METRIC MULTIDIMENSIONAL SCALING AND COMMUNICATION: THEORY AND IMPLEMENTATION

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ABSTRACT

METRIC MULTIDIMENSIONAL SCALING AND COMMUNICATION:
THEORY AND IMPLEMENTATION

By

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This investigation attempts to examine the historical and methodological roots of multidimensional scaling in a measurement-theoretical framework and propose areas and methods for adoption in communication studies. Specifically, this work discusses scaling rigor, dimensionality, and isomorphism as criteria for the comparison of measurement techniques. The various contributions to the evolution of multidimensional scaling are examined with regard to these criteria and current problems identified. The thesis introduces metric multidimensional scaling as a response to these problems and argues for its application to longitudinal and aggregation situations.

Discussion of the relative conceptual measurement applications of ordinal and ratio scaling is presented. Four levels of comparison are generated from this discussion in conjunction with an examination of unidimensionality and multidimensionality. These levels are contrasted on the basis of isomorphism between data and numbering systems.

Using the scaling discussion as a framework for comparison, the historical developments of mathematical transformation, factor analysis, and multidimensional scaling are traced. Multidimensional scaling is shown to draw on the mathematics of astronomy and the theory of psychophysics, relying heavily on the contributions of Pearson, Garnett, and Hotelling and the seminal work of Richardson, Gulliksen, and Torgerson.

Torgerson's model is contrasted with the later nonmetric work of Shepard and others.

From these theoretical and historical foundations, metric multidimensional scaling is discussed in depth. The problems of judgement unreliability, violations of linearity, and unknown dimensionality are shown to be overcome by this methodological approach. Further, the use of this technique to measure longitudinal, process variables and to examine conceptual relationships as a function of communicative interaction is developed. The mathematical and theoretical components of the metric approach are detailed utilizing examples drawn from on-going political communication research. Implications for further research are discussed.

The Galileo computer package which supplies the necessary software to implement and facilitate the use of metric multidimensional scaling is appended.

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Chapter 1

Introduction

Powerful and compelling new techniques of data analysis have been and are being developed in the varied branches of social science; while taking many forms, they are intended to meet a common goal: to better explain, predict, and control the exigencies of human behavior. In the field of communication study it is the goal of the methodologist to find techniques that provide both accuracy and parsimony for exploring the dynamic, transactional processes that comprise human interaction. Among the most exciting, and potentially most valuable, of these is the technique of metric multidimensional scaling.

It is the purpose of this thesis, to (a) examine, in general, the historical development of multidimensional techniques, both theoretically and mathematically, and (b) introduce Galileo, a metric multidimensional scaling algorithm, and show its advantages over more well-known techniques, particularly for longitudinal analysis of communication processes.

Scaling Theory: A Conceptualization

It is the object of measurement to classify and compare observations in a meaningful way such that the representational measure and its transformations are indicative of those observations and their changes in reality. Our reality consists of constructs (such as, people, attitudes, relationships, and beliefs). These constructs are referred to by Torgerson (1958) as <u>systems</u>, the "things" which make up our conceptual universe. However, it is not the system which we measure, rather the properties of a system comprise the observable aspects of characteristics of that system which are present in the empirical universe.

The act of measurement is one of assigning a numerical set to correspond with the properties of a system. The rigor of this process is expressed in the attempt to attain an isomorphism between properties and the systems which they describe. As many texts indicate (e.g., Carnap, 1959; Coombs, 1964; McNemar, 1969; Blalock, 1972) we can describe <u>levels</u> of scaling which express this degree of isomorphism, as it is achieved by a particular measuring technique or device, as a function of ordinality, linearity, and origin; the closer a scale conforms to these criteria, the greater the likelihood it will achieve a one-to-one correspondence between the properties of a system and the numbers used to represent those properties.

Perhaps the most well-known expression of the relative isomorphism of scaling levels can be seen in the organization of transformation groups by Stevens (1951). These serve to reduce the arbitrariness of selecting a numerical system to represent a property by the quality of transformation which may be imposed upon that numerical system:

<u>Ordinal scaling</u>. If objects can be ordered only on the basis of the <u>relative</u> position or magnitude of some property, they then lack the distinctiveness desirable for achieving sophisticated mathematical transformation. Since the numbers are assigned such that they are order-preserving, the ordinal scale is said to determine relationships to within a monotonic-increasing transformation (Figure 1a). Such "scales" are the minimum expression of relationship between two or more variables

(excluding the notion of nominal scales which are not truly measures of properties but rather categorical representations for the classification of the systems themselves).

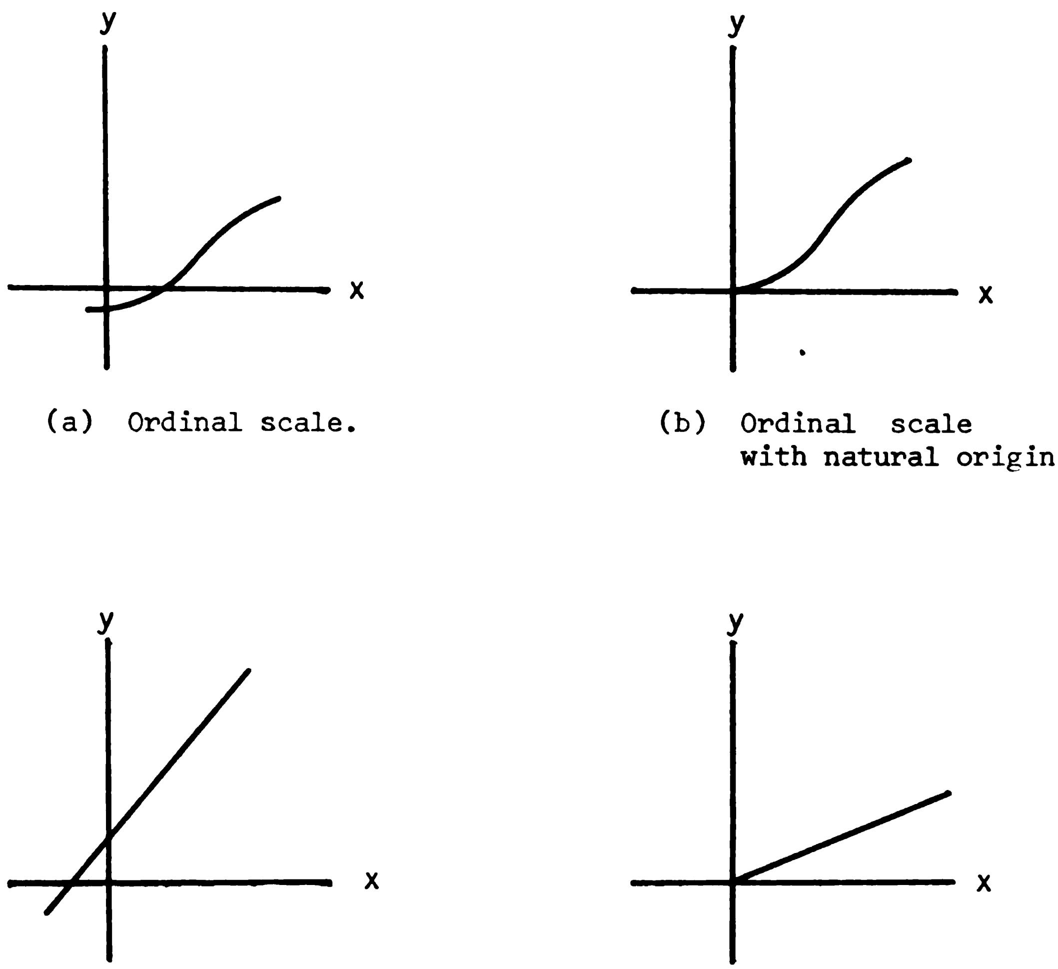
Ordinal scaling with a natural origin. If, in addition to the monotonic transformation described above, the scale has a unique point of origin, the ordered relationship of two or more systems can be more accurately specified. This allows us to indicate from the scale value of zero that there is an absence of any amount of the property being observed. Thus, according to Stevens' scheme, this type of scale can only generate those transformations which leave the origin unchanged (Figure 1b).

Interval scaling. If the scale lacks an absolute point of origin but the numerical differences reflect equal intervals between finite amounts of a property, then the relationship between two or more systems on the basis of that property can be specified exactly. However, this does not yet describe a ratio scale since the absolute magnitude cannot be expressed. As is indicated by Stevens, this does yield a scale that is not affected by a transformation of the form:

$$y = ax + b,$$

where a is any positive real number and b is any real number. This simply describes the basic form of a linear transformation. As an example, the difference, or interval, of two units on the low end of a scale will be equal to the interval of two units on the high end of the scale, and the slope of a line defined by these units will remain constant (Figure 1c).

Ratio scaling. A ratio scale is any scale which meets the criteria for an interval scale, and in addition, satisfies the linear transformation such that b=0, or in other words, has a natural origin (Figure 1d). Such a scale has a unique quality such that any set of properties may be expressed as a ratio of the magnitude of one to another, and the absence of any magnitude of a property is represented by a value of zero.



(a) Ordinal scale.

(b) Ordinal scale with natural origin.

(c) Interval scale.

(d) Ratio scale.

Figure 1. Examples of transformations for each of the four types of scales satisfying the Stevens scheme. If the abscissa x is the linear continuum of possible observations, the values on the ordinate y will fulfill the requirements of transformation to the scale indicated. (Adopted from Torgerson, 1958).

The most important issue of scaling is thus one of achieving an isomorphic correspondence between the actual properties of systems and the numerical structure used to represent those properties. It is obvious that the more rigid the criteria for transforming a scale the better that scale is able to represent the state of a system with regard to any single property. Two major consequences of seeking isomorphism exist. First, the closer measurement of the properties of a system comes to achieving one-to-one correspondence with the "reality" of that system the more adequately we can describe the structure of the system. Second, the selection of numerical representations that are more highly defined (as are those used to underlie higher level scales)* will allow us to perform more mathematically rigorous operations, upon the number set, which are equivalent to operations which might be performed upon the properties of the system under study.

Toward Isomorphism: Multidimensional Scaling

Historically, the majority of measuring devices have sought simplicity, at the expense of isomorphism, in the form of unitary scales. During

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*It should be noted that with highly defined (linear) scales the transformation process of subtraction will always yield a ratio scale of change scores (Woelfel, personal discussion). When performed upon interval and ratio scales this is consistent with our desire to generate scales capable of measuring processional changes in attitudes as functions of communicative behavior. It is also notable that this transformation to ratio scaling is readily observable in, and probably derived from, the physics of motion which uses the principle of relativity to deal with constant changes occuring in the universe (Einstein, 1923; Hempel, 1952). Further, this method of deriving ratio scales is not without foundation in the behavioral sciences; the various unfolding techniques use dissimilarities as a second generation transformation to arrive at better quantifiable measures (Coombs, 1964; Coombs and Kao, 1954, 1955).

the development of psychophysical measurement, it became apparent that judgement responses could not always be arrayed on a single undimensional scale (Thurstone, 1927; Richardson, 1938) and that the measurement excess must be attributed either to error or multiple influences. Klingberg (1941) demonstrated that by interpreting the results as a multidimensional configuration rather than as a unitary judgement factor error was reduced considerably and that distinct bases (dimensions) for judgements could be identified.

From these early presentations of multidimensional scaling the notion of representing judgements of the relationships between stimulus-objects as distances made it possible to conceive of conceptual structure as analogous to Euclidean real space. In this format, dimensions, angles, and distances could be used to express data relationships more directly and more accurately than by representing the single largest component of each interrelationship solely. A scale of this nature would, additionally, be capable of measuring very accurately changes in relationships which do not appear to occur on the single unitary factor.

Currently, the majority of behavior measurement techniques subsumed under the rubric "multidimensional scaling" are static designs, structurally oriented. They are neither parsimonious with the intent of measuring "distance," failing to overcome difficulties in meeting the assumptions of measuring physical distance, nor process-oriented, failing to provide a sound scale against which to measure change. It is precisely the static nature which many of these design have that Roger Shepard describes, in the introduction to his major work on multidimensional scaling (Shepard, Romney, Nerlove, 1972:1) as rationale for these methods of analysis:

> The unifying purpose that these techniques share, despite their diversity, is the double one (a) of somehow getting hold of whatever pattern or structure may otherwise lie hidden in a matrix of empirical data and (b) of representing that structure in a form that is much more accessible to the human eye--namely, as a geometrical model or picture. The objects under study (whether these be stimuli, persons, or nations) are represented by points in the spatial model in such a way that the significant features of the data about these objects are revealed in the geometrical relations among the points.

To find a methodological technique satisfactory to the process demands of communication research, quantitatively more accurate than the present techniques of communication research, and sufficiently elegant to stand alone or easily enmesh with existing techniques requires that the technique meet, or attempt to meet, the following criteria:

(a) it should be of the highest possible level of scaling, utilizing wholly ratio scales with natural origins,

(b) it should be able to measure <u>changes</u> in the relationship of the variables being scaled, with precision,

(c) it should be able to clearly and simply represent those relationships to the researcher while maintaining a format readily transformable for less obvious means of analysis,

(d) it should operate on the bases of theoretical and mathematical assumptions which do not <u>force</u> the loss of information through transformation, and

(e) it should achieve isomorphism between the properties being measured and the characteristics of the system used to describe those properties such that transformations and operations performed upon the descriptive

> system are equivalent to transformations and operations which might be performed on the properties of the real system being represented.

These are neither simple nor easily attainable criteria. The non-metric or "Shepard-Kruskal" approach (Kruskal, 1964b), while not meeting these requirements exactly, does come closer than any existing technique in standard usage among communication researchers. Examining the foundations of the non-metric approach however yields a highly practical scaling model for communication which does satisfy the criteria: metric multidimensional scaling. Metric multidimensional scaling is a technique for the construction of spatial representations of interrelationships from ratio judgement data. Judgements of dissimilarity (distance) between concepts are arrayed to depict the structure of all possible concepts, simultaneously, in a configuration analogous to Euclidean real space. This allows us to examine and describe structure, represent change, and operate on the model in ways parallel to operations in reality without distortion of our original data measurements.

The primary importance of multidimensional scaling (MDS) to communication research is that it can provide an analytic tool for measuring and interpreting processes and change oriented hypotheses. It has been suggested (Berlo, 1969; Smith, 1966; Dance, 1970; Mortensen, 1972; Miller and Steinberg, 1974) that a major component of the concept of communication is that it is a dynamic, on-going process. As such, it is necessary to seek ways to examine communication as a process rather than as a series of discrete events. Metric multidimensional scaling, by reliance on ratio scaling and the ability to generate a latent structure that is analogous to Euclidean continuous space, allows us to manipulate processes and observe change with a high degree of accuracy.

It will be useful to the understanding of the relationship of the process variables found in communication to examine the differences that accrue from the application of unidimensional and multidimensional scales conjointly with the differences between the use of ordinal level and interval (ratio) level scaling.

Unidimensionality Versus Multidimensionality

The purpose of using unidimensional scaling is to measure single attributes or properties of a system. The unidimensional scale achieves correspondence by establishing a continuum of points on which the magnitude of the attribute is represented by a point in the continuum (Russell, 1938). The primary test for unidimensional continua is the order proposition of transitivity. Meeting Huntington's postulates is, thus, the first necessary condition for identifying the existence of a single underlying dimension in a set of measurements. Those postulates are (Stevens, 1951:14):

1. If $a \neq b$, then either $a < b$ or $b < a$.
2. If $a < b$, then $a \neq b$.
3. If $a < b$, and $b < c$, then $a < c$.

However, a second condition must be met to sufficiently satisfy the condition of unidimensionality. Given a set of points, P, the selection of any three of those points, P_j, P_k, and P_l, where it is known that P_k lies between P_j and P_l, must yield the equation:

$$d_{jk} + d_{kl} = d_{jl}.$$

If any three points of the continuum fail to satisfy this equation, (Thurstone, 1927) the dimensionality is of a higher order (or is imaginary) and we would posit two or more properties influencing the measurement of the system.*

The limitation of a unidimensional scale is, by definition, its ability to represent only one property or attribute. This limitation is manifested in two ways: (a) if our purpose is to measure a unidimensional attribute and our results fail to satisfy the distance equation, then we must choose between the possible explanation that measurement error exists or that multiple properties were measured; (b) if our purpose is to measure all properties of a system and identify those which are salient to interpretation, then we must construct numerous unidimensional scales, all of which face the first limitation, and any of which may be inconsistent with the others such that they are incomparable directly.

The primary advantage of constructing a metric multidimensional scale is that it can overcome these problems by directly incorporating

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*The concept of distance is introduced at this point as a criterion for examining the rigor of scaling types. To satisfy this criterion we should limit our discussion to levels of scaling which have a known distance function (e.g., interval and ratio scales). Since much of social and psychological measurement is based upon ordinal "scales" and since it is our purpose to examine the limitations of this, so-called, method of scaling, it is of some utility to note that what we previously (p. 2) referred to here as ordinal scales do not exist, as such. While the property of order may exist for a set of magnitudes, ordering itself does not constitute measurement because it lacks the ability to be represented spatially; it lacks a distance component. More correctly we should refer to ordinal scales as ordinal relationships. We should distinguish, separately, ordinal scales which are in actuality interval scales which possess the quality of order while assuming that distances exist; this distance is, however, unknown and no attempt has been made to measure it. For a more lengthy philosophical discussion of this principle and an underlying rationale, the reader may examine Descartes (1685), Kant (1755), Newton (1686), Mach (1893), and others.

them into the scale design. Since it is intended to measure many attributes at once, the first problem of measuring unitary properties becomes the purpose of multidimensional scaling. The second problem does not arise because, by constructing a single scale, attributes are treated as directly comparable.

For communication variables, or other process-oriented variables, the problem of dimensionality is actually a problem of complexity; it is obvious that the ability to compare many attributes simultaneously increases our predictive or explanatory capabilities greatly. A research design can be simplified considerably if changes in many variables can be measured with a single multidimensional instrument rather than with many lesser scales.

A greater problem for communication research, which metric multidimensional scaling will be shown to overcome, is the effect of utilizing ordinal, distanceless, measures as opposed to interval/ratio measures. Because the ordinal scale represents only the order of properties and not their magnitude it can only reveal transpositions in order. It is easily seen that much change can occur without disturbing the order of a set of stimulus-objects; without being able to measure that change, however, comparison and prediction are impossible. Torgerson (1958:31) underscores this view of measurement:

> The interval and ratio scales are by far the most useful measurement scales employed in science. As a matter of fact, the term measurement is often restricted to these kinds of scales, both in the ordinary use of the term and in the more advanced discussions of the topic. (See, for example,

Carnap, 1959, p. 9; Hempel, 1952, p. 58; and Campbell in Ferguson, 1940, p. 347.) It might further be noted, that, in discussion of the nature of measurement, the distinction between fundamental and derived measurement is also commonly made only in terms of interval and ratio scales.

An example, drawn from Myron Wish's "Tea-Tasting" experiment (Carroll, 1972), should clarify this discussion. We may imagine several different cups of tea, as stimuli to be judged on the basis of first one and then many properties, and scaled using different levels of sophistication.

In the first case, we can imagine four cups of tea which vary in sweetness according to the amount of sugar present. If we present the subjects with pairs of teacups and ask them to judge which cup is sweeter we will arrive at an order list from least to most sweet such as:

1. cup b
2. cup c
3. cup a
4. cup d.

If we know that the stimuli are cup a = 2 teaspoons of sugar, cup b = 0 teaspoon, cup c = 1 teaspoon, and cup d = 3 teaspoons, then the result shown above would be considered correct within a monotonic transformation. However, we would not be able to recognize, in subsequent presentations of the stimuli, whether or not changes in the sugar levels had actually produced changes in judgements unless the order of the list was transposed.

Again, imagine that we present these four stimulus-objects. However, this time we use an interval or ratio scaling technique such as dissimilarity judgements (Woelfel, 1972) or an ordered metric (Coombs, 1964:80-4) so that values reported will be proportional on a scale of equal intervals with a real distance component. This information might be represented:*

```
cup        cup        cup        cup
 b          c          a          d
 | -------- | -------- | -------- | ---------------
 0          u         2u         3u
```

where u represents a known but arbitrary unit of measurement.

A longitudinal aspect of the experiment becomes available to us when the component of distance is added. As we have shown, unless the order of the stimulus-objects becomes transposed when changes in the system are introduced, the change can not be measured by an ordinal technique. However, with our ability to observe exact differences in distance along the scale from times, t_1 to t_2, longitudinal measurement becomes a meaningful activity allowing us to add a class of process-oriented variables to our experiments.

