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REPLICATING NATURAL TREE STAND PATTERNS IN A NORTHERN MICHIGAN ROCK OUTCROP LANDSCAPE: A FRACTAL BASED METHOD AND APPLICATION FOR REFORESTING A RECLAIMED MICHIGAN SURFACE MINE

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Wade J Lehmann

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Abstract

REPLICATING NATURAL TREE STAND PATTERNS IN A NORTHERN MICHIGAN ROCK OUTCROP LANDSCAPE: A FRACTAL BASED METHOD AND APPLICATION FOR REFORESTING A RECLAIMED MICHIGAN SURFACE MINE

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Landscape planners and designers are interested in replicating natural landscape patterns to reclaim degraded landscapes to match existing conditions. One approach that shows promise is the use of fractal geometry to create natural landscape patterns. While the measurement of the actual fractal dimension of an object is difficult, the box-counting method (developed at Agrocampus Ouest, Angers, France) approximates the fractal dimension of an object. This process is illustrated by measuring and replicating a stand of trees in the Upper Peninsula of Michigan and applying the method for a planting plan on a Northern Michigan surface mine. The estimated fractal dimension of each tree is; 0.329 for *Tsuga Canadensis Carrière*, 0.674 for *Thuja occidentalis L.*, 0.607 for *Acer rubrum L*, 0.345 for *Acer saccharum Marshall*, 0.442 for *Pinus strobus L.*, and 0.359 for *Picea glauca (Moench) Voss*.

Dedicated to my parents Bill Lehmann and Holly Weller

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INTRODUCTION

Surface mine reclamation is an important subject which involves land planning, ecology, landscape design, and site engineering. Reclaiming surface mines is the process of successfully converting a material resource exhausted environment into one that can accomplish a new land use (Burley, 2001). Mine reclamation has become an area of interest in the past half decade possibly because of increased environmental awareness. The Surface Mining Control and Reclamation Act of 1977 mandated that all abandoned surface mines be reclaimed. The western United States alone houses over 500,000 abandoned and active mines, spanning millions of acres (Berger, 2008). The amount of surface mines in the United States and the harmful effect of abandoned mines require attention from landscape planners and designers.

Surface mine reclamation can utilize many different end results. According to Burley (2001), a successful reclamation process includes; recognizing the traditional land use of the pre mining environment, and attempting to return the post mining landscape to this condition or another acceptable land use. Typical post mining land uses include but are not limited to; agriculture, housing development, parks and recreation, pasture, wildlife habitat, and forested land. According to Berger (2008), "mine sites enable designers to speculate over a landscape that is not bound by, nor indebted to, historical filters, aesthetic tradition, or strict contextuality… reclamation can act as a laboratory for experimentation."

The process of reforesting reclaimed landscapes is typically achieved by mass plantings of the most commercially viable trees for a particular site. This study investigates a new fractal based procedure for replicating natural patterns found in the landscape.

Landscape planners, designers, and environmental specialists are concerned in evaluating the spatial composition of landscape features such as composition of vegetation, forms of water bodies, and shape of terrain to unify disturbed landscapes with natural ones. However, natural looking assemblies were difficult to mathematically duplicate. Typical techniques used to replicate natural systems include the gestalt methods and ecological field methods (Fleurant, et al., 2009). The gestalt method was heuristic in nature where one would creatively merge and combine patterns together, until a desired condition was achieved. The ecological field laboratory method used scientific measures such as frequency, density, and size to construct patterns. A new approach has evolved which utilizes fractals to calculate spatial patterns in the landscape (Fleurant et al., 2009). A fractal designates an irregular or fragmented shape that can be divided into parts, each of which is approximately a smaller copy of the entire shape (Foroutan-pour et al., 1999).

1.1 ORIGIN OF FRACTALS

Fractals were originally noticed at the end of the 19^{th} century. However, the term "fractal" was coined later, the Peano curves appear to be the first example of fractal objects, first explained by Guiseppe Peano. The Peano curves could fill a void through a series of iterations utilizing only a few simple rules (Mandelbrot, 1982).

Fractals have been explored more thoroughly in the latter half of the 20^{th} century most notably by the French mathematician Benoit Mandelbrot. Mandelbrot, while researching "econometry" (mathematics applied to the economy), found that there were no difference in the slopes of curves predicting short-term and long-term market prices. He compiled an extensive description of the curves and created the term fractal (from the Latin word *fractus,* meaning broken) to describe the objects where irregularity separates them from typical Euclidian geometry curves. Upon the discovery of fractals, their use and application has broadened. Mandelbrot (1982) expresses the applications for fractals as follows, "Nature exhibits a high level of complexity in which typical Euclidian geometry classifies as formless, these irregular and fragmented patterns around us can be found using fractal geometry". This is an explanation of why fractals are used today in such sciences as biology, ecology, and geology.

1.2 FURTHER DESCRIPTIONS ILLUSTRATING FRACTALS

To demonstrate the concept of fractals, picture the rugged and rocky French coastline of Brittany. What is the real length of the coastline? To determine the length one could examine two forms of resolution.

1. An aerial image from 10,000 meters high and calculating the visible length of the coast.

2. Another aerial image from 500 meters high and measuring the details of the coast one meter at a time.

When measuring the length, one will determine that the coastline is longer in the second case, and also more accurate. If one were to examine the coastline at an even finer resolution, the overall length would increase again. The more defined the system of measurement, the greater the length of the coastline will increase. The complexity of the Brittany coastline (unable to be described with Euclidian geometry) makes it a fractal object (Mandelbrot, 1982). As expressed by Mandelbrot previously, fractals are everywhere in nature.

Fleurant et al.(2009) give a practical definition of the concept of fractals as a "geometrical shape resulting from infinite regular fragmentation of a given form". It is also proper to describe a fractal as a recurrence of the same form on each part of the curve. If one looked closely at any one part of a curve, it would resemble the entire curve itself (Fleurant et al., 2009). Cantor's Dust illustrates this property. Cantor's Dust is an image which results from Cantor's set, "a collection into a whole, of definite, well distinguished objects of our perception or thought." (Kamke, 1950) Cantor's dust has the geometric property where as the construction iteration process increases towards infinity, the total length L increases towards infinity. Imagine a straight line, then the same line with the middle 1/3rd removed. This process is continued to infinity and eventually the divisions become so small they are unobservable by the human eye (Figure 1.1). (Barnsley, 1988) The rings of Saturn are a real world example of this phenomenon. Saturn's ring was originally thought to be one solid entity, upon closer examination with higher powered telescopes it became clear that the ring was actually comprised of many small rings.

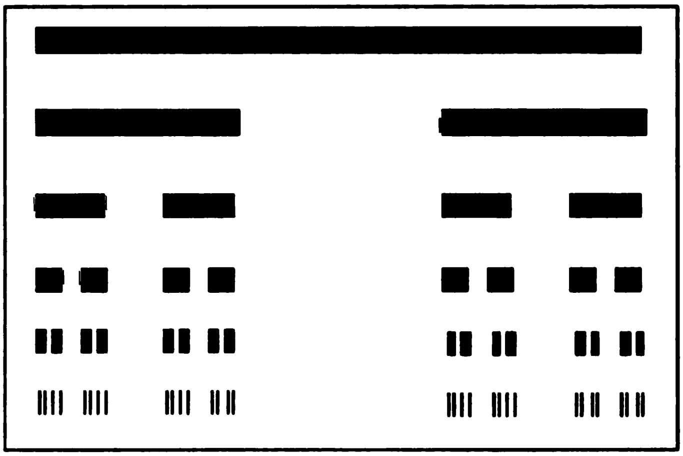

Figure 1.1. Cantor's Dust fractal object

1.3 GEOMETRIC PROPERTIES OF FRACTALS

Geometric properties of fractals are utilized in a number of different sciences and numerous models. For example, in geology fractals can be used for identifying fractures in rocks, which threaten their structural integrity (Velde et al., 1990). In economics, fractals are used to predict complex random fluctuations in the stock market (Mandelbrot, 1982). In computer sciences, fractals are used to retrieve patterns in image processing (Liangbin et al., 2005). In medicine, fractals can predict a patient's susceptibility to osteoporosis based on their bone mineral density structure (Harrar and Hamami, 2007). In chemistry, they are used to design new materials. The fractal nature of these materials allows them extraordinary properties, such as high thermal cooling power (Fleurant et al., 2009).

