CONCAVE FILLINGS AND BRANCHED COVERS

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ABSTRACT

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This dissertation contains two results. The first result involves concave symplectic structures on a neighborhood of certain plumbing of symplectic surfaces, introduced by D. Gay. We draw the contact surgery diagram of the induced contact structure on boundary of a concave filling, when the induced open book is planar. We show that every Brieskorn sphere admits a concave filling in the sense of D. Gay and the induced contact structure on it is overtwisted. We also show that in certain cases a (-1)-sphere in Gay's plumbing can be blown down to obtain a concave plumbing of the same type. The next result examines the contact structure induced on the boundary of the cork W_1, induced by the double branched cover over a ribbon knot. We show this contact structure is overtwisted in a specific case.

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Chapter 1

Introduction

This Dissertation contains two results. The first result involves concave fillings of contact 3-manifolds. In [8] D. Gay introduced a procedure for handle-by-handle construction of a concave symplectic structure on a neighborhood of a certain type of plumbing of symplectic surfaces- called a positive plumbing- in a symplectic manifold (X, ω). His method also specifies the induced contact structure on the boundary of this type of plumbing by its compatible open book. We demonstrate how to draw the contact surgery diagram of these contact structures in the case when the compatible open book is planar (by looking at the induced open book on the Brieskorn sphere $\Sigma(2,3,5)$ as an example). A natural question is which 3-manifolds can be presented in this way as boundary of concave fillings. We show that every Brieskorn manifold has a surgery diagram as a positive plumbing and therefore its neighborhood in a symplectic manifold carries a concave structure. We show that the contact structure induced on Brieskorn spheres by their concave fillings is always overtwisted. We also show that in certain cases a (-1)-framed sphere in the concave filling of a contact 3-manifold can be blown down to obtain a smaller concave filling of the same contact 3-manifold.

Any 3-manifold presented as a regular p-fold branched cover of (S^3, ξ_{st}) over a transverse knot K, can be assigned a natural contact structure induced by the cover. Our next result involves such a contact structure on the boundary of the Akbulut cork W_1, induced by considering the boundary as a double branched cover over a ribbon knot K and considering a specific transverse realizations of K obtained by braiding. We show that for certain braiding of the ribbon knot K the induced contact structure is overtwisted. Our motivation for this problem was to consider a naturally induced contact structure on ∂W_1 and examine its tightness. Using the right-veering criteria for monodromy of open books, we show that in a certain case the induced contact structure is in fact overtwisted.

Chapter 2

Background

In this chapter we review the background results that are needed for our main results.

2.1 Contact structures

For an introduction to contact structures and open books on 3-manifolds, the reader is advised to [1].

Definition 2.1.1. Suppose Y is a $(2n+1)$-dimensional manifold. A 1-form $\alpha \in \Omega^1(Y)$ is called a *contact form* if $\alpha \wedge (d\alpha)^n$ is nowhere zero. A $2n$-dimensional distribution ξ is called a *contact structure* if it locally can be written as $\xi = \ker\alpha$.

We will only work with contact 3-manifolds and from now on most of our definitions and examples involving contact manifolds will be limited to this case only.

Example 2.1.2. The standard contact structure ξ_{st} on $\mathbb{R}^{2n+1}$ with coordinates $(x_1, y_1, ..., x_n, y_n, z)$ is given as $\ker(dz + \Sigma_1^n x_i dy_i)$.

Example 2.1.3. The standard contact structure ξ'_{st} on S^3 thought of as the unit sphere in $\mathbb{C}^2$ is defined as $\xi_{st} = TS^3 \cap i(TS^3)$. Using coordinates $(r_1 e^{i\theta}1, r_2 e^{i\theta}2)$ on $\mathbb{C}^2$, we can also describe this contact structure as $\ker(r_1^2 d\theta_1 + r_2^2 d\theta_2)$.

Definition 2.1.4. Two contact $3-$manifolds (Y, ξ) and (Y', ξ') are called *contactomorphic* if there is a diffeomorphism $f : Y \to Y'$ such that $f_\star(\xi) = \xi'$. If $\xi = \ker\alpha$ and $\xi' = \ker\alpha'$, this is equivalent to existence of a nowhere zero function g on Y such that $f^*(\alpha') = g\alpha$.

Two contact structures ξ and ξ' on a manifold Y are said to be *isotopic* if there is a contactomorphism $h:(Y,\xi)\to(Y,\xi')$ which is isotopic to the identity.

Example 2.1.5. One can show that for a point $p\in S^3$, $(S^3-\{p\},\xi'_{st})$ is contactomorphic to $(\mathbb{R}^3,\xi_{st})$.

Example 2.1.6. Let $\alpha' = dz+xdy-ydx = dz+r^2d\theta$ and ξ_{sym} =kerα'. We call ξ_{sym} the symmetric contact structure on $\mathbb{R}^3$ (One can see that the contact planes are symmetric with respect to the z-axis). This contact structure is contactomorphic to the standard contact structure ξ_{st} on $\mathbb{R}^3$. Refer to [1] for more details. We will come back to this contact structure later in 2.1.9..

Definition 2.1.7. Suppose that (Y,ξ) is a given contact 3-manifold. A knot $K\subset Y$ is *Legendrian* if the tangent vectors TK satisfy $TK\subset\xi$. In other words $\alpha(TK)=0$ for the contact $1-$form α defining ξ. The knot K is *transverse* if TK is transverse to ξ along the knot K, i.e. if $\alpha(TK)$ is nonzero. The contact framing of a Legendrian knot is defined by he normal of ξ along K. Equivalently, we can take the framing obtained by pushing K off in the direction of the vector field transverse to K which stays inside the contact planes. This framing is called the *Thurston-Bennequin framing* of the Legendrian knot K denoted by $tb(K)$. Another invariant of a Legendrian knot, *rotation number* $rot(K)$ can be defined by trivializing ξ_{st} along K and then taking winding number of TK. For this invariant to be well-defined we need to orient K and then the result will change sign when orientation is reversed.

We can study Legendrian knots in standard contact $\mathbb{R}^3$ (or S^3) via their *front projection.* Namely, for ξ_{st} =ker$(dz+xdy)$ and a Legendrian knot $K\subset(\mathbb{R}^3,\xi_{st})$, we consider its projection onto the yz-plane. Notice that the front projection has no vertical tangencies as $\dfrac{dz}{dy}=-x\neq\infty$. For the same reason, at a crossing the strand with smaller slope is in front.

For instance the following is the front projection of two different Legendrian unknots:

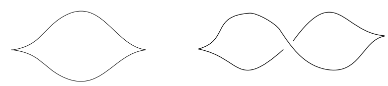

Figure 2.1: front projection of two Legendrian knots

Lemma 2.1.8. *For a Legendrian knot $K \in (\mathbb{R}^3, \xi_{st})$, we have the following formula for the Thurston-Bennequin number of K: $tb(K) = w(K) - \frac{1}{2}c(K)$, where $w(K)$ is the writhe of K and $c(K)$ is the number of cusps in the front projection of K.*

Proof. We note that the vector $\frac{\partial}{\partial z}$ is transverse to $\xi = ker(dz + xdy)$ so that $tb(K)$ is just the linking number $lk(K, K')$ where K' is the push-off of K in the direction of this vector. Now for the linking number we count the number of crossings of K' and K with sign. It is easy to see that a self-crossing of K will result in a crossing of K' and K of the same sign. A cusp on the left will give a negative crossing of K' under K and a cusp on the right will give a crossing of K' over K. The result follows because the number of left and right cusps are equal.

