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THREE ESSAYS ON COMPETITION AND INTERACTIONS

By

Jaesoo Kim

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ABSTRACT

THREE ESSAYS ON COMPETITION AND INTERACTIONS

By

Jaesoo Kim

This dissertation consists of three essays exploring competition and interaction among economic agents. I find several new arguments for and against competition and study how to foster competition and facilitate cooperation among agents in the firm level, group level, and within-group level in each paper.

In the first chapter, I study the firm-level competition in a model of product differentiation. In this type of model, competition occurs as a localized phenomenon. Virtually, firms compete for marginal consumers who are indifferent between two products. As a result, the location and the number of marginal consumers are very crucial to measure the intensity of market competition. This essential feature has been paid little attention to in most papers. This chapter develops a new way to handle a general distribution of consumer preferences, and explains why it matters in analyzing the effects of various strategies adopted by firms.

In the second chapter, I turn to the problem of the within-group competition. When a principal combines team rewards and competition to motivate multiple agents, what is the optimal combination of these two compensation schemes? Team rewards can mitigate a discouragement effect in competition, and serve the purpose of managing competition. This result is a strong and fundamental challenge to the efficiency of a winner-take-all competition. In addition, the effects of strategic interactions between

agents such as collusion and precommitment can be beneficial to the principal only when team rewards are greater than competitive compensation. These are new rationales for the merit of team rewards.

Finally, in the last chapter, I study the collective action problem in two potentially important environments: competition between groups and internal conflict within a group. We shape both intergroup and intragroup competition in a general manner. We include a heterogeneity of individuals within a group and consider a general function of collective action to reflect a possible complementary effect or different contributory roles between individuals' efforts. The interplay between internal and external competition turns out to be very crucial in the analysis of the collective action problem. We will explain how each group's internal conflict influences its chance of winning in the external competition and how a change in power inequality within a group affects collective action.

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TABLE OF CONTENTS

LIST OF FIGURES

Chapter 1

Competition Intensity and Strategies in a Market with Differentiated Products

1.1 Introduction

In order to analyze a market with differentiated products, we extensively use a Hotelling type model of spatial competition. In particular, when we consider a consumer's choice between competing products, a spatial competition model is necessary. A typical assumption in this type of model is that consumer preferences are uniformly distributed. This restriction is simply imposed to ensure closed-form solutions and analytical tractability.

However, this assumption is not only too restrictive to obtain generally valid results, it also fails to reflect the essential and foundational feature in a spatial model, which is that competition is *localized*. Firms compete for marginal consumers who are indifferent between two products. Each firm's incentive to cut its price depends on the number of marginal consumers that it can steal from its rival firm. Thus, when consumer preferences are uniformly distributed along the Hotelling line, firms perceive identical competitive pressure at every location of marginal consumers. In contrast, unless the distribution function is uniform, the perception of competitive pressure is different across the locations of marginal consumers. Moreover, two firms perceive competitive pressure differently. For example, in a bell-shaped distribution, one firm may obtain a smaller and smaller density of consumers with slight price cuts. On the other hand, the other firm captures a higher and higher density of consumers with small price reductions. As a result, the assumption of the uniform distribution assumes away all the interesting and important outcomes which possibly stem from different perceptions of competitive pressure.[1]

The purpose of this paper is to explain how we generalize the distribution of consumer preferences in a reasonable way and why we need to do it. The paper provides a new general way to analyze the effect of a transformation of the consumer distribution on equilibrium prices, which will be very useful for other research using the Hotelling model. More importantly, the paper shows that competition intensity measured by the dispersion of the consumer distribution affects firms' choice of business strategies significantly.

As for the first question, I take a look at how the dispersion of the consumer distribution corresponds to the intensity of market competition. I order a family of distribution functions by mean-preserving spread (MPS), and impose the monotone likelihood ratio property (MLRP) on the distribution functions for the sake of smooth dispersion.[2] A MPS represents that a greater proportion of consumers have a higher relative preference for one good over the other. I find that a MPS under the MLRP rotates reaction functions outward and raises firms' equilibrium prices. In a nutshell, the more dispersed the distribution is, the less intense market competition becomes. Hence, a mean-preserving transformation stands for a change in the intensity of competition in Hotelling type models.[3]

Why is it important to consider the dispersion of the consumer distribution? There are two primary reasons. First, the dispersion of the consumer distribution changes the equilibrium location of marginal consumers. This implies that competition intensity changes a firm's relative position within its industry such as profit difference between two competing firms. In particular, as the consumer distribution is more concentrated, the difference between two firms' equilibrium profits is getting larger. This result explains that when a firm adopts a particular strategy to achieve the relative advantage over its rival, the firm can choose different strategies in different competition environments. As an example, I study how competition intensity influences incentives to innovate. I find that firms make more investments for R&D in a more competitive product market.

The second reason for considering a general distribution is that firms have to face different competitive pressures by different strategies. This simple property can have enormous impacts on firms' choice of strategies and their effects. Thus, firms may choose a particular strategy to soften market competition by competing on a less competitive basis. It will be shown how non-uniform distributions can drastically reverse some well-known results derived from the uniform distribution in the literature of price discrimination and information sharing.

In fact, most papers measure the degree of competition in the Hotelling model by transportation costs or the size of product differentiation, assuming the uniform distribution.[4] However, so different are these two approaches that the equilibrium location of marginal consumers does not change with the size of transportation costs, but with the dispersion of the consumer distribution. Thus, transportation costs fail to capture the two important effects that I have described. It sounds obvious that competition intensity changes firms' behavior, but this aspect has been surprisingly neglected in most papers by assuming the uniform distribution. This point will be central throughout the paper.

There are several papers which study the properties of the equilibrium in Hotelling-style models with a general distribution of consumer preferences. Caplin and Nalebuff (1991) provide the proof of the existence of a pure-strategy price equilibrium for any number of firms producing any set of products. Perloff and Salop (1985) show the uniqueness of the price equilibrium if there do not exist mass points in the density function. Bester (1992) further shows the stability of price competition and its uniqueness under asymmetric distributions as long as consumer preferences are sufficiently dispersed. Thus, these papers provide a safe environment for my study. In addition, Neven (1986) and Anderson, Goeree, and Ramer (1997) study the firms' location choice under non-uniform distributions.[5] None of these authors notices the importance of the relationship between competition intensity and strategies.

In a different vein, this paper is closely related to Johnson and Myatt (2006). They show that the dispersion of consumers' willingness-to-pay leads to rotations of a demand curve. A remarkable result is that a monopolist prefers low dispersion when serving a mass market, while it prefers high dispersion when pursuing a small niche market. I believe that this paper and my paper can be viewed as complementary in understanding the effect of consumer preferences in firms' strategies. They find that the dispersion of consumers' *absolute* valuations induces demand rotations, and study some implications in the monopolist's strategies. On the other hand, my paper finds that the dispersion of consumers' *relative* preferences between competing firms results in softening market competition, and studies implications in terms of competing firms' strategies.

This paper proceeds as follows. Section 2 introduces our basic model of product differentiation and characterizes the price equilibrium. I define the MPS and MLRP in the distribution of consumer preferences to study how the dispersion of the consumer distribution affects equilibrium prices. Section 3 and 4 explain why and how the dispersion of the consumer distribution affects firms' business strategies. Section 5 concludes the paper.

1.2 Basic Model

Consider a simple market with two competing firms which produce good A and B with a constant marginal cost of c_A and c_B per unit respectively. Each consumer buys either one unit of a good from only one firm or nothing. A consumer's value for each product is $v + \theta_i$, where $\theta_i \in [0, \overline{\theta}]$ and $i = A$ or B. v is sufficiently large so that the market is fully covered. A consumer is indexed by (θ_A, θ_B).[6] Now let us define

$$\theta = \theta_B - \theta_A.$$

θ represents a consumer's relative preference for the product B over A, where $\theta \in [\underline{\theta}, \overline{\theta}]$ and $\underline{\theta} = -\overline{\theta} < 0$.[7] I assume that θ follows the cumulative distribution function $F(\theta)$ which is twice continuously differentiable in θ. The density function $f(\theta)$ is symmetric at zero. To ensure nice demand curves, I further assume that a hazard rate $\frac{f(\theta)}{1-F(\theta)}$ is strictly increasing in θ. By this assumption, the second order condition is automatically satisfied by the first order condition.[8]

Assumption 1 *Monotone hazard rate (MHR)*

$$\frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right) \leq 0 \text{ and } \frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right) \geq 0.$$

The symmetry of $f(\theta)$ ensures the second inequality. Now I index a family of distribution functions by the parameter $k \in K = [k_L, k_H]$. I assume that $F_k(\theta)$ is continuously differentiable in k. In an alternative way, to compare two possible distribution functions, I use $F_k(\theta)$ and $F_{k+1}(\theta)$. They are ordered by the mean-preserving spread as follows.[9]

Assumption 2 *Mean-preserving spread (MPS). For $\theta \in [\underline{\theta}, \overline{\theta}]$ and* $\int_{\underline{\theta}}^{\overline{\theta}} \theta f_k(\theta) d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \theta f_{k+1}(\theta) d\theta$,

$$\int_{\underline{\theta}}^{\theta} F_k(x)dx \leq \int_{\underline{\theta}}^{\theta} F_{k+1}(x)dx.$$

Assumption 2 means that a high order distribution is a MPS of a low order distribution. The economic interpretation of the mean-preserving spread (contraction) is that the density of consumers with strong preference for one good over the other increases (decreases) in k. To put it differently, the proportion of loyal consumers grows. Figure 1.1 represents some examples of possible density functions and corresponding distribution functions.

They may cross more than once. But, I consider the case in which a MPS moves density from the center toward the both tails smoothly so that all possible distribution functions cross only once at $\theta = 0$, as presented in Figure 1.1. Formally, to ensure a smooth change of the MPS, I impose the monotone likelihood ratio property on the sequence of distribution functions.

Assumption 3 *Monotone likelihood ratio property (MLRP)*

$$\frac{f_{k+1}(\theta_0)}{f_k(\theta_0)} \leq \frac{f_{k+1}(\theta_1)}{f_k(\theta_1)} \text{ for } \theta_0 \leq \theta_1 \in [0, \overline{\theta}] \text{ and}$$
$$\frac{f_{k+1}(\theta_0)}{f_k(\theta_0)} \geq \frac{f_{k+1}(\theta_1)}{f_k(\theta_1)} \text{ for } \theta_0 \leq \theta_1 \in [\underline{\theta}, 0].$$

When a MPS arise, we have a stochastically larger density of consumers with higher relative preferences.[10] It will be shown that this property leads to rotations in firms' reactions functions and monotonic changes in equilibrium prices. The following result will be repeatedly used throughout the rest of the paper.

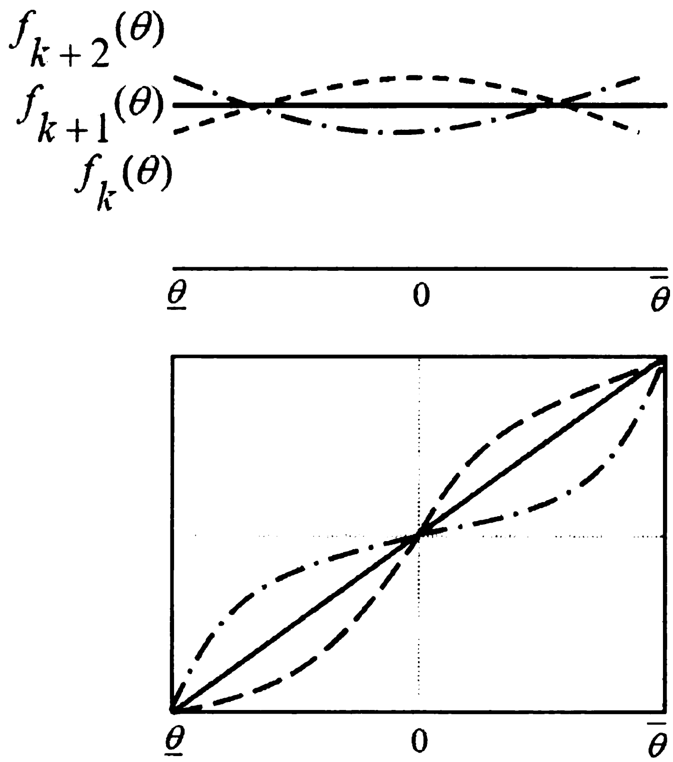

Figure 1.1: Density and distribution functions

Proposition 1 *The MLRP yields the following stochastic orders.*

$$
\begin{array}{ccccc}
\text{For } \theta & \in & [\underline{\theta}, 0] & \text{For } \theta \in [0, \overline{\theta}] & \\
\dfrac{1-2F_k(\theta)}{f_k(\theta)} & \geq & \dfrac{1-2F_{k+1}(\theta)}{f_{k+1}(\theta)} & \dfrac{1-2F_k(\theta)}{f_k(\theta)} \leq \dfrac{1-2F_{k+1}(\theta)}{f_{k+1}(\theta)} & (1.1) \\
\dfrac{F_k(\theta)}{f_k(\theta)} & < & \dfrac{F_{k+1}(\theta)}{f_{k+1}(\theta)} & \dfrac{1-F_k(\theta)}{f_k(\theta)} < \dfrac{1-F_{k+1}(\theta)}{f_{k+1}(\theta)} & (1.2) \\
F_k(\theta) & \leq & F_{k+1}(\theta) & F_k(\theta) \geq F_{k+1}(\theta) & (1.3)
\end{array}
$$

Proof. See Appendix.

The inequalities in (1.2) are often called reverse hazard rate and hazard rate dominance respectively. The inequalities in (1.3) show the first-order stochastic dominance (FOSD) relationship. The MLRP implies (reverse) hazard rate dominance, which implies the FOSD. Note that a high order distribution dominates a low order distribution in the sense of the FOSD for positive θ, whereas the opposite relationship holds for negative θ. Thus the MPS with the MLRP results in the single crossing property of distribution functions at $\theta = 0$.

From now on, in order to economize on notations, I often use F and f without subscript as a representative of distribution and density function respectively. The consumer of type θ_i enjoy net utility $v + \theta_i - p_i$ when purchasing from firm i at price p_i and 0 when not buying any good. Thus, consumers $\theta < \widetilde{\theta} = p_B - p_A$ choose to buy good A, whereas consumers $\theta \geq \widetilde{\theta}$ choose to buy good B. Normalizing the number of consumers to one, the market share of firm A and firm B are $F(\widetilde{\theta})$ and $1 - F(\widetilde{\theta})$ respectively. The profit function of each firm is given as

$$\pi_A = (p_A - c_A)F(\widetilde{\theta}) \text{ and } \pi_B = (p_B - c_B)(1 - F(\widetilde{\theta})).$$

The first-order conditions are $\frac{\partial \pi_A}{\partial p_A} = F(\widetilde{\theta}) + (p_A - c_A)f(\widetilde{\theta}) = 0$ and $\frac{\partial \pi_B}{\partial p_B} = (1 - F(\widetilde{\theta})) - (p_B - c_B)f(\widetilde{\theta}) = 0$. A simple manipulation yields the following reaction functions,

$$p_A(p_B) = c_A + \frac{F(\widetilde{\theta})}{f(\widetilde{\theta})} \text{ and } p_B(p_A) = c_B + \frac{1 - F(\widetilde{\theta})}{f(\widetilde{\theta})}.$$

Subtracting the first equation from the second equation, we obtain the equilibrium condition, and the equilibrium prices are characterized as follows.

Proposition 2 *The equilibrium prices are represented by the ratio of each firm's market share and the density on the location of marginal consumers in equilibrium.*

$$\begin{aligned} p_A^* &= c_A + \frac{F(\theta^*)}{f(\theta^*)} \textit{ and } p_B^* = c_B + \frac{(1 - F(\theta^*))}{f(\theta^*)}, \\ \textit{where } \theta^* - \Delta &= \frac{1 - 2F(\theta^*)}{f(\theta^*)} \textit{ and } \Delta = c_B - c_A. \end{aligned} \tag{1.4}$$

The corresponding equilibrium profits are $\pi_A = \frac{F(\theta^*)^2}{f(\theta^*)}$ and $\pi_B = \frac{(1-F(\theta^*))^2}{f(\theta^*)}$. θ^* indicates the location of marginal consumers in equilibrium. In the uniform distribution, $f(\theta^*)$ becomes a constant. This implies that we must suffer the loss of two potentially important economic forces: the equilibrium location of marginal consumers and the density at that point. In other words, the uniform distribution ignores the issues such as who will be the marginal consumers and how many consumers are at the margin.

An increase in $f(\theta^*)$ forces both prices to fall, given everything equal. Why does an increase in the proportion of marginal consumers lead to more aggressive competition? The intuition to understand this result is simple. When a firm charges a slightly lower price than its rival, it can steal a greater number of consumers. Thus, the incentives to reduce prices are greater as the density of marginal consumers becomes larger. In this sense, I refer to $f(\theta^*)$ as the competitive pressure on the equilibrium location of

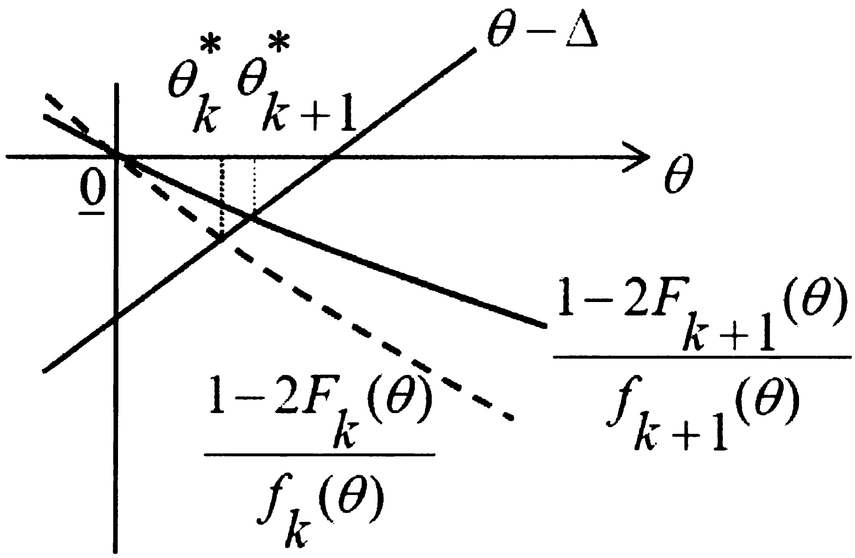

Figure 1.2: The location of marginal consumers

marginal consumers.

I illustrate the equilibrium condition (1.4) in Figure 1.2. $\frac{1-2F(\theta)}{f(\theta)}$ is decreasing in θ by Assumption 1, and goes through the origin at $\theta = 0$. On the other hand, since $\theta - \Delta$ is increasing in θ, we must have a unique θ^*. Without loss of generality, I assume that firm A is more efficient than firm B by $\Delta = c_B - c_A > 0$. The equilibrium location of marginal consumers θ^* is greater than 0. This implies that the more efficient firm set a lower price, i.e., $p_A^* < p_B^*$. In contrast, the more efficient firm can impose a higher mark-up, i.e., $\frac{F(\theta^*)}{f(\theta^*)} > \frac{(1-F(\theta^*))}{f(\theta^*)}$. Moreover, θ^* is increasing in Δ. As the cost difference between the two firms is larger, the location of marginal consumers will be closer to $\overline{\theta}$. Finally, as Δ increases, it can easily be shown that firm A's mark-up rises, while firm B's mark-up decreases.

$$\frac{\partial}{\partial \Delta}\left(\frac{F(\theta^*)}{f(\theta^*)}\right) > 0 \text{ and } \frac{\partial}{\partial \Delta}\left(\frac{1 - F(\theta^*)}{f(\theta^*)}\right) < 0.$$

I turn now to the important question of how the dispersion of the consumer distribution changes the equilibrium prices. I conduct comparative statics in terms of k on the equilibrium prices.

Proposition 3 $p_A(\theta_k^*) < p_A(\theta_{k+1}^*)$ *and* $p_B(\theta_k^*) < p_B(\theta_{k+1}^*)$. *If the cost asymmetry is not large enough, the mean-preserving spread raises equilibrium prices.*

Proof. See Appendix.

