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AN EMPIRICAL COMPARISON OF TESTS OF THE
HYPOTHESIS OF NO FIXED MAIN EFFECTS
IN THE MIXED MODEL

Thesis for the Degree of Ph. D.
MICHIGAN STATE UNIVERSITY
William G. Darnell
1966



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thesis entitled

AN EMPIRICAL COMPARISON OF TESTS OF THE HYPOTHESIS

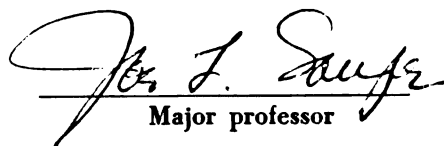
OF NO FIXED MAIN EFFECTS IN THE MIXED MODEL

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ABSTRACT

AN EMPIRICAL COMPARISON OF TESTS OF THE HYPOTHESIS OF NO FIXED MAIN EFFECTS IN THE MIXED MODEL

By William G. Darnell

This study considered the possibility of using an F-ratio statistic, mean square treatment/mean square interaction, to test the hypothesis of no fixed main effects in the mixed model when the variance-covariance matrix for means is nonhomogeneous. The F-ratio and exact T^2 statistic are compared with the appropriate tabled F distribution under the null hypothesis and with each other for power.

A Monte Carlo routine involving a composite random generator, exponential approximation to the normal, and a factorial structure was used to generate the basic score matrix. Several correlation matrices and variance sets were considered. The empirical probabilities of the statistics were compared with tabled F distributions with appropriate degrees of freedom. The empirical power of the two statistics was plotted and compared for several sets of main effects.

Under the null hypothesis the F-ratio proved to be susceptible to dispersion in the correlation matrix and particular sets of variance. The T^2 statistic remained slightly conservative for all comparisons. The liberal effect on the F-ratio proved to be a monotonic function of the amount of dispersion in the correlations. The power comparisons demonstrated the F-ratio to be well behaved and generally more powerful than the erratic T^2 statistic. The T^2 statistic was sensitive to patterns as well as magnitude of the fixed effects when the variance-covariance matrix was nonhomogeneous.

It was concluded that there are only a few circumstances under which the F-ratio should not be considered as an alternative test of the hypothesis. If the intermean correlations are expected to approach 1.00 or if radical variance patterns are expected, the T^2 statistic should be used. In all other cases the F-ratio is recommended although minor modifications of the test may be necessary.

AN EMPIRICAL COMPARISON OF TESTS OF THE HYPOTHESIS
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by
William G. Darnell

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CHAPTER I: THE PROBLEM AND PURPOSE OF THIS INVESTIGATION

Introduction

The mixed model analysis of variance is the appropriate analysis procedure for a wide variety of experimental situations commonly found in the behavioral sciences, education in particular. The mixed model is an analysis of mean differences similar to the common analysis of variance except that one of the categorical variables (factors) is sampled. An example should point out the basic differences between the mixed model analysis of variance and the common or fixed effects analysis of variance.

Consider a situation in which there are 25 schools and 12 age levels and the experimenter wishes to measure some ability and determine whether differences exist as a function of the schools or as a function of the levels or, possibly, if some interaction of schools and levels is the important contributor to the variance existing in the means. If he employed all schools and all age groups in a completely crossed experiment, the results could be analyzed using fixed or common analysis of variance. But it may be economically or practically infeasible to use all of the schools and/or all levels and it may be necessary to sample the schools and/or levels used in the study and to generalize the results to all schools and levels.

If both factors are sampled, that is say ten schools and five age levels are considered, a situation exists which demands components of variance analysis, a random effects analysis of variance model. If only one of the factors is sampled, say schools, and all categories of the other factor are included, the appropriate analysis procedure would

involve the mixed model. The fixed effects or common analysis of variance should be used in this situation only if all possible categories for each factor had been included.

A common application of the mixed model analysis of variance in education is the analysis of profile data. In particular, consider the situation in which the observations take the form of a battery of tests and the problem is to test for differences in group profiles. The tests comprise a fixed, exhausted experimental factor and the individuals or groups of individuals the other. Individuals is the random factor, since we wish to generalize our results not simply to the selected group of individuals under investigation but to all individuals in the population from which they were randomly sampled. (Greenhouse-Geisser 1959)

Consider another example which illustrates an application of the mixed model. Suppose an experimenter is interested in the effect of three different tasks on a subject's ability to learn lists of paired associates in a verbal learning study. Recognizing that the variance in individuals on such tasks is large, the experimenter controls for individual differences by using the same set of subjects for each treatment application. Furthermore, the experimenter is aware that some systematic trial related change may occur. In an attempt at further control he measures each subject three times on parallel forms of the dependent variables after each task is performed. The order in which the tasks are assigned each subject is randomized and the subjects are selected at random from some specified population. The tasks are the fixed factor in this study since three and only three are included in the experiment. The subjects comprise a random factor since the experimenter wishes to generalize

his results to some population from which the subjects can be considered a sample. Since a fixed and a random factor are employed in the above study, analysis must be performed under the assumptions of the mixed model.

These few examples demonstrate the wide applicability of the mixed model and illustrate the basic characteristics of an experiment which would fit the mixed model. It has two factors. (Generalizations to more than two factors are available. See Scheffe 1959, pp. 275-289.) One of the factors is fixed; it exhausts all possible values of the factor, and generalizations from the data will not be made beyond the categories investigated. The other factor is a random factor, the levels of the factors are a random sample of those to which the results will be generalized.

The Model

The statistical model for the mixed model as developed by Scheffe (1956a) contains a restriction and an assumption which are of interest. The assumption, basic to most parametric statistics, is that errors are independent. The restriction permits one to use an F-ratio of mean squares to test the hypothesis of null fixed treatment effects only when the variance-covariance matrix of treatment means over levels is highly symmetric.¹ It is the restriction that has received major attention in this investigation.

¹ A highly symmetric matrix is one in which all variances are equal and covariances are equal.

Scheffe (1956a) assumes that the k'th unit observation of the ij'th cell of the score matrix is represented by the structure

$$(1.1) \quad y_{ijk} = m_{ij} + e_{ijk},$$

where:

(1.2) the "errors" $\{e_{ijk}\}$ are independently distributed with zero means and variance σ_e^2 , and are independent of the "true" means $\{m_{ij}\}$.

M_j is a vector random variable on an I variate multinormal distribution with variance-covariance matrix, V, and mean vector,

$U = (\mu_1, \mu_2, \dots, \mu_i, \dots, \mu_I)$. μ_i is defined to be the mean of the i'th component, $\mu = \mu$. and $\mu_j = \sum_i m_{ij} / I$. The following effects are defined

$$\alpha_i = \mu_i - \mu.$$

$$b_j = \mu_j - \mu.$$

$$c_{ij} = m_{ij} - \mu_i - \mu_j + \mu.$$

The "true" mean m_{ij} is the non-error portion of equation (1.1) and may now be represented as

$$(1.3) \quad m_{ij} = \mu + \alpha_i + b_j + c_{ij}.$$

The notion of "true" mean will be important in subsequent discussions.

The restriction under consideration involves the true mean m_{ij} . For a specific i, m_{ij} is a random variable with expected value, μ_i and a variance σ_{ii} which is a function of the variance of the random effects b_j and c_{ij} as well as the covariance of these two variables, $\text{Cov}(b, c_i)$. Similarly for i', $m_{i'j}$ is a random variable with some variance $\sigma_{i'i'}$, and expected value $\mu_{i'}$. The restriction requires that

$$(1.4) \quad \text{for all } i, (i = 1 \dots I) \quad \sigma_{ii} = \sigma^2$$

and

$$(1.5) \quad \text{for } i \neq i' \quad \sigma_{ii'} = \text{Cov}(m_i, m_{i'}) = \rho\sigma^2$$

where ρ is the population correlation coefficient. That is, the covariances for all intertreatment sets of means are equal and for all treatments the variance of the means within the treatment category over levels must be constant.

When the errors are uncorrelated and conditions (1.4) and (1.5) are satisfied the analysis is quite straight forward and requires no difficult computation. $F = \text{mean square fixed effects} / \text{mean square interaction}$ is the appropriate test of the hypothesis of null fixed effects. But this is seldom the case since correlations among the cell means, m_{ij} , are quite apt to exist. It is absurd to assume that these intertreatment correlations will be equal in all situations involving the mixed model. As a matter of fact, unequal correlations will be the usual case. When the intermean correlations are unequal the F-ratio is no longer exactly distributed as F in spite of the independence of the two mean squares and identical expected value under the null hypothesis (Scheffe 1956a, p. 32). When (1.4) and (1.5) are not satisfied the exact test of the hypothesis is multivariate. Hotelling's T^2 (Hotelling 1931), as recommended by Scheffe, or the more general multivariate analysis of variance (Rao 1952), could be used here.

Discussion

The multivariate procedures provide theoretically exact tests but are cumbersome and unwieldy when the calculations are attempted. Calculation of these statistics involves matrix inversion or, at best,

the calculation of several determinants and requires the use of a high speed computer when the number of treatments and hence the dimension of the codeviance matrix exceeds two. Further, the use of multivariate methods, in particular the employment of mean square ratios and multivariate procedures in the analysis of one set of data, is an unfamiliar procedure. The analysis of variance procedure remains appropriate for testing the nullity of the levels and interaction effects. The exact test is therefore avoided by many researchers familiar with common analysis of variance. Even when multivariate tests are attempted by the researcher the general problem of interpreting high powered statistics leaves the user in a quandary.

The multivariate test has other limitations. It requires that the number of categories of the random factor exceed or at least equal the number of categories of the fixed factor. Imhoff (1962) has indicated that the T^2 statistic has low power in general and particularly low power when the number of levels is close to the number of treatments. This low power should be expected since the degrees of freedom associated with the denominator of the test are $(R - C + 1)$, a very small number when R and C are close to being equal. For small degrees of freedom in the denominator, the statistic F has a very large standard deviation, and for fixed 'alpha' power is inversely related to the size of the standard deviation. Further, "If the approximate test is used and the hypothesis is rejected we could follow it with an approximate S or T method of multiple comparisons..." (Scheffe 1959, p. 271).

Computational facility, familiarity, and ability to handle cases not fitting the multivariate statistic make the analysis of variance procedure a very desirable method. The problem as noted before is that

the F-ratios do not have an exact F-distribution when conditions (1.4) and (1.5) are not satisfied. Errors will be incurred if the analysis of variance procedures are used to test the hypothesis of null treatment effects when there exist non-equal correlations between treatment means. Scheffe (1956a, 1959) indicated that "...it is not clear at present whether in practice the use of this exact test instead of the approximate F-test...based on referring MS_A/MS_{AB} to F-tables with $I-1$ and $(I-1), (J-1)$ d.f. is worth the extra computational labor involved." (1959, pp. 270-271) He also stated after a discussion of the simplicity of the F-ratio and the possibility of its use as an approximate test that "A justification of this would be welcomed by the practitioner, because the computations are simpler and more familiar than those with Hotelling's T^2 , but until numerical investigations² are made which indicate the errors involved are tolerable, the practice should be suspect in the present case." Such a numerical investigation is not in evidence in the literature.

If conditions (1.4) and (1.5) are satisfied, that is if the means for any treatment over levels have equal variances and there is mutual independence across treatments or, at worst, equal correlations, the test of the hypothesis of null fixed treatment effects is a ratio of mean squares. This ratio is exactly distributed as F with appropriate degrees of freedom much like the test for common or fixed effects analysis of variance. The computational formulas employed in obtaining the mean square estimates are identical to those of the fixed model.

² Italics added.

This procedure is easy to use and involves no difficult analysis procedures and may tempt the practitioner to employ the test even when the conditions are not satisfied. Sensing this possibility Scheffe sounds the following note of caution, "We do not recommend that the assumption ...ordinarily be made in applications, where there usually exists no real symmetry corresponding to it." (Scheffe 1959, p. 264)

The Problem

Given that the data fits the mixed model but the intertreatment correlations and/or variances in the means are unequal, a common situation, what procedures does the experimenter follow for analysis? Does he use one of the theoretically exact but cumbersome multivariate methods? Does he use the familiar, arithmetically simple but questionable analysis of variance? Both of the above are with varying degrees of justification available in the literature, but the question remains unanswered since there has been no serious attempt at validation. Information must be provided to reduce the problem of decision. It is the problem of providing such information which gave rise to this investigation.

The Purpose

It was the purpose of this investigation to determine whether the errors incurred by using the univariate F test in this special case of the mixed model are tolerable.

This study was designed to compare the F-ratio and T^2 tests on the basis of power for a few selected sets of conditions as well as to investigate the deviation of the distribution of mean square ratios

under the null hypothesis from what would be the exact distribution if there were equal correlations and variances in the means for treatments over levels.

Comparisons were empirical, based on a large number of samples from specified populations. The data were generated by means of a Monte Carlo sampling technique on a Control Data Corporation 3600 computer. The procedure and the conditions employed will be discussed in the following chapter.

CHAPTER II: CONDITIONS UNDER INVESTIGATION AND METHODS OF DATA COLLECTION

Introduction

The object of the investigation is not to support or invalidate a theory, but to obtain useful information which can guide the working statistician when he suspects that he is outside theory. As noted earlier, it is often doubtful whether real data collected from real subjects actually fits the assumptions of the statistic desired. In the case of the mixed model seldom, if ever, will the experimenter be able to state conclusively that conditions (1.4) and (1.5) are met since whenever levels and interaction effects exist the intertreatment mean variances and covariances may not be equal. The basic question therefore is, "What happens to the distribution of the two statistics under consideration when this restriction is violated?" Further, might there exist other conditions in the data which will effect the seriousness of this violation?

It might be helpful to clarify these basic questions and the problem at hand with a parallel example from the fixed effects model which has been thoroughly investigated. Suppose one was asked, "In the fixed effects model analysis of variance, what is the effect upon the results of the analysis if the assumption of normally distributed errors with equal variance in all cells is violated?" Without any difficulty one would answer, possibly citing Scheffe (1959), Norton (1952), or a reliable text like Hays (1963), that violation of these assumptions is not serious if one has sufficiently large sample sizes and, the sample sizes are equal. Notice that the statement concerning the tolerability of errors is qualified with a statement

of conditions which are related to, yet external to, the assumption. In an attempt to discover whether such qualifying conditions exist in the case under consideration certain conditions external to the assumptions, yet felt to have possible bearing on the tolerability of errors when the assumptions are violated, were included. The basic question was investigated by the inclusion of several cases of unequal correlation.

