INSIDE PRACTICE-BASED TEACHER EDUCATION:
A STUDY OF ONE TEACHER EDUCATOR'S PRACTICE

By

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ABSTRACT

INSIDE PRACTICE-BASED TEACHER EDUCATION: A STUDY OF ONE TEACHER EDUCATOR'S PRACTICE

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This dissertation builds on research about practice-based teacher preparation (e.g., Ball & Cohen, 1999; Ball, Sleep, Boerst, & Bass, 2009; Graziani, 2005; Heibert, Morris, Berk, & Jansen, 2007; Lampert, 2010), formative feedback (e.g., Black & Wiliam, 1998; Carless, Salter, Yang, & Lam, 2011; Evans, 2013; Hattie & Timperley, 2007; Kluger & DeNisi, 1996), teacher noticing, (e.g., van Es, 2011; van Es & Sherin, 2008; Sherin, Jacobs, & Philipp, 2011), and positioning theory (e.g., Esmonde, 2009; Gresalfi & Cobb, 2006; Harré & van Langenhove, 1991; Herbel-Eisenmann & Wagner, 2010; Herbel-Eisenmann, Wagner, & Cortes, 2010; Wagner & Herbel-Eisenmann, 2009). In particular, I focus on *how* prospective elementary mathematics teachers are prepared, through a study of one teacher educator's practice.

This dissertation is a self-study during a practice-based mathematics methods course for prospective elementary school teachers. Study participants consist of prospective elementary teachers (PTs) who were seniors enrolled in a mathematics methods course at a large Midwestern university. The data for this dissertation includes my lesson plans, videos of each class meeting, prospective teachers' assignments including videotaped lessons, the feedback I provided prospective teachers on assignments and teaching, and my own reflections on teaching.

This dissertation contains an introductory chapter, concluding chapter, and three stand-alone manuscripts. Each manuscript examined a different aspect of my practice as a teacher educator. Each manuscript included a review of relevant literature, data collection, analysis, results, and discussion. Research on practice-based teaching viewed through the lens of

examining my own practice provides the thread that sews the manuscripts in this dissertation together.

The first manuscript details a lesson-planning tool co-edited with study participants. The tool helped PTs focus attention on student ideas and learning, the mathematical point of the lesson, and the facilitation of mathematically rich discussions. The tool draws liberally on research about lesson planning, orchestrating discussions, and attending to student thinking. The PT collaboration on this tool helped me to see the power of including the PTs' voices in their learning.

The second manuscript is an empirical study examining the characteristics of effective feedback in teacher education. I argue that teacher education is a hybrid space where feedback practices bridge both K-12 and higher education contexts. I analyze the feedback I provided to prospective teachers, the characteristics of feedback that participants took up and used to further their learning, and the characteristics of feedback that closed down opportunities for further learning.

The third manuscript is an empirical study that draws on the Learning to Notice Framework (van Es, 2011), which I combine with positioning theory (e.g. Harré & van Langenhove, 1991). These theories guide my analysis of student interviews collected in my methods course. Positioning theory points out the links between the ways PTs position students in their written analysis of the interviews and through their instructional decisions. I identify and define both explicit and implicit positioning and argue that static explicit positioning influences PTs' instructional choices and limits the opportunities PTs' students have to learn. I propose a framework for learning to notice positioning in teacher preparation—both for teacher educators and PTs.

For Dixie
who believed in me
when I had difficulty believing in myself

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You cannot teach another human being anything;

You can only design opportunities for others to create their own learning.

(Adapted from Galileo)

CHAPTER 1: INTRODUCTION

This dissertation focuses on *how* teacher education prepares prospective elementary mathematics teachers, through a study of one educator's practice—my own, as a mathematics teacher educator (MTE) teaching an elementary mathematics methods course. It charts, in part, my own evolution as both a MTE and scholar. Performing this research led me to ask many questions about how I could better prepare teachers to face the challenges of actual K-12 classrooms. It was the genesis of my current, practice-focused view of teacher education.

Contents of this Dissertation

This dissertation consists of an introductory chapter, three manuscripts, and a concluding chapter. In this introductory chapter, I will frame how I came to this study, both personally and through a synthesis of the literature on practice-based teacher preparation. This introduction concludes with a detailed description of the course at the center of this dissertation, a pilot practice-based mathematics methods course I developed for prospective elementary school teachers.

Each of the manuscripts in this dissertation is a stand-alone manuscript, drawn from data on the same elementary mathematics methods course, with complete literature reviews revealing gaps in the literature the study addresses, data, data analysis, methods, and discussion. My lesson plans, videos of the class meetings, prospective teachers' assignments (including videotaped lessons), the feedback I provided prospective teachers on assignments and teaching, and my own reflections on teaching provide data for the three manuscripts. Each manuscript examines a different slice of my practice and as such draws on additional literature to support the aspect of practice-based teaching I explore within the manuscript. The course itself as well as research on practice-based teaching connect the three manuscripts, such that together they create a comprehensive picture of the course and the insights I gained about teacher preparation. Taken as a whole, the literature suggests teacher education should focus on the practice of teaching, as should future research.

I wrote the first manuscript with an audience of MTEs in mind. It describes a tool that supports lesson planning that I designed and then edited with PTs in my methods course. It was designed to help PTs in my methods course focus attention on student ideas and learning needs, the mathematical point of the lesson, and the facilitation of mathematically rich discussions. The tool draws liberally on current research in the field about lesson planning, orchestrating discussions, and attending to student thinking. Co-editing this tool helped me to see the importance of PTs having a voice in their own learning.

The second manuscript is an empirical study examining the characteristics of effective feedback in teacher education. I use the literature on the feedback from K-12 PTs and higher education (HE) as well as the data from this study to develop a perspective about the nature and characteristics of effective feedback for teacher preparation. I argue that MTEs should consider teacher preparation a hybrid space that spans both K-12 and HE settings, and as such should include effective practices and perspectives from both settings. I analyzed the feedback I provided PTs in my methods course on lesson planning, their instructional practice, and mathematics to determine the types of feedback and the characteristics of feedback that was used by PTs to make shifts in their practice. My analysis uncovered my own use of practices with respect to feedback that spanned both K-12 and HE settings. The intended audience for this manuscript includes both TEs and teacher education researchers. Since providing feedback is one of the most time-consuming aspects of teaching, understanding which types of feedback had the most influence on PTs can help TEs develop effective feedback practices.

The third manuscript is an empirical study that draws on the Learning to Notice Framework (van Es, 2011) and adds a layer from positioning theory. I use positioning theory to analyze the data from a student interview assignment collected in my methods course to identify links between the ways PTs position students and student learning in their written analysis of student the interviews and through their instructional decisions. I identify and define both explicit and implicit positioning in this data and argue that static explicit positioning influences PTs' instructional choices, and in fact, limits the opportunities students have to learn. PTs position students both explicitly through their descriptions of students and implicitly through pedagogical choices. Using these themes, the Learning to Notice Framework (van Es, 2011), and the mathematics education literature, I propose a framework for learning to notice positioning in teacher preparation—both for MTEs and PTs. Teachers and MTEs may find the tool useful to heighten their awareness of and to disrupt accepted storylines about teaching and learning mathematics by examining positioning.

In the concluding chapter, I focus on the implications of the three studies for current work in practice-based teacher education, research in teacher education, and the practices of TEs. I examine the limitations of my dissertation research as well as my learning as a TE and a researcher throughout the dissertation writing process. I also examine the implications for future research in mathematics teacher education.

How I Came to This Dissertation Study

In this section, I will describe several experiences that led to the proposal of my dissertation study that influenced my thinking and dispositions about teaching and learning. These experiences influenced the choices I made in graduate school about the projects I wanted to work on and the courses I asked to teach. They occurred when I was the project director of a large mathematics and science partnership in New York City public schools, during which time I was responsible for the professional development of hundreds of secondary mathematics and science teachers. I had been an administrator for a short period after six years as a classroom teacher and had mentored several prospective secondary mathematics teachers from two universities in NYC. My own teacher preparation had influenced my philosophy of teaching. I believed that all students can learn. I tried to meet my students where they were in their learning and guide them to higher levels of achievement. I did not consider myself a teacher, but rather a facilitator of learning. The environment that I created with my students, the trust that we developed, and the relationship we built on that trust were the factors that ensured that learning could thrive. I did not assume the students in my care would learn if I presented them with material, modeled a few examples, and asked them to mimic what I had done. I believed real understanding came from making connections—to ideas and concepts within mathematics as well as in their everyday lives. I knew I could not make the connections for them, at best could provide the scaffolding for students to make these connections themselves. I believed learning was the responsibility of each individual.

The Power of Real Time Feedback

In order to address an immediate problem in the school system, I went into secondary mathematics and science classrooms in schools where students were struggling to pass the state end-of-course exams in mathematics. These practicing teachers held degrees in mathematics and science and performed well on content exams. In their classrooms, however, I observed problems. Most did not have realistic ideas about how students would engage with content. Many had difficulty building connections between student thinking about mathematics and the desired lesson outcomes. I began to notice that the content of the mathematics as well as the classroom learning environment played a large role in students' engagement. Students' behavior reflected drastic differences in engagement from one classroom to another. I began to see that each teachers' disposition was crucial to whether he or she could engage students. It appeared that teachers who really listened to students, who tried to begin their teaching at the students' level, who students perceived as fair and as caring about them, stimulated engagement.

It also became apparent to me that teachers were not aware of the same things that I recognized in their classrooms. After multiple debriefing sessions with teachers following a lesson observation, it seemed unproductive to talk about things that happened in the classroom after the fact—despite my best efforts, I always felt teachers received my comments as criticisms of their teaching as opposed to constructive feedback. It did not seem to matter how many wonderful things I said about student learning or teachers' practice; most teachers seemed to fixate on the aspects of their teaching that could be improved or opportunities they missed. Teachers could not go back and change how they had behaved, and they felt my feedback was therefore useless. Thus, I decided to try real-time feedback in which I would call teachers' attention to students' lack of understanding or misconceptions *during class time* in ways that were appropriate and productive. I was, of course, unwilling to do anything in the classroom that would make students lose confidence in their teacher, but it seemed necessary to give feedback in real time.

I began working with a teacher I will call Sophia. Sophia was getting her master's degree in mathematics education from a local university but had come to teaching by way of Teach for America. She did not have a deep understanding of mathematics yet, but she was deeply student-focused. She wanted desperately to help her students perform better in mathematics, but she was unsure how to go about it. When I began working with her, less than 20% of her previous students passed the New York State end-of-course Regents' exam.

After several debriefing sessions, I asked Sophia how she would feel if I raised my hand and asked a question in her class, or if I found ways to talk with her during class time about my observations of students' engagement and difficulties with the content. She enthusiastically agreed that it would be better than feeling as if she could have made better choices during the class and feeling overwhelmed with missed opportunities. We began by setting up some parameters—if I thought she had not accurately assessed a student's ideas then I would raise my hand and ask a question that I thought captured the student's thinking, and she would let the student respond or respond herself. I would be careful to let her maintain control of the room. If she felt stuck, then it would be up to her to ask me if I had an idea about how to respond to a given student, but I would keep my responses short. This worked fairly well for a while, but before long Sophia described feeling under pressure to figure out a way to support student learning, especially when she did not have any ideas of her own.

After some discussion, Sophie and I decided that we would divide the class in half and give them a task to complete. While they completed the task, we would circulate in our own half of the room, make notes about what we noticed relative to student thinking and meet in the middle of the class to share our observations. While the students continued working, we shared ideas about possible approaches to take, made decisions about how to interact with students about their progress, and then we switched sides of the classroom and each tried to implement what we had decided on together. After working with students and observing the impact of our interventions, we met again in the middle of the room to discuss the challenges and successes and made decisions about what to do next. Sometimes at this point, we decided to demonstrate a common error or misconception and bring the class together to discuss it; other times, we modified a task we had given students. When only a few students were struggling, we crafted interventions specific to their needs.

Forming a partnership with Sophia was a very powerful learning experience for me, and she said she felt the same way. This approach felt far more constructive than giving feedback after the fact. Through repeated use of in-class feedback, Sophia learned to trust her instincts and to carefully listen to her students. She began to see that she could ask her students to explain their thinking and that by carefully listening she would be able to follow their thinking and find effective ways to respond. She learned that there was no one right way to respond, but multiple, effective ways to take up student ideas. Sophia was more purposeful in her interactions with students and based her decisions on these interactions.

I learned from the experience as well. Approaching coaching as a partner in the classroom was enormously useful. Slowing down teaching and giving feedback made Sophia feel supported instead of criticized. The power of those real-time interactions has influenced my perspectives about teaching PTs and my perspectives about how and when to provide feedback.

The Importance of Mathematical Goals

It became apparent to me through my observations that many practicing teachers as well as PTs demonstrated a lack of intentionality in coordinating and guiding students' learning with respect to the purpose of a lesson. They would write a goal on the board and then proceed through a lesson that only tangentially connected to that goal. In a particularly extreme example, a teacher in a large high school classroom wrote the following goal (given to her by her department head) on the board: “Students will discover the formula for the area of a triangle based on their knowledge of the area of a rectangle.” As the lesson started, the teacher wrote the formula for the area of a triangle on the board and then had students practice using it with a variety of applications, ranging from naked math to contextualized problem situations. Rather than *discovering* the formula, they received it. When I spoke with the teacher afterward, she said she felt she had successfully reached the goal of the lesson. From the teacher’s perspective, the goal described the area of a triangle and the activities students engaged in required students to work on finding the area of triangles, so students met the goal.

It was clear during our discussion that the teacher did not see the same mismatch I saw. Later I wondered how the teacher would describe the purpose of lesson goals and what connections there were between the lesson goal and the lesson. I believed the role of the teacher was to guide a class based on the mathematical goals of a lesson, but exchanges such as this, led me to realize that my belief was not a universally accepted instructional norm. I came to understand that many practicing and prospective teachers needed to have more opportunities to learn to see the connections between their interpretation of the goals of a lesson and the instructional decisions they make during a class period.

Distinguishing Theory From Practice

I realized the limitations of detailed planning through interactions with elementary mathematics methods students in two consecutive classes in the two years before this study. In this case, I was their instructor, but did not regularly observe their classrooms; all of these PTs were teaching in K-5 settings. When I observed their classrooms, I realized that my mathematical instruction had not translated into practical effectiveness. For instance, I had provided manipulatives to my PTs to use in solving tasks and modeled norms and procedures for using manipulative, but never talked about why I made the decisions or procedures. I had not considered why highlighting the importance of some of my instructional decisions (such as not putting the manipulatives in front of them until I was ready for them to use them) would be of such importance to PTs' learning. When I observed lessons in their placement schools, every PT who used manipulatives had major behavioral issues arise in their classes. In every classroom session I observed, as the PT brought out manipulatives, I could see by the students' excitement that they were likely to cause problems and take children off task—something the PTs did not recognize before it was too late. I realized how vital it was to address practical issues to better equip my students to serve their own students.

The following year, I asked my students to write lessons that would teach students about different forms of representations of data, based on eliciting student ideas and guiding them to a mathematical point. I gave them feedback on these plans and they revised them. The written lessons used correct mathematical reasoning and language that was appropriate for the tasks. It was my first experiment with asking PTs in my class to do microteaching or peer teaching, and I was very optimistic. When my PTs implemented these lessons in our class, I was disappointed and a little surprised. The moment that the lessons became interactive—when they had to respond to a statement or question, the PTs leading the lesson seemed to forget all their mathematics knowledge and all of their planning, and responded in ways that were misleading at best, and often mathematically incorrect. To add to my surprise, I noted that many of the PTs had anticipated these questions arising in their lesson plans and had written out responses I considered excellent. Somehow their planning had not translated into effective practice.

This experience helped me see the difference between planning lessons and teaching them. Even though PTs planned lessons that I thought were likely to reach the mathematical point, when they actually taught them, they were unsuccessful. This experience made me commit to having my PTs rehearse teaching frequently and to incorporate my use of in-the-moment feedback to help them develop their practice and their confidence in their planning.

An Emphasis on Practice-Based Teacher Education

The pedagogies in the teacher education classes I instruct have shifted as a result of my experiences. I have begun to emphasize *doing* alongside writing or reflecting. By having my students practice and receive feedback in the university classroom before they teach in K-12 settings, I am able to provide feedback to improve the ways they enact lessons. The PTs in my classroom teach during most class sessions and get feedback on their practice of teaching in addition to their planning, reflections, analysis, and discussions of observed teaching.

Both for this dissertation and in reforming my course, I have read extensive literature on practice-based teaching, microteaching, and high-leverage practices. This has revealed that research in mathematics education has generally been divided between studies that focus on either the practices or theories of teaching and learning. My own experience has revealed that neither theory nor practice is sufficient; effective math educators must understand and attend to the theories of teaching and learning *in practice* in order to understand the practice of teaching. The next section lays out the literature and the case for a greater focus on the practice of teaching in teacher education curriculum, the need for more research on the practices of teacher education and MTEs, and the gaps in the literature that this study addresses.

A Practice-Oriented Perspective on Teacher Preparation

A growing number of teacher education scholars have proposed reconceptualizing the work of teacher preparation around "practice." Like the introduction of the term "pedagogical content knowledge," the uptake of this rhetorical and curricular turn in university-based teacher education is strong and growing stronger. For as long as teacher education has existed in this country, a disequilibrium between how much to emphasize the practice of teaching as compared to the subject matter knowledge necessary to teach has kept the field roiling.

Shulman (1986) provided a psychological perspective on research on teaching from the 1950s to the early 1980s in his review of the literature. He described a trajectory that placed an emphasis on investigations of teacher characteristics in the 1950-60s era, followed by research on teacher behaviors in the 1970-80s era, and finally research on teacher cognition characterized by research on teacher's knowledge and beliefs, in the 1990-2000s era. Cochran-Smith and Fries (2005) characterized the history of research on teacher education from the 1950s to the 1980s as focused on teacher education as a *training*[1] *problem,* which assumed teachers could be *trained* to perform and follow basic scripts once they had mastered a small set of skills. Cochran-Smith and Fries described how behaviorist perspectives on teaching in the 1950s to 1980s spawned practices such as competency-based teacher education, microteaching, and laboratory models, which grew from process-product research. A cognitive revolution in teacher education research that focused on more cognitive aspects of teaching and learning followed in 1990s and early 2000s. They indicated, research on teacher education shifted to address a *learning problem*. It

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[1] I use the term *training* in this dissertation only when it is included in direct quotes or when the meaning implied by the source intentionally surfaces behaviorist perspectives that exclude engaging, reflective practice. The term *training* to me implies instruction that is based on replication of a particular skill and is undertaken without thought or reflection. It implies exactly the opposite of what I argue is needed in teacher education, namely, rich reflective practice that emerges only from a deep understanding of mathematics, people (as individuals), and how people learn.

focused, Cochran-Smith and Fries (2005) write, on understanding "teachers' knowledge development, sources and use of knowledge, beliefs, and attitudes; the teacher education pedagogies that prompted knowledge development; and how people learned to teach over time" (p. 84). This responded, they explained, to "*policy problems* that created a shift to accountability, teacher quality, teacher preparation, and "outcomes" (Cochran-Smith & Fries, 2005).

Contemporary research on the professional education of teachers is reinvigorating some of the behaviorist era's learnings, shoring them up with research from the cognitive revolution in the midst of the policy concerns of jurisdictional challenges about who has the right and responsibility to prepare teachers. Although Shulman (1986) highlighted many significant contributions from the era of microteaching, explaining that research in teacher education aligned with current thinking in the field of psychology, he also criticized it for the focus on "what worked, rather than why it worked" (p. 13) noting that such research paradigms lacked a theoretical framing. Ball and Forzani (2009, p. 508) citing Shulman (1986) among others, wrote that microteaching, has been "criticized for representing teaching as a set of decontextualized and atomized practices." They place research from the cognitive revolution, such as "the integration of subject-matter knowledge for teaching and the capacity for discretionary adaptation and judgment with discrete behaviors and actions" (p. 508), at the center of "the next generation of practice-oriented teacher education pedagogies" that some branches of contemporary research support. Researchers claim that contemporary research into teacher education brings a wider lens to reconceptualizing teacher education than previous incarnations and emphasizes slightly different practices (Lampert, et al., 2013). Based on this research, I propose that it may not be possible to create a concise list of practices that can be taught during teacher preparation to prepare teachers to be successful within the content areas while simultaneously addressing every problem earlier efforts have faced.

This literature review situates my study in relation to existing literature by exploring the practice-focused reform of the past and comparing it to recent research on practice-based teaching. I will examine three practice-based efforts to reform the teacher preparation curriculum: the normal schools movement, competency-based teacher education (CBTE), and microteaching. I will highlight the lessons of these efforts to reform the curriculum and examine why these forays into practice-based teaching have not drastically altered the curriculum of teacher education. I will then summarize the current state of the field, analyzing the current conception of practice-based teaching, in particular high-leverage practices, and identifying areas in which the field might benefit from an examination of historical efforts to make the teacher preparation curriculum practice-based.

Normal Schools, Competency-Based Teaching, and Microteaching

Up through the 1830s, teachers had to complete schooling up through the highest grade level to be taught (Labaree, 2008) and be able to maintain order in the classroom (Sedlak, 1989) in order to qualify to teach. At that time, the common school movement began the systematizing of public education. The movement advocated for teacher education, and as a result, summer teacher institutes developed to improve the skills of teachers in both content and pedagogical knowledge (Mattingly, 1975).

Normal schools. The first normal school, which was dedicated to preparing teachers, opened in Massachusetts to prepare teachers in 1839 (Herbst, 1989). Curricula in normal schools focused on both subject matter and what Herbst (1989) called the "arts of teaching" (p. 292). Normal school initially focused on practice (Forzani, 2014). Students at normal schools would receive model lesson plans that dictated what to say and ask, and that provided alternatives if things did not proceed as expected. They studied these lesson plans in their coursework and both observed and practiced teaching in schools. Novices received a performance assessment at the end of a year of instruction that took the form of a lesson taught in the school with feedback provided by the "critic teacher." Thus, all of the focus was on practice in what was called object teaching.

Object teaching was widely criticized for being overly prescriptive and routinized, and it fell out of use by the early 20th century (Forzani, 2014). Other practice-focused instruction succeeded object teaching. Ideas such as demonstration and practice teaching, and model schools connected to normal schools and universities were being used in other parts of the world, and educationist and graduate students alike brought these practices back to the United States.

The Herbartian movement took hold at Illinois State Normal University (ISNU) through the work of Richard Edwards, principal, and before long ISNU was described as the "the largest, best known, and most influential normal school in America" (Harper, 1935, p. 59, as cited in Forzani, 2011). According to Forzani (2014), the Herbartian approach was another example of an early attempt to focus the curriculum of teacher education on teachers' daily practice. The illustrative lessons taught by master teachers served as a centerpiece for discussions about pedagogy, content, and novices' analyses. These critique sessions provided opportunities for TEs to instruct novices on current teaching techniques as well as instructional decisions. Children attended classes at the normal school, which served both as a school for children and a laboratory for learning to teach (Forzani, 2014).

Each of these developments within the normal school movement reflected a succession of discreditation of the prior technique (Forzani, 2014). Yet they remained fairly consistent in the use of observation by novice teachers, explication by master teachers, and immediate practice in classrooms in normal schools. In contrast to contemporary approaches, they do not include significant presence of subject- or grade level–specific pedagogy. The techniques that normal schools embraced seem to suggest that teaching is not complex or demanding, but rather straightforward and easily mimicked. The curricula did not reference students except in the object lessons in which students' interest dictated content and pacing. Discussions of children's learning or prior knowledge were absent from these descriptions of teaching, and the approach emphasized drill and practice, with little opportunity for exploration.

Labaree (2008) described the tug of war between normal schools and school superintendents' eagerness to hire, which eventually doomed the system:

> From the very beginning, [the normal school movement] was caught in a classic bind between quality and quantity. It could provide a few model teachers with a high degree of professional training, or it could provide the large number of teachers needed for the expanding common school system by skimping on professional preparation. … It should surprise no one that normal school leaders ended up choosing relevance over rigor. (p. 293)

The need to prepare many teachers overtaxed and ultimately destroyed the normal school movement with the help of the rise of the university and competition for student tuition. Labaree described an inevitable consolidating of normal schools into the university structure as the beginning of an "uneasy relationship" between teacher education and universities (2008, p. 290).

Many normal schools became liberal arts colleges since it was financially prudent to broaden the scope of these institutions to attract students who were not preparing to teach. Specialized institutions for preparing teachers slowly disappeared, leaving liberal arts colleges in their place (Labaree, 2008). The work of teacher preparation came under the umbrella of universities, where by the early 1900s, one could earn a doctoral degree in curriculum and instruction as well as other fields of education. Unfortunately, universities viewed teacher preparation as "vocational education," and teacher education programs became increasingly marginalized within their own institutions, and programs of teacher education became viewed as a fall back career, if other pursuits did not come to fruition (Labaree, 2008).

Process-product research. After World War II, policymakers began to criticize public education as soft and mediocre, a stance that was fueled by the Soviet launch of Sputnik in 1957. Public outcry called on public schools to produce the next generation of American scientists (Lagemann, 2000). In 1963 former president of Harvard University, James Bryant Conant released his book, The Education of American Teachers, colloquially called The Conant Report. Conant described U.S. teacher education offerings as "Mickey Mouse courses" and teachers' educational requirements as deficient. The report recommended more emphasis on liberal arts and humanities courses in the preparation of teachers and less focus on pedagogy and methods. James Koerner's study (1963), The Miseducation of American Teachers, swiftly followed, characterizing teachers as having inferior intellect to the general public and education courses as "vague, insipid, time-wasting adumbrations of the obvious" (p. 56). These studies led to the creation of new, federally funded programs in the 1960s and 1970s to support teacher development and recruitment

Due to the large amount of funding available to reform education, scores of competing reform efforts were undertaken to reconceptualize the preparation of teachers for this new era. Proposals to reform teacher education grew exponentially. Process-product research on the relationships of processes (teaching) and products (student learning) began to appear in the late

1950s and early 1960s (Gage, 1989). Efforts were made to identify teacher *behaviors* that were shown to be effective, design various procedures and teacher development programs to *train* teachers to exhibit these *behaviors*, and recruit teachers who were most likely to embody these *behaviors* (Cochran-Smith & Fries, 2005). This behaviorist approach to teacher education posits that learning takes place when knowledge is broken into small pieces, such as effective teacher behaviors, called *processes* hereafter. This approach is built on the idea that if PTs learn the requisite processes and the sum of all the processes, then they should become effective teachers. Learning in this approach is highly mechanical.

Although the United States established many new programs in the 1960s and 1970s, the teacher education curriculum actually changed very little (Cochran-Smith & Fries, 2005). The emphases on CBTE and pedagogy for practice-based teaching, called microteaching, which are part of the current movement for a practice-based education curriculum, have, in fact, remained the same throughout this period. These practices received significant criticism (see Shulman, 1986), yet current work in teacher education on practice-focused teaching reflects their continued influence.

Competency-based teacher education. In the 1970s, multiple universities heralded the CBTE movement as the next great wave of teacher education reform. In a review of CBTE programs, Roth (1977) found 215 programs operating in American universities. Some had more than one program, but most implemented experimental courses or pilot programs using CBTE instead of embracing it throughout their program.

The goal of CBTE was to design teacher education around a discrete set of specific, precise learning objectives, defined in behavioral terms and designed to cut across all areas of teacher preparation (Houston & Howsam, 1972). As Sykes (1984) explains:

> Steps in the process called for, first, the decomposition of teaching into a set of discrete competencies, empirically validated through their connection to learning outcomes; then development of a training program within which students would learn the appropriate behaviors, via practice and coaching; close evaluation of students to determine their mastery of the competencies, with additional training prescribed for those who required it; and finally, certification based on mastery of the mandated competencies. Proponents of this approach did not claim that teaching is simply a finite bundle of skills, nor did they deny that teaching is an artistic activity involving improvisation. Rather they argued that effective teachers exhibit skills that can be precisely identified and transmitted. (p. 1294)

Some universities incorporated a self-paced program of study that could be tailored to every prospective teacher's needs (Sykes, 1984). Ultimately, certification required mastery in all of the competencies their program had identified. During this period, most research on teacher *training* focused on evaluating and identifying the competencies and testing instructional methods that teacher *training*[2] programs were using (Lanier & Little, 1986; Wideen, Mayer-Smith, & Moon, 1998; Zeichner, 1999). This movement received a great deal of criticism for its connection to a behaviorist model of teacher education (Shulman, 1986; Sykes, 1984).

Sykes (1984) writes that the CBTE approach was abandoned "for its proliferation of competencies, for its inability to validate competencies, and for its narrow instrumental view of teaching" (p. 175). Shulman (1986) argues the CBTE movement's focus on what worked rather than why it worked—"causes were sought in behaviors, not in theoretically meaningful mechanisms or explanations" (p. 13)—led to its demise. In fact, CBTE had many critics who

[2] Explicitly references a behaviorist perspective about teachers being *trained* (see footnote 1).

described the approach as reductive, making prospective teachers competent in a few aspects of teaching but failing to prepare them for the range of complexities of teaching, including its dynamic aspects. Teachers needed to be able to respond to a range of situations and CBTE prepared them to behave in particular ways but not to think about situations and problem-solve in the classroom (Sykes, 1984).

Microteaching was among the pedagogies employed in the CBTE movement, and it received significant attention in teacher education research. It also has some connections to the current incarnations of practice-based teaching, as I will discuss.

Microteaching. Microteaching was first introduced in 1963 at Stanford University and quickly established itself in the United States, Australia, and the United Kingdom as well as other countries (e.g., Hargie & Maidment, 1979; Turney, 1970; Turney, Clift, Dunkin, & Trail, 1973; Ward, 1970). It was developed as a practice-based pedagogy for teacher education and grew out of the CBTE movement. The goal of microteaching was to break down complex teaching situations into specific, learnable, practicable teaching skills and competencies, and to allow novice teachers to practice these in a university setting (Allen & Ryan, 1969). The particular skills chosen to practice were linked to student achievement through process-product research (Allen & Richard, 1965; Allen, 1966). The teacher educators would introduce a teaching skill through modeling, direct instruction, or through the analysis of a video of a master teacher enacting this skill in the classroom. In the next phase, prospective teachers would develop plans for teaching that included this skill, practice them in the university classroom with peer observers, and receive feedback from a TE on their success in demonstrating the skill (Allen, 1966; Allen & Richard, 1965). It was common to video record lessons for analysis, which might occur with the PT present or not. Allen and Richard (1966) specified five key features of microteaching: first, it was actual teaching; second, it reduced the complexities of teaching to enhance the PT's learning; third, it focused on mastery of very specific tasks; fourth, it required controlled practice settings; and finally, feedback was a key feature (1966). The microteaching environment had reduced class sizes, shortened lessons, and rendered tasks less complex. MacLeod (1987) reviewed the research on microteaching and found schools of education and researchers around the world had taken up the method. Microteaching was popular among teacher education programs and widely researched an ERIC search provides over 180 citations in the ERIC database for microteaching in 1970 alone (MacLeold, 1987).

Although it was very widespread, microteaching also received a great deal of criticism. Critics claimed that microteaching created a complex situation for novices since they were torn between their identities as students, novice teachers, and peer observers, and that practicing it in the university setting meant it was unlikely to change the way PTs taught (Bell, 2007; Wagner, 1973). He and Yan (2011) claimed that microteaching did not provide real classroom teaching experience, and Metcalf (1993) charged it was inauthentic. Others called microteaching difficult to organize (Brown & Armstrong, 1975), artificial (Brown & Armstrong, 1975), and insensitive to individual PTs' needs to acquire particular skills (Seidman, 1969). Research has depicted microteaching as atheoretical and has disparaged it as reflecting a discredited behaviorist model of learning (Nash & Agne, 1971). Microteaching has been criticized for its lack of attention to the social aspects of teaching (Nash & Agne, 1971), and Brown (1975) condemned microteaching as likely to "produce homogenized teachers with standard smiles and procedures" (p.17), which reduced diversity. Despite these criticisms, microteaching is still in practice today, often, but not always in a more cognitive instantiation then in the early 1960s and early 1970s (e.g., Fernandez, 2010; Molina, Fernandez, & Nisbet, 2013). Later versions of microteaching (in the 1990s) focused on more cognitive outcomes, such as teacher reasoning or content knowledge for teaching (Grossman, 2005).

Like CBTE, microteaching focused on a specific set of skills. Researchers posited that less complex rehearsals of teaching would support effective teaching among novices. Nonetheless, microteaching is still in use in many university courses.

Clearly, practice-oriented curricula for teacher education is not a new idea. Current efforts to reform the curriculum of teacher education include efforts to establish a list core competencies or high leverage practices (Graziani, 2005; Lampert, 2005; Lampert, Beasely, Ghousseini, Kazemi, & Franke, 2010; Lampert & Graziani, 2009; Leinhardt & Steele, 2005) to emphasize the practice-based aspects of teaching builds on this long history, but questions remain: Is this approach to reforming the curriculum of the professional education of teachers truly different from previous reform efforts? Have the criticisms of the past been successfully addressed with these new reform agendas?

The Turn From Knowledge to Practice

The cognitive emphasis that has emerged starting in the 1980s in teacher education research influences the current efforts to reform the curriculum to have a greater practice-focus, shoring up the behaviorist approach of earlier eras. Ball and Cohen (1999) advocated for research and design in teacher education to focus on classroom practice in 1999 and other researchers concurred (e.g., Little, Gearhart, Curry, & Kafka, 2003). Over the course of the current century, many research groups have turned their attention to a practice-focus for research and design in teacher education.

In order to understand the current movement to design a practice-focused curriculum for teacher education, it is important to understand how researchers are defining practice. The central focus of a practice-based curriculum, according to Ball and Forzani (2011), must be actual activities and tasks that teachers do and, to that end, the knowledge novices must acquire is "that which is used in practice" (p. 21). Lampert (2010), like Ball and Forzani, emphasized the action inherent in teacher preparation when she described it as helping prospective teachers "learn *teaching*" (Lampert, 2010). Practice, in this case, could have several meanings, Lampert pointed out. It could mean practice in the sense that is used in contrast with theory, but it could also mean practicing, that is, doing something over and over. Practice was meant to invoke action, not just imagining what one might do, writing about what someone has done, or planning to do something. Practice also invoked things teachers "do, constantly and habitually" (Lampert, 2010, p. 25)—create routines. This definition of practice was very similar to the ideas of CBTE and the work in the normal schools. Ball and Forzani (2011) posited that it was important for PTs to practice engaging in the real work of teachers. PTs needed to both learn *and* practice using the knowledge of content, teaching, and students, and receive feedback about their teaching. CBTE also shared the ideas of focusing teacher preparation on the work of teachers, and microteaching was a good example of a pedagogy that included these repetitive cycles of planning, teaching, and feedback. It seemed that the key aspect of the cognitive movement that Ball and Forzani (2011) foregrounded was the work of unpacking and defining the content knowledge needed for teaching.

