THREE ESSAYS ON COMPETITION UNDER UNCERTAINTY

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ABSTRACT

THREE ESSAYS ON COMPETITION AND REGULATION

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Chapter 1: Designing Contests with Heterogeneous Agents

This paper studies how to design a contest between agents with heterogeneous abilities under the uncertainty of their performances. We find that the level effort crucially depends on marginal winning probabilities of effort, more rigorously, on the probability density of the expected output gap. In particular, we emphasize that the contest mechanism with heterogeneous agents should be qualitatively different from that with homogeneous ones. The principal often chooses to adopt a worse monitoring technology, to assign less positively correlated tasks, and to announce a winner only if the agent outdoes the rival conspicuously. These schemes are beneficial to the principal only when the agent's abilities are sufficiently different.

Chapter 2: Entry Decision with Tying under Quality Uncertainty and Switching Costs

This paper studies primary market monopolist's entry decision into the competitive subsidiary market through tying strategy, in which both primary and subsidiary goods are non-depreciating durables with periodically upgraded. Under the quality uncertainty and switching costs in the subsidiary market, we show that, as switching cost goes up, a

primary market monopolist would be more likely to make an early-entry into the subsidiary market to capture future profits from periodic upgrades. From a policy perspective, this result implies that, when antitrust authorities decide whether or not to prohibit primary market monopolist's tying behavior, they have to consider a technical aspect of the good as well that determines the size of switching cost. If switching cost is high, they need to scrutinize tying behavior more strictly at the early stage of market evolution. On the contrary, if the switching cost is low they need to pay more attention as the market progresses more.

Chapter 3: Customer Return Policy as a Signal of Quality

This paper presents a signaling model in which the length of return period is used as a signaling device for product quality. Without consumer's interim benefit, we show that there exist multiple separating equilibria, where a seller with a high-quality good offers a longer return period than a specific minimum level while a seller with a low-quality good does not offer return service. All the separating equilibria satisfy the Cho-Kreps intuitive criterion. We find, however, no pooling equilibrium exists since any pooling strategy would be dominated by high-quality seller's deviation to the strategy offering a perfect-information price and a maximum refund period. With interim benefits there could be multiple separating equilibria, but the smallest return period among them satisfies the intuitive criterion. Multiple pooling equilibria could also exist and not all of them would necessarily be eliminated by the intuitive criterion.

Dedicated to my parents, sisters and beloved Jin Young

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Chapter 1

Designing Contests with Heterogeneous Agents

1.1 Introduction

A rank-order payment scheme has been widely used when a principal interacts with multiple agents. Compared with piece-rate payments, it has several virtues as a relative evaluation scheme. When there are common shocks, a rank-order payment scheme dominates a piece-rate scheme by filtering them out (Green and Stokey; 1983). It also reduces principal's incentive to betray agents by undervaluing their performances or reneging on the contract *ex post* (Malcomson; 1984). Moreover, intrinsically, some competitions cannot be specified without a tournament or contest mechanism, such as promotion, R&D races, or sport events.

When a principal adopts a rank-order reward scheme, she endeavors to design a contest in a way to enhance agents' efforts, thereby to increase total outputs. A main purpose of this paper is to study how the principal can manage competition to induce more favorable outcomes under this reward scheme. We set up a model, in which two agents with heterogeneous abilities compete to win the prizes with some uncertainties of their performances and the principal has imperfect observability on their efforts. The important finding from this basic model is that the level of agents' effort is determined by their marginal winning probability of effort, or, more specifically, it depends crucially on the probability density of the output gap on their competitive front. Moreover we find that total outputs are proportional to the level of output gap at the equilibrium. Therefore the principal can induce more outputs by adjusting factors that affect the probability density of the output gap.

Keeping those results in mind, we made a further analysis of three representative ways that could affect contest mechanism, and found some interesting and empirically testable results. The most striking result is related to the choice of monitoring technology. We argue that principal's attempt to improve monitoring technology, which reduces uncertainty of the contest, does not necessarily increase total outputs. In other words, asymmetric agents may make more efforts when they are monitored less intensively rather than more intensively, especially when they have a large enough gap in their abilities. Actually the improvement of monitoring technology affects the contest mechanism in two ways. On the one hand it increases principal's valuation of agents' efforts. This means that, with a little more efforts, agents can persuade the principal more convincingly than before that he deserves to receive a winner's prize. Therefore both high-ability and low-ability agents would have an incentive to make more efforts. On the other hand, it makes the outcome of the contest more obvious. This implies that the high-ability agent does not need to dominate his contestant anymore with a large output margin while the low-ability agent gives up too quickly. Therefore both agents would have an incentive to make less effort as well. We call the former a substitution effect and the latter an income effect. When the two agents are sufficiently heterogeneous, a negative income effect dominates a positive substitution effect. Therefore a better monitoring technology may reduce agents' efforts and bring less total outputs to the principal. This result is sharply contrasting with the idea of traditional moral hazard literature, in which a moral hazard problem stems basically from the unobservability on agent's action. In this paper, however, we argue that reducing uncertainty by better observing agents' action can lead to a more severe moral hazard problem, especially when there is large heterogeneity in agents' abilities.

Another interesting result is on how a principal assigns jobs to competing agents. When the principal has various tasks, she may choose to assign positively, negatively or independently correlated tasks to them. We show that the principal always prefers to assign more positively correlated tasks to symmetric agents and less positively correlated tasks to asymmetric agents. This is because assigning highly positively correlated tasks leads to increasing observability on the difference between two agents. Technically, assigning more positively correlated tasks to them is equivalent to choosing the better monitoring technology. Thus, by the same reason above, it would be better for the principal to assign less positively correlated tasks if agents are more asymmetric in their abilities.

The last result we found is on what we call 'the margin rule.' Sometimes a principal requires agents to outdo her competitors sufficiently to be the winner. In other words, to win the contest, an agent must do better than their rivals by a large enough margin. Unscheduled promotion or a huge bonus for outstanding job performance among agents can be exemplified in this context. We show that this margin rule may increase the efforts of heterogeneous agents while it is not optimal for symmetric agents. The reason is that it stimulates a low-ability agent by giving him a relative advantage while spurring a high-ability agent by penalizing his shirking. All these three applications, that is, choosing a monitoring technology, assigning jobs and using the margin rule, are at times observed in the real world and cannot be explained without considering the essential characteristics of contest between heterogeneous agents. We explain more in detail in remaining sections.

Our model is developed based on Lazear and Rosen (1981). This seminal paper addresses the contest mechanism in the presence of costly monitoring for worker's efforts. They show that a rank-order payment scheme can be optimal if agents are homogeneous while it is no longer true if the agents have heterogeneous abilities.[1] Meanwhile they pay little attention

[1] Yet Bhattacharya and Guasch (1988) show that if wages can be made contingent on performance the tournament can again attain the first-best outcome. A crucial premise in their model is that a principal is assumed to know the heterogeneity of abilities across agents. That is, the principal knows who is a high-ability agent or a low-ability agent, and can offer

to the problem of designing a better contest mechanism, which is the issue we focus on in this paper. In the rent-seeking literature, there are several papers that discuss implications of player asymmetry; Katz et al. (1990), Baik (1994), and Nti (1999). In most of them the source of asymmetry is players' different valuations, not their abilities themselves. Moreover they study the properties of the Nash equilibrium efforts and payoffs in different settings rather than an issue of designing a contest mechanism. Regarding Section 3 to 5, there are also a lot of papers in the literature of contest design. For example, Che and Gale (1998) analyze the effect of caps on bidding and Moldovanu and Sela (2001) study the way of allocating prizes. Compared to them, the issues addressed in this paper are the change of monitoring technology, job assignment, and the margin rule. The brief review of each issue and our contribution to the literature will be discussed below.

The remaining parts of this paper are as follows. In section 2, we analyze basic model, focusing on the difference between symmetric and asymmetric cases. In section 3, we study the choice of monitoring technologies linked with moral hazard issues and provide a striking result different from traditional moral hazard literature. In section 4 and 5, we explore job assignment and the margin rule in turn. In section 6, we present concluding remarks.

1.2 Model

There are two agents, $i = A, B$, who contest fixed prizes. A principal awards v^w to the winner and v^L to the loser. The output of agent i is $q_i = \alpha_i x_i + \epsilon_i$, where x_i indicates the level of effort. Each agent's ability is parameterized by α_i, the marginal products of an effort.[2] Each α_i is known to both agents and a principal, but the principal does not know which agent has the higher ability than the other. A random shock, $\epsilon_i \in (-\infty, \infty)$, is

different prizes to different agents. On the other hand, we still assume that a principal can offer only a uniform and fixed prize to them irrespective of their types. This assumption is more reasonable in the example of promotion and R&D race.

[2]Alternatively, we can also represent the heterogeneity of agents' abilities with different marginal costs of exerting their efforts. All the results below are equivalent in both ways.

drawn from a known symmetric distribution with mean 0 and variance σ^2, and distributed independently and identically across the agents. What ϵ_i implies here is that the principal can observe the level of each agent's effort with some uncertainty. It may come from agent's production luck or principal's measurement error.

The winner of the contest is the agent who produces more outputs than the other. Then the probability for the agent A to win can be written as

$$\begin{aligned} \Pr(q_A > q_B) &= \Pr(\alpha_A x_A - \alpha_B x_B > \epsilon_B - \epsilon_A) = \Pr(\alpha_A x_A - \alpha_B x_B > \epsilon) \\ &= G(\alpha_A x_A - \alpha_B x_B) \equiv G(\theta) \end{aligned}$$

where θ is defined as the difference in outputs between agent A and B, or simply an output gap, and $G(\cdot)$ is the symmetric distribution of $\epsilon \equiv (\epsilon_B - \epsilon_A)$. Since ϵ_A and ϵ_B are *i.i.d.*, we obtain $E(\epsilon) = 0$ and $Var(\epsilon) = \sigma_\epsilon^2 \equiv 2\sigma^2$. Correspondingly, the probability for the agent A to lose is $[1 - G(\alpha_A x_A - \alpha_B x_B)]$. Both agents have the same cost function $C(x)$ with $C' > 0$ and $C'' > 0$. Without loss of generality, we consider the following cost function, $C(x_i) = \frac{\gamma}{2} x_i^2$, whose marginal cost is $C'(x_i) = \gamma x_i$. Then agent A's maximization problem at a given degree of uncertainty can be written as

$$\underset{x_A}{Max} \qquad G(\alpha_A x_A - \alpha_B x_B)v^w + [1 - G(\alpha_A x_A - \alpha_B x_B)]v^l - \frac{\gamma}{2} x_A^2.$$

The first-order condition is

$$\alpha_A (v^w - v^l) g(\alpha_A x_A - \alpha_B x_B) - \gamma x_A = 0.$$ [3]

From this condition, we can find four factors that determine agent's optimal level of efforts. Obviously, the effort of agent A is increasing in $(v^w - v^l)$, the winner's gain, and decreasing in

[3] The second-order condition is $\alpha_A^2 (v^w - v^l) g'(\alpha_A x_A - \alpha_B x_B) - \gamma < 0$. This condition is always satisfied if we further assume $g'(\theta)$ is positive for $\theta < 0$ and negative for $\theta > 0$, which holds actually in most of well-known symmetric densities.

γ, the parameter of marginal costs. Moreover, it depends crucially on the marginal winning probability of effort, $\alpha_A g(\alpha_A x_A - \alpha_B x_B)$, which can be decomposed into α_A , the parameter for agent's ability, and $g(\theta)$, the probability density of output gap. Actually $g(\theta)$ stands for the marginal winning probability of output gap, that is, the measurement of the change in agent A's winning probability caused by a marginal change of output gap at a given level. The condition above shows that, as the density increases, the agent exerts more efforts, in other words, competition becomes more aggressive. Similarly, the first-order condition for agent B is

$$\alpha_B(v^w - v^l)g(\alpha_A x_A - \alpha_B x_B) - \gamma x_B = 0.$$

Comparing two conditions, we can see that the high-ability agent always makes more efforts than the low-ability agent. This result comes from the fact that the marginal gain of a unit increase in effort is greater to the high-ability agent than the low-ability agent.

Combining two first-order conditions, we can get the following condition that holds at the equilibrium.

$$g(\theta^*) = \frac{\gamma\theta^*}{(\alpha_A^2 - \alpha_B^2)(v^w - v^l)} \quad \text{where } \theta^* = \alpha_A x_A^* - \alpha_B x_B^*. \tag{1.1}$$

θ^* indicates the equilibrium level of output gap, which also represents the location of competitive front on the equilibrium. We can immediately observe that θ^* is increasing in the degree of asymmetry in agents' abilities. Also, it is increasing in the winner's gain and decreasing in the marginal cost parameter γ. Figure 1.1 illustrates these features well. Thus we have the following proposition.

Proposition 1 *θ^* is increasing in $(\alpha_A^2 - \alpha_B^2)$ and $(v^w - v^l)$, but decreasing in γ.*

Now we find the total output in equilibrium. Using the first-order conditions, we can

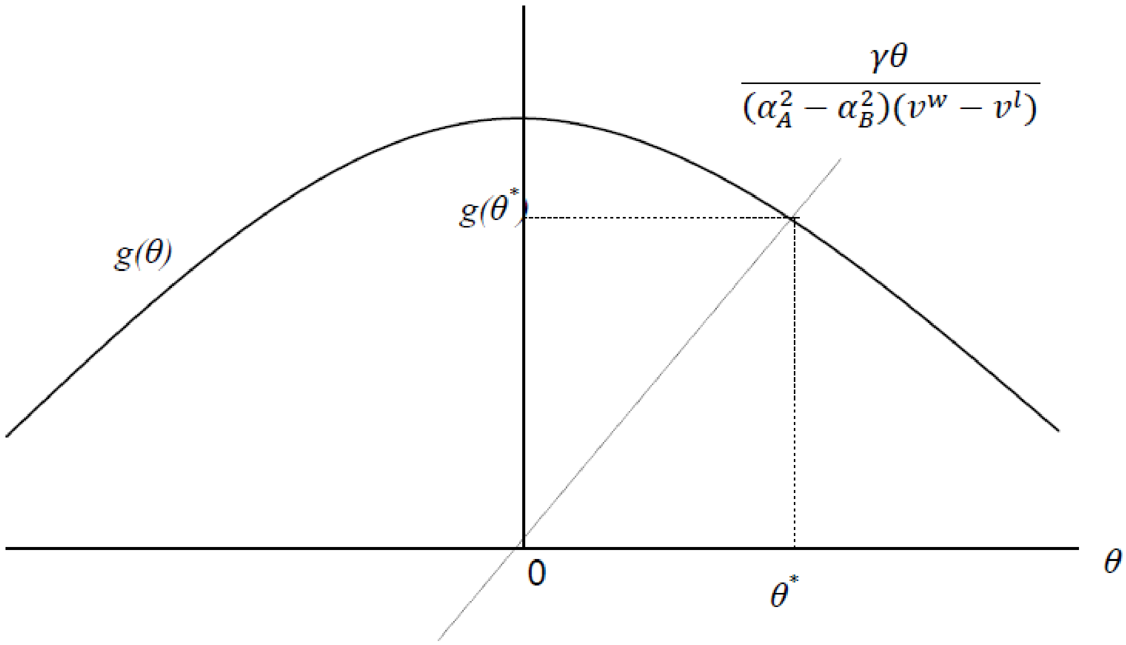

Figure 1.1: Equilibrium Effort

represent the expected total outputs as follows.

$$E\left[q_A^* + q_B^*\right] = \alpha_A x_A^* + \alpha_B x_B^* = \frac{\alpha_A^2 + \alpha_B^2}{\gamma}(v^w - v^l)g(\theta^*) \tag{1.2}$$

The total expected output is increasing in agents' abilities and the winner's gain, and is decreasing in the marginal cost. In particular, in equilibrium the total expected output depends crucially on $g(\theta^*)$, a marginal winning probability of output gap. Substituting the equilibrium condition (1.1) into (1.2), the expected total output can be rewritten as

$$\alpha_A x_A^* + \alpha_B x_B^* = \frac{\alpha_A^2 + \alpha_B^2}{\alpha_A^2 - \alpha_B^2} \cdot \theta^*.$$

This shows that the total output is increasing in θ^*. The next proposition summarizes the results above.

Proposition 2 *In equilibrium, the total output is increasing in the equilibrium output gap between agents and the marginal winning probability of the gap, given everything else remains.*

These two propositions are very useful for the future analysis.[4] In what follows, we will consider three representative ways of managing a contest to increase the level of agents' efforts and thereby total outputs.

1.3 Moral Hazard and Monitoring

The issue what we address in this section is whether and when the better monitoring technology on agents' efforts would increase or decrease their efforts. In traditional principal-agent literature, agent's moral hazard comes from imperfect observability on his behavior. So intensified monitoring over agents' efforts is considered as a very good way to prevent it. However what we argue is that it may have an adverse effect when the degree of asymmetry in agent's ability is large enough. To show this, we focus on how a change in the uncertainty of monitoring affects each agent's efforts.

Suppose that there are two monitoring technologies that are represented by two possible distribution functions with the same mean, but with different variances. The better monitoring technology allows ϵ to have the lower variance, σ_l, while the worse monitoring technology to have the higher variance, σ_h, where $\sigma_h > \sigma_l$. This means that we consider two distribution functions, $G(\epsilon, \sigma_h)$ and $G(\epsilon, \sigma_l)$, that can be ordered by the mean-preserving spread (MPS) such that $\int \epsilon g(\epsilon, \sigma_l) d\epsilon = \int \epsilon g(\epsilon, \sigma_h) d\epsilon$ and $\int G(\epsilon, \sigma_l) d\epsilon \leq \int G(\epsilon, \sigma_h) d\epsilon$. The MPS in this model implies that the principal has more uncertainty in monitoring the agents' effort levels, indicating the less accurate monitoring technology. We further assume the following *Single Crossing Property.*

$$
\begin{aligned}
g'(\epsilon, \sigma_l) &> g'(\epsilon, \sigma_h) \text{ for } \epsilon \in (-\infty, 0) \text{ and} \\
g'(\epsilon, \sigma_l) &< g'(\epsilon, \sigma_h) \text{ for } \epsilon \in (0, \infty).
\end{aligned}
$$

[4] Sometimes, the principal may want to maximize the effort of high-ability agent, for example, in R&D race. Both Proposition 1 and 2 in our model still hold in this case.

This assumption ensures that the MPS moves the probability mass from the center toward both tails smoothly, so two distribution functions must be crossing only once at 0. In other words, $G(\epsilon, \sigma_l)$ has first-order stochastic dominance over $G(\epsilon, \sigma_h)$ for $\epsilon \in (-\infty, 0]$, while $G(\epsilon, \sigma_h)$ does over $G(\epsilon, \sigma_l)$ for $\epsilon \in [0, \infty)$. This assumption also guarantees that two density functions cross once in each positive and negative region.[5] We define these unique crossing point as $\overline{\theta}$ and $-\overline{\theta}$, where $\overline{\theta} > 0$, such that

$$g(\overline{\theta}, \sigma_l) = g(\overline{\theta}, \sigma_h) = g(-\overline{\theta}, \sigma_l) = g(-\overline{\theta}, \sigma_h).$$

To economize on notation, we often denote a representative distribution and density function by $G(\cdot)$ and $g(\cdot)$ respectively. For simplicity, we ignore any cost the principal might incur in changing her monitoring technology.

As a benchmark, let us begin with the case where agents have symmetric abilities, i.e., $\alpha_A = \alpha_B = \alpha$. From (1.1), the level of symmetric equilibrium efforts is

$$x_A^* = x_B^* = x^* = \alpha(v^w - v^l)g(0)/\gamma.$$

In this case, an output gap is zero regardless of the degree of uncertainty. Since we have $g(0, \sigma_l) > g(0, \sigma_h)$, competition becomes more intense under less uncertainty, in other words, under the better monitoring technology. This result is well consistent with common sense in that the agents are forced to work harder since the principal can monitor them better. Put it differently, the improvement in monitoring technology reduces the moral hazard problem of the agents.

However, this result can dramatically change if agents are asymmetric in their abilities. Without loss of generality, we now assume that agent A has better ability than agent B, i.e.,

[5] Again, this assumption is fairly general in that most known distribution functions satisfy this property. For example, for a Normal distribution with the density function $f(\theta) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{\theta^2}{2\sigma^2})$, we obtain $\overline{\theta} = \sqrt{\left(\ln\frac{\sigma_h}{\sigma_l}\right) \Big/ \left(\frac{\sigma_h^2 - \sigma_l^2}{2\sigma_h^2\sigma_l^2}\right)}$.

