MONEY AND WEALTH IN OPEN AND INTERDEPENDENT ECONOMIES

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Anthony Koo
Major professor

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ABSTRACT

MONEY AND WEALTH IN OPEN AND
INTERDEPENDENT ECONOMIES

By

Louis Raymond Eeckhoudt

This dissertation is concerned with two recent developments in economic theory. One deals with the macroeconomic aspect of international trade and balance of payments adjustment; the other one is related to the definition of a truly static macroeconomic equilibrium. The main contribution of this thesis lies in an attempt to integrate these developments in a model of two and then three open and interdependent economies.

Some developments in economic theory have brought into question the traditional static macroeconomic equilibrium condition: saving equals investment. The objection is on the ground that positive investment implies a positive rate of growth of output, and hence for a constant money stock, continuously falling prices which are inconsistent with a static equilibrium.

Quite recently it has been proposed to write the truly static equilibrium condition as: saving equals investment equals zero.

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In this dissertation the implication of this approach in the case of two and then of three interdependent economies are examined. The interdependence between the economies is taken into account not only for the exchange of goods and services but also for that of capital. As a result both income and interest rates will play a role in the balance of payments adjustment.

The analysis is carried out in comparative statics as well as in a dynamic framework, viewing saving and investment as means by which the private sector of the economy tends to correct a disequilibrium between its desired and actual levels of wealth.

The most important result of this analysis is that in the comparison of two equilibrium situations, monetary policy cannot affect the level of income even if it can change the equilibrium rate of interest. However, it is found that monetary policy can cause fluctuations of income in the path towards equilibrium.

It is also found that three interdependent economies are more likely to be stable than two.

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By

Louis R. Eeckhoudt

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LIST OF SYMBOLS

Up to the beginning of part III a primed variable will refer to country II, an unprimed one to country I. The changes occurring in part III will be mentioned in the text.

Y : national income. Y_E stands for the equilibrium level

G_0: autonomous component of government expenditures

X_0: autonomous component of exports

X : total exports

Im: total imports

t : tax rate

T : total tax receipts

m : marginal propensity to import

i : interest rate

$\tilde{W}$: total desired wealth of the private sector

W : total existing wealth of the private sector

M_0: total initial money stock

B_0: total initial stock of bonds

K_0: total initial stock of capital

I : investment

S : saving

S^f: trade surplus

S_g: government surplus

v : fractional reserve requirement of the central bank

P : net deficit or surplus of the balance of payments

B : net inflow of bonds into country I

INTRODUCTION

This dissertation is concerned with two recent developments in economic theory. One deals with the macroeconomic aspect of international trade and balance of payments adjustment; the other one is related to the definition of a truly static macroeconomic equilibrium.

It is well accepted that--broadly speaking--three possible balance of payments adjustment mechanisms (price-specie flow mechanism, exchange rate changes and income changes) have been considered in the economic literature (41, pp. 47-55). Although some attempts have been made to incorporate these three mechanisms in a more general theory it is still usual to deal with each of them separately.[1] Because of our concomitant interest in the definition of a truly static macroeconomic equilibrium we shall focus our attention upon the income changes adjustment mechanism although we fully recognize that a more global approach is ultimately desirable. Thus for the sake of simplicity the price level in all countries and the exchange rates between them will be assumed to be constant.

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[1]For instance, by assuming that the price level can fluctuate while income and the exchange rate are constant.

Among adherents of the "income" approach there has been some divergence on the basic assumptions of the analysis. The pioneering work of Metzler and Machlup dealing with the foreign trade multiplier (17, 20, 21) considered two or more countries which were supposed to be open and interdependent. That is, the exports of any country depended upon the level of income of its partners. Additionally, in these models the interest rate was assumed to remain constant so that no attention was paid to the financial arrangements underlying transfers between countries. In order to make the constancy of the interest rate in each country more plausible, Metzler (20) studied the case of:

> Two countries on gold standards with sufficient reserves so that central bank policies are not affected by gold movements. The consistency of domestic changes with the maintenance of external equilibrium will then depend upon the way in which variations of investment and consumption in one country react upon investment and consumption in others. The effects of such changes are isolated by setting up a model of trade between two countries in which variations of prices, interest rates and the rate of exchange are impossible.

Of course as these models were concentrated exclusively upon the analysis of the commodity market equilibrium condition, the incidence of capital movements on other markets (for instance bonds and money) had to be left aside.

In subsequent studies (15, 27, 39) the restrictive assumption about interest rates was dropped but this gain was achieved at the cost of neglecting interdependence between economies.

The assumption of a "small country" became widespread in the literature. As an example, A. O. Krueger (15) writes:

> It is assumed that the rest of the world is unaffected by any development within the country: in particular that foreign interest rates are constant and that the terms of trade and foreign prices are unaffected by the level of domestic real income or prices. (15, p. 199)

Consequently the level of exports of the "small" country could be treated as a completely exogenous variable while its monetary policy did not affect the level of the interest rate in the rest of the world. The capital movements between the "small" country and the rest of the world--when they were considered--were made a function of the national interest rate only, given the (exogenous) interest rate prevailing in the other countries. It is interesting to note that the "assignment problem" for an open economy has been solved around this set of assumptions [see Mundell (27)].

Quite recently, however, R. N. Cooper has shown that both the assumption of interdependence and flexibility of national and foreign interest rates can be incorporated into a macroeconomic model of open economies. In most parts of this dissertation these assumptions will be maintained because they can lead to interesting results while being at the same time more realistic. Consequently interdependence not only in the markets of goods and services but also in those of capital will be considered.

All the works cited up to this point share one thing in common: in the Keynesian tradition they all define the equilibrium condition in the commodity market as planned saving equals planned investment. Recently, however, this position has come under strong attack. In 1965, Mundell showed that the intersection between the I-S and L-M curves does not determine a truly static equilibrium since:

> Positive savings-investment implies a growing stock of wealth and capital goods and a positive rate of growth of output. The growth effects are usually dismissed as magnitudes of the second order in the time interval relevant to the short-run equilibrium under consideration. . . . (However) the rates of change of the capital stock, financial assets and the money supply are dimensionally equivalent to the rate of interest and cannot be disregarded. Neglect of them in the literature . . . has concealed results of great interest. (27, p. 61)

In trying to solve the above mentioned problem, Mundell used a growth model. As a result he rejected the static equilibrium analysis of the Keynesian models.

The possibility of keeping the static analysis which is interesting in itself and avoiding the contradiction pointed out by Mundell was studied in 1968 by R. McKinnon (18, 1968). His goal was to:

> Remove the logical difficulty so neatly posed by Mundell and at the same time retain the analytical simplicity of a static Keynesian framework.

The solution to this problem is found:

> When one recognizes that saving out of current income represents the desire to accumulate real and financial assets. Once the accumulation of these assets reaches a certain desired level vis à vis current income and the rate of interest, saving out of current income can be assumed to cease. (16, 1968, p. 207)

Starting from this observation McKinnon developed a comparative statics macro model characterized by the following equilibrium conditions:

- For the commodity market: a flow equilibrium condition as well as a stock condition.

- For both the money and bond markets, a stock equilibrium condition.

Although this dissertation will follow McKinnon's basic line of argument it will differ from it in two important respects:

- First, McKinnon separated the equilibrium conditions for each component of wealth and did not pay attention to their total. Obviously if each component of wealth is in equilibrium so is the total but this approach implicitly tends to emphasize the substitution effects between different forms of wealth. In this dissertation, we shall follow a suggestion of Jones (12) and focus on the total wealth equilibrium condition leaving aside the "partial conditions." In this way we hope to be able to dissociate the wealth effects from the substitution effects which have been analyzed by McKinnon. Of course the wealth effects themselves have already been studied in the economic literature [see, for instance, (24) and (30)]. However, the analysis has been limited to the existing stock of wealth. For example, Patinkin (30) has studied the effect on the demand for bonds, money and commodities of an increased stock of real balances.

In contrast with this view, the position taken here will be that what matters is not so much the existing level of total wealth or of any of its components, but rather the difference between desired and actual wealth.

- Although the comparative statics analysis is interesting, it is not sufficient. McKinnon himself speculated about the out-of-equilibrium behavior of his models but his speculations were not derived from an explicit dynamic model. Hence a dynamic model, where saving and investment take place in disequilibrium situations to increase or decrease the actual level of wealth and bring it in line with the desired one will be a part of this dissertation. The distinction made in the comparative statics analysis between total wealth equilibrium and partial equilibrium for each component will be carried over to the dynamic analysis.

As this dissertation is dealing with rather new concepts and approaches we shall proceed very cautiously in the analysis, starting from a simple case in Part I and reaching a more elaborate model in Part II, considering only two interdependent economies.

In Part I the equilibrium condition between the desired and actual level of wealth is assumed to be automatically satisfied. Thus only the equilibrium condition between the flows of commodities in each country is considered. Using these conditions multipliers of different monetary, fiscal or commercial actions on income are studied both from a comparative statics and a dynamic point of view. The results are then compared with those of the more traditional analysis.

In Part II the wealth equilibrium condition is explicitly introduced along with equations describing the flows of commodities.

By making imports a function of both income and wealth, a direct link is introduced between the two types of equilibrium conditions. Finally a dynamic model is presented which incorporates saving and investment functions whenever the desired and actual levels of wealth differ.

The first two parts of this dissertation deal with the traditional two countries model while the last part extends the analysis to three countries. In this way it can be seen to what extent a given country is affected by changes occurring in one or two of its partners. Some comparisons are also drawn with the "two-countries" case.

As indicated at the beginning of the introduction, it will be assumed throughout the analysis that the exchange rate between the countries is constant.

PART I

NO EXPLICIT WEALTH EQUILIBRIUM CONDITION

THE TWO COUNTRIES CASE

CHAPTER 1

DESCRIPTION OF THE MODEL

Consider two interdependent economies, each of them with private and government activities. These economies exchange goods and services at a fixed exchange rate and they are supposed to be in underemployment so that it is safe to assume a constant price level in each country.

In analyzing the effect of change of autonomous variables on endogenous ones, we shall introduce a balance sheet equilibrium condition (namely that in the private sector desired wealth equals existing wealth) and compare the results of such a model with the more traditional approach where balance sheet relationships are neglected.

Section 1.1: Basic Assumptions

(1) In each country the existing level of wealth is always equal to the desired one and does not affect any variable in the commodity market. These assumptions will be relaxed in Part II when the wealth equilibrium condition is explicitly introduced.

(2) The exports of one country depend upon the level of income in the other country plus an autonomous component.

It is assumed that the marginal propensity to import is constant for each country.

(3) While government expenditures are not a function of income, taxes are a function of income with a constant marginal rate and there is no autonomous component.

Obviously for (2) and (3) many variants could be discussed (e.g. government expenditures could also be made a function of income) but as they would change only the algebra and not the essential features of the model, they will not be considered here.

Section 1.2: Equilibrium Conditions

If saving and investment decisions result from a disequilibrium between desired and existing wealth, the consequence of assumption (1) is that in equilibrium all incentives to save or invest have disappeared.

As a result, the equilibrium between the flow of commodities produced and demanded, which in traditional models is:

$$I + X + G = S + Im + T$$

becomes simply $X + G = Im + T$ or $T - G = X - Im$, as $I = S = 0$ in a true stationary state equilibrium where I = investment, X = exports, G = government expenditures, S = savings, Im = imports and T = taxes. To emphasize, G consists only of consumption expenditures.

Using now assumptions (2) and (3) the equilibrium conditions can be written:

(1.1) Country I : $tY - G_0 = X_0 + m'Y' - X_0' - mY$

(1.2) Country II: $t'Y' - G_0' = X_0' + mY - X_0 - m'Y'$

where $Im = X_0' + mY$, $X = X_0 + m'Y'$ and $Im = X$ since there are only two countries.

CHAPTER 2

ANALYSIS OF THE MODEL

Section 2.1: Comparative Statics Analysis

We have a system of two equations with two unknowns Y and Y' which gives the following solutions:

$$(1.3) \qquad Y_E = \frac{G_0(t' + m') + G_0'(m') + X_0(t') - X_0'(t')}{tt' + mt' + m't}$$

$$(1.4) \qquad Y_E' = \frac{G_0(m) + G_0'(m + t) - X_0(t) + X_0'(t)}{tt' + mt' + m't}$$

Some characteristics of these results are worth noticing:

(a) If t,t', m and m' are all positive then all multipliers have a well defined sign: the multipliers of G_0 $(= \frac{t' + m'}{tt' + m't + mt'})$ and of G_0' $(= \frac{m'}{tt' + m't + mt'})$ in country I are always positive. The multiplier effect is likely to be greater in the country which initiates the increase of government expenditures since in general $t' + m' > m$. Furthermore the effect of changes of G_0 upon the national income of country I depends *inter alia* on t' and m', the marginal propensities to tax and import in country II.

(b) For a given country, the multiplier of G_0 is greater than that of X_0 since $t' + m' > t'$. The economic reason for this difference is that ΔG_0 has only expansionary effects in country II while ΔX_0 first reduces income in the other country so that the induced component of exports of I will be lower than when G_0 is increased.

(c) An increase of exports by one country tends to reduce income in the other one.

(d) The effects of changes in G_0 and X_0 on the trade surplus (S^f) can also be computed from the definition of $S^f = X + m'Y' - X' - mY$.

It is easily found that

$$\frac{\Delta S^f}{\Delta G_0} = \frac{-\,mt'}{tt' + mt' + m't} < 0$$

$$\frac{\Delta S^f}{\Delta X_0} = \frac{tt'}{tt' + mt' + m't} > 0$$

These multipliers are smaller than unity in absolute value.

Comparison with the Traditional Analysis

As indicated in the introduction, the analysis made here is similar in one respect to that of the pioneering work of the "foreign trade multiplier" in the sense that interest rates are left out of the picture. Consequently a detailed comparison with Metzler's models of international trade (20-23) seems in order.

A first difference with his models is more apparent than real. Metzler is mostly concerned with the problem of transfers between two economies and makes a distinction between three kinds of transfers:

(1) Transfers which affect income directly in the paying country.