What conditions should one impose on the data? Of the many variables which could affect the distribution of the F-ratio, which ones should be investigated? To what extent should each variable selected be scrutinized? These are the problems of this section.

It was difficult to rank the variates in order of importance or even decide upon which should be included and which should not be included in this initial investigation. It was decided to include as many variates as time would permit, each variate to be crossed with the other variates under consideration in an attempt to discover interactions which might affect conclusions about the tolerability of errors. Over 100 sets of conditions were considered in the investigation of the distributions of the F-ratio and T^2 under the null hypothesis. The power comparisons, while involving fewer actual sets of conditions, involved over 100 cases since 10-14 sets of means were used in each comparison. It is felt that the data presented here permits a substantial first look at the problem of the mixed model in the case of non-symmetric variance-covariance matrix of the means.

Those variates selected for investigation are presented below. Following the initial presentation of the variates, a brief discussion of each is included.

Questions for Investigation

1. Does the inequality of correlation (Covariance) between means over levels introduced intolerable differences between the actual distribution of F-ratios and the tabled F-distributions with $(I-1)$ and $(I-1)(J-1)$ degrees of freedom? Does it affect the power relationship of the F-ratio and T^2 based tests?

2. Does the lack of homogeneity of variance in the means introduce intolerable differences between the actual distribution of F-ratios and the tabled F-distribution with $(I-1)$ and $(I-1)(J-1)$ degrees of freedom? Does it affect the power relationship of the F-ratio and T^2 based tests?

3. Is the number of units per cell a factor influencing the distribution of the F-ratios when the symmetry assumption is not satisfied? Does it affect the power relationship of the F-ratio and T^2 based tests?

4. Is the number of levels a factor influencing the distribution of the F-ratios when the symmetry assumption is not satisfied? Does it affect the power relationship of the F-ratio and T^2 based tests?

5. Does the level of significance chosen affect the power relationship of the F-ratio and T^2 based tests?

Attached to each of the above questions is a corollary. If the variable affects the distribution or the power of one of the tests, what is the magnitude and direction of the effect?

After having observed the partial answers to the above questions, the most important question of all must be answered. What are the implications of this collection of facts for the practitioner, the user of statistics? From the combination of data bearing on the distribution of the test statistics under the null hypothesis and the information gleaned from the power comparison can it be determined whether the errors incurred by violation of the symmetry assumption are tolerable?

Specific Conditions Considered

In this investigation three treatments have been considered. Initially it was the intention of the investigator to include more

than three treatments and cross number of treatments with all other factors under consideration. This intention was not followed up for two reasons. First, time became prohibitive. Each time a new treatment total is applied the number of cases to be investigated increases substantially. Secondly, considering number of treatments other than three loses meaning if the effect of number of treatments cannot be analyzed for a given covariance matrix or set of covariance matrices that can be considered equivalent. Similarly equivalent sets of variances must be available. The same variance-covariance matrices cannot be considered for sets of three, four, and five means.

Correlations -- This is the most important question asked in this investigation. In order to determine the effect of unequal correlations in the treatment means upon the distributions of the F-ratio several sets of correlations were considered. An attempt was made to look at both the magnitude and dispersion of the correlations. The correlation matrices included in this investigation are identified in table 2.1. Four sets of correlations might be considered the substructure of this factor since they were crossed with almost every other combination of factors in this study. They are identified below as Low, Medium, High, and Disperse I. It was from these correlation matrices that the basic data indicating effects of magnitude and amount of dispersion in the correlations was to be obtained. The effect of the second aspect of this dual problem was further investigated by the inclusion of the matrix identified below as Disperse II. The case of independent means, no correlation, and several cases of equal correlation, were also included in

order to check out the contention of the theory that the F-ratio is an acceptable test if the correlations are equal. Further, two sets of data were included which involved negative correlations. The case of perfectly correlated means was also included in expectation that it might provide unique considerations. Correlation matrices, Medium and Disperse I were included in the power comparison.

Variance -- Homogeneity of variance in the means is the lesser part of the assumption which has given rise to this investigation and hence is interesting in its own right. In addition the crossing of the variance vectors and the correlation matrices provides an opportunity for isolating covariance.

For almost all cases of the other variables, two sets of variances were considered in order to determine whether homogeneity was a significant factor. In the case of three treatments the variances imposed on the treatment means were (100, 100, 100) in the case of homogeneous variance and (225, 100, 25) to establish the case of unequal variance. Other sets of variances were considered in order to investigate problems of unequal variance independent of the other factors. The sets of variances considered for this purpose were, (assigned respectively to treatments as were the above cases) (25, 100, 225); (00, 100, 00); (100, 100, 00); (2500, 10000, 25000); and (10000, 00,00). For the case of four treatments similar sets of equal and unequal variances were considered. (100, 100, 100) and (225, 100, 25) were compared for their affect on the power of the statistics.

Table 2.1 - Correlation Matrices Included in This Investigation

Name	Correlations									Covariance When V= (100, 100, 100)			Covariance When V= (225, 100, 25)			Direct			Tanh ⁻¹ Transformation		
	1,2	1,3	2,3	1,2	1,3	2,3	1,2	1,3	2,3	1,2	1,3	2,3	1,2	1,3	2,3	μ_p	σ_p	$\mu_{z'}$	$\sigma_{z'}$	μ_p'	σ_p'
Low	.35	.21	.15	.35	.21	.15	.35	.21	.15	.52	.16	.8	.24	.09	.24						
Med.	.63	.54	.42	.63	.54	.42	.63	.54	.42	.95	.40	.21	.53	.09	.60						
High	.86	.74	.63	.86	.74	.63	.86	.74	.63	.128	.55	.32	.74	.09	.99						
Disperse I	.88	.48	.16	.88	.48	.16	.88	.48	.16	.132	.36	.8	.51	.29	.69						
Disperse II	.50	.25	.75	.50	.25	.75	.50	.25	.75	.75	.19	.38	.51	.20	.59						
EQ (.25)	.25	.25	.25	.25	.25	.25	.25	.25	.25	.38	.17	.12									
EQ (.50)	.50	.50	.50	.50	.50	.50	.50	.50	.50	.75	.38	.25									
EQ (.75)	.75	.75	.75	.75	.75	.75	.75	.75	.75	.112	.56	.38									
EQ (1.00)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.150	.75	.50									
P/N I	.88	-.48	-.16	.88	-.48	-.16	.88	-.48	-.16	.132	-.36	-.8									
P/N II	-.50	.25	-.75	-.50	.25	-.75	-.50	.25	-.75	-.75	.19	-.38									

Number of Units Per Cell -- Two and ten observations per cell were the usual cases of this factor considered in this study. All cases of the other variables were compared for ten observations per cell. Two and five observations per cell were considered for the purpose of replication and to confirm an expectation that this variable would not affect the F-ratio. The interesting case of one observation per cell was considered for a limited number of cases. For those covariance matrices which resulted in divergence from tabled probabilities additional analysis were performed using 25 observations per cell. For power comparisons, 2, 5, and 10 observations per cell were considered.

Number of Levels -- The usual case in this analysis involved five levels although liberal consideration was given to the case of three and ten levels for those variance and covariance matrices which indicated a possible serious discrepancy between the actual distribution of the F-ratio and the tabled values. From the inclusion of these three cases it was expected that any pattern existing would appear. Number of levels was expected to affect the power of the statistics since it directly influences degrees of freedom. Three, five, and ten levels were considered for the case of independent correlation.

Levels of Significance -- All of the conditions above were compared for "alpha" equal to .10, .05, .025, .01, .001.

Effects -- All power comparisons involved 10 to 14 sets of means representing different noncentrality parameters. The sets of means considered are listed in table 2.2 along with their sum of squares. Care was taken to select mean vectors which were linearly independent.

Table 2.2 - The Fixed Effects Considered for Power Comparisons

Mean Vector ($\alpha_1, \alpha_2, \alpha_3$) α_i = Effect of i'th Treatment	σ_i^2
(.75, -.05, -.70)	1.1
(-2, .25, 1.75)	7.1
(3, -1, -2)	14
(4, -1, -3)	26
(5, 0, -5)	50
(7, -5, -2)	78
(9, -1, -8)	146
(13, -1, -12)	314
(19, -13, -6)	566
(20, 0, -20)	800
(24, -22, -2)	1064
(30, -20, -10)	1400
(+50, 0, -50)	5000
(75, -10, -65)	9945

Procedure

This study involved the calculation of a large number of F-ratios and T^2 based F-statistics (1,000 for each set of conditions in both the power and null hypothesis phase of the investigation) based on samples from population distributions with specified characteristics. Following generation the resulting empirical distributions of the F-ratio and T^2 based F were compared with each other and with the tabled F-distribution with appropriate degrees of freedom. For the power comparisons the power of the statistics was determined for selected sets of mean differences for each of the conditions considered. The power of the two statistics was then plotted and inspection of the power curves and the tabled power values used to determine the relative merit of the two statistics. Results of the two phases were merged in order to draw some conclusions concerning the tolerability of errors incurred under the null hypothesis.

Samples were generated and the F and T^2 statistics were calculated on a Control Data Corporation 3600 computer at Michigan State University's Computer Center. The procedure described in parts 1, 2, and 3 below is commonly called the Monte Carlo technique. The unfamiliar reader may wish to consult Green (1963), Guetzkow (1962), or Kahn (1956).

1. Generation of uniformly distributed random numbers. Although the term random number is used here it should be noted that the numbers are truly pseudo-random numbers since they have a finite period and can recycle. This first step of the procedure is extremely important to the outcome of the investigation. If an inappropriate generator is selected the results will be invalidated. There are three popular computer based techniques for generating pseudo-random numbers (Green 1963, pp. 163-164). An additive technique which involves adding the preceding random number to a random number generated earlier and retaining a fractional part of the resulting sum as the new random number. The representational formula of this method would be

$$(2.1) \quad x_j = x_{j-1} + x_{j-n} \pmod{1}$$

where x_j represents the random number being generated. The modulus value usually represents the capacity of a machine register. Mod 1 is purely symbolic. It indicates that a remainder is saved as the random number.

The multiplicative method of random number generation uses the preceding random number as the basic unit. This random number is then multiplied by a constant C and a fractional part of the product is kept as the new random number. This procedure may be symbolically represented as follows

$$(2.2) \quad x_j = Cx_{j-1} \pmod{1}$$

Where again x_j represents the new random number.

The composite method involves a multiplicative and an additive operation in the generation of new instances. The preceding random number is multiplied by a preselected odd integer C and an odd constant or another random number x_{j-2} is added to this product. As in the case of the above generators a remainder is kept as the new random number x_j . The symbolic formula for this routine would be

$$(2.3) \quad x_j = Cx_{j-1} + x_{j-2} \pmod{1}$$

Other methods such as entering a table or using an electronic roulette wheel have been attempted as random number generators. These techniques have met with little success since most mechanical devices have biasing irregularities and tables lack completeness (Brown 1951). Such procedures predated the use of the high speed computing machine and have been replaced by modern computerized procedures such as those discussed above.

Empirical studies by Green, Smith, and Klem (1959) have indicated that many additive routines exhibit serial correlations in the numbers generated. When the number of random numbers used to start the routine is small, the data generated by an additive routine failed to pass the "runs test." For larger samples of starter numbers the runs test was not significant. In order to protect against serial correlations, large sets of starter numbers are required each time the generator is employed. The additive generator was not used since serial correlations could appear and confound the results of the study. Further, thousands of starter sets would be required. It was decided that one of the faster multiplicative or safer composite routines should be employed even though Green (1963, p. 167) notes that "...the runs test is very sensitive and failure to pass this test does not mean the numbers are badly awry."

Green (1963, p. 165) recommends the use of a multiplicative generator for obtaining random numbers on the basis of empirical tests by Greenberger (1961) which indicate that the multiplicative generator passes all of the tests of randomness including the runs test. Further, it has been found to have a period of 2^{33} , over eight billion numbers. It is a faster routine than the composite generator discussed below and has a period which is shorter although this seems unimportant since its period as noted is tremendous. This method was not incorporated in this study for reasons to be discussed below, but is very desirable because of its large period and relatively short computation time.

The composite generator introduced by Rotenberg (1960) was selected as the method of random number generation to be used in this investigation. It has all of the desirable characteristics of a random number generator and an extremely large period of 2^{35} . That is, it generates all the numbers which can be represented by 35 bits before cycling, over 32 billion random numbers. Green's (1963, p. 168) only criticism of this generator is that it takes more time than the multiplicative generator. This is a valid criticism and one which would have caused the investigator to select the multiplicative generator if another aspect of the programming had not required otherwise.

All random number generators require at least one starter number, and in the case of the additive generator n such numbers are required. The selection of starter numbers can be a problem if the program must be used many times in a given study. Several common practices are followed in the selection of these starter numbers. Random numbers

might be selected from a table of random numbers and fed into the machine or, in order to make the program self contained, an internal figure such as the date might be used. The computer's time clock was used in this investigation since a self contained program was desired and the time clock reading in millionths of a second would feed the same number into the generator only one time in many million runs. If the same number is used on two runs the exact same set of random numbers will be generated.

The use of the time clock incurs difficulties when one uses a multiplicative generator. What would happen to the multiplicative generator if the clock had just turned over and it read all zeros or the majority of the dials were zero? The results would be consecutive generation of zeros, not very random. Further, is it not possible that one of the random numbers generated by the routine will be zero? The composite generator employed protects against this difficulty by providing a constant which is added to the starter number of the preceding random number. For this reason the composite generator was selected in preference to the faster multiplicative generator. It was preferred to the additive generator because of its tendency to generate data with serial correlations built in if $n \leq 16$ starter numbers are employed and the difficulty of selecting large sets of starter numbers for each of the over 200,000 runs employed in this study.

