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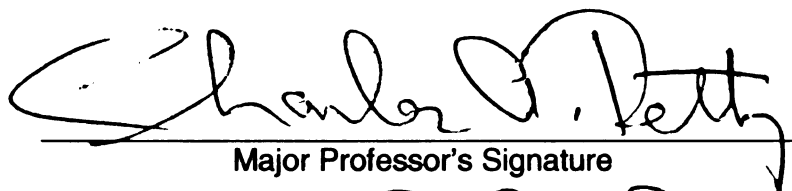
UNIVERSAL REALIZABLE ANISOTROPIC PRESTRESS
CLOSURE FOR THE NORMALIZED REYNOLDS STRESS

presented by

KARUNA SREE KOPPULA

has been accepted towards fulfillment
of the requirements for the

Ph.D degree in Chemical Engineering


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UNIVERSAL REALIZABLE ANISOTROPIC PRESTRESS CLOSURE
FOR THE NORMALIZED REYNOLDS STRESS

By

KARUNA SREE KOPPULA

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

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Chemical Engineering

2009

The Reynolds-averaged Navier-Stokes (RANS) equations are a set of time-averaged equations that relate the mean velocity field to the mean Reynolds stress. The continuity equation and the statistical closure for the Reynolds stress are the primary closure models for RANS. In the new theory, the normal stress is modeled as a linear dyadic-valued, algebraic function of the mean turbulent time scale, the mean velocity gradient, and the mean velocity. This stems from an analysis of the turbulent time scale of a constant property. The resulting closure can be used to model the class of benchmark turbulent flows.

The foregoing low-order closure model is based on research by Parks (1997) and is a simple model of turbulent fluctuating velocity correlations. The turbulent correlations have

ABSTRACT

UNIVERSAL REALIZABLE ANISOTROPIC PRESTRESS CLOSURE FOR THE NORMALIZED REYNOLDS STRESS

By

Karuna Sree Koppula

The Reynolds averaged Navier-Stokes (RANS-) equation is an exact, albeit unclosed, equation that relates the mean velocity field to the mean pressure field and the Reynolds stress. The continuity equation and a Reynolds stress model provide a low-order statistical closure for the RANS-equation. This research has developed a new algebraic closure model for the Reynolds stress that is *realizable* for all turbulent flows. In the new theory, the normalized Reynolds (NR-) stress is a solution to an implicit, non-linear, dyadic-valued, algebraic equation that depends on the relative importance of a local turbulent time scale, a local viscous time scale, a local time scale related to the mean field velocity gradient, and a time scale associated with the frame of reference. The theory stems from an analysis of the dynamic equation governing the fluctuating velocity field of a constant property Newtonian fluid in a rotating frame of reference. Therefore, the resulting closure can be applied in either inertial or non-inertial frames regardless of the class of benchmark flows used to determine the phenomenological closure parameters.

The foregoing low-order closure model for the RANS-equation generalizes earlier research by Parks (1997) and Weispfennig (1997) based on an integral analysis of turbulent fluctuating velocity fields and the physical assumption that all space-time turbulent correlations have finite memories. In this research, the Parks-Weispfennig

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The URAPS-closure is
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approach is extended to non-inertial frames. A preclosure equation shifts the turbulence closure problem from the NR-stress to a normalized prestress. The prestress is caused by pressure fluctuations and fluctuations in the instantaneous Reynolds stress. A self-consistent hypothesis, similar to the one for the pressure/strain rate correlation, is used to relate the prestress to the NR-stress. In the present research, a closure for the prestress is developed and combined with the preclosure equation for the NR-stress to produce a universal realizable anisotropic prestress (URAPS-) closure for the NR-stress. A critical review of other algebraic closure models in the literature indicates that the URAPS-closure provides an answer to one of the key questions in turbulence modeling: Can a low-order closure model for the NR-stress be formulated that is *realizable* for all turbulent flows independent of the specific benchmark flows used for calibration?

The URAPS-closure is formulated as a mapping of a non-negative operator into itself. The mapping depends on the rotational operator $\underline{\underline{\Omega}}$ associated with the frame of reference, a local scalar-valued turbulent transport time scale τ_R , and an operator $\underline{\underline{F}} > (\equiv \nabla < \underline{u} > + 2\underline{\underline{\Omega}})$:

$$\underline{\underline{\mathfrak{R}}}(\underline{\underline{R}}, \tau_R < \underline{\underline{F}} >) = \underline{\underline{R}} \quad , \quad \underline{\underline{R}} \equiv \frac{\underline{\underline{u}}' \underline{\underline{u}}'}{\text{tr}(\underline{\underline{u}}' \underline{\underline{u}}')}.$$

The URAPS-closure is used to predict the components of the NR-stress for three benchmark flows: rotating homogeneous decay, rotating homogeneous shear, and spanwise rotating fully-developed channel flows. The URAPS-predictions are consistent with complementary direct numerical simulations of these flows and, thereby, partially supports its use as a closure model for the RANS-equation.

dedicated to my parents and my husband

First and foremost,

Thank you for his guidance

and research. I would also

thank Professor Krishnan

and Dr Brigitte Rosen

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this document.

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Fig. 45 URAPS-
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Fig. 46 Influence
 $\tilde{\Gamma} = -20$

Fig. 47 URAPS-
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Fig. 17 URAPS-CL

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Σ Symbols

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LIST OF NOTATION

English Symbols

$\underline{\underline{A}}$	= preclosure operator, $\underline{\underline{A}} \equiv \left(\underline{\underline{I}} + \underline{\underline{K}} \right)^{-1}$, dimensionless
$\underline{\underline{B}}$	= normalized prestress, $\frac{\langle \underline{\underline{f}}' \underline{\underline{f}}' \rangle}{\text{tr}(\langle \underline{\underline{f}}' \underline{\underline{f}}' \rangle)}$
$\underline{\underline{B}}_{n+1}$	= (n+1)-iteration for the normalized prestress defined by the APS-model
$\underline{\underline{b}}$	= anisotropic component of $\underline{\underline{R}}$, $\underline{\underline{b}} \equiv \underline{\underline{R}} - \frac{1}{3} \underline{\underline{I}}$
$\underline{\underline{C}}$	= $\text{adj}(\underline{\underline{I}} + \underline{\underline{K}})$, preclosure operator (adj: adjugate, i.e. transpose of the minor)
C_n	= Cayley-Hamilton coefficients
C_p	= “production” coefficient in the ε – equation
C_D	= “dissipation” coefficient in the ε – equation
C_μ	= dimensionless turbulent transport time scale
	$C_e = C_e \left(\frac{k^2}{\nu \varepsilon}, \frac{k}{\varepsilon} \left\ \langle \underline{\underline{F}} \rangle \right\ \right)$
C_{cc}	= correlation coefficient (Schwartz inequality),

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$$0 \leq C_{cc} \equiv \sqrt{\frac{R_{yz}^2}{R_{yy}R_{zz}}} \leq 1$$

$\underline{e}_1, \underline{e}_2, \underline{e}_3$ = Euclidean base vectors

$\langle \underline{\underline{F}} \rangle$ = mean field kinematic variable in the preclosure operator $\underline{\underline{A}}$,
 $\nabla \langle \underline{\underline{u}} \rangle + 2 \underline{\underline{\Omega}}, 1/t$

$f_F, f_{F\text{ HP}}, f_{F\text{ LP}}$ = friction factor, based on the average, high pressure side and low pressure side shear stresses respectively.

$-\rho \underline{\underline{f}}'$ = fluctuating force per unit volume induced by fluctuations in the instantaneous turbulent stress $\underline{\underline{f}}' \equiv \nabla \cdot \left(\frac{\underline{\underline{p}}'}{\rho} \underline{\underline{I}} + \underline{\underline{u}}' \underline{\underline{u}}' - \langle \underline{\underline{u}}' \underline{\underline{u}}' \rangle \right), L/t^2$

$\underline{\underline{g}}$ = acceleration due to gravity, L/t^2

$G_v(\underline{\underline{x}}, t | \underline{\underline{\hat{x}}}, \hat{t})$ = Green's function associated with the mean field convective/viscous differential operator, $1/L^3$

I_b, II_b, III_b = invariants of $\underline{\underline{b}}$: $\text{tr}(\underline{\underline{b}}), \text{tr}(\underline{\underline{b}} \cdot \underline{\underline{b}}), \text{tr}(\underline{\underline{b}} \cdot \underline{\underline{b}} \cdot \underline{\underline{b}})$, resp.

I_R, II_R, III_R = invariants of $\underline{\underline{R}}$: $\text{tr}(\underline{\underline{R}}), \text{tr}(\underline{\underline{R}} \cdot \underline{\underline{R}}), \text{tr}(\underline{\underline{R}} \cdot \underline{\underline{R}} \cdot \underline{\underline{R}})$, resp.

I_S, II_S, III_S = $\text{tr}(\langle \underline{\underline{S}} \rangle \cdot \langle \underline{\underline{S}} \rangle), \text{tr}(\langle \underline{\underline{S}} \rangle \cdot \langle \underline{\underline{S}} \rangle \cdot \langle \underline{\underline{S}} \rangle)$

$\underline{\underline{J}}_k$ = sum of the viscous and turbulent flux of turbulent kinetic energy relative to the mean velocity, $\underline{\underline{J}}_k \equiv -(\nu \underline{\underline{I}} + \tau_R \langle \underline{\underline{u}}' \underline{\underline{u}}' \rangle) \cdot \nabla k$, L^3/t^3

$\underline{\underline{J}}_\varepsilon$ = $\underline{\underline{J}}_\varepsilon \equiv -(\nu \underline{\underline{I}} + \tau_R \langle \underline{\underline{u}}' \underline{\underline{u}}' \rangle) \cdot \nabla \varepsilon$, L^3/t^4

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k	$=$ turbulent kinetic energy, $k \equiv \frac{\text{tr} \langle \underline{u}' \underline{u}' \rangle}{2}$, L^2/t^2
k^+, k_{HP}^+, k_{LP}^+	$= \frac{k}{u_*^+}, \frac{k}{u_{HP}^+}, \frac{k}{u_{LP}^+}$, , normalized by wall parameters.
$\underline{\underline{K}}$	$= \tau_R (\nabla \langle \underline{u} \rangle + 2 \underline{\underline{\Omega}})$, dimensionless kinematic operator.
M_R	$=$ dimensionless memory kernel
N_F	$= \frac{k}{\varepsilon} \ \langle \underline{\underline{F}} \rangle\ $, $\frac{\text{turbulent time scale}}{\text{kinematic time scale}}$
N_Γ	$= \tau_R \Gamma_{yz} = C_{Rl} \tilde{\tau}_R \tilde{\Gamma}$, $\frac{\text{turbulent time scale}}{\text{flow time scale}}$
N_Ω	$= 2 \tau_R \Omega_x = 2 C_{Rl} \tilde{\tau}_R \tilde{\Omega}$
$\mathcal{P}$	$= -\langle \underline{u}' \underline{u}' \rangle : \langle \underline{\underline{S}} \rangle = -2 k \underline{\underline{R}} : \langle \underline{\underline{S}} \rangle$, “production” of turbulent kinetic energy, L^2/t^3
p'	$=$ fluctuating pressure, $M/L \cdot t^2$
$-\rho \underline{\underline{\mathbb{P}}}$	$=$ “production” of Reynolds stress, $\underline{\underline{\mathbb{P}}} \equiv -\langle \underline{u}' \underline{u}' \rangle \cdot \nabla \langle \underline{u} \rangle - (\nabla \langle \underline{u} \rangle)^T \cdot \langle \underline{u}' \underline{u}' \rangle$, L^2/t^3
$\underline{\underline{R}}$	$=$ normalized Reynolds stress, $\frac{\langle \underline{u}' \underline{u}' \rangle}{\text{tr}(\langle \underline{u}' \underline{u}' \rangle)}$
$\underline{\underline{R}}_{n+1}$	$=$ (n+1)-iteration for the normalized stress defined by the APS-model

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$2\rho\underline{\underline{\mathbb{R}}}$	= turbulent pressure/strain rate correlation, $\underline{\underline{\mathbb{R}}} \equiv + < \frac{\underline{\underline{p}}'}{\rho} [\nabla \underline{\underline{u}}' + (\nabla \underline{\underline{u}}')^T] >, L^2/t^3$
Re	$= \frac{\delta u_b}{\nu}, \frac{\text{viscous time scale}}{\text{flow time scale}}, \text{Reynolds number}$
Re^+	$= \delta^+ = \frac{\delta u^*}{\nu}, \text{Reynolds number based on friction velocity.}$
Re_{HP}^+	$= \frac{\delta u_{\text{HP}}^*}{\nu}, \text{Reynolds number based on friction velocity on the high pressure side.}$
Re_{LP}^+	$= \frac{\delta u_{\text{LP}}^*}{\nu}, \text{Reynolds number based on friction velocity on the high pressure side.}$
Ro	$= \frac{\delta \underline{\underline{\Omega}} }{u_b}, \frac{\text{flow timescale}}{\text{frame time scale}}, \text{Rossby number}$
Ro^+	$= \frac{\delta \underline{\underline{\Omega}} }{u^*}$
$<\underline{\underline{S}}>$	= mean strain rate, $<\underline{\underline{S}}> \equiv \frac{\nabla <\underline{\underline{u}}> + (\nabla <\underline{\underline{u}}>)^T}{2}, 1/t$
$<\tilde{\underline{\underline{S}}}>$	= dimensionless mean strain rate, $<\tilde{\underline{\underline{S}}}> \equiv \frac{k}{\varepsilon} <\underline{\underline{S}}>$
t	= time, t
$\hat{t}$	= time, $t - \hat{t} \geq 0$, instantaneous memory variable in space-time turbulent correlation, 1/t

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$\rho \underline{\underline{T}}$	= turbulent flux of turbulent momentum flux, $\underline{\underline{T}} \equiv + \langle \underline{u}' \underline{u}' \underline{u}' \rangle + \frac{1}{\rho} (\langle \underline{u}' p' \rangle \underline{\underline{I}} + \underline{\underline{I}} \langle \underline{u}' p' \rangle), L^3/t^3$
$\underline{u}$	= instantaneous velocity, L/t
$\underline{u}'$	= fluctuating velocity, L/t
$\langle \underline{u} \rangle$	= Reynolds average velocity (ensemble average)
u_x, u_y, u_z	= components of the instantaneous velocity in a Cartesian coordinate system
u^*	= $\sqrt{\frac{\tau_w}{\rho}}$ friction velocity (based on the average wall shear stress incase of rotating channel, $\sqrt{(u_{HP}^* + u_{LP}^*)/2}$), L/t
u_{HP}^*	= $\sqrt{\frac{\tau_{w HP}}{\rho}}$, friction velocity of the high pressure side in rotating channel flow, L/t
u_{LP}^*	= $\sqrt{\frac{\tau_{w LP}}{\rho}}$, friction velocity of the low pressure side in rotating channel flow, L/t
$-\langle \underline{u}' \underline{u}' \rangle$	= kinematic Reynolds stress, L^2/t^2
$+\langle \underline{u}' \underline{u}' \rangle$	= kinematic Reynolds momentum flux, L^2/t^2
$-\rho \langle \underline{u}' \underline{u}' \rangle$	= Reynolds stress, $M/L-t^2$
$+\rho \langle \underline{u}' \underline{u}' \rangle$	= Reynolds momentum flux, $M/L-t^2$

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$\hat{V}$ = spatial domain in Green's function representation, L^3

$\langle \underline{w} \rangle$ = mean vorticity, $\langle \underline{w} \rangle \equiv \nabla \wedge \langle \underline{u} \rangle$, $1/t$

$\langle \underline{\underline{W}} \rangle$ = mean rotational strain rate associated with the mean vorticity,
 $\langle \underline{\underline{W}} \rangle \equiv \frac{\nabla \langle \underline{u} \rangle - (\nabla \langle \underline{u} \rangle)^T}{2}$, $1/t$

$\langle \underline{\underline{\tilde{W}}} \rangle$ = mean intrinsic vorticity, $\langle \underline{\underline{\tilde{W}}} \rangle \equiv \langle \underline{\underline{W}} \rangle + \underline{\underline{\Omega}}$, $1/t$

$\underline{x}$ = position vector, L

$\underline{y}_{Bi}$ = eigenvector associated with $\underline{b}$

$\underline{y}_{Ri}$ = eigenvector associated with $\underline{R}$

$[^+]$ = (+ superscript), normalized by wall parameters, e.g.
 $k^+ = \frac{k}{u_*^2}$, $\varepsilon^+ = \frac{\varepsilon}{(u_*^4 / \nu)}$

Greek Symbols

α = anisotropic coefficient of the first kind in the URAPS closure

β = anisotropic coefficient of the second kind in the URAPS closure

ε = turbulent dissipation, $\varepsilon \equiv \nu \langle (\nabla \underline{u}') : (\nabla \underline{u}')^T \rangle$, L^2/t^3

ε^+ , ε_{HP}^+ , ε_{LP}^+ = $\frac{\varepsilon}{(u_*^4 / \nu)}$, $\frac{\varepsilon}{(u_{HP}^*{}^4 / \nu)}$, $\frac{\varepsilon}{(u_{LP}^*{}^4 / \nu)}$, normalized by wall parameters.

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$\underline{\underline{\mathcal{E}}}$	= dyadic-valued turbulent dissipation, $\underline{\underline{\mathcal{E}}} \equiv \nu [\langle (\nabla \underline{\mathbf{u}}') \cdot (\nabla \underline{\mathbf{u}}')^T \rangle + \langle (\nabla \underline{\mathbf{u}}')^T \cdot (\nabla \underline{\mathbf{u}}') \rangle], L^2/t^3$
$\underline{\underline{\varepsilon}}$	= permutation triadic $\underline{\underline{\varepsilon}} \equiv \underline{\mathbf{e}}_1 \underline{\mathbf{e}}_2 \underline{\mathbf{e}}_3 + \underline{\mathbf{e}}_3 \underline{\mathbf{e}}_1 \underline{\mathbf{e}}_2 + \underline{\mathbf{e}}_2 \underline{\mathbf{e}}_3 \underline{\mathbf{e}}_1 - \underline{\mathbf{e}}_3 \underline{\mathbf{e}}_2 \underline{\mathbf{e}}_1 - \underline{\mathbf{e}}_2 \underline{\mathbf{e}}_1 \underline{\mathbf{e}}_3 - \underline{\mathbf{e}}_1 \underline{\mathbf{e}}_3 \underline{\mathbf{e}}_2$
κ	= wave number
$\underline{\underline{\Phi}}_n$	= nth order integrity base associated with $\underline{\mathbf{b}}$ for S-hypothesis, Eq.(2.11)
Ψ	= Optimization variable, (see Eq. 4.9)
λ_{bi}	= eigenvalue associated with $\underline{\mathbf{b}}$, dimensionless
λ_{Ri}	= eigenvalue associated with $\underline{\mathbf{R}}$, dimensionless
ν	= kinematic viscosity of the fluid, L^2/t
ν_e	= “eddy” viscosity, [=] L^2/t
ρ	= mass density of the fluid, M/L^3
τ_t	= turbulent transport time scale, $\tau_t = C_\mu \frac{k}{\varepsilon}$, $1/t$
τ_R	= turbulent relaxation time defined as follows $\tau_R = \int_{-\infty}^t M(\underline{\mathbf{x}}, t \underline{\mathbf{x}}, \hat{t}) [\int_{\hat{\mathbf{V}}} G(\underline{\mathbf{x}}, t \underline{\hat{\mathbf{x}}}, \hat{t}) d\hat{\mathbf{V}}] d\hat{t}$
$\underline{\chi}(\underline{\mathbf{X}}, t)$	= motion of a material particle, [=], L

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$$\Gamma_{yz} \equiv \underline{e}_z \underline{e}_y : \nabla < \underline{u} >$$

$$\tilde{\Gamma} = [\underline{e}_z \underline{e}_y : \nabla < \underline{u} >] \frac{k}{\varepsilon} = \Gamma_{yz} \frac{k}{\varepsilon}$$

$$\underline{\underline{\Omega}} = \text{rotational operator, } \Omega_x [\underline{e}_y \underline{e}_z - \underline{e}_z \underline{e}_y] \text{ of a non-inertial rigid body frame, } 1/t$$

$$\underline{\Omega} = \text{angular velocity of a rigid body frame, } \Omega = \Omega_x \underline{e}_x, \ 1/t$$

$$\Omega_x \equiv \underline{e}_z \underline{e}_y : \underline{\underline{\Omega}}$$

$$\Omega_x^+ = \Omega_x v / (u^*)^2$$

$$\tilde{\Omega} = \Omega_x \frac{k}{\varepsilon}$$

Mathematical operators

$$< \underline{\underline{A}} > = \text{Reynolds average (ensemble average) of } \underline{\underline{A}}$$

$$\text{tr}(\underline{\underline{A}}) = \sum_{i=1}^3 A_{ii} \text{ trace of a dyadic-valued operator } \underline{\underline{A}}$$

$$(\underline{\underline{A}})^T = \underline{\underline{A}}^T = \left(\sum_{i=1}^3 \sum_{j=1}^3 A_{ij} \underline{e}_i \underline{e}_j \right)^T = \sum_{i=1}^3 \sum_{j=1}^3 A_{ij} \underline{e}_j \underline{e}_i \text{ transpose of a dyadic-valued operator } \underline{\underline{A}}$$

$$(\underline{a}) \cdot (\underline{b}) = \text{scalar product between two vectors } \underline{a} \text{ and } \underline{b}$$

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$\underline{\underline{\mathbf{A}}} : \underline{\underline{\mathbf{B}}}$ = double contraction (Gibbs convention) between two dyadic-valued operators $\underline{\underline{\mathbf{A}}}$ and $\underline{\underline{\mathbf{B}}}$

$\|\underline{\underline{\mathbf{A}}}\|$ = norm of a real dyadic-valued operator,
 $\|\langle \underline{\underline{\mathbf{A}}} \rangle\| \equiv \sqrt{\langle \underline{\underline{\mathbf{A}}} \rangle : \langle \underline{\underline{\mathbf{A}}} \rangle^T}$

$\underline{\underline{\mathbf{I}}}$ = dyadic-valued identity operator, $\underline{\underline{\mathbf{I}}} \cdot \underline{\mathbf{a}} = \underline{\mathbf{a}}$

$\mathcal{L}$ = convective/diffusive mean field differential operator,
 $\mathcal{L} \equiv \frac{\partial}{\partial t} + \langle \underline{\mathbf{u}} \rangle \cdot \nabla - \nu \nabla^2, 1/t$

$\frac{\partial}{\partial t}$ = time derivative, $1/t$

$\nabla, \nabla^2 \equiv \nabla \cdot \nabla$ = gradient operator, $1/L$ and Laplacian operator, $1/L^2$

(1) Rate of change of $\mathbf{F}$

This research

is for rotating and

is governed by the

equation for $\mathbf{F}$, $\mathbf{F}$ is

$$\frac{d\mathbf{F}}{dt} = -\nabla \cdot \mathbf{F} - \frac{\mathbf{F}}{\rho}$$

$$\nabla \cdot \mathbf{F} = 0$$

$$\text{where } \mathbf{F} = \nabla \mathbf{u} - 2\mathbf{Q}$$

condition satisfies

$$-\nabla \cdot \mathbf{F} = \frac{\mathbf{F} \cdot \mathbf{Q}}{\rho} - \frac{\mathbf{F} \cdot \mathbf{Q}}{2}$$

The independent $\mathbf{v}$

equations are defined

$\mathbf{Q}$ is related to the

rotation triadic

$$\text{with } \mathbf{Q} = 0$$

For large R

appropriate boundary

initial conditions

CHAPTER 1

INTRODUCTION

1.1 Rationale for Turbulence Modeling Research

This research is based on the premise that the instantaneous velocity and pressure fields for rotating and non-rotating turbulent flows of constant property Newtonian fluids are governed by the Navier-Stokes (NS-) equation and the continuity equation (see Greenspan, 1968; Piquet, 1999):

$$\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \underline{\underline{F}} = -\nabla \left(\frac{p}{\rho} - \frac{\underline{x} \cdot (\underline{\underline{\Omega}} \cdot \underline{\underline{\Omega}}^T) \cdot \underline{x}}{2} \right) + \nu \nabla^2 \underline{u} + \underline{g} \quad (1.1)$$

$$\nabla \cdot \underline{u} = 0 \quad (1.2)$$

where $\underline{\underline{F}} \equiv \nabla \underline{u} + 2\underline{\underline{\Omega}}$. Eqs.(1.1) and (1.2) imply that the instantaneous pressure distribution satisfies a Poisson equation:

$$-\nabla^2 \left(\frac{p}{\rho} - \frac{\underline{x} \cdot (\underline{\underline{\Omega}} \cdot \underline{\underline{\Omega}}^T) \cdot \underline{x}}{2} \right) = \nabla \cdot (\underline{u} \cdot \underline{\underline{F}}) \quad (1.3)$$

The independent variables $(\underline{x}, t)$ and the dependent variables $(\underline{u}, p)$ in the above equations are defined relative to a non-inertial frame-of-reference. The rotation operator $\underline{\underline{\Omega}}$ is related to the angular velocity of the frame by $\underline{\underline{\Omega}} \equiv \underline{\underline{\varepsilon}} \cdot \underline{\underline{\Omega}}$, where $\underline{\underline{\varepsilon}}$ denotes the permutation triadic. The same notation is used for an inertial frame-of-reference for which $\underline{\underline{\Omega}} = \underline{0}$.

For large Reynolds numbers, solutions to Eqs.(1.1) and (1.2) subject to appropriate boundary conditions are unsteady, spatially three dimensional, and sensitive to initial conditions. A direct numerical simulation (DNS) of these equations is

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An ensemble

equations for $\langle \underline{u} \rangle$ and

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computationally intensive (see Chapter 1 and p. 349 in Pope, 2000) and is limited to relatively simple geometries. This situation has motivated the development and use of low-order statistical methods to study ensembles of solutions associated with Eqs.(1.1) and (1.2), rather than individual solutions.

An ensemble average of Eqs.(1.1) and (1.2) yields exact, albeit unclosed, equations for $\langle \underline{u} \rangle$ and $\langle p \rangle$ that depend on the turbulent momentum flux $\rho \langle \underline{u} \underline{u}' \rangle$. The fluctuating velocity $\underline{u}'$ is defined as $\underline{u}' \equiv \underline{u} - \langle \underline{u} \rangle$. An analysis of turbulent flows based on low-order statistical moments has been an active area of research for more than a century. Previous studies have produced numerous and significant insights into the behavior of the Reynolds momentum flux $\rho \langle \underline{u} \underline{u}' \rangle$ or, equivalently, the Reynolds stress, $-\rho \langle \underline{u} \underline{u}' \rangle$ (see Chen and Jaw, 1999; Piquet, 1999; and, Pope, 2000). The Reynolds stress is fundamentally important in determining the behavior of all turbulent flows, including non-rotating flows with streamline curvature and rotating flows encountered in turbo-machinery, mixers, and fans (Gupta et al., 1984; Salhi and Cambon, 1997).

1.2 Turbulence Closure Problem

Unclosed Reynolds-Averaged Navier-Stokes Equation

The low-order statistical properties of turbulence are governed by the Reynolds-averaged Navier-Stokes (RANS-) equation and the Reynolds-averaged continuity equation. Ensemble averages of Eqs. (1.1), (1.2) and (1.3) yield the following unclosed equations:

$$\frac{\partial \langle \underline{u} \rangle}{\partial t} = - \langle \underline{u} \cdot \nabla \rangle \langle \underline{u} \rangle + \frac{\nabla \cdot \underline{\tau}}{\rho}$$

$$\tau_{ij} = \mu \frac{\partial \langle u_i \rangle}{\partial x_j}$$

$$\frac{\partial \langle \underline{u} \rangle}{\partial t} = - \frac{\nabla \cdot \underline{\tau}}{\rho} + \frac{\nabla \cdot \underline{Q}}{\rho}$$

The equations are for

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$$\langle \underline{u} \rangle = \nabla \cdot \langle \underline{u} \rangle + 2 \langle \underline{u} \rangle$$

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$$\langle \underline{u} \cdot \underline{u} \rangle = \langle \underline{u} \rangle \cdot \langle \underline{u} \rangle$$

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$$\frac{\partial \langle \underline{u} \rangle}{\partial t} + \langle \underline{u} \rangle \cdot \langle \underline{F} \rangle = -\nabla \left(\frac{\langle p \rangle}{\rho} - \frac{\underline{x} \cdot (\underline{\underline{\Omega}} \cdot \underline{\underline{\Omega}}^T) \cdot \underline{x}}{2} \right) + \underline{g} + \nu \nabla^2 \langle \underline{u} \rangle - \nabla \cdot \langle \underline{u}' \underline{u}' \rangle \quad (1.4)$$

$$\nabla \cdot \langle \underline{u} \rangle = 0 \quad (1.5)$$

$$-\nabla^2 \left(\frac{\langle p \rangle}{\rho} - \frac{\underline{x} \cdot (\underline{\underline{\Omega}} \cdot \underline{\underline{\Omega}}^T) \cdot \underline{x}}{2} \right) = +\nabla \cdot [\langle \underline{u} \rangle \cdot \langle \underline{F} \rangle + \nabla \cdot \langle \underline{u}' \underline{u}' \rangle]. \quad (1.6)$$

These equations are fundamentally different from Eqs. (1.1), (1.2) and (1.3) due to the presence of the Reynolds stress, $-\rho \langle \underline{u}' \underline{u}' \rangle$. The fluctuating velocity is $\underline{u}'(\underline{x}, t)$ and $\langle \underline{F} \rangle (\equiv \nabla \langle \underline{u} \rangle + 2 \underline{\underline{\Omega}})$ is an average kinematic operator. The ensemble-average operator is linear, commutes with spatial and temporal derivatives, and has the property that $\langle \langle \underline{u} \rangle \rangle = \langle \underline{u} \rangle$; consequently, $\langle \underline{u}'(\underline{x}, t) \rangle = \underline{0}$. Clearly, the use of Eqs.(1.4) and (1.5) for engineering design and analysis requires a closure model for the specific momentum flux $\langle \underline{u}' \underline{u}' \rangle$. This closure problem is a major challenge for turbulence research and is the primary focus of this dissertation. Unlike the NS-equation, solutions to a closed RANS-equation at high Reynolds numbers are temporally and spatially smooth due to the presence of the Reynolds stress. The mean velocity field may be unsteady and may have three components that vary in all three spatial directions. However, for specific boundary conditions and initial conditions on the mean velocity and the mean pressure fields, the RANS-equation may also have periodic solutions, unique statistically stationary (i.e., steady state) solutions, multiple statistically stationary solutions, and statistically homogeneous solutions.

More than a century ago, Boussinesq (1877) used a phenomenological closure for the Reynolds stress that assumes that the turbulent momentum flux depends on the local mean strain rate and the local statistical properties of the fluctuating velocity. Turbulent

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models based on this idea are often referred to as “eddy” viscosity closures (see, p. 359, Pope, 2000) wherein

$$-\rho \langle \underline{u}' \underline{u}' \rangle = -\frac{2}{3} \rho k \underline{I} + 2 \rho \nu_e \langle \underline{S} \rangle, \quad \text{tr} \langle \underline{S} \rangle = 0. \quad (1.7)$$

In the above equation for the Reynolds stress, k is the turbulent kinetic energy ($k \equiv \text{tr}(\langle \underline{u}' \underline{u}' \rangle) / 2$). The parameter $\rho \nu_e$ is a scalar-valued turbulent “eddy” viscosity, which can be related to a local characteristic length scale and a local characteristic velocity scale or, equivalently, to the local turbulent kinetic energy and a local characteristic time scale:

$$\nu_e = \ell_c u_c = (\tau_c \sqrt{k}) \sqrt{k} = \tau_c k. \quad (1.8)$$

The transport time scale $\tau_c (\equiv C_v k / \varepsilon)$ depends on the local turbulent time scale k / ε , where ε is the local dissipation of turbulent kinetic energy, $\varepsilon \equiv \nu \text{tr}(\langle (\nabla \underline{u}') \cdot (\nabla \underline{u}')^T \rangle)$. Thus, with the turbulent Reynolds number defined as $Re_t \equiv (k / \varepsilon) / (\nu / k)$, the turbulent “eddy” viscosity can be formally expressed as $\nu_e / \nu = C_v Re_t$. In the outer region of non-rotating, fully-developed channel flows, $Re_t > 30$; thus, in some regions, the “eddy” viscosity ν_e may be several orders of magnitude larger than the molecular kinematic viscosity ν . Deep in the viscous sublayer near a solid/fluid interface, the “eddy” viscosity is less than the molecular kinematic viscosity. The no slip boundary condition implies that $\nu_e \rightarrow 0$ near a solid/fluid interface.

In order to complete the Boussinesq (B-) closure for the RANS-equation, additional closure models are needed for the turbulent kinetic energy k , the turbulent dissipation ε , and the dimensionless transport time scale $\tau_c \varepsilon / k \equiv C_v$. Numerous closure models for

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these three statistical properties have emerged over the past forty years in support of computational fluid dynamic simulations based on the RANS-equation (see, esp., Chen and Jaw, 1999). The dimensionless group C_v is usually assumed to be an algebraic function of the local statistical state of the turbulence. Dimensional reasoning suggests that C_v depends on three distinct time scales: a viscous time scale, ν/k , a turbulent time scale, k/ε , and mean field time scales related to the two nontrivial independent invariants of the local mean strain rate $\langle \underline{\underline{S}} \rangle$ (i.e., $II_S \equiv \text{tr}(\langle \underline{\underline{S}} \rangle \cdot \langle \underline{\underline{S}} \rangle)$ and $III_S \equiv \text{tr}(\langle \underline{\underline{S}} \rangle \cdot \langle \underline{\underline{S}} \rangle \cdot \langle \underline{\underline{S}} \rangle)$). Based on this hypothesis, C_v depends on three independent dimensionless groups: the turbulent Reynolds number $(k/\varepsilon)/(\nu/k)$; and, the two dimensionless time scales: $(k/\varepsilon)II_S^{1/2}$ and $(k/\varepsilon)III_S^{1/3}$.

Transport Equations for the Turbulent Kinetic Energy and the Turbulent Dissipation

Although an exact, albeit unclosed, equation for turbulent kinetic energy can be developed from the NS-equation (see Appendix B), the following phenomenological transport equation for the kinetic energy is used to support the research developed hereinafter (see Hanjalic, 1994):

$$\frac{\partial k}{\partial t} + \langle \underline{u} \rangle \cdot \nabla k = +\nabla \cdot \{ \underline{\underline{D}}_v \cdot \nabla k \} + \{ -\langle \underline{u}' \underline{u}' \rangle : \langle \underline{\underline{S}} \rangle \} - \{ \varepsilon \}. \quad (1.9)$$

The three contributions in braces on the right-hand-side of Eq.(1.9) that cause changes in the turbulent kinetic energy include: 1) the viscous flux and the turbulent flux of kinetic energy relative to the mean velocity; 2) the “production” of kinetic energy due to the coupling between the Reynolds stress and the mean strain rate; and, 3) the dissipation of kinetic energy. The “production” term may be either positive or negative, depending on

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1. *Introduction*

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the local coupling between the Reynolds stress and the mean strain rate (see p. 180 in Pope, 2000; Nishino et al., 1996). The dissipation of energy is positive for all turbulent flows inasmuch as $\varepsilon \equiv \nu \text{tr}(\langle (\nabla \underline{u}') \cdot (\nabla \underline{u}')^T \rangle) > 0$. The flux of turbulent kinetic energy, $-\underline{\underline{D}}_{\nu} \cdot \nabla k$, acts like mixing for all turbulent flows provided the dispersion operator $\underline{\underline{D}}_{\nu}$ satisfies the following conditions:

$$\underline{\underline{D}}_{\nu} \equiv \nu \underline{\underline{I}} + \tau_R \langle \underline{u}' \underline{u}' \rangle, \quad \tau_R > 0, \quad \underline{\underline{D}}_{\nu} : \nabla k \nabla k > 0, \quad \forall \nabla k \ni \|\nabla k\| > 0. \quad (1.10)$$

Although an exact, albeit unclosed, equation for turbulent dissipation can also be developed from the NS-equation (see Appendix C), the following complementary phenomenological transport equation for the dissipation is used to support the research developed hereinafter (see Appendix C):

$$\frac{\partial \varepsilon}{\partial t} + \langle \underline{u} \rangle \cdot \nabla \varepsilon = + \nabla \cdot \{ \underline{\underline{D}}_{\nu} \cdot \nabla \varepsilon \} + C_P \left\{ \frac{- \langle \underline{u}' \underline{u}' \rangle : \langle \underline{S} \rangle}{\tau_R} \right\} - C_D \left\{ \frac{\varepsilon}{\tau_R} \right\}. \quad (1.11)$$

The three contributions in braces on the right-hand-side of Eq.(1.11) that cause changes in the turbulent dissipation include: 1) the viscous flux and the turbulent flux of turbulent dissipation relative to the mean velocity; 2) the “production” of turbulent dissipation; and, 3) the dissipation of turbulent dissipation. Rodi and Mansour (1993) give an assessment of the individual contributions to the exact equation for ε based on DNS results for non-rotating fully-developed channel flows. For a derivation of the exact dissipation equation, see Appendix C.

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1.3 The Normalized Reynolds Stress

The foregoing results show that the Reynolds stress appears in the equations that govern the behavior of $\langle \underline{u} \rangle$ and $\langle p \rangle$. The normalized Reynolds (NR-) stress, defined as follows

$$\underline{\underline{R}} \equiv \frac{\langle \underline{u}' \underline{u}' \rangle}{\text{tr}(\langle \underline{u}' \underline{u}' \rangle)} \quad , \quad \underline{\underline{R}}^T = \underline{\underline{R}} \quad \text{tr}(\underline{\underline{R}}) = 1, \quad (1.12)$$

has real eigenvalues and real eigenvectors that are mutually orthogonal. Most significantly, the NR-stress is a non-negative, dyadic-valued operator inasmuch as its quadratic form $Q_R(\underline{z})$ is non-negative for arbitrary unit vectors in a Euclidean three-dimensional vector space:

$$Q_R(\underline{z}) \equiv \underline{\underline{R}} : \underline{z} \underline{z} = \frac{(\underline{z} \cdot \underline{u}')(\underline{u}' \cdot \underline{z})}{\text{tr} \langle \underline{u}' \underline{u}' \rangle} \geq 0 \quad , \quad \forall \underline{z} \in E^3 \ni \|\underline{z}\| = 1. \quad (1.13)$$

Ineq.(1.13), which is equivalent to Schwartz's inequalities for the components of the NR-stress, implies that the eigenvalues of the NR-stress are non-negative and satisfy the following inequalities: $0 \leq \lambda_{R1} \leq \lambda_{R2} \leq \lambda_{R3} \leq 1$ (cf., Schumann, 1977; Lumley, 1978; Parks, 1997; and, p. 51 and p. 401 in Pope, 2000). Closure models for the NR-stress that produce solutions that satisfy Ineq.(1.13) for all rotating and non-rotating turbulent flows are *universal* and *realizable*, but they may not be accurate or practical.

The diagonal components of the NR-stress represent the fraction of turbulent kinetic energy associated with each component of the instantaneous fluctuating velocity. This information is often displayed on a triangular diagram (or energy simplex), which represents a subset of positive points in a hyperplane of a three-dimensional Euclidean

space for which $\sum_{i=1}^3 R_{ii} = 1$. For states with energy equally distributed among the

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$$\underline{I}_2 = \text{tr}(\underline{b} \underline{b}^T) = 0 \quad \text{if } \underline{b} = 0$$

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velocity components, $R_{11} = R_{22} = R_{33} = 1/3$. This is a necessary condition for an *isotropic state* of turbulence, but it is not sufficient inasmuch as the eigenvalues of the NR-stress may not be the same even if the diagonal components of $\underline{\underline{R}}$ are the same. If the eigenvalues of $\underline{\underline{R}}$ are equal (i.e., $\lambda_{R1} = \lambda_{R2} = \lambda_{R3} = 1/3$), then the turbulent state is *isotropic*.

Figure 1.1, which is similar to the pseudo-triangular graphs introduced by Lumley and others (see Lumley, 1978; Reynolds, 1987; and, p. 401 in Pope, 2000), defines the states of turbulence in terms of the invariants of the anisotropic operator:

$$\underline{\underline{b}} \equiv \underline{\underline{R}} - \frac{1}{3}\underline{\underline{I}} \quad , \quad \underline{\underline{b}}^T = \underline{\underline{b}} \quad , \quad \text{tr}(\underline{\underline{b}}) = 0. \quad (1.14)$$

Table 1.1 defines the boundaries of the Lumley (L-) diagram in terms of the non-trivial invariants of the anisotropic operator and the eigenvalues of the NR-stress. For a derivation of the inequalities presented in Table 1.1, see Parks (1997). In this research, the anisotropic invariants are defined by the following three scalar parameters:

$$I_b \equiv \text{tr}(\underline{\underline{b}}) = 0 \quad , \quad II_b \equiv \text{tr}(\underline{\underline{b}} \cdot \underline{\underline{b}}) \quad , \quad III_b \equiv \text{tr}(\underline{\underline{b}} \cdot \underline{\underline{b}} \cdot \underline{\underline{b}}) \quad (1.15)$$

Figure 1.1 provides a means to compare different anisotropic states associated with normalized, symmetric, non-negative operators. The eigenvalues of the NR-stress corresponding to specific anisotropic states on the L-diagram are located in the 1st Sextet of the eigenvalue simplex (see Table 1.1 and the inset on Figure 1.1). Table 1.1 identifies the type of states on each boundary of the L-diagram. For example, the quadratic forms associated with the AB- and AC-boundaries are oblate ellipsoids and prolate ellipsoids, respectively. The quadratic forms associated with the BC-boundary are planar ellipses inasmuch as one of the eigenvalues of the NR-stress is zero. The quadratic forms

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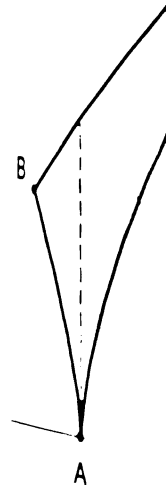


Fig. 1.1 Realizable

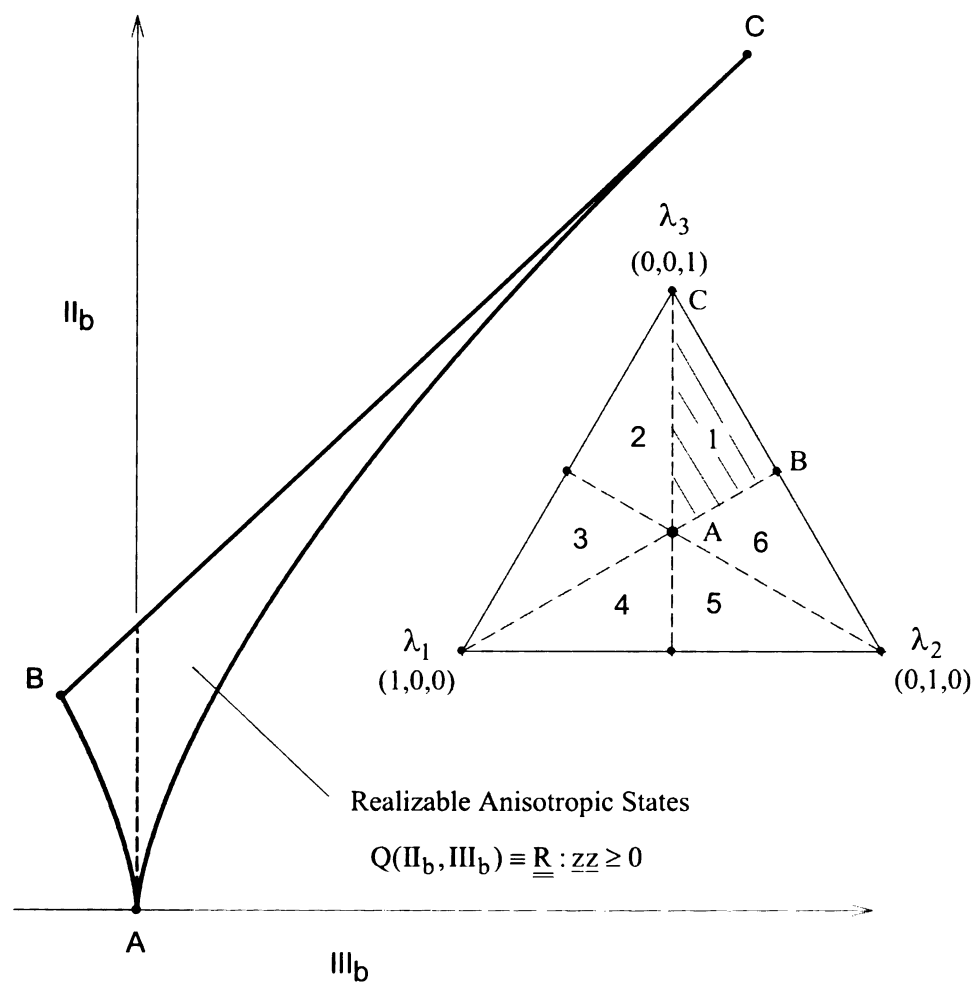


Figure 1.1 Realizable States of the NR-Stress (Lumley, 1978).

Table Boundary
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System State	Q-II _A -III-
A	3D-isotropic
B	2D-isotropic
C	1D-isotropic
AB boundary	Onset
BC boundary	Planar
AC boundary	Prolate

Table 1.1 Boundary Characteristics of the L-Triangle (Parks, 1997; Weispfennig, 1997).

Anisotropic States	$Q(\Pi_b, \text{III}_b)$	Eigenvalues of $\underline{\underline{R}}$	Invariants of $\underline{\underline{b}}$
A	3D-Isotropic	$\lambda_{R1} = \lambda_{R2} = \lambda_{R3} = 1/3$	$\text{III}_b = 0, \Pi_b = 0$
B	2D-Isotropic	$\lambda_{R1} = \lambda_{R2} = 1/2, \lambda_{R3} = 0$	$\text{III}_b = -1/36, \Pi_b = 1/6$
C	1D-Isotropic	$\lambda_{R1} = 1, \lambda_{R2} = \lambda_{R3} = 0$	$\text{III}_b = 2/9, \Pi_b = 2/3$
AB boundary	Oblate	$0 \leq \lambda_{R1} \leq 1/3 \leq \lambda_{R2} = \lambda_{R3} \leq 1/2$	$\Pi_b = 6(-\text{III}_b/6)^{2/3},$ $-1/36 \leq \text{III}_b \leq 0$
BC boundary	Planar	$0 = \lambda_{R1} \leq \lambda_{R2} \leq 1/2 \leq \lambda_{R3} \leq 1$	$\Pi_b = 2/9 + 2\text{III}_b$ $-1/36 \leq \text{III}_b \leq 8/36$
AC boundary	Prolate	$0 \leq \lambda_{R1} = \lambda_{R2} \leq 1/3 \leq \lambda_{R3} \leq 1$	$\Pi_b = 6(\text{III}_b/6)^{2/3}$ $0 \leq \text{III}_b \leq 8/36$

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associated with the states at the A-vertex are spheres; the quadratic forms associated with the states at the B-vertex are circles; and, the quadratic forms associated with the states at the C-vertex are infinitely elongated needles. The anisotropic states for which $0 \leq \Pi_b \leq 2/9$ and $\text{III}_b = 0$ are associated with an NR-stress with eigenvalues that satisfy the following conditions: $0 \leq \lambda_{R1} \leq \lambda_{R2} = 1/3 \leq \lambda_{R3} \leq 1$. Anisotropic states that satisfy all of the Schwartz inequalities are either on the boundary or located within the L-diagram. Models that predict anisotropic states with (Π_b, III_b) outside the L-diagram are unrealizable and, thereby, unphysical.

1.4 Objectives of This Research

The primary objective of this research is to develop an algebraic closure model for the NR-stress that supports the practical use of the RANS-equation (see Eq.(1.4) for rapid design and analysis of engineering processes. The new closure model is referred to hereinafter as the universal realizable anisotropic prestress (URAPS-) closure. The emphasis in this dissertation is on the theoretical development, calibration, and validation of the URAPS-closure. The implementation of the new closure in a computational code and the verification of the resulting computer code are not addressed in this research. The principal focus is on the further development of an algebraic closure model for the NR-stress initiated earlier by Parks (1997) for non-rotating homogeneous shear and by Weispfennig (1997) for non-rotating fully-developed channel flows.

The underlying premise of the research stems from the phenomenological idea that the Reynolds stress is determined by the local statistical properties of the fluctuating velocity field and the local mean field kinematics. This algebraic idea is similar to the

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hypothesis proposed by Boussinesq in 1877, which has evolved into the class of “eddy” viscosity closures exemplified by Eqs.(1.7), (1.9), and (1.11) above. In what follows, the NR-stress is formulated as a non-negative mapping of $\underline{\underline{R}}$ into itself with the following property:

$$\mathfrak{R}(\underline{\underline{R}}, \underline{\underline{K}}) = \underline{\underline{R}} \quad , \quad \text{if } \underline{\underline{K}} = \underline{\underline{0}} \text{ , then } \underline{\underline{R}} = \frac{1}{3} \underline{\underline{I}} . \quad (1.16)$$

where $\underline{\underline{K}} \equiv \tau_R < \underline{\underline{F}} >$ In the above hypothesis,

$$< \underline{\underline{F}} > = \nabla < \underline{\underline{u}} > + 2 \underline{\underline{\Omega}} \quad \text{and} \quad \tau_R = \tilde{\tau}_R (\text{Re}_t, \tilde{N}_F) \frac{k}{\varepsilon} . \quad (1.17)$$

The ratio of the turbulent time scale k/ε and the mean field time scale is $\tilde{N}_F \equiv k \| < \underline{\underline{F}} > \| / \varepsilon$ where $\| < \underline{\underline{F}} > \|^2 \equiv \text{tr}(< \underline{\underline{F}} > \cdot < \underline{\underline{F}} >^T)$. The specific goal of this dissertation is to develop a closure for the NR-stress that satisfies Eq.(1.16) and, most significantly, produces solutions that satisfy Ineq.(1.13) for all rotating and non-rotating turbulent flows.

Clearly, the pseudo-steady state hypothesis expressed by Eq.(1.16) cannot explain turbulent phenomena related to the finite time required for the NR-stress to relax to an isotropic state in an inertial frame, if $\underline{\underline{K}} \rightarrow \underline{\underline{0}}$ instantaneously. Therefore, for the theory developed hereinafter, temporal changes in the NR-stress occur implicitly through temporal changes in the gradient of the mean velocity and the turbulent time scale k/ε governed by Eqs.(1.9) and Eq.(1.11) above. Furthermore, Eq.(1.16) cannot explain the weak anisotropy that occurs on the symmetry plane of non-rotating, fully-developed channel flows where $\underline{\underline{K}} = \underline{\underline{0}}$, yet $0 < R_{xx} = R_{yy} < 1/3 < R_{zz} < 1$. However, Eq.(1.16)

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does imply that for rotating homogeneous turbulent flows for which $\underline{K} = 2 \underline{\Omega}$, turbulence decay is not isotropic (see Chapter 5 hereinafter).

In addition to the development and calibration of the URAPS-closure, the “production” and dissipation coefficients C_P and C_D in Eq. (1.11) are estimated by using the new NR-stress model together with benchmark experimental data and DNS results related to non-rotating asymptotic homogeneous shear flows (Tavoularis and Karnik, 1989; Parks, 1997), and non-rotating homogeneous decay (see, esp., Parks, 1997; and, Pope, 2000). The dimensionless relaxation time $\tilde{\tau}_R$, formally introduced by Eq.(1.16), is related to the space-time structure of the local turbulence and is determined by using previous theoretical and computational results related to rotating homogeneous decay (Park and Chung, 1999).

As indicated above, the goal is to develop a new algebraic closure model for the NR-stress that is aligned with previous turbulent modeling principles. This dissertation addresses four fundamental issues related to this goal: 1) *realizability* of the closure model; 2) the selection of appropriate benchmark flows for calibration; 3) the practical development of approximate solutions to the URAPS-closure; and, 4) an assessment of model predictions related to the influence of frame rotation on the redistribution of energy.

In summary, with the discovery of an algebraic closure for the NR-stress that produces realizable solutions in rotating and non-rotating frames of reference for all turbulent flows, this research provides a unified and practical closure for the RANS-equation that may have a significant impact on engineering design, research and education.

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1.5 Outline of the Dissertation

A review of the relevant literature that supports the objectives of this dissertation is given in Chapter 2. Previously developed algebraic models are discussed relative to realizability and redistribution of energy. Chapter 2 also includes a brief critique of Eq.(1.7) and other anisotropic models that relate the mean strain rate and the intrinsic mean vorticity to the NR-stress. The class of truncated explicit algebraic Reynolds stress models developed recently by Gatski and Jongen (2000) are also reviewed in Chapter 2. The predictions of a realizable algebraic model by Shih et al. (1994) are discussed and the results are used to partly justify the approach developed in this research. A summary of the hydrodynamic preclosure equation previously developed by Parks (1997) and by Weispfennig (1997) is also given in Chapter 2.

In Chapter 3, the preclosure result developed earlier by Parks et al. (1998) is generalized to include frame rotation. This is a key step in the development that directly links the NR-stress to another non-negative operator referred to hereinafter as the prestress. The preclosure mapping includes the hydrodynamic/kinematic operator $\underline{\underline{K}}$ (see Eq.(1.16) above) and satisfies the condition that if the prestress is a positive operator, then the NR-stress is a positive operator, which means that Ineq.(1.13) is satisfied. This *a priori* theoretical result, which does not depend on any calibration, holds for all rotating and non-rotating turbulent flows. In Chapter 3, the prestress is assumed to be caused by the NR-stress and the Cayley-Hamilton (CH-) theorem is used to relate the eigenvalues of the NR-stress and the prestress. The phenomenological CH-coefficients associated with the “extra” anisotropy of the prestress must be determined by benchmark flows, but Ineq.(1.13) provides a means to identify *universal* bounds on these parameters. Thus, the

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resulting URAPS-closure is a non-linear mapping of the NR-stress into itself for all rotating and non-rotating turbulent flows, provided the *universal* bounds on the CH-coefficients are observed. This result provides a means to significantly broaden the use of the RANS-equation as a diagnostic tool for analyzing turbulent flows.

The URAPS-closure coefficients are estimated in Chapter 4. This includes two CH-coefficients, α and β , implicit in the hypothesis expressed by Eq.(1.16); two coefficients, C_P and C_D , associated with the ε -equation defined by Eq.(1.11); and, the parameters implicit in the turbulent transport group $\tilde{\tau}_R(\text{Re}_t, \tilde{N}_F)$, introduced by Eq.(1.17) above. For large Re_t , $\tilde{\tau}_R(\infty, \tilde{N}_F)$ depends on three coefficients: C_{R1} , C_{R2} , and C_{R3} . The foregoing seven URAPS-closure coefficients are estimated by using the following three canonical benchmark flows: 1) non-rotating homogeneous decay; 2) rotating homogeneous decay; and, 3) non-rotating asymptotic homogeneous shear.

The calibrated URAPS-equation can be solved by the method of successive substitution. This attractive numerical feature is demonstrated in Chapter 4 for simple shear flows. The surprising role of the “extra” anisotropy operator in solving the URAPS-equation is illuminated in Chapter 4.

Chapter 5 examines the influence of rotation on the NR-stress for homogeneous decay. The URAPS-closure predicts that for rotating homogenous decay, the anisotropic states are all on the AB-boundary of the L-diagram (see Figure 1.1). Whence, the coupling between the velocity fluctuating field and the Coriolis acceleration causes an initially isotropic state to become anisotropic. For long times, the kinetic energy is transferred to the fluctuating velocity component that is co-linear with the rotation axis. This result clearly demonstrates that a return-to-isotropy upon the sudden removal of the

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relatively with the

non-rotating flow

results. Thus, the ne

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Eq. (16).

mean velocity gradient only occurs in an inertial frame. This Coriolis-induced anisotropy may also have implications on scattering of electromagnetic fields in the atmosphere.

The influence of rotation on asymptotic homogeneous shear as well as on the development of these states is presented in Chapter 6. The URAPS-predictions are qualitatively consistent with DNS results and with other closure models in the literature.

In Chapter 7, the URAPS-closure is used to predict the redistribution of energy for non-rotating fully-developed channel flows ($\Gamma_{yz} = \underline{e}_z \underline{e}_y : \nabla < \underline{u} >$). The URAPS-closure is also used to predict the components of the NR-stress in the region of zero intrinsic vorticity for fully-developed channel flows with spanwise rotation ($\underline{\Omega} = \Omega_x \underline{e}_x$).

DNS results for $\tilde{N}_F \equiv k \|\langle \underline{F} \rangle\| / \varepsilon$ are used to predict the URAPS-results. It is noteworthy that the normal components predicted by the URAPS-closure agree qualitatively with the DNS results for non-rotating flows: $R_{yy} < R_{xx} < R_{zz}$; and, for spanwise rotating flows, $R_{zz} < R_{xx} < R_{yy}$. This behavior is also indicated in the DNS results. Thus, the new algebraic closure shows that the Coriolis coupling with the fluctuating velocity field causes a transfer of energy from the longitudinal component of the fluctuating velocity field to the cross flow (or transverse) component of the fluctuating velocity field. This prediction supports the conclusion that the URAPS-closure has captured the essential relationship between the NR-stress and the hydrodynamic/kinematic operator $\underline{\underline{K}}$, as anticipated by the closure hypothesis expressed by Eq.(1.16).

Chapter 8 gives a summary discussion of salient conclusions related to the URAPS-closure and Chapter 9 identifies additional research needed before the URAPS-closure can be integrated into a CFD code.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In this chapter, direct numerical simulation (DNS) and experimental results related to the following canonical benchmark flows are reviewed: 1) rotating and non-rotating, fully-developed, turbulent channel flows (Sections 2.2 and 2.3); 2) rotating and non-rotating, homogeneous, turbulent shear flows (Section 2.4); and, 3) rotating and non-rotating, homogeneous turbulent decay (Section 2.5). Figure 2.1 illustrates the three benchmark flows used in this study. DNS results from the literature are used in Chapter 3 to guide the development of the URAPS-closure and in Chapter 4 to estimate the phenomenological closure coefficients introduced by the theory. In Chapters 5-7, DNS benchmark results are used to evaluate the predictions of the NR-stress based on the URAPS-closure. A critique of current algebraic closure models for the NR-stress is given in Section 2.6. The chapter concludes by restating the objectives of this research relative to the prior art.

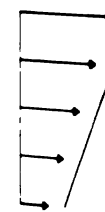
2.2 Non-Rotating Fully-Developed Channel Flows

RANS-Equation

DNS results for non-rotating, fully-developed, channel flows have been developed by Kim et al.(1987), Moser et al.(1999), Iwamoto et al. (2002), Alamo and Jimenez (2003), Alamo et al.(2004), Hoyas and Jimenez (2006) and many others (see, esp., Chapter 7 in Pope, 2000). The mean velocity has one component (i.e.,



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$$\frac{y}{\delta} =$$

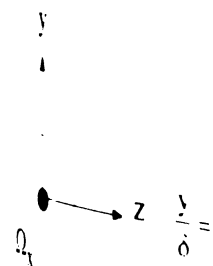
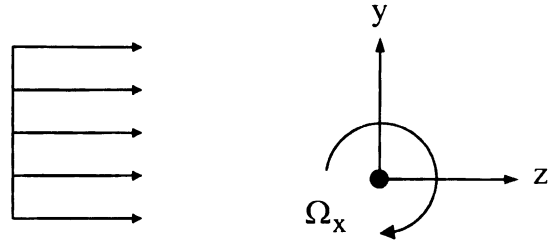
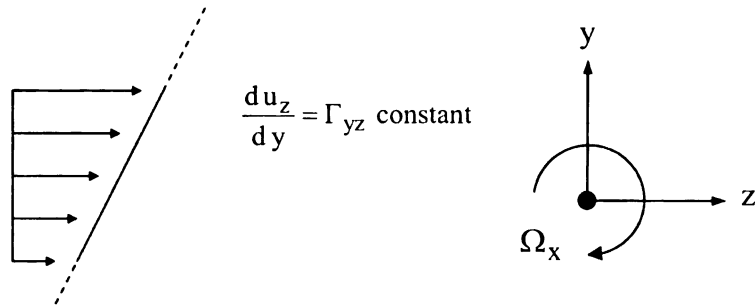


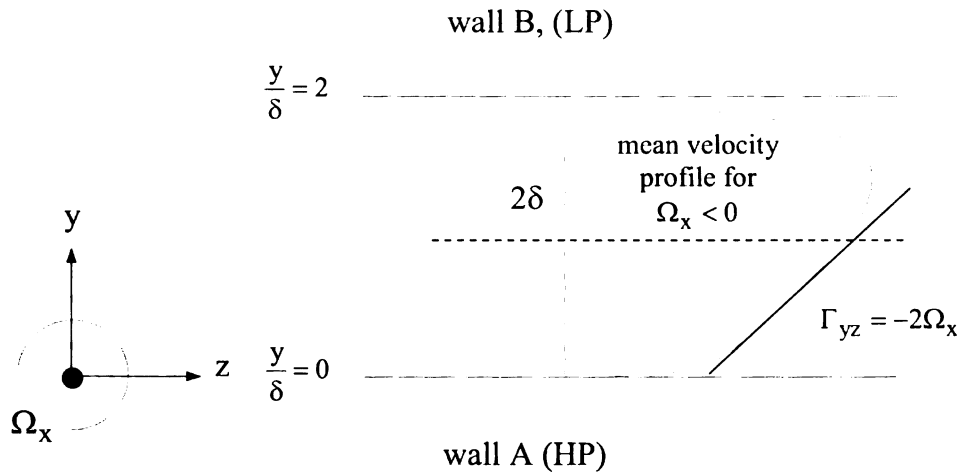
Fig. 21. Definitions of



Homogeneous decay with spanwise rotation



Homogeneous shear with spanwise rotation



Channel flow with spanwise rotation

Figure 2.1. Definitions of Benchmark Flows.

$u_2 = \langle u_2 \rangle (y/\delta_z)$ that d

Eq. (2.1). The mean press

Eq. (2.1), the symmetry

are located at $y = 0$

towards the solid fluid inte

$\hat{e}_1, \hat{e}_2$, and $\nabla \cdot \underline{u} = 1/y$

NAVStokes equation, defined by

$$-\frac{1}{\rho} \frac{\partial \langle p_D \rangle}{\partial y} = \frac{d}{dy} \left[-2k \mu \frac{d}{dy} \right]$$

$$-\frac{1}{\rho} \frac{\partial \langle p_D \rangle}{\partial z} = \frac{d}{dy} \left[\nu \frac{d}{dy} \right]$$

the above equations, $\langle p_D \rangle$

mean velocity and the mean

$$u_2(0) = \langle u_2 \rangle (2\delta) = 0$$

Eq. (2.1) implies that

$$\langle p_D \rangle(y, z) - \langle p_D \rangle(0, z)$$

Due to no-slip on so

$$R_{yy}(0) = R_{yy}(2\delta) = 0$$

the symmetry plane, max

Eq. (2.2) and Eq.

the coordinate y :

$\langle \underline{u} \rangle = \langle u_z \rangle(y) \underline{e}_z$) that depends only on the transverse (cross flow) coordinate y (see Figure 2.1). The mean pressure depends on y and z : $\langle p \rangle = \langle p \rangle(y, z)$. As indicated in Figure 2.1, the symmetry plane of the channel is located at $y = \delta$; the walls of the channel are located at $y = 0$ and 2δ . Mean momentum is transported across the flow towards the solid/fluid interfaces by viscous and turbulent stresses. With $\underline{\underline{\Omega}} = \underline{\underline{0}}$, $\underline{g} = g_z \underline{e}_z$, and $\nabla \langle \underline{u} \rangle = \Gamma_{yz}(y) \underline{e}_y \underline{e}_z$, the non-trivial components of the unclosed RANS-equation, defined by Eq. (1.4), are

$$0 = -\frac{1}{\rho} \frac{\partial \langle p_D \rangle}{\partial y} + \frac{d}{dy} [-2k R_{yy}] \quad , \quad 0 \leq y \leq 2\delta \quad (2.1)$$

$$0 = -\frac{1}{\rho} \frac{\partial \langle p_D \rangle}{\partial z} + \frac{d}{dy} \left[v \frac{d \langle u_z \rangle}{dy} - 2k R_{yz} \right] \quad , \quad 0 \leq y \leq 2\delta. \quad (2.2)$$

In the above equations, $\langle p_D \rangle(y, z) \equiv \langle p \rangle(y, z) - z \rho g_z$. Boundary conditions for the mean velocity and the mean pressure are

$$\langle u_z \rangle(0) = \langle u_z \rangle(2\delta) = 0 \quad , \quad -\frac{1}{\rho} \frac{\partial \langle p_D \rangle}{\partial z} = \text{constant} . \quad (2.3)$$

Eq. (2.1) implies that

$$\langle p_D \rangle(y, z) - \langle p_D \rangle(0, z) \equiv \Delta_{pD}(y) = -2\rho k(y) R_{yy}(y) \leq 0 \quad , \quad 0 \leq y \leq 2\delta. \quad (2.4)$$

Due to no-slip on solid/fluid interfaces and continuity, $k(0) = k(2\delta) = 0$ and $R_{yy}(0) = R_{yy}(2\delta) = 0$; therefore, Eq. (2.4) shows that $\langle p_D \rangle(0, z) = \langle p_D \rangle(2\delta, z)$. On

the symmetry plane, $\max |\Delta_{pD}| = +\rho \langle u_y' u_y' \rangle(\delta) \neq 0$.

Eq. (2.2) and Eq. (2.3) imply that the total stress is a linear function of the cross flow coordinate y :

$$\frac{d\langle u_x \rangle}{dy} - 2k R_{yz} = -\left\{ -\frac{1}{\rho} \right\}$$

at $y=0$ and 2δ , the shear

implies that

$$\left[\frac{1}{\rho} \frac{\partial \langle p \rangle}{\partial z} \right] 2\delta = \nu \frac{d \langle u_x \rangle}{dy}$$

The friction velocity u^* , defined

equivalently, to the average

2.2.5. DNS results

The DNS results

$$\Gamma_{yz} = \frac{d\langle u_x \rangle}{dy} = \Gamma_{yz}^+ (u^*)$$

say δ . Figure 2.2 shows the

nonlinear function of ξ :

$$\Gamma_{yz} - 2k^+ R_{yz} = \Gamma_{yz}^+ (0) - \xi$$

$$\frac{\partial \langle u_x \rangle}{\partial y} = \delta^+ (\equiv \delta u^* / \nu) \approx 3$$

$\Gamma_{yz}^+(0)=1$. It is noteworthy

that the important non-

Reynolds stress determines

total stress is anti-symmetric

$$\nu \frac{d \langle u_z \rangle}{dy} - 2k R_{yz} = - \left[- \frac{1}{\rho} \frac{\partial \langle p_D \rangle}{\partial z} \right] y + \nu \frac{d \langle u_z \rangle}{dy} \Big|_{y=0}. \quad (2.5)$$

For $y = 0$ and 2δ , the shear components of the Reynolds stress are zero; therefore, Eq.

(2.5) implies that

$$+ \left[- \frac{1}{\rho} \frac{\partial \langle p_D \rangle}{\partial z} \right] 2\delta = \nu \frac{d \langle u_z \rangle}{dy} \Big|_{y=0} - \nu \frac{d \langle u_z \rangle}{dy} \Big|_{y=2\delta} \equiv 2(u^*)^2 > 0. \quad (2.6)$$

The friction velocity u^* , defined by Eq. (2.6), is related to the axial pressure gradient or, equivalently, to the average wall shear stress.

Total Stress

The DNS results used herein are consistent with Eq. (2.5). With $\Gamma_{yz}^+ \equiv du^+ / dy^+ = \Gamma_{yz} \nu / (u^*)^2$, $\Omega_x^+ \equiv \Omega_x \nu / (u^*)^2$, $k^+ \equiv k / (u^*)^2$, $y^+ \equiv y u^* / \nu$, and $\xi \equiv y / \delta$, Figure 2.2 shows that the total dimensionless stress, defined as $\Gamma_{yz}^+ - 2k^+ R_{yz}$, is a linear function of ξ :

$$\Gamma_{yz}^+ - 2k^+ R_{yz} = \Gamma_{yz}^+(0) - \xi, \quad 0 \leq \xi \leq 1, \quad \forall \delta^+ > \delta_c^+, \quad \Omega_x^+ = 0. \quad (2.7)$$

For $Re^+ = \delta^+ (\equiv \delta u^* / \nu) \cong 300$ and $\Omega_x^+ = 0$, the DNS results imply that $u_b / u^* \cong 17$ and

$\Gamma_{yz}^+(0) = 1$. It is noteworthy that the viscous component of the total stress is

Quantitatively important near the solid/fluid interface (viscous sublayer) and that the **Reynolds** stress determines momentum transport in the outer region of the flow field. The **total** stress is anti-symmetric about the symmetry plane located at $\xi = 1$. These results are

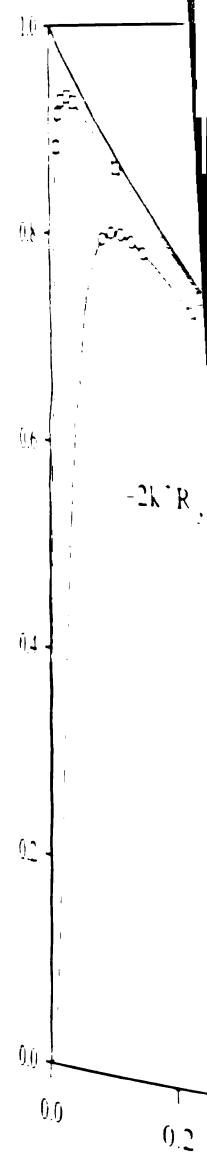


Fig. 22. Total Shear Flows

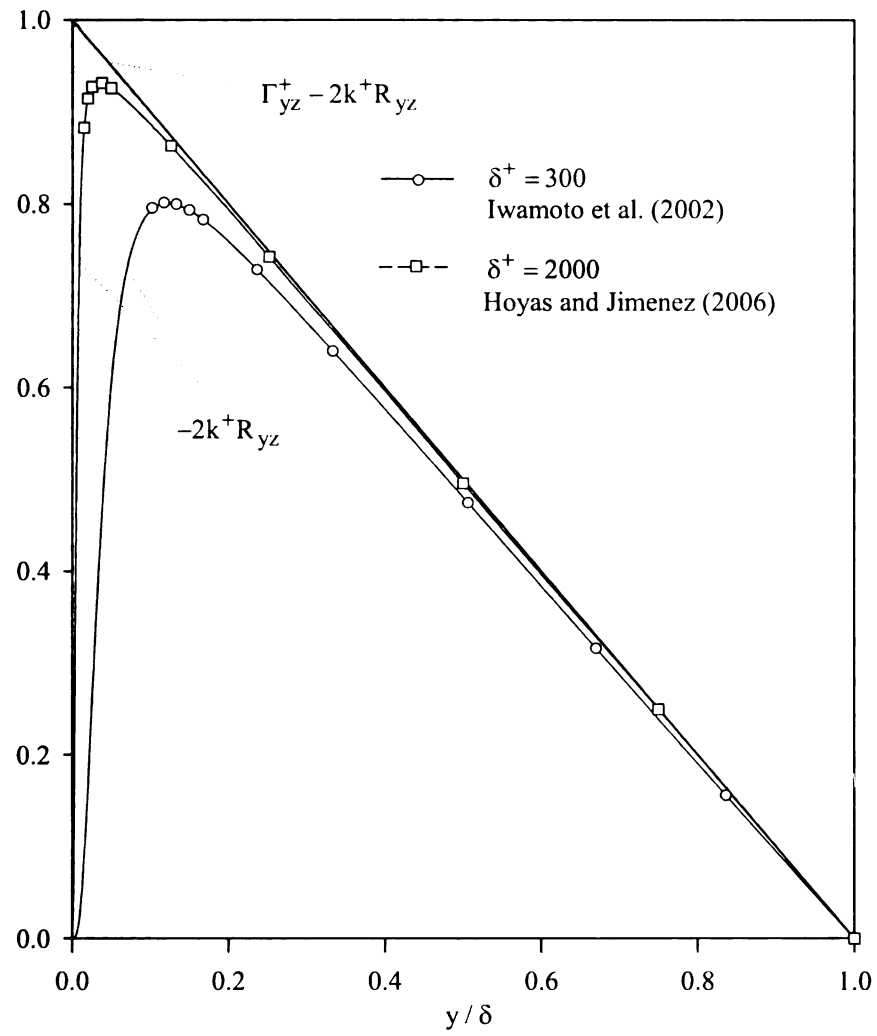


Figure 2.2. Total Shear Stress Profile for Non-Rotating, Fully-Developed, Channel Flows

constant with the 1938

angular duct (see p 165)

Flow State and Friction

Figure 2.3 shows that

lies all lie within the 2nd

is low, the NR-stress has

the coordinate y^* ($= y u^*$)

$$\underline{\underline{t}} = R_{xx} \underline{\underline{e}}_x \underline{\underline{e}}_x + R_{yy} \underline{\underline{e}}_y \underline{\underline{e}}_y + R_{zz} \underline{\underline{e}}_z \underline{\underline{e}}_z$$

Due to no slip at a solid frust

for $y^* = 0$ and $y^* = 2\delta$

components of the NR-stress

in Menin and Yaglom

$$R_{xx}(0) < R_{zz}(0) < 1, \quad R_{yy}(0) = 0$$

the quadratic form

surface is a planar ellipsoid

ellipsoid:

$$R_{xx}(\delta^*) = R_{yy}(\delta^*) < 1, \quad R_{zz}(\delta^*) = 0$$

DNS results show that

fully-developed, cl

R_{xx} and R_{zz} to R_{yy}

consistent with the 1938 experiments of H. Reichardt for turbulent flow of air in a rectangular duct (see p.165 of Bird et al., 2007).

Energy States and Anisotropic Invariants

Figure 2.3 shows that the energy states for non-rotating, fully-developed, channel flows all lie within the 2nd Sextet of the energy simplex: $0 < R_{yy} \leq R_{xx} < R_{zz} < 1$. For this flow, the NR-stress has three independent components that depend only on the cross flow coordinate y^+ ($\equiv y u^* / \nu$):

$$\underline{\underline{R}} = R_{xx} \underline{e}_x \underline{e}_x + R_{yy} \underline{e}_y \underline{e}_y + R_{zz} \underline{e}_z \underline{e}_z + R_{yz} \underline{e}_y \underline{e}_z + R_{zy} \underline{e}_z \underline{e}_y. \quad (2.8)$$

Due to no slip at a solid/fluid interface, all of the components of the Reynolds stress are zero for $y^+ = 0$ and $y^+ = 2\delta^+$. Continuity and the no-slip condition imply that the components of the NR-stress have the following properties at a solid/fluid interface (see p.279 in Monin and Yaglom, 1965, Weispfennig et al. 1999):

$$0 < R_{xx}(0) < R_{zz}(0) < 1, \quad R_{yy} \propto (y^+)^2 \text{ and } R_{yz} \propto y^+ \text{ as } y^+ \rightarrow 0 \text{ and } y^+ \rightarrow 2\delta^+. \quad (2.9)$$

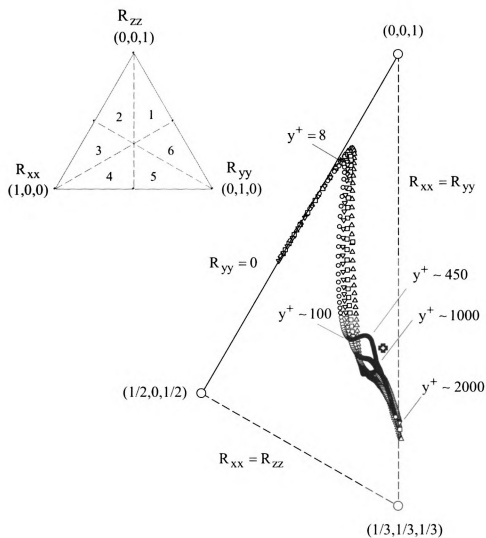
Thus, the quadratic form associated with the NR-stress (QNR-form) at a solid/fluid interface is a planar ellipse. On the symmetry plane at $y^+ = \delta^+$, the QNR-form is a **prolate** ellipsoid:

$$R_{xx}(\delta^+) = R_{yy}(\delta^+) < 1/3 < R_{zz}(\delta^+) \quad , \quad R_{yz}(\delta^+) = 0. \quad (2.10)$$

DNS results show that four qualitatively distinct energetic regions occur in a non-rotating, fully-developed, channel flow. Near the wall ($0 < y^+ < 8$), energy is transferred from R_{yy} and R_{zz} to R_{xx} as $y^+ \rightarrow 0$. A highly anisotropic region occurs for

R₁₃
11111

Page 23



(DNS Results)

- $\triangle$ $\delta^+ = 395$ Moser et al., 1999
- $\square$ $\delta^+ = 550$ Alamo and Jimenez, 2003
- ∇ $\delta^+ = 950$ Alamo et al., 2004
- $\circ$ $\delta^+ = 2000$ Hoyas and Jimenez, 2006

(Experiments, AHS)

- $\oplus$ Tavoularis and Karnik, 1989

Figure 2.3. Energy States for Non-Rotating, Fully-Developed, Channel Flows

$\epsilon_{ij} \leq 30^\circ$ w

process as

$\mu(\gamma) \leq 0.7$

$R_p \geq 1.3$ (8)

$R_{\perp}(0) = R_{\perp}$

$\epsilon(\gamma) = \text{grad}$

$R_{\perp}(0) = R_{\perp}$

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$8 < y^+ < 30$ with $\max(R_{zz}) \cong 0.85$ and $\max(\Gamma_{yz}^+) \cong 18$. For $30 < y^+ < 100$, $\Gamma_{yz}^+(y^+)$ decreases as y^+ increases and energy is transferred from R_{zz} to R_{xx} and R_{yy} . For $100 < y^+ < \delta^+$, energy continues to be transferred from R_{zz} to R_{yy} as $y^+ \rightarrow \delta^+$, but $R_{xx} \cong 0.3$ (see Figure 2.3). At the solid/fluid interface, $R_{yy}(0) = 0$, $R_{xx}(0) \cong 0.3$, $R_{zz}(0) = 1 - R_{xx} \cong 0.7$, and $R_{yz}(0) = 0$. At the symmetry plane (i.e., $y^+ = \delta^+$), the mean velocity gradient is zero and the components of the NR-stress are $R_{xx}(\delta^+) = R_{yy}(\delta^+) \cong 0.28$, $R_{zz}(\delta^+) = 1 - 2R_{xx}(\delta^+) \cong 0.44$, and $R_{yz}(\delta^+) = 0$. This weak anisotropic phenomenon on the symmetry plane is consistent with the classical 1938 experiments of H. Reichardt for turbulent flow of air in a rectangular duct (see p.165, Bird et al., 2007). A similar phenomenon is observed in axisymmetric pipe flows. This anisotropic energy state is contrary to the hypothesis expressed by Eq. (1.16) and may be related to the anisotropy in the normal components of the fluctuating pressure gradient correlation for wall bounded flows (i.e., $\langle \nabla p' \nabla p' \rangle$ for $y^+ = \delta^+$), or to an incomplete development of the flow field. It is noteworthy, however, that the fraction of energy in the spanwise component of the fluctuating velocity near the symmetry plane is comparable to the fraction of energy in the spanwise fluctuating velocity deep within the viscous sublayer (i.e., $R_{xx} \cong 0.3$). Clearly, the transverse redistribution of kinetic energy among the three components of the fluctuating velocity is significant.

Figure 2.4 shows the DNS results for the second and third invariants of the anisotropic component of the NR-stress for non-rotating, fully-developed channel flows. The anisotropic states, defined by II_b and III_b , are all to the right of the anisotropic states



Fig. 24. Anisotropy

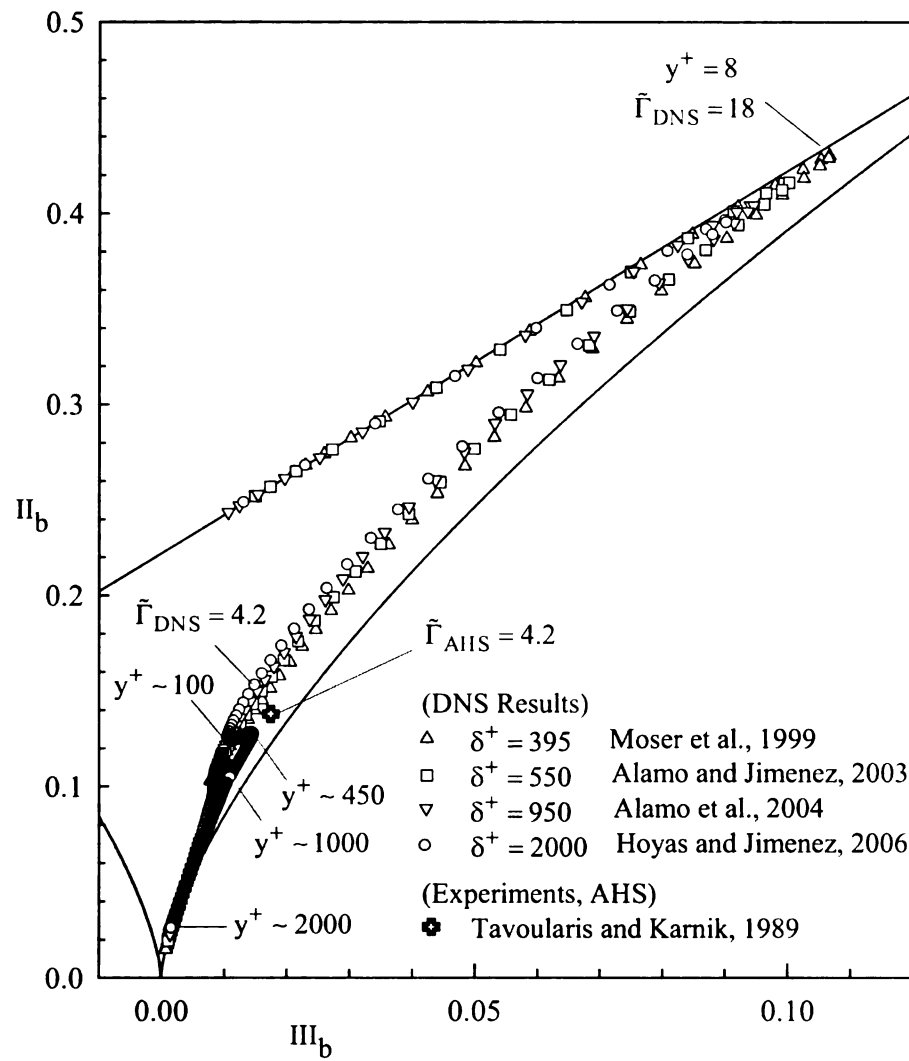


Figure 2.4. Anisotropic States for Non-Rotating, Fully-Developed, Channel Flows

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Appendix I

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for which $\text{III}_b = 0$. These prolate-like states are parameterized by $y^+ (\equiv y u^* / \nu)$ and $\tilde{\Gamma}_{\text{DNS}} (\equiv \Gamma_{yz} k / \varepsilon \equiv \Gamma_{yz}^+ k^+ / \varepsilon^+; \Gamma_{yz}^+ \equiv du^+ / dy^+)$. In the outer region (i.e., $100 < y^+ < \delta^+$), the anisotropic states are associated with prolate-like ellipsoidal quadratic forms. Figure 2.4 shows that the anisotropic state characteristic of non-rotating, asymptotic homogeneous shear (see Tavoularis and Karnik, 1989; and, Section 2.4 below) is located near the edge of the equilibrium core region in a channel flow (i.e., $\tilde{\Gamma}_{\text{DNS}} \cong 4.2$ for $y^+ \cong 100$). This observation partly motivates the use of non-rotating asymptotic homogeneous shear as a benchmark flow for model calibrations (see Chapter 4).

Across the core region (i.e., $30 < y^+ < 100$), the quadratic forms associated with the NR-stress (QNR-forms) change from a prolate-like structure to an elongated ellipsoidal structure as the turbulent kinetic energy increases near the wall. Within the viscous sublayer, viscous transport of momentum causes the elongated ellipsoidal QNR-form at $y^+ = 8$ to collapse to a planar elliptical form as $y^+ \rightarrow 0$. The DNS results used to construct Figures 2.3 and 2.4 are summarized in tabular form in Table I.1 and Table I.2 in Appendix I.

Turbulent Kinetic Energy and Turbulent Dissipation

Figure 2.5 shows the spatial distribution (inner and outer scaling) of turbulent *kinetic* energy (i.e., $k^+ \equiv k / (u^*)^2$) for δ^+ equal to 300 and 2,000. Figure 2.6 shows the *spatial* distribution (inner and outer scaling) of turbulent dissipation (i.e., $\varepsilon^+ \equiv \varepsilon \nu / (u^*)^4$)

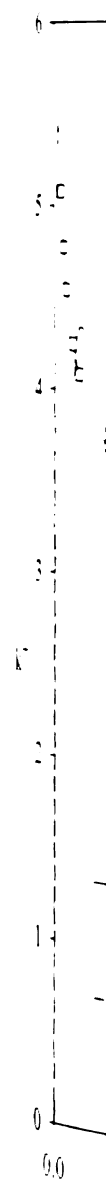


Fig. 25. Kin

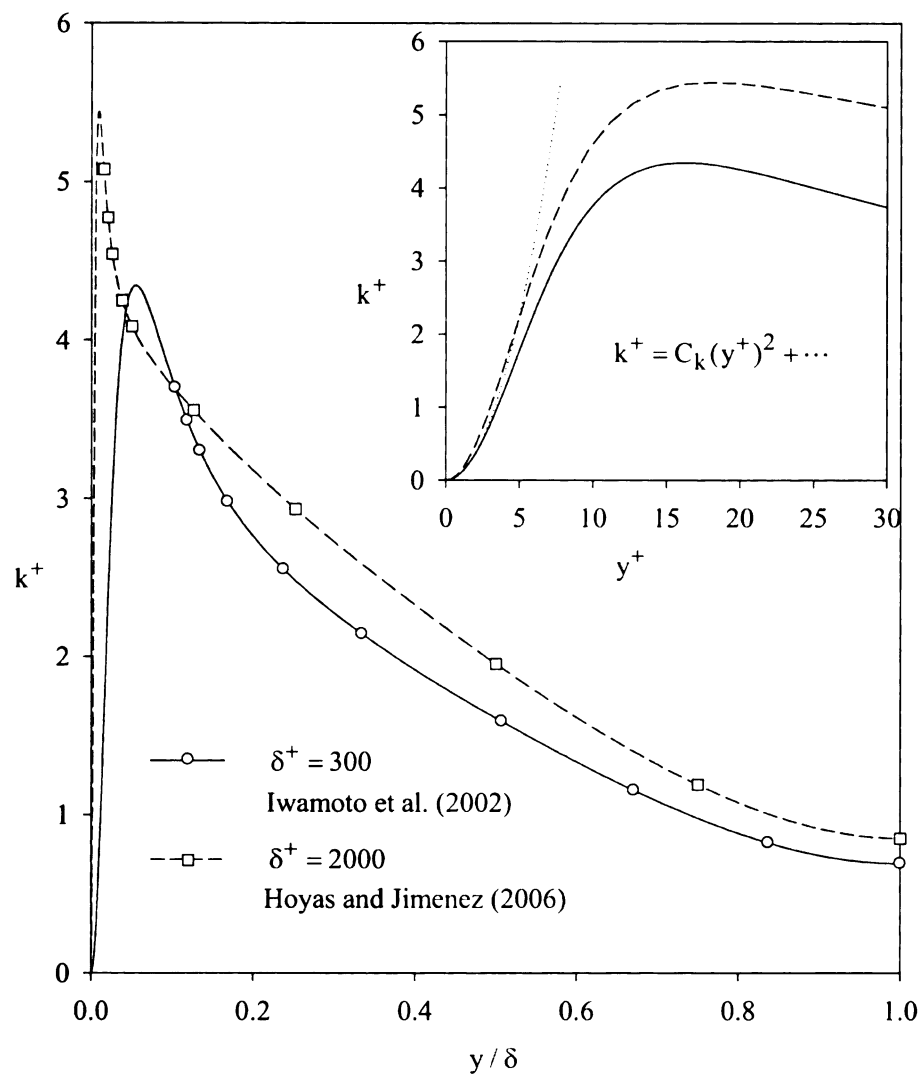


Figure 2.5. Kinetic Energy for Non-Rotating, Fully-Developed, Channel Flows

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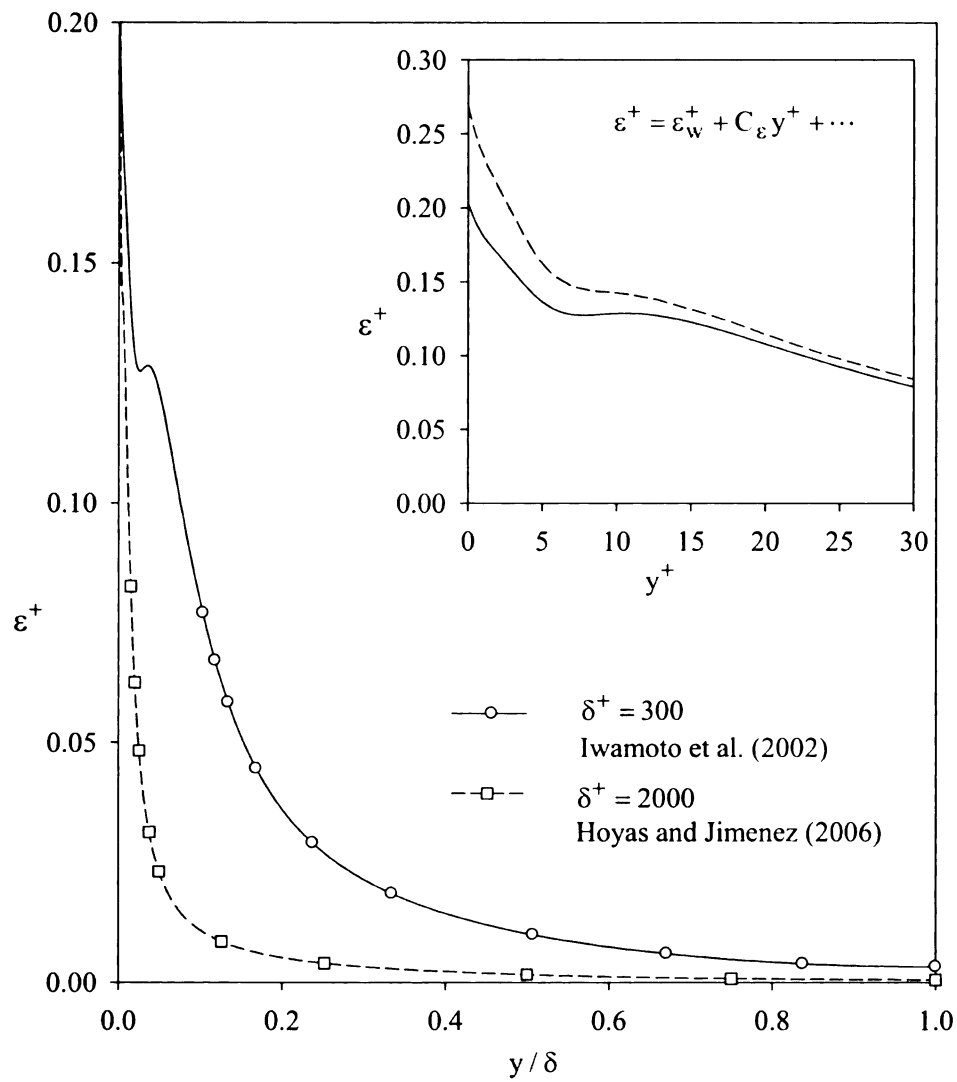


Figure 2.6. Dissipation for Non-Rotating, Fully-Developed, Channel Flows.

of δ^* equal to

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Table 12 of A

Continued

$\hat{\beta} = \hat{\beta}_0 + C_1 \gamma$

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for δ^+ equal to 300 and 2,000. The DNS results for k^+ and ε^+ at different values of y^+ over the half width of the channel (i.e., $0 < y^+ < \delta^+$) are listed in Table I.1a and Table I.2a of Appendix I.

