the IRE pattern, some teachers, such as Ms. Albert, Ms. DePalma, Ms. Mendoza, and Ms. James, incorporated student presentations into their look-back phases. There were also instances of mathematical discussions facilitated, but not completely directed (as in IRE), by teachers. There were also instances of multiple discourse patterns within the same look-back phase (e.g., a teacher who starts with IRE interactions but then shifts to more open discourse).

Teachers also varied in how they segmented the look-back phase. Sixteen of the 37 enacted mathematical tasks involved a single, contiguous look-back segment at the conclusion of the enactment.<sup>21</sup> The remainder included multiple segments of looking back, each usually focusing on a portion of the written task. For example, Ms. Wyncott spent three minutes in the middle of her second task enactment to go over what the students had accomplished thus far and address any confusions that had arisen. Then, the students worked for approximately six more minutes until Ms. Wyncott used the last five minutes of the class period to "wrap this up" and have students share out their answers. It was not uncommon, however, for at least one segment of the look-back phase to occur during the next class period after the beginning of the task enactment.

In the next sections, I present the results of the coding of cognitive demand and participatory demand across the phases of mathematical task enactment. Before focusing on cognitive demand and participatory demand separately, however, I address the question of the

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<sup>21</sup> These 16 instances do not precisely correspond with the 16 instances of contiguous set-up phases.

extent to which the constructs of cognitive demand and participatory demand capture different aspects of the enacted curriculum.

### **Independence of Cognitive Demand and Participatory Demand**

In Chapter 3, I argued from a theoretical basis that the constructs of cognitive demand and participatory demand are distinct. Now, it is appropriate to examine whether they were empirically confirmed to be distinct. To do so, I used the rankings, which are presented in subsequent sections, of the 12 sets of lessons according to each construct. With these two rankings, I calculated Spearman's rank correlation coefficient  $\rho$ . If cognitive demand and participatory demand were overlapping or dependent constructs, one would expect  $\rho$  to be positive and statistically significant, with values closer to 1 indicating a stronger relationship between the constructs. The null hypothesis is that the two constructs are unrelated or at least functionally independent. Based on the results of analysis described below, the rank correlation coefficient was  $\rho = 0.112$  ( $df = 10$ ,  $t = 0.356$ ,  $p = 0.365$ ), which is both weak and statistically insignificant. Thus, the null hypothesis that the two constructs are functionally independent is maintained.

There were also instances of sets of lessons where the independence of cognitive demand and participatory demand was particularly evident. For example, Mr. Johnson enacted four mathematical tasks and each one involved at least one phase of enacted with a high level of student participation (H). Three of the four enactments actually involved high mathematical student participation (H / M). This placed him third in the participatory demand ranking. Yet, this student participation was not always around cognitively demanding mathematical ideas or processes. In fact, only one of the four tasks involved the maintenance of high cognitive demand through the look-back phase. In the other tasks, Mr. Johnson had a tendency to indicate to

students the next step to take or to ask students to explain only what they did and what answer they got, rather than attending to why they did what they did and why the answer made sense.

These results confirm that the constructs of cognitive demand and participatory demand are operationally distinct, and thus we proceed to examine each separately with that knowledge in mind.

### **Enacted Curriculum Coding Results**

#### **Cognitive Demand**

Although the majority (26 out of 37) of written tasks observed in this study were of high cognitive demand for students at the middle school level being introduced to variables, many of the teachers did not fully enact the tasks in highly demanding ways during the set-up and look-back phases (with the working phase being excluded from analysis). Eight (31%) of the 26 high-level tasks were enacted at a low level during both of those two phases. Another eight (31%) of the 26 were set-up at a high level but had look-back phases that were low level. Also, two (8%) high-level written tasks dipped to low demand during the set-up phase but then returned to a high level in the look-back phase. Thus, overall, 69% of the high-level tasks declined for at least one of the coded phases of enactment. Seven (27%) high-level tasks were enacted at a high level through all phases. (There was one high-level task in Ms. Mendoza's classroom, AY 2004–2005, which was enacted without a look-back phase.) It should be noted, however, that a portion of the 27% which were seen in this study to maintain a high level of cognitive demand may have had declines during the working phase, which was not coded. The 11 low-level written tasks, in line with past findings (Stein et al., 1996), generally remained at a low level throughout enactment,

with only one exception.<sup>22</sup> Appendix C contains images of the cognitive demand levels for all task enactments.

Using the procedure outlined in Chapter 3, the 12 sets of lessons were ranked according to the prevalence of cognitive demand, which yielded the ordering contained in Table 6. Ms. Albert had the highest cognitive demand ranking. She used the *Mathematics in Context* textbook and was the only teacher to maintain a high level of cognitive demand throughout all phases of enactment for both of her tasks. Ms. DePalma, who also used *Mathematics in Context*, was ranked second, setting up both tasks at a high level and looking back at a high level for one of the tasks. Mr. Milson, ranked at the bottom of the list, enacted four tasks from *Connected Mathematics*, three of which were coded high level as written, but had a low level of cognitive demand in all phases of enactment. This ranking is used below in the correlational analysis with respect to student learning.

Table 6

*Cognitive demand ranking*

Rank	Set of Lessons
1	Ms. Albert
2	Ms. DePalma
3	Ms. Cavillon (AY 2002–2003)
4	Ms. Mendoza (AY 2004–2005)
5	Ms. Mendoza (AY 2003–2004)
6	Ms. Wyncott

<sup>22</sup> The exception was the last task from Ms. Mendoza (AY 2002–2003) which was coded as low level as a written task and set-up phase, but during the look-back phase she pressed students to connect their symbolic equations to the problem context and also asked them to draw connections between different students' presented solutions. This led to a high-level code.

Table 6 (cont'd)

7	Mr. Johnson
8	Ms. Cesky
9	Ms. Mendoza (AY 2002–2003)
10	Ms. Cavillon (AY 2004–2005)
11	Ms. James
12	Mr. Milson

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## Participatory Demand

In general, the written tasks did not contain any explicit expectations for how students were to interact as they engaged with the task. The sole exception to this rule was the first task in the *Mathematics in Context* (1998) textbook. On page 32 of that textbook, the authors write that students are supposed to “discuss strategies for solving this problem” in groups. Note that students are not simply expected to solve the problem but to discuss strategies for how they might solve the problem. On that same page, the authors included a numbered item in the written task that read as follows: “Share your group’s method with the other members of your class.” These passages were the only instances in any of the written tasks where the forms and foci of student participation were explicitly addressed.<sup>23</sup>

The level of student participation was generally low or medium during the set-up phases of enactment. In seven (19%) instances, there was no student participation in that the set-up phase consisted entirely of teacher talk or, occasionally, student silent reading. In eight (22%)

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<sup>23</sup> This absence of explicit participatory demand in written tasks is not unexpected when one considers that teachers are active creators of the enacted curriculum (Remillard, 2005). Teachers may shape participatory demand around written tasks, perhaps based on material in the teacher’s guide, but also in consideration of particular students in particular classroom contexts.

instances, the level of participation was low and the focus of that participation was either non-mathematical or semi-mathematical. The most common form of participatory demand during the set-up phase was medium semi-mathematical (D/S). For example, Ms. Cavillon's set-up phases typically involved her describing the directions of the task and asking students closed questions about the context of the problem and about past mathematical topics or facts that would come into play in the present task. Some of these closed questions elicited choral responses, involving most of the students in the class, but because the student responses were single words or short phrases, this was coded as a medium (D) level of participation. Furthermore, because some of the student talk was about the non-mathematical context, and students uttered some mathematical terms but did not speak sentences containing mathematical semantic relations, the focus of this participation was semi-mathematical (S). Figure 12 contains a brief excerpt of set-up from Ms. Cavillon's classroom, illustrating such semi-mathematical focus of student participation.

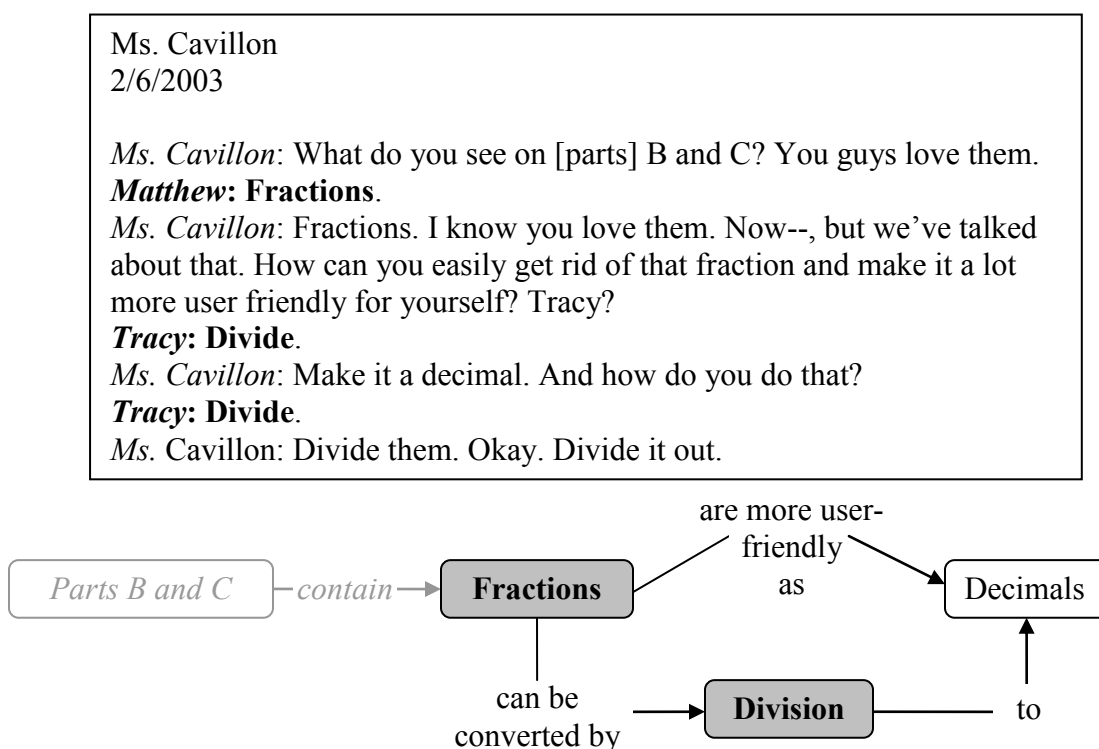


Figure 12. Semi-mathematical participatory demand in a set-up phase of enactment.



There were also three (8%) instances of medium non-mathematical (D / N) participatory demand in the set-up phase. The remaining instances involved some form of high participatory demand. For example, in setting up Investigation 4.2 from *Connected Mathematics* which involves time and distance, Mr. Johnson facilitated a discussion in which many students shared thoughts about distances between various places in their region of the country. This alone would have constituted high semi-mathematical (H / S) participatory demand, but I felt that Mr. Johnson then achieved high mathematical (H / M) participatory demand by shifting the discussion toward students' experiences estimating how long it would take to reach a familiar destination at various speeds, thus soliciting student sentences containing mathematical relations such as those between time and distance at various speeds. This discussion contained primarily qualitative judgments (e.g., traveling faster over a distance reduces the travel time), whereas the written task provided an opportunity to look more quantitatively at the specific relations.

The look-back phases exhibited a wide variety of forms of participatory demand, though there was some consistency within sets of lessons. Ms. Albert had high mathematical (H / M) participatory demand in each of her look-back phases and Ms. Cavillon had high (H / M or H / S) participatory demand in all but one of her look-back phases over two academic years. Mr. Johnson, Ms. James, and Mr. Milson also had more than one enacted task with high (H / M or H / S) participatory demand in the look-back phase. Indeed, the look-back phase overall involved the highest levels of student participation as well as the most mathematically-focused participation. There were 14 (38%) tasks for which the look-back phase was the only phase of enactment that involved high participatory demand, and 10 of these 14 were high mathematical participatory demand. There were also, however, teachers who tended not to have high

participatory demand in their look-back phases, such as Ms. Cesky, Ms. Mendoza (AY 2003–2004), and Ms. Wyncott. Appendix D contains the participatory demand codes for all mathematical task enactments.

Table 7 shows the ranking of the 12 sets of lessons according to prevalence of high mathematical (H / M) participatory demand. Ms. Albert was ranked highest as her task implementations (working phases and look-back phases) were all high mathematical participatory demand. Mr. Johnson was ranked second highest because three of his four task enactments involved high mathematical participatory demand and the fourth had a working phase with high semi-mathematical participatory demand. Ms. Mendoza (AY 2003–2004) was ranked lowest because she did not have any instances of high participatory demand or mathematically-focused participatory demand, let alone high mathematical participatory demand. Interestingly, however, Ms. Mendoza’s other sets of lessons involved several phases of enacted tasks that were high mathematical participatory demand, ranking third and eighth in Table 7.

Table 7

*Participatory demand ranking*

Rank	Set of Lessons
1	Ms. Albert
2	Ms. Cavillon (AY 2004–2005)
3	Mr. Johnson
4	Ms. Cavillon (AY 2002–2003)
5	Ms. Mendoza (AY 2004–2005)
6	Mr. Milson
7	Ms. Mendoza (AY 2002–2003)
8	Ms. James
9	Ms. DePalma
10	Ms. Wyncott
11	Ms. Cesky
12	Ms. Mendoza (AY 2003–2004)

With these rankings of cognitive demand and participatory demand in hand, I now turn to the analysis of the relationship between these aspects of the enacted curriculum and students' learning as measured by the standardized gain score on the pre- and post-test.

## **Enacted Curriculum and Student Learning**

### **Student Learning**

Out of 27 possible points, the class averages on the pre-test ranged from 5.4 to 14.3 points with a mean of 9.29 and a median of 9.48 across the 12 sets. The standard deviation of the pre-test class averages was 2.82, which was used, together with the mean, to compute a standardized score for each class. For example, Ms. DePalma's class averaged 12.9 points on the pre-test, a deviation of 3.61 points from the mean, and so her class's standardized score was approximately 1.28. On the post-test, which also consisted of 27 possible points, the class averages generally improved, ranging from 7.5 to 16.3 with a mean of 10.98 and a median of 10.69. The standard deviation of the post-test averages was 2.51 and was used to again compute standardized scores for each class. For example, on the post-test, Ms. DePalma's class averaged 13.2 points or 0.89 on the standardized scale. I then used the standardized scores on the pre- and post-test to determine the standardized gain for each class. Again using Ms. DePalma as an example, her class average shifted from 1.29 to 0.89 on the standardized scales, meaning her standardized gain was approximately  $-0.40$ . It should be noted that a negative standardized gain score does not imply there was an absence of mathematical learning. Ms. DePalma's students improved from an average of 12.9 points to an average of 13.2 points. A negative standardized gain score simply means that the class average did not increase enough to maintain the same (or better) position on the standardized scale relative to the other classes. Table 8 contains the

ranking for standardized gain scores. This ranking will be used in the calculations of Spearman's rank correlation coefficient in the subsequent sections.

Table 8

*Standardized gain score ranking*

Rank	Set of Lessons	STD Gain
1	Mr. Johnson	0.97
2	Ms. Mendoza (AY 2004–2005)	0.62
3	Ms. Cavillon (AY 2002–2003)	0.61
4	Ms. Cesky	0.59
5	Ms. Albert	0.36
6	Mr. Milson	0.12
7	Ms. Mendoza (AY 2002–2003)	–0.19
8	Ms. Mendoza (AY 2003–2004)	–0.27
9	Ms. DePalma	–0.40
10	Ms. James	–0.53
11	Ms. Cavillon (AY 2004–2005)	–0.67
12	Ms. Wyncott	–1.22

### **Cognitive Demand and Student Learning**

Using the cognitive demand ranking from Table 6 and the standardized gain score ranking from Table 8, Spearman's rank correlation coefficient is calculated to be  $\rho = 0.273$  ( $df = 10$ ,  $t = 0.896$ ,  $p = 0.1956$ ). This relationship is relatively weak and there is a nearly 20% chance that the relationship observed is due to random variation in the variables. In other words, the rank of a set of lessons with respect to cognitive demand is not a strong predictor of rank by standardized gain scores. Figure 13 depicts the cognitive demand ranks in comparison with the standardized gain score rank and illustrates the lack of a strong relationship.