Ball and Forzani (2011) emphasized the need to shift teachers' thinking from knowledge and beliefs to actions and judgments. These shifts implied a shift in the pedagogy for teacher education from having novices analyze, describe, write, and talk about teaching, to actually implement teaching. Ball and Forzani asserted, "The professional curriculum would emphasize repeated opportunities to do the interactive work of teaching and to receive feedback—not just to talk about the work" (2011, p. 19). This work has implications for the role of the TEs.

The Implications of Focusing on the Practice of Teaching in Teacher Education

The turn in the field toward practice is not simply semantic. Teacher education as an enterprise—what content would be covered, the pedagogies involved, who would be qualified to teach courses, how those courses would be organized and ordered—might be fundamentally different if organized to teach actions instead of to impart knowledge. Grossman, Hammerness, and McDonald (2009) underscore the substantive changes in teacher education that a focus on the practice of teaching implies:

> Taking clinical practice seriously will require us to add pedagogies of enactment to our existing pedagogies of reflection and investigation. In order to make the shift, we also argue that teacher educators will need to undo a number of historical divisions that underlie the education of teachers. These include the curricular divide between foundations and methods courses, as well as the separation between universities and schools. (p. 274)

The current turn in the field addresses the need for new pedagogies as well as a restructuring of the divisions between content courses and methods courses and between K-12 schools and universities. This implies radical change. A full turn toward practice-based teaching requires more than pilot programs and experimental courses. It would in fact change the ways universities partner with schools in the sense that teacher education would need to occur in large part in classrooms with children. A turn toward practice-based teaching would involve changing promotion and tenure processes at the university to support TEs who reflect this value. It would also involve valuing the school knowledge teachers and administrators bring to university/K-12 partnerships to prepare teachers. To shift the curriculum of teacher education would require more than a mere change in the pedagogies in teacher preparation from pedagogies of inquiry to pedagogies of enactment. An effort to reform teacher preparation would require a systemic change in the ways teachers are prepared to teach.

Scholars have pointed to the need to develop a shared language or framework to discuss teaching and teacher education (Ball & Forzani, 2009; Grossman & McDonald, 2008) that focused on the clinical and cognitive aspects of teaching. Indeed, researchers and TEs did not share a common language, and as Grossman et al. (2009) notes, this makes it difficult to decompose, approximate, or implement a set of practices. As Roth (2007) reported, a lack of consensus about what CBTE really meant, how to measure it, or how to compare programs made it impossible to make sweeping claims about the CBTE movement and programs. Teachers (the people actually engaged in the practice) may not have language (shared or otherwise) to describe what they do. Of course, there are many ways in which people can and do discuss teaching, but scholars and educators need to work together to develop a shared language to talk about the nuances of teaching from a practice perspective. Relatedly, no agreed upon set of practices upon which to build a curriculum exists.

Researchers have pointed to a need to design a common curriculum for teacher education around the practice of teaching (Ball & Forzani, 2009, 2011; Grossman et al., 2009; Lampert, 2010). Current efforts seek to use the cognitive movement's language and purposes to describe the CBTE ideas, to place current movements of practice-based teacher education in K-12 settings, and to focus learning on responsiveness to the complexities of teaching. Grossman et al. (2009) call on TEs to become coaches for PTs, requiring TEs to tap into knowledge and experience from professional development and bringing it into teacher education. Some professional development programs prepare TEs to work with teachers in the classroom, reflecting a turn toward clinical experiences that also underscore a need to instruct TEs in these facets of clinical work. Current research suggests a need to expand the role of the TE from providing feedback about analysis and reflection to also giving feedback on the actual teaching novices perform. Giving feedback in the moment about teaching requires a different skill set than providing feedback on written work. This, too, will require a substantive shift in the preparation of TEs.

Ball and Forzani (2009) highlighted the need for "foundational knowledge, but designed and developed differently from its usual treatment in teachers' preparation" (p. 503), and Lampert (2010) called for further research to support the development of a theory of learning from practice and the requisite pedagogies. Current reformers recognize the need to create new pedagogies and new theories of learning to support the new categories of knowledge and practice.

Although current reform efforts attend to the criticisms of the past in order to infuse teacher preparation with a practice-based perspective, there are many more questions than answers about how best to do this. In the next section, I synthesize disparate conceptualizations of the current work of practice-based reforms for teacher education and highlight their similarities and differences to provide snapshot of current thinking and identify areas of shared belief, and areas where thought diverges.

Conceptualizations of Practice-Based Curriculum for Teacher Education

Two research studies have been instrumental in creating the frames for the research on practice-based teacher education that supports high-leverage practices. In the first study, Lampert and Graziani (2009) examined the novice teacher education program, "Basic Formation," at

Dilit-International House in Rome. Its major contributions to the literature consist of conceptions of routines of practice, set instructional activities, novice teachers' rehearsals of these activities with TEs serving as coaches, and novices' field-based performance of these activities. The second study, by Grossman et al. (2009), provides a framework for thinking about practice-based teaching. It is a comparative case study of professional education across three relational professions: the clergy, clinical psychology, and teaching. Through their study of the eight programs, they distilled their analysis to three structures used across the settings that seemed to offer the most leverage in learning how to practice each profession. The first structure, *representations of practice*, comprised the various methods educators use to provide a picture of practice. These include using video, modeling strategies, looking at artifacts of practice, and so on, with a view to what each of these makes visible for PTs in their teacher education. The second structure, *decomposition of practice*, involved "breaking down complex practice into its constituent parts for the purposes of teaching and learning" (p. 2069) to "enable students [meaning PTs] to see and enact elements of practice more effectively" (p. 2069). The third structure, *approximations of practice,* involved providing novices with university-based opportunities to actually engage in professional practice. Approximations of practice are organized so that they gradually approach the demands of actual practice over time.

Defining a core set of practices. Researchers working to reform the enterprise of teacher education to reflect a more practice-based stance are looking at defining *high-leverage practices* (HLPs). These researchers are also seeking productive ways to revise the curriculum for teacher preparation by taking up a different view of content knowledge, restructuring teacher education programs, defining new theories of learning, creating new pedagogies of teacher education, creating new roles for TEs, and designing new perspectives on the tasks involved in preparing PTs.

Scholars have proposed different terms and different examples in pursuit of defining “practice.” Candidates include a “core set of practices” (Grossman, Hammerness, & McDonald, 2009), “generative practices” (Franke & Chan, 2006; Franke & Kazemi, 2001), “routine instructional activities” (Graziani, 2005; Lampert, 2005; Leinhardt & Steele, 2005; Lampert, Beasely, Ghousseini, Kazemi, & Franke, 2010; Lampert & Graziani, 2009), and “high-leverage practices” (Hatch & Grossman, 2009; Sleep, Boerst, & Ball, 2007; Ball et al., 2009; Windschitl, Thompson, & Braaten, 2009). All of these groups share a basic agreement on the definition of HLPs, which I define next.

According to the *TeachingWorks* website from the University of Michigan (UM), HLPs are central to teaching. When a PT learns an increasing number of HLPs, s/he increases the likelihood that his or her teaching will support all students’ learning. HLPs should apply across content, grades, and contexts, and they are focused on supporting students in meaningful work within each content area. Since UM has been transforming its teacher education program to prepare teachers to learn through practice, other criteria for HLPs speak to the ease of use at the university level—that is, the HLPs need to be conceptually accessible to preservice teachers and to be easily articulated and taught. PTs need to be able to practice these HLPs in university and K-12 settings, and the HLPs that UM has adopted are designed so that prospective teachers work on a continuum of complexity, authenticity, and grain size (Ball et al., 2009, p. 460).

The research group *Learning in, from & for Teaching Practice* (LTP) described HLPs in the same way as UM did, with two additions: HLPs are generative, meaning they allow novices to learn more about students and about teaching; and HLPs are research-based (Grossman, et al., 2009, p. 277[3]). In fact, the HLPs that UM described conform to these descriptions, although the *TeachingWorks* website did not articulate these attributes. Windschitl, Braaten, and colleagues, a group of science researchers from the Universities of Washington, Seattle and of Wisconsin, Madison (see Windschitl et al., 2009) are working on defining a core set of practices in science which they define as part of *ambitious teaching*. In their writing, these scholars adopted the UM definition of HLPs with the addition of generative practices, as well as stating that HLPs must allow "significant time for novices to develop beginning instantiations of each of these practices" (Windschitl et al., 2009, p. 8). Windschitl, Braaten, and colleagues described a goal of preparing PTs in ambitious teaching that demands that each HLP play a clear role in a larger, coherent system of instruction, since no single HLP can address its broad agenda (Windschitl et al., 2009, pp. 8–9). Windschitl, Braaten, and colleagues did not suggest how their practices could span content areas other than science.

Identifying high-leverage practices. Windschitl, Braaten, and colleagues and UM both give lists of the HLPs they are testing, and LTP researchers have identified candidates for HLPs in published manuscripts, although they do not provide a comprehensive list of what practices fit their conceptualization. Windschitl, Braaten, and colleagues described a "core set" of HLPs that is imbedded within a coherent system of instruction to support novices through the planning, enacting, and assessment phases of teaching that they call the Science Learning Framework (Windschitl et al., 2009). In keeping with this terminology, Windschitl, Braaten, and colleagues (2009) suggest different disciplines require different sets of practices. While acknowledging that other researchers should scrutinize their list, Windschitl, Braaten, and colleagues, explain that

[3] See also: Franke, Grossman, Hatch, Richert, & Schultz, 2006; Kazemi & Hintz, 2008; Kazemi, Lampert, & Ghousseini, 2007.

"fully support[ing] the development of effective, caring, and reflective practitioners" requires a list as robust as theirs (p. 18). Pointing out the complexities of learning to teach, they propose a set of four core practices for adoption, arguing that without a set of core practices, teacher education can be isolated, individual, and haphazard. Current practices in many teacher preparation programs, they charge, are an "eclectic assortment of programs in which individual practitioners' developmental trajectories cannot be supported by clear standards" and the "instructional mission" of teaching preparation remains underserved (Windschitl et al., 2009, p. 18).

Researchers at UM have redesigned the curriculum, the structure of their courses, and how time is spent both on campus and in K-12 schools based on the 19 HLPs listed in the first column of Table 1, which are detailed on the *Teaching Works* website. This set of practices is applicable to a range of subjects that PTs might be prepared to teach. Beyond practices that encompass teaching, it includes activities that teachers frequently do that are related to teaching, such as conversations with parents and communicating with other professionals.

The program structure at UM has shifted from a course-based program to a module-based program that offers more flexibility for students. This allows flexibility for the length of the module as well as the intensity of the workload. UM is experimenting with various clinical settings, both on campus and in local K-12 schools, to provide opportunities for interns to observe, study, and practice instruction, and to receive detailed coaching (University of Michigan, n.d.).

Table 1

High-leverage practices proposed by three different research groups

Teaching Works, through UM	Windschitl, Braaten, and colleagues	LTP
Making content explicit through explanation, modeling, representations, and examples.	Constructing the big ideas.	Leading a guided reading lesson in elementary reading.
Eliciting and interpreting individual students' thinking.	Eliciting students' ideas to adapt instruction.	Engaging students in choral counting in mathematics.
Leading a whole-class discussion.	Helping students make sense of material/activity.	Reading aloud to children.
Establishing norms and routines for classroom discourse central to the subject-matter domain.	Pressing students for evidence-based explanation.	Leading a classroom discussion (i.e., on literature).
Recognizing particular common patterns of student thinking in a subject-matter domain.		Providing clear instructional explanations.
Identifying and implementing an instructional response to common patterns of student thinking.		Developing a classroom culture.
Teaching a lesson or segment of instruction.		Establishing structures for students' independent and collaborative activities.
Implementing organizational routines, procedures, and strategies to support a learning environment.		Eliciting student thinking during interactive teaching.
Setting up and managing small group work.		Anticipating student responses.
Engaging in strategic relationship-building conversations with students.		Eliciting further thinking.
Setting long- and short-term learning goals for students referenced to external benchmarks.		
Appraising, choosing, and modifying tasks and texts for a specific learning goal.		
Designing a sequence of lessons toward a specific learning goal.		
Selecting and using particular methods to check understanding and monitor student learning.		
Composing, selecting, interpreting, and using information from methods of summative assessment.		

(continued)

Table 1 (cont'd)

Teaching Works, through UM	Windschitl, Braaten, and colleagues	LTP
Providing oral and written feedback to students on their work.		
Communicating about a student with a parent or guardian.		
Analyzing instruction for the purpose of improving it.		
Communicating with other professionals.		

Defining new pedagogies. The LTP research team's list of high leverage practices provide two major contributions to our understanding of HLPs. First, it identifies a particular pedagogy, called Cycles of Enactment and Engagement (Lampert et al, 2013), for practice-based teaching that resonates with microteaching. Second, it lists a small set of instructional activities they have designed that maintain the complexity of instruction that PTs learn through a unique rehearsal structure. The group addresses critics of microteaching directly by clarifying how their rehearsals differ from the highly criticized pedagogy of microteaching:

> Our analysis highlights the differences between rehearsals in our three sites and another approximation of practice common in teacher education, microteaching. In microteaching, teachers practice an instructional segment, typically between 5 and 15 min. in length, without interjection or intervention by peers or a TE. The practice is followed by self-assessment, peer discussion, and TE feedback. In rehearsals, NTs [novice teachers] and TEs work together to try to realize ambitious practices in the moment (Grossman, 2005). Our rehearsals involve almost equal amounts of NT rehearsing and TE/NT exchange. Feedback and discussion is not saved to the end of rehearsals. Our rehearsals typically involved 14 TE/NT exchanges. In addition to ongoing intervention within rehearsal, [rehearsal] also differs from microteaching in that it is embedded in Cycles of Enactment and Engagement with the goal of supporting NTs to

> create a frame for developing the complex relationships among students and content that are required in teaching. In these cycles, rehearsal is surrounded by observing, analyzing, and planning to use an IA [instructional activity] before it is rehearsed, then teaching it to students and debriefing that teaching afterward before trying it again. (Lampert et al., 2013, p. 229)

This description reveals the role of a coach who interjects or intervenes while the PT is rehearsing, which, as the designer, is very different from microteaching. The embedding of the rehearsals in a cycle of enactment that does not attempt to reduce classroom interactions into discrete events that can be taught also distinguishes the LTP approach from microteaching. It attends to the complexities of teaching while supporting PTs in making decisions in the moment.

The LTP group's description of its practices seems to suggest that by learning how to do a few instructional activities well, and by repeating these activities over and over while receiving feedback, novice teachers will be able to use these in classrooms effectively. They make no claims, however, about whether novices learn an embedded set of skills from these instructional activities or how the learning from these activities impacts novel situations. The group has published four accounts of instructional activities and has indicated that their teacher education programs use additional instruction activities (see Lampert et al., 2013). Their published research described a combination of practices, principles, and mathematical knowledge that serve as the scope of instruction in the methods courses across three universities, where the pedagogy of rehearsal has been used. These practices are not content specific and could apply to any teacher preparation course. The principles speak to issues of equity, learning as a social activity, and responsive teaching in that the group asserts that the "principles, practices, and mathematics knowledge cannot stand alone, but must be used in relation to one another and in relationships among teacher, students, and the content to be learned" (Lampert et al., 2013, p. 228).

Conceptions of practice-based teaching vary widely among UM, Windschitl, Braaten, and colleagues, and LTP, yet they share significant overlap, much as the CBTE programs of the 1970s did. As in the CBTE movement, TEs are attempting to delineate a list of practices that can be taught to novice teachers in an effort to focus the professional education of teachers on the work of teaching. The current movement is distinct from the CBTE movement in several key ways, however. First, proponents of the current movement to focus teacher education on the practice of teaching expressly limit the number of practices, whereas the CBTE era saw the identification of hundreds of competencies (Sykes, 1984). Second, current approaches propose a collection of practices that are interconnected to construct a composite whole, rather than having novice teachers master particular skills in isolation.

Challenges and Future Research

A large part of the body of literature about defining and implementing core or high-leverage practices has focused on developing a shared language and frameworks to talk about the practice of teaching. The LTP framework (Grossman et al., 2009) represents an initial effort to create a shared language to guide both the discussion and analysis of the pedagogy of professional education. The field needs to continue to develop a more nuanced language for the practice of teaching, including a language to help assess the enactment of practice. Researchers from LTP, University of Washington, University of Wisconsin, etc. meet periodically to discuss how to better share a language for practice.

Although research groups seem to have widespread agreement on the definition of a high-leverage practice, interpretations of the work on decomposition of practices differ in several ways. How do we know what constitutes a meaningful decomposition of practice, such that it will help to ensure well-prepared beginning teachers? The variation of the grain size of the core practices across research groups raises questions for researchers. What constitutes a reasonable grain size for a practice? The number of practices that novice teachers can actually learn during their professional preparation is another challenge for TEs. Does the grain size of the practice change, given core practice may include discrete and intermediate-level practices as well as techniques? What is the balance between practices that speak to disciplinary knowledge and practices that are not connected to particular subject area knowledge? What representations of practice have been created for each of these types of practices? Long-term and short-term research are both needed to determine the impact on both prospective teacher learning and K-12 student learning.

Clearly TEs play a key role in the development of the novice teachers; it is one of the pivotal findings of research from the CBTE movement (i.e. Shulman, 1986; Ball & Cohen, 1999). TEs need to be able to provide feedback to novices on the enactment of practice and not just reflection on practice. This means identifying and articulating, in-the-moment during a rehearsal, the novice's strengths and skills. It also includes spontaneously finding opportunities for growth. Determining what constitutes various levels of sophistication of enactment in the absence of a clear language to describe practice is a challenging task. We need common tools that can be shared across contexts for assessing practice. We need research to determine how to prepare TEs to take up this work. Researchers have made a compelling case for the curriculum of teacher education to focus on the practice of teaching.

One goal of this study was to understand the skills and knowledge that would improve my efficacy in teaching a practice-based methods class. I felt confident I had already amassed a good deal of that knowledge in observing and coaching teachers, but I wanted to better understand how to decompose HLPs depending on their grain size, and therefore, I selected six practices in a range of sizes. It seemed likely that grain size would affect the types of activities and skills TEs like me would need to develop in order to support PTs in a particular HLP.

In the next chapter, I describe the complexities of making my instructional decisions visible to PTs.

CHAPTER 2: COLLABORATIVELY DESIGNING A TOOL FOR LESSON PLANNING

One of the most reliable findings of research on teaching and learning is that students learn what they are given opportunities to learn (Hiebert, 2003, p. 10).

As Hiebert (2003) points out, students learn what they have opportunities to learn (OTL). The job of a teacher is to ensure the quality of these opportunities. According to Hiebert, providing students meaningful OTL involves

> setting up the conditions of learning that take into account students' entry knowledge, the nature and purpose of the tasks and activities, [and] the kind of engagement required. … Such *engagement* might include listening, talking, writing, reasoning, and a variety of other intellectual processes. (Hiebert, 2003, p. 10)

By describing the "conditions of learning" that a teacher has to set up, Hiebert noted that choosing a task or activity is only the starting point when creating OTL for students. One must consider the purpose of the task or activity in light of what students already know. One has to consider the ways students will need to engage with the task to ensure that they will wrestle with the intended mathematics in meaningful ways. In other words, one has to consider how a specific task or activity will prompt students to think about the intended mathematical ideas and the kinds of intellectual practices that will compel students to engage with the mathematical ideas of the task. Hiebert italicized the word *engagement* purposefully. Engagement implies participation and becoming involved with something. If a teacher wants students to acquire particular knowledge or skills, then he/she must provide students with opportunities to become involved with the content, to participate in conversations about, to write about, and to reason about the content. This implies teachers need to do more than just select tasks.

In this paper, I present a problem of practice: As a teacher educator, how do I both model powerful planning and decision-making practices in my course and make these practices accessible and visible to PTs? To do this, I analyzed lesson planning tools and activities that I provided to my prospective teachers, my instructional choices during teaching, how I assessed whether my PTs had learned what I had hoped they would learn, and my decision-making process when I found evidence that they had not learned something.

I will use the literature on lesson planning to situate this paper in current knowledge in the field about both common and effective practices. I examined relevant tools designed to support teachers to plan mathematics lessons that are likely to create opportunities for students to learn meaningful mathematics in the way Hiebert advocates, in order to understand the affordances and limitations of the tools as well as the thinking that went into their design. I then provide a brief episode from my own classroom that highlights a moment of disequilibrium about what it meant to teach with the mathematical goal of the lesson in mind. This episode underscores the complexity of teaching PTs to plan meaningful lessons and provided the motivation to create a planning tool with and for my PTs that I present in this chapter.

Lesson Planning Literature

I turn to the literature on expert-novice lesson planning to better understand the differences between how experienced and novice teachers plan. In their review of the literature on planning, Clark and Peterson (1986) found a variety of descriptive studies about experienced teachers' lesson planning practices that documented teacher-planning practices through the use of observations, surveys, interviews, and analyses of written plans. A typical type of lesson planning these authors found involved four steps: (1) write out objectives; (2) choose the tasks or activities; (3) determine a flow for the activities; and (4) plan assessment. This model focused on what the teacher and students did but did not address either students' or teachers' thinking.

Several studies have reported that experienced teachers did not actually follow these steps in this order and, instead, that they often started their planning by identifying the content area and choosing an activity. Experienced teachers then moved on to other aspects of planning like selecting materials, writing goals, and choosing modes of assessment (Borko & Niles, 1987; Clark & Peterson, 1986). Borko and Niles (1987) also noted that many teachers did not write objectives because they believed that the activity itself determined the objectives of the lesson. Clark & Peterson (1986) expressed that experienced teachers often wrote the goals at the end of the planning stage, if at all.

Leinhardt (1993) identified predominant differences between expert and novice teachers planning practices. She highlighted the differences in the *agendas* of expert and novice teachers, which she defined as “an operational plan that is concise, focused, and descriptive of the general set of goals and actions in which the teacher intends to engage for the next 40 to 50 minutes” (p. 19). She found that expert teachers predicted the struggles students might have understanding the lesson content and payed attention to student thinking in order to assess the impact of a lesson. Expert teachers were able to think about their own thoughts and actions as well as the students’ thinking and understanding. In contrast, novice teachers did not describe their intentions to monitor how students responded to a lesson nor did they usually describe attending to student thinking in reflection on the lesson. Novices were not able to attend to both their own actions and students’ thinking simultaneously. Leinhardt argued that teacher’s agendas, but not necessarily their written plans, were closely related to the teacher’s implementation of the lesson (Leinhardt, 1993).

Researchers (Leinhardt & Steele, 2005; Livingston & Borko, 1990) described three essential practices of lesson planning. First, teachers need to articulate and understand the concepts to be discussed, not just the processes or procedures used. Second, teachers need to anticipate how students might engage with a task or activity. This includes considering both student solution strategies and common misconceptions or errors students might make. Third, teachers must purposefully plan questions to elicit student ideas that will subtly guide student thinking and talk toward the mathematical goals of the lesson (e.g., Leinhardt & Steele, 2005; Livingston & Borko, 1990; Smith & Stein, 2011; Stein, Smith, Henningsen, & Silver, 2000, 2009). Novice teachers, as newcomers with limited experience and exposure to children's ways of thinking about content, are in a difficult position. They likely understand how they themselves would approach and solve a task, but it might be difficult for them to think about how children might do so. Researchers have found PTs to be limited in their knowledge of the ways students might approach content and tasks, misconceptions students might have, areas of content that might be difficult for students to understand, and alternative solution strategies students might present (Livingston & Borko, 1990). These limitations could have an impact on the ways PTs are able to plan lessons that attend to student thinking.

Novice teachers' planning determines to a large extent what happens in classrooms during lessons (Livingston & Borko, 1990). Expert teachers may have a stronger purchase than novices on the ways students approach mathematical tasks and the kinds of misconceptions students might hold. Novice teachers, through their inexperience, possess limitations in their knowledge about how children interact with content. Essential components of teachers' lesson plans that provide students meaningful opportunities to learn include knowledge of the conceptual ideas of the content, anticipation of student solution strategies, and questions to elicit student thinking. Armed with this knowledge, researchers have given thoughtful attention to the kinds of supports and tools that might help novice teachers to learn to plan in ways that minimize the impact of their inexperience. I turn to this research next.

Tools to Support Teachers to Plan Meaningful Mathematics Lessons

Mathematics education researchers and teacher educators have provided detailed tools for teachers to use to plan thoughtful lessons that are likely to result in student learning. Work on the QUASAR Project, for example, resulted in several tools to help teachers be purposeful in their planning for the use of mathematical tasks. One tool from this work, The Mathematical Tasks Framework, described several phases of a lesson. The framework addresses how the tasks teachers use in the classroom have an impact on the opportunities students have to learn. The teachers' decisions during instruction impact those opportunities too (Smith & Stein, 1998; Stein et al., 2000). These authors describe how tasks evolve through three phases in a lesson: first, tasks appear in curriculum materials or are created by teachers and written down in some form for students (the "written task"); second, teachers launch or set up tasks in the classroom; and third, students work on tasks and the tasks become something slightly different as students talk about and solve them. Each of these phases of the task influences students' learning (Smith & Stein, 1998; Stein et al., 2000) because the cognitive demand of a task can be maintained or diminished by moves the teachers and students make in the classroom.

A second tool designed by these authors, the Factors Associated with Maintenance and Decline (Smith & Stein, 1998; Stein et al., 2000, 2009), illustrates the ways in which teachers maintain or reduce cognitive demand of the task. For example, the teacher, when she launches the task, could set up a preferred solution strategy for students, which would lower the cognitive demand of the task. When students work on the task, they might ask questions about how to solve it. Teachers can answer these questions with a solution strategy or hints about the directions; however, these answers could also diminish the cognitive demand of the task. This framework provides teachers and teacher educators some language to use to talk about the use of tasks in classrooms.

The third tool from these researchers, Task Analysis Guide (Smith & Stein, 1998; Stein et al., 2000), provides teachers ways of identifying important features of written tasks that relate to the cognitive demand required to solve them—memorization, procedures without connections, procedures with connections, and doing mathematics. Tasks that require only memorization are characterized as having the lowest cognitive demand and tasks described as doing mathematics as requiring the highest cognitive demand. The purpose of the framework is not to rate tasks and use only tasks of high cognitive demand, but rather to help teachers become intentional about the tasks they use so that task selection is consistent with lesson goals. Again, this tool provides language to talk about aspects of tasks that have an impact on students' opportunities to learn.

A related, and more recent, tool for planning is the Thinking Through a Lesson Protocol (TTLP; Smith, Bill, & Hughes, 2008; Stein et al., 2000). This planning tool is different from the typical models for planning mentioned earlier that followed a linear process and focused on what teachers and students do. The TTLP encourages teachers to think and capture in writing aspects of the lesson that will help the teacher focus on student thinking. The protocol is divided into three parts: selecting and setting up the mathematical task; supporting students' exploration of the task; and sharing and discussing the task. Within each of these sections of the TTLP, the authors provide a series of questions for teachers to consider important aspects of the lesson. For example, the second section of the protocol includes the following questions: "What questions will you ask to help students get started or make progress on the task? What questions will you ask to assess students thinking around the key mathematical ideas in the task? What assistance will you give or what questions will you ask a student or group that becomes quickly frustrated and requests more direction in solving the task?" (Smith, Bill, & Hughes, 2008, p. 134). As teachers think about and write out the responses to the questions in the protocol, they are positioned to maintain the cognitive demand of the task and to focus on student thinking.

Stein et al. (2008) noted that what teachers could *actively do* during discussions received little attention in this work. A consequence of this lack of attention to the role of the teacher was the impression that students must be left to their own devices to work on tasks and that very little guidance from the teacher was recommended (Chazen & Ball, 2001; Lobato, Clarke, & Ellis, 2005). This prompted researchers to begin "second generation practice" research that focused on using student thinking as "the launching point of whole-class discussions in which the teacher actively shapes the ideas that students produce to lead them toward more powerful, efficient, and accurate mathematical thinking" (Stein et al., 2008, p. 320). The more active role of the teacher, with attention to how a teacher can shape student ideas and support students to think in more powerful ways about the mathematics, shaped the research on orchestrating productive discussions.

A Tool for Planning: 5 Practices for Orchestrating a Discussion

Stein et al. (2008) brought the findings from research on several aspects of planning together in their 5 Practices Framework in order to support teachers who were new to having mathematics discussions in their classrooms, including novice teachers. These practices, they posit, will help to reduce the impromptu decision-making, by allowing teachers to plan the direction of a discussion in advance. The practices they articulate are:

(1) anticipating probable student responses to mathematical tasks,

(2) monitoring students' responses to the tasks,

(3) selecting particular students to present their mathematical thinking and work,

(4) purposefully sequencing the student responses that will be used in the discussion, and

(5) helping students to find and make mathematical connections among different student solution strategies, and between these strategies and the key mathematical ideas of the task (Stein et al., 2008, p. 321).

Smith and Stein (2011) provided a more elaborate description of the 5 Practices, along with cases from classrooms, to support teachers in understanding how to use these practices to facilitate a productive discussion. Smith and Stein (2011) annotate the TTLP to highlight how the 5 Practices Framework is connected to the TTLP. They make a case for continuing to use the questions from the TTLP that are not represented in the 5 Practices Framework, such as, “How will you ensure students are engaged in the task?” and “What will you see or hear that lets you know that *all* students in the class understand the mathematical ideas you intended for them to learn?” (p. 81). They posit that a teacher who spends the time thinking about these issues will be prepared to deal with much of what happens as the lesson unfolds.

Conclusions

The literature reviewed above presents challenges to the work of preparing prospective teachers to design effective lessons. Recall, for example, that research showed PTs were limited by their lack of experience in anticipating likely student solutions, understanding common misconceptions students may hold, and knowing how students might interact with the content ideas. After selecting a task, teachers using the 5 Practices Framework can focus on student ideas—first anticipating student solutions and then using those anticipated ideas to design a plan that builds from those solution strategies and thinking. PTs experienced a reduced sense of efficacy in supporting student learning because of their limited interactions with students (Borko & Livingston, 1989). PTs can easily become frustrated and feel that successfully planning lessons for discussion is beyond their reach (Borko & Livingston, 1989; Heaton, 2000;

Schoenfeld, 1998; Sherin, 2002). Since, according to the research, what is written into novices' lesson plans dictated what happened in the classroom, teacher preparation must find ways to help support PTs in building their understanding of how students think, possible misconceptions students may hold, and how to develop a detailed mental image of the likely flow of the lesson from the student perspective. Teacher educators must find ways to create learning opportunities for PTs to develop an agenda that includes student thinking, common misconceptions, and attention to student ideas. To that end, the remainder of this chapter documents one teacher educator's efforts to develop ways to support PTs in building a lesson planning practice that revolves around student thinking and foregrounds the engagement of students with the mathematical concepts in the planning phases. This chapter comprises a self-study in which I reflect on and analyze my practice as a teacher educator while learning how to create meaningful opportunities for my PTs to learn how to plan *for their own involvement in the lesson* in order to help students reach a mathematical goal.

The Framing of My Methods Course

I conceptualize teaching as a complex, interactive activity in which the responsibility of the teacher is to create opportunities for students to develop their own understanding about content. I designed my elementary mathematics methods course by drawing on my understanding of the high-leverage practice literature (e.g., Ball et al., 2009; Lampert, 2010; Lampert et al., 2010; Sleep et al., 2007). I chose to focus on practices that were ubiquitous in teaching, provided opportunities for PTs to rehearse these practices, and designed experiences for PTs I hoped would allow them to become more nuanced in their ability to "see" how effective teachers' practices result in student learning.

The practices I focus on in the mathematics methods course have all been documented by research to be necessary for effective teaching and include: (a) designing meaningful, measurable, mathematically specific goals, which can be explored through the use of high cognitive demand tasks (e.g., Hiebert & Grouws, 2007; Joyce & Hartoounian, 1964; Smith & Stein, 1998, 2011; Stigler & Thompson, 2009); (b) eliciting student thinking about the mathematics embedded in a learning goal (e.g., Grossman & McDonald, 2008; Smith & Stein, 2011; Stein et al., 2008); (c) assessing student thinking for mathematical underpinnings (e.g., Ball et al., 2009; Smith & Stein, 2011); and (d) responding to student thinking in ways that support students in articulating their ideas and moving them toward the mathematical goals of the lesson (e.g., Ball, et al., 2009; Sleep, 2009; Smith & Stein, 2011). Hiebert and colleagues asserted that much of the intellectual work of teaching can occur outside the K-6 classroom, including specifying learning goals, anticipating student responses that demonstrate acquisition of the learning goals, and evaluating lessons in order to map the evidence of student learning (Hiebert et al., 2007; Hiebert et al., 2003). I designed many activities in my methods class to support PTs so they could plan meaningful lessons and identify aspects of planning that influenced what happened in the classroom.

For this chapter, I will focus on two pivotal events early in the semester that created some disequilibrium for me. As a MTE, I have learned to pay attention to moments where what I anticipated would happen, does not happen. This is an indicator that what I intended to teach, and in fact thought I had taught, was actually not taken up by my PTs in the ways I intended. I analyze the instructional decisions I made to offer them as opportunities for other MTEs to reflect on their own practice and begin to notice similar events in their classes. I begin with my reflection on the PTs' work on a class assignment that surprised me. I analyze the discussion where I intentionally probed to determine what the PTs were thinking and then analyze the instructional decisions I made while teaching to help move PTs closer to my intended goals. Again, I do this to provide one MTE's experiences making in-the-moment decisions and reflecting on practice because instructional decisions are often hidden behind closed doors making it difficult to learn from the practice of others. I provide evidence to demonstrate the shifts in planning and thinking that occurred for me as well as for my PTs.