To represent this, suppose that we add one teaspoon of sugar to cup c, one half teaspoon to cup a, and no sugar to either cup b or cup d. Our scale (assuming the judgement values were proportional to actual sweetness levels, an empirical question) would now appear like this:

```
cup                  cup    cup    cup
 b                    c      a      d
 | ------------------ | ---- | ---- | ----------------.
 0
```

*In addition to the representation of distance we are now able to visualize the order relationship expressed in our earlier "measurement."

If the original distance from cup b to cup c had been 10 scale units, the judged change in sweetness would yield different results for the two types of scales discussed. Obviously, a significant event with respect to the overall structure of the stimulus-objects and the relationship of the changed stimuli has occurred; however, only the interval scale takes into account enough information to explain that event, or even report that the event had occurred (figure 2).

	Interval Scale			Ordinal Scale		
	t_1	t_2	ΔSweetness	t_1	t_2	ΔSweetness
cup a	20	25	+5	3	3	No change
cup b	0	0	0	1	1	No change
cup c	10	20	+10	2	2	No change
cup d	30	30	0	4	4	No change

Figure 2. Changes in sweetness represented on two scales.

This example may now be expanded to the multidimensional case. We have sought to represent the property of sweetness as a measure of the difference between cups of tea, yet this difference might be portrayed as well by measuring the properties of temperature, strength, color, or age. Logically, if all of these and, potentially, other attributes can be used as measures of the difference between stimuli then some combination of measures should increase the accuracy of description.

One way in which we might achieve greater accuracy would be to list all the possible attributes and create a measuring instrument for each; numerous problems are incurred with this approach however, not the least of which is the fact that such a procedure requires that all attributes along which the teas may differ must be known to the investigator in advance. A second method would be to measure the stimulus-objects by a direct technique, such as having the subjects judge dissimilarities without regard to any specified attribute. In this way, it is necessary for the judgement to be made on the basis of those attributes which are salient to the judge. This is the thrust of a multidimensional approach to scaling attributes.

The procedure for generating a multidimensional scale (which will be discussed in more detail in a later chapter) entails the derivation of a judgement of relationship for all possible pairs of stimulus-objects and the transformation of the judgement matrix into a matrix of loadings, or projections, on orthogonal axes of a real Euclidean space.* From these projections we may identify the axes (or dimensions), examine structure and, if the judgements are metric, use the scale to observe change over time.**

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*It may also entail projections in imaginary space. Hypotheses about the imaginary component will not be dealt with in this work. For a mathematical treatment of this problem, see Wilkinson (1961).

**Other models exist for the interpretation of multidimensional data, such as the "city-block" (Householder and Landahl, 1945; Atteneave, 1950) and hierarchical cluster analysis (Johnson, 1967), however both rely on measures of distance and the "real-world" conception of Euclidean space for their particular representations. As such, the Euclidean model supplies both a simpler and more fundamental system for the examination of concept relationships.

Returning to the tea experiment, we may examine measurement differences for the multidimensional case (Carroll, 1972):

> Twenty-five hypothetical cups of tea were described in terms of pairs of description words or phrases. The first set of descriptions referred to the temperature of the cup of tea. Five temperature related words of phrases--ice cold, cold, lukewarm, hot, steaming hot--were used. The second set of descriptions specified the amount of sugar: no sugar, 1/2 teaspoon, 1 teaspoon, 2 teaspoons, 4 teaspoons. Subjects were shown a standard size styrofoam cup, in which they were to imagine tea of "moderate strength" with no cream or lemon. All 25 possible combinations of the two sets of descriptions were used to define the basic (verbally described) stimuli. The 300 possible pairs of stimuli were generated in a random order to form the basic questionnaire. There were a total of 12 subjects, divided randomly into two sets of subjects which responded to the items in opposite orders. Each of the 12 subjects was asked to give . . . a rating of dissimilarity (called "degree of difference") of the pair on a scale from 0 (for an indistinguishable pair) to 9 (for an extremely dissimilar pair).

The ordinal data from this procedure were then analyzed according to one of the major multidimensional algorithms:*

> INDSCAL (individual differences scaling) was applied to the dissimilarities, producing the group stimulus space shown in [Figure 3]. . . . Note that the basic lattice structure embodied in the factorial design used to generate the stimuli is quite clearly in evidence (though a bit distorted) in the stimulus space, and, furthermore, that the "sides" of the lattice are very nicely parallel to the two coordinate axes. These axes are exactly as they came out of the analysis; no rotation whatsoever has been done.

From this example, we can distinguish two levels of scaling rigor, ordinal and interval, for the multidimensional measurement approach. Ordinal, or nonmetric, multidimensional scales are generated by transformations based on an "unknown distance function" (Shepard, 1962a). That is, by using a fixed monotonic function based on identifiable relationships in ordinal data such as those generated by dominance measures (Coombs, 1964; Carroll, 1972) or profile measures

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* INDSCAL, developed by Carroll and Chang (1970), is a multidimensional scaling program for deriving the product matrix by the Eckart-Young rank reduction algorithm (Eckart and Young, 1936). Additionally, it generated weighting factors and a canonical decomposition of data to produce a space which accounts for individual differences in reporting raw judgements. INDSCAL is limited by the normalization to unity of both scalar products and solution matrices which eliminates the original distances reported.

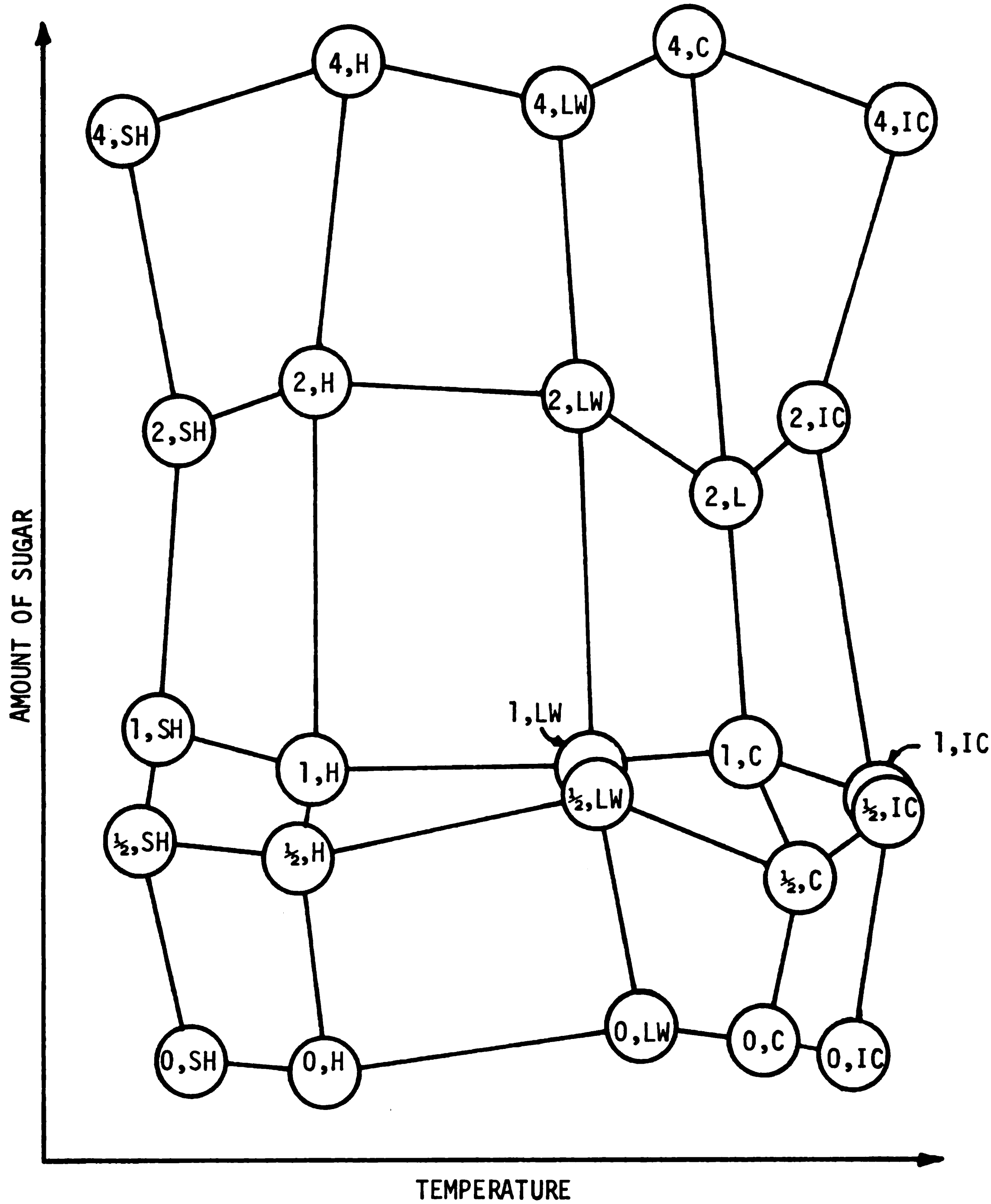

Figure 3. The unrotated two-dimensional solution (n=12) of the Wish data using INDSCAL on dissimilarities data. In this figure the coding: 0, 1/2, 1, 2, 4 refers to the number of teaspoons of sugar specified and for temperature: IC = ice cold, C = cold, LW = lukewarm, H = hot, and SH = steaming hot. (Adopted from Carroll, 1972.)

(Shepard and Carroll, 1966) rather than actual distance estimates, a configuration approximating the latent structure of the data can be found. Metric multidimensional scales are generated directly from unidimensional judgements made with interval or ratio scales. Measures for this type of scale are usually in the form of proxemities data (Torgerson, 1951, 1952; Woelfel, 1972); however, other forms such as frequency scores and individual differences may be found.*

The metric procedure essentially asks the subject to make judgements about how each stimulus-object differs from all other stimulus-objects. Since the attributes on which the stimuli are to be judged are not specified, the individual is able to use those most important for distinguishing each pair. This results in a dissimilarities matrix

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*Some controversy exists as to which component of the technique, the original data or the multidimensional computational algorithms, constitutes the basis for discriminating metric and nonmetric scales. Traditionally, the computation and transformation processes have been used to judge the quality of the scale (Shepard, 1962a, 1962b; Kruskal, 1964, Guttman, 1968); those requiring an iterative procedure to adjust discrepancies are considered nonmetric, those using a direct derivation of the latent structure of a numerical set (such that differences between numbers in the set are maintained within a linear transformation) are considered metric. This assumes the ability to treat the data by rules of order and transformation (Hempel, 1952) which apply to more sophisticated numbering systems than we have actually utilized with our original measurement procedure. On this basis it would seem more natural to distinguish multidimensional scaling levels on the basis of data gathered. Obviously, the computational criterion is then also met if the appropriate transformation is applied to that data. A less rigid transformation would yield a nonmetric solution for interval type data, due to the standardization necessary to compute a nonmetric transformation, while the use of a metric algorithm for ordinal data would only occur under strong theoretical scrutiny and with rigid restriction on the approximation of interval numbers.

in which all of the information about the data interrelationships is present but much of it (i.e. the attributed used to make judgements) is in a latent form. Generating the multidimensional scale, then, has two functions: (a) it attempts to derive the most parsimonious description (i.e., the smallest set of numbers to represent all of the information) and (b) it allows us to make explicit some of the information that is latent in the original dissimilarities matrix.

From the tea experiment it can be seen that a similar result for ordinal and interval data will be produced. Had the multidimensional transformations been performed upon a dissimilarities matrix derived from an interval scaling procedure, the result would have been a configuration based upon reported distances between the concepts scaled rather than a configuration based upon distances estimated from correlational strength.

Figure 3 is a representation of the data in two dimensions; the configuration closely resembles an array of the stimulus-objects on the basis of actual physical properties. It is expected that the metric judgement procedure described would have further increased parsimony between the stimulus set and the final coordinate values.

In either case, the quality of the data representation is improved to reflect the overall structure of the relationship rather than the order and/or distance of any single dimension of the relationship. However, the multidimensional ordinal <u>configuration</u>, like undimensional ordering is not comparable over time. The structure is an artifact of a monotonic transformation which provides only one of many possible interpretations.

The problem of representing change over time multidimensionally is that it requires both angle and distance. Since most nonmetric procedures rely on standardization of the data, they remove the distances reported and substitute correlations. Correlation between any two concept values is r_{ij}; further, $r_{ij} = \cos\alpha_{ij}$, where α is the angle between two vectors representing the concepts i and j. Thus, the changes occurring over time may result in higher or lower correlations which will be represented as changes in angle (direction); however, changes in magnitude will not be represented at all.

Figure 4 represents the same data set at two different points in time for which the magnitudes are changed. In both cases the angles remain constant and the correlation matrix, R_1 is:

$$R_1 = \begin{bmatrix} 1.0 & 1.0 & 0.0 & -1.0 & 0.0 \\ . & 1.0 & 0.0 & -1.0 & 0.0 \\ . & & 1.0 & 0.0 & -1.0 \\ . & & & 1.0 & 0.0 \\ 0.0 & . & . & . & 1.0 \end{bmatrix} .$$

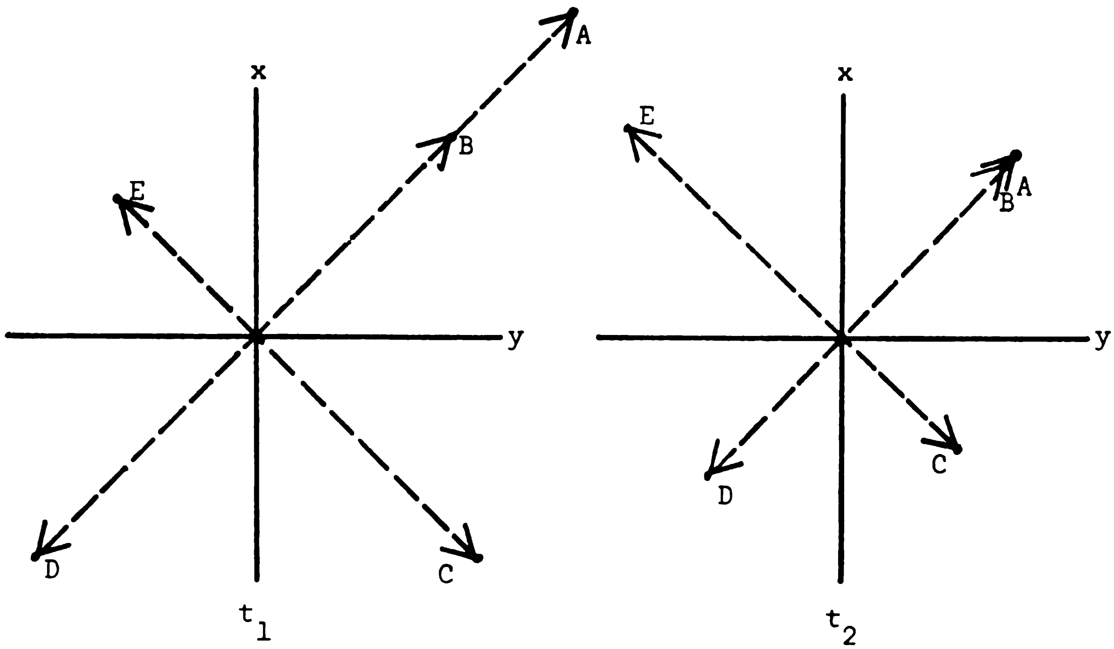

Figure 4. A multidimensional configuration at two points in time (magnitudes changed, correlations remain constant).

An interval configuration has both angle and magnitude which may be compared directly to determine longitudinal differences. It is then possible to introduce measures of change (such as velocity, accelaration, and displacement) and posit change agents in mathematical terms (force, inertia, and momenta). It is the conceptual analogy to motion that we will refer to as the behavioral characteristics of process.

Objectives of the Thesis

Torgerson seeks to remind us that science and measurement can be conceived simply (1958:2):

> Science can be thought of as consisting of theory on the one hand and data (empirical evidence) on the other. The interplay between the two makes science a going concern. The theoretical side consists of constructs and their relations to one another. The empirical side consists of the basic observable data. Connecting the two are rules of correspondence which serve the purpose of defining or partially defining certain theoretical constructs in terms of observable data.

It is the purpose of this thesis to examine this view of science, and specifically, to apply the discussion of measurement toward the development of a more rigorous application of the principle of isomorphism in science.

In the following chapters, the notion of high level scaling in the behavioral sciences will be considered and an application to communication science developed. The thesis will make two main arguments: 1) that the complex and multi-faceted nature of communication phenomena requires a multidimensional approach to measurement, and 2) that the processual character of communication phenomena requires interval or higher level scaling. Finally it will contend that these two requirements can be met by metric multidimensional scaling techniques. One such system, Galileo, along with complete computer software, will be presented as an example.

Chapter II

Historical Development of Multidimensional Scaling

The development of multidimensional scaling appears to be culminating in the form of a broad and powerful technique for measuring self-conception and influences upon the self-conception on a micro level, and cultural processes on a larger scale. It is a technique which specifically for communication study, holds great potential for understanding the effects of communicative acts, and their subsumed components, on cognitive aspects of the act. Historically, many attempts have been made to achieve this goal, and these attempts have contributed directly, and indirectly, to the development of multidimensional scaling.

It will be useful to a more complete understanding and appreciation of how MDS can provide such a powerful interpretation of data to examine the development of the mathematical and theoretical components involved. Further, it will be briefly placed into the context of the more fundamental sciences from which it derives.

Pre-Psychophysical Influences on Multidimensional Scaling

Multidimensional scaling as a psychological measuring technique can be attributed primarily to the work of Torgerson (1951, 1952, 1958). It also draws heavily on the theoretical construction of Gulliksen (1946) and Thurstone (1927a), and the mathematic contributions of Hotelling (1933), Young and Householder (1938), and Garnett (1919a). However, it is useful to examine the more basic scientific roots of the technique and place it into the perspective of science as a whole before considering the more technical aspects of the multidimensional "model".

The mathematical history of multidimensional scaling is derived, appropriately, from the mathematics of astronomy and specifically celestial mechanics. Since MDS attempts to treat the self-concept or culture as an analogy to Euclidean real space, the structure and generation of such space may be examined with regard to the context from which it developed.

The notion of "space" was originally treated, philosophically, by Plato, and developed conceptually by Aristotle, as the universe of objects and abstractions (intelligence) in which man functions. As a pure scientific construct, Euclid, in the third century B.C., proposed space as the context for objects in relationship to one another. To express this, Euclid proposed the geometry, a formulation of mathematical rules to define physical relationships according to distance and direction (angle).

The most cogent presentation of Euclid's geometry for deducing relationships from information that is incomplete or not in its most representative form, was its application to celestial description by Aristarchus of Samos (310-230 B.C.). It was Aristarchus' contention that the universe was heliocentric, and that despite the observable motion of the sun and other bodies in the sky the earth's motion would describe a circle around the sun. To argue his model, Aristarchus used a viewpoint outside of this system, where he could "see" the earth and the other known celestial bodies as comparable globes in space. He then intersected these bodies and their shadow cones with planes, drew intersections as circles and triangles, and applied Euclid's rigid methods described in the Elements to demonstrate the relative positions and the most likely paths of movement. In addition, by treating the problem as a geometric lemma, Aristarchus established that the hypothetical structure of the spatial relationships could be derived from a known subset of the interrelationships of the celestial bodies (which would intuitively suggest a different solution).

The Greek views of science, particularly astronomy, mathematics, and philosophy, dominated Western thought until the fifteenth century and later. Among the first of the major challenges to this trend, it was Descartes' notion of separating the abstract reality of the entelechy from physical reality that provided much of the stimulus to change. In his Principia Philosophiae (1685), Descartes stated, "I will explain the results by their causes, and not the causes by their results." With this he initiated a set of theories of space and forces of motion and maintenance in space which made it possible to predict observable phenomena.

It was this theorizing by Descartes which Newton was responding to when he introduced the laws of motion and initiated the classical mechanics (Pannekoek, 1961). Both Newton and Descartes, however, had in common the notion of a Euclidean model as the reference system in which to examine and describe the forces and effects that they postulated.

