PRACTICAL LIMITATIONS TO ANALYTIC SYNTHESIS OF A FOUR-LINK MECHANISM FUNCTION GENERATOR

Thesis for the Degree of Ph. D.

MICHIGAN STATE UNIVERSITY

Harold L. Dowell, Jr.

1965

THESIS

LIBRARY
Michigan State
University

This is to certify that the

thesis entitled

PRACTICAL LIMITATIONS TO ANALYTIC SYNTHESIS

OF A FOUR-LINK MECHANISM FUNCTION GENERATOR

presented by

Harold L. Dowell, Jr.

has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Mech. Engr.

Rollin T. Humble

Major professor

Date November 29, 1965

O-169

ABSTRACT

PRACTICAL LIMITATIONS TO ANALYTIC SYNTHESIS
OF A FOUR-LINK MECHANISM FUNCTION GENERATOR

by Harold L. Dowell, Jr.

The problem was to investigate certain practical limitations to synthesis of a four-bar mechanism function generator. The limitations investigated were:

a) Branching: Some of the precision points may lie on one branch of closure and some on the other, meaning that a joint must be separated and reconnected in a different configuration of the mechanism to complete the range.

b) Closure: Moving the links beyond the first or last precision position, but still within the range may require the mechanism to enter a region beyond its limits of motion.

c) Change Points: The link lengths may be such that as the links pass through a folded configuration the follower is able to move in either direction.

d) Transmission Angle: During operation within the range of motion, the acute angle between centerlines of coupler and follower link may become so small as to preclude effective force transmission.

e) Excessive Link Length Ratio: Two of the links may be much longer than the other two making the operation of the mechanism impossible because of deflection of the longer links or because of space requirements.

The method of investigation was experimental. Several commonly used functions and their ranges were chosen; for each, a series of four-bar

mechanisms to generate the function were synthesized. When one of the limitations was encountered, the particulars were recorded and the next mechanism was synthesized. The syntheses and analyses were performed with the aid of a digital computer; 4050 function generators were synthesized for each of eight functions. Since attention was concentrated on the design failures attributable to any one of the above limitations, the sucessful designs were discarded.

For this work, the analytic synthesis was performed with three unknowns (the three link length ratios) and four arbitrary parameters to give the greatest variety of solutions consistent with reasonably accurate function approximation. The Freudenstein Equation with three precision points was used. The arbitrary parameters were the starting angles (corresponding to the first precision point) and range angles for input and output links. The aim was to find a pattern for each limitation in terms of these four arbitrary parameters; if such a pattern is known, then the parameters can be chosen so as to avoid or minimize encountering the limitations.

The results of the investigation are based on choices of 40 degrees for minimum transmission angle in the range of operation and six for maximum link length ratio. On this basis, about 92 percent of the attempts at synthesis failed because of one of the limitations. Choice of ranges from 30 degrees to 150 degrees had little influence on whether or not one of the practical limitations appeared.

Making the starting angles differ will reduce the probability of

branching for input and output links rotating in the same direction if the function has positive slope or for input and output links rotating in opposite directions if the function has negative slope. Avoiding the third quadrant for the starting angle of output link will reduce the probability of branching for input and output links rotating in opposite directions if the function has positive slope or for input and output links rotating in the same direction if the function has negative slope. These same measures will also reduce the likelihood of problems with closure.

Patterns for the transmission angle limitation were found and are given in the thesis. These patterns depend on whether input and output links rotate in the same or opposite directions and on whether the first and second derivatives of the function have like or unlike signs.

As long as the starting angles of input and output links were not exactly the same, change points were not a practical limitation.

No significant generalization can be formed from this work which will aid the linkage designer in avoiding the limitation of excessive link length ratio.

PRACTICAL LIMITATIONS TO ANALYTIC SYNTHESIS OF A FOUR-LINK MECHANISM FUNCTION GENERATOR

By

Harold L. Dowell, Jr.

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Mechanical Engineering

1965

Copyright by

HAROLD LEE DOWELL, JR.

1966

ACKNOWLEDGMENTS

During the academic year 1964-65, I was a Science Faculty Fellow of the National Science Foundation; the support and confidence signified by that award have been important to my work.

Professor Palmer J. Reiten, Chairman of Mechanical Engineering, University of North Dakota, relieved me of teaching duties for two consecutive years, thereby expressing his confidence in me.

While a student at Michigan State University, I have frequently received inspiration and encouragement — exemplary and direct — from my professors.

Many of my hours were spent in the Computer Laboratory of Michigan State University; the Director, Professor Lawrence von Tersch, made special arrangements essential to the completion of this task. Mrs. Elizabeth Unger, with her staff of Programming Consultants, frequently helped me out of bewilderment; herself an engineer, she showed genuine interest in my problem. Mr. Robert Sparbel wrote (and named) the tape handling function, KLUNKER, and advised me on other programming matters.

At the time I was writing computer programs, I shared an office with Professor Mahlon C. Smith, who applied his talents and experience in my behalf, often and generously.

Mr. Michael Hooth made half of the seventy-two plots, a tedious chore which he executed with speed and care.

The members of my Graduate Committee gave good counsel — technical and administrative. They are Professors William A. Bradley, Rolland T. Hinkle,

George H. Martin, Edward Nordhaus and Howard L. Womochel. Professor Hinkle served as chairman of this committee.

My interest in kinematics of mechanisms was kindled by Professor Hinkle's excellent textbook on the subject several years ago; my personal association with him has been rewarding.

By her tactful and constant encouragement, my wife, Johnnie, has shared with me the burdens of the task; she has helped to make it pleasant work.

With care and devoted persistence, in spite of my interruptions, Mrs. Joyce Medalen typed the manuscript.

TABLE OF CONTENTS

Page

ACKNOWLEDGMENT ii

LIST OF FIGURES vii

INTRODUCTION 1

The Four-Bar Planar Mechanism As a Function Generator 1

DESCRIPTION 1

FUNCTIONS WHICH CAN BE MECHANIZED APPROXIMATELY 3

APPLICATIONS 4

A REVIEW OF THE DEVELOPMENT OF DESIGN TECHNIQUES 4

THE PROBLEM 9

Statement of Problem 9

THE BRANCHING LIMITATION 13

THE CLOSURE LIMITATION 14

THE CHANGE POINTS LIMITATION 16

THE TRANSMISSION ANGLE LIMITATION 17

THE LINK LENGTH RATIO LIMITATION 17

THE METHOD OF INVESTIGATION 19

Statement of Method 19

THE FOUR ARBITRARY PARAMETERS 21

FUNCTIONS AND RANGES 24

ORDERING THE LIMITATIONS 24

COMPUTER OUTPUT 26

A Preliminary Study 26

The Computer Programs 29

Page

MAIN PROGRAM FOR FINAL STUDY 29

SUBROUTINES 30

MAIN PROGRAM FOR PRELIMINARY STUDY 34

Displaying the Results 34

THE INDEX I 35

THE PLOT 37

THE RESULTS 40

Number of Design Failures by Kind 40

Plots 40

Summaries of Plots 40

Sample Output from Preliminary Study 59

INTERPRETATION OF RESULTS 74

Branching 74

Closure 74

Transmission Angle 75

FUNCTIONS HAVING LIKE SIGNS FOR FIRST AND SECOND DERIVATIVES 75

FUNCTIONS HAVING UNLIKE SIGNS FOR FIRST AND SECOND DERIVATIVES 75

Link Length Ratio 75

CONCLUSIONS 76

APPENDIX I 78

Derivation of Equations 78

THE SYNTHESIS EQUATION 78

THE ANALYSIS EQUATION 80

Page

APPENDIX II 82

Computer Programs in Fortran 83

MAIN PROGRAM FOR FINAL STUDY 83

SUBROUTINES 86

MAIN PROGRAM FOR PRELIMINARY STUDY 99

LIST OF REFERENCES 104

LIST OF FIGURES

Figure	Page	Figure	Page	Figure	Page
1	2	16	36	31	54
2	2	17	38	32	55
3	12	18	41	33	56
4	13	19	42	34	57
5	13	20	43	35	58
6	15	21	44	36	60
7	15	22	45	37	61
8	18	23	46	38	62
9	21	24	47	39	63
10	21	25	48	40	64
11	23	26	49	41	65
12	25	27	50	42	66
13	28	28	51	43	67
14	32	29	52	44	68
15	32	30	53	45	79