Lemma 2.1.9. *The rotation number $rot(K)$ of a Legendrian knot K is given by the formula: $rot(K) = \frac{1}{2}(c_d(K) - c_u(K))$, where c_d and c_u are the number of down and up cusps in the projection.*

Proof. The vector field $\frac{\partial}{\partial x}$ gives rise to a trivialization of ξ_{st}, hence the rotation number can be counted as the winding number with respect to this vector field. We have to count the number of times the tangent of K passes the vector field as we traverse K. Define $l_\pm$ (resp. $r_\pm$) as the number of left (resp. right) cusps where the knot K is oriented upward or downward. Then we can see that $rot(K) = l_- - r_+$. Counting with respect to $-\frac{\partial}{\partial x}$ we get $rot(K) = r_- - l_+$ and taking the average gives the result.

For instance, for the two Legendrian knots on the left and the right in figure 2.1 we have $tb(K) = 0 - \frac{1}{2}(2) = 1$ and $tb(K') = -1 - \frac{1}{2}(2) = -2$ respectively. Similarly for the rotation numbers we have: $rot(K) = 0$ and $rot(K') = \pm 1$ depending on the orientation on the knot K'.

Definition 2.1.10. An embedded disk $D \subset (Y, \xi)$ is an *overtwisted* disk if $\partial D = K$ is a Legendrian knot with $tb_D(K) = 0$, i.e. the contact framing of K coincides with the framing given by the disk D. A contact manifold (M, ξ) is called overtwisted if it contains an overtwisted disk; (Y, ξ) is called *tight* otherwise.

According to a fundamental result of Eliashberg, overtwisted contact structures on 3-manifolds can be classified up to homotopy of plane fields, as in the following theorem:

Theorem 2.1.11. *Two overtwisted contact structures are isotopic, if and only if they are homotopic as oriented* 2*-plane fields. Moreover, every homotopy class of* 2*-plane fields contains an overtwisted contact structure.*

Therefore, the classification of overtwisted contact structures reduces to a homotopy theoretic problem which is not hard to solve. For more discussion on the above theorem the reader can consult [2].

We will need to represent transverse knots as braids. Let us consider the symmetric version of the standard contact structure (S^3, ξ_{sym}) with $\xi_{sym} = ker(dz + xdy - ydx)$. Given a closed braid B braided about the z-axis, we can isotopy it through closed braids so that it is far from the z-axis. As $\xi_{sym} = span\{x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}, x\frac{\partial}{\partial z} - \frac{\partial}{\partial y}\}$, away from the z-axis the planes that make up ξ_{sym} are almost vertical. Thus the closed braid B represents a transverse knot. The opposite of the above theorem is also true:

Theorem 2.1.12 Any transverse knot is transversely isotopic to a closed braid.

Refer to [9] for a proof of the above theorem.

2.2 Open book decompositions

Definition 2.2.1. Suppose there is a link L in a 3-manifold Y that the complement $Y - L$ fibers as $\pi : Y - L \to S^1$ such that fibers are interiors of Seifert surfaces for L. Then (L, π) is called an *open book decomposition* of Y. Each fiber $\Sigma = \pi^{-1}(t)$ is called a *page* and L the *binding* of the open book.The monodromy of fibration π is called the *monodromy* of the open book decomposition.

A theorem of Alexander states that every 3-manifold admits an open book decomposition. Refer to [1] or [2] for a proof and further discussion.

Example 2.2.2. Consider S^3 as the unit circle in $\mathbb{C}^2$. Define $D = \{(r_1, \theta_1, r_2, \theta_2) \in S^3 : r_2 = 0\}$. The fibration $\pi : S^3 - D \to S^1$ given by $\pi((r_1, \theta_1, r_2, \theta_2)) = \theta_2$ gives rise to an open book on S^3 with page a disk and monodromy equal to the identity.

Definition 2.2.3. Given an open book decomposition (Σ, ϕ), we attach a 1-handle to the surface Σ connecting two points on $\partial\Sigma$ to obtain a new surface Σ'. Let α be a closed curve in Σ' going over the new 1-handle once, as in the following figure. The new open book $(\Sigma', \phi o t_\alpha)$ is called a positive stabilization of the original open book (where t_α denotes a positive Dehn twist about α).

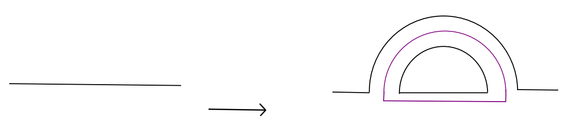

Figure 2.2: positive stabilization of an open book

Definition 2.2.4. An open book decomposition is said to be compatible with the contact structure ξ on Y if ξ can be represented by a contact form α such that the binding is a transverse link, $d\alpha$ is a volume form on every page and orientation of the transverse binding induced by α agrees with boundary orientation of the pages.

The conditions $\alpha > 0$ on the binding and $d\alpha > 0$ on the pages can be thought of strengthening of the contact condition $\alpha \wedge d\alpha > 0$ in the presence of an open book on M.

Example 2.2.5. We can see that the trivial open book for S^3 in the previous example is compatible with ξ_{st} as follows: the tangent to the binding is given by $\dfrac{\partial}{\partial \theta_1}$ and the contact form is $d\theta_1$ restricted to $r_2 = 0$. Therefore the binding is transverse to the contact structure ξ_{st}. The contact form restricted to a page is $r_1^2 d\theta_1$ and thus $d(r_1^2 d\theta_1) = 2r_1 dr_1 \wedge d\theta_1$ is a volume form.

The following theorem of Giroux states that open books up to positive stabilization correspond to contact structures up to isotopy:

Theorem 2.2.6. *(a) For a given open book decomposition of Y there is a compatible contact structure ξ on Y. Contact structures compatible with a fixed open book decomposition are isotopic.(b) For a contact structure ξ on Y there is a compatible open book decomposition of Y. Two open book decompositions compatible with a fixed contact structure admit common positive stabilizations.*

For a complete proof of the above theorem the reader can refer to [1].

2.3 Criteria for overtwistedness

Let Σ be a compact connected oriented surface with boundary. Define the mapping class group of Σ to be the isotopy classes of orientation-preserving self-diffeomorphisms of the surface Σ which restrict to the identity on $\partial\Sigma$ and denote it by $MCG(\Sigma, \partial\Sigma)$. In [4], Honda-Kazez-Matic introduced the notion of right-veering dif- feomorphisms and the monoid $Veer(\Sigma, \partial\Sigma) \subset MCG(\Sigma, \partial\Sigma)$ of right-veering diffeomorphisms of Σ. We recall these notions.

Definition 2.3.1. Let α and β be two properly embedded arcs with a common initial point $x \in \partial\Sigma$. Isotope α and β fixing the endpoints so that they intersect transversely with the least possible number of points and that they are transverse to $\partial\Sigma$. We say that "β is to the right of α" if $\alpha = \beta$ or the tangent vectors (β', α') give the orientation of Σ at x.

Definition 2.3.2. A diffeomorphism $h : \Sigma \to \Sigma$ is called right-veering if for every choice of basepoint $x \in \partial\Sigma$ and every choice of properly embedded arc α based at x, $h(\alpha)$ is to the right of α at x. It is easy to see that for two isotopic self diffeomorphisms h_1 and h_2 of Σ, h_1 is right-veering if and only if h_2 is right-veering. Therefore, one can talk about right veering mapping classes. The subset of $MCG(\Sigma, \partial\Sigma)$ consisting of right-veering elements is denoted by $Veer(\Sigma, \partial\Sigma)$. It follows that $Veer(\Sigma, \partial\Sigma)$ is a monoid. In [4], it was shown that the monoid $Dehn^{+}(\Sigma, \partial\Sigma) \subset MCG(\Sigma, \partial\Sigma)$ consisting of products of right Dehn twists is a submonoid of $Veer(\Sigma, \partial\Sigma)$. The main result of [4] is the following theorem:

Theorem 2.3.3. *A contact structure (M, ξ) is tight if and only if all of its compatible open book decompositions (Σ, h) have right-veering monodromy.*

Therefore in order to prove a contact structure is overtwisted, we only have to find a compatible open book for which the monodromy is not right-veering.