2. Once generated the uniform random numbers were transformed into random deviates from a distribution closely resembling a normal distribution with mean zero and a standard deviation of one by means of a logarithmic function which is the inverse of an exponential

approximation to the intractable cumulative normal distribution function. The exponential approximation to the normal employed was offered by Kahn (1956, p. 43) and is given below.

$$(2.4) \quad f(x) = Be^{Bx} / (1 + e^{Bx})^2 \quad B > 0,$$

where x has mean 0 and variance $\pi^2/3B^2$. The transformation employed was the inverse of (2.4)

$$(2.5) \quad x = F^{-1}(R) = -1/B \cdot \ln (1/R - 1) \quad B > 0,$$

where R is the generated random number and $1/B = \sqrt{3}/\pi = .568234601$.

The value of $1/B$ was selected since it generates data which is near normal with a mean, median, and mode near zero and a variance of one. The approximation function (2.5) was thoroughly tested before it was employed. The distribution of 10,000 scores was found to be slightly non-normal when compared with theoretical probabilities using the χ^2 goodness of fit test. This minor difference ($\chi^2: p < .10$ for $n=10,000$) has little practical significance since any statistic susceptible to such minor violation of the normality assumption could never be employed with real data (see figure 2.1). There existed the possibility that the transformed variables might no longer be random. Hence, the 10,000 scores were subjected to the runs test in order to double check the generator as well as the transformation. The obtained Z for the large sample runs test was .0175 indicating that there existed no serious positive or negative cycles in the data.

3. The generation of the score matrix from which the statistics under consideration were calculated involved two steps. First, an $L \times C$ matrix of means was generated, L being the number of levels and C being the number of treatments under consideration. C sets of L means were generated in such a way that they represented samples of

size L from a C variate population with specified variances and covariances. For the null-hypothesis considerations, each of the populations had mean zero. Hence, any differences in the means of the C samples would represent chance treatment effects only. For the power comparisons the vector of treatment effects was added to the means of each treatment group at this point.

The desired intertreatment correlations were generated by means of linear transformations which parallel the theory of factor analysis (Harmon, 1965). Specifying the coefficients to be used in the linear transformation is identical to selecting the matrix of factor loadings in the factor analysis model. The population intercorrelations are completely determined by these factor loadings and can be represented as a product of the matrix of factor loadings and its transpose.

If we let M represent the matrix of means, A the matrix of pre-specified factor loadings, and Q the matrix of factor scores (random $N(0,1)$ deviates), then it is well known that $M = AQ$ and $R = AA'$ where R is the matrix of intermeasure or in this case intertreatment correlations. The result of this transformation is an $L \times C$ matrix of scores with the following properties L = number of levels, n = number of observations per cell ($n = 1$ when generating means), C = number of treatments. The i 'th ($i = 1 \dots C$) column of this resulting matrix represents the means for the first treatment over levels. The values in each of the L rows represent the means for the L levels over treatments. That is, element a_{ji} is the mean for level j and treatment i . The intercolumn correlations are specified by AA' . The variance of the means within any treatment group are also specified by the

coefficients selected for the linear transformation. The routine was checked by generating several score matrices and correlating the observations by means of an independent correlation program.

The means having been generated it was then necessary to generate n error units per cell in order to complete the score matrix. The error units in this investigation are assumed to be independent observations from normal populations with mean zero and variance one. Hence, for each error term desired it was only necessary to generate a random uniform deviate and transform it by means of equation (2.5) into a random deviate from the desired normal distribution. Once generated the error term was added to the mean for a given cell. This process was repeated n times until the cells of the score matrix were filled. Each observation y_{ijk} can be represented therefore as a linear function of its "true mean", m_{ij} , and an error term, e_{ijk} .

4. The F and T^2 statistics were calculated using scores of the data matrix. The calculation formulas for the mean squares of the F -ratio are identical to those of the common univariate analysis of variance. The calculation formulas for the T^2 based test are similar to those found in Hotelling's original publication (1931) or presented with example by Rao (1952, pp. 237-246). The specific method to be followed for the mixed model is presented by Scheffe, (1956a, 1959).

5. As noted in section 3 of this discussion, the desired theoretical fixed treatment effects were added directly to the cell means when they are required for power comparisons. Care was taken when selecting the means to select mean vectors which were linearly independent.

6. The empirical distributions resulting from the analyses were hand tabulated initially in order to check on the program. In the later phases of the investigation a sub-routine was added to the basic program which automatically tabulated the values of the statistics as they were produced.

In summary, random samples from normally distributed populations with specified conditions imposed on the means were generated. The F-ratio and T^2 statistics were then calculated for each of 1,000 sets of data for each set of conditions. The resulting empirical distributions were then tabulated for values of special interest and appropriate comparisons made.

The results obtained from this investigation and a discussion of these results are presented in the following chapter. The basic data are frequency counts for values of the statistics specified by the theoretical distributions.

CHAPTER III: PRESENTATION AND ANALYSIS OF RESULTS

Introduction

This investigation concerns the tolerability of the errors incurred when the F-ratio statistic is used to test the hypothesis of null fixed treatment effects under the mixed model instead of using the theoretically exact T^2 based F statistic when conditions (1.3) and (1.4) are not satisfied. The question is not one which asks whether the distribution of the F-ratio is identical to the distribution of F with $(I-1)$ and $(I-1)(J-1)$ degrees of freedom; it is expected that differences do exist. Hence, a statistical test of no difference or goodness of fit is not appropriate. The analysis of the data is, for the most part, logical and based on the assumption that the observed empirical probabilities closely approximate the true but unknown probabilities.

For each set of conditions the distribution of the F is approximated by 1,000 F-ratios and 1,000 T^2 based statistics. The resulting probabilities estimates are very accurate for all points of the corresponding distribution and are particularly good when large or small probabilities are estimated. In the majority of the cases considered, the probabilities of interest are either large or small.

The one situation in which moderate probabilities are of interest is in the case of power comparisons. But in this case the moderate probabilities will occur between the points of inflection of the power curves where the curves are relatively stable and have a high positive slope. Between the points of inflection the power

curves are almost straight lines when plotted on semi-log paper against the noncentrality parameter or the sum of squared fixed effects. (The graphs reported in this investigation are on 4 cycle by 10 to the inch semilogarithmic graph paper.) Since the curves are expected to be straight, any deviations from the true probability between the inflection points would be very much in evidence.

The standard error of estimating a true probability is a function of the probability being estimated and the size of the sample from which the probability is estimated. For all sets of conditions considered in this study the number of observations in the sample is large and constant, $N = 1,000$. The other variable affecting the standard error of estimate, size of probability being estimated, will in general be either very large, greater than .90, or very small, less than .10. The most serious difficulty in estimating the true probability as noted above will be in the case of power comparisons where the probability being estimated is near .50. Below is tabled the standard error of estimating the probability for several true probability values.

Table 3.1 - Standard Error of Estimating P for
Several True Probability Values

True Probability (P)*	Standard Error of Estimating the True Probability (N=1,000)
.01	.00316
.05	.00686
.10	.00949
.20	.01265
.30	.01449
.40	.01549
.50	.01581

*If $P > .50$ use $1-P$ to determine the standard error of the sampling distribution.

It should be noted that the poorest case, $P = .50$ has a standard error of only .01581. This means that if the true probability were .50 over 99 percent of the observed values would fall between .4526 and .5474 and over 95 percent of the values would fall between .4684 and .5316. This situation improves rapidly as the true probability tends away from .50. For the case of $P = .90$ we have 99 percent of the scores falling between .87153 and .92847, and over 95 percent of the observed probabilities falling between .88102 and .91898. Hence, whatever the true probability, the estimates in the following tables can be considered close approximations.

Probabilities Under the Null Hypothesis

Introduction

Probability points of interest were calculated for a variety of treatment mean intercorrelations. The case of three treatments was of primary interest and the number of levels and observations per cell were varied. Within treatment variances were also varied with interest centering on the following sets of standard deviations; (10, 10, 10) and (15, 10, 5). In addition, some special cases were considered for their unique interest. Several cases were replicated in order to verify results and to check upon the reliability of the estimates.

The conditions considered and the resulting probabilities are presented in Table I of Appendix A. Due to the variety of conditions the table may prove cumbersome when first inspected but should be clarified by the discussion in this chapter.

For each set of conditions the estimated true probability and the tabled probability are compared for the following probability points: $\{ F \mid P(f > F) \leq .10; .05; .025; .01 \}$. The probability in the appendix table is the proportion of empirical values of the statistic which exceed the value of the statistic normally associated with a given probability. For each run, column 1 specifies the number of levels, treatments, and units considered in that run. Column 2 specifies the variances for the means of respective treatments over levels. Columns 3 and 4 specify the covariance and correlation of the means. (In Columns 3 and 4, 1, 2 identifies the correlation or covariance for the means of treatment groups one and two.) Columns 5 and 6 are the empirical probabilities of the two statistics compared against the tabled probabilities.

Results

Correlation

Is the magnitude of the correlations ~~or~~ the amount of dispersion of the correlations a factor influencing the acceptability of the F-ratio as an approximate test of the hypothesis of no fixed main effects in the mixed model? Using three treatments, five levels and ten observations per cell as a base, twelve sets of correlations with varying signs and magnitudes were considered.

Magnitude -- In runs 53, 54, and 55 the size of the correlations are increased while holding dispersion in the coefficients fairly constant. Low and medium correlations do not have a liberalizing

effect on the F-ratio. The set of high correlations appear to have a slight effect which is probably a result of chance rather than true effect. When the other runs in which these correlation matrices are considered are inspected, it becomes clear that if an effect exists it is at best minor. Table 3.2 below summarizes the runs in which these matrices were crossed with equal variance. Less than 3 percent of all of the F-ratio probabilities considered were more than two standard deviations away from the tabled probability; none were greater than three standard deviations away. Further, an inspection of table 3.2 shows very little difference in the F-ratio and T^2 based statistic.

Table 3.2 - The Average Emperical Probability Associated with Correlation Sets High, Med and Low when Variances are Equal

Correlation Matrix	F-Ratio: $\alpha =$				T^2 : $\alpha =$			
	.100	.050	.025	.010	.100	.050	.025	.010
High	.104	.054	.028	.012	.098	.047	.023	.008
Med	.092	.048	.028	.012	.099	.053	.027	.010
Low	.096	.044	.021	.009	.097	.049	.022	.009
Most Liberal Run								
Run 96 - High	.118	.061	.032	.016				

Spread or Dispersion -- In runs 54, 56, and 57 the average magnitude of the correlation array is held constant and the dispersion in the coefficients is varied. These matrices have been identified as Med, Disperse II an Disperse I for the sake of discussion (see table 2.1).

These three matrices have approximately the same average magnitude, approximately .50. The dispersion in the coefficients as indicated by a transformed variance coefficient is as follows: Med - 12, Disperse II - 30, and Disperse I - 51.

An inspection of the empirical probabilities in the main table indicated that as dispersion increases the F-ratio becomes increasingly liberal. Run 54, as indicated earlier, indicates that matrix Med has little effect on the distribution of the F-ratio. Disperse I and Disperse II, runs 56 and 57, have a definite positive bias on the distribution of the F-ratio. For each set of conditions considered Disperse I and Disperse II continue to affect the statistic. The size of the effect is related to the amount of dispersion. Disperse I is most spread and has the greatest effect. For medium dispersion none of the F-ratio probabilities were more than two standard deviations above the tabled probability. The effect of Disperse II falls between that of Med and Disperse I. The T^2 statistic does not reflect the effect of disperse correlations.

Table 3.3 summarizes the runs which involve these matrices when the variances are equal.

Table 3.3 - The Average Empirical Probability Associated with Correlation Sets Med, Disperse I, and Disperse II When Variances are Equal

Correlation Matrix	F-Ratio: $\alpha =$				$T^2: \alpha =$			
	.100	.050	.025	.010	.100	.050	.025	.010
Disperse I	.132	.079	.048	.023	.098	.049	.026	.012
Disperse II	.122	.062	.036	.016	.096	.048	.025	.009
Med	.092	.048	.028	.012	.097	.049	.022	.009

Most Liberal Run

Run 42-Dis.I. .147 .089 .059 .027

Equal Correlation -- The above discussion of dispersion in the correlations almost precludes the need for discussing the problem of equal correlation. In this case dispersion is at a minimum and the resulting lack of effect on the distribution of the F-ratio follows logically. The full range from EQ(.00) through EQ(1.00) was considered and no effect was found as long as the variances were equal.

When unequal variance was cross with perfect correlation (1.00), the F-ratio reflects a definite positive bias. This effect was one of the most severe found in the investigation. It was not in evidence for any other cases of high correlation (see discussion of unequal variance).

Table 3.4 - The Average Empirical Probability Associated With Correlation Sets EQ(.00), EQ(.25), EQ(.50), EQ(.75), EQ(1.00) When Variances are Equal

Correlation Matrix	F-Ratio: $\alpha =$				T ² : $\alpha =$			
	.100	.050	.025	.010	.100	.050	.025	.010
EQ(.00)	.097	.048	.025	.009	.097	.046	.021	.006
EQ(.25)	.082	.031	.016	.008	.093	.039	.020	.012
EQ(.50)	.092	.042	.020	.008	.094	.053	.029	.013
EQ(.75)	.083	.042	.020	.008	.075	.035	.011	.007
EQ(1.00)	.098	.048	.025	.010	.098	.048	.024	.010

Most Liberal Run

Run 45-EQ(1.00) .104 .049 .023 .010

Negative Correlation -- Runs 64, 65, 70, 80 represent four special runs with negative correlations. The effect on the F-ratio closely resembles the results of positive runs. It appears as though the negative correlation tends to suppress the liberalizing effect of the disperse coefficients. There is insufficient evidence to identify a general result. See Chapter IV for discussion of limitations of this study.

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Variance

Does non-homogeneous variance in the means affect the distribution of the F-ratio? For the majority of cases considered the effect of inequality of variance was tested using a moderate set of variances (225, 100, 25). Five special cases were also considered (runs 82 through 85 and 90).

