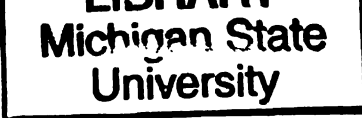


thesis
2
2009



This is to certify that the
dissertation entitled

**MATH WARS: A RHETORICAL ANALYSIS OF THE TERMS
OF DEBATE**

presented by

Irfan Muzaffar

has been accepted towards fulfillment
of the requirements for the

Ph.D.

degree in

Curriculum, Teaching, and
Educational Policy

A handwritten signature in dark ink, consisting of a large, stylized 'C' followed by a series of loops and a horizontal stroke.

Major Professor's Signature

26 April 2009

Date

MSU is an Affirmative Action/Equal Opportunity Employer

PLACE IN RETURN BOX to remove this checkout from your record.
TO AVOID FINES return on or before date due.
MAY BE RECALLED with earlier due date if requested.

DATE DUE	DATE DUE	DATE DUE
APR 21 2013		

MATH WARS: A RHETORICAL ANALYSIS OF THE *TERMS OF DEBATE*

By

Irfan Muzaffar

A DISSERTATION

Submitted to
Michigan State University
in partial fulfilment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Curriculum, Teaching, and Educational Policy

2009

ABSTRACT

MATH WARS: A RHETORICAL ANALYSIS OF TERMS OF DEBATE

By

Irfan Muzaffar

This study concerns itself with the conflict in mathematics education—popularly known as *math wars*—in the United States. More specifically, it investigates the *terms of debate* in this conflict to develop insights into the varied, and sometimes conflicting, relationships between the perceived nature of mathematics and its pedagogy. It also extends the use of rhetorical analysis to understand the ways in which the rhetoric of *standards* was at odds with the idea of *mathematical power for all*.

The study centres its analysis on terms—such as *mathematical power* or *proficiency*—that became the sites of contestations. It explores the internal logic of particular clusters of terms, and the relations between them in several significant texts about mathematics and mathematics education. To perform this analysis, I have chosen the image of drama offered by Kenneth Burke due to its emphasis on conflict. Image of drama is invoked because of its relevance to an understanding of conflict. This image invokes particular *scenes*, populated with particular *agents* engaged in *purposeful acts* using *means* available to them. As an analytical device to study mathematics education texts, it implies the recognition that any text having to do with mathematics education implicitly or explicitly would have some description of *background* or *scene* against which mathematicians, teachers of mathematics, or students [*actors*, that is] would appear as engaged in purposive *acts* of doing, teaching, or learning mathematics.

There are two kinds of texts examined in this dissertation. The first kind of texts are those written long before the current conflicts around mathematics education reforms—i.e. the texts by well-known mathematicians such as Rene Descartes, Bertrand Russell, and the 19th century American mathematician Benjamin Pierce. Second kind of texts examined in this dissertation were generated as part of the mathematics education reforms, which were constituted in the wake of crisis calls to reform education in the early to mid 1980s. The dramatistic analysis of these texts suggests that terms such as *mathematical power for all* and *mathematical proficiency for all* were bound up in two different conceptions of mathematics and its teaching and learning. While the former represented a merger of mathematics and pedagogy, the latter emphasized their separation. The study also discusses the implication of these suggestions for professional claims of mathematics education.

In the final chapter, the study uses dramatistic analysis together with the notion of *American Jeremiad*—a reference to perpetual announcements of impending doom and calls for reform in American history—to develop insights about the relative strength of reform texts in a scene set up by the Jeremiad of *A Nation at Risk*. The insights so developed suggest that mathematics education reforms' strength as well as their vulnerability accrued from their reliance on the rhetoric of *standards*.

Copyright by
Irfan Muzaffar
2009

ACKNOWLEDGEMENTS

I will start by thanking my advisors Helen Featherstone and Lynn Fendler without whom this dissertation could never be completed. Thank you, Helen and Lynn for being two amazing mentors. You remained an inexhaustible source of knowledge, critical comments, ideas, and tons of sympathy through my trials and tribulations as a doctoral student. The numerous conversations I had with you always helped orient and focus my intellectual gyroscopes as I groped my way through this dissertation.

My work is also indebted in a great measure to my committee members Suzanne Wilson and Brian Delany. Suzanne helped me immensely in understanding the intricacies of mathematics education reforms, always pushing me to think harder and clearer with her penetrating question. Brian, interested and sympathetic, read the drafts of my chapters and helped me refine the analysis with his feedback. I also thank Nathalie Sinclair for being a shaping influence in the early part of this project as part of my guidance committee, and also for continuing to provide me with her insights and comments on my drafts even after leaving the Michigan State for Simon Fraser University.

I will be remiss if I do not mention Jay Featherstone who led me to several critical sources on the history of educational reforms in the United States and responded to my ideas and drafts. I also want to thank Cleo Cherryholmes for initiating me to curriculum theory and for remaining supportive of my work even after retirement. There are many other people to whom I would like to acknowledge my debt. They are Sandra Crespo, Joyce Grant, Ajay Sharma and Punya Mishra.

I was also immensely fortunate to be part of the Critical Studies Group, an amazing collection of faculty and graduate students. I would particularly like to thank Don

Moore, Steve Tuckey, Brett Merritt, Kelly Merritt, Jeanne Meier, Ann Lawrence and Adam Greteman.

I am also indebted to the help provided by the Spencer Foundation Research Training Grant, which helped me immensely in finding my way through a maze of theoretical ideas, many of which are ultimately reflected in this work.

My appreciation also goes to my Mom who trusted me and kept me in her thoughts and prayers, my little brother Kamran who lost his life while I was writing this dissertation, and other members of my extended clan. And finally, but in the least, it was my wife and friend Shamaila, who treated me with indulgence through out this project, and without whose love, support, patience and care, I could never have completed this work.

Table of Contents

List of Tables	ix
Chapter I: Introduction.....	1
The Math Wars at a glance	5
Understanding the terms of the debates	9
The Methodological Considerations	11
Outline of the Dissertation	17
Assumptions about Relations between Action, Conflict, and Drama.....	20
The Analysis	37
Concluding thoughts: Analyzing the Relations between Terms of the Pentad.....	40
Chapter III: Mathematics, God, and Pedagogy.....	43
The Mountain Range: Mathematics as the Scene in Classical Texts.....	50
The Power to attain Natural Light of Reason: The Agency and the Purpose	68
The Relationship between Mathematics and its Teaching and Learning in Classical Texts	69
The Drama of Reform Texts	74
The Reform Texts	76
The Scene of Mathematics Education.....	80
Mathematical Power: Examining Act and Agency in the Reform Texts.....	95
Purpose: the Mathematically Empowered, Democratic Citizen	102
Concluding Thoughts.....	105
Chapter V: The Drama of Counter Reforms: Implication for Mathematics Education..	107
Counter Reforms	111
Standards as Pathways and Milestones: The Scene of Mathematical Proficiency	115
Separating Content and Pedagogy: Contesting the Claims of Mathematics Education	129
Concluding Thoughts.....	132
Chapter VI: The American Jeremiad and Reforms: The Dramatistic Vulnerabilities of Reform Texts	134
The American Jeremiad	136
From Jeremiads to Jeremiads: Transformations from Sacred to Secular	144
Similarities between Theologized and Secularized Jeremiads	147
American Jeremiad and Mathematics Education Debates.....	150
The Scene Described and Acts Called for by A Nation at Risk: The Scene-Act Ratio for Mathematics Education Reforms.....	154
Echoes of Jeremiad: The Expressions Scene-Act Ratio in Reforms and Counter- Reforms.....	159
Reform Texts, Comedy and Tragedy: The Affirmation of scene-act Ratio	171
Concluding Thoughts.....	175

References.....	177
-----------------	-----

List of Tables

Table 1: Pentad at a Glance—Focus on Classical Texts.....	43
Table 2: Pentad at a Glance—Focus on Reform-based Mathematics Education Texts ...	73
Table 3: Strands and Unifying Ideas at various levels in the 1992 <i>Framework</i>	79
Table 4: Verbs of Mathematical Power	98
Table 5: Pentad at a Glance—Focus on Counter-reform Texts.....	107
Table 6: Standard for the Number Sense Strand in grades K-3	120
Table 7: Number of Standards associated with 1999 Framework	121
Table 8: The Dramatistic Elements in American Jeremiad	139
Table 9: Comparison between the Puritan and Secularized Jeremiads.....	147
Table 10: Binaries of <i>Math Wars</i>	149
Table 11: Comparison of <i>Scene-Act</i> ratio in <i>A Nation at Risk</i> , <i>NCTM Standards</i> , and 1999 <i>Framework</i>	169

Chapter I: Introduction

If the world around us is at least partially a matter of how it is framed, how the situation is defined, then the reader, the observer, the audience can best understand how the procedure operates by getting outside the box of his own logics.

(Gusfield, 1989, p. 7)

Through this dissertation, I study the *terms of debate* in mathematics education. Specifically the study concerns itself with the conflict in mathematics education in the United States in the last two decades or so. In this introductory chapter, I will provide you with a description of why the terms of debate in mathematics education are worthy of a dissertation length investigation. This chapter will also describe the research questions as well as the key methodological decisions taken by me as both a reader of the *terms of the debate* and as the writer of this report.

Viewed from outside, especially from a vantage point in a foreign institution, the United States of the 1990s appear as an exporter of terms such as *constructivist teaching and learning, teaching for understanding, child-centeredness, pedagogical content knowledge, and professional development schools*.¹ Half understood, but largely welcomed, these terms are reinterpreted and recontextualized in educational reform discourses of other cultures without any reference to conditions of their production.

When these words and descriptions produced in one culture traveled to distant cultures—

¹ I have opened this conversation using these terms without highlighting their subtleties and multiple meanings. For example, while all versions of constructivism see the student as active constructors of knowledge, there are considerable variations among them (See, for example, Phillips, 1995; Wilson, 2003, pp. 40-41). My purpose is not to survey these variations in meaning here, but to tell you that to an outsider these terms come across not as carriers of these subtleties but as recipes for action.

as from the American continent to South Asian Sub-continent where I encountered them as a mathematics teacher and a teacher educator—they come across as pretty secure descriptions of teaching and learning generally, and of mathematics teaching and learning particularly.²

As a physics major in my undergraduate and early graduate work, I was committed to a view of mathematics—usually attributed to Galileo Galilei (1564-1642)³—as the language in which the universe reveals its truths. My early encounters with a constructivist view of learning and teaching did not interfere with this belief about the nature of mathematics. I encountered terms mentioned above as exclusively about teaching and learning and not as a set of statements about the nature of mathematics as a discipline. For example, whereas I understood the term constructivism as a basis for thinking about *how children learn* with implications about *how they should be taught*, this understanding did not push me to think about *what mathematics is* or *should be*.

Not so, after arriving in the United States. Here, the constructivist discourse in the talk about school mathematics was not just about the teaching and learning of mathematics. It also impinged profoundly upon beliefs about the nature of mathematics. In fact, mathematics assumed “many faces” with questions being raised about whether mathematics educators and mathematicians talked about the same thing when they used the term mathematics (Sfard, 1998, p. 491). As I was to learn later—and more

² Gita Steiner-Khamsi has used the title “traveling reforms” to talk about the lending and borrowing of educational discourses across the world (Popkewitz & Steiner-Khamsi, 2004).

³ Galileo thought of universe as “...written in mathematical language, and its characters are triangles, circles, and other geometrical figures; without these it is humanly impossible to understand a word of it, and one wanders around pointlessly in a dark labyrinth”(Galilei & Finocchiaro, 2008, p. 183). This assumption continues to be widely held by the theoretical physicists (see, for example, Eugene, 1960; Tegmark, 2007)

thoroughly through the analysis of terms of mathematics education debate in this study—one could identify various different, but internally coherent,⁴ texts with each containing different and conflicting descriptions of the nature of mathematics and its relationship with teaching and learning. The groups adhering to one or the other text about mathematics and its pedagogy seemed to be hard at work, at times with religious zeal, to mark themselves off from what they were not (see, Wilson, 2003, pp. 48-49). This difference appeared as a raging conflict in the policy arena.

The conflicts were not just in mathematics education. The *figure of war* appeared frequently to refer to conflicts in education. Terms such as culture wars, social studies wars, language wars, and reading wars were commonplace. But, being a mathematics educator and a believer in the *mainstream* view of mathematics as certain and a priori,⁵ I found the phrase *Math Wars*, to echo Alan Schoenfeld, “oxymoronic, a category error” (Schoenfeld, 2004, p. 253). Yet, here, the proponents and detractors seemed to have locked horns with each other in this spectacular battle. Stepping into the territory of mathematics education in the United States was much like becoming part of a territorial jurisdiction, complete with its citizens, friends, and enemies.

Also, as an outsider, I did not immediately notice that American educational discourse frequently appeared to be associated with a *reform movement*. *Reforms* and *Reformers* are a ubiquitous feature of educational discourse in America in a way that they seldom are outside of the United States. Before I came to the United States, I was not

⁴ My reference to internal coherence merely restates the observations made by other scholars as well that formal documents have the appearance of a consensus, which masks differences if any between their authors while appearing to be internally consistent. (See, for example, Wilson, 2003, p. 48.)

⁵ Hersh (1999) uses the term *mainstream* to speak of the pervasive view of mathematics as a bastion of certainty and as consisting of preexisting mathematical forms. I will use this distinction again in Chapter 3.

accustomed to seeing the term reform used for matters considered to be traditionally under the control of professionals (teachers, teacher educators). As a traveler from a distant culture it took me some time before I realized that the work I was doing as a professional mathematics educator in graduate school was also part of a reform movement. I had done similar work and used the same progressive discourses and materials, without ever thinking of myself as anything more than a professional mathematics educator.

Insiders in the field of education in the United States may not recognize it due, perhaps, to their proximity with the term *reform*, that the commingling of professional with evangelical as exemplified by the insertion of *reforms* and *reformers* within the discourse of professional fields such as education is a peculiarly American conjoining of secular with sacred.⁶ My reference is not to the form and content of particular reforms. Rather, I stress the availability of the terms “reforms” and “reformers” as locations to be inhabited by individuals and groups. In this discourse, the individuals and groups come across as reformers, anti-reformers, un-reformed, to-be-reformed, or reformed. American scholarship on educational reforms typically raises questions about factors that promote or hinder reform, but assumes the ubiquity of reforms without question. To me the idea of reform is not something familiar that needs to be made strange in order to interrogate it; it is already strange.

Why have there always been *reforms* of one kind or the other in the United States? And why are the reforms and reformers prone to getting into trouble? These questions

⁶ The peculiar mixing of sacred and secular emerges in the unique form of political prose that the Sacvan Bercovitch terms *American Jeremiad* (Bercovitch, 1978). The jeremiads produce reforms by directing public attention toward risk and decline, and calling for reforms to stem decline. I will discuss this idea at greater length in Chapter 6.

are not the central questions that guide the inquiry in this project, but they keep shadowing me throughout this project, pushing me to account for them within the context of mathematics education debates.

The recent conflicts in mathematics education are also battles over reforms. They are not new disagreements over teaching and learning of mathematics but instead are the most recent volcanic eruptions from a conflict simmering beneath the surface of American education at least since the middle of the 19th century (Wilson, 2003, p. 5). Below, I will first describe the sequence of events in the math wars briefly. This description will be followed by my rationale for this study, a discussion of assumptions that I take to this analysis, and finally an outline of the dissertation.

The Math Wars at a glance

I do not intend to provide a full description of the recent math wars, but will only provide a chronology of events and some details that I think are important to develop my argument. The reader may find many important details of math wars omitted in this drastically reduced summary. For this description and many of my data sources, I have borrowed heavily from Wilson's history of the *math wars* (Wilson, 2003).

"*Math Wars*" is the media title for the debates around school mathematics that began in the wake of the calls for reforms made in the report *A Nation at Risk* (NCEE, 1983). I will discuss this report and its implications for reforms in more detail in Chapter 6. Here it suffices to note that this report is widely believe to have spawned the large scale reform effort to improve student achievement in K-12 mathematics and other school subjects by developing and implementing national standards—the so called standards-based reforms.

The National Council of Teachers of Mathematics (NCTM) took the lead in publishing *Curriculum and Evaluation Standards* (NCTM, 1989) for K-12 mathematics. Subsequently, the NCTM also published *Professional Teaching Standards* (NCTM, 1991) and *Assessment Standards* (NCTM, 1995).⁷ All of these three volumes were addressed to teachers, school administrators, teacher educators, policy makers and the wider public and they responded to the call for change with new ideas about appropriate goals for school mathematics teaching.

From the perspective of this study, it is important to note that NCTM Standards assumed that all students could be *mathematically empowered*. The standards defined the term *mathematical power for all* as “an individual's abilities to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve non-routine problems.” (NCTM, 1989). The term *mathematical power* was repeated and reinforced in the 1992 *Mathematics Framework for California Public Schools*⁸ as well as several other texts produced around the same time (see, for example, NRC, 1989; NRC, 1993).

Development of *mathematical power* was seen as associated with an increased emphasis on learning the meaning of operations, operation sense, mental computation, estimation and the reasonableness of answers, use of calculators for complex computation, and thinking strategies for basic facts. At the same time, the standards deemphasized traditional activities—such as complex paper-and-pencil computations, traditional algorithms, rote memorization of number facts—as expressions of

⁷ In this dissertation, unless stated otherwise, I will use the shorthand NCTM Standards to refer to *Curriculum and Evaluation Standards* (1989).

⁸ In this dissertation, unless stated otherwise, I will use the shorthand 1992 *Framework* to refer to *Mathematics Framework for California Public Schools* (1992).

mathematical power. Broadly speaking the NCTM standards, attempted to shift mathematics curriculum and instruction away from ‘drill and kill’ and teaching of ‘paper and pencil algorithms” towards a vision that placed more emphasis on communication, problem-solving, and *invented* instead of *already known* or *standard* algorithms for solving mathematical problems. The standards conceptualized mathematics as a product of human activity and made the capacities to produce mathematics—the so called *mathematical powers*—the objects of pedagogical intervention.

An important feature of the standards and documents associated with them was their claim to be an expression of *professional consensus* by the mathematics education community (Carl & Frye, 1991; Crosswhite, Dossey, & Frye, 1989; Frye, 1990).

The NCTM Standards were received warmly by the advocates and criticized scathingly by the critics. Through its publication of 1992 *Framework*, California took the lead in aligning its K-12 mathematics education policy with NCTM Standards. Fierce debates followed the efforts to align the practices of teaching and learning in school districts with the precepts of the NCTM *Standards* and 1992 *Framework*. The opponents emphasized what was deemphasized by the reform texts; the traditional ways of teaching and learning mathematics such as drill, paper and pencil computation, learning standard algorithms, and automatic recall of basic mathematical facts.

These debates, which spanned most of the 1990s, turned into a bitter controversy that by the middle of 1990s had embroiled parents, math educators, mathematicians, education reformers, local school boards, federal and state policymakers, and political pundits in a struggle over the content and teaching of mathematics. The figure of war began to dominate these debates as they turned more and more acrimonious. As Suzanne

Wilson puts it: "...the language of war crept into many debates about mathematics education" (Wilson, 2003, p. 2). A scene dominated by the figure of war tends to divide humans into warriors, peace makers, and spectators. The language in which the debates were projected by the 'warriors,' especially those resisting the reforms, sometimes mirrored the language of civil war. This language of war permeated up and down and across the system, as evidenced from the calls by the U.S. Secretary of Education Riley for cessation of hostilities and a return to "civil and constructive discourse" in his address to the Mathematical Association of America in 1998.

Meanwhile, in California, the *math wars* led to a revision of 1992 *Framework* and publication of a new set of standards to replace the NCTM standards. In 1999 the revised *Mathematics Framework for California Public Schools* together with revised Content Standards (CDE, 1999) were endorsed by California's legislature.

Important from the standpoint of my project is the shift in vocabularies between the 1992 and 1999 *Framework*. The 1999 *Framework* replaced the language of *mathematical power for all* with that of *mathematical proficiency for all*. It defined *mathematical proficiency* as a measurable construct whose presence/absence could be indicated by the students' scores on standardized tests. The new standards were an effort at defining in unambiguous terms what the students needed to know and be able to do in order to be declared proficient. They came across as precise statements of several desirable strands for school mathematics and of precise benchmarks on each one of those strands. Becoming *mathematically proficient*, according to the 1999 *Framework*, required all students to cover all the benchmarks in all the strands at each grade level.

This shift and its implications for mathematics education are discussed more fully in Chapter 5.

What needs to be mentioned in this introduction is that after the shift in vocabulary of mathematics education in California, a different kind of consensus emerged and found its way into the major NCTM and other publications. For instance, in the year 2000, NCTM published the *Principles and Standards of School Mathematics* (PSSM) (NCTM, 2000). PSSM dropped the term *mathematical power* from its repertoire of terms in its description of the principles and standards of school mathematics. Likewise, the most recent NCTM publication titled *Focal Points for Curriculum* (NCTM, 2006) also makes no reference to the term *mathematical power*. Meanwhile, several documents have emerged at the national level refining the concept of *mathematical proficiency* as the new consensus (Ball, 2003; Kilpatrick, Swafford, & Findell, 2001; USDOE, 2008).

Understanding the terms of the debates

I began this chapter with an account of my encounter with a blizzard of *terms*—such as *constructivist teaching and learning*, *teaching for understanding*, *child-centeredness*, *pedagogical content knowledge*—in different contexts. The point of this brief narrative of my encounter with similar terms in different contexts was to highlight that the meaning of these terms—some of which, such as *mathematics*, we take for granted—are not stable and same across different texts. I have come to believe that *terms* do not exist in a necessary and inextricable relationship with particular worldviews. When the terms that I have talked about, left their discursive places of origin and traveled to the distant lands where I encountered them, they did not necessarily carry with them the trace of their origin. In their new contexts, ideas such as *child-centeredness* or *pedagogical*

constructivism were not part of a *progressive* worldview as they are taken to be in the United States. They remained the same, yet were different. In general, we may assume that when terms travel from one discursive community to the other, they are recontextualized in their new discursive habitats.

I believe it is also important to recognize that we use terms that are perpetually constituted by the conflicts. When I began my work as a mathematics educator, terms such as *constructivism*, *teaching for understanding*, *mathematical empowerment*, worked as guides for our practice. But, as I have observed in the previous section, the discourse defined by these terms was challenged. For the workers in the field of mathematics education it is important to recognize that terms which shape their work are not always expressions of scientific progress, but residues of rhetorical conflicts in education. This project—much like cartography—is about understanding conflict by drawing maps of specific relations between the terms the terms used in mathematics education in the conflicting discourses.

The understanding of terms in conflicting discourses does not assume a necessary relationship between the use of particular terms and the worldviews of the users as a strategy to avoid what Wilson refers to as “facile simplifications.” (Wilson, 2003, p.xii). In her documentation of the *math wars* in California and, more generally, in the United States, Wilson suggests the need to avoid facile simplifications in thinking about reforms and counter reforms. About the *math wars*, Wilson concludes that:

This wasn't a debate between the *Republican-Conservative-Traditionalist-Positivist-Math-as-Skills-Direct-Instruction-Social-Efficiency* Camp and the *Democratic-Progressive-Constructivist-Interpretivist-Math-as-Conceptual-*

Understanding-Child-Centered-Instruction-Democratic-Equality Camp (Wilson, 2003, p. 165).

I interpret Wilson as suggesting that it is not easy, not even desirable, to draw the lines in ways that declare *conservatives* as against democratic equality or *progressives* as essentially against *social mobility*. The facile simplifications tend to conflate political and pedagogical orientations in misleading ways. An example from a Californian mathematician documented in the Notices of the Mathematical Association of America (MAA) is in order here. A mathematics professor who was concerned about some mathematical aspects of the reform texts was reported as having been stunned when invited to speak at a local Republican convention. “Mathematicians tend to jump into such issues with both feet,” she says, “and then they find themselves labeled as right wing conservatives. And it’s pretty hilarious. I don’t know any mathematicians who are right-wing conservatives” (Abigail Thompson, quoted in Jackson, 1997, p. 820). Yet, when we direct our attention to competing goals of education and on conflict in terms of worldviews signified by such terms as *progressives* and *conservatives*,⁹ we foreclose other ways of thinking about debates in mathematics education.

The Methodological Considerations

When exploring a particular cluster of terms, I do not work like a social scientist. That is to say, I am not using a particular conceptual repertoire and applying it to conflict in mathematics education. Nor am I developing a grounded theory of particular conflicts

⁹ For an explanation of conflicts in terms of worldviews, see, Apple, 1982, 1986, 1993; Shor, 1986

in the tradition of anthropologists. Rather, I read the texts that I chose to study in a manner similar to literary analysis.

The argument of this dissertation is based on exploring the internal logic of particular clusters of terms, and the relations between them. I expect my readers will apply a similar mode of reading to follow my argument. To perform this analysis, I have chosen the image of drama offered by Kenneth Burke (Burke, 1945) due to its emphasis on conflict. Kenneth Burke's dramatism—whose theoretical underpinnings I develop in more detail in Chapter 2—works for me as it takes ambiguity in the meaning of terms describing human action as central to the conflict. As Burke puts it: “Since no two things or acts or situations are exactly alike, you cannot apply the same term to both of them without thereby introducing a certain margin of ambiguity, an ambiguity as great as the difference between the two subjects that are given the identical title” (Burke, 1945, p. xix). This dramatistic ambiguity is echoed in Sfard's question about whether mathematicians and mathematics educator mean the same thing when they use the term mathematics (Sfard, 1998).

As someone who has acquired an awareness of the shifting meanings of the terms through actually going back and forth between different linguistic communities, I saw terms—such as the ones designating the subject matter of mathematics and its relationship with teaching and learning mathematics—within particular texts as the very sites of conflict in the mathematics education debates. For example—as I mentioned in my brief history of the *math wars*—the term *mathematical power for all* was a site of conflict inasmuch as the opponents of the reform texts containing it sought to substitute it with *mathematical proficiency*. An exploration of internal logic of the discourses reveals,

as I will show in more detail subsequently, that these terms are bound up with very different notions of what constituted *mathematics* and mathematical ways of knowing. Thus, considering the *terms of mathematics education debates* as sites of the conflict, I am interested in ways in which these terms function to provide us with a structure of understanding the subject matter of mathematics and of motivation for doing, teaching, and learning mathematics.

There are two kinds of texts examined in this dissertation. First kind consists of texts that came into being long before the present conflict between reformers and counter reformers, texts created by well-known mathematicians within a broad time frame ranging from the 17th to late 19th century. These texts give us a chance to see some continuities as well as discontinuities of perspective between the discourse associated with some iconic figures in the history of mathematics and the current debates in mathematics education.

In choosing these texts, I follow the distinctions between *mainstream* and *humanist* mathematical texts offered by Reuben Hersh (1999) in his book *What is Mathematics, Really?* Hersh's distinctions work for me because they are not set in fixed time periods. What he calls *Mainstream* can be seen as distributed from ancient Pythagoreans to modern day Platonist mathematicians (Davis & Hersh, 1981). As he puts it:

For the Mainstream, mathematics is superhuman—abstract, ideal, infallible, eternal. So many great names: Pythagoras, Plato, Descartes, Spinoza, Leibniz, Kant, Frege, Russell, Carnap...Humanists see mathematics as a human activity, a human creation. Aristotle was a humanist in that sense, as were Locke, Hume, and Mill. Modern philosophers outside the Russell tradition—mavericks—include Peirce, Dewey, Roy

Sellars, Wittgenstein, Popper, Lakatos, Wang, Tymoczko, and Kitcher (Hersh, 1999, p. 92).

Like Hersh, I do not assume progress from *mainstream* to *humanist* notions of mathematics. I imagine these distinctions as existing in texts across times and spaces. From a rhetorical standpoint, there seems to be no difference in the motivation of a Pythagorean or a modern day mathematicians who believes—like Pythagoras—that mathematical objects are not constructed by humans but exist independently of them.

Specifically, I explore the descriptions of the nature of mathematics in René Descartes' (1596-1650) *Meditations*, in archived letters of Bertrand Russell (1809-1880), and the writing of Benjamin Peirce (1809-1880). I then make use of these descriptions to think about the possibilities of *acts* of teaching and learning mathematics that become thinkable in relation to those conceptions of mathematics.

The second kind of texts examined in this study are those that directly relate to the conflict in mathematics education in the last two decades. As described earlier in this chapter, I call these the reform and counter reform texts in this dissertation. The reform texts examined in this dissertation are the NCTM Standards and the 1992 *Framework*. The NCTM Standards and 1992 *Framework* may be imagined as a single textual complex which was ultimately replaced by the California Mathematics Standards and 1999 *Framework* (CDE, 1999), which I refer to as the counter reform texts.

On what grounds can we compare the texts so disparate and distant in nature and what insights can we gain from such comparison between the *terms of debate* in mathematics education? To constitute the grounds of comparison between the texts I use the image of drama as employed by rhetorician Kenneth Burke (Burke, 1945). Why

drama? Drama suggests organization of texts that deal with human actions and their motivation in terms of descriptions of *actors*—in my case reformers, mathematicians, counter reformers, and learners—as engaged in *purposive acts* in specific *scenes* using *agencies* [means]. Using the figure of drama as an analytical device implies the recognition that any text—whether Pythagorean, Cartesian, or Reform texts of the last two decades—would have some description of *background* or *scene* against which mathematicians, teachers of mathematics, or students [*actors*, that is] would appear as engaged in purposive *acts* of doing, teaching, or learning mathematics.

When I am occupied with finding out what *school mathematics* might mean in particular texts, I do not see—following Wilson’s phrase that I cited above and repeat here—it as defined primarily in terms of a conflict between *Republican-Conservative-Traditionalist-Positivist-Math-as-Skills-Direct-Instruction-Social-Efficiency* Camp and the *Democratic-Progressive-Constructivist-Interpretivist-Math-as-Conceptual-Understanding-Child-Centered-Instruction-Democratic-Equality* Camp. My reference point becomes the drama rather than the particular worldview or a theoretical notion that may be seen to be at the heart of it. By using drama as a reference point, I hope to show that the texts about mathematics attributed to Descartes, Peirce and Russell may be centuries apart but may still be dramatically similar inasmuch as they contain similar descriptions of nature of mathematics and actions of mathematicians. Similarly, mathematics education counter reform texts may bear no direct relation to Cartesian texts, and yet be seen as enacting a similar drama. As far as the drama is concerned, what matters is not the traditional label of the text but the ways it describes the context, the agents, their purposive acts, and the means with which they act.

A rhetorical analysis of the texts that I describe above is offered in response to the following question:

What can we learn about the terms of mathematics education debates of the 1990s by looking closely at some of the terms that figure prominently in key mathematics texts, mathematics education reform texts, and mathematics education counter-reform texts?

To respond to this I take the following set of questions to each set of texts¹⁰:

1. Scene: How is the context of mathematical activity described in classical mathematical texts and in key reform and counter reform texts??
2. Act: How are mathematical acts described in these different texts?
3. Agent: Who is described as entitled to act, that is, to do mathematics in these different texts?
4. Agency: What means are available to act, that is, to do mathematics in these different texts?
5. Purpose: What is described as the purpose of mathematical activity in these different texts?

The five categories mentioned before the statement of question are the elements that organize descriptions in dramatism. Dramatism, as I have mentioned earlier, borrows the metaphor of drama to organize descriptions of actors—in my case mathematicians, reformers, counter reformers, learners, and so on—as engaged in *purposive acts* in specific *scenes* using *agencies* [means] made available to them.

¹⁰ The questions provided here are adapted from five questions that Kenneth Burke takes to descriptions of human actions and their motivations. As Burke put it, “...any complete statement about motives will offer *some kind* of answers to these five questions: what was done (act), when or where it was done (scene), who did it (agent), how he did it (agency), and why (purpose)” (Burke, 1945, p. xv).

Outline of the Dissertation

Chapter 2 provides a primer to Burke's dramatism. My readers should know at the outset that Burke uses terms of dramatism in ways that may sound unfamiliar. Chapter 2, therefore, is my attempt to clarify the terms that will organize the analysis throughout this dissertation.

In Chapter 3, I take the research questions mentioned above to selected classical texts on mathematics. In particular, I reach for descriptions of mathematics and mathematical activity in Rene Descartes' *meditations*, in the writing of the 19th century American mathematician Benjamin Peirce, and in excerpts from the letters of logician Bertrand Russell. By examining the texts that are temporally distant from the reform and counter-reform texts, I want to make a rhetorical point about the vast differences in meaning that inhabit the same words. Terms such as *mathematics*, *mathematical powers*, and *mathematics teaching and learning* carry quite different meanings and implications when articulated in recent reform documents and in these classical mathematical texts.

The drama of the classical mathematical texts unveiled in this chapter will become a dramatisitic reference point for comparison with the more contemporary discourses.

In Chapter 4 and 5, I will examine the reform (Chapter 4) and counter-reform (Chapter 5) texts using the same set of questions. In all of these texts, I observe the ways in which the texts constrain the meaning of mathematics and of what it means to learn and teach mathematics.

Chapter 6 shifts the scene and views of both reforms and counter reforms as competing *acts* on a stage prepared by the calls for reforms. The argument in this chapter highlights the ways in which the language of standards embedded in the calls for reform,

and, by corollary, in the responses to such calls, works to undermine the progressive reforms.

Chapter II: Understanding Dramatism

If man-as-symbol-user, then action; if action, then conflict, if conflict, then drama. And if drama, then you must find a critical language that deals with drama.

(Booth, 1974)

This chapter is designed to introduce the reader to key ideas of Dramatism (Burke, 1945), which is a technique of analysis of language and thought as basically modes of action rather than as means of conveying information. Burke's oeuvre is huge. In this chapter, I will only be concerned with the ideas that I believe my readers must know before they proceed to read this argument. Specifically, the following points will form the body of this chapter:

1. I will describe assumptions about connection between language, conflict and drama. Next, I will describe the five elements of what has been called dramatic Pentad. These are stated succinctly by Booth in the passage with which I begin this chapter, and I will repeat and expand them in the first part of the chapter.
2. I will describe the elements of Dramatism, which are mapped onto the set of questions that I take to each text in Chapters 3, 4, 5, and 6 and the relations between them.

In what follows, I will first discuss the assumptions that undergird Dramatism and then I will explain the technique of analysis Dramatism provides and the key terms that will help in understanding the subsequent analysis.