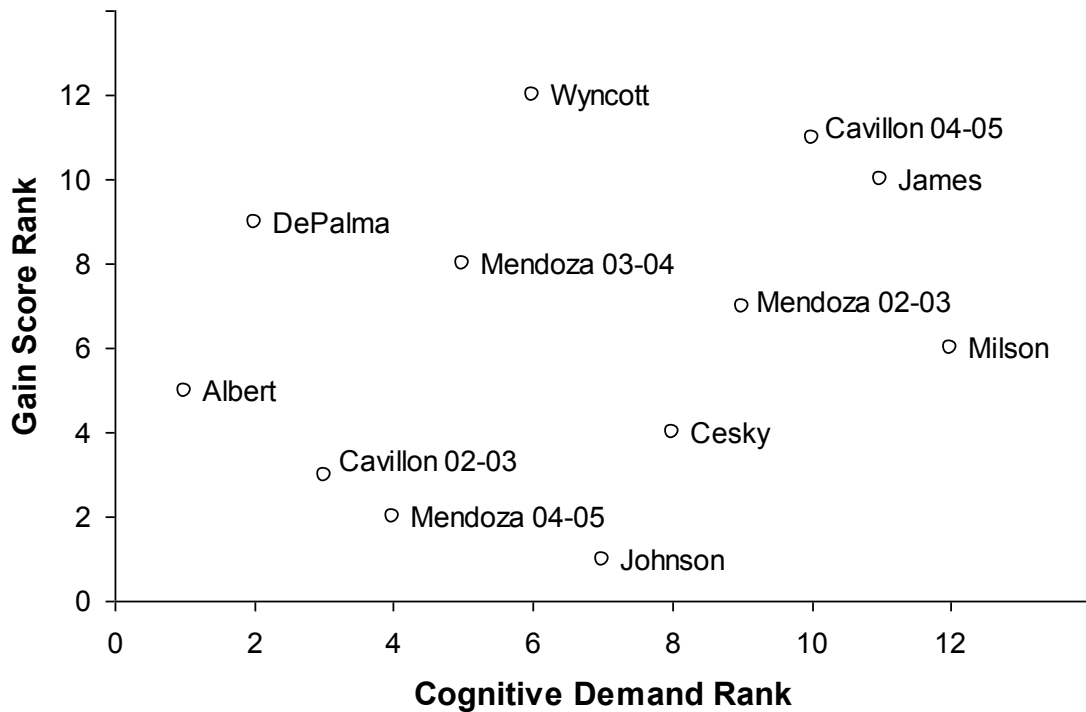


Figure 13. A comparison of cognitive demand rank to standardized gain score rank.

### Participatory Demand and Student Learning

Using the participatory demand ranking from Table 7 and the standardized gain score ranking from Table 8, Spearman's rank correlation coefficient between these two variables is calculated to be  $\rho = 0.357$  ( $df = 10$ ,  $t = 1.207$ ,  $p = 0.128$ ). This relationship is slightly stronger than that found for cognitive demand, though by no means strong and there is a nearly 13% chance that it could be the result of random variation in the variables. Figure 14 depicts the participatory demand ranks in comparison with the standardized gain score rank.

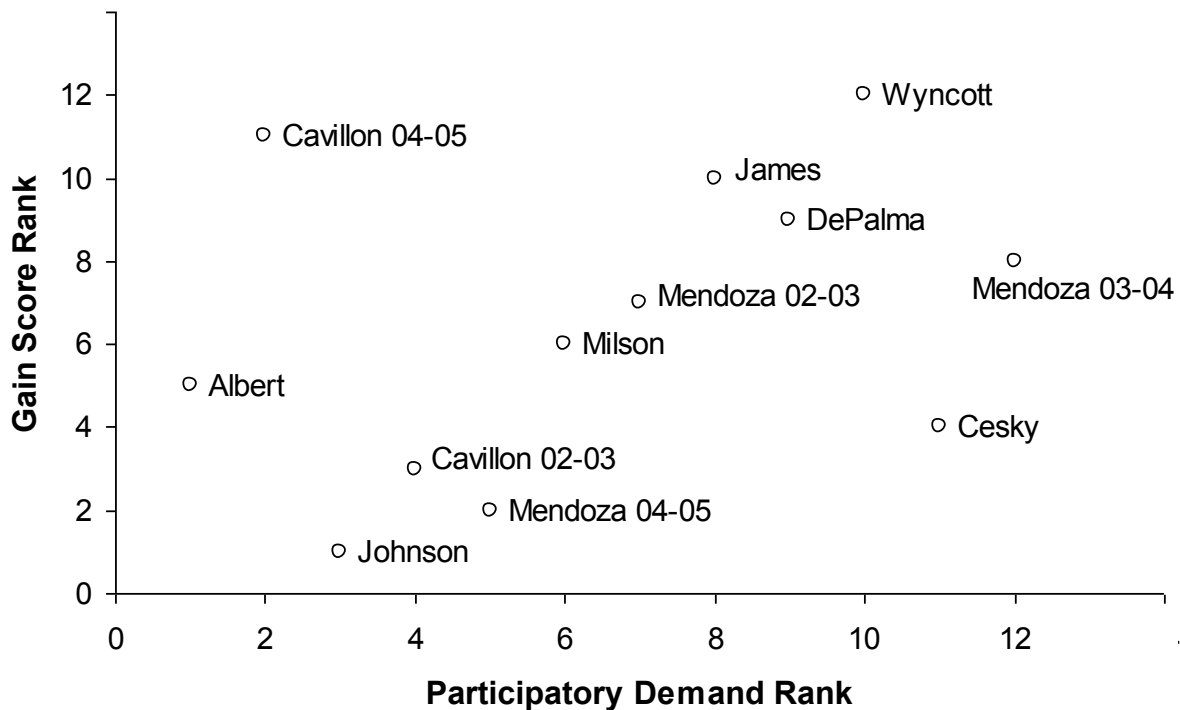


Figure 14. A comparison of participatory demand rank to standardized gain score rank.

Interestingly, more than 70% of the squared deviations used to compute  $\rho$  come from only 2 of the 12 data points, Ms. Cesky and Ms. Cavillon (AY 2004–2005). Ms. Cesky was ranked second to last with respect to participatory demand but had the fourth highest standardized gain score, whereas Ms. Cavillon was ranked second highest with respect to participatory demand (AY 2004–2005) but had the second lowest standardized gain score. Because of the relatively small sample used in this statistical analysis, such outlying data points can have a substantial impact on the quantitative findings. A small sample, however, also makes it difficult to say definitively that a single data point is an outlier. Omitting Ms. Cesky or Ms. Cavillon (AY 2004–2005) and recalculating  $\rho$  based on the rankings of the other 11 sets of lessons yields  $\rho$ -values above 0.6 in both cases, and  $p$ -values below 5% and 1%, respectively.

This finding warrants further attention to Ms. Cesky and Ms. Cavillon, as they may be viewed as exceptions to the rule with regard to participatory demand.

**Considering Ms. Cesky and Ms. Cavillon.** Recall that Ms. Cesky, at the time of the data collection, was a Texas 8th grade teacher from the Delta school district (see Table 3 above) who had six years of mathematics teaching experience. She used the *Math Thematics* (Billstein et al., 1999) textbook series and, in fact, was the only teacher in the present study who enacted tasks from this textbook, as Ms. Mendoza instead used self-supplied activities. Ms. Cesky enacted two tasks addressing the focal learning objective, spending approximately 30 minutes on each task. The first task (pp. 179–181) was enacted in a unique manner because Ms. Cesky led the whole class through the task in a manner that merged working and looking back on the problems. In doing so, Ms. Cesky lowered the cognitive demand from high on the written task to low throughout the enactment because Ms. Cesky undertook a substantial part of the thinking entailed in the problem and her emphasis was on the steps and procedures of the task without attention to why the procedures made sense. She also incorporated some aspects of memorization as she asked the entire class for facts as she worked and the class responded in chorus.

The participatory demand of this enactment was low semi-mathematical during the set-up phase and medium semi-mathematical during the working/look-back combined phase. Several students did contribute vocal turns during the first task enactment, and many of them were mathematical in nature, but they were overwhelmingly of the single-word variety in response to Ms. Cesky's questions. There were a few instances of students speaking full sentences, but they were procedural in nature (as is to be expected given the cognitive demand of the enactment) and not numerous enough to warrant a mathematical code for participatory demand.

The second task (pp. 189–192) was also written with a high level of cognitive demand and was maintained at this level during the set-up phase. Then, as with the first task, the cognitive demand declined during implementation. In contrast to the first task, the set-up phase of the second task involved a high level of student participation, with the focus of this participation being the non-mathematical context of the problem. During the look-back phase, which followed an 8-minute working phase that was not coded, the participatory demand was low and semi-mathematical as Ms. Cesky looked-back with the entire class over that first portion of work. The last segment of the task enactment remained in the whole-class format as Ms. Cesky led the students, frequently using the IRE interaction pattern, through the remainder of the task.

Ms. Cavillon, in AY 2004–2005, had more than 9 years mathematics teaching experience and was working with 7th graders in Delaware school district Gamma. She used the *Connected Mathematics* (Lappan et al., 1998b) curriculum materials and enacted three investigations over the course of seven class periods. These class periods were 50 minutes long, but she typically spent 15–20 on an unrelated warm-up activity, so there was approximately 3 hours total spent on the focal tasks. Ms. Cavillon’s high participatory demand ranking can be traced to her look-back phases, which involved a high level of student participation in every case and was either mathematical or semi-mathematical in focus. Ms. Cavillon’s high level of student participation included contributions from a large percentage of the class, and she was able to achieve this by spending extraordinarily large amounts of time in the look-back phase of enactment. From Figure 11, one can see that Ms. Cavillon (AY 2004–2005), at 74%, spent a larger portion of time in the look-back phase than any other teacher. When one considers the cognitive demand in Ms. Cavillon’s case, however, it is seen to be low in all of these long look-back phases. There is a



substantial focus on answers to the task and, at times, the sharing of answers seems to carry on for many more minutes than is necessary. The ideas are brought together in the public space in an unsystematic way, another characteristic of low levels of cognitive demand.

Therefore, Ms. Cesky and Ms. Cavillon provide glimpses at different ends of a spectrum with respect to enactment. Ms. Cesky's enactments did not exhibit high mathematical participatory demand and declined in cognitive demand, yet her class's standardized gain score between the pre- and post-test was near the top of ranking. Ms. Cavillon's enactments, on the other hand, did exhibit high mathematical participatory demand but this student participation occurred in protracted look-back phases that were of low cognitive demand. Her class's standardized gain score were near the bottom of the ranking. An analysis beyond the scope of this study would be required to uncover the mechanisms of student learning in this case, but with respect to the research questions of this study, Ms. Cavillon may serve as a reminder that participatory demand without cognitive demand may be unfruitful, whereas Ms. Cesky may serve as an existence proof of the fact that cognitive demand and participatory demand do not tell the whole story when it comes to the enacted mathematics curriculum.

### **Preliminary Discussion**

To summarize the findings above, based on significance tests of Spearman's rank correlation coefficient, cognitive demand and participatory demand were not significantly related to standardized gain scores, though, with the exceptions of Ms. Cesky and Ms. Cavillon, who achieved high gain scores with low participatory demand and low gain scores with high participatory demand, respectively, participatory demand was a significant predictor of standardized gain scores.

Any discussion of these results should begin with caveats about the form of analysis that was used. In particular, such correlational analysis does not establish causal links between variables. In the case of participatory demand and gain scores, high mathematical participatory demand causing students' mathematical learning is only one possible explanation for the observed phenomena. There are several others. For example, the causality may flow in the opposite direction—it may be that students who are learning successfully tend to speak more often during class and, when they do speak, tend to vocalize mathematical relations. Another possibility is that teachers who engender high mathematical participatory demand also tend to ask good mathematical questions and it is actually these questions that cause the student learning. In fact, there may be many teaching practices that covary with high mathematical participatory demand and it may be some combination of those practices that causes the learning. A final example of an alternative explanation would be that the correlation arose merely by chance and the variables are, in fact, unrelated. The analysis presented above does not support any of these explanations over the others (although the last is statistically improbable).

Nevertheless, both of the correlational results presented in this chapter are worthy of further consideration. With respect to cognitive demand, the lack of a positive relationship to the standardized gain scores is surprising, given past findings (e.g., Stein & Lane, 1996). Let us, in deference to these past studies, again consider alternative explanations for the lack of relationship found here. First, it could be that there truly was no relationship between cognitive demand and students' learning in these particular settings and with this particular mathematical content. This seems unlikely, however, because of the wealth of evidence in the research literature related to the importance of cognitive demand as a construct in mathematics education, much of which is situated within middle school contexts. Instead, it may have been

methodological limitations that prevented the uncovering of a relationship. In particular, the modesty of the sample size in the present study, with the unit of analysis being sets of lessons ( $n = 12$ ) rather than mathematical tasks ( $n = 37$ ) or students ( $n = 246$ ), meant that rank correlation had to be used with a lower level of statistical power than more sophisticated types of analysis such as regression. Additionally, the written assessments used as a measure of students' mathematical learning may not have been sensitive enough to illuminate the changes in students' performance that are linked to cognitive demand. Although the test was designed around the focal learning objective of the task enactments and did contain both high- and low-demand items, it was not specifically designed with regard to cognitive demand. Stein and Lane (1996), for example, had to refine their outcome measures substantially to capture cognitive demand, which allowed them to arrive at the findings they did.

Perhaps the most important reason to consider for the evident lack of relationship between cognitive demand and class gain score was the fact that cognitive demand in this study was operationalized differently than in past studies, meaning that this was not a true replication study. In the present study, sets of lessons were ranked based on the prevalence of tasks that involved at least one phase of enactment at a high level of cognitive demand. Enacting multiple phases of a task at a high level only came into play when breaking ties between sets of lessons. Moreover, the working phase of implementation was not coded. Within studies such as Stein, Grover, and Henningsen (1996) or Stein and Lane (1996), on the other hand, the focus was on the maintenance or decline of cognitive demand, not the prevalence of high cognitive demand. In fact, a cursory glance at the trajectories of cognitive demand within this study's task enactments (see Appendix C) reveals several task enactments which involved a high-level set-up but a low-level working phase and look-back phase. Such enactments would be considered declining

trajectories of cognitive demand but, in this study, were counted as an enactment that did involve high cognitive demand.

Similarly, with respect to participatory demand, the ranking process focused on the prevalence of high mathematical (H / M) verbal student participation but did not make distinctions about where the high mathematical participatory demand occurred within the task enactment or between other forms of participatory demand. As can be seen in Appendix D, the majority of instances of high mathematical participatory demand occurred during the look-back phase. Given the fact that such participatory demand was found to be related to student learning, I now move on to examine two teacher's look-back phases in more detail. Further implications and issues surrounding the findings from this chapter are addressed in Chapter 6.

## **CHAPTER 5**

### **TWO CASES OF ENACTMENT**

In this chapter, I focus on the same aspects of the enacted curriculum as in Chapter 4—namely, the cognitive demand and participatory demand of mathematical task enactments—but instead of coding across all 12 classrooms I present narrative cases of two teachers’ classrooms to illuminate those aspects in a different way, that is, with representations of classroom interactions and a discussion of those particular interactions. Moreover, I focus these narrative cases primarily on the look-back phase of enactment because this phase is new to the MTF and it is also the phase where high mathematical participatory demand most commonly occurred. Recall from Chapter 3 that these specific classrooms were selected because they were in the same district and used the same curriculum materials but differed in the ways that cognitive demand and participatory demand played out during the task enactments. Additionally, the classes differed in their standardized gain scores.

Because this study is not solely an examination of the enacted curriculum but of the enacted curriculum and its relationship with student learning, it is important to note the way in which student learning arises in this chapter. In contrast with the previous chapter in which student learning was operationalized as the gain scores on written pre- and post-tests, and thus situated as an outcome of enactment, this chapter involves student learning as embedded within the enacted curriculum itself. As explained previously, the relationship between the enacted curriculum and student learning is bidirectional and student learning is inseparable from students coming to participate in the discourse community of their mathematics class. Thus, when the interactions and contributions of students are included in the narrative cases below, this

simultaneously represents the enacted curriculum—what is happening in the classroom at the moment—and provides insight into features of student learning.

