THE EFFECTS OF VERTICAL VIBRATION ON THE TRANSMISSIBILITY OF FORCES THROUGH A UNIFORM STACK OF PACKAGES

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ABSTRACT

THE EFFECTS OF VERTICAL VIBRATION ON THE TRANSMISSIBILITY OF FORCES THROUGH A UNIFORM STACK OF PACKAGES

By

Richard G. Holubik

In the past few years, the attention given Packaging and the environment through which products travel has increased immensely. One of the main areas of the environment receiving this attention is vibration, and the damage it causes.

The problem under investigation here is to observe the effects of vertical vibration on a stack of packages. The main area of concern is to see what effects the input g level, frequency of vibration, and cushioning used has on the response acceleration felt by a particular package in the stack. By varying the number of packages in the stack from one to four, many comparisons may be made concerning the effects of positioning within the stack.

To obtain the data necessary to evaluate such a study, a device that could input a monitored frequency and amplitude of vibration to yield a desired input g

level was required. A vibration table system capable of vibrating over a range of 0-100 cycles per second (Hz), with a maximum stroke of 6 inches was used. The packages were placed on the table and restricted from excessive horizontal sliding and vibration by a corrugated chute.

With the packages on the table and the chute secured, the test is ready to begin. The table was monitored to vibrate at a desired frequency. The amplitude of vibration was then regulated to yield the desired input acceleration level. When this input level was reached, the data for the study could be recorded.

There were two main pieces of data needed to evaluate the effects of this vibration test. The first form of data was a ratio of the response acceleration, which was felt by the product, over the input acceleration, at which the table was being monitored. This ratio is called the transmissibility of the system. It shows how the acceleration input is magnified (or attenuated) over a frequency range of tests. With the ratio recorded, and the input acceleration known, the response acceleration felt by the product for any reading can be found by multiplying the input acceleration times the ratio.

The second important piece of data was the frequency of vibration at which a particular ratio was obtained. This means that a ratio was obtained for each frequency tested.

To make the evaluation of the data easier, it can be plotted on a graph with the ratios on the vertical axis, and the frequency on the horizontal axis. In this study, the actual data was not plotted in most cases, however, a couple of transmissibility curves are given. A plot of these data sets would show that some sets have one and some two pronounced peaks over the tested frequency range.

An evaluation of the data yields four major findings.

1. The position of a package in a stack does affect the response acceleration it feels.

2. Even at a low g input of 0.1, the low damping of the (4-4) system caused transmissibility to occur.

3. The transmissibility of a system outside the frequency range of the normal vehicle suspension, and especially after 50 cycles per second, is very low. This indicates that very little damage from high g level inputs should occur in this range.

4. The pound per square inch (psi) loading on the cushioning of an individual product in a package, and the loading on the package tested affects the transmissibility. As the psi loading increases, the transmissibility was found to decrease.

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Dedicated to

My Family

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CHAPTER I

INTRODUCTION

> During vibration, every package reacts differently--depending on its center of gravity, friction between the package and the surface on which it is resting, among other things. Understanding how a package acts, and the forces acting upon it, during vibration can lead to improved structural package design.[1]

The purpose of this study is to observe the effects of vertical vibration on the transmissibility of forces through a stack of packages.

> The ratio of the acceleration of the mass to the applied base acceleration is called the transmissibility of a system. The transmissibility ratio states the magnification (or attenuation) of the input accelerations and forces transmitted through the spring from the base to the mass.[2]

The study is broken down into four sections.

The test begins by placing a product, to which an accelerometer has been attached, on the vibration table, which also has an accelerometer mounted to it. A chute is then placed around the package to help control horizontal sliding and vibration to a certain extent.

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[1] Stan Gaynes, "What Makes the Package Jump?" Package Engineer, March, 1961, p. 54.

[2] Effects of Vertical Dynamic Loading on Corrugated Fiberboard Containers, USDA Forest Service, Research Paper, FPL, July, 1968, p. 3.

> The range of most probable vibration inputs are from 0.2 to 0.8 g at 3 to 10 Hz for rail transportation, and from 0.1 to 0.8 g at 2 to 20 Hz for truck.[3]
>
> In addition to these accelerations, higher frequency oscillations were measured which attained values up to 1-g within a frequency-range of up to 1000 Hz. These high-frequency accelerations, however, were all absorbed by the outer packing of the consignments.[4]

At this point it may be worthwhile to define the term, g.

> The rate of acceleration due to gravity is always very close to 32.2 ft./sec./sec. This acceleration caused by gravity is called 1g. Thus, for any fixed mass, 1g unit of force is a force which will accelerate that mass to 32.2 ft./sec./sec.[5]

After the setup is ready to test, the vibration table is set to run at a frequency of approximately 2 cycles per second. The testing was started at this frequency so that the entire range of "most probable vibration frequencies" could be observed.

The amplitude of the vibration is then regulated so that the input to the table is 1g peak-to-peak for a sine wave signal, or 0.5g zero-to-peak for the same signal. The 0.5g input was chosen because it was near the middle of the g level range for both modes of transportation. Figure 1 will illustrate this condition.

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[3] *Ibid.*, p. 8.

[4] K. J. Pentinga, "Effects of Vibration in Rail Transport," *Packaging*, November, 1966, p. 118.

[5] *Op. cit.*, p. 55.

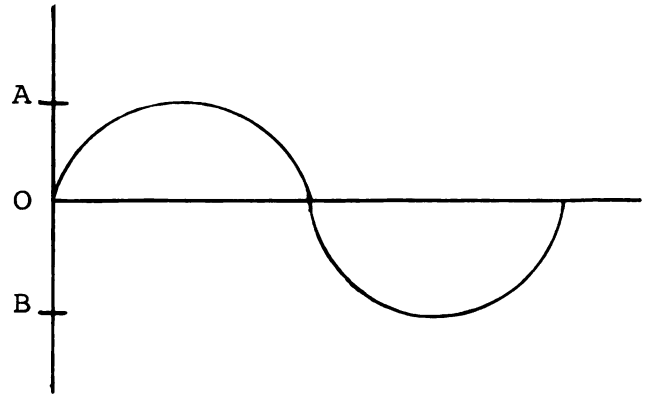

Figure 1.--Sine wave pulse used to monitor g level input.

The above signal is like the one used to monitor the g level input for the tests. If a g level of 0.5 was used this means a pulse of amplitude OA corresponded to the desired acceleration if properly calibrated. This value is called the zero-to-peak vibration level. The 1g peak-to-peak value is measured from A to B, and is twice as large a value as the OA portion of the pulse.

A look at the formula for calculating the acceleration of a vibrating system will show the factors that affect the g level. The formula is $\ddot{X} = -A(2\pi f)^2$, where $\ddot{X}$ is the acceleration in inches/sec^2, A is the amplitude of vibration in inches, and f is the frequency of vibration in cycles/sec. Since $\ddot{X}$ is in inches/sec^2, this value will have to be divided by 386 inches/sec^2 to convert this value to g's. Looking at the formula it can be seen that acceleration depends on the square of the frequency, f, and varies directly with the amplitude of vibration. Therefore, as the amplitude increases, the frequency must decrease to keep a constant g level, and vice versa. In this study, a test over a known range of frequencies was desired. The input acceleration was also known. Therefore by setting the number of Hz, the amplitude was regulated to give the desired input acceleration level.

After attaining the desired input g level, the reading that was showing the acceleration level the product was being subjected to was taken. Knowing the two values, a ratio was set up between the response acceleration, which the package was feeling, to the input acceleration, at which the table was being monitored. The frequency was then increased from 2 to 20 Hz in intervals of 1 Hz, and from 20 to 40 Hz in intervals of 2 Hz. A constant g level input of 0.5 was used throughout this section of tests.

By plotting these ratios over the entire range of frequencies, a transmissibility curve was obtained. This curve is a plot of the attained ratios, on a vertical axis, against the corresponding frequencies on the horizontal axis. The most important point on this curve shows the peak value of the ratios, and also the frequency at which the peaks occur.

A point worth noting here is that as the response/input acceleration ratio reaches a value where the monitored package is feeling 1.0g, this package will begin to bounce. This bouncing will continue over the range of frequencies until this ratio decreases to a point where the monitored one is no longer feeling 1.0g. Over this range of the test, a true picture of the transmissibility is not possible since the bouncing breaks up the normal pattern of magnification.

For this reason, the actual values of the peaks that occur during this bouncing, while they are "true" transmissibility values felt by the package, do not correspond to the theoretical conditions. This is the case since the bouncing introduces discontinuity into the system, making it non-linear.

When this test was completed, an identical package was placed on the vibration table, and the package with the accelerometer was set on top of it. Other than this change, the equipment setup was the same as the one just described. The same test was performed again, at the same g level, and a new set of ratios was obtained. This test was repeated until four packages were in the stack, and until it had been run with the accelerometer in each of the positions possible.