INTRODUCTION

The Four-Bar Planar Mechanism As a Function Generator

DESCRIPTION

A function generator is a passive device which has a prescribed functional relationship between its inputs and its output. The four-bar mechanism with pinned joints when used as a function generator has only a single input. The functional relationship may be accomplished by coordinating rotations of input and output shafts or by coordinating derivatives of these rotations. Use of a rack and pinion transforms these rotations to linear motion.

Thus, if the function be designated f, the relationship is: (Output) = f(Input). The input and output for the mechanical function generators considered here will be restricted to rotations. The constraint of the four-bar mechanism (Fig. 1) provides the necessary unique relationship between rotations of input (driver) and output (follower), these bars being joined by the frame and coupler. All bars are considered to be rigid bodies and the pinned joints assure coplanar motion. So if the rotations of the input and output, each measured by an angle from some radial datum fixed to the frame, are called P and S, respectively, then the desired function if exactly or ideally mechanized would be characterized by: $S = f(P)$.

Shaffer and Cochin (5)[1] have given a means to determine whether a particular function can be exactly mechanized by a four-bar mechanism.

---

[1] Numbers in parentheses indicate References.

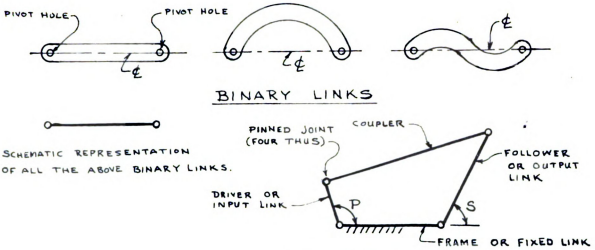
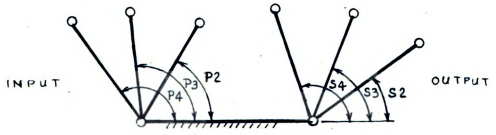


FIGURE 1



POSITIONS OF INPUT LINK AND OUTPUT LINK

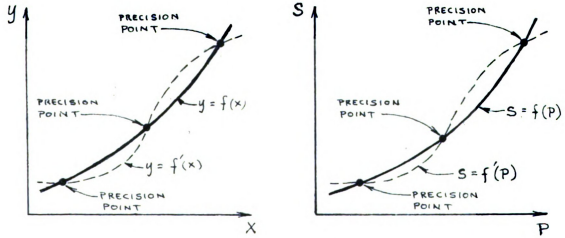


FIGURE 2

Very few of the commonly required functions can be exactly mechanized by a four-bar mechanism, but many of them can be approximately mechanized satisfactorily.

The usual procedure is to obtain point-approximation for several positions of input and output links or some combination of point- and order-approximation. In point-approximation the mechanized function will match the intended (ideal) function at three, four or five different positions in the range while at other positions the mechanized function will deviate from the ideal function. These points of agreement have been called "precision points" (Fig. 2). Order-approximation consists of matching first, second, third, etc. derivatives of the function at each of several precision points.

Accordingly, an error E usually exists between the ideal function f and the mechanized function f' : $E(P) = f'(P)-f(P)$. Through a certain range of motion of the mechanism, satisfactory approximation (meaning acceptably small error) is often possible.

FUNCTIONS WHICH CAN BE MECHANIZED APPROXIMATELY

The kinds of functions which can thus be satisfactorily mechanized by a four-bar mechanism, and their ranges, are limited. Freudenstein (7,10) has discussed these limitations.

They are:

Linear functions should be avoided.

The function should be monotonic.

The function should not be symmetric about the mid-point of the range.

The slope of the function should change gradually.

Ranges of input and output should correspond to less than 120 degrees of shaft rotation.

APPLICATIONS

Four-bar mechanisms have been used as function generators in mechanical analog computers and control devices (1,10,11). Their use would be more common if it were easier to design them since linkage mechanisms possess advantages that other mechanical function generators do not have. Cams, non-circular spur gears, wires used as belts with variable pitch-radius pulleys and sliders guided in curved grooves have been used for function generation. But these devices are relatively expensive to manufacture and their performance may be unfavorably affected by large friction forces or non-positive driving. They can be replaced by four-link mechanisms designed to have good force transmission characteristics.

A REVIEW OF THE DEVELOPMENT OF DESIGN TECHNIQUES

The first successful efforts to design linkage mechanisms as function generators were graphical. Until electronic digital computers became available graphical methods were faster than analytic techniques; judging from published literature, graphical synthesis is being replaced by wholly analytic methods. Not only is the computer solution faster than the graphical, but the use of a digital plotter for some of the computer output retains the advantage of graphic procedures.

During World War II, concentrated linkage design work resulted in practical mechanical analog computers. Svoboda's report on this work (1) has been cited frequently since its appearance in 1948. He presented graphical design methods using nomograms and overlays and discussed various means for refinement of the design to reduce error.

In Germany, Hain (3) used graphical construction (i.e., successive inversion of the mechanism) with his "point-position-reduction" to synthesize four-bar and six-bar function generators. Point-position-reduction is the choice of the starting angle for input or output link so as to obtain angular symmetry with respect to the frame for pairs of the specified positions. In the United States, Nickson (4) elaborated on the overlay method discussed by Svoboda, a direct graphical approach to function generator design.

In 1954, Shaffer and Cochin (5) obtained a differential equation to test a function for exact synthesis by a four-bar mechanism and, if it can be exactly mechanized, to determine for what range of values such mechanization is possible. Freudenstein (6, 7) presented an analytic approach to the design of four-bar function generator mechanisms and developed synthesis equations for three, four, and five positions of input-output as well as equations for specified derivatives of displacement up to seventh order at a single position. His equation for three positions now appears in many textbooks for undergraduate engineering students.

Using complex numbers to represent vectors in a plane and combining them to form the quadrilaterals of the four-bar linkage for three and four specified positions ("loop equations"), Sieker (8) obtained sets of simultaneous equations. He noted that these are equations for the "circle-point-curve" and the "center-point-curve". He obtained synthesis equations for a function generator for three and four positions as well as for fewer positions with their derivatives. Several numerical examples were presented.

Allen (9) applied Hain's point-position-reduction to the graphical design of four-bar mechanisms as function generators. Freudenstein (10) tabulated dimensions of numerous four-bar mechanisms to generate various functions using his equations for 5, 6, and 7 precision points. Each mechanism is a minimum error approximation; he solved his equations (7) with a digital computer. Also, he mentioned some applications of mechanical function generators and discussed the suitability of functions for mechanization.

Michalec (11) gave a summary of types of elements used in mechanical analog computers based on his experience with a United States manufacturer of these computers.

Starting with the graphical construction employing point-position-reduction, Worthley and Hinkle (12) derived analytic equations for design of four-link function generators. And, to reduce the error of the approximation, Hahn (13) proposed four-bar mechanisms in series; the first mechanism is designed using one of the analytic schemes and its error curve is plotted. Then, from a collection of error curves which he prepared, one chooses a second four-bar mechanism such that the errors of both when superposed will result in reduced error for the combination.

