IMPEDANCE - LOADED RECEIVING ANTENNAS WITH MINIMUM BACKSCATTERING AND MAXIMUM RECEIVED POWER

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THESIS

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thesis entitled

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WITH MINIMUM BACKSCATTERING
AND MAXIMUM RECEIVED POWER

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ABSTRACT

IMPEDANCE-LOADED RECEIVING ANTENNAS
WITH MINIMUM BACKSCATTERING
AND MAXIMUM RECEIVED POWER

by Howard Joseph Deck

In addition to serving its recognized intended purpose, a conventional receiving antenna of almost any variety acts inherently as a scatterer, re-radiating a portion of the electromagnetic energy impinging on it. In many practical applications this re-radiation is inconsequential in its effect on the environmental surroundings of the antenna. In other situations, however, such secondary-source effects are undesirable and, in fact, may be highly detrimental to overall system performance. This is true, for example, in situations where several receiving antennas (presumably operating independently) exist in close proximity to each other.

The purpose of this thesis is to achieve a reduction in the amount of electromagnetic power backscattered from a dipole receiving antenna, utilizing techniques of impedance loading. This approach involves inserting lumped impedances directly

into the antenna itself in an effort to modify the distribution of induced current on the antenna in such a way as to effect the reduced scattering desired.

With two such identical "auxiliary impedances" placed symmetrically about the center-load impedance, an approximate closed-form solution is found for the antenna current distribution. The condition of zero broadside backscattering is established in the form of a constraint equation between the auxiliary impedances Z and the center-load impedance Z_L. Using this constraint to eliminate either Z or Z_L, the received power delivered to Z_L is expressed in terms of a single impedance parameter and is subsequently maximized with respect to this single complex variable. This procedure yields values of Z and Z_L for an <u>optimum receiving antenna</u> whose backscattered field in the broadside direction vanishes and (by virtue of the negative real parts of the <u>optimum</u> auxiliary impedances obtained) which produces an infinite amount of received power.

Several cases are investigated in which one of the four real variables contained in $Z = R + jX$ and $Z_L = R_L + jX_L$ is arbitrarily specified. In each case the remaining unspecified impedance parameter values are determined for a <u>near-optimum receiving antenna</u> for which the condition of zero backscattering is satisfied but for which the received power is finite and maximum.

In general, using a purely reactive auxiliary impedance (R specified as zero), the backscattering constraint between Z and Z_L leads to a trivial receiving antenna for which $R_L = 0$. However, a very interesting phenomenon occurs at a specific position of the auxiliary load. At one particular frequency, with the value and placement of the auxiliary reactance properly selected, the resulting antenna is not only invisible to electromagnetic illumination at normal incidence, but is incapable of delivering any power to the center impedance Z_L. Furthermore, this invisible frequency-rejection characteristic is independent of the choice of Z_L.

Relaxing the backscattering constraint between Z and Z_L, it becomes possible to utilize reactive loading to achieve minimum backscattering. Two such cases are considered: one in which Z_L is selected as the complex conjugate of the input impedance of the antenna, and another in which R_L is arbitrarily specified while X_L is chosen as the negative of the input reactance. Corresponding to the latter case, a fairly extensive correlation between theory and experiment is given for a fixed-length antenna for several different positions of loading.

Experimental results are also presented which substantiate the existence of the invisible frequency-rejection characteristic predicted theoretically.

IMPEDANCE-LOADED RECEIVING ANTENNAS
WITH MINIMUM BACKSCATTERING
AND MAXIMUM RECEIVED POWER

By

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INTRODUCTION

In addition to serving its recognized intended purpose, a conventional receiving antenna of almost any variety acts inherently as a scatterer, re-radiating a portion of the electromagnetic energy impinging on it. In many practical applications this re-radiation is inconsequential in its effect on the environmental surroundings of the antenna. In other situations, however, such secondary-source effects are undesirable and, in fact, may be highly detrimental to overall system performance. This is true, for example, in situations where several receiving antennas (presumably operating independently) exist in close proximity to each other.

The purpose of this thesis is to achieve a reduction in the amount of electromagnetic power backscattered from a dipole receiving antenna, utilizing techniques of impedance loading. This approach involves inserting lumped impedances directly into the antenna itself in an effort to modify the distribution of induced current on the antenna in such a way as to effect the reduced scattering desired.

On the presumption that this objective can be accomplished without seriously degrading the primary function of the antenna, the results might well provide a new impetus to the area of antenna design. Not only would such a capability be a potential aid in relieving the interference problem between independently-acting receiving dipoles, but would provide a means for making such structures invisible to radar. Moreover, an impedance loading technique which reduces the broadside re-radiation from individual dipole structures might very well provide a practical means for reducing the mutual coupling between the elements of transmitting antenna arrays, as well as receiving arrays.

I THE DOUBLE-LOADED ANTENNA

1.1 Formulation of the Problem

The configuration of the antenna fundamental to this study is shown in Figure 1.1. The antenna assumed is a perfectly conducting cylinder of radius a and length 2h. A coordinate system is chosen such that the cylinder lies along the z-axis with its center at the origin. At locations $z = \pm d$ two identical impedances Z are inserted "in-line" into infinitesimal gaps in the cylinder. These symmetrically placed loads will be referred to as "auxiliary impedances." When operating as a receiving antenna, it is assumed that the illuminating electromagnetic field is a plane wave, normally incident with its E-field parallel to the z-axis. In this mode, a receiving load impedance Z_L is inserted into the antenna at $z = 0$.

The dimensions of interest are

$$h < \lambda_o \qquad \text{and} \qquad \beta_o a << 1 \tag{1.1}$$

where λ_o is the free space wavelength and $\beta_o = 2\pi / \lambda_o$ is the wave number. For antennas longer than two wavelengths the approximating form which is used for the antenna current distribution

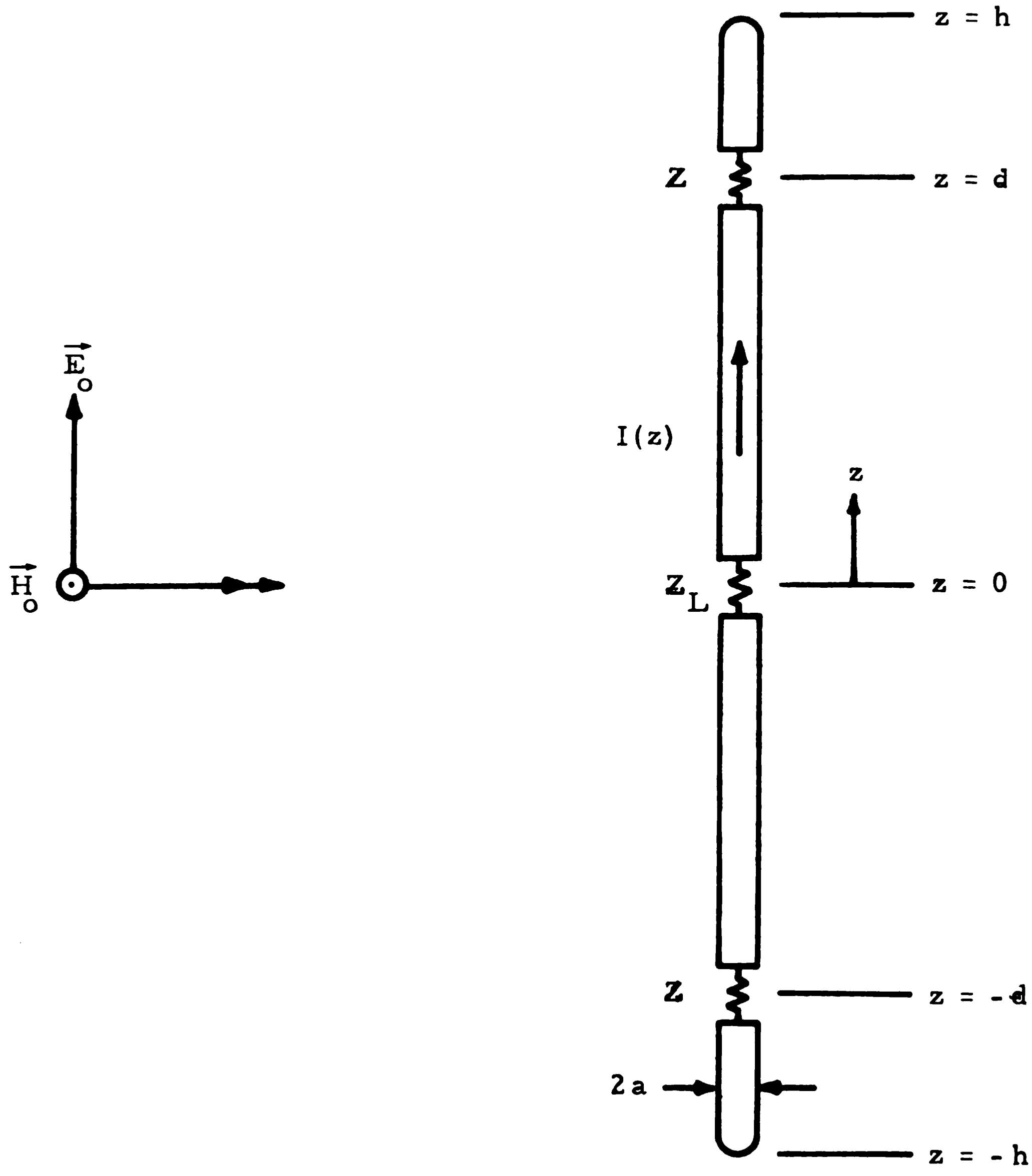


Figure 1. 1. The double-loaded receiving antenna

becomes too inaccurate. The second restriction given in (1.1) implies that the cylinder is thin so that it can be assumed that only an axial current is induced.

1.2 Induced Current on the Receiving Antenna

1.2.1 Integral Equation for the Induced Current

The electric field $\vec{E}_o$ illuminating the receiving antenna is assumed to be normally incident and parallel to the antenna. It is further assumed that E_o varies sinusoidally with time. Since the tangential component of the total electric field must be continuous at the surface of the cylinder, the following equation is valid on the surface.

$$E_z^a + E_o = Z_L I(0)\delta(z) + ZI(d)\delta(z-d) + ZI(-d)\delta(z+d) \quad (1.2)$$

E_z^a is the tangential electric field maintained by the current and charge on the cylinder, $I(p)$ is the induced current at $z = p$, and $\delta(z-p)$ is the Dirac delta function which is identically zero for $z \neq p$. Equation (1.2) indicates that the total tangential electric field vanishes on the surface of the cylinder while maintaining voltages $ZI(-d)$, $Z_L I(0)$, and $ZI(d)$ across the infinitesimal gaps located respectively at $z = -d$, $z = 0$, and $z = d$.

The electric field produced on the surface of the antenna by the antenna itself is given by

$$E_z^a = -\frac{\partial\phi}{\partial z} - j\omega A_z \tag{1.3}$$

where ϕ is the scalar potential at the surface maintained by the antenna charge and A_z is the axial component of the vector potential at the surface supported by the antenna current. The scalar potential ϕ can be eliminated from (1.3) by using the Lorentz condition

$$\frac{\partial A_z}{\partial z} = -j\omega\mu_o\epsilon_o\phi \tag{1.4}$$

where the symbols ω, μ_o, and ϵ_o represent, respectively, the angular frequency of the incident electric field, the permeability of free space, and the permittivity of free space. Differentiating both sides of (1.4) with respect to z and substituting the resulting expression for $\frac{\partial\phi}{\partial z}$ into (1.3) gives the following equation for E_z^a.

$$E_z^a = -\frac{j\omega}{\beta_o^2}\left[\frac{\partial^2 A_z}{\partial z^2} + \beta_o^2 A_z\right] \tag{1.5}$$

Combining equations (1.2) and (1.5) and using the fact that by symmetry I(-d) = I(d), the following differential equation is obtained for A_z.

$$\frac{\partial^2 A_z}{\partial z^2} + \beta_o^2 A_z = \frac{j\beta_o^2}{\omega}\left[Z_L I(0)\,\delta(z) + ZI(d)\{\delta(z-d) + \delta(z+d)\} - E_o\right] \tag{1.6}$$

The general solution of (1.6) is given by

$$A_z(z) = \frac{-j}{v_o} [C_1 \cos\beta_o z + C_2 \sin\beta_o z$$

$$+ E_o/\beta_o - \frac{1}{2} Z_L I(0) \sin\beta_o |z| - \frac{1}{2} Z I(d) \{\sin\beta_o |z-d| + \sin\beta_o |z+d|\}] \tag{1.7}$$

where v_o is the velocity of light in free space and is equal to $1/\sqrt{\mu_o \epsilon_o}$. The first two terms on the right side of equation (1.7) correspond to the complementary solution; C_1 and C_2 are complex constants. The last three terms are particular integrals stemming from the inhomogeneity of the differential equation.

In addition to (1.7) another equation may be written for the tangential component of the vector potential in which $A_z(z)$ is expressed in terms of the induced current which produces it; viz.,

$$A_z(z) = \frac{\mu_o}{4\pi} \int_{-h}^{h} I(z') K_a(z, z') dz' \tag{1.8}$$

where the kernel $K_a(z, z')$ is given by

$$K_a(z, z') = \frac{e^{-j\beta_o \sqrt{(z-z')^2 + a^2}}}{\sqrt{(z-z')^2 + a^2}} \ . \tag{1.9}$$

Based upon the evenness of the function $I(z)$, it can readily be demonstrated from (1.8) that $A_z(z)$ is also an even function. This implies that $C_2 = 0$ in equation (1.7).

Equating (1.7) with (1.8) yields the following integral equation for the induced current. Notice that $\mu_o v_o = 120\pi$.

$$\int_{-h}^{h} I(z')K_a(z,z')dz' =$$

$$\frac{-j}{30}\left[C_1 \cos\beta_o z + \frac{E_o}{\beta_o} - \frac{Z_L I(0)}{2}\sin\beta_o|z| - \frac{ZI(d)}{2}\{\sin\beta_o|z-d| + \sin\beta_o|z+d|\}\right] \tag{1.10}$$

It is expedient at this point to express C_1 in terms of the vector potential at some fixed point on the antenna. This potential can later be conveniently determined from equation (1.8). Selecting z = h as the fixed point and substituting this value of z into (1.7),

$$jv_oA_z(h) = C_1\cos\beta_o h + E_o/\beta_o - \frac{1}{2}Z_L I(0)\sin\beta_o h - \frac{1}{2}ZI(d)\{2\sin\beta_o h\cos\beta_o d\} \tag{1.11}$$

or

$$C_1 = \sec\beta_o h\{jv_oA_z(h) - E_o/\beta_o + \frac{1}{2}Z_L I(0)\sin\beta_o h + ZI(d)\sin\beta_o h\cos\beta_o d\}. \tag{1.12}$$

The result of putting this expression for C_1 back into equation (1.10) is

$$\int_{-h}^{h} I(z')K_a(z,z')dz' = \frac{-j}{30}[\sec\beta_o h\{jv_oA_z(h) - E_o/\beta_o + \frac{1}{2}Z_L I(0)\sin\beta_o h$$
$$+ ZI(d)\sin\beta_o h\cos\beta_o d\}\cos\beta_o z + E_o/\beta_o - \frac{1}{2}Z_L I(0)\sin\beta_o|z|$$
$$- \frac{1}{2}ZI(d)\{\sin\beta_o|z-d| + \sin\beta_o|z+d|\}]. \tag{1.13}$$

A more convenient form of the integral equation is obtained by substituting z = h into (1.13) and subtracting the result from (1.13).

This procedure yields

$$\int_{-h}^{h} I(z')K_d(z, z')dz' =$$

$$\frac{-j}{30} \sec\beta_o h[(jv_o A_z(h) - E_o/\beta_o)(\cos\beta_o z - \cos\beta_o h) + \frac{1}{2} Z_L I(0) \sin\beta_o(h - |z|)$$

$$+ \frac{1}{2} ZI(d)\, g(z)] \tag{1.14}$$

where the new kernel is

$$K_d(z, z') = K_a(z, z') - K_a(h, z') \tag{1.15}$$

and $$g(z) = 2\sin\beta_o h \cos\beta_o d \cos\beta_o z - \cos\beta_o h\{\sin\beta_o|z-d| + \sin\beta_o|z+d|\}. \tag{1.16}$$

Equation (1.14) is an integral equation for the induced current on the double-loaded receiving antenna, valid for $-h \leq z \leq h$. Unlike equation (1.13) the integral in (1.14) is proportional to vector potential difference; as such, the latter equation is automatically satisfied at the ends of the antenna. Notice that $A_z(h)$, $I(0)$, and $I(d)$ on the right side of (1.14) are functions of the induced current being sought and are still unknown.

1.2.2 Solution for the Induced Current

Although an exact, closed form solution to integral equations of the type represented in (1.14) has never been found, a number of methods for obtaining approximate solutions have been advanced in the last several years--notably schemes devised by

King[1, 2, 3], King and Middleton[1], Chen[4, 5, 6], Duncan and Hinchey[7], and Mei[8]. Most of these methods are inherently numerical in nature, based upon techniques such as iteration, numerical integration, or solution by series. Such methods are unsuitable here since the solutions would be too complicated to make further theoretical development feasible.

The technique chosen for obtaining an approximate solution to equation (1.14) is based on a recent method of King[3]. This procedure yields a solution in closed form as desired, and provides a suitable degree of accuracy. The rationale for this method will be indicated briefly as it is used.

The key to the solution technique afforded by King's modified method lies in closely examining the difference kernel $K_d(z, z')$ appearing in integral equation (1.14). The kernel is written in terms of a real and an imaginary part and expanded as follows:

$$K_d(z, z') = K_{dR}(z, z') + j\, K_{dI}(z, z') \qquad (1.17)$$

where

$$K_{dR}(z, z') = \beta_o \left[\frac{\cos \beta_o \sqrt{(z-z')^2 + a^2}}{\beta_o \sqrt{(z-z')^2 + a^2}} - \frac{\cos \beta_o \sqrt{(h-z')^2 + a^2}}{\beta_o \sqrt{(h-z')^2 + a^2}} \right] \qquad (1.18)$$

and

$$K_{dI}(z, z') = \beta_o \left[\frac{\sin \beta_o \sqrt{(h-z')^2 + a^2}}{\beta_o \sqrt{(h-z')^2 + a^2}} - \frac{\sin \beta_o \sqrt{(z-z')^2 + a^2}}{\beta_o \sqrt{(z-z')^2 + a^2}} \right] . \qquad (1.19)$$

It can be readily shown from (1.19) that $|K_{dI}(z, z')|/\beta_o < 2$. On the other hand examination of (1.18) indicates that as z' approaches z, $|K_{dR}(z, z')|$ becomes very large. Indeed, at z' = z, the latter quantity increases without bound as the radius of the antenna approaches zero. This sharp peak in $K_{dR}(z, z')$ indicates that the principal contributions to the part of the integral that has $K_{dR}(z, z')$ as its kernel come from elements of current very near z'= z. Likening $K_{dR}(z, z')$ to a Dirac delta function, moreover, suggests, in view of the right side of equation (1.14), that the form of the current distribution be assumed as

$$I(z) = C_c(\cos\beta_o z - \cos\beta_o h) + C_s \sin\beta_o(h - |z|) + C_i g(z) \quad (1.20)$$

where C_c, C_s, and C_i are complex constants to be determined. Notice that this assumed form for I(z) satisfies the boundary conditions at the ends of the antenna. The substitution of (1.20) and (1.17) into integral equation (1.14) yields

$$\int_{-h}^{h} \{C_c(\cos\beta_o z' - \cos\beta_o h) + C_s \sin\beta_o(h-|z'|) + C_i g(z')\}\{K_{dR}(z, z') + jK_{dI}(z, z')\}dz' = \frac{-j}{30} \sec\beta_o h[(jv_o A_z(h) - E_o/\beta_o)(\cos\beta_o z - \cos\beta_o h) + \frac{1}{2} Z_L I(0)\sin\beta_o(h-|z|) + \frac{1}{2} ZI(d)g(z)]. \quad (1.21)$$

Finally equation (1.21) is broken up into the following three equations:

$$\int_{-h}^{h} C_s \sin\beta_o(h-|z'|)K_{dR}(z,z')dz' = \frac{-j}{30} \sec\beta_o h\left[\frac{1}{2} Z_L I(o)\sin\beta_o(h-|z|)\right] \tag{1.22}$$

$$\int_{-h}^{h} C_i g(z')K_{dR}(z,z')dz' = \frac{-j}{30} \sec\beta_o h\left[\frac{1}{2} Z I(d) g(z)\right] \tag{1.23}$$

$$\int_{-h}^{h} C_c(\cos\beta_o z' - \cos\beta_o h)K_{dR}(z,z')dz' + \int_{-h}^{h} \{C_c(\cos\beta_o z' - \cos\beta_o h)$$
$$+ C_s \sin\beta_o(h-|z'|) + C_i g(z')\}\{jK_{dI}(z,z')\}dz' = \frac{-j}{30} \sec\beta_o h[(jv_o A_z(h)$$
$$- E_o/\beta_o)(\cos\beta_o z - \cos\beta_o h)]. \tag{1.24}$$

The justification for the first two equations in this group is based on the characteristics of the function $K_{dR}(z, z')$ mentioned above. The sharp peak in $K_{dR}(z, z')$ at $z' = z$ also accounts for the presence of the first integral on the left side of equation (1.24). Notice that the remaining terms from the integrand of (1.21) not already accounted for are included in the second integral in (1.24). The justification for the presence of this latter integral is based on numerical considerations. It can be shown numerically that the results obtained by integrating the products $(\cos\beta_o z' - \cos\beta_o h)K_{dI}(z, z')$, $\sin\beta_o(h-|z'|) \cdot K_{dI}(z, z')$, and $g(z')K_{dI}(z, z')$ over the length of the antenna closely approximate, in each case, a function which is proportional to the shifted cosine function $(\cos\beta_o z - \cos\beta_o h)$.

Three complex definite integrals which arise in the process of solving for the constants in equation (1.20) are defined here.

$$T_{cd} = \int_{-h}^{h} (\cos\beta_o z' - \cos\beta_o h) K_d(0, z')dz' \qquad (1.25)$$

$$T_{sd} = \int_{-h}^{h} (\sin\beta_o(h - |z'|) K_d(0, z')dz' \qquad (1.26)$$

$$T_{id} = \int_{-h}^{h} g(z') K_d(0, z')dz' \qquad (1.27)$$

The letter "R" or "I" appended to any of these "T-integrals" will be understood to denote "the real part of" or "the imaginary part of," respectively.