In all, ten different sets of ratios were collected from this portion of the study. The purpose of this section was twofold. The first was to see if and how the value of the ratios observed for each test differed.

Secondly, it was important to see if the peak values of these ratios occurred at a different frequency.

The theory was that after finding the frequency and peak value of the ratio for the single package system, it would be repeated, possibly attenuated, for all the other package combinations. This means that for any vibration system composed of more than one package, two pronounced peaks will be obtained. One of these peaks will be caused by the spring and cushioning properties of the other packages in the system. The other peak is probably due to the actual cushioning of the package containing the accelerometer.

The second section of the study was used to observe how a different input g level affected the ratios. The same products were used, they were cushioned to the same pound per square inch (psi) loading, and were placed in the same type of container.

The test as described in section one was then repeated with the exception that a 0.3g input was used. At the completion of this test, it was repeated with a 0.1g input. By looking at the peak values and the frequencies at which they occur, a comparison is made possible between each of the three input levels. This comparison should show how much the different input levels affect an identical stack of packages.

The third section of the study was performed to see how packages are affected by frequencies outside the normal range of frequencies related to vehicle suspension in rail or truck transportation. Since that range is from 3 to 20 Hz, the test was run over the range of 30 to 80. This range was chosen because frequencies of 50 to 60 Hz often occur in rail transportation. This point is brought out in the Forest Products study.

The same input acceleration of 0.5g is used and the products are cushioned and packaged exactly as before. A set of ratios and corresponding frequencies was also obtained for this portion of the study.

The final section of the study was performed to see if and how the psi loading on a cushion affects the transmissibility. In this case a psi loading twice as high as the one used so far in the test was utilized. The input level was again 0.5g.

For this section of the study, two package arrangements were used. First, the setup with only one package. The second one used was the one with the four packages and the accelerometer in the top package.

By running these four series of tests, it is possible to further speculate as to the effects of certain other combinations of g level input, cushioning and frequencies on a stack of packages. The study was not intended to give the final answer to all vibration and transmissibility problems, or develop a complex theory as to why some things happen.

CHAPTER II

EQUIPMENT

In order to complete this study on vibration and transmissibility of a stack, it was necessary to use a number of different pieces of equipment. The equipment was utilized in such a way so as to enable the operator to measure the acceleration levels of the vibration table and products that were placed on the table, over a large range of frequencies.

The first piece of equipment is the vibration table system, Figure 2. This is a table mounted on an MTS, Model 205.31 hydraulic cylinder. The piston on which the table rests is 2.25 square inches in area and allows a stroke of 6 inches in the vertical direction.

The movements of this vibration table system are controlled by an MTS, Model 481.01, Material Test Control Panel, Figure 3. Three different areas of this control system were utilized to perform my study.

One of the components used was a Model 413.50, Control Panel. Its function was to turn the flow of hydraulic fluid, which operated the piston, on and off. This panel was also used to start and stop the vibration

Figure 2.--Vibration table system.

Figure 3.--Control panel.

of the table. Another component was a Model 410.26 Function Generator. This was used to regulate the number of Hz at which the table was to vibrate. The final component was a Servac, Model 401.01, Universal Input Module. Its function was to regulate the amplitude of the stroke realized by the vibration table.

Two accelerometers were also necessary to perform the study. The two used were Endevco, Model 2265-20, Piezoresistive Accelerometers. One of these is shown in Figure 4.

An amplifier was another vital piece of equipment, because the signals had to be amplified to allow them to be read. An Endevco, Model 2995, Signal Monitor Panel was used.

Two filters were necessary to the operation. They were used to "cut out" certain frequencies that were not desired for a particular reading. Krohn-Hite, Model 3750 Filters were used to fulfill this need.

The final piece of technical equipment used was an oscilloscope. A Tektronix, Type 502 Dual-Beam Oscilloscope was utilized. A dual channel scope was needed because two signals had to be monitored at the same time. These last three pieces of equipment described are shown in Figure 5.

In addition to the above mentioned equipment, four identical products were utilized to simulate actual

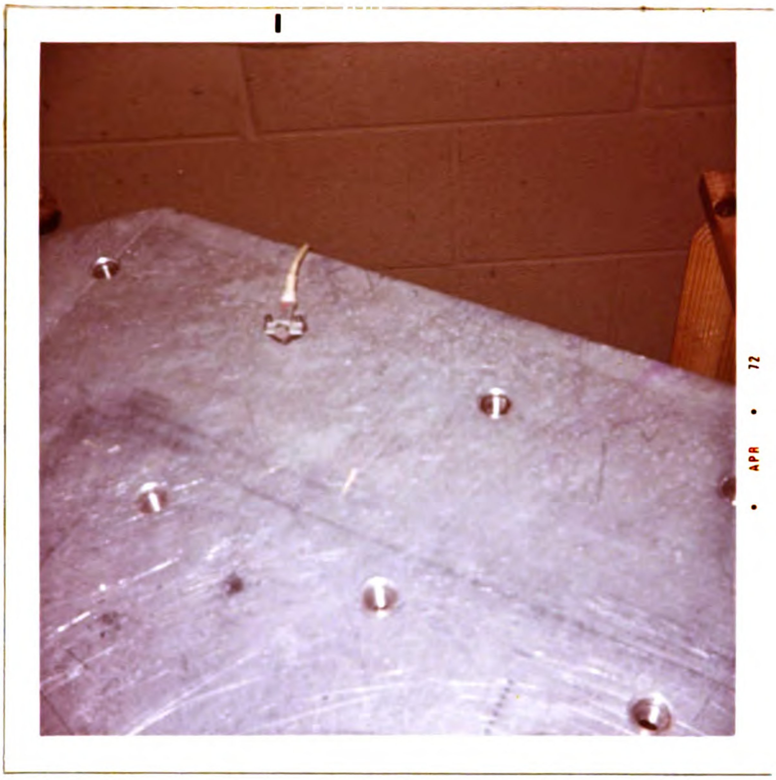

Figure 4.--Accelerometer.

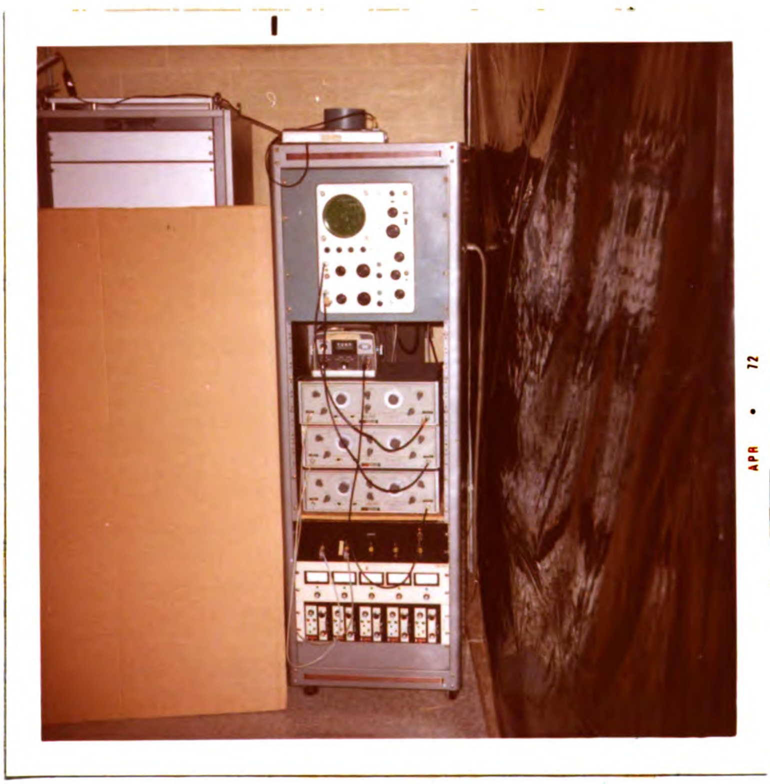

Figure 5.--Amplifier, filters, and oscilloscope.

products. These products were identical in that they were blocks of wood 15" x 15" x 8", and they each weighed 20 pounds. Cushioning was also used for these simulated products. Two pound foam polyethylene was chosen as the cushioning material, and two different static psi loadings were utilized during the test. Finally, the four products were placed in separate regular slotted fiberboard containers (RSC). These containers were made of 200-pound test, C-Flute, corrugated board, with inside dimensions of 15" x 15" x 12". This assured a tight fit for the sides of the product inside the container, and allowed cushions 2 inches thick to be placed on the top and bottom of the product.

Some type of chute was necessary to control the amount of horizontal movement of the packages. This chute was made of the same type of corrugated board as that used on the containers. The inside dimensions were 1/16" greater than the outside dimensions of the packages.