Taking the four-bar mechanisms used in preparation of Hrones and Nelson's atlas of coupler curves (2), Vidosic and Johnson (14) developed a set of 146 pairs of curves as suggested earlier by Nickson (4):

1) The trace curve; normalized output vs. normalized input.
2) The deviation curve; (normalized output-normalized input) vs. normalized input.

By making trace and deviation curves for a function which is to be

mechanized and comparing them with the trace and deviation curves of their collection, one can select a suitable mechanism. Their collection of curves was prepared with the aid of an electronic digital computer.

Also using the digital computer, Dunk and Hanson (15) wrote a program to analyze the performance of a six-bar mechanism as a function generator. By iteration with the aim of reducing error, design is possible.

Hinkle (16) suggested using his angular velocity ratio theorem in selecting a four-bar mechanism configuration; by studying a plot of the function the orientation of the coupler with respect to the frame at any station in the range can be predicted. The electronic analog computer was used by Keller (22) as a design aid for four-link mechanisms; he obtained graphical display with the cathode ray oscilloscope.

Equations for synthesis of four-bar mechanism function generators (loop equations) were presented by McLarnan (17, 19); he gave equations for three, four, and five conditions and Fortran programs to perform synthesis and analysis. Also, he gave (19) a complete discussion of theory and limitations, including comments about "branching", to be considered below. McLarnan and Hagan (18) proved that no more than five constraints are allowed if scale factors for input and output are to be arbitrarily chosen. They noted that seven different combinations of precision points and derivatives are possible, and that the linkage having two links very long relative to the other two links can be replaced by an inversion of the slider-crank. Their development

resulted in a sixth degree equation with three trivial solutions and either one or three real, nontrivial solutions.

Finally, two modern textbooks contain rather complete discussions, to date, of design techniques for function generators. One is by Hirschhorn (20); the other by Hartenberg and Denavit (21).

This review of the development of design techniques has been selective rather than complete; the intention was to indicate the variety of approaches to the problem of function generation using a four-bar mechanism, and to show the movement away from graphical methods toward analytic methods augmented by the electronic digital computer.

THE PROBLEM

Statement of the Problem

The problem confronted in this thesis is to investigate the practical limitations to analytic synthesis of a four-link mechanism function generator with the aim of finding some pattern for their occurrence and thus a means to avoid them.

In spite of the amount of work and the diversity of attack on the problem of designing a four-link mechanism as a function generator, the solutions obtained are often not usable. While the solutions do satisfy the theoretical requirements at the precision points, they do not always satisfy the mechanical requirements.

a) Some of the precision points may lie on one branch of closure and some on the other, meaning that a joint must be separated and reconnected in a different configuration to complete the range of operation. This limitation is discussed as "branching" below.

b) Moving the links beyond the first or last precision point, but still within the range, may cause the mechanism to enter a region beyond its limits of motion where it is impossible for the links to remain connected. This is treated below as "closure".

c) The synthesized link-lengths may be such that as the links pass through a folded configuration the follower is able to move in either direction: it may continue to move in the same direction that brought it into the folded configuration or it may move in the opposite direction. This is discussed under "change points", below.

d) During operation within the range of motion, the acute angle between centerlines of coupler and follower link may become so small

as to preclude effective force transmission. This limitation is discussed below as "transmission angle".

e) Two of the links may be relatively much longer than the other two making the operation of the mechanism physically impossible because of deflection of the longer links, or because of space requirements. This limitation is referred to as "link length ratio".

Except for the transmission angle and link length ratio, these limitations are not matters of quality or degree; they are more in the nature of dichotomies. They are decisive. Graphical synthesis reveals these limitations as they arise; analytic synthesis can obscure their presence until after the solution is completed and a sketch or model of the linkage has been made. When such a limitation appears, new values for the arbitrary parameters must be chosen and another solution begun. It is impossible to predict such failures of solution.

Concerning branching, Hartenberg and Denavit declare (21), p. 233, "No systematic way is known to the authors for avoiding the trouble that was encountered." Again, McLarnan (19) has this to say about selecting arbitrary parameters, p. 14, "Normally the designer would want to make a number of choices of u_2 and u_3 [rotations of coupler] in order to have a number of alternative solutions to examine before selecting the best one for use. The proper selection of values u_2 and u_3 has not been studied extensively enough with regard to optimizing the solution to permit any guide lines to be laid down at this time."

Of course, the error or the deviation between the mechanized function and the ideal function may also be regarded as a practical limitation

to design but the recent literature is replete with error-reducing techniques to be applied after a generally usable configuration has been determined. The five practical limitations mentioned above must be satisfied prior to such refinement.

The problem for this thesis is an investigation of these practical limitations to the design of four-bar mechanism function generators.

THE BRANCHING LIMITATION

Given four bars identified by their lengths A, B, C, and F, where these are input link, coupler, output link and fixed link, respectively, let the position of the input link be specified. Then the coupler and output link can close the loop of four links in either of two different ways or branches as shown in Fig. 3.

The output angle SB_1 corresponds to branch 1; output angle SB_2 to branch 2. Each branch may be thought of as a reflection of the other about the diagonal D-D.

As an example of the branching difficulty, suppose that Fig. 4 represents a three precision-point approximation to some function. Use of the synthesis equation with P2 = 45 deg., S2 = 60 deg., range of P 90 deg. and range of S 60 deg. gives the linkage A, B, C, and frame shown in heavy solid lines in Fig. 5. For the first position, the four links are connected so that Link A is inclined at angle P2 and Link C at angle S2. As Link A is moved from angle P2 counterclockwise to angle P3, Link C moves to occupy position b_1; when A is moved to P4 then C moves to c_1. But the desired second and third locations for C are b_2 and c_2, on the other branch. If after being connected at position a_1, Link A is moved