(2) Transfers which affect income directly in the receiving country only.

(3) Transfers which affect income directly in both countries.

Of course nothing has been said specifically about "transfers" in this dissertation but it should be obvious that the three cases analyzed by Metzler can be incorporated in this model without any difficulty.

If country I is the receiving country and II the paying one, Metzler's case (1) corresponds to an increase of G_0 in country I for an amount equal to that of the transfer while nothing occurs in country II. Case (2) corresponds to a decrease of G_0' equal to the transfer from II to I, nothing occurring in I. Finally case (3) can be represented by a concomitant increase of G_0 in I and equal decrease of G'_0 in country II. Thus the transfer problem can easily be introduced into our model. It is simply a matter of reading G_0 or G_0' as "the amount transferred" rather than as "autonomous government expenditures."

What then is the difference between Metzler's analysis and ours? It will be recalled that all the multipliers studied in this model have a definite sign. On the contrary

Metzler's results are not so obvious: to determine the signs of the multipliers, the stability conditions have first to be determined (see (21) pp. 413-414).

The reason for this difference is that Metzler's equations allow for positive saving and investment in equilibrium while in this model $S = I = 0$, in the final equilibrium position. The existence of positive saving and investment in Metzler's models implies that a consumption function is introduced into the equilibrium conditions for each country. Then the signs of the multipliers remain undetermined if some stability restrictions are not put on the magnitude of the marginal propensities to consume domestic goods in each country.

Consequently the failure to recognize that in full equilibrium saving and investment should both equal zero gives rise to different multipliers not only in absolute value but even in sign as long as some stability restrictions have not been put on some coefficients of Metzler's model.

Section 2.2: Dynamic Analysis

To carry out the dynamic analysis it will be assumed that:

- current tax receipts depend upon current income.
- current imports depend upon the previous year income.

Given these assumptions the equilibrium conditions can be written:

(1.5) $$X_{0t} + m'Y'_{t-1} - X'_{0t} - mY_{t-1} = tY_t - G_{0t}$$

(1.6) $$X'_{0t} + mY_{t-1} - X_{0t} - m'Y'_{t-1} = t'Y'_t - G'_{0t}$$

where X_{0t}, X'_{0t}, G_{0t} and G'_{0t} stand for the values of the autonomous components at time t.

Before proceeding to the analysis it must be recalled that the wealth equilibrium condition is supposed to be satisfied in all periods so that no investment or saving take place. This assumption will be relaxed in Part II.

After the necessary transformations, equations (1.5) and (1.6) can be rewritten in matrix form:

(1.7) $$\begin{bmatrix} Y_t \\ Y'_t \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{m}{t} & \frac{m'}{t} \\ \frac{m}{t'} & -\frac{m'}{t'} \end{bmatrix}}_{A} \begin{bmatrix} Y_{t-1} \\ Y'_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{X_{0t} - X'_{0t} + G_{0t}}{t} \\ \frac{X'_{0t} - X_{0t} - G'_{0t}}{t'} \end{bmatrix}}_{B}$$

The solution of this system of two linear difference equations is - given Y and Y' as initial conditions:

$$\begin{bmatrix} Y_t \\ Y'_t \end{bmatrix} = A^t \cdot \begin{bmatrix} Y_0 \\ Y'_0 \end{bmatrix} + (I - A^t) \cdot (I - A)^{-1} \cdot B$$

The stability condition will be satisfied if A^t tends to ϕ, the null matrix, then t tends to infinity. If this condition is satisfied $\begin{bmatrix} Y_t \\ Y'_t \end{bmatrix}$ will converge to $(I - A)^{-1} \cdot B$ which is the vector made of Y_E and Y'_E as given in (1.3) and (1.4).

As the matrix A is singular, one of its characteristic roots is zero. The other one is simply $-(\frac{m}{t} + \frac{m'}{t'})$. For the system to converge towards equilibrium it is thus necessary and sufficient that $\frac{m}{t} + \frac{m}{t'} < 1$.

This condition implies that in each country the marginal rate of taxation should at least be as great as the marginal propensity to import[1] and the boundary diagram is as follows

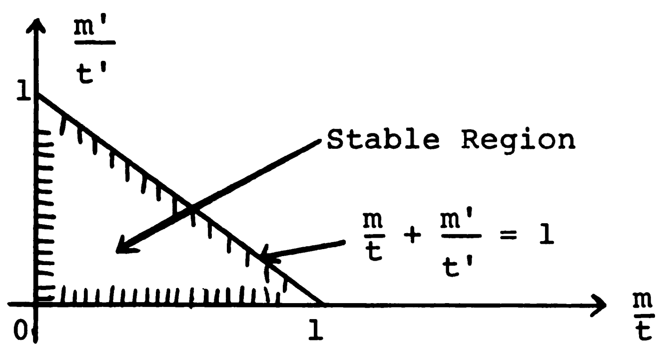

Figure 1

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[1] This condition is necessary, not sufficient.

Again a comparison with Metzler's model should be mentioned. His stability conditions (18, p. 102) involve such terms as the marginal propensity to consume domestic or foreign goods and the marginal propensity to invest in each country. The marginal rate of taxation does not appear in the stability conditions since his model does not include any explicit government activity.

In this model, only the marginal propensities to consume foreign goods (and the marginal rates of taxation) are relevant for stability. Thus, in contrast to the traditional models no attention need be paid here to the marginal propensities to invest or consume domestic goods.

In order to determine how this system behaves through time either towards equilibrium or away from it, the following formula (10, p. 231) can be used:

$$A^t = \left(-\frac{m}{t} - \frac{m'}{t'}\right)^{t-1} \cdot A$$

Then

$$\begin{bmatrix} Y_t \\ Y'_t \end{bmatrix} = \left(-\frac{m}{t} - \frac{m'}{t'}\right)^{t-1} \cdot A \cdot \begin{bmatrix} Y_0 \\ Y'_0 \end{bmatrix} + \left(I - \left(-\frac{m}{t} - \frac{m'}{t'}\right)^{t-1} A\right) \begin{bmatrix} Y_E \\ Y'_E \end{bmatrix}$$

Calling $-\frac{m}{t} - \frac{m'}{t'} = a$, the expression for Y_t and Y'_t becomes

$$(1.8) \quad Y_t = a^{t-1}\left(-\frac{m}{t} Y_0 + \frac{m'}{t} Y'_o\right) + \left(1 + \frac{m}{t} a^{t-1}\right) Y_E - \frac{m'}{t} a^{t-1} Y'_E$$

$$(1.9) \quad Y'_t = a^{t-1}\left(\frac{m}{t'} Y_0 - \frac{m'}{t'} Y'_0\right) - \frac{m}{t'} a^{t-1} Y_E + \left(1 + \frac{m'}{t'} a^{t-1}\right) Y'_E$$

or by rearranging terms:

$$(1.10) \quad Y_t = Y_E + a^{t-1}\left(\frac{m}{t}(Y_E - Y_0) - \frac{m'}{t}(Y'_E - Y'_0)\right)$$

$$(1.11) \quad Y'_t = Y'_E + a^{t-1}\left(-\frac{m}{t'}(Y_E - Y_0) + \frac{m'}{t'}(Y'_E - Y'_0)\right)$$

As a is necessarily negative Y_t and Y'_t will always have an oscillatory behavior either converging to Y_E, the equilibrium value of income, if $\frac{m}{t} + \frac{m'}{t'} < 1$ or diverging from it if $\frac{m}{t} + \frac{m'}{t'} > 1$.

The formulation of (1.10) and (1.11) also gives an interesting property of the behavior of Y_t with respect to Y'_t.

Consider for instance an increase of autonomous government expenditures such that $G_{0t} = G_{00} + \Delta G_0$ (that is an increase maintained in all periods). Then (1.10) and (1.11) can be rewritten as:

$$(1.12) \quad Y_t = Y_E + a^{t-1}\left(\frac{mt'}{t(tt' + mt' + m't)} \Delta G_0\right)$$

$$(1.13) \quad Y'_t = Y'_E + a^{t-1}\left(\frac{-m}{tt' + mt' + m't} \Delta G_0\right)$$

It appears that the variations of Y_t will lead those of Y'_t by one period: for a given t, if Y_t is increasing, Y'_t must be decreasing and conversely since the quantities inside the brackets in (1.12) and (1.13) are of opposite signs.

Furthermore country I will experience stronger fluctuations of income than country II if $\frac{t'}{t} > 1$. Indeed the terms which represent the fluctuations of income in (1.12) and (1.13) are respectively

$$a^{t-1} \frac{mt' \cdot \Delta G_0}{t(tt' + mt' + m't)} \qquad \text{and} \qquad a^{t-1} \frac{- m \, \Delta G_0}{tt' + mt' + m't}$$

Taking the ratio of these two terms a^{t-1}, ΔG_0, and $tt' + mt' + m't$ will cancel out so that the ratio of the fluctuations will simply be given by $\frac{t'}{t}$ in absolute value. If $\frac{t'}{t}$ happened to be equal to unity the amplitude of the fluctuations would be the same in both countries.

Numerical Examples

A. Stable Case

In the initial period $G_{00} = 60$, $X_{00} = 40$, $G'_{00} = 100$ and $X'_{00} = 40$. It is also assumed that $t = .4$, $m = .1$, $m' = .2$ and $t' = .4$. Given these conditions the initial equilibrium values of Y are: $Y_0 = 200$ and $Y'_0 = 200$, which is found by using (1.3) and (1.4). If G_0 is now increased to 100 (that is $G_{0t} = 100$ for $t = 1,2,\ldots\ldots$), coeteris paribus, the behavior of Y, E, T, G, X and Im in both countries is as follows:

TABLE 1

	Y	E	T	G	X	M	Y'	E'	T'	G'	X'	M'
t=0	200	120	80	60	80	60	200	120	80	100	60	80
t=1	300	180	120	100	80	60	200	120	80	100	60	80
t=2	275	165	110	100	80	70	225	135	90	100	70	80
t=3	294	176.4	117.6	100	85	67.5	206	123.5	82.4	100	67.5	85

which can be shown graphically as Figure 2.

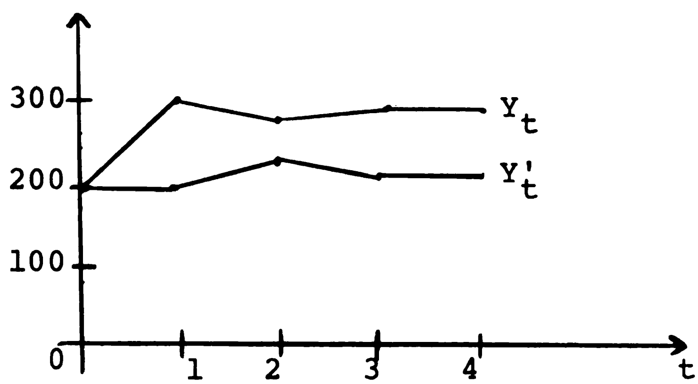

Figure 2

The final equilibrium values are: $Y_E = 288$ and $Y'_E = 214$. The trade surplus of country I which was 20 to start with falls to 14 in the final equilibrium position.

B. Unstable Case

We assume now that $G_{00} = 80$, $G'_{00} = 40$, $X_{00} = X'_{00} = 50$ and that $t = .2$, $m = .4$, $t' = .4$ and $m' = .2$ so that $\frac{m}{t} + \frac{m'}{t'} = 1$. If G_{00} becomes 100 and remains at this level the fluctuations will be:

TABLE 2

	Y	E	G	X	M	T	Y'	E'	G'	X'	M'	T'
t=0	200	160	80	90	130	40	200	120	40	130	90	80
t=1	300	240	100	90	130	60	200	120	40	130	90	80
t=2	100	80	100	90	170	20	300	180	40	170	90	120
t=3	600	480	100	110	90	120	50	30	40	90	110	20

Graphically the evolution is pictured in Figure 3.

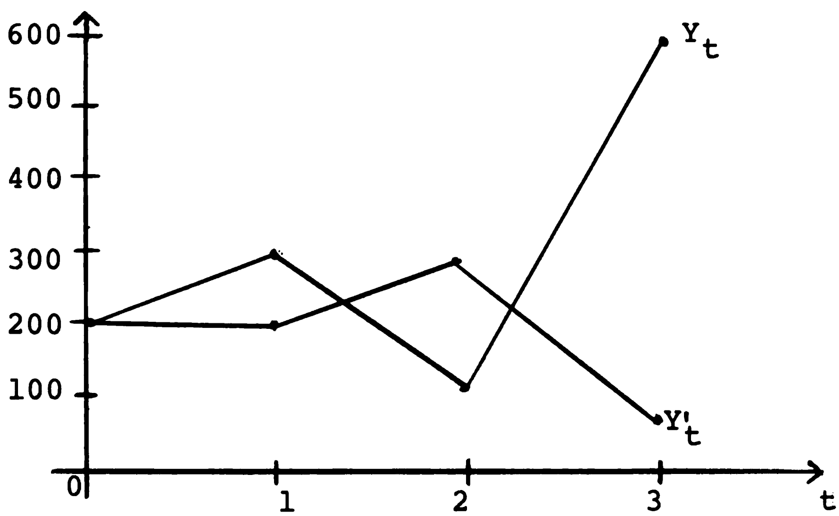

Figure 3

Although ΔG_0 was relatively small and m, m', t and t' had been chosen so that the model was just unstable it appears that the fluctuations of Y_t (and to a lesser extent those of Y'_t) become quickly very strong. Should this system converge towards equilibrium, the comparative statics equilibrium values would be 243 for Y_E and 228 for Y'_E.

PART II

THE WEALTH EQUILIBRIUM CONDITION EXPLICITLY CONSIDERED

THE TWO COUNTRIES CASE

INTRODUCTION

In the first part it has been assumed that the equilibrium condition between total desired wealth and its actual level was always satisfied.