### **Background**

The two narrative cases presented in this chapter involve Ms. Wyncott and Ms. Albert. Ms. Wyncott is presented as a case of cognitive demand and participatory demand that were not followed through to their full potential as defined by the written task and the task as set-up by Ms. Wyncott herself. Ms. Albert, on the other hand, constitutes a case of high mathematical participatory demand occurring in conjunction and in support of the maintenance of high levels of cognitive demand. Before presenting the cases, I use this section to describe various aspects of the classroom setting that are common to both teachers' classrooms including the timeframe, the school district, and the curriculum materials. I do this in order to set the broader, shared context in which the narrative cases are to be interpreted.

As shown in Table 3, Ms. Wyncott and Ms. Albert both taught grade 8 mathematics in the Alpha school district in Delaware during the 2003–2004 academic year, which was year 2 of the *Project 2061* professional development program. The fact that this was year 2 of the professional development meant that both teachers had attended the summer sessions in which *Project 2061* staff worked with teachers related to representations of relationships between varying quantities. In particular, *Project 2061* guided teachers to identify what mathematical idea was being represented, whether it was being identified accurately, whether it was likely to be comprehensible to students, and how instruction might help students in considering the strengths and limitations of the representation (DeBoer et al., 2004). With respect to the Alpha school district, Ms. Wyncott and Ms. Albert taught in the same middle school, which currently has a student population of approximately 770 students (grade 6–8). Although the district population

has been growing for the past decade, it is unclear whether the population at this particular school was higher or lower in AY 2003–2004 because, at that time, there were only two middle schools whereas there are three presently. The ethnic breakdown of the district in AY 2009–2010 was 56% white, with African Americans constituting the largest minority group at 34%. The district is located within the most affluent county in Delaware, but the state Department of Education reports that in AY 2009–2010 there were 29% of students in the district classified as “low income” (see Table 3).

Ms. Wyncott and Ms. Albert used the *Mathematics in Context* (National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1998) textbook series in their teaching. For the focal lessons in this study, they enacted tasks from Section E, which was entitled “Solving Equations.” This section followed a section in which students explored graphed lines and wrote equations for those lines. In the “Solving Equations” section, the students moved on to writing equations and developing strategies for solving those equations. The first portion of this section involves a relationship between varying quantities—that is, the number of hops taken by frogs and their resulting position—and the use of variables to represent this relationship. The teachers and *Project 2061* staff identified this section to be most closely aligned with the focal learning goal of using variables to represent relationships of change. In particular, the first task begins with a frog named Alice sitting 8 decimeters from a path and another frog named Fred sitting 18 decimeters from the same path. The students are asked to consider what information they would need to be able to determine their new distances from the path. This question draws attention to the role of the jump length, which is not yet known. In the next question, students are told to assume that Alice and Fred’s jumps are all of the same length and the textbook authors introduce a diagrammatic representation that is used throughout the section (see Figure 15).

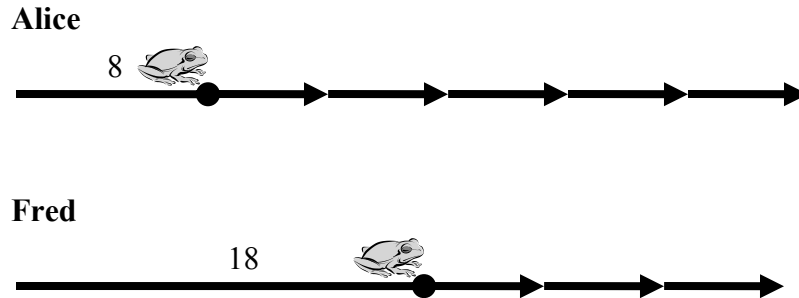


Figure 15. A diagram of two frogs' jumps from *Mathematics in Context* (1998).

The written task continues by providing specific values for the jump lengths and asking students to determine Alice and Fred's final positions. Next, the written task reverts to an unknown jump length and instead asks students to solve the following problem, which I have paraphrased: If Alice and Fred take five jumps and three jumps, respectively, and finish at the same distance from the path, how long was each jump? The written task explicitly directs students to “discuss strategies for solving this problem” (p. 32) and then share their group's method with the rest of the class. A goal for this task is to have students use a variable to represent the unknown jump length and write algebraic expressions for each of the frogs, which can then be set equal to one another to solve the problem. The textbook section then has students work with a variety of frog scenarios, translating them into algebraic expressions, while also asking them to take algebraic expressions and generate the associated frog diagrams. Explanations are commonly requested from the students and the textbook authors also include prompts such as, “How can you be sure that your answers are correct?” (p. 34).

With respect to cognitive demand, the written tasks included multiple representations as well as explicit prompts to draw connections between these representations and the problem context. The written task also asked students to consistently explain their thinking, compare methods with one another, and make sense of the steps of a solution process. These are features



of a high-level task. With respect to participatory demand, this written task was the only one in the entire study to explicitly set expectations for student participation as it asks students to discuss approaches to the problem in groups, not just solve the problem, and then share out to the entire class.

I now turn to the narrative cases of Ms. Wyncott's and Ms. Albert's enactment of these written tasks. For each, I first remind the reader of the teacher's background characteristics and then describe the set-up and working phases of enactment before moving on to the heart of the case, which is the look-back phases of enactment. Throughout the cases, issues related to cognitive demand and participatory demand are central and my interpretations of the classroom interactions are woven into the narrative as, indeed, any telling of events involves implicit interpretation, which I am choosing to pair with explicit interpretations.<sup>24</sup>

### **The Case of Ms. Wyncott**

Ms. Wyncott was in her fourth year teaching mathematics during AY 2003–2004. She had received her teaching certification in elementary education with a specialization in middle school mathematics. Her grade 8 mathematics class used the *Mathematics in Context* textbook series and this case centers on her enactment of the initial tasks within the “Solving Equations” section of the textbook, which is Section E. During these lessons, there were 25 students in attendance (though only 21 students completed both the pre- and post-test; see Chapter 3). With regard to the physical arrangement of the classroom, Ms. Wyncott had her students positioned in clusters of four desks each, essentially forming small-group tables throughout the room. There

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<sup>24</sup> The cases involve excerpts of classroom transcripts. For details on my treatment of transcripts, see Appendix B.

were six such four-desk tables arranged in two rows across the room and a seventh four-desk table in a back corner. A blackboard was located on the front wall with a screen pulled down in the center for use with an overhead projector, which was stationed between the front board and the front-center small-group table. From the perspective of a student facing the front board, Ms. Wyncott's desk was located in the front-left corner and a bulletin board and filing cabinet were located in the front-right corner. The door to the classroom was in the right wall and an exterior window was in the left.

### **Setting Up the First Task**

After a warm-up problem in which students were reminded of the size of a decimeter (i.e., 10 centimeters), Ms. Wyncott shifted to the written task from the textbook by noting that the students were “going to be using decimeters a lot in this [textbook] section.” She then directed students to open their books to page 31 and to get out a clean sheet of paper, labeling it “Section E” at the top. After a few moments, Ms. Wyncott had a male student read aloud the first two introductory paragraphs on page 31 and a female student read aloud the third paragraph located just above problem 1 on the same page. After the students finished reading, Ms. Wyncott said, “Okay, I need someone to summarize. What’s this about? What’s the situation here?” She called on another male student, who responded that the situation involves frogs that are jumping away from a path.

Ms. Wyncott then read the first problem aloud, “Number one, ‘What information would you need to find their new distances from the path?’” She explained that the textbook had left out several pieces of information. “If we want to know now how far away they are from the path, what kinds of things would we need to know?” Various students responded with “the distance of

their jumps,” “how many times they jump,” and “direction.” Ms. Wyncott revoiced<sup>25</sup> these responses and recorded them on an overhead transparency, reminding the students that they should also be recording them on their papers.

Moving on to page 32, Ms. Wyncott again had a student read aloud. After the student finished reading, Ms. Wyncott asked, “What did we find out now that Patricia read us that new information? What do we know that maybe we didn’t know before?” A male student responded, “How many times they jumped.” Ms. Wyncott affirmed this response and then asked, “Anything else?” A different male student said, “Where they stopped.” Ms. Wyncott replied, “Well, we’re going to try to figure out where they stopped, but if we don’t know how big their jump is, which is something we still don’t know, it’s going to be hard to figure out where they stopped.” She then pointed out herself that, based on the diagram on page 32 (see Figure 15), the frogs were traveling in the same direction, that is, away from the path. During this time, Ms. Wyncott began handing out a worksheet to the students. The worksheet simply contained five copies of the diagram on page 32—one for the students to use as they worked on problem 2A, another for problem 2B, and three extra diagrams. Ms. Wyncott directed, “I want you in your groups to work on 2 A and B. Label your diagrams based on answering those questions, 2 A and B.” She continued, “I’m assuming this won’t take any more than, you know, three or four minutes. So

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<sup>25</sup> Revoicing is a teacher discourse move in which the teacher repeats or rephrases a student contribution and checks back with the original student regarding the accuracy of the rephrasing (Chapin, O’Connor, & Anderson, 2003). This move may serve many different functions in the classroom (Herbel-Eisenmann, Drake, & Cirillo, 2009).

keep working, 2A and B.” Ms. Wyncott then moved from the front of the room and crouched next to a student who had raised his hand.

### **Working and Looking Back on the First Task**

Following the set-up phase, and contrary to Ms. Wyncott’s estimate, the students worked in small groups for 11 minutes before Ms. Wyncott called them back together as a whole class. This work time consisted of most students sitting quietly and working on their own individual papers for the first 5 minutes, with Ms. Wyncott circulating to several groups to help them clarify what it was they were supposed to be doing for 2A and 2B.<sup>26</sup> This clarification typically involved Ms. Wyncott drawing the students’ attention to the diagram on the worksheet and how the information from the problem can be represented on the diagram. During the next 6 minutes of work time, the general volume of talking in the room increased but it appeared as though some of the groups were engaging in off-task discussions. Ms. Wyncott moved among the groups, spending approximately 90 seconds at each, but frequently calling out to other groups, asking them to “get back to work.” The last few groups that Ms. Wyncott visited had already completed the problems.

Next, Ms. Wyncott turned off the classroom lights and turned on the overhead projector. She addressed the entire class, “Lights are out, all eyes should be up front.” After a few moments during which the students settled down, Ms. Wyncott began to look back with the class on their work from 2A and 2B, beginning with 2A.

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<sup>26</sup> Problem 2A asked students to find the distances from the path of Alice and Fred if their jumps were of length 4 decimeters. Problem 2B involved the frogs jumping between 2 decimeters and 6 decimeters each time and asked the students what they can conclude in this scenario.

1 Ms. Wyncott: First I want to talk about 2A. They had told us the distance was four  
 2 per jump. So, how did you label your picture to show that? Dennis.  
 3 Dennis: From, like, the eight I counted up four for each little arrow, so I got  
 twelve, sixteen, twenty, twenty-four, twenty-eight. *[[Ms. Wyncott writes these numbers on the top portion of the diagram on the overhead projector.]]* For the bottom one it would be eighteen to  
 twenty-two, twenty-six, thirty. *[[Ms. Wyncott writes these numbers on the bottom portion of the diagram.]]*  
 4 Ms. Wyncott: So their ultimate distance is right here at the end. *[[Ms. Wyncott circles the 30 and the 28 on the diagram.]]* Okay. Okay. *[[Ms. Wyncott turns on the classroom lights.]]* Did anyone label their  
 picture differently? I'll use a different color [marker].  
 5 Trayvon: I just put my answer at the end.  
 6 Ms. Wyncott: You just put your answer at the end? Okay. Anybody else label it  
 differently? *[[pause]]* Patricia?  
 7 Patricia: I put the plus four on the jumps.  
 Ms. Wyncott: Okay, you put plus four on each of the jumps. *[[Ms. Wyncott writes "+4" on each arrow in the diagram.]]* Good. Anybody else? I think  
 that pretty much covers the two ways that people might have drawn  
 there. Now in 2B they're telling you that these frogs can jump as  
 short as two decimeters but their jump can also be as long as six  
 decimeters, and they want to get an idea of the different distances  
 that the frogs could have been away from the path based on different  
 sized jumps. So can someone give me an example of what you  
 might have drawn for one of your examples in 2B? Rodney.  
 8 Rodney: Plus two plus three plus four plus five plus six.  
 9 Ms. Wyncott: Here [at Alice's starting position], plus two plus three plus four plus  
 five plus six. *[[Ms. Wyncott writes "+2" and so forth on a new copy of the diagram.]]* Well, when you do that you're taking away the  
 assumption that they have to travel the same distance for each jump.  
 Right? So instead of labeling plus two plus three plus four plus five,  
 how could we change that so that each jump is the same distance in  
 our picture? Dennis?  
 10 Dennis: Um, you could have, like I did, for eight I did, um, plus two plus  
 five plus six plus two plus four.  
 11 Ms. Wyncott: Again, when--, all in one picture you're doing that? *[[Dennis nods.]]*  
 Because when you do that you're now not--, you're not assuming  
 that they've jumped the same distance in each jump.  
 12 Dennis: But I thought you said not to assume that they jumped the same  
 distance.  
 13 Ms. Wyncott: No, [the textbook] says up at the top [of page 32], "Alice and Fred  
 travel the same distance," and what Rodney did was he did *[[Ms. Wyncott points at the diagram]]* plus two plus three plus four plus  
 five, so each jump was a different size.  
 14 Dennis: So you have to do, like, plus five plus five plus five plus five plus  
 five? *[[Ms. Wyncott writes "+5" on each arrow in the top portion of*

- the diagram.*]]
- 15 Ms. Wyncott: There we go. And I know some of you out there did that, you must not be volunteering. So what distance would that be? ... What would [Alice] ultimately get to? *[[pause]]* Robby?
- 16 Robby: Thirty-two.
- 17 Ms. Wyncott: Okay *[[Ms. Wyncott writes "32" at the end of Alice's diagram]]*. And if she jumped plus five plus five plus five plus five, we're going to assume that Fred also jumped plus five plus five plus five *[[Ms. Wyncott writes "+5" on the arrows in Fred's diagram]]*. *[[Kevin raises his hand.]]* Kevin?
- 18 Kevin: I did plus two.
- 19 Ms. Wyncott: Okay, that's going to be another example that we can draw down here. What's the ultimate distance when you jump plus five plus five plus five?
- 20 Student 1: Twenty-five.
- 21 Student 2: I got thirty-three.
- 22 Student 3: I got thirty-three, too.
- 23 Ms. Wyncott: Yeah, *[[Ms. Wyncott writes "33" on Fred's diagram and also writes "33" replacing the "32" on Alice's diagram]]* actually this should be thirty-three, too. Twenty-five plus eight is thirty-three. Now, I'm going to give you two more minutes *[[Ms. Wyncott holds up two fingers]]*. You need to come up with examples other than four or five that they could have jumped and fill in your other two pictures *[on the worksheet]*. Two minutes, that's all you've got. If you need to talk to your group, that's fine, but it shouldn't take too long. Two minutes.

Within the exchanges above, Ms. Wyncott had students share how they labeled their frog diagrams for 2A and then she clarified for students that, in 2B, the jump lengths had to remain constant within a diagram even if they varied between diagrams. Students stated and Ms.

Wyncott recorded the final distances in the case of jumps of length 5 decimeters.

This look-back on the first task contrasts with the high level of the task as written because the focus was on the way in which diagrams were labeled rather than the thinking behind the labeling (see turns 1–6) and the issue of what it meant for the jumps to be between 2 decimeters and 6 decimeters was handled in terms of a dictated assumption (see turn 13) rather than a conceptual discussion of how the problem would be different under different constraints.

The final turn in the transcript excerpt above is a brief set-up of an additional segment of the working phase. As in the previous segment of work time, the students spent the first few minutes working relatively quietly. Many students appeared to be working independently on their own paper rather than interacting around the task with the classmates at their table. For the last two minutes or so of this six-minute work segment, Ms. Wyncott went to different small-group tables and had the students copy one of their new examples of jump lengths onto her overhead transparency. During this time, several groups were laughing and seemed to be off task. Ms. Wyncott asked one group of female students in particular to “stop fooling around, please.” She then called the whole class back together by returning to the front of the room and turning on the overhead projector.

24 Ms. Wyncott: Alright, there’s some examples that your groups have come up with. *[[Ms. Wyncott displays a transparency with two diagrams filled in from before and three additional diagrams now filled in.]]* We already talked about jumping four decimeters each jump. We talked about jumping five decimeters each jump. Now I’ve had three different people in the room put up if he jumped three decimeters each jump *[[Ms. Wyncott gestures to the “+3” diagram]]*, or six decimeters each jump *[[Ms. Wyncott gestures to the “+6” diagram]]*, or two decimeters each jump. That includes our range of saying the frog can’t hop any shorter than two or any further than six.