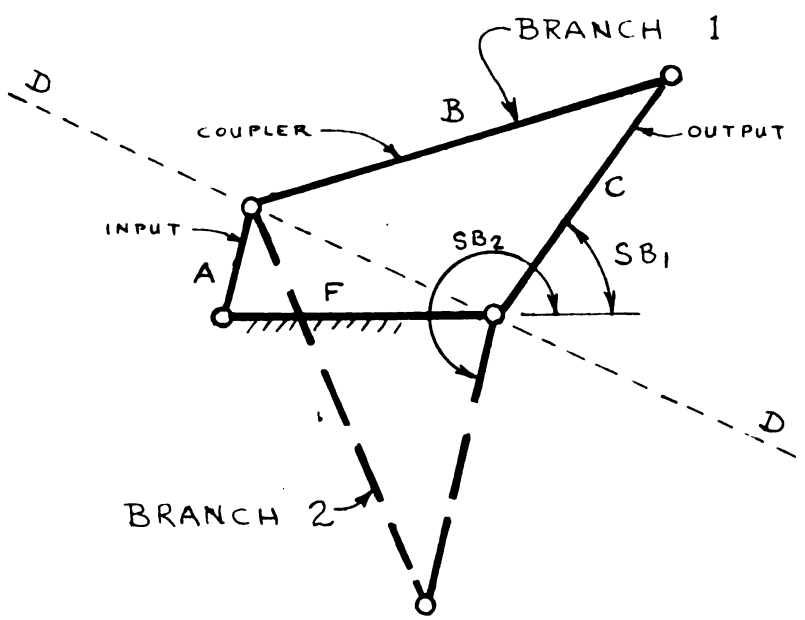


FIGURE 3

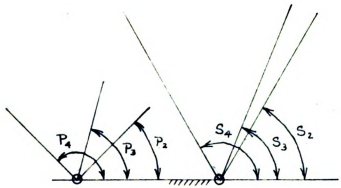


FIGURE 4

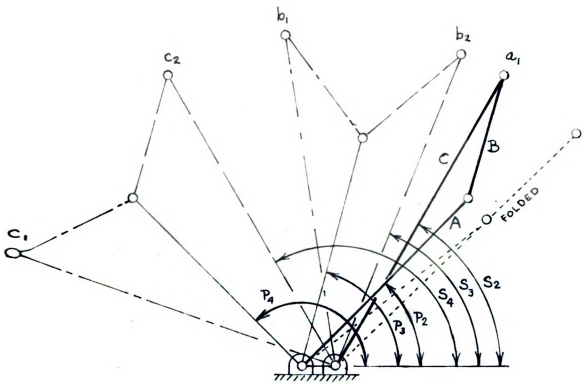


FIGURE 5

clockwise to the folded position, Fig. 5, then moved counterclockwise again with an open rather than a crossed configuration, positions b_2 and c_2 will be satisfied when A is at P3 and P4, respectively, but then position a_1 will not be satisfied with A at P2.

THE CLOSURE LIMITATION

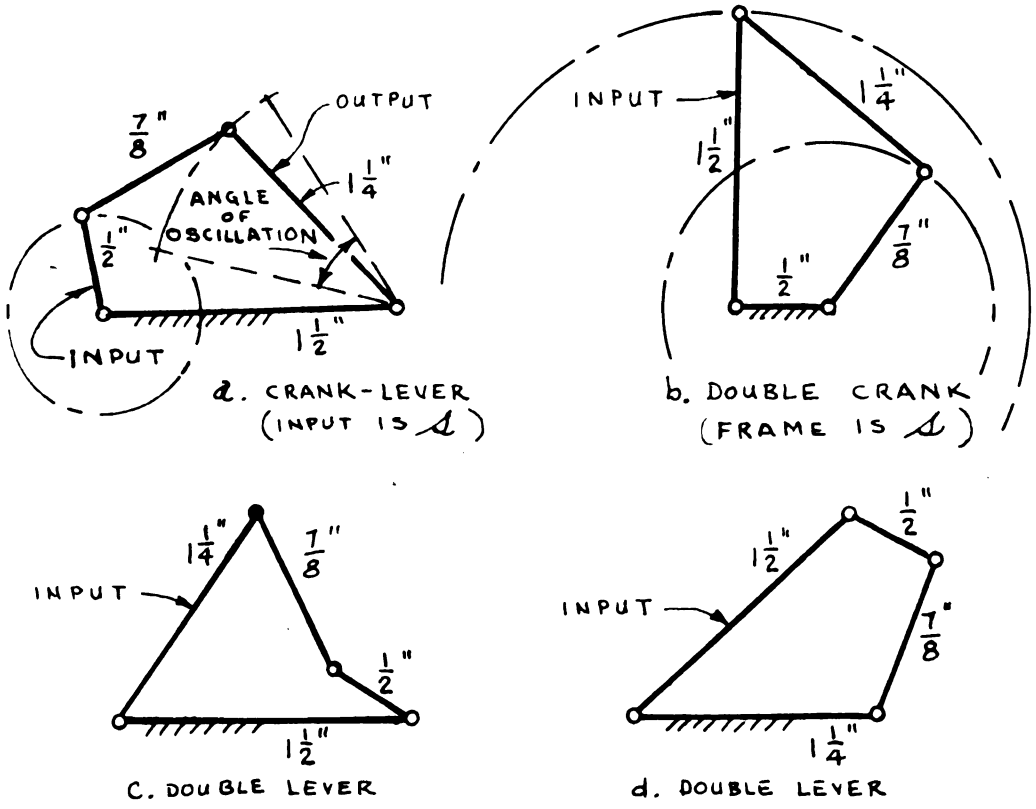
The input link and output link of a four-bar mechanism may not be able to rotate through 360 deg. (i.e., turn continuously) but, instead, one or both may be restricted to oscillate within some angle. A link which can turn continuously is referred to as a crank; one which can only oscillate is called a lever. This matter has been thoroughly investigated and the particulars are well-known. One good discussion is in Hartenberg and Denavit's book (21); part of their discussion is followed below to establish some terminology that is needed. Let the length of the shortest link be called s; the length of the longest link, l; the lengths of the other two links can be called p and q.

Then, if $s+l < p+q$, the linkage is termed a Grashof linkage (Fig. 6) for which three cases exist:

a) If link s is the input, the input link is a crank and the output link is a lever. The mechanism is called a crank-lever. Two of these are possible, since either link adjacent to s can be fixed.

b) If link s is fixed (i.e., if s is the frame) both input and output links are cranks; the mechanism is called a drag-link or a double-crank mechanism.

c) Otherwise, input and output links are levers. This is called a double-lever mechanism and there are three possibilities. Note that when s is the output link of a Grashof linkage s could turn continuously if the output were the driver rather than driven, but this is precisely the converse meaning of "output".



MECHANISMS FROM THE GRASHOF LINKAGE

FIGURE 6

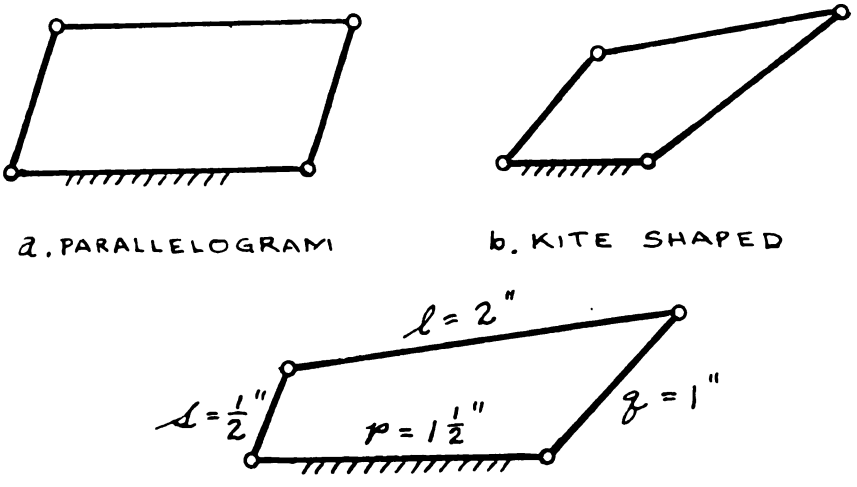


FIGURE 7

On the other hand, if $s+l>p+q$, only double-lever mechanisms are possible. Or, if $s+l=p+q$, change points occur; this is considered below.

Note in Fig. 6-a, for example, that the output lever is restricted to move within its angle of oscillation and that it occupies the extreme values of this angle when input link and coupler centerlines become colinear. In a synthesized linkage, the precision points will lie within this angle of oscillation but it often happens that the range angle specified in the synthesis problem must lie partly outside one of the extremes of the angle of oscillation. Where this happens the mechanism is not a solution to the synthesis problem. Closure is not possible. Similar considerations apply to the input link of a double lever mechanism.

THE CHANGE POINTS LIMITATION

In the discussion of closure, one of the possibilities was shown to be: $s+l=p+q$. For this case, if $s=p$ or if $s=q$, the linkage is a parallelogram or kite-shaped as the equal-length links are opposite or adjacent to each other, respectively (Fig. 7). But even if s is equal neither to p nor to q, as long as $s+l=p+q$, change points occur when the linkage is folded flat (i.e., when the centerlines of all links coincide). While the indecisiveness of change points can be corrected by inertia forces or by mechanical stops to prevent an undesired change, and even though the range of operation of the function generator may never include a change point, the possibility of change points is regarded herein as a design failure.