It is important to understand there are two different categories of fractals; theoretical fractals, and real fractals. Theoretical fractals, such as the Peano curve and Cantor's dust mentioned prior, exhibit self similarity and the dimensions can be mathematically calculated to infinite (Mandelbrot, 1982). Real fractals, such as objects found in nature, are not self similar, and do not continue to infinite. To determine the dimensions of real fractals, one must employ an estimation process such as the box-counting method (Foroutan-Pour et al., 1999).

1.4 FRACTAL DIMENSIONS

In Euclidian geometry, the point has a dimension of 0. Line and curves have a dimension of 1. Areas have a dimension of 2, such as a triangle or circle. Volumes have a dimension of 4, such as a cylinder or sphere. Fractal objects also have dimensions (Mandelbrot, 1982).

Fractal dimensions have values which cannot be expressed by a simple point or line. Objects such as those found in nature cannot be explained by Euclidian geometry, but can be expressed using fractals. Barnsley (1993) affirms this idea by stating,

> "Fractal dimensions can be attached to clouds, trees, coastlines, feathers, networks of neurons in the body, dust in the air at an instant in time, the clothes you are wearing, the distribution of frequencies of light reflected by a flower, the colors emitted by the sun, and the wrinkled surface of the sea during a storm."

Fractal dimensions attempt to quantify a subjective feeling which we have about how densely the fractal object fills the space in which it lies. They also provide a means for comparing the complexity of different fractals (Fleurant et al., 2009).

To demonstrate fractal dimensions, reconsider the Brittany coastline. If one were to calculate a 1 m length of a relatively straight line with a 20 cm ruler, the ruler will be used 5 times, 10 times for a 10 cm ruler, or 20 times with a 5 cm ruler. If one were to measure the same distance along the coastline, the total length will be underestimated due to the irregular pattern of the coast. The smaller the ruler used to measure the coast the more accurate the estimated length. To evaluate this phenomenon mathematically, one can declare that the result is more accurate when using a smaller ruler that fits the curvature of the line. If one can divide the length of the ruler of an infinite small size by "n", one has to use this ruler "n" times more. This property can define the topological dimension of the curve (Figure 1.2):

$$D_{topological} = \frac{\log(n)}{\log(n)} = 1$$

Figure 1.2 Equation for the fractal dimension of a line

Replicating this process again using a surface, one can use a square where the length of the side is L. To measure its area, one can use a smaller square where the length of one side is L/2, then one will need 4 squares, 16 squares using L/4, and so on. If the length of the side of the measuring square is divided by "n", the number of such squares used is multiplied by "n" (Figure 1.3):

$$D_{topological} = \frac{\log(n^2)}{\log(n)} = 2 \times \frac{\log(n)}{\log(n)} = 2$$

Figure 1.3 Equation for the fractal dimension of an area

Similar results can be obtained for volumes and the topological dimension of a Euclidian geometric object with a fractal dimension of 3 (Fleurant et al., 2009).

In the moderately simple case of self-similar fractal objects (meaning they appear the same no matter which zooming factor is used), resulting in a constant iterative factor "k", the fractal dimension is (Figure 1.4):

$$D_{fractal} = \frac{\log(n)}{\log(k)}$$

Figure 1.4 Equation for the fractal dimension of self-similar objects

Where:

n = the number of subsets counted during the scaling process using a factor 1/k (self-similarity factor).

k = number of iterations

Cantor's Dust illustrates how to calculate the fractal dimension of self-similar fractal objects. Consider a single line with a length of L. If one were to remove the middle 1/3rd of that line they would be left with two lines where L equals 1/3. One can continue to remove the middle 1/3rd of every line formed by the previous division (the dust presents an infinite number of "lines" with each iteration). This process can be carried on indefinitely. Then, using the same reasoning one can calculate the fractal geometry of Cantor's Dust (Figure 1.5):

$$D_{fractal} = \frac{log2}{log3} = 0.6309$$

Figure 1.5 Equation for the fractal dimension of Cantor's Dust

Therefore, one can conclude that the fractal dimension of this strange curve is not 1 as any of the classic linear geometrical curves. Cantor's Dust has a topological dimension equal to 0 (it's a broken line), but has a fractal dimension of greater than 0, which is not an integer but a real number.

The previous equations (figures; 1.2, 1.3, 1.4, and 1.5) are utilized to calculate the dimension of theoretical fractals. These equations cannot be used to determine the dimension of real fractals due to random elements present in the natural setting (Foroutan-Pour et al., 1999). Instead one must employ a more appropriate method to estimate the fractal dimension. One such method that is commonly utilized is the box-counting method (Foroutan-Pour et al., 1999).

1.5 INVERSE BOX-COUNTING METHOD: A TOOL FOR REPLICATING LANDSCAPES

The fractal dimension is not easy to calculate but can be estimated using several methods. The box-counting method is one of the simpler and most popular methods to utilize. The box-counting method was developed by Duchesne et al.

(2002) and computed by Durandet (2003) in the Landscape Department of the National Institute of Horticulture and Landscape Angers, France, now the Unite de RecherchePaysage; AgroCampusOuest. The natural object is covered with a grid of size r and one counts the number of boxes, N(r) that contain some part of the object. The value of "r" is progressively reduced and N(r) is similarly re-measured. As "r" tends to be very small values (0 in a theoretical way) one finds that $\frac{\log(N(r))}{\log(\frac{1}{r})}$ becomes the fractal dimension of the object (Fleurant et al., 2009).

The box-counting method is a simple tool to calculate the complexity of a landscape using the value of its fractal dimension. The greater an objects fractal dimension (2 is the maximum value in a plane), the less complex the arrangement of the planting pattern (in terms of scale, structure, alignment, etc.)(Fleurant et al., 2009). By utilizing this method, one is able to control the randomness of plantings or other landscape features with certain parameters: the fractal dimension (D), the average minimum distance between two trees (ϵ_{min}) and the average maximum size of the boxes (ϵ_{max}).

1.6 PLANNING AND DESIGN APPLICATIONS

There is a belief that fractals may have the ability to re-create complex landscape patterns that are hard to replicate with Euclidian geometry because the landscape is full of fractals: rivers, trees, landscape networks in general (Barnsley, 1993). Fractals are extremely detailed, complex geometric shapes and a measure of their complexity is the fractal dimension (Mandelbrot 1982). Accordingly, a number of professionals have examined fractals in landscape planning and design including studies by Diaz-Delgado et al., (2005); DiBari (2007); Griffith et al., (2000); Li (2000); Milne (1991); Palmer (1988); and Thomas et al., (2007). However, the use of fractals seems to be looking for a more practical application. For example, in landscapes it has always been relatively simple to describe an existing pattern, but hard to replicate that pattern. Presented in this paper is an approach to replicate landscape patterns and a practical approach towards the use of fractals.

Application of the inverse box-counting method to a reclaimed surface mine has the potential to accurately depict the natural vegetation patterns for a Northern Michigan rock outcrop.

METHODOLOGY

This study examines the application of fractals in the planting pattern of trees in the Upper Peninsula of Michigan in Dickinson County. The area selected for the study, located in Dickinson County (Figure 2.1), was selected on a rocky and dry xeric northern forest (Figure 2.2), an environment similar to waste rock piles on a surface mine where the fractal planting plan might be appropriate (Curtis, 1959). Trees equaling 3 inches dbh (diameter at breast height) or greater were recorded by a remote gps (global positioning system) unit.