Context for this Study

I taught two sections of a mathematics methods course for prospective elementary teachers in the spring of 2013. In the first three weeks of the course, my PTs spent time writing, analyzing, and discussing how to write goals; planned and taught brief lesson segments using the 5 Practices Framework; analyzed videos for evidence of student thinking relative to the goals; and identified teacher moves that they thought had an impact on student learning in written and video cases. It is important to note that by week three, I felt I had *taught* my PTs how to write meaningful goals, plan by using goals, and identify goals in video episodes. I also thought they had *learned* to do this and believed I had evidence to show it. In the next section, I describe the activity I gave students in week three. The PTs' responses made me realize I had *not taught* them what they needed to know in order to identify whether students met the goals of a lesson. I provide this as an example of what I noticed and why I paused instruction to uncover the misunderstandings, so that other MTEs might use it to reflect on their own practice and perhaps write about their experiences as well.

Identifying Instructional Decisions

I designed an activity based on a written case, "The Case of Darcy Dunn" (Smith & Stein, 2011, pp. 21–30) with three purposes in mind: first, to assess my PTs' ability to identify evidence from the classroom to support a claim they were making about teaching; second, to give them ways to think about student approaches to tiling tasks; and third, to identify how the 5 Practices Framework was evident in a written case of teaching. The mathematical task in the case was a typical tiling problem that appears in many standards-based curriculum materials. Students had to determine the number of tiles on the border of the patio as the garden area in the center grew. The garden area started out with a measure of one square foot and increased by one square foot in each new step. Ms. Dunn wanted her students to understand three mathematical ideas:

> (1) linear functions grow at a constant rate; (2) there are different but equivalent ways to write a rule that defines a relationship between two variables; (3) the rate of change of a linear function can be highlighted in different representation forms: successive differences in a table, as the slope of an equation, and as the slope of a function when graphed (Smith & Stein, 2011, p. 22).

The written case described how Mrs. Dunn helped her students to reason about how the geometric patterns led to algebraic equations using two variables. In the case, the students in the class came up with three different equations that Mrs. Dunn wrote on the board. Mrs. Dunn ran out of time before she could talk about the rate of change, so although a discussion about constant rates could have followed from the student work, she did not have time to discuss rate. Mrs. Dunn asked students for homework to determine whether all three equations the class found could be correct. She did not have time during class, however, to address the fact that the three equations were equivalent ways of representing the relationships observed in the task. The students' representations all involved using the geometric patterns of the patios to create equations, which was part of Mrs. Dunn's third goal, but she did not have time to discuss representations of the relationships in the form of tables or graphs. She had situated herself well to do this in the next class or two.

On week three, as the last activity for the day, students read, analyzed, and discussed the case in small groups in order to document evidence for how Mrs. Dunn built her discussion on student thinking to move students toward the three mathematical goals of the lesson. I gave PTs the following writing prompts:

Analyzing the Case (30 minutes)

1. Darcy Dunn has three goals for this lesson: List them.
2. Starting from her goals, make a brief roadmap of the strategies she chooses to share and the moves she makes to highlight the goals of her lesson on an 8 ½-x-11 piece of paper.
3. Does she reach her goals? Show evidence either way.
4. If you do not think she reached her goals, what could she have done differently?

PTs handed in their work on this task before they left class. I expected my PTs would have little difficulty finding evidence to support their claims and expected them to have few issues with questions 1–3. I assumed this activity would serve to strengthen my PTs' understanding of using evidence to make claims and that we would be able to dig in and unpack how the five practices appeared in the lesson tool we had used previously on several occasions.

When I reviewed the work my PTs created, everyone correctly identified the mathematical goals of the lesson. All the PTs, however, determined that Ms. Dunn had met the lesson goals, even though Smith and Stein (2011) concluded that Ms. Dunn did not reach her goals. Smith and Stein even offered some commentary about how Ms. Dunn had laid the groundwork to reach the intended goals of this lesson on subsequent days, drawing further attention to the fact that the students had did not met the goals on that day.

The evidence PTs used to substantiate their claims seemed to conflate moves the teacher used with evidence for student understanding of the learning goals. For example, one group wrote: “Yes, she did reach her goals because she allowed the class to understand through discussion, questioned what her students were doing, and made adjustments when needed.” This group identified some key features of responsive teaching in their statement, namely, Ms. Dunn was working to build student “understanding” through a “discussion” in which she “questioned what her students were doing” and “made adjustments” as needed. When reviewing this response, I noted that it did not clearly address the question I had posed, “Does she reach her goals?” but I was unsure why not. It seemed as though the PTs answered a question about whether or not this lesson fit with the models of responsive teaching we had previously discussed.[4] Even more surprising to me, every group had a similar response to the question. I was expecting them to use examples of student work, thinking, and talk as evidence to determine whether students understood the intended goals. I knew my PTs were thinking about the task and believed that what they wrote answered my question. This confused me because their responses did not fit my perspective on the kinds of evidence that demonstrated whether or not students met the goals. I wondered why my PTs responses did not directly address the question I had posed. When something like this happened, I tried to determine what went wrong in my teaching: Was the question unclear? Did I give them some indication that I wanted them to write about something different? Did they not understand the question? Was the question worded in such a way that they focused on the teacher decisions and the mathematics instead of the student work and talk? The question itself does make the teacher the center of their reflection and work.

[4] After reviewing the literature on teacher noticing, this example makes sense in a different way (I will examine teacher noticing in depth in Chapter 4 of this dissertation), but for now, I will say only that the PTs’ noticing focused primarily on what the teacher was doing and did not focus on the students in the interaction.

By using words such as “she” and “her” I draw attention to the teacher and not student thinking. Perhaps a better question might have been, “Is there evidence that students learned what the teacher hoped they would learn?”

The PTs’ responses in the next class session helped me understand the lens they were bringing to the case. I began the discussion by asking about their understanding of lesson goals, and my PTs very articulately demonstrated their understanding by drawing on readings and research we had read in class to date to describe how mathematical goals needed to be meaningful, measurable, and mathematically specific. They were able to provide examples of goals that would meet those criteria. When I asked my PTs to “help me understand why you believe Darcy Dunn met her learning goals, convince me that she met them,” I began to understand why the work they had turned in was so different from what I had expected.

In the first section, a female elementary teacher candidate with a mathematics specialization spoke first:

Kayla[5] I think I approached it from my point of view and not their point of view. Because like, I can see that it’s growing at a constant rate when I look at that equation and I’m like, oh, duh, that’s what they’re seeing. But maybe they’re not because maybe they haven’t learned it yet. So, it’s kind of like what we talked about before, where we can give a student a division problem even if they don’t know the algorithm for it because they will be able to figure it out by breaking it in groups or something. It’s kind of the same way, like, we can see that stuff, but they can’t see it yet. So, they’re making the equations, from the same

[5] All names are pseudonyms.

problem, but they might see that the equations look really different and it might not occur to them that they are equivalent.

ME How many of you agree with what Kayla is saying? That you thought about whether or not the mathematics made sense to you and that since the math made sense, you decided she had met her goals? [most hands raised] Oh, this is making more sense now. OK.

Kayla called to my attention that she had a particular lens through which she viewed the class, that of her own understanding of the mathematics. She mentioned that, upon reflection, she might have made these connections because of her mathematics knowledge, but that the students might not have the same knowledge she did, and therefore it might not have been as obvious to them. She referenced some of our work with *Children's Mathematics: Cognitively Guided Instruction* (Carpenter, Fennema, Franke, Levi, & Empson, 1999) materials that addressed how not knowing the division algorithm did not mean students did not understand the concept of division. The statement from Kayla seemed to imply that what she was thinking about was how the task aligned with the learning goals, but not what evidence there was to show whether or not students understood the key ideas.

When I taught my second section that afternoon, we had a similar conversation; however, it took a little longer for me to understand what they were thinking:

Dillon We were thinking that since students found three different equations for the relationship that she met the goal [goal 2 from Darcy Dunn's lesson]. She didn't do much with them, but they found them so we thought that she had met the second goal.

James So, the three equations are equal—they're just different ways of writing the relationship or thinking about the relationship, so I still think she met her second goal.

Dillon claimed that since students found the equations, Ms. Dunn had met the goal. She added that she noticed that the teacher "didn't do much with them," but the fact that students had found the equations was enough evidence for Dillon to believe Ms. Dunn met the goals of her lesson. James then added the idea of equivalence into the conversation. His conclusion was rooted in his own mathematical knowledge. Since James knew the three equations were equivalent, the teacher met the goals.

During the conversation in the omitted section of this transcript, lines 8–17, we established for the entire class that the equations were actually equivalent. A PT demonstrated that the equations were equivalent using algebraic manipulation. Following this conversation, we turned back to the question of whether students met goal two of the lesson:

ME So, it's clear to you as prospective teachers and seniors in college that these equations are equivalent, but the question you need to ask yourselves in order to determine if the learning goals were met is: Did the *students* understand that the three equations were equivalent ways to represent the relationship? If you believe they did, then I'd just like to see some evidence that shows students talking about that idea. Just flip through the case and find me a quote or two. [15-second pause as students flipped through the pages of the case seeming to skim the text]

ME James? Do you have a quote?

James No, not yet.

ME It's OK. I'll wait for you to find one. [23-second pause, students continue flipping pages and reading]

Brenda I was in James's group, and I agreed that she had met this goal, but now I see what you are saying. It was never made explicit for the students, and they never discussed it—even in the small groups, we don't have evidence that this goal was met for the students.

James So, I guess I was thinking that the teacher had all the math there and it was true, they are equivalent, but the students might not know it.

ME So, can we say the students walked away from class knowing that there are different ways of writing equations that define relationships between two variables?

Class No.

ME Why not? Mollie?

Mollie Because we don't have any evidence that the students understood it.

In lines 18–22 of the transcript, I pointed out that there were differences between our understanding of the mathematics and the students' understanding of the mathematics. When I directed the PTs to find a quote to support their claims, I interpreted the long pauses as evidence that they were not entirely convinced the teacher had not met the goals and so they were flipping through the pages to find their evidence. It is also possible they were just doing what I asked. Brenda joined the conversation and made an important connection that since there was never explicit conversation about the goals, "not even in the small groups," we did not have evidence that students understood that the equation(s) students found were equivalent. James's comments underscored the need for students to be aware of the goals of a lesson, and Mollie made a succinct statement about the need for evidence that students understood the goals before we could claim that students met the goals.

I could have just marked the responses on the homework as incorrect or written feedback about how their responses did not answer the question I provided. If I had done this, I would have missed this opportunity to help my PTs think critically about the evidence needed to claim that a lesson achieved the intended goals. Since I had already spent several class sessions developing my PTs' understanding about goals, I could have just moved on. If I had done that, my PTs would have left my class thinking that as long as the mathematics in their lessons made sense to them and was correct, that was all that was required to effectively teach. Responsive teaching focuses intensely on student work and ideas. Without having come back to this issue, my PTs would have completely missed a big idea in my course, that student thinking is central to teaching and that we cannot say teaching has occurred unless we have evidence students have learned it. In other words, I would have prepared them to go into their own classrooms and do "show and tell" (Ball, 1990).

Making the Connection to Planning

The conversations that followed this activity centered around the other two goals from Ms. Dunn's class and making evidence-based claims about what students understood. PTs then tried to map out Ms. Dunn's lesson in small groups to create a skeletal sketch of the lesson using the goals, teacher moves, and evidence from student work to show how Ms. Dunn had purposefully guided her students' understanding toward the goals, even though she did not completely realize them during the episode captured in the case. This task proved more difficult than I had expected. In my mind, I expected my PTs to create a flow chart or outline on which they grouped each goal and respective key supporting mathematical ideas together, indicated the subsequent student ideas that connected to each goal, and provided evidence from the transcript to show students making sense of these ideas. I had created my own flow chart when preparing the lesson. I expected that this activity would make it very clear that Ms. Dunn’s lesson did not address two of the goals that demonstrated linear functions have constant rates of change or how the rates appeared in different representations of data. There was no evidence to support these ideas since the class never examined charts or tables and they never discussed rate of change.

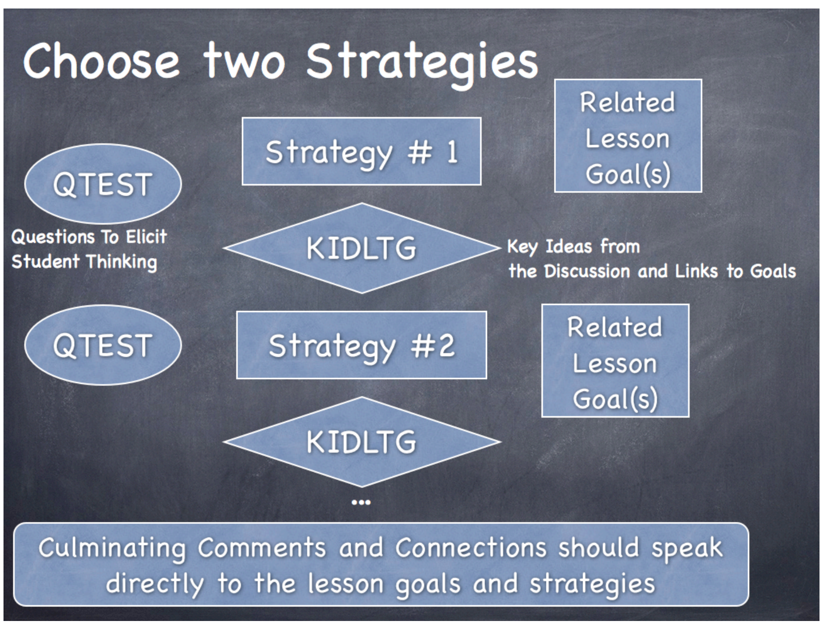
As I observed and listened in on my PTs’ conversations, I saw that they placed the goals at the top of the page and then started flipping pages, quietly, for about 10 minutes. Occasionally they talked but wrote very little. When I asked them to talk about why this task was difficult, this conversation followed:

Ben	Well, there’s nothing here.
ME	Can you say more about what you mean?
Ben	The students don’t talk about rate, there are no tables, they never even talk about slope. There is not conversation about equivalent expressions, either.
ME	So, what do you think that means?
Emily	That the teacher didn’t reach her goals with the kids.
ME	But it was a great class, right? Kids were talking about the math. Kids were doing math. So was the class pointless because Ms. Dunn never got to her goals?
Sarah	I don’t think so. I think there were lots of great things happening, but they just don’t address these goals.
ME	So, what goals do you think this lesson did address? Can you spend a few minutes in your groups writing some of those out?

I had anticipated their difficulty and came to class prepared to help them think through how to parse out supporting ideas related to the goals of the lesson, how to anticipate student thinking relative to the content, and how to respond to student ideas in ways that helped students develop an understanding of the important mathematical ideas. My PTs did return from this exercise with meaningful understandings of the content that students explored. We were able to parse these out and talk about how Mrs. Dunn supported student learning for each of the intended goals of the lesson and laid the groundwork to move toward those goals in subsequent classes.

I had come to class with a PowerPoint slide that I planned to use to illustrate how each of the five practices interacted in a purposefully planned lesson (see Figure 1).

Figure 1. PowerPoint Slide of Original Lesson Map



I used this slide to talk through the connections teachers make between the goals and student strategies to facilitate a discussion that would subtly guide students toward an understanding of the goals. The four sections of this map that are repeated: Questions to Elicit Student Thinking (QTEST), Key Ideas from the Discussion and Links to Goals (KIDLTG), Related Lesson

Goal(s), and Strategy, were meant to show the purposeful planning that went into discussing each strategy PTs chose to discuss. The STRATEGY section of the map was a place for PTs to write out the anticipated student solutions—one at a time. I provided the RELATED GOALS box on the far right for PTs to explicitly describe the particular aspect of the goal the strategy would illuminate. The diamond shape in the middle was a placeholder for the key ideas and explicit connections to the goals (KIDLTG) PTs would plan to make to connect the strategy to the goal, and to connect each strategy to one another. This sequence was repeated for every strategy the PTs discussed. I provided the long rectangular shape at the bottom to help PTs remember to plan for the culminating ideas, connections, and questions they would ask to ensure that students were aware of and engaged in building connections toward the goals.

After we talked about each section of the mapping tool, I modeled how I planned a lesson and used the tool to highlight my intentionality using another task in the 5 Practices book (Smith & Stein, 2011). My PowerPoint slides included each section of the map and which ideas I was intentionally planning to use to support building students' understanding of the lesson goals. I then modeled the lesson I planned in class with PTs. I did not set out to create a tool for them to use to plan their lessons; I just created a graphic to help them visualize the connections between related ideas in a lesson.

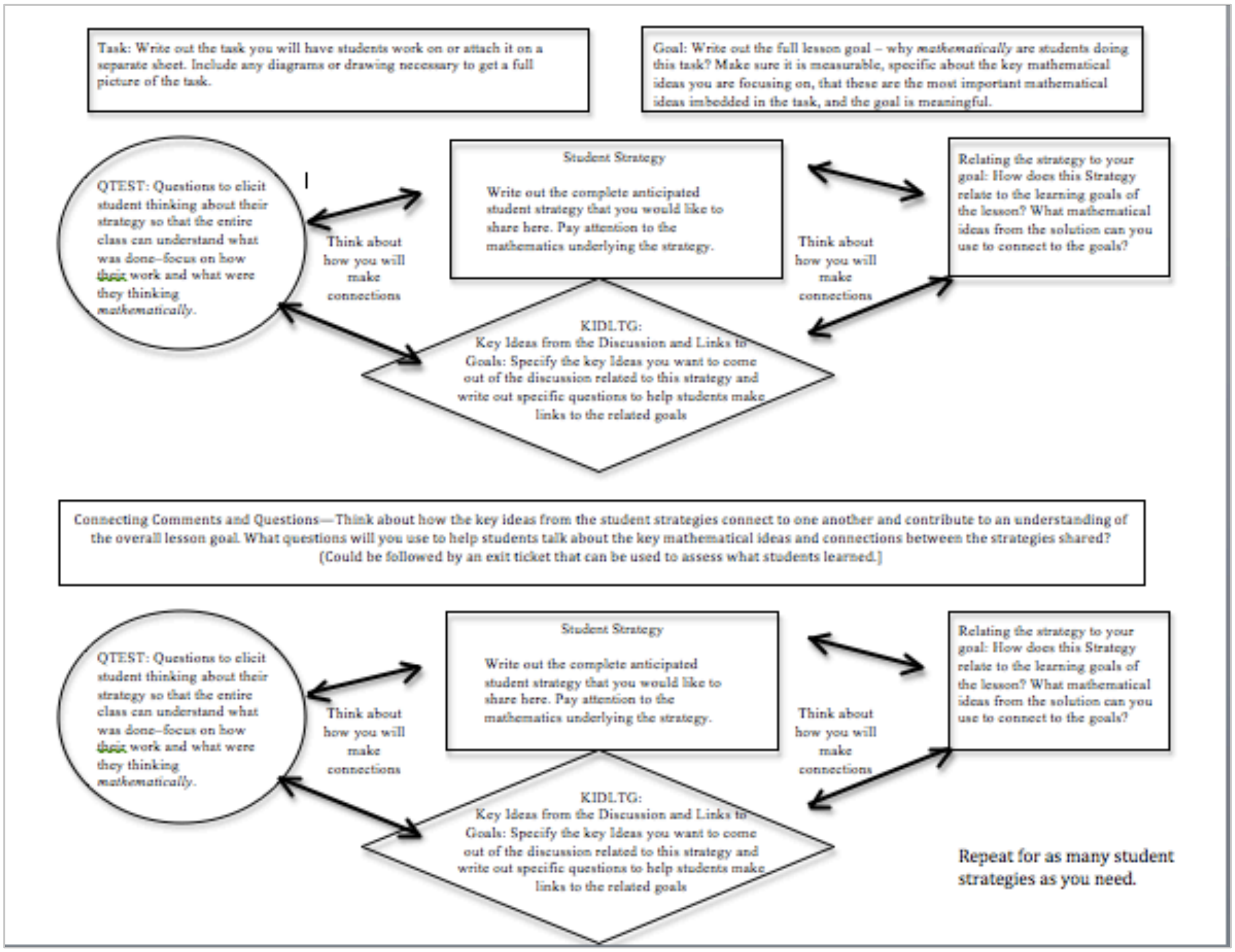
The PTs liked this graphic idea and wanted to try using it to plan a 10-minute activity they would teach in the university classroom the following week. After using the organizer, PTs provided feedback about things that were missing—for example, there was no place on this map that described the task or the original, overarching goal. They wanted a place to write the full lesson goal so that they could use the "Related Lesson Goals" boxes on the right-hand side of the map to identify which aspect of the goals spoke directly to each strategy. Over the course of the semester, we revised the map several times and eventually adopted it as our go-to lesson-planning tool.

Lesson Mapping Tool

The Lesson Mapping Tool (LMT) shown in Figure 2 was the result of several rounds of revisions to the original map from the PowerPoint slide on week four of my course. This map did not replace the other tools (e.g., TTLP, 5 Practices Framework) in the class, but PTs voted to use it because they felt it was a good way to help them keep track of where they were going in the lesson and how they made planning decisions. It also provided an additional lens for them to clearly see how they would purposely plan to use evidence of student thinking to subtly guide students to the goals of the lesson.

We revised the LMT to support purposeful planning and ensure that PTs explicitly *linked* the learning goal to the discussions of the student strategies they drew on, as well as to *connect those strategies* to one another. The overall structure of the LMT remained the same as the original PowerPoint slide; however, the final version had several additions (see Appendix A).

Figure 2. Lesson Mapping Tool (LMT)



The rectangular boxes in the middle column of the tool in Figure 2 provided a space for PTs to write out anticipated student strategies. The LMT included prompts for PTs to write out questions that PTs planned to use to elicit student thinking about their strategy so that the entire class could understand what was done, with particular focus on what students were thinking *mathematically* (QTEST-section of LMT—circle on Figure 2). The PTs then needed to *connect the strategy* in writing to a particular facet of the *lesson goal* that the strategy illuminated. They also needed to articulate the key ideas to include in the discussion that would help build links to their learning goals. They needed to record these explicit connections on the KIDLTG (Key Ideas for Discussion and Links to Goals) section of the LMT (diamond on Figure 2). The prompt written on the LMT directs PTs to include questions in this section. The last section of the LMT, the long rectangle in the middle, was included to ensure PTs wrote specific questions to connect the strategies to one another and back to the lesson goal (CCQ—*Connecting comments and questions* section of LMT in Figure 2). The PTs repeated this sequence for each anticipated student strategy.

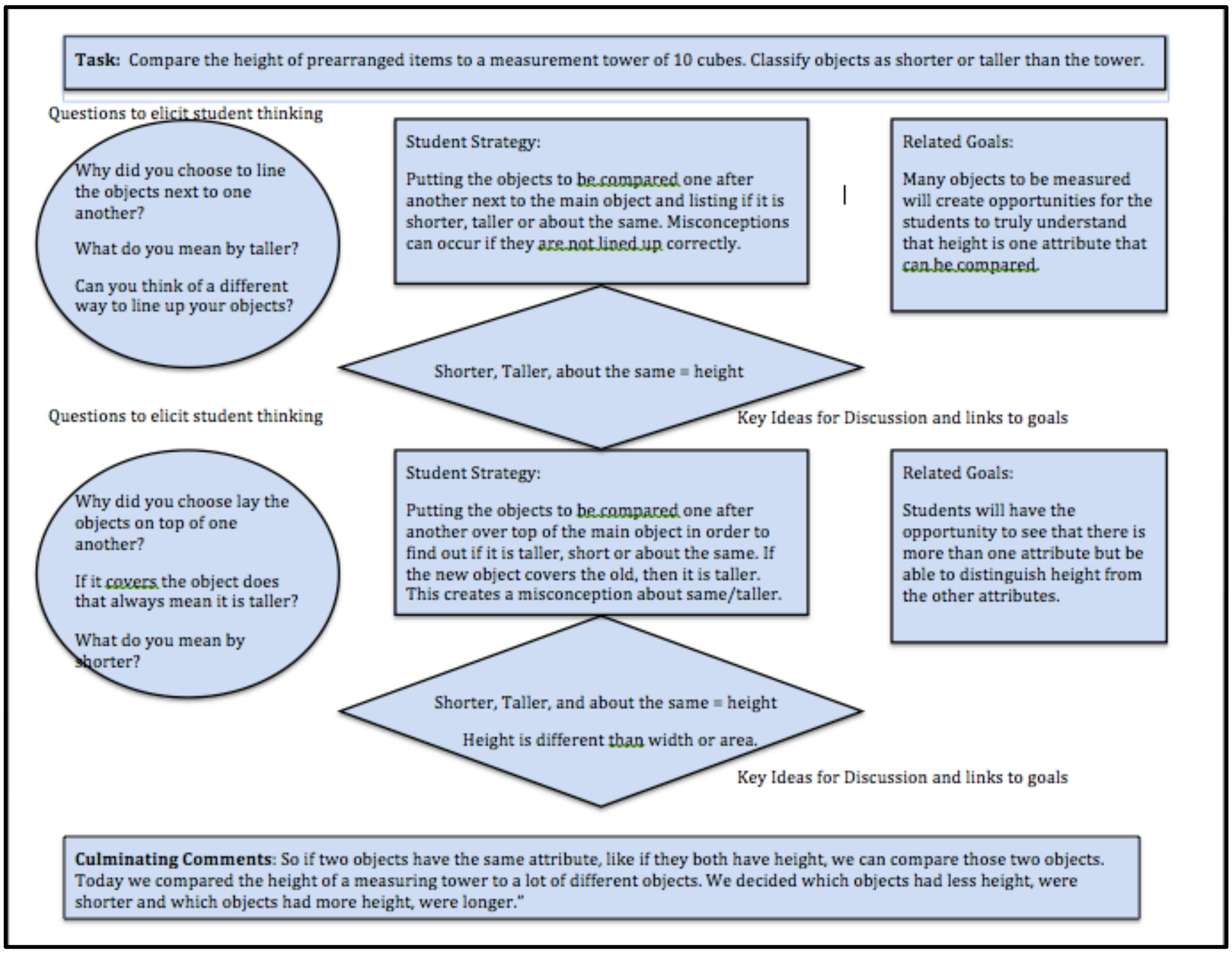
After writing out an explicit measurable mathematical goal, PTs were asked to describe the task they used to help build student understanding toward this lesson goal. The LMT provided space for PTs to anticipate the student solutions to the task and record these strategies. As Smith and Stein (2011) suggested, anticipating means more than just writing out the solution, but also considering how one might respond to these strategies and which strategies are likely to be most useful in moving students toward understanding the original lesson goal. The prompts on the LMT emphasize that PTs need to think of the *mathematical ideas* the strategies help to uncover, rather than the process alone.

Once PTs had anticipated several correct student strategies, they needed to determine an order for discussing the strategies that they felt would be most helpful in addressing the mathematics. We designed the LMT to support PTs in becoming explicit about threading the key mathematical ideas throughout the lesson. PTs needed to be able to draw on their own content knowledge as well as their knowledge of students in order to fill out the tool in their planning phase.

I did not assign the LMTs a grade. I provided feedback, and PTs revised their maps with an emphasis on becoming more mathematically specific and explicit in their thinking and planning. After they revised their work, PTs were able to use the maps to inform their teaching of the task. I provided feedback to support them as they learned how to make the mathematical thinking of their students visible and to use this thinking to support students in understanding the mathematical point of the lesson (Ball & Bass, 2009; Sleep, 2009). I analyze this feedback in the third chapter of this dissertation.

I have included a sample map (Figure 3) that one PT created for the 10-minute activity in

Figure 3. Sample First Grade Map from Week 5



the week five class. This was representative of the initial attempts of PTs as they tried to think through how to plan a short lesson segment that was connected to the goals of the lesson. The related goals connected to the ultimate goal of comparing the heights of two objects. This PT was able to tease out how identifying the attribute of an object was an essential precursor to making comparisons. She linked vocabulary as a piece of the essential learning in order to make comparisons. This PT also embedded ideas of measurement through her questions and thinking about possible student solutions to this task. The PT thought about possible misconceptions students might hold about covering an object verses comparing the heights.

Reflections on the LMT as a Tool for Learning

My first key learning was that, as teacher educators, the power of creating a space for the voice and ideas of PTs cannot be over emphasized. Developing a sense of *shared ownership* was a turning point in my classes. One of the most powerful aspects of this process for my learning was developing the tool with and for the PTs. I had not planned for my PowerPoint slide to change the way PTs planned 10-minute lessons. I, too, used the LMT to plan short activities for class to serve as models and to better understand the process PTs went through as they planned. The PTs took ownership of this tool in ways I did not anticipate. They offered suggestions to improve the tool. They shared their ideas with one another and shared their finished LMTs. The LMT became a part of our work together because the PTs *wanted* to use it. That was a powerful experience for me and helped me to better understand how to create shared ownership.

By providing PTs with a voice on how to plan lessons, they actively engaged in critically analyzing both the tool and the process of planning. As examples, PTs asked to add possible student misconceptions to the student strategy section so they would be sure to attend to these. In the early iterations of the LMT, PTs asked to have a section that explicitly required them to explain how they were going to help students make connections between the lesson goals and the strategies they anticipated.

My second key learning moment was that PTs want to understand *why* we, as teacher educators, ask them to plan in certain ways. My PTs were open to questioning why certain sections were included and what additional information would be helpful to them in planning.

We continued to use the longer TTLP protocol for full class lessons, but for the 10-minute lessons we planned for our classroom use, the PTs used the LMT throughout the course.

My third key learning moment was that, as a teacher educator, when I noticed something was off, such as the responses to the assignment about Darcy Dunn's class, I needed to stop and figure out why the PTs responded the way they did. I needed to let the PTs come to their own conclusions by asking pointed questions. I learned an important lesson about paying close attention when PTs did not respond in ways I had anticipated. By taking the time to reflect on their work mapping the Darcy Dunn case (Smith & Stein, 2011) and returning to it in the classroom, I was able to identify and address a major misunderstanding. If I had only given them feedback and moved on, I might not have noticed that we did not share a perspective on determining whether a lesson met the intended goals. I might have spent the rest of the semester frustrated that PTs did not make connections to evidence of student learning and consequently spent hours of my time giving them feedback on their lesson plans.

My fourth key learning moment was that developing a community of practice, where PTs provided feedback to one another, supported PTs' growth in ways I could not have predicted. The PTs grew in their ability to provide critical feedback to one another about the connections between their lesson goals and their anticipated student solutions. The numbers of different strategies PTs included on their LMTs increased as well, sometimes PTs used three or four copies of the LMT for one 10-minute lesson. PTs could be heard asking, "What do you think a third grader would do with this task?" The PTs' lesson planning practices improved throughout the semester, and they remained focused on student thinking as the evidence for meeting their learning goals.

My fifth key learning moment as a teacher educator was the realization that starting lesson planning with a meaningful, measurable mathematical goal and explicitly planning how to thread it through the lesson is a complex practice for PTs. Embedded in this practice are ideas about student engagement with the mathematics, misconceptions, multiple solution strategies, connection building, and being purposeful in lesson planning (Sleep, 2009). To do this effectively, PTs needed to develop multiple skills and understandings. My work as a teacher educator must explicitly attend to each of these ideas.

CHAPTER 3: CHARACTERISTICS OF EFFECTIVE FEEDBACK IN TEACHER PREPARATION

Giving and receiving feedback is a ubiquitous practice in teaching. There is an extensive research base on feedback for teaching in K-12 schools and in higher education (HE), but no comparable literature base for feedback in teacher education. This study builds on the research on feedback in K-12 and HE settings and situates teacher preparation as existing in a hybrid space between these two areas of education. Thus, this chapter will examine the overlaps between feedback in K-12 setting and HE to identify types and characterizations of feedback appropriate for teacher educators who not only provide feedback to PTs, but also prepare PTs to provide feedback to their students. An analysis of one teacher educators' feedback practices in a practice-based mathematics methods course will drive this chapter's examination of the types of feedback PTs received that had an impact on their lesson planning practices and were visible in the instructional decisions PTs made. By better understanding the research on effective feedback practices of teacher education as well as how PTs use the feedback to make changes in their planning and instructional practices, teacher educators will be better positioned to provide feedback that will have a high likelihood of being used by PTs to grow in their practice.

Background Literature

The literature on feedback has provided a definition of formative feedback. After assembling this definition, this section will examine the research on effective feedback practices in K-12 and HE, reflecting an understanding that teacher preparation involves an interplay between these contexts. I demonstrate that teacher preparation is a hybrid space and that synthesizing the characteristics of effective feedback in the two areas of literature is a way to formulate a theory of effective feedback for PTs.

Formative feedback is a subcategory of formative assessment that is focused on supporting learners through the process of instruction (Back & Wiliam, 1998). Formative feedback has the aim of changing learners' conceptions of learning, enhancing learner responsiveness, and helping them become independent learners. The intended purpose of formative feedback has five different categories: correction, reinforcement, diagnosis, benchmarking, and longitudinal development (Price, Handley, Milar & O'Donovan, 2010). Researchers studying feedback on teaching in K-12 settings emphasize the types, quality, and effectiveness of feedback and hold the *teacher* primarily accountable for the impact or effectiveness of the feedback provided (Black & Wiliam, 1998; Hattie & Timperley, 2007; Moreno, 2004; Shute, 2008). In HE, the literature builds on the research on feedback from the K-12 teaching literature, but researchers place the responsibility on the *student* in the feedback interaction, rather than on the instructor (Carless & Yang, 2012; Evans 2013; Radloff, 2010). In K-12, the teacher has the responsibility of making feedback effective, whereas the obligation shifts to the student to make use of feedback in HE. PTs are in HE and therefore have responsibility to use feedback, but they need models and examples of effective K-12 feedback types so that they will be ready to assume the responsibility of making feedback effective.

