SIGNAL ANALYSIS IN GUIDED WAVE STRUCTURAL HEALTH MONITORING

 $\mathbf{B}\mathbf{y}$

Oleksii Karpenko

A THESIS

Submitted to Michigan State University in partial fulfillment of the requirements for the degree of

Electrical Engineering - Master of Science

2013

ABSTRACT

SIGNAL ANALYSIS IN GUIDED WAVE STRUCTURAL HEALTH MONITORING

By

Oleksii Karpenko

In recent decades, Guided waves (GW) have emerged as the most promising modality for Structural Health Monitoring (SHM). In SHM applications with distributed sensor networks, GWs can be efficiently actuated with the help of surface-bonded or embedded piezoelectric elements. Nevertheless, the analysis of GW signals and accurate identification of damage sites still remains a challenge. Algorithms for locating defects, such as Probability Diagnostic Imaging, usually assume unimodal GW propagation. However, in many cases selective mode excitation could be hardly accomplished in practice. From this perspective, decomposition of the measured signal into its constituent components is a critical requirement for accurate damage detection. This work presents a signal processing method that combines timefrequency representation (TFR) with Matching Pursuit (MP) for decoupling of GW modes. The proposed TFR is designed on the basis of the reassigned spectrogram whose kernel is substituted with a Chirp-Z transform in order to improve the resolution without increasing its computational complexity. The MP dictionary is constructed of atoms, which numerically simulate the propagation of wave packets corresponding to different GW modes in the sample. The dictionary also accounts for the effect of bonding between the piezoelectric element and the structure. Performance of the algorithm is demonstrated on aluminum plates and woven composite samples for cases when two fundamental modes S_0 and A_0 are simultaneously actuated.

TABLE OF CONTENTS

LIST (OF TABLES	v
LIST (OF FIGURES	vi
CHAP	TER 1 Structural Health Monitoring	1
1.1	Introduction	1
1.2	SHM Techniques and Sensing Modalities	4
	1.2.1 Ultrasonic Testing and Guided Wave SHM	6
1.3	Guided Wave Theory	7
	1.3.1 Symmetric Solution	2
	1.3.2 Antisymmetric Solution	.3
	1.3.3 Phase Velocity and Group Velocity Dispersion Curves	4
	1.3.4 Mode Shapes	.6
	1.3.5 Transduction of Guided Waves	7
	1.3.5.1 PZT Wafers and Piezoelectric Governing Equations 1	.8
	1.3.5.2 Anisotropic Piezoelectric Transducers	20
1.4	Actuation of Guided Waves with Surface-bonded Piezoelectric Transducers . 2	21
	1.4.1 Shear Lag Model	21
	1.4.2 Ideal Bonding Assumption	26
	1.4.3 Voltage Output of the PZT Sensor	28
CHAP	TER 2 Signal Analysis in Guided Wave SHM	9
2.1	Overall Approach to Damage Detection	29
2.2	Probability Diagnostic Imaging	31
2.3	Challenges in Signal Processing of Guided Waves	32
2.4	Characteristic Features for Mode Identification	\$4
2.5	Time-Frequency Representations in GW SHM	35
	2.5.1 Linear Time-Frequency Representations	36
	2.5.1.1 Short Time Fourier Transform $\ldots \ldots \ldots \ldots \ldots \ldots \ldots $	36
	2.5.1.2 Continuous Wavelet Transform	39
	2.5.1.3 Chirplet Transform and Warping Operators	0
	2.5.2 Quadratic TFRs $\ldots \ldots 4$	2
	$2.5.2.1$ Wigner-Ville Distribution $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 4$	13
	2.5.3 Time-Frequency Reassignment	4

2.6	Reassigned Spectrogram with Chirp Transform Kernel	47	
2.7	Matching Pursuit (MP) algorithms in Guided wave signal processing	53	
	2.7.1 Gabor, Chirplet and Dispersion-based Dictionaries for MP	54	
	2.7.2 Generation of Atoms of the Dispersion-based Dictionary	55	
2.8	Hybrid Algorithm based on RSCT and Matching Pursuit with Dispersion-		
	based Dictionary for Mode Decomposition of Guided Wave Signals	58	
	2.8.1 Efficient Implementation of the MD Algorithm	59	
CHAP	TER 3 Experimental Studies	61	
3.1	Defect Localization in Metal Structures	61	
3.2	3.2 Defect Localization in Woven Composite Structures		
CHAP	TER 4 Conclusions and Future Work	74	
APPE	NDICES	76	
	Appendix A: Expression of Stresses, Strains and Displacements in terms of		
	Wave Potentials		
Appendix B: Relationship between Stiffness Constants, Young's Moduli and			
	Poisson's Ratios for Orthotropic Materials	79	
BIBLI	OGRAPHY	81	

LIST OF TABLES

Table 3.1	Length of possible wave paths and corresponding Times-of-Flight for S_0 and A_0 modes	65
Table 3.2	Comparison of analytic and MD-extracted Times-of-Flight for S_0 and A_0 modes.	65

LIST OF FIGURES

Figure 1.1	SHM with distributed sensor networks. Overall approach and main stages [2]. For interpretation of the references to color in this and all other figures, the reader is referred to the electronic version of this	
	thesis	3
Figure 1.2	Shear vertical and pressure waves in solids	10
Figure 1.3	Isotropic plate of infinite length and thickness $2d$. Wave propagates in <i>x</i> -direction	10
Figure 1.4	Phase velocity dispersion curves of 2-mm thick a luminum plate. $\ .$.	15
Figure 1.5	Group velocity dispersion curves of 2-mm thick a luminum plate. $\ .$.	15
Figure 1.6	Displacements of S_0 mode	16
Figure 1.7	Displacements of A_0 mode	17
Figure 1.8	Transducers for actuation and sensing of Guided waves	18
Figure 1.9	Piezoelectric effect: a).neutral state; b). strain conversion; c). voltage conversion.	19
Figure 1.10	Anisotropic Piezocomposite Transducers.	21
Figure 1.11	Actuation using surface-bonded PZT	22
Figure 1.12	Actuation using collocated PZT pairs	23
Figure 1.13	Shear lag model: surface stress.	25
Figure 1.14	Shear lag model: surface strain.	25

Figure 1.15	Actuation using surface-bonded PZT	27
Figure 2.1	PZT sensor configurations.	30
Figure 2.2	Approach to signal processing	33
Figure 2.3	Narrowband actuation waveform	34
Figure 2.4	Dispersion curves of a 2-mm thick aluminum plate corresponding to the spectrum of the Morlet wavelet.	35
Figure 2.5	Time-frequency tiling of STFT	38
Figure 2.6	Time-frequency tiling using dyadic wavelets.	39
Figure 2.7	Warped tiling of time-frequency plane	41
Figure 2.8	Ideal TFR and Wigner-Ville distribution of the signal	45
Figure 2.9	Comparison of smoothed WVD and STFT of the signal (window length = 413 points)	46
Figure 2.10	Comparison of reassigned smoothed WVD and reassigned spectro- gram of the signal (window length = 413 points)	46
Figure 2.11	Comparison of Discrete Fourier transform with Chirp-Z transform	50
Figure 2.12	Implementation of Chirp-Z transform algorithm.	50
Figure 2.13	Experimental set-up for obtaining GW signal: $600 \times 600 \times 2$ mm aluminum plate.	51
Figure 2.14	FFT of GW signal.	51
Figure 2.15	CZT of GW signal. Zooming into 220-320 kHz range	51
Figure 2.16	Lamb wave signal (10240 samples).	52
Figure 2.17	STFT (window size 350 samples)	52

Figure 2.18	Reassigned spectrogram with CZT	52
Figure 2.19	Atoms of conventional MP dictionaries: Gabor and two chirplets with different scales.	55
Figure 2.20	Generated atoms of A_0 mode propagating in 2 mm thick aluminum plate at 0.25 m, 0.5 m, 1.0 m and 1.5 m from the actuator. Actuation waveform: 250 kHz, 5 cycle Hann windowed sinusoid	56
Figure 2.21	Influence of PZT strain transfer conditions on the actuated GW modes.	57
Figure 2.22	Spectra of S_0 and A_0 modes generated by surface-bonded PZT	57
Figure 2.23	Flowchart of the two stage MD algorithm	58
Figure 3.1	Experimental set-up for damage detection in a luminum plates. $\ .$.	62
Figure 3.2	Group velocity dispersion curves of the aluminum plate used in the experiment.	62
Figure 3.3	Possible wave paths from the actuator to the receiving PZT. $\ . \ . \ .$	63
Figure 3.4	GW actuation using single surface-bonded PZT-1	63
Figure 3.5	Reassigned spectrogram with CZT	63
Figure 3.6	Collocated actuation of Guided waves	63
Figure 3.7	MD decomposed A_0 components	64
Figure 3.8	MD decomposed S_0 components	64
Figure 3.9	MD decomposed shear wave components	64
Figure 3.10	Damage detection in 2-mm thick aluminum plate	66
Figure 3.11	Signal from PZT 1-2 sensor pair	66
Figure 3.12	Signal from PZT 2-3 sensor pair	66

Figure 3.13	Phase velocity dispersion curves generated for 5-mm thick 8-layer composite plate with the help of Transfer Matrix method	69
Figure 3.14	Damage detection in 2-mm thick aluminum plate	70
Figure 3.15	Variation of the group velocity of A_0 mode actuated at 70 kHz with respect to the direction of propagation in a 5-mm thick 8-layer woven composite plate.	71
Figure 3.16	Composite samples used in experimental studies	72
Figure 3.17	Probability Diagnostic Imaging on composite samples	72
Figure 3.18	Baseline and differenced signal obtained by PZT-6 and PZT-2 sensor pair on composite plate	72

CHAPTER 1

Structural Health Monitoring

1.1 Introduction

In recent years, there has been an increasing demand for methods providing in-situ Structural Health Monitoring (SHM) of safety-critical systems. Numerous disastrous events such as disintegration of space shuttle Challenger (Canaveral National Seashore, 1986), San Bruno pipeline explosion (California, 2006), Hoan Bridge failure (Wisconsin, 2001) and various flight accidents [1] caused by undetected structural defects, have increased public concerns about the need for more reliable NDE methods. Current industrial practice involves wellestablished techniques such as eddy current, optical and ultrasonic testing due to their ability to furnish precise information about structural damage, its location and severity. However, these NDE inspections are required at scheduled intervals to assess the integrity of a component during its downtime. Moreover, it is always possible that damage might occur in service and maintenance schedules will not be adequate to detect an impending hazard. From this perspective, the transition from regular maintenance to condition-based maintenance is envisioned as an efficient solution to minimizing catastrophic failures of safety-critical structures.

In Chapter 1 we consider main concepts of Structural Heath Monitoring and present a brief overview of sensing modalities used in modern SHM. Chapter 1 also describes the key elements and operation of GW monitoring systems, which interrogate the structure with piezoelectric transducers. Additionally, we provide the essential background for solving problems in Guided wave signal analysis, such as Rayleigh-Lamb equations governing the propagation of Guided waves in plates and a theory of GW actuation with the help of surface-bonded wafers.

Structural Health Monitoring can be viewed as an efficient approach to continual diagnostics of complex structures with the help of structurally integrated sensor networks. SHM is initially aimed at detecting damage and degradation of constituent materials caused by sudden or dynamic stresses or respectively, fatigue, corrosion and environmental changes experienced by the structure in service. However, its global objective is to provide a comprehensive estimate of structural integrity as well as to generate data required for statistical analysis and long term prognosis.

Schematic of a typical SHM application is demonstrated in Fig. 1.1. In a monitoring system, sensors can be strategically placed (embedded or surface-mounted) in the critical areas of the structure in order to collect signals which would be informative enough for damage detection. Hardware components of the system also include autonomous low footprint devices or *sensor nodes* designed to control the data acquisition process and to transmit the data to remotely installed *base station*. Base station is usually a personal computer that supports real-time signal processing applications. The purpose of signal analysis is the extraction of characteristic features from the acquired signals, which helps locate damaged



Figure 1.1: SHM with distributed sensor networks. Overall approach and main stages [2]. For interpretation of the references to color in this and all other figures, the reader is referred to the electronic version of this thesis.

sites and classify detected defects. It should be noted that modern SHM applications also take advantage of wireless data transmission and therefore, they additionally require development of specialized communication protocols that would enable reliable data transfer and realization of different network topologies. Finally, SHM systems may store baselines, material databases and threshold values in order to establish an accurate damage identification criterion. Clearly, SHM technology has many advantages and it offers a great potential to overcome the shortcomings of conventional NDE methods. Firstly, preventive damage detection provides an early warning and allows for the immediate repair in order to prolong the lifespan of structural components. Secondly, continual monitoring is cost-efficient because it reduces the downtime of the system.

1.2 SHM Techniques and Sensing Modalities

The analysis of recent works on SHM shows that various sensing schemes can be efficiently employed in a monitoring system. However, the selection of a sensing modality highly depends on a particular industrial application. Generally, sensing methods in SHM can be broadly classified as passive and active depending on the actuation capabilities of the system.