Proceeding with the determination of C_c, C_s, and C_i, equations (1.22), (1.23), and (1.24) are first rewritten with z set to zero.

$$\int_{-h}^{h} C_s \sin\beta_o(h - |z'|) K_{dR}(0, z')dz' = \frac{-j}{30} \sec\beta_o h[\frac{1}{2} Z_L I(0) \sin\beta_o h] \qquad (1.28)$$

$$\int_{-h}^{h} C_i g(z') K_{dR}(0, z')dz' = \frac{-j}{30} \sec\beta_o h[ZI(d)\sin\beta_o(h-d)] \qquad (1.29)$$

$$\int_{-h}^{h} C_c(\cos\beta_o z' - \cos\beta_o h) K_d(0, z')dz' + \int_{-h}^{h} \{C_s \sin\beta_o(h - |z'|) + C_i g(z')\}\{jK_{dI}(0, z')\}dz' = \frac{-j}{30} \sec\beta_o h[(jv_o A_z(h) - E_o/\beta_o)(1 - \cos\beta_o h)] \qquad (1.30)$$

C_s and C_i are found immediately from equations (1.28) and (1.29) as

$$C_s = \frac{-j}{30} \sec\beta_o h \left[\frac{Z_L I(0) \sin\beta_o h}{2T_{sdR}} \right] \tag{1.31}$$

$$C_i = \frac{-j}{30} \sec\beta_o h \left[\frac{Z I(d) \sin\beta_o (h-d)}{T_{idR}} \right] . \tag{1.32}$$

Using (1.31) and (1.32) equation (1.30), after rearrangement, yields

$$C_c = \frac{-j}{30} \sec\beta_o h \left\{ \frac{1}{T_{cd}} \left[(jv_o A_z(h) - E_o/\beta_o)(1 - \cos\beta_o h) - j \left(\frac{Z_L I(0) T_{sdI} \sin\beta_o h}{2\, T_{sdR}} + \frac{Z I(d) T_{idI} \sin\beta_o (h-d)}{T_{idR}} \right) \right] \right\} . \tag{1.33}$$

Substitution of these constants into equation (1.20) produces the following expression for the induced antenna current.

$$I(z) = \frac{-j}{30} \sec\beta_o h \left\{ \frac{1}{T_{cd}} \left[(jv_o A_z(h) - E_o/\beta_o)(1 - \cos\beta_o h) - j \left(\frac{Z_L I(0) T_{sdI} \sin\beta_o h}{2\, T_{sdR}} + \frac{Z I(d) T_{idI} \sin\beta_o (h-d)}{T_{idR}} \right) \right] \cdot [\cos\beta_o z - \cos\beta_o h] + \left[\frac{Z_L I(0) \sin\beta_o h}{2\, T_{sdR}} \right] \sin\beta_o (h - |z|) + \left[\frac{Z I(d) \sin\beta_o (h-d)}{T_{idR}} \right] g(z) \right\} \tag{1.34}$$

Notice that the quantities $A_z(h)$, I(0), and I(d) on the right side of equation (1.34) are still unknown. The currents I(0) and I(d) can be determined from (1.34) itself by successively letting z = 0 and z = d and solving the resulting two equations simultaneously. Making the indicated substitutions in equation (1.34) gives the system of equations

$$K_1 I(0) + K_2 I(d) = F_1 \tag{1.35}$$

$$K_3 I(0) + K_4 I(d) = F_2 \tag{1.36}$$

where

$$K_1 = j30\cos\beta_o h + \frac{Z_L \sin\beta_o h}{2T_{cd}T_{sdR}}\left[jT_{sdI}(1-\cos\beta_o h) - T_{cd}\sin\beta_o h\right] \tag{1.37}$$

$$K_2 = \frac{Z\sin\beta_o(h-d)}{T_{cd}T_{idR}}\left[jT_{idI}(1-\cos\beta_o h) - 2T_{cd}\sin\beta_o(h-d)\right] \tag{1.38}$$

$$K_3 = \frac{Z_L\sin\beta_o h}{2T_{cd}T_{sdR}}\left[jT_{sdI}(\cos\beta_o d - \cos\beta_o h) - T_{cd}\sin\beta_o(h-d)\right] \tag{1.39}$$

$$K_4 = j30\cos\beta_o h + \frac{Z\sin\beta_o(h-d)}{T_{cd}T_{idR}}\left[jT_{idI}(\cos\beta_o d - \cos\beta_o h) - 2T_{cd}\cos\beta_o d\sin\beta_o(h-d)\right] \tag{1.40}$$

$$F_1 = \frac{(jv_o A_z(h) - E_o/\beta_o)(1-\cos\beta_o h)^2}{T_{cd}} \tag{1.41}$$

$$F_2 = \frac{(jv_o A_z(h) - E_o/\beta_o)(1 - \cos\beta_o h)(\cos\beta_o d - \cos\beta_o h)}{T_{cd}} . \quad (1.42)$$

Solving (1.35) and (1.36) yields

$$I(0) = \frac{K_4 F_1 - K_2 F_2}{K_1 K_4 - K_2 K_3} \quad (1.43)$$

$$I(d) = \frac{K_1 F_2 - K_3 F_1}{K_1 K_4 - K_2 K_3} . \quad (1.44)$$

Equations (1.43) and (1.44) are used to eliminate I(0) and I(d) from equation (1.34). After a considerable amount of algebraic manipulation the expression for the antenna current becomes

$$I(z) = K_5[jv_o A_z(h) - E_o/\beta_o][K_c(\cos\beta_o z - \cos\beta_o h) + K_s \sin\beta_o(h - |z|)$$
$$+ K_i g(z)] \quad (1.45)$$

where

$$K_5 = \frac{-j(1 - \cos\beta_o h)}{30\, T_{cd} T_{idR} T_{sdR}(K_1 K_4 - K_2 K_3)} \quad (1.46)$$

$$K_c = Z[Z_L\{\sin\beta_o h \sin\beta_o d \sin^2\beta_o(h-d)\} - j60 T_{sdR} \cos\beta_o d \sin^2\beta_o(h-d)]$$
$$+ Z_L[-j15\, T_{idR} \sin^2\beta_o h] - 900\, T_{idR} T_{sdR} \cos\beta_o h \quad (1.47)$$

$$K_s = \{-\sin\beta_o h\}\{ZZ_L(1-\cos\beta_o d)\sin^2\beta_o(h-d) + Z_L[-j15\, T_{idR}(1-\cos\beta_o h)]\}$$
$$(1.48)$$

$$K_i = \left\{\frac{\sin\beta_o(h-d)}{2}\right\} \{ZZ_L \sin\beta_o h[\sin\beta_o h(1-\cos\beta_o d) - \sin\beta_o d(1-\cos\beta_o h)]$$

$$+ Z[j60T_{sdR}(\cos\beta_o d - \cos\beta_o h)]\} . \qquad (1.49)$$

The vector potential at the end of the antenna is the only remaining unknown in equation (1.45). Another expression relating $A_z(h)$ to the induced antenna current is obtained by setting $z = h$ in equation (1.8).

$$A_z(h) = \frac{\mu_o}{4\pi} \int_{-h}^{h} I(z') K_a(h, z')dz' \qquad (1.50)$$

Substituting equation (1.45) into (1.50) yields

$$A_z(h) = \frac{\mu_o K_5}{4\pi} [jv_o A_z(h) - E_o/\beta_o][K_c T_{ca} + K_s T_{sa} + K_i T_{ia}] \qquad (1.51)$$

where the new complex constants introduced here are defined as

$$T_{ca} = \int_{-h}^{h} (\cos\beta_o z' - \cos\beta_o h) K_a(h, z')dz' \qquad (1.52)$$

$$T_{sa} = \int_{-h}^{h} \sin\beta_o(h - |z'|) K_a(h, z')dz' \qquad (1.53)$$

$$T_{ia} = \int_{-h}^{h} g(z') K_a(h, z')dz' . \qquad (1.54)$$

Multiplying equation (1.51) by jv_o, subtracting E_o/β_o from each side, and rearranging,

$$[jv_o A_z(h) - E_o/\beta_o] = \frac{-E_o/\beta_o}{1 - j30K_5[K_c T_{ca} + K_s T_{sa} + K_i T_{ia}]} . \tag{1.55}$$

Putting (1.55) back into equation (1.45) completely determines the antenna current. After rearrangement and simplification, the induced current on the receiving antenna may finally be expressed as

$$I(z) = K[K_c(\cos\beta_o z - \cos\beta_o h) + K_s \sin\beta_o(h-|z|) + K_i g(z)]. \tag{1.56}$$

The complex coefficients in equation (1.56) are functions of the antenna dimensions, the value of the center-load impedance, and the value and position of the symmetrically placed auxiliary impedances. K_c, K_s, and K_i are given explicitly in terms of these parameters in equations (1.47), (1.48), and (1.49). The function g(z) is given by equation (1.16). Factor K can be expressed as

$$K = \frac{-\left(\frac{j}{15}\right)(1 - \cos\beta_o h)}{Z(AZ_L + B) + CZ_L + D} \cdot \frac{E_o}{\beta_o} \tag{1.57}$$

where

$$A = -\sin\beta_o h \sin\beta_o(h-d)\{2\sin\beta_o(h-d)[F_4 \sin\beta_o d + T_{sa}(1 - \cos\beta_o h)(1 - \cos\beta_o d)]$$
$$- T_{ia}F_3(1 - \cos\beta_o h) + j\cos\beta_o h[T_{idI}F_3 - 2T_{sdI}(1 - \cos\beta_o d)\sin\beta_o(h-d)]\} \tag{1.58}$$

$$B = j60T_{sdR}\sin\beta_o(h-d)\{2F_4\cos\beta_o d\sin\beta_o(h-d)$$

$$+ [\cos\beta_o d - \cos\beta_o h][T_{ia}(1-\cos\beta_o h) - jT_{idI}\cos\beta_o h]\} \qquad (1.59)$$

$$C = j30T_{idR}\sin\beta_o h\{F_4\sin\beta_o h + [1-\cos\beta_o h][T_{sa}(1-\cos\beta_o h)$$

$$- jT_{sdI}\cos\beta_o h]\} \qquad (1.60)$$

$$D = 1800\,T_{idR}T_{sdR}F_4\cos\beta_o h \qquad (1.61)$$

$$F_3 = \sin\beta_o h - \sin\beta_o d - \sin\beta_o(h-d) \qquad (1.62)$$

$$F_4 = (T_{ca} + T_{cd})\cos\beta_o h - T_{ca} \qquad (1.63)$$

1.3 Current Distribution on the Transmitting Antenna

In the design techniques developed in the next three chapters for optimizing receiving antenna performance it will be quite desirable to have at hand an expression for the input impedance of the double-loaded antenna. In order to determine the ratio of applied voltage to current at the driving point of the transmitting antenna, it is necessary to first solve for the current distribution on the driven antenna. The solution for this current is included here since the mathematical development closely parallels the technique of the preceding section used in finding the induced current on the receiving antenna.

When operating as a transmitting antenna the center-load Z_L shown in Figure 1.1 is replaced by a sinusoidal voltage driver having a specified terminal voltage V. Proceeding exactly as before the continuity of the tangential component of the total electric field at the surface of the antenna is expressed as

$$E_z^a = -V\delta(z) + ZI^t(d)\delta(z-d) + ZI^t(-d)\delta(z+d). \quad (1.64)$$

As before E_z^a represents the tangential electric field maintained on the antenna surface by the current and charge on the antenna, $I^t(p)$ is the current at $z = p$, and $\delta(z-p)$ is the Dirac delta function.

This equation is quite similar to equation (1.2) which gives E_z^a for the double-loaded receiving antenna. In fact the latter equation is obtainable from the former simply by setting the incident field to zero and replacing the factor $Z_L I(0)$ with $-V$. Since these factors are not functions of the z coordinate and since the symmetry condition $I^t(-d) = I^t(d)$ applies in the case of the transmitting antenna as well as for the receiving antenna, it follows immediately that the integral equation for the current on the transmitting antenna is identical to that found for the receiving antenna with $Z_L I(0)$ replaced with $-V$ and $E_o = 0$.

Thus, from equation (1.14),

$$\int_{-h}^{h} I^t(z')K_d(z,z')dz' =$$

$$\frac{-j}{30}\sec\beta_o h[(jv_o A_z(h))(\cos\beta_o z - \cos\beta_o h) - \frac{1}{2} V\sin\beta_o(h-|z|)$$
$$+\frac{1}{2} ZI^t(d)g(z)]. \qquad (1.65)$$

Again the current is assumed to be of the form

$$I^t(z) = C_c(\cos\beta_o z - \cos\beta_o h) + C_s \sin\beta_o(h-|z|) + C_i g(z), \qquad (1.66)$$

given in the previous section as equation (1.20).

It follows, therefore, that the results of the previous section prior to the elimination of I(0) are, subject to the simple modification indicated above, immediately applicable here. Thus, substituting -V for $Z_L I(0)$ and setting $E_o = 0$ in equation (1.34), the current on the transmitting antenna is

$$I^t(z) = \frac{-j}{30}\sec\beta_o h\left\{\frac{1}{T_{cd}}\left[(jv_o A_z(h))(1-\cos\beta_o h) - j\left(\frac{-VT_{sdI}\sin\beta_o h}{2T_{sdR}}\right.\right.\right.$$
$$\left.\left.+\frac{ZI^t(d)T_{idI}\sin\beta_o(h-d)}{T_{idR}}\right)\right]\cdot[\cos\beta_o z - \cos\beta_o h]$$
$$\left.+\left[\frac{-V\sin\beta_o h}{2T_{sdR}}\right]\sin\beta_o(h-|z|) + \left[\frac{ZI^t(d)\sin\beta_o(h-d)}{T_{idR}}\right] g(z)\right\}. \qquad (1.67)$$

The terms $I^t(d)$ and $A_z(h)$ are eliminated from the right side of equation (1.67) by the same process as was used in the previous section. $I^t(d)$ is determined by substituting $z = d$ into equation (1.67) after which equation (1.50) is used to determine $A_z(h)$. When the details of this procedure are carred out, equation (1.67) becomes

$$I^t(z) = K^t[K_c^t(\cos\beta_o z - \cos\beta_o h) + K_s^t \sin\beta_o(h-|z|) + K_i^t g(z)] \quad (1.68)$$

where

$$K^t = \frac{V \tan\beta_o h}{BZ + D} \quad (1.69)$$

$$K_c^t = Z\sin^2\beta_o(h-d)[(1 - \cos\beta_o h)(T_{ia} - 2T_{sa}\cos\beta_o d) - j\cos\beta_o h(T_{idI} - 2T_{sdI}\cos\beta_o d)] + 30T_{idR}\cos\beta_o h[T_{sdI}\cos\beta_o h + jT_{sa}(1 - \cos\beta_o h)] \quad (1.70)$$

$$K_s^t = Z\sin\beta_o(h-d)[-2F_4\cos\beta_o d\sin\beta_o(h-d) + (\cos\beta_o d - \cos\beta_o h)(jT_{idI}\cos\beta_o h - T_{ia}\{1 - \cos\beta_o h\})] + j30F_4T_{idR}\cos\beta_o h \quad (1.71)$$

$$K_i^t = Z\sin\beta_o(h-d)[F_4\sin\beta_o(h-d) + (\cos\beta_o d - \cos\beta_o h)(T_{sa}\{1 - \cos\beta_o h\} - jT_{sdI}\cos\beta_o h)]. \quad (1.72)$$

1.4 Input Impedance of the Transmitting Antenna

An analytical form for the input impedance of the double-loaded transmitting antenna is readily obtained from the result of Section 1.3. The current at the center of the transmitting antenna is found by substituting z = 0 into equation (1.68).

$$I^t(0) = K^t[K_c^t(1 - \cos\beta_o h) + K_s^t \sin\beta_o h + K_i^t \{2\sin\beta_o(h-d)\}] \quad (1.73)$$

After substituting the expressions for K^t, K_c^t, K_s^t, and K_i^t defined in the previous section and simplifying, equation (1.73) becomes

$$I^t(0) = V\left[\frac{AZ + C}{BZ + D}\right] . \quad (1.74)$$

Thus, the input impedance of the double-loaded transmitting antenna is given by

$$Z_{in} = \frac{V}{I^t(0)} = \frac{BZ + D}{AZ + C} \quad (1.75)$$

where the factors A, B, C, and D, defined by equations (1.58), (1.59), (1.60), and (1.61), are functions of the antenna dimensions and the position of the auxiliary impedances.

1.5 Reduction to the Unloaded Dipole Antenna

Closed form solutions for the current distributions on the double-loaded transmitting and receiving dipole antennas were developed in the preceding sections. These results can be readily reduced for application to the ordinary unloaded dipole antenna simply by setting the values of the auxiliary impedances to zero or by stipulating that these impedances be located at the very ends of the antenna. Reduction of the results to the unloaded dipole is included here for the sake of completeness.

1.5.1 Current Distribution on the Unloaded Receiving Antenna

The current on the unloaded receiving dipole antenna is found by specifying Z = 0 in equation (1.56). Thus,

$$I(z) = K(Z=0)[K_c(Z=0)(\cos\beta_o z - \cos\beta_o h) + K_s(Z=0)\sin\beta_o(h-|z|) + K_i(Z=0)g(z)] \quad (1.76)$$

where the complex coefficients are given by (1.47), (1.48), (1.49), and (1.57) with Z set to zero. After inserting the expressions for these constants and simplifying, equation (1.76) becomes

$$I(z) = K_o[(-Z_L \sin^2\beta_o h + j60T_{sdR}\cos\beta_o h)(\cos\beta_o z - \cos\beta_o h) + Z_L \sin\beta_o h(1 - \cos\beta_o h)\sin\beta_o(h-|z|)] \quad (1.77)$$

where K_o =

$$\frac{(1 - \cos\beta_o h)\left(\frac{E_o}{\beta_o}\right)}{\left[\begin{array}{c} j30Z_L \sin\beta_o h[F_4 \sin\beta_o h + (1-\cos\beta_o h)(T_{sa}\{1-\cos\beta_o h\} - jT_{sdI}\cos\beta_o h)] \\ + 1800\, T_{sdR} F_4 \cos\beta_o h \end{array}\right]} \quad (1.78)$$

Equation (1.77) gives the current induced on an ordinary receiving dipole antenna with center-load Z_L situated parallel to the field E_o of a normally incident electromagnetic plane wave. This expression is very similar to the result determined by Chen and Liepa[4], differing only in the use of T_{sdR} in place of T_{sd} and the inclusion of the term in the denominator of K_o involving the factor T_{sdI}. It is expected that equation (1.77) is a better representation of the true current distribution than the result of Chen and Liepa. This statement is based upon the results of a direct comparison of the two methods (the technique of Chen and Liepa as compared with King's modified method) of separating the integral equation for the current on the transmitting antenna. This test consists simply of evaluating both sides of the integral equation numerically at several points on the antenna--first using one solution form and then the other. For antennas of lengths one wavelength or less this comparison indicates a substantial margin of accuracy in favor of King's method.

1.5.2 Current Distribution on the Unloaded Transmitting Antenna

The current on the unloaded transmitting antenna is found by specifying Z = 0 in equation (1.68). Thus,

$$I^t(z) = K^t(Z=0)[K_c^t(Z=0)(\cos\beta_o z - \cos\beta_o h) + K_s^t(Z=0)\sin\beta_o(h-|z|)$$
$$+ K_i^t(Z=0)g(z)] \qquad (1.79)$$

where the complex coefficients are given by (1.69), (1.70), (1.71), and (1.72) with Z set to zero. After inserting the expressions for these constants and simplifying, equation (1.79) becomes

$$I^t(z) = K_o^t[(T_{sdI}\cos\beta_o h + jT_{sa}\{1-\cos\beta_o h\})(\cos\beta_o z - \cos\beta_o h) + jF_4\sin\beta_o(h-|z|)] \qquad (1.80)$$

where
$$K_o^t = \frac{V\tan\beta_o h}{60T_{sdR}F_4}. \qquad (1.81)$$

Equation (1.80) gives the current distribution on an ordinary transmitting dipole antenna center-driven by a slice generator of potential difference V. This expression is in exact agreement with King's result[3].

1.5.3 Input Impedance of the Unloaded Antenna

As the values of the auxiliary impedances are reduced to zero the expression for the input impedance of the double-loaded transmitting antenna found in Section 1.4 becomes that of an ordinary unloaded dipole. Thus, substituting Z=0 in equation (1.75),

$$Z_{in} = \frac{D}{C} = \frac{60T_{sdR}F_4\cos\beta_o h}{\left[\begin{array}{c}\sin\beta_o h[(T_{sdI}\cos\beta_o h + jT_{sa}\{1-\cos\beta_o h\})(1-\cos\beta_o h) \\ + jF_4\sin\beta_o h]\end{array}\right]}. \qquad (1.82)$$

Alternatively, (1.82) may be obtained as the ratio $V/I^t(0)$ using equation (1.80).

II THE OPTIMUM RECEIVING ANTENNA

Traditionally the load impedance connected to the terminals of an ordinary dipole receiving antenna is chosen purely from the standpoint of absorbing as much power as possible from the antenna. The fact that the receiving antenna acts as a scattering element re-radiating in virtually every direction some portion of the energy intercepted from the incident field is noted as an inherent secondary effect in the system and is generally given no further consideration.

It is well known that the load impedance which will result in the extraction of maximum power from a dipole receiving antenna of specified dimensions is equal to the complex conjugate of the input impedance of the antenna. In recognition of this fact and in the interest of simplicity and expediency, one of the most popularly used dipole receiving antennas is the nominal half-wave dipole, corresponding in length to the so-called first resonance condition in which the reactive part of the input impedance vanishes. With an antenna length corresponding to one of the resonant lengths the conjugate match condition is achieved with a purely resistive load, obviating the need for reactive tuning at the antenna terminals. In addition to the obvious physical advantage of being the shortest antenna in the resonant length class the half-wave dipole exhibits a reasonably convenient value of input resistance--approximately 72 ohms.

In the half-wave dipole example it is noteworthy that no real effort has been given to truly maximize the received power with respect to each of the available design parameters. Rather, the length of the antenna is selected for reasons not directly related to power expectations; not until after the physical dimensions of the antenna are specified is an attempt made to maximize the power receiving capability.

Chen[4, 6], in his efforts to reduce the radar cross section of thin metallic cylinders, has demonstrated that the backscattered power from objects of this shape can be reduced very substantially using a central impedance loading technique. Although Chen's final objectives in this study are distinctly different from the considerations associated with the design of a receiving antenna, the physical system in his work may, nevertheless, be properly regarded as an ordinary dipole receiving antenna. By controlling the distribution of the induced current on such cylinders through his choice of values for the center-load impedance, Chen is able to theoretically achieve the condition of zero broadside backscattering. Although the load impedances corresponding to the zero backscatter condition predicted by his work are complex and, in fact, passive for cylinders less than one wavelength long, cylinders loaded in this manner do not make effective receiving antennas. Compared with a cylinder having the same dimensions but with a conjugate-matched center load, the receiving power capabilities of the former are on the order of 20 dB less efficient. Moreover, this is probably a grossly conservative estimate of the actual inefficiency; when the work of Chen is recast using the more accurate approximation to the induced cylinder current afforded by King's modified method, it turns out that the resistive part of the center load required for the condition of zero backscatter vanishes altogether. This result is shown in Section 2.3 where it follows as a reduced case in the study of the double-loaded receiving antenna.

It is clear from the above discussion that the specification of zero backscattered power and the simultaneous requirement of maximum received power, or at least a power receiving capability comparable to that obtained in the conjugate-match case, are mutually incompatible objectives insofar as an ordinary dipole antenna is concerned. What is needed in order to accomplish these goals simultaneously is the inclusion of some additional degrees of freedom in the antenna system. This added flexibility is provided by the double-loaded receiving antenna introduced in

Chapter 1. The three new real parameters corresponding to the resistive and reactive components of the auxiliary impedance Z and the distance d at which these impedances are symmetrically placed about the center of the antenna allow for considerable control over the antenna current distribution. Consequently, by adapting this antenna configuration the designer gains rather wide latitudes in his ability to demand a particular scattering characteristic while simultaneously insisting that his antenna deliver a respectable amount of power to the receiving load.

It is appropriate at this point to lay down some specific performance requirements for the double-loaded receiving antenna and to proceed with the determination of the parameters in the system which will produce these desired characteristics. The first and most stringent set of performance specifications to be considered is expressed explicitly in the following definition of the optimum receiving antenna.

Definition: The Optimum Receiving Antenna is defined as a double-loaded dipole receiving antenna of given dimensions and having the placement d of the auxiliary impedances Z specified for which the broadside backscattered power vanishes and for which, subject to the first constraint, the power delivered to the center-load Z_L is maximized with respect to each of the impedance parameters Z and Z_L.

Although the received power is clearly a function of not only Z and Z_L but also the antenna dimensions a and h and the placement d of the auxiliary impedances, notice that the latter group of parameters is held fixed in the definition of the optimum receiving antenna. This rather artificial limitation of variables reduces to a manageable task the analytical work involved in determining the optimum values of Z and Z_L without actually narrowing in any way the scope of the investigation. Were it attempted to retain any combination of the parameters a, h, and d as variables in this definition, the procedure for maximizing the received power would become extremely difficult due to the highly implicit manner in which these parameters appear in the equation for the induced antenna current. On the other hand the influence which these quantities have on the power delivered to Z_L can eventually be determined rather easily by displaying graphically the manner in which the received power of the optimum receiving antenna changes as one or more of these parameters is varied over some appropriate range of values.

With both Z and Z_L completely variable it turns out theoretically that it is possible to obtain infinite received power for any choice of the parameters a, h, and d. Thus, the problem of choosing from the set of all optimum receiving antennas the one yielding the most received power does not arise. In Chapter 3, however, some one of the impedance parameter values is assigned, thus reducing by one the number of degrees of freedom in the maximization of received power. For the resulting near-optimum receiving antennas (to be defined almost exactly the same as the optimum antenna but with some one of the impedance parameters fixed), the maximum obtainable value of received power varies as a, h, and d are varied and so the distinction of a precise definition for a near-optimum receiving antenna becomes much more important.