Feedback in K-12 Teaching: A Focus on the Teacher

The purpose of using formative feedback in the K-12 classroom is to enhance students' knowledge, skills, and understanding with respect to content and general skills (Shute, 2007). Researchers generally agree that feedback is vital to both improving student knowledge and the acquisition of skills (Black & Wiliam, 1998; Hattie & Timperley, 2007; Moreno, 2004; Shute, 2008). Researchers of K-12 teaching have found that providing effective feedback is central to student learning (Black & Wiliam, 1998; Cohen, 1995; Hattie et al., 1996; Hattie, 1999; Hattie &

Jaeger, 1998). Research indicated that feedback is *effective*, when it is written clearly and offers students a sense of what they did well and areas in need of improvement. (Hounsell, 2003; Cohen, 1995). Classroom have two main types of formative feedback: directive and facilitative (Black & William, 1998). Teachers use directive feedback to highlight specific places for improvement. Teachers use facilitative feedback to guide students in their revisions and conceptualization (Shute, 2007). Formative feedback can be both corrective and facilitative. Effective formative feedback should provide information that learners can use to identify and correct inappropriate learning strategies, errors, and misconceptions (Mason & Bruning, 2001).

Hounsell (2003) provided four key components of feedback that are crucial to student improvement. Students need a clear picture of what is expected; information about what they have done well; a description of how the status of student work measures up to expectations; and guidance about how to close the gap between the expected and current status of their work (Black & Wiliam, 1998; Hattie & Jaeger, 1998). Gibson and Simpson (2004) highlighted that feedback should be understandable, timely, and actionable by students. Cohen (1995) suggested that teachers should tell students when they demonstrated confusion about key concepts or making dubious assumptions in their work. Wiggins (1998) advocated for “educative assessments” in the form of scoring rubrics that have been shown to be effective as tools for students to learn about their own learning. Feedback is a powerful tool in teaching when it communicates, clearly and in a timely fashion, what students have done well, what is lacking in their current work, and what next steps they must take.

Not all feedback, however, will support students’ learning effectively. Kluger and DeNisi (1996) characterized feedback that hampered learning as feedback that drew attention closer to the individual and away from the task. The literature has highlighted summative forms of assessment as having a negative impact on learning, particularly when students perceived these as critical and controlling (Kluger & DeNisi, 1996). Other forms of feedback that impede learning were providing grades or overall scores indicating the student's relative standing with respect to peers and coupling such normative feedback with low levels of specificity (Williams, 1997).

Multiple studies have shown that the most common type of feedback K-12 teachers provide is unspecific verbal or written praise such as "Okay", or "Good" (Bond, Smith, Baker, & Hattie, 2000; Cohen, 1995; Pauli, 2010; Voerman, Paulien, Meijer, Korthagen, & Simons, 2012). Hattie and Timperley (2007) found that this kind of vague, non-specific feedback often had the effect of stopping continued learning, since students had no direction to continue to grow or advance their understanding. Cohen (1995) added that while this type of feedback may engender a good feeling in students, it does little to promote learning. Vorman et al. (2012) studied types of effective and ineffective feedback as well as the frequency of feedback interventions and found that teachers "seldom provide the types of feedback interventions identified as effective in enhancing learning" (p. 1107) while they are teaching. Frequent, timely, and specific formative feedback was most effective. Although feedback is central to teaching, teachers did not often use specific feedback on performance.

The K-12 literature on feedback practices has many implications for teacher education. Feedback should be specific enough to help students understand where they stand in their learning and performance (Hounsell, 2003; Evans, 2013). According to Cohen (1995), teachers' feedback must include comments about what students know and can do as well as areas for improvement. Researchers have pointed out that effective feedback from teachers provided next steps to bridge the gap between current and desired levels of understanding. Housnell (2003) indicated that efficacy in feedback depended on timely delivery and the use of language students understood. These studies have suggested, as Kluger and DeNisi (1996) articulated directly, that K-12 teachers have the responsibility to provide feedback that relates to the task, students' processes, or the ways in which students can learn from their own work. The literature has described the K-12 teacher as the main actor in both providing feedback and ensuring students benefit from it, yet it has noted that teachers often fall short in providing highly effective formative feedback. By contrast, as the following section describes, the literature on feedback in HE settings shifts the onus from teacher to student.

Feedback for Teaching in Higher Education Literature—Focus on the Student

The literature on feedback for teaching in HE is not as expansive as the literature on feedback for K-12 teaching. In the university setting, students must *seek out* feedback or *find out* how the instructor assesses learning (Carless & Yang, 2012). For example, feedback may take place in peer of study groups and assessment plans that instructors provide that guide students to assess their own learning (Carless & Yang, 2012; Evans, 2013). In keeping with this type of model, the HE literature focuses on students' ability to use feedback to learn rather than on the ability of instructors to provide effective feedback (Evans, 2013; Radloff, 2010). When surveying HE students' satisfaction with the effectiveness of feedback, studies (Carless & Yang, 2012; Radloff, 2010) have found this to be one of the least satisfactory aspects of students' university experience. Carless (2006) reported that HE students found feedback to be inadequate in helpfulness, timeliness, consistency, specificity, and clarity. Carless suggested that this is because their K-12 experiences did not prepare them for an environment in which the onus fell on them to act on feedback. Carless and Yang (2012) identified the barriers of large class sizes in most HE environments, suggesting that university students may not be able to obtain timely and helpful dialogue about their progress even if they did seek it out.

There is no widely agreed upon definition of feedback in HE research. Most use the term assessment feedback, which Evans (2013) defined as

> all *feedback exchanges* generated within assessment design, occurring within and beyond the immediate learning context, being overt or covert (actively and/or passively sought and/or received), and importantly, drawing from a range of sources. (p. 71)

Thus, Evans implicitly denoted the emphasis on the student role in describing how assessment was “sought” or “received” and not “provided or given,” which were the more common terms in the K-12 literature. She described feedback in HE as covert or subtle. Research in HE has often used the terms *feed-forward* and *feed-up* (Hounsell, McCune, Hounsell, & Litjens, 2008) to describe feedback that served to support current learning as well as future learning beyond the HE context.

Evans (2013) extracted 12 key principles of effective assessment feedback for HE students from her exhaustive review of the literature addressing such feedback, and only one principle related to the actual metric of good feedback from instructors: “clear and focused feedback on how students can improve their work including signposting the most important areas to address” (p. 79). The remaining 11 principles described the instructor role as merely laying out the plan for assessment (where students’ grades will come from), ensuring that students understood the assessment plan (making sure they knew what assignments and tests would count towards their grade), and making sure resources were in place for students to use—all of which could be outlined in the course syllabus. None of the 11 remaining principles describe any action on the part of the instructor. All responsibility for growth is placed with the students. She wrote that researchers in this area widely recognized a need for students to operationalize the feedback and use it to further their learning and described failures as a *feedback gap* for which students were responsible. For example, Evans (2013) described students in the studies she reviewed as having an "inability to benefit from assessment feedback" (p. 94) rather than as subject to inadequate feedback. Evans attributed this inability among students to their beliefs about learning, dispositions with regard to power differences between students and instructors, feedback networks (feedback in the form of a failing grade might mean students need to go to office hours or seek out a tutor), information about the feedback process (when are office hours and where can one find a tutor on campus), and inadequate tools for students to monitor how they use feedback to grow and develop deeper understanding.

In spite of the emphasis on student responsibility with respect to feedback, HE literature has suggested that instructors should articulate clearly how learning will be assessed; provide resources for self and peer assessment; pay attention to issues of power, beliefs, and status that could affect whether feedback practices alienate students; and emphasize the student responsibility in the feedback process (Carless & Yang, 2012; Evans, 2013; Hounsell, McCune, Hounsell, & Litjens, 2008).

Feedback for Teacher Education—A Dual Focus

The research base on feedback in teacher education is limited; all of the researchers in this area of the literature studied feedback exchanges between supervisors and student teacher. The findings paralleled the research on feedback for K-12 students. For example, in a review of 10 studies of feedback given to student teachers, Scheeler, Ruhl, and McAfee (2004) concluded:

> (a) feedback is better than no feedback; (b) immediate feedback is better than delayed feedback, and (c) feedback that is immediate, specific, positive, and corrective holds the most promise for bringing about lasting change in teacher behavior. (p. 405)

They acknowledged a limitation in that there were only 10 studies, but posited, “supervisors should provide feedback focused on specific teaching behaviors and provide clear and concise directions for desired behavior change” (p. 405). These recommendations echoed the K-12 literature on feedback. The supervisor *gave* feedback, immediately, not just by providing resources for student teachers to self-assess or outline the assessment plan. Similarly, Jonsson and Svingby (2007) performed a review of the use of scoring rubrics in teaching and found that rubrics were helpful in allowing students to learn from feedback, but also showed improved learning through rubrics as a self-assessment.

A more recent review of the literature on feedback in teacher education (Thurlings, Vermeulen, Bastiaens, & Stijnen, 2013) echoed the results of Scheeler, et al. (2004) that feedback should be specific, clear, and unbiased and added that “feedback should support [teachers] to further improve their learning processes and…should be given frequently” (p. 13). This emphasis on frequency also echoes the K-12 literature, as does the emphasis on giving feedback instead of receiving it properly. Thurlings et al. (2013) stressed that effective feedback “elaborates, explains, and justifies the errors and that it encourages the learners to actively engage in repairing” (p. 13). In other words, supervisors had an obligation, like K-12 teachers, to ensure feedback is effective, unlike HE instructors.

Several researchers described what supervisors of student teachers do in practice and provided anecdotal examples of their feedback practices (e.g. Feiman-Nemser, 2001; Stanulis, 1994). They highlighted the need for feedback to be specific, direct, and timely, but did not define it. However, these studies were not based on testing the efficacy of feedback techniques in improving student teachers’ performance or satisfaction.

The Hybrid Space

PTs and thus should make sense of their own assignments and assessment practices and take ownership of their own learning. They have experience using feedback and should have responsibility to seek out and use feedback for their own learning. Yet they are also students of teaching. Thus, they need to learn how to give feedback that is effective in the K-12 setting, and therefore need models of effective feedback practices. This implies that teacher educators need to balance feedback to ensure it is specific and timely and that it highlights gaps in PT knowledge and practice but is not overly directive. I argue that feedback practices in teacher preparation must occupy a hybrid space and that teacher educators need to consider the dual role they play as modelers of effective feedback practice and providers of feedback to individuals with agency in their learning and teaching.

Thus, teacher educators pursue a delicate balance of giving sufficient instruction while supporting autonomy. PTs are still very new to the practice of teaching. Therefore, for example, while teacher educators are helping PTs to develop effective lesson planning practices, they should bear in mind that PTs need to feel a responsibility for their own learning and planning choices. Given that PTs' lesson plans largely determine their instructional decisions (Clark & Yinger, 1977), it may be difficult for their supervisors to restrain the desire to control their choices. Yet PTs must learn from their own mistakes.

As teacher educators support PTs in developing their lesson planning practices, a common practice is to include feedback. In order for feedback to be useful for PTs to develop their lesson planning practices, teacher educators have to leave room for PTs to take responsibility for their planning decisions. To explore how teacher educators might best support their students, the current study examines the following research questions:

1. What types of feedback on lesson planning practices does one teacher educator provide PTs?
2. What are the characteristics of the feedback that PTs are able to use to increase the specificity of planning?
3. How do PTs use the specific TE feedback to shift their language around practice?

To answer these questions, I: (a) analyzed the feedback I provided to PTs on their lesson plans; (2) characterized the types of feedback I provided on their lesson plans; (3) described the characteristics of the feedback PTs used to revise their planning; (4) analyzed how PTs made use of the feedback provided to revise their lesson plans; and (5) provided the characteristics of feedback that PTs did not use to revise their lesson plans. I elaborate these methods in the following section.

Method

This dissertation study was originally conceived as a design experiment (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Brown, 1992; Design-Based Research Collective, 2003). Design-Based Research (DBR) was particularly salient for this study because it allowed me to address complex problems in real, authentic contexts using a process of research and development through continuous cycles of design, enactment, analysis, and redesign that aligned with offering PTs repeated opportunities to move through the cycles of planning, feedback, rehearsals, and feedback. Using each of these cycles of enactment and feedback to test and revise conjectures about the types of feedback that foster PTs' learning provided me, as the instructor, the opportunity to analyze the patterns of thinking and enactment that emerge in order to construct new hypotheses about how PTs learn from practice and through feedback.

Broader Program and Course Context

The teacher preparation program at the large midwestern university where the study took place included two courses devoted to teaching mathematics content and methods, both of which I teach. All students in the Elementary Teacher Preparation Program take one during their senior year and another during a year-long internship that takes place after they graduate. This study focused on those enrolled in the methods courses during their senior year, on the PTs enrolled in the two sections of the course I taught in the Fall 2012. All of the PTs had previously taken methods courses with a science and social studies content focus, and all PTs were concurrently enrolled in an ELA methods course. All participants had written lesson plans in other content areas before enrolling in the course and about half of the PTs had taught a full class lesson in their placement classroom prior to our first class meeting. In addition to 15 hours of elementary classroom time, PTs were required to spend 2 hours every week in a local elementary school field placement. The course was designed to have PTs investigate the mathematical thinking and understandings of students using Cognitively Guided Instruction methodologies (Carpenter, Fenema, Franke, Levi, & Empson, 1999); develop mathematical reasoning through problem solving using the mathematics task framework (Smith & Stein, 1998); and engage in systematic planning, studying, and improving mathematics lessons using the *5 Practices* framework (Smith & Stein, 2011).

I incorporated Smith and Stein's (2011) 5 Practices framework to help PTs develop a meaningful lesson goal using a mathematics task with high cognitive demand. The five practices consist of (1) anticipating student solutions, (2) monitoring student work, (3) selecting particular student strategies to discuss as a class, (4) sequencing these strategies to build an effective discussion, and (5) connecting the student strategies throughout the discussion. This framework was designed to support the learning of complex knowledge and skills through social interaction (Lave & Wenger, 1991) where students retain authorship of their ideas and are held accountable for reasoning about key mathematical concepts.

Teacher preparation courses widely use *Thinking Through a Lesson Protocol* (TTLP) as an example of a detailed lesson planning protocol (Smith, Bill, & Hughes, 2008). The TTLP divided the planning into three parts: selecting and setting up a mathematical task, supporting students' exploration of the task, and sharing and discussing the task. A feature of the TTLP is that each section had extensive lists of questions PTs should ask themselves as they plan lessons to help PTs consider aspects of planning lessons that will support their consideration of student thinking. This helps to focus PTs on the interactions that will occur in the lesson. This helps PTs deepen their mental representations of the lesson, to which Berliner (1988) referred, and to effectively plan how to respond to learners.

Mathematics Methods Course

I brought 11 years' experience teaching mathematics methods courses at the university level to the course and this research project. This included piloting some practice-based methods and materials in a methods course the semester previous to this study. In DBR it was crucial for the research, design, and implementation to be well aligned. Given my dual roles as MTE and researcher, this was challenging. Studying my own teaching raised the possibility that the data and decisions I would make in my role as a researcher would not support my actions as an instructor. Nonetheless, I strived to select the study design based on the data rather than my intuition as an instructor.

The study was a design experiment in two senior-level mathematics methods courses with approximately 25 PTs in each. Students had a 2-hour per week field placement in an elementary mathematics classroom and spent 15 three-hour class periods in the university-based classroom. The instruction, planning materials, revisions, and instructor journal were available as data sources. All student work including assignments, notebooks, and performance videos, served as sources of data as well.

I conceptualized teaching as a complex, interactive activity where the responsibility of the instructor—both my own in my methods course and the PTs in their field placements—was to create opportunities for students to develop their own understanding about content. I believed the mechanisms of learning combined my curriculum, planning, pedagogy, reflections, and revisions as well as PTs' engagement with the material. I designed my elementary mathematics methods course by drawing on my understanding of the high-leverage practice literature (e.g., Ball et al., 2009; Lampert, 2010; Lampert et al., 2010; Sleep et al., 2007). Based on this literature and my observations of practicing and prospective teachers in the field, I chose seven HLPs that are ubiquitous in teaching to study.

The larger grain size HLPs (Ball et al., 2009; Lampert, 2010; Lampert et al., 2010; Sleep et al., 2007) I focused on were:

a) Designing meaningful, measurable, mathematically specific goals and choosing tasks of high cognitive demand. This provided opportunities for PTs to grapple with the content imbedded in the goal (e.g., Hiebert & Grouws, 2007; Joyce & Hartoounian, 1964; Smith & Stein, 1998, 2011; Stigler & Thompson, 2009).
b) Eliciting student thinking about the mathematics embedded in a learning goal (e.g., Grossman et al., 2009; Smith & Stein, 2011; Stein, Engle, Smith, & Hughes, 2008).
c) Assessing student thinking for understanding of the mathematical underpinnings (e.g., Ball et al., 2009; Smith & Stein, 2011).

d) Responding to student thinking in ways that help students to articulate their ideas and move them toward the mathematical goals of the lesson (e.g., Ball et al., 2009; Sleep, 2009; Stein & Smith, 2011).

I provided opportunities for PTs to rehearse these practices and designed experiences to help them understand how effective teachers' practices in these areas result in student learning.

The methods course incorporated a cycle of planning, rehearsals, feedback, and re-teaching to support PTs in learning to make instructional decisions in the midst of the complex activity of teaching. My classroom observations had also indicated that the ability to "read the room," or notice students' levels of engagement often contributed to many PTs difficulties. I wanted to support PTs' ability to notice what was happening and equip them to make informed decisions in the midst of teaching.

Hiebert and colleagues asserted that much of K-6 teachers' intellectual work, which includes specifying learning goals, anticipating student responses that demonstrate acquisition of the learning goals, and evaluating lessons in order to map the evidence of student learning, occurs outside the classroom (Hiebert et al., 2007; Hiebert, Morris, & Glass, 2003). I designed many activities in my methods class to support PTs as they developed their ability to plan meaningful lessons and identify aspects of planning that influenced what happened in the classroom.

In the context of this course, a rehearsal consisted of PTs practicing to lead a mathematical discussion that met predetermined lesson goals in the university classroom. During each rehearsal, one PT took the role of a teacher. The remaining participants had two roles, as elementary students at a particular grade level and as observers of the lesson with the intention of providing feedback to the PT presenting. I could interrupt the rehearsal with a question or redirection, but the PTs provided feedback only at the conclusion of the presentation. The rehearsal was carried out either in a full class setting or in small groups. The content of the rehearsal was based on a particular mathematical task or activity that had already been explored through a mathematics homework assignment as well as the presentation of a sample lesson that I modeled or via a written or video recording of a practicing teacher. The sample lesson was presented and then discussed in order to distill it into particular features central to the activity, such as launching the task, soliciting student strategies, or connecting student strategies.

The rehearsals provided opportunities for PTs to become familiar with routines that frequently occur in mathematics classrooms, such as launching or introducing a task, monitoring student work on that task, selecting and sequencing student responses to share in a discussion, and making connections across these solutions in order to move the class toward reaching the mathematical goals of the lesson. Rehearsals also ensured that PTs would experience the in-the-moment decision-making that is ubiquitous in teaching. Throughout the course, the rehearsals were focused on two main objectives: teaching PTs to hold a discussion in a mathematics class and teaching PTs how to teach a lesson that would have a high probability of achieving the lesson goals. The first rehearsal in the course helped develop my PTs' ability to establish and maintain a classroom presence, but thereafter rehearsals engaged the PTs in facilitating elementary mathematics discussions with an individual, with a small group of their peers role-playing as elementary students, or with an entire class of their peers role-playing as elementary students.

These rehearsals were different from microteaching in several ways. One role of the TE was to act as an elementary student responding to the PT's lesson with the characteristics and knowledge of an elementary student. A second role of the TE was to act as a coach for the rehearsals. The TE stopped and started the rehearsal in order to interject questions of the PT leading the lesson or to hold a brief discussion with the entire group's participants. The other PTs played a role of participant-observer—participating as they imagine elementary students would and observing the lesson in order to provide specific feedback to the PT. As the course progressed, PTs were expected to identify for themselves the prior knowledge and limits on knowledge typical of a particular grade level, depending on the grade level of the task they were implementing. Participant-observers then used that information to decide how to respond as students during the lesson.

PTs were not working on different skills each week, but rather on developing more nuanced notions about what it meant to elicit, assess, and respond to student thinking in ways that opened up a conversation and moved students toward understanding the goal of the activity. I did not provide a list of competencies that students needed to demonstrate proficiency; rather, I used performance assessments to assess the quality of teaching with respect to the main course goals. Skills were not isolated and taught individually. Rather, PTs increased their ability to notice the nuances in teaching over the course of the semester, whether through practice or my feedback or modeling. PTs' skills improved collectively as they worked toward holding a meaningful discussion in a mathematics classroom.

I sought to model the practices I wanted PTs to learn while I taught the methods class and followed each teaching activity with explicit discussions about the instructional decisions I made before and during teaching. These discussions focused on four main ideas: how PTs experienced the activity as learners, what PTs noticed about my teaching that had an impact on their learning, what they believed motivated my instructional decisions, and what planning they imagined I had undertaken in order to lead the activity. I provided PTs with my lesson plans for each of our class sessions in advance of the class. After teaching the lesson, I modified future lesson plans based on our progress toward the goals of the course. PTs could access these lesson plans, and the lesson plans were discussed frequently during class sessions.

Participants

The sample for this study consisted of 49 prospective elementary teachers from two sections of the senior-level mathematics methods course of the teacher education program. Participants were enrolled during the Spring semester of 2013. I invited PTs from both of my course sections to participate and 49 out of 51 students enrolled in the two sections consented. There were 46 female PTs and three male PTs. Most of them were between 21 and 23 years of age, but one of the men was 36 years old and pursuing his second career as an elementary teacher. Almost half had specializations: six specialized in mathematics, four in special education, and 12 were members of an urban education cohort program.

Data Sources

The primary data for this study came from the 147 lesson plans PTs created through the regular enactment of the course using the LMT described in the previous article (Chapter 2) of this dissertation study. PTs taught these lessons in the university classroom. PTs' lesson plans from week 5, 9, and 13 of the course served as the data set for this study, one for each PT. I chose these weeks to get a sense of how the PTs' planning practices developed over time. Week five served as a baseline, since it was the first lesson created after the introduction of the LMT, week 9 was midway between weeks 5 and 13, and the plans from week 13 were the last plans collected during the course. I collected the final lesson plan each week along with each revision and used these as the data set for this analysis. In some cases, PTs lesson plans went through two revisions. This usually occurred when the original lesson plans lacked specificity.

Data Analysis

For the analysis, I used a modified version of theoretical sampling (Corbin & Strauss, 2008) to characterize the feedback provided to PTs on their written lesson plans. I first compiled a spreadsheet that included every comment I wrote on a lesson plan, original and revised. From this list, I looked for themes in the purposes of the feedback I had provided and then created codes to reflect the categorization (see Table 2).

In developing the codes for types of feedback, I drew on the literature about feedback from K-12 and HE regarding the types of feedback that was both considered effective and ineffective and performed an initial analysis (Glaser & Strauss, 1967) from a sample of 12 LMTs, four from each sample period. I began by generating initial category descriptions for the

Table 2

Categorizations of Feedback

Description of the category	**Code**
Format and grammar	FG
Non-specific praise	NSP
Specific reinforcing praise	SRP
Use of the tool	TOOL
Identifying opportunities to confront possible student conceptions of the mathematics	ISMCONC
Specificity of core mathematical ideas	SPCM
Specificity of the connections across core mathematical ideas	SPCON
Specificity about how the discussion of the task is connected to the lesson goals	SPDLG
Specificity of the expected mathematical learning/thinking	SPEXML
Specificity of potentially problematic mathematical language	SPML
Specificity about how the task/activity develops the mathematical ideas	SPTM
Non-mathematical specificity	NMSP

feedback provided such as requests for greater specificity, specific reinforcing praise, non-specific praise, and format and grammar. I found that “requests for greater specificity” was too broad a category to describe the type of feedback provided, so I further deconstructed this category by examining the topic of the request. For example, I coded requests for greater specificity about how the discussion of the task connected to the lesson goals as different from requests for more specificity about how PTs would confront possible student misconceptions. Through this process, I realized I also needed to capture descriptions of the PTs’ lesson maps across revisions, so I included these in my research notes (Glaser 1978). I used the rubric provided to PTs to self-assess their work (see Appendix B) to rate the original and final LMTs using the scale described.

Types of feedback provided. Most of the codes were instances where I prompted the PTs to be more specific in some aspect of their lesson planning. The majority of the feedback was directive—meant to highlight an area in need of improvement or direct the PT to correct an error. Facilitative feedback often took the form of a question, which I noted in the descriptions below as well as in the findings section. I summarized the codes for the categorization of the feedback in Table 2 and I have provided more detail in the following sections.

Format and grammar (FG). Occasionally PTs would write in incomplete sentences or unclear language. When it was not clear what they meant or the structure of the sentences was confusing, I coded these as FG. An example of this was, “I’m not sure what you mean here. Could you put it in sentence form so I have a better idea what you are thinking?”

Praise (NSP, SRP). There were two types of praise offered: non-specific praise and specific praise that reinforced PTs’ ideas. Examples of nonspecific praise included comments such as “yes,” “OK,” or “good.” Other examples of praise were more focused. For example, "You have captured what the student is doing and how the student is thinking," provided specific reinforcing praise.

Feedback about the use of the tool (TOOL). This code was used to provide clarity about what to put in a particular section of the tool. An example of this from the QTEST was, "This is really the launch of your task. This section should have questions about what and how the student was thinking to make it available to the whole class." This category of feedback provided direction about what the section on the LMT should include and how it was different from the launch of the activity.

Identifying opportunities to confront student conceptions of the math (ISMCONC). This code addressed instances where I prompted a PT to be more explicit about how to use the anticipated prior knowledge and thinking to move students towards the mathematical point of the lesson. In the student *strategy* section of the LMT, many PTs included solutions that reflected incorrect or incomplete student prior knowledge, but then did not show how they would leverage these ideas in the discussion following the presentation of the student strategy. As an example, a PT planned a lesson to help students use what they knew about the area of a rectangle to find the area of a parallelogram. One expected solution showed students using the length of the shorter side of the parallelogram as they would the width in a rectangle to calculate area. The PT wrote out this direction: "Identify the length and width on a parallelogram." I thought that the PT had established an opportunity to unpack the difference in calculating the area of a rectangle from the area of a parallelogram, but that having students identify the length and width of the parallelogram would not address that difference. Therefore, I wrote, "You seem to be overlooking why the student might be confusing the width and the height. Perhaps a question about how the width of the rectangle is similar and different from the height of a parallelogram would help address that misunderstanding."

Specifying core mathematical ideas (SPCM). Feedback categorized with this code always included a mathematical term *and* the words "ideas," "key," "conceptually," or "core." From the goals section of an LMT, "be more specific about how you are working on the *ideas* related to measurement" was representative of this code where *measurement* was the mathematical term used in conjunction with *ideas*.

Specifying the connections across core mathematical ideas (SPCON). Some of the feedback encouraged PTs to make connections between what they had already written and how they could build stronger connections to the most important mathematical ideas of the lesson. This type of feedback generally included the words *why* and/or *goal.* As an example, in the *key ideas* section of the LMT, these questions, "*Why* is this particular method important to reaching your *goals*? What does it provide as a 'way in' to talk about your *goals*?" were both coded as SPCON. From the QTEST section of the LMT, where PTs were asked to write out some questions to elicit student thinking, "*Why* is this important mathematically?" received the same code since the feedback prompted PTs to think about how to connect the student thinking back to the key mathematical ideas of the lesson.

Specifying how the discussion of the task is connected to the lesson goals (SPDLG). This type of feedback was given only on the key idea and connections sections of the LMT. This feedback prompted PTs to make explicit connections to the learning goals. For example, one of the goals on an LMT was to have students understand area through visualizing it as *tilin*g or *covering.* I coded this feedback, "Can you connect Mandy's strategy to tiling and covering?" as SPDLG.

Specify the expected mathematical learning/thinking (SPEXML). When I prompted PTs to make the expected student solutions more mathematically specific, I used this code. For example, “How does Sammy do this? Does he use a formula? Add the sides? Double top and bottom and then add?” These prompts requested information about the process students used, rather than the key ideas that undergirded the strategy.

Specifying potentially problematic mathematical language (SPML). This code was used when the feedback drew on issues specifically related to the use of mathematical language or terminology. For example, “Do you think the language here is accessible to your students?”

Specifying how the task/activity develops the mathematical ideas (SPTM). This code was used to capture feedback that attended specifically to the mathematical affordances of the task. For example, in an LMT on clock time the PT described a task where students were telling time using an analog clock accurate to 15 minutes. In the *strategy* section of the LMT one of the solutions used language that revealed the student was looking only at the hour hand. The feedback stated, “It is unclear if you are using a one-handed clock, a two-handed clock, or both. How does the use of one or the other of these types of clocks help students get to the ideas you are working on?” This feedback signaled to the PT to be more specific about how the use of the different types of clocks related to the big ideas about measurement involved in telling time.

Non-mathematical specificity (NMSP). Feedback that was coded as requests for specificity, which were not mathematical in nature, included requests for *what* and *how*. For example, on the LMT section for the student strategy, feedback such as, “Can you be more specific about what the student actually does—think about what and how.” When these instances were mathematical in nature, they were coded with one of the mathematically specific codes described above.

Uptake of feedback. This phrase, “uptake of feedback,” refers to how the PTs used the feedback to inform their planning practices. To examine the second research question about the types of feedback PTs used to change their planning practice, I drew on the literature about feedback in HE, and analyzed the uptake of feedback with the expectation that PTs would use the feedback to improve their work. I again used a modified version of theoretical sampling (Corbin & Strauss, 2008) to characterize the uptake of the feedback provided to PTs on their written lesson plans. I developed a three-point scale to document how closely PTs took up feedback concerning specificity of the mathematical idea and how well the revisions aligned to the feedback given. I then categorized the changes in relation to the feedback using the three-point scale—full uptake, partial uptake, and no uptake of the feedback. Upon completion of this analysis I worked to ensure the key characteristics of typical LMTs had been captured so that I was sure that the range of practices was "saturated" (Corbin and Strauss, 2008, p.14).

As I analyzed the kinds of feedback PTs took up, I noticed that when PTs revised twice, the first revisions often showed major revisions to the *strategy* section of the LMT, and then the second revision PTs moved on to revise the *questions* and *key ideas* in that order. I analyzed the lesson plans for instances of PTs learning from their own planning work as well as how they used the feedback in one section to show changes in another section of the lesson plan. In determining whether and how PTs used the feedback, I created a table highlighting changes in the wording of each section from the original to the revised LMT and compared the changes to the feedback provided. I then wrote descriptions of the changes in each of these sections with a focus on the match between the purpose of the feedback and the changes in wording. Determining how to code the uptake of feedback was more complicated than the previous coding of ideas since evidence of the uptake of feedback sometimes occurred in a second revision, was addressed in other sections of the LMT, or was no longer appropriate once the PT made a different choice in their lesson planning. I ended up looking across the entire LMT to find evidence of a PT taking up feedback and if the feedback was addressed elsewhere I considered that she or he had taken it up. I also looked across the entire set of revisions. In the findings section, I break out LMTs requiring a second revision and compare the uptake of feedback from the original through the second revision.

Full uptake. When the PT was able to take the ideas embedded in the feedback and make changes to their LMT that captured the entirety of the main purpose of the feedback, I used the code FUT. As an example, on an original LMT in week 9 under the goal section, a PT wrote, “Students will understand that there are different strategies to complete division measurement word problems.” The following feedback was provided: “Do you want students to know the different strategies or to make connections between the strategies? What is the focus of your lesson? All strategies? Only a few? Be specific.” I coded the original feedback itself with four separate codes requesting more 1) SPDLG—students knowing the strategies vs. making connections; 2) NMSP—identifying the focus of the lesson; 3) SPCM—determining the number of strategies on which to focus; and 4) NMSP—the specificity of the written goal. For example, one PT changed the goal section on the revised LMT to state, “Students will begin to build connections between various counting and grouping strategies to create some meaning for what “division” might mean.” I coded this as four instances of full uptake of the feedback since the PT was able to clarify that she did not want to focus on students knowing the strategies, but on making connections; this also clarified the purpose of the lesson; the PT specified the particular strategies, counting and grouping, that she intended to focus on; and it was written specifically.

Partial uptake. The code PUT was used when the PT partially took up the feedback. It indicated an intention to use the ideas embedded in the feedback and make changes to their LMT that captured the main purpose of the feedback, but for a variety of possible reasons: a) disagreement with the feedback; b) confusion about what was meant; c) inability to do what I was asking; or d) the choice to take up some features but not others. As an example, in a week 5 map, a PT wrote in the QTEST section, “Why are your answers different?” The PT seemed to intend to prompt students to compare two solution strategies for a task about area. The first strategy showed a student incorrectly using perimeter ideas; the second strategy showed a student using area ideas. I provided the following feedback for this section: “What if students can’t answer this question right away? What else might you ask to help them get there?” In revising, the PT eliminated the original question, “Why are your answers different?” in favor of questions to help individual students talk through their thinking. The idea of comparing the two strategies was absent from the first revision to the LMT, but the questions the PT proposed could help students to see more clearly how the thinking behind the two strategies was different. On this basis, I coded this as PUT, which called for a second revision. The feedback on the first revision returned to the comparison of strategies contained in the original LMT: “What about some questions that link, connect, or contrast what Sammy did with what Jessie did? Could that help you make sure the difference between A (area) & P (perimeter) stands out?” In the second revision of the LMT the PT included the question, “How is this different from perimeter in Sammy’s solution?” The feedback from the first revision was then coded as full uptake.

No uptake. This meant that there was no evidence on any part of the LMT where the PT responded to the feedback I provided by making a change to their lesson plan.

Characteristics of feedback taken up and not taken up. Using the categorizations outlined above regarding the types of feedback provided as well as the codes for how PTs made revisions to their LMTs by taking up the ideas embedded in the feedback, I created two tables. In the first table I collected all of my feedback that PTs used to change their lesson plans. In the second table, I recorded the feedback that PTs did not use to make changes in their lessons. I analyzed each of these to determine if there were particular formats or content of the feedback that PTs used and did not use to make revisions. I organized the findings using the research questions.

Findings

Research question 1: What types of feedback did the teacher educator provide PTs on their lesson planning practices?

Each of the categories of feedback appeared in every section of the LMT, but not necessarily in each sample period. In week 5, I provided a more detailed description of each portion of the initial LMTs for the sample period, since this sample of LMTs provided the baseline for PTs' lesson writing practice. I provided a very detailed description of the characteristics of the week 13 LMTs since these were the final LMTs collected. I highlighted the types of feedback provided by using a few typical, but salient examples and documented the shifts in language and specificity PTs made through the feedback and revision process. In weeks 9 and 13, I provided descriptions and examples that highlight how the nature of feedback provided shifted over time in response to the work PTs submitted.