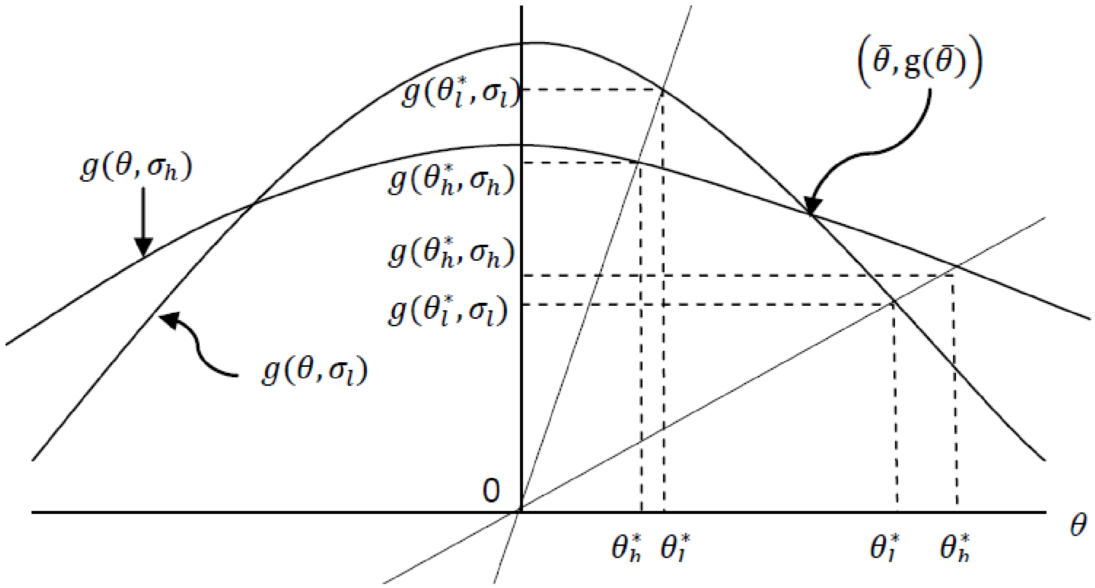

Figure 1.2: Equilibrium Effort under Different Monitoring Technologies

$\alpha_A > \alpha_B$. Then θ_k^* is positive, which represents the competitive front under the distribution function with the variance σ_k, where $k = h, l$. Now we compare two different competitive fronts and their intensities of competition under two different levels of uncertainty. As illustrated in Figure 1.2, we can easily identify that $g(\theta_l^*, \sigma_l) \gtreqless g(\theta_h^*, \sigma_h)$ and, correspondingly, $\theta_l^* \gtreqless \theta_h^*$ according to $\theta_k^* \lesseqgtr \overline{\theta}$. Then, by Proposition 2, the total output is greater under less uncertainty if the output gap is relatively small, and so it is under more uncertainty if the output gap is large enough. Then the following proposition summarizes the previous two results.

Proposition 3 *For symmetric agents, total outputs always increase under less uncertainty. However, for asymmetric agents, total outputs increase under less uncertainty if* $\overline{\theta} > \theta_l^* > \theta_h^*$ *and under more uncertainty if* $\overline{\theta} < \theta_l^* < \theta_h^*$.

Interestingly, the better monitoring does not always result in the better outcome. Intuition behind this surprising result is as follows. When a principal improves her monitoring technology, it affects agents' efforts in two ways. On the one hand, from the relationship between a principal and agents, the improvement of monitoring technology makes both agents' effort more valuable than before since it brings a more honest result to the agents. Putting it differently, with a little more efforts, agents could persuade the principal more convincingly that he deserves to receive a winner's prize. This brings a positive effect on agent's efforts. We call this *a substitution effect* in that the value of their efforts appreciated by the principal is heightened by the improvement of monitoring technology. Actually this is the very principal-agent problem that traditional moral hazard literature addresses. A thing worthy of note here is that this substitution effect widens the output margin between two agents due to the gap in their abilities.

On the other hand, from the relationship between agents, the improvement of monitoring technology makes the outcome of the contest more predictable, or even obvious. Putting it differently, the principal would be more likely to declare the high-ability agent as a winner since she could be more convinced that the output gap results from the difference in abilities between the agents. If the high-ability agent knows that his winning probability is sufficiently high, he does not need to dominate his competitor by a large output margin anymore to convince the principal that he deserves to win the contest. Similarly, if the low-ability agent knows his winning chances are sufficiently low, he has no more reason to exert an effort to catch up with his opponent, thus would give up the contest very quickly. This brings a negative effect on both agents' efforts. We call this *an income effect* in that the change of total winning probability causes both agents to shirk. This income effect is the unique characteristic, which is present only in contest framework. Again we need to note that the income effect always narrows the output gap. One more thing to mention is that the size of income effects definitely depends on the degree of asymmetry in agents' abilities, thereby on the level of their output gap.

Consequently, we can expect that the total effect of adopting better monitoring technology is determined by the relative size of the substitution effect and the income effect on the given competitive front. When the output gap is small enough, the substitution effect is greater than the income effect because the latter is relatively small. In this case better monitoring always brings a favorable result to the principal. However, when the output gap is large enough, the income effect dominates the substitution effect. In this case the better monitoring causes the agents to reduce their efforts while the worse monitoring induces them to exert more efforts.

According to the Proposition 1, the output gap in equilibrium is determined by the difference of agents' abilities, given the prize. Moreover, by Proposition 3, the principal chooses different monitoring technology depending on the size of difference of the abilities. In particular, if agents' abilities are sufficiently different, the principal prefers the worse monitoring. The next proposition summarizes it.

Proposition 4 *As* $(\alpha_A^2 - \alpha_B^2) \gtreqless \frac{\gamma\bar{\theta}}{(v^w - v^l)g(\bar{\theta})}$, $\theta_l^* \lesseqgtr \theta_h^*$. *If the difference of abilities is large (small) enough, the principal prefers worse (better) monitoring.*

The most striking point from this proposition is that when the difference of agents' abilities is sufficiently large, principal's endeavor to intensify the monitoring on agent's effort with a view to reaping more outputs from them can bring an adverse effect.

This result is sharply contrasting with the idea of traditional moral hazard literature. Many papers, followed by Holmstrom (1979) and Grossman and Hart (1983), argue that a moral hazard problem basically stems from unobservability on agent's actions, thus a worse monitoring system leads to a severe moral hazard problem. It is obvious that they pay attention only to the substitution effect in arriving at this conclusion. In this paper, however, we argue that less uncertainty in observing agent's action can lead to a more severe moral hazard problem, especially when there is large heterogeneity in agents' abilities, due to the income effect. One of few exceptions is Cowen and Glazer (1996). They show that better monitoring can induce less effort. However, there are several differences between their paper and our work. They consider one agent case, and the paper is mainly based on the graphical analysis without analytical treatment. Since their model is based on the dynamic environment, time inconsistency is an important problem, which is not an issue in our paper. By contrast, we have studied the contest model. Dubey and Wu (2001) and Dubey and Haimanko (2003) also drive a similar result to this paper that worse scrutiny could result in a better performance, but they use very specific multi-period performance evaluation scheme. Meanwhile we use more general framework, which is a lot applicable.

1.4 Job Assignment

In this section, we address how the principal would assign tasks between agents. The principal can make competing agents work on various sets of tasks. The outputs of the tasks may be positively correlated, negatively correlated, or independent. For example, if the principal makes two salesmen work in the same area, their performance would be more likely to have positive correlation at given levels of effort. By contrast, if they work in different areas, the outcomes would be less likely to be correlated. Interestingly this job correlation plays a crucial role in configuring a winner. This implies that the possible correlation between tasks assigned to asymmetric contestants can be considered as a very important strategic tool to increase outputs.

To see this, let us change agent's output function slightly into $q_i = \alpha_i x_i + (1+\rho)\epsilon_i$. Here ρ captures a potential correlation between both agents' outputs facing a common shock.[6] Suppose that the principal can choose the tasks with various correlations.[7] We denote such different situations with ρ_h and ρ_l respectively where $\rho_h > \rho_l$. Then we can obtain $Var(\epsilon) = \sigma_\epsilon^2 = 2(1-\rho_k)^2\sigma^2$, $k = h \; or \; l$. Given the same monitoring technology, the variance

[6] We should be very careful in understanding the nature of this correlation. It comes from the possible correlation from the shock, not from the differences between agents abilities.

[7] Actually tasks are very highly and positively correlated in most of the contests, such as sport competitions, arts concourses, or essay contests. Nevertheless in some contests like the promotion within the company, we can often witness negative correlation or independence between tasks.

can differ only in the correlation of tasks that the agents perform. Putting it differently, considering these different correlations between tasks is equivalent to treating two possible distribution functions with different variances. For example, if the principal assigns more positively correlated tasks to the agents, she can observe more correctly that the output gap truly comes from the agents' heterogeneity, not from output shocks.

Now, without loss of generality, we can set $\hat{g}(\epsilon, \rho_l) = g(\epsilon, \sigma_h)$ and $\hat{g}(\epsilon, \rho_h) = g(\epsilon, \sigma_l)$. Then all the previous analysis can be applied immediately. We find $\hat{g}(\theta_h^*) \gtreqless \hat{g}(\theta_l^*)$ according to $\theta_k^* \lesseqgtr \overline{\theta}$, where θ_k^* is the equilibrium competitive front under $\hat{g}(\epsilon, \rho_k)$. Correspondingly, we obtain $\theta_h^* \gtreqless \theta_l^*$ according to $\theta_k^* \lesseqgtr \overline{\theta}$. The next proposition summarizes it.

Proposition 5 *If the difference of abilities are large (small) enough, the principal assigns less (more) correlated tasks.*

Waldman(1984) and Meyer (1994) study the way of assigning tasks in different settings. The common theme of the papers is that the principal attempts to gather information about agent's types through designing a different way of task assignments. Compared to these papers, we study the way of task assignments to induce most efforts, based on the degree of heterogeneity in agents' abilities.

1.5 Margin Rule

In this section we address an issue on what we call 'the margin rule.' In many cases, the principal awards the prize only when the winning is conspicuous. For example, a CEO picks up an employee as an officer only when the employee shows an extraordinary performance in a given project compared with the others. This kind of unscheduled promotion happens often in many organizations. If the employee has performed relatively-well, the CEO would not make such a big decision. We study why the contest designer requires such a huge margin for the agent to be awarded the prize.

Suppose that the principal announces the winner only if the agent's output is greater than her rival by a positive margin, Δ. Then there are three possible outcomes. Agent A will be a winner if $q_A > q_B + \Delta$, and a loser if $q_A < q_B - \Delta$. For $|q_A - q_B| < \Delta$, the game will end up with a draw and both agents divide the prizes evenly. Setting $v^L = 0$ for simplicity, we can write the agent's problem as follow.

$$
\begin{aligned}
\underset{x_A}{Max} \quad & G(\alpha_A x_A - \alpha_B x_B - \Delta)v^w \\
& +[G(\alpha_A x_A - \alpha_B x_B + \Delta) - G(\alpha_A x_A - \alpha_B x_B - \Delta)]\frac{v^w}{2} - \frac{\gamma}{2}x_A^2 \\
= \quad & [G(\alpha_A x_A - \alpha_B x_B + \Delta) + G(\alpha_A x_A - \alpha_B x_B - \Delta)]\frac{v^w}{2} - \frac{\gamma}{2}x_A^2
\end{aligned}
$$

One interpretation for the second equation is that the agent competes with both an advantage of Δ and a disadvantage of $-\Delta$ at the same time. The first-order condition is

$$
[g(\alpha_A x_A - \alpha_B x_B + \Delta) + g(\alpha_A x_A - \alpha_B x_B - \Delta)]\,\frac{\alpha_A}{2}v^w - \gamma x_A = 0.
$$

This shows that the agent competes on two competitive fronts, at $g(\alpha_A x_A - \alpha_B x_B + \Delta)$ and at $g(\alpha_A x_A - \alpha_B x_B + \Delta)$. This means the marginal winning probability of effort depends on the average of probability densities at these two points. Now the equilibrium condition is written as

$$
g(\widetilde{\theta}^* + \Delta) + g(\widetilde{\theta}^* - \Delta) = \frac{2\gamma\widetilde{\theta}^*}{(\alpha_A^2 - \alpha_B^2)v^w} \quad \text{where } \widetilde{\theta}^* = \alpha_A x_A^* - \alpha_B x_B^*. \tag{1.3}
$$

Let us study the symmetric case first. Both agents choose the same effort levels, and thereby $\tilde{\theta}^* = 0$. Since $g(\Delta) = g(-\Delta)$ by the symmetry of the density function, the Nash equilibrium effort level of each agent is

$$
x^* = x_A^* = x_B^* = \alpha v^w g(\Delta)/\gamma.
$$

Although the agents are symmetric, they compete virtually at $\theta = \Delta$. However, $g(\Delta)$ is maximized when $\Delta = 0$. Thus the contest designer will never use the margin rule for symmetric agents.

On the other hand, when the agents are asymmetric, the expected total outputs are

$$\alpha_A x_A^* + \alpha_B x_B^* = \frac{\alpha_A^2 + \alpha_B^2}{\gamma} v^w \frac{\left[g(\widetilde{\theta}^* + \Delta) + g(\widetilde{\theta}^* - \Delta)\right]}{2}.$$

Therefore, if $\frac{1}{2}\left[g(\widetilde{\theta}^* + \Delta) + g(\widetilde{\theta}^* - \Delta)\right] > g(\widetilde{\theta}^*)$, the total outputs increase with the margin rule. The following proposition summarizes the results above and provides the condition under which there exists a positive Δ that increases the total outputs.

Proposition 6 *For symmetric agents, the principal prefers no margin rule. For asymmetric agents, there exists an optimal margin that maximizes the total output as long as* $g'(\widetilde{\theta}^* + \Delta) > g'(\widetilde{\theta}^* - \Delta)$. *Under the Normal distribution, the principal prefers the margin rule if the difference of agents' abilities is large enough.*

Proof. By Proposition 2, the principal wants to maximize $\widetilde{\theta}^*$. Applying implicit differentiation to equation (1.3), we obtain

$$\frac{d\widetilde{\theta}^*}{d\Delta} = -\frac{g'(\widetilde{\theta}^* + \Delta) - g'(\widetilde{\theta}^* - \Delta)}{g'(\widetilde{\theta}^* + \Delta) + g'(\widetilde{\theta}^* - \Delta) - \frac{2\gamma\widetilde{\theta}^*}{(\alpha_A^2 - \alpha_B^2)v^w}}.$$

The denominator is always negative. Thus, as long as $g'(\widetilde{\theta}^* + \Delta) > g'(\widetilde{\theta}^* - \Delta)$,[8] the principal can increase the total efforts.[9] With the Normal distribution, $\theta \sim N(0, \sigma_\epsilon^2)$, it is shown that the sign of $\frac{d\widetilde{\theta}^*}{d\Delta}$ is the sign of $-\frac{1}{\sqrt{2\pi\sigma^3}}\left[\exp\left(-\frac{(\widetilde{\theta}^* + \Delta)^2}{2\sigma^2}\right)(\widetilde{\theta}^* + \Delta) - \exp\left(-\frac{(\widetilde{\theta}^* - \Delta)^2}{2\sigma^2}\right)(\widetilde{\theta}^* - \Delta)\right]$.

[8] Note that this is exactly the same condition as $\frac{1}{2}\left[g(\widetilde{\theta}^* + \Delta) + g(\widetilde{\theta}^* - \Delta)\right] > g(\widetilde{\theta}^*)$.

[9] Note that Δ cannot be greater than $\widetilde{\theta}^*$. Otherwise, $g'(\widetilde{\theta}^* - \Delta)$ becomes positive, the total output must be reduced.

Then $\frac{d\widetilde{\theta}^*}{d\Delta} \gtreqless 0$ corresponds to

$$\exp\left(\frac{\widetilde{\theta}^* + \Delta}{\widetilde{\theta}^* - \Delta}\right)^2 \lesseqgtr \frac{\widetilde{\theta}^* - \Delta}{\widetilde{\theta}^* + \Delta}. \tag{1.4}$$

While the left-hand side is decreasing in $\widetilde{\theta}^*$, the right-hand side is increasing in $\widetilde{\theta}^*$. This implies that when $\widetilde{\theta}^*$ is greater than a threshold value, the presence of Δ induces more efforts. In other words, the optimal choice of Δ to maximize efforts must satisfy equation (1.4). ■

Implementing the margin rule has the same effect as giving an advantage to a low-ability agent and a disadvantage to a high-ability agent at the same time. In other words, this margin rule makes the result of the contest more uncertain. Therefore, by the similar logic in the previous sections, if agent's abilities are more asymmetric the principal can make the contest more productive by implementing the margin rule.

In this context, the margin rule is closely related to the handicapping theory, in which a principal confers a relative advantage or disadvantage on agents to make an uneven contest more competitive. For example, Baye et al (1993) show that a politician can maximize political rents by excluding the lobbyist who values the prize most. Che and Gale (2003) show that, when contestants are asymmetric, it is optimal to handicap the strongest contestant through imposing a maximum on an allowable prize.[10] Fu (2006) analyzes the effect of affirmative action in admission policy, showing that favoring the weaker group increases

[10] In our setting, it is easy to show that the principal designs a biased contest toward the low-ability agent. For example, let λ represent the relative advantage on the agent B. Then agent A maximizes $G(\alpha_A x_A - \alpha_B x_B - \lambda)v^w - \frac{\gamma}{2}x_A^2$, and agent B maximizes $G(\alpha_B x_B - \alpha_A x_A + \lambda)v^w - \frac{\gamma}{2}x_B^2$. The equilibrium with the asymmetric agents is characterized by $g(\widehat{\theta}^* - \lambda) = \frac{\gamma\widehat{\theta}^*}{(\alpha_A^2 - \alpha_B^2)v^w}$ where $\widehat{\theta}^* = \alpha_A x_A^* - \alpha_B x_B^*$.The total output is again written as $\alpha_A x_A^* + \alpha_B x_B^* = \frac{\alpha_A^2 + \alpha_B^2}{\gamma}v^w g(\widehat{\theta}^* - \lambda)$. There is the unique optimal λ that maximizes $\widehat{\theta}^*$. The principal sets λ to satisfy $g(\widehat{\theta}^* - \lambda) = g(0)$, which attains the maximum of the marginal winning probability.

competition and, thereby, investments in education. However, most of the papers assume that the principal knows which agent is a high-ability agent or a low-ability agent. Therefore the principal could induce the first-best outcome through offering different prizes or conferring a relative advantage or disadvantage according to agent's type. Meanwhile the premise of this paper is that the principal does not know which agent has higher ability than the other although she recognizes the degree of heterogeneity. A notable exception is the study by Meyer (1991), in which she shows that, even though a principal could not differentiate agent's types, handicapping the contest favoring the first-period winner in the second period competition provides better information on agent's types, thus results in better outcome. However, the margin rule can induce better performance from agents even without a multi-period framework or any other devices. In this sense, the margin rule can be used more flexibly in a variety of settings.

1.6 Concluding Remarks

This paper has examined how agent's optimal level of efforts and the total output are determined under the regime of contest when there is uncertainty in their performance. We find that the marginal winning probability of effort, more specifically, the probability density of output gaps, plays a crucial role in determining them. Thus the principal can design the contest better by adjusting the factors that affect on the density.

Based on this result, we also provided three ways of improving the mechanism of the contest: choosing monitoring technology, assigning tasks, and using the margin rule. One major theme throughout the paper is that the contest mechanism with asymmetric agents should be qualitatively different from that with symmetric ones. At first glance, it appears that more monitoring, assigning highly correlated tasks, and not using the margin rule can induce more efforts. This is true when the agents are symmetric. However, we challenge this intuition in the case of heterogeneous agents. The principal prefers to adopt the counterintuitive alternatives such as less monitoring, assigning less correlated tasks, and using the margin rule, as the agents are more heterogeneous in their abilities.

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Chapter 2

Entry Decision with Tying under Quality Uncertainty and Switching Costs

2.1 Introduction

Tying strategy has been widely used when a primary good monopolist tries to extend his monopolistic power to a relevant subsidiary market. By tying his subsidiary good to the primary good, the primary good monopolist can force consumers to buy his subsidiary good and exclude the rival effectively out of the competitive subsidiary market. Especially in software market, this strategy has been very successful. For example, Microsoft could exclude its rivals in application software markets by tying its applications, such as *Internet Explorer* or *Windows Media Player*, to its operating system, *Windows*.[1] Microsoft cases have triggered fierce debates among economists and antitrust authorities. However, most of

[1]The *Internet Explorer* case was filed in 1994 by the Department of Justice of the United States, and settled in 2001 by Microsoft's agreement on not tying *Internet Explorer* to *Windows*. In *Windows Media Player* case, the European Union ruled that Microsoft had abused its dominant position in the operating system to exclude its competitors out of the multimedia player market, and fined 479 million euro in 2004.

debates have focused on traditional market structures without considering important features that are unique in software industry.

Software has a few characteristics that make it differentiated from other goods. First, although an operating system and an application program are closely related as complementary goods, the relative status between them is not symmetric. An operating system has intrinsic stand-alone value even without an application program, but the latter is of no use without the former. Second, software is a durable good without physical depreciation, but with periodic upgrades. This implies that consumers can make a choice between an old version and a new version. Putting it differently, the new generation of software has to compete with the old generation. Third, software-users incur costs when they switch between two alternative softwares, and the size of switching cost is different across the kinds of softwares. For instance, a user of one word-processing program has to incur a relatively high switching cost in moving to another word-processing program since they have to learn a lot of new function keys until getting used to it. Meanwhile, a user of one music player program can easily move to another music player program with incurring a low switching cost since those programs can be played well with several common function keys.