25 Michael: The higher the numbers the more Alice gains.

26 Ms. Wyncott: Okay, the higher the jump the more further Alice gains on him. Interesting, interesting. The more she gains. When you say “gains,” what do you mean by that?

27 Michael: The more she catches up.

28 Ms. Wyncott: The more she catches up. Does she ever end up further than him?

29 Kevin: Yeah.

30 Dennis: At six.

31 Ms. Wyncott: At six, okay. Okay, keep that in mind as you go ahead--, actually, we’re going to answer this one [problem 3] together as a class. Can I have a volunteer to read at the bottom of page 32. *[[Ricky raises his hand.]]* Ricky, go ahead.

32 Ricky: *[[Ricky reads from the textbook]]* “Suppose the frogs finish their jumps at exactly the same distance from the path, and you want to know the distance of each jump and each frog’s final distance from

- the path. In groups, discuss strategies for solving this problem.”
- 33 Ms. Wyncott: Okay, so it says suppose the frogs finish their jumps at exactly the same distance, exactly the same distance. Think about how you could figure out how big their jump was if you want them to end up exactly the same distance. Any ideas? *[[Several students raise their hands.]]* Dennis.
- 34 Dennis: You’d have it at plus five because, um, they both end up at thirty-three.
- 35 Ms. Wyncott: Okay, we actually have an example where it worked out, don’t we? Right here *[[Ms. Wyncott points to the “+5” diagram]]*, plus five plus five plus five. So how did we figure that out? How did we end up with that plus five plus five plus five originally, Dennis?
- 36 Dennis: Because, uh, we were--, we had said that they could jump anywhere between two and six.
- 37 Ms. Wyncott Mm hmm.
- 38 Dennis: And we were just picking numbers, and we had five and I guess we just accidentally came up with them the same.
- 39 Ms. Wyncott: Um, okay, okay. It just happened in this case to end up the same. We also had Michael point out that Alice is always trying to catch up here *[[Ms. Wyncott points to the diagrams]]*, with a distance of two, with a distance of three, with a distance of four. But with a distance of six she was further along. So somewhere in between there they must have met up and been at the same distance, because all of a sudden, instead of being shorter, she’s further along from the path.

In the preceding transcript excerpt, students noted that Alice gained more on Fred when the jump lengths were longer. Ms. Wyncott then went through problem 3 as a whole class, with Dennis articulating that a jump length of 5 decimeters was a solution to that problem. Ms. Wyncott, in turn 39, summarized that this solution occurred in between the cases where Alice ended closer to the path than Fred and the case where Alice ended farther from the path. It is noteworthy that, in looking back on the first task, the solution to what could have been a reasoned problem, that is, problem 3, turned out to simply be one of the previous trial situations (see turns 34 and 38). Because of this, the solution was “accidentally” found, which represents a lower level of cognitive demand than if they had reasoned their way to the solution.



Although the student's version of the written task explicitly mentioned the type of participation that was expected during the task, Ms. Wyncott did not enact these participation structures. She instead led the class through a single solution to problem 3, which was essentially to notice that the case of "plus five plus five plus five" from problems 2A and 2B yielded the correct answer (see turns 33–39). The participatory demand during this portion of the enactment was of a moderate level as a few students spoke in full sentences. The focus was semi-mathematical as students did use mathematical terms (e.g., numbers, "plus five" as a single thematic item), however, these numbers were most often in relation to the frogs or to students themselves, as when the students recounted an action that they had taken or to produce the number, rather than talking about mathematical relations.

Following the work on the first task, Ms. Wyncott had the students turn to page 33 in the textbook, which remained in the same problem context of the jumping frogs but shifted toward an explicit focus on the use of variables and the writing of algebraic expressions and equations.

### **Setting Up the Second Task**

As before, Ms. Wyncott began by asking a student to read aloud the paragraph at the top of page 33. The paragraph mentioned that the symbol  $x$  can be used to represent the unknown jump length. Ms. Wyncott said, "We have talked about an unknown before, but we haven't actually called it an unknown." She explained how the pattern numbers they had used before, symbolized by  $n$ , were now being called unknowns. Ms. Wyncott said that the "formal way, especially when you're using graphing calculators, is to use the  $x$  and the  $y$ ." She added, "another name for it can be a variable." Next, Ms. Wyncott moved the class on to problem 4 of the written task. She read aloud from the book, "Explain how the equation  $[8 + 5x = 18 + 3x]$  describes the diagram in Box A." Then Ms. Wyncott gave students two minutes to discuss this problem in

their small groups and write their answers on the back of the worksheet they had worked on previously. She repeated, “Two minutes, that’s all you’ve got,” and then left the front of the room as students started working.

### Working and Looking Back on the Second Task

Ms. Wyncott gave the students approximately four and a half minutes of work time in their small groups. Most students appeared to be writing independently or talking with one another about non-mathematical topics. Ms. Wyncott visited several groups and asked how they were doing on problem 4, asking some of them about the meaning of particular components of the equation (e.g., “What does this 5 mean?” “What does the  $x$  stand for?”).<sup>27</sup> Ms. Wyncott then pulled the whole class back together in the midst of this task enactment.

- 40 Ms. Wyncott: Alright, I’m getting some mixed ideas here about what exactly this eight plus five  $x$  stands for. Now, I know that we’ve got a different situation here, and it’s frogs jumping and we haven’t really seen frogs jumping before, but we’ve written equations like this [[Ms. Wyncott points to the diagram and equation on the overhead transparency]] many times before, like in Patterns and Figures with the hair growth. So we set up that the unknown is  $x$ , that’s our unknown. Our unknown is what in this situation? What are we trying to find out? [[Brandon raises hand.]] Brandon?
- 41 Brandon: How many jumps, or the--, what equals the whole jumps.
- 42 Ms. Wyncott: Say that again.
- 43 Brandon: The distance between our jumps.
- 44 Ms. Wyncott: The distance between the jumps? Okay, so how big a jump is. Okay,

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<sup>27</sup> In several of Ms. Wyncott’s interactions, even students who correctly identified  $x$  in the diagram talk about it, as did Ms. Wyncott, as meaning “jumps” when a more precise phrasing would be that the  $x$  represents the length of each jump, which distinguished it more clearly from the coefficients of  $x$  which are the number of jumps. This distinction is brought up later in a whole-class setting.

- that's our unknown. So  $x$  is how big that actual jump is. So what's this five telling me here? [*Trayvon raises his hand.*] Trayvon?
- 45           Trayvon: How many times the frog jumped.
- 46           Ms. Wyncott: How many times Alice jumped. So some of you guys, as I walked around and heard your conversations, you were thinking--, you were thinking that the five was telling you already how big the jump is. The five is telling us that Alice has jumped five times, and we're going to try to figure out how big one of those jumps are.

In this brief whole-class exchange, Ms. Wyncott worked to clarify the use of  $x$  in the problem situation and to relate it to students' previous experiences using letter symbols in the context of sequential patterns. Brandon's responses to her questions may not have been entirely clear, but Ms. Wyncott revoiced them in turn 44 to explicate the fact that  $x$  represented the jump length. She then asked what one of the coefficients of  $x$  represented and Trayvon responded that it was the number of jumps. Ms. Wyncott employed the IRE interaction pattern twice (turns 40–44 and 44–46), which aligns with a generally low level of participatory demand as only two students were involved and they uttered only phrases. These phrases constituted a semi-mathematical focus, because the teacher supplied the first portions of the semantic relations in question and the students supplied only the latter portions.

Next, Ms. Wyncott drew the students' attention to the other diagrams and equations displayed on page 33 of the textbook. These depicted in three boxes—Box B, Box C, and Box D (Box A contained the original equation)—the equations  $8 + 2x = 18$ ,  $2x = 10$ , and  $x = 10/2 = 5$  as well as the arrow diagrams associated with each of these equations. Ms. Wyncott stated to the class that each of these diagram and equation pairs represented steps in solving problem 4. Ms. Wyncott set the students back to work for five minutes by saying, "So just follow along and see what happens here [throughout the steps]. I'm going to give you probably about five minutes in your group to examine what's happening here--, maybe three minutes to examine what's

happening in each box. I'm going to come around to the groups as much as I can. But talk with your group. Other people have ideas, you need to hear them."

The students spend approximately six minutes working through problems 5, 6, and 7 in the textbook. These problems ask the students to "explain the equation[s]" and "describe how the diagram was changed" from step to step. Several of the groups appeared at this time to be talking about the task, although there were a few that were audibly talking about nonmathematical topics. Ms. Wyncott circulated the room and asked students what they were thinking about the diagrams and equations and encouraged two groups to talk with their group about their ideas.

The last three minutes of the in-class task enactment were spent again looking back as a whole class. As mentioned above, Box B contained the diagram and equation for  $8 + 2x = 18$  and Box C contained the diagram and equation for  $2x = 10$ . To begin this segment of the look-back phase, Ms. Wyncott moved to the front of the room.

- 47 Ms. Wyncott: I need all eyes back up front. We are quickly running out of time. I see a lot of you guys heading in the right direction, but I'm also seeing a lot of you off task and we only have five minutes to wrap this up. We talked about what the eight and the five  $x$ 's are [from the equation  $8 + 5x = 18 + 3x$ ], what the eighteen and the three  $x$ 's are. What happened from Box A to Box B? What happened from Box A to Box B? [*Trayvon raises his hand.*] Trayvon?
- 48 Trayvon: They just showed the first two jumps of Alice will--, the two jumps will equal the first part.
- 49 Ms. Wyncott: Okay, Alice's distance and her two jumps equals where Fred starts. So basically what [the textbook authors have] done is they cut this part of the picture [Alice and Fred's last three jumps] out... [*Ms. Wyncott crosses out the three jumps on the original diagram.*] so that they can show that Alice's distance and two of her jumps equals Fred's distance that he started at. Okay, so then what happens from Box C--, from Box B to Box C?
- 50 Trayvon: They, uh--.
- 51 Ms. Wyncott: Not necessarily Trayvon but anybody. What happens from Box B to Box C? Chonda?
- 52 Chonda: There's less, um, hops. To start--.
- 53 Ms. Wyncott: There's less hops here [*Ms. Wyncott points to Box C on the overhead transparency*] than there are here [*Ms. Wyncott points to*

- Box B on the overhead transparency*]]? You said there's less hops, I'm not sure what you mean.
- 54 Chonda: There was--,  $x$  is like--, there is two  $x$ 's.
- 55 Ms. Wyncott: Okay, there are two  $x$ 's here *[[Ms. Wyncott points to Box B]]* but I still see two  $x$ 's here *[[Ms. Wyncott points to Box C]]*. What is different from B to C? B to C? Trayvon already told us about A to B. B to C? What do you see happening? Patricia? *[[Patricia shakes her head.]]* Dennis?
- 56 Dennis: Well, from B to C, that each, um--, each of the little  $x$ 's is five because if two of them--, if two  $x$  equals ten, then--.
- 57 Ms. Wyncott: You know this already from box C? How did they get, though, from Box B to Box C?
- 58 Dennis: They, um--, *[[pause]]*.
- 59 Ms. Wyncott: What did they change to get from Box B to Box C? That's all we're asking to look at here. *[[James raises his hand.]]* James.
- 60 Student 4: They took out eight.
- 61 James: They took out the starting amount.
- 62 Ms. Wyncott: They took out the starting amount for the first one. *[[Ms. Wyncott crosses out the initial 8 units from Box B.]]* And I can also go up here [in Box A] and cross out eight. *[[Ms. Wyncott crosses out the initial 8 units from Box A.]]* So now I'm left with two jumps of Alice equals Fred's distance of ten. Now what did they change to get from Box C to Box D. *[[Several students raise hands.]]*
- 63 Monique: They divided by half.
- 64 Ms. Wyncott: Why did they divide it by half?
- 65 Courtney: Because two jumps is ten.
- 66 Monique: Yeah, what Courtney said.
- 67 Ms. Wyncott: So we know what two jumps is and if we get rid of one of the jumps *[[Ms. Wyncott crosses out the right half of the diagram in Box C]]*, now we know what one jump is. One jump covers a distance of five. *[[Ms. Wyncott removes the transparency from the overhead projector.]]*

Within these exchanges, Ms. Wyncott asked students to identify changes between the various diagrams. When they did so (i.e., turns 48, 52, 54, 60, 63), Ms. Wyncott sometimes asked for further clarification (e.g., turn 53) or for students to focus on specific changes rather than other features of the diagrams (e.g., turns 57 and 59). With respect to the rationales for the changes, Ms. Wyncott provided one (i.e., turn 49), asked students to provide one (i.e., turn 64), and, in one

case, asked a student to withhold from sharing a rationale (i.e., turn 59) until a later time. Ms. Wyncott ended this set of interactions by removing the written task from the overhead projector.

In the previous segment of the look-back phase (turns 40–46), Ms. Wyncott was working to clarify the connection between the  $x$  symbol and the problem situation as represented in the diagrams. The interpretation of  $x$  is a key conceptual link in the task, but in this final segment of the look-back phase, the key conceptual links between steps in the solution process (e.g., from Box B to Box C) were not attended to in the same way. Instead, Ms. Wyncott asked students to describe what changed between the consecutive boxes as opposed to *why* the changes were productive or how the steps were building toward a solution to the problem. The solution was formed, but not at a high level of cognitive demand.

Also, in this final segment of the look-back phase, several of the verbally participating students are those who have also participated previously (e.g., Trayvon, Dennis).<sup>28</sup> There were several full sentences uttered by students but, again, they often consisted semantically of actions being taken rather than mathematical relations. In this case, the actor was an abstract “they” because the steps were depicted in the textbook (e.g., “They took out eight,” “They divided by half”). The participatory demand did not generally consist of students verbalizing mathematical relations between the steps of the solution process. Such participatory demand may have simultaneously impacted the cognitive demand described above as low during this phase of enactment. The exception to this pattern was Dennis’s contribution in turn 56 when he worked to

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<sup>28</sup> One might also notice that a large majority of the students who contribute verbally are male. It should be noted that Ms. Wyncott had significantly more male students than female students, but a more detailed analysis of participation based on gender is beyond the scope of this thesis.

articulate a mathematical relation between  $2x = 10$  and the solution of  $x = 5$ . This turn, however, was somewhat unendorsed by Ms. Wyncott who, in her next two turns, shifted attention back to a description of the changes between Box B and Box C rather than the mathematical relations between Box C and Box D.

To conclude the task, Ms. Wyncott assigned several problems from the remainder of the written task for students to complete as homework. These problems were not explicitly looked back upon during this or the subsequent class period when they moved on to the next task in the textbook.

### **Summary: A Case of Declining Cognitive and Participatory Demand**

Ms. Wyncott's task enactments form a case of cognitive demand and participatory demand that were not followed through to their full potential, where this potential was indicated by the written task and by Ms. Wyncott herself. As described above, the cognitive demand declined in various ways during the look-back phases of enactment.<sup>29</sup> With respect to participatory demand, the student talk during the set-up phases consisted primarily of non-mathematical thematic items related to the context, which is not unexpected because this was the

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<sup>29</sup> Although there is evidence of low cognitive demand during the look-back phase, it might also have been the case, especially with the first task, that the cognitive demand declined during the working phase because Ms. Wyncott provided too much time for the students to complete problems 2A and 2B, as evidenced by the fact that students increased over time in their off-task behavior and, when visited by Ms. Wyncott, had already completed the problems. Nevertheless, it is noteworthy that the cognitive demand remained low during the look-back phase when it is conceivable that it could have shifted back up.

students' first encounter with the problem situation of the jumping frogs. The look-back phases, however, also lacked high mathematical participatory demand as the students tended to provide single mathematical thematic items or recount experience rather than form full mathematical relations. Overall, then, Ms. Wyncott's case involves declines in cognitive demand as well as a lack of high mathematical participatory demand, with the set-up phases somewhat non-mathematically focused, the working phases marked by independent or off-task activity, and the look-back phases typically characterized by moderate semi-mathematical participatory demand. This case raises the question of whether a push for high mathematical participatory demand, perhaps during the look-back phases, could have simultaneously impacted the cognitive demand of the students' mathematical work in positive ways.