Assumptions about Relations between Action, Conflict, and Drama

Understanding the idea of action and how Burke distinguishes it from motion is central to understanding Dramatism. Burke makes a distinction between *Action* and *Motion*. For him the terms explaining *motion* describe unambiguously stated concepts, such as, for example, those dealt with in physics. However, human *action* is different from physical [according to Burke, even biological or behavioral] motion, and involves ambiguities. For example, descriptions of human comedies, tragedies, and conflicts defy unambiguous conceptualizations in the manner of science. The broad question to which the language of Dramatism responds is posed by him in these terms: “What is involved, when we say what people are doing and why they are doing it”(Burke, 1945, p. xv). The answer to this question is not a theory of *motives*, but a rhetoric of *forms*. Five forms of thought, he argues, are *necessarily* present in any description of action: “...any complete statement about motives will offer *some kind* of answers to these five questions: what was done (act), when or where it was done (scene), who did it (agent), how he did it (agency), and why (purpose)” (Burke, 1945, p. xv).

Wayne Booth explains the need for Burke’s dramatistic method succinctly: “*if man-as-symbol-user, then action; if action, then conflict, if conflict, then drama. And if drama, then you must find a critical language that deals with drama*” (Booth, 1974, p. 8). This phrase contains the assumptions about *language as a mode of action* and the conflict and drama that becomes visible when we assume language as such. Dramatism, as will become clearer in the following discussion, cannot work without these assumptions. That is to say, if we do not assume the language to be *symbolic action*, we lose sight of the

ambiguity and the possibility of conflict that characterizes drama. Below, I take each part of Wayne Booth's assertion and discuss the assumed relations between the action, conflict, and drama suggested by it.

If Man-as-symbol-user Then Action

I will first discuss what is entailed in regarding language as a mode of action. The first point in that direction is to recognize the ubiquity of language. As Burke puts it:

...can we bring ourselves to realize...just how overwhelmingly much of what we mean by "reality" has been built up for us through nothing but our symbol systems? Take away our books, and what little do we know about history, biography, even something so "down to earth" as the relative position of seas and continents? What is our "reality" for today (beyond the paper-thin line of our own particular lives) but all this clutter of symbols about the past combined with whatever things we know mainly through maps, magazines, newspapers, and the like about the present? In school, as they go from class to class, students turn from one idiom to another. The various courses in the curriculum are in effect but so many different terminologies. And however important to us is the tiny sliver of reality each of us has experienced firsthand, the whole overall "picture" is but a construct of our symbol systems. To meditate on this fact until one sees its full implications is much like peering over the edge of things into an ultimate abyss (Burke, 1966, p. 5).

An example from sociologist Joseph Gusfield will make Burke's point clearer. While conceptualizing his research on auto deaths and alcoholism, Gusfield points toward two distinct terminologies, the *drinking-driver* and *drinking-driving*. The first, as he puts it,

“directs attention to the agent (*driver*) as the source of the act, the second frames the experience as an event...The first, *drinking-driver*, is a call to transform the motorist. The second, *drinking-driving*, directs attention to the auto, the road, the event” (Gusfield, 1989, p. 15). Extending this point to mathematics education debates, I take terms—which will be discussed more fully in the next three chapters—such as *mathematical power* and *mathematical proficiency* as associated with particular descriptions of learning and teaching of mathematics.

Burke tells us that symbols, or terms as I call them, are all we have available to apprehend things and motives. The identities we assume, the roles we accept, the reality of the stars we gaze at, all reach us as words about reality. The support for this claim is not restricted to Kenneth Burke. For example, in the 1950s, linguists Edward Sapir and Benjamin Whorf put forward their eponymous hypothesis that “language shapes thought.” Also, the philosopher of language, John Searle agrees that language is constitutive of thought:

In order that something can be money, property, marriage, or government, people have to have appropriate thoughts about it. But in order that they have these appropriate thoughts, they have to have the devices for thinking those thoughts, and those are essentially symbolic or linguistic devices (Searle, 2006, p. 95).

Similarly, Ian Hacking claims that dividing people into categories in order to count them (as in the case of censuses) created human kinds. He writes about the emergence of ways of counting people, as constitutive of human types:

Even the decennial censuses in the different states amazingly show that the categories into which people fall change every ten years. This is partly because

social change generates new categories of people, but I think the countings were not mere reportings. They were part of an elaborate, well-meaning, indeed innocent creating of new kinds of ways for people to be, and people innocently “chose” to fall into these new categories (Hacking, 2002, p. 49) .

Philosopher of linguistics, J.L. Austin (1962) made a similar distinction between performative utterances and statements. Austin said, “the issuing of a [performative] utterance is the performing of an action—it is not normally thought of as just saying something” (Austin, 1962, pp. 6-7).

Hacking, Searle, Austin, and Burke all make the same point about language as providing individuals with ways of acting—seeing, saying, doing, and thinking etc. Finally, I exemplify the influence of terms that we use by applying this idea to my self-perception as initially simply a mathematics educator, and eventually a reformer mathematics educator.

The idea of language as *doing* something is also exemplified beautifully in Garry Wills’ rhetorical analysis of the Lincoln’s famous Gettysburg Address. Speaking about the meaning of the Civil War, Wills writes, “The Civil War is, to most Americans, what Lincoln wanted it to *mean*” (Wills, 1992, p. 35). And Wills attributes Lincoln’s success in defining the meaning of this conflict for future generations to the Gettysburg Address. Lincoln’s speech, then, is *action*, or a *performative utterance* inasmuch as it alters [or constructs] the reality of civil war for most Americans.

Let me elaborate this further by giving two examples from the mathematics education debates in the United States. First, consider the ways in which language of *counter-reform texts* worked to undermine the term “*progressive*” or *reform* by associating it with

disparaging terms such as *faddist*, *dogmatic*, *unscientific*, *touchy feely*, *un-rigorous*, *fuzzy*, and *permissive*.. At the same time, the term “traditional” in those texts was loaded with an attitude of *flexibility*, *balance*, *rigor*, and *discipline*. While there is no necessary relationship between any label and the terms used to define an attitude toward it, such relationships are strategically constructed in the public discourse to achieve an effect, and, therefore, ought to be examined as *modes of action*.

If Action then Conflict

The assumption about *language as a mode of action* can be extended to an understanding of conflict in general, and conflicts in education particularly. For example, consider two statements about mathematics and their significance as *modes of action*. When we want teachers to act in certain ways in their math classrooms, we say, ‘*mathematics is reasoning*.’ This statement, together with the rhetoric explaining and supporting it, is a way of acting on teachers, schools, and curriculum. Conversely, when we push our audience to accept *mathematics as consisting of apriori objects* then we push them to *act* in a way that makes them appear to be in search of those *apriori objects*. Conflict may be thought of as an engagement between these two *performative utterances*, to use J.L. Austin’s phrase, to define the meaning of mathematics.

In these terms, Chapters 3, 4, and 5 in this dissertation delineate three different *modes of action* and spell out the ways in which they attempt to define the meaning of mathematics and its teaching and learning.

Two aspects of *language as a mode of action* contribute to conflict. The first is the need to come up with definitions to work with. For example, as I will consider in more detail in the subsequent chapters, mathematics is conceptualized and defined. Particular

conceptions of mathematics are then associated with descriptions of mathematical practices or its teaching and learning. That is to say, a *performative utterance* or a *mode of action* involves constructing definitions or, working with existing ones. Second, definitions are inhabited with ambiguity. To understand this, consider the example of the term *standards* in education. As I will discuss in detail in Chapter 6, what counted as the meaning of term *standards* varied depending on its location in various competing discourses. Different actors defined *standards* based on the concepts available to, and valued by, them. Below, I will discuss the conceptual connection between definitions, ambiguity, and conflict, which becomes available when we think of *language as action*.

Definitions and Antagonisms: Defining a term marks off its discursive boundaries.

Setting the boundaries implies an inside and an outside: certain meanings and implications are included in the term – they are declared to be part of its meaning – while others are excluded. Exclusions may be thought of as a prerequisite for conflict. It follows that conflicts accompany the process of setting up clear definitions. Burke—and almost three centuries before him, Spinoza¹¹—acknowledged this “inevitable paradox” in setting up definitions. As Burke puts it:

To tell what a thing is, you place it in terms of something else. This idea of locating, or placing, is implicit in our very word for definition itself: to *define*, or *determine* a thing, is to mark its boundaries, hence to use terms that possess, implicitly at least, contextual reference (Burke, 1945, p. 24).

¹¹ This notion of any definition of a *thing* as making sense only in terms of *what it is not*, i.e. in terms of its context in which it appears to be rooted is associated with Spinoza’s notion of “determination is negation”(For a discussion on this, see, Burke, 1945; Morgan, 2002, p. 892).

Setting up definitions, standpoints, and perspectives is, as the above quote suggests, also an act of *marking off* the boundaries, thus creating an inside and outside. This can set up the *inside* and the *outside* as each others' antagonist. The point I am making is that definitions, standpoints, perspectives etc. become possible through defining an outside. As Burke puts it, "To tell what a thing is, you place it in terms of something else. This idea of locating, or placing, is implicit in our very word for definition itself: to *define*, or *determine* a thing, is to mark its boundaries, hence to use terms that possess, implicitly at least, contextual reference. We here take the pun seriously because we believe it to reveal an *inevitable* paradox of definition..." (Burke, 1945, p. 24).

Given above is indeed a very geometric description of definition. That is to say, just as in coordinate geometry specific coordinates in space are defined in relation to a *frame of reference* which positions them in relation to a fixed set of coordinates. We also place the coordinates of *language as action* in relation the coordinates of *language as not action*. Burke describes, and also complicates this relation as follows:

Contextual definition might also be called "positional," or "geometric," or "definition by location." The embarrassments are often revealed with particular clarity when a thinker has moved to a high level of generalization, as when motivational matters are discussed in terms of "heredity and environment," or "man and nature," or "mind and matter," or "mechanism and teleology," where each of the paired terms is the other's "context" in the universe of discourse (Burke, 1945, p. 26),

Take for example the first pair, "heredity and environment". Burke seems to be saying that these binaries reveal embarrassments, because of what I interpret to be,

mutual exclusions. That is, when one generalizes human behavior in terms of heredity, excluding environment, then one forgets that environment is needed for the definition of heredity. That is to say, it is only through excluding the environment [Read NOT heredity] that heredity becomes possible. Hence, in a certain important sense, heredity is *because of* what it is not. Likewise, the *progressive/conservative* binary may also be treated similarly. *Progressive* defines or positions itself *in relation* to *conservative*, and would lose its identity if there was no such reference point. That is to say, *progressive* is defined partly in terms of what it is not.

To wrap up this discussion on the effects of definition on the possibility of *conflicts*: Defining something is construed in this discussion as an *action* that works to set the boundaries of a concept. Setting up boundaries, Burke lets us see, inevitably involves exclusions and inclusions. The exclusions haunt the definition from the outside. It is interesting that any definition—including definitions of identities such as *progressives* or *conservatives*—becomes possible only by marking off, and thus defining an outside. But their condition of possibility also works to undermine them. The conflict is only to be expected from what is excluded in the process of formulating a definition or an identity. For example, while it is impossible to even think about *progressive* without marking it off from *conservative*, the former is also threatened by the latter and vice-versa. Since they are necessarily based on selections, definitions both reflect and deflect aspects of what they attempt to define. Burke refers to these clusters of terms as *terministic screens*. “Terministic screen” is Burke’s term for a sort of grid of intelligibility, consisting of a cluster of terms, through which we make sense of the world. When it is definite, it would, Burke argues, select some aspects of the reality it is describing and

deflect others. That is to say, Burke is denying the possibility of totalizing theoretical constructs about human motives. In this dissertation, the descriptions of relations between mathematics and its teaching and learning in mutually agonistic discourses are treated as terministic screens.

Men seek for vocabularies that will be faithful *reflections* of reality. To this end, they must develop vocabularies that are *selections* of reality. And any selection of reality must, in certain circumstances, function as a *deflection* of reality. Insofar as the vocabulary meets the needs of reflection, we can say that it has the necessary scope. In its selectivity, it is a reduction. Its scope and reduction become a deflection when the given terminology, or calculus, is not suited to the subject matter which it is designed to calculate (Burke, 1945, p. 59).

Thus, conflict appears to follow from earnest efforts to define, refine, and perfect a discourse, which ends up in exclusions and inclusions. As Burke puts it, “we are rotten with perfection” (Burke quoted in, Booth, 1974, p. 12). I interpret “rottenness” here to be indicating the ultimate paradox of positioning oneself rigidly and attempting to perfect that position. It reminds me of the plot of the movie *Quills* (2000). The plot [or *scene* to use the language of Dramatism] contains the famous French character Marquis de Sade held as an inmate in a French insane asylum in the 1790s and his reformer, a priest. The scene unfolds as a long engagement between the reformer and the to-be-reformed. But the perfect act of reform is symbolized by elimination of Marquis de Sade from the scene, signifying the elimination of all the evil he stood for. However, ironically, as the priest eliminates de Sade, he himself assumes the character of de Sade. In the Burkean sense,

the priest “rots with perfection,” through a paradoxical operation involving the elimination of what he loathed.

As mentioned above, the Chapter 3, 4, and describe three *different modes of action*. By the same taken, they are also descriptions of three different ways of defining mathematics and its teaching and learning. The argument in these chapters will emphasize these texts as presenting terministic screens about the reality of mathematics and its teaching and learning. To summarize, the terministic screen is a sort of cluster of terms that collectively afford as well as constrain the object of their description. In chapters three, four, and five, terministic screens constitute the cornerstone of analysis. In these chapters, I use the pentad to describe the cluster of terms for *agent*, *act*, *scene*, *purpose*, and *agency*. These descriptions constitute the terministic screen because they define the ways in which the drama of mathematics and its teaching and learning is permitted to unfold. And by virtue of being definitive, they are also exclusive.

The dialectic of ambiguity and clarity: We define things because we want to be understood clearly. Yet, the story of conflict, tragedy, and even comedy is one based on misunderstanding, or understanding differently, the terms which are otherwise assumed to be stated clearly. From the *math wars* we know that even the term *mathematics*, when caught in two different discourses—that of mathematics and education—is rendered ambiguous. The associated ambiguity is reflected in the following observation on the mathematics education reform texts by the Berkeley mathematician Hsi-Wu:

The shock comes from the discovery that what passes for mathematics in these publications bears scant resemblance to the subject of our collective professional life. Mathematics has undergone a re-definition, and the ongoing process of

promoting the transformed version in the mathematics classrooms of K-14 (i.e., from kindergarten to the first two years of college) constitutes the current mathematics education reform movement (Wu, 1998, p. 1).

Viewed in the light of the above discussion on the relations between *defining* as a *mode of action* and *conflict*, Wu's expression of dismay on not recognizing his professional discipline in the definitions of reform texts is also a refusal of the system of relations within the reform texts, which, according to him, shapes and defines what mathematics could be. That mathematicians and mathematics educators do not talk about the same thing when they refer to mathematics is also observed by some prominent mathematics educators (see, for example, Sfard, 1998). When mathematics educators defined *mathematics* without including the perspective of folks like Hsi-Wu, ambiguity ruled, and debates followed. Ambiguity, thus, has the power to move debates in action. As my analysis in Chapters 3 and 4 will show, the term mathematics and the notions about its learning and teaching mean different things, depending on who speaks it, to which audience, in what context, and for what purposes.

So same terms may have different meaning because of being situated in two different dramas. As Burke puts it: "Since no two things or acts or situations are exactly alike, you cannot apply the same term to both of them without thereby introducing a certain margin of ambiguity, an ambiguity as great as the difference between the two subjects that are given the identical title" (Burke, 1945, p. xix). Below, I change this quote slightly to illustrate the ways in which the assertions about ambiguity are relevant to my study: "Since no two *situations in which the term mathematics is spoken* are exactly alike, you cannot apply the term *mathematics* to both of them without thereby introducing a certain

margin of ambiguity, an ambiguity as great as the difference between the two subjects that are given the identical title.” The utterance *mathematics* is ambiguous, and when ambiguous it is conflictual. Action, drama, and conflict are brought into a relation because of the contingent ambiguity that accompanies our efforts to define and clarify concepts.

Based on the discussion above, I understand ambiguity as a suggestion that words cannot shift from one discursive neighborhood to another without losing or changing their meaning. Burke speaks quite dramatically of these one-to-many relationships between the words and their particular investments by asserting that we do not even know what was said when we hear someone uttering a word as simple as ‘yes’: “Let us suppose that I ask you: “What did the man say?” And that you answer: “He said ‘yes.’” You still do not know what the man said. You would not know unless you knew more about the situation and about the remarks that preceded his answer... For there is a difference in style or strategy, if one says “yes” in tonalities that imply “thank God” or .. in tonalities that imply “alas!”” (Burke, 1966, p. 77).

Thus, Dramatism assumes, privileges, looks for, and describes ambiguities associated with specific terms:

A perfectionist might seek to evolve terms free of ambiguity and in consistency (as with the terministic ideals of symbolic logic and logical positivism)... We take it for granted that, insofar as men cannot themselves create the universe, there must remain something essentially enigmatic about the problem of motives, and that this underlying enigma will manifest itself in inevitable ambiguities and inconsistencies among the *terms for motives*. Accordingly, what we want *is not*

terms that avoid ambiguity, but terms that clearly reveal the strategic spots at which ambiguities necessarily arise (Burke, 1945, pp. xviii, italics mine).

If Conflict then Drama

So how does drama follow from the conflict? Drama happens on a stage, and involves actors who appear to be acting purposefully. These actors, love, hate, kill, die, laugh, cry, and do many other things when playing their part. Production of drama requires presence of *motives* that must remain essentially ambiguous. For example, in Shakespeare's plays ambiguity is said to play an important role when the playwright makes his characters say something whose significance cannot be grasped with reference only to the time of their utterance. "For what they say might have two meanings. The one meaning which the speaker has in mind refers to the momentary situation, but the other meaning may point beyond this moment to other issues of the play" (McDonald, 2004, p. 51).

This theme about the function of ambiguity in conflict and drama will be revisited in Chapter 6, where I will dwell upon the ways in which the utterances of the reform texts respond to a momentary scene as well as to other moments in the historical scene. When reformers uttered the term *standards* they responded to a current scene which called for *measurable standards* as a large part of the educational reforms. However, they filled the term standards with a meaning that went beyond the contemporary historical context, thus making the term standards ambiguous. Conflict and drama followed, when different people used the term *standards* but meant different things.

Ambiguity characterizes drama and sets it apart from what Burke terms scientific explanations. Scientific explanations work to eliminate ambiguity. To the contrary, drama thrives on ambiguity.

The dramatic/scientific distinction is complemented by action/motion distinction. In the spirit of definition by placement, Burke sets up Dramatism as a means of analysis of human action, while leaving the analysis of motion to science. The concept of action works to delineate motivated and purposeful human acts from mechanical responses to situations. As Burke puts it: “As for “act,” any verb, no matter how specific or how general, that has connotations of consciousness or purpose falls under this category.” (Burke, 1945, p. 14). This Burkean distinction between motion and action, and identification of action with language and rhetoric is stressed in contemporary writings on Burke. Robert Wess elaborates this distinction by linking motion to life and death as biological: “Rhetoric is linguistic action, but language-users live and die in the biological realm of motion” (Wess, 1996, p. 112).

In this section, I have argued that Dramatism becomes thinkable against a background of assumptions about language as action, about the paradoxical nature of framing definitions of ideas and ambiguity of terms and their clusters. As Burke puts it, Dramatism as “a method of analysis and a corresponding critique of terminology is designed to show that the most direct route to the study of human relations and human motives is via a methodical inquiry into cycles or clusters of terms and their functions” (Burke, 1968, p. 445).

To summarize, in this section, I described ways in which assuming *language as a mode of action* helps us see conflict and debate as arising from attendant ambiguities.

To analyze the *terms* in mathematics education debate, I have used the five part framework of Dramatism—also called the *Pentad*—which is outlined in Burke’s *Grammar of Motives* (Burke, 1945). I will turn to its description in the next section.

Elements of Dramatism

At its simplest, a dramatistic analysis is built around a response to the following broad question and its specific variations: “What is involved, when we say what people are doing and why they are doing it?” (Burke, 1945, p. xv). By way of response, Burke offers what he argues to be:

...basic forms of thought which, in accordance with the nature of the world as all men necessarily experience it, are exemplified in the attributing of motives. These forms of thought can be *embodied profoundly or trivially, truthfully or falsely*.

They are equally present in systematically elaborated metaphysical structures, in legal judgments, in poetry and fiction, in political and scientific works, in news and in bits of gossip offered at random (Burke, 1945, p. xv).

The five elements of Dramatism are referred to as forms because of their emptiness, generative principles of investigation because of their capacity to generate dramatistic analysis, and sometime simply the elements of *elements of Dramatism*. For any particular narrative these elements are signified by:

... some word that names the *act* (names what took place, in thought or deed), and another that names the *scene* (the background of the act, the situation in which it occurred); also, you must indicate what person or kind of person (*agent*) performed the act, what means or instruments he used (*agency*), and the *purpose*.

Men may violently disagree about the purposes behind a given act, or about the character of the person who did it, or how he did it, or in what kind of situation he acted; or they may even insist upon totally different words to name the act itself. But be that as it may, any complete statement about motives will offer *some kind* of answers to these five questions: what was done (act), when or where it was done (scene), who did it (agent), how he did it (agency), and why (purpose).
(Burke, 1945, p. xv)

Act, Scene, Agent, Agency, and Purpose, then are the *forms* that form the system of reasoning that Dramatism offers us. The next important point to explicate is that these terms do not refer to fixed categories. For example, an agent does not have to always be a person, and agency does not always refer to some intrinsic ability to act freely in this world. A somewhat longer quote from Burke provides us with a glimpse of the malleability of these terms:

A portrait painter may treat the body as a property of the agent (an expression of personality), whereas materialistic medicine would treat it as "scenic," a purely "objective material"; and from another point of view it could be classed as an agency, a means by which one gets reports of the world at large. Machines are obviously instruments (that is, Agencies); yet in their vast accumulation they constitute the industrial scene, with its own peculiar set of motivational properties. War may be treated as an Agency, insofar as it is a means to an end; as a collective Act, sub divisible into many individual acts; as a Purpose, in schemes proclaiming a cult of war. For the man inducted into the army, war is a Scene, a

situation that motivates the nature of his training; and in mythologies war is an Agent, or perhaps better a super-agent, in the figure of the war god.

(Burke, 1945, p. xx)

So although the analysis works with the five elements of drama mentioned above, they are not used as ‘unvarying, frozen, literal categories...but as fluid reagents, applicable in different “ratios”¹²—to be explained shortly—for different problems. What is one agent’s action is another agent’s scene. A given agent can be of someone else’s agency—a tool to other ends—or he can be, again, a part of someone’s scene ”(Booth, 1974, p. 11). Also notice that I have used this malleability in the characterization of the elements of pentad to organize this chapter. For example, to explain Dramatism--which you may simply define as a method or technique of analysis—I have thought of it as an agency. The assumptions about *language as a mode of action*, then, form the *scene* against which the *agency* of Dramatism makes sense.

Dramatistic use of these terms is further exemplified by the following quote in which Burke is examining an excerpt from John Dewey’s *Intelligence and Modern World*.

...though Dewey stresses the value of "intelligence" as an instrument (agency, embodied in "scientific method"), the other key terms in his casuistry, "experience" and "nature," would be the equivalents of act and scene respectively.

We must add, however, that Dewey is given to stressing the overlap of these two terms, rather than the respects in which they are distinct, as he proposes to

¹² I should acknowledge that Burke’s use of the term ratio will sound very unconventional to the readers familiar with the term in mathematics. Ratio is not used here in its usual mathematical sense, but in the sense of logic of the terms. While in mathematics ratio appears as a relation between two similar magnitudes in respect of quantity, determined by the number of times one contains the other (integrally or fractionally), Oxford English Dictionary also defines ratio as reason. Dramatism’s meaning of the term *ratio* is closer to this latter sense.

"replace the traditional separation of nature and experience with the idea of continuity. (Burke, 1945, pp. xxi-xx)

The sense in which Dewey might have deployed experience in relation to nature, then, inserts them as *act* and *scene* respectively in the space of Dramatism.

The Analysis

A scholar using Dramatism to examine a text does more than simply decompose the texts in terms of pentadic categories; he also uses Dramatism to develop insights about relations between the various pentadic terms. I will elaborate these points through examples of some texts.

Example of texts with stress on one or the other Dramatistic Element

Almost a year ago, as part of my many efforts to learn about the history of American public education, I stumbled on an obscure text, written by a prominent name in American educational history. I am speaking of William Bagley's book 'A century of universal school' (Bagley, 1937). Bagley's book was an appraisal of universal schooling at the turn of the century, not just in the US, but also in other countries of the world. I was particularly struck by a passage on Iraq. Bagley's observations on Iraq pointed toward a facet of Iraqi society that resonates so with current accounts of the tribal configurations in Iraq:

Another serious educational problem in Iraq illustrates very clearly how dangerous it is to assume that generalizations regarding human affairs can be applied to all peoples. We hear a great deal today about the importance of reducing the sentiments that attach to the idea of nationalism. But Iraq needs a

healthy growth of the feeling of nationalism as much as she needs literacy. The loyalties of the people are tribal loyalties. Intertribal warfare has always been taken for granted among the nomads. It has persisted over the ages--and for a reason that is biologically sound. It is a means of keeping the population within the limits of a very precarious food-supply. When the tribes settle, however, a different situation develops; and yet, just as the powerful mores against the use of water persist, so the tribal loyalties and enmities persist, making very difficult the enforcement of law and order—making impossible, indeed, an effective national government on a democratic basis (Bagley, 1937, p. 50).

On this view, the tribal motivations for internecine warfare were explained in terms that may be regarded as ‘natural.’ If we accept this account as a valid explanation of the Iraqi situation, then the Iraqi people would come across as not really engaged in a tribal warfare, but as merely moved to perform a natural function, that of keeping the population in check in the face of a precarious food supply. This naturalist account, then, does not stress Iraqi people as *agents*. Rather it stresses the ‘precarious food supply’ as an element of *scene* that determines their motivation to kill. Furthermore, with internecine strife as part of the scene marked by precarious food supply, Bagley declares a democratic government in Iraq as impossible. Thus, in this account, the precarious supplies of food, the internecine strife, and the impossibility of democracy are clustered together in such a way as to stress the *scene* and construe the *acts* of as completely determined by a *scene* characterized by a precarious food supply. The tribes do not *act* to kill but follow a law of nature that explains conflict as a means to regain a balance between the availability of food and size of population.

Contrast the above mentioned account of Iraqis with another account borrowed from Kenneth Burke in *Grammar of Motives* (Burke, 1945) ¹³:

Many people in the Great Britain and the United States think of these nations as "vessels" of democracy. And democracy is felt to reside in us, intrinsically, because we are "a democratic people". *Democratic acts are, in this mode of thought, derived from democratic agents*, agents who would remain democratic in character even though conditions required the temporary curtailment or abrogation of basic democratic rights (Burke, 1945, p. 17).

Notice that, in contrast to Bagley's text, this construes the *scene* [of democracy] as emanating from the nature of the *agents*.

Burke extended this analysis to all major philosophical narratives of his time, proposing that materialism, idealism, pragmatism, mysticism, and realism all feature one or the other terms of the dramatistic pentad: The discourse of materialism—which explains the universe sufficiently on the assumption of body and matter is aligned with scene; Idealism—which accounts for universe as the work of reason and mind—features agent¹⁴; Pragmatism—with its concern for instruments to act—features agency¹⁵; Mysticism—with its concern for the valid ends for human existence—is aligned with the

¹³ The quoted passage does not present a theoretical position that Burke necessarily subscribes to. Rather, he has used this passage as an example to clarify a distinction between the discourses that privilege *agent* compared with the discourses that feature *scene*.

¹⁴ For details, see Burke (1945, p. 131 & 171).

¹⁵ Burke's association of Pragmatism with agency seems due to pragmatists' theory of knowledge as means to desirable ends. He says: "In accordance with our thesis, we here seize upon the reference to *means*, since we hold that Pragmatist philosophies are generated by the featuring of the term, Agency. We can discern this genius most readily in the very title, *Instrumentalism*, which John Dewey chooses to characterize his variant of the pragmatist doctrine. Similarly William James explicitly asserts that Pragmatism is "a method only" (Burke, 1945, p. 275).

element of purpose¹⁶; Finally, act is featured in realism.¹⁷ This idea of prominence of any one term from dramatistic pentad in major philosophical themes is important to my subsequent analysis.

Concluding thoughts: Analyzing the Relations between Terms of the Pentad

When a text stresses *scene* or *agent*—as in the case of Bagley’s texts from which I quoted above, it also shapes the relations between various terms of the pentad. The analytic stress, then, is not just on the terms of pentad but also on the relations between them. Burke calls these relationships “ratios”. A ratio, according to Burke, is a formula indicating a relation of precedence from one term to another. For example, a *scene-act* or a *scene-agent* ratio gives priority to the *scene*, or background, in relation to the *act* or *agent*. That is to say, the *scene* sets the parameters of what *agents* and *acts* can be.

So we cannot speak of an *act* or an *agent*, without also speaking of the *scene*. Speaking in terms of ratios such as *scene-act* or *scene-agent* appears useful when assigning relative weights to the elements of the pentad. The use of these ratios in dramatistic analysis is exemplified by the explanations for existence of democracy in some countries in the above quote from Burke in the last section. Bagley’s text stressed

¹⁶ According to Burke, mystical philosophies appear in times of social chaos or confusion about human purpose. “They are a mark of transition, flourishing when one set of public presuppositions about the *ends* of life has become weakened or disorganized, and no new public structure, of sufficient depth and scope to be satisfying, have yet taken its place. Thus, precisely at such times of general hesitancy, the mystic can compensate for his own particular doubts about human purpose by submerging himself in some vision of a *universal*, or *absolute* or *transcendent* purpose, with which he would identify himself” (Burke, 1945, p. 288).

¹⁷ This association, according to Burke, is rooted in realism’s concern with actual embodying a sort of Act-Actual ratio. Another example: if one replaces Act and Actual by God and Universe respectively in this formulation, the real universe would appear to follow from a supreme act of creation—so it does in the discourse of creationists.

scene more as compared with the *act*. By featuring the *scene* it set up a *scene-agent* and *scene-act* ratio which worked to explain Iraqi people and their actions in terms of contextual factors such as scarcity of food. On the other hand, the example that Burke gives [which, I emphasize, is not his own theoretical stance] derives *democratic acts* from the characteristics of an inherently democratic people. Notice that the text which stresses *scene* defines *acts* in relation to the context [definition by placement] and the text which stresses acts describes scene as derived from acts [definition by derivation].

In an important way, then, a dramatistic ratio signifies a move from *potential* to *actual*. As Burke puts it: "...a mode of thought in keeping with the *scene-agent* ratio would situate in the *scene* certain potentialities that were said to be actualized in the *agent*. And conversely, the *agent-scene* ratio would situate in the agent potentialities actualized in the scene" (Burke, 1945, p. 262).

Insofar as men's actions are to be interpreted in terms of the circumstances in which they are acting, their behavior would fall under the heading of a "scene-act ratio." But insofar as their acts reveal their different characters, their behavior would fall under the heading of an "agent-act ratio." For instance, in a time of great crisis, such as a shipwreck, the conduct of all persons involved in that crisis could be expected to manifest in some way the motivating influence of the crisis. Yet, within such a "scene-act ratio," there would be a range of "agent-act ratios," insofar as one man was "proved" to be cowardly, another bold, another resourceful, and so on (Burke, 1978, pp. 332-333).

It is important to recognize that a ratio, in bringing together a potential-actual relation, is not necessarily a cause-effect relation. As Burke put it: "It is not easy to know

just when one is deriving potentialities from actualities and the reverse” (Burke, 1945, p. 260).

Let me now briefly allude to how these insights are put to use - and also complicated - in the analysis of mathematics education texts. In Chapter 3, the texts that I examine feature mathematics as *scenic* thus setting up a scene-agent and scene-act ratios. That is to say, mathematicians and their mathematical acts are completely determined by the scene. When mathematics is construed to be *a priori*, all the agents can do is discover it. However, if we reverse these ratios, as it happens in the texts examined in Chapter 4, we will also need to reconfigure the nature of mathematics to keep it coherent with the act-scene ratio. Mathematics, in such a text, will need to be conceived as *being constructed* instead of *being out there*.

Throughout this dissertation, the analysis involves exploring the consequences of shifting the values of the dramatistic place holders, i.e. the *scene, agent, act, agency, and purpose*. What changes in the relations between *agent, acts, agencies, and purposes* come into play as the scene shifts? The analysis will sort the texts in terms of the dramatistic categories first, examining the ways in which these categories relate to each other in particular texts, and answering the question about what insights can be developed about mathematics and mathematics education based on these relations.

Chapter III: Mathematics, God, and Pedagogy

Dramatistic Pentad	Classical Texts	Reform Texts	Counter-Reform Texts
Scene	<i>Mathematics</i>	Mathematics Education—Based on NCTM Standards	Standards
Act	<i>Following, Reading, Discovering mathematics</i>	Constructing Mathematics	Following, Reading, demonstrating fluency
Agent	<i>Mathematicians</i>	Children, Teachers	Children
<u>Agency</u>	<u>Mathematical Powers</u>	<u>Mathematical Powers</u>	<u>Mathematical proficiency</u>
Purpose	<i>Illumination by natural light of reason</i>	Empowered citizenry	Numerate Citizenry

Table 1: Pentad at a Glance—Focus on Classical Texts

Reuben Hersh (1999) in his book *What is Mathematics, Really?* identifies two parallel descriptions of mathematics, labeling them *Mainstream* and *Humanist*:

For the Mainstream, mathematics is superhuman—abstract, ideal, infallible, eternal. So many great names: Pythagoras, Plato, Descartes, Spinoza, Leibniz, Kant, Frege, Russell, Carnap... Humanists see mathematics as a human activity, a human creation. Aristotle was a humanist in that sense, as were Locke, Hume, and Mill. Modern philosophers outside the Russell tradition—mavericks—include Peirce, Dewey, Roy Sellars, Wittgenstein, Popper, Lakatos, Wang, Tymoczko, and Kitcher (Hersh, 1999, p. 92).