It is very straightforward to show the result for symmetric firms, that is $c_A = c_B = c$. The equilibrium prices increase by the MPS from $F_k(\theta)$ to $F_{k+1}(\theta)$. Let θ_k^* and θ_{k+1}^* represent the equilibrium location of marginal consumers when firms face the distribution function $F_k(\theta)$ and $F_{k+1}(\theta)$, respectively. By symmetry, we obtain $\theta_k^* = \theta_{k+1}^* = 0$. The firms' corresponding equilibrium prices are $c + \frac{1}{2f_k(0)}$ and $c + \frac{1}{2f_{k+1}(0)}$. Because $f_{k+1}(0) \leq f_k(0)$, the equilibrium prices are greater in more dispersed distributions. This result corroborates my basic claim that the distribution of consumer preferences corresponds to the intensity of competition. More dispersed (concentrated) distributions lead to less (more) intense market competition.

Alternatively, let us explain how a MPS under the MLRP influences each firm' reaction function. Inequalities (1.2) in Proposition 1 implies that while firm A's reaction curve rotates outward for $\theta = p_B - p_A \leq 0$, firm B's reaction curve rotates outward for $\theta = p_B - p_A \geq 0$. This is the reason why the equilibrium prices increase with a MPS on the consumer distribution when the cost difference is not large enough.

1.3 Competition Intensity and Relative Position

The most important notion is that competition is occurring as a localized phenomenon in this type of spatial model. Virtually, the firms compete for the density of consumers

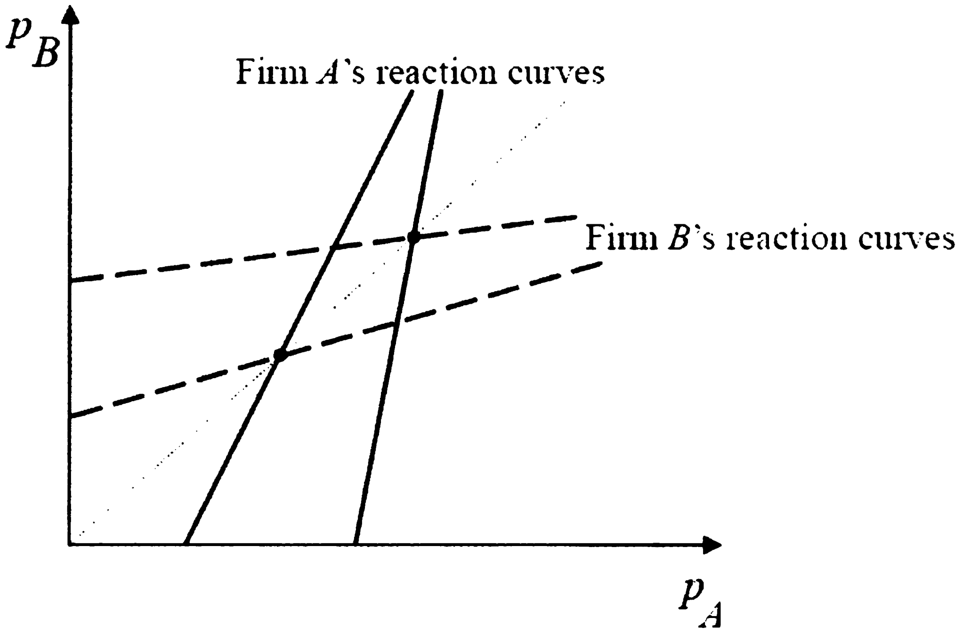

Figure 1.3: The rotation of reaction curves

on the competitive front which is the location of marginal consumers. Thus, unless the distribution is uniform, the two competing firms perceive competitive pressure differently. For example, in a bell-shaped distribution, the less efficient firm can steal a higher density of consumers through price reduction than the more efficient firm.

Therefore, the dispersion of the consumer distribution changes each firm's perception of competitive pressure and the equilibrium location of marginal consumers. I find that a MPS shifts the location of marginal consumers θ^* to the right.

Proposition 4 $\theta_k^* < \theta_{k+1}^*$. *As the distribution is transformed by a MPS (as k increases), the equilibrium location of marginal consumers is farther away from the center.*

$$\begin{aligned} \theta_{k+1}^* - \theta_k^* &= \left(\frac{1 - 2F_{k+1}(\theta_{k+1}^*)}{f_{k+1}(\theta_{k+1}^*)}\right) - \left(\frac{1 - 2F_k(\theta_k^*)}{f_k(\theta_k^*)}\right) > 0 \\ &\iff \left[p_B(\theta_k^*) - p_A(\theta_k^*)\right] < \left[p_B(\theta_{k+1}^*) - p_A(\theta_{k+1}^*)\right] \end{aligned}$$

Proof. See Appendix.

Proposition 1 and 2 lead to this result as seen in Figure 1.2. The intuition is that firm A has more incentives to cut its price relative to firm B when it faces $F_{k+1}(\theta)$ rather than $F_k(\theta)$ because firm A can capture a stochastically larger proportion of consumers. As a consequence, the difference between the two firms' equilibrium prices is getting larger.

In fact, most papers measure the intensity of market competition by transportation costs or the size of product differentiation with the assumption of the uniform distribution. However, since competitive pressure is unvarying over all locations, the equilibrium location of marginal consumers is invariable with transportation costs. Using $f(\theta) = \frac{1}{2\overline{\theta}}$ in the uniform distribution, we find that the equilibrium prices are $p_A = \overline{\theta} + \frac{2c_A + c_B}{3}$ and $p_B = \overline{\theta} + \frac{c_A + 2c_B}{3}$. Note that $\overline{\theta}$ plays an identical role as transportation costs in a typical Hotelling model.[11] The equilibrium location of marginal consumers is $\theta^* = \frac{\Delta}{3}$, which is independent of $\overline{\theta}$.

The result further implies that firm A's relative advantage over firm B decreases as market becomes less competitive. Note that the difference between two firms' equilibrium profits is getting smaller.

$$
\begin{aligned}
\pi_A(\theta_k^*) - \pi_B(\theta_k^*) &= \frac{2F_k(\theta_k^*) - 1}{f_k(\theta_k^*)} = \Delta - \theta_k^* \\
&> \Delta - \theta_{k+1}^* = \frac{2F_{k+1}(\theta_{k+1}^*) - 1}{f_{k+1}(\theta_{k+1}^*)} = \pi_A(\theta_{k+1}^*) - \pi_B(\theta_{k+1}^*).
\end{aligned}
$$

In contrast, under the uniform distribution, each firm's equilibrium profit is $\pi_A = 2\overline{\theta}(\frac{1}{2} + \frac{\Delta}{6\overline{\theta}})^2$ and $\pi_B = \overline{\theta}(\frac{1}{2} - \frac{\Delta}{6\overline{\theta}})^2$, and accordingly $\pi_A - \pi_B = \frac{2}{3}\Delta$. The relative advantage of the more efficient firm does not depend on $\overline{\theta}$.

Proposition 5 $\left[\pi_A(\theta_k^*) - \pi_B(\theta_k^*)\right] > \left[\pi_A(\theta_{k+1}^*) - \pi_B(\theta_{k+1}^*)\right]$. *As the distribution is transformed by a MPS (as k increases), the difference between the two firms' equi-*

librium profits is getting smaller.

A firm's relative advantage over its rival depends on the intensity of competition shaped by the distribution of consumer tastes. In fact, since many types of strategies aim to gain a competitive advantage, the result implies that competition intensity can have a significant impact on firms' choice of strategy. Nevertheless, by assuming the uniform distribution, this issue is left behind completely in most papers. As a simple example, I will show that R&D investment will be different depending on the intensity of market competition.

1.3.1 Incentives for Innovation

I extend the basic model by incorporating the possibility of R&D. In the first stage, two firms invest in innovation. In the second stage, price competition follows. I assume symmetric costs for simplicity, thereby $c_A = c_B = c$. The firms conduct process innovations which reduce marginal production costs from c to $c - \Delta$.[12] This R&D competition is a winner-take-all contest. Hence, we need to formulate the winner's payoff and loser's payoff, which are denoted by π^W and π^L respectively. Following the previous analysis, each payoff can be represented as

$$\begin{aligned} \pi^W &= \frac{F(\theta^*)^2}{f(\theta^*)} \text{ and } \pi^L = \frac{(1 - F(\theta^*))^2}{f(\theta^*)}, \\ \text{where } \theta^* &= \Delta + \frac{1 - 2F(\theta^*)}{f(\theta^*)}. \end{aligned}$$

Let us consider the following simple R&D competition. I_A and I_B are each firm's investment level respectively. Define $p(I_A, I_B)$ as the probability that firm A wins, while $1 - p(I_A, I_B)$ as the probability that it loses. Then $1 - p(I_A, I_B)$ is firm B's winning probability and $p(I_A, I_B)$ is B's corresponding losing probability. To ensure an interior solution, I assume $\frac{\partial p(I_A, I_B)}{\partial I_A} > 0$, $\frac{\partial^2 p(I_A, I_B)}{\partial I_A^2} < 0$, $\frac{\partial p(I_A, I_B)}{\partial I_B} < 0$, and

$\frac{\partial^2 p(I_A, I_B)}{\partial I_B^2} > 0$. Each firm's expected profit is

$$\begin{aligned} \pi_A &= p(I_A, I_B)\pi^W + (1 - p(I_A, I_B))\pi^L - I_A \text{ and} \\ \pi_B &= (1 - p(I_A, I_B))\pi^W + p(I_A, I_B)\pi^L - I_B. \end{aligned}$$

The first-order conditions are $\frac{\partial p(I_A, I_B)}{\partial I_A}(\pi^W - \pi^L) - 1 = 0$ and $-\frac{\partial p(I_A, I_B)}{\partial I_B}(\pi^W - \pi^L) - 1 = 0$. Accordingly, given that the model is symmetric, we have $I_A^* = I_B^* = I^*$ satisfying $\pi^W - \pi^L = \frac{1}{p'(I^*, I^*)}$ in equilibrium. [13]

This equilibrium condition implies that $(\pi^W - \pi^L)$ can be thought of as R&D incentives. Therefore, we need to analyze how R&D incentives are changed by the transformation of the consumer distribution. The difference between the winner's and loser's payoffs can be simplified as

$$\pi^W - \pi^L = \frac{2F(\theta^*) - 1}{f(\theta^*)} = \Delta - \theta^*.$$

$(\pi^W - \pi^L)$ becomes smaller as the order of distribution rises, because we have $\Delta - \theta_k^* > \Delta - \theta_{k+1}^*$ as shown in Proposition 5. The firms' R&D incentives are greater in the more competitive market. In other words, more aggressive competition in product market fosters more R&D competition.[14]

Proposition 6 *R&D incentives are greater as market competition becomes more intense.*

1.4 Competitive Pressure on Competition Fronts

In the previous section, it became clear that competition intensity changes the equilibrium location of marginal consumers and the firms' relative position. A direct implication is that competition intensity has impact on strategies to obtain a relative advantage such as R&D investment.

Even if a certain business strategy does not have the purpose of changing the relative position, most business strategies may move the competition front in a spatial model. This may not produce any effect in the uniform distribution simply because the firms perceive identical competitive pressure regardless of the equilibrium location of marginal consumers. However, unless the distribution is uniform, the firms face different competitive pressures by the choosing different strategies. Thus, the firms may choose a particular strategy to mitigate competition. This characteristic leads inevitably to a reconsideration of well-known conclusions in the literature.

This feature should not be considered as a minor technical issue. In the following examples, we will see that the competitive pressure on the competition front can be the main driving force behind some well-known results in the literature of price discrimination and information sharing. Nevertheless, most papers have assumed the uniform distribution in a spatial competition model without noticing the significance of competitive pressure.

1.4.1 Preference-based Price Discrimination

Recently, preference or behavior-based price discrimination has been studied intensely. The Hotelling model has a good nature to analyze this issue because this type of price discrimination is based on consumers' brand preference. The prevailing literature on this issue shares one important result that price discrimination based on consumer preferences is not a profitable strategy.[15] However, I will show that this result may not hold in a general distribution. Firms can increase their profits from the preference-based price discrimination in intense competition where consumer preferences follow a bell-shaped distribution. In particular, this type of price discrimination can soften market competition by letting firms compete on less competitive fronts.

For simplicity, I assume that the firms are symmetric, and marginal costs are zero, $c_A = c_B = 0$. As a benchmark, I summarize the symmetric equilibrium prices and profits without price discrimination as below.

$$p_i^* = \frac{1}{2f(0)} \text{ and } \pi_i^* = \frac{1}{4f(0)} \tag{1.5}$$

Now, suppose that the firms are able to observe whether a consumer has more preference for its good or its rival's, i.e., whether θ is greater or smaller than 0. Let us refer to the region $\theta \in [-\overline{\theta}, 0]$ as firm A's turf and the region $\theta \in [0, \overline{\theta}]$ as firm B's turf. The both firms offer different prices to different turfs. As in Bester and Petrakis (1996), this scenario can be thought of as targeted coupons offered to the rival's turf. Also, this can be interpreted as the second-period poaching competition in Fudenberg and Tirole (2000).

Let $\overline{p}_i$ denote the price offered to consumers in its own turf, while $\widehat{p}_i$ represents the poaching price offered to consumers in its rival's turf. Let us analyze price competition in firm B's turf, first. The marginal consumers are $\widetilde{\theta} = \overline{p}_B - \widehat{p}_A$. The profit functions are written as

$$\widehat{\pi}_A = \widehat{p}_A(F(\widetilde{\theta}) - \frac{1}{2}) \text{ and } \overline{\pi}_B = \overline{p}_B(1 - F(\widetilde{\theta})).$$

The equilibrium prices and profits are

$$\begin{aligned}
\widehat{p}_A &= \frac{F(\theta^*) - \frac{1}{2}}{f(\theta^*)} \text{ and } \overline{p}_B = \frac{1 - F(\theta^*)}{f(\theta^*)}, \text{ and} \\
\widehat{\pi}_A &= \frac{(F(\theta^*) - \frac{1}{2})^2}{f(\theta^*)} \text{ and } \overline{\pi}_B = \frac{(1 - F(\theta^*))^2}{f(\theta^*)}, \\
\text{where } \theta^* &= \frac{\frac{3}{2} - 2F(\theta^*)}{f(\theta^*)}.
\end{aligned} \tag{1.6}$$

If consumers are uniformly distributed, we can easily show that $\overline{p}_i = \frac{2\overline{\theta}}{3}$ and $\widehat{p}_i = \frac{\overline{\theta}}{3}$. Both prices are lower than the non-discrimination symmetric equilibrium price, $p_i^* = \overline{\theta}$. Evidently, the firms are worse-off than they would have been without price discrimination. To explain this result, Armstrong (2006) writes "discrimination acts to intensify competition ... when firms differ in their view of which markets are strong and which are weak." Similarly, Corts (1996) uses the term "best response asymmetry". On the other hand, Anderson and Leruth (1993) says "firms compete on more fronts". This last statement captures the idea that firms have to compete more aggressively simply because they compete for a greater number of marginal consumers on more fronts.

However, if $f(\theta^*)$ is sufficiently small compared to $f(0)$, the discriminating prices can be greater than the non-discriminating price. Then, we have a possibility that the equilibrium profits are greater with price discrimination. Since the model is symmetric, we will have a symmetric outcome in the analysis for competition in the region $\theta \in [-\overline{\theta}, 0]$, firm A's turf. Thus, the equilibrium profits with price discrimination can be thought of the sum of $\widehat{\pi}_A = \frac{(F(\theta^*)-\frac{1}{2})^2}{f(\theta^*)}$ and $\overline{\pi}_B = \frac{(1-F(\theta^*))^2}{f(\theta^*)}$, that are given by

$$\pi_i^{PD} = \frac{2F(\theta^*)^2 - 3F(\theta^*) + \frac{5}{4}}{f(\theta^*)}. \tag{1.7}$$

Proposition 7 *A sufficient condition for $\pi_i^{PD} > \pi_i^*$ is $f(0) > 2f(\theta^*)$. The firms' equilibrium profits can be greater with the preference-based price discrimination in a bell-shaped distribution.*

Proof. See Appendix.

The sufficient condition $f(0) > 2f(\theta^*)$ is very intuitively appealing. The total number of marginal consumers does matter for the comparison of equilibrium profits. Without price discrimination, the density of marginal consumers is $f(0)$, whereas it is $2f(\theta^*)$ with price discrimination. It is not difficult to imagine that a bell-shaped distribution can yield the situation in which the preference-based price discrimination is a profitable strategy. Competition for the entire market is very intense, but competition in the segmented markets becomes less intense. Following Anderson and Leruth's words, we can say "firms compete on less competitive fronts" through price discrimination. This result stands in sharp contrast to the previous literature.

Example. *Consider the following density function* $f(\theta) = -\frac{1}{\overline{\theta}^2}|\theta| + \frac{1}{\overline{\theta}}$. *The corresponding distribution function is* $F(\theta) = \frac{1}{2} + \frac{\theta^2}{2\overline{\theta}^2} + \frac{1}{\overline{\theta}}\theta$ *for* $\theta < 0$ *and* $F(\theta) = \frac{1}{2} - \frac{\theta^2}{2\overline{\theta}^2} + \frac{1}{\overline{\theta}}\theta$ *for* $\theta > 0$. *Without price discrimination, the symmetric equilibrium prices and profits are* $p_i^* = \frac{\overline{\theta}}{2}$ *and* $\pi_i^* = \frac{\overline{\theta}}{4}$. *With price discrimination, equation (1.6) becomes* $\theta^* = \frac{3}{4}\overline{\theta} - \sqrt{\left(\frac{3}{4}\overline{\theta}\right)^2 - \frac{1}{4}}$. *It can be shown that the sufficient condition* $f(0) > 2f(\theta^*)$ *holds if* $\frac{2}{3} < \overline{\theta} < \frac{\sqrt{2}}{2}$.

1.4.2 Information Sharing

Here, I investigate how competition intensity influences incentives to share private information. There are many papers which address this issue. For example, Vives (1990) and Gal-Or (1985, 1986) show that the incentives for information sharing depend on the nature of competition (Cournot or Bertrand) and the nature of the information structure (demand or costs). In particular, regarding private information about costs, Gal-Or (1986) shows that information sharing is a dominant strategy with Cournot competition and concealing is a dominant strategy with Bertrand competition. One critical point in the literature is that the analyses are based on a linear demand curve. This is a crucial driving force behind their results. However, I will show, in the simplest form, that their results can be reversed.

Consider the following textbook example. Suppose firm B's marginal cost is c, while firm A's marginal cost is uncertain. It can be either c_H or c_L with equal probability, where $c_L < c_H$. Information is asymmetric. While firm A knows its own marginal cost and firm B's, firm B knows its cost and only that firm A's marginal cost is either c_H or c_L with equal probability.[16] All other things are common knowledge. I assume $(c_H - c) = (c - c_L)$ so that firm A does not have any *ex ante* cost advantage. I define marginal consumers in each state as $\widetilde{\theta}_H = p_B - p_A(c_H)$ and $\widetilde{\theta}_L = p_B - p_A(c_L)$.