### **The Case of Ms. Albert**

Ms. Albert had taught mathematics for 7 years at the time that data were collected for this study, which was AY 2003–2004. She was certified in middle school mathematics and had already received a Master's degree at this time. Ms. Albert's grade 8 class used the same *Mathematics in Context* textbook as Ms. Wyncott's and this case centers on the same tasks. During these lessons, there were approximately 25 students in attendance (of whom 24 completed both the pre- and post-test; see Chapter 3). The physical arrangement of the students' desks was in five clusters of six desks each. A green chalkboard was located at the front of the room along with an overhead projector and pulled-down screen. Ms. Albert's desk was located in the front-right corner of the room, the door to the classroom was located in the opposite corner, and an exterior window was located in the right wall.



## Setting Up the First Task

Ms. Albert began the task enactment by having two student volunteers read the opening paragraphs of the written task on page 31 and then providing the class with approximately 18 seconds to think independently about what information would be needed to find the frogs' new distances from the path. She then asked the students to share what information they thought was needed. Students stated such things as in what direction they are jumping, "how far they jumped each time," and "how many times they jumped." Ms. Albert repeated each of these responses and evaluated them by saying "good" or "excellent." One student responded, "What distance they stopped at," and Ms. Albert clarified that this is what the question is asking for rather than a piece of information that is needed.

Next, Ms. Albert asked students to turn to page 32 and she then read aloud from the written task. Ms. Albert prompted a student to explain to the class how the diagram (see Figure 15) showed the number of jumps taken by each frog and then Ms. Albert noted on the overhead transparency that she drew curved arrows rather than straight arrows to represent the jumps, though the length measure is still straight along the ground. Ms. Albert then said, "Okay, what we're going to do is [problem] 2A in your groups. And let me give you some directions for what I'd like you to do for 2A." She then had a student read aloud problem 2A and stated that she wanted them to draw the diagram for 2A "to scale." She proceeded to pass out graph paper and rulers and to ask the students about what scale would be appropriate to use with the graph paper (i.e., 1 grid length = 1 decimeter). This portion of the task set-up ended when Ms. Albert asked students to work with their tools on problem 2A, which is described in the next section.

After working and looking back on problem 2A, Ms. Albert set up the work on problem 2B by, first, having a student volunteer read the problem aloud. She then explained that they

would not be drawing the diagrams “to scale” in every instance, even though they did so for problem 2A. In particular, she said it would be fine to “just sketch” the new situations. Following this monologue, Ms. Albert asked the students to work in small groups on problems 2B and 3.

### **Working and Looking Back on the First Task**

Students worked for approximately 9 minutes on problem 2A. The small groups appeared to be generally on task, as evidenced by their use of the rulers and graph paper, their gestures to one another as they worked, and Ms. Albert’s reactions to the groups as she circulated. Many of the students interacted verbally with one another and these exchanges seemed to be centered on the mathematical task. As she circulated, Ms. Albert mentioned to a few groups that she would like them to be “ready to share” their work and she gave them transparency sheets to copy their work onto.

The first segment of the look-back phase involved looking back on the work of problem 2A. Ms. Albert returned to the front of the room and turned on the overhead projector for a student, Darlene, who was also at the front to present her work.

- 1 Ms. Albert: Okay, everybody, Darlene is going to show us what she did, if you would take a look up here, please.
- 2 Darlene: *[[Darlene places her transparency on the overhead projector.]]*  
Okay. For mine, I started out as each block is the thing and then I labeled the starting point with the, um, the little lines, and I made each jump four blocks and I labeled the--, uh, each number so I knew where I was at. And then I ended up--, I did three blocks for Fred--, er, yeah, three times four for--, that’s twelve blocks for Fred, and then I did five jumps, or four times five blocks [for Alice].
- 3 Ms. Albert: Good. And so, Darlene, can you circle exactly where Fred and Alice started jumping, please.
- 4 Darlene: Okay. *[[Darlene circles 18 and 8 on the paths.]]*
- 5 Ms. Albert: Good. So that’s where the frogs began, okay? Because Alice started at eight decimeters away from the path *[[Ms. Albert points to the diagram]]* and Fred started eighteen decimeters away from the path. Any questions for Darlene?
- 6 John: I got twenty-nine for Fred.
- 7 Ms. Albert: You got twenty-nine for Fred? What did you do differently than

- what Darlene did?
- 8 John: I did the same thing.
- 9 Ms. Albert: Maybe--.
- 10 Student 1: I got twenty-eight.
- 11 Ms. Albert: Did anyone else get twenty-eight?
- 12 Students: No.
- 13 Ms. Albert: Why don't you recheck your, um--, well, for Fred, eighteen plus twelve is--, do you agree that it's eighteen plus twelve?
- 14 Students: Yeah.
- 15 John: I get it.

Within these exchanges, a student, Darlene, explained to the class how she represented the problem situation on graph paper as well as her answer to the question of how far Alice and Fred ended up in relation to the path. Ms. Albert asked Darlene to highlight the starting positions in her diagram, and then Ms. Albert and the students compared and checked the exact values of the final frog positions.

At this point, a few other students made side comments about Fred's final distance from the path and Ms. Albert excused Darlene from the front of the room and turned off the overhead projector. Ms. Albert then asked the class if the diagram on page 32 (see Figure 15 above) was an accurate representation of the problem they had just discussed. A male student remarked that it seemed pretty accurate, but Ms. Albert pressed further.

- 16 Ms. Albert: Because Darlene just told us that Alice jumped twenty-eight decimeters and Fred jumped thirty decimeters. *[[Ms. Albert holds up her textbook and points at the diagram.]]* This is the picture that I'm referring to, that picture in your book. Is that a good picture of what we just looked at? *[[Several students raise their hands.]]* Bethany?
- 17 Bethany: It shows the jumps but it shows that Alice jumped farther than Fred.
- 18 Ms. Albert: Right, exactly. It does show that Alice made five jumps and Fred made three, but if you look at the total length of the line there *[[Ms. Albert holds up her textbook and points to the lines]]*, Alice's is a little bit longer, and that's not what we just found out. We found out that Fred jumped a little bit longer. So maybe this is referring to something else that we're going to get to a little later on the page but it does not--, it's not a good picture for the situation that we just looked at.

This interaction involved Ms. Albert, with Bethany, pointing out that the diagram in the textbook is not accurate with respect to the relative ending positions of Alice and Fred, although it is accurate in the sense that it represents five jumps for the former and three jumps for the latter.

Next, Ms. Albert moved on to set up problem 2B, as described above. This second segment of the set-up phase led to an 18-minute work time in which the students focused on problems 2B and 3. Again, several of the groups seemed to be engaging with the mathematical task for the majority of the time. There were certainly moments in which students laughed or appeared to be talking about something other than the mathematics at hand, but there are also consistent instances of students working on their papers, gesturing to one another's work, and speaking about the task. As Ms. Albert circulated between the groups, she often asked where the students were in their work and helped to clarify the problem, such as the assumption that Alice and Fred were still starting at distances of 8 and 18 decimeters from the path, respectively. She also silently observed some small groups. Most groups at some point can be seen or heard comparing diagrams of different jump lengths, and near the end of the work time a trio of males can be observed discussing the strategy for problem 3 that Alice would have to jump a distance of 5 twice to make up the 10 that is the difference between Alice starting at 8 decimeters and Fred starting at 18 decimeters.

Ms. Albert gave a two-minute warning to alert students to the end of this work time and then began a second segment of looking back approximately two-and-a-half minutes later by turning off the lights and turning on the overhead projector. She had selected a few groups to write or draw their work on transparency sheets.

19            Ms. Albert:    Okay, everybody. I heard a lot of great conversations. *[[Some students are still talking in their groups.]]* Everybody ready? I heard

a lot of great conversations in your groups about this problem. I'm going to ask a few people to come up and share their strategies. Um, Evan, are you close?

20 Evan: Almost.

21 Ms. Albert: Clark, why don't you start us off. Are you ready? *[[Ms. Albert moves to the side of the room.]]*

22 Clark: *[[Clark brings a transparency sheet to the projector.]]* Okay, I found out that, if each jump was between two and six decimeters, Fred will finish between twenty-four and thirty-six decimeters because three times six decimeters equals eighteen [in the maximum case] because Fred jumps three times at six decimeters *[[Clark points to his written work]]*, equals eighteen, and eighteen plus eighteen is thirty-six, because Fred starts at eighteen. And two times three [in the minimum case] because Fred jumps two times and three decimeters, it equals six, plus eighteen equals twenty-four. And then Alice will finish eighteen decimeters through thirty-eight because she jumps five times *[[Clark points to his written work]]* and it--, five times two for two decimeters equals ten, plus eight equals eighteen [in the minimum case]. And five--, er, five times six equals thirty. She jumps five times at six decimeters, plus eight is thirty-eight. Eight is the starting distance for Alice and eighteen is the starting distance for Fred.

23 Ms. Albert: Very good, Clark, very good. *[[Students applaud as Clark sits down.]]* Any questions for Clark? *[[pause]]* And this group also--, I noticed Dave and Clark talk about this, you also made another interesting observation. Can you share that with us, too?

24 Clark: *[[Dave and Clark both stand up at their desks.]]* Okay, for number three, um, Fred--.

25 Ms. Albert: Do you want to draw up here to show us?

26 Clark: Yeah. *[[Dave and Clark move to the projector. Clark takes out a marker.]]* We'll just say that this [line segment drawn on the transparency] is eighteen, so eighteen decimeters *[[Clark labels the bottom segment in the diagram; see Figure 16]]*, and this is eight for Alice's starting point *[[Clark draws and labels a shorter segment above the first]]*. Now, eighteen minus eight equals ten, so this [piece] is ten decimeters right here. *[[Clark marks the diagram.]]* So ten decimeters, and we found out that if Alice jumps five decimeters two times it meets up with, uh, Fred's starting point. *[[Clark draws two jumps in Figure 16.]]* So she's already jumped twice, and Fred has to jump three times, so it turns out to be even. *[[Clark finishes Figure 16.]]* Because Alice still jumps five and Fred still jumps three.

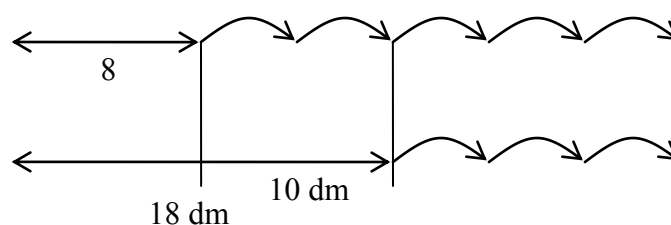
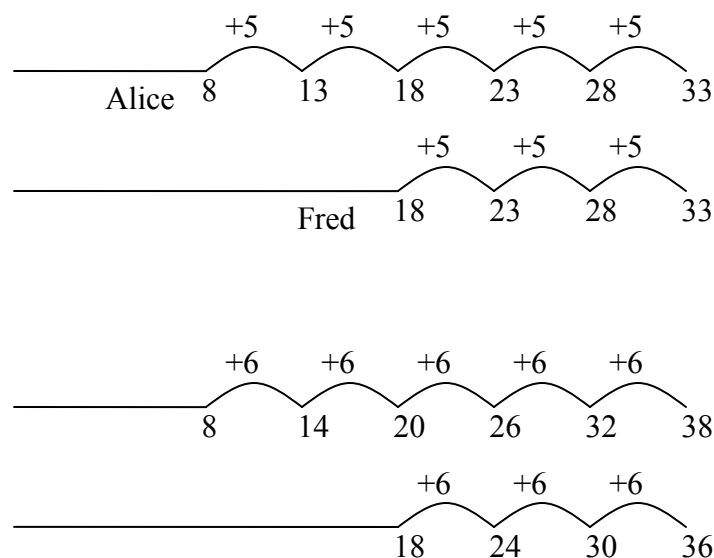


Figure 16. The diagram that accompanied Clark's explanation.

- 27 Ms. Albert: Comments or questions for Clark? *[[A few students applaud again. Dave and Clark sit down.]]* Good job, Clark. Thank you for that. Um, Evan, would you come show us what your group did? *[[Evan and Phil bring their transparency to the projector.]]* And Evan's group was having a conversation about--, they couldn't decide whether one frog was always going to end up ahead or not. And, um, they realized when they tried out the jumps of five and six that something interesting happens when they got to the jump sizes of
- 28 Evan: So, like, the same thing that Clark said, we found it equaling when it starts here at eight *[[Evan points to their first diagram; see Figure 17]]* and it jumps twice [with jump length five] from eight it gets to eighteen, and it jumps three more times and that's the same as Fred's.
- 29 Ms. Albert: So when the jump size is five decimeters they end up at the same place. Very good. And then at six, what happened there?
- 30 Phil: Well, um, at six, um, eight plus six is fourteen decimeters *[[Phil points to their second diagram; see Figure 17]]*, then twenty decimeters, then twenty-six decimeters, then thirty-two decimeters, then thirty-eight decimeters. And Fred, since he only made three jumps, he only got to thirty-six decimeters, and so Alice jumped two more decimeters than Fred.
- 31 Student 2: Ohhhh.
- 32 Ms. Albert: So there's where, um, Alice finally overtakes Fred. Nicely done. Thank you guys very much. *[[Students applaud as Evan and Phil return to their seats and Ms. Albert comes to the front.]]*



*Figure 17.* The diagram that accompanied Evan and Phil’s explanation.

With respect to participatory demand thus far, note that in Darlene’s presentation of her representation for problem 2A the semantic relations in her utterances (see turns 2 and 4) were primarily between herself as actor and mathematical objects such as aspects of the diagram. She also verbalized some relations between mathematical thematic items and non-mathematical thematic items (e.g., between blocks in the diagram and frog jumps). She did not, however, voice any purely mathematical relations. Other students in this segment also verbalized relations that included themselves as actors or subjects of the sentences (see turns 6, 8, and 10). Subsequently, however, several students had extended turns (see turns 22, 26, 28, 30, and 43, 45, and 47) that involved mathematical thematic items as well as some mathematical relations. For example, Clark moved away from “I” as the actor after the beginning of turn 22 and stated mathematical relations as well as relations between mathematical thematic items and the problem context (e.g., “if each jump... because three times six decimeters equals eighteen”). Also, in his next explanation in turn 26, he formed mathematical relations such as the line segment representing

18 decimeters before shifting back to a human agent (i.e., “we”) in relation to the objects being acted upon.

Continuing forward from the previous transcript excerpt, in which Clark explained from the front of the room how to determine the final positions in the cases of the minimum and maximum jump lengths, Ms. Albert next asked Clark to explain how his group had solved problem 3. Next, two students from a different group also presented their solutions for jump lengths of 5 decimeters and 6 decimeters, with the former being connected back to Clark’s explanation. From this point, Ms. Albert shifted to an explicit focus on looking back at problem 3 (although Clark’s second presentation in turn 26 and Evan’s presentation in turn 28 were already related to problem 3).

- 33 Ms. Albert: Okay, what about number three? What kind of conversations did  
you guys have about number three?
- 34 Student 3: Wasn’t that number three [that we just discussed]?
- 35 Student 4: We couldn’t do it.
- 36 Ms. Albert: Let’s read through--, let’s read through the paragraph above number  
three... Let’s read that paragraph above number three together.  
Ashley?
- 37 Ashley: *[[Ashley reads from the textbook.]]* “Suppose the frogs finish their  
jumps at exactly the same distance from the path, and you want to  
know the distance of each jump and each frog’s final distance from  
the path. In groups, discuss strategies for solving this problem.”
- 38 Ms. Albert: So what’s the key piece of information we have here that wasn’t  
always true when we looked at all the jumps in [problem] 2B? Go  
ahead, Tina.
- 39 Tina: Because now we want the frogs to end at the same distance.
- 40 Ms. Albert: Okay, excellent. So now we’re talking about when the frogs end up  
at the same place. How would you solve a problem like that? Again,  
remember, we know how many jumps each frog is taking and we  
know their starting distance. We know they end up at the same  
place, we just don’t know what that total distance is... What would  
be one, um, strategy? How could you think about solving a problem  
like this in general? Alivia, I don’t mean to put you on the spot, I’m  
sorry, but I know that you had some things to say about that in your  
group. Would you mind sharing a little bit of that?
- 41 Alivia: Okay. *[[Alivia giggles and looks at her paper.]]* Oh yeah, okay. For  
Alice, like, what you do--, do you want me to say that, what we did?