Among the humanists, Hersh also includes psychologist Jean Piaget, and mathematics educators Paul Ernest and Anna Sfard. Some mathematicians have also made it onto Hersh's list of humanists, prominent among them Henry Poincaré and George Polya.

From these lists we may make two important observations. First, the *mainstream* description of mathematics extends all the way from Pythagoras to our times (Davis & Hersh, 1981; Sfard, 1998). Second, that the *mainstream/humanist* distinction also suggests a schism between most mathematicians and most mathematics educators.

Hersh says that these descriptions of mathematics are not just diachronic. That is to say, we cannot assume a linear progress from a *mainstream* view of mathematics to a *humanist* view of mathematics. Rather, the *mainstream* and *humanist* views exist side by side. As someone who has moved past all of these discourses, while changing careers from physics to mathematics education, I agree! Mathematics has changed its meaning for me several times in a lifetime. As a student in elementary and high school, I experienced mathematics as an abstract and meaningless but strictly rule-driven play of symbols. Formalists, I learned later, have something similar to say about the nature of mathematics. As a university student and later a teacher of physics, I experienced mathematics as a collection of mysterious objects, which, if applied appropriately and wisely, spoke, somehow, nothing but truth about the physical universe at scales unimaginably large and small. As a mathematics educator, I came to see mathematics as a product of human activity and socio-culturally constructed.

The mainstream and humanist approaches to mathematics practices are usually so insulated that each appears distant to the other. That is, as a physicist, I had no clue of, or even felt the need for a socio-cultural dimension of mathematics. But when working as a

teacher and mathematics educator, I came to work within a perspective on mathematics grounded in the context of schooling. While the classical perspective on mathematics—*mainstream*, to use Hersh’s phrase—projected mathematics as a bastion of absolute and objective certainty, in the socio-cultural perspective—*humanist* in Hersh’s terms—as Sfard puts it, there was “no absolute truth any longer” (Sfard, 1998, p. 491). Ultimately on the educational landscape the distinction between the *humanist* and *mainstream* plays out, as I will discuss in more detail in Chapters 4 and 5, in terms of *mathematics as invention* and *mathematics as discovery*, with the former inscribed in the reform and the latter in the counter-reform texts.

The difference between the mathematicians’ and mathematics educators’ descriptions of the nature of mathematics has assumed a renewed importance in the wake of recent mathematics education debates, and hence are the focus of this dissertation.

This chapter explores the descriptions of the nature of mathematics in some selected *mainstream* texts, organizing these descriptions in terms of the five elements of Burke’s (1945) dramatism, each defined by the following questions:

1. *Scene*: How is the context of mathematical activity described?
2. *Act*: How are *mathematical acts* described?
3. *Agent*: Who is described as entitled to *act*, that is, to do mathematics?
4. *Agency*: What means are available to *act*, that is, to do mathematics?
5. *Purpose*: What is described as the *purpose* of mathematical activity?

The Texts: To explore the answers to these questions, I do not survey all the texts produced by all the authors in Hersh’s list, and I include one mathematician—the 19th

century Harvard mathematician Benjamin Peirce¹⁸—who, curiously enough, has not made it to Hersh *mainstream* listings. Specifically, I explore the writings of René Descartes (1596-1650) because of a widely held perception about him as the father of modernity, Benjamin Peirce (1809-1880), and Bertrand Russell (1872-1970). While Descartes may not need much of an introduction here, Benjamin Peirce is not a name that one frequently encounters, as is evident from his absence Hersh's list of *mainstream* mathematical thinkers. However, given his half a century long tenure at Harvard, Peirce was uniquely positioned to influence American mathematics. Peirce was Hollis Professor of Mathematics at Harvard University from 1833 until his death in 1880. He also served as president of the American Association for Advancement of Science for seven years from 1847 to 1954. Historian Daniel Cohen sums up Peirce's influence on the American mathematics community as follows:

Holding a position at Harvard for a pivotal half-century in which the university shifted its character away from clerical training toward a new research model, he trained and guided the work of two generations of mathematicians who would go on to train and guide countless others. Personally Peirce was responsible for numerous theories and applications, and his linear Associative Algebra (1870) was a landmark both for its new form of algebra and for its statements regarding the nature of mathematics itself (Cohen, 2007, p. 42).

Bertrand Russell is usually known to the world of philosophy of mathematics as the logicist *par excellence*. His monumental work with Alfred North Whitehead (1861-1947) *Principia Mathematica* (Whitehead & Russell, 1997) is usually known as an attempt to find logical foundations to all mathematical truths, i.e. to reduce mathematics to logic. However, for the purpose of analysis in this chapter, I was interested in Russell's

¹⁸ Benjamin Peirce was father of pragmatist philosopher Charles Sanders Peirce.

statements about the nature of mathematics. I have used what I found in *Selected Letters of Bertrand Russell* (Griffin, 2002).

The particular texts in which I searched for statements about the nature of mathematics, mathematical practices, and mathematical purposes are Descartes' *Fifth Meditation* as well as the documented objections to it. For Descartes' writings, I have consulted Cottingham's compilation of Descartes' philosophical works (Descartes, 1984) and translations of Descartes by Wollaston (Descartes, 1960). I access Benjamin Peirce's ideas through his book *Ideality in Physical Sciences* (Peirce, 1881) and his textbook *Linear Associative Algebra* (Peirce, 1882). I have also looked at the writings of Peirce's colleagues and students, and drawn upon a recent historical survey of the connections between religious faith and mathematics in 19th century (Cohen, 2007), looking for clues to answer the questions I posed above.

Below, I foreshadow the argument that I develop in the rest of the chapter.

Scene: The nature of mathematics in both Descartes and Peirce can be read as grounded in a peculiar mix of theology and intellect, which renders mathematics as existing in a heavenly realm in the mind of God. These mathematicians' descriptions of mathematics stress *scene*—the existence of a preexisting mathematical landscape.

Acts like discovering, reading, following: When *scene* is a priori mathematics, *acts* are described by the dramatistic *scene-act* ratio. That is to say, if the *scene* is already given, then *acts* cannot construct it. Thus the mathematician is positioned in a math land in whose construction he does not play any role. Contrast this with the Hersh's *humanists* whose description of mathematics as a human activity will need a reversal of this ratio.

Agent: The mathematician comes across as a seeker in the math land, or a prophet receptive to the mind of the God.

Agency: Powers to practice mathematics lie beyond normal human faculties. If mathematics *exists in the mind of God*, the powers to do mathematics are like spiritual powers needed to provide access to the mind of God.

Purpose: To *know the mind of God*

Examining the implications of this description for teaching and learning of mathematics, I argue that mainstream mathematics is incompatible with the humanist mathematics practiced by math educators. That is, if we accept the arguments of Descartes, Peirce, and Russell, mathematics learning and teaching cannot be thought about in the ways in which we educators think about it in the public school contexts. When mathematical truths exist in the mind of God as *a priori* truths, the capacity of individuals to do mathematics, the so-called *mathematical powers* must also be understood as gifts of God.

This point may be illustrated metaphorically by considering a mountain range as a *scene*, climbers as *agents*, and climbing as an *act*. Climbers do not invent or construct the peaks that they scale. We believe that the job of climbers is to find the best routes and use them to reach the summits. Once a particular summit has been scaled, the beaten paths come on record, and become available for future generations of climbers. These climbers may also discover new, more efficient, and safer paths to reach the summit. Yet, they, and those before or after them, cannot change the shape of the mountain.

Like the mountain range and its many summits, mathematics comes across in the classical texts as *a priori*. Just as the mountain range *as a scene* contains the summits

and the climbers, as well as their attempt to scale the summits, mathematics also works as a scene. The texts examined in this chapter are those that assume the existence of a *priori* math world—a universe populated with mathematical forms. On this view, the mathematicians *act*—that is, they do mathematics—to gain access to the pre-existing world of mathematics. Since the world of mathematical forms pre-exists us, it constrains what can or cannot be done as mathematics. In these texts the preexisting world of mathematics is described in two ways. First, in the Cartesian *Meditations* mathematics is conflated with the mind of God thus commingling religion and mathematics. Second, in the Platonist texts (Russell) it is construed as existing in a metaphysical realm independent of mental and physical reality.

The idea of mathematics as conveyed in these texts can be analyzed according to Burke's pentad as follows: The mathematician [*agent*] reads the mind of God or discovers mathematical objects, or follows the paths already tread by other mathematicians, in a preexisting mathematical landscape [*scene*]. Since mathematics is either in the mind of God or existing in a metaphysical math world, the powers to do mathematics come across as abilities to read, comprehend, and follow the mind of God. Those with the gift are the elect who can gain to access the math world. Such specification of the nature of mathematics implies an elitist approach to knowledge focused on identifying the gifted. I make no claims that such is the perspective of all contemporary mathematicians. I also make no claims about whether such a perspective is inherently good or bad. Rather, I perform this rhetorical analysis in order to help identify a *cluster of terms*—that is, a *terministic screen*—that is just as internally coherent as the one that motivates us mathematics educators, teacher educators, and reformers.

However, the mainstream conception operates with a set of truths about mathematics that is different from that of the humanists, and by implication their respective assumptions about teaching and learning are also different from one another.

The Mountain Range: Mathematics as the Scene in Classical Texts

There are striking similarities between the perspectives of René Descartes (1596-1650) and Benjamin Peirce (1809-1880) inasmuch as both preserve the nature of mathematical reason as the *God-given light of the mind* and the connection of this form of rationality with the heavenly and divine. Both construe mathematics as *a priori* and divine. By virtue of being *a priori* and divine this perspective frames mathematics not just as preexisting but also as containing the totality of mathematical objects that can be accessed by the mathematicians. It is this description of mathematics that, I argue, provides it with the connotation of *scene* in the sense in which Burke uses the term, as containing both the *agent* and the *act*, and when mathematics is the scene—the given—then learning mathematics becomes a discovery process, and the role of the teacher becomes that of forebear.

Recall from Chapter 2 that the *scene* contains the *agents* and their *acts*. Furthermore, there are narratives that stress the *scene*, while there are others that stress *agents*. In a particular narrative on the nature of mathematics that stresses mathematics as a scene, we will expect the terms for mathematicians and mathematical practice to exhibit *scene-agent* and *scene-act* relationships. What a mathematician *can* or *cannot* do will depend on what kinds of agents and acts can be contained in the scene without destroying the

coherence of the narrative.¹⁹ For example, if the narrative describes mathematical objects as existing independently of the human acts, then mathematics must be discovered; mathematics can never be invented in such a narrative.

In the texts of Descartes and Peirce mathematics comes across as belonging to a divine realm. This may be as surprising to others as it was for me. The name of Descartes is usually spoken in relation to rationalism. Rationalism, rationality, and reason have come to be understood as in opposition to religious dogma. When Descartes is read as the father of modern rationalism, the relation of the divine and religious with Cartesian reason is not remembered. Reading mathematical rationality as at the roots of the modern scientific enterprise, which boasts to be secular, obscures the cordial relationship that mathematics and religion enjoyed until the late nineteenth century. It was significant that I could recall nothing from my early encounters with mathematics that suggested a deep relation between mathematics and divine. Yet, the relationship between mathematics and divinity remained alive and well until very recently (Cohen, 2007). Professionalization is a kind of secularization, and at the end of the nineteenth century, the professionalization of mathematics served to sever the link between religion and mathematics (Cohen, 2007).

Yet, as expressed through the claims earlier, Platonism reemerges from this severing of the link between religion and mathematics. Theologizing mathematics had only located it in the mind of God. Modern Platonism released it from God's house only to relocate it (at least for some mathematicians) in some a metaphysical realm. As Hersh observes, "the trouble with today's Platonism is that it gives up God, but wants to keep

¹⁹ Burke uses the term *narrative* here where others might use the terms philosophy or epistemology.

mathematics a thought in the mind of God” (Hersh, 1999, p. 135). A dramatistic analysis is useful for educators because it helps us see that substituting the theological basis for mathematics with a Platonist base is not likely to change the *scene-act* or the *scene-agent* ratios. That is to say, whether the mathematical forms are creations of God or they are assumed to exist in some a priori realm, the key mathematical *act* remains *discovery*.

Let me begin with an analysis of selections from Descartes’ *Meditations* (Descartes, 1960). Throughout this analysis, I focus on the ways in which, from a dramatistic standpoint, the Cartesian *Meditations* talk as not as if mathematics were an act, but rather as if mathematics were the *scene* in which actors might take a position.

René Descartes and mathematics as scene. I read Descartes’ perspective on mathematics through a reading of his *Fifth Meditation*.²⁰ Descartes’ famous argument comes across in four moves. First, he isolates the properties of the sensible world that are susceptible to proportional reasoning, the most significant being the discreteness (lending itself to counting), extension (lending itself to the measure of degree of extent), and duration (lending itself to the measure of change), and calls them the ‘quantifiable dimensions.’ Second, he invokes the idea of *reason*, which he also calls *natural light of mind*, as capable of illuminating the quantifiable dimensions. Third, after identifying the particular geometrical forms in the sensible world he decides that he did not invent this form, and that it existed *a priori* in the deepest reaches of his soul or *Cogito*, as he calls it. Cogito is mind or spirit, which contains ideas as forms; and the cogito is separate from the body and sensations. Cogito, and not the world of sensations, becomes the spring

²⁰ In the *Fifth Meditation* Descartes attempted a mathematical proof of the existence of God. He based this truth on the geometry of the triangle. The certainty of triangle was used in this argument to imply the certainty of religion.

from which the dimensions emanate. Fourth, he decides that since the form was already there and it could not have been invented by him, that there must be a perfect being *who knows*, and that *Cogito* is but the extension of that perfect being. This is the meaning of “*cogito ergo sum*.” Let me retrace these moves in some more detail.

The dimensions (sometimes called essences) were written in Descartes’ text as quantitative, or quantifiable, properties of matter, the magnitudes and dimensions that could ultimately be isolated and subject to analysis by mathematical reason. Number could be assigned to counting parts, fixing a degree of extent, and assigning duration. As Descartes put it:

Now, in the first place, I have a distinct image of that quantity which philosophers commonly call continuous quantity, the extension, that is to say, in length, breadth, and depth, of that quantity, or rather of the thing to which quantity is attributed. In addition, I can count its several parts, attributing to each various sizes, figures, locations, and movements; and, finally, I can assign duration, in varying degrees, to these movements (Descartes, 1960, pp. 144-145).

If continuous and discrete quantities are the dimensions, or quantifiable attributes of things in the world, then what might their source be? Descartes locates the source of these attributes of extension and of magnitude in the *Cogito*—the thinking substance:

And all this I know not only distinctly, when I consider it in general, but I have only to apply my attention to become aware of a host of particulars regarding numbers, figures, movements, and so on, of which the truth appears with so much evidence, and seems so connatural with my mind, that it does not seem so much that *I am learning something new as that I am recalling what I knew before*, or

perceiving what I already had in my mind, although I had not yet turned my thoughts in that direction (Descartes, 1960, p. 145).

The *a priori* nature of the mathematical truth is stated clearly in this passage. The truth of, say, a geometrical figure such as a triangle, must preexist its demonstration; the triangle exists before it can appear in the world. It is already there, much before it is mapped onto the materiality of a physical shape. A shape such as a triangle possesses its own true and unalterable nature—immutable and eternal like the metaphorical mountain I alluded to earlier. However, even if the truth of triangle precedes its mapping on a sensible triangle, where did it come from? Who invented it? According to Descartes, the triangle could not possibly be invented by *him*. Humans can only conceive of a triangle, more or less distinctly. So even when it was possible to demonstrate various properties of the triangle, such as its three angles and their sum as equivalent to the sum of two right angles, and so on, Descartes regarded these properties to be contained in the true and immutable form of triangle. As he put it: “when I first imagined a triangle, I had no thought of these properties, which cannot therefore have been invented by me” (Descartes, 1960, p. 145).

In sum (!) then, the properties of which the ideas of physical things are composed, namely extension, shape, position, and movement, cannot exist formally in us since we are merely a thinking being. Those ideas are the forms, “the garments,” as he puts it, “in which substance is clad.” The form, existing independently of us, is supplied with content from facts of experience to account both of a priority and applicability of mathematics (Shabel, 2007).

Descartes' account of mathematics then assigns a certain ideality and *a priority* to mathematics. But there is another factor involved here. Descartes was a practicing Jesuit, and he worked to make his philosophy rationally coherent with his religious beliefs (see, e.g., Toulmin, 1990). This ideal and *a priori* nature of mathematics is consistent with his religious beliefs. The *apriority* of mathematical truths follows from their being "connatural" with his mind. That is to say, ideas belong to the realm of Cogito (the spiritual world), and not the realm of the world, the body, or the sensations. The expression and application of mathematical truths involves discovering and eliciting the *natural light of mind*.

However, Descartes adds to the complexity of Cogito as he extends his argument to prove the existence of God. Cogito in this sense may be read as relating God and Man, and by the same token, the infinite and perfect with the finite and imperfect, the same way that *scene* is related to what it contains. The imperfect and the finite are contained in the perfect and infinite. God as infinite and perfect contains what is finite and imperfect. God becomes the scene of all scenes:

the idea of God is particularly clear and distinct, and *contains* [italics mine] in itself more objective reality than any other, there is none that is more true in itself, and less open to the suspicion that it is false. This idea, I say, of a sovereignly perfect and infinite being is wholly true. A pretense can be made that no such being exists; there can be no pretence that its idea represents nothing real to me, as I have said was the case with the idea of cold. And the idea of God, being particularly clear and distinct, *contains wholly within itself* all that I perceive to be

real and true and as having some degree of perfection (Descartes, 1960, p. 128, emphasis mine).

Finally, Descartes' views on the relation between mathematics and God come out even more explicitly in exchanges with those objecting to the claims in his *Meditations*, chief among them the mathematician and French theologian, philosopher, priest, and astronomer Pierre Gassendi (1592-1655). Gassendi was not happy with the idea of granting to the triangle an immutable existence prior to its perception in the physical world. Gassendi's objection to the *Fifth Meditation* argues:

It seems very hard to propose that there is any 'immutable and eternal nature' apart from almighty God. You will say that all that you are proposing is the scholastic point that the natures or essences of things are eternal, and that eternally true propositions can be asserted of them. But this is just as hard to accept. (quoted in, Descartes, 1984, pp. 221-222)

Gassendi then objects explicitly to the triangle as "out there" and argues that the essence of a triangle is elicited from the experience of a triangle, an approach that Descartes—and as I will show in the next section, also Benjamin Peirce—obviously reject in forming a relation between mathematics and God. For Gassendi, the term *triangle* is a mental construct which is put to use to sort triangles from other shapes in the physical world, and as one it has been induced from material experience with triangles. Gassendi explained his objection this way:

The Triangle is a kind of mental rule that you use to find out whether something deserves to be called a triangle. But we should not therefore say that such a triangle is something real. For it is the intellect alone which, after seeing material

triangles, has formed this nature and made it a common nature... It follows that we should not suppose that properties demonstrated of material triangles belong to them because they derive them from the ideal triangle. Rather, they themselves possess these properties in their own right, and it is the ideal triangle which does not possess except in so far as the intellect, after inspecting the material triangles, has attributed such properties to it, only to give them back to the material triangles to again in the course of the demonstration. (quoted in, Descartes, 1984, p. 223)

Descartes, defends the immutability of mathematical truths in response to this objection by claiming that the objection would have been worthy if he was claiming the immutability of mathematical truths as apart from God:

Just as the poets suppose that fates were originally established by Jupiter, but that after they were established he bound himself to abide by them, so I do not think that the essences of things, and the mathematical truths which we can know concerning them, are independent of God. Nevertheless I do think they are immutable and eternal, since the will and decree of God willed and decreed that they should be so. Whether you think this is hard or easy to accept, it is enough for me that it is true (Descartes, 1984, p. 261).

Descartes reinforced this opinion in a letter to one of his contemporary French mathematicians, theologian Marin Mersenne (1588-1648):

The mathematical truths which you call eternal have been laid down by God and depend on Him entirely no less than the rest of his creatures. Indeed, to say that these truths are independent of God is to talk of Him as if He were Jupiter or Saturn and to subject Him to the Styx and the Fates. Please do not hesitate to

assert and proclaim everywhere that it is God who has laid down these laws in nature just as a king lays down laws in his kingdom. (Descartes, Quoted in Kenny, 1970, p. 693)

To summarize—using the dramatistic image of a stage, agent, act, purpose and agency—Cartesian descriptions depict mathematics as a scene on which Descartes the mathematician also appears as a worshipper, who does mathematics to simultaneously connect with the mind of God through his notion of *ego*, and illuminate the workings of the world by shining on it the light of mathematical reason.

Benjamin Peirce and mathematics as scene. For half a century (from 1833 until his death in 1888) Peirce held the Hollis Professorship of Mathematics at Harvard University. He also served as the President of the American Association for Advancement of Sciences from 1852 to 1854. He was often called the “Father of pure mathematics in America” (Cohen, 2007, p. 42).

Peirce was an impressive, influential, firebrand, and hard to follow. A witty *New York Times* correspondent covering his presidential address at the sixth annual meeting of AAAS in Washington, D.C. in 1854, had this to say:

...the professor’s sentences were long and full of metaphors, similes, allusions, and the like, which no mortal man might hope to set down. They could be no more sketched, than the shapes of a puff of smoke. They came out regularly and beautifully, but they widened and enlarged, and they went up, and if you looked too long at one you lost the next dozen. It was magnifying of geometry—a glorification of pure mathematics (Anonymous, 1854).

For Peirce's views on mathematics, I consulted his books *Ideality in Physical Sciences* (1881) and *Associative Linear Algebra* (Peirce, 1881, 1882) and also searched for what was written about him by his contemporaries. However, while in the middle of writing this chapter, I serendipitously came across a recent historical study of Victorian mathematics by historian Daniel Cohen, which he aptly titled *Equations from God* (Cohen, 2007). Given the analytical resonance between what I was trying to accomplish in this chapter and Cohen's work, I also draw heavily on his work to support my argument.

To begin a discussion on Peirce's ideas on the nature of mathematics, I first draw on his Harvard colleague, Andrew Peabody (1811-1893), a mathematician and Professor of Christian Morals at Harvard. The following quote from Peabody, whose fundamental premises are repeated in the writings of Peirce, stresses the *a priori* nature of mathematical thought and its *applicability* to the physical universe. Peabody's text is as much an eulogy of Peirce, as it is a statement of belief about mathematics as the language in which the universe seems to have been written. He says:

Two years ago, my colleague, Professor Peirce, who in his own department has no superior among living men, delivered from this platform a course of Lectures, in which he constructed a theoretical universe. He took his stand outside of the visible creation, assumed merely the existence of brute matter and certain fundamental mathematical laws, and determined by a masterly line of *a priori* reasoning what the proportions and relations of a universe constructed in accordance with those laws must have been. The result was the coincidence,

point for point, of this universe of theory with the actually existing universe
(Peabody, 1864, p. 196).

According to Peabody, proceeding deductively and step by step from a few first principles, mathematics recreates the universe, seemingly mimicking the work of the Creator. What else would this coincidence of mathematically deduced universe with “actually existing universe” signify, if not the privileging of a connection between the realms of pure thought: between the Cartesian Cogito, and the divine?²¹

Peirce’s perspective, given in the following quotation from *Ideality in Physical Sciences* is similarly poised toward the connection between mathematics as *a priori*:

Ideality is pre-eminently the foundation of the mathematics. Observation supplies fact. Induction ascends from fact to law. *Deduction, applying the pure logic of mathematics, reverses the process and descends from law to fact* [Italics mine].

The facts of observation are liable to the uncertainties and inaccuracies of the human senses; and the first inductions of law are rough approximations to the truth. The law is freed from the defects of observation and converted by the speculations of the geometer into exact form. But it has ceased to be pure induction, and has become an ideal hypothesis. Deductions are made from it with syllogistic precision, and consequent facts are logically evolved without immediate reference to the actual events of Nature (Peirce, 1881, p. 165).

²¹ However, as I will argue later, the *a priority* of mathematics is not disturbed after the exclusion of God from this perspective. Not surprisingly, the dualism that separates mathematics from the ‘real’ universe continues to dominate fields such as theoretical physics. The effectiveness of mathematics in predicting “real phenomena” is acknowledged and assumed by theoretical physicists, but also termed “unreasonable” and mysterious. See, for example, (Eugene, 1960).

By accounting for the coincidence of thought with actual facts of creation, it may be argued that Peirce believed his mathematical work to be an engagement with the heavenly and divine:

“The loftiest conceptions of transcendental mathematics have been outwardly formed, in their complete expression and manifestation, in some region or other of the physical world... They are the reflections of the divine image of man’s spirit from the clear surface of the eternal fountain of truth.” (Peirce (1854), quoted by Cohen, 2007, p. 55)

This perspective on mathematics as the reflection of divine image was reinforced in the nineteenth century by the successes of classical celestial mechanics: How could mathematics be anywhere but existing in the mind of God if it could be used to *predict* the existence of a planet in the farthest reaches of the solar system, starting from some basic mathematical laws? Just to give an idea of the wide extent of the belief in the connection between mathematics and God, I will quote briefly Thomas Hill (1818-1891)²², a Unitarian clergyman who served as president of Harvard University from 1862-1868.

But in the pursuit of mathematical knowledge men began, at an early age, to invent and investigate a priori laws, laws of which they had not received any suggestion from nature. And the intellectual origin of the forms of nature was made still more manifest when these *a priori laws*, of man’s invention, were, in many cases, afterwards discovered to have been truly embodied in the universe

²² Thomas Hill is known to have had a lifelong friendship with Benjamin Peirce and shared his view of the relationship between mathematics and divinity. Daniel Cohen speaks about Thomas Hill as a great popularizer of Peirce’s work and an “exemplar of affinity between pure mathematics and idealist Unitarianism in nineteenth century.” (See Cohen, 2007, p. 57.)

from the beginning; as, for example, Plato's conic sections in the forms and orbits of the heavenly bodies, and Euclid's division in extreme and mean ratio (Hill, 1874, pp. 4-5).

According to Thomas Hill, Newton and Descartes were led by nature's own guidance to their discoveries of calculus and algebra respectively, delightful discoveries indeed:

When we remember how intense the delight which man feels in the discovery of mathematical truths...we may surely own, with gratitude, the marks of divine wisdom and love, in this *gift* to man, of the *power* to penetrate space, and apply to it the laws of time. (Hill, 1874, p. 21, emphasis added)

These mathematicians have influenced two generations of mathematicians directly, and countless generations indirectly. I take the perspectives of Benjamin Peirce and others like Thomas Hill and Andrew Peabody—all at Harvard University—on mathematics to be stressing *scene*. Mathematics, under this perspective came across as a preexisting expression of divine. It is transcendent and potentially capable of recreating an entire universe. Only few, which have the God's *gift* to man, or as Thomas Hill put it in the above quote, the power to penetrate, could access mathematical truths and, of course, take delight in this spiritual journey across the divine math world.

Russell's math world. Bertrand Russell (1872-1970) is credited along with Gottlob Frege and Alfred North Whitehead as a pioneering proponent of logicism—a story that reduces mathematics to the terms of logic. Logicism also resonates with the works of Descartes and Peirce on the nature of mathematics.

In a letter written to Helen Thomas²³ in 1901, Russell expressed his out-of-this-world conception of mathematics:

In the same passage Russell also betrays a longing for engagement with *pure reason* and a feeling of shame in engagement with the *transient things*:

I have come to feel a certain shame in thinking of transient things, and to regard a year spent, as this year has been, in human sympathy, as something weak and slightly contemptible. But the life of pure reason remains a remote aspiration, which (fortunately, you probably think) I do not find myself attaining (Griffin, 2002, p. 218).

Russell's description of the mathematical *world* as a cold, passionless, and eternal realm of pure reason apparently puts him in the *mainstream* camp. However, unlike Descartes, who is in conversation with seventeenth century Jesuits, Russell does not connect mathematics to God. The common denominators between Descartes, Russell, and Peirce are *a priority and immutability* of mathematical truths. The relations between the elements of mathematical drama—i.e. the *scene-act* ratio discussed above—are the same for all of these mathematicians. For all three of these mathematicians, mathematics is a scene that can be discovered by actors. Mathematics as a scene cannot be invented, and cannot be affected by actors.

This common denominator is important because it connects, as Hersh argues, all the various shades of the *mainstream* in mathematics. So let us dwell on the ways in which

²³ Helen Thomas was Carey Thomas's sister, and was at Bryn Mawr College around the same time when Carey Thomas was president of the college. Both sisters were a cousin to Bertrand Russell's first wife Alys Pearsall Smith.

Descartes, Peirce and Russell are similar. Russell is working in a setting in which the theological strand which connected mathematics and divinity, and which was prominent in the works of Descartes and Peirce, had lost out to professionalization in the mathematics departments (See, Cohen, 2007). However, as Russell's description suggests, this shift apparently only involved a disengagement of Descartes' Cogito from the divine²⁴ and its re-engagement with Plato's ideal world of mathematics that preceded the conception of a monotheistic God. An important aspect of modern Platonism is that unlike the reading-the-mind-of-God that characterizes Descartes' and Peirce's projects, the modern Platonist is identified as being simultaneously God and human. On Russell's description, humans become Godlike when we think about mathematics, yet we must also enter the perfect math world that other mathematicians have created. The earlier version of Plato seems to have been separated from the modern version of Plato by two interregna: Christianity (the belief that Christ is the coming of God into the world of man) and the Age of Reason inaugurated by democratic revolutions in the West.

Whether it wells up from the Cartesian Cogito representing the *natural light of mind*, or accessed from the independent abstract Platonist math world, mathematics is *not invented* but can only be *discovered* in both of these views.

²⁴ This disengagement is inscribed as a forgetting of the religious motivation of Cartesian rationalism. If we see Descartes' Cogito as the origin of modern science, then from our present perspective we do not entertain the possibility that Descartes' Cogito was conceived in response to questions about the existence of God.

Climbing the Peaks: Mathematician (*Agents*) and Mathematical Practice (*Acts*)

What do the conceptions of Descartes and later Peirce and Russell imply for the *act* of doing mathematics?

First, the *a priori* nature of mathematical truths is a common thread in these texts. As a physicist, I was initiated into such a conception of mathematics. All great equations in physics were spoken about as discoveries. The list of these discoveries was long: Newton discovered laws of motion; Hamilton discovered quaternions; Maxwell discovered laws of electromagnetism; Einstein discovered relativity, Erwin Schrödinger discovered wave mechanics, etc. *Discover* was the verb that described the act in this discourse.

From the perspective of the dramatistic analytical scheme (and as I have said before) the verb *discover* suggests a strict containment of the *act* within the *scene*, the so-called scene-act ratio. The *act* of doing mathematics is completely contained in a mathematical universe. Of course, one cannot discover a mathematical object that does not exist, even though we only find out after the fact that it did indeed exist.

Second, mathematics as a relation to divine, leads to the notion of acting as reading the book of God. An exercise in pure thought, as proceeding from first principles, and deductively creating the universe, it signifies that accessing mathematical language is tantamount to reading the mind of God. Since mathematical conceptions are thoughts

from God's mind, engaging in mathematics is tantamount to "adoration, praise, or prayer."²⁵

Adoration, prayer, and praise are not the verbs that found their way in twentieth century discourse of mathematical practices; as I have discussed earlier, the reference to God was eliminated. The mathematician replaced God, indeed as suggested by Russell's quotation. For instance, when Russell enthusiastically assigned a combination of beauty, an inhuman coldness, purity, sublimity, dignity, timelessness, and privileged access to mathematics, he also placed the privileged mathematicians in a realm that had previously belonged properly to God alone.

Russell the mathematician—one with the access rights to this territory—*acts* by traveling around in this [pre-existing] mathematical universe. Like Descartes and Peirce, doing mathematics on the stage so set is like finding one's feet, and ultimately, one's way on this mathematical landscape. Russell, or any other mathematician for that matter, has the license to be there, only to find his way around and tell others what they found. The agent does not act to create, but it is the scene—the timeless world of mathematics—that determines what the agent's act can be.

Russell's description, like that of Descartes and Peirce, both affords, and limits, what a mathematician could be, as someone endowed with powers—mathematical powers—to follow the contours of the paradisiacal place called mathematics. Once again, *discover*, not *invent* or *construct*, is the verb that describes his act. While the mathematician may be seen as representing mathematics, the act seems to involve appropriately describing

²⁵ The quotation refers to a statement made by a friend of Ichabod Nichols (1784-1859), a pioneering liberal theologian who was also a mathematician, and a formative influence on Benjamin Peirce. For a detailed exposition see Cohen, (2007, p. 51).

the maps he forms of a pre-existing math-world, thus conflating production with discovery.

These aspects of mathematical acts are also exemplified in the perspectives that support theoretical physics and other mathematical sciences. Induction and empiricism reign in the experimental sciences with their reliance on experiential; however, mathematical physicists continue to preserve a belief in a preexisting mathematical language in which the universe is written (See, for example, Tegmark, 2007).

This does not mean, however, that there are no mathematical texts other than the one that belongs to the divine (or Platonist) realm. Of course, mathematical treatises are made available to the students to understand and learn about the existing maps of the math world. However, acting here also means reading and following the text.

I exemplify this aspect of mathematical acts by taking an example from a nineteenth century English economist William Stanley Jevons (1835-1888). In an attempt to translate a book written by his French contemporary Augustin Cournot, Jevons reported that he was unable to follow all aspects of Cournot's analysis, and attributed this situation to his own lack of mathematical powers (Fisher, 1898). If by power we mean an *ability to act*, then a mathematical *ability to act*, in Jevons's text, is an ability to *comprehend and follow* the mathematical reason. I read Jevons' purpose—the desire to follow Cournot—as akin to an urge to occupy the same vantage point as occupied by Cournot, from where the latter illuminated the nature of economics with mathematical reason.

In sum, then, the *mathematician* and his *mathematical practice* is constrained by a perspective shaped by the *a priori* nature of mathematics. Under Descartes, the *a priori* nature of mathematics is elicited from Cogito, which is an extension of the divine.

Likewise, under Peirce, engaging with mathematics is engaging with the divine.

However, under the contemporary Platonists—as described by Davis and Hersh—the engagement is with a mysteriously, but still *a priori*, math world. In all of these perspectives, *reading, following, discovering, comprehending* are the terms that describe mathematical acts.