Passive sensing accrues information about the current state of the structure from its vibrations or deformation under stress. Such an approach to data acquisition enables the implementation of many damage detection techniques based on mode shape, mode curvature, impedance or Fourier analysis. For instance, UT sensors can be used for continual monitoring of helicopter engine shafts: if defect occurs the resonant frequencies of the collected signals may change significantly [10]. One more example of a passive sensing technique is Acoustic Emission (AE). Recently, the use of AE-based SHM systems has been reported in nuclear power industry [11]. Usually, AE sensors are strategically placed in safety-critical areas of the plant; the sensors stay in idle mode and continuously "listen" to possible acoustic signatures emitted by newly appearing defects. Another growing trend in passive techniques is the increasing popularity of FBG sensors for SHM of composite materials. Minimal FBG strain causes the spectrum of light passing through it to shift or attenuate in frequency domain. And since composites are susceptible to matrix cracking and delamination between the layers, the high sensitivity of FBGs to minor deviations of strain in damaged areas is crucial for reliable defect detection. However, despite the simplicity of data analysis, all passive methods share one major drawback – their sensors have a very local range meaning that if damage occurs at some distance, it will never be detected. Therefore, such SHM systems require large density of receivers per unit area, which in turn increases the number of acquisition nodes and conditioning circuits. As a consequence, passive sensing is generally worth implementing for monitoring of the most failure-prone components such as dynamically loaded structures, cohesively bonded or riveted elements, composite stiffeners, surfaces subjected to impacts, etc. But whenever passive techniques are used for large-scale applications, some defects can be missed and only a "global" assessment can be obtained. This is due to the sparsity of sensor arrays, which limits the area of coverage.

Unlike passive methods, active modalities utilize the actuators to perturb the structure and examine it at any desired time. Therefore, it is always possible to perform an automatic inspection in repeatable manner that follows a maintenance schedule. Moreover, critical areas can be scanned locally if necessary, since active SHM systems provide full control of their excitation sources. Depending on damage detection scheme, actuators can be excited in groups or sequentially in order to interrogate the region of interest. Then the response is measured by sensors and processed by signal processing software. Data corresponding to current and previous states of the structure are often compared to identify damage presence, its location and severity. It is worth mentioning that conventional NDE incorporates many active methods, such as radiography, eddy current, microwave tomography, thermo-optical scans, etc.; but only few of them can be efficiently adopted in SHM applications. This is due to a number of design constraints, the most important of which is the need for small and lightweight transducers driven by compact devices with on-board processor. In addition, the whole system should be low cost and have the capability of being easily deployed on the structure. From this perspective, the technologies showing most amount of promise for active SHM belong to the field of ultrasonics.

1.2.1 Ultrasonic Testing and Guided Wave SHM

Active ultrasonic methods can be broken down into bulk wave inspection and guided waves. Unfortunately, bulk wave NDE have shown to be somewhat impractical for SHM purposes, since it involves the use of wedge-mounted transducers with grease couplants. Moreover, it requires a tedious data collection procedure: bulk waves are excited by surface "tapping" and propagate through the thickness of the structure providing only point-by-point measurements.

In contrast to bulk waves, Guided wave inspection has recently emerged as a prominent modality that is easily transitioned to SHM systems. Guided waves are elastic waves that follow the boundaries of the media in which they propagate. The main advantage of GWs is their ability to travel long distances without high energy losses, which provides large area coverage of the components to be monitored. The capability of GWs to penetrate through the thickness of the structure makes them sensitive to different imperfections such as cracks, impact damage and delaminations, essential in real-time SHM.

Guided waves exist in waveguides whose one particular dimension is much smaller than the others. The simplest examples of such geometries would include rods, plates, shells and pipes. If the wavelength is bigger than the thickness of the structure, a guided wave is confined between the boundaries through multiple reflections and propagates with complicated particle displacement patterns. GWs can be excited in many possible modes having different velocity and shape. Hence, the selection of the appropriate mode is important for maximizing damage detection capabilities of the system. Next section presents a brief introduction to Guided wave theory, since understanding of wave propagation in solids is essential for solving problems of GW signal analysis.

1.3 Guided Wave Theory

Guided wave theory dates back to the beginning of the twentieth century, when the main concepts of elastic wave propagation in solids were studied by Horace Lamb [12]. The British mathematician was the first to explain special properties of waves propagating in plates of infinite extent. Consequently, guided waves have been also called Lamb waves. In 1967, Viktorov used programmed digital device to analyze Lamb's equations and generate the dispersion curves: the non-linear relationship between the wave speed and actuation frequency for different propagation modes. Later he published his comprehensive study of GWs in isotropic plates highlighting their potential for NDE [13].

However, the truly broad use of guided waves in numerous applications was inspired by the advent of computer era. The advantage of high computational power opened the opportunity to implement guided wave technology for the inspection of structures with complex geometries and materials. It also facilitated the development of finite element modeling and signal processing algorithms for wave propagation analysis and damage detection. Works having the highest impact on guided wave NDE and SHM were reported by Rose, Nayfeh, Graff, Alleyne and many others [14, 15, 16].

This section introduces the fundamental equations describing Guided waves in isotropic plate-like structures. Since the topic is well documented in the literature, only the key results are presented in this section and the intermediate steps are summarized in the Appendix A. The case of fiber-reinforced composites is outlined in Chapter 3.

The constitutive relations describing Guided waves in plates can be thought of as a special case of elastic wave propagation in unbounded media. The differential equation of motion

in three-dimensional solids is known as Navier's equation [17]:

$$\mu \left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}\right) u_x + (\lambda + \mu) \frac{\partial}{\partial x} \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right) u_x = \rho \frac{\partial u_x}{\partial t^2}$$

$$\mu \left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}\right) u_y + (\lambda + \mu) \frac{\partial}{\partial x} \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right) u_y = \rho \frac{\partial u_y}{\partial t^2}$$

$$\mu \left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}\right) u_z + (\lambda + \mu) \frac{\partial}{\partial x} \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right) u_z = \rho \frac{\partial u_z}{\partial t^2}$$

$$(1.1)$$

Eq. (1.1) can be concisely expressed in vector form:

$$\mu \nabla^2 \mathbf{u} + (\lambda + \mu) \nabla (\nabla \cdot \mathbf{u}) = \rho \frac{\partial \mathbf{u}}{\partial t^2}$$
(1.2)

where $\mathbf{u} = \mathbf{i}u_x + \mathbf{j}u_y + \mathbf{k}u_z$ is the particle displacement vector, ρ is the material density, μ and λ are the Lame constants, the ∇ is a *del* differential operator and ∇^2 is a vector Laplacian.

It should be noted that Navier's differential equations governing wave propagation are based upon three fundamental relationships from linear elasticity theory. These include strain-displacement relations (Eq. (1.3)), equation of motion also known as Newton's generalized second law (Eq. (1.4)) and constitutive stress-strain relations or Hooke's law (Eq. (1.5)). Their expressions in tensor notation are:

$$\epsilon_{ij} = \frac{1}{2}(u_{j,i} + u_{i,j}) \tag{1.3}$$

$$\frac{\partial \sigma_{ij}}{\partial x_i} = \rho \frac{\partial u_i}{\partial t^2} \tag{1.4}$$

$$\sigma_{ij} = c_{ijkl} \epsilon_{kl} \tag{1.5}$$

where i, j = 1, 2, 3; ϵ_{ij} – the strain tensor and c_{ijkl} – the stiffness tensor; repeated in-

dexes imply summations and commas imply partial derivatives. Therefore, Navier's equation combines three equilibrium equations, six strain-displacement equations, and six constitutive equations from Eq. (1.3)–Eq. (1.5). Navier's equation can be solved with the help of Helmholtz vector and scalar potentials **H** and Φ :

$$\mathbf{u} = \nabla \Phi + \nabla \times \mathbf{H} \tag{1.6}$$

$$\nabla \cdot \mathbf{H} = 0 \tag{1.7}$$

Upon the substitution of Eq. (1.5) and Eq. (1.6) into Eq. (1.1) we get

$$(\lambda + 2\mu)\nabla^2 \Phi - \rho \nabla \ddot{\Phi} = 0 \tag{1.8}$$

$$\mu \nabla^2 \mathbf{H} - \rho \ddot{\mathbf{H}} = 0 \tag{1.9}$$

The rearrangement of (1.8-1.9) yields the equation of wave propagation in 3D homogeneous solids in terms of scalar and vector potentials

$$c_l^2 \nabla^2 \Phi = \ddot{\Phi}$$

$$(1.10)$$

$$c_s^2 \nabla^2 \mathbf{H} = \ddot{\mathbf{H}}$$

where $c_l = (2\mu + \lambda)/\rho$ and $c_s = \mu/\rho$. Double dot operator represents the second order time derivative. It can be shown that the total solution for displacement vector **u** has three components [17]:

$$\mathbf{u} = \mathbf{u}_{\mathbf{L}} + \mathbf{u}_{\mathbf{SH}} + \mathbf{u}_{\mathbf{SV}} \tag{1.11}$$

where $\mathbf{u_L}$ corresponds to *longitudinal* or *pressure* wave whose particle displacement is parallel to the direction of propagation, $\mathbf{u_{SH}}$ and $\mathbf{u_{SV}}$ define the *shear horizontal* and *shear vertical* waves. Motion of particles for these solutions is perpendicular to the direction of wave propagation. The snapshots of S-wave and P-wave are shown in Fig. 1.2.



Figure 1.2: Shear vertical and pressure waves in solids.

Shear vertical and pressure wave solutions are of particular importance if Navier's equations are solved for practical cases when the structure has some geometric bounds. In particular, the interaction of shear vertical and pressure waves in thin plates and shells gives rise to guided waves. In this section we consider the derivation of Lamb wave equations for two-dimensional plates of infinite length. For simplicity of analysis it is possible to assume that the wave potentials are invariant to the z-direction along the wave front [17].



Figure 1.3: Isotropic plate of infinite length and thickness 2d. Wave propagates in x-direction.

In this case $\partial/\partial z = 0$ and $\mathbf{u_{SH}}$ accepts only shear displacement u_z . In contrast, $\mathbf{u_L}$ with $\mathbf{u_{SV}}$ accept u_x and u_y displacements, which depend only on scalar potential Φ and a z-component of a vector potential H_z . Therefore, Navier's equation in terms of scalar and vector potentials for $\mathbf{u_L}$, $\mathbf{u_{SV}}$ possible solutions and the geometry in Fig. 1.3 takes the form:

$$\begin{cases} c_l^2 \nabla^2 \Phi = \ddot{\Phi} \\ c_s^2 \nabla^2 H_z = \ddot{H}_z \end{cases}$$
(1.12)

where c_l and c_s are longitudinal and shear wave speeds.

Introducing the simpler notation $\Phi = \phi$ and $H_z = \psi$ we get a classical formulation for Lamb wave equations in the potential form:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\omega^2}{c_p^2} \phi = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\omega^2}{c_s^2} \psi = 0$$
(1.13)

Assuming harmonic solution $e^{-i(\omega t - \xi x)}$, the system of equations Eq. (1.13) becomes:

$$\frac{\partial^2 \phi}{\partial y^2} + \left(\frac{\omega^2}{c_p^2} - \xi^2\right)\phi = 0$$

$$\frac{\partial^2 \psi}{\partial y^2} + \left(\frac{\omega^2}{c_s^2} - \xi^2\right)\psi = 0$$
(1.14)

In Eq. (1.14), $\xi = \omega/c$ defines a wavenumber. At this point we can introduce the more compact notation for convenience:

$$p^{2} = \frac{\omega^{2}}{c_{p}^{2}} - \xi^{2}$$

$$q^{2} = \frac{\omega^{2}}{c_{s}^{2}} - \xi^{2}$$
(1.15)

Hence,

$$\frac{\partial^2 \phi}{\partial y^2} + p^2 \phi = 0$$

$$\frac{\partial^2 \psi}{\partial y^2} + q^2 \psi = 0$$
(1.16)

The general solution for the above system takes the form:

$$\phi = A_1 \sin py + A_2 \cos py$$

$$\psi = B_1 \sin qy + B_2 \cos qy$$
(1.17)

where A_1, A_2, B_1 and B_2 are constants to be determined from the boundary conditions.

1.3.1 Symmetric Solution

Symmetric solution implies that u_x component of the displacement vector and shear stresses are symmetric about the midplane across the thickness of the plate, namely

$$u_{x}(x,-d) = u_{x}(x,d) \qquad \tau_{yx}(x,-d) = -\tau_{yx}(x,d)$$

$$u_{y}(x,-d) = -u_{y}(x,d) \qquad \tau_{yy}(x,-d) = \tau_{yy}(x,d)$$
(1.18)

Symmetric boundary conditions also include traction free surfaces

$$\tau_{yx}(x,-d) = -\tau_{yx}(x,d) = 0$$

$$\tau_{yy}(x,-d) = \tau_{yy}(x,d) = 0$$
(1.19)

Using the general solution for potentials in Eq. (1.17) and formulas for displacements and stresses from the Appendix A, Eq. (1.19) can be expressed as

$$\begin{cases} -2i\xi A_2 \sin pd + B_1(\xi^2 - q^2) \sin qd = 0\\ A_2(\xi^2 - q^2) \cos pd - 2i\xi B_1 \cos qd = 0 \end{cases}$$
(1.20)

A non-trivial solution for the above linear system of equations exists if the determinant in Eq. (1.20) vanishes:

$$\Delta_S = (\xi^2 - q^2)^2 \sin q d \cos p d + 4\xi^2 p q \cos q d = 0 \tag{1.21}$$

Rearranging Eq. (1.21) yields the dispersion relation for symmetric modes:

$$\frac{\tan pd}{\tan qd} = -\frac{(\xi^2 - q^2)^2}{4\xi^2 pq}$$
(1.22)

where p and q are given in Eq. (1.15).