Due to these features, tying behavior in software market shows a couple of interesting patterns. First, most of tying cases that have been issues among antitrust authorities occurred when a market matures to some extent, not in the early stage of the market.[2] When a firm launches a new software, it faces a substantial risk since a new software turns out quickly to be one of two extremes, a big hit or a big failure, due to the network externalities. As long as the late-entry into the subsidiary market after observing consumer's response is possible, a primary good monopolist would prefer it to avoid this risk. In this sense primary good monopolist's tying strategy for early-entry could be less criticized than that for late-entry in view of risk-taking, even though both tying behaviors have negative effect on social welfare as shown in section 4. Second, most of tying cases, especially for late-entry,

[2]For example, Microsoft began to tie *Internet Explorer* to its operating system from *Windows 95* onwards.

have been related with the softwares of which switching costs are relatively small.[3] At the beginning of the market evolution, a primary good monopolist can surely take up the relevant subsidiary market by simply tying the new software with his operating system. At the later stage, however, he has to compensate consumers for switching costs to succeed in making a new entry into the subsidiary market even though he ties them. In this sense firm's intertemporal entry decision crucially depends on the probability of succeeding in late-entry and the probability, in turn, would be determined by the size of switching costs. If switching cost is sufficiently high compared to the upgrade benefit of primary good, consumers would not switch to the newly-introduced subsidiary good of the primary good monopolist. Thus the monopolist is less likely to succeed in late-entry and fails to exclude his competitor out of the market, and vice versa otherwise. Therefore the smaller the switching costs are, the more likely a primary good monopolist is to make a late-entry. In this paper we present the economic explanation for these featured patterns; that is, what determines firm's decision on whether or not to tie, and how to choose the optimal timing of tying? Here we focus on the role of switching costs in determining them.

To identify monopolist's optimal entry-decision through tying, we set up a three-period entry model where a primary good monopolist has to decide whether or not to enter into the subsidiary market in each period, with or without tying respectively. The main results are as follows. In the presence of uncertainty in consumer's valuation on a subsidiary good, a primary good monopolist decides when to enter into the subsidiary market with tying by comparing the of probability of the good being valued high and the chances of succeeding in late-entry. If switching costs are sufficiently low a primary good monopolist would always prefer to enter late into the subsidiary good market rather than to enter early. If switching costs fall on intermediate range the firm would prefer a late-entry rather than an early-entry

[3]For example, Microsoft tied *Internet Explorer* and *Windows Media Player* to *Windows* and it was very successful. Actually, both application softwares incur very little or no switching cost. Meanwhile, in more complicated software like word-processing software, Microsoft does not tie *MS-Word* to its operation system. Especially in Korea, Microsoft didn't try to tie *MS-Word* to *Windows* even though it has been dominated by *the Hangul* of Haansoft in word-processing software market.

with a positive probability and the probability is decreasing in the size of switching costs. If switching costs are sufficiently large the firm would prefer a late-entry with a positive probability but the probability is fixed at such a low level.

These results have important policy implications. When antitrust authorities determine whether a firm's tying behavior is anti-competitive or not, they consider several factors together. Typically, indices on market structure are used; such as concentration ratio, market shares, etc. This paper suggests that they should also consider technical aspects of the goods and the status of market evolution as well. That is, if switching costs are high enough antitrust authorities should pay more attention at the early stage of market maturity because after that stage exclusive power of tying decreases gradually. On the contrary, if switching costs are relatively small they should pay more attention as the market develops more.

There is voluminous literature on tying as an exclusion strategy. Traditional leverage theory sees tying as a strategic device for a monopolist in one market to extend his monopoly power to the other market. However, well-known Chicago School argument refutes it stating there is no incentive for the monopolist to tie them since he could extract entire rents without tying by charging a sufficiently high price on the primary good. This argument made the previous leverage theory vulnerable. In response to Chicago School argument, Winston (1990) shows that, in a differentiated independent good case, tying can be a profitable strategy since a monopolist could deter potential entry by committing to tie in the future. For a complementary good case, Choi and Stefanadis (2001) shows that, under risky R&D, tying can also deter entry by reducing the expected return of entry in each market since entry is profitable only when entries into both markets are all successful. Nalebuff (2004) also analyzes an integrated firm facing different competitors in each market and shows the firm can be benefited by tying through larger market shares. Meanwhile Carlton and Waldman (2002) studies tying incentives on the presence of economies of scope.

However these studies do not consider periodic upgrades and switching costs, which play a crucial role in software market. Carlton and Waldman (2005) showed that, assuming continuous demand space, switching costs and upgrades, the monopolist's incentive to tie is intensified because it guarantees future profits resulting from upgrades. Kim (2007) also draws a similar conclusion assuming unit demands for heterogeneous consumers. Nevertheless, both papers are less realistic in that they assume both goods can be useful only when they are consumed together. In software market a primary product generally has its own intrinsic value without joint use of subsidiary goods, while not vice versa. Moreover in both papers entry decision always takes place at the first period in two-period model, which is a very restrictive assumption. The analysis presented here considers stand-alone value of the primary good and the possibility of late-entry.

The remaining parts of this paper are as follows. In section 2, we set up a basic model, and, in section 3, analyze it comparing monopolist's profits between under an early-entry and under a late-entry. Section 4 provides welfare analysis and policy implications. Section 5 gives concluding remarks.

2.2 Model

There are two firms, i and j, and two non-depreciating durable goods, A and B. Firm i produces both A and B, but firm j produces B only. A is of use by itself, but B is of use only if used together with A. For example, in software industry, we may think A as an operating system and B as an application program. In this sense, we call A a primary good and B a subsidiary good.

We consider three-period competition, $t = 1, 2, 3$. Both A and B are newly introduced at $t = 1$ and upgraded periodically at $t = 2$ and 3 respectively. Here we assume any upgrade of B is compatible to the previous version of A. This means that any upgrade of A have to compete with its older versions to capture potential buyers for the upgrade of B.

Both firms have to incur research and development (R&D) costs in developing and upgrading the products. The costs are R^A and R^B in initially developing A and B respectively. Upgrades of both goods also require the same R&D costs as those of initial development. Here we introduce ε as R&D cost savings in developing B and upgrading it at the same time, where $\varepsilon < R^B$. This means the costs of developing and upgrading B simultaneously at $t = 2$ are smaller than those of developing it at $t = 1$ and upgrading it at $t = 2$ subsequently. The existence of ε also implies that there is no case in which a firm would invest R&D costs in a given period without being able to sell the product in that period. Marginal costs are all normalized to zero.

Consumers, whose size is normalized to one, are homogeneous in their preference for the goods and have a unit demand for each good. Good A gives consumers an intrinsic stand-alone value of V^A per-period and each upgrade gives them an additional per-period benefit of V^A. Thus entire per-period benefit of the second-period version would be $2A$ and that of the third-period version would be $3A$.

Good B_i and B_j give consumers per-period benefits of V_i^B and V_j^B respectively only when used together with A. To study a primary good monopolist's tying incentive to extend its monopoly power to the relevant competitive subsidiary market where his product has inferior quality, we endow firm j with the quality superiority as follows:

$$V_i^B \equiv V^B \text{ and } V_j^B - V_i^B = \Delta > 0.$$

Meanwhile, the upgrades of B give consumers the same additional per-period benefit, V^B, across the firms in subsequent periods. This means that the quality difference would be maintained throughout the periods. Again, this upgrade benefit could be obtained only when consumed with good A, but it does not matter which version of A is combined.[4] Furthermore we introduce the following uncertainty on consumer's valuation of V^B before firm i's investing R&D costs in B in the first period:

$$V^B \in \left\{ \underline{V}^B, \overline{V}^B \right\} \text{ where } \Pr(\underline{V}^B) = 1 - \rho \text{ and } \Pr(\overline{V}^B) = \rho.$$

[4]In reality, a new version of application software often functions better under new operating system than under the old one. However, this simplification doesn't affect main results at all.

The true value of V^B is realized just before being released in the market in the first period.

Consumers choose either B_i or B_j at $t = 1$ and can switch to the other at $t = 2$ with incurring switching costs, $s \in [0, \infty)$. We further assume that there is no switching at $t = 3$. This indicates that if one firm prevails in the second period it will do so in the third period as well. This assumption reflects the reality that the longer consumers use one good the more they are accustomed to it so that they would be unlikely to switch to the other. Moreover this keeps the focus on firm's entry decision only between period 1 and 2.

In addition, to simplify the model and prevent trivial cases, we will use the following assumption throughout the analysis.

Assumption 1. $2V^A > (2\Delta - s)$.
Assumption 2. $\frac{R^B}{3} - \Delta < \underline{V}^B < \frac{R^B}{3}$ and $\overline{V}^B > (2\Delta + s + 3R^B) - \varepsilon$.
Assumption 3. Tie-breaking rule: tying strategy will be used if it gives at least the same profit as separate-selling does.
Assumption 4. If neither firm enters into the subsidiary market in a given period, the market will collapse and no firm enters onwards.

By Assumption 1, as shown later, we can exclude the case in which the quality superiority of B_j is extremely high so that firm j can make a new entry into the market B at $t = 2$ in spite of firm i's tying. By Assumption 2, as shown later as well, the supports of $\underline{V}^B$ and $\overline{V}^B$ are defined sufficiently low and high respectively such that if $V^B = \underline{V}^B$ investment in market B is unprofitable in any case while if $V^B = \overline{V}^B$ even late-entry into market B might be profitable. Assumption 3 can be justified by the fact that, for firm i to sell its subsidiary good, the less stringent incentive compatibility constraints are required with tying than without tying. Assumption 4 means, if neither firm enters, no future profits would be expected at all.

In this situation, in the first period, firm j invests R&D costs and enters into the market B. At the same time, firm i decides whether or not to enter into the market B. If firm i decides to enter, it can choose whether or not to tie its subsidiary good to the primary good.

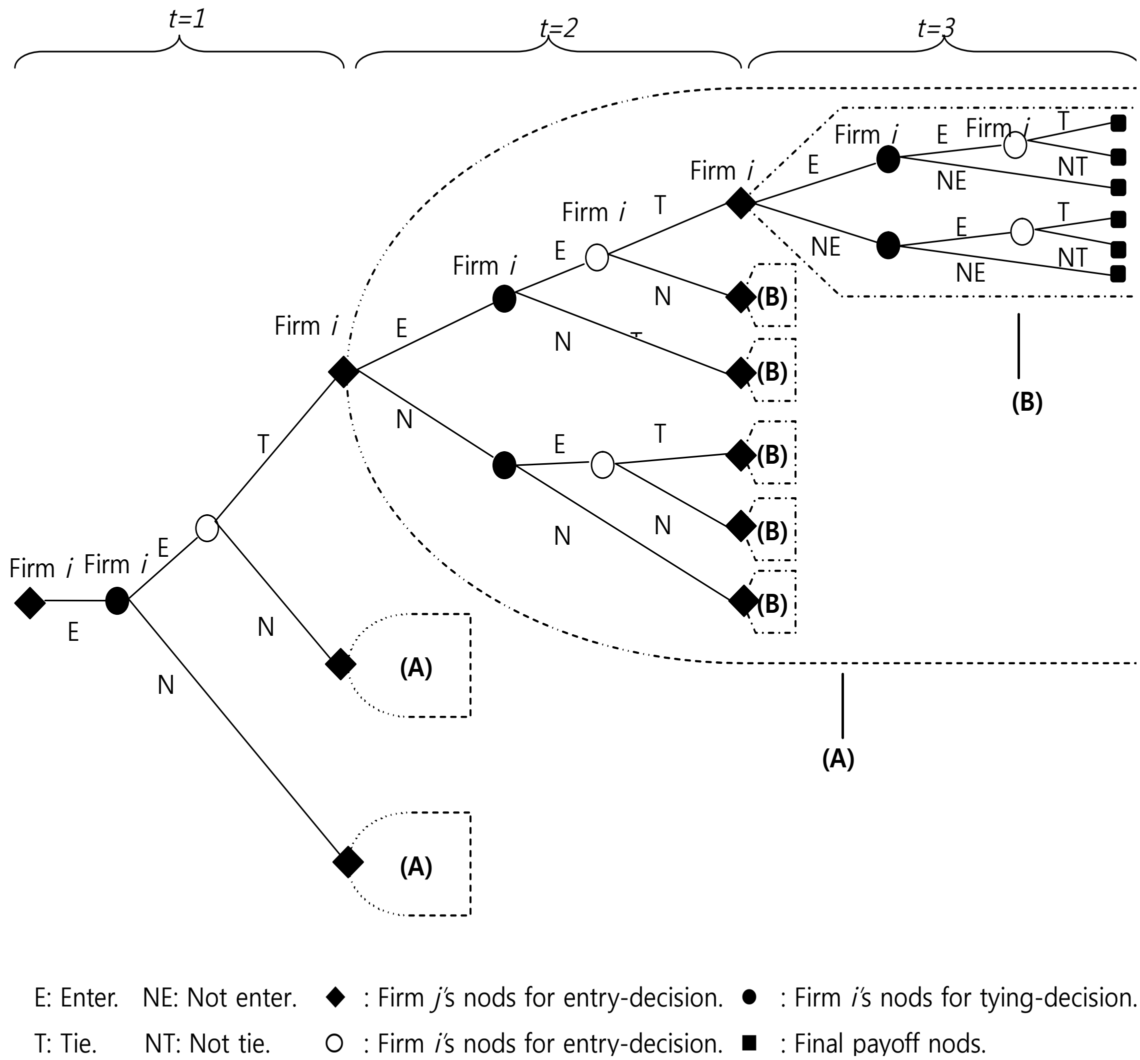

Figure 2.1: Game Tree

Depending on the first period outcome, both firms decide whether or not to enter into the market in the subsequent periods and firm i chooses whether or not to tie as well. To make a new entry in the second or the third period, each firm has to release the corresponding upgraded products. Finally they compete in a *Bertrand* way under the restriction of non-negative price. An extensive-form representation of the entire game is shown in Figure 2.1.

2.3 Analysis

In this section, we solve the model and find a subgame perfect Nash equilibrium that specifies both firms' entry decision and firm i's tying decision. If V^B turns out to be $\underline{V}^B$ in the first period, neither firm invests in market B anymore. So, for the analysis in the second and third periods, we restrict our attention to the case that V^B was revealed as $\overline{V}^B$ in the first period. Then, for the analysis in the first period, we will consider both cases together.

2.3.1 In the third period

In market A, firm i will invest R^A and release the final upgrade version of his primary good with the quality of $3V^A$ at the price of V^A $(= 3V^A - 2V^A)$, gaining a profit of $(V^A - R^A)$. In market B, meanwhile, the equilibrium configuration depends on which firm prevailed in the previous period. Since there is no switching in this period, the firm who prevailed in the previous period will invest R^B and release the final upgrade version of B at the price of V^B, with appropriating the whole monopoly profit of $(V^B - R^B)$.[5] The following lemma summarizes firm's equilibrium strategies and profits at $t = 3$.

Lemma 1 *Suppose that $V^B = \overline{V}^B$ and no switching happens in the third period. Then a firm that prevailed at $t = 2$ prevails at $t = 3$ as well. Profits at $t = 3$ are*

$$\begin{aligned} \widetilde{\pi}_i^3 &= (V^A - R^A) + (\overline{V}^B - R^B) \text{ and } \widetilde{\pi}_j^3 = 0 \quad \text{if } B_i \text{ sold at } t = 2, \\ \widetilde{\pi}_i^3 &= (V^A - R^A) \text{ and } \widetilde{\pi}_j^3 = (\overline{V}^B - R^B) \qquad \text{if } B_j \text{ sold at } t = 2. \end{aligned}$$

[5] Alternatively, if we assume the possibility that consumers switch in the last period as well, the equilibrium characterization in the last period would be changed in a way to reduce firm i's incentive to prevail in the second period because it would lower the expected future profit from the last period's upgrade. So it lowers the critical level of switching cost that allows firm i to enter late in the second period. However this modification does not change the entire analysis qualitatively as long as a positive third-period profit is expected. If we assume infinite periods instead of three periods, no switching assumption in the last period could be relaxed.

We have a couple of things to notice here. First, contrary to the Chicago School argument, the release of primary good upgrade does not allow firm i to appropriate the surplus in market B by charging a high price on the primary good because the upgrade of the primary good has to compete with its old version to attract potential buyers. Second, at $t = 3$, tying strategy is of no use in firm i's entering into the market B because no switching happens in this period.

2.3.2 In the second period

The second period equilibrium also depends on the first period equilibrium configuration. We will first solve two cases separately, depending on which firm prevailed at $t = 1$, and then provide overall equilibrium configuration at $t = 2$.

(Case 1: when B_j sold at $t = 1$.)

In this sub-section we analyze a case that firm j prevailed in the first period. We have four sub-cases to consider depending on both firms' entry decision into the market B at $t = 2$; both firms enter, either i or j enters, or neither does. In the last case, both firms would get zero profit in market B, so we will analyze the remaining three sub-cases one by one.

First, suppose that only firm i enters into the market B at $t = 2$. Then firm i would newly introduce a good B with an upgraded per-period quality of $2\overline{V}^B$, investing $(2R^B - \varepsilon)$. Since all consumers were using B_j in the previous period, they can either keep using it without upgrade with getting a per-period benefit of $(\overline{V}^B + \Delta)$, or switch to a newly-introduced B_i with getting a per-period benefit of $2\overline{V}^B$ minus switching cost, s. So incentive compatibility for consumers to switch to B_i is

$$2 \times 2\overline{V}^B - s - p_i^B \geq 2 \times (\overline{V}^B + \Delta) \Longleftrightarrow p_i^B \leq 2\overline{V}^B - (2\Delta + s).$$

Note that $2\overline{V}^B$ and $(\overline{V}^B + \Delta)$ are multiplied by two because consumers have two remaining periods onwards. Since $2\overline{V}^B > (2\Delta + s)$ by Assumption 2, firm i would sell B_i at

$\left[2\overline{V}^B - (2\Delta + s)\right]$ in market B and make a profit of $\left[2\overline{V}^B - (2\Delta + s) - (2R^B - \varepsilon)\right]$. In market A, firm i would sell an upgrade of A with a quality of $2V^A$ at the price of $2V^A$, which comes from $2 \times (2V^A - V^A)$, and make a profit of $\left(2V^A - R^A\right)$. Moreover, from Lemma 1, he will take the whole profits from upgrade at $t = 3$ in both markets as well. Thus the overall profits for $t = 2$ through 3 would be

$$\left(\pi_i^2 + \widetilde{\pi}_i^3\right) = (3V^A - 2R^A) + (3\overline{V}^B - 2\Delta - s - 3R^B) + \varepsilon \text{ and } \left(\pi_j^2 + \widetilde{\pi}_j^3\right) = 0. \tag{2.1}$$

Note that firm i makes the same profit whether he would tie or not because he is the only firm in market B. By tie-breaking rule, however, firm i sells both goods in a bundle.

Second, suppose that only firm j enters at $t = 2$. In this case there is no switching in market B. Therefore firm j would sell its upgrade of B at the price of $2\overline{V}^B$ and firm i would sell its upgrade of A at $2V^A$ respectively. Then overall profits for $t = 2$ through 3 are

$$\left(\pi_i^2 + \widetilde{\pi}_i^3\right) = 3V^A - 2R^A \text{ and } \left(\pi_j^2 + \widetilde{\pi}_j^3\right) = 3\overline{V}^B - 2R^B. \tag{2.2}$$

Finally, suppose that both firms enter into the market B at $t = 2$. Now consumers can either keep using B_j and upgrade it, or move to a newly-introduced B_i. In this case the market outcome hinges crucially on firm i's tying decision. *With firm i's tying*, consumer's incentive compatibility to switch to B_i turns out

$$2 \times (V^A + \overline{V}^B - \Delta) - s - p_i^{AB} \geq 2 \times \overline{V}^B - p_j^B \Longleftrightarrow p_i^{AB} \leq p_j^B + 2V^A - 2\Delta - s,$$

where p_i^{AB} is the price of firm i's bundle of an upgrade of A and a newly-released B_i. Note that, under tying regime, if consumers choose to keep using B_i, they cannot upgrade A. If $2V^A > 2\Delta + s$, firm i would sell its bundled products, but firm j would sell nothing. The resulting second period profits would be $\left[2V^A - (2\Delta + s) - R^A - 2R^B + \varepsilon\right]$ for firm i and $-R^B$ for firm j. Meanwhile, if $2V^A < 2\Delta + s$, firm j could sell the upgrade of its product in market B, while firm i could sell nothing in both markets. The profits would be

$\left[-(R^A + 2R^B) + \varepsilon\right]$ for firm i and $\left(2\Delta + s - 2V^A - R^B\right)$ for firm j. Then overall profits for $t = 2$ through 3 are

$$\left(\pi_i^2 + \widetilde{\pi}_i^3\right) = \begin{cases} (3V^A - 2R^A) + (\overline{V}^B - 2\Delta - s - 3R^B) + \varepsilon & \text{if } 2V^A > 2\Delta + s \\ (V^A - 2R^A) - 2R^B + \varepsilon & \text{if } 2V^A < 2\Delta + s \end{cases}$$
$$\left(\pi_j^2 + \widetilde{\pi}_j^3\right) = \begin{cases} -R^B & \text{if } 2V^A > 2\Delta + s \\ \overline{V}^B + (2\Delta + s) - 2V^A - 2R^B & \text{if } 2V^A < 2\Delta + s. \end{cases} \tag{2.3}$$

Without firm i's tying, firm i could not sell his subsidiary good in market B due to the quality inferiority and non-negative price constraints. So, consumer's incentive compatibility to switch to B_i turns out

$$2 \times (\overline{V}^B - \Delta) - s - p_i^B \geq 2 \times \overline{V}^B - p_j^B \Longleftrightarrow p_j^B \geq p_i^B + (2\Delta + s)\,.$$

In market B, therefore, firm j would sell the upgrade of B_j at $(2\Delta + s)$ and make a profit of $\left(2\Delta + s - R^B\right)$, while firm i make $\left(-2R^B + \varepsilon\right)$. In market A, firm i would sell the upgrade of A at $2V^A$, resulting in a profit of $\left(2V^A - R^A\right)$. Then overall profits for $t = 2$ and 3 are

$$\left(\pi_i^2 + \widetilde{\pi}_i^3\right) = (3V^A - 2R^A) - 2R^B + \varepsilon \text{ and } \left(\pi_j^2 + \widetilde{\pi}_j^3\right) = \overline{V}^B + 2\Delta + s - 2R^B. \tag{2.4}$$

Comparing firm i's profits in (2.3) and (2.4) under Assumption 2, we can verify that firm i would tie the goods if $2V^A > 2\Delta + s$, otherwise he would sell them separately. Therefore the profits for $t = 2$ through 3 are as follows:

$$\left(\pi_i^2 + \widetilde{\pi}_i^3\right) = \begin{cases} (3V^A - 2R^A) + (\overline{V}^B - 2\Delta - s - 3R^B) + \varepsilon & \text{if } 2V^A > 2\Delta + s \\ (3V^A - 2R^A) - 2R^B + \varepsilon & \text{if } 2V^A < 2\Delta + s \end{cases}$$

$$\left(\pi_j^2 + \widetilde{\pi}_j^3\right) = \begin{cases} -R^B & \text{if } 2V^A > 2\Delta + s \\ \overline{V}^B + 2\Delta + s - 2R^B & \text{if } 2V^A < 2\Delta + s. \end{cases} \tag{2.5}$$

Now we can characterize an overall equilibrium strategy and expected profits at $t = 2$ when firm j prevailed in the previous period as the following lemma.