THE TRANSMISSION ANGLE LIMITATION

Whether the coupler pushes or pulls the output link during operation of the mechanism, the optimum angle between centerlines of the coupler and the output link for force transmission is 90 degrees. The transmission angle, TA, is defined as the angle between centerlines of the coupler and follower link or its supplement, whichever is smaller. See Fig. 8. While the optimum transmission angle is well-defined, the minimum value is a matter of judgement.

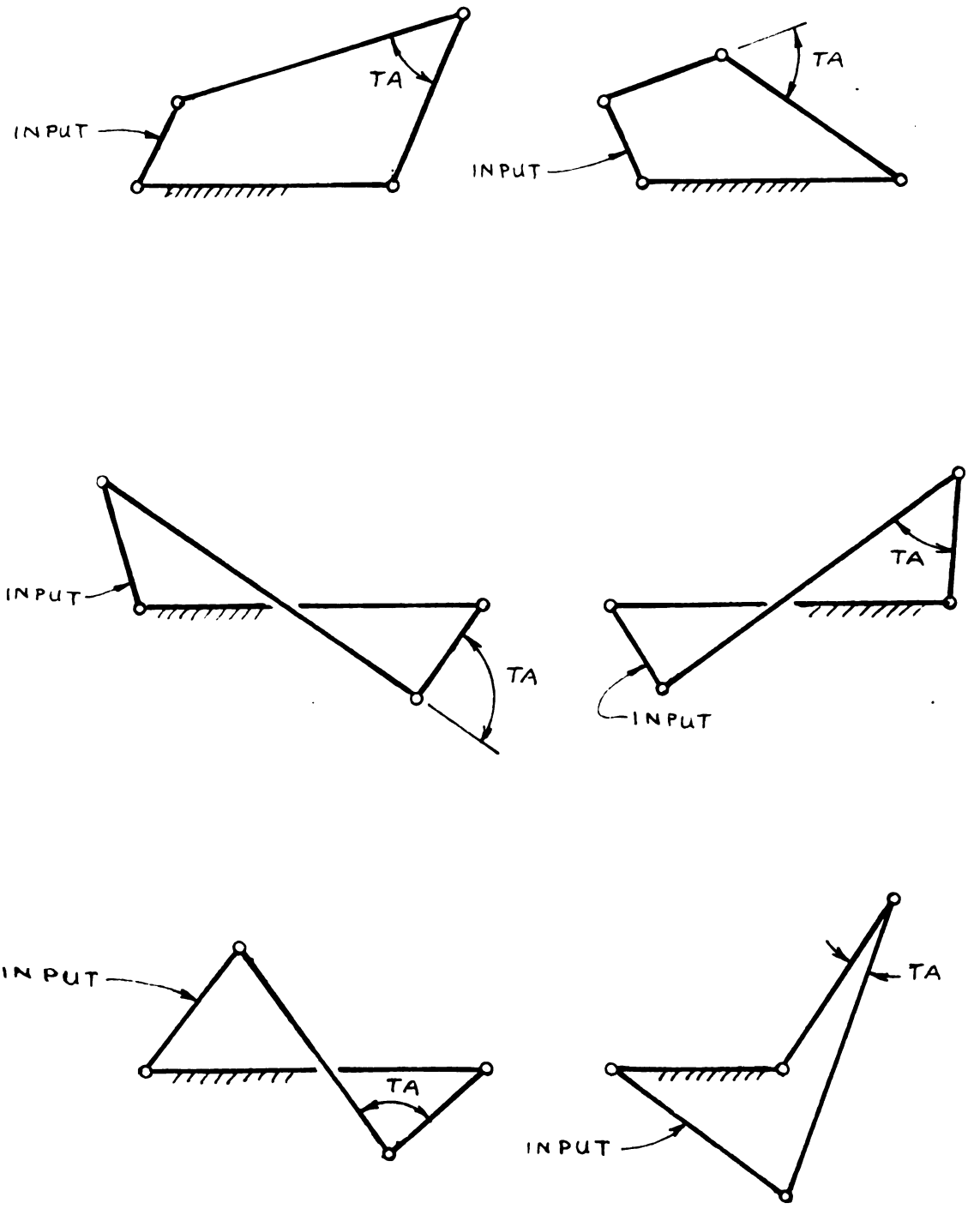
A quotation from Hirschhorn (20), p. 257, is pertinent:

> "Alt, who first suggested the use of this parameter as a quality index for the transmission of motion, recommended a minimum value of 40 deg. for low-speed and 50 deg. for high-speed applications. Other kinematicians have reiterated these values. However, it is extremely doubtful whether such generalizations should be accepted uncritically."

For the purposes of this study, 40 degrees was taken as a minimum in accord with recommendations in most references. So, if at any position within the range the transmission angle becomes less than 40 degrees, the design is regarded as a failure.

THE LINK-LENGTH RATIO LIMITATION

Choosing a maximum link-length ratio, RATIO = $\frac{\ell}{s}$, is again a judgement. For this work, the maximum link-length ratio was arbitrarily chosen to be 6.



THE TRANSMISSION ANGLE, TA

FIGURE 8

THE METHOD OF INVESTIGATION

Statement of the Method

The investigation was basically experimental: several commonly used functions and their ranges were chosen; for each, a number of mechanisms to generate the functions were synthesized using a well-known analytic technique. When a practical limitation was encountered, the particulars were recorded and a different synthesis was performed. The syntheses and analyses were done with the aid of a digital computer; in this way 4050 function generators were synthesized and analyzed for each of eight functions.

For the analytic synthesis of a four-link mechanism, seven variables are involved: three link-length ratios, the starting positions for the input link and for the output link, and the range angles for the input link and for the output link. Theoretically, all seven of these variables may be treated as unknowns; actually, the mathematical difficulties are such that ordinarily only three, four, or five unknowns are attempted, values for the remaining variables being arbitrarily assumed as parameters.

For this work, the analytic synthesis was performed with three unknowns and four arbitrarily chosen parameters to give the greatest variety of solutions consistent with reasonably accurate approximation. With three unknowns, three precision points may be prescribed; obviously, with more precision points the error of the approximation can be reduced. But even with only three precision points, errors of less than one percent of the range of output are possible. Furthermore, as stated previously, error reduction methods are plentiful.

The Freudenstein Equation (derived in the Appendix) was used for synthesis at three precision points. Here the arbitrary parameters were taken as the positions of input link and output link for the first precision point and the range angles of input link and output link. The unknowns were taken as the three link-length ratios, based on the fourth link which was assumed to have a length of unity. Hence the following notation (See Figs. 9 and 10.):

A = Length of input link

B = Length of coupler link

C = Length of output link

F = Length of fixed link (frame) = 1

P2 = Angular position of input link at first precision point configuration.

S2 = Angular position of output link at first precision point configuration.

RP = Range angle for input link

RS = Range angle for output link

THE FOUR ARBITRARY PARAMETERS

For a function with positive slope, the input and output links can rotate in the same direction. But Hinkle (16), p.286 , points out that a different mechanism results when the input and output links are assumed to rotate in opposite directions; he calls it the alternate solution. Similarly, if the function has negative slope, the input and output links can rotate in opposite directions or — for the alternate solution — in the same direction. In this work, the alternate solution is accomplished by using negative values of one of the range angles: RS was given both positive and negative values.