Then, firm A's profit function in each state is written as $\pi_A(c_H) = (p_A(c_H) - c_H)F(\widetilde{\theta}_H)$ and $\pi_A(c_L) = (p_A(c_L) - c_L)F(\widetilde{\theta}_L)$. Firm B anticipates that firm A's price will be $p_A(c_H)$ or $p_A(c_L)$. Firm B solves $E\pi_B = \frac{1}{2}(p_B - c)(1 - F(\widetilde{\theta}_H)) + \frac{1}{2}(p_B - c)(1 - F(\widetilde{\theta}_L))$. The Bayesian Nash equilibrium is characterized by

$$\theta_H^I = c - c_H + \frac{1 - 2F(\theta_H^I)}{2f(\theta_H^I)} \text{ and } \theta_L^I = c - c_L + \frac{1 - 2F(\theta_L^I)}{2f(\theta_L^I)}, \tag{1.8}$$

where $\theta_L^I = -\theta_H^I > 0$ by symmetry. Note that this implies $f(\theta_L^I) = f(\theta_H^I)$ and $F(\theta_L^I) = 1 - F(\theta_H^I)$. Equilibrium prices are $p_A(c_H) = c_H + \frac{F(\theta_H^I)}{f(\theta_H^I)}$, $p_A(c_L) = c_L + \frac{F(\theta_L^I)}{f(\theta_L^I)}$, and $p_B = c + \frac{(1-F(\theta_H^I))+(1-F(\theta_L^I))}{f(\theta_H^I)+f(\theta_L^I)}$. Now, we are interested in firm A's *ex ante* expected profit, which is given by

$$E\pi_A^I = \frac{1}{2}\frac{F(\theta_H^I)^2}{f(\theta_H^I)} + \frac{1}{2}\frac{F(\theta_L^I)^2}{f(\theta_L^I)} = \frac{F(\theta_L^I)^2 + \left(1 - F(\theta_L^I)\right)^2}{2f(\theta_L^I)}. \tag{1.9}$$

Let us consider the other case in which firm A shares the information about its marginal cost. Firm A's *ex ante* expected profit is simply the average of equilibrium profits in each state under complete information. Then, the equilibrium will be a replication of the basic model, which can be represented by

$$\theta_H^S = (c - c_H) + \frac{1 - 2F(\theta_H^S)}{f(\theta_H^S)} \text{ and } \theta_L^S = (c - c_L) + \frac{1 - 2F(\theta_L^S)}{f(\theta_L^S)}. \tag{1.10}$$

Firm A's expected profit from information sharing is

$$E\pi_A^S = \frac{1}{2}\frac{F(\theta_H^S)^2}{f(\theta_H^S)} + \frac{1}{2}\frac{F(\theta_L^S)^2}{f(\theta_L^S)} = \frac{F(\theta_L^S)^2 + \left(1 - F(\theta_L^S)\right)^2}{2f(\theta_L^S)}. \tag{1.11}$$

Proposition 8 *$\theta_H^I < \theta_H^S < 0 < \theta_L^S < \theta_L^I$ because $\frac{1-2F(\theta)}{2f(\theta)} \gtreqless \frac{1-2F(\theta)}{f(\theta)}$ as $\theta \gtreqless 0$ by comparing (1.8) to (1.10). Information sharing can be preferred in price competition in a U-shaped distribution.*

Information sharing makes firms compete on farther within fronts in the Hotelling line.[17] Thus, the shape of the distribution function obviously affects firm A's decision of whether to share its private information. In the uniform distribution, competitive pressure does not come into play. In this case, we immediately obtain $E\pi_A^I = \frac{\bar{\theta}}{2} + \frac{(c_H - c_L)^2}{32\bar{\theta}} > \frac{\bar{\theta}}{2} = E\pi_A^S$. Concealing information is the dominant strategy. However, when firm A faces a U-shaped distribution of consumer preferences, information sharing allows it to compete on less competitive fronts. In other words, if $f(\theta_L^S)$ is sufficiently lower than $f(\theta_L^I)$ in a U-shaped distribution, firm A can prefer information sharing.

Example. *Consider the following density function $f(\theta) = \frac{1}{\bar{\theta}^2}|\theta|$. The corresponding distribution function is $F(\theta) = \frac{1}{2} - \frac{\theta^2}{2\bar{\theta}^2}$ for $\theta < 0$ and $F(\theta) = \frac{1}{2} + \frac{\theta^2}{2\bar{\theta}^2}$ for $\theta > 0$. In this case, equation (1.8) becomes $\theta_H^I = \frac{2}{3}(c - c_H)$ and $\theta_L^I = \frac{2}{3}(c - c_L)$, while equation (1.10) becomes $\theta_H^S = \frac{1}{2}(c - c_H)$ and $\theta_L^S = \frac{1}{2}(c - c_L)$. It can be easily shown that $E\pi_A = \frac{1}{2}\frac{F(\theta_H)^2}{f(\theta_H)} + \frac{1}{2}\frac{F(\theta_L)^2}{f(\theta_L)} = \frac{1}{4}\left(\frac{\theta_L}{\bar{\theta}^2} + \frac{\bar{\theta}^2}{\theta_L}\right)$. Therefore, $E\pi_A^I \gtreqless E\pi_A^S$ corresponds to $\bar{\theta}^2 \lesseqgtr \sqrt[4]{\frac{c-c_L}{3}}$.*

1.5 Concluding Remarks

I have set forth a model of product differentiation which relates the distribution of consumer preferences to the intensity of competition. The analyses in this article highlight the importance of taking into account non-uniform distributions because competition intensity has significant impacts on the effects of firms' strategies. Through examples, I emphasized how important competitive pressure is on the equilibrium location of marginal consumers to the extent that non-uniform distributions of consumer preferences can dramatically reverse some well-known results in the literature. In this sense, the uniform distribution typically assumed in the spatial model might be too restrictive.

It is expected that there will be many possible extensions of my paper. The imposition of MPS and MLRP on the distributions provides a new way of analyzing the systematic change of equilibrium prices according to the dispersion of the distribution function. This framework will be very useful for further research on various topics, in particular, such as studying firms' strategies to change consumers' preferences. Another important issue to be explored in future is the implications for social welfare.

I conclude the paper by presenting another interesting perspective on the distribution of consumer preferences. In fact, in the Hotelling (horizontal differentiation) model, each consumer's valuations for the two products are perfectly negatively correlated. In contrast, they are perfectly positively correlated in the Bertrand (vertical differentiation) model. These two familiar models can be thought of as limiting cases of a general model in which each consumer's valuations are independently distributed. Chen and Riordan (2008) represent this general model with a rectangular area. Similarly, we can also imagine another rectangular model in which products are differentiated in two horizontal dimensions. Nevertheless, this extension does not change the nature of the model fundamentally. Competition is still locally occurring for marginal consumers. In fact, the only difference is having a different distribution of consumer preferences. At last, the model with a general distribution of consumer preferences is more generalized than the model with multiple dimensions of consumer preferences in the Hotelling type model.

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Chapter 2

Team Incentives for Managing Competition and Exploiting Strategic Interactions

2.1 Introduction

In order to motivate individuals within a team, a principal can relate compensation to one another's performance. The principal offers rewards to agents based on either team performance, relative performance, or both. In fact, the combination of team rewards and competition is commonly observed in many organizations. For example, workers have to compete against each others for promotion or bonus. At the same time, they receive team-based rewards such as profit-sharing and employee stock options.[18]

The question is, then, what will be the optimal combination of team rewards and competitive compensation? This is a basic and important, but yet unanswered question. We can continue to ask the following related questions. Is there a tension between team and competition incentives, and if so, what is it? How will this combination change depending on the ability of agents? In other words, should the principal provide a more competitive or less competitive environment for higher ability agents? In addition, a more fundamental question is why many organizations motivate employees with team rewards? This is a challenging question because there is the essential weakness in team incentives, which is the free-rider problem. This paper aims to answer these questions by focusing on providing new rationales for team rewards.

In this paper, "team" is defined as a group of individuals who works for a same principal. In fact, in most papers, the definition of team is not always clearly defined and restricted to a number of persons working on teamwork consisting of complementary tasks.[19] The contribution of this paper starts from our broader definition of team. It is intuitive and well-known that team rewards are necessary to improve the performance of teamwork. On the other hand, the situation to be considered here is that agents undertake independent tasks or even perfectly substitutable tasks in the perspective of their principal. This is a situation where team rewards seem to be needless. In this sense, I believe that the following two arguments are the most fundamental rationales for team rewards.

First, the principal fine-tunes the effectiveness of competition with team incentives. I find that there is a discouragement effect in competition. As competitive compensation increases, an agent will work harder and the other agent will work harder as well. The expected increase in effort of the opponent has an *ex ante* effect of depressing each other's effort by reducing the probability of being ranked first and winning competitive compensation. Hence, the principal encounters a tension between the free-rider problem in team rewards and the discouragement effect in competitive compensation. Obviously, the principal has to balance the trade-off between the two compensation schemes. In other words, the principal always prefers to combine team rewards and competitive compensation because team incentives avoid the discouragement effect in competition and serve the purpose of managing competition.

This result is a strong challenge to the efficiency of a winner-take-all competition such as contest or tournament. Without team rewards, my model turns out to be a winner-take-all competition in which competition is the only force to induce effort from agents. However, this cannot be the optimal mechanism because the discouragement effect is not controlled. In fact, many believe that the winner-take-all competition is the way to maximize agents' efforts at given costs. This is why we run the patent system which gives an exclusive ownership to inventors despite the *ex post* efficiency loss from monopoly. However, remarkably, this paper suggests a possibility that granting multiple patents as team rewards are able to increase firms' incentives to innovate.[20]

In addition, somewhat counterintuitively, the principal offers a less competitive atmosphere for the high ability agents than for the low ability agents. As competition becomes intense due to an increase in the ability of agents, the discouragement effect becomes more severe, but the free-rider problem is mitigated. Thus, the principal should provide a larger proportion of team incentives for the higher ability agents.

Second, team rewards help the principal exploit the effects of strategic interactions between agents such as collusion and precommitment. I will show that agents put forth more effots by collusion or playing sequentially, which is obviously a better outcome for the principal. If the principal offers team rewards greater than competitive compensation, agents perceive the tasks as complementary even though two tasks are identical and perfectly substitutable in their physical functions. In this case, collusion enables them to avoid the complement problem and internalize the externality of their effort on each other's payoff. Likewise, agents voluntarily play a sequential-move game and the strategic precommitment effect is endogenously induced.

This paper is related to two strands of literature. First, there is a large literature on tournament or contest, followed by Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983), which is the study of compensation based on the winner-take-all competition. Second, there are several papers that address the team incentive scheme as a principal's optimal strategy. For example, the non-pecuniary benefits of more social interaction and less boring work in Rosen (1986), fostering help among agents in Itoh (1991), peer pressure in Kandel and Lazear (1992), complementarities in production among workers in Lazear (1998), and peer sanction induced from long-term relationship in Che and Yoo (2001) explain the reasons for the presence of team rewards.

This paper connects these two literatures by studying the interaction between two types of compensation schemes. The novel contribution of this paper is to clarify what is the trade-off between team and competition incentives and to analyze the interplay between the free-rider problem and the marginal discouraging problem. As a result, compared to the contest literature, I find that there is a diminishing returns trap in competition incentives and suggest that competition within a team should be managed by providing team incentives.[21] On the other hand, compared to the literature on team rewards, this paper discovers other important rationales, which are managing competition and exploiting the positive effect of collusion or precommitment. These explanations are contrasting, and unique, to the literature on team incentives.

In a different vein, this paper is related to Marino and Zábojník (2004) and Gershkov, Li, and Schweinzer (forthcoming). Both papers show that internal competition can solve the free-rider problem in team-based rewards. While they center their attention on the intrinsic problem in team incentives and propose competition as a solving method, I point out a potential problem of competition and suggest supplying team incentives as a solution. Consequently, both apporaches together imply that there is some degree of complementarity between competition incentives and team incentives to motivate workers.

The main strength of my model lies in its ability to analyze the effect of a marginal change of the wage scheme and how competition between agents is characterized. These are the reason why I can find the optimal combination of the two incentives which is determined by the ability level of agents. The model can also explain such characteristics of competition as whether tasks are (gross) substitutes or (gross) complements and whether tasks are strategic substitutes or strategic complements. These are important features in analyzing strategic interactions between agents, which make our study of collusion and precommitment possible. In contrast, these have not been studied in the previous literature, perhaps because many papers studying a principal multiple-agent model are based on the framework in which agents are allowed to choose discrete actions, in particular binary actions such as work and shirk or high effort and low effort.

The remainder of the paper is organized in the following way. Section 2 introduces the basic model. In this section, I characterize competition between agents and explore incentive issues without a principal for the moment. In Section 3, I analyze the principal's problem. I mainly focus on finding the optimal combination of team and competition incentives. Section 4 extends the model by incorporating strategic interactions between agents. Section 5 discusses applications such as nonexclusive patent system and joint liability in group lending. Section 6 concludes.

2.2 Incentives: To be a Winner or Not to be a Loser

There are two agents, A and B. Each agent is given an identical task to complete. The outcome of this task can be either success or failure. Each agent's probability of success is $p(I_i)$, where I_i is the effort levels of $i = A$ or B. This probability function is concave, i.e., $p'(I_i) > 0$ and $p''(I_i) < 0$. This concavity ensures the existence of equilibrium. I also assume the following inequality for the sake of the global stability and uniqueness of equilibrium.[22]

$$p''(I_A)p''(I_B)p(I_A)p(I_B) - \left[p'(I_A)p'(I_B)\right]^2 > 0.$$

As a result of effort, four different situations can occur: both agents succeed with probability $p(I_A)p(I_B)$, agent A succeeds but agent B fails with $p(I_A)(1 - p(I_B))$, agent B succeeds but agent A fails with $(1 - p(I_A))p(I_B)$, and neither agent succeeds with $(1-p(I_A))(1-p(I_B))$. Corresponding payoffs of agent A and B will be represented by (v^S, v^S), (v^W, v^L), (v^L, v^W) , and (v^O, v^O). For example, when agent A succeeds but agent B fails in the second case, agent A receives v^W and agent B obtains v^L. The agents' cost function is $C(I_i) = \frac{1}{\alpha}I_i$ and the marginal cost of effort is $\frac{1}{\alpha}$ so that $\alpha > 1$ represents the ability of the agents. For the moment, the agents are assumed to be risk neutral. Agent A's maximization problem can be written as follows.

$$\begin{aligned}\underset{I_A}{Max} \qquad & V_A = p(I_A)p(I_B)v^S + p(I_A)(1 - p(I_B))v^W \\ & +(1 - p(I_A))p(I_B)v^L + (1 - p(I_A))(1 - p(I_B))v^O - \tfrac{1}{\alpha}I_A\end{aligned}$$

In this contest-like situation, what are incentives to put forth effort? The agents strive against each other, not only to win but also not to lose. At first glance, winning seems to be another expression of not losing. Yet, in terms of incentives, we can distinguish winning from not losing and this distinction leads to an important implication.

Agent A chooses an effort level to maximize the expected payoffs, given that the other agent will act optimally. The first-order condition for a maximum is given by

$$p'(I_A)\left[p(I_B)\Delta^{SL} + (1 - p(I_B))\Delta^{WO}\right] - \frac{1}{\alpha} = 0, \tag{2.1}$$

where $\Delta^{SL} = (v^S - v^L)$ and $\Delta^{WO} = (v^W - v^O)$. This equation shows that incentives to expend effort depend on two terms, Δ^{SL} and Δ^{WO}, given the other agent's effort level. Δ^{SL} represents the net loss that a loser must bear when one's rival succeeds, whereas Δ^{WO} indicates the net gain that a winner can earn when one's rival fails. In this sense, I will refer to Δ^{SL} as the loser's loss and Δ^{WO} as the winner's gain. In this model, incentives are not simply measured by the prize accruing to the winner, i.e., its post-success payoffs. When the rival succeeds, the agent works not to be the loser. When the rival fails, the agent works to be the winner. After all, incentives are represented by a weighted average of the loser's loss and the winner's gain.

The loser's loss and winner's gain do not necessarily need to be equal. The relative size of one to the other depends on a particular type of game. I will refer to the offensive game as the one where the winner's gain is greater than the loser's loss because to be a winner is a more important goal of agents than not to be a loser. In a similar sense, the game in which the loser's loss is greater than the winner's gain will be called the defensive game.

Before finding a Nash equilibrium, let us see the slope of reaction functions. Totally differentiating equation (2.1), we obtain

$$\frac{dI_B}{dI_A} = -\frac{p''(I_A)(p(I_B)\Delta^{SL} + (1 - p(I_B))\Delta^{WO})}{p'(I_A)p'(I_B)(\Delta^{SL} - \Delta^{WO})}. \tag{2.2}$$

The numerator is negative by the second-order condition. Hence the sign of the slope of reaction functions is determined by the sign of $(\Delta^{SL} - \Delta^{WO})$ in the denominator. When $\Delta^{SL} > \Delta^{WO}$, the reaction functions are upward sloping. By contrast, when $\Delta^{SL} < \Delta^{WO}$, the reaction functions are downward sloping. That is to say, the tasks are strategic complements and strategic substitutes respectively.[23]

Proposition 9 *If the loser's loss is greater than the winner's gain ($\Delta^{SL} > \Delta^{WO}$, defensive game), the tasks are strategic complements. Conversely, if the winner's gain* *is greater than the loser's loss ($\Delta^{SL} < \Delta^{WO}$, offensive game), the tasks are strategic substitutes.*

Now, through well-known oligopoly models, I provide some examples to show different types of competition in which being a winner is more important than not being a loser, and vice versa. I consider a case in which firms conduct a cost-reducing innovation, which reduces production costs from c to $c-\lambda$. Then, in examples below, both Δ^{WO} and Δ^{SL} are functions of λ, i.e., $\Delta^{WO}(\lambda)$ and $\Delta^{SL}(\lambda)$. We will see that these firms play the offensive or defensive game depending on a market environment.

Example. *(Cournot Model) Consider an duopoly market with a linear demand $Q(p) = a - p$. When firms involve the process innovation as described above, the corresponding profits for each case are given by $\pi^S = \frac{(a-c+\lambda)^2}{9}$, $\pi^W = \frac{(a-c+2\lambda)^2}{9}$, $\pi^L = \frac{(a-c-\lambda)^2}{9}$, and $\pi^O = \frac{(a-c)^2}{9}$. The loser's loss and winner's gain are calculated easily as $\Delta^{SL} = \frac{4}{9}(a-c)\lambda$ and $\Delta^{WO} = \frac{4}{9}(a-c+\lambda)\lambda$. Note $\Delta^{SL} - \Delta^{WO} = -\frac{4}{9}\lambda^2 < 0$. In the simple Cournot model with a linear demand and constant marginal costs, firms play the offensive game in R&D competition.*

Example. *(Perfect Complements) Consider a case that two goods form a system as prefect complements. A consumer type $x \in (-\infty, 0]$ has a valuation of $V - p_A - p_B + x$ for the system. This generates the following profit function, $\pi_i = (p_i - c)(V - p_i - p_j)$. The firms have the same chance of the process innovation which can reduce production costs. Then we find $\pi^S = (\frac{V-2(c-\lambda)}{3})^2$, $\pi^W = \pi^L = (\frac{V-2c+\lambda}{3})^2$, and $\pi^O = (\frac{V-2c}{3})^2$. The winner's profit equal the loser's profit ($\pi^W = \pi^L$). In the case of perfect complements, the benefit of innovation is equally shared by two firms because of externalities. This yields $\Delta^{SL} - \Delta^{WO} = \frac{2}{9}\lambda^2 > 0$, where $\Delta^{SL} = \frac{3\lambda^2+2\lambda(V-2c)}{9}$ and $\Delta^{WO} = \frac{\lambda^2+2\lambda(V-2c)}{9}$. Firms providing complements play the defensive game in which not losing is more important than winning.*

Example. *(Hotelling Model) Suppose that a consumer is indexed by $\theta \in [\underline{\theta}, \overline{\theta}]$, where $\underline{\theta} = -\overline{\theta} < 0$. θ represents a consumer's relative preferences for the product B over A. Consumers are distributed by cumulative distribution function F over θ. We assume F is symmetric about zero and the monotone hazard rate $\frac{f(\theta)}{1-F(\theta)}$ is strictly increasing in θ.*[24] *Consumers $\theta < \tilde{\theta} = p_B - p_A$ choose to buy A, whereas consumers $\theta \geq \tilde{\theta}$ choose B. Then, the profit function of each firm is given by $\pi_A = (p_A - c)F(\tilde{\theta})$ and $\pi_B = (p_B - c)(1 - F(\tilde{\theta}))$ respectively. The symmetric outcome of the Hotelling model displays $\pi^S = \pi^O = \frac{F(0)^2}{f(0)} = \frac{1}{4f(0)}$. On the other hand, when only one firm, say firm A, succeeds, the profit functions are written by $\pi_A = (p_A - c - \lambda)F(\tilde{\theta})$ and $\pi_B = (p_B - c)(1 - F(\tilde{\theta}))$. The Nash equilibrium must satisfy $\theta^* = \lambda + \frac{1-2F(\theta^*)}{f(\theta^*)}$.*[25] *With this equilibrium condition, we can represent $\pi^W = \frac{F(\theta^*)^2}{f(\theta^*)}$ and $\pi^L = \frac{(1-F(\theta^*))^2}{f(\theta^*)}$. A simple calculation yields $\Delta^{SL} - \Delta^{WO} = \frac{1}{f(0)} - \frac{(2F(\theta^*)-1)^2+1}{f(\theta^*)}$, whose sign depends on the distribution of consumers. They play the defensive game when $f(0)$ is sufficiently small as in a U-shaped distribution. Otherwise, they play the offensive game. In this example, it should be noted that innovations can be strategic complements or strategic substitutes depending on the market demand curve.*

Example. *(Entry Game) Suppose that there is a monopolist in a market with a potential entrant. In order to enter the market, the entrant must be a sole winner in R&D competition to cover large fixed costs, K. That is, I consider $\frac{(a-c+\lambda)^2}{9} < K < \frac{(a-c+2\lambda)^2}{9}$. The entrant's profits are $\pi_E^W = \frac{(a-c+2\lambda)^2}{9} - K$ and $\pi_E^S = \pi_E^L = \pi_E^O = 0$. The incumbent's corresponding profits are $\pi_I^S = \frac{(a-c+\lambda)^2}{4} = \pi_I^W$, $\pi_I^L = \frac{(a-c-\lambda)^2}{9}$, and $\pi_I^O = \frac{(a-c)^2}{4}$. We immediately observe $\Delta_E^{SL} < \Delta_E^{WO}$ for the entrant and $\Delta_I^{SL} > \Delta_I^{WO}$ for the incumbent. Therefore, the entrant plays the offensive game, whereas the incumbent plays the defensive game.*

The agent B's maximization problem yields the symmetric first-order condition with equation (2.1) as $p'(I_B)\left[p(I_A)\Delta^{SL} + (1 - p(I_A))\Delta^{WO}\right] - \frac{1}{\alpha} = 0$. Given that the model is symmetric and the condition for global stability is met, we have $I^* = I_A^* = I_B^*$ in equilibrium satisfying

$$\frac{1}{\alpha p'(I^*)} = p(I^*)\Delta^{SL} + (1 - p(I^*))\Delta^{WO}. \tag{2.3}$$

Evidently, the higher ability agents exert more effort, i.e.,

$$\frac{dI^*(\alpha)}{d\alpha} > 0. \tag{2.4}$$

This is straightforward by applying implicit function theorem to (2.3). Since the equilibrium effort is strictly increasing in the ability of the agents, I often interpret an increase or decrease in the equilibrium effort as a change of the ability.