2.1 The Condition for Zero Backscattering--The Constraint Equation

The electric field produced in the broadside direction and in the radiation zone of a thin cylindrical radiating element in free space is given by

$$E^s = -j30\beta_o \frac{e^{-j\beta_o R_o}}{R_o} \int_{-h}^{h} I(z')dz' \; . \qquad (2.1)$$

This is a well known result from elementary antenna theory. In equation (2.1) E^s is the electric field at an observation point located a distance R_o from the center of the radiating cylinder; $I(z')$ represents the complex current distribution on the cylinder of length 2h. As before $\beta_o = 2\pi/\lambda_o$ is the wave number associated with the free space wavelength λ_o.

The power density at this same far-zone observation point is

$$P_s = \frac{1}{2\zeta_o} |E^s|^2 \tag{2.2}$$

or, substituting (2.1) for E^s,

$$P_s = \frac{1}{2\zeta_o} \left(\frac{30\beta_o}{R_o}\right)^2 \left| \int_{-h}^{h} I(z')dz' \right|^2 . \tag{2.3}$$

Here, $\zeta_o = \sqrt{\mu_o/\epsilon_o}$ is the impedance of free space and is equal to 120π ohms.

If the radiating element in question happens to be a dipole receiving antenna which is being illuminated by a parallel electric field at normal incidence, as illustrated in figure 1.1, then equation (2.3) represents the power density of the backscattered field maintained by this receiving antenna. Specifically, when (1.56) is substituted for I(z') in equation (2.3) the backscattered power density from the double-loaded receiving antenna is found to be:

$$P_s = \frac{1}{2\zeta_o} \left(\frac{30\beta_o}{R_o}\right)^2 \left| \int_{-h}^{h} K[K_c(\cos\beta_o z' - \cos\beta_o h) + K_s \sin\beta_o(h - |z'|) + K_i g(z')]dz' \right|^2 . \tag{2.4}$$

Performing the indicated integration, this becomes

$$P_s = \frac{15}{\pi}\left(\frac{|K|}{R_o}\right)^2 \left| K_c(\sin\beta_o h - \beta_o h\cos\beta_o h) + K_s(1-\cos\beta_o h) + 2K_i(\cos\beta_o d - \cos\beta_o h)\right|^2 . \qquad (2.5)$$

After substituting the defining relations for the K-coefficients from Chapter 1 into (2.5) and carrying out a moderate amount of re-arrangement and simplification, the backscattered power density in the far-zone of the double-loaded receiving antenna can be written:

$$P_s = \frac{(1-\cos\beta_o h)^2 (E_o/\beta_o)^2}{15\pi R_o^2} \left| \frac{Z(FZ_L+G)+HZ_L+L}{Z(AZ_L+B)+CZ_L+D} \right|^2 \qquad (2.6)$$

where A, B, C, and D are defined in (1.58) through (1.61) and where

$$F = \sin\beta_o h \sin\beta_o(h-d)[\sin\beta_o(h-d)\{\sin\beta_o d(\sin\beta_o h-\beta_o h\cos\beta_o h) - (1-\cos\beta_o h)(1-\cos\beta_o d)\} + F_3(\cos\beta_o d-\cos\beta_o h)] \qquad (2.7)$$

$$G = j60T_{sdR}\sin\beta_o(h-d)[(\cos\beta_o d-\cos\beta_o h)^2-\cos\beta_o d(\sin\beta_o h -\beta_o h\cos\beta_o h)\sin\beta_o(h-d)] \qquad (2.8)$$

$$H = j15T_{idR}\sin\beta_o h[(1-\cos\beta_o h)^2-\sin\beta_o h(\sin\beta_o h-\beta_o h\cos\beta_o h)] \qquad (2.9)$$

$$L = -900T_{idR}T_{sdR}\cos\beta_o h(\sin\beta_o h-\beta_o h\cos\beta_o h) . \qquad (2.10)$$

To make the backscattered field vanish the numerator of (2.6) is set to zero, imposing the following condition on the ultimate selection of the impedance parameters Z and Z_L.

$$Z[FZ_L + G] + HZ_L + L = 0 \qquad (2.11)$$

Once the antenna dimensions and the placement of the auxiliary impedances have been specified, Z and Z_L must be chosen subject to this constraint equation if the condition of zero backscattering is to be achieved.

The fact that in equation (2.11) the coefficients F and L are real while G and H are imaginary places one rather severe restriction on the implementation of a zero-backscattering receiving antenna: specifying Z as being purely reactive in the constraint equation leads ultimately to an antenna which is incapable of delivering any power whatsoever to the receiving load Z_L. This will be demonstrated in Chapter 3 in connection with the investigation of some near-optimum cases.

2.2 The Condition for Maximum Received Power

The received power delivered to the center-load Z_L of a dipole receiving antenna is given by

$$P_R = \frac{1}{2} |I(0)|^2 R_L \qquad (2.12)$$

where I(0) is the complex current at the center of the antenna and R_L is the real part of the center load Z_L.

The current existing at the center of the double-loaded receiving antenna is obtained from equation (1.56) with $z = 0$.

$$I(0) = K[K_c(1 - \cos\beta_o h) + K_s \sin\beta_o h + 2K_i \sin\beta_o (h-d)] \tag{2.13}$$

After substituting the defining expressions for the K-coefficients and simplifying, equation (2.13) becomes

$$I(0) = \frac{J(MZ + N)}{Z(AZ_L + B) + CZ_L + D} \tag{2.14}$$

where the new coefficients appearing in the numerator of I(0) are defined as

$$J = -4T_{sdR} \cos\beta_o h(1 - \cos\beta_o h)(E_o/\beta_o) \tag{2.15}$$

$$M = (1 - \cos\beta_o d)\sin^2\beta_o(h-d) \tag{2.16}$$

$$N = -j15T_{idR}(1 - \cos\beta_o h) . \tag{2.17}$$

The received power yielded by the double-loaded antenna is obtained by substituting (2.14) into equation (2.12).

$$P_R = \frac{R_L}{2} \left| \frac{J(MZ + N)}{Z_L(AZ + C) + BZ + D} \right|^2 \tag{2.18}$$

For an ordinary unloaded dipole antenna equation (2.18) reduces to

$$P_R = \frac{R_L}{2} \left| \frac{JN}{CZ_L + D} \right|^2 , \quad \text{or} \tag{2.19}$$

$$P_R = \frac{R_L}{2} \left| \frac{V'}{Z_L + Z'_g} \right|^2 \qquad (2.20)$$

where $$V' = JN/C \quad \text{and} \quad Z'_g = D/C \; . \qquad (2.21)$$

In Equation (2.20) V' and Z'_g can be interpreted as the voltage driver and its series impedance in an equivalent circuit representation of the receiving antenna system as viewed from the antenna terminals. The maximum power transfer theorem from classical circuit theory, applicable in this situation since V' and Z'_g are independent of Z_L, requires that Z_L be chosen as the complex conjugate of Z'_g in order to effect maximum power transfer to the load Z_L. As evidenced by comparing (2.21) with (1.82) in Chapter 1, Z'_g in Equation (2.20) corresponds to the input impedance of the unloaded antenna; thus, this is just a reiteration of the well known conjugate-match condition for an ordinary dipole receiving antenna.

It should be emphasized that in the application of the maximum power transfer theorem, it is presumed that the specified generator impedance Z'_g is passive; the theorem accordingly predicts the value of a second passive impedance Z_L which, when connected across the terminals of the equivalent network composed of the series combination of V' and Z'_g, results in the absorption of maximum power by Z_L. Should the real part of Z'_g be negative, on the other hand, this theorem per se has no applicability to this simple generator-load network; for the delivery of maximum power to Z_L in the latter case one should obviously choose $Z_L = -Z'_g$.

Acknowledgment of the implications of the above statements is helpful in visualizing the unique power delivering capabilities of a multiply-loaded dipole receiving antenna. Notice that the expression given in (2.18) for the received power of a double-loaded antenna can also be cast in the form of (2.20) wherein

$$V' = J(MZ + N)/(AZ + C) \text{ and } Z'_g = (BZ + D)/(AZ + C). \qquad (2.22)$$

As before, Z'_g corresponds to the antenna's input impedance--this time, of the double-loaded antenna, as evidenced by comparing (2.22) with equation (1.75) in Chapter 1. With the value of Z fixed in (2.22) exactly the same arguments hold regarding the selection of Z_L as were made above in connection with the unloaded antenna. However, whereas the input impedance of an ordinary unloaded antenna is inherently passive, so that the value of Z_L can always be selected meaningfully on the basis of the maximum power transfer theorem, the use of auxiliary impedances having negative real parts in the implementation of a multiply-loaded antenna immediately admits the possibility of producing a receiving antenna having an input impedance with a negative real part. In this situation one can theoretically obtain an infinite amount of received power by choosing $Z_L = -Z_{in}$.

That the received power of the double-loaded antenna can be expressed, innately, in the form shown in (2.20) has not been established; this follows from the fact that equation (2.18) for P_R was derived using only an approximate solution for the induced antenna current. It is possible, however, to demonstrate the validity of (2.20) for the double-loaded receiving antenna on the basis of fundamental considerations, in a manner which requires no foreknowledge of the current distribution. This is shown in the Appendix.

2.3 Selection of the Optimum Parameters

In accordance with the definition for the optimum receiving antenna, values for the impedance parameters Z and Z_L are sought, subject to the constraint that the broadside backscattered field maintained by the antenna vanishes, such that the received power delivered to Z_L is maximized with respect to each of the parameters Z and Z_L. Elimination of the backscattered field is achieved by imposing the condition of constraint equation (2.11) on the relationship between Z and Z_L. Equation (2.11) can be rearranged to give either

$$Z = -\left[\frac{HZ_L + L}{FZ_L + G}\right] \quad \text{or} \tag{2.23}$$

$$Z_L = -\left[\frac{GZ + L}{FZ + H}\right] \quad . \tag{2.24}$$

The expression formulated in Section 2.2 for the received power produced by the double-loaded antenna is repeated below for convenience.

$$P_R = \frac{R_L}{2}\left|\frac{J(MZ + N)}{Z_L(AZ + C) + BZ + D}\right|^2 \tag{2.18}$$

Upon substituting constraint (2.24) into equation (2.18), P_R becomes

$$P_R = \frac{R_L}{2}\left|\frac{J\{\mathcal{K}Z^2 + \mathcal{A}Z + \mathcal{B}\}}{\mathcal{C}Z^2 + \mathcal{D}Z + \mathcal{E}}\right|^2 \tag{2.25}$$

where

$$\left.\begin{aligned} \mathcal{K} &= FM \\ \mathcal{A} &= FN + HM \\ \mathcal{B} &= HN \\ \mathcal{C} &= AG - BF \\ \mathcal{D} &= AL + CG - BH - DF \\ \mathcal{E} &= CL - DH \end{aligned}\right\} \quad . \tag{2.26}$$

As is evident from equation (2.25) the optimum values of Z occur at the poles of the current I(0) and are determined as the solutions of the complex quadratic equation

$$\mathcal{C}[Z]_{op}^{2} + \mathcal{B}[Z]_{op} + \mathcal{E} = 0 \, . \tag{2.27}$$

The corresponding optimum values of the center-load impedance, $[Z_L]_{op}$, are obtained from constraint equation (2.24). Notice that forcing the denominator of P_R to vanish in (2.18) or (2.25) is equivalent to requiring that $Z_L = - Z_{in}$, as discussed in the previous section, where from equation (1.75)

$$Z_L = - Z_{in} = - \left[\frac{BZ + D}{AZ + C} \right] \, . \tag{2.28}$$

Thus, the quadratic expression for $[Z]_{op}$ in (2.27) can be formulated, alternatively, simply by equating the right-hand sides of (2.24) and (2.28).

There are several considerations in regard to the optimum receiving antenna which deserve some comment at this point. First of all, there is no guarantee that the numerical values of the optimum parameters found from equations (2.27) and (2.24) will make sense physically; only if the real part of $[Z_L]_{op}$ is positive will these results be meaningful insofar as a receiving antenna is concerned. Whether the optimum values of R_L are positive or negative depends entirely upon the placement d of the auxiliary impedances and the specified dimensions of the antenna. Furthermore it would be naive indeed to expect the real parts of $[Z]_{op}$ and $[Z_L]_{op}$ to be positive simultaneously in view of the unique power delivering capability required of this antenna.

Secondly, notice that although the denominator in equation (2.6) for the backscattered power density vanishes at the optimum values of Z and Z_L, P_s remains zero because the condition of constraint equation (2.11) is satisfied; that the denominator of (2.6) approaches zero as the impedance parameters approach their optimum values is merely indicative of the fact that the current distribution on the antenna is accordingly increasing without bound.

Finally, in the interest of completeness and in order to provide some degree of corroboration of the results obtained here with the work of other investigators in the area of scatter reduction from thin metallic cylinders, the constraint equation for zero backscattering derived in Section 2.1 is reduced to yield the following two special cases. First, specifying Z = 0 in (2.24) gives the zero-backscatter value of impedance for a center-loaded cylinder.

$$Z_L = -\frac{L}{H} = \frac{-j60T_{sdR}[1-\beta_o h\cot\beta_o h]}{2\cos\beta_o h - 2 + \beta_o h\sin\beta_o h} \tag{2.29}$$

Except for the factor T_{sd} in place of T_{sdR}, (2.29) is in exact agreement with the impedance value predicted by Chen[4,6] for eliminating the broadside backscattered field from a center-loaded cylinder. That the form given in (2.29) differs in its real part from Chen's result is a consequence of the difference between Chen's method and King's modified method for approximating the induced antenna current.

In the second special case center-load Z_L is set to zero, corresponding to a cylinder loaded with two impedances Z placed symmetrically at distances d about the center. For this situation constraint equation (2.23) is used and

$$Z = -\frac{L}{G}$$

$$= \frac{-j15T_{idR}\cos\beta_o h(\sin\beta_o h-\beta_o h\cos\beta_o h)}{\sin\beta_o(h-d)[(\cos\beta_o d-\cos\beta_o h)^2-\cos\beta_o d(\sin\beta_o h-\beta_o h\cos\beta_o h)\sin\beta_o(h-d)]} . \tag{2.30}$$

Again, by virtue of the T-integral term (T_{id} instead of T_{idR}), Chen's[5] value for Z, in addition to having a reactive component identical to (2.30), also possesses a resistive component.

2.4 Theoretical Results

Numerical values of the optimum impedance parameters are given in Figures 2.1 and 2.2 and Table 2.1 for several representative antenna lengths, according to the formulas developed in Section 2.3. Whereas the quadratic equation (2.27) yields two values for $[Z]_{op}$, it will be noted that only one set of optimum parameter values is indicated in the graphical and tabular data which follow. The reason for this is that, invariably, over the range of antenna lengths investigated, the real part of one of the two values of $[Z_L]_{op}$ obtained from (2.24) and (2.27) has an extremely small magnitude--typically, in the range between 10^{-3} and 10^{-9} ohms. The magnitude of the second value of $[R_L]_{op}$ obtained from these formulas, on the other hand, is much larger--in most cases, greater than one ohm. Therefore, for practical reasons, the value of $[Z_L]_{op}$ with the smaller real part is discarded, and all numerical results shown here correspond to the larger value of $[R_L]_{op}$.

The curves in Figure 2.1 show the optimum center-load resistance $[R_L]_{op}$ for all possible positions of the auxiliary impedances $[Z]_{op}$. Notice, in each case, that $[R_L]_{op}$ is positive over only a portion of the complete range of loading positions. Since the results corresponding to negative values of $[R_L]_{op}$ have no meaning in the case of a receiving antenna, a tabular, rather than graphical, format has been chosen for representing only those sets of optimum parameter values which have applicability to a receiving antenna.

In addition, one complete set of optimum parameter specifications is furnished by the curves in Figure 2.2, corresponding to the case of $\beta_o h = 2.5$. This is included merely to provide some indication of the manner in which the complement set of parameters--$[R]_{op}$, $[X]_{op}$, and $[X_L]_{op}$--vary as the loading position d ranges over all possible values.

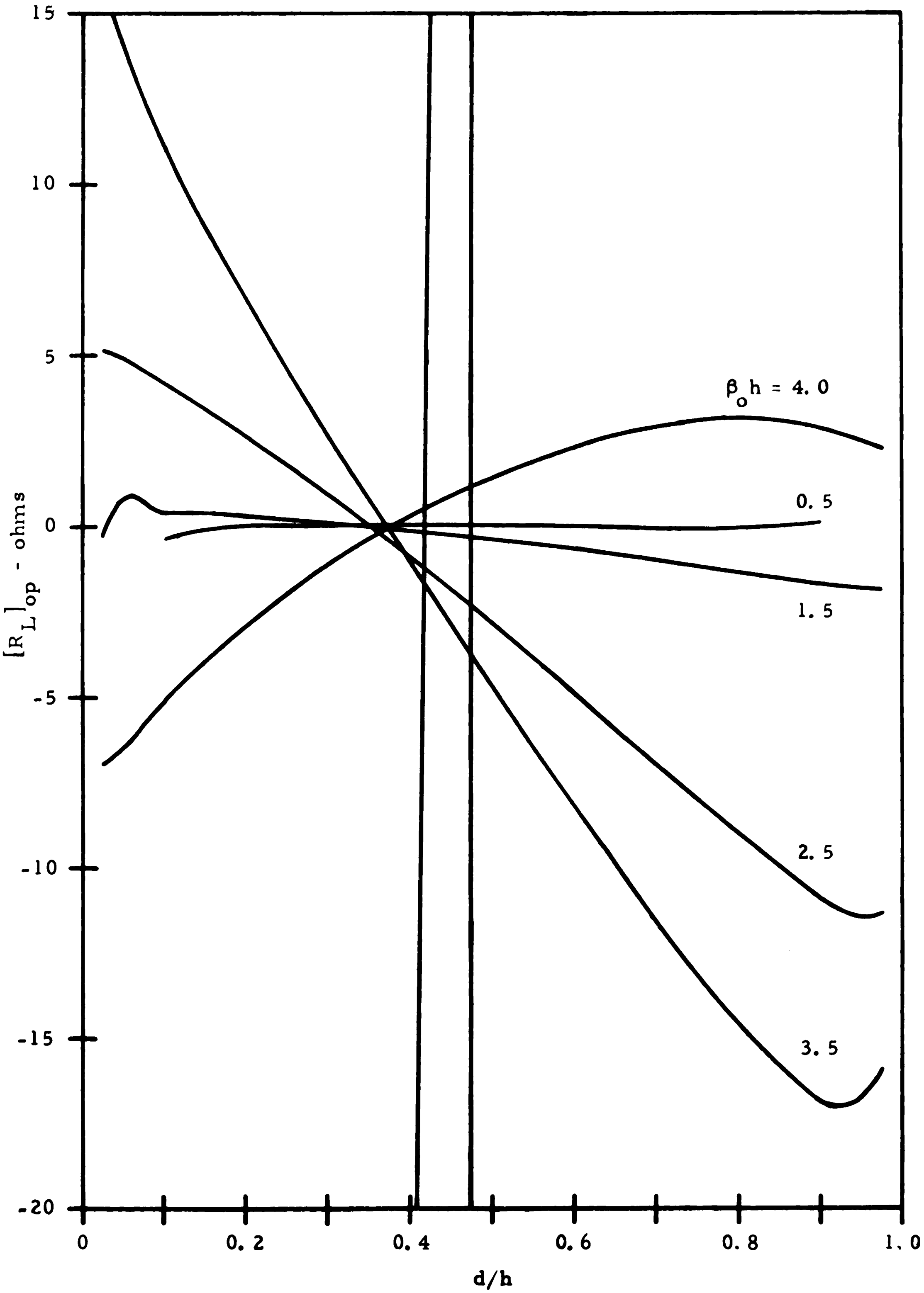


Figure 2. 1. Center-load resistance for the Optimum Receiving Antenna ($\beta_o a$ = 0. 001)

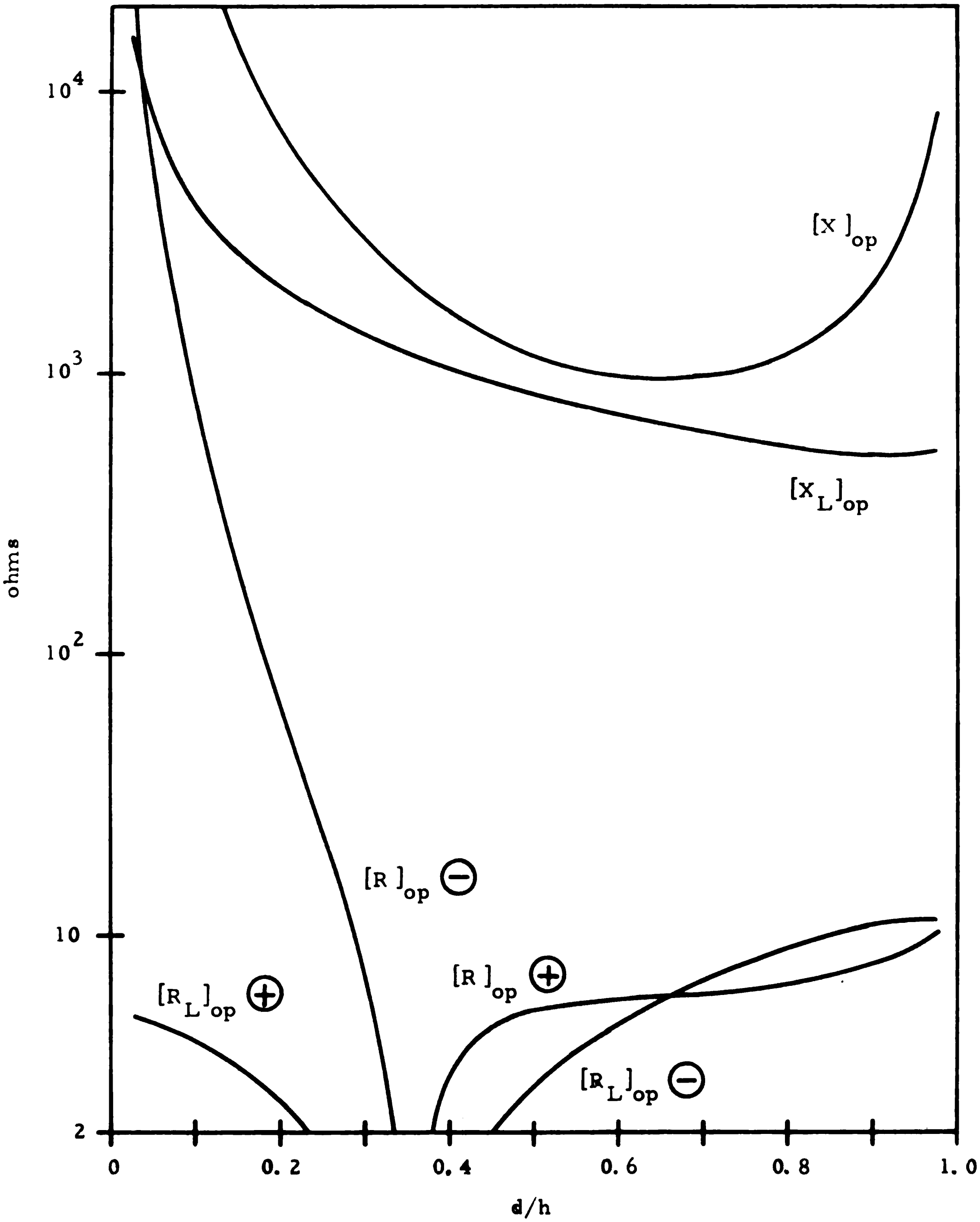


Figure 2.2. Optimum Receiving Antenna parameter values for $\beta_o h = 2.5$ ($\beta_o a = 0.001$)

Table 2.1. Parameter values for the Optimum Receiving Antenna in ohms ($\beta_o a = 0.001$)

$\beta_o h$	d/h	$[R_L]_{op}$	$[X_L]_{op}$	$[R]_{op}$	$[X]_{op}$
0.5	0.2	0.00380	7560	-0.0508	20800
	0.3	0.00714	5450	-0.0423	10600
	0.6	0.0130	3340	-0.0187	4530
	0.9	0.0931	2640	-0.0548	7380
1.0	0.1	0.0967	8020	-5.66	44300
	0.2	0.0777	4340	-1.10	12200
	0.3	0.0159	3110	-0.0964	6080
1.5	0.1	0.405	5850	-28.1	35100
	0.2	0.295	3130	-4.62	9200
	0.3	0.0866	2210	-0.551	4410
2.0	0.1	1.56	4690	-151	32700
	0.2	0.955	2460	-18.0	7820
	0.3	0.311	1720	-2.15	3520
2.5	0.1	4.18	3990	-768	37000
	0.2	2.59	2040	-67.6	7380
	0.3	0.894	1380	-7.11	2980
3.0	0.1	11.2	4130	-10700	74800
	0.2	6.85	2030	-300	8320
	0.3	2.52	1330	-22.3	2700
3.5	0.1	11.0	2230	-61700	-350000
	0.2	6.58	1030	-4810	19400
	0.3	2.60	632	-66.9	2800
4.0	0.4	0.238	245	-1.90	1020
	0.5	1.39	126	-3.32	515
	0.6	2.28	45.2	-2.77	375
	0.7	2.86	-11.3	-2.48	362
	0.8	3.10	-48.6	-2.55	469
	0.9	2.88	-61.4	-3.23	979

Table 2.1 (continued)

$\beta_o h$	d/h	$[RL]_{op}$	$[X_L]_{op}$	$[R]_{op}$	$[X]_{op}$
4.5	0.4	0.601	-6.13	-8.53	1070
	0.5	39.4	-120	-92.8	385
	0.6	73.2	-197	-72.3	248
	0.7	102	-249	-67.9	230
	0.8	125	-283	-81.1	304
	0.9	134	-291	-134	678
5.0	0.5	220	-466	-403	471
	0.6	443	-451	-309	203
	0.7	577	-374	-276	175
	0.8	636	-294	-320	269
	0.9	634	-252	-458	714
5.5	0.5	1400	-1070	-266	1020
	0.6	1220	502	-726	714
	0.7	734	600	-643	582
	0.8	534	539	-403	702
	0.9	450	462	-224	996
6.0	0.5	20900	4240	-22.6	1100
	0.6	296	1450	-219	1290
	0.7	141	882	-186	1050
	0.8	91.9	644	-57.5	716
	0.9	70.0	496	-23.3	822

III NEAR-OPTIMUM RECEIVING ANTENNAS

In this chapter and the next, several special cases of the double-loaded receiving antenna are considered in which one or more of the impedance parameters--R_L, X_L, R, X--is arbitrarily specified. Although, by definition, the optimum receiving antenna of Chapter 2 is unattainable when a pre-condition exists on Z or Z_L, by proper utilization of the remaining unspecified parameters, it is possible to achieve performance characteristics which, although less spectacular than those of the optimum receiving antenna, may be very desirable for certain applications.