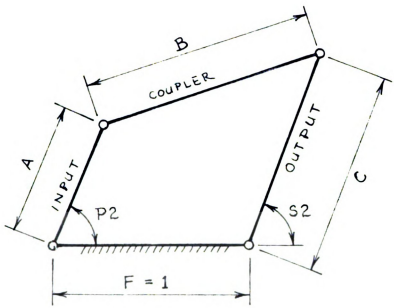


FIGURE 9

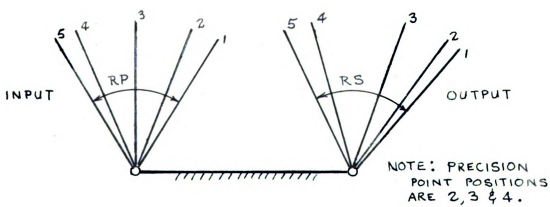


FIGURE 10

Hence, the schedule for selection of arbitrary parameters was as follows:

	Value of Parameter, Degrees									
P2	10	50	90	130	170	210	250	290	330	
S2	20	60	100	140	180	220	260	300	340	
RP	30	60	90	120	150					
RS	30	-30	60	-60	90	-90	120	-120	150	-150

Angles P2 and S2, corresponding to the first precision point, are always measured counterclockwise from a datum ray; range angles RP and RS are magnitudes. Two hypothetical functions, having a positive slope and the other having negative slope, are shown in Fig. 11, where the meaning of the four arbitrary parameters is illustrated. Parameters P2 and S2 fix a point in the P-S plane; parameters RP and RS act as scale factors, the sign of RS having its further significance mentioned above.

Also, note in Fig. 11 that the first and last precision points (P2 and P4) are distinct from the ends of the range RP (P1 and P5). This distinction is necessary for re-spacing the precision points to optimize the error. However, of course, P2 and P1, for example, might have the same value. In the programs, precision points P2, P3, and P4 have been calculated from the values of X2, X3, and X4, taken as input data. And X2, X3, and X4 were determined by Chebyshev spacing in the range of X; see Hartenberg and Denavit (21), p. 140, or Hinkle (16), p. 284.

While parameters P2 and S2 were chosen in steps of 40 degrees, the data shows steps of 20 degrees for them. In solving the synthesis equation, link lengths A and C may have emerged as negative values whereupon they were changed to positive values and angles P (if A was negative) and S (if C was negative) were advanced 180 degrees. Since 180 degrees consists

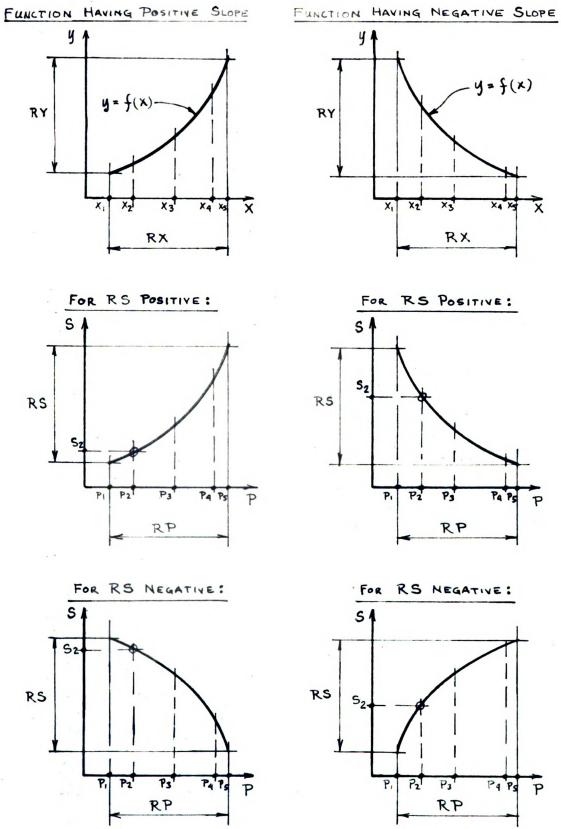

FUNCTION HAVING POSITIVE SLOPE
FUNCTION HAVING NEGATIVE SLOPE
y
y = f(x)
RY
X
RX
FOR RS POSITIVE:
FOR RS NEGATIVE:
S
RS
P
RP

FIGURE II

of four-and-a-half 40 degree intervals, the 20 degree steps were introduced.

The difference of 10 degrees (or integral multiples of 10 degrees) between P2 and S2 was introduced to prevent starting a solution with all four links having colinear centerlines. Incidentally, this precluded parallelogram linkages.

FUNCTION AND RANGES

The functions used for this study are listed in Fig. 12, which also gives a function identification number, the range for the approximation and the precision point spacing: X2, X3, X4.

ORDERING THE LIMITATIONS

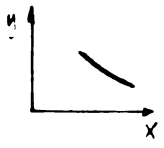
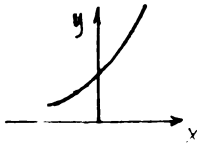
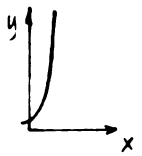
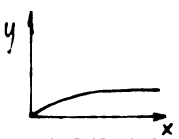
Unavoidably, the order of apprehending the practical limitations had the effect of assigning priority to them. Since it is not known that these limitations are exclusive, failures attributed to one kind of practical limitation might have been attributed to some other kind had the order of testing been different. From the viewpoint of the mechanism designer this is not important but it does impede developing a theory of failure for a particular kind of limitation.

Even so, certain ordering criteria prevail:

a) Over-riding influence of certain limitations.
   (For example, if branching occurs then the maximum link length ratio and minimum transmission angle are immaterial.)

b) Consquence of the limitations.
   (For example, the change points limitation can be corrected by other means, so it has low priority.)

c) Time of computation.

Nº	FUNCTION	RANGE	PRECISION POINTS X_2	X_3	X_4
1	$y = \frac{1}{x}$ 	$1 \leq X \leq 2$	1.067	1.5	1.933
2	$y = e^x$ 	$-1 \leq X \leq 1$	-.866	0	.866
3	$y = e^x$ 	$0 \leq X \leq 10$	.667	5.0	9.33
4	$y = \text{LOG}_{10} X$	$1 \leq X \leq 10$	1.603	5.5	9.397
5	$y = \text{SINH } X$	$0 \leq X \leq 2$	.134	1.0	1.866
6	$y = \text{COSH } X$	$0 \leq X \leq 2$	.134	1.0	1.866
7	$y = \text{TAN } X$	$0 \leq X \leq 1.5$	.10	.75	1.4
8	$y = \text{ARCTAN } X$ 	$0 \leq X \leq 10$	.667	5.0	9.33
9	$y = x^2$	$0 \leq X \leq 10$	.667	5.0	9.33

FUNCTIONS AND RANGES

FIGURE 12

According to these criteria, the following order was used:

1. Maximum link length ratio exceeding 15.
2. Branching.
3. Closure.
4. Minimum transmission angle less than 40 degrees.
5. Maximum link length ratio exceeding 6.
6. Change points.

COMPUTER OUTPUT

For each mechanism synthesis which failed to be usable, the kind of practical limitation responsible for the failure and the corresponding values of the four arbitrary parameters were recorded on magnetic computer tape. Later, a sorting program read this tape, consolidated the data as explained below in Displaying the Results and printed the results in a form convenient for plotting.

A Preliminary Study

Prior to extensive examination of the practical limitations to analytic synthesis, an intensive preliminary study was made; in addition to recording the failures, certain performance features of each usable mechanism were determined and recorded. In the preliminary study, the practical limitations were discovered using the same methods as for the final study except that 10 degrees rather than 40 degrees was taken for the minimum value of the transmission angle.