The Power to attain Natural Light of Reason: The *Agency* and the *Purpose*

Mathematical power is a term that I will discuss in more detail in the following chapter as related to mathematics educators' perspectives on mathematics. In the classical texts, power seldom comes across as a term, but is, arguably, implied. In the dramatistic pentad outlined above, agency is interpreted as the means that an agent uses to act purposefully. When mathematical objects appear as *natural light of mind* or as inhabiting a Platonist world, existing *a priori* in both cases, the *agency* can be construed as *power* to navigate this pre-existing math world, the *privilege* that Russell speaks about, and the *mathematical powers* that Jevons bemoans as lacking.

The idea of *power* is also found in Descartes' *Meditations*. Descartes' rationalism does not imply that the ultimate source of the "natural light of reason" is inherent in self-contained human subjects. Cogito, as he puts it, is always up for growth in its quest to approach the divine perfection. *Power* is what is needed to keep the Cogito in motion toward the divine:

Perhaps I am something greater than I take myself to be; perhaps all these perfections I attribute to the nature of a God, are in me potentially, even though they have not yet been realized or shown themselves in act. For my experience

teaches me that my knowledge increases and grows more perfect little by little, and I see no reason why it should not grow to infinity and, being thus augmented and made perfect, acquire for me all the remaining perfections that belong to the Divine nature; and if I have this *power, the power to acquire these attributes*, that should be sufficient for my own mind to produce the ideas of them (Descartes, 1960, p. 128).

So Cogito becomes a vehicle to potentially access the infinite wisdom of God only if sufficiently powered.

Dramatistically speaking, then, this narrative speaks of doing mathematics as an engagement with divine or with an independent and pre-existing (but unexplained) ontological reality. On this view, when an agent (*mathematician*) acts (*does mathematics*), it is an exercise of mathematical powers through the use of right methods (*Agencies*) that lets him be at one with the 'paradisiacal' (as Russell would have us believe) world of mathematics by *discovering, reading, and following* what is already given.

The Relationship between Mathematics and its Teaching and Learning in Classical Texts

At the risk of repeating, I have argued that in none of the descriptions given above—Descartes, Russell, Peirce, Jevons, and Wigner—does the practice of mathematics come across as an act of creation. Classical mathematicians do mathematics when they *discover, read, and follow* either as yet unknown mathematical truths, or, as students, those that are already known. Not enough data are available to reflect on the implications of foregoing analysis for the teaching of mathematics. We do not know much about

Descartes' or Russell's teaching philosophies. Yet, dramatistic ratios can help us complete the picture. The question is if mathematics is believed to be a preexisting *scene*, then what could we say about teaching by imagining it in terms of a dramatistic *scene-act* ratio?

Since the motivations for a specific kind of mathematical pedagogy are well documented for Peirce and his colleagues, one way to answer this question would be to see if Peirce's teaching conforms to a *scene-act* logic with scene defined by mathematics. Peirce was well known as an eminent educational reformer (See, Cohen, 2007, p. 42). S.R. Peterson (1955) in an essay titled *Benjamin Peirce: Mathematician and Philosopher* has documented Peirce's disposition toward teaching. I will consider what has been said about Peirce by his students. Through these views, teaching comes across as not about making all students learn mathematics but about filtering away those who cannot—perhaps reforms urged by Peirce at Harvard may be some of the first sorting practices based on elective courses in American education; learning appears as an exercise in silently following the great master's work; students appear in this perspective as either gifted or not.

Peirce was contemptuous of teaching any but a gifted few students. The reforms he introduced at Harvard University were designed to offer mathematics to a select few. His move to have mathematics made into an elective discipline for all the four years of college at Harvard University is widely interpreted as a step to ward off the unworthy from his classes:

One compelling motive for this action may have been his intense dislike of teaching any but the most gifted students. When mathematics was made an

elective, the students stayed away in droves, and the mathematics department became known as small, difficult, and unpopular. Generations of unhappy students have recorded what they have suffered at Peirce's hands, a combination of respect for his enthusiasm and genius with a total befuddlement as to what he was trying to say (Peterson, 1955, p. 93)²⁶.

Charles Eliot, the president of Harvard University from 1869 to 1909, and member of the famous Committee of Ten, was Peirce's student from 1849 to 1853. Eliot reminisced about Peirce's teaching in the following words:

He was no teacher in the ordinary sense of that word. His method was that of the lecture or monologue, his students never being invited to become active themselves in the lecture room. He would stand on a platform raised two steps above the floor of the room, and chalk in hand cover the slates which filled the whole side of the room with figures, as he slowly passed along the platform; but his scanty talk was hardly addressed to the students who sat below trying to take notes of what he said and wrote on the slates. No question ever went out to the class, the majority of whom apprehended imperfectly what professor Peirce was saying (Eliot, Lowell, Byerly, Chace, & Archibald, 1925, p. 2).

Peirce's conduct as a teacher is consistent with the demands of the *scene-act* ratio. He may be looked upon as a bad teacher when viewed from our perspective as educators. But when mathematics is the scene, then it makes sense to narrate the accounts from one's adventures and discoveries in the math world, then to target individual learner's capacities to learn mathematics. Peirce was not there to teach mathematics, but—as a

²⁶ Also see Cohen (2007) for a similar description of justification for declaring mathematics an elective subject.

forbear—to provide images of the math world to those who were adequately gifted. As Eliot wrote:

If a question [asked by a student] interested him, he would praise the 'questioner, and answer it in a way, giving his own interpretation to the question. If he did not like the form of the student's question, or the manner in which it was asked, he would not answer it at all, but sometimes would address an admonition to the student himself which went home (Eliot, et al., 1925, p. 2).

The *acts* of the 'unrepentant elitist' as a teacher are consistent with a description of mathematics as *scenic*. For the true believer in the connection between mathematics and divine, reforming education meant getting rid of those without mathematical powers, not inculcating them in those who did not possess them. Perhaps Peirce, even Descartes, and Russell would be horrified if we brought them into a room and posed to them the problem of teaching mathematics to all children in the primary school. Slogans like "Mathematical power for all," might appear to them as incomprehensible gibberish.

If access to natural light of reason is a gift of God as Descartes would have us believe, and if mathematics is indeed out there, existing in a divine realm which can be accessed only by few, then mathematical powers have a less psychological and more spiritual connotation. The questions, such as "how do humans learn to reason?" did not arise in a setting defined by mathematics as a divine and heavenly habitat open only to a privileged few. Pedagogy, in the terms in which educators and educational psychologists might render it, did not seem to have a place on the registers of Cartesian and Platonist mathematics.

Chapter IV: Revisiting Mathematical Power for all

To some extent, everybody is a mathematician...School mathematics must endow
all students with a realization that doing mathematics is a common human
 activity.
 (NCTM, 1989, p. 8)

Truth and beauty, utility and application frame the study of mathematics like the
 muses of Greek theater. Together, they define mathematical power, the objective
 of mathematics education.
 (NRC, 1989, p. 43)

Dramatistic Pentad	Classical Texts	Reform Texts	Counter-Reform Texts
Scene	Mathematics	<i>Mathematics Education—Based on NCTM Standards</i>	Standards
Act	Following, Reading, Discovering mathematics	<i>Constructing Mathematics</i>	Following, Reading, demonstrating fluency
Agent	Mathematicians	<i>Children, Teachers</i>	Children
Agency	Mathematical Powers	<i>Mathematical Powers</i>	Mathematical proficiency
Purpose	Illumination by natural light of reason	<i>Empowered citizenry</i>	Numerate Citizenry

Table 2: Pentad at a Glance—Focus on Reform-based Mathematics Education Texts

I would like to remind the reader at the beginning of this chapter what I wrote in the introduction to this dissertation project. As an outsider to mathematics education discourse in the United States, and a mathematics educator, I was introduced to a set of terms—like *mathematical power*, *constructivism*, *child-centered pedagogy*, and so on—which were part of progressive mathematics educators' repertoire in the United States. I

encountered those terms as part of ‘traveling reforms’.²⁷ When I arrived in the United States, the only major change—albeit very significant one—inasmuch as these terms were concerned was the recognition that they were part of a progressive reform package—something that did not travel with them to the distant lands. This move did not change the terms that I used. I just saw them as part of a different scene. Later, I recognized this *scene* to be fiercely contested and in a state of conflict with its detractors.

This chapter is about the *scene* of mathematics education and the drama it entails.

The Drama of Reform Texts

The drama of reform texts unfolds at two different levels. At one level, the scene is the field of mathematics education itself with several independent but interrelated discourses reverberating within it, and constituting the descriptions of its *actors*, *acts*, *agencies*, and *purposes*. At the second level, the scene of mathematics education also permits an *act-scene* ratio. That is to say, it construes mathematics as produced due to human acts.

In this drama, the key term is *mathematical power* that table-2above indicates as occupying the place of *agency* in the dramatistic pentad. It occupies the same place, as the table shows, in the case of classical texts as well. However, as the analysis in this chapter will suggest, the dramatistic ratio associated with the term *mathematical power* is different in these two cases. In the reform texts *mathematical power* follows the logic of *act-scene* ratio. That is to say *agents* (children) are described as exercising their *mathematical powers* to engage in mathematical practice. Unlike the classical texts,

²⁷ Traveling reforms is the term used by Gita Steiner-Khamsi, a scholar of comparative and international education, to describe the import and export of educational discourses across cultures (Popkewitz & Steiner-Khamsi, 2004).

mathematics does not preexist to be discovered and thus discovered in the due course, but is actively constructed through mathematical practices (thus the *act-scene* ratio).

However, these mathematical practices (with *act-scene* ratio) are themselves enabled by the *scene* of mathematics education. This *enabling* is, in an important sense, a certain kind of democratizing of mathematics. *Mathematics education* works as a scene for this democratizing impulse much in the same way as the modern *liberal democratic* state works as a scene for democratic processes in the society at large.

A study of this scene of mathematics education reveals a contingent coming together of independent but related discourses in the wake of the calls for reforms in the 1980s. Three distinct streams—pedagogical progressivism deriving from as Wilson puts it, “Dewey-ian conception of the child and curriculum as two sides of the same coin” (Wilson, 2003, p. 18), the constructivist learning theories, and the quasi-empiricist philosophy of mathematics attributed to Imre Lakatos—come together and spread into a policy space under the banner of NCTM standards—to be described shortly—and other related reform texts. The task this chapter does is to map this scene of mathematics education and the ways in which it enables *agents* with *mathematical powers* as their *agencies*.

To do this I will recycle the questions that were applied to the classical texts in the last chapter.

Scene: How is the *context* of mathematical activity described?

Act: How are mathematical *acts* described?

Agent: Who is described as entitled to *act*, that is, to do mathematics?

Agency: What means are available to *act*, that is, to do mathematics?

Purpose: What is described as the *purpose* of mathematical activity?

In what follows, I will first describe the reform texts that I have used in this analysis.

Apart from those texts, the analysis also draws heavily on academic writing in mathematics education to map the coordinates of the *scene* of mathematics education and other elements of the dramatistic pentad.

The Reform Texts

The research reported in this chapter has involved chasing similar terms in the reform documents produced in the late 1980s and early 1990s, and then collecting and sorting the texts according to the dramatistic pentad. Around 1990 there was a heavy outpouring of reform documents from many different directions, by many different organizations, and at different levels. The notable documents are:

- National Research Council's *Everybody Counts* (NRC, 1989)
- NCTM's *Curriculum and Evaluation Standards* (NCTM, 1989)
- The Mathematical Sciences Education Board's *Reshaping School Mathematics: A Philosophy and Framework for Curriculum* (MSEB, 1990)
- *Professional Standards, and* (NCTM, 1991)
- California's Department of Education's *Mathematics Framework for California Schools* (CDE, 1992) (henceforth called the *Framework*).
- *Measuring What Counts* (NRC, 1993)
- *Measuring Up: Prototypes for Mathematics Assessment* (MSEB, 1993)
- *Assessment Standards* (NCTM, 1995)

Of these documents, I draw primarily on NCTM *Curriculum and Evaluation Standards*²⁸ and *Mathematics Framework for California Public Schools*. I chose these documents because these texts have been at the center of the drama of the so called reforms and counter reforms in mathematics education. These documents are also amenable to a dramatistic analysis because together they make up the dramatistic structure of motivations about mathematical practice, teaching and learning. I describe them in some detail below.

NCTM Standards

NCTM projects the standards as “one facet of the *mathematics education community*’s response to the *calls for reform* in the teaching and learning of mathematics” (NCTM, 1989, p. 1, emphasis added). That is to say, mathematics education in these texts does not signify just a field of practice, but also a professional community. I will discuss the implications of this professional posture in detail later in this chapter. Here it suffices to note that this set of documents is projected as representing the professional consensus of the mathematics education community.

The *Curriculum and Evaluation Standards* [to be called NCTM Standards from this point onward in this chapter] published in 1989 make recommendations for improving and updating the mathematics curriculum and the evaluation of students’ achievement. The NCTM standards were part of, and in a sense a catalyst for, a broader standards-based reform movement called into existence by the report *A Nation at Risk*, which made an urgent call for reforms for higher and more rigorous standards (NCCE, 1983). The Bush

²⁸ The term *standards* is used by both reform texts and counter-reform texts. In both sets of texts, however, like other terms, standards are dramatistically different. In this chapter, when the term *standards* is used, it refers only to the NCTM standards. In other chapters, when I use the term *standards*, I will call the attention of the reader to the text in which it is located.

administration (1989–1993) used NCTM’s standards “as a free-market model for educational reform” (Ravitch, 1995, p. 28). Following their example, the U.S. Department of Education provided grants for creation of similar “voluntary national standards in science, history, geography, foreign languages, the arts, English, and civics” (Ravitch, 1995, pp. 28-29). Yet, the standards, as well as the policy documents and curricula based on them, were contested by critics who accused them of being devoid of mathematical content. (For a detailed documentation of mathematics education debates, see, Wilson, 2003; also, see, Wu, 1998; Wu, 2000).²⁹

Mathematics Framework for California Public Schools

Publication of Mathematics Framework for California Public *Schools* answers a call from ...the National Council of Teachers of Mathematics, the Mathematical Sciences Education Board, and the teachers and mathematics educators who served on the Framework Committee. That call is to change what mathematics we teach, how we teach it, and to whom (CDE, 1992, p. vii).

The 1992 *Framework* is an important document because of its proximity with NCTM standards. By proximity here, I mean that many of the writers who are part of the NCTM were also members of the team of authors who wrote 1992 *Framework* (NRC, 1989). Defining *Mathematical Power* in terms similar to those of the NCTM, the 1992 *Framework* devotes an entire chapter—one of its four main chapters—to describing and explaining this notion. The remaining four chapters are entitled *Developing Mathematical Power in the Classrooms*, *Structure and Content of the Mathematics*

²⁹ Eventually, NCTM published a revised document entitled *Principles and Standards of School Mathematics* (PSSM) in the year 2000. PSSM no longer used the phrase *mathematical power*.

Program, Mathematical Content in Kindergarten Through Grade Eight and Mathematical Content and Course Structure in Grades Nine Through Twelve.

The 1992 *Framework* works with the terms *strands* and *unifying ideas*. *Strands* refer to the mathematical content that is covered at each level and *unifying ideas* refer to the terms such as patterns, proportional reasoning and so on. The unifying ideas are assumed to cut across strands. There are eight strands namely *number, measurement, geometry, functions, statistics and probability, logic and language, algebra, and discrete mathematics* and several different unifying ideas at different levels. An example of strands and unifying ideas is given in table 3 below

Strands	Unifying Ideas		
	Elementary	Middle Grades	High School
Functions Algebra Geometry Statistics and probability Discrete mathematics Measurement Number Logic and language	How many? How much? Finding, making, and describing patterns Representing quantities and shapes	Proportional relationships Multiple representations Patterns and generalization	Mathematical modeling Variation Algorithmic thinking Mathematical argumentation Multiple representations

Table 3: Strands and Unifying Ideas at various levels in the 1992 Framework

The 1992 *Framework* requires curriculum units to be created in ways that bring together the strands and unifying ideas through experiences that would result in the development of *mathematical power*. The framework urges that these curriculum units be *investigations* that enable the exercise of *mathematical power*.

Both NCTM Standards and the 1992 *Framework* appeal to the notion of empowering *all* students mathematically and justify this emphasis on the basis of multiple objectives

of ensuring equity, meeting the demand for a mathematically literate work force, and the need for a numerate electorate.

The Scene of Mathematics Education

In dramatistic analysis, the *scene* is a container of the *agent* and his/her *acts*. The scene, as discussed in Chapter 2, is not a physical landscape. In particular narratives it can be anything that could be construed as *containing* the *agent* and the *act*.

I will map the *scene* of reform mathematics first by contrasting it with the scene of the classical texts. Examining the classical texts in the last chapter, I argued that some notable texts produced about mathematics between the sixteenth and nineteenth centuries described mathematics as existing in the mind of God or in a metaphysical Platonist realm. I also argued that from the classical perspectives on mathematics as *a priori*, teaching made sense as a project to identify the gifted few. This may sound elitist, and surely different from what we, as mathematics and teacher educators, have come to believe as the purpose of teaching these days. Yet, identifying the mathematically powerful—i.e., gifted—students was wholly consistent with a mathematical practice that was based on epistemological *discovery* because mathematical truths were assumed to have been derived from a divine or Platonist realm. In those classical narratives, then, it made sense to aim for identifying the gifted few.

So, in the classical texts mathematics came across as *scenic* in the dramatistic sense because, by being *out there*, it framed the mathematical practice as well as its purpose. With mathematics as the scene, doing mathematics was described by verbs such as *discovering*, *reading*, and *following*. In the mathematics education reform-texts, as I will discuss in detail in this chapter, it is the field of mathematics education which works as

the scene. Dramatistically, I take mathematics education to mean a frame—not a collection of individuals or a phrase signifying the teaching of mathematics—which legitimates the meaning of the terms such as *mathematics* and *mathematical power*.

As the frame—or the *scene*—mathematics education defines what mathematics is. The reform texts define the nature of mathematics as being produced as a result of purposive human action with the ‘real’ world.³⁰ When the scene shifts from classical to recent reform texts, the wanderer in the math world becomes a creator of mathematical landscapes. While the term *mathematical power* in the classical texts comes across as the ability to discover, read, and follow, in the reform texts, under a different scene, it becomes capacity to construct, conjecture, communicate, and validate.

In what follows I will first map two critical features of the *scene* of mathematics education. First, the *scene* of mathematics education is constituted by an emergent professionalization of the field in the early 1980s. Second, I present, borrowing heavily from Batarce and Lerman (Batarce & Lerman, 2008), an argument about an inherent issue with the term *mathematics education* that invites intervention from mathematicians to reclaim the *mathematics* part of mathematics education.

Mathematics Education and the Rhetoric of Professionalism

With the publication of NCTM’s *Curriculum and Evaluation Standards* (NCTM, 1989), the air seemed filled with the talk of the professional status of a mathematics education community. The professionalization of mathematics education can be described as a scene through an analogy with the profession of medicine.

Professionalization creates particular roles for the physician, the patient, and the relation

³⁰ This view of mathematics as being inductively elicited from the ‘world’ sounds much like one embodied in Gassendi’s objections to Descartes’ *Fifth Meditation*. See previous chapter for details.

between these entities with respect to the sickness. Put differently, the *acts* of a physician as well as a patient are circumscribed by professionalism. Likewise, mathematics education, in the spate of reforms in the last two decades, reached out to all aspects of professional practice like never before. Professionalization involved defining mathematics, mathematical practice, teaching, learning, and the roles and responsibilities of teachers and learners, and curricula in particular ways. It is in this sense that I take mathematics education discourse as assuming a *scenic* connotation, namely that of professionalism.

Mathematics education was also seen by its participants as a young field still in search of a professional identity (Sfard, 1998; Sierpiska & Kilpatrick, 1998). Indeed, at the turn of the 20th century, it may have been difficult to imagine a separation between mathematicians and mathematics educators. Concerns regarding the learning of arithmetic had belonged traditionally to educational psychology, a field that already had a status as a professional community.

Mathematics education—as a field of research and practice as an academic group—had never led a huge reform effort until the mid 1980s. For instance, the *New Math* reforms of the 1960s that erupted in the wake of the launching of Sputnik were almost wholly organized by mathematicians. The pronouncements from NCTM officials in the wake of their publication of the *Curriculum and Evaluation Standards* (NCTM, 1989) lent support to the above claim about the emerging professional status of mathematics education in the 1980s. In an essay outlining the vision for implementation of these standards, F. Joe Crosswhite, one the past presidents of NCTM, described the standards project as “the most ambitious, and certainly the most expensive, program ever”

undertaken by the council (Crosswhite, Dossey, & Frye, 1989, p. 513). The standards, said Crosswhite, would also be “remembered as a prototype for the development of a *professional consensus*...And none has ever had the prepublication endorsement of so many professional groups” (Crosswhite, et al., 1989, p. 513, emphasis mine). The image of a professional identity for mathematics education was further reinforced by then president of NCTM, Shirley M. Frye. She wrote:

In this next decade *our profession* has an unparalleled opportunity to revitalize mathematics education and to make that mathematics education effective for all students. Our initial step in developing the *Standards* was unique! No other effort of this kind had ever been undertaken in any discipline by a professional organization (Crosswhite, et al., 1989, p. 518).

Thus, Frye’s rhetoric positioned NCTM within the rank of professional organizations in a uniquely triumphant manner. Mathematics educators, speaking through their professional organization NCTM, seemed to have a solid grasp of what needed to be done to fix mathematics education in the United States. And they seemed poised to do it. Sociologist of professions Andrew Abbott has defined the term *jurisdiction* as a relation that a group of professionals establishes with the task it sets for itself (Abbott, 1988). For mathematics educators, this relationship was embedded in the standards document that they would strive hard to implement in the next decade or so. Through the standards, the mathematics education community spelled out what mathematics and its learning and teaching could be. It was now only to be implemented through policy shifts. Significantly enough, Frye spelled out ways in which the standards construed mathematics by calling attention to a set of verbs:

Our challenge is to translate the *Standards* into action by raising expectations and by bringing vitality and vigor into our classrooms. The verbs used in the *Standards* vividly describe the behavior our instruction should aim to achieve. Words like *explore*, *communicate*, *construct*, *use*, and *represent* stress the involvement of students in the active "doing" of mathematics. Words like *collaborate*, *question*, *express*, *value*, *share*, and *enjoy* bring a new flavor to the work of the students. Words like *reflect*, *appreciate*, *connect*, *apply*, and *extend* build a new attitude toward mathematics and its uses...Mathematics itself is changing, and with it, the entire school mathematics curriculum is entering a period of unprecedented change (Crosswhite, et al., 1989, pp. 519-520).

I should also observe that professionalism among educators, like educational reform, was the order of the day in the 1980s. Mathematics educators were not the only group attempting to cross professional thresholds in the mid and late eighties. The efforts to turn teaching "from an occupation into a genuine profession" were also underway (See, for details, Holmes Group, 1986).

Mathematics under erasure

Second, the term *mathematics and education* was not a simple conjunction of two words. Rather it came across as an agonistic pairing during the math wars. The term *education*—here my focus is on the meanings associated with education—brings a set of concerns to mathematics that may be different from the concerns that shape professional mathematics communities. For example, the professional—that is university—mathematicians do not have as their top priority the education of *all* students. Mathematicians' conceptions of mathematics arise from a different set of disciplinary

concerns, namely academic research and the preparation of the next generation of professional mathematicians. In contrast, *mathematics education*'s concerns arise from its domain that takes into account the concerns of teachers and school classrooms as well as mathematics. The calls for a *different* mathematics in the reform texts (NCTM, 1989, p. 1) can be seen as an example of this recontextualization.

The rearrangement of the meaning of mathematics and mathematical practice within the folds of mathematics education, as I will discuss in the subsequent sections, reflects some aspects of mathematics and deflects its other aspects, thus working as a sort of *terministic screen* (See Chapter 2). This redefinition of mathematics circulates through other terms of mathematics education reforms—such as *mathematical power*—both affording and constraining what mathematics and mathematical practice could be.

A simple comparison with the drama of classical texts described in the previous chapter tells us that mathematics education involves knocking mathematics off from the top row in the table that describes pentad at a glance (See table 1). The displacement of mathematics by mathematics education entails a rearrangement of the table. This rearrangement of the table is signified by the emergence of a new professional jurisdiction as I have discussed in the previous section.

By displacing mathematics from its place, *mathematics education* works to efface the possibility of any role of mathematicians in this professional enterprise. Mathematicians must first become mathematics educators, before they can be insiders, if we go strictly by the logic of the table. Thus mathematics education decides both what *mathematics* is and what *mathematics education* can be.

Batarce and Lerman (2008), while interpreting the conjunction of terms *mathematics* and *education*, propose to read mathematics education as ~~mathematics~~ education (Batarce & Lerman, 2008), with the strike-through being read as putting mathematics under erasure. They argue that the term *mathematics education* cannot be imagined prior to both *mathematics* and *education*. *Mathematics Education* must contain the traces of both of its constituent terms. However when the identity of *mathematics education* is given precedence, the hierarchy is disrupted in a way that undermines the *mathematics* (Batarce & Lerman, 2008, p. 45). This is expressed in *mathematics education* through a simultaneous mistrust as well as a dependence on mathematical knowledge. As Batarce and Lerman put it:

On the one hand, mathematics education, at some point, must mistrust the nature of mathematical knowledge (for instance, for not being adequate for the mathematics teacher education); on the other hand, mathematics education, when it attempts to delimit itself and institute its episteme (for instance, by its notion of methods of research), must ask for help from the nature of mathematical knowledge (Batarce & Lerman, 2008, pp. 46-47).

Thus, the term *mathematics education* is in perpetual debate—it is, as Burke (1945) would term it, agonistic. The terms for *mathematics* and *education* need each other, yet are agonistic. It “effaces the ‘mathematics’ word written within the term ‘mathematics education’” (Batarce & Lerman, 2008, p. 47). The studies in the psychology of mathematics education did not draw such debate because psychology of mathematics education did not concern itself with, or raise epistemological questions about, the nature of mathematical knowledge. Reading mathematics under erasure in mathematics

education is significant, and expresses a democratizing impulse. When mathematics is recontextualized in a way that deemphasizes authoritative knowledge and places in the hands of a child the authority to create mathematical knowledge, mathematics is democratized. Whether it is a good or a bad thing is not a question that I answer. However, I only suggest that when the scene shifts from *mathematics* to *mathematics education*, it also inserts a democratic impulse into the dramatistic configuration.

The Elements of the Scene of Mathematics Education

The professional stance of mathematics education shapes a scene which is supported by a contingent merger of three distinct strands with remarkable dramatistic similarity. The first emanates from Piagetian advances in mathematics education, namely constructivism. The second approach can be described as proceeding from John Dewey's notion of experience. The third may be loosely described as Imre Lakatos' articulation of Popper's philosophy of science into the field of mathematics (Lakatos, 1979), which sees mathematics advancing as a dialectic of proofs and refutations.

Constructivism: Constructivism was a theory of *learning* that spun off from the neo-Piagetian work in mathematics education (Steffe & Kieren, 1994). Not everyone agrees on what constructivism *is*.³¹ I am not as much concerned with the range of meaning associated with the term *constructivism* as with its dramatistic implications. So, while there is considerable variation in meaning, all versions of constructivism regard students as active constructors of knowledge rather than as passive recipients thereof (Wilson, 2003, p. 40).

³¹ For a range of thinking about constructivism in mathematics education, see, Confrey, 1990; Davis, 1990; Glaserfeld, 1990; Goldin, 1990; Noddings, 1990; Phillips, 1995; Simon, 1995.

This theory reverberates strongly through mathematics education texts. As I mentioned earlier, when I had first encountered the term *constructivism*, it came across to me as a learning theory. Yet, in the reform texts, constructivism crisscrosses, the pedagogical and epistemological domains. Overall the reform texts appealed to constructivism as their perspective on learning but insisted this theoretical recognition be expressed in one's teaching. For instance, the NCTM standards articulated an unambiguous commitment to constructivism while insisting: "constructive, active view of the learning process must be reflected in the way much of mathematics is taught" (NCTM, 1989). The text perceived children as "active individuals who construct, modify, and integrate ideas by interacting with the physical world, materials, and other children" (NCTM, 1989). The 1992 Framework also echoed this emphasis in assumptions about children as active creators of knowledge rather than its passive absorbers. The reign of constructivism was extended from the domain of learning theory—as I will discuss in more detail in the next section—to epistemology with the admission of Imre Lakatos's view of mathematics as fallible (Ernest, 1991; Lakatos, 1979). Not surprisingly, as I will discuss in more detail in the next chapter, the counter-reform texts contested this amalgamation of constructivism and epistemology by reestablishing a clear distinction between content and pedagogy.

Pedagogical Progressivism: I see this strand as proceeding from John Dewey's lamentations about educational sectarianism in America as emerging from a bogus separation between the child and the curriculum (Dewey, 1902). Dewey had talked about two "sects" of education who were fighting over the curriculum. One sought to "subdivide each topic into studies; each study into lessons; each lesson into specific facts

and formulae. Let the child proceed step by step to master each one of these separate parts, and at last he will have covered the entire ground.” For the other sect, the “child is the starting point, the center, and the end” (Dewey 1902). Dewey sought to collapse this binary distinction by claiming both child and curriculum as the two sides of the same coin. Dewey’s lamentation and his campaign to collapse the binary distinction between the child and curriculum may be seen as constituting what has been described as progressivism by some and pedagogical progressivism by others.

Pedagogical progressivism appears as inseparable from constructivism in the reform texts in mathematics education, as well as in progressive reform texts of the last two decades in all subject areas. As Labaree (2005) puts it, pedagogical progressivism means:

Basing instruction on the needs, interests and developmental stage of the child; it means teaching students the skills they need in order to learn any subject, instead of focusing on transmitting a particular subject; it means promoting discovery and self-directed learning by the student through active engagement; it means having students work on projects that express student purposes and that integrate the disciplines around socially relevant themes; and it means promoting values of community, cooperation, tolerance, justice and democratic equality...this adds up to ‘child-centered instruction’, ‘discovery learning’ and ‘learning how to learn’.

And in the current language of American education schools there is a single label that captures this entire approach to education: constructivism (Labaree, 2005, p. 277).

In mathematics education pedagogical progressivism seeks to be faithful to both mathematics and the child. Thus, it seeks a sort of mathematical authenticity as an element of classroom practices. As Ball puts it, in her attempts to build a teaching practice that responds to this challenge, “With my ears to the ground, listening to my students, my eyes are focused on the mathematical horizon” (Ball, 1993, p. 376).

The point I am trying to make is that constructivism and pedagogical progressivism are distant cousins brought together by historical happenstance to form a scene in which students are seen as constructors of authentic mathematical knowledge. It is also a happy happenstance that these strands are met midway with a particular streak in epistemology of mathematics, the so-called quasi-empirical tradition of Imre Lakatos, which I describe below.

Imre Lakatos and mathematics education

In *Proofs and Refutations*, Lakatos (1979) depicted the history of evolution of a famous conjecture—known as Euler-Descartes Formula³²—as a conversation in a classroom between fictional characters. The book is narrated as a story of a conversation between a fictional teacher and his students. The teacher presents a particular proof of the Euler-Descartes formula to his students. The students begin to offer counter examples—which are drawn from the writings of the actual mathematicians—to dislodge the proof offered by the teacher. The result is a conversation—spread over not a single lesson but over many centuries of mathematicians’ struggles with the proof of the Euler-Descartes formula—that helps shape the form of the proof, constantly modifying it through a process of proofs and refutations. Thus, the proposals and counter proposals

³² The formula is $V - E + F = 2$; in this formula, E = number of edges of the polyhedra, V = number of vertices of the polyhedral, and F = number of faces of the polyhedra.

spread over many centuries appear as a classroom conversation moderated by a teacher. Lakatos' is not just a story of a particular proof, but a conception of mathematics as fallible and progressing through a dialectic of proofs and refutations similar to the logic of scientific discovery proposed by Popper. Lakatos also makes a distinction between the informal and formal mathematics. That is, from the coded mathematics resulting after the dialectic of proofs and refutations is exhausted, one cannot tell the intense back and forth and zigzagging which allowed a proof to grow in maturity and strength before it was finally formalized and validated as mathematical knowledge.

This zigzagging behavior of mathematical progress resonates with the ways in which the reform texts construct mathematics. For example, conjecturing, refuting, and validating are the verbs that define the mathematical *power* of students in these texts and not their abilities to read, and follow the already existing and coded mathematics—as was suggested in classical texts. Lakatos' epistemology, which constructs mathematics as fallible, incomplete and uncertain, therefore, could be inserted comfortably into the repertoire of reform texts.

Let me emphasize this point about the discursive proximity between reform texts and Lakatos' epistemology. By construing the object of its investigation as fallible, and, therefore, *in process*, Lakatos' view of informal mathematical discourse support the conception of a classroom environment in which knowledge could be seen as being in the process of construction through application of higher order thinking skills. Interestingly, the higher order skills are not catalogued in response to ways of knowing mathematics proposed by Lakatosian epistemology. The psychological imperatives underlying higher order thinking skills do not necessarily make any reference to Lakatosian epistemology.

The reform texts' emphasis on the students' construction of their own mathematical knowledge, and their use of higher order thinking skills to do this does not directly follow from a philosophy of mathematics. Rather, Lakatosian epistemology is comfortably absorbed into this structure because it supports the tenets of pedagogical preferences of reform texts. In other words, the reform texts pick and chose the nature of mathematics as a perfect fit within an existing *terministic screen*.³³

Thus, in the reform texts the philosophy of mathematics of Lakatos became the privileged vantage point from which the traditional classroom practices—such as memorizing, practicing, and giving the one right answer—appear as undesirable aspects of traditional mathematics teaching and learning that reform texts wish to exclude from their approach to teaching mathematics.

Below, I provide another extended quote from Lampert to reinforce two points. First, reform texts, through a sort of synecdochical move³⁴ identify mathematics with Lakatosian epistemology. Second, through this identification reform texts stake a claim as enacting practices of the discipline of mathematics in the classroom.³⁵ In the words of Lampert (1990), then:

³³ In the 1960s, the discourse of new math reforms exhibited a search for an educational psychology to fit with the preferences of the new math. As Steffe and Kieren document this: "In a sense, the mathematicians who have guided the recent curriculum reforms have been waiting to be shown that psychological theories of learning and intelligence have something relevant to say about how mathematics shall be taught in the schools. These reformers (and I speak now not only of MSG) have been so successful in teaching relatively complex ideas to young children, and thus doing considerable violence to some old notions about readiness, that they have become highly optimistic about what mathematics can and should be taught in the early grades" (Steffe and Kieren, 1994, p. 712).