1.3.2 Antisymmetric Solution

Antisymmetric solution requires that displacements and stresses are antisymmetric with respect to the midplane

$$u_{x}(x,-d) = -u_{x}(x,d) \qquad \tau_{yx}(x,-h) = \tau_{yx}(x,h)$$

$$u_{y}(x,-d) = u_{y}(x,d) \qquad \tau_{yy}(x,-h) = -\tau_{yy}(x,h)$$
(1.23)

The antisymmetric boundary conditions are the following

$$\tau_{yx}(x,-d) = \tau_{yx}(x,d) = 0$$

$$\tau_{yy}(x,-d) = -\tau_{yy}(x,d) = 0$$
(1.24)

Using the general solution for the potentials in Eq. (1.17) and formulas for the displacements and stresses from the Appendix A, Eq. (1.24) can be expressed as

$$\begin{cases} 2i\xi A_1 \cos pd + B_2(\xi^2 - q^2) \cos qd = 0\\ A_1(\xi^2 - q^2) \sin pd + 2i\xi B_2 \cos qd = 0 \end{cases}$$
(1.25)

A non-trivial solution exists if determinant in Eq. (1.25) vanishes:

$$\Delta_A = (\xi^2 - q^2)^2 \sin pd \cos qd + 4\xi^2 pq \cos qd \sin qd = 0$$
 (1.26)

Finally, we obtain Lamb wave equation for antisymmetric modes:

$$\frac{\tan pd}{\tan qd} = -\frac{4\xi^2 pq}{(\xi^2 - q^2)^2} \tag{1.27}$$

It can be noticed that the right-hand side of Eq. (1.22) is reciprocal to that of Eq. (1.27).

1.3.3 Phase Velocity and Group Velocity Dispersion Curves

It can be noticed that solutions of Rayleigh-Lamb equations in Eq. (1.22) and Eq. (1.27) define the dispersion curves or the relationship between the phase velocity, c_{ph} and the actuation frequency, ω . Since p and q also depend on ω , the phase velocity should be evaluated numerically at each frequency step for a fixed value of a plate thickness 2h.



Figure 1.4: Phase velocity dispersion curves of 2-mm thick aluminum plate.



Figure 1.5: Group velocity dispersion curves of 2-mm thick aluminum plate.

$$\frac{\partial c}{\partial (fd)} \cong \frac{\Delta c}{\Delta (fd)} \tag{1.28}$$

The example of phase velocity dispersion curves is presented in Fig. 1.4. It follows that the solution is not unique, and at high frequency-thickness products multiple symmetric and antisymmetric modes exist in the plate. However, the knowledge of group velocity dispersion curves (Fig. 1.5) can be considered even more important, since the group velocity, c_{gr} determines how fast the wavefront of each mode propagates. This enables one to calculate the Time-of-Flight (ToF) and distance traveled by wave packets in the structure, which is essential for locating damage sites. Group velocity dispersion curves can be obtained from the phase velocity data using the finite difference formula (Eq. (1.28)).

1.3.4 Mode Shapes

The across-thickness particle motion or mode shape can be considered another important aspect of reliable damage detection in GW SHM. In fact, the mode shape determines the sensitivity of a particular mode to different defects at some actuation frequency. For ex-



Figure 1.6: Displacements of S_0 mode.

ample, the dominance of u_y component indicates the increased sensitivity to surface defects, and big displacements u_x along the direction of wave propagation provide good sensitivity to cracks. Fig. 1.6 and Fig. 1.7 illustrate displacement components of S_0 and A_0 modes that propagate in 1-mm thick aluminum plate.



Figure 1.7: Displacements of A_0 mode.

1.3.5 Transduction of Guided Waves

In recent decades, many devices have been developed to excite guided waves in conventional NDE applications. For example, angle beam transducers Fig. 1.8(a) have been extensively utilized to test the integrity of welds and plate-like structures. These transducers introduce a refracted shear or longitudinal wave into a test piece and the selection of appropriate incidence angles allows for excitation of guided waves. Other types of transducers, such as comb actuators, EMATs and magnetostrictive tapes Fig. 1.8(c) have been used primarily for GW inspection of pipes and tubing. The main principle of their operation are well understood and documented in literature [13].

However, all the above-mentioned devices are too large to be permanently installed or embedded in the structure. Besides, they are driven at relatively high voltages which is not suitable for monitoring systems with power limitations. Therefore, the research in recent decades was mainly focused on the alternative method for guided wave excitation with the help of surface-mounted piezoelectric elements (PZT).



(c) Magnetostricitive transducer [19].

(d) PZT wafer [20].

Figure 1.8: Transducers for actuation and sensing of Guided waves.

PZT Wafers and Piezoelectric Governing Equations 1.3.5.1

PZT transducers for guided wave SHM are cheap, light weight and usually available in rectangular or round shapes Fig. 1.8(d). Additionally, piezo wafers can be designed to have a small thickness of about 0.2 mm or less. As a consequence, such sensors are unobtrusive and barely affect the dynamics of the system being monitored. PZTs are usually bonded to the surface of the structure with a temperature resistant epoxy in order to ensure reliable contact between them. In this coupled electromechanical system, PZT generates guided waves through surface "pinching". The detailed description of such interaction will be presented in the next section. The action of a PZT is based upon the direct and the



Figure 1.9: Piezoelectric effect: a).neutral state; b). strain conversion; c). voltage conversion.

inverse piezoelectric effects. By virtue of piezoelectric effect, mechanical stress induces an electric potential between certain faces of the crystal. Conversely, when the electric field is applied across the faces, the crystal undergoes mechanical strain (Fig. 1.9). This important property makes it possible to use the same wafer as an actuator and a receiver. Piezoelectric effect can be well described with the help of a coupled system of equations. According to the IEEE standard on piezoelectricity [21], the constitutive relations in strain-charge form are

$$\{\mathbf{D}\} = [\mathbf{e}]\{\mathbf{S}\} + [\varepsilon^S]\{\mathbf{E}\}$$
(1.29)

$$\{\mathbf{T}\} = [\mathbf{C}^E]\{\mathbf{S}\} + [\varepsilon^T]\{\mathbf{E}\}$$
(1.30)

where \mathbf{E} and \mathbf{D} – the electric field and the electric displacement vectors; \mathbf{T} and \mathbf{S} – the stress and strain vectors; $[\mathbf{C}^E]$ – stiffness matrix, $[\mathbf{e}]$ – piezoelectric stress coefficients and $[\varepsilon^S]$ – permittivity at a constant strain.

Eq. (1.29) implies the conversion of electric energy into mechanical energy (actuation), and Eq. (1.30) corresponds to the inverse energy transfer (sensing). Piezoelectric elements, most widely used in guided wave SHM, are manufactured from ceramic perovskite materials such as Lead Zirconate Titanate. In order to introduce the desired piezoelectric properties, wafers are subjected to siltering and polarization right after machining process. Most frequently, polarization is carried out across the thickness of the element and this direction is referred to as direction 3. Piezoelectric wafers have electrode layers on top and bottom faces, across which the input electric field is applied. This direction also coincides with the polarization axis in order to provide the highest strain response. Hence, the unidirectional actuation implies that E_1 and E_2 components of the electric field as well as D_1 and D_2 components of the electric displacement vector are negligibly small. This fact simplifies the analysis of interaction between the PZT and the structure in SHM.

However, despite numerous advantages of monolithic piezo-elements, there exist a major drawback, which is their brittleness. Low conformability could be a critical limitation for SHM of shells and pipe-like structures. Recently, this problem have been addressed by the development of a new generation of wafers called Anisotropic Piezocomposite Transducers.

1.3.5.2 Anisotropic Piezoelectric Transducers

The Anisotropic Piezoelectric Transducers (APT) consist of active piezoelectric fibers embedded into a soft epoxy matrix. Such engineering design makes the element more flexible than a monolithic PZT. Therefore, the APT can be easily deformed to match the shape of the curved surface. Another important property of APTs is anisotropy of electromechanical properties. In contrast to regular PZT, whose response as a strain sensor is independent of orientation, anisotropic transducers can be highly directional. In fact, the electric response of APT largely depends on the angle between the applied strain component and the orientation of its fibers. The most common types of fiber piezoelectric wafers are Active Fiber Composite (AFC) transducers (Fig. 1.10(b)) designed by Bent and Hagood [22] and Macro-Fiber Composite (MFC) transducers (Fig. 1.10(a)) developed by NASA. Both types use





(a) Macro-Fiber Composite transducer (b) Active Fiber Composite trans-[23]. ducer [22].

Figure 1.10: Anisotropic Piezocomposite Transducers.

interdigitated electrodes connected to fibers, which provides the highest piezoelectric coupling coefficient in the intended direction of actuation. The peculiarities of AFC and MFC design can be employed to detect the direction of the incident guided wave or to actuate specific GW modes.

1.4 Actuation of Guided Waves with Surface-bonded Piezoelectric Transducers

1.4.1 Shear Lag Model

The analytical solution for the actuation of guided waves with surface-mounted piezoelectric elements is important for GW signal processing, because the interaction between the wafer and structure determines the modes excited, their relative amplitudes and cut-off frequencies. In SHM, piezoelectric elements are bonded to the structure with epoxy, therefore the actuation and sensing largely depend on properties of the adhesive layer Fig. 1.11.



Figure 1.11: Actuation using surface-bonded PZT.

$$\tau(x) = \tau_0 e^{j\omega t} \tag{1.31}$$

When the oscillatory voltage is applied across the electrodes, PZT induces in-plane strain through d_{31} coupling coefficient making the bonding layer act in shear (Eq. (1.31)). The strain generated be the piezoelectric wafer is related to the applied voltage, V and thickness of the element, t_a through the equation

$$\varepsilon_v = d_{31} \frac{V}{t_a} \tag{1.32}$$

However, the strain γ in the adhesive layer and the strain transferred to the surface, ε , in general, will have different distributions compared to ε_v . This is influenced by the thickness of the bonding layer t_b , as well as the elastic properties of the adhesive and the structure, namely, the shear modulus G_b and the Young's modulus E (Fig. 1.11). The problem of determining the stress and strain distributions along the surface of the structure was first addressed by Crawley [24] and later revisited by Giurgiutiu [17]. It was shown that the surface strain generated by a single surface-bonded element can be represented as a superposition of surface strains induced by two collocated actuator pairs (Fig. 1.12). If the wafers are driven in-phase (the applied electric field has the same polarity at both wafers) the structure is subjected to linear extension. Similarly, if the polarity of the voltage across the electrodes is different, the structure is subjected to bending moment. By examining the free body



(b) Out-of-phase actuation

Figure 1.12: Actuation using collocated PZT pairs.

diagrams of the piezoelements in Fig 1.13 one can construct the equilibrium equations for the cases of symmetric and antisymmetric excitation:

$$t_a \frac{\partial \sigma_a}{\partial x} - \tau = 0 \tag{1.33}$$

$$t\frac{\partial\sigma}{\partial x} - \alpha\tau = 0 \tag{1.34}$$

Eq. (1.33) corresponds to the equilibrium equation of the PZT, while Eq. (1.34) describes the equilibrium of the structure. Note that the above equations are defined for symmetric $(\sigma_A; \alpha_A)$ and antisymmetric cases $(\sigma_S; \alpha_S)$ separately. In general, coefficient α needs to be determined based on the actuation frequency and mode shapes. However, Giurgiutiu showed that under the simplifying assumptions of uniform stress distribution for symmetric case and linear stress distribution for the antisymmetric case, the coefficient α takes on the values of 1 and 3. Further, the shear lag solution can be evaluated as the superposition of symmetric and antisymmetric excitations: $\sigma = \sigma_A + \sigma_S$ and $\alpha = \alpha_A + \alpha_S = 4$. Taking this into account, Eq. (1.33) and Eq. (1.34) can be further expressed in terms of strains to give the following system

$$\frac{\partial^4 \varepsilon_{act}}{\partial x^4} - \Gamma^2 \frac{\partial^2 \varepsilon_{act}}{\partial x^2} = 0 \tag{1.35}$$

$$\frac{\partial^4 \varepsilon_{st}}{\partial x^4} - \Gamma^2 \frac{\partial^2 \varepsilon_{st}}{\partial x^2} = 0 \tag{1.36}$$

where $\Gamma = \sqrt{G_b(\alpha + \psi)/E_a t_a t_b \psi}$ is a shear-lag constant and $\psi = Et/E_a t_a$ is a function of geometry and material properties.