- 42 Ms. Albert: You certainly can, sure.
- 43 Alivia: Okay. We're like, for the first one, Alice, we did two times five because, um, the jump could have been between two and six decimeters or whatever.
- 44 Ms. Albert: Right.
- 45 Alivia: And the five came from how many jumps she took, and then we get ten plus eight, which is where her starting point's at, and you get eighteen.
- 46 Ms. Albert: Okay, and, um, what about the part where you talked about that they end up at the same place?
- 47 Alivia: Oh. Um, for Alice it would be five times five pl--, equals twenty-five, plus eight equals thirty-three. And then for Fred it would be five times three.
- 48 Ms. Albert: Okay, *[[Ms. Albert draws a new diagram on an overhead transparency]]* so in that case we knew what the length of the jump was, right? We knew it was going to be five decimeters. So one strategy is, if you know the length of the jump, then you can figure out what the total distance jumped was, which turned out to be thirty-three decimeters. Conversely, if you have the total length jumped, then you could also use that information if you didn't know the jump length. You could go back and find the jump length. Alright, thank you, Alivia. Good job. *[[A few students applaud.]]* Okay, lots of applause today.

In the excerpt above, Ms. Albert had a student read aloud the text around problem 3 and then she asked about the key difference between problem 3 and what had been done in the previous problems. Alivia then attempted to answer Ms. Albert's question about how to solve this problem by describing what her group did; her first few turns (i.e., 43 and 45), however, were not about the solution to problem 3 but about the case of a jump length of 2 decimeters. In turn 46, Ms. Albert redirected back toward problem 3, but Alivia answered in turn 47 by describing the calculations her group made rather than explaining how they went about solving the problem, as had been originally asked by Ms. Albert in turn 40.

In turn 48, Ms. Albert made summary remarks and, with that, ended work on the first task. The second task, which is the same as in the case of Ms. Wyncott, moved on to the use of

variables in the writing of algebraic expression and equations that represent various frog jumping situations.

### **Setting Up the Second Task**

Ms. Albert started the second task in a similar fashion to the first, by having a student read aloud from the textbook. Ms. Albert then asked the class, “What’s another word for unknown?” A student responded, “Variable,” and Ms. Albert said, “Excellent. Very good.” She then drew the students’ attention to the diagram and equation in Box A, that is,  $8 + 5x = 18 + 3x$ . Ms. Albert stated that the diagram was associated with the equation and that it was drawn to scale. Then she asked the students to talk in their groups for “just a second” about “how that equation is represented by the picture.” After approximately one minute in which the students talked immediately and consistently about the prompt, Ms. Albert asked students to share out their ideas. Many students raised their hands and Ms. Albert elicited the following points: the number of  $x$ ’s in the algebraic expressions correspond to the number of arrows in the diagram, the constant values correspond to the frog’s starting points, the left-hand expression corresponds to Alice and the right-hand expression corresponds to Fred, and the equality of the expressions corresponds to the fact that the frogs’ arrows end at the “same spot.” Throughout this sharing out, Ms. Albert clarified the terms “equation” and “expression.”

Next, Ms. Albert asked the students what was the same for Fred and Alice. Students stated that the jump lengths and the direction of the jumps were the same, and Ms. Albert added that the ending distance was also the same. She then asked what was different for Fred and Alice. Responses were their starting distances and their number of jumps. Following this interaction, Ms. Albert told the class that they would be explaining several different diagrams and equations, presented on a hand-out, in much the same way that they had just explained the diagram and

equation in Box A. They would have to identify changes between consecutive diagrams and equations and explain those changes in both representations. Her distribution of the hand-out initiated the working phase of this task enactment.

### Working and Looking Back on the Second Task

Most students appeared to begin working on the hand-out as soon as they receive it. There was a mix of students who worked independently and those who worked together with a partner at their small-group table. Ms. Albert circulated for several minutes, primarily observing students as they worked but occasionally clarifying the directions of the task for students and answering students' questions about whether they were on the right track in their work. She also pushed groups to "talk in your groups to make sure everyone understands." This working phase lasted for slightly longer than 10 minutes.

Ms. Albert then began the look-back phase by turning on the overhead projector, displaying a blank transparency, and drawing students' attention to the shift from Box A (i.e.,  $8 + 5x = 18 + 3x$ ) to Box B (i.e.,  $8 + 2x = 18$ ). Ms. Albert called on Jamie's group first.

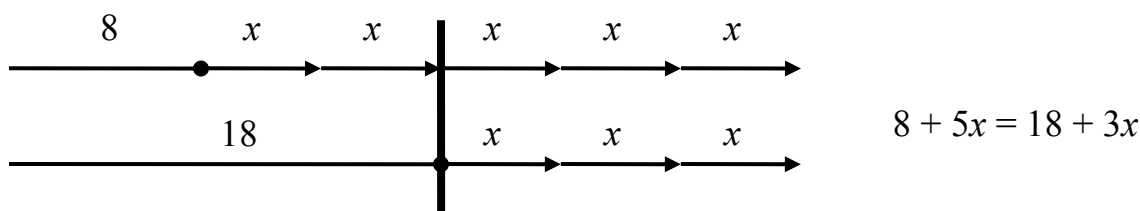


Figure 18. A diagram that aided the thinking of Jamie's group.

- 49 Ms. Albert: Jamie, you drew this line here. *[[Ms. Albert draws a line in Box A; see Figure 18.]]* You said that helped you to think about the problem. Can you tell me why?
- 50 Jamie: Because we knew they had three jumps and it made it easier to see that they were equal.
- 51 Ms. Albert: Okay, so basically what has happened from Box A to Box B is that you've taken three jumps off of Alice and also three jumps off of Fred. And then you're left with just the eight plus two  $x$  and

- eighteen. Okay? What happens from Box B to C? [*Several students raise their hands.*] Jared?
- 52           Jared: Um, they take off eight from each.
- 53           Ms. Albert: Okay, so they've taken eight off the top and also eight away from the eighteen, so they get down here [in Box C] to ten equals--, is equal to the two  $x$ 's. What now happens from Box C to D? [*Several students raise their hands.*] Ashley?
- 54           Ashley: They divided by two.
- 55           Ms. Albert: Okay, why'd they divide by two?
- 56           Ashley: Because it's showing that  $x$  equals five.
- 57           Ms. Albert: Yeah, because if two jumps are equal to ten, then one jump must be equal to five.

This transcript excerpt began with Ms. Albert asking Jamie to articulate some of the rationale for the diagram they had used in their work. Ms. Albert then stated (i.e., turn 51) or revoiced (i.e., turn 53) the changes from box to box and then asked for the reason for the change between the last two boxes (i.e., turn 55) and revoiced (i.e., turn 57) Ashley's response.

At this point, Ms. Albert asked if there were any questions from the students. After pausing a few seconds and seeing no hands raised, Ms. Albert closed the window on questions and ended the task enactment.

### **Summary: A Case of High Mathematical Participatory Demand and the Maintenance of High Cognitive Demand**

Ms. Albert's task enactments constitute a case of high mathematical participatory demand in conjunction with the maintenance of high levels of cognitive demand. With respect to cognitive demand, Ms. Albert enacted the first task in a manner that saw the students doing the cognitive work of connecting the diagrams to the problem situation and that allowed for multiple forms of student thinking to come out. Ms. Albert had more than one student express their thinking about problem 3 (see, for example, turns 26 and 43–47). During the set-up phase of the second task, as well, Ms. Albert maintained a high level of cognitive demand by asking students

to tie particular pieces of the algebraic representation (e.g., coefficients, constants) to the diagram as well as larger components of the algebraic representation (e.g., expressions). During the working phase of this second task, Ms. Albert refrained from lightening the cognitive load and pressed students to understand one another, and even during the fairly brief look-back phase, which was not particularly rich, she presented a students' thinking aid (see turn 49) and asked a "why" question (see turn 55).

With respect to participatory demand, the set-up phases were not mathematically focused but, as with Ms. Wyncott's case, this is not unexpected because the students were being introduced to a new problem context. Thus, in Ms. Albert's case, the set-up of the first task involved a moderate amount of student participation but it was focused on the context, which was non-mathematical. In setting up the second task, perhaps because students were already somewhat familiar with the problem context, the participatory demand remained moderate but was semi-mathematical in focus as much of the students' talk involved relations between mathematical thematic items (e.g., pieces of algebraic expressions) and aspects of the problem context. Her look-back phases, however, involved many extended turns from the students that included recounting language as well as full mathematical relations voiced by students. In other words, there was high mathematical participatory demand.

Although Ms. Albert achieved high mathematical participatory demand at various points in the task enactments, it is important to also mark some ways in which the student participation remained limited with regard to the potential opportunities that existed. At several points (see turns 5, 23, and 27) Ms. Albert asked the class if they had questions or comments for a student that had just offered an explanation. In none of these instances did students actually engage in student-to-student discourse around the mathematical ideas, instead, typically applauding

because the student had apparently completed his or her presentation. One might also wonder if the look-back phase of the second task could have developed into high mathematical participatory demand if given more time to develop. For example, Ms. Wyncott only asked about the rationale behind the changes between boxes in the last instance (i.e., turn 55) but she could have also asked about the rationales behind the other changes and not just for descriptions of what changed.

Overall, Ms. Albert's case involves task enactments that remain fairly consistent with the cognitive demands of the written tasks and exhibits high mathematical participatory demand in both the working and look-back phases. This case raises the question of whether the high mathematical participatory demand was instrumental in the maintenance of high cognitive demand or vice versa.

### **Preliminary Discussion**

The previous chapter presented a broad view of results across 12 classrooms with respect to cognitive demand and participatory demand of task enactments. This chapter included two narrative cases that provide a more detailed view of classroom practice in which to explore similar issues. Both focal distances raise issues with regard to the interactions between cognitive demand and participatory demand throughout enactment and these issues are taken up in the concluding chapter. Here, I outline some of the caveats that should accompany any interpretation of the cases above.

First, the fact that Ms. Wyncott and Ms. Albert came from the same school district and used the same textbook series made for an ease of comparison, but also limits the generality of claims that can be drawn from the cases. This is not to say, however, that the cases are not of general value because they may be seen as representative of a larger category—being a case of

something—and they may resonate with others’ experiences. Second, within these two particular teachers’ classrooms, the cases only constitute a small slice of practice. They are not meant to characterize the teachers’ instruction overall but rather are meant to simply illustrate two particular sets of enactment of particular tasks for the purpose of exploring cognitive and participatory demand. Third, even within these small slices of practice, I have only scratched the surface of the dynamics and wide variety of phenomena at play in the enacted curriculum. Indeed, as mentioned in Chapter 3, one of the primary benefits of narrative cases is that they may be repurposed (Shulman, 1992). Educators may look at these cases and focus on completely different areas than I have here, or they may look at cognitive demand and participatory demand differently than I have, and this is not detrimental to this analysis but desirable.

## **CHAPTER 6**

### **LOOKING BACK**

This study of the enacted curriculum in the context of middle school algebra aimed to balance attention between the mathematical thought processes expected of students and their verbal interactions within a classroom community. The former dimension was viewed through the lens of cognitive demand (Stein et al., 1996), a well-known construct in mathematics education, whereas the latter was achieved through the new construct of participatory demand, which stemmed from views of learning that highlight the process of coming to participate in a discourse community (Lave & Wenger, 1991; Lemke, 1990). Classroom observations were used as data of the enacted curriculum, specifically, the enactment of mathematical tasks. The study, however, also dealt with the relationship between task enactments and student learning, with written pre- and post-tests used as a measure of student learning.

The particular questions guiding this study were the following: How does the enacted curriculum—specifically, the levels of cognitive demand and the forms of student participation during the phases of mathematical task enactments—relate to student learning? In what ways do the levels of cognitive demand and forms of student participation interact during the look-back phase of mathematical task enactments? In this chapter, I first summarize the ways in which this study has addressed the research questions. Next, I connect various findings to past research that formed the background of this study. Finally, I enumerate some implications for practice and future research.

#### **Returning to the Research Questions**

RQ1 deals with the link between the enacted mathematics curriculum and students' mathematical learning. Based on past research (e.g., Stein & Lane, 1996), I originally anticipated



that the prevalence of high cognitive demand during mathematical task enactments would be positively correlated with class's standardized gain scores on the pre- and post-tests. This relationship, however, did not emerge from the data and thus did not confirm past findings, although, as described in Chapter 4, this study does not necessarily disconfirm those findings, either. I also expected the prevalence of high mathematical participatory demand to be positively related to student learning, with this expectation grounded in philosophy and a discourse-oriented perspective on learning and on past empirical research, such as Nystrand's (2006) review of discourse and reading comprehension. I was unsure of whether or not a relationship would surface in this particular analysis, however, because of the individual, written nature of the learning measure in contrast with the interactive, verbal nature of the participatory demand construct, coupled with the relatively small sample size of 12 sets of enacted lessons. Although not statistically significant, non-parametric correlational analysis revealed a moderately positive relationship between participatory demand and student learning ( $p = 0.128$ ). There were two teachers, Ms. Cesky and Ms. Cavillon, whose classes moved against the suggested trend by achieving fairly high standardized gain scores without high mathematical participatory demand or by failing to achieve high standardized gain scores with high mathematical participatory demand, respectively. In fact, by excluding either teacher, the other 11 classes exhibited a highly significant correlation between participatory demand and standardized gain scores. Thus, a discussion of Ms. Cesky and Ms. Cavillon's task enactments is included below with regard to implications for practice.

Both of the analyses just described were correlational in nature and so do not warrant causal claims directed from the enacted curriculum to student learning. For example, a possible explanation of the correlation between participatory demand and student learning is that students

were learning a great deal during the enactment and so contributed more substantive, mathematical utterances to the classroom discourse. Yet, the goal of this study was not to determine causality with regard to the enacted curriculum and student learning but to explore relationships in any form. One conclusion that is supported by the findings is that actively engaging these students in mathematical discourse was not detrimental to their paper-and-pencil learning. Furthermore, this study left unexamined some of the more direct ways in which participatory demand might relate to students' mathematical learning, such as their development of communication skills and their identity development as legitimate mathematical agents.

RQ2 deals with interrelationships between cognitive demand and participatory demand and the look-back phase as a particular component of mathematical task enactments. First, with regard to the look-back phase, this study confirmed in several ways that there is value in giving particular attention to the look-back phase of enactment, included in the modified version of the MTF (see Figure 3). In particular, the look-back phases typically involved whole-class talk formats and shifting structures of discourse from previous phases, which confirms earlier work (Gibbons, 2009; Herbel-Eisenmann & Cirillo, 2009; Otten, 2010a). The differences in language demands between various talk formats is nontrivial and has implications for mathematics teaching and learning (Gibbons, 2009; Herbel-Eisenmann & Cirillo, 2009). Additionally, a detailed analysis of potential shifts in cognitive demand between the working and look-back phases, which could not occur in this study but is suggested for future work, would contribute to past work on the shifts between the written task and the set-up phase or between the set-up phase and implementation overall (e.g., Henningsen & Stein, 1997; Stein et al., 1996). Finally, in this study it was valuable to treat the look-back phase separately because this was a common phase of task enactments for high mathematical participatory demand to occur.

With regard to cognitive demand and participatory demand considered together, the findings from both Chapter 4 and Chapter 5 make contributions to addressing this part of RQ2. In Chapter 4, I showed that cognitive demand and participatory demand were separable constructs within the enacted curriculum in terms of their lack of correlation with one another. In other words, teachers that enacted mathematical tasks at high levels of cognitive demand did not necessarily employ high mathematical participatory demand and vice versa. In Chapter 5, however, I presented two cases of teacher's task enactments that raised questions about the potential relationships between the constructs. Many mathematics educators assert that in order to have students engage in rich mathematical discourse you need to have them work on a rich mathematical task so that they have something to talk about. In the case of Ms. Albert, one could see some truth to this assertion. Ms. Albert enacted high level tasks in ways that generally maintained the cognitive demand on the students, and high mathematical participatory demand was also common in her enactments, particularly in the look-back phase of the first task enactment. Within this case, these two phenomena did not seem to me to be separate but, in fact, that the high mathematical participatory demand was completely interlaced with the maintenance of high cognitive demand. Conversely, Ms. Wyncott enacted the same mathematical tasks in ways that saw declines in the cognitive demand and she also did not achieve high participatory demand with her students. Both of these cases suggest at least a potential, and mutually supportive, link between cognitive demand and participatory demand in the enacted mathematics curriculum.