In this part we shall consider explicitly the conditions under which the wealth equilibrium requirement can be fulfilled. Such an analysis will have two implications:

- In the first part of this chapter monetary policy could not play any role since neither the stock of money or the interest rate were included in the equilibrium equation. The introduction of wealth will give some power to monetary actions since outside money is a component of wealth. Chapter 3 deals with these problems.

- Secondly, when the wealth equilibrium condition is disrupted people have an incentive to save (dissave) in order to accumulate (consume) wealth and restore equilibrium between the desired and actual levels. The saving and investment functions absent from the analysis in Part I will thus reappear here in the context of the dynamic model (see Chapter 4).

Before writing down the new equilibrium conditions it is necessary to specify the components of wealth and describe how they are influenced by changes in income or interest rates.

CHAPTER 3

COMPARATIVE STATICS ANALYSIS

Section 3.1: Basic Characteristics of the Model and Equilibrium Conditions

(1) In each country there are three components of wealth: outside money (to be explained later), outside bonds (government or foreign bonds or both) and the stock of capital, human and non-human. Inside bonds and money are not components of wealth and unless redistribution effects are involved (e.g. debtors tend to have a higher propensity to import than creditors) they will not affect the economy. These redistribution effects will be neglected here and thus inside money or bonds will not be considered in the analysis.

(2) Outside money is issued by the central bank either to cover a government deficit or against an inflow of international reserves. The creation of money resulting from an inflow of reserves is not limited, however, to that amount of reserves. It will be assumed that the central banks can create an amount of money greater than the inflow of reserves, the ratio of these values being denoted $v (v \geq 1)$ for country I or v' for country II.

The central banks thus have two policy instruments: either change M directly or change v, the fractional reserve requirement. A value of v equal to unity would imply a 100 per cent reserve requirement while a value of v between zero and one would mean that the central bank accumulates reserves without creating money. Throughout the analysis it will be assumed that v and v' are both greater than or at least equal to unity unless otherwise mentioned.

(3) The government has three ways of financing a deficit: taxation, borrowing from the central bank or borrowing from abroad. A fourth possibility--borrowing from the public--will not be considered here for the sake of simplicity.

(4) Besides exchanging goods the countries will now be exchanging bonds at a fixed exchange rate and consequently it is important to examine the effects of these capital movements upon the level of wealth in each country.[1]

Suppose first that v = v' = 1. Then if the private sector of country I lends to country II, II incurs a debt to I but at the same time experiences a payments surplus of the same magnitude. As v' = 1, an amount of outside money

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[1]The analysis of the "transfer" problem (11, 21, 22) has usually been limited to the impact of the transfer upon the level of planned expenditures in each country involved. In fact the introduction of capital movements will also determine the influence of the transfer upon the level of wealth in each country.

equal to that of the surplus is issued in II so that the two effects (increase of the external debt and the addition to outside money) exactly neutralize each other and wealth remains constant in country II. This is also true for country I since the acquisition of an asset is exactly compensated for by a payments deficit which generates an equal decrease in the stock of outside money (recalling that v is assumed to be unity).

Once v and v' are different from unity, lending abroad will no longer leave wealth unaffected. If country I lends to country II its wealth will decrease because the reduction of the stock of outside money will more than offset the addition to wealth due to the creation of a new asset. These relationships will become more clear when the equilibrium conditions are written down.

Contrary to the common assumptions made in the literature, the capital markets in the two countries considered here will be assumed to be neither perfectly connected nor completely isolated. If they were perfectly connected there would be one interest rate common to both countries and if they were isolated no capital movement would take place.

Equilibrium Conditions

(2.1.1.) $tY - G_0 = X_0 + m'Y' - X'_0 - mY$

(2.1.2.) $t'Y' - G'_0 = X'_0 + mY - X_0 - m'Y'$

$$(2.1.3.) \quad \tilde{W}(Y,i) = M_0 + B_0 + K_0 + vP + B(i,i')$$

$$(2.1.4.) \quad \tilde{W}(Y',i') = M_0' + B_0' + K_0' - v'P - B(i,i')$$

$$(2.1.5.) \quad P \equiv X_0 + m'Y' - X_0' - mY - B(i,i')$$

where $\tilde{W}$ = desired wealth in the private sector of the economy, P = balance of payments deficit or surplus, B = net inflow of bonds into country I, B_0 = initial stock of bonds, B_0 = initial stock of bonds, K_0 = initial stock of capital, M_0 = initial stock of money and i = interest rate.

Some comments on these equations are in order:

(a) As mentioned above $B(i,i')$, the net inflow of bonds into country I influences the existing level of wealth in I in two ways: directly as a component of wealth in equation (2.1.3.) and indirectly as a component of P in (2.1.5.). These two effects have opposite signs as indicated earlier and compensate each other if $v = v' = 1$.

(b) It is assumed that a deficit (surplus) in the balance of payments gives rise to an immediate and equivalent loss (gain) of reserves.

(c) The total money stock in each country can vary for two reasons:

- "active" monetary policy, that is an autonomous change of M or M'.

- "passive" monetary policy: an inflow of reserves gives rise to an increase in the stock of money. This change does not however reflect a totally passive behavior of the central bank since $v(v')$ is one of its instruments.

(d) The signs of most partial derivatives need little comment: $B_i < 0$, $B_{i'} > 0$, $\tilde{W}_Y > 0$, $\tilde{W}'_Y > 0$ where the subscripts refer to the argument of the function for which the partial differentiation is carried out. Only the signs of $\tilde{W}_i$ and $\tilde{W}'_{i'}$ are less clearcut.

From standard economic theory it is known that at lower interest rates people are willing to increase their holdings of money and physical goods and reduce those of bonds. It can then be argued that as physical goods and outside money represent a very large proportion of total wealth, people are also willing to hold more wealth; in other words, the decrease in the amount of bonds the public is willing to hold is more than compensated for by an increase in the demand for money and physical goods when interest rate falls. A more elaborate version of this argument is given in Appendix I. Furthermore it will be shown later on that a sufficient condition for stability of the model is that $\tilde{W}_i$ and $\tilde{W}'_{i'}$ be both negative.

(e) A word of caution is necessary about the exact meaning of P and B(i,i') in equations (2.1.3.) and (2.1.4.). $\tilde{W}(Y,i)$ is the demand for a stock and M_0, B_0 as well as K_0 are existing stocks. Thus P and B(i,i') cannot be considered as flow per period: they represent, in fact, the total deficit (for P) and the total movement of capital (B(i,i')) necessary to restore equilibrium between actual and desired wealth after a given disturbance. Thus, as will become apparent in the dynamic analysis, P is the sum of all the yearly payments surpluses or deficits necessary to restore the equilibrium between desired and actual wealth. In other words, $P = \sum_{t=1}^{n} P_t$, where P_t is the annual deficit or surplus at time t, and n is the number of periods necessary to return to equilibrium. A similar interpretation must be applied to B(i,i'), namely that $B = \sum_{t=1}^{n} B_t$.

Section 3.2: Properties of the Model

(1) The most important property of this model is that it is dichotomized: equations (2.1.1.) and (2.1.2.) determine the level of income regardless of the money stock and once the level of income is determined in these equations the burden of adjustment between desired and actual wealth in both countries is left to the two interest rates.

This result is exactly equivalent to that arrived at by authors who have introduced balance-sheet considerations or any form of budget restraint into macroeconomic models. For instance by introducing a government budget constraint into a model of a closed economy, D. Ott and A. Ott conclude about the role of monetary policy:

> Monetary policy under such conditions exerts no permanent effect on income. Changes in the parameter of the investment or consumption function affect the rate of interest but not income. (29, p. 75)

Extending this analysis to an open economy (with fixed exchange rates) Oates arrived at a similar conclusion: "In the open economy, monetary policy still remains ineffective in altering the equilibrium level of income." (28, p. 494.) It should be stressed that these conclusions are only valid in a comparative static analysis. In the dynamic model of Chapter 4 these results will be reconsidered.

(2) In a model where only the total equilibrium condition for wealth is considered while no specific attention is paid to its components, open market operations do not affect interest rates. Indeed an open market operation will increase B_0 and decrease M_0 (or vice versa) by the same amount and thus leave the initial existing wealth constant so that nothing happens to the interest rates.

Obviously if the equilibrium conditions for each component of wealth were introduced the interest rate would be affected. However as we continue to try to concentrate on the total wealth effect (which in general has been left aside in the literature) we do not consider here the substitution effects generated by open market operations.

(3) In a comparative static analysis (but not in a dynamic one) the multipliers of Y and Y' with respect to G_0, G_0', X_0 and X_0' are the same as in the previous model since equations (2.1.1.) and (2.1.2.) are unchanged. Thus the only interesting multipliers at this stage are those relating to changes in the equilibrium rates of interest.

A. Effects of a Change in M_0

To analyze the effects of an autonomous increase in the stock of money, we differentiate totally (2.1.3.), (2.1.4.) and (2.1.5.), keeping B_0 and K_0 constant and remembering that Y and Y' have already been determined in (2.1.1.) and (2.1.2.). We get:

$$\tilde{W}_i di = dM_0 + v\,(- B_i di - B_{i'} di') + B_i di + B_{i'} di'$$

$$\tilde{W}'_{i'} di' = - v'\,(- B_i di - B_{i'} di') - B_i di - B_{i'} di'$$

Collecting terms and using matrix notation we find:

$$\begin{bmatrix} \tilde{W}_i + vB_i - B_i\,, & v\,B_{i'} - B_{i'} \\ - v'B_i + B_i\,, & \tilde{W}'_{i'} - v'B_{i'} + B_{i'} \end{bmatrix} \begin{bmatrix} di \\ di' \end{bmatrix} = \begin{bmatrix} dM \\ 0 \end{bmatrix}$$

As a result $\frac{di}{dM_0} = \frac{d}{\Delta'}$ and $\frac{di'}{dM_0} = \frac{-c}{\Delta'}$ where

$$d = \tilde{W}'_{i'} - v'B_{i'} + B_{i'} < 0\;,\quad c = - v'B_i + B_i > 0$$

and Δ' is the determinant of the matrix of coefficients of di and di'.

Expanding Δ', it is found to be necessarily positive and as d is negative and c positive, an expansionary monetary policy will reduce the interest rate not only in the initiating country but also abroad.

The economic rationale behind this result is the following one: if the central bank of country I increases the stock of outside money and thus of wealth, the interest rate in I has to fall to restore equilibrium between the desired and actual level of wealth since $\tilde{W}_i$ is negative. But the fall of the interest rate in I which follows the disequilibrium between desired and existing wealth in the same country calls for capital flows between the two countries: the private sector of I engages in lending to that of II so that the stock of outside money and of wealth increases in II and puts a downward pressure on the interest rate there. From this argument a condition for having $\frac{di'}{dM_0} = 0$ is easily found. If country II chooses $v' = 1$ then its level of wealth is not altered by the monetary action of country I and then $\frac{di'}{dM_0} = 0$. This result could also be derived directly from the mathematical expression for $\frac{di'}{dM_0}$ given on page 32. Indeed if $v' = 1$ then $c = 0$.

At this stage some further comments can be made upon the signs of $\tilde{W}_i$ and $\tilde{W}'_{i'}$. It is argued on page 29 that both should be negative. However the possibility of positive $\tilde{W}_i$ and $\tilde{W}'_i$ cannot definitely be ruled out (see Appendix I). Should this case occur then $\frac{di}{dM_0}$ and $\frac{di'}{dM_0}$ could be positive or negative.

To say more about the comparative statics properties of the model when $\tilde{W}_i$ and $\tilde{W}'_{i'}$ are positive, a digression in dynamics is necessary. Suppose that in each country the interest rate increases when there is an excess demand for wealth,[1] that is:

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[1]For additional comments about this specification see pages 51 and 52. Note also that a more complete formulation should follow Beckmann and Ryder (4) but once two countries are involved it becomes unpractical.

$$\frac{di}{dt} = k_1(\tilde{W} - W) \qquad \text{and} \qquad \frac{di'}{dt} = k_2(\tilde{W}' - W')$$

where k_1 and k_2 are the speeds of adjustment and $\tilde{W}$, $\tilde{W}'$, W and W' are all functions of i and i' (They are also a function of Y and Y' but as these variables do not respond to a change in M_0 they can be neglected).[1]

The necessary and sufficient conditions for stability are given by (33, p. 449):

$$\frac{\partial f_1}{\partial i} + \frac{\partial f_2}{\partial i'} < 0 \qquad \text{and} \qquad \frac{\partial f_1}{\partial i} \cdot \frac{\partial f_2}{\partial i'} - \frac{\partial f_1}{\partial i'} \cdot \frac{\partial f_2}{\partial i} > 0$$

where

$$f_1 = f_1(i,i') = k_1(\tilde{W} - W)$$

$$f_2 = f_2(i,i') = k_2(\tilde{W}' - W')$$

Applying this result to the case studied here we find that the stability conditions are:

$$\text{(i)} \quad \frac{\partial f_1}{\partial i} + \frac{\partial f_2}{\partial i'} \equiv k_1[\tilde{W}_i + (v-1)B_i] + k_2[\tilde{W}'_{i'} - (v'-1)B_{i'}] < 0$$

$$\text{(ii)} \quad \frac{\partial f_1}{\partial i} \frac{\partial f_2}{\partial i'} - \frac{\partial f_1}{\partial i'} \frac{\partial f_2}{\partial i} \equiv k_1 \cdot k_2[\tilde{W}_i\tilde{W}'_{i'} + \tilde{W}'_{i'}(v-1)B_i - \tilde{W}_i(v'-1)B_{i'}] > 0 .$$

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[1]The reader's attention is called to a shift from difference equations into differential equations. Refer to a footnote on page 79 for the reasons of this shift.

Consequently, if $\tilde{W}_i$ and $\tilde{W}'_{i'}$ are both negative, as they are very likely to be, the stability conditions are always satisfied. If they are both positive the stability condition will be fulfilled if Δ' is positive and $\tilde{W}_i$ + $(v - 1)B_i$ as well as $W'_{i'} - (v' - 1)B_{i'}$ are both negative.