Lemma 2 *Suppose that $V^B = \overline{V}^B$ and B_j has been sold at $t = 1$. (i) If $2V^A > 2\Delta + s$, firm i would enter into both market at $t = 2$ and sell its tied products while firm j would not enter into the market B. Profits for $t = 2$ through 3 are*
$\left(\widetilde{\pi}_i^2 + \widetilde{\pi}_i^3\right) = \left[(3V^A - 2R^A) + 3\overline{V}^B - (2\Delta + s + 3R^B) + \varepsilon\right]$ *and* $\left(\widetilde{\pi}_j^2 + \widetilde{\pi}_j^3\right) = 0.$
(ii) If $2V^A < 2\Delta + s$, firm i would enter into only the market A while firm j would sell its upgraded product in market B. Profits for $t = 2$ through 3 are
$\left(\widetilde{\pi}_i^2 + \widetilde{\pi}_i^3\right) = \left(3V^A - 2R^A\right)$ *and* $\left(\widetilde{\pi}_j^2 + \widetilde{\pi}_j^3\right) = \left(3\overline{V}^B - 2R^B\right).$

Proof. From (2.1), (2.2), and (2.5), we can construct the following payoff matrix.

	Firm j enter	Firm j not enter
Firm i enter	$\{(3V^A - 2R^A) + (\overline{V}^B - 2\Delta - s - 3R^B)$ $+\varepsilon,\ -R^B\}$ if $2V^A > 2\Delta + s$ $\{(3V^A - 2R^A) - 2R^B + \varepsilon,\ \overline{V}^B + 2\Delta$ $+s - 2R^B\}$ if $2V^A < 2\Delta + s$	$\{(3V^A - 2R^A) + 3\overline{V}^B$ $-\left(2\Delta + s + 3R^B\right) + \varepsilon,\ 0\}$
Firm i not enter	$\left\{3V^A - 2R^A,\ 3\overline{V}^B - 2R^B\right\}$	$\left\{3\overline{V}^B - 2R^B,\ 0\right\}$

Finding Nash equilibria in each case directly proves the lemma. ■

This lemma implies that, for a primary good monopolist to succeed in making a late-entry into the subsidiary market, the value of primary good upgrade should be sufficiently high compared to the sum of quality superiority of his competitor's good and switching cost.

(Case 2: when B_i sold at $t = 1$.)

In this sub-section we analyze a case that firm i prevailed in the first period. We will consider the same sub-cases as in *Case 1*.

First, supposing firm i enters only, he would sell an upgrade of B at the price of $2\overline{V}^B$ and an upgrade of A at the price of $2V^A$. By the similar calculation to that in *Case 1*, firm i's profit at $t = 2$ is $(2V^A - R^A) + (2\overline{V}^B - R^B)$. Then, from Lemma 1, overall profits for $t = 2$ through 3 are

$$\left(\pi_i^2 + \widetilde{\pi}_i^3\right) = (3V^A - 2R^A) + (3\overline{V}^B - 2R^B) \text{ and } \left(\pi_j^2 + \widetilde{\pi}_j^3\right) = 0. \tag{2.6}$$

It does not matter whether he would tie or not since firm i is the only seller in both market. By tie-breaking rule, however, firm i will use tying strategy.

Second, supposing firm j only enters at $t = 2$, firm j would introduce a product B with the quality of $(2\overline{V}^B + \Delta)$ in market B, with investing R^B. Now consumers can either keep using B_i without upgrade and get a per-period benefit of $\overline{V}^B$, or switch to a newly-introduced B_j and get a per-period benefit of $(2\overline{V}^B + \Delta)$ with incurring a switching cost, s. So consumer's incentive compatibility to switch to B_j turns out

$$2 \times (2\overline{V}^B + \Delta) - s - p_j^B \geq 2 \times \overline{V}^B \Longleftrightarrow p_j^B \leq 2\overline{V}^B + 2\Delta - s.$$

That is, firm j would sell his product with an upgraded quality at the price of $\left(2\overline{V}^B + 2\Delta - s\right)$. Thus overall profits for $t = 2$ through 3 are

$$\left(\pi_i^2 + \widetilde{\pi}_i^3\right) = (3V^A - 2R^A) \text{ and } \left(\pi_j^2 + \widetilde{\pi}_j^3\right) = (3\overline{V}^B - 2R^B) + (2\Delta - s). \tag{2.7}$$

Finally, suppose that both firms enter into the market B at $t = 2$. *With firm i's tying*, consumer's incentive compatibility to switch to B_j is,

$$2(V^A + \overline{V}^B) - p_i^{AB} \geq 2(\overline{V}^B + \Delta) - s - p_j^B \Longleftrightarrow p_i^{AB} \leq p_j^B + 2V^A - (2\Delta - s),$$

where p_i^{AB} is the price of firm i's bundle of upgrades. Since $2V^A > (2\Delta - s)$ by Assumption 1, firm i could sell its bundled products at the price of $2V^A - (2\Delta - s)$. So profits at $t = 2$ are $\left[2V^A - (2\Delta - s) - (R^A + R^B)\right]$ for firm i and $-R^B$ for firm j. Thus total profits for $t = 2$ through 3 are

$$\left(\pi_i^2 + \widetilde{\pi}_i^3\right) = (3V^A - 2R^A) + \overline{V}^B - (2\Delta - s) - 2R^B \text{ and } \left(\pi_j^2 + \widetilde{\pi}_j^3\right) = -R^B. \tag{2.8}$$

Without tying, consumer's incentive compatibility to switch to B_j turns out

$$2\overline{V}^B - p_i^B \geq 2(\overline{V}^B + \Delta) - s - p_j^B \Longleftrightarrow p_j^B \geq p_i^B + (2\Delta - s).$$

If $2\Delta > s$, firm j would sell a good in market B. So profits at $t = 2$ are $(2V^A - R^A - R^B)$ for firm i and $\left[(2\Delta - s) - R^B\right]$ for firm j. If $2\Delta < s$, firm i would sell a good in market B. So profits are $(2V^A - R^A) + (s - 2\Delta) - R^B$ for firm i and $-R^B$ for firm j. Thus total profits for $t = 2$ through 3 are

$$\begin{aligned}
\left(\pi_i^2 + \widetilde{\pi}_i^3\right) &= \begin{cases} (3V^A - 2R^A) - R^B & \text{if } 2\Delta > s \\ (3V^A - 2R^A) + \overline{V}^B - (2\Delta - s) - 2R^B & \text{if } 2\Delta < s \end{cases} \\
\left(\pi_j^2 + \widetilde{\pi}_j^3\right) &= \begin{cases} \overline{V}^B + (2\Delta - s) - 2R^B & \text{if } 2\Delta > s \\ -R^B & \text{if } 2\Delta < s. \end{cases}
\end{aligned} \tag{2.9}$$

Comparing firm i's profits in (2.8) and (2.9), we can easily verify that, when both firms enter the market B, firm i would always make a higher profit under tying strategy because $\overline{V}^B - (2\Delta - s) - 2R^B > 0$ by Assumptions 2 and 3. The equilibrium profits are those in (2.8).

Now the following lemma describes an equilibrium at $t = 2$ when firm i prevailed in the first period.

Lemma 3 *Suppose that $V^B = \overline{V}^B$ and B_i has been sold at $t = 1$. Then firm i would sell its bundled upgrades of A and B at $t = 2$ and 3 while firm j would not enter into the market B at $t = 2$. Profits for $t = 2$ through 3 are $\left(\widetilde{\pi}_i^2 + \widetilde{\pi}_i^3\right) = (3V^A - 2R^A) + (3\overline{V}^B - 2R^B)$ and $\left(\widetilde{\pi}_j^2 + \widetilde{\pi}_j^3\right) = 0$.*

Proof. From (2.6), (2.7), and (2.8), we can construct the following payoff matrix.

	Firm j enter	Firm j not enter
Firm i enter	$\{(3V^A - 2R^A) + \overline{V}^B - (2\Delta - s) - 2R^B,$ $-R^B\}$	$\{(3V^A - 2R^A)$ $+(3\overline{V}^B - 2R^B),\ 0\}$
Firm i not enter	$\left\{(3V^A - 2R^A),\ (3\overline{V}^B - 2R^B) + (2\Delta - s)\right\}$	$\left\{(3V^A - 2R^A),\ 0\right\}$

Finding Nash equilibria in each case directly proves the lemma. ■

This lemma is simply saying that, if the primary good monopolist prevailed in the subsidiary market in the first period, he could take the whole market for the rest of the period as well.

From Lemma 2 and 3, we can get the following proposition.

Proposition 7 *Under the given Assumptions 1 to 4, if firm B_i prevailed at $t = 1$ it will take the whole market for the rest of the periods. If firm B_j prevailed at $t = 1$ firm i can take the market B away from firm j only if $s < 2(V^A - \Delta)$.*

This proposition indicates that the only case where firm i could not sell his subsidiary good at $t = 2$ onwards is that firm i did not enter into the market B at $t = 1$ and the switching cost at $t = 2$ is sufficiently high.

2.3.3 In the first period

We analyze a subgame perfect Nash equilibrium for the entire game in this subsection. In the first period, firm i has two alternative strategies on entering into the subsidiary market; to enter in the first period, or to enter in the second period when it is possible and profitable.

First, suppose that firm i does not enter into market B at $t = 1$ and makes a new entry at $t = 2$. Since, at $t = 1$, good B is of no use without good A, firm i can exploit the entire surplus from both markets. So firm i's first period profit would be $\left[3(V^A + \overline{V}^B + \Delta) - R^A\right]$ if V^B turns out $\overline{V}^B$ with a probability of ρ or $\left[3(V^A + \underline{V}^B + \Delta) - R^A\right]$ if V^B turns out $\underline{V}^B$ with a probability of $(1 - \rho)$. Meanwhile firm j's profits would be $-R^B$. In this case the second period problem corresponds to the *Case 1*. Thus, from Lemma 2, total profits for the entire game are

$$\Pi_i = \begin{cases} \left.\begin{array}{ll} (6V^A - 3R^A) + (6\overline{V}^B - 3R^B) + \Delta - s + \varepsilon & \text{if } s < 2\left(V^A - \Delta\right) \\ (6V^A - 3R^A) + 3(\overline{V}^B + \Delta) & \text{if } s > 2\left(V^A - \Delta\right) \end{array}\right) & \text{with } \rho \\ (6V^A - 3R^A) + 3(\underline{V}^B + \Delta) & \text{with } (1-\rho), \end{cases}$$

$$\Pi_j = \begin{cases} \left.\begin{array}{ll} -R^B & \text{if } s < 2\left(V^A - \Delta\right) \\ 3\overline{V}^B - 3R^B & \text{if } s > 2\left(V^A - \Delta\right) \end{array}\right) & \text{with } \rho \\ -R^B & \text{with } (1-\rho). \end{cases}$$

Then expected profits are

$$E(\widetilde{\Pi}_i) = \begin{cases} (6V^A - 3R^A) + \rho\left[(6\overline{V}^B - 3R^B) + \Delta - s + \varepsilon\right] + (1-\rho)\left[3(\underline{V}^B + \Delta)\right] \\ \qquad \text{if } s < 2\left(V^A - \Delta\right) \\ (6V^A - 3R^A) + \rho\left[3(\overline{V}^B + \Delta)\right] + (1-\rho)\left[3(\underline{V}^B + \Delta)\right] \\ \qquad \text{if } s > 2\left(V^A - \Delta\right). \end{cases} \tag{2.10}$$

$$E(\widetilde{\Pi}_j) = \begin{cases} -R^B & \text{if } s < 2\left(V^A - \Delta\right) \\ \rho\left[3\overline{V}^B - 3R^B\right] - (1-\rho)R^B & \text{if } s > 2\left(V^A - \Delta\right). \end{cases}$$

Second, suppose that firm i enters into market B in the first period. In this case firm i's entering at $t = 1$ without tying is always dominated by entering at $t = 2$, due to the existence of ε, R&D cost savings. So, if firm i enters in the first period, it always accompanies tying strategy. With tying firm i's profit at $t = 1$ would be $(3V^A - R^A) + (3\overline{V}^B - R^B)$ with a probability of ρ and $(3V^A - R^A) + (3\underline{V}^B - R^B)$ with a probability of $(1 - \rho)$ while firm j's profit would be $-R^B$. In this case the second period problem corresponds to the *Case 2*. Thus, from Lemma 3, total profits for the entire periods are

$$\Pi_i = \begin{cases} (6V^A - 3R^A) + (6\overline{V}^B - 3R^B) & \text{with } \rho \\ (6V^A - 3R^A) + (3\underline{V}^B - R^B) & \text{with } (1-\rho), \end{cases} \qquad \text{and} \quad \Pi_j = -R^B.$$

Then firm's expected profits are

$$\begin{aligned} E(\widetilde{\Pi}_i) &= (6V^A - 3R^A) + \rho(6\overline{V}^B - 3R^B) + (1-\rho)(3\underline{V}^B - R^B) \qquad (2.11) \\ E(\widetilde{\Pi}_j) &= -R^B. \end{aligned}$$

Now comparing firm i's expected profits between (2.10) and (2.11), we can find firm i's optimal entry decision. When $s < 2\left(V^A - \Delta\right)$,

$$\rho(6\overline{V}^B - 3R^B) + (1-\rho)(3\underline{V}^B - R^B) \lesseqgtr \rho\left[(6\overline{V}^B - 3R^B) + \Delta - s + \varepsilon\right] + (1-\rho)\left[3(\underline{V}^B + \Delta)\right]$$
$$\Longleftrightarrow \rho \lesseqgtr \frac{R^B + 3\Delta}{R^B + s + 2\Delta - \varepsilon} \equiv \rho_1.$$

That is, firm i makes an entry at $t = 1$ rather than at $t = 2$ if $\rho > \rho_1$. Meanwhile when $s > 2\left(V^A - \Delta\right)$,

$$\rho(6\overline{V}^B - 3R^B) + (1-\rho)(3\underline{V}^B - R^B) \lesseqgtr \rho\left[3(\overline{V}^B + \Delta)\right] + (1-\rho)\left[3(\underline{V}^B + \Delta)\right]$$
$$\Longleftrightarrow \rho \lesseqgtr \frac{R^B + 3\Delta}{3\overline{V}^B - 2R} \equiv \rho_2.$$

That is, firm i makes an early-entry if $\rho > \rho_2$.

Then the following proposition describes the subgame perfect equilibrium of the entire game.

Proposition 8 *Subgame perfect equilibrium of the game is as follows. (i) When* $s < 2\left(V^A - \Delta\right)$*, firm* i *enters into the market* B *at* $t = 1$ *if* $\rho > \rho_1$ *but enters at* $t = 2$*, otherwise. (ii) When* $s > 2\left(V^A - \Delta\right)$*, firm* i *enters into the market* B *at* $t = 1$ *if* $\rho > \rho_2$ *but enters at* $t = 2$*, otherwise. (iii) Equilibrium expected profits,* $E(\widetilde{\Pi}_i)$ *and* $E(\widetilde{\Pi}_j)$*, are those defined in (2.10) if firm* i *enters at* $t = 2$ *and those defined in (2.11) if firm* i *enters at* $t = 1$*.*

By entering in the first period, firm i have to take the risk of B not being successful. On the contrary, if B is valued high by consumers it can secure the future profits coming from periodic upgrades. So this proposition states that the higher the probability of B's being successful, the more incentive firm i has to enter in the first period.

Moreover, from the relation between ρ_1, ρ_2 and s, we get the following proposition.

Proposition 9 *The probability for the firm* i *to enter into the market* B *at* $t = 1$ *is*

$$\begin{cases} 0 & \text{if } s \in [0, \Delta) \\ (1 - \rho_1) & \text{if } s \in \left[\Delta, 2\left(V^A - \Delta\right)\right) \\ (1 - \rho_2) & \text{if } s \in \left[2\left(V^A - \Delta\right), \infty\right), \end{cases}$$

where ρ_1 *is increasing in* s *but* ρ_2 *is independent of* s*, such that* $(1 - \rho_1) < (1 - \rho_2)$

Note that $\rho_1 > \rho_2$ by Assumption 2. If switching costs are sufficiently low, firm i has no incentive to enter into the market B in the first period. This is because, in spite of late-entry, firm i can always take the market away from firm j while reducing the risk of B's being unsuccessful. For the intermediate switching costs, the probability is monotonously increasing in s because high switching costs reduce firm i's second period profit even when firm i succeeds in taking the market away from its rival. Finally, if switching costs are

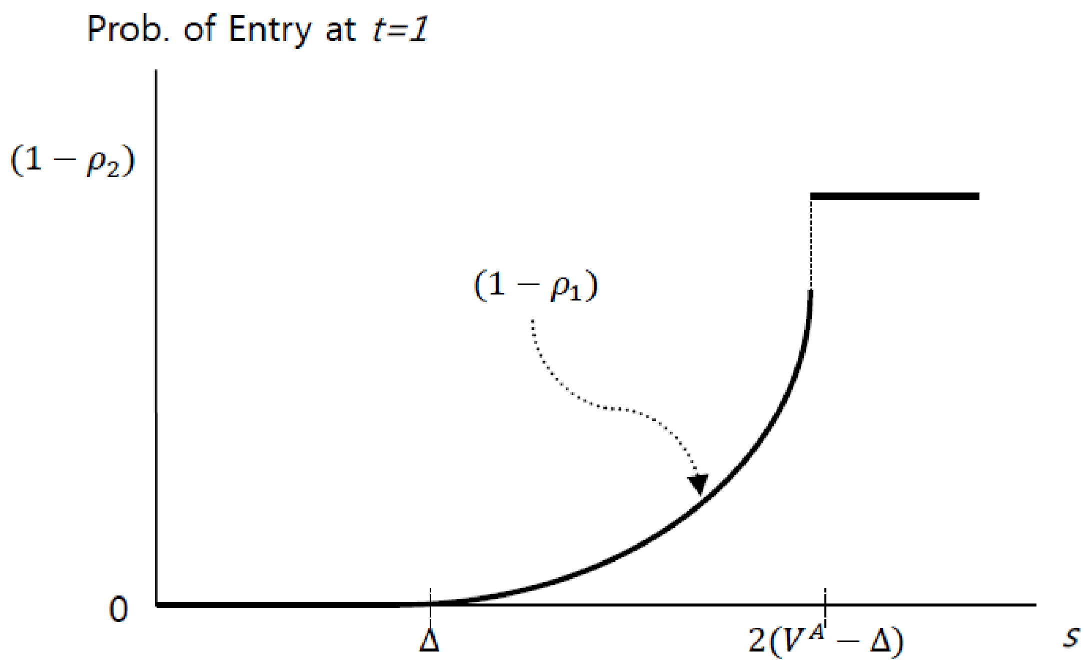

Figure 2.2: Switching Cost and the Probability of Firm i's Entry at $t = 1$.

sufficiently high, the probability is fixed at $(1 - \rho_2)$, which is always higher than $(1 - \rho_1)$. The reason is that firm i's profit does not depend on switching costs anymore, since in this case firm i's choice would be constrained between two alternatives, entering in the first period or not entering at all. Figure 2.2 plots this relationship.