As shown below, when only one of the four impedance parameters is specified beforehand, it is always possible to achieve the condition of zero backscattering; furthermore, unless the specified pre-condition is that R = 0 or $R_L = 0$, one degree of freedom will remain with respect to which the received power can be maximized. Antennas such as these will be called near-optimum receiving antennas, since they produce no backscattering, but have only a finite power delivering capability.

The condition for zero backscattering found in Section 2.1 is repeated below for convenience.

$$Z[FZ_L + G] + HZ_L + L = 0 \tag{2.11}$$

As pointed out in Section 2.3, constraint equation (2.11) can be rearranged to give either

$$Z = -\left[\frac{HZ_L + L}{FZ_L + G}\right] \tag{2.23}$$

or

$$Z_L = -\left[\frac{GZ + L}{FZ + H}\right] . \tag{2.24}$$

Given that either Z or Z_L is partially or totally specified beforehand, the condition for zero backscattering can be enforced by choosing the remaining free impedance value according to (2.23) or (2.24). However, constraint equation (2.11) contains one rather important implication about the interrelation of Z and Z_L which is not obvious from any of the forms above; not until the right-hand sides of (2.23) and (2.24) are written in such a way that the real and imaginary parts can be distinguished, does this feature become apparent.

Noting from the defining relations (2.7) through (2.10) that the coefficients in (2.11) are not strictly complex (F and L are real, while G and H are imaginary), equations (2.11), (2.23), and (2.24) can be expanded and simplified to give:

$$fRR_L - X(fX_L + g') - h'X_L + \ell = 0 \tag{3.1a}$$

$$fXR_L + R(fX_L + g') + h'R_L = 0 \tag{3.1b}$$

$$Z = -\left\{\frac{(f\ell + g'h')R_L}{(fR_L)^2 + (fX_L + g')^2}\right\} + j\left\{\frac{-fh'(R_L^2 + X_L^2) + (f\ell - g'h')X_L + g'\ell}{(fR_L)^2 + (fX_L + g')^2}\right\} \tag{3.2}$$

$$Z_L = -\left\{\frac{(f\ell + g'h')R}{(fR)^2 + (fX + h')^2}\right\} + j\left\{\frac{-fg'(R^2 + X^2) + (f\ell - g'h')X + h'\ell}{(fR)^2 + (fX + h')^2}\right\} \tag{3.3}$$

where Z and Z_L have been represented as $Z = R + jX$ and $Z_L = R_L + jX_L$, and where the convention of designating the imaginary part of a complex number by a prime and the real part without a prime has been adopted in writing

$$F = f, \quad G = jg', \quad H = jh', \quad L = \ell. \tag{3.4}$$

The important consideration revealed by (3.3) is that the arbitrary specification of $R = 0$ implies, in general, that $R_L = 0$. The single exception to this implication is considered in Section 3.1.

It is clear from these preliminary comments that the condition of zero backscattering can always be satisfied while arbitrarily specifying any <u>one</u> of the impedance parameters--R_L, X_L, R, or X--(although the particular pre-condition that $R = 0$ or, of course, $R_L = 0$ leads to a trivial receiving antenna). On this basis, the following definition is given for a near-optimum receiving antenna.

Definition: A <u>Near-Optimum Receiving Antenna</u> is defined as a double-loaded dipole receiving antenna of given dimensions, having the placement d of the auxiliary impedances Z specified, and having no more than one of the impedance parameters--R_L, X_L, R, X--arbitrarily pre-selected, for which the broadside backscattered power vanishes and for which, subject to the above constraints, the power delivered to the center-load Z_L is maximized with respect to each of the impedance parameters Z and Z_L.

3.1 Pure Reactive Loading

With the pre-condition that R = 0, the auxiliary impedances are purely reactive and the backscattering constraint as represented by the two real equations in (3.1) reduces to

$$(fX_L + g')X + h'X_L - \ell = 0 \qquad (3.5a)$$

$$(fX + h')R_L = 0 \; . \qquad (3.5b)$$

Equation (3.5b) is satisfied for either $R_L = 0$ or $X = -h'/f$. Discarding the first as trivial, and substituting the second into (3.5a) yields the following result.

$$(f\ell + g'h') = 0 \qquad (3.6)$$

Since the terms in (3.6) are independent of the impedance parameters, but are functions only of the antenna dimensions and the placement of the auxiliary loads, (3.6) is not satisfied, in general, for an arbitrary loading position; it must be concluded, therefore, according to the definition, that a near-optimum antenna does not exist in the case of pure reactive loading.

The implications of (3.6), nevertheless, are rather interesting. If a, h, and d can be chosen in such a way as to satisfy this equation, a receiving antenna can be produced for which the backscattered field automatically vanishes independent of the choice of Z_L, simply by choosing $X = -h'/f$. Not at all surprising, however, is the fact that these very same conditions simultaneously force the current at the center of the antenna to vanish; this is readily demonstrated by using (3.6) in the numerator of equation (2.14) for I(0) with $Z = -jh'/f$.

In spite of the fact that (3.6) leads to a trivial receiving antenna at the design frequency (corresponding to $\beta_o = 2\pi/\lambda_o$), such a loading scheme suggests a practical means for providing signal suppression at some other frequency. A double-loaded receiving antenna requiring only reactive auxiliary loading which has the ability to reject a given (unwanted) signal frequency from the receiving load, while at the same time achieving the condition of radar invisibility at the same frequency, might well be desirable in certain applications. The condition which (3.6) places on the loading position d for such an antenna is investigated below.

Using the defining relations (2.7) through (2.10), the left-hand side of (3.6), after simplifying, can be written

$$\ell\ell + g'h' = 900\, T_{idR} T_{sdR} \sin\beta_o h \sin\beta_o (h-d) \cdot \{(\sin\beta_o h - \beta_o h \cos\beta_o h)\sin\beta_o(h-d) - (1-\cos\beta_o h)(\cos\beta_o d - \cos\beta_o h)\}^2 . \tag{3.7}$$

The result of setting the right side of (3.7) to zero indicates that, given the antenna dimensions a and h, loading position d must be chosen such that

$$Q\cos\beta_o d + S\sin\beta_o d + T = 0 \tag{3.8}$$

where

$$\left.\begin{aligned} Q &= (\sin\beta_o h - \beta_o h\cos\beta_o h)\sin\beta_o h - (1-\cos\beta_o h) \\ S &= -(\sin\beta_o h - \beta_o h\cos\beta_o h)\cos\beta_o h \\ T &= (1-\cos\beta_o h)\cos\beta_o h \end{aligned}\right\} . \tag{3.9}$$

The substitution of $\cos\beta_o d = \sqrt{1 - \sin^2\beta_o d}$ in (3.8) leads to the following quadratic equation in $\sin\beta_o d$.

$$\sin^2\beta_o d + \left(\frac{2\,ST}{S^2 + Q^2}\right) \sin\beta_o d + \left(\frac{T^2 - Q^2}{S^2 + Q^2}\right) = 0 . \tag{3.10}$$

In general, equation (3.10) produces only one meaningful result for d; a second value, obvious from inspection of (3.7) is the trivial solution d = h.

The value and the position of the auxiliary reactance required in the realization of a double-loaded receiving antenna having a rejection characteristic at the frequency corresponding to β_o are shown in Figure 3.1 as a function of antenna length. The fact that these results are based on only an approximate solution for the antenna current should be appreciated. It might well be, for example, that in practice a non-zero value of R is required to implement such an antenna. Moreover, it is possible that the conditions determined here for making I(0) vanish are merely indicative of a minimum of I(0) and not a true null.

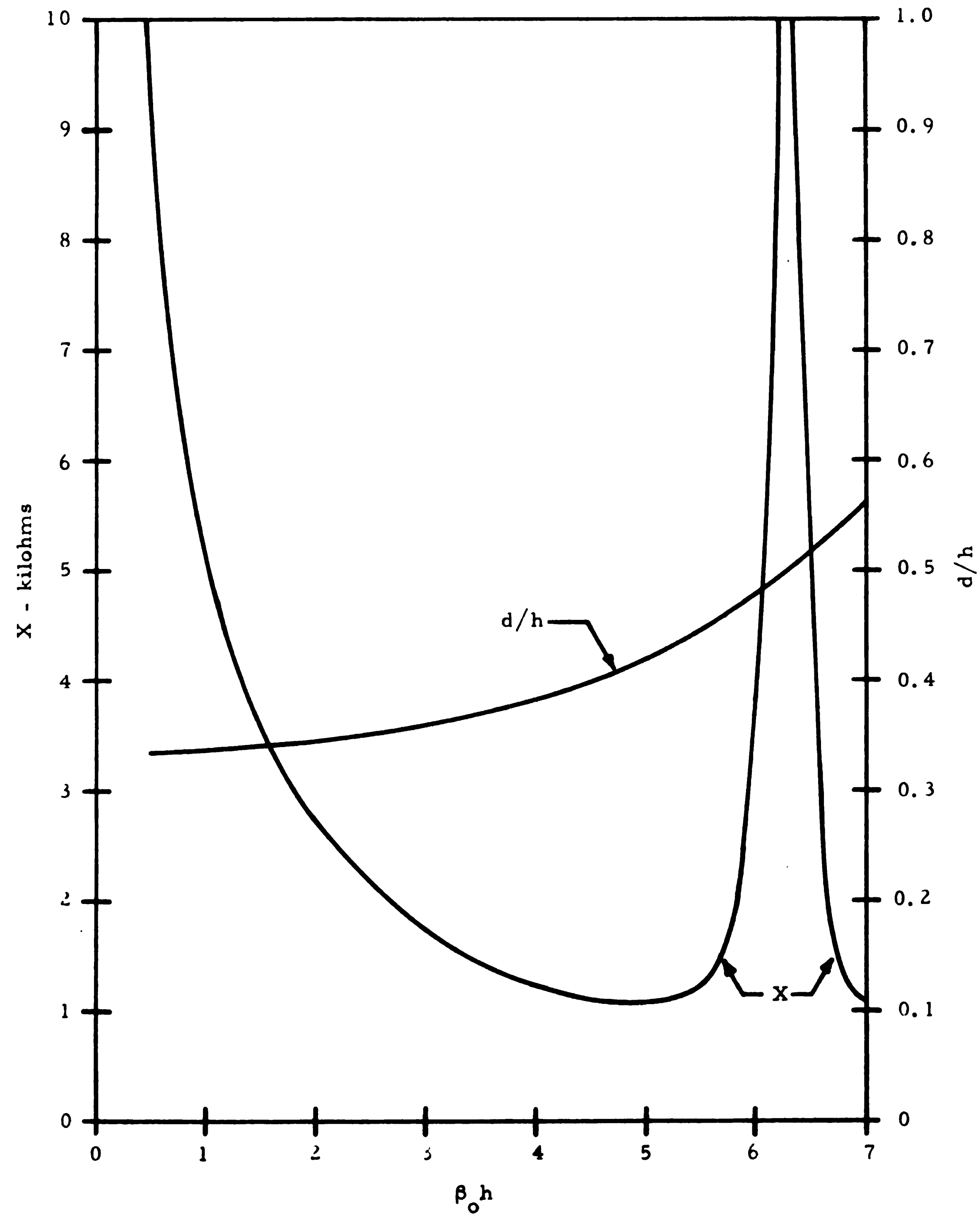


Figure 3.1. Auxiliary reactance and loading position for a receiving antenna with a frequency-rejection characteristic at $\omega = \beta_o v_o$ $(\beta_o a = 0.001)$

3.2 Pure Resistive Loading

When the auxiliary impedance is specified to be of the form $Z = R + j0$, the broadside backscattered field will automatically vanish if, setting $X = 0$ in (3.3), the center-load impedance is chosen as

$$Z_L = -\left\{\frac{(f\ell + g'h')R}{(fR)^2 + h'^2}\right\} + j\left\{\frac{-fg'R^2 + h'\ell}{(fR)^2 + h'^2}\right\} . \qquad (3.11)$$

From (2.25) in Section 2.3, the power received by an antenna loaded in this manner is

$$P_R = \left[\frac{R_L(Z)}{2}\left|\frac{J\{\mathcal{X}Z^2 + \mathcal{A}Z + \mathcal{B}\}}{\mathcal{C}Z^2 + \mathcal{D}Z + \mathcal{E}}\right|^2\right]_{Z=R+j0} . \qquad (3.12)$$

Using $R_L(Z = R)$ from (3.11), equation (3.12) becomes

$$P_R = \frac{|J|^2}{2}\left|\frac{\mathcal{X}R^2 + \mathcal{A}R + \mathcal{B}}{\mathcal{C}R^2 + \mathcal{D}R + \mathcal{E}}\right|^2 \cdot \left\{\frac{-(f\ell + g'h')R}{(fR)^2 + h'^2}\right\} . \qquad (3.13)$$

In (3.13) the received power is a function of the single variable R. At this point, according to the definition of a near-optimum receiving antenna, R is sought such that the function P_R is maximum. Before this maximization procedure can be carried out, however, the right side of (3.13) must be rearranged somewhat.

After breaking up the complex terms into real and imaginary parts:

$$\left.\begin{aligned} \mathcal{K} &= k + j0 & \mathcal{C} &= c + jc' \\ \mathcal{A} &= 0 + ja' & \mathcal{D} &= d + jd' \\ \mathcal{B} &= b + j0 & \mathcal{E} &= e + je' \end{aligned}\right\}, \tag{3.14}$$

equation (3.13) can be expanded and simplified to the form

$$P_R = \frac{|J|^2}{2}\left[\frac{u_4R^4 + u_2R^2 + u_0}{v_4R^4 + v_3R^3 + v_2R^2 + v_1R + v_0}\right]\left[\frac{w_1R}{y_2R^2 + y_0}\right] \tag{3.15}$$

where

$$\left.\begin{aligned} u_4 &= k^2 & v_4 &= c^2 + c'^2 \\ u_2 &= a'^2 + 2bk & v_3 &= 2(cd + c'd') \\ u_0 &= b^2 & v_2 &= d^2 + d'^2 + 2(ce + c'e') \\ w_1 &= -(f\ell + g'h') & v_1 &= 2(de + d'e') \\ y_2 &= f^2 & v_0 &= e^2 + e'^2 \\ y_0 &= h'^2 \end{aligned}\right\} . \tag{3.16}$$

After carrying out the multiplication indicated in (3.15), this equation can be put into the form*

$$P_R = \frac{w_1|J|^2}{2}\left\{\frac{\sum_{n=0}^{6} a_nR^n}{\sum_{n=0}^{6} b_nR^n}\right\} \tag{3.17}$$

*Rational functions similar to the one in (3.17) are encountered in the next section and in Chapter 4. As in the case of (3.17), these functions are to be maximized or minimized with respect to a single variable. Thus, although the numerator in (3.17) is actually a fifth degree polynomial, it is purposely indicated as sixth degree, so that the results obtained here in connection with maximizing $P_R(R)$ can be applied again later by simply re-defining the coefficients a_i and b_i in (3.18).

where

$$\left.\begin{array}{llll} a_6 = 0 & & b_6 = v_4 y_2 \\ a_5 = u_4 & & b_5 = v_3 y_2 \\ a_4 = 0 & & b_4 = v_4 y_0 + v_2 y_2 \\ a_3 = u_2 & & b_3 = v_3 y_0 + v_1 y_2 \\ a_2 = 0 & & b_2 = v_2 y_0 + v_0 y_2 \\ a_1 = u_0 & & b_1 = v_1 y_0 \\ a_0 = 0 & & b_0 = v_0 y_0 \end{array}\right\} . \qquad (3.18)$$

To find the maxima of the function in (3.17), it is sufficient to differentiate the quantity within the braces; the critical points of P_R occur at the zeros of the resulting rational function of R. The differentiation is carried out most readily if the summing notation indicated in (3.17) is retained in the process--the result is

$$\frac{d}{dR}\left\{\frac{\sum\limits_{n=0}^{6} a_n R^n}{\sum\limits_{n=0}^{6} b_n R^n}\right\} = \frac{\sum\limits_{i=0}^{6}\sum\limits_{n=1}^{6} n[a_n b_i - a_i b_n] R^{i+n-1}}{\left\{\sum\limits_{n=0}^{6} b_n R^n\right\}^2} . \qquad (3.19)$$

Setting the right side of (3.19) to zero implies that

$$\left.\begin{array}{l} \sum\limits_{m=0}^{11}\left\{\sum\limits_{n=1}^{6} n[a_n b_{m-n+1} - a_{m-n+1} b_n]\right\} R^m = 0 \\ \\ (a_i,\ b_i \equiv 0 \quad \text{for} \quad i < 0 \quad \text{and} \quad i > 6) \end{array}\right\} \qquad (3.20)$$

where the summation indices have been altered in order to show each term A_iR^i of the polynomial explicitly. The leading coefficient A_{11} vanishes identically so that the degree of the polynomial in (3.20) is actually only ten.

Although clearly not an odd function of R, $P_R(R)$ exhibits characteristics not unlike an odd function: given that the quantity $(f\ell + g'h')$ is positive, it is evident from inspection of (3.13) that R and P_R have unlike signs (that, in general, $(f\ell + g'h')$ is positive follows from (3.3) in view of the fact that in Chapter 2 $[R]_{op}$ and $[R_L]_{op}$ have unlike signs); moreover, P_R approaches zero as R becomes infinitely large through either positive or negative values; finally, in numerically extracting the roots of the polynomial in (3.20), only two real critical points of the function P_R are obtained, and in every case considered, these occur as $\pm|R_c|$. It follows, therefore, that $+|R_c|$ corresponds to the only minimum of $P_R(R)$, and $-|R_c|$ corresponds to the only maximum. Of the two roots, only $R = -|R_c|$ leads to a receiving antenna which is meaningful; $R = +|R_c|$ yields negative values of P_R by virtue of requiring negative values of center-load resistance.

Accordingly, there exists but one set of impedance parameter values for the resistive-loaded near-optimum receiving antenna. These results are presented graphically for antennas of several different lengths in the figures which follow. The curves for Z_L represent plots of equation (3.11) with $R = -|R_c|$. The received power is shown by curves of power gain, in which the performance of this near-optimum antenna is compared with that of an ordinary dipole antenna of the same dimensions having a conjugate-matched center load. The pronounced dip in each of the curves of R_L and P_R occurs at the particular loading position d for which (3.6) is satisfied. In order to sustain the condition of zero backscattering, the equations in (3.1) demand that R_L vanish for this placement of the auxiliary loads.