A second reason for the chute was to simulate the friction that would be produced between two stacks of packages during vibration in transportation. Figure 6 shows the stack of packages on the vibration table, and Figure 7 shows the chute mounted, over the products, to the vibration table.

Figure 6.--Stack of packages.

Figure 7.--Chute secured around packages.

CHAPTER III

EQUIPMENT SETUP AND USE

A simple diagram will be very helpful in trying to explain how the actual components were linked together to perform the study. This diagram is shown in Figure 8.

For illustration, a stack of four packages is shown on the vibration table, however, the number varied from one to four over the entire experiment. All of the equipment used in the diagram are the ones described above.

The first step is to mount the accelerometers. One of them is mounted directly to the vibration table. This will be needed to measure and regulate the g level that the table is delivering to the stack. The second accelerometer is mounted to one of the wooden products so that its response accelerations in the vertical direction can be measured.

The vibration is started by turning on the hydraulic system and setting the desired frequency with the function generator. The desired g level of input to the table is then obtained by adjusting the amplitude of vibration.

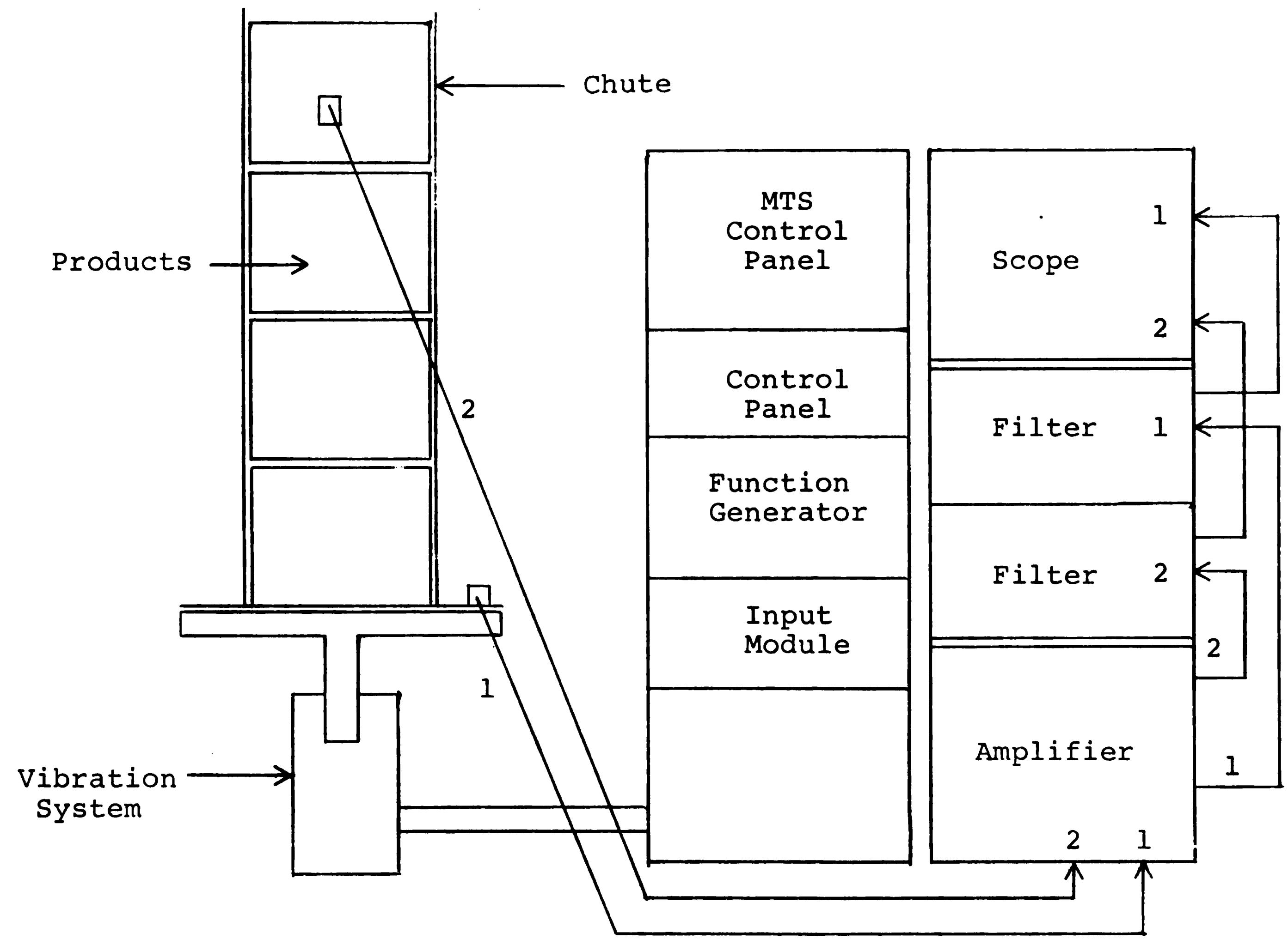

Figure 8.--Diagram of the equipment setup that was used to perform the study.

When the vibration begins, a signal is sent from each of the accelerometers, through a cable, to the amplifier. Here the amplifier sets the value of the signal by allowing .625 volts to correspond to 1g peak-to-peak for a sine wave pulse.

From the amplifier, the signal from each accelerometer is sent to one of the filters, via cable. These filters are set so as to "cut out" the frequencies that are not relevant to the study and that distort the signals. If the table is vibrating at 10 Hz, the filter was set at approximately 20 Hz.

At this point the signal from each accelerometer is sent to a separate channel of the oscilloscope. Once the signal reaches the scope, the actual value or magnitude of the signal was read. This value is in volts, and must be converted to g's to be used in the study.

The first signal to look at is the one being sent from the accelerometer on the vibration table. This is the input signal traveling via cable 1. The desired value for the system is known and is monitored through this accelerometer. For the major portion of the study this input value is 0.5, zero-to-peak for a sine wave pulse. However, for one section it is set at 0.3 and then 0.1g.

After the desired input for the system is found, the g level realized by the product being tested can be read. To obtain this value, the channel monitoring the signal coming from the package had to be varied from 0.2 volts per division, up to as high as 2 volts per division.

When actually performing the study, the function generator was set at approximately 2 Hz. The amplitude was then adjusted so that the desired input g level was reached. Once the correct input was attained, the value of acceleration corresponding to the package was read. The number of Hz were then increased from 2 to 20, in intervals of 1 Hz. Starting at 20 cycles, the interval increases were 2 Hz, until the final frequency level was reached.

CHAPTER IV

TESTING AT 0.5g INPUT AND 0.5 PSI LOADING

To more effectively evaluate the results of each section, it will be necessary to further divide these broad areas.

A summary of the data compiled in section one of the study is appropriate. This data is listed in Table 1. The actual numbers in the table are the ratios of the response over the input accelerations for the tests. Each row of values corresponds to the frequencies at which the table was vibrating when the reading was made for each particular stacking arrangement. Each column gives the readings of one stacking arrangement over the entire range of frequencies.

A coding system is used to describe each individual stacking arrangement. This code has the general form (A-B), where A is the number of packages in the stack and B is the position of the packaged accelerometer in the stack. Therefore, a code of (4-2) would correspond to four packages in the stack, and the accelerometer in the second one from the bottom.

TABLE 1.--response/input ratios for 0.5g and 0.5 psi.