Another criterion is given by by Goodman in [11] to detect overtwistedness of a contact structure. We call an open book decomposition overtwisted if the contact structure compatible with this open book is overtwisted. Let $\alpha, \beta \subset \Sigma$ be properly embedded oriented arcs which intersect transversely on an oriented surface F. The algebraic intersection number $i_{alg}(\alpha, \beta)$ is the oriented sum over interior intersections. The geometric intersection number $i_{geom}(\alpha, \phi(\alpha))$ is the count of interior intersections regardless of sign, minimized over all boundary fixing isotopies of α and β. The boundary intersection number $i_{\partial}(\alpha, \beta)$ is half of the oriented sum over the boundary intersections after minimizing interior intersections fixing the boundary.

Definition 2.3.4. A properly embedded arc $\alpha \subset \Sigma$ is called a sobering arc for a monodromy ϕ, if $i_{alg}(\alpha, \phi(\alpha)) + i_{geom}(\alpha, \phi(\alpha)) + i_{\partial}(\alpha, \phi(\alpha)) \leq 0$, and α is not isotopic to $\phi(\alpha)$.

In particular, since $i_{\partial} \geq -1$ and each positive intersection contributes twice to the sum of intersection numbers, there can be no interior intersections with positive sign. Therefore we can reinterpret the definition as follows: an arc α is sobering if and only if, after minimizing geometric intersections, $i_{\partial} \leq 0$, there are no positive (internal) intersections of α with $\phi(\alpha)$, and α is not isotopic to $\phi(\alpha)$.

The importance of sobering arcs is in the following theorem (refer to [11]):

Theorem 2.3.5. *If there is a sobering arc $\alpha \subset \Sigma$ for ϕ, then the open book (Σ, ϕ) is overtwisted.*

Example 2.3.6. The open book decomposition (S^3, h) induced by negative Hopf link H_- with fiber surface F_-. The arc α in figure 2.3 is a sobering arc for the monodromy h which is a left-handed Dehn twist. We observe that $i_{\partial}(\alpha, h(\alpha)) = -1$ and $i_{alg}(\alpha, h(\alpha)) = i_{geom}(\alpha, h(\alpha)) = 0$. Therefore the induced open book is overtwisted.

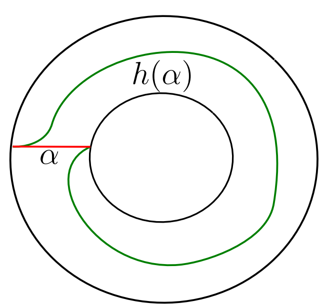

Figure 2.3: A sobering arc

2.4 Contact structure induced by the branched cover

For a transverse link $L \subset (S^3, \xi_{st})$, the 3-manifold Y obtained by the p-fold branched cover over L can be equipped with a natural contact structure ξ_L. Roughly speaking, this contact structure is obtained by lifting the standard contact structure on the knot complement to its p-fold cover and extending it to a neighborhood of the branch set L upstairs. We describe this construction in some detail.

Let L be a transverse knot in (S^3, ξ_{st}) (if L is a link we treat each component separately).Using Darboux theorem for transverse knots, a neighborhood of L embeds into $\mathbb{R}^2 \times S^1$ via the coordinates (r, θ, z), where (r, θ) are polar coordinates on $\mathbb{R}^2$, $z \in S^1$ and $L = r = 0$ and the contact structure can be given as kernel of $dz + r^2 d\theta$. In this neighborhood the covering map $p : Y \to S^3$ is given by $p((w, z)) = (w^p, z)(w = re^{i\theta})$. Let $\xi_p = dz + pr^{2p} d\theta$ be the kernel of the pull-back form. But this 1-form fails to be a contact form along L. To resolve this issue, we define a new contact form by interpolating between the form $dz + r^2 d\theta$ and the pull-back form in a small tubular neighborhood of L. Let $\epsilon_1, \epsilon_2 < r$ where r is the radius of the neighborhood above and $\epsilon_1^2 < p\epsilon^{2p}$. Now set $\xi_L = dz + f(r)d\theta$ where $f(r) = r^2$ for $r < \epsilon_1$ and $f(r) = pr^{2p}$ for $r > \epsilon_2$ and $f'(r) > 0$ in between. It is clear that ξ_L is a contact form. It turns out this contact structure is independent of the choices. The reader is referred to [3] for the details.

We can also describe the contact structure ξ_L on Y via open books. We represent L as a braid of index n which intersects a generic page of the trivial open book for S^3 at n points. Then the generic page of the open book compatible with ξ_L will be a surface which is the p-fold cover of the disk branched over n points. To determine the monodromy, we need to determine how the half-twist generators of the braid L lift to the branched cover. For details of this construction we refer the reader to [3]. We only describe the monodromy of this open book.

If L has a (transverse) braid representation as $\sigma = \sigma_{i_1}...\sigma_{i_k} \in B_n$ so that σ_{i_j} are some standard generators of the braid group B_n, then the contact manifold (Y, ξ_L) is compatible with an open book (Σ, ϕ). Here Σ is the Seifert surface for the (n,p)-torus link. The lift $\hat{\sigma}_i$ of $\sigma_i \in B_n$ is $t_1^i...t_{p-1}^i$, which t_j^i is a Dehn twist about the curve α_j^i as in the figure below (for $n = p = 4$). The monodromy of the open book is $\phi = (t_1^{i_k}...t_{p-1}^{i_k})...(t_1^{i_1}...t_{p-1}^{i_1})$.

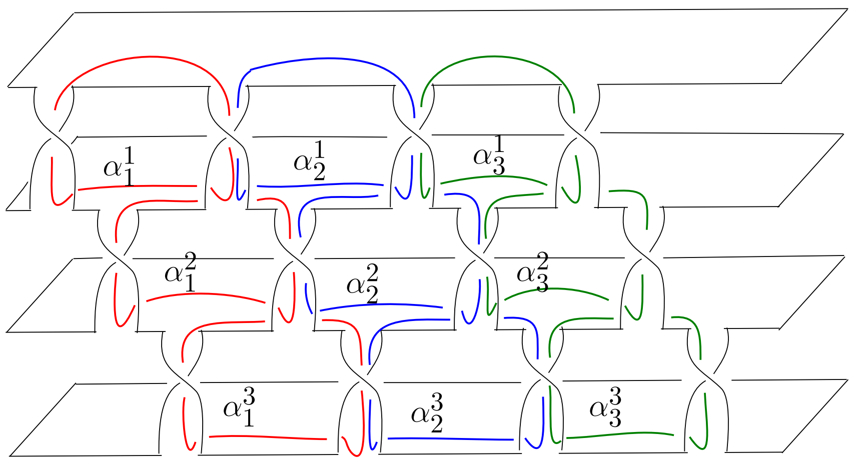

Figure 2.4: Seifert surface of a $(4,4)$-torus link

2.5 Homotopy invariants of contact structures

A contact structure ξ regarded as an oriented 2-plane field on a 3-manifold Y induces a spin^c structure which we denote by t_ξ. Let $p : \pi_0(\Xi(Y)) \to spin^c(Y)$ be the map associating t_ξ to ξ. We briefly review the classification of oriented 2-plane fields on Y. By trivializing TY and considering the oriented normal of a plane field, we associate a map $Y \to S^2$ to ξ. For the case of $Y = S^3$ the oriented 2-plane fields are in one-to-one correspondence with elements of $[S^3, S^2] = \pi_3(S^2) = \mathbb{Z}$. By the Pontryagin-Thom construction, the space $[Y, S^2]$

can be identified with framed cobordism classes of framed 1-manifolds in Y. Homotopies outside a disk (in other words spin^c structures) can be parametrized by 1-manifolds in Y up to cobordism which corresponds to elements of $H_1(Y;\mathbb{Z})$. Note there is a $[S^3, S^2] = \mathbb{Z}$ action on the fiber $p^{-1}(t)$ for a spin^c structure t, by twisting by n the given framing of the framed link corresponding to the oriented 2-plane field. We can also see this action from a different point of view. Consider oriented 2-plane fields (or the corresponding orthogonal vector fields) inducing a specified spin^c structure t to be identical outside a disk in Y. Then $\mathbb{Z}$ acts on $p^{-1}(t)$ by connect summing (Y, v) (v is a nonzero vector field on Y) with (S^3, w), where w is a nonzero vector field on S^3.