Holding all other factors equal and comparing parallel runs it is apparent that moderate inequality of variance does not influence the F-ratio as long as the correlations remain less than one.

Table 3.5 below summarizes the runs with moderate inequality of variance. The probabilities in the table closely resemble those achieved when the variances were equal (tables 3.3 and 3.4) in all cases except EQ(1.00). Dispersion in the correlation coefficients continues to have an effect. Cases of moderate dispersion and equal variance do not disturb the distribution of the F-ratio. The T^2 statistic is again unaffected and continues to be slightly conservative even in the case of EQ(1.00).

Table 3.5 - The Average Empirical Probability Associated With All Positive Correlation Sets When the Variances Are Unequal V(225, 100, 25)

Correlation Matrix	F-Ratio: $\alpha =$				T^2 : $\alpha =$			
	.100	.050	.025	.010	.100	.050	.025	.010
Low	.102	.054	.026	.012	.099	.049	.023	.008
Med	.099	.051	.028	.012	.098	.047	.023	.008
High	.107	.058	.032	.014	.096	.048	.026	.009
Disperse II	.118	.067	.037	.018	.091	.045	.032	.012
Disperse I	.128	.076	.047	.022	.098	.047	.025	.010
EQ(.00)	.103	.054	.029	.012	.094	.045	.020	.008
EQ(.25)	.099	.051	.030	.013	.092	.047	.020	.008
EQ(.50)	.102	.055	.027	.014	.100	.049	.026	.010
EQ(.75)	.104	.058	.026	.012	.084	.034	.016	.004
EQ(1.00)	.146	.093	.060	.034	.096	.047	.023	.009
Most Liberal Run Run 18-EQ(1.00)	.172	.113	.074	.042				

Runs 82 through 85 and 90 involve cases of extremely non-homogeneous variances. In each case the variances differ radically. The effect these cases had on the F-ratio was of approximately the same magnitude as the interaction of EQ(1.00) and V(225, 100, 25). One case, V(100, 100, 0) did not follow the rule and must be explained as a chance phenomenon since V(100, 100, 1) and V(0, 100, 0) did have an effect. The effect is not due to the dispersion in the variances but rather must be due to the pattern of variances since the effect of V(0, 100, 0) is greater than V(10,000; 100; 0).

Covariance

The seriousness of the interaction of unequal variance and perfect correlation leads to the suspicion that covariance is the critical factor since perfect correlation has no effect when the variances are equal. The interaction of EQ(1.00) and unequal variance involves covariances more disperse than cases Disperse I and Disperse II with equal variance, and has a correspondingly more liberal effect on the F-ratio.

That covariance is the factor and not correlation is not reasonable. Consider the cases of High and Med correlation with unequal variance. They have a covariance dispersion greater than Disperse I or Disperse II with equal variance yet do not liberalize the F-ratio. Similarly Disperse I with equal variance has a less disperse covariance than with unequal variance; yet, the bias is greater with the smaller covariance.

Number of Units Per Cell

Do the number of observations per cell influence the deviation of the F-ratio from the tabled F distribution? It might be expected

that some of the niceties of large sample statistics may wipe out differences in distribution. This is not the case, nor should it be. The problem is not with the error term which is unit bound and hence related to sample size but, rather, a problem of means. Sample size is related to standard error of estimating a cell parameter and is normally reflected in the distribution of the statistic by the degrees of freedom. The degrees of freedom of the statistics under consideration are a function of the number of levels and the number of treatments only and not sample size. Alternatively, an error in setting up the generating function may have suppressed any indirect effect of this factor since the within cell variance was small compared to the variance in the treatment means.

The effect of sample size was examined by holding all conditions constant and varying sample size. The comparison is most complete for the case of three treatments and five levels although some comparisons are available in Appendix A for the case of three treatments and three levels and for the case of three treatments and ten levels. For the case of three treatments and three levels, sample sizes 1, 2, 5, 10, 25 were considered.

Sample size did not prove to have an effect on the distribution of the F-ratio. The probabilities associated with the distribution of the F-ratio for the case of Disperse I are reproduced in table 3.5 below. A similar lack of effect was apparent for all other correlation matrices.

Table 3.6 - Probabilities of the F-Ratio Associated With the Case of Disperse I for Varying n (Variances Equal).

L x T x n	F-Ratio			
	.100	.050	.025	.010
5 x 3 x 1	.130	.077	.044	.023
5 x 3 x 2	.147	.089	.057	.024
5 x 3 x 5	.147	.089	.059	.027
5 x 3 x 10	.142	.092	.051	.028
5 x 3 x 25	.141	.081	.049	.033

Varying sample size has essentially no effect on the distribution of the F-ratio. While not as obvious, the lack of trend for the case of Disperse II supports this conclusion.

Table 3.7 - Probability of the F-Ratio Associated With the Case of Disperse II for Varying n (Variance Equal).

L x T x n	F-Ratio			
	.100	.050	.025	.010
5 x 3 x 1	.120	.058	.028	.009
5 x 3 x 2	.119	.057	.033	.015
5 x 3 x 5	.124	.066	.041	.019
5 x 3 x 10	.144	.077	.046	.026
5 x 3 x 25	.116	.057	.035	.013

In general, inspection of the tables indicates a remarkable similarity in distributions for all conditions irrespective of the sample size, including samples of size one.

Number of Levels

The number of levels may be the underrated variable in this study. It is possible that the number of levels could play much the same role in the case of means as sample size does when considerations involve unit error. The effect is not in evidence in the null hypothesis phase of this investigation but is apparent when power is considered.

For the case of three treatments; three, five and ten levels were considered. Tabled below are the results of this comparison for the interesting correlation matrix Disperse I. The probabilities have been averaged over sample size for all runs with equal variance.

Table 3.8 - Average Probability Associated With F-Ratio for Case Disperse I With Varying Number of Levels (Variances are Equal).

No. of Levels	F-Ratio			
	.100	.050	.025	.010
3	.132	.077	.046	.024
5	.139	.083	.049	.026
10	.123	.073	.046	.018

Summary

The size of the correlations in the means does not bias the distribution of the F-ratio. Dispersion in the correlations is a factor. The discrepancy between the distribution of the F-ratio and the tabled F with (I-1) and (I-1)(J-1) degrees of freedom increases monotonically with the amount of dispersion. The deviations are predictable and seem to have an upper limit.

Inequality of variance is a factor worthy of consideration only if the differences in the variances are severe or if the correlation in the means is 1.00.

Size of sample and number of levels did not prove to have any effect on the distribution of the F-ratio.

The T^2 statistic was unaffected by the varying conditions and proved to be a conservative test of the hypothesis.

Power Comparisons

Introduction

The use of a statistical test involves two decisions. The distribution of the statistic must be considered under the null hypothesis, and the distribution under alternative hypotheses must also be considered. Part II of this investigation considers probabilities under alternative hypotheses. The results are presented below.

The graphed power comparisons of the F-ratio statistic follow. For each set of conditions considered the power of the statistic was determined for five commonly chosen levels of significance: .10, .05, .025, .01, .001. Levels .10, .025 and .001 are included in the graphs and all probabilities are tabled in Appendix B.

The ordinate of each graph is the probability of rejecting the null hypothesis. The abscissa represents the sum of the squared fixed effects. For each statistic there are three plots representing three common levels of significance. The graphs have been prepared on semi-logarithmic graph paper (4 cycles by 10 to the inch).

Results

Number of Levels -- Figures 3.1, 3.2, 3.3 represent the power of the statistics when all of the conditions of the mixed model are satisfied. The treatment means are independent over levels, variances are equal, and normality and independence assumptions are satisfied. In each case, three treatments and ten observations per cell are considered, and number of levels is varied. Fig. 3.1 represents the case of three levels, Fig. 3.2 - five levels, and Fig. 3.3 - ten levels.

Figure 3.1 Power Comparison of F-Ratio and τ^2 When There Exists Independent Means, 3 Levels, 10 Observations, Equal Variance

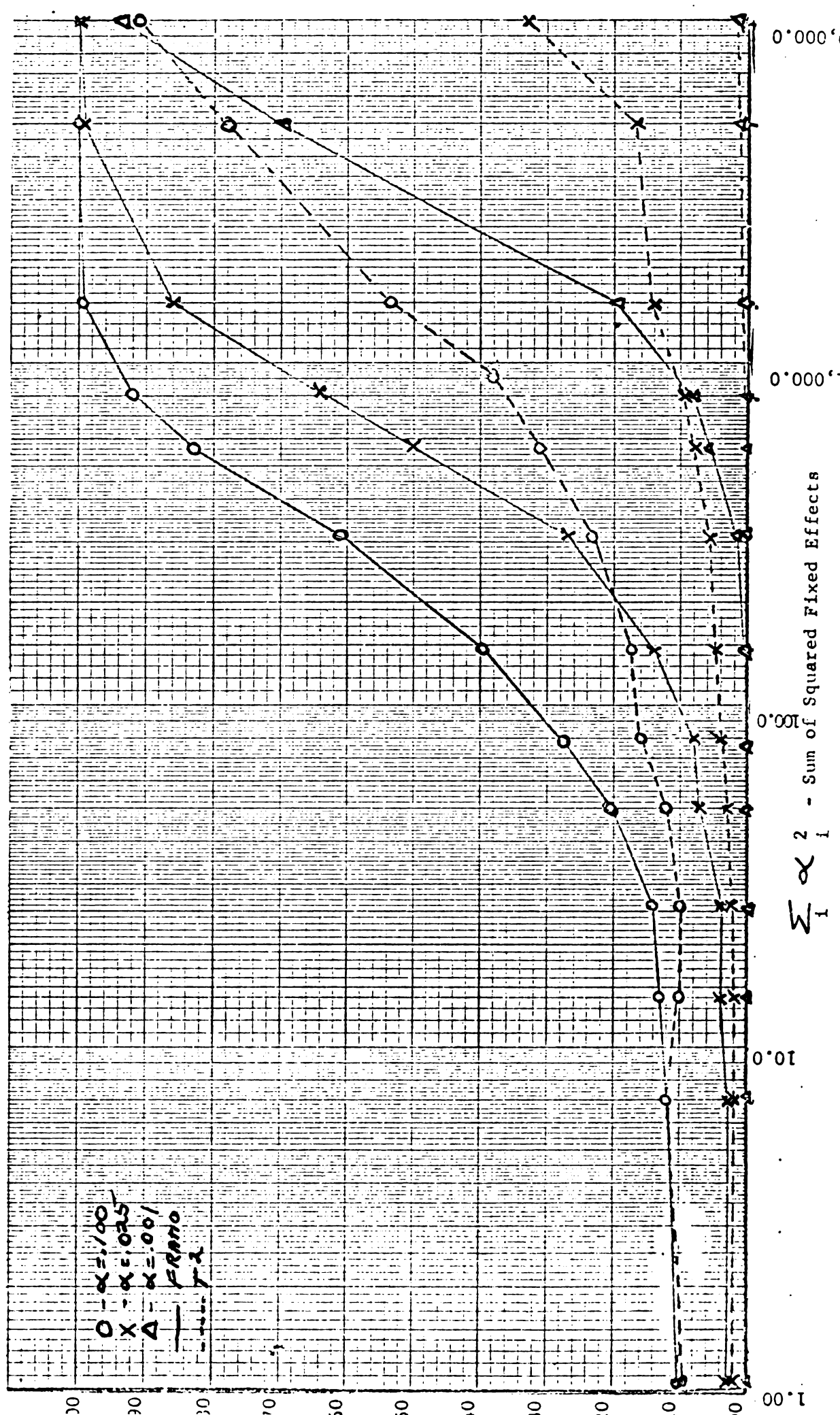


Figure 3.2 Power Comparison of F-Ratio and T^2 When There Exists Independent Means
5 Levels, 10 Observations, Equal Variance