Week 5 LMT cycle. This sample of LMTs serves as the baseline for understanding how PTs used their knowledge of mathematics, students, and lesson planning to plan lessons to reach a mathematical goal using the LMT. However, 11.1% of the feedback provided in this sample period focused on clarifying what the PTs should record in each section of the LMT, because they had some confusion. An example of feedback of this type was: "In this section you need to elicit students' ideas about what they did and what they were thinking in order to share the ideas with the class. Your questions in this section should reflect that." Week 5 is the only sample period in which the code TOOL appeared. Thus, after handing back the LMTs with feedback comments, I reviewed the purposes of each section of the LMT in week 6 and clarified many questions about the purpose for each section. I also handed out a rubric for PTs to assess their own LMTs and directed them to two completed LMTs on the course website for reference. PTs handed in the revised LMTs over a period of 2 weeks. A lack of mathematical specificity in the goal, only one anticipated student solution, or questions that did not map onto the student strategies anticipated prompted me to require that 38% of the LMTs revise twice.

Goals. On the original LMTs, PTs wrote goals that lacked both measurability and specificity. Most goals did not highlight the most important mathematical ideas embedded in the tasks. An example of a goal on an original LMT was, "telling time using an analog clock, based on where the hour hand is, students can tell what minute it is." In this case, the PT did not connect telling time to measurement in meaningful ways and essentially interpreted the task and described an inexplicit process a student might use to tell time.

The feedback I provided prompted the PT to be more specific about the key mathematical ideas, to consider the expected mathematical learning, and to specify how the task might develop the mathematical ideas. I provided some suggestions about ways to think about the measurement ideas imbedded in the task with the following feedback:

> You are missing the key ideas related to measurement. How will students tell time? To what level of accuracy will they be expected to tell time? Are you looking for an estimate? One key idea is that the hour hand moves 5 minute "marks" every 12 minutes, but the minute hand moves one "mark" for every minute. How will you help students to see this measurement idea?

The PT made the following revisions to her goals:

> Students will be able to draw a representation of time accurate to the minute when given a digital representation of time and either a clock with the hour hand only, or a blank clock face. Students will be able to describe how the position of the hour hand is related to the position of the minute hand.

The PT addressed the issue of accuracy and became more specific about the actual task—including using a clock face with one hand as well as one with two hands instead and created a statement that made visible the key measurement ideas.

I gave every type of feedback on the goals section of the original week 5 LMTs except prompts asking PTs to specify how the discussion of the task was connected to the lesson goals. Feedback on the goals section comprised 25% of the feedback I provided on the LMT in week 5. Revised LMTs, after a cycle of feedback and revision, showed an increase in PTs' ability to write the goals in mathematically specific and measurable ways when incorporating changes in their lesson plans based on feedback provided.

Anticipated student strategies. The PTs generally incorporated one correct solution, which they frequently described using process-oriented language, and included one or more incorrect solutions or solutions with misconceptions embedded. Most solutions described what the PTs expected students to do in very general ways, such as, "students measure a table length, book length, and a pencil length using paper clips and pencils." This lesson activity focused on using non-standard units to measure the length of a given object. The PT's strategy gives little indication that she has anticipated the students' conceptions of measuring, conceptions of length as an attribute of the objects used, or their conception of the unit as having the attribute of length. Many solutions lacked the specificity necessary to really understand student thinking.

The feedback I provided was:

> Think about the different aspects of measurement that students need to understand in order to measure. How do students understand what it means to measure? How do they choose the attribute? How do they make sense of the attributes of the unit, and do they understand what it means to compare units to the length of the objects they are to measure? Be explicit about what and how students to [sic] this.

The feedback drew attention to the key mathematical ideas related to measurement—in particular, measurement as a comparison of an attribute of an object to units with the same attributes. The feedback also focused the PT's attention on the expected student thinking and current understanding and prompted the PT to be explicit in describing what and how she anticipates students thinking about measurement. The PT changed the student strategy section as follows:

> Student measures by placing as many "unit" size pieces along the attribute of the object they are measuring to determine the length of the object. Student iterated one object repeatedly to measure a table length, book length, and pencil length. Some students may leave gaps, not line up the units in a straight line, or overlaps the units demonstrating a lack of understanding of what it means to measure.

This strategy suggested that the PT had considered what it meant to measure and was thoughtful about how insufficient understanding might affect student work and thinking in the activity. This section earned the highest average on the original week 5 LMTs and also showed improvement with revisions.

Questions to elicit student thinking. This section of the LMT seemed to be confusing to half (~52%) of the PTs in week 5. PTs provided only 1-3 questions, and generally included a big question that drove straight for the goal with little or no discussion rather than unpacking student thinking. As an example, in a kindergarten activity designed to help students make comparisons using the terms *longer, shorter,* and *same*, the PT posed these questions: “What is longer, shorter, same? If something is bigger does it make it longer?” The PT started by asking for definitions for the terms and then followed this up with a question that underscored a common misconception. This PT did not provide questions that addressed how students were thinking about objects, attributes, length, or to how students decided what made an object longer or shorter than another. There were very limited opportunities to explore student thinking. The PT revised this LMT twice. The initial feedback provided was:

> How do these questions relate to the strategies? These questions should focus on eliciting what a student was thinking and how the student performed the task. Are you trying to tease out the difference between bigger vs. longer? Why would this be important?

The feedback spoke to the purpose of this section on the tool as well as the connections the PT made to the expected student thinking and the key ideas of measurement. On the first revision, the PT changed this section to read, “What part of each object are you thinking about calling length?”—an initial attempt to speak to the idea of length as an attribute of an object. She also wrote, “how did you decide which was longer or shorter?” which was likely to elicit students’ thinking about how they made the comparison. The feedback on the revision said:

> Could you ask questions that help the student see there are different ways to look at an object and therefore different attributes to consider? Look at the KIDLG [Key Ideas from the Discussion and Links to Goals]. Could students have done this in different ways? Could they get different answers?

This feedback prompted the PT to again consider how the students might think about attributes, length, and measuring. The PT was also referred to the KIDLG section of her lesson plan where she wrote:

> when you are measuring, you must specify the attribute you are measuring. Are all of the aspects of the objects the same size? How are they different? So, when you measure an object what is important to say about your measurement so that others understand what you are describing as the length?

The KIDLG section of the LMT showed that the PT had thought about the importance of selecting an attribute to measure. Prompting the PT to look at this helped the PT to revise the QTEST section in this way:

> How did you decide that ___ was longer? What if you positioned them differently and decided to measure the height of the book? Is the book still longer than the pencil? So, when you say the book is longer than the pencil, can you be more precise? What aspect of the book is longer? Why is it important to be so specific?

In this final revision to the QTEST, the PT had developed questions that were likely to elicit student thinking about the key ideas of measurement and how students demonstrated their understanding when measuring. The PT provided a "what if" question that served to challenge students to be more articulate and precise about their understanding of measurement as well as the ideas of longer and shorter.

Key ideas and links to goals. In the *key ideas and links to goals* section, PTs generally wrote a statement or two about the key idea but used very few questions and did not make explicit connections with the anticipated student strategies or the responses to questions posed. For example, on an LMT designed to help teach students how to tell time accurate to 15 minutes, a PT wrote this in the *key ideas* section: “look at both hands to tell time; which hand to look at first? (hour hand).” In this example, the PT focused on the *process* of telling time, but not the big ideas of *measurement*. In a few cases PTs used this section to restate their goals. Although PTs made many changes to this section of the LMT, writing out *connections* to the strategies, questions, and key ideas proved challenging. I provided feedback to nearly all the LMTs in week 5 that challenged them to write some questions to underscore the key mathematical ideas and prompt students to make connections between strategies.

Connecting comments and questions. Nearly one quarter of PTs left blank the final section of the LMT, connecting comments and questions. Those who did write in this section wrote general statements, sometimes an exact restatement of the goals. PTs frequently failed to reference the student strategies or student thinking in this section. Almost one third, 32%, of the PTs included an exit ticket or some formative assessment to collect information about student learning. The revisions to this section provided improvements, but detail remained scant. Little or no feedback was provided when this section was left blank or if the PT merely restated the goals and as a consequence, only 10% of all provided feedback applied to this section. More than half of this feedback (~59%) was coded as TOOL, which was the default code when students left it blank or did not use it to summarize the lesson in some way.

Praise. Only 6% of the feedback on the original LMTs was in the praise category with 4% coded as specific praise to reinforce PTs’ thinking. On the final revision of the week 5 LMTs, 64.5% of the feedback was praise, of which 68.9% consisted of specific praise to reinforce PTs’ ideas. Specific praise in week 5 was often paired with additional feedback. For example, in a week 5 revision LMT, the feedback given was, “These are clearly written. Now, think about what in particular you want to build up to as a big math idea students will take away—what math ideas do these strategies represent?" In this example, the praise is directed to the PTs' ability to describe the anticipated student strategies explicitly. What was left unaddressed was how the discussion would include this understanding, what key ideas the PT planned to connect, and how.

Summary of week 5 LMT feedback. Across all sections of the week 5 LMTs, 76.9% of all feedback provided addressed issues of mathematical specificity. Each category of feedback appeared in each of the sections of the LMT multiple times. Feedback for the use of the tool only appeared in this sample period and represented about 10% of all feedback provided. Nearly 25% of all feedback in week 5 spoke to the connections across sections, prompting PTs to connect student strategies and goals to the key mathematics, expected student work, and the lesson goals. Each of the sections showed increases in the explicitness of the PTs' thinking, in line with the feedback provided. PTs were able to make changes in the LMT that reflected the essence of the feedback provided.

Week 9. In week 9 PTs showed a marked improvement in their ability to be more mathematically explicit than in week 5. The LMTs overall were more detailed and provided more depth of content knowledge than the initial LMTs from week 5. There were half as many feedback comments on week 9 LMTs than week 5. One quarter of the LMTs in this sample period required two revisions.

Goals. PTs' goals in week 9 focused more explicitly on the main mathematical ideas of the tasks than in week 5, but the connections to students' new learning about the key ideas through work on the task was not explicit. PTs' lesson goals often lacked measurability and many PTs wrote the lesson goal without making explicit connections to what students already knew and how they would build on that knowledge through the task. 21% of the feedback I provided in week 9 was on this section of the LMT where I asked for revisions to make the goals measurable and make explicit the connections PTs wanted to highlight as a result of using the task.

As a typical example, on the original LMT a PT wrote, “Students will understand that there are different strategies to complete division measurement word problems.” This PT was able to identify the key idea that the task involved measurement division word problems. She realized that understanding the different strategies for solving them was at the heart of the task. But she did not indicate which strategies students would use to complete these tasks or make a case for why knowing the strategies would be a reasonable goal. The feedback provided was:

> Do you want students to know the different strategies or make connections between the strategies? What is the focus of your lesson? All strategies? Only a few? Be specific. Usually problems of this type are given before students have an algorithm for solving them. Think about the connections that can be made using skills students already possess.

The feedback prompted the PT to be more explicit about the lesson goals, to consider the ways she would leverage the task to highlight these goals, and to think about the knowledge and skills students brought to the work that could be used as a foundation. Feedback about the measurability of the goal was not provided. She revised the goal section to say, “Students will begin to build connections to various counting and grouping strategies to create some meaning for what ‘division’ might mean.” Although the goal still lacked measurability, it was mathematically specific and identified the prior knowledge and skills that the PT would leverage to understand the new content of division.

Anticipated student strategies. PTs provided 2-15 complete and correct solution strategies in week 9 as compared with one correct strategy and one or two strategies that incorporated student errors in week 5. Nearly a quarter (~23%) of the feedback provided addressed inconsistences in what students might be expected to know at a particular grade level or a mismatch between the expected student solutions and the expectations of the task. For example, in a first-grade activity where students created their own rulers and used them to measure objects, the student solutions provided only addressed how the students might make the rulers using ideas of measurement but did not include how students might use the ruler to measure.

Questions to elicit student thinking. I provided feedback to PTs who were still finding it difficult to formulate questions that would push students to make connections and explain their thinking. In week 9, nearly 28% of the feedback provided was on this section of the LMT. On average, PTs wrote eight questions to elicit student thinking in their original week 9 version of the LMT as compared with an average of four questions in week 5. Generally, the questions spoke both to what and how students solved the task with one or two questions about the meaning of the solution or connections to other strategies and key ideas. Nearly every LMT had at least one question that asked students to justify their answers. This suggests that feedback given in week 5 about writing questions to elicit the key mathematical ideas or to help students make connections was taken up by many PTs by week 9.

Although PTs wrote many more questions in this section, many questions lacked a connection to the big mathematical ideas and the purpose of the task. I provided feedback to help focus PTs' thinking on the purpose of the tasks and questions. As an example, "You are asking students to predict, so what is this idea of predicting getting at? That should help you with your goal and strategy. What do you want students to notice here?" I wanted the PT to focus on why they were having students predict where numbers would appear on a hundreds chart in an attempt to help the PT consider the predictable patterns that result from using a base-ten number system.

A feedback comment that came up on all of the LMTs that required two revisions was, "Once you spend more time thinking about the big ideas you want students to walk away with, this section will be clearer. Connect the strategies to the goals." This feedback highlighted the PTs struggle to write explicitly about the big mathematical ideas. When PTs did not have a clear understanding of the big mathematical ideas, the questions they wrote also lacked specificity.

Key ideas for discussion and links to goals. The feedback I provided on this section of the LMT was greatly reduced from week 5. In the feedback I provided in week 9 LMTs, 19% was found in the KIDLTG section. Although PTs demonstrated considerable growth in their ability to make connections, many PTs continued to struggle to focus this section on the key mathematical ideas and help students to make connections. My feedback in these instances was often more detailed in another section of the LMT but offered PTs suggestions on how to make improvements once the key ideas were solidified. As examples, "Again, clarity on your goals and strategies will help you with this section" and "Again, this goes beyond counting. Be specific about the connections you want students to make here. Highlight the mathematical ideas you want to come out with your questions."

Connecting comments and questions. PTs wrote many of these entries in the form of a question and then added a parenthetical note to themselves that said for example, "get this idea to come out." These ideas tied each aspect of the lesson together and served as formative assessment opportunities at the end of the lesson. PTs were able to see that more than just a summary statement, this presented an opportunity to bring everything that happened in the lesson to a culmination; most of the PTs put the responsibility on the students to talk about the key connections and learning. Some of the feedback I provided on this section of the LMT also underscored the struggles PTs experienced when they were still unclear about the key mathematical ideas and how these could be connected to move students towards the lesson goals. As examples of feedback in week 9 that attended to the lack of specificity of the goals, “The second part of this is the most important. Spend some time teasing that out more –how do the problems connect? What ideas are imbedded in certain solutions? and insert it into the other sections of the plan.” When PTs were not clear about the key mathematical ideas, the lack of specificity could be found in nearly every other section of their LMT.

Summary of week 9 LMT feedback. Feedback in the form of praise appeared primarily in the revised LMTs. Instances of nonspecific praise and specific praise were both present. Twice as many code instances prompted PTs to be more specific about the mathematics than prompted them to make connections to goals, strategies, or key mathematical ideas. Four times the amount of feedback attended to mathematically explicit concerns than to nonmathematical ideas. This suggests that feedback and instruction provided since the week 5 LMTs may have had a positive impact on the PTs’ planning practices. In general, in week 9, LMTs were much more detailed, specific about the mathematics, and more complete. No section on the LMT was left blank in week 9. PTs had more clarity about how to use the tool, as indicated by the absence of the code TOOL. The number and quality of the solution strategies and question showed marked improvement from the week 5 LMTs. Every section of the LMT showed increases in the quality of their work in comparison to the week 5 LMT.

Week 13. The LMTs in this sample period served as a summative assessment for PTs’ lesson writing skills. They prepared a lesson in week 13 that they would teach in their placement classrooms that would be videotaped for a performance assessment. Consequently, I provided very little formative feedback on these lessons. I asked PTs to turn in the LMTs after selecting the task, writing the goals for the lesson, and starting to anticipate student strategies. So, the feedback was limited to those sections and was given only if there was a major omission. The content focus of these lessons was often outside the realm of the content emphasis of the course since it was more closely matched to the K-6 school calendar.

Half of the feedback provided on these LMTs prompted PTs to spend more time elaborating possible student solutions. Since they had only just learned to incorporate student solutions, however, I treated this section as an early draft. The goals were concrete, focused on the key mathematical ideas, called on students' prior knowledge that they could leverage in the task, and were written in correct mathematical language. Some PTs wrote multiple correct strategies using explicit, mathematically correct language, while others focused on one or two strategies to find the correct answer and then wrote strategies using misconceptions or misunderstandings students might hold. There were a few feedback comments directed at awkward use of mathematics language or terminology that needed elaboration.

There were a few exceptions to the overall trends. One PT (who was placed in a self-contained special education classroom), chose a teacher-centered task at a procedural level. She did not take up feedback about revising or changing the task. A second PT wrote a lesson that was teacher-centered. The task he used was high cognitive demand, but the questions he wrote diminished the level of the task very quickly as they would lead students directly to an answer. This PT asked questions that did not require students to explain their thinking, but rather questions requiring a specific one word answer such as "yes," "no," or "four-tenths." On week 13 LMTs, 40% of the feedback was not taken up by PTs and these two cases accounted for 70% of that feedback.

With the exceptions of feedback on the *tool* and *format and grammar*, I used the categories of feedback provided in the data analysis section of this paper in every sample period on each section of the LMT. Praise generally related to revisions to the LMTs and there were examples of both specific and nonspecific praise in each sample period. The largest percentage of codes across all sample periods prompted PTs to be more mathematically explicit. Week 9 LMTs included a higher percentage of feedback comments about making connections between student conceptions, mathematics, and the discussion than in week 5. Twice as many prompts for PTs to specify their expectations around student thinking appeared in week 9 as did in week 5.

Research Question 2: What are the characteristics of the feedback that PTs are able to use to increase the specificity of planning?

Similar to what has been documented in the literature, I found that PTs used specific, open-ended, timely, and frequent feedback to improve their lesson planning records. This study also illuminated four additional characteristics, that were not exclusive of the previously mentioned characteristics, of feedback PTs were able to use: feedback in the form of a question; feedback that referenced a familiar resource; feedback that prompted the PTs to consider students; and feedback that made connections across the LMT. Each of these is described below with examples from the data.

Feedback in the form of a specific, open-ended question. Nearly half (~48%) of all of the feedback PTs used in the LMTs was posed in the form of a question. Examples were:

- Which mathematical patterns are you looking for in particular?
- Are you thinking about any particular characteristics?
- What are you hoping to get from this question?
- What will you do with [student] answer?

- If students estimated in different ways, then how can you pull those ways together to help support a meaningful goal?

Each of these examples were specific to the PTs' entry in the LMT and had a narrow focus, as indicated by these words or phrases: "particular," "this question," "those ways," and "her answer." The language usage in the questions put the responsibility on the PT to address the question.

The following feedback prompts also fall into this category, but lack the pronoun "you":

- How do the answers to these questions get to the ideas of area?
- Why is this important?
- Could they have done it different ways?
- Would other placements give different answers?

These are also all open-ended questions with a specific focus connected to the ideas PTs wrote about in their LMTs. These questions also give the PT the responsibility to address the question, but since there are several possible responses to the questions, the feedback elicited the PTs' work and thinking. It was possible that PTs responded to these questions because they felt that I would expect it in a methods course.

In each of these cases PTs revised their work and the revisions reflected a response to the question asked. For example, in a lesson designed to have students understand how to determine whether an object was longer or shorter than a given object, the PT listed one possible student solution: "Student puts two objects next to one another on the table and decides which is longer or shorter." She then followed up with this question: "How did you decide which was longer or shorter?" The feedback was "Could they have done it different ways?" The student revised her strategies thus:

> Student lays a pencil next to a book and says the book is longer than the pencil. Student holds two objects in the air and says one is longer or shorter than the other. Student stands the pencil and the eraser up on the table and says the eraser is shorter than the pencil.

Although the PT had not yet demonstrated a perspective about *how* the student might determine which is longer or shorter (i.e. one extends beyond the other), she did include *other ways* students could perform the task—laying them side-by-side, holding them in the air, and standing them up—which is what the feedback prompted her to do. This suggests feedback in the form of open-ended questions was effective in supporting PTs to add specificity to LMTs.

Feedback that referenced a familiar resource. Often in the feedback I provided a direct reference to something that was discussed in class or a section of text PTs had read. In this way, PTs were able to go to the source or their notes and make adjustments to their plans, based on the information in these other resources. Some examples of this feedback appear below, where "VDW" and "Van de Walle" referred to a text we used in class by Van de Walle, Karp, and Bay-Williams (2010). Van de Walle and colleagues provided four ways to estimate: using benchmarks, chunking, subdivisions, and iteration (p. 390). It also described a three-step process to measuring: choose the attribute, select the unit, and then compare the units (p. 370). Van de Walle, Karp, and Bay-Williams provided insight into how students learn to measure by making comparisons, using physical models of units, and using measuring instruments (p. 371-371). Here are three examples of feedback and the revision they prompted which suggest how PTs took up the language of Van de Walle et al. (2010) when revising their LMT:

1. Feedback: Think about the 4 ways to estimate according to Van de Walle. This should help you think through these strategies and make them more specific. What

would a student do? How would they do it?

Revision: Student measures by placing as many "unit" size pieces along the attribute of the object they are measuring. Student iterates one object repeatedly to measure a table length, book length, and pencil length using paper clips and pencils.

2. Feedback: How would the student measure using the string? Think of the different ways VDW (Van de Walle, 2010) talked about measuring.

   Revision: Student took string and repeatedly copied the length of the string to determine the length of the wall.

3. Feedback: Are there other ways students could measure and/or estimate? See VDW (Van de Walle, 2010).

   Revision: Estimate distances by using a copy of a standard unit of measure mentally and iterating it. Student thinks of the space as chunks that are equal to a given number of units. Student divides the distance to be measured in sections of a particular number of units. Student took string and repeatedly iterated the length of the string to determine the length of the wall.

PTs were able to refer to the text and find the aspects of estimating and measuring that were related to their particular task. Van de Walle et al. (2010) provided explicit examples of the trajectories students might follow when learning to measure and estimate. The text provided PTs a resource to consult to find the kind of information that could be included in the *strategy* section of the LMT. PTs used this type of feedback to envision how students might approach the tasks of estimation or measuring. Each of the examples above also prompted PTs to consider a student perspective and the feedback was often in two parts, as the examples above showed. One part was typically a question in the form of "how did" or "how would." The question was typically coupled with a reference to a familiar event, idea, or resource from class. The references occurred without the paired question very rarely. Thus, feedback that drew on a familiar resource, or directed PTs to a familiar reference, was helpful to PTs in adding mathematical strategies to their LMTs. All of the feedback also referenced the student: "What would a student do?" "How would the student measure…?" "Are there other ways students could measure…?" The next category of feedback highlights evidence suggesting this practice enhances feedback's effectiveness.

Feedback that prompts the PT to consider students. Making the connection to children or students rather than just to the mathematics seemed to provide PTs a way into the revisions. The examples below remind PTs that they are teaching *children* about mathematics and spending time considering how children might respond to particular questions. In the first example, the PT wrote in the first person, perhaps imagining herself enacting a teacher-centered lesson. First, the feedback reminder that she was eliciting ideas from *students* resulted in a shift to second person pronouns. Second, the PT shifted the language from process oriented to "where do I start and stop" to more open-ended questions that are likely to elicit student thinking.

Original entry:	Where do I start and stop? Where should I look to find the "measurement"? Why is it labeled that way?
Feedback:	Remember you are eliciting students' ideas about what and how they did what they did to share it with the class. Your questions should reflect that.
Revision:	How did you use the paper clips to measure the object? Could you fit more paper clips along the edge of the book? If I did the same thing would I get the same number of paper clips?

The example below also couples two ideas in the feedback—a prompt for additional questions and the request to consider what she expected a student to say in response. The PT generated four questions that focused on the student thinking and actions.

Original entry:	How did you find the area of each rectangle?
Feedback:	Any additional questions you might ask? Think about what you expect the students to say in response to your questions.
Revision:	What method did you use to find your answer? What are your units? How are you thinking about area, can you describe it? So, by adding the lengths of the sides why could this not be a measure of area?

These feedback and revision exchanges allowed the PTs to consider how thinking about students' perspectives might influence their planning. Perhaps focusing on the students as participants in the eventual lesson allowed PTs to step back and think about implementation as they planned.

Feedback that made connections across different portions of the LMT. There were 210 instances in which an entry in one section of an LMT directly referenced another section. For example, in Table 3, in the first row, third column, there is a reference to the KIDLG section of the LMT in the feedback provided on the GOAL section of the LMT. References such as these this made coding difficult since the reasons for revisions were not necessarily clear. It was necessary to look across sections to examine the feedback provided and compare the feedback to the revisions on all the sections on the LMT.

Table 3 provides one PT's written work for one strategy on a revised LMT, together with the feedback provided, the revisions made, and an additional round of feedback. The gray scale shading links the feedback from one section to revisions to the same section. The italicized words in the second column are the additions to the LMT after feedback. The italicized words in the third column are the elements of the feedback that the PT seemed to take up and use in her revisions. Feedback on the goal seemed to lead to changes to the goal, the QTEST, and the KIDLG sections in the second revision—the bold phrases highlighted in those cells on Table 3. An LMT wrote about how a student used a *covering strategy* showed up in the other three sections, and I argue the feedback provided in the goal section prompted this.

In addition to the direct references, there were many subtle or indirect references made across the sections of the LMTs through feedback. This also made it difficult to determine why a PT made a particular change in a specific section of the LMT. Since each of the sections of the

Table 3

Excerpt From LMT Highlighting Cross Section Connections

1st revision	PT Writing	Feedback Provided
Goal	Distinguish the difference between perimeter and area	*Perhaps you could say what in particular you want them to distinguish like what you stated in KIDLG and in thinking about* ***what new ideas Jessie's solution offers in terms of the larger lesson goals***
Strategy	Jessie (using grid paper model) counts the amount of square units in the rectangle to come up with the area.	Good description, Specific. Easy to tell her method.
QTEST	Why did you decide to count the amount of squares in the rectangle?	*What are you hoping to get from this question? How would Jessie answer it? What will you do with her answer? What about some questions that link, connect, or contrast what Sammy did with what Jessie did? Could that help you make sure the difference between A & P stand out?*
KIDLG	Perimeter and area are not the same. Perimeter is the total distance outside of a 2D shape. To find the perimeter add up the dimensions of all four sides	Good. You are clearly talking about the difference between A&P. *And are you ready to define area here using Jessie's method?*
2nd Revision	PT Writing	Feedback Provided
Goal	*Area can be determined by finding the* ***number of square units that cover the 2D shape***	Excellent. You have broken the larger goal into pieces that fit better with the strategy under discussion.

Table 3 (cont'd)

Strategy	Jessie (using grid paper model) counts the amount of square units in the rectangle to come up with the area.	
QTEST	*How does counting all of the square units show the area of the shape?* *How are you thinking about area?* ***So, if you are thinking about using the units to cover the shape, what is area?*** *How is this different from perimeter from Sammy's solution?*	These questions are specific to the strategy used, make connections to the big ideas of the lesson, and connect the previous solution as well.
KIDLG	Perimeter and area are not the same. Perimeter is the total distance outside of a 2D shape. To find the perimeter add up the dimensions of all four sides. ***Area can be thought of as covering the shape with units.***	Good. You are clearly talking about the difference between A&P. Good way to link Jessie's method to the main goal of the lesson and in particular what her strategy contributes.

LMT were connected to the overall discussion PTs were planning, it makes sense that the feedback and revisions would be intertwined. It is not possible to talk about *questions to elicit student thinking* about the strategy if you don't actually reference the student *strategy*. PTs used feedback that drew on different sections and that prompted them to make connections across the LMT to make revisions.

The unclear role of praise. Praise was given more often on revisions than the original LMTs for every sample period. This included nonspecific and specific reinforcing praise. In week 5 on the original LMTs, 6% of all feedback instances were coded as praise, of which two-thirds was nonspecific praise. On the revised LMTs in week 5 there were two feedback comments giving praise for every one comment providing formative feedback distributed in a 1:2 ratio between nonspecific and specific reinforcing praise. In week 9 there was only one instance of nonspecific praise given on the original LMTs and no specific praise. On the revisions, however, there were equal amounts of nonspecific and specific reinforcing praise and the ratio of praise to formative feedback comments was 1:2. In week 13, 32% of feedback was coded as praise on the original LMT with a ratio for feedback coded as nonspecific to specific praise of 1:3. On the final LMTs 94% of feedback was coded as praise.

When specific praise was provided with no additional formative feedback to a given section, PTs made revisions 5.4% of the time and 6.3% of the time for weeks 5 and 9 respectively. For example, the *strategy* section of the original LMT in Table 3 remained unchanged and specific reinforcing praise was the only category of feedback offered. In week 13 the LMTs received feedback very early in the process and every instance of specific praise also included at least one formative feedback comment or question.

When praise was offered in conjunction with a request for more specificity, PTs revised their LMTs in line with the feedback provided. I am including a lengthy feedback and revision exchange with a PT planning a lesson for her first-grade students to create a ruler and measure objects. In Table 3, I have transcribed three sections—*Strategy*, QTEST, and KIDLG—of a week 9 revised LMT, together with the feedback, and the second revision of each section after feedback. Specific reinforcing praise coupled with a prompt for the PT to make some revisions to the LMT was given in every section on the LMT. Reinforcing praise alone did not produce changes in the LMT (for an example see Table 4). I do not have the data to make a claim about whether the PT would have made changes if the reinforcing praise was not provided; however, there is evidence to support the fact that in each of these couplings (similar to the examples provided in Table 3), PTs went on to revise the LMT in line with the suggested prompts.

The entries in Table 4 are *italicized* to highlight where the PT revised her plan in response to the feedback; the PT deleted the bold phrases and sentences upon revision. Under the *strategy* section of the LMT, "Student measures objects by counting the number of colored strips the object covers," the PT revised this to read, "Student measures objects by *comparing the* *length of an object's attribute* and then counting the number of whole colored *strips that correspond to the length*."

Table 4

Week 9 Revised and Second Revision LMT Sections to Highlight the Role of Praise

1st Revision	**PT Writing**	**Feedback Provided**
Strategy	Student glues strips (either vertically or horizontally) without spaces, overlaps and in a straight line; Student glues strips (either horizontally or vertically) with some spaces and overlaps or in a crooked line; *Student measures objects by counting the number of colored strips the object covers;* **Student measures using the middle of the ruler; Student measures by placing the end of the ruler at the end of the object.**	Good job! You have complete strategies that relate to the tool creation as well as some key measurement ideas. I want you to think about what each of these strategies might mean about student understanding. Maybe you should make a ruler and try and measure objects. This might help you be more specific about what in particular might come up as students use the collection of units to measure objects.
QTEST	Why did you decide to put your strips on that way? How will you use the tool to measure? Can you demonstrate your measuring process with this object? What do you mean when you say it is "4"? Can you use the tool to measure the same object in a different way?	Your questions about the tool creation are great. Maybe add some questions about how the design of the tool impacts measurement and how students use the tool to measure. Again, doing this task yourself—creating a ruler—might help you with this.
KIDLG	Why do we have different measurements for the same object? What kinds of things might give us different answers? How could we make sure everyone in the class got the same answer when we measured?	Good. So, you're clearly making the point about the differences in the design of the rulers. Also, you'll want to make sure that students connect previous ideas about what a "unit" is to the idea of the collection of units you have with a ruler.
2nd Revision	**PT Writing**	
Strategy	Student glues strips (either vertically or horizontally) without spaces, overlaps and in a straight line; Student glues strips (either horizontally or vertically) with some spaces and overlaps or in a crooked line; *Student measures objects by comparing the length of an object's attribute and then counting the number of whole colored strips that correspond to the length; Student counts the lines rather than spaces as units; Student measures by placing the end of the unit at the end of the object; Student places the object in the middle of a colored strip indicating he/she does not understand the idea of a unit.*	

Table 4 (cont'd)

2nd Revision	PT Writing
QTEST	Why did you decide to put your strips on that way? *(Holding up a ruler that has overlaps or gaps)* and *ask will this give me the same measurement as your ruler*? *Why or why not? What is important if we all want to get the same measurement when we measure something? How will you use the tool to measure?* Can you demonstrate your measuring process with this object? What do you mean when you say it is "4"? Can you use the tool to measure the same object in a different way? *(start at the beginning of a unit, but not necessarily the beginning of the ruler)*
KIDLG	Why do we have different measurements for the same object? What kinds of things might give us different answers? How could we make sure everyone in the class got the same answer when we measured? *What does a measurement of 5 mean? Are there different ways of getting a measurement of 5 using our rulers?*

The following feedback provided in the KIDLG section seemed to have had an impact on the strategy entry:

> Good. So, you're clearly making the point about the differences in the design of the rulers. Also, you'll want to make sure that students *connect previous ideas* about what a *"unit" is* to the idea of the *collection of units* you have with a ruler.

The feedback I provided that began with a statement of nonspecific praise followed by an instance of specific praise. The revised version was a more succinct and mathematically accurate statement of the process of measurement that connects students' previous experiences with measurement to the next experience of using a collection of units.

The feedback in both the *strategy* and QTEST sections about making a ruler and trying the task on her own was taken up by the PT, who brought in several homemade rulers before class and sat with her peers to figure out how students might conceptualize measuring with respect to the units on the ruler. The feedback from both the *strategy* and QTEST sections also began with a combination of nonspecific and specific praise: "Good job! You have complete strategies that relate to the tool creation as well as some key measurement ideas." And "Your questions about the tool creation are great." The specific praise across these two sections highlighted that the PT had strategies and questions that effectively included ideas related to the making of the ruler. The continued feedback prompts for more clarity on connections between the strategy and student thinking: "I want you to think about what each of these strategies might mean about student understanding*"* and this feedback from the QTEST section: "Maybe add some questions about how the design of the tool impacts measurement and how students use the tool to measure" seemed to influence the additions to the QTEST section as well as the revisions to the strategy section:

> QTEST additions*: (Holding up a ruler that has overlaps or gaps)* and *ask will this give me the same measurement as your ruler*? *Why or why not? What is important if we all want to get the same measurement when we measure something? How will you use the tool to measure?... (start at the beginning of a unit, but not necessarily the beginning of the ruler)*
>
> STRATEGY additions*: Student measures objects by comparing the length of an object's attribute and then counting the number of whole colored strips that correspond to the length; Student counts the lines rather than spaces as units; Student measures by placing the end of the unit at the end of the object; Student places the object in the middle of a colored strip indicating he/she does not understand the idea of a unit.*

Again, it was difficult to determine exactly how the feedback influenced PTs' revisions since it often bled across sections of the LMT, but these revisions appear to connected back to the ideas contained in the feedback comments.