2.4 Policy Implication

In this model, all consumers demand both one primary good and one subsidiary good in each period, whether they are initial products or upgrades. Moreover the entire value of the product is distributed between consumers and producers in the form of consumer's surplus and firm's revenue. Then the total welfare is the sum of the product values sold in both markets net of switching costs and R&D costs. First, suppose that tying is prohibited. Then the subsidiary good with high quality would always prevail in the market and no switching happens. So the total welfare is would be

$$(6V^A - 3R^A) + \rho(6\overline{V}^B + 3\Delta - 3R^B) + (1-\rho)(3\underline{V}^B + 3\Delta - R^B). \tag{2.12}$$

Second, suppose that tying is allowed and firm i enters in the first period. Then firm i would prevail in market B for the entire period and firm j would only incur R&D cost in the first period. Thus total welfare would be

$$(6V^A - 3R^A) + \rho(6\overline{V}^B - 3R^B) + (1-\rho)(3\underline{V}^B - R^B) - R^B. \tag{2.13}$$

Third, suppose that tying is allowed and firm i does not enter in the first period. Then firm j would prevail in the first period. From the second period, firm j would prevail for the rest of the period as well if switching cost is sufficiently high, but firm i would prevail otherwise. Thus total welfare would be, if $s > 2\left(V^A - \Delta\right)$,

$$(6V^A - 3R^A) + \rho(6\overline{V}^B + 3\Delta - 3R^B) + (1-\rho)(3\underline{V}^B + 3\Delta - R^B) \text{ and} \tag{2.14}$$

and, if $s < 2\left(V^A - \Delta\right)$,

$$(6V^A - 3R^A) + \rho(6\overline{V}^B + \Delta - 3R^B) - (s + R^B - \varepsilon) + (1-\rho)(3\underline{V}^B + 3\Delta - R^B). \tag{2.15}$$

From (2.12) and (2.13), it is apparent that if firm i's entry into the market B through tying from the first period it brings negative effect on social welfare by $-(3\Delta + R^B)$. This reflects the fact that consumers have to bear utility loss due to the use of low-quality good and there is waste of resources resulting from overlapping investments between two firms. From (2.12) and (2.14), we can see that there is no loss in social welfare if firm i did not enter in the first period and switching cost is sufficiently high. The reason is that in this case there is no switching and no overlapping investments since high switching cost prevent firm i from entering late. However, from (2.12) and (2.15), if firm i didn't enter in the first period and switching cost is not sufficiently high, tying has again negative effect on social welfare by $-\rho(2\Delta + s + R^B - \varepsilon)$. In this case the welfare loss comes from the quality inferiority of the good, consumer's switching costs, and firm's overlapping investment.

The above welfare analysis verifies again the fact that primary good monopolist's tying strategy may have negative effect on social welfare. This negative effect asks antitrust authorities to pay careful attention in scrutinizing this type of tying behavior. In this respect, the results in the previous section provide important policy implication. It suggests another criterion that should be considered in judging whether monopolist's tying behavior is anti-competitive or not. So far most of the criteria have focused on indices related with market structure, such as concentration ratio, market shares, etc. This paper suggests that they have to consider both technical aspects of the goods that determine the size of switching cost and the status of market evolution as well.

If a subsidiary product incurs high enough switching costs so that late-entry into a subsidiary market through tying is difficult, a primary good monopolist is more likely to enter early with tying than to enter late. In this case antitrust authorities should pay more attention in the early stage of market maturity since after that the monopolist incentive to enter into the subsidiary market decreases gradually. Meanwhile if a subsidiary product incurs relatively low switching costs, the monopolist is more likely to enter late through tying than to enter early. In this case the authorities should pay more attention as the market becomes more matured.

2.5 Concluding Remarks

We study an issue on primary good monopolist's entry decision to expand its monopoly power to the relevant subsidiary market. A focus is taken on the timing of entry with tying strategy. When the quality of the subsidiary good is uncertain, the firm's entry decision depends on the size of switching cost. If switching costs are sufficiently low, the firm always makes a late-entry. As the switching costs go up, the probability for the firm to make an early-entry also increases. If the switching cost is sufficiently high, this probability is fixed at certain high level. Once the monopolist succeeds in entering into the subsidiary market, the firm can take up the market forever and exploit entire expected future profits from upgrades. Therefore antitrust authorities need to scrutinize firm's tying behavior more strictly in the early stage of market evolution if switching costs are high, vice versa if those are low.

In this paper, I analyze a case of selling, while Carlton and Waldman (2005) considers cases of both selling and renting. If both firms rent the product instead of selling, the result is quite simple. In this case the new version does not need to compete with its old version anymore. Then the monopolist will not tie the products. Instead he exploits whole market profit by charging a high price on the primary good, as addressed by Chicago School argument.

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Chapter 3

Customer Return Policy as a Signal of Quality

3.1 Introduction

When you place an order for a good, especially online, you may not be completely sure of product quality. When you receive the item, its true quality might be higher or lower than anticipated. If the expected value of the good is greater than your willingness to pay, you will buy it in spite of the quality uncertainty. Otherwise, the uncertain quality of the product may prevent purchase, even though true quality is high. In the latter case customer return policy, which allows customers to return the item within a specific period, may attract potential buyers by reducing customer risk. After receiving the item, product quality becomes known. If you are satisfied with the quality, you would keep and use it; otherwise you could simply ship it back to the seller and get a refund of it. In this sense a seller's return policy enables customers to defer their purchase decision until more is known about product quality.

We can observe a variety of return periods across sellers and types of goods. One feature in common with them is that most of the sellers with high reputation on product quality offer a relatively long return period while smaller retailers or private sellers offer a relatively short one. For example, on Ebay where various qualities of goods are sold, private or used-good sellers offer shorter return period than commercial or new-good sellers.[1] In this paper, we address economic rationale for this variation of return periods, focusing on its role as a signaling device for product quality.

Consumers always benefit by a longer return period since they can collect more information on product quality. Meanwhile seller profits are affected by a longer return period in two different ways. On the one hand, a longer return period increases the depreciation loss from the returned items. On the other hand, it increases the chances that consumers get information about true product quality. The former unilaterally lowers seller's profits. The latter, however, brings different effects on seller's profits depending on the qualities of the products they sell. Since the precision of information is positively correlated with the length of return period, a longer return period leads to a high return rate for a low-quality seller and a low return rate for a high-quality seller. Therefore the high-quality seller would benefit whereas the low-quality seller would be harmed. In this sense a length of return period can be used as an efficient signaling device for product quality. That is, by offering a longer return period that cannot be imitated by a low-quality seller, a high-quality seller can differentiate himself from a low-quality seller.

To address this issue, we set up a model where a seller offers both a price and a length of return period. We adopt the specific information structure, in which only true information on product quality is revealed during the return period and precision of information is increasing function of the length of return period.[2] In other words, if customers receive a signal on the product quality during return period, they can perfectly figure out the true quality of the product, but a probability to get the signal is in proportion with the length of the period. Taking this information structure into consideration, a seller quotes a price and a return

[1] On Ebay, conditions of an item are labeled as 'New, New others, Used, and For parts or not working' according to the quality of the product. See more details on Ebay website. (http://pages.ebay.com/help/sell/contextual/condition_11.html)

[2] The loss of depreciation comes not only from actual damage of being used, but also from the devaluation by being re-categorized as an open box item after the return is accepted. In this case the loss of depreciation may not be proportional to the length of return period. Meanwhile throughout the paper we assume time-proportional depreciation loss.

period. After observing the offer and updating their belief on product quality, customers decide whether or not to accept the offer.

The main results are as follows. First, without consumer's interim benefits[3] during the return period, there exist multiple separating equilibria, where a high-quality seller offers a positive length of return period that is longer than a specific critical level, but a low-quality seller does not offer return service. By doing so, a high-quality seller can fully convince customers of the quality of the product he sells. Interestingly, at the separating equilibria, both types of sellers charge the same price as that of perfect information. Moreover, all the separating equilibria satisfy the Cho-Kreps intuitive criterion.[4] However, we find that there is no pooling equilibrium in which both types choose the same level of price and return period. This is because any pooling strategy is dominated by high-quality seller's deviation to the strategy in which he offers a perfect information price and a maximum refund period. Second, with consumer's interim benefits, both separating and pooling equilibria could exist depending on the specific forms of information and depreciation function. Even if there exist multiple separating equilibria, the unique separating equilibrium that satisfies the intuitive criterion is the one with the smallest return period. Some of pooling equilibria could also survive the intuitive criterion.

This paper builds on two veins of literature; the one is return policy and the other is signaling quality. There is a lot of economic literature on customer return policy. Che (1996) studies customer return policy with experience goods and shows that a seller will adopt return policy if customers are highly risk-averse or retail costs are high. Risk-averseness plays a crucial role in seller's adopting return policy. In this paper, however, we assume risk-neutral customers and implement a potential depreciation loss from the returned items as a restriction on seller's behavior. Ben-Shaha and Posner (2010) assume depreciation loss

[3]Consumer's interim benefits are the utilities from the trial use of the items during the return period.

[4]Cho and Kreps (1987) provide a way to refine equilibrium concept by restricting beliefs off-the-equilibrium path. That is, if a deviation is observed, the receiver believes that the deviation is not made by a type for whom the deviation is equilibrium-dominated.

and information structure similar to our paper[5] and address the effect of implementing a mandatory return policy. They propose that the mandatory return policy should be neither too strict nor too generous to balance consumer protection and seller's depreciation loss due to customer's abuse of the policy. Meanwhile we address the return policy in view of seller's signaling device.

There is a vast amount of literature on signaling quality. Among them, the closest signaling device to customer return policy is seller's warranties for durability of the product. For example, Spence (1977) shows that there is a unique separating equilibrium in a competitive market where a high-quality seller offers better warranty than a low-quality seller because the low-quality seller has to incur higher costs when an actual break-down happens. Grossman (1981) also draws a similar result assuming a single seller, in which warranty service is provided according to *ex post* verifiable events that depend on product quality. Meanwhile Gal-Or (1989) shows that, in a duopoly model, the signaling effect of warranty could be limited if product durabilities net of warranty are either too close or too different. However, as pointed out by Moorthy and Srinivasan (1995), warranty is somewhat different from return service or money-back guarantee; it takes the focus on performance-based warranties, it induce consumer's moral hazard due to relatively long warranty periods, and it compensates only partial value of the good. Moreover, the ultimate purchasing-decision is made at the initial purchase stage under warranties, while the decision would be made at the end of return period under customer return policy. In this sense, customer return policy gives better protection for consumers, thereby stronger signaling effect than warranty. Beside them, there are so many papers studying a variety of signaling devices; uninformative advertising in Milgrom and Roberts (1986), high and declining prices in Bagwell and Riordan (1991), product compatibility in Kim (2002), and bundling in Choi (2003). Recently Bourreau and Lethiais (2007) evaluated the use of free contents as a signaling device, using the same information structure as ours, and show that at separating equilibria a high-quality

[5] The information structure in Ben-Shaha and Posner (2010) is a little bit different from that in ours in that a low-quality seller can send either a true or a false signal. In this paper, however, we assume signals are always true.

seller provides positive amount of free contents. In this paper, we study the length of return period instead.

The remaining parts of the paper are as follows. In section 2, we present a signaling model, focusing on the information structure. In section 3, we solve basic maximization problems for a high-quality seller and a low-quality seller respectively. In section 4, we explain separating and pooling equilibria without consumer's interim benefits. In section 5, we address separating and pooling equilibria with consumer's interim benefits. In section 6, we give concluding remarks.

3.2 Model

There is one firm who sells an experience good. The quality of the good, chosen by nature, can be either v_H or v_L, where $0 < v_L < v_H$ and $v_H \leq 2v_L$, meaning the quality difference is not extremely large. A seller with a type of $i \in \{H, L\}$ quotes a price, p_i, to a buyer and at the same time offers a return period, t_i, as well. So seller's strategy profile is $s_i = (t_i, p_i)$. If the item is returned at the end of the return period, the seller incurs loss caused by depreciation damage, $\delta(t)v_i$, which is continuous function of depreciation ratio. We assume that $\delta'(t) > 0$ and $\delta''(t) < 0$, which means the depreciation ratio is increasing in t at a diminishing rate

There is a continuum of consumers whose preferences for the good are heterogeneous. Preference for quality, θ, is assumed to be uniformly distributed between 0 and 1, i.e., $\theta \sim u[0, 1]$. Before the seller makes an offer, consumers have common prior belief on the quality of the good, denoted by $\lambda = \Pr(v_i = v_H)$ and $(1 - \lambda) = \Pr(v_i = v_L)$, thus consumer's *ex ante* expected quality is $\lambda v_H + (1 - \lambda) v_L \equiv \bar{v}$. Consumers have a unit demand for the good, so they will buy it if the net utility is greater than 0, that is,

$$U = (\theta v - p) > 0.$$

So consumers *ex ante* expected net utility is $(\theta\bar{v} - p)$. After purchasing the good, consumers can get information about product quality by inspecting or using it during the offered return period. If they are not satisfied with the realized quality or their *ex post* expected net utility is negative, they can return it to the seller at no cost. We assume no buyer's interim benefit during the return period and no discount factor in consumer utility. We further assume that during the return period consumers do not incur any opportunity cost in keeping the item. This assumption enables them to make a purchase decision at the end of the return period, not during the period, so simplifies subsequent analysis.

Information structure is as follows. During the return period, consumers receive a signal, σ_H or σ_L depending on seller types, with a probability of ρ, which contains true information on the product quality. That is, if they receive a signal they can figure out the quality of the good for sure. Meanwhile if they do not receive a signal, which denoted by $\varnothing$, with a probability of $(1-\rho)$, they will maintain their prior belief on product quality. We assume that the precision of the information, which is equal to the probability to receive a signal, is increasing in the length of return period; $\rho \equiv \rho(t)$ is continuous in t such that $\rho'(t) > 0$ and $\rho''(t) < 0$, which means the probability is increasing in t at a diminishing rate. We further assume $\rho(0) = 0$ and $\rho(\bar{t}) = 1$, where $\bar{t}$ is sufficiently large so that consumers always receive a signal with a probability of 1. This information structure can be formalized as follows:

$$\Pr(\sigma_i|v_j) = \begin{cases} \rho(t) & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \text{ and } \Pr(\varnothing|v_j) = 1 - \rho(t),$$

where $i \in \{H, L\}$ and $j \in \{H, L\}$, and

$$\Pr(v_H|\sigma_H) = 1,\ \Pr(v_H|\sigma_L) = 0, \text{ and } \Pr(v_H|\varnothing) = \lambda.$$

The timing of the game is as follows. After the quality of the good determined by nature, a seller with v_i offers $s_i = (t_i, p_i)$ depending on his type. After observing seller's offer, consumers update their beliefs on the quality of the product and choose whether or not to accept the offer. After purchasing the good, consumers receive a signal σ_i during the return period and decide whether or not to return it at the end of the return period. If return is claimed by consumers, the seller will accept it and incur depreciation loss.

3.3 Basic Analysis

Under perfect information, a consumer with θ such that $(v_i\theta - p) \geq 0$, where $i = \{H, L\}$, will buy the good. The location of the critical consumer who is indifferent between buying and not buying is $\theta_i = \frac{p}{v_i}$. So a seller will charge $p_i = \frac{v_i}{2}$ and obtain a profit of $\pi_i = \frac{v_i}{4}$ depending on his type. Under the quality uncertainty, if there is no customer return policy, a critical consumer locates at $\theta_{\bar{v}} = \frac{p}{\bar{v}}$, and a seller will charge $\frac{\bar{v}}{2}$ and get a profit of $\frac{\bar{v}}{4}$ regardless of his type. So a seller with a low-quality good will benefit from quality uncertainty, while a seller with a high-quality good will be harmed since $v_L < \bar{v} < v_H$.

Now suppose that a seller adopts a customer return policy. Let Θ denote the location of the critical consumer who is indifferent between buying and not buying. We need to consider two cases separately according to the location of Θ. First, suppose that $\Theta \in [\theta_{\bar{v}}, \theta_L)$.[6] Under a given length of return period, if consumers do not receive a signal, with probability of $(1 - \rho(t))$, consumers who purchased the good will expect a net utility of $(\bar{v}\theta - p)$. If they receive a signal, with a probability of $\rho(t)$, their net utility would be $(v_H\theta - p)$ if the signal is σ_H, or 0 if σ_L. Therefore the total expected utility will be

$$U = (1 - \rho(t))(\bar{v}\theta - p) + \rho(t)\lambda(v_H\theta - p),$$

and the critical consumer locates at

$$\Theta = \frac{1 - (1 - \lambda)\rho(t)}{1 - (1 - \lambda)\rho(t)\frac{v_L}{\bar{v}}} \cdot \frac{p}{\bar{v}} < \frac{p}{\bar{v}} = \theta_{\bar{v}}.$$

So Θ cannot be in between $\theta_{\bar{v}}$ and θ_L. Second, suppose that $\Theta \in [\theta_H, \theta_{\bar{v}})$. If consumers

[6]Note that a critical consumer should always be between θ_H and θ_L.

do not receive a signal, those who purchased a good will return it and get a utility of 0. If they receive a signal, their net utility would be $(v_H\theta - p)$ if the signal is σ_H, but 0 if σ_L. Therefore the total expected utility will be $U = \rho(t)\lambda(v_H\theta - p)$, and the critical consumer locates at $\Theta = \theta_H$.

consumer location |————————|————————-|————————|————————|

$0 \qquad \Theta = \theta_H \qquad \theta_{\bar{v}} \qquad \theta_L \qquad 1$

Therefore, under customer return policy, consumers with $\theta \in [\theta_H, 1]$ will buy the good. The intuition is quite obvious. Since there is no return fee and no cost in keeping the item, consumers who can get at least positive net utility from the best possible quality will purchase the good.

Then we can characterize consumer responses profile as follows:

$$\text{Consumer responses} \begin{cases} \theta \in [0, \theta_H)\text{: Not buy at all.} \\ \theta \in [\theta_H, \theta_{\bar{v}})\text{: Buy, and return if not receiving } \sigma_H\text{, otherwise retain.} \\ \theta \in [\theta_{\bar{v}}, \theta_L)\text{: Buy, and return if receiving } \sigma_L\text{, otherwise retain.} \\ \theta \in [\theta_L, 1]\text{: Buy, and always retain.} \end{cases}$$

Here we have four segments which have different response profiles respectively. Consumers in the first segment will not buy the good at purchase stage since their preferences for the good is sufficiently low while consumers in the last segment will buy and never return since their preference is sufficiently high. Consumers in the second segment have relatively-low preference, so they will retain the good only if they are sure of high quality. Meanwhile, consumers in the third segment have relatively-high preference, so they will retain the good as long as they are not sure that the quality is low.

Based on these consumer responses, the seller faces different demand and profit functions depending on the quality of the product he sells. In the remaining parts of this section, we solve each seller's profit maximization problem when a return period is given and the belief is fixed at λ, and find an equilibrium price and profit as a function of return period.

3.3.1 Seller with a low-quality good

Suppose that a seller provides a low-quality good, that is $v = v_L$. When he offers (p_L, t_L), consumers with $\theta \in [\theta_H, 1]$ would buy the good at purchase stage. At the end of the offered return period, consumers with $\theta \in [\theta_H, \theta_{\bar{v}}]$ would return it for sure and those with $\theta \in [\theta_{\bar{v}}, \theta_L]$ would return it with a probability of $\rho(t)$. Thus a low-quality seller's expected profit after the return period ends is

$$\Pi_L = \begin{cases} p\left[(1-\rho(t))\left(1-\theta_{\bar{v}}\right)\right] - \delta(t)v_L\left[\rho(t)\left(1-\theta_{\bar{v}}\right) + \left(\theta_{\bar{v}} - \theta_H\right)\right] & \text{if } \theta_L > 1, \\ p\left[(1-\theta_L) + (1-\rho(t))\left(\theta_L - \theta_{\bar{v}}\right)\right] & \\ \qquad - \delta(t)v_L\left[\rho(t)\left(\theta_L - \theta_{\bar{v}}\right) + \left(\theta_{\bar{v}} - \theta_H\right)\right] & \text{if } \theta_L \leq 1, \end{cases} \tag{3.1}$$

where Π_L denotes the profit for a seller with a low-quality goods.[7] Solving profit maximizing problem, we can find seller's equilibrium price and profit, given the length of return period, and characterize consumer's equilibrium response profile. The following lemma summarizes it.

Lemma 4 *Suppose that consumers are uncertain on the quality of the product and a seller offers both a price and a return policy. Then, when consumer's belief on the quality is fixed at λ given the length of return periods, a low-quality seller offers p_L^* and consumers with $\theta \in [\theta_H, 1]$ will purchase the good. At the end of the return period consumers with $\theta \in [\theta_H, \theta_{\bar{v}}(p_L^*))$ would return it for sure, those with $\theta \in [\theta_{\bar{v}}(p_L^*), \theta_L(p_L^*))$ would do so if they received a signal, and the rest of them would keep it. An equilibrium price and seller's profit*

[7] The case that $\theta_{\bar{v}} > 1$ is of no interest since no one would buy and keep the good.

are

$$
\begin{aligned}
p_L^*(t) &= \frac{\bar{v}}{2} \cdot \frac{v_L}{\rho(t)\bar{v} + (1-\rho(t))v_L} \left[1 - \delta(t) \frac{\rho(t)\bar{v} + (1-\rho(t))\, v_L - v_L \left(\frac{v_L}{v_H}\right)}{\bar{v}} \right], \\
\Pi_L^*(t) &= \frac{\bar{v}}{4} \cdot \frac{v_L}{\rho(t)\bar{v} + (1-\rho(t))v_L} \left[1 - \delta(t) \frac{\rho(t)\bar{v} + (1-\rho(t))\, v_L - v_L \left(\frac{v_L}{v_H}\right)}{\bar{v}} \right]^2.
\end{aligned} \tag{3.2}
$$

Proof. *See Appendix.* ■

From this lemma, we can verify that an equilibrium price and a profit are decreasing in t, since both $\frac{\partial}{\partial t}\left[\frac{v_L}{\rho(t)\bar{v}+(1-\rho(t))v_L}\right]$ and $\frac{\partial}{\partial t}\left[1-\delta(t)\frac{\rho(t)\bar{v}+(1-\rho(t))v_L-v_L\left(\frac{v_L}{v_H}\right)}{\bar{v}}\right]$ have negative signs. The intuitions are as follows. As for the decreasing-price in t, a low-quality seller would face higher returns if he offers a longer return period while the amount of sale is fixed at $(1-\theta_H)$. This, in turn, would force a low-quality seller to lower the price in order to reduce the returns. As for the decreasing-profit in t, the better information harms a low-quality seller in two ways; it lowers final demand for the good and increases depreciation loss. Actually the profit of a low-quality seller decreases from $\Pi_L^*(0) = \frac{\bar{v}}{4}$ to $\Pi_L^*(\bar{t}) = \frac{v_L}{4}\left[1-\delta(\bar{t})\frac{v_H(\bar{v}-v_L)+v_L(v_H-v_L)}{\bar{v}v_H}\right]^2$ as t increases from 0 to $\bar{t}$. If t is sufficiently large, a low-quality seller would make a profit lower than that of perfect information case, which provides a basis on which a high-quality seller can differentiate himself from a low-quality sell by using return period as a signaling device for product quality.