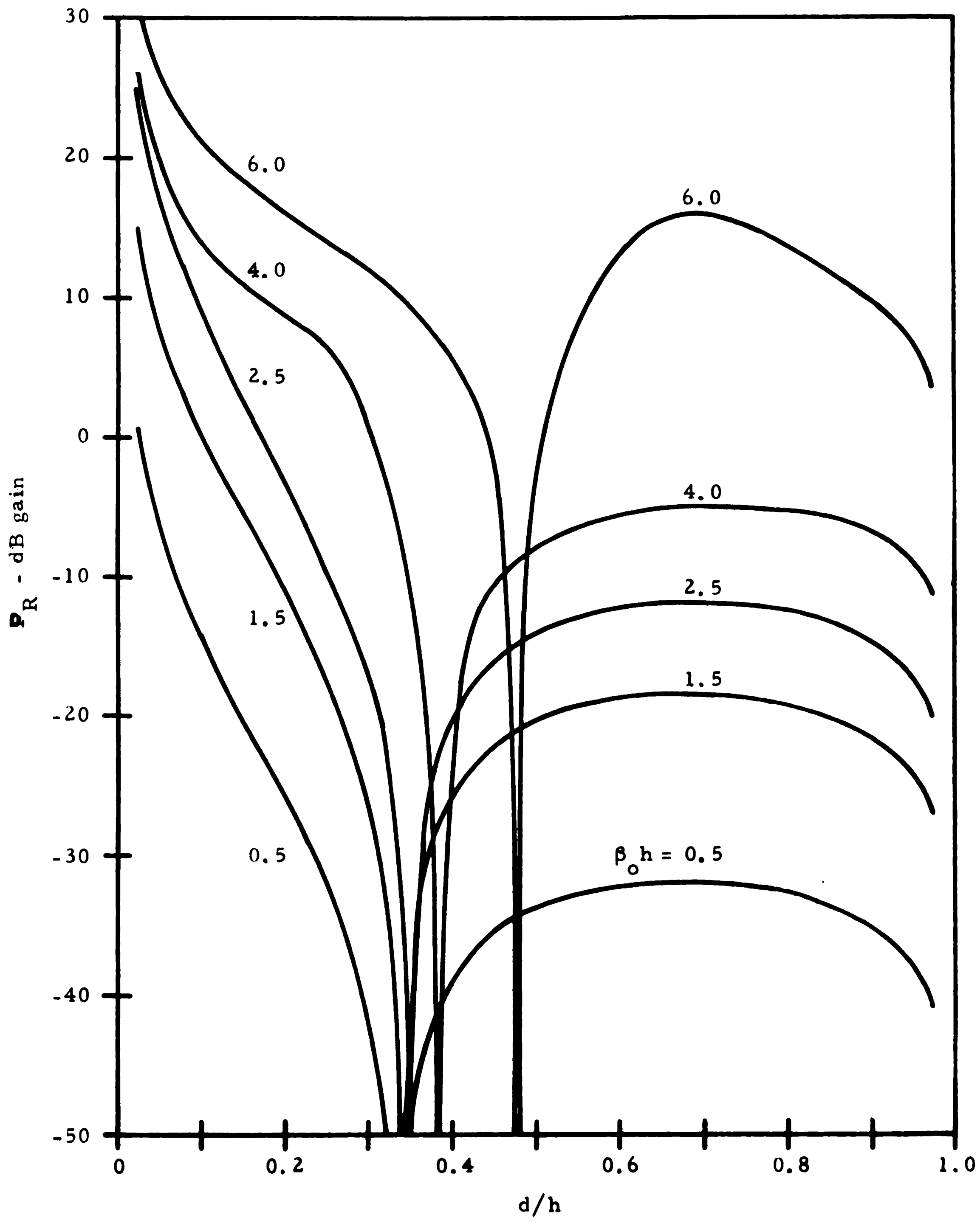


Figure 3.2. Received power for the resistance-loaded Near-Optimum Receiving Antenna ($\beta_o a = 0.001$)

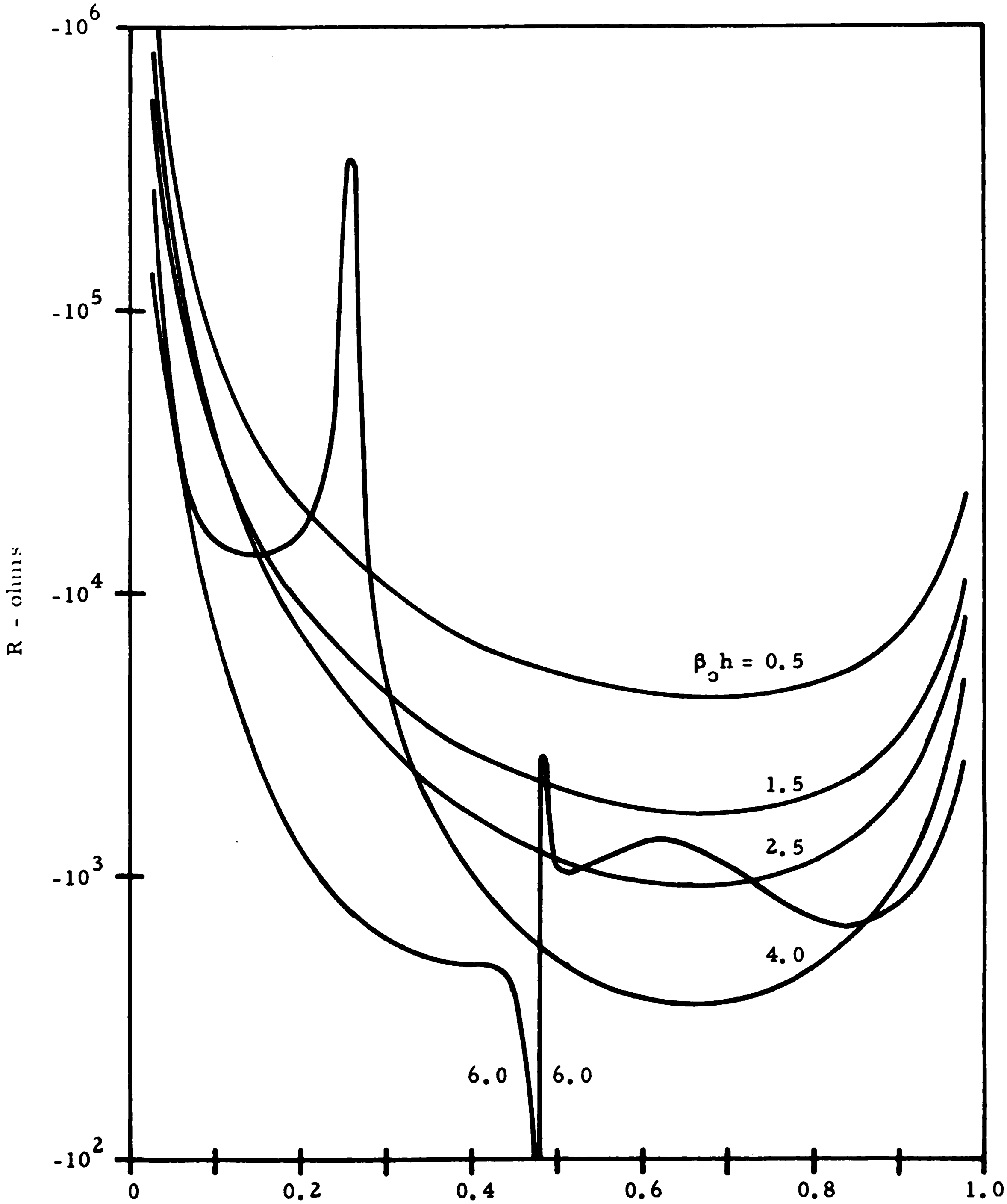


Figure 3.3. Auxiliary resistance for the resistance-loaded Near-Optimum Receiving Antenna ($\beta_o a = 0.001$)

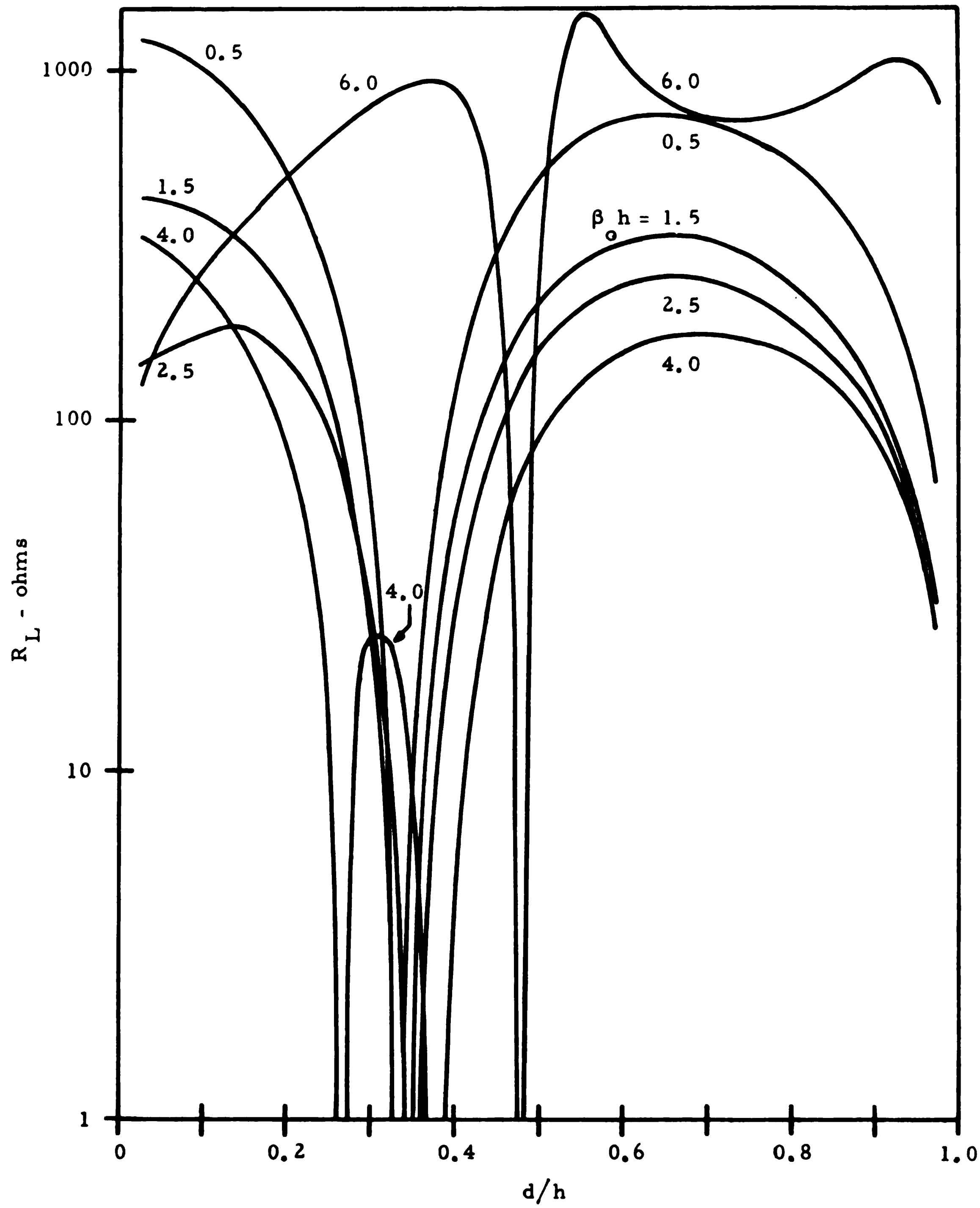


Figure 3. 4. Center-load resistance for the resistance-loaded Near-Optimum Receiving Antenna ($\beta_o a = 0.001$)

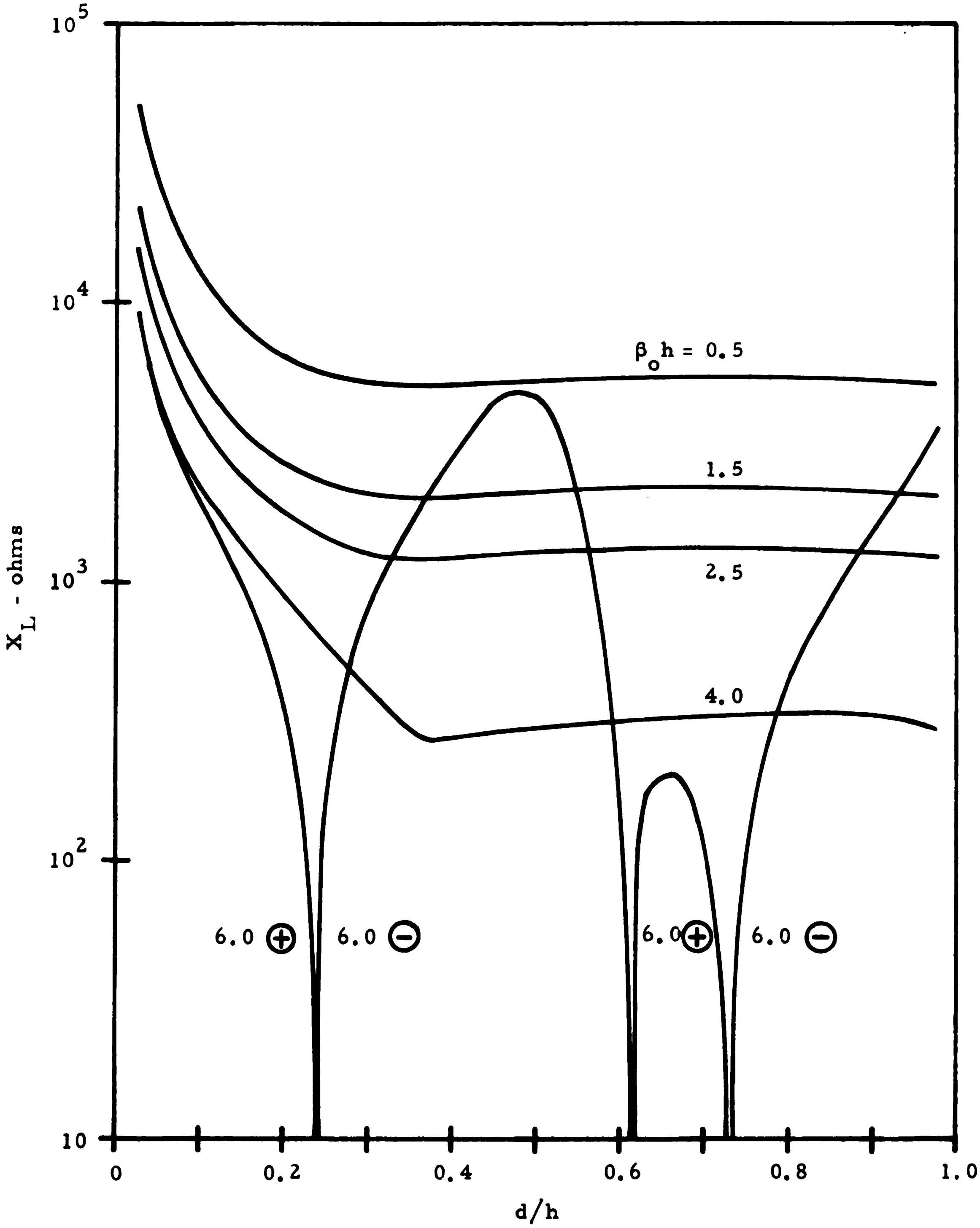


Figure 3.5. **Center-load reactance for the resistance-loaded Near-Optimum Receiving Antenna** ($\beta_o a = 0.001$)

3.3 Center-Load Resistance Specified

With R_L arbitrarily specified, the condition of zero back-scattering is achieved by choosing Z according to constraint equation (2.23), repeated below in the expanded form given in (3.2).

$$Z = -\left\{\frac{(f\ell + g'h')R_L}{(fR_L)^2 + (fX_L + g')^2}\right\} + j\left\{\frac{-fh'(R_L^2 + X_L^2) + (f\ell - g'h')X_L + g'\ell}{(fR_L)^2 + (fX_L + g')^2}\right\} \qquad (3.2)$$

Using either of these forms for Z, the received power of the double-loaded antenna can be expressed as a function of Z_L alone; substituting (2.23) into equation (2.18) for P_R gives

$$P_R = \frac{R_L}{2}\left|\frac{J\{\tilde{\mathcal{A}}Z_L + \tilde{\mathcal{B}}\}}{\tilde{\mathcal{C}}Z_L^2 + \tilde{\mathcal{D}}Z_L + \tilde{\mathcal{E}}}\right|^2 \qquad (3.21)$$

where

$$\left.\begin{aligned} \tilde{\mathcal{A}} &= FN - HM &&= 0 + ja' \\ \tilde{\mathcal{B}} &= GN - LM &&= b + j0 \\ \tilde{\mathcal{C}} &= AH - CF &&= c + jc' \\ \tilde{\mathcal{D}} &= AL + BH - CG - DF &&= d + jd' \\ \tilde{\mathcal{E}} &= BL - DG &&= e + je' \end{aligned}\right\} . \qquad (3.22)$$

The tilde distinguishes the script symbols which appear in (3.21) from the otherwise identical notation used in (2.25). Note, however, that (for convenience) the very same symbols which were introduced previously in (3.14) are used again in (3.22) for designating the real and imaginary parts of these script coefficients.

After expanding and simplifying, equation (3.21) can be put into the same form as (3.17), which was formulated in connection with the resistance-loaded antenna; viz.,

$$P_R = \frac{R_L |J|^2}{2} \left\{ \frac{\sum_{n=0}^{6} a_n X_L^n}{\sum_{n=0}^{6} b_n X_L^n} \right\} \tag{3.23}$$

where

$$\left.\begin{array}{llll} a_6 = 0 & & b_6 = 0 \\ a_5 = 0 & & b_5 = 0 \\ a_4 = 0 & & b_4 = c^2 + c'^2 \\ a_3 = 0 & & b_3 = 2(cp_1 + c'q_1) \\ a_2 = a'^2 & & b_2 = p_1^2 + q_1^2 + 2(cp_0 + c'q_0) \\ a_1 = -2a'b & & b_1 = 2(p_1 p_0 + q_1 q_0) \\ a_0 = a'^2 R_L^2 + b^2 & & b_0 = p_0^2 + q_0^2 \end{array}\right\} \tag{3.24}$$

and

$$\left.\begin{array}{ll} p_1 = 2c'R_L + d' & q_1 = -(2cR_L + d) \\ p_0 = -(cR_L^2 + dR_L + e) & q_0 = -(c'R_L^2 + d'R_L + e') \end{array}\right\} . \tag{3.25}$$

With R_L arbitrarily specified in (3.23), P_R is a function of the single variable X_L. Determination of the particular value of X_L for which $P_R(X_L)$ is maximum establishes the near-optimum receiving antenna for this case. Since (3.23) has exactly the same form as (3.17), it follows that the results obtained in Section 3.2 for maximizing the function in (3.17) are also applicable here. Thus, replacing R with X_L in (3.20), the equation whose solutions correspond to the critical points of the function in (3.23) is

$$\left.\begin{array}{l} \sum_{m=0}^{11} \left\{ \sum_{n=1}^{6} n[a_n b_{m-n+1} - a_{m-n+1} b_n] \right\} X_L^m = 0 \\ (a_i,\ b_i \equiv 0 \quad \text{for} \quad i < 0 \quad \text{and} \quad i > 6) \end{array}\right\} . \tag{3.26}$$

In view of the coefficients which are zero in (3.24), (3.26) actually reduces to the fifth degree equation

$$\sum_{m=0}^{5} A_m X_L^m = 0 \tag{3.27}$$

where A_m is still given by the summation within the braces in (3.26).

Several comments can be made at this point about the behavior of the power function $P_R(X_L)$. On the basis of equations (3.21) and (3.23) and the presumption that R_L is positive, $P_R(X_L)$ is a non-negative function which tends to zero as $|X_L|$ becomes infinitely large. Furthermore, unless R_L is specified as having the value $[R_L]_{op}$, the function remains finite. It follows that at least one of the critical points contained in (3.27) corresponds to a maximum of $P_R(X_L)$.

In the numerical work undertaken in connection with this particular antenna, parameter R_L has been assigned, successively, the values of 1, 5, 10, 50, and 72 ohms; for each case the real solutions of (3.27) and the corresponding values of $P_R(X_L)$ have been computed for each of several antennas with lengths corresponding to the range $0.5 \leq \beta_o h \leq 7.0$, over the entire range of possible loading positions. Although it is difficult to draw any general conclusions about the behavior of $P_R(X_L)$ on the basis of the numerical results, a remark about the number of real solutions obtained from equation (3.27) can be made: in none of the cases considered do more than three real solutions to (3.27) occur, and with R_L equal to 50 ohms and 72 ohms, only the one real solution guaranteed by the odd degree of the equation is obtained.

Shown in the following figures is the complete set of impedance parameter values corresponding to the near-optimum receiving antenna for which $R_L = 50$ ohms. Using the real solution of (3.27), curves for the auxiliary impedance are obtained from equation (3.2). As in Section 3.2, the received power is expressed as power gain relative to an ordinary dipole antenna of the same dimensions having a conjugate-matched center load. Similar to the situation described in connection with the resistive-loaded antenna of Section 3.2, the received power vanishes at loading positions which satisfy equation (3.6). This time $R_L \neq 0$, but $I(0) = 0$; this case corresponds exactly to the frequency rejection antenna of Section 3.1.

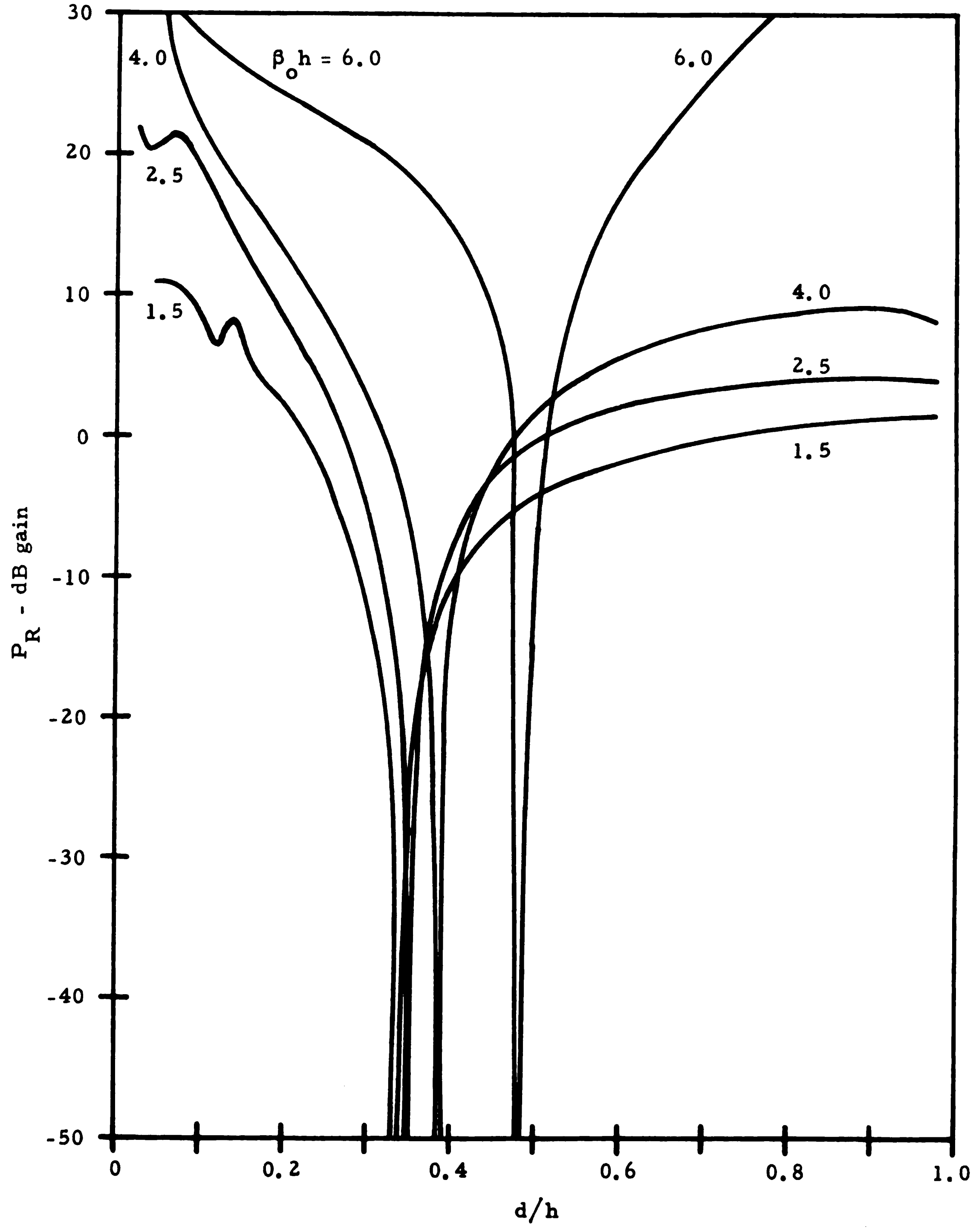


Figure 3.6. Received power for the Near-Optimum Receiving Antenna for which the load resistance is specified--R_L = 50 ohms ($\beta_o a$ = 0.001)