Note that Δ^{SL} and Δ^{WO} affect the equilibrium effort levels differently. Δ^{SL} raises the slope of the right-hand side term in (2.3) and the equilibrium effort is increasing in Δ^{SL}. On the other hand, interestingly, an increase in Δ^{WO} raises the equilibrium effort through lifting the intercept but reduces it by lowering the slope. Put differently, an increase in the winner's gain not only encourages the agents, but also discourages them to make effort.

The intuition to understand this result is as follows. An agent can receive Δ^{WO} when he succeeds and the opponent fails. As Δ^{WO} increases, the rival increases his effort, and accordingly raising the probability of the opponent's achievement. In turn, this lessens the agent's probability of receiving the winner's gain. In this way, the winner's gain can affect the agents' supply of effort indirectly and negatively.[26]

Now, let us compare the marginal effect of the loser's loss to that of the winner's gain. In equation (2.3), an increase in $p(I^*)$ places less weight on the effect of the winner's gain and more weight on the effect of the loser's loss. Thus, the relative effectiveness of the loser's loss to the winner's gain is increasing in $p(I^*)$.

Proposition 10 $-\left.\frac{d\Delta^{WO}}{d\Delta^{SL}}\right|_{I^*} = \frac{\partial I^*/\partial\Delta^{SL}}{\partial I^*/\partial\Delta^{WO}} = \frac{p(I^*)}{1-p(I^*)}$. *The ratio of the marginal effect of the loser's loss to that of the winner's gain is increasing in the equilibrium probability* $p(I^*)$.

We also obtain

$$\frac{\partial I^*}{\partial\Delta^{SL}} \gtreqless \frac{\partial I^*}{\partial\Delta^{WO}} \text{ as } p(I^*) \gtreqless \frac{1}{2}.$$

The loser's loss has a higher marginal effect on agents' equilibrium effort levels than the winner's gain if the equilibrium probability of success is greater than $1/2$, and vice versa. Colloquially speaking, a stick (loser's loss) is a better motivator than a carrot (winner's gain) if agents are more likely to succeed or they are relatively capable. Conversely, a carrot is a better motivator if agents are less likely to succeed or they are relatively incapable.

2.3 Optimal Team and Competition Incentives

In this section, I include a risk neutral principal and modify the model to make it more appropriate for a principal-agent setup. The agents are risk averse: each agent's utility function is $u(x)$ with $u'(x) > 0$ and $u''(x) < 0$. To have an interior solution, I assume $u'(0) = \infty$. Consider that there is a common environmental shock. When agents undertake similar tasks or even independent tasks in the same working environment, they are exposed to a common shock. The common shock is good with probability σ. In this case, both agents can succeed the tasks without exerting any effort. By contrast, it is bad with probability $1-\sigma$. Then the probabilities of success are $p(I_A)$ and $p(I_B)$ as before. The presence of the common shock necessarily leads to the correlation between performance of the two agents.[27] Obviously, the principal is not able to observe the agents' effort level, but she can observe whether each agent accomplishs the task or not, separately.

Now, $\mathbf{v} = (v^S, v^W, v^L, v^O)$ is considered as the wages that the principal offers to agents in each situation. Following the literature, I define a wage scheme $\mathbf{v}$ exhibits joint performance evaluation (JPE) if $(v^S, v^L) > (v^W, v^O)$, independent performance evaluation (IPE) if $(v^S, v^L) = (v^W, v^O)$, and relative performance evaluation (RPE) if $(v^S, v^L) < (v^W, v^O)$. If the other agent performs well, the principal rewards the agent under JPE, whereas the principal penalizes the agent under RPE.

One immediate result is that the agents' perception of tasks depends on whether the wage scheme follows either JPE or RPE. How does a more aggressive strategy by agent j affect agent i's payoffs? The direction of the effect is the conventional definition of (gross) substitutes and (gross) complements. My model displays

$$\frac{\partial V_i}{\partial I_j} = p'(I_j)\left[p(I_i)(u(v^S) - u(v^W)) + (1 - p(I_i))(u(v^L) - u(v^O))\right] \begin{array}{l} > 0 \text{ under JPE} \\ < 0 \text{ under RPE.} \end{array} \tag{2.5}$$

While the agents perceive the tasks as complements under JPE, they perceive the tasks as substitutes under RPE, irrespective of whether tasks are truly complementary or substitutable in their physical functions. For instance, suppose that both agents pursue to develop an identical technology. This is a situation where the principal makes the same amount of revenue between when only one agent succeeds and when both agents succeed. In this regard, the tasks are identical and perfectly substitutable in the perspective of the principal. Nonetheless, the agents take the two tasks as complementary under JPE.

Proposition 11 *The agents perceive that the tasks are complements under JPE, while the tasks are substitutes under RPE.*

Let us formulate the principal's problem. I consider the most unfavorable case for the principal to provide team rewards to agents, as the situation where the tasks undertaken by agents are identical and perfectly substitutable with each other.[28]

The revenue R is realized when either of agents achieves the task with probability $\sigma + (1-\sigma)p(I^*)(2 - p(I^*))$.[29] Thus the problem of the principal is

$$\underset{v^S, v^W}{Max} \; [\sigma + (1-\sigma)p(I^*)(2 - p(I^*))]\, R - 2[\sigma + (1-\sigma)p(I^*)^2]v^S \qquad (2.6)$$
$$- 2(1-\sigma)p(I^*)(1-p(I^*))(v^W + v^L) - 2(1-\sigma)(1-p(I^*))^2 v^O$$

subject to

(IC) $\frac{1}{(1-\sigma)\alpha p'(I^*)} = p(I^*)[u(v^S) - u(v^L)] + (1-p(I^*))[u(v^W) - u(v^O)]$

(IR) $V_i^* = \sigma u(v^S) + (1-\sigma)\begin{bmatrix} p(I^*)^2 u(v^S) \\ +p(I^*)(1-p(I^*))\{u(v^W) + u(v^L)\} \\ +(1-p(I^*))^2 u(v^L) \end{bmatrix} - \frac{1}{\alpha}I^* \geq \overline{V}$

(LL) v^S, v^W, v^L, and $v^O \geq 0$

First of all, given (v^S, v^W, v^L, v^O) offered by the principal, it is optimal for the agents to choose the effort level I^* according to the incentive compatibility (IC) constraint. The IC constraint is the non-cooperative equilibrium outcome that we have derived from the maximization problem of each agent. Second, we impose a limited liability (LL) or wealth constraint which does not allow the wage to be negative.

Lemma 1. $v^L = v^O = 0$.

All the proofs are in Appendix. Intuitively, there is no reason that the principal has to reward an agent who does not perform well. As a result, the principal has the two choice variables (v^S, v^W). We will refer to v^S as team rewards, because the agents can receive v^S when all team member carry out the tasks successfully. Likewise, v^W will be called competitive compensation because an agent can receive this when he performs better than the other agent. Also, by setting $v^L = v^O = 0$, note that the agents play an offensive game under RPE, while they play a defensive game under JPE.

Last, the individual rationality (IR) constraint is that the agents' expected utility in equilibrium should be at least as high as his outside option utility which is denoted by $\overline{V}$. Without loss of generality, we set $\overline{V} = 0$. In the following lemma, we show that the IR constraint is always satisfied. This implies that the principal can implement any effort level.

Lemma 2. *The IR constraint is not binding.*

As in Grossman and Hart (1983), it is convenient to think of this problem in two stages. First, as the implementation problem, the principal finds the best wage scheme, given the effort level. In other words, the principal finds the optimal combination of $(\widetilde{v}^S, \widetilde{v}^W)$. Second, as the effort selection problem, the principal chooses the agents' optimal effort level $\widetilde{I}^*$ for the choice of $(\widetilde{v}^S, \widetilde{v}^W)$.

$\widetilde{I}^*$ can be induced by various combinations of (v^S, v^W) which satisfy the IC constraint, where $I^* = \widetilde{I}^*$. Then, what is the optimal combination of $(\widetilde{v}^S, \widetilde{v}^W)$ to induce $\widetilde{I}^*$ at the lowest cost? At the optimum, the slope of the isocost line must be equal to the slope of the IC constraint. The optimal incentive scheme is set to satisfy

$$\frac{\sigma + (1-\sigma)p(\widetilde{I}^*)^2}{(1-\sigma)p(\widetilde{I}^*)(1-p(\widetilde{I}^*))} = \frac{p(\widetilde{I}^*)}{1-p(\widetilde{I}^*)}\frac{u'(v^S)}{u'(v^w)} = -\left.\frac{dv^W}{dv^S}\right|_{\widetilde{I}^*}$$

$$i.e., \frac{\sigma}{(1-\sigma)p(\widetilde{I}^*)^2} + 1 = \frac{u'(v^S)}{u'(v^w)}. \tag{2.7}$$

In this model, the slope of the IC constraint accounts for the trade-off between the free-rider problem in team rewards and the discouragement effect in competition. Regarding team rewards, efficiency requires that an individual is compensated with the full marginal return of one's effort. However, the free-rider problem arises because the agent's marginal return depends on the other agent's success probability. The agents supply insufficient effort due to that the agent is not able to receive team rewards when the other agent fails with probability $1 - p(\widetilde{I}^*)$. On the other hand, the discouragement effect stems from that the agent is not able to realize competitive compensation when the other agent succeeds with probability $p(\widetilde{I}^*)$.

As $\widetilde{I}^*$ increases, while the free-rider problem is mitigated, the marginal discouragement effect becomes severe. As a result, the principal wants to provide the larger proportion of team incentives.

Proposition 12 $\widetilde{v}^S/\widetilde{v}^W$ *is increasing in* $\widetilde{I}^*$. *The proportion of team rewards relative to competitive compensation should be increasing in the equilibrium effort.*

Note that the optimal wage scheme is RPE in this model as long as σ is positive. Since the left-hand side in (2.7) is greater than 1, we must have $\widetilde{v}^S < \widetilde{v}^W$.[30] Although competition force is greater to motivate the agents, the relative importance of team incentives should increase as the equilibrium effort rises. In this respect, team rewards serve the purpose of managing competition. In addition, since the equilibrium effort is increasing in α, the principal provides a larger proportion of team incentives for a high ability group than a low ability group.

It is worthwhile to point out how my model is related to the existing literature. In the absence of the common shock, the IPE $\widetilde{v}^S = \widetilde{v}^W$ is the optimal scheme. This is what Green and Stockey (1983) and Mookherjee (1984) study. Both papers compare the rank order tournaments to individual contracts. They show that piece rates may dominate tournaments when agents are risk averse, while the dominance can be reversed by the presence of a common shock as in our model.[31] While these papers compare tournament versus piece rates, this paper shows that the principal can do better in motivating workers by attuning competition with team incentives and how this tuning should change with the equilibrium effort or the ability of the agents.

For completeness, consider what effort levels the principal should induce. The optimal effort level $\widetilde{I}^*$ will be set to maximize the principal's expected payoff (2.6) with two constraints, the IC constraint and the outcome of the implementation problem (2.7). One obvious result is that $\widetilde{I}^*$ is increasing in R. The principal wants to induce a higher effort level as R rises. Therefore, the proportion of team incentives is greater in R as well.

2.4 Strategic Interactions between Agents

In the previous section, I have suggested a new perspective on team rewards, which is that team incentives manage competition between individuals within a team. Yet, the principal still prefers a RPE wage scheme in which competition incentives are greater than team incentives so long as there is a positive common shock. In this section, I turn to the analysis of two types of strategic interactions between the agents such as collusion by effort coordination and precommiment by the endogenous choice of timing of actions. We will find that those voluntary interactions of the agents lead the principal to switch from RPE to JPE. Namely, team incentives become more important in motivating workers.

2.4.1 Collusion

I consider a possibility that the agents cooperate by coordinating each other's effort levels. We can think of several possible scenarios for collusion to take place. First, if the agents interact over a long period, they are able to collude by the grim trigger strategy. Second, the principal allows the agents to sign side contracts to coordinate their actions. Last, we can also think of a side contract offered to the agents by a third party who maximizes the sum of the agents' expected payoffs. Since the purpose of this paper is not studying contractual details, in order to make the problem simple, I assume that collusion emerges voluntarily as long as it is beneficial to themselves.[32] In what follows, the premise is that each agent can observe the other agent's choice of effort so that the agents can write a side contract contingent on their effort choice.[33]

Recall that since we set $v^L = v^O = 0$, RPE is associated with the offensive game with downward sloping reaction functions, while JPE accompanies the defensive game with upward sloping reactions functions. In addition, the isopayoff curves are increasing and decreasing around the reaction functions under RPE, and decreasing and increasing under JPE. This can be easily verified by inspecting the slope of isopayoff curves in the space of I_i and I_j as follows.

$$\left.\frac{dI_j}{dI_i}\right|_{\overline{V}_i} = -\frac{(1-\sigma)p'(I_i)\left[p(I_j)u(v^S) + (1-p(I_j))u(v^W)\right] - \frac{1}{\alpha}}{(1-\sigma)p'(I_j)p(I_i)(u(v^S) - u(v^W))}$$

The numerator is the well-behaved first-order condition. Thus, the isopayoff curve is U-shaped under JPE, and it is reversed under RPE.

Their equilibrium payoffs are greater as isopayoff curves move down under RPE and as isopayoff curves move up under JPE. Therefore, given v^S and v^W, the symmetric collusive effort level is smaller than non-cooperative equilibrium level of effort under RPE, but it is greater under JPE. Formally, to find the best collusive outcome, we need to maximize the sum of expected payoffs of the two agents. Equivalently, each agent maximizes the following collusive payoff function,

$$V^C = [\sigma + (1-\sigma)p(I)^2]u(v^S) + (1-\sigma)p(I)(1-p(I))u(v^W) - \frac{1}{\alpha}I.$$

Then, the collusive level of effort I^C satisfies

$$\frac{1}{(1-\sigma)\alpha p'(I^C)} = 2p(I^C)u(v^S) + (1 - 2p(I^C))u(v^W). \tag{2.8}$$

Proposition 13 *$I^C \gtreqless I^*$ if and only if $v^S \gtreqless v^W$. The symmetric collusive level of effort is greater than the non-cooperative equilibrium level of effort under JPE, and vice versa under RPE.*

The proof can be shown promptly by comparing (2.8) to the IC constraint from the non-cooperative equilibrium. Compared to the non-cooperative equilibrium, the agents collusively put forth more effort under JPE and less effort under RPE.[34] Thus, the principal benefits from collusion under JPE. The driving force behind this result is as follows. Under JPE, the agents feel like working on complementary tasks even if two tasks are identical and perfectly substitutable in their physical functions. Thus, they face the classical double marginalization problem of complements. However, through collusion, the agents are able to internalize the externality of one's effort on each other's payoff, thereby avoiding this double marginalization problem. This is the reason why the agents exert more effort through collusion under JPE.[35]

There are a few papers which study the effect of collusion under a JPE scheme, for example, Che and Yoo (2001) and Laffont and Martimort (1998). In their models only with two actions or two states, such as work and shirk, collusion is defined as both agents' shirking regardless of whether a wage scheme follows RPE or JPE. In other words, they exclude the possibility that the agents are better off by making more efforts. This is the reason why both papers argue that collusion does not come into play under JPE.[36] This is showing a potential drawback of the principal-agent model formulated with two actions.

For the principal's problem, we replace the previous non-cooperative (collusion-free) IC constraint with the collusion IC constraint (2.8). The principal maximizes (2.6) subject to (2.8). As seen in Figure 2.1, the collusion IC constraint (2.8) is more relaxed than the collusion-free IC constraint for $v^S > v^W$ and vice versa for $v^S < v^W$. Thus, there is a possibility that a JPE scheme can be adopted under the agents' collusion. Again, the relative price of $\tilde{v}^S$ and $\tilde{v}^W$ should be equal to the slope of the collusion IC constraint (2.8) at the optimal combination of $(\tilde{v}^S, \tilde{v}^W)$ as follows.

$$\frac{\sigma + (1-\sigma)p(\tilde{I}^C)^2}{(1-\sigma)p(\tilde{I}^C)(1-p(\tilde{I}^C))} = \frac{2p(\tilde{I}^C)}{1-2p(\tilde{I}^C)} \frac{u'(v^S)}{u'(v^w)}. \tag{2.9}$$

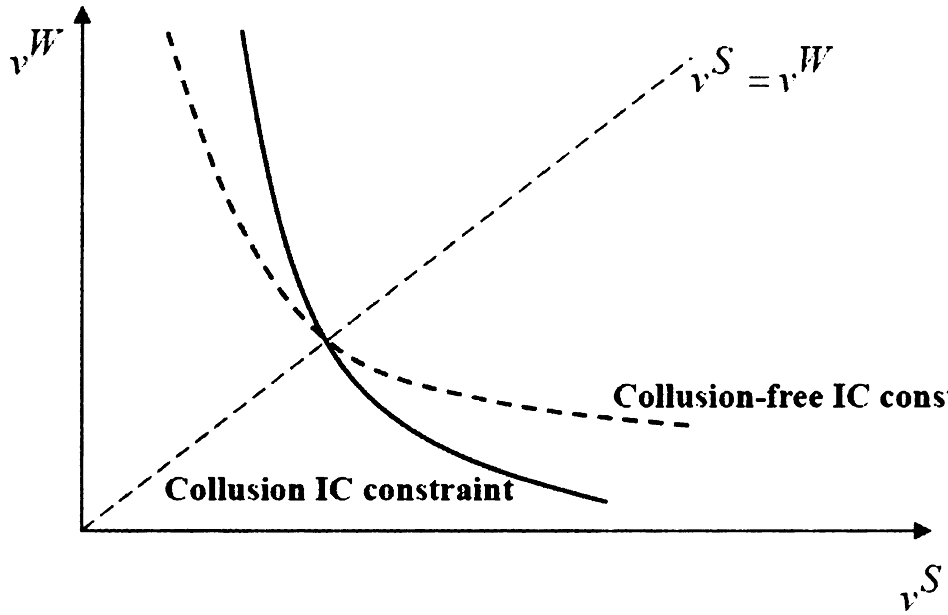

Figure 2.1: Collusion-free versus collusion

If $u'(v^S)/u'(v^w)$ is less than 1, JPE is the optimal scheme, and vice versa.[37]

Proposition 14 *When the agents collude, the principal prefers JPE if* $\widetilde{I}^C > \widehat{I}$ *and RPE if* $\widetilde{I}^C < \widehat{I}$, *where* $p(\widehat{I}) = \frac{-\sigma+\sqrt{\sigma}}{(1-\sigma)}$.

The principal selects a JPE scheme when she wants to induce a relatively high level of effort from the agents, or equivalently, when the agents have relatively high abilities or the principal's revenue is relatively large. In particular, if the equilibrium probability is greater than 1/2, only team rewards are provided because the right-hand side in (2.9) is negative.