3.3.2 Seller with a high-quality good

Suppose that a seller provides a high-quality good, that is $v = v_H$. When he offers (p_H, t_H), consumers with $\theta \in [\theta_H, 1]$ will buy the good at purchase stage. At the end of the offered return period, consumers with $\theta \in [\theta_H, \theta_{\bar{v}}]$ will return it with a probability of $(1-\rho(t))$ and the rest of them will keep it. Thus high-quality seller's expected profit after the return period ends is

$$\Pi_H = \begin{cases} p\left[\rho(t)\left(1-\theta_H\right)\right] - \delta(t)v_H(1-\rho(t))\left(1-\theta_H\right) & \text{if } \theta_{\bar{v}} > 1, \\ p\left[(1-\theta_{\bar{v}}) + \rho(t)\left(\theta_{\bar{v}}-\theta_H\right)\right] - \delta(t)v_H(1-\rho(t))\left(\theta_{\bar{v}}-\theta_H\right) & \text{if } \theta_{\bar{v}} < 1, \end{cases} \tag{3.3}$$

where Π_H denotes the profit for a seller with high-quality good.[8] Solving profit maximizing problem again, we can find seller's equilibrium price and profit, given the length of return period, and characterize consumer's equilibrium response profile. The following lemma summarizes it.

Lemma 5 *Suppose that consumers are uncertain on the quality of the product and a seller offers both a price and a return policy. Then, when consumer's belief on the quality is fixed at λ given the length of return periods, a high-quality seller offers p_H^* and consumers with $\theta \in [\theta_H(p_H^*), 1]$ will purchase the good. At the end of the return period consumers with $\theta \in [\theta_H(p_H^*), \theta_{\bar{v}}(p_H^*))$ will return it if they did not receive a signal, and the rest of them will keep it. An equilibrium price and the seller's profit are*

$$\begin{aligned} p_H^*(t) &= \frac{\bar{v}}{2} \cdot \frac{v_H}{\rho(t)\bar{v} + (1-\rho(t))v_H}\left[1 - \delta(t)\frac{(1-\rho(t))\left(v_H - \bar{v}\right)}{\bar{v}}\right], \\ \Pi_H^*(t) &= \frac{\bar{v}}{4} \cdot \frac{v_H}{\rho(t)\bar{v} + (1-\rho(t))v_H}\left[1 - \delta(t)\frac{(1-\rho(t))\left(v_H - \bar{v}\right)}{\bar{v}}\right]^2. \end{aligned} \tag{3.4}$$

Proof. *See Appendix.* ■

On the contrary to the low-quality seller's case, we cannot unilaterally determine the movement of equilibrium price and profit as t changes. While $\frac{\partial}{\partial t}\left[\frac{v_H}{v_H - \rho(t)(v_H - \bar{v})}\right]$ is always positive, $\frac{\partial}{\partial t}\left[1 - \delta(t)\frac{(1-\rho(t))(v_H-\bar{v})}{\bar{v}}\right]$ can be positive or negative depending on whether $\frac{\rho'(t)}{1-\rho(t)}$ is greater or smaller than $\frac{\delta'(t)}{\delta(t)}$. This reflects the fact that a high-quality seller faces

[8] The case that $\theta_H > 1$ is of no interest since no one would buy the good.

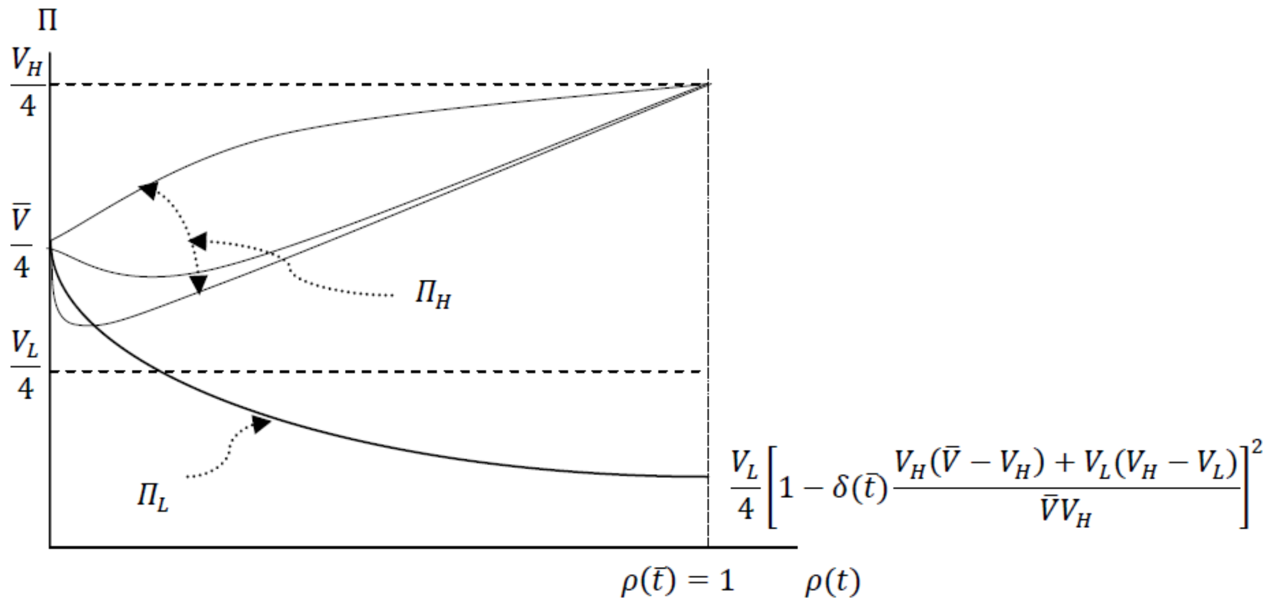


Figure 3.1: Seller's Profits

trade-off between revealing quality and incurring depreciation loss when he increases t. So the entire sign of $\frac{\partial P^*_H(t)}{\partial t}$ and $\frac{\partial \Pi^*_H(t)}{\partial t}$ will be determined by the specific functional forms of both $\rho(t)$ and $\delta(t)$. Intuitively, when t is relatively small, the effect of depreciation loss dominates the effect of revealing quality. Specifically if the good incurs a huge depreciation loss in earlier period, the profits might decrease from $\Pi^*_H(0) = \frac{\bar{v}}{4}$. However, as t increases, the quality revealing effect would dominates depreciation loss effect, so the profit would eventually increase and converge to $\Pi^*_H(\bar{t}) = \frac{v_H}{4}$, which provides an incentive for a high-quality seller offer longer return period.

In Figure 3.1, three possible profit curves of high-quality seller are shown, coupled with that of low-quality seller.

3.4 Equilibrium in Signaling Game

In this section we analyze equilibrium in signaling game, in which a return period is used as a signaling device. We will look into the existence of both separating and pooling equilibrium, and refine them by Cho-Kreps 'intuitive criterion.'

3.4.1 Separating Equilibrium

In this sub-section we find separating equilibria. Suppose that there exists a separating equilibrium, where a high- and a low-quality seller offer $s_H = (t_H, p_H^*(t_H))$ and $s_L = (t_L, p_L^*(t_L))$ respectively. At any separating equilibria consumers can perfectly differentiate seller's types, that is consumer's posterior beliefs after observing seller's offer are $\Pr(v_H; s_H) = 1$ and $\Pr(v_H; s_L) = 0$. Substituting $\lambda = 0$ into $p_L^*(t)$ and $\Pi_L^*(t)$ in Lemma 4, low-quality seller's price and profit are $p_L^*(t_L) = \frac{v_L}{2}\left[1 - \delta(t_L)\frac{v_H - v_L}{v_H}\right]$ and $\Pi_L^*(s_L) = \frac{v_L}{4}\left[1 - \delta(t_L)\frac{v_H - v_L}{v_H}\right]^2$ on the separating equilibria. This implies that the low-quality seller would set $t_L = 0$ and $p_L^*(t_L) = \frac{v_L}{2}$, resulting a profit of $\Pi_L^*(s_L) = \frac{v_L}{4}$. Meanwhile, substituting $\lambda = 1$ into $p_H^*(t)$ and $\Pi_H^*(t)$ in Lemma 5, high-quality seller's price and profit at the separating equilibria are fixed at $p_H^*(t_H) = \frac{v_H}{2}$ and $\Pi_L^*(s_H) = \frac{v_H}{4}$. Then the next proposition characterizes the separating equilibria.

Proposition 10 *(i) There exist multiple separating equilibria such that* $s_L = (t_L, p_L^*(t_L)) = (o, \frac{v_L}{2})$ *and* $s_H = (t_H, p_H^*(t_H)) = (t_H \in \{t_{se}, \bar{t}\}, \frac{v_H}{2})$ *where* t_{se} *satisfies*

$$\delta(t_{se})\left(\rho(t_{se}) + \frac{v_L}{v_H}\right)\left(1 - \frac{v_L}{v_H}\right) + \sqrt{\rho(t_{se}) + (1 - \rho(t_{se}))\frac{v_L}{v_H}} = 1.$$

The resulting profits are $\Pi_H^*(s_H) = \frac{v_H}{4}$ *and* $\Pi_L^*(s_L) = \frac{v_L}{4}$. *At any separating equilibria consumer's posterior belief is* $\Pr(v_H; s_H) = 1$ *and* $\Pr(v_L; s_L) = 0$. *(ii) All the separating equilibria satisfy the Cho-Kreps 'intuitive criterion.'*

Proof. See Appendix. ■

Proposition 10 shows that we have multiple separating equilibria, in which a seller with a high-quality good offers a positive return period beyond a certain specific level while a seller

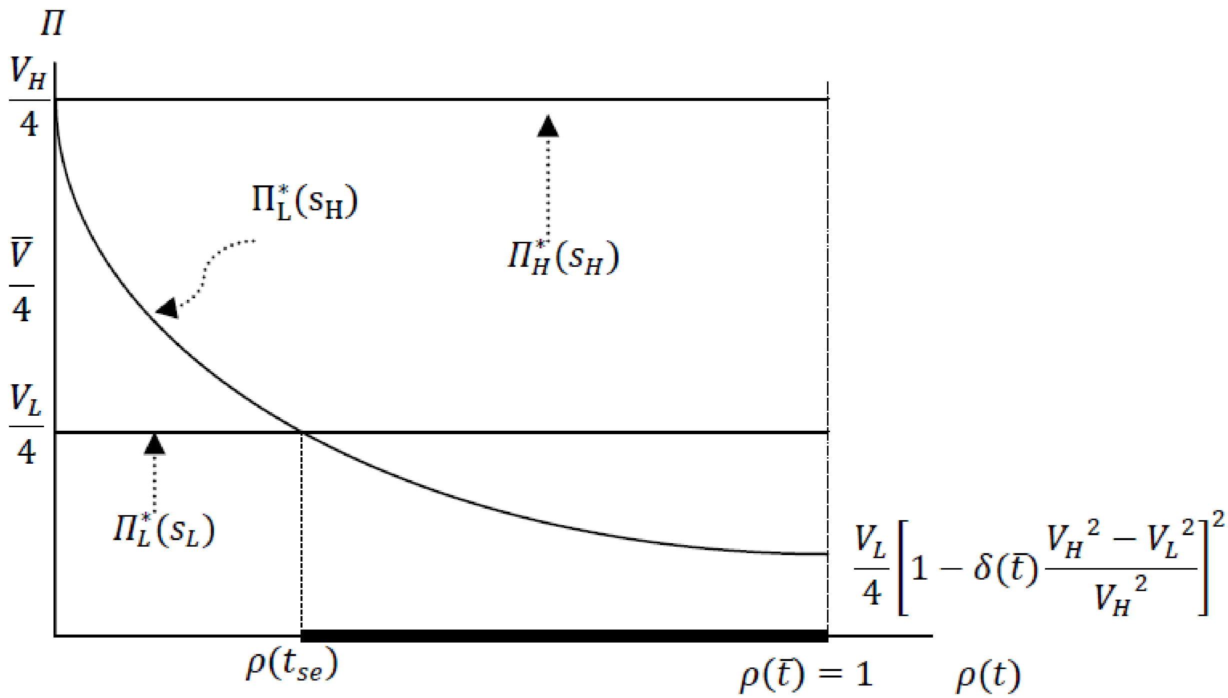


Figure 3.2: Separating Equilibria

with a low-quality good provides no return service. As we've seen in the previous section, low-quality seller's profit is decreasing in t and smaller than that of perfect information if $t > t_{se}$. Therefore, by offering return period longer than t_{se}, a high-quality seller can prevent low-quality seller's imitation and convince consumers that he is selling a high-quality good. Interestingly, the prices and the profits are the same as those under perfect information. The reason is that at the separating equilibria seller's type is perfectly inferred by customers, so no return happens. This also guarantees that at any separating equilibria the intuitive criterion is always satisfied since all of the profits on the equilibrium path are the highest ever profit a high-quality seller could make.

From the welfare perspective, at separating equilibria, the social welfare is optimal since the outcome is the same as that of perfect information and, moreover, no return happens.

3.4.2 Pooling Equilibrium

In this sub-section we look into the pooling equilibria. Suppose that there exists a pooling equilibrium, where $s_H = s_L = \tilde{s} = (\tilde{t}, \tilde{p})$. Consumers do not update their beliefs, so their *ex post* beliefs are still $\Pr(v_H; \tilde{s}) = \lambda$ and $\Pr(v_L; \tilde{s}) = 1 - \lambda$. As for the belief on off-the-equilibrium path, we assume that $\Pr(v_H; s \neq \tilde{s}) = 0$. Then the best off-the-equilibrium profit for the low-quality seller is

$$\underset{t}{Max}\Pi_L^*(s \neq \tilde{s}) = \underset{t}{Max}\frac{v_L}{4}\left[1 - \delta(t)\frac{v_H - v_L}{v_H}\right]^2,$$

and the maximized profit is $\frac{v_L}{4}$ at $t = 0$. Similarly, the best off-the-equilibrium profit for the high-quality seller is

$$\underset{t}{Max}\Pi_H^*(s \neq \tilde{s}) = \underset{t}{Max}\frac{v_L}{4} \cdot \frac{v_H}{\rho(t)v_L + (1 - \rho(t))v_H}\left[1 - \delta(t)\frac{(1 - \rho(t))(v_H - v_L)}{v_L}\right]^2,$$

and the maximized profit is $\frac{v_H}{4}$ at $t = \bar{t}$. Note that $\frac{v_H}{4}$ is the best profit that the high-quality seller could get under perfect information. This means that, at any pooling equilibrium, the profit for the high-quality seller cannot exceed the best off-the-equilibrium profits, where the seller charges $\frac{v_H}{2}$ and offers the return period of $\bar{t}$. Thus we have the following proposition.

Proposition 11 *There exists no pooling equilibria, such that $s_H = s_L = (\tilde{t}, \tilde{p})$.*

That is, at any pooling strategy, the high-quality seller has an incentive to deviate to the strategy offering a price of perfect information and a maximum refund period. Thus any pooling strategy cannot be supported as equilibrium.

The result of the analysis in this section can be summarized that, without consumer's interim benefits, there exist only multiple separating equilibria in signaling game and all of them satisfy intuitive criterion. This may not seem to be consistent with the reality in that it implies that sellers would be indifferent between t_{se} and $\bar{t}$. In reality, however, the seller seems to prefer shorter return period if other things are equal, especially when the return actually happens. This difference basically comes from the assumption that there is no interim benefit for consumers.[9] On the contrary, if we assume consumer's interim benefits, the smallest t among separating equilibria satisfies the intuitive criterion, as will be discussed in next section.

3.5 Extension: Equilibrium with Consumer's Interim Benefits

In the previous section, under the assumption of no consumer's interim benefit, we found that there exist multiple separating equilibria that satisfy intuitive criterion. Now we relax that assumption and show that, with consumer's interim benefits, the separating equilibria that satisfy the intuitive criterion boils down to the unique one with the smallest return period among them. When there exist consumer's interim benefits, a seller would not offer excessively long return period because it results in a surge of interim benefit-poaching consumers, who have no intension of retaining the good. Actually the maximum return period, $\hat{t}$, is limited to the level at which the profit maximizing price is non-negative, specifically $\delta(\hat{t}) = \frac{1}{2}$ as shown later. Thus we restrict our attention to $t \in [0, \hat{t}]$ in this section.

Suppose that there are consumer's interim benefits and, for simplicity, the size of interim benefit is the same as that of depreciation loss. At purchase stage, all consumers would buy the product, that is $\Theta = 0$, whether the quality of the product is high or low. This is because consumers can get at least positive interim benefits without incurring return fee or any other opportunity cost in keeping it. At the end of return period, consumers would compare the remaining expected value of the good with the price he paid, and keep the good if the former is greater than the latter, or return it otherwise. Then we can identify new

[9]Meanwhile consumer's return fee doesn't affect the previous two propositions qualitatively. It only shifts the t_{se} to the right as discussed in next section.

locations of critical consumers, similar in section 3, as follows:

$$\dot{\theta}_H \equiv \frac{p}{(1-\delta(t))v_H} \leq \dot{\theta}_{\bar{v}} \equiv \frac{p}{(1-\delta(t))\bar{v}} \leq \dot{\theta}_L \equiv \frac{p}{(1-\delta(t))v_L}.$$

Then consumers with $\theta \in [\dot{\theta}_L, 1]$ would not return it in any case, those with $\theta \in [\dot{\theta}_{\bar{v}}, \dot{\theta}_L]$ would return it if receive σ_L, those with $\theta \in [\dot{\theta}_H, \dot{\theta}_{\bar{v}}]$ would return it if not receive σ_H, and those with $\theta \in [0, \dot{\theta}_H]$ would return it for sure.

In this case seller's profits would be

$$\dot{\Pi}_L = p\left[(1-\dot{\theta}_L) + (1-\rho(t))\left(\dot{\theta}_L - \dot{\theta}_{\bar{v}}\right)\right] - \delta(t)v_L\left[\rho(t)\left(\dot{\theta}_L - \dot{\theta}_{\bar{v}}\right) + \dot{\theta}_{\bar{v}}\right] \tag{3.5}$$

for a low-quality seller[10], and

$$\dot{\Pi}_H = p\left[(1-\dot{\theta}_{\bar{v}}) + \rho(t)\left(\dot{\theta}_{\bar{v}} - \dot{\theta}_H\right)\right] - \delta(t)v_H\left[(1-\rho(t))\left(\dot{\theta}_{\bar{v}} - \dot{\theta}_H\right) + \dot{\theta}_H\right] \tag{3.6}$$

for a high-quality seller[11]. Solving maximization problem respectively, we have the following

[10] In this section we pay attention only to the case that $\dot{\theta}_L \leq 1$ since, otherwise, local maximum doesn't exist. Formally, supposing $\dot{\theta}_L > 1$, $\dot{\Pi}_L = p\left[(1-\rho(t))\left(1-\dot{\theta}_{\bar{v}}\right)\right] - \delta(t)v_L\left[\rho(t)\left(1-\dot{\theta}_{\bar{v}}\right) + \dot{\theta}_{\bar{v}}\right]$. Solving the maximization problem, and substituting $\dot{p}_L^*$ into $\dot{\theta}_L$, $\dot{\theta}_L = \frac{\bar{v}}{2v_L}\left[1-\delta(t)(1+\frac{v_L}{\bar{v}})\right] \leq 1$ since $v_H \leq 2v_L$. This contradicts to the given assumption.