	Stacking Arrangements									
Freq	1-1	2-1	2-2	2-1	3-2	3-3	4-1	4-2	4-3	4-4
2	1.09	1.08	1.10	1.09	1.10	1.09	1.09	1.15	1.15	1.12
3	1.09	1.09	1.10	1.09	1.10	1.10	1.09	1.15	1.20	1.23
4	1.09	1.09	1.14	1.10	1.15	1.20	1.09	1.15	1.28	1.31
5	1.11	1.10	1.18	1.12	1.26	1.31	1.14	1.34	1.60	1.60
6	1.11	1.12	1.28	1.14	1.39	1.44	1.15	1.41	1.68	1.76
7	1.11	1.14	1.28	1.15	1.45	1.71	1.21	1.84	2.88	2.20
8	1.11	1.14	1.68	1.18	1.68	1.74	1.28	2.52	2.84	3.00
9	1.12	1.18	1.68	1.34	2.88	2.20	1.21	2.32	2.80	4.80
10	1.14	1.20	1.92	1.35	2.75	4.70	1.21	1.76	2.78	4.48
11	1.15	1.25	2.08	1.28	2.64	4.52	1.20	1.68	2.76	4.21
12	1.30	1.35	4.05	1.23	2.56	4.47	1.15	1.28	2.72	4.08
13	1.31	1.91	4.45	1.21	2.24	4.00	1.12	0.96	2.40	3.36
14	1.32	2.10	3.69	1.17	2.00	3.60	1.10	0.74	1.92	3.06
15	1.33	1.76	3.36	1.10	1.60	3.20	1.09	0.64	1.84	2.80
16	1.38	1.60	3.20	1.10	1.36	2.88	1.09	0.69	1.76	2.24
17	1.42	1.44	3.04	1.10	1.12	2.56	1.10	0.70	1.44	2.08
18	1.60	1.09	2.28	1.09	0.82	2.40	1.17	0.90	0.96	1.60
19	1.76	1.15	1.05	1.09	0.64	2.10	1.18	0.96	0.72	1.36
20	1.79	1.18	0.91	1.15	0.56	1.84	1.25	1.08	0.64	1.28
22	1.84	1.20	0.60	1.21	0.78	1.60	1.38	1.28	0.90	1.44
24	2.04	1.25	0.68	1.32	0.90	1.51	1.50	2.16	1.40	1.48
26	2.20	1.34	0.72	1.47	1.28	1.38	1.34	2.40	1.87	1.52
28	2.22	1.50	0.79	1.58	1.73	1.64	1.20	1.84	1.92	2.40
30	2.96	1.30	1.02	1.60	1.92	1.55	1.18	1.44	1.84	2.32
32	2.80	1.18	0.89	1.55	1.90	1.47	1.14	1.12	1.76	2.08
34.	2.68	1.06	0.71	1.45	1.67	1.44	1.10	0,80	1.64	1.84
36	2.44	1.02	0.53	1.40	1.62	1.28	1.05	0.72	1.60	1.52
38	2.40	1.01	0.51	1.38	1.57	1.04	1.02	0.69	1.44	1.49
40	2.24	1.00	0.47	1.31	1.52	0.80	1.01	0.63	1.36	1.43

*The underlines values are the peak ratios for each individual stacking arrangement.

The ratios given in this table can be plotted on a graph. The coordinates for these graphs are the same as those used on the table. The frequencies are plotted along the horizontal axis and the ratios along the vertical axis.

For illustration, two of these graphs are shown. Figure 9 is a graph for the transmissibility of a vibration system composed of only one package. The data for this plot is that of the (1-1) system shown in Table 1. One pronounced peak is obtained over the range of 0 to 40 Hz.

Figure 10 is the same type of graph, however, more than one package is being subjected to the test. The data for this plot is found in Table 2, under the 0.1g input. This data was used because a response g level of 1.0g was never reached over the entire tested frequency range, as is the case for "true" transmissibility curves. Above 1.0g of response the package will begin to bounce, so as to break-up the normal transmissibility pattern.

In this case, two pronounced peaks are obtained over the same frequency range. It is believed that one of these peaks is due to the other packages acting as an additional spring system, and the second one is due to the cushioning properties of the individual package that contains the accelerometer.

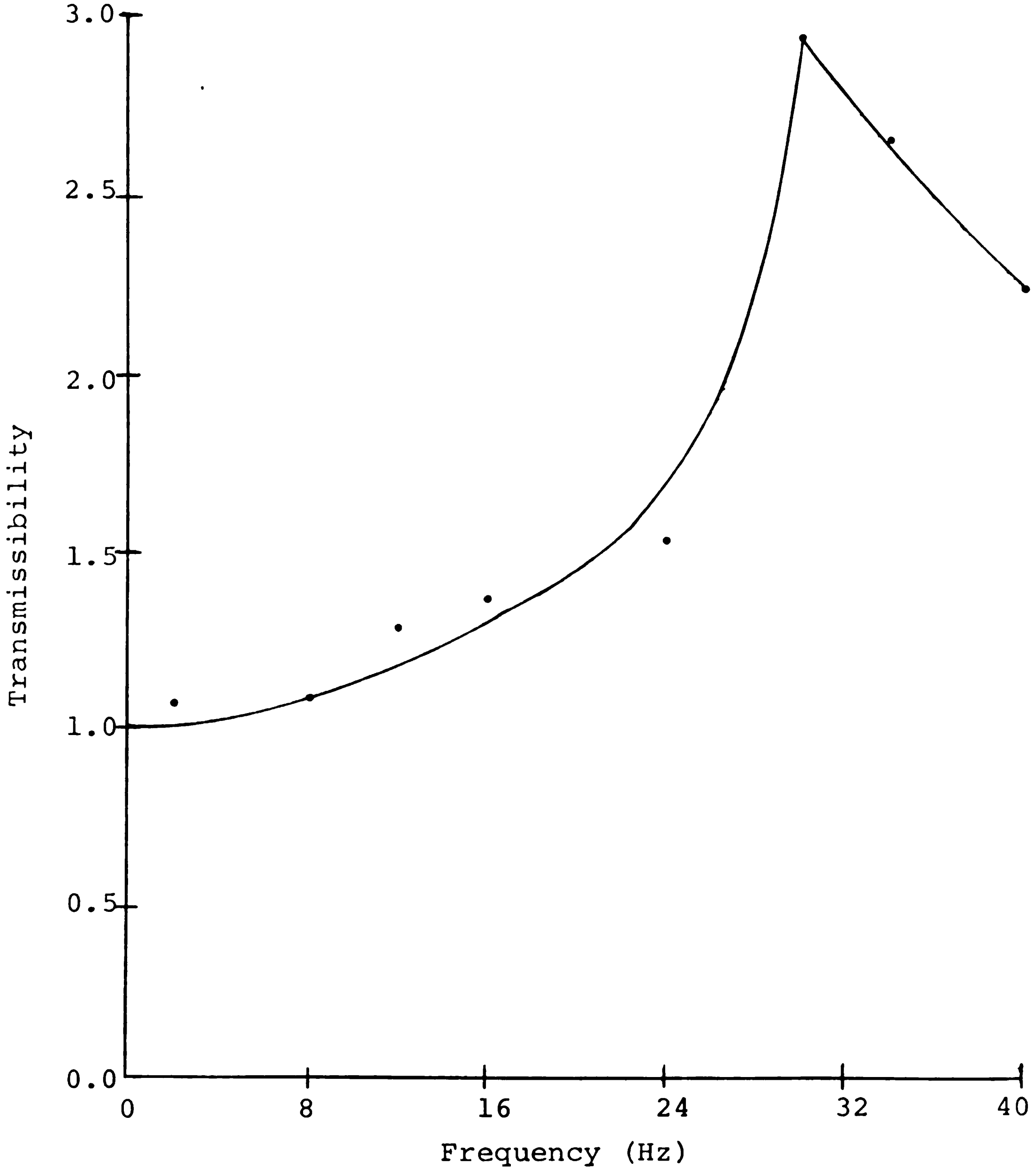

Figure 9.--Transmissibility curve for a single package system. Data used is from the (1-1) arrangement with a 0.5g input.