From this point in scientific evolution, the disciplines of astronomy and mathematics began to develop simultaneously and mutually, in the form of celestial mechanics (Hagihara, 1970). It was also at this juncture that the science of the mind began to develop separately, first in the form of philosophy, and later in the forms of psychology and psychophysics.

To express his theories of mechanics, Newton simultaneously and independently with Leibniz also invented the first forms of differential and integral calculus; these mathematical aspects were refined by others to perpetuate the study of mechanics and developed to provide working algorithms for manipulating spatial concepts. For the later psychometric techniques of factor analysis and multidimensional scaling this meant that the relationship of data expressed by points would be interpretable by the mathematics of mechanics.

Theoretically, the notion of representing points in space was formalized by Pierre Simon de Laplace who is credited with the founding of celestial mechanics as a major function of astronomy. Laplace, with Lagrange is responsible for the validation of Newtonian mechanics at a point in time when the model was about to be abandoned (Pannekoek, 1961). In the late eighteenth century, they measured the accelaration and retardation of planetary motion, explaining perturbations and apparent inconsistencies of force as functions of distance and elliptical motion. This action gave rise to the more extensive mathematical developments which followed in the nineteenth century.

During the nineteenth century, much of the mathematics of psychometrics was developed in the context of astronomy. Following Laplace's advances and publications (1796), numerous computational methods were developed by Gauss, Jacobi, Bravias, and Seidel (Wilkinson, 1965;

Jacobi, 1846; Harmon, 1967; Pannekoek, 1961) and refined by others (notably astronomers and mathematicians such as Bessel, Encke, Olbers, Cauchy, Hansen, Leverier, Poincaré, and many others).

Several important contributions which affect the conception of multidimensional scaling were presented by:

1. C. F. Gauss. In 1804 he originated the "method of least squares" which, "by the condition that the sum total of the squares of the remaining errors shall be a minimum, the 'most probable' value of the unknown quantity is found (Pannekoek, 1961)." Geometrically and statistically, this is interpreted to mean the sum of the squared distances from the projections of points on a line when minimized yields the line of best fit to the configuration of values. It is this foundation upon which equations for factors and dimensions are based.

2. C. G. J. Jacobi. The problems inherent in interpreting matrices (Jacobi, 1846; Wilkinson, 1965) led to the development of the diagonalization method for deriving the eigen roots. This not only provided the technique for generating the latent structure for an unlimited number of points in space; it also provided a rapid convergence algorithm upon which many high speed computer routines are based.

3. A. L. Cauchy. In his work on determinants (Kowalewski, 1909), Cauchy provides a proof of the reality of roots and the determination of the number of underlying dimensions. This treatment of dimensionality, which is basically dealt with in discussion of quadratic surfaces in analytic geometry, influenced Hotelling (1933) in his development of factor analytic procedures and the restriction of interpretations

made from results of those procedures.

Near the end of the nineteenth century, pure mathematics and the mathematics of celestial mechanics began to diverge. Computational devices, in conjunction with improved technology for astronomical measurement allowed the astronomer to develop theory and concentrate less on improving and developing more accurate algorithms. Conversely, mathematics ceased to be devoted exclusively to the development of computational methods, and began to gain recognition as the language of scientific theory. Figure 5 represents these developments graphically.

Early Factor Analysis

At the same time that mathematics began to diverge from its roots in astronomy, psychology began to seek and develop more rigorous methods of measuring ability and behavior and expressing theory. Early twentieth century psychologists such as Spearman and Thomson began to draw on statistical science to express their theories probabilistically.

In 1901, Karl Pearson, a mathematician and statistician, published the paper "On Lines and Planes of Closest Fit to Systems of Points in Space." In it he presents the method of principal axes as a technique for deriving the line of best-fit through a system of points in two, three, or n dimensions. This method was significant because it allowed points to be represented by a vector from the origin of a coordinate system to some point in the space defined by the coordinate system. (Frequently the vector is represented only by its endpoint). Further, this vector is a function of the configuration itself. It is obtained

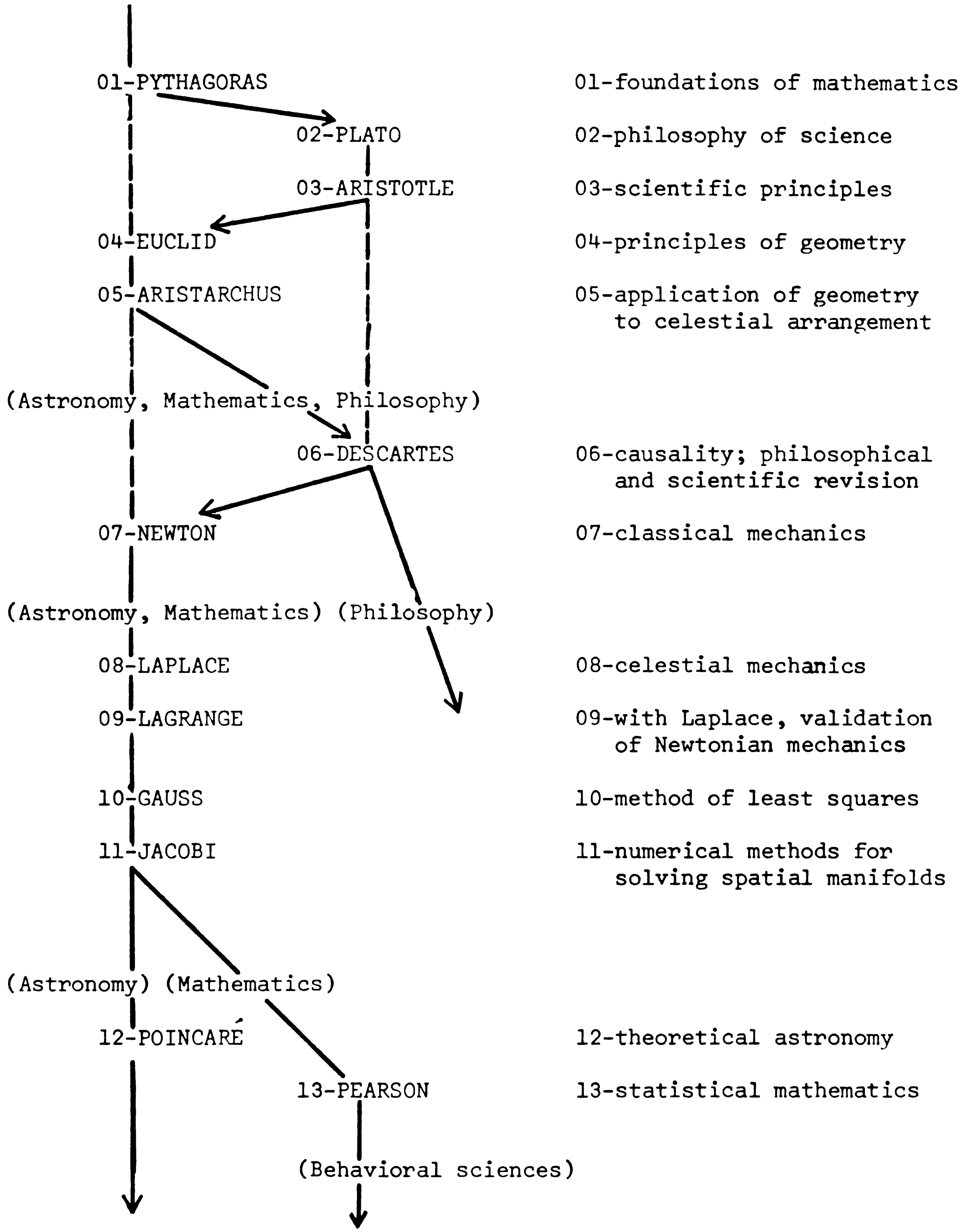

Figure 5. Chronological development of pre-factor analytic contributions to the development of multidimensional scaling.

by minimization of the sum of squares of the perpendiculars, or projections (Figure 6) of a set of points representing any data configuration.

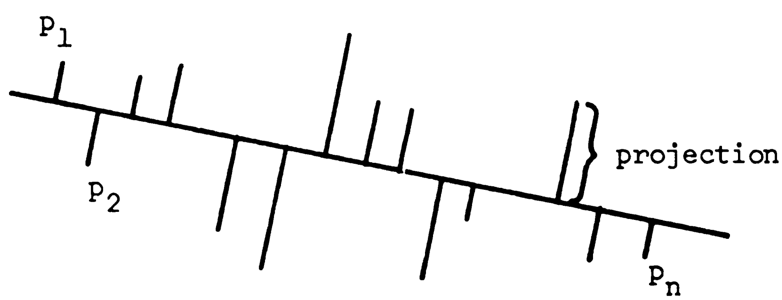

Figure 6. Line of least squares best-fit.

Earlier astronomical techniques allowed for the representation of celestial points on an arbitrarily designated coordinate system derived from Euclidean distances. By Pearson's technique, the endpoint of the vector (the least squares line of best-fit) could be placed in a coordinate system or the line itself could be designated as one axis of the coordinate system. This was useful, statistically, when the purpose of the representation was not to predict from one variable to another but to observe the interrelationships of a number of variables. By this, Pearson notes, "we observe x and y and seek a unique functional relationship between them."

Utilizing this geometric representation of data, and the means, standard deviations, and correlation coefficients used to generate it, Spearman (1904) initiated his theory of the general factor. With his theory he posited that ability was correlated with intelligence and that from the measurement of a set of abilities that a factor, or component, common to all abilities could be identified.

To test his theory, Spearman developed the "criterion of hierarchy" or method of tetrad differences. If for four tests, or all combinations of four tests, the intercorrelations could be accounted for by a single source of variation (or factor) then the general factor was identified for those tests. This was achieved when the coefficients in any combination of two columns of the intercorrelation matrix were found to be proportional; that is, the following equations were satisfied:

$$r_{13}/r_{23} = r_{14}/r_{24}$$
$$r_{12}/r_{32} = r_{14}/r_{34}$$
$$r_{12}/r_{42} = r_{13}/r_{43}.$$

Spearman found that for a considerable number of the psychophysical tests the intercorrelations satisfied the proportionality criterion and could be accounted for by the general factor. Additionally, he postulated that every test which satisfies the proportionality criterion also contains a second factor that is specific to a given test and, statistically, represents that portion of the variance that does not correlate with the other tests. Thus by Spearman's theory, a perfectly reliable test would have two components, g (the general factor) and s (the specific factor), such that $g^2 + s^2 = 1.00$. Fruchter (1954)

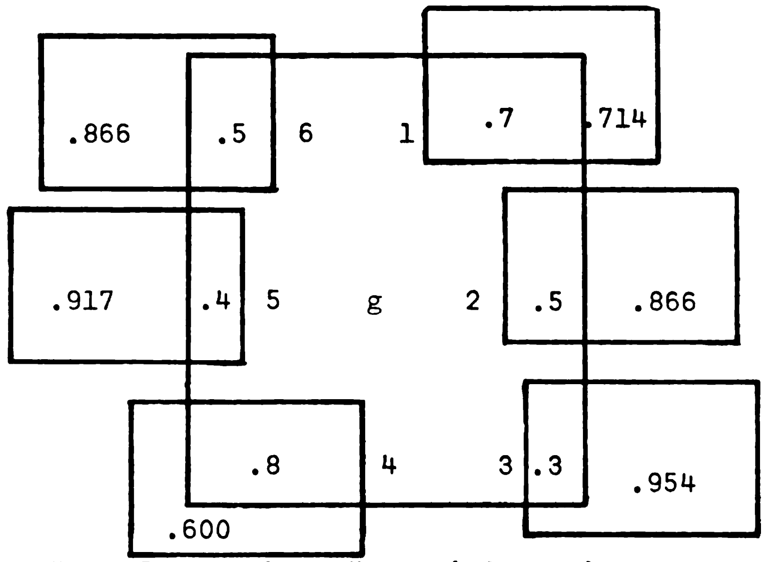

Figure 7. The "two-factor theory" applied to six tests. The values of the overlapping g factor are the correlations of each test with that factor. The s factor is represented by the residuals ($s^2 = 1.0 - g^2$).

Table 1. Coefficients for the six tests represented in Figure 7.

	g	s_1	s_2	s_3	s_4	s_5	s_6	h^2
1	.7	.714						.49
2	.5		.866					.25
3	.3			.954				.09
4	.8				.600			.64
5	.4					.917		.16
6	.5						.866	.25

suggested the schematic presentation of this "two-factor theory" shown in Figure 7 and Table 1.

In a number of articles (Spearman, 1904, 1914a, 1914b, 1920, 1922, 1923; Krueger and Spearman, 1906; Hart and Spearman, 1912), the two factor theory was considered, defined, and developed. Further statistical tests for the general factor, such as satisfaction of the equation:

$$r_{ac.g} = \frac{r_{ac} - r_{ag}r_{cg}}{k_{ag}k_{cg}} = 0.0$$

where $k_{ag} = 1.0 - r_{ag}^2$ and $k_{cg} = 1.0 - r_{cg}^2$, were proposed to strengthen the argument for a two-factor theory. However, it became readily apparent to Spearman that the two-factor theory was insufficient.

Spearman's adaptation of Pearson's (1901) method for deriving the projections of points on a least squares line of best-fit involved deriving the communality, h^2, such that an individual test loading on the general factor, a_{e0}, was equal to $\sqrt{h_e^2}$, where e is the element of the factor being computed. Given the criterion of proportionality, the equation for the loading is:

$$a_{e0}^2 = h_e^2 = \frac{\Sigma(r_{ej}r_{ek};\ j, k = 1, 2, \ldots, n;\ j, k \neq e;\ j < k)}{\Sigma(r_{jk};\ j, k = 1, 2, \ldots, n;\ j, k \neq e;\ j < k)} .$$

For example, the square of the general factor coefficient (loading) for the first variable in a set of five would be:

$$a_{10}^2 = \frac{r_{12}r_{13} + r_{12}r_{14} + r_{12}r_{15} + r_{13}r_{14} + r_{13}r_{15} + r_{14}r_{15}}{r_{23} + r_{24} + r_{25} + r_{34} + r_{35} + r_{45}} .$$

Harmon (1967) suggests the more convenient computational form:

$$a_{e0}^2 = \frac{\left(\sum_{j=1}^{n} r_{ej}\right)^2 - \left(\sum_{j=1}^{n} r_{ej}^2\right)}{\left(\sum_{j<k=1}^{n} r_{jk} - \sum_{j=1}^{n} r_{ej}\right)} \quad \text{(e is fixed, } j \neq e\text{).}$$

However, this technique for deriving the general factor had severe computational difficulties, the most significant of which was the inability to derive more than one general factor. Early in the development of the two-factor approach it was suggested that group factors might exist (Krueger and Spearman, 1906) which would explain the "error" occurring in the tetrad differences; however, theoretical development of this proposition was not possible with the existing transformational algorithm.

A series of experiments and articles debating the general theory of intelligence and the two-factor theory (Spearman, 1914a, 1916; Thomson, 1916, 1919a, 1920a, 1920b; Garnett, 1919a, 1919b; Garnett and Thomson, 1919) forced the development of more advanced mathematical theories of factor analysis.

Thomson, using dice throws to randomly generate and group data, was able to show that the general factor was obtainable by means other than the criterion of hierarchy (proportionality). Spearman claimed that Thomson's substitution of artificial data for theoretically grounded results was "arbitrary, undefined, and extremely improbable by chance." However, Garnett provided a stronger challenge by citing Bravais (1846) to show that the normal correlational surface could be expressed as a function of many small variables distributed normally to explain the larger variable's value.*

Both of these challenges provided the impetus to develop methods for deriving the "group" factors, or residuals, that would account for the correlational component which the two-factor theory attributed to error. The primary conception of the "group" factor was of an uncorrelated (with

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*This is more recently a foundation for multiple regression (and coincidentally, the limitations of multiple regression).

the general factor) subset of loadings among the variables. Garnett (1919b) further expanded this conception by proposing that any normal variable, q, could be expressed by n coordinates mutually at right angles in an n-space, such that:

$$q_1 = r_{e_1q_1}e_1 + r_{e_2q_1}e_2 + r_{e_3q_1}e_3 + \ldots + r_{e_nq_1}e_n.$$

This equation is the basis for Garnett's "cosine law" which states that the correlation between the vector q_1 and the e coordinates (where e denotes the independent elements) is equal to the cosine of the angle between the vector and the coordinates. All e's are uncorrelated and may be represented as orthogonal axes. Further, each coordinate, or independent factor, e, contributes an amount, r_{eq}^2, to the determination of the vector, or dependent variable, q, by the equation:

$$r_{e_1q_1}^2 + r_{e_2q_1}^2 + r_{e_3q_1}^2 + \ldots + r_{e_nq_1}^2 = 1.0.$$

Additionally, the cosine law states that the correlation between any two observed variables, q_1 and q_2 is expressed in the formula:

$$r_{q_1q_2} = r_{e_1q_1}r_{e_1q_2} + r_{e_2q_1}r_{e_2q_2} + \ldots + r_{e_nq_1}r_{e_nq_2}$$

which, in turn, is equal to the cosine of the angle between the two vectors.

This equation provided the earliest conception of representing multiple vectors in an n-space to be found in psychology. It suggested that this geometric interpretation of correlation could be solved by deriving sequentially less of the variance by computing the partial correlation for each variable on each orthogonal axis. The later ramifications for multidimensional scaling were noted in Dodd's (1928) comment that this was "a tool of possibly immense value for the quantitative analysis of psychological data."

It was not until Hotelling (1933), with the guidance of T. L. Kelley (1928, 1931), developed his method of principal components that Garnett's cosine law (and his speculation about simultaneous derivation of underlying factors) became a true expression of multiple factors. In the interim, Spearman (1927) with Holzinger developed the bi-factor theory.

The bi-factor theory (later Holzinger's bi-factor theory) was the product of expressed belief that the residuals of the two-factor method, in addition to being interpreted as specific factors and error, could be used to compute the group factors. While Garnett (1919a) proposed that the group factors could be extracted directly from the correlation matrix, the Holzinger method (Harmon, 1967) suggested obtaining a residual matrix, $\dot{R}$, by the equation:

$$\dot{r}_{jk} = r_{jk} - a_{j0}a_{k0} \quad (j,k = 1,2,\ldots,n)$$

and recomputing by the two-factor method with the general factor removed. From this residual matrix, a common factor (represented by high loadings for a subgooup of the variables) could be derived. A second residual matrix, $\dot{R}_2$, could be computed from $\dot{R}_1$ by the same method, and a second common factor derived, and so on until all elements, $\dot{r}_{jk}$ were reduced to zero. The principal assumption of the original two-factor theory was preserved by this method; Spearman posited that the two-factor representation would hold if the elements of the first residual matrix, $\dot{r}_{jk}$, were immediately reduced to zero.

An adaptation of Fruchter's (1954) presentation of this approach is demonstrated schematically in Figure 8 and Table 2.

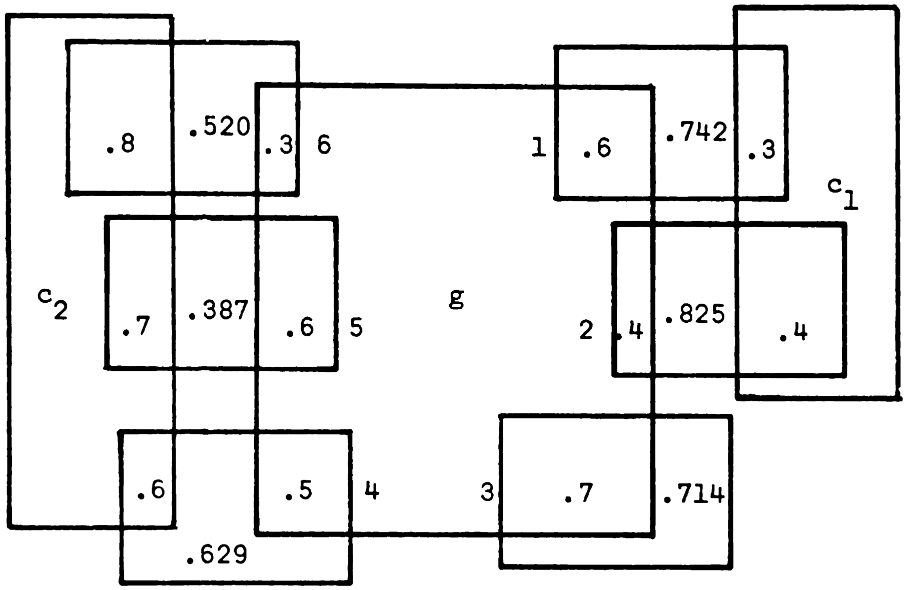

Figure 8. The bi-factor theory applied to six tests. The values of the overlapping g factor are the correlation of each test with that factor. The values of the overlapping group factors are the correlation of a specific test with a common factor for a subset of the tests with the general factor removed. The s factor is represented by the residuals.