Since I did not collect interview data, this paper cannot say if the praise itself (or lack of it) had an impact on the revisions, but evidence supported the claim that when praise was connected to an instance of formative feedback, the feedback influenced the revisions to the LMT.

The value of a second revision. PTs had some difficulty at the beginning of the course because they had little experience writing lesson plans for mathematics or using the LMT. They also were novices in the midst of learning the language of the practice of teaching. They were not yet accustomed to the nuances of writing goals and questions to elicit student thinking specific to mathematics. Up to this point PTs had never written a lesson for mathematics, and as such, week 5 represented their first attempt at taking on this aspect of practice. Each revision of the LMT showed gains in the use of mathematically specific language. The PTs wrote more with each revision as well, resulting in longer and more detailed LMTs. As was previously reported, the number of correct student strategies PTs incorporated into the LMT increased over the data collection period, demonstrating the PTs had increased their ability to solve tasks from different perspectives.

Some PTs required additional feedback and a little more time to revise their LMT. There were three LMTs from the week 5 sample and two LMTs from week 9 that required a second revision. Providing PTs with more feedback and additional time made it possible for PTs to incorporate more feedback from the original LMT. Nearly all the feedback that PTs partially took up in the first revision fully took it up after a second revision in week 9. In week 9 the PTs took up all feedback in the first revision.

In general, the PTs were able to use four types of feedback to revise their LMTs. Feedback in the form of a question seemed to encourage PTs to respond to the questions in their revisions, especially when the question asked them what they thought would improve the lesson.

Feedback that referenced a familiar resource also allowed PTs to make revisions that strengthened their use of mathematical terminology. These resources showed up in the revisions as a greater clarity to predict and describe student conceptions about mathematical ideas and responses to a task. PTs used feedback that helped them to consider students to elaborate their LMTs to include more detailed descriptions of what students might think and how this revealed student thinking. When feedback helped make connections across the LMT, the revision to the document gave more overall consistency to the mathematical connections and how those connections might come out in discussion. It was unclear what role praise played in the revisions of the LMTs since revisions were made in instances both where praise was given and withheld. Praise alone, however, seemed to limit PTs' revisions. Opportunities to revise LMTs multiple times with repeated feedback resulted in LMTs that were more explicit, detailed, and mathematically sound.

Research Question 3: How do PTs use the specific TE feedback to shift their language around practice?

PTs used the feedback provided to shift their language around the practice of planning mathematics lessons in the following ways: by taking phrases from the feedback to revise statements and questions in the LMT; by putting the answer to questions feedback raised into the LMT; and by looking up the references cited in the feedback and incorporating language and strategies from these resources into the LMT. I will describe these categories and include examples.

Taking phrases from the feedback to revise statements and questions in the LMT. Table 3 shows a direct connection between feedback on the QTEST section and the revised QTEST. The original question in the QTEST section, "Why did you decide to count the amount of squares in the rectangle?" was given the following feedback:

> What are you hoping to get from this question? How would Jessie answer it? What about some questions that link, connect, or contrast what Sammy did with what Jessie did? Could that help you make sure the difference between A & P stand out?

The first feedback question prompted the PT to consider whether the response to the question would help her move students towards the goal. Her original question was modified slightly to get her revised question: "How does counting all of the square units show the area of the shape?" She changes "Why did you decide to" to "how does"; the idea of counting the squares remains consistent since this is Jessie's strategy; and the PT changes "in the rectangle" to "the area of the shape." The revised question connected the strategy to area directly and reflected the suggested revisions from the feedback.

The second feedback question was a prompt for the PT to think like the imaginary student, Jessie. Notice the third question in the revised section used the phrase, "how are you thinking." It seems reasonable to assume that this phrase comes from the feedback "how would Jessie answer it." PTs incorporated feedback by borrowing phrases and ideas to revise their work.

Putting the answer to feedback questions into the LMT. Sometimes the feedback was in the form of a question that the PT thought about and then incorporated the answer into the LMT. Continuing to use the example provided in Table 3, the third feedback question, "What will you do with her answer?" was a prompt for the PT to consider how this would play out in the discussion—how she would use the answer to move students towards her goals. The answer to that question required the PT to first imagine the student response, which was likely something about "covering the" shape with unit squares. Note how the PT reiterated this imaginary response in the question she added to the LMT: “So, if you are thinking about using the units to cover the shape, what is area?” The feedback question prompted the PT to think about the answer and then make an addition or revision to the LMT.

In Table 3, the last question in the second revision of the QTEST, “How is this different from perimeter from Sammy’s solution?” apparently linked to the following feedback prompts: “What about some questions that link, connect, or contrast what Sammy did with what Jessie did? Could that help you make sure the difference between A & P stand out?” Using the original question in the LMT, it seemed reasonable to think that the PT intended to draw Jessie (or the class) out to compare the area solution Jessie just described to Sammy’s solution for perimeter (distinguishing between area and perimeter was also a goal of the lesson, although Table 3 does not include this goal). Again, I provided questions for the PT to answer and make a decision, which might help make the connection. Revisions were generally made in cases similar to this one.

Looking up the references cited and incorporating language from resources. Under the types of feedback that PTs took up in the previous section, the category of *feedback that was directive and referenced a familiar resource* provided examples of how PTs took up the language from Van de Walle et al. (2010) and used it in their LMTs. In addition, feedback referenced *Cognitively Guided Instruction* materials (Carpenter et al., 1999), the case studies in the *5 Practices* book (Smith & Stein, 2011), videos that we watched together in class, and articles that we read. In each case, similar to the Van de Walle example, PTs found ideas, language, or suggestions from these resources that ended up in their LMTs.

The analysis of the feedback and revisions provided evidence about how the PTs used the feedback to shift their planning practices and in particular the language used to plan lessons. PTs often took words, phrases, and ideas from the feedback to revise statements and questions in the LMT. Evidence showed that the terminology used in feedback comments and questions appeared in the revisions PTs made to their LMTs. In some instances, PTs responded to questions that were posed in the feedback and incorporated their answers into the LMT. These questions ranged from what they expected students to do or say in relation to a question as well as how PTs would make a particular mathematical point stand out in the discussion. PTs also made use of reference materials and resources that were cited in the feedback. PTs incorporated language and strategies from these resources into the LMT.

Discussion

The interactive formative feedback process taken up by PTs reflected a critical component of learning to teach. Providing effective feedback is central to student learning (Hattie et al., 1996; Hattie, 1999; Black & Wiliam, 1998; Hattie & Jaeger, 1998). The findings in this study supported the findings of other research in that specific, targeted feedback led PTs to make meaningful revisions (Gibson & Simpson, 2004; Hattie & Timperley, 2007). This study added to the literature on effective formative feedback practices in teacher education and in particular in mathematics education in several key ways as described below.

Formative Feedback to Support Teaching to the Mathematical Point

Written feedback during teacher preparation was used to focus PTs' attention on the mathematical point during the planning phases. Through receiving focused formative feedback provided to support them to connect the goals to expected student strategies and questioning techniques, PTs were able to develop a language to articulate how the mathematics in a task could be intentionally threaded through every aspect of lesson planning. Feedback that supported PTs to make revisions to lesson plans that had the likelihood of meeting the mathematical point for all students fell into these categories: identifying opportunities to confront possible student conceptions of the math; specifying core mathematical ideas; specifying the connections across core mathematical ideas; specifying how the discussion of the task was connected to the lesson goals; specifying the expected mathematical learning/thinking; specifying potentially problematic mathematical language; specifying how the task/activity developed the mathematical ideas; and identifying teaching moves that would highlight the mathematics,.

Questions in the feedback that prompted PTs to consider the student perspective or to focus more specifically on the key mathematical ideas a particular student strategy brought to light served to support PTs in planning to teach to the mathematical point. Sleep (2009) designed a framework that outlined what it would mean to focus on *the mathematical point* of a lesson and to follow that mathematical point through enactment. By *the mathematical point*, Sleep referenced a combination of mathematical goals and instructional purposes. From her perspective, knowing the mathematical point involved not only knowing and using specified learning goals, but also, knowing how instruction can be designed to help students make progress towards these goals. Sleep's approach rested on the idea that the mathematical learning goals must first be appropriate for the students, the mathematics, and the activity. The analytical framework of this study aligned with Sleep's (2009) conception of teaching to the mathematical point and I have evidence that providing various kinds of feedback can support PTs' development in planning with the mathematical goals in mind.

The Power of Formative Feedback Over Summative

Providing formative feedback on the written plans allowed PTs to learn from their own planning practices during teacher preparation. When specific praise was provided without any formative comments, PTs did not make revisions. In line with Kluger and DeNisi (1996), this finding added to our understanding that summative feedback may not provide further learning opportunities and may, in fact, have a negative impact on learning. Researchers (Hiebert, et al, 2007; Hiebert, et al., 2003; Putnam & Leinhardt, 1986; Stigler & Thompson, 2009) have hypothesized about the appropriate time and possible approaches to teach novice teachers how to use lesson goals to guide instruction. Hiebert and colleagues (Hiebert, Morris, Berk, & Jansen, 2007; Hiebert, Morris, & Glass, 2003), for example, asserted that much of the intellectual work of teaching at the K-6 level can occur outside the classroom, including specifying learning goals, anticipating student responses that demonstrate acquisition of the learning goals, and then evaluating lessons in order to map the evidence of student learning. I provided evidence that PTs can learn about teaching outside of the K-12 classroom, in this case from feedback on their planning practices, as evidenced by the shifts in language found in the revisions to their LMTs. Summative feedback did not provide the opportunity for PTs to speak back to the feedback.

Although providing the type of feedback this study has described was time-intensive, summative assessments cannot match the opportunities it offered me to formatively assess the needs and current skills of the PTs and to respond in productive ways. The feedback exchanges presented insights into the content understanding of PTs as well as their conceptions of students and teaching that I would not have otherwise seen.

Leveraging PTs' Understanding of Student Strategies

In general, when PTs were able to articulate their thinking about the student strategy or explicitly write out the questions they would ask to elicit student thinking, they were in a better position to develop stronger lesson plans with well-articulated goals and connections to these goals throughout the lesson plan. Across the sample periods, the strategy section of the LMT received the highest scores. When this section of the LMT was weak, PTs often had to revise twice, which necessitated more detailed feedback. If PTs understood what it meant to develop different correct solutions for a task and were equipped with the knowledge and skills to do this, they would be in a position to plan meaningful mathematics lessons. When PTs spent time articulating the strategy section of the LMT this work led to a better understanding of the goal as measured by PTs' work, illuminating connections that could be made between strategies, and clarify important questions they should definitely ask in order to make student thinking available to the larger class. This type of growth resulted in less feedback on their LMTs. Revisions to the student strategy section of the LMT resulted in stronger goals and links to goals on the second revision.

Implications for Mathematics Teacher Educators

The demands on instructors of method courses are already very high. Adding time-intensive feedback exchanges and opportunities for revision may seem to unreasonably tax educators' time; however, the literature on feedback repeatedly highlights the value of formative feedback that is specific, timely, and purposeful. The powerful effects of feedback on PTs' learning necessitates that teacher educators find room in their schedules to support PTs with immediate, specific, and frequent formative feedback and create opportunities for PTs to rewrite and revise. For these reasons, this study's findings suggest that teacher educators should incorporate repeated formative feedback exchanges with opportunities for PTs to revise their work in place of providing a single instance of feedback. Findings suggest teacher educators should spend time in methods courses supporting PTs' development of the patterns of thinking and skills necessary to solve tasks in multiple ways in order to facilitate deep thinking about mathematical connections between different approaches to tasks.

Implications for Teacher Education Researchers

This study's findings suggested that PTs' ability to write lesson plans with explicit mathematical goals and specify the key mathematical connections builds over time. Having written mathematically specific and comprehensive lesson plans through the process of revision positioned them to write stronger original lessons. This study, however, did not follow PTs into their own classrooms. One assumption of the work in practice-based teaching is that using these pedagogies of practice will enable PTs to take what they have learned and use it in their classroom teaching. A necessary next step in the work would be to answer the question: in what ways does such a pedagogy of practice make its way into teachers' when they have their own classroom?

The research base on feedback for teacher education is not as formidable as it is for K-12 and HE. More research needs to be done in the area of teacher education to unpack the practices for written and verbal feedback that prompt PTs to improve their lesson plans the most. Research on feedback for courses as well as in student teaching needs will lead to critical improvements in how teacher education prepares teachers for practice.

CHAPTER 4: EXPLICIT AND IMPLICIT POSITIONING OF STUDENTS

The ways teachers describe learners, learning, and teaching are impacted by cultural and societal norms, norms of teacher education, norms of schooling, and norms of mathematics (Parks, 2010). Not only do these broader norms impact the language and terminology that teachers use, what teachers *do* in the classroom is directly related to the assumptions they hold about students and student learning, and these assumptions are shaped by these broader norms. Whether conscious or not, the mathematics education community often describes children as being "behind" or "ahead" of some constructed standard demarcated by grade level and endorsed by the state. This kind of language requires comparing, ranking, and ordering children (Parks, 2010); these processes make it almost impossible to see difference, readiness, or achievement in any positive light. Teachers are implicitly taught to view conformity as the way to success, and children who do not conform are viewed as deficient in some way. When teachers are asked to compare, rank, or order children, it impacts both what teachers notice and how they notice. This in turn drives the decisions teachers make when teaching.

Wagner and Herbel-Eisenmann (2009) posit that positioning theory provides a lens through which researchers, teacher educators, and teachers can begin to change the ways mathematics is discussed. In this chapter, I take up the challenge of considering the impacts of positioning in my work as an instructor of an elementary mathematics methods course. Although I do not consider myself an expert at identifying and responding to various positionings of students, I feel compelled to find effective ways in my own work to begin to shift my language and practice as well as encourage PTs to do the same. I argue that PTs' explicit and implicit positionings of students influence the opportunities they provide students to learn. I will examine the ways PTs position students and compare these positionings to their instructional choices. In the next section I turn to the research on teacher noticing to make a case that what someone notices has an influence on their actions and reactions.

Teacher Noticing

Researchers examining teacher noticing hold a particular conception about teaching, that is, teaching is a complex activity with many issues calling for teacher's attention, and teachers respond to any number of these issues in a variety of ways. Mathematics educators studying teacher noticing have defined noticing in different ways. Some studies defined noticing as the focus of teachers' attention measured through the teachers' talk after observing an episode of teaching and how they talked about the aspects that caught their attention. For instance, what did teachers talk about—students, the teacher, class behavior, mathematics, student thinking—and how did they describe the events? For example, did they evaluate what they observed, base their observations on evidence, or interpret these events (e.g., Sherin, Russo, & Colestock, 2011; Star & Stickland, 2008)? Other researchers broadened the definition of noticing to include how teachers *made sense* of what they saw, meaning how they reasoned about and interpreted what they saw (e.g., Sherin, 2007; Sherin & Han, 2004; van Es & Sherin, 2008). This second definition of noticing, which includes teachers' interpretations of what was noticed, allowed researchers to examine teachers' assumptions about mathematics, learning, teaching, and students. This definition served as a bridge for me from teacher noticing to positioning because I was particular interested in why my PTs may have noticed particular aspects of students' thinking, but not others.

Jacobs, Lamb, Philipp, and Schappelle (2011) defined *making sense* as including both *interpreting* and *deciding how to respond* to what drew their attention. They posited that since noticing happens in the moment, as does deciding how to respond, it should also be included in the conception of noticing. Making the connections to teachers' actions seemed a fertile area of research to examine *why* teachers make the decisions they do in a classroom. Without also looking at the influences on teachers' decisions one is left with just descriptions of *what* and *how* with little attention to teacher agency.

Findings from Research on Teacher Noticing

Researchers submit that, teachers' ability to notice is related to teachers' classroom experience: more experienced teachers notice more, and make different kinds of observations, than prospective or inexperienced teachers (e.g., Ball & Bass, 2009; Berliner et al., 1988; Sherin & van Es, 2009). Multiple studies have shown that PTs can develop their noticing skills (e.g., Star & Strickland, 2008) and argue that improving one's ability to notice should be an explicit focus of teacher preparation (Sherin & van Es, 2005) since observation of others is a learned skill (Berliner et al., 1988). Some research focused attention on what caught prospective teachers' attention and what they missed (e.g., Star & Stickland, 2008). Other studies reported only on a particular aspect of classroom interaction, such as what teachers noticed with respect to children's mathematical thinking (e.g., Borko, Jacobs, Eiteljorg, & Pittman, 2008; Jacobs, Lamb, & Philipp, 2010).

Sherin and van Es have studied teacher learning extensively through the use of video clubs with teachers (Sherin, 2001, 2007; Sherin & Han, 2004; Sherin & van Es, 2005, 2009; van Es & Sherin, 2002, 2006, 2008). In a study based on data from two yearlong video clubs, Sherin and van Es (2009) presented evidence of teachers developing *selective attention*, meaning that over time teachers were able to focus on mathematical thinking depicted in the videos even though at first teachers focused primarily on teacher pedagogy. Van Es and Sherin (2009) also found that teachers demonstrated more *knowledge-based reasoning*, meaning that over time teachers' discussions shifted from merely restating what students did to trying to make sense of student ideas by discussing the meaning of individual student's work and then drawing connections between the ideas of multiple students. One key finding from this study was that van Es and Sherin (2009) found that this shift in noticing was also evident in the classroom interactions for some participants and were not just exhibited in the study group sessions. Thus, it is possible to develop one's ability to notice student thinking and that these new ways of noticing can influence teachers' classroom practice. The notion that noticing could change teachers' practice has potentially powerful implications for a more equitable view of teaching and learning. If by developing teachers' ability to notice, one could support teachers toexamine their assumptions about mathematics, teachers, learning, and students, noticing could be used as a lens to change the normed "stories" about these ideas and eventually to change how we do mathematics (Wagner & Herbel-Eisenmann, 2009).

Research on noticing generally focused on what teachers notice and/or how teachers notice but did not often examine why teachers noticed particular aspects of children's thinking. The majority of studies used video to examine noticing, and researchers' findings described shifts in teachers' noticing from describing events to interpreting them; from attending to the teacher to attending to students; from making general descriptive statements to interpreting what was observed; and from attending to climate to attending to mathematical thinking; and from making general observations to more specific foci (Borko, Jacobs, Eiteljorg, & Pittman, 2008; Hess, 2004; Rosaen, Lundeberg, Cooper, Fritzen, & Terpstra, 2008; Sherin, 2004; Sherin & Han, 2004; Sherin & van Es, 2007; Stockero, 2008; van Es & Sherin, 2002, 2008). A few studies characterized shifts in teachers learning by offering a framework for learning to notice (van Es, 2011; van Es & Sherin, 2002, 2008). Several studies focused on PTs' learning to notice

(Calandra, Gurvitch, & Lund, 2008; Jacobs, Lamb, & Philipp, 2010; Sherin & van Es, 2005; Star & Strickland, 2008; Stockero, 2006, 2008; van Es & Sherin, 2002).

How What Teachers Notice Influences Teacher Decision-Making

Erickson (2011) described seven propositions about teacher noticing. One of those propositions addressed how teachers' *pedagogical commitments*, or philosophy of practice influenced what and how teachers noticed. He described these commitments as assumptions that teachers bring with them into the classroom that can be implicitly or explicitly observed in a teachers' practice. Teachers may place value on students' efforts, which could be captured in language that describes how willing students are to perform tasks or characterizing students as lazy or unwilling to try. Teachers' views about the nature of mathematics can also be implicitly observed in practice. For example, a teacher might characterize a task or topic as easy or hard, interesting or boring. The language teachers use to describe mathematics and students may provide insight into their pedagogical commitments.

Erickson (2011) found that the ways teachers framed their noticing of classroom interactions "were profoundly influenced by their pedagogical commitments" (p. 28), that pedagogical commitments varied from teacher to teacher, and that teachers' interpretations of events "differed markedly along the lines of different pedagogical commitments" (p. 21). Research on *why* and *how* teachers' pedagogical commitments influenced teacher noticing and responses to students' thinking might be a first step in revealing and understanding the hidden assumptions teachers have about mathematics, teaching, learning, and students so that the field can begin to develop ways to disrupt unproductive assumptions. The construct of pedagogical commitments calls to mind positioning theory research. When examining the pedagogical commitments that PTs' noticing invokes, positioning theory provides a helpful lens because it adds the *why* to the *what* and *how* of noticing, making it possible to explore the hidden assumptions teachers hold.

Positioning Theory as a Lens for Examining Teacher Noticing

Positioning theory seeks to explain what motivates people to act and react in particular ways by studying the verbal and nonverbal exchanges during conversation (Harré & van Langenhove, 1991). In any interactions, including classroom interactions, there are multiple storylines in play at any given moment (Wagner & Herbel-Eisenmann, 2009). *Storylines* refer to the broader normative narratives that inform how and why people interact in the ways they do. These can be "ongoing repertoires that are already shared culturally or they can be invented as participants interact" (Herbel-Eisenmann, Wagner, Johnson, Suh, & Figueras, 2015, p. 6). One storyline for mathematics is the metaphor of mathematics as a path (Parks, 2010) that is noticeable in practices such as comparing, ranking, and ordering children. These storylines influence teachers' assumptions about mathematics, teaching, learning, and students, and can be seen in their pedagogical commitments, which, in turn, influence what they notice. Positioning theory assumes that all conversation have storylines and that "the positions people take will be linked to those storylines" (Harré & van Langenhove, 1991, p. 396). This positioning is often implicit, as illustrated by the following example:

> If Jones says to Smith: "Please, iron my shirts," then both Smith and Jones are positioned by that utterance. Jones as somebody who has the moral right (or as someone who thinks he has the moral right) to command Smith, and Smith as someone who can be commanded by Jones (Harré & van Langenhove, 1991, p. 396).

One possible positioning and storyline from this example could be that, although you do not hear Jones *say* that he has or thinks he has the moral right to tell Smith to iron his shirts, or that Smith is someone who can be told what to do by Jones, it is implicit in the utterance; Jones has invoked a storyline where he is in such a position. Effectively Jones has *positioned* Smith as someone who can be instructed to perform the task of ironing, while positioning himself as someone who has the power to direct Smith.

Each individual might draw on and use different but overlapping storylines and act or react in ways that can reinforce or challenge others' storylines. Individuals can position one another through their language or assignment of positions, and how those positions are taken up, challenged, or rejected determines whether or not the storyline invoked is accepted or rejected by those interacting. Researchers using positioning theory often write about ideas of status, authority, and power (e.g., Esmonde, 2009) since these constructs are often implicitly rather than explicitly invoked and researchers use positioning theory to uncover these hidden aspects of interaction.

Researchers have studied participants' use of language and interactions to make sense of how participants view themselves, the content, and others (in the moment) by the choice of words they use when speaking and the decisions made during interactions that occur. Researchers have used positioning theory in mathematics education as a means of unpacking what is happening in mathematics classrooms by attending to the storylines and interactions between children and teachers (Esmonde, 2009; Gresalfi & Cobb, 2006; Herbel-Eisenmann & Wagner, 2010; Herbel-Eisenmann, Wagner, & Cortes, 2010; Wagner & Herbel-Eisenmann, 2009.

Although Parks (2010) did not use positioning theory when she unpacked the metaphor of mathematics as a "path," she used discourse from elementary classrooms, university classrooms, textbooks, and standards documents to bring this hidden metaphor into the light. Still educators have one “path” and one measuring stick for determining whether or not children are following “the path” at an appropriate rate. It is no wonder deficit positioning occurs in teaching and teacher education. The issue is that the deficit lens for viewing everyone not progressing on the path at the correct rate can impact what teachers do in practice. The response to this perceived deficiency implicitly impacts what and how teachers notice children’s thinking and their responses to this thinking.

I see connections between teacher noticing and the work that is necessary to identify explicit and implicit positioning. Categorizing, labeling, and ranking children is ubiquitous in education circles. Identifying and intentionally acting on positioning means pushing against the normative discourse in mathematics education. One needs to learn how to attend to positioning. So, I turned to the research on teacher noticing to better understand how teachers learn to notice classroom practice. I then use the literature on teacher noticing to build connections between teachers learning to notice and learning to notice positioning. Building on these relationships I present a draft framework to serve as a tool to help teachers and teacher educators to begin to explore these relationships and the implications for student learning.

Explicit positioning is characterized by language that directly categorizes, describes, or defines students or student thinking. The language use leaves nothing to the imagination. As examples: *John is a low-achieving student. Samantha is a bright child.* The terms *low achieving* and *bright* convey that a comparison has taken place, these children have been labeled, ranked, and categorized. The label conveys a static, non-negotiable classification of these children.

I see implicit positioning appear through teachers’ pedagogical decisions. The effects of these pedagogical choices position students in particular ways. For example, providing solution strategies to children as they work on mathematical tasks, whether through hints or the use of leading questions, positions students implicitly, and perhaps unintentionally, as incapable of thinking mathematically. Asking probing questions of a student to uncover her ideas also implicitly positions that student as capable of thinking mathematically. These aspects of positioning are not always obvious to observers or teachers themselves, so it is necessary to think about how one begins to attend to the positioning of students.

Examining the van Es Framework through Positioning Theory

I return to the Learning to Notice Framework (van Es, 2011) and use positioning theory to highlight how assumptions about student learning have implications for this framework and research in noticing. Van Es in her Learning to Notice Framework (see Table 5) created

Table 5

Van Es (2011) Learning to Notice Framework

Baseline	Mixed	Focused	Extended
	What Teachers Notice		
Attend to whole class environment, behavior, and learning and to teacher pedagogy	Primarily attend to teacher pedagogy	Attend to particular students' mathematical thinking	Attend to the relationship between particular students' mathematical thinking and between teaching strategies and student mathematical thinking
	Begin to attend to particular students' mathematical thinking and behaviors		
	How Teachers Notice		
Form general impressions of what occurred	Form general impressions and highlight noteworthy events	Highlight noteworthy events	Highlight noteworthy events
Provide descriptive and evaluative comments	Provide primarily evaluative with some interpretive comments	Provide interpretive comments	Provide interpretive comments
Provide little or no evidence to support analysis	Begin to refer to specific events and interactions as evidence	Refer to specific events and interactions as evidence	Refer to specific events and interactions as evidence
		Elaborate on events and interactions	Elaborate on events and interactions
			Make connections between events and principles of teaching and learning

Table 5 (cont'd)

Baseline	Mixed	Focused	Extended
			On the basis of interpretations, propose alternative pedagogical solutions

dimensions for what and how teachers notice across a continuum from baseline noticing to extended noticing. In her descriptions of *what* teachers notice, she pays attention to the topic of the noticed thing (NT). She depicts a narrowing lens for noticing that begins with a wide view of the classroom environment, behavior and learning very generally, and teacher pedagogy. Van Es described the trajectory as shifting from noticing whole class issues to noticing a combination of teacher pedagogy and some individual student thinking and behaviors, then exclusive attention to individual student thinking, and finally to the relationships between student thinking and teaching strategies.

Examining the framework categories (Table 5) of *how* teachers notice, one sees several shifts in the trajectory. When describing how teachers notice at the baseline level[6], van Es documented that teachers "form general impressions, provide descriptive and evaluative comments, and provide little or no evidence to support claims" (p. 139, 2011). As I read this comment with a positioning lens, I noticed these evaluative comments often took the form of static positioning of students. When a teacher's analysis of teaching rested on general statements and evaluative comments, and provided little or no evidence to support claims, I posited that this might indicate that a teacher's practice simultaneously communicated that students and student thinking are of little consequence to teaching and learning. Further, teachers who noticed at the

[6] The irony of using the word *level* in this section on positioning and throughout the chapter is not lost on me. As educators, we need better ways of describing and evaluating teaching in order to eliminate these implicit positionings of individuals.

extended level "attend[ed] to the relationships between particular students' mathematical thinking and between teaching strategies and student mathematical thinking" (van Es, 2011, p. 139). I considered that it might imply that those teachers who notice at the extended level actually shared a different view of teaching than the teachers who noticed at the baseline level, and conjectured that this view could likely be observed in their practice. In this study, I explored the connections between the levels of noticing from this framework and the PTs teaching practices. If a PT who noticed at the baseline level also used practices in their teaching that were parallel to the baseline descriptions in the framework, this would indicate a need to spend more time in teacher preparation examining underlying assumptions about students' learning. In the remainder of this paper, I report on research to answer this question: How did participants' levels of noticing seem to relate to the ways they positioned students?

Method

Data for this study was collected during a mathematics methods course for prospective elementary teachers at a large Midwestern university during the spring of 2013. The sample for this study consisted of 49 prospective elementary teachers from two sections of the senior-level mathematics methods course of the teacher education program. Participants were enrolled during the Spring semester of 2013. I invited PTs from both of my course sections to participate and 49 out of 51 students enrolled in the two sections consented. There were 46 female PTs and three male PTs. Most of them were between 21 and 23 years of age, but one of the men was 36 years old and pursuing his second career as an elementary teacher. Almost half had specializations: six specialized in mathematics, four in special education, and 12 were members of an urban education cohort program. All of the students had previously passed methods courses with a science and social studies content focus, and all students were concurrently enrolled in an English language arts methods course.

This study is a qualitative study where I used a grounded theory approach to identify the potential relationships between PT noticing and positioning of students. Data analysis followed a constant comparative approach in which I adopted some of the characteristics of grounded theory (Glaser & Strauss, 1967). I analyzed the data sources (described in detail below) for evidence of teacher noticing and coded this data using the van Es (2011) Learning to Notice Framework in order to determine the level of PTs' noticing in their written work. I used the results of this coding to do a secondary analysis of the data in which I looked for relationships between the levels of noticing and how participants positioned students. Finally, I did a tertiary analysis of the data to develop categories of teacher positioning. I then used research from mathematics education to add the details to a framework for identifying positioning in teaching. I present this finding as well.

Data Sources

Each of the 49 participants videotaped a series of three student interviews and chose one to analyze for the Student Interview Assignment[7] (see Appendix C). Eleven PTs wrote about all three students using all three tasks. Two PTs wrote about one student with one task. The rest of the PTs wrote about one student solving three tasks. In total, I examined 209 video episodes for this study along with the PTs' written assignments. The PTs' interviews were analyzed based on individual tasks given to students during the interview. Although each PT was expected to provide three tasks to each of three students, they were only asked to write about their

[7] The structured student interview, often called a clinical interview (Hunting, 1997) is a staple in mathematics methods courses (e.g., Cross & Hynes, 1994; Moyer, 2004; Moyer & Moody, 1998; Sleep & Boerst, 2012).

interactions with one child. Some PTs only recorded one task with each student; others used two or three tasks per student.

The Assignment Description (Appendix C) required PTs to create a plan for the interview that included: the mathematical tasks the students would work on, a rationale for choosing the tasks, any materials the PT would provide the student, the PT's expectations for how the students would engage with the tasks, and a list of potential follow-up questions they could use if the students' thinking was not clear from their work. They turned their plans in for my approval, and I gave feedback regarding appropriateness of the tasks (see Appendix C). The PTs were given the following prompts to write their analysis:

> After having watched the video recordings multiple times, select one student to write about in detail. Your analysis should include the following three parts:
>
> > **Part I:** Write an analysis of your assessment of the child's mathematical thinking about the tasks provided. Provide evidence from the interview (transcribe several exchanges between you and your interviewee to show this).
> >
> > **Part II:** Write an analysis on how well you felt you elicited student thinking during the interview. Again, provide evidence from the interview (transcribe several exchanges between you and your interviewee to show this).
> >
> > **Part III:** If you were not happy with your eliciting of student ideas, provide some ideas about what you could have done differently. (Student Interview Assignment, Appendix C).

The PTs were instructed to watch the interview repeatedly in order to select one child about whom to write. They then analyzed the mathematical thinking of the child interviewed as well as self-assessed their own efforts to elicit student thinking. In the weeks prior to the interview assignment, PTs were given three resources for designing questions: Herbel-Eisenmann and Breyfogle's (2005) article on questioning patterns, Boaler and Brodie's (2004) categories of questioning, and a PDF of unknown origin that provided 24 different question stems for them to use that would help students reason, make connections, or conjecture. The PTs turned in the analysis papers and videos, and I returned them with feedback so they could revise and resubmit the assignment.

Data Analysis

I organized the data analysis section of the paper by describing my process of data analysis for each of the primary, secondary, and tertiary analyses under individual subheadings. The findings section of the paper will follow the same structure. So, before I discuss the findings from the primary analysis, I describe the secondary analysis, and so on. The findings for each of the phases of analysis will be presented in the findings section accordingly.

Primary Analysis—Coding with the Learning to Notice Framework

The interview was one-on-one, and the PTs commented on the student's work, the student's talk, the immediate surroundings, and the PT's own questioning strategies. The PTs were expected to transcribe particular sections of the interview and use these transcripts in their analysis paper as evidence of what students knew and understood about the particular mathematical tasks that were provided. Every PT used transcribed portions of the interview in the writing of their analysis paper. I explain how I coded the data using the van Es (2011) framework and provide examples of each of the levels of coding on the framework with a rationale for how they were coded.

I began by coding the PTs' noticing using the Learning to Notice Framework (van Es, 2011) given in Table 5. The trajectory for noticing has four basic levels from baseline to extended. The category for *what teachers notice* captures both the *people* as well as the *topic* of their focus. From Table 5, as teachers become more advanced in their noticing, their focus shifts from whole classroom, to teaching, to student learning, and finally to noticing connections between teaching and learning. The category of *how teachers notice* examines the analytic stance the teachers take as they analyze episodes of teaching and the level of depth the teachers use to interpret or evaluate the teaching episode. My unit of analysis was one episode in a PT's written analysis of one mathematical task (van Es, 2011). Some episodes were quite short, lasting two or three minutes, while others were over 10 minutes in length.