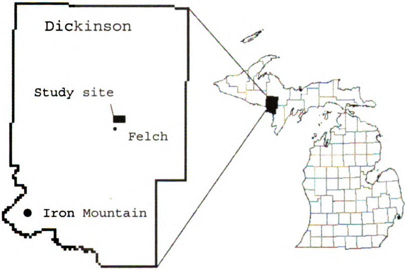

Figure 2.1. Location of the study area in Michigan

Figure 2.2. Forest stand at the study site (notice the rocky terrain and exposed bedrock)

To further express the relationship between the study site and the application site a pre-settlement image of the study area (figure 2.3) has been created from an interpretation of the 1816-1856 general land office surveys by Albert and Comer (2008).

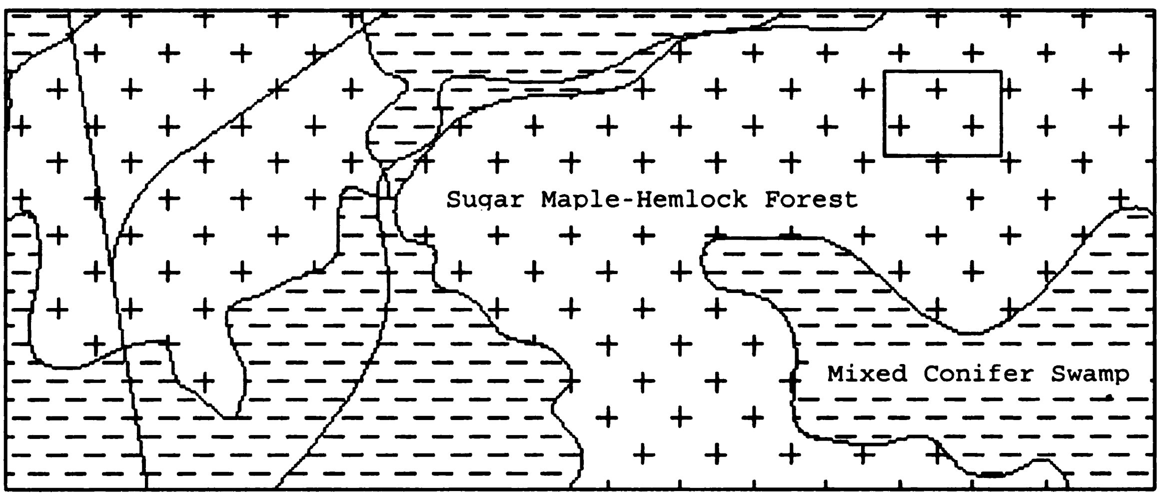

Figure 2.3. Pre-settlement vegetation map (study area is outlined by the rectangle in the upper right of image)

The map of pre-settlement vegetation suggests that the study area was originally a Sugar Maple-Hemlock forest. This forest type was the most predominant upland system in the Upper Peninsula, and also consisted of large numbers of White Pine. Soils associated with this cover type can be steep and rocky, including exposures of basalt and granite bedrock (Albert & Comer, 2008).

Another examination of the study area revealed a more detailed analysis of the soil conditions. According to the Soil Survey of Dickinson County, Michigan (United States Department of Agriculture, Soil Conservation Service) the study area consists of a Pemene-rock outcrop complex (Linsemier, 1989). This complex consists of 35-65% Pemene soil and 15-20% rock outcrop on slopes of 18-35%. Trees to be planted on this complex include (but are not limited to); White Pine, White Spruce, and Sugar Maple (Linsemier, 1989).

The location of trees can be placed on a map derived from remote sensing field survey. This set of points (location of trees) can be viewed as a complex and fractal object in nature.

Points were gathered as X, Y data (latitude and longitude) by a remote global positioning system (gps) unit. Points were collected on an entire rock outcrop, and mapped (globally) using ArcGIS software. The map of points was then projected into UTM's (universal transverse Mercator) for the application of trial grids to a two-dimensional surface. The resulting map was then exported to an Autocadd (computer aided drafting) program for creation of the trial grids.

Selection criteria for the size of the trial grids were determined by the size of the rock outcrop. The entire outcrop measured approximately 60 meters by 80 meters. Thus a trial grid of 50 meters was selected to encompass the entire site with a series of trials occurring at random placements within the study site. A total of five different trials were completed for each species of tree on the rock outcrop, resulting in 30 (50 by 50 meter) trials (see Appendix). Each trial was then subject to the box-counting method.

The box-counting process starts with the pairs of values r and the number of boxes N(r), the starting value of r is 50 meters, and the starting value of N(r) is one. Then r is divided in half and the value of r becomes 25 meters, while N(r) can range from 1 to 4, depending on the number of boxes which contain trees. The pairs of numbers for the regression analysis includes the first pair where at least one box becomes empty, and continues with successive pairs at smaller sizes until every box contains either one or no trees (Fleurant et al., 2009). In total there were five 50 meter by 50 meter boxes for every species of tree recorded in the study area with a count of greater than one.

RESULTS

The tree species tallied on site include; Eastern Hemlock (*Tsuga Canadensis Carrière*), Northern White Cedar (*Thuja occidentalis L.*), Red Maple (*Acer rubrum L.*), Sugar Maple (*Acer saccharum Marshall*), White Pine (*Pinus strobus L.*), and White Spruce (*Picea glauca (Moench) Voss*).

Out of the 30 trials, 113 dependent and independent variables for the regression analysis were derived (Table 3.1). The regression analysis revealed an adjusted r-square of 0.444, with a significant p-value of 0. The slope of the line expressed in the regression equation is 0.578. This suggests that the fractal dimension is between a point and a line in typology (Figure 3.1).

$$Ln(N(r)) = 0.578Ln\left(\frac{1}{r}\right) + 3.107$$

Figure 3.1. Fractal dimension equation for all species

Where: N(r)= number of boxes with trees

r = length of one side of the box

Of the 30 trials, 6 species were identified as having their own fractal dimension (see Table 3.2 for statistical information). Each species of tree has the same number of trials (5) but they have differing pairs of numbers.

Table 3.1. Dependent and independent variables for regression analysis

Species Plot	Ln(1/r)	Ln(N(r))
Eastern Hemlock 1	-3.219	0.693
	-2.526	1.386
	-1.833	1.609
	-1.139	1.792
Eastern Hemlock 2	-3.219	1.099
	-2.526	1.386
	-1.833	1.386
	-1.139	1.792
Eastern Hemlock 3	-3.219	0.693
	-2.526	0.693
	-1.833	1.099
Eastern Hemlock 4	-3.219	0.693
	-2.526	1.099
	-1.833	1.099
	-1.139	1.386
	-0.447	1.609
Eastern Hemlock 5	-3.219	1.099
Northern White Cedar 1	-3.219	1.099
	-2.526	2.079
	-1.833	2.639
	-1.139	2.833
Northern White Cedar 2	-2.526	2.079
	-1.833	2.565
	-1.139	2.708
Northern White Cedar 3	-2.526	2.197
	-1.833	2.398
	-1.139	2.773
Northern White Cedar 4	-3.219	1.099
	-2.526	1.792
	-1.833	2.303
	-1.139	2.565
Northern White Cedar 5	-2.526	1.946
	-1.833	2.303
	-1.139	2.639
Red Maple 1	-2.526	2.197
	-1.833	2.773
	-1.139	3.091
	-0.447	3.296

TABLE 3.1 CONT.