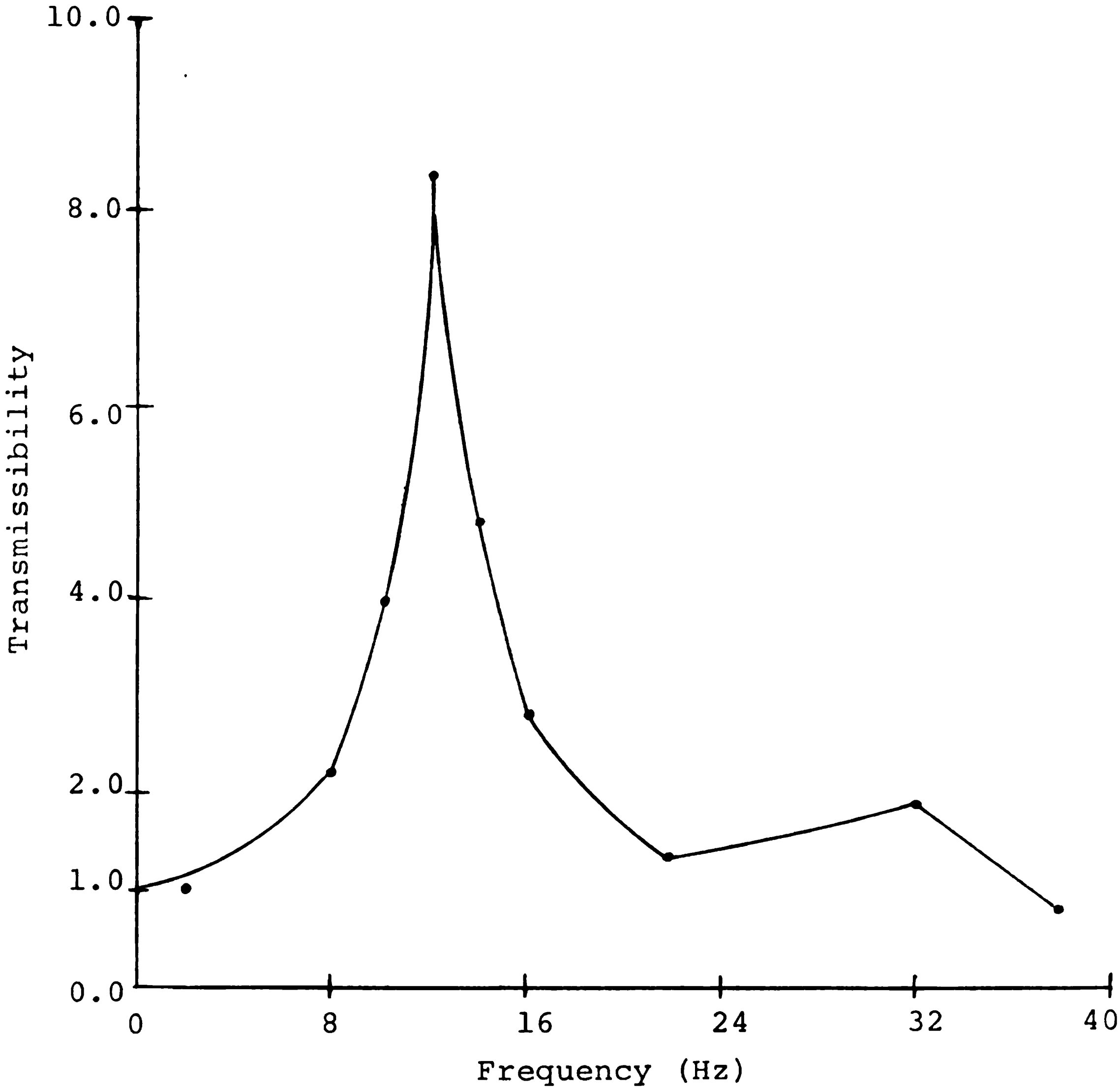

Figure 10.--Transmissibility curve for a multiple package system. Data used is from the (4-4) arrangement with a 0.1g input.

TABLE 2.--Response/input ratios. (Variable g level and 0.5 psi).

Freq.	(4-4) Stacking Arrangement					
	0.5g Input		0.3g Input		0.1g Input	
	g Level (Resp.)	Ratio	g Level (Resp.)	Ratio	g Level (Resp.)	Ratio
2	0.56	1.12	0.32	1.06	0.10	1.00
4	0.65	1.31	0.40	1.30	0.12	1.20
6	0.88	1.76	0.48	1.60	0.16	1.60
8	1.50	3.00	0.76	2.53	0.22	2.20
9	2.40	<u>4.80</u>				
10	2.24	4.48	1.92	<u>6.40</u>	0.40	4.00
12	2.04	4.08	1.28	4.27	0.84	<u>8.40</u>
14	1.53	3.06	0.92	3.07	0.48	4.80
16	1.12	2.24	0.64	2.13	0.28	2.80
18	0.80	1.60	0.46	1.53	0.22	2.20
20	0.64	1.28	0.42	1.40	0.16	1.60
22	0.72	1.44	0.43	1.43	0.13	1.30
24	0.74	1.48	0.44	1.47	0.14	1.40
26	0.76	1.52	0.45	1.50	0.14	1.40
28	1.20	<u>2.40</u>	0.50	1.66	0.15	1.50
30	1.16	2.32	0.76	2.53	0.16	1.60
32	1.04	2.08	0.78	<u>2.60</u>	0.19	<u>1.90</u>
34	0.92	1.84	0.54	1.80	0.16	1.60
36	0.76	1.52	0.41	1.36	0.12	1.20
38	0.74	1.49	0.32	1.06	0.08	0.80

*The underlined values are the peak ratios for each individual stacking arrangement.

The first major division of this study is concerned with the tests run at a constant input of 0.5g and with the cushioning of each product at 0.5 psi. Of major concern for these tests are the peak values of the response over input ratios, and the corresponding number of Hz at which they occur. This major division is now divided into three more specific areas.

Tests Monitoring the Bottom Package of a Stack

The first subdivision is concerned with the tests on stacking arrangement with the accelerometer in the bottom package. These setups have the codes of (1-1), (2-1), (3-1), and (4-1), the 1 meaning the accelerometer is always on the bottom of the stack.

The arrangement with the code of (1-1) was the first one tested. The purpose of this was to find the peak value of the response over input ratio, and the frequency at which this peak occurs. A look at Figure 9 shows that the peak value was 2.96 and it occurred at 30 Hz. This means that the transmissibility or magnification of the input was 2.96. Therefore, with a 0.5g input, the product felt an acceleration of 2.96 x 0.5g, or 1.48g's.

Once the values of this arrangement were known, it was possible to see the effects of having more than one package in the stack being tested. If these sets of data from multiple package stacks are now plotted, two pronounced peaks will be obtained. This plot would look something like the one illustrated in Figure 10.

The values obtained in the rest of this subdivision will be dealt with together, however, it may be worthwhile to look at one of these arrangements separately to see how the additional package affects the peak values. The one considered will be the one with two packages and the accelerometer in the bottom one. The code for this set of data is (2-1). A look at the data in Table 1 for this stacking arrangement shows that two pronounced peaks do occur over the tested range.

The first peak of (2-1) has a ratio value of 2.1, and occurs at 14 Hz. This means that with the 0.5g input, the product is being subjected to 1.05g's. This value is probably due to the spring action of the second package added to the system.

The second peak occurs at 28 Hz and has a ratio of 1.5. A comparison can be made between the peak for (1-1), and the second peak of (2-1). The values are quite a bit different, 2.96 and 1.5. However, they occur at frequencies that are very close to each other, one being 30 and the other 28. This seems to indicate that there is a possibility that the second peak of (2-1) corresponds to the peak of (1-1), slightly attenuated.

By looking at all four sets of data for this subdivision, found in Table 1, it is possible to see a trend developing. Consider first the peak of (1-1), and the second peaks of the other sets of data. They show quite a bit of variation in peak value, since they range from 2.96 for (1-1) to 1.5 for (2-1), however, they occur at frequencies very close to 30. This seems to give further evidence to the possibility that the second peak is really due to the individual package cushioning properties.

The peak value for (1-1), and the first peaks of the other three data sets are now compared. A definite trend is developing here. The (1-1) arrangement has a peak ratio of 2.96 and occurs at 30 Hz. The (2-1) setup has a peak of 2.1 at 13 Hz. The (3-1) arrangement had a peak of 1.35 at 10 Hz. The first peak for (4-1) is 1.28 at 8 Hz.

The general trend shows two things. First, as the number of packages on top of the bottom one, which contains the accelerometer, increases, the peak value of the ratios decrease at a decreasing rate. Also, as the number of packages increase, the number of Hz at which this peak occurs decreases at a decreasing rate. This phenomenon occurs because of the increased psi loading subjected to the bottom package, and also the spring action of the other packages. This theory will be considered again later in the study. This concludes the analysis of the data of subdivision one.

Tests Monitoring the Middle Packages of a Stack

The second subdivision of this first major portion of the study is concerned with the results of having the monitored package in the middle of the stack. This means that at least one package is above and below the package containing the accelerometer.

For this analysis, four sets of ratios are again used. The (1-1) arrangement is again utilized as a standard. The other three arrangements are (3-2), (4-2), and (4-3). The ratios and corresponding frequencies can be found in Table 1.

The first comparison will be between the peak of (1-1), and the second peaks of the other data. The peak values vary from 2.96 for (1-1) to 1.92 for (4-3), however, they again occur at frequencies close to each other, two of them being 26 and the other two at 30. This seems to add further support to the theory that the original peak for (1-1) is repeated, slightly attenuated, for each different arrangement.

The second comparison of this subdivision is going to be further divided into two parts. First of all it is worthwhile to compare the data from (3-2) and (4-2).

This will show the effects of having an additional package on top of the packaged accelerometer.

For the (3-2) setup, the first peak value is 2.88 and occurs at 9 Hz. The (4-2) setup has its first peak of 2.52 at 8 Hz. This shows that increasing the number of packages on top of the accelerometer in position two changes the frequency at which the peaks occur slightly, however, it decreases the transmissibility of forces that the package in position two feels quite a bit. Both of these changes are probably due to the increased psi loading and additional spring action on the package being monitored.

The next comparison will show the effects of being in position two as opposed to position three in a four package system. In this case, the arrangements (4-2) and (4-3) are used. The values of peak one for (4-2) are 2.52 at 8 Hz and 2.88 at 7 Hz for the (4-3) arrangement. Here it is apparent that as the accelerometer moves from position two to three, the ratios increase, however, the cycles per second at which these values occur decrease slightly. This gives further evidence that psi loading and the "springs" present are relevant to this or any transmissibility study.

A comparison is also possible between the data of subdivision one and this subdivision. In general it can be seen that the transmissibility of acceleration increases as the monitored package is moved from the bottom position of the stack to a position in the middle of the system. There also seemed to be a slight decrease in the frequency at which these peak values occurred. This seems to indicate that more damage due to vibration will occur in packages in the middle of the stack as opposed to those on the bottom of the stack.