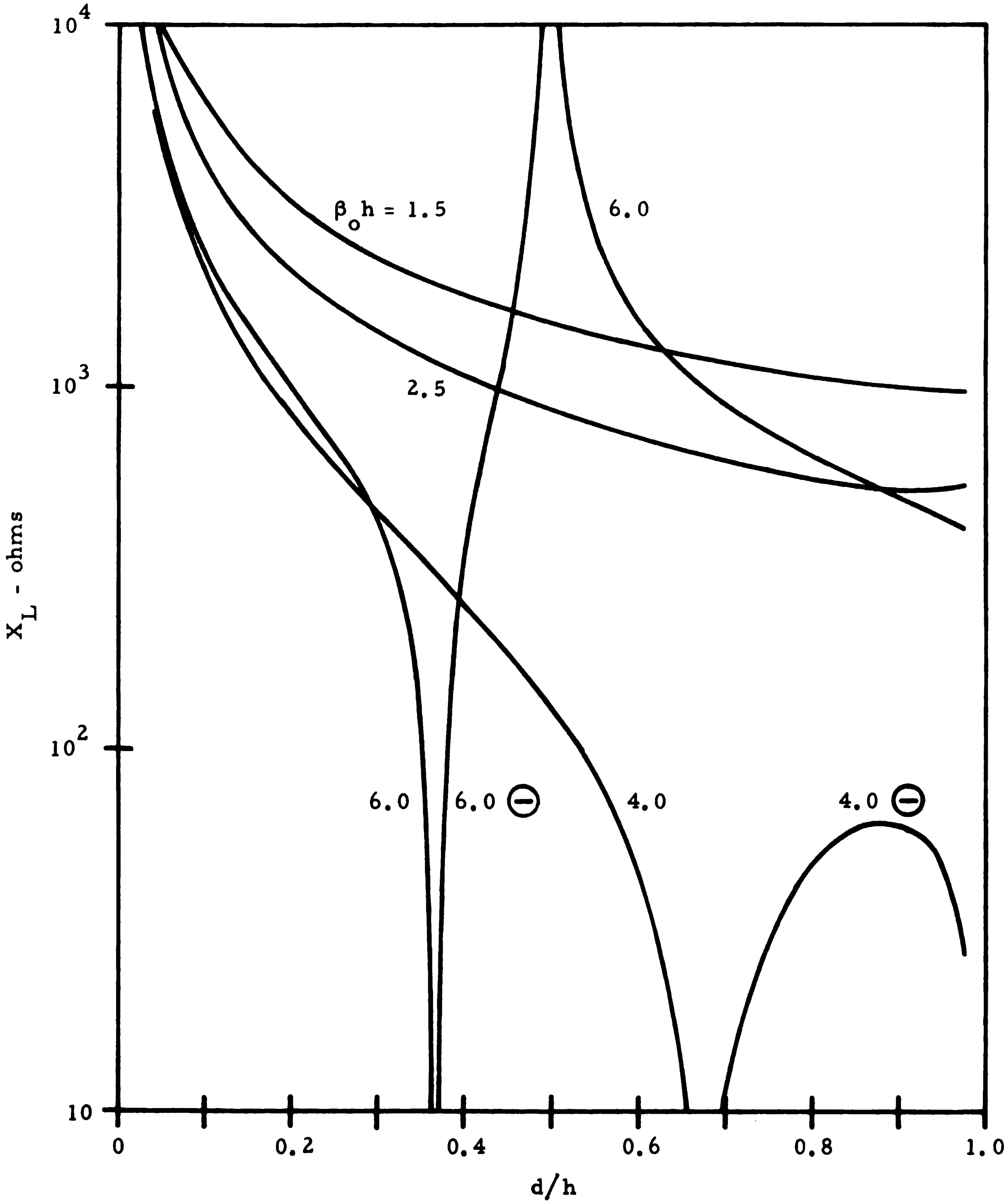


Figure 3. 7. Center-load reactance for the Near-Optimum Receiving Antenna for which the load resistance is specified-- R_L = 50 ohms ($\beta_o a$ = 0. 001)

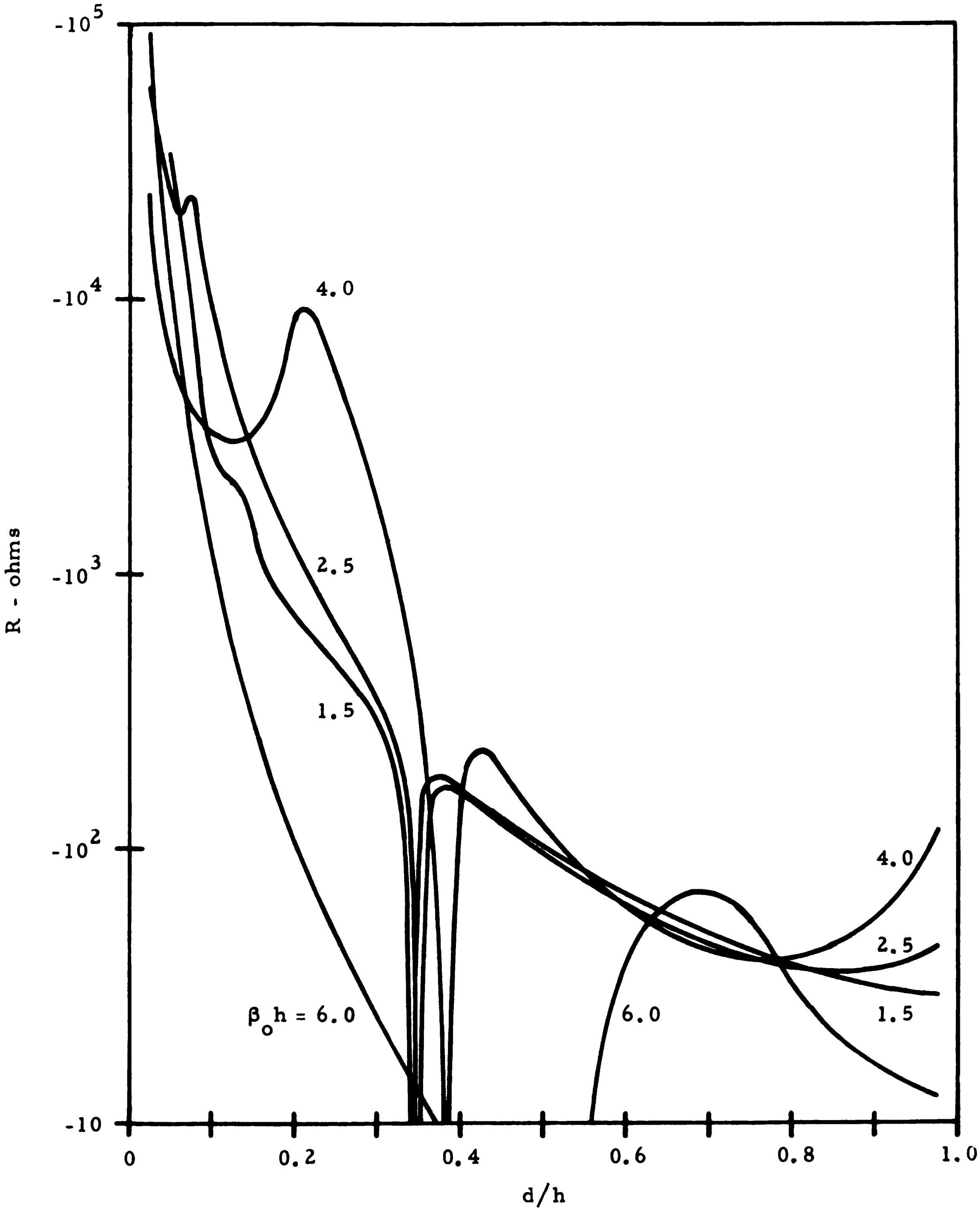


Figure 3.8. Auxiliary resistance for the Near-Optimum Receiving Antenna for which the load resistance is specified-- R_L = 50 ohms ($\beta_o a$ = 0.001)

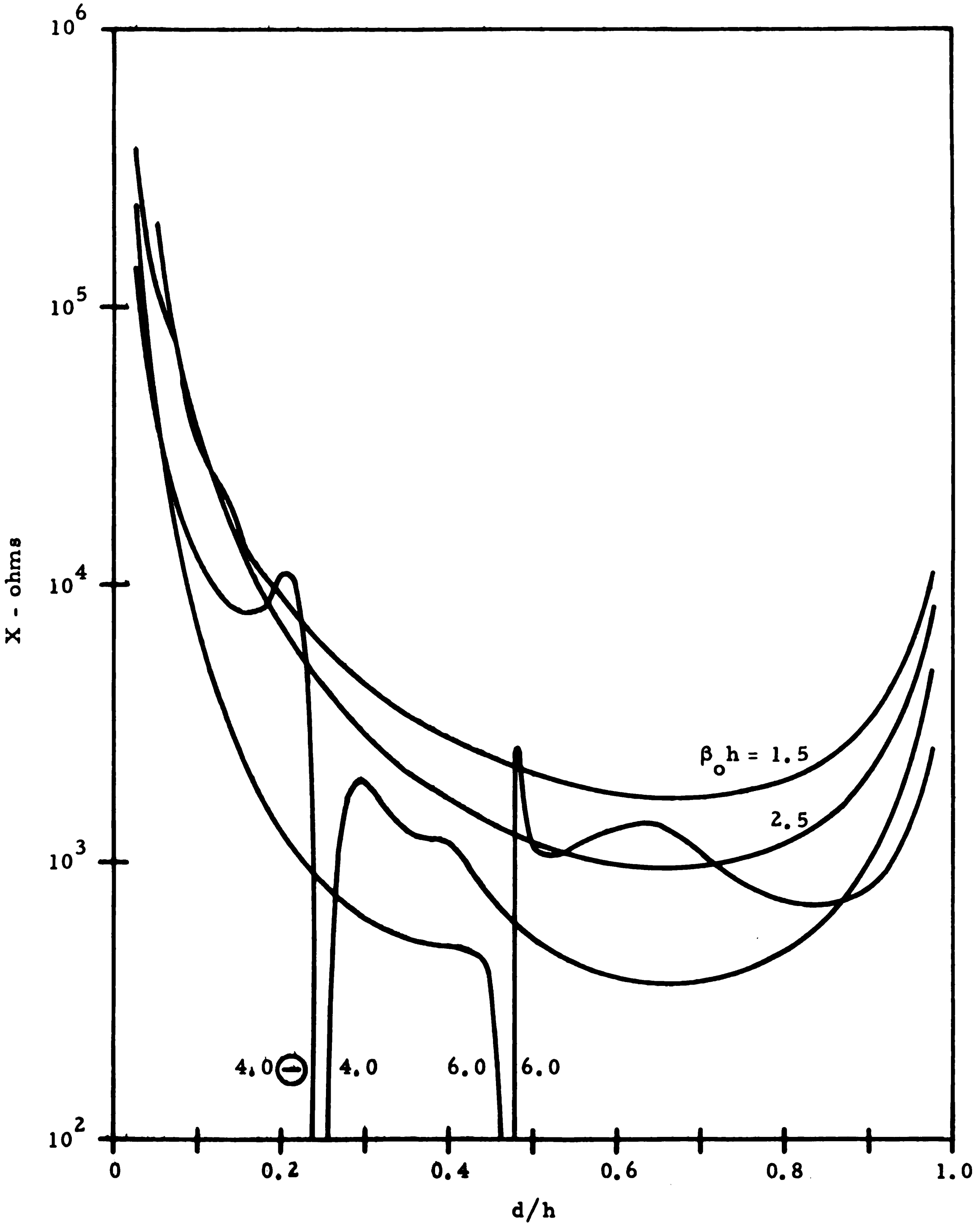


Figure 3.9. Auxiliary reactance for the Near-Optimum Receiving Antenna for which the load resistance is specified-- R_L = 50 ohms ($\beta_o a = 0.001$)

IV REACTANCE-LOADED RECEIVING ANTENNAS WITH MINIMUM BACKSCATTERING

As pointed out in Section 3.1, with Z restricted to the form $Z = 0 + jX$, insistence on the condition of zero broadside back-scattering leads to the trivial receiving antenna for which $R_L = 0$. However, if the backscattering requirement is relaxed somewhat, so that Z and Z_L are no longer constrained by equation (2.11), the case of pure reactive loading, clearly of practical interest, can be reconsidered.

With $R = 0$ and no pre-conditions on the remaining three impedance parameters, an entirely new problem arises in which the fundamental quantities describing the antenna performance are of the form

$$P_R = P_R(R_L, X_L, X) \tag{4.1a}$$

$$P_s = P_s(R_L, X_L, X) \ . \tag{4.1b}$$

In spite of the many degrees of freedom existing at the outset, it is not obvious how one should go about using them in order to yield the receiving antenna with the best performance. More to the point, it is not clear just what the best performance characteristics might be.

Certainly the ultimate selection of R_L, X_L, and X must provide an antenna which has a power delivering capability which is at least comparable to that of its ordinary conjugate-loaded counterpart,and at the same time must achieve a reduction in the backscattered power which can be regarded as significant. Beyond this, the particular approach to be used for "maximizing" the function of three variables in (4.1a) while simultaneously "minimizing" the function of the same three variables in (4.1b) is somewhat arbitrary. Clearly, unless the maximum value of (4.1a) and the minimum value of (4.1b) occur for the same parameter values, there is no "solution" for this problem which could be termed unique.

A "best" antenna could probably be found from inspection and comparison of the two functions in (4.1) for all possible combinations of values for R_L, X_L, and X. In dealing with functions of three variables, however, such an approach would be, at best, highly impractical. The alternative is to choose a method which offers a more limited scope for studying the variations of P_R and P_s. Such a method is provided, for example, if (on the basis of some reasonable criteria), the number of variables involved can be reduced at the outset. In the case of the functions in (4.1), this can be accomplished by the immediate elimination of Z_L by stipulating that for each and every value of X, Z_L be chosen to yield maximum received power. The auxiliary reactance can be subsequently selected in such a way that with $R_L = R_L(X)$ and $X_L = X_L(X)$, the function in (4.1b) is minimized. Although this procedure may very well fail to yield the "best" reactance-loaded receiving antenna, it does lead to an antenna which achieves the stated objectives with at least moderate effectiveness. Moreover, this process for selecting the impedance parameters is well suited for purposes of experimentation, particularly since R_L can be arbitrarily specified, if desired.

4.1 The Conjugate-Match Condition

Given the value of auxiliary reactance X, the center-load impedance which yields maximum received power is $Z_L = Z_{in}^*(X)$ (the asterisk denotes the complex conjugate). This follows from the maximum power transfer theorem when the real part of Z_{in} is positive, as discussed in Section 2.2. With this condition on Z_L, the backscattered power density becomes a function of X alone, and from (2.6) is given by

$$P_s = \left[\frac{(1-\cos\beta_o h)^2 (E_o/\beta_o)^2}{15\pi R_o^2} \left| \frac{Z(FZ_L+G)+HZ_L+L}{Z(AZ_L+B)+CZ_L+D} \right|^2 \right]_{\substack{Z_L = Z_{in}^*(Z) \\ Z = jX}} \tag{4.2}$$

where, by equation (1.75) for Z_{in},

$$Z_L = \left[\frac{B(jX) + D}{A(jX) + C} \right]^* . \tag{4.3}$$

After substituting (4.3) into (4.2) and rearranging, P_s can be written as

$$P_s = \frac{(1-\cos\beta_o h)^2 (E_o/\beta_o)^2}{15\pi R_o^2} \cdot \left| \frac{(u_2 X^2 + u_1 X + u_o) + j(v_2 X^2 + v_1 X + v_o)}{2(w_2 X^2 + w_1 X + w_o)} \right|^2 . \tag{4.4}$$

With A, B, C, and D written as

$$\left.\begin{array}{lll} A = a + ja' & & C = c + jc' \\ B = b + jb' & & D = d + jd' \end{array}\right\} , \tag{4.5}$$

the coefficients in (4.4) are

$$\left.\begin{array}{l} u_2 = a'g' + bf \\ u_1 = -a'\ell + bh' - cg' + d'f \\ u_0 = c\ell + d'h' \\ v_2 = ag' - b'f \\ v_1 = -a\ell - b'h' + c'g' + df \\ v_0 = -c'\ell + dh' \\ w_2 = ab + a'b' \\ w_1 = ad' - a'd + bc' - b'c \\ w_0 = cd + c'd' \end{array}\right\} \tag{4.6}$$

where f, g', h' and ℓ are as defined in (3.4). Finally, after expanding and collecting terms, (4.4) can be put into the form

$$P_s = \frac{(1-\cos\beta_o h)^2 (E_o/\beta_o)^2}{60\pi R_o^2} \left\{ \frac{\sum_{n=0}^{6} a_n X^n}{\sum_{n=0}^{6} b_n X^n} \right\} \tag{4.7}$$

where

$$\left.\begin{aligned}
a_6 &= 0 & b_6 &= 0 \\
a_5 &= 0 & b_5 &= 0 \\
a_4 &= u_2^2 + v_2^2 & b_4 &= w_2^2 \\
a_3 &= 2(u_1 u_2 + v_1 v_2) & b_3 &= 2w_1 w_2 \\
a_2 &= u_1^2 + v_1^2 + 2(u_0 u_2 + v_0 v_2) & b_2 &= w_1^2 + 2w_0 w_2 \\
a_1 &= 2(u_0 u_1 + v_0 v_1) & b_1 &= 2w_0 w_1 \\
a_0 &= u_0^2 + v_0^2 & b_0 &= w_0^2
\end{aligned}\right\} . \tag{4.8}$$

Using the results of Section 3.2, the critical points of the function $P_s(X)$, according to (3.20), correspond to the real solutions of the following equation.

$$\left.\begin{aligned}
&\sum_{m=0}^{11} \left\{ \sum_{n=1}^{6} n[a_n b_{m-n+1} - a_{m-n+1} b_n] \right\} X^m = 0 \\
&(a_i, b_i \equiv 0 \text{ for } i < 0 \text{ and } i > 6)
\end{aligned}\right\} \tag{4.9}$$

Because a_6, a_5, b_6, and b_5 are zero and since the coefficient of X^7 vanishes identically, (4.9) reduces to the sixth degree equation

$$\sum_{m=0}^{6} A_m X^m = 0 \qquad (4.10)$$

where A_m is still given in (4.9) by the summation within the braces.

When the backscattered power density function is computed at each of the real solutions of (4.10), the minimum value sought is readily determined by inspection of the numerical results. These computations have been made for several different antennas with lengths in the range of $0.5 \leq \beta_o h \leq 6.0$ and over the entire range of possible loading positions. In general, equation (4.10) produces either two or four real values of X. However, there are intervals of load-placement, for the shorter antennas, where all six solutions of (4.10) are complex.

Table 4.1 shows the results of the numerical investigation for a few selected antenna lengths. The fact that the "best" case often shifts from one minimum of (4.7) to another as the position of the auxiliary load is varied results in a definite lack of continuity in the "best antenna" values of X, Z_L, P_R, and P_s. Principally for this reason, a tabular, rather than graphical, format has been chosen for representing the findings in this study. Entries for P_s in this table are given in terms of power gain, and represent a normalization of this minimum value of backscattered power density with respect to that produced by an ordinary dipole antenna having a conjugate-matched center-load. The corresponding values of received power, computed from (2.18), are represented similarly. Values of Z_L are obtained from (4.3).

Of the nine loading positions considered (for each antenna length, d/h = 0.1, 0.2, . . . , 0.9), only those cases which resulted in a decrease in P_s of 5 dB or more and which simultaneously produced a loss in received power no greater than 5 dB are shown in Table 4.1.

Although not apparent from the data in Table 4.1, as d approaches the particular loading position predicted in (3.10) and shown in Figure 3.1, this antenna approaches the invisible frequency-rejection antenna described in Section 3.1.

Table 4.1. Reactance-loaded receiving antennas with minimum backscattering--the conjugate-match condition ($\beta_o a = 0.001$)

$\beta_o h$	d/h	X (ohms)	R_L (ohms)	X_L (ohms)	P_R (dB gain)	P_s (dB gain)
1.5	0.2	9290	0.00214	3120	-4.70	-13.8
	0.4	2840	0.0196	1740	-4.42	-7.85
	0.5	2130	0.0417	1450	-4.38	-6.55
2.5	0.1	37100	0.0190	3990	-0.597	-17.0
	0.2	7380	0.0376	2040	-3.77	-23.4
4.0	0.1	-12700	0.0774	1980	+7.83	-9.12
	0.2	-16200	0.500	823	+9.63	-4.54
	0.4	1020	0.0491	245	-2.74	-30.0
	0.5	515	0.251	126	+3.54	-20.0
	0.6	375	0.423	45.5	+5.29	-17.4
	0.7	361	0.581	-10.8	+6.59	-16.2
	0.8	469	0.725	-48.1	+7.92	-15.4
	0.9	978	0.853	-60.9	+9.59	-14.8
6.0	0.1	6340	32.0	2300	+24.4	-1.57
	0.2	1240	52.2	1020	+23.6	-7.54
	0.3	611	90.8	422	+20.7	-13.9
	0.4	495	183	-342	+16.3	-22.5

4.2 Load Resistance Specified

When the value of R_L is arbitrarily specified, conjugate-matched loading cannot be achieved. In this case, on the basis of a corollary of the maximum power transfer theorem, maximum received power occurs when X_L is chosen as the negative of the (series) input reactance $X_{in}(X)$.

The backscattered power density from a reactance-loaded receiving antenna having a center-load impedance selected in this manner is, from (2.6),

$$P_s = \left[\frac{(1-\cos\beta_o h)^2 (E_o/\beta_o)^2}{15\pi R_o^2} \left| \frac{Z(FZ_L + G) + HZ_L + L}{Z(AZ_L + B) + CZ_L + D} \right|^2 \right]_{\substack{Z_L = R_L - jX_{in}(Z) \\ Z = jX}} \tag{4.11}$$

where, by equation (1.75) for Z_{in},

$$Z_L = R_L - j\,\mathcal{I}m \left[\frac{B(jX) + D}{A(jX) + C} \right] . \tag{4.12}$$

When A, B, C, and D are written as shown in (4.5) and the result is simplified, (4.12) becomes

$$Z_L = \frac{[\{p_2X^2 + p_1X + p_0\}R_L] + j[q_2X^2 + q_1X + q_0]}{p_2X^2 + p_1X + p_0} \tag{4.13}$$

where

$$\left.\begin{array}{llll} p_2 = a^2 + a'^2 & & q_2 = -ab' + a'b \\ p_1 = 2(ac' - a'c) & & q_1 = ad + a'd' - bc - b'c' \\ p_0 = c^2 + c'^2 & & q_0 = -cd' + c'd \end{array}\right\} . \qquad (4.14)$$

After substituting (4.13) into (4.11) and rearranging, P_s can be written as

$$P_s = \frac{(1-\cos\beta_o h)^2 (E_o/\beta_o)^2}{15\pi R_o^2} \cdot$$

$$\left| \frac{[u_3X^3 + u_2X^2 + u_1X + u_0] - j[v_3X^3 + v_2X^2 + v_1X + v_0]}{[w_3X^3 + w_2X^2 + w_1X + w_0] - j[y_3X^3 + y_2X^2 + y_1X + y_0]} \right|^2 \qquad (4.15)$$

where

$$\left.\begin{array}{l} u_3 = fq_2 + g'p_2 \\ u_2 = fq_1 + g'p_1 + h'q_2 - \ell p_2 \\ u_1 = fq_0 + g'p_o + h'q_1 - \ell p_1 \\ u_0 = h'q_0 - \ell p_0 \end{array}\right\} \qquad (4.16)$$

$$\left.\begin{array}{l} v_3 = (fp_2)R_L \\ v_2 = (fp_1 + h'p_2)R_L \\ v_1 = (fp_0 + h'p_1)R_L \\ v_0 = (h'p_0)R_L \end{array}\right\} \qquad (4.17)$$

$$\left.\begin{aligned}
w_3 &= aq_2 + (a'R_L + b')p_2 \\
w_2 &= aq_1 + c'q_2 + (a'R_L + b')p_1 - (cR_L + d)p_2 \\
w_1 &= aq_0 + c'q_1 + (a'R_L + b')p_0 - (cR_L + d)p_1 \\
w_0 &= c'q_0 - (cR_L + d)p_0
\end{aligned}\right\} \qquad (4.18)$$

$$\left.\begin{aligned}
y_3 &= -a'q_2 + (aR_L + b)p_2 \\
y_2 &= -a'q_1 + cq_2 + (aR_L + b)p_1 + (c'R_L + d')p_2 \\
y_1 &= -a'q_0 + cq_1 + (aR_L + b)p_0 + (c'R_L + d')p_1 \\
y_0 &= cq_0 + (c'R_L + d')p_0
\end{aligned}\right\} . \qquad (4.19)$$

The terms f, g', h', and ℓ appearing in (4.16) through (4.19) are defined in (3.4). When (4.15) is expanded and simplified, P_s can finally be put into the form:

$$P_s = \frac{(1-\cos\beta_o h)^2 (E_o/\beta_o)^2}{15\pi R_o^2} \left\{ \frac{\sum_{n=0}^{6} a_n X^n}{\sum_{n=0}^{6} b_n X^n} \right\} \qquad (4.20)$$

where

$$\left.\begin{aligned}
a_6 &= u_3^2 + v_3^2 \\
a_5 &= 2(u_2u_3 + v_2v_3) \\
a_4 &= u_2^2 + v_2^2 + 2(u_1u_3 + v_1v_3) \\
a_3 &= 2(u_0u_3 + v_0v_3 + u_1u_2 + v_1v_2) \\
a_2 &= u_1^2 + v_1^2 + 2(u_0u_2 + v_0v_2) \\
a_1 &= 2(u_0u_1 + v_0v_1) \\
a_0 &= u_0^2 + v_0^2
\end{aligned}\right\} \qquad (4.21)$$

$$\left.\begin{aligned}
b_6 &= w_3^2 + y_3^2 \\
b_5 &= 2(w_2 w_3 + y_2 y_3) \\
b_4 &= w_2^2 + y_2^2 + 2(w_1 w_3 + y_1 y_3) \\
b_3 &= 2(w_0 w_3 + y_0 y_3 + w_1 w_2 + y_1 y_2) \\
b_2 &= w_1^2 + y_1^2 + 2(w_0 w_2 + y_0 y_2) \\
b_1 &= 2(w_0 w_1 + y_0 y_1) \\
b_0 &= w_0^2 + y_0^2
\end{aligned}\right\} . \qquad (4.22)$$

By (3.20) in Section 3.2, the critical points of the function $P_s(X)$ correspond to the real solutions of the following equation.

$$\left.\begin{aligned}
&\sum_{m=0}^{11} \left\{ \sum_{n=1}^{6} n\left[a_n b_{m-n+1} - a_{m-n+1} b_n\right] \right\} X^m = 0 \\
&(a_i,\ b_i \equiv 0 \ \text{for} \ \ i < 0 \ \text{and} \ i > 6)
\end{aligned}\right\} \qquad (4.23)$$

Inasmuch as the coefficient of X^{11} vanishes identically, (4.23) is actually the tenth degree equation:

$$\sum_{m=0}^{10} A_m X^m = 0 . \qquad (4.24)$$

Similar to the conjugate-matched case of Section 4.1, P_s is computed at each of the real solutions of (4.24), and the minima of this function are found from inspection of these results. As in the previous section, the particular local minimum of P_s which provides the best balance between P_R and P_s is selected as the "best" case. For each of four values of assumed load resistance (1, 10, 100, and 1000 ohms), these "best" cases are shown in Figure 4.2 for antennas of several different lengths. Only those loading positions are included at which $P_s < -5$ dB for at least one of these values of R_L and, at the same time, where $P_R > -5$ dB for at least one of these values of R_L. As before, P_R and P_s are indicated as gain quantities, normalized to the performance of the ordinary conjugate-loaded receiving antenna. Values of X_L are obtained from (4.12).