³⁴ The term synecdoche implies taking the part for the whole. The point I am making here is that while Lakatos' epistemology illuminates some aspects of mathematics, it obscures other aspects of it.

³⁵ Ironically—and as the reaction of mathematicians during the math wars suggest—mathematicians did not view the reform texts as included in this dispensation.

At every level of schooling, and for all students, reform documents recommend that *mathematics students should be making conjectures, abstracting mathematical properties, explaining their reasoning, validating their assertions, and discussing and questioning their own thinking and the thinking of others.*

These activities do not fit within the tasks that currently define mathematics lessons. *Moreover, they require both teachers and students to think differently about the nature of mathematical knowledge.* Little research has examined what the intellectually generative sort of mathematical activities espoused by NCTM or MSEB might look like in classrooms or the role that the classroom culture plays in the social construction of a view of mathematical knowledge; studies of this sort are needed if we are to understand what it will take to transform *discipline-derived standards* into school practice... (Lampert, 1990, pp. 32-33, emphasis added)

Thus positioned, reform texts construed the ideals of Lakatos about mathematical practice as contrasting “sharply with the way in which knowing mathematics is viewed in popular culture and in most classrooms” (Lampert, 1990, pp. 31-32).

However, what does not come across in the above quotation is that the ideas of Lakatos also contrasted sharply with those of a significant number of mathematicians. Lakatos was not directly debated, but mathematicians’ emphasis on axioms and mathematical truths as the starting line for a deductive approach toward proofs did not suggest Lakatos’ epistemology as the cornerstone of the totality of mathematical practices. Hung-Hsi Wu’s quotation below suggests this to be the case:

On the one hand, logical deduction—proof—is the backbone of mathematics...On the other hand, it would be a grave mistake to insist that every statement in elementary mathematics, up to and including calculus, be given a proof. There is no reason to impose the kind of training designed for future professional mathematicians *on the average student*...What is important, however, is to give students adequate training in making logical deductions...A reasonable mathematics education should aim for at least this much (Wu, 1996, p. 4).

To summarize, constructivism, pedagogical progressivism, and Lakatosian quasi-empiricist mathematical philosophy are brought together as allies under the banner of standards-based reforms in mathematics education. In Chapter 6, I will discuss the ways in which this threesome was both made possible as well as undermined by the language of standards. Here, I want to speak of it as part of the scene of mathematics education, constituting a drama in which the *agents* (children) are provided with mathematical *powers*, which they can use to construct *authentic* mathematics.

To wrap up this conversation, in this section I have made the case that mathematics education from the mid 1980s worked as a scene with two overarching characteristics. First, the texts of mathematics education loudly pronounced a professional jurisdiction over school mathematics, and second, mathematics education was projected as an agonistic term in which mathematics was reshaped, reconstituted and rearranged. This reconstitution, I have argued, happened through a contingent coming together of three independent but, ultimately, interrelated discourses of constructivism, pedagogical progressivism, and Lakatos' philosophy of mathematics.

This reconstitution of mathematics is coalesced in the term *mathematical power* in the reform texts, which I will discuss in the following section.

Mathematical Power: Examining Act and Agency in the Reform Texts

The development of *mathematical power* in all students was mobilized as the overarching goal of school mathematics in the mathematics education reform texts in the last two decades of the twentieth century (See, MSEB, 1990; NCTM, 1989, 1991, 1995; NRC, 1993) until its near replacement recently by *Mathematical Proficiency for all* (Ball, 2003; California Department of Education, 1999, 2006; Kilpatrick, Swafford, & Bradford Findell, 2001).

When in use, *mathematical power* worked as an umbrella term for a (reform-based) school mathematics that signaled creative construction of mathematical knowledge through engagement with the problems in “real world” involving investigational work, discussion and argumentation in the classrooms. (See, California Department of Education, 1985, 1992; NCTM, 1989, 1991, 1995; NRC, 1989). In these documents the need to *empower* all students *mathematically* was also seen as part of the attempts to foster creation of mathematical communities in K-12 classrooms. The image of *mathematical power*, as one commentator noted, went way beyond “the usual minimalist prescriptions...by including performance criteria more associated with professional mathematicians in the mathematical sciences” (Bishop, 1990, p. 362).³⁶

³⁶ After remaining the focus of reforms proposed by the NCTM Standards (NCTM, 1989, 1991, 1995) for a little over a decade, the term *mathematical power* completely disappeared from the more recent NCTM publications (NCTM, 2000, 2006). Likewise, the mathematics framework for K-12 education published by California’s Department of Education in 1985 and 1992 articulated *mathematical power for all* as a central goal of school mathematics. However, the 1999 *Framework*, which I will examine in more detail in the

What do the reform texts mean by “doing mathematics”? From the terms (mainly verbs) that define this activity, I infer the parameters within which mathematics is conceived in these texts. In what follows, I will describe the ways in which reform texts define *mathematical power*. Then, isolating the terms that describe mathematical acts and, thus, expressions of mathematical power, I will, suggest the ways in which these texts both afford and constrain what can be viewed as mathematics and ways of knowing it.

Mathematical Power in *Standards and Framework*:

Standards articulate *mathematical power* in terms of five goals for *all* students:

...K-12 standards articulate five general goals for all students: (1) that they learn to value mathematics, (2) that they become confident in their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically. (NCTM, 1989, p. 6).

The NCTM Standards take these goals to imply educational experiences for the students in which these goals could be met. Such experiences are expected to encourage students to “*value* the mathematical enterprise, to develop mathematical habits of mind, and to understand and appreciate the role of mathematics in human affairs” (NCTM, 1989, p. 6). The standards also use verbs such as *explore*, *guess*, *conjecture*, *test* and *build arguments* to describe student actions as they undergo the experiences prepared for them. With a voice sounding a professional consensus, the NCTM Standards exclaim,

next chapter, purged this term, establishing *mathematical proficiency for all students* as the new consensus. I will examine the texts that take the idea of mathematical proficiency for all as the goal in more detail in the next chapter.

“We are convinced that if students are exposed to the kinds of experiences outlined in the *Standards*, they will gain *mathematical power*” (NCTM, 1989, p. 6).

The 1992 *Framework* defines *mathematical power* in terms that are similar to NCTM Standards. Mathematically powerful students, according to the 1992 *Framework*, are those who *think and communicate drawing on mathematical ideas and using mathematical tools and techniques*. It, then, defines thinking and communicating in terms of other *acts* that could be used to indicate the *doing* of mathematics, and *ideas, tools, and techniques* in terms of what the programs make available to students as mathematical knowledge, technological tools such as computers and calculators, as well as algorithms. As stated in the Framework:

- *Thinking* refers to intellectual activity and includes analyzing, classifying, planning, comparing, investigating, designing, inferring and deducing, making hypotheses and mathematical models, and testing and verifying them.
- *Communication* refers to coherent expression of one's mathematical processes and results.
- *Ideas* refer to content: mathematical concepts such as addition, proportional relationships, geometry, counting, and limits.

Tools and techniques extend from literal tools such as calculators and compasses and their effective use to figurative tools such as computational algorithms and making visual representations of data.

Expressions of mathematical powers: What does a student do when expressing her mathematical powers? To answer this question, I have created lists of verbs that reform texts mobilize as expressions of mathematical powers by the individuals.

The various ways *mathematical power* is described in the reform texts are restricted to the terminological cluster of verbs mentioned in the table below. The 1999 *Framework* describes mathematically powerful students as those who do all of the things mentioned in the table above. That is, “*think and communicate, drawing mathematical ideas and using mathematical tools and techniques.*” The verbs in the table are also sorted in terms of thinking and communicative verbs. In addition some *verbs* also appear to show students’ attitude toward mathematics.

Type of Verb	Verbs
Thinking Verbs	<i>Analyze</i> <i>Classify</i> <i>Compare</i> <i>Deduce</i> <i>Design</i> <i>Estimate</i> <i>Hypothesize</i> <i>Infer</i> <i>Investigate</i> <i>Model (construct patterns)</i> <i>Plan</i> <i>Validate</i> <i>Construct</i>
Communicative Verbs	<i>Communicate</i> <i>Present</i> <i>Share</i>
Attitudinal Verbs	<i>Value mathematics</i> <i>Appreciate its role in human affairs</i>

Table 4: Verbs of Mathematical Power

Furthermore, the thinking skills mentioned in the table above are also nominated as higher-order thinking skills which are marked off from the *other kind of thinking skills* by the following characteristics:

- Higher-order thinking is non-algorithmic. That is, the path of action is not fully specified in advance.

- Higher-order thinking tends to be complex. The total path is not visible (mentally speaking) from any single vantage point.
- Higher-order thinking often yields multiple solutions, each with costs and benefits, rather than unique solutions.
- Higher-order thinking involves nuanced judgment and interpretation.
- Higher-order thinking involves the application of multiple criteria, which sometimes conflict with one another.
- Higher-order thinking often involves uncertainty. Not everything that bears on the task at hand is known.
- Higher-order thinking involves self-regulation of the thinking process. We do not recognize higher order thinking in an individual when someone else “calls the plays” at every step.
- Higher-order thinking involves imposing meaning, finding structure in apparent disorder.
- Higher-order thinking is effortful. There is considerable mental work involved in the kinds of elaborations and judgments required (CDE, 1992, p. 21).

If by doing mathematics the students *must* exercise higher order thinking skills, then mathematics ought to be envisaged as the site of this exercise. That is to say, mathematical activity, to be consistent with the characteristics of higher order skills, must also appear as non-algorithmic and complex; it should appear to yield multiple solutions with multiple criteria for judgment; should be uncertain; it should involve self-regulation with no one else ‘calling the play’; it should require effortful investigative work. Thus, the reform texts framed the nature of mathematics in ways that made it consistent with

the definitions of higher order thinking skills, which must be exercised in the classroom settings to make the whole enterprise of reforms internally coherent. As one of the reform texts put it:

Real mathematics is rarely prestructured or marked with key words. Real situations seldom look like recipes; more often, they are complex and ambiguous. A single task can encompass many problems, often not clearly defined. There may be many ways to go about finding a solution or even deciding what constitutes a solution. Completing a task may take hours, weeks, or even years of sustained, persistent work (California Department of Education, 1992, p. 16).

The 1992 *Framework*, for example, regarded the student work in mathematics as complete only if it demonstrated all of the four dimensions of mathematical power: “when a student successfully finishes a large-scale project that demonstrates all the dimensions of mathematical power, that accomplishment will be referred to as *complete mathematical work*” (CDE, 1992, p. 5). While, the dimensions of mathematical power involved reasoning and use of higher order thinking skills to *investigate, conjecture, justify, validate*, so on and so forth, they de-emphasize other skills such as memorization, automatic recall of number facts, and so on.

In order to make school mathematics compatible with a child engaged in practices defined by the above-mentioned terms, mathematics would have to be conceptualized in such ways as to offer possibilities for such higher-order thinking acts to exist. This conceptualization would also be such as to exclude the use of other schools. The extreme manifestation of this posture toward mathematics is shown poignantly in the following passage from Wilson’s documentation of mathematics education debates in California.

As a participant-observer in one professional development seminar, when asked to solve a math problem, I used the quadratic formula. When asked to present my results, I did, surprised to discover the hostility that erupted among my colleagues: “You *can’t* use formulas. You have to *think*” (Wilson, 2003, p. 49).

As I have noted above, it is happy coincidence that the reform texts eventually found support for their notion of “real mathematics” in a particular epistemology of mathematics, which, like reform texts, viewed mathematics as progressing when mathematicians made and refuted conjectures. Armed with Lakatos, mathematics education could now see mathematics classrooms as images of mathematical communities and children as little mathematicians.

To wrap up the conversation in this section, the reform texts construe mathematics as an object of higher order thinking skills expressed through action verbs given in table 3. *Agents* (students) do mathematics as they apply higher order thinking skills purposefully to solve ‘real world’ problems. Learning mathematics is the same as doing mathematics, which is the same as using higher thinking skills. Equating learning with doing mathematics enables the reform texts to project the learning communities as mathematical communities engaged in mathematical practice.

This image is consistent with the broader tenets of pedagogical progressivism and constructivism. Pedagogical progressivism’s concern with the separation of child and curriculum finds a resolution, when on one hand constructivism empowers the child to construct mathematics and on the other hand Lakatos provides support to this *constructability* of mathematics from an epistemological standpoint.

At this point, I would like to call the attention of the reader to the contrasts between the drama of classical texts and the reform texts that I have examined in this chapter. The coincidence of pedagogical progressivism, constructivism, and Lakatos' philosophy may be a narrow terministic screen highlighting some aspects of mathematics and mathematical practice. Yet, it works to displace mathematics from the mind of God or in a mysterious heavenly realm, and places it in the hands of a human *agent*. Its imagery of *mathematical power* is quintessentially democratic inasmuch as it is shifting the locus of authority from the heavens to earth. Just as democratic governance displaced the sovereign in the society, the conjunction of pedagogical progressivism, constructivism and Lakatosian philosophy works to create a similar effect in the classroom.

Purpose: the Mathematically Empowered, Democratic Citizen

Before going any further let me digress to emphasize that the reform-based texts make individual responsibility central to empowerment. This emphasis on individual responsibility lets us read empowerment as a technology for governing conduct. The California Mathematics Framework, for example, which declares *mathematical power for all students* as the central goal of K-12 school mathematics, makes a loud pronouncement about what it requires from the students in terms of taking responsibility for their own learning:

Students are also members of the school community; they *must* change as well.

Like their teachers, students will learn to work in different ways and to play different roles. And like their teachers, they will need “staff” development in these new ways of working and studying...To be ready [read *mathematically powerful*], California students need to develop—and everyone needs to nurture—

a mathematical work ethic that requires *self-discipline* and efforts to meet a high-quality standard (CDE, 1992, pp. 12-13. Italics mine).

At this point, I would like to draw attention to an analogy with the emergence of constitutional democracies. In this emergence, the influence of sovereign monarchies was curtailed in developments that diffused the power throughout the social body. This is expressed as an emphasis on individual responsibility (See, for example, CDE, 1992). As I mentioned earlier, while the reform texts permit an *act-scene* ratio—i.e., children as active constructors of knowledge--this enabling is itself circumscribed, almost enforced, by the scene of *mathematics education*. The *act-scene* relation, then, exists within a *scene-act* relation in which the *scene*, as argued earlier, is mathematics education. The counter reforms targeted this scene. When the scene is replaced, as it happened in the case of California's 1999 *Framework*—a complete set of relations, signified by the term *mathematical power*, disappeared with it.

Agency: *Mathematical Power* as an Exercise of Power

In the beginning of my discussion on *mathematics education* as a scene I used Batarce and Lerman's (2008) argument to suggest the undermining of the term *mathematics* in mathematics education. In a discussion of *mathematical power* as an agency, I use a similar argument to argue that even though mathematical power works as agency in the reform texts, it does so in a way that may undermine both *mathematics* and the intention to *empower* students.

Here it is useful to refer to the analysis of classical texts in the previous chapters. *Mathematical power* was situated in a different structure of motivations in the classical texts. People were either gifted with *mathematical powers* or not. Those who were thus

gifted were able to glimpse a divine order. However, in the reform texts mathematical *power* is an expression of teachable higher order thinking skills. That is to say, it is a site of pedagogical intervention. It is assumed that by creating appropriate educational experiences, teachers will enable their students to be mathematically empowered.

However, the purpose of *empowerment* here is not just mathematical empowerment, but about the possibility of introducing a democratic attitude toward mathematics. By reversing the dramatistic ratio, the reform texts are mapping the drama of democracy in the mathematics classrooms. This is happening, at the cost of undermining the notion of mathematics as a bastion of certainty, de-authorizing it from its traditional sanctioning authorities, and re-authorizing the children as authentic denizens of a math world. The point I am making is that empowerment is loaded with larger societal and unmathematical concerns and can be construed as preparation for a democratic citizenship a lot more than merely mathematical preparation for social efficiency.

The *mathematically powerful* mathematician did not have to be a great citizen just because s/he was mathematically empowered. The mathematicians like Benjamin Peirce were not interested in initiating all students into mathematics. The schoolteachers of the nineteenth century perhaps did not think they were creating a community of mathematicians in their classrooms. Unlike classical texts, however, the reform texts suggested the possibility of creating 'genuine' mathematical communities in the classrooms. The texts sought to do so with reference to the language of empowerment. Thus was created a scene in which the teachers had to come across as the repositories of power. This scene made possible both the powerful and the to-be-empowered. The drama engaged the powerful in acts of empowering those who were less powerful. The

power in such a scene comes across as the capacity of one actor to alter the behavior of another actor. However, the empowerment itself becomes an exercise of power. That is to be empowered, the students must do some things and not do others. That is to say, it is assumed that being mathematically powerful must mean being able to investigate, conjecture, refute, validate, communicate, and all the other things that I discussed in the previous sections. Yet, the skill to form and communicate a conjecture may not always be needed in real life situations. Some situations may need it, but others may need automaticity with computational skills. When found in such situations, the students who used to associate mathematical power with a well defined and narrow set of skills will not necessarily feel powerful.

Concluding Thoughts

This chapter took the dramatistic pentad and applied it to the reform texts. The drama of reform through this analysis appeared at two levels. At the first level, it is defined by a dramatistic *act-scene* ratio. A contrast with classical texts will help me summarize this. In the classical texts, *mathematical* powers were described as a gift of God, and enabled one who possesses it entry to the preexisting math world. In contrast, in the mathematics education reform texts the idea of *mathematical power* conjures up the image of a democratized math world in which a human can think and produce mathematics, revalidate it. That is to say, in contrast with the classical texts, the *mathematical power* in reform texts becomes a capacity to construct, not discover, mathematics. Thus, on this stage *agents* (children) appear as working—like mathematicians—in mathematical communities, actively constructing their mathematical knowledge in this process.

At the second level, *empowering* itself appears as an exercise of *power*. That is to say, the enabling of *mathematical power* appears as an *act* that is sanctioned by the *scene* of mathematics education. The legitimating discourses of mathematical power, which contingently come together in this *scene*, are pedagogical progressivism, constructivism, and Lakatos' philosophy of mathematics. I have explicated this discursive merger by using the arguments of Batarce and Lerman (2008), showing that it undermines the term *mathematics* in mathematics education, and *mathematical* in mathematical power.

Thus, the registers on which *mathematical power* makes its appearance are vastly different in classical texts and the texts of reform mathematics. That is, the notion of *mathematical power* responds to a different structure of motivations, is mobilized by a different set of actors, and is directed toward different purposes.

Chapter V: The Drama of Counter Reforms: Implication for Mathematics Education

[W]hat passes for mathematics in these publications bears scant resemblance to the subject of our collective professional life. Mathematics has undergone a re-definition, and the ongoing process of promoting the transformed version in the mathematics classrooms of K-14 (i.e., from kindergarten to the first two years of college) constitutes the current mathematics education reform movement.

(Wu, 1998, p. 1)

If there was ever a time in the United States when no one cared about mathematics education, it certainly has not been the past couple years. Mathematics education has been written about in the local, regional, and most important national newspapers and magazine. Reports have also appeared on radio and national television. The focus of attention has been the so-called “Math Wars” that center on reform in school mathematics curriculum and its teaching.

(Becker & Jacob, 1998)

Dramatistic Pentad	Classical Texts	Reform Texts	Counter-Reform Texts
Scene	Mathematics	Mathematics Education—Based on NCTM Standards	<i>Standards</i>
Act	Following, Reading, Discovering mathematics	Constructing Mathematics	<i>Following, Reading, demonstrating fluency</i>
Agent	Mathematicians	Children, Teachers	<i>Children</i>
Agency	Mathematical Powers	Mathematical Powers	<i>Mathematical proficiency</i>
Purpose	Illumination by natural light of reason	Empowered citizenry	<i>Numerate Citizenry</i>

Table 5: Pentad at a Glance—Focus on Counter-reform Texts

The landscape of mathematics education may be in flux in the middle of the ongoing mathematics education debates. I argue this based on my observations of the shifts in the

scene and the relations between various elements—*agents, acts, agencies, and purposes*—of the drama of mathematics education. When the *scene* shifts, the definitions of mathematics also shift together with the descriptions of what constitutes the *act* of learning mathematics as well as the learners. This chapter, through an analysis of the counter reform texts, reveals shifts in the relations between the mathematics and its learning and teaching.

Before proceeding I would like to call attention to how this dissertation is advancing its argument by recalling the central concerns of the last two chapters.

In those chapters, the ultimate purpose of dramatistic analysis has been to answer the questions adapted from the dramatistic *pentad* and use those answers to develop insights about the relations between the subject matter of mathematics and its learning *as they appear in particular texts*.³⁷

In Chapter 3, the historical mathematical texts that I examined projected mathematics as a *scene*. That is to say, mathematics was projected in those texts as *a priori* and existing independently of human cognition. The mathematical practice of mathematicians could be described, in metaphorical terms, as finding their way in a sprawling terra incognita. In dramatistic terms, the mathematical practice was described in terms of a strict *scene-act* ratio. Once discovered, the mathematical truths no longer remained in terra incognita but available for others to read, comprehend, follow, and use. The guiding terms for *mathematical acts*, therefore, were discovering, comprehending,

³⁷ The specific questions guiding the dramatistic inquiry are:
Scene: How is the context of mathematical activity described?
Act: How are *mathematical acts* described?
Agent: Who is described as entitled to *act*, that is, do mathematics?
Agency: What means are available to *act*?
Purpose: What is described as the *purpose* of mathematical activity?

and following the preexisting mathematical forms. Mathematical powers became thinkable in these texts as abilities to discover, follow, and comprehend.

Chapter 4, in which I examined the mathematics education reform texts, suggested a shift in the *scene*. While the classical texts posited mathematics itself as *scenic*, in reform texts the *scene* was mathematics education. But this *scene* of mathematics education also contained within it the *act-scene* ratios expressed through a redefinition of mathematics as a human construction—the *act-scene* ratio here implied that it was human *acts* that preceded *mathematics* and not vice-versa. Consistent with this view of mathematics, the reform texts construed mathematical powers as abilities to construct, and also as an object of pedagogical intervention: teaching mathematics well meant expanding students’ power to do—construct, invent—mathematics.

In classical texts, pedagogy was not important because access to *mathematics* was described as a gift of God. In reform texts, pedagogy was important because mathematical power was conceptualized as an object of intervention and development, hence the slogan “mathematical power for all.”

In this chapter, I examine one of the counter reform texts, the *Mathematics Framework for California Public Schools* (CDE, 1999) and the California content standards (which were published together with the 1999 *Framework*). I have selected these documents, not because they are the only example of counter reform texts, but because they replaced a reform-based framework (CDE, 1992). From this point on, I will refer to both the 1999 *Framework* and the content standards published with it as the 1999 *Framework*.

Through this analysis, I will show that the 1999 *Framework* constitutes the scene for the learners in a similar way as mathematics was *scenic* for mathematicians in the classical texts (as discussed in Chapter 3). This is not to suggest that the 1999 *Framework* was derived from a Platonist perspective on mathematics. Rather I mean to indicate that classical texts and counter-reform texts share a dramatistic similarity; both the classical texts and the counter reform texts appear as dramas in which the *agents* (mathematicians in the former and children in the latter) and their mathematical practices are constrained to wander about in a preexisting math land. This contrasts sharply with the drama of reforms in which the *scene* of mathematics education redefined mathematics as a product of human activity and spoke of children as *constructing* mathematics.

The classical texts described the *mathematician* as a discoverer in the preexisting math land. The content standards associated with the 1999 *Framework* work to place students on a narrow set of pathways and benchmarks with *mathematical proficiency* signaling an individual's incremental progress along those pathways. Curriculum, instruction, and assessment are called forth to ensure that *all* students stay on course, that is to say, no child is left behind.

I further argue that the dramatistic arrangement of the 1999 *Framework*, by reversing the relations between mathematics and pedagogy described by the reform texts, also tended to map onto content and pedagogy a separation that exists between the fields of mathematics and education. Since the reform texts were premised on merging content and pedagogy, taking them apart works to contest the claims of professional mathematics educators and to raise questions about what mathematics education is as it emerges from the debris of the math wars.

What follows is a description of the 1999 *Framework*, followed by its dramatic analysis.

Counter Reforms

The *Framework* and the associated content standards that this chapter examines were published in 1999 after the counter-reformers in California were able to reverse the 1992 mathematics education policy in California that had been based on the *Mathematics Framework for California Public Schools* (CDE, 1992) and *Curriculum and Evaluation Standards* (NCTM, 1989).

The 1992 *Framework*, the 1989 NCTM *Standards*, and the curricula based on these documents, had all become targets of intense criticism by different groups for different reasons. The heat generated by these ‘debates’ earned them the title of “math wars” (for a detailed description of math wars in California and nationally, see, Wilson, 2003).

When the curtain finally dropped in California in 1999, the framework had changed hands. The 1999 *Framework*, while ‘accurately’ laying down what students needed to know at each level, raised the flag of *Mathematical Proficiency for All* as its goal. The notion of *Mathematical Power*—which was central to earlier framework and NCTM standards—was expunged from the *framework* and *proficiency* inserted in its place. Beginning with the 1999 *Framework*, the calls for *mathematical proficiency for all* have been pouring into the system nationally and regularly. Notable among the publications defining and building on this notion are *Adding it Up* (Kilpatrick, et al., 2001), *Mathematical Proficiency for All Students* (Ball, 2003), and *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (USDOE, 2008). Notable NCTM publications such as *Principles and Standards of School Mathematics* and

Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics (NCTM, 2000, 2006) also dropped the language of *mathematical power*.

When I first read the 1999 *Framework*, I did not read it with a focus on terms such as *mathematical proficiency*. In fact, in my first reading of both the reform and counter reform texts, I was simply looking for ways in which these texts constructed mathematics, its teachers and learners, without reference to terms such as *power* or *proficiency*. However, the shifts in vocabulary felt irresistible, almost seducing me to focus on the texts which stirred the controversy and those that eventually replaced *power* by *proficiency*.

This shift seemed significant, and, perhaps, a key to understanding the terms of debates. Part of the reason this seemed so was, as I discussed in the previous chapter, that the language of *mathematical power* was part of the professional claims of the field of mathematics education. But the claim of reform texts, were also perceived as undermining the mathematicians' perspective on mathematics. Some mathematicians sprang to action. As mathematician Hung-Hsi Wu,³⁸ who was an influential critic of the reform texts, put it: "One positive outcome of the current mathematics education reform may very well be the revival of the idea that mathematics is important in discussions of mathematics education. The battle over the standards is a stunning illustration of this

³⁸ I often cite Hung-Hsi Wu because of his passionate involvement in mathematics education debates, and possible influence on both 1999 *Framework* and several reports produced nationally after the publication of 1999 *Framework*. Wu was a critic reforms initiated by the 1992 framework in California and one of the authors of the 1999 *Framework*. After 1999, he has been part of the NRC committee along with Jeremy Kilpatrick, Deborah Ball, Hyman Bass and others. This committee published the report *Adding it Up* which brings the notion of *mathematical proficiency for all* in the national conversation about mathematics education. More recently, he has also been part of the *National Mathematics Advisory Panel* (NMAP) commissioned by President George W. Bush along with other notable mathematics educators. The final report of NMAP published in March 2008 captures many of the themes initiated by the 1999 *Framework*, including the ideas of *mathematical proficiency* and of all school mathematics as a preparation for a culminating experience in Algebra I and Algebra II.

fact” (Wu, 2000, p. 27). For Wu, the battle over the standards was a battle over correct representation of mathematics in school mathematics.

1999 Framework

The 1999 *Framework* is a 351-page document, nearly half of which (Chapters 2 and 3) is taken up by a description of mathematics content standards; 28% of the document is five appendices consisting of samples of instructional profiles, three lesson plans, and math problems; and the remaining 22% contains small chapters on Instructional Strategies; Assessment; Universal Access; Responsibilities of Teachers, Parents, Students, and Administrators; Professional Development; Use of Technology; and Criteria for Evaluating Instructional Resources.

Standards in this document are defined as *essential content* for all students, identifying what all students should know and be able to do at each grade level. For kindergarten through grade seven, the statements describing the standards are distributed vertically across the grade levels and horizontally across five content strands, *Number Sense; Algebra and Functions; Measurement and Geometry; Statistics, Data Analysis, and Probability; and Mathematical Reasoning*. A particular strand keeps becoming increasingly dense as the grade level advances. The standards for grades eight through twelve are not organized in strands. Referring to them as ‘discipline names, the framework rearranges them as Algebra 1 & 2, Geometry, Trigonometry, Mathematical Analysis, Linear Algebra, Calculus, and Advanced Placement Probability and Statistics. Each standard is followed up by its component parts giving the appearance of a hierarchical results framework. The 1999 *Framework* also cites research to support its emphases. However, the citations show a clear preference for experimental research.

Wilson observes this preference when describing the 1999 *Framework*, “When research is cited (in the 1999 *Framework*), it is research that fits-for the most part-a narrower paradigm of experimental and quasi-experimental research. Research is presented as definitive and authoritative. No mention is made of disputes over certain practices (like tracking). No mention is made of highly regarded research (among mathematics educators) that draws on other research traditions that are more qualitative” (Wilson, 2003, p. 173).

The framework does not specify how the curriculum should be delivered explicitly leaving pedagogical choices to teachers: “Teachers may use direct instruction, explicit teaching, or knowledge-based discovery learning; investigatory, inquiry-based, problem-solving-based, guided discovery, set-theory-based, traditional, or progressive methods; or other ways in which to teach students the subject matter set forth in these standards” (CDE, 1999, p. 19).

Yet, the 1999 *Framework* also contains three sample lesson plans as appendices. The lesson plans provide precise statements of the goals of the lesson, a list of materials to be used, the plan of activities, and precise steps to be taken by the teachers, sometimes also suggesting the exact phrases that the teacher may use while addressing students.

Whereas the 1992 *Framework* appealed to higher order thinking skills and constructivism as a theory of learning, I did not find any appeal to developmental appropriateness or any other psychological considerations for justification of mathematical content. Rather, the 1999 *Framework* appears to be building up K-7 mathematics as an increasingly complex set of knowledge and skills along a clearly laid

out roadmap with all strands leading to the culminating experiences in Algebra I, Geometry, and Algebra II in grade 8-12.

Standards as Pathways and Milestones: The Scene of Mathematical Proficiency

In Chapters 3 and 4, I have discussed narratives in which the stress on the scene varied. The classical texts presented a *scene-act* ratio stressing *mathematics as scenic*—being out there to be discovered—and mathematicians as wandering around in this preexisting math land. The reform texts presented an *act-scene* ratio, stressing the *agent* as having the *power* to construct mathematics. The 1999 *Framework*, as I argue in detail below, presents a *scene-act* ratio, stressing the *mathematics content standards* embedded in it as scenic.

The 1999 *Framework* sets development of *mathematical proficiency* in all students as the ultimate goal of mathematics education.³⁹ Mathematical Proficiency is said to be gained when all students “can solve meaningful, challenging problems; demonstrate both a depth and breadth of mathematical understanding; and perform both simple and complex computations and mathematical procedures quickly and accurately, with and without the aid of computational tools” (CDE, 1999, p. vi). What solving, demonstrating understanding, and performing computations might mean is stipulated in a set of

³⁹ I would also like to stress that my analysis of the term *mathematical proficiency* in this chapter is limited to the text of the 1999 *Framework*. I should observe that the language that seems to have emerged from California has an interesting trajectory. For instance, the language of *mathematical power* was first used in the 1985 framework. It then reappeared as part of a more emphatic professional consensus in the 1989 NCTM standards, and was later re-emphasized in the 1992 California Framework. Likewise, *mathematical proficiency*, after appearing in the 1999 Framework has found its way into many influential documents. Notable are NRC’s *Adding it Up* (Kilpatrick, et al., 2001), Deborah Ball’s monograph *Mathematical Proficiency for All* (Ball, 2003), and more recently published report *Foundations for Success: The Final Report of the National Mathematical Advisory Panel* (USDOE, 2008). These documents have refined this idea after its first appearance in the 1999 *Framework*.

California mathematics content standards that replace NCTM's standards as the basis for the framework.

In this section, my central thrust is to direct your attention to the work that standards do as a *scene*. As *scenic*, standards embedded in the 1999 *Framework* work to describe the entire mathematical landscape for school mathematics with its pathways and benchmarks in a way that is akin to the mountain range metaphor I used in Chapter 3 to describe mathematics as a *scene* in the classical texts. As such, mathematical *proficiency* is contained by this scene as the ability of the student to reach the benchmarks prescribed by the standards.

Standards as Pathways and Benchmarks

Let me revisit the mountain range metaphor I used in Chapter 3 to talk about the relationships between the mathematics and mathematical acts suggested by the classical texts. The classical texts presented mathematics, like the mountain range, as existing *a priori*. The mathematician, like the mountain trekker/climber, could not create mathematics, but only discover it. Just as the job of climbers was to find the best routes to ³⁴Platonists at the turn of the 20th century. In the case of both Cartesians and Platonists, a large body of mathematics existed in *terra incognita* which became gradually accessible to those with mathematical powers. The high priests alone—the mathematicians who also happened at times to be the clergymen—could access this *terra incognita* and draw its maps for others to understand, read, and follow.

In what ways does this description contribute to an understanding of the 1999 *Framework* and the standards as scenic? By foregrounding the mountain range metaphor, I am certainly not implying an exact parallel between the classical and counter reform

texts. I am *not* claiming that perspectives of those contemporary mathematicians who agitated against the reform texts, and who authored or validated the *1999 Framework*, are similar to nineteenth century mathematicians. The link between these two very different kinds of texts becomes obvious only when one resorts to metaphorical thinking to understand the scenic stress in both documents. That is to say, the classical texts and the contemporary counter reform texts such as the *1999 Framework* are similar only inasmuch as both stress mathematics as existing externally, in the heavenly or mysterious domains in the case of the former, and in the mathematically rigorous standards in the case of the latter. Let me elaborate.