Finally, by solving Eq. (1.36) we obtain the expressions for induced surface stresses and strains

$$\tau(x) = \frac{t_a}{a} \frac{\psi}{\alpha + \psi} E_a \varepsilon_v (\Gamma a \frac{\sinh \Gamma x}{\cosh \Gamma a})$$
(1.37)

$$\varepsilon(x) = \frac{\psi}{\alpha + \psi} \varepsilon_v (1 - \frac{\cosh \Gamma x}{\cosh \Gamma a}) \tag{1.38}$$

Eq. (1.35) defines new boundary conditions for Rayleigh-Lamb equations by applying surface traction forces along the bonding layer. Therefore, energy transfer conditions for symmetric and antisymmetric guided wave modes can be derived at this point. Giurgiutiu [17] demonstrated that the total solution for displacement and strains across



Figure 1.13: Shear lag model: surface stress.



Figure 1.14: Shear lag model: surface strain.

the thickness of the interrogated structure can be obtained in the Fourier domain resulting in the following relations

$$\varepsilon_{x}(x,t) = \frac{1}{2\mu} \sum_{\xi S} \frac{\tilde{\tau}(\xi^{S}) N_{S}(\xi^{S})}{D_{S'}(\xi^{S})} e^{i(\xi^{S}x - \omega t)} + \frac{1}{2\mu} \sum_{\xi A} \frac{\tilde{\tau}(\xi^{A}) N_{A}(\xi^{A})}{D_{A'}(\xi^{A}S)} e^{i(\xi^{A}x - \omega t)}$$

$$u_{x}(x,t) = \frac{1}{2\mu} \sum_{\xi S} \frac{1}{\xi^{S}} \frac{\tilde{\tau}(\xi^{S}) N_{S}(\xi^{S})}{D_{S'}(\xi^{S})} e^{i(\xi^{S}x - \omega t)} + \frac{1}{2\mu} \sum_{\xi A} \frac{1}{\xi^{A}} \frac{\tilde{\tau}(\xi^{A}) N_{A}(\xi^{A})}{D_{A'}(\xi^{A}S)} e^{i(\xi^{A}x - \omega t)}$$
(1.39)

where the determinants D_A ; D_S and the quantities N_A ; N_A as well as D_S' and D_A' are defined as

$$N_A = \xi q (\xi^2 - q^2) \sin p h \sin q h \tag{1.40}$$

$$D_A = (\xi^2 - q^2)^2 \sin ph \cos qh + 4\xi^2 pq \cos ph \sin qh$$
(1.41)

$$D_{S'} = \frac{\partial D_S}{\partial \xi}; \quad D_{A'} = \frac{\partial D_A}{\partial \xi}$$
 (1.42)

where ξ – wavenumber, h – half plate thickness; p and q are defined in Eq. (1.15).

1.4.2 Ideal Bonding Assumption

It can be inferred from Fig. 1.13 that the stress and strain induced by the piezoelectric transducer is largely concentrated near its edges. Hence, for the simplicity of analysis stress can be approximated with reasonably good accuracy by two delta functions positioned at the corresponding coordinates:

$$\tau(x) = a\tau_a[\delta(x-a) - \delta(x+a)] \tag{1.43}$$

Considering the latter, the amplitude of individual frequency components transferred to the structure through the ideal adhesive layer can be obtained by assuming the harmonic excitation of the PZT:

$$\tau(x) = a\tau_0[\delta(x-a) - \delta(x+a)]e^{j\omega t}$$
(1.44)

Upon substitution of Eq. (1.44) to Eq. (1.39) we obtain the solution for surface strain and displacement

$$\varepsilon_{x}(x,t) = -\frac{a\tau_{0}}{\mu} \left[\sum_{\xi S} \sin\xi^{S} a \frac{\tilde{\tau}(\xi^{S}) N_{S}(\xi^{S})}{D_{S'}(\xi^{S})} e^{i(\xi^{S}x - \omega t)} + \sum_{\xi A} \sin\xi^{A} a \frac{\tilde{\tau}(\xi^{A}) N_{A}(\xi^{A})}{D_{A'}(\xi^{A}S)} e^{i(\xi^{A}x - \omega t)} \right] \\ u_{x}(x,t) = -\frac{a\tau_{0}}{\mu} \left[\sum_{\xi S} \frac{\sin\xi^{S} a}{\xi^{S} a} \frac{\tilde{\tau}(\xi^{S}) N_{S}(\xi^{S})}{D_{S'}(\xi^{S})} e^{i(\xi^{S}x - \omega t)} + \sum_{\xi A} \frac{\sin\xi^{A} a}{\xi^{A} a} \frac{\tilde{\tau}(\xi^{A}) N_{A}(\xi^{A})}{D_{A'}(\xi^{A}S)} e^{i(\xi^{A}x - \omega t)} \right]$$
(1.45)

The displacement across the thickness of the structure u_y as well as the stresses τ_{yx} and τ_{yy} can be found upon substitution of the same constants obtained from the boundary conditions in the corresponding equations (see Appendix A).



Figure 1.15: Actuation using surface-bonded PZT.

Fig. 1.15 illustrates the surface strain induced by the PZT along the x-direction for the fundamental S_0 and A_0 modes that exist at all frequencies. The relations are evaluated for the case of 2-mm thick aluminum plate with 7-mm square PZT ideally bonded to its surface. Interestingly, at some frequencies such as 390 kHz the two modes are actuated with high amplitudes. However, some frequencies reject both modes (e.g. 710 kHz). This fact makes
it possible to use so called *mode tuning* in order to amplify some modes with respect to the others and achieve mode tuning.

1.4.3 Voltage Output of the PZT Sensor

Earlier in this section the transduction of guided waves to the structure was analyzed. However, in experiments one usually measures voltage output of the PZT sensor. Therefore, we need to consider the parameters on which the voltage depends. Piezo-sensor response to vibration was derived by Ragavan and Cesnik [25]. Voltage between the electrodes of the PZT is proportional to the superposition of strains across the area of its bottom face

$$V = \frac{Q}{C} = \frac{Y^{11} h_c g_{31}}{S_c (1 - \nu)} \int_S \varepsilon_{ii} dS$$
(1.46)

where Q – electric charge accumulated by the piezoelement; C – capacitance of the PZT; S_c – surface area of the PZT; h_c – thickness of the piezo; g_{31} – corresponding entry from the matrix of piezoelectric constants; Y^{11} – the in-plane Young's modulus of the sensor and ν – the Poisson's ratio; ε_{ii} – the sum of in-plane surface strains.

In the case of rectangular actuator Eq. (1.46) takes the form

$$V = \frac{Y^{11}h_cg_{31}}{4s_1s_2(1-\nu)} \int_{x_c-s_1}^{x_c+s_1} \int_{y_c-s_2}^{y_c+s_2} \left[\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2}\right] dx_1 dx_2 \tag{1.47}$$

where s_1 and s_2 are half-length and half-width of rectangular PZT; x_c and y_c are coordinates of the geometric center of the PZT. Clearly, V is linear with respect to the summation of the in-plane surface strains.

CHAPTER 2

Signal Analysis in Guided Wave SHM

2.1 Overall Approach to Damage Detection

Guided wave SHM with distributed PZT sensor networks employs pitch-catch and pulseecho damage detection methods (Fig. 2.1). In pitch-catch mode, one piezoelement actuates guided waves in order to interrogate the structure and the collocated element measures the response. If the wavelength of the GW mode is comparable to the size of the defect, the incident wave will be reflected and captured by the receiving PZT. Compared to pitchcatch, pulse-echo utilizes the same transducer to perform excitation and sensing. This is accomplished by electronic switching between the corresponding circuits of the hardware node.

Once the signals from all sensor pairs in the network are recorded, one needs to isolate Lamb wave scattering from anomalies in contrast to reflections from various boundaries in the complex structure. For pulse-echo mode damage reflections could be easily identified in



(b) Pulse-echo

Figure 2.1: PZT sensor configurations.

the case when anomalies are located closer to the transducer rather than to the edges of the structure. However, in pitch-catch configuration this can be accomplished by subtracting the actual sensor response from the baseline obtained on healthy sample. The baseline is required because the reflection might be relatively weak compared to the incident wave packets and edge reflections. Once the measurement from damage is isolated, the analysis algorithm should be capable of extracting damage-related features, such as the size, depth and location of the defect. For this purpose, Probability Diagnostic Imaging (PDI) has been widely adopted in recent works on guided wave SHM.

2.2 Probability Diagnostic Imaging

Essentially, PDI estimates the most probable location of defect from the TOF of individual reflection [26]. The method may combine pulse-echo and pitch-catch configurations of sensors in an active network to acquire signal features associated with damage. Assuming the GW signal contains a single mode, damage location could be easily identified by measuring the Time-of-Flight of a wave packet reflected by the anomaly. This ToF includes the time required for the wave packet to travel from the actuator to the defect and the time to reach from defect to the receiver. Hence, the ToF corresponding to the reflection from damage can be computed using the equation

$$ToF = \frac{L_{A-D} + L_{D-R}}{c_{gr}} \tag{2.1}$$

It follows that damage must be located on the locus of an ellipse defined by each actuatorreceiver pair. Since measurement errors are always present, PDI assigns the 2D Gaussian probability density function to each pixel on every ellipse of the array

$$F(z_i) = \int_{-\infty}^{z_i} f(z_i) dz \tag{2.2}$$

$$I(x_m, y_n) = 1 - [F(z_i) - F(-z_i)]$$
(2.3)

where $I(x_m, y_n)$ – illumination function, (x_m, y_n) – coordinates of the specific point on the grid, $F(z_i)$ – Gaussian cumulative probability function, $f(z_i)$ – Gaussian probability density function.

As a result, the regions of the image where more of the ellipses overlap are illuminated with higher intensity, indicating the most probable location of the damage.

2.3 Challenges in Signal Processing of Guided Waves

Damage detection algorithms employed in SHM systems with active PZT sensor networks assume unimodal propagation of guided waves. For instance, PDI requires the knowledge of group velocity for accurate damage detection, that follows from Eq. (2.1). Actuation of a single GW mode can be accomplished with the help of collocated PZT pair or frequency tuning as discussed in previous sections. However, the use of collocated actuators is somewhat impractical for SHM, since it doubles the number of transducers in the network. Moreover, both surfaces of the structure may not be always accessible. On the other hand, mode tuning requires only one piezoelement for selective mode excitation. Nevertheless, this approach does not guarantee the complete rejection of other modes with respect to the dominant mode. This follows from the fact that in practice the bonding layer between the PZT and structure is not ideal as compared to theoretical assumptions made in derivation of Eq. (1.39). In addition, one needs to consider mode shapes which would maximize the sensitivity of the system to specific types of defects, but in general the optimal mode shape may correspond to frequencies at which the sweet spot does not exist. In the case when actuation is not strictly mode selective, multiple modes will propagate in the structure with different amplitudes and velocities, interfering with each other as well as with reflections from edges and structural defects. Given the monolitic PZT elements are equally sensitive to u_1 and u_2 displacements (Eq. (1.47)), the collected signals will contain reflections from all directions of wave propagation across the surface of the structure. In terms of damage detection, all of the above makes the interpretation of GW signals a fairly complicated problem. It is apparent



Figure 2.2: Approach to signal processing.

that GW signals should be subjected to some pre-processing stage or *Mode Decomposition* (MD) in order to decouple the contributions from different modes and facilitate the correct performance of diagnostic imaging. Hence, the overall approach to signal processing in SHM systems with distributed PZT sensor networks can be now illustrated by the Fig. 2.2.

As will be seen next, development of MD algorithms requires the selection of features to classify the modes and forward model to simulate wave propagation in the structure. Moreover, the MD algorithm should process signals automatically and it should be fast enough to be used in real-time monitoring systems. This study focuses on the development of decomposition algorithms which could be established with the help of time-frequency analysis and iterative methods such as matching pursuits. In the next section, a brief overview of the above methods will be presented.

2.4 Characteristic Features for Mode Identification

The narrowband actuation of guided waves (e.g. Morlet wavelet, Fig. 2.3(a)) on any non-flat region of dispersion curves introduces a difference between the velocities of lower and higher frequency components of the generated wave packets. Consider the group delay of a single wave packet [38]:

$$\tau_g(\omega) = \frac{\partial \varphi(\omega)}{\partial \omega} = \frac{\partial k(\omega) x_0}{\partial \omega} = \frac{x_0}{\partial \omega / \partial k(\omega)} = \frac{x_0}{c_{qr}}$$
(2.4)

where $\varphi(\omega)$ – phase of the actuation waveform, $k(\omega)$ – wavenumber, x_0 – propagation distance.



Figure 2.3: Narrowband actuation waveform.

The group delay is in inverse relation with the group velocity, therefore it can be an informative feature about modal content of a particular GW signal. For example, for a given narrowband actuation frequency range of Morlet wavelet (Fig. 2.3(b)), the group delay slope of S_0 mode is positive and that of A_0 is negative if GW propagates in a 2-mm thick aluminum



Figure 2.4: Dispersion curves of a 2-mm thick aluminum plate corresponding to the spectrum of the Morlet wavelet.

plate with dispersion curves shown in Fig. 2.4. It follows that information about the mode can be extracted from the slopes of the instantaneous frequency components of the signal on a time-frequency plane.