Red Maple 2	-2.526	2.485
	-1.833	2.890
	-1.139	3.258
	-0.447	3.367
Red Maple 3	-2.526	2.079
	-1.833	2.565
	-1.139	2.944
Red Maple 4	-2.526	2.398
	-1.833	2.944
	-1.139	3.219
	-0.447	3.367
Red Maple 5	-3.219	1.099
	-2.526	1.792
	-1.833	2.565
	-1.139	2.773
	-0.447	2.944
Sugar Maple 1	-3.219	1.099
	-2.526	1.609
	-1.833	1.792
	-1.139	1.792
	-0.447	1.946
Sugar Maple 2	-3.219	1.099
	-2.526	1.386
	-1.833	1.792
	-1.139	1.946
Sugar Maple 3	-3.219	1.099
Sugar Maple 4	-3.219	1.099
	-2.526	1.386
	-1.833	1.386
	-1.139	1.792
Sugar Maple 5	-3.219	0.693
	-2.526	1.099
	-1.833	1.099
	-1.139	1.609
White Pine 1	-2.526	2.197
	-1.833	2.485
	-1.139	2.565
	-0.447	2.639
White Pine 2	-2.526	2.303
	-1.833	2.708
	-1.139	3.091
	-0.447	3.135

TABLE 3.1 CONT.

White Pine 3	-2.526	2.485
	-1.833	3.135
	-1.139	3.135
	-0.447	3.219
White Pine 4	-2.526	2.485
	-1.833	2.833
	-1.139	2.890
	-0.447	2.944
White Pine 5	-3.219	1.099
	-2.526	2.079
	-1.833	2.485
	-1.139	2.565
White Spruce 1	-3.219	1.099
	-2.526	1.946
	-1.833	2.303
	-1.139	2.303
	-0.447	2.398
White Spruce 2	-3.219	1.099
	-2.526	1.609
	-1.833	1.792
	-1.139	1.946
White Spruce 3	-3.219	1.099
	-2.526	1.099
	-1.833	1.099
	-1.139	1.386
	-0.447	1.609
White Spruce 4	-2.526	1.946
	-1.833	2.079
	-1.139	2.303
White Spruce 5	-3.219	0.693
	-2.526	1.792
	-1.833	1.792
	-1.139	1.946

Species	Plots	EH	NWC	RM	SM	WP	WS
	1	6	17	27	7	14	11
	2	6	15	29	7	23	7
	3	3	16	19	3	25	5
	4	5	13	29	6	19	10
	5	3	14	19	5	13	7
Mean		**4.6**	**15**	**24.6**	**5.6**	**18.8**	**8**
Standard Deviation		**1.52**	**1.58**	**5.18**	**1.67**	**5.31**	**2.45**

Table 3.2. Mean and standard deviations for each species trial. EH-Eastern Hemlock, NWC-Northern White Cedar, RM-Red Maple, SM-Sugar Maple, WP-White Pine, WS-White Spruce

1. Eastern Hemlock consists of 17 pairs of numbers. The regression analysis revealed an adjusted r-square of 0.580, with a significant p-value of 0, and a significant t-value of 4.807. The slope of the line expressed in the regression equation is 0.329, suggesting that the fractal dimension is between a point and a line in typology (Figure 3.2).

$$Ln(N(r)) = 0.329Ln\left(\frac{1}{r}\right) + 1.936$$

Figure 3.2. Fractal dimension equation for Eastern hemlock

2. Northern White Cedar consists of 17 pairs of numbers. The regression analysis revealed an adjusted r-square of 0.845, with a significant p-value of 0, and a significant t-value of 9.382. The slope of the line expressed in the regression equation is 0.674, suggesting that the fractal dimension is between a point and a line in typology (Figure 3.3).

$$Ln(N(r)) = 0.674Ln\left(\frac{1}{r}\right) + 3.582$$

Figure 3.3. Fractal dimension equation for Northern white cedar

3. Red Maple consists of 20 pairs of numbers. The regression analysis revealed an adjusted r-square of 0.768, with a significant p-value of 0, and a significant t-value of 7.994. The slope of the line expressed in the regression equation is 0.607, suggesting that the fractal dimension is between a point and a line in typology (Figure 3.4).

$$Ln(N(r)) = 0.607Ln\left(\frac{1}{r}\right) + 3.689$$

Figure 3.4. Fractal dimension equation for Red maple

4. Sugar Maple consists of 18 pairs of numbers. The regression analysis revealed an adjusted r-square of 0.681, with a significant p-value of 0, and a significant t-value of 6.106. The slope of the line expressed in the regression equation is 0.345, suggesting that the fractal dimension is between a point and a line in typology (Figure 3.5).

$$Ln(N(r)) = 0.345Ln\left(\frac{1}{r}\right) + 2.168$$

Figure 3.5. Fractal dimension equation for Sugar maple

5. White Pine consists of 20 pairs of numbers. The regression analysis revealed an adjusted r-square of 0.554, with a significant p-value of 0, and a significant t-value of 4.962. The slope of the line expressed in the regression equation is 0.442, suggesting that the fractal dimension is between a point and a line in typology (Figure 3.6).

$$Ln\big(N(r)\big) = 0.442Ln\left(\frac{1}{r}\right) + 3.342$$

Figure 3.6. Fractal dimension equation for White pine

6. White Spruce consists of 21 pairs of numbers. The regression analysis revealed an adjusted r-square of 0.387, with a significant p-value of 0.002, and a significant t-value of 3.689. The slope of the line expressed in the regression equation is 0.359, suggesting that the fractal dimension is between a point and a line in typology (Figure 3.7).

$$Ln(N(r)) = 0.359Ln\left(\frac{1}{r}\right) + 2.387$$

Figure 3.7. Fractal dimension equation for White spruce

APPLICATION AND DISCUSSION

To apply the inverse box-counting method to the reclaimed landscape one would follow these procedures:

1. Divide the landscape to be planted in 50 meter grids.
2. Divide each 50 meter grid into grids with sides equal to 1.563 meters (the size of the smallest boxes).
3. Use a random number generator to fill the grid with numbers from 1-1024 for each tree species. Fill any box which contains a number that is less than or equal to the mean number of trees (from table 3.2) for each trial. Next, count each box that contains a tree and make sure the total falls within one standard deviation (from table 3.2). The resulting grid represents that particular species planting plan. Repeat this process for each species of tree recorded, and then combine the grids of all species onto one grid of the same size for the overall planting plan (Figure 4.1). The number of trees per grid can be increased proportionally if the mortality rate of the trees is known.

This approach is illustrated with figures 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, and 4.7. This process generated seven different fractal patterns, one for each of the tree species examined and one for all species combined (figure 4.1).