Continuity and the no-slip conditions imply that $k^+ = C_k(y^+)^2 + \dots$ and $\varepsilon^+ = \varepsilon_w^+ + C_\varepsilon y^+ + \dots$ near the wall, which is consistent with the DNS results shown on the inset of Figures 2.5 and 2.6. ($C_k \cong 0.09$ and $C_\varepsilon \cong -0.03$) As y^+ increases, k^+ increases to $\max(k^+) \cong 4.3$ and 5.4 respectively at $y^+ \cong 16$ and 18 for $\delta^+ = 300$ and $2,000$. However, as expected, the peak kinetic energy occurs at slightly different values of $\xi \equiv y/\delta$. For $\delta^+ = 300$, $k^+ \cong 3.7$ and $\varepsilon^+ \cong 0.077$ at $y^+ = 30$ or, equivalently, at $\xi = 0.1$. For $\delta^+ = 2,000$, $k^+ \cong 5.1$ and $\varepsilon^+ \cong 0.08$ at $y^+ = 30$ or, equivalently, at $\xi = 0.015$. Towards the center of the channel, $k^+ \cong 1$ and $\varepsilon^+ \cong 0.01$. The maximum value of ε^+ occurs at the wall ($\varepsilon_w^+ \cong 0.2$ and $\varepsilon_w^+ \cong 0.27$ respectively for the two δ^+ cases). For $y^+ \sim 10$, ε^+ reaches a plateau (see inset of Figure 2.6) and then decreases monotonically to a positive minimum at the center of the channel. The plateau phenomenon related to the dissipation occurs in the region where the turbulent kinetic energy is a maximum.

Figure 2.7 compares the ratio of the local turbulent time scale k/ε and the local *mean* field time scale (i.e., $\tilde{\Gamma}_{\text{DNS}} \equiv \Gamma_{yz} k/\varepsilon$) for $0 < \xi < 1$ at two different Reynolds numbers ($\text{Re}^+ = \delta^+ = 300$ and $2,000$). The inset on Figure 2.7 shows the behavior of the

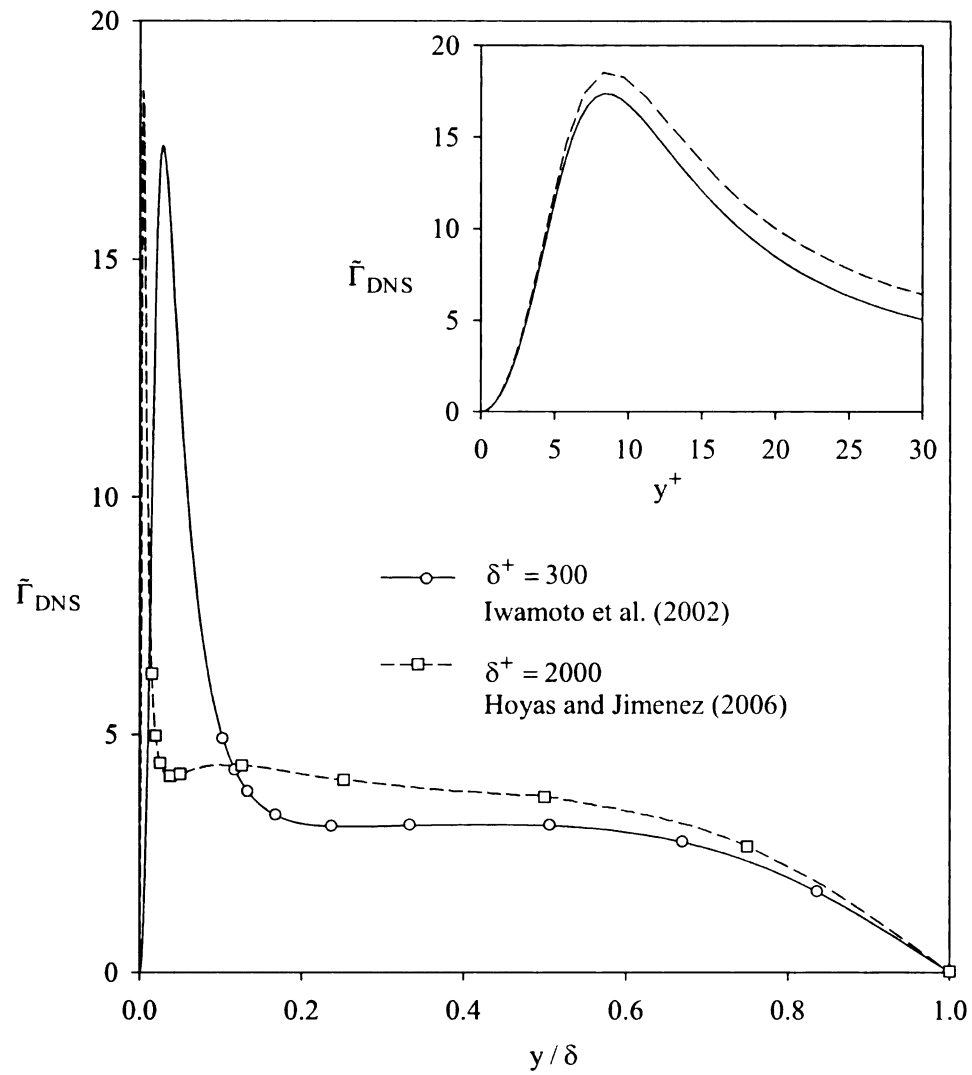


Figure 2.7. Mean Dimensionless Strain Rate $\tilde{\Gamma}_{DNS}$ for Non-Rotating, Fully-Developed Channel Flows.

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dimensionless shear rate group near the solid/fluid interface: $\tilde{\Gamma}_{\text{DNS}} \propto k^+ \sim (y^+)^2$. For $0 < y^+ < 10$, $\tilde{\Gamma}_{\text{DNS}}$ increases and peaks at $y^+ \cong 10$, where $\tilde{\Gamma}_{\text{DNS}} \cong 18$. For $y^+ > 10$, $\tilde{\Gamma}_{\text{DNS}}$ decreases. At $y^+ = 30$, $\tilde{\Gamma}_{\text{DNS}} \cong 4.9$ for $\delta^+ = 300$ and $\tilde{\Gamma}_{\text{DNS}} \cong 6.3$ for $\delta^+ = 2,000$. For $0.2 < \xi < 0.5$, $\tilde{\Gamma}_{\text{DNS}} \cong 4$. In this region, the local dissipation and the local production of turbulence kinetic energy approximately balance one another (see p. 281 in Pope, 2000). At the center of the channel, $\tilde{\Gamma}_{\text{DNS}} = 0$ inasmuch as the mean velocity gradient is zero due to symmetry. Table 2.1 tabulates the transverse variation of the dimensionless shear group $\tilde{\Gamma}_{\text{DNS}}$ and the components of the NR-stress for $30 \leq y^+ \leq \delta^+$. This information is used in Chapter 6 to evaluate predictions of the NR-stress for non-rotating fully-developed channel flow for $0 \leq \tilde{\Gamma} \leq 6$.

2.3 Rotating Fully-Developed Channel Flows

DNS results for fully-developed channel flows with spanwise rotation have been developed by Wu and Kasagi (2002, 2004), Alvelius (1999), and Grundestam et al. (2008). Simulations with and without spanwise rotation are performed with the same constant axial pressure drop or, equivalently, for the same average wall shear stress. Consequently, the friction velocity, defined by Eq. (2.18) below, is the same for all simulations. Comprehensive low-order statistical properties for the mean velocity, the components of the NR-stress, the turbulent kinetic energy, and the turbulent dissipation *are available* for $\delta^+ \equiv \delta u^* / \nu \cong 296$ and $\Omega_x^+ (\equiv \Omega_x \nu / (u^*)^2) = -0.0042$. The results of Wu *and Kasagi* (2004) are used in Chapter 7 to benchmark the predictions of the URAPS-

Table 2.1 Cross Flow Variation of $\tilde{\Gamma}_{\text{DNS}}$ and the Components of the NR-Stress for Non-Rotating, Fully-Developed, Channel Flow: (a) $\delta^+ \cong 300, \Omega_x^+ = 0$ (Iwamoto et al. 2002) ; (b) $\delta^+ \cong 2000, \Omega_x^+ = 0$ (Hoyas and Jimenez, 2006).

(a)

$[y^+]$	y/δ	$\tilde{\Gamma}_{\text{DNS}}$	R_{xx}	R_{yy}	R_{zz}	R_{yz}
30	0.10	4.90	0.21	0.09	0.70	-0.11
35	0.12	4.24	0.22	0.11	0.67	-0.11
40	0.13	3.79	0.23	0.12	0.65	-0.12
45	0.15	3.49	0.24	0.14	0.62	-0.13
50	0.17	3.30	0.25	0.15	0.60	-0.13
70	0.24	3.06	0.27	0.18	0.55	-0.14
100	0.33	3.09	0.28	0.19	0.53	-0.15
150	0.51	3.08	0.28	0.21	0.51	-0.15
200	0.67	2.73	0.27	0.23	0.50	-0.14
250	0.84	1.69	0.27	0.26	0.47	-0.09
300	1.00	0.00	0.27	0.28	0.45	0.00

(b)

$[y^+]$	y/δ	$\tilde{\Gamma}_{\text{DNS}}$	R_{xx}	R_{yy}	R_{zz}	R_{yz}
30	0.015	6.28	0.23	0.08	0.69	-0.09
40	0.02	4.98	0.25	0.10	0.65	-0.10
50	0.03	4.40	0.27	0.12	0.61	-0.10
75	0.04	4.13	0.27	0.14	0.59	-0.11
100	0.05	4.17	0.27	0.15	0.58	-0.11
250	0.13	4.35	0.25	0.16	0.59	-0.12
500	0.25	4.05	0.24	0.18	0.58	-0.13
1000	0.50	3.69	0.25	0.20	0.55	-0.13
1500	0.75	2.65	0.26	0.23	0.51	-0.10
2000	1.00	0.03	0.27	0.26	0.47	0.00

Page: 10

10

10

For span

$\langle p \rangle = \langle p \rangle_{\text{avg}}$

$p = 1$ and the so

relation is $\Omega =$

regular velocity

$\langle p \rangle = \nabla \cdot \langle u \rangle$

and component

can be as

$\langle u \rangle = \langle u \rangle_{\text{avg}}$

$\langle u \rangle = \frac{\langle u \rangle_{\text{avg}}}{\rho}$

In the abo

$\langle u \rangle = \langle u \rangle_{\text{avg}}$

boundary condition

$\langle u \rangle = \langle u \rangle_{\text{avg}}$

closure. Hereinafter, a nominal value of $2\delta^+ = 600$ is used to refer to the DNS results for $2\delta^+ = 592$.

RANS-Equation

For spanwise rotation of a fully-developed channel flow, $\langle \underline{u} \rangle = \langle u_z \rangle (y) \underline{e}_z$ and $\langle p \rangle = \langle p \rangle (y, z)$. As illustrated by Figure 2.1 above, the symmetry plane is located at $y = \delta$ and the solid/fluid interfaces are located at $y = 0$ and 2δ . The angular velocity of the frame is $\underline{\Omega} = \Omega_x \underline{e}_x$, and the rotation operator is $\underline{\underline{\Omega}} = \underline{\underline{\varepsilon}} \cdot \underline{\Omega} = \Omega_x (\underline{e}_y \underline{e}_z - \underline{e}_z \underline{e}_y)$. The angular velocity of the frame is co-linear with the mean vorticity $\langle \underline{w} \rangle \equiv \nabla \wedge \langle \underline{u} \rangle = \langle w_x \rangle \underline{e}_x$. With $\underline{g} = g_z \underline{e}_z$ and $\nabla \langle \underline{u} \rangle = \Gamma_{yz}(y) \underline{e}_y \underline{e}_z$, the two non-trivial components of the unclosed RANS-equation (see Eq. (1.4)) for this flow can be written as

$$-2 \langle u_z \rangle \Omega_x = -\frac{\partial}{\partial y} \left[\frac{\langle p_D \rangle}{\rho} \right] + \frac{d}{dy} [-2k R_{yy}] \quad , \quad 0 \leq y \leq 2\delta \quad (2.11)$$

$$0 = -\frac{\partial}{\partial z} \left[\frac{\langle p_D \rangle}{\rho} \right] + \frac{d}{dy} \left[v \frac{d \langle u_z \rangle}{dy} - 2k R_{yz} \right] \quad , \quad 0 \leq y \leq 2\delta \quad (2.12)$$

In the above equations, the dynamic pressure $\langle p_D \rangle (y, z)$ is defined as

$$\langle p_D \rangle (y, z) \equiv \langle p \rangle (y, z) - \rho \frac{(y^2 + z^2) \Omega_x^2}{2} - z \rho g_z \quad (2.13)$$

Boundary conditions for the mean velocity and the mean axial pressure gradient are

$$\langle u_z \rangle (0) = \langle u_z \rangle (2\delta) = 0 \quad , \quad -\frac{\partial \langle p_D \rangle}{\partial z} = \text{constant} \quad (2.14)$$

The model

Key idea

Quadratic

page 100

transverse

derivation

Fig. 10.1

The model

Key idea

transverse

$\Omega = 0$

Fig. 10.1

Fig. 10.1

$\Omega < 0$

The no-slip condition at solid/fluid interfaces implies that all the components of the Reynolds stress are zero on the channel walls. Therefore, for spanwise rotation, the Coriolis force, $2\rho \langle \underline{u} \rangle \wedge \underline{\Omega}$, acts in the transverse (cross flow) direction and, thereby, plays an explicit role in the force balance expressed by Eq. (2.11). As a consequence, the transverse component of the Reynolds stress and the Coriolis stress cause a cross flow variation in the dynamic pressure:

$$\begin{aligned} \langle p_D \rangle(y, z) - \langle p_D \rangle(0, z) &\equiv \Delta_{PD}(y) \\ &= -2\rho k(y) R_{yy}(y) + 2\rho \Omega_x \int_0^y \langle \hat{u}_z \rangle(\hat{y}) d\hat{y} \end{aligned} \quad (2.15)$$

The no-slip condition at a solid/fluid interface implies that the components of the Reynolds stress are zero at $y = 0$ and 2δ ; therefore, unlike the non-rotating case, the dynamic pressure difference between the two solid/fluid interfaces is non-zero for $\Omega_x \neq 0$:

$$\Delta_{PD}(2\delta) = +2\rho \Omega_x \int_0^{2\delta} \langle u_z \rangle(\hat{y}) d\hat{y} \equiv +4\delta \rho \Omega_x u_b = [4\Omega_x^+ \delta^+ u^* / u_b] \rho u_b^2. \quad (2.16)$$

Eq. (2.16) shows that $\langle p_D \rangle(2\delta, z)$ is smaller than $\langle p_D \rangle(0, z)$ if $\langle u_z \rangle(y) \geq 0$ and $\Omega_x < 0$. For this configuration, the solid/fluid interface at $y = 2\delta$ is the low pressure (LP-) interface and the solid/fluid interface at $y = 0$ is the high pressure (HP-) interface.

For $\delta^+ = 300$ and $\Omega_x^+ \cong -0.0042$, the simulations predict that the bulk average velocity

equals $u_b \cong 18 u^*$ and the pressure difference across the channel is

$$\Delta_{PD}(2\delta) = -0.275 \rho u_b^2.$$

Eq. (2.12) and E.

pressure coordinate y^*

$$\frac{d\omega_{yz}}{dy} - 2k R_{yz} = -[-$$

For $y=0$ and 2δ , the shear

$$\left[\frac{1}{\rho} \frac{\partial \tau_{yz}}{\partial z} \right] 2\delta = \nu \frac{d\omega_{yz}}{dy} \Big|_{y=0}^{y=2\delta}$$

As with the no-rotation case

the total pressure gradient

$$\bar{U} = \sqrt{(\tau_{xy})_{yp}^2 + (\tau_{yz})_{yp}^2} / \rho$$

Total Shear Stress

The DNS results of

at 2000 are consistent

$$\bar{U} \equiv k(u^*)^2, \text{ and } y^* \equiv y/\delta$$

near function of $\xi \equiv y/\delta$

$$\frac{d\omega_{yz}}{dy} - 2k^* R_{yz} = + \Gamma_{yz}^*$$

for $\delta^* = 300$ and $\Omega_{\lambda}^* \equiv$

$\bar{U}_0/\delta_0 \approx 1.38$. Figure 2.8

Eq. (2.12) and Eq. (2.14) imply that the total stress is a linear function of the transverse coordinate y :

$$\nu \frac{d \langle u_z \rangle}{dy} - 2k R_{yz} = - \left[-\frac{1}{\rho} \frac{\partial \langle p_D \rangle}{\partial z} \right] y + \nu \frac{d \langle u_z \rangle}{dy} \Big|_{y=0}. \quad (2.17)$$

For $y = 0$ and 2δ , the shear components of the Reynolds stress are zero; therefore,

$$\begin{aligned} + \left[-\frac{1}{\rho} \frac{\partial \langle p_D \rangle}{\partial z} \right] 2\delta &= \nu \frac{d \langle u_z \rangle}{dy} \Big|_{y=0} - \nu \frac{d \langle u_z \rangle}{dy} \Big|_{y=2\delta} \\ &\equiv \frac{(\tau_w)_{HP} + (\tau_w)_{LP}}{\rho} = 2(u^*)^2 \end{aligned} \quad (2.18)$$

As with the no-rotation case, the friction velocity u^* for spanwise rotation is related to the axial pressure gradient or, equivalently, to the average wall shear stress (i.e.

$$u^* = \sqrt{[(\tau_w)_{HP} + (\tau_w)_{LP}]/(2\rho)} = \left[-\frac{\delta}{\rho} \frac{\partial \langle p_D \rangle}{\partial z} \right]^{1/2}.$$

Total Shear Stress

The DNS results of Wu and Kasagi (2004) for $\delta^+ (\equiv \delta u^* / \nu = Re^+)$ equal to 300 and 2,000 are consistent with Eq. (2.17). For example, with $\Gamma_{yz}^+ \equiv du^+ / dy^+$, $k^+ \equiv k / (u^*)^2$, and $y^+ \equiv y u^* / \nu$, Figure 2.8 shows that $\Gamma_{yz}^+ - 2k^+ R_{yz}$ is approximately a linear function of $\xi \equiv y / \delta$ for $0 \leq \xi \leq 2$:

$$\Gamma_{yz}^+(\xi) - 2k^+ R_{yz} = +\Gamma_{yz}^+(0) - \xi, \quad 0 \leq \xi \leq \delta^+, \quad \forall \delta^+ > \delta_c^+, \quad -\infty < \Omega_x^+ < +\infty. \quad (2.19)$$

For $\delta^+ = 300$ and $\Omega_x^+ \cong -0.0042$, the DNS results predict that $u_b / u^* \cong 18$ and

$\Gamma_{yz}^+(0) \cong 1.38$. Figure 2.8 also illustrates that the viscous component of the total shear

1.5

1.0

0.5

0.0

0.5

0.0

2.8

Total

$\delta \approx$

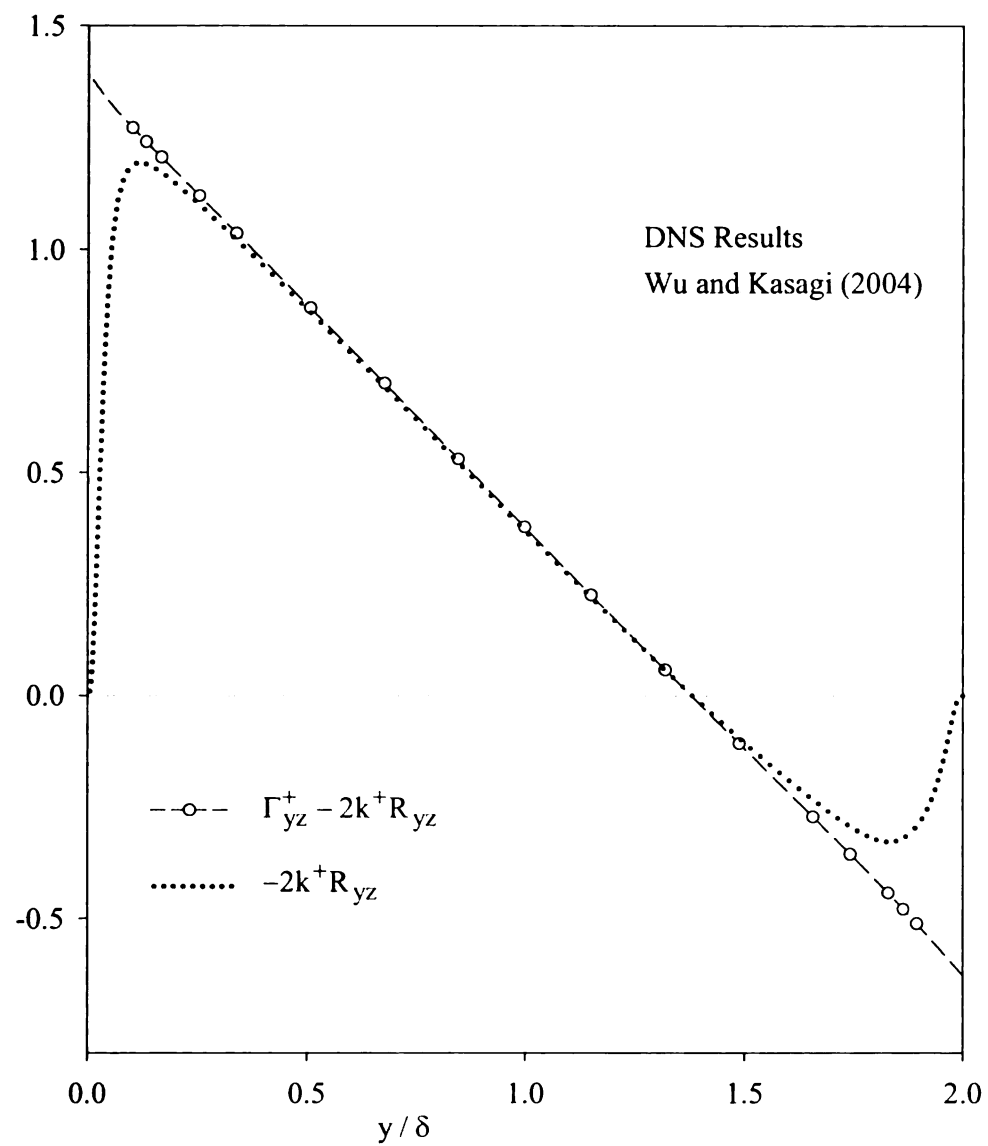


Figure 2.8 Total Mean Shear Stress Profile Across the Rotating Channel for $\delta^+ \cong 300$ and $\Omega_x^+ \cong -0.0042$.

stress is quantitatively in

examines transport of

(17) (or, equivalently,

(18) for no rotation, the

stress, albeit linear in

$y=\delta$ (or, $\xi=1$) for Ω^2

shows that the total shear

$y=\delta$ for no rotation

stress is directly influen

rotation for the devel

Stress, Mean Vorticity

As indicated

$\bar{u} \equiv \langle u_z \rangle > \bar{u}^*$ is n

works on the low pres

which is consistent w

$\xi=2$) is less than

$\bar{\Gamma}_{yz}(0) > \bar{\Gamma}_{yz}(2)$.

The mean veler

equivalently, the mean

field (also, see, p.

stress is quantitatively important near the solid/fluid interface and that the Reynolds stress determines transport of momentum in the outer region of the flow field. Although Eq. (2.17) (or, equivalently, Eq. (2.19)) for spanwise rotation is formally the same as Eq. (2.5) for no rotation, the DNS results summarized by Figure 2.8 show that the total shear stress, albeit linear in the transverse coordinate, is no longer anti-symmetric about $y = \delta$ (or, $\xi = 1$) for $\Omega_x^2 > 0$ (i.e., $\Gamma_{yz}^+(0) \cong 1.38$ and $\Gamma_{yz}^+(2) \cong -0.627$). Figure 2.8 also shows that the total shear stress is zero at $y_0 (\equiv \Gamma_{yz}^+(0)\delta) = 1.38\delta$ for $\Omega_x^+ \cong -0.0042$; and $y_0 = \delta$ for no rotation inasmuch as $\Gamma_{yz}^+(0) = 1$. Eq. (2.19) implies that the Reynolds stress is directly influenced by the Coriolis force. This phenomenon provides ample motivation for the development of the URAPS-closure in Chapter 3.

Absolute Mean Vorticity

As indicated by Figure 2.9, the transverse (cross flow) variation of $u^+ (\equiv \langle u_z \rangle / u^*)$ is not symmetric about the symmetry plane. The maximum velocity occurs on the low pressure side of the flow field in the vicinity of $y^+ = 415$ ($\xi \cong 1.38$), which is consistent with the result that the wall shear stress at the LP-interface (i.e., $\xi = 2$) is less than the wall shear stress at the HP- interface (i.e., $\xi = 0$), i.e., $|\Gamma_{yz}^+(0)| > |\Gamma_{yz}^+(2)|$.

The mean velocity profile shows that the mean *intrinsic* vorticity operator (or, equivalently, the mean absolute vorticity) is approximately zero over a finite region of the flow field (also, see, p. 617 in Piquet, 1999; Hamba, 2006; and Grundestam et al., 2008).

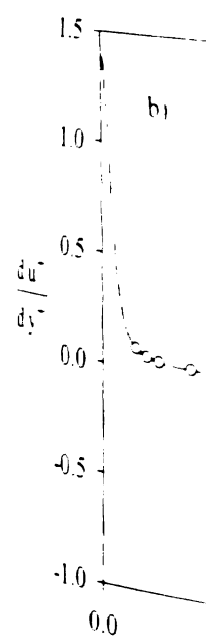
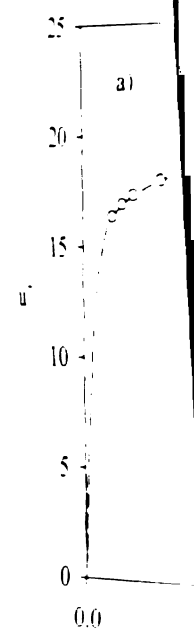


Figure 2.9 Mean Vel
 $\Omega_x^+ \cong -0.0$

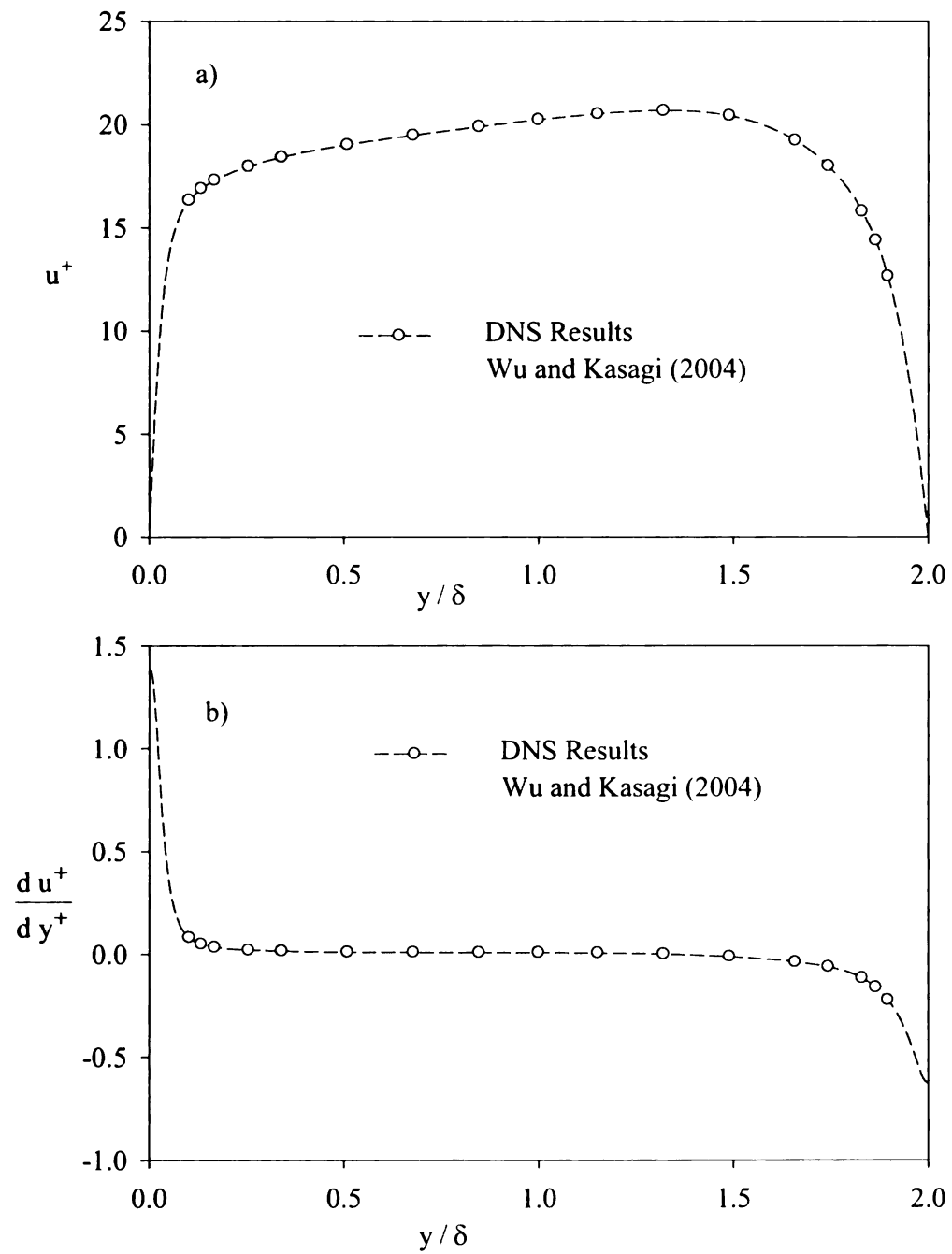


Figure 2.9 Mean Velocity and the Mean Velocity Gradient for $\delta^+ \cong 300$ and $\Omega_x^+ \cong -0.0042$.

for spanwise rotation

$$\underline{\underline{\Omega}} \equiv \underline{\underline{W}} - \underline{\underline{\Omega}} \equiv \underline{\underline{0}}$$

for $\delta^+ = 300$ and $\Omega_\lambda^+ \cong -$

finite region where the

$$\frac{du^+}{d\eta^+} = \frac{1}{\delta^+} \frac{du^+}{d\eta^+} \cong -2\Omega_\lambda^+$$

Free States and Transitions

Similar to the no

depend only on the

solid fluid interfaces

$$\eta y^+ = 0 \text{ and } y^+ = 2\delta^+$$

continuity and no-slip in

characteristics:

$$0 < R_{\lambda\lambda}(0) < R_{zz}(0) < 1$$

$$0 < R_{\lambda\lambda}(2\delta^+) < R_{zz}(2\delta^+)$$

for $\delta^+ \cong 300$ and $\Omega_\lambda^+ \cong$

the HP-wall; and, $R_{\lambda\lambda}$

quadratic forms associated

typical at the HP- and

For spanwise rotation, the intrinsic mean vorticity operator is

$$\langle \underline{\tilde{W}} \rangle \equiv \langle \underline{W} \rangle + \underline{\Omega} \equiv \underline{\varepsilon} \cdot \left(\frac{1}{2} \langle \underline{w} \rangle + \underline{\Omega} \right) = \left(\frac{1}{2} \frac{d \langle u_z \rangle}{dy} + \Omega_x \right) (\underline{e}_y \underline{e}_z - \underline{e}_z \underline{e}_y). \quad (2.20)$$

For $\delta^+ = 300$ and $\Omega_x^+ \cong -0.0042$, the DNS results (see Figure 2.9) show that there exist a finite region where the absolute mean vorticity is approximately zero:

$$\frac{du^+}{dy^+} = \frac{1}{\delta^+} \frac{du^+}{d\xi} \cong -2\Omega_x^+ \cong +0.0084, \quad 0 < \xi^{\min} \cong 0.5 < \xi < \xi^{\max} \cong 1 < 2 \quad (2.21)$$

Energy States and Anisotropic Invariants

Similar to the no rotation case, the NR-stress has three independent components that depend only on the transverse coordinate $y^+ (\equiv y u^* / \nu)$. Due to the no slip condition at solid/fluid interfaces, all of the components of the Reynolds stress are zero for $y^+ = 0$ and $y^+ = 2\delta^+$. Deep in the viscous sublayers near the HP-wall and LP-wall, continuity and no-slip imply that the components of the NR-stress have the following characteristics:

$$0 < R_{xx}(0) < R_{zz}(0) < 1, \quad R_{yy} \propto (y^+)^2, \quad R_{yz} \propto y^+, \quad y^+ \rightarrow 0, \quad (2.22)$$

$$0 < R_{xx}(2\delta^+) < R_{zz}(2\delta^+) < 1, \quad R_{yy} \propto (2\delta^+ - y^+)^2, \quad (2.23)$$

$$R_{yz} \propto (2\delta^+ - y^+), \quad y^+ \rightarrow 2\delta^+.$$

For $\delta^+ \cong 300$ and $\Omega_x^+ \cong -0.0042$, Figure 2.10 shows that $R_{xx}(0) \cong 0.3$ and $R_{zz}(0) \cong 0.7$ at the HP-wall; and, $R_{xx}(600) \cong 0.2$ and $R_{zz}(600) \cong 0.8$ at the LP-wall. Thus, the quadratic forms associated with the NR-stress (QNR-form) are planar anisotropic (elliptical) at the HP- and LP-walls.

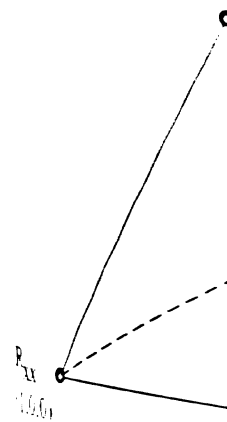


Figure 2.10 Energy St
and $\Omega_{\lambda}^* \cong$

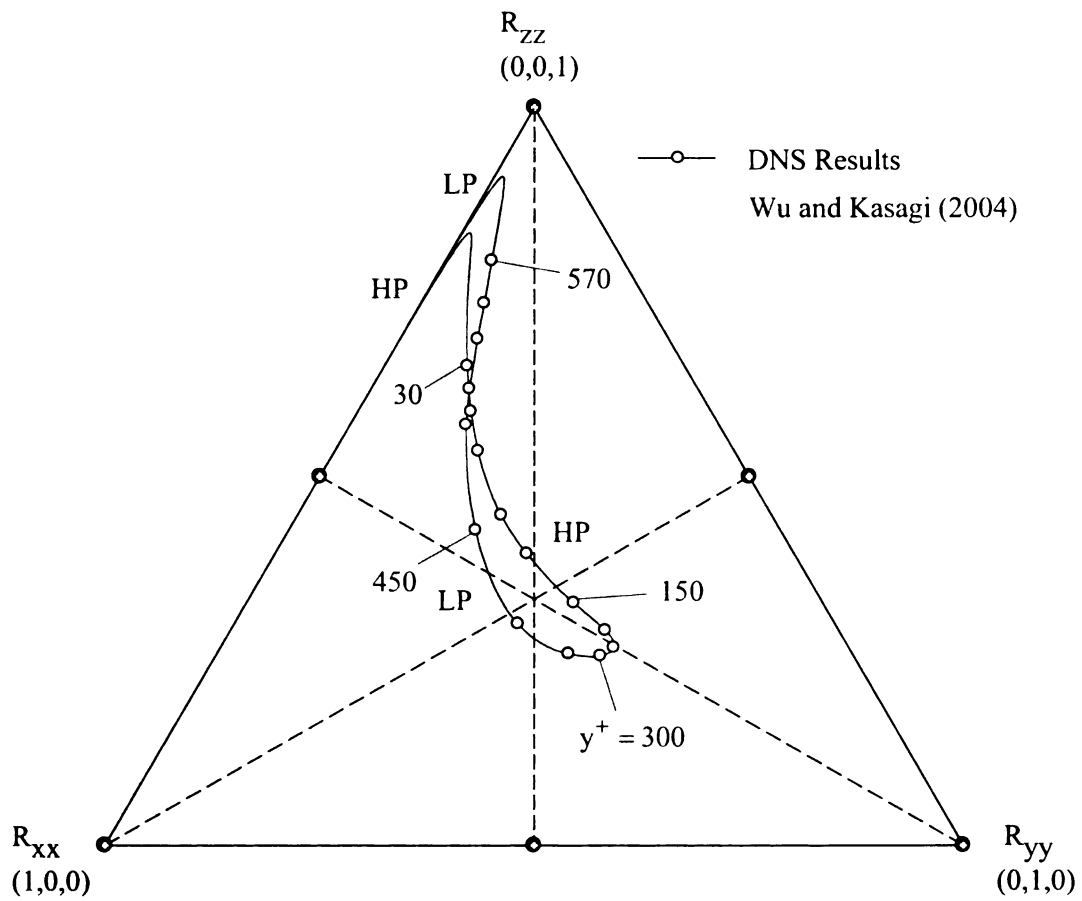


Figure 2.10 Energy States for Rotating, Fully-Developed, Channel Flow ($\delta^+ \cong 300$ and $\Omega_x^+ \cong -0.0042$).

For $\delta^* \approx 300$ and

$\bar{y} \approx 10$ near the HP-wall

rotation case (see Fig

energy simplex. On the

$\bar{y} \approx 1$, the energy state

$R_{xx}(300) = 0.26$. At $\bar{y} \approx$

are $R_{yz}(300) = -0.135$

are at $\bar{y} \approx 415$. The ma

and $R_{yz}(415) \approx 0$. As mo

at $\delta^* \approx 415$ are low

$R_{xx}(415) = 0.37$, $R_{yy}(415)$

Figure 2.10 shows

components of the Re

$(\bar{y} \approx 415)$ and the I

near the HP-wall within

viscous sublayer ($595 <$

$\bar{y} \rightarrow 0$ and 600, respec

$\bar{y} \approx 30$ and $570 <$

are $R_{zz} \approx 0.8$ on the H

For $\delta^+ \cong 300$ and $\Omega_x^+ \cong -0.0042$, Figure 2.10 shows that $\max(R_{zz}) \cong 0.8$ at $y^+ \cong 10$ near the HP-wall and $\max(R_{zz}) \cong 0.9$ at $y^+ \cong 590$ near the LP-wall. Unlike the no-rotation case (see Figure 2.3), the energy states are distributed in all six sextets of the energy simplex. On the symmetry plane of the rotating channel (i.e., $y^+ = \delta^+ \cong 300$, or $\xi = 1$), the energy state is in the 5th Sextet: $R_{xx}(300) = 0.29$, $R_{yy}(300) = 0.45$, and $R_{zz}(300) = 0.26$. At $y^+ \cong 300$, the cross correlation component of the NR-stress is non-zero: $R_{yz}(300) = -0.135$. As noted above, the total shear stress (i.e., $\Gamma_{yz}^+ - 2k^+ R_{yz}$) is zero at $y^+ \cong 415$. The mean velocity profile (see Figure 2.9) shows that $\Gamma_{yz}^+(415) \cong 0$ and $R_{yz}(415) \cong 0$. As indicated by Figure 2.10, the normal components of the NR-stress at $\delta^+ \cong 415$ are located in the 4th Sextet of the energy simplex: $R_{xx}(415) = 0.37$, $R_{yy}(415) = 0.29$, and $R_{zz}(415) = 0.34$.

Figure 2.10 shows that the energy is distributed among the three normal components of the Reynolds stress differently on the HP-side of the channel ($0 \leq y^+ \leq 415$) and the LP-side of the channel ($415 \leq y^+ \leq 600$). As indicated above, near the HP-wall within the viscous sublayer ($0 < y^+ < 5$) and the LP-wall within the viscous sublayer ($595 < y^+ < 600$) energy is transferred from R_{yy} and R_{zz} to R_{xx} as $y^+ \rightarrow 0$ and 600, respectively (2nd Sextet of the energy simplex). Within the regions $5 < y^+ < 30$ and $570 < y^+ < 595$, highly anisotropic QNR-forms develop with $\max(R_{zz}) \cong 0.8$ on the HP-side of the channel and $\max(R_{zz}) \cong 0.9$ on the LP-side of the

parallel (2nd Sextet) of

$\omega_{cy} < 5^\circ$, energy

HP-wall and LP-wall in

ing the normal compo

$\omega_{cy} \approx 300$. Figure 2.10

distributed over the 5°

in this region. This

stress in this region of th

Figure 2.11 show

$\bar{\omega} \approx 300$ and $\Omega_{\lambda}^* \approx -0$

shows some similarities

stress is zero, the QN

$\bar{\omega} \approx 300$ whereas for

$\bar{\omega} \approx 415$. Figure 2.4 s

and Figure 2.11 show

$\Pi_1, \Pi_2 \approx (-0.03, -0)$

kinetic energy peaks (i.

and $\Pi_2 \approx -0.06$ (see

kinetic energy peaks

$\Pi_1 \approx -0.44$, and $\Pi_2 \approx$

channel (2nd Sextet of energy simplex). Within the regions $30 < y^+ < 150$ and $440 < y^+ < 570$, energy is transferred from R_{zz} to R_{xx} and R_{yy} as the distance from the HP-wall and LP-wall increases. Within the region $150 < y^+ < 440$, energy is redistributed among the normal components R_{xx} , R_{yy} , and R_{zz} with the result that $\max(R_{yy}) \cong 0.45$ at $y^+ \cong 300$. Figure 2.10 shows that the energy states in the region $150 < y^+ < 300$ are distributed over the 5th and 6th -Sextets. The mean absolute vorticity is approximately zero in this region. This result is used in Chapter 7 to predict the components of the NR-stress in this region of the flow field.

Figure 2.11 shows the anisotropic states associated with the NR-stress for $\delta^+ \cong 300$ and $\Omega_x^+ \cong -0.0042$. A comparison with Figure 2.4 for the no-rotation case shows some similarities and some differences. Near the plane where the total mean shear stress is zero, the QNR-form for the no-rotation case is nearly prolate elliptical for $y^+ \cong 300$ whereas for the rotation case the QNR-form is nearly oblate elliptical for $y^+ \cong 415$. Figure 2.4 shows that for all y^+ , the invariant III_b is positive. On the other hand, Figure 2.11 shows that for $y^+ \cong 440$ on the LP-side the invariant III_b is negative: $(\text{II}_b, \text{III}_b) \cong (+0.03, -0.001)$. It is noteworthy that on the HP-side of the channel, the kinetic energy peaks (i.e., $\max(k) \cong 5.9$) at $y^+ \cong 15$ for which $\tilde{\Gamma}_{\text{DNS}} = 23$, $\text{II}_b \cong +0.33$, and $\text{III}_b \cong +0.06$ (see Figure 2.12 below). However, on the LP-side of the channel, the kinetic energy peaks (i.e., $\max(k) \cong 2.7$) at $y^+ \cong 582$ for which $\tilde{\Gamma}_{\text{DNS}} = -33$, $\text{II}_b \cong +0.44$, and $\text{III}_b \cong +0.11$ (see Figure 2.12 below). In the case of no rotation, the

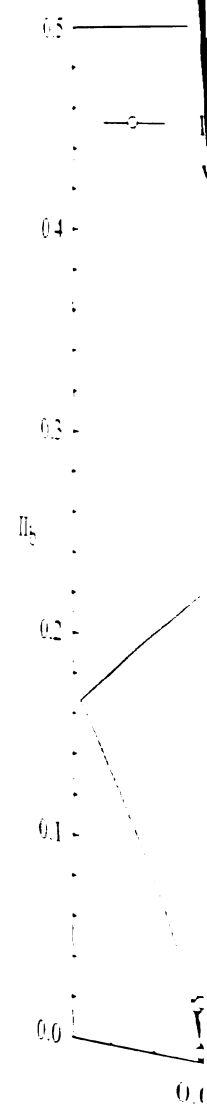


Fig. 2.11 Anisotropy
 $(\delta^* \cong 30^\circ)$

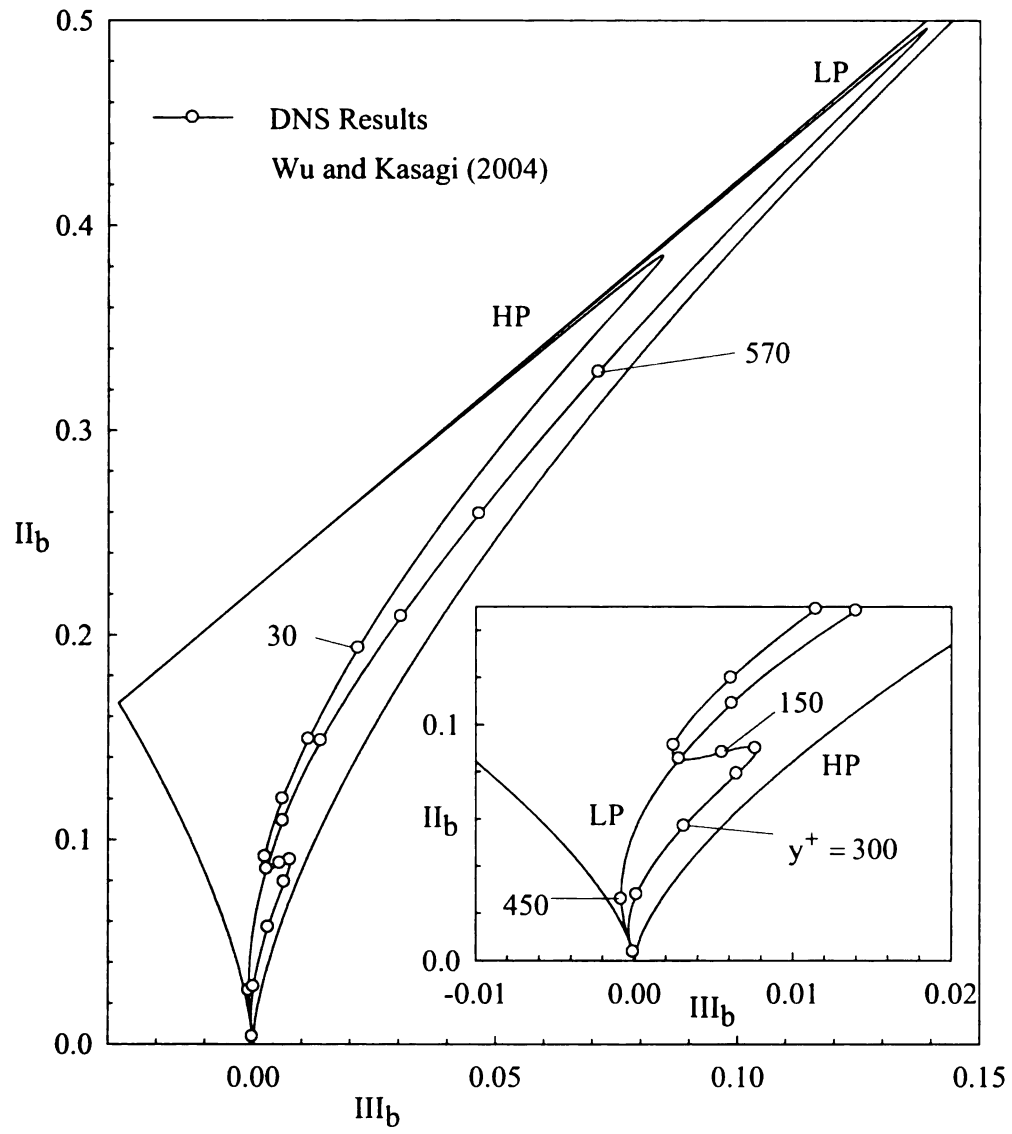


Figure 2.11 Anisotropic States for Rotating, Fully-Developed, Channel Flows
($\delta^+ \cong 300$ and $\Omega_x^+ \cong -0.0042$)

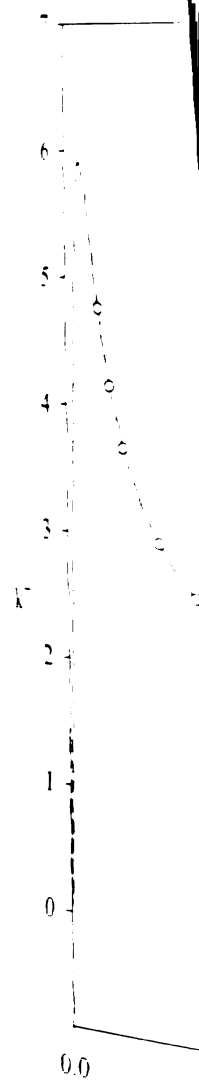


Figure 2.12 Kinetic
and Ω^2

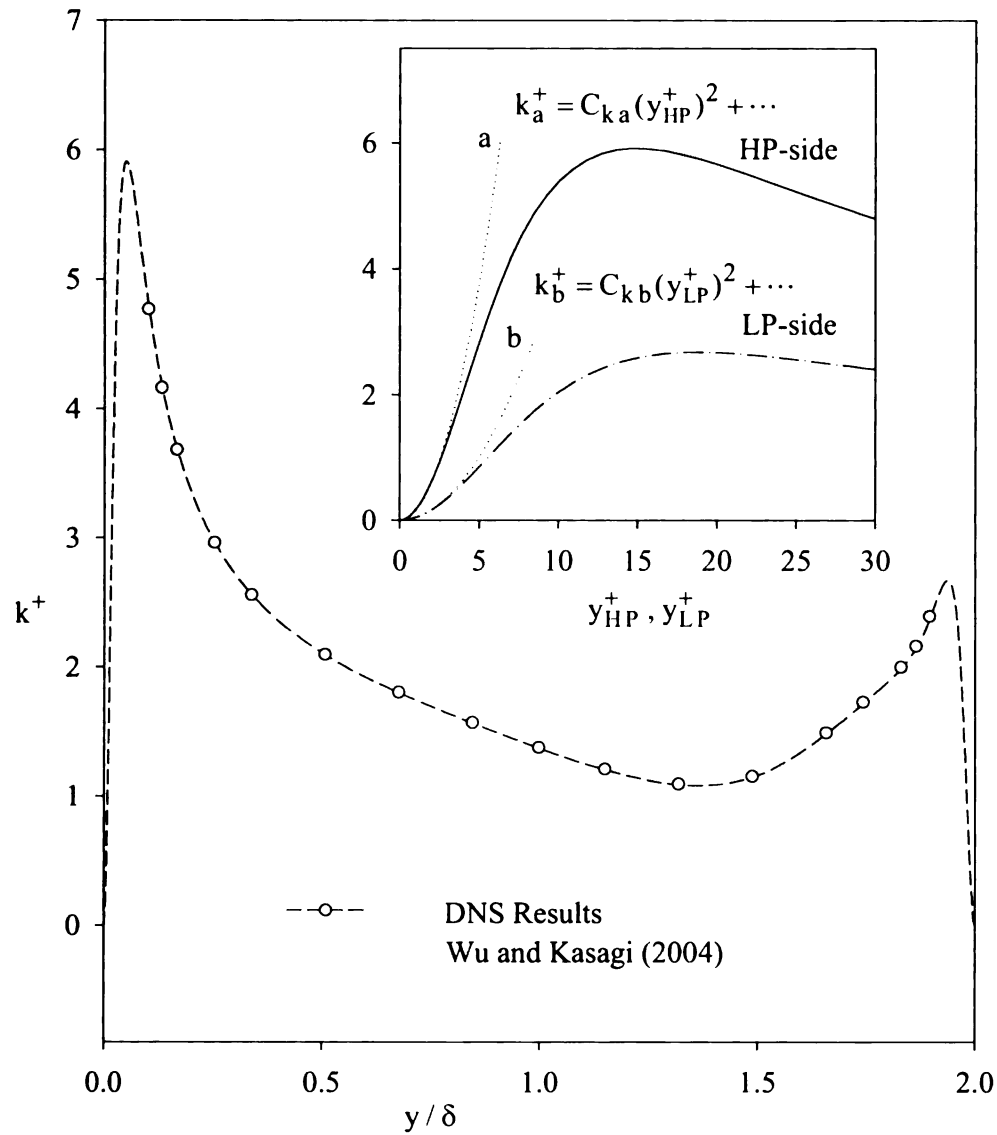


Figure 2.12 Kinetic Energy for Rotating, Fully-Developed, Channel Flows ($\delta^+ \cong 300$ and $\Omega_x^+ \cong -0.0042$).

Article number 2017

1, $\beta = 0.38$, α

Figure 1

Figure

the turbulent

for $\Omega_1 \approx 0.1$

results by using

HP- and LP-

values of y^*

Center

$K = C_k(y^*)$

$\Omega_{HP} = 0.1$

$\Omega_{LP} = 0.0$

$\Omega_{LP} \approx 5$

2.13 shows

where the tu

the total

and 2.13 sho

kinetic energy peaks (i.e., $\max(k) \cong 4.3$) at $y^+ \cong 16$ and $y^+ \cong 584$ for which $\bar{\Gamma}_{\text{DNS}} = 12$, $\Pi_b \cong +0.38$, and $\text{III}_b \cong +0.09$ (see Figure 2.4 above).

Turbulent Kinetic Energy and Turbulent Dissipation

Figures 2.12 and 2.13 show the spatial distributions (inner and outer scaling) of the turbulent kinetic energy, $k^+ \equiv k/(u^*)^2$, and the turbulent dissipation, $\varepsilon^+ \equiv \varepsilon \nu/(u^*)^4$, for $\Omega_x^+ \cong -0.0042$ for $\delta^+ = 300$. The results near the HP- and the LP-walls are shown as insets by using inner scaling parameters. The inset scale refers to the distance from the HP- and LP-walls in viscous wall units. The DNS results for k^+ and ε^+ at different values of y^+ are tabulated in Table I.4 of Appendix I.

Continuity and the no-slip conditions at a solid/fluid interface imply that $k^+ = C_k (y^+)^2 + \dots$, and $\varepsilon^+ = \varepsilon_w^+ + C_\varepsilon y^+ + \dots$. At the HP-wall, $(C_k)_{\text{HP}} = 0.15$, $(\varepsilon_w^+)_{\text{HP}} = 0.18$, and $(C_\varepsilon)_{\text{HP}} \cong -0.02$; and, at the LP-wall, $(C_k)_{\text{LP}} = 0.04$, $(\varepsilon_w^+)_{\text{LP}} = 0.05$, and $(C_\varepsilon)_{\text{LP}} \cong -0.008$. Near the HP-wall at $y^+ \cong 15$ ($\xi = 0.05$), $\max(k^+) \cong 5.9$; and, near the LP-wall at $y^+ \cong 582$ ($\xi = 1.94$), $\max(k^+) \cong 2.7$. Figure 2.13 shows (see inset) that the turbulent dissipation profile has a plateau in the region where the turbulent kinetic energy has a local maximum. As noted above (see Figure 2.8), the total shear stress is zero at $\delta^+ = 415$ ($\xi = 1.38$). At this position, Figures 2.12 and 2.13 show that $\min(k^+) \cong 1.1$ and $\min(\varepsilon^+) \cong 0.0025$.

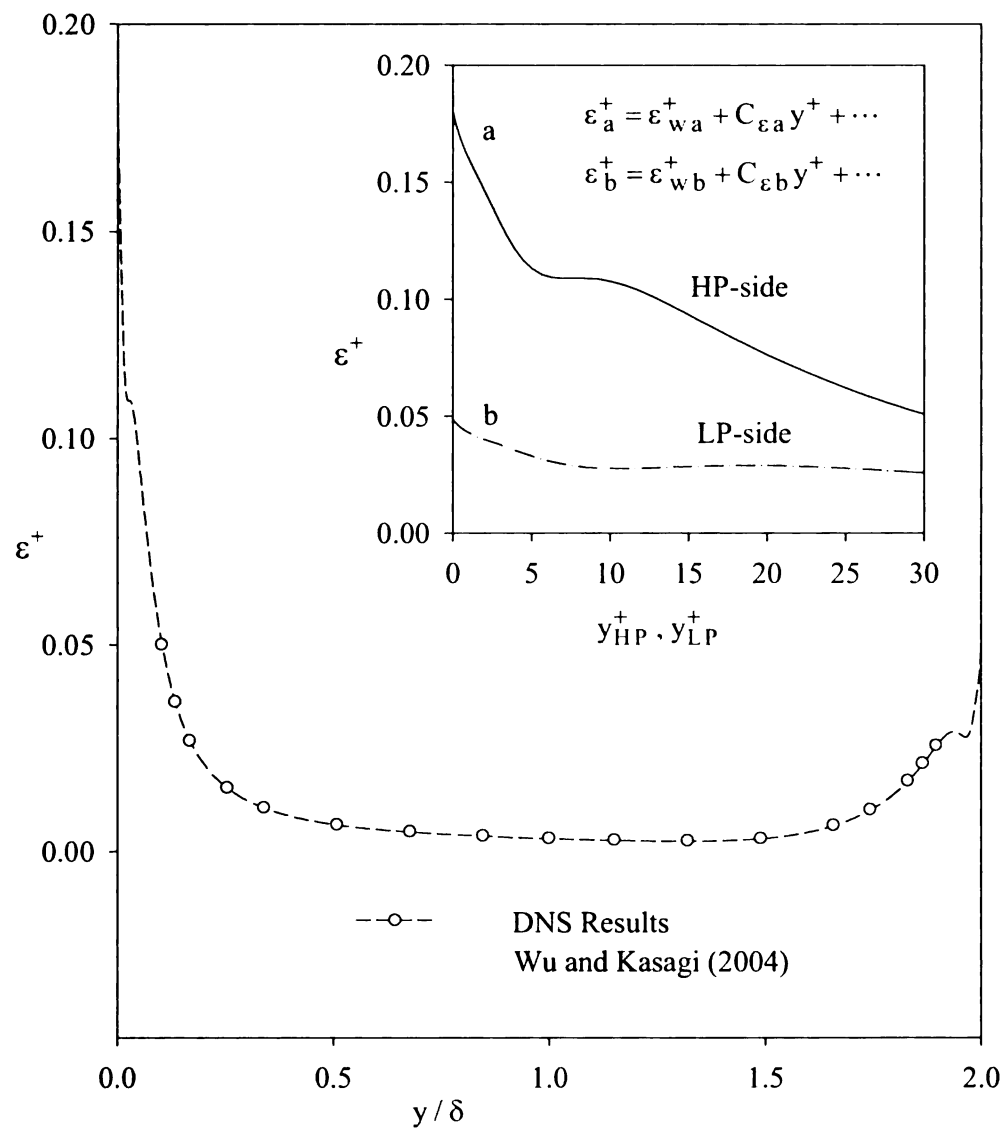


Figure 2.13 Dissipation for Rotating, Fully-Developed, Channel Flows ($\delta^+ \cong 300$ and $\Omega_x^+ \cong -0.0042$).

For δ

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$$\hat{\Gamma}_{\text{DYS}} = \Gamma_{\text{yz}} \frac{1}{2}$$

acts on the

in HP-wall

the HP- and

$$\Gamma_{\text{yz}}(3\delta) =$$

For

$$\min \hat{\Gamma}_{\text{DYS}} \geq$$

$$\hat{\Gamma}_{\text{DYS}}(2) =$$

side of the fl

mean absolute

$$1.34 < \xi < 2.$$

components

and in Chap

for rotating fi

Product Pr

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approximatio

For $\delta^+ = 300$ and $\Omega_x^+ \cong -0.0042$, Figures 2.14 and 2.15 show the behavior of the turbulent time scale k/ε made dimensionless with the mean velocity gradient, $\tilde{\Gamma}_{\text{DNS}} \equiv \Gamma_{yz} k/\varepsilon$, and with the angular velocity of the frame, $\tilde{\Omega}_{\text{DNS}} \equiv \Omega_x k/\varepsilon$. The insets on the figures use inner scaling to highlight the behavior of $\tilde{\Gamma}_{\text{DNS}}$ and $\tilde{\Omega}_{\text{DNS}}$ near the HP-wall ($0 < y^+ < 30$) and the LP-wall ($570 < y^+ < 600$). The group $\tilde{\Gamma}_{\text{DNS}}$ is zero at the HP- and LP-walls because $k(0) = k(2\delta) = 0$. It is also zero at $\xi \cong 1.38$ because $\Gamma_{yz}(1.38\delta) = 0$.

For $y^+ \cong 12$ ($\xi \cong 0.04$), $\max(\tilde{\Gamma}_{\text{DNS}}) \cong +40$. At $y^+ \cong 588$ ($\xi \cong 1.96$), $\min(\tilde{\Gamma}_{\text{DNS}}) \cong -40$. Near the HP-wall, the mean absolute vorticity, defined as $(\tilde{\Gamma}_{\text{DNS}}/2) + \tilde{\Omega}_{\text{DNS}}$, is positive. For $0.5 < y/\delta < 1$, the mean absolute vorticity on the HP-side of the flow field is approximately zero. And, on the LP-side of the flow field, the mean absolute vorticity is negative inasmuch as $\tilde{\Gamma}_{\text{DNS}}$ and $\tilde{\Omega}_{\text{DNS}}$ are both negative for $1.34 < \xi < 2$. Table 2.2 tabulates the cross flow variation of $\tilde{\Gamma}_{\text{DNS}}$, $\tilde{\Omega}_{\text{DNS}}$, and the components of the NR-stress for $0.1 \leq \xi^+ \leq 1.9$ ($30 \leq y^+ \leq 570$). This information is used in Chapter 7 to evaluate predictions of the NR-stress based on the URAPS-closure for rotating fully-developed channel flow.

Turbulent Production/Dissipation Ratio

Production is approximately balanced by dissipation in the outer region of the non-rotating channel. This feature is often applied in modeling for obtaining algebraic approximation for the Reynolds stress transport equation (this will be further discussed in

60 -
40 -
20 -
0 -
-20 -
-40 -
-60 -
0

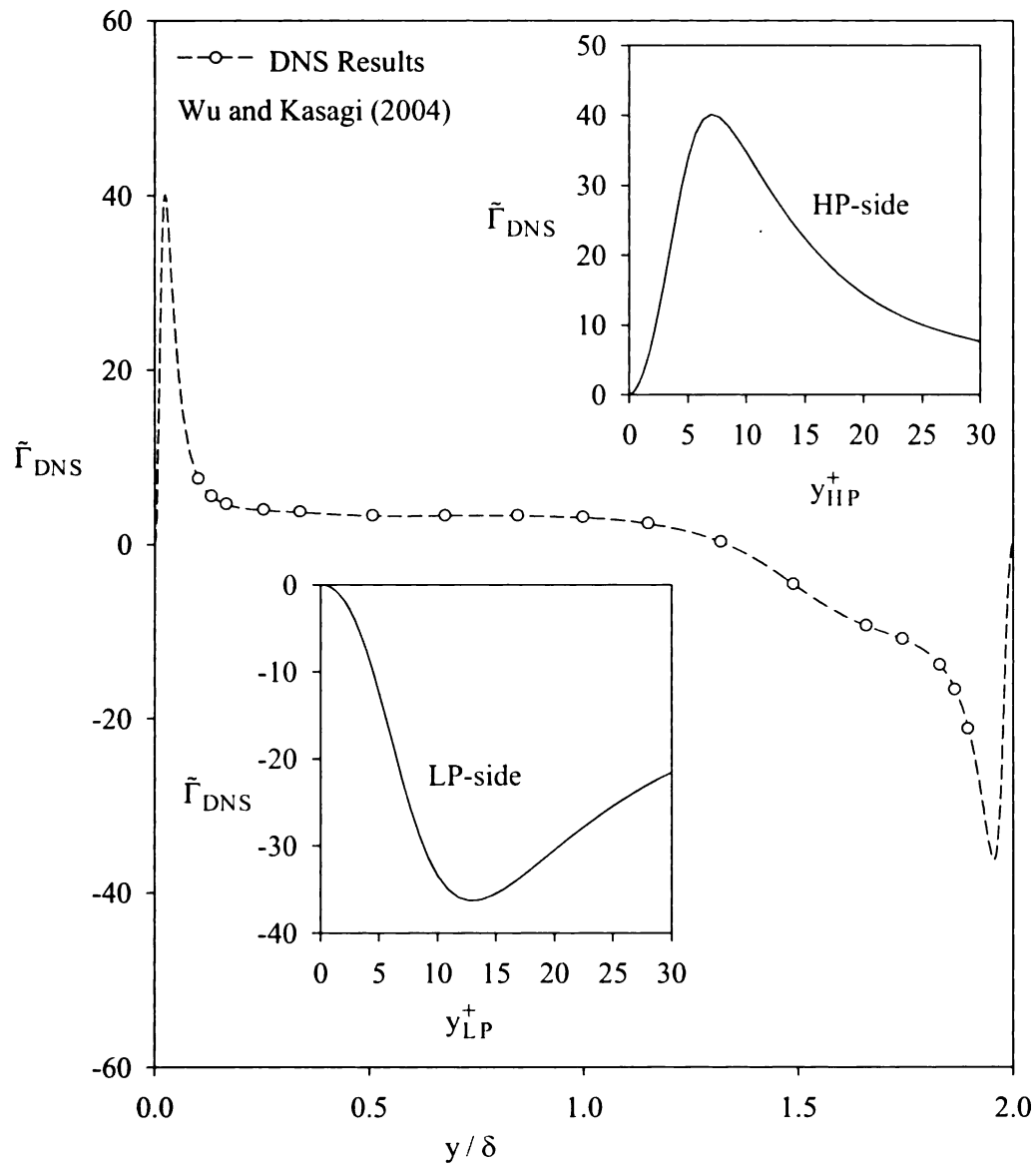


Figure 2.14 Transverse Profiles of $\tilde{\Gamma}_{DNS}$ for Rotating, Fully-Developed, Channel Flows ($\delta^+ \cong 300$ and $\Omega_x^+ \cong -0.0042$).

0.0

-0.5

-1.0

$\hat{Q}_{288}$

-1.5

-2.0

-2.5

0

Figure 2.15

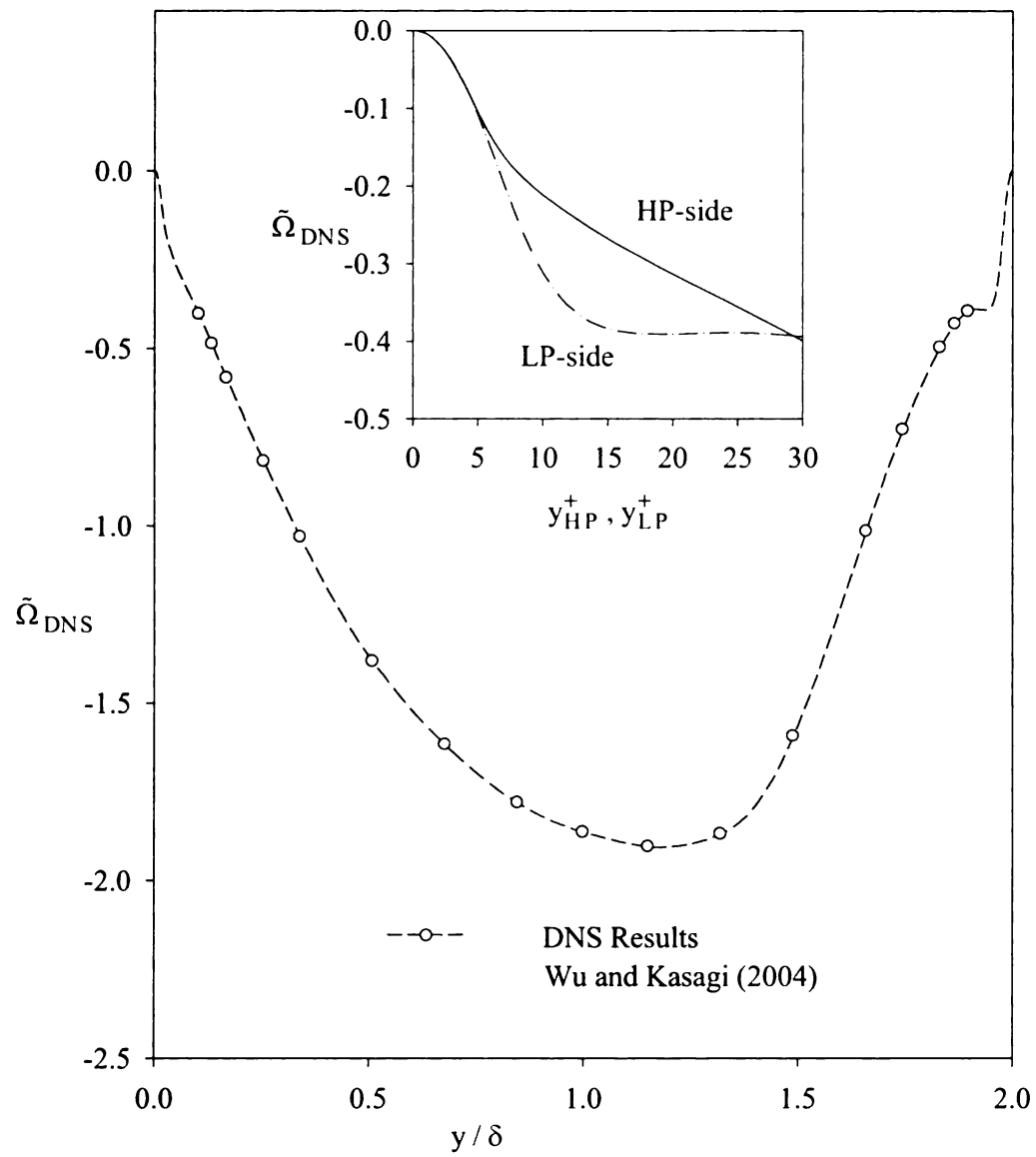


Figure 2.15 Transverse Profiles of $\tilde{\Omega}_{DNS}$ for Rotating, Fully-Developed, Channel Flows ($\delta^+ \cong 300$ and $\Omega_x^+ \cong -0.0042$).

Table 2.2 Cross Flow Variation of $\tilde{\Gamma}_{\text{DNS}}$, $\tilde{\Omega}_{\text{DNS}}$, and the Components of the NR-Stress for Rotating, Fully-Developed, Channel Flow: $\delta^+ \cong 300$, $\Omega_x^+ \cong -0.0042$ (Wu and Kasagi, 2004).

$[y^+]$	y/δ	$\tilde{\Gamma}_{\text{DNS}}$	$\tilde{\Omega}_{\text{DNS}}$	R_{xx}	R_{yy}	R_{zz}	R_{yz}
30	0.10	7.47	-0.40	0.25	0.10	0.65	-0.12
40	0.13	5.48	-0.49	0.28	0.13	0.59	-0.14
50	0.17	4.57	-0.58	0.30	0.17	0.53	-0.16
75	0.26	3.91	-0.82	0.31	0.24	0.45	-0.19
100	0.34	3.67	-1.03	0.31	0.29	0.40	-0.20
150	0.51	3.25	-1.38	0.29	0.38	0.33	-0.20
200	0.68	3.25	-1.62	0.27	0.44	0.29	-0.19
250	0.85	3.27	-1.78	0.27	0.46	0.27	-0.17
300	1.00	3.07	-1.86	0.29	0.45	0.26	-0.13
350	1.15	2.34	-1.90	0.33	0.41	0.26	-0.09
400	1.32	0.24	-1.87	0.37	0.33	0.30	-0.03
450	1.49	-4.62	-1.59	0.35	0.22	0.43	0.04
500	1.66	-9.40	-1.01	0.29	0.14	0.57	0.08
550	1.83	-13.92	-0.50	0.22	0.09	0.69	0.08
560	1.87	-16.74	-0.43	0.19	0.07	0.74	0.07
570	1.90	-21.23	-0.40	0.15	0.05	0.80	0.06

Section 2.6

rotating chain

$\mathcal{P} \cong \mathbb{R}^3$

$\Omega_1 \cong \mathbb{R}^3$

and then de

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24 Hom

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self-similar

$\mathbb{R}^3 \cong \mathbb{R}^3 \frac{k}{a}$

$\mathbb{R}^3 \xrightarrow{z \rightarrow x}$

Section 2.6). The production to dissipation ratio, $\mathcal{P}/\varepsilon$, for the rotating and the non-rotating channel is shown in Figure 2.16. For $\delta^+ = 300$ and $\Omega_x^+ = 0$, a maximum value of $\mathcal{P}/\varepsilon (\cong 1.8)$ occurs in the region where k^+ is a maximum. For $\delta^+ = 300$ and $\Omega_x^+ \cong -0.0042$, the production/dissipation ratio has a maximum value of 4.2 at $\xi = 0.04$ and then decreases to zero at $\xi = 1.38$, where the total shear stress is zero. Near the LP-side, the peak value of $\mathcal{P}/\varepsilon$ is about 2.6 at $\xi = 1.92$.

2.4 Homogeneous Simple Shear

Non-Rotating Frame of Reference

Experimental measurements of low order statistical properties of homogenous simple shear flows by Harris et al. (1977), Rohr et al. (1988), Tavoularis and Karnik (1989)) and others have been used to calibrate turbulent models (see Pope, 2000; and, esp., Parks, 1997). As illustrated by Figure 2.1, this statistically stationary flow in a non-inertial frame of reference (i.e., $\Omega_x = 0$) is caused by a screen with a transverse resistance that produces an approximate homogenous shear environment downstream of the grid: $\nabla \langle \underline{u} \rangle : \underline{e}_z \underline{e}_y = \Gamma_{yz} = \text{constant}$. As $z \rightarrow \infty$, the turbulent kinetic energy and the turbulent dissipation become unbounded (i.e., $k \rightarrow \infty$ and $\varepsilon \rightarrow \infty$) and the flow becomes self-similar with the following characteristics:

$$\tilde{\Gamma} \equiv \Gamma_{yz} \frac{k}{\varepsilon} \xrightarrow{z \rightarrow \infty} \tilde{\Gamma}_{\text{AHS}} = 4.18 \quad (2.24)$$

$$\underline{\underline{R}} \xrightarrow{z \rightarrow \infty} \underline{\underline{R}}^{\text{AHS}} = 0.236 \underline{e}_x \underline{e}_x + 0.197 \underline{e}_y \underline{e}_y + 0.567 \underline{e}_z \underline{e}_z - 0.165 \underline{e}_y \underline{e}_z - 0.165 \underline{e}_z \underline{e}_y \quad (2.25)$$

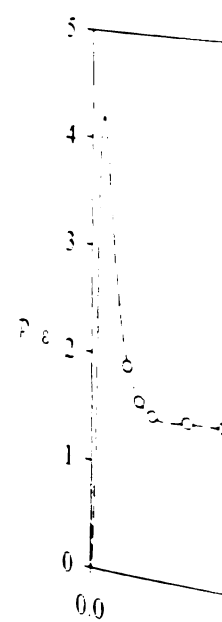
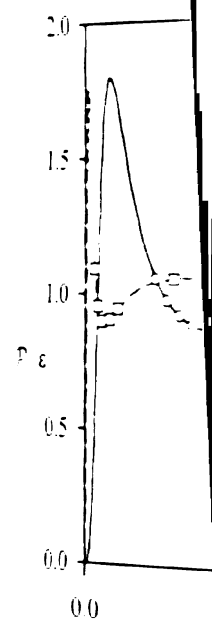


Figure 2.16 Transverse
Ratio, P_e vs.
Flows (σ)

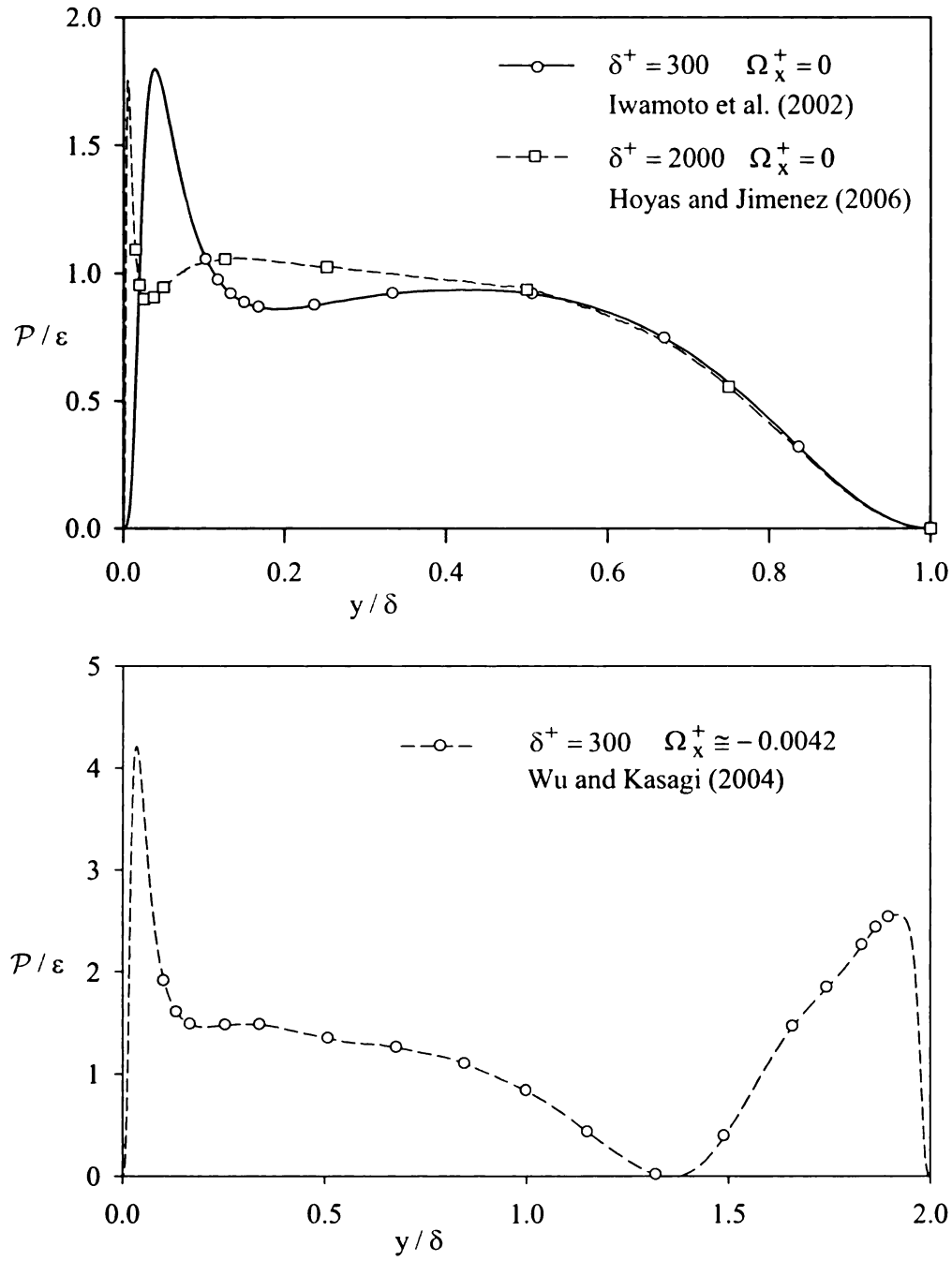


Figure 2.16 Transverse Profiles for Turbulent Production/Dissipation Ratio, $\mathcal{P}/\varepsilon \equiv -2 R_{yz}^{\text{DNS}} \tilde{\Gamma}_{\text{DNS}}$ for Rotating, Fully-Developed, Channel Flows ($\delta^+ \cong 300$ and $\Omega_x^+ \cong -0.0042$).

The z -point on the
 diagram (see Figure 2)
 defined by Eqs.(2.24) is
 rotating, fully-developed
 This observation justifies
 benchmark flow. Eqs (2.24)
 URAPS-closure develop

Rotating Frame of Reference

Bardina et al. (1980)
 1987 used large eddy simulation
 effect of spanwise rotation
 homogeneous mean shear
 "small" dimensionless
 rotation numbers: $\tilde{\Omega} = \Omega / \epsilon$
 allowed to develop
 $1 \leq z < u_z > (0)$. The
 summarized in Table 2
 the ability of the URAPS
 Brethouwer (2000)
 this study, each simulation
 homogeneous turbulent

The ☆-point on the energy simplex (see Figure 2.3 above) and on the anisotropic diagram (see Figure 2.4 above) shows that the asymptotic state for homogeneous shear, defined by Eqs.(2.24) and (2.25) above, is close to the turbulent states formed in non-rotating, fully-developed, channel flows near the outer edge of the equilibrium region. This observation justifies the use of asymptotic homogenous shear flow statistics as a benchmark flow. Eqs.(2.24) and (2.25) are used in Chapter 4 below to calibrate the URAPS-closure developed in Chapter 3.

Rotating Frame of Reference

Bardina et al. (1983) and others (see, esp., Speziale 1989; Salhi and Cambon, 1997) used large eddy simulations (LES) of the Navier-Stokes equation to determine the effect of spanwise rotation on the statistical properties of a statistically stationary, homogeneous mean shear flow (see Figure 2.1 above). The results were developed for an “initial” dimensionless shear rate of $\tilde{\Gamma}(0) \cong 3.3$. Simulations are available for three rotation numbers: $\tilde{\Omega}/\tilde{\Gamma} \equiv \Omega_x/\Gamma_{yz} = -1/2, -1/4$, and 0. The computations were allowed to develop for a dimensionless space-time of $t_u \Gamma_{yz} = 5$, where $t_u \equiv z/\langle u_z \rangle(0)$. The components of the NR-stress and $\tilde{\Gamma}$ for $t_u \Gamma_{yz} = 5$ are summarized in Table 2.3. In Chapter 6, these LES results will be used to partially assess the ability of the URAPS-closure to predict the influence of rotation on the NR-stress.

Brethouwer (2005) also developed DNS results for $\tilde{\Omega}/\tilde{\Gamma} = -1/2, -1/4$, and 0. In this study, each simulation was initiated by suddenly setting $\tilde{\Gamma}(0) \cong 18$ in an otherwise homogenous turbulent field. The simulations were conducted over a finite development

Tab. 23 DNS an
DNS (B
Speziale

	$\tilde{\Omega}$	$\tilde{\Gamma}$	
DNS	0		1
	-0.25		1
	-0.5		1
LES	0		3
	-0.25		3
	-0.5		3

Table 2.3 DNS and LES Results for Asymptotic Rotating Homogeneous Shear:
 DNS (Brethouwer, 2005); LES (Bardina et al., 1983 (as reported by
 Speziale , 1989; Salhi and Cambon, 1997)).

	$\tilde{\Omega}/\tilde{\Gamma}$	$\tilde{\Gamma}_o$ (appx)	$\tilde{t}$ (appx)	$\tilde{\Gamma}(8)$ (appx)	R_{xx}	R_{yy}	R_{zz}	R_{yz}
DNS	0	18	12	12	0.210	0.120	0.670	-0.130
	-0.25	18	8	12	0.280	0.370	0.350	-0.320
	-0.5	18	14	12	0.394	0.553	0.053	-0.065
LES	0	3.3	5	-	0.294	0.073	0.633	-0.15
	-0.25	3.3	5	-	0.224	0.383	0.393	-0.35
	-0.5	3.3	5	-	0.344	0.583	0.073	-0.10

time, $0 \leq t_0 \Gamma_{yz} < 15$.

give the experimental

data to attain a self-sim

approximate asymptotic

culated results show

the streamwise compon

component. Note that

distribution flips and R_y

simulations for spanwise

are absolute mean vort

Brethouwer (200

homogeneous shear flow

rotation numbers $\tilde{\Omega}$

$\tilde{\Omega} \tilde{\Gamma}_{min} < (\tilde{\Omega} - \tilde{\Gamma}) < (9$

The DNS results dev

similar behavior inasm

all the components of

size $\tilde{r}$ increases, but

zero as indicated by T

$\tilde{\Omega} \tilde{\Gamma} = +1.4$ the turbu

to zero as the develop

shear produces two qu

time, $0 \leq t_u \Gamma_{yz} < 15$. For $\Omega_x = 0$ and $t_u \Gamma_{yz} = 12$, $\tilde{\Gamma}(12) = 12$, which is significantly above the experimental value of $\tilde{\Gamma}_{AHS} = 4.2$. The simulation time may have been too short to attain a self-similar state. The components of the NR-stress listed in Table 2.3 are approximate asymptotic values extrapolated from the developing statistical results. The tabulated results show that as $\tilde{\Omega}/\tilde{\Gamma}$ becomes more negative, the energy is shifted from the streamwise component of the fluctuating velocity to the crossflow (or transverse) component. Note that for $\tilde{\Omega}/\tilde{\Gamma} = 0$, $R_{yy} < R_{zz}$; but, for $\tilde{\Omega}/\tilde{\Gamma} = -1/2$, the energy distribution flips and $R_{yy} > R_{zz}$. This redistribution of energy was also noted in the DNS simulations for spanwise rotation of channel flows (see Figure 2.4 above) in the region of zero absolute mean vorticity (see Eq. (2.21) above).

Brethouwer (2005) recently developed DNS simulations for spanwise rotation of homogeneous shear flows and showed that self-similar solutions occur over a finite range of rotation numbers $\tilde{\Omega}/\tilde{\Gamma}$:

$$(\tilde{\Omega}/\tilde{\Gamma})_{\min} < (\tilde{\Omega}/\tilde{\Gamma}) < (\tilde{\Omega}/\tilde{\Gamma})_{\max}. \quad (2.26)$$

The DNS results developed for $\tilde{\Omega}/\tilde{\Gamma}$ between $(\tilde{\Omega}/\tilde{\Gamma})_{\min}$ and $(\tilde{\Omega}/\tilde{\Gamma})_{\max}$ show self-similar behavior inasmuch as the turbulent kinetic energy, the turbulent dissipation, and all the components of the Reynolds stress increase as the dimensionless development time $\tilde{t}$ increases, but $\tilde{\Gamma}$ and all the components of the NR-stress approach a self-similar state as indicated by Table 2.3. Brethouwer (2005) also noted that for $\tilde{\Omega}/\tilde{\Gamma} = -3/4$ and $\tilde{\Omega}/\tilde{\Gamma} = +1/4$ the turbulent kinetic energy k and the turbulent dissipation ε both decrease to zero as the development time increases. Therefore, spanwise rotation of homogeneous shear produces two qualitatively distinct flow regimes. A decay regime, defined as

$$\lim_{t_0 \rightarrow \infty} (k, \varepsilon) = (0, 0) \Rightarrow$$

for $(\tilde{\Omega} > 1)$

And a self-similar regime

$$\lim_{t_0 \rightarrow \infty} (k, \varepsilon) = (\infty, \infty) \Rightarrow$$

for $(\tilde{\Omega} < 1)$

In Chapter 6 and in Sec.

homogeneous shear flow

of different closure models

25 Homogeneous Decay

Rotating Frames

Experimental

homogeneous decay in

Corrin (1971) and

fundamental ideas related

and Parks, 1997). DNS

homogeneous decay and

1997; Yu et al. 2000

$\varepsilon \approx \nu \langle \underline{u}' \underline{u}' \rangle / 2$ and

not positive, are related

$$\frac{d\varepsilon}{dt} = -\varepsilon.$$

$$\lim_{t_u \rightarrow \infty} (k, \varepsilon) = (0, 0) \ni \lim_{t_u \Gamma_{yz} \rightarrow \infty} \tilde{\Gamma} = \infty \quad (2.27)$$

for $(\tilde{\Omega}/\tilde{\Gamma}) < (\tilde{\Omega}/\tilde{\Gamma})_{\min}$ or $(\tilde{\Omega}/\tilde{\Gamma}) > (\tilde{\Omega}/\tilde{\Gamma})_{\max}$

And, a self-similar regime, defined as

$$\lim_{t_u \rightarrow \infty} (k, \varepsilon) = (\infty, \infty) \ni \lim_{t_u \Gamma_{yz} \rightarrow \infty} \tilde{\Gamma} = \tilde{\Gamma}_{AHS} < \infty \quad (2.28)$$

for $(\tilde{\Omega}/\tilde{\Gamma})_{\min} < (\tilde{\Omega}/\tilde{\Gamma}) < (\tilde{\Omega}/\tilde{\Gamma})_{\max}$

In Chapter 6 and in Section 2.6 below, the existence of self-similar states for rotating homogenous shear flows provides a means to validate and compare specific predictions of different closure models for the NR-stress.

2.5 Homogeneous Decay

Non-Rotating Frames of Reference

Experimental measurements of the low-order statistical properties of homogeneous decay in an inertial (non-rotating) frame of reference by Comte-Bellot and Corrsin (1971) and many others (see, esp., pg. 160, Pope 2000) confirm many fundamental ideas related to turbulent velocity fluctuations (see, Chapter 3, Piquet, 1999; and Parks, 1997). DNS and lattice Boltzmann simulations have also been developed for homogenous decay and are consistent with experimental results (see, Mansour and Wray, 1993 ; Yu et al. 2005). For this flow (see Figure 2.1), the turbulent kinetic energy $k (\equiv \text{tr} < \underline{u}' \underline{u}' > / 2)$ and the turbulent dissipation $\varepsilon (\equiv \nu \text{tr} < (\nabla \underline{u}') \cdot (\nabla \underline{u}')^T >)$, which are both positive, are related by (see Eq. (1.9) above)

$$\frac{dk}{dt} = -\varepsilon. \quad (2.29)$$

Figure 2.17 gives a s

(97) in the form of a

decay curves are par

viscous decay of turbu

implies that $\bar{k} \bar{\epsilon} \rightarrow \infty$

equations (see Eqs.(1.9)

elaborate the phenom

dissipation (see Chapter

$$\frac{d\bar{\epsilon}}{dt} = -C_D \frac{\bar{\epsilon}}{\tau_R}$$

Rotating Frame of Reference

Experimental resu

has reported by Ibbetso

in 1990. They noted th

energy is decreased alon

observed in the integral

reported. Complementary

(1985). Speziale et al. (1

other simulations were

total rotation number

developed from an initia

Figure 2.17 gives a summary of the experimental results (Comte-Bellot and Corrsin, 1971) in the form of a phase plane graph of $\tilde{k}(\tilde{t}) \equiv k(t)/k(0)$ and $\tilde{\varepsilon}(\tilde{t}) \equiv \varepsilon(t)/\varepsilon(0)$. The decay curves are parameterized by the dimensionless time $\tilde{t} \equiv t\varepsilon(0)/k(0)$. The non-viscous decay of turbulent kinetic energy has the feature that $0 \leq \tilde{\varepsilon}(\tilde{t}) \leq \tilde{k}(\tilde{t}) \leq 1$, which implies that $\tilde{k}/\tilde{\varepsilon} \rightarrow \infty$ as $\tilde{t} \rightarrow \infty$. All NR-stress closure models that use $k - \varepsilon$ transport equations (see Eqs.(1.9) and (1.11)) use the results portrayed in Figure 2.17 to partially calibrate the phenomenological relaxation time τ_R governing the decay of turbulent dissipation (see Chapter 4 below):

$$\frac{d\varepsilon}{dt} = -C_D \frac{\varepsilon}{\tau_R} . \quad (2.30)$$

Rotating Frame of Reference

Experimental results for homogenous decay in a rotating frame of reference have been reported by Ibbetson and Tritton (1975), Weigand and Naguib (1978) and Jaquin et al. 1990. They noted that in the presence of rotation, rate of decay of turbulent kinetic energy is decreased along with a decrease in the dissipation. Although anisotropy was observed in the integral length scales, there was no significant Reynolds stress anisotropy reported. Complementary DNS and LES results were developed also by Bardina et al. (1985), Speziale et al. (1987) and Mansour et al. (1992) (see pg. 59 Gatski 1992). These earlier simulations were limited to relatively small decay times (i.e., $\tilde{t} < 10$) and low initial rotation numbers, $\tilde{\Omega}_0 = \Omega_\chi k(0)/\varepsilon(0) < 75$). A weak prolate anisotropic state developed from an initial rotation number of $\tilde{\Omega}_0 \sim 1$; however, no discernable anisotropy

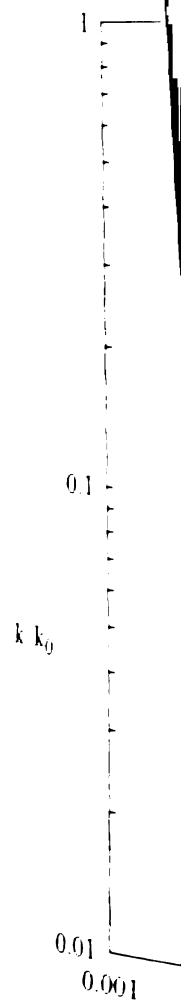


Fig. 2.17 Experiment
(1997)

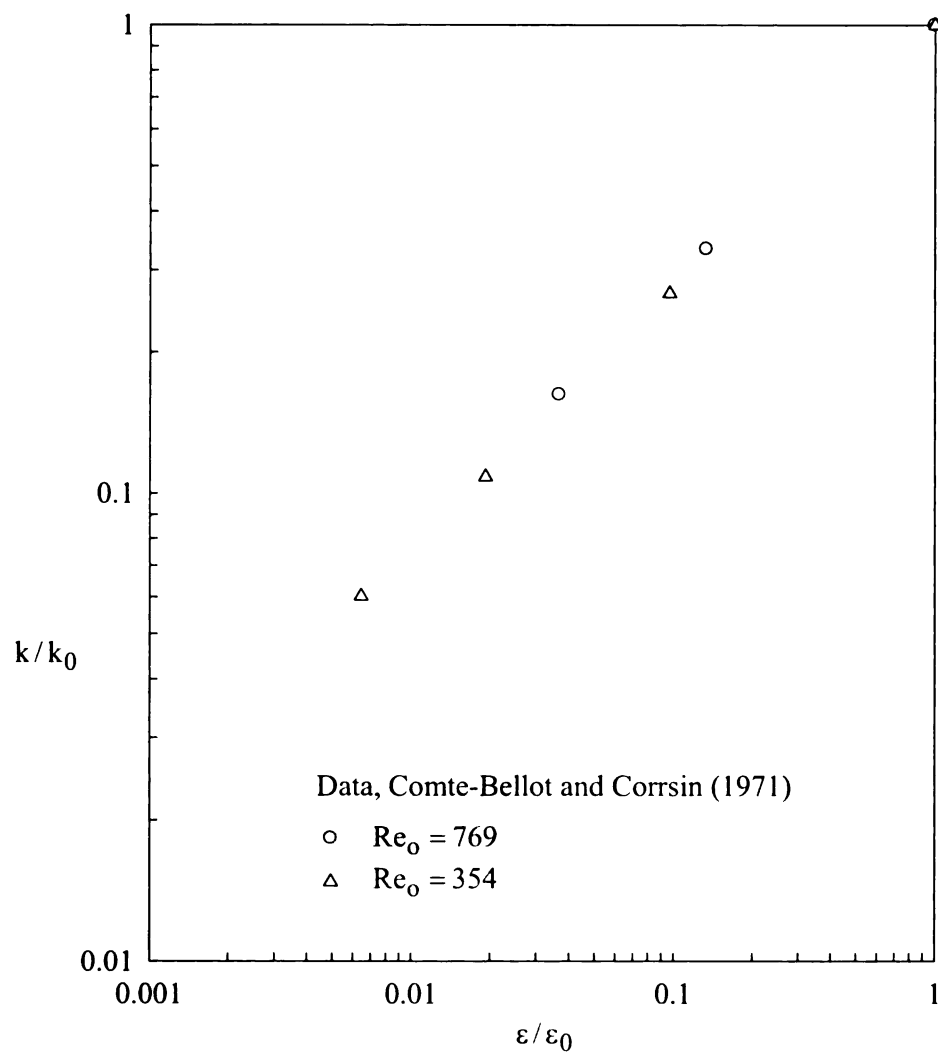


Figure 2.17 Experimental data of Comte-Bellot and Corrsin (1971) (also see Parks, 1997)

developed for large

Boltzmann, LES, and I

kinetic energy among

1993, Bartello et al.,

Domaradzki, 2004). Ho

Domaradzki (2004) use

long simulation time

1997) show that the Q^*

depending on the initial

Garbon et al., 1997; and

The influence of

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research to determine s

Appendix F provides a

of the URAPS-closure i

26 Algebraic Model

An algebraic clo

rates the normalized

statistical properties of

The relationship may

developed for large rotation rates. However, recent simulations based on lattice Boltzmann, LES, and DNS methods have indicated that rotation can redistribute turbulent kinetic energy among the three components of the fluctuating velocity (see Yu et al., 2005; Bartello et al., 1994; Cambon et al., 1997; Mornishi et al. 2001; and Yang and Domaradzki, 2004). However, significant anisotropy only occurs after $\tilde{t} \gg 10$. Yang and Domaradzki (2004) used LES to show that an anisotropic state eventually develops after a long simulation time for rotation numbers ($\tilde{\Omega}_0 > 20$) and the results of Cambon et al. (1997) show that the QNR-form may be either an oblate ellipsoid or a prolate ellipsoid, depending on the initial conditions and the development time (see, esp., Figure 7 in Cambon et al., 1997; and Yang and Domaradzki, 2004).