Again, with X a free variable in a situation involving backscatter reduction, this antenna approaches the invisible frequency-rejection antenna of Section 3.1 as d/h nears the value shown in Figure 3.1.

Table 4.2. Reactance-loaded receiving antennas with minimum backscattering--load resistance specified ($\beta_o a=0.001$)

$\beta_o h$	d/h	R_L (ohms)	X (ohms)	X_L (ohms)	P_R (dB gain)	P_s (dB gain)
1.5	0.8	1	2090	941	-2.30	-2.76
		10	1890	1230	-1.12	-8.57
		100	2120	920	-14.9	-26.0
		1000	692	-252	-4.51	-13.3
	0.9	1	3560	646	-5.00	+4.32
		10	3470	698	-1.29	-3.39
		100				
		1000	2880	262	-2.03	-7.26
2.5	0.4	1	284	1190	-33.3	-72.6
		10	284	1190	-23.3	-52.6
		100	284	1190	-13.4	-32.7
		1000	284	1180	-4.22	-13.5
	0.5	1	264	1150	-32.5	-71.1
		10	264	1150	-22.5	-51.1
		100	264	1150	-12.6	-31.2
		1000	264	1140	-3.60	-12.2
	0.6	1	284	1100	-31.9	-70.0
		10	284	1100	-21.9	-50.0
		100	284	1100	-12.0	-30.1
		1000	283	1080	-3.13	-11.2
	0.7	1	358	1060	-31.4	-69.1
		10	358	1060	-21.4	-49.1
		100	358	1060	-11.6	-29.2
		1000	358	1040	-2.76	-10.4
	0.8	1	564	1020	-31.0	-68.4
		10	564	1020	-21.0	-48.4
		100	564	1020	-11.1	-28.5
		1000	564	990	-2.44	-9.82
	0.9	1	1350	988	-30.6	-67.7
		10	1350	988	-20.6	-47.9
		100	1350	985	-10.8	-28.0
		1000	1350	892	-2.16	-9.35

Table 4.2 (continued)

$\beta_o h$	d/h	R_L (ohms)	X (ohms)	X_L (ohms)	P_R (dB gain)	P_s (dB gain)
4.0	0.2	1	6930	1200	-21.6	-46.8
		10	6930	1200	-11.8	-27.0
		100	6730	1190	-3.55	-8.85
		1000	1640	534	-2.81	-8.36
	0.3	1	-1570	361	-18.3	-43.9
		10	-1570	361	-8.58	-24.2
		100	-1520	373	-0.729	-6.34
		1000	884	543	-10.8	-25.9
	0.5	1	514	126	+1.64	-15.9
		10	-510	348	-18.5	-42.3
		100	-510	347	-8.78	-22.6
		1000	-513	303	-1.21	-5.04
	0.6	1	374	46.2	+4.48	-14.5
		10	373	47.8	-2.58	-11.8
		100	-609	462	-10.4	-25.5
		1000	-612	415	-2.19	-7.28
	0.7	1	361	-10.2	+6.21	-14.2
		10	357	-5.44	0.00	-10.7
		100	-1250	587	-11.5	-27.4
		1000	-1260	542	-3.02	-8.91
	0.8	1	468	-47.5	+7.74	-14.2
		10	465	-41.7	+2.06	-10.1
		100	19100	711	-12.5	-28.9
		1000	18400	669	-3.78	-10.2
	0.9	1	978	-60.1	+9.43	-14.2
		10	973	-54.7	+4.19	-9.61
		100	2570	816	-13.4	-29.9
		1000	2560	775	-4.53	-11.1

Table 4.2 (continued)

$\beta_o h$	d/h	R_L (ohms)	X (ohms)	X_L (ohms)	P_R (dB gain)	P_s (dB gain)
6.0	0.1	1	6350	2300	+15.2	-25.8
		10	6170	2300	+22.0	-8.49
		100	3640	2460	+16.9	-1.44
		1000	1210	3120	+10.7	-0.117
	0.2	1	1250	1020	+12.3	-36.0
		10	1240	1020	+20.9	-17.4
		100	1180	1020	+22.3	-5.58
		1000	639	1090	+14.6	-3.41
	0.3	1	612	421	+7.04	-47.1
		10	612	421	+16.2	-27.9
		100	606	422	+20.5	-13.5
		1000	476	433	+15.0	-9.25
	0.4	1	496	-342	-0.308	-61.8
		10	496	-342	+9.28	-42.2
		100	495	-342	+15.9	-25.5
		1000	464	-341	+13.4	-18.3

V EXPERIMENT AND RESULTS

In this chapter experimental results for the reactance-loaded receiving antenna are presented and correlated with the theoretical work of the previous chapters. A fairly extensive comparison of theory and experiment is made for an antenna of one particular length, in which, for several positions of auxiliary loading and with a specified load resistance of 100 ohms, the theoretical and experimental values of received and backscattered power are compared, as the value of the auxiliary reactance is varied over a rather wide range. In addition, an experimental search for the invisible frequency-rejection receiving antenna described in Section 3.1 is conducted for antennas of several different lengths.

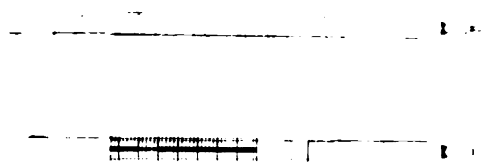
5.1 The Experimental Reactance-Loaded Receiving Antenna

All of the experimental work has been performed using a specially designed reactance-loaded monopole antenna, mounted on a rectangular ground plane (6' x 8') which forms one wall of a completely enclosed anechoic chamber. The choice of a test frequency of 1.5 GHz allows the receiving monopole to be operated in the far zone of the transmitting monopole on the given aluminum ground plane, and, at the same time, permits construction of an electrically thin experimental receiving antenna from metallic tubing large enough to make the fabrication reasonably easy. At 1.5 GHz, corresponding to a wavelength of 20 centimeters in free space, brass tubing with an outside diameter of 1/4 inch gives an electrical thickness of $\beta_o a = 0.1$.

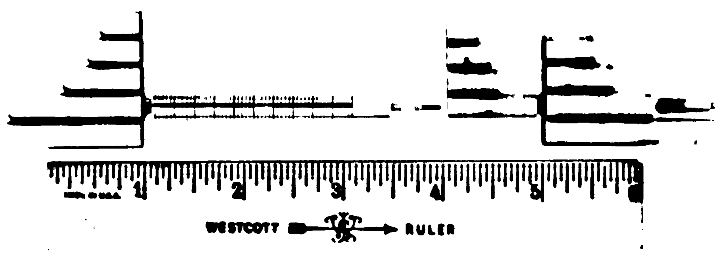
Photographs of the experimental receiving monopole are shown in Figure 5.1. The antenna sections in Figure 5.1b appearing in multiple lengths are necessary so that loading position d can be varied continuously over a wide range of values. A variable auxiliary reactance is provided across the gap containing the white teflon spacer by the longitudinally-slotted cylindrical section of tubing immediately adjacent and to the left of the gap. This slotted section contains a sliding brass disc which is threaded to a 1/16 inch brass rod running the entire length of the section. Viewed from the gap end, this section represents a shorted length of coaxial transmission line, and effects a reactance across the gap of

$$X = R_o \tan \beta_o \ell_s \qquad (5.1)$$

where R_o is the characteristic resistance of the lossless coaxial transmission line and ℓ_s is the distance between the shorting disc and the left edge of the gap. The characteristic resistance



(a)



(b)

Figure 5.1. The experimental reactance-loaded receiving antenna

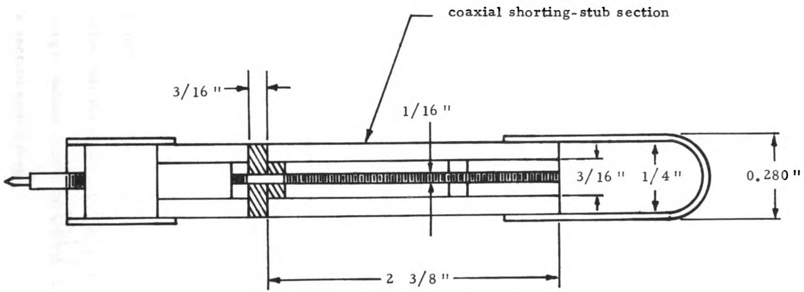
of an air-filled transmission line composed of concentric cylinders is well known and is given by

$$R_o = \frac{\zeta_o}{2\pi} \ln \frac{d_2}{d_1} \tag{5.2}$$

where $\zeta_o = 120\pi$ ohms is the characteristic impedance of free space, d_2 is the inside diameter of the outer cylinder, and d_1 is the outside diameter of the inner cylinder. Using the nominal values of diameter indicated in the assembly drawing of Figure 5.2, equation (5.2) gives $R_o = 66$ ohms. However, when the actual measured diameter of the threaded inner conductor is used, (5.2) gives $R_o = 71$ ohms. Moreover, if the effective value of d_1 is approximated as the average diameter (on the basis of assuming a perfect V-cut thread), equation (5.2) yields a characteristic resistance of almost 80 ohms.

Lack of knowledge of the amount of stray capacitance existing directly across the teflon spacer adds a further complication to the problem of predicting the exact value of auxiliary reactance effected across the gap, as a function of the stub length ℓ_s. Means for indirectly determining approximate values of R_o and this parallel gap capcitance from the experimental results are discussed in Section 5.3.1.

Some mention of the criterion used for selecting the (fixed) length of the coaxial shorting-stub section should be made.



material is brass except
the shaded items which
are teflon

Figure 5. 2. Assembly drawing of the experimental reactance-loaded receiving antenna

Disregarding the effect which the gap capacitance has on the total value of the auxiliary reactance, an adjustable stub with a maximum length of one-half wavelength (10 cm at 1.5 GHz) would be required in order to provide the capability of achieving all values of X ($-\infty < X < \infty$). A section of this length would immediately restrict the applicability of the experimental receiving monopole to antennas for which $\beta_o h > \pi$, even before provisions for mounting and implementing a means for varying d were considered. The particular length chosen for this section (6 cm) allows the line to be tuned over all positive values of X, according to equation (5.1), and a large portion of the negative values. The presence of the stray capacitance across the gap at location d extends the range of achievable negative values of true reactance even more.

With the length of the shorting-stub section finalized, the remaining sections of the experimental antenna assembly have been carefully designed (with a minimum number of parts in each set of like parts) to provide a range of applicability of $3.0 \leq \beta_o h \leq 5.0$, and such that the position of the auxiliary loading can be varied continuously over the range $0.2 \leq d/h \leq 0.9$ (with the exception that the lower limit on d/h is about 0.35 for $\beta_o h = 3.0$). The larger values of d/h are obtained by reversing the stub tuner section end-for-end.

5.2 The Experimental Setup

A block diagram of the experimental setup used in the measurement of the received and backscattered power of the reactance-loaded test antenna is shown in Figure 5.3. Figure 5.4 contains photographs of the anechoic chamber.

Operating at a frequency of 1.5 GHz, the experimental receiving antenna described in Section 5.1 is located 36 inches (nearly five wavelengths) from the transmitting antenna. A third element positioned on a line with the transmitting and receiving monopoles and lying 18 inches beyond the receiving antenna will be referred to as a scattering detector, and is used in the measurement of the backscattered power density produced by the test receiving antenna.

Measurement of the backscattering is accomplished by a simple cancellation method. With only the scattering detector and the transmitting antenna in the chamber, the signal from the scattering detector is added to a second signal of equal amplitude but opposite phase derived from the same oscillator which drives the transmitting antenna. The relative amplitude of this sum is monitored by a heterodyne measurement scheme. When the test receiving antenna is inserted in place, the scattering detector responds to what can be considered the sum of two electric fields: one equal to the field which existed before the test antenna was

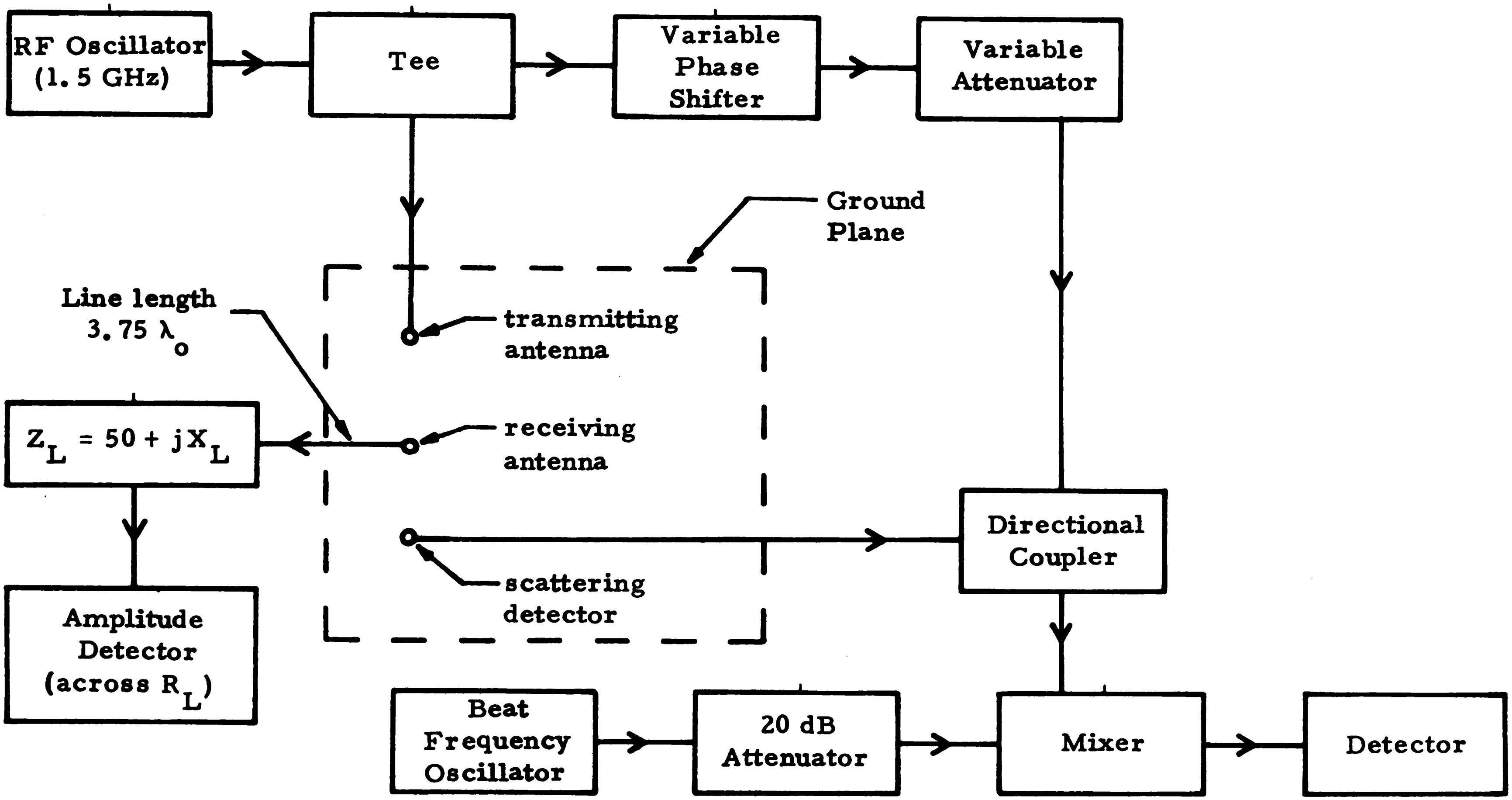
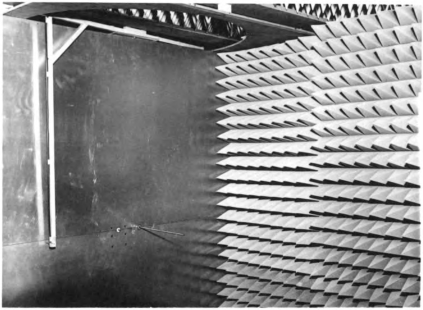
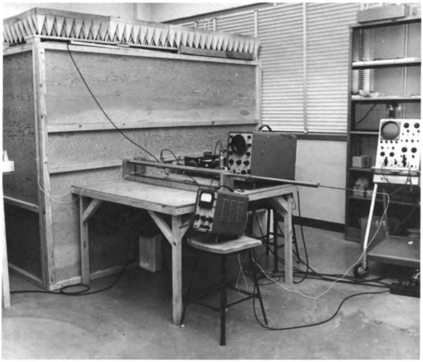


Figure 5.3. Block diagram of the experimental equipment setup



(a)



(b)

Figure 5.4. The anechoic chamber

added to the chamber, and one caused directly by the broadside re-radiation of the test antenna. By first establishing a reference level of scattering with an ordinary unloaded monopole of the same dimensions as the test antenna, this scheme provides a very efficient means for determining the test antenna's back-scattered power gain.

The measurement of received power is complicated somewhat by the practical impossibility (using commercially available cylindrical transmission line test equipment) of locating the center-load impedance right at the ground plane. In the experimental search for the invisible frequency rejection antenna, the load impedance which appears at the driving point of the receiving monopole is not critical, since this phenomenon is purportedly independent of Z_L. On the other hand, in an experimental study of the receiving antenna of Section 4.2, a means for insuring a constant specified value of R_L at the driving point is essential.

The problem of transforming what is inherently a parallel combination of resistance and adjustable reactance (using commercially available resistive terminations, stub reactance tuners, air lines, and tees), occurring at some distance away from the ground plane, into a desired series combination right at the driving-point is resolved as follows.

The input impedance to a lossless air transmission line of length ℓ_t terminated in an impedance Z_p is given by

$$Z_\ell = R_o \left[\frac{Z_p \cos\beta_o \ell_t + jR_o \sin\beta_o \ell_t}{R_o \cos\beta_o \ell_t + jZ_p \sin\beta_o \ell_t} \right] \tag{5.3}$$

where R_o is the characteristic resistance of the line. If ℓ_t is selected such that for n a positive integer

$$\ell_t = \frac{\lambda_o}{4} + n\left(\frac{\lambda_o}{2}\right) , \tag{5.4}$$

then

$$\beta_o \ell_t = \frac{\pi}{2} (1 + 2n) \tag{5.5}$$

and, using (5.5), equation (5.3) becomes

$$Z_\ell = \frac{R_o^2}{Z_p} . \tag{5.6}$$

Now if Z_p consists of resistance R_p in parallel with reactance X_p, (5.6) can be expanded and written as

$$Z_\ell = \frac{R_o^2}{R_p} - j \frac{R_o^2}{X_p} . \tag{5.7}$$

Furthermore, notice that

$$Z_\ell = R_o - j(R_o^2/X_p) \quad \text{for } R_p = R_o . \tag{5.8}$$

In the experimental study of the reactance-loaded antenna of Section 4.2, using a tee and an adjustable coaxial air line of 50 ohms characteristic resistance, a 50 ohm termination (corresponding to R_p in (5.8)) can be connected in parallel with a reactance tuning stub to achieve a center-load impedance of

$$\left.\begin{aligned} Z_L &= 50 + jX_L \\ (X_L &= -2500/X_p) \end{aligned}\right\} \qquad (5.9)$$

at the driving point of the receiving antenna, simply by choosing the length of the adjustable air line according to (5.4).

5.3 Experimental Results and Comparison with Theory

5.3.1 Performance of the Reactance-Loaded Receiving Antenna as a Function of the Auxiliary Reactance

For a fixed length antenna corresponding to $\beta_o h = 4.0$, gain measurements of backscattered power density and received power have been made for each of the loading positions: $d/h = 0.2, 0.3, 0.4, \ldots, 0.9$. In each case, with $R_L = 50$ ohms and the center-load reactance adjusted for maximum received power, the auxiliary reactance has been varied over the complete range of attainable values. The measured values of P_s and P_R are shown in the following figures for several different positions of auxiliary loading. Curves based upon the theoretical work in Section 4.2 are shown for each case.

As pointed out in Section 5.1, the actual value of auxiliary reactance is the combination of the reactance given by equation (5.1) in parallel with some unknown stray reactance, owing to the stray capacitance across the gap at the position of the load. In order to correlate the length of the coaxial shorting stub ℓ_s with the total value of reactance existing across this gap, this stray reactance must somehow be determined first. Knowing the length $[\ell_s]_{min}$ for which the experimental value of P_s is minimum, this problem is resolved simply by assuming the experimental value of minimizing auxiliary reactance to be the same as the theoretical value, and, by using $[\ell_s]_{min}$ in (5.1), choosing the value of parallel stray reactance accordingly.

Since the configuration of the conducting surfaces of the test antenna changes as the loading position is varied, the amount of stray reactance also varies. For this reason, the assumed value of stray reactance, as computed by the above scheme, is different in each of the cases shown.

As it turns out, the above correlation procedure is relatively insensitive to small changes in the assumed value of the characteristic resistance of the coaxial shorting-stub section. In all cases, R_o has been taken as 75 ohms.