For each usable mechanism found in this preliminary study, the error was optimized by re-spacing the precision points in the manner described by Hinkle (16) and by Hartenberg and Denavit (21) and the mechanism was

classified as crank-lever, double-lever, double-crank, etc. according to the index NGRAS in Fig. 13. Then all results of interest were printed in the following order:

Identification number of usable mechanism . . . NPLOT

Classification number . . . . . NGRAS

Error, percent . . . . . EP

Minimum transmission angle . . . . . TAM

Value of angle P corresponding to TAM . . . . . PTM

Maximum link length ratio . . . . . RATIO

Number of passes in SPACE subroutine . . . . . NSPAC

Length of links . . . . . A,B,C

Indicator for negative A or C . . . . . NEGP, NEGS

Precision points as re-spaced X2, X3, X4

Values of the four arbitrary parameters P2,S2,RP,RS

For each non-usable mechanism found in this preliminary study, appropriate results were printed. A portion of the printed output is given in The Results, pp. 70 and 71. The interpretation of this printed output will be aided by the following notes:

1. In the line beginning "RATIO EXCEEDS 15.", the statement LARGE = 1 appears. This indicates that the maximum link length ratio, while greater than 15, did not exceed the numerical capacity of the computer. If it ever did, the statement LARGE = 2 would appear.
2. "UNABLE TO CLOSE LOOP" means that a failure due to the closure limitation occurred.
3. "NSPAC = 100" means that after 100 passes in the SPACE subroutine the test for optimizing the error had not

NGRAS	CLASS
1	CRANK - LEVER
2	DOUBLE - CRANK
3	DOUBLE - LEVER, BUT OUTPUT COULD ROTATE CONTINUOUSLY IF IT WERE MADE THE DRIVER
4	DOUBLE - LEVER
5	CHANGE POINTS
6	CHANGE POINTS, KITE - SHAPED
7	CHANGE POINTS, PARALLELOGRAM

THE INDEX NGRAS

FIGURE 13

been satisfied, so the effort was abandoned for the mechanism.

The function used for the preliminary study was the reciprocal function, $y = 1/x$, in the range $1 \leq x \leq 2$. Sketches of each usable mechanism in the configuration for the first precision point were drawn by the digital plotter. A few of the 922 sketches obtained are shown in The Results, pp. 72 and 73.

The preliminary study confirmed certain aspects of the procedure and sharpened the pursuit. Most of the subroutines developed for the preliminary study were used for the final study. Furthermore, the sketches of mechanisms made by the plotter provided the writer much satisfaction and encouragement.

The Computer Programs

Although a full listing of all digital computer programs and subroutines appears as Appendix II, this section contains concise descriptions of these programs to give the reader a comprehensive summary.

MAIN PROGRAM FOR FINAL STUDY

Nine passes in this main program were required to collect the data for the eight functions (including two different ranges of the exponential function) listed in Fig. 12. Automatic advance through the nine passes was planned but an unknown total run-time and limited availability of the computer required a provision for orderly interruption before the time required exceeded the time allowed. Such an interruption should occur between passes and should leave the tape on which the data was being recorded in proper condition for a re-start. The indexes I1, I2, I3, I4, I5, and NPLOT and subroutine COPY were intended to be used for such

a re-start. However, as it happened, the total run-time was within the time allowed, so this feature was not used.

Index NP was used to identify each pass. For each pass, the data input was the five values of X (i.e., X1, X2, X3, X4, and X5). Proper choice of each function in subroutines FERD3 and ERROR was also regulated by index NP.

Next, the four arbitrary parameters were set for one of the 4050 combinations. For this combination, a mechanism was synthesized by subroutine FERD3. Then this mechanism was checked for maximum link length ratio by subroutine MAMI, for branching by subroutine BRANCH, for minimum transmission angle by subroutine ERROR, again for maximum link length ratio and finally for change points by subroutine GRAS. Following this, certain parameter restorations were made, if necessary, and the next one of the 4050 parameter combinations was set. At several points during the above procedure, a failure may have been recorded.

SUBROUTINES

Subroutine FERD3

Input: X(J), J=1, 2, ...5, P(2), S(2), RP, RS

Output: Y(J), J=1, 2, ...5, P(1), P(3), P(4), P(5),
S(1), S(3), S(4), S(5), RX, RY, A, B, C,
NEGP, NEGS, LARGE

(The index NP was also used as an input parameter during the final study to permit changing the functions for successive runs.)

Calculate: A, B, C using the Synthesis Equation. (See Appendix I for a derivation of this equation.) If A is negative, rotate input link 180 degrees and make A positive.

Set NEGP = 2. If C is negative, rotate output link 180 degrees and make C positive. Set NEGS = 2. Initially, LARGE = 1. If A or C is quite large, set LARGE = 2. If loop-closure is impossible, set LARGE = 3. (This never happened in FERD3.)

This subroutine calculates the link lengths for the mechanism using the Synthesis Equation.

Subroutine MAMI

Input: A, B, C

Output: LS, LL, LE1, LE2, LE3

This subroutine determines which of A, B, C and F = 1 is maximum and which of these is minimum, finds the maximum link length ratio, RATIO, and sets indexes according to the schedules shown in Fig. 14.

Subroutine BRANCH

Input: P(J), J=2, 3, 4, S(J), J=2, 3, 4, A, B, C,

Output: NS, NSD

Calculate: SSP(J), J=2, 3, 4 as the output angle for the mechanism when the positive square root is used in the Analysis Equation.

Calculate: SSM(J), J=2, 3, 4 as the output angle for the mechanism when the negative square root is used in the Analysis Equation.

Compare: SSP(J) with S(J), J=2, 3, 4. If all three agree (within ± 3 deg.) set NS = 1 and NSD = 1 and return to main program. If not, . . .

Compare: SSM(J) with S(J), J=2, 3, 4. If all three agree (within

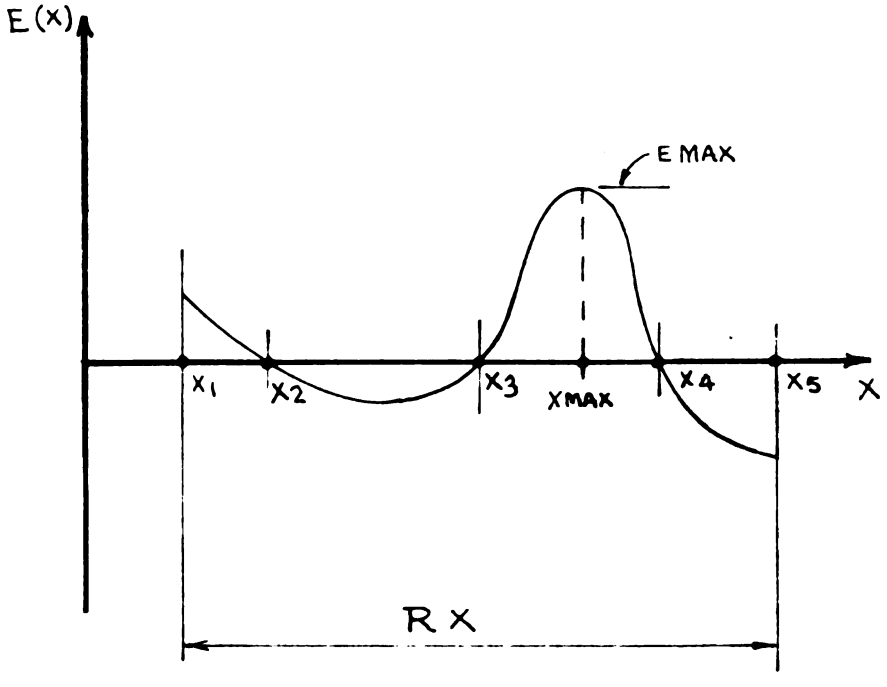
LS: INDEX FOR SHORTEST LINK, s				
s	A	B	C	1
LS	1	2	3	4

LL: INDEX FOR LONGEST LINK, l				
l	A	B	C	1
LL	1	2	3	4

INDEXES FOR EQUAL LINK LENGTHS (INITIALLY LE1 = LE2 = LE3 = 1)						
	A=B	A=C	A=1	B=C	B=1	C=1
LE1	2	3	4			
LE2				2	3	
LE3						2

INDEXES FOR SUBROUTINE MAMI

FIGURE 14



THE ERROR FUNCTION

FIGURE 15

± 3 deg.) set NS = 2 and NSD = 1 and return to main program. If not, set NSD = 2 and return to main program. This subroutine determines if branching occurs (NSD = 2) and, if not, it also determines whether the positive or the negative square root should be used in the Analysis Equation.