Relatedly, we can consider a situation where the principal can design the wage scheme with the organizational structure or work environment at the same time. For example, the principal is able to create two different work environments: competitive organization or cooperative organization. The competitive organization means that the principal can effectively prevent the agents from coordinating each other's effort levels. On the other hand, the principal facilitates the agents to collaborate with each other in the cooperative organization. Then, in our model, this situation is equilvalent to solving the principal's problem with either the collusion-free IC constraint or the collusion IC constraint. If the collusion IC constraint is binding when the agents are of relatively high ability, the principal organizes the task environment in cooperative manner with a JPE wage scheme. Otherwise, the principal designs a competitive task environment with a RPE wage scheme.

2.4.2 Endogenous Timing of Actions

The agents are now allowed to choose the order of moves to be a leader or a follower in competition. To this end, I add the stage in which the agents select the timing of their moves before choosing the effort level, as in Hamilton and Slutsky (1990). The agents choose to play the first or to play the second in the first stage. If the agents make the same decision, the simultaneous game proceeds in the second stage. If they make different decisions, the Stackelberg game follows, in which the follower can observe the effort chosen by the leader.[38] To keep the analysis as simple as possible, I assume that the agents are risk neutral and there is no common shock. Then, we have already known that the principal is indifferent between any combination of (v^S, v^W) satisfying the IC constraint unless strategic interaction is involved.

By backward induction, we solve the second stage problem first. Since the simultaneous game has already been analyzed in Section 2, we solve the sequential-move game here. Without loss of generality, suppose that agent A moves first and B follows. We solve the Stackelberg game where agent A maximizes his payoff function subject to agent B's first-order condition.

$$\underset{I_A}{Max} V_A = p(I_A)p(I_B)v^S + p(I_A)(1 - p(I_B))v^W - \tfrac{1}{\alpha} I_A$$

$$s.t. \quad p'(I_B)\left[p(I_A)v^S + (1 - p(I_A))v^W\right] - \tfrac{1}{\alpha} = 0$$

The first-order condition of this problem is summarized by

$$p'(I_A)\left[p(I_B)v^S + (1-p(I_B))v^W\right] + \frac{dI_B}{dI_A}p'(I_B)p(I_A)(v^S - v^W) = 0. \qquad (2.10)$$

Note that the second term is always positive by $\frac{\partial I_B}{\partial I_A} \gtreqless 0$ as $v^S \gtreqless v^W$ in Proposition 1. Comparing (2.10) to (2.1), the first-order condition in the simultaneous game. Given I_B, I_A is always greater in the Stackelberg game than in the simultaneous game. Thus, in equilibrium, I_B is lower than the symmetric equilibrium effort levels I^* in the offensive game because the reaction function is downward sloping. By the same token, I_B should be greater than I^* in the defensive game.

Proposition 15 *$I_A^1 > I^* > I_B^2$ under RPE. $I_A^1 > I^*$ and $I_B^2 > I^*$ under JPE.*

Alternatively, equation (2.10) can be written as

$$\frac{dV_i}{dI_i} = \frac{\partial V_i}{\partial I_i} + \frac{\partial V_i}{\partial I_j}\frac{dI_j}{dI_i}.$$

This represents the effect of precommitment. The first term is zero by the envelope theorem. The second term is the product of (2.2) and (2.5), that are whether a wage scheme is RPE or JPE and whether the game is offensive or defensive. As a result, the effect of precommitment can be summarized by using Fudenberg and Tirole (1984)'s terminologies as follows.

	JPE	RPE
Offensive game	– (lean and hungry)	+ (top dog)
Defensive game	+ (fat cat)	– (puppy dog)

Indeed, this is the main issue that Dixit (1987) studies. He shows that the symmetric agents have no incentive to precommit in the contest model. However, my model describes that the effect of precommitment depends on (a) whether they play the offensive or defensive game and (b) whether team incentives are greater or smaller than competition incentives.

Denote the expected payoffs in the simultaneous-move subgame by (V_k^N, V_k^N) and those in the leader-follower subgame by (V_k^1, V_k^2). The superscripts 1, 2, and N indicate the first-mover, the second-mover, and the simultaneous-mover respectively. The subscript k is O or D which represents the offensive game and the defensive game respectively. The first stage game is represented by the following normal-form representation.

		Agent B	
		First	*Second*
Agent A	*First*	V_k^N, V_k^N	V_k^1, V_k^2
	Second	V_k^2, V_k^1	V_k^N, V_k^N

Then we can easily see

$$V_O^1 > V_O^N > V_O^2 \qquad \text{under RPE and}$$
$$V_D^2 > V_D^N \text{ and } V_D^1 > V_D^N \qquad \text{under JPE.}$$

This result is immediate from equation (2.5). Agent i's payoff is decreasing in agent j's effort level I_j in the offensive game, and vice versa in the defensive game. In fact, it is well-known that the agents have the first-mover advantage when having downward sloping reaction functions, while the potential second-mover has the disadvantage compared to the simultaneous move game.[39] Thus, there exists the dominant strategy which is to choose the first, and so the two agents simultaneously choose the effort level in the second period.

In the defensive game with upward sloping reaction functions, the agents have the second-mover advantage.[40] Yet, it does not mean that the first-mover becomes worse-off than in the simultaneous move game, because the first-mover's expected payoff is greater than when he moves simultaneously. Hence, they do not have a dominant strategy. There are two pure subgame perfect Nash equilibria. Given that one agent chooses the first (the second), the other agent chooses the second (the first). One becomes the leader and the other becomes the follower.[41]

Proposition 16 *(1) Under RPE, the both agents choose to be the first, and so they move simultaneously in choosing the effort level. (2) Under JPE, there are multiple equilibria in which one agent chooses the first and the other chooses the second.*

Proposition 7 and 8 together suggest that the principal is better off resorting to a JPE scheme. Under JPE, the agents voluntarily play the sequential move game, thereby exerting more efforts. In fact, it can be shown that $I_A^1 + I_A^2 > 2I^*$ under RPE. That is, the effect of precommitment is good for the principal in both regimes. However, only JPE allows precommitment to be accommodated by the other agent and to be the endogenous equilibrium outcome. Therefore, collusion and precommitment share the same spirit that all the principal has to do is to allow the agents to interact with each other freely under JPE.

2.5 Discussions

2.5.1 Nonexclusive Patent System

One interesting perspective on the model in this paper is that our model allows for dual winners. This feature is different from most contest models with the winner-take-all feature. Having a possibility of dual winners is equivalent to offering team rewards even in contest-like situations. What I have shown is that offering team rewards enables the principal to induce the optimal effort level at lower costs.[42] This result implies that selecting the sole and exclusive winner may not be the best way. In this sense, we can pose a question about the current patent system which does not allow dual patent holders for an identical technology.

Recently, several papers raise a possibility of nonexclusive patents. Maurer and Scotchmer (2002) argue that when the value of the invention is sufficiently larger than R&D cost and the inventor is over-rewarded, granting the patent to an another independent inventor can improve social welfare. Leibovitz (2002) argues in depth how a nonexclusive patent system would work in practice. Manna et al (1989) suggests a simple way of implementing it by accepting all applications up to the date in which a first inventor is awarded the patent, with the provision that the Patent Office keeps the technical details of patents secret. Ayres and Klemperer (1999) also study several methods to restrict patentee's monopoly power. One method suggested is duopoly auction, which is that the government would auction the patent right to one additional firm.

The reason why they propose a nonexclusive patent system is that nonexclusive patents can realize the efficiency gain from the *ex post* market after patents have been issued. For example, more competition will follow among rival patent holders and more reasonable licensing will increase. Obviously, one important concern in a nonexclusive patent system would be the possible efficiency loss from the *ex ante* market. In other words, granting patents to multiple independent inventors may not provide enough incentives for innovation.[43] In fact, it is a textbook lesson that we have the exclusive patent system to protect innovation incentives at the cost of *ex post* inefficiency.

However, this paper suggests that the nonexclusive patent system is able to increase innovation incentives, instead. The Patent Office may be able to give different rewards by setting different patent lengths in the case of having a single inventor and of having dual (or more) independent inventors. This means that the Patent Office virtually offers team incentives to competing firms and can manage the discouragement effect of competition.

One problem in applying our model to the patent system is that while firms' investment should be regarded as social costs, the agents' effort is not reflected in the cost to the principal. Nonetheless, keep in mind that a potential concern of multiple patents is not excessive investment but insufficient investment. Thus, our paper can suggest at least that nonexclusive patents may not harm firms' innovation incentives.[44]

2.5.2 Group Lending with Joint Liability

Group lending programs such as the Grameen Bank have been working successfully to lend to poor people without any collateral. One distinctive feature of these programs is asking borrowers to form a group in which all borrowers are jointly liable for each other's loans. This joint liability has been celebrated as the successful mechanism to solve the adverse selection problem.[45] Now, our results suggest an important implication to the moral hazard problem.

Suppose that a person must borrow capital from a bank to perform a project. If a borrower succeeds in the given projects, he can make revenue Y. The bank asks two borrowers to form a group.[46] Borrowers have to pay an interest rate r by a standard loan contract. A condition of joint liability specifies that a borrower must pay an additional amount c to the bank if his partner is unable to repay his loan.[47] Then, similarly to the basic model, a risk averse borrower's expected payoff is represented by

$$V_A = [\sigma + (1-\sigma)p(I_A)p(I_B)]u(Y-r) + (1-\sigma)p(I_A)(1-p(I_B))u(Y-r-c) - \frac{1}{\alpha}I_A.$$

We can also define the bank's problem in like manner as the principal's. It is not difficult to see that the analysis will be equivalent to our previous model. One can think of $Y-r$ as v^S and $Y-r-c$ as v^W respectively. A slight difference is that the principal has chosen her own costs which increase agents' incentives for effort in the previous model, whereas the bank's choice variables, r and c, are the source of income which reduces borrowers' incentives for effort. In addition, in this model, a positive joint liability ($c > 0$) means that the bank adopts a JPE scheme always.

As a result, $\widetilde{c}/\widetilde{r}$ is increasing in $\widetilde{I}^*$. As the borrowers are more capable or the revenue of the project is greater, the bank should offer a contract with a lower interest rate and higher joint liability. In fact, increasing joint liability in this model is lessening competition incentives, not strengthening team incentives, but the relative importance of team incentives increases.

However, the result of this paper suggests that the joint liability is not the best mechanism to provide proper incentives to solve the moral hazard problem. According to the result in this paper, strategic interactions are crucially necessary for the joint liability as JPE to be adopted. In fact, recently, many group-lending programs are turning to individual-lending. The reason can be related to our result. Armendáriz and Morduch (2007) observe that "[T]he emerging new contracts do not necessarily involve groups... Individual-lending approaches also have appeal in sparsely populated regions, areas with heterogenous populations, and areas marked by social divisions, where poor monitoring costs are high and social punishments for noncompliance more difficult to implement." In developed countries or urban areas, borrowers are less likely to know each others so that they cooperate or interact with each others. In this case, the joint liability is inferior in terms of solving the moral hazard problem.

2.6 Concluding Remarks

I have presented a simple model of the principal multiple-agent problem and have studied what is the principal's optimal choice of team and competition incentives. Competition is the major force to motivate agents, but the relative weight on team incentives to competition should be increasing in the equilibrium effort in order to manage the effectiveness of competition. Moreover, the importance of team incentives should be much greater when we consider strategic interactions between agents such as collusion and precommitment. I have also discussed the implications of our results in patent system and group-lending program.

These results offer empirically testable hypotheses regarding how the ability of the agents governs the principal's choice of compensation schemes. First, is there a positive relationship between the abilities of agents and the relative proportion of team rewards to competitive compensation? (Proposition 5) Second, does the principal choose a JPE scheme in a team with high ability agents and a RPE scheme with low ability agents? (Proposition 7) These are, I believe, worthwhile to study.

The central insight of this paper that I would like to repeat is that the agents place a weight on team incentives with each other's success probability, and on competition incentives with each other's failure probability. This leads to an important extension to the study of agents with heterogeneous abilities. When agents are of different abilities, another interesting trade-off comes into play for the principal's choice of the optimal wage scheme. A high-ability agent is more likely to be responsive to competition incentives relative to a low-ability agent, because the low-ability agent's failure probability is greater. Likewise, the low-ability agent will be more responsive to team incentives relative to the high-ability agent, because the high-ability agent's success probability is greater. Thus, the trade-off between team and competition incentives that the principal wants to balance is the difference between the high-ability and low-ability agent's incentives to work. This issue is left for future research.

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Chapter 3

Collective Action and Power Inequality

3.1 Introduction

This paper studies the collective action problem in two potentially important environments: competition between groups and internal conflict within a group.[48] When two or more individuals within a group work collectively for a certain goal, they often encounter competition from another group which has a similar interest. [Hardin (1995)] At the same time, individuals confront intragroup competition to determine how to divide the prize. In other words, many economic or non-economic agents engage in intergroup and intragroup competition at the same time.

Interest groups compete for rents created by government policies and individuals within a group compete for the spoils of the victory. Firms producing a system good as complements have to compete against another system and they have to divide profits among themselves. The same issue resides in joint R&D ventures. Countries in an alliance compete against another alliance and they have to decide how to share the burden of costs. Even in nature, many species compete for a limited amount of resources within and between species simultaneously. In these examples, the nature of internal conflict characterizes the shape of external competition, in particular, through collective action between agents within a group. Thus, this paper analyzes competition between groups with internal conflicts.

The main question we address is how these two environments shape the collective action problem. We attempt to treat both intergroup and intragroup competition in a general manner. We include a heterogeneity of individuals within a group and consider a general function to define collective action. Then, the interplay between internal and external competition turns out to be very crucial in the analysis of the collective action problem.

More specifically, we will answer the following two questions. First, how does each group's internal conflict influence its chance of winning in external competition? There are two contrasting views about this question. One view suggests that a group with less internal conflict has an advantage in external competition against a rival group [Deutsch (1949)]. The other view is that intragroup competition is more conducive in eliciting efforts from group members for external competition [Lüschen (1970)]. We show that both views have some validity in them by clarifying the interaction between intergroup and intragroup competition.

We measure the severity of internal conflicts within groups by the rate of rent dissipation. Not surprisingly, as group members have similar power, internal conflict is more aggressive. In this sense, a more (less) conflictive group is defined as one where the distribution of power is more concentrated (dispersed). Interestingly, a more (less) conflictive group competes better, in particular when collective action requires complementary (substitutable) works between group members.

Here is the intuition. Each member's incentive to contribute to collective action depends on one's equilibrium share of the prize in the internal conflict. Thus, when we compare the weak persons within groups, the person in a more conflictive group is willing to contribute to collective action more than the one in a less conflictive group vis-a-vis their respective persons in power. As a result, if individiuals' efforts are relatively complementary in creating collective action, people in a more conflictive group face the free-rider problem less severely. By the same token, when the contributory role of the person with less (more) power is relatively important in the formation of collective action, the winning probability of the more (less) conflictive group is greater.

Second, how does a change in power inequality within a group affect collective action? One famed argument in Olson (1965) is that the redistribution of wealth in favor of inequality can make individuals contribute to collective action more, because an individual who gains a significant proportion of total benefits from public goods has more incentive to contribute. We study this issue in terms of power distribution. Again, the interplay between the two layers of competition determines when collective action is facilitated or mitigated by a more dispersed power distribution. This issue will be quite interesting in the perspective of a team organizer. We can answer how the organizer wants to distribute power within a team in order to maximize the probability of winning.

When individuals' efforts are relatively substitutable or incumbents (persons in power) play a significant role in collective action, the redistribution of power toward incumbents facilitates collective action. This result is consistent with Olson's argument, because the driving force is that strong individuals have more incentives to contribute to collective action. By contrast, this result is sharply reversed when individuals' efforts are relatively complementary or the outsiders turn out to be an important person in generating collective action. Thus, in this case, a more equal distribution of power fosters collective action. In addition, the power distribution in a rival group affects the amount of collective action in a similar way.

Our paper is related to two strands of literature: collective action and contest literature. Katz, Nitzan, and Rosenberg (1990), Nitzan (1991), Katz and Tokatlidu (1996) Wärneryd (1998), Esteban and Ray (2001), Konrad (2004), Niou and Tan (2005), and Cheikbossian (2008), among many others, study the collective action problem in the environment of group contests. While most papers focus on issues related to contest design that maximizes overall contest effort, our paper addresses the issue about how heterogeneity within groups characterizes group competition.

The existing literature shares either or both of the following two assumptions. First, collective action is assumed to be merely a sum of each individual's effort. This assumption ignores any possibility of complementary effect in collective action.[49] We believe that there are a wide variety of situations in which collective action cannot be treated as the sum of individual members' effort. For example, Scully (1995) states "[p]layers interact with one another in team sports. The degree of interaction among player skills determines the nature of the production function."[50] Thus, we extend collective action from the additive functional form to a general CES function.

Second, these papers assume that each group consists of homogenous individual members who have equal power in internal conflict. Under this symmetry assumption, there is no unique solution for individual members' effort levels.[51] Because of this, we must suffer the loss of any link between intergroup and intragroup competition. On the other hand, in our model with heterogenous individuals, individual's incentive to contribute to collective action depends on their equilibrium power. This paper is the first attempt to relax these assumptions in the model of collective action and group contest.[52]

We believe that our model contributes to the understanding of the effect of within-group characteristics on intergroup competition. For example, our model can be adjusted to analyze how domestic politics affect international conflict [(Garfinkel (1994)], how the structure of interest groups affect lobbying competition, how profit-sharing rules of joint R&D ventures affect R&D race, and so forth. In this sense, this paper complements Garfinkel (2004a), who considers a domestic contest for power in order to redistribute future income within a country in the presence of external threat of terrorism.[53] While she studies the influence of external conflict on internal conflict, we analyze how the internal distribution of power within a group influences external competition against a rival group.

The remainder of the paper is organized as follows. Section 2 lays out basic features of the model. Section 3 characterizes the equilibrium for internal competition within groups and for external competition between groups. Then, in Section 4, we analyze the competition of collective action by the degree of complementarity and by the relative contributory role of group members. In Section 5, we study some possible extensions. Section 6 concludes.

3.2 Basic Model: Collective Action and Conflict Technologies

There are two groups, A and B, that compete for a prize whose value is given by R. Each group G consists of two individuals, $G1$ and $G2$, where $G = A, B$. The way the prize is allocated between the two groups depends on the relative collective efforts put forth by each group. A group's share of the prize is further contested by the members of each group. Thus, members of the same group have a common interest and cooperate in external competition against the rival group, but they are competitors against each other in the division of the spoils.

Each individual has I units of initial endowment to allocate among three different activities: contributing to collective activity for intergroup conflict, contesting a given share of the prize within groups, or earning income from other opportunities. An individual i in group A (B) allocates a_i (b_i) units of effort toward internal conflict and α_i (β_i) units of effort for collective action toward external conflict. Then this individual's earned income is $w(I - a_i - \alpha_i)$, where w denotes the wage rate from outside markets. We assume that w is sufficiently large to have an interior solution. All individuals make their decisions simultaneously.

Internal Competition. Individuals within a group are heterogeneous by ability or power, where power is defined in terms of advantage conferred in internal conflict. Without any loss of generality, we designate individual 1 of each group to be the one who has more power and thus has advantage in internal conflict. More specifically, this advantage is represented by the following conflict technology. Let $p(x_1, x_2)$ be the probability of individual 1 (the person in power) to win in the internal contest when x_1 and x_2 are the internal effort level exerted by individual 1 and 2, respectively. Alternatively, it can be interpreted as a share that individual 1 receives. Then, internal conflict is resolved by Tullock style contest. The contest success funtion in group G is given by

$$\begin{aligned} p(x_1, x_2; \theta_G) &= \frac{f(x_1)}{f(x_1) + \theta_G f(x_2)} = \frac{1}{1 + \tau_G}, \\ \text{where } f(x_i) &= x_i^m \text{ and } \tau_G = \theta_G \left(\frac{x_2}{x_1}\right)^m, \ \theta_G \in [0, 1] \text{ and } G = A, B. \end{aligned}$$

The parameter θ_G represents asymmetry in power distribution within group G, with a higher θ_G implying more even power distribution. One way to interpret this function is that individual 1 is the incumbent in power and plays the role of a defender whereas individual 2 is an outsider in their respective group and engages in offense. [Grossman and Kim (1995)] If the defender has an advantage, the internal conflict technology would exhibit an asymmetry as specified above. For instance, if $\theta_G = 1$, the power is evenly distributed between the incumbent and the outsider, whereas if $\theta_G = 0$, all the power in internal conflict is possessed by the incumbent with $p(x_1, x_2; \theta_G) = 1$. Without any loss of generality, assume $\theta_A < \theta_B$: the power is more asymmetrically distributed in group A than in group B. This implies that the incumbent in group A has relatively more power than the incumbent in group B vis-a-vis their respective outsiders. We will refer to an increase in θ_G as the dispersion of power and a decrease in θ_G as the concentration of power.