By pulling back the generator of $H^2(S^2;\mathbb{Z})$ by the map $f_\xi : Y \to S^2$ associated to $\xi \in \Xi(Y)$ we get a second cohomology class $\Gamma_\xi \in H^2(Y;\mathbb{Z})$. This shows there is also a $H^2(Y;\mathbb{Z})$-action on spin$^c(Y)$. Now regarding ξ as a complex line bundle we have $c_1(\xi) = f_\xi^*(c_1(TS^2))$ which show that $c_1(\xi) = 2\Gamma_\xi$. Therefore as long as $H^2(Y,\mathbb{Z})$ has no 2-torsion, $c_1(\xi)$ determines the spin^c structure t_ξ of ξ.

Therefore the homotopy type of a 2-plane field is determined by the induced spin^c structure and the framing of the corresponding 1-manifold in Y. This latter invariant is generally hard to work with except in the case of torsion $c_1(t_\xi)$. In this case the set of framings can be lifted to $\mathbb{Q}$ and is calculated as $d_3(\xi) = \frac{1}{4}(c_1^2(X,J) - 3\sigma(X) - 2\chi(X))$, where (X, J) is an almost complex manifold such that $\partial X = Y$ and ξ is homotopic to the oriented 2-plane field of complex tangencies along ∂X. $\sigma(X)$ and $\chi(X)$ are signature and Euler characteristic of the manifold X respectively. The rational number d_3 is called the 3-dimensional invariant of ξ.

For more discussion and proofs of the above results the reader can refer to [2], chapter 6.

Now we show how to calculate the homotopy invariants from a contact surgery diagram for a contact 3-manifold.

Definition 2.5.1. Let K be a Legendrian knot in a contact manifold (Y, ξ). By a *contact r-surgery* on (Y, ξ) along K we mean an r-surgery on K such that the framing is measured with respect to the contact framing. It can be shown that the surgered manifold $Y_r(K)$ also admits a contact structure naturally. Refer to [2], section 11.2. for more details.

We will only deal with (± 1)-contact surgery on Legendrian knots (i.e. the topological framing of the surgery is $tb(K) \pm 1$). According to the following theorem, every contact manifold admits a contact surgery diagram in (S^3, ξ_{st}):

Theorem 2.5.2. *For any closed contact manifold (Y, ξ) there is a Legendrian link $L = L^+ \cup L^-$ in (S^3, ξ_{st}) such that contact surgery on $L^\pm$ with framing (± 1) with respect to the contact framings provides (Y, ξ).*

Refer to [5] for a proof of this theorem.

Recall that two oriented 2-plane fields ξ_1 and ξ_2 on a 3-manifold M are homotopic if and only if their induced spin^c structures t_{ξ_i} and 3-dimensional invariants $d_3(\xi_i)$ are equal. When $c_1(t_\xi)$ is torsion, the d_3 invariant can be lifted to $\mathbb{Q}$ and can be computed as $d_3(\xi_i) = \frac{1}{4}(c_1^2(X_i, J_i) - 3\sigma(X_i) - 2\chi(X_i))$, where (X_i, J_i) are almost complex 4-manifolds with $\partial X_i = M$ such that 2-plane fields of complex tangencies of J_i are homotopic to ξ_i along ∂X_i. Now we explain how to obtain the almost complex manifold X. Suppose $L = L^+ \cup L^-$ is the surgery diagram for (M, ξ) and let X' be the 4-manifold defined by the diagram. X' admits an achiral Lefschetz fibration (refer to [2], section 10.2. for the proof). We consider the 2-plane field of tangents to the fibers away from the critical points. By taking orthogonal complement with respect to some metric, we can define an almost complex structure J on $X' - C$ by counterclockwise 90 degree rotation on these planes. This complex structure extends to critical points corresponding to (-1) surgeries and can be extended to points corresponding to (+1)-surgeries by connect-summing with $\mathbb{C}P^2$. For more details refer to [2]. Therefore $X = X'\#q\mathbb{C}P^2$ (q is the number of components of L^+) with extended almost complex structure is our choice of (X, J) for (M, ξ).

Theorem 2.5.3. *The first Chern class $c_1(X, J) \in H^2(X; \mathbb{Z})$ of the almost complex structure discussed above evaluates on the surgery curve K as a homology class as it rotation number: $c_1(K) = rot(K)$.*

Refer to [2], chapter 11 for a proof of the above theorem.

Theorem 2.5.4. *Suppose that the contact 3-manifold (Y, ξ) is given by contact (± 1)-surgery along the link $L = L^+ \cup L^- \subset (S^3, \xi_{st})$. Let X_1 denote the 4-manifold defined by the diagram and suppose $c \in H^2(X; Z)$ is given by $c([\Sigma_K]) = rot(K)$ on $[\Sigma_K] \in H_2(X_1; Z)$, where Σ_K is the surface corresponding to the surgery curve $K \subset L$. If the restriction $c|_{\partial X_1}$ to the boundary is torsion and L^+ has q components then: $d_3(\xi) = \frac{1}{4}(c^2 - 3\sigma(X_1) - 2\chi(X_1)) + q$.*

Proof. The formula is a direct result of the above discussion and noting that $\chi(X_1) = \chi(X_1 - \{x_1, ..., x_q\})$ for the critical points $\{x_1, ..., x_q\}$ of the achiral Lefschetz fibration $X_1 \to D^2$ which lie on the incorrectly oriented charts.

2.6 Concave fillings of contact manifolds

Definition 2.6.1. A vector field V on a symplectic manifold (W, ω) is a symplectic dilation or a *Liouville vector field* if $L_V\omega = 0$. We say that a compact symplectic manifold (W, ω) is a *convex filling* of closed contact manifold (M, ξ) if $\partial W = M$ as oriented manifolds and there exists a Liouville vector field V defined in a neighborhood of M, pointing out of W along M and satisfying $\xi = ker(\iota_V\omega|M)$. In this case (M, ξ) is said to be the convex boundary of (W, ω). On the other hand if V points into W along M, then we say that (W, ω) is a *concave* *filling* of (M, ξ).

We have the following important fact about concave fillings:

Theorem 2.6.2. *Every contact manifold admits a concave filling.*

Refer to [6] for a proof of this theorem.

Definition 2.6.3. Let the symplectic manifold (X, ω) be the convex filling of the contact manifold (Y, ξ). By a *convex-to-concave* 2-handle H we mean a 2-handle attached symplectically to $\partial X = Y$ (i.e. the symplectic structure on X extends to $X' = X \cup H$) along a transverse knot K in (Y, ξ) so that X' is the concave filling of the new boundary $\partial X' = \partial(X \cup H)$.

The existence of convex-to-concave 2-handles was proved in [7]. More specifically, they show that if K is a transverse boundary component of an open book for (Y, ξ) as convex boundary of (X, ω), then a 2-handle attached along K with a framing greater than the page framing of K (the framing induced on K as boundary of page of the open book), is in fact a convex-to-concave 2-handle. We will discuss these 2-handles in the next section to describe concave fillings constructed in [8] for specific plumbed 3-manifolds.