Tests Monitoring the Top Package of a Stack

The third subdivision of the first major section of tests is concerned with the stacks where the package with the accelerometer is on top of the stack. Four stacking arrangements are considered here. They are (1-1), (2-2), (3-3), and (4-4). The data for these arrangements can be found in Table 1.

The first comparison will be made of the (1-1) peak value and the second peaks of the other data. Again the peak value of these ratios are slightly attenuated from the value attained for (1-1). The values range from 2.96 at 30 Hz to 1.02 at 30 Hz for (2-2). Looking at the frequencies you will see that most of these peaks occur near 30 Hz. The theory that the peak of (1-1) is repeated for every other system seems to gain added assurance here.

Next, the peak value of (1-1) is compared to the first peaks of the other data sets. The values range from 2.96 at 30 Hz for (1-1) to 4.8 at 9 Hz for (4-4).

In general the peak values are increasing at a decreasing rate. That is, for each additional package added to the stack, the ratio is increasing, but it is increasing by a smaller amount each time.

The other important area of concern is the frequency at which these first peaks occur. As the height of the stack increases, the value in Hz where the peaks occur decrease at a decreasing rate.

To calculate the "natural frequency" of a system, the formula $f_n = 1/2\pi\sqrt{KG/W}$ may be used, where f_n is the natural frequency of the system in Hz, G is the acceleration of gravity, or 386 in/sec^2, W is the weight of the product in pounds, and K is the spring constant in pounds per inch. For the setups with the accelerometer in the top package, the value of G and W are constant. Therefore, a change in f_n will be the result of a change in K.

A look at the setups used in this part of the study shows that as each package is added to the stack, essentially, another spring is added to the system. In a situation like this you have springs in a series. When springs are put together in a series, "an equivalent single spring having the same value for springs in a series is, $K_{eq} = 1/(1/K_1 + 1/K_2 + 1/K_3)$."[6] In this

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[6]James B. Vernon, Linear Vibration Theory (New York: John Wiley and Sons, Inc., 1967), p. 60.

equation the K_1, K_2, and K_3 are the values of each additional spring added to the system.

Since all the packages added to these tests are the same, it might be assumed that all these K's are equal. If this is the case, the above formula becomes $K_{eq} = K/n$ where K_{eq} is the equivalent spring constant for all springs, K is the spring constant for one package, and n is the number of packages under the top one. This K is the slope of the line shown in Figure 11, however, because of the difficulty and high possibility of error in finding the correct data, it will not be used in any calcualtions. If its value could have been accurately determined, it would have been used in the calculations.

The data from Table 1 shows that the first peak frequencies for the multiple package systems are 13 for (2-2), 10 for (3-3), and 9 for (4-4). The frequency value for (2-2) should conform to the formula $f_n = 1/2\pi\sqrt{KG/W}$, where K is the spring constant for the package on the bottom of the stack.

The K_{eq} for (3-3) should be K/2 according to the above equation for equal K's in a series. Assuming the frequency for (2-2) is correct, the one for (3-3) should be able to be computed by the formula, $f_n = 1/2\pi\sqrt{(K/2)G/W}$. This means that the natural frequency for this system should be less than the natural frequency for the (2-2)

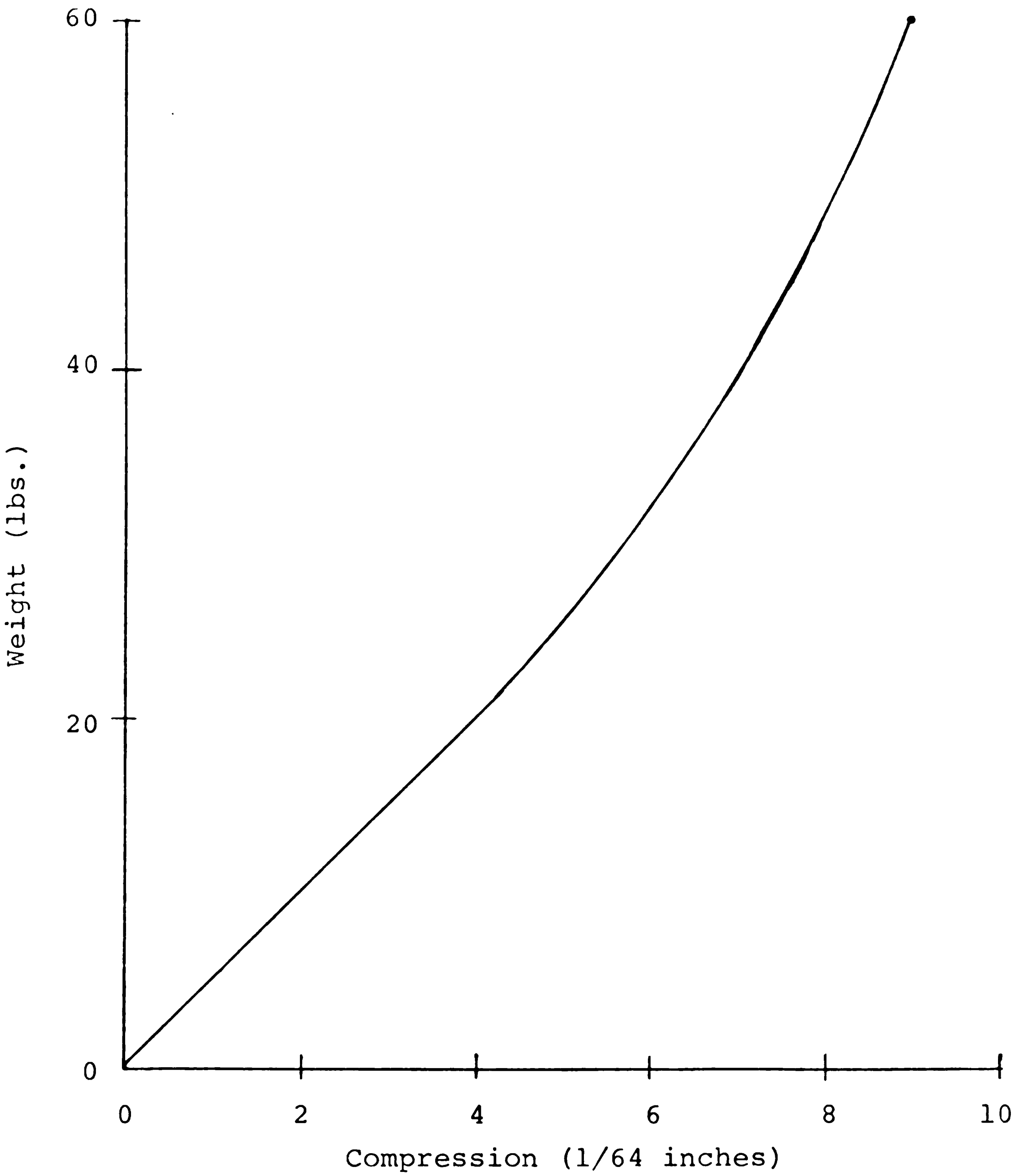

Figure 11.--A plot to obtain the spring constant, K, for a single package. K equals the slope of the line at the desired weight level.

system by a factor of the square root of 2 or 1.414. Dividing 13 by 1.414 yields approximately 9.

The K_{eq} for (4-4) will be K/3 according to the above theory. Again assuming the value of f_n for (2-2) is correct, the natural frequency for the (4-4) setup should be less than the value of f_n for (2-2) by a factor of the square root of 3, or 1.73. Dividing 13 by 1.73 yields approximately 8.

Looking at the actual and predicted natural frequencies shows that they vary a little. This might be explained by the fact that the K's are not all equal. If the K's are not the same, the formula for the equivalent K for a series of 2 springs is, $K_{eq} = (K_1K_2)/(K_1 + K_2)$, and 3 springs is, $K_{eq} = (K_1K_2K_3)/(K_2K_3 + K_1K_3 + K_1K_2)$. This means that these new K_{eq}'s would be used in the formula for f_n, instead of the one derived for the case of equal K's. Since the actual and predicted values are very close, it seems as though these results are very logical. This above analysis indicates that as the stack reaches a certain height, additional packages will have less effect on the frequency at peak, and the data supports this finding.

A few general, but very helpful things can be learned from this portion of the study. First, as has been stated in each subdivision, the peak of the initial one package system seems to be repeated for every test.

This could mean that should this second peak fall inside the normal operating frequency range for a mode of transportation, it may cause damage that is unexplained by the peak due to the entire system.

Also, unlike the damage due to static loading which is most severe at the bottom of a stack, damage due to vibration increases at the top of the stack as each additional package is added to the system. It also shows that as the stack reaches a certain height, the peak values level off. In all probability, increasing the height to five packages would not change the numerical value of the peak ratio noticeably, however, the number of Hz at which the peak occurs will decrease a little, as can be seen by the change in K_{eq}. This above analysis of frequency can be applied to all first peaks for the entire study.