The *1999 Framework* speaks of the standards as a “*context* for a coordinated effort to enable all California students to achieve rigorous, high levels of mathematics proficiency” (CDE, 1999, p. vi, emphasis added). It is the description of the nature of standards as a strict frame in this text, which stresses them as *scenic* from a dramatist perspective. And standards as a scene, I argue here, are similar to the mountain range metaphor inasmuch as both conjure the image of an externally existing landscape. But unlike the classical math texts, the counter-reform mountain range has been converted into a national park with well defined pathways, trail markers, and milestones. Indeed, the authors of the *1999 Framework* pride themselves on their clarity and explicitness.

To understand this claim, consider a student entering a kindergarten classroom modeled on the *1999 Framework*. Much of what is contained in the standards is embedded in the *1999 Framework* is *terra incognita* for the entering student. Yet, there is a preexisting map of mathematical knowledge and skills spread out in time and space

from kindergarten to grade 12—a sort of math world—prepared for him by mathematicians.⁴⁰

Consider now the relationship between the *acts* of the students and standards and compare it with the one that I argued to have existed between a mathematician and a Platonist or a divine world of mathematics. Along the contours of the metaphorical mountain range, all the pathways and benchmarks are already in place in the standards. The student is placed on these pathways upon entering kindergarten is required to cover the distance marked out by every path. In fact, once placed on the path, the student must find herself guided along the pathways, nudged forward if left behind, by the external forces such as those wielded by the curriculum, instruction, and assessment. In much the same way that the individual members of a climbing team are roped in so that no one falls in a crevasse, there are forces and regimens—such as the legislation *No Child Left Behind*—put in place to ensure that students stay on course. That is, this map does not merely preexist but has strictly regulated pathways on which succeeding benchmarks cannot be reached by any other means except first reaching the preceding benchmarks. As the authors of the 1999 *Framework* put it, “Mastery of *almost all the material* at each level depends on mastery of *all* the basic material at all previous levels” (CDE, 1999, p. 198). As mathematics educator Sinclair points out, “this image, by its very nature, discourages connective paths, circuitous paths, short cuts... because such things would

⁴⁰ I stress *mathematicians* among a great variety of actors who may have contributed to the 1994 *Framework* and Standards, primarily because the standards is what mathematicians targeted for their involvement and ‘jurisdiction’. Although, the NCTM standards had the endorsement of mathematical organizations such as MAA and AMS. However, many mathematicians considered that endorsement to be a mistake (see, Klein, 1997) and sought direct involvement in the creation and sanction of content standards, and the sanction of professional community of mathematicians. As Wu put it: “It is often forgotten in the war of words that mathematics education has a substantive component: mathematics” (Wu, 2000, p. 27).

make it almost impossible to judge who is left behind, and also makes the presences of crevasses harder to see” (Sinclair, N., personal correspondence, Aug 14, 2008).

The strands—number sense, algebra and functions, measurement and geometry, statistics and probability, and mathematical reasoning—may be imagined as the major pathways on which the student must remain as s/he advances to the next level. On each strand, within each grade level, standards are stated as benchmarks to be achieved. With the scene set up in this manner, *mathematical proficiency* can be imagined as *the achievement of* what is stated in the benchmarks. The following table shows the ways in which standards are stated as increasingly complex but precise statements about what students must know and be able to do for the first standard (labeled 1.0 in each standard, followed by its component standards) in the ‘number sense’ strand for the kindergarten to grade 3.

The students advance along the pathways by displaying proficiency in each one of these standards. A cursory glimpse on this table shows the path for developing the number sense as laid out carefully. Students begin by simply comparing sets of objects to determine their comparative size. These comparisons are replaced by ordering numbers and using the symbolic notations of less than ($<$), equal to ($=$), or greater than ($>$) as they advance to the next level. As they keep advancing, the place value of the numbers they must be able to count is pushed up by one place.

Kindergarten
1.0 Students understand the relationship between numbers and quantities (i.e., that a set of objects has the same number of objects in different situations regardless of its position or arrangement): 1.1 Compare two or more sets of objects (up to ten objects in each group) and identify which set is equal to, more than, or less than the other 1.2 Count, recognize, represent, name, and order a number of objects (up to 30)
Grade 1
1.0 Students understand and count up to 100 1.1 Count, read, and write whole numbers to 100 1.2 Compare and order whole numbers to 100 by using symbols for less than, equal to, or greater than ($<$, $=$, $>$) 1.3 Represent equivalent forms of the same number through the use of physical models, diagrams, and number expressions 1.4 Count and group objects in ones and tens 1.5 Identify and know the value of coins and show different combinations of coins that equal the same value
Grade 2
1.0 Students understand the relationship between numbers, quantities, and place value in whole numbers up to 1,000. 1.1 Count, read, and write whole numbers to 1000 and identify the place value for each digit 1.2 Order and compare whole numbers to 1000 by using symbols $<$, $=$, $>$
Grade 3
1.0 Students understand the place value of whole numbers 1.1 Count, read, and write whole numbers to 10,000 1.2 Compare and order whole numbers to 10,000 1.3 Identify the place value for each digit in numbers to 10,000 1.4 Round off numbers to 10,000, to the nearest ten, and use expanded notation to represent numbers

Table 6: Standard for the Number Sense Strand in grades K-3

I have tabulated the number strands standards above to illustrate the way they describe the signposts on a path defined by a particular strand. There are 391 standards statements for K-7 in all, distributed across strands as follows:

Strands	Number of Standards
Number Sense	122
Algebra and Functions	60
Statistics, Data Analysis and Probability	55
Mathematical Reasoning	95
K-8 through 12	
Algebra I and II	59
Geometry	22
Trigonometry	21
Mathematical Analysis	10
Linear Algebra	12
Advance Placement Probability and Statistics	8
Calculus	34
Total	391

Table 7: Number of Standards associated with 1999 Framework

With these signposts prescribed by the standards crossed, and the content specified at each signpost mastered, the goal for mathematical proficiency in grades K-12 stands achieved.

Thus, from a dramatistic perspective, the standards in the 1999 *Framework* are *scenic* in largely—but not exactly—the same ways that mathematics was for the mathematicians in the classical texts. That is to say, that they are scenic similarly inasmuch as both refer to an externally existing *a priori* entity. However, it is important to point out that mathematics, as it is projected in classical texts as a creation of God or mysteriously in a Platonist realm, is qualitatively different than *standards* constructed by humans for the purpose of providing a purposive educational experience at the K-12 level. It is important to realize, however, that similarity is dramatistic. That is to say, both Platonist mathematics and Standards-based mathematics is experienced as preexisting. While the *scenes* are different, the *scene-act* ratios are similar.

They are also different since students—unlike the early mathematicians of classical texts—are always already in an environment in which they must do things as prescribed (not just preexisting): commit facts to memory, learn and apply procedures, solve problems according to well defined heuristics, and so forth, and do all of this under the gaze of a teacher.

Standardized Tests as Aspects of the Scene

With the scene laid out in terms of strictly regulated pathways and milestones, the assessment comes across as an aspect of the scene as well and following from the *path-dependent* nature of mathematical progress. The 1999 *Framework* construes progress in achieving the goal of mathematical proficiency as almost entirely *history-* and *path-dependent*. As it is put in the 1999 *Framework*:

A feature unique to mathematics instruction is that new skills are almost entirely built on previously learned skills. If students' understanding of the emphasis topics from previous years or courses is faulty, then it will generally be impossible for students to understand adequately any new topic that depends on those skills. For example, problems with the concept of large numbers as introduced in kindergarten and the first grade may well go unnoticed until the fifth grade, when they could cause students severe difficulty in understanding fractions. The biggest problem facing mathematics assessment is, therefore, how to devise comprehensive methods to detect the mastery of these basic learned skills (CDE, 1999, p. 197).

The framework proposes this detection of mastery (or its absence) to take place before, during and after the instructional intervention. That is to say, the assessment is

performed to establish baseline data, monitor progress, and evaluate *mathematical proficiency* developed after the intervention—exposure to curriculum and instruction in this case—has been completed. Taken together, these forms of assessment “provide a road map that leads students to mastery of the essential mathematical skills and knowledge described in the *Mathematics Content Standards*” (CDE, 1999, p. 196, emphasis added).

Another aspect of assessment as *scenic* is the emphasis on assessing and grouping students according to proficiency levels for comparisons, as well as for assigning them to different diagnostic groups for treatment. If proficiency is to be interpreted in terms of a measure of whether or not students have reached a specific benchmark (characterized by specific knowledge and skills), then assessment becomes thinkable as a mechanism to discover the extent to which the knowledge and skills required at the benchmark, are present or not. Furthermore, if the framework is to guide all students to gain proficiency in standards, then the assessment must appear as part of one or the other risk management strategy that identifies those who are behind so that an appropriately adjusted instructional treatment is provided. The framework also recommends use of assessment to identify those who are above the norm.

It may be useful to contrast briefly the description of assessment in the 1999 *Framework* with the one given in the 1992 framework. In the 1992 framework, the sites of student assessment were focused on assessing expressions of mathematical power—construed more as *acts* than *facts*. For example, the 1992 framework emphasized the *acts* of investigating, conjecturing, and communicating more than automatic recall of number facts on a timed multiple-choice test. In order to collect information about *acts*, the

sources of assessment information in this case moved away from the traditional timed tests to more open-ended questions (tasks that could take a few minutes or be spread over several weeks): observations of students at work; and *Student Portfolios* (see, CDE, 1992, pp. 66-72). This recourse to different sources of information for assessment data was also supported by NCTM *Curriculum Standards*:

An assessment method that stresses only one kind of task or mode of response does not give an accurate indication of performance, nor does it allow students to show their individual capabilities. For example, a timed multiple-choice test that rewards the speedy recognition of a correct option can hamper the more thoughtful, reflective student, whereas unstructured problems can be difficult for students who have had little experience in exploring or generating ideas (NCTM, 1989, p. 202).

While the 1999 *Framework* acknowledges the existence of other ways of collecting assessment data—as the one mentioned in the 1992 *Framework* and cited above—it nevertheless accords *timed tests* a privileged place in measuring mathematics proficiency. Furthermore, a preference for timed tests is justified with appeals to the nature of mathematics.

All of these techniques can provide the teacher and the student with valuable information about their knowledge of the subject. However, they also represent a serious misunderstanding of what mathematics is and what it means to *understand mathematical concepts*. *Assessment methods such as timed tests play an essential role in measuring understanding—especially for the basic topics, the ones that must be emphasized* (CDE, 1999, p. 198, emphasis original).

The timed tests are justified with appeals to automaticity as a requirement for mathematical understanding. The argument rests on the assumption that if students are not able to *quickly* recall knowledge and answer questions, then their knowledge is *superficial*. Students need to master all standards at each level in order to meet the standards at the next level. And since they cannot move to the next level if their knowledge is *superficial*, it is important to detect and eliminate *superficiality*. And, according to the 1999 *Frameworks*, the only way quick recall and automaticity (or its absence) can be detected is through the *standardized timed tests* (CDE, 1999).

In 1999 *Framework*, then, the other sources of assessment information—such as open-ended questions, portfolios, and student observations—privileged by 1992 framework—are deemphasized. The timed tests are reemphasized and justified on the basis of an appeal to the nature of mathematics and its learning. This may be understood better by extending the metaphor of strands as pathways and standards as benchmarks to include assessment as a recording mechanism (assessment sources) placed on various points along the path with the task of activating triggers. The triggers are activated when the recording mechanism records a score. This sounds akin to the ways in which a prescription is triggered by the medical discourse that follows the diagnosis of an ailment. Diagnosis is a sort of interpretation of the body's score on various scales. Likewise the score on timed tests is also interpreted in terms of a diagnostic regimen.

This brings us to the path-dependent nature of mathematical instruction and learning suggested in the framework. As teachers and teacher educators we know that education is complex, uncertain, and dynamic. However, assessment on timed tests simplifies the management of this complexity, uncertainty, and dynamism by simply locating the

coordinates of students on narrow pathways. These coordinates at a given time place students as either behind, on the benchmark, or as gone past it. A trigger is activated when an individual is not recorded as doing all s/he must to reach the benchmark, or if an individual reaches the benchmark too fast. This metaphorical trigger is meant to reroute the slow (or fast) individuals to a processing facility.

In sum, then, assessment, as it is set up in the 1999 *Framework*, can be construed as setting up recording devices (the timed tests), which activate triggers for treatment according to fixed scoring algorithms. That is to say, it activates a remedial program for those performing two standard deviations below the mean score and an accelerating program for those performing two standard deviations above the norm (the gifted or advanced learners). Nothing needs to be done for those students whose score hovers around the mean.

The 1999 *Framework* identifies three such groupings depending on the individual's test score on entry level and formative assessments. These are: Benchmark group (containing children whose score is around the mean score); Strategic group (one or two standard deviations below the mean score); and Intensive group (more than two standard deviations below the mean). The treatments, which the 1999 *Framework* suggests after the students have been labeled as *Benchmark*, *Strategic*, or *Intensive* are broadly described in the document. They are also not important to my argument. My purpose in this conversation has been to highlight those aspects of standards that render them scenic from a dramatistic standpoint. I have argued that in the 1999 *Framework* standards work to structure the space in terms of strands (pathways) and benchmarks for each strand.

Mathematical proficiency comes across in this scene as the ability to reach and get past

the benchmarks in each strand, without getting caught in the *strategic* or *intensive* net. The recording devices (mostly timed tests) are positioned at various points (at the entry level, throughout the pathway, and at the end) to generate data about whether the students are meeting standards (gaining proficiency). The scores on these devices are associated with triggers that, once activated, must send students into different streams for differential treatments aimed at ensuring that *all* move on to the next level.

From a dramatic standpoint, the discussion above suggests a *scene-act* ratio, in which standards completely contain the pathways (strands) as well as the benchmarks in relation to which the notion of *mathematical proficiency* becomes operative. Once placed on these pathways, children are sorted into falling behind, going in the middle, or leading, depending on the diagnosis of their proficiency levels.

The *scene-act* ratio of the 1999 *Framework* is a reversal of *act-scene* ratio expressed in 1992 *Framework*. That is to say, in the 1999 *Framework* the content of mathematics is laid out in advance and *learning, knowing, or doing* school mathematics is contained in this layout, whereas in the 1992 *Framework*, existing mathematics was viewed as a tool with which students could engage in constructing their (new) mathematical knowledge (CDE, 1992).

Rigor and Scene

So far, I have argued that the 1999 *Framework* stresses a scene which is composed of 1) content standards (containing the pathways as well as the milestones), 2) the *mechanisms* for documenting the individual's proficiencies in terms of their placement on the pathways, and 3) the triggers to sort students according to such placement. Before advancing the discussion on the ways in which the element of rigor is inserted into this

scene, I would like to digress to a contrast between classical and reform texts once again. In the classical texts the *scene* is mathematics. In the counter-reform texts, however, the *scene* is a construction, a layout. In the classical texts the mathematical landscape is beyond and independent of humans. In the 1999 *Framework*, however, the scene is a selected representation of mathematics neatly laid out by humans. Thus, even the notion of rigor works differently in these two domains. The classical drama, for example, permits Fermat's theorem whose rigorous proof can wait for two centuries. Fermat's theorem—much like a revelation—does not have to be defined by existing definitions. In the drama of the 1999 *Framework* the notion of rigor, as I will explain below, spells out a technology of control.

The 1999 *Framework* frequently appeals to this qualifier to bolster its exhortations about standards, curriculum, instruction, and assessment. The 1999 *Framework*, unlike NCTM standards and the 1992 *Framework*, does not appeal as much to the psychological arguments—such as constructivism and ideas about higher order thinking skills—as to what it terms as the rigorous mathematical content standards containing essential mathematical content that must be learned by all students. At the same time, the term *rigorous* does not just refer to mathematics but spills over to other domains of mathematics education. It works as an additional qualifier for mathematical proficiency, curriculum, and instruction. For example, calls are made for rigorous levels of mathematical proficiency, rigorous mathematics standards, rigorous and challenging mathematics program, rigorous and demanding curriculum and instruction (pp. 1-2); engagement with rigorous mathematics is offered as a recipe to solve classroom management issues (p. 210).

As a qualifier for standards, rigor is linked with the need to be explicit. The need for being rigorous and explicit is projected as an aid to learning and motivations: since, “students are not biologically prepared to learn much of this material on their own, nor will all of them be inherently motivated to learn it...*explicit and rigorous standards*, effective teaching, and well-developed instructional materials are so important” (p. 185, emphasis original).

Rigor, then, permeates the scene of the 1999 *Framework* as its qualifier and as its appeal and justification. Rigor is not just about mathematics. As I have shown above, rigor is ubiquitous and works as a qualifier for curriculum, instruction, and assessment. Rigor, as I will discuss at length in the next chapter, was used in the rhetoric of counter reforms as a symbolic weapon which aimed at associating the counter reform texts with the powerful cultural values of science, while at the same time dissociating reform texts from these values. Rigor as an element of the scene is extended into the domain of teaching, assessment and research. By corollary, then, in the drama of counter reforms, the rigor is projected on *acts* [of students and teachers] as well as *agency* [the *mathematical proficiency*].

Separating Content and Pedagogy: Contesting the Claims of Mathematics Education

In this discussion so far, I have called your attention to a *scenic* stress in 1999 *Framework*, implying that students and teachers must move along prescribed pathways and milestones—what the 1999 *Framework* calls rigorous and essential mathematical content. In this section, I suggest that this arrangement works to separate *content* from *pedagogy* and that that separation of content and pedagogy works to undermine the professional claims of mathematics educators contained in the reform texts.

In the 1999 *Framework*, separation of content from pedagogy is stated explicitly and is achieved by regarding the content as essential. Mathematical proficiency is stated in terms of mastery of essential mathematical content defined by standards. With proficiency so defined, the zone of pedagogical intervention is separated from definition of content. Teachers are called upon to use any methods that they think would work to achieve the standards. As the framework puts it:

The standards do not specify how the curriculum should be delivered. Teachers may use direct instruction, explicit teaching, or knowledge-based discovery learning; investigatory, inquiry-based, problem-solving-based, guided discovery, set-theory-based, traditional, or progressive methods; or other ways in which to teach students the subject matter set forth in these standards (CDE, 1999, p. 19).

The NCTM *Curriculum and Evaluation Standards* and 1992 *Framework* had blurred the distinction between content and pedagogy by regarding knowing mathematics as doing it (see, NCTM, 1989, p. 7), conceptualizing mathematical power in terms of aspects of doing, and ultimately focusing pedagogical attention on development of mathematical power. Therefore, once enacted this separation between content and pedagogy contests and undermines the professional claims of mathematics educators established through the reform texts signaling standards as a zone of influence for mathematicians. It signals standards as formal space whose contents need professional mathematician's involvement and sanction.

I am not arguing that the shifts signaling separation of content and pedagogy are good or bad. Rather, I am indicating, as I suggested at the beginning of this dissertation, that policy is an extension of the ways in which debates are concluded. To give you an

extreme example, the manner in which World War II concluded shaped the geography of Europe for the next 50 years. Likewise, the manner in which the recent round of mathematics education debates appear to be concluding may be shaping the geography of mathematics education. This constitution, as I have argued in this section, may not require educators to think hard about what mathematics is before deciding how to teach it. It wants them to be competent in the discipline as it is defined by the professional community of mathematicians and turn their efforts on how best to teach it. This proposal to reshape the geography of relations between mathematics and mathematics education is repeated in several documents (CDE, 1999; USDOE, 2008; Wu, 1998, 2007). Berkeley mathematician Hung-Hsi Wu proposes that mathematics education be considered akin to ‘mathematical engineering.’ This phrase, according to Wu, is not offered as a metaphor for mathematics education. As he puts it, “I am not making an analogy. I am not using “engineering” as a metaphor. Rather, I am giving a precise description of what mathematics education really is” (Wu, 2007). Wu appeals to the notion of Engineering as involving customization of abstract scientific principles to satisfy human needs. Like the chemical engineer’s work with the *science* of chemistry to develop products such as “the plexi-glass tanks in aquariums, the gas you pump into your car, shampoo, Lysol...,” mathematical engineers [read educators] have to develop from abstract mathematics a “mathematics that meets the needs of students and teachers in the k-12 classrooms” (Wu, 1998).

This proposal is premised on a separation of content and education. At this point, I would like to recall Nell Noddings’ reflections about mathematics education (Noddings, 1990). Although Noddings is thinking about the methodological and pedagogical aspects

of constructivism, her comments also speak to the analysis I am offering in this section. Noddings suggests that the assumption that all knowledge is constructed did not resolve the matter of locus of power in mathematics, “If p is a mathematical statement, we are more likely to accept it if George Polya or John von Neumann is its source than if, say, Ronald Reagan or a local high school student came up with it. The mathematicians have an authority that the other two do not have” (Noddings, 1990, p. 11). In other words, Noddings is suggesting what this dramatistic analysis identifies as a vulnerability in the reform discourse. Noddings was pointing toward the cultural reality about mathematicians’ power over what defining what counts as correct irrespective of what the mathematics educators define as correct.

Nonetheless, in reform discourse, mathematics educators redefined mathematics by combining content and pedagogy. By becoming part of the counter reform discourse the mathematicians contributed toward redefining *mathematics education* by separating content and pedagogy again.

Concluding Thoughts

The school mathematics conceptualized in the 1999 *Framework*, I have argued in this chapter, can be described by a dramatistic scene-act ratio. The content standards contained in the 1999 *Framework* define the entire landscape of school mathematics as divided in pathways—the strands of number sense, algebra and functions, measurement and geometry, statistics and probability, and mathematical reasoning—and milestones to be reached on each one of these pathways. A student’s journey through these pathways is supposed to be strictly regulated by the milestones. Standardized timed tests work to determine the coordinates of students in relation to milestones (criterion reference) and in

relation to the mean scores. Students are distributed into benchmark, strategic, or intensive categories depending on the standard deviation of their scores from the mean.

This analysis suggests an irony in thinking about the possible relations between mathematics and school mathematics. Mathematics educators claimed to create authentic mathematical communities in the classrooms (Chapter 4). The professional claims of mathematics educators redefined mathematics and mathematical powers as objects of pedagogical intervention. However, these efforts, the reforms, constituted a *terministic screen* with important inclusions and exclusions. The irony is contained in the observation that the strongest challenge to attempts to construct authentic mathematical communities came from the mathematicians. Ultimately, when mathematicians turned to school mathematics, they rendered it a highly rationalized system of pathways and benchmarks in which the progress in mathematics is defined strictly and linearly. While the reformers talked in terms of authentic practice, and counter-reformers in terms of mathematical content, both suffer from the paradox of definitions that Burke had suggested in his term *terministic screen*. Although both reform and counter-reform texts seek vocabularies that they hope to be faithful *reflections* of mathematical reality, to this end they must develop vocabularies that are *selections* of reality. “And any selection of reality must, in certain circumstances, function as a *deflection* of reality” (Burke, 1945, p. 59).

Chapter VI: The American Jeremiad and Reforms: The Dramatistic Vulnerabilities of Reform Texts

[I]n a time of great crisis, such as a shipwreck, the conduct of all persons involved in that crisis could be expected to manifest in some way the motivating influence of the crisis. Yet, within such a "scene-act ratio," there would be a range of "agent-act ratios," insofar as one man was "proved" to be cowardly, another bold, another resourceful, and so on.

(Burke, 1978, pp. 332-333)

A Nation at Risk: The Imperative for Educational Reform

(NCEE, 1983)

In his penetrating analysis of early religious and political sermons in New England, Sacvan Bercovitch identifies a particular rhetorical trope that worked to generate in its audience a deep anxiety by showing them the images of an impending damnation, while at the same time promising redemption through proper *acts* (Bercovitch, 1978).

Bercovitch called it the *American Jeremiad* to highlight the peculiarity of this trope as New England Puritans' unique variation on the religious sermons of the Hebrew prophet Jeremiah.

I find the trope of the American Jeremiad together with the elements of dramatism particularly generative as a way of understanding the drama of mathematics education reforms and counter reforms as *acts* within a *scene* set up by jeremiads. From the standpoint of dramatism, as I will discuss in detail subsequently, jeremiads come across as narratives that stress *scene*, albeit one that is always characterized by decline and

crisis. That is to say, the jeremiads declare their audience to be in a state of deep crisis and doomed if they did not *act* to transform the *scene* containing the crisis. As such, the jeremiads constitute possibilities for reforms as *acts* seeking to transform the crisis ridden scene. In an important way, then, jeremiads appear to be the generators of reforms.⁴¹ In this chapter, I mobilize this rhetorical relation—between the jeremiads as *scenes* describing decline and potential damnation and *acts* [of reforms] promising salvation—to think about both reforms and counter reforms as *acts* within a scene set up by a jeremiad.

The jeremiad in question here is *A Nation at Risk* (NCEE, 1983), the report was produced by the National Commission for Excellence in Education (NCEE). The report is famous for its phrase, “...the educational foundations of our society are presently being eroded by a *rising tide of mediocrity* that threatens our very future as a Nation and a people” (NCEE, 1983, p. 113). From a dramatistic standpoint, *A Nation at Risk*, as I will explain at length in this chapter, describes the *scene* of a crisis in terms of declining test scores, and thus circumscribe possible reforms as efforts to establish higher and more rigorous standards for the students to eventually reverse the decline. I will use Burke’s pentad, in particular the notion of the *scene-act ratio* to account for the vulnerability of the rhetorical strategies employed by mathematics education reform texts to respond to *A Nation at Risk*.

With standards becoming the terms in which the possibilities of reforms were stated, the *scene-act* logic demanded production of *standards* as the proper reform *acts*. And indeed, in the decade following *A Nation at Risk*, the air was filled with the talk of standards. In school mathematics—as well as in other subjects—the reform proposal for

⁴¹ The ubiquity of jeremiads taken together with ubiquity of reforms in America may also explain the absence of these terms in the cultures that do not possess jeremiads as a dominating rhetorical trope.

higher standards was justified with reference to *call for reforms* by *A Nation at Risk* and other reports that reinforced the jeremiad (CDE, 1985, 1992; NCTM, 1989; NRC, 1989).

This chapter consists of two main sections. In the first section, I survey the notion of American jeremiad as a rhetorical trope that stresses a *scene* of crisis. I will provide examples of earlier jeremiads to show the ways in which they, as descriptions of *scene*, circumscribe the *acts* of reforms. I will then elucidate how *A Nation at Risk* works as an instance of American jeremiad, and as such, constitutes the scene for both mathematics education reforms and counter reforms. This discussion creates the analytical space for consideration of both reform and counter-reform texts in mathematics education as competing responses to the same calls for *action* constituted by *A Nation at Risk*.

In the second section, I examine the rhetorical strategies of the reform-texts, chiefly those produced by the NCTM, as vulnerable affirmations of the *scene-act* ratio. I argue that the reform texts in mathematics education used the term “standards” but infused the term with a meaning that was shown by counter reformers to be incompatible with the *acts* called for by the scene that the jeremiad *A Nation at Risk* had described. That is, if the scene is a jeremiad, then reform texts could not be heard on the stage because of their simultaneous presence in a different scene, that of *progressive* discourse.

The American Jeremiad

The term *jeremiad* is derived from the name of the Hebrew Prophet Jeremiah who lived circa seventh century BCE. Jeremiah is considered to be one of the Major Prophets (with Isaiah and Ezekiel, and sometimes Daniel) in the books of the Bible that Christians call “the old testament.” Jeremiah’s sermons were crafted to warn ancient Israel of its impending doom, and urged repentance for its sins so that it could be redeemed and

restored to its proper place. The Oxford English Dictionary shows the term jeremiad as in use since the late eighteenth century. James Jasinski's *Sourcebook of Rhetoric* also corroborates this claim highlighting its use as a form of public discourse that echoed the key themes of the religious sermons of prophet Jeremiah (Jasinski, 2001).

Elements of Jeremiad

The eighteenth century American jeremiad was a ritualized evaluation of the state-of-the-covenant ritual. It had three recognizable parts. First, it emphasized the covenant between the Puritans and God, affirming the belief that God had chosen Puritans as the historic instrument of His will. This relationship between Puritans and God seemed to be based on a balanced compact implying that “if they kept their part of the bargain, God would keep His promise and grant the Puritans salvation” (Jasinski, 2001, p. 335). Second, jeremiads routinely depicted the state-of-the-covenant as characterized by Puritans’ deviation from the covenant. This deviation was used to account for the calamities that had befallen them in the present and to warn them of a terrible future if the earlier robust state-of-covenant was not restored. “What was happening, Puritans were told over and over again, was a process of *decline* or a falling away from the terms of the promise” (Jasinski, 2001, p. 336, emphasis in original). As such then, the jeremiads worked to rhetorically alter people’s perceptions about tragedy, disease, and other calamities, real or perceived. Through the announcements of jeremiad, they appeared as expressions of the declining state of covenant between Puritans and God.⁴²

⁴² In rhetorical studies Jeremiad is considered part of what is broadly called the *epideictic* genre. Booth describes epideictic genre as consisting of rhetorical attempts to “reshape views of the present” (Booth, 2004, p. 17). This *effect* seeking rhetorical genre includes a wide range of rhetorical activities including

The third element of the Puritan jeremiad was a prophetic reassurance to its audience that God had not forsaken them: That even though God had let wolves loose on them, they were still His sheep, and that redemption would ultimately follow if appropriate reforms were undertaken. “The *only* way to avoid the shipwreck,” announced Winthrop, “is to follow the counsel of Micah: to do justly, to love mercy, to walk humbly with our God” (Quoted in, King, 2004, p. 206, emphasis added). The ultimate purpose of this sermon was, as Bercovitch puts it, to “direct an imperiled people of God toward the fulfillment of their destiny, to guide them individually toward salvation, and collectively toward the American city of God” (Bercovitch, 1978, p. 9).

There are three important aspects of this description from the standpoint of dramatistic grammar. First, there is a stress on the existence of an imperiled people together with the conditions of their imperilment, which comes across as the description of a scene. Second, the imperiled people are directed to escape this state of imperilment to fulfill their destiny, which is an exhortation to act or reform. Finally, and more importantly from the standpoint of the argument in this chapter, the jeremiads define the terms of the reforms, in effect constituting what could be said or done as reforms—“do justly, love mercy, walk humbly with our God” or “increase the church staff” etc.

This last element of the jeremiad, then, by asking for a rhetorical resonance between the *scene* and the *act*, can also be seen as constituting the possibilities of debate about what could be an *appropriate* response to the calls of reform. I imagine debates between various reform positions as affirmations of this *scene-act* ratio. Conceivably, the *acts* that identify themselves with the *calls for reform* in a particular jeremiad are expected to

education (Sullivan, 1994), funeral oratory (Wills, 1992), even poetry. The notion of education being an epideictic rhetoric is not, however, a universally accepted notion.

play on the terministic grounds prepared by the jeremiad. Below, I have shown the alignment of these early Jeremiads with elements of Dramatism.

Elements of Pentad	The Early American Jeremiad
<i>Scene</i>	<i>State of the Covenant as in Decline</i>
<i>Act</i>	<i>Reforms</i>
<i>Agent</i>	<i>Reformers</i>
<i>Agency</i>	<i>Church</i>
<i>Purpose</i>	<i>Restore the State of Covenant</i>

Table 8: The Dramatistic Elements in American Jeremiad

To flesh out this description of the rhetoric of American jeremiad and to further highlight its relevance to an understanding of the terms of debate in education, I provide below an example from a late seventeenth century New England's Synod—a commission of Puritan clergymen,⁴³ who were called upon to inquire into the causes of the perceived decline of New England and to suggest reforms to rid the Puritans of maladies that afflicted them.⁴⁴ The Synod was appropriately titled *The Reforming Synod* (Walker, 1893, p. 10).

Two questions defined the work of the synod:

1. What are the evils that have provoked the Lord to bring His judgments on
New England? (Walker, 1893, p. 426, Caps as in original text)

⁴³ Synod's similarity with its more recent equivalents, the special and presidential commissions such as the *National Commission for Excellence* that authored the report *A Nation at Risk* in 1983, is quite pronounced

⁴⁴ The descriptions of the context of New England's synod and quotes from its final report have been documented in several archival works. The descriptions that I use are compiled by church historian Williston Walker (1860-1922). Williston Walker was Titus Professor of Ecclesiastical History at Yale University from 1901 until his death in 1922.

2. What is to be done so these evils may be reformed? (Walker, 1893, p. 433)

While the first question suggests the *scene* populated with *evil* and, therefore, one that needs to be cleansed, the second leads to a description of *acts* [reforms].

Scene: Past as a Place of Bliss, Present that of Crisis. The scene constructed by the jeremiad is New England's glorious past and current calamities. The synod also justifies its existence as a means to provide an explanation for the calamities, and to prescribe a way out and back to the glory of the past. The explanations bolster the scene by adding to it the elements whose presence is advanced as the reason for decline. The glorious past—characterized by a healthy state-of-the-covenant between the Puritans and God—appears to cast a nostalgic shadow on the present providing the motivational influence for the restoration of a glory. The past is mobilized as a place of bliss, delight and peace and characterized by a healthy state-of-the-covenant between the Puritans and God.

Describing this scene, its authors observe:

The good will of him that dwelt in the bush hath been upon the head of those that were separated from their Brethren: and the Lord hath (by turning a Wilderness into fruitful land) brought us into a wealthy place; he hath planted a Vine, having cast out the Heathen, prepared Room for it, and caused it to take deep rooting, and to fill the land, which hath sent out its boughs unto the Sea, and its branches to the River. If we ask of the dayes that are past, and look from the one side of heaven to the other, where can we' find the like to this great thing which the Lord hath done? (Walker, 1893, pp. 423-424, spellings as in original)

But, then, as Walker puts it, "this course of prosperity was rudely interrupted..." by wars and natural disasters. The Jeremiad came into play to account for the wars and

disasters by rhetorically identifying these calamities as expressions of the ‘wrath of God’:⁴⁵

But that we have in too many respects, been forgetting the Errand upon which the Lord sent us hither...And therefore we may not wonder that God hath changed the tenour of his Dispensations towards us, turning to doe us hurt, and consuming us after that he hath done us good. If we had continued to be as once we were, the Lord would have continued to doe for us, as once he did. (Walker, 1893, p. 424, spellings as in original)

Explanation of the Calamities. The second element of the *scene*, as explained above, is a description of the ways in which New Englanders were seen as deviating from the covenant. In the synod’s report, these descriptions are offered as responses to the first question above. Throughout these descriptions, the synod seeks to persuade its audience by first describing what it observed as sins of the Puritans and then specifying why they are sufficient reasons for God’s wrath and punishment. Below, I briefly describe the sins documented by the synod in order for you to see the manner in which they form a *scene* for the proposed reforms. These are: “a visible decay of the power of Godliness among many Professors” in the churches; neglect of church fellowship and other divine institutions; Sabbath breaking; sinful indulgence of masters of servants and parents of children; “sinful heats and hatreds” among the members of the church; the frequent “heathenish and Idolatrous practice of” drinking; promise breaking and lying; inordinate

⁴⁵ In rhetorical studies, the narratives that seek to alter the sense of reality of their audience are part of what is called the *epideictic* genre. Booth describes epideictic genre as consisting of rhetorical attempts to “reshape views of the present” (Booth, 2004, p. 17). This effect-seeking rhetorical genre includes a wide range of rhetorical activities including education (Sullivan, 1994), funeral oratory (Wills, 1992), even poetry.

affection of the world; opposition to the work of reformation; an absence of public spirit⁴⁶; and finally sinning against the Gospel (Walker, 1893, pp. 427-432).