2.6.1 Concave filling of positive plumbings

We briefly review the construction in [8] of the concave fillings for specific class of plumbed 3-manifolds.

Definition 2.6.1.1. Suppose (X, ω) is a symplectic 4manifold. By a *symplectic configuration* in a symplectic 4-manifold we mean a union $C = S_1 \cup ... S_n$ of closed symplectic surfaces embedded in (X, ω) such that all intersections between surfaces are ω-orthogonal. A *symplectic configuration graph* is a labeled graph G with no edges from a vertex to itself and with each vertex v_i labeled with a tuple (g_i, m_i), where g_i is the genus of the symplectic surface S_i associated to the vertex v_i and m_i is the self-intersection of the surface S_i. A symplectic configuration graph is called positive if $m_i + d_i > 0$, where d_i is the degree of the vertex v_i. An example is given by the graph on the top of figure 2.6.

The goal of the next theorem is to explicitly construct a symplectic structure $\omega(G)$ on a neighborhood $N(G)$ of a positive symplectic configuration graph handle-by-handle. Then by the sympelectic neighborhood theorem, it follows that there is a neighborhood of any positive symplectic configuration $(\nu(C), \omega)$ in a symplectic manifold (X, ω) which is symplectomorphic to $(N(G), \omega(G))$. Moreover, the open book compatible with the contact structure on the boundary induced by the concave filling will be determined.

Theorem 2.6.1.2. *Let $C = S_1 \cup ... S_n \subset (X, \omega)$ be a positive configuration of symplectic surfaces. Then there is a sympelectomorphism $f : (\nu(C), \omega) \to (N(G), \omega(G))$, where G is the configuration graph associated to C and $(N(G), \omega(G))$ is constructed as in the proof of the theorem.*

Proof. We only give a sketch here. For more details the reader can refer to [8]. We explain our construction by looking at the example in figure 2.5. We begin with disks and positive Hopf links as pages of open books for different copies of $(S^3, \xi_{st}) \subset (B^4, \omega st)$. The disks are used in order to construct individual surfaces upon them and the Hopf links to construct plumbing of the surfaces. Then we attach (4-dimensional) 1-handles with feet on boundary of these disks or Hopf links to different copies of (S^3, ξ_{st}) which contain them, as in the bottom of figure 2.5. Then in the same manner we attach extra 1-handles if necessary to raise the genus of the surfaces. These 1-handles are attached to the right hand side of disks. To this end each surface has one boundary component. To increase the number of boundary components if necessary, we attached more 1-handles to the left of the disks. We note that up until this point, we have constructed a convex neighborhood of the surfaces (by attaching 1-handles). Before attaching 1-handles to the left, we have an open book for (S^3, ξ_{st}) corresponding to each vertex with $pf(K) = cf(K) - d_i$. After attaching $2g$ 1-handles to to the right, still the we have $pf(K) = cf(K) - d_i$. After attaching $m_i + d_i - 1$ 1-handles on the lower left, the surface inside will have one more component for each handle attached, which satisfy: $pf(K) = cf(K)$. Now attach 2-handles to each binding component with framing $pf(K) + 1$. This step will turn convex filling to concave ones. We will have closed surfaces so that the self-intersection of each component F_i is $\Sigma_{K \in \partial F_i}(pf(K)+1) - cf(K) = (m_i + d_i - 1)(1) + (1 - d_i) = m_i$. This finishes the construction of the concave filling. By the symplectic neighborhood theorem, a neighborhood of a positive configuration in a symplectic manifold should be symplectomorphic to $(N(G), \omega(G))$.

For more details of the proof refer to [8].

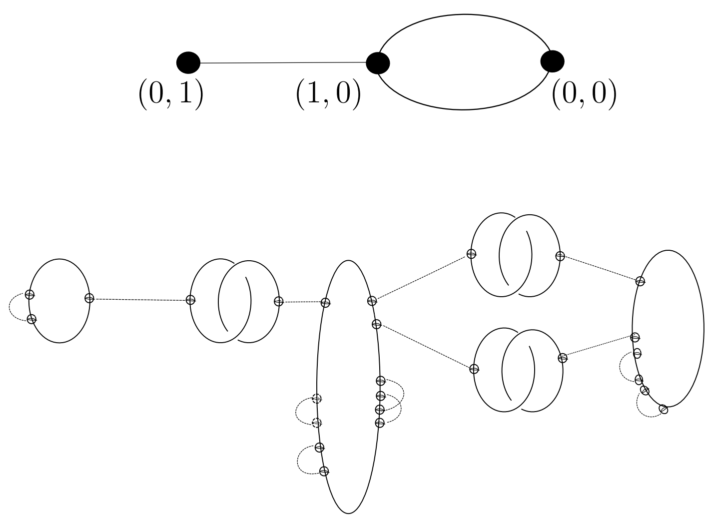

Figure 2.5: construction of concave filling

2.7 Corks

Definition 2.7.1. A cork is a pair (W, f), where W is a compact contractible Stein manifold and $f : \partial W \to \partial W$ is an involution which extends to a self-homeomorphism of W but it does not extend to a self-diffeomorphism of W. We say that W is a cork of X if $W \subset X$ and cutting W out of X and re-gluing it by f produces an exotic copy X' of X (a smooth manifold homeomorphic but not diffeomorphic to X). This means that we have the following decomposition: $X = Y \cup_{id} W$ and $X = Y \cup_f W$, where $Y = X - int(W)$.

It can be shown that any exotic copy X' of a closed simply-connected 4-manifold X differs from its original copy by a cork (refer to [12], chapter 10 for more information). Figure 2.6 shows a family of corks W_n, where the involution f is defined as the zero and dot exchange on their underlying symmetric links. For instance, W_1 is a cork of $E(2)\#\mathbb{C}\bar{P}^2$ (refer to [12] and references therein for further discussion). We will encounter the cork W_1 in section where we look at a specific contact structure on its boundary.

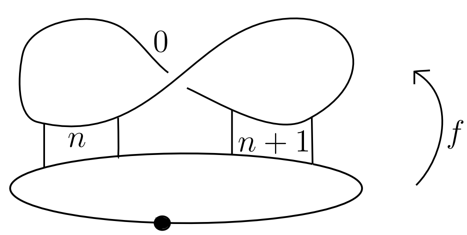

Figure 2.6: The family of corks W_n

Chapter 3

Main results

3.1 Concave fillings

We recall the definition of a positive plumbing from last chapter. Consider a plumbing of closed symplectic surfaces $P = S_1 \cup ... S_n$ in a symplectic manifold (X, ω). To this Plumbing we associate a plumbing graph consisting of a vertex for each symplectic surface and an edge between two vertices if the two symplectic surfaces are plumbed together. Let m_i be the self-intersection and d_i the degree of the vertex v_i. This plumbing is called positive if $m_i + d_i > 0$ for each vertex v_i. In [8] it was proved that such a plumbing of symplectic surfaces has a neighborhood that is a concave filling of its boundary and the induced contact structure on the boundary is compatible with an open book as follows: the generic page is a surface obtained by connect-summing the surfaces S_i as in the plumbing configuration and there are $m_i + d_i$ boundary components for each surface S_i. The monodromy consists of one positive Dehn twist about each boundary curve and one negative Dehn twist about each neck of the connect-sum.

We demonstrate this with an example. Look the plumbing graph of symplectic surfaces in figure 3.1, where (g_i, m_i) for each vertex v_i means the surface S_i has genus g_i and self-intersection equal to m_i. Then induced contact structure on its boundary is compatible with the following open book with monodromy $\phi = \Pi \delta_i \Pi \sigma_j^{-1}$ where δ_i are Dehn twists about blue curves (about boundary components) and σ_j are Dehn twists about the red curves (about the neck) as in the figure 3.1.