The influence of frame rotation on the energy cascade in homogeneous decay has been studied theoretically by Zhou (1994) and Zeman (1995). Park and Chung (1999) developed a relationship between the relaxation time τ_R , defined by Eq. (2.29), and the rotation number $\tilde{\Omega} \equiv \Omega_x k / \varepsilon$. The results of Park and Chung (PC-) are used in this research to determine several calibration parameters associated with the URAPS-closure. Appendix F provides a brief summary of the PC-paper needed to support the calibration of the URAPS-closure in Chapter 4 below.

2.6 Algebraic Models for the NR-Stress

An algebraic closure model for the NR-stress in a non-inertial frame of reference relates the normalized Reynolds stress to the local mean velocity gradient, to the local statistical properties of the fluctuating velocity, and to the angular velocity of the frame. The relationship may be either *implicit* or *explicit*. An *implicit* model is a non-linear

algebraic mapping of

as follows:

$$\underline{\underline{S}}^I(\underline{\underline{R}}, \underline{\underline{K}}) \rightarrow \underline{\underline{R}}, \quad \underline{\underline{K}}$$

In general, the scalar

dimensionless groups re

local statistical state of

An explicit model prov

$\underline{\underline{K}}$ and the NR-stress $\underline{\underline{R}}$

$$\underline{\underline{S}}^I(\underline{\underline{K}}) \rightarrow \underline{\underline{R}},$$

Both *implicit* and *explicit*

local isotropic state can

$\underline{\underline{K}} = \underline{\underline{0}}$ in either Eq. (2.3)

the possibility of a weak

channel flows (see Fig

component for $\underline{\underline{K}} = \underline{\underline{0}}$ i

However, it is consiste

predict a zero total str

component of the NR-st

and 2.9 above).

In Chapter 1, se

Sections 2.2-2.5, e

benchmark flows were

algebraic mapping of $\underline{\underline{R}}$ into itself. A formal representation of this idea can be expressed as follows:

$$\mathfrak{R}^I(\underline{\underline{R}}, \underline{\underline{K}}) \rightarrow \underline{\underline{R}}, \quad \underline{\underline{K}} \equiv \tau_R \langle \underline{\underline{F}} \rangle, \quad \langle \underline{\underline{F}} \rangle \equiv \nabla \langle \underline{\underline{u}} \rangle + 2\underline{\underline{\Omega}}, \quad \tau_R \equiv C_R k / \varepsilon. \quad (2.31)$$

In general, the scalar-valued dimensionless closure function C_R depends on two dimensionless groups related to the relative importance of time scales associated with the local statistical state of the turbulence, viz., $Re_t \equiv (k / \varepsilon) / (\nu / k)$ and $\tilde{N}_F \equiv (k / \varepsilon) \|\langle \underline{\underline{F}} \rangle\|$.

An *explicit* model provides a direct link between the hydrodynamic/kinematic operator $\underline{\underline{K}}$ and the NR-stress $\underline{\underline{R}}$:

$$\mathfrak{R}^E(\underline{\underline{K}}) \rightarrow \underline{\underline{R}}. \quad (2.32)$$

Both *implicit* and *explicit* algebraic models employ the assumption that deviations from a local isotropic state can only occur by a coupling with an external field. Therefore, if $\underline{\underline{K}} = \underline{\underline{0}}$ in either Eq. (2.31) or Eq. (2.32) above, then $\underline{\underline{R}} = \underline{\underline{I}}/3$. This hypothesis excludes the possibility of a weak anisotropic state on the symmetry plane for fully-developed channel flows (see Figure 2.3). It also excludes the possibility of a non-zero shear component for $\underline{\underline{K}} = \underline{\underline{0}}$ in fully-developed annular flows (see Churchill and Chan, 1995). However, it is consistent with DNS results for spanwise rotating channel flows that predict a zero total stress on the LP-side of the symmetry plane where the shear component of the NR-stress is zero and the mean velocity is a maximum (see Figures 2.8 and 2.9 above).

In Chapter 1, several theoretical properties for the NR-stress were identified and, in Sections 2.2-2.5, experimental and DNS results for the NR-stress for specific benchmark flows were noted. In this section, these observations are used to critique

several algebraic clos

equation (see Eq. (1)

available in the book b

following nine charac

asure models:

C1: The NR-s

C2: The eigen
flows.

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several algebraic closure models presently used to support an analysis of the RANS-equation (see Eq. (1.4)). Additional information related to these specific models is available in the book by Pope (2000) and in the references cited in the bibliography. The following nine characteristics of the NR-stress will be used to benchmark algebraic closure models:

- C1: The NR-stress is real, symmetric ($\underline{\underline{R}}^T = \underline{\underline{R}}$), and normalized ($\text{tr}(\underline{\underline{R}}) = 1$).
- C2: The eigenvalues of the NR-stress are non-negative real numbers for all flows.
- C3: For non-rotating, fully-developed, channel flows the normal components of the NR-stress are in the 2nd Sextet of the energy simplex (see Figure 2.3).
- C4: For spanwise rotation of fully-developed, channel flows, the normal components of the NR-stress are distributed in all six sextets of the energy simplex (see Figure 2.10).
- C5: In the absence of rotation, the QNR-form for fully-developed, channel flows near the isotropic state is nearly prolate (i.e., $\text{III}_b > 0$; also see Figure 2.4).
- C6: For spanwise rotation of fully-developed, channel flows, the QNR-form near the isotropic state is nearly oblate (i.e., $\text{III}_b < 0$; also see Figure 2.11).
- C7: For non-rotating homogenous shear, the NR-stress is self-similar and the dimensionless turbulent time scale, $\tilde{\Gamma} \equiv (k/\varepsilon)\Gamma_{yz}$, is self-similar for sufficiently large development times.
- C8: For spanwise rotation of homogeneous shear, the NR-stress and $\tilde{\Gamma}$ are self-similar for sufficiently large development times for a finite range of rotation numbers:

$$-\infty < (\Omega_x / \Gamma_{yz})_{\min} < (\Omega_x / \Gamma_{yz}) < (\Omega_x / \Gamma_{yz})_{\max} < +\infty .$$

- C9: For rotating homogeneous decay, the energy distribution is anisotropic and, for long decay times, the QNR-form is stretched into a prolate or

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Closure Models Based on a Phenomenological Hypothesis

An “eddy” viscosity model for the Reynolds stress is analogous to the closure employed for the molecular stress of a Newtonian fluid. This idea, which is often referred to as the Boussinesq (B-) closure (see p. 162 in Bird et al., 2007; p. 93 in Pope, 2000; and, Boussinesq, 1877), has been used by numerous researchers to relate the anisotropic component of the Reynolds stress to the mean strain rate. The B-closure is an example of an *explicit* model for the NR-stress inasmuch as (see Eq. (1.7) above):

$$\underline{\underline{\mathbf{R}}} = \mathfrak{R}^E(\underline{\underline{\tilde{\mathbf{S}}}}) = \frac{1}{3}\underline{\underline{\mathbf{I}}} + \underline{\underline{\mathbf{b}}}, \quad \underline{\underline{\mathbf{b}}} \equiv -C_v \underline{\underline{\tilde{\mathbf{S}}}}, \quad \underline{\underline{\tilde{\mathbf{S}}}} \equiv \frac{k}{\varepsilon} \underline{\underline{\mathbf{S}}}. \quad (2.33)$$

The dimensionless scalar-valued “eddy” viscosity coefficient C_v links the mean strain rate and the turbulent time scale k/ε to the anisotropic component of the NR-stress. In general, C_v depends on the turbulent Reynolds number, $Re_t = (k/\varepsilon)/(\nu/k)$, and the invariants of $\underline{\underline{\tilde{\mathbf{S}}}}$. Eq. (2.33) is widely used to support computational analyses of the RANS-equation (see Eq. (1.4)). The “eddy” viscosity closure for the NR-stress has the following characteristics.

- Eq. (2.33) clearly satisfies characteristic C1 above.
- The eigenvalues of $\underline{\underline{\tilde{\mathbf{S}}}}$ and $\underline{\underline{\mathbf{R}}}$ are related by $\lambda_{Ri} = -C_v \tilde{\lambda}_{Si} + 1/3$. Eq. (2.33)

is *realizable* for all flows and, therefore, satisfies characteristic C2 above provided the invariants of $\underline{\underline{\mathbf{b}}}$ are within the L-diagram (see Figure 1.1). This requires a phenomenological coefficient C_v that satisfies the following

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inequality $0 \leq -C_v \tilde{\lambda}_{Si} + 1/3 \leq 1$ for all flows. A specific form for C_v has been identified by Shih et al. (1995) using benchmark statistics for non-rotating simple shear, but a recalibration is needed for rotating simple shear flows. Therefore, Eq. (2.33) satisfies characteristic C2 provisionally inasmuch as the B-closure is not *universal* (i.e., a different “eddy” viscosity coefficient is needed for rotating flows).

- For non-rotating homogeneous shear and for non-rotating fully-developed channel flows, the B-closure predicts that the turbulent kinetic energy is equally distributed among the three components of the fluctuating velocity. Therefore, Eq. (2.33) is qualitatively inconsistent with C3 above.
- Eq. (2.33) can be “sensitized” to rotation through the turbulent time scale k/ε ; however, the B-closure applied to a rotating simple shear still predicts that the energy is equally divided among the three normal components of the NR-stress. This theoretical prediction is qualitatively inconsistent with C4 above.
- Eq. (2.33) predicts that all the *realizable* anisotropic states for non-rotating simple shear flows have invariants that satisfy the following conditions: $0 \leq \Pi_b \leq 2/9$ and $\text{III}_b = 0$. This result is qualitatively inconsistent with C5 (see Figure 2.4). If Eq. (2.33) were used for rotating channel flows, then $\text{III}_b = 0$ would still be predicted for all anisotropic states. This prediction is qualitatively inconsistent with characteristic C6 above.
- The “production” term, $-\langle \underline{u}' \underline{u}' \rangle : \langle \underline{S} \rangle$, in the k-equation (see Eq. (1.9)) may be either positive or negative (see Pope, p.180, 2000; and Nishino et al.

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1996). However, if the *realizable* B-closure (Shih et al., 1995) is used as a model for the Reynolds stress, energy “production” is positive for all flows inasmuch as $C_v > 0$ and

$$-\langle \underline{\underline{u}}' \underline{\underline{u}}' \rangle : \langle \underline{\underline{S}} \rangle = -2\varepsilon \underline{\underline{R}} : \langle \underline{\underline{S}} \rangle = +2\varepsilon C_v \langle \underline{\underline{S}} \rangle : \langle \underline{\underline{S}} \rangle \geq 0.$$

Speziale (1991) extended the *explicit* “eddy” viscosity model for the NR-stress to an *explicit* second-order model for the NR-stress by using the Cayley-Hamilton theorem of linear algebra with the result that

$$\underline{\underline{R}} = \frac{1}{3} \underline{\underline{I}} + \underline{\underline{b}} = \frac{1}{3} \underline{\underline{I}} + C_1 \langle \underline{\underline{S}} \rangle + C_2 (\langle \underline{\underline{S}} \rangle \cdot \langle \underline{\underline{S}} \rangle - II_S \underline{\underline{I}}/3). \quad (2.34)$$

The phenomenological coefficients $C_1(II_S, III_S)$ and $C_2(II_S, III_S)$ may depend on the invariants of $\langle \underline{\underline{S}} \rangle$, defined as $II_S \equiv \text{tr}(\langle \underline{\underline{S}} \rangle \cdot \langle \underline{\underline{S}} \rangle)$ and $III_S \equiv \text{tr}(\langle \underline{\underline{S}} \rangle \cdot \langle \underline{\underline{S}} \rangle \cdot \langle \underline{\underline{S}} \rangle)$.

If $C_2(II_S, III_S) = 0$, then Eq. (2.34) reduces to Eq. (2.33). Clearly, Eq. (2.34) satisfies C1 above. Moreover, the phenomenological coefficients $C_1(II_S, III_S)$ and $C_2(II_S, III_S)$ could be selected so that $\underline{\underline{R}}$ is *realizable* (characteristic C2); however, Eq. (2.34) predicts that for simple shear flows, $R_{zz} = R_{yy}$. This result clearly does not agree with C3, which requires $0 \leq R_{yy} \leq R_{xx} \leq R_{zz} \leq 1$. Apparently, the mean strain rate is not an appropriate kinematic operator for the NR-stress.

Rivlin (1955) used the classical Cayley-Hamilton (CH) theorem of linear algebra to develop a general class of constitutive stress models for fluids that satisfy certain invariance properties. Pope (1975) and others (see p.452, Pope, 2000; Gatski, 2000) applied a similar approach to develop phenomenological models for the normalized Reynolds stress by assuming that $\underline{\underline{b}}$, the anisotropic component of $\underline{\underline{R}}$, depends on the

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$\langle \underline{\underline{\dot{W}}} \rangle$. With $\underline{\underline{b}} = \underline{\underline{B}}$

Erzer et al., 1966(1)

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dimensionless strain rate operator, $\langle \underline{\tilde{S}} \rangle$, and the dimensionless intrinsic vorticity operator, $\langle \underline{\tilde{W}} \rangle \equiv \langle \underline{\tilde{W}} \rangle + \underline{\tilde{\Omega}}$. Thus, a further generalization of Eq (2.33) assumes that the NR-stress in rotating and in non-rotating frames depends independently on $\langle \underline{\tilde{S}} \rangle$ and $\langle \underline{\tilde{W}} \rangle$. With $\underline{b} = \underline{B}(\langle \underline{\tilde{S}} \rangle, \langle \underline{\tilde{W}} \rangle)$, an application of the Cayley-Hamilton theorem (see Frazer et al., 1960) yields the following explicit irreducible representation for the anisotropic component of the NR-stress (see p. 453 in Pope, 2000):

$$\underline{R} = \frac{1}{3} \underline{I} + \underline{b} = \frac{1}{3} \underline{I} + \sum_{n=1}^{10} C_n \underline{\Phi}_n(\langle \underline{\tilde{S}} \rangle, \langle \underline{\tilde{W}} \rangle). \quad (2.35)$$

The scalar-valued CH-coefficients C_n in Eq. (2.35) depend on the non-trivial invariants of the so-called *integrity* operators $\underline{\Phi}_n$ (see Appendix D). The dimensional operators $\langle \underline{S} \rangle$ and $\langle \underline{\tilde{W}} \rangle$ are *objective*, symmetric, and traceless hydrodynamic/kinematic operators. If k/ε is assumed to be an *objective* scalar field, then all ten *integrity* operators $\underline{\Phi}_n$ are *objective* and the ten CH-coefficients are scalar-valued *objective* functions.

Comment on Objectivity: The eigenvalues and the magnitude of the eigenvectors associated with an *objective* operator do not depend on the rotation of the frame of reference. DNS simulations in non-inertial frames clearly show that the NR-stress depends on the angular velocity of the frame. Therefore, the NR-stress is not *objective*. Theoretical support for this conclusion follows from the fact that the fluctuating velocity field $(\underline{u}^*)'$ in a $*$ -frame (i.e., rotating frame) is related to the fluctuating velocity $\underline{u}'$ and the fluctuating motion $\underline{\chi}'$ in a non-rotating frame

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by the following kinematic equation $(\underline{u}^*)' = \underline{\underline{Q}} \cdot \underline{u}' + \dot{\underline{\underline{Q}}} \cdot \underline{\underline{\chi}}'$, where $\underline{\underline{Q}}$ is an arbitrary, time-dependent, orthogonal operator. An earlier impression that unfortunately still persist in the turbulence modeling literature (see, esp., Durst 2001, Gatski, 2004, Thais et al. 2005, Dafalias and Younis, 2007) is that the Reynolds stress is an *objective* statistical property of a turbulent ensemble of flows. This idea stems from an earlier theoretical argument by Speziale (1998) that $\langle (\underline{u}^*)' (\underline{u}^*)' \rangle = \underline{\underline{Q}} \cdot \langle \underline{u}' \underline{u}' \rangle \cdot \underline{\underline{Q}}^T$. This relationship between the Reynolds stress in two different equivalent frames is not correct inasmuch as the fluctuating motion $\underline{\underline{\chi}}'$ of a turbulent ensemble of flows is not zero.

If $C_n = 0$ for $n \geq 2$, then Eq. (2.35) reduces to Eq. (2.33) with $\underline{\underline{\Phi}}_1 = \langle \tilde{\underline{\underline{S}}} \rangle$. If

$C_n = 0$ for $n \geq 3$, then

$$\underline{\underline{R}} = \frac{1}{3} \underline{\underline{I}} + \underline{\underline{b}} = \frac{1}{3} \underline{\underline{I}} + C_1 \langle \tilde{\underline{\underline{S}}} \rangle + C_2 [\langle \tilde{\underline{\underline{S}}} \rangle \cdot \langle \tilde{\underline{\underline{W}}} \rangle + \langle \tilde{\underline{\underline{W}}} \rangle^T \cdot \langle \tilde{\underline{\underline{S}}} \rangle]. \quad (2.36)$$

The bi-linear model defined by Eq. (2.36) was introduced by Shih et al. (1994) as a means to account for the anisotropic structure of the NR-stress. The model is clearly symmetric and reduces to $\underline{\underline{R}} = \underline{\underline{I}}/3$ if $\langle \tilde{\underline{\underline{S}}} \rangle = \underline{\underline{0}}$. The coefficients C_1 and C_2 can be specified so the resulting NR-stress is *realizable* (see Appendix E). For simple shear flows, Eq. (2.36) predicts that the energy states are in the 2nd Sextet of the energy simplex: $0 < R_{yy} \leq R_{xx} \leq R_{zz} < 1$. However, for simple shear flows, Eq. (2.36) also predicts that $R_{xx} = 1/3$ and $\text{III}_b = 0$ for all values of $\tilde{\Gamma} \equiv \Gamma_{yz} k / \varepsilon$, which is not consistent with the energy distribution and the anisotropic invariants associated with fully-developed channel flows (see Figures 2.18; also see Appendix E). Clearly, a

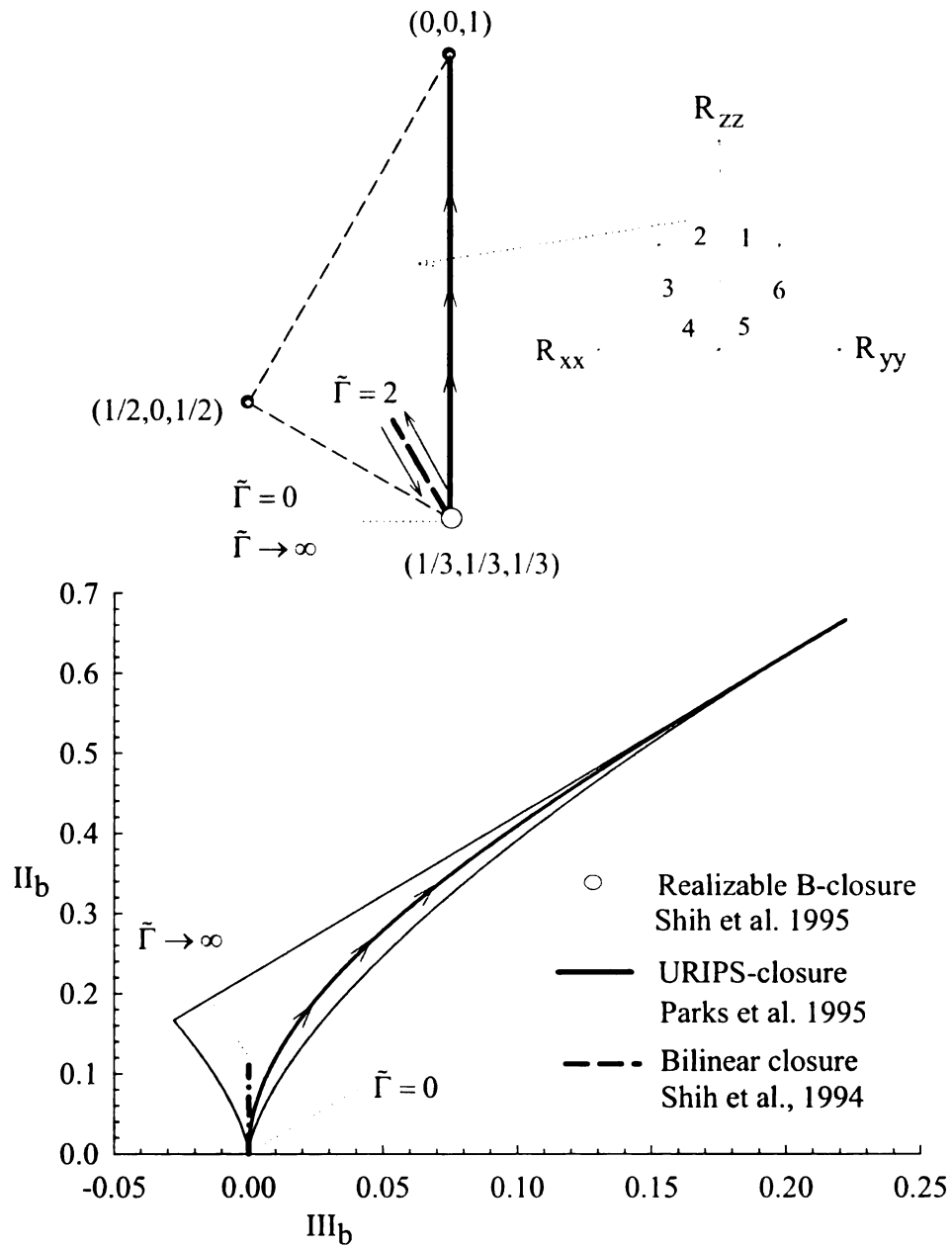


Figure 2.18 Energy Distribution and Anisotropic States for Non-Rotating Simple Shear Predicted by the Generalized Eddy Viscosity Closure, the Bilinear Closure, and the URIPS Closure for increasing values of $\tilde{\Gamma}$.

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phenomenological approach based on the hypothesis that $\underline{\underline{b}} = \underline{\underline{B}}(<\underline{\underline{\tilde{S}}}>, <\underline{\underline{\tilde{W}}}>)$ does not capture the low-order statistical phenomena of turbulent flows.

Higher order closure models have been developed by retaining additional terms in Eq. (2.35). For example, Craft et al. (1996) and Speziale (1996) developed a closure for the NR-stress that includes contributions from the cubic terms. The resulting six CH-coefficients were selected based on statistical properties related to flows through curved channels, impinging jet flows, and rotating pipe flows. Although the Craft-closure can reproduce the flows used to select the model parameters, it does not ensure *realizable* behavior for other flows.

Recently, Rahman and Siikonen (2006) developed an explicit non-linear algebraic stress model that considered the first four terms in Eq. (2.35). This quadratic model uses a scalar-valued turbulent transport time scale τ_R that accounts for the anisotropic turbulent structure near a solid/fluid interface. The authors assume that the CH-coefficients depend on the ratio of energy “production” and energy dissipation, $\mathcal{P}/\varepsilon = -2 \varepsilon \underline{\underline{R}} : <\underline{\underline{\tilde{S}}}>$. The closure model is consistent with characteristic C2 above for boundary layers and for homogeneous shear flows. It is also *realizable* for mean field planar-extensional flows and for axisymmetric-extensional flows. However, because the model coefficients were selected based on specific flows, *realizability* may not occur for other flows.

Comment on the Phenomenological Approach: Eq. (2.35) stems from the assumption that the NR-stress is an *objective* operator (see Speziale, 1987, 1989, 1991). This conjecture, which was inspired by a similar assumption regarding the molecular stress of fluids, is not supported by DNS results (see Chen and Song, 1997; El-Samni and Kasagi, 2001;

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Abid and Habibi, 2003; Grundestam 2004; Brethouwer, 2005; Grundestam and Wallin, 2008; and Grundestam et al., 2008). Nevertheless, many researchers continue to use *objectivity* as a guiding principle to improve low-order turbulence closure models (see, for example, Gatski, 2000; p.19 in Piquet, 1999; Durst, 2001; Weis and Hutter, 2003; Girimaji 2004; Dafalias and Younis, 2007; Hamba, 2006). Unfortunately, the *explicit* representation of the NR-stress in terms of *objective* operators has not provided a practical closure for the RANS-equation. The primary weakness has been characteristic C2 above. In Chapter 3 below, this problem is addressed by formulating a non-negative mapping of $\underline{\underline{R}}$ into itself that does not depend on specific benchmark flows used to calibrate closure parameters. Thus, once the URAPS-closure is calibrated, it does not need to be recalibrated to ensure *realizable* behavior for other flows. This theoretical approach has the potential of transforming the use of the RANS-equation as a diagnostic tool for engineering design and education.

Closure Models Based on a Weak Equilibrium Hypothesis

The need for an algebraic closure model for the Reynolds stress motivated Rodi (1972) (see pg. 449, Pope 2000; pg. 281, Piquet, 1999) and many others over the past 30 years to seek an algebraic model based on hydrodynamic principles. An unclosed, albeit exact, differential equation for the second-order moment of the velocity distribution functional can be developed from the Navier-Stokes equation (see p.387 in Pope, 2000).

The resulting momentum

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provides an explicit

$$\langle \underline{f} \rangle = \nabla \cdot \underline{u} > + 2 \underline{u} \cdot \underline{u}$$

re-written as follows

$$\frac{\partial}{\partial t} \langle \underline{u} \rangle + \nabla \cdot (2 \underline{k} \underline{R})$$

$$- \{ \nabla^2 \langle \underline{u} \cdot \underline{u} \rangle$$

$$- \{ - \langle \underline{u} \cdot \underline{u} \rangle \}$$

$$- 2 \left[\langle \frac{\underline{p}}{\rho} \cdot \underline{S} \rangle \right]$$

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$$\frac{\partial}{\partial t} \langle \underline{u} \rangle + \nabla \cdot (2 \underline{k} \underline{R}) \approx$$

The resulting moment equation requires three statistical closure models: one for “mixing”, one for “redistribution”, and another for “dissipation”. The “production” term provides an explicit coupling to the frame rotation operator through the operator $\langle \underline{\underline{F}} \rangle (\equiv \nabla \langle \underline{u} \rangle + 2 \underline{\underline{\Omega}})$ and is already in closed form. The unclosed moment equation can be written as follows (see Appendix A for a derivation):

$$\begin{aligned}
 \left(\frac{\partial}{\partial t} + \langle \underline{u} \rangle \cdot \nabla \right) (2k \underline{\underline{R}}) = & \\
 & \underbrace{+ [\nu \nabla^2 \langle \underline{u}' \underline{u}' \rangle - (\nabla \langle \underline{u}' \frac{\underline{p}'}{\rho} \rangle)^T - (\nabla \langle \frac{\underline{p}'}{\rho} \underline{u}' \rangle) - \nabla \cdot \langle \underline{u}' \underline{u}' \underline{u}' \rangle]}_{\text{"mixing"}} \\
 & + \underbrace{[- \langle \underline{u}' \underline{u}' \rangle \cdot \langle \underline{\underline{F}} \rangle - \langle \underline{\underline{F}} \rangle^T \cdot \langle \underline{u}' \underline{u}' \rangle]}_{\text{"production"}} \\
 & + 2 \underbrace{[\langle \frac{\underline{p}'}{\rho} \underline{\underline{S}}' \rangle]}_{\text{redistribution}} - 2 \underbrace{[\nu \langle (\nabla \underline{u}')^T \cdot (\nabla \underline{u}') \rangle]}_{\text{dissipation}}
 \end{aligned} \tag{2.37}$$

Numerous closure models for the foregoing three statistical correlations in Eq. (2.37) have appeared in the literature (see Pope, 2000). Each set of closure models produce a separate transport equation for the NR-stress that must be calibrated. Consequently, many calibrated RSM transport models are available to support the RANS-equation. Unfortunately, a general assurance that any of these models produce *realizable* operators is not available. Realizability issues are generally handled *a posteriori* for specific classes of flows when they are deemed to be important.

Rodi (1972) reduced Eq. (2.37) to an *implicit* algebraic equation for the NR-stress by using the following *weak equilibrium hypothesis* (also see Gatski, 2000) together with Eq. (1.9) without the mixing contribution:

$$\left(\frac{\partial}{\partial t} + \langle \underline{u} \rangle \cdot \nabla \right) [2k \underline{\underline{R}}] \cong 2 \underline{\underline{R}} \left(\frac{\partial}{\partial t} + \langle \underline{u} \rangle \cdot \nabla \right) [k] \cong 2 (-2 \underline{\underline{R}} : \langle \underline{\underline{S}} \rangle - 1) \varepsilon \underline{\underline{R}}. \tag{2.38}$$

Eq. (2.38) is partially

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$$(-2\underline{\underline{R}} < \underline{\underline{S}} > - 1) \underline{\underline{R}} =$$

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$$\frac{\langle \underline{\underline{P}} \underline{\underline{S}} \rangle}{\rho \epsilon} = \underline{\underline{M}} \underline{\underline{R}} < \underline{\underline{S}} >$$

Solutions to Eqs.(2.39

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Eq. (2.38) is partially justified by the observation that in fully-developed channel flows, the NR-stress is approximately constant in the outer region. The following *implicit* algebraic Reynolds stress model (IARSM) follows by neglecting the mixing term in Eq. (2.37) and using an isotropic closure for the dissipation term:

$$(-2\underline{\underline{R}}:\underline{\underline{\tilde{S}}} - 1)\underline{\underline{R}} = -(\underline{\underline{R}}\cdot\underline{\underline{\tilde{F}}} + \underline{\underline{\tilde{F}}}^T\cdot\underline{\underline{R}}) + \frac{\langle p'S' \rangle}{\rho\varepsilon} - \frac{1}{3}\underline{\underline{I}}. \quad (2.39)$$

The IARSM closure is completed by using a closure for the pressure/strain-rate correlation (Speziale et al. 1991, Girimaji, 1996; Craft and Launder, 1996):

$$\frac{\langle p'S' \rangle}{\rho\varepsilon} = \mathfrak{T}(\underline{\underline{R}}, \underline{\underline{\tilde{S}}}, \underline{\underline{\tilde{W}}}). \quad (2.40)$$

Solutions to Eqs.(2.39) and (2.40) are generally difficult to develop and, according to Rung et al. (1999), are *unrealizable* for flows with strong streamline curvature.

Gatski (2000) addressed the dilemma associated with the IARSM type closures by developing an explicit algebraic Reynolds stress model (EASRM) for the NR-stress based on Eq. (2.35). In principle, the ten CH-coefficients in Eq. (2.35) can be determined by using “benchmark” flows generated by a closed moment equation for the NR-stress. EASRM closure models vary depending on the number of “integrity” operators used in the representation defined by Eq. (2.35) and the closure models used for Eq. (2.37) above. The RSM “benchmark” flows used in the calibration strategy include flows in rotating frames, flows with curvature, and other flows with difficult features. Unfortunately, according to Rung et al.(1999) and Gatski (see, esp., Gatski,1998; Gatski and Jongen, 2000; Gatski, 2004; and Gatski and Rumsey, 2006) a fully calibrated IASRM model may still produced *unrealizable* behavior. The IASRM and the EASRM class of

algebraic NR-stress

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Quantum Models Base

In a rotating

continuity equation, ∇

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

In the above equation,

pressure fluctuations are

$$p' = \nabla \cdot \left(\frac{\rho}{\rho_0} \mathbf{I} + \mathbf{u}' \mathbf{u}' \right)$$

A formal solution to the

and fluctuations due to

Green's function assoc

left hand side of Eq. (2)

$$p(\mathbf{x}, t) = - \int_{-\infty}^t dt' \iiint_V dV'$$

for non-inertial frame

derived the following

based on a smoothing

$$\hat{\mathbf{z}} = \frac{\mathbf{A}^T \cdot \mathbf{B} \cdot \mathbf{A}}{\text{tr}(\mathbf{A}^T \cdot \mathbf{B} \cdot \mathbf{A})}$$

algebraic NR-stress closure models do not provide a practical closure for the RANS-equation in support of engineering design and analysis.

Closure Models Based on a Space-Time Smoothing Hypothesis

In a rotating frame of reference, turbulent velocity fluctuations satisfy the continuity equation, $\nabla \cdot \underline{u}' = 0$, and the following dynamic equation (Parks et al., 1998):

$$\frac{\partial \underline{u}'}{\partial t} + \langle \underline{u} \rangle \cdot \nabla \underline{u}' - \nu \nabla^2 \underline{u}' = -\underline{u}' \cdot \langle \underline{F} \rangle - \underline{f}'. \quad (2.41)$$

In the above equation, the vector $\underline{f}'$ represents the fluctuating acceleration caused by pressure fluctuations and fluctuations in the *instantaneous* Reynolds stress:

$$\underline{f}' \equiv \nabla \cdot \left[\frac{\underline{p}'}{\rho} \underline{I} + \underline{u}' \underline{u}' - \langle \underline{u}' \underline{u}' \rangle \right]. \quad (2.42)$$

A formal solution to the above equation that neglects fluctuations due to initial conditions and fluctuations due to boundary conditions can be expressed in terms of a mean field Greens function associated with the convective/viscous parabolic operator defined by the left hand side of Eq. (2.41) above:

$$\underline{u}'(\underline{x}, t) = - \int_{-\infty}^t dt \iiint_V d\hat{V} \left\{ G(\underline{x}, t; \hat{\underline{x}}, \hat{t}) [\underline{u}'(\hat{\underline{x}}, \hat{t}) \cdot \langle \underline{F} \rangle(\hat{\underline{x}}, \hat{t}) + \underline{f}'(\hat{\underline{x}}, \hat{t})] \right\} \quad (2.43)$$

For non-inertial frames (i.e., $\langle \underline{F} \rangle = \nabla \langle \underline{u} \rangle$), Parks et al. (1998) used Eq. (2.43) and derived the following non-negative mapping between the prestress and the NR-stress based on a smoothing approximation:

$$\underline{\underline{R}} = \frac{\underline{\underline{A}}^T \cdot \underline{\underline{B}} \cdot \underline{\underline{A}}}{\text{tr}(\underline{\underline{A}}^T \cdot \underline{\underline{B}} \cdot \underline{\underline{A}})} \quad , \quad \underline{\underline{A}} \equiv [\underline{I} + \tau_R \langle \underline{F} \rangle]^{-1} \quad , \quad \underline{\underline{B}} \equiv \frac{\langle \underline{f}' \underline{f}' \rangle}{\text{tr} \langle \underline{f}' \underline{f}' \rangle} . \quad (2.44)$$

Clearly, if the prestress

the satisfies C2 for a

basis for the developm

Parks (1997) a

operator that

its time derivative:

$$\underline{\underline{B}} = \frac{1}{3} \underline{\underline{I}} - \frac{\underline{\underline{S}}}{\epsilon} < \underline{\underline{S}} > . (1)$$

A generalized convect

of turbulent flows:

$$\frac{\partial_2 < \underline{\underline{S}} >}{\partial t} = \frac{\partial < \underline{\underline{S}} >}{\partial t} + <$$

The above operator is

(2.46) reduces to the c

If $\underline{\underline{B}} = \frac{1}{3} \underline{\underline{I}}$, t

prestress (URIPS-) c

introduced by Boussin

$$\underline{\underline{\tau}} = \frac{\underline{\underline{A}}^T \cdot \underline{\underline{A}}}{\text{tr}(\underline{\underline{A}}^T \cdot \underline{\underline{A}})} .$$

For simple shear flo

boundary between

$$1 \leq R_{xx} = R_{yy} \leq 1/3$$

Clearly, if the prestress operator is non-negative, then Eq. (2.44) produces a NR-stress that satisfies C2 for all flows (see above model characteristics). This result provides the basis for the development of the URAPS-closure in Chapter 3.

Parks (1997) and Weispfennig (1997) assumed that the prestress operator $\underline{\underline{B}}$ is an *objective* operator that depends on the turbulent time scale k/ε , the mean strain rate, and its time derivative:

$$\underline{\underline{B}} = \frac{1}{3} \underline{\underline{I}} + \underline{\underline{B}} \left(\frac{k}{\varepsilon} \langle \underline{\underline{S}} \rangle, \left(\frac{k}{\varepsilon} \right)^2 \frac{\delta_a \langle \underline{\underline{S}} \rangle}{\delta t} \right). \quad (2.45)$$

A generalized convective derivative was used to account for mean field memory effects of turbulent flows:

$$\begin{aligned} \frac{\delta_a \langle \underline{\underline{S}} \rangle}{\delta t} \equiv & \frac{\partial \langle \underline{\underline{S}} \rangle}{\partial t} + \langle \underline{\underline{u}} \rangle \cdot \nabla \langle \underline{\underline{S}} \rangle + \langle \underline{\underline{W}} \rangle \cdot \langle \underline{\underline{S}} \rangle + \langle \underline{\underline{S}} \rangle \cdot \langle \underline{\underline{W}} \rangle^T \\ & + \zeta [\langle \underline{\underline{S}} \rangle \cdot \langle \underline{\underline{S}} \rangle + \langle \underline{\underline{S}} \rangle \cdot \langle \underline{\underline{S}} \rangle] \end{aligned} \quad (2.46)$$

The above operator is *objective* (pg. 181, Parks, 1997) for $-\infty < \zeta < +\infty$. For $\zeta = 0$, Eq. (2.46) reduces to the classical Jaumann derivative (see pg. 250 in Bird et al., 2007).

If $\underline{\underline{B}} = \underline{\underline{I}}/3$, then Eq. (2.44) reduces to the universal, realizable, isotropic, prestress (URIPS-) closure and provides a significant generalization for Eq. (2.33) introduced by Boussinesq more than 130 years ago:

$$\underline{\underline{R}} = \frac{\underline{\underline{A}}^T \cdot \underline{\underline{A}}}{\text{tr}(\underline{\underline{A}}^T \cdot \underline{\underline{A}})} \quad , \quad \underline{\underline{A}} \equiv [\underline{\underline{I}} + \underline{\underline{K}}]^{-1} \quad , \quad \underline{\underline{K}} \equiv \tau_R \langle \underline{\underline{F}} \rangle \quad , \quad \text{tr}(\underline{\underline{K}}) = 0 \quad . \quad (2.47)$$

For simple shear flows, the energy states for the URIPS-closure are all located on the boundary between the 1st and 2nd Sextets of the energy simplex: $0 \leq R_{xx} = R_{yy} \leq 1/3 \leq R_{zz} \leq 1$. For $\underline{\underline{K}} = \underline{\underline{0}}$, the NR-stress is isotropic, $\underline{\underline{R}} = \underline{\underline{I}}/3$. For no

rotation and as $\|\underline{\underline{K}}\| \rightarrow \infty$, the turbulent energy is shifted to the axial component of the fluctuating velocity (i.e., $R_{zz} \rightarrow 1$). Also, for no rotation, the locus of anisotropic states predicted by the URIPS-closure follows the DNS results in the core region of the flow field: $10 \leq y^+ \leq 100$. For $\tilde{\Gamma} = 0$, the turbulence is isotropic (see Point A of Figure 1.1). As $\tilde{\Gamma} \rightarrow \infty$, the QNR-form is elongated into an infinite needle (see Point C of Figure 1.1).

If $\underline{\underline{K}} = \underline{\underline{0}}$, Eq. (2.47) implies that $\underline{\underline{R}} = \underline{\underline{I}}/3$. If $0 < \|\underline{\underline{K}}\| \ll 1$, then a representation of the operator in terms of a power series of $\underline{\underline{K}}$ can be expressed as (see Kantorovich and Akilov, 1964)

$$\underline{\underline{A}} = \underline{\underline{I}} - \underline{\underline{K}} + \underline{\underline{K}} \cdot \underline{\underline{K}} + \dots \quad (2.48)$$

For small $\|\underline{\underline{K}}\|$, Eqs.(2.47) and (2.48) imply that

$$\underline{\underline{R}} = \frac{\underline{\underline{A}}^T \cdot \underline{\underline{A}}}{\text{tr}(\underline{\underline{A}}^T \cdot \underline{\underline{A}})} \xrightarrow{\|\underline{\underline{K}}\| \rightarrow 0} \frac{1}{3} - \frac{2}{3} \tau_R < \underline{\underline{S}} > . \quad (2.49)$$

The Coriolis acceleration formally drops out of the representation at first order; therefore, the URIPS-closure reduces to the B-closure for $\|\underline{\underline{K}}\| \ll 1$.

2.7 Conclusions

DNS results and experimental data for channel flows with and without rotation show significant primary and secondary normal stress differences. In contrast to this evidence, the traditional B-closure predicts that the primary and secondary normal stress differences for simple shear are zero for non-rotating and for rotating simple shear flows. For non-rotating channel flows, the energy states are all in the 2nd Sextet Of the energy

simplex. However, for rotating simple shear flows, the normal components of the NR-stress are distributed among all six sextets in the energy simplex. For rotating simple shear, the anisotropic states may be prolate-like with $\text{III}_b > 0$, or oblate-like with $\text{III}_b < 0$. For rotating channel flows, the variation of $\tilde{\Gamma}$ is not symmetric about the center of the channel. On the HP-side of a fully-developed channel flow there is a region where the mean velocity profile is a linear function of the crossstream coordinate and the intrinsic absolute mean vorticity is zero. On the LP side of the channel, the maximum velocity occurs at the same position where the total shear stress is zero. On the LP-side of the channel, both $\tilde{\Gamma}$ and $\tilde{\Omega}$ are negative and the anisotropic states near the isotropic state are oblate-like.

DNS results clearly show that the NR-stress is not *objective*; therefore, the eigenvalues of the NR-stress depend on the angular velocity of a non-inertial frame. Closure models built on the hypothesis that the NR-stress is *objective* and, thereby, represented in terms of objective basis operators are inconsistent with the physical nature of a fluctuating velocity field governed by the Navier-Stokes equation.

The preclosure theory developed by Parks et al. (1998) is a fundamentally different approach which relates the NR-stress to the mean field based on a Green's function representation of the fluctuating velocity. The isotropic assumption for the prestress which lead to the URIPS-closure is realizable for all flows. The preclosure theory can be further developed to include anisotropy and also preserve the non-negative feature of the prestress to develop a universal and realizable algebraic closure for the NR-NR-stress.

CHAPTER 3

URAPS-CLOSURE FOR THE NORMALIZED REYNOLDS STRESS

3.1 Introduction

The preclosure theory by Parks et al. (1998) discussed in Chapter 2 provides an explicit relation for the NR-stress in terms of the prestress. This theory (see Eq. 2.44) is extended for rotating flows and a summary of the preclosure theory for non-inertial frames is presented in Section 3.2 below. This Chapter is focused on developing a closure for the prestress that appears in the preclosure equation. A hypothesis will be presented for closing the prestress in the following Section 3.3. An irreducible representation based on the Cayley- Hamilton theorem for the prestress resulting in an implicit algebraic relation for the NR-stress is developed. Further, the eigenvalues of the prestress closure are analyzed to understand the conditions of realizability for all flows. From a study of the various boundaries on the NR-eigenvalue simplex, a domain for the coefficients that appear in the prestress is identified such that the CH-mapping of the NR-stress into the prestress is non-negative for any choice of the coefficients within the domain. This ensures that the NR-stress from the preclosure theory is also realizable. A numerical strategy to solve the implicit closure relation is also discussed. The scalar-valued turbulent transport time scale introduced by the preclosure theory is used as the phenomenological transport time in the equation for the turbulent kinetic energy and in the equation for the turbulent dissipation.

3.2 Hydrodynamic Preclosure

Preclosure theory (Parks et al. 1998) briefly discussed in Chapter 2 is presented in detail for non-inertial frames in this section. Although the derivations are clearly presented in Parks (1997), and Weispfennig (1997), the derivation is also discussed in detail in this Chapter for non-inertial frames to understand the kinematic operators associated with rotating flows. Equation for the fluctuating velocity in Eq. (2.41) for rotating flows remains the same except for a term that involves the frame rotation dyad through $\langle \underline{\underline{F}} \rangle (\equiv \nabla \langle \underline{u} \rangle + 2 \underline{\underline{\Omega}})$. The fluctuations within the flow domain in Eq. (2.41) are caused by (1) a convective coupling between the mean velocity gradient and the fluctuating velocity, (2) fluctuations in Coriolis forces (3) pressure fluctuations and (4) fluctuations in instantaneous Reynolds stress. A formal representation for the fluctuating velocity in terms of a Green's function has been presented in Eq. (2.43) discussed in Chapter 2. An unclosed equation for the fluctuating correlation $\langle \underline{u}' \underline{u}' \rangle (\underline{x}, t)$ (i.e. Reynolds momentum flux) can be developed from Eq. (2.43).

Unclosed Integral Equation for the Reynolds Momentum Flux

An unclosed integral equation for $\langle \underline{u}' \underline{u}' \rangle (\underline{x}, t)$ can be developed from Eq. (2.43) by first multiplying with the fluctuating velocity and then averaging the result over the turbulent ensemble of realizations as shown in Eq. (3.1) below.

$$\langle \underline{u}' \underline{u}' \rangle (\underline{x}, t) = - \int_{-\infty}^t dt \iiint_V d\underline{\underline{V}} G_v(\underline{x}, t; \underline{\hat{x}}, \hat{t}) [\langle \underline{u}' \underline{\hat{u}}' \rangle \cdot \langle \underline{\underline{\hat{F}}} \rangle + \langle \underline{u}' \underline{\hat{f}}' \rangle] . \quad (3.1)$$

Eq. (3.1) shows that $\langle \underline{u}' \underline{u}' \rangle (\underline{x}, t)$ is caused by $\langle \underline{u}' \underline{\hat{u}}' \rangle (\underline{x}, t; \underline{\hat{x}}, \hat{t}) \cdot \langle \underline{\underline{\hat{F}}} \rangle (\underline{\hat{x}}, \hat{t})$ and $\langle \underline{u}' \underline{\hat{f}}' \rangle (\underline{x}, t; \underline{\hat{x}}, \hat{t})$. The convective/viscous Green's function, $G_v(\underline{x}, t; \underline{\hat{x}}, \hat{t})$ accounts for

the transport of momentum from $(\underline{\hat{x}}, \hat{t})$ to $(\underline{x}, t)$. The utility of Eq. (3.1), which is unclosed, is that it clearly identifies two space-time turbulent correlations (i.e., $\langle \underline{u}' \underline{\hat{u}}' \rangle(\underline{x}, t; \underline{\hat{x}}, \hat{t})$ and $\langle \underline{u}' \underline{\hat{f}}' \rangle(\underline{x}, t; \underline{\hat{x}}, \hat{t})$) as important factors in determining the local behavior of $\langle \underline{u}' \underline{u}' \rangle(\underline{x}, t)$. In addition, and most significantly, Eq. (3.1) also shows that $\langle \underline{\underline{F}} \rangle (\equiv \nabla \langle \underline{u} \rangle + 2 \underline{\underline{\Omega}})$ is the primary connection between the mean field and the Reynolds stress. An analogous representation for the correlation $\langle \underline{\hat{f}}' \underline{u}' \rangle(\underline{x}, t)$ follows directly from Eq. (3.1):

$$\langle \underline{\hat{f}}' \underline{u}' \rangle(\underline{x}, t) = - \int_{-\infty}^t dt \iiint_V dV G_v(\underline{x}, t; \underline{\hat{x}}, \hat{t}) [\langle \underline{\hat{f}}' \underline{\hat{u}}' \rangle \cdot \langle \underline{\hat{F}} \rangle + \langle \underline{\hat{f}}' \underline{\hat{f}}' \rangle]. \quad (3.2)$$

Representations similar to Eq. (3.2) for the turbulent flux of a passive scalar near a solid/fluid interface have been used by Petty (1975), Hill and Petty (1996), and Parks and Petty (1999). Petty and Lyons (1985) also used a result analogous to Eq. (3.2) to study the effect of dilute polymer solutions on the Reynolds momentum flux.

Smoothing approximation

The single-point point correlations $\langle \underline{u}' \underline{u}' \rangle$ and $\langle \underline{\hat{f}}' \underline{u}' \rangle$ given by Eqs. (3.1) and (3.2) depend on the space/time relaxation of the Green's function and the space/time relaxation of specific turbulent correlations. In a frame of reference moving with the mean velocity, the convective/viscous Green's function is spatially peaked for small values of $|\underline{x} - \underline{\hat{x}}|$. The Green's function acts like a delta distribution on a time scale for which turbulent correlations become uncorrelated. If the space/time turbulent correlations have finite memories and if a single scalar-valued memory function $M_R(\underline{x}, |\underline{x} - \underline{\hat{x}}|)$ or time

scale is used to characterize all space/time correlations, then the application of a *spatial smoothing approximation* to Eqs. (3.1) and (3.2) yields the following algebraic equations for $\langle \underline{u}'\underline{u}' \rangle$ and $\langle \underline{f}'\underline{u}' \rangle$ (see Parks et al., 1998):

$$\langle \underline{u}'\underline{u}' \rangle(\underline{x}, t) = -\tau_R [\langle \underline{u}'\underline{u}' \rangle \cdot \langle \underline{\underline{F}} \rangle + \langle \underline{u}'\underline{f}' \rangle] \quad (3.3)$$

$$\langle \underline{f}'\underline{u}' \rangle(\underline{x}, t) = -\tau_R [\langle \underline{f}'\underline{u}' \rangle \cdot \langle \underline{\underline{F}} \rangle + \langle \underline{f}'\underline{f}' \rangle]. \quad (3.4)$$

where τ_R is the turbulent relaxation time defined as below in Eq. (3.5)

$$\tau_R \equiv \int_{-\infty}^t d\hat{t} \{ M_R(\underline{x}, |t - \hat{t}|) \iiint_V d\hat{V} G_v(\underline{x}, t; \hat{\underline{x}}, \hat{t}) \}. \quad (3.5)$$

Combining Eqs. (3.3) and (3.4), a relation between $\langle \underline{u}'\underline{u}' \rangle$ and $\langle \underline{f}'\underline{f}' \rangle$ can be obtained as in Eq. (3.6) below.

$$\langle \underline{u}'\underline{u}' \rangle = \tau_R^2 (\underline{\underline{A}}^T \cdot \langle \underline{f}'\underline{f}' \rangle \cdot \underline{\underline{A}}) \quad (3.6)$$

where the operator $\underline{\underline{A}} (\equiv [\underline{\underline{I}} + \tau_R \langle \underline{\underline{F}} \rangle]^{-1})$ is related to the turbulent relaxation time τ_R , the mean velocity gradient and the frame rotation rate. Rearranging the Eq. (3.6) yields the preclosure relation presented in Eq. (2.44).

As discussed in Chapter 2, both the operators $\underline{\underline{R}} (\equiv \frac{\langle \underline{u}'\underline{u}' \rangle}{\text{tr} \langle \underline{u}'\underline{u}' \rangle})$ and

$\underline{\underline{B}} (\equiv \frac{\langle \underline{f}'\underline{f}' \rangle}{\text{tr} \langle \underline{f}'\underline{f}' \rangle})$ are symmetric and non-negative and have a unit trace. A relation for

the prestress should essentially be formulated to be symmetric and non-negative and would result in a realizable NR-stress closure. The kinematic operator $\underline{\underline{K}}$ and the preclosure operator $\underline{\underline{A}} \equiv [\underline{\underline{I}} + \underline{\underline{K}}]^{-1}$ are important to understand the applicability of the

closure. The general preclosure operator for the benchmark flows discussed in Chapter 2 can be written as below (see Appendix G for details)

$$\begin{aligned}\underline{\underline{K}} &= \tau_R \Gamma_{yz} \underline{e}_y \underline{e}_z + 2 \tau_R \Omega_x (\underline{e}_y \underline{e}_z - \underline{e}_z \underline{e}_y) \\ &= N_\Gamma \underline{e}_y \underline{e}_z + N_\Omega (\underline{e}_y \underline{e}_z - \underline{e}_z \underline{e}_y)\end{aligned}\quad (3.7)$$

where $N_\Gamma = \tau_R \Gamma_{yz}$ and $N_\Omega = 2 \tau_R \Omega_x$. The preclosure operator is,

$$\underline{\underline{A}} = \frac{(1 + N_\Gamma N_\Omega + N_\Omega^2) \underline{e}_x \underline{e}_x + \underline{e}_y \underline{e}_y + \underline{e}_z \underline{e}_z - (N_\Gamma + N_\Omega) \underline{e}_y \underline{e}_z + N_\Omega \underline{e}_z \underline{e}_y}{(1 + N_\Gamma N_\Omega + N_\Omega^2)}.\quad (3.8)$$

The operator $\underline{\underline{A}}$ above as shown has an inverse in the case when the denominator

$$(1 + N_\Gamma N_\Omega + N_\Omega^2) \neq 0. \text{ For } N_\Omega = \frac{-1 \pm \sqrt{N_\Gamma^2 - 4}}{2}, \text{ the inverse of } \underline{\underline{A}} \text{ does not exist.}$$

Example of such points would be $(N_\Gamma, N_\Omega) = (3, -\frac{1}{2} \pm \frac{\sqrt{5}}{2}), (4, -\frac{1}{2} \pm \sqrt{3})$ etc. for rotating simple shear flows. However, to avoid this, the adjugate operator $\underline{\underline{C}} = \text{adj}(\underline{\underline{I}} + \underline{\underline{K}})$ is used so that the determinant is cancelled in the numerator and denominator in the limiting case of the determinant $\det(\underline{\underline{C}}) = 0$. This is used in the subsequent numerical implementation of the preclosure theory.

$$\underline{\underline{R}} = \frac{\underline{\underline{A}}^T \cdot \underline{\underline{B}} \cdot \underline{\underline{A}}}{\text{tr}(\underline{\underline{A}}^T \cdot \underline{\underline{B}} \cdot \underline{\underline{A}})} = \frac{\underline{\underline{C}}^T \cdot \underline{\underline{B}} \cdot \underline{\underline{C}}}{\text{tr}(\underline{\underline{C}}^T \cdot \underline{\underline{B}} \cdot \underline{\underline{C}})} \quad ; \quad \underline{\underline{C}} = \text{adj}(\underline{\underline{I}} + \underline{\underline{K}})\quad (3.9)$$

The above operator for the specific benchmark flows will be discussed in the Chapters 5, 6 and 7.

3.3 Prestress Closure and “Extra” Anisotropy

The preclosure equation expressed by Eq. (3.9) shifts the closure problem from the normalized Reynolds stress $\underline{\underline{R}}$ to the normalized prestress $\underline{\underline{B}}$. Most significantly, $\underline{\underline{B}}$ must be modeled to be non-negative. So a closure for $\underline{\underline{B}}$, such that it is realizable and satisfies $Q(\underline{z}) \equiv \underline{\underline{B}} : \underline{z}\underline{z} = \frac{(\underline{f}' \cdot \underline{u}')(\underline{f}' \cdot \underline{z})}{\text{tr} \langle \underline{u}'\underline{u}' \rangle} \geq 0$; $\text{tr}(\underline{\underline{B}}) = 1$ should be developed.

Closure Hypothesis

As noted in Chapter 2, $\underline{f}'$ represents the fluctuating acceleration caused by pressure fluctuations and fluctuations in the *instantaneous* Reynolds stress. The prestress $\underline{\underline{B}}$ is intrinsically related to the pressure fluctuations. As noted in Eq. (3.3), the correlation $\langle \underline{f}'\underline{f}' \rangle$ is related to $\langle \underline{u}'\underline{f}' \rangle$ through mean field operator $\langle \underline{F} \rangle$. It is interesting to note from the equation for fluctuating velocity that the correlation ($\langle \underline{u}'\underline{f}' \rangle + \langle \underline{u}'\underline{f}' \rangle$) is linked to the pressure/strain-rate (pressure redistribution) term as shown in Eq. (3.10) below.

$$\begin{aligned}
 & (\langle \underline{u}'\underline{f}' \rangle + \langle \underline{f}'\underline{u}' \rangle) \\
 &= \langle -\frac{1}{\rho} \nabla \cdot [(\underline{u}'\underline{p}')\underline{\underline{I}} + \underline{\underline{I}}(\underline{u}'\underline{p}')] + \frac{\underline{p}'}{\rho} [\nabla \underline{u}' + (\nabla \underline{u}')^T] - \nabla \cdot \underline{u}'\underline{u}'\underline{u}' \rangle \quad (3.10) \\
 &= [-(\nabla \langle \underline{u}'\frac{\underline{p}'}{\rho} \rangle)^T - (\nabla \langle \frac{\underline{p}'}{\rho} \underline{u}' \rangle) - \nabla \cdot \langle \underline{u}'\underline{u}'\underline{u}' \rangle] + 2[\langle \frac{\underline{p}'}{\rho} \underline{\underline{S}}' \rangle]
 \end{aligned}$$

The first term in the LHS is related to the turbulent mixing and the second term is the pressure/strain-rate term in the transport equation for fluctuating Reynolds stress (see Eq. (2.37)). Because $\underline{\underline{B}}$ is closely linked to $\langle \underline{u}'\underline{f}' \rangle$ and hence to the pressure/strain

correlation (see Eq. 3.10), the prestress operator can be modeled using similar kinematic variables as for the pressure/strain-rate correlation.

In second order closure models, the pressure/strain-rate terms have also been modeled to be functions of the mean strain rate and the Reynolds stress itself as discussed in Chapter 2 (see esp. Eq. 2.40). A key hypothesis in this research is that the pre-stress operator $\underline{\underline{B}}$ is a dyadic-valued function of $\underline{\underline{R}}$. With $\underline{\underline{B}} = \underline{\underline{\mathfrak{B}}}(\underline{\underline{R}})$, which is similar to the Rotta-conjecture (Rotta, 1951) for the pressure/strain rate, the following CH representation forms the prestress closure. The hypothesis that $\underline{\underline{B}} = \underline{\underline{\mathfrak{P}}}(\underline{\underline{R}})$ may provide a practical closure to the preclosure theory.

Cayley – Hamilton theorem and non-negative mapping

If $\underline{\underline{B}}$ is a continuous function of $\underline{\underline{R}}$, then an irreducible representation of $\underline{\underline{B}}$ in terms $\underline{\underline{R}}$ and its invariants can be developed by the application of the Cayley- Hamilton theorem from linear algebra (Frazer et al., 1960). An uncalibrated closure for the prestress can be written as

$$\underline{\underline{B}} = C_0 \underline{\underline{I}} + C_1(\underline{\underline{R}}) + C_2(\underline{\underline{R}} \cdot \underline{\underline{R}}) \quad (3.11)$$

C_0, C_1 and C_2 can be functions of the invariants of $\underline{\underline{R}}$. Since $\text{tr}(\underline{\underline{B}}) = 1$ and $\text{tr}(\underline{\underline{R}}) = 1$, one of the three coefficients can be eliminated and with two coefficients remaining.

$$\underline{\underline{B}} = \frac{\underline{\underline{I}}}{3} + C_1(\Pi_R, \text{III}_R) \left(\underline{\underline{R}} - \frac{\underline{\underline{I}}}{3} \right) + C_2(\Pi_R, \text{III}_R) \left(\underline{\underline{R}} \cdot \underline{\underline{R}} - \Pi_R \frac{\underline{\underline{I}}}{3} \right) \quad (3.12)$$

$$\Pi_R \equiv \sum_{i=1}^3 \lambda_{R,i}^2 \quad \text{and} \quad \text{III}_R \equiv \sum_{i=1}^3 \lambda_{R,i}^3 \quad (3.13)$$

The **eigenvalues** of these two real, symmetric, and non-negative operators are related to each **by** the following equation:

$$\lambda_{B,i} = \frac{1}{3} + C_1(\lambda_{R,i} - \frac{1}{3}) + C_2(\lambda_{R,i}^2 - \frac{\Pi_R}{3}) \quad , \quad i = 1, 2, 3 \quad (3.14)$$

The **three** eigenvalues of $\underline{\underline{B}}$ and $\underline{\underline{R}}$ satisfy the following conditions: $0 \leq \lambda_1 \leq \lambda_2 \leq \lambda_3 \leq 1$ and $\lambda_1 + \lambda_2 + \lambda_3 = 1$. The non-negative mapping of $\underline{\underline{R}}$ into $\underline{\underline{B}}$ is equivalent to a mapping of **the** 1st Sextet into itself, which also corresponds to a mapping of the L-diagram (see Figure 1.1) into itself. It follows from the above equation that the eigenvalues of $\underline{\underline{R}}$ on the **vertices** and in the center of the λ_R – triangle , map onto similar points on the λ_B – **triangle** (see Figure 3.1). The three boundaries of the λ_R – triangle are mapped onto **corresponding** boundaries of the λ_B – triangle , provided Eq. (3.15) below is **satisfied**.

$$C_1 + C_2 \Pi_R = 1 \quad (\text{only on the boundaries of the } \lambda_R \text{ – triangle}). \quad (3.15)$$

One of the eigenvalues is zero on each of the boundaries of the λ_R – triangle . In order to map **the** boundaries of λ_R – triangle onto λ_B – triangle plane, then Eq. (3.15) results in Eq. (3.16) below by direct substitution of $\lambda_{B,3} = \lambda_{R,3} = 0$ (for one of the boundary **lines**).

$$\underline{\underline{B}} = \underline{\underline{R}} + C_2(\underline{\underline{R}} \cdot \underline{\underline{R}} - \Pi_R \underline{\underline{R}}). \quad (3.16)$$

From the above result, without loss of generality, the mapping from λ_R – triangle to λ_B – triangle can now be expressed as in Eq. (3.17) where C_2 could be a function of the **invariants** as below.

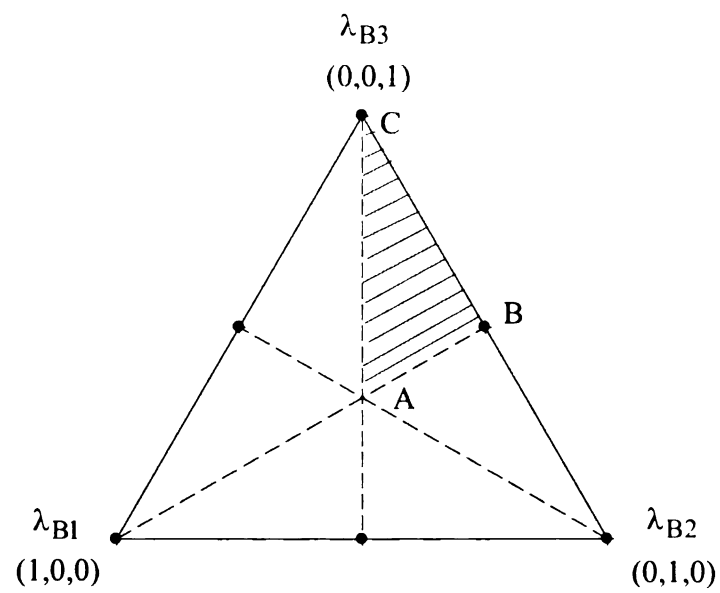
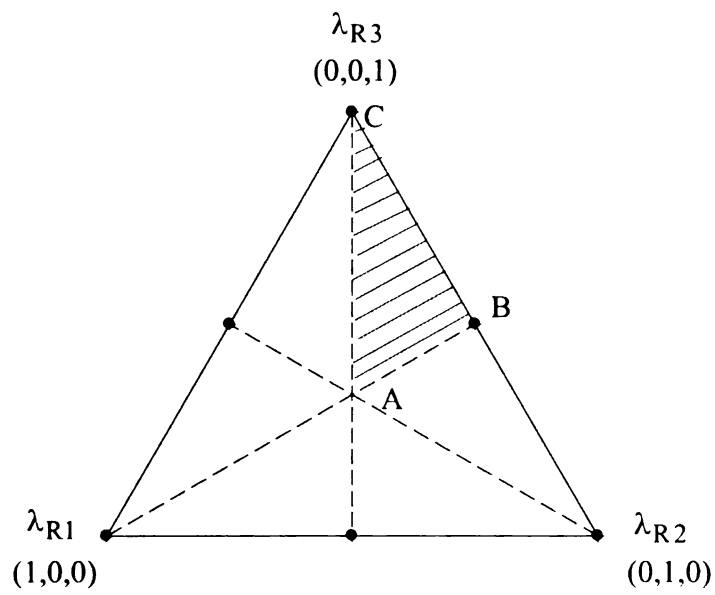


Figure 3.1 Eigenvalue mapping from the λ_R – triangle to λ_B – triangle

$$\underline{\underline{B}} = \underline{\underline{R}} + C_2(\underline{\underline{R}} \cdot \underline{\underline{R}} - \Pi_R \underline{\underline{R}}) + \beta(\underline{\underline{R}} - \frac{1}{3} \underline{\underline{I}}) 27 \det(\underline{\underline{R}}) \quad (3.17)$$

The coefficient C_2 is chosen to be a linear function of the second invariant Π_R such that deviation of normal components of $\underline{\underline{R}}$ from isotropic states is along the $R_{xx} = R_{yy}$ line for simple shear flows. This is because DNS results for channel flow as discussed in Chapter 2 shows that the results close to the center of the channel are along $R_{xx} = R_{yy}$.

$$C_2 = -\alpha(\Pi_R - \frac{1}{3}) \quad (3.18)$$

$$\underline{\underline{B}} = \underline{\underline{R}} - \alpha(\Pi_R - \frac{1}{3})(\underline{\underline{R}} \cdot \underline{\underline{R}} - \Pi_R \underline{\underline{R}}) + \beta(\underline{\underline{R}} - \frac{1}{3} \underline{\underline{I}}) 27 \det(\underline{\underline{R}}). \quad (3.19)$$

The above representation is therefore associated with linear and nonlinear terms. The following discussion is related to developing the coefficients α and β such that Eq. (3.19) above is realizable.

Realizability Domain:

The eigenvalue relation for the prestress relation, Eq. (3.19) is,

$$\lambda_{Bi} = \lambda_{Ri} + \beta[27 \det(\underline{\underline{R}})(\lambda_{Ri} - \frac{1}{3})] - \alpha[(\Pi_R - \frac{1}{3})(\lambda_{Ri}^2 - \Pi_R \lambda_{Ri})] \quad , \quad i = 1, 2, 3. \quad (3.20)$$

In the following sections, the lower and upper bounds on α and β are determined from an analysis of Eq. (3.20) on the boundaries of the 1st sextet.

Lower bound on α :

The CH-mapping of $\underline{\underline{R}}$ into $\underline{\underline{B}}$ for planar anisotropic states (see BC-boundary of Figure 3.1) implies that

$$\lambda_{B3} = \lambda_{R3} - \alpha \left[\left(\Pi_R - \frac{1}{3} \right) (\lambda_{R3}^2 - \Pi_R \lambda_{R3}) \right] , \quad \Pi_R = (1 - \lambda_{R3})^2 + \lambda_{R3}^2 . \quad (3.21)$$

On the BC-boundary, $\lambda_{R1} = 0$, $\lambda_{R2} = 1 - \lambda_{R3}$, and $1/2 \leq \lambda_{R3} \leq 1$. By direct substitution, it follows from Eq. (3.21) that Point B is mapped into itself for all values of α ; and, Point C is mapped into itself for all values of the α . However, some interior points on the BC-boundary may not map onto the BC-boundary of the 1st sextet if α is too small or too large. Eq. (3.22) can be used to determine a critical value of α that would give $\lambda_{B3} = 1$ for specific values of λ_{R3} :

$$\alpha_{\min} \equiv \frac{\lambda_{R3} - 1}{(\Pi_R - 1/3)(\lambda_{R3}^2 - \Pi_R \lambda_{R3})} , \quad \lambda_{R3} \xrightarrow{\text{CH-mapping}} \lambda_{B3} = 1. \quad (3.22)$$

As indicated by Figure 3.2, a plot of $\alpha_{\min}$ vs. λ_{R3} shows that $\max(\alpha_{\min}) = -3/2$, which is an *a priori* universal lower bound on the α -coefficient.

Upper bound on α :

Eq. (3.21) can also be used to eliminate redundancy in the CH-mapping. For example, a critical value of α can be determined that would give $\lambda_{B3} = 1/2$ for specific values of λ_{R3} :

$$\alpha_{\max} \equiv \frac{\lambda_{R3} - 1/2}{(\Pi_R - 1/3)(\lambda_{R3}^2 - \Pi_R \lambda_{R3})} , \quad \lambda_{R3} \xrightarrow{\text{CH-mapping}} \lambda_{B3} = 1/2. \quad (3.23)$$

As indicated by Figure 3.3, a plot of $\alpha_{\max}$ vs. λ_{R3} shows that $\min(\alpha_{\max}) = +9$, which is an *a priori* universal upper bound on the α -coefficient. It is noteworthy that for $\alpha = 9$ the CH-mapping gives

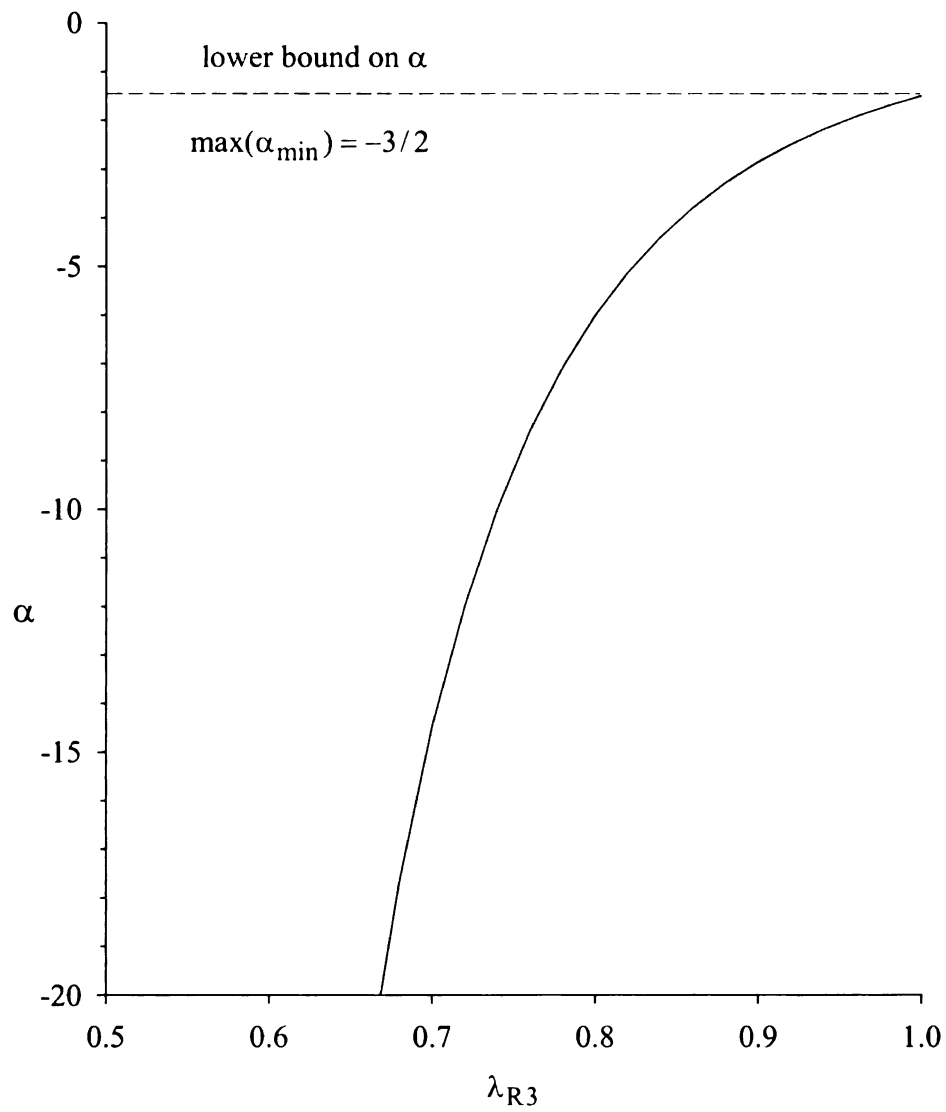


Figure 3.2 The influence of λ_{R3} on $\alpha_{\min}$ for $\lambda_{B3} = 1$.

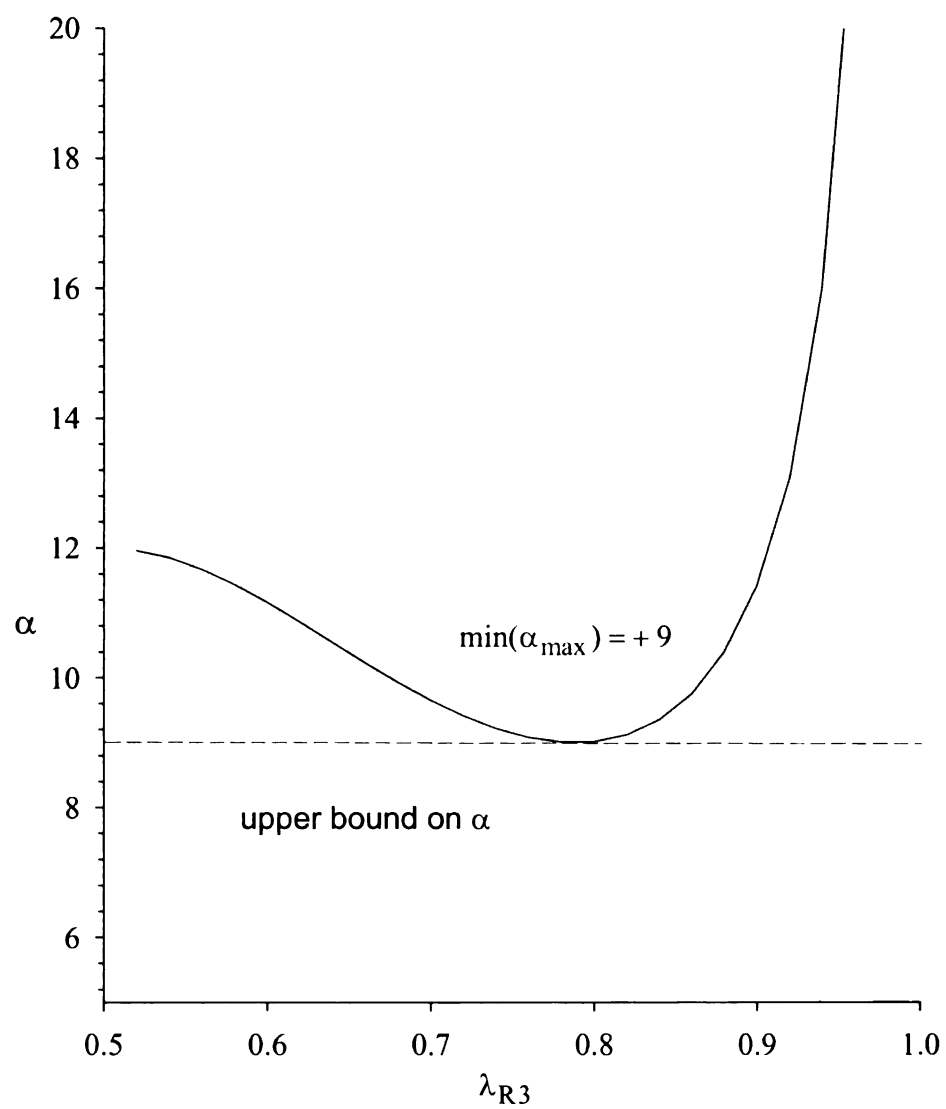


Figure 3.3 The influence of λ_{R3} on $\alpha_{\max}$ for $\lambda_{B3} = 1/2$.

$$(\lambda_{R1}, \lambda_{R2}, \lambda_{R3}) = (0, 0.2, 0.8) \xrightarrow{\alpha=9} (0, 0.5, 0.5) = (\lambda_{B1}, \lambda_{B2}, \lambda_{B3}). \quad (3.24)$$

For $\alpha > 9$, points on the BC-boundary of the 1st Sextet would be mapped into planar anisotropic states associated with the 6th Sextet. Thus, the upper bound on the α -coefficient eliminates redundant non-negative states whereas the lower bound on the α -coefficient eliminates unrealizable states.

Lower bound on β

The CH-mapping of $\underline{\underline{R}}$ into $\underline{\underline{B}}$ for oblate anisotropic states (see AB-boundary of Figure 3.1) implies that

$$\begin{aligned} \lambda_{B3} &= \lambda_{R3} - \alpha \left[\left(\Pi_R - \frac{1}{3} \right) (\lambda_{R3}^2 - \Pi_R \lambda_{R3}) \right] + \beta \left[27 \det(\underline{\underline{R}}) \left(\lambda_{R3} - \frac{1}{3} \right) \right] \\ \Pi_R &= 2 \left(\frac{1 - \lambda_{R3}}{2} \right)^2 + \lambda_{R3}^2, \quad \det(\underline{\underline{R}}) = \left(\frac{1 - \lambda_{R3}}{2} \right)^2 \lambda_{R3} \end{aligned} \quad (3.25)$$

On the AB-boundary (see Figure 3.1), $\lambda_{R2} = \lambda_{R3}$, $\lambda_{R1} = 1 - 2\lambda_{R3}$, and $1/3 \leq \lambda_{R3} \leq 1/2$. By direct substitution, it follows from Eq. (3.25) that Point B is mapped into itself for all values of α and β ; and, Point A is mapped into itself for all values of the α and β . However, for $\alpha_{\min} \leq \alpha \leq \alpha_{\max}$, some interior points on the AB-boundary may not map onto the AB-boundary of the 1st Sextet if β is too small or too large. For a specific value of α and λ_{R3} , Eq. (3.25) implies that a minimum value of β for $\lambda_{B3} = 1/3$ is given by

$$\beta_{\min} = \frac{1/3 - \lambda_{R3} + \alpha [(\Pi_R - \frac{1}{3})(\lambda_{R3}^2 - \Pi_R \lambda_{R3})]}{[27 \det(\underline{\underline{R}})(\lambda_{R3} - \frac{1}{3})]} \quad (3.26)$$

$$\lambda_{R3} \xrightarrow[\alpha_{\min} < \alpha < \alpha_{\max}]{\text{CH-mapping}} \lambda_{B3} = 1/3$$

As indicated by Figure 3.4, a plot of $\beta_{\min}$ vs. λ_{R3} shows that $\max(\beta_{\min}) = -1$ for all α in the interval $\alpha_{\min} < \alpha < \alpha_{\max}$. The parameter $\max(\beta_{\min})$ is an *a priori* universal lower bound on the β -coefficient. If $\beta = -1$, then

$$(\lambda_{R1}, \lambda_{R2}, \lambda_{R3}) \rightarrow (1/3, 1/3, 1/3) \Rightarrow (\lambda_{B1}, \lambda_{B2}, \lambda_{B3}) \rightarrow (1/3, 1/3, 1/3). \quad (3.27)$$

However, if $\beta < -1$, then some oblate anisotropic states on the AB-boundary between the 1st and 6th Sextets will be mapped into prolate anisotropic states between the 3rd and 4th Sextets. The lower bound on β removes the redundancy of the CH mapping.

Upper bound on β

For $\alpha_{\min} \leq \alpha \leq \alpha_{\max}$, some interior points on the AB-boundary may not map onto the AB-boundary of the 1st Sextet if β is too large. Eq. (3.25) implies that for a specific value of α and λ_{R3} , a maximum value of β occurs for $\lambda_{B3} = 1/2$. Therefore, with $\lambda_{B3} = 1/2$, Eq. (3.25) can be used to define $\beta_{\max}$ as follows:

$$\beta_{\max} = \frac{1/2 - \lambda_{R3} + \alpha [(\Pi_R - \frac{1}{3})(\lambda_{R3}^2 - \Pi_R \lambda_{R3})]}{[27 \det(\underline{\underline{R}})(\lambda_{R3} - \frac{1}{3})]} \quad (3.28)$$

$$\lambda_{R3} \xrightarrow[\alpha_{\min} < \alpha < \alpha_{\max}]{\text{CH-mapping}} \lambda_{B3} = 1/2$$

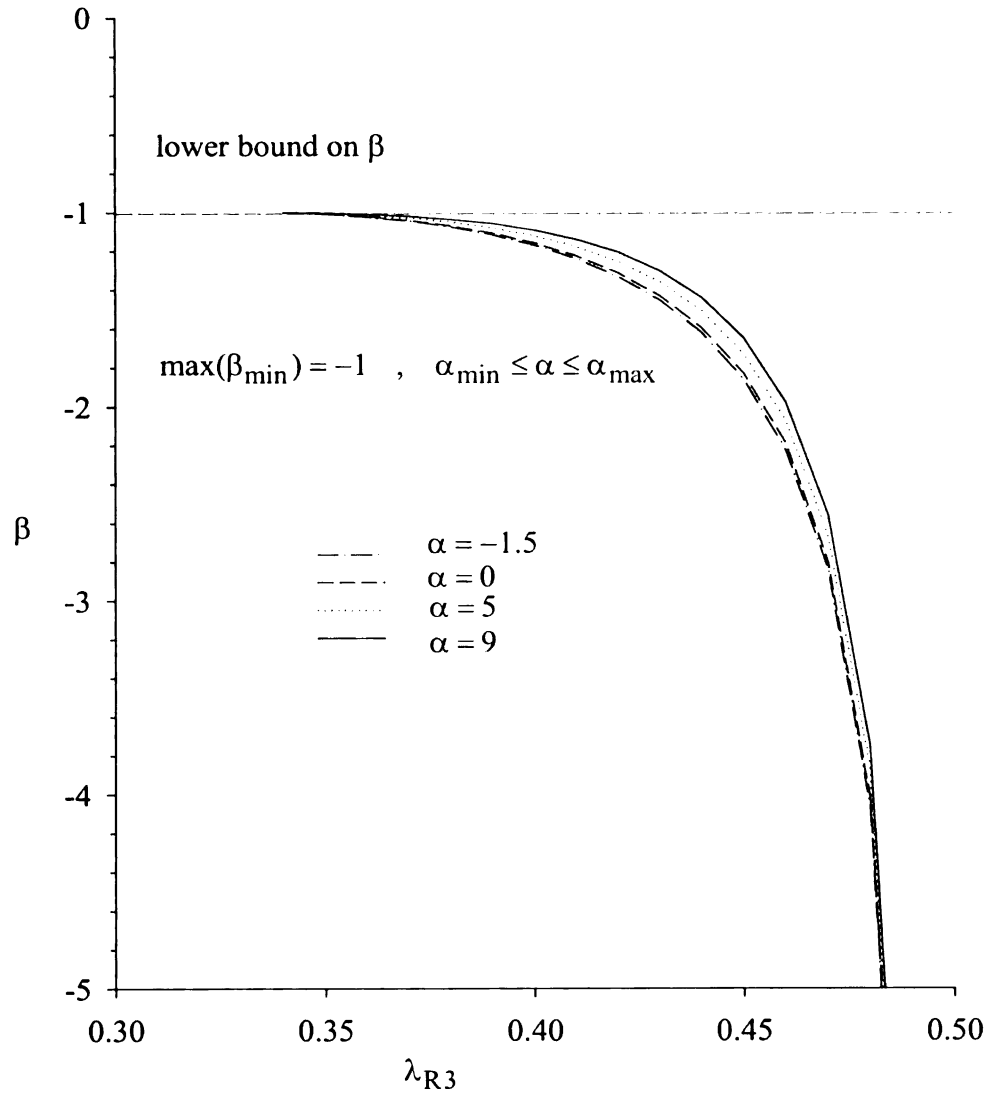


Figure 3.4 The influence of λ_{R3} on $\beta_{\min}$ for $\lambda_{B3} = 1/3$ and $\alpha_{\min} < \alpha < \alpha_{\max}$.

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Figure 3.5 shows how $\beta_{\max}$ depends on λ_{R3} for specific values of α . Clearly, the minimum upper bound (i.e., $\min\beta_{\max}$) occurs at $\lambda_{R3} \rightarrow 1/2$ and depends on α . The results shown in Figure 3.5 can be represented by a linear relation:

$$\min(\beta_{\max}) = \alpha / 27 + 4/9 \quad , \quad \alpha_{\min} \leq \alpha \leq \alpha_{\max} . \quad (3.29)$$

The parameter $\min\beta_{\max}$ is an *a priori* universal upper bound on the β -coefficient. If $\beta > \min\beta_{\max}$, then a point on the AB-boundary of the 1st Sextet will be mapped into an unrealizable state.

In summary, Eq. (3.20) maps the 1st sextet of the eigenvalue simplex into itself provided α and β satisfy the following two inequalities:

$$-3/2 < \alpha < 9 \quad (3.30)$$

$$-1 < \beta < \alpha / 27 + 4/5 . \quad (3.31)$$

The above theoretical inequalities are universal. Since $\underline{\underline{B}}$ has non negative eigenvalues, $\underline{\underline{R}}$ is *a priori* realizable with non-negative eigenvalues. Figure 3.6 identifies universal bounds for the “extra” anisotropy functions $\alpha(\text{II}_R, \text{III}_R)$ and $\beta(\text{II}_R, \text{III}_R)$, which must be determined from experimental and DNS results for benchmark flows. The physical and mathematical significance of Figure 3.6 is that the CH-mapping of the NR-stress into the prestress is non-negative for all turbulent flows provided the “extra” anisotropy functions are within the realizable region developed. This conclusion does not depend on the benchmark flow selected to calibrate the closure. The bounds are universal and apply for all inertial and non-inertial frames of reference.

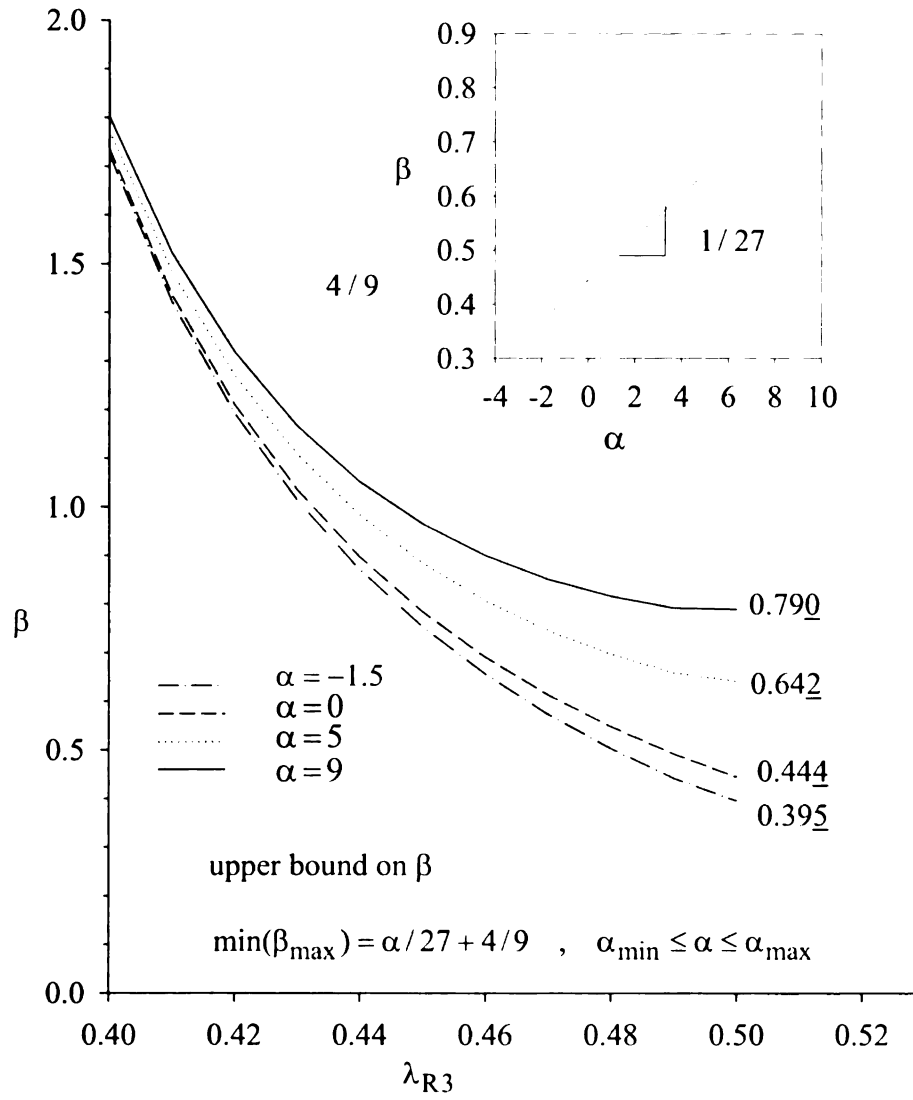


Figure 3.5 The influence of λ_{R3} on $\beta_{\max}$ for $\lambda_{B3} = 1/2$ and $\alpha_{\min} < \alpha < \alpha_{\max}$.