As a result if $\tilde{W}_i$ and $\tilde{W}'_{i'}$ are positive, to guarantee stability they must be outweighted by $(v - 1)B_i$ and $(v' - 1)B_{i'}$, respectively. Then $\frac{di}{dM_0}$ and $\frac{di'}{dM_0}$ would remain negative.

Having analyzed the impact of dM_0 on the equilibrium level of the interest rate in each country, its incidence upon P, the payments balance, can be easily found.

Differentiating (2.1.5.) we get:

$$\frac{dP}{dM_0} = - B_i \frac{di}{dM_0} - B_{i'} \frac{di'}{dM_0}$$

$$\frac{dP}{dM_0} = - B_i \frac{\tilde{W}'_{i'} - v'B_{i'} + B_{i'}}{\Delta'} - B_{i'} \frac{v'B_i - B_i}{\Delta'}$$

$$\frac{dP}{dM_0} = - \frac{B_i \tilde{W}'_{i'}}{\Delta'} ,$$

which is necessarily negative and smaller than unity in absolute value.

A last question to be solved is the effect of monetary changes upon the final equilibrium value of wealth. Obviously in a first stage an autonomous increase of M_0 increases existing wealth also. However as payments deficits take place as a consequence of the increase of M_0 it is not clear _a priori_ what the final result will be.

Using (2.1.3.) we find

$$\frac{dW}{dM_0} = 1 + v\frac{dP}{dM_0} + B_i\frac{di}{dM_0} + B_{i'}\frac{di'}{dM_0}$$

$$\frac{dW}{dM_0} = 1 + (v - 1)\frac{dP}{dM_0}$$

As $\frac{dP}{dM_0}$ is negative $\frac{dW}{dM_0}$ could be negative and this is more likely when v increases. In the special case where v = 1 $\frac{dW}{dM_0}$ = 1 and the process culminates in the first stage just mentioned since a fractional reverse requirement of 100 per cent prevents the reduction of wealth due to lending abroad.

B. Effects of a Change in G_0

The results for ΔM_0 were very similar to those of traditional macroeconomic models where an increase of M_0 also puts a downward pressure on interest rates. As far as ΔG_0 is concerned it is usually expected that an increase of autonomous government expenditures, _coeteris paribus_, raises the equilibrium level of the interest rate. As this section will show this result is not always valid when wealth effects in interdependent economies are considered.

To derive $\frac{di}{dG_0}$ and $\frac{di'}{dG_0}$ from (2.1.3.) and (2.1.4.) the values of $\frac{dY}{dG_0}$, $\frac{dY'}{dG_0}$, $\frac{dS^f}{dG_0}$ and $\frac{dS^{f'}}{dG_0}$ found in Part I must be used.

Then it is found that:

$$\begin{bmatrix} \tilde{W}_i + vB_i - B_i & , vB_{i'} - B_{i'} \\ -vB_i + B_i & , \tilde{W}'_{i'} - v'B_{i'} + B_{i'} \end{bmatrix} \begin{bmatrix} \frac{di}{dG_0} \\ \frac{di'}{dG_0} \end{bmatrix} = \begin{bmatrix} v \frac{ds^f}{dG_0} - \tilde{W}_Y \frac{dY}{dG_0} \\ - \tilde{W}'_{Y'} \frac{dY'}{dG_0} - v' \frac{ds^f}{dG_0} \end{bmatrix}$$

As the sign of the second component of the right hand vector is undetermined the effect of dG_0 on i and i' can be positive or negative. Indeed it can be shown that

$$\frac{di}{dG_0} = \frac{d\left(v \frac{ds^f}{dG_0} - \tilde{W}_Y \frac{dY}{dG_0}\right) - b\left(- \tilde{W}'_{Y'} \frac{dY'}{dG_0} - v' \frac{ds^f}{dG_0}\right)}{\Delta'}$$

and

$$\frac{di'}{dG_0} = \frac{a\left(- \tilde{W}'_{Y'} \frac{dY'}{dG_0} - v' \frac{ds^f}{dG_0}\right) - c\left(v \frac{ds^f}{dG_0} - \tilde{W}_Y \frac{dY}{dG_0}\right)}{\Delta'}$$

where $a = \tilde{W}_i + vB_i - B < 0$, $b = vB_{i'} - B_{i'} > 0$ and $\frac{ds^f}{dG_0} < 0$.

For the reason just explained (indeterminacy of the sign) the sign of these multipliers depends on the values of $\tilde{W}_Y$, $\frac{dY}{dG_0}$, $\frac{dY'}{dG_0}$ (and so on) but also on those of v and v':

- The greater v, <u>coeteris paribus</u>, the greater $\frac{di}{dG_0}$ and $\frac{di'}{dG_0}$ and thus also the greater the chance that they will be positive. This result makes intuitive sense since when G_0 is increased a deficit in the trade balance of country I is created and if v is large the money supply is strongly reduced in I so that the interest rate tends to increase more than when v is small.

- The greater v' the lower $\frac{di'}{dG_0}$ and $\frac{di}{dG_0}$. Again this result can be justified by economic reasoning: if the trade surplus of country II is accompanied by a large v', the money supply in country II expands rapidly and this increase tends to lower interest rates in both countries since $\frac{di}{dM_0}$ and $\frac{di'}{dM_0}$ are both negative.

The impact on P of an increase of G_0 is obviously uncertain since $\frac{di}{dG_0}$ and $\frac{di'}{dG_0}$ can be positive or negative. Thus if an increase of autonomous government expenditures creates a trade deficit (see Part I) it does not necessarily imply a payments deficit.

C. Effects of Changes in X_0

Applying the same technique as in B. we get:

$$\begin{bmatrix} \tilde{W}_i + vB_i - B_i \,, & vB_{i'} - B_{i'} \\ \\ -v\,B_i + B_i \,, & \tilde{W}'_{i'} - v'B_{i'} + B_{i'} \end{bmatrix} \begin{bmatrix} \frac{di}{dX_0} \\ \\ \frac{di'}{dX_0} \end{bmatrix} = \begin{bmatrix} v\,\frac{dS^f}{dX_0} - \tilde{W}_Y\,\frac{dY}{dX_0} \\ \\ -\,\tilde{W}'_{Y'}\,\frac{dY'}{dX_0} - v'\,\frac{dS^f}{dX_0} \end{bmatrix}$$

As in B. the signs of $\frac{di}{dX_0}$ and $\frac{di'}{dX_0}$ are undetermined but for slightly different reasons. Now the sign of the two components of the right hand vector (and not only one) may be positive or negative. The economic reason for the different effects on i and i' caused by dG_0 and dX_0 (where $dG_0 = dX_0$) will be discussed below (see conclusions of the comparative static analysis).

D. Effect of a Greater Propensity to Lend Abroad

Suppose that people in country I decide to lend more abroad than they did before for all combinations of i and i'. According to the traditional view the equilibrium foreign interest rate should fall and the domestic interest rate should rise. Do these propositions remain valid when interdependent economies and wealth effects are considered?

To analyze this point we rewrite B(i,i') as B(i,i',α) where α stands for preferences and $B_\alpha = \frac{\partial B}{\partial \alpha} = 1$. The effect on the interest rates of a change of α is given by:

$$\frac{di}{d\alpha} = \frac{-(v-1)\tilde{W}'_{i'}}{\Delta'} \quad \text{and} \quad \frac{di'}{d\alpha} = \frac{(v'-1)\tilde{W}_i}{\Delta'}$$

If v and v' are both greater than unity $\frac{di}{d\alpha}$ will be positive and $\frac{di'}{d\alpha}$ negative. Thus, these results are those of the traditional analysis. However the reasons which underlie them are entirely different:

- In the traditional view interest rates change because a higher propensity to lend abroad modifies in each country the existing ratio between monetary and non-monetary assets. In order to induce the public to hold these new ratios the interest rates have to change. The emphasis is on substitution effects [see for instance (24)].

- In this model where total wealth and not its components are considered the fact that country I lends more abroad reduces its total wealth (if v > 1 as it has been assumed) and to restore equilibrium between the desired and actual level of wealth the interest rate must rise.

In country II the obverse is true and the increase in wealth causes the interest rate reduction.

Should v and v' both equal unity then $\frac{di}{d\alpha}$ and $\frac{di'}{d\alpha}$ would equal zero. Indeed, in this case a higher propensity to lend abroad would not redistribute wealth between the two countries so that no adjustment in interest rates would take place.

Conclusions of the Comparative Statics Analysis

(1) As in all models with a balance-sheet equilibrium condition (e.g. wealth or government budget) no direct relationship between Y and M_0 has been found. However, once the wealth equilibrium condition is explicitly considered, changes in the amount of outside money will affect interest rates in both countries.

(2) In traditional macroeconomic theory equal changes of G_0 or X_0 have the same effect on output and interest rates, an increase of G_0 or X_0 raising both the equilibrium values of Y and i.

In Part I it has already been shown that this proposition was not true for Y in the case of two interdependent economies since the multipliers of G_0 and X_0 were found to be different. As far as interest rates are concerned it appears that they also are affected differently by changes of G_0 or X_0. The reason for this is that increases of G_0 and X_0 have different effects on the trade balance (see Part I, page 12). In turn, a different impact on the trade balance implies different changes in the existing level of wealth and thus finally different adjustments in the interest rate.

(3) It could not be emphasized enough that this analysis is confined to wealth effects and not the substitution effects. It does not imply at all that the substitution effects within changes in wealth are unimportant.

Section 3.3: An Extension of the Comparative Statics Model

As pointed out earlier the basic model in this second part is dichotomized in the sense that an increase of the money stock does not affect income directly when positions of final equilibrium are compared. This proposition was valid for both countries.

This characteristic can be removed in two ways which are not mutually exclusive:

- Either by assuming that tax receipts are a function not only of current income but also of wealth. This assumption would be particularly valid if wealth were extensively used as a base for taxation.

- Or by making a similar assumption for imports. Only this second possibility will be considered here. The basic equations (2.1.1.) to (2.1.5.) remain essentially the same with the exception that now

$$Im = X_0' + m_1 Y + m_2 W \quad \text{and} \quad Im' = X_0 + m_1' Y' + m_2' W'$$

where m_1 and m_2 are the marginal propensities to import out of income and wealth respectively.

To avoid confusion the equilibrium conditions will be explicitly rewritten under these new assumptions:

(2.2.1) $tY - G_0 = X_0 + m_1'Y' + m_2'W' - X_0' - m_1Y - m_2W$

(2.2.2) $t'Y' - G_0' = X_0' + m_1Y + m_2W - X_0 - m_1'Y' - m_2'W'$

(2.2.3) $W = \tilde{W}(Y,i) = M_0 + B_0 + K_0 + vP + B(i,i')$

(2.2.4) $W' = \tilde{W}'(Y',i') = M_0' + B_0' + K_0' - v'P - B(i,i')$

(2.2.5) $P \equiv X_0 + m_1'Y' + m_2'W' - X_0' - m_1Y - m_2W - B(i,i')$

We have a system of seven equations with seven unknowns: Y, Y', i, i', W, W' and P. Differentiating totally and considering only a change in M_0--coeteris paribus--it is found:

(2.2.6)

$$\begin{bmatrix} t+m_1, & -m_1', & m_2, & -m_2', & 0, & 0, & 0 \\ -m_1, & t'+m_1', & -m_2, & m_2', & 0, & 0, & 0 \\ -\tilde{W}_Y, & 0, & 1, & 0, & -\tilde{W}_i, & 0, & 0 \\ 0, & -\tilde{W}'_{Y'}, & 0, & 1, & 0, & -\tilde{W}'_{i'}, & 0 \\ 0, & 0, & 1, & 0, & -B_i, & -B_{i'}, & -V \\ 0, & 0, & 0, & 1, & B_i, & B_{i'}, & v' \\ m_1, & -m_1', & m_2, & -m_2', & B_i, & B_{i'}, & 1 \end{bmatrix} \begin{bmatrix} dY \\ dY' \\ dW \\ dW' \\ di \\ di' \\ dP \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ dM_0 \\ 0 \\ 0 \end{bmatrix}$$

Obviously it would be very tedious and improductive to compute multipliers such as $\frac{dY}{dM_0}$, $\frac{dY'}{dM_0}$, $\frac{dW}{dM_0}$ and so on. Furthermore the application of Lancaster's method of "qualitative comparative statics" (16) to this problem did not yield any definite result about the signs of $\frac{dY}{dM_0}$, $\frac{dY'}{dM_0}$, $\frac{dW}{dM_0}$ and so on. However a special limiting case is worth mentioning since it can give some idea of what occurs when M_0 is changed.

As we are interested in the effect of an autonomous increase of outside money on the equilibrium values of Y, Y', W .... we shall consider that v = v' = 0. In this way no endogenous change of the stock of money can take place. Furthermore to avoid complications from the fact that the exchange of bonds between countries involves redistribution of wealth between these countries it will also be assumed that the capital markets of each country are completely isolated. Under these simplifications P and B(i,i') drop out of equations (2.2.3) and (2.2.4) so that dW = dM in country I and dW' = 0 in country II. Consequently equation (2.2.5) is no longer necessary.

As a result of all these assumptions it is found that

$$\frac{dY}{dM_0} = \frac{-\ m_2 t'}{tt' + m_1 t' + m_1' t} < 0$$

$$\frac{dY'}{dM_0} = \frac{tm_2}{tt' + t'm_1 + m_1' t} > 0$$

$$\frac{di}{dM_0} = \frac{1}{\tilde{W}_i}\left(1 - \tilde{W}_Y \frac{dY}{dM_0}\right) < 0$$

by equation (2.2.3) and the assumptions of a fractional reserve requirement equal to zero and the absence of capital movements so that $dM = dW = \tilde{W}_Y dY + \tilde{W}_i di$.

$$\frac{di'}{dM_0} = \frac{\tilde{W}'_{Y'}}{\tilde{W}'_{i'}} \bullet \frac{dY'}{dM_0} \qquad > 0$$

These results should not be given more importance than they deserve since they depend completely on the assumptions which underlie them. However by comparison with previous results and as a first approximation they can clarify the very intricate case presented at the beginning of this chapter.