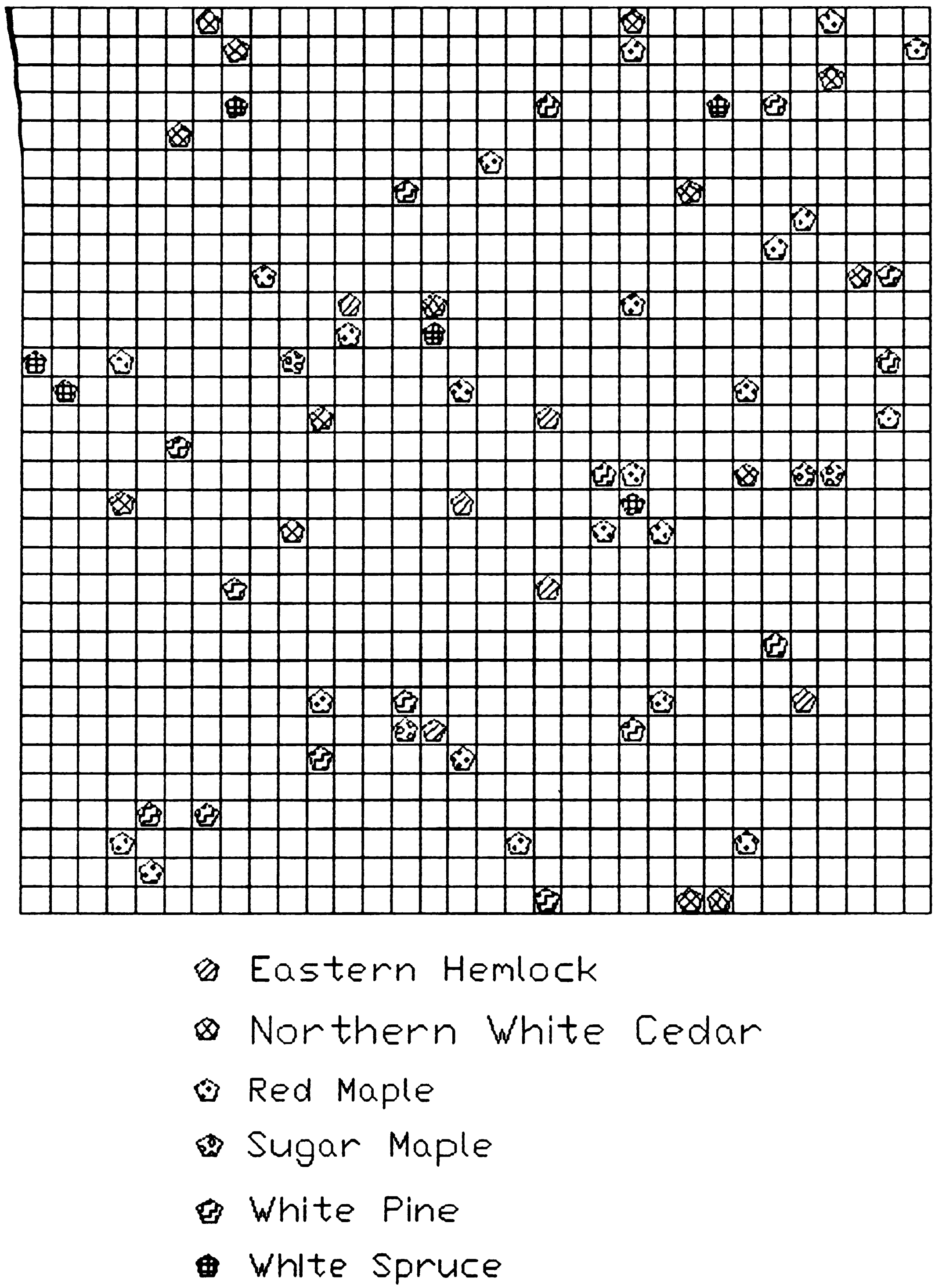

Figure 4.1. Fractal based planting plan for all species

EASTERN HEMLOCK

The process generated 6 boxes for planting trees (figure 4.2). 6 boxes are within one standard deviation (± 1.52) of the average of 4.6, so the 6 boxes were deemed acceptable.

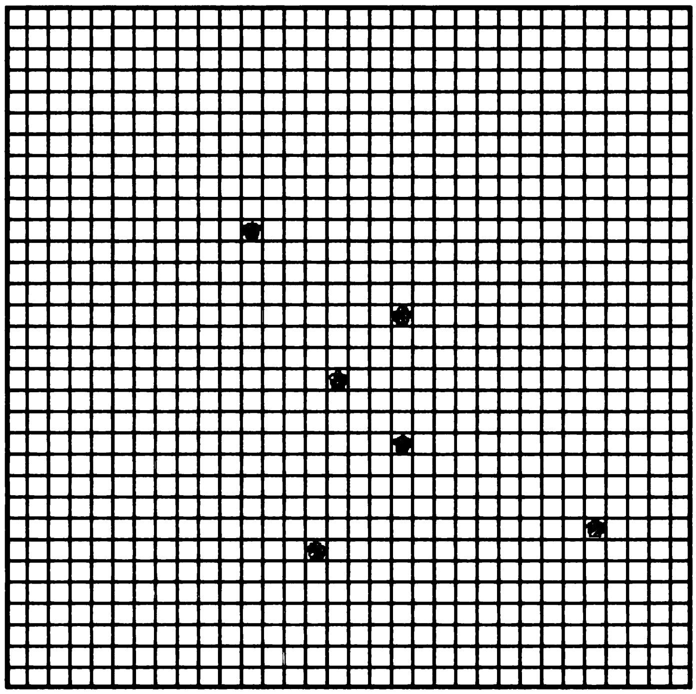

Figure 4.2. Fractal based planting plan for Eastern Hemlock

NORTHERN WHITE CEDAR

The process generated 14 boxes for planting trees (figure 4.3). 14 boxes are within one standard deviation (±1.58) of the average of 15, so the 14 boxes were deemed acceptable.

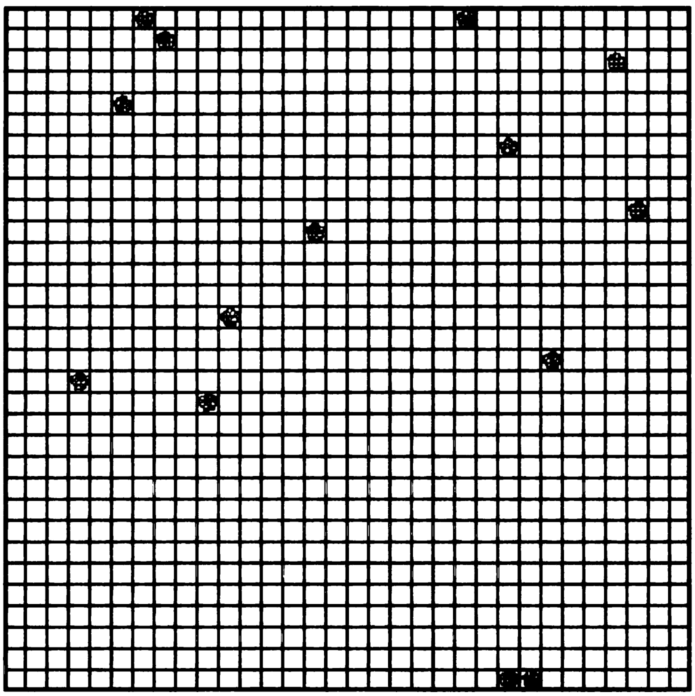

Figure 4.3. Fractal based planting plan for Northern White Cedar

RED MAPLE

The process generated 23 boxes for planting trees (figure 4.4). 23 boxes are within one standard deviation (±5.18) of the average 24.6, so the 23 boxes were deemed acceptable.

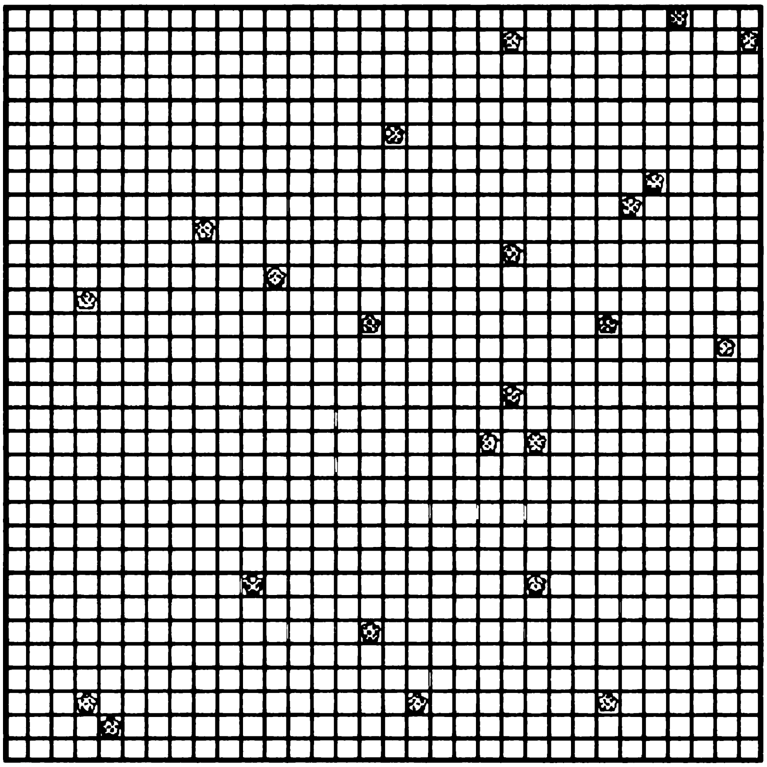

Figure 4.4. Fractal based planting plan for Red Maple

SUGAR MAPLE

The process generated 4 boxes for planting trees (figure 4.5). 4 boxes are within one standard deviation (± 1.67) of the average 5.6, so the 4 boxes were deemed acceptable.