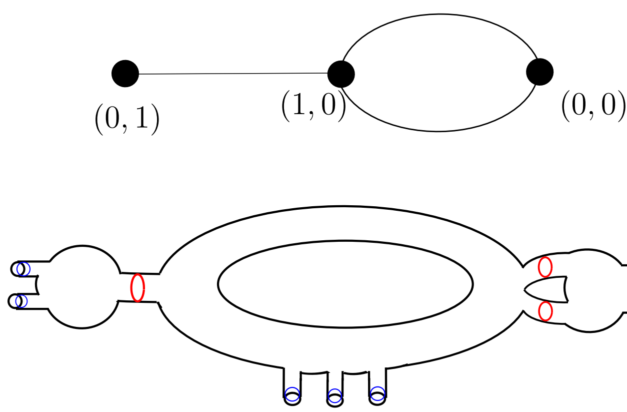

Figure 3.1: a concave plumbing graph and its corresponding open book

In this section we show that in some cases a (-1)-framed symplectic sphere in a positive plumbing can be blown down to obtain another positive plumbing with the same induced contact structure on the boundary. We then present an algorithm to construct a positive concave filling for each Brieskorn manifold (but the concave filling we obtain is not unique). Then we look at an example $\Sigma(2, 3, 5)$ and draw the contact surgery diagram for the contact structure induced by this concave filling. We compute homotopy invariants of this contact structure and compare them to those of the standard Milnor fillable one. We then show that any concave filling of a Brieskorn manifold constructed by the above algorithm, induces an overtwisted contact structure on it.

Theorem 3.1.1. *Suppose we have a positive cofiguration of symplectic surfaces as in figure* 3.2*. By blowing down the middle* (-1)*-sphere, we obtain another positive configuration with the same contact boundary (i.e. both configurations are concave fillings of the same contact* 3*-manifold).*

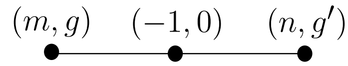

Figure 3.2: A special positive plumbing graph

Proof. According to the lantern relation, we have $t_a t_b t_c t_d = t_\alpha t_\beta t_\gamma$ as in figure 3.3 below:

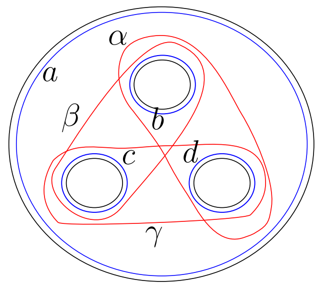

Figure 3.3: the lantern relation

Now we consider a 4-holed sphere as figure 3.4 below. This type of picture of a 4-holed sphere will be useful for our argument. From the lantern relation we obtain the relation $t_a t_b t_\alpha^{-1} = t_c^{-1} t_d^{-1} t_\beta t_\gamma$. The left and right pictures in figure 3.4 correspond to the curves involved in left and right hand sides of this relation. By destabilizing the monodromy (removing a positive Hopf band) we obtain the last picture in the figure:

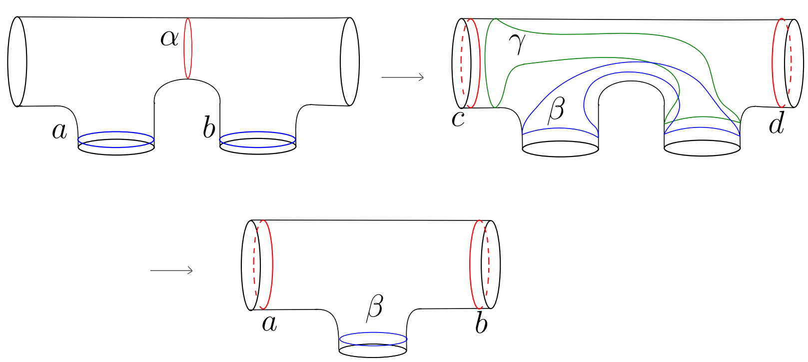

Figure 3.4: removing a positive Hopf band

Now we prove the theorem by considering a specific example as in the figure 3.5 where $m = 2$, $n = 1$ and $g = g' = 0$, but our argument applies to the general case as well.

The induced open book on the boundary has a page as in the top picture in the figure 3.5 and the monodromy is $\phi = t_{\delta_1} t_{\delta_2} t_{\delta_3} t_{\sigma_1}^{-1} t_{\delta_4} t_{\sigma_2}^{-1} t_{\delta_5} t_{\delta_6}$. Now if we trace the pictures in the figure backwards, we conclude that this open book is equivalent to the bottom picture as the page and monodromy $\phi' = t_{\delta_1} t_{\delta_2} t_{\delta_3} t_{\delta_4} t_{\sigma_1}^{-1} t_{\delta_5} t_{\delta_6} t_{\delta_7}$.

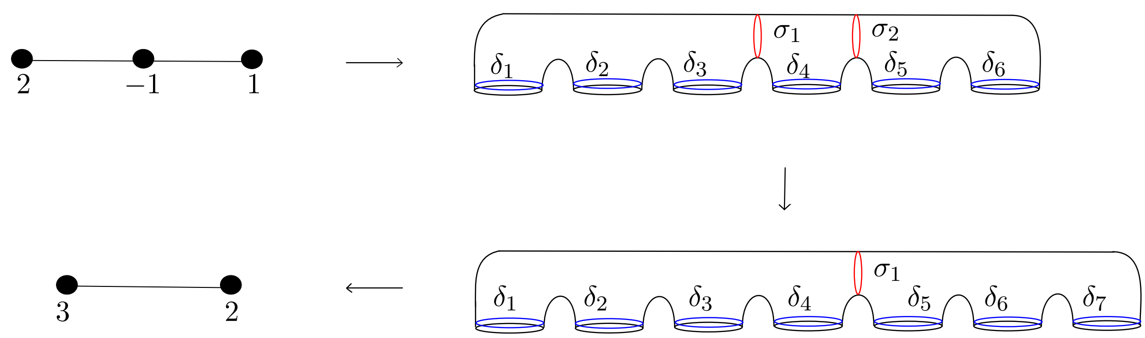

Figure 3.5: Blowing down and the corresponding open books

It is easy to see that this new open book corresponds to the concave plumbing graph below, which proves the theorem in our special case. The general case is similar.

Next we show that all Brieskorn manifolds are boundary of positive concave plumbings.

Lemma 3.1.2. *Each Brieskorn manifold $\Sigma(p, q, r)$ admits a positive concave plumbing.*

Proof. We know that a Brieskorn manifold $\Sigma(p, q, r)$ is a Seifert fibered space with three singular fibers and the base a genus zero surface (refer to [13], theorem 2.1. for a proof). Thus it has a plumbing diagram as in figure 3.6. If for the central vertex $m_i < -2$ or if there is a middle vertex with $m_i < -1$ or an end vertex with $m_i < 0$, we blow up a $+1$-sphere between this vertex and a vertex next to it. This will increase increase self-intersection of the two old vertices by one and the new $+1$-sphere already satisfies the condition $m_i + d_i > 0$ (in this case $1 + 2 > 0$). It is easy to check that after a finite steps this will give us a plumbing graph as desired. We demonstrate this with and example.

Example 3.1.3. $\Sigma(2, 3, 5)$ has the following surgery description: $15b_1 + 10b_2 + 6b_3 = 1$

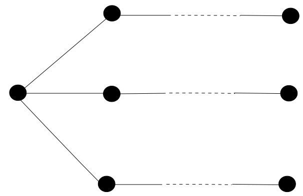

Figure 3.6: plumbing diagram of a Brieskorn manifold

with $b_1 = -1$, $b_2 = 1$ and $b_3 = 1$ (refer to [14], theorem 6.7. for obtaining surgery diagram of a Brieskron manifold).