(1) In contrast to the results obtained in traditional macroeconomic models an expansion of the money stock in one

country tends to have contractionary effects on the income of the initiating country but not in the rest of the world. Indeed the expansion of the money stock in country I increases wealth in this country and thus also the demand for imports which reduces income.

(2) When imports are a function of income only an expansionary monetary policy will reduce interest rates in both countries. This result no longer holds under the present circumstances for country II. In country II the monetary expansion of country I raises income $\frac{dY'}{dM_0} > 0$ and thus also the demand for wealth. As the existing stock of wealth remains constant, to keep equilibrium the interest rate in country II must increase.

(3) What would happen if the restrictive assumptions made before were relaxed? When M_0 is increased the immediate effect is to increase existing wealth in the initiating country. The higher level of existing wealth in I at first stimulates the imports of this country and the exports of the other one, leading to an increase of income in II and a reduction in I. So far the results coincide with those arrived at under the simplifying assumptions. However in their absence the process does not stop here: once v and v' are not zero the increase of imports by country I and exports by country II change P and thus also M in each country in such a way that the stock of money declines in I and increases in II. Furthermore as Y and Y' have also

changed the desired level of wealth is no longer in line with the existing one so that interest rates have to adjust.

Once B(i,i') is different from zero, the resulting capital movements affect P (and thus M) further and the existing level of wealth. All these interacting forces are summarized in (2.2.6). If they are taken into consideration nothing can be said *a priori* about the final results. The assumptions made in this chapter enabled us to look at a special limiting case and to show what would be the "first round" effect of an autonomous increase in the stock of outside money.

At this stage it is clear that a detailed dynamic analysis is called for by the above literary description. This is the purpose of the next chapter.

CHAPTER 4

THE DYNAMIC ANALYSIS

Section 4.1: The Dynamic Model

So far it has been necessary to maintain the condition $S = I = 0$ in the flow of commodities equation. However as a dynamic model makes possible the analysis of disequilibrium situations, saving and investment functions can now be introduced.

Remembering that in this model, as in McKinnon's one, saving and investment are viewed as means by which a disequilibrium situation between desired and actual wealth is corrected it is natural to write the saving and investment functions as follows:

(2.3.1) $$S_t = k_1(\tilde{W}_t - W_t)$$

(2.3.2) $$I_t = k_2(\tilde{W}_t - W_t)$$

where k_1 and k_2 are positive constants and the subscript t refers to the time period considered. This formulation can be criticized on two grounds:

(1) k_1 and k_2 are in fact probably not constant: for instance when the interest rate is low k_2 is likely to increase and k_1 to fall. Indeed the lower the interest rate the slower will savers tend to correct a disequilibrium

between desired and actual wealth while on the other hand when the interest rate is low investors will be prompt to raise the actual level of the capital stock to the desired one.

In a first approximation this problem will be ignored but later on the effect of dropping this assumption will be analyzed.

(2) While S_t depends on the difference between the total desired and actual wealth, I_t strictly speaking should be made a function of the difference between the desired and actual physical capital stock. As in this thesis however no disaggregation of total wealth between its components is made this refinement cannot be incorporated. The formulation in (2.3.2) could then be justified by saying that the disequilibrium in the capital stock is a fixed proportion of the disequilibrium between desired and actual total wealth [for a similar approach, see Scitovsky (35, pp. 49-54)].

For the rest of the commodity market (governmental and foreign sectors) the dynamic formulation of Part I is maintained, namely: current tax receipts depend upon current income while imports are a function of the previous year income. The dynamic equations for the whole commodity are thus:

$$(2.3.3)\quad k_1(\tilde{W}_t - W_t) + tY_t + X'_{0t} + mY_{t-1} = k_2(\tilde{W}_t - W_t) + G_{0t} + X_{0t} + m'Y'_{t-1}$$

$$(2.3.4)\quad k'_1(\tilde{W}'_t - W'_t) + t'Y'_t + X_{0t} + m'Y'_{t-1} = k'_2(\tilde{W}'_t - W'_t) + G'_{0t} + X'_{0t} + mY_{t-1}$$

where the left hand side of (2.3.3) or (2.3.4) represents $S_t + IM_t + T_t$ while the right hand side is simply $I_t + X_t + G_t$. As equations (2.3.3) and (2.3.4) make clear it is also necessary to define the behavior of $\tilde{W}_t$ and W_t.

From the analysis of the beginning of Part II it appears that $\tilde{W}_t$ and $\tilde{W}'_t$ are functions of income and the interest rate. Taking a linear approximation, the functions become in dynamic terms:

$$(2.3.5)\qquad \tilde{W}_t = \alpha_0 Y_{t-1} - \alpha_1 i_{t-1}$$

$$(2.3.6)\qquad \tilde{W}'_t = \alpha'_0 Y'_{t-1} - \alpha'_1 i'_{t-1}$$

where α_0, α'_0, α_1 and α'_1 are positive constants.

The amount of wealth existing at any moment of time is given by the following identities for each country:

$$(2.3.7)\quad W_t \equiv W_{t-1} + I_{t-1} + vP_{t-1} - \gamma_0 i_{t-1} + \gamma_1 i'_{t-1}$$

$$(2.3.8)\quad W'_t \equiv W'_{t-1} + I'_{t-1} - v'P_{t-1} + \gamma_0 i_{t-1} - \gamma_1 i'_{t-1}$$

These equations indicate that the existing wealth at time t is the wealth existing at the beginning of the previous period ($W_{t-1} = M_{t-1} + B_{t-1} + K_{t-1}$) plus the addition or subtraction made during the previous period as a result of investment (I_{t-1}), balance of payments surpluses or deficits (vP_{t-1}), and capital movements ($-\gamma_0 i_{t-1} + \gamma_1 i'_{t-1}$) where γ_0 and γ_1 represent the partial derivatives of $B(i,i')$ with respect to i and i' respectively.

The definitions of W_t and $\tilde{W}_t$ involve new variables P_t, i_t and i'_t the behavior of which must now be explained. The equation for P_t is already implicit in the previous analysis since imports and exports of goods and services as well as capital have already been defined in equations (2.3.3), (2.3.4), (2.3.7) and (2.3.8). Thus:

$$(2.3.9) \quad P_t \equiv X_{0t} + m'Y'_{t-1} - X'_{0t} - mY_{t-1} + \gamma_0 i_t - \gamma_1 i'_t$$

The meaning of P_t (for $t > 0$) is different from that of P in the final equilibrium position of the comparative statics analysis. In fact $P = \sum_{t=0}^{n} P_t$ where n is the number of periods necessary to restore equilibrium. Similarly for capital movements $B = \sum_{t=0}^{n} B_t = \sum_{t=0}^{n} (-\gamma_0 i_t + \gamma_1 i'_t)$.

As far as the behavior of interest rates is concerned many possible dynamic assumptions exist. I shall stick to a simple one, namely that the behavior of interest rates in each country depends only upon the excess demand for wealth in the same country and not at all upon the situation in the other country. This assumption gives rise to the following formulation:

$$(2.3.10) \qquad i_t - i_{t-1} = k_3(\tilde{W}_t - W_t)$$

$$(2.3.11) \qquad i'_t - i'_{t-1} = k'_3(\tilde{W}'_t - W'_t)$$

The signs of k_3 and k'_3 are not a priori obvious. An excess demand for wealth can take many forms, some of them being indicated in the following table.

TABLE 3

	Outside Money	Outside Bonds	Physical Capital	Change in Interest Rate
(1)	Excess demand ↑	Excess supply ↑	Excess demand ↑	↑
(2)	Excess demand ↑	Excess supply ↑	Excess supply ↓	?
(3)	Excess supply ↓	Excess demand ↓	Excess supply ↓	↓
(4)	Excess supply ↓	Excess demand ↓	Excess demand ↑	?

In economic theory it is usually assumed that an excess demand (supply) for capital goods raises (lowers) the interest rate and so does an excess demand for money. An excess demand for bonds however has the contrary effect. The arrows in Table 3 indicate the effect of the different excess demands and supplies upon the interest rate.

If the excess demand for wealth manifests itself in the form of (1) then it will put an upward pressure upon the interest rates since each component works in the same direction, pushing the interest rate up.

In case (3) the three components still work in the same direction but downward so that the excess demand for wealth tends now to reduce interest rates. Finally in cases (2) and (4) the various effects are contradictory.

As a result it cannot be determined a priori how an excess demand for wealth will affect interest rates and k_3 as well as k_3' might be positive, negative or zero.

In view of this situation we shall assume in the numerical example (see Section II) that k_3 and k_3' are both positive since the case investigated there is that of an increase in the stock of outside money. However other assumptions could not be definitely rejected as unrealistic.

Incidentally this discussion makes clear that the benefits derived from the analysis of wealth as a whole can only be gained at some cost, namely the information provided by the situation in the market for each component of wealth.

Summarizing, the model so far consists of equations (2.3.3) to (2.3.11) which represent a system of nine independent equations with nine unknowns: W_t, W_t', $\tilde{W}_t$, $\tilde{W}_t'$, Y_t, Y_t', i_t, i_t' and P_t. This system can be solved once the initial condition for each of the variables are known. For practical reasons an analytical solution as the one found in Part I is no longer possible. As a result, numerical values will be assigned to the parameters and the corresponding solution will be sought. From the results obtained in this way some more general conclusions will be derived about the properties of the model.

Section 4.2: Analysis of the Model

The dynamic model analyzed here will rest upon the following values of the parameters:

- In equations (2.3.3) and (2.3.4),

  $k_1 = .2$, $t = .4$, $m = .2$

  $k_2 = .1$, $m' = .1$, $k_1' = .1$, $t' = .3$, $k_2' = 15$.

- In (2.3.5) and (2.3.6),

  $\alpha_0 = 5.5$, $\alpha_1 = 10{,}000$, $\alpha_0' = 4.3$, $\alpha_1' = 5{,}000$.

- In (2.3.7) and (2.3.8),

  $v = 1$, $\gamma_0 = 1{,}000$, $\gamma_1 = 1{,}200$, $v' = 1$.

- In (2.3.10) and (2.3.11),

  $k_3 = \frac{1}{50{,}000}$ and $k_3' = \frac{1}{25{,}000}$.

The values of α_1, γ_0, γ_1, k_3 and k_3' may seem unreasonable as compared to the others. The explanation is simply that they refer to interest rates which will be expressed in per cent. The initial equilibrium values at time $t = 0$ are assumed to be: $Y_0 = 1{,}000$, $Y_0' = 1{,}000$, $W_0 = \tilde{W}_0 = 5{,}000$, $W_0' = \tilde{W}_0' = 4{,}150$, $i_0 = .05$ and $i_0' = .03$.

Y_0 and Y_0' are obtained from the values of the autonomous components of government expenditures and exports, namely:

$$X_{0t}' = 100 \ , \ G_{0t} = 500 \ , \ X_{0t} = 100 \quad \text{and} \quad G_{0t}' = 200 \ .$$

Suppose now that as a result of an increase in the total stock of outside money, existing wealth in country I becomes 5,500 instead of the initial 5,000. The effect of this change is analyzed in detail for each country in Table 4 but the basic reactions can be summarized as follows: once existing wealth increases in country I from its initial equilibrium value it exceeds the desired level of wealth so that people start dissaving and disinvesting. As k_1 is smaller than k_2 the fall of saving in country I is greater than the reduction of investment. Consequently income rises and the interest rate falls in country I, which has the following effects:

- The desired level of wealth in country I increases and tends now to exceed the existing one itself reduced by capital movements and the trade deficit.

- Exports of country II to country I increase so that income in II is also affected. The propagation of the effect of the change in country I's stock of money on country II also takes place through changes in the balance of payments. These effects are minor relative to the increase of exports.

Only the first steps can be described verbally because later on all the effects become interrelated. A better account of them is given in Table 4.

TABLE 4

Variables	Time					
	t = 0	t = 1	t = 2	t = 3	t = 4	t = 5
Country I						
Y	1,000	1,125	905	1,262	510	1,908
$\tilde{W}$	5,000	5,000	5,788	4,498	6,621	2,455
W	5,000	5,500	5,400	5,290	5,247	5,088
i	.05	.04	.048	.032	.035	.030
P	/	-96	-108	-43	-159	0
Country II						
Y'	1,000	1,000	1,068	930	1,020	620
$\tilde{W}'$	4,150	4,150	4,150	4,462	4,000	4,256
W'	4,150	4,150	4,242	4,357	4,400	4,559
i'	.03	.03	.026	.027	.026	.025
P'	/	+96	+108	+43	+159	0

Should the model be stable the final comparative static equilibrium values would be: Y = 1,000; Y' = 1,000, i = .00, i' = .03 (i' remains unchanged in equilibrium because of the assumption that v' = 1--see pages 27 and 28). The equilibrium value of P is -50.

Results of the Analysis

(1) The Incidence of Monetary Changes

While the comparative statics analysis showed that monetary policy does not affect income (unless imports are a function of both income and wealth) the dynamic analysis on the contrary restores its usefulness.

According to the dynamic model it appears that an increase of the stock of outside money affects the economy by changing the existing level of wealth and causing flows of (dis)saving and (dis)investment.

It is important to notice that the effectiveness of this wealth effect does not require that consumption or investment depend upon the level of real balances. [Since in this model the consumption and investment functions do not respond to the level of real balances.] Furthermore the argument that "an increase of the stock of outside money induces only a very small percentage change in total wealth and is thus unimportant" can be dismissed. It obviously remains true that a change in the stock of outside money does not result in a large relative change in the stock of wealth, however, the dynamic analysis shows that this is not the relevant comparison to make.

Indeed the change of M_0 creates a disequilibrium between desired and actual wealth which in turn affects saving and investment. These changes however are commensurate with the level of exports and government expenditures and thus cannot be neglected. In fact the first period multiplier of ΔM_0 in country I can be very high and is given by $\frac{\Delta Y}{\Delta M_0} = \frac{k_1 - k_2}{t}$ which in the above example gives 1/4.