Subroutine ERROR

Input: P1, P2, S2, RP, RS, X1, Y2, RX, RY, NS, A, B, C

Output: E(J), J=1,2,. . . 101, EMAX, XMAX, EP, TAM, PTM, NE.

(The index NP was also used as an input parameter during the final study to permit changing the functions for successive runs.)

This subroutine builds the error function, E(x), consisting of 101 ordinates (Fig. 15), determines the maximum error, EMAX, the corresponding value of X, XMAX, and the maximum percent error, EP, based on the range of y. This subroutine also finds the minimum value of the transmission angle in the range of operation, TAM, and the corresponding value of P, PTM. Finally, it checks for closure throughout the range of operation. If closure is a limitation, the index, NE, is set to 3 and control is returned to the main program.

Subroutine GRAS

Input: A, B, C, LS, LL, LE1, LE2, LE3

Output: NGRAS

This subroutine determines to which of the classifications in Fig. 13, p. 28, the mechanism belongs.

Subroutine ATQ

Input: Y, X

Output: V

This subroutine determines the angle V= Arctan Y/X, in the correct quadrant.

Subroutine SPACE (Not used during the final study.)

Input: X(J), J=1, 2, ...5, RX, RY, XMAX,
E(K), K=1, 2, ... 101.

Output: X(J), J=2, 3, 4, NOPT, NSPAC

This subroutine re-spaces the precision points X(2), X(3) and X(4) until the extrema of the error function agree within a tolerance of $\pm$(.001)(RY). A count of passes through this subroutine, using index NSPAC, is kept. NOPT = 1 means: Error function not yet optimized. NOPT = 2 means: Error function is optimized.

Subroutine SCALE (Not used during the final study.)

Input A, B, C, LS, P(2), G
(G is the angle of inclination of the coupler link for the configuration of the first precision point, measured from the datum ray. See Fig. 45.)

Output: X2, X3, X4, Y2, Y3

This subroutine scales link lengths preparatory to plotting so that the shortest link has a length of unity. Also, horizontal and vertical components of the pinned joints are determined for input to the plotter.

MAIN PROGRAM FOR PRELIMINARY STUDY

Only one range of one function was handled by this program but the interrupt and re-start feature was needed often because of the long run-time (more than six hours of computer time). But here, the interruption had to be between any two parameter combinations among the 4050. Much

printed output was obtained; in addition, certain output data was recorded on tapes 5 and 6, although the data on tape 6 was never used.

The plotting was done with the aid of an MSU Computer Center library subroutine, PLOT. Another library subroutine, CHAR, was used to plot the small circles indicating the pinned joints and to plot the identification number, NPLCT, for each sketch. Otherwise, this program was similar to the main program for the final study.

Displaying the Results

Since the purpose of the work was to investigate the practical limitations to analytic synthesis of linkage function generators, the occurrences of these limitations had to be displayed in such a way as to show a pattern, if such a pattern exists. For each limitation studied, it was to be determined if that limitation was dependent on the function being generated and if so, in what way. Also, the influence of the four arbitrary parameters on each of these limitations had to be revealed by the display.

In view of the thousands of items of data stored on tape,the anticipated manner of display affected the way in which this data was to be read from the tape and printed. Graphical display on transparent paper was indicated because of the possibility of stacking sheets of paper together and looking through them, somewhat in the manner of a contour map.

THE INDEX I

In order to reduce the number of variables involved and still preserve distinction, two of the four arbitrary parameters, the range angles RP and RS, were combined into a single index, I. Referring to Fig. 16, it is seen that any pair of values for RP and RS gives a distinct value of

FOR RS (−): $I = \frac{RP-30}{6} + \left|\frac{RS}{30}\right|$

RS \ RP	30	60	90	120	150
30	1	6	11	16	21
60	2	7	12	17	22
90	3	8	13	18	23
120	4	9	14	19	24
150	5	10	15	20	25

FOR RS (+): $I = 25 + \frac{RP-30}{6} + \frac{RS}{30}$

RS \ RP	30	60	90	120	150
30	26	31	36	41	46
60	27	32	37	42	47
90	28	33	38	43	48
120	29	34	39	44	49
150	30	35	40	45	50

THE INDEX I

FIGURE 16

the index I, from 1 through 50. For example, for RP = 60, RS = -120, I = 9 and for RP = 90, RS = +60, I = 37.

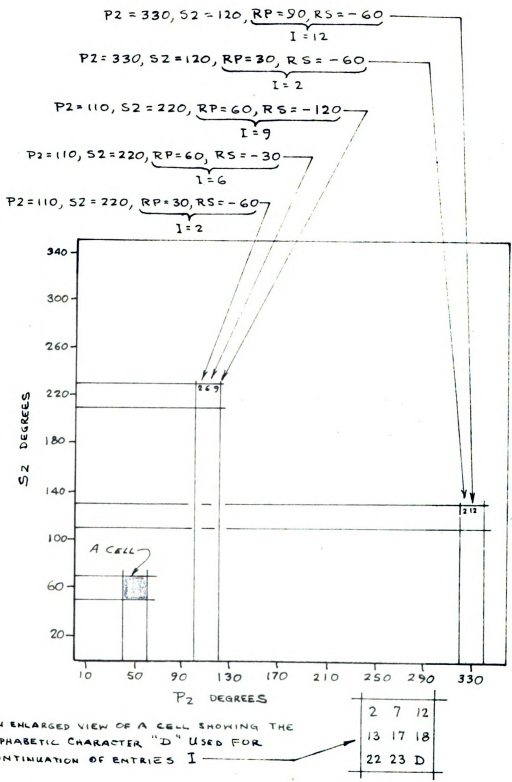
THE PLOT

Then, for a single function and its range, using P2 as abscissa and S2 as ordinate, the event of a failure due to branching, say, could be indicated by placing the number I in the cell given by coordinates (P2, S2). It would then always be possible, though perhaps tedious, to recapture all values of the four parameters for which branching was a limitation for that function by reading the number I and its associated coordinates from the plot. See Fig. 17 for some examples of the plot.

This procedure has the disadvantage of subduing the effect of changes in the range angles RP and RS. But some initial plots indicated that the limitations being studied were not sensitive to variation of RP and RS, so the procedure was adopted.

Initial plots also revealed similarities in pattern for plots belonging to RS(positive) and similarities in pattern for plots belonging to RS(negative) but no similarities when plots belonging to RS(positive) were compared with plots belonging to RS(negative). Thus, two families of patterns were quickly identified: the RS(+) family and the RS(-) family, or the family for the solutions and the family for the alternate solutions.

This knowledge simplified the plot procedure since it reduced from