Table 2. Coefficients for the six tests represented in Figure 8.

	g	c_1	c_2	s_1	s_2	s_3	s_4	s_5	s_6	h^2
1	.6	.3		.742						.45
2	.4	.4			.825					.32
3	.7					.714				.49
4	.5		.6				.625			.61
5	.6		.7					.387		.85
6	.3		.8						.520	.73

While this approach was of interest to the traditional factor analysts, the quality of the solution, with regard to its isomorphic relationship with known properties of correlation, and the basic complexity of the algorithm, was much less than could be achieved by the method suggested by Garnett (1919a, 1919b).

It is important, before discussing the actual development of multidimensional scaling, to consider Garnett's work in greater depth. From the cosine law, we find that by a linear transformation, two variables (or concepts in MDS terms), q_s and q_t, can be shown to depend upon only two independent variables from a set y_i (i - 1,2,...,n).

The linear equation for a variable, q_s, is:

$$q_s = l_1x_1 + l_2x_2 + \ . \ . \ . + l_nx_n$$

where l_j (j = 1,2,...,n) is the value or loading on the j-th element of the set of all possible dimensions, x. By the cosine law, this equation can be replaced by the geometric formula:

$$q_s = y_1\cos{}_s + y_2\sin{}_s.$$

With q_t computed in a similar manner and in the same coordinate frame, the correlation, r_{st}, will be equal to the cosine, $\cos\alpha_{st}$. In terms of measurement, this means that if we scale any subject along two axes, Oy_1 and Oy_2, at right angles to one another (two uncorrelated variables), and plot the point P (P = y_1,y_2) corresponding to the subject, so that y_1 and y_2 are the projections of P on Oy_1 and Oy_2, and if we draw a line through the origin and P, the cosine of the angle between Oy_1 and this axis of q_s will be the correlation between q_s and the test or scale represented by Oy_1.

If a second variable, q_t, is also plotted as a function of y_1 and y_2, it can then be correlated with q_s as a function of the angle between the two vectors. Garnett noted that this correlation "represents the average deviation in q_s (or q_t) corresponding to a unit deviation in q_t (or q_s)." This is represented graphically in figure 9.

Subsequently, three variables may be conceived of in terms of two independent factors (orthogonal dimensions), four variables in three

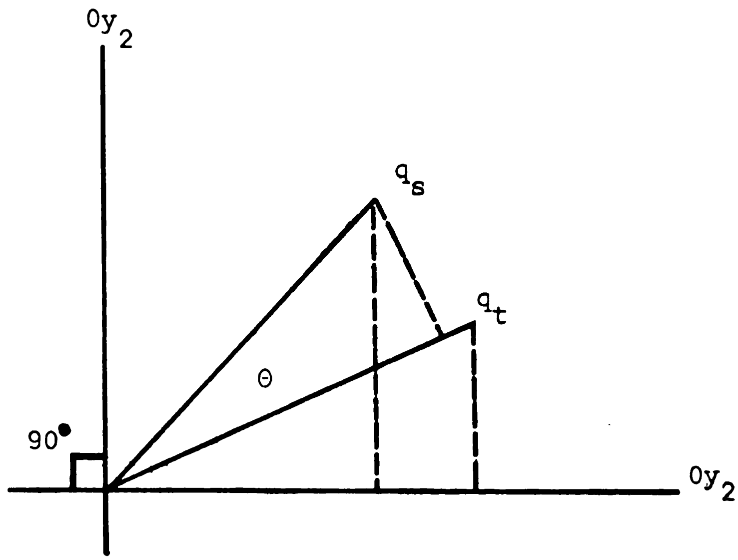

Figure 9. Projections of two variables on an independent coordinate system. Correlation between the two is represented by $\cos\alpha_{st}$.

dimensions, and so on. The condition that three quantities, q_1, q_2, and q_3, should be expressible in terms of two independent factors follows at once from the theorem:

$$\cos^{-1} r_{23} + \cos^{-1} r_{31} + \cos^{-1} r_{12} = 0$$

or its correlary:

$$r_{23}^{2} + r_{31}^{2} + r_{12}^{2} - 2r_{23}r_{31}r_{12} = 1.$$

Wolfle (1940), in his overview of the development of factor analysis, provides a good summary of this conception:

> It is convenient to think of the problem of factor analysis geometrically (Thomson, 1939; Thurstone, 1935). The scores of a number of individuals on two tests may be plotted in two dimensions. The two coordinates of each point represent the scores made by one individual on the two tests. Such a plot is commonly known as a "scatter plot" or a "correlation plot." If three tests have been given, it is possible, though physically more difficult, to plot the

scores in three dimensions. Here each of the three coordinates of any point represents one of the three test scores. If four or more tests have been given, it is no longer possible actually to plot a point representing all four scores of each individual, but it is possible to treat the scores mathematically as if they had been so plotted. Conceptually, it is possible to think of points plotted in four or more dimensions by extending our ideas of three-dimensional geometry into a larger number of dimensions.

After such a plot is made, actually in a space of three or fewer dimensions or theoretically in a space of four or more, it is sometimes found that the points occupy fewer dimensions than the space in which they were originally plotted. In the simplest case, points representing a perfect correlation will all lie on a line even though they were plotted on a plane.

Wolfle continues:

Geometrically the problem of factor analysis is one of finding the minimum number of dimensions in which the distribution of scores can be accurately described. This new set of dimensions, or coordinates, represents the factors. Before the factor analysis, the location of each point could be stated in terms of a number of coordinates, each of which represents a test; after the analysis, the location of each point can be stated in terms of a new set of coordinates, each of which represents a factor.

The techniques of factor analysis observed the greatest thrust of development during the 1930's following the publication of theories by Spearman (1927), Thurstone (1927), and Holzinger (1930). Spearman

(1928, 1930, 1931, 1933, 1938) and Holzinger (1938a, 1938b, 1944, 1949) continued to seek explanations in the general factor, with the purpose of reproducing as much data with as simple an explanation as possible. Tryon (1932a, 1932b, 1935, 1939a, 1939b, 1939c, 1957, 1958a, 1958b) and later, Cattell (1944), Peatman (1947), and Johnson (1967) developed alternative methods for dealing with the original raw data (and correlation matrix) in the form of cluster analysis. Thurstone in numerous experiments and publications (1931, 1933a, 1933b, 1934, 1935, 1936a, 1936b, 1937, 1938a, 1938b, 1945, 1947) emphasized the development of methods for finding "meaningful psychological factors" by multiple factor analysis and various rotation and translation schemes.

By far the most meaningful development in this later phase of the evolution of factor analysis, as a foundation for multidimensional scaling, was the introduction by Hotelling (1933, 1936) of the method of principal components. This algorithm, based on Garnett's (1919a, 1919b) cosine law, and incorporating work by Kelley (1935), and Thurstone (1933) and later modified by Burt (1938), introduces the procedure by which factors are derived in decreasing order of importance.

By this method, the first axis, or coordinate, is located so that it accounts for the maximum possible variance in a distribution of scores (Pearson's principal axis, 1901). The second axis is then placed orthogonally to the first so that it accounts for the maximum fraction of the remaining variance. A third axis is placed orthogonal to both the first and second such that the greatest amount of the residual variance remaining after the removal of the first two axes is accounted for. This procedure is continued until all of the variance is accounted for and the tests (variables, concepts) can be satisfactorily reproduced by the factor structure in the coordinate system thus generated; this will produce, potentially, n-1 or fewer factors, where n is the number of variables in the configuration.

Mathematically, the procedure for arriving at these axes is already described for the three-dimensional case in the work of the nineteenth century mathematician and astronomer, Jacobi (1846). By deriving the characteristic equation for the intercorrelation matrix (in multidimensional scaling, the scalar products matrix will be substituted), through expansion of the determinant, n eigenroots will be produced. These roots, λ_j ($j = 1,2,\ldots,n$), when multiplied by an associated column vector, k_j, will yield the same product as the original matrix post-multiplied by k_j. These vectors, k_j, are the columns of a matrix, K, in which each element k_{ij} ($i = 1,2,\ldots,n$; $j = 1,2,\ldots,n-1$) is the value or loading of the i-th variable on the j-th axis of the coordinate system. The vector, Λ, contains the values, λ_j, associated with the columns of K.

The derivation of the vector (Van de Geer, 1971), which it may be seen is also the axis or underlying dimension being sought, is given by the equation:

$$Ak - \lambda k = 0$$

or:

$$(A - \lambda I)k = 0.$$

Solution of the vector requires only the expansion of the determinant. Practically, however, we are limited to solving matrices with the rank of two, since the solution of a determinant is given by the polynomial equation for that determinant, and the highest order directly solvable polynomial has an order equal to two (by the quadratic equation). Alternatively, many forms of iteration have been proposed and adopted to arrive at the factor solution; of these, the Jacobi, or diagonalization, method described by Hotelling and the direct iteration methods (Wilkinson, 1965) are the most common.

Development of Multidimensional Scaling

The introduction and proliferation of multidimensional scaling is generally attributed to Torgerson (1951, 1952, 1958); it is in his work on psychophysics and scaling theory that MDS is first treated as a viable approach to psychological measurement. However, it is possible to identify several major contributions which have focused the development of multidimensional scaling, both mathematically and theoretically.

The initiative for recognizing multidimensional scales as part of measurement may be found in Thurstone's (1927) law of comparative judgement (and probably in Weber's law and Fechner's law, to which the law of comparative judgement pertains). In it Thurstone states that the judgement task when applied by a method such as paired comparisons, triadic comparisons, or tetrad differences may be expressed as a proportionality and that the set of proportionalities of judgement may be laid out on a unidimensional scale. If the proportionalities, treated as distances, do not satisfy the criterion for linearity:

$$d_{jk} + d_{kl} = d_{jl}$$

then the scores are adjusted (on the assumption that inability to fit is a function of measurement error) to unidimensionality by a procedure for expanding and shrinking the relative values.

From this, Richardson (1938) extrapolated the notion that the error might be attributable, not to the measurement, but to the judgement task itself, as a function of multiple influences on judgement. He posited that if judgements involved several independent components the effect of these components could be demonstrated by extending the linear representation to a spatial or hyperspatial representation.

At this same time Gulliksen, who had been working with problems of factor analysis (1936) began working on development of a multiple dimension model to account for overlap in judgement data. Richardson at this time also sought to develop a model for psychophysical measurement using observed "distance" from which to generate a scale. It was in response to Gulliksen and Richardson that Young and Householder (1938) invented their classical algorithm and proof for the derivation of a configuration from a matrix of interpoint distances.

The Young and Householder scheme relies on the notion that the post-multiplication of a set of coordinates will reproduce the scalar products of the original distance matrix:

> Consider a set of n points, and let $a_i = 1 \ldots n-1$, be the vector from point n to point i. Let a_{ij} be the component of a_i along the j-th axis of an orthogonal coordinate system with origin at point n and let A denote the matrix (a_{ij}). The dimensionality of the point set is equal to the rank of A and to the rank of $B = AA'$. The elements of B are given by $b_{ij} = a_i \cdot a_j$. The vector from point i to point j is $v_{ij} = a_j - a_i$, and by taking the scalar product of each side with itself there results the familiar 'cosine law':
>
> $$d_{ij}^2 = d_{jn}^2 + d_{in}^2 - 2a_i \cdot a_j,$$
>
> where d_{ij} is the distance between points i and j. From this it follows at once that
>
> $$b_{ij} = (d_{in}^2 + d_{jn}^2 - d_{ij}^2)/2, \qquad (1)$$

so that AA' is expressable in terms of the mutual distances only. Thus

(I) The dimensionality of a set of points with mutual distances d_{ij} is equal to the rank of the n-1 square matrix B whose elements are defined by (1).

(II) The dimensionality of a set of points with mutual distances d_{ij} is [two*] less than the rank of the n+1 square matrix F given by (2).

$$F = \begin{bmatrix} 0 & d_{12}^2 & \cdots & d_{1n}^2 & 1 \\ d_{21}^2 & 0 & \cdots & d_{2n}^2 & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ d_{n1}^2 & d_{n2}^2 & \cdots & 0 & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \qquad (2)$$

(III) A necessary and sufficient condition for a set of numbers $d_{ij} = d_{ji}$ to be the mutual distances of a real set of points in Euclidean space is that the matrix B whose elements are defined by equation (1) be positive semi-definite; and in this case the set of points is unique apart from a Euclidean transformation.

With these theorems, Gulliksen (1946), and later Torgerson (1951), began to build a multidimensional technique whereby measurement of

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*According to Klingberg (1941), the word "two" was inadvertantly left out of the Young and Householder article. It is worthwhile to note that after making this correction, Klingberg, himself, misuses the theorem by indicating that a matrix with the rank seven, for example, could have no more than five dimensions. He fails to observe that the theorem is expressed in terms of a bordered matrix which has a rank of n, plus 1. More fundamentally, his statement, which appears to have mislead other early attempts at multidimensional analysis, violates the basic Euclidean theorem that a set of n points will always define a space of n-1 or fewer dimensions.

interpoint distances could be made, and from these distances a spatial configuration generated. The generation of the configuration could be carried out simply by the factorization of the squared distance matrix B, using any standard algorithm (specifically a principal components routine, or if an Eckart-Young rank reduction was done, by one of the earlier direct solutions).

The Gulliksen model (1946) presented multidimensional scaling of paired comparison judgement data as an alternative to unidimensional scales when the internal consistency check (Thurstone, 1927) for the law of comparative judgements was not satisfied. Further, in defense of multidimensional "scales,"* he reaffirmed the linear relationship of the multidimensional configuration to the original judgement data and set forth a procedure for <u>intentionally</u> carrying out multidimensional analysis.

Torgerson's earliest model (1951), which subsumes the work of Gulliksen, presents three salient aspects for an MDS procedure:

> In the first step, a scale of comparative distances between all pairs of stimuli is obtained. This scale is analogous to the scale of stimuli obtained in the traditional paired comparison-type methods. In this scale, however, instead of locating each stimulus object on a given continuum, the distances between each pair of stimuli are located on a distance continuum. As in paired comparisons, the procedures for obtaining a scale of comparative distances

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*Challenges had earlier been leveled against the use of multidimensional scales in either physical or behavioral sciences by Campbell (1920, 1928). It was his contention that they violated the extensive, or additive, criterion for judging numerical representation systems.

leave the true zero point undetermined. Hence, a comparative distance is not a distance in the usual sense of the term, but is a distance minus an unknown constant. When the unknown constant is obtained, the comparative distances can be converted into absolute distances. In the third step, the dimensionality of the psychological space necessary to account for these absolute distances is determined, and the projections of stimuli on axes of this space are obtained.

These three steps, as discussed by Torgerson, integrated the many developments in factor analysis, comparative judgements, and multiple dimension scaling, drawing upon the modern refinements of each. One addition, based on Young and Householder's theorems, was the transformation of the scalar products matrix by double centering so that the axes would join at the centroid of the configuration. This increased the interpretability of the dimensions, speeded up convergence in the eigen routine, and (as will be demonstrated with a later model, Galileo), facilitated the translation of multiple measures for longitudinal analysis. A summary of the exact procedures is presented in Torgerson (1952).

Several basic problems related to the intervality and the dimensionality of the scale lead to the development of the "nonmetric" type of multidimensional scale prevalent in psychometrics today.

First, the notion that subjects could make reliable interval or ratio judgements was not readily accepted. It is axiomatic that the reliability of the judgement is inversely proportional to the difficulty of the judgement task. Since the model was attempting to describe individual differences in cognitive arrangement, or psychological distance, it was believed that the technique should be adapted to function with ordinal judgement data which simplify the subjects' task and increase the reliability of the findings reported.

Second, the use of the "additive constant" approach was considered suspect since its use with ratio scaling violates the assumption of absolute magnitude and its use with interval scaling required seperation of systematic and random error to get the data to fit a Euclidean real space.* Richardson (1938) made the assumption, which Torgerson defends, that the data was fallible and represented a foreshortening of the differences which when strictly interpreted would yield triangle inequalities and add unnecessary dimensions. The nonmetric approach (Shepard, 1962a, 1962b; Kruskal, 1964a, 1964b) avoided the problem rather than attempting to deal with it by (1) eliminating the distance component from the procedure, (2) artificially generating a configuration in a Euclidean real space of m dimensions (m is less than n and is determined from the data) which could be adjusted to fit the dissimilarities relationship, and (3) reporting the degree of monotonicity between the scale distance and the reported dissimilarities (stress).

Third, the process of determing dimensionality was itself challenged as contradictory to the goal of simplicity. Shepard (1964a, 1972) states that the purpose of multidimensionality in scaling should be to produce an expression of interrelationship which is readily interpretable. Techniques built upon this viewpoint tended to emphasize rank reduction at the price of isomorphism of the solution to the raw data, and actually limit the interpretability by introducing approximations into the analysis.

* Cf. chapter one, pp. 9-10, on the relationship between distance measures and the dimensionality and characteristics of the space in which those distances are arrayed.

This last point, on dimensionality, will be of particular importance to the presentation of the Galileo technique. Shepard comments on the nonmetric view of dimensionality (1972:2):

> In most cases one seeks a representation of the lowest possible dimensionality consistent with the data. Clearly, a lower-dimensional representation is more parsimonious in that it represents the same data by means of a smaller number of numerical parameters (the spatial coordinates of the points). Moreover, to the extent that fewer parameters are estimated from the same data, each is generally based upon a larger subset of the data and, so, will have greater statistical reliability. Finally, and perhaps most significantly, a picture or model is much more accessible to human visualization if it is confined to two or, at most, three spatial dimensions.
>
> On the other hand, one cannot reduce dimensionality arbitrarily without running the risk of doing some violence to the data. A representation of one or even two dimensions just may not be rich enough (in total degrees of freedom) to accommodate the full complexity of the relations in the given data. Still, it is a fact of decisive practical significance that most applications of multidimensional scaling have yielded interpretable and sometimes even enlightening representations in no more than three and, indeed quite often, in only two spatial dimensions.

Following the early metric work by Torgerson and others (Abelson, 1954; Gulliksen, 1954; Messick, 1954, 1956; Messick and Abelson, 1956), considerable effort was put into the nonmetric approach. Put forth by the Bell Telephone Laboratories group (Shepard, 1962a, 1962b, 1966, 1972; Kruskal, 1964a, 1964b, 1965, 1972; Carroll, 1968, 1969; Johnson, 1967; Kruskal and Carroll, 1968; Shepard and Kruskal, 1964; and Carroll and Chang, 1964a, 1964b, 1970), and others (Guttman, 1968; Lingoes, 1963, 1966, 1971; McGee, 1966, 1968; Young, 1968a, 1968b, 1968c, 1972; Coombs, 1964; Tucker, 1960, 1963, 1964; Tucker and Messick, 1963), the nonmetric approach has produced a substantial set of techniques for the analysis of individual ordinal spaces. A review of this branch of MDS is presented by Shepard, Romney, and Nerlove (1972, volume I) and Romney, Shepard, and Nerlove, (1972, volume II).

In the next chapter, the problems suggested in this discussion and the historical developments detailed here will be used as a framework for discussing the reintroduction of metric multidimensional scaling as a socio-metric technique. In it, I will present the mathematical and theoretical considerations of Galileo, a completely metric approach utilizing aggregate and longitudinal data.

Chapter III

Galileo: A Procedure for Metric Multidimensional Analysis

Chapters I and II enumerate criteria for methodological rigor and suggest problems which have arisen in attempts to satisfy those criteria. In chapter III, we will examine the Galileo model proposed by Woelfel (1972, 1974a, 1974b), and the solutions, both mathematical and theoretical, which it offers.

In summary, the criteria proposed in chapter I for an advanced methodology are:

(a) it should be of the highest possible level of scaling,

(b) it should be able to measure change,

(c) it should be parsimonious and yet complete,

(d) it should not force the loss of information, and

(e) it should achieve isomorphism between reality and the numerical representation of reality.