At this stage the role of inside money can also be analyzed although it will not be done in a formal way. Suppose that there is an increase in the stock of outside money of country I, <u>coeteris paribus</u>. Inside money is not a component of wealth and thus there is no direct effect analogous to that for outside money. However the relative increase of the total stock of money with respect to other liquid assets will affect the interest rate (presumably lowering it) and the demand for wealth will change so that a dynamic behavior similar to that described above will be initiated, the demand for wealth exceeding the existing level.

An increase in the stock of outside money is more powerful than an equivalent increase of the stock of inside money. If both involve a substitution effect which affects the interest rate and thus the demand for wealth, outside money has a property that inside money does not share: it can immediately affect the existing level of wealth.

(2) The Stability of the System

As mentioned above no general result can be found even for local stability when many equations and unknowns are involved. However as the first time periods in the numerical example indicate such systems can easily be unstable. The instability of the whole model exists although the stability conditions of Part I (namely $0 < \frac{m}{t} + \frac{m'}{t'} < 1$) is satisfied by the numerical values chosen for m, m', t and t'. The reason for this instability is inter alia the link between the commodity market and the wealth equilibrium condition which takes place through saving and investment.[1] The greater the difference between k_1 and k_2 the stronger the effect on the commodity market of an increase in the stock of outside money or more generally of a disequilibrium between desired and actual wealth. On the other hand the instability is also aggravated by a large value of α_0: the greater α_0, the more desired wealth changes in response to fluctuations in income.

The likelihood of having a stable system can be increased in two ways:

- One is "built-in" to the economic system. If k_1 and k_2 instead of being constant were functions of the interest rate prevailing in the previous period, that is:

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[1]This link did not exist in Part I since there we assumed S = I = 0.

$$(2.3.12) \qquad k_1 = k_1(i_{t-1}) \quad \text{with} \quad \frac{dk_1}{di} > 0$$

$$(2.3.13) \qquad k_2 = k_2(i_{t-1}) \quad \text{with} \quad \frac{dk_2}{di} < 0$$

then the fluctuations of income would be reduced.

Indeed a quick look at Table 4 reveals that in country I increases in income are accompanied by decreases of the interest rate and conversely so that one variable leads the other one by one period of time. This behavior of income and interest rates results from the dynamic assumptions of the model. Consider for instance in a given period t that there is an excess of desired over existing wealth. Such a situation implies at the same time that: - S and I are positive (by equation 2.3.1) but as k_1 is greater than k_2 by assumption saving exceeds investment and income falls.

- As k_3 is assumed to be positive, it is clear from equation (2.3.10) that i_t will be greater than i_{t-1} and thus the interest rate reaches a peak.

As in period t the interest rate is very high it appears that in t + 1, k_1 will increase and k_2 decrease (by equations 2.3.12 and 2.3.13) so that the difference between them is reduced which in turn implies less fluctuation of saving, investment and thus income.

- Another factor which could increase the likelihood of stability might result from changes of the fractional reserve requirement, v, which is with M_0 a policy tool available to the central bank.

By manipulating the fractional reserve coefficient the central bank can indeed modify the existing level of wealth and bring it closer to the desired one. An assumption on which Table 4 is built is that v and v' remain constant and equal to unity but such an assumption is not necessary.

Whenever the desired level of wealth tends to exceed the actual one (which occurs at t = 2 and t = 4--see Table 4) v should be lowered if the previous period's balance of payments exhibited a deficit and increased in the contrary case: a negative P reduces existing wealth (see equation 2.3.7) while a positive P has the opposite effect. In this way the existing level of wealth would be closer to the desired one and a source for instability would disappear. The instability introduced into the system by using one tool of monetary policy, namely M_0, can thus be reduced by a proper use of the other tool available to the monetary authority.

PART III

THE THREE COUNTRIES CASE

INTRODUCTION

The purpose of Part III is to give a generalization of the previous model confined to only two countries and answer the following question: to what extent will policy measures affecting only countries I and II influence economic variables in country III.

The approach will be similar to the one adopted before. A model with no explicit wealth condition will be studied first and after that the wealth equilibrium equation will be explicitly introduced.

CHAPTER 5

NO EXPLICIT WEALTH EQUILIBRIUM CONDITION

Section 5.1: Comparative Statics

In order to analyze the three countries case some notational change is in order:

- m_{ij} will represent the marginal propensity to import from country i by country j.

- $X_{0,ij}$ will represent the autonomous component of exports from country i to country j.

- Y_i is the total income in country i.

Furthermore--and more importantly--the exports of one country will no longer be the imports of the other one since each country can import from (export to) two other countries. Thus the total exports of a country will be a function (assumed to be linear) of the national incomes of its two partners. The total marginal propensity to import of country j is $\sum_{i \neq j} m_{ij}$. For example the marginal propensity to import of country I is given by:

$$m_1 = m_{21} + m_{31}$$

As it is assumed (as for the two countries case) that wealth is always in equilibrium no saving or investment takes place so that for each country the equilibrium condition remains as before: budget surplus (deficit) = trade deficit (surplus).

More explicitly:

(3.1.1) $$X_{0'12} + X_{0'13} + m_{12}Y_2 + m_{13}Y_3 - X_{0'21} - X_{0'31} - m_1Y_1 = t_1Y_1 - G_{01}$$

(3.1.2) $$X_{0'21} + X_{0'31} + m_{21}Y_1 + m_{23}Y_3 - X_{0'12} - X_{0'32} - m_2Y_2 = t_2Y_2 - G_{02}$$

(3.1.3) $$X_{0'31} + X_{0'32} + m_{31}Y_1 + m_{32}Y_2 - X_{0'13} - X_{0'23} - m_3Y_3 = t_3Y_3 - G_{03}$$

which are the equilibrium conditions for country I, II and III respectively. These conditions can be rewritten more conveniently in matrix form:

$$\overset{A}{\begin{bmatrix} t_1+m_1, & -m_{12}, & -m_{13} \\ -m_{21}, & t_2+m_2, & -m_{23} \\ -m_{31}, & -m_{32}, & t_3+m_3 \end{bmatrix}} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \overset{B}{\begin{bmatrix} G_{01}+X_{0'12}+X_{0'13}-X_{0'21}-X_{0'31} \\ G_{02}+X_{0'21}+X_{0'23}-X_{0'12}-X_{0'32} \\ G_{03}+X_{0'31}+X_{0'32}-X_{0'13}-X_{0'23} \end{bmatrix}}$$

Many multipliers could be derived from this system of equations but only two will be considered here since they can validly represent all the other ones:

A. $\frac{dY_i}{dG_{01}}$: <u>The Multiplier Effect for each Country of an Increase of G_0 in Country I</u>

(1) $$\frac{dV_1}{dG_{01}} = \frac{(t_2 + m_2)(t_3 + m_3) - m_{23}m_{32}}{\det A}$$

As $m_2 \geqslant m_{32}$ and $m_3 \geqslant m_{23}$ the numerator is definitely positive. The sign of the denominator is less obvious but after some simplifications it can be shown to be positive so that $\frac{dY_1}{dG_{01}}$ is necessarily positive.

(2) Similar operations yield:

$$\frac{dY_2}{dG_{01}} = \frac{m_{21}(t_3 + m_3) + m_{23}m_{31}}{\det A}$$

$$\frac{dY_3}{dG_{01}} = \frac{m_{31}(t_2 + m_2) + m_{21}m_{32}}{\det A}$$

Both are also positive. Consequently the basic result of the two countries case, namely that an increase of government expenditures in one country affects income in both positively, is maintained in the three countries case. Furthermore if in the two countries case the multiplier was most likely to be greater in the initiating country, this statement still holds true for the three countries case. Indeed m_{21} and m_{31} are likely to be smaller than $t_2 + m_2$ and $t_3 + m_3$ respectively.

It is also interesting to note that the numerator of any multiplier--say $\frac{dY_2}{dG_{01}}$--is greater the lower the interdependence between countries I and III because then m_{31}

is smaller and m_{21} greater. This result makes intuitive sense since the greater the interdependence between two countries the more the policy effects in one of them are felt in the other one, leaving aside the third country.

B. $\frac{dY_i}{dX_{0,12}}$ = The Multiplier Effect of an Increase of Autonomous Exports from Country I to Country II

$$(1) \quad \frac{dY_1}{dX_{0,12}} = \frac{(t_2+m_2)(t_3+m_3)-m_{12}(t_3+m_3)-m_{13}m_{32}-m_{23}m_{32}}{\det A}$$

Comparing this multiplier with $\frac{dY_1}{dG_{01}}$ it can be seen that an increase of exports in one country has a less expansionary effect upon its income than an equal increase of government expenditures. Nevertheless $\frac{dY_1}{dX_{0,12}}$ is always positive. The reason why the two multipliers differ is simply that in the first stage an increase of exports from country I to country II reduces income in country II so that the induced exports from I to II are reduced. This result is exactly equivalent to the one obtained in the two countries case with however a slight difference: the magnitude of the multiplier (but not its sign) is influenced not only by parameters pertaining to the two countries directly involved (I and II) but also by the marginal propensities to import and to tax in the third country.

$$(2) \quad \frac{dY_2}{dX_{0,12}} = \frac{-(t_1+m_1)(t_3+m_3)+m_{21}(t_3+m_3)+m_3(m_{31}+m_{13})}{\det A}$$

As in the two countries case this multiplier is necessarily negative but its magnitude is also influenced by m_{13}, m_{31} and t_3.

(3) The most interesting multiplier is obviously $\frac{dY_3}{dX_{0,12}}$. Indeed $dX_{0,12}$ means a change of autonomous exports from country I to country II so that the third country is not directly affected. After some obvious transformations it is found that:

$$\frac{dY_3}{dX_{0,12}} = \frac{m_{31}t_2 - m_{32}t_1}{\det A}$$

As a result this multiplier could be positive, negative or even zero. For it to be nil it is not necessary that country III has no commercial relationship with countries I and II. Indeed a multiplier equal to zero can result from values of the coefficients such that $\frac{m_{31}}{t_1} = \frac{m_{32}}{t_2}$. Of course such a situation is very unlikely and it is safe to expect that the third country will be affected by changes occurring directly between its two partners.

A better understanding of the multiplier can be reached by investigating the circumstances under which it is positive or negative. It has been shown in (1) and (2) that $\frac{dY_1}{dX_{0,12}}$ was positive and $\frac{dY_2}{dX_{0,12}}$ negative. Thus when m_{31} is large as compared to m_{32} the increase of induced exports from III to I will be greater than the decrease of induced exports from III to II. As induced exports from country III increase it is normal that $\frac{dY_3}{dX_{o,12}}$ should be positive as it is indicated by the expression given on page 67.

So far only the multipliers for Y_i have been analyzed. What about the effect of changes in G_{01} and $X_{0,12}$ on the trade balance of each country? Before proceeding to the analysis it will be recalled that in the two countries case an increase of government expenditures in a given country worsened its trade balance while an increase of autonomous exports improved it.

In the three countries case the following results are obtained:

$$\frac{dS^f}{dG_{01}} = m_{13}\frac{dY_3}{dG_{01}} + m_{12}\frac{dY_2}{dG_{01}} - m_1\frac{dY_1}{dG_{01}}$$

As this expression involves positive and negative terms and no simplification is possible the sign of $\frac{dS^f}{dG_{01}}$ remains indeterminate. Similarly

$$\frac{dS_1^f}{dX_{0,12}} = 1 + m_{13}\frac{dY_3}{dX_{0,12}} + m_{12}\frac{dY_2}{dX_{0,12}} - m_1\frac{dY_1}{dX_{0,12}}$$

is indeterminate since $\frac{dY_3}{dX_{0,12}}$ itself is. Consequently the introduction of a third country prevents any <u>a priori</u> information on the sign of $\frac{dS_1^f}{dG_{01}}$ and $\frac{dS_1^f}{dX_{0,12}}$ in contrast to the two countries case.

Section 5.2: Dynamic Analysis

The dynamic analysis for the three countries proceeds along the same line as the dynamic analysis for two countries without an explicit wealth equilibrium condition. Its purpose is to determine if a multilateral interdependence between countries tends to make economic systems more stable than the simple bilateral interdependence of Part I.

To make the comparison meaningful the same assumptions as in Part I will prevail:

- In each country current taxes depend upon current income.

- Current imports are a function of the previous year income.

As a result of these assumptions and after some transformations the three countries dynamic model can be compactly written as:

$$(3.1.5) \quad \begin{bmatrix} Y_{1,t} \\ Y_{2,t} \\ Y_{3,t} \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{m_1}{t_1}, & \frac{m_{12}}{t_1}, & \frac{m_{13}}{t_1} \\ \frac{m_{21}}{t_2}, & -\frac{m_2}{t_2}, & \frac{m_{23}}{t_2} \\ \frac{m_{31}}{t_3}, & \frac{m_{32}}{t_3}, & -\frac{m_3}{t_3} \end{bmatrix}}_{A'} \begin{bmatrix} Y_{1,t-1} \\ Y_{2,t-1} \\ Y_{3,t-1} \end{bmatrix}$$

(3.1.5) (Cont'd)

$$+\underbrace{\begin{bmatrix} \dfrac{G_{01,t} + X_{0,12,t} + X_{0,13,t} - X_{0,21,t} - X_{0,31,t}}{t_1} \\ \dfrac{G_{02,t} + X_{0,21,t} + X_{0,23,t} - X_{0,12,t} - X_{0,32,t}}{t_2} \\ \dfrac{G_{03,t} + X_{0,31,t} + X_{0,32,t} - X_{0,23,t} - X_{0,13,t}}{t_3} \end{bmatrix}}_{B'}$$

To determine the stability conditions the characteristic roots of A' have to be found.