Collective Action. $F(y_1, y_2)$ will be referred as a production function that represents collective action of a group in external competition when the incumbent and the outsider contribute y_1 and y_2, respectively. The two groups have an identical production function. We allow the possibility that efforts by individual 1 and 2 are not perfect substitutes. We consider a general CES production function,

$$F(y_i, y_j) = [ky_i^r + (1-k)y^r]^{\frac{1}{r}},$$

where $F(y_i, y_j)$ is concave: $F_i(y_1, y_2) > 0$, $F_{ii}(y_1, y_2) < 0$, and $F_{ij}(y_1, y_2) > 0$, where $i, j = 1, 2$ and $i \neq j$. Collective action is increasing in each member's contribution, but at a diminishing rate. This production funtion is constant returns to scale. $r \in (-\infty, 1]$ represents complementarity between individuals' efforts.

External Competition. The external conflict technology is also assumed to be of the additive form. The crucial point here is that it depends on collective contributions by individual members of each group. Let $q(F(\alpha_1, \alpha_2), F(\beta_1, \beta_2))$ denote the probability that group A wins in external competition. Alternatively, we can interpret it as a share of the prize that group A will receive. We assume:

$$q(F(\alpha_1, \alpha_2), F(\beta_1, \beta_2)) = \frac{F(\alpha_1, \alpha_2)}{F(\alpha_1, \alpha_2) + F(\beta_1, \beta_2)}$$

To economize on notation, we will often use $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$ and $\boldsymbol{\beta} = (\beta_1, \beta_2)$. For instance, $q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta})) = \frac{F(\boldsymbol{\alpha})}{F(\boldsymbol{\alpha})+F(\boldsymbol{\beta})}$.

3.3 Equilibrium Analysis

3.3.1 Internal Conflict within Groups

The incumbent and outsider in group A maximize the objective functions represented by

$$\begin{aligned} V_{A1} &= p(a_1, a_2; \theta_A) q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))R + w(I - a_1 - \alpha_1) \\ V_{A2} &= [1 - p(a_1, a_2; \theta_A)] q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))R + w(I - a_2 - \alpha_2). \end{aligned}$$

Our formulation assumes that each individual makes a decision on his choice of effort in internal and external conflicts simultaneously if we interpret $q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))$ as the probability that group A wins in a winner-take-all external contest. However, if we take the alternative, non-probabilistic interpretation of $q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))$ as the share of A's contested resource, the timing does not matter; we will have the same result when we consider a two-stage competition in which the two groups compete for the prize first and then the intragroup competition takes places for the share of each group.

Similarly, the objective functions for the incumbent and the outsider in group B are given by

$$\begin{aligned} V_{B1} &= p(b_1, b_2; \theta_B)[1 - q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))]R + w(I - b_1 - \beta_1) \\ V_{B2} &= [1 - p(b_1, b_2; \theta_B)][1 - q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))]R + w(I - b_2 - \beta_2). \end{aligned}$$

We first derive an invariance result that the each member's winning probability (or each member's share) in their internal conflict is independent of the level of their contributions to external conflict $(\boldsymbol{\alpha}, \boldsymbol{\beta})$: it is constant and depends only on their respective group's power distribution parameter θ_G. This result considerably simplifies our analysis.

Lemma 3 *In equilibrium, both the incumbent and the outsider of group G choose the same level of efforts for internal conflict ($a_1^* = a_2^*$ and $b_1^* = b_2^*$) As a result, the winning probabilities for the incumbent and the outsider are constant and depend only on θ_G. More precisely, $p(a_1^*, a_2^*; \theta_A) = \frac{1}{1+\theta_A}$ and $p(b_1^*, b_2^*; \theta_B) = \frac{1}{1+\theta_B}$.*

All the proofs are in Appendix. To investigate the relationship between the resources dissipated in internal conflict and power distribution within each group, let us define $\lambda_A = \frac{a_1^*+a_2^*}{q(F(\alpha),F(\beta))R}$ and $\lambda_B = \frac{b_1^*+b_2^*}{[1-q(F(\alpha),F(\beta))]R}$. The denominator of λ_G represents the expected value of the collective prize for group G in the external conflict whereas the numerator of λ_G is the total resources used up in internal conflict. Thus, λ_G is the equilibrium rate of rent dissipation in internal conflict and measures the level of resources used up for internal conflict *relative* to the expected value of collective prize for group G. The next lemma shows that the group whose power is more evenly distributed dissipates proportionately more resources out of their respective collective prize in internal conflict. In this sense, the group with more even power distribution (group B) is more conflictive.[54]

Lemma 4 $\lambda_A < \lambda_B$. Group B is more conflictive than group A.

The severity of internal conflict within a group depends on the distribution of power across individual members. As individual members have similar power, they compete more aggressively.

3.3.2 External Conflict between Groups

Now let us study how the intergroup contest is shaped by the intensity of internal competition or the distribution of power within each group. Power should not be necessarily interpreted as political power or competitive advantage in intragroup competition. Note that the role of power disparity in this model is nothing but the division rule of the prize. In this sense, one can think of our study as how an unequal sharing rule affects collective action. With the invariance result we derived earlier, we can now state each individual's objective function in relation to their contribution to external conflict can be written as follows for group A members. For notation simplicity, we denote the equilibrium winning probability by $p(\theta_G)$.

$$\begin{aligned} V_{A1} &= p(\theta_A)q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))R + w(I - a_1^* - \alpha_1) \\ V_{A2} &= (1 - p(\theta_A))q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))R + w(I - a_2^* - \alpha_2). \end{aligned}$$

For external conflict, individual i in group A maximizes his payoff function V_{Ai} by choosing α_i, where $i = 1, 2$, given that all individuals will act optimally. We can derive similar conditions for group B members who choose β_i. The first-order conditions can be expressed as

$$\frac{F_1(\boldsymbol{\alpha})F(\boldsymbol{\beta})}{[F(\boldsymbol{\alpha}) + F(\boldsymbol{\beta})]^2}\frac{R}{w} = \frac{1}{p(\theta_A)} = (1 + \theta_A), \tag{3.1}$$

$$\frac{F_2(\boldsymbol{\alpha})F(\boldsymbol{\beta})}{[F(\boldsymbol{\alpha}) + F(\boldsymbol{\beta})]^2}\frac{R}{w} = \frac{1}{1 - p(\theta_A)} = (\frac{1 + \theta_A}{\theta_A}), \tag{3.2}$$

$$\frac{F(\boldsymbol{\alpha})F_1(\boldsymbol{\beta})}{[F(\boldsymbol{\alpha}) + F(\boldsymbol{\beta})]^2}\frac{R}{w} = \frac{1}{p(\theta_B)} = (1 + \theta_B), \text{ and} \tag{3.3}$$

$$\frac{F(\boldsymbol{\alpha})F_2(\boldsymbol{\beta})}{[F(\boldsymbol{\alpha}) + F(\boldsymbol{\beta})]^2}\frac{R}{w} = \frac{1}{1 - p(\theta_B)} = (\frac{1 + \theta_B}{\theta_B}). \tag{3.4}$$

They can be further manipulated and summarized in the following way.

$$
\begin{aligned}
\frac{F_1(\boldsymbol{\alpha})}{F_2(\boldsymbol{\alpha})} &= \frac{1-p(\theta_A)}{p(\theta_A)} = \theta_A, &(3.5)\\
\frac{F_1(\boldsymbol{\beta})}{F_2(\boldsymbol{\beta})} &= \frac{1-p(\theta_B)}{p(\theta_B)} = \theta_B, &(3.6)\\
\frac{F_1(\boldsymbol{\alpha})}{F_1(\boldsymbol{\beta})}\frac{F(\boldsymbol{\beta})}{F(\boldsymbol{\alpha})} &= \frac{p(\theta_B)}{p(\theta_A)} = \left(\frac{1+\theta_A}{1+\theta_B}\right), \text{ and} &(3.7)\\
\frac{F_2(\boldsymbol{\alpha})}{F_2(\boldsymbol{\beta})}\frac{F(\boldsymbol{\beta})}{F(\boldsymbol{\alpha})} &= \frac{1-p(\theta_B)}{1-p(\theta_A)} = \left(\frac{\theta_B}{\theta_A}\right)\left(\frac{1+\theta_A}{1+\theta_B}\right) &(3.8)
\end{aligned}
$$

Equations (3.5) and (3.6) tell us the relationship between the *marginal* contributions of the incumbent and the outsider in the generation of collective action in each group. In each group, the outsider's marginal contribution to the collective action is greater than the incumbent's in equilibrium. This is because the outsider with less internal power is expected to receive a smaller share of the prize in external competition.

This asymmetry in the relative marginal contributions of the incumbent and the outsider translates into the asymmetry in the relative total contributions. Each member's incentive to contribute to collective action depends on one's equilibrium power in the internal conflict. The incumbent's contribution relative to the outsider's is greater in the less conflictive group A.

Proposition 17 $\frac{\alpha_1^*}{\alpha_2^*} > \frac{\beta_1^*}{\beta_2^*}$ *when* $\theta_A < \theta_B$. *This implies that the incumbent's relative contribution to external conflict vis-a-vis the outsider's is higher in group* A *where power is more asymmetrically distributed.*

One important implication of this result is that the two groups exhibit different patterns of inefficiency. First of all, the generation of collective action in each group is inefficient, because efficiency requires that an individual is compensated with full marginal return of one's effort. Now, the inefficiency in terms of the outsider is more pronounced for group A in which the internal power distribution is more asymmetric. Likewise, the inefficiency in terms of the incumbent is more pronounced for group B.

3.4 The Properties of Collective Action

A basic, but unanswered, question is which group has a higher winning probability in external competition. We can answer this question by figuring out whether $\frac{F(\alpha_1^*,\alpha_2^*)}{F(\beta_1^*,\beta_2^*)}$ is greater than 1 or not. This is equivalent to whether $q(F(\boldsymbol{\alpha}), F(\boldsymbol{\beta}))$ is greather than 1/2 or not. Equations (3.7) and (3.8) together result in

$$\frac{F(\alpha_1^*,\alpha_2^*)}{F(\beta_1^*,\beta_2^*)} = \left(\frac{1+\theta_B}{1+\theta_A}\right)\left(\frac{\theta_A}{\theta_B-\theta_A}\right)\left[\frac{F_2(\alpha_1^*,\alpha_2^*)}{F_2(\beta_1^*,\beta_2^*)} - \frac{F_1(\alpha_1^*,\alpha_2^*)}{F_1(\beta_1^*,\beta_2^*)}\right]. \tag{3.9}$$

This shows that the answer hinges on the ratio of marginal contributions between incumbents or outsiders in equilibrium. Thus, the way in which collective action is generated through individual contributions is crucial to predict which group will be winning. In addition, it is worthwhile to study how each group's winning probability is changed by the distribution of power within a group.

3.4.1 Complementarity between Individual Contributions

An important factor in collective action, which has been neglected in the literature, is a possible complementarity between individual members' contributions. This can be measured by r in our CES function. As is well-known, the elasticity of substitution is

$$\frac{d\ln(y_2/y_1)}{d\ln MRS} = \frac{1}{1-r},$$

which is a measure of complementarity or substitutability between individual members' contributions. As r increases, the incumbent and outsider's contributions become less complementary (more substitutable).[55] To focus on the effect of complementarity, we assume $F(y_i, y_j)$ is symmetric at $y_i = y_j$ by setting $k = 1/2$.

Proposition 18 *When* $F(y_i, y_j) = (y_i^r + y_j^r)^{\frac{1}{r}}$*, the ratio of collective action between the two groups is given by*

$$\frac{F(\alpha_1^*, \alpha_2^*)}{F(\beta_1^*, \beta_2^*)} = \left[\frac{p(\theta_A)^\rho + (1-p(\theta_A))^\rho}{p(\theta_B)^\rho + (1-p(\theta_B))^\rho}\right]^{\frac{1}{\rho}}, \text{ where } \rho = \frac{r}{1-r}.$$

$F(\alpha_1^*, \alpha_2^*) \gtreqless F(\beta_1^*, \beta_2^*)$ *as* $r \gtreqless 1/2$. *If the individuals' contributions is relatively complementary in the generation of collective action, the winning probability of more conflictive group is greater, and vice versa.*

At first sight, the result appears to be counter-intuitive. When collective action requires complementary efforts, the individuals in the more conflictive group contribute to collective action more than in the less conflictive group. People are more likely to believe that competition harms cooperation. However, in our model, we show that competition coexists with cooperation in harmony.

Interestingly, a solution for $\frac{F(\alpha_1^*,\alpha_2^*)}{F(\beta_1^*,\beta_2^*)}$ provides us with a similar functional form of the CES function. We will see the similar pattern from the Cobb-douglas function below. This enables us to conduct comparative statics in terms of power distribution to study whether the internal redistribution of power increase or decrease the group's winning probability. We obtain $\frac{\partial}{\partial p(\theta_A)} \frac{F(\alpha_1^*,\alpha_2^*)}{F(\beta_1^*,\beta_2^*)} \gtreqless 0$ and $\frac{\partial}{\partial p(\theta_B)} \frac{F(\alpha_1^*,\alpha_2^*)}{F(\beta_1^*,\beta_2^*)} \lesseqgtr 0$ as $r \gtreqless 1/2$. The dispersion of power inequality increases (decreases) the group's probability of winning when the individuals' contributions is relatively substitutable (complementary).

First of all, this result confirms the intuition of Olson (1965) in a general setting. He argues that more inequality can facilitate collective action when collective action is defined as the sum of individuals' efforts ($r = 1$). This result is valid to the extent that r is greater than $1/2$. In contrast, it should be also emphasized that the result can be sharply reversed if the individuals' contribution is relatively complementary as the case of $r < 1/2$. In addition, a change in power distribution in the other group affects the group's probability of winning in the opposite way.

The following examples are two extreme cases to make our arguments clearer.

Example. *Suppose $F(y_i, y_j) = y_i + y_j$. Cooperation is performed by the sum of individuals' efforts. In this case, outsiders are completely free-riding in producing collective action.*

$$\alpha_1^* = \frac{p(\theta_A)^2 p(\theta_B)}{[p(\theta_A) + p(\theta_B)]^2} \frac{R}{w} > \frac{p(\theta_A) p(\theta_B)^2}{[p(\theta_A) + p(\theta_B)]^2} \frac{R}{w} = \beta_1^* \text{ and } \alpha_2^* = \beta_2^* = 0.$$

Note the winning probability in the external conflict depends only on the incumbents' effort level. Thus the less conflictive group's winning probability is always greater.

Example. *Suppose $F(y_i, y_j) = \min(y_i, y_j)$. This case represents that a minimum effort between group members establishes the level of collective action. In fact, it is well-known that there are multiple equilibria, but we focus on the most efficient outcome. Then, we obtain*

$$\alpha_1^* = \alpha_2^* = \frac{[1 - p(\theta_A)]^2 [1 - p(\theta_B)]}{[1 - p(\theta_A) + 1 - p(\theta_B)]^2} \frac{R}{w} < \frac{[1 - p(\theta_A)] [1 - p(\theta_B)]^2}{[1 - p(\theta_A) + 1 - p(\theta_B)]^2} \frac{R}{w} = \beta_1^* = \beta_2^*.$$

Contrary to example 1, collective action is virtually determined by the outsider, because the incumbents merely make the same effort as much as the outsiders. In this case, the more conflictive group's winning probability is always greater.

3.4.2 Relative Role between Individual Contributions

Another typical assumption in defining collective action in the literature of group contest is that the way every group member contributes to collective action is identical. However, one can take on a more important role than other members. In our CES function, $k/(1-k)$ can be interpreted as the relative importance of the incumbent's contribution vis-a-vis the outsider's contribution in producing collective action. We define the marginal rate of substitution between individual contributions as the ratio of the marginal productivity of the incumbent's contribution and that of the outsider's contribution, as in the standard production theory.

$$MRS(y_1, y_2) = \frac{F_1(y_1, y_2)}{F_2(y_1, y_2)} = \frac{k}{1-k}\left(\frac{y_1}{y_2}\right)^{r-1}.$$

As $k/(1-k)$ increases, the incumbent's contribution becomes more important in collective action. To focus on the effect of asymmetry in the roles of the incumbents' and the outsiders' contributions, we consider the case $r = 0$. That is, we assume that the production function is a Cobb-Douglas function such as $F(y_i, y_j) = y_i^k y_j^{1-k}$.

Proposition 19 *When $F(y_i, y_j) = y_i^k y_j^{1-k}$, the ratio of collective action between the two groups is given by*

$$\frac{F(\alpha_1^*, \alpha_2^*)}{F(\beta_1^*, \beta_2^*)} = \frac{p(\theta_A)^k(1-p(\theta_A))^{1-k}}{p(\theta_B)^k(1-p(\theta_B))^{1-k}} = \left(\frac{1+\theta_B}{1+\theta_A}\right)\left(\frac{\theta_A}{\theta_B}\right)^{1-k}.$$

$F(\alpha_1^*, \alpha_2^*) \gtreqless F(\beta_1^*, \beta_2^*)$ *as* $k \gtreqless \widehat{k}$, *where* $\widehat{k} = \log_{\left(\frac{\theta_B}{\theta_A}\right)}\left(\frac{1+\theta_B}{1+\theta_A}\right)$. *If the incumbents' contribution is relatively more important in the generation of collective action, the winning probability of less conflictive group is greater, and vice versa.*

When the relative role of incumbent over outsider in the production function is over the threshold, group A's winning probability is greater. In other words, when the incumbent is considerably responsible for collective action, the less conflictive group has an advantage. On the contrary, when the outsider's contribution is relatively important to generate collective action, group B's winning probability is greater.

We also obtain $\frac{\partial}{\partial p(\theta_A)}\frac{F(\alpha_1^*,\alpha_2^*)}{F(\beta_1^*,\beta_2^*)} \gtreqless 0$ as $\frac{p(\theta_A)}{1-p(\theta_A)} \lesseqgtr \frac{k}{1-k}$. Each group's winning probability is maximized at $\frac{p(\theta_G)}{1-p(\theta_G)} = \frac{k}{1-k}$. This result is also quite intuitive. When the production function requires the relative importance of the incumbent's contribution vis-a-vis the outsider's contribution with the ratio of $\frac{k}{1-k}$, a group can maximize its winning probability when the incumbent's relative share of the prize to the outsider is the same ratio.

3.5 Discussion and Extensions

3.5.1 Competition between species: coexistence or extinction?

One interesting example is interspecific and intraspecific competition in ecology. [(Vandermeer, 1975)] Competition within and between species arises for a limited amount of resources such as space, food, or mates. One important issue in the literature is to explain when different species can coexist or when one species becomes extinct. Let us interpret the prize R in our model as the given space for which two species are competing. Then, as a result of competition, each individual species occupy a portion of the space by $\frac{1}{1+\theta_A}q(F(\boldsymbol{\alpha}^*), F(\boldsymbol{\beta}^*))$, $\frac{\theta_A}{1+\theta_A}q(F(\boldsymbol{\alpha}^*), F(\boldsymbol{\beta}^*))$, $\frac{1}{1+\theta_B}(1 - q(F(\boldsymbol{\alpha}^*), F(\boldsymbol{\beta}^*)))$, and $\frac{\theta_B}{1+\theta_B}(1 - q(F(\boldsymbol{\alpha}^*), F(\boldsymbol{\beta}^*)))$.

Now, suppose that there is a minimum size of space for survival. Then we can predict that the extinction of one species arises when $q(F(\boldsymbol{\alpha}^*), F(\boldsymbol{\beta}^*))$ is sufficiently small or large. That is to say, the outcome of interspecific competition should be extreme to the extent that one of species can occupy a minimal space. We can have this situation when two groups are very heterogeneous in the sense that θ_B is sufficiently greater than θ_A, and the individual species efforts are either almost perfectly substitutable or complementary. Otherwise, two different species may be able to coexist while each individual residing in an enough space. For the case of coexistence, we can predict two different situations. If both θ_A and θ_B are relatively large, every individual may coexist. However, if both θ_A and θ_B are considerably small, only the superior individual in each species can survive and coexist.