It will be argued from this explanation of the Galileo procedure that this technique more closely satisfies these criteria than any existing multidimensional scaling approach. Galileo couples the mathematically pure approach developed by Torgerson (1958), the scaling rigor demanded by Stevens (1951, 1959) and by Suppes and Zinnes (1963), and the scientific principles expressed by Einstein (1961) for the purpose of accurately measuring social psychological phenomena (conception, culture, and communication).

Galileo procedures have three main aspects: (a) the collection of linearly transformable data by an interval or ratio judgement technique, (b) aggregation of the judgement data, and (c) computation of the spatial configuration for the concepts, or stimulus-objects, in a multidimensional coordinate system determined by the data.

A Method of Ratio Judgements

As we have dealt with factor analytic and multidimensional scaling procedures in the previous chapters, the problems of development have been identified as chiefly mathematical. However, the kind of data on which we operate with these procedures is of central importance to the hypotheses which we deal with and the interpretations that we might make from such a technique.

The Galileo model is designed for the analysis of dissimilarities data drawn, primarily, from paired judgements. The judgement task is one of perceiving and identifying concepts*, which itself is basically a process of differentiation on the basis of dissimilarities for one or more underlying attributes (Torgerson, 1958). For example, we might distinguish two individuals on the basis of height or weight, and do so in terms of a specific amount of each attribute. Therefore, when asked which person is taller, we could respond appropriately and even indicate _how much_ taller one was over the other.

Most procedures for identifying perceived or conceptual relationships may be viewed as some form of dissimilarities judgement by utilizing a number of salient attributes against which to judge a particular stimulus-object; that object may be discriminated from all other objects in the conceptual universe. In most cases this amounts to the

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*The term "concept" refers to any perceived physical or psychological object or abstraction. This is analogous to Torgerson's "system" (cf. chapter I, pp.1-2).

characterization of several stimuli in terms of having or not having certain attributes.

For example, we might distinguish between two political candidates by comparing them with regard to a number of categories. At the most primitive level we may indicate that a particular category, or attribute, applies (1) or does not apply (0) to the candidate:

	Democrat	Republican	Liberal	Honest	Active	Incumbent
$Candidate_1$	1	0	1	1	1	0
$Candidate_2$	0	1	0	1	1	1

Obviously, a great deal of information about the difference between the two candidates is lost in this representation; by categorization we are limited to knowledge of the presence or absence of a condition, and by prescribing the attributes we fail to account for all possible distinctions between the two. Even if we use unitary scales which tell us <u>how much</u> of each attribute is present, our ability to discriminate is limited by our ability to select concepts which we think will be important to the individual making the judgement.

The purpose of this type of conception is to arrive at an aggregate of the dissimilarities as an index of overall difference. However, we might seek ways of obtaining greater isomorphism to the judgement process without violating the parsimony of judging a small number of salient attributes. One such method would be to ask the respondent to provide the index of overall difference, directly (e.g., report, on a scale of overall differences, how different two stimuli are).

Differences among concepts could then be represented by a continuous scale of the magnitudes of dissimilarities, with a natural origin representing the condition of pairs being completely identical. These scale values can be presented in a matrix of all possible combinations for a set of concepts by the element, d_{ij}, representing the distance between the i-th and j-th elements:

$$D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & & \vdots \\ d_{n1} & d_{n2} & \cdots & d_{nn} \end{bmatrix} .$$

So far, we have dealt with the fundamental notion of dissimilariy, or distance, not as a scale that is tied to any single attribute but rather as a characteristic of all scales on which attributes may be measured. This is seen most clearly in the physical sciences where space is measured in terms of the three dimensions of height, width, and depth; all three of these are expressions of distance (stated with regard to direction). Psychological distance is, of course, represented in all of the properties of a concept, which if they could be treated independently (i.e., also by direction) could yield a structure similar to that found in the physical world.

The problem becomes how to measure psychological distance. The answer is suggested by Einstein (1961) who proposes that the unit of measurement is wholly arbitrary, but that once it is chosen, it becomes the standard for all measures which are to be compared. In physical measurement we choose two points on a rigid body, construct according to the rules of geometry a line segment S between the points, and compare other line segments as ratios of the known line segment S to the unknown line segments. The numerical expression of the length of a measured segment is given by the number of times S is applied.

If we accept that it is a cognitive judgement of a concept rather than the concept itself with which we are dealing, then we may apply Einstein's treatment of physical distances to psychological distances also. By specifying the distance between any two concepts as a standard, we may compare the perceived distance between any other pair of concepts as a ratio of the perception of the standard pair to the measured pair.

With the Galileo technique, this is accomplished by simply asking a respondent the question:

"If x and y are u units apart, how far apart are a and b?"

This has the force of producing an unbounded ratio scale of non-negative real numbers that may be considered continuous across its entire range. Further, because the scale is viewed as continuous, and requires ratio judgements, we have also eliminated the additive constant problem raised by Torgerson. The Galileo technique assumes, from the rationale provided by Einstein, that distances measured against an absolute standard will need no adjustment of magnitude to be made to lie in Euclidean space.

Aggregation

As we have shown, the technique for gathering the data meets the general criteria for scaling rigor at a very high level. However, the problem arises that the reliability of the judgements is very low. Due to the difficulty and complexity of the judgement task an individual mav over or under estimate the distances thus introducing considerable error into the paired judgement set. This was a problem that led Torgerson

(1951) to the use of artificially constructed distances and Shepard, Coombs, and others (Shepard, 1972) to the use of ordinal data and non-metric algorithms.

Since, in communication, we are interested in the identification of cognitive and behavioral regularities, it is advantageous to us to have techniques whereby we may summarize the whole of many interactions, behaviors, and attitudes. This allows us an option which Galileo utilizes; we may aggregate the date and arrive at a matrix of mean distances for a sample of distance judgements.

We know, by the psychometric axiom that the reliability of any scale is inversely proportional to the complexity of the judgement task used to derive that scale. Hence, the judgements made in response to a Galileo instrument will be of low reliability for the individual. We also know that for a series of measures, random error will be normally distributed. Given these conditions, the central limit theorem and law of large numbers indicates that the scores reported will be normally distributed about the sample mean and that the sample mean will converge upon the true score for the population as the n grows large.

Aggregation has been considered a useful and desireable procedure for conducting communication research (Rosenthal, 1973). In this context, we may utilize it to overcome a problem (unreliability of judgement) that has inhibited the development of precision in other applications of multidimensional scaling. The aggregation technique used with MDS is straight-forward; we may take the individual distance matrices and compute a single mean distance matrix, $\overline{D}$, by the formula:

$$\overline{D} = (\overline{d}_{ij.}) = \frac{\sum_{k=1}^{n_{ij}} d_{ijk}}{n_{ij}}$$

where $\overline{d}_{ij.}$ is the ij-th element of the means matrix and the sample n is variable across elements.

The utility of aggregation is shown by expansion of the previous example in which political attributes were judged. In that example a single set of responses was used to evaluate the two candidates. However, gathering multiple response sets allows the aggregation to be performed; the arithmetic means can be computed and a new set of values reported. The data for Candidate_1 might appear as follows:

	Democrat	Republican	Liberal	Honest	Active	Incumbent
Subject_1	1	0	1	1	1	0
Subject_2	1	0	0	0	0	1
Subject_3	1	0	1	0	1	1
Subject_4	1	0	0	0	1	1

The aggregated values for the sample would then be:

Candidate_1	1.00	0.00	0.50	0.25	0.75	0.75

These aggregate values have greater practicable meaning and provide a more parsimonious representation than the original table of responses. This also yields distinctions not apparent in the single response set; the original values indicate only the perceived existence of an attribute for the concept (in this example, the candidate) while the aggregate values for attributes can be compared across concepts. Because the attributes are dichotomous the concepts' differences can be represented by the percentage of respondents in agreement on each of the attributes.

The aggregated values are, therefore, representative of the population as a whole.

Two changes occur with the shift from nominal judgement data to data collected using higher levels of scaling. First, the values reported for the judgements become more precise, thus increasing the rigor of operations which may be performed upon the data. Second, the reliability decreases; this produces a greater demand for the use of analytic techniques which will compensate for unreliability. Thus, the introduction of aggregation facilitates the use of more rigorous analyses while stabilizing the values and making them more representative of the population of scores from which they were drawn.

The application of aggregation is often associated with and may suggest interesting and complex applications to methods for inferring from the population to the individual (Goodman, 1959; Boudon, 1963). However, the greatest utility of the procedure is the representation of socially or culturally grouped relational conceptions. Rather than deriving the individual cognitive arrangement of concepts we may deal with the group's cognitive structure (Woelfel, 1972; Barnett, 1972) as it is represented by the aggregation of attribute scores for a set of concepts. Stated more parsimoniously, the aggregate data matrix represents a subset of <u>everything</u> known by a population; if this knowledge could be scaled by some exhaustive procedure it would provide a map of the "culture" (Woelfel, 1972).

Transformation to the Spatial Manifold

By the procedures described for gathering judgement data and aggregation, Gailieo poses a theoretical alternative to the first and second major problems that diverted the evolution of multidimensional scaling from the metric model to the Shepard-Kruskal methodological disjuncture: scale adjustment and reliability. In this section, the algorithm for generating a multidimensional scale from the matrix of dissimilarities will be outlined, and the notion of isomorphism will be supplemented for parsimony in the expression of dimensionality.

The Galileo procedure begins by gathering data of the form suggested earlier in this chapter in the section on ratio judgements. For each respondent, we will have a matrix of all possible paired judgements for a list of concepts. These distance matrices will then be aggregated to produce the matrix $\overline{D}$ from which the multidimensional solution will be derived. For example, the following matrix has been generated for a set of political concepts:

	Nixon	McGovern	Prosperity	Taxes	Employment	Me
Nixon	0.0	130.4	43.0	31.5	45.8	114.2
McGovern	130.4	0.0	37.6	38.7	26.3	39.2
Prosperity	43.0	37.6	0.0	56.4	14.1	34.9
Taxes	31.5	38.7	56.4	0.0	22.9	37.1
Employment	45.8	26.3	14.1	22.9	0.0	15.0
Me	114.2	39.2	34.9	37.1	15.0	0.0

The judgements for this matrix were made by a random sample of 116 respondents drawn from the voter registration lists of Champaign county, Illinois in June, 1972. The values represent the mean distances, or dissimilarities, for the concepts as judged by the sample.

To obtain the multidimensional coordinate space in which to represent the concepts, the procedure, beyond the aggregation step, is a fairly straight-forward variation on factor analytic techniques. As shown above, the original data yields a concepts by concepts by persons matrix which is then averaged for each cell over persons (cf. Tucker, 1964 for alternative procedures) to produce a concepts by concepts square symmetric matrix, $\overline{D}$, as described in the section on aggregation.

This matrix may then be transformed into a scalar products matrix, B, which is normally computed:

$$B = \overline{D}' \, \overline{D} ,$$

or by the method from Young and Householder (1938):

$$B = (b_{jk}) = (d_{ij}^{\,2} + d_{ik}^{\,2} - d_{jk}^{\,2})/2 \qquad \begin{matrix}(j,k = 1,2,\ldots,n)\\(j,k \neq i)\end{matrix}$$

where the element, b_{jk}, is the scalar product of the vectors (i,j) and (i,k).

Galileo uses an adaptation of Young and Householder's method as proposed by Torgerson, for "double-centering" the origin of the space at the centroid of the distribution (configuration). This centroid scalar products matrix is thus computed in a one-step procedure where the elements of $\tilde{B}$ are given by the equation:

$$\tilde{b}_{ij} = \frac{1}{2}\left(\frac{\sum_{i=1}^{n} d_{ij}^{\,2}}{n} + \frac{\sum_{j=1}^{n} d_{ij}}{n} + \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}^{\,2}}{n^2} - d_{ij}^{\,2}\right).$$

By Garnett's cosine law, these elements, $\tilde{b}_{ij}$, represent the unstandardized vector lengths (as opposed to the standardized form which would be applied in a factor analysis) multiplied together with the cosine of the angle between the vectors such that:

$$\tilde{b}_{ij} = p_i p_j \cos\alpha ij$$

where p_i and p_j are the vector lengths.

The scalar products matrix for the political data described above would appear, by the Torgerson equation, as follows:

$$\underset{\sim}{B} = \begin{bmatrix} 4529.4 & -5042.7 & 1344.6 & 1748.9 & 876.7 & 3456.9 \\ -5042.7 & 2389.3 & 492.2 & 426.1 & 509.6 & 1225.5 \\ 1344.6 & 492.2 & 8.8 & -1605.8 & -434.2 & 194.5 \\ 1748.9 & 426.1 & -1605.8 & -39.4 & -621.1 & 91.2 \\ 876.7 & 509.6 & -434.2 & -621.1 & -678.4 & 347.4 \\ -3456.9 & 1225.5 & 194.5 & 91.2 & 347.4 & 1598.3 \end{bmatrix} .$$

This centroid scalar products matrix may now be transformed linearly by any routine factorization to arrive at a matrix of coordinate values for the set of concepts. The particular routine utilized in the Galileo software (Galileo 2.3, CDC 6000 series FORTRAN) is a direct iterative solution proposed by Van de Geer (1971) and suggested as a slow but highly accurate alternative for the Jacobi-type eigen routine (Wilkinson, 1965). This type of solution has the advantages of simultaneously deriving the root and the vector for any given axis and yielding the Euclidean components of both real <u>and</u> <u>imaginary</u> space. The Van de Geer technique has been further adapted to produce rapid convergence during the iteration procedure. In addition, Galileo provides any imaginary dimensions present in the data; this represents triangle inequalities present in most judgement data sets. A left-right reversal*

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* Since the spatial configuration is derived from the interpoint distances of the concepts, the valences for any dimension are arbitrary and are independent of other dimensions. Thus, while consistency of sign must be maintained among the loadings along a dimension, it does not change the distances if all negative values are made positive and all positive values are made negative. Further, the change of sign on one dimension will not affect the distance along other dimensions; proof for this is derived from the Pythagorean theorem.

of spatial position is also provided by Galileo, so that distances can be minimized during translation and rotation for longitudinal comparisons.

The procedure, which is treated in depth by Van de Geer (1971), is summarized here:

Matrix $\tilde{B}_1$, the original centroid scalar products matrix, is multiplied by a vector of ones, v_{11}, with the rank n to produce a second vector, v_{12}:

$$\tilde{B}_1 v_{11} = v_{12}.$$

This procedure is repeated to produce v_{13}, $v_{14}, \ldots$, $v_{1,p+1}$, such that $v_{1,p+1}$, when divided by its largest value will be equal to v_{1p}. Therefore, when v_{1p} pre-multiplied by the matrix is equal to a vector which divided by its largest scalar gives back v_{1p}, the routine has converged on the unnormalized eigenvector (or axis).

After converging on the vector v_{1p}, the largest value in $v_{1,p+1}$ is taken to be the root, and v_{1p} is normalized to that root. This yields the first coordinate axis of the spatial manifold, and may be interpreted as the projections of the n concepts on that axis.

The normalized eigenvector, which we refer to as f_1 is then post-multiplied by its transpose to yield an outer products matrix. The outer products matrix (Veldman, 1967) is then subtracted from $\tilde{B}_1$ (added, if the root is negative and the associated vector is imaginary) to produce the residual scalar products matrix, $\tilde{B}_2$:

$$\tilde{B}_2 = \tilde{B}_1 - f_1 f_1' \, .$$

The entire procedure is repeated for the residual matrix, $\tilde{B}_2$, starting with v_{21}, to arrive at v_{2p}; the second axis, f_2, is computed and the new residual matrix, $\tilde{B}_3$, is separated from the previous one. The procedure continues until all n roots and their associated vectors are computed.

For the paired judgements on the six political concepts described above, the coordinate solution contains three real and two imaginary dimensions with the following loadings:

	1	2	3	4*	5	6
Nixon	75.6	-.3	-.6	.2	-.2	34.5
McGovern	-53.9	2.1	-16.3	-.2	-.0	28.1
Prosperity	5.1	-27.8	-3.3	.0	-6.8	-27.5
Taxes	9.4	28.4	-1.7	.0	-5.6	-30.2
Employment	1.7	-2.1	.7	.0	14.6	-21.8
Me	-37.9	-.3	21.2	-.1	-2.0	16.8

A two dimensional representation of the concepts (which accounts for 94 percent of the distance in the configuration) is given in Figure 10.

This routine is made competitive with the various diagonalization methods (principal components factorization), which provide optimum speed for computing, but usually limit the solution to positive factors and may terminate when an acceptable number of factors is reached. By repeated pre-multiplication of the scalar products matrix to raise it to its fifth power, Galileo substantially increases speed of convergence with the direct iterative method. This is of some consequence when it is considered that most of the existing factorization methods were developed

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* The values for this dimension are artificial and represent rounding error in the computer algorithm. Greater precision can be achieved by reducing the tolerance level for comparison functions; however, the increase in computation costs seems unwarranted.

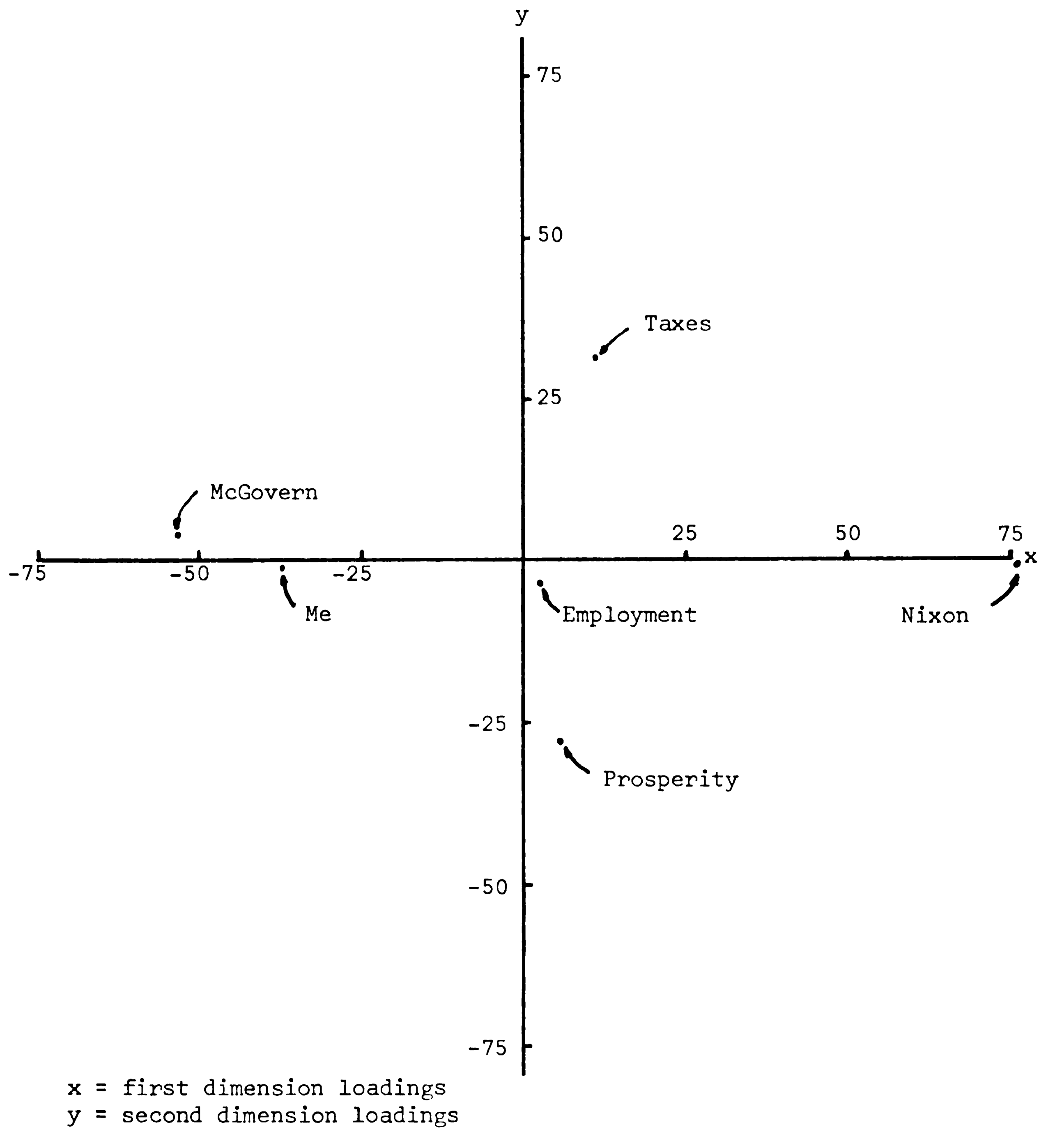


Figure 10. Two-dimensional representation of six political concepts scaled by the Galileo procedure.

before high speed computers were introduced and required considerable expenditures of time, effort, and money to perform the factor routines. The representation of only a few factors in most MDS solutions, which is argues for by Shepard and others on the grounds of parsimony, is more likely an artifact of the earlier difficulties of deriving a full solution matrix.