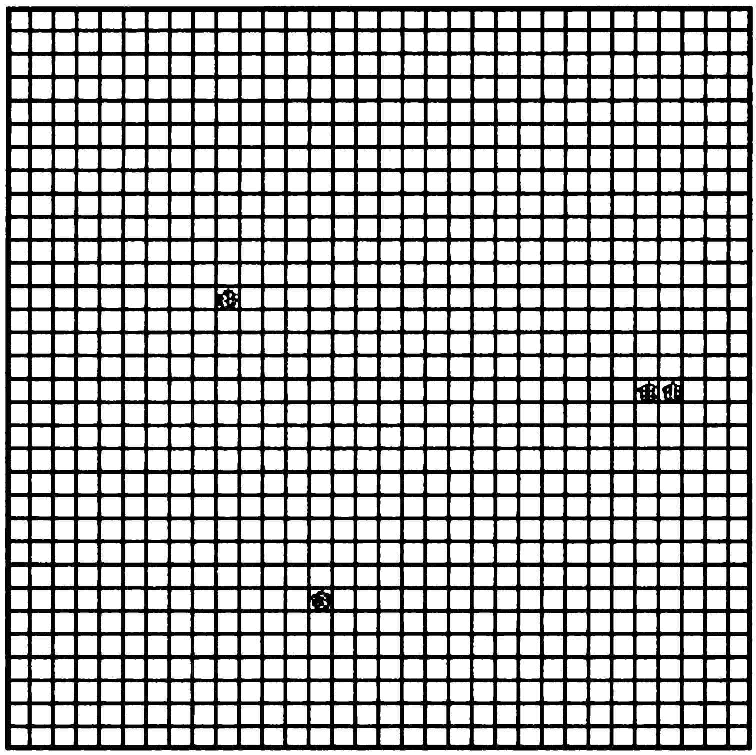

Figure 4.5. Fractal based planting plan for Sugar Maple

WHITE PINE

The process generated 15 boxes for planting trees (figure 4.6). 15 boxes are within one standard deviation (±5.31) of the average 18.8, so the boxes were deemed acceptable.

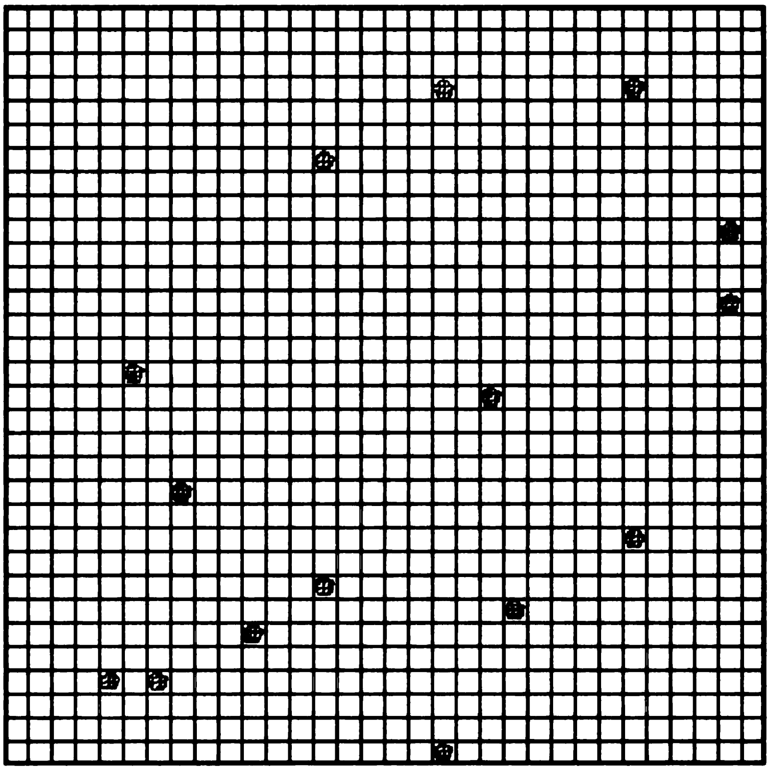

Figure 4.6. Fractal based planting plan for White Pine

WHITE SPRUCE

The process generated 6 boxes for planting trees (figure 4.7). 6 boxes are within one standard deviation (±2.45) of the average 8, so the boxes were deemed acceptable.

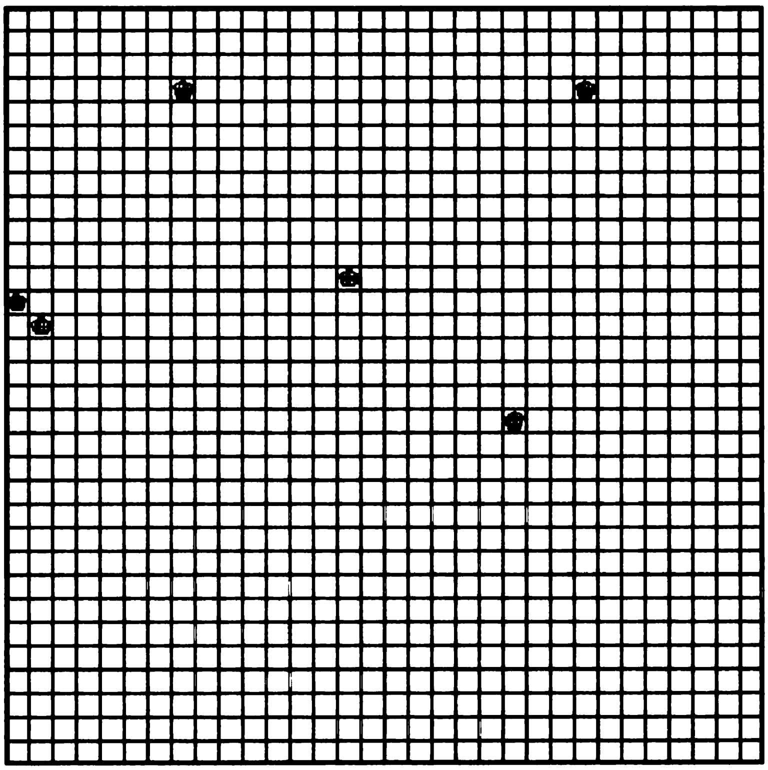

Figure 4.7. Fractal based planting plan for White Spruce

In the Upper Peninsula of Michigan, a typical mine site contains waste rock, with environmental conditions similar to xeric forest sites in the region (figure 4.8). The planting method can be completed with seedlings being planted by hand or machine, as long as the tree is planted in the correct designated box.

Figure 4.8. Example of a waste rock pile in Michigan's Upper Peninsula

The composition of trees in the study are similar to those specified by Curtis, dominant trees however vary from typical northern xeric forest. This is not a rare condition as stated by Curtis (1959)

"Vegetation… is a chaotic mixture of communities, each composed of a random assortment of species, each independently adapted to a particular set of external environmental factors. Rather there is a certain pattern to the vegetation, with more or less similar groups of species re-occurring from place to place."