2.5 Time-Frequency Representations in GW SHM

Time-frequency representation (TFR) is an important aspect of signal processing in Guided wave SHM, since it provides tools for mode identification. Moreover, TFRs can be used in a combination with some post-processing such as ridge or skeleton extraction in order to isolate wave packets and locate their time-frequency centers. Unlike well established Fourier transform, which is suitable for analysis of stationary signals only, TFRs allow for determining the instantaneous frequency components of the signal as a function of time. In other words, the TFRs are capable of analyzing a given evolutionary time series in joint time and frequency domain. TFRs can be broadly classified as atomic decompositions (linear TFRs) and energy distributions (quadratic TFRs). It should be mentioned that both classes have been employed in different applications involving GW signal analysis. A brief overview of TFRs is given in the next section.

2.5.1 Linear Time-Frequency Representations

2.5.1.1 Short Time Fourier Transform

Linear TFRs also called *atomic representations* decompose original signal into a weighted sum of elementary functions. One of the simplest and most intuitive examples of a linear TFR is the Short Time Fourier Transform (STFT). The GW signal to be processed is multiplied by a window function with a short time support. Then the Fourier Transform of the signal within the window is taken as it moves along the time axis. This results in a two-dimensional time-frequency representation

$$STFT_x(t,\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} s(\tau)h(\tau-t)e^{-j\omega t}d\tau$$
(2.5)

where x(t) – original signal; ω – radian frequency; τ – time lag variable.

Window function is always chosen to be shorter compared to the length of the signal in order to provide localized information about its spectrum. Additionally, the window is designed to suppress the original signal at the ends of the selected support in order to prevent spectrum leakage [27]. However, the main drawback of STFT is a trade-off between time and frequency resolution. Considering the signal which is perfectly localized in time

$$x(t) = \delta(t - t_0) \tag{2.6}$$

the time resolution of STFT will be affected by the length of the analysis window

$$STFT_x(t,\omega) = h(t-t_0)e^{-j\omega t_0}$$
(2.7)

Similar relation holds for frequency resolution indicating that long analysis windows yield less frequency spreading. This result may also be interpreted from the perspective of timebandwidth product theorem. The time-bandwidth product theorem, or more generally, the *uncertainty principle* is a fundamental concept of time-frequency analysis based upon the properties of Fourier transform pairs. The uncertainty principle states that the joint timefrequency resolution is always limited and the TFR cannot provide perfect localization in time and frequency simultaneously. The above concept can be well illustrated by considering the energy density of the signal $|x(t)|^2$ and its Fourier transform $|X(\omega)|^2$ as probability distributions [28]. Assuming the signal has finite energy

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt < +\infty,$$
 (2.8)

one can calculate the average time t_m at which the signal is centered and the average frequency ω_m of its spectrum

$$t_m = \frac{1}{E} \int_{-\infty}^{+\infty} t |x(t)|^2 dt$$
 (2.9)

$$\omega_m = \frac{1}{E} \int_{-\infty}^{+\infty} \omega |X(\omega)|^2 d\omega \qquad (2.10)$$

Then the time and frequency spreading can be defined as variances

$$\sigma_t^2 = \frac{4\pi}{E} \int_{-\infty}^{+\infty} (t - t_m)^2 |x(t)|^2 dt$$
 (2.11)

$$\sigma_{\omega}^2 = \frac{4\pi}{E} \int_{-\infty}^{+\infty} (\omega - \omega_m)^2 |S(\omega)| d\omega \qquad (2.12)$$

where σ_t – time variation and σ_{ω} – bandwidth. It follows that the energy of the signal is concentrated in the area proportional to the time-bandwidth product $\sigma_t \sigma_{\omega}$, and moreover

$$\sigma_t^2 \sigma_\omega^2 \ge \frac{1}{4} \tag{2.13}$$

Depending on selected window, STFT uniformly tiles time-frequency plane in rectangles schematically shown in Fig. 2.5. Eq. (2.13) holds for signals pre-multiplied by Gaussian



Figure 2.5: Time-frequency tiling of STFT.

windows and therefore, it also determines the time-frequency resolution of STFT. It should be

noted that STFT was employed in some works on guided wave signal processing [29], but in general, it does not provide the best capability to resolve overlapped multimodal reflections. Hence, methods for reducing the spread of STFT is discussed later in this section.

2.5.1.2 Continuous Wavelet Transform

In recent decades, Continuous and Discrete Wavelet Transforms have been widely adopted in GW signal processing. In contrast to the STFT, which decomposes a given signal in terms of sinusoids with infinite duration, wavelet transform projects the signal on a family of oscillatory functions (wavelets) with limited time support

$$CWT_{x}(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{+\infty} x(t)\psi^{*}(\frac{t-b}{a})dt$$
(2.14)

Wavelet families can be easily constructed from an elementary function $\psi(t)$ called mother wavelet by dilations and translations using scaling factor a > 0 and time shift b. CWT



Figure 2.6: Time-frequency tiling using dyadic wavelets.

operates with continuous time, but time-scale parameters are often sampled on dyadic grid, which allows for efficient implementation of the algorithm

$$CWT_{j,k} = CWT\{x(t); a = 2^j, b = k2^j\} \ j,k \in \mathbb{Z}$$
 (2.15)

Clearly, the resulting distribution expresses the signal with respect to translation and scale rather than time and frequency. One of the characteristic features of CWT is the way the time-frequey plane is tiled (Fig. 2.6). It can be noticed that the resolution of CWT turns out to be scale dependent: lower frequency components of the analyzed signal can be well resolved in scale but poorly in time; conversely, higher frequency components are better localized in time, but the corresponding scale resolution is low. As a consequence, CWT can be useful for multiresolution analysis of broadband GW signals. Additionally, its discrete version (Discrete Wavelet Transform) is shown to be efficient in applications which require denoising [30]. However, CWT is still subject to uncertanity principle and its joint resolution in time and scale is also limited.

2.5.1.3 Chirplet Transform and Warping Operators

Chirplet Transform and TFRs employing warping operators are natural extension of previously discussed atomic decompositions. Chirplet transform expands a signal in terms of a basis of multi-scale wavelets with linear frequency modulation also called chirplets.

$$CT_{x} = \int_{-\infty}^{+\infty} x(t) g_{t_{0},\omega_{0},s,q,p}^{*}(t) dt$$

$$g_{t_{0},\omega_{0},s,q,p}(t) = T_{t0} F_{\omega_{0}} S_{s} Q_{q} P_{p} h(t)$$
(2.16)

where h(t) is window function, T_{t0} – time shift of the chirplet, F_{ω_0} – frequency shift, S_s – scaling parameter, Q_q – frequency shear and P_p – time shear parameters.



Figure 2.7: Warped tiling of time-frequency plane.

The additional degrees of freedom, namely shear parameters, control the chirp rate of generated functions and enable rotation of elementary time-frequency cells. In fact this property is useful when some prior information about localization of signal components on a TF plane is known. Further, in recent works on GW signal processing warping operators have been introduced. The detailed description of the concept can be found in [31]. TFRs based on warping functions allow for tiling of time-frequency plane in a fashion that matches the group velocity curves of different materials. Many works employing such TFRs were focused on obtaining the dispersion properties of the structure using broadband excitation of guided waves. However, this is not very practical for online monitoring applications that require minimal signal spreading and narrowband actuation.

2.5.2 Quadratic TFRs

Quadratic representations provide valuable information about time and frequency localization of the energy carried by the GW signal. Naturally, such TFRs can be understood as a joint energy density function that satisfies

$$E_x = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \rho_x(t,\omega) \, dt d\omega \tag{2.17}$$

Another important properties of quadratic TFRs include the marginals

$$\int_{-\infty}^{+\infty} \rho_x(t,\omega) dt = |X(\omega)|^2$$

$$\int_{-\infty}^{+\infty} \rho_x(t,\omega) d\omega = |x(t)|^2$$
(2.18)

It can be shown that these are satisfied by *energy* distributions and more generally, by TFRs of *Cohen's* class [32]. However, many GW signal processing applications for SHM of metal and composite structures involve *spectrograms* and *scaleograms* defined as a squared modulus of STFT and CWT correspondingly:

$$S_x(t,\omega) = |STFT(t,\omega)|^2 \tag{2.19}$$

$$C_x(t,\omega) = |CWT(t,a)|^2 \tag{2.20}$$

The above representations are non-negative and always real-valued. However, they do not satisfy conditions in Eq. (2.18) and therefore can be considered as transition groups [32] from atomic decompositions to properly defined energy distributions. This can be explained by the fact that spectrograms and scaleograms mix the energy of the window or wavelet function with that of the original signal. Another interesting property of these representations is a quadratic superposition principle, which means that the spectrogram or scaleogram of the sum of two signals is not the sum of the two separate representations

$$x(t) = y(t) + z(t)$$
 (2.21)

$$S_{x}(t,\omega) = S_{y}(t,\omega) + S_{z}(t,\omega) + 2Re\{S_{y}(t,\omega)S_{z}^{*}(t,\omega)\}$$
(2.22)

However, it is true that for the above distributions the interference terms appear only at points where the frequency components of the signal cross on a TF plane [28, 32]. Hence, this drawback is not very critical. Finally, the time-frequency resolution of spectrograms and scaleograms is limited exactly as it happens in the case of STFT or CWT.

2.5.2.1 Wigner-Ville Distribution

The first and most important TFR which belongs to the class of energy distributions is Wigner-Ville distribution (WVD)

$$W_x(t,\omega) = \int_{-\infty}^{+\infty} x(t+\frac{\tau}{2}) x^*(t-\frac{\tau}{2}) e^{-j\omega\tau} d\tau \qquad (2.23)$$

where x(t) – original signal; τ – time lag variable.

Wigner-Ville provides perfect time and frequency resolution, however it suffers from issues related to cross-terms. Unlike spectrograms and scleograms, interference terms of WVD are non-zero regardless of the time-frequency distance between the signal terms. Moreover, WVD can yield negative values, which makes it difficult to interpret the results of decomposition. The influence of interference terms can be alleviated by using special smoothing kernels in the ambiguity domain [32]. However, this approach reduces the resolution of the representation.

2.5.3 Time-Frequency Reassignment

The reassigned TFRs define a different group of representations that has recently gained attention in GW signal analysis. The reassignment procedure substantially reduces the blurriness of the TFR and improves its mode identification capabilities. Moreover, reassignment suppresses the contributions from cross-terms and makes the TFR amenable to post-processing. In general, it is known that any distribution of Cohen's class can be expressed in terms of a 2D-convolution of the WVD with some kernel function

$$C_x(t,\omega;\Pi) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \Pi(t-s,\omega-\xi) \ W_x(s,\xi) \ ds \ d\xi$$
(2.24)

where $\Pi(t-s,\omega-\xi)$ – kernel, corresponding to a specific TFR; $W_x(s,\xi)$ – Wigner-Ville distribution of the signal.

Considering the above equation, it is noticed that essentially this operation performs weighted averaging of the WVD of the signal around (t,ω) point [28]. Moreover, (t,ω) defines the *geometrical* center of the local TF region limited by s and ξ . However, it would be more logical to assign the local average to the *center of gravity* of the region. This is the idea underlying time-frequency reassignment: reassignment procedure moves a particular value of the original TFR from point (t,ω) to local center of gravity of WVD centroid at $(\hat{t},\hat{\omega})$. Consequently, the reassigned coordinates are given by

$$\hat{t}(x;t,\omega) = \frac{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} s\Pi(t-s,\omega-\xi) \ W_x(s,\omega) \ ds \ d\omega}{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \Pi(t-s,\omega-\xi) W_x(s,\xi) \ ds \ d\xi}$$
(2.25)

$$\hat{\omega}(x;t,\omega) = \frac{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \xi \Pi(t-s,\omega-\xi) W_x(s,\xi) \, ds \, d\xi}{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \Pi(t-s,\omega-\xi) W_x(s,\xi) \, ds \, d\xi}$$
(2.26)

The resulting TFR takes the form

$$CR_x(t',\omega';\Pi) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} C_x(t,\omega;\Pi) \,\delta(t' - \hat{t}(x;t,\omega))\delta(\omega' - \hat{\omega}(x;t,\omega)) \,dt \,d\omega \quad (2.27)$$

In the same manner, the reassignment procedure can be also applied to the time-scale



Figure 2.8: Ideal TFR and Wigner-Ville distribution of the signal.

energy distributions. It is possible to illustrate the performance of reassignment method on specific examples. Fig. 2.8(a) demonstrates the synthetic signal generated in the timefrequency domain with the help of Matlab Time-Frequency Toolbox [28]. The designed signal consists of a linear and a polynomial chirp autoterms, which in some way approximate real world GW signals. In turn, Fig. 2.8(b) – Fig. 2.10(b) present the comparison of different



(a) STFT of the signal

(b) Smoothed WVD of the signal

Figure 2.9: Comparison of smoothed WVD and STFT of the signal (window length = 413 points).



(a) Reassigned Smoothed WVD of the sig- (b) Reassigned spectrogram of the signal nal

Figure 2.10: Comparison of reassigned smoothed WVD and reassigned spectrogram of the signal (window length = 413 points).