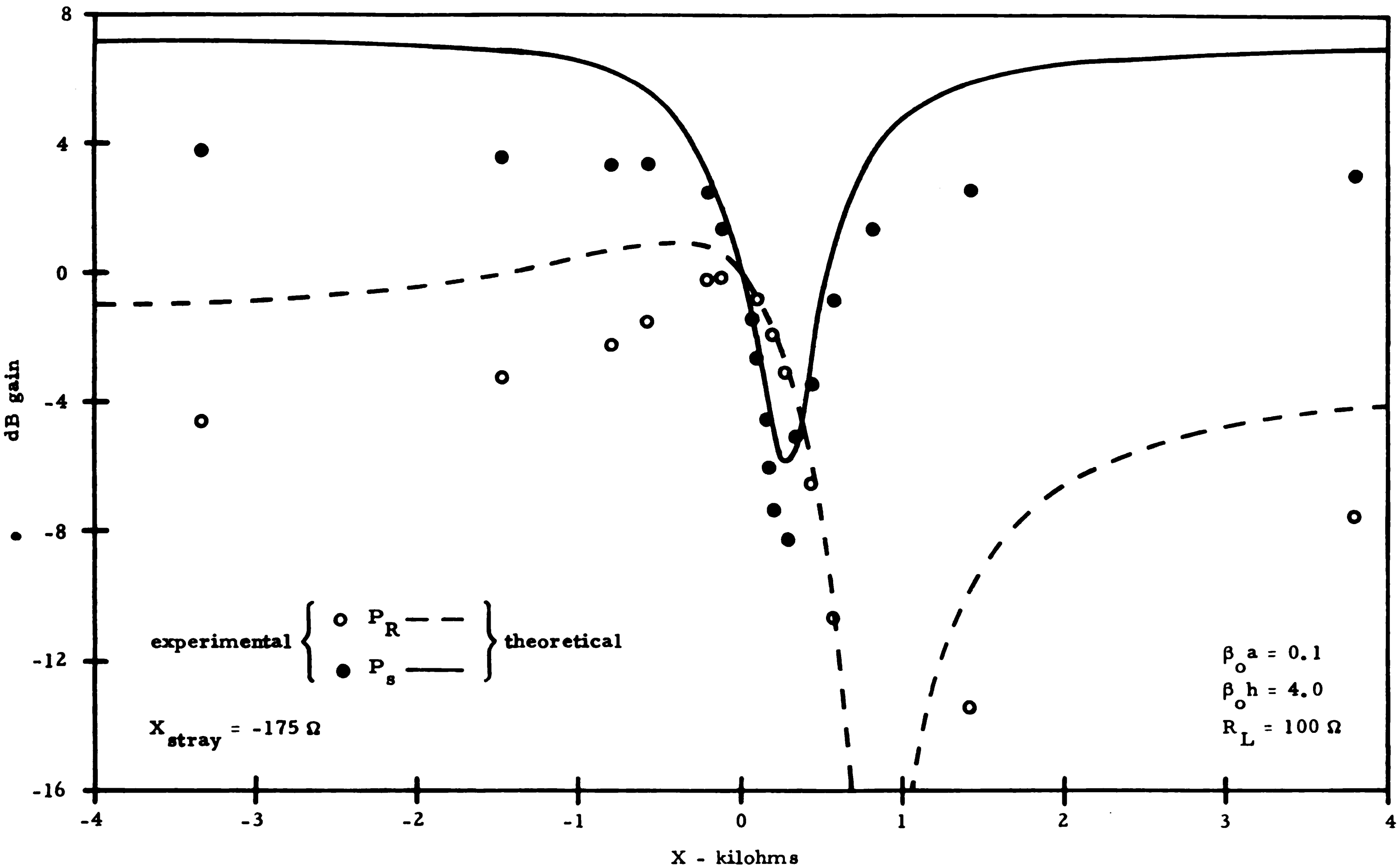


Figure 5.5. Experimental and theoretical results for a reactance-loaded receiving antenna with $X_L = -X_{in}(X)$ and $d/h = 0.3$

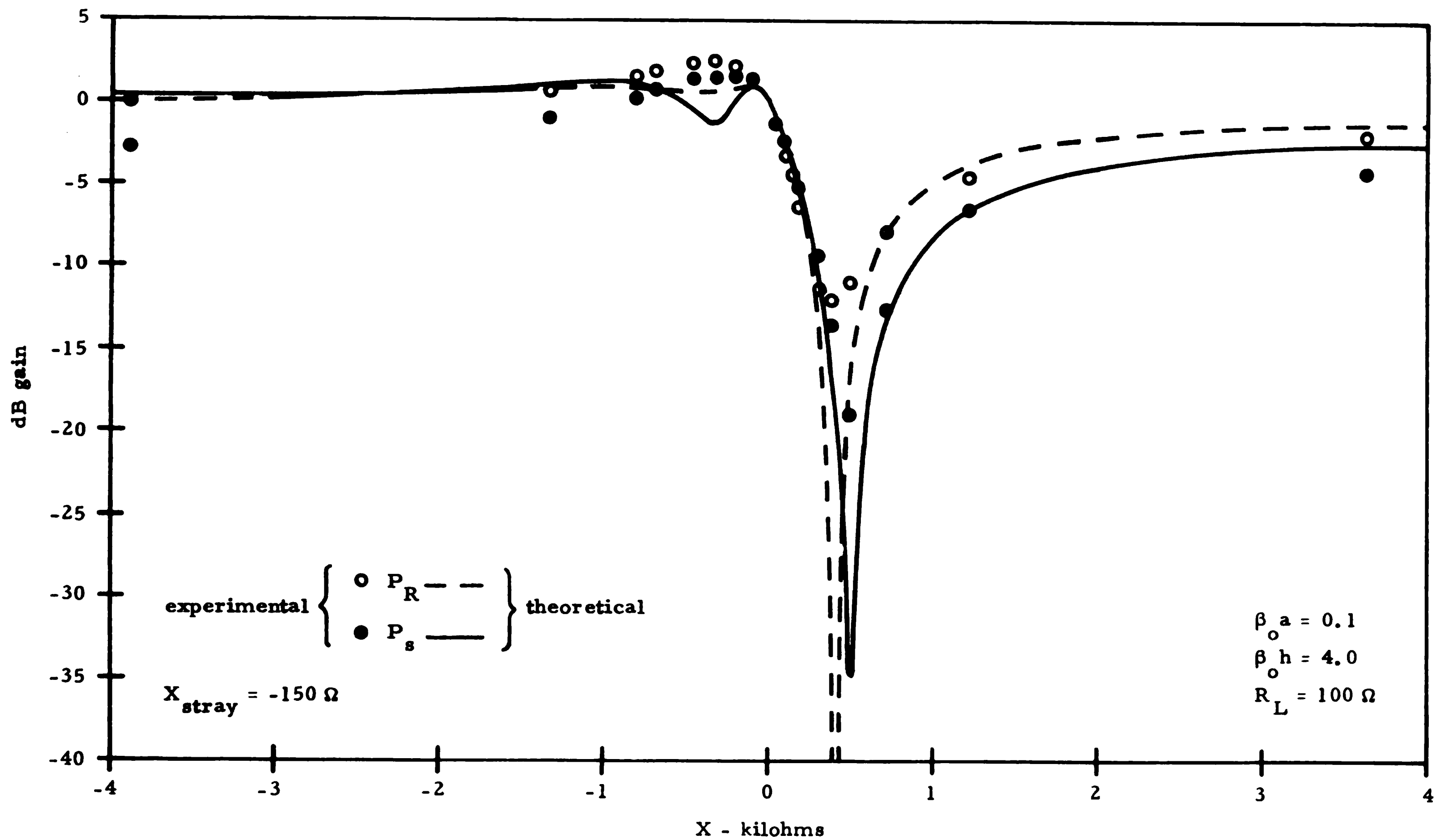


Figure 5.6. Experimental and theoretical results for a reactance-loaded receiving antenna with $X_L = -X_{in}(X)$ and $d/h = 0.4$

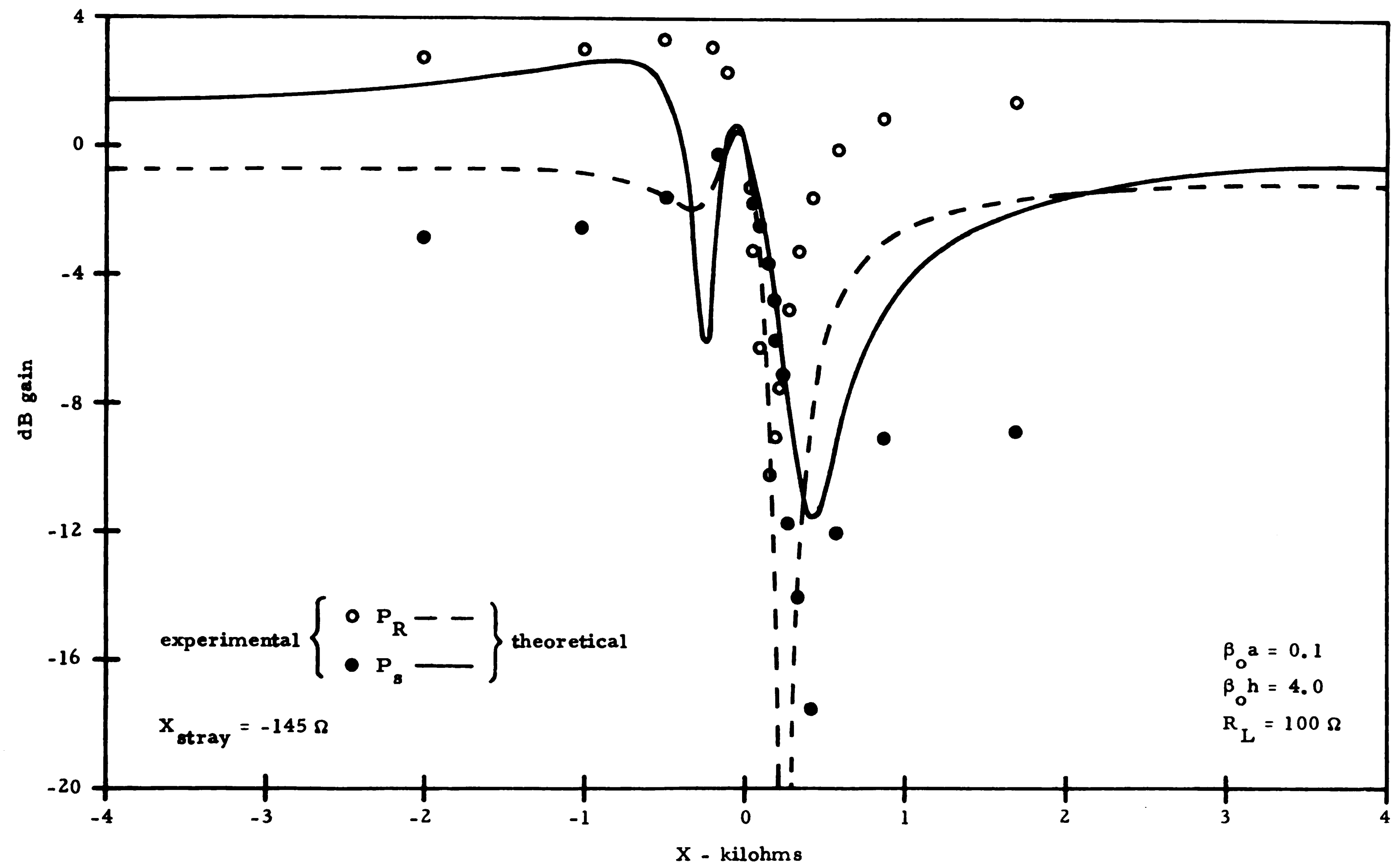


Figure 5. 7. Experimental and theoretical results for a reactance-loaded receiving antenna with $X_L = -X_{in}(X)$ and $d/h = 0.5$

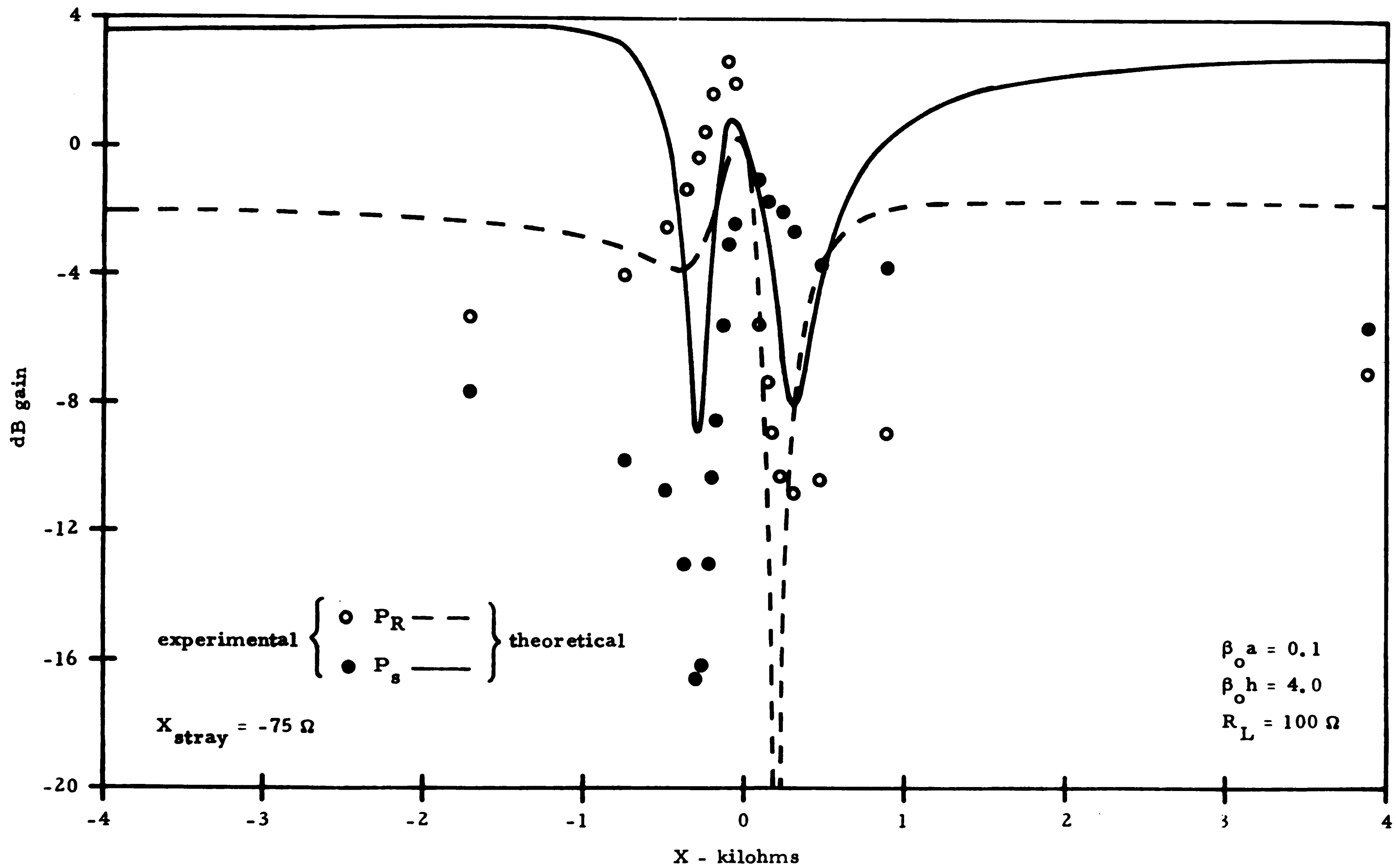


Figure 5.8. Experimental and theoretical results for a reactance-loaded receiving antenna with $X_L = -X_{in}(X)$ and $d/h = 0.6$

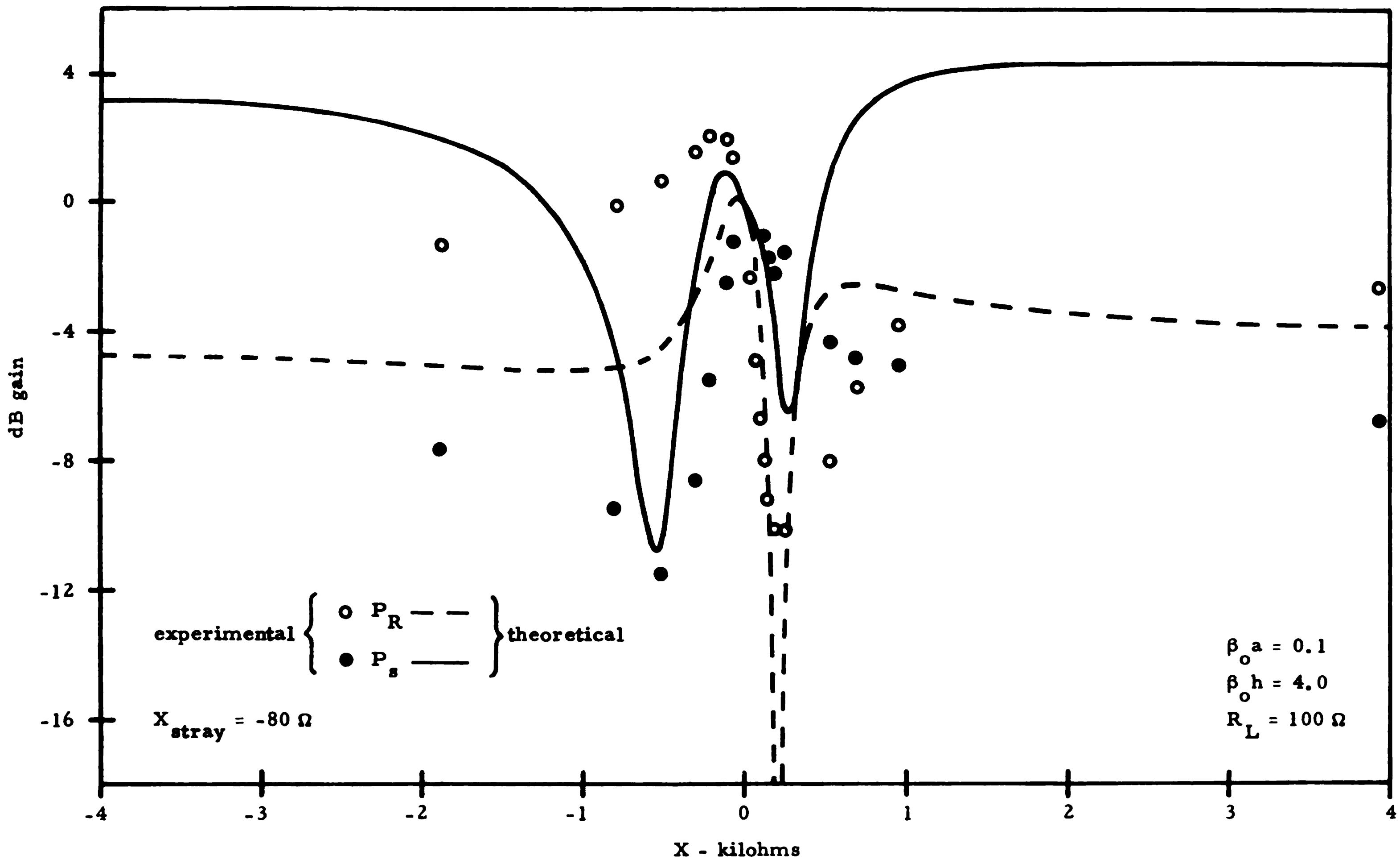


Figure 5.9. Experimental and theoretical results for a reactance-loaded receiving antenna with $X_L = -X_{in}(X)$ and $d/h = 0.7$

5.3.2 Experimental Verification of the Invisible Frequency-Rejection Receiving Antenna

According to the theoretical work in Section 3.1, there exists a particular value of loading position d such that, with the proper value of auxiliary reactance, the backscattered field and the current at the center of the antenna will vanish simultaneously, independent of the value of Z_L. Using the same setup as in the previous experiment, and with Z_L arbitrary but adjustable, a search for this invisible frequency-rejection antenna has been made. With the initial position of the load selected according to the information in Figure 3.1, both the position and the reactance of the auxiliary load have been carefully adjusted to achieve minimum values of P_R and P_s simultaneously. The results of this investigation are shown in Table 5.1. With the center-load reactance tuned for maximum received power in each case (i.e., with $X_L(X, d) = -X_{in}(X, d)$), the entries for P_R and P_s shown in this table represent the simultaneous minimum values of these power gain quantities as a function of the two variables X and d. Except for the case of $\beta_o h = 5.0$, where no minimum of P_R could be found, these results are clearly indicative of a phenomenon almost exactly like that predicted in Section 3.1.

Table 5.1. **Experimental results of the search for the invisible frequency-rejection antenna.**

$\beta_o h$	theoretical	experimental		
	d/h	d/h	P_R (dB gain)	P_s (dB gain)
3.5	0.369	0.371	-20.0	-24.0
4.0	0.382	0.390	-12.7	-25.5
4.5	0.399	0.405	-2.5	-21.0
5.0	0.420	0.427		-15.5

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APPENDIX

RECEIVED POWER OF THE DOUBLE-LOADED ANTENNA

Based on the continuity of the tangential component of the total electric field at the antenna surface, the vector potential $A_z(z)$ maintained on the surface of the double-loaded receiving antenna by the induced antenna current I(z) must satisfy:

$$\frac{\partial^2 A_z}{\partial z^2} + \beta_o^2 A_z = \frac{j\beta_o^2}{\omega} [Z_L I(0)\,\delta(z) + Z I(d)\,\{\delta(z-d) + \delta(z+d)\} - E_o]. \tag{A.1}$$

The symbols in this equation are carefully defined in Chapter 1 where (A.1) is derived as equation (1.6).

Since (A.1) is a linear equation it can be written as the sum of the following two equations.

$$\frac{\partial^2 A_z^R}{\partial z^2} + \beta_o^2 A_z^R = \frac{j\beta_o^2}{\omega} [Z I^R(d)\{\delta(z-d) + \delta(z+d)\} - E_o] \tag{A.2}$$

$$\frac{\partial^2 A_z^T}{\partial z^2} + \beta_o^2 A_z^T = \frac{j\beta_o^2}{\omega} [Z_L I(0)\delta(z) + Z I^T(d)\{\delta(z-d) + \delta(z+d)\}] \tag{A.3}$$

where $$A_z(z) = A_z^R(z) + A_z^T(z) \tag{A.4}$$

and $$I(d) = I^R(d) + I^T(d) \tag{A.5}$$

Noting that equation (A.1) applies to the receiving antenna system depicted in Figure 1.1, comparison of the right side of this equation, in turn, with the right-hand sides of (A.2) and (A.3) readily indicates the physical significance of the latter two equations: in equation (A.2), $A_z^R(z)$ is the surface vector potential supported by the induced current $I^R(z)$ on a double-loaded receiving antenna for which the center impedance is specified as zero; equation (A.3), on the other hand, relates the surface vector potential $A_z^T(z)$ to the current distribution $I^T(z)$ existing on a double-loaded transmitting antenna which is being center-driven by a voltage source having the value

$$V = -I(0)Z_L \, . \tag{A.6}$$

Using equation (1.8), the vector potential functions $A_z^R(z)$ and $A_z^T(z)$ are individually expressed in terms of the current distributions supporting them.

$$A_z^R(z) = \frac{\mu_o}{4\pi} \int_{-h}^{h} I^R(z')\, K_a(z, z')dz' \tag{A.7}$$

$$A_z^T(z) = \frac{\mu_o}{4\pi} \int_{-h}^{h} I^T(z')K_a(z, z')dz' \tag{A.8}$$

Adding these equations, according to (A.4), gives

$$A_z(z) = \frac{\mu_o}{4\pi} \int_{-h}^{h} \{I^R(z') + I^T(z')\}\, K_a(z, z')dz' \, . \tag{A.9}$$

In view of equation (1.8), it follows from (A.9) that

$$I(z) = I^R(z) + I^T(z) . \qquad (A.10)$$

Thus, in particular,

$$I(0) = I^R(0) + I^T(0) . \qquad (A.11)$$

Now, in terms of the quantities defined in connection with the transmitting antenna described by equation (A.3), the input impedance of the double-loaded antenna can be written

$$Z_{in} = \frac{V}{I^T(0)} . \qquad (A.12)$$

By combining (A.12) with (A.6), V is eliminated to give

$$I^T(0) = \frac{-I(0)Z_L}{Z_{in}} . \qquad (A.13)$$

After substituting (A.13) into (A.11) and rearranging, I(0) becomes

$$I(0) = \frac{Z_{in}I^R(0)}{Z_L + Z_{in}} . \qquad (A.14)$$

Equation (A.14) can be rewritten in the form:

$$I(0) = \frac{V'}{Z_L + Z'_g} \qquad (A.15)$$

where $Z'_g = Z_{in}$ and $V' = Z_{in}I^R(0)$, (A.16)

so that the received power of the double-loaded antenna can be expressed as

$$P_R = \frac{R_L}{2} \left| \frac{V'}{Z_L + Z'_g} \right|^2 . \qquad (A.17)$$

The terms V' and Z'_g appearing in (A.17) are properly interpreted as the voltage driver and its series impedance in an equivalent circuit representation of the double-loaded receiving antenna system as viewed from the antenna terminals. As is evident from (A.16), these two quantities, although clearly functions of the auxiliary impedances Z, are totally independent of the center-load impedance Z_L.

That equation (A.17) applies, innately, in the case of a double-loaded receiving antenna constructed of perfectly conducting material, is therefore established. Significant in its own right, this result is contributory to the work in Section 2.2.