3.5.2 Management of Internal Conflict

Until now, we have been assuming that the prize is distributed within groups entirely by internal conflict. However, group members may be able to make binding commitments to share a portion of the prize on egalitarian grounds. Then, the adjusted conflict technology is given by

$$p(x_1, x_2) = \frac{\phi_G}{2} + (1 - \phi_G)\frac{x_1}{x_1 + \theta_G x_2}, \text{ where } G = A, B.$$

ϕ_G represents the effectiveness of conflict management within a group. In other words, each group does not have to have internal conflict for this portion of the prize. On the other hand, they still have to contest for the other portion, $(1 - \phi_G)$, of the prize.

Another interesting interpretation is that a team organizer can control internal conflict in a way that the incumbent and outsider share a portion of the prize equally. In particular, while ϕ_G can be thought of as the portion of team rewards, $(1 - \phi_G)$ as the portion of the competitive prize.

The share of the prize of the incumbent in each group is respectively

$$p(\theta_G) = \frac{\phi_G}{2} + (1 - \phi_G)\frac{1}{1 + \theta_G}.$$

The winning probability of the incumbent is decreasing in ϕ_G, while the winning probability of the outsider is increasing in ϕ_G, i.e., $\frac{\partial p(\theta_G)}{\partial \phi_G} < 0$ and $\frac{\partial [1 - p(\theta_G)]}{\partial \phi_G} > 0$. Then, what would be the commitment level ϕ_G^* to maximize the group's winning probability?

Given the symmetric CES function, we obtain $\phi_G^* = \begin{cases} 0 & \text{if } r>1/2 \\ 1 & r<1/2 \end{cases}$. If group members' contributions are relatively complementary, higher ϕ_G increases the group's winning probability, and vice versa. Therefore, the team organizer wants to distribute the prize equally within a team if collective action is relatively complementary. On the other hand, he prefers to allow group members to fight for the prize, otherwise.

3.5.3 Group Size Paradox

Another celebrated argument of Olson (1965) is that the free-rider problem makes the smaller group more effective. It is well-known that this result holds under two assumptions. One is that collective action is additive, and the other is that individuals are homogeneous. Then, in a smaller group, individuals can have a larger share of the prize and the free-rider problem is weaker. These are the reason why the smaller group could be better in facilitating collective action.

However, in our model in which individuals are heterogeneous, this argument regarding the size of the group is almost futile. When individuals' efforts are perfectly substitutable, the only most powerful individual within a group contributes to collective action. Thus, regardless of the size of groups, a comparison between the most powerful individuals across groups can tell us which group is more effective in intergroup competition. Likewise, when individuals' efforts are perfectly complementary, the least powerful individual determine the extent of collective action within a group.

3.6 Concluding Remarks

We have developed the model of collective action in which intergroup and intragroup competition interplay with each other. This interaction turns out to be important in answering what group is more active in collective action and how the power distribution in each group affects the winning probability. We conclude with discussing potential worthwhile extensions based on some limitations of our model.

First, our analysis suggests that the heterogeneity of individuals will be important factors to study the endogenous formation of groups. Most papers that study the group formation deal with homogeneous individuals and focus on finding a stable formation. On the other hand, when individuals are heterogeneous, another interesting problem will be to find matching patterns.

Second, the two groups share the same production function in our model, but this is not necessary in many cases. It will be interesting to analyze the competition between groups with different production functions. Recently Clark and Konrad (2007) study the case where an attacker has the best-shot function and a defender has the weakest-link function. It would be worthwhile to extend their model to be a group contest.

Last, in our model, the groups compete for a fixed prize. It will be interesting to study the case where the prize is endogenously determined. In other words, we can consider a situation where individuals allocate their resources between productive activity and conflictual activity. Then, the prize will be each other's outcome of production.

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Notes

[1]This feature of the localized competition is more than a technical issue. Many marketing decisions try to target marginal consumers who will change their purchase decision rather than to satisfy all consumers. Therefore, the number of marginal consumers is central to the choice of marketing strategies.

[2]The MLRP is widely used in the literature of contract theory and auction theory. Its important role is ensuring the optimal compensation scheme to be monotonically increasing and allowing comparison between bidding prices across different bidders or expected revenues across various types of auction. Likewise, the smooth dispersion by MLRP enables us to have a monotonic change in equilibrium prices.

[3]It is helpful to compare two extreme cases. If all consumers are indifferent between two products, we have a degenerate distribution which is localized at the middle point in the Hotelling line. In this case, products are homogenous and the market is perfectly competitive. By contrast, if consumers have a significantly strong preference for one good over another, we can have a two-point distribution such that consumers are located only at two endpoints. Then, competition disappears and firms are able to set monopoly prices for their loyal consumers. Therefore, one can hypothesize that any intermediate level of distribution of consumer preferences may represent an intermediate degree of competition between monopoly and perfect competition.

[4]As is well-known, equilibrium prices are increasing in transportation costs. In particular, when transportation costs are zero, the model exhibits perfect competition between homogeneous goods. On the contrary, when it is significantly high, two firms become local monopolists.

[5]Hotelling (1929)'s location choice model has been extended in various ways. For example, multi-characteristics space in Irmen and Thisse (1998), income heterogeneity in Peitz (1999), and uncertainty of consumer tastes in Meagher and Zauner (2005) are interesting extensions.

[6]If $\theta_A = \theta_B$, the model becomes a vertical differentiation model. In contrast, if the sum of θ_A and θ_B is a constant, the model turns out to be a horizontal differentiation model.

[7]In a typical Hotelling model, consumers lying at the middle of the Hotelliing line have the lowest willingness to pay. However, in our model, they are people who are perfectly indifferent between the two products regardless of their willingness to pay.

[8]Assumption 1 also ensures upward reaction functions as well, i.e., $\frac{\partial^2 \pi_i}{\partial p_j \partial p_i} \geq 0$. That is to say, the two goods are strategic complements.

[9]Assumption 2 can be written, equivalently, as $\frac{\partial}{\partial k} \int_{\underline{\theta}}^{\theta} F_k(x)dx \geq 0$.

[10]Equivalently, Assumption 3 can be written as $\frac{\partial}{\partial k}\left(\frac{f_k(\theta_1)}{f_k(\theta_0)}\right) \geq 0$ for $\theta_0 \leq \theta_1 \in [0, \overline{\theta}]$, and $\frac{\partial}{\partial k}\left(\frac{f_k(\theta_1)}{f_k(\theta_0)}\right) \leq 0$ for $\theta_0 \leq \theta_1 \in [\underline{\theta}, 0]$.

[11]In a typical Hotelling model, the size of product differentiation is normalized as a unit, and transportation costs are measured by a parameter. On the other hand, our model can be thought of as normalizing transportation costs as a unit, but denotes the size of product differentiation by $\overline{\theta}$. Anyway, both ways are equivalent.

[12]Alternatively, we can think of product innovations which improve the quality of products. All the results below will be equivalent.

[13]Consequently, each firm has an equal chance of winning R&D at the symmetric equilibrium, i.e. $p(I_A, I_B) = \frac{1}{2}$.

[14]In fact, there are some other ways to study the effect of market competition on R&D incentives. A typical way is considering the number of firms in a market. For example, Loury (1979) and Lee and Wilde (1980) study how the equilibrium level of investment is changed by the number of firms in a market. Several papers compare Bertrand competition to Cournot competition because the Bertrand model yields more competitive market outcome than the Cournot model does. For example, Delbono and Denicolo (1990), Bester and Petrakis (1993), and Bonanno and Haworth (1998) compares incentives for innovation between Cournot and Bertrand

competition.

[15]For example, see Anderson and Leruth (1993), Bester and Petrakis (1996), Corts (1996), Chen (1997), Fudenberg and Tirole (2000), and Armstrong (2006).

[16]One justification for this situation is that firm A may be a new entrant, or may employ a new technology.

[17]Under Cournot competition where reaction functions are downward sloping, information sharing allows firms to compete on farther away fronts in the sense that the low cost type firm produces more and the high cost type firm produces less. This is why Gal-Or (1986) finds that information sharing is a dominant strategy with Cournot competition.

[18]Promotion is one of the most common form of pay for performance in most organizations. Likewise, more than 70 percent of U.S. major companies are adopting some type of team-based rewards. Prendergast (1999) surveys the literature on these incentive schemes.

[19]For example, Holmstrom (1982) defines team as a group of individuals who are organized so that their productive inputs are related."

[20]Recently, several authors study and propose a nonexclusive patent system. This paper provides a significant support to this literature. I will explain more in detail in Section 5.

[21]In the marketing literature, it is often said that competition dampens team spirit or cooperation between team members. However, my model incorporates neither team production nor cooperation, and the agents are conducting their own tasks independently. In addition, keep in mind that competition raises the discouragement effect in the marginal sense. An increase in competitive compensation boosts agents' effort always in our model, but the increase in effort is decreasing. Therefore, precisely speaking, competition stifles marginal team performance.

[22]This is a sufficient condition for the following stability condition, $\left(\frac{\partial^2 V_A}{\partial I_A^2}\right)\left(\frac{\partial^2 V_B}{\partial I_B^2}\right) >$

$\left(\frac{\partial^2 V_A}{\partial I_B \partial I_A}\right)\left(\frac{\partial^2 V_B}{\partial I_A \partial I_B}\right)$. That is, the slope of agent A's reaction curve is greater than that of agent B's.

[23]Bulow et al. (1985) define strategic substitutes and strategic complements by whether a more aggressive strategy by one player lowers or raises the other player's "marginal" payoffs.

[24]The symmetry of F then implies that the reverse hazard rate $\frac{f(\theta)}{F(\theta)}$ is strictly decreasing in θ. These assumptions ensure nice demand curves for both firms so that the second-order conditions are always satisfied.

[25]The analysis of the equilibrium in the Hotelling model with non-uniform distributions is studied by my working paper, Kim (2008).

[26]Nonetheless, the direct and positive effect is always greater than the negative effect because the probability functions are increasing.

[27]All the previous results hold with these modifications.

[28]Suppose that the agents undertake independent or complementary tasks. According to the analysis below, in equilibrium, the proportion of team incentives must be greater than the current case.

[29]One concern regarding this assumption might be that duplication costs are unavoidable when multiple agents work on the identical and perfectly substitutable tasks. Hence, one may think that there is a possibility that principal prefers to contract with only one agent. However, according to the informativeness principle developed by Holmström (1979), the principal should include any measure of performance that reveals information about the agent's choice of effort level. Thus, the contract with multiple agents dominates that with a single agent, because the output of one agent is correlated with the action of the other agent.

[30]As is well-known, the superiority of RPE is that it exposes agents to less risk by filtering out the common shock. In addition, as σ increases, the principal's preference to RPE is more skewed in a sense that the proportion of competitive compensation

relative to team rewards should be greater.

[31]In addition, if agents are risk neutral and there is no common shock ($\sigma = 0$), the isocost line coincides with the slope of the IC constraint. Hence, the principal is indifferent between any combination of (v^S, v^W). Substituting the constraint into the objective function, the reduced problem is maximizing $p(I^*)(2 - p(I^*))R - 2\frac{p(I^*)}{p'(I^*)}$. It becomes the unconstrained maximization problem. This means that $\widetilde{I}^*$ can be induced by any combination of (v^S, v^W) which satisfies the IC constraint, where $I^* = \widetilde{I}^*$.

[32]This assumption abstract from the self-enforcing mechanism to collude with each other. Though this is rather strong, many papers such as Tirole (1986), Holmström and Milgrom (1990) and Itoh (1993) make the full-side contract assumption. As one justification, Itoh (1993) states "[I] make the full-side-contract assumption that agents can costlessly write side contracts based on information commonly observable to them. These side contracts may not be enforceable explicitly but instead implicitly through promises that are self-enforcing within the group. This assumption is clearly extreme. However, it also appears extreme to assume that no promise can be honored. In fact, there is experimental evidence that people have 'words of honor'."

[33]Otherwise, as pointed out by Varian (1990), a side contract merely adds constraints to the principal's problem which cannot make the principal better off.

[34]On the other hand, collusion makes no difference under IPE. In this sense, offering $v^S = v^W$ is collusion-proof in that IPE does not give rise to a side contract between the agents. In other words, the principal's cost incurred by the agents' collusion under RPE is that the principal has to choose the IPE scheme.

[35]There are a few papers which also show that collusion between agents can improve the principal's welfare. In Holmström and Milgrom (1990), collusion reduces agents' total risk exposures by risk sharing. In Varian (1990), collusion increases agents' peer monitoring through information sharing. Itoh (1993) is very similar to this paper.

He argues that the principal is better off under collusion not by mutual risk sharing, but by monitoring and coordinating each other's effort. However, the paper does not study the principal's wage scheme explicitly, and it is not emphasized enough that JPE is a sufficient condition for the principal being better off. In addition, the logic and mechanism to reach the result are quite different in this paper. I highlight that the agents' perception of complementarity in tasks leads to pareto-improving collusion under JPE.

[36]Che and Yoo (2001) argue that the cost of using the RPE scheme rises when the players are allowed to collude, while that of the JPE scheme remains unchanged. I come to the same result for RPE, but the players' collusion makes the principal better off under JPE.

[37]When $\sigma = 0$, the JPE scheme is always optimal. But, as σ rises, the region in which the JPE scheme is preferred shrinks down. The reason is that the merit of RPE, which is filtering out the common shock, becomes greater as σ rises.

[38]Although the effect of precommitment or Stackelberg game is considerably studied in the field of industrial organization, it has not been paid attention to in the principal-agent model with multiple agents. The reason might be that the most papers adopt the framework in which agents are allowed to make a discrete effort decision, particularly, between only binary actions.

[39]See Gal-Or (1985) for the first-mover or second-mover advantage in an oligopoly model.

[40]In this sense, the offensive game can be thought of as a race, while the defensive game can be thought of as a waiting game.

[41]For the game between the offensive player and defensive player, we obtain $V_O^1 > V_O^1 > V_O^2$, $V_D^2 > V_D^N$ and $V_D^1 > V_D^N$. In this case, the offensive player still has the dominant strategy, which is choosing the first. Given the offensive player's choice, the defensive player chooses the second. Recall, in Example 4 above, the incumbent

firm plays defensively, while the entrant plays offensively. If we apply our finding to this example, the entrant starts to develop innovation first and then the incumbent follows. Indeed, we often observe that new technologies are announced by entrants. Similarly, we can predict who brings the lawsuit in litigations and who starts early on campaign trail in political elections. The offensive players do.

[42]One way of modifying our model to have the winner-take-all feature is simply to set $v^S = v^W/2$. When both agents succeed, each can receive the winner's prize with equal probability. In this case, equation (2.7) implies that the winner-take-all contest is always dominated by the case where the principal can offer team incentives.

[43]Maurer and Scotchmer (2002) attempt to answer this question. In their model, they assume that a new innovation can be developed by investing the fixed costs. They show that a possibility of duplication does not jeopardize the first inventor's ability to recover R&D costs as long as the costs of duplication are not substantially low.

[44]In addition, our discussion of strategic interactions also suggests that collusion among competing firms in the R&D stage can increase firms' investment if multiple patentees' competition profit is greater than a single patentee's monopoly profit. This JPE type incentive scheme can be designed by setting patent length for multiple inventors far longer than that for a single inventor. When a technology is easy to develop or it creates a huge revenue, we can consider this JPE type patent length.

[45]According to the literature, the joint liability solves the adverse selection problem by peer monitoring (Varian; 1990), self-selection (Ghatak; 1999), and screening mechanism (Ghatak; 2000). For example, when borrowers know each other's characteristic, the voluntary group formation results in groupings of the same type of borrowers under the joint liability. Based on this self-selection, the bank is able to screen a group of safe and/or capable borrowers by offering a contract with low interest rates and high joint liability.

[46]I assume that two borrowers in a group are identical. Indeed, the literature found that self-selection leads to that the same type of borrowers are matched when they know the characteristics of each other.

[47]In fact, the typical form of joint liability is denying future loans to all group members if a group member is default. Thus Ghatack and Guinnane (1999) justify the model in the way of that "this c can be interpreted as the net present discounted value of the cost of sacrificing present consumption in order to pay joint liability for a partner."

[48]The study of collective action has long and extensive history followed by Olson (1965). See Sandler (1992), Ostrom (2000) and Sandler and Hartley (2001) for the literature review.

[49]In fact, in the early literature of voluntary contributions to a public good, Hirshleifer (1983) studies the case that the aggregate effort level can be the smallest contribution within a group, which is assuming the perfect complements between individuals' efforts. The paper acknowledges a possible complementary effect in collective action. However, the effect of complementary efforts between group members has not been thoroughly studied in a model of group contests.

[50]Borland (2007) also suggests that while the production function in baseball is nearly additive in the sense that hitting and pitching are separate activities, players' efforts are almost perfect complements in american football.

[51]One exception is Baik (2008). He studies the case where individuals have different valuations for the public-good prize and shows that only the highest-valuation players expend positive effort and the rest expend zero effort. This is equivalent to Example 1 below.

[52]In fact, the excellent survey paper about economics of conflict, Garfinkel and Skaperdas (2006) suggests this way of study as further research topic. They state in page 52 "[I]n addition, the literature has paid scant attention to asymmetries. Yet,

the heterogeneity of individuals raises some important and interesting issue about the composition of alliances and about resolving conflicts therein, provided that a stable structure exists at all."

[53]She derives conditions under which an increased threat of terrorism reduces the expected value of the contest prize and thus lessens the severity of the domestic conflict. One limitation is that her model treats the external threat as an *exogenous* shock and conducts a comparative statics analysis of the effect of a heightened external threat on domestic conflict. In contrast, the level of external threat arises *endogenously* in our model as a result of contest between groups.

[54]This does not mean that members in group B spend more resource for internal conflict. Since the total efforts depend on the size of contestable prize, people in group A may expend more efforts if group A's winning probability is much larger in the external competition.

[55]While we obtain the linear (perfect substitutes) function as r approaches 1, we obtain the Leontief (perfect complements) function as r approaches $-\infty$. The Cobb-douglas function is also a special case of $r = 0$.

Appendix A

Proofs of Proposition and Lemmas

A.1 Appendix of Chapter 1

The proof of Proposition 1.

For $\theta \in [0, \overline{\theta}]$, the definition of MLRP gives us

$$f_{k+1}(\theta_0) f_k(\theta_1) \leq f_{k+1}(\theta_1) f_k(\theta_0). \tag{A.1}$$

Integrating both sides over θ_0 from 0 to θ_1, we get

$$\left(F_{k+1}(\theta_1) - \frac{1}{2}\right) f_k(\theta_1) \leq f_{k+1}(\theta_1) \left(F_k(\theta_1) - \frac{1}{2}\right).$$

This inequality can be rewritten as

$$\frac{1 - 2F_k(\theta)}{f_k(\theta)} \leq \frac{1 - 2F_{k+1}(\theta)}{f_{k+1}(\theta)}.$$

Similarly, integrating both sides in (A.1) over θ_1 from θ_0 to $\overline{\theta}$, we obtain

$$f_{k+1}(\theta_0)\left(1 - F_k(\theta_0)\right) \leq \left(1 - F_{k+1}(\theta_0)\right) f_k(\theta_0).$$

This inequality is called hazard rate dominance,

$$\frac{1-F_k(\theta)}{f_k(\theta)} < \frac{1-F_{k+1}(\theta)}{f_{k+1}(\theta)}.$$

Let us define the hazard rate of F by $\mu_k(\theta) \equiv \frac{f_k(\theta)}{1-F(\theta)}$. If we write $-\mu_k(\theta) = \frac{d}{d\theta}\ln(1-F(\theta))$, then the distribution function can be written as $F_k(\theta) = 1 - \exp(-\int_0^{\theta}\mu_k(x)dx)$. It is straightforward to show the first order stochastic dominance as follows.

$$F_{k+1}(\theta) = 1 - \exp(-\int_0^{\theta}\mu_{k+1}(x)dx) \leq 1 - \exp(-\int_0^{\theta}\mu_k(x)dx) = F_k(\theta).$$

In the same way, we can show the corresponding stochastic orders for $\theta \in [\underline{\theta}, 0]$.

The proof of Proposition 3.

The proof is given in the following two steps.