TFRs applied to the time domain version of Fig. 2.8(a). Analyzing the obtained representations, one may notice that the spectrogram and smoothed WVD have significant amount of blurring, while reassigned smoothed WVD and reassigned spectrogram are more localized than other TFRs and yield nearly same results. However, real-time SHM applications require the TFR to be computationally efficient and easy to implement. From this perspective, reassigned spectrogram computed with the help of Nelson's method [33] can be considered the best candidate for GW mode identification. Besides, the reassigned spectrogram also satisfies the time and frequency shifts covariance and it is perfectly localized on linear chirp signals and impulses

$$x(t) = Ae^{j(\omega_0 t + \alpha t^2/2)}$$
(2.28)

$$\hat{\omega} = \omega_0 + \alpha \hat{t} \tag{2.29}$$

$$x(t) = A\delta(t - t_0) \tag{2.30}$$

$$\hat{t} = t_0 \tag{2.31}$$

Finally, the performance of the reassigned spectrogram can be further enhanced by using Chirp-Z transform instead of computing the FFT for each window. This approach is discussed in the next section.

2.6 Reassigned Spectrogram with Chirp Transform Kernel

At this point we develop the TFR based on reassigned spectrogram, which will be used for mode decomposition algorithm discussed in this work. First, it is possible to derive an explicit expression for re-mapped time and radian frequency coordinates in Eq. (2.25) and Eq. (2.26). STFT can be described in terms of the Gabor Transform:

$$x(t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \epsilon(t,\omega) \cdot \Psi^*(t,\tau,\omega) d\tau \ d\omega$$
(2.32)

The coefficients of the transform are defined as

$$\epsilon(t,\omega) = \int_{-\infty}^{+\infty} x(t) \cdot \Psi^*(t,\tau,\omega) dt$$
(2.33)

The window function of STFT is expressed in terms of

$$\Psi^*(t,\tau,\omega) = h(t-\tau)e^{-j\omega(\tau-t)}$$
(2.34)

The coefficients of the Gabor Transform have amplitude and phase modulation

$$\epsilon(t,\omega) = \int_{-\infty}^{+\infty} x(t)h(\tau-t)e^{j\omega(\tau-t)}dt = e^{j\omega\tau} \int_{-\infty}^{+\infty} x(t)h(\tau-t)e^{-j\omega t}dt = A(\tau,\omega)e^{j\Phi(\tau,\omega)}$$
(2.35)

The original signal can be reconstructed as

$$x(t) = \int_{-\infty}^{+\infty} A(\tau,\omega)h(\tau-t)e^{j[\Phi(\tau,\omega)-\omega\tau+\omega t]}$$
(2.36)

The reassignment method that identifies the local energy centers, can be also explained by the phase stationarity principle. In the case of Lamb wave signals, Eq. (2.36) represents the oscillatory integral over a function that fluctuates rapidly under a slowly varying envelope. The significant contributions to Eq. (2.36) come from the vicinity of points where the phase is stationary, that is, where it goes through zero [34], which yields

$$\begin{aligned} x(t) &= \frac{\partial}{\partial \omega} [j(\Phi(\tau, \omega) - \omega\tau + \omega t)] \\ x(t) &= \frac{\partial}{\partial \tau} [j(\Phi(\tau, \omega) - \omega\tau + \omega t)] \end{aligned} \tag{2.37}$$

Hence, the re-mapped time and frequency coordinates of the spectrogram can be computed as

$$\hat{t}(\tau,\omega) = \tau - \frac{\partial \Phi(\tau,\omega)}{\partial \omega}$$

$$\hat{\omega}(\tau,\omega) = \frac{\partial \Phi(\tau,\omega)}{\partial \tau}$$
(2.38)

where $-\partial \Phi(\tau,\omega)/\partial \omega$ is a local group delay and $\partial \Phi(\tau,\omega)/\partial \tau$ is the instantaneous frequency.

In this work, the reassigned spectrogram was implemented using the method, proposed by Nelson [33]. It should be noted that the algorithm takes advantage of mathematically defined cross-spectral surfaces to compute the coordinates in Eq. (2.38). In addition, the reassigned spectrogram gets more informative when the window step is small and the FFT window length is large. Therefore, the redundancy of the representation can be reduced and the resolution can be further improved by using Chirp-Z transform (CZT) instead of FFT. CZT (Fig. 2.11) is an algorithm frequently used to compute any set of equally spaced samples of the Discrete Fourier Transform (DFT) for an arbitrary contour on a Z-plane [35, 36]. Or, equivalently, it provides zooming into a narrowband frequency region of the signal in order to interpolate the details (Fig. 2.15). Let us denote a signal x[n] and its Fourier transform $X(\omega)$. Consider evaluation of a circular segment on a Z-plane having M equally spaced points

$$\omega = \omega_0 + k\Delta\omega, \qquad k = 0, 1, 2...M - 1$$
(2.39)

The DFT of the signal can be expressed in terms of the chirp $W = e^{j\Delta\omega}$:

$$X(e^{j\omega k}) = \sum_{n=0}^{N-1} x[n] \cdot e^{j\omega kn} = \sum_{n=0}^{N-1} \cdot e^{j\omega_0 n} W^{nk}$$
(2.40)

Using the identity $nk = \frac{1}{2}[n^2 + k^2 - (k - n)^2]$, we can derive the final expression for CZT

$$X(e^{j\omega k}) = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\omega_0 n} W^{n^2/2} W^{k^2/2} W^{-(k-n)^2/2} = W^{k^2/2} (\sum_{n=0}^{N-1} g[n] W^{-(k-n)^2/2})$$
(2.41)

CZT can be implemented using Eq. (2.41). The proposed time-frequency representation



(a) Linear chirp and a unit circle on a (b) CZT allows for evaluating of DFT on a complex plane specific segment

Figure 2.11: Comparison of Discrete Fourier transform with Chirp-Z transform.



Figure 2.12: Implementation of Chirp-Z transform algorithm.

or Reassigned Spectrogram with Chirp Transform kernel (RSCT) can be adjusted to zoom into the narrowband range around the peak frequency of the actuation Morlet wavelet and do not compute the FFT coefficients that are not of interest (Fig. 2.12). The reassignment procedure developed in this section was applied to experimental data. Fig. 2.17 and Fig. 2.18 demonstrate the original STFT and RSCT of Lamb wave signal (Fig. 2.16) obtained on 2-mm thick aluminum plate (Fig. 2.13). Comparing the obtained results with STFT



Figure 2.13: Experimental set-up for obtaining GW signal: $600 \times 600 \times 2$ mm aluminum plate.



Figure 2.15: CZT of GW signal. Zooming into 220-320 kHz range.

representation, it can be noticed that STFT is too blurred, but RSCT is capable of extracting the information about the group delay (or slopes of the chirps corresponding to wave packets of different modes). As expected from the previous section, the slope of S_0 mode is positive and that of A_0 is negative. Interestingly, the amplitude-frequency centers presented in RSCT are clearly different for S_0 and A_0 modes. This can be explained by frequency-dependent strain transfer conditions from the PZT to the plate, discussed in Chapter 1.



Figure 2.16: Lamb wave signal (10240 samples).



Figure 2.17: STFT (window size 350 samples).



Figure 2.18: Reassigned spectrogram with CZT.

In general, Time-Frequency Representations allow for determining of GW modes with well separated ToFs. However, the superposition of wave packets in time domain may result in cancellation of some frequency components contained in the spectra of the original wave packets. This creates some interference frequency transitions, that could be misinterpreted as modes. Therefore, in this work we propose to complement time-frequency analysis with Matching Pursuit method for mode decomposition.

2.7 Matching Pursuit (MP) algorithms in Guided wave signal processing

Matching pursuit is an iterative algorithm which decomposes the signal into a linear combination of functions (atoms) from an overcomplete frame D, called a dictionary [37]:

$$x = \sum_{n=0}^{N-1} \alpha_n g_{\gamma n} \tag{2.42}$$

where x – input GW signal, α_n – weight coefficients corresponding to selected atoms from the dictionary, $g_{\gamma n}$ – frame functions, N – number of dictionary elements required for sparse decomposition.

Given a dictionary with unit norm atoms, let R_n be the residual at n^{th} approximation of a signal x. MP projects the signal onto the elements of D in order to find the element that has the highest correlation with it. In a classical formulation by Mallat and Zhang [37], this iterative scheme consists of the following steps:

- 1. Specify initial conditions $Rx = x_0$.
- 2. Find the atom with the highest inner product $g_{\gamma n} = argmax \langle R_{n-1}, g_{\gamma n} \rangle$.
- 3. Subtract chosen atom from the signal and update the residual: $R_n = R_{n-1} \langle R_n, g_{\gamma n} \rangle$.

4. Stop the algorithm when the residue energy is sufficiently small or the maximum number of iterations n_{max} is exceeded.

Essentially, the redundancy of the dictionary allows MP to find the best projections onto D thus making the original signal sparse in a coefficient domain. This property makes it useful for solving complex minimization problems in Compressed Sensing and many other applications. However, one of the key properties of MP that makes it useful for GW SHM, is its ability to decompose the overlapped ultrasonic wave packets with the help of a properly designed dictionary. In addition MP is robust to noise, which follows from the fact that the dictionary contains only predefined noise-free functions. MP always aims at finding the sparsest decomposition of the signal (e.g. linear combination requiring the least number of frame-functions). For a properly constructed tight frame, the solution would be sparser if more atoms are used. However, this increases the complexity of the problem and in general, it is non-polynomial hard. Therefore, the choice of dictionary plays an essential role in MP decomposition.

2.7.1 Gabor, Chirplet and Dispersion-based Dictionaries for MP

In GW analysis, elements of D are chosen to have shapes similar to those of ultrasonic wave packets. Since Gabor and chirplet functions are very similar to oscillatory waveforms with short time support, the matching pursuit algorithm with these dictionaries has been explored by a number of researchers [38, 39, 40]. A unit energy chirplet can be expressed as

$$g(s, u, \omega, c) = \frac{1}{\sqrt{s}} g(\frac{t-u}{s}) \cdot e^{j[\omega(t-u) + \frac{c}{2}(t-u)^2 + \varphi]}$$
(2.43)

where s – scaling factor, u – translation; c – chirp rate; g(x) – Gaussian modulation function; φ – initial phase.



Figure 2.19: Atoms of conventional MP dictionaries: Gabor and two chirplets with different scales.

If c = 0, the atom degrades to a Gabor function (Fig. 2.19). However, the implicit assumption in these works is that the signals are either non-dispersive or have a linear phase velocity. In general, the atoms of such dictionaries are ill-suited for analyzing dispersive wave packets at long propagation distances from the actuator, where the envelope of the wave packet is not necessarily Gaussian-shaped. Furthermore, numerically computed atoms are not tiled to the propagation distance explicitly. As a result, MP algorithm needs to compute a large number of inner products along all possible time samples in order to find the best projection. With such dictionaries, the decomposition tends to be slow (redundant dictionary that includes all possible shifts of atoms), inaccurate (coarse dictionary) or, finally, it may require interpolation after each iteration. From this perspective, dispersion based dictionary offers more precise analysis of the signal's mode content.

2.7.2 Generation of Atoms of the Dispersion-based Dictionary

Dispersion based dictionary for MP decomposition of ultrasonic signals was first proposed by Hong [41]. In such a dictionary, atoms correspond to snapshots of wave packets at a distance x_n from the actuator. Each atom can be generated in the frequency domain and then transformed into the time domain:

$$g_{\gamma n} = iFT\{M(\omega) \cdot e^{-jk(\omega)x_n}\}$$
(2.44)

where $g_{\gamma n}$ – specific atom from the dictionary, $M(\omega)$ – Fourier Transform of the Morlet wavelet and $k(\omega)$ – wavenumber, x_n – distance traveled by a wave packet.

Figure 2.20: Generated atoms of A_0 mode propagating in 2 mm thick aluminum plate at 0.25 m, 0.5 m, 1.0 m and 1.5 m from the actuator. Actuation waveform: 250 kHz, 5 cycle Hann windowed sinusoid.

Eq. (2.44) assigns different phase velocities to the Fourier components of the actuation waveform by considering the corresponding wavenumbers. As a result, the atoms realistically simulate the dispersion of Lamb waves (Fig. 2.20). However, in the case of surface-bonded piezoelectric wafer sensors, the magnitude of each frequency component transferred to the structure depends not only on the shape of Morlet wavelet, but also on the geometry and material properties of PZT. Hence, the compensation for energy transfer conditions can be included into the atoms to achieve more accurate decomposition (Fig. 2.21). In this case Eq. (2.44) becomes

$$g_{\gamma n}^{\varepsilon} = iFT\{M(\omega) \cdot \varepsilon(\omega) \cdot e^{-jk(\omega)x_n}\}$$
(2.45)

where $\varepsilon(\omega)$ is a frequency dependent function describing strain transfer of ideally bonded



Figure 2.21: Influence of PZT strain transfer conditions on the actuated GW modes.



Figure 2.22: Spectra of S_0 and A_0 modes generated by surface-bonded PZT.

PZT for each GW mode. Taking closer look at Fig. 2.22 it is possible to notice that at some frequencies $\varepsilon(\omega)$ acts as a notch filter for A_0 mode, which distorts the Gaussian shaped envelope of wave packets. As will be seen in the next Chapter, atoms generated using Eq. (2.45), which accounts for strain transfer conditions, simulate the actual shapes of experimental wave packets most accurately.