[11] Similarly we pay attention only to the case that $\dot{\theta}_{\bar{v}} \leq 1$ since, otherwise, local maximum doesn't exist. Formally, supposing $\dot{\theta}_{\bar{v}} > 1$, $\dot{\Pi}_H = p\left[\rho(t)\left(1-\dot{\theta}_H\right)\right] - \delta(t)v_H\left[(1-\rho(t))\left(1-\dot{\theta}_H\right) + \dot{\theta}_H\right]$. Solving the maximization problem, and substituting $\dot{p}_L^*$ into $\dot{\theta}_{\bar{v}}$, $\dot{\theta}_{\bar{v}} = \frac{v_H}{2\bar{v}}\left[1-\delta(t)\left(1+\frac{v_L}{v_H}\right)\right] \leq 1$ since $v_H \leq 2v_L$. This contradicts to the given assumption.

equilibrium prices and profits, without updating belief, similar to those in section 3,

$$\begin{aligned}
\dot{p}_L^* &= \frac{\bar{v}}{2}\frac{(1-\delta(t))v_L}{\rho(t)\bar{v}+(1-\rho(t))\,v_L}\left[1-\delta(t)\frac{\rho(t)\bar{v}+(1-\rho(t))\,v_L}{(1-\delta(t))\bar{v}}\right], \\
\dot{\Pi}_L^* &= \frac{\bar{v}}{4}\frac{(1-\delta(t))v_L}{\rho(t)\bar{v}+(1-\rho(t))\,v_L}\left[1-\delta(t)\frac{\rho(t)\bar{v}+(1-\rho(t))\,v_L}{(1-\delta(t))\bar{v}}\right]^2,
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
\dot{p}_H^* &= \frac{\bar{v}}{2}\frac{(1-\delta(t))v_H}{\rho(t)\bar{v}+(1-\rho(t))\,v_H}\left[1-\delta(t)\frac{\rho(t)\bar{v}+(1-\rho(t))\,v_H}{(1-\delta(t))\bar{v}}\right], \\
\dot{\Pi}_H^* &= \frac{\bar{v}}{4}\frac{(1-\delta(t))v_H}{\rho(t)\bar{v}+(1-\rho(t))\,v_H}\left[1-\delta(t)\frac{\rho(t)\bar{v}+(1-\rho(t))\,v_H}{(1-\delta(t))\bar{v}}\right]^2.
\end{aligned} \tag{3.8}$$

Note that $\dot{\theta}_L(\dot{p}_L^*) < 1$ for a high-quality seller and $\dot{\theta}_{\bar{v}}(\dot{p}_H^*) < 1$ for a low-quality seller[12], thus the local maximums, which are global maximum as well, exist respectively. Comparing (3.2) and (3.7), and (3.4) and (3.8), we can easily verify that both sellers' profits decrease when there exist consumer's interim benefits. This is because the number of consumers who end up with buying and retaining the good eventually is smaller than that of without interim benefits. In other words, more sales but more returns reduce seller's profits.

Now let us find separating equilibria. Substituting $\lambda = 0$ into $\dot{p}_L^*(t)$ and $\dot{\Pi}_L^*(t)$ in (3.7), low-quality seller's price and profit are $\dot{p}_L^*(t_L) = \frac{v_L}{2}(1-2\delta(t_L))$ and $\dot{\Pi}_L^*(s_L) = \frac{v_L}{4}\frac{(1-2\delta(t_L))^2}{1-\delta(t_L)}$ on the separating equilibria. Note that, from $\dot{p}_L^*(t_L) = \frac{v_L}{2}(1-2\delta(t_L)) \geq 0$, $\delta(t_L)$ should be smaller than $\frac{1}{2}$ and, for $\delta(t_L) \in \left[0, \frac{1}{2}\right]$, $\Pi_L^*(s_L)$ is decreasing in t. Thus the low-quality seller would set $t_L = 0$ and $\dot{p}_L^*(t_L) = \frac{v_L}{2}$, resulting a profit of $\dot{\Pi}_L^*(s_L) = \frac{v_L}{4}$, which is the same as that without interim benefits. Meanwhile substituting $\lambda = 1$ into $\dot{p}_H^*(t)$ and $\dot{\Pi}_H^*(t)$ in (3.8), high-quality seller's price and profit are $\dot{p}_H^*(t_H) = \frac{v_H}{2}(1-2\delta(t_H))$

[12] With a similar logic in the proofs of Lemma 4 and 5, for a low-type seller, $\dot{\theta}_L(\dot{p}_L^*) = \frac{1}{2}\frac{\bar{v}}{\rho(t)\bar{v}+(1-\rho(t))v_L}\left\{1-\delta(t)(\left[1+\rho(t)+(1-\rho(t))\frac{v_L}{\bar{v}}\right]\right\} \leq 1$, and, for a high-type seller, $\dot{\theta}_{\bar{v}}(\dot{p}_H^*) = \frac{1}{2}\frac{v_H}{\rho(t)\bar{v}+(1-\rho(t))v_H}\left\{1-\delta(t)\left[1+\rho(t)+(1-\rho(t))\frac{v_H}{\bar{v}}\right]\right\} \leq 1$.

and $\dot{\Pi}^*_H(s_H) = \frac{v_H}{4}\frac{(1-2\delta(t_H))^2}{1-\delta(t_H)}$ at separating equilibria. For $\delta(t_L) \in \left[0, \frac{1}{2}\right]$, $\dot{\Pi}^*_H(t)$ is also decreasing in t. Then next proposition characterizes the separating equilibria.

Proposition 12 *Suppose there are interim benefits for consumers. Let us define* $\dot{t}^L_{se}$ *such that* $\delta(\dot{t}^L_{se})\left[1+\rho(\dot{t}^L_{se})+\left(1-\rho(\dot{t}^L_{se})\right)\frac{v_L}{v_H}\right] + \sqrt{(1-\delta(\dot{t}^L_{se}))\left[\rho(\dot{t}^L_{se})+(1-\rho(\dot{t}^L_{se}))\frac{v_L}{v_H}\right]} = 1$, *and* $\dot{t}^H_{se}$ *such that* $\frac{\left(1-2\delta(\dot{t}^H_{se})\right)^2}{1-\delta(\dot{t}^H_{se})} = \frac{v_L}{v_H}$. *(i) Then, if* $\dot{t}^L_{se} < \dot{t}^H_{se}$, *there exist multiple separating equilibria, in which* $s_L = \left(o, \frac{v_L}{2}\right)$ *and* $s_H = \left(t_H \in \left[\dot{t}^L_{se}, \dot{t}^H_{se}\right], \frac{v_H}{2}(1-2\delta(t_H))\right)$, *resulting in* $\dot{\Pi}^*_L(s_L) = \frac{v_L}{4}$ *and* $\dot{\Pi}^*_H(s_H) = \frac{v_H}{4}\frac{(1-2\delta(t_H))^2}{1-\delta(t_H)}$. *If* $\dot{t}^L_{se} = \dot{t}^H_{se}$, *there exists a unique separating equilibrium, in which* $s_L = \left(o, \frac{v_L}{2}\right)$ *and* $s_H = \left(\dot{t}^L_{se}, \frac{v_H}{2}\left(1-2\delta(\dot{t}^L_{se})\right)\right)$, *resulting in* $\dot{\Pi}^*_L(s_L) = \frac{v_L}{4}$ *and* $\dot{\Pi}^*_H(s_H) = \frac{v_H}{4}\frac{\left(1-2\delta(\dot{t}^L_{se})\right)^2}{1-\delta(\dot{t}^L_{se})}$. *Otherwise, there exists no separating equilibrium. (ii) Even if there exist multiple separating equilibria, the unique one that satisfies the Cho-Kreps 'intuitive criterion' is* $s_L = \left(o, \frac{v_L}{2}\right)$ *and* $s_H = \left(\dot{t}^L_{se}, \frac{v_H}{2}\left(1-2\delta(\dot{t}^L_{se})\right)\right)$.

Proof. See Appendix. ■

The first part of the proposition shows that the existence of consumer's interim benefit puts an upper bound on the return period of separating equilibria. Without consumer's interim benefits, a high-quality seller has no incentive to deviate. So we have an incentive compatibility constraint only for a low-quality seller. However, with the interim benefits, a high-quality seller also has an incentive to deviate when the return period is high enough. Compared to the case without interim benefit, more consumers purchase initially but less consumers end up with final purchase under a given return period and the size of final purchasers is decreasing in the return period.[13] This brings negative effect on the seller providing longer return period in two ways. First it encourages strategic consumers who poach only interim benefits but have no intention to purchase finally. Consumers with

[13]Note that $\dot{\theta}_H$, $\dot{\theta}_{\bar{v}}$, and $\dot{\theta}_L$, which are increasing in t, are larger than θ_H, $\theta_{\bar{v}}$, and θ_L respectively.

$\theta \in [0, \dot{\theta}_H)$ correspond to these strategic consumers. Second it deters some of potential buyers from making final purchase due to lowered residual values. This restricts high-quality seller's incentive to provide excessively long return period. Therefore, in this case, we have incentive compatibility constraints for both types of sellers. The second part of the proposition is saying that, since consumers are sufficiently sophisticated to make use of interim benefits during the return period, the high-quality seller would offer the shortest return period among separating equilibria to minimize depreciation loss and, thereby, maximize profits as long as he can differentiate himself from a low-quality seller.

Now let us look into pooling equilibria. At the pooling equilibrium, both sellers offer $\tilde{s} = (\tilde{t}, \tilde{p}) = \left(\tilde{t}, \frac{(1-\delta(t))\bar{v}}{2}\right)$. Substituting this into (3.5) and (3.6), the profits on-the-equilibrium path are

$$\begin{aligned} \dot{\Pi}_L(\tilde{s}) &= \tilde{p}\left[1 - \left(\frac{\tilde{p} - \delta(t)v_L}{1-\delta(t)}\right)\left(\frac{\rho(t)}{v_L} + \frac{1-\rho(t)}{\bar{v}}\right)\right], \\ \dot{\Pi}_H(\tilde{s}) &= \tilde{p}\left[1 - \left(\frac{\tilde{p} - \delta(t)v_H}{1-\delta(t)}\right)\left(\frac{\rho(t)}{v_H} + \frac{1-\rho(t)}{\bar{v}}\right)\right]. \end{aligned}$$

Again, the best off-the-equilibrium profits are for the low-quality seller is $\frac{v_L}{4}$ at $t = 0$. For the high-quality seller, the best off-the-equilibrium profit is

$$Max\ \dot{\Pi}_H(s \neq \tilde{s}) = \underset{t}{Max} \frac{v_H}{4} \frac{(1-\delta(t))}{\rho(t) + (1-\rho(t))\frac{v_H}{v_L}} \left[1 - \delta(t)\frac{\rho(t) + (1-\rho(t))\frac{v_H}{v_L}}{(1-\delta(t))}\right]^2.$$

Then the next proposition characterizes the pooling equilibria under consumer's interim benefits.

Proposition 13 *Suppose there are interim benefits for consumers. Then, there exist multiple pooling equilibria,* $\tilde{s} = (\tilde{t}, \tilde{p})$*, if* $(\tilde{t}, \tilde{p})$ *satisfies*

$$\dot{\Pi}_L(\tilde{s}) \geq \frac{v_L}{4} \quad \textit{and } \dot{\Pi}_H(\tilde{s}) \geq Max\ \dot{\Pi}_H(s \neq \tilde{s}).$$

In case of pooling equilibria, we cannot specify the critical values of return period without further assuming specific functional forms on $\rho(t)$ and $\delta(t)$. Moreover the intuitive criteria cannot eliminate all the pooling equilibria; that is, the degree of elimination depends on the range of return period that supports pooling or separating equilibria respectively and the size of each equilibrium profits.

In summary, when there is interim benefit during the return period, there may exist multiple separating equilibria, but the minimum return period among them could survive the intuitive criterion. There could also exist multiple pooling equilibria. The intuitive criterion could eliminate some of them, but not all necessarily.

Throughout the paper, we assume that there is no return fee for consumers. Relaxing the assumption, however, does not change the previous results qualitatively. It only changes the critical level of return period that can support the separating equilibria. Suppose there is a return fee. Then consumer's incentive to buy and return would decrease and the location of the critical consumer moves to the right, meaning fewer consumers would purchase. The specific effects of this shift on both seller's profits depend on the location of Θ. Intuitively, we can conjecture that a low-quality seller benefits more from it than a high-quality seller. This is because most of withdrawn buyers belong to for-sure returners, who might harm the low-quality seller more if they purchased. This narrows the profit margin between two sellers, and, in turn, shifts the minimum return period supporting separating equilibria to the right.

3.6 Concluding Remarks

In this paper we show that the length of return period can be used as an effective signaling device. Without interim benefits, we show that there exist multiple separating equilibria, where a high-quality seller offers a positive length of return period above a specific level, while a low-quality seller does not provide customer return policy. All the separating equilibria satisfy the intuitive criterion. We also find that there is no pooling equilibrium since the high-quality seller always has an incentive to deviate to perfect information price and the maximum return period. With interim benefits, in the meanwhile, there could be multiple separating equilibria, but the smallest return period survives the intuitive criterion. Multiple pooling equilibria could also exist and not all of them would be necessarily eliminated by the intuitive criterion.

The result drawn in this paper reflects the reality well. As shown in the example of Ebay, the seller with high-quality good offers a longer return period than the seller with low-quality good. Moreover, as a signaling device, customer return policy is more effective than warranty in that warranty generally does not guarantee a full refund.

APPENDIX

Appendix

■ Proofs omitted in the text

Proof of Lemma 4. We first find a local equilibrium when $\theta_L < 1$ and $\theta_L > 1$ respectively. Then, comparing both profit functions, we show that the global equilibrium always corresponds to the case in which $\theta_L < 1$.

(Case 1: $\theta_L < 1$**)**

From (3.1), the profit function is

$$\begin{aligned}\Pi_{L1} &= p\left[(1-\theta_L) + (1-\rho(t))\left(\theta_L - \theta_{\bar{v}}\right)\right] - \delta(t)v_L\left[\rho(t)\left(\theta_L - \theta_{\bar{v}}\right) + \left(\theta_{\bar{v}} - \theta_H\right)\right] \\ &= p\left[1 - \left(\frac{\rho(t)}{v_L} + \frac{1-\rho(t)}{\bar{v}}\right)p\right] - \delta(t)v_L\left(\frac{\rho(t)}{v_L} + \frac{1-\rho(t)}{\bar{v}} - \frac{1}{v_H}\right)p,\end{aligned}$$

and the first-order condition is

$$\frac{\partial \Pi_{L1}}{\partial p} = 1 - 2\left(\frac{\rho(t)}{v_L} + \frac{1-\rho(t)}{\bar{v}}\right)p - \delta(t)v_L\left(\frac{\rho(t)}{v_L} + \frac{1-\rho(t)}{\bar{v}} - \frac{1}{v_H}\right) = 0.$$

The second-order condition is satisfied; $-2\left(\frac{\rho(t)}{v_L} + \frac{1-\rho(t)}{\bar{v}}\right) < 0$. Then the profit maximizing price and the corresponding profit are

$$\begin{aligned}p^*_{L1}(t) &= \frac{\bar{v}}{2}\cdot\frac{v_L}{\rho(t)\bar{v} + (1-\rho(t))v_L}\left[1 - \delta(t)\frac{\rho(t)\bar{v} + (1-\rho(t))\,v_L - v_L\left(\frac{v_L}{v_H}\right)}{\bar{v}}\right], \\ \Pi^*_{L1}(t) &= \frac{\bar{v}}{4}\cdot\frac{v_L}{\rho(t)\bar{v} + (1-\rho(t))v_L}\left[1 - \delta(t)\frac{\rho(t)\bar{v} + (1-\rho(t))\,v_L - v_L\left(\frac{v_L}{v_H}\right)}{\bar{v}}\right]^2.\end{aligned}$$

For this to be a local maximum, it must be the case $\theta_L(p^*_L(t)) < 1$. Because $\theta_L(p^*_L(t)) = \frac{p^*_L}{v_L}$,

$$\theta_L(p^*_{L1}(t)) = \frac{1}{2}\cdot\frac{\bar{v}}{\rho(t)\bar{v} + (1-\rho(t))v_L}\left[1 - \delta(t)\frac{\rho(t)\bar{v} + (1-\rho(t))\,v_L - v_L\left(\frac{v_L}{v_H}\right)}{\bar{v}}\right],$$

Note that

$$0 \leq \left[1 - \delta(t)\frac{\rho(t)\bar{v} + (1-\rho(t))\,v_L - v_L\left(\frac{v_L}{v_H}\right)}{\bar{v}}\right] \leq 1,$$

since $0 \leq \left[\rho(t)\bar{v} + (1-\rho(t))\,v_L - v_L\left(\frac{v_L}{v_H}\right)\right]/\bar{v} \leq 1$. Then we need to show $\bar{v}/\left[\rho(t)\bar{v} + (1-\rho(t))v_L\right] \leq 2$. Rearranging it, the condition is simplified as $v_L/\bar{v} \geq \left[1 - 2\rho(t)\right]/\left[2(1-\rho(t))\right]$. The minimum of the left-hand side is $\frac{1}{2}$ by the assumption of $v_H \leq 2v_L$, and the maximum of the right-hand side is $\frac{1}{2}$ at $\rho(t) = 0$. So the condition always holds.

(Case 2: $\theta_L > 1$)

From (3.1), we have a profit function as follows;

$$\begin{aligned} \Pi_{L2} &= p\left[(1-\rho(t))\,(1-\theta_{\bar{v}})\right] - \delta(t)v_L\left[\rho(t)\,(1-\theta_{\bar{v}}) + (\theta_{\bar{v}} - \theta_H)\right] \\ &= (1-\rho(t))p\left(1 - \frac{p}{\bar{v}}\right) - \delta(t)v_L\left[\rho(t)\left(1 - \frac{p}{\bar{v}}\right) + \left(\frac{p}{\bar{v}} - \frac{p}{v_H}\right)\right], \end{aligned}$$

and the first-order condition is

$$\frac{\partial \Pi_{L2}}{\partial p} = (1-\rho(t))\left(1 - 2\frac{p}{\bar{v}}\right) - \delta(t)v_L\left(\frac{1-\rho(t)}{\bar{v}} - \frac{1}{v_H}\right) = 0.$$

The second-order condition is satisfied; $-2(1-\rho(t))\frac{2}{\bar{v}} < 0$. Then the profit maximizing price and the corresponding profit are

$$\begin{aligned} p^*_{L2}(t) &= \frac{\bar{v}}{2}\left[1 + \delta(t)\left(\frac{1}{1-\rho(t)}\frac{v_L}{v_H} - \frac{v_L}{\bar{v}}\right)\right], \\ \Pi^*_{L2}(t) &= \frac{\bar{v}}{4}(1-\rho(t))\left[1 + \delta(t)\left(\frac{1}{1-\rho(t)}\frac{v_L}{v_H} - \frac{v_L}{\bar{v}}\right)\right]^2 - \delta(t)\rho(t)v_L. \end{aligned}$$

For this to be a local maximum, it must be the case that $\theta_L(p^*_L(t)) > 1$. Because $\theta_L(p^*_L(t)) = \frac{p^*_L}{v_L}$,

$$\theta_L(p^*_L(t)) = \frac{\bar{v}}{2v_L}\left[1 + \delta(t)\left(\frac{1}{1-\rho(t)}\frac{v_L}{v_H} - \frac{v_L}{\bar{v}}\right)\right].$$

So the local equilibrium exists if and only if

$$\frac{\bar{v}}{2v_L}\left[1+\delta(t)\left(\frac{1}{1-\rho(t)}\frac{v_L}{v_H}-\frac{v_L}{\bar{v}}\right)\right]>1$$

$$\Longleftrightarrow \delta(t)\left(\frac{1}{1-\rho(t)}\frac{v_L}{v_H}-\frac{v_L}{\bar{v}}\right)>\frac{2v_L}{\bar{v}}-1.$$

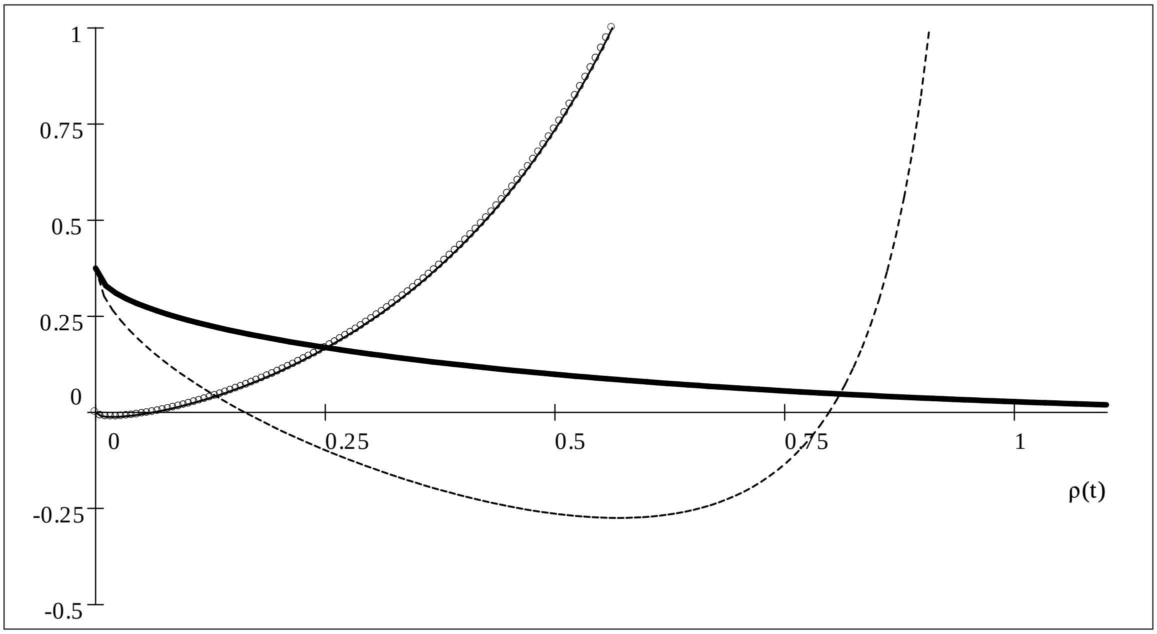
Let's denote $\delta(t)\left(\frac{1}{1-\rho(t)}\frac{v_L}{v_H}-\frac{v_L}{\bar{v}}\right)$ as $\Omega_L(t)$. We will show that there exist a unique t such that $\Omega_L(t)=\frac{2v_L}{\bar{v}}-1$. Note that $\left(\frac{2v_L}{\bar{v}}-1\right)\in[0,1]$, and $\Omega_L(0)=0$ and $\Omega_L(\bar{t})=\infty$. $\frac{\partial\Omega(t)}{\partial t}$ would be negative when t is very small since $\frac{v_L}{v_H}\leq\frac{v_L}{\bar{v}}$, but should be unilaterally positive for t such that $\rho(t)>1-\frac{\bar{v}}{v_H}$, which is equivalent to $\Omega_L(t)>0$. This means that $\Omega_L(t)$ should be increasing in t for $\Omega_L(t)\geq 0$. So there exists a unique critical value, $t_L^{\underline{B}}$, such that $\Omega_L(t_L^{\underline{B}})=\frac{2v_L}{\bar{v}}-1$. Therefore there exists a local maximum if $t\in\left[t_L^{\underline{B}},1\right]$.