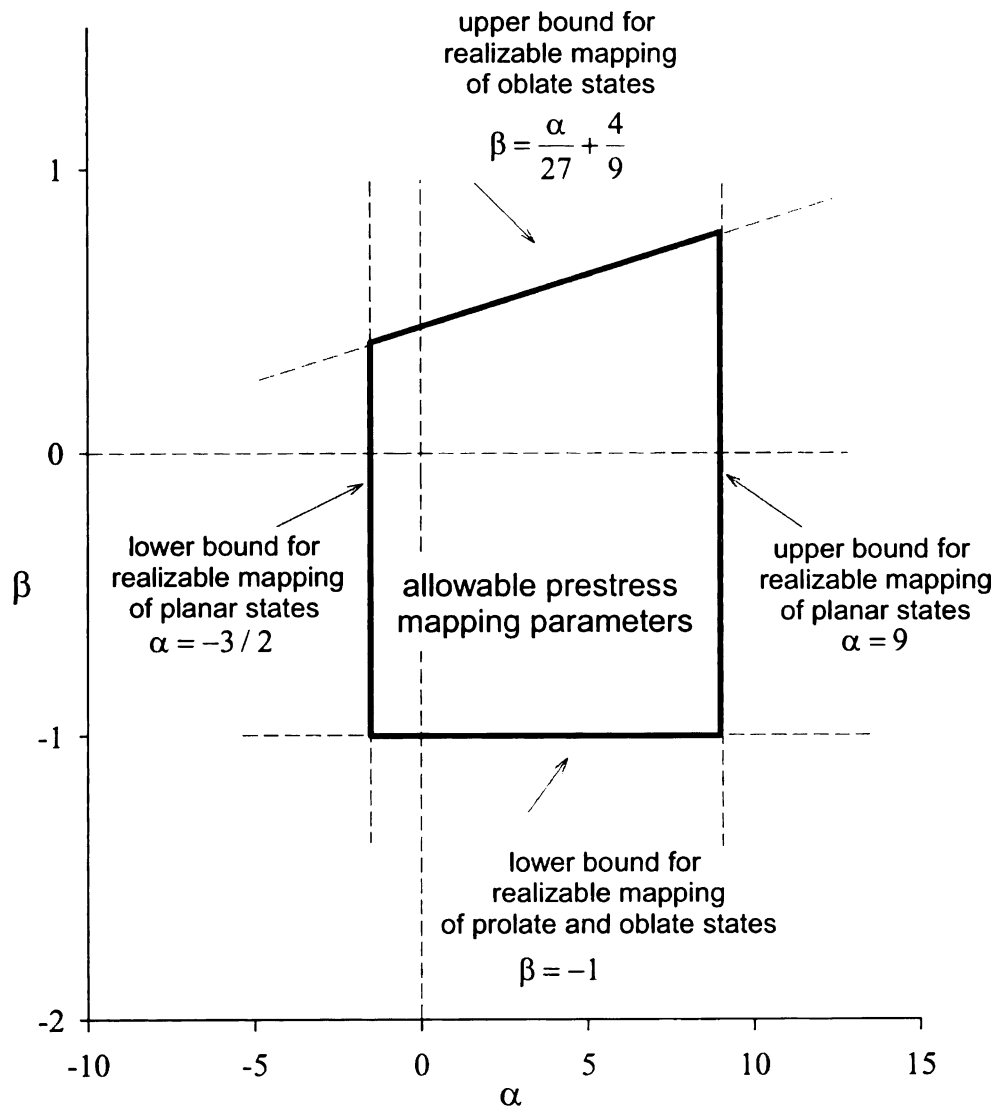


Figure 3.6 Universal Bounds for the “Extra” Anisotropy Functions $\alpha(\text{II}_R, \text{III}_R)$ and $\beta(\text{II}_R, \text{III}_R)$.

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Different classes of turbulent flows may have different “extra” anisotropy functions, but they must all conform to the universal bounds developed herein. In Chapter 4, non-rotating asymptotic homogeneous shear flow is used as a benchmark flow to estimate α and β . These estimates are used to predict the NR-stress for rotating asymptotic homogeneous shear in Chapter 6. All of the NR-stress solutions to the URAPS-closure are realizable.

3.4 URAPS-Closure

The preclosure equation (Eq. (3.9)) and the prestress closure (Eq. (3.12)) can be combined to give a non-linear, implicit, algebraic equation for the NR-stress, referred to herein as the universal realizable anisotropic prestress (URAPS-) closure. The URAPS-closure is a significant generalization of the universal, realizable, isotropic prestress (URIPS-) closure. The resulting URAPS-closure can be formally expressed as

$$\underline{\underline{\mathbf{R}}} = \frac{\underline{\underline{\mathbf{A}}}^T \cdot \underline{\underline{\mathfrak{B}}}(\underline{\underline{\mathbf{R}}}) \cdot \underline{\underline{\mathbf{A}}}}{\text{tr}(\underline{\underline{\mathbf{A}}}^T \cdot \underline{\underline{\mathfrak{B}}}(\underline{\underline{\mathbf{R}}}) \cdot \underline{\underline{\mathbf{A}}})} = \mathfrak{R}(\underline{\underline{\mathbf{K}}}, \underline{\underline{\mathbf{R}}}) . \quad (3.32)$$

The k - and ε - transport equations adopted for the URAPS-closure are briefly presented in the next section. An important requirement of these scalar-valued partial differential equations is that they should produce positive solutions inasmuch as the turbulent kinetic energy k and the turbulent dissipation ε are by definition positive within the flow domain.

The URAPS-closure is *universally realizable* inasmuch as the predicted NR-stress is *realizable* for all rotating and non-rotating rigid-body frames of reference. A calibrated URAPS-closure provides a means to predict the NR-stress for bounded and for

unbounded flows as well as for statistically stationary and non-stationary flows governed by the RANS-equation. Accuracy of the model for a specific class of flows will depend on the previous theoretical ideas and on the benchmark flows used to determine the closure parameters.

3.5 Turbulent Relaxation Time scale

A single time scale, τ_R has been introduced in the preclosure theory and the k and ε equations. Dimensional reasoning suggests that τ_R depends on three local time scales: a viscous time scale, ν/k ; a turbulent time scale, k/ε ; and, a mean field time scale, $\|\langle \underline{\underline{F}} \rangle\|^{-1}$ that includes the time scale associated with the frame rotation.

$$\tau_R \equiv C_{R1} \frac{k}{\varepsilon} \tilde{\tau}_R, \quad \text{where} \quad \tilde{\tau}_R = f_R(\text{Re}_t, N_F) \quad (3.33)$$

Thus, $\tau_R = C_{R1} \tilde{\tau}_R(\text{Re}_t, N_F) k/\varepsilon \xrightarrow{\text{Re}_t \gg 1} C_{R1} \tilde{\tau}_R(\infty, N_F) k/\varepsilon$, where $\text{Re}_t \equiv k^2/(\nu\varepsilon)$,

$N_F \equiv \frac{k}{\varepsilon} \|\underline{\underline{F}}\| = \frac{\text{turbulent time scale}}{\text{mean field time scale}}$, and $\|\langle \underline{\underline{F}} \rangle\| \equiv \sqrt{\langle \underline{\underline{F}} \rangle : \langle \underline{\underline{F}} \rangle^T}$. For turbulent flows

near a solid/fluid interface, the influence of the local Reynolds number will have an important impact on $\tilde{\tau}_R$. A model and calibration for $\tilde{\tau}_R$ will be presented in Chapter 4.

3.6 Transport Equations for Turbulent Kinetic Energy and Dissipation

An unclosed transport equation for the kinetic energy (see Appendix A) and unclosed equation for dissipation equation (see Appendix B) can be derived from the dynamic equation for the fluctuating velocity. However, in the k equation (see Eq. 1.9) the dispersion (or mixing) terms must be modeled as discussed in Chapter 1. Modeled

transport equation presented in Eq. (1.11) is used in this research. The URAPS-closure relies on positive solutions for the k and ε equations for all flows.

The dispersion operator in the k -equation and the ε -equation are the same. An interesting consequence of this assumption is that these equations support the existence of self-similar behavior. That is if $k(\underline{x}, t)$ is proportional to $\varepsilon(\underline{x}, t)$ within a finite space-time domain, then

$$\frac{\tau_R}{k/\varepsilon} = \frac{C_P \mathcal{P} - C_D \varepsilon}{\mathcal{P} - \varepsilon} \quad , \quad \text{provided } \frac{k}{\varepsilon} = \text{constant} . \quad (3.34)$$

The calibration for the “production” and dissipation coefficients, C_P and C_D , as well as the turbulent transport time τ_R must be developed so that solutions to transport equations Eqs. (1.9) and (1.11) yield non-negative solutions for all turbulent flows. In the following chapter, these are determined from benchmark flows discussed in Chapter 2.

3.7 Fixed-Point Mapping and Solution Strategy of the URAPS-closure

Solutions to a calibrated URAPS-equation can be developed by successive substitution. Thus, if $\underline{\underline{R}}_n$ is a positive operator, then the following iterative algorithm yields another positive operator $\underline{\underline{R}}_{n+1}$ that converges to a fixed point of the closure equation.

$$\underline{\underline{R}}_{n+1} = \frac{\underline{\underline{A}}^T \cdot \underline{\underline{\mathfrak{B}}}(\underline{\underline{R}}_n) \cdot \underline{\underline{A}}}{\text{tr}(\underline{\underline{A}}^T \cdot \underline{\underline{\mathfrak{B}}}(\underline{\underline{R}}_n) \cdot \underline{\underline{A}})} = \underline{\underline{\mathfrak{R}}}(\tau_R < \underline{\underline{F}} >, \underline{\underline{R}}_n) . \quad (3.35)$$

An initial guess starting at the isotropic distribution of energy, the prestress for the IPS closure would also be isotropic.

$$\underline{\underline{R}}_0 = \frac{\underline{\underline{I}}}{3} \Rightarrow \quad (3.36)$$

$$\underline{\underline{B}}_1 = \underline{\underline{R}}_0 + C_1(\underline{\underline{R}}_0 - \frac{1}{3}\underline{\underline{I}}) + C_2(\underline{\underline{R}}_0 \cdot \underline{\underline{R}}_0 - \Pi_{\underline{\underline{R}}_0} \underline{\underline{R}}_0) = \frac{\underline{\underline{I}}}{3}$$

$$\underline{\underline{R}}_1 = \frac{\underline{\underline{A}}^T \cdot \underline{\underline{B}}_1(\underline{\underline{R}}_0) \cdot \underline{\underline{A}}}{\text{tr}[\underline{\underline{A}}^T \cdot \underline{\underline{B}}_1(\underline{\underline{R}}_0) \cdot \underline{\underline{A}}]} = \frac{\underline{\underline{A}}^T \cdot \underline{\underline{A}}}{\text{tr}[\underline{\underline{A}}^T \cdot \underline{\underline{A}}]} \quad (\equiv \underline{\underline{K}}) \Rightarrow \quad (3.37)$$

$$\underline{\underline{B}}_2 = \underline{\underline{K}} + C_1(\underline{\underline{K}} - \frac{1}{3}\underline{\underline{I}}) + C_2(\underline{\underline{K}} \cdot \underline{\underline{K}} - \Pi_{\underline{\underline{K}}} \underline{\underline{K}})$$

In subsequent iterations, re-distribution of energy starts to develop for $\underline{\underline{R}}$ which as a result yields anisotropic distribution in $\underline{\underline{B}}$.

$$\underline{\underline{R}}_2 = \frac{\underline{\underline{A}}^T \cdot \underline{\underline{B}}_2(\underline{\underline{R}}_1) \cdot \underline{\underline{A}}}{\text{tr}[\underline{\underline{A}}^T \cdot \underline{\underline{B}}_2(\underline{\underline{R}}_1) \cdot \underline{\underline{A}}]} \Rightarrow \quad (3.38)$$

$$\underline{\underline{B}}_3 = \underline{\underline{R}}_2 + C_1(\underline{\underline{R}}_2 - \frac{1}{3}\underline{\underline{I}}) + C_2(\underline{\underline{R}}_2 \cdot \underline{\underline{R}}_2 - \Pi_{\underline{\underline{R}}_2} \underline{\underline{R}}_2)$$

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The iterations are proceeded till the successive substitution has converged when each of the components of the Reynolds stress remain the same up to certain significant figures.

$$\underline{\underline{R}}_{n+1} = \frac{\underline{\underline{A}}^T \cdot \underline{\underline{B}}_{n+1}(\underline{\underline{R}}_n) \cdot \underline{\underline{A}}}{\text{tr}[\underline{\underline{A}}^T \cdot \underline{\underline{B}}_{n+1}(\underline{\underline{R}}_n) \cdot \underline{\underline{A}}]} \quad (3.39)$$

$$\underline{\underline{B}}_{n+1} = \underline{\underline{R}}_n + C_1(\underline{\underline{R}}_n - \frac{1}{3}\underline{\underline{I}}) + C_2(\underline{\underline{R}}_n \cdot \underline{\underline{R}}_n - \Pi_{\underline{\underline{R}}_n} \underline{\underline{R}}_n) \quad (3.40)$$

$$\lim_{n \rightarrow \infty} \underline{\underline{R}}_n = \underline{\underline{R}}^\star \ni \Re(\tau_R < \underline{\underline{F}} >, \underline{\underline{R}}^\star) = \underline{\underline{R}}^\star$$

In the following Chapter 4, the iteration procedure is illustrated to converge to a fixed final solution independent of the realizable three component initial condition given, for simple shear flows.

3.8 Conclusions

1. Figure 3.7 presents a summary of the above URAPS-closure. The various steps in development of the implicit algebraic closure are shown starting from the Navier Stokes equation. An integral analysis of the equation for fluctuating velocity using Green's function is used to relate turbulent corrections to the mean field kinematics. A smoothing approximation based on the assumption that turbulent correlations have finite memories yields the preclosure theory.
2. The preclosure theory for non-inertial frames relates the Reynolds stress to the prestress through a kinematic operator, $\underline{\underline{K}} = \tau_R (\nabla \langle \underline{u} \rangle + 2 \underline{\underline{\Omega}})$ that takes into account both the mean velocity gradient and the angular velocity of the rotating frame. The preclosure time scale τ_R depends on the turbulent time scale, the mean field time scale, the time scale associated with frame rotation, and a viscous time scale.
3. An implicit closure for the Reynolds stress is obtained by relating the prestress to the NR-stress stress itself. Based on this self-consistent hypothesis, the CH-theorem has been used to find an irreducible representation for the prestress (see Eq.(3.19)). In order for the resultant closure to provide realizable solutions for the NR-stress, CH-coefficients in the prestress relation must remain within the realizable bounds given by Ineq.(3.30) and Ineq.(3.31). The boundaries of the realizable domain are identified by analyzing the eigenvalue relation for $\underline{\underline{R}}$ and $\underline{\underline{B}}$ (see Eq. (3.20)) on the prolate and oblate anisotropic boundaries. The URAPS-closure is thus *a priori* realizable for all flows.

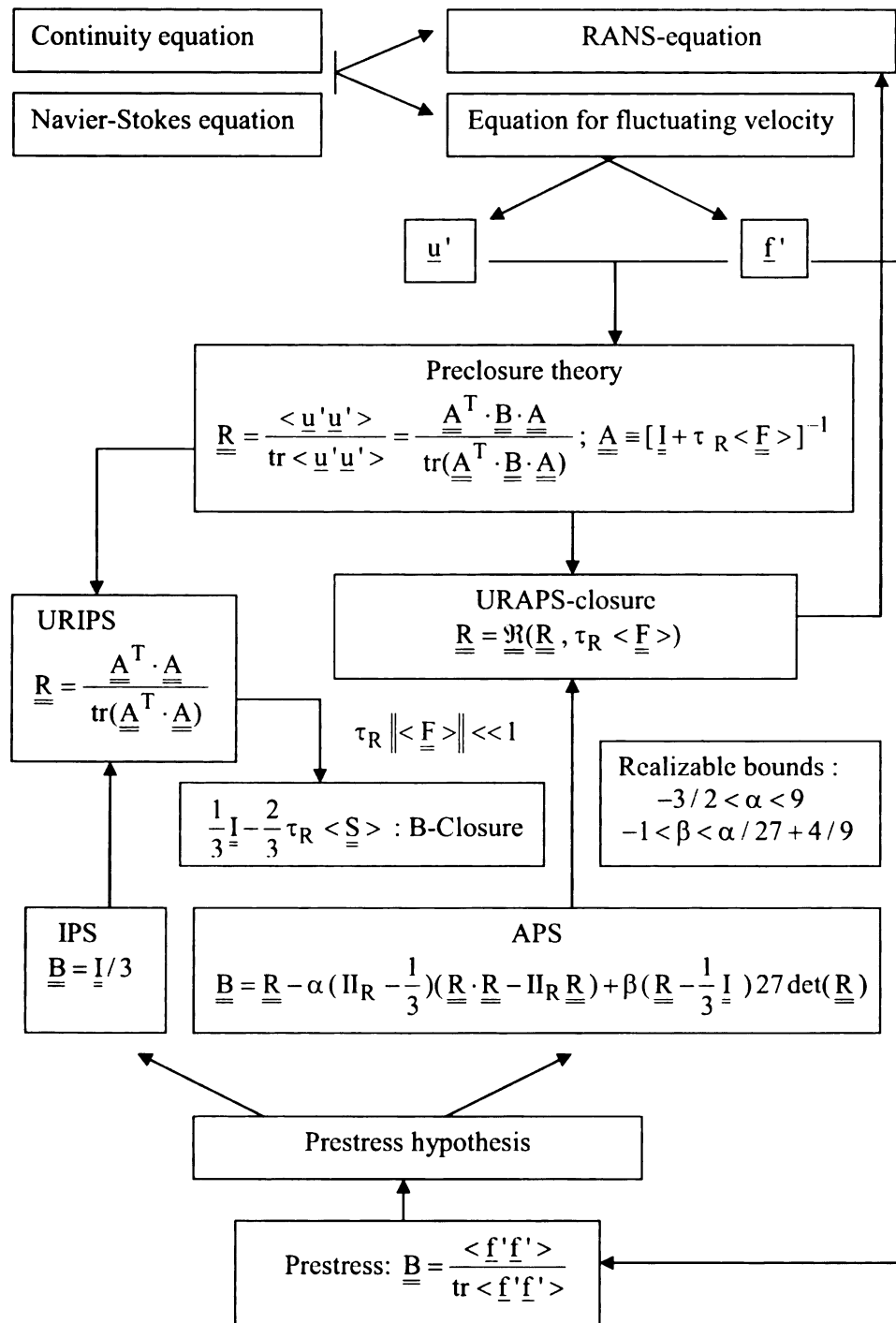


Figure 3.7 Summary of URAPS-Closure

4. Successive substitution method can be used to solve the URAPS-closure starting with an initial state say, isotropic state. The results for the prestress and subsequently the NR-stress are updated until a converged state for the NR-stress is obtained based on the convergence criteria used.

CHAPTER 4

MODEL CALIBRATION

4.1 Introduction

The URAPS-closure requires the determination of the following four dimensionless phenomenological parameters: α , β , C_P , C_D and a model for the turbulent timescale τ_R . In this chapter, the parameters for the URAPS-closure are estimated from the following benchmark flows. 1) non-rotating homogeneous decay; 2) rotating homogeneous decay; and, 3) non-rotating asymptotic homogeneous shear as discussed in Chapter 2. Transport equations for k and ε are simplified for homogeneous flows because of the absence of production and diffusive transport. The coefficient of dissipation is estimated from this flow as described in Section 4.2. From the theoretical analysis of energy spectrum in rotating homogeneous flows, the dimensionless turbulent time scale $\tilde{\tau}_R$ in Eq. (3.33) is formulated in Section 4.3, along with the specification of the coefficients. The coefficients α and β in the prestress relation are estimated using the components equations for the URAPS-closure and from the knowledge of asymptotic states described in Chapter 2 (see Eq. (2.25)) for homogeneous shear using an optimization procedure. Optimization also yields the value of C_{R1} , associated with model for τ_R in Eq. (3.33). Finally, the coefficient C_P is also evaluated from the asymptotic equation for homogeneous shear flows.

4.2 Dissipation Coefficient

For non-rotating isotropic homogeneous decay at high initial turbulent Reynolds numbers, the NR-stress is isotropic and the decay of turbulent kinetic energy and turbulent dissipation are governed by the differential equations, Eqs.(2.29) and (2.30) as described in Chapter 2. The turbulent time scale for this flow is

$$\tau_R = C_{R1} \frac{k}{\varepsilon} \quad \text{for } Re_o \equiv \frac{k_0^2}{\nu \varepsilon_0} \gg 1 \quad (\text{i.e., } \underline{\Omega} = \underline{0}), \quad N_F = 0, \quad \tilde{\tau}_R(N_F) = 1). \quad \text{If } C_D \text{ and } C_{R1}$$

are constant, then the autonomous Eqs. (2.29) and (2.30) imply that

$$\frac{d\varepsilon}{dk} = + \frac{C_D}{C_{R1}} \frac{\varepsilon}{k} \Rightarrow \ln\left(\frac{\varepsilon}{\varepsilon_0}\right) = \frac{C_D}{C_{R1}} \ln\left(\frac{k}{k_0}\right), \quad (4.2)$$

where $k(0) = k_0$ and $\varepsilon(0) = \varepsilon_0$. Figure 4.1 shows the decay process in the k - ε plane parameterized by the dimensionless decay time $\tilde{t} \equiv t \varepsilon_0 / k_0$. The classical experimental

data of Comte-Bellot and Corrsin (1971) for $Re_o = 354$ and 769 (see Figure 2.17)

described in Chapter 2 are used to estimate the coefficient C_D / C_{R1} . The DNS results of

Mansour and Wray (1994) and experimental results of Comte-Bellot and Corrsin (1971)

imply that $C_D / C_{R1} \doteq 1.83$. Parks (1997) incorporated a Reynolds number dependence in

the coefficient C_D / C_{R1} based on a comprehensive temporal analysis of the classical

Kármán-Howarth equation together with the experimental data of Tavoularis et al. (1978)

and the DNS results of Mansour and Wray (1992). In this research, C_D / C_{R1} is assumed

to be a constant for $Re_t \gg 1$. For high Reynolds number homogeneous decay,

$k(t) \geq \varepsilon(t) \rightarrow 0$ and $Re_t \rightarrow \infty$ as $\tilde{t} \rightarrow \infty$. The temporal development of k and ε

reconstructed for homogenous flows as well as predictions for rotating homogeneous

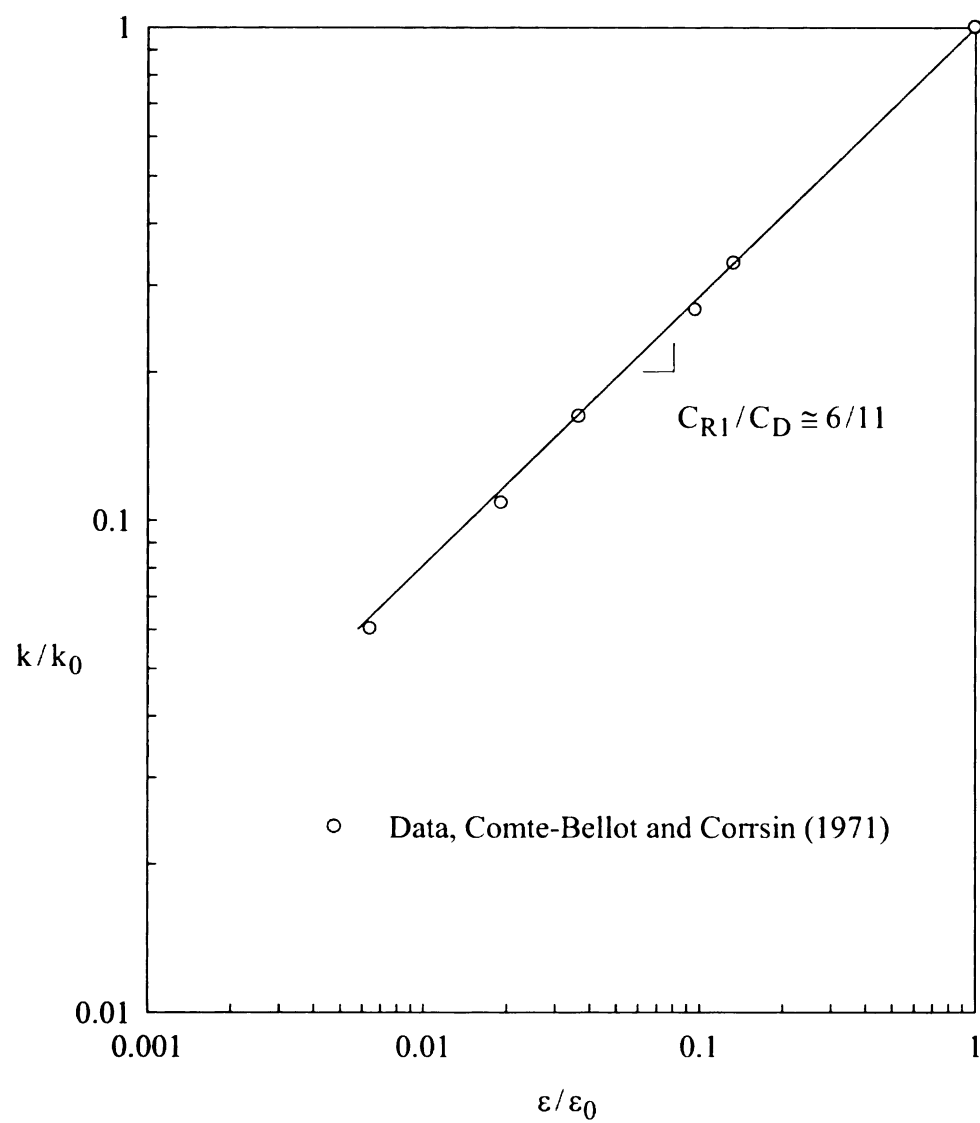


Figure 4.1 Isotropic Homogeneous Decay in an Inertial Frame of Reference

flows which include this coefficient $C_D / C_{R1} \doteq 1.83$, are presented in Chapter 5. In the following section 4.3, the model for turbulent transport time is discussed.

4.3 Turbulent Transport Time

Turbulent relaxation time scale τ_R is which appears in the preclosure theory in Eq. (3.33) is described as a function of the characteristic time scales of turbulence using dimensional reasoning. In Eq. (3.33), the dimensionless turbulent transport time $\tilde{\tau}_R(\text{Re}_t, N_F)$ is defined so that $\tilde{\tau}_R(\infty, 0) = 1$. $\tilde{\tau}_R$ needs to be determined so that solutions to the k -equation and the ε -equation are non-negative functions for all turbulent flows. Analysis of the energy spectrum provides information on the dependence of $\tilde{\tau}_R$ on rotation rate. For the case of rotating homogeneous turbulence, the energy spectrum deviates from the Kolmogorov's $-5/3$ dependence on wave number κ in inertial subrange and shows a κ^{-2} dependence instead (see Zeman, 1995 and Zhou, 1994). In the inertial subrange, the spectrum is represented as

$$E(\kappa) = C_K (\varepsilon)^{2/3} \kappa^{-5/3} \quad \kappa \gg (\Omega^3 / \varepsilon)^{1/2} \quad (4.3)$$

$$E(\kappa) = C_\Omega (\varepsilon \Omega)^{1/2} \kappa^{-2} \quad \kappa \ll (\Omega^3 / \varepsilon)^{1/2} \quad (4.4)$$

where κ is the wave number, C_K is Kolmogorov's constant and $C_\Omega = C_K^{3/4}$. From the spectrum equations (Eqs. 4.3 and 4.4), Park and Chung, (1999) derived an equation for k by integrating the energy spectrum over the entire range of wave length and also an equation for ε by differentiating the k equation with time. This analysis of the spectrum, identified the variation of the C_D / τ_R for the dissipation equation to vary with

dimensionless rotation rate $|\Omega|k/\varepsilon$. The complete summary of the derivation of the key results are presented in Appendix F. Based on Eq. (F.42) for the coefficient of dissipation, a similar dependence on rotation rate is adopted into the model for turbulent time scale. A, non-unique, empirical equation for $\tilde{\tau}_R$ is proposed:

$$\tilde{\tau}_R = \frac{(1 + C_{R3} N_F^n)}{(1 + C_{R2} N_F^n)}. \quad (4.5)$$

The coefficients are assumed to be universal constants and can be determined by well-defined benchmark flow and applied to any other flow. In the limiting cases, $\tilde{\tau}_R(N_F \rightarrow 0) = 1$ and $\tilde{\tau}_R(N_F \rightarrow \infty) = C_{R3}/C_{R2}$. The dependence $\tilde{\tau}_R$ on the group

$\frac{k}{\varepsilon} \parallel F \parallel$ can be deduced from Eq.(F.42) to be $3/2$ since for homogeneous flows,

$$\frac{k}{\varepsilon} \parallel F \parallel = 2\sqrt{2} |\Omega| k / \varepsilon.$$

Figure 4.2 shows the variation of $\frac{C_D}{C_{R1}} \frac{1}{\tilde{\tau}_R}$ in Eq. (F.42) with $|\Omega|k/\varepsilon$. The following are

the noticeable features of the Figure 4.2.

$$\frac{C_D}{C_{R1}} \frac{1}{\tilde{\tau}_R} = \begin{cases} \frac{11}{6} & , \text{ for } |\Omega|k/\varepsilon = 0 \quad (N_F = 0) \\ 2 & , \text{ for } |\Omega|k/\varepsilon = 1 \quad (N_F = 2\sqrt{2}) \\ \frac{8}{3} & , \text{ for } |\Omega|k/\varepsilon = \infty \quad (N_F \rightarrow \infty) \end{cases}. \quad (4.6)$$

(Note that from the above points, it can be seen that $\frac{C_D}{C_{R1}} \frac{(1 + C_{R2}(2\sqrt{2})^{3/2})}{(1 + C_{R3}(2\sqrt{2})^{3/2})} = 2$ and

$\frac{C_D}{C_{R1}} \frac{C_{R2}}{C_{R3}} = \frac{8}{3}$). From Eq. (4.6), with $\frac{C_D}{C_{R1}} = \frac{11}{6}$, the estimates for the coefficients are:

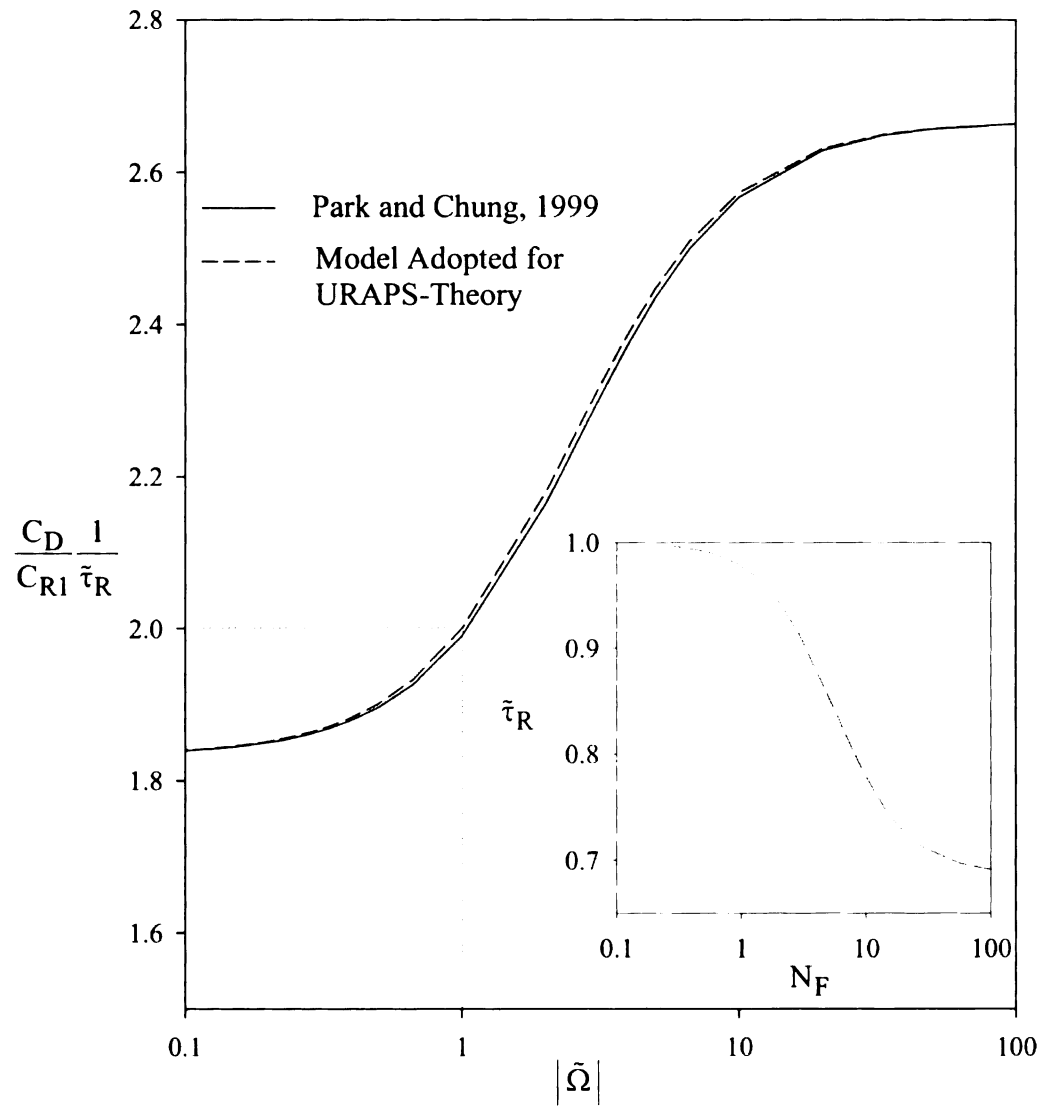


Figure 4.2 The Influence of $|\tilde{\Omega}|$ ($\equiv |\Omega|k/\varepsilon$) on the Turbulent Transport Time Scale

$$(C_{R2}, C_{R3}) = (0.0764, 0.0526) \quad (4.7)$$

The variation of $\tilde{\tau}_R$ with N_F with the above coefficients and $n = 3/2$ is shown in the Figure 4.2. For $0 < N_F < \infty$, $\tilde{\tau}_R$ varies as $0.69 < \tilde{\tau}_R < 1$ with smaller values of $\tilde{\tau}_R$ corresponding to larger values of N_F . The model reconstruction in the phase plane of k and ε for the limiting cases discussed in Eq. (4.6) are discussed in Chapter 5 along with predictions for NR-stress distribution for rotating homogeneous flows using the above calibrated model for turbulent time scale. In the forgoing Section 4.4, the two CH-coefficients related to the “extra” anisotropy terms in the prestress are determined.

4.4 Coefficients of First and Second-Order “Extra” Anisotropy (α and β)

In the following analysis, first the ratio of β/α is obtained from the analysis of the component equations of URAPS-equation for homogeneous shear and using the experimental results described in Table 4.1. All the NR-stress components from the experimental data for asymptotic states cannot be exactly satisfied by the URAPS closure. So an optimization strategy is used to find the coefficients (α and C_{R1}) that closely recover the experimental values.

For non-rotating asymptotic homogeneous shear, the algebraic URAPS-equation depends on three dimensionless groups: $N_\Gamma (\equiv C_{R1} \tilde{\tau}_R \tilde{\Gamma})$; α ; and, β . For simple shear, a relation for the “extra” anisotropy coefficients and the components of the NR-stress can be obtained from an analysis of the component equations of URAPS-equation as shown in Eq. (4.8) below. For details of the derivation, see Appendix G (Eq. G.30)

Table 4.1 Optimum URAPS Closure Coefficients and Stress Components Compared with the Experimental Results of Asymptotic States for Homogeneous Shear.

Model Statistic →	N_F	R_{xx}	R_{yy}	R_{zz}	$-R_{yz}$	N_Γ	α	$-\beta$	C_{R1}
experimental data (Tavoularis and Karnik , 1989)	4. <u>2</u>	0.2 <u>4</u>	0.20 <u>0</u>	0.5 <u>6</u>	0.17 <u>0</u>	n/a	n/a	n/a	n/a
APS-closure	4. <u>2</u>	0.2 <u>5</u>	0.20 <u>0</u>	0.5 <u>5</u>	0.19 <u>0</u>	0.01 <u>3</u>	0.10 <u>0</u>	0.010 <u>0</u>	0.0036 <u>0</u>

$$\frac{\beta}{\alpha} = +3 \frac{(\Pi_R - \frac{1}{3})}{27 \det(\underline{\underline{R}})} \frac{R_{xx} R_{yy}}{(R_{xx} - R_{yy})} (R_{xx} - R_{yy} - \frac{R_{yz}^2}{R_{yy}}). \quad (4.8)$$

where $\Pi_R = \underline{\underline{R}} : \underline{\underline{R}} = R_{xx}^2 + R_{yy}^2 + R_{zz}^2 + 2R_{yz}^2$ and $\det(\underline{\underline{R}}) = R_{xx} R_{yy} R_{zz} - R_{xx} R_{yz}^2$

Eq.(4.8) is used herein to estimate β/α from self-similar (asymptotic) homogeneous shear measurements of Tavoularis and Karnik (1989) with $\tilde{\Gamma} = N_F = 4.2$ summarized in Table 4.1. The components of the Reynolds stress listed in Table 4.1 imply that $\lambda_{R1} = 0.134$; $\lambda_{R2} = 0.236$; and, $\lambda_{R3} = 0.630$. Thus, $\det(\underline{\underline{R}}) = 0.020$ and $\Pi_R = 0.47$. Therefore, the CH-ratio, defined by Eq.(4.8), is $\beta/\alpha = -0.096$. The important conclusion from this estimate is that $\beta/\alpha < 0$; and, $\beta/\alpha \cong -0.1$. For non-rotating simple shear flows, the primary normal stress difference, $R_{zz} - R_{yy}$, is positive; and, the secondary normal stress difference, $R_{yy} - R_{xx}$, is negative (i.e., $R_{yy} < R_{xx} < R_{zz}$) as discussed in Chapter 2. This observation implies that the CH-coefficients α and β for homogeneous simple shear satisfy the following inequalities: $\alpha > 0$ and $\beta < 0$.

The remaining two groups N_F and α (with $\beta = -0.1\alpha$) are estimated by minimizing the following optimization metric over a two dimensional domain where the components of the NR-stress are $R_{xx}^{\text{exp}} = 0.24$, $R_{yy}^{\text{exp}} = 0.2$, $R_{zz}^{\text{exp}} = 0.56$, and $R_{yz}^{\text{exp}} = -0.17$ listed in Table 4.1.

$$\Psi(N_\Gamma, \alpha, \beta) \equiv \left[\left(\frac{R_{xx}^{APS} - R_{xx}^{exp}}{R_{xx}^{exp}} \right)^2 + \left(\frac{R_{yy}^{APS} - R_{yy}^{exp}}{R_{yy}^{exp}} \right)^2 + \left(\frac{R_{zz}^{APS} - R_{zz}^{exp}}{R_{zz}^{exp}} \right)^2 + \left(\frac{R_{yz}^{APS} - R_{yz}^{exp}}{R_{yz}^{exp}} \right)^2 \right]^{1/2} \quad (4.9)$$

A search is conducted over a two dimensional domain defined as $0 \leq N_\Gamma \leq 1$, $0 < \alpha \leq 9$, $\beta = -0.1\alpha$. Figure 4.3 shows several graphs of how $\Psi(N_\Gamma, \alpha, \beta)$ depends on N_Γ for a fixed value of α . The search is conducted subject to the constraint implied by Eq.(4.9) above. The method of successive substitution is used to find the solution to the URAPS-equation for each value of N_Γ . Figures 4.3 and 4.4. show how the optimization is done. Figure 4.3 shows the influence of the “extra” anisotropy coefficient α on the constrained optimization metric Ψ_{min}^α , defined as follows

$$\Psi_{min}^\alpha \equiv \min_{N_\Gamma} (\Psi|_\alpha) \quad , \quad \beta = -0.1\alpha = \text{constant} . \quad (4.10)$$

For each fixed value α , the URAPS-equation for the NR-stress was solved for several values of N_Γ in order to determine Ψ_{min}^α . Figure 4.3 shows an example of various curves constructed with $\alpha = 0.25, 0.1, 1, 5$ and $0 < N_\Gamma < 1$. There appears a Ψ_{min}^α for each α . The $(N_\Gamma)_{min}^\alpha$ corresponding to each of those Ψ_{min}^α are also shown in the figure. Figures 4.3 and 4.4 show that the minimum of $\Psi(N_\Gamma, \alpha, \beta)$ subject to the constraint defined by Eq.(4.9) is almost the same for α in the range $0 < \alpha < 1$ with $\min(\Psi_{min}^\alpha) \cong 0.153 \pm 0.05$.

Figure 4.5 shows the locus of anisotropic states of the NR-stress predicted by the URAPS-closure for four different values of the second-order “extra” anisotropy

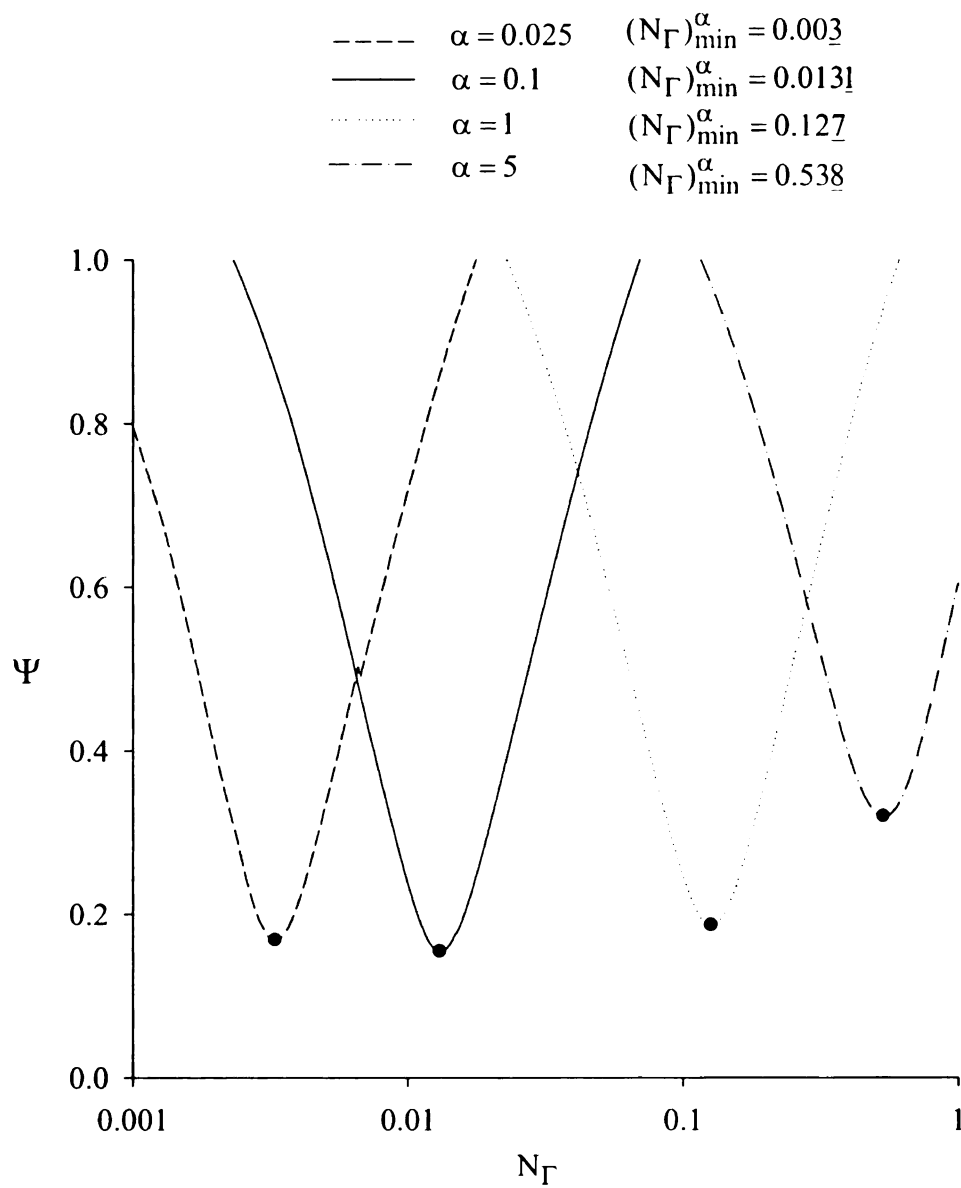


Figure 4.3 Constrained Optimization Strategy for Determining the “Extra” Anisotropy Coefficients, α and β , and the Dimensionless Transport Group, N_Γ , for Non-Rotating Asymptotic Homogeneous Shear.

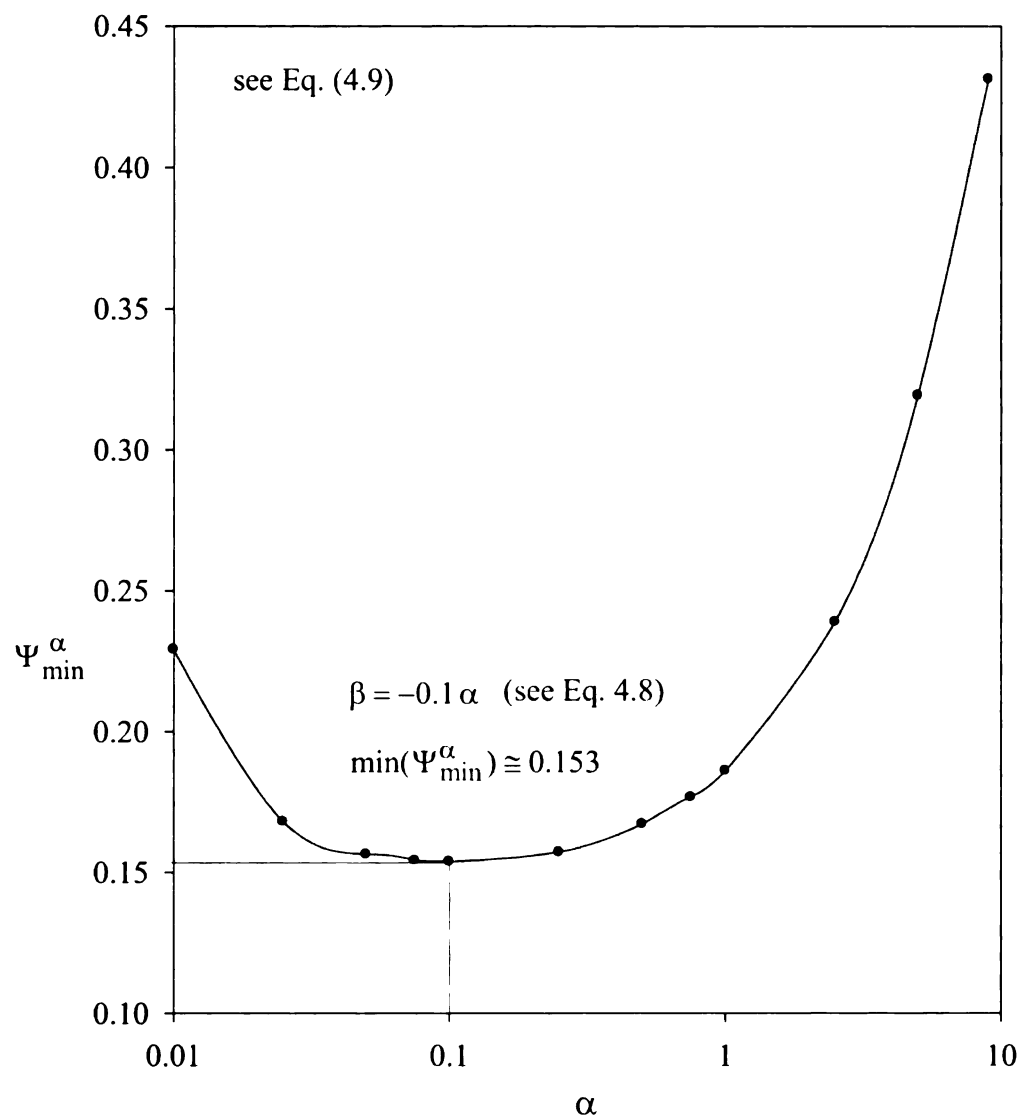


Figure 4.4 The Influence of the “Extra” Anisotropy Coefficient α on the Optimization Metric $\Psi_{\min}^{\alpha}$

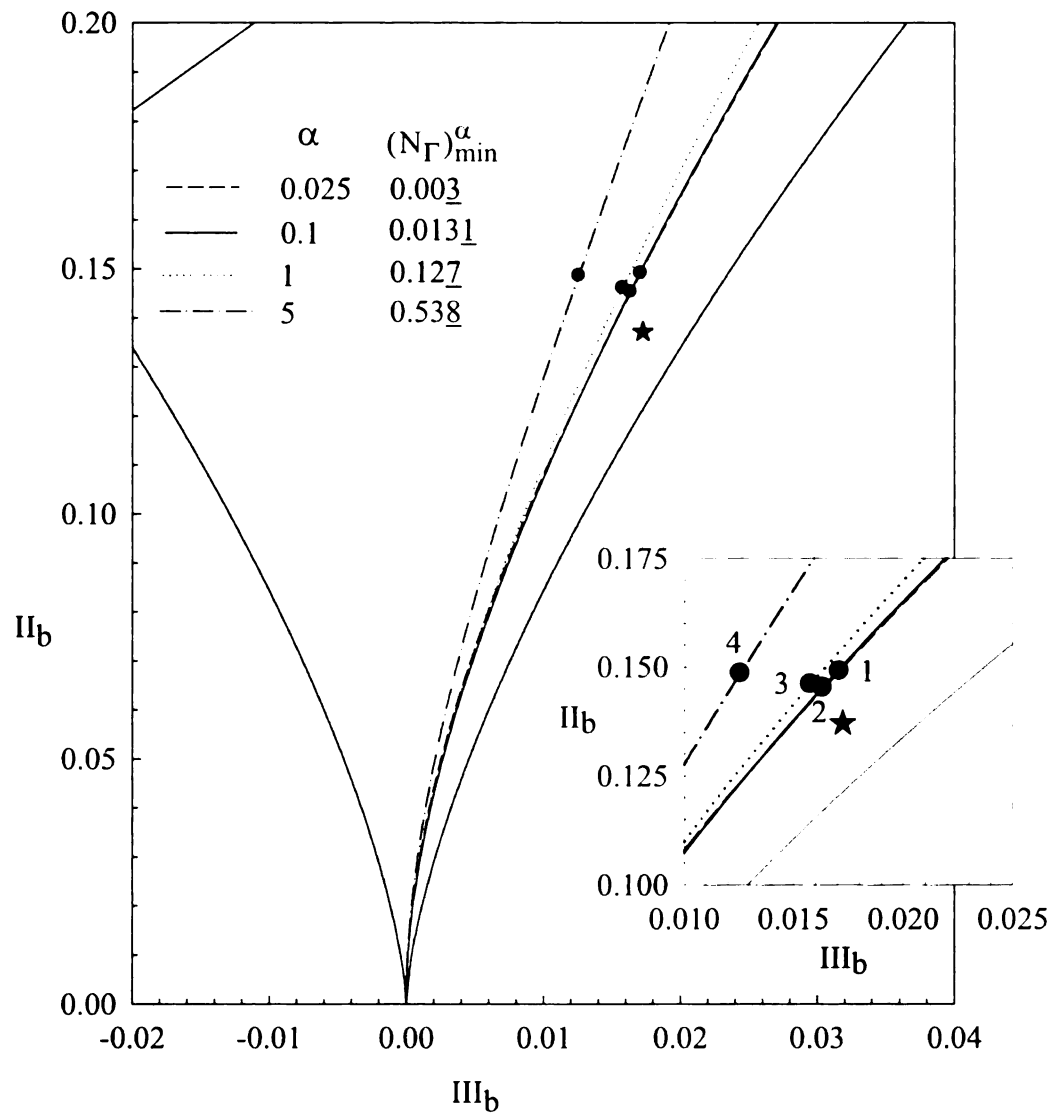


Figure 4.5 Parametric Sensitivity of the Anisotropic States of the NR-Stress Predicted by the URAPS-Closure for Different Values of the Second-Order “Extra” Anisotropy Coefficient (☆-state, experiments, Tavoularis and Karnik, 1989;

coefficient α . Each α -curve is parameterized by $N_\Gamma = C_{R1} \tilde{\tau}_R \tilde{\Gamma}$. The values of N_Γ identified in the caption correspond to the local minima of the optimization metric $\Psi(N_\Gamma)$ defined by Eq.(4.9). Each α -curve has a minimum value of Ψ . The ‘ $\star$ ’ state corresponds to the experimental results for asymptotic homogeneous shear in a non-rotating frame of reference. By definition, $\Psi = 0$ at the ‘ $\star$ ’ state. The inset in Figure 4.5 shows the behavior of the α -curve for $0 \leq N_\Gamma \leq 1$.

The parameter optimization strategy yields the following result:

$$\min_{N_\Gamma, \alpha} \Psi(N_\Gamma, \alpha) \cong 0.153 \Rightarrow \alpha \cong +0.1, \quad \beta \cong -0.01, \quad N_\Gamma \cong +0.013. \quad (4.11)$$

Table 4.1 compares the components of the NR-stress predicted by the calibrated URAPS-closure with the experimental results for asymptotic homogeneous shear in a non-rotating frame of reference. The URAPS-closure optimized state differs from the experiments by 0.01 for the normal components and by about 0.02 for the shear component.

The coefficient C_{R1} can be estimated from the following relationship between N_Γ and $\tilde{\Gamma}$.

$$\begin{aligned} N_\Gamma &= C_{R1} \tilde{\Gamma} \tilde{\tau}_R(\infty, N_F) \\ &= C_{R1} \tilde{\Gamma} \frac{(1 + C_{R3} N_F^n)}{(1 + C_{R2} N_F^n)}. \end{aligned} \quad (4.12)$$

For homogenous shear with no rotation, $N_F = \tilde{\Gamma} = 4.2$ (see Table 4.1) and $N_\Gamma = 0.013$ (from optimization). Eq.(4.12) implies that $C_{R1} = 0.0036$. The three coefficients C_{R1} , α , and β are assumed to be universal constants. In the section below, the coefficient of production in dissipation equation is determined.

4.5 Production Coefficient

The k and Equations for the k and ε yield a relation for the turbulent time scale as a function of the production to dissipation ratio as described earlier in Eq. (3.34). This relation is used to determine the model coefficient C_P / C_{R1} inasmuch as

$$C_P / C_{R1} = \frac{C_D / C_{R1} + ([\mathcal{P} / \varepsilon] - 1) \bar{\tau}_R(\infty, N_F)}{[\mathcal{P} / \varepsilon]}, \quad [\mathcal{P} / \varepsilon] \equiv -2 N_F R_{yz}^{APS}(N_\Gamma) \quad (4.13)$$

Eq.(4.13) and the experimental data in Table 4.1 for $N_F = 4.2$ indicate that $\bar{\tau}_R(\infty, 4.2) = 0.88$. With $N_\Gamma = 0.013$, $R_{yz}^{APS} = -0.19$ (see Table 4.1), it follows directly from Eq.(4.13) that $[\mathcal{P} / \varepsilon] = 1.596$. Finally, with $C_D / C_{R1} = 1.8$, Eq.(4.13) implies that $C_P / C_{R1} = 1.5$.

In summary, Table 4.2 lists the eight URAPS closure coefficients for $Re_t \rightarrow \infty$. The specific benchmark flows used for evaluation along with the calibrated coefficients are shown. In the forthcoming Chapters 5, 6, and 7 on application of the URAPS closure, the above closure coefficients are used assuming them to be universal constants.

4.6 Contraction Mapping and “Extra” Anisotropy

The following two cases for shear flow are illustrated. These cases appear in Ch.6 and the significance of anisotropy will be discussed in Ch.6, for this specific flow case along with the relevant equations.

Case 1: $\tilde{\Omega} / \tilde{\Gamma} = 0$ and $\tilde{\Gamma} = 4$;

Case2: $\tilde{\Omega} / \tilde{\Gamma} = -5/8$ and $\tilde{\Gamma} = 11$.)

Table 4.2 Summary of calibration for the URAPS closure parameters.

Parameter	Benchmark Flow
$\frac{C_D}{C_{R1}} \doteq 1.8$	Non-rotating homogeneous decay. Parameter selected using high Reynolds number, initial decay data (Batchelor and Townsend, 1948, Comte-Bellot and Corrsin, 1971). Independent of other calibration parameters.
$n = \frac{3}{2}$	Rotating homogeneous decay. Parameter selected based on theoretical analysis of energy spectrum (Park and Chung, 1999).
$(C_{R2}, C_{R3}) = (0.076, 0.053)$	Rotating homogeneous decay. Parameters selected based on energy spectrum analysis (Park and Chung, 1999). Depends only on an estimate for C_D / C_{R1} .
$\frac{\beta}{\alpha} = -0.1$	Non-rotating simple shear. Parameter selected based on URAPS closure and the components of $\underline{\underline{R}}$ for asymptotic homogeneous shear (Tavoularis and Karnik, 1989). Independent of other parameters.
$\alpha = 0.1$	Non-rotating homogeneous simple shear. Parameter selected based on the components of $\underline{\underline{R}}$ for asymptotic homogeneous shear (Tavoularis and Karnik, 1989) Depends on all the above parameter estimates.
$C_{R1} = 0.0036$	Non-rotating homogeneous simple shear. Parameter selected based on APS transport time consistent with the R-eigenvalues of the asymptotic anisotropic state (Tavoularis and Karnik, 1989). Depends on all the above parameter estimates.
$\frac{C_P}{C_{R1}} = 1.5$	Non-rotating homogeneous simple shear. Parameter selected based on asymptotic behavior of k and ε (Tavoularis and Karnik, 1989). This parameter does not depend on the estimate for C_{R1} , but it does depend on all the other parameters listed above.

The components of the prestress, the NR-stress, and the extra-anisotropic operator **are** listed in Table 4.3 for non-rotating simple shear flows with $\tilde{\Omega}/\tilde{\Gamma} = 0$ and $\tilde{\Gamma} = 4$. For **this** case, $N_F = 4$, $\tilde{\tau}_R = 0.8818$ and $N_\Gamma = 0.01259$. Figure 4.6 shows the locus of **anisotropic** invariants (II_b, III_b) associated with several solution paths produced by **URAPS**-theory. The invariants of the fixed-point (i.e., $\star$ on Figure 4.6) are $II_b^\star = 0.1389$ and $III_b^\star = 0.01525$, which corresponds to the anisotropic state for asymptotic homogeneous shear for case 1. The following initial operators were selected to **produce** the four solution paths illustrated in Figure 4.6:

$$\underline{\underline{R}}_0 \Rightarrow \frac{1}{3} \underline{e}_x \underline{e}_x + \frac{1}{3} \underline{e}_y \underline{e}_y + \frac{1}{3} \underline{e}_z \underline{e}_z, \quad II_b = 0, III_b = 0, \text{ isotropic state} \quad (4.14)$$

$$\underline{\underline{R}}_0 \Rightarrow \frac{9}{20} \underline{e}_x \underline{e}_x + \frac{9}{20} \underline{e}_y \underline{e}_y + \frac{9}{20} \underline{e}_z \underline{e}_z, \quad II_b = 0.0817, III_b = -0.0095, \text{ oblate state}; \quad (4.15)$$

$$\underline{\underline{R}}_0 \Rightarrow \frac{1}{10} \underline{e}_x \underline{e}_x + \frac{1}{10} \underline{e}_y \underline{e}_y + \frac{8}{10} \underline{e}_z \underline{e}_z, \quad II_b = 0.3267, III_b = 0.0762, \text{ prolate state}; \quad (4.16)$$

$$\begin{aligned} \underline{\underline{R}}_0 \Rightarrow & 0.2 \underline{e}_x \underline{e}_x + 0.3 \underline{e}_y \underline{e}_y + 0.5 \underline{e}_z \underline{e}_z + 0.1 \underline{e}_x \underline{e}_y + 0.1 \underline{e}_y \underline{e}_x \\ & - 0.3 \underline{e}_x \underline{e}_z - 0.3 \underline{e}_z \underline{e}_x - 0.2 \underline{e}_y \underline{e}_z - 0.2 \underline{e}_z \underline{e}_y; \quad , \text{ ellipsoidal state.} \quad (4.17) \\ & II_b = 0.3267, III_b = 0.0582 \end{aligned}$$

The eigenvalues of $\underline{\underline{R}}_0$ associated with the ellipsoidal state are $\lambda_1^R = 0.0127$, $\lambda_2^R = 0.2000$, and $\lambda_3^R = 0.7873$. Figure 4.6 shows that all the solution paths are in the **vicinity** of the fixed-point in about 120 iterations. More than 600 iterations are needed to **satisfy** the convergence criterion $\left| (\underline{\underline{R}}_{n+1} - \underline{\underline{R}}_n) : \underline{e}_i \underline{e}_j \right| \leq 10^{-5} \underline{\underline{R}}_n : \underline{e}_i \underline{e}_j$ for $i = x, y, z$ and $j = x, y, z$. However, within about 200 iterations, all the solutions are close to **a** single unique solution. The specific number of iterations depend on the initial

Table 4.3. Components of $\underline{\underline{B}}$, $\underline{\underline{R}}$, and $\underline{\underline{\Delta}}$ ($\tilde{\Gamma} = 4$ and $\tilde{\Omega}/\tilde{\Gamma} = 0$)

component	xx	yy	zz	yz	χ_B and χ_R
$\underline{\underline{B}}$	+ 0.253705	+ 0.202491	+ 0.543804	− 0.185129	+ 0.557892
$\underline{\underline{R}}$	+ 0.252520	+ 0.201545	+ 0.545935	− 0.186801	+ 0.563148
$\underline{\underline{\Delta}} = \underline{\underline{B}} - \underline{\underline{R}}$	+ 0.001185	+ 0.000946	− 0.002131	+ 0.001672	n/a

$$\chi_R \equiv |R_{yz}| / \sqrt{R_{yy}R_{zz}} ; \chi_B \equiv |B_{yz}| / \sqrt{B_{yy}B_{zz}}$$

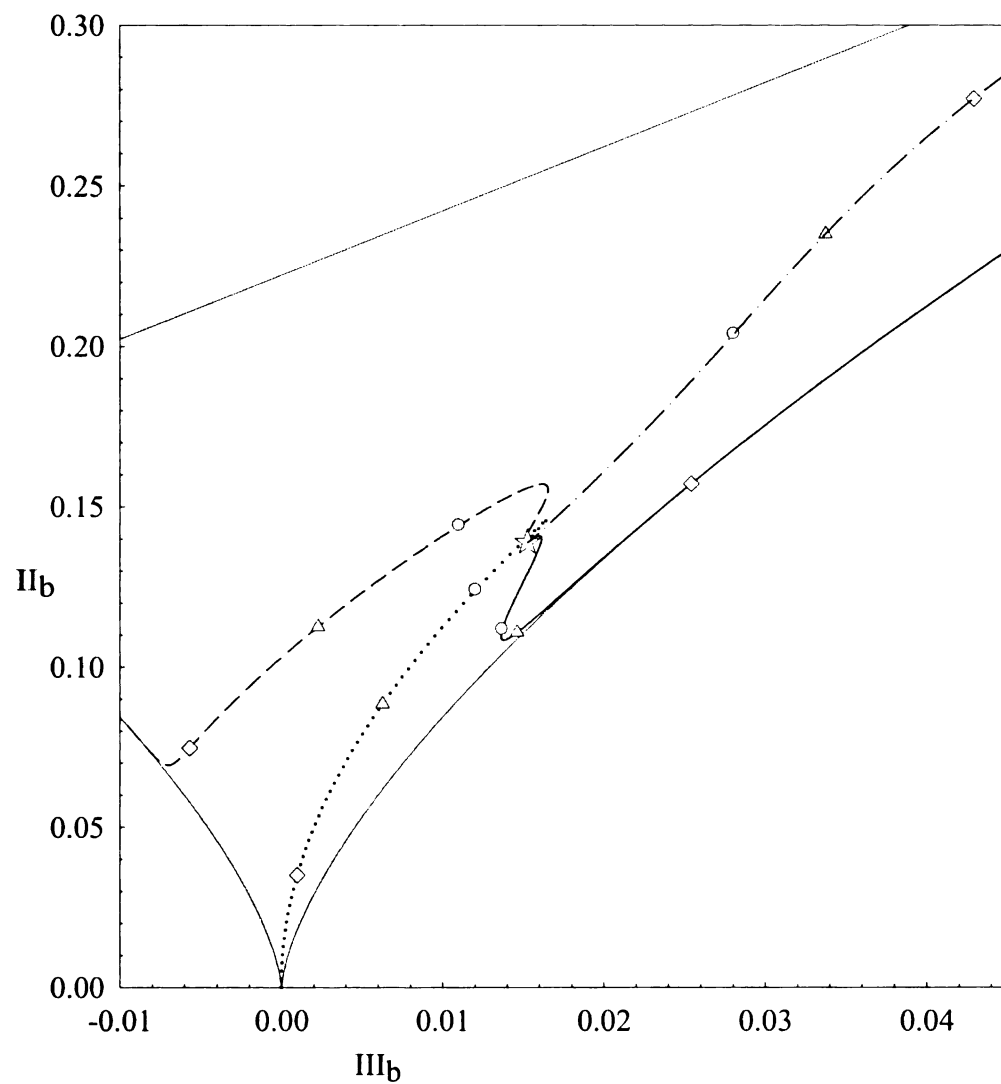


Figure 4.6 The Development of Anisotropic Invariants of the NR-stress for $\tilde{\Gamma} = 4$ and $\tilde{\Omega}/\tilde{\Gamma} = 0$ from Different Initial States (initial states: isotropic $\cdots\cdots$; oblate $-----$; prolate $——$; ellipsoidal $- \cdot - \cdot -$; iterations: 40 $\diamond$; 80 $\triangle$; 120 $\circ$; $\Pi_b^\star = 0.1389$; $\Pi_b^\star = 0.01525$).

condition. Figure 4.7, shows solution trajectories from the initial conditions in Eqs. (4.14) to (4.17) for case 2. The components of $\underline{B}$, $\underline{R}$, and $\underline{\Delta}$ are listed in Table 4.4. The components of $\underline{\Delta}$ are of the order of 10^{-3} . For this case ($\tilde{\Omega}/\tilde{\Gamma} = -5/8$ and $\tilde{\Gamma} = 11$), $N_F = 14$, $\tilde{\tau}_R = 0.7506$, $N_\Gamma = 0.02948$, and $N_\Omega = -0.03685$. Although the trajectories seem to intersect at some point for the case 2, the components are all unique and finally reach single solution.

4.7 Conclusions

The URAPS-closure coefficients have been estimated using benchmark flows described in Chapter 2 assuming them to be universal constants. Following is a summary of the calibration described in Table 4.2.

1. The ratio C_D/C_{R1} , which appears in the in the ε -equation, has been calibrated to be 1.8 from experimental data (see Figure 2.17) for non-rotating homogeneous flow for high initial Reynolds number. In this research, C_D/C_{R1} is assumed to be a universal constant

2. The turbulent time scale for the URAPS closure is based on an empirical

formulation for the dimensionless timescale $\tilde{\tau}_R = \frac{(1+C_{R3} N_F^{3/2})}{(1+C_{R2} N_F^{3/2})}$ which has

been proposed from the understanding derived from the analysis of Park and Chung (1999) presented in Appendix F. The analysis of energy spectrum for homogeneous flows in the presence of rotation shows the dependence of dissipation equation on rotation (see Eq.(F.42)). This information is used to derive

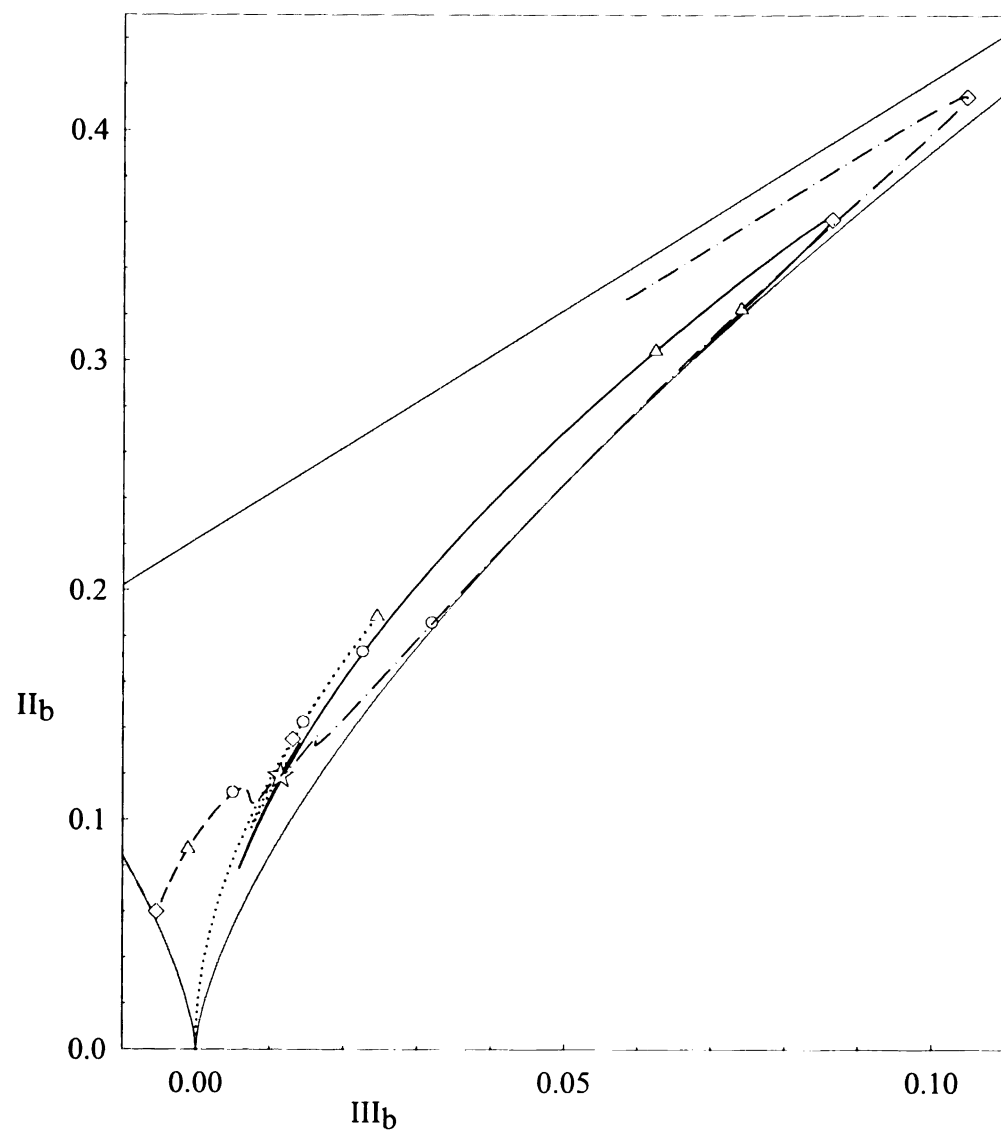


Figure 4.7. The Development of Anisotropic Invariants of the NR-Stress for $\tilde{\Gamma} = 11$ and $\tilde{\Omega}/\tilde{\Gamma} = -5/8$ from Different Initial States (same as Figure 4.6); $\Pi_b^\star = 0.1190$; $\text{III}_b^\star = 0.01144$).

Table 4.4. Components of $\underline{\underline{B}}$, $\underline{\underline{R}}$, and $\underline{\underline{\Delta}}$ ($\tilde{\Gamma} = 11$ and $\tilde{\Omega}/\tilde{\Gamma} = -5/8$)

component	xx	yy	zz	yz	χ_B and χ_R
$\underline{\underline{B}}$	+ 0.264508	+ 0.592243	+ 0.143249	− 0.066540	+ 0.228447
$\underline{\underline{R}}$	+ 0.263520	+ 0.594788	+ 0.141692	− 0.067148	+ 0.231303
$\underline{\underline{\Delta}} = \underline{\underline{B}} - \underline{\underline{R}}$	+ 0.000988	− 0.002545	+ 0.001557	+ 0.000608	n/a

$$\chi_R \equiv \left| R_{yz} \right| / \sqrt{R_{yy} R_{zz}} \; ; \; \chi_B \equiv \left| B_{yz} \right| / \sqrt{B_{yy} B_{zz}}$$

the coefficients $(C_{R2}, C_{R3}) = (0.0764, 0.0526)$ in order to include the dependence on mean field time scale. The power on the group N_F , i.e. $n = 3/2$, is adopted from the dependence of time scale on the rotation rate observed in the energy spectrum analysis. For $0 < N_F < \infty$, $\tilde{\tau}_R$ varies between the limits $0.69 < \tilde{\tau}_R < 1$.

3. The URAPS relation for simple shear and experimental data for asymptotic states flows has been used to estimate the ratio $\beta/\alpha \cong -0.1$. URAPS-closure component equations cannot exactly satisfy all the experimental data in Table 4.1 for asymptotic homogeneous shear. Consequently, an optimization strategy was used to minimize the metric defined by Eq. (4.9). The resulting coefficients $\alpha = +0.1$ and $C_{R1} \cong 0.0036$ minimized the error between the model predictions and the experimental data. The coefficient of production ($C_P/C_{R1} \cong 1.5$) was estimated from the asymptotic relation for the turbulent time scale. The summary of the parameters listed in Table 4.2 will be used for the application of URAPS-closure for predicting the NR-stress. Figure 4.8 shows the realizable choice of $\alpha = +0.1$ and $\beta = -0.01$ from the calibration procedure to be within the realizable bounds discussed in Chapter 3.
4. Successive substitution method for finding the URAPS-closure solution is illustrated for two cases of simple shear flows: Case 1: $\tilde{\Omega}/\tilde{\Gamma} = 0$ and $\tilde{\Gamma} = 4$; and, Case 2: $\tilde{\Omega}/\tilde{\Gamma} = -5/8$ and $\tilde{\Gamma} = 11$. It is shown that for different initial anisotropic states in Eqs. (4.14) to (4.17) show a convergence towards a unique final ‘☆’ state shown in Figures 4.6 and 4.7 for a specific convergence criteria.

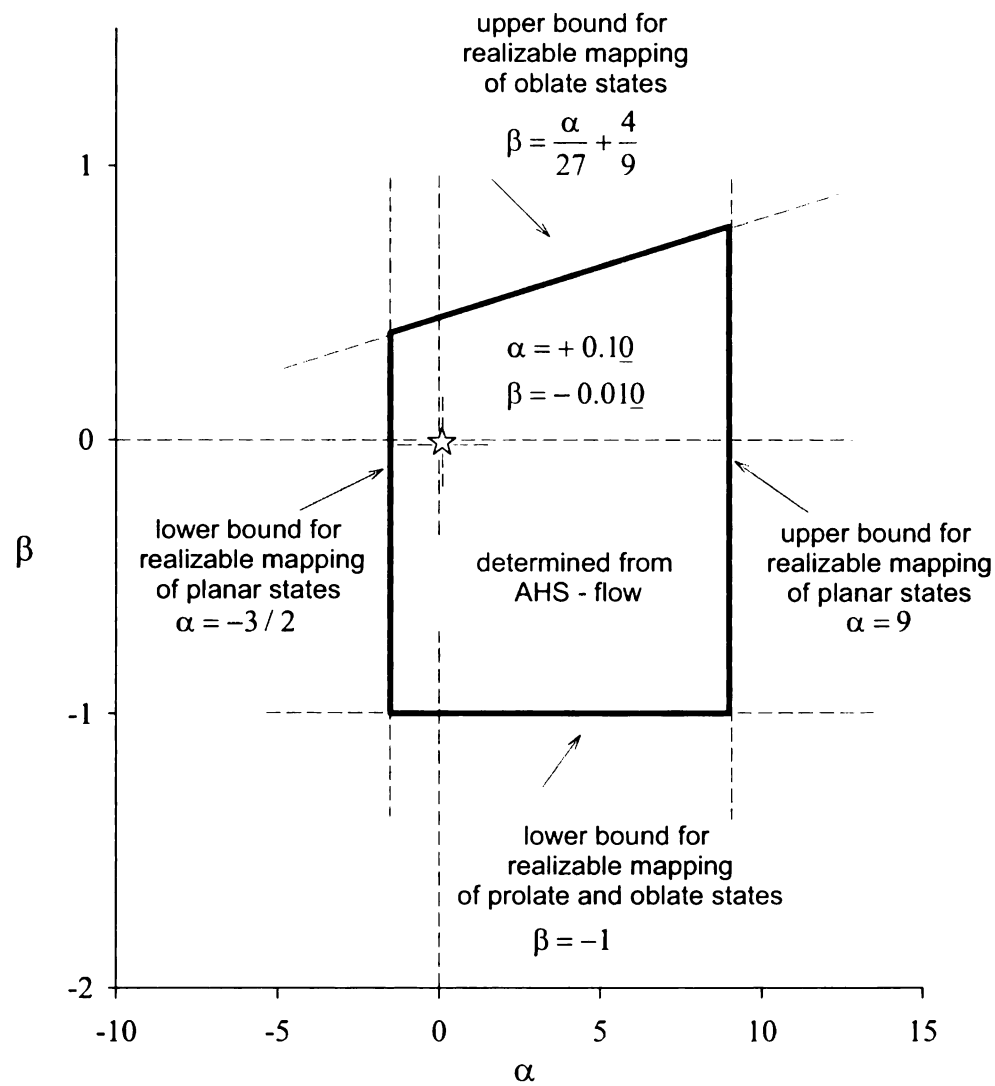


Figure 4.8 Prestress Coefficients α and β Determined From Asymptotic Homogeneous Shear flow.

CHAPTER 5

HOMOGENEOUS DECAY

5.1 Introduction

In the absence of spatial gradients in the mean velocity, turbulence decays. This flow field which is referred to as homogeneous decaying turbulence, is often used as a benchmark flow for studying turbulence models. Eq. (2.29) and Eq. (2.30), which are autonomous non-linear dynamic equations for $\tilde{k}(\equiv k/k_0)$ and $\tilde{\varepsilon}(\equiv \varepsilon/\varepsilon_0)$, define the decay dynamics as discussed in Chapter 4. The dimensionless turbulent time scale $\tilde{\tau}_R$ described in Eq. (4.5) has been modeled in Chapter 4 to include the time scale associated with the rotation of the frame through the group $N_F \equiv k \|\underline{\underline{F}}\|/\varepsilon$. In Section 5.2, the reconstruction of some of the results used in the calibration along with a further parametric analysis for various important turbulent quantities like the turbulent time scale $\tilde{k}/\tilde{\varepsilon}$, and the turbulent dispersion $\tilde{k}^2/\tilde{\varepsilon}$ are discussed to understand the temporal development of $\tilde{\Omega}(\equiv \tilde{\Omega}_0 \tilde{k})/\tilde{\varepsilon}$. The focus of Chapter 5 is on the development of the Reynolds stress anisotropy in the presence of rotation for high initial turbulent Reynolds numbers ($k_0^2/(\nu\varepsilon_0) \gg 1$). The URAPS-closure component equations are simplified for this case because of no shear. These are discussed in Section 5.3. The temporal development of the NR-stress for the URAPS-closure estimated from the temporal variation of $\tilde{\Omega}$ is discussed in Section 5.4. The temporal development of the ‘extra’ anisotropy terms is studied in Section 5.5 to understand their role in energy redistribution.

5.2 Turbulent Energy and Turbulent Dissipation: Results and Discussion

The dimensionless time $\tilde{t}$ is defined as $\varepsilon_0 t / k_0$ and the rotational operator is related to the angular velocity by $\underline{\underline{\Omega}} = \Omega_x (\underline{e}_y \underline{e}_z - \underline{e}_z \underline{e}_y)$. For $\tilde{t} = 0$, $\tilde{k} = 1$ and $\tilde{\varepsilon} = 1$, the initial value problem (defined by Eq. (2.29) and (2.30)) was solved for a wide range of rotation numbers by using a fourth-order Runge-Kutta algorithm implemented in MatLab® (Appendix H provides the flowchart and Matlab program). Summary of the influence of $\tilde{\Omega}_0$ on the decay process as discussed below.

In the presence of rotation for decay of homogeneous turbulent flow, the rate of decay of k is hindered by rotation with a decrease in dissipation (Bardina et al. 1985, Speziale et al. 1987, Cambon et al. 1997) as discussed in the literature in Chapter 2. Figure 5.1 shows the decay dynamics for $0 < \tilde{\Omega}_0 < 10$ all lie between the two curves. Although turbulence decays monotonically for all rotation rates, the decay rate of k decreases with increasing rotation. During the initial stage (i.e., $0 < \tilde{t} < 0.1$), an enhanced rate-of-decay of $\tilde{\varepsilon}$ in Eq. (2.30) arises due to a decrease in the dimensionless turnover time $\tilde{\tau}_R$ as the rotation number $N_F (= 2\sqrt{2} \tilde{\Omega}_0 \tilde{k} / \tilde{\varepsilon})$ increases as discussed in Chapter 4 (see Figure 4.2). The influence of rotation on $\tilde{k}$ is relatively small during the initial stage. However, during the intermediate stage (i.e., $0.1 < \tilde{t} < 4$), the rate of decay of $\tilde{k}$ is mitigated by the smaller dissipation that is developed during the initial stage. For the final (non-viscous) decay period (i.e., $\tilde{t} > 4$), the rate-of-decay of $\tilde{\varepsilon}$ decreases due to the persistence of a relatively large value of $\tilde{k}$ as $\tilde{k} / \tilde{\varepsilon} \rightarrow \infty$. Thus, for $\tilde{t} \gg 4$, $N_F \rightarrow \infty$ and $\tilde{\tau}_R \rightarrow C_{R3} / C_{R2} = 11/16$. Figure 5.1b shows that the dissipation for $\tilde{\Omega}_0 = 10$ eventually

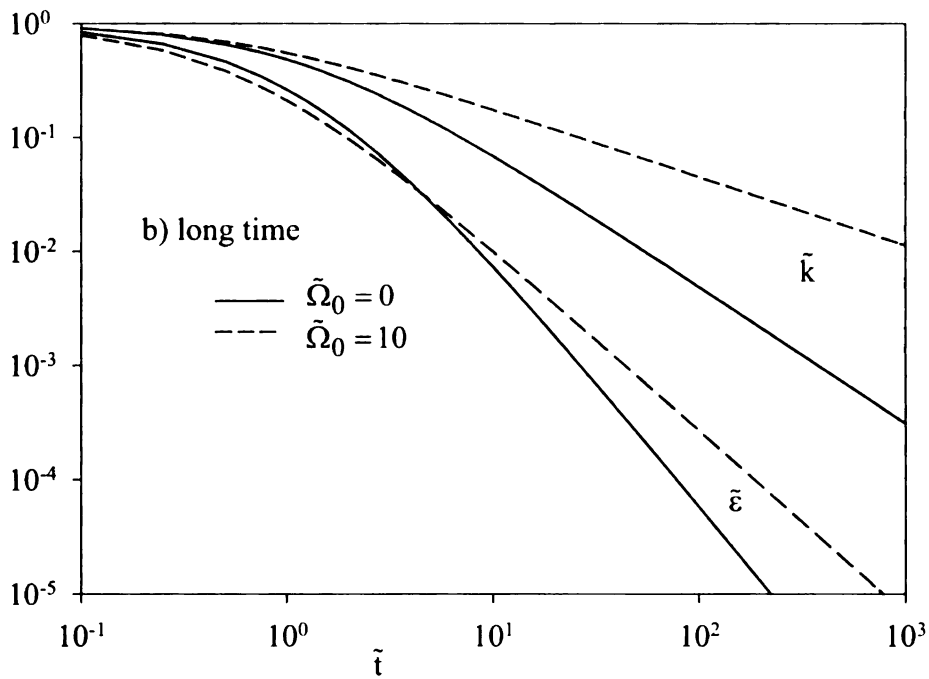
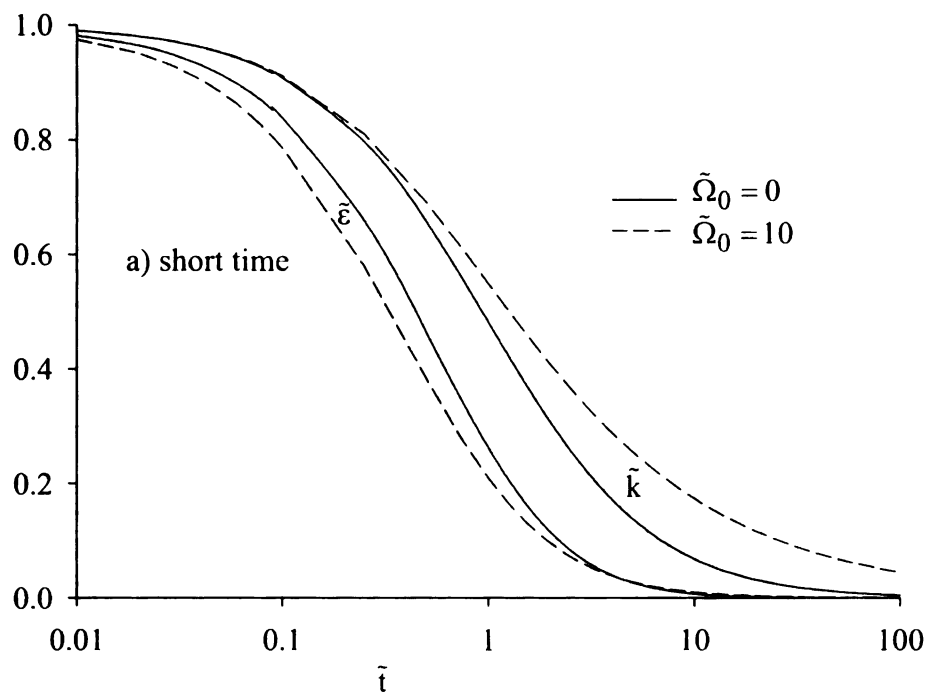


Figure 5.1. The Influence of Rotation Number on the Decay of $\tilde{k}$ and $\tilde{\epsilon}$.

exceeds the dissipation for $\tilde{\Omega}_0 = 0$. This feature, which was not noted by Park and Chung (1999) shows that rotation actually retards the rate-of-decay of dissipation during the final stage of the decay process.

Figure 5.2 shows the decay dynamics in a phase plane. The decay process starts at $\tilde{k} = \tilde{\varepsilon} = 1$ and decays with time asymptotically approaching zero with $\tilde{k} \geq 0$, and $\tilde{\varepsilon} \geq 0$ for all times. During the initial decay period, the slope of the curves in the phase plane shown in this figure are related to the coefficients in Eq.(5.1) below, which is obtained by combining Eqs. (2.29) and (2.30).

$$\left. \frac{d \tilde{\varepsilon}}{d \tilde{k}} \right|_{\tilde{t}=0} = -\frac{C_D}{C_{RI}} \frac{1}{\tilde{\tau}_R} \frac{\tilde{\varepsilon}}{\tilde{k}} = -\frac{C_D}{C_{RI}} \frac{1}{\tilde{\tau}_R} \quad (5.1)$$

For $\tilde{\Omega}_0 = 0$, $\frac{C_D}{C_{RI}} \frac{1}{\tilde{\tau}_R} = \frac{11}{6}$ since $\tilde{\tau}_R(N_F \rightarrow 0) = 1$ for no rotation. For very large rotation

rates, where $\tilde{\Omega}_0 \rightarrow \infty$, the slope is $\frac{C_D}{C_{RI}} \frac{1}{\tilde{\tau}_R} \rightarrow \frac{8}{3}$ since $\tilde{\tau}_R(N_F \rightarrow \infty) \rightarrow 11/16$. These

limiting values are a consequence of the model for $\tilde{\tau}_R$ in Chapter 4. Figure 5.3 shows that the turbulent time scale increases monotonically for increasing rotation rates. The initial slopes of the curves are positive and continue to remain so during the entire decay process as described by Eq. (5.2) for turbulent time scale below.

$$\left. \frac{d}{d \tilde{t}} \left(\frac{\tilde{k}}{\tilde{\varepsilon}} \right) \right|_{\forall \tilde{t}} = -1 + \frac{C_D}{C_{RI}} \frac{1}{\tilde{\tau}_R} \quad (5.2)$$

$\frac{C_D}{C_{RI}} \frac{1}{\tilde{\tau}_R} > 1 \quad \forall \tilde{\Omega}_0$ and so $-1 + \frac{C_D}{C_{RI}} \frac{1}{\tilde{\tau}_R} > 0$ i.e. positive slope for any rotation rate.

The decay dynamics in the presence of rotation has a qualitatively different response compared with its non-rotating counterpart. Figure 5.4 shows that turbulent

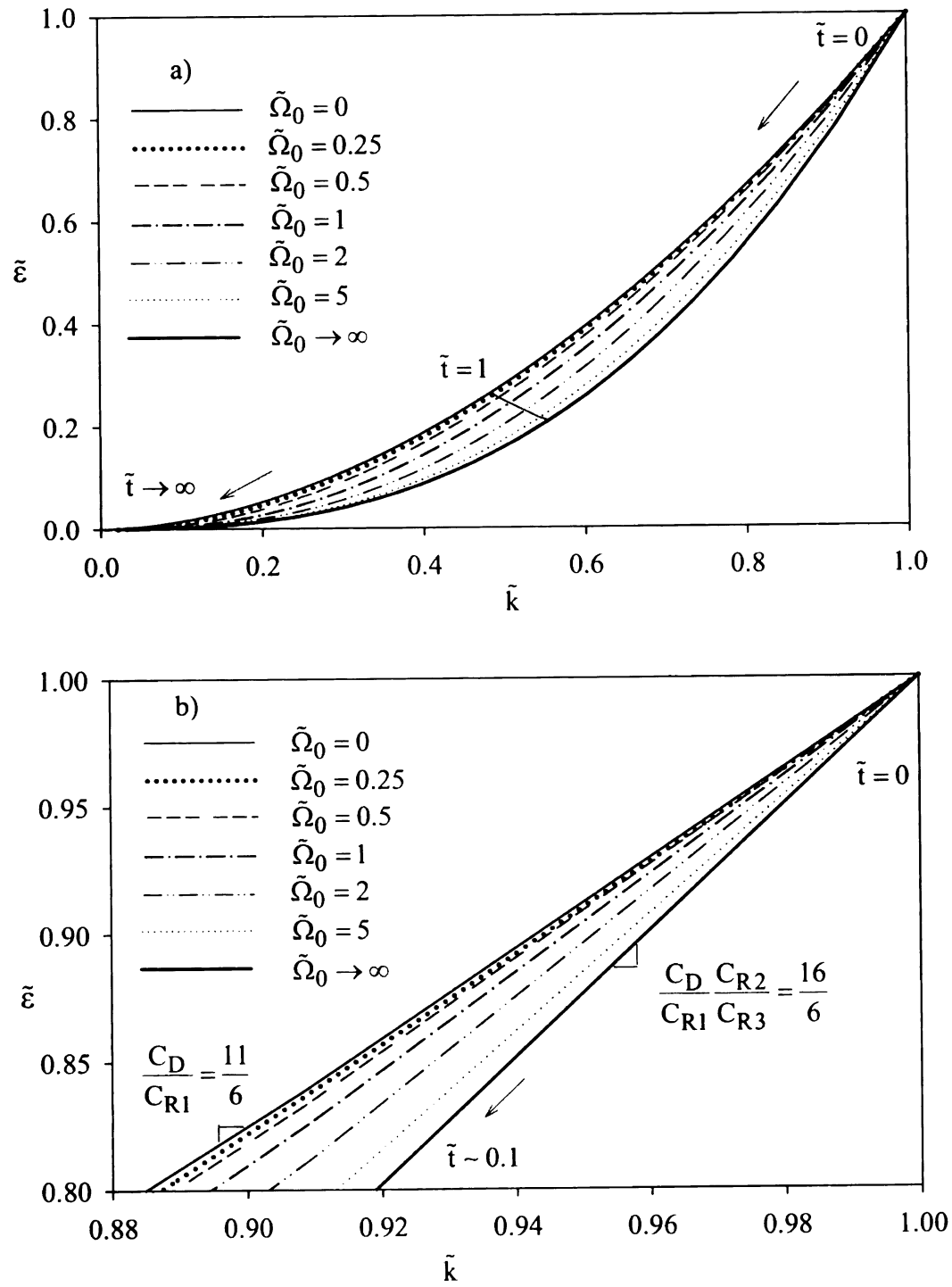


Figure 5.2. $\tilde{k}$ and $\tilde{\epsilon}$ Phase Plane Diagrams for Rotating Homogeneous Decay.