### **Connecting to Background Literature**

One of the foundational assumptions of this study was that an analysis of the enacted curriculum would be more robust if it attended to the complementary dimensions of students'

individual thought processes and their participation in a classroom community (Cobb et al., 1992; Sfard, 1998). This assumption appears to have been confirmed as the cognitive demand and participatory demand constructs captured different aspects of the task enactments and both seem to have associations with student learning—cognitive demand based on years of past research and educators’ experiences and participatory demand based on tenets of sociocultural theory and the correlational analysis in this study.<sup>30</sup> The cases in Chapter 5 also suggest that the two dimensions complement one another through their interaction, as well. With that being said, the operationalizations in this study are limited and there are certainly more facets of both the individual and the collective dimensions that are worthy of attention. Hence I turn now to place this work into dialogue with past research.

In examining the student participation, I focused on the thematic items that students’ uttered and the semantic relations that they verbalized. This allowed me to classify the expectations for students’ participation in the mathematics discourse of the classroom but did not trace the type of discussion that may have been taking place with regard to the instruction or the development of mathematical ideas. In particular, Staples and Colonis’s (2007) notion of collaborative discussions versus sharing discussions highlights a different feature of the classroom discourse. Taken together with participatory demand, one can illuminate the mathematical thematics being verbalized by students but also the ways in which students are building on one another’s mathematical ideas or not. These issues are particularly salient in the

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<sup>30</sup> Student participation in general has also been linked to student learning by research and educators’ experience, but the participation has not been conceived in precisely the way I have in this study through the construct of participatory demand.

look-back phases of task enactments when teachers often attempt to lead a whole-class discussion to summarize key ideas from the tasks, perhaps making moves that open up or close down the discourse in particular ways (Wagner & Herbel-Eisenmann, 2008). In light of the present study, one might consider whether the discourse is being opened up for higher or more mathematical forms of participatory demand or for collaborative sense-making between students, or both.

Also on the topic of the look-back phase of mathematical task enactments, Shimizu (2006) has analyzed the summary statements that teachers often make to conclude lessons (or task enactments). His work, in conjunction with the notion of participatory demand, allows one to trace the mathematical semantic relations as verbalized by students (or not) during the look-back phase and the relationship between these relations and the way in which they may (or may not) show up in the teacher's concluding summary statement. This tracing of thematics would seem to have implications for the establishment of taken-as-shared mathematical knowledge (Cobb et al., 1997). Furthermore, the mathematical ideas that teachers choose to extend or restrict during the look-back phase and the way in which they do so may be related to the teacher's own knowledge (Cengiz, Kline, & Grant, 2011) or their beliefs about discourse in mathematics classrooms (Nathan & Knuth, 2003).

Shifting from the teacher back to the students, this study found an association between the nature of students' verbal participation and their learning as measured on a paper-and-pencil assessment. This study did not address, however, the potential relationship between participatory demand and students' motivation to learn mathematics and engage in solving mathematical tasks. Jansen's (2006, 2008) work has shown that motivation is an important mediator with respect to students' participation in middle school mathematics classrooms. Might it also be the case that

motivation is a mediator between participatory demand and student learning? In particular, one might hypothesize that high mathematical participatory demand placed on students increases their motivation to participate which, in turn, has positive effects on their mathematical learning, or instead that a certain degree of motivation is necessary to achieve high mathematical participatory demand in the enacted curriculum. Even if these hypotheses turn out to be overly simplistic, it suggests that motivation is an important factor to consider when interpreting the relationship between participatory demand and student learning identified in this study. Furthermore, it is important to remember that this participation and motivation is taking place in a middle school setting, which has implications as a turning point in students' mathematical trajectories (Eccles et al., 1993).

I would be remiss in this section if I did not mention the body of research that was perhaps the most substantial basis for the present study, and that is the work around the Mathematical Tasks Framework. I have mentioned above how this study confirmed the value of including the look-back phase as part of the framework. The other primary contribution of my work to the existing body of research in this area has to do with the construct of participatory demand, which I fashioned as a sort of partner to the construct of cognitive demand, with both being traced throughout the phases of enactment. I described above how these two constructs were empirically complementary, but I hope that they can be practically complementary as well. Boston and Smith (2009) have documented how the original MTF and the construct of cognitive demand can contribute to actual changes in teachers' practice. In particular, they found that secondary mathematics teachers who had experienced professional development focused on the MTF and cognitive demand chose to enact more high-level tasks and better maintained the cognitive demand during enactment than did control teachers. Ideally, professional development

that also incorporated participatory demand as conceptualized in this study would have similar or expanded success in effecting change in teachers' practice. Teachers might not only select more high-level tasks to enact but they may capitalize on a mutually beneficial relationship between cognitive demand and participatory demand, maintaining high cognitive demand so that there is a great deal to talk about and eliciting high mathematical participation so that the mathematical ideas are public, which assists in the maintenance of cognitive demand.<sup>31</sup> Such a practice has the potential to assist students as they grow not only as mathematical thinkers but also as active contributors to the classroom discourse and the mathematics classroom community.

This discussion of practical implications of the present study continues in the next section.

### **Looking Forward**

#### **Implications for Practice**

Another facet of the original work around the MTF that has been used substantially in teacher preparation and teacher education (Smith, Bill, & Hughes, 2008; Stein et al., 2000, 2009) is the set of factors associated with the decline or maintenance of cognitive demand (Henningsen & Stein, 1997). Some of these factors, such as giving an appropriate amount of time and pressing for explanations and meaning, are likely to have implications for participatory demand as well as

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<sup>31</sup> Indeed, Chapin and O'Connor (2004) have reported that, at the upper-elementary level, mathematics instruction focused on sense-making, justification, and reasoning (i.e., high cognitive demand activities) and a "heavy emphasis" (p. 2) on students' vocabulary and language development yielded high student performance on standardized tests, with post-hoc analysis tracing the achievement to the instructional intervention. Moreover, the gains appeared to extend beyond mathematics to students' English and Language Arts skills.

cognitive demand. Teacher educators, therefore, may want to consider both dimensions of enactment when they discuss these factors with pre-service and in-service teachers. Additionally, there may be factors that are unique to the management of participatory demand, and these may be identified through future research (described below).

As teachers engage with ideas of cognitive demand and participatory demand and make connections to their practice, it is important to note that the goal is not a curriculum consisting entirely of high-level tasks or phases of enactment that are always highly demanding and involve high mathematical participatory demand. Rather, the goal is an awareness of the potential benefits along both of these dimensions but always in relation to the teacher's own instructional goals. As Stein and colleagues (2000) articulated, they do not suggest "that all tasks used by a teacher should engage students in cognitively demanding activity, since there may be some occasions on which a teacher might have other goals for a particular lesson, goals that would be better served by a different kind of task" (pp. 14–15). In other words, there are cases in which memorization or procedures-with-connections tasks are perfectly appropriate. I echo this sentiment with regard to participatory demand. A teacher's instructional goal may lead them to require that students work independently and in silence or that student-to-student interaction remain at a minimum. Even within a single task enactment, a teacher may have good reasons to give a monologue during the set-up phase and then lead the class through the look-back phase in a way that involves a semi-mathematical focus of student participation, perhaps using the IRE interaction pattern (Mehan, 1979) to maintain teacher control and efficiency (Herbel-Eisenmann, Cirillo, & Steele, in preparation). If, however, cognitively demanding tasks are entirely absent and students are never expected to verbalize mathematical relations in their discourse, teachers should be aware of the implications that this may have on various dimensions of student learning.



One legitimate concern that teachers and other practitioners (such as principals) may have with regard to task enactments is that pushing students to work at a high level of cognitive demand or involving students actively in the classroom discourse will use up great amounts of time. The increases in duration may stem from the sharing of multiple solution strategies or from the risk inherent in high-level tasks and high levels of student participation that side-tracked, erroneous, or unclear student ideas may surface. Indeed, a close inspection of the two cases presented in Chapter 5, which involved enactments of the same written tasks, reveals that Ms. Wyncott did progress through the tasks in less time than did Ms. Albert. This study, however, contributes to a body of research that suggests that such risks in duration are worth it, as long as the time is well spent. Ms. Albert, for example, was ranked in the top half with respect to standardized gain scores whereas Ms. Wyncott was at the bottom of the ranking. Thus, Ms. Albert represents a case suggestive of the power of using class time in productive ways. On the other hand, Ms. Cavillon (AY 2004–2005), described in detail in Chapter 4, serves as a warning to the unproductive use of class time as her extraordinarily lengthy look-back phases seemed to have little payoff. With effective enactment, however, the risk may be worth it particularly because the potential causes of delay may be precisely the things that are powerful sources of student learning as students articulate their ideas, engage with one another's reasoning, and, in so doing, provide formative assessment information to the teacher, which can be a powerful force for learning (Black & Wiliam, 1998).

This potential for learning pay-off is one of the reasons that NCTM's (1991) call for teaching practices that cultivate rich mathematics discourse in classrooms at all levels remains vibrant today, although the challenge of achieving this call seems to be ever present. One root of the challenge is the fact that teaching in ways that incorporate cognitively demanding tasks and

rich student discourse is difficult (Henningsen & Stein, 1997; Herbel-Eisenmann & Cirillo, 2009; National Center for Education Statistics, 2003) and not typical practice in the United States (Hiebert et al., 2003; Stigler & Hiebert, 1999). As long as the possibility exists of garnering student achievement score gains through instruction based on carefully executed teacher explanations and rote student practice, there may be many teachers who do not desire to change their practice. In this study, the outlying instance of Ms. Cesky reiterates past work (e.g., Smith, Silver, & Stein, 2005) in demonstrating that this possibility exists. Ms. Cesky ranked second to last in prevalence of high mathematical participatory demand and in the lower half in prevalence of high cognitive demand, yet her class's standardized gain scores on the pre- and post-test were fourth highest out of 12 classes. Some might take the existence of Ms. Cesky's data to mean that it is not necessary to adopt a cognitively demanding or student-participation-oriented teaching approach because student gains can be achieved without it. There is decades of evidence, however, that an instructional approach in which students are passive recipients of information and engage only in rote exercises tends to work only for a minority of the school mathematics population, and the students who succeed under this paradigm were likely to have been successful under an alternative paradigm as well. Adding to this body of evidence, the present study suggests that an interpretation of Ms. Cesky's class as a confirmation of such teaching is unwise as Ms. Cesky was the exception to the general trend and a fixation on her class's performance clouds the evidence that this study provides in support of, particularly, instruction that incorporates high mathematical participatory demand.<sup>32</sup>

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<sup>32</sup> It should be noted that this study only involved teachers who used some form of the launch-explore-summarize lesson structure. On one hand, this means that findings do not necessarily

## **Future Directions of Research**

The curriculum framework presented in Chapter 1 (see Figure 1) provides a way to organize future research that may build on the present study. First, with regard to student learning, I have already alluded several times to the fact that this study measured student learning through a written pre- and post-test, which was designed to capture the students' knowledge and understanding associated with the focal learning goal but did not do so perfectly and also failed to capture other facets of learning such as attitudes and beliefs about what it means to do and learn mathematics and about student agency in that process. Other researchers have worked in such areas (e.g., Cobb et al., 2009; Lester, Garofalo, & Kroll, 1989; Ma & Kishor, 1997; Muis, 2004; Wagner & Herbel-Eisenmann, 2009), but not from the perspective of the MTF and in conjunction with cognitive demand, which allows for student participation to be considered throughout various phases of a task enactment and in relation to the thought processes entailed in solving that task. Additionally, student learning is embedded within the enacted curriculum, not solely separate from it, and thus the current operationalization of participatory demand and the use of analytic tools such as thematic analysis (Chapman, 2003; Herbel-Eisenmann & Otten, 2011; Lemke, 1990) might be employed further to track in greater detail students' appropriation of mathematical forms of language use.

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extend to other forms of instruction such as the common exposition-example-exercise structure (Schoenfeld, 1992), but, on the other hand, it may be that the findings in favor of high mathematical participatory demand are even more pronounced if they were to be examined across these different lesson structures.

With regard to instructional materials, which relate to the enacted curriculum as well as the teacher's intended curriculum and student learning, it was not a goal of this study to examine the relationship of the various textbooks to the task enactments or student performance on the pre- and post-test. This does not mean, however, that the instructional materials played a minor role in either case. On the contrary, textbooks and other forms of written curriculum materials are known to impact teacher's planning and the enacted curriculum (Chávez-López, 2003; Cohen, Raudenbush, & Ball, 2003). In fact, Herbel-Eisenmann (2007, 2009) has revealed that textbooks are not neutral entities with respect to authority (and thus participation) and that textbooks can become present in the classroom discourse in various ways. In this study's classrooms, for example, students and teachers frequently read aloud from their textbook and there were also many instances of the pronoun "they," which may have referred to the textbook authors.

As I alluded above, one major area of research that can build on the present study has to do with the teacher's role in setting and shaping participatory demand. Some work that has already been done may be reinterpreted through the lens of participatory demand. For example, Ellis (2011) identified specific actions taken by a teacher-researcher with middle school students that promoted mathematical generalization as a socially-situated process. Might these actions also have shaped the participatory demand of the tasks in particular ways? Similarly, Staples (2007) studied a high school classroom over time to identify the teacher's role in spurring on collaborative interactions. Particular aspects of the teacher's role, such as supporting students in making contributions, seem to have clear connections to participatory demand throughout a task enactment and also over multiple enactments. It may also be that specific teacher questions (Boaler & Brodie, 2004) or teacher talk moves (Chapin et al., 2009) influence participatory demand in specific ways. For example, the Mathematics Discourse in Secondary Classrooms

(<http://www.mdisc.org>) development project is designing curriculum for use with secondary mathematics teachers to guide them in digging deeply into their classroom discourse practices (Herbel-Eisenmann, Steele, & Cirillo, under review). Professional development with these materials would make an ideal site to investigate the relationship between teacher's discourse and students' discourse as well as the potential to effect change in practice.

Another potential area of future research has to do with the classroom norms, culture, and expectations for student participation. Norms such as providing justification for claims and being open to critique of one another's reasoning are related to the maintenance of high cognitive demand (Henningsen & Stein, 1997), and there much research that exists related to classroom norms and issues of student participation (e.g., Herbel-Eisenmann & Wagner, 2010; Herbel-Eisenmann et al., 2010; Yackel & Cobb, 1996), but the present study lays a foundation for the consideration of both issues together over a common framework, which may be a fruitful way forward. The lens of norms and expectations may also be a way in which to reveal more nuances with regard to the interrelations between cognitive demand and participatory demand.

In closing, I wish to step back to an aspect of the original conceptualization of this study that, for various reasons (see Chapter 3), did not materialize—that is, the standards for mathematical practice (National Governors Association & Council of Chief State School Officers, 2010). The standards for mathematical practice have clear connections to cognitive demand as the highest level of cognitive demand is termed “doing mathematics” and the practices are precisely those activities (in addition to some others) that the writers of the *Common Core State Standards for Mathematics* perceive to be characteristic of mathematical thinking and activity. Future research could be undertaken to illuminate these links between cognitive demand and the practices within the enacted curriculum.

I strongly believe, however, that it is not sufficient for students to engage in the practices (although if all students experienced at least this much, I would be content), students must have opportunities to step back and realize that they are engaging in these practices and realize, as well, that what they are experiencing is quintessentially mathematical (Cobb, Wood, & Yackel, 1991; Rittenhouse, 1998). The look-back phase in the modified MTF seems to be a natural place to provide students with such opportunities to reflect on their mathematical activity. Do teachers use look-back phases for such purposes and, if so, how? In what ways are students involved in the practices and in what ways do the practices surface in the students' verbal discourse? How might students' reflection on the mathematical practices surface in the students' verbal discourse and how might such discourse be different than discourse during the practice? What these questions point to for me is that the role of participatory demand during both the engagement in the practices and the reflection on the practices is worth studying.

## **APPENDICES**

## APPENDIX A

### EXAMPLE OF PRE- AND POST-TEST

#### Multiple Choice Section

DIRECTIONS: Please mark your answers on the Multiple Choice Answer Sheet by circling the letter of the correct answer.

1. What is the value of  $\square$  in this equation?

$$43 = \square - 28$$

- A. 15
- B. 25
- C. 61
- D. 71

2. Mary has some trading cards. Julie has 3 times as many trading cards as Mary. They have 36 trading cards in all.

Which of these equations represents their trading card collection?

- A.  $3x = 36$
- B.  $x + 3 = 36$
- C.  $x + 3x = 36$
- D.  $3x + 36 = x$

3. There are  $n$  Girl Scouts marching in a parade. There are 6 girls in each row. Which expression could you use to find out how many rows of Girl Scouts are marching in the parade?