The Galileo solution achieves isomorphism with the distance judgement data when all of the distance between points is accounted for in m dimensions (n - 1 or less). Since interpretation of the full solution is limited only by one's ability to read a table of coordinate values and not by some artificial requirement that the solution fit a three-dimensional (or less) graphic representation, there is no violence done to the data. For this reason, the direct iterative approach used by Galileo is recommended over other methods of factorization.

Longitudinal Data and Rotation

The final major advantage of the Galileo model over existing techniques, both metric and nonmetric, is the incorporation of a temporal component for longitudinal comparison. This makes the technique particularly advantageous for the analysis of communicative acts which are by definition processual and thus produce continuously changing effects rather than discrete outcomes.

This continuous change which characterizes process may be observed by examination of the spatial model over a period of time. Since the fundamental variable in all metric multidimensional analysis is distance, and since motion is the description of alternative states or interrelationships measured as differential quantities of distance, change may be conceived of as motions in the cognitive or cultural space. Motion is measured and expressed as velocity, a change in distance over a change in time,

$$V_1 = \frac{\Delta d}{\Delta t} = \frac{d_1 - d_0}{t_1 - t_0} \quad .$$

Additionally, the precision of describing change may be increased by the expression of acceleration (the change in velocity over time) and the change in acceleration.

Describing change in a process analysis assumes that measurements are taken at successive intervals in time and compared to obtain measures of difference. When using a nonmetric multidimensional procedure to make these measurements, a problem of comparability occurs. The nonmetric algorithms yield static, monotonic configurations which cannot be suitably compared with configurations generated at other points in time. The metric configuration, however, may be viewed as the representation of a changing set of relationships that have been frozen at a single point in time. To describe the changes which occur it is necessary to "unfreeze" the configuration. Ideally, this would involve continually measuring the set of relationships and plotting the trajectories of the concepts as they move over time. In practice, this means measuring the concept distances at regular intervals of time and approximating the trajectories by fitting curves which best represent their motions.

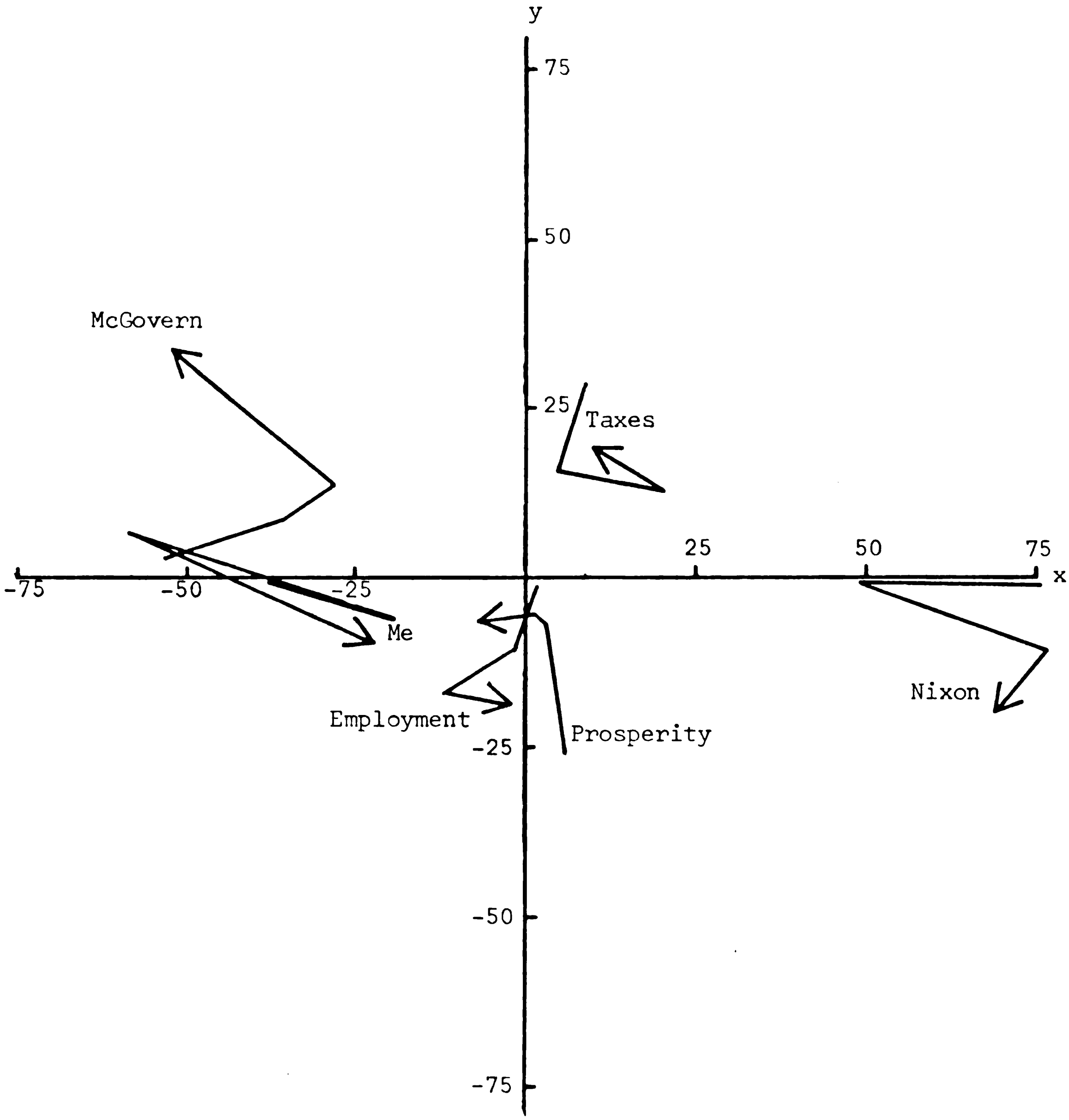
The Galileo technique extends traditional metric multidimensional analysis by incorporating time through the use of translation and rotation. Translation is facilitated in the computation of scalar products where, for each configuration, the origin is set at the centroid of the distribution. The centroid of one distribution may be considered to be superimposed upon the centroid of another distribution. The axes are then rotated pairwise so that the distances between all concepts, on all possible pairs of axes, are minimized by a Gaussian least squares (squared distances of the projections from each other) best fit. This is given by the formula:

$$\text{Min} \sum_{i=1}^{n} \sum_{j=2}^{m} \sum_{k=1}^{m-1} \left[\Theta_{ijk(t)} - \Theta_{ijk(t-1)} \right]^2 \qquad (j<k),$$

where i is the subscript for concepts in the set of n concepts, j and k are the dimensions being paired for comparison, and m is the rank or dimensionality of the space.

This procedure (in effect) reduces the representation from multiple spaces at different points in time to a single space in which the trajectories of motion are plotted. Figure 11 is an extension of the two dimensions represented in Figure 10 showing changes in the six political concepts across four points in time (June, 1972 to June, 1973).

The concept of motion implies change which may be described in terms of velocity, acceleration, duration, and other variables drawn from physics and mechanics. In addition, derived explanatory variables such as mass and force can be applied to motions in the cognitive space. In many cases these formal connections of model and observable reality have correlates in the behavioral sciences, and in communication particularly. They may be found in the operationalization of constructs such as attitude stability (balance) and cognitive consistency (Heider, 1946; Newcomb, 1963), rates of exposure (Klapper, 1960; Schramm, 1960), persuasive messages (Bettinghaus, 1973), and information processing (Shannon and Weaver, 1949). Applications of spatial variables in communication theory are



x = first dimension loadings
y = second dimension loadings
arrows indicate direction of change over time

Figure 11. Representation of movement of six political concepts scaled at four points in time.

discussed at length by Arundale (1971), Woelfel (1974c), and Woelfel and Saltiel (1974).

In summary, the Galileo approach to multidimensional scaling adopts Torgerson's theoretical foundation for metric spatial representations, and through a matrix of ratio judgements, projects a continuous Euclidean space. Using both real and imaginary components of the complex number system (from the direct iterative solution) the distance between any two concepts, treated as points, is accounted for and presented in the coordinate solution. This solution, coupled with the rotation of spaces to a least squares best fit, allow us to postulate the principles of motion. Analysis of motion provides a foundation for the interpretation of communicative acts and principles, and for the development of precise models for explaining the cognitive and cultural aspects of communication behavior.

Conclusion

In the behavioral sciences it is often argued that the nature of humanity is such that the development of a complete and comprehensive general theory of behavior is impossible. The most fundamental weakness, leading to this conclusion, is the lack of adequate linkages between observable "reality" and models used to explain human behavior. It is my contention that this is not a failure of theory but rather a failure, quantitatively, to produce results worth interpreting. Without useful research. the behavioral sciences have been unable to produce the necessary linkages from which to generate a substantial theory.

From a philosophical perspective, science may be perceived as having two components which interact to produce the quality of predictive ability:

theory and methodology. Theory provides us with a structure for explanation while methodology provides the tools for comparing that explanation with the reality which we are studying. It is a prevailing belief in the behavioral sciences that theory must dictate the selection of methodology; failure to conform to this belief dooms the theory to stagnate or be consumed by the techniques used to test it.

This thesis has argued that methodology and theory may be treated as independent components of scientific study. It has operated from the basic assumption that, while the two must necessarily be related to conduct research, the origination of a methodology and the development of a theory are not de facto tied together. It is common, as in pure mathematics, to develop a method or technique without consideration for some specific application. And it often occurs that the theories we develop go untested.

The failures of the behavioral sciences do not occur from developing bad theory, and they are only partially related to the lack of adequate methodologies. The most significant problem has been the use of weak and inappropriate methods and research strategies which are incapable of either supporting or rejecting the hypotheses we are testing. The problem is similar to that of trying to shovel snow with a tablespoon; even after many attempts, very little is accomplished.

The historical presentation of developments leading to the use of a technique such as the Galileo version of multidimensional scaling is an attempt to induce awareness of methodological considerations in dealing with the problem. We are not ignoring theory; rather we are dealing with considerations of criteria and appropriateness for selection of methodologies.

Theory may dictate the need for certain methodologies; some of these exist, while others need to be invented. In the case of communication research, our existing methods have allowed us to delineate areas of study without providing the concomitant data base necessary to turn hypotheses into theory. The development of longitudinal metric multidimensional scaling is one attempt to bridge this gap. Historically, it is part of a long series of methodological advances which have come as responses to the needs of different sciences. These advances have drawn on existing techniques, adding, modifying, and rejecting parts. The evolution of multidimensional scaling has come through astronomy, mechanics, and psychology; it is now receiving consideration for communication study. Until a more appropriate technique which is able to satisfy rigorous requirements is developed, Galileo provides a framework with which to deal with the exigencies of human interaction and information systems. Its use in communication should serve to improve our knowledge of human complexities. When it fails to do this, or if some other technique proves more fruitful, it should be replaced or altered so that our theories can advance unencumbered by the tools we use to support or reject them.

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APPENDICES

APPENDIX A

APPENDIX A

Table A-1. Coordinate values for six political concepts in a multidimensional space (June, 1972).

	Dimensions					
	1	2	3	4	5	6
Nixon	75.64	-.27	-.61	.21	-.15	34.54
McGovern	-53.94	2.13	-16.31	-.15	-.02	28.13
Prosperity	5.10	-27.84	-3.25	.01	-6.81	-27.51
Taxes	9.45	28.44	-1.67	.02	-5.60	-30.15
Employment	1.61	-2.13	.68	.00	14.56	-21.76
Me	-37.87	-.33	21.17	-.10	-1.97	16.75
	Eigenroots					
	10184.32	1593.44	728.82	.08	-293.93	-4404.72

Table A-2. Coordinate values for six political concepts in a multidimensional space (August, 1972).

	Dimensions					
	1	2	3	4	5	6
Nixon	48.88	-6.33	-.88	-.83	.03	20.57
McGovern	-37.81	-23.39	-2.04	.98	.03	16.59
Prosperity	3.90	7.90	-4.67	9.98	-.02	-12.33
Taxes	4.14	-10.06	14.16	-1.23	-.03	-19.95
Employment	-.76	1.18	-11.08	-6.77	-.03	-19.15
Me	-18.34	30.69	4.53	-2.13	.02	14.27
	Eigenroots					
	4189.40	1694.78	370.72	153.30	-.00	-1819.48

Table A-3. Coordinate values for six political concepts in a multidimensional space (November, 1972).

	Dimensions					
	1	2	3	4	5	6
Nixon	80.02	-1.98	-1.71	.08	-.11	35.28
McGovern	-24.26	-8.05	21.30	-.02	1.79	5.74
Prosperity	.80	-.37	-7.99	.00	9.17	-24.97
Taxes	15.67	-21.02	4.29	.01	-4.03	-30.07
Employment	-8.25	-18.75	-7.59	-.00	-5.87	-23.24
Me	-63.98	8.13	-8.29	-.06	-.93	37.27
	Eigenroots					
	11401.74	929.04	665.54	.01	-139.09	-4735.83

Table A-4. Coordinate values for six political concepts in a multidimensional space (June, 1973).

	Dimensions					
	1	2	3	4	5	6
Nixon	72.73	-11.03	.66	-.19	-.07	-27.75
McGovern	-60.34	-24.90	3.60	.26	-.06	-24.76
Prosperity	-2.55	14.22	-1.46	7.72	.02	8.34
Taxes	4.21	-19.63	-12.58	-1.81	.09	35.37
Employment	4.20	7.71	17.68	-2.32	.06	24.79
Me	-18.25	33.64	-7.89	-3.65	-.04	-16.00
	Eigenroots					
	9306.92	2520.98	549.10	81.85	-.02	-3574.91