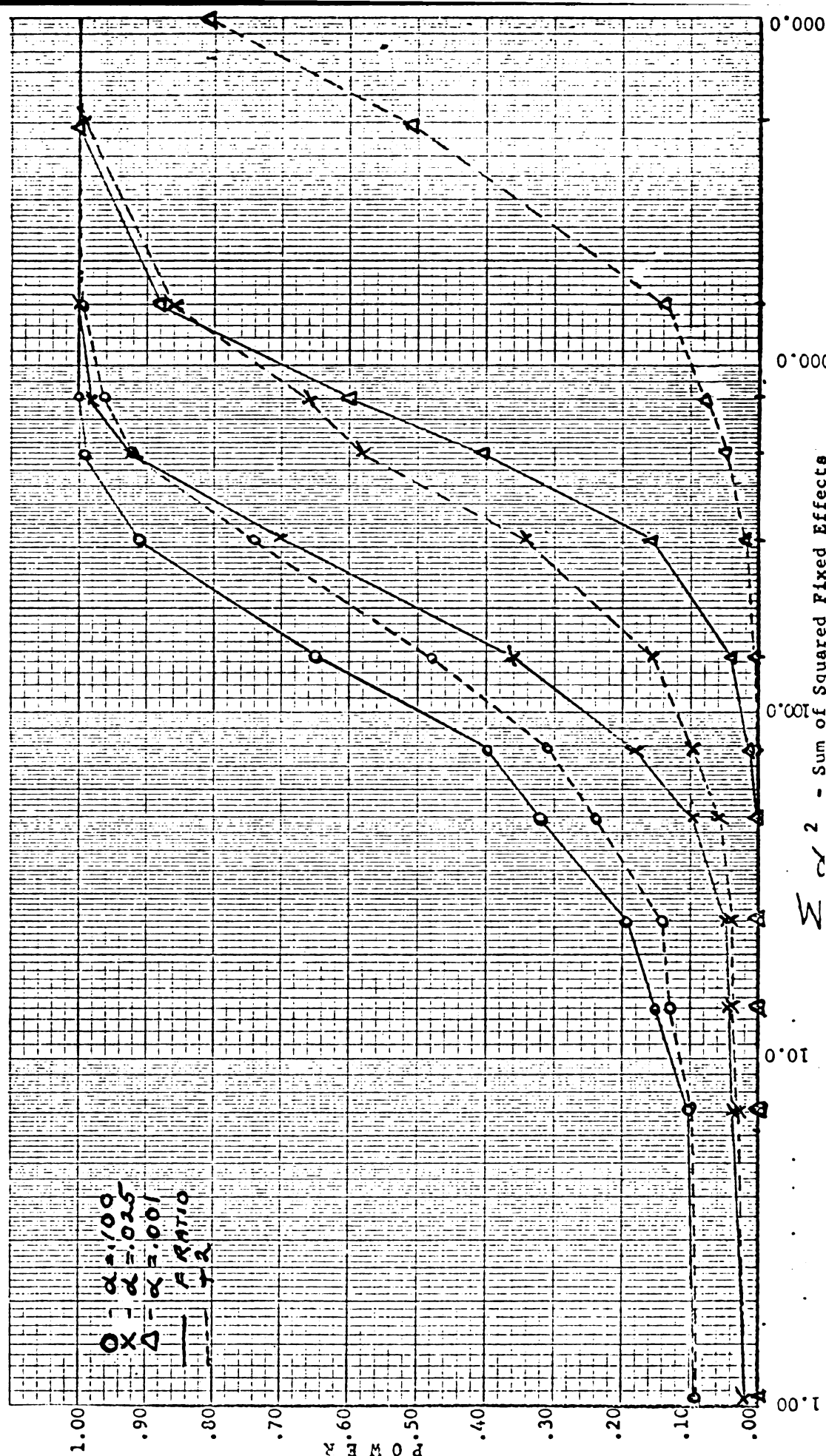
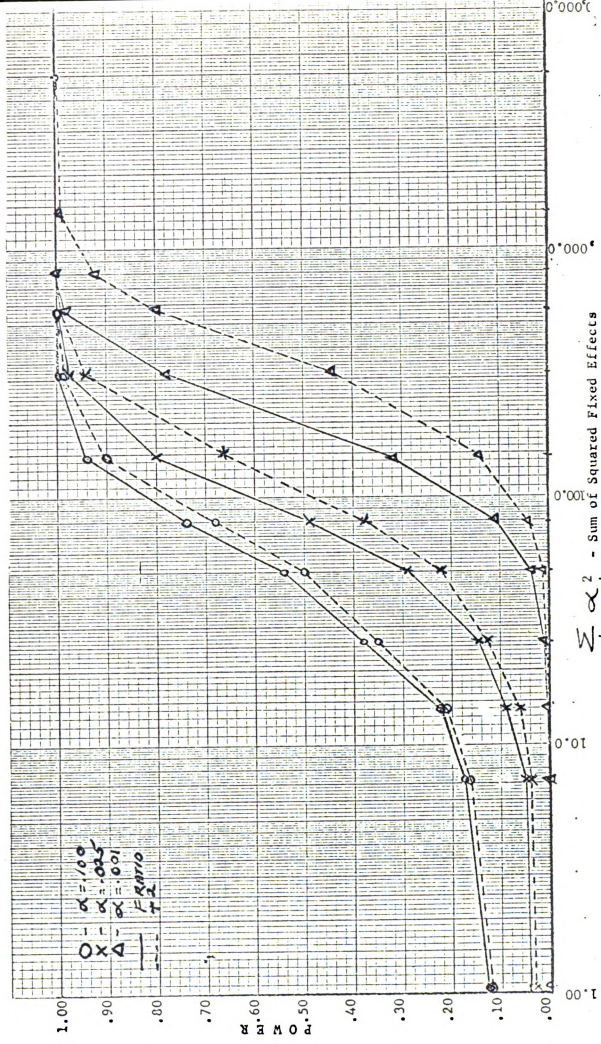


Figure 3.3 Power Comparison of F-Ratio and T^2 When There Exists Independent Means
10 Levels, 10 Observations, Equal Variance



An investigation of the three graphs yields the following observations:

1. First, and most obvious, the power of the statistic is a monotonically increasing function of the sum of the main effects squared. That is, the greater the difference in treatment means the greater the probability that the statistic will recognize these differences. For each curve there are two points of inflection between which the power statistic increases most rapidly. These characteristics are common to most of the curves which follow and represent the typical power curve of a statistic whose test is one tailed.

2. Throughout the three sets of conditions the T^2 statistic has less power than the F-ratio. The difference between the power of the F-ratio and the T^2 statistic is most serious when "alpha" is fixed at .001 and least different for "alpha" set at .10. For the case of three levels the power of T^2 is essentially zero when the power of the F-ratio is 1.00 for $\alpha = .001$. For the same conditions the power of T^2 is approximately .80 when the F-ratio reaches 1.00 for $\alpha = .10$.

3. The power of the T^2 statistic as well as the power of the F-ratio increases as the number of levels considered increases. Similarly, the difference between the power of the two statistics decreases as the number of levels increases.

Sample Size -- Figures 3.4, 3.5, and 3.6 demonstrate the effect of varying sample size on the power of the statistics. In each case there are moderate correlations imposed on the means and the usual error assumptions are satisfied. Three treatments and five levels are considered and the sample size is varied.

Figure 3.4 Power Comparison of F-Ratio and τ^2 When There Exists Medium Correlation in the Means, 5 Levels, 2 Observations, Equal Variance

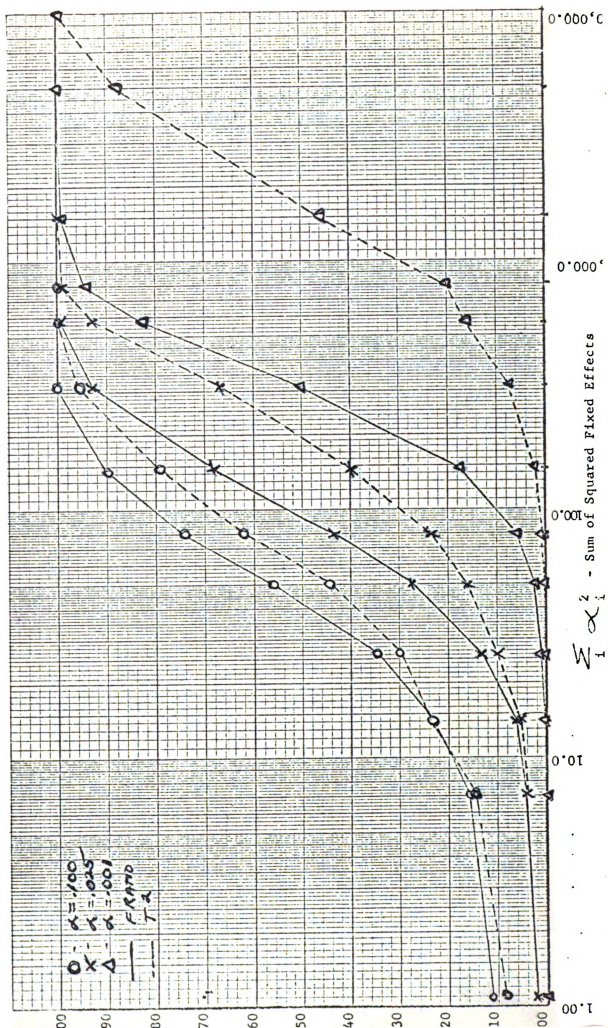


Figure 3.5 Power Comparison of F-Ratio and T^2 When There Exists Medium Correlation
5 Levels, 5 Observations, Equal Variance

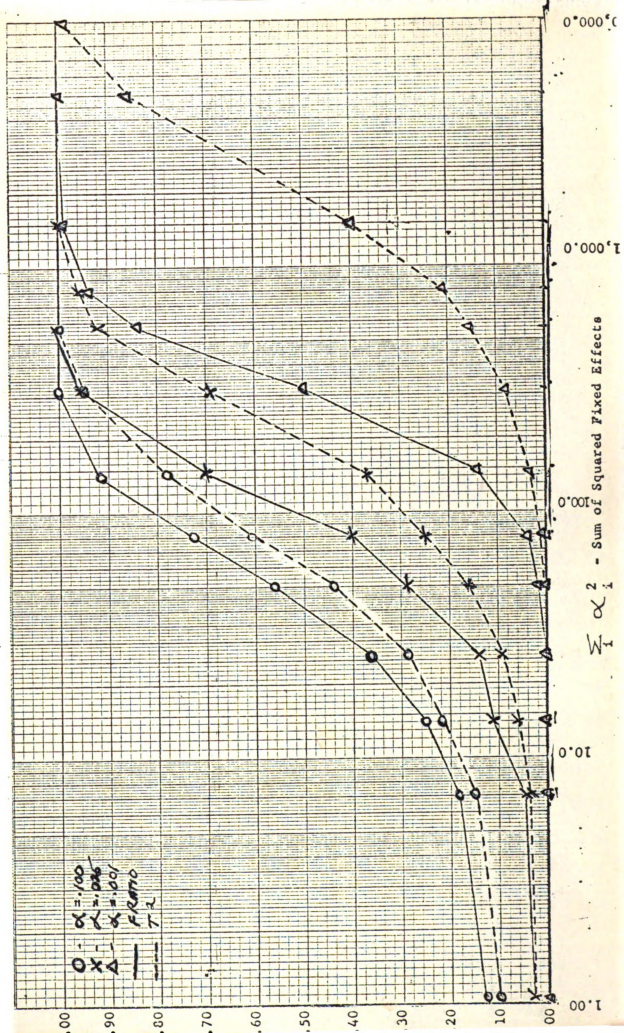
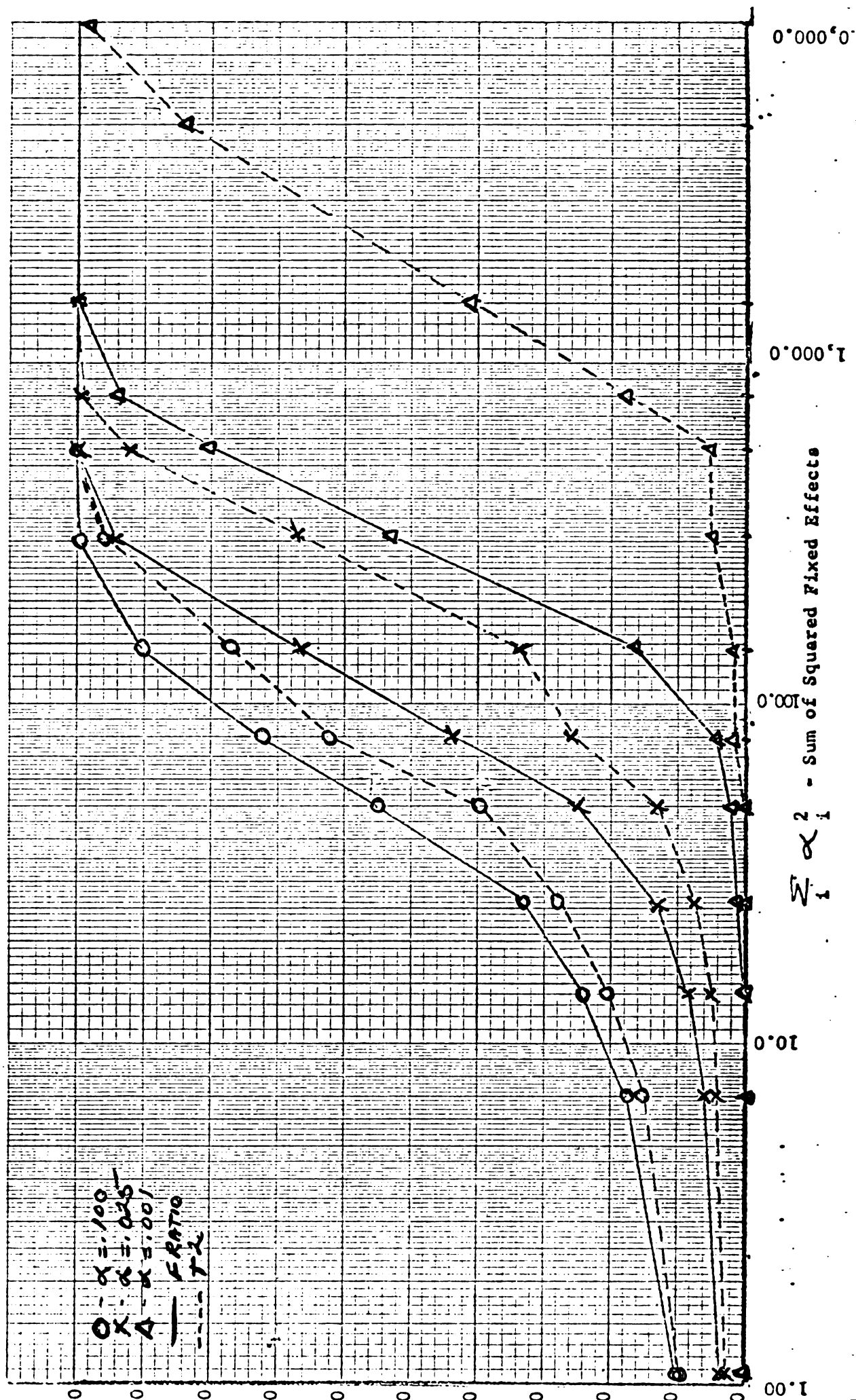


Figure 3.6 Power Comparison of F-Ratio and T^2 When There Exists Medium Correlation
5 Levels, 10 Observations, Equal Variance



1. The power curves are again monotonically increasing, but the smoothness of the T^2 statistic power curve is interrupted to a small degree at approximately the same point on the three power curves. The F-ratio power function continues with smooth predictability. (The points of interruption in the T^2 curve are $\sum \alpha^2 = 149$ and 800.)