This explanation from Curtis can also be attributed to cover change over time. According to Albert and Comer, the existing tree species composition is different from the pre-settlement vegetation according to an interpretation of the 1816-1856 general land office surveys (2008).

Results of the data collection process reveal a consistent vegetation type by those described by Curtis (1959), Linsemier (1989), and Albert & Comer (2008). There were a number trees not indentified by these sources, however changes to composition and introduction of new species by humans can attribute these changes. It is also important to remember that each area has its own unique set of environmental conditions which can affect the composition of vegetation within a given cover type (Curtis, 1959). One constant that holds true throughout these investigations is soil conditions, rocky and steep terrain with exposed bedrock. It can be concluded that most tree species identified by these investigations will be appropriate for reforestation of surface mine reclamation projects within the Upper Peninsula of Michigan.

Statistical analysis of the fractal dimensions of each species, and the combined analysis reveal that all species have similar patterns. The total fractal dimension of all species revealed a slope of 0.587. The fractal dimension of each species ranged from 0.329 to 0.674, revealing that each species has the same Euclidian dimension of 0, but their own distinctive fractal dimension. The intercept value of all species was 3.107. The intercept value of each species ranged from 1.936 to 3.582, revealing that each species indeed has their own pattern and the overall species composition falls within the parameters of these patterns. The investigation of the fractal dimension of each species reveals numbers which are similar to that of Cantor's Dust. This result suggests that these fractal patterns may be expressed at different scales (100 meter by 100 meter, 1 mile by 1 mile, etc.). Further research is needed to determine if it is possible to apply these findings to areas larger than 50 meters by 50 meters.

A limitation of this study is the scale of application as specified above. This investigation used 50 by 50 meter square grids, most reclamation projects are larger than this. To be able to apply these findings at a larger scale is an area of further investigation one may choose to explore. The box-counting method expresses this scale based limitation in the upper and lower limits of the regression line. As the regression line continues past the boundaries of the box-counting method, the line is skewed. The upper limit of the regression line flattens, while the lower limit of the regression line is abnormally steep. To scale the results of this study without the proper mathematical function would yield an unreliable result. Another limitation of the box-counting method also relates to scale, specifically the maximum size of the grid. The fractal dimension estimate is highly correlated to the size of the largest box (Kenkel & Walker, 1996). Site limitations which caused this experiment to utilize 50 meter grids may have ultimately affected the fractal dimension estimated. The estimated fractal dimension of all species was relatively low when compared to the previous investigations of Fleurant et al. (2009) and does not meet the standards for a set of unaligned points in a two dimensional plane. According to Kenkel & Walker (1996) the fractal dimension of a two dimensional point pattern should be $\geq 1 \leq 2$.

Another limitation of this study is the site of application. This study focused solely on the vegetation of a Northern Michigan rock outcrop. To apply these findings anywhere but a Northern Michigan surface mine, one would have to conduct their own survey of a natural area they wished to replicate. This investigation determined the fractal pattern of trees, while this is not a limitation, further research is needed to determine if this process can be used for other landscape features such as; topography, or water networks.

In conclusion, it is determined that the box-counting method can be used to estimate the fractal dimension of an individual species of tree within a vegetation stand. The inverse box-counting method can then be applied to re-create the fractal patterns found in the landscape. Successfully replicating natural tree stands is important to many disciplines outside of mine reclamation. This method can be applied for many projects including; reforestation after forest fire (or other natural disaster), restoration, or any project which attempts to blend in with the surrounding vegetative community. Why then choose reclamation to utilize this method? Returning to a quote from Berger (2008), "reclamation can act as a laboratory for experimentation."