The Future Scene. The third element of Jeremiad's scene is an affirmation of Puritan's destiny. The ways in which the rhetoric of synod affirms the status of Puritans as God's *sheep* is sprinkled all over in the jeremiad of the Synod. Nowhere in the report, is even a hint of God's intention to break the Covenant because of the Puritan's sins. The Puritans could be punished and chastised by God but they were not abandoned. "We have therefore cause to fear that the Wolves which God in His holy Providence hath let loose upon us, have been sent to chastise *His Sheep* for their dividings and strayings one from another..." (Walker, 1893, p. 427, emphasis added). This affirmation of a special and unbreakable bonding between the God and Puritans together with existence of sins forms a *scene* which rationalizes the *acts* of reforms.

Acts

Finally, the Synod responds to the question of what needs to be done to reform and restore the covenant. With eradication of the sins as the telos, the Synod prescribed the Church as a major site of reforms describing it as in need of more discipline, more staff and more encouragement. Interestingly, the Synod also mentioned schools of learning as important sites for reform. "As an expedient for Reformation," it says, "it is good that

⁴⁶ The synod does not seem to understand public in the same sense in which we understand it. Describing the absence of public sentiment as an affliction of New England, the synod says, "all seek their own, not the things that are Jesus Christ's. Matters pertaining to the Kingdome of God, are either not regarded, or not in the first place" (Walker, 1893, p. 432). In this report, the symbol of *Jesus Christ and Kingdome of God* combine to assign to the term *public* its collective meaning. Interestingly, and in the next breath, the synod also associates lack of *public* sentiment with the *languishing state* of the *schools of learning*. The synod clearly recognizes the importance of school for a collective allegiance to the image of Jesus Christ. Comparing these lines with the place of the symbol of *America* in the contemporary rhetoric, one may see the symbol of *America* may be doing for the *secular* what *Jesus* did for the *sacred*.

effectual care should be taken, respecting Schools of Learning. The interest of Religion and good Literature have been wont to rise and fall together” (Walker, 1893, p. 437).

The parallel between the *Reforming Synod*’s report with the modern day Jeremiads such as *A Nation at Risk* are unmistakable and striking. Replace, for instance, the *Covenant* by the *Nation* and *Church* by *School*, and the similarity comes in full view. This similarity and its implications, I will discuss later in this chapter.

The operation of scene-act logic in jeremiad

Here, I want to emphasize the dramatistic logic of *scene-act* ratio as applicable to any act of reform that refers to jeremiad for its justification. The reforms in the wake of the synod’s report would be constrained as efforts to strengthen the church and the ‘schools of learning.’ Imagine a hypothetical reform proposal devised as a response to the synod’s report. Suppose that this hypothetical program sought to restore the covenant and justified itself with reference to and as a response to the synod’s report, but instead of being a program to strengthen the church or the schools of learning, as suggested by the synod, it decided to strengthen the prison houses and lunatic asylums. Just because it justified its existence with respect to the findings and prescriptions of the Synod, its acts (reform prisons and lunatic asylums) would be perceived as out of synch with the purposes (restore the covenant) they intended to serve. Alternatively, reforming prisons will need to be defended as a better alternative than strengthening the Church in order to achieve the same purpose (restore the covenant). However, if the terms of reforms did not follow the prescription of the synod, or did not defend them with reference to the *purposes* of reform, they would be seen as a rhetorical misfit due to the violation of *scene-act* ratio.

As I will argue in the next section, the *scene-act* ratio unites the earlier religious with the contemporary secular jeremiads.

From Jeremiads to Jeremiads: Transformations from Sacred to Secular

Although, I do not intend to summarize the whole history of the jeremiad as a form of American political prose, it makes sense for the purposes of this argument to survey the major shifts in jeremiad, while highlighting that they perpetuate the same dramatistic *scene-act* ratio. One important shift is the secularization of jeremiads.

Horace Mann's lectures delivered in the early nineteenth century can be seen as part of this ongoing transformation. Like the earlier jeremiads, Mann's rhetoric too directs the attention of its audience toward a growing menace and decline. But the source of impending doom is not God but (humankind's) unschooled mind. In one of his lectures delivered in 1837, he said:

The mobs, the riots, the burnings, the lynching, perpetrated by the men of the present day, are perpetrated, because of their vicious or defective education, when children. We see, and feel, the havoc and the ravage of their tiger-passions, now, when they are full grown; but it was years ago that they were whelped and suckled. And so, too, if we are derelict from our duty, in this matter, our children, in their turn, will suffer. If we permit the vulture's eggs to be incubated and hatched, it will then be too late to take care of the lambs (Mann, 1845, p. 13).

The theme of damnation and salvation is preserved in Mann's rhetoric. However, unlike the jeremiad of the *Reforming Synod*, the focus, and object, of reform is no longer the church but the common school. Tyack and Cuban also read Horace Mann's rhetoric similarly, when they see him as taking his "audience to the edge of the precipice to see

the social hell that lay before them if they did not achieve salvation through the common school” (Tyack & Cuban, 1995, p. 1). Mann, like the synod I discussed earlier, persuades his audience by altering their perceptions about business as usual without reforms. When his audience see themselves as teetering at “the edge of the precipice,” his calls for action also assume urgency. Although common school replaced church in these rhetorical substitutions common school replace church as the focus of attention, Horace Mann’s case for common schools is similar to 17th century Puritan jeremiads inasmuch as it also contains warnings of an impending doom. Furthermore, like its predecessors, it also seeks salvation through reform. The agency of redemption, however, is not church but common school.

The secularization of the American jeremiad over the past two centuries also entailed other rearrangements. For example, jeremiads were transformed from the earlier state-of-the-covenant sermons to the state-of-the-American Dream statements. This was also exemplified in the famous “I Have a Dream” jeremiad by Martin Luther King Jr. Like the Synod’s report, Martin Luther King’s speech also mentioned a *promise* and depicted it as at risk. Martin Luther King’s promise, however, was not to be traced back to the scriptures but to the *emancipation proclamation*, a covenant not with God but with the people. It then moved on to depict America’s aberration from this covenant, and ended with a prophecy of what his American dream—a replacement for the term destiny in earlier Jeremiads—would look like, if only we rededicated ourselves to the spirit of emancipation proclamation. *Emancipation proclamation* then can be interpreted as the secularized version of the *Covenant*.

The rhetorical substitutions of religious with secular symbols entailed by the secularization of American Jeremiad (Jasinski, 2001) is succinctly summarized by Jasinski in the following quote from *Sourcebook of Rhetoric*:

First, the idea that Puritans were the chosen people was replaced by the idea that the American people (or more abstractly, America) were somehow special... Through this substitution or replacement, “America” becomes a condensation symbol—a verbal cue that embodies our most basic beliefs and values—and the promise contained in the Puritans’ covenant with God becomes our collective promises to each other embodied in the idea of the “American Dream.” Second, the Bible and the lives of the saints were replaced by new texts and new heroes—the Declaration of Independence, the Constitution, Jefferson, Washington, Lincoln, and so on—as the sources from which secular Jeremiads could draw to instruct their audiences on the appropriate policies and practices for realizing America’s special role in the world (Jasinski, 2001, p. 336).

The promise or destiny, frozen in the symbol of America, would be continually rhetorically affirmed in the expressions of the fear of losing them and in the calls for subsequent frenetic efforts—i.e. reforms—to restore to the symbol its earlier vitality. The perpetual state of *being at risk* will come to characterize the state of American dream and destiny and calls for reforms would surface as *acts* needed to restore it.

Elements of Pentad	The Puritan Jeremiad	Secularized Jeremiad
<i>Scene</i>	<i>State of the Covenant as in Decline</i>	<i>State of the American Dream as in Decline</i>
<i>Act</i>	<i>Reforms</i>	<i>Reforms</i>
<i>Agent</i>	<i>Reformers</i>	<i>Reformers</i>
<i>Agency</i>	<i>Church</i>	<i>Varied, Schools in the case of Horace Mann</i>
<i>Purpose</i>	<i>Restore the State of Covenant</i>	<i>Restore the State of American Dream</i>

Table 9: Comparison between the Puritan and Secularized Jeremiads

Similarities between Theologized and Secularized Jeremiads

I have one more aspect of jeremiad to highlight before moving on to a discussion of jeremiad as a *scene* for mathematics education debates in the last two decades. So far my discussion of jeremiads has delineated them as belonging to a rhetorical genre that aims at shaping the perception of its audience about a present *declining state of things* before grounding its calls for reforms in this perception. I would like to examine the dramatic implications of the jeremiadic reference to the term *state* (as in the phrase “state of being”). Under a dramatic translation, the term *state* assumes a scenic connotation. But the *state* at any given time can also be imagined as an enactment. The term *state* in most narratives is also indicated by a particular arrangement of things. Let me elaborate this by considering the 17th Synod report again. When the Synod directed the attention of its audience toward a bad *state-of-the-covenant*, it depicted the *scene* as characterized by a proliferation of sins. Sinning, however, is an *act*. But in the synod’s report it characterizes the state of Puritans, and hence has a scenic connotation.

Dramatically speaking, *acts* and *scenes* can be interpreted to be in a symbiotic relationship. *Scenes* can be seen as derived from *acts*, and vice versa. As descriptions of

scenes, the 17th century Jeremiads depicted the state of the covenant between Puritans and God. The point I am making is that it is possible to view Jeremiads as acts. I could do it if I wanted to, but since I am concerned more with reforms as acts, and since jeremiads appeared to frame the reforms, I prefer to emphasize the scenic connotation of jeremiads in my argument.

To sum up, the jeremiads work to mix lamentations about a present, and a declining, state of the covenant (with *God* for 17th century jeremiads and with the *Nation* for the secularized jeremiads) with prophecies about eventual salvation and fulfillment of a covenant. The perpetuation of a similar *scene-act* ratio across religious and secular jeremiads renders them dramatically similar.

The *acts* sought by the jeremiads are expected to exorcise the scene by removing from it the elements that it identifies as at the root cause of anxiety. The *purpose* of jeremiad, as Bercovitch puts it, is to “exorcise, revitalize, and consolidate” (Bercovitch, 1978, p. 169).

Whether the acts appeal to the audience as serious and effective, or comic and ineffective affirms the logic of *scene-act* ratio. Recall from Chapter 2 that there is a relation of consistency between the *scene* and the *act*:

It is a *principle* of drama that the nature of acts and agents should be consistent with the nature of the scene. And whereas comic and grotesque works may deliberately set these elements at odds with one another, audiences make allowances for such liberty, which reaffirms the same principle of consistency in its very violation (Burke, 1945, p. 3, emphasis added).

Burke points toward the possibility of understanding the *comic* as a consequence of the violation of the principle of consistency.

My interpretation of reform texts in mathematics education (*as acts*) is based on the evaluation of this *scene-act* ratio. It is commonplace to think of the *Math Wars* as between conservatives and progressives. As such, the overarching labels associated with a set of binaries are given in the following table.

Conservative	Progressive
Mathematical proficiency	Mathematical power
Traditionalist	Constructivist
Math as skills	Math as conceptual understanding
Direct instruction	Child-centered instruction
Social efficiency	Democratic equality
Positivist	Interpretivist

Table 10: Binaries of *Math Wars*

While this table highlights the terms of difference between the so-called progressive and conservative responses to calls for reforms, it leaves out the terms that united both in their claims as valid responses to those calls. As practicing mathematics educators we know that progressive reforms were associated with a description of mathematics education that articulated all the terms in the right hand column of the table 10 above as various facets of reform. However, not much attention is paid to the implications of the

language of standards that was common to both sides in the reforms, and the relation of the language of standards with the other categories in this list. But when we see the reforms as *acts* formulated in response to a jeremiad, the common denominator of standards, as I will describe in more detail below, is sent into sharp relief. Thus the lens of dramatism helps us see the rhetoric of reforms as arising from the requirements of a *scene-act* ratio set up by the jeremiad.

As I will discuss below in detail, the term *standards*, qualified by the term *measurable*, was foremost in the jeremiadic pronouncements that were used as justification by the reformers. The analysis in the next section will outline the ways in which the reformers' mobilization of the term *standards* together with other terms in the right column of the table above worked as a double edged sword.

Even as it worked to qualify the reform discourse—as a proper act, it ultimately worked to undermine its own rhetoric. Ultimately, the reform texts were made to appear in the public discourse as comic by the traditionalist activists in a way that affirmed the consistency of a *scene-act* ratio by making the claims of reform-text appear as its violation.

American Jeremiad and Mathematics Education Debates

The mathematics education reform and counter-reform texts that this dissertation is concerned with are largely seen as part of the so called tidal wave of reforms generated in the wake of the report *A Nation at Risk* by the National Commission for Excellence (NCEE) in Education in 1983.

The NCEE was formed by U.S. Secretary of Education Terrel Bell in 1981, to study perceived problems in education and prescribe ways of resolving them. Echoing the tone

of earlier jeremiads, *A Nation at Risk* warned that U.S. society was losing its economic preeminence to other competitors and that its security, prosperity, and democracy were at grave risk due primarily to falling quality of American public schools. Briefly, the report described American schools as faltering in their mission due to the falling test scores, linked this pattern to an eventual, urgently called for and then spawned a nation wide movement for higher standards.

For the next decades or so, talk of standards filled the air. With the eruption of the national standards in all subject areas in the hindsight, *A Nation at Risk* can be seen as constituting the *scene* within which the drama of standards-based reforms would unfold in the next two decades or so. This *scene*, then, framed both reform and counter-reform texts⁴⁷ in mathematics education as competing *acts*.⁴⁸

A Nation at Risk had its share of detractors and supporters. The critics called it part of a burgeoning “conservative restoration” that erupted a decade after the civil rights movement (see, for example, Apple, 1982, 1993, 1996; Shor, 1986). The report has also been critiqued for its flawed use of evidence and accused of joining the wave of attacks on public schools (Berliner & Biddle, 1995). My analysis of *A Nation at Risk* does not pigeonhole it as either a *progressive* or *conservative* document. I am interested in thinking about *A Nation at Risk* as an instance of American Jeremiad. Jeremiads have not

⁴⁷ I am not defending the use of the terms *reform* and *counter-reform* texts on any essential grounds. From a dramatisitic perspective they are both regarded as competing *acts* within a *scene* that calls for them. Reform and reformers is merely a place created by the scene and, as such, can be claimed by multiple *agents*. I, however, use the terms as they are used conventionally.

⁴⁸ I would like to emphasize that I do not make a simplistic assertion about *A Nation at Risk* as the only motivating influence behind mathematics education reforms. Because of its formal similarity with the jeremiads I discussed earlier, I am interested in thinking about what this *scene-act* relation means for understanding the conflict between various competing responses to this report.

been a monopoly of conservatives alone (Bercovitch, 1978).⁴⁹ In each case, however, they have worked to constitute a context for subsequent reforms.

The relation between jeremiad and reform. Theorizing *A Nation at Risk* as an instance of jeremiad permits an understanding of the mathematics education reforms (as *acts*) in relation to a *scene* filled with crisis. Central to the analysis provided in the preceding section is an understanding of jeremiads as constitutive of *scenes* in which something of critical value to the society is described as eroding or declining. But erosion, decline, and other such nouns presuppose a past state as a datum, which by necessity should be free of these characteristics. That is to say, the descriptions of erosion and decline in the present simultaneously evoke a past and a future state free of them. The reforms, then, appear as *acts* sandwiched between an impending damnation—a *scene* that calls for them—and a promised salvation—a *scene* that is expected to result from them.

Dealing out the hand. Furthermore, a jeremiad is always announced from a high pulpit.⁵⁰ As such, then, the cards dealt out by the jeremiads are expected to be picked up and played. The jeremiads thereby indicate general directions that reforms ought to follow in order to appear as proportionate responses to them. For example, the report of the *reforming synod* describes reforms as actions needed to strengthen the Church, Horace Mann defines them as making common school accessible to all children, and Martin Luther King Jr. calls for making good the promises contained in the emancipation

⁴⁹ For instance, even those who were talking of ‘conservative restoration’ in the United States in the 1980s, had their own jeremiad going on. The progressives have their own jeremiad. In fact, the particular battles in the *math wars* may, then, be viewed as a polemic that pits jeremiad against jeremiads.

⁵⁰ I find it significant that jeremiads are not documented as the speech of the ordinary public. It is prophets, saints, synods, political leaders, or the commissions—such as the one that authored *A Nation at Risk*—that can speak the jeremiad.

proclamation. So the purposes and the natures of *reform-acts* are dealt out by the jeremiad like hands in a card game. That is to say, the players—reformers in this case—are constrained to play with the cards dealt out to them.

However, I am not arguing for a simplistic, necessary, and strict relation of correspondence between terms in which calls for reforms and responses to such calls are stated. Rather, I see such a *scene-act* relation as generating ambiguities, and, therefore, also drama and conflict. As we will see in more detail, the jeremiad of *A Nation at Risk* called for reform in terms of *standards*. That is to say, *standards* were dealt out by *A Nation at Risk* as the hand in the game of reforms. To be of relevance on the *scene* created by *A Nation at Risk*, both the so-called conservatives and progressives picked up this card. As such, then, the conflict that followed in mathematics education in particular—and to some extent in other school subjects—can also be seen as a battle over the meaning of the term *standards*. As highlighted in Chapters 4 and 5, both the mathematics education reforms and counter-reform texts were based on, supported by, and repeated the term *standards*; the former associated the *standards* with the terms *mathematical power* while the latter with *mathematical proficiency* for all students.

Revitalizing function. If we view *A Nation at Risk* as merely a politically conservative document then we will not be able to make sense of the ways in which the so called progressive reform documents, such as the NCTM Standards and the 1992 California Mathematics Framework, used it as their justification. Alternatively, if we adopt a rhetorical approach, we can see that a jeremiad such as *A Nation at Risk* had a little something for all sides in the arena of education. In other words, it provided the

context for both the emergence of the so-called *progressive* as well as *conservative* reform texts in mathematics education by providing rhetorical openings to both.

The Scene Described and Acts Called for by A Nation at Risk: The Scene-Act Ratio for Mathematics Education Reforms

A Nation at Risk, like a characteristic jeremiad, emphasized a *promise*, declared it to be at *risk*, called for the reforms for its restoration, and ended on a prophetic note stressing an eventual success with past glory as the guiding light for future destiny. The parallels between the synod's report and *A Nation at Risk* are striking. Like the synod's report *A Nation at Risk* also set out to describe a dismal scene. While the synod had described the *scene* in terms of the moral pitfalls of Puritans, the commission described it in terms of *falling test scores*. While the synod held the *church* responsible for both the past glories and the present predicament of Puritans and called for reforms to strengthen it, *A Nation at Risk* held the *schools* responsible for both the American success and its current *risks* and demanded for reforms that strengthened schools and made them accountable for raising educational standards. While the ultimate purpose of the synod was to rid the Puritans of the social evils and their consequences (sins against the covenant with God), *A Nation at Risk* saw the telos of maintaining national preeminence in terms of raising what appeared to be falling—the *standards* and the *standardized test scores*. I provide this comparison between the two jeremiads to direct your attention to the way role of *A Nation at Risk*, like all jeremiads, works to generate as well as to limit the reforms by providing them with the *terms* in which they must be conceptualized.

The terms in which the *scene* of crisis was constituted by *A Nation at Risk* were *standards* and *standardized test scores*. Any subsequent reforms, which were to refer to

A Nation at Risk as their justification would then be constrained to defend them as proposals to improve what *A Nation at Risk* had declared to be in decline—i.e. the standards and standardized test scores. The report conceptualized both the problem of equity as well as excellence in terms of declining standards. Test scores alone constituted the evidence of the *risk* to the nation at large (NCEE, 1983, p. 113). The extended quote below shows the emphasis on test scores in setting the stage on which the *scene* of crisis was laid out by *A Nation at Risk*:

- International comparisons of student achievement, completed a decade ago, reveal that on 19 academic tests American students were never first or second and, in comparison with other industrialized nations, were last seven times.
- Some 23 million American adults are functionally illiterate by the simplest tests of everyday reading, writing, and comprehension.
- About 13 percent of all 17-year-olds in the United States can be considered functionally illiterate. Functional illiteracy among minority youth may run as high as 40 percent.
- Average achievement of high school students on most standardized tests is now lower than 26 years ago when Sputnik was launched.
- The College Board's Scholastic Aptitude Tests (SAT) demonstrate a virtually unbroken decline from 1963 to 1980. Average verbal scores fell over 50 points and average mathematics scores dropped nearly 40 points.
- Both the number and proportion of students demonstrating superior achievement on the SATs (i.e., those with scores of 650 or higher) have also dramatically declined (NCEE, 1983, p. 115).

A Nation at Risk set up its descriptions of the *risk* in terms of declining test scores and, in keeping with this description, called for higher and measurable *standards*. If the scene of decline is characterized by results on standardized tests, and if the excellence is articulated in terms of higher standards, then the calls to establish *measurable standards* together with ways of meeting them appears to be a proportionate response.⁵¹

Another term that found its way into both reform and counter-reform documents is *all students*. The term *all* obviously appealed to concerns about equity. In attempting to trace the use of the term *all students*, I looked for similar usage in the previous waves of reforms in the United States, but did not find it stated as explicitly and directly as it was in *A Nation at Risk* and the reforms texts spawned by it. The post Sputnik reforms, for example, articulated a rhetoric of quality and excellence and not equity.⁵² However, *A Nation at Risk* foregrounded the term *all*: “All, regardless of race or class or economic status, are entitled to a fair chance and to the tools for developing their individual powers of mind and spirit to the utmost” (NCEE, 1983, p. 115, emphasis added). This emphasis on *all* was inscribed in both the competing responses in mathematics education as mathematical power for all and mathematical proficiency for all.

⁵¹ Not everyone agreed with the conclusions and recommendations of *A Nation at Risk*. But even its critics played on the terministic grounds prepared by this report. If the report claimed that test scores had declined, the critics disputed the evidence without disputing the measure itself (see, for example, Berliner & Biddle, 1995). Thus even the critiques repeated the terms in which the scene of educational decline was set up.

⁵² To direct your attention to the inscriptions of the *scene* set up by *A Nation at Risk* in the subsequent *acts*, I observe that the term *all* was not part of the vocabulary of post-sputnik wave of reforms. The post sputnik reforms squarely focused on what Jerome Bruner worded as “a concern for quality and intellectual aims of education—but without abandonment of the ideal that education should serve as a means for training well balanced citizens for a democracy” (Bruner, 1977, p. 1). The focus on *all* children that became a hallmark of reforms constituted in response to *A Nation at Risk* points to *scene-act* logic.

Setting up excellence for all as the ultimate *purpose* of the reforms, the document shaped a scene in which reforms would need to ultimately articulate the expectations and goals—ultimately to be understood as standards:

Excellence characterizes a school or college that *sets high expectations and goals for all learners, then tries in every way possible to help students reach them.*

Excellence characterizes a society that has adopted these policies, for it will then be prepared through the education and skill of its people to respond to the challenges of a rapidly changing world. Our Nation's people and its schools and colleges must be committed to achieving excellence in all these senses.

(NCEE, 1983, emphasis added)

With excellence described in these terms, the *scene* prepared by *A Nation at Risk* called for the *acts* [of reforms] to frame *high expectations and goals* [read standards], a mechanism for achieving them, and ways of assessing progress toward them. Furthermore, consistent with the terms in which the indicators for decline were stated, *A Nation at Risk* qualified standards as *measurable*:

We recommend that schools...adopt more *rigorous and measurable standards*, and higher expectations, for academic performance and student conduct...This will help students do their best educationally with challenging materials in an environment that supports learning and authentic accomplishment.

(NCEE, 1983, p. 125)

It also suggested regular conduct of standardized testing as a device to keep students on course:

Standardized tests of achievement...should be administered at major transition points from one level of schooling to another and particularly from high school to college or work. The purposes of these tests would be to: (a) certify the student's credentials; (b) identify the need for remedial intervention; and (c) identify the opportunity for advanced or accelerated work. The tests should be administered as part of a nationwide (but not Federal) system of State and local standardized tests. This system should include other diagnostic procedures that assist teachers and students to evaluate student progress. (NCEE, 1983, p. 125)

To summarize, *A Nation at Risk* used the rhetorical form of jeremiad to take its audience—to use the terms used by Tyack and Cuban to describe Horace Mann's jeremiad—to the edge of the precipice and to show them the social and economic hell that lay ahead if the decline in public education was not reversed. Like all jeremiads, it also translated the terms of *crisis-scene*—declining scores and achievement gaps—in the corresponding terms for *reform-acts*—higher and measurable standards and accountability linked to improved achievement on such standards. From a dramatic standpoint it set up the expectations from the reforms in terms of a *scene-act* ratio.

In what follows, I will focus particularly on various responses [acts] to the calls for standards, evaluating their conformity with the expected *scene-act* ratio. For each of these responses, I would suggest to the reader to imagine them as *acts* in response to a jeremiad. In this sense, the reform and counter-reform texts, the politicians, and the critics all appear as reinforcing, the terms of reform set up by *A Nation at Risk*. The *scene-act* ratio will remain the anchor of the analysis that follows. Through this

dramatistic ratio, I will also attempt to explain the ways in which the rhetoric of the progressive reforms was rendered vulnerable.

Echoes of Jeremiad: The Expressions Scene-Act Ratio in Reforms and Counter-Reforms

Politicians and the Discourse of Standards

The terms in which political leaders responded to the jeremiad of *A Nation at Risk* can be seen as conforming to the *scene-act* ratio outlined in the previous section. My evidence for this claim is based on data gathered from archives of the *New York Times* and the periodical *Education Week*. I searched for and picked up pieces of news reports that described the pronouncements of the political leaders in the decade following *A Nation at Risk*, hoping to see the ways in which the terms of the jeremiad—measurable standards, declining test scores, standardized tests as evidence of progress—were repeated [or not] in the political pronouncements about education.

About the same time *A Nation at Risk* was released, another task force consisting mainly of governors was reflecting on "Education for Economic Growth." The jeremiadic tone of this task force echoed that of *A Nation at Risk*. It also spoke with a similar urgency as *A Nation at Risk* in calling for a deep and lasting change in the American education to put the country on a par with other economic competitors. "If we are serious about economic growth in America—about improving productivity, about recapturing competitiveness in our basic industries and maintaining it in our newer industries, about guaranteeing to our children a decent standard of living and a rewarding quality of life, then we must get serious about improving education" (Fiske, 1983). Like *A Nation at Risk*, the governors also drew the motivation for educational reform by

identifying the imperative of economic growth with that of educational reform. One of the governors was reported in the New York Times as saying, "The fate of the country is in the balance...Education is the issue, and this country will rise or fall with it." (Hechinger, 1983).

A Nation at Risk's calls for higher and measurable standards were faithfully echoed by the political actors. In particular, the media reports suggest a repetition and reinforcement of the messages of *A Nation at Risk* in governors' pronouncements in the 1980s. The issue of educational standards appeared so frequently in pronouncements from the governors that one of the New York Times reports found it apt to describe it as "Gubernatorial activism" (Hechinger, 1983). The governors repeated *A Nation at Risk's* emphatic association of state-of-the-economy/Nation/People with state-of-education. Also reverberating through their talk was the notion of higher and measurable standards as instruments of public accountability.

The politicians' call for more dollars for education was linked to demands for accountability. With the measurable standards being seen as instruments of accountability, the talk of reforms was transfixed by the discourse of measurable standards. Just a year after the publication of *A Nation at Risk*, political and business leaders were reported as pushing for evidence about whether or not increased financing of elementary and high schools was producing results. The interpretation of standards as measuring sticks reverberated through this discourse, often expressed as calls to "measure the condition of education across the country by developing a set of standards similar to the gross national product, the Consumer Price Index and other indicators published by the Government to describe the state of the economy" (Fiske, 1984).

To summarize, the discourse of political leaders in the decade after the publication of *A Nation at Risk* was less ambiguous than, as I will show in the next section, that of mathematics educators. They repeated and reinforced *A Nation at Risk's* call for measurable standards and measuring instruments as the backbone of the much needed reforms.

Standards in Mathematics Education Reform Texts: The *scene-act ratio* +

The *Standards* may be remembered as the first attempt by any teachers' organization to specify national, professional standards for school curricula in their discipline.

(Crosswhite, et al., 1989, p. 513)

The mathematics education community signaled a rhetorical identification with the *scene-act* ratio constituted by *A Nation at Risk* by fully employing the term *Standards*. I will refer to NCTM's *Curriculum and Evaluation Standards* (NCTM, 1989) as CES to distinguish them from other uses of the term *standards*.

The process leading to CES is described succinctly by Alan Schoenfeld as follows:

In 1986, the NCTM's board of directors established the Commission on Standards for School Mathematics, chaired by Thomas Romberg. The following year, NCTM President John Dossey appointed a team of 24 writers to produce the *Standards*. The group produced a draft in the summer of 1987, obtained feedback on the draft during the 1987-1988 working year, and revised the draft in the summer of 1988. NCTM published the *Standards* in the fall of 1989 and began a major effort to bring the work to the attention of its membership. Copies of the *Standards* were mailed to all members, and various aspects of the *Standards*

became the themes for regional and national NCTM meetings (Schoenfeld, 2004, p.265).

CES were received in an environment in which *A Nation at Risk*'s rhetoric of restoration and revitalization of the nation through reforming schools, and reforming schools through putting in place *higher and measurable standards* had already been repeated, reinforced, and entrenched by a plethora of reports coming about the same time as *A Nation at Risk*. "Between 1983 and 1984," as documented by Wilson, "approximately twenty national commissions reported on the ills of American schools" (Wilson, 2003, pp. 121-122).

The CES were warmly received by a constituency already prepared for reception of *standards*. As Diane Ravitch puts it:

The NCTM standards quickly became a dynamic force in changing staff development, instructional practices, teacher education, textbooks, technology, and assessment. Every new commercial mathematics textbook or instructional program that entered the market since 1989 has claimed to incorporate the NCTM standards. According to the NCTM, 30% to 40% of the nation's mathematics teachers were using the new standards by 1992. Even the federally sponsored National Assessment of Educational Progress (NAEP) began to change in response to the NCTM standards. (Ravitch, 1993)

Some even spoke of CES as a national model for standards in other subject areas (Winston & Royer, 2003, p. 193). The enthusiasm with which the CES were received reinforces the point made earlier about the strength of *scene-act* ratio under the

jeremiads.⁵³ The language of standards as a model of reform was a rhetorical fit within the *scene-act* ratio set up by the jeremiad of *A Nation at Risk*. Not everyone rejoiced, and some scholars cautioned that suggesting a certain vagueness in the CES was kept there so “that powerful groups or individuals who would otherwise disagree can fit under the umbrella” (Apple, 1992, p. 413).

However, the CES enunciated by the mathematics education reforms were also contained simultaneously by multiple scenes. That is to say, while the rhetoric of *standards*, particularly the CES, worked well within an air filled with the talk of *standards*, it dropped the qualifier *measurable*. The *scene-act* ratio set up by the jeremiad had called for higher and *measurable* standards for all. Dropping the qualifier of *measurable* created the necessary space for the articulation of a conception of mathematical power that was more consistent of a Dewey-ian conception of the relation between content and pedagogy (I have discussed this in more detail in Chapter 4).

While the language of “standards” in the scene was constituted by the jeremiad, the idea of *mathematical power for all* was circumscribed by a different historical *scene*, that of *pedagogical progressivism*.⁵⁴ While the CES endorsed the scene constituted by *A Nation at Risk* by using the language of *standards* for their proposals, it also incorporated the elements of pedagogical progressivism. The CES did this by changing the definition of what it meant to succeed in school mathematics, and, therefore, transforming the objectives of reforms (Rothman, 1989). As a part of an explanation of the connection between ambiguity and dramatism, I had referred to Russ McDonald’s observation about

⁵³ In fact the standards based reforms may be seen as the primary mode of reforms with other reforms being spawned within the framework of standards.

⁵⁴ The term *pedagogical progressivism* is used by David Labaree to distinguish the Dewey-ian tradition in education from the *administrative progressives* (Labaree, 2005).

Shakespeare's use of ambiguities to heighten the drama thereby bolstering the explanation of connection between ambiguity and dramatism. Russ McDonald observes that in critical moments of a Shakespearean play, his characters say utterances that cannot be grasped with reference to the scene in which they take place. "For what they say might have two meanings. The one meaning which the speaker has in mind refers to the momentary situation, but the other meaning may point beyond this moment to other issues of the play" (McDonald, 2004, p. 51). Likewise, I suggest that reformers' rhetorical strategies involved using the language of standards to mean two things; as a reference to momentary situation—i.e., as responses to calls for standards by a jeremiad: as a site for reawakening the progressive discourses in American education, bolstered by, as I have discussed in Chapter 4, by constructivism as a learning theory and availability of Lakatos' philosophy of mathematics as a fallible human activity⁵⁵. While the reform texts' use of the term *standards* had the initial effect of strengthening their claims as proper responses to *A Nation at Risk*, the meanings that reform texts infused into the term standards diverged from the other competing references to the standards as *measuring sticks*, i.e., as enablers of precise and measurable statements about students' learning outcomes.