Baseline noticing. Since the setting of the teaching was a clinical interview, there was no whole class instruction, so the baseline framework as written from van Es (2011) was not an appropriate fit for this assignment because it included attention to the whole class environment, and there was nothing else but the student to focus on. When reading the student interviews, however, I found that many PTs in this one-on-one setting described student behavior primarily rather than student thinking, so I modified that first cell in the framework to read, "Attend to student behavior and to teacher pedagogy." The emphasis on teaching in their reflections was expected since the assignment itself asked them to analyze and reflect on their teaching.

Baseline noticing was characterized by a focus on student behavior and pedagogy written in descriptive and evaluative language that provided little or no evidence. I coded PTs' noticing as baseline when PTs provided an overview of what happened but did not draw on evidence from the video. Comments such as, "I think the interview went well," were interpreted as general impressions. Statements such as, "He was correct," were considered evaluative. As an example of an episode I coded as baseline:

> J used a strategy that I have seen Mr. T use in his class many times. She put __ + __ = __. She had the right idea, but the numbers she had, including the answer, were all in the wrong blanks, which made her answer incorrect.[8]

This PT wrote an analysis that gives several evaluative statements, “She had the right idea,” “the wrong blanks,” and “made her answer incorrect.” The PT provided a general impression of what happened with no focus on detail.

Mixed noticing. The PTs with mixed noticing began to attend to student thinking and also wrote about their own pedagogy. This was to be expected because PTs were asked in the assignment to reflect and comment on their teaching as well as student thinking. They continued to write using general impressions and evaluative statements, but also wrote about particular events in the solving of the task that stood out to them, provided some details about student thinking that were missing in episodes coded as baseline, and offered limited interpretation of events. As an example of a PT who wrote in a way that was categorized as mixed noticing:

> Overall, I was really impressed with P’s ability to tell me what he was thinking. I feel there were strong parts of my interview, and also weak ones. One way I feel I was successful was in eliciting student ideas. … I felt my strongest display of getting P to explain to me what happened was in problem three. … I saw him use his fingers and getting frustrated. He seemed to just spit out an answer when he got stuck. Patrick counted out three blocks, and then 7. “He said he has 3, she has 7. She has four more than

[8] Note that all of the data included in the Methods and Findings sections are transcribed verbatim from PTs’ analysis papers and not reformatted for APA in order to maintain authenticity. <I did change numerals to written-out numbers and vice versa, and added minimal punctuation only where I felt it was necessary to maintain clarity, but I stetted everything else. If you feel these changes affect the authenticity of the quotes, please feel free to ignore those edits.>

> him." … I felt that by asking him this leading question, I was able to get to exactly what he was thinking.

The mixed level of noticing is characterized by a focus on teacher pedagogy with some attention to student thinking and behaviors. The PT primarily focuses on herself, indicated by phrases such as, "I feel there were strong parts of my interview and weak ones;" "I felt my strongest display of getting P to explain;" and "I felt that by asking him this leading question." Her impressions were general, which is another characteristic of mixed noticing, but she did point out a few things she noticed about how the student solved the task: "I saw him use his fingers," and "He counted out three blocks and then 7;" "He said he has 3 and she has 7. She has four more than him." The PT, however, never talked about what the evidence might mean about the student's thinking.

Focused noticing. The PTs with focused noticing attended to particular students' mathematical thinking, highlighted noteworthy events, interpreted what they saw, referred to specific events as evidence, and elaborated on what the interactions might mean about student thinking. As an example of a PT noticing at the focused level:

> I chose this particular task because the fourth-grade students are familiar with single-digit division problems, but I wanted to go one step further and see if Dominic could solve a word problem. … Dominic wrote down 4x6 = 24, so I asked him again how many sandwiches the restaurant could make. He covered up the 4 and 24 in his equation and said, "So they have 4 sandwiches and 24 pieces of cheese. That's the answer (pointing to 6)." I asked, "So, what's the answer?" Dominic replied, "6." I then asked him if he could explain to me how he got that answer. He said, "You said there's 4 pieces of cheese on each sandwich and you said they have 24 slices of cheese, so they can make 6 sandwiches because 6x4 is 24." I wondered if Dominic just interpreted this as a

> multiplication problem from the start, or if he saw the relationship between the numbers, or something else. I then elicited Dominic's thinking more by asking, "How did you know to do 4x6?" He said, "Because you said there were 4 pieces of cheese on each sandwich and there's 24 and I thought to myself, well okay, 4x6 is 24, so I just left out the numbers you said and got 6." Here, Dominic was referring to 4 and 24 as the numbers that he "left out" when solving the task. He did not really leave the numbers out because he needed them to find the answer of 6, but he knew that they were already given to him, so they were ultimately not his answers. Dominic was aware of the numbers he was working with, so his goal was to find the missing number, which was 6. … I discovered that Dominic used the number of objects in each group (4 pieces of cheese) to measure the total number of objects (24 pieces of cheese) and find the unknown quantity (6 sandwiches).

Although this PT retold events, she also interspersed her narrative with writing that interpreted and elaborated on the events and interactions. For example, when retelling Dominic's solution strategy, she elaborated on the event, writing, "I wondered if Dominic just interpreted this as a multiplication problem from the start, or if he saw the relationship between the numbers, or something else." She noticed not only what Dominic wrote, but also what he did, and that allowed her to elaborate on the events she retold. The end of the episode shows her elaboration on his thinking. During this elaboration, she made sense of what he said by uncovering likely thinking and wrote about how his work indicated he could have solved it in two ways based on his interactions.

Extended noticing. The PTs noticing at the extended level wrote analyses that attended to the relationships between students' mathematical thinking and teaching strategies. Extended noticing was characterized by interpretation of and elaboration on interactions. The PTs with extended noticing refer to specific events and interactions as evidence for their claims. As an example of a PT with extended noticing:

> Jose counted out the starting number of chips (10). When revisiting the video, I noticed something that I had not noticed when Jose was doing the problem initially. When counting out the original/starting 10 counters, Jose organized the chips in two rows of 5. This was very interesting to me, because it meant that Jose not only knew that there were two sets of 5 that went into 10, but it meant that he could also easily separate 4 from the group as the result, making it very easy for him to find the "change," if he already knew what a group of 4 looked like. Jose pushes the "changed" number of chips away from the remaining, or "result," or 4. He counts the chips he pushed away, and deduces that 6 students went to line up, leaving 4 remaining. Jose then fills out the blank box problem according to this deduction.
>
> It is important to note that Jose correctly identified the "Start," "Change," and "Result" numbers when filling in the problem, placing 10 in the "start" box, a subtraction sign in the sign box (indicating a Separate Problem), a 6 in the "change," and, finally, a 4 in the "result" box. When asked to explain why he put the 6 where he did, Jose responded with, "6 was how many kids left, and 4 were left." I was delighted by this thinking, which demonstrated his reasoning that I [had] previously wondered if kindergarten students would be able to use.
>
> This is a problem that I had anticipated to be very difficult for my students, because it is not a problem type [that] kindergarten students are typically familiar with. I predicted in my student interview draft that my students would have a difficult time solving this

problem effectively for two reasons, because: (1) it is a problem type they are unfamiliar with, and (2) many of my students have a difficult time with subtraction problems. Jose's performance made me rethink my expectations for kindergarteners, and made me wonder whether modeling was a strategy that could be used more often in kindergarten to help support students' developing thinking about number concepts.

This PT attended to the relationship between the child's mathematical thinking and the teaching strategies used both in the interview and in the class at large. She highlighted specific events to support her claims. The PTs with extended noticing reflected on the interview and identified aspects of the interview not previously noticed. In this episode, the PT noticed upon review of the video that Jose had organized the chips into two rows of five. The PT provided interpretations about why Jose might have this and connected the use of manipulatives to the child's solution strategy. She also commented on how the modeling strategy might be a useful tool for past tasks as well as future tasks.

When coding the PTs' analyses using the van Es (2011) framework, I began to notice some static positioning of students by PTs who noticed at baseline and mixed levels but did not find this positioning with PTs who noticed at focused and extending levels. I wondered whether or not there might be a connection between how PTs noticed and the ways they positioned students. I ungrouped the episodes coded using the van Es framework, and then used a positioning lens to analyze the written interviews. In the next section, I describe how I identified features of the PTs' interviews where their positioning of students was visible.

Secondary Analysis—Identifying Positioning

Three fairly distinct facets of the PTs' written analyses emerged as fruitful areas to examine positioning: how PTs described students, the types of questions and questioning strategies PTs used, and who did the cognitive work for the task solutions. In order to classify the positioning of PTs, I started with the task descriptions from the mathematical task framework (Smith & Stein, 1998) and added a positioning lens. For this analysis, I classified PTs' positioning of students as incapable or capable of thinking deeply about mathematics. In the next sections, I elaborate on each of these facets of the interviews and how I used the categories for positioning to compare the ways PTs positioned students.

How PTs described students. Explicit positioning was identifiable in the ways many PTs described students. Some PTs described students using explicit positionings that incorporated binary language such as in high/low-level groups, high/low achieving, bright/difficult, advanced/behind, and so on. When PTs described students in binary terms with static positioning language, I classified the lower end of the binary as positioning students as incapable of thinking deeply about mathematics. When PTs described how they expected students to approach the task, students whom they categorized as low-level thinkers were paired with words such as "guess," "random," "mediocre," and "struggle," and PTs wrote that they expected incorrect responses. Students categorized at high levels by PTs were paired with words such as "confident," "smart," and "fast," and were expected to use a particular procedure to solve the task. These positionings were static, essentially holding students in one group or the other, and did not leave room for what students knew or could do. As an illustrative example, one PT wrote:

> D, T, and J were the three students that were picked for my student interview. D and T are in the high-level math group while J is in the low-level math group. Prior to completing my student interview, I anticipated that J would have the most trouble completing my task because she is in low-level math. I thought T would struggle next

> because he has trouble paying attention and grasping concepts. I believe that D would have little to no trouble completing my math task because he is one of the strongest students in the high-level group.

This PT used the binary high/low-level group categorizations and, based on those categorizations, predicted that the student in the "low-level group" would struggle because she is in "low-level math." Although T was in the higher-level math group, the PT compared her expectations for him with the other students and decided he would "struggle next," meaning that on a continuum, T would struggle less than J, but more than D, since D was "one of the strongest students in the high-level group." One storyline the PT invoked here seemed to relate to the narratives about comparing and ranking that are connected with standardized testing and tracking in schools since she is using the idea of students' membership in a particular institutionally-sanctioned group. The storyline she uses to position T shifts, and now the PT invokes a storyline of how students who focus are more successful than students like T who have difficulty paying attention.

For purposes of comparison, I provide the ways two PTs described their students that did not use static positionings.

> PT1: For my student interview, I asked three questions to three different third-grade students in my placement classroom. One question that could be solved by multiplication and two were based on division ideas.
>
> PT2: Jose, a student from CES was chosen to participate in the student interview. Jose was given two problems to work through; these included a Separate-Change Unknown problem, and a Join-Result Unknown problem.

These two PTs did not provide any categorization or classification of students. Neither PT included a description of the students; the second provided only a pseudonym for the student. Both PTs describe the kinds of tasks chosen. The language choice of the PT in the first example, “could be solved by,” made it clear that while multiplication and division ideas were imbedded in the tasks, there was no expectation that the students would solve these using multiplication or division. These PTs positioned students as capable of choosing their own method and thus as capable mathematical thinkers. The language they used seemed to indicate that the PTs would welcome any approach.

The types of questions and questioning strategies. Examining the question types and the questioning strategies PTs used proved a fruitful way to examine implicit positioning of students. My initial ideas about how questions might position students were from Boaler and Brodie’s (2004) articulation of focusing and funneling questions, where she posited that funneling students to an answer could be a marker that the PT was positioning students as incapable of mathematical thinking. Boaler and Brodie (2004) described how funneling and narrowing questions show that one doesn’t take “learner’s mathematical thinking seriously” (p. 10). Herbel-Eisenmann and Breyfogle (2005) further showed how a teacher could turn funneling interactions into focusing interactions so that students would be supported in understanding the mathematical goals of the lesson.

For example, when interviewing a kindergarten student who used his fingers to add four and three, one PT asked the following questions in succession: “How did you find that answer?” “What did you do first?” “Then you added what?” “And then you said the answer was?” This interaction showed a step-by-step documentation of what happened. There were no probing questions that unpacked what the child was thinking, for instance, how he knew that he could use his fingers, what told him it was an addition problem, or how he got from four to seven when he added three more fingers. Often, PTs did little analysis after documenting a step-by-step process. Perhaps they felt that the description was self-explanatory. There was also no indication that the PT recognized that the questions did little to assess student thinking, and in fact may have done the cognitive work for the child.

In contrast to the PTs described above, who only asked additional questions of students they positioned as “high-level,” some PTs asked probing questions or exploring mathematical meanings questions (Boaler & Brodie, 2004) to better understand both the strategies used and student thinking. For example, these PTs asked questions such as, “Why did you set the problem up the way you did?” “What were you thinking that helped you make that choice?” “Why do you say it’s not a subtraction or an addition problem? What kind of problem is it?” Each of these questions served to elicit student thinking. These types of questions were coded as PTs positioning students as capable of thinking mathematically.

Of note was that many of the questions PTs asked (which were apparent in the videos) did not appear in their written analysis. These questions were often short prompts that could be and often were answered with one word—yes/no or a number—for example: “How much money did you have left?” “Does that equal one dollar?” “How many does he have?” and “Are you counting on the number line now?” These prompts were also often leading prompts, or funneling questions (Herbel-Eisenmann & Breyfogle, 2005). I also identified question types that represented recall, a focus on procedure, and questions asking for justification.

Who did the cognitive work. I based my initial ideas about how PTs positioned students on Smith and Stein (1998) articulation of the ways that the cognitive demand of a task could be reduced. Stein and Smith’s ideas about how the teacher expressed explicit procedures through questions or prompts; how the teacher took over the task, telling students how to do the problem; or how the teacher emphasized correct or complete answers over the understanding of concepts. I analyzed the interviews with respect to positioning by considering how reducing the cognitive demand of the task positioned students. In some cases, the PT essentially solved the task for the student by using hints or direct instruction. In other cases, the PT provided hints only when students were stuck, and less frequently, the PT provided no hints or direct prompts about how to solve a task. I transcribed the following exchange from the video of one PT's student interview:

S So, I'm going to take out 52 [Student counts out 52 blocks]

S I've got 52 right here.

PT So put them in four groups.

S OK, I'm going to put them into four groups. [Student moves counters one at a time to four different groups. He stops when he has 12 in each group.]

S I've got four left, so…

PT So, put one of those in each group. [Student takes 30 seconds to move counters one at a time to each group.]

PT So how many groups do you have?

S I have four groups

PT And how many are in each group?

By interjecting comments or questions, this PT provided hints or direction to the student as she solved the task, thus lowering the cognitive demand on the student. I recorded this event as positioning students as incapable of thinking mathematically.

I documented instances in which a pedagogy of "telling" was used as positioning students as incapable of thinking mathematically (when the telling was not about mathematical conventions or arbitrary facts) and instances where the PT allowed students to retain ownership of their ideas as a way of positioning them as capable of thinking mathematically. When a PT maintained the cognitive demand of the task, I coded these instances as positioning students as capable of thinking mathematically. When the PT provided step-by-step hints and thereby reduced the cognitive demand of the task, I coded these instances as positioning students as incapable of thinking mathematically. In the next section, I describe the tertiary analysis that led me to draft a framework for position.

Tertiary Analysis—Designing a Framework for Noticing Positioning

My tertiary analysis consisted of reanalyzing all of the interview videos, coding the videos for observed positioning using the coding scheme from the secondary analysis, and matching the pedagogical practices PTs used in the video to the practices described in their interview paper. This analysis helped me to determine the veracity of what PTs wrote about in their papers and the potential links I might make to their actual practice.[9] This level of analysis served to verify my previous results and layer the noticing framework with the observed positionings. The resulting coding scheme was then arranged in a Framework for Learning to Notice Positioning (see Table 6 in the discussion section). The four categories for positioning, with respect to how PTs described students, their questioning strategies, and the maintenance or decline of the cognitive demand were: positions students as incapable of thinking mathematically, positions students as capable of following procedures, positions students as capable mathematical thinkers, and positions students as capable of doing mathematics.[10] In the

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[9] There were five papers where PTs misrepresented what happened in the actual interview. For example, PTs described asking questions that they did not ask, or misrepresented students' responses to tasks. These four interviews were excluded from my analysis. Each PTs said they didn't think I would check the video, so they fabricated evidence.

[10] The "doer of mathematics" language comes from the math task framework (Smith & Stein, 1998)

next sections, I will describe how I layered PTs' noticing and PTs' positionings of students to analyze the interplay between the PTs' ability to notice and the subsequent positionings of students uncovered in their video episodes and written analyses.

Positions students as incapable of thinking mathematically. When describing students, PTs' analyses that were classified as positioning students as incapable of thinking mathematically incorporated binary language with static positioning language, such as in high/low-level group, high/low achieving, bright/difficult, advanced/behind, and so on. The questions PTs used in their videos were almost always recall questions or questions that required one-word responses, and when PTs wrote their analyses, these question types were either overlooked or their importance was minimized. Video and/or written analyses in which the PT provided hints or prompts to the student as he/she solved the task, but either did not notice or acknowledge this in writing, were classified as instances of positioning students as incapable of thinking mathematically.

Positions students as capable of following procedures. The analyses and videos in which the PTs used questions that focused on procedures or routines (e.g., "what" questions, or "what did you do next?"), but did not acknowledge in the analyses that these questions led students to a solution, were classified as positioning students as capable of following procedures. Analyses and videos in which PTs suggested possible solution strategies for students to use, but did not notice or acknowledge this in the analysis, were also classified as positioning students as capable of following procedures. These analyses were also characterized by descriptions of students using language that focused on students' characteristics (e.g., hard-working, tries hard) or what students cannot do (e.g., student cannot solve multiplication problems).

Positions students as capable mathematical thinkers. When PTs used language in their analyses that focused on what students did without ranking, comparing, or classifying them, they were coded as positioning students as capable mathematical thinkers. These analyses and videos included some recall or procedural questions but provided alternative questions that PTs could have asked in their written analyses. Analyses that were coded as positioning students as capable of thinking mathematically included questions that focused on how students approached the mathematics and required some justification. These videos and analyses also showed that students were primarily responsible for solution strategies, but the PT may have provided scaffolding when students were stuck.

Positions students as capable of doing mathematics. When PTs' analyses used language that focused on what students knew and could do mathematically, they were coded as positioning students as capable of doing mathematics (e.g., T understood that the problem could be modeled using cubes). Although these analyses and videos occasionally incorporated some recall or procedural questions, the PTs always provided alternative questions in their written analyses that were more open-ended. These analyses and video episodes also included PTs asking questions that focused on why a particular strategy worked and required students to justify and describe their reasoning. In the videos, students were observed solving the tasks independently or in collaboration with other students, but without the PT prompting them.

Findings

The most significant findings from this study are a result of the secondary and tertiary analyses. Research on what and how PTs notice is well documented. For that reason, I will discuss the findings from the primary analysis only briefly and will spend more time on the findings from the other analyses.

Primary Analysis—Coding with the van Es Framework

I coded 32 of the episodes as PTs having baseline noticing; 147 as mixed noticing; 18 as focused noticing; and 12 as extended noticing. With an exception of four PTs, PTs' level of noticing did not change between the different tasks analyzed. Four PTs noticed at different levels for different episodes. Each of these PTs' noticing was coded at either baseline or mixed levels. It is not lost on my that through my analysis, I am also positioning PTs.

Consistent with other literature, in general, PTs without specialized mathematics methods instruction were limited in their ability to notice student thinking and connections between student thinking and teacher pedagogy. Eighty percent of the written analyses were coded as baseline or mixed levels, with 86% of all episodes reflecting noticing at these levels. Since ten PTs noticed at focused or extended levels, however, my findings seem to imply that it might not be experience alone that determines what or how PTs notice. I now turn to the secondary analysis.

Secondary Analysis—Relationships Between Noticing and Positioning

In the secondary analysis, I examined the relationships between PTs' levels of noticing and the ways they positioned students. This section is broken into the three themes that I found in the analysis: how PTs described students and their expectations for student approaches to mathematics tasks, what types of questions PTs asked, and who did the cognitive work of solving the task.

How PTs described students. The PTs' responses that received baseline classification also had the shortest videos (2–2.5 min) and written analysis (1–1.5 pages). Those with analyses receiving a baseline classification also always included a statement or two of static positioning and included evaluative statements such as, "he got the right answer" or "her approach was wrong." PTs' analyses receiving a baseline classification generally began with a statement of their impression of the interviews such as, "I think the interview went well."

All of the PTs who noticed at a "baseline level" (van Es, 2011) interpreted the instructions in the planning section to mean that they needed to describe students in some way. As a consequence, in the first few paragraphs of these student interviews, PTs described students using binary language such as high/low-level group or high/low achieving. Those PTs who noticed at the mixed level used some of this binary language that ranked or compared students, but the majority of PTs who noticed at mixed level described students as people who "tried hard" or who they "knew would talk about their thinking." A few PTs who noticed at the focused level carried over language that positioned students as "hard working," and the majority of PTs noticing at the focused level provided the pseudonyms of the students but did not describe the students themselves. The PTs noticing at the extended level did not describe students and only provided their pseudonyms.

The types of questions asked. After writing about the presentation of the task and the subsequent answer, most PTs followed up the student work with a question asking, "How did you get that answer?" If the student responded with a vague response, like "thinking" or "doing math," *and* the student was previously categorized by the PT as a low-level student, the PT said something such as "OK" and then read the next question. The PTs whose analyses were coded as noticing at baseline or mixed levels gave up quickly on students who they had previously described or positioned as low level or struggling. If a student was categorized as a high-level student, PTs followed up their "how" questions with additional questions about strategies or particular aspects of the student work. The PTs at the mixed level asked questions that closely followed the procedure used by students.

The PTs who noticed at the focused or extended levels asked probing questions or exploring mathematical meanings questions (Boaler & Brodie, 2004). Those who noticed at the extended level asked questions that required students to justify their solution strategies, positioning them as capable of thinking mathematically. The PTs at the baseline level asked gathering information questions that required only one or two words in response, and PTs using these questions positioned students as capable of restating facts. The PTs categorized with mixed noticing used questions about procedures or a step-by-step process marked by "what" or "What did you do next?" Those who used these types of questions positioned students as able to follow procedures. Those with focused and extended noticing asked questions that prompted students to reason, using words such as "how" and "why." Despite the expectations that PTs were supposed to probe in the interviews but not teach, PTs at the baseline and mixed levels often used leading or funneling questions to get the students to solve a task, or they provided hints for students so that they could be successful. The PTs noticing at the focused or extended levels did not insert themselves into the solution process and rather positioned students as capable of explaining their thinking.

Where solutions for tasks come from. The third dimension of the coding scheme addresses from where the solutions to tasks come. My expectations for the student interview were that PTs would observe and analyze what the students were thinking and how they approached problems, but PTs were not expected to teach students. More interesting than the fact that the PTs actually led students step-by-step through a solution was how they noticed what happened and recorded it in the retelling. There were several instances when the video and the retelling of what happened were drastically different. In particular, this was representative of several PTs whose analyses were classified at baseline levels. When I read one PT's paper, I assumed that the student solved this task and stated her reasoning using the set of steps described as they progressed through the problem. When I watched the video, however, I had a very different perspective about what happened. Perhaps the PT did not notice that she actually did the thinking for the student, or she has been socialized to think that errors should be avoided and therefore did not want to acknowledge that she did the thinking for the student. This type of retelling in which the PT left out critical aspects of the interaction was characteristic of all of the PTs noticing at the baseline and mixed levels. These PTs conveyed an implicit message in their interactions with students that the students were either not capable of solving the task on their own or capable only of following directions.

PTs noticing at focused and extended levels included the transcribed portions where they led students, but added commentary demonstrating that they noticed what they were doing when watching the video and provided alternatives types of questions that they could have asked. As an example, one PT gave a student a task that required estimating the location of various fractions on a number line. In the middle of the interview, she asked the student two questions and transcribed those. A little later she asked another question, which she documented as well:

> Steve said, "I put it right here because I have to go here (pointing to the X) and take away two … well I have to figure out the middle and go two up from the middle." Trying to elicit more information, I asked, "What is the middle? What does the middle signify?" At this point, Steve marked a line showing the middle and said that the X was a "fair one move over." Trying to get him to put a numeric value on the "middle," such as 1 or 8/8, I asked him again why he put the mark in the middle, but he did not change his explanation and said again that he looked at the middle and went up one space. It confused me as to why Steve switched from saying that the X was two spaces up from

> the half to only one space. … I should have asked Steve to clarify his explanation using numbers or asked him what measurement intervals he used since this would have helped me to understand his thinking better without making inferences. During the interview, I was hesitant to ask those questions because I thought it might persuade him to use a certain method.

This PT interjected questions into the interview while the student was working. She did not give the answers to the tasks as did the PT in the previous example, and her student did not ask her direct questions. The PT asked probing questions to be able to understand what the student was thinking. At the end of episode analysis, the PT described a different approach that might have helped the student clarify his thinking. She also noted that she felt she was making inferences.

These three themes—descriptions including static or dynamic positionings, the types of questions asked, and the source of the solutions of the tasks—were used along with research in mathematics education to design a framework for Learning to Notice Positioning that is described in the discussion section. From these themes, I make an argument about the potential positionings of students by the PTs and provide insight into the possible reasons for their instructional choices.

Tertiary Analysis—Trajectory for Positioning

Using the findings from the primary and secondary analysis, I layered the classifications of noticing from PTs' analyses over the positioning that was evidenced in both the written analyses and the video episodes. I will address this layering using the following classifications: positions students as incapable mathematically, positions students as capable of following procedures, positions students as capable mathematical thinkers, and positions students as capable of doing mathematics

Positions students as incapable mathematically. When a PT noticed the baseline level, the interviews contained positionings that used a categorical lens. Explicitly, this appeared in the interview in the language used to describe students and the attention given to teasing out student thinking. Implicitly, the pedagogy used by the PT also demonstrated deficit positionings. For example, students perceived as less capable were asked only gathering information questions that tended to lead students to a particular solution path. Even if the PT did not write about these questions, the questions were still evident when watching the video. Here is an example of all of the writing one PT did for one task with a student described as "low level" in the interview plan:

> Monica has six marbles. How many more does she need to have 13 marbles altogether?
>
> To begin the task, Katie counted out six red counters and placed them in a row. Then she continued to grab yellow counters until she arrived at a total of 13 counters altogether. After reaching 13, she totaled up the yellow counters and said the answer was seven. She was correct.

In the video, I observed the child counting and recounting, and aligning the red counters with the yellow counters in two different ways. The PT asked several gathering information questions: "How many yellow counters are there?" and "How many counters total?" The PT prompted the child twice in the interview, saying, "Why don't you use yellow for the number of marbles she adds?" and "Do you want to compare them by putting them in two rows?" The description ended with an evaluative statement, "She was correct." Through her interjections into the problem solving, the PT positioned the child as incapable of thinking mathematically since she provided the student with little hints or suggestions throughout the interview.

The PTs who noticed at the baseline level used language that characterized the tasks as "easy," "hard," "less difficult," or "more difficult." The use of this language conveyed tacit assumptions the PTs held about mathematics and student learning (e.g., the bigger the number, the more difficult the task). When reflecting on students' work during the interview, these positionings of mathematics and student learning appeared in the writing again when a student solved a task in a way the PT had not expected, or if students did not use the traditional strategy or when s student solved a task in a way the PT had not expected. In these cases, the PT positioned the student as deficient in some way and remarked on what the child could not do or did not do with the task. As examples:

> Although these problems should be able to be solved in the child's head since they are common facts, she was not able to do so. Because of this, she resorted to using her fingers.
>
> For the final problem, she chose to use addition when in fact, multiplication is the correct method. It seems as though she did not take much time to think about the problem and assumed it was addition right away.

These statements provide some insight into how the pedagogical commitments (Erickson, 2011) of the PTs may have influenced what they noticed. The PTs were expecting to see particular methods of solution and were prepared to elicit student ideas about those methods. When a child solved a task using a different method, the PT's attention and interpretation of the events were often focused on making the child's work reflect the expected solution and not necessarily on understanding what the child was thinking. When the PT used the words, "resorted to" and "it seems as though she didn't take much time," the PT frames the strategies the student used as lesser than what the PT expected the student to do—sending a subtle messages are that what the student did was not particularly valued by the PT.

Positions students as capable of following procedures. When PTs noticed at a mixed level, the interviews and analyses demonstrated an expectation for a particular mathematical solution strategy. When students did not solve the task using the strategy PTs expected, PTs frequently made reference to this, using a deficit positioning of the student's thinking or lack of attention in class where the given strategy had been presented. Sometimes the PT ended the interview once students answered the "How did you find that?" question, while some PTs directed students to use the strategy they expected. The excerpt below falls into this latter category:

> The first problem I gave Jillian was a Join Result Unknown problem. I gave her the problem: "Jane had 10 pieces of candy. Jim gave her 6 more pieces. How many pieces of candy does Jane have now?" She replied "seven" without using any manipulative strategies. I repeated the question again, saying all parts of the problem slowly. Jillian then used a Counting strategy, which I predicted she might do in the plan for my student interview. I was surprised that she chose to use a Counting strategy before Direct Modeling. She used her fingers to "Count on" from 10 and whispered "11, 12, 13…" as she held up each finger. Although Jillian had the right idea, she miscounted and answered "15." I then asked Jillian how she got her answer and she replied by saying that she counted six more using "these" (which were her fingers). This tells me that she understood which number she needed to begin with (which was 10) although when I explicitly asked her which number she started with, she answered "15." She then finished

> my sentence as I was trying to elicit her thinking: "Then you said you added…" and "six more" was Jillian's response.
>
> To see if she could use the "Joining all" Direct Modeling strategy, I asked Jillian if the pieces of candy could help her. She replied "yep" and I went on to repeat the first part of the question. She quickly recognized that she could use the pieces of candy to count and help her solve the problem. She understood how to use the Workmat and counted out loud "1, 2, 3…" every time she placed a piece of candy on the mat until she reached 10. I then repeated the second part of the question and without prompting, Jillian added six pieces of candy but this time did not count out loud. This tells me that Jillian was able to correctly set up the problem, however still ended up giving me an answer of "15" when I finally repeated the last part of the question. She did not join the two sets together that she had created. Looking back at my interview now, I wish I would have asked Jillian more thought-provoking questions to direct her to explain how she got her answer and if she was counting in her head while she was placing the pieces of candy on the Workmat or if she just knew based on the base-10 grids on the mat.

In this interview, the PT expected the student to use a direct modeling strategy since she had the Starburst candies on the worktable. Even though the PT understood the strategy used by the child as evidenced by her detailed description in the paper, she insisted the student use a direct modeling strategy. At first, it seemed the PT wanted the student to use the direct modeling strategy as a means of finding a correct answer to the task, but when the interview ended once the child had directly modeled and still had not found 16 as her answer, it was clear that the PT really was pushing the child to use a particular strategy, direct modeling, and not necessarily to find an answer that worked. The PT wrote about how her other students used the direct modeling strategy and that she thought it was, “the best way to do the problems.”

Although this PT described wishing that she had asked Jillian more questions to support her in explaining how she got her answer, I coded this excerpt as positioning the student as capable of following procedures because the PT did not articulate her questions in the text. This was an example of how the categories in my coding overlapped slightly with the next level.

The step-by-step questioning illustrated in this example was also characteristic of PTs who noticed at the mixed level. The PT positions the student as capable of following procedures by asking step-by-step questions: “What number did you start with?” “Then you said you added…” and “Your final answer was …?” PTs who were categorized as doing baseline noticing often suggested strategies. When telling the child to use a particular strategy, the PT positioned the child as not capable of thinking about the mathematics on her own or stated that the student’s ways of solving were inadequate because they were not the strategies the PT would have used or endorsed. Leading the child step-by-step also served to position the child as incapable of thinking mathematically because the PT provided the steps and prevented the students from articulating their thinking. The PT does position the student as capable of following steps or procedures.

Positions students as capable mathematical thinkers. Those PTs noticing at focused levels positioned students as capable of thinking about mathematics. This is seen not only in the ways students are described—as hard workers, for example, but also in the pedagogical choices the PTs made in the interview. In the following example, Lucy is described as a “third grade student” who “tries hard” and was “working on multiplication in class.”

A fourth-grade class needs four leaves each day to feed its two caterpillars. How many leaves would the students need each day for 10 caterpillars?

In Lucy's first attempt to solve the problem, she began by grouping four linking cubes together multiple times. However, not all of the groups had exactly four cubes, one group had five cubes and another group had six cubes. After making 11 groups, she counted all the cubes and found the answer was 46, when in reality the total number of cubes was 47. When I asked, "How did you get that?" Lucy said while pushing the cubes into the pile, "I put four cubes 10 times and I counted them. I think that's how I did it." Because I realized that this explanation did not equal 46, I asked her to "show me how 46 represents four sets of 10 cubes."

At this point in the interview, the student had made several errors. The PT did not point them out, but rather probed the student for an explanation about how she solved the task. When the student again made an error by saying 46 is four sets of 10, the PT positioned her as capable of finding her mistake and thinking about place value or grouping by tens when she asked the child to show her how 46 represented the four groups of 10.

At this point she started over again, and she realized that she had too many cubes and moved some to the side when arranging her 10 groups of four cubes. Lucy then explained, "I put all of these together and counted how much I had." I asked, "How many was that?" Lucy responded with, "I think 64." Then she counted all of the cubes again and said, "40."

The PT allowed the child time to find an answer that she was happy with and never positioned her as someone who was not capable of thinking about mathematics. Even when the child arrived at her answer, and it was not correct, the PT continued to probe to find out how the child was making meaning of the task and the answer.

> After she gave me her answer, I asked, “So what does your answer mean?” She said, “10 caterpillars would eat 40 leaves.” And I then asked, “So, does your answer make sense with the first statement in the problem that two caterpillars eat four leaves?”

The PT continued to probe and did not express doubt that the child could figure out the scaling up of the leaves and caterpillars. The PT chose a phrase in the task that would likely help the student make sense of her response and built a question from it.