(Overall equilibrium)

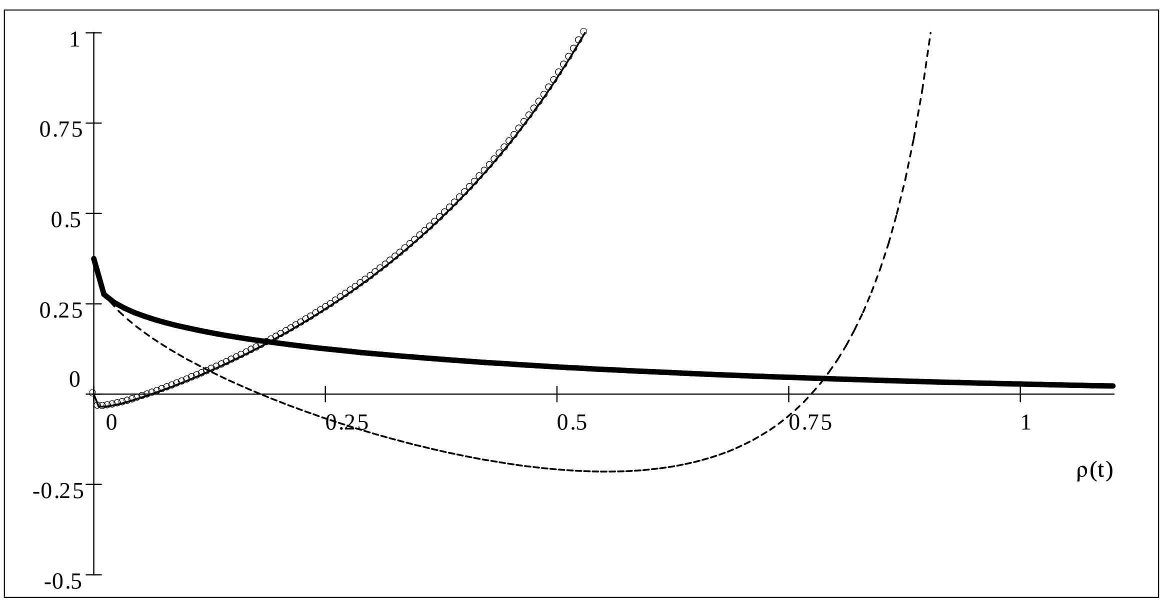
For $t\in[0,t_L^{\underline{B}})$, $\Pi_L^*(t)=\Pi_{L1}^*(t)$ since there is no local maximum in *Case 2*. For $t\in[t_L^{\underline{B}},1]$, since there exists a local maximum in *Case 2*, we need to compare both profits for the given interval. However we cannot explicitly compare two profits, due to the complexity of the model, without assuming specific functional forms on $\rho(t)$ and $\delta(t)$. Nevertheless we can intuitively explain that the seller would always choose the strategy in *Case 1*.

Choosing a strategy in *Case 2* means that the seller wants to extract surplus from the small size of high-preference consumer group, by charging high price to the state-contingent buyers rather than by charging a low price and increasing for-sure buyers. Thus to be benefited from this strategy, the seller needs to charge a sufficiently high price. $p_{L2}^*(t)$ decreases in t when t is relatively small and then starts to increase as t goes up. However, the price he could charge is restricted by the value of the product. The maximum price he can charge is $\bar{v}$, since the seller is a low type. From $\frac{\bar{v}}{2}\left[1+\Omega_L(t)\right]\leq\bar{v}$, we can find $t_L^{\overline{B}}$ such that $\Omega_L(t_L^{\overline{B}})=1$. So only $t\in\left[t_L^{\underline{B}},t_L^{\overline{B}}\right]$ can be supported as an equilibrium in *Case 2*. However note that $\Pi_{L2}^*(t_L^{\overline{B}})<0$. This implies that for $t\in\left[t_L^{\underline{B}},t_L^{\overline{B}}\right]$, $\Pi_{L1}^*(t)>\Pi_{L2}^*(t)$. Therefore the global equilibrium is $p_L^*=p_{L1}^*(t)$ and $\Pi_L^*(t)=\Pi_{L1}^*(t)$.

The examples are based on the assumption that $v_H = 2$, $v_L = 1$, $\lambda = \frac{1}{2}$. Dashed, dotted and solid lines are $\Pi^*_{L1}(t)$, $\Pi^*_{L2}(t)$, and $\Omega_L(t)$ respectively.



(a) $\rho(t) = \sqrt{\frac{t}{\bar{t}}}$, $\delta(t) = \sqrt{\frac{t}{\bar{t}}}$



(b) $\rho(t) = \sqrt{\frac{t}{\bar{t}}}$, $\delta(t) = \sqrt[4]{\frac{t}{\bar{t}}}$

■

Figure A1: Example

Proof of Lemma 5. We first find a local equilibrium when $\theta_{\bar{v}} < 1$ and $\theta_{\bar{v}} > 1$ respectively. Then, comparing both profit functions, we show that the global equilibrium always corresponds to the case in which $\theta_{\bar{v}} < 1$.

(Case 1: $\theta_{\bar{v}} < 1$**)**

From (3.3), the profit function is

$$\begin{aligned}
\Pi_H &= p\left[(1-\theta_{\bar{v}}) + \rho(t)\left(\theta_{\bar{v}} - \theta_H\right)\right] - \delta(t)v_H(1-\rho(t))\left(\theta_{\bar{v}} - \theta_H\right) \\
&= p\left[1 - \left(\frac{1-\rho(t)}{\bar{v}} + \frac{\rho(t)}{v_H}\right)p\right] - \delta(t)v_H(1-\rho(t))\left(\frac{1}{\bar{v}} - \frac{1}{v_H}\right)p,
\end{aligned}$$

and the first-order condition is

$$\frac{\partial \Pi_H}{\partial p} = 1 - 2\left(\frac{1-\rho(t)}{\bar{v}} + \frac{\rho(t)}{v_H}\right)p - \delta(t)v_H(1-\rho(t))\left(\frac{1}{\bar{v}} - \frac{1}{v_H}\right) = 0.$$

The second-order condition is satisfied; $2\left(\frac{1-\rho(t)}{\bar{v}} + \frac{\rho(t)}{v_H}\right) < 0$. Then, the profit maximizing price and the corresponding profit are as follows,

$$\begin{aligned}
p^*_{H1} &= \frac{\bar{v}}{2} \cdot \frac{v_H}{\rho(t)\bar{v} + (1-\rho(t))v_H}\left[1 - \delta(t)\frac{(1-\rho(t))\left(v_H - \bar{v}\right)}{\bar{v}}\right], \\
\Pi^*_{H1} &= \frac{\bar{v}}{4} \cdot \frac{v_H}{\rho(t)\bar{v} + (1-\rho(t))v_H}\left[1 - \delta(t)\frac{(1-\rho(t))\left(v_H - \bar{v}\right)}{\bar{v}}\right]^2.
\end{aligned}$$

For this to be a local maximum, it must be the case that $\theta_{\bar{v}}(p^*_H) < 1$. From $\theta_{\bar{v}}(p^*_H) = \frac{p^*_H}{\bar{v}}$,

$$\theta_{\bar{v}}(p^*_{H1}) = \frac{1}{2} \cdot \frac{v_H}{\rho(t)\bar{v} + (1-\rho(t))v_H}\left[1 - \delta(t)\frac{(1-\rho(t))\left(v_H - \bar{v}\right)}{\bar{v}}\right].$$

Note that $0 \le \left[1 - \delta(t)\frac{(1-\rho(t))(v_H-\bar{v})}{\bar{v}}\right] \le 1$ since $0 \le \frac{v_H - \bar{v}}{\bar{v}} \le 1$. Then all we have to show is $\frac{v_H}{\rho(t)\bar{v}+(1-\rho(t))v_H} \le 2$. Rearranging it, the condition is simplified as $\frac{\bar{v}}{v_H} \ge \left(1 - \frac{1}{2\rho(t)}\right)$. The minimum of the left-hand side is $\frac{1}{2}$ by the assumption of $v_H \le 2v_L$, but the maximum of the left-hand side is $\frac{1}{2}$ at $\rho(t) = 1$. So the condition always holds.

(Case 2: $\theta_{\bar{v}} > 1$)

From (3.3), we have a profit function as follows:

$$\begin{aligned}\Pi_H &= p\left[\rho(t)\left(1-\theta_H\right)\right] - \delta(t)v_H(1-\rho(t))\left(1-\theta_H\right) \\ &= p\left[\rho(t)\left(1-\frac{p}{v_H}\right)\right] - \delta(t)v_H(1-\rho(t))\left(1-\frac{p}{v_H}\right),\end{aligned}$$

and the first-order condition is

$$\frac{\partial \Pi_H}{\partial p} = \rho(t)\left(1-2\frac{p}{v_H}\right) + \delta(t)(1-\rho(t)) = 0.$$

The second-order condition is satisfied; $-\frac{2\rho(t)}{v_H} < 0$. Then, the profit maximizing price and the corresponding profit are

$$\begin{aligned}p^*_{H2} &= \frac{v_H}{2}\left[1+\delta(t)\frac{(1-\rho(t))}{\rho(t)}\right], \\ \Pi^*_{H2} &= \frac{v_H}{4}\rho(t)\left[1-\delta(t)\frac{(1-\rho(t))}{\rho(t)}\right]^2.\end{aligned}$$

Now we need to check if $\theta_{\bar{v}}(p^*_H) > 1$. From $\theta_{\bar{v}}(p^*_H) = \frac{p^*_H}{\bar{v}}$,

$$\theta_{\bar{v}}(p^*_{H2}) = \frac{v_H}{2\bar{v}}\left[1+\delta(t)\frac{(1-\rho(t))}{\rho(t)}\right].$$

So the local equilibrium exist if and only if

$$\frac{v_H}{2\bar{v}}\left[1+\delta(t)\frac{1-\rho(t)}{\rho(t)}\right] > 1 \Longleftrightarrow \delta(t)\frac{(1-\rho(t))}{\rho(t)} > \frac{2\bar{v}}{v_H} - 1.$$

Let $\delta(t)\frac{(1-\rho(t))}{\rho(t)}$ denote as $\Omega_H(t)$. On the contrary to the case of a low-quality seller, the local maximum does not always exist. It depends on the relative size of growth rates in $\rho(t)$ and $\delta(t)$. Note that $\left(\frac{2\bar{v}}{v_H}-1\right) \in [0,1]$, and $\Omega_H(0) = \Omega_H(\bar{t}) = 0$ and $\lim_{t\longrightarrow 0}\Omega_H(t) = \frac{\delta'(0)}{\rho'(0)}$ by *l'Hospital's rule.* $\frac{\partial \Omega_H(t)}{\partial t} = \frac{\delta'(t)(1-\rho(t))}{\rho(t)} - \delta(t)\frac{\rho'(t)}{\rho(t)^2} \gtrless 0$ depending on $\frac{\delta'(t)}{\rho'(t)} \gtrless \frac{\delta(t)}{\rho(t)(1-\rho(t))}$, but $\frac{\partial^2 \Omega_H(t)}{\partial t^2} < 0$ by the assumption that $\rho''(t) < 0$ and $\delta''(t) < 0$. This means $\Omega_H(t)$ could increase or decrease in t when t is relatively small, but should eventually decrease and converge to 0 as t increases. If $\frac{\delta'(0)}{\rho'(0)} > 1$, that is, the depreciation rate increases faster than the information precision rate, there always exists a unique $t_H^{\overline{B}}$ such that $\Omega_H(t_H^{\overline{B}}) = \frac{2\bar{v}}{v_H} - 1$. So we could always find an interval, $t \in [0, t_H^{\overline{B}}]$, in which the local maximum exists. If $\frac{\delta'(0)}{\rho'(0)} < 1$, however, the existence of local maximum depends on the size of $\left(\frac{2\bar{v}}{v_H} - 1\right)$, and, if exists, the local maximum exist for $t \in [\max(0, t_H^{\underline{B}}), t_H^{\overline{B}}]$.

(Overall equilibrium)

Even though there exists a local maximum in *Case 2*, it cannot be the global equilibrium since, for the interval that satisfies price constraint, $\Pi^*_{H1}(t)$ is always greater than $\Pi^*_{H2}(t)$. Note that in *Case 2* the price cannot exceed v_H. That is $p^*_{H2} = \frac{v_H}{2}\left[1 + \frac{\delta(t)(1-\rho(t))}{\rho(t)}\right] < v_H$, which is equivalent to $\Omega_H(t) \leq 1$. Now let us compare $\Pi^*_{H1}(t)$ and $\Pi^*_{H2}(t)$;

$$\begin{aligned}
\Pi^*_{H1}(t) &= \frac{v_H}{4} \cdot \frac{\bar{v}}{\rho(t)\bar{v} + (1-\rho(t))v_H}\left[1 - \delta(t)\frac{(1-\rho(t))\,(v_H - \bar{v})}{\bar{v}}\right]^2 \\
\Pi^*_{H2}(t) &= \frac{v_H}{4}\rho(t)\left[1 - \delta(t)\frac{(1-\rho(t))}{\rho(t)}\right]^2.
\end{aligned}$$

First compare the terms in the squared brackets. We can easily verify $\left[1 - \delta(t)\frac{(1-\rho(t))(v_H - \bar{v})}{\bar{v}}\right]^2 \geq \left[1 - \delta(t)\frac{(1-\rho(t))}{\rho(t)}\right]^2$ since $\frac{v_H - \bar{v}}{\bar{v}} \leq 1$, $\frac{1}{\rho(t)} > 1$ and both terms in the brackets are positive due to the restriction that $\Omega_H(t) \leq 1$. Next, compare the rest terms. The condition that $\frac{\bar{v}}{\rho(t)\bar{v} + (1-\rho(t))v_H} \geq \rho(t)$ is equivalent to $\frac{1}{(v_H/\bar{v})-1} - \rho(t) > 0$, which always hold since $\frac{1}{(v_H/\bar{v})-1} \in [1, \infty)$. This, in turn, means that $\Pi^*_{H1}(t) \geq \Pi^*_{H2}(t)$ for all t. Therefore, global equilibrium price and profit are $p^*_H(t) = p^*_{H1}(t)$ and $\Pi^*_H(t) = \Pi^*_{H1}(t)$. ■

Proof of Proposition 10. For a high-quality seller, he has no incentive to deviate in any case since $\Pi_H^*(s_L) = \frac{v_L}{4} < \frac{v_H}{4}$. For a low-quality seller, incentive compatibility condition is

$$\begin{aligned}(IC_L)\ \Pi_L^*(s_H) &= \frac{v_L}{4}\frac{v_H}{\rho(t)v_H + (1-\rho(t))v_L}\left[1-\delta(t)\left(\rho(t)+\frac{v_L}{v_H}\right)\left(1-\frac{v_L}{v_H}\right)\right]^2 \\ &< \frac{v_L}{4} = \Pi_L^*(s_L),\end{aligned}$$

$$\Longleftrightarrow \delta(t)\left(\rho(t)+\frac{v_L}{v_H}\right)\left(1-\frac{v_L}{v_H}\right) + \sqrt{\rho(t)+(1-\rho(t))\frac{v_L}{v_H}} \equiv \Phi_{se}^L(t) > 1.$$

Note that $\Phi_{se}^L(t)$ is increasing in $\rho(t)$ and $\Phi_{se}^L(0) = \sqrt{\frac{v_L}{v_H}} \leq 1$ and $\Phi_{se}^L(\bar{t}) = \delta(\bar{t})\frac{v_H - v_L}{v_H} + 1 \geq 1$. So there exists a unique t_{se} such that $\Phi_{se}^L(t_{se}) = 1$. Thus, for $t \in [t_{se}.\bar{t}]$, the separating equilibria can be supported. Finally, since on-the-equilibrium path the profits of both types of seller are fixed with the levels of $\Pi_L^*(s_L) = \frac{v_L}{4}$ and $\Pi_H^*(s_H) = \frac{v_H}{4}$ respectively, all the separating equilibria survive the intuitive criterion. ■

Proof of Proposition 12. In this case, to support separating equilibria, we have to consider incentive compatibilities of both sellers as follows:

$$\begin{aligned}(IC_L)\ \dot{\Pi}_L^*(s_H) &= \frac{v_L}{4}\frac{v_H(1-\delta(t))}{\rho(t)v_H + (1-\rho(t))\,v_L}\left[1-\delta(t)\frac{\rho(t)v_H + (1-\rho(t))\,v_L}{(1-\delta(t))v_H}\right]^2 \\ &\leq \frac{v_L}{4} = \dot{\Pi}_L^*(s_L), \\ (IC_H)\ \dot{\Pi}_H^*(s_L) &= \frac{v_L}{4} \leq \frac{v_H}{4}\frac{(1-2\delta(t))^2}{1-\delta(t)} = \dot{\Pi}_H^*(s_H),\end{aligned}$$

which are simplified as

$$\begin{aligned}&(IC_L)' \quad \delta(t)\left[1+\rho(t)+(1-\rho(t))\,\frac{v_L}{v_H}\right] + \sqrt{(1-\delta(t))\left[\rho(t)+(1-\rho(t))\frac{v_L}{v_H}\right]} \equiv \dot{\Phi}_{se}^L(t) \geq 1, \\ &(IC_H)' \quad \frac{(1-2\delta(t))^2}{1-\delta(t)} \equiv \dot{\Phi}_{se}^H(t) \geq \frac{v_L}{v_H}.\end{aligned}$$

Note that $\dot{\Phi}_{se}^L(t)$ is increasing in t but $\dot{\Phi}_{se}^H(t)$ is decreasing in t;

$$\frac{\partial \dot{\Phi}^L_{se}(t)}{\partial t} = \delta'(t) + \underbrace{\frac{\rho'(t) + (1-\rho'(t))\frac{v_L}{v_H}}{2\sqrt{(1-\delta(t))\left[\rho(t) + (1-\rho(t))\frac{v_L}{v_H}\right]}}}_{(+)}$$

$$+ \underbrace{\left\{\delta'(t)\left[\rho(t) + (1-\rho(t))\frac{v_L}{v_H}\right] + \delta(t)\left[\rho'(t) + \left(1-\rho'(t)\right)\frac{v_L}{v_H}\right]\right\}}_{(+)}$$

$$\times \underbrace{\left[1 - \frac{1}{2\sqrt{(1-\delta(t))\left[\rho(t) + (1-\rho(t))\frac{v_L}{v_H}\right]}}\right]}_{(+) \text{ for } \delta(t)\in\left[0,\frac{1}{2}\right]} > 0$$

$$\frac{\partial \dot{\Phi}^H_{se}(t)}{\partial t} = -\underbrace{\left[\frac{\delta'(t)\,(1-2\delta(t))\,[3-2\delta(t)]}{[1-\delta(t)]^2}\right]}_{(+) \text{ for } \delta(t)\in\left[0,\frac{1}{2}\right]} < 0$$

So, similarly in Proposition 10, we can verify that there exist a unique $\dot{t}^L_{se}$ and a $\dot{t}^H_{se}$ such that $\dot{\Phi}^L_{se}(\dot{t}^L_{se}) = 1$ and $\dot{\Phi}^H_{se}(\dot{t}^H_{se}) = \frac{v_L}{v_H}$ respectively. Then the existence of the separating equilibria counts on the size of two critical values, which in turn depend on the specific functional forms of $\rho(t)$ and $\delta(t)$. Therefore there would be multiple separating equilibria if $\dot{t}^L_{se} < \dot{t}^H_{se}$, a unique separating if $\dot{t}^L_{se} = \dot{t}^H_{se}$, and no separating equilibrium otherwise. Finally, if separating equilibria exist, the unique separating equilibria that satisfies the intuitive criterion is $(\dot{t}^L_{se}, \dot{p}^*_H(\dot{t}^L_{se}))$ since $\dot{\Pi}^*_H(s_H)$ is decreasing in t. ■

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