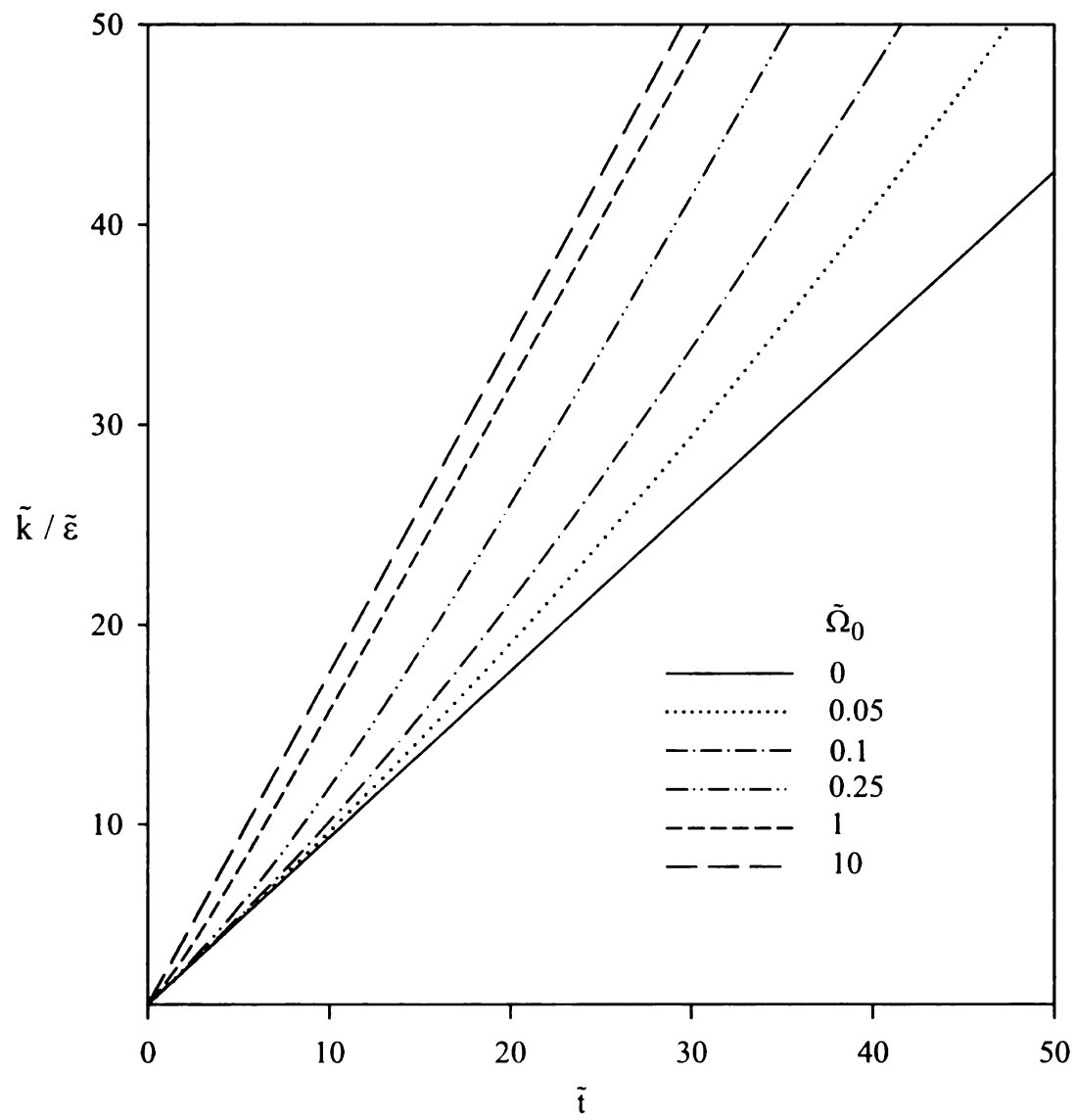


Figure 5.3. The Influence of Rotation on the Dimensionless Turbulent Time Scale.

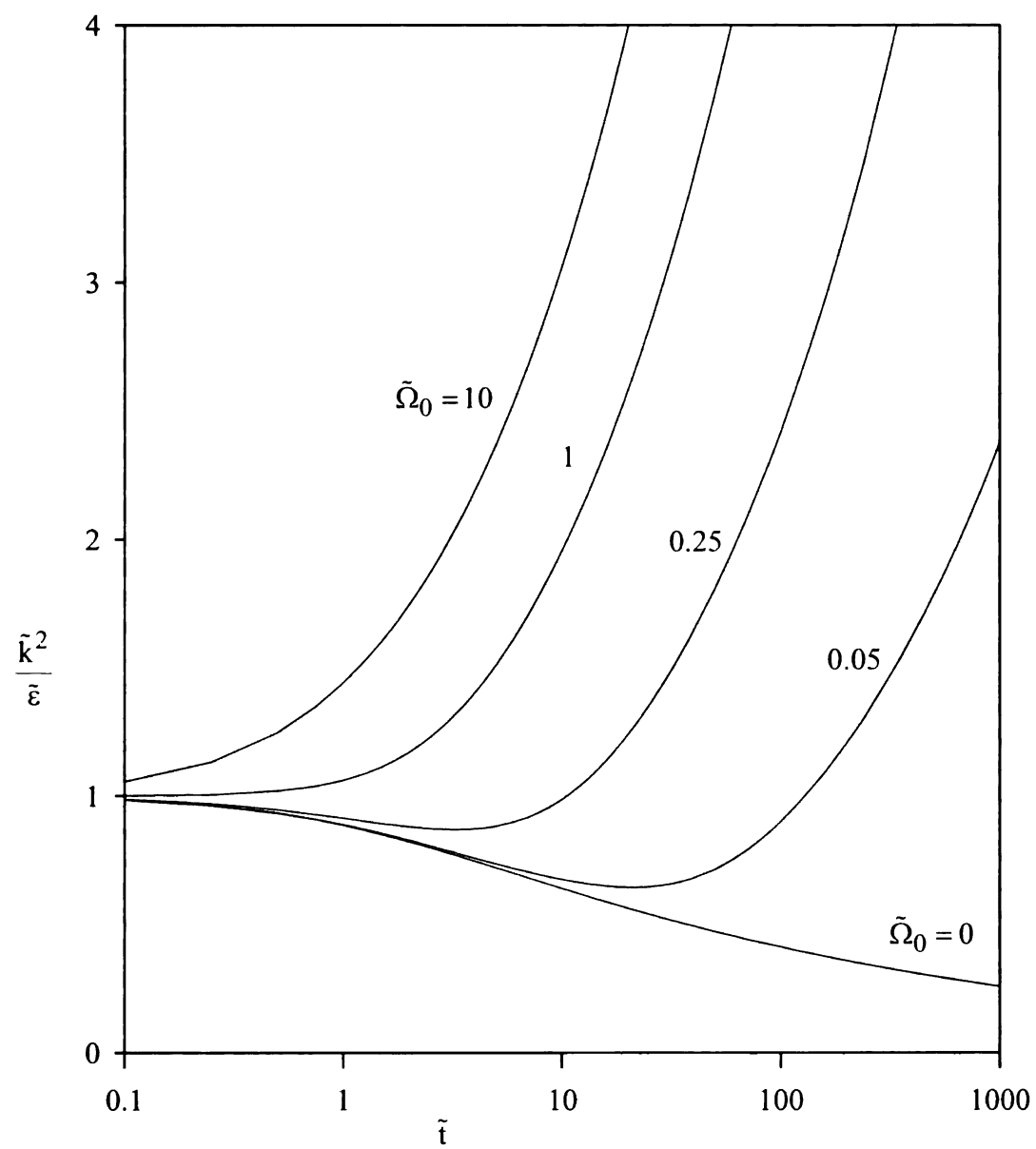


Figure 5.4. Influence of Rotation on $\tilde{k}^2 / \tilde{\epsilon}$.

dispersion (i.e., $\tilde{k}^2/\tilde{\epsilon}$) variation with $\tilde{t}$ for various rotation rates. In the absence of rotation, $\tilde{k}^2/\tilde{\epsilon}$ decreases monotonically as $\tilde{k}^2$ decreases faster than $\tilde{\epsilon}$. However, in the presence of rotation, although $\tilde{k}$ and $\tilde{\epsilon}$ decrease monotonically, $\tilde{k}^2/\tilde{\epsilon}$ becomes unbounded for large rotation numbers (see Figure 5.4). For $0 < \tilde{\Omega}_0 < 1$, $\tilde{k}^2/\tilde{\epsilon}$ first decreases and then increases and becomes unbounded for long times while for $\tilde{\Omega}_0 \geq 1$, there is a monotonic increase. This can be explained by the Eq. (5.3) below for the rate of change of turbulent dispersion which gives the initial slope of $\tilde{k}^2/\tilde{\epsilon}$ vs. $\tilde{t}$.

$$\left. \frac{d}{d\tilde{t}} \left(\frac{\tilde{k}^2}{\tilde{\epsilon}} \right) \right|_{\tilde{t}=0} = -2 + \frac{C_D}{C_{R1}} \frac{1}{\tilde{\tau}_R}. \quad (5.3)$$

The initial slope is negative for $0 < \tilde{\Omega}_0 < 1$ since $\frac{11}{6} < \frac{C_D}{C_{R1}} \frac{1}{\tilde{\tau}_R} < 2$. For $\tilde{\Omega}_0 \geq 1$, the initial slope is positive as $\frac{C_D}{C_{R1}} \frac{1}{\tilde{\tau}_R} > 2$ and so $\tilde{k}^2/\tilde{\epsilon}$ increases monotonically.

From the above information on the $\tilde{k}$ and $\tilde{\epsilon}$, the group $\tilde{\Omega} = \tilde{\Omega}_0 \tilde{k}/\tilde{\epsilon}$ increases for all rotation numbers during the decay process. Viscous effects are negligible for all initial rotation numbers $\tilde{\Omega}_0 \geq 1$ and for $0 < \tilde{\Omega}_0 < 1$ during long times since $\tilde{k}^2/(\nu\tilde{\epsilon})$ increases monotonically in this region as discussed above. The following Section illustrates the simplified URAPS-closure component equations and the Reynolds stress anisotropy predicted by the URAPS-equation.

5.3 Preclosure Operator and URAPS-Closure

For rotating homogeneous flows, the preclosure operator $\underline{\underline{A}}$ is defined by the following equations Eq.(5.4) and Eq.(5.5). (See Appendix G. for more detailed derivations).

$$\underline{\underline{A}}^{-1} = \underline{e}_x \underline{e}_x + \underline{e}_y \underline{e}_y + \underline{e}_z \underline{e}_z + N_\Omega \underline{e}_y \underline{e}_z - N_\Omega \underline{e}_z \underline{e}_y \quad (5.4)$$

$$\underline{\underline{A}} = \frac{(1 + N_\Omega^2) \underline{e}_x \underline{e}_x + \underline{e}_y \underline{e}_y + \underline{e}_z \underline{e}_z - N_\Omega \underline{e}_y \underline{e}_z + N_\Omega \underline{e}_z \underline{e}_y}{(1 + N_\Omega^2)} \quad (5.5)$$

where $N_\Omega = 2 C_{R1} \tilde{\tau}_R \frac{\tilde{k}}{\tilde{\varepsilon}} \tilde{\Omega}_0 = 2 C_{R1} \tilde{\tau}_R \tilde{\Omega}$. For $\nabla \cdot \underline{u} = 0$ and $0 < \tilde{\Omega}_0 < \infty$, an analysis

URAPS-closure predicts that if $B_{xy} = 0$ and $B_{xz} = 0$, then $R_{xy} = 0$ and $R_{xz} = 0$.

Furthermore, if $B_{yz} = 0$ and $B_{yy} = B_{zz}$, then $R_{yz} = 0$. Therefore, for rotating homogeneous decay, URAPS closure can be written in the component form (See Appendix G. Case3).

$$\kappa R_{xx} = (1 + N_\Omega^2)^2 B_{xx} \quad (5.6)$$

$$\kappa R_{yy} = B_{yy} + N_\Omega^2 B_{zz} \quad (5.7)$$

$$\kappa R_{zz} = N_\Omega^2 B_{yy} + B_{zz} \quad (5.8)$$

$$\kappa \equiv (1 + N_\Omega^2)(1 + N_\Omega^2 B_{xx}) \quad (5.9)$$

The components of the prestress operator for rotating homogeneous decay are also presented in Appendix G. Eqs.(5.7) and (5.8) imply that if $B_{yy} = B_{zz}$, then $R_{yy} = R_{zz}$.

This is consistent with the closure expressed by URAPS-theory. Therefore, with

$B_{yy} = B_{zz} = (1 - B_{xx})/2$ and $R_{yy} = R_{zz} = (1 - R_{xx})/2$, URAPS-equations implies that

$$R_{xx} = \frac{(1 + N_{\Omega}^2)B_{xx}}{(1 + N_{\Omega}^2 B_{xx})}. \quad (5.10)$$

Based on the simplified URAPS-closure result in Eq. (5.10), the NR-stress variation with rotation is discussed in the following Section.

5.4 The NR-Stress: Results and Discussion

Eq. (5.10) shows that for $N_{\Omega} = 0$, $R_{xx} = B_{xx}$. For the non-rotating case, three solutions to URAPS equations exist: $R_{xx} = 0; 1/3$; and 1 . For $N_{\Omega} = \pm \infty$, Eq.(5.10) implies that $R_{xx} = 1$; therefore, $B_{xx} = 1$ also. Thus, as the rotation number increases, the URAPS-closure predicts that the Coriolis force causes a transfer of energy from the fluctuating velocity components in the plane orthogonal to the rotating axis (i.e. y and z components) into the fluctuating velocity aligned with the angular velocity (i.e. x component). If $-\infty < N_{\Omega} < \infty$, prolate anisotropic states also exist. The prolate anisotropic form produced is characterized by the following inequality: $0 \leq R_{yy} = R_{zz} < 1/3 < R_{xx} \leq 1$.

Figures 5.5 and 5.6 summarize the solutions to URAPS-equations for different rotation numbers, $\tilde{\Omega}_0$. Temporal variation of the rotation group, $N_{\Omega} = 2C_{Rl} \tilde{\tau}_R (\Omega_0 \tilde{k}) / \tilde{\epsilon}$ are calculated based on the decay dynamics discussed earlier. Thus from Eq. (5.10), the URAPS-closure predicts that the Coriolis force causes a redistribution of energy from an initial isotropic state to a prolate state as the turbulent kinetic energy decays. The algebraic nature of the URAPS-closure causes a rapid (instantaneous) redistribution of energy at $\tilde{t} = 0$ followed by a slower transfer of energy

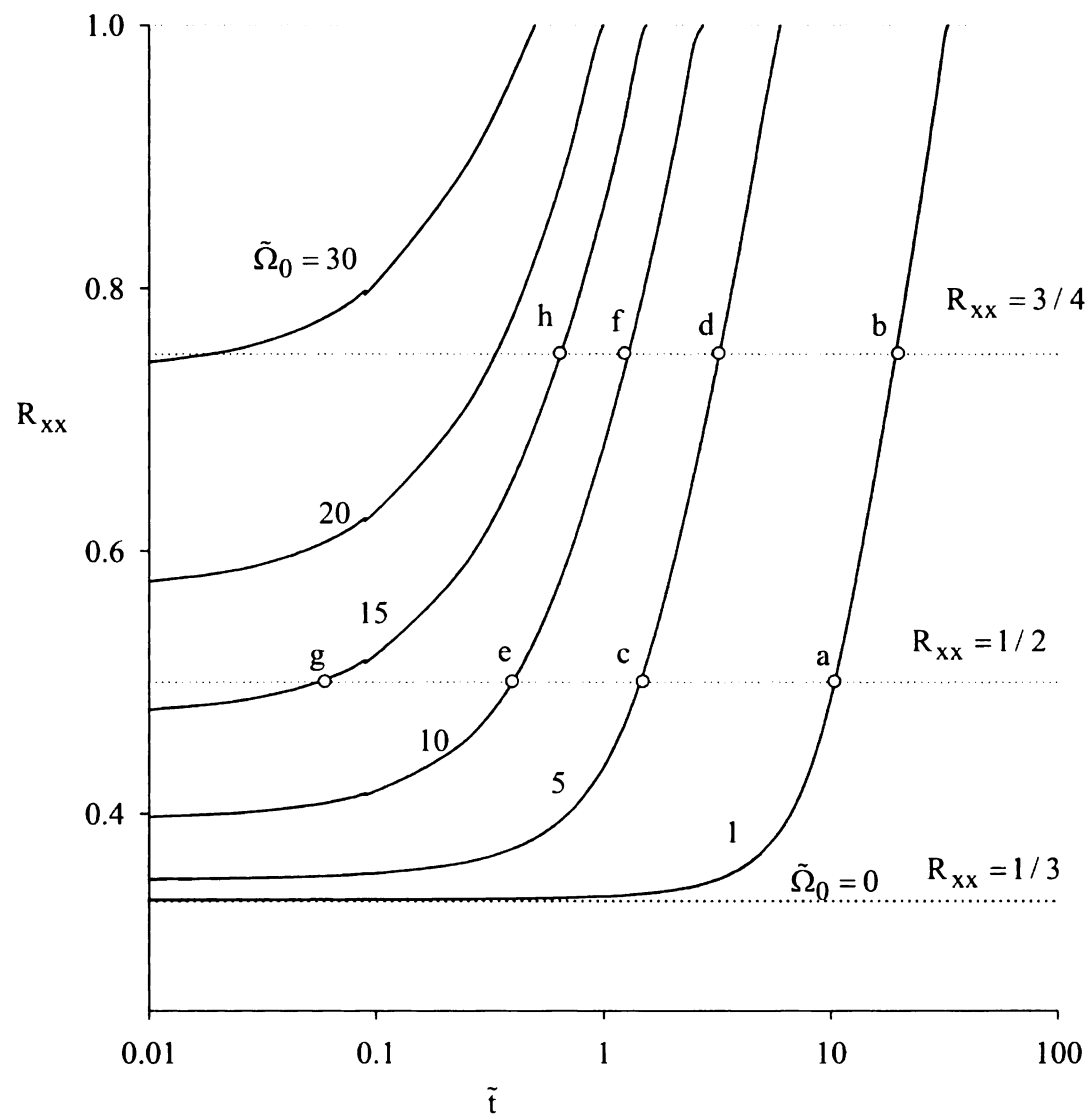


Figure 5.5. The Influence of Rotation on the Temporal Variation of R_{xx} Component of Reynolds Stress for Homogeneous Decay (see Tables 5.1a,b).

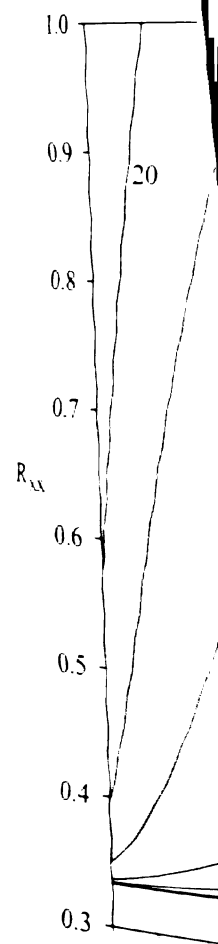


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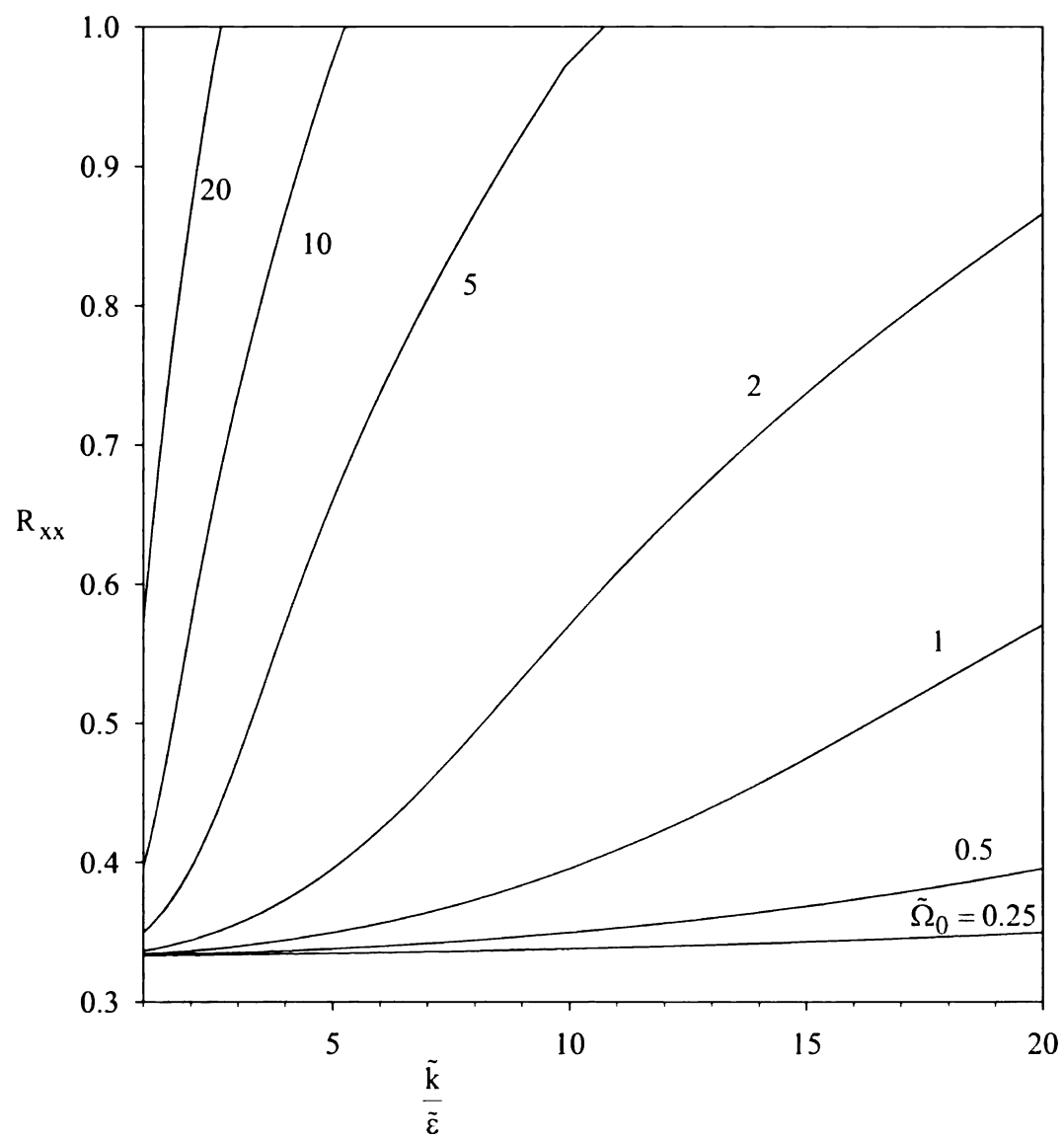


Figure 5.6. The Influence of Rotation on the Redistribution of Kinetic Energy with Respect to the Turbulent Time Scale.

for $i > 0$. During the
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for $\tilde{t} > 0$. During the decay process, the quadratic form associated with the NR-stress changes from a spherical isotropic form to a prolate ellipsoidal form with an aspect ratio of $2R_{xx}/(1-R_{xx})$. Table 5.1 lists the anisotropic states identified as ‘a’, ‘c’, ‘e’, and ‘g’ in Figure 5.5 for which $R_{xx} = 1/2$ all have quadratic forms with an aspect ratio of 2:1. The anisotropic states identified as ‘b’, ‘d’, ‘f’, and ‘h’ in Figure 5.5 for which $R_{xx} = 3/4$ all have quadratic forms with an aspect ratio of 6:1. (See Table 5.1 for the details of values of each of URAPS variables for these states). As the turbulent transport time increases (i.e., $\tau_R \gg \Omega_x^{-1}$), the energy redistribution process continues and is effectively completed (i.e., $R_{xx} \cong 1$) in a finite amount of time $\tilde{t}_f = t_f \varepsilon_0 / k_0 < \infty$. Figure 5.5 shows that the time needed to shift the energy to the R_{xx} component depends on the rotation number $\tilde{\Omega}_0$. For $\tilde{\Omega}_0 = 1$, the transfer time is significant inasmuch as $\tilde{t}_f \cong 33$. However, for $\tilde{\Omega}_0 = 15$, $\tilde{t}_f \cong 1.8$.

From the above discussion, it can be seen that for URAPS-closure, if the initial state is isotropic ($R_{xx} = R_{yy} = R_{zz} = 1/3$) and homogeneous, then prolate anisotropic states ($0 < R_{yy} = R_{zz} < 1/3 < R_{xx} < 1$) develop due to strong non-linear interactions among the velocity components (i.e., $\|\underline{f}'(\underline{x}, t)\| \cong \|\underline{u}'(\underline{x}, t) \cdot \underline{\underline{\Omega}}\|$). The action of the intrinsic acceleration $\underline{f}'(\underline{x}, t)$ is fundamental in this reorganization of turbulent kinetic energy. Transport equation for the Reynolds stress shows that the effect of rotation is purely reorganization of turbulent kinetic energy into an axisymmetric form, without effecting the equation for k directly (see Piquet, 1999) This is consistent with the knowledge of literature presented in Chapter 2, indicating that the Reynolds anisotropy is

Table 5.1a.

Table 5.1b.

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a . c , e
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Table 5.1a. The Influence of Rotation on the NR-Stress for Homogeneous Decay (see Figure 5.5 For Prolate States).

$\tilde{\Omega}_0$	$R_{xx} \cong 1$ $\tilde{t}_f$	prolate state	$\tilde{t}$	$\tilde{k} / \tilde{\varepsilon}$	N_F	$\tilde{\tau}_R$
1	<u>33</u>	a	<u>11</u>	<u>16</u>	<u>46.2</u>	<u>0.70</u>
		b	<u>20</u>	<u>32</u>	<u>87.5</u>	<u>0.69</u>
5	<u>6.0</u>	c	<u>1.5</u>	<u>3.3</u>	<u>46.2</u>	<u>0.70</u>
		d	<u>3.3</u>	<u>6.2</u>	<u>87.5</u>	<u>0.69</u>
10	<u>2.8</u>	e	<u>0.40</u>	<u>1.6</u>	<u>46.2</u>	<u>0.70</u>
		f	<u>1.3</u>	<u>3.0</u>	<u>87.5</u>	<u>0.69</u>
15	<u>1.8</u>	g	<u>0.060</u>	<u>1.1</u>	<u>46.2</u>	<u>0.70</u>
		h	<u>0.65</u>	<u>2.0</u>	<u>87.5</u>	<u>0.69</u>

Table 5.1b. The Influence of Rotation Number on the NR-Stress, the Prestress, and the “Extra” Anisotropy for Homogeneous Decay Predicted by The URAPS-Closure.

prolate states (see Figure 5.5)	N_Ω	R_{xx}	B_{xx}	Δ_{xx}
a , c , e , g	+ 0.082	+ 0.500	+ 0.498	– 0.0017
b , d , f , h	+ 0.15	+ 0.750	+ 0.746	– 0.0042

sensitive to the initial

indicated $R_{xx} > 1.3$

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5.5 "Extra" Anis

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and $\Pi_{\Delta 2} \equiv \underline{\underline{\Delta}}_2 : \underline{\underline{\Delta}}_2$

5.7, although Π_{Δ}

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sensitive to the initial conditions and may be either oblate or prolate. DNS results indicated $R_{xx} > 1/3$ for simulations with $\tilde{\Omega}_0 = 25$ (see page 71, Gatski, 1992). As demonstrated by Cambon et al. (1997) and Yang and Domaradzki (2004), anisotropic distribution among the normal components of NR-stress could be either prolate or oblate for long time simulations. Rapid Distortion Theory (RDT) theory which indicates no anisotropy for pure rotation case is incorrect (see Piquet, 1999).

Thus, for three-dimensional rotating homogeneous decay, the Reynolds stress is clearly anisotropic and the URAPS-closure predicts anisotropy shifting the energy to R_{xx} component in a finite time (although long time $\tilde{t}_f \cong 33$ for small rotation rates $\tilde{\Omega}_0 = 1$).

5.5 “Extra” Anisotropy: Results and Discussion

The normalized prestress and the NR-stress have anisotropic components. The “extra” anisotropy $\underline{\underline{\Delta}}$ in the CH-representation is defined by

$$\begin{aligned}\underline{\underline{\Delta}} &\equiv \underline{\underline{B}} - \underline{\underline{R}} = C_1 (\underline{\underline{R}} - \underline{\underline{I}}/3) + C_2 (\underline{\underline{R}} \cdot \underline{\underline{R}} - \Pi_R \underline{\underline{R}}) \\ &= \beta (\underline{\underline{R}} - \frac{1}{3} \underline{\underline{I}}) / 27 \det(\underline{\underline{R}}) - \alpha (\Pi_R - \frac{1}{3}) (\underline{\underline{R}} \cdot \underline{\underline{R}} - \Pi_R \underline{\underline{R}}). \\ &= \underline{\underline{\Delta}}_1 + \underline{\underline{\Delta}}_2\end{aligned}\tag{5.11}$$

The second invariants of the “extra” anisotropic operators are defined as $\Pi_{\Delta_1} \equiv \underline{\underline{\Delta}}_1 : \underline{\underline{\Delta}}_1^T$ and $\Pi_{\Delta_2} \equiv \underline{\underline{\Delta}}_2 : \underline{\underline{\Delta}}_2^T$. The second invariant associated with $\underline{\underline{\Delta}}$ is $\Pi_{\Delta} \equiv \underline{\underline{\Delta}} : \underline{\underline{\Delta}}^T$. In Figure 5.7, although Π_{Δ} is small compared with the second invariants of the anisotropic components of the normalized prestress and the NR-stress, the presence of $\underline{\underline{\Delta}}$ in

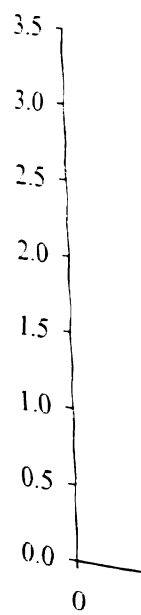
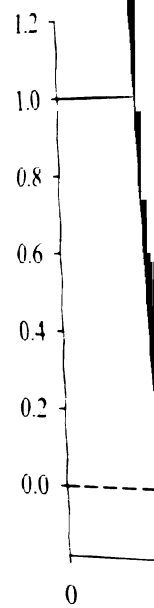


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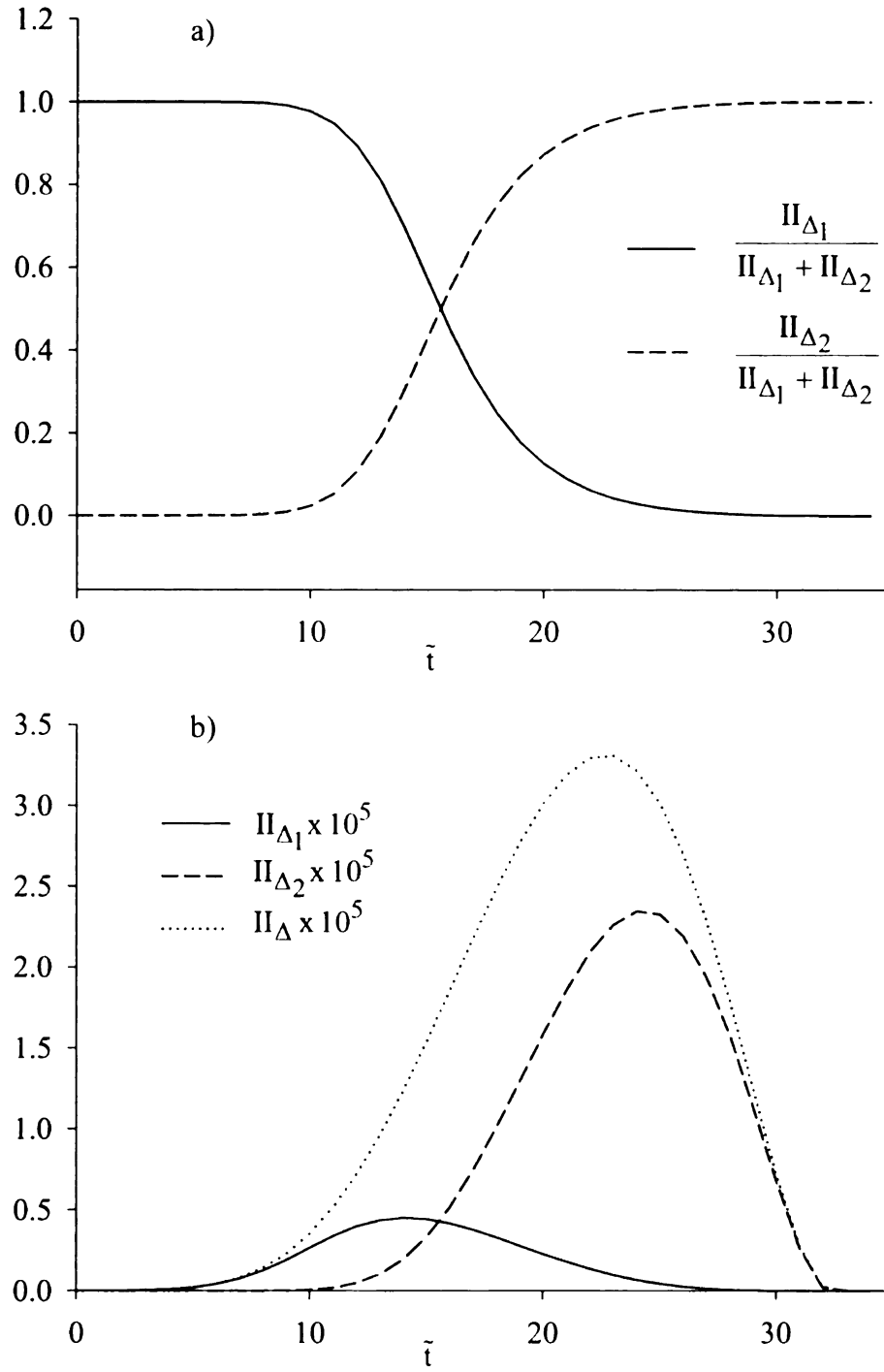


Figure 5.7. The Development of “Extra” Prestress Anisotropy During the Decay Process for $\tilde{\Omega}_0 = 1$: a) Dynamic Response of the Second Invariants for the First Order and Second Order CH-Contributions to the “Extra” Prestress Anisotropy. b) Relative Changes of the First Order and Second Order “Extra” Prestress Anisotropies.

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5.6 Conclusions

In this chapter

$\bar{\epsilon}$, $\bar{k}$, $\bar{\epsilon}$ and $\bar{k}^2 / \bar{\epsilon}$.

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Eq.(5.11) is nevertheless fundamentally important. It is noteworthy that if $\underline{\underline{\Delta}} = \underline{\underline{0}}$, then the only prolate solution is $\underline{\underline{R}} = \underline{\underline{B}} = \underline{\underline{e}}_x \underline{\underline{e}}_x$. Figure 5.7 also shows that the relative importance of the first-order and the second-order anisotropic operators on the local anisotropic state. During the initial stage of the decay process ($\tilde{t} \ll 1$), the first-order anisotropic operator $\underline{\underline{\Delta}}_1$ controls the redistribution of energy (see Figure 5.7a and Figure 5.7b). However, during the final stage ($\tilde{t} \gg 1$), the second-order anisotropic operator $\underline{\underline{\Delta}}_2$ controls the transfer of energy from the transverse plane to the longitudinal axis.

5.6 Conclusions

In this chapter, the NR-stress distribution, along with the dynamic behavior of $\tilde{k}$, $\tilde{\varepsilon}$, $\tilde{k}/\tilde{\varepsilon}$ and $\tilde{k}^2/\tilde{\varepsilon}$, for rotating homogeneous flows is studied for various rotation rates.

1. The decay of $\tilde{k}$ and $\tilde{\varepsilon}$ shown in the phase plane indicates that both $\tilde{k}$ and $\tilde{\varepsilon}$ decay for all rotation rates. The decay dynamics in the initial stage indicate that there is decreased dissipation rate with increased rotation. During the initial stage (i.e., $0 < \tilde{t} < 0.1$), an enhanced rate-of-decay of $\tilde{\varepsilon}$ is observed because of the decrease in $\tilde{\tau}_R$. However, during the final stage for $\tilde{t} > 4$, where rotation mitigates the rate of decay of dissipation, dissipation for $\tilde{\Omega}_0 = 10$ eventually exceeds the dissipation for $\tilde{\Omega}_0 = 0$ (see Figure 5.1b).
2. The turbulent time scale and dispersion monotonically decrease for non-rotating homogeneous decay. However, in the presence of rotation, turbulent dispersion, $\tilde{k}^2/\tilde{\varepsilon}$ becomes unbounded for all rotation rates (see Figure 5.4) with the initial

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slopes defined in Eq. (5.3), to be either positive or negative depending the initial rotation number $\tilde{\Omega}_0$.

3. In rotating homogeneous decay, the anisotropic quadratic form associated with the NR-stress is elongated by a coupling between velocity fluctuations and the Coriolis acceleration. The component equation for R_{xx} (see Eq.(5.10)) shows that prolate anisotropic states exist for the rotation group $-\infty < N_{\Omega} = 2C_{R1}\tilde{\tau}_R\tilde{\Omega} < \infty$.
4. For $\tilde{\Omega}_0 < 1$, energy is shifted slowly to the R_{xx} component ($R_{xx} = 1$). The transfer time is significant inasmuch as $\tilde{t}_f > 30$. However, for rotation rates $\tilde{\Omega}_0 > 1$, the redistribution of energy from an isotropic initial state is attained in a relatively short time (c.g. $\tilde{\Omega}_0 = 15$, $\tilde{t}_f \cong 1.8$).
5. The extra-anisotropy terms in the preclosure play the role of this transfer of energy in URAPS-closure. During the initial and final decay stages, first and second order anisotropic contributions to the prestress control the transfer process (see Figure 5.7).

6.1 Introduction

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CHAPTER 6

HOMOGENEOUS SHEAR

6.1 Introduction

Statistically homogenous shear flow is characterized by constant or uniform mean shear independent of spatial position. Self similar solutions obtained for this flow observed in experiments (Rohr et al., 1988, Tavoularis and Karnik, 1989) as well as DNS and LES simulations (see Salhi and Cambon, 1997, Brethouwer, 2005) for rotating homogeneous shear discussed in Chapter 2, are often used in turbulence modeling to calibrate and validate closures for low-order statistical properties. Figure 6.1 illustrates the idea of a homogeneous shear with rotation. The shear rate Γ_{yz} is constant and the rotation rate is $\underline{\underline{\Omega}} = \Omega_x \mathbf{e}_x$. In this chapter, the dynamic equation for $\tilde{\Gamma} = \Gamma_{yz} k / \varepsilon$ and the URAPS-closure component equations are presented in Sections 6.2 and 6.3. In Section 6.4, the asymptotic states for different values of Ω_x / Γ_{yz} are determined by solving the dynamic equation for $\tilde{\Gamma} = \Gamma_{yz} k / \varepsilon$ and the URAPS-equation. The predictions are also compared with the literature discussed in Chapter 2. The NR-stress results for various initial conditions (i.e., $\tilde{\Gamma}_0$) are discussed in Section 6.6. Sections 6.5 and 6.7 are focused on understanding the contribution of the “extra” anisotropy terms in the URAPS-closure.

6.2 Dynamic Equation for $\tilde{\Gamma}$

As discussed in Chapter 3, the k and epsilon equations support the existence of self-similar behavior. That is, if $k(\underline{x}, t)$ is proportional to $\varepsilon(\underline{x}, t)$ within a finite space-

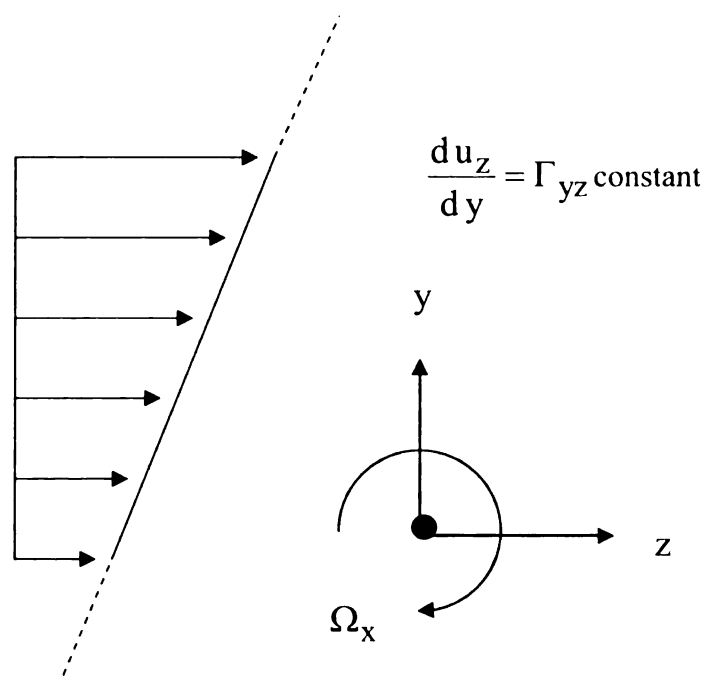


Figure 6.1 Schematic of Rotating Homogeneous Shear Flow

time domain (i.e., $k \ll \omega$)

field time scale $\tilde{\Gamma}$ sat

$$\frac{d\tilde{\Gamma}}{dt} = \frac{1}{\tilde{\Gamma}} - 2\tilde{\Gamma} R \equiv \frac{1}{\tilde{\Gamma}} - \frac{1}{\tilde{\tau}_P}$$

$$= ([P] - \epsilon) - 1 - \frac{1}{\tilde{\tau}_P}$$

where $\tilde{\Gamma} \equiv \tilde{\Gamma}_0 \frac{\tilde{k}}{\tilde{\epsilon}}$, $\tilde{\Gamma}_0 \equiv \frac{1}{\tilde{\tau}_P}$

This flow shows an

equation can be rearranged

$$\dot{\tilde{\Gamma}}_R(x, N_F) = \frac{\tilde{C}_P [P]}{[P]}$$

where $\tilde{C}_P \equiv C_P + C_R$

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6.3 Preclosure C

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which the angular

time domain (i.e., k/ε is a constant), then the ratio of the turbulent time scale to the mean field time scale $\tilde{\Gamma}$ satisfies the following non-linear algebraic equation:

$$\begin{aligned} \frac{d\tilde{\Gamma}}{d\tilde{t}} &= \left\{ -2\tilde{\Gamma} \underline{\underline{R}} : \underline{\underline{e}}_z \underline{\underline{e}}_y - 1 \right\} - \left\{ \tilde{C}_P \frac{-2\tilde{\Gamma} \underline{\underline{R}} : \underline{\underline{e}}_z \underline{\underline{e}}_y}{\tilde{\tau}_R} - \tilde{C}_D \frac{1}{\tilde{\tau}_R} \right\} \\ &= ([\mathcal{P}/\varepsilon] - 1) - \frac{1}{\tilde{\tau}_R} (\tilde{C}_P [\mathcal{P}/\varepsilon] - \tilde{C}_D) = 0 \end{aligned} \quad (6.1)$$

$$\text{where } \tilde{\Gamma} \equiv \tilde{\Gamma}_0 \frac{\tilde{k}}{\tilde{\varepsilon}}, \quad \tilde{\Gamma}_0 \equiv \Gamma_{yz} \frac{k_0}{\varepsilon_0}, \quad \tilde{t} \equiv \Gamma_{yz} t_u \quad \text{and} \quad t_u \equiv \frac{z}{\langle u_z \rangle(0)}.$$

This flow shows an asymptotic solution that $\frac{d\tilde{\Gamma}}{d\tilde{t}} = 0$, (i.e. $\tilde{\Gamma}^a = \text{constant}$) The above equation can be rearranged as below in Eq. (6.2)

$$\tilde{\tau}_R(\infty, N_F) = \frac{\tilde{C}_P [\mathcal{P}/\varepsilon] - \tilde{C}_D}{[\mathcal{P}/\varepsilon] - 1}, \quad [\mathcal{P}/\varepsilon] = -2\tilde{\Gamma} (\underline{\underline{R}} : \underline{\underline{e}}_z \underline{\underline{e}}_y) \quad (6.2)$$

where $\tilde{C}_P \equiv C_P / C_{R1}$, $\tilde{C}_D \equiv C_D / C_{R1}$ and $\mathcal{P}/\varepsilon$ is the ratio of production of turbulent kinetic energy to dissipation. It should be noted that $\mathcal{P}/\varepsilon$ is also a constant for the asymptotic states. The above Eq. (6.1) however, needs the information of the shear component of NR-stress that can be computed from the URAPS-closure. Eq. (6.1) together with the URAPS-closure can be solved for developing homogeneous flows so that Eq. (6.2) is satisfied in the asymptotic region. The URAPS-closure equations are discussed in the following section.

6.3 Preclosure Operator and URAPS-Closure

Asymptotic homogeneous shear (AHS-) flow in a rotating frame of reference for which the angular velocity is co-linear with the mean field vorticity has the following

URAPS equations (>

case). The preclosure

$$\underline{\underline{A}}^{-1} = \underline{\underline{I}} + \tau_R < \underline{\underline{F}} > =$$

$$\underline{\underline{A}} = \frac{\underline{\underline{C}}}{\det(\underline{\underline{I}} + \tau_R < \underline{\underline{F}} >}$$

$$= \frac{(1 + N_\Gamma N_\Omega + N_\Omega^2}$$

where $N_\Gamma \equiv \bar{\tau}_R \bar{\Gamma}$ and

$\bar{\Gamma}$ and $\bar{\Omega}$ or for asymm;

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N_Ω were determined

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6.4 Asymptotic S

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solving the above

URAPS equations (see Appendix G for URAPS equations with $N_\Gamma \neq 0$ and $N_\Omega \neq 0$ case). The preclosure operator for this flow is,

$$\underline{\underline{A}}^{-1} = \underline{\underline{I}} + \tau_R \langle \underline{\underline{F}} \rangle = \underline{e}_x \underline{e}_x + \underline{e}_y \underline{e}_y + \underline{e}_z \underline{e}_z + (N_\Gamma + N_\Omega) \underline{e}_y \underline{e}_z - N_\Omega \underline{e}_z \underline{e}_y \quad (6.3)$$

$$\begin{aligned} \underline{\underline{A}} &= \frac{\underline{\underline{C}}}{\det(\underline{\underline{I}} + \tau_R \langle \underline{\underline{F}} \rangle)} \\ &= \frac{(1 + N_\Gamma N_\Omega + N_\Omega^2) \underline{e}_x \underline{e}_x + \underline{e}_y \underline{e}_y + \underline{e}_z \underline{e}_z - (N_\Gamma + N_\Omega) \underline{e}_y \underline{e}_z + N_\Omega \underline{e}_z \underline{e}_y}{(1 + N_\Gamma N_\Omega + N_\Omega^2)} \end{aligned} \quad (6.4)$$

where $N_\Gamma \equiv \tilde{\tau}_R \tilde{\Gamma}$ and $N_\Omega \equiv 2 \tilde{\tau}_R \tilde{\Omega}$. URAPS-closure can be fully defined by specifying $\tilde{\Gamma}$ and $\tilde{\Omega}$ or for asymptotic homogeneous shear. N_Γ and N_Ω defined above are used to simplify the component equations. Through the developing flow region towards asymptotic states, the ratio $\frac{1}{2} \frac{N_\Omega}{N_\Gamma} (= \frac{\tilde{\Omega}}{\tilde{\Gamma}} = \frac{\Omega_x}{\Gamma_{yz}})$ remains constant although N_Γ and N_Ω change with time. Solutions to URAPS-closure equations for a given value of N_Γ and N_Ω were determined by successive substitution. For rotating simple shear flows, the algorithm converges to a unique solution for any initial non-negative operator (i.e., $n = 0$) with invariants within the L-diagram (see Figures 4.6 and 4.7)

6.4 Asymptotic States for $\tilde{\Gamma}$ and the NR-Stress: Results and Discussion

The self-similar solutions summarized in this section were developed numerically by simultaneously solving Eq.(6.1) and the URAPS-closure equations for long times such that RHS of Eq. (6.1) is essentially zero. Appendix H has the flowchart and program for solving the above equations. Figure 6.2 shows that non-trivial, self-similar solutions

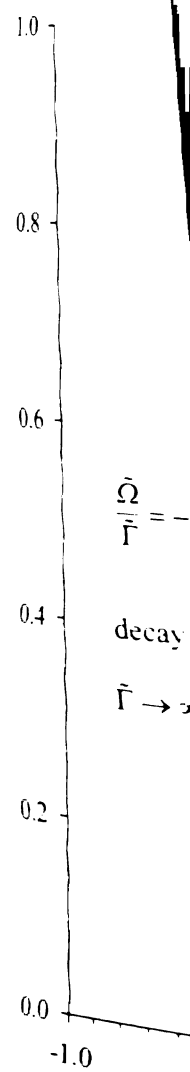


Figure 6.2

Value

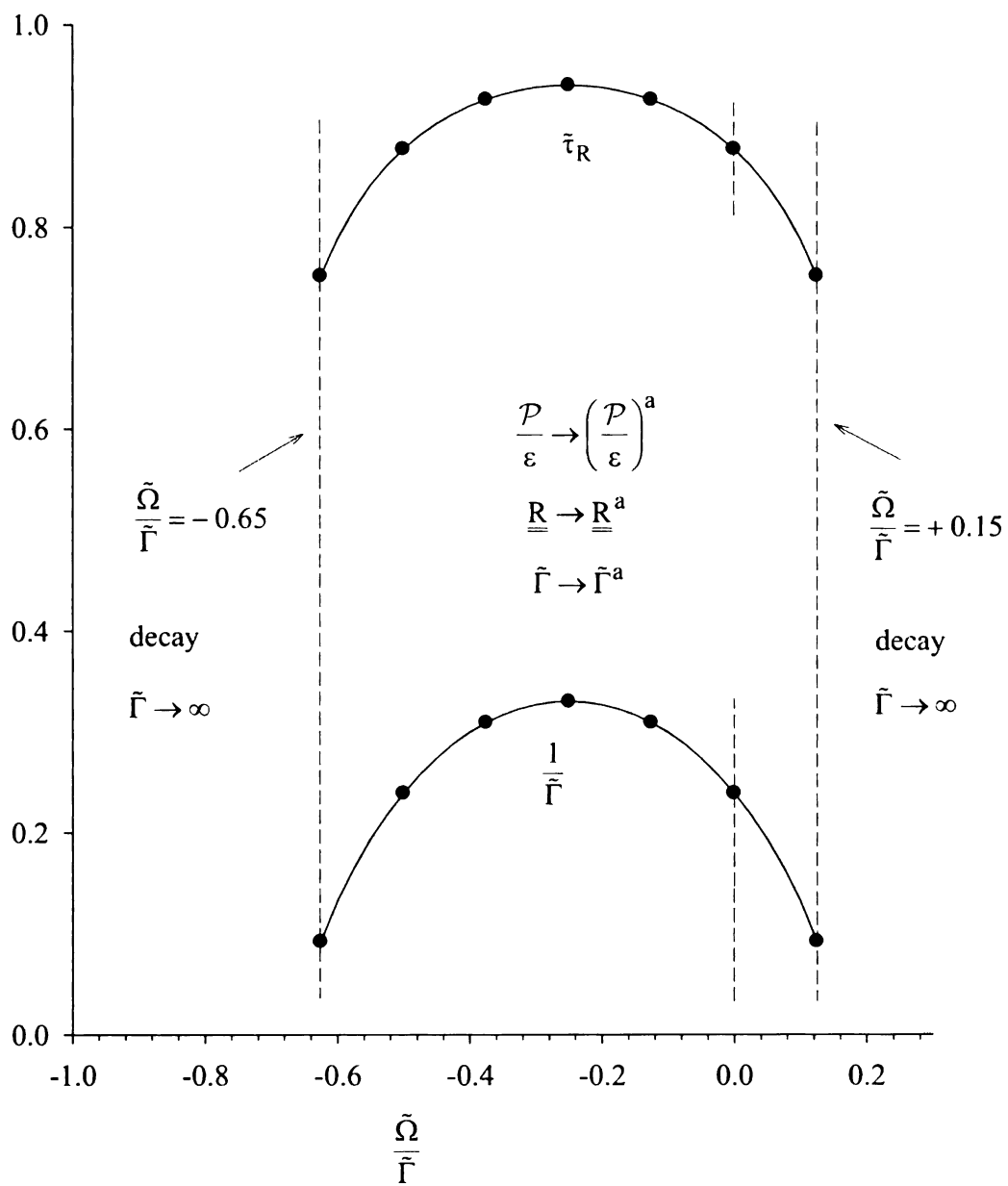


Figure 6.2 Values of $\tilde{\tau}_R$ and $1/\tilde{\Gamma}$ for the Asymptotic States Characterized by $\tilde{\Omega}/\tilde{\Gamma}$.

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Piquet, p. 213, 199

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Figure 6.3).

occur for a finite range of rotation numbers: $-0.65 = (\frac{\tilde{\Omega}}{\tilde{\Gamma}})_{\min} < \frac{\tilde{\Omega}}{\tilde{\Gamma}} < (\frac{\tilde{\Omega}}{\tilde{\Gamma}})_{\max} = +0.15$.

The lower curve in Figure 6.2 shows the effect of the rotation number on $\tilde{\Gamma}^{-1}$ and the upper curve shows the effect of $\tilde{\Omega}/\tilde{\Gamma}$ on the dimensionless transport time $\tilde{\tau}_R$. Clearly, the rotation number has a large effect on $\tilde{\Gamma}$. For example, with $\tilde{\Omega} = 0$, $\tilde{\Gamma} = 4.2$. However, near the upper and lower bounds on $\tilde{\Omega}/\tilde{\Gamma}$ (see States 1 and 7), $\tilde{\Gamma}$ is about 11. As $\tilde{\Omega}/\tilde{\Gamma}$ decreases from 0 to $-1/4$, $\tilde{\Gamma}$ decreases from 4.2 to 3, which is a local minimum.

Figure 6.3 illustrates the effect of rotation on the redistribution of the normalized turbulent kinetic energy and on the anisotropic invariants. The URAPS-closure predicts that all the anisotropic states are prolate-like (i.e., $\text{III}_b > 0$). For $\tilde{\Omega}/\tilde{\Gamma} > 0$, the energy is removed from the cross-flow and span-wise components of the fluctuating velocity and shifted into the stream-wise component. For $\tilde{\Omega}/\tilde{\Gamma} = 0$, R_{zz} is about 0.55 and increase to about 0.60 as $\tilde{\Omega}/\tilde{\Gamma}$ increases to $+1/8$. However, as $\tilde{\Omega}/\tilde{\Gamma}$ decreases to $-8/5$, R_{zz} decreases from 0.55 to 0.14; and, R_{yy} increases from 0.20 to 0.60. As shown in Figure 6.3, the predicted shift in the turbulent energy by the URAPS-closure is qualitatively consistent with results observed in DNS and LES studies (see Bardina et al., 1983; Piquet, p. 213, 1999; Brethouwer, 2005) (The solution is extrapolated from the short time simulations). The widely used B-closure predicts an equipartition of energy among the three components of the fluctuating velocity for all values of the rotation group (see Figure 6.3).

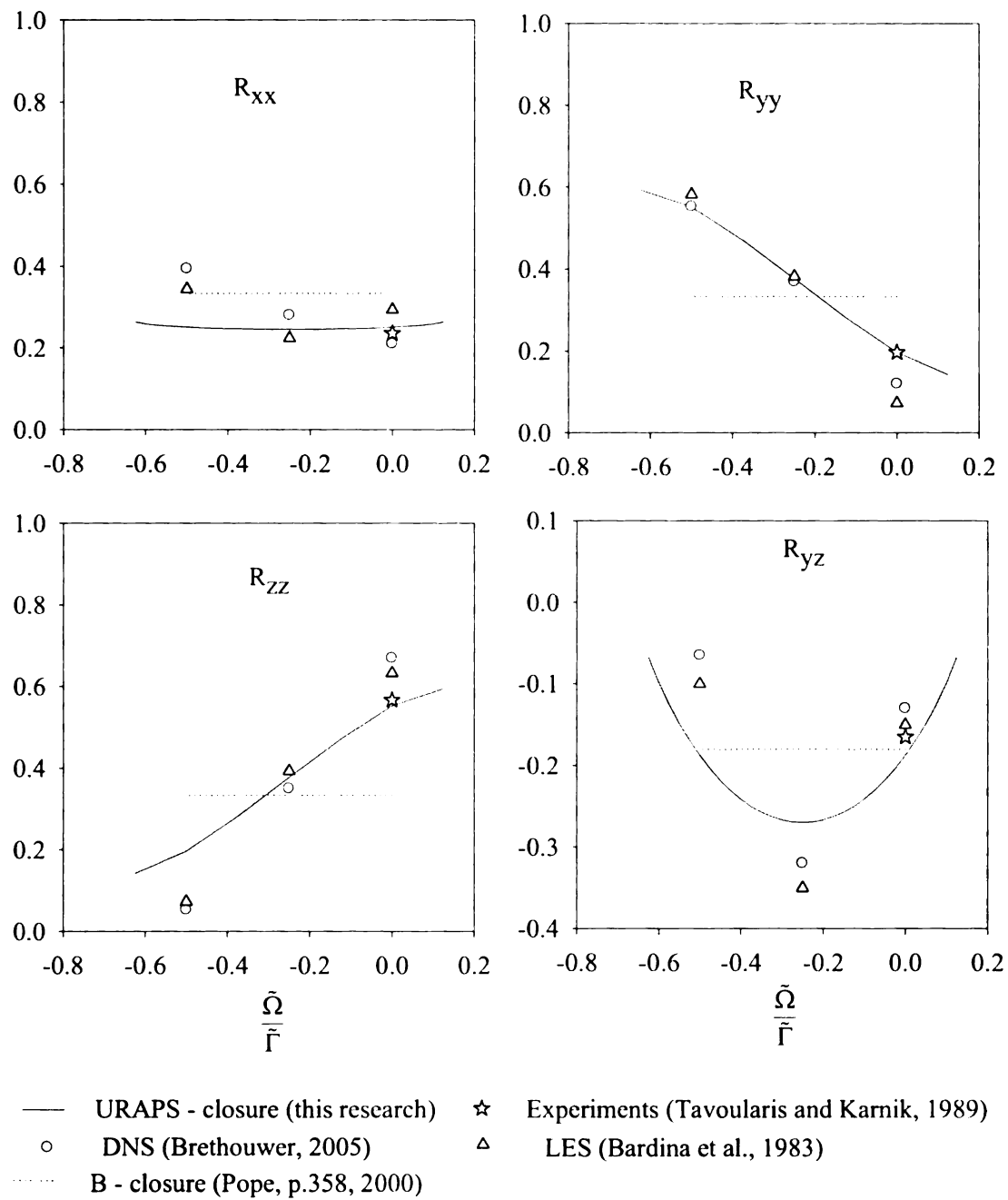


Figure 6.3 Asymptotic States of the Components of $\underline{\underline{R}}$ (Comparison with DNS and LES data)

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absolute vorticity di
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Piquet, p.259, 1999

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It is especially interesting to note that $\tilde{\Omega}/\Gamma = -0.5$ corresponds to the zero mean absolute vorticity discussed in Chapter 2. This region occurs in the HP side of spanwise rotating channel (see Chapter 7). The results for the asymptotic solution in this region and the results in rotating channel (see Table I.4, Appendix I) show the same trend of energy distribution with $R_{yy} > R_{zz}$.

URAPS-closure predictions are qualitatively consistent with the second-order closures which predict asymptotic solutions for $-0.53 = (\frac{\tilde{\Omega}}{\Gamma})_{\min} < \frac{\tilde{\Omega}}{\Gamma} < (\frac{\tilde{\Omega}}{\Gamma})_{\max} = +0.09$ (Speziale et al., 1996, Piquet, p.259, 1999). Rapid Distortion Theory (RDT) predicts asymptotic states in the region of about $-0.6 < \frac{\tilde{\Omega}}{\Gamma} < 0.2$ (see Salhi and Cambon, 1997, Piquet, p.259, 1999). Standard B-closure based models cannot predict this bifurcation region which distinguishes region of self-similar solutions with regions where there are no self similar solutions. Linear and Bilinear Shih models (Shih et al. 1994, Shih et al. 1995) predict self similar solutions for $-\infty < \frac{\tilde{\Omega}}{\Gamma} < \infty$. For all values of $\frac{\tilde{\Omega}}{\Gamma}$, the value of $\tilde{\Gamma}$ approaches a constant within a finite time, which is not consistent with the knowledge of DNS and LES results. In case of URAPS-closure, asymptotic states are observed for a finite range of rotations. The effect of rotation on the Eq. (6.1) through the production term which causes this phenomenon of asymptotic states for finite rotation rates is captured. Further details of why the asymptotic states occur for URAPS and not for some of the algebraic models will be clearer at the end of this chapter after studying developing states for all rotation rates.

In this section

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6.5 Asymptotic “Extra” Anisotropy: Results and Discussion

In this section, first the convergence of URAPS closure theory and the role of extra anisotropy in attaining a unique state is discussed for two of the asymptotic states. Later, the extra anisotropy contributions for all the asymptotic states are discussed. (Note that the fixed point mapping trajectory for the two cases presented below has been illustrated in Chapter 4, Section 4.7, for various initial conditions. Case 1: $\tilde{\Omega}/\tilde{\Gamma} = 0$ and $\tilde{\Gamma} = 4$; Case2: $\tilde{\Omega}/\tilde{\Gamma} = -5/8$ and $\tilde{\Gamma} = 11$.)

The components of the prestress, the NR-stress were discussed in earlier Chapter in Table 4.3, and the extra-anisotropic operator are listed in Table 6.1a for non-rotating simple shear flows with $\tilde{\Omega}/\tilde{\Gamma} = 0$ and $\tilde{\Gamma} = 4$. For this case, $N_F = 4$, $\tilde{\tau}_R = 0.8818$ and $N_\Gamma = 0.01259$. The results show that the components of the extra-anisotropic operator are more than two orders of magnitude smaller than the components of the prestress and the NR-stress. The correlation coefficients for $\underline{\underline{B}}$ (i.e., $\chi_B = |B_{yz}| / \sqrt{B_{yy}B_{zz}} = 0.558$) and for $\underline{\underline{R}}$ (i.e., $\chi_R = |R_{yz}| / \sqrt{R_{yy}R_{zz}} = 0.563$) are comparable. Table 6.1 gives the eigenvalues for $\underline{\underline{B}}$, $\underline{\underline{R}}$, and $\underline{\underline{\Delta}}$. Although the eigenvalues of the extra-anisotropic operator $\underline{\underline{\Delta}}$ are two orders of magnitude smaller than the eigenvalues of $\underline{\underline{B}}$ and $\underline{\underline{R}}$, the $\underline{\underline{\Delta}}$ -operator nevertheless plays a fundamental role in determining the fixed-point solutions to URAPS as shown by Figures 6.4 and 6.5. As illustrated by Figure 6.4, the eigenvalues of the extra-anisotropic operators, $\underline{\underline{\Delta}}_1$ and $\underline{\underline{\Delta}}_2$, determine the fine structure of the solution path near the fixed-point. Figure 6.4a shows that the eigenvalues of $\underline{\underline{\Delta}}_1$ begin to appear after a few iterations whereas the eigenvalues of $\underline{\underline{\Delta}}_2$ do not appear until after 20 iterations. If

Table 6.1a. Eigenvalue

eigenvalue	λ_i
i=1	+ 0.12
i=2	+ 0.25
i=3	+ 0.62

Table 6.1b. Eigenvalue

eigenvalue	λ_{iB}
i=1	+ 0.133
i=2	+ 0.26
i=3	+ 0.60

Table 6.1a. Eigenvalues of $\underline{\underline{B}}$, $\underline{\underline{R}}$, and $\underline{\underline{\Delta}}$ ($\tilde{\Gamma} = 4$ and $\tilde{\Omega}/\tilde{\Gamma} = 0$)

eigenvalue	λ_i^B	λ_i^R	$\lambda_i^{\Delta} \times 10^3$
i=1	+ 0.121361	+ 0.119680	+ 1.681
i=2	+ 0.253705	+ 0.252520	+ 1.185
i=3	+ 0.624934	+ 0.627800	− 2.866

Table 6.1b. Eigenvalues of $\underline{\underline{B}}$, $\underline{\underline{R}}$, and $\underline{\underline{\Delta}}$ ($\tilde{\Gamma} = 11$ and $\tilde{\Omega}/\tilde{\Gamma} = -5/8$)

eigenvalue	λ_i^B	λ_i^R	$\lambda_i^{\Delta} \times 10^3$
i=1	+ 0.133596	+ 0.131950	+ 1.646
i=2	+ 0.264508	+ 0.263520	+ 0.988
i=3	+ 0.601896	+ 0.604530	− 2.634

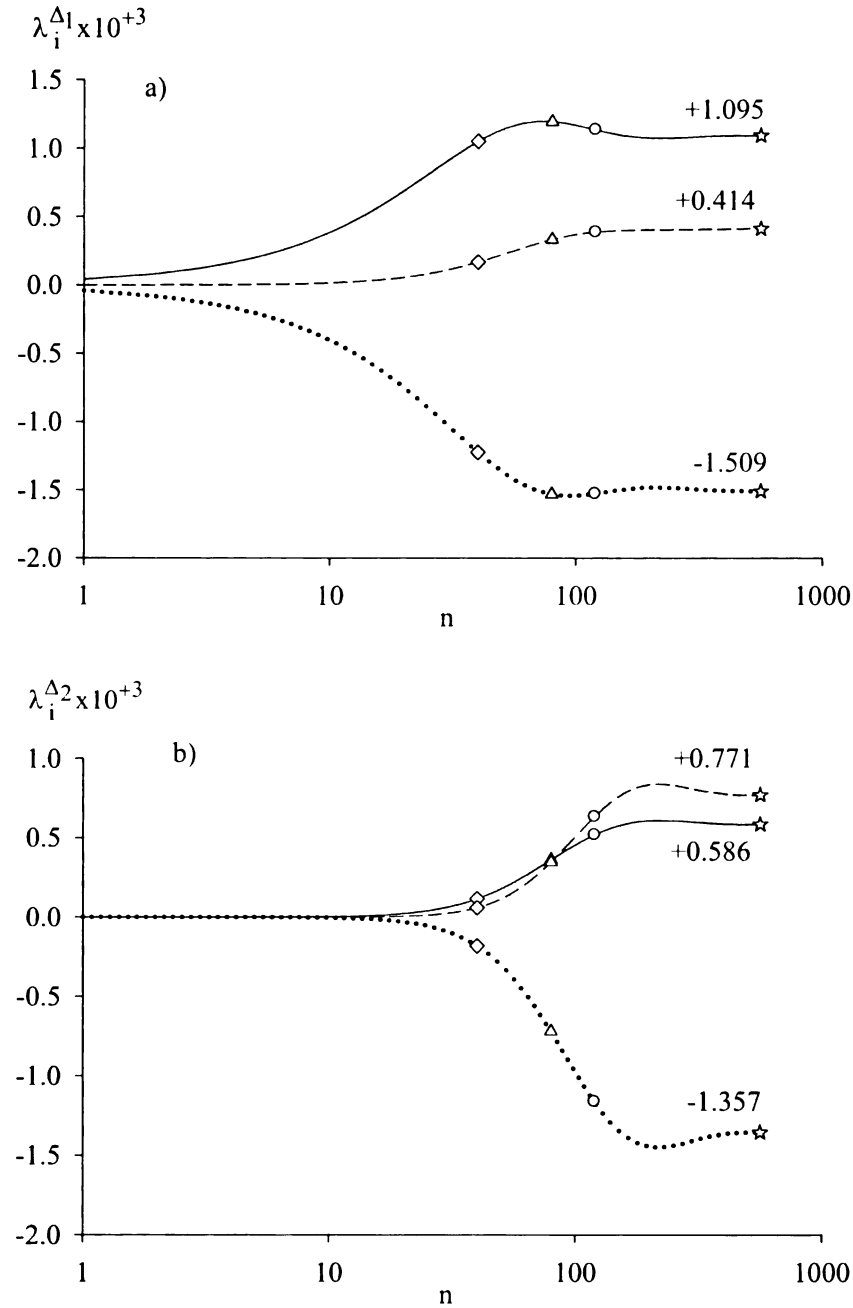


Figure 6.4 The Development of Extra-Anisotropy Eigenvalues for $\tilde{\Gamma} = 4$ and $\tilde{\Omega}/\tilde{\Gamma} = 0$ from an Isotropic State ((a) $\underline{\Delta}_1$ -eigenvalues; (b), $\underline{\Delta}_2$ -eigenvalues; λ_1 —; λ_2 ----; λ_3 ; iterations: 40 $\diamond$; 80 $\triangle$; 120 $\circ$; fixed point, $\star$).

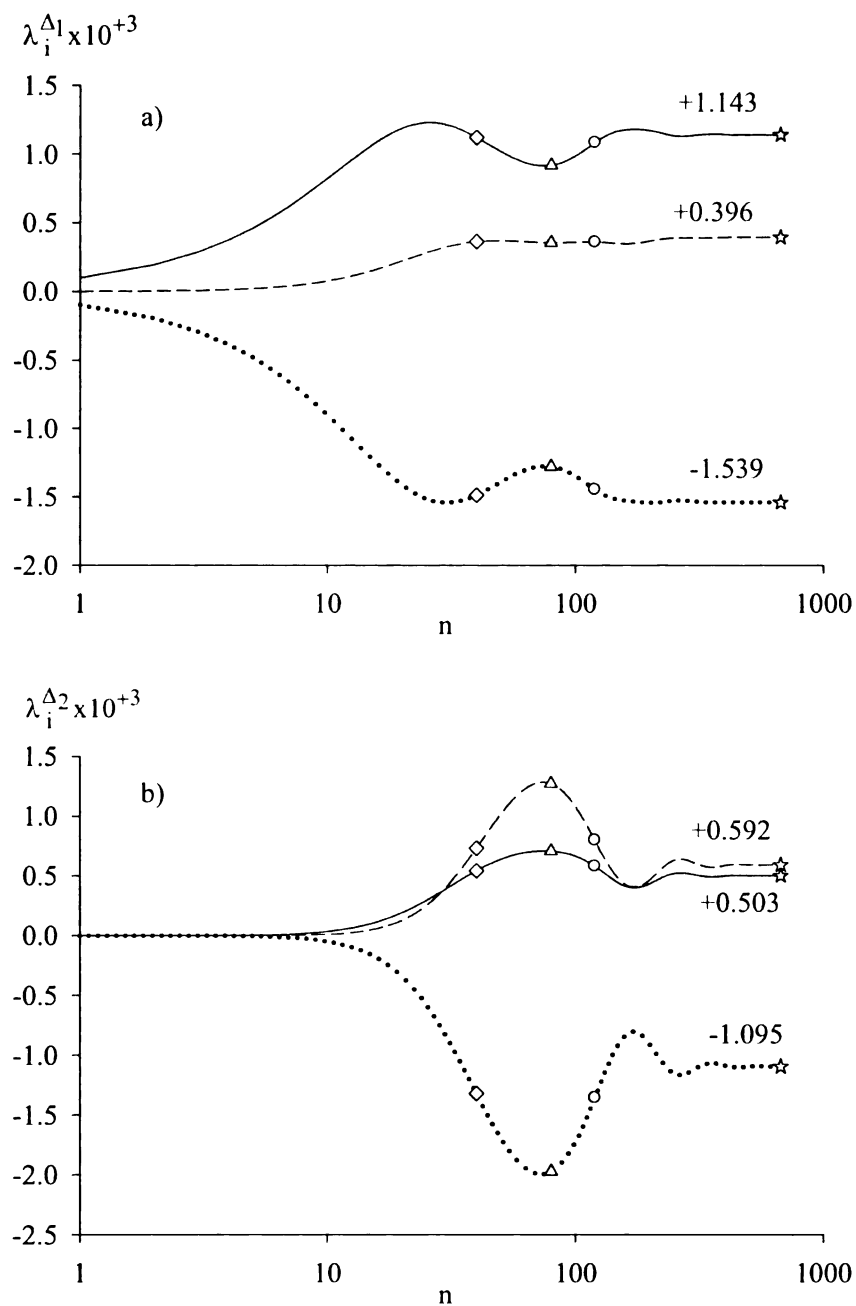


Figure 6.5. The Development of Extra-Anisotropy Eigenvalues for $\tilde{\Gamma}=11$ and $\tilde{\Omega}/\tilde{\Gamma} = -5/8$ from an Isotropic State (see Figure 6.4 for notation).

the extra-anisotropic operator were turned off for $n > 120$ (i.e., $\alpha = 0$ and $\beta = 0$), a fixed-point to URAPS still exist, but it moved to a different ellipsoidal state with $\Pi_b = 0.6662$ and $\text{III}_b = 0.2220$. This is significantly different than the $\star$ -point for which $\Pi_b^\star = 0.1389$ and $\text{III}_b^\star = 0.01525$ with $\alpha = 1/10$ and $\beta = -1/100$. Thus, the prestress extra-anisotropy, albeit small, plays an important quantitative role in determining the prestress and, thereby, the NR-stress.

The components of $\underline{\underline{B}}$, $\underline{\underline{R}}$, and $\underline{\underline{\Delta}}$ are listed earlier in Table 4.4 for rotating simple shear flows with $\tilde{\Omega}/\tilde{\Gamma} = -5/8$ and $\tilde{\Gamma} = 11$. For this case, $N_F = 14$, $\tilde{\tau}_R = 0.756$, $N_\Gamma = 0.02948$, and $N_\Omega = -0.03685$. The results in Figure 6.5 also show that the fixed-point components of the extra-anisotropic operator are more than two orders of magnitude smaller than the components of the prestress and the NR-stress. The correlation coefficients for $\underline{\underline{B}}$ and $\underline{\underline{R}}$ are comparable (i.e., $\chi_B = 0.228$ and $\chi_R = 0.231$), but significantly smaller than the non-rotating example above. Table 6.1b gives the eigenvalues for the rotating example. Like the non-rotating case, the eigenvalues of $\underline{\underline{\Delta}}$ are two orders of magnitude smaller than the eigenvalues of $\underline{\underline{B}}$ and $\underline{\underline{R}}$; and, as illustrated by Figures 6.4 and 6.5, the $\underline{\underline{\Delta}}$ -operator plays a fundamental and quantitative role in determining the NR-stress.

The eigenvalue contributions from the extra-anisotropy $\underline{\underline{\Delta}}$ are of the order of 10^{-3} for all the asymptotic states as can be seen in Figure 6.6. Two of the eigenvalues are positive while the other is negative. One of the eigenvalues remains almost approximately the same for all the rotation variations at about 1.5×10^{-3} . The variation in

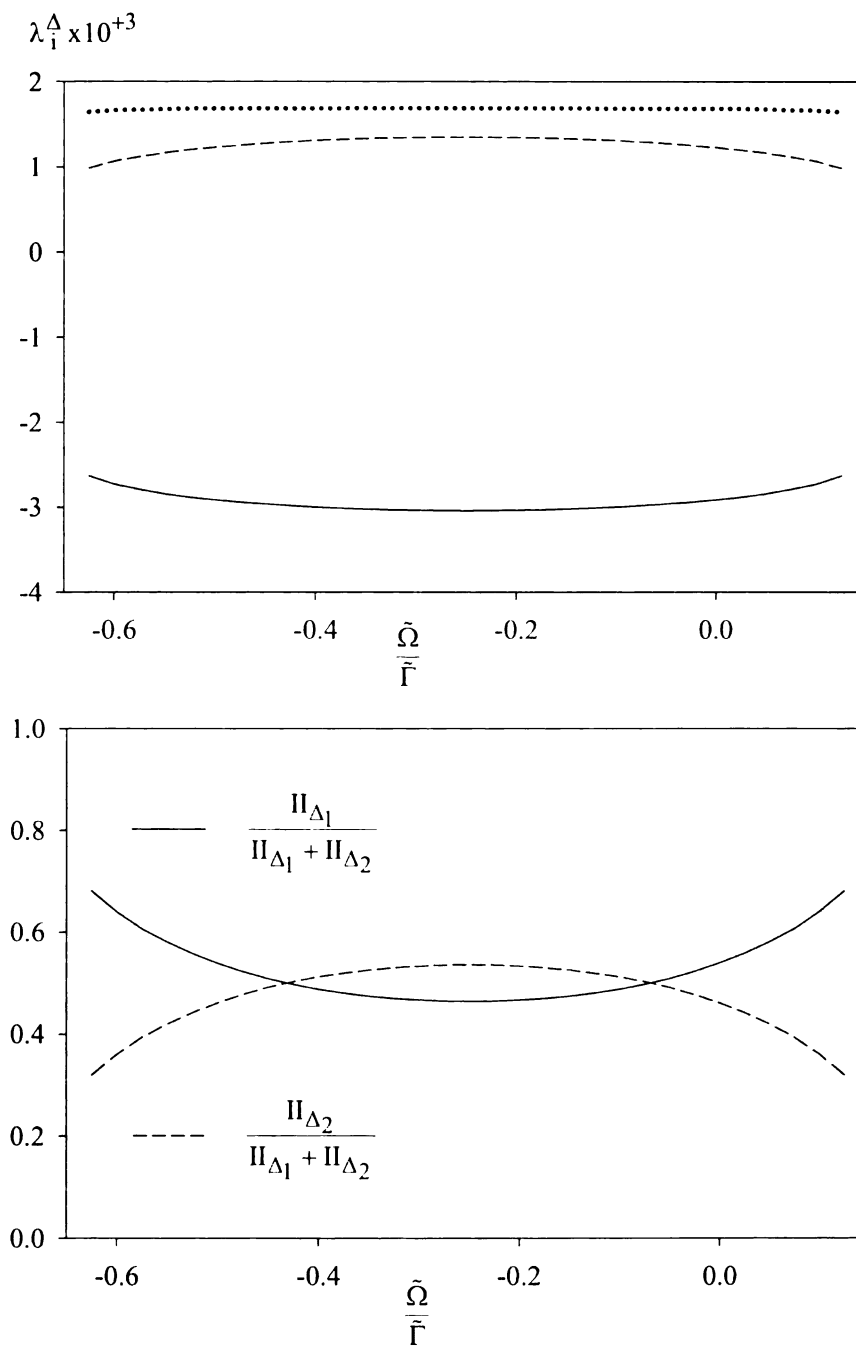


Figure 6.6 Eigenvalues of the Extra-Anisotropy $\underline{\underline{\Delta}}$ and Relative Contributions from $\underline{\underline{\Delta}}_1$ and $\underline{\underline{\Delta}}_2$ for Asymptotic Solutions.

the other two although small, causes different energy states for the NR-distribution. The normal components of NR are different for $\frac{\tilde{\Omega}}{\tilde{\Gamma}} = 0$ and -0.5 because of the different energy contributions in the R_{yy} and R_{zz} . However, the eigenvalues associated with these states are the same as there is a shift in the energy states without change in the values. This is reflected in the eigenvalues of the extra anisotropy too as the eigenvalue distribution is symmetric about $\frac{\tilde{\Omega}}{\tilde{\Gamma}} = -0.25$, with the same eigenvalues for 0 and -0.5 etc. The relative contributions from $\underline{\underline{\Delta}}_1$ and $\underline{\underline{\Delta}}_2$ can be seen in Figure 6.6. The relative values of the second invariant contributions show that contributions from $\underline{\underline{\Delta}}_2$ are significant in the range between $-0.45 < \frac{\tilde{\Omega}}{\tilde{\Gamma}} < 0.05$. At these limits, both the contributions are almost equal. Outside this range, $\underline{\underline{\Delta}}_1$ eigenvalues are more important for the anisotropic distribution.

6.6 Developing $\tilde{\Gamma}$, $\mathcal{P}/\varepsilon$ and the NR-Stress (shear component) for Homogeneous Shear

Eq. (6.2) along with the URAPS-closure discussed in Sections 6.2 and 6.6 are studied for different initial conditions of $\tilde{\Gamma}_0 = 0.5, 1, 2, 5, 10$ and 15 and for different rotation rates to understand the development towards asymptotic states which occurs only for the finite range of $\frac{\tilde{\Omega}}{\tilde{\Gamma}}$ discussed in Section 6.4. Figure 6.7 presents variation of the dimensionless shear $\tilde{\Gamma}$ from different initial values of $\tilde{\Gamma}_0$. Although at $\tilde{t} = 0$, the states

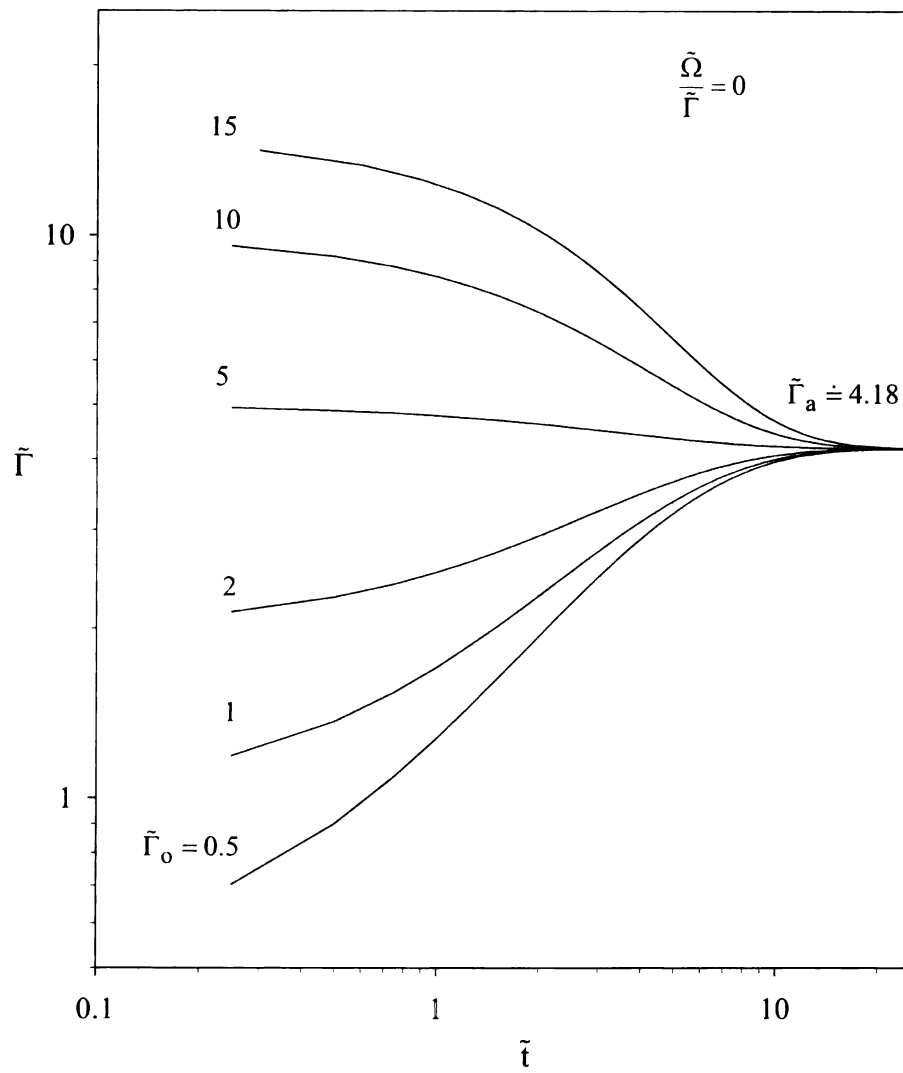


Figure 6.7 Developing $\tilde{\Gamma}$ Towards Asymptotic States for Various Initial $\tilde{\Gamma}_0$.

are all different, all the curves rapidly attain a final asymptotic state of $\tilde{\Gamma}_a \cong 4.2$. Independent of the initial condition, $\tilde{\Gamma}$ attains an asymptotic state that does not vary any further with the downstream distance in $\tilde{t} \cong 10$. Development of the shear component for different initial values of $\tilde{\Gamma}_0$ mentioned above are presented in Figure 6.8. Depending on the initial shear, as shown in the small graph, the initial R_{yz} is different. For an increase of $\tilde{\Gamma}_0$ from 0.5 to 5, there is an increase in the initial value ($\tilde{t} < 0.3$) of the shear component ($-R_{yz}$) from 0.08 to 0.19, while for $\tilde{\Gamma}_0$ variation from 5 to 10, the initial values of ($-R_{yz}$) are smaller (see Figure 6.8). However, independent of the initial values, the curves rapidly attain a final asymptotic state of $-R_{yz}^a \cong 0.19$ (in $\tilde{t} \sim 11$). As shown in Figure 6.9, the ratio of production to dissipation ($\mathcal{P}/\varepsilon$) also approaches a constant value in a finite time. The initial values range from 0 to 5 for the initial values of $\tilde{\Gamma}_0$ chosen. The final states of $(\mathcal{P}/\varepsilon)^a \cong 1.57$ are attained for all the initial conditions.

The development of NR-stress to asymptotic states is presented for high initial shear $\tilde{\Gamma}_0 = 18$ in Figure 6.10. DNS results from Brethouwer (2005) are also shown in this figure for comparison. The URAPS closure predicts NR stress distribution instantaneously to imposed shear like other algebraic models. So the initial values of NR-stress distribution is anisotropic compared to the isotropic initial condition in the DNS simulations. The simulations show development towards constant values, which however may need more simulation time steps to further develop into asymptotic states. The extrapolation of DNS results seem to be directed towards the final states predicted by the URAPS-closure (see Figure 6.10).

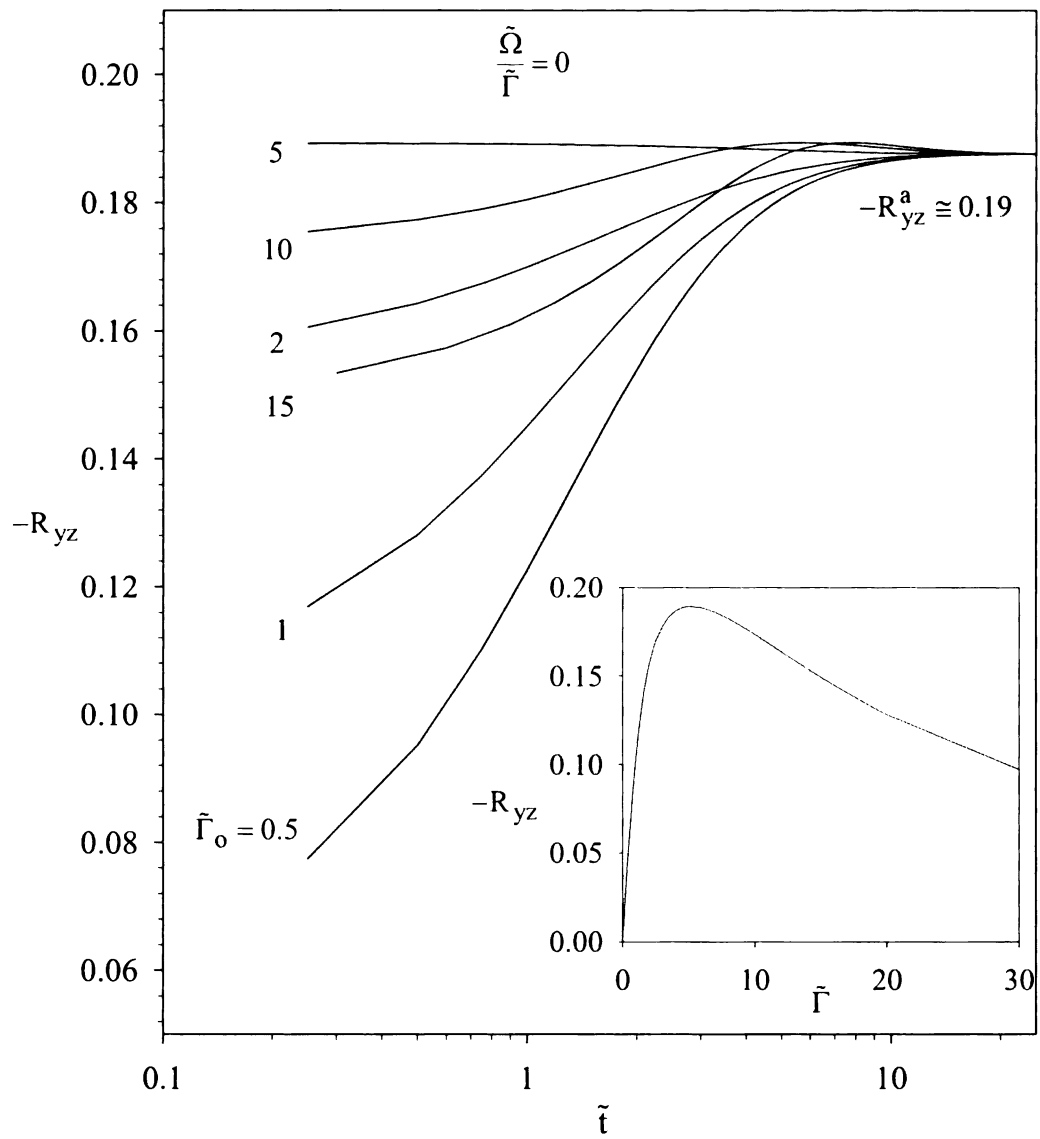


Figure 6.8 Temporal Development of $-R_{yz}$ for Different Initial $\tilde{\Gamma}_o$.

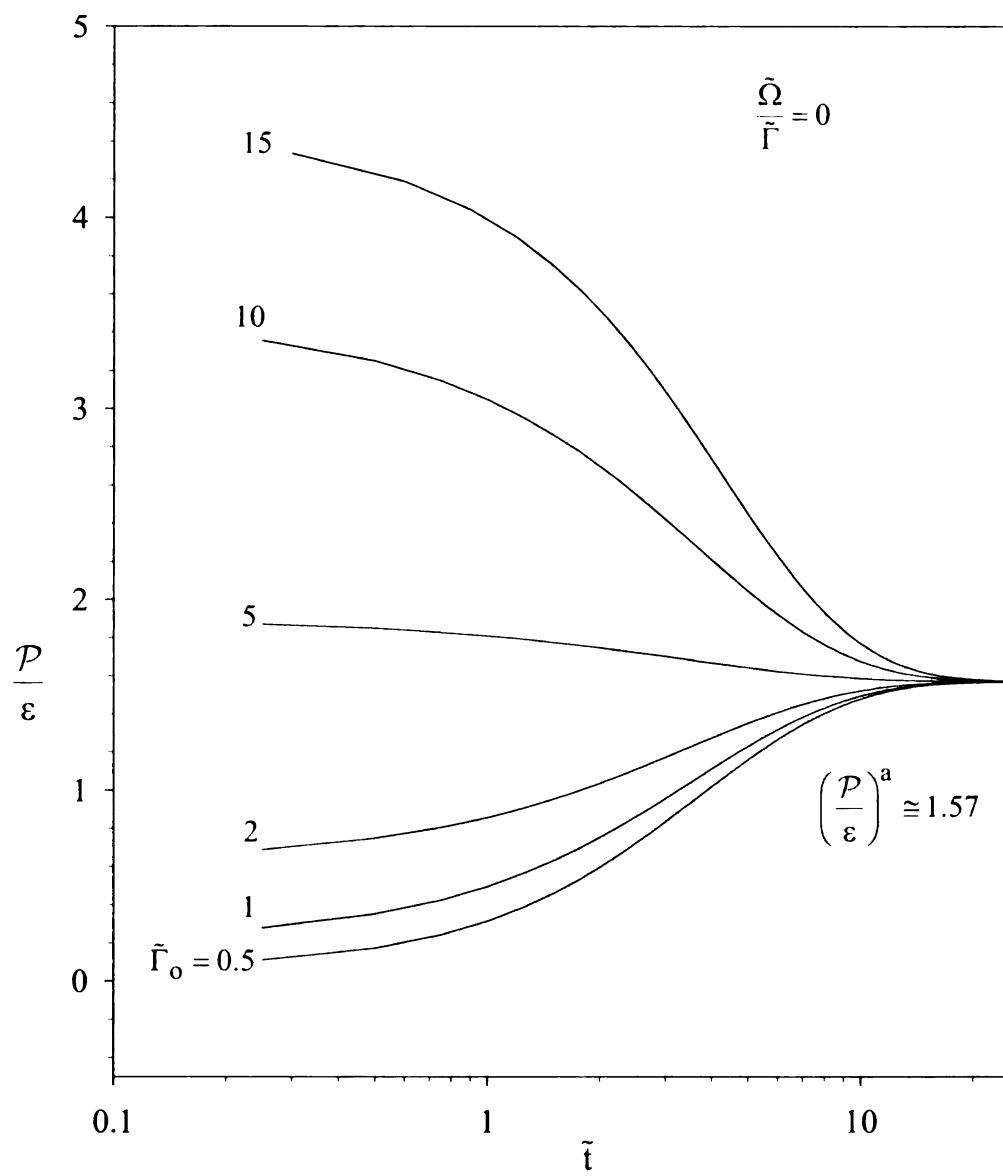


Figure 6.9 Temporal Development of Production to Dissipation Ratio for Different Initial $\tilde{\Gamma}_o$.