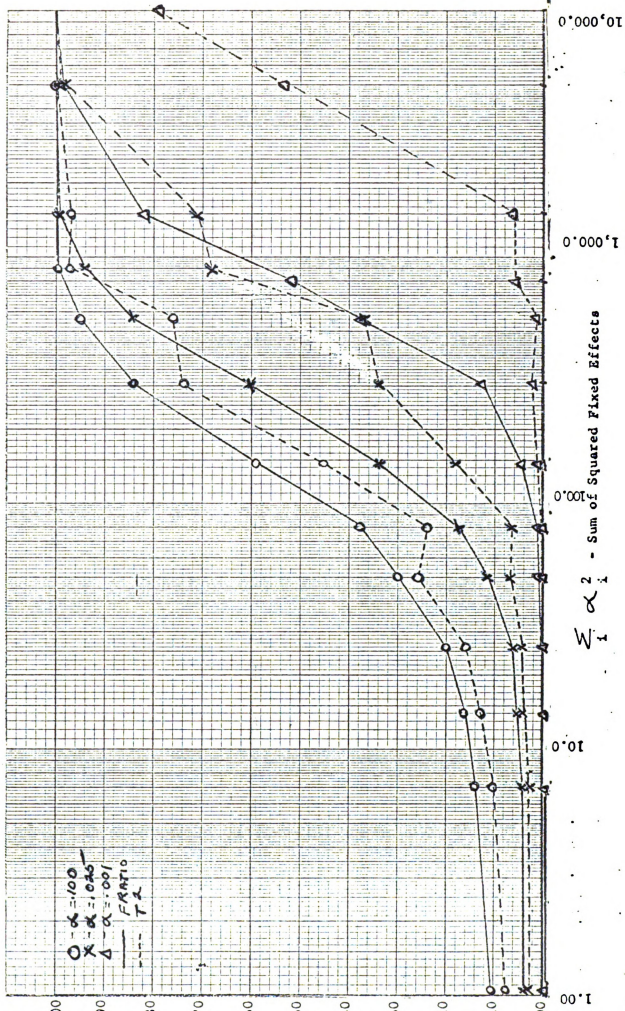
2. Comparing the three graphs it can be seen that varying the sample size has no effect on the power of the statistics. For the three sample sizes considered the three power curves are nearly identical.

3. If figure 3.6 is compared with figure 3.2 it is apparent that the statistics are more powerful when the means are moderately correlated than when there exists no correlation in the means.

Variance -- Figure 3.7 represents the empirical power of the statistics when the within treatment variance in the means is not equal for all treatments. Three treatments, five levels and ten observations per cell are considered. There are no correlations imposed on the means and all of the usual assumptions concerning errors are satisfied.

1. The power curves are again basically monotonically increasing except for a few inversions in the curve of the T^2 statistic. This tends to confirm the exaggerations noted in the power curve for the case of medium correlations and leads to the suspicion that the power of the T^2 is not a monotonic function of mean differences. It appears to be sensitive to deviations from homogeneity in the variance-covariance matrix. If this is the case the T^2 is impure as a test of mean differences and becomes a test of gross distribution differences. The F-ratio continues to behave in a predictable manner.

Figure 3.7 Power Comparison of F-Ratio and T^2 When There Exists Independent Means
5 Levels, 10 Observations, Unequal Variance



2. A comparison of figures 3.2 and 3.7 indicates that the power of the F-ratio is not affected by non-homogeneous variance. The curve of the T^2 is affected as noted above but the power of the statistic not otherwise increased.

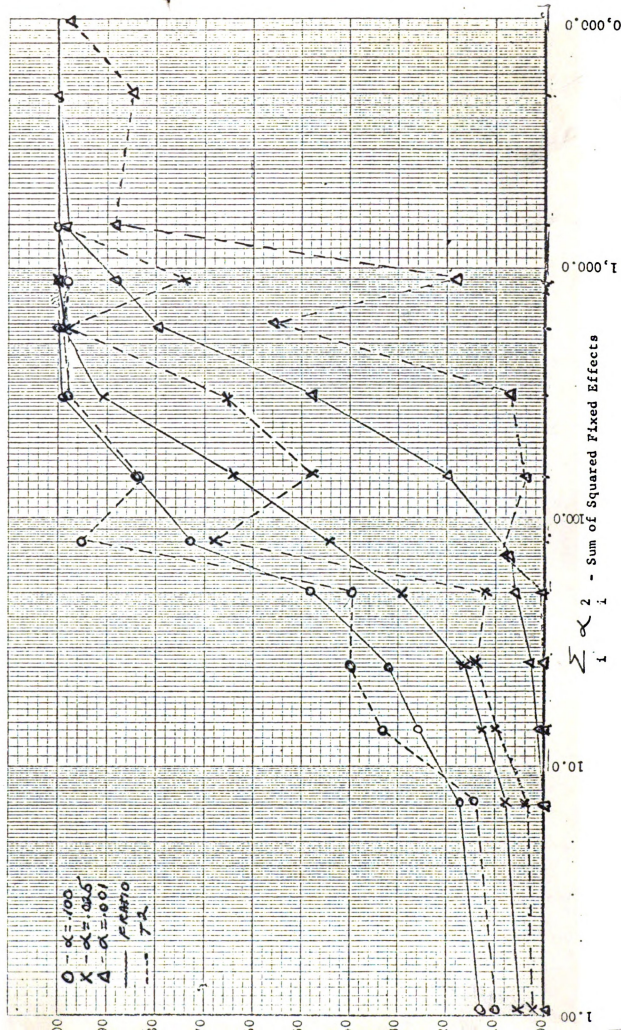
3. The power of the F-ratio continues to exceed that of the T^2 based statistic.

Correlations -- Figure 3.8 is a graphic comparison of the two statistics under the case of extremely disperse correlations in the means. Three treatments, five level and ten observations per cell are considered. The mean variances are held constant and the usual error assumptions are satisfied.

1. The power of the F-ratio is not affected by disperse correlations in the means. When figures 3.8 and 3.6 are compared very little difference exists in the power curves of the F-ratio indicating that the increased dispersion has not altered the distribution substantially. This is surprising since increased dispersion had a measurable effect under the null hypothesis. One of two explanations is possible. Either dispersion has no effect on the power and magnitude of the correlations does (Med and Disperse I have same average magnitude), or when alternative hypotheses are considered the small amount of dispersion in evidence in Med was sufficient to exert the effect of disperse correlations on the F-ratio when the null hypothesis is false. (Item 3, p.49)

2. The effect on the power of the T^2 is severe. Figures 3.4, 3.5, 3.6, and 3.7 indicates that the T^2 might be sensitive to lack of homogeneity in the variance-covariance matrix and 3.7 indicates that possibly the power function of this statistic was not monotonically

Figure 3.8 Power Comparison of F-Ratio and t^2 When There Exists Disperse Correlation, 5 Levels, 10 Observations, Equal Variance



related to increases in mean differences under all conditions. All of these indications are confirmed by this situation. The power function is grossly non-monotonic. It is evident that the statistic is sensitive to deviations other than those treated in the hypothesis tested. This condition was replicated in order to reconfirm these results. The erratic pattern held up under replication.

3. In this case the power of the T^2 statistic exceeds the power of the F-ratio in selected cases, but these cases follow no predictable pattern. The power of the T^2 statistic deviates from the normal pattern of a one-tailed statistic while the F-ratio remains predictably smooth.

4. It was suspected that the nature of the fixed effects might have caused the erratic performance of the T^2 statistic's power curve. The fixed effects used for the power comparisons (table 2.2) were checked to determine if there might be some explainable reason for the behavior of the T^2 statistic. The drops in the power curve occurred when $\sum \alpha^2 = 50, 146, \text{ and } 800$ and the curve peaked when $\sum \alpha^2 = 78$ and 566. The T^2 power curve similarly misbehaved in the earlier situations also. It is not clear but it appears as though the pattern of the fixed effects may affect the power of the T^2 . Very possibly this effect is an interaction of the treatment mean vector and the matrix of treatment mean variances and covariances since T^2 is a product of this vector and matrix. In this case the power decreased when the effect of the second treatment was negligible with respect to the effect of the first and third treatment and the power increased when the effect of the third treatment was small relative to the first two treatments.

Summary

The power comparison of the F-ratio and T^2 based statistics indicates that the F-ratio is in general more powerful than the T^2 based statistic and has a much better behaved power function.

The power of the F-ratio is most superior for small "alpha" and a small number of levels. As the number of levels and the size of the type one error increase, the power advantage of the F-ratio decreases but remains in the favor of the F-ratio. The number of units per cell has no effect on either power function.

When non-homogeneous variances and correlations were considered some strange results were discovered. Moderately unequal variance had no effect on the power of the F-ratio and the effect of correlations on the power of the F-ratio appeared to be an effect of magnitude rather than dispersion; a reversal of the findings in Part I.

The power function of the T^2 was disturbed by both unequal variances and unequal correlations. The function became non-monotonic and appeared to be influenced by the pattern of the fixed effects when the variance-covariance matrix is non-homogeneous. This effect is reasonable since the power of T^2 is a quadratic function, the size of which changes as a function of both the length and direction of the main effect vector.

CHAPTER IV: DISCUSSION AND CONCLUSIONS

Summary

The mixed model analysis of variance is a useful technique of behavioral science research. Its use has been restricted because the test procedures are complex and difficult to interpret. The appropriate test of the hypothesis of null-fixed effects is a multivariate technique unfamiliar to most educational researchers. This investigation considered the possibility of replacing this multivariate procedure with a familiar F-ratio test.

The approximate F-ratio, $F = MS_a / MS_{ab}$, was compared with the exact T^2 based test for a variety of conditions under the null hypothesis and several alternative hypotheses. The comparison was based on 1,000 observations of each statistic for each case considered. The data from which the statistics were calculated were generated using a computerized Monte Carlo procedure.

Results

Under the null hypotheses the F-ratio proved to be moderately liberal for selected conditions. The T^2 based statistic remained slightly conservative for all conditions considered. Those conditions which liberalized the F-ratio were disperse correlation patterns, interaction of perfect correlation and unequal variance, and variance patterns which deviated radically from homogeneity.

The number of observations per cell, number of treatments, number of levels, magnitude of correlation, and moderate non-homogeneity of variance had no effect on the distribution of the F-ratio under the null hypothesis.

Magnitude of correlation, number of levels, and level of significance did affect the power comparison of the F-ratio and T^2 based statistic. Unequal variance also caused deviations in the power curve of the T^2 based F.

The F-ratio had superior power under almost all conditions, and was particularly superior when the number of levels and "alpha" were small. The T^2 became very erratic when the dispersion of the correlations increased. In this erratic condition it exceeded the power of the F-ratio for a few selected sets of means.

Discussion

It must be concluded that the F-ratio, mean square of fixed effects/mean square of interaction effects, is an acceptable alternate test of the hypothesis of null fixed main effects under the mixed model.

The errors incurred by using the F-ratio instead of the exact T^2 based F test are predictable, generally small, and appear to have an upper limit. In addition to simplicity and wider applicability the advantages of the F-ratio include superior power and a well behaved power function.

The effect of disperse correlations of the F-ratio is monotonically related to the amount of dispersion in the coefficients. The liberalizing effect can be counteracted by appropriately adjusting the critical value required for rejecting the hypothesis. The most severe deviation due to disperse correlations could be corrected by selecting the critical value associated with the next common "alpha level" below the desired "alpha level." For example, selecting the critical F associated with $\alpha = .100, .050, .010, .001$ will provide a conservative test

for $\alpha = .100, .050, .025, .010$ respectively. In many cases no adjustment is required or a lesser adjustment would be satisfactory.

The F-ratio has superior power and in most cases remains nearly as powerful as the T^2 based statistic after adjustment. (See power charts - Chapter III)

An adjustment such as the one described above should counteract the problem of severe deviations in the mean variances as well.

The interaction of perfect correlations and unequal variance has a low probability of occurrence. If it is suspected that this condition exists in the data, the T^2 based statistic should be used.

Exactly what adjustment is required is not clear since only one set of variances was considered. Further, the magnitude of the adjustment required to correct the F-ratio may bring about severe loss of power.

Questions for Further Investigation

This investigation has demonstrated that it is reasonable to consider the F-ratio statistic as a test of the hypotheses of no fixed main effects.

The problem of negative correlations was not satisfactorily answered. In this study negative correlation was confounded with disperse correlations. The effect of the negative correlations was to suppress the effect of disperse correlations indicating that negative correlation may cause the F-ratio to be conservative. This effect requires further exploration.

The interaction of unequal variance and perfect correlation requires further examination. Only one set of variance was considered. Would other variance sets cause an even greater effect?

What is it that caused the variance to have an effect when radical patterns were considered? The results were mixed and confusing.

Might number of levels prove interesting in the case of the null hypothesis if a greater number, 25 or 50, were considered?

Further examination of the sensitivity of the T^2 to pattern of mean effects when the variance-covariance is non-homogeneous should be very interesting.

The question of generalization of results to more than three treatments has not been answered. Increasing the number of treatments makes possible variance and covariance patterns not considered here. Unique considerations may evolve when more than three treatments are considered.

All of these questions require further investigation. In particular the general question involving the use of univariate tests in circumstances requiring multivariate techniques requires definite consideration.

Summary Conclusion

Unless the inter-treatment mean correlations are perfect or near perfect, the mean square F-ratio may be used to test the hypothesis of no fixed treatment effects. This test may require a simple adjustment depending upon the dispersion in the correlations or the pattern of variances.

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APPENDIX A

The following table is a summary of conditions and the empirical probability of rejecting a true null hypothesis for the F-ratio and T^2 statistics.

For each set of conditions the estimated true probability and the tabled probability are compared for the following probability points $\{ F/P(f > F) \leq .10, .05, .025, .01 \}$. The probability in the following table is the proportion of empirical values of the statistic which exceed the value of the statistic normally associated with a given probability. For each run, column 1 specifies the number of levels, treatments, and units considered in that run. Column 2 specifies the variances for the means of respective treatments over levels. Columns 3 and 4 specify the covariance and correlation of the means. (1,2 identifies the correlation or covariance for the means of treatment groups one and two.) Columns 5 and 6 are the empirical probabilities of the two statistics compared against the tabled probabilities.