APPENDIX B

```
      SUBROUTINE GALILEO
C
C          GALILEO IS AN INTEGRATED PROGRAMMING PACKAGE FOR
C          METRIC MULTIDIMENSIONAL SCALING UTILIZING PAIRED
C          DISTANCE JUDGEMENT DATA.  GALILEO WILL ACCEPT DATA
C          IN THE FORM OF RAW DISTANCE SCORES, AGGREGATE MEANS
C          SCORES IN THE FORM OF A SQUARE SYMMETRIC MATRIX, OR
C          A CENTROID SCALAR PRODUCTS MATRIX.  THE DATA SHOULD
C          HAVE THE FOLLOWING FORMAT:
C
C                RAW DATA       8 COLUMNS OF IDENTITY FIELD
C                               72 COLUMNS OF (I2,I2,I5)
C                               WHICH CONTAIN THE ROW AND COLUMN
C                               ADDRESS (LOW VALUE FIRST) AND A
C                               VALUE REPORTED FOR THAT ADDRESS
C                               (0-99999, BLANK ADDRESS FOR
C                               NO RESPONSE)
C
C                MEANS MATRIX   80 COLUMNS OF (F10.4) WHICH CON-
C                               TAIN 8 VALUES FROM A MATRIX ROW,
C                               THE ENTIRE ROW IS PUNCHED, A NEW
C                               CARD INDICATES A NEW ROW
C
C                SCALAR PRODUCTS
C                               EACH RECORD IS (6F12.3) WHICH CON-
C                               TAIN 6 VALUES FROM A MATRIX ROW,
C                               THE ENTIRE ROW IS PUNCHED, A NEW
C                               CARD INDICATES A NEW ROW
C
C          FURTHER INFORMATION ON USING THE GALILEO PACKAGE IS
C          AVAILABLE IN THE GALILEO USERS GUIDE BY WOELFEL AND
C          SEROTA OR BY CONTACTING KIM SEROTA, DEPT. OF COM-
C          MUNICATION, MICHIGAN STATE UNIVERSITY.
C
      DIMENSION SCALAR(40,40),FAXT(2500),RUN(5),
     1          LABELS(40,4),CRIT(5),OPT(7)
      COMMON FAXT
      INTEGER OPT
      DATA SCALAR,OPT,NDIM,MAIN,MIDDLE,LABELS,THETA/
     1 1600*0.0,7*0,3,0,0,160*8H        ,0/
      DATA MAXVAL/99999/
C
C          CONTROL CARD READ-IN AND JOB SET-UP
C
      CALL CONCARD (RUN,LABELS,CRIT,OPT,MSIZE,NSETS,KJOB,
     1 NDIM,MAIN,MIDDLE,THETA,MAXVAL)
      PRINT 1001,(RUN(K),K=1,5),MSIZE,NSETS
 1001 FORMAT (*1*/10X,*PROGRAM GALILEO (VERSION 2.3) *////
     110X,*KIM SEROTA -- JOSEPH WOELFEL*/
     210X,*DEPARTMENT OF COMMUNICATION*/
     310X,*MICHIGAN STATE UNIVERSITY*/
     410X,*COPYRIGHT 1973*///
     510X,*GALILEO -- A PROGRAM OF METRIC MULTIDIMENSIONAL SCALING*/////
     6/////////45X,5A8,//50X,*AN ANALYSIS OF *,I2,* CONCEPTS*/
     750X,*UTILIZING *,I3,* SETS OF DATA*)
      PRINT 1002
 1002 FORMAT (*2*,61X,*BEGIN GALILEO*)
      IF (KJOB-91) 90,91,92
   90 ASSIGN 9000 TO JOB
      GO TO 95
   91 ASSIGN 9100 TO JOB
      GO TO 95
   92 ASSIGN 9200 TO JOB
   95 CONTINUE
C
C          CALLING ROUTINES
C
      GO TO JOB (9000,9100,9200)
 9000 CALL MTRIX (MSIZE,NSETS,CRIT,RUN,LABELS,0,MAXVAL)
      GO TO 929
9100  IF (OPT(1)-1) 9102,9101,9102
9101  CALL MTRIX (MSIZE,NSETS,CRIT,RUN,LABELS,1,MAXVAL)
9102  IF (OPT(2)-1) 9103,9104,9103
9103  DO 9105 INCMENT=1,NSETS
      CALL CENTRIX (SCALAR,OPT,MSIZE,INCMENT,RUN)
      CALL EIGTRIX (SCALAR,OPT,MSIZE,INCMENT,KJOB)
9105  CONTINUE
      GO TO 929
9104  DO 9106 INCMENT=1,NSETS
      CALL EIGTRIX (SCALAR,OPT,MSIZE,INCMENT,KJOB)
9106  CONTINUE
```

```
      GO TO 929
9200  IF (OPT(1)-1) 9202,9201,9202
 9201 CALL MTRIX (MSIZE,NSETS,CRIT,RUN,LABELS,1,MAXVAL)
      GO TO 9203
 9202 IF (OPT(6)-1) 9203,9204,9203
9203  DO 9205 INCMENT=1,NSETS
      CALL CENTRIX (SCALAR,OPT,MSIZE,INCMENT,RUN)
      CALL EIGTRIX (SCALAR,OPT,MSIZE,INCMENT,KJOB)
9205  CONTINUE
 9204 CALL COMPARE (OPT,LABELS,MSIZE,NDIM,NSETS,MAIN,MIDDLE,THETA)
929   CONTINUE
      PRINT 1004, (RUN(K),K=1,5)
C
C         TERMINATION
C
 1004 FORMAT (*A*//////50X,*END PROGRAM GALILEO*//46X,*END *,5A8)
      PRINT 1006
 1006 FORMAT (*-*///////////////20X,*THANK YOU FOR USING THE GALILEO SYSTE
     1M*/21X,*IF YOU HAVE ANY PROBLEMS, SUGGESTIONS, COMPLAINTS,*/22X,
     2*OR DESIRE ANY FURTHER INFORMATION ON METRIC-MDS*/23X,
     3*PLEASE CONTACT THE AUTHORS --*,/28X,
     4*KIM SEROTA -- JOSEPH WOELFEL*/29X,
     5*DEPT OF COMMUNICATION*/30X,
     6*MICHIGAN STATE UNIVERSITY*/31X,
     7*EAST LANSING, MICH.  48824*///50X,*THANKS AGAIN*)
      RETURN
      END
      SUBROUTINE CONCARD (RUN,LABELS,CRIT,OPT,MSIZE,NSETS,KJOB,
     1 NDIM,MAIN,MIDDLE,THETA,MAXVAL)
C
C         SUBROUTINE CONCARD READS GALILEO CONTROL CARDS AND
C         SETS UP THE JOB FOR OPERATION ON DATA
C
      DIMENSION RUN(5),LABELS(40,4),CRIT(5),OPT(7),TEMP(5),KEMPTY(5),
     1          ITEMP(5),INT(5)
      INTEGER OPT,CODE
      EQUIVALENCE (TEMP(1),ITEMP(1))
      DATA KEMPTY,INT/5*0,5*0/
C
C         READ CARDS,SET PARAMETERS,AND PRINT CONTROL LIST
C
C              MAIN PARAMETERS
C
      READ 10,(RUN(I),I=1,5)
   10 FORMAT (15X,5A8)
      PRINT 405, (RUN(I),I=1,5)
  405 FORMAT (*1*,30X,* GALILEO CONTROL LIST *,//////////,16X,
     1*RUN NAME IS....................*,5A8)
      READ 20, TEMQ
   20 FORMAT (15X,F10.0)
      MSIZE=TEMQ
      PRINT 420, TEMQ
  420 FORMAT (* *,15X,*NUMBER OF CONCEPTS IS.........*,F3.0)
      READ 20, TEMQ
      NSETS=TEMQ
      PRINT 430, TEMQ
  430 FORMAT (* *,15X,*NUMBER OF SETS OF DATA IS.....*,F3.0)
C
C              OPTIONAL LABELS
C
   40 READ 30, CODE,(ITEMP(K),K=1,5)
   30 FORMAT (A10,5X,5A8)
   50 IF (CODE.NE.10HCRITERION ) GO TO 60
      DO 51 K=1,5
   51 CRIT(K)=TEMP(K)
      PRINT 440, (CRIT(K),K=1,5)
  440 FORMAT (* *,15X,*CRITERION PAIR IS.............*,5A8)
      KEMPTY(2)=1
      GO TO 100
   60 IF (CODE.NE.10HCONLABELS ) GO TO 70
      DO 61 K=1,4
   61 LABELS(1,K)=ITEMP(K)
      DO 62 K=2,MSIZE
      READ 63, (LABELS(K,L),L=1,4)
   63 FORMAT(15X,4A8)
   62 CONTINUE
      DO 451 J=1,MSIZE
      PRINT 450, J,(LABELS(J,K),K=1,4)
  450 FORMAT (* *,15X,*CONCEPT *,I2,* IS..................*,4A8)
  451 CONTINUE
      KEMPTY(3)=1
```

```
      GO TO 100
C
C               OPERATION TO BE PERFORMED
C
70    IF (CODE.NE.10HOPERATIONS) GO TO 90
      PRINT 460, (ITEMP(K),K=1,5)
  460 FORMAT (* *,15X,*OPERATION SPECIFIED IS........*,5A8)
      IF (ITEMP(1).NE.8HDISTANCE) GO TO 710
      ASSIGN 9000 TO JOB
      KJOB=90
C
C               SPECIFICATIONS AND OPTIONS
C
      READ 735, CODE,(TEMP(K),K=1,5)
  735 FORMAT (A10,5X,5(F1.0,1X))
      IF (CODE.NE.10HSPECIFICAT) GO TO 702
      KSPECS=TEMP(1)
      PRINT470,KSPECS
      READ 736, CODE,TEMQ
  736 FORMAT (A10,5X,F10.0)
      IF (CODE.EQ.10HMAXIMUM VA) GO TO 737
      PRINT 101
      GO TO 103
  737 MAXVAL=TEMQ
      PRINT 738, MAXVAL
  738 FORMAT (* *,15X,*MAXIMUM ALLOWABLE VALUE IS....*,I5)
      GO TO 71
  710 IF (ITEMP(1).NE.8HCOORDINA) GO TO 720
      ASSIGN 9100 TO JOB
      KJOB=91
      GO TO 71
  720 IF (ITEMP(1).NE.8HCOMPARIS) GO TO 790
      ASSIGN 9200 TO JOB
      KJOB=92
   72 READ 73, CODE,ITEMP(1)
   73 FORMAT (A10,5X,I1)
   74 IF (CODE.NE.10HSPECIFICAT) GO TO 75
      PRINT 470, ITEMP(1)
  470 FORMAT(* *,15X,*SPECIFICATIONS LISTED.........*,I1)
      IF (ITEMP(1)) 171,71,171
  171 IT = ITEMP(1)
      DO 76 J=1,IT
      READ 77,CODE,TEMP(1)
   77 FORMAT (A10,5X,F10.2)
      IF (CODE.NE.10HNDIMENSION) GO TO 177
      NDIM=TEMP(1)
      PRINT 480, TEMP(1)
  480 FORMAT (* *,15X,*NUMBER OF DIMENSIONS IS.......*,F2.0)
      GO TO 76
  177 IF (CODE.NE.10HMAINSPACE ) GO TO 277
      MAIN=TEMP(1)
      PRINT 481
  481 FORMAT(* *,15X,*FIRST SPACE HAS BEEN DESIGNATED AS THE CRITERION*)
      GO TO 76
  277 IF (CODE.NE.10HMIDPOINT  ) GO TO 377
      MIDDLE=TEMP(1)
      PRINT 482, MIDDLE,(LABELS(MIDDLE,L),L=1,4)
  482 FORMAT (* *,15X,*CENTROID HAS BEEN TRANSLATED TO CONCEPT *,I2,
     12X,4A8)
      GO TO 76
  377 IF (CODE.NE.10HDEGREES   ) GO TO 75
      THETA=TEMP(1)
      PRINT 483, THETA
  483 FORMAT (* *,15X,*ANGLE OF ROTATION INCREMENT...*,F4.2,* DEGREES*)
   76 CONTINUE
      GO TO 71
   75 PRINT 78
   78 FORMAT (*1*,10X,*COMPARISON ROUTINE WAS INDICATED, BUT NO SPECIFIC
     1ATIONS CARD WAS FOUND -- GALILEO TERMINATES*)
      GO TO 103
   71 READ 701, CODE,(TEMP(K),K=1,5)
  701 FORMAT (A10,5X,5(F1.0,1X))
  702 IF (CODE.NE.10HOPTIONS   ) GO TO 790
      DO 49 K=1,5
      INT(K)=TEMP(K)
   49 CONTINUE
      PRINT 490, (INT(K),K=1,5)
  490 FORMAT (* *,15X,*OPTION NUMBERS ARE............*,5(I1,1X))
      IF (TEMP(1)) 300,250,300
300   N=5
      DO 301 M=2,5
```

```
      IF (TEMP(M).EQ.0) N=M-1
      IF (N.NE.5) GO TO 302
301   CONTINUE
  302 GO TO JOB (9000,9100,9200)
9000  GO TO 250
9100  DO 310 M=1,N
      IF (TEMP(M).EQ.1.0) OPT(1)=1
      IF (TEMP(M).EQ.2.0) OPT(2)=1
      IF (TEMP(M).EQ.3.0) OPT(3)=1
      IF (TEMP(M).EQ.4.0) OPT(4)=1
      IF (TEMP(M).EQ.5.0) OPT(5)=1
310   CONTINUE
      GO TO 250
9200  DO 410 M=1,N
      IF (TEMP(M).EQ.1.0) OPT(1)=1
      IF (TEMP(M).EQ.2.0) OPT(6)=1
      IF (TEMP(M).EQ.3.0) OPT(7)=1
410   CONTINUE
250   CONTINUE
      KEMPTY(5)=1
      GO TO 100
790   PRINT 791
791   FORMAT (*1*,10X,*OPERATIONS OR OPTIONS CARD IS MISSING -- GALILEO
     +TERMINATES*)
      GO TO 103
C
C         ASSIGN DEFAULT VALUES AND RETURN TO GALILEO DRIVER
C
90    IF (CODE.EQ.10HREAD DATA ) GO TO 200
      PRINT 101
  101 FORMAT (*1*,20X,*NO INSTRUCTION HAS BEEN READ, PLEASE CHECK YOUR C
     1ONTROL CARDS AND TRY AGAIN*//80X,*THANK YOU*)
  103 CALL EXIT
  100 GO TO 40
  200 PRINT 491
  491 FORMAT (* *,15X,*READ DATA *)
      IF (KEMPTY(2)) 220,202,220
  202 CRIT(1)=8HCRITERIO
      CRIT(2)=8HN PAIR W
      CRIT(3)=8HAS NOT S
      CRIT(4)=8HPECIFIED
      CRIT(5)=8H
  220 IF (KEMPTY(3))230,203,230
  203 LABELS(1,1)=8HCONCEPTS
      LABELS(1,2)=8H ARE NOT
      LABELS(1,3)=8H LABELED
      LABELS(1,4)=8H
  230 IF (KEMPTY(5)) 260,240,260
  240 GO TO 790
  260 CONTINUE
      RETURN
      END
      SUBROUTINE MTRIX (MSIZE,NSETS,CRIT,RUN,LABELS,OPTION,MAXVAL)
C
C         SUBROUTINE MTRIX WILL READ IN RAW DATA PAIRS FROM
C         ANY GALILEO QUESTIONNAIRE USING STANDARDIZED FORMAT.
C         IT WILL COMPUTE THE MEAN VALUES MATRIX FOR THAT
C         DATA AND CAN PRINT THE MATRIX AND A SAMPLE SIZE
C         TABLE.  IN ADDITION MTRIX WILL PUNCH THE MEAN VALUES
C         MATRIX FOR FURTHER GALILEO PROCESSING NOT REQUIRING
C         RAW DATA INPUT.   THE USER SHOULD REFER TO THE GALILEO
C         ABSTRACT FOR INSTRUCTIONS ON USING THE SUBROUTINE
C
      DIMENSION FAXT(2500), RUN(5),LABELS(40,4),CRIT(5),JC(8),JR(8),
     1          DIST(8),SMEAN(40,40),Y(40,40)
      COMMON FAXT
      INTEGER OPTION,Y
C
C         READ AND CLEAN DATA - PERFORM SUMMING OPERATIONS
C
      M=0
      DO 611 K4NSETS=1,NSETS
      DO 5 K=1,MSIZE
      DO 5 L=1,MSIZE
      SMEAN(K,L)=0.0
      Y(K,L)=0
    5 CONTINUE
      PRINT 10,  (RUN(I),  I=1,5),K4NSETS
   10 FORMAT (*1*,38X,5A8,5X,*SET NUMBER*,I3,//)
      KLEAN=0
 1000 READ 20,   TEMP,(JC(I),JR(I),DIST(I),I=1,8)
```

```
   20 FORMAT (A8,8(2I2,F5.0))
  120 M=M+1
      IF (TEMP.EQ.8HENDOFSET) GO TO 110
      DO 130 I=1,8
      J=JC(I)
      K=JR(I)
      IF (J.EQ.0.AND.K.EQ.0) GO TO 130
      IF (J.GT.0.AND.J.LE.MSIZE) GO TO 128
      PRINT 30, MSIZE,M,TEMP
   30 FORMAT (* *,*A MATRIX DIMENSION LE ZERO OR GT*,I5,* WAS ENCOUNTERE
     1D ON CARD NUMBER*,I8,*, ID IS *,A8)
      KLEAN=KLEAN+1
      GO TO 130
  128 IF (K.GT.0.AND.K.LE.MSIZE) GO TO 127
      PRINT 30, MSIZE,M,TEMP
      KLEAN=KLEAN+1
      GO TO 130
  127 IF  (J.LT.K) GO TO 129
      PRINT 40, J,K,M,TEMP
   40 FORMAT (* *,*A VALUE WAS READ INTO A DIAGONAL ELEMENT OR INTO THE
     1UPPER PORTION OF THE MATRIX AT *,2I2,* FROM CARD *,I8,* ID IS *,
     2A8)
      KLEAN=KLEAN+1
      GO TO 130
  129 IF (DIST(I)-MAXVAL) 140,140,141
  141 PRINT 142, DIST(I),K,J,M,TEMP
  142 FORMAT (* *,*THE EXTREME VALUE *,F5.0,* WAS DELETED FROM MATRIX LO
     1CATION *,2I2,* FROM CARD NUMBER *,I8,*, ID IS *,A8)
      GO TO 130
  140 SMEAN(K,J)=SMEAN(K,J)+DIST(I)
      Y(K,J)=Y(K,J)+1
      IF (DIST(I)-1000) 130,130,135
  135 PRINT 136, DIST(I),K,J,M
  136 FORMAT (* *,*THE EXTREME VALUE *,F5.0,* WAS READ INTO THE MATRIX L
     1OCATION *,2I2,* FROM CARD NUMBER *,I8)
  130 CONTINUE
      GO TO 1000
  110 IF (KLEAN)111,112,111
  112 PRINT 113, K4NSETS
  113 FORMAT (*-*,10X,*DATA IN SET *,I3,* WAS PROPERLY STORED*)
C
C         CALCULATION OF MEANS MATRIX
C
  111 DO 210 J=1,MSIZE
      DO 210 K=1,J
      IF (J-K) 209,210,209
  209 ASUM=SMEAN(J,K)
      AN=Y(J,K)
      IF (AN) 231,230,231
  230 SMEAN(J,K)=0.0
      GO TO 210
  231 SMEAN(J,K)=ASUM/AN
  210 CONTINUE
C
C         STORAGE OF MATRIX AND CALCULATION OF SYMMETRY
C
      IF (OPTION) 301,301,305
  301 DO 302 J=1,MSIZE
      DO 302 K=1,J
      SMEAN(K,J)=SMEAN(J,K)
  302 CONTINUE
      GO TO 401
  305 LODEX=(MSIZE**2)*(K4NSETS-1)
      DO 310 J=1,MSIZE
      DO 310 K=1,J
      INDEX=(((J*(J-1))/2)+K)+LODEX
      FAXT(INDEX)=SMEAN(K,J)=SMEAN(J,K)
  310 CONTINUE
C
C         MATRIX PRINTOUT OPERATION
C
  401 LSIZE=MSIZE/11
      IF ((LSIZE*11).LT.MSIZE) LSIZE=LSIZE+1
      DO 410 L=1,LSIZE
      K1=11*(L-1)+1
      K2=K1+10
      IF (K2.GT.MSIZE) K2=MSIZE
      PRINT 50, (RUN(JK),JK=1,5),K4NSETS
   50 FORMAT (*1*,35X,5A8,2X,*--GALILEO MEANS MATRIX*,22X,*SET NO. *,I3)
      PRINT 60, (K,K=K1,K2)
   60 FORMAT (*0*,11(1X,I11)/)
```

```
      DO 410 J=K1,MSIZE
      K3=J
      IF (K3.GT.K2) K3=K2
      PRINT 80, J,(SMEAN(J,K),K=K1,K3)
   80 FORMAT (* *,I2,11(1X,F11.3))
  410 CONTINUE
C
C         SAMPLE SIZE TABULATION PRINTOUT OPERATION
C
      DO 510 L=1,LSIZE
      J1=11*(L-1)+1
      J2=J1+10
      IF (J2.GT.MSIZE) J2=MSIZE
      PRINT 90, (RUN(JK),JK=1,5)
   90 FORMAT (*1*,32X,5A8,2X,*-- SAMPLE SIZE FOR EACH PAIR*)
      PRINT 60,(J,J=J1,J2)
      DO 510 K=J1,MSIZE
      J3=K
      IF (J3.GE.J2) J3=J2
      PRINT 95, K,(Y(K,J),J=J1,J3)
   95 FORMAT (* *,I3,11(1X,I11))
  510 CONTINUE
C
C         MATRIX PUNCHING OPERATION
C
      DO 610 K=1,MSIZE
      PUNCH 11, (SMEAN(J,K),J=1,MSIZE)
   11 FORMAT (8F10.4)
  610 CONTINUE
  611 CONTINUE
      PRINT 620, (RUN(K),K=1,5),MSIZE,NSETS,(CRIT(K),K=1,5)
  620 FORMAT (*1*,48X,5A8,//,44X,*AN ANALYSIS OF *,I2,* CONCEPTS IN *,I3
     1,* SETS OF DATA*,//,36X,*THE CRITERION PAIR IS-- *,5A8)
      PRINT 630
  630 FORMAT (*-*,5X,*THE CONCEPTS ARE--*,/)
      DO 640 K=1,MSIZE
      PRINT 650, K,(LABELS(K,L),L=1,4)
  650 FORMAT (* *,6X,I2,3X,4A8)
  640 CONTINUE
      RETURN
      END
      SUBROUTINE CENTRIX (SCALAR,OPT,MSIZE,INCMENT,RUN)
C
C         SUBROUTINE CENTRIX WILL COMPUTE A DOUBLE CENTERED