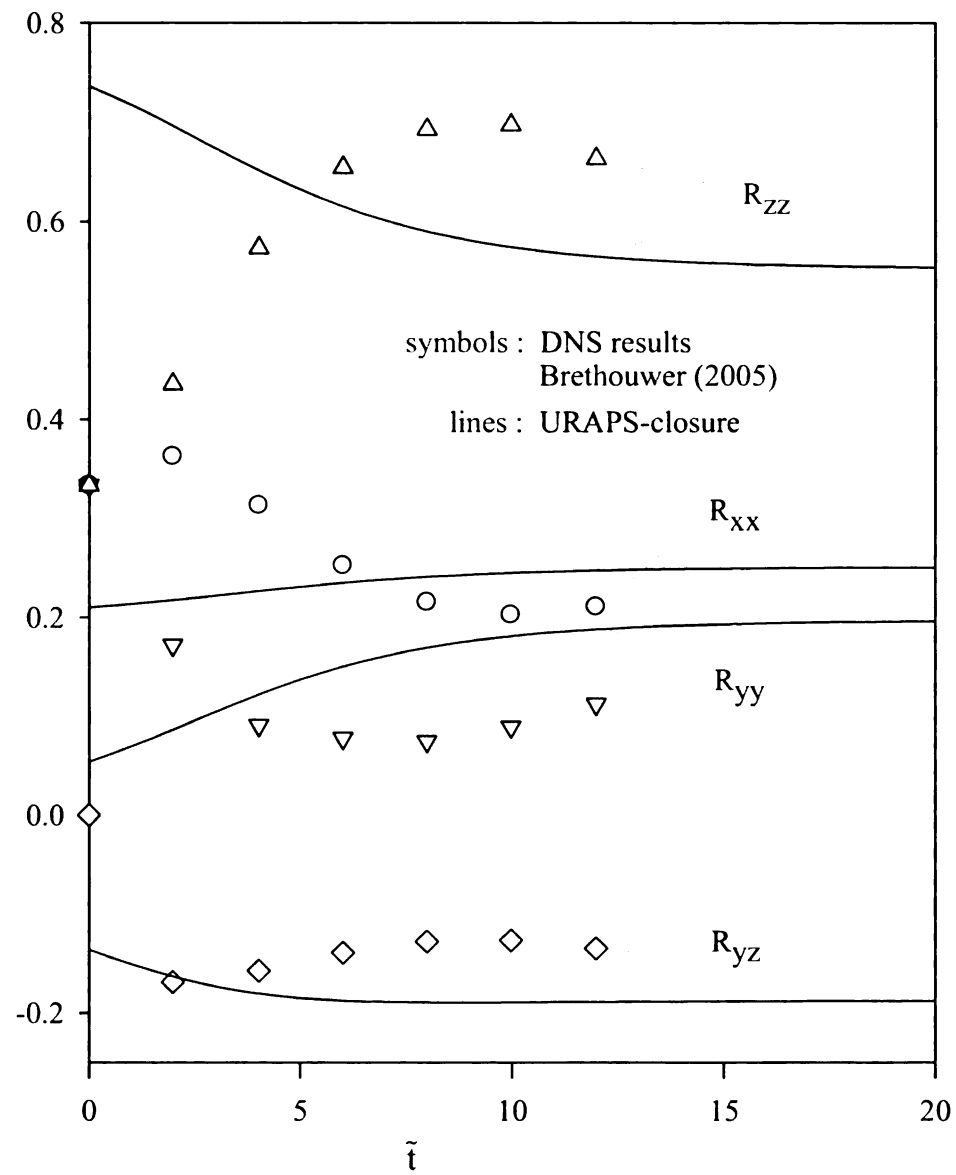


Figure 6.10 Temporal Development of NR-Stress Components Compared with DNS Results (Brethouwer, 2005) for High Initial Shear $\tilde{\Gamma}_0 = 18$ ($\tilde{\Omega}/\tilde{\Gamma} = 0$).

The asymptotic states exist for $\tilde{\Omega}/\tilde{\Gamma} = 0$ for all initial values described above. The following discussion shows the temporal development for various values of $\tilde{\Omega}/\tilde{\Gamma} = -0.75, -0.5, -0.25, -0.1, 0, 0.1, 0.25, 1$, covering beyond the range for which asymptotic solutions exist (AS-range). The initial value is fixed at $\tilde{\Gamma}_0 = 1$ for all the above cases of rotation. Interesting features below explain why there are no asymptotic states in a certain region. Figure 6.11 describes the downstream development of dimensionless shear for a fixed initial $\tilde{\Gamma}_0$, for different values of rotation rates. As discussed in the Section 6.4 of this Chapter $\frac{\tilde{\Omega}}{\tilde{\Gamma}} = -0.5, -0.25, -0.1, 0$, and 0.1 , have asymptotic states as they are range between the maximum and minimum discussed previously. However, the asymptotic states are different with $\tilde{\Gamma}^a$ being the minimum for -0.25 . For the range of $\frac{\tilde{\Omega}}{\tilde{\Gamma}} = -0.75, 0.25, 1$, and 2 , which are outside the AS-range, $\tilde{\Gamma}$ increases monotonically, with slopes equal to Eq.(6.5) below, where the slopes depend on the rotation rate through $\tilde{\tau}_R$.

$$\left. \frac{d\tilde{\Gamma}}{d\tilde{t}} \right|_{\tilde{t} \rightarrow \infty} = \left(\frac{\tilde{C}_D}{\tilde{\tau}_R} \right) - 1 \quad (6.5)$$

Eq. (6.5) is a consequence of understanding that $R_{yz} \rightarrow 0$ as $\tilde{\Gamma} \rightarrow \infty$ as can be seen in Figure 6.12. For $R_{yz} \rightarrow 0$, Eq. (6.1) becomes Eq. (6.5). Figure 6.12 describes the downstream development of the shear component of the Reynolds Stress. The initial values seem to be different for the same $\tilde{\Gamma}_0 = 1$ because of the rotation rates. R_{yz} attains different asymptotic states in a finite time for the rotation rates within the AS-range

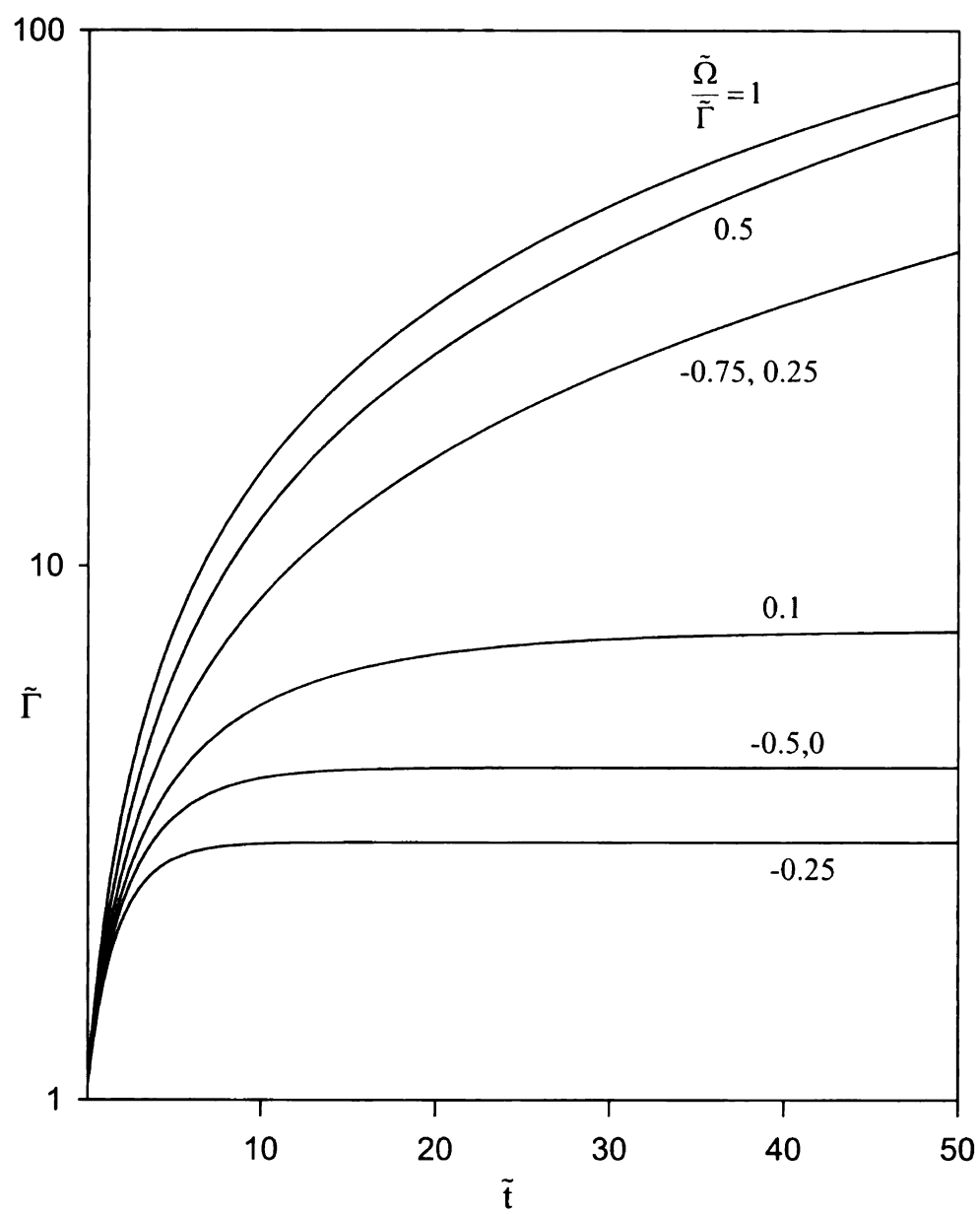


Figure 6.11 Developing $\tilde{\Gamma}$ for Different Rotation Numbers for $\tilde{\Gamma}_0 = 1$.

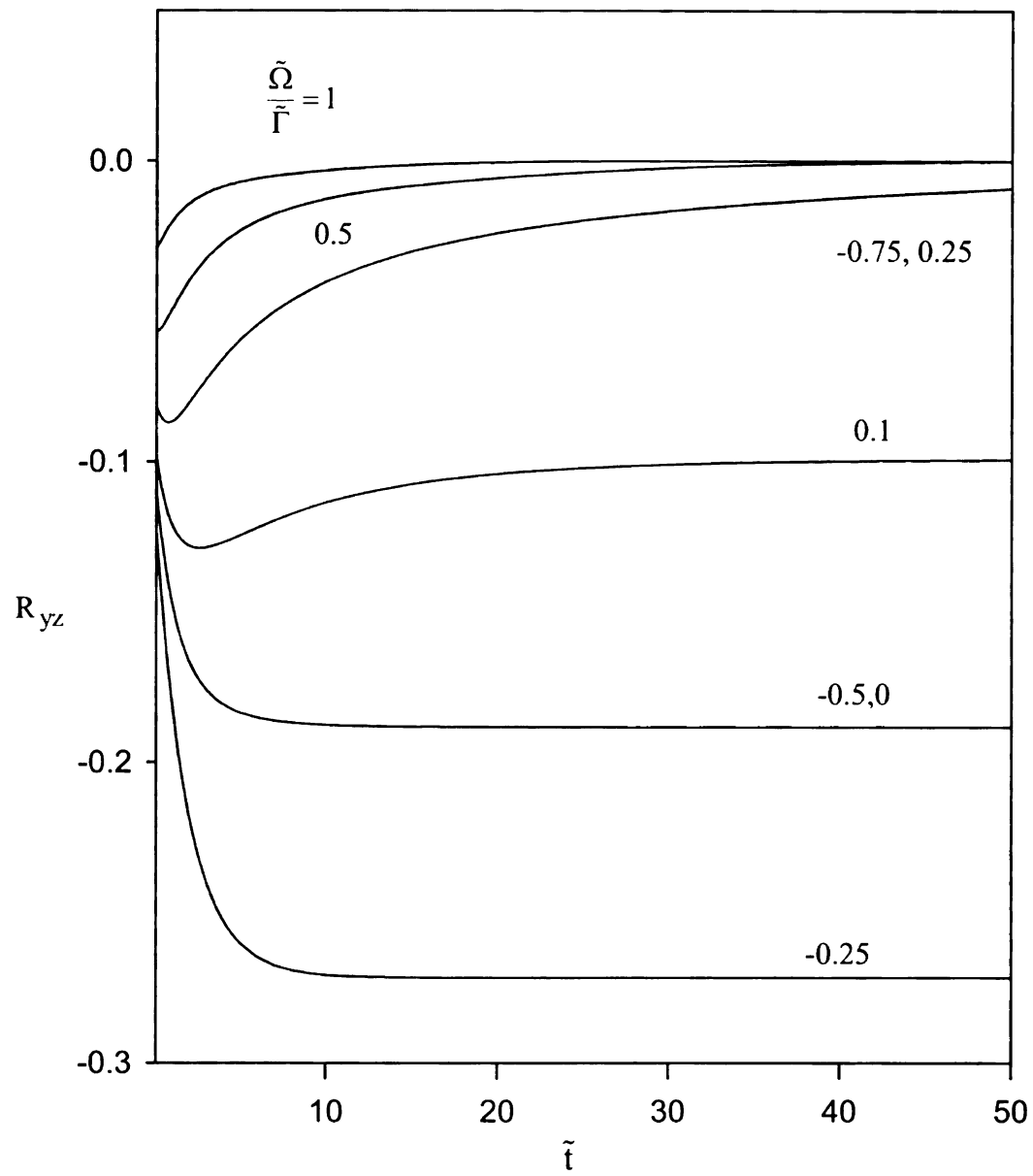


Figure 6.12 Developing R_{yz} for Different Rotation Numbers for $\tilde{\Gamma}_0 = 1$.

described earlier. Outside this range, $R_{yz} \rightarrow 0$. The development of the $\mathcal{P}/\varepsilon$ ratio is shown in Figure 6.13 for various rotations for an initial $\tilde{\Gamma}_0 = 1$. Within the AS-range, $\mathcal{P}/\varepsilon$ approaches a constant (The asymptotic states are all between 1.45 and 1.65 as shown in the inset to Figure 6.13. For the rotation rates outside the asymptotic range, $\mathcal{P}/\varepsilon$ approaches zero because R_{yz} approaches zero.

From the knowledge of the above discussion for rotation rates outside the AS-range, it can be seen that since URAPS closure and Eq. (6.1) yield $R_{yz} \rightarrow 0$ and so

$$\mathcal{P}/\varepsilon (= 2 \tilde{\Gamma} R_{yz}) \rightarrow 0. \quad \left. \frac{d \tilde{\Gamma}}{d \tilde{t}} \right|_{\tilde{t} \rightarrow \infty} \text{ approaches a constant value depending on the rotation}$$

rate as described in Eq. (6.5). For rotation rates within the AS-range, Eq. (6.1) proceeds

till the rate of change $\frac{d \tilde{\Gamma}}{d \tilde{t}} = 0$. For the bilinear model by Shih et al. (1994) and some of

the other non linear models (see Piquet 1999) which cannot predict effect of rotation through the R_{yz} component predict that all the range of rotation rates have asymptotic states and that $\mathcal{P}/\varepsilon$ and the normal components of NR-stress always approach a constant. Realizable model by Shih et al. (1995) can predict variation of the normal components of NR-stress with rotation but not the shear component as shown in Appendix E.

6.7 Developing “Extra” Anisotropy: Results and Discussion

The extra anisotropy in the URAPS-closure is studied for the following case of developing flow. $\tilde{\Gamma}_0 = 1$ and $\tilde{\Omega}/\tilde{\Gamma} = 0$. Figure 6.14 shows the eigenvalues of the extra-

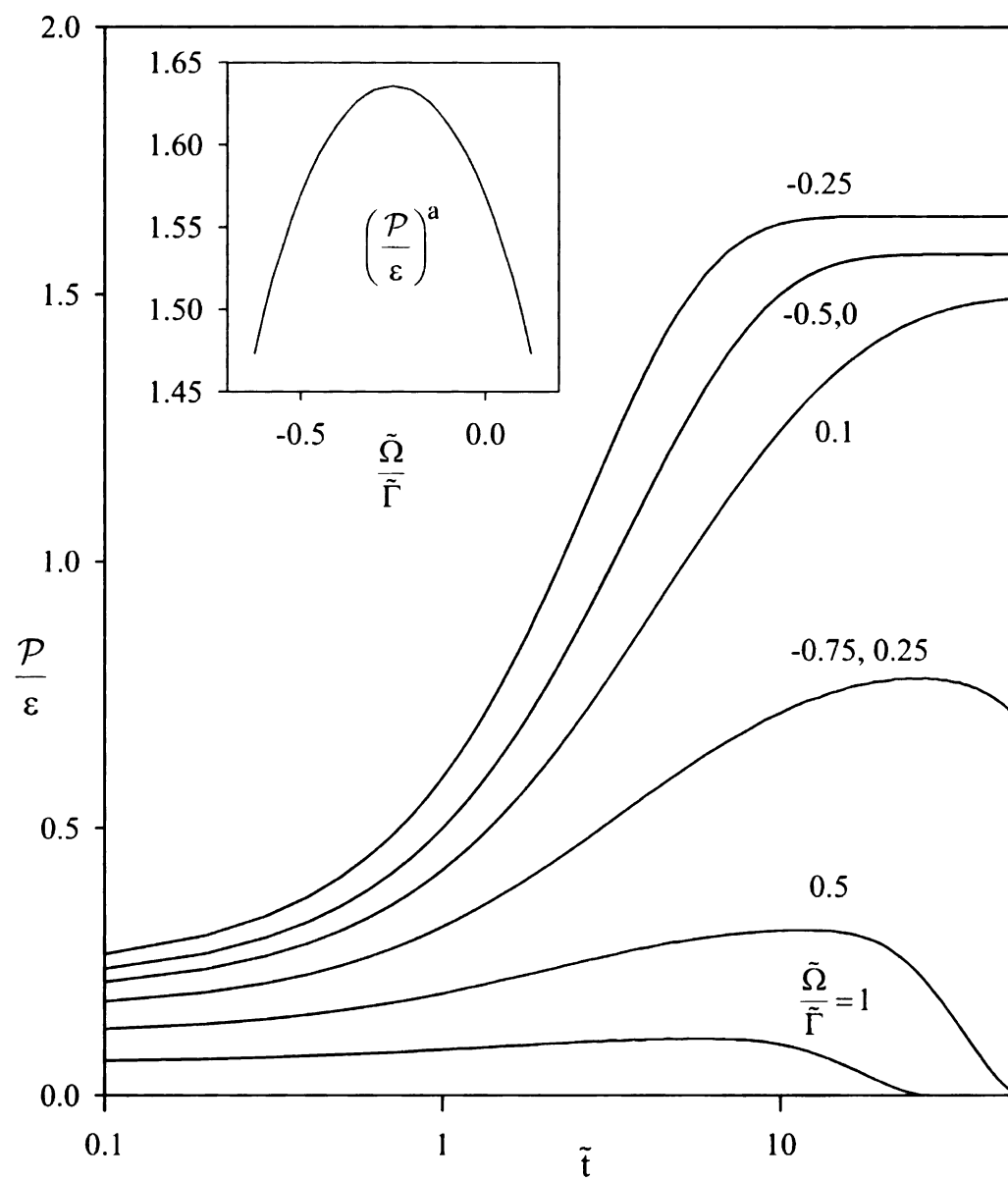


Figure 6.13 Developing $\mathcal{P}/\varepsilon$ Ratio for Various Rotation Numbers for $\tilde{\Gamma}_0 = 1$

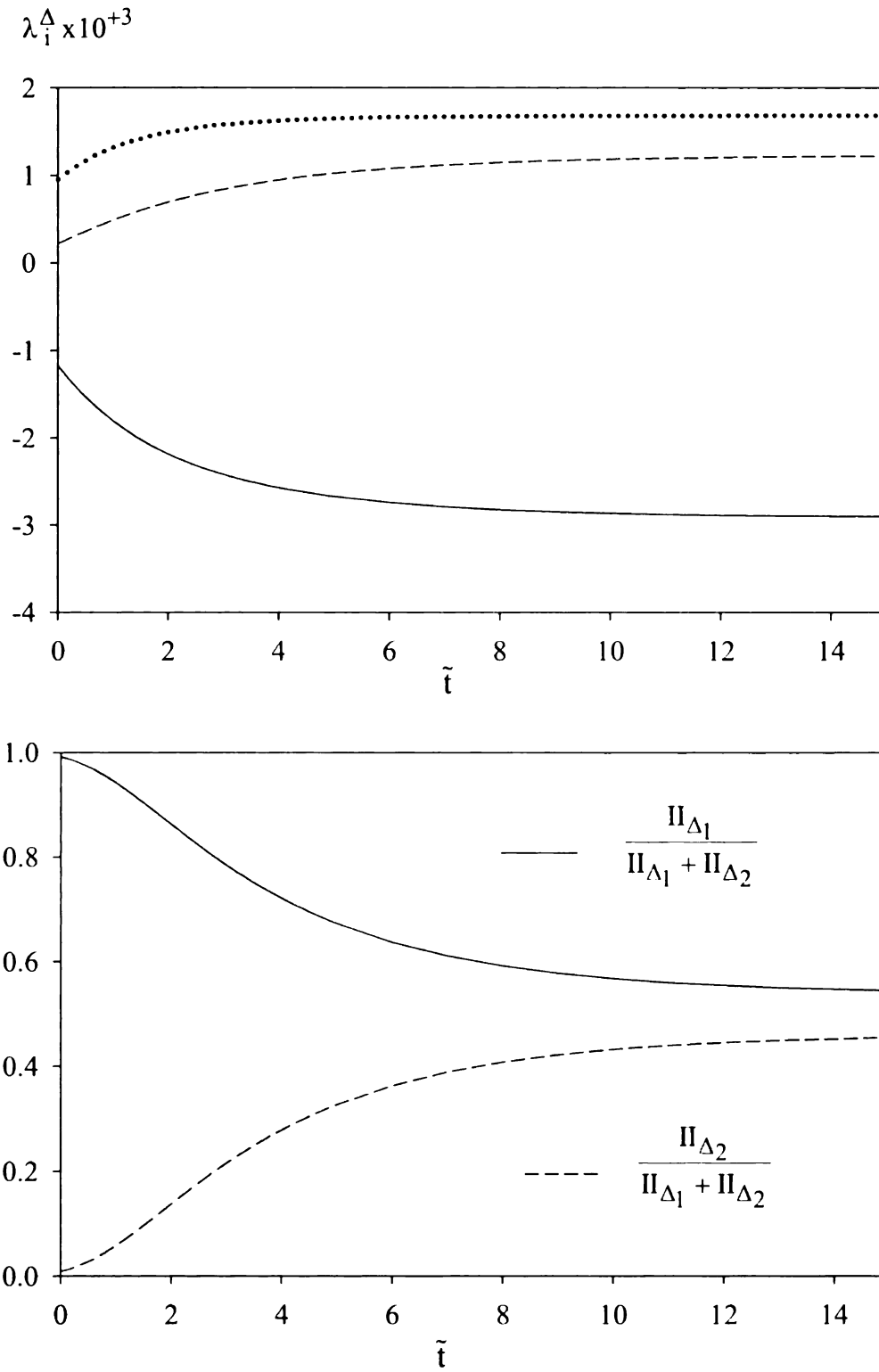


Figure 6.14 Eigenvalues of the Extra-Anisotropy $\underline{\underline{\Delta}}$ and Relative Contributions from $\underline{\underline{\Delta}}_1$ and $\underline{\underline{\Delta}}_2$ for Developing Homogeneous Shear for $\tilde{\Gamma}_0 = 1$ and $\tilde{\Omega}/\tilde{\Gamma} = 0$.

anisotropy $\underline{\underline{\Delta}}$ start with an initial distribution corresponding to the initial $\underline{\underline{R}}$ at $\tilde{t} = 0$ and $\tilde{\Gamma}_0 = 1$. (i.e. $\underline{\underline{\Delta}} \times 10^3 = 0.22 \underline{e}_x \underline{e}_x + 0.22 \underline{e}_y \underline{e}_y - 0.44 \underline{e}_z \underline{e}_z + 0.10 \underline{e}_y \underline{e}_z + 1.01 \underline{e}_z \underline{e}_y$ and the eigenvalues are $\lambda_i^\Delta \times 10^3 = (-1.17, 0.22, 0.95)$). As the time proceeds, with the variation in $\tilde{\Gamma}$ towards the asymptotic state, the eigenvalues of $\underline{\underline{\Delta}}$ also approach the asymptotic state as $\underline{\underline{\Delta}}$ approaches a constant ($\underline{\underline{\Delta}} \times 10^3 = 1.23 \underline{e}_x \underline{e}_x + 0.97 \underline{e}_y \underline{e}_y - 2.2 \underline{e}_z \underline{e}_z + 0.10 \underline{e}_y \underline{e}_z + 1.67 \underline{e}_z \underline{e}_y$ and eigenvalues are $\lambda_i^\Delta \times 10^3 = (-2.91, 1.23, 1.69)$). The eigenvalues contribution from the second order terms $\underline{\underline{\Delta}}_2$ are almost zero initially as can be seen from the ratio of the second invariants in Figure 6.14b. However, both become significant during the final development time ($\tilde{t} > 15$) as $\underline{\underline{\Delta}}$ approaches its asymptotic values (see in Figure 6.14), with the ratio $\Pi_{\Delta_1} / (\Pi_{\Delta_1} + \Pi_{\Delta_2})$ being about 0.539. This value is also evident in earlier Figure 6.6 which showed the asymptotic states for several rotation rates. The values for $\tilde{\Omega} / \tilde{\Gamma} = 0$ on Figure 6.6 are the final states of $\tilde{t} > 15$ in Figure 6.14.

6.8 Conclusions

The transport equations for k and ε , along with the URAPS-closure predict self-similar states for homogeneous shear. The range of rotations for which asymptotic states exist is qualitatively consistent with known literature results (Brethouwer 2005).

1. The observed anisotropy for the URAPS-closure shows a transfer of energy from

R_{yy} to R_{zz} for increasing $\frac{\tilde{\Omega}}{\tilde{\Gamma}}$ in the range

$-0.65 = (\frac{\tilde{\Omega}}{\tilde{\Gamma}})_{\min} < \frac{\tilde{\Omega}}{\tilde{\Gamma}} < (\frac{\tilde{\Omega}}{\tilde{\Gamma}})_{\max} = +0.15$, which is similar to the trend in DNS and

LES simulations (see Figure 6.3b). For this range of rotation rates the dynamic states approach asymptotic values, $\tilde{\Gamma} \rightarrow \tilde{\Gamma}^a$, $\mathcal{P}/\varepsilon \rightarrow (\mathcal{P}/\varepsilon)^a$ and $\underline{\underline{\mathbf{R}}} \rightarrow \underline{\underline{\mathbf{R}}}^a$. This occurs because Eq. (6.2) balances in the asymptotic range (AS-). However, outside the AS- range, the turbulent kinetic energy, the turbulent dissipation, and the shear component of the NR-stress relax to zero. (i.e., $\mathbf{R}_{yz} \rightarrow 0$). The ratio $\mathcal{P}/\varepsilon$ also decays. So the dynamic equation for $\tilde{\Gamma}$ for $t \rightarrow \infty$ becomes Eq. (6.5) resulting in monotonically increasing values of $\tilde{\Gamma}$.

- 2 The effect of rotation predicted by the URAPS-closure agrees with the DNS and LES results (see Figure 6.3b), with the energy shift from the $\mathbf{R}_{zz}$ to $\mathbf{R}_{yy}$ component as $\tilde{\Omega}/\tilde{\Gamma}$ changes from + 0.15 to -0.65. The distribution of energy in the case of $\tilde{\Omega}/\tilde{\Gamma} = -0.5$, which corresponds to the zero absolute mean vorticity, with $\mathbf{R}_{yy} > \mathbf{R}_{zz}$, is consistent with the knowledge of NR-stress distribution in spanwise rotating channel in similar region (see Table I.4). Distinctly larger with maximum values of $1/\tilde{\Gamma}^a \cong 0.33$, $(\mathcal{P}/\varepsilon)^a \cong 1.64$, and $-\mathbf{R}_{yz}^a \cong 0.27$ for $\tilde{\Omega}/\tilde{\Gamma} = -0.25$ occur in the AS-region, which is also consistent with literature (Brethouwer 2005). The URAPS- closure accounts for the effect of rotation on the $\mathbf{R}_{yz}$ component which is important for the finite range of asymptotic states observed.

3. The development towards asymptotic states occurs within $\tilde{t} \sim 11$ for various initial $\tilde{\Gamma}_0$ ranging from 0.5 to 15 for the dimensionless shear rate, NR-stress and the “production” to dissipation ratio. For the case of $\tilde{\Gamma}_0 = 1$ and $\tilde{\Omega}/\tilde{\Gamma} = 0$, during the initial time of development towards AS-states, $\underline{\underline{\Delta}}_1$ contributions to the NR-stress are large compared to $\underline{\underline{\Delta}}_2$. However, both the contributions are almost equal towards the asymptotic solution. In the asymptotic range, for $-0.45 < \tilde{\Omega}/\tilde{\Gamma} < 0.05$, $\underline{\underline{\Delta}}_2$ is important compared to $\underline{\underline{\Delta}}_1$.

CHAPTER 7

FULLY-DEVELOPED CHANNEL FLOWS

7.1 Introduction

The RANS-equation and important flow characteristics for non-rotating and spanwise rotating channel flow for a constant streamwise pressure gradient have been discussed in Chapter 2. Low-order statistics ($\langle \underline{u} \rangle^+$, k^+ , ε^+ , Γ_{yz}^+) from DNS results available in literature have also been discussed in Chapter 2 for $\delta^+ = 300$ and 2000 (see Figures 2.5, 2.6 and 2.7) for non-rotating channel and also for $\delta^+ = 300$ with rotation number $\Omega^+ = -0.0042$ (see Figures 2.9 to 2.14). The mean statistics are not symmetric about the center of the channel in the case of rotating channel. These results in the core region of the channel have been tabulated in Appendix I (Tables I.1 to I.3). Apart from these, the linear velocity region results for various rotations have also been presented in Appendix I (Table I.4) and discussed in Section 7.6. The DNS results for the NR-stress are compared with the NR-stress predicted by the URAPS-theory. In this chapter, the URAPS-closure predictions for NR-stress in a fully developed channel flow with and without rotation are presented. DNS results are used to test the closure performance. Influence of rotation on the NR-stress is also compared with the DNS results in the zero absolute mean vorticity region and in the core region of a spanwise rotating channel. The closure prestress and its components are studied to understand their contributions to the NR-stress predictions. Figure 7.1 presents a schematic of fully-developed channel flow with z as the streamwise direction and y as the crossflow direction ($\langle \underline{u}_z \rangle(y)$). The

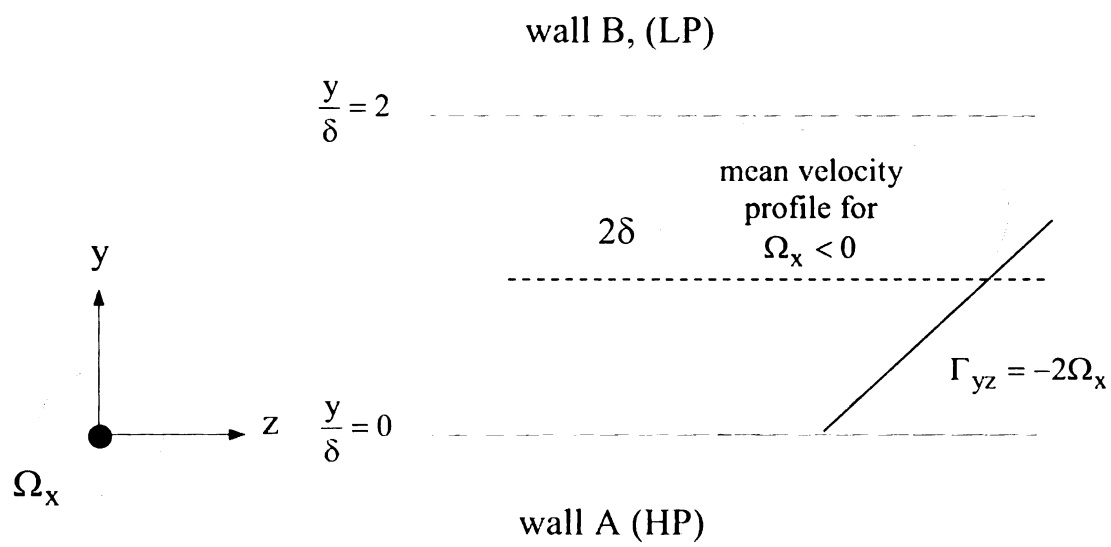


Figure 7.1 Schematic of Fully Developed Channel Flow with Spanwise Rotation.

rotation vector is aligned along the vorticity direction ($\underline{\Omega} = \Omega_x \underline{e}_x$). The cross length of the channel is from $y/\delta = 0$ to $y/\delta = 2$ where δ is the half-width of the channel.

7.2 Preclosure Operator and URAPS-Closure

The preclosure operator for URAPS for non-rotating simple shear has been presented in Appendix G, Eq. (G.7). Eqs. (G.12)-(G.20) are the URAPS-closure NR-stress component equations. For simple shear flow without rotation, the simplified URAPS equations have been presented in Case 2 (i.e. $N_\Gamma \neq 0$ & $N_\Omega = 0$). The equations show that there is a mechanism to transfer energy from one component to the other through the prestress components depending on the shear group N_Γ . Case 4 represents the case when $N_\Gamma = -N_\Omega$ (i.e. $\Gamma_{yz}^+ = -2\Omega_x^+$). It is interesting to note that the preclosure operator for Case 2 (i.e. non rotation simple shear) and Case 4 (i.e. linear velocity profile region of rotating channel) are the transpose of each other (compare Eqs. (G.22) and (G.44)). The resulting URAPS component equations for NR-stress are also the similar except for a change in the yy and zz component equations (see Eqs.(G.24)-(G.27) and Eqs.(G.46)-(G.49) in Appendix G).

7.3 Locus of States Predicted by URAPS-Closure for Simple Shear Flows

Solution to the URAPS-equation for the NR-stress can be obtained by successive substitution, as described in Chapter 4. Locus of states predicted by URAPS-closure equations for a range of $\tilde{\Gamma}$ and $\tilde{\Omega}$ are presented below. The study can be simplified by fixing the ratio $\tilde{\Omega}/\tilde{\Gamma}$ and varying $\tilde{\Gamma}$. Figure 7.2a shows the locus of URAPS-closure

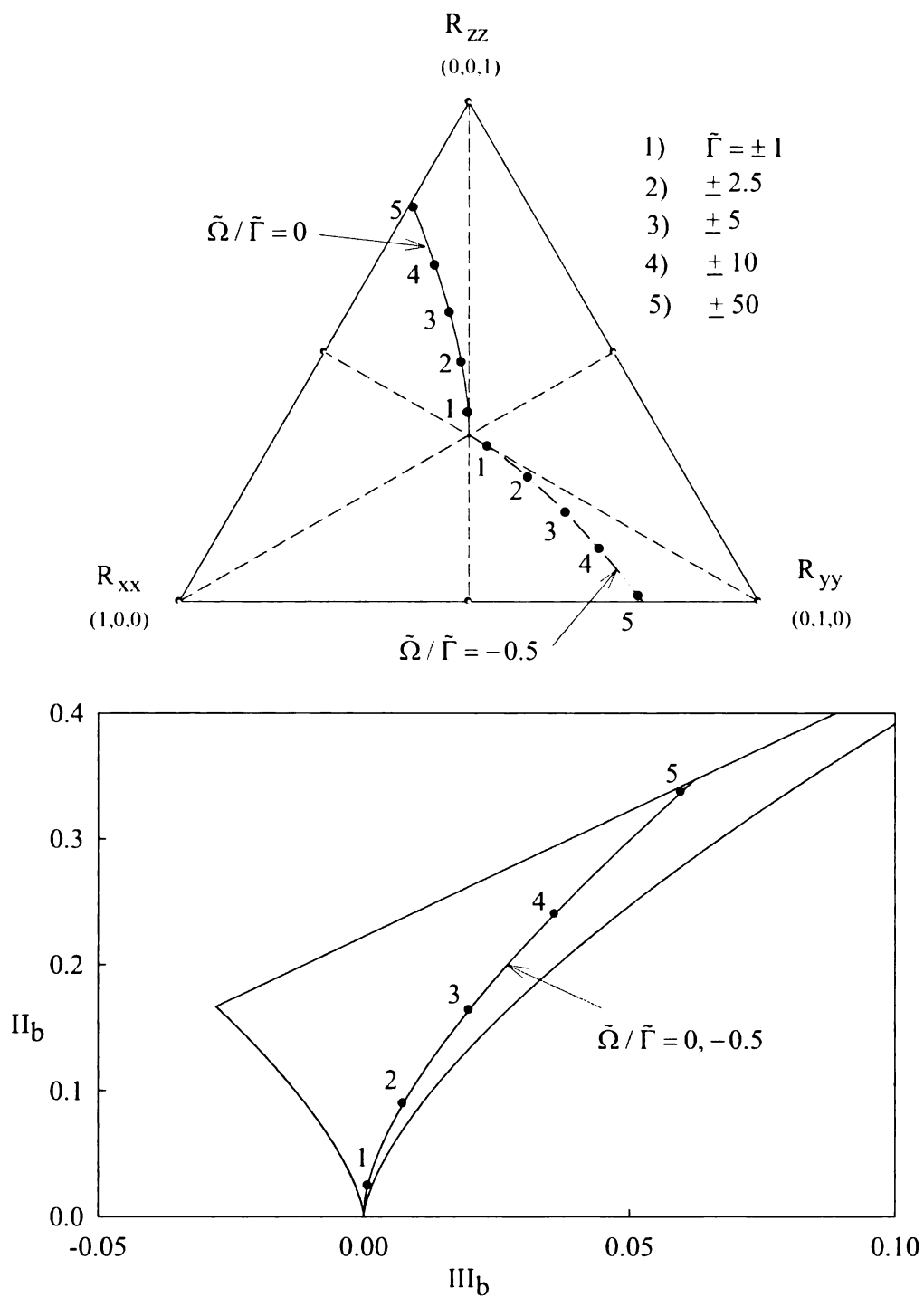


Figure 7.2a Locus of URAPS-Closure States Parameterized by $\tilde{\Gamma}$ for $\tilde{\Omega}/\tilde{\Gamma} = 0$ and $\tilde{\Omega}/\tilde{\Gamma} = -0.5$ (Zero Absolute Mean Vorticity, see Section 7.6).

theory for non-rotation case (i.e. $\tilde{\Omega}/\tilde{\Gamma} = 0$) and for the case $\tilde{\Omega}/\tilde{\Gamma} = -0.5$. As discussed earlier, since the preclosure operator $\underline{\underline{A}}$ for the above cases is the transpose of one other, URAPS-equations predict same anisotropic states, except for a flip in energy states R_{yy} and R_{zz} for the two cases. The energy simplex reflects the above feature with states in the 2nd sextet and 5th sextet respectively for the two cases. Figure 7.2b complements Figure 7.2a showing the component distribution with variation in $\tilde{\Gamma}$. Components of $\underline{\underline{R}}$ with a variation in the dimensionless shear $-\infty < \tilde{\Gamma} (= \Gamma_{yz}k / \varepsilon) < +\infty$, show that for $\tilde{\Gamma} = 0$, there is equi-distribution of energy with no shear component to the NR-stress. But as $|\tilde{\Gamma}| \rightarrow \infty$, the energy is transferred from the R_{xx} and R_{yy} components to the R_{zz} components. For negative values of $\tilde{\Gamma}$, there is a positive R_{yz} and for positive $\tilde{\Gamma}$ negative R_{yz} occurs with peak value of about $|R_{yz}| = 0.19$ occurring at $|\tilde{\Gamma}| \cong 5$. Typically the normal components are close to the two component energy state $R_{zz} = 0.8$ and $R_{yy} = 0.2$ for $|\tilde{\Gamma}| \cong 50$. However, the shear component relaxes to zero asymptotically for $|\tilde{\Gamma}| \gg 50$. In the case of $\tilde{\Omega}/\tilde{\Gamma} = -0.5$, the energy distribution approaches the two component state where $R_{zz} = 0.2$ and $R_{yy} = 0.8$, for $|\tilde{\Gamma}| \cong 50$. However, the anisotropic states for both the cases are the same because of the values being the same.

Locus of states for $\tilde{\Omega}/\tilde{\Gamma} > 0$ and $\tilde{\Omega}/\tilde{\Gamma} < -0.5$ are presented in Figures 7.3a and 7.3b. For $\tilde{\Omega}/\tilde{\Gamma} > 0$, the energy distribution shifts from the 2nd and 3rd sextet and approaches the single component state $R_{xx} = 1$ with increasing values of $|\tilde{\Gamma}|$. As noted

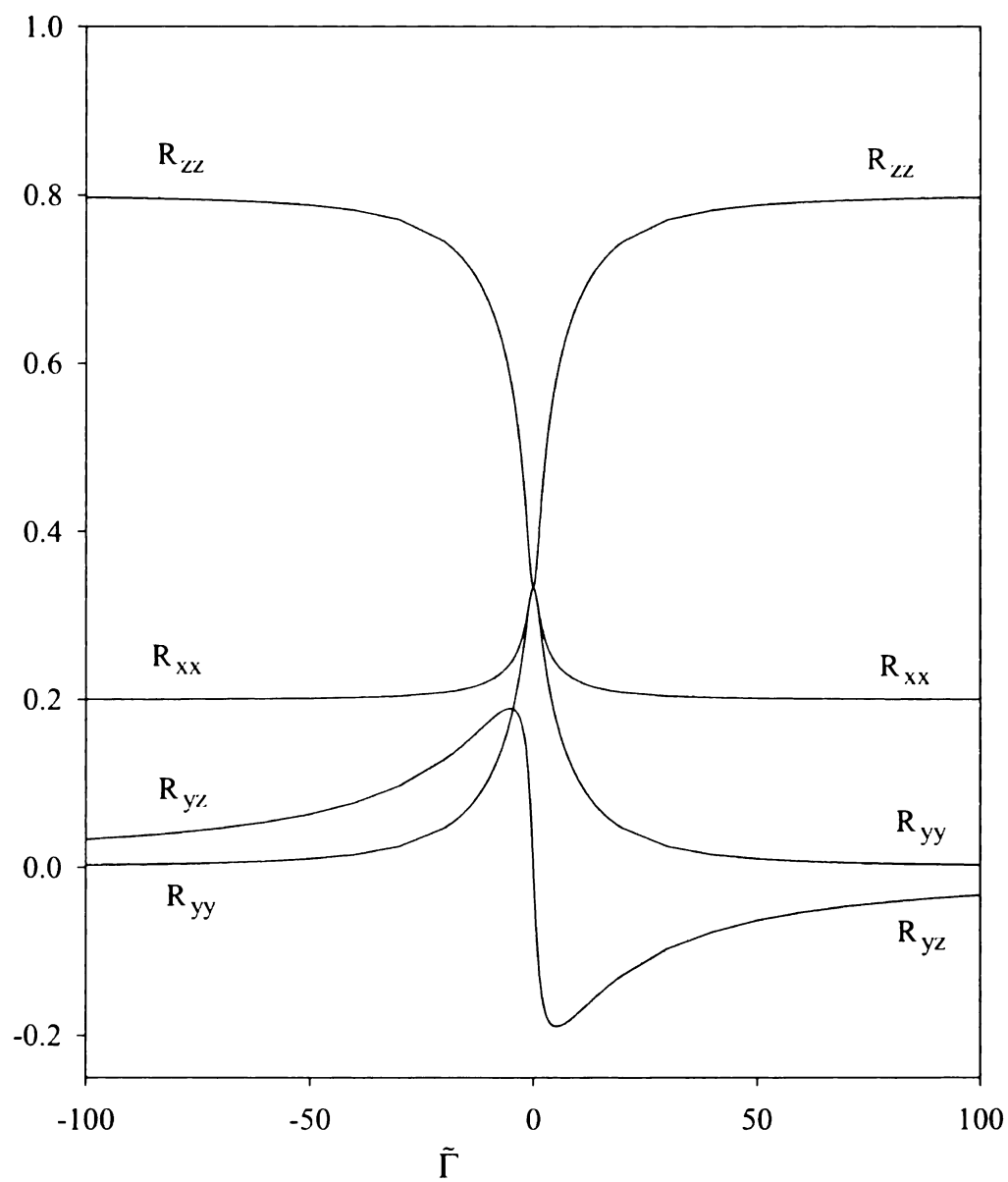


Figure 7.2b Influence of $\tilde{\Gamma}$ on the Components of NR-Stress Distribution for URAPS-Closure ($\tilde{\Omega}/\tilde{\Gamma} = 0$).

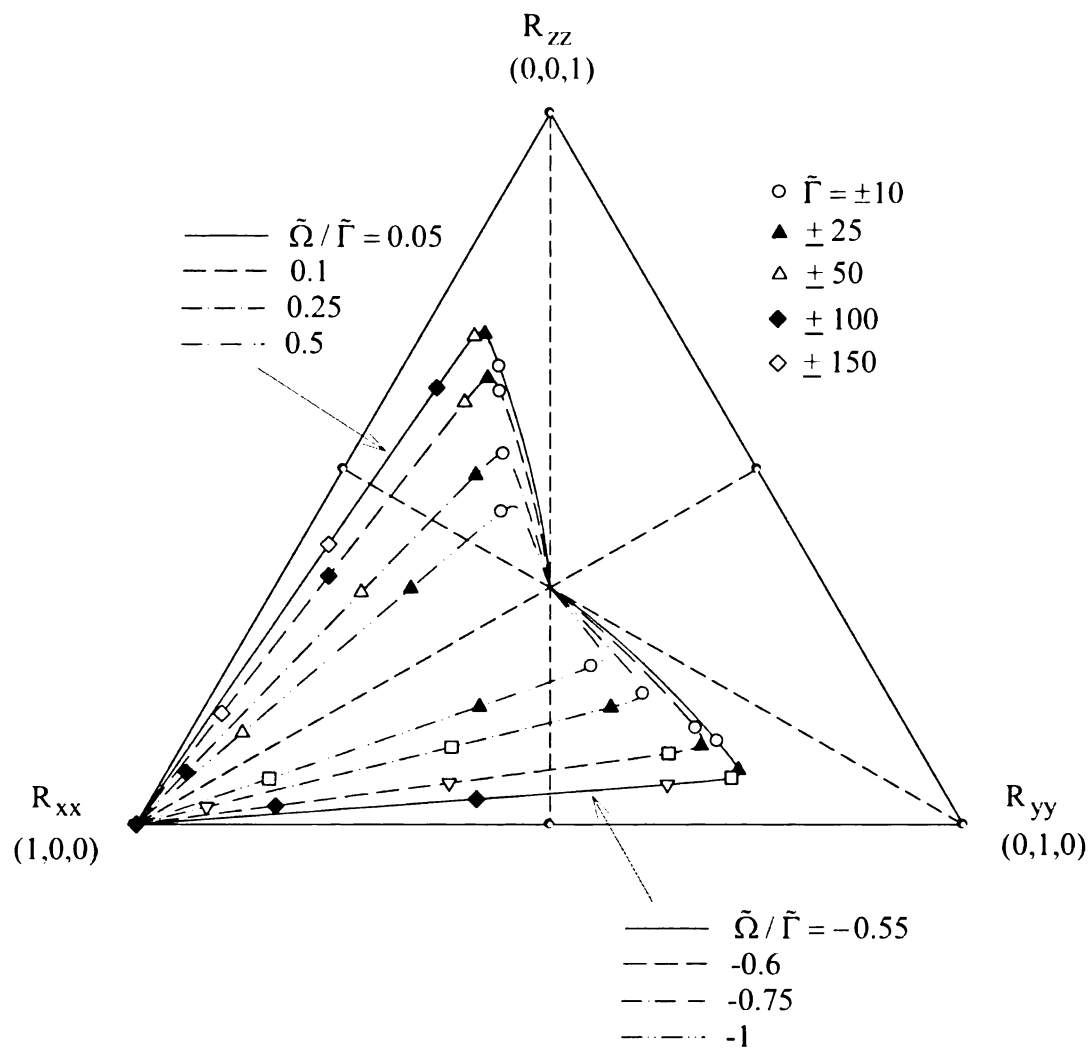


Figure 7.3a The Influence of $\tilde{\Omega}/\tilde{\Gamma}$ and $\tilde{\Gamma}$ on the Energy States for $\tilde{\Omega}/\tilde{\Gamma} < -0.5$ and $\tilde{\Omega}/\tilde{\Gamma} > 0$.

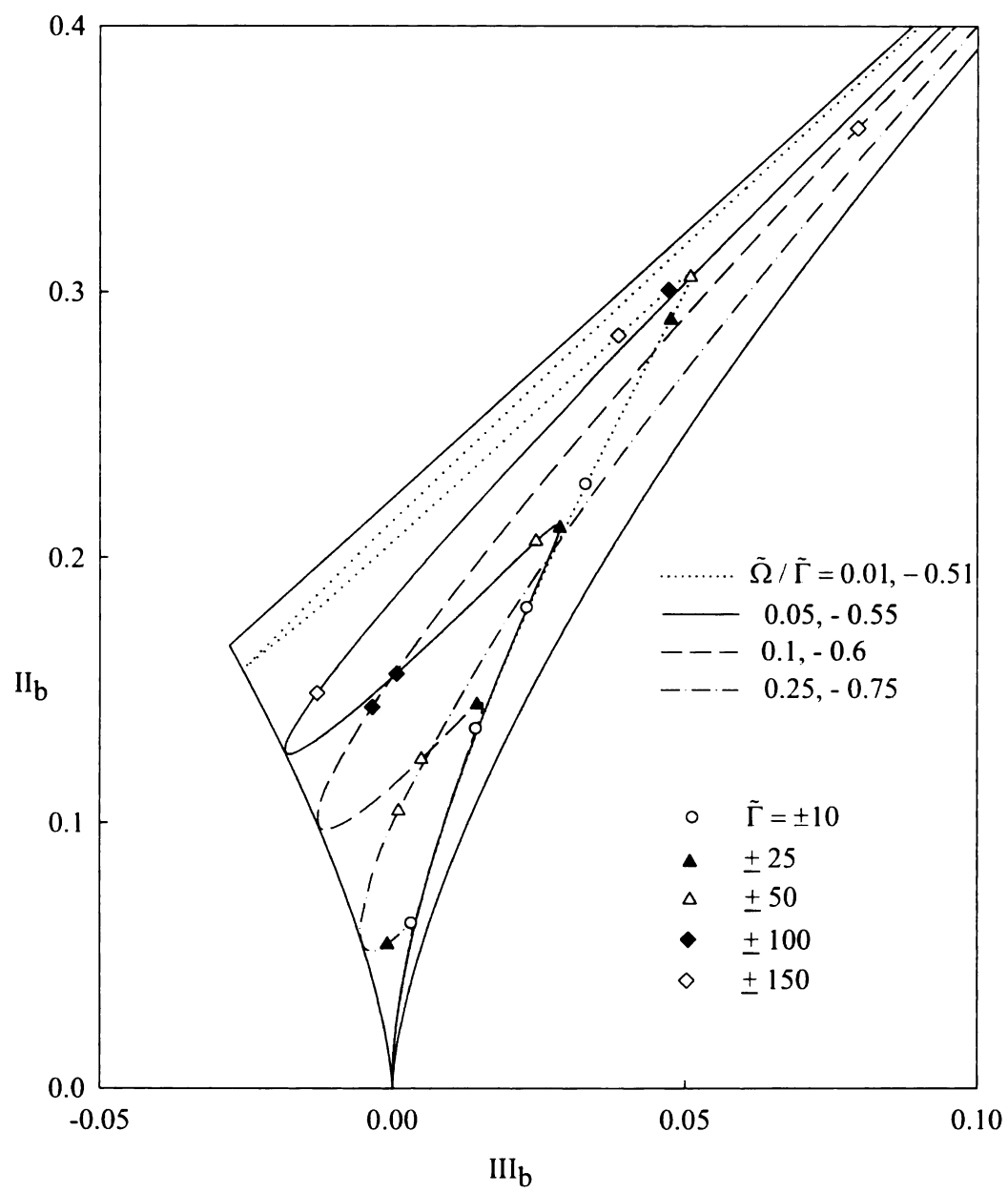


Figure 7.3b The Influence of $\tilde{\Omega}/\tilde{\Gamma}$ and $\tilde{\Gamma}$ on the Anisotropic States for $\tilde{\Omega}/\tilde{\Gamma} < -0.5$ and $\tilde{\Omega}/\tilde{\Gamma} > 0$.

earlier, if $\tilde{\Gamma} > 0$, then $R_{yz} < 0$ and vice versa. However, the magnitude of the shear component is the same. So as shown in the Figure 7.3a, for both positive and negative values of $\tilde{\Gamma}$, the locus of states remain the same in the energy simplex. For $\tilde{\Omega}/\tilde{\Gamma} < -0.5$, the energy shifts from the 5th sextet to the 4th sextet approaching $R_{xx} = 1$ for large values of $|\tilde{\Gamma}|$. The anisotropic states for each value of $\tilde{\Omega}/\tilde{\Gamma} > 0$, remains the same as for $-0.5 - \tilde{\Omega}/\tilde{\Gamma}$. For example, the anisotropic states are the same for $\tilde{\Omega}/\tilde{\Gamma} = 0.05$ and $\tilde{\Omega}/\tilde{\Gamma} = -0.55$ (see states Figure 7.3b). This is because although there is a shift in the energy distribution, the eigenvalues are the same. It can be seen that the states can be either prolate like distribution ($\text{III}_b > 0$) or towards oblate like states ($\text{III}_b < 0$) depending on the magnitude of $|\tilde{\Gamma}|$. Each of the constant $\tilde{\Omega}/\tilde{\Gamma}$ lines start at the isotropic state for $|\tilde{\Gamma}| = 0$ and pass through the oblate state, which typically is associated with $\tilde{\Gamma}$ corresponding to the $R_{xx} = R_{zz}$ and $R_{xx} = R_{yy}$ in energy plane for $\tilde{\Omega}/\tilde{\Gamma} > 0$ and $\tilde{\Omega}/\tilde{\Gamma} < -0.5$, respectively. This is because of the shear component being small compared to the normal components. Unlike other eddy viscosity based linear and bilinear closure models (see Chapter 2), the URAPS-closure states occupy both $\text{III}_b > 0$ and $\text{III}_b < 0$ states depending on the rotation rate and magnitude of the shear rate, which is expected from the previous knowledge of DNS results for rotating simple shear flows.

For $-0.5 < \tilde{\Omega}/\tilde{\Gamma} < 0$, the locus of states are restricted to $R_{xx} < 1/3$ in the energy simplex (see Figure 7.4). The locus of states starting at the equi-distribution of energy approach $R_{xx} = 0$ for increasing values of $|\tilde{\Gamma}|$ from 0 to 20 as shown in the Figure 7.4. For $\tilde{\Omega}/\tilde{\Gamma} = -0.25$, $R_{yy} = R_{zz}$. All the states for $\tilde{\Omega}/\tilde{\Gamma} > -0.25$ are in the region

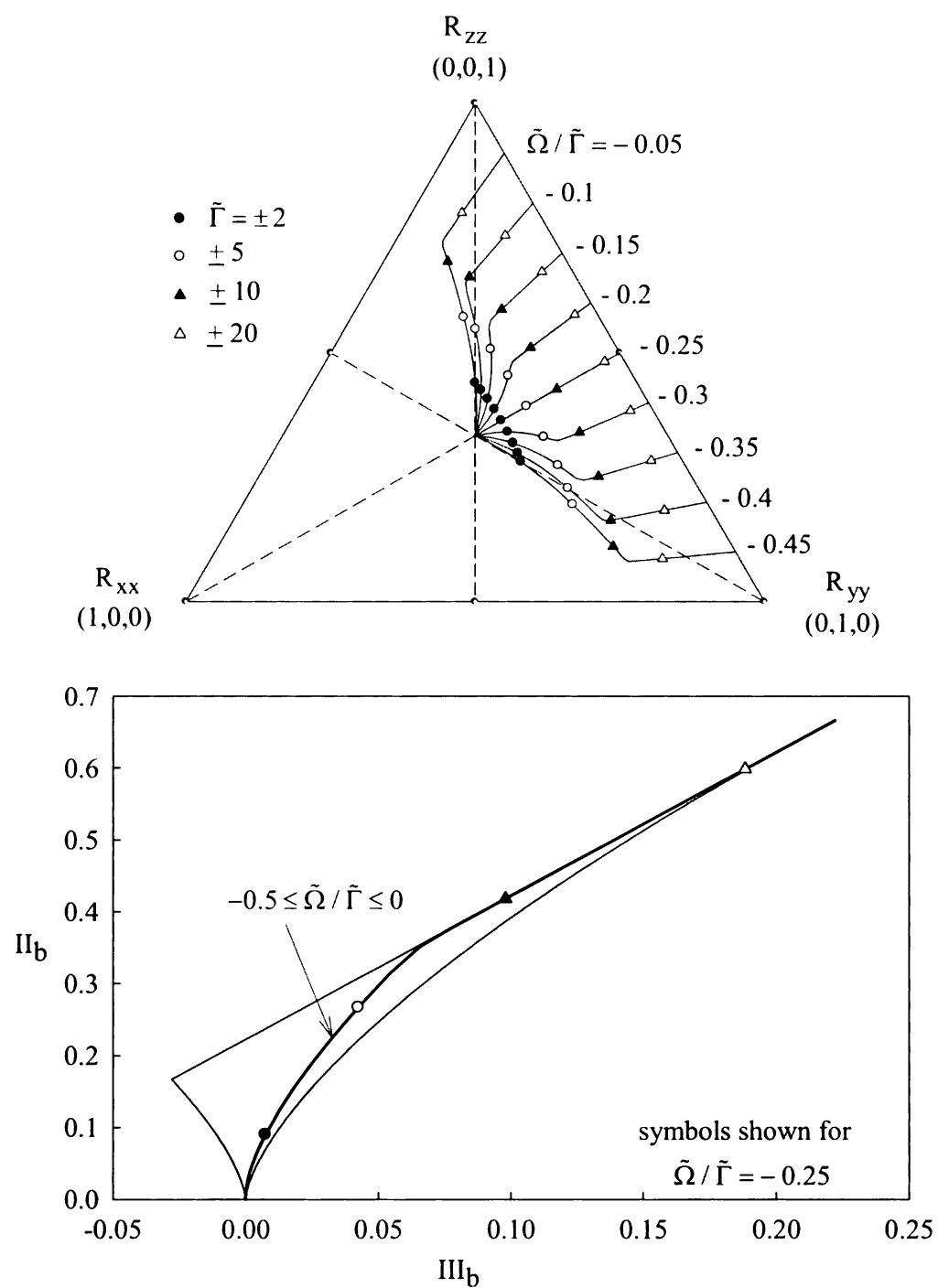


Figure 7.4 The Influence of $\tilde{\Omega}/\tilde{\Gamma}$ and $\tilde{\Gamma}$ on the Energy and Anisotropic States for $-0.5 < \tilde{\Omega}/\tilde{\Gamma} < 0$.

$R_{yy} < R_{zz}$. For $\tilde{\Omega}/\tilde{\Gamma} < -0.25$, the states are in $R_{zz} < R_{yy}$ of the energy plane. However, interestingly, for all $\tilde{\Omega}/\tilde{\Gamma}$ in this range, the anisotropic states follow a single curve in the Lumley's diagram. Each $\tilde{\Omega}/\tilde{\Gamma}$ is characterized by different $|\tilde{\Gamma}|$ for any given state along this curve.

7.4 NR-Stress in the Outer Region with $\tilde{\Omega} = 0$: Results and Discussion

The URAPS-equation needs the group $N_{\Gamma} (\equiv \tau_R \Gamma_{yz} = C_{RI} \tilde{\tau}_R \tilde{\Gamma})$ to give the NR-stress distribution for non rotating simple shear. This can be calculated from the $\tilde{\Gamma}_{DNS} (= \Gamma_{yz} k / \varepsilon)_{DNS}$ group from the DNS simulations as a preliminary analysis without solving the boundary value problem. $\tilde{\Gamma}_{DNS}$ calculated from the DNS statistics has been presented in Chapter 2 (see Figure 2.7 and Table 2.1 discussed in Chapter 2). From this the $\tilde{\tau}_R$ and N_{Γ} for URAPS theory are calculated.

Figure 7.5 presents the URAPS closure predictions for components of R with a variation in the dimensionless shear $0 < \tilde{\Gamma} < +\infty$. DNS results corresponding to each $\tilde{\Gamma}$ are also shown for comparison. For $\tilde{\Gamma} = 0$, there is equi-distribution of energy with no shear component to the NR-stress, while DNS results show weak anisotropy (with $R_{zz} > R_{yy} = R_{xx}$). The qualitative shift of energy to R_{zz} component for $\tilde{\Gamma} > 0$, which is seen in the DNS results, is also observed in the URAPS-closure results. For $\delta^+ = 300$ and 2000, there is a sudden change in the energy distribution occurring at $\tilde{\Gamma} \cong 3$ and 4 respectively. In this region, DNS results show different NR-stress variations for the same values of $\tilde{\Gamma}$. Although URAPS- closure predicts unique values of NR-stress distribution

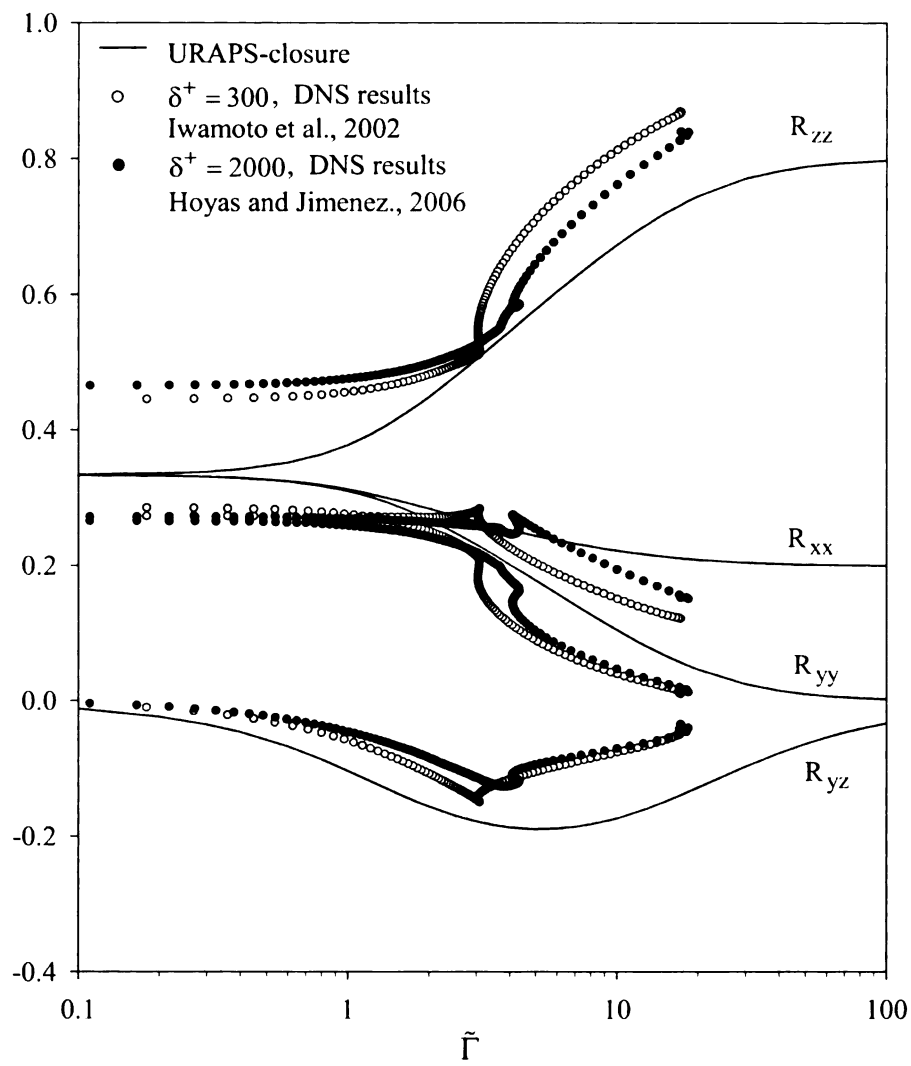


Figure 7.5 The Influence of $\tilde{\Gamma}$ on the Components of NR-Stress for Non-Rotating Fully-Developed Channel Flows.

for each $\tilde{\Gamma}$, the values in this region agree closely with DNS results. URAPS-closure for $\tilde{\Gamma} = 4$ predicts $R_{xx} = 0.25$, $R_{yy} = 0.20$, $R_{zz} = 0.55$ and $R_{yz} = -0.187$. DNS results show $R_{xx} = 0.25$, $R_{yy} = 0.18$, $R_{zz} = 0.57$ and $R_{yz} = -0.13$ for $\delta^+ = 2000$. The minimum value of the shear component R_{yz} in the DNS results is of the order of -0.15 for $\delta^+ = 300$ while the URAPS closure has minimum of about -0.19. There is a significant qualitative agreement with the DNS results for the NR-stresses. Typically the normal components are close to the two component energy state $R_{zz} = 0.8$ and $R_{yy} = 0.2$ for $\tilde{\Gamma} \cong 50$. $(R_{yz})_{\text{DNS}}$ relaxes to zero for $\tilde{\Gamma} < 50$, while for the URAPS-closure, the shear component relaxes to zero asymptotically for $\tilde{\Gamma} > 100$.

The shear group $N_{\Gamma}(=C_{RI}\tilde{\tau}_R\tilde{\Gamma})$, which appears in the URAPS-closure equation, determines the NR-stress variation. So the variation of the group along with the $\tilde{\tau}_R$ is presented in Figure 7.6 with respect to the wall coordinates. For y/δ , where $\tilde{\Gamma} = 0$, $\tilde{\tau}_R = 1$. Away from the center of the channel (i.e., $0 < y/\delta < 1$), the dimensionless time is influenced by the increasing shear rate. For $\tilde{\Gamma} > 0$ the mean field time scale affects the timescale associated with turbulent transport. These effects are maximum when $\tilde{\Gamma} \cong 18$ (see Figure 2.7, Chapter 2) which occurs at $y^+ \sim 10$ for $y/\delta < 0.05$ (Note that $\tilde{\tau}_R$ has not been calibrated to include the Re_t dependence and so the near wall effects are not discussed). N_{Γ} typically reflects the $\tilde{\Gamma}$ variation with increasing values from $y/\delta = 0$ to $y/\delta = 0.05$. The URAPS-closure predictions of the NR-stress, calculated from the group $\tilde{\Gamma}_{\text{DNS}}$ (for $\delta^+ \cong 300$), are presented in Figure 7.7. The redistribution of energy with

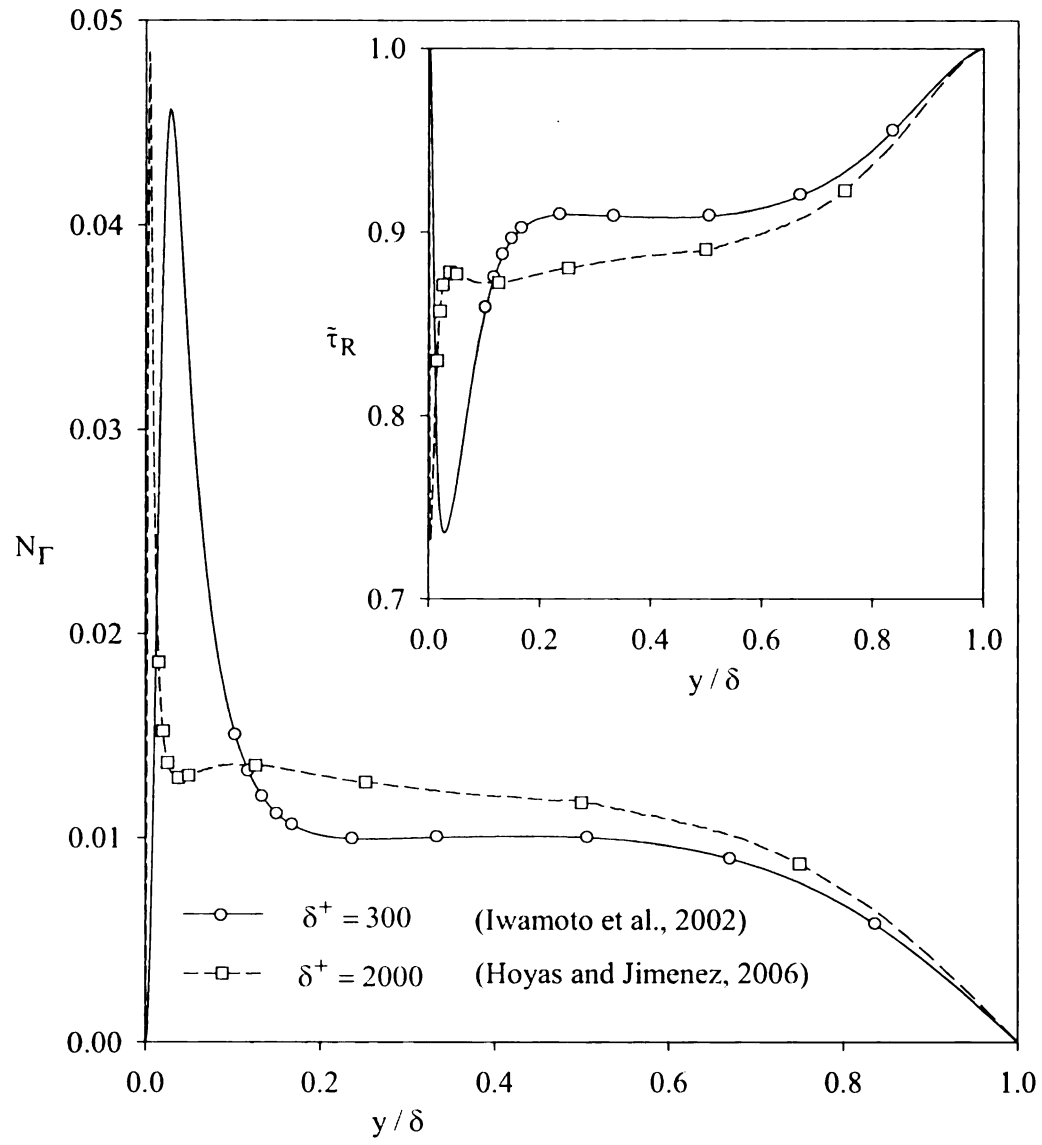


Figure 7.6 Spatial Variation of N_Γ and $\tilde{\tau}_R$ based on $\tilde{\Gamma}_{DNS}$ for Non-Rotating Fully-Developed Channel Flow

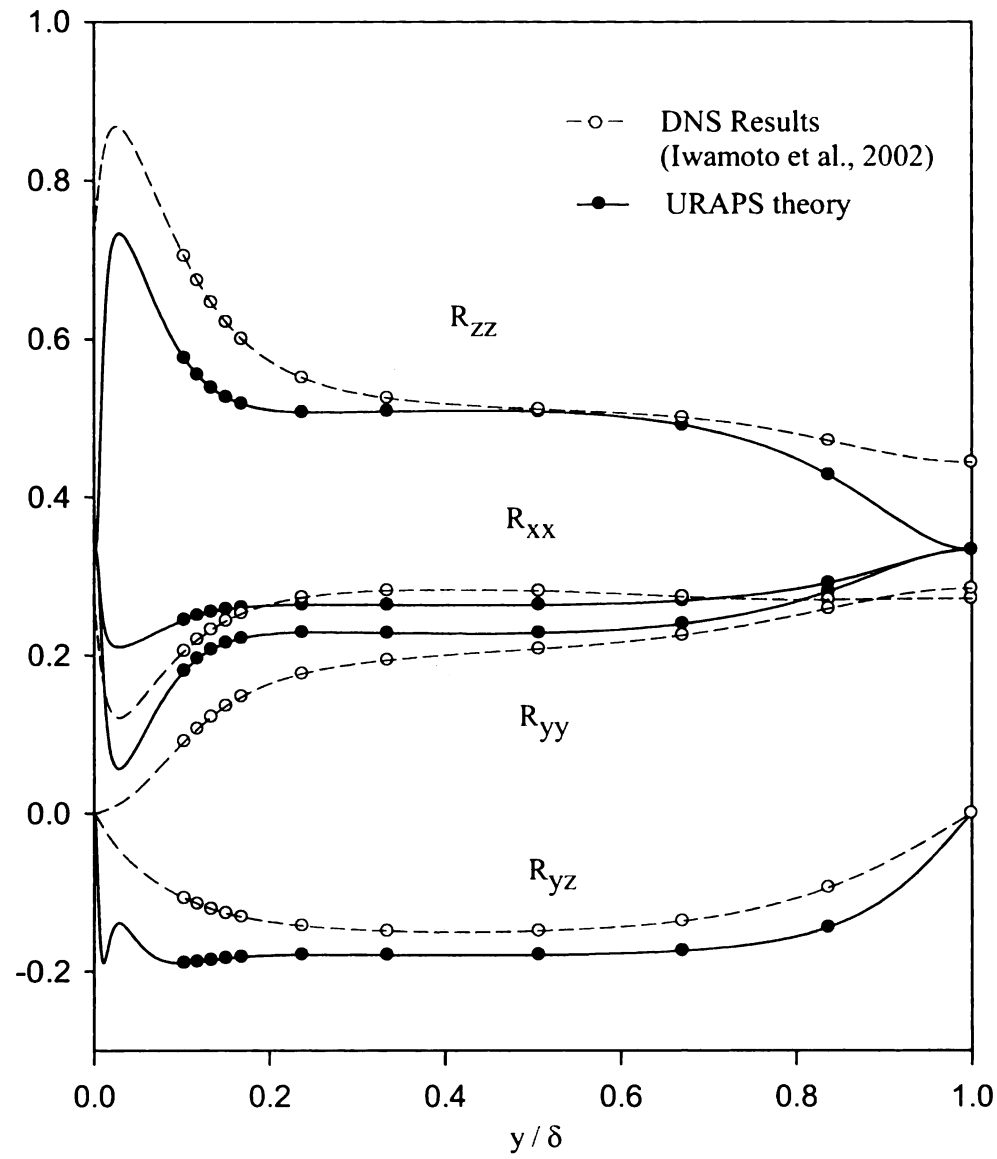


Figure 7.7 Spatial Variation of NR-Stress Components Predicted by URAPS-Closure based on $\tilde{\Gamma}_{DNS}$ for Non-Rotating Fully-Developed Channel Flows with $\delta^+ = 300$.

$R_{zz} > R_{xx} > R_{yy}$ is predicted by the URAPS-closure. At the center of the channel for $\tilde{\Gamma}_{\text{DNS}} = 0$, there is no anisotropy predicted unlike the DNS results, but the states in the core region of the channel remain in close agreement with the DNS results. The symbols correspond to the specific points presented in Table I.1c (Appendix I). R_{yz} component is slightly higher for the URAPS compared to the DNS results (See Table I.1c for the values). The statistics of the URAPS closure for Reynolds stress components for $\delta^+ \cong 2000$ are also presented in Figure 7.8. The overall predictions in the core region of the channel flow are in qualitatively agreement with the DNS results. The $|R_{yz}|$ predicted by URAPS closure has a maximum of 0.19 while the DNS data has 0.15 for this case. The near wall region needs further modeling.

7.5 NR-Stress in the Outer Region with $\tilde{\Omega} \neq 0$: Results and Discussion

DNS results by Wu and Kasagi, 2004 for spanwise rotating channel is presented in Table I.3. in Appendix I. for $Re^+ \cong 300$, $\Omega^+ \cong -0.0042$ and the dimensionless time scales associated with the shear and rotation numbers, $\tilde{\Gamma}$ and $\tilde{\Omega}$, are discussed in Chapter 2. The shear and rotation groups $N_{\Gamma} = C_{R1}\tilde{\tau}_R\tilde{\Gamma}$ and $N_{\Omega} = 2C_{R1}\tilde{\tau}_R\tilde{\Omega}$ give the relative degree of each of the effects (i.e shear and rotation) in the URAPS-equations. As shown in Figure 7.9, N_{Γ} values vary between 0.02 and -0.04 in the core region of the channel while N_{Ω} values reach to a maximum of about -0.012. So the effect of rotation may be small. $\tilde{\tau}_R$ as seen on the Figure 7.9 has a smaller value at $y/\delta = 1.5$ compared to $y/\delta = 0.5$. Since both $\tilde{\Gamma}$ and $\tilde{\Omega}$ are negative at $y/\delta = 1.5$, the value of N_F is larger and

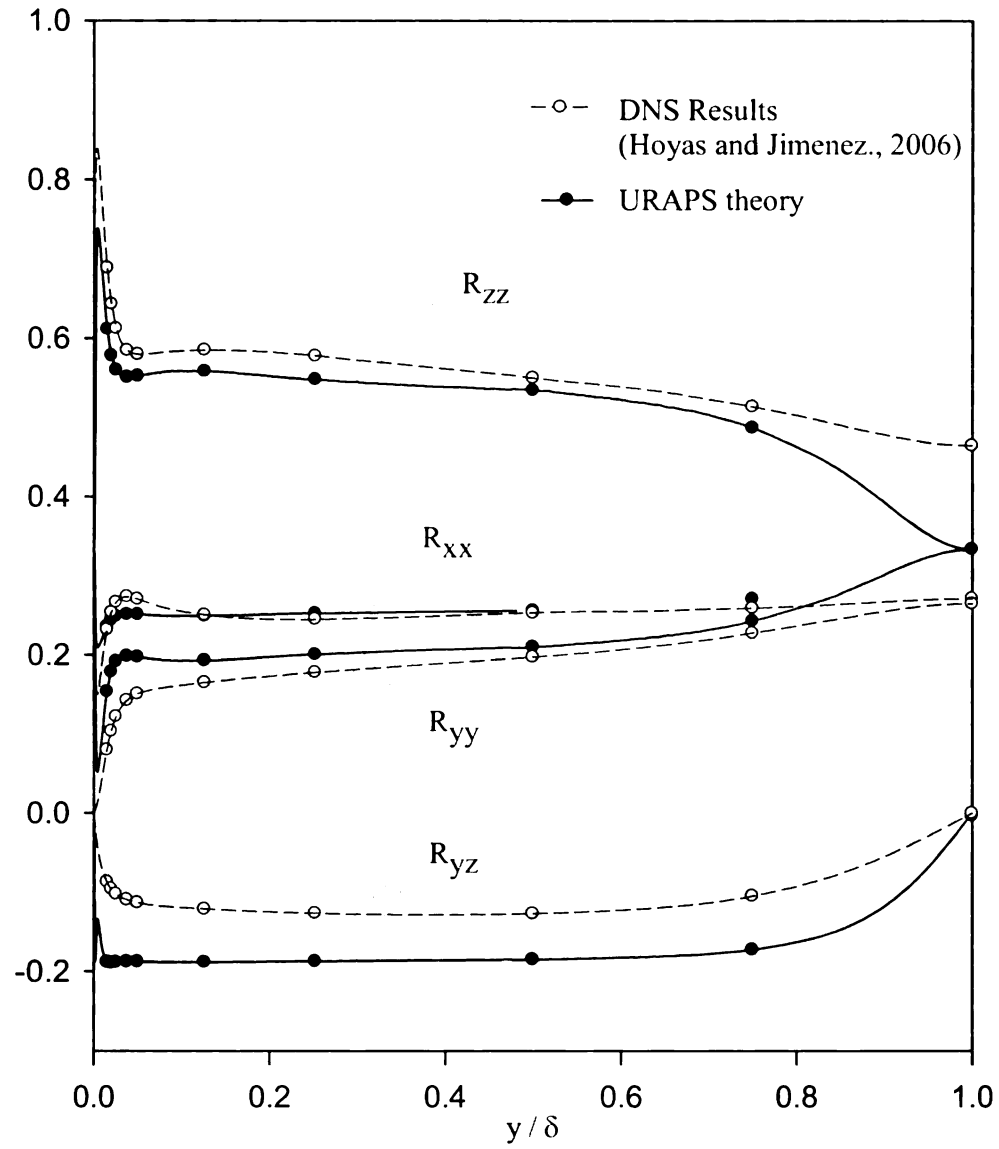


Figure 7.8 Spatial Variation of NR-Stress Components Predicted by URAPS-Closure based on $\tilde{\Gamma}_{\text{DNS}}$ for Non-Rotating Fully-Developed Channel Flows with $\delta^+ = 2000$.

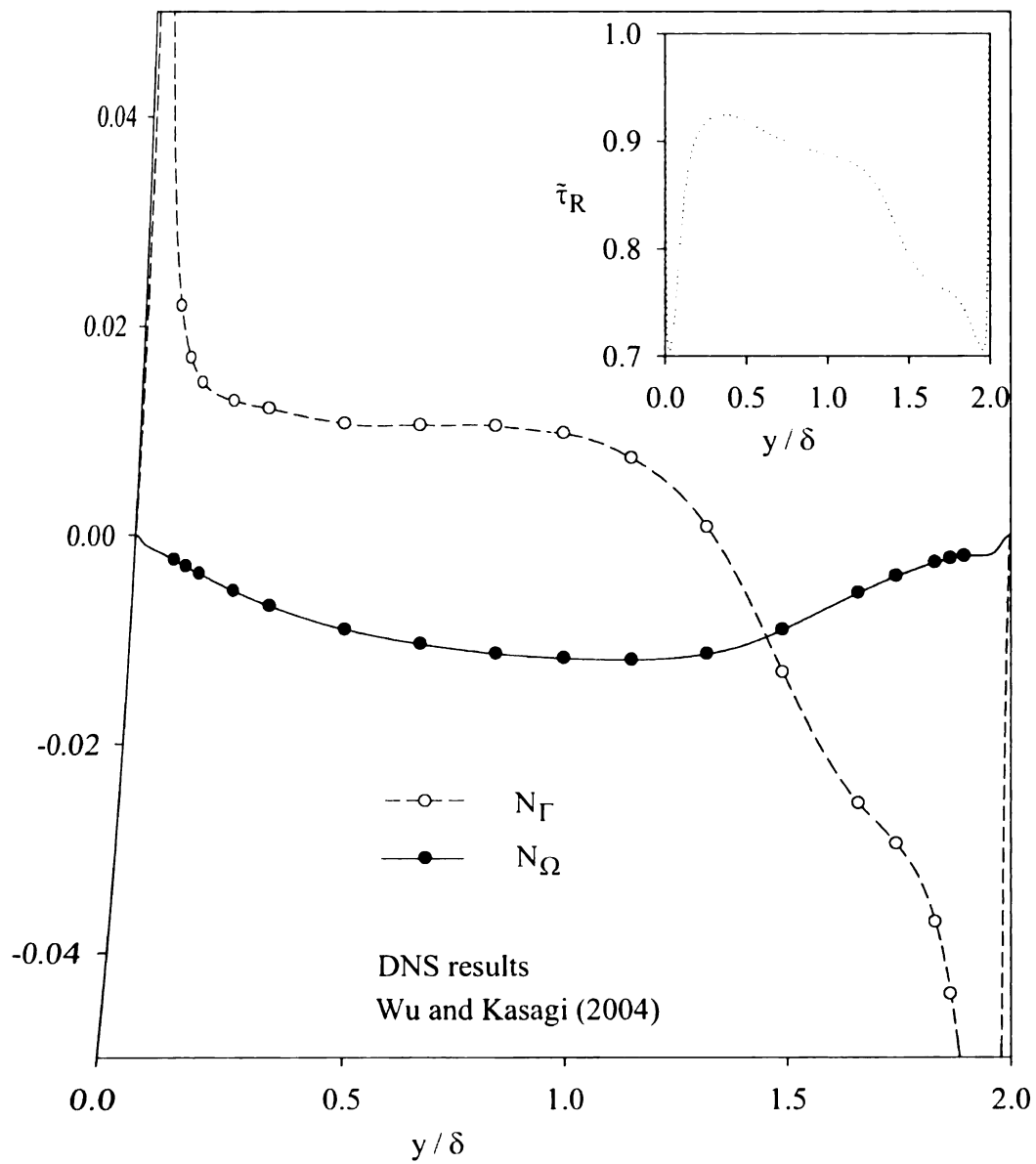


Figure 7.9 Spatial Variation of N_Γ , N_Ω and $\tilde{\tau}_R$ for Fully-Developed Spanwise Rotating Channel based on DNS Results for $\tilde{\Gamma}_{\text{DNS}}$ and $\tilde{\Omega}_{\text{DNS}}$ with $\delta^+ \cong 300$, $\Omega^+ \cong -0.0042$.

the value of $\tilde{\tau}_R$ is smaller compared to the corresponding values at $y/\delta = 0.5$. URAPS-closure can essentially be computed from the knowledge of the two groups $\tilde{\Gamma}_{\text{DNS}}$ and $\tilde{\Omega}_{\text{DNS}}$. The URAPS-closure states can be computed by successive substitution. The converged solution predicts a distribution of turbulent kinetic energy among the normal components. The predictions of R_{yy} , R_{zz} and R_{yz} are shown in Figures 7.10, 7.11 and 7.12 respectively. The results for each of the components in the LP-side of the channel (i.e. $1 < y/\delta < 2$) are in close agreement with the DNS results. This is the region where both $\tilde{\Gamma}$ and $\tilde{\Omega}$ are negative. Qualitative agreement is also observed for the normal components in HP side of the channel (i.e. $0.1 < y/\delta < 1$). At $y/\delta \cong 0.75$, it can be seen that $R_{yy} > R_{zz}$ unlike the case of channel without rotation. However deviation from the DNS predictions seem to occur at $y/\delta = 0.1$ ($y^+ \sim 30$), possibly due to influence of wall (viscous effects).

7.6 NR-Stress for Zero Absolute Mean Intrinsic Vorticity: Results and Discussion

Data presented by Grundestam et al. (2008) in the core region of the channel is compared with the URAPS-closure predictions. Results reported by Grundestam et al. (2008) in this region which falls mostly in the $0 < y/\delta < 1$ region (HP-side where kinetic energy peaks) are presented in Table I.4 of Appendix I. It is interesting to note that asymptotic homogeneous shear discussed earlier in Chapter 6 for $\tilde{\Omega}/\tilde{\Gamma} = -0.5$ also represents zero absolute mean vorticity region and represents similar features of NR-stress distribution which will be discussed below.

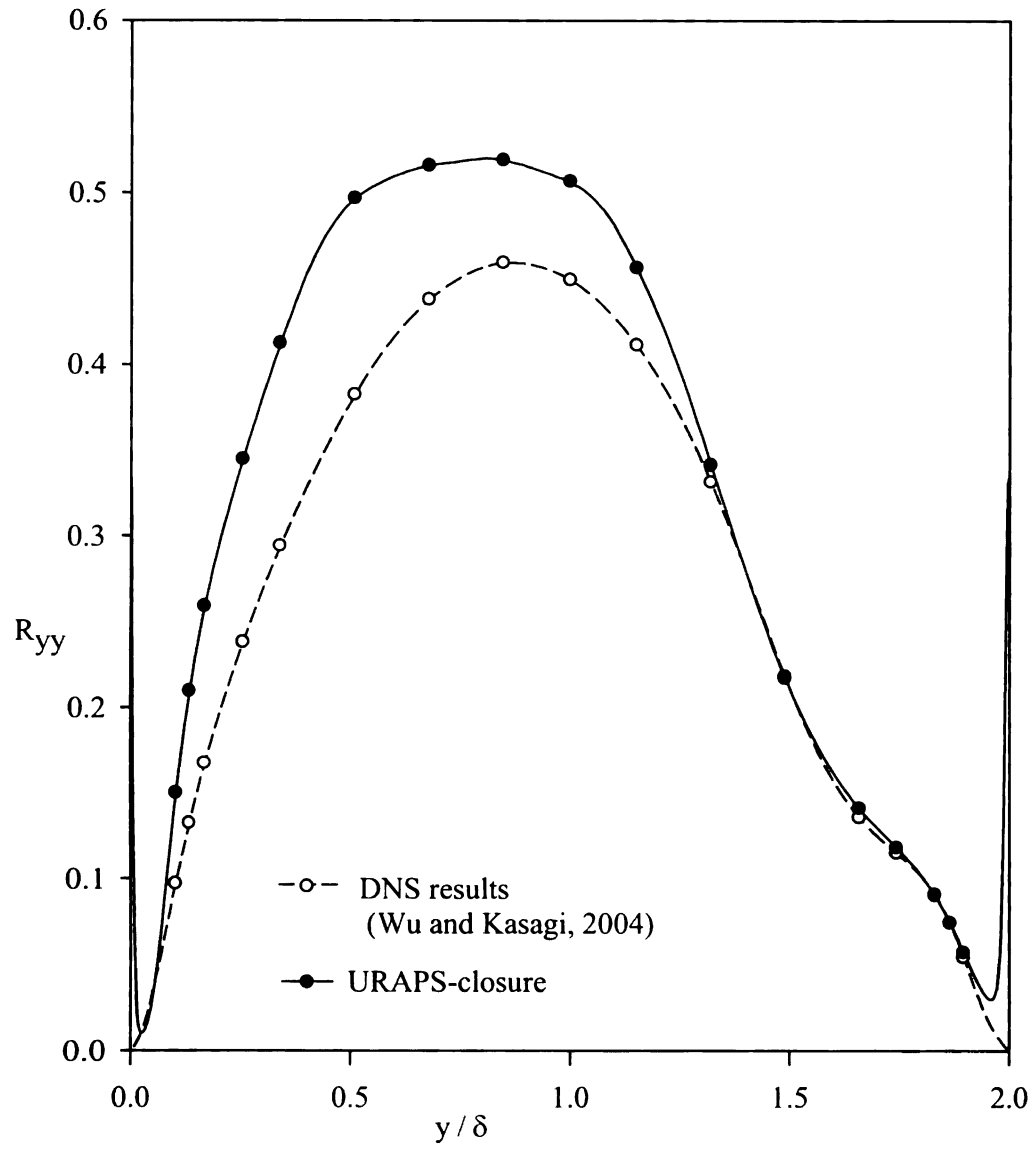


Figure 7.10 URAPS-Closure Predictions for the R_{yy} Component of the NR-Stress for Fully-Developed Spanwise Rotating Channel ($\delta^+ \cong 300$, $\Omega^+ \cong -0.0042$).

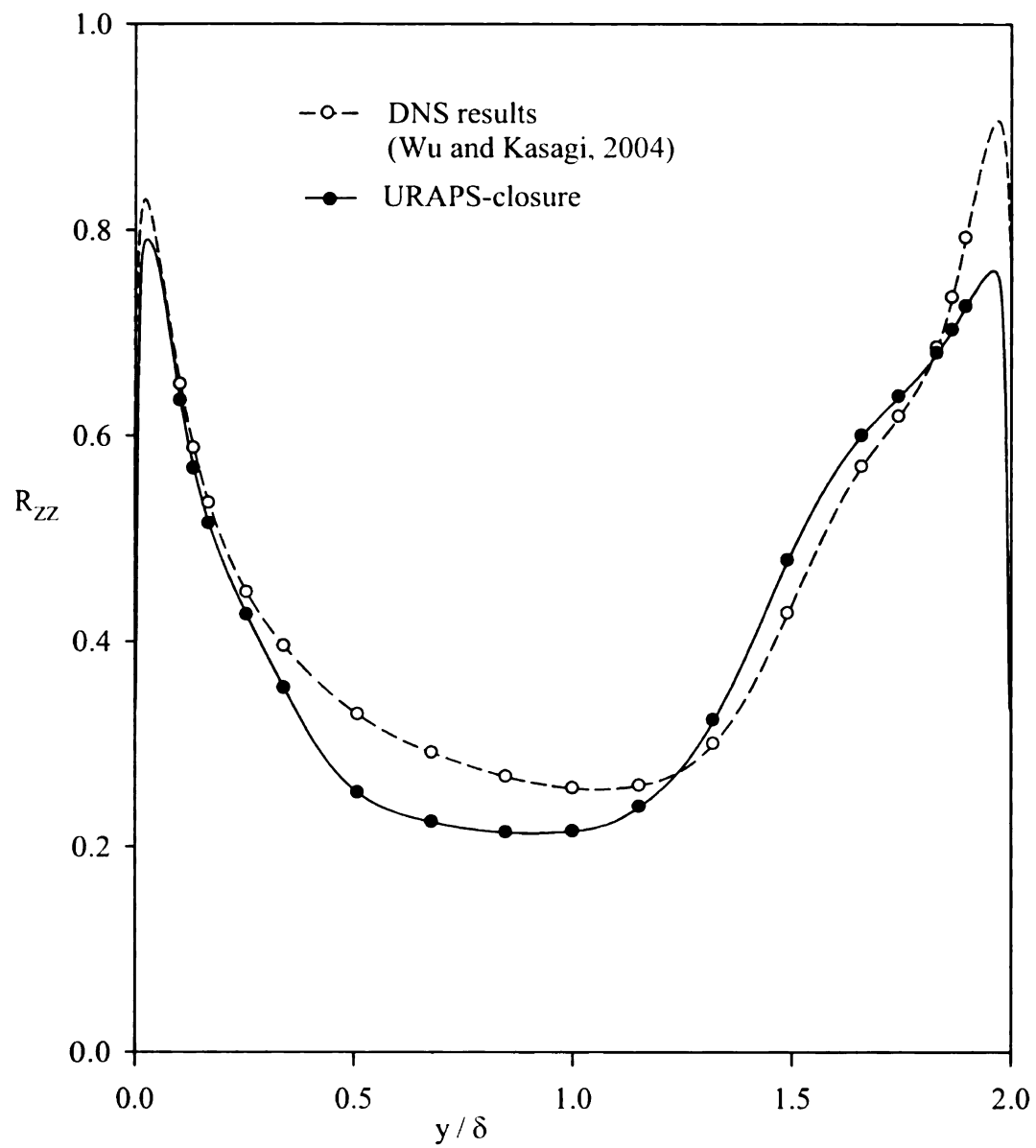


Figure 7.11 URAPS-Closure Predictions for the R_{zz} Component of the NR-Stress for Fully-Developed Spanwise Rotating Channel ($\delta^+ \cong 300$, $\Omega^+ \cong -0.0042$).