2.8 Hybrid Algorithm based on RSCT and Matching Pursuit with Dispersion-based Dictionary for Mode Decomposition of Guided Wave Signals

Efficient implementation of Mode Decomposition algorithm is critical for online monitoring applications that require minimal time for signal processing. Since MP is iterative it becomes computationally intensive. However, the structure of dispersion-based dictionary can be exploited to substantially decrease its complexity. In fact, each element of such a dictionary has a short time support L compared to the length of the signal N in samples.



Figure 2.23: Flowchart of the two stage MD algorithm.

Therefore, the speed-up can be achieved by local computation of inner products corresponding to the support L only. Furthermore, the set of computed correlations between two consecutive iterations of MP remains the same everywhere except at the support of the last atom chosen. As a result, previous comparisons between the inner products can also be stored in a tournament tree to make the search for the maximum in implementation of Eq. (2.42) faster. MP decomposition should be robust with respect to deviations between the assumed material properties of the structure and their real values.

The difference in material's density, Young's modulus and Poison's ratio can affect the dispersion relations. Hence, atoms of the dictionary are allowed to have the offset of h samples to the left and to the right from the simulated propagation distance to compensate for minor discrepancies in the group velocities. Therefore, each element of the dictionary in such decomposition generates a sequence of 2h + 1 inner products that can be efficiently computed using FFT-based convolution. The computational cost of a single iteration of this MP algorithm requires approximately *LlogL* computations.

2.8.1 Efficient Implementation of the MD Algorithm

The performance of MP can be further improved by utilizing the mode identification capabilities of RSCT. A schematic of such a two-step MD method is presented in Fig. 2.23. The initial information about the mode of the wave packet is extracted from the slope of the group delay on the RSCT-image, which is evaluated at the peaks of the signal's envelope. The knowledge of the mode provides the opportunity to exclude the atoms corresponding to other modes during MP decomposition, thereby making it faster.

Unfortunately, linear superposition of more than two wave packets arriving at the receiver with close TOFs, could distort the instantaneous frequency and, in this case, the slope of the group delay may not be a reliable measure of the mode. Therefore, MP starts with the RSCT-supplied modes first, but then checks the quality of projection by evaluating the angle φ between the atom-function and the corresponding support of the signal. If the latter is small enough, the RSCT-proposed projection is subtracted from the residual, otherwise, all possible atoms are considered.

CHAPTER 3

Experimental Studies

3.1 Defect Localization in Metal Structures

Performance of the proposed MD method was tested on 2-mm thick 6061-aluminum alloy plate with a notch (Fig. 3.10(a)). Lamb waves were generated with a pair of collocated $7 \times 8 \times 0.2$ mm transducers PZT1-1, PZT1-2 and were sensed by the receiver PZT-2. Morlet wavelet excitation (10V) at 250 kHz center frequency was applied to PZT-1 using the arbitrary waveform generator 33220A (Agilent, Inc.) in order to activate S_0 and A_0 modes in the plate. Data were sampled from the output of the charge amplifier with the help of the digital oscilloscope DSO1004 (Agilent, Inc.).

For a given plate thickness, both S_0 and A_0 modes have high energy, therefore neither of them can be ignored in the analysis. In the set-up in Fig. 3.1(b), S_0 mode gets reflected from the defect before A_0 reaches the receiver, hence their wave packets arrive with close TOFs. In order to test the MD, Lamb waves are first actuated with a single surface bonded



(a) $600 \times 600 \times 2$ mm aluminum plate

(b) Experimental set-up

Figure 3.1: Experimental set-up for damage detection in aluminum plates.

PZT1-1 and then, with the collocated PZT pair. Out-of-phase activation of PZT-1 reduces the energy of S_0 mode and forces the received signal to become nearly unimodal A_0 . This fact is later used to verify the decomposition results.



Figure 3.2: Group velocity dispersion curves of the aluminum plate used in the experiment.

Given the group velocities $V_{S0} = 5150 \ m/s$, $V_{A0} = 2788 \ m/s$, $V_{SH} = 3070 \ m/s$ (Fig. 3.2), it is possible to predict the TOF of wave packets analytically for this simple geometry. The shortest possible wave paths are shown in Fig. 3.3 and the corresponding TOFs are presented in Table 1. Two signals (10240 samples each) corresponding to a single and collocated



Figure 3.3: Possible wave paths from the actuator to the receiving PZT.



Figure 3.4: GW actuation using single surface-bonded PZT-1.







Figure 3.6: Collocated actuation of Guided waves.
PZT actuation, were subjected to the MD algorithm, described in the previous sections. Dictionary used for MP included 482 atoms: the Gabor atom for shear waves, $241-A_0$ and $261-S_0$ atoms to simulate possible propagation of wave packets for a maximum distance corresponding to the time-length of the signal. Each atom was allowed to have a shift of 70 samples from its simulated position. The threshold for the cosine of the angle between the TFR-proposed wave-packets and the local support of the signal was set to 0.9. MP converged after 7 iterations to the residual energy (5 per cent of initial level).



Figure 3.7: MD decomposed A_0 components.



Figure 3.8: MD decomposed S_0 components.



Figure 3.9: MD decomposed shear wave components.

Wave path	Length, cm	Time-of-Flight S0, μs	Time-of-Flight A0, μs
1	14.5	28.4	52.0
2 (defect)	36.5	71.5	130.5
3	53.1	104.1	_
4	62.0	121.4	_
5	62.0	130.2	_

Table 3.1: Length of possible wave paths and corresponding Times-of-Flight for S_0 and A_0 modes.

Number of wave path	1	2	3	4	5
SO Analytical / MD ToF, μs	28.4/30.2	71.5/69.5	104.1/102.1	121.4/-	130.2/-
SO Experim. / MD Dist., cm	14.5/15.5	36.5/37.2	53.1/52.6	62.0/-	66.5/-
AO Analytical / MD ToF, μs	52.0/50.5	130.9/132	_	_	_
AO Experim.l / MD Dist., cm	14.5/15.2	36.5/35.9	_	_	_

Table 3.2: Comparison of analytic and MD-extracted Times-of-Flight for S_0 and A_0 modes.

Separated modal components of the signal obtained with a single PZT actuation (Fig. 3.4) are presented in Fig. 3.7–Fig. 3.9. Comparing the results with analytical calculations (Table 3.1), it is possible to notice that MD has correctly extracted the TOF of S_0 and A_0 defect reflections and, at the same time, identified the incident S_0 , A_0 and shear mode wave packets. The signal reconstructed as a superposition of A_0 -selected atoms, correlates well with the experimental waveform obtained with the collocated PZT set-up (Fig. 3.7 and Fig. 3.6). Hence, the MD algorithm can be utilized to provide diagnostic imaging in the case when more than one high-energy mode is present in the structure. In particular, such an approach can be implemented without a baseline if the defect is large enough to be considered a strong reflector. Otherwise, the algorithm should be applied to the differenced signal.

The MD-based PDI was tested on the similar 60×60 cm and 2-mm thick aluminum plate, but with a different positioning of sensors shown in Fig. 3.10(a). The method used Lamb wave







(a) $600 \times 600 \times 2$ mm aluminum plate

(b) PDI-generated ellipses

(c) Most probable damage location identified with the help of PDI

Figure 3.10: Damage detection in 2-mm thick aluminum plate.



Figure 3.11: Signal from PZT 1-2 sensor pair.



Figure 3.12: Signal from PZT 2-3 sensor pair.

data recorded for each pair of sensors in pitch-catch configuration. Reflections of S_0 mode were chosen to identify the defect location. Typical sampled signals are presented in Fig. 3.11 and Fig. 3.12. It should be noted that S_0 reflection from damage had minimal overlap with other wave packets in the signal obtained from sensor pair PZT1-PZT2. However, for a sensor pair PZT1-PZT3, it merged with A_0 incident wave packet. In this case, the capabilities of MD algorithm were fully exploited to extract information about the overlapped modes and their ToFs. The S_0 mode components resulting from MD were utilized as an input for a standard PDI algorithm. Ellipses computed for each sensor pair were merged and thresholded to obtain the most probable damage location. The coordinates identified for the damage location was within a deviation of less than 1 cm from its true geometric coordinates of the machined defect (Fig. 3.10(c)).

3.2 Defect Localization in Woven Composite Structures

Over the last decade, composite materials have become widely used in aerospace, civil, automotive and other industries. A rapid development of composites for various applications can be explained by their advantages such as high strength, fracture toughness and light weight. Composite laminates are manufactured from multiple components such as fiber and matrix. However, the main challenge in engineering problems involving such materials is anisotropy: mechanical properties differ with respect to the direction considered for analysis. Moreover, composites usually undergo complicated deformations when a tensile force is applied in a particular direction. For instance, stretching of composite plate can cause shear deformations, bending and even twisting. In a Classical Laminate Theory (CLT), mechanical behavior of a single-layered laminate can be described by the contracted system of elasticity equations

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ & & C_{33} & C_{34} & C_{35} & C_{36} \\ & & & C_{44} & C_{55} & C_{56} \\ & & & & C_{55} & C_{56} \\ & & & & & C_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6 \end{pmatrix}$$
(3.1)

The coefficients of Eq. (3.1) are grouped into the stiffness matrix $|\mathbf{C}|$, whose entries relate stresses σ_{ij} applied to the structure with the corresponding strains ε_i (note that $\gamma_i = 2\varepsilon_i$ for i = 4, 5, 6). The entries of the stiffness matrix C_{ij} uniquely define material properties in all possible directions. Additionally, the stiffness matrix is symmetric and contains 36 elements, at most 21 of which can be independent. It should be noted that knowledge of stiffness matrix is required for accurate GW signal processing, because it allows one to calculate the phase and group velocity dispersion curves of a composite material used in experimental studies. The latter can be accomplished using a theory of Guided wave propagation in general anisotropic media and Transfer Matrix Method developed by Nayfeh [42]. Woven composites used in this work were manufactured with layers having woven fibers oriented in two principle directions x_1 and x_2 . Such design significantly reduces the anisotropy of mechanical properties in the corresponding plane. However, in this case CLT provides only an approximate representation of the stiffness matrix. Nevertheless, the Young's moduli and Poisson's ratios were determined from tensile tests: $E_{11} = 30$ GPa, $E_{22} = 30$ GPa, $G_{12} = 10$ GPa, $\nu_{12} = 0.4$ (ν_{13} and ν_{23} were assumed to be equal to ν_{12} , as well as $G_{12} = G_{13} = G_{23}$) and later the stiffness matrix was assumed to have the form of an orthotropic material. The



Figure 3.13: Phase velocity dispersion curves generated for 5-mm thick 8-layer composite plate with the help of Transfer Matrix method.

expressions of C_{ij} entries of the above matrix in terms of Young's moduli and Poisson's ratios are presented in Appendix B.

$$\begin{pmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{pmatrix}$$
(3.2)

The dispersion relations were obtained with the help of Transfer Matrix method assuming a single 5-mm thick layer of composite material, since it was woven and cured (Fig. 3.13). In general, dispersion curves can be also determined experimentally with the help of broadband laser actuation [43] and processing of collected signals with specially designed TFRs as discussed in Chapter 2. Alternatively, the elastic constants can be estimated using Genetic Algorithm based inversion of ToF data obtained from multiple sensors and different directions of GW propagation [44].



(a) Laser vibrometer set-up for measuring (b) Woven composite plate and points the ToF of A_0 mode wave packets



for taking laser vibrometer measurements

Figure 3.14: Damage detection in 2-mm thick aluminum plate.

In the case of composite materials we additionally need to determine the variation of the group velocity with respect to the propagation angle to ensure that the performance of PDI after Mode Decomposition is still accurate. For this purpose, a laser vibrometer can be utilized to measure the ToF of wave packets that propagate in different directions from the actuating PZT (Fig. 3.14(b)). The vibrometer involved in this work was capable of sensing displacements along the direction of the laser beam only, hence vertical orientation of the head provided measurements of out-of-plane displacements corresponding to antisymmetric modes. Guided waves were actuated in 5-mm thick composite plate with a $7 \times 8 \times 0.2$ mm PZT wafer using Morlet wavelet centered at 75 kHz. Fig. 3.15 illustrates the extracted group velocities of the A_0 mode from the collected GW signals. Difference of A_0 mode group velocity at various propagation angels is less than 8% at frequencies less than 200



Figure 3.15: Variation of the group velocity of A_0 mode actuated at 70 kHz with respect to the direction of propagation in a 5-mm thick 8-layer woven composite plate.

kHz. It follows that the Probability Diagnostic Imaging can be still applied to locate damage in woven composite materials without major modification. Therefore, it is possible to use the approach for damage detection described in the previous section. However, composite samples introduce more intensive scattering of Guided waves than aluminum plates and therefore, baseline subtraction is needed to identify reflection from damage.

The experimental set-up for identifying defects in composite samples is presented in Fig. 3.17(a). $60 \times 45 \times 5$ mm woven composite plate with six surface mounted PZT wafers was subjected to impact damage Fig. 3.17(b). Using the same procedure as in the previous section, Guided waves were actuated at 75 kHz and pitch-catch approach was employed to collect baselines and signals containing damage signatures from all sensor pairs. The example



- (a) Composite plate with impact damage
- (b) Impact damage

Figure 3.16: Composite samples used in experimental studies.



.

(a) PDI generated ellipses on composite sample

(b) Most probable damage location

Figure 3.17: Probability Diagnostic Imaging on composite samples.