We apply the above algorithm to find a concave plumbing graph from this surgery diagram. The result is shown in figure 3.7.

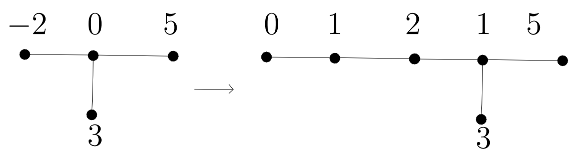

Figure 3.7: A positive plumbing graph for $\Sigma(2, 3, 5)$

According to [8] the concave plumbing graph on the right induces the following open book on its boundary. The red curves correspond to negative Dehn twists and the blue curves to positive Dehn twists. In order to be able to realize the surgery curves as Legendrian curves we present this surface as in figure 3.8.

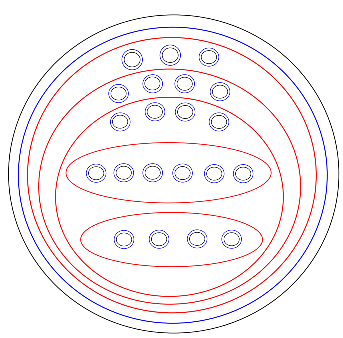

Figure 3.8: the open book on $\Sigma(2, 3, 5)$

We notice that the first five Dehn twists from the top can be removed. The first two

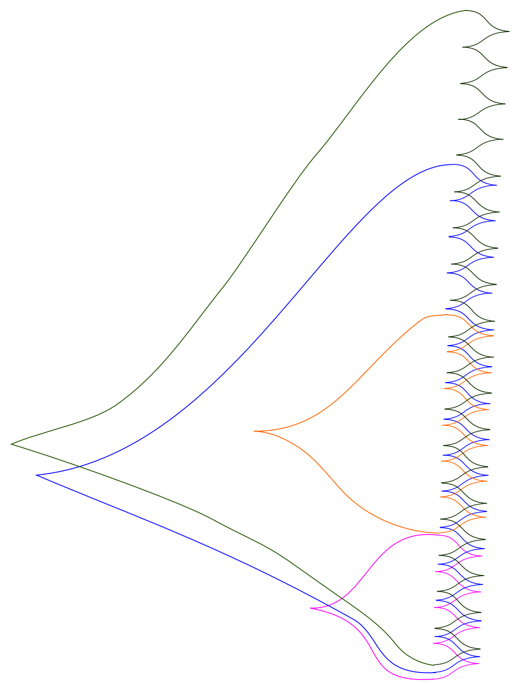

Figure 3.9: A Legendrian diagram for $\Sigma(2,3,5)$

since they are positive and negative twists about the same curve and the next three positive twists since they correspond to positive stabilizations. Then the surgery diagram of the contact structure compatible with this open book will be given as in figure 3.9. Since we perform negative Dehn twists about all the red curves, the framing on each surgery curve is a (+1)-contact framing:

Now we calculate the homotopy invariants of the this contact structure ξ. Since $Y = \Sigma(2,3,5)$ is a homology sphere, the first obstruction $d_2(\xi) \in H^2(Y;Z) = 0$. Now recall that the next obstruction is $d_3(\xi) = \dfrac{1}{4}(c^2 - 3\sigma(X) - 2\chi(X)) + q$, where X is the handlebody obtained by attaching $2-$handles to D^4 along the surgery curves, q is the number of +1-surgery curves, $c \in H^2(X;Z)$ is given by $c([\Sigma_i]) = rot(k_i)$ on $[\Sigma_i] \in H_2(X;Z)$ where Σ_i is the Seifert surface corresponding to a component k_i of the diagram. Finally $\sigma(X)$ and $\chi(X)$ are the signature and Euler characteristic of X.

The linking matrix is as follows:

$$\begin{bmatrix} -17 & -14 & -6 & -4 \\ -14 & -13 & -6 & -4 \\ -6 & -6 & -5 & 0 \\ -4 & -4 & 0 & -3 \end{bmatrix}$$

We compute $\sigma(X) = -2$ and $\chi(X) = 5$. Now we look at the long exact sequence of the pair $(X, \partial X)$: $0 \to H_2(\partial X; Z) \to H_2(X; Z) \to^{\phi_1} H_2(X, \partial X; Z) \to^{\phi_2} H_1(\partial X; Z) \to 0$.

The maps ϕ_1 and ϕ_2 are calculated as follows: $\phi_1([\Sigma_i]) = \Sigma lk(k_i, k_j)[N_j]$ and $\phi_2([N_i]) = [\mu_i]$ (N_i is a disc bounding the meridian μ_i). Now $PD(c) = \Sigma rot(k_i)[N_i] = 17[N_1]+13[N_2]+5[N_3]+3[N_4]$. We find the solution to $\phi_1(C) = PD(c)$ as $C = 17[\Sigma_1]-47[\Sigma_2]+37[\Sigma_3]+41[\Sigma_4]$ (for negative choice of rotation numbers). Thus $c^2 = C^2 = 14$ and $d_3(\xi) = \frac{1}{4}(14 - 3(-2) - 2.5) + 4 = \frac{13}{2}$.

We can compare the result to homotopy invariants of the standard contact structure ξ_{st} on $\Sigma(2,3,5)$ induced by its Milnor fiber $-E_8$. Let the complex polynomial $F : \mathbb{C}^3 \to \mathbb{C}$ be given by $F(x,y,z) = x^2 + y^3 + z^5$ and U be a connected open subset of $\mathbb{C}^3$ containing the origin. If $w_0 \in F(U)$ is a regular value, then the compact smooth manifold with boundary $F^{-1}(w_0) \cap B^6$ is the Milnor fiber $\Phi = \Phi(2,3,5)$ of the Brieskorn manifold $\Sigma(2,3,5)$. It can be shown that Φ has a plumbing description as in figure 3.10 called the $-E_8$ plumbing (refer to [12], chapter 12 for further discussion). As the Milnor fiber Φ is the pre-image of a regular value under F, its normal bundle $\nu\Phi$ is trivial, as is $T\mathbb{C}^3|\Phi$. Since $\nu\Phi := \frac{T\mathbb{C}^3|\Phi}{T\Phi}$, we conclude the tangent bundle $T\Phi$ is trivial and therefore $c_1(\Phi) = c_1(\xi_{st}) = 0$. By looking at the plumbing $-E_8$, we conclude that $\chi(\Phi) = 9$ and $\sigma(\Phi) = -8$. Thus $d_3(\xi_{st}) = \frac{1}{4}(0 - 2.9 - 3(-8)) = \frac{3}{2}$. Therefore we conclude that these two contact structures are not homotopic.

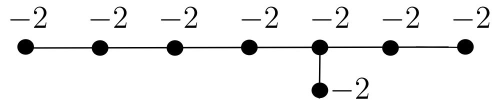

Figure 3.10: The $-E_8$ plumbing

In fact we can prove that the contact structure induced by the concave filling is overtwisted. (The contact structure induced by $-E_8$ is tight since $-E_8$ is a Stein filling of the Poincare sphere.)

As mentioned above, there are many positive concave plumbings that fill a fixed Brieskorn sphere. We prove that the contact structure induced on a Brieskorn sphere, by a positive concave plumbing according to the above algorithm, is always overtwisted (i.e. it does not matter which plumbing we choose). We prove this by using the sobering arc technique which was mentioned in section 2.3..