As the rank of A' is two, one of the roots is necessarily zero and the constant term in the characteristic equation, which is the determinant of A', is also zero. The characteristic equation can be shown to be:

$$x^3 + bx^2 + cx = 0 \text{ or equivalently for } x \neq 0:$$

$$x^2 + bx + c = 0 \text{ where } b = \frac{m_1}{t_1} + \frac{m_2}{t_2} + \frac{m_3}{t_3} \text{ and}$$

$$c = \frac{t_1(m_2m_3-m_{23}m_{32}) + t_2(m_1m_3-m_{13}m_{31}) + t_3(m_1m_2-m_{12}m_{21})}{t_1 \cdot t_2 \cdot t_3}$$

From the discussion above b and c appear to be both positive. The three characteristic roots are thus

$$0, \ \frac{-b + \sqrt{b^2 - 4c}}{2}, \ \frac{-b - \sqrt{b^2 - 4c}}{2}$$

and the stability conditions are:

$$1 + b + c > 0 \; ; \; 1 - c > 0 \quad \text{and} \quad 1 - b + c > 0$$

As both b and c are always positive the first condition for stability is automatically satisfied so that only the other two are relevant here. They can be represented graphically as follows:

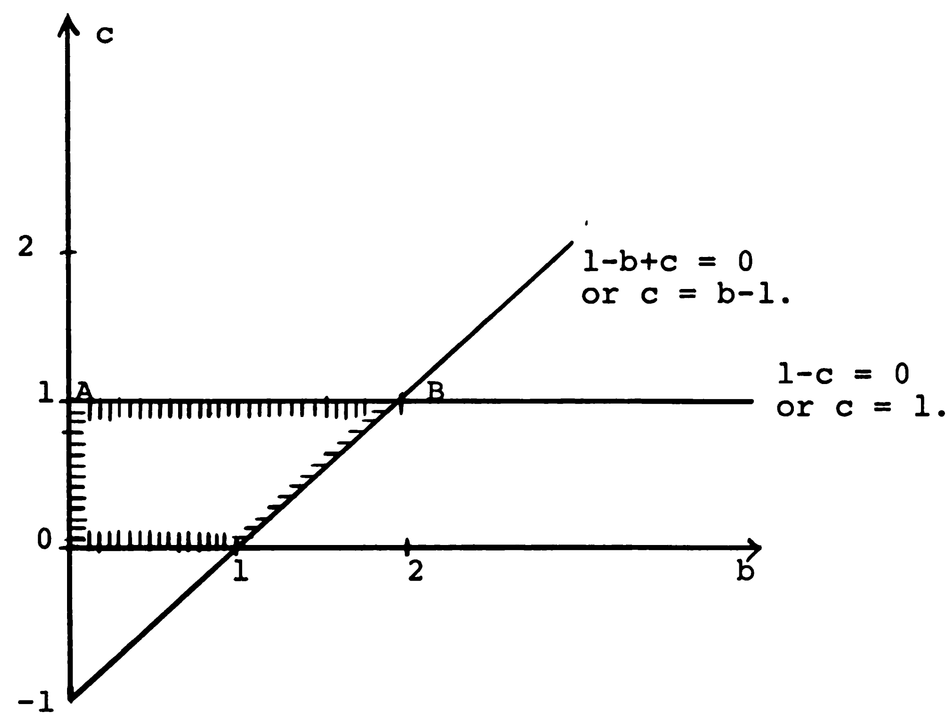

Figure 4

where the stability region is represented by the area OABC.

How do these conditions compare with those of the two countries case?

(1) First of all the stability conditions depend not only upon the total marginal propensities to import (m_1,

m_2 and m_3) as compared to the marginal rates of taxation (t_1, t_2 and t_3) but also upon the allocation of each propensity to import between each country (the m_{ij} terms). It will be recalled that such terms did not exist in the two countries case.

(2) As for stability in this case it is necessary and sufficient that $1 - c > 0$ and $1 - b + c > 0$, adding these two inequalities a sufficient condition for stability is found, that is $2 - b > 0$ or as b is always positive $0 < b < 2$.

But $0 < b < 2$ implies that $0 < \frac{m_1}{t_1} + \frac{m_2}{t_2} + \frac{m_3}{t_3} < 2$ in the three countries case while the corresponding condition was (see p. 19) $0 < \frac{m_1}{t_1} + \frac{m_2}{t_2} < 1$ for the two countries case.

A comparison of these two expressions indicates immediately that a three countries model is more likely to be stable than a similar two countries model. Indeed in the two countries case no country may have a marginal propensity to import greater than its marginal rate of taxation otherwise the system is unstable. In the three countries case one country can have $m_i > t_i$ and yet the system might still be stable provided that the marginal rates of taxation in the other two countries substantially exceed the marginal propensities to import.

As a result multilateral interdependence seems to produce more stable economic systems than bilateral interdependence. At this stage it is very interesting to compare this result with the one arrived at by Metzler in (16). Despite the fact that he introduces the consumption function in his equilibrium conditions, he notices that going from a "one country" case to a "two countries" case the likelihood of having stability tends to increase. Indeed:

> A country which would be unstable when left to itself may be perfectly stable in a two economies world because of the dampening influence of low propensities in the other country. (20, p. 103)

Thus the fact that multilateral interdependence facilitates stability tends to remain true even when the models on which the dynamic analysis is based are slightly different.

(3) As b and c are both positive the two roots (other than zero) are necessarily negative (see (3) page 220). As a result the fluctuations in the three countries will be oscillatory as in the two countries model.

CHAPTER 6

AN EXPLICIT WEALTH EQUILIBRIUM CONDITION

In the two countries model the introduction of the wealth equilibrium condition made possible the study of the effect of monetary policy upon the equilibrium level of the interest rate in each country. This same objective will be pursued here for the three countries case so that the similarities and differences with the previous results can be analyzed.

As in Part II (two countries case) the dichotomy between the wealth equilibrium condition and the commodities equations is still present: the level of income is determined by the equilibrium conditions upon the markets for commodities as indicated in the first chapter of Part III and then the interest rates have to adjust in each country so that all the wealth equilibrium conditions are simultaneously satisfied.

The introduction of a third country however rules out the simplification that the exports of capital by one country are the imports of the other one. Furthermore the situation is still worse for capital movements than for commodities. Indeed for commodities the exports of country I to country II depended only upon the level of income in country II and similarly for any other pair of countries. For capital movements however the interrelationships between countries are stronger: the amount that country I plans to lend to country II depends not only upon the interest rate prevailing in country II but also the interest rates in I and III.

Formally capital movements will be represented as follows:

$$B_{ij} = B_{ij}(i_1, i_2, i_3) \text{ for i and j} = 1,2,3$$

where i_1, i_2 and i_3 are the interest rates respectively in countries I, II and III and B_{ij} is the amount of lending from i to j.

Taking for example B_{12} it is found that $\frac{\partial B_{12}}{\partial i_1} < 0$, $\frac{\partial B_{12}}{\partial i_2} > 0$ and $\frac{\partial B_{12}}{\partial i_3} < 0$ since residents of country I will lend more to country II when the interest rate is high in country II relative to that prevailing in other countries. In general thus $\frac{\partial B_{ij}}{\partial i_k} > 0$ if $j = k$ and $\frac{\partial B_{ij}}{\partial i_k} < 0$ if $j \neq k$.

Section 6.1: Comparative Statics Analysis

Equilibrium Conditions

(1) Commodities Market

As briefly mentioned on page 74 the equilibrium conditions for each of these markets are the same as before so that the equations (3.1.1) to (3.1.3) of page 64 are still valid.

(2) Wealth Equilibrium Conditions

While the desired level in each country depends only upon the interest rate prevailing in this country the existing wealth is a function of three interest rates because of the interdependence of the capital markets indicated above.

Furthermore if for two countries the balance of payments of one is the negative of that of the other this is no longer valid for three countries. The only constraint upon the balance of payments is that their sum adds up to zero. A similar argument also holds for the balance of trade.

Taking these remarks into account the equilibrium conditions can now be written as:

$$\text{(3.2.1)} \quad \tilde{W}_1(Y_1,i_1) = M_{01} + B_{01} + K_{01} + B_{12}(i_1,i_2,i_3) + B_{13}(\ ) - B_{21}(\ ) - B_{31}(\ ) + v_1P_1$$

$$\text{(3.2.2)} \quad \tilde{W}_2(Y_2,i_2) = M_{02} + B_{02} + K_{02} + B_{21}(\ ) + B_{23}(\ ) - B_{12}(\ ) - B_{32}(\ ) + v_2P_2$$

$$\text{(3.2.3)} \quad \tilde{W}_3(Y_3,i_3) = M_{03} + B_{03} + K_{03} + B_{31}(\ ) + B_{32}(\ ) - B_{13}(\ ) - B_{23}(\ ) + v_3P_3$$

where ( ) stands for (i_1, i_2, i_3).

The balance of payments surpluses or deficits are defined as:

$$P_1 \equiv S_1^f + B_{21}(\ ) + B_{31}(\ ) - B_{12}(\ ) - B_{13}(\ )$$

$$P_2 \equiv S_2^f + B_{12}(\ ) + B_{32}(\ ) - B_{21}(\ ) - B_{23}(\ )$$

and $$P_1 + P_2 + P_3 \equiv 0$$

Analysis of the Model

We are now in a position to verify if an increase of M_{01}, the stock of outside money of country I, will still put a downward pressure upon all the interest rates as it happened in the "two-countries" case.

Differentiating totally (3.2.1) to (3.2.3), using the definitions for the P_i's and keeping in mind that the Y_i's are given in the solution of the equations (3.1.1) to (3.1.3) it is found:

(3.2.4)

$$\begin{bmatrix} \frac{\partial \tilde{W}_1}{\partial i_1} + (v_1-1)\left(\frac{\partial B_{12}}{\partial i_1} + \frac{\partial B_{13}}{\partial i_1} - \frac{\partial B_{21}}{\partial i_1} - \frac{\partial B_{31}}{\partial i_1}\right), & (v_1-1)\left(\frac{\partial B_{12}}{\partial i_2} + \frac{\partial B_{13}}{\partial i_2} - \frac{\partial B_{21}}{\partial i_2} - \frac{\partial B_{31}}{\partial i_2}\right), & (v_1-1)\left(\frac{\partial B_{12}}{\partial i_3} \cdots - \frac{\partial B_{31}}{\partial i_3}\right) \\ (v_2-1)\left(\frac{\partial B_{21}}{\partial i_1} + \frac{\partial B_{23}}{\partial i_1} - \frac{\partial B_{12}}{\partial i_1} - \frac{\partial B_{32}}{\partial i_1}\right), & \frac{\partial \tilde{W}_2}{\partial i_2} + (v_2-1)\left(\frac{\partial B_{21}}{\partial i_2} \cdots - \frac{\partial B_{32}}{\partial i_2}\right), & (v_2-1)\left(\frac{\partial B_{21}}{\partial i_3} \cdots - \frac{\partial B_{32}}{\partial i_3}\right) \\ (v_3-1)\left(\frac{\partial B_{31}}{\partial i_1} + \frac{\partial B_{32}}{\partial i_1} - \frac{\partial B_{13}}{\partial i_1} - \frac{\partial B_{23}}{\partial i_1}\right), & (v_3-1)\left(\frac{\partial B_{31}}{\partial i_2} \cdots - \frac{\partial B_{23}}{\partial i_1}\right), & \frac{\partial \tilde{W}_3}{\partial i_3} + (v_3-1)\left(\frac{\partial B_{31}}{\partial i_3} \cdots - \frac{\partial B_{23}}{\partial i_3}\right) \end{bmatrix} \begin{bmatrix} di_1 \\ di_2 \\ di_3 \end{bmatrix} = \begin{bmatrix} dM_0 \\ 0 \\ 0 \end{bmatrix}$$

A

In a given row the terms into brackets are the same except that in the different columns partial differentiation is carried out with respect to a different interest rate.

All the diagonal elements are definitely negative. Consider for instance a_{11}, the first diagonal element: we have $\frac{\partial \tilde{W}_1}{\partial i_1} < 0$; $\frac{\partial B_{12}}{\partial i_1} < 0$; $\frac{\partial B_{13}}{\partial i_1} < 0$; $- \frac{\partial B_{21}}{\partial i_1} < 0$; and $- \frac{\partial B_{31}}{\partial i_1} < 0$ so that a_{11} is negative.[1] However all the non diagonal elements have undefined signs. Consider a_{12} again as an example: $\frac{\partial B_{12}}{\partial i_2} > 0$; $\frac{\partial B_{13}}{\partial i_2} < 0$; $- \frac{\partial B_{21}}{\partial i_2} > 0$; $- \frac{\partial B_{31}}{\partial i_2} > 0$ so that in a_{12} as in all non diagonal elements, three terms are positive and one is negative. The ambiguity of the sign of a_{12} stems from the only negative term which indicates how capital flows from country I to country III are affected by changes in the interest rate of country II.[2]

As a result six out of nine terms in the matrix A have an indeterminate sign which means that there are 3^6 (= 729) possible combinations of signs in the matrix. In this case even the application of Lancaster's method of qualitative comparative statics [see (12)] to determine

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[1]As all the diagonal elements of A are negative, A is "potentially stable" (see (33), page 208).

[2]A similar situation is encountered by Brainard and Tobin (see (40) especially pp. 78-80). Indeed one of the coefficients of the Jacobian matrix they are using has an undefined sign.

the sign of $\frac{di_1}{dM_{01}}$, $\frac{di_2}{dM_{01}}$ and $\frac{di_3}{dM_{01}}$ would be a formidable task.

To be able to say more about $\frac{di_1}{dM_{01}}$, $\frac{di_2}{dM_{01}}$ and $\frac{di_3}{dM_{01}}$ it is necessary to look at the stability conditions of the model as Tobin and Brainard did under similar circumstances [see (40)]. Assume that in each country the interest rate increases when desired wealth exceeds existing wealth (for a full discussion of this assumption see pages 51 and 52). The dynamic system can then be written as:

(3.2.5) $$\frac{di_j}{dt} = k_j(\tilde{W}_j - W_j) \text{ for } j = 1,2,3.$$ [1]

where the k's are positive constants which can be made equal to unity by an appropriate change of units.