APPENDIX

Images of each trial of existing tree species

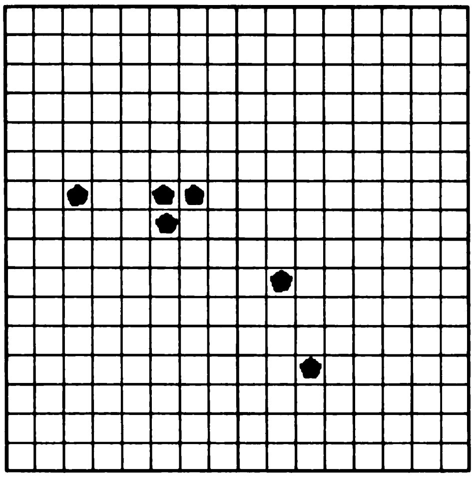

Figure A.1. Eastern Hemlock trial 1

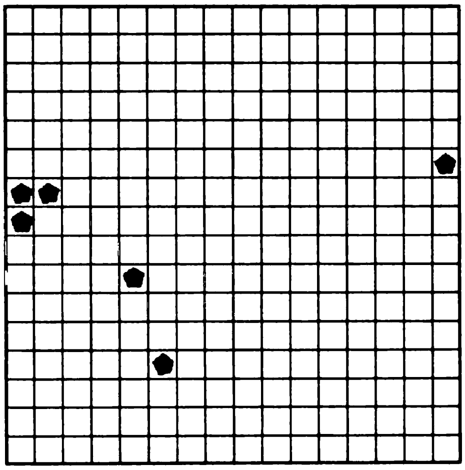

Figure A.2. Eastern Hemlock trial 2

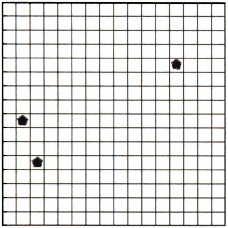

Figure A.3. Eastern Hemlock trial 3

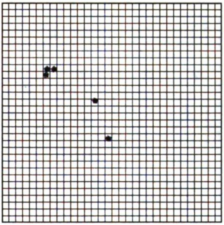

Figure A.4. Eastern Hemlock trial 4

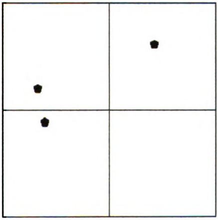

Figure A.5. Eastern Hemlock trial 5

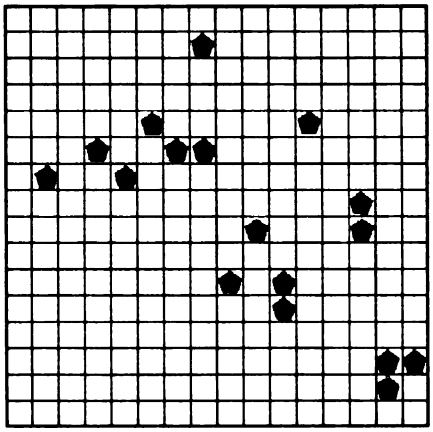

Figure A.6. Northern White Cedar trial 1

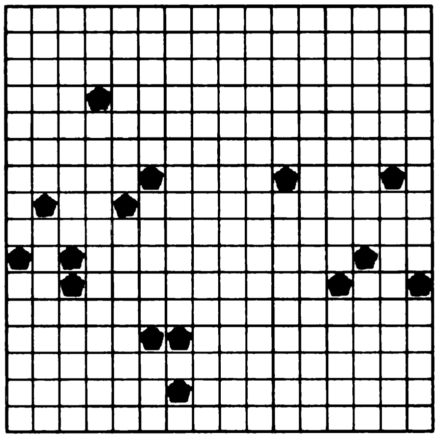

Figure A.7. Northern White Cedar trial 2

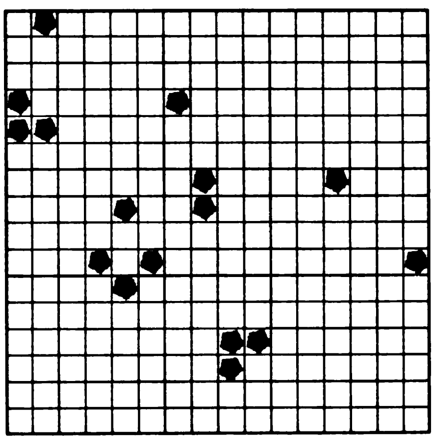

Figure A.8. Northern White Cedar trial 3

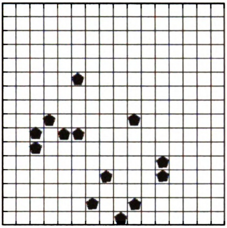

Figure A.9. Northern White Cedar trial 4

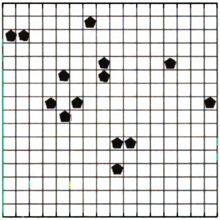

Figure A.10. Northern White Cedar trial 5

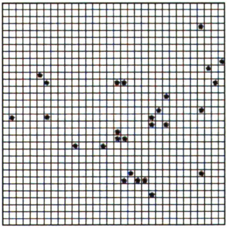

Figure A.11. Red Maple trial 1

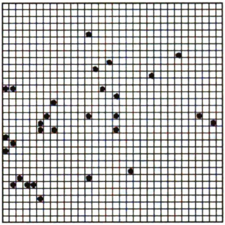

Figure A.12. Red Maple trial 2

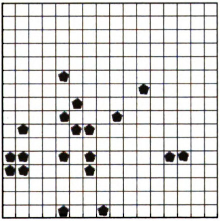

Figure A.13. Red Maple trial 3

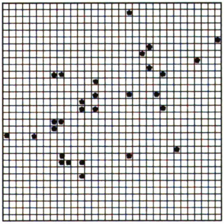

Figure A.14. Red Maple trial 4

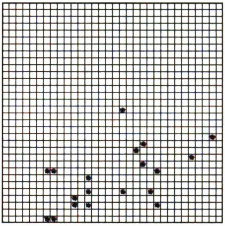

Figure A.15. Red Maple trial 5

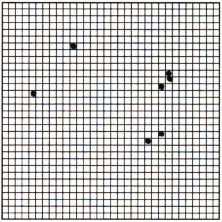

Figure A.16. Sugar Maple trial 1

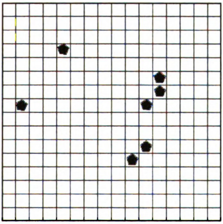

Figure A.17. Sugar Maple trial 2

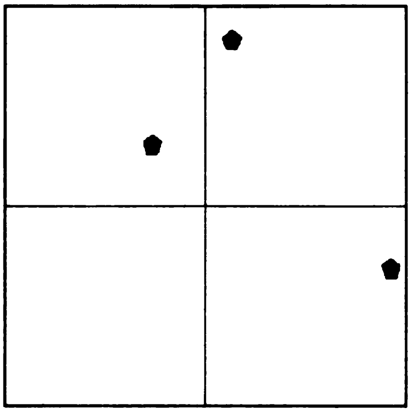

Figure A.18. Sugar Maple trial 3

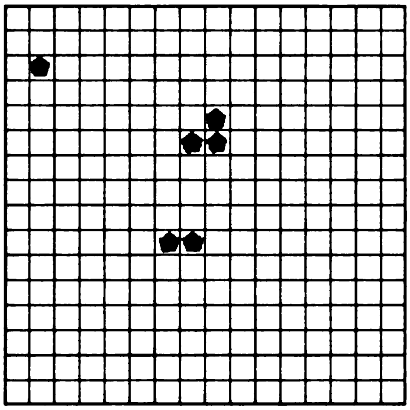

Figure A.19. Sugar Maple trial 4

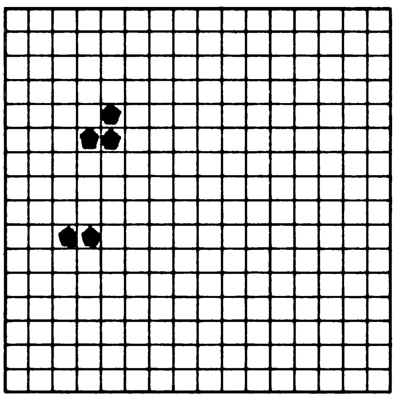

Figure A.20. Sugar Maple trial 5

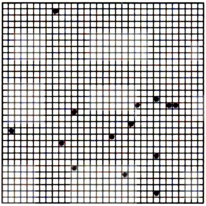

Figure A.21. White Pine trial 1

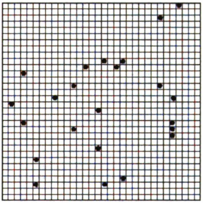

Figure A.22. White Pine trial 2

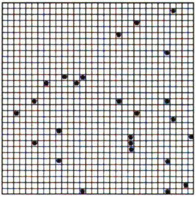

Figure A.23. White Pine trial 3

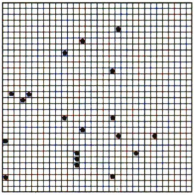

Figure A.24. White Pine trial 4

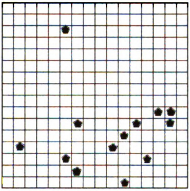

Figure A.25. White Pine trial 5

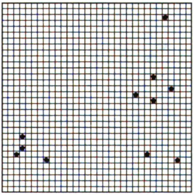

Figure A.26. White Spruce trial 1

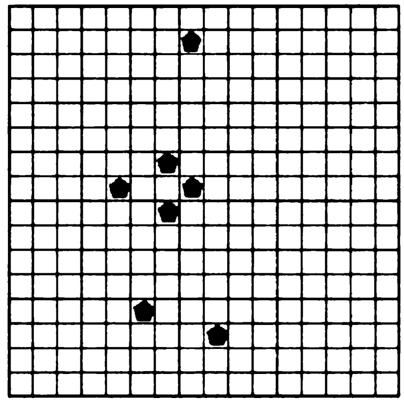

Figure A.27. White Spruce trial 2

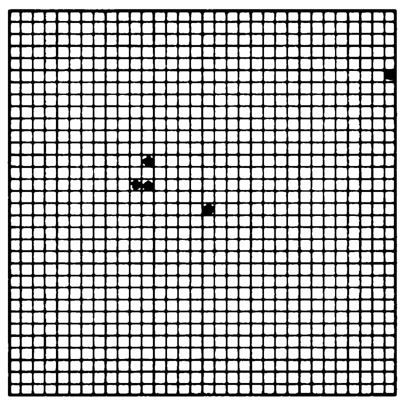

Figure A.28. White Spruce trial 3

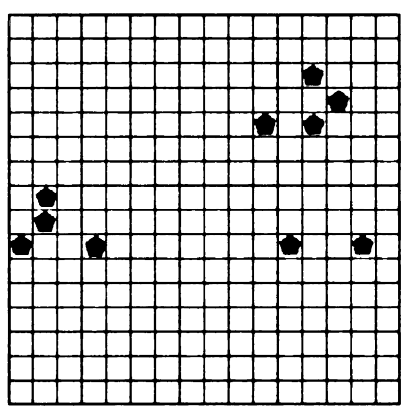

Figure A.29. White Spruce trial 4

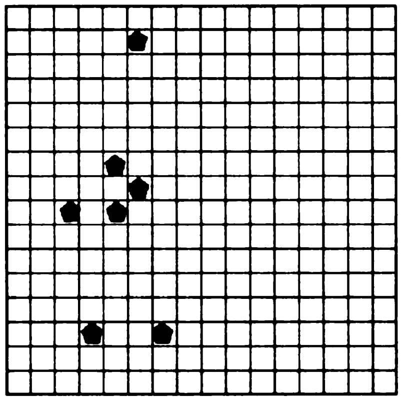

Figure A.30. White Spruce trial 5

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