Theorem 3.1.4. *A positive concave plumbing constructed by the above algorithm, always induces an overtwisted contact structure on a Brieskorn sphere.*

Proof. We first construct a concave filling of Brieskorn sphere as in lemma 3.2. We then construct the open book as described at the beginning of this section. We modify the open book using the move in figure 3.5. We start with one of the end vertices v_1 and remove all but one of the original boundary components corresponding to this vertex. Again we demonstrate with an example. We can use a similar argument for a general graph. Suppose one branch of the plumbing ends with vertices as follows:

Figure 3.11: A branch in the plumbing

Then the corresponding open book has one end as below and we apply the move in figure 3.4 to remove its boundary components one by one:

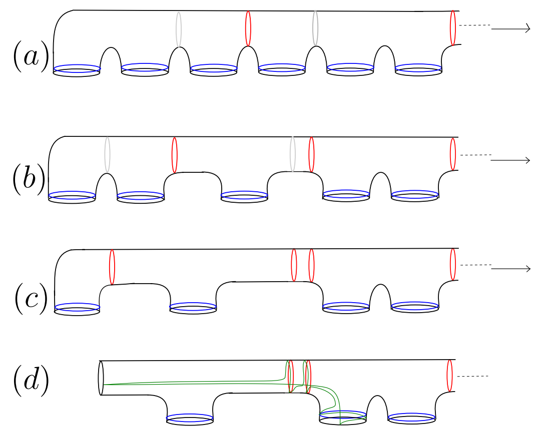

Figure 3.12: modifying the open book

The grey curves on each surface mean that there will be a negative Dehn twist about the curve in the next picture. Look at figure 3.12 (c). We observe that the negative and positive Dehn twists about the first boundary component from right cancel each other. Thus we get figure 3.12 (d) where there is no Dehn twist about this component. In the same figure we have sketched an arc α from this boundary component to another one, together with its image under the monodromy. We can check that the arc α is a sobering arc: All the interior intersections are negative and and the boundary intersection is equal to zero. Therefore the contact structure compatible with this open book is overtwisted.

3.2 A contact structure on the boundary of the cork

The cork W_1 can be seen as double branched cover over a ribbon disk (refer to [12], chapter 11 for further discussion). Its boundary $M = \partial W_1$ is the double branched cover over the ribbon knot K as shown in figure 3.13.

In this section we examine boundary of the cork as the contact manifold (M, ξ) that arises as the double branched cover over K realized as a transverse knot in $(S^3; \xi_{st})$. We construct the open book compatible with this contact structure using the techniques discussed in section 2.4 and we use the right-veering criterion to prove that it is overtwisted.

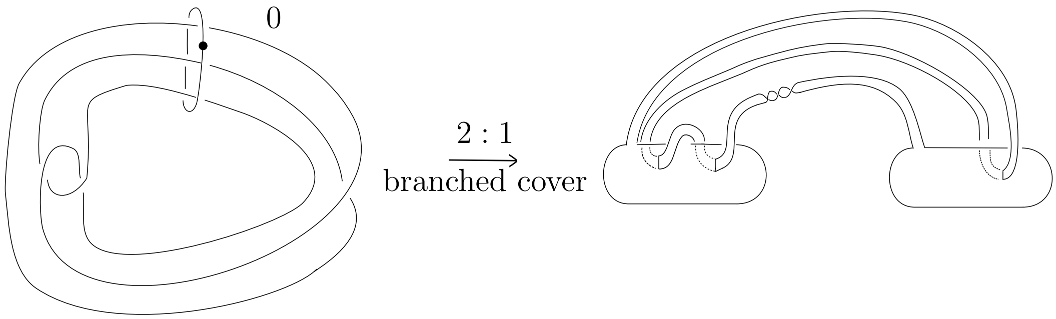

Figure 3.13: Cork as branched cover over a ribbon disk

Theorem 3.2.1. *The ribbon knot K has the following presentation as a braid in B_4:*

$\sigma = \sigma_3^{-1}\sigma_2\sigma_1^{-2}\sigma_2^{-1}\sigma_3\sigma_1\sigma_2^{-1}\sigma_1^{-1}\sigma_2\sigma_3\sigma_2^{-1}\sigma_3^2\sigma_2^{-1}$

Let $(\partial W_1, \xi)$ be boundary of the cork equipped with the contact structure ξ induced by the double branched cover over the transverse knot K corresponding to the braid σ. Then ξ is overtwisted.

Proof. Using the techniques discussed in section 2.4. we construct an open book compatible with the contact structure ξ. Then using the right-veering criterion, we prove that this contact structure in overtwisted. Recall from section 2.4. the contact structure ξ is supported by an open book (Σ, ϕ) such that a generic page Σ is equal to the Seifert surface of a $(2,4)$-torus link ($p = 2$ and $n = 4$ n this case) which is a twice-punctured torus, and its monodromy is given as below:

$\phi = t_b^{-1}t_c^2t_b^{-1}t_ct_bt_a^{-1}t_b^{-1}t_at_ct_b^{-1}t_a^{-2}t_bt_c^{-1}$

Refer to figure 3.14 for the surface Σ and the curves a, b and c on it. For our convenience we rewrite this monodromy as follows:

$\phi = (t_c^{-1}t_b^{-1}t_c)t_c(t_b^{-1}t_ct_b)(t_a^{-1}t_b^{-1}t_a)t_c(t_b^{-1}t_a^{-2}t_b)$

$= t^{-1}_{t_c^{-1}(b)}t_ct_{t_b^{-1}(c)}t^{-1}_{t_a^{-1}(b)}t_ct^{-2}_{t_b^{-1}(a)} = t_ct_{t_b^{-1}(c)}t^{-1}_{t_a^{-1}(b)}t_ct^{-2}_{t_b^{-1}(a)}t^{-1}_{t_c^{-1}(b)}$

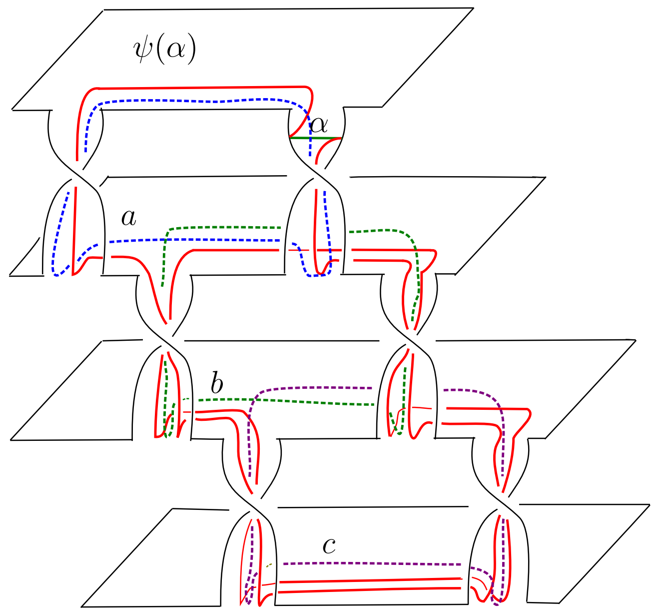

Figure 3.14: α and $\psi(\alpha)$

Now if $\phi \in Veer(\Sigma, \partial\Sigma)$, composing it with $t_{t_c^{-1}(b)} t^2_{t_b^{-1}(a)}$ would give us another right-veering diffeomorphism (since $Veer(\Sigma, \partial\Sigma)$ is a monoid according to the previous section). We show that $\psi = \phi o t_{t_c^{-1}(b)} t^2_{t_b^{-1}(a)} = t_c t_{t_b^{-1}(c)} t^{-1}_{t_a^{-1}(b)} t_c$ is not right-veering. Therefore ϕ cannot be right-veering either and (M, ξ) is overtwisted. Figure below shows an arc α and its image $\psi(\alpha)$ on the surface Σ. It is clear that ψ is not right-veering for this arc. Therefore the result follows.

Remark 3.2.2. It might be possible to check directly that ϕ is not right-veering, by examining its effect on some arc in Σ. But since ϕ contains a lot of words, we decided to proceed as above as a shortcut.

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BIBLIOGRAPHY

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