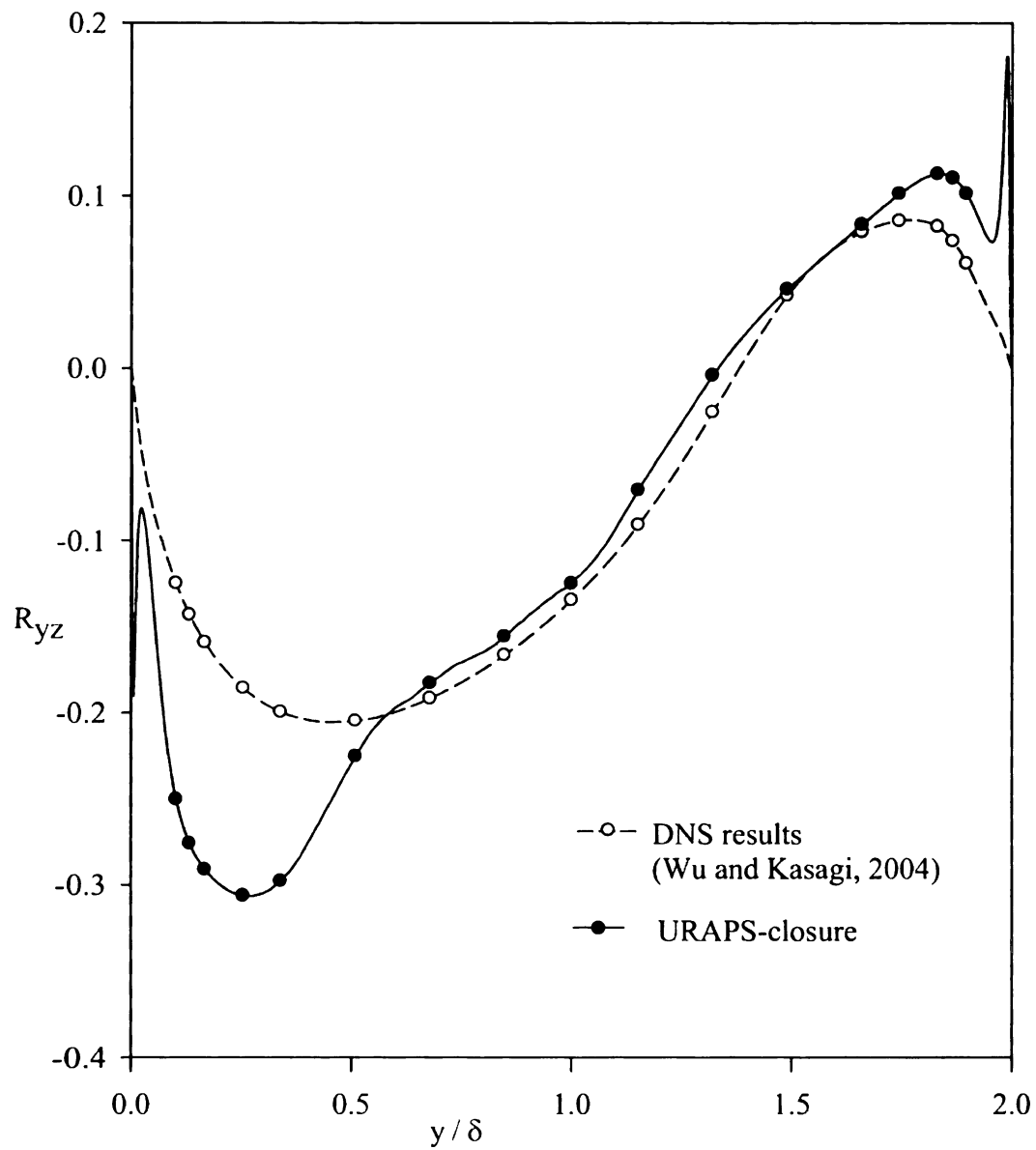


Figure 7.12 URAPS-Closure Predictions for the Shear Component of the NR-Stress for Fully-Developed Spanwise Rotating Channel ($\delta^+ \cong 300$, $\Omega^+ \cong -0.0042$).

The zero absolute mean vorticity has the feature discussed in Case 4 of Appendix G. The URAPS closure equations for this case also show a similarity with the non-rotating channel equations as discussed earlier in Figure 7.2. The predictions in this case shown in Figure 7.13 indicate that energy is concentrated in the R_{yy} component unlike the R_{zz} component for the non-rotating case. For $\tilde{\Omega} = 0$, there is equi-distribution of energy and this shifts to R_{yy} component when $|\tilde{\Omega}| > 0$. For about $|\tilde{\Omega}| \cong 50$, the energy is all in the R_{yy} and R_{xx} components. The shear component is negative for negative values of $\tilde{\Omega}$ and vice versa with a maximum value of 0.19 occurring at $|\tilde{\Omega}| \cong 2.5$. As summarized in Figure 7.14, for the case when $\tilde{\Gamma} = -2\tilde{\Omega}$, with increasing values of $\tilde{\Gamma}$ (or $-2\tilde{\Omega}$), the energy is transferred from R_{zz} to R_{yy} . DNS results from literature (open symbols) for rotating channel from different simulations with different rotation rates in this region show that $R_{yy} > R_{zz}$. Also note that the previous results for asymptotic homogenous shear (see Figure 6.3) discussed in Chapter 6, with $\tilde{\Omega}/\tilde{\Gamma} = -0.5$ also show that $R_{yy} > R_{zz}$. Corresponding predictions by URAPS closure (closed symbols in Figure 7.14) show that the redistribution of energy consistent with the trend observed in DNS results.

7.7 “Extra” Anisotropy with $\tilde{\Omega} = 0$: Results and Discussion

Figure 7.15 indicates that as the value of $\tilde{\Gamma}$ increases from 0 to 5, the magnitude of the eigenvalues of the extra anisotropy tensor also increase. From $\tilde{\Gamma} = 5$ to 50, one of the eigenvalues decreases to zero. This is the region where the NR-stress approaches the

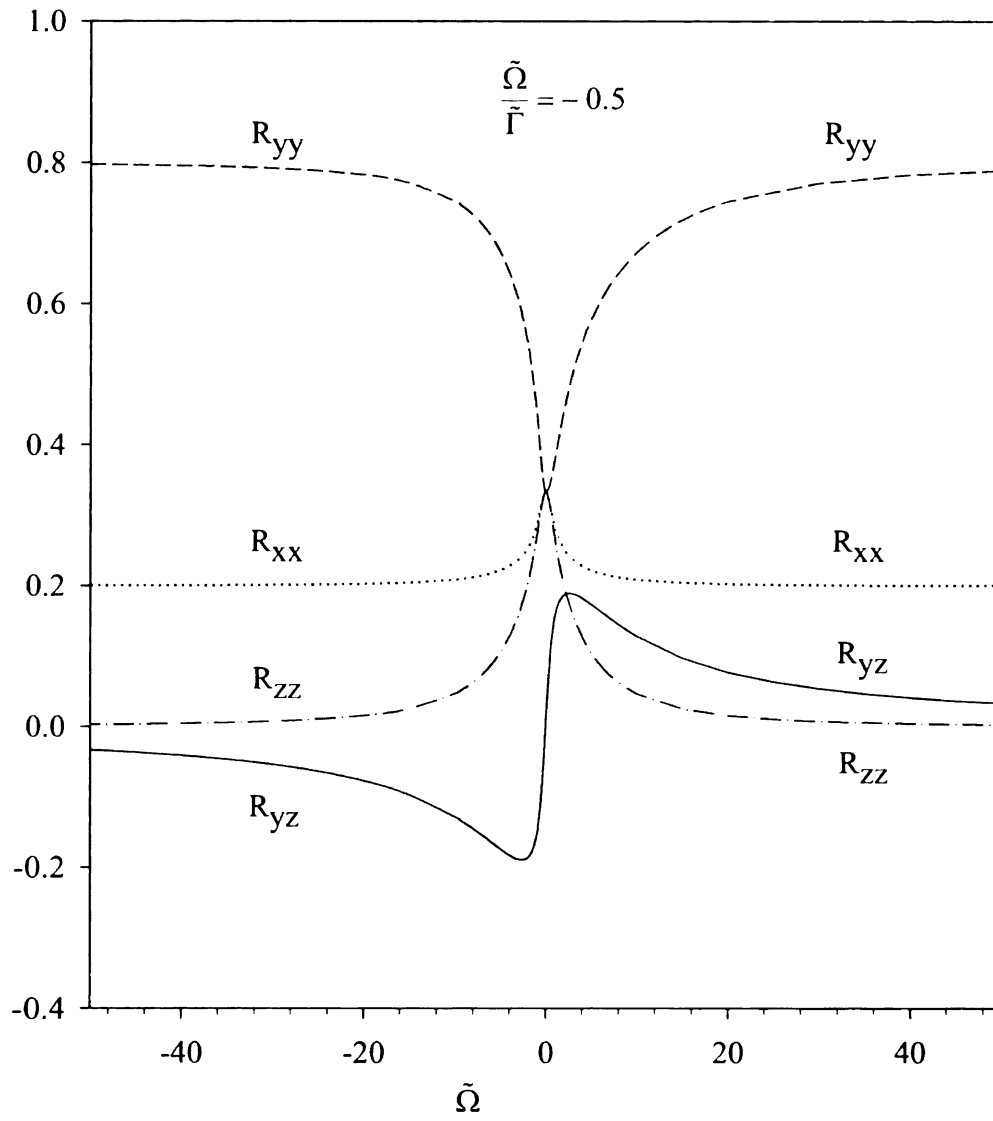


Figure 7.13 Influence of $\tilde{\Omega}$ on the NR-Stress Distribution for URAPS-Closure when $\tilde{\Gamma} = -2\tilde{\Omega}$

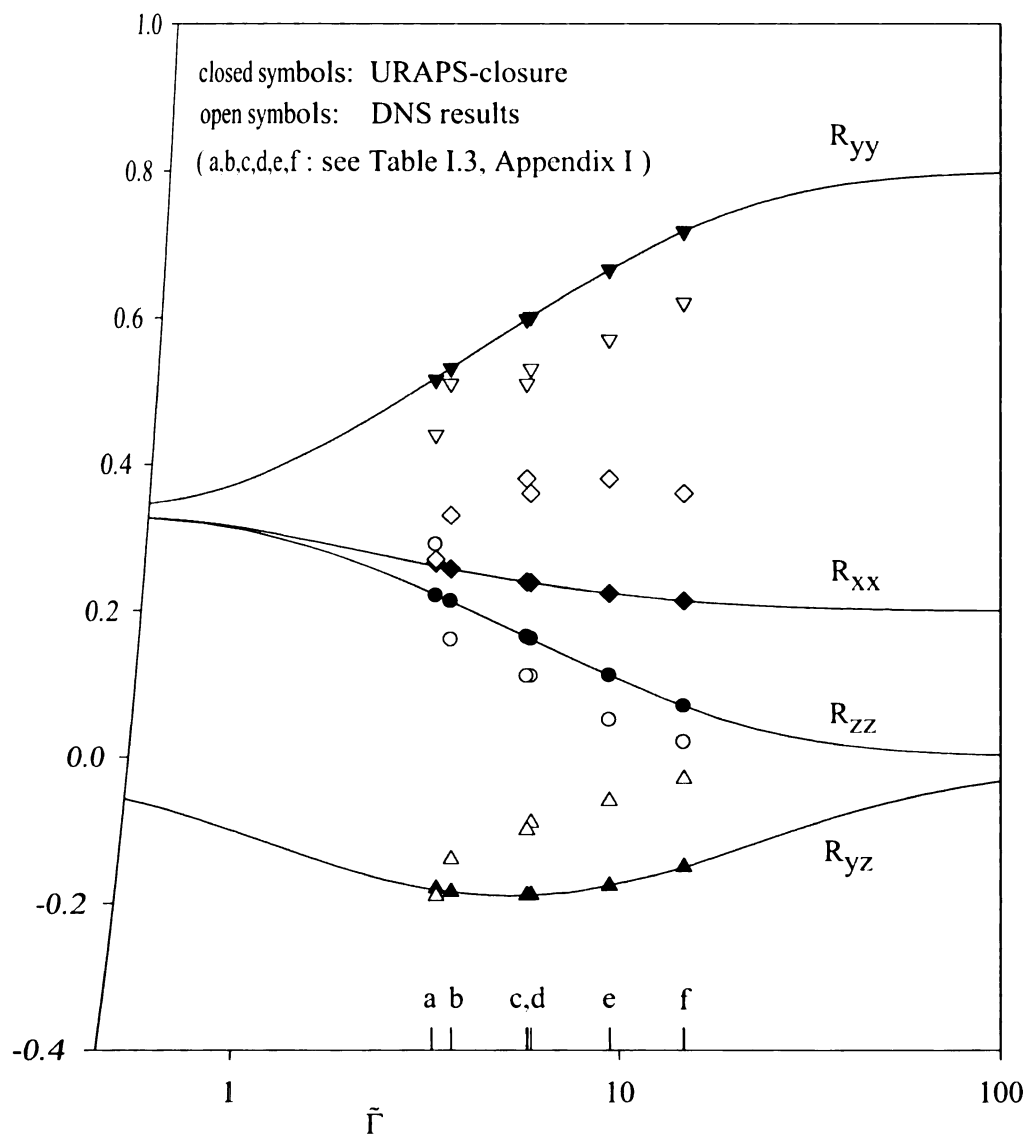


Figure 7.14 URAPS-Closure Predictions in Comparison with DNS Results for Spanwise Rotating Channel, in the Linear Velocity Region where $\tilde{\Omega} / \tilde{\Gamma} = -0.5$ (Zero Absolute Mean Vorticity).

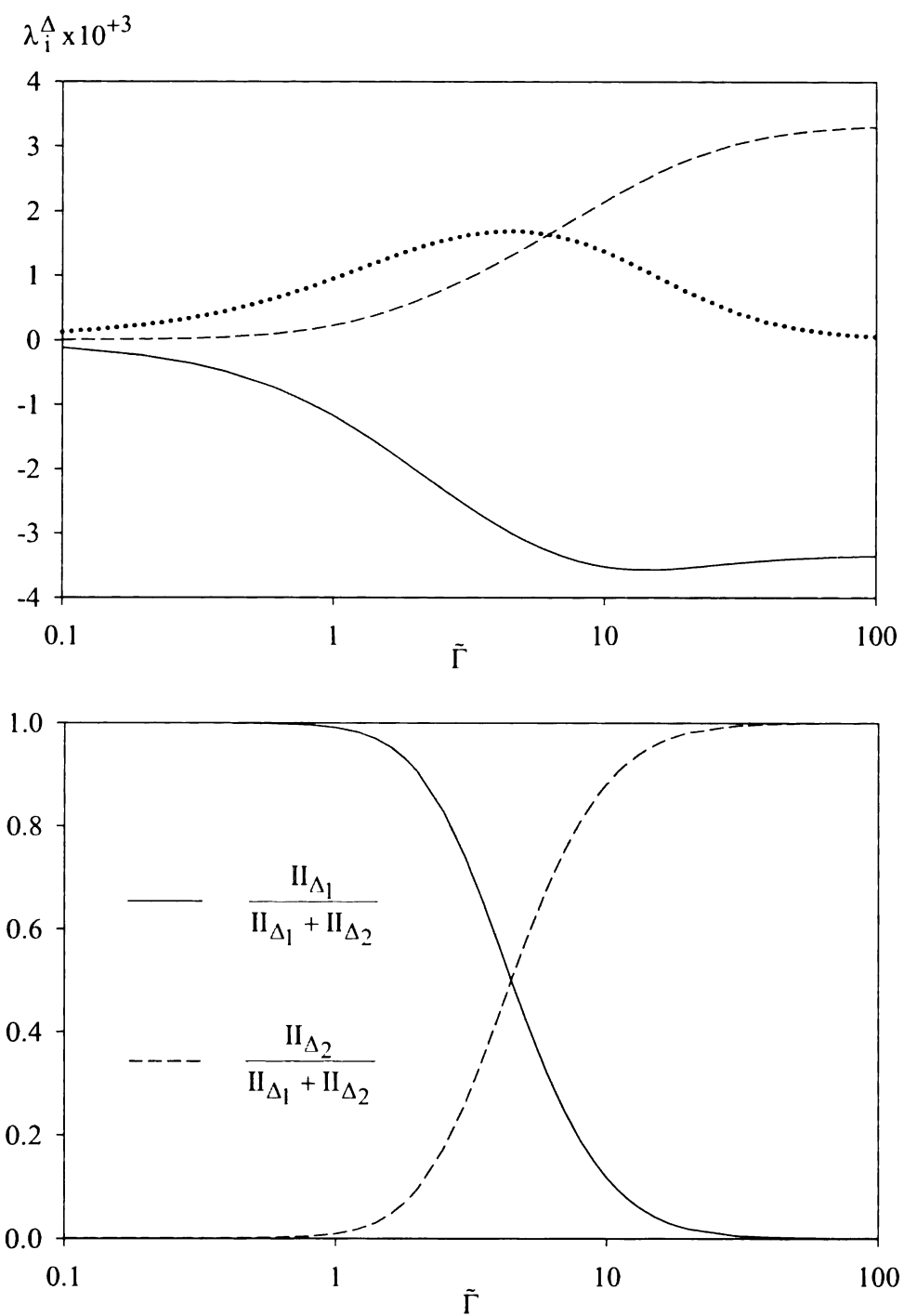


Figure 7.15 Extra Anisotropy for Simple Shear Flow for $\tilde{\Omega} = 0$.

two component anisotropic state where the shear component eventually decreases to zero. The first order linear term of the extra anisotropy is important for smaller values of $\tilde{\Gamma}$ up to about 5. For $\tilde{\Gamma} > 5$, the second order extra anisotropy has a dominating role in controlling the NR-stress anisotropy towards the two component anisotropic states.

7.8 Conclusions

The URAPS-closure with the parameters calibrated in Chapter 4 has been used to predict the NR-stress distribution for simple shear flows using DNS results for the dimensionless shear time and the dimensionless rotation time. Key features for this flow are as below.

1. For simple shear flows without rotation, the energy distribution predicted by the URAPS-closure is consistent with the DNS results in the core region of the channel. The anisotropic states $\text{III}_b > 0$ starting with the isotropic state for $\tilde{\Gamma} = 0$ and reaching a two component state for $\tilde{\Gamma} > 10$ are also consistent with the DNS results. Near wall results indicate that further evaluation is needed so the NR-stress near a solid/fluid interface is qualitatively consistent with current knowledge.
2. As expected, in the case of rotating simple shear, the states predicted by the URAPS- closure are within the L-diagram for all rotations. For the range of $-0.5 < \tilde{\Omega}/\tilde{\Gamma} < 0$, the locus of states are in the region for which $R_{xx} < 1/3$ in the energy simplex and the anisotropic states fall on the curve as for the case of no rotation with $\text{III}_b > 0$, although the $\tilde{\Gamma}$ is different for each specific state. For

rotation rates $\tilde{\Omega}/\tilde{\Gamma} < -0.5$ and $\tilde{\Omega}/\tilde{\Gamma} > 0$, the locus of anisotropic states span the entire L-diagram with $\text{III}_b > 0$ as well as with $\text{III}_b < 0$.

3. URAPS-closure results for rotating channel flows with $\tilde{\Gamma}_{\text{DNS}}$ and $\tilde{\Omega}_{\text{DNS}}$ from DNS results for $\text{Re}^+ \cong 300$, $\Omega^+ \cong -0.0042$ show a close agreement with the DNS NR-stress results on the LP-side of the channel where both $\tilde{\Gamma}$ and $\tilde{\Omega}$ are negative. Qualitative agreement is also observed on the HP-side of the channel although the predicted values for the normal components differ by about 0.1 for the R_{yy} when compared with DNS results. The shear component in this region needs improvement by studying the near wall viscous effects and taking into account for the URAPS-closure through the turbulent relaxation time.
4. Zero absolute mean vorticity region of the channel which is towards the HP-side of the channel, for various rotation rates show a consistent trend that $R_{yy} > R_{zz}$ as indicated through DNS results for Ω^+ ranging from -0.004 to -0.3 (Table I.4, Appendix I). This is also observed in the asymptotic region of rotating homogeneous shear (see Figure 6.3b). URAPS-closure correctly predicts this feature (see Figure 7.14).
5. First order and second order extra-anisotropy terms are important for the convergence of URAPS-closure states. The first order anisotropy contribution is more important for smaller values of $\tilde{\Gamma} < 5$ while second order terms are more important for larger values, for the case of non-rotating simple shear flows (see Figure 7.15).

CHAPTER 8

CONCLUSIONS

A universal realizable anisotropic prestress (URAPS-) closure for the normalized Reynolds (NR-) stress has been formulated for the first time as part of this research. This result has much potential as a *practical* and *dependable* closure model for the Reynolds averaged Navier-Stokes (RANS-) equation. *Practicality* stems from the algebraic nature of the URAPS-equation. The new closure is formulated as a non-negative algebraic mapping of the NR-stress into itself. Consequently, the method of successive substitution yields a local solution to the URAPS-equation that is *realizable*. A calibrated URAPS-closure is *dependable* inasmuch as all solutions are non-negative operators. This feature holds for all turbulent flows in inertial and non-inertial frames of reference.

Unlike other commonly encountered algebraic models for the NR-stress, *realizability* for the URAPS-closure does not depend on the selection of benchmark flows used to determine phenomenological coefficients in the closure. Once the model has been benchmarked against a class of turbulent flows, it yields a *realizable* NR-stress for all other flows. Thus, the assurance of *realizability* (dependability feature) together with the simplicity of constructing solutions to a non-linear equation by using a successive substitution algorithm (practicality feature) strongly support the use of the URAPS-closure as a turbulence model for the RANS-equation.

Synopsis of the URAPS-Closure

The URAPS-closure depends on two fundamentally important physical ideas: 1) all turbulent space-time correlations have finite memories; and, 2) the normalized prestress operator is phenomenologically determined by the normalized Reynolds stress. As demonstrated in Chapter 3, the first idea supports the use of a *smoothing approximation* to obtain a local algebraic preclosure mapping between the NR-stress and the normalized prestress:

$$\underline{\underline{R}} = \frac{\underline{\underline{A}}^T \cdot \underline{\underline{B}} \cdot \underline{\underline{A}}}{\text{tr}(\underline{\underline{A}}^T \cdot \underline{\underline{B}} \cdot \underline{\underline{A}})} \quad , \quad \underline{\underline{A}} = [\underline{\underline{I}} + \tau_R <\underline{\underline{F}}>]^{-1} \quad , \quad <\underline{\underline{F}}> = \nabla <\underline{\underline{u}}> + 2\underline{\underline{\Omega}}. \quad (8.1)$$

In the above equation, the NR-stress $\underline{\underline{R}}$ and the normalized prestress $\underline{\underline{B}}$ are defined as follows:

$$\underline{\underline{R}} \equiv \frac{<\underline{\underline{u}}'\underline{\underline{u}}'>}{\text{tr}(<\underline{\underline{u}}'\underline{\underline{u}}'>)} \quad , \quad \underline{\underline{B}} \equiv \frac{<\underline{\underline{f}}'\underline{\underline{f}}'>}{\text{tr}(<\underline{\underline{f}}'\underline{\underline{f}}'>)} \quad , \quad \underline{\underline{f}}' \equiv \nabla \cdot \left[\frac{\underline{\underline{p}}'}{\rho} + \underline{\underline{u}}'\underline{\underline{u}}' - <\underline{\underline{u}}'\underline{\underline{u}}'> \right]. \quad (8.2)$$

It is noteworthy that the NR-stress and the normalized prestress are symmetric, normalized, and non-negative operators. Eq.(8.1) is clearly a non-negative mapping of $\underline{\underline{B}}$ into $\underline{\underline{R}}$.

The second idea noted above provides an additional equation that completes the formal URAPS-closure:

$$\underline{\underline{B}} = \underline{\underline{\mathfrak{B}}}(\underline{\underline{R}}) = \underline{\underline{R}} + C_1 \left(\underline{\underline{R}} - \frac{1}{3} \underline{\underline{I}} \right) + C_2 \left(\underline{\underline{R}} \cdot \underline{\underline{R}} - \Pi_R \underline{\underline{R}} \right). \quad (8.3)$$

The coefficients in Eq.(8.3) may be functions of the invariants of the NR-stress and, without loss of generality, can be expressed as (see Chapter 3 for a rationale)

$$C_1 \equiv +\beta \, 27 \det(\underline{\underline{R}}) \quad , \quad C_2 \equiv -\alpha \left(\Pi_R - \frac{1}{3} \right). \quad (8.4)$$

A major finding in Chapter 3 is the discovery that Eq.(8.3) is also a non-negative mapping of $\underline{\underline{R}}$ into $\underline{\underline{B}}$ provided the dimensionless coefficients α and β in Eq.(8.4) satisfy the following universal inequalities:

$$-3/2 < \alpha < 9 \text{ and } -1 < \beta < \alpha / 27 + 4/5. \quad (8.5)$$

Specific functional forms for $\alpha(\Pi_R, III_R)$ and $\beta(\Pi_R, III_R)$ can be determined by appropriate benchmark flows. However, they must satisfy the upper and lower universal bounds to insure that all solutions to the URAPS-closure are non-negative operators.

About fifty-eight years ago, Rotta (see p. 422 in Pope, 2000) used a self-consistent approximation wherein the pressure/strain rate correlation was assumed to depend on the anisotropic component of the Reynolds stress. Rotta's conjecture provided an algebraic closure for a correlation that appears in the second-order moment equation for the NR-stress (see the redistribution term in Eq.(A.11)). The research developed herein uses a similar strategy for the normalized prestress. Thus, Eq.(8.3) completes the closure for the preclosure equation given by Eq.(8.1) above. The URAPS-closure generalizes previous work of Parks (1997) and of Weispfennig (1997) by assuming that the prestress is phenomenologically connected to the NR-stress and that the preclosure operator $\underline{\underline{A}}$, defined by Eq.(8.1) above, provides an appropriate coupling between the NR-stress and the normalized prestress for all rigid body frames of reference.

As noted in Chapter 2, other phenomenological algebraic closures for the NR-stress incorrectly relate the NR-stress to *objective* operators, such as the mean strain rate and the intrinsic vorticity operator. This approach misrepresents the underlying physics of the simplest turbulent flows and should be avoided. Therefore, extensions of the prestress closure hypothesis employed in this dissertation should not be based on representation

theorems based on *objective* operators, as proposed earlier by Parks (1997) and, by Weispfennig (1997). However, an appropriate extension of the URAPS-closure could be made based on the URIPS-closure (see Eq.(2.47)) developed previously by Parks et al. (1998). For example,

$$\underline{\underline{B}} = \underline{\underline{\mathfrak{B}}}(\underline{\underline{R}}, \underline{\underline{R}}^0) \quad , \quad \underline{\underline{R}}^0 \equiv \frac{\underline{\underline{A}}^T \cdot \underline{\underline{A}}}{\text{tr}(\underline{\underline{A}}^T \cdot \underline{\underline{A}})} . \quad (8.6)$$

This hypothesis is not developed in the research reported herein.

URAPS-Closure Calibration

The URAPS-closure for the NR-stress requires an estimate of five phenomenological statistical properties: a turbulent transport time τ_R ; the “extra” anisotropic prestress coefficients α and β ; and, the dissipation and production coefficients, C_D and C_P , in the transport equation for the turbulent dissipation. Table 4.3 gives a summary of the calibration results.

In this research, a scalar-valued turbulent transport time τ_R was defined in terms of the space-time structure of the local turbulent state (see Eq. (3.5)). The theoretical development suggests that τ_R depends on three distinct time scales associated with the local turbulent state: a viscous time scale, ν/k ; a turbulent time scale, k/ε ; and, a mean field time scale, $1/\|\underline{\underline{F}}\|$. An analysis of the energy spectrum for rotating and non-rotating homogeneous decay shows that $\tau_R \varepsilon/k$ depends on the dimensionless group $N_F \equiv (k/\varepsilon)\sqrt{\underline{\underline{F}}:\underline{\underline{F}}^T}$ (see Section 4.3; Appendix F; and, Koppula et al., 2009). The three coefficients C_{R2} , C_{R3} , and the exponent ‘n’ in Eq.(4.3) were determined by recovering

the results of Park and Chung (1998) for rotating homogeneous decay. The dissipation coefficient C_D/C_{R1} was estimated by using experimental and DNS results related to non-rotating homogeneous decay.

The “extra” anisotropy coefficients α and β , the coefficient C_{R1} , and the production coefficient C_P are calibrated based on self-similar states in non-rotating homogeneous shear. Optimal values of the coefficients are chosen so that the NR-stress components for self-similar states are recovered with minimum deviation from the experimental data used. The optimized choice of $\alpha = +0.1$ and $\beta = -0.01$ from the realizable domain ensures that the predicted normalized Reynolds stress is non-negative for all flows.

Solution Strategy for the URAPS-equation: Successive Substitution

The mathematical significance of the URAPS-closure for the NR-stress is that a solution can be obtained by successive substitution. If the hydrodynamic/kinematic operator $\tau_R < \underline{\underline{F}} >$ is known locally, then a solution to the URAPS-closure can be determined by the application of the following algorithm: $\underline{\underline{R}}_{n+1} = \mathfrak{R}(\underline{\underline{R}}_n)$. The algorithm converges within 600 iterations (i.e., $|(\underline{\underline{R}}_{n+1} - \underline{\underline{R}}_n) : \underline{\underline{e}}_i \underline{\underline{e}}_j| \leq 10^{-5} \underline{\underline{R}}_n : \underline{\underline{e}}_i \underline{\underline{e}}_j$) for rotational simple shear flows. This result occurs for any initial non-negative operator $\underline{\underline{R}}_0$. If this solution method converges for all flows (conjecture), then the URAPS-closure will provide a practical closure of the RANS-equation in rotating and in non-rotating frames of reference.

Homogeneous Decay

For rotating homogeneous decay, the Coriolis acceleration couples with the fluctuating velocity field and causes a redistribution of turbulent kinetic energy into the fluctuating velocity component co-linear with the angular velocity of the frame. The URAPS-closure predicts this redistribution of energy from an initial isotropic state. The resulting quadratic form associated with the NR-stress is a prolate ellipsoid. The prolate anisotropic states ($0 < R_{zz} = R_{zz} < 1/3 < R_{xx} < 1$) exist for all non-zero rotation numbers. This phenomenon is consistent with DNS and LES simulations of rotating homogeneous decay. Thus, there is no return to isotropy in non-inertial frames. For small rotation rates (i.e., $\tilde{\Omega}_0 < 1$), the URAPS-closure predicts that energy is shifted slowly to the R_{xx} component with a large transfer time of $\tilde{t}_f > 30$. For larger rotation rates (i.e., $\tilde{\Omega}_0 = 15$), the redistribution of energy occurs for $\tilde{t}_f \cong 1.8$. The “extra” anisotropic operator $\underline{\underline{\Delta}}_1$ triggers the reorganization of energy during the initial stage of decay ($\tilde{t} \ll 4$) and the “extra” anisotropic operator $\underline{\underline{\Delta}}_2$ completes the energy transfer during for the final stage of the decay ($\tilde{t} \gg 4$).

Unbounded Rotating Homogeneous Shear

For unbounded rotating homogeneous shear, the transport equations for k and ε together with the URAPS-closure for the NR-stress have self-similar solutions for $-0.65 = (\tilde{\Omega}/\tilde{\Gamma})_{\min} < (\tilde{\Omega}/\tilde{\Gamma}) < (\tilde{\Omega}/\tilde{\Gamma})_{\max} = +0.15$. As the rotation group decreases from 0 to $-1/2$, energy is shifted from R_{zz} to R_{yy} . For $\tilde{\Omega}/\tilde{\Gamma} = -1/4$, $1/\tilde{\Gamma}^a$, $(P/\varepsilon)^a$, and

$-R_{yz}^a$ show a maximum compared to other rotation numbers. The URAPS-predictions are qualitatively consistent with DNS and LES results. Algebraic models that cannot account for the rotation dependence on the shear component of the NR-stress cannot predict the finite asymptotic region and the energy redistribution phenomenon.

For rotating homogeneous shear flows, the eigenvalues associated with the “extra” anisotropy operators $\underline{\underline{\Delta}}_1$ and $\underline{\underline{\Delta}}_2$ are several orders of magnitude smaller than the eigenvalues associated with the anisotropic component of the NR-stress. Nevertheless, these operators play an important role in attaining the self-similar states. For $\tilde{\Omega}/\tilde{\Gamma} = 0$ and $\tilde{\Gamma}_0 = 1$, the $\underline{\underline{\Delta}}_1$ -operator is prominent during the early stage of development towards a self-similar state; however, during the final stage of development, both “extra” anisotropic operators are important.

Fully-Developed Channel Flows with Spanwise Rotation

For simple shear flow, there are no energy states in the 1st -Sextet or the 2nd -Sextet of the energy simplex (see Figure 7.3a) for rotation rates $\tilde{\Omega}/\tilde{\Gamma} < -0.5$ and $\tilde{\Omega}/\tilde{\Gamma} > 0$. However, the anisotropic states span the entire L-diagram (III_b may be either positive or negative) as indicated by Figure 7.3b. However, for $-0.5 < \tilde{\Omega}/\tilde{\Gamma} \leq 0$, there are no energy states in the 3rd -Sextet and the 4th -Sextet (see Figure 7.4). The third invariant of the anisotropic stress is always positive and all anisotropic states, $(\text{II}_b, \text{III}_b)$, fall on the same curve within the L-diagram (see Figure 7.4). URAPS-closure predictions for non-rotating and rotating fully-developed channel flows depend on two groups $\tilde{\Gamma}$ and $\tilde{\Omega}$ that vary across the channel. Both groups are zero at a solid/fluid interface. The shear

group $\tilde{\Gamma}$ is zero where the mean velocity is a maximum. The magnitude of the rotation group (i.e., $|\tilde{\Omega}|$) is non-zero within the flow domain inasmuch as $k/\varepsilon > 0$.

For non-rotating flows ($\tilde{\Omega}=0$), the energy states are all in the 2nd-Sextet and the second invariant of the anisotropic stress is positive for all states (i.e., $\text{III}_b > 0$). These predictions are in good qualitative agreement with DNS results for fully-developed channel flows (see Figures 7.5, 7.7, and 7.8). The invariants of the “extra” anisotropy operators $\underline{\underline{\Delta}}_1$ and $\underline{\underline{\Delta}}_2$ indicate that $\underline{\underline{\Delta}}_1$ is more important for $\tilde{\Gamma} < 5$, while $\underline{\underline{\Delta}}_2$ is more important for larger values of the local shear group (see Figure 7.15).

As indicated by Figure 7.14, the URAPS-closure predictions in the region where $\tilde{\Gamma} = -2\tilde{\Omega}$ are in good qualitative agreement with DNS results. Note that the Coriolis phenomenon has shifted the energy to the transverse component in the region where the intrinsic vorticity is zero. This is a significant conclusion and strongly supports the use of the URAPS-closure as a practical and physical closure for the RANS equation.

As indicated by Figures 7.10, 7.11, and 7.12, the local components of the NR-stress predicted by the URAPS-closure based on the local values of $\tilde{\Gamma}_{\text{DNS}}$ and $\tilde{\Omega}_{\text{DNS}}$ are qualitatively consistent with the DNS results. The URAPS-closure predictions on the LP-side of the channel are in close agreement with the DNS results for $\text{Re}^+ \cong 300$ and $\Omega^+ \cong -0.0042$. Qualitative agreement is also observed on the HP-side of the channel. The shear component on the HP-side (see Figure 7.12) needs to be improved by either modifying τ_R near the solid/fluid interface and/or by modifying the “extra” anisotropic prestress coefficients near the solid/fluid interface. In conclusion, the remarkable qualitative agreement between the URAPS-closure and the DNS results strongly supports

the conclusion that Eq. (3.32) will provide a low-order closure model for the RANS-equation.

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CHAPTER 9

RECOMMENDATIONS

The following three problems will complement the research results developed herein and will support the use of the URAPS-closure as a low-order closure model for the RANS-equation.

9.1 Turbulent Transport Time for Low Turbulent Reynolds Numbers

The turbulent transport time τ_R introduced by the preclosure equation (see Eq.(3.5)) depends on the turbulent Reynolds number $Re_t = (k/\varepsilon)/(\nu/k)$ and the shear group $N_F \equiv (k/\varepsilon) \|\underline{\underline{F}}\|$. In this research (see Section 4.3), the URAPS-closure for the NR-stress was calibrated for unbounded turbulent flows for which

$$(k/\varepsilon) \gg (\nu/k) \text{ and } \tau_R = C_{R1} \bar{\tau}_R(\infty, N_F) k/\varepsilon.$$

For wall-bounded flows, turbulent transport time will be influenced significantly by the turbulent Reynolds number. A method for including a Reynolds number dependence in the preclosure equation was proposed earlier by Weispfennig (1997) for the URIPS-closure. This approach can also be used for the URAPS-closure.

For fully-developed channel flows, the turbulent Reynolds number Re_t is large in the outer region and small near the walls. Viscous transport of mean momentum compared with turbulent transport of mean momentum is negligible in the core of the flow field; however, close to the wall, viscous effects dominate the transport of mean momentum as can be seen in the total shear stress profiles portrayed by Figures 2.2 and 2.8. The near wall influence of the turbulent Reynolds number can be included in the

turbulent transport time by developing a functional dependence of $\tilde{\tau}_R$ on Re_t . In the near wall region (i.e., $0 < y^+ < 30$) major differences between DNS and the URAPS-closure are observed for the shear stress for both rotating and non-rotating flows (see Figure 7.7, 7.8 and 7.12). Accurate predictions of the shear component of the NR-stress in the near wall region are critical for predicting the mean velocity profile. Hence, the following analysis based on a previous study by Weispfennig (1997) may provide a framework to include the influence of the turbulent Reynolds number on the turbulent transport time: $\tilde{\tau}_R = \tilde{\tau}_R(Re_t, N_F)$.

Near a solid/fluid interface, continuity and the no-slip boundary condition imply that (see Monin and Yaglom, 1965; Weispfennig, 1997):

$$\langle u_x' u_x' \rangle = (u^*)^2 [C_{xx}(y^+)^2 + \dots] \quad (9.1)$$

$$\langle u_y' u_y' \rangle = (u^*)^2 [C_{yy}(y^+)^4 + \dots] \quad (9.2)$$

$$\langle u_z' u_z' \rangle = (u^*)^2 [C_{zz}(y^+)^2 + \dots] \quad (9.3)$$

$$\langle u_y' u_z' \rangle = -(u^*)^2 [C_{yz}(y^+)^3 + \dots] \quad (9.4)$$

$$k^+ = C_k (y^+)^2 + \dots \quad (9.5)$$

$$\varepsilon^+ = \varepsilon_w^+ + C_\varepsilon y^+ + \dots \quad (9.6)$$

DNS results for rotating and non-rotating channel flows are consistent with Eqs.(9.1)-(9.6). Eqs. (9.4) and (9.5) imply that

$$-R_{yz} = C_{yz} y^+ + \dots \quad (9.7)$$

Figure 9.1 shows the variation of $-R_{yz}$ with $N_\Gamma (\equiv \tau_R \Gamma_{yz})$ for the URAPS-closure. For $N_\Gamma \gg 1$, it can be seen that $-R_{yz} \propto 1/N_\Gamma$. Thus, Eq. (9.7) implies that

$$N_\Gamma \propto 1/y^+ \text{ as } y^+ \rightarrow 0. \quad (9.8)$$

An empirical expression for $\tilde{\tau}_R(\text{Re}_t, N_F)$ consistent with this behavior of N_Γ is

$$N_\Gamma \equiv \tau_R \Gamma_{yz} = C_{R1} \tilde{\tau}_R(\text{Re}_t, N_F) \frac{k}{\varepsilon} \Gamma_{yz} \quad \text{where} \quad (9.9)$$

$$\tilde{\tau}_R(\text{Re}_t, N_F) = (1 + C_W \text{Re}_t^{-3/4}) \frac{(1 + C_{R3} N_F^{3/2})}{(1 + C_{R2} N_F^{3/2})}$$

Near wall statistical data can be used to determine the dependence of the wall function C_W on Re_t and N_F subject to the following two conditions:

$$C_W(\text{Re}_t, N_F) \rightarrow \begin{cases} C_W(0, 0) = C_W^0 < \infty \\ C_W(\infty, N_F) = 0 \end{cases}. \quad (9.10)$$

9.2 “Extra” Anisotropy Coefficients

Although the components of the NR-stress predicted by the high Reynolds number URAPS-closure are qualitatively consistent with DNS, an improved quantitative agreement is nevertheless needed in the near wall region where the anisotropy associated with the NR-stress is large. The “extra” anisotropic coefficients α and β in the URAPS-theory (see Section 3.3) were calibrated in this research based on experimental data associated with asymptotic homogeneous shear (see Section 4.4), which is a relatively weak anisotropic state by comparison with the near-wall region. The near wall URAPS-results portrayed in Figure 7.12 were calculated based on the assumption that α and β are universal constants. This assumption is probably not correct and needs to be

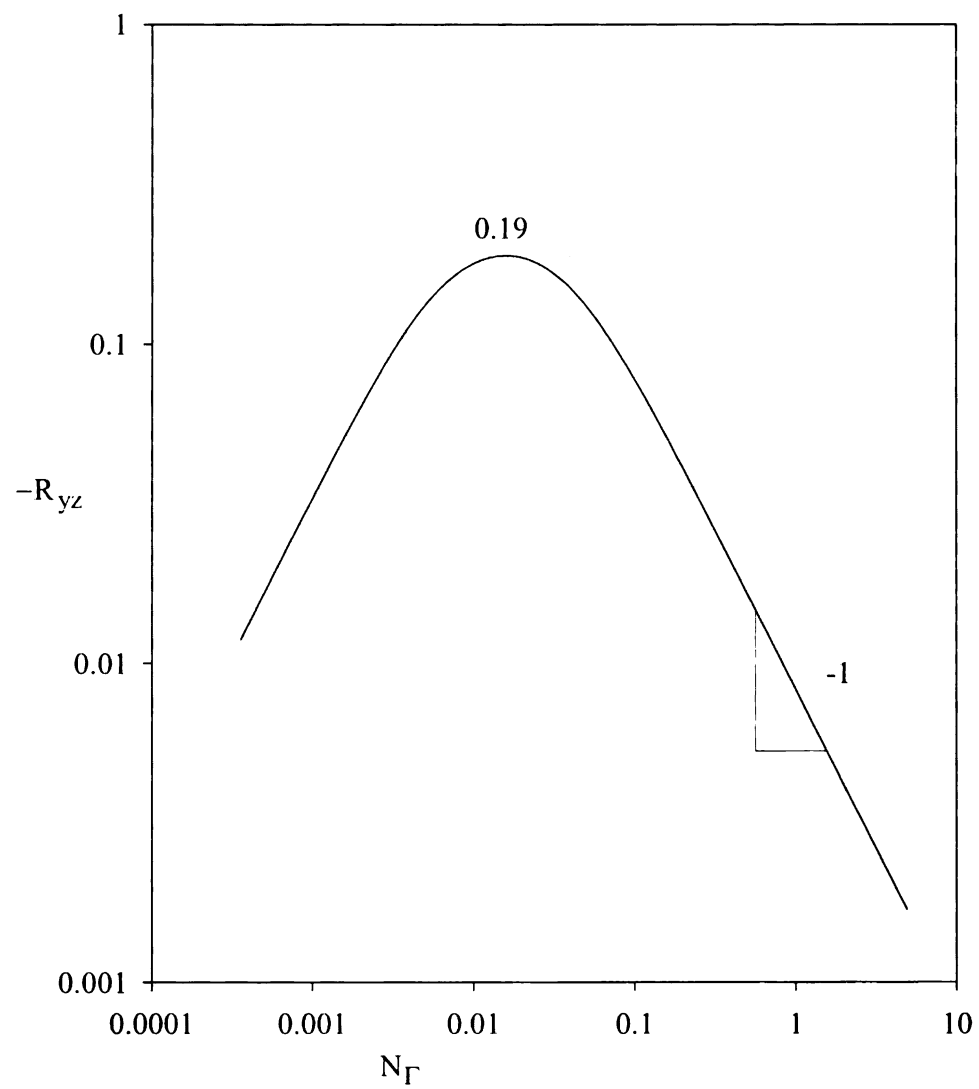


Figure 9.1 URAPS-Closure Predictions of $-R_{yz}$ for Non-Rotating Simple Shear.

examined carefully. An approach forward based on the theoretical finding portrayed by Figure 3.6 is proposed below.

For simple shear flows in an inertial frame of reference, the maximum value of $-R_{yz}$ predicted by the URAPS-closure is about $+0.19$ for $\tilde{\Gamma} \cong 5$. This is the maximum value for all δ^+ . This is too large. DNS results show that $\max(-R_{yz}) = +0.15$ for $\tilde{\Gamma} \cong 3$ and $\delta^+ = 300$. For $\delta^+ = 2000$, DNS results show that $\max(-R_{yz}) = +0.13$ for $\tilde{\Gamma} \cong 4$. The URAPS-prediction of this statistical property can be improved by further exploring the assumption that the “extra” anisotropy coefficients α and β depend on the local invariants of the NR-stress: $\alpha(\Pi_R, III_R)$ and $\beta(\Pi_R, III_R)$. The only theoretical restriction on these coefficients is the universal bounds portrayed by Figure 3.6. This type of point re-calibration could also improve the near wall predictions of the URAPS-closure inasmuch as the invariants Π_R and III_R change significantly across a wall-bounded flow field. Figure 9.2 shows the URAPS-prediction for the energy distribution and the anisotropic states for fully-developed channel flows with spanwise rotation based on the assumption that α and β are constants. A comparison of Figure 9.2 with Figures 2.10 and 2.11 provides ample motivation for dropping the assumption that the “extra” anisotropic coefficients α and β are universal constants. The proposed re-calibration under the constraint expressed by Figure 3.6 should improve the quantitative agreement between the DNS results and the URAPS-closure predictions.

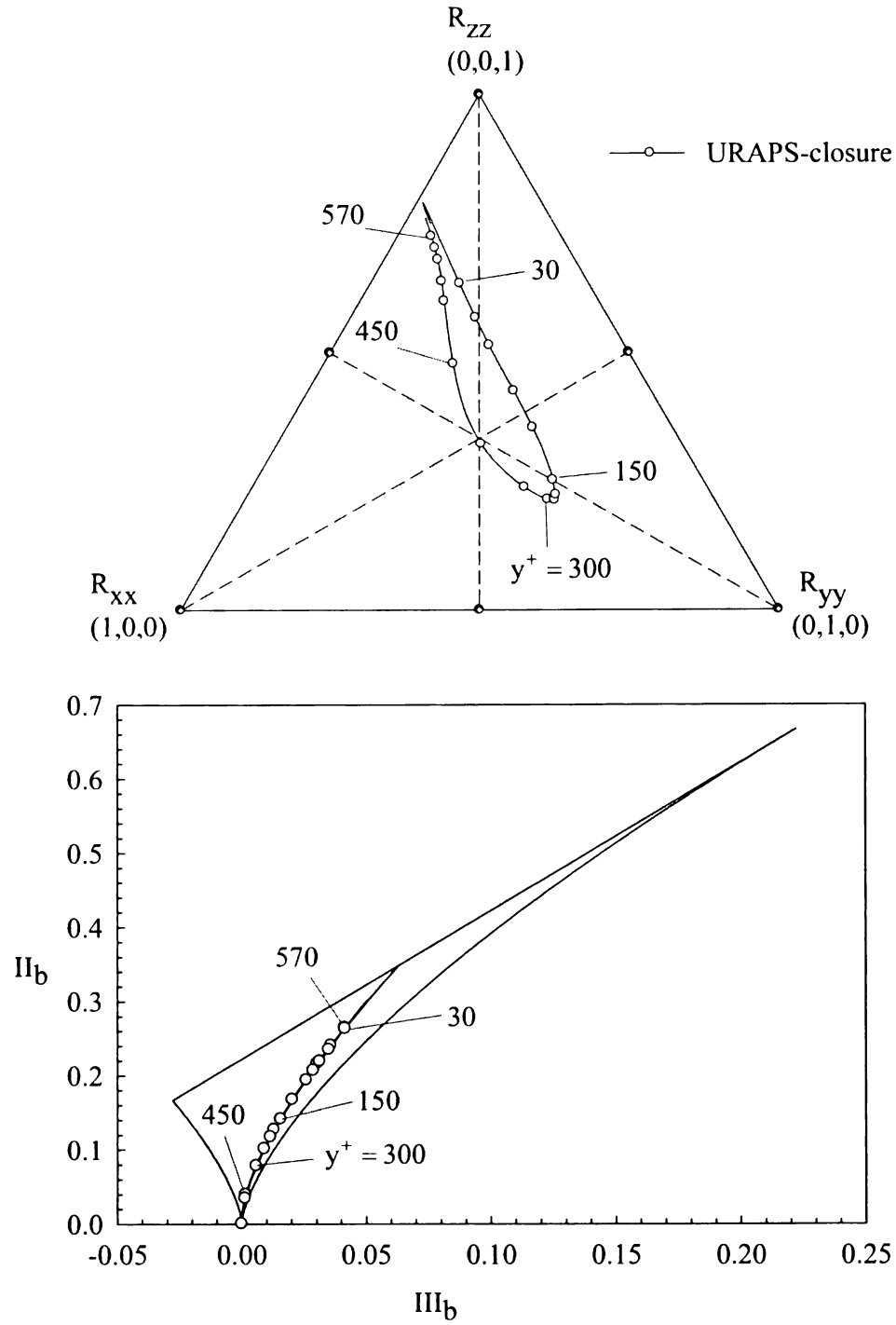


Figure 9.2 URAPS-Closure Predictions of the Energy and Anisotropic States for Fully-Developed Spanwise Rotating Channel ($\delta^+ \cong 300$, $\Omega^+ \cong -0.0042$)

9.3 *Explicit* URAPS-Closure

The URAPS-closure, defined by Eq.(3.32), is an *implicit* equation for the NR-stress. As demonstrated in this research, Eq.(3.32) can easily be solved by using a method of successive substitution. This result occurs in part because the URAPS-closure is a mapping of the NR-stress into itself for all turbulent flows (i.e., all $\tau_R < \underline{\underline{F}} >$). The convergence of the successive substitution algorithm depends on the choice of the initial operator in the space of non-negative operators. For simple shear flows with spanwise rotation, this research demonstrated that any initial non-negative operator will converge to a unique NR-stress by applying a successive substitution algorithm to Eq.(3.32). This global contraction property is highly desirable and would support the need for rapid solution methods in CFD applications. However, for more complex flows, the choice of the initial condition for the successive substitution algorithm to converge may be restricted to a “neighborhood” of the NR-stress operator that satisfies Eq.(3.32). For example, this will surely occur for statistically stationary flows with multiple solutions. This issue can be addressed by formulating an *explicit* URAPS-closure wherein the non-negative prestress operator is directly related to the hydrodynamic/kinematic operator $\tau_R < \underline{\underline{F}} >$ by a non-linear algebraic equation. Previous research by Parks (1997) and Weispfennig (1997) used the assumption that the normalized prestress could be approximated as an isotropic operator, but this approach could not explain the energy distribution for non-rotating, fully-developed channel flows. A proposed alternative to the Parks-Weispfennig conjecture is to use the URIPS-stress as an appropriate hydrodynamic/kinematic operator. This idea is briefly discussed below.

An example of an *explicit* representation for the NR-stress follows by combining the preclosure equation (see Eq.(2.44) and Eq.(3.9)) with the following hypothesis:

$$\underline{\underline{B}} = \underline{\underline{\mathfrak{B}}}(\underline{\underline{R}}^{\text{IPS}}) \quad , \quad \underline{\underline{R}}^{\text{IPS}} \equiv \frac{\underline{\underline{A}}^T \cdot \underline{\underline{A}}}{\text{tr}(\underline{\underline{A}}^T \cdot \underline{\underline{A}})} = \frac{\underline{\underline{C}}^T \cdot \underline{\underline{C}}}{\text{tr}(\underline{\underline{C}}^T \cdot \underline{\underline{C}})} . \quad (9.11)$$

The application of the non-negative Cayley-Hamilton mapping theorem developed in this research yields the following representation for the prestress (cf., Eq.(3.19)),

$$\begin{aligned} \underline{\underline{B}} = \underline{\underline{R}}^{\text{URIPS}} - \alpha(\text{II}_{\text{R}^{\text{IPS}}}, \text{III}_{\text{R}^{\text{IPS}}}) \left[\text{II}_{\text{R}^{\text{IPS}}} - \frac{1}{3} \right] \left[\underline{\underline{R}}^{\text{IPS}} \cdot \underline{\underline{R}}^{\text{IPS}} - \text{II}_{\text{R}^{\text{IPS}}} \underline{\underline{R}}^{\text{IPS}} \right] \\ + \beta(\text{II}_{\text{R}^{\text{IPS}}}, \text{III}_{\text{R}^{\text{IPS}}}) 27 \det(\underline{\underline{R}}^{\text{IPS}}) \left[\underline{\underline{R}}^{\text{IPS}} - \frac{1}{3} \text{I} \right] . \end{aligned} \quad (9.12)$$

The calibration of the two “extra” anisotropic prestress coefficients with appropriate benchmark flows is subject to the following universal bounds (see Figure 3.6):

$$\begin{aligned} -\frac{3}{2} < \alpha(\text{II}_{\text{R}^{\text{IPS}}}, \text{III}_{\text{R}^{\text{IPS}}}) < +9 \\ -1 < \beta(\text{II}_{\text{R}^{\text{IPS}}}, \text{III}_{\text{R}^{\text{IPS}}}) < \frac{1}{27} \alpha(\text{II}_{\text{R}^{\text{IPS}}}, \text{III}_{\text{R}^{\text{IPS}}}) + \frac{4}{9} \end{aligned} . \quad (9.13)$$

The proposed *explicit* URAPS-closure is defined by

$$\underline{\underline{R}} = \frac{\underline{\underline{A}}^T \cdot \underline{\underline{\mathfrak{B}}}(\underline{\underline{R}}^{\text{IPS}}) \cdot \underline{\underline{A}}}{\text{tr}(\underline{\underline{A}}^T \cdot \underline{\underline{\mathfrak{B}}}(\underline{\underline{R}}^{\text{IPS}}) \underline{\underline{A}})} . \quad (9.14)$$

The above EURAPS-closure does not require a numerical search for a solution. Once $\tau_{\text{R}} < \underline{\underline{F}} >$ has been specified locally, then Eq.(9.14) yields a prediction of the local NR-stress.

APPENDICES

APPENDIX A

UNCLOSED EQUATION FOR THE REYNOLDS STRESS

The equation for fluctuating velocity can be derived by subtracting the ensemble average equation from the instantaneous equation. A transport equation for the Reynolds stress can be developed by multiplying with the fluctuating velocity and ensemble averaging it. An unclosed equation for the Reynolds stress in a rotating frame of reference is presented in this appendix. The governing equation for fluid flow in a non-inertial frame of reference is as follows:

$$\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \underline{F} = -\nabla \left(\frac{p}{\rho} - \frac{\underline{x} \cdot (\underline{\Omega} \cdot \underline{\Omega}^T) \cdot \underline{x}}{2} \right) + \nu \nabla^2 \underline{u} + \underline{g} \quad (\text{A.1})$$

$$\nabla \cdot \underline{u} = 0 \quad (\text{A.2})$$

Upon ensemble averaging,

$$\frac{\partial \langle \underline{u} \rangle}{\partial t} + \langle \underline{u} \rangle \cdot \langle \underline{F} \rangle = -\nabla \left(\frac{\langle p \rangle}{\rho} - \frac{\underline{x} \cdot (\underline{\Omega} \cdot \underline{\Omega}^T) \cdot \underline{x}}{2} \right) + \underline{g} + \nu \nabla^2 \langle \underline{u} \rangle - \nabla \cdot \langle \underline{u}' \underline{u}' \rangle \quad (\text{A.3})$$

$$\nabla \cdot \langle \underline{u} \rangle = 0 \quad (\text{A.4})$$

The equation for fluctuating velocity can be obtained by subtracting Eq.(a.3) from Eq. (A.1) as shown in Eq. (A.5)

$$\begin{array}{ccccccc} \frac{\partial \underline{u}'}{\partial t} & + & \langle \underline{u} \rangle \cdot \nabla \underline{u}' & - & \nu \nabla^2 \underline{u}' & = & -\underline{u}' \cdot \langle \underline{F} \rangle - \nabla \cdot \left(\frac{p'}{\rho} \underline{I} + \underline{u}' \underline{u}' - \langle \underline{u}' \underline{u}' \rangle \right) \\ \text{I} & & \text{II} & & \text{III} & & \text{IV} \quad \text{V} \quad \text{VI} \quad \text{VII} \end{array} \quad (\text{A.5})$$

where $\langle \underline{F} \rangle = \nabla \langle \underline{u} \rangle + 2 \underline{\Omega}$. Upon pre-multiplication and post multiplication of the above equation with $\underline{u}'$ the resulting equations are

$$\underline{u}' \left[\frac{\partial \underline{u}'}{\partial t} + \langle \underline{u} \rangle \cdot \nabla \underline{u}' - \nu \nabla^2 \underline{u}' \right] = \underline{u}' \left[-\underline{u}' \cdot \langle \underline{F} \rangle - \nabla \cdot \left(\frac{p'}{\rho} \underline{I} + \underline{u}' \underline{u}' - \langle \underline{u}' \underline{u}' \rangle \right) \right] \quad (\text{A.6})$$

$$\left[\frac{\partial \underline{u}'}{\partial t} + \langle \underline{u} \rangle \cdot \nabla \underline{u}' - \nu \nabla^2 \underline{u}' \right] \underline{u}' = \left[-\underline{u}' \cdot \langle \underline{F} \rangle - \nabla \cdot \left(\frac{p'}{\rho} \underline{I} + \underline{u}' \underline{u}' - \langle \underline{u}' \underline{u}' \rangle \right) \right] \underline{u}' \quad (\text{A.7})$$

Adding the above two equations, the equation for fluctuating momentum can be obtained.

$$\begin{aligned} \text{I:} \quad & \left(\underline{u}' \frac{\partial \underline{u}'}{\partial t} + \frac{\partial \underline{u}'}{\partial t} \underline{u}' \right) = \frac{\partial}{\partial t} (\underline{u}' \underline{u}') \\ \text{II:} \quad & \underline{u}' (\langle \underline{u} \rangle \cdot \nabla \underline{u}') + (\langle \underline{u} \rangle \cdot \nabla \underline{u}') \underline{u}' = \langle \underline{u} \rangle \cdot \nabla (\underline{u}' \underline{u}') \\ \text{III:} \quad & -\nu [\underline{u}' (\nabla^2 \underline{u}') + (\nabla^2 \underline{u}') \underline{u}'] = -\nu \nabla^2 (\underline{u}' \underline{u}') + 2\nu [(\nabla \underline{u}')^T \cdot (\nabla \underline{u}')] \\ \text{IV:} \quad & \underline{u}' (-\underline{u}' \cdot \langle \underline{F} \rangle) - (\underline{u}' \cdot \langle \underline{F} \rangle) \underline{u}' = -\underline{u}' \underline{u}' \cdot \langle \underline{F} \rangle - \langle \underline{F} \rangle^T \cdot \underline{u}' \underline{u}' \\ \text{V:} \quad & -\underline{u}' \nabla \cdot \left(\frac{p'}{\rho} \underline{I} \right) - \left(\nabla \cdot \left(\frac{p'}{\rho} \underline{I} \right) \right) \underline{u}' = \\ & -\frac{1}{\rho} \nabla \cdot [(\underline{u}' p') \underline{I} + \underline{I} (\underline{u}' p')] + \frac{p'}{\rho} [\nabla \underline{u}' + (\nabla \underline{u}')^T] \\ \text{VI:} \quad & -\underline{u}' \nabla \cdot (\underline{u}' \underline{u}') - (\nabla \cdot (\underline{u}' \underline{u}')) \underline{u}' = -\nabla \cdot \underline{u}' \underline{u}' \underline{u}' \\ \text{VII:} \quad & \underline{u}' \nabla \cdot \langle \underline{u}' \underline{u}' \rangle + (\nabla \cdot \langle \underline{u}' \underline{u}' \rangle) \underline{u}' = \nabla \cdot (\underline{u}' \langle \underline{u}' \underline{u}' \rangle) \end{aligned} \quad (\text{A.8})$$

The resultant equation is the transport equation for instantaneous Reynolds stress

$$\begin{aligned} \frac{\partial}{\partial t} (\underline{u}' \underline{u}') + \langle \underline{u} \rangle \cdot \nabla (\underline{u}' \underline{u}') - \nu \nabla^2 (\underline{u}' \underline{u}') = & \\ \text{I} \quad & \text{II} \quad \text{III}_a \quad \text{III}_b \quad \text{IV} \quad \text{V}_a \quad \text{V}_b \quad \text{VI} \quad \text{VII} \\ & -2\nu [(\nabla \underline{u}')^T \cdot (\nabla \underline{u}')] - [(\underline{u}' \underline{u}') \cdot \langle \underline{F} \rangle + \langle \underline{F} \rangle^T \cdot (\underline{u}' \underline{u}')] \\ & -\frac{1}{\rho} \nabla \cdot [(\underline{u}' p') \underline{I} + \underline{I} (\underline{u}' p')] + \frac{p'}{\rho} [\nabla \underline{u}' + (\nabla \underline{u}')^T] \\ & -\nabla \cdot \underline{u}' \underline{u}' \underline{u}' + \nabla \cdot (\underline{u}' \langle \underline{u}' \underline{u}' \rangle) \end{aligned} \quad (\text{A.9})$$

Ensemble averaging Eq. (A.9) results in Eq. (A.10) below. Note that

$$\text{VII: } \langle \nabla \cdot (\underline{u}' \langle \underline{u}' \underline{u}' \rangle) \rangle = \nabla \cdot (\langle \underline{u}' \rangle \langle \underline{u}' \underline{u}' \rangle) = 0.$$

$$\begin{aligned} \frac{\partial}{\partial t} \langle \underline{u}' \underline{u}' \rangle &+ \underbrace{\langle \underline{u} \rangle \cdot \nabla \langle \underline{u}' \underline{u}' \rangle}_{\text{II}} - \underbrace{\nu \nabla^2 \langle \underline{u}' \underline{u}' \rangle}_{\text{III}_a} = \\ &\underbrace{- \langle \underline{u}' \underline{u}' \rangle \cdot \langle \underline{F} \rangle - \langle \underline{F} \rangle^T \cdot \langle \underline{u}' \underline{u}' \rangle}_{\text{IV}} \\ &- \underbrace{\frac{1}{\rho} \nabla \cdot (\langle \underline{u}' \underline{p}' \rangle \underline{I} + \underline{I} \langle \underline{u}' \underline{p}' \rangle)}_{\text{V}_a} + \underbrace{\langle \frac{\underline{p}'}{\rho} [\nabla \underline{u}' + (\nabla \underline{u}')^T] \rangle}_{\text{V}_b} \\ &\underbrace{- \nabla \cdot \langle \underline{u}' \underline{u}' \underline{u}' \rangle}_{\text{VI}} - \underbrace{2\nu [(\nabla \underline{u}')^T \cdot (\nabla \underline{u}')]}_{\text{III}_b} \end{aligned} \quad (\text{A.10})$$

Eq. (A.10) can be further re-arranged showing the physical significance of each term as

$$\begin{aligned} \frac{\partial}{\partial t} \langle \underline{u}' \underline{u}' \rangle &+ \langle \underline{u} \rangle \cdot \nabla \langle \underline{u}' \underline{u}' \rangle = \\ &\underbrace{[\nu \nabla^2 \langle \underline{u}' \underline{u}' \rangle - (\nabla \langle \underline{u}' \frac{\underline{p}'}{\rho} \rangle)^T - (\nabla \langle \frac{\underline{p}'}{\rho} \underline{u}' \rangle) - \nabla \cdot \langle \underline{u}' \underline{u}' \underline{u}' \rangle]}_{\text{"mixing"}} \\ &+ \underbrace{[- \langle \underline{u}' \underline{u}' \rangle \cdot \langle \underline{F} \rangle - \langle \underline{F} \rangle^T \cdot \langle \underline{u}' \underline{u}' \rangle]}_{\text{"production"}} \\ &+ 2 \underbrace{[\langle \frac{\underline{p}'}{\rho} \underline{S}' \rangle]}_{\text{redistribution}} - \underbrace{2[\nu \langle (\nabla \underline{u}')^T \cdot (\nabla \underline{u}') \rangle]}_{\text{dissipation}} \end{aligned} \quad (\text{A.11})$$

Eq. (A.11) has been discussed in the context of algebraic closures derived from the Reynolds stress transport equation (based on a weak equilibrium hypothesis, see Eq. (2.38)) and also in deriving the equation for turbulent kinetic energy.

APPENDIX B

UNCLOSED EQUATION FOR KINETIC ENERGY

The exact equation for turbulent kinetic energy ($k = \langle \underline{u}' \cdot \underline{u}' \rangle / 2$) can be obtained by taking the trace of the exact transport equation for Reynolds stress (A.11). The resulting transport equation in a rotating frame of reference is presented in this appendix. The individual terms after taking the trace of Eq. (A.10) are as below.

$$\text{I:} \quad \frac{\partial}{\partial t} \langle \underline{u}' \cdot \underline{u}' \rangle = 2 \frac{\partial k}{\partial t}$$

$$\text{II:} \quad \langle \underline{u} \rangle : \nabla \langle \underline{u}' \underline{u}' \rangle = 2 \langle \underline{u} \rangle : \nabla k$$

$$\text{III}_a: \quad -\nu \nabla^2 \langle \underline{u}' \cdot \underline{u}' \rangle = -2 \nu \nabla^2 k$$

$$\text{III}_b: \quad -2\nu [(\nabla \underline{u}')^T : (\nabla \underline{u}')] = 2 \varepsilon$$

$$\begin{aligned} \text{IV:} \quad -\langle \underline{u}' \underline{u}' \rangle : \langle \underline{\underline{F}} \rangle - \langle \underline{\underline{F}} \rangle^T : \langle \underline{u}' \underline{u}' \rangle &= \\ &= -\langle \underline{u}' \underline{u}' \rangle : (\nabla \langle \underline{u} \rangle + 2 \underline{\underline{\Omega}}) - ((\nabla \langle \underline{u} \rangle)^T - 2 \underline{\underline{\Omega}}) : \langle \underline{u}' \underline{u}' \rangle \\ &= -\langle \underline{u}' \underline{u}' \rangle : (\nabla \langle \underline{u} \rangle) - (\nabla \langle \underline{u} \rangle)^T : \langle \underline{u}' \underline{u}' \rangle \\ &= -2 \langle \underline{u}' \underline{u}' \rangle : \langle \underline{\underline{S}} \rangle \end{aligned}$$

$$(\text{Note: } \langle \underline{u}' \underline{u}' \rangle : 2 \underline{\underline{\Omega}} = 2 \underline{\underline{\Omega}} : \langle \underline{u}' \underline{u}' \rangle = 0)$$

$$\text{V}_a: \quad \left(-\frac{1}{\rho} \nabla : (\langle \underline{u}' \underline{p}' \rangle \underline{\underline{I}} + \underline{\underline{I}} \langle \underline{u}' \underline{p}' \rangle)\right) = -2 \frac{1}{\rho} \nabla \cdot (\langle \underline{u}' \underline{p}' \rangle + \langle \underline{u}' \underline{p}' \rangle)$$

$$\text{VI:} \quad -\nabla : \langle \underline{u}' \underline{u}' \underline{u}' \rangle = -\nabla \cdot \langle \underline{u}' \underline{k} \rangle$$

$$\text{V}_b: \quad \text{tr} \left[\langle \frac{\underline{p}'}{\rho} (\nabla \underline{u}' + (\nabla \underline{u}')^T) \rangle \right] = \langle \frac{\underline{p}'}{\rho} (\nabla \cdot \underline{u}' + (\nabla \cdot \underline{u}')^T) \rangle = 0$$

Dividing by 1/2, the transport equation for turbulent kinetic energy is thus,

$$\begin{array}{cccccc}
\frac{\partial k}{\partial t} + \langle \underline{u} \rangle \cdot \nabla k - \nu \nabla^2 k & = & -\nu \langle (\nabla \underline{u}') : (\nabla \underline{u}')^T \rangle & - \langle \underline{u}' \underline{u}' \rangle : \langle \underline{\underline{S}} \rangle & & \\
& & \text{(dissipation rate)} & \text{(production due to mean strain)} & & \\
\text{I} & \text{II} & \text{III}_a & \text{III}_b & \text{IV} & \\
& & -\frac{1}{\rho} \nabla \cdot \langle \underline{u}' \underline{p}' \rangle - \nabla \cdot \langle \underline{u}' \frac{(\underline{u}' \cdot \underline{u}')}{2} \rangle & & & \\
& & \text{V} & \text{VI} & &
\end{array} \tag{B.1}$$

Eq. (B.1) is exact and unclosed. A model for $\langle \underline{u}' \frac{(\underline{u}' \cdot \underline{u}')}{2} \rangle$ is needed. This can be simplified as defining $\phi = \frac{(\underline{u}' \cdot \underline{u}')}{2}$ (can be considered as fluctuating part of kinetic energy).

$$\langle \underline{u}' \frac{(\underline{u}' \cdot \underline{u}')}{2} \rangle = \langle \underline{u}' k \rangle + \langle \underline{u}' \phi \rangle = \langle \underline{u}' \rangle k + \langle \underline{u}' \phi \rangle = \langle \underline{u}' \phi \rangle \tag{B.2}$$

So closure for $\langle \underline{u}' \phi \rangle$ is needed. Equation for ϕ can be written by subtracting the equation for $k = \langle \underline{u}' \underline{u}' \rangle / 2$ from $(\underline{u}' \cdot \underline{u}') / 2$. Taking the trace of Eq. (B.4),

$$\begin{aligned}
\frac{\partial}{\partial t} \frac{(\underline{u}' \cdot \underline{u}')}{2} + \langle \underline{u} \rangle \cdot \nabla \frac{(\underline{u}' \cdot \underline{u}')}{2} - \nu \nabla^2 \frac{(\underline{u}' \cdot \underline{u}')}{2} &= -(\underline{u}' \underline{u}') : \langle \underline{\underline{S}} \rangle - \frac{1}{\rho} \nabla \cdot [(\underline{u}' \underline{p}')] \\
&\quad - \nabla \cdot \underline{u}' \frac{(\underline{u}' \cdot \underline{u}')}{2} - \nu [(\nabla \underline{u}') : (\nabla \underline{u}')^T] + \nabla \cdot (\underline{u}' k)
\end{aligned} \tag{B.3}$$

Subtracting Eq. (B.6) from Eq. (B.8),

$$\begin{aligned}
\frac{\partial \phi}{\partial t} + \langle \underline{u} \rangle \cdot \nabla \phi - \nu \nabla^2 \phi &= -(\underline{u}' \underline{u}' - \langle \underline{u}' \underline{u}' \rangle) : \langle \underline{\underline{S}} \rangle - \frac{1}{\rho} \nabla \cdot (\underline{u}' \underline{p}') - \nabla \cdot \left(\underline{u}' \frac{(\underline{u}' \cdot \underline{u}')}{2} \right) \\
&\quad - \nu \langle (\nabla \underline{u}') : (\nabla \underline{u}')^T \rangle + \underline{u}' \cdot \nabla k
\end{aligned} \tag{B.4}$$

Now, re-arrange Eq. (B.9), with the explicit Laplace term,

$$\mathcal{L}(\phi) = \underline{u}' \cdot \nabla k - f_k'$$

$$\mathcal{L} \equiv \frac{\partial}{\partial t} + \langle \underline{u} \rangle \cdot \nabla - \nu \nabla^2 \quad (\text{B.5})$$

$$f_k' = (\underline{u}' \underline{u}' - \langle \underline{u}' \underline{u}' \rangle) : \underline{\underline{S}} + \frac{1}{\rho} \nabla \cdot (\underline{u}' p') - \nabla \cdot (\underline{u}' \frac{(\underline{u}' \cdot \underline{u}')}{2})' + \nu \langle (\nabla \underline{u}') : (\nabla \underline{u}')^T \rangle$$

Now applying the Green's function analysis and smoothing approximation as discussed in Chapter 3 (also see Weispenning, 1997),

$$k'(\underline{x}, t) = - \int_{-\infty}^t d\hat{t} \iiint d\hat{V} \underbrace{G(\underline{x}, t; \hat{\underline{x}}, \hat{t})}_{\uparrow} [\underline{u}' \cdot \nabla k - f_k'](\hat{\underline{x}}, \hat{t}) \quad (\text{B.6})$$

$$\langle \underline{u}' k' \rangle = - \int_{-\infty}^t d\hat{t} \iiint d\hat{V} \underbrace{G(\underline{x}, t; \hat{\underline{x}}, \hat{t})}_{\uparrow} [\underline{u}' \cdot \nabla k - f_k'](\hat{\underline{x}}, \hat{t})$$

Applying smoothing approximation and introducing a time scale,

$$\langle \underline{u}' k' \rangle = -\tau_R \langle \underline{u}' \underline{u}' \rangle \cdot \nabla k - \tau_R \langle \underline{u}' f_k' \rangle \quad (\text{B.7})$$

Similarly,

$$\langle \underline{u}' f_k' \rangle = -\tau_R \langle \underline{u}' f_k' \rangle \cdot \langle \underline{\underline{F}} \rangle - \tau_R \langle \underline{f}' f_k' \rangle \quad (\text{B.8})$$

Rearranging, Eqs. (B.7) and (B.8),

$$\langle \underline{u}' k' \rangle = -\tau_R \langle \underline{u}' \underline{u}' \rangle \cdot \nabla k - \tau_R^2 \langle \underline{f}' f_k' \rangle \cdot \underline{\underline{A}} \quad (\text{B.9})$$

$$\langle \underline{u}' k' \rangle \approx -\tau_R \langle \underline{u}' \underline{u}' \rangle \cdot \nabla k. \quad (\text{B.10})$$

The unclosed exact equation presented in Eq. (B.1) together with Eq. (B.10) are used in describing the transport of turbulent kinetic energy in a rotating frame of reference (see Eqs. (1.9) and (1.10)).

APPENDIX C

UNCLOSED EQUATION FOR DISSIPATION

Exact turbulent dissipation equation can be derived from the transport equation for fluctuating velocity (see Eq. (A.5)) (Eq. (A.5) is presented again in Eq. (C.1) below for convenience).

$$\begin{array}{ccccccc} \frac{\partial \underline{u}'}{\partial t} & + & \langle \underline{u} \rangle \cdot \nabla \underline{u}' & - & \nu \nabla^2 \underline{u}' & = & - \underline{u}' \cdot \langle \underline{F} \rangle - \nabla \cdot \left(\frac{p'}{\rho} \underline{I} + \underline{u}' \underline{u}' - \langle \underline{u}' \underline{u}' \rangle \right) \\ \text{I} & & \text{II} & & \text{III} & & \text{IV} \quad \text{V} \quad \text{VI} \quad \text{VII} \end{array} \quad (\text{C.1})$$

where $\langle \underline{F} \rangle = \nabla \langle \underline{u} \rangle + 2 \underline{\underline{\Omega}}$. The Roman numerals represent individual terms. The following operations are performed in the order, on the above equation to finally derive the equation for ε . 1) gradient operator is applied 2) scalar multiplication with $(\nabla \underline{u}')^T$ is carried 3) trace operator is applied 4) multiplied by 2ν and 5) ensemble average is performed.

$$\text{I:} \quad (\nabla \underline{u}')^T : \nabla \frac{\partial \underline{u}'}{\partial t} = \frac{1}{2} \nu \frac{\partial}{\partial t} ((\nabla \underline{u}')^T : \nabla \underline{u}')$$

$$2\nu \langle \frac{1}{2} \nu \frac{\partial}{\partial t} ((\nabla \underline{u}')^T : \nabla \underline{u}') \rangle = \frac{\partial \varepsilon}{\partial t}$$

$$\begin{aligned} \text{II:} \quad (\nabla \underline{u}')^T : \nabla (\langle \underline{u} \rangle \cdot \nabla \underline{u}') &= (\nabla \underline{u}')^T : (\nabla \langle \underline{u} \rangle \cdot \nabla \underline{u}') + (\nabla \underline{u}')^T : (\langle \underline{u} \rangle \cdot \nabla \nabla \underline{u}') \\ &= (\nabla \underline{u}')^T : (\nabla \langle \underline{u} \rangle \cdot \nabla \underline{u}') + \frac{1}{2} \langle \underline{u} \rangle \cdot \nabla (\nabla \underline{u}' : (\nabla \underline{u}')^T) \\ &= (\nabla \underline{u}') \cdot (\nabla \underline{u}')^T : \nabla \langle \underline{u} \rangle + \frac{1}{2} \langle \underline{u} \rangle \cdot \nabla (\nabla \underline{u}' : (\nabla \underline{u}')^T) \end{aligned}$$

$$\text{Note that } (\nabla \underline{u}') \cdot (\nabla \underline{u}')^T : \nabla \langle \underline{u} \rangle = (\nabla \underline{u}') \cdot (\nabla \underline{u}')^T : \nabla \langle \underline{u} \rangle$$

$$\begin{aligned}\Rightarrow \text{II:} \quad 2\nu < (\nabla \underline{u}') \cdot (\nabla \underline{u}')^T : < \underline{\underline{S}} > + \frac{1}{2} < \underline{u} > \cdot \nabla (\nabla \underline{u}') : (\nabla \underline{u}')^T > \\ &= 2\nu < (\nabla \underline{u}') \cdot (\nabla \underline{u}')^T > : < \underline{\underline{S}} > + \nu < \underline{u} > \cdot \nabla \varepsilon\end{aligned}$$

$$\begin{aligned}\text{III:} \quad & (\nabla \underline{u}')^T : \nabla (-\nu \nabla^2 \underline{u}') = -\nu (\nabla \underline{u}')^T : (\nabla \nabla^2 \underline{u}') \\ &= -\nu \nabla \cdot ((\nabla \underline{u}')^T : (\nabla \nabla \underline{u}')) + \nu (\nabla \cdot (\nabla \underline{u}')^T : (\nabla \nabla \underline{u}')) \\ &= -\frac{1}{2} \nu \nabla^2 ((\nabla \underline{u}')^T : \nabla \underline{u}') + \nu \nabla (\nabla \underline{u}')^T : \nabla (\nabla \underline{u}')^T \\ &2\nu < -\frac{1}{2} \nu \nabla^2 ((\nabla \underline{u}')^T : \nabla \underline{u}') + \nu \nabla (\nabla \underline{u}')^T : \nabla (\nabla \underline{u}')^T > \\ &= -\nu \nabla^2 \varepsilon + 2\nu^2 \nabla < (\nabla \underline{u}')^T : \nabla (\nabla \underline{u}')^T >\end{aligned}$$

$$\begin{aligned}\text{IV:} \quad & (\nabla \underline{u}')^T : \nabla (-\underline{u}' \cdot < \underline{\underline{F}} >) = -(\nabla \underline{u}')^T : \nabla (\underline{u}' \cdot < \underline{\underline{F}} >) \\ &= -(\nabla \underline{u}')^T : [\nabla \underline{u}' \cdot < \underline{\underline{F}} > + \underline{u}' \cdot \nabla < \underline{\underline{F}} >] \\ &= -(\nabla \underline{u}')^T : (\nabla \underline{u}' \cdot < \underline{\underline{F}} >) - (\nabla \underline{u}')^T : (\underline{u}' \cdot \nabla < \underline{\underline{F}} >) \\ &= -(\nabla \underline{u}')^T : (\nabla \underline{u}' \cdot < \underline{\underline{F}} >) - ((\nabla \underline{u}')^T \underline{u}' \cdot \nabla) : < \underline{\underline{F}} > \\ &= -[(\nabla \underline{u}')^T \cdot (\nabla \underline{u}')] : < \underline{\underline{F}} > - ((\nabla \underline{u}')^T \underline{u}' \cdot \nabla) : < \underline{\underline{F}} >\end{aligned}$$

$$\begin{aligned}\text{Note that} \quad & [(\nabla \underline{u}')^T \cdot (\nabla \underline{u}')] : < \underline{\underline{F}} > = [(\nabla \underline{u}')^T \cdot (\nabla \underline{u}')] : (\nabla < \underline{u} > + 2 \underline{\underline{\Omega}}) \\ &= [(\nabla \underline{u}')^T \cdot (\nabla \underline{u}')] : \nabla < \underline{u} > \quad (\because < (\nabla \underline{u}')^T \cdot (\nabla \underline{u}') > : 2 \underline{\underline{\Omega}} = 0)\end{aligned}$$

$$\text{Also,} \quad -\nu < (\nabla \underline{u}')^T \cdot (\nabla \underline{u}') > : \nabla < \underline{u} > = -\nu < (\nabla \underline{u}')^T \cdot (\nabla \underline{u}') > : (\nabla < \underline{u} >)^T$$

$$\begin{aligned}\Rightarrow \text{IV:} \quad &= -((\nabla \underline{u}')^T \cdot (\nabla \underline{u}')) : < \underline{\underline{F}} > - ((\nabla \underline{u}')^T \underline{u}' \cdot \nabla) : < \underline{\underline{F}} > \\ &= -((\nabla \underline{u}')^T \cdot (\nabla \underline{u}')) : < \underline{\underline{S}} > - ((\nabla \underline{u}')^T \underline{u}' \cdot \nabla) : (\nabla < \underline{u} > + 2 \underline{\underline{\Omega}}) \\ &2\nu < -((\nabla \underline{u}')^T \cdot (\nabla \underline{u}')) : < \underline{\underline{S}} > - ((\nabla \underline{u}')^T \underline{u}' \cdot \nabla) : (\nabla < \underline{u} > + 2 \underline{\underline{\Omega}}) > \\ &= -2\nu < (\nabla \underline{u}')^T \cdot (\nabla \underline{u}') > : < \underline{\underline{S}} > - 2\nu < (\nabla \underline{u}')^T \underline{u}' \cdot \nabla > : (\nabla < \underline{u} > + 2 \underline{\underline{\Omega}})\end{aligned}$$

$$\begin{aligned}
V: \quad (\nabla \underline{u}')^T : \nabla(-\nabla \cdot (\frac{\underline{p}'}{\rho} \underline{I})) &= -(\nabla \underline{u}')^T : \nabla \nabla (\frac{\underline{p}'}{\rho}) \\
&= -(\nabla \nabla (\frac{\underline{p}'}{\rho})) : (\nabla \underline{u}')^T = -(\nabla \cdot (\nabla (\frac{\underline{p}'}{\rho})) : (\nabla \underline{u}')^T)
\end{aligned}$$

$$2\nu < \nabla \cdot (\nabla (\frac{\underline{p}'}{\rho})) : (\nabla \underline{u}')^T > = 2\nu \nabla \cdot < (\nabla (\frac{\underline{p}'}{\rho})) : (\nabla \underline{u}')^T >$$

$$\begin{aligned}
VI \quad -(\nabla \underline{u}')^T : \nabla(\nabla \cdot \underline{u}' \underline{u}') &= -(\nabla \underline{u}')^T : \nabla(\underline{u}' \cdot \nabla \underline{u}') \\
&= -[(\nabla \underline{u}')^T \cdot (\nabla \underline{u}')] : (\nabla \underline{u}') - (\nabla \underline{u}')^T : (\underline{u}' \cdot \nabla (\nabla \underline{u}')) \\
&= -((\nabla \underline{u}')^T \cdot (\nabla \underline{u}')) : (\nabla \underline{u}') - \frac{1}{2} \underline{u}' \cdot \nabla ((\nabla \underline{u}')^T : (\nabla \underline{u}')) \\
2\nu < -((\nabla \underline{u}')^T \cdot (\nabla \underline{u}')) : (\nabla \underline{u}') &- \frac{1}{2} \underline{u}' \cdot \nabla ((\nabla \underline{u}')^T : (\nabla \underline{u}')) > \\
&= 2\nu < (\nabla \underline{u}')^T \cdot (\nabla \underline{u}') : (\nabla \underline{u}') > - \nu < \underline{u}' \cdot \nabla ((\nabla \underline{u}')^T : (\nabla \underline{u}')) > \\
&\approx 2\nu < (\nabla \underline{u}')^T \cdot (\nabla \underline{u}') : \underline{\underline{S}}' > - \nu \nabla \cdot < \underline{u}' ((\nabla \underline{u}')^T : (\nabla \underline{u}')) >
\end{aligned}$$

$$\begin{aligned}
VII: \quad (\nabla \underline{u}')^T : \nabla(\nabla \cdot < \underline{u}' \underline{u}' >) &= (\nabla \underline{u}')^T : \nabla < \underline{u}' \cdot \nabla \underline{u}' > \\
2\nu < (\nabla \underline{u}')^T : \nabla < \underline{u}' \cdot \nabla \underline{u}' > > &= 2\nu (\nabla < \underline{u}' >)^T : \nabla < \underline{u}' \cdot \nabla \underline{u}' > \\
&= 0
\end{aligned}$$

Arranging all the above terms to obtain the exact equation for the dissipation,

$$\begin{aligned}
& \frac{\partial \varepsilon}{\partial t} + \langle \underline{u} \rangle \cdot \nabla \varepsilon - \nu \nabla^2 \varepsilon = \\
& \quad \text{I} \quad \quad \text{II}_b \quad \quad \text{IIIa} \\
& \quad - 2\nu \langle (\nabla \nabla (\frac{\underline{p}'}{\rho}) : (\nabla \underline{u}')^T \rangle - \nu \langle \underline{u}' \cdot \nabla ((\nabla \underline{u}')^T : (\nabla \underline{u}')) \rangle \\
& \quad \quad \quad \text{V} \quad \quad \quad \text{VIb} \\
& \quad - 2\nu \langle (\nabla \underline{u}')^T \cdot (\nabla \underline{u}') \rangle : \underline{\underline{S}} \rangle - 2\nu \langle (\nabla \underline{u}') \cdot (\nabla \underline{u}')^T \rangle : \underline{\underline{S}} \rangle \quad (C.2) \\
& \quad \quad \quad \text{IV}_a \quad \quad \quad \text{II}_a \\
& \quad - 2\nu \langle (\nabla \underline{u}')^T \underline{u}' \cdot \nabla \rangle : (\nabla \langle \underline{u} \rangle + 2 \underline{\underline{\Omega}}) - 2\nu \langle (\nabla \underline{u}')^T \cdot (\nabla \underline{u}') \rangle : \underline{\underline{S}}' \rangle \\
& \quad \quad \quad \text{IVb} \quad \quad \quad \text{VIa} \\
& \quad - 2\nu^2 \nabla \langle (\nabla \underline{u}')^T : \nabla (\nabla \underline{u}')^T \rangle \\
& \quad \quad \quad \text{IIIb}
\end{aligned}$$

Upon rearranging:

$$\begin{aligned}
& \underbrace{\frac{\partial \varepsilon}{\partial t} + \langle \underline{u} \rangle \cdot \nabla \varepsilon}_{\text{rate of change of dissipation relative to the mean velocity}} = \\
& - \nabla \cdot \left\{ \underbrace{-\nu \nabla \varepsilon}_{\text{viscous flux of dissipation}} + \underbrace{2\nu \langle \nabla (\frac{\underline{p}'}{\rho}) \cdot \nabla \underline{u}' \rangle + \nu \langle \underline{u}' [(\nabla \underline{u}') : (\nabla \underline{u}')^T] \rangle}_{\text{turbulent flux of dissipation}} \right\} \\
& \underbrace{-2\nu \langle (\nabla \underline{u}') \cdot (\nabla \underline{u}')^T \rangle : \underline{\underline{S}} \rangle}_{\text{production due to mean strain rate}} \underbrace{-2\nu \langle (\nabla \underline{u}')^T \cdot (\nabla \underline{u}') \rangle : \underline{\underline{S}} \rangle}_{\text{mixed production due to mean strain rate}} \quad (C.3) \\
& \underbrace{-2\nu \langle (\nabla \underline{u}')^T \underline{u}' \cdot \nabla \rangle : (\nabla \langle \underline{u} \rangle + 2 \underline{\underline{\Omega}})}_{\text{gradient production due to the mean field}} \underbrace{-2\nu \langle (\nabla \underline{u}') \cdot (\nabla \underline{u}')^T : \underline{\underline{S}}' \rangle}_{\text{production due to fluctuating strain rate}} \\
& \underbrace{-2\nu^2 \langle \nabla (\nabla \underline{u}')^T : \nabla (\nabla \underline{u}')^T \rangle}_{\text{dissipation of dissipation}}
\end{aligned}$$

Often the following terms are modeled as: (see Launder and Spalding, 1972; Rodi and Mansour, 1992)

$$\nu \langle 2(\nabla \underline{u}')^T \cdot \nabla (\frac{\underline{p}'}{\rho}) \rangle - \nu \langle \underline{u}' [(\nabla \underline{u}') : (\nabla \underline{u}')^T] \rangle = -\tau_R \langle \underline{u}' \underline{u}' \rangle \cdot \nabla \varepsilon \quad (C.4)$$

$$\begin{aligned}
-2\nu < (\nabla \underline{u}') \cdot (\nabla \underline{u}')^T > : \underline{\underline{S}} > - 2\nu < (\nabla \underline{u}')^T \cdot (\nabla \underline{u}') > : \underline{\underline{S}} > \\
- 2\nu < (\nabla \underline{u}')^T \underline{u}' > : \nabla \nabla < \underline{u} > - 2\nu < (\nabla \underline{u}') \cdot (\nabla \underline{u}')^T : \underline{\underline{S}}' > = C_p \frac{- < \underline{u}' \underline{u}' > : \nabla < \underline{u} >}{\tau_R} \quad (C.5)
\end{aligned}$$

$$2\nu^2 < \nabla (\nabla \underline{u}')^T : \nabla (\nabla \underline{u}')^T > = + C_D \frac{\varepsilon}{\tau_R} \quad (C.6)$$

A transport equation resulting from Eqs. (C.3) to (C.6), which is similar to the transport equation for turbulent kinetic energy is used for describing the turbulent dissipation (see Eq. (1.11)).