Lemma 3.1 *As long as p_A rises, p_B has to increase.*

Firm A and B's reaction functions are given by $p_A(p_B) = c_A + \frac{F_k(\theta)}{f_k(\theta)}$ and $p_B(p_A) = c_B + \frac{1-F_k(\theta)}{f_k(\theta)}$ respectively. I perform a comparative static analysis by totally differentiating two reaction functions. Using the matrix form, I summarize it as

$$\begin{bmatrix} 1+\frac{\partial}{\partial\theta}\frac{F_k(\theta)}{f_k(\theta)} & -\frac{\partial}{\partial\theta}\frac{F_k(\theta)}{f_k(\theta)} \\ \frac{\partial}{\partial\theta}\frac{1-F_k(\theta)}{f_k(\theta)} & 1-\frac{\partial}{\partial\theta}\frac{1-F_k(\theta)}{f_k(\theta)} \end{bmatrix} \begin{bmatrix} \frac{dp_A}{dk} \\ \frac{dp_B}{dk} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial k}\frac{F_k(\theta)}{f_k(\theta)} \\ \frac{\partial}{\partial k}\frac{1-F_k(\theta)}{f_k(\theta)} \end{bmatrix}.$$

By using Cramer's rule, the effects of a MPS on p_A and p_B are represented by

$$\frac{dp_A}{dk} = \frac{\begin{vmatrix} \frac{\partial}{\partial k}\frac{F_k(\theta)}{f_k(\theta)} & -\frac{\partial}{\partial \theta}\frac{F_k(\theta)}{f_k(\theta)} \\ \frac{\partial}{\partial k}\frac{1-F_k(\theta)}{f_k(\theta)} & 1-\frac{\partial}{\partial \theta}\frac{1-F_k(\theta)}{f_k(\theta)} \end{vmatrix}}{\begin{vmatrix} 1+\frac{\partial}{\partial \theta}\frac{F_k(\theta)}{f_k(\theta)} & -\frac{\partial}{\partial \theta}\frac{F_k(\theta)}{f_k(\theta)} \\ \frac{\partial}{\partial \theta}\frac{1-F_k(\theta)}{f_k(\theta)} & 1-\frac{\partial}{\partial \theta}\frac{1-F_k(\theta)}{f_k(\theta)} \end{vmatrix}} \text{ and } \frac{dp_B}{dk} = \frac{\begin{vmatrix} 1+\frac{\partial}{\partial \theta}\frac{F_k(\theta)}{f_k(\theta)} & \frac{\partial}{\partial k}\frac{F_k(\theta)}{f_k(\theta)} \\ \frac{\partial}{\partial \theta}\frac{1-F_k(\theta)}{f_k(\theta)} & \frac{\partial}{\partial k}\frac{1-F_k(\theta)}{f_k(\theta)} \end{vmatrix}}{\begin{vmatrix} 1+\frac{\partial}{\partial \theta}\frac{F_k(\theta)}{f_k(\theta)} & -\frac{\partial}{\partial \theta}\frac{F_k(\theta)}{f_k(\theta)} \\ \frac{\partial}{\partial \theta}\frac{1-F_k(\theta)}{f_k(\theta)} & 1-\frac{\partial}{\partial \theta}\frac{1-F_k(\theta)}{f_k(\theta)} \end{vmatrix}}.$$

The denominator becomes $1 + \frac{\partial}{\partial \theta}\frac{F_k(\theta)}{f_k(\theta)} - \frac{\partial}{\partial \theta}\frac{1-F_k(\theta)}{f_k(\theta)}$, which is always positive. Thus, for both $\frac{dp_A}{dk}$ and $\frac{dp_B}{dk}$ to be positive, we need

$$\frac{\frac{\partial}{\partial k}\frac{F_k(\theta)}{f_k(\theta)}}{\frac{\partial}{\partial k}\frac{1-F_k(\theta)}{f_k(\theta)}} \geq -\frac{\frac{\partial}{\partial \theta}\frac{F_k(\theta)}{f_k(\theta)}}{1-\frac{\partial}{\partial \theta}\frac{1-F_k(\theta)}{f_k(\theta)}} \tag{A.2}$$

and

$$\frac{\frac{\partial}{\partial k}\frac{F_k(\theta)}{f_k(\theta)}}{\frac{\partial}{\partial k}\frac{1-F_k(\theta)}{f_k(\theta)}} \geq \frac{1+\frac{\partial}{\partial \theta}\frac{F_k(\theta)}{f_k(\theta)}}{\frac{\partial}{\partial \theta}\frac{1-F_k(\theta)}{f_k(\theta)}}. \tag{A.3}$$

The RHS of both inequalities is negative. Comparing two inequalities, the second inequality always holds as long as the first inequality holds.

Lemma 3.2 *Both p_A and p_B increase, if $\Delta < \Delta^\dagger$, where $\theta^\dagger = \Delta^\dagger + \frac{1-2F(\theta^\dagger)}{f(\theta^\dagger)}$ and $\frac{\partial}{\partial k}\frac{F_k(\theta^\dagger)}{f_k(\theta^\dagger)} = 0$.*

For $\theta_k^* > 0$, while we have $\frac{\partial}{\partial k}\frac{1-F_k(\theta_k^*)}{f_k(\theta_k^*)} > 0$ by Proposition 1, the sign of $\frac{\partial}{\partial k}\frac{F_k(\theta_k^*)}{f_k(\theta_k^*)}$ is not clear. Thus, if $\frac{F_k(\theta_k^*)}{f_k(\theta_k^*)}$ is increasing in k, i.e., $\frac{\partial}{\partial k}\frac{F_k(\theta_k^*)}{f_k(\theta_k^*)} \geq 0$, p_A must increase. Let us define $\theta^\dagger > 0$, where $\theta^\dagger$ satisfies $\frac{\partial}{\partial k}\frac{F_k(\theta^\dagger)}{f_k(\theta^\dagger)} = 0$. Thus, as long as $\theta_k^* < \theta^\dagger$, a MPS always raises both prices, p_A and p_B.

We can also define $\Delta^\dagger > 0$, $\Delta^\dagger$ satisfies $\theta^\dagger = \Delta^\dagger + \frac{1-2F(\theta^\dagger)}{f(\theta^\dagger)}$. This ensures the one to one relationship between $\Delta^\dagger$ and $\theta^\dagger$. Thus, $\Delta < \Delta^\dagger$ is a sufficient condition for a MPS to raise both prices, p_A and p_B.

The proof of Proposition 4.

Suppose $\theta^*_{k+1} < \theta^*_k$. We obtain

$$\frac{1-2F_k(\theta^*_k)}{f_k(\theta^*_k)} < \frac{1-2F_k(\theta^*_{k+1})}{f_k(\theta^*_{k+1})} < \frac{1-2F_{k+1}(\theta^*_{k+1})}{f_{k+1}(\theta^*_{k+1})}.$$

The first inequality holds by Assumption 1, and the second inequality holds by Proposition 1. However, $\frac{1-2F_k(\theta^*_k)}{f_k(\theta^*_k)} < \frac{1-2F_{k+1}(\theta^*_{k+1})}{f_{k+1}(\theta^*_{k+1})}$ contradicts $\theta^*_{k+1} < \theta^*_k$ by the equilibrium condition (1.4). Therefore, we must have $\theta^*_k < \theta^*_{k+1}$.

The proof of Proposition 7.

$\pi_i^{PD} = \frac{2F(\theta^*)^2 - 3F(\theta^*) + \frac{5}{4}}{f(\theta^*)} > \frac{1}{4f(0)} = \pi_i^*$ can be rewritten as

$$4\left[2\left(F(\theta^*) - \frac{3}{4}\right)^2 + \frac{1}{8}\right] > \frac{f(\theta^*)}{f(0)}.$$

The minimum of the LHS is $\frac{1}{2}$ at $F(\theta^*) = \frac{3}{4}$. Thus, the sufficient condition for $\pi_i^{PD} > \pi_i^*$ is $f(0) > 2f(\theta^*)$.

A.2 Appendix of Chapter 2

The proof of Lemma 3.

We are going to show that any positive v^L or v^O always reduce the agents' incentives to work. With a slight abuse of notation, here we denote $\Delta^{SL} = u(v^S) - u(v^L)$ and $\Delta^{WO} = u(v^W) - u(v^O)$. Using the implicit function theorem to the IC constraint, we obtain

$$\frac{\partial I^*}{\partial \Delta^{SL}} = -\frac{p'(I^*)p(I^*)}{\left[\left(p''(I^*)p(I^*) + (p'(I^*))^2\right)(\Delta^{SL} - \Delta^{WO}) + p''(I^*)\Delta^{WO}\right]}$$

and

$$\frac{\partial I^*}{\partial \Delta^{WO}} = -\frac{p'(I^*)(1 - p(I^*))}{\left[\left(p''(I^*)p(I^*) + (p'(I^*))^2\right)(\Delta^{SL} - \Delta^{WO}) + p''(I^*)\Delta^{WO}\right]}.$$

We need to show that the denominator is always negative. $p''(I^*)p(I^*) + (p'(I^*))^2$ is negative by the stability condition. If $\Delta^{SL} - \Delta^{WO}$ is a large negative number, there may be a possibility of having a positive denominator, but this violates the second-order condition. Rearranging the denominator in terms of the second-order condition, we get $p''(I^*)(p(I^*)\Delta^{SL} + (1-p(I^*))\Delta^{WO}) + (p'(I^*))^2(\Delta^{SL} - \Delta^{WO})$. For this to be positive, $\Delta^{SL} - \Delta^{WO}$ must be a large positive number, but it violates the stability condition. This means that the denominator is always negative regardless of the sign of $\Delta^{SL} - \Delta^{WO}$ as long as both the stability condition and the second-order condition are satisfied. Since we get $\frac{\partial I^*}{\partial \Delta^{SL}} > 0$ and $\frac{\partial I^*}{\partial \Delta^{WO}} > 0$, the princiapl sets $v^L = v^O = 0$.

The proof of Lemma 4.

Since we set $v^L = v^L = 0$ by Lemma 1, the IP constraint becomes

$$V_i^* = \sigma u(v^S) + (1-\sigma)\left[p(I^*)p(I^*)u(v^S) + (1 - p(I^*))u(v^W)\right] - \frac{1}{\alpha}I^* \geq 0.$$

This can be rewritten as $\sigma u(v^S) + p(I^*)/p'(I^*) - \frac{1}{\alpha}I^* \geq 0$ using the IC constraint. Even when $v^S = 0$, let us show this constraint is not binding by proving $p(I^*)/p'(I^*) > I^*$. The derivative of $p(I^*)/p'(I^*)$ is $\frac{p'(I^*)^2 - p(I^*)p''(I^*)}{p'(I^*)^2}$, which is increasing in I^* and always greater than 1 because $p''(I_i) < 0$. Therefore, $p(I^*)/p'(I^*)$ must be above I^*.

A.3 Appendix of Chapter 3

The proof of Lemma 3.

The first order conditions with respect to internal conflict in group A are given by

$$\begin{aligned}
\frac{\partial V_{A1}}{\partial a_1} &= \frac{\theta_A f'(a_1) f(a_2)}{[f(a_1) + \theta_A f(a_2)]^2} q(F(\alpha_1, \alpha_2), F(\beta_1, \beta_2))R - w = 0 \\
\frac{\partial V_{A2}}{\partial a_2} &= \frac{\theta_A f'(a_2) f(a_1)}{[f(a_1) + \theta_A f(a_2)]^2} q(F(\alpha_1, \alpha_2), F(\beta_1, \beta_2))R - w = 0.
\end{aligned}$$

The first-order conditions can be summarized by

$$\frac{f(a_1)}{f'(a_1)} = \frac{f(a_2)}{f'(a_2)} = \frac{\theta_A}{(1+\theta_A)^2} q(F(\alpha_1, \alpha_2), F(\beta_1, \beta_2)) \frac{R}{w}. \tag{A.4}$$

Since $\frac{f(x)}{f'(x)}$ is nondecreasing in x, condition (A.4) implies that both the incumbent and the outsider in group A choose the same level of efforts, $a_1^* = a_2^*$ for internal conflict regardless of their possibly different choice of α_1 and α_2 for external conflict. By proceeding in a similar manner, we can also derive that

$$\frac{f(b_1)}{f'(b_1)} = \frac{f(b_2)}{f'(b_2)} = \frac{\theta_B}{(1+\theta_B)^2} [1 - q(F(\alpha_1, \alpha_2), F(\beta_1, \beta_2))] \frac{R}{w} \tag{A.5}$$

This implies that $b_1^* = b_2^*$. That is, both the incumbent and the outside in each group spend the same level of resources for the internal conflicts. The equilibrium condition (A.4) and (A.5) lead us to the result that the incumbent and the outsider's winning probabilities in Group A internal conflict are $p(a_1^*, a_2^*; \theta_A) = \frac{1}{1+\theta_A}$ and $1 - p(a_1^*, a_2^*; \theta_A) = \frac{\theta_A}{1+\theta_A}$. A similar result holds for group B internal conflict with $p(b_1^*, b_2^*; \theta_B) = \frac{1}{1+\theta_B}$.

The proof of Lemma 4.

Putting $\frac{f(x_i)}{f'(x_i)} = \frac{x_i}{m}$ into equation (A.4) and (A.5), we immediately obtain λ_A

$= \frac{\theta_A}{(1+\theta_A)^2}\frac{2m}{w}$ and $\lambda_B = \frac{\theta_B}{(1+\theta_B)^2}\frac{2m}{w}$. Given $\theta_A < \theta_B$, we must have $\frac{\theta_A}{(1+\theta_A)^2} < \frac{\theta_B}{(1+\theta_B)^2}$.

The proof of Proposition 17.

$F(y_i, y_j)$ is a homothetic function. α_1^* (β_1^*) must have a linear relationship with α_2^* (β_2^*). This means that the slopes of the level sets of $F(y_i, y_j)$ are the same along rays coming from the origin. Let us define those as $s_A = \alpha_2^*/\alpha_1^*$ and $s_B = \beta_2^*/\beta_1^*$. Equations (1.1) and (1.2) can be written as

$$\frac{F_1(1, \alpha_2^*/\alpha_1^*)}{F_2(1, \alpha_2^*/\alpha_1^*)} = \frac{F_1(1, s_A)}{F_2(1, s_A)} < \frac{F_1(1, s_B)}{F_2(1, s_B)} = \frac{F_1(1, \beta_2^*/\beta_1^*)}{F_2(1, \beta_2^*/\beta_1^*)},$$

because $F_i(y_i, y_j)$ and $F_j(y_i, y_j)$ are homogeneous degree of 0. Note that $\frac{F_1(1,s_G)}{F_2(1,s_G)}$ is increasing in s_G under $F_{jj}(y_i, y_j) < 0$ and $F_{ij}(y_i, y_j) > 0$ as follows.

$$\frac{\partial}{\partial s_G}\left[\frac{F_1(1, s_G)}{F_2(1, s_G)}\right] = \frac{F_{12}(1, s_G)F_2(1, s_G) - F_1(1, s_G)F_{22}(1, s_G)}{[F_2(1, s_G)]^2} > 0.$$

Therefore we must have $s_A = \alpha_2^*/\alpha_1^* < \beta_2^*/\beta_1^* = s_B$.

The proof of Proposition 18.

Equation (3.5), (3.6), (3.7), and (3.8) correspond to

$$\frac{\alpha_1}{\alpha_2} = \left(\frac{1 - p(\theta_A)}{p(\theta_A)}\right)^{\frac{1}{r-1}}, \tag{5'}$$

$$\frac{\beta_1}{\beta_2} = \left(\frac{1 - p(\theta_B)}{p(\theta_B)}\right)^{\frac{1}{r-1}}, \tag{6'}$$

$$\left(\frac{\alpha_1}{\beta_1}\right)^{r-1}\frac{\beta_1^r + \beta_2^r}{\alpha_1^r + \alpha_2^r} = \frac{p(\theta_B)}{p(\theta_A)} \text{ and} \tag{7'}$$

$$\left(\frac{\alpha_2}{\beta_2}\right)^{r-1}\frac{\beta_1^r + \beta_2^r}{\alpha_1^r + \alpha_2^r} = \frac{1 - p(\theta_B)}{1 - p(\theta_A)}. \tag{8'}$$

Putting equation (5') and (6') into (7'), we obtain $(\frac{\alpha_1}{\beta_1}) = \frac{p(\theta_B)}{p(\theta_A)} \cdot \frac{1+\left(\frac{p(\theta_A)}{1-p(\theta_A)}\right)^{\frac{r}{r-1}}}{1+\left(\frac{p(\theta_B)}{1-p(\theta_B)}\right)^{\frac{r}{r-1}}}$.

Plugging this into (7') again, we get $\frac{\alpha_1^r+\alpha_2^r}{\beta_1^r+\beta_2^r} = \left(\frac{1+\left(\frac{p(\theta_A)}{1-p(\theta_A)}\right)^{\frac{r}{r-1}}}{1+\left(\frac{p(\theta_B)}{1-p(\theta_B)}\right)^{\frac{r}{r-1}}}\right)^{1-r} \left(\frac{p(\theta_A)}{p(\theta_B)}\right)^r$.
Using this, we can further manipulate the equation as follows.

$$
\begin{aligned}
\frac{F(\alpha_1^*, \alpha_2^*)}{F(\beta_1^*, \beta_2^*)} &= \left(\frac{\alpha_1^r + \alpha_2^r}{\beta_1^r + \beta_2^r}\right)^{\frac{1}{r}} = \left(\frac{1 + \left(\frac{p(\theta_A)}{1-p(\theta_A)}\right)^{\frac{r}{r-1}}}{1 + \left(\frac{p(\theta_B)}{1-p(\theta_B)}\right)^{\frac{r}{r-1}}}\right)^{\frac{1-r}{r}} \left(\frac{p(\theta_A)}{p(\theta_B)}\right) \\
&= \left(\frac{p(\theta_A)^{\frac{r}{1-r}} + (1-p(\theta_A))^{\frac{r}{1-r}}}{p(\theta_B)^{\frac{r}{1-r}} + (1-p(\theta_B))^{\frac{r}{1-r}}}\right)^{\frac{1-r}{r}}
\end{aligned}
$$

Now, $F(\alpha_1^*, \alpha_2^*) \gtreqless F(\beta_1^*, \beta_2^*)$ is comparable to $p(\theta_A)^\rho + (1-p(\theta_A))^\rho \gtreqless p(\theta_B)^\rho + (1-p(\theta_B))^\rho$. Let us define the function,

$$g(x) = x^\rho + (1-x)^\rho, \text{ where } x \geq 1/2.$$

This function is increasing in $\rho > 1$ and decreasing in $\rho < 1$, because $g'(x) = \rho(x^{\rho-1} - (1-x)^{\rho-1})$. Note that ρ must be greater than 0 for $r < 1$. Therefore, since $p(\theta_A) > p(\theta_B)$, $F(\alpha_1^*, \alpha_2^*) \gtreqless F(\beta_1^*, \beta_2^*)$ must correspond to $\rho \gtreqless 1$, which is again equivalent to $r \gtreqless 1/2$.

The proof of Proposition 19.

Equations (3.5), (3.6), (3.7), and (3.8) correspond to $\frac{\alpha_2}{\alpha_1} = \left(\frac{1-k}{k}\right)\theta_A$, $\frac{\beta_2}{\beta_1} = \left(\frac{1-k}{k}\right)\theta_B$, $\frac{\beta_1}{\alpha_1} = \left(\frac{1+\theta_A}{1+\theta_B}\right)$, and $\frac{\beta_2}{\alpha_2} = \left(\frac{\theta_B}{\theta_A}\right)\left(\frac{1+\theta_A}{1+\theta_B}\right)$ respectively. We can immediately observe $\alpha_1^* > \beta_1^*$, $\alpha_2^* < \beta_2^*$, and $\frac{\alpha_1^*}{\alpha_2^*} > \frac{\beta_1^*}{\beta_2^*}$. Solving four equation simultaneously, we obtain

$$\frac{F(\alpha_1^*, \alpha_2^*)}{F(\beta_1^*, \beta_2^*)} = \frac{p(\theta_A)^k(1-p(\theta_A))^{1-k}}{p(\theta_B)^k(1-p(\theta_B))^{1-k}} = \left(\frac{1+\theta_B}{1+\theta_A}\right)\left(\frac{\theta_A}{\theta_B}\right)^{1-k}.$$

Thus, after the simple algebra, one can show $F(\alpha_1^*, \alpha_2^*) \gtreqless F(\beta_1^*, \beta_2^*)$ as $k \gtreqless \widehat{k}$, where $\widehat{k} = \log_{\left(\frac{\theta_B}{\theta_A}\right)}\left(\frac{1+\theta_B}{1+\theta_A}\right)$. The winning probability of the less conflictive group is increasing in k, and the vice-versa is true for the more conflictive group.