- A.  $n - 6$
- B.  $\frac{n}{6}$
- C.  $6n$
- D.  $\frac{6}{n}$

4. Jacob writes the following rule:

If  $a$  and  $b$  represent any two numbers,  $a + b = b + a$ .



Which of the following describes Jacob's rule in words?

- A. Equals added to equals are equal.
- B. Order doesn't matter when adding two numbers.
- C. The sum of two whole numbers is a whole number.
- D. When adding three numbers, it doesn't matter how the numbers are grouped.

5.

A	B
12	3
16	4
24	6
40	10

What is the rule used in the table to get the numbers in column B from the numbers in column A?

- A. Add 9 to the number in column A.
  - B. Subtract 9 from the number in column A.
  - C. Multiply the number in column A by 4.
  - D. Divide the number in column A by 4.
6. Which of the following statements is NOT TRUE about the equation  $y = 2t$ , if  $t$  is a positive number?
- A. It shows how  $y$  changes for different values of  $t$ .
  - B. It shows a linear relationship between  $y$  and  $t$ .
  - C. It shows that the value of  $y$  is independent of the value of  $t$ .
  - D. It shows that as  $t$  increases,  $y$  also increases.

7. The table shows values for the equation  $y = 2x + 5$ .

$x$	$y$
1	7
2	9
3	11
4	13

Which sentence describes the change in the  $y$  values compared to the change in  $x$  values?

- A. The  $y$  values increase by 6 as the  $x$  values increase by 1.
- B. The  $y$  values increase by 7 as the  $x$  values increase by 1.
- C. The  $y$  values increase by 2 as the  $x$  values increase by 1.
- D. The  $y$  values increase by 5 as the  $x$  values increase by 2.

## Short Response Section

DIRECTIONS: Answer Questions 8 through 15 in this Test Booklet. Be sure to show all your work in the Test Booklet.

8. Tachi is exactly one year older than Bill.

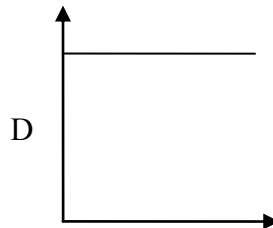
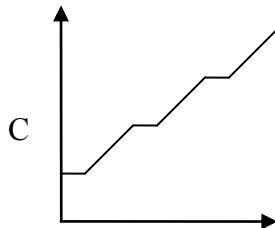
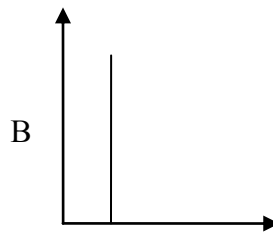
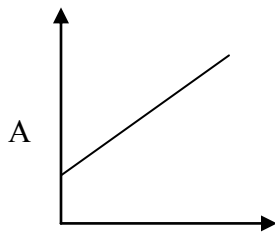
Let  $T$  stand for Tachi's age and  $B$  stand for Bill's age.

Write an equation to compare Tachi's age to Bill's age.

9.  $a = b - 2$  is a true statement when  $a = 3$  and  $b = 5$ .

Find a different pair of values for  $a$  and  $b$  that also make this a true statement.

10. A small boy was raising a flag up a flagpole.



Write the letter of the graph that best represents the height of the flag above the ground as the small boy raises the flag.

Explain why you chose this graph.

11. The table represents a relationship between  $A$  and  $B$ .

$A$	$B$
8	3
12	5
20	9

32	15
?	23

Based on this relationship, what is the missing number in column A?

12.

Age of car (in years)	Value of car
0	\$20,000.00
1	\$10,000.00
2	\$5,000.00
3	\$2,500.00

Circle the correct choice (**is** or **is not**) BELOW and complete the statement.

The relationship between the age of the car and the value of the car **is** / **is not** linear because...

13. Stella has a phone plan. She pays \$10.00 each month plus \$0.10 each minute for long distance calls.

One month she made 100 minutes of long distance calls and her bill was \$20.00

The next month she made 300 minutes of long distance calls and her bill was \$40.00.

Stella said, "If I talk three times as long it only costs me twice as much!"

Will Stella's rule **always** work? Show or explain why or why not.

14. Maria sells  $k$  donuts. Jinko sells five times as many donuts as Maria. They sell donuts for 25 cents each.

The number of donuts Maria sells is a variable.

- A. Name another variable in the problem.
- B. Name something in the problem that is NOT a variable.

15. Find the value(s) of  $x$  that make the equation true. Show how you got your answer.

$$19 = 3 + 4x$$

## Patio Patterns

DIRECTIONS: All of the questions in the next set are based on the same story. Some of the questions may seem very simple, while others more difficult.

Read the story. Then turn the page to read the first question.

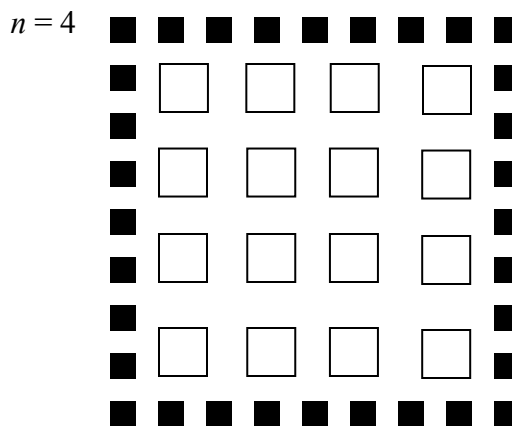
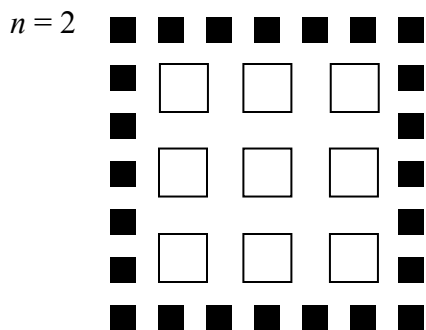
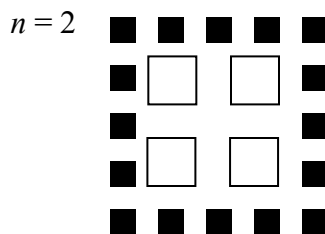
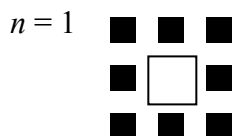
16. Pablo plans a pattern for paving a patio with bricks and stones. His pattern has flat white stones in the center and square bricks around the border.

Here you see a diagram of different sized patios, where you can see the pattern of stones and bricks:

■ = brick

□ = stone

$n$  = number of rows of stones



(A) How many bricks are in a patio with 2 rows of stones?

(B) Complete the table. ( $n$  = number of rows of stones)

$n$	Number of stones	Number of bricks
1	1	8
2	4	
3		
4		
5		

(C) Look at the table. You might notice that the number of stones can be found by using the formula  $n \cdot n$ . The number of bricks can be found by using the formula  $8 \cdot n$ . Remember,  $n$  is the number of rows of stones.

There is a value of  $n$  for which the number of stones equals the number of bricks. Find that value of  $n$ .

Explain how you found your answer.

(D) Suppose Pablo wants to make a much larger patio with many rows of stones. As he makes the patio bigger, which will increase more quickly, the number of stones or the number of bricks?

Explain how you found your answer.

## APPENDIX B

### TRANSCRIPTION

Although the present study proceeded primarily from untranscribed video data, transcription played a role not only in the excerpts included in the dissertation itself but also in my notes and visualizations of the words being spoken that I considered as I viewed the video. No transcription method records the actual discourse that takes place but is always partial in nature, regardless of the level of detail, and captures an interpretation and representation of that discourse (Ochs, 1999). Thus transcription is a phase of analysis and researchers are required to consider what aspects of the discourse are necessary for the purposes of their particular inquiry and how best to go about including those aspects in written form. Because my purposes included the use of thematic analysis, it was necessary to encode the actual words that were spoken. Other aspects of discourse, such as tone, speed, and facial movements, were not attended to, although some gestures played a part in communicating the meaning of an utterance. I also noted the objects of ambiguous references (e.g., this, that, s/he). With regard to the development of the narrative cases, however, further information was needed. Thus, when reviewing video and revising all the transcripts, I also included brief records of classroom events such as the position of the teacher (e.g., at the front board), notable actions by the teacher (e.g., writing on the front board), and notable actions by students (e.g., coming up to the overhead projector). In this way, the transcripts of the classroom episodes contained not only what was said and by whom, but also a form of written record about who was doing what at various points in time.

Below is a table of my transcription conventions.

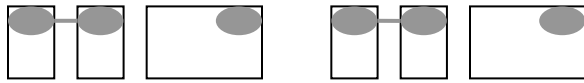
Student X:	Unidentified student speaker (X is a number incrementing by 1)
...	Omitted speech
[[ ]]	Action (including timed pauses)
[ ]	Clarified reference (e.g., "that")
—.	Verbal fill-in-the-blank
--,	Self interruption
--.	Interruption by another speaker

## APPENDIX C

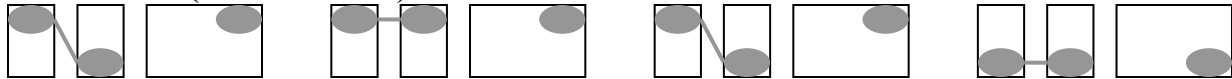
### TRACING COGNITIVE DEMAND THROUGH ENACTMENT

The representations below are organized by the 12 sets of lesson. The groups of three boxes signify the phases of the MTF (i.e., written, set-up, implementation) and the gray oval paths signify the level of cognitive demand (i.e., high, low) in each phase, with the implementation phase divided into the working and look-back sub-phases (see Figure 3 in Chapter 1), though the working phase was not coded in this study. Each mathematical task enacted within the set of lessons is represented. An oval is omitted when the phase was not included in the task enactment, and a question mark is used to indicate incomplete data.

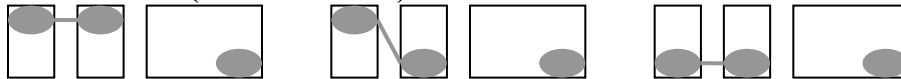
#### Ms. Albert



#### Ms. Cavillon (AY 2002–2003)



#### Ms. Cavillon (AY 2004–2005)



#### Ms. Cesky



#### Ms. DePalma



#### Ms. James



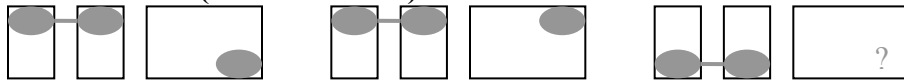
#### Mr. Johnson



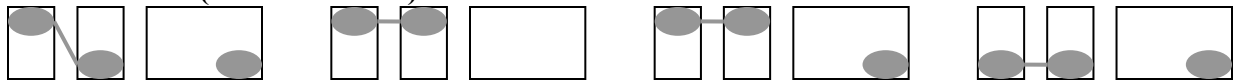
**Ms. Mendoza (AY 2002–2003)**



**Ms. Mendoza (AY 2003–2004)**



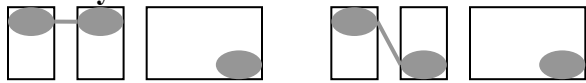
**Ms. Mendoza (AY 2004–2005)**



**Mr. Milson**



**Ms. Wyncott**



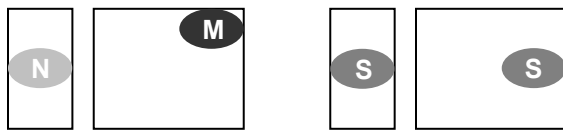


## APPENDIX D

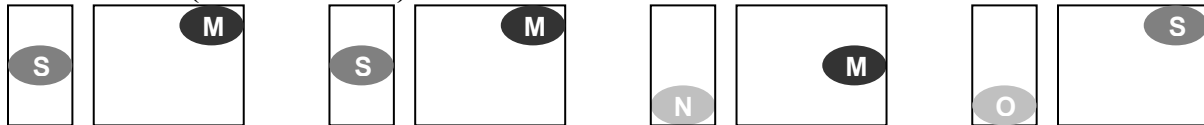
### TRACING PARTICIPATORY DEMAND THROUGH ENACTMENT

The representations below are organized by the 12 sets of lesson. Because only one written task explicitly mentioned expectations for student participation, the written phase is excluded and groups of only two boxes are used to represent the phases of mathematical task enactment, with the working and look-back phases both contained within task implementation. Because participatory demand is a two-dimensional construct (i.e., level and focus of student participation), it cannot be presented in as straightforward a manner as was cognitive demand (see Appendix C). In this case, however, the level of student participation is represented by the placement of the gray ovals (high, medium, and low, respectively) and the focus of student participation is encoded within the ovals. The types of focus are mathematical (M), semi-mathematical (S), non-mathematical (N), or absent (O).

#### Ms. Albert



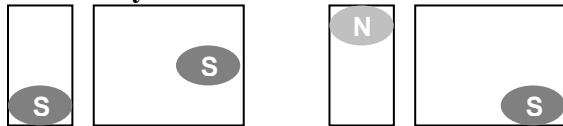
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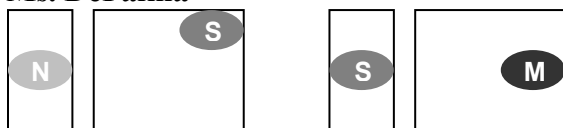
#### Ms. Cavillon (AY 2004–2005)



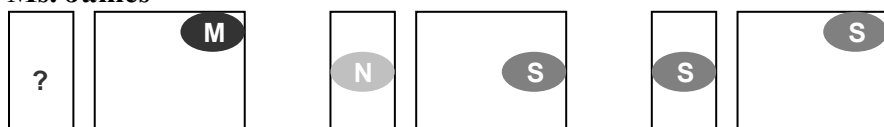
#### Ms. Cesky



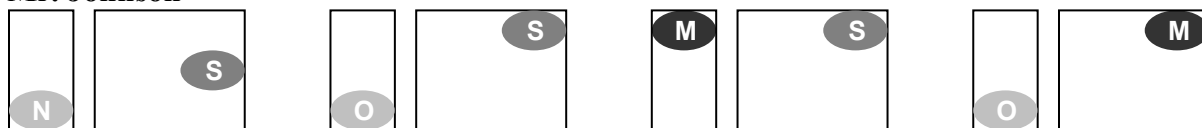
#### Ms. DePalma



**Ms. James**



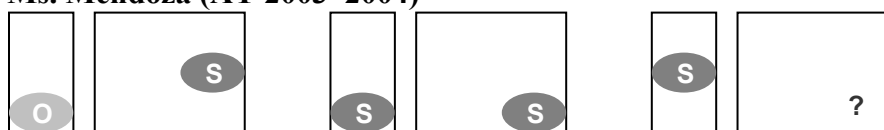
**Mr. Johnson**



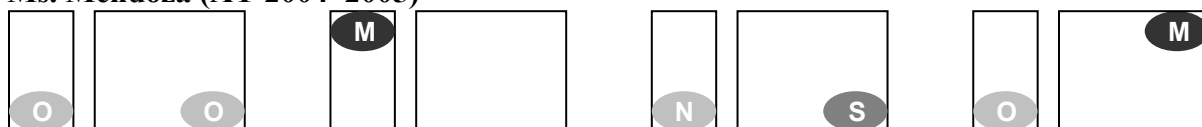
**Ms. Mendoza (AY 2002–2003)**



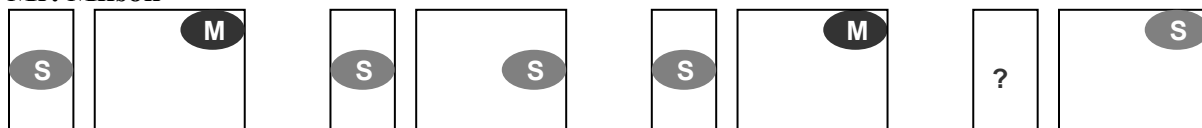
**Ms. Mendoza (AY 2003–2004)**



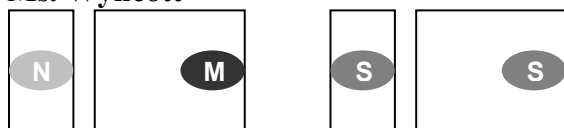
**Ms. Mendoza (AY 2004–2005)**



**Mr. Milson**



**Ms. Wyncott**



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