CHAPTER V

TESTING AT 0.3 AND 0.1g INPUTS AND 0.5 PSI LOADING

The second major section of this study concentrated on the effects of variable g level inputs to the vibration system. To do this, all other experimental variables were the same as those used in section one with the exception that the tests were run at a g level of 0.3 and then at 0.1. Once these values are obtained a comparison of peak values can be made for a certain stacking arrangement at three different acceleration inputs.

Since everything but the input g level is held constant, it would be useless to run these tests for every stacking arrangement. For this reason, only the stacking arrangement that afforded the severest conditions was utilized. This was the one with four packages and the accelerometer in the top package.

The values for the 0.5g level were obtained in the last set of tests for this (4-4) setup. Therefore, the first test to run was at the 0.3g level. Two pronounced peaks were again realized over the tested frequency range. Table 2 contains the ratios obtained and used in this study. This table is set up somewhat like Table 1. Each row still corresponds to the reading at a particular frequency, and the columns are the reading for a particular arrangement over the entire range of the test. The main difference in the two tables is that Table 2 also shows the actual g level the product was feeling throughout the entire test. The ratios in this table could be plotted to obtain a graph like the one in Figure 10.

A look at the first peaks for the (4-4) arrangement at 0.5g and 0.3g shows that the transmissibility, or magnification of g level, is noticeably higher for the lower g level. The first peak of (4-4) with the 0.5g input is 4.8 at 9 cycles per second. The corresponding peak at the 0.3 g input is 6.4 at 10 cycles per second.

The reason for the higher transmissibility may be due to the fact that some magnification occurs at any g level, however, as the acceleration is increased, the cushion may absorb a larger percentage of the input being transferred. As far as the frequency at peak for each, they should not have been different, since only the input level was different, and this does not enter into the calculation of the natural frequency of a system. However, since they were different by only one cycle per second, the variation is probably due to a slight misreading of the equipment.

The second peaks of each set of data show the same phenomenon. For the 0.5g input, the transmissibility is 2.4, whereas, for the 0.3g input it is 2.62. The peak value again occurs at a lower frequency for the higher input, however, they are so close that the difference is probably due to inaccuracy of the equipment.

Further study of these two second peaks gives more support to the repetition of the single product peaks at somewhat equal frequencies. These values again fall very near 30 Hz, namely 28 and 32.

The second part of this test was performed to see if further reduction of the g level input would affect the transmissibility. As can be seen from Table 2, the first peak value for the 0.1g data is 8.36. This is an increase over the other 0.3g data, however, it is a smaller increase. This seems to show that the increase in transmissibility increases at a decreasing rate as the g level is dropped. Therefore, at some g level the transmissibility will probably stabilize.

A look at the second peaks for this set of data shows that it does not follow the trend of increasing somewhat steadily. However, the frequency at which the peak occurs is again near 30 Hz, namely 32.

So far in this part of the study, only the magnification of transmissibility of the accelerations has been considered. However, the g level the product is feeling may be more important. Table 2 shows the g level subjected to the product all along the test range.

Even though the transmissibility is higher for the lower g inputs, the response g level is higher for the larger inputs. This means that even though the magnification of the input is lower for the 0.5 g input, more damage will be incurred at this level than at either of the other two. The second peaks for each of these three sets of data further supports this fact.

CHAPTER VI

TESTING OUTSIDE FREQUENCY RANGE OF VEHICLE SUSPENSION (0.5g, 0.5 PSI LOADING)

The first two portions of this study have been concerned with tests that relate directly to two modes of transportation, namely rail and truck. This means that the tests were mainly interested in seeing what would happen under a simulated transportation environment. As was stated earlier, 3 to 20 Hz at 0.1 to 0.8g covered the conditions found in both of these types of transportation.

However, in much of the data on vibration received from actual in-transit tests, signals in the range of 50 to 60 Hz are frequently recorded. It is felt by many people that these frequencies are due to noise, and cause very little if any damage to the product. In this section of my study, the same products utilized throughout the study will be used to pursue this idea.

In order to test this area of interest, only one stacking arrangement was used. Again the one with the most severe conditions was chosen. This is the (4-4) setup. For this test, a constant 0.5g input was used, and cushioning was the same.

As was stated earlier, these frequencies are usually in the range 50 to 60 Hz. Therefore, to obtain a certain margin of safety, the test will be performed over the range of 30 to 80 Hz. The actual data is not presented in a table, however, a transmissibility curve is given, Figure 12, showing the plot of the response--input ratios at each corresponding frequency.

By observing Figure 12 it can readily be seen that as the frequency is increased from 30 to 50 Hz, the ratios decrease rather quickly. However, it should be noted that they are decreasing at a decreasing rate.

At 30 Hz the ratio is 2.15. Since the input is 0.5g, the package is being subjected to 1.08g's. Looking at the ratio at 50 cycles you can see that it is approximately 0.3. This means that the package is feeling 0.15g. This is over 300% less than the input g level.

The above analysis seems to say that after 30 Hz, and especially from 40 to 50 cycles, most of the input level is not transmitted to the product. This seems to strongly indicate the fact that much less damage will occur in this range due to vibration than in the 0-30 Hz range.

Now looking at the graph from 50 to 80 Hz, an interesting occurrence can be noted. In this range, the product feels almost none of the input g level. This data seems to support the idea that frequencies outside the

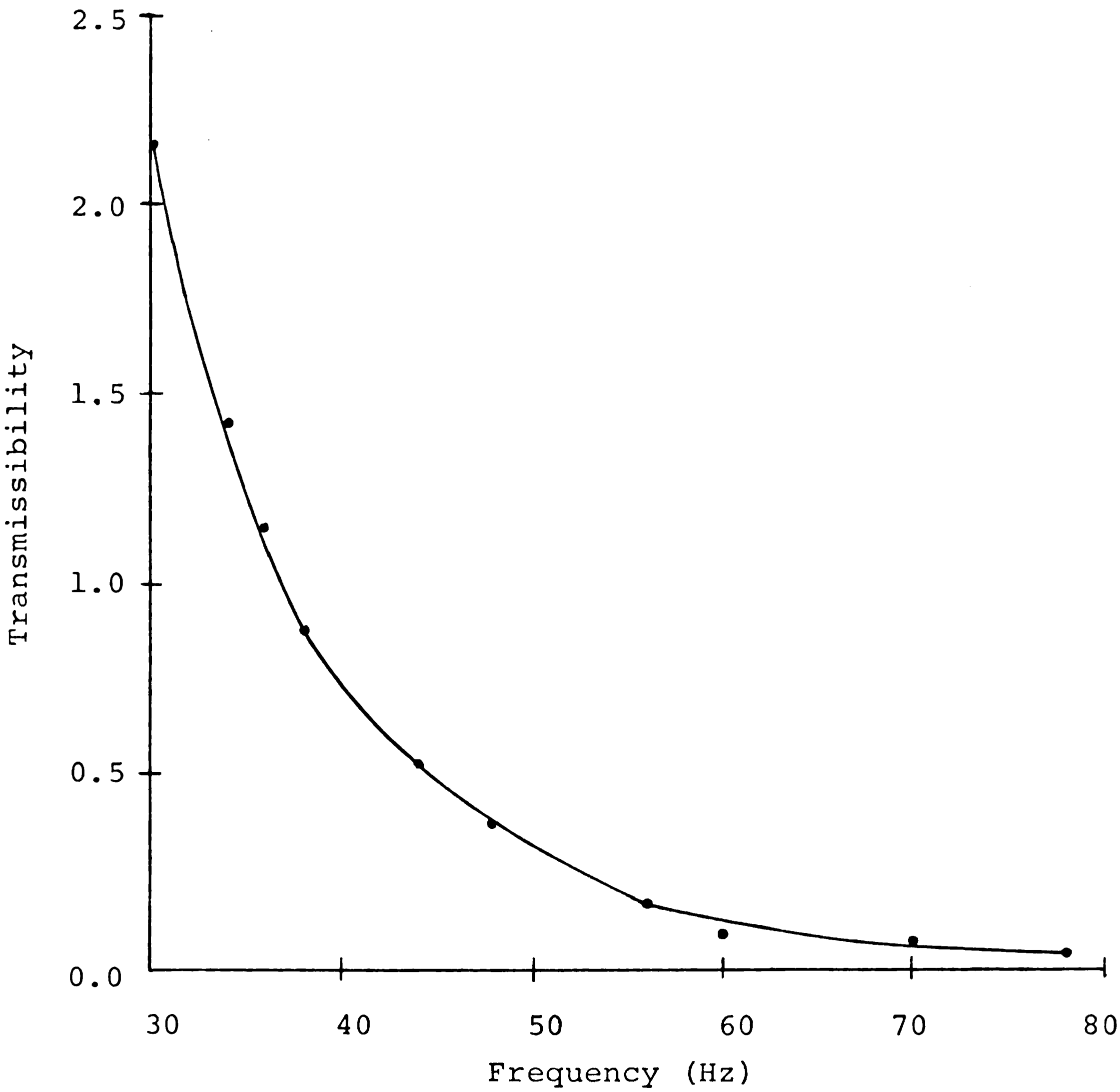

Figure 12.--Transmissibility curve for a multiple package system outside the frequency range related to normal vehicle suspension.

normal range of truck or rail cause very little if any damage to the products.