TABLE I: Summary of Conditions for Each Run and the Empirical Probability of Rejecting A True Null Hypothesis for the F-Ratio and T^2 Based Statistics*

Run	LxTxn	Variances			Covariances			Correlations			α : F-Ratio			α : T ²				
		1	2	3	1,2	1,3	2,3	1,2	1,3	2,3	.100	.050	.025	.010	.100	.050	.025	.010
1.	3x3x2	100	100	100	88	48	16	.88	.48	.16	.129	.070	.045	.022	.094	.051	.028	.006
2.	3x3x2	100	100	100	100	100	100	1.00	1.00	1.00	.096	.046	.025	.009	.094	.042	.024	.007
**3.	3x3x2	100	100	100	100	100	100	1.00	1.00	1.00	.103	.049	.022	.006	.108	.055	.026	.008
4.	3x3x2	225	100	25	128	55	32	.86	.74	.63	.101	.055	.034	.016	.096	.056	.030	.011
5.	3x3x2	225	100	25	95	40	21	.63	.54	.42	.103	.058	.033	.012	.097	.053	.027	.011
6.	3x3x2	225	100	25	52	16	8	.35	.21	.15	.109	.061	.026	.009	.102	.053	.024	.006
7.	3x3x2	225	100	25	132	36	8	.88	.48	.16	.134	.083	.051	.018	.103	.056	.028	.011
8.	3x3x2	225	100	25	150	75	50	1.00	1.00	1.00	.148	.099	.064	.036	.092	.045	.028	.015
9.	3x3x10	100	100	100	86	74	63	.86	.74	.63	.105	.056	.032	.017	.119	.061	.038	.014
10.	3x3x10	100	100	100	63	54	42	.63	.54	.42	.104	.055	.029	.012	.130	.080	.036	.011
11.	3x3x10	100	100	100	35	21	15	.35	.21	.15	.092	.047	.025	.006	.104	.055	.029	.012
12.	3x3x10	100	100	100	88	48	16	.88	.48	.16	.134	.083	.048	.026	.111	.053	.028	.019
13.	3x3x10	100	100	100	100	100	100	1.00	1.00	1.00	.096	.056	.030	.009	.093	.047	.022	.012
14.	3x3x10	225	100	25	128	55	32	.86	.74	.63	.103	.057	.025	.011	.100	.046	.024	.009
15.	3x3x10	225	100	25	95	40	21	.63	.54	.42	.089	.042	.027	.013	.100	.048	.027	.011
16.	3x3x10	225	100	25	52	16	8	.35	.21	.15	.110	.054	.025	.014	.102	.055	.030	.014
17.	3x3x10	225	100	25	132	36	8	.88	.48	.16	.130	.077	.050	.027	.107	.051	.025	.011
18.	3x3x10	225	100	25	150	75	50	1.00	1.00	1.00	.172	.113	.074	.042	.091	.049	.021	.009
19.	5x3x1	100	100	100	86	74	63	.86	.74	.63	.111	.063	.031	.014	.096	.046	.023	.008
20.	5x3x1	100	100	100	63	54	42	.63	.54	.42	.079	.038	.024	.009	.085	.041	.020	.009
21.	5x3x1	100	100	100	88	48	16	.88	.48	.16	.118	.065	.041	.019	.094	.051	.029	.016
**22.	5x3x1	100	100	100	88	48	16	.88	.48	.16	.130	.077	.044	.023	.089	.039	.022	.012
23.	5x3x1	100	100	100	50	25	75	.50	.25	.75	.110	.058	.028	.009	.085	.041	.015	.004

*The critical values of the F-ratio correspond to $F_{\alpha:(T-1)(L-1)}$; and for the T^2 they correspond to $F_{\alpha:(T-1)(L-T+1)}$.

**A replicated run.

TABLE I (Continued)

Run	LxTxn	Variances			Covariances			Correlations			α : F-Ratio			α : T ²				
		1	2	3	1,2	1,3	2,3	1,2	1,3	2,3	.100	.050	.025	.010	.100	.050	.025	.010
24.	5x3x1	225	100	25	00	00	00	.00	.00	.00	.110	.051	.034	.010	.080	.036	.015	.007
25.	5x3x2	100	100	100	86	74	63	.86	.74	.63	.091	.050	.025	.015	.096	.048	.018	.006
26.	5x3x2	100	100	100	88	48	16	.88	.48	.16	.147	.089	.057	.024	.113	.056	.026	.013
27.	5x3x2	100	100	100	50	25	75	.50	.25	.75	.119	.057	.033	.015	.106	.058	.030	.015
28.	5x3x2	100	100	100	00	00	00	.00	.00	.00	.090	.046	.020	.008	.096	.041	.014	.004
29.	5x3x2	100	100	100	100	100	100	1.00	1.00	1.00	.099	.043	.028	.012	.093	.051	.023	.008
30.	5x3x2	225	100	25	128	55	32	.86	.74	.63	.099	.057	.035	.014	.090	.050	.027	.010
31.	5x3x2	225	100	25	95	40	21	.63	.54	.42	.095	.045	.024	.008	.102	.050	.023	.006
32.	5x3x2	225	100	25	52	16	8	.35	.21	.15	.084	.048	.024	.010	.098	.047	.025	.011
33.	5x3x2	225	100	25	132	36	8	.88	.48	.16	.126	.074	.048	.022	.090	.052	.024	.013
34.	5x3x2	225	100	25	00	00	00	.00	.00	.00	.120	.064	.025	.009	.106	.055	.026	.009
*35.	5x3x2	225	100	25	00	00	00	.00	.00	.00	.094	.054	.030	.018	.094	.050	.022	.010
36.	5x3x2	225	100	25	38	17	12	.25	.25	.25	.096	.050	.030	.011	.098	.051	.023	.008
37.	5x3x2	225	100	25	75	38	25	.50	.50	.50	.093	.054	.028	.014	.101	.047	.024	.009
38.	5x3x2	225	100	25	112	56	38	.75	.75	.75	.107	.057	.028	.014	.095	.034	.014	.004
39.	5x3x2	225	100	25	150	75	50	1.00	1.00	1.00	.130	.083	.058	.035	.094	.039	.014	.002
40.	5x3x2	225	100	25	75	19	38	.50	.25	.75	.115	.075	.038	.017	.082	.042	.022	.014
41.	5x3x5	100	100	100	86	74	63	.86	.74	.63	.088	.043	.020	.006	.083	.032	.017	.010
42.	5x3x5	100	100	100	88	48	16	.88	.48	.16	.147	.089	.059	.027	.097	.048	.027	.009
43.	5x3x5	100	100	100	50	25	75	.50	.25	.75	.124	.066	.041	.019	.101	.047	.023	.007
44.	5x3x5	100	100	100	100	100	100	1.00	1.00	1.00	.088	.046	.020	.012	.095	.052	.027	.013
*45.	5x3x5	100	100	100	100	100	100	1.00	1.00	1.00	.104	.049	.023	.010	.090	.043	.021	.009
46.	5x3x5	225	100	25	128	55	32	.86	.74	.63	.115	.060	.030	.014	.096	.048	.025	.013
47.	5x3x5	225	100	25	95	40	21	.63	.54	.42	.097	.050	.021	.009	.102	.048	.025	.010
48.	5x3x5	225	100	25	52	16	8	.35	.21	.15	.109	.054	.032	.011	.093	.044	.022	.010
49.	5x3x5	225	100	25	75	19	38	.50	.25	.75	.124	.063	.032	.019	.100	.044	.023	.009
50.	5x3x5	225	100	25	132	36	8	.88	.48	.16	.131	.069	.043	.014	.095	.039	.022	.008
51.	5x3x5	225	100	25	00	00	00	.00	.00	.00	.099	.050	.025	.011	.089	.043	.024	.008
52.	5x3x5	225	100	25	150	75	50	1.00	1.00	1.00	.161	.095	.066	.036	.105	.056	.025	.010

*Replicated Run.

TABLE I (Continued)

Run	LxTxn	Variances			Covariances			Correlations			α : F-Ratio			α : T^2				
		1	2	3	1,2	1,3	2,3	1,2	1,3	2,3	.100	.050	.025	.010	.100	.050	.025	.010
53.	5x3x10	100	100	100	86	74	63	.86	.74	.63	.113	.055	.037	.008	.089	.045	.023	.007
54.	5x3x10	100	100	100	63	54	42	.63	.54	.42	.093	.046	.022	.009	.081	.034	.016	.007
55.	5x3x10	100	100	100	35	21	15	.35	.21	.15	.096	.041	.020	.010	.086	.041	.012	.004
56.	5x3x10	100	100	100	88	48	16	.88	.48	.16	.142	.092	.051	.028	.106	.058	.032	.008
57.	5x3x10	100	100	100	50	25	75	.50	.25	.75	.144	.077	.046	.026	.102	.052	.030	.009
58.	5x3x10	100	100	100	00	00	00	.00	.00	.00	.098	.050	.027	.008	.092	.044	.027	.008
*59.	5x3x10	100	100	100	00	00	00	.00	.00	.00	.102	.049	.025	.012	.104	.054	.022	.005
60.	5x3x10	100	100	100	25	25	25	.25	.25	.25	.082	.031	.016	.008	.093	.039	.020	.012
61.	5x3x10	100	100	100	50	50	50	.50	.50	.50	.092	.042	.020	.008	.094	.053	.029	.013
62.	5x3x10	100	100	100	75	75	75	.75	.75	.75	.083	.042	.017	.007	.075	.035	.011	.007
63.	5x3x10	100	100	100	100	100	100	1.00	1.00	1.00	.100	.050	.024	.012	.112	.049	.026	.013
64.	5x3x10	100	100	100	-50	25	-75	-.50	.25	-.75	.108	.061	.033	.011	.079	.032	.015	.007
65.	5x3x10	100	100	100	88	-48	-16	.88	-.48	-.16	.130	.088	.056	.032	.097	.053	.023	.006
66.	5x3x10	225	100	25	128	55	32	.86	.74	.63	.113	.063	.027	.013	.097	.047	.023	.011
67.	5x3x10	225	100	25	95	40	21	.63	.54	.42	.109	.058	.034	.017	.092	.041	.014	.003
68.	5x3x10	225	100	25	52	16	8	.35	.21	.15	.098	.050	.023	.009	.097	.046	.015	.004
69.	5x3x10	225	100	25	132	36	8	.88	.48	.16	.126	.073	.042	.024	.104	.050	.019	.005
70.	5x3x10	225	100	25	75	19	38	.50	.25	.75	.115	.064	.041	.017	.091	.052	.039	.012
71.	5x3x10	225	100	25	00	00	00	.00	.00	.00	.099	.044	.023	.007	.084	.041	.020	.007
*72.	5x3x10	225	100	25	00	00	00	.00	.00	.00	.097	.063	.036	.018	.107	.042	.016	.007
73.	5x3x10	225	100	25	38	17	12	.25	.25	.25	.102	.053	.029	.015	.085	.043	.017	.009
74.	5x3x10	225	100	25	75	38	25	.50	.50	.50	.111	.056	.026	.014	.098	.051	.028	.010
75.	5x3x10	225	100	25	112	56	38	.75	.75	.75	.103	.048	.027	.011	.069	.026	.010	.002
*76.	5x3x10	225	100	25	112	56	38	.75	.75	.75	.102	.056	.029	.012	.088	.043	.023	.009
77.	5x3x10	225	100	25	150	75	50	1.00	1.00	1.00	.135	.090	.053	.035	.108	.051	.019	.009
*78.	5x3x10	225	100	25	150	75	50	1.00	1.00	1.00	.140	.087	.049	.026	.087	.044	.023	.011
79.	5x3x10	225	100	25	-75	19	-38	-.50	.25	-.75	.109	.050	.027	.011	.096	.052	.025	.010
80.	5x3x10	225	100	25	132	-36	8	.88	-.48	-.16	.127	.073	.044	.019	.081	.042	.022	.013

*A replicated run.

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TABLE I (Continued)

Run	LxTxn	Variances			Covariances			Correlations			α : F-Ratio			α : T ²				
		1	2	3	1,2	1,3	2,3	1,2	1,3	2,3	.100	.050	.025	.010	.100	.050	.025	.010
81.	5x3x10	25	100	225	44	36	24	.88	.48	.16	.147	.083	.055	.027	.091	.041	.019	.009
82.	5x3x10	100	100	1	25	25	2.5	.25	.25	.25	.123	.068	.033	.012	.103	.047	.030	.013
83.	5x3x10	10,000	100	0	00	00	00	.00	.00	.00	.133	.087	.059	.030	.088	.040	.018	.009
84.	5x3x10	0	100	0	00	00	00	.00	.00	.00	.157	.104	.075	.045	.101	.042	.023	.010
85.	5x3x10	100	100	0	00	00	00	.00	.00	.00	.107	.051	.029	.009	.090	.046	.026	.007
86.	5x3x25	100	100	100	86	74	63	.86	.74	.63	.095	.046	.025	.008	.092	.041	.019	.006
87.	5x3x25	100	100	100	88	48	16	.88	.48	.16	.141	.081	.049	.033	.103	.049	.026	.018
88.	5x3x25	100	100	100	50	25	75	.50	.25	.75	.116	.057	.035	.013	.090	.040	.024	.009
89.	5x3x25	225	100	25	150	75	50	1.00	1.00	1.00	.138	.086	.058	.026	.098	.047	.029	.010
90.	5x3x25	10,000	100	0	00	00	00	.00	.00	.00	.140	.093	.060	.031	.089	.043	.020	.011
91.	10x3x2	100	100	100	86	74	63	.86	.74	.63	.108	.060	.031	.018	.097	.049	.026	.010
92.	10x3x2	100	100	100	88	48	16	.88	.48	.16	.123	.079	.053	.026	.092	.046	.025	.011
93.	10x3x2	225	100	25	128	55	32	.86	.74	.63	.105	.051	.037	.014	.099	.048	.029	.006
94.	10x3x2	225	100	25	132	36	8	.88	.48	.16	.129	.076	.049	.026	.094	.048	.031	.011
96.	10x3x10	100	100	100	86	74	63	.86	.74	.63	.118	.061	.032	.016	.109	.053	.024	.006
97.	10x3x10	100	100	100	63	54	42	.63	.54	.42	.093	.053	.035	.016	.099	.048	.022	.008
98.	10x3x10	100	100	100	35	21	15	.35	.21	.15	.101	.043	.019	.010	.101	.052	.027	.011
99.	10x3x10	100	100	100	88	48	16	.88	.48	.16	.115	.069	.043	.009	.082	.039	.016	.010
*100.	10x3x10	100	100	100	88	48	16	.88	.48	.16	.130	.072	.043	.019	.092	.042	.022	.006
101.	10x3x10	225	100	25	128	55	32	.86	.74	.63	.111	.063	.038	.015	.104	.043	.023	.005
102.	10x3x10	225	100	25	95	40	21	.63	.54	.42	.100	.056	.029	.013	.093	.043	.022	.005
103.	10x3x10	225	100	25	52	16	8	.35	.21	.15	.105	.059	.028	.018	.102	.050	.020	.003
104.	10x3x10	225	100	25	132	36	8	.88	.48	.16	.122	.078	.046	.023	.096	.046	.023	.010

*A replicated run.