Thus, the standards based on the conjunction of pedagogical progressivism, constructivism, and Lakatos were a response to the call for standards, but one that used the language of standards to challenge the *scene of decline* itself by changing the definition of what it meant to succeed in mathematics. These transformed objectives were stated in terms of developing mathematical power in all students.

⁵⁵ For the sake of brevity, I will use *Pedagogical Progressivism* as an umbrella term for the conjunction of *progressivism*, *constructivism*, and *Lakatos' philosophy of mathematics* that I have described in Chapter 4.

What it meant to succeed in mathematics, that is, to develop *mathematical power*, as I argued in Chapter 4, was articulated largely as an open ended engagement with mathematics in the classrooms. Such an engagement between the learner and subject matter belonged to a *scene* which was constituted, not by *A Nation at Risk*, but by John Dewey's lamentations at the turn of the century about the educational sectarianism in America, as resulting from a bogus separation between the child and the curriculum (Dewey, 1902). Dewey had talked about two "sects" of education who were fighting over the curriculum. One, according to him, sought to "subdivide each topic into studies; each study into lessons; each lesson into specific facts and formulae. Let the child proceed step by step to master each one of these separate parts, and at last he will have covered the entire ground" (Dewey 1902, p. 12). For the other sect, the "child is the starting point, the center, and the end" (Dewey 1902, p. 13). Dewey sought to collapse this binary distinction by claiming both child and curriculum as the two sides of the same coin. Dewey's lamentation and his suggestion to collapse the binary distinction between the child and curriculum is seen by many as constituting the basis of pedagogical progressivism (Labaree, 2005). However, it is important to note at this point that some commentators observe that pedagogical progressivism lost out in the longer struggle for American curriculum (Labaree, 2005; Lagemann, 2000).

Yet, just as the jeremiad of *A Nation at Risk* sought to restore and revitalize American schools by calling for higher and measurable standards, the pedagogical progressivism sought to respond to the challenge thrown by John Dewey. Indeed, some notable reformers interpreted the NCTM standards and the ideas it contained, as creating opportunities to imagine a resolution of the century old lamentations of John Dewey

about the separation of child and curriculum. As Ball observed: “Across school subjects, current proposals for educational improvement are replete with notions of “understanding,” “authenticity,” and “community,” about building bridges between the experience of the child and the knowledge of the expert” (Ball, 1993, p. 374). Ball’s observation resonates with the terms of reform discourse—such as, *Teaching for Understanding* and *Authentic Assessment*—that circulated in the educational discourse through the 1990s. In Chapter 4, I argued that reform texts’ way of building bridges between the child and curriculum was expressed through a collapsing of content and pedagogy. In making that claim, I am not as much referring to the construct of Pedagogical Content Knowledge (Shulman, 1988)—a specialized knowledge base of teaching—or its recent incarnation in mathematics education that goes by the name of Mathematical Knowledge for Teaching (MKT). Rather, I wish to direct your attention to statements that sought to narrowly redefine mathematics (See Chapter4). This redefinition was expressed in the reform discourse through phrases such as mathematics as reasoning, communication, problem solving, etc.

Reverberating through this scene, as Ball (1993) also suggests, are articulations of Schwab’s idea about the possibility, and desirability, of a school curriculum that approximates not just the knowledge but also the ways of knowing in a particular discipline (Schwab, 1964), and Bruner’s hypothesis that any subject can be taught effectively in some intellectually honest form to any child at any stage of development (Bruner, 1977). Ball speaks of this tension succinctly in describing her stance toward teaching: “With my ears to the ground, listening to my students, my eyes are focused on the mathematical horizon” (Ball, 1993, p. 376). But the scene in which a practice which

requires teachers to be both grounded in their students with their eyes fixed at the disciplinary horizons at the same time does not entertain measurable standards in a straight forward way.

This point can be understood with reference to Dewey's, observations on the term standards—though made in relation to the critical judgment of the works of arts—is relevant to this discussion. Dewey defined standards as *precisely measurable attributes* that could be used for comparison of physical attributes of things. Using such a notion of standards to compare works of art, thought Dewey, could be misleading:

When, therefore, the word “standard” is used with respect to judgment of works of art, nothing but confusion results, unless the radical difference in the meaning now given standard from that of standards of measurement is noted. The critic is really judging, not measuring physical fact. He is concerned with something individual, not comparative—as is all measurement. His subject matter is qualitative, not quantitative. There is no external or public thing, defined by the law to be the same for all transactions, that can be physically applied (Dewey, 1959, p. 307).

But it is individuals, and groups that measurable standards in education would ultimately seek to compare. Yet, the progressive reformers, nonetheless, used the language of standards to find a place in the scene constituted by the talk of measurable standards. They also simultaneously attempted to respond to Dewey's challenge to eliminate the bogus distinction between the child and curriculum. Within the *scene-act* framework this could be done—as Dewey had suggested in the case of debates about

standards of judging the works of art—by rearticulating the meaning of the term *standards*.

Indeed the meaning of the term standards multiplied in the reform discourse, with all meanings carefully avoiding the connotation of standards as measures. Thomas Romberg, for instance, interpreted them as a vision and a set of criteria for judging the quality of curriculum and instruction (Romberg, 1992). Standards were also called the rallying flag for teachers (Ball, 1991; Romberg, 1992, 1993). The official interpretations of the term *standards* offered in the reform texts also kept changing over time. NCTM's *Curriculum and Evaluation Standards* (CES) defined them as a project to “guide the revision of the school mathematics curriculum and its associated evaluation toward this vision” (NCTM, 1989, p. 1) and also as “statements of criteria for excellence in order to produce change...” (NCTM, 1989, p. 1). The CES, under this view, were to be viewed as facilitators of reform. They were also portrayed as based on a grand consensus of the professional mathematics education community about the nature of mathematics and its teaching and learning (Carl & Frye, 1991; Crosswhite, et al., 1989).

The *Principle and Standards of School Mathematics* (NCTM, 2000), discussed standards as instruments of improvement in school mathematics by providing opportunities for debates and improvement:

As with the previous NCTM *Standards*, *Principles and Standards* offers a common language, examples, and recommendations to engage many groups of people in productive dialogue. Although there will never be complete consensus within the mathematics education profession or among the general public about the ideas advanced in any standards document, the Standards provide a guide for

focused, sustained efforts to improve students' school mathematics education (NCTM, 2000, p. 5).

Calling them a vision, criteria, professional consensus, rallying flag for teachers of mathematics, or a trigger for debates, these reform documents kept holding the language of standards in a tight embrace until recently. The most recent document published by the NCTM—*The Curriculum Focal Points: A Quest for Coherence*—replaced the language of *standards* with that of the *focal points* (NCTM, 2006). The NCTM focal points are statements about a small number of significant mathematical “targets” for each grade level. These statements attempt to distinguish them from the “commonly accepted notions of goals, standards, objectives, or learning expectations” (NCTM, 2006, p. 1). Rather, the document claims to be containing descriptions of the most significant mathematical concepts and skills that must be attained at each grade level.

I construe the shifts in the definitions of standards and ultimate departure from this language as signaling a vulnerability that can be accounted for with reference to the *scene-act* ratio constructed by a jeremiad.

	<i>Jeremiad of A Nation at Risk</i>	<i>NCTM Standards</i>	<i>1999 Framework</i>
<i>Scene</i>	<i>Declining Test Scores</i>	<i>Declining Test Scores</i>	<i>Declining Test Scores</i>
<i>Act</i>	<i>Reforms: Put in Place Higher and Measurable Standards</i>	<i>Reforms: Put in Place Higher Standards</i>	<i>Reforms: Put in Place Higher and Measurable Standards</i>
<i>Agent</i>	<i>A wide range of stakeholders, including what ultimately became reformers and counter reformers</i>	<i>Reformers</i>	<i>Counter-Reformers</i>
<i>Purpose</i>	<i>Excellence for all</i>	<i>Mathematical Power for All</i>	<i>Mathematical Proficiency for All</i>

Table 11: Comparison of Scene-Act ratio in *A Nation at Risk*, *NCTM Standards*, and *1999 Framework*

Standards after the NCTM standards: The Affirmation of *scene-act* Ratio

“What if these standards are wrong” asked Chester E. Finn (1993). Acknowledging the initial success of standards Finn suggested what I interpret as violation of *scene-act* ratio, indicating what he thought was the misunderstanding between the *content* standards and *student-performance* standards:

The content standards describe what schools should teach and—presumably—their pupils should learn...Content standards are about curriculum: its goals, frameworks, scope and sequence, etc. They are intended mostly for educators...Student-performance standards are something else. They involve how well youngsters must do in order to be said to have met the expectations of the content standards (Finn, 1993).

Finn observed that while NCTM had provided us with Curriculum Standards, it did not provide student performance standards needed so that “students (and parents) see how well they—and their schools—are doing vis-à-vis what's expected of them. Student-performance standards are truly about results and outcomes” (Finn, 1993).

Details of the math debates are messy and, though not irrelevant, are not needed for the analysis I am rendering here. Taking into consideration the fact that a progressive agenda was reversed in California, I was interested in using dramatism to account for the rhetorical vulnerabilities associated with using the term standards by the progressive reformers, when the *scene-act* ratio was constituted by a jeremiad.

Arguably, the drama, its scenes acts and actors, its situation of comedy and tragedy may be seen as constituted in affirmation and violation of dramatistic ratios. If the *scene* calls for crying, and the actor laughs out loud, the latter can be interpreted as a violation

of *scene-act* ratio. The *act* of laughing when the scene calls for an expression of sadness could make sense as insanity or, perhaps, a move to sweeten the air of a bitter scene by doing something unexpected. If the *actor* is able to alter the *scene* the ratios may reversed. If not, the *act* would not appear to be contained by the scene. In the case of mathematics education reform texts, the standards were infused with a meaning that was not anticipated, or called for, by the jeremiad of *A Nation at Risk*. Not so, however, with the counter-reform texts.

In Chapter Five, I examined the 1999 *Framework* and the embedded California Content Standards. I would like to repeat my findings from Chapter 5 briefly to argue that 1999 *Framework* appears perfectly aligned with the *scene-act* ratio set up by *A Nation at Risk* and endorsed and repeated by politicians.

The 1999 *Framework* sets up standards as precise statements about strands and benchmarks, specifying in precise and measurable terms what the students should know and be able to do. The idea of mathematical proficiency for all is advanced as a measurable construct, and a regimen of standardized testing is put in place to assess progress, make diagnostic judgments, and place students in diagnostic groups accordingly. The idea of rigor is introduced to bolster the rhetoric of measurable standards. Students are supposed to learn ‘rigorous mathematics,’ are to be assessed ‘rigorously,’ and ‘rigorous’ research is needed to evaluate interventions that are more likely to help students achieve the standards.

Reform Texts, Comedy and Tragedy: The Affirmation of scene-act Ratio

The violation of scene-act ratio by the reform texts created the opportunity for the comic and the satire. Funny images of reformers and reform texts began to fill the air,

already contaminated by the language of *war*. Nearly all of these depicted the reform texts as working against the reforms. I am not interested in whether these renderings of the reforms were right or wrong. Rather I am thinking about the possibility of comedy in this drama in which the scene was set up by a very serious jeremiad stating the nation to be in jeopardy due to falling test scores. Yet, the discourses that took the lead in occupying the reform field appeared to be working. This was akin to doing something that the public believing in the jeremiad did not expect. For example, one column written by Deborah Saunders⁵⁶, a columnist writing for *San Francisco Chronicle*, suggested that the reforms called for by *A Nation at Risk* had been hijacked by the ‘edu-crats’.⁵⁷ Quoting *A Nation at Risk*, she wrote:

More than a decade ago, a national commission issued a devastating report, "A Nation at Risk." It warned, "If an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war." In the wake of the report, reforms were promised. Some improvements resulted, but also some horrific trends accelerated full bore. No surprise: The very people [the reformers] behind the "mediocre educational performance" were piloting the so-called reforms (Saunders, 1995).

The detractors rhetorically identified reformers and reform-texts with the very ‘sin’ that the *jeremiad* of *A Nation at Risk* had identified for elimination—the falling standards

⁵⁶ Deborah Saunders wrote frequently, launching a ruthless tirade against the progressive reforms. Between 1995 and 2000, she wrote over 25 columns in the *San Francisco Chronicle*.

⁵⁷ Wilson has documented what she terms as the labels assigned conservative journalists for the reformers. The list included. “pedagogical imperialists,” “educrats,” “the educational establishment,” or “curriculum Nazis” (Wilson, 2003, p. 145),

and the test scores. What began may be termed as another jeremiad in the public sphere against the reforms and reformers. This jeremiad was played out by the influential parents, conservative politicians, conservative journalists, and like all jeremiads, it did not need accurate evidence for its claims.⁵⁸ Where *A Nation at Risk* had called for reforms, this jeremiad called for war against the progressives ‘masquerading’ as reformers. This jeremiad set up a scene of its own with its own *scene-act* ratio. A scene described as one in which an ‘evil’ power had come to dominate the policy space and victimize children and parents. The scene called for its own heroes.

The figure of war was used as an *act* by the counter reformers to identify them as rescuers and saviors of the ‘oppressed’ children. What would be more effective as a rhetorical figure than Abraham Lincoln’s 271-word address given on the dedication ceremony of the Gettysburg National Cemetery? I quote from the take off from the Lincoln’s address put up as the dedication of the *warriors* on *mathematically correct*,⁵⁹ one of the first and best known of the websites maintained by the counter reformers:

Over four score and seven decades ago philosophers brought forth into this world a new mathematics, conceived in correct computational formulae and dedicated to the proposition that two plus two equals four.

Now we are engaged in a great educational war, testing whether Algebra I or any form of mathematics so conceived and so dedicated can long endure. We are met on a great virtual battlefield of that war. We have come to dedicate a portion of

⁵⁸ This rhetoric depended for its sayability, not on evidence but on mobilizing the figure of war and repeatedly identifying reformers with denigrating titles (for details of this counter-reform campaign, see, Wilson, 2003, pp. 132-165).

⁵⁹ I first accessed *mathematically correct* in the beginning of 1996, a few days after setting my feet in the United States. It is still online and can be accessed at <http://www.mathematicallycorrect.com>

that field to those who are giving up the quality of their education so that California's Math Framework might live. It is altogether fitting and proper that we should do this.

But in a larger sense, we cannot dedicate, we cannot consecrate, we cannot hallow their loss. The brave children who now must struggle to learn math outside of the classroom have consecrated it far above our power to add or subtract. The world will little note nor long remember the actions of a few irate parents, but it can never forget what fate has befallen the children. It is for us, the mathematically competent, rather to be dedicated here to the unfinished work, to the *battle to save basic math skills* that has thus far been so nobly advanced. It is rather for us to be here dedicated to the great task remaining before us—that from *these honored children* we take increased devotion to the cause for which they gave up their weekends and vacation time—that we highly resolve that these children shall not have suffered in vain, that this state shall have a rebirth of computational skills, and that a mathematics of Algebra I, Geometry, and Algebra II shall not perish from our schools (Clopton, 1995, italics mine).

This takeoff from the Gettysburg Address substitutes “a new nation, conceived in Liberty, and dedicated to the proposition that all men are created equal” with “a new mathematics, conceived in correct computational formulae and dedicated to the proposition that two plus two equals four,”; the “great civil war” with “great educational war”; and “the honored dead” with “these honored children”. It’s a rhetorical move that sets up the counter reformers as rescuers and saviors of the *mathematics*.

The *scene* of math wars, so constructed in the rhetoric of counter-reforms, came complete with *innocent victims* (the children and parents) victimized by villains (*reformers*), and in need for heroic rescue by those who were competent and cared enough to rise to the occasion. The reform texts were projected as harbingers of tragedy and destruction in a scene that had called for reforms to suppress the ‘rising tide of mediocrity’ (see, NCCE, 1983). In sum, the counter-reform criticism projects reformers as *mathematically incompetent* villains whose victims, children and parents stood in need of rescue, and the *mathematically competent* as heroes under obligation to rescue them.

Furthermore, the language used by the critics of reform texts often invoked the important cultural notions of ‘fundamentals’ and ‘rigor,’ a rhetoric apparently well suited to engage the sympathy of scientists and mathematicians. The reformers’ views were projected as compromising the aspects of *fundamentals* and *rigor* and replacing them with what critics termed as fuzzy notions of mathematics and mathematical practice. Reform texts were satirized as *Weapons of Math Destruction*, reformer as *educrats* and *educational establishment*.

All of this undermined what mathematics educators termed as the professional consensus (see, Carl & Frye, 1991; Crosswhite, et al., 1989; Frye, 1990) by making their public claims about the content and pedagogy of school mathematics appear as falling short on such culturally important criteria as, say, *rigor*.

Concluding Thoughts

In the previous chapters, I had described the drama of reform and counter reform texts mapping it out internally. That is to say, I presented to my reader the respective scenes within which the terms of both reforms and counter reforms made sense. In this

chapter, I went at a level of dramatistic analysis that sets up both reforms and counter-reforms as *acts* within a *scene* constituted by the jeremiad of *A Nation at Risk*.

I have argued that this was a drama in which both reforms and counter-reforms were responses to calls for reform embedded in *A Nation at Risk*. Both, therefore, were constrained to use the language of standards. Reform texts, however, were not a straightforward response to the calls for *higher and measurable* standards. Beyond the scene constituted by the jeremiad of *A Nation at Risk* they were also a response to another influential text, namely Dewey's lamentation about the bogus separation of the *child* and *curriculum*. The math education debates, then, were battles that pitted one seminal educational text against the other. The reformers were at a disadvantage primarily because they set themselves an impossible task: to respond to both of these key documents at the same time.

Since 1999, the descriptions of standards have seen some shifts that can be interpreted as reassertion of the *scene-act* ratio constituted by the jeremiad of *A Nation at Risk*. Nearly all documents released by the NCTM and other panels and groups, including the recently published report by National Mathematics Advisory Panel (NMAP) endorse the notion of measurable standards (Ball, 2003; Kilpatrick, et al., 2001; USDOE, 2008). The cumulative effect of the *reforms* and *counter reforms*, so far, has been to reinforce and affirm the drama set in motion by the Jeremiad.

References

- Abbott, A. (1988). *System of Professions*. Chicago, IL: University of Chicago Press.
- Anonymous (1854, April, 29). American Association for Advancement of Science: Sixth Annual Meeting. *New York Times*,
- Apple, M. W. (1982). Cultural and Economic Reproduction in Education: Essays on Class, Ideology, and State. New York: Routledge & Kegan Paul.
- Apple, M. W. (1986). National Reports and the Construction of Inequality. *British Journal of Sociology of Education*, 7(2), 171-190.
- Apple, M. W. (1992). Do the Standards Go Far Enough? Power, Policy, and Practice in Mathematics Education. *Journal for Research in Mathematics Education*, 23(5), 412-431.
- Apple, M. W. (1993). Official knowledge, democratic education in a conservative age. New York: Routledge.
- Apple, M. W. (1996). *Cultural politics and education*. New York: Teachers College Press.
- Austin, J. L. (1962). *How to do things with words*. Cambridge, MA: Harvard University Press.
- Bagley, W. C. (1937). *A Century of the Universal School*. New York: The Macmillan Company.
- Ball, D. L. (1991). *Implementing the NCTM Standards: Hopes and Hurdles*. Paper presented at the Conference on Telecommunications as a Tool for Educational Reform.
- Ball, D. L. (1993). With an Eye on the Mathematical Horizon: Dilemmas of Teaching Elementary School Mathematics. *Elementary School Journal*, 93(4), 373-397.
- Ball, D. L. (2003). *Mathematical Proficiency for All Students*. Arlington, VA: Rand Corporation.
- Batarce, M., & Lerman, S. (2008). Mathematics and Mathematics Education--Deconstructing the Math Wars. In E. d. Freitas & K. Nolan (Eds.), *Opening the Research Text*. New York: Springer.

- Becker, J. P., & Jacob, B. (1998). Math War Developments in the United States (California) Retrieved 5/17, 2008, from <http://www.mathunion.org/ICMI/bulletin/44/index.html>
- Bercovitch, S. (1978). *The American Jeremiad*. Madison, WI: University of Wisconsin Press.
- Berliner, D., & Biddle, B. J. (1995). *The Manufactured Crisis: Myths, Fraud, And The Attack On America's Public Schools* New York: Perseus Books.
- Bishop, A. J. (1990). Essay Reviews: Mathematical Power to the People. *Harvard Educational Review*, 60(3), 357.
- Booth, W. C. (1974). Kenneth Burke's Way of Knowing. *Critical Inquiry*, 1(1), 1-22.
- Booth, W. C. (2004). *The Rhetoric of Rhetoric: The Quest for Effective Communication*. New York: Blackwell Publishing.
- Bruner, J. (1977). *The Process of Education*. Cambridge, MA: Harvard University Press.
- Burke, K. (1945). *A Grammar of Motives*. New York: Prentice Hall Inc.
- Burke, K. (1966). *Language as Symbolic Action*. Berkeley, CA: University of California Press.
- Burke, K. (Ed.) (1968) *International Encyclopedia of the Social Sciences*. New York: Macmillan and the Free Press.
- Burke, K. (1978). Questions and Answers about the Pentad. *College Composition and Communication*, 29(4), 330-335.
- California Department of Education (1985). *Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve*. Sacramento, CA: Author.
- California Department of Education (1992). *Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve*. Sacramento: Author.
- California Department of Education (1999). *Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve*. Sacramento: Author.
- California Department of Education (2006). *Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve*. Sacramento: Author.
- Carl, I., & Frye, S. M. (1991). President's Report: The NCTM's Standards: New Dimensions in Leadership. *Journal for Research in Mathematics Education*, 22(5), 432-440.

- CDE (1985). *Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve*. Sacramento, CA: Author.
- CDE (1992). *Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve*. Sacramento: Author.
- CDE (1999). *Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve*. Sacramento: Author.
- Clopton, P. (1995). Four Score and Seven: Mathematically Correct Dedication Retrieved 07/01, 2007, from <http://mathematicallycorrect.com/lincoln.htm>
- Cohen, D. J. (2007). *Equations from God: Pure Mathematics and Victorian Faith*. Baltimore, MD: John Hopkins University Press.
- Crosswhite, F. J., Dossey, J. A., & Frye, S. M. (1989). NCTM Standards for School Mathematics: Visions for Implementation. *Journal for Research in Mathematics Education*, 20(5), 513-522.
- Davis, P. J., & Hersh, R. (1981). *The Mathematical Experience*. Boston: Houghton Mifflin Company.
- Descartes, R. (1960). *Descartes Discourse on Method and other Writings* (A. Wollaston, Trans.). Baltimore, MD: Penguin.
- Descartes, R. (1984). *The Philosophical Writings of Descartes* (J. Cottingham, R. Stoothoff & D. Murdoch, Trans. Vol. II). Cambridge, UK: Cambridge University Press.
- Dewey, J. (1902). *Child and Curriculum*. Chicago, IL: University of Chicago Press.
- Dewey, J. (1959). *Art as Experience*. New York: Perigee Trade.
- Eliot, C. W., Lowell, A. L., Byerly, W. E., Chace, A. B., & Archibald, R. C. (1925). Benjamin Peirce. *The American Mathematical Monthly*, 32(1), 1-30.
- Ernest, P. (1991). *The Philosophy of Mathematics Education*. London: Falmer Press.
- Eugene, P. W. (1960a). The unreasonable effectiveness of mathematics in the natural sciences. Richard courant lecture in mathematical sciences delivered at New York University, May 11, 1959 (Vol. 13, pp. 1-14).
- Eugene, P. W. (1960b). The unreasonable effectiveness of mathematics in the natural sciences. Richard courant lecture in mathematical sciences delivered at New York University, May 11, 1959. *Communications on Pure and Applied Mathematics*, 13(1), 1-14.

- Finn, C. E. (1993). What if those Standards are Wrong. *Education Week*. Retrieved from <http://www.edweek.org/ew/articles/1993/01/20/17finn.h12.html?qs=what+is+standards+are+wrong>
- Fisher, I. (1898). Cournot and Mathematical Economics. *The Quarterly Journal of Economics*, 12(2), 119-138.
- Frye, S. M. (1990). President's Report: Maintaining Momentum as We Reach New Heights. *Journal for Research in Mathematics Education*, 21(5), 427-436.
- Griffin, N. (Ed.). (2002). *The selected Letters of Bertrand Russell: The Private Years, 1884-1914*. London: Routledge.
- Gusfield, J. (1976). The Literary Rhetoric of Science: Comedy and Pathos in Drinking Driver Research. *American Sociological Review*, 41(1), 16-34.
- Gusfield, J. (1989). Introduction. In J. Gusfield (Ed.), *On Symbols and Society*. Chicago: University of Chicago Press.
- Gusfield, J. R. (1981). *The culture of public problems : drinking-driving and the symbolic order*. Chicago: University of Chicago Press.
- Gusfield, J. R. (1996). *Contested Meanings: The construction of alcohol problems*. Madison, WI: The University of Wisconsin Press.
- Hacking, I. (2002). *Historical Ontology*. Cambridge: Harvard University Press.
- Hersh, R. (1999). *What is Mathematics? Really?* London: Oxford University Press.
- Hill, T. (1874). *Geometry and Faith: A Fragmentary Supplement to the Ninth Bridgewater Treatise*. New York: G. P. Putnam's Sons.
- Holmes Group (1986). *Tomorrow's Teachers*. East Lansing, MI: Holmes Group.
- Jackson, A. (1997). The Math Wars: California Battles it Out Over Mathematics Education Reform, Part II. *Notices of American Mathematical Society*, 44(7).
- Jasinski, J. (2001). *Sourcebook on Rhetoric: Key Concepts in Contemporary Rhetorical Studies*. Thousand Oaks: Sage Publications Inc.
- Kenny, A. (1970). The Cartesian Circle and the Eternal Truths. *The Journal of Philosophy*, 67(19), 685-700.
- Kilpatrick, J., Swafford, J., & Bradford Findell (Eds.). (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academies Press.

- King, D. W. (2004). *The Bible in History: How the Texts Have Shaped the Times*. London: Oxford University Press.
- Klein, D. (1997). Withdraw Endorsement of NCTM Standards. *Notices of the AMS*, 44(3), 310.
- Labaree, D. F. (2005). Progressivism, Schools and Schools of Education: An American Romance. *Pedagogica Historica*, 41(1), 275-288.
- Lagemann, E. C. (2000). *An Elusive Science: The Troubling History of Education Research*. Chicago: University of Chicago Press.
- Lakatos, I. (1979). *Proofs and Refutations: The Logic of Mathematical Discovery*. Cambridge, UK: Cambridge University Press.
- Lampert, M. (1990). When the Problem is Not the Question and the Solution is Not the Answer: Mathematical Knowing and Teaching. *American Educational Research Journal*, 27(1), 29-63.
- Machamer, P. (1998). *The Cambridge Companion to Galileo*. Cambridge, UK: Cambridge University Press.
- Mann, H. (1845). *Lectures on Education*. Boston: WM. B. Fowle and N. Capen.
- McDonald, R. (2004). *Shakespeare: An Anthology of Criticism and Theory 1945-2000*. Oxford: Blackwell Publishing Ltd.
- McLemee, S. (2001). A Puzzling Figure in Literary Criticism is Suddenly Central. *The Chronicle of Higher Education*. Retrieved from <http://chronicle.com/free/v47/i32/32a02601.htm>
- Morgan, M. L. (Ed.). (2002). *Spinoza: Complete Works*. Indianapolis, IN: Hackett Publishing Company, Inc.
- MSEB (1990). *Reshaping School Mathematics: A Philosophy and Framework for Curriculum*. Washington, D.C.: National Academies Press.
- MSEB (1993). *Measuring Up: Prototypes for Mathematics Assessment*. Washington D.C.: National Academies Press.
- NCCE (1983). *A Nation at Risk: The Imperative for Educational Reform*. Washington D.C.: US Government, Printing Office.
- NCEE (1983). A Nation at Risk: The Imperative for Educational Reform. *The Elementary School Journal*, 84(2), 113-130.

- NCTM (1989). *Curriculum and Evaluation Standards*. Reston, VA: Author.
- NCTM (1991). *Professional Teaching Standards*. Reston, VA: Author.
- NCTM (1995). *Assessment Standards*. Reston, VA: Author.
- NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.
- NCTM (2006). *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*. Reston, VA: Author.
- Noddings, N. (1990). Chapter 1: Constructivism in Mathematics Education. *Journal for Research in Mathematics Education. Monograph, 4*, 7-210.
- NRC (1989). *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*. Washington, D.C.: National Science Academies.
- NRC (1993). *Measuring What Counts: A Conceptual Guide to Mathematics Assessment*. Washington, D.C.: National Academies Press.
- Peabody, A. (1864). *Christianity the Religion of Nature*. Boston: Gould and Lincoln.
- Peirce, B. (1881). *Ideality in Physical Science*. Boston: Little, Brown, and Company.
- Peirce, B. (1882). *Linear Associative Algebra*. New York: D. Von Nostrand Publisher.
- Peterson, S. R. (1955). Benjamin Peirce: Mathematician and Philosopher. *Journal of the History of Ideas, 16*(1), 89-112.
- Phillips, D. C. (1995). The Good, the Bad, and the Ugly: The many faces of constructivism. *Educational Researcher, 24*(7), 5-12.
- Popkewitz, T. S., & Steiner-Khamsi, G. (2004). *The Global Politics of Educational Borrowing and Lending*. New York: Teachers College Press.
- Ravitch, D. (1993). Launching a Revolution in Standards and Assessments. *Phi Delta Kappan, 74*.
- Ravitch, D. (1995). *National Standards in American Education: A Citizen's Guide*. Washington D.C.: Brookings Institution Press.
- Romberg, T. A. (1992). Further Thoughts on the Standards: A Reaction to Apple. *Journal for Research in Mathematics Education, 23*(5), 432-437.
- Romberg, T. A. (1993). NCTM's Standards: A Rallying Flag for Mathematics Teachers. *Educational Leadership, 50*(5), 36-41.

- Rothman, R. (1989). Math Group Issues 'Standards' To Update Curricula. Retrieved from http://www.edweek.org/ew/articles/1989/03/22/08220025.h08.html?qs=mathematical_power
- Said, E. W. (1976). Roads Taken and Not Taken in Contemporary Criticism. *Contemporary Literature*, 17(3), 327-348.
- Saunders, D. J. (1995, August 7, 1995). Educrats Need Not Apply. *San Francisco Chronicle*, from <http://www.sfgate.com/cgi-bin/article.cgi?f=/c/a/1995/08/07/ED69820.DTL&hw=debra+saunders+new+new+math&sn=046&sc=409>
- Schoenfeld, Alan H. 2004. The Math Wars. *Educational Policy* 18 (1):253-286.
- Schwab, J. (1964). Structure of Disciplines: Meaning and Significances. In G. Ford & L. Pugno (Eds.), *The Structure of Knowledge and the Curriculum* (pp. 31-49). Chicago, IL: Rand McNally.
- Searle, J. (2006). Freedom and Neurobiology: Reflections on Free Will, Language, and Political Power. New York City: Columbia University Press.
- Sfard, A. (1998). The Many Faces of Mathematics: Do Mathematicians and Researchers in Mathematics Education Speak About the Same Thing? In M. d. Guzman & M. Niss (Eds.), *Mathematics Education as Research Domain: A Search for Identity: An ICMI Study, Book 2* (Vol. 4). Boston: Kluwer Academic Publishers.
- Shabel, L. (2007). Apriority and Application: Philosophy of Mathematics in the Modern Period. In S. Shapiro (Ed.), *The Oxford Handbook of Philosophy of Mathematics and Logic*. London: Oxford University Press.
- Shor, I. (1986). Culture Wars: School and Society in the Conservative Restoration 1969-1984. Boston: Routledge & Kegan Paul.
- Shulman, L. (1988). Theory, Practice, and the Education of Professionals. *The Elementary School Journal*, 98(5), 511-519.
- Sierpiska, A., & Kilpatrick, J. (Eds.). (1998). *Mathematics Education as a Research Domain: A Search for Identity*. Boston: Kluwer Academic Publishers.
- Steffe, L. P., & Kieren, T. (1994). Radical Constructivism and Mathematics Education. *Journal for Research in Mathematics Education*, 25(6), 711-733.
- Sullivan, D. L. (1994). A Closer Look at Education as Epideictic Rhetoric. *Rhetoric Society Quarterly*, 23(3/4), 70-89.

- Tegmark, M. (2007). The Mathematical Universe. *arxiv*. Retrieved from http://arxiv.org/PS_cache/arxiv/pdf/0704/0704.0646v1.pdf
- Tyack, D., & Cuban, L. (1995). *Tinkering Towards Utopia: A Century of Public School Reforms*. Cambridge: Harvard University Press.
- USDOE (2008). Foundations for Success: The Final Report of the National Mathematics Advisory Panel. Washington, D.C: U.S. Department of Education.
- Walker, W. (1893). *The Creeds and Platforms of Congregationalism*. New York: Charles Scribner's Sons.
- Wess, R. (1996). *Kenneth Burke: Rhetoric, Subjectivity, Post Modernism*. New York: Cambridge University Press.
- Whitehead, A. N., & Russell, B. (1997). *Principia Mathematica*. Cambridge, UK: Cambridge University Press.
- Wills, G. (1992). *Lincoln at Gettysburg: The Words that Remade America*. New York: Simon and Schuster.
- Wilson, S. M. (2003). *California Dreaming: Reforming Mathematics Education*. New Haven, CT: Yale University Press.
- Winston, D. A., & Royer, J. M. (2003). *Mathematical Cognition*. New York: IAP.
- Wu, H.-H. (1996). The Mathematician and Mathematics Education Reforms. *Notices of American Mathematical Society*, 43, 1531-1537.
- Wu, H.-H. (1998). The mathematics education reform: What is it and why should you care? Retrieved 03/21/03, 2003, from <http://math.berkeley.edu/~wu/reform3.pdf>
- Wu, H.-H. (2000). The 1997 Mathematics Standards War in California. In S. Stotsky (Ed.), *What's at Stake in the K-12 Standards Wars: A Primer for Educational Policy Makers*. New York: Peter Lang.
- Wu, H.-H. (2007). What is Mathematics Education? Text of a plenary presentation at the 2007 NCTM Annual Meeting,. Retrieved from <http://math.berkeley.edu/~wu/C49.pdf>

MICHIGAN STATE UNIVERSITY LIB



3 1293 03062 7362