APPENDIX D INTEGRITY OPERATORS

Complete Set of Symmetric, Anisotropic, Objective Operators $\underline{\underline{\Phi}}_n$ described in Eq. (2.35) (see Pope 2000) are presented in this appendix.

n	$(\underline{\underline{\Phi}}^{(n)})^T = \underline{\underline{\Phi}}^{(n)}$ symmetric; $\text{tr}(\underline{\underline{\Phi}}^{(n)}) = 0$ anisotropic; $\underline{\underline{Q}}^T \cdot [\underline{\underline{\Phi}}^{(n)}] \cdot \underline{\underline{Q}} = \underline{\underline{\Phi}}^{(n)}$
1	$\underline{\underline{\Phi}}^{(1)} = \langle \underline{\underline{\tilde{S}}} \rangle$
2	$\left[\langle \underline{\underline{\tilde{S}}} \rangle \cdot \langle \underline{\underline{\tilde{S}}} \rangle - 1/3 \text{tr}(\langle \underline{\underline{\tilde{S}}} \rangle \cdot \langle \underline{\underline{\tilde{S}}} \rangle) \underline{\underline{I}} \right]$
3	$\left[\langle \underline{\underline{\tilde{S}}} \rangle \cdot \langle \underline{\underline{\tilde{W}}} \rangle - \langle \underline{\underline{\tilde{W}}} \rangle^T \cdot \langle \underline{\underline{\tilde{S}}} \rangle \right]$
4	$\left[\langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{W}}} \rangle - 1/3 \text{tr}(\langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{W}}} \rangle) \underline{\underline{I}} \right]$
5	$\left[\langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{S}}} \rangle \cdot \langle \underline{\underline{\tilde{S}}} \rangle + \langle \underline{\underline{\tilde{S}}} \rangle^T \cdot \langle \underline{\underline{\tilde{S}}} \rangle^T \cdot \langle \underline{\underline{\tilde{W}}} \rangle^T \right]$
6	$\langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{S}}} \rangle + \langle \underline{\underline{\tilde{S}}} \rangle^T \cdot \langle \underline{\underline{\tilde{W}}} \rangle^T \cdot \langle \underline{\underline{\tilde{W}}} \rangle^T$ $- 2/3 \text{tr}(\langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{S}}} \rangle) \underline{\underline{I}}$
7	$\left[\langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{S}}} \rangle \cdot \langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{W}}} \rangle + \langle \underline{\underline{\tilde{W}}} \rangle^T \cdot \langle \underline{\underline{\tilde{W}}} \rangle^T \cdot \langle \underline{\underline{\tilde{S}}} \rangle^T \cdot \langle \underline{\underline{\tilde{W}}} \rangle^T \right]$
8	$\left[\langle \underline{\underline{\tilde{S}}} \rangle \cdot \langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{S}}} \rangle \cdot \langle \underline{\underline{\tilde{S}}} \rangle + \langle \underline{\underline{\tilde{S}}} \rangle^T \cdot \langle \underline{\underline{\tilde{S}}} \rangle^T \cdot \langle \underline{\underline{\tilde{W}}} \rangle^T \cdot \langle \underline{\underline{\tilde{S}}} \rangle^T \right]$
9	$\langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{S}}} \rangle \cdot \langle \underline{\underline{\tilde{S}}} \rangle + \langle \underline{\underline{\tilde{S}}} \rangle^T \cdot \langle \underline{\underline{\tilde{S}}} \rangle^T \cdot \langle \underline{\underline{\tilde{W}}} \rangle^T \cdot \langle \underline{\underline{\tilde{W}}} \rangle^T$ $- 2/3 \text{tr}(\langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{S}}} \rangle \cdot \langle \underline{\underline{\tilde{S}}} \rangle) \underline{\underline{I}}$
10	$\langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{S}}} \rangle \cdot \langle \underline{\underline{\tilde{S}}} \rangle \cdot \langle \underline{\underline{\tilde{W}}} \rangle \cdot \langle \underline{\underline{\tilde{W}}} \rangle + \langle \underline{\underline{\tilde{W}}} \rangle^T \cdot \langle \underline{\underline{\tilde{W}}} \rangle^T \cdot \langle \underline{\underline{\tilde{S}}} \rangle^T \cdot \langle \underline{\underline{\tilde{S}}} \rangle^T \cdot \langle \underline{\underline{\tilde{W}}} \rangle^T$

APPENDIX E

REALIZABLE SHIH CLOSURE FOR THE NR-STRESS

Shih et al. (1994) developed a realizable bilinear eddy viscosity closure. A linear version of this model (Shih et al., 1995) is widely used in commercial codes. This appendix summarizes the model and also presents the scope or locus of the states for simple shear flows in rotating and non-rotating cases.

$$\underline{\underline{S}} = \frac{1}{2} [\nabla \underline{u} + (\nabla \underline{u})^T] \quad (E.1)$$

$$\underline{\underline{W}} = \frac{1}{2} [\nabla \underline{u} - (\nabla \underline{u})^T] \quad (E.2)$$

$$\underline{\underline{R}} = \frac{1}{3} \underline{\underline{I}} - C_\mu \frac{k}{\varepsilon} \underline{\underline{S}} + C_2 \frac{k^2}{\varepsilon^2} [-\underline{\underline{S}} \cdot \underline{\underline{W}} + \underline{\underline{W}} \cdot \underline{\underline{S}}] \quad (E.3)$$

where the coefficients are as below:

$$C_\mu = \frac{1}{A_0 + A_s (U \frac{k}{\varepsilon})}, \quad A_0 = 6.5, \quad C_2 = \frac{\sqrt{1 - 9C_\mu^2 (\frac{Sk}{\varepsilon})^2}}{C_0 + 6(\frac{Sk}{\varepsilon})(\frac{Wk}{\varepsilon})}, \quad C_0 = 1, \quad A_s^* = \sqrt{6} \cos \phi,$$

$$3\phi = \cos^{-1}(\sqrt{6}X), \quad X = \frac{\text{tr}(\underline{\underline{S}} \cdot \underline{\underline{S}} \cdot \underline{\underline{S}} \cdot \underline{\underline{S}})}{(S)^{3/2}}, \quad S = \sqrt{\underline{\underline{S}} \cdot \underline{\underline{S}}}, \quad W = \sqrt{\underline{\underline{W}} \cdot \underline{\underline{W}}},$$

$$\text{and } U = \sqrt{\underline{\underline{S}} \cdot \underline{\underline{S}} + \underline{\underline{W}} \cdot \underline{\underline{W}}}.$$

In *non-inertial* frames, the model is modified. $\underline{\underline{W}}$ is replaced by $\underline{\underline{\tilde{W}}}$ defined by

$$\underline{\underline{\tilde{W}}} = \underline{\underline{W}} + \underline{\underline{\Omega}} \quad (E.4)$$

Apart from this, U is modified particularly as $\tilde{U}$

$$\tilde{U} = \sqrt{\langle \underline{\underline{S}} \rangle : \langle \underline{\underline{S}} \rangle + (\langle \underline{\underline{\mathcal{W}}} \rangle + 2 \underline{\underline{\Omega}}) : (\langle \underline{\underline{\mathcal{W}}} \rangle + 2 \underline{\underline{\Omega}})^T} \quad (\text{E.5})$$

The K and epsilon equations for this model are:

$$(\frac{\partial}{\partial t} + \langle \underline{u} \rangle \cdot \nabla - \nu \nabla^2) k = \frac{\nu_t}{\sigma_k} \nabla^2 k - \langle \underline{u}' \underline{u}' \rangle : \langle \underline{\underline{S}} \rangle - \varepsilon \quad (\text{E.6})$$

$$(\frac{\partial}{\partial t} + \langle \underline{u} \rangle \cdot \nabla - \nu \nabla^2) \varepsilon = \frac{\nu_t}{\sigma_\varepsilon} \nabla^2 \varepsilon - C_{\varepsilon 1} \frac{\varepsilon}{k} \langle \underline{u}' \underline{u}' \rangle : \langle \underline{\underline{S}} \rangle - C_{\varepsilon 2} \frac{\varepsilon^2}{k} \quad (\text{E.7})$$

$$C_{\varepsilon 1} = 1.44; C_{\varepsilon 2} = 1.92; \sigma_k = 1; \sigma_\varepsilon = 1.3$$

Component equations for NR-stress for simple shear flows

$$R_{zz} = \frac{1}{3} + C_2 \frac{1}{2} \tilde{\Gamma} (\tilde{\Gamma} + 2\tilde{\Omega}) \quad (\text{E.8})$$

$$R_{yy} = \frac{1}{3} - C_2 \frac{1}{2} \tilde{\Gamma} (\tilde{\Gamma} + 2\tilde{\Omega}) \quad (\text{E.9})$$

$$R_{xx} = \frac{1}{3} \quad (\text{E.10})$$

$$R_{yz} = -C_\mu \frac{1}{2} \tilde{\Gamma} \quad (\text{E.11})$$

where $\tilde{\Gamma} = \frac{k}{\varepsilon} \frac{d \langle u_z \rangle}{dy}$. It is evident that the states on an energy diagram would be on the

line of $R_{xx} = 1/3$. Further more, the coefficients reduce to

$$X = 0 \Rightarrow \phi = \frac{\pi}{6} \Rightarrow A_s^* = \sqrt{6} \cos(\frac{\pi}{6}) = \frac{3}{\sqrt{2}}, \quad C_\mu = \frac{1}{6.5 + \frac{3}{\sqrt{2}} (\frac{U^* k}{\varepsilon})}, \quad C_2 = \frac{\sqrt{1 - 9C_\mu^2 (\frac{Sk}{\varepsilon})^2}}{1 + 6(\frac{Sk}{\varepsilon})(\frac{W^* k}{\varepsilon})},$$

$$U^* \frac{k}{\varepsilon} = \sqrt{\frac{\tilde{\Gamma}^2}{2} + \frac{(\tilde{\Gamma} + 6\tilde{\Omega})^2}{2}}, \quad \frac{Sk}{\varepsilon} = \frac{\tilde{\Gamma}}{\sqrt{2}} \quad \text{and} \quad \frac{W^* k}{\varepsilon} = \frac{1}{\sqrt{2}} (\tilde{\Gamma} + 2\tilde{\Omega}).$$

In an inertial frame, as $\tilde{\Gamma} \rightarrow 0$, $R_{xx}, R_{yy}, R_{zz} \rightarrow 1/3$ and $R_{yz} \rightarrow 0$. As $\tilde{\Gamma} \rightarrow \infty$,

$$C_\mu \rightarrow \frac{\sqrt{2}}{3} \frac{1}{\tilde{\Gamma}}, \quad C_2 \rightarrow 0, \quad R_{xx}, R_{yy}, R_{zz} \rightarrow 1/3 \quad \text{and} \quad R_{yz} \rightarrow -\frac{1}{3\sqrt{2}} \quad (\text{III}_b = 0 = 0, \quad \text{II}_b =$$

$$2 R_{yz}^2 = 1/9 = 0.111).$$

The above closure based on the hypothesis that NR-stress is *objective*, has been discussed in Section 2.6 (see Chapter 2). The locus of energy and anisotropic states for the above closure for non-rotating simple shear are shown in Figure 2.18.

APPENDIX F

SPECTRAL ANALYSIS FOR ROTATING HOMOGENEOUS DECAY

This appendix presents details of the derivations of equations in Park and Chung (1999) that are used in modeling $\tilde{\tau}_R(N_F)$ for the relaxation time for the URAPS theory. The model energy spectrum in the presence of rotation is represented by Eqs. (F.1), (F.2), and (F.3). (Speziale, 1998, Zhou, 1995 and Zeman, 1994). (See Figure F.1). In the following discussion, κ_L and κ_η are wave numbers associated with the inverse of energy containing scale $(k^{3/2}/\varepsilon)$ and the Kolmogorov scale $(\nu^3/\varepsilon)^{1/4}$. In the small wave number range or the energy containing scales ($\kappa < \kappa_L$), the spectrum is

$$E(\kappa) = B\kappa^m \quad (F.1)$$

In the inertial subrange ($\kappa_L < \kappa < \kappa_\eta$), the spectrum is represented as

$$E(\kappa) = C_\Omega (\varepsilon \Omega)^{1/2} \kappa^{-2} \quad \kappa \ll (\Omega^3/\varepsilon)^{1/2} \quad (F.2)$$

$$E(\kappa) = C_K (\varepsilon)^{2/3} \kappa^{-5/3} \quad \kappa \gg (\Omega^3/\varepsilon)^{1/2} \quad (F.3)$$

where $C_K = 25/9$ is the Kolmogorov constant and $C_\Omega = C_K^{3/4}$. Let κ_M be the wave number where the two functions in Eq.(F.2) and (F.3) meet ($\kappa_M = C_K^{-3/4} \kappa_\Omega$). In the absence of rotation, Eq. (F.3) holds in the inertial subrange as well as dissipation range of the spectrum. The turbulent kinetic energy for homogeneous flows can be obtained by the integral of the energy spectrum over the entire wave number space.

$$k = \int_0^\infty E(\kappa) d\kappa \quad (F.4)$$

Case I: No rotation

$$k = \int_0^{\kappa_L} B \kappa^m + \int_{\kappa_L}^{\kappa_\eta} C_K(\varepsilon)^{2/3} \kappa^{-5/3} \quad (F.5)$$

$$k = \frac{B \kappa_L^{m+1}}{m+1} - \frac{3}{2} C_K(\varepsilon)^{2/3} (\kappa_\eta^{-2/3} - \kappa_L^{-2/3}) \quad (F.6)$$

In the region where the two spectrum functions Eq. (F.1) and Eq. (F.3) meet,

$$C_K(\varepsilon)^{2/3} \kappa_L^{-2/3} = B \kappa_L^{m+1} \quad (F.7)$$

Substituting the (F.7) in the third term of Eq. (F.6),

$$k = \frac{(3m+5)}{2(m+1)} B \kappa_L^{m+1} - \frac{3}{2} C_K(\varepsilon)^{2/3} (\kappa_\eta^{-2/3}) \quad (F.8)$$

Eq. (F.7) can be rearranged as below for a representation for κ_L .

$$\Rightarrow (\kappa_L)^{m+1} = \left(\frac{C_K(\varepsilon)^{2/3}}{B} \right)^{\frac{3(m+1)}{(3m+5)}} \quad (F.9)$$

Also, $\kappa_\eta = \frac{1}{\eta} = \left(\frac{v^3}{\varepsilon} \right)^{-1/4}$. So inserting κ_L and κ_η in Eq. (F.8),

$$k = \frac{(3m+5)}{2(m+1)} B \left(\frac{C_K(\varepsilon)^{2/3}}{B} \right)^{\frac{3(m+1)}{(3m+5)}} - \frac{3}{2} C_K(\varepsilon)^{2/3} \left(\frac{v^3}{\varepsilon} \right)^{1/6} \quad (F.10)$$

Rearranging to get a relation between k and ε ,

$$k = \frac{(3m+5)}{2(m+1)} B \left(\frac{C_K}{B} \right)^{\frac{3(m+1)}{(3m+5)}} (\varepsilon)^{\frac{2(m+1)}{(3m+5)}} - \frac{3}{2} C_K(v)^{1/2} (\varepsilon)^{1/2} \quad (F.11)$$

Now differentiating the above equation,

$$\frac{\partial k}{\partial t} = \left(B \left(\frac{C_K}{B} \right)^{\frac{3(m+1)}{(3m+5)}} (\varepsilon)^{\frac{2(m+1)}{(3m+5)}-1} - \frac{3}{4} C_K (\nu)^{1/2} (\varepsilon)^{-1/2} \right) \frac{\partial \varepsilon}{\partial t} \quad (F.12)$$

Comparing the above equation with the standard model equations in homogeneous decaying turbulence,

$$\frac{\partial k}{\partial t} = -\varepsilon \quad (F.13)$$

$$\frac{d\varepsilon}{dt} = -C_{\varepsilon 2} \frac{\varepsilon^2}{k} \quad (= -\frac{C_D}{C_{R1}} \frac{1}{\tau_R} \frac{\varepsilon^2}{k}) \quad (F.14)$$

Substituting F. (13) and F.(14) (and also (F.9) for simplification), in the L.H.S and R.H.S of Eq. (12),

$$C_{\varepsilon 2} = \frac{\left(\frac{(3m+5)}{2(m+1)} B \kappa_L^{m+1} - \frac{3}{2} C_K (\nu)^{1/2} (\varepsilon)^{1/2} \right)}{(k)(\varepsilon)} \quad (F.15)$$

Replacing k in the denominator of Eq. (15) with Eq. (11),

$$C_{\varepsilon 2} = \frac{\left(\frac{(3m+5)}{2(m+1)} B \kappa_L^{m+1} - \frac{3}{2} C_K (\nu)^{1/2} (\varepsilon)^{1/2} \right)}{\left(B \kappa_L^{m+1} - \frac{3}{4} C_K (\nu)^{1/2} (\varepsilon)^{1/2} \right)} \quad (F.16)$$

Eq. (16) can be rearranged to be a function of turbulence Reynolds number as following.

Substituting Eq. (F.7) and dividing numerator and denominator by $C_K (\varepsilon)^{2/3} \kappa_L^{-2/3}$,

$$C_{\varepsilon 2} = \frac{\left(\frac{(3m+5)}{2(m+1)} - \frac{3}{2} (\nu)^{1/2} (\varepsilon)^{-1/6} \kappa_L^{2/3} \right)}{\left(1 - \frac{3}{4} C_K (\nu)^{1/2} (\varepsilon)^{-1/6} \kappa_L^{2/3} \right)} \quad (F.17)$$

Now substitute, $\kappa_L = \frac{\varepsilon}{k^{3/2}}$

$$C_{\varepsilon 2} = \frac{\left(\frac{(3m+5)}{2(m+1)} - \frac{3}{2} (v)^{1/2} (\varepsilon)^{1/2} k^{-1} \right)}{\left(1 - \frac{3}{4} (v)^{1/2} (\varepsilon)^{1/3} k^{-1} \right)} \quad (F.18)$$

Replacing $k^2/(v\varepsilon)$ with Re_t ,

$$C_{\varepsilon 2} = \frac{\left(\frac{(3m+5)}{2(m+1)} - \frac{3}{2} Re_t^{-1/2} \right)}{\left(1 - \frac{3}{4} Re_t^{-1/2} \right)} \quad (F.19)$$

Case II: Intermediate rotation rates

$$k = \int_0^{\kappa_L} B \kappa^m + \int_{\kappa_L}^{\kappa_M} C_{\Omega} (\varepsilon \Omega)^{1/2} \kappa^{-2} + \int_{\kappa_M}^{\kappa_{\eta}} C_K (\varepsilon)^{2/3} \kappa^{-5/3} \quad (F.20)$$

$$k = \frac{B \kappa_L^{m+1}}{m+1} - C_{\Omega} (\varepsilon \Omega)^{1/2} (\kappa_M^{-1} - \kappa_L^{-1}) - \frac{3}{2} C_K (\varepsilon)^{2/3} (\kappa_{\eta}^{-2/3} - \kappa_M^{-2/3}) \quad (F.21)$$

In the region where the two spectrum functions Eq. (F.1) and (F.2) meet,

$$B \kappa_L^m = C_{\Omega} (\varepsilon \Omega)^{1/2} \kappa_L^{-2} \quad (F.22)$$

In the region where the two spectrum functions Eq. (F.2) and (F.3) meet,

$$C_{\Omega} (\varepsilon \Omega)^{1/2} \kappa_M^{-2} = C_K (\varepsilon)^{2/3} \kappa_M^{-5/3} \quad (F.23)$$

Substituting Eqs. (F. 22) and (F. 23) in Eq. (F. 21),

$$k = \frac{m+2}{m+1} B \left[\frac{C_{\Omega} (\varepsilon \Omega)^{1/2}}{B} \right]^{m+1} + \frac{1}{2} C_{\Omega} (\varepsilon \Omega)^{1/2} \kappa_M^{-1} - \frac{3}{2} C_K (\varepsilon)^{2/3} (\kappa_{\eta}^{-2/3}) \quad (F.24)$$

Now, substituting , the wave numbers, $\kappa_M = C_K^{-3/4} \kappa_\Omega$, $\kappa_\Omega = (\frac{\Omega^3}{\varepsilon})^{1/2}$, and

$$\kappa_\eta = \frac{1}{\eta} = (\frac{v^3}{\varepsilon})^{-1/4},$$

$$k = \frac{m+2}{m+1} B \left[\frac{C_\Omega(\Omega)^{1/2}}{B} \right]^{\frac{m+1}{m+2}} (\varepsilon)^{\frac{1}{2} \frac{(m+1)}{(m+2)}} + \frac{1}{2} C_\Omega C_K^{3/4} (\Omega)^{-1} (\varepsilon) - \frac{3}{2} C_K v^{1/2} (\varepsilon)^{1/2}. \quad (F.25)$$

Now differentiating the above equation for k,

$$\frac{\partial k}{\partial t} = \left(\frac{1}{2} B \left[\frac{C_\Omega(\Omega)^{1/2}}{B} \right]^{\frac{m+1}{m+2}} (\varepsilon)^{\frac{1}{2} \frac{(m+1)}{(m+2)} - 1} + \frac{1}{2} C_\Omega C_K^{3/4} (\Omega)^{-1} - \frac{3}{4} C_K v^{1/2} (\varepsilon)^{-1/2} \right) \frac{\partial \varepsilon}{\partial t} \quad (F.26)$$

Comparing the above equation with the standard model equations in homogeneous decaying turbulence Eqs (F.13) and (F.14),

$$C_{\varepsilon 2} = \frac{\left(\frac{m+2}{m+1} B \left[\frac{C_\Omega(\Omega)^{1/2}}{B} \right]^{\frac{m+1}{m+2}} (\varepsilon)^{\frac{1}{2} \frac{(m+1)}{(m+2)}} + \frac{1}{2} C_\Omega C_K^{3/4} (\Omega)^{-1} (\varepsilon) - \frac{3}{2} C_K v^{1/2} (\varepsilon)^{1/2} \right)}{\left(\frac{1}{2} B \left[\frac{C_\Omega(\Omega)^{1/2}}{B} \right]^{\frac{m+1}{m+2}} (\varepsilon)^{\frac{1}{2} \frac{(m+1)}{(m+2)} - 1} + \frac{1}{2} C_\Omega C_K^{3/4} (\Omega)^{-1} - \frac{3}{4} C_K v^{1/2} (\varepsilon)^{-1/2} \right) \varepsilon} \quad (F.27)$$

Now substituting, $B \kappa_L^{m+2} = C_\Omega (\varepsilon \Omega)^{1/2}$ (derived from Eq. F.22),

$$C_{\varepsilon 2} = \frac{\left(\frac{m+2}{m+1} B [\kappa_L]^{m+1} + \frac{1}{2} C_\Omega C_K^{3/4} (\Omega)^{-1} (\varepsilon) - \frac{3}{2} C_K v^{1/2} (\varepsilon)^{1/2} \right)}{\left(\frac{1}{2} B [\kappa_L]^{m+1} + \frac{1}{2} C_\Omega C_K^{3/4} (\Omega)^{-1} (\varepsilon) - \frac{3}{4} C_K v^{1/2} (\varepsilon)^{1/2} \right)} \quad (F.28)$$

Now dividing both numerator and denominator by $B \kappa_L^{m+1} = C_\Omega(\varepsilon\Omega)^{1/2} \kappa_L^{-1}$ and

substituting, $\kappa_L = \frac{\varepsilon}{k^{3/2}}$,

$$C_{\varepsilon 2} = \frac{\left(\frac{m+2}{m+1} + \frac{1}{2} C_K^{3/4} \left(\frac{\varepsilon}{\Omega k} \right)^{3/2} - \frac{3}{2} C_K \left(\frac{\varepsilon}{\Omega k} \right)^{1/2} \left(\frac{k^2}{v\varepsilon} \right)^{-1/2} \right)}{\left(\frac{1}{2} + \frac{1}{2} C_K^{3/4} \left(\frac{\varepsilon}{\Omega k} \right)^{3/2} - \frac{3}{4} C_K \left(\frac{\varepsilon}{\Omega k} \right)^{1/2} \left(\frac{k^2}{v\varepsilon} \right)^{-1/2} \right)} \quad (F.29)$$

Replacing $k^2/(v\varepsilon)$ with Re_t and $\varepsilon/\Omega k$ with Ro_l

$$C_{\varepsilon 2} = \frac{\left(\frac{m+2}{m+1} + \frac{1}{2} C_K^{3/4} (Ro_l)^{3/2} - \frac{3}{2} C_K (Ro_l)^{1/2} (Re_t)^{-1/2} \right)}{\left(\frac{1}{2} + \frac{1}{2} C_K^{3/4} (Ro_l)^{3/2} - \frac{3}{4} C_K (Ro_l)^{1/2} (Re_t)^{-1/2} \right)} \quad (F.30)$$

Case III: For high rotation rate

In this case, the inertial range of energy spectrum function Eq.(F.2) extends till the dissipation range of spectrum.

$$k = \int_0^{\kappa_L} B \kappa^m + \int_{\kappa_L}^{\kappa_\eta} C_\Omega(\varepsilon\Omega)^{1/2} \kappa^{-2} \quad (F.31)$$

$$k = \frac{B \kappa_L^{m+1}}{m+1} - C_\Omega(\varepsilon\Omega)^{1/2} (\kappa_\eta^{-1} - \kappa_L^{-1}) \quad (F.32)$$

At κ , where the two functions in F.2 and F.3 meet,

$$B \kappa_L^m = C_\Omega(\varepsilon\Omega)^{1/2} \kappa_L^{-2} \quad (F.33)$$

From Eq. (33), it can be seen that $C_\Omega(\varepsilon\Omega)^{1/2} \kappa_L^{-1} = B \kappa_L^{m+1}$ and also eliminating

$\kappa_L^{m+2} = (C_\Omega(\varepsilon\Omega)^{1/2})/B$. So using these relations in Eq. (F.32),

$$k = \frac{m+2}{m+1} B \left[\frac{C_{\Omega}(\varepsilon\Omega)^{1/2}}{B} \right]^{\frac{m+1}{m+2}} - C_{\Omega}(\varepsilon\Omega)^{1/2} \kappa_{\eta}^{-1} \quad (\text{F.33})$$

Substituting, the wave numbers, $\kappa_{\eta} = \frac{1}{\eta} = \left(\frac{\nu^3}{\varepsilon}\right)^{-1/4}$ in Eq. (F.33),

$$k = \frac{m+2}{m+1} B \left[\frac{C_{\Omega}(\varepsilon\Omega)^{1/2}}{B} \right]^{\frac{m+1}{m+2}} - C_{\Omega}(\varepsilon\Omega)^{1/2} \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} \quad (\text{F.34})$$

Rearranging for a relation between k and ε

$$k = \frac{m+2}{m+1} B \left[\frac{C_{\Omega}(\Omega)^{1/2}}{B} \right]^{\frac{m+1}{m+2}} (\varepsilon)^{\frac{1}{2} \frac{(m+1)}{(m+2)}} - C_{\Omega}(\Omega)^{1/2} (\nu^3)^{1/4} (\varepsilon)^{1/4} \quad (\text{F.35})$$

Differentiating the above equation for k,

$$\frac{\partial k}{\partial t} = \left(\frac{1}{2} B \left[\frac{C_{\Omega}(\Omega)^{1/2}}{B} \right]^{\frac{m+1}{m+2}} (\varepsilon)^{\frac{1}{2} \frac{(m+1)}{(m+2)} - 1} - C_{\Omega}(\Omega)^{1/2} (\nu)^{3/4} (\varepsilon)^{-3/4} \right) \frac{\partial \varepsilon}{\partial t} \quad (\text{F.36})$$

Comparing the above equation with the standard model equations in homogeneous decaying turbulence Eqs (F.13) and (F.14),

$$C_{\varepsilon 2} = \frac{\left(\frac{m+2}{m+1} B \left[\frac{C_{\Omega}(\Omega)^{1/2}}{B} \right]^{\frac{m+1}{m+2}} (\varepsilon)^{\frac{1}{2} \frac{(m+1)}{(m+2)}} - C_{\Omega}(\Omega)^{1/2} (\nu)^{3/4} (\varepsilon)^{1/4} \right)}{\left(\frac{1}{2} B \left[\frac{C_{\Omega}(\Omega)^{1/2}}{B} \right]^{\frac{m+1}{m+2}} (\varepsilon)^{\frac{1}{2} \frac{(m+1)}{(m+2)} - 1} - C_{\Omega}(\Omega)^{1/2} (\nu)^{3/4} (\varepsilon)^{-3/4} \right) \varepsilon} \quad (\text{F.37})$$

The above equation can be simplified and rearranged using, Eq. (F.33) as following

$$C_{\varepsilon 2} = \frac{\left(\frac{m+2}{m+1} B[\kappa_L]^{m+1} - C_{\Omega}(\Omega)^{1/2} (v)^{3/4} (\varepsilon)^{1/4} \right)}{\left(\frac{1}{2} B[\kappa_L]^{m+1} - C_{\Omega}(\Omega)^{1/2} (v)^{3/4} (\varepsilon)^{1/4} \right)} \quad (F.38)$$

Now dividing both numerator and denominator by $B \kappa_L^{m+1} = C_{\Omega}(\varepsilon \Omega)^{1/2} \kappa_L^{-1}$ and

substituting, $\kappa_L = \frac{\varepsilon}{k^{3/2}}$,

$$C_{\varepsilon 2} = \frac{\left(\frac{m+2}{m+1} - \left(\frac{v\varepsilon}{k^2} \right)^{3/4} \right)}{\left(\frac{1}{2} - \left(\frac{v\varepsilon}{k^2} \right)^{3/4} \right)} \quad (F.39)$$

$$C_{\varepsilon 2} = \frac{\left(\frac{m+2}{m+1} - Re_t^{-3/4} \right)}{\left(\frac{1}{2} - Re_t^{-3/4} \right)} \quad (F.40)$$

Summary: Results from Cases I, II and III, (See Eq. (F. 19), (F.30) and (F.40) have been used to formulate the following relation by Park and Chung for all rotation rates for $m=2$.

$$C_{\varepsilon 2} = \left(\frac{C_D}{\tau_R} \right)_{URAPS} = \frac{4}{3} + \frac{\left(\frac{4}{3} + C_K^{3/4} Ro_l^{3/2} \right)}{\left(1 + 2C_K^{3/4} Ro_l^{3/2} \right)} \quad (F.41)$$

This can be re-arranged as

$$\left(\frac{C_D}{\tau_R} \right)_{URAPS} = \frac{4}{3} + \frac{\left(1 + \frac{2}{3} C_K^{-3/4} (|\Omega| k / \varepsilon)^{3/2} \right)}{\left(1 + \frac{1}{2} C_K^{-3/4} (|\Omega| k / \varepsilon)^{3/2} \right)} \quad (F.42)$$

The asymptotic behavior of the above equation is as shown in

$$\frac{d\varepsilon}{d\tilde{t}} = \begin{cases} -\frac{11}{6} \frac{\varepsilon^2}{k} & , \text{ for } Ro_l = \infty \\ -2 \frac{\varepsilon^2}{k} & , \text{ for } Ro_l = 1 \\ -\frac{16}{6} \frac{\varepsilon^2}{k} & , \text{ for } Ro_l = 0 \end{cases} \quad (F.43)$$

This asymptotic behavior in Eq. (F.42) is incorporated into the model for $\tilde{\tau}_R$ in the URAPS theory (see Chapter 4).

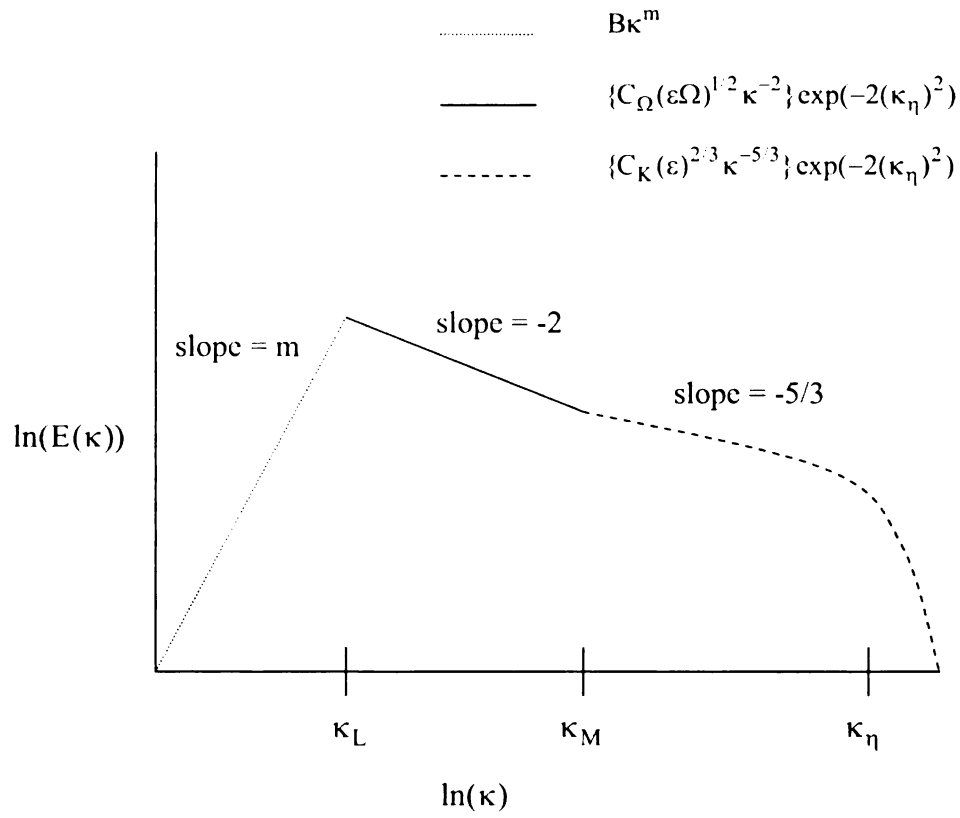


Figure F.1 Model Spectrum in the Presence of Rotation (Park and Chung, 1999)

APPENDIX G

COMPONENT EQUATIONS FOR THE URAPS-CLOSURE

The URAPS-closure component equations are described for specific benchmark flows (see Chapters 5, 6 and 7) in this appendix. For rotating simple shear flows with z as the flow direction and y as the cross flow direction, and with axis of rotation aligned in the direction of vorticity, the URAPS closure equations can be simplified as shown in this appendix.

$$\nabla < \underline{u} > \xrightarrow{\text{simple shear flows}} \frac{d < u_z >}{dy} \underline{e}_y \underline{e}_z = \Gamma_{yz} \underline{e}_y \underline{e}_z \quad (\text{G.1})$$

$$\underline{\underline{\Omega}} = \underline{\underline{\varepsilon}} \cdot \underline{\underline{\Omega}} \xrightarrow{\underline{\underline{\Omega}} \text{ is colinear with } < \underline{w} >} \Omega_x (\underline{e}_y \underline{e}_z - \underline{e}_z \underline{e}_y) \quad (\text{G.2})$$

$$\nabla < \underline{u} > + 2 \underline{\underline{\Omega}} = \Gamma_{yz} \underline{e}_y \underline{e}_z + 2 \Omega_x (\underline{e}_y \underline{e}_z - \underline{e}_z \underline{e}_y) \quad (\text{G.3})$$

The inverse of the preclosure operator defined in Eq. (2.44) is related to the mean field kinematics as shown in Eq. (G.4) below.

$$\begin{aligned} \underline{\underline{A}}^{-1} &= \underline{\underline{I}} + \tau_R (\nabla < \underline{u} > + 2 \underline{\underline{\Omega}}) = \underline{\underline{I}} + \underline{\underline{K}} \\ &= \underline{e}_x \underline{e}_x + \underline{e}_y \underline{e}_y + \underline{e}_z \underline{e}_z + (N_\Gamma + N_\Omega) \underline{e}_y \underline{e}_z - N_\Omega \underline{e}_z \underline{e}_y \end{aligned} \quad (\text{G.4})$$

where the two groups appearing in the above Eq. (G.4) are

$$N_\Gamma \equiv \tau_R \Gamma_{yz} = C_{RI} \tilde{\tau}_R \tilde{\Gamma} \quad (\text{G.5})$$

$$N_\Omega \equiv 2 \tau_R \Omega_x = 2 C_{RI} \tilde{\tau}_R \tilde{\Omega} \quad (\text{G.6})$$

Then, Eq. (G.2) can be rewritten in terms of the adjugate operator $\underline{\underline{C}} \equiv \text{adj}(\underline{\underline{I}} + \tau_R < \underline{F} >)$

$$\begin{aligned}\underline{\underline{A}} &= \frac{\underline{\underline{C}}}{\det(\underline{\underline{I}} + \tau_{\text{R}} < \underline{\underline{F}} >)} \\ &= \frac{(1 + N_{\Gamma} N_{\Omega} + N_{\Omega}^2) \underline{e}_x \underline{e}_x + \underline{e}_y \underline{e}_y + \underline{e}_z \underline{e}_z - (N_{\Gamma} + N_{\Omega}) \underline{e}_y \underline{e}_z + N_{\Omega} \underline{e}_z \underline{e}_y}{(1 + N_{\Gamma} N_{\Omega} + N_{\Omega}^2)}\end{aligned}\quad (\text{G.7})$$

Also, the normalized turbulent relaxation time scale is defined as below.

$$\tilde{\tau}_{\text{R}} = \frac{(1 + C_{\text{R}3} N_{\text{F}}^{3/2})}{(1 + C_{\text{R}2} N_{\text{F}}^{3/2})} \quad (\text{G.8})$$

$$N_{\text{F}} = \| < \underline{\underline{F}} > \| \text{ k } / \varepsilon = +\sqrt{(\tilde{\Gamma} + 2\tilde{\Omega})^2 + (2\tilde{\Omega})^2} \quad (\text{G.9})$$

where the two dimensionless timescales are

$$\tilde{\Gamma} \equiv \Gamma_{\text{yz}} \text{ k } / \varepsilon \quad (\text{G.10})$$

$$\tilde{\Omega} \equiv \Omega_{\text{x}} \text{ k } / \varepsilon \quad (\text{G.11})$$

$$\text{The component equations from the URAPS closure } \underline{\underline{R}} = \frac{\underline{\underline{A}}^{\text{T}} \cdot \underline{\underline{B}} \cdot \underline{\underline{A}}}{\text{tr}(\underline{\underline{A}}^{\text{T}} \cdot \underline{\underline{B}} \cdot \underline{\underline{A}})} = \frac{\underline{\underline{C}}^{\text{T}} \cdot \underline{\underline{B}} \cdot \underline{\underline{C}}}{\text{tr}(\underline{\underline{C}}^{\text{T}} \cdot \underline{\underline{B}} \cdot \underline{\underline{C}})}$$

follow as in Eq (G.12) to Eq. (G.16)

$$\kappa R_{\text{xx}} = (1 + N_{\Gamma} N_{\Omega} + N_{\Omega}^2)^2 B_{\text{xx}} \quad (\text{G.12})$$

$$\kappa R_{\text{yy}} = B_{\text{yy}} + 2 N_{\Omega} B_{\text{yz}} + N_{\Omega}^2 B_{\text{zz}} \quad (\text{G.13})$$

$$\kappa R_{\text{zz}} = (N_{\Gamma} + N_{\Omega})^2 B_{\text{yy}} - 2(N_{\Gamma} + N_{\Omega}) B_{\text{yz}} + B_{\text{zz}} \quad (\text{G.14})$$

$$\kappa R_{\text{yz}} = B_{\text{yz}} - (N_{\Gamma} + N_{\Omega}) B_{\text{yy}} - (N_{\Gamma} + N_{\Omega}) N_{\Omega} B_{\text{yz}} + N_{\Omega} B_{\text{zz}} \quad (\text{G.15})$$

The parameter κ is defined as follows

$$\kappa \equiv (1 + N_{\Gamma} N_{\Omega} + N_{\Omega}^2)^2 B_{\text{xx}} + (1 + (N_{\Gamma} + N_{\Omega})^2) B_{\text{yy}} + (1 + N_{\Omega}^2) B_{\text{zz}} - 2 N_{\Gamma} B_{\text{yz}}. \quad (\text{G.16})$$

Furthermore, the CH-mapping of $\underline{\underline{R}}$ into $\underline{\underline{B}}$ (see Eq.(4.1)) implies that

$$B_{xx} = R_{xx} + C_1(R_{xx} - \frac{1}{3}) + C_2(R_{xx}^2 - \Pi_R R_{xx}) \quad (G.17)$$

$$B_{yy} = R_{yy} + C_1(R_{yy} - \frac{1}{3}) + C_2(R_{yy}^2 + R_{yz}R_{zy} - \Pi_R R_{yy}) \quad (G.18)$$

$$B_{zz} = R_{zz} + C_1(R_{zz} - \frac{1}{3}) + C_2(R_{zz}^2 + R_{zy}R_{yz} - \Pi_R R_{zz}) \quad (G.19)$$

$$B_{yz} = [1 + C_1 + C_2(R_{yy} + R_{zz} - \Pi_R)]R_{yz} . \quad (G.20)$$

where $C_1 \equiv 27\beta \det(\underline{\underline{R}})$, $C_2 \equiv -\alpha(\Pi_R - \frac{1}{3})$. Four specific cases of the above general set of equations for simple shear flows in the presence of rotation are discussed below. Case1: $N_\Gamma = 0$ and $N_\Omega = 0$; Case 2: $N_\Gamma = 0$ and $N_\Omega \neq 0$ Case 3: $N_\Gamma = 0$ and $N_\Omega \neq 0$ and Case 4: $N_\Gamma = -N_\Omega$. Case 4 here refers to the zero absolute mean vorticity region discussed in Chapters 2, 6, and 7.

Case 1: $N_\Gamma = 0$ and $N_\Omega = 0$

This case is for Homogeneous flows in a non-inertial frame where $\underline{\underline{K}} = \underline{\underline{0}}$ and $\underline{\underline{A}} = \underline{\underline{I}}$. It follows directly from URAPS-equations that $\underline{\underline{R}} = \underline{\underline{B}}$ and $\underline{\underline{\Delta}} = \underline{\underline{0}}$. Consequently, $\underline{\underline{R}} = \underline{\underline{I}}/3$. The following cases illustrate the equations for rotating homogeneous flows and shear flow with and without rotation.

CASE 2: $N_\Gamma \neq 0$ and $N_\Omega = 0$;

The equations below apply to homogeneous shear and for simple shear flows without rotation.

$$\underline{\underline{K}} = N_\Gamma \underline{\underline{e}}_y \underline{\underline{e}}_z \quad (G.21)$$

$$\underline{\underline{A}} = \underline{e}_x \underline{e}_x + \underline{e}_y \underline{e}_y + \underline{e}_z \underline{e}_z - N_\Gamma \underline{e}_y \underline{e}_z \quad (\text{G.22})$$

$$N_F = \left\| \langle \underline{\underline{F}} \rangle \right\| k / \varepsilon = +\sqrt{\tilde{\Gamma}^2} \quad (\text{G.23})$$

The preclosure equations in Eq.(G.12) to Eq.(G.16) can be simplified as below

$$\kappa R_{xx} = B_{xx} \quad (\text{G.24})$$

$$\kappa R_{yy} = B_{yy} \quad (\text{G.25})$$

$$\kappa R_{zz} = N_\Gamma^2 B_{yy} - 2 N_\Gamma B_{yz} + B_{zz} \quad (\text{G.26})$$

$$\kappa R_{yz} = B_{yz} - N_\Gamma B_{yy} \quad (\text{G.27})$$

The parameter κ is defined as follows

$$\kappa \equiv 1 + N_\Gamma^2 B_{yy} - 2 N_\Gamma B_{yz}. \quad (\text{G.28})$$

It should be noted that in this case for shear flows without rotation (homogeneous or simple shear), the URAPS-equations have a feature that

$$\frac{R_{xx}}{R_{yy}} = \frac{B_{xx}}{B_{yy}} = \frac{R_{xx} + C_1(R_{xx} - \frac{1}{3}) + C_2(R_{xx}^2 - \Pi_R R_{xx})}{R_{yy} + C_1(R_{yy} - \frac{1}{3}) + C_2(R_{yy}^2 + R_{yz} R_{zy} - \Pi_R R_{yy})}. \quad (\text{G.29})$$

This results in the following equation for the ratio the two CH-related extra anisotropy coefficients

$$\frac{\beta}{\alpha} = +3 \frac{(\Pi_R - \frac{1}{3})}{27 \det(\underline{\underline{R}})} \frac{R_{xx} R_{yy}}{(R_{xx} - R_{yy})} (R_{xx} - R_{yy} - \frac{R_{yz}^2}{R_{yy}}). \quad (\text{G.30})$$

Case 3: $N_\Gamma = 0$ and $N_\Omega \neq 0$;

This case if for homogeneous flows in inertial frames (i.e. rotating flows)

$$\underline{\underline{K}} = N_{\Omega} (\underline{e}_y \underline{e}_z - \underline{e}_z \underline{e}_y) \quad (G.31)$$

$$\underline{\underline{A}} = \frac{(1 + N_{\Omega}^2) \underline{e}_x \underline{e}_x + \underline{e}_y \underline{e}_y + \underline{e}_z \underline{e}_z - N_{\Omega} \underline{e}_y \underline{e}_z + N_{\Omega} \underline{e}_z \underline{e}_y}{(1 + N_{\Omega}^2)} \quad (G.32)$$

$$N_F = \left\| \langle \underline{\underline{F}} \rangle \right\| k / \varepsilon = +\sqrt{(2\tilde{\Omega})^2 + (2\tilde{\Omega})^2} = +2\sqrt{2\tilde{\Omega}^2} \quad (G.33)$$

For this case,

$$\nabla \langle \underline{u} \rangle = \underline{\underline{0}}, \text{ and } \underline{\underline{\Omega}} = \underline{\underline{\varepsilon}} \cdot \underline{\underline{\Omega}} = \Omega_x (\underline{e}_y \underline{e}_z - \underline{e}_z \underline{e}_y). \quad (G.34)$$

Furthermore, if $B_{yz} = 0$ and $B_{yy} = B_{zz}$, then $R_{yz} = 0$, (URAPS closure prediction for this flow field),

$$\kappa R_{xx} = (1 + N_{\Omega}^2)^2 B_{xx} \quad (G.35)$$

$$\kappa R_{yy} = B_{yy} + N_{\Omega}^2 B_{zz} \quad (G.36)$$

$$\kappa R_{zz} = N_{\Omega}^2 B_{yy} + B_{zz} \quad (G.37)$$

The parameter κ is defined as follows

$$\kappa \equiv (1 + N_{\Omega}^2)^2 B_{xx} + (1 + N_{\Omega}^2) B_{yy} + (1 + N_{\Omega}^2) B_{zz}. \quad (G.38)$$

$$B_{xx} = R_{xx} + C_1(R_{xx} - 1/3) + C_2(R_{xx}^2 - \Pi_R R_{xx}) \quad (G.39)$$

$$B_{yy} = R_{yy} + C_1(R_{yy} - 1/3) + C_2(R_{yy}^2 - \Pi_R R_{yy}) \quad (G.40)$$

$$B_{zz} = R_{zz} + C_1(R_{zz} - 1/3) + C_2(R_{zz}^2 - \Pi_R R_{zz}). \quad (G.41)$$

In the above equations, $\Pi_R \equiv \underline{\underline{R}} : \underline{\underline{R}} = R_{xx}^2 + R_{yy}^2 + R_{zz}^2$ inasmuch as the shear components of $\underline{\underline{R}}$ are zero. Eqs.(G.26) and (G.27) imply that if $B_{yy} = B_{zz}$, then

$R_{yy} = R_{zz}$ which means that $B_{yy} = B_{zz} = (1 - B_{xx})/2$ and $R_{yy} = R_{zz} = (1 - R_{xx})/2$.

So, Eq.(G.26) results in

$$R_{xx} = \frac{(1 + N_{\Omega}^2)B_{xx}}{(1 + N_{\Omega}^2 B_{xx})}. \quad (G.42)$$

Case 4: $N_{\Gamma} = -N_{\Omega}$.

This case is specifically occurs in the zero absolute mean vorticity region of channel.

$$\underline{\underline{K}} = N_{\Gamma} \underline{e}_y \underline{e}_z + N_{\Omega} (\underline{e}_y \underline{e}_z - \underline{e}_z \underline{e}_y) \quad (G.43)$$

$$\underline{\underline{A}} = \underline{e}_x \underline{e}_x + \underline{e}_y \underline{e}_y + \underline{e}_z \underline{e}_z + N_{\Omega} \underline{e}_z \underline{e}_y \quad (G.44)$$

It can be noted that the pre-closure operator for Case 2 non-rotating channel is the transpose of the operator for rotating channel where the velocity profile is linear.

$$N_F = \left\| \langle \underline{\underline{F}} \rangle \right\| k / \varepsilon = +\sqrt{(2\hat{\Omega})^2} \quad (G.45)$$

$$\kappa R_{xx} = B_{xx} \quad (G.46)$$

$$\kappa R_{yy} = B_{yy} + 2 N_{\Omega} B_{yz} + N_{\Omega}^2 B_{zz} \quad (G.47)$$

$$\kappa R_{zz} = B_{zz} \quad (G.48)$$

$$\kappa R_{yz} = B_{yz} + N_{\Omega} B_{zz} \quad (G.49)$$

The parameter κ is defined as follows

$$\kappa \equiv 1 + (N_{\Omega}^2) B_{zz} - 2 N_{\Gamma} B_{yz}. \quad (G.50)$$

It is important to note that the equations for Case 2 and Case 4 are quite similar, except for a change/ flip in R_{yy} and R_{zz} equations. The component equations of the URAPS-closure for the specific cases described in this appendix are used in Chapter 5 (Case 2:

$N_\Gamma = 0$ and $N_\Omega \neq 0$) and in Chapters 6 and 7 (Case 3: $N_\Gamma = 0$ and $N_\Omega \neq 0$; and Case 4: $N_\Gamma = -N_\Omega$).

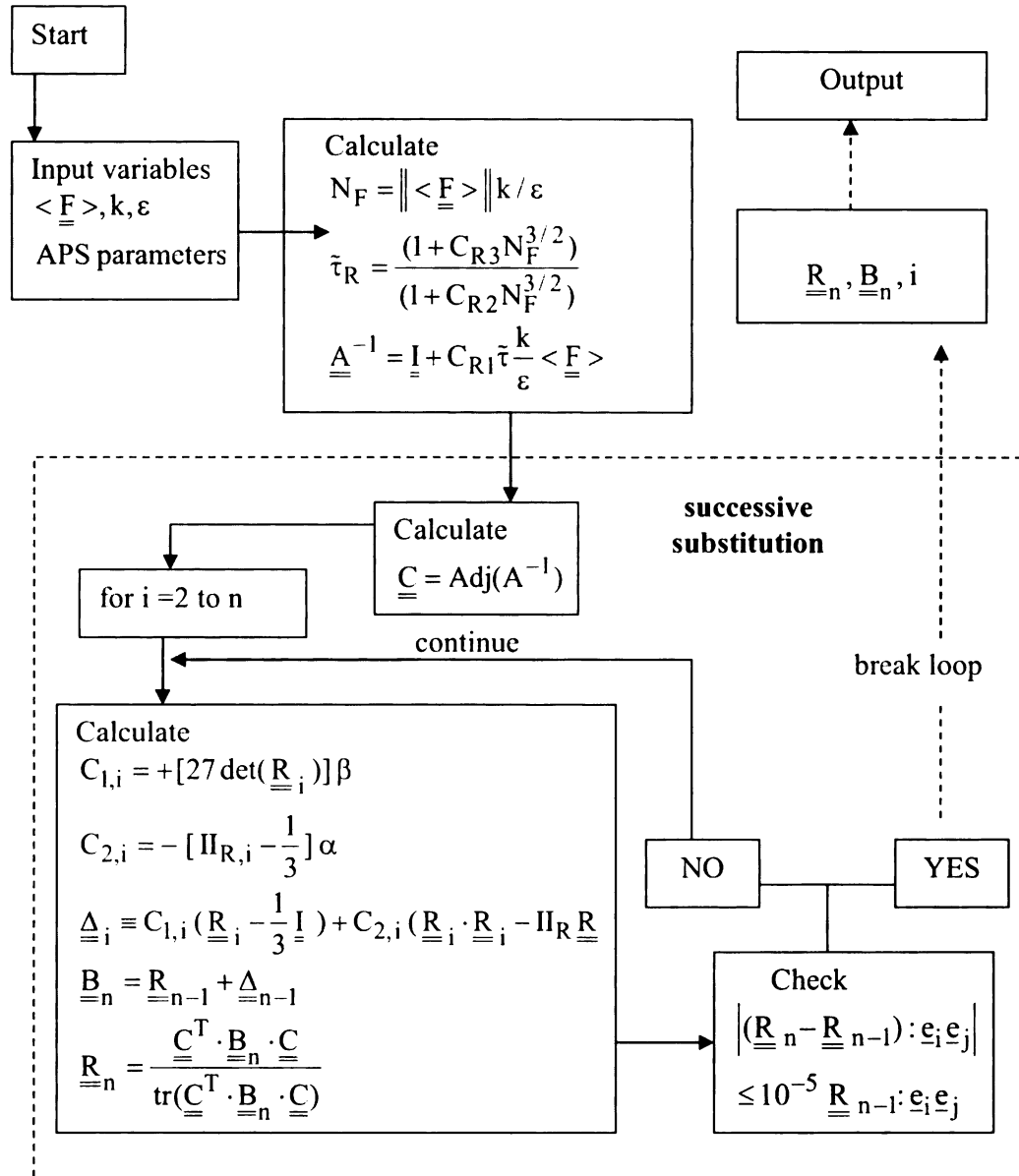
APPENDIX H

PROGRAMS AND FLOW CHARTS

The following programs are used in different chapters of the dissertation:

- H.1 General URAPS Closure Calculation Subprogram
- H.2 Optimization of APS Parameters
- H.3 Homogeneous decay
- H.4 Homogeneous Shear

H.1 General URAPS Closure Calculation Subprogram



```

% Input variable needed
% F=[ 0 0; 0 0 Gam_yz; 0 0 0] (in the form a matrix)
% k (kinetic energy)
% eps (dissipation)
% APSpar=[0.1, -0.01, 0.00357, 0.0764, 0.0526, 3/2] (APS parameters)
% Call the following matlab program
% [R,B,it,Tau]=aps_gen_fun1(F,k,eps,APSpar);

function[R,B,it,Tau]=aps_gen_fun1(F,k,eps,APSpar)

alpha= APSpar(1);
beta= APSpar(2);
cr1= APSpar(3);
cr2= APSpar(4);
cr3= APSpar(5);
n= APSpar(6);

I=[1 0 0; 0 1 0; 0 0 1];
NF=k/eps*sqrt(trace(F*(F)'));
T2=(1+cr2*(NF)^(n));
T3=(1+cr3*(NF)^(n));
Tau=T3/T2;
TauR=cr1*Tau*(k/eps);

% A inverse
Ain=I+TauR*F;

%Calling the successive substitution function
[R,B,it]=aps_gen_fun2(Ain,alpha,beta);

end

function [R,B,i]=aps_gen_fun2(Ain, alpha, beta)

I=[1 0 0; 0 1 0; 0 0 1];
Cxx=Ain(2,2)*Ain(3,3)-Ain(3,2)*Ain(2,3);
Cxy=(-1)*(Ain(2,1)*Ain(3,3)-Ain(3,1)*Ain(2,3));
Cxz=Ain(2,1)*Ain(3,2)-Ain(3,1)*Ain(2,2);
Cyx=(-1)*(Ain(1,2)*Ain(3,3)-Ain(3,2)*Ain(1,3));
Cyy=Ain(1,1)*Ain(3,3)-Ain(3,1)*Ain(1,3);
Cyz=(-1)*(Ain(1,1)*Ain(3,2)-Ain(3,1)*Ain(1,2));
Czx=Ain(1,2)*Ain(2,3)-Ain(2,2)*Ain(1,3);
Czy=(-1)*(Ain(1,1)*Ain(2,3)-Ain(2,1)*Ain(1,3));
Czz=Ain(1,1)*Ain(2,2)-Ain(2,1)*Ain(1,2);

rxx(1)=1/3;
ryy(1)=1/3;
rzz(1)=1/3;
ryz(1)=0;
rxy(1)=0;
rxz(1)=0;

% (note that C adjugate which is Transpose of the minor
C=[Cxx Cxy Cxz; Cyx Cyy Cyz; Czx Czy Czz]';

```

```
R=[rxx(1) rxy(1) rxz(1); rxy(1) ryy(1) ryz(1); rxz(1) ryz(1) rzz(1)];
```

```
for i=2:4000
```

```
    IIR = trace(R*R);
    detr=det(R);
    c1=beta*(27*detr);
    c2=-alpha*(IIR -1/3);
    B = R + c1*(R-I/3)+ c2*(R*R - IIR*R);
```

```
    CT=C';
    D=(CT*B)*C;
    tr=trace(D);
    R=D/tr;
```

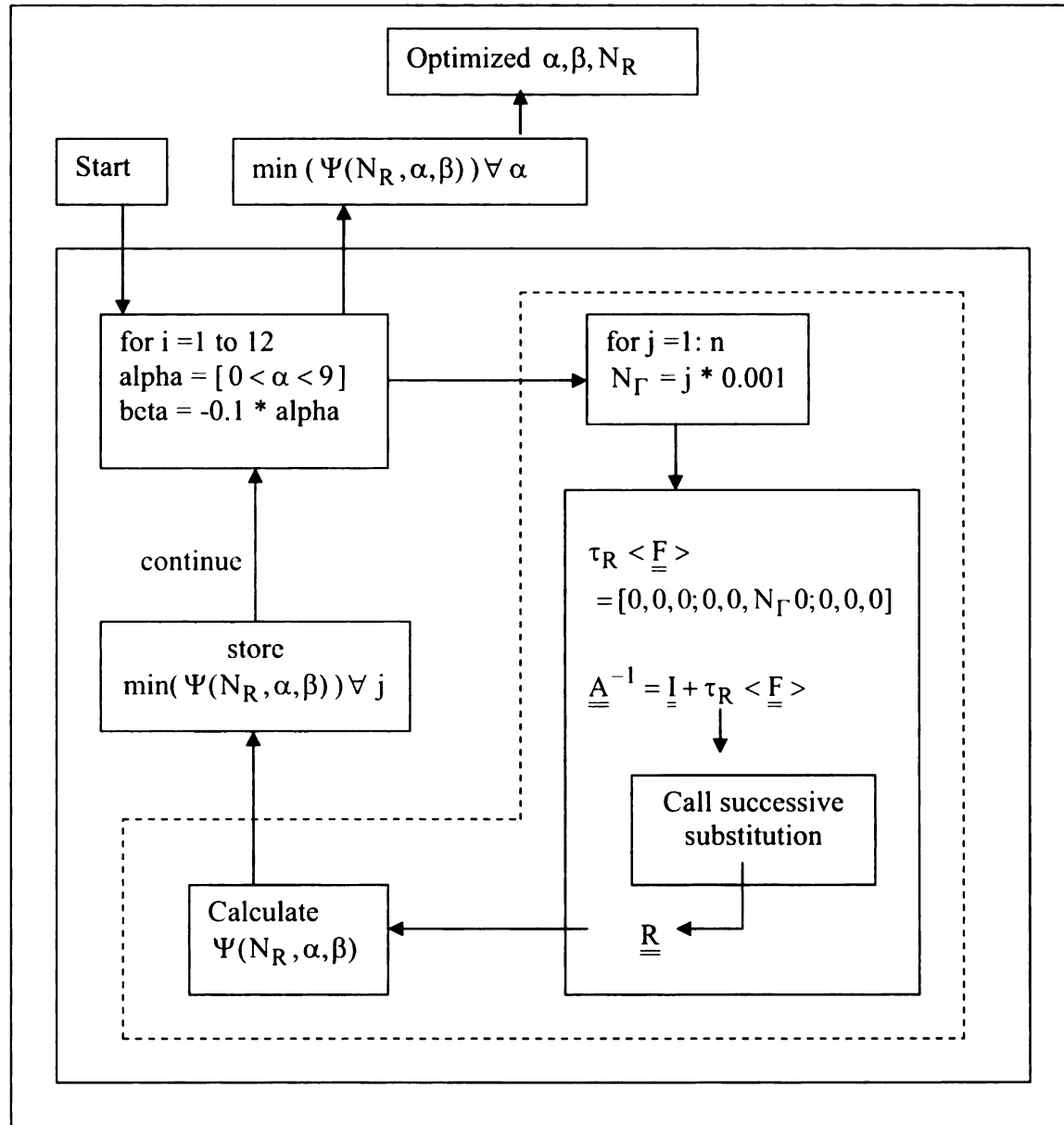
```
        rxx(i)=R(1,1);
        ryy(i)=R(2,2);
        rzz(i)=R(3,3);
        rxy(i)=R(1,2);
        rxz(i)=R(1,3);
        ryz(i)=R(2,3);
```

```
    err1= abs((rxx(i)-rxx(i-1))/rxx(i-1));
    err2= abs((ryy(i)-ryy(i-1))/ryy(i-1));
    err3= abs((rzz(i)-rzz(i-1))/rzz(i-1));
    err4= abs((rxy(i)-rxy(i-1))/rxy(i-1));
    err5= abs((rxz(i)-rxz(i-1))/rxz(i-1));
    err6= abs((ryz(i)-ryz(i-1))/ryz(i-1));
```

```
        if ((err1<0.00001) && (err2<0.00001) && (err3<0.00001) &&
            (err4<0.00001) && (err5<0.00001) && (err6<0.00001))
            break
        end
    end
```

```
end
```

H.2 Optimization of APS Parameters



Note:

$$\Psi(N_R, \alpha, \beta) \equiv$$

$$\left[\left(\frac{R_{xx}^{APS} - R_{xx}^{exp}}{R_{xx}^{exp}} \right)^2 + \left(\frac{R_{yy}^{APS} - R_{yy}^{exp}}{R_{yy}^{exp}} \right)^2 + \left(\frac{R_{zz}^{APS} - R_{zz}^{exp}}{R_{zz}^{exp}} \right)^2 + \left(\frac{R_{yz}^{APS} - R_{yz}^{exp}}{R_{yz}^{exp}} \right)^2 \right]^{1/2}$$

```

I=[1 0 0; 0 1 0; 0 0 1];
for a=1:12
    alpha =[0.01,0.025,0.05,0.075,0.1,0.25,0.5,0.75,1,2.5,5,9];
    beta(a)=-0.1*alpha(a);

    if (alpha(a)<1)
        step=0.0001;
    else if (alpha(a)>=1)
        step=0.001;
    end
    end
    for i=1:1001
        Ng(i)=(i-1)*step;
        Ngam=[0 0 0; 0 0 Ng(i); 0 0 0];
        I=[1 0 0; 0 1 0; 0 0 1];
        Ain=I + Ngam;
        [R,B,it]=aps_gen_fun2(Ain,alpha(a),beta(a));
        Rxx_ss=R(1,1);
        Ryy_ss=R(2,2);
        Rzz_ss=R(3,3);
        Ryz_ss=R(2,3);
        d(i) = sqrt( ((Rxx_ss-0.236)/0.236)^2
                    + ((Ryy_ss-0.197)/0.197)^2
                    + ((Ryz_ss+0.165)/0.165)^2
                    + ((Rzz_ss-0.567)/0.567)^2 );

        data(i)=d(i);
        data2(i)=Ng(i);
        data3(i)=i;
        data4(i)=Rxx_ss;
        data5(i)=Ryy_ss;
        data6(i)=Rzz_ss;
        data7(i)=Ryz_ss;

    end

    mini(a,1)=alpha(a);
    [y,v]=min(data); %y is the value and v is the index
    mini(a,2)=data2(v);
    mini(a,3)=data(v);
    mini(a,4)=y;

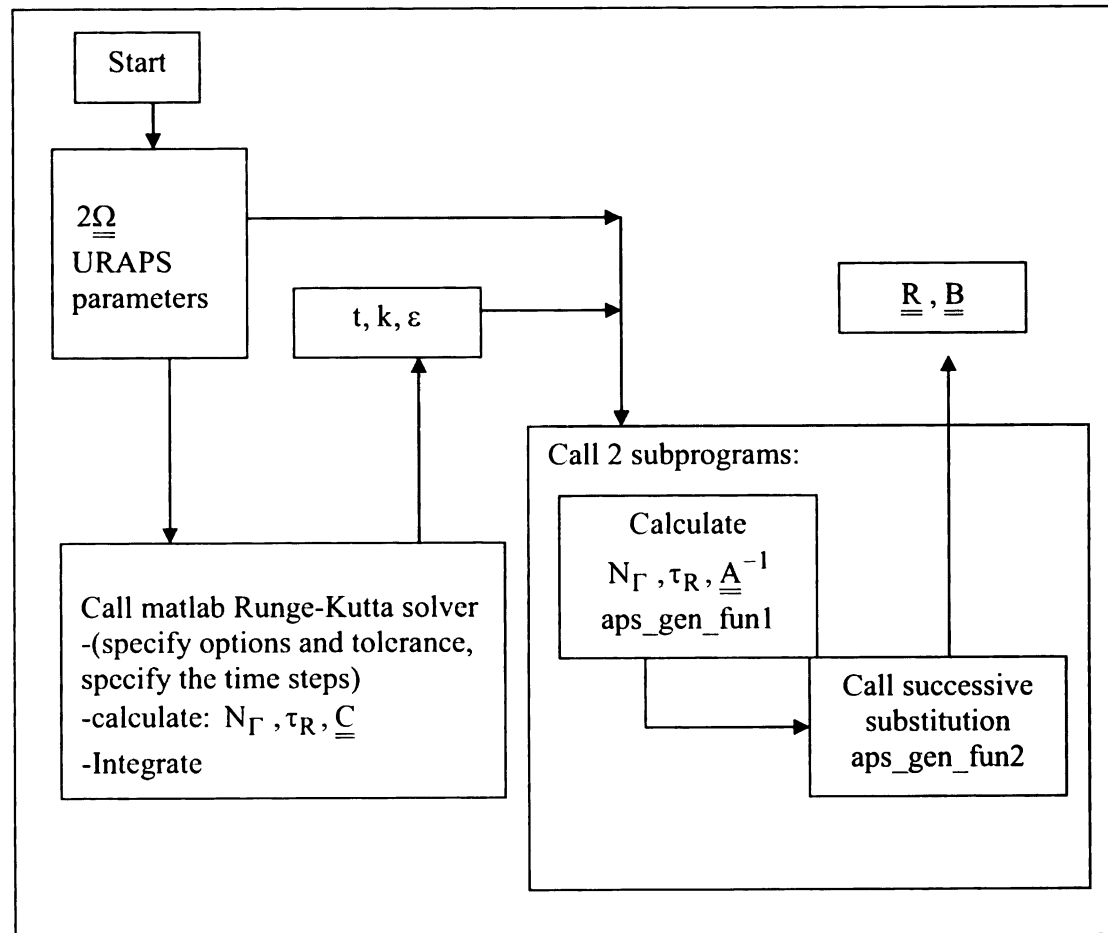
    mini(a,7)=data4(v);
    mini(a,8)=data5(v);
    mini(a,9)=data6(v);
    mini(a,10)=data7(v);

    R=[data4(v) 0 0; 0 data5(v) data7(v); 0 data7(v) data6(v)];
    b=R-I/3;
    IIb(a)=trace(b*b);
    IIIb(a)=trace(b*b*b);
    x(i)=(data6(v)+2*data5(v))/2;
    y(i)=0.5*sqrt(3)*data6(v);

end

```

H.3 Homogeneous Decay



```

APSpar=[0.1, -0.01, 0.00357, 0.0764, 0.0526, 3/2];

omg_x=0.25;
yz=2*omg_x;
zy=-2*omg_x;
F=[0 0 0; 0 0 yz; 0 zy 0];
I=[1 0 0; 0 1 0; 0 0 1];

[t,y]=homogeneous_decay(F,APSpar);

for i=1:101
k =y(i,1);
eps=y(i,2);

[R,B,it,Tau]=aps_gen_fun1(F, k, eps, APSpar);

Rxx(i)=R(1,1); Ryy(i)=R(2,2); Rzz(i)=R(3,3);
Rxy(i)=R(1,2); Rxz(i)=R(1,3); Ryz(i)=R(2,3);

B=R-I/3;
IIb_aps(i)=trace(B*B);
IIIB_aps(i)=trace(B*B*B);

Bxx(i)=B(1,1); Byy(i)=B(2,2); Bzz(i)=B(3,3);
Bxy(i)=B(1,2); Bxz(i)=B(1,3); Byz(i)=B(2,3);
end

function[t,y]=homogeneous_decay(F,APSpar);

Cd_crl=11/6; Cp_crl=1.486;

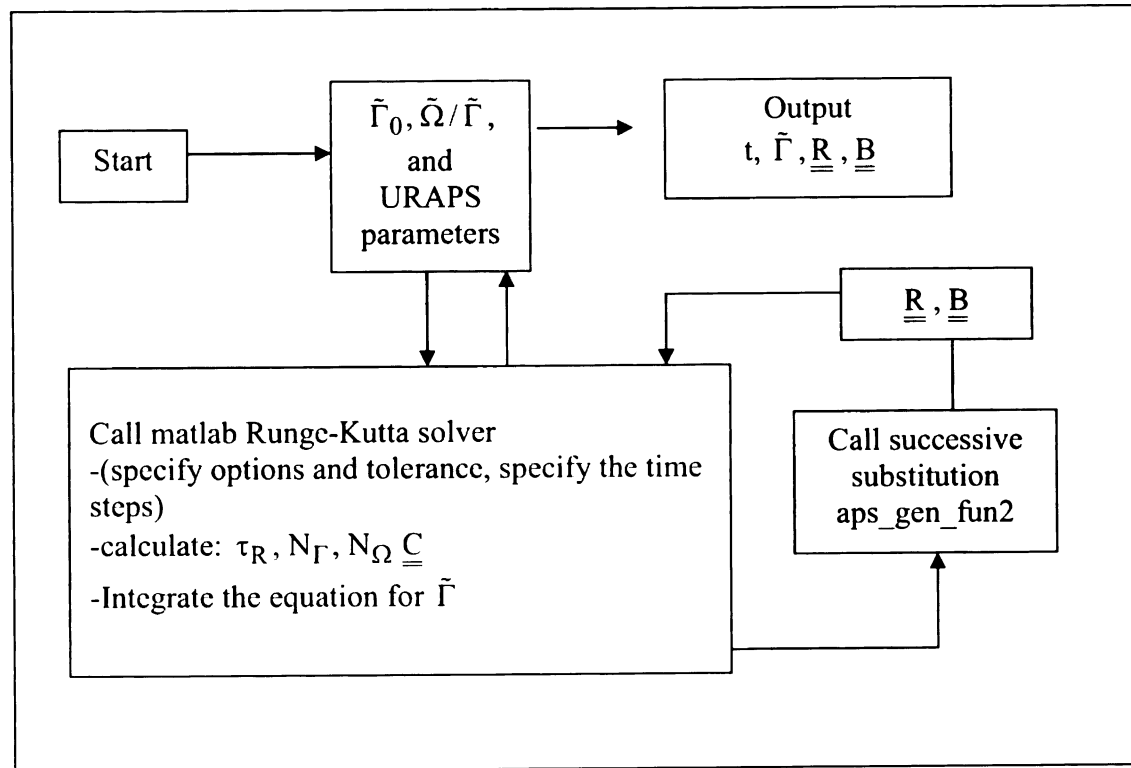
alpha= APSpar(1);
beta= APSpar(2);
crl= APSpar(3);
cr2= APSpar(4);
cr3= APSpar(5);
n= APSpar(6);

options = odeset('RelTol',1e-4,'AbsTol',[1e-6 1e-6]);
[t,y]=
ode45(@isodecay1,[0:0.1:1,1.2:0.2:5,5.5:0.5:25,26:1:40,45:5:60,70:10:17
0],[1 1],options);

function dydt = isodecay1(t,y)
dydt = zeros(2,1); % a column vector
NF=(y(1)/y(2)*sqrt(trace(F*(F') +0.00000000001)));
T2=(1+cr2*(NF)^(n));
T3=(1+cr3*(NF)^(n));
Tau=(y(1)/y(2))*T3/T2;
dydt(1) = -y(2);
dydt(2) = -Cd_crl/(Tau)* y(2);
end
end

```


H.4 Homogeneous Shear



```

APSpar=[0.1, -0.01, 0.00357, 0.0764, 0.0526, 3/2];
Omg_Gam =-0.25;
Gamo=1;

Cd_crl=11/6;
Cp_crl=1.486;
alpha= APSpar(1);
beta= APSpar(2);
crl= APSpar(3);
cr2= APSpar(4);
cr3= APSpar(5);
n= APSpar(6);

[t,y]=homogeneous_shear(Gamo,Omg_Gam,APSpar);

num=size(t);

for i=1:num

Gam=y(i);
I=[1 0 0; 0 1 0; 0 0 1];
D=sqrt((1+2*Omg_Gam)^2+(2*Omg_Gam)^2);
NF=Gam*D;
T2=(1+cr2*(NF)^(n));
T3=(1+cr3*(NF)^(n));
Tau=crl*T3/T2;
yz=1+2*Omg_Gam;
zy=-2*Omg_Gam;
Fke=y(i)*[0 0 0; 0 0 yz; 0 zy 0];
Ain=I+Tau*Fke;

[R,B,iter]=aps_gen_fun2(Ain, alpha, beta);

    Rxx(i)=R(1,1); Ryy(i)=R(2,2); Rzz(i)=R(3,3);
    Rxy(i)=R(1,2); Rxz(i)=R(1,3); Ryz(i)=R(2,3);

end

```

```

function [t,y]=homogeneous_shear(Gamo,Omg_Gam,APspar);
Cd_crl=11/6;
Cp_crl=1.486;

alpha= APSpar(1);
beta= APSpar(2);
crl= APSpar(3);
cr2= APSpar(4);
cr3= APSpar(5);
n= APSpar(6);

D=sqrt((1+2*Omg_Gam)^2+(2*Omg_Gam)^2);

[t,y] = ode45(@isodecay1,[0:0.5:1000],[Gamo]);

function dydt = isodecay1(t,y)
Gam=y;
I=[1 0 0; 0 1 0; 0 0 1];
NF=Gam*D;
T2=(1+cr2*(NF)^(n));
T3=(1+cr3*(NF)^(n));
Tau=crl*T3/T2;
%Ta=R-Tau*y;
% A inverse
yz=1+2*Omg_Gam;
zy=-2*Omg_Gam;
Fke=y*[0 0 0; 0 0 yz; 0 zy 0];
Ain=I+Tau*Fke;

[R,B,i]=aps_gen_fun2(Ain, alpha, beta);

Rxx=R(1,1); Ryy=R(2,2); Rzz=R(3,3);
Rxy=R(1,2); Rxz=R(1,3); Ryz=R(2,3);
dydt= (-2*y*Ryz-1)-(Cp_crl*crl*(-2*y*Ryz)/Tau-Cd_crl*crl/(Tau));

end

yf=y(2001);
zer=(-2*Ryz*yf)-(Cd_crl*crl-Tau)/(Cp_crl*crl-Tau);
end

```

APPENDIX I

BENCHMARK STATISTICS

The following Tables are the DNS results for non-rotating and rotating channel used in Chapters 2 and 7. The information of $\tilde{\Gamma}$ and $\tilde{\Omega}$ from the DNS results is used in the URAPS-closure equations to predict the NR-stress. A comparison of DNS results and URAPS-closure results is presented in Chapter 7.

Table I.1a: DNS data in the core region of non rotating channel (Iwamoto, 2002, Iwamoto et al. 2002).

$[y^+]$	y^+	y/δ	p^+	u^+	$\frac{du^+}{dy^+}$	k^+	ε^+	$\tilde{\Gamma}$	Re^+	$\mathcal{P}/\varepsilon$
30	30.72	0.10	0.56	13.64	0.102	3.70	0.077	4.90	177.64	1.05
35	35.18	0.12	0.67	14.04	0.082	3.49	0.067	4.24	181.62	0.97
40	39.91	0.13	0.75	14.39	0.067	3.30	0.058	3.79	186.59	0.92
45	44.92	0.15	0.80	14.70	0.057	3.13	0.051	3.49	192.48	0.88
50	50.21	0.17	0.85	14.98	0.049	2.98	0.045	3.30	199.12	0.87
70	70.74	0.24	0.88	15.82	0.035	2.55	0.029	3.06	224.20	0.87
100	99.65	0.33	0.83	16.70	0.027	2.14	0.018	3.09	248.48	0.92
150	151.07	0.51	0.66	17.86	0.019	1.59	0.010	3.08	254.39	0.92
200	199.84	0.67	0.52	18.67	0.014	1.16	0.006	2.73	222.71	0.75
250	249.37	0.84	0.43	19.23	0.008	0.82	0.004	1.69	173.70	0.32
300	297.90	1.00	0.39	19.43	0.000	0.69	0.003	0.00	147.44	0.00

Table I.1 b.: DNS data of Table I.1a continued.

y/δ	R_{xx}	R_{yy}	R_{zz}	R_{yz}	λ_{R1}	λ_{R2}	λ_{R3}	Π_b	Π_b
0.10	0.21	0.09	0.70	-0.11	0.07	0.21	0.72	0.039	0.236
0.12	0.22	0.11	0.67	-0.11	0.08	0.22	0.70	0.031	0.206
0.13	0.23	0.12	0.65	-0.12	0.10	0.23	0.67	0.024	0.182
0.15	0.24	0.14	0.62	-0.13	0.10	0.24	0.66	0.020	0.162
0.17	0.25	0.15	0.60	-0.13	0.11	0.25	0.64	0.016	0.146
0.24	0.27	0.18	0.55	-0.14	0.13	0.27	0.60	0.010	0.116
0.33	0.28	0.19	0.53	-0.15	0.14	0.28	0.58	0.008	0.104
0.51	0.28	0.21	0.51	-0.15	0.15	0.28	0.57	0.007	0.094
0.67	0.27	0.23	0.50	-0.14	0.17	0.27	0.56	0.006	0.080
0.84	0.27	0.26	0.47	-0.09	0.22	0.27	0.51	0.004	0.046
1.00	0.27	0.28	0.45	0.00	0.28	0.27	0.45	0.001	0.018

Table I.1c: URAPS predictions

y/δ	$\bar{\tau}$	N_Γ	R_{xx}	R_{yy}	R_{zz}	R_{yz}	λ_{R1}	λ_{R2}	λ_{R3}	Π_b	Π_b
0.10	0.859	0.015	0.24	0.18	0.58	-0.19	0.10	0.24	0.66	0.019	0.162
0.12	0.875	0.013	0.25	0.20	0.55	-0.19	0.12	0.25	0.63	0.016	0.145
0.13	0.888	0.012	0.25	0.21	0.54	-0.19	0.12	0.25	0.63	0.014	0.133
0.15	0.896	0.011	0.26	0.22	0.52	-0.18	0.14	0.26	0.60	0.013	0.124
0.17	0.902	0.011	0.26	0.22	0.52	-0.18	0.14	0.26	0.60	0.012	0.118
0.24	0.910	0.010	0.26	0.23	0.51	-0.18	0.14	0.26	0.60	0.010	0.110
0.33	0.909	0.010	0.26	0.23	0.51	-0.18	0.14	0.26	0.60	0.011	0.111
0.51	0.909	0.010	0.26	0.23	0.51	-0.18	0.14	0.26	0.60	0.011	0.111
0.67	0.920	0.009	0.27	0.24	0.49	-0.17	0.15	0.27	0.58	0.009	0.098
0.84	0.955	0.006	0.29	0.28	0.43	-0.14	0.20	0.29	0.51	0.003	0.055
1.00	1.000	0.000	0.33	0.33	0.34	0.00	0.33	0.33	0.34	0.000	0.000

Table I.2a: DNS data in the core region of non rotating channel (Hoyas and Jimenez, 2006).

$[y^+]$	y^+	y/δ	u^+	$\frac{du^+}{dy^+}$	k^+	ε^+	$\tilde{\Gamma}$	Re^+	$\mathcal{P}/\varepsilon$
30	30.56	0.015	13.49	0.102	5.08	0.083	6.28	312.05	1.09
40	40.39	0.02	14.28	0.065	4.77	0.063	4.98	364.64	0.95
50	51.27	0.03	14.88	0.047	4.54	0.048	4.40	427.23	0.90
75	75.86	0.04	15.79	0.030	4.25	0.031	4.13	576.87	0.91
100	100.16	0.05	16.44	0.024	4.09	0.023	4.17	722.93	0.95
250	252.71	0.13	18.75	0.010	3.56	0.009	4.35	1488.97	1.06
500	505.16	0.25	20.61	0.005	2.93	0.004	4.05	2161.31	1.02
1000	1002.11	0.50	22.61	0.003	1.95	0.002	3.69	2327.16	0.94
1500	1503.02	0.75	23.82	0.002	1.19	0.001	2.65	1777.16	0.56
2000	2004.30	1.00	24.29	0.000	0.85	0.001	0.03	1333.08	0.00

Table I.2 b: DNS data of Table I.2a continued.

y/δ	R_{xx}	R_{yy}	R_{zz}	R_{yz}	λ_{R1}	λ_{R2}	λ_{R3}	Π_b	Π_b
0.015	0.23	0.08	0.69	-0.09	0.07	0.23	0.70	0.030	0.216
0.02	0.25	0.10	0.65	-0.10	0.09	0.25	0.66	0.019	0.173
0.03	0.27	0.12	0.61	-0.10	0.10	0.27	0.63	0.014	0.148
0.04	0.27	0.14	0.59	-0.11	0.12	0.27	0.61	0.011	0.127
0.05	0.27	0.15	0.58	-0.11	0.12	0.27	0.61	0.011	0.124
0.13	0.25	0.16	0.59	-0.12	0.13	0.25	0.62	0.014	0.128
0.25	0.24	0.18	0.58	-0.13	0.14	0.25	0.61	0.015	0.124
0.50	0.25	0.20	0.55	-0.13	0.16	0.25	0.59	0.011	0.104
0.75	0.26	0.23	0.51	-0.10	0.19	0.26	0.55	0.007	0.071
1.00	0.27	0.26	0.47	0.00	0.27	0.27	0.46	0.002	0.026

Table I.2c: URAPS predictions

y/δ	$\bar{\tau}$	N_Γ	R_{xx}	R_{yy}	R_{zz}	R_{yz}	λ_{R1}	λ_{R2}	λ_{R3}	Π_b	Π_b
0.015	0.830	0.019	0.24	0.15	0.61	-0.19	0.08	0.24	0.68	0.025	0.190
0.02	0.857	0.015	0.24	0.18	0.58	-0.19	0.10	0.24	0.66	0.020	0.163
0.03	0.871	0.014	0.25	0.19	0.56	-0.19	0.11	0.25	0.64	0.017	0.150
0.04	0.878	0.013	0.25	0.20	0.55	-0.19	0.12	0.25	0.63	0.016	0.142
0.05	0.877	0.013	0.25	0.20	0.55	-0.19	0.12	0.25	0.63	0.016	0.144
0.13	0.872	0.014	0.25	0.19	0.56	-0.19	0.11	0.25	0.64	0.017	0.148
0.25	0.881	0.013	0.25	0.20	0.55	-0.19	0.12	0.25	0.63	0.015	0.140
0.50	0.891	0.012	0.26	0.21	0.53	-0.18	0.13	0.26	0.61	0.014	0.130
0.75	0.923	0.009	0.27	0.24	0.49	-0.17	0.15	0.27	0.58	0.008	0.095
1.00	1.000	0.000	0.33	0.33	0.34	0.00	0.33	0.33	0.34	0.000	0.000

Table I.3a: DNS data for Spanwise rotating channel in the linear velocity profile region.

a: Wu and Kasagi, 2002, Wu and Kasagi, 2004.

b,c: Alvclius, 1999 (as reported by Grundestam et al. 2008)

d,e,f :Grundestam et al., 2008:

	a	b	c	d	e	f
$Ro \equiv \frac{\delta \Omega_x}{u_b}$	-0.0687	-0.215	-0.385	-0.49	-0.75	-1.03
$Re_b = \frac{u_b \delta}{\nu}$	5378	~3100	~3470	4026	6592	9605
$Ro^+ \equiv \frac{\delta \Omega_x}{u^*}$	-1.25	-3.71	-7.41	-11	-27.5	-55
$Re^+ \equiv \frac{\delta u^*}{\nu}$	295.5	180	180	180	180	180
$\frac{y}{\delta}$	0.7	0.5	0.5	0.25	0.25	0.25
$\langle u'_z u'_z \rangle^+$	1.00	1.1	1.05	1.1	0.6	0.1
$\langle u'_y u'_y \rangle^+$	1.55	3.6	4.9	5.3	6.3	3.6
$\langle u'_x u'_x \rangle^+$	0.95	2.3	3.3	3.9	4.2	2.1
$\langle u'_y u'_z \rangle^+$	-0.67	-0.95	-0.85	-1.05	-0.7	-0.2
k^+	1.75	3.5	4.625	5.15	5.55	2.9
ε^+	0.0045	0.04	0.065	0.11	0.18	0.12
$\frac{du^+}{dy^+} = 2 \left \Omega_x^+ \right $	0.0084	0.04	0.08	0.12	0.31	0.61
$\tilde{\Gamma} = \frac{du^+}{dy^+} \frac{k^+}{\varepsilon^+}$	3.30	3.61	5.86	5.72	9.42	14.77
$\tilde{\Omega} = \Omega_x \frac{k^+}{\varepsilon^+}$	-1.65	-1.80	-2.93	-2.86	-4.71	-7.38
$\frac{\Omega_x}{\Gamma_{yz}}$	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5

Table I.3b: DNS data for spanwise rotating channel in the linear velocity profile region and the corresponding APS predictions for the Reynolds stress components.

		a	b	c	d	e	f
DNS	$\tilde{\Gamma} = \frac{du^+}{dy^+} \frac{k^+}{\varepsilon^+}$	3.30	3.61	5.86	5.72	9.42	14.77
	$\tilde{\Omega} = \Omega_x \frac{k^+}{\varepsilon^+}$	-1.65	-1.80	-2.93	-2.86	-4.71	-7.38
	R_{zz}	0.29	0.16	0.11	0.11	0.05	0.02
	R_{yy}	0.44	0.51	0.53	0.51	0.57	0.62
	R_{xx}	0.27	0.33	0.36	0.38	0.38	0.36
	R_{yz}	-0.19	-0.14	-0.09	-0.10	-0.06	-0.03
URAPS	R_{zz}	0.22	0.21	0.16	0.16	0.11	0.07
	R_{yy}	0.51	0.53	0.60	0.60	0.67	0.72
	R_{xx}	0.27	0.26	0.24	0.24	0.22	0.21
	R_{yz}	-0.18	-0.18	-0.19	-0.19	-0.18	-0.15

Table I.4a: DNS data for Spanwise rotating channel (Wu and Kasagi, 2002, Wu and Kasagi, 2004).

$[y^+]$	y^+	y/δ	p^+	u^+	$\frac{du^+}{dy^+}$	k^+	ε^+	$\tilde{\Gamma}$	$\tilde{\Omega}$
570	560.53	1.90	2.09	12.65	-0.223	2.39	0.026	-21.23	-0.40
560	551.41	1.87	2.09	14.38	-0.162	2.16	0.021	-16.74	-0.43
550	541.20	1.83	2.06	15.79	-0.117	1.99	0.017	-13.92	-0.50
525	515.53	1.74	1.88	17.98	-0.063	1.72	0.010	-10.91	-0.73
500	490.34	1.66	1.62	19.23	-0.039	1.49	0.006	-9.40	-1.01
450	440.45	1.49	1.13	20.42	-0.012	1.15	0.003	-4.62	-1.59
400	390.49	1.32	0.97	20.66	0.001	1.09	0.002	0.24	-1.87
350	340.45	1.15	1.08	20.50	0.005	1.20	0.003	2.34	-1.90
300	295.50	1.00	1.25	20.21	0.007	1.37	0.003	3.07	-1.86
250	250.55	0.85	1.42	19.88	0.008	1.56	0.004	3.27	-1.78
200	200.52	0.68	1.60	19.47	0.009	1.80	0.005	3.25	-1.62
150	150.55	0.51	1.81	19.01	0.010	2.09	0.006	3.25	-1.38
100	100.66	0.34	2.12	18.40	0.015	2.55	0.010	3.67	-1.03
75	75.47	0.26	2.35	17.96	0.021	2.96	0.015	3.91	-0.82
50	49.80	0.17	2.65	17.30	0.034	3.68	0.027	4.57	-0.58
40	39.59	0.13	2.76	16.89	0.049	4.16	0.036	5.48	-0.49
30	30.47	0.10	2.81	16.32	0.080	4.76	0.050	7.47	-0.40

Table I.4b: DNS data of Table I.4a continued.

y/δ	Ret	P/e	R_{xx}	R_{yy}	R_{zz}	R_{yz}	λ_{R1}	λ_{R2}	λ_{R3}	Π_b	Π_b
1.90	222.86	2.53	0.15	0.05	0.80	0.06	0.05	0.15	0.80	0.07	0.33
1.87	219.42	2.43	0.19	0.07	0.74	0.07	0.07	0.19	0.74	0.05	0.26
1.83	234.19	2.26	0.22	0.09	0.69	0.08	0.08	0.22	0.70	0.03	0.21
1.74	296.48	1.84	0.27	0.12	0.61	0.09	0.10	0.27	0.63	0.01	0.15
1.66	356.79	1.46	0.29	0.14	0.57	0.08	0.12	0.29	0.58	0.01	0.11
1.49	432.43	0.39	0.35	0.22	0.43	0.04	0.21	0.35	0.44	0.00	0.03
1.32	481.39	0.01	0.37	0.33	0.30	-0.03	0.29	0.35	0.37	0.00	0.00
1.15	541.56	0.43	0.33	0.41	0.26	-0.09	0.22	0.33	0.45	0.00	0.03
1.00	603.95	0.83	0.29	0.45	0.26	-0.13	0.19	0.29	0.52	0.00	0.06
0.85	658.33	1.10	0.27	0.46	0.27	-0.17	0.17	0.27	0.56	0.01	0.08
0.68	687.73	1.26	0.27	0.44	0.29	-0.19	0.16	0.27	0.57	0.01	0.09
0.51	683.05	1.35	0.29	0.38	0.33	-0.20	0.15	0.29	0.56	0.01	0.09
0.34	622.05	1.48	0.31	0.29	0.40	-0.20	0.14	0.31	0.55	0.00	0.09
0.26	571.60	1.47	0.31	0.24	0.45	-0.19	0.13	0.31	0.56	0.00	0.09
0.17	506.61	1.48	0.30	0.17	0.53	-0.16	0.11	0.30	0.59	0.01	0.12
0.13	478.37	1.60	0.28	0.13	0.59	-0.14	0.09	0.28	0.63	0.01	0.15
0.10	454.12	1.91	0.25	0.10	0.65	-0.12	0.07	0.25	0.68	0.02	0.19

Table I.4c: URAPS predictions

y/δ	$\bar{\tau}$	N_Γ	N_Ω	R_{xx}	R_{yy}	R_{zz}	R_{yz}
1.90	0.723	-0.055	-0.002	0.22	0.06	0.72	0.101
1.87	0.735	-0.044	-0.002	0.22	0.07	0.71	0.110
1.83	0.746	-0.037	-0.003	0.23	0.09	0.68	0.113
1.74	0.760	-0.030	-0.004	0.24	0.12	0.64	0.101
1.66	0.766	-0.026	-0.006	0.26	0.14	0.60	0.083
1.49	0.797	-0.013	-0.009	0.30	0.22	0.48	0.046
1.32	0.854	0.001	-0.011	0.34	0.34	0.32	-0.004
1.15	0.880	0.007	-0.012	0.31	0.46	0.23	-0.071
1.00	0.888	0.010	-0.012	0.28	0.51	0.21	-0.125
0.85	0.894	0.010	-0.011	0.27	0.52	0.21	-0.156
0.68	0.904	0.011	-0.010	0.26	0.52	0.22	-0.183
0.51	0.918	0.011	-0.009	0.25	0.50	0.25	-0.225
0.34	0.924	0.012	-0.007	0.23	0.41	0.36	-0.298
0.26	0.918	0.013	-0.005	0.23	0.34	0.43	-0.306
0.17	0.893	0.015	-0.004	0.23	0.26	0.51	-0.291
0.13	0.866	0.017	-0.003	0.22	0.21	0.57	-0.276
0.10	0.822	0.022	-0.002	0.22	0.15	0.63	-0.250

(P.T.O. Table I.4c contd.)

Table I.4c continued: URAPS predictions

y/δ	λ_{R1}	λ_{R2}	λ_{R3}	Π_b	Π_b
1.90	0.04	0.22	0.74	0.04	0.26
1.87	0.06	0.22	0.72	0.04	0.24
1.83	0.07	0.23	0.70	0.03	0.22
1.74	0.10	0.24	0.66	0.02	0.17
1.66	0.13	0.26	0.61	0.01	0.13
1.49	0.21	0.30	0.49	0.00	0.04
1.32	0.32	0.34	0.34	0.00	0.00
1.15	0.22	0.31	0.48	0.00	0.03
1.00	0.17	0.28	0.55	0.01	0.08
0.85	0.15	0.27	0.58	0.01	0.10
0.68	0.14	0.26	0.60	0.01	0.12
0.51	0.12	0.25	0.63	0.02	0.14
0.34	0.08	0.23	0.68	0.03	0.19
0.26	0.08	0.23	0.69	0.03	0.21
0.17	0.07	0.23	0.70	0.03	0.22
0.13	0.06	0.22	0.72	0.03	0.24
0.10	0.04	0.22	0.74	0.04	0.26

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Table I.5 DNS Results for Developing Homogeneous Shear (Brethouwer, 2005)($\tilde{\Omega}/\tilde{\Gamma} = 0$); $\tilde{\Gamma}_o = 18$

$\tilde{t}$	R_{xx}	R_{yy}	R_{zz}	R_{yz}
0	0.333	0.333	0.334	0.000
2	0.436	0.172	0.392	-0.169
4	0.573	0.091	0.336	-0.157
6	0.654	0.078	0.268	-0.139
8	0.693	0.075	0.232	-0.128
10	0.697	0.089	0.214	-0.127
12	0.663	0.112	0.225	-0.135

Table I.6 Self Similar States for Homogeneous Shear (DNS- Brethouwer, 2005; LES- Bardina et al., 1983 (see Salhi and Cambon, 1997))

	$\tilde{\Omega}/\tilde{\Gamma}$	$\tilde{\Gamma}_o$ (appx)	$\tilde{t}$ (appx)	$\tilde{\Gamma}(8)$ (appx)	R_{xx}	R_{yy}	R_{zz}	R_{yz}
DNS	0	18	12	12	0.210	0.120	0.670	-0.130
	-0.25	18	8	12	0.280	0.370	0.350	-0.320
	-0.5	18	14	12	0.394	0.553	0.053	-0.065
LES	0	3.3	5	-	0.294	0.073	0.633	-0.15
	-0.25	3.3	5	-	0.224	0.383	0.393	-0.35
	-0.5	3.3	5	-	0.344	0.583	0.073	-0.1

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