APPENDIX B

The following tables are a summary of conditions and the empirical probability of rejecting a false null hypothesis for the F-ratio and the T^2 based statistic. The tables correspond to respective figures in Chapter III. Figure 3.1 is a partial graphic representation of the results in Table 1. The critical values of the F-ratio correspond to $F_{\alpha : (T - 1); (T - 1)(L - 1)}$ and for the T^2 they correspond to $F_{\alpha : (T - 1); (L - T + 1)}$. For each cell in the tables below, the top number represents the empirical power of F ratio and the lower figure represents the empirical power of T^2 . In all cases three treatments have been considered.

Listed below are the conditions which were imposed on the population represented in each table:

Table 1 - Independent Means
3 Levels
10 Observations
Equal Variance

Table 2 - Independent Means
5 Levels
10 Observations
Equal Variance

Table 3 - Independent Means
10 Levels
10 Observations
Equal Variance

Table 4 - Med. Correlation in the Means
5 Levels
2 Observations
Equal Variance

Table 5 - Med. Correlation in the means
5 Levels
5 Observations
Equal Variance

Table 6 - Med. Correlation in the Means
5 Levels
10 Observations
Equal Variance

Table 7 - Independent Means
5 Levels
10 Observations
Unequal Variance

Table 8 - Disperse Correlation in the Means
5 Levels
10 Observations
Equal Variance

TABLE 1

$\sum \alpha_i^2$	.100	.050	.025	.010	.001
.75, -.05, -.70	.107	.061	.030	.008	.000
1.055	.087	.035	.019	.006	.001
-2, .25, 1.75	.105	.054	.027	.013	.001
7.125	.102	.050	.022	.010	.002
3, -1, -2	.129	.067	.039	.013	.001
14	.095	.048	.020	.008	.000
4, -1, -3	.142	.066	.034	.017	.002
26	.096	.043	.023	.008	.001
5, 0, -5	.208	.125	.072	.023	.003
50	.125	.068	.034	.010	.001
7, -5, -2	.273	.149	.075	.039	.007
78	.157	.081	.040	.014	.002
9, -1, -8	.393	.231	.139	.067	.005
146	.174	.103	.045	.016	.000
13, -1, -12	.611	.422	.269	.130	.011
314	.236	.105	.055	.025	.002
19, -13, -6	.832	.676	.495	.299	.055
566	.319	.164	.079	.021	.001
20, -0, -20	.924	.809	.637	.397	.081
800	.381	.191	.088	.034	.004
30, -20, -10	.991	.959	.862	.661	.192
1400	.531	.286	.138	.057	.008
50, -0, -50	1.000	1.000	1.000	.990	.670
5000	.785	.478	.264	.114	.006
75, -10, -65	1.000	1.000	1.000	1.000	.934
9945	.908	.584	.335	.132	.010

[illegible]

TABLE 2

$\Sigma \alpha_i^2$	.100	.050	.025	.010	.001
.75, -.05, -.70	.092	.046	.022	.010	.000
1.055	.087	.051	.022	.008	.002
-2, .25, 1.75	.124	.063	.036	.018	.003
7.125	.122	.056	.028	.015	.000
3, -1, -2	.151	.080	.043	.018	.003
14	.130	.074	.036	.012	.000
4, -1, -3	.190	.092	.049	.023	.001
26	.142	.080	.038	.012	.001
5, 0, -5	.321	.179	.099	.049	.007
50	.240	.119	.064	.023	.002
7, -5, -2	.397	.293	.184	.098	.015
78	.312	.182	.094	.028	.002
9, -1, -8	.651	.483	.361	.221	.047
146	.482	.290	.158	.082	.007
13, -1, -12	.910	.821	.705	.537	.157
314	.746	.526	.346	.167	.023
19, -13, -6	.988	.959	.924	.833	.402
566	.920	.787	.585	.314	.053
20, -0, -20	.999	.993	.976	.930	.596
800	.964	.851	.664	.435	.082
24, -22, -2	.999	.997	.991	.974	.768
1064	.938	.930	.791	.560	.103
30, -20, -10	1.000	1.000	1.000	1.000	1.000
1400	.997	.961	.862	.659	.138
75, -10, -65	1.000	1.000	1.000	1.000	1.000
9945	1.000	1.000	1.000	.999	.802

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TABLE 3

Σx_i^2	.100	.050	.025	.010	.001
.75, -.05, -.70 1.055	.124 .116	.073 .061	.041 .028	.016 .013	.000 .001
-2, .25, 1.75 7.125	.175 .162	.101 .081	.052 .041	.022 .019	.002 .003
3, -1, -2 14	.224 .213	.147 .109	.091 .065	.041 .027	.006 .004
4, -1, -3 26	.384 .353	.251 .213	.152 .133	.090 .060	.010 .008
5, 0, -5 50	.539 .500	.410 .349	.289 .221	.170 .122	.035 .014
7, -5, -2 78	.738 .680	.615 .517	.488 .377	.319 .225	.113 .048
9, -1, -8 146	.941 .902	.874 .803	.801 .667	.673 .496	.318 .147
13, -1, -12 314	.999 .992	.995 .981	.985 .946	.965 .870	.795 .448
19, -13, -6 566	1.000 1.000	1.000 .999	1.000 .997	.999 .981	.980 .808
20, -0, -20 800	1.000 1.000	1.000 1.000	1.000 1.000	1.000 .996	1.000 .924
30, -20, -10 1400	1.000 1.000	1.000 1.000	1.000 1.000	1.000 1.000	1.000 .999
50, -0, -50 5000	1.000 1.000	1.000 1.000	1.000 1.000	1.000 1.000	1.000 1.000

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TABLE 4

$\sum_{i=1}^2$	.100	.050	.025	.010	.001
.75, -.05, -.70	.115	.055	.021	.004	.000
1.055	.080	.044	.023	.008	.000
-2, .25, 1.75	.155	.080	.036	.015	.004
7.125	.143	.081	.044	.021	.002
3, -1, -2	.215	.118	.069	.031	.001
14	.219	.124	.064	.031	.004
4, -1, -3	.344	.215	.127	.057	.016
26	.297	.181	.096	.039	.001
5, 0, -5	.571	.413	.259	.142	.026
50	.445	.291	.161	.074	.011
7, -5, -2	.726	.573	.431	.239	.058
78	.625	.420	.238	.113	.014
9, -1, -8	.899	.810	.679	.504	.167
146	.789	.585	.396	.212	.025
13, -1, -12	.998	.973	.935	.840	.499
314	.964	.852	.672	.414	.074
19, -13, -6	1.000	1.000	.996	.984	.829
566	.999	.978	.929	.721	.165
20, -0, -20	1.000	1.000	1.000	.998	.932
800	1.000	.992	.947	.776	.194
24, -22, -2	1.000	1.000	1.000	1.000	.985
1064	1.000	.997	.988	.919	.334
30, -20, -10	1.000	1.000	1.000	1.000	.997
1400	1.000	.999	.997	.965	.467
50, -0, -50	1.000	1.000	1.000	1.000	1.000
5000	1.000	1.000	1.000	.999	.878
75, -10, -65	1.000	1.000	1.000	1.000	1.000
9945	1.000	1.000	1.000	1.000	.988

TABLE 5

$\sum \alpha_i^2$	.100	.050	.025	.010	.001
.75, -.05, -.70 1.055	.127 .104	.064 .060	.027 .026	.012 .016	.002 .003
-2, .25, 1.75 7.125	.185 .158	.099 .090	.055 .053	.023 .025	.003 .001
3, -1, -2 14	.249 .223	.145 .111	.076 .063	.036 .024	.007 .001
4, -1, -3 26	.360 .291	.226 .152	.130 .086	.069 .053	.012 .005
5, 0, -5 50	.559 .438	.407 .269	.274 .157	.170 .080	.032 .005
7, -5, -2 78	.726 .612	.559 .415	.398 .250	.214 .121	.040 .011
9, -1, -8 146	.913 .780	.811 .590	.701 .364	.502 .170	.043 .027
13, -1, -12 314	.995 .954	.984 .856	.953 .685	.864 .429	.496 .081
19, -13, -6 566	1.000 1.000	1.000 .988	.998 .992	.990 .741	.842 .157
20, -0, -20 800	1.000 1.000	1.000 .999	1.000 .975	.999 .741	.951 .157
24, -22, -2 1064	1.000 .997	1.000 .997	1.000 .991	.999 .925	.982 .311
30, -20, -10 1400	1.000 1.000	1.000 1.000	1.000 .996	1.000 .960	.999 .406
50, -0, -50 5000	1.000 1.000	1.000 1.000	1.000 1.000	1.000 1.000	1.000 .868
75, -10, -65 9945	1.000 1.000	1.000 1.000	1.000 1.000	1.000 1.000	1.000 .986

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the ninety-sixth is the fact that the
the ninety-seventh is the fact that the
the ninety-eighth is the fact that the
the ninety-ninth is the fact that the
the hundredth is the fact that the

TABLE 6

$\sum \alpha_i^2$	.100	.050	.025	.010	.001
.75, -.05, -.70	.120	.062	.024	.004	.000
1.055	.117	.061	.028	.012	.001
-2, .25, 1.75	.175	.099	.047	.017	.002
7.125	.151	.074	.033	.012	.005
3, -1, -2	.241	.157	.078	.031	.004
14	.213	.109	.049	.021	.003
4, -1, -3	.329	.205	.132	.066	.009
26	.280	.158	.077	.029	.001
5, 0, -5	.548	.389	.244	.143	.026
50	.393	.234	.130	.054	.004
7, -5, -2	.724	.586	.443	.260	.045
78	.627	.437	.260	.134	.018
9, -1, -8	.906	.803	.668	.494	.169
146	.773	.569	.341	.165	.019
13, -1, -12	.994	.981	.946	.879	.529
314	.965	.890	.673	.418	.051
19, -13, -6	1.000	1.000	.996	.978	.812
566	1.000	.983	.923	.725	.152
20, -0, -20	1.000	1.000	1.000	.999	.938
800	.999	.988	.940	.760	.192
24, -22, -2	1.000	1.000	1.000	1.000	.985
1064	1.000	.996	.990	.916	.333
30, -20, -10	1.000	1.000	1.000	1.000	.997
1400	1.000	1.000	1.000	.957	.428
50, -0, -50	1.000	1.000	1.000	1.000	1.000
5000	1.000	1.000	1.000	1.000	.845
75, -10, -65	1.000	1.000	1.000	1.000	1.000
9945	1.000	1.000	1.000	1.000	.987

[illegible]

TABLE 7

$\sum \alpha_i^2$	.100	.050	.025	.010	.001
.75, -.05, -.70 1.055	.106 .075	.062 .039	.035 .021	.014 .006	.002 .000
-2, .25, 1.75 7.125	.140 .105	.074 .047	.033 .020	.015 .014	.002 .001
3, -1, -2 14	.166 .128	.082 .067	.052 .038	.024 .019	.003 .001
4, -1, -3 26	.201 .161	.132 .085	.075 .044	.033 .016	.004 .002
5, 0, -5 50	.299 .262	.194 .134	.116 .066	.065 .028	.014 .001
7, -5, -2 78	.377 .241	.250 .129	.169 .072	.099 .023	.014 .002
9, -1, -8 146	.590 .456	.455 .289	.337 .181	.208 .095	.048 .012
13, -1, -12 314	.842 .729	.723 .553	.606 .337	.441 .167	.134 .025
19, -13, -6 566	.947 .762	.907 .561	.843 .376	.724 .200	.379 .018
20, -0, -20 800	.993 .978	.979 .879	.935 .681	.864 .420	.520 .065
30, -20, -10 1400	1.000 .975	.998 .887	.992 .714	.981 .461	.824 .068
50, -0, -50 5000	1.000 1.000	1.000 1.000	1.000 1.000	1.000 .994	.999 .537
75, -10, -65 9945	1.000 1.000	1.000 1.000	1.000 1.000	1.000 1.000	1.000 .793

[illegible]

TABLE 8

Σd_i^2	.100	.050	.025	.010	.001
.75, -.05, -.70 1.055	.137 .091	.087 .041	.057 .019	.029 .013	.004 .000
-2, .25, 1.75 7.125	.166 .145	.111 .075	.072 .034	.040 .016	.011 .005
3, -1, -2 14	.259 .333	.180 .189	.129 .101	.083 .043	.017 .005
4, -1, -3 26	.321 .403	.221 .253	.153 .134	.094 .065	.029 .005
5, 0, -5 50	.485 .396	.378 .239	.287 .115	.198 .053	.049 .008
7, -5, -2 78	.734 .954	.587 .843	.431 .670	.265 .429	.069 .076
9, -1, -8 146	.834 .830	.744 .633	.642 .472	.510 .253	.199 .038
13, -1, -12 314	.975 .963	.950 .872	.906 .657	.806 .416	.480 .070
19, -13, -6 566	1.000 1.000	.999 1.000	.994 1.000	.976 .990	.798 .565
20, -0, -20 800	1.000 .999	.999 .986	.998 .932	.991 .739	.884 .171
30, -20, -10 1400	1.000 1.000	1.000 1.000	1.000 1.000	1.000 1.000	.987 .884
50, -0, -50 5000	1.000 1.000	1.000 1.000	1.000 1.000	1.000 .989	1.000 .856
75, -10, -65 9945	1.000 1.000	1.000 1.000	1.000 1.000	1.000 1.000	.990 .994

[illegible]

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