The system will be locally stable if the characteristic roots of A in (3.2.4) have negative real parts since A is nothing else than the matrix of partial derivatives of $[\tilde{W}_j - W_j]$ (for j = 1,2,3) with respect to i_1, i_2 and i_3. These conditions are fulfilled if and only if:[2]

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[1]Ideally we should work with difference equations since discrete changes seem to fit better the economic events. This is the reason why the dynamic analysis of sections 2.2 and 4.2 have been carried out in the form of difference equation.

Unfortunately the stability conditions for a system of difference equations are harder to generalize to a great number of equations than those of an equivalent system of differential equations [see (31)]. Consequently when stability conditions are studied in order to derive comparative statics properties of a model, economists usually turn to differential equations. As a result they are used here and also on page 34.

[2]See [(30), p. 450].

(1) $m_1 > 0$, $m_2 > 0$, $m_3 > 0$ where $m_i = (-1)^i \times$ sum of the i^{th} order principal minors

(2) $m_1 m_2 - m_3 > 0$

By inspection of the matrix A it is found that m_1 is always greater than zero since all diagonal elements of A are known to be negative. Consequently the requirement $m_1 > 0$ is always satisfied and adds nothing to the knowledge of the comparative statics properties of the model. Fortunately however the conditions $m_2 > 0$ and $m_3 > 0$ will be of great help. Indeed $m_3 > 0$ implies that the determinant of the matrix A (denoted det. A) must be negative for local stability and $m_2 > 0$ will certainly be satisfied if:

$$\det \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} > 0 \qquad \det \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} > 0$$

and

$$\det \begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix} > 0$$

With this information available some conclusions can be drawn about the behavior of interest rates in the three countries case when the stock of outside money is increased in one country.

(1) $\frac{di_1}{dM_{01}} = \frac{\det \begin{bmatrix} a_{22} & 2_{23} \\ a_{32} & a_{33} \end{bmatrix}}{\det A}$. The numerator has been shown to be positive and the denominator negative so that

$\frac{di_1}{dM_{01}}$ is negative. Thus an expansionary monetary policy reduced interest rates in the initiating country provided that the stability conditions are satisfied.

$$(2)\quad \frac{di_2}{dM_{01}} = \frac{-\det \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}}{\det A} = \frac{-a_{21}a_{33} + a_{31}a_{23}}{\det A}$$

As the numerator has only one element the sign of which is defined (a_{33}) the sign of the whole expression is also indeterminate.

$$(3)\quad \frac{di_3}{dM_{01}} = \frac{\det \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}}{\det A} = \frac{a_{21}a_{32} - a_{22}a_{31}}{\det A}$$

and the same conclusion as in (2) is reached.

If nothing can be said for sure about $\frac{di_2}{dM_{01}}$ and $\frac{di_3}{dM_{01}}$ some conjectures are however possible. It is shown above (see page 78) that for each a_{ij} $(i \neq j)$ three components are positive and one is negative. Thus each a_{ij} is likely to be positive since after all the negative component represents only an "indirect" effect, namely the effect of a change in i_j upon capital movements between countries other than country j. If this is the case, as it is likely to be, then $\frac{di_2}{dM_{01}}$ and $\frac{di_3}{dM_{01}}$ will both be negative and an expansionary monetary policy in one country will put a downward pressure on all interest rates everywhere as it was already the case for two countries. Thus for an expansionary

monetary policy to reduce interest rates in the rest of the world when three countries are involved two conditions must be satisfied:

1. The system is stable so that det. A is negative.

2. The indirect effect of changes of the interest rate of any country on capital movements between the other two is small relative to the direct effects.

Consequently it can be said that in general the introduction of a third country does not vitiate the basic result for two countries that an increase of M_0 in one country reduces the interest rate in the rest of the world. However it should be kept in mind that for two countries the result is always true while for three countries it is subject to the two restrictions just mentioned.

Section 6.2: An Extended Dynamic Model

In this dissertation dynamic analysis has been extensively used in different contexts:

- When the wealth equilibrium conditions were assumed to be automatically satisfied (Part I and Part III Chapter 1) the dynamic analysis was used to determine under which conditions we would return to an equilibrium situation on the commodity market after an initial disturbance.

- Within the wealth equilibrium conditions themselves a dynamic analysis has been carried out to discover some properties of the comparative statics models. This was the case in Part II Chapter 1 and Part III Chapter 2.

- Finally a dynamic model has been developed to study the interrelationships between the wealth and commodities equations in disequilibrium situations. This analysis has been made in Part II Chapter 2 and has enabled us to introduce saving and investment into the commodities market equilibrium condition.

So far in Part III only the first two kinds of dynamic analysis have been used and nothing has been said about saving and investment. It is the purpose of this section to indicate how such an analysis should be carried out.

In Part II, saving and investment have been considered as means by which a disequilibrium between desired and actual wealth could be corrected. This assumption will be kept here so that for each country we have:

$$S_{jt} = k_{j1}(\tilde{W}_{jt} - W_{jt}) \quad \text{and} \quad I_{jt} = k_{j2}(\tilde{W}_{jt} - W_{jt})$$

where $j = 1,2,3$ stands for the j^{th} county.

All the other assumptions of Part II are also maintained so that we would have a system of 14 independent equations with 14 unknowns since three countries instead of two are involved.

Obviously a numerical solution as in Part II could be found but it would be tedious and of little interest since all the mechanisms are the same as in Part II. They are briefly described here for the three countries case so that the similarity with the two countries case will be obvious:

- Once the stock of wealth is increased in country I because of an increase of the stock of outside money, existing wealth exceeds desired wealth so that a flow of investment and saving takes place which increases income in I.

- The increase of income in I stimulates exports of countries II and III to country I so that income also increases in these countries. But because of this increase of income people now desire to accumulate more wealth so that a flow of saving and investment takes place in II and III as well.

- The increased exports from II and III to I also influence the balance of payments of each country which implies changes in the endogenous component of the stock of money and the existing level of wealth.

- All these effects interact with each other so that after a few stages a literary description becomes impossible. All the links are in fact the same as in page 54 except that now three countries instead of two are involved.

CONCLUSION

At the end of this dissertation the basic results of the analysis and their consequences for policy decisions should be stressed. It will be recalled that the goals of the thesis were:

- To determine the implications of viewing saving and investment as resultants of a disequilibrium between desired and actual wealth.

- To analyze the wealth effects of monetary, fiscal or commercial actions rather than their substitutions effects emphasized by the economic literature in general and McKinnon's model in particular. Furthermore by considering open and interdependent economies the redistribution of wealth between countries which takes place through balance of payments surpluses or deficits has been analyzed.

These two objectives have been carried out in the context of both comparative statics and dynamics.

When the equilibrium conditions on the commodities markets of each country are written as $S = I = 0$ rather than simply $S = I$ it appears that the comparative statics multipliers of G_0 and X_0 have positive signs even in the case of two interdependent economies, what is not always true in traditional macroeconomic models. Besides after the explicit introduction of the wealth equilibrium condition the model remains dichotomized in the sense that the level of income in each country is determined exclusively by the equilibrium condition on the market for commodities. Thus, an increase of the stock of outside money cannot affect the level of income. However, in a dynamic context where saving and investment can be different from zero an increase of M_o will cause fluctuations of income because it creates a disequilibrium between the existing and desired stock of wealth.

The attention paid in this dissertation to the wealth effects of monetary actions rather than the substitution effects enabled us to notice that even if both happened to have the same impact on interest rates, the reasons for these results could be different (see for instance the impact of a greater M or a higher propensity to lend abroad). In some other cases (increase of G_o or open market operations) the focus on the wealth equilibrium condition as a whole rather than on its "partial" compenents can yield results different from those of traditional economic theory.

The interdependence between the economies--as opposed to the traditional analysis of an open but small country--gave interesting results mostly in the comparison between the two and three countries case. First, it appears that even when three countries are considered, none of them can avoid the effects of policy measures taken by any other one as long as its marginal propensity to import is different from zero. Furthermore, three countries are more likely to achieve the conditions of dynamic stability than only two countries since the effects of policy measures taken by one member can be spread between two partners rather than one.

As far as policy implications are concerned, a distinction must be made between the comparative statics and the dynamic parts of the analysis:

- When situations of stationary equilibrium are compared, budget policy can affect income positively while its effects upon the interest rate and the balance of payments are uncertain. In contrast, monetary policy has no effect at all upon the equilibrium level of income but an increase (decrease) in the stock of money will lower (raise) interest rates and worsen (improve) the balance of payments, by an amount less than the increase in the stock of money. Thus, it seems that in a comparative statics framework budget policy should be directed to the achievement of employment and national income goals while its sometimes perverse effects upon the balance of payments can be offset by an appropriate monetary policy.

- In a dynamic setting, on the other hand, this way of assigning policy tools to the objectives is less imparative. Indeed an increase of the stock of money in the first period can cause an increase of income, the magnitude of which depends upon the relative speeds of adjustment of saving and investment in reaction to a disequilibrium between desired and actual wealth. But this increase of income is accompanied by a deficit of payments which can be judged undesirable and consequently, monetary policy alone cannot attain both objectives. In fact there is no general way of assigning the tools to the goals because the choice of the tools will depend upon the values of the parameters. If they are such that ΔG_o raises income and reduces the payments deficit then budgetary policy should be the only tool used. On the other hand, if ΔG_o has a perverse effect on the external balance the simultaneous achievement of internal and external balance can be reached only by a mix of expansionary fiscal policy and restrictive monetary policy.

x
x x.

This dissertation with its emphasis on stock equilibrium conditions may seem strange to the traditional economic theorist used to Keynesian models in the macroeconomic theory of international trade.

To see the relevance of this approach and its prospect for the future, the best alternative available is to quote H. G. Johnson's conclusion to a conference on "Monetary Problems of the International Economy" (see (12), pp. 398-399):

> International monetary economists have built models of international adjustment and policy problems which have employed as building blocks quite naive Keynesian models of the national economy . . . in which no attention has been paid to recent developments in the theories of consumption and investment which have stressed the function of saving and investment as a means of adjusting actual to desired stocks of wealth. It is high time for international monetary theorists to take account of these developments and to integrate them into international monetary theory. It is true that the applicability of these developments at present is rather remote . . . but the framework is both more fundamental economically and more elegant intellectually than the Keynesian apparatus of purely flow relationships on the real side of the analysis and it is a matter mainly of time and effort to develop its implications for short-run disequilibrium situations. This, while McKinnon's paper may have seemed . . . to require much more intellectual effort than its fruit would justify, I would not be surprised if in ten years time most of us will be talking McKinnon's language as a matter of course.

It is our hope that this dissertation will be a step--however small it may be--in this direction.

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APPENDIX

APPENDIX I

In this appendix the exact conditions for $\tilde{W}_i$ and $\tilde{W}_i'$ to be smaller than zero are studied in greater detail.

As the total desired stock of wealth is simply the sum of the three components, desired stock of outside money, outside bonds and physical capital we have:

$$\tilde{W}(Y, i) = \tilde{K}(Y,i) + \tilde{B}(Y,i) + \tilde{M}(Y,i)$$

Thus, $\tilde{W}_i = \tilde{K}_i + \tilde{B}_i + \tilde{M}_i$ where the subscripts stand for the argument of the function for which partial differentiation is carried out.

Multiplying both sides by $\frac{i}{W}$ we find:

$$\tilde{W}_i \frac{i}{\tilde{W}} = \tilde{K}_i \frac{i}{\tilde{W}} + \tilde{B}_i \frac{i}{\tilde{W}} + \tilde{M}_i \frac{i}{\tilde{W}}$$

The left hand side is simply the elasticity of the desired stock of wealth with respect to the interest rate and will be denoted $e_{\tilde{W},i}$. The three terms on the right hand side can also be transformed into elasticities. Indeed:

$$\tilde{K}_i \frac{i}{\tilde{W}} = \tilde{K}_i \frac{i \cdot \tilde{K}}{\tilde{K} \cdot \tilde{W}} = e_{\tilde{K},i} \cdot \frac{\tilde{K}}{\tilde{W}} \quad \text{where}$$

$e_{\tilde{K},i}$ = elasticity of the desired capital stock with respect to i and $\frac{\tilde{K}}{\tilde{W}}$ is the share of the capital stock in the desired wealth. Similar expressions can be derived for M and B so that:

$$e_{\tilde{W},i} = e_{\tilde{K},i} \cdot \frac{\tilde{K}}{\tilde{W}} + e_{\tilde{B},i} \cdot \frac{\tilde{B}}{\tilde{W}} + e_{\tilde{M},i} \frac{\tilde{M}}{\tilde{W}} .$$

From economic theory it is known that $e_{\tilde{K},i} \leq 0$ and $e_{\tilde{M},i} \leq 0$ while $e_{\tilde{B},i}$ is positive so that the sign of $e_{\tilde{W},i}$ is not <u>a priori</u> clearcut: it will depend upon the absolute values of the elasticities and the relative shares.

Suppose, at worst, that $e_{\tilde{M},i} = 0$ so that a bias is introduced against a negative sign for $e_{\tilde{W},i}$. Then $e_{\tilde{W},i}$ will be negative if and only if

$$\left| e_{\tilde{K},i} \cdot \frac{\tilde{K}}{\tilde{W}} \right| > \left| e_{\tilde{B},i} \cdot \frac{\tilde{B}}{\tilde{W}} \right|$$

If $e_{\tilde{K},i}$ happens to be zero then $e_{\tilde{W},i}$ will be positive every time that $e_{\tilde{B},i}$ is positive. However, the empirical evidence available [6] suggests that $e_{\tilde{K},i}$ is not zero and if it is smaller in absolute value than $e_{\tilde{B},i}$ this is more than compensated by the fact that the share of capital (physical) in total wealth exceeds by far that of outside bonds.

Furthermore, the available evidence also indicates that $e_{\tilde{M},i}$ is negative, thus reinforcing the belief that $\tilde{W}_i$ and $e_{\tilde{W},i}$ are negative.