It might be worthwhile to look at one of the ratios in this range and see what the actual response g level is. At 58 Hz the ratio is 0.1. This means that with a 0.5g input, the package is only feeling 0.05g. This is 20 times less than the pull of gravity, and would be almost disregarded in the transportation environment.

An overall evaluation of the entire curve from 30 to 80 cycles seems relevant. The curve asymptotically approaches a response over input ratio of zero, however, it will probably never be exactly zero. This supports the theory that frequencies in this range are next to meaningless. These frequencies are probably absorbed by the packaging material as those in the 1000 Hz range described earlier.

This single test does not ultimately prove that these frequencies are to be totally disregarded. However, by looking at all of the data collected and reported on thus far, it seems obvious that most of the work on vibration of packages should be concentrated on the frequency range of 0 to 30 Hz. This is the case since the highest transmissibility and possibility of damage seems to occur in this range.

CHAPTER VII

TESTING AT 0.5g INPUT AND 1 PSI LOADING

The final section of this study is concerned with seeing how the cushioning of a product affects the transmissibility of forces through a stack. The primary difficulty in evaluating the actual affect of the cushioning material arises because of the particular product/package system used. In the system under consideration, the 20 pound wooden products are cushioned with 2 pound foam polyethylene, however, they are then placed in corrugated containers. Therefore, two cushions are present, the polyethylene and the corrugated board. For this reason, the single effect of the polyethylene cushioning will not be apparent. However, if the polyethylene has any effect at all, it should be shown in this test.

In this portion of the study, the same equipment, products and containers are used. All previous test packages have had 40 square inches of 2 inch foam polyethylene on the top and bottom of the products. This had the effect of causing a 0.5 psi loading on the cushioning for a particular, single product. In this final test, the amount of cushioning in the monitored package will be half as much as was previously used. This means that 20 square inches of 2 inch polyethylene will be utilized. This will result in a static loading of 1 psi, twice as high as before.

In this series of tests, as in the last two series, only a few of the stacking arrangements were used. The single package system, or the (1-1) setup, was chosen as one of the arrangements. This will show how the cushioning will affect a single package. A comparison of the data from this test, with the data obtained for the same one package system, but with a 0.5 psi cushion loading is now possible.

The final test will be with the arrangement that has the severest transmissibility ratio, namely (4-4). Once obtaining this data, a comparison is made possible between these results, and the data obtained for the same setup with a 0.5 psi loading.

The single package system had a peak ratio of 2.24 at 26 Hz. This data is shown in Table 3. For the same (1-1) system with a 0.5 psi loading, the peak ratio was 2.96 at 30 Hz. Converting these values to g's, the package with the higher psi loading felt 1.12g's, whereas, the other one felt 1.48g's. This is quite a substantial decrease realized by increasing the psi

TABLE 3.--Response/input ratios for 0.5g and 1.0 psi.

Freq.	0.5 psi				1.0 psi			
	(1-1)		(4-4)		(1-1)		(4-4)	
	g Level (Resp.)	Ratio	g Level (Resp.)	Ratio	g Level (Resp.)	Ratio	g Level (Resp.)	Ratio
2	0.54	1.09	0.56	1.12	0.54	1.08	0.58	1.16
4	0.54	1.09	0.65	1.31	0.54	1.08	0.64	1.28
6	0.55	1.11	0.88	1.76	0.54	1.08	0.88	1.76
8	0.55	1.11	1.50	3.00	0.54	1.08	1.84	3.69
9			2.40	4.80			2.40	4.80
10	0.57	1.14	2.24	4.48	0.64	1.28	2.16	4.32
12	0.65	1.30	2.04	4.08	0.71	1.42	1.60	3.20
14	0.66	1.32	1.53	3.06	0.73	1.46	1.20	2.40
16	0.69	1.38	1.12	2.24	0.74	1.48	1.00	2.00
18	0.80	1.60	0.80	1.60	0.78	1.56	0.84	1.68
20	0.89	1.79	0.64	1.28	0,80	1.60	0.77	1.54
22	0.92	1.84	0.72	1.44	1.04	2.08	0.72	1.44
24	1.02	2.04	0.74	1.48	1.08	2.16	0.84	1.68
26	1.10	2.20	0.76	1.52	1.12	2.24	0.88	1.76
28	1.11	2.22	1.20	2.40	1.08	2.16	0.80	1.60
30	1.48	2.96	1.16	2.32	1.06	2.12	0.70	1.40
32	1.40	2.80	1.04	2.08	1.04	2.08	0.56	1.12
34	1.34	2.68	0.92	1.84	0.94	1.88	0.48	0.96
36	1.22	2.44	0.76	1.52	0.86	1.72	0.27	0.54
38	1.20	2.40	0.74	1.49	0.80	1.60	0.24	0.48

*The underlines values are the peak ratios for each individual stacking arrangement.

loading. Also, the number of Hz at which the peaks occurred were reduced a little by increasing the cushion-loading. This conforms to the formula, $f_n = 1/2\pi\sqrt{KG/W}$, since a smaller K value will be obtained with the increased psi loading, and this will yield a smaller natural frequency, which was the case.

The second test of this section shows some very important results. The values for the (4-4) setup with 1 psi can be found in Table 3. The values for the (4-4) setup with a 0.5 psi loading are also in Table 3. Since these systems have more than one product, two peaks were again found over the tested frequency range. The first things to look at are the initial peaks for each data set.

Table 3 shows that the peak for this test at 1 psi loading had a ratio of 4.8 and occurs at 9 cycles per second. The other (4-4) setup also has a peak of 4.8 at 9 cycles per second. Since these two values are the same, and the only difference in the systems is the cushioning of the monitored products, it may be stated that the first peaks are undoubtedly caused by the other packages. This is the case because the change in psi loading in the top package did not affect the ratios or frequencies at which they occurred.

The most interesting results of this test occurred at the second peaks for each set of data. For the (4-4) setup with a 1 psi loading, the peak ratio is 1.76 at 26

Hz. The other (4-4) system has a peak ratio of 2.4 and occurs at 28 Hz. Again the number of Hz are decreased because of the increased psi loading on the cushioning on the monitored package, which decreases the K value, causing a lower natural frequency. Converting the ratios to g's of response you can see that the (4-4) setup with 1 psi loading only feels 0.88g as opposed to 1.2 g's for the other one. This adds further support to the theory that cushioning influences the transmissibility.

However, despite these findings, a more important point may be brought out in this test. Earlier it was theorized that these second peaks were a repetition of the single product peak value, and that the first peak was due to the cushioning action of the entire product system. Since the first peak for each were the same, it may be stated that these peaks were due to the bottom three packages acting as a spring. The theory about the second peaks also seems probable, since a change in the cushioning here did change this reading, even though it did not change the first peaks.

Also, the second peak did occur at a number of Hz near that of the single product system. Therefore it seems logical to say that indeed this second peak is a slightly attenuated version of the single peak ratio.

CHAPTER VIII

CONCLUSIONS

Overall, the results obtained from the study were very interesting and enlightening. A few assumptions that were made seemed to be reassured. In particular the assumption that the peak of the (1-1) system is repeated for all other systems seems to be almost certain. Also, the assumption that frequencies outside the normal transportation frequency range cause very little damage from vibration was strongly supported by the testing in section three of the study.

In addition to these assumptions, a number of trends were observed over the entire test. Such things as the changing of the position in the stack, the g level, and cushioning did show that they did affect the results in a patterned way in many cases.

In light of these tests, a few recommendations for future testing may be helpful. First, the frequency range of the 0 to 30 Hz should receive the brunt of the future testing. Second, the g level of inputs should be raised and lowered beyond those values used to see if the transmissibility ratios do in fact level off at both ends of the g level inputs. A third possibility might be to vary the weights of the packages. Here, a uniform set of packages that weigh half as much as the ones used for this study may be tested. Also, a stack of products that are of different weights may yeild interesting results.

As a final recommendation, some test should be devised to find the definite trend followed by the magnification of the second peaks.

In conclusion the test results were very satisfying and will be helpful as a reference for future testing. Also, before any definite, concrete statements are made concerning this study, more tests should be run to verify the results.

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