Figure 3.18: Baseline and differenced signal obtained by PZT-6 and PZT-2 sensor pair on composite plate.

of differenced signal is demonstrated in Fig. 3.18. It can be noticed that only A_0 mode is sensitive to surface defects. Thus, the signal contains a single A_0 wave packet reflecting from damage, which can be also identified by MD algorithm. However, in this case the problem simplifies and the ToF of A_0 mode can be extracted directly from the peak of its envelope. Analyzing signals from multiple sensor paths, Probability Diagnostic Imaging is applied to construct ellipses corresponding to the most probable damage location Fig. 3.17(a). Finally, similar to the case of aluminum plates, damage site can be correctly identified by thresholding on the image Fig. 3.17(b) or computing its geometric mean.

CHAPTER 4

Conclusions and Future Work

This work presented a new approach for processing of multi-modal Lamb wave signals, which is a critical requirement for accurate identification of damage sites with the help of distributed PZT sensor networks. Two-stage Mode Decomposition algorithm, employing the Reassigned Spectrogram with Chirp Transform kernel and Matching Pursuit with dispersionbased dictionary, was proposed to identify and separate high energy S_0 and A_0 modes propagating in aluminum and woven composite plates. The MP dictionary was designed to account for strain transfer conditions between the PZT and the structure, which provided a good agreement between the generated dictionary atoms and experimentally obtained wave packets. It was demonstrated that MD-extracted information about the mode, amplitude, Time-of-Flight (ToF) and distance traveled by each reflection could be efficiently used to identify the location of the defect with the help of Probability Diagnostic Imaging.

Future work will focus on the extension of the proposed method to cases when more then two fundamental modes are present in the monitored sample. Moreover, the application of the proposed signal processing technique to structures with more complicated geometries and multiple defects is of a great interest in Guided wave SHM. Finally, the dispersion-based dictionary, which includes strain transfer conditions of the PZT, can be also exploited to solve minimization problems in Compressed Sensing for SHM with distributed sensor networks.

APPENDICES

Appendix A: Expression of Stresses, Strains and Displacements in terms of Wave Potentials

In the case of z-invariant propagation of elastic waves the components of displacement vector are given by

$$u_{x} = \frac{\partial \Phi}{\partial x} + \frac{\partial H_{z}}{\partial y}$$
$$u_{y} = \frac{\partial \Phi}{\partial y} - \frac{\partial H_{z}}{\partial x}$$
$$u_{z} = \frac{\partial H_{y}}{\partial x} - \frac{\partial H_{x}}{\partial y}$$
(A.1)

The stresses induced by pressure and shear vertical waves are expressed as

$$\sigma_{yx} = \mu \left(2 \frac{\partial^2 \Phi}{\partial x \partial y} - \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2}\right)$$

$$\sigma_{xx} = \lambda \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2}\right) + 2\mu \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 H_z}{\partial x \partial y}\right)$$

$$\sigma_{yy} = \lambda \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2}\right) + 2\mu \left(\frac{\partial^2 \Phi}{\partial y^2} - \frac{\partial^2 H_z}{\partial x \partial y}\right)$$

$$\sigma_{zz} = \lambda \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2}\right)$$
(A.2)

The stresses caused by shear horizontal waves are defined in the following way

$$\sigma_{yz} = \mu \left(\frac{\partial^2 H_z}{\partial x \partial y} - \frac{\partial^2 H_z}{\partial y^2}\right)$$

$$\sigma_{zx} = \mu \left(\frac{\partial^2 H_z}{\partial x^2} - \frac{\partial^2 H_z}{\partial x \partial y}\right)$$
(A.3)

The strains are defined as spatial derivatives of displacement components

$$\varepsilon_x = \frac{\partial u_x}{\partial x}$$

$$\varepsilon_y = \frac{\partial u_y}{\partial y}$$

$$\varepsilon_z = \frac{\partial u_z}{\partial z}$$
(A.4)

Appendix B: Relationship between Stiffness Constants, Young's Moduli and Poisson's Ratios for Orthotropic Materials

The components of stiffness matrix are given in terms of compliances

$$C_{11} = \frac{(S_{22}S_{33} - S_{23}S_{23})}{K} \qquad C_{22} = \frac{(S_{33}S_{11} - S_{13}S_{13})}{K}$$
(B.1)

$$C_{12} = \frac{(S_{13}S_{23} - S_{12}S - 33)}{K} \qquad C_{13} = \frac{(S_{12}S_{23} - S_{13}S_{22})}{K}$$
(B.2)

$$C_{33} = \frac{(S_{11}S_{22} - S_{12}S_{12})}{K} \qquad C_{23} = \frac{(S_{12}S_{13} - S_{23}S_{11})}{K}$$
(B.3)

$$C_{44} = \frac{1}{S_{44}} \qquad C_{55} = \frac{1}{S_{55}} \qquad C_{66} = \frac{1}{S_{66}} \tag{B.4}$$

$$K = S_{11}S_{22}S_{33} - S_{11}S_{23}S_{23} - S_{22}S_{13}S_{13} - S_{33}S_{12}S_{12} + 2S_{12}S_{23}S_{13}$$
(B.5)

The compliances can be expressed in terms of Young's moduli and Poisson's ratios

$$S_{11} = \frac{1}{E_1}$$
 $S_{12} = \frac{-v_{12}}{E_1}$ $S_{13} = \frac{-v_{13}}{E_1}$ (B.6)

$$S_{22} = \frac{1}{E_2}$$
 $S_{23} = \frac{-v_{23}}{E_2}$ $S_{33} = \frac{1}{E_3}$ (B.7)

$$S_{44} = \frac{1}{G_{23}}$$
 $S_{55} = \frac{1}{G_{13}}$ $S_{66} = \frac{1}{G_{12}}$ (B.8)

BIBLIOGRAPHY

BIBLIOGRAPHY

- National Transportation Safety Board. Aircraft Accident Report-Aloha Airlines, Flight 243, Boeing 737-200, N73711, near Maui, Hawaii, April 28, 1988. NTSB/AAR-89/03, 1989.
- [2] Crossbow Technology, Inc. IRIS Wireless Measurement System. Document Part Number: 6020-0124-01 Rev A, 2010.
- [3] K. Diamanti and C. Soutis, Structural Health Monitoring Techniques for Aircraft Composite Structures. Progress in Aerospace Sciences, 46:342-352, 2010.
- [4] P. C. Chang, A. Flatau and S. C. Liu. Review Paper: Health Monitoring of Civil Infrastructure. Structural Health Monitoring, 2(3):257-267, 2003.
- [5] E. Keller and A. Ray. Real-time Health Monitoring of Mechanical Structures. Structural Health Monitoring, 2(3):191-203, 2003.
- [6] J. P. Carmo and J. E. Ribeiro. New Advances in Vehicular Technology and Automotive Engineering. InTech, 2012.
- [7] C. J. Stulla, C. J. Earlsb and P. S. Koutsourelakisb. Model-based Structural Health Monitoring of Naval Ship Hulls. Computer Methods in Applied Mechanics and Engineering, 200:1137-1149, 2011.
- [8] D. Inaudi and B. Glisic. Fiber Optic Sensing for Innovative Oil and Gas Production and Transport Systems. 18th International Conference on Optical Fiber Sensors, 2006.
- Crossbow Technology, Inc. Imote2 High-performance Wireless Sensor Network Node. Document Part Number: Document Part Number: 6020-0117-02 Rev A, 2010.

- [10] S. Stupar, A. Simonovic, M. Jovanovic. Measurement and Analysis of Vibrations on the Helicopter Structure in Order to Detect Defects of Operating Elements. Scientific Technical Review, 62:58-63, 2012.
- [11] M. Lee, J. Lee and J. Kim. Condition Monitoring of a Nuclear Power Plant Check Value Based on Acoustic Emission and a Neural Network. J. Pressure Vessel Technol. 127(3):230-236, 2005.
- [12] H. Lamb, On Waves in an Elastic Plate. Proceedings of the Royal Society, Physical and Engineering Sciences, 93:114-128, 1917.
- [13] I. Viktorov. Rayleigh and Lamb Waves: Physical Theory and Applications. Springer, 1995.
- [14] J. L. Rose. Ultrasonic Waves in Solid Media. Cambridge University Press, 1999.
- [15] K. F. Graff. Wave Motion in Elastic Solids. Dover Publications, 1991.
- [16] A.H. Nayfeh. Wave Propagation in Layered Anisotropic Media: With Applications to Composites. North Holland, 1995.
- [17] V. Giurgiutiu. Structural Health Monitoring with Piezoelectric Wafer Active Sensors. Academic Press, 2008.
- [18] http://www.olympus-ims.com/
- [19] http://www.ndt.net/
- [20] http://www.steminc.com/
- [21] An American National Standard. IEEE Standard on Piezoelectricity. ANSI/IEEE Std 176-1987, 1988.
- [22] A. Bent and N.W. Hagood. Piezoelectric Fiber Composites with Interdigitated Electrodes. Journal of Intelligent Material Systems and Structures, 8 (11):903-919, 1997.
- [23] http://www.smart-material.com/MFC-product-main.html

- [24] E. F. Crawley and J. de Luis, Use of Piezoelectric Actuators as Elements of Intelligent Structures. American Institute of Aeronautics and Astronautics (AIAA), 26:1373-1385, 1987.
- [25] A. Raghavan and E. S. Cesnik. Finite-dimensional Piezoelectric Transducer Modeling for Guided Wave Based Structural Health Monitoring. Smart. Mater. and Struct., 14(6):1448-1461, 2005.
- [26] C. Zhou, Z. Su and L. Chen. Probability-based Diagnostic Imaging Using Hybrid Features Extracted from Ultrasonic Lamb Wave Signals. Smart. Mater. Struct., 20, 2011.
- [27] E. Zurich. The Discrete Fourier Transform, Part 4: Spectral Leakage. Journal of Object Technology, 8(7):23-34, 2009.
- [28] F. Auger, P. Flandrin, P. Goncalves and O. Lemoine. *Time-Frequency Toolbox for Use with MATLAB.*, 1996.
- [29] V. Giurgiutiu and L. Yu. Comparison of Short-time Fourier Transform and Wavelet Transform of Transient and Tone Burst Wave Propagation Signals For Structural Health Monitoring. 4th International Workshop on Structural Health Monitoring, Stanford University, Stanford, CA, September 15-17, 2003.
- [30] V. Amaravadi. Structural Health Monitoring Using Wavelet Transform. Smart. Mater. and Struct., 4327:258-269, 2001.
- [31] L. De Marchi, A. Marzani, S. Caporale and N. Speciale. A New Warped Frequency Transformation (WFT) for Guided Waves Characterization. Health Monitoring of Structural and Biological Systems, 7295, 2009.
- [32] L. Cohen. *Time-Frequency Analysis*. Prentice Hall, 1995.
- [33] D. J. Nelson. Cross-spectral Methods for Processing Speech. J. Acoust. Soc. Am. 110(5):2575-2592, 2001.
- [34] A. Rihaczek. Signal Energy Distribution in Time and Frequency. IEEE Transactions on Information Theory. 14:369-384, 1968.
- [35] A. V. Oppenheim, R. W. Schafer and J. R. Buck. Discrete-time Signal Processing. Prentice Hall, 1999.

- [36] L. R. Rabiner and R. W. Schafer. *The Chirp-Z Transform*. IEEE Transactions on Audio and Electroacoustics. AU-17(2):86-92, 1969.
- [37] S.G. Mallat and Z. Zhang. Matching Pursuits with Time-Frequency Dictionaries. IEEE Trans. Signal Process. 41:3397-3415, 1993.
- [38] B. Xu, V. Giurgiutiu and L. Yu. Lamb Waves Decomposition and Mode Identification Using Matching Pursuit Method. Sensors and Smart Structures Technologies for Civil, Mechanical, and Aerospace Systems. 7292, 2009.
- [39] A. Raghavan and C. Cesnik. Guided-wave Signal Processing Using Chirplet Matching Pursuits and Mode Correlation for Structural Health Monitoring. Smart. Mater. Struct. 16:255-366, 2007.
- [40] Y. Li and J. Michaels. Numerical Implementation of Matching Pursuit for the Analysis of Complex Ultrasonic Signals IEEE Trans. Ultrason. Ferroelectr. Freq. Control. 55:173-182, 2008.
- [41] J. C. Hong, K. H. Sun and Y. Y. Kim Waveguide Damage Detection by the Matching Pursuit Approach Employing the Dispersion-based Chirp Functions. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. 53:592-605, 2006.
- [42] A. H. Nayfeh The General Problem of Elastic Wave Propagation in Multilayered Anisotrpic Media. J. Acoust. Soc. Am. 89(4):1521-1531, 1991.
- [43] M. Niethammer and L. J. Jacobs. Time-frequency Representation of Lamb Waves using the Reassigned Spectrogram J. Acoust. Soc. Am. 107(5):19-24, 2000.
- [44] J. Vishnuvardhan, C Krishnamurthy and K. Balasubramaniam. Genetic Algorithm based Reconstruction of Elastic Moduli using an Ultrasonic Guided Wave and Single Transmitter-multiple Receiver SHM Array Systems. Smart. Mater. Struct. 16:1639, 2007.