> She replied, “I think so. I have four leaves in each group, like the four leaves for the two caterpillars. Since there are now 10 caterpillars, I made 10 groups.” I replied, “Ok, I see what you did there.”

This PT noticed what the child did, but also made connections to what it might mean for the child’s mathematical thinking and understanding. She referenced specific events in the interview to highlight student thinking. The PT used appropriate mathematical language when describing the child’s work and also the mathematical ideas that served as the foundation for this task, which helped her to make connections to the child’s mathematical thinking. Of note is that this PT did not just end the interview when the child gave her first answer to the task, but also pressed the child to consider her thinking and her answer. The PT also wrote without evaluating Lucy’s work; she described it and interpreted it to unpack the student’s thinking. In her writing, this PT demonstrated that she perceived Lucy as being capable of thinking mathematically, finding her own errors, and assigning meaning to her work.

The PTs noticing at focused levels commonly spent significant time trying to understand what the students were doing and allowed them to work out solutions for themselves. The PTs did not lead students, but rather, worked to understand the students' ideas. These interview analyses were characterized by PTs' interjection of their thinking and reasoning in the midst of the writing about how a child solved a task. Even when a child did not find a correct answer, the PTs' writing focused on the students' thinking, not on the incorrect responses.

Positions students as knowers and doers of mathematics. The PTs who noticed at extended levels connected teaching and learning, provided elaboration on interactions, and often proposed reasonable alternative pedagogical strategies in their written analysis. Those who noticed at this level reflected on student's thinking and on how pedagogical decisions impacted that thinking. The PTs noticing at this level credited students for thinking and when there were issues, the PTs focused attention on the inadequacy of the teaching, rather than the students. The PTs whose work is categorized as positioning students as knowers and doers of mathematics used language that focused on what the student knew and could do. When the PTs questioned the students, their questions asked for justification and reasoning.

The following example highlighted the attention to students' thinking and connections to pedagogy—both in the larger class where the PT observed and in her own pedagogical choices:

> The second question I asked Grace was, "A store has 26 basketballs, which they want to pack into boxes so that there are three balls in each box. If they fill as many boxes as possible, how many balls will be left over?" Grace solved this problem by long division which is the strategy her class has primarily been using.

Here the PT connected the strategy the child was using back to the classroom practices.

> From Grace's work, it was evident that she understood the process of long division and got the correct answer of 2. However, from looking at her work, I noticed that she had placed the 8 above the 2 in 26 instead of above the 6 where she wrote the remainder of 2.

The PT noticed that although the child had solved the task and found a correct answer, her placement of the numbers in the long division algorithm did not show an understanding of place value. The PT made a connection back to the classroom and connected a strategy that her cooperating teacher used to help address this error.

> This showed that Grace didn't fully understand place value as the 3 went into 26 eight times, not 3 going into 2 eight times. Because this is a common problem occurring in her class, the teacher has had them position the loose leaf so that the lines are going up and down so that the students can use these columns as place holders when solving problems to try and avoid errors such as the one Grace made with lining up the numbers correctly. I think this technique might have helped Grace although she may still need to understand the concept of place values.

The PT's next move demonstrated that she positioned the child as a doer of mathematics. By creating a problem on the spot to help the child confront her misunderstanding about place value, the PT allowed the student to keep ownership of her ideas, and the student rose to the expectation.

> So, I decided to give her another problem that was already solved, but had the same misunderstanding she showed in her problem. It was just a long division problem not a word problem. I solved the problem 23 divided by 4 and put the 5 over the 2 and the remainder of 3 over the 4 and asked her to find my mistake.

The PT created a problem that presented the same issues Grace had confused earlier.

> Grace looked at the problem and said it was mostly right. When I asked her what she meant my "mostly" she said that I needed to put an "R" next to the 3 to show that it was the remainder. When I drew the R in between the 5 and the 3, she giggled and said, "no

> not like that" and erased the 5, the R, and the 3 and placed them where they should go.
>
> So, I asked her why she moved everything and she told me that the 5 was too big and wouldn't go into the 2, but it would go into 23, so it had to go over the 3.

This PT could have stopped when the student solved the original task, but she continued to investigate this students' learning and positioned Grace as someone capable of talking about place value, finding mistakes, and even explaining her thinking.

Only PTs who noticed at extended levels positioned students as knowers and doers of mathematics. These PTs made connections in writing between teaching and learning, often elaborated on their interactions, and frequently provided alternative instructional moves and strategies in their written analysis. These PTs reflected on student's thinking and also on how their instructional choices impacted that thinking. PTs noticing at this level focused attention on how they could have improved questioning, rather than how the students could have responded differently. The PTs whose work is categorized as positioning students as knowers and doers of mathematics used language that focused on what the student knew and could do. PTs' questions frequently asked for justification and reasoning.

Discussion

As a result of the third level of analysis, I propose a Framework for Learning to Notice Positioning, which brings together how PTs notice student thinking and how they position students (see Table 6). The themes described in the secondary analysis are the categories for each of the rows. The headings of this framework help identify the positionings inherent in each of the cells. There was consistency between the levels of noticing and the ways PTs positioned students, however, these categories were not completely distinct. I found that some PTs wrote about their questions and identified them as leading or recall questions, but then provided suggestions for higher-level questions they could have asked instead. Other PTs did not provide alternative questions, perhaps because they did not notice their questions were all recall or process-oriented questions. I thought this was a crucial aspect of learning to notice teaching and positioning, being able to identify the questions and provide alternatives. I wanted the analysis to capture PTs' ability to identify question types and provide alternative questions. I revised the codes to include this aspect of noticing and recoded the written interviews accordingly.

Table 6

Framework for Learning to Notice Positioning

	Positions students as incapable mathematically	Positions students as capable of following procedures	Positions students as capable mathematical thinkers	Positions students as capable of doing mathematics
How teachers describe students and their expectations for student approaches to the mathematics	Uses categorical language that has a deficiency prospective (i.e., high/ low-level or high/low-achieving)	Language focuses on student characteristics or what students cannot do	Language focuses on what students do, but without classifying or categorizing them	Language focuses on what students know and can do mathematically
The kinds of questions teachers ask to elicit student thinking, and the teachers' attention to the impact of these types of questions	Uses questions that involve recall only or one-word responses (i.e., yes/no; specific number), but overlooks them in the retelling or minimizes their importance	Uses questions that focus on procedures or routines ("what" questions), but does not recognize that these questions lead students to a solution	Uses some recall or procedural questions, but provides alternative questions in analysis. Uses questions that focus on how one approached the mathematics – some justification required	Occasionally uses some recall or procedural questions, but provides alternative questions in analysis. Uses questions focused on why this strategy works – justification and reasoning required
Where solutions for tasks come from, and the PTs' attention to the impact of their pedagogy	Teacher solves task for students and tells them how to do it, but does not notice this in the retelling	Suggests possible solution strategies for students to use, but does not notice this in the analysis	Students primarily responsible for solution strategies, but teacher provides scaffolding when students are stuck	Students solve tasks independently or in collaboration with other students

Some PTs asked questions that required students to justify their solution strategies, positioning them as capable of thinking mathematically. Those noticing at the baseline level asked gathering information questions that required only one or two words in response, and PTs using these questions positioned students as capable of restating facts. The PTs categorized with mixed noticing used questions about procedures or a step-by-step process marked by “what” or “What did you do next?” The PTs who used these types of questions positioned students as able to follow procedures. The PTs with focused and extended noticing asked questions that prompted students to reason, such as “how” and “why.” Despite having set the parameters for interviews to probe but not teach, PTs at the baseline and mixed levels often used leading or funneling questions to get the students to solve a task or provided hints for students so that they could be successful. The PTs noticing at the focused or extended levels did not insert themselves into the solution process and rather positioned students as capable of explaining their thinking. Some PTs positioned students as unable to use a particular strategy, but then suggested a less sophisticated strategy the PT expected the student to use. These PTs often wrote out possible strategies any student might use in the prediction section of their analyses, but did not link these strategies to expectations for individual students.

Much like the van Es (2011) framework, there was some overlap between levels of noticing in my data. For example, van Es (2011) described baseline noticing and mixed noticing as having similar characteristics in terms of how teachers noticed. Teachers noticing at the baseline level provided descriptive and evaluative comments. At the mixed level, van Es described teacher noticing as primarily evaluative with some interpretive comments. At the focused and extended levels, only interpretive comments were described (p. 139). A similar overlap existed in the ways PTs positioned students and how aware they were in their analysis about their practices. I drew on my analysis of PTs’ noticing, research in mathematics education on questioning strategies, and research on positioning to create the final descriptions.

The video proved to be an important aspect of the assignment both for the assessment of PTs' work and in the analyses for this chapter. First, one inherent value of using video in teacher education is the ability to watch a segment repeatedly and perhaps view it with a different lens on subsequent viewings. Second, since the written paper captured what PTs noticed and wrote about, having the video helped me to identify what PTs either did not notice in their interviews or did not write about in their analyses. Third, when analyzing the data for this paper, the videotapes were helpful in supplementing the written analyses to verify the categories of the secondary analysis and collect information about the PTs' practices. There was evidence in this study that PTs' noticing might be related to the ways they positioned students and that the ways they positioned students influenced their practices. As teacher educators, there is evidence from this study to suggest that PTs working in certain levels of noticing might be more likely to position students in unproductive ways.

I offer this positioning framework as a way to build on research about teacher noticing and to connect positioning theory to other existing research in mathematics education. This framework draws on research about teacher noticing, cognitive demand, and questioning strategies to help educators better understand how to attend to the explicit and implicit positioning of students. Unintentional, deficit positionings of students can impact students' opportunities to learn mathematics. This study suggests that the types of questions used in classrooms can inadvertently send a message to students that we believe they are not capable of knowing and doing mathematics. Decreasing the cognitive demand required for a task may also have the effect of positioning students as incapable of doing mathematics without these hints or guidance.

This study suggests continued investigations about how explicit and implicit positioning may unconsciously impact PTs' instructional choices. This study also supports continued research into the use of video for teacher preparation. Many PTs in this study reported having a different perspective of the events in the student interview after they looked at the video. This study also suggests continued research into how to develop PTs' abilities to notice student thinking and the connections between student thinking and instructional choices.

CHAPTER 5: DISCUSSION AND IMPLICATIONS OF THIS STUDY

Since research using the conception of defining a core set of practices (e.g., Ball et al., 2009; Lampert, 2010; Lampert et al., 2010; Sleep et al., 2007) is emerging, there is a need for research on how these conceptualizations impact the work of teacher educators in these contexts. This study offers perspectives from one MTE's practice while working to operationalize current conceptions of practice-based teaching.

In the first manuscript,[11] I presented a tool, Lesson Mapping Tool, LMT I designed in collaboration with my prospective teachers to support them in learning to plan to teach by focusing on student thinking in order to thread the mathematical goal throughout their lessons. The mathematics education community has found that focusing on students' mathematical thinking during planning is a critical aspect of part of improving instruction and student achievement (e.g., Lampert et al., 2010; Stigler & Hiebert, 2009). The Lesson Mapping Tool drew from this research and was designed to help prospective teachers "see" the flow of a lesson in order to focus on the mathematical objectives while planning using anticipated student thinking. The tool helped prospective teachers to focus on threading the goal of the lesson through each phase of lesson planning. While there are many tools to assist practicing and prospective teachers to think through the necessary steps of planning mathematics lessons (TTLP; Smith, Bill, & Hughes, 2008; Stein et al., 2000), many are quite long and involved. Although there is an emphasis on the mathematical goal, and the types of questions teachers could ask to help students engage with the mathematical goal, these tools are often overwhelming for prospective teachers due to their length and the number of steps a teacher must

[11] Since each of the chapters in this dissertation study includes a full discussion section, I limit my comments here to avoid redundancy.

take to plan. *The 5 Practices for Orchestrating Productive Discussions* (Smith & Stein, 2011) begins with describing the first practice of anticipating what students will do with the task. In my experience, however, many practicing and prospective teachers write a goal and then proceed with the activities they are going to *do* with students, without allowing these goals to directly impact each of the instructional decisions they make in a lesson. I concur with Smith and Stein (2011) when they wrote,

> Some of the teachers with whom we have worked have argued that determining the mathematical goal for the lesson should be "practice 0," suggesting that it is the foundation on which the five practices are built. We agree that setting the goal for the lesson is indeed an a priori practice-it must occur before enacting the five practices. (p. 13).

Many of the lessons I observed in my methods classes as well as with practicing teachers, only tangentially connect to the previously written goal and focus more on *doing* the activity, but not *why* teachers are choosing that activity in connection to the goal. The LMT was designed to be brief and focus primarily on the connections between the goals and all other aspects of the lesson: student strategies, the mathematical connections to these strategies, key mathematical ideas, and the questions prospective teachers ask, to make their instructional decisions explicitly connected to the lesson goal. The focus of the LMT, which sets it apart from other tools for lesson planning, is that it focuses explicitly on the connections between the goal of a lesson and all of the other facets of a lesson. It could perhaps be used with both practicing and prospective teachers to understand this one essential aspect of lesson planning.

A next step would be to create tools for observation and reflection that explicitly reference the connections to the mathematical goals at every point during the lesson. These tools could be used when debriefing a lesson to help practicing or prospective teachers to understand where opportunities exist to make the connections to the goals explicit. Another helpful reflection tool could be created for after planning but before teaching, that would help practicing and prospective teachers think about how explicitly their plans reflect the key mathematical ideas. When teachers, such as the one I describe in the introduction, teach lessons without understanding how the goal could and should drive instructional choices in the lesson, students lose out on powerful learning opportunities and often miss the point of a lesson.

In the second study, I argued that teacher education is a hybrid space between K–12 and higher education with respect to feedback practices. In higher education settings students must *seek out* feedback or *determine* how the instructor assesses learning (Carless & Yang, 2012). Formative feedback in the K-12 classroom, however, is expected to be *given by the teacher* (Shute, 2007). I posit that prospective teachers need to see MTEs model effective feedback practices during teacher preparation, in other words, PTs need to receive the feedback from MTEs, be supported by MTEs to use it, perhaps through revisions, and then taught how to provide feedback to their own students by MTEs.

I analyzed the characteristics of the feedback my prospective teachers found useful in revising their lesson plans and found that the content and form of the feedback was essential for it to be used by prospective teachers in revising their plans. My findings aligned with research on feedback practices in both higher education and K-12 contexts. The feedback most often used by PTs to make revisions was clear and focused feedback about PTs could improve their work (Evans, 2013; Cohen, 1995). As I examined the feedback I provided, I studied the characteristics of feedback that prospective teachers took up and used to make revisions as well as how they used that feedback to edit their lesson plans and found that my research confirmed finding in research in both the K–12 and higher education literature on effective feedback practices. I found that formative feedback had a more immediate effect than summative feedback on prospective teachers' revision process. In line with other research, (e.g. Voerman, Paulien, Meijer, Korthagen, & Simons, 2012), I found that praise—both specific and nonspecific seemed to have little influence on PTs' planning practices. When prospective teachers were given multiple opportunities with specific feedback connected to the purpose of the lesson to anticipate multiple solution strategies, they seemed to articulate measurable goals and plan how to make connections between student ideas and the goals of the lesson. This process of providing formative feedback and offering opportunities for revision is supported by Shute (2007). Some characteristics of feedback that were not described by previous research included feedback in the form of a question; feedback that referenced a familiar resource; and feedback that prompted the PTs to consider students. These findings add to the research base by providing the characteristics of feedback in teacher education settings that influenced PTs to make revisions.

The expectation of teacher education is to prepare prospective teachers to be well-started beginners. The practice of providing formative feedback (Black & Wiliam, 1998) to students about their growth relative in understanding the key mathematical ideas, should be a ubiquitous feature of teacher preparation programs. The findings in this study demonstrate that specific, targeted feedback led PTs to make meaningful revisions. This study's findings suggest that teacher educators should incorporate repeated formative feedback exchanges with opportunities for PTs to revise their work. Knowing that effective formative feedback has several crucial features—is open-ended, is written in question form, invites further exploration—is important for all teacher educators to understand and learn how to model with prospective teachers.

In the third manuscript, I used the existing framework for Learning to Notice (van Es, 2011) and added a lens of positioning to examine the relationship between the types of positionings my prospective teachers used and their instructional practice. The research on positioning theory (Harré & van Langenhove, 1991) provides the opportunity for teacher educators to examine their own practice. This study suggests that the types of questions used in classrooms can inadvertently send a message to students that we believe they are not capable of knowing and doing mathematics. By adding positioning theory to the Learning to Notice Framework (van Es, 2011), MTEs can prepare PTs not only to ask questions, but to help them consider how their questions might unintentionally position students. There was evidence in this study suggesting that PTs' noticing might be related to the ways they positioned students and that the ways they positioned students may influence their practice. As teacher educators, there is evidence from this study to suggest that PTs working in some levels of noticing might be more likely to position students in unproductive ways.

I offered a Framework for Learning to Notice Positioning as a way to build on research about teacher noticing and to connect positioning theory to other existing research in mathematics education. This framework draws on research about teacher noticing (e.g. Sherin & van Es, 2005; Star & Strickland, 2008), cognitive demand (e.g. Smith & Stein, 1998), and questioning strategies (Boaler & Brodie, 2004) to help educators better understand how to attend to the explicit and implicit positioning of students. I propose that mathematics teacher educators spend time understanding how positioning theory could support noticing their own, and prospective teachers' unconscious positioning of and learn how to use these opportunities to address issues of equity as they occur in the moment since unintentional, deficit positionings of students can impact students' opportunities to learn mathematics. This study suggests that the types of questions used in classrooms can inadvertently send a message to students that we believe they are not capable of knowing and doing mathematics. Supporting prospective teachers to notice the implicit messages questions send to students is vital to address issues of equity. When a prospective teacher is aware why decreasing the cognitive demand required for a task may have the effect of positioning students as incapable of doing mathematics without these hints or guidance, I argue that they are more likely to attend to these moments in their practice.

Implications of this Work

For Researchers

This study suggests continued investigations are necessary in order to understand how explicit and implicit positioning (Erickson, 2011) may unconsciously impact PTs instructional choices. This study also supports continued research into the use of video in teacher preparation (e.g. Sherin, 2007; Sherin, & Han, 2004). Many PTs in this study reported having a different perspective of the events in the student interview after they looked at the video. Additional research about how to use video effectively to support prospective teachers in analyzing their own teaching as well as to learn from the teaching of others is suggested by this study. This study underscores the need for continued research into how to develop PTs' ability to notice student thinking and the connections between student thinking and instructional choices.

When teaching classes using a core set of practices (e.g. Windschitl et al., 2009) as the focal point, research on how teacher educators address issues of equity would be helpful—at the goal level as well as at the activity level. For example, one issue I confronted in this study was that when my PTs rehearsed in my university classroom, they rehearsed their lessons with one another; however, my PTs did not represent a wide range of backgrounds and experiences. As future teachers of elementary school students who already know algebra, they were hard-pressed to find alternative solution strategies students might use. Talking with peers was not particularly helpful since their peers had similar experiences. So, practicing teaching in the university classroom and writing lessons for teaching in the university classroom can be very different than planning and teaching in K–6 classrooms. The ways I approached this in my classroom were unsatisfactory and, in some instances, even served to essentialize and stereotype students' experiences. The field needs research that focuses on how teacher educators use practice-based methods while addressing concerns about equity.

In many ways, this study is really a starting point for future research. There were two tools generated through this dissertation study. Both of these tools were designed by analyzing the work of prospective teachers, meaning that the work of prospective teachers provided ideas for the tools, but was not used to validate the tool. How might the tools provide useful ways to analyze data from the same assignment from teacher educators within this same university? Could the tools be used for similar assignments from other universities? How can this tool be used by other teacher educators to support planning efforts? How can the Learning to Notice Positioning Tool be used for additional quantitative research on the ways prospective or practicing teachers position students? How can the Learning to Notice Positioning Framework be used outside teacher education in the classes of arts and science faculty classes, or in practicing teachers' classrooms?

For Teacher Educators

My work provides practical tools that mathematics teacher educators can use in their own practice. My experience working with both experienced teachers in classrooms and prospective teachers underscores the difficulty many teachers have teaching lessons that begin with a meaningful, measurable goal and leverages student thinking to subtly guide students toward understanding. The Lesson Mapping Tool is an example of a tool to help support prospective teachers to think about the overall purpose of the lesson and to intentionally plan connections to the goal throughout the lesson. The LMT presented in this study could be used by MTEs to help practicing and prospective teachers focus on the key mathematical concepts, allowing the mathematical ideas to drive instruction (Sleep, 2009). The visualization of the flow of a lesson and how each of the components of a lesson connect back the goal may help PTs to keep the mathematical goal in the forefront of their mind when planning. Researchers (Hiebert, et al, 2007; Putnam & Leinhardt, 1986; Stigler & Thompson, 2009) have hypothesized about the appropriate time and possible approaches to teach novice teachers how to use lesson goals to guide instruction, the LMT from this study provides a template to help MTEs support PTs to visualize how to make the connections to the goals stand out. Since every aspect of the LMT is designed to maintain a focus on the key mathematical ideas, prospective teachers could begin to understand the difference between *covering content* and *facilitating students' learning* of the key mathematics. Perhaps having a brief, big-picture view of a lesson that maps to goals rather than tasks, activities, or discussions (TTLP; Smith, Bill, & Hughes, 2008; Stein et al., 2000) could provide the opportunity for teachers to remain focused on the goal while in the midst of teaching. Given that little research examines the use of the TTLP in methods courses and its influence on teacher learning and practice, continued empirical research on this form of lesson planning could provide powerful insights to the field.

The LMT tool could likely be adapted to other content areas and used by both prospective and practicing teachers as a way of thinking about lessons that revolves around moving student thinking toward the goals of lessons. The student strategy section might work well in science classes, and teachers could modify this section to connect better to humanities classrooms when considering student perspectives. Using the Lesson Mapping Tool before, during, and after planning could help teachers be proactive in planning lessons that reach their intended goals as well as analyzing why lessons might not have reached the intended goals.

Teacher educators might adapt and use the second tool provided through this study—the Framework for Learning to Notice Positioning of Students—when working with prospective or practicing teachers as well. Through this study, I found that I was not particularly aware of the ways I positioned my own students through my feedback. This lack of awareness is likely true for other practicing teachers and teacher educators as well. Having practicing and prospective teachers examine their practice for hidden positionings could provide opportunities to talk about practice in different ways. For example, consider how the types of questions asked impact students' opportunities to learn (Boaler & Brodie, 2004). Teacher educators could use the ideas in this framework to shift the conversations with prospective teachers from categorizing question types to talking about why it is important to choose the types of questions that will help students engage meaningfully with the content.

My work demonstrates a need in the field for more research on the practices of teacher educators and especially in relationship to studying the impact of their work on the practice of prospective teachers. When teacher educators use current literature and tools in classrooms and in their own teaching, it would be helpful to offer these tools to other teacher educators so as to enable them to learn from their practice. How, for example, have other teacher educators tackled the problem of teachers missing the mathematical point of a lesson when teaching? What tools do they use? How do they use them? I would be interested in knowing what tools teacher educators use to support prospective teachers to consider how the tasks they select or the questions they pose might have an impact on their future students. This study helped me to see that I really want to know more about the practices of other teacher educators. Sharing these resources openly would be a powerful way to build on and improve individual and collective practices for teacher education.

Limitations of this Study

A major limitation of this study is that I did my work alone. I designed the course, taught the classes, assessed PTs' learning, and analyzed the data by myself. At times, it was difficult to separate my voice as a researcher from my voice as a teacher educator. How did I value particular ways of knowing and doing that biased my analyses? How did my experiences and purchase on the research in mathematics education limit the choices I made when teaching and researching? Working with a team of educators to study the practices of teacher education would provide a broader lens through which to view my teaching practice. Having others to code the data would provide a stronger case for the trustworthiness of my findings.

This study drew on two sections of PTs from one university. They were seniors and had many previous experiences inside the university that were beyond the scope of the study. These facts could account for some of my findings but investigating these were beyond the scope of my study.

My Learning as a Researcher and Teacher Educator

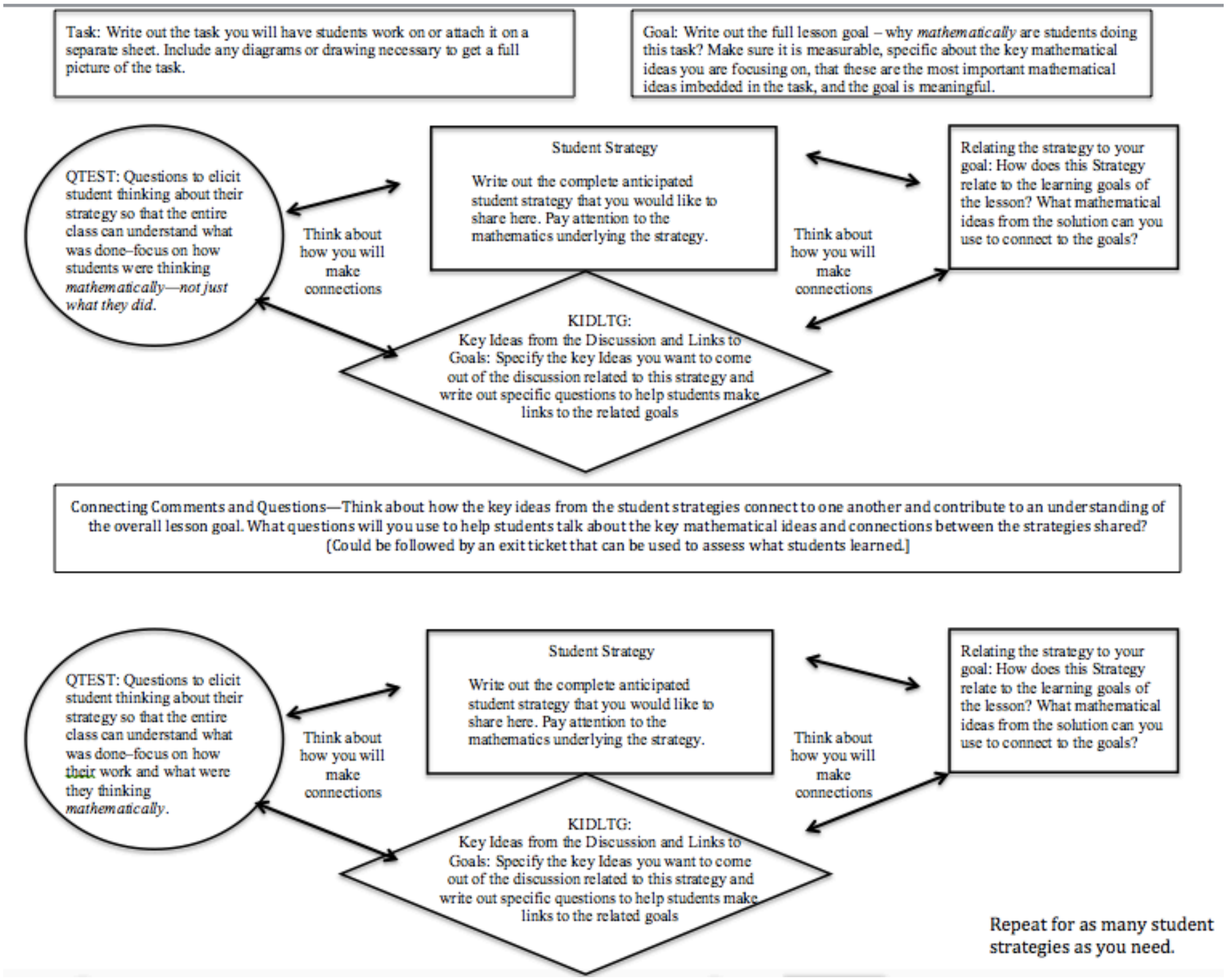
Examining my practice left me feeling vulnerable at times. When I had concrete evidence, for instance, of how I used praise in the classroom, I was shocked and a bit disappointed with myself. Writing about aspects of my practice that were not what I had hoped or envisioned was difficult. This also gave me the opportunity to look at my practice and know that this was a snapshot of it and provided me opportunities to grow in my work as a teacher educator. Considering the ways PTs positioned their students made me very aware of the ways I positioned my PTs. I did not consider how my work affected my own students as deeply as I needed to in order to complete this study, but by studying my own practice, I found opportunities to become more effective by using this lens of positioning.

Writing an argument using a critical lens toward research that I have read and felt resonated with my philosophy of teaching made me question my assumptions about teaching and learning. I selected the work on noticing and high-leverage practices, for instance, because I thought both of these areas were fascinating and provided a lot of useful information that informed my practice. Having spent time reviewing the literature in these areas and then analyzing my own practice through a critical perspective forced me to examine what I really believed and why. This critical perspective did not allow me to let myself or other researchers off the hook easily. I began to see how the perspective of a researcher and a teacher educator were different yet informed one another.

I have always enjoyed teaching and did not think that research was my "thing," which is why I thought that researching my own practice would be a fruitful place for me to put my efforts. This study helped me to see the value of stepping back and analyzing my own practice, but also stepping back and seeing the artifacts from my class as objects of study. Although that transition was difficult, I think it was a powerful experience and helped me to understand how research on teaching might actually inform teacher education practice. This study also helped me to see the power of encouraging prospective teachers have a voice and take ownership of the process of learning to teach.

APPENDICES

APPENDIX **A: LMT Template**



APPENDIX B: Rubric for PTs to Self-Assess their LMTs

POINTS	GOAL	STRATGEY	QTEST	KIDLG	CCQ
0	Goal does not encompass important mathematical ideas. Goal is not specific about the mathematical ideas you are trying to elicit. Goal is not measurable.	Only one or no reasonable student strategies provided, and no details are given about what students were thinking.	No questions related to the student strategies are provided or questions do not help elicit student thinking.	Key mathematical ideas for the discussion are and links to the goals are missing. No questions are provided to elicit these ideas from students.	Only a restatement of the goal is included, but no questions eliciting how the student strategies connect to the goals of the lesson are provided.
1	Goal encompasses important mathematical ideas. Goal is somewhat specific to the mathematical ideas you are trying to elicit. Goal is not clearly measurable.	1–2 student strategies provided, but few details are included about student thinking or how student arrived at this particular strategy.	Questions are specific to the individual strategies provided. Questions serve to elicit processes, but not student thinking. No consideration is given to how students might respond to previous questions.	Key mathematical ideas for the discussion are highlighted, but links to the goals are not clear. Questions are included, but connections to the goals are not clear. Questions focus on process only.	Some statements and questions eliciting how the student strategies connect to the goals of the lesson are provided. Questions will likely serve to link to the goals of the lesson.
2	Goal encompasses important mathematical ideas. Goal is specific to the mathematical ideas you are trying to elicit. Goal is measurable.	2 or more student strategies provided, and clear details about student thinking are included in the student strategy.	Questions are specific to the individual strategies provided. Questions serve to elicit student thinking and not just processes. Questions reflect thoughtfulness about how students might respond to previous questions.	Key mathematical ideas for the discussion are highlighted with clear links to the goals of the lesson. Questions that will help to elicit these ideas from students are included.	Questions eliciting how the student strategies connect to the goals of the lesson are provided. Questions will likely serve to have students discuss the key mathematical ideas across student strategies and how these ideas link to the goals of the lesson.

APPENDIX C: TE 402 Student Interview Assignment

One major thread of this seminar is the examination of the ways children think as they build an understanding of the number system. One way to learn about children's thinking is to listen carefully to individuals as they articulate their thoughts while doing mathematical tasks and then asking appropriate questions to elicit their thinking.

The math interview is one strategy for obtaining evidence about children's thinking. Doing interviews helps you develop a sense of the kinds of responses that most students at a certain age or grade might give. An interview also helps you delve into the thinking of an individual child, to get underneath the surface of the child's responses to better understand his or her reasoning.

Our class textbook (*Children's Mathematics: Cognitively Guided Instruction*) showcases a particular approach to conducting student interviews—often called "clinical interviews." In class, we have been referring to them as CGI interviews. One important characteristic of these kinds of interviews is that the interviewer attempts to stay neutral and does not attempt to steer students toward a particular mathematical conclusion or influence their thinking in any way. Instead the goal is to allow the children to show and share what they know and can do with a given task. The interviewers' role is to listen and to ask follow-up questions if and when they are not certain they understand what the child is doing.

This assignment will give you an opportunity to work on your skills for listening to and analyzing children's thinking in the manageable setting of a one-on-one interview. Insights you gain from this interview experience will help you to better analyze and assess children's thinking in a whole class setting. We are not suggesting that a teacher should interview all students about every major concept. Rather, the interview can be an assessment tool used periodically with a small sample of your students to give you a sense of the ideas and issues that most children in your class are trying to figure out. Interview techniques can be used with individuals or small groups during independent or small group work times. They can also help you figure out how to support struggling students.

You will select three tasks/problems from our class textbooks, videotapes we have watched, or from activities we have done together in class to try out with at least three students in your field placement.

Although you will need to plan questions and tasks in advance, you will also need to follow carefully what the student does and says during the interview, so that you can follow up with questions or problems that seem appropriate. **Keep in mind that your job in the interview is to find out as much as you can about the student's ideas, <u>not</u> to try to teach the student anything. Videotape the interview highlighting the student workspace so it can be clearly seen in the video.**

STUDENT INTERVIEW PLAN: Draft is DUE by Jan 30 … then revise (if needed) and include along with your Student interview write-up due by Feb 13)

Include the following:

- The tasks the students will work on in the interview
- Why you chose to try these tasks
- Materials you will have available for students to use
- Who you are planning to interview (use pseudonyms)
- Expectations you have for how each of your focus students might approach the tasks/problems and questions
- List of potential follow-up questions you might ask the students, if their thinking was not clear to you, that will help you to better understand their thinking

STUDENT INTERVIEW WRITE-UP and VIDEO SUBMISSION: Due (By February 13 before class with revised interview plan)

After having ***watched the video recordings multiple times***, select one child to write about in detail.

Your analysis should include the following three parts:

- Part I: Write an analysis of your assessment of the child's mathematical thinking about the tasks provided. Provide evidence from the interview (transcribe several exchanges between you and your interviewee to show this).
- Part II: Write an analysis of how well you felt you elicited student thinking during the interview. Again, provide evidence from the interview (transcribe several exchanges between you and your interviewee to show this).
- Part III: If you were not happy with your eliciting of student ideas, provide some ideas about what you could have done differently?

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