C         CENTROID SCALAR PRODUCTS MATRIX ACCORDING TO THE
C         ALGORITHM PROVIDED BY YOUNG AND HOUSEHOLDER (1938)
C         AND ADAPTED BY TORGERSON (1958)
C
      DIMENSION FAXT(2500), SCALAR(40,40),OPT(7),XMEAN(8),ZMEAN(40),
     1          SMEAN(40,40),SQJ(40),SQK(40),RUN(5)
      COMMON FAXT
      INTEGER OPT
      DO 5 K=1,MSIZE
      SQJ(K)=0.0
      SQK(K)=0.0
      DO 5 L=1,MSIZE
      SCALAR(K,L)=0.0
      SMEAN(K,L)=0.0
    5 CONTINUE
C
C         READ DATA AND/OR ARRAY MEANS MATRIX
C
      IF (OPT(1)) 10,10,11
   10 KRAP=MSIZE/8
      KOMP=KRAP*8
      IF (MSIZE.GT.KOMP) KRAP=KRAP+1
      DO 20 K=1,MSIZE
      DO 30 L=1,KRAP
      READ 100,(XMEAN(LA),LA=1,8)
  100 FORMAT (8F10.4)
      DO 30 LB=1,8
      LC=(8*(L-1))+LB
      ZMEAN(LC)=XMEAN(LB)
   30 CONTINUE
      DO 20 J=1,MSIZE
      SMEAN(J,K)=SMEAN(K,J)=ZMEAN(J)
   20 CONTINUE
      GO TO 12
C
   11 LODEX=(MSIZE**2)*(INCMENT-1)
      DO 15 J=1,MSIZE
```

```
      DO 15 K=1,J
      INDEX=((J*(J-1))/2)+K+LODEX
      SMEAN(J,K)=SMEAN(K,J)=FAXT(INDEX)
   15 CONTINUE
C
C          SQUARE ALL MATRIX ELEMENTS
C
   12 DO 50 J=1,MSIZE
      DO 50 K=1,MSIZE
      SMEAN(J,K)=SMEAN(J,K)**2
   50 CONTINUE
C
C          COMPUTE SUM OF SQUARES FOR THE ROWS
C
      SSQ=0.0
      DO 60 J=1,MSIZE
      DO 60 K=1,MSIZE
      SSQ=SSQ+SMEAN(J,K)
      SQJ(J)=SQJ(J)+SMEAN(J,K)
   60 CONTINUE
C
C          COMPUTE SUM OF SQUARES FOR THE COLUMNS
C
      IF (OPT(3)) 611,611,610
  610 PRINT 603,  (RUN(L),L=1,5),INCMENT,SSQ
  603 FORMAT (*1*,20X,5A8,* - CENTROID SCALAR PRODUCTS MATRIX FOR DATA S
     1ET *,I3,/////////30X,* SUM OF SQUARES FOR ALL MATRIX VALUES EQUAL
     2S *,F15.4)
  611 DO 70 K=1,MSIZE
      DO 70 J=1,MSIZE
      SQK(K)=SQK(K)+SMEAN(J,K)
   70 CONTINUE
C
C          COMPUTE EACH ELEMENT OF THE MATRIX
C
      N=MSIZE
      NSQD=N**2
      DO 80 J=1,MSIZE
      DO 80 K=1,MSIZE
      SCALAR(J,K)=((SQJ(J)/N)+(SQK(K)/N)-(SSQ/NSQD)-SMEAN(J,K))/2
   80 CONTINUE
C
C          STORE OR OUTPUT THE CENTROID SCALAR PRODUCTS MATRIX
C
      IF (OPT(4)) 613,613,612
  612 DO 90 K=1,MSIZE
      PUNCH 200,  (SCALAR(J,K),J=1,MSIZE)
  200 FORMAT (6F12.3)
   90 CONTINUE
C
  613 IF (OPT(3))615,615,614
  614 LSIZE=MSIZE/11
      IF ((LSIZE*11).LT.MSIZE) LSIZE=LSIZE+1
      DO 110 L=1,LSIZE
      K1=(11*(L-1))+1
      K2=K1+10
      IF (K2.GT.MSIZE) K2=MSIZE
      PRINT 300,INCMENT
  300 FORMAT (*1*,40X,*CENTROID SCALAR PRODUCTS MATRIX - DATA SET *,I3/)
      PRINT 400,  (K,K=K1,K2)
  400 FORMAT (*0*,11(1X,I11),/)
      DO 110 J=K1,MSIZE
      K3=J
      IF (K3.GT.K2) K3=K2
      PRINT 500, J,(SCALAR(J,K),K=K1,K3)
  500 FORMAT (* *,I2,11(1X,F11.3))
  110 CONTINUE
  615 CONTINUE
      RETURN
      END
      SUBROUTINE EIGTRIX (SCALAR,OPT,MSIZE,INCMENT,KJOB)
C
C          SUBROUTINE EIGTRIX WILL COMPUTE CHARACTERISTIC
C          ROOTS AND VECTORS OF A MATRIX OF UNSTANDARDIZED
C          VALUES USING THE ITERATIVE ADAPTATION OF THE DETERMINANT
C        METHOD SUPPLIED BY VAN DE GEER (1972).  ALL NEGATIVE
C        EIGENVALUES ARE MADE AVAILABLE TO THE USER. (THE USER
C          SHOULD NOTE THAT THIS IS AN EXACT SOLUTION.  ITERATION
C          PROVIDES A MORE RAPID AND ECONOMICAL METHOD FOR
C          DERIVING THE LATENT STRUCTURE THAN DOES POLYNOMIAL
C        EXPANSION.)
```

```
C
      COMMON FAXT
      INTEGER OPT
      DIMENSION SCALAR(40,40),FAXT(2500),OPT(7),FAX(40,40),ROOT(40),
     1          CENTX(6),VT(40),C(40),D(40),Y(40,40),Z(40,40),ITER(40),
     2          PTOT(40),PREAL(40)
      DO 5 K=1,MSIZE
      DO 5 L=1,MSIZE
      FAX(K,L)=0.0
      Y(K,L)=0.0
      Z(K,L)=0.0
    5 CONTINUE
C
C          READ AND/OR ARRAY SCALAR PRODUCTS MATRIX
C
      IF (OPT(2))601,601,600
  600 LSIZE=MSIZE/6.+.9
      DO 20 K=1,MSIZE
      DO 20 L=1,LSIZE
      READ 30, (CENTX(J),J=1,6)
   30 FORMAT (6F12.3)
      DO 20 JA=1,6
      JB=(6*(L-1))+JA
      IF(JB-MSIZE)200,200,20
  200 SCALAR(K,JB)=CENTX(JA)
   20 CONTINUE
  601 KOUNT=1
C
C          QUINQUE-EXPONENTIATE THE MATRIX
C
 1000 DO 13 J=1,4
      DO 12 K=1,MSIZE
      DO 12 L=1,MSIZE
      Y(K,L)=0.0
      DO 12 M=1,MSIZE
      IF(J-1)210,205,210
  205 Z(K,M)=SCALAR(K,M)
  210 Y(K,L)=Y(K,L)+Z(K,M)*SCALAR(M,L)
   12 CONTINUE
      DO 13 K=1,MSIZE
      DO 13 L=1,MSIZE
      Z(K,L)=Y(K,L)
   13 CONTINUE
C
C          ITERATE TO EQUIVALENCE PRODUCT VECTORS
C
   26 DO 40 J=1,MSIZE
      VT(J)=1.0
   40 CONTINUE
      ITER(KOUNT)=1
  120 DO 50 M=1,MSIZE
      C(M)=0.0
      DO 50 N=1,MSIZE
      C(M)=C(M)+Z(M,N)*VT(N)
   50 CONTINUE
      DIV=C(1)
      ITER(KOUNT)=ITER(KOUNT)+1
      NT=MSIZE-1
      DIV2=ABS(DIV)
      DO 60 J=1,NT
      IF(ABS(C(J+1))-DIV2)60,57,57
   57 DIV=C(J+1)
      DIV2=ABS(DIV)
   60 CONTINUE
      DO 70 K=1,MSIZE
      D(K)=C(K)/DIV
   70 CONTINUE
      T=.0001
      KA=1
   81 IF (D(KA)+T.GE.VT(KA).AND.D(KA)-T.LE.VT(KA)) GO TO 80
      GO TO 90
   80 IF(KA-MSIZE)85,110,85
   85 KA=KA+1
      GO TO 81
   90 DO 100 K=1,MSIZE
      VT(K)=D(K)
  100 CONTINUE
      GO TO 120
C
C          RETRIEVE ROOT AND NORMALIZE VECTOR
C
```

```
  110 DIV=SIGN(ABS(DIV)**.2,DIV)
      ROOT(KOUNT)=DIV
      SUMC=0.0
      DO 130 K=1,MSIZE
      SUMC=SUMC+D(K)**2
  130 CONTINUE
      P=SQRT(SUMC)
      DO 140 K=1,MSIZE
      D(K)=(D(K)/P)*SQRT(ABS(DIV))
      FAX(K,KOUNT)=D(K)
  140 CONTINUE
C
C          COMPUTE RESIDUAL MATRIX
C
      DO 150 K=1,MSIZE
      DO 150 L=1,MSIZE
      IF (DIV.LT.0.0) GO TO 151
      SCALAR(K,L)=SCALAR(K,L)-(FAX(K,KOUNT)*FAX(L,KOUNT))
      GO TO 150
  151 SCALAR(K,L)=SCALAR(K,L)+(FAX(K,KOUNT)*FAX(L,KOUNT))
  150 CONTINUE
      KOUNT=KOUNT+1
      IF(KOUNT-MSIZE)1000,1000,215
C
C          SORT VECTORS FROM ABSOLUTE TO REAL ORDER
C
  215 NUMLES1=MSIZE-1
      NDEX=NUMLES1
      DO 4000 IXD=1,NDEX
      DO 3000 IXE=1,NUMLES1
      IF (ROOT(IXE).GE.ROOT(IXE+1)) GO TO 3000
      TMP=ROOT(IXE)
      ROOT(IXE)=ROOT(IXE+1)
      ROOT(IXE+1)=TMP
      ITMP=ITER(IXE)
      ITER(IXE)=ITER(IXE+1)
      ITER(IXE+1)=ITMP
      DO 2050 J=1,MSIZE
      TMP=FAX(J,IXE)
      FAX(J,IXE)=FAX(J,IXE+1)
 2050 FAX(J,IXE+1)=TMP
 3000 CONTINUE
      NUMLES1=NUMLES1-1
 4000 CONTINUE
C
C          COMPUTE CUMULATIVE PERCENTAGES OF DISTANCE
C
      SREAL=0.0
      STOT=0.0
      DO 525 K=1,MSIZE
      STOT=STOT+ROOT(K)
      IF(ROOT(K))525,525,524
  524 SREAL=SREAL+ROOT(K)
  525 CONTINUE
      PTOT(1)=(ROOT(1)/STOT)*100.0
      PREAL(1)=(ROOT(1)/SREAL)*100.0
      DO 530 K=2,MSIZE
      PTOT(K)=PTOT(K-1)+((ROOT(K)/STOT)*100.0)
      PREAL(K)=PREAL(K-1)+((ROOT(K)/SREAL)*100.0)
  530 CONTINUE
C
C          STORE AND OUTPUT COORDINATE MATRIX
C
      IF (KJOB-91) 190,91,92
  190 ASSIGN 9000 TO JOB
      GO TO 95
   91 ASSIGN 9100 TO JOB
      GO TO 95
   92 ASSIGN 9200 TO JOB
   95 CONTINUE
      GO TO JOB (9000,9100,9200)
 9200 LODEX=(MSIZE**2)*(INCMENT-1)
      DO 625 J=1,MSIZE
      DO 625 K=1,MSIZE
      INDEX=((J-1)*MSIZE)+K+LODEX
      FAXT(INDEX)=FAX(J,K)
  625 CONTINUE
 9000 CONTINUE
 9100 IF (OPT(7)) 610,610,611
610   LSIZE=MSIZE/11
      IF((LSIZE*11)-MSIZE)281,290,290
```

```
  281 LSIZE=LSIZE+1
  290 DO 300 L=1,LSIZE
      KA=(11*(L-1))+1
      KB=KA+10
      IF(KB-MSIZE)291,291,292
  292 KB=MSIZE
  291 PRINT 240, MSIZE,INCMENT
  240 FORMAT (*1*,25X,* GALILEO COORDINATES OF *,I2,* VARIABLES IN A MET
     1RIC MULTIDIMENSIONAL SPACE FOR DATA SET *,I3/)
      PRINT 250, (M,M=KA,KB)
  250 FORMAT (*0*,11(1X,I11)/)
      DO 301 J=1,MSIZE
      PRINT 62, J,(FAX(J,K),K=KA,KB)
   62 FORMAT (* *,I2,11(1X,F11.3))
  301 CONTINUE
      PRINT 61, (ROOT(K),K=KA,KB)
   61 FORMAT (/,* *,5X,*EIGENVALUES (ROOTS) OF EIGENVECTOR MATRIX-- *,/,
     12X,11(1X,F11.3))
      PRINT 63, (ITER(K),K=KA,KB)
   63 FORMAT (/,* *,5X,* NUMBER OF ITERATIONS TO DERIVE THE ROOT-- *,/,
     1    2X,11(5X,I4,3X))
      PRINT 64,(PREAL(K),K=KA,KB)
   64 FORMAT(/,* *,5X,* CUMULATIVE PERCENTAGES OF REAL DISTANCE ACCOUNTE
     1D FOR--1*,/,2X,11(1X,F11.3))
      PRINT 65, (PTOT(K),K=KA,KB)
   65 FORMAT (/,* *,5X,* CUMULATIVE PERCENTAGES OF TOTAL (REAL AND IMAGI
     1NARY) DISTANCE ACCOUNTED FOR-- *,/,2X,11(1X,F11.3))
  300 CONTINUE
C
C          PUNCH MATRICES
C
      DO 400 K=1,MSIZE
      PUNCH 270, (FAX(K,J),J=1,MSIZE)
  270 FORMAT (6F12.4)
  400 CONTINUE
      IF (OPT(5)) 611,611,650
  650 DO 500 K=1,MSIZE
      PUNCH 280, (FAX(K,J),J=1,3),K
  280 FORMAT (3(F10.4,5X),33X,I2)
  500 CONTINUE
  611 CONTINUE
      RETURN
      END
      SUBROUTINE COMPARE (OPT,LABELS,MSIZE,NDIM,NSETS,MAIN,MIDDLE,THETA)
C
C          SUBROUTINE COMPARE WILL PROVIDE COORDINATES OF SPACES
C          ROTATED TO LEAST SQUARES CRITERIA (SUBJECT TO REVISION)
C
      COMMON FAXT
      INTEGER OPT
      DIMENSION FAXT(2500),OPT(7),LABELS(40,4),TEMP(6),CSPACE(40,9),
     1PSPACE(40,9)
      DATA CSPACE,PSPACE/360*0.0,360*0.0/
C
C          SET PARAMETERS AND INPUT TARGET MATRIX
C
      IF (THETA) 30,31,30
   31 THETA=1.0
   30 IF (MIDDLE.EQ.0) GO TO 3
      PRINT 362, MIDDLE,THETA
  362 FORMAT (*1*,10X,*THE ORIGIN OF THE SPACE IS CENTERED ON CONCEPT NU
     1MBER*,I3,///,10X,*THE ANGLE OF ROTATIONAL INCREMENT IS*,F5.2,
     2* DEGREES*)
      GO TO 4
    3 PRINT 5, THETA
    5 FORMAT (*1*,10X,*THE ORIGIN IS AT THE CENTRIOD*,///,10X,*THE SOLUT
     1ION IS ACCURATE TO WITHIN *,F5.2,* DEGREES*)
    4 THETA=THETA/57.2957795
      KOUNT=1
      IF (OPT(6)-1) 500,502,500
  500 DO 501 J=1,MSIZE
      DO 501 K=1,NDIM
      INDEX=((J-1)*MSIZE)+K
      CSPACE(J,K)=FAXT(INDEX)
  501 CONTINUE
      GO TO 550
  502 LSIZE=MSIZE/6
      IF ((LSIZE*6).LT.MSIZE) LSIZE=LSIZE+1
      NSIZE=NDIM/6
      IF ((NSIZE*6).LT.NDIM)  NSIZE=NSIZE+1
      JSIZE=LSIZE-NSIZE
```

```
      DO 505 J=1,MSIZE
      DO 504 K=1,NSIZE
      READ 506,(TEMP(L),L=1,6)
  506 FORMAT (6F12.4)
      DO 503 L=1,6
      NDEX=((K-1)*6)+L
      IF (NDIM-NDEX) 504,507,507
  507 CSPACE(J,NDEX)=TEMP(L)
  503 CONTINUE
  504 CONTINUE
      IF (JSIZE) 508,516,508
  508 DO 515 M=1,JSIZE
      READ 509, (TEMP(L),L=1,6)
  509 FORMAT (6F12.4)
  515 CONTINUE
  516 CONTINUE
  505 CONTINUE
  550 PRINT 6
    6 FORMAT (*1*,35X,*THE COORDINATES OF SPACE NUMBER 1*,///)
      DO 7 J=1,MSIZE
      PRINT 90, J,(LABELS(J,N),N=1,4),(CSPACE(J,M),M=1,NDIM)
   90 FORMAT (* *,1X,I2,1X,4A8,5X,9F9.4)
      PUNCH 91, (CSPACE(J,K),K=1,3),J
   91 FORMAT (3(F10.4,5X),33X,I2)
    7 CONTINUE
C
C          INPUT MATRIX TO BE ROTATED
C
   10 IF (KOUNT.GE.NSETS) GO TO 20
      KOUNT=KOUNT+1
      IF (OPT(6)-1) 600,602,600
  600 LODEX=(MSIZE**2)*(KOUNT-1)
      DO 601 J=1,MSIZE
      DO 601 K=1,NDIM
      INDEX=((J-1)*MSIZE)+K+LODEX
      PSPACE(J,K)=FAXT(INDEX)
  601 CONTINUE
      GO TO 650
  602 DO 605 J=1,MSIZE
      DO 604 K=1,NSIZE
      READ 606,(TEMP(L),L=1,6)
  606 FORMAT (6F12.4)
      DO 603 L=1,6
      NDEX=((K-1)*6)+L
      IF (NDIM-NDEX) 604,607,607
  607 PSPACE(J,NDEX)=TEMP(L)
  603 CONTINUE
  604 CONTINUE
      IF (JSIZE) 608,616,608
  608 DO 615 M=1,JSIZE
      READ 609, (TEMP(L),L=1,6)
  609 FORMAT (6F12.4)
  615 CONTINUE
  616 CONTINUE
  605 CONTINUE
C
C          CALL ROTATE-- LEAST SQUARES ALGORITHM
C
  650 CALL ROTATE (CSPACE,PSPACE,THETA,MSIZE,NDIM)
C
C          OUTPUT ROTATED SPACE
C
      PRINT 201, KOUNT
  201 FORMAT (*1*,35X,*THE COORDINATES OF SPACE NUMBER *,I2,///)
      DO 660 J=1,MSIZE
       PRINT 90,J,(LABELS(J,N),N=1,4),(PSPACE(J,M),M=1,NDIM)
      PUNCH 91, (PSPACE(J,K),K=1,3),J
  660 CONTINUE
C
C          COMPUTE AND PRINT INTERVALS OF CHANGE
C
      IF (MAIN) 671,671,670
  670 JOUNT=1
      GO TO 672
  671 JOUNT=KOUNT-1
  672 PRINT 14, JOUNT,KOUNT
   14 FORMAT (*1*,35X,*DISTANCES MOVED IN THE INTERVAL BETWEEN TIME *,I2
     1,* AND TIME *,I2,////////)
      DIST=0.0
      DO 15 J=1,MSIZE
      SUMP=0.0
```

```
      DO 16 K=1,NDIM
      PYTH=((CSPACE(J,K)-PSPACE(J,K))**2)
      SUMP=SUMP+PYTH
   16 CONTINUE
      THAG=SQRT(SUMP)
      PRINT 17, J,(LABELS(J,L),L=1,4),THAG
   17 FORMAT (* *,10X,*CONCEPTS *,I2,* (*,4A8,*) MOVED *,F10.4,* UNITS*)
      DIST=DIST+THAG
   15 CONTINUE
      DIST=DIST/MSIZE
      PRINT 18, JOUNT,KOUNT,DIST
   18 FORMAT (* *,//,5X,*THE MEAN DISTANCE BETWEEN ALL POINTS IN SPACE *
     1,I2,* AND THEIR COUNTERPARTS IN SPACE *,I2,* IS *,F10.5)
      IF (MAIN) 25,25,10
   25 DO 19 J=1,MSIZE
      DO 19 K=1,NDIM
      CSPACE(J,K)=PSPACE(J,K)
   19 CONTINUE
      GO TO 10
   20 CONTINUE
      RETURN
      END
      SUBROUTINE ROTATE (TARGET,MATRIX,ALPHA,M,N)
C
C         SUBROUTINE ROTATE SUPPLIES THE ALGORITHM NECESSARY
C         TO ROTATE SPACES TO THE LEAST SQUARES CRITERIA
C         AXES ARE ROTATED PAIR-WISE IN THEIR OWN PLANE
C
      DIMENSION MATRIX(40,9),TARGET(40,9)
      REAL MATRIX
      COSA=COS (ALPHA)
      SINA=SIN (ALPHA)
      MCHK=0
  700 SDSQ=0.0
      SDSQO=0.0
C
C         COMPUTE SUM OF SQUARED DISTANCES
C
      L=N-1
      DO 710 J=1,L
      JJ=J+1
      DO 702 K=JJ,N
      DO 703 I=1,M
      DSQ=(TARGET(I,J)-MATRIX(I,J))**2+(TARGET(I,K)-MATRIX(I,K))**2
703   SDSQO=SDSQO+DSQ
      GO TO 705
706   SDSQO=SDSQ
      SDSQ=0.0
705   DO 704 I=1,M
      AP=MATRIX(I,J)*COSA-MATRIX(I,K)*SINA
      BP=MATRIX(I,J)*SINA+MATRIX(I,K)*COSA
      MATRIX(I,J)=AP
      MATRIX(I,K)=BP
      DSQ=(TARGET(I,J)-MATRIX(I,J))**2+(TARGET(I,K)-MATRIX(I,K))**2
      SDSQ=SDSQ+DSQ
704   CONTINUE
C
C         COMPARE AND MINIMIZE SUMS OF SQUARES
C
      IF(SDSQ.LT.SDSQO)GO TO 706
707   SDSQO=SDSQ
      SDSQ=0.0
      DO 709 I=1,M
      AP=MATRIX(I,J)*COSA+MATRIX(I,K)*SINA
      BP=MATRIX(I,K)*COSA-MATRIX(I,J)*SINA
      MATRIX(I,J)=AP
      MATRIX(I,K)=BP
      DSQ=(TARGET(I,J)-MATRIX(I,J))**2+(TARGET(I,K)-MATRIX(I,K))**2
      SDSQ=SDSQ+DSQ
709   CONTINUE
      IF(SDSQ.LT.SDSQO) GO TO 707
      SDSQO=0.0
      SDSQ=0.0
702   CONTINUE
710   CONTINUE
      IF (MCHK) 40,50,40
C
C         RIGHT-LEFT REVERSAL TO MINIMIZE DISTANCES
C
   50 DO 720 K=1,N
      BSQ=0.0
```

```
      SBSQ=0.0
      BNSQ=0.0
      SBNSQ=0.0
      DO 730 J=1,M
      BSQ=(TARGET(J,K)-MATRIX(J,K))**2
      SBSQ=SBSQ+BSQ
      BNSQ=(TARGET(J,K)+MATRIX(J,K))**2
      SBNSQ=SBNSQ+BNSQ
  730 CONTINUE
      IF (SBSQ-SBNSQ) 720,720,60
   60 DO 740 J=1,M
      MATRIX(J,K)=(-MATRIX(J,K))
  740 CONTINUE
  720 CONTINUE
      MCHK=1
      GO TO 700
   40 CONTINUF
      RETURN
C
C     ******************************************************************
C          END OF GALILEO SUBROUTINE FILE
C     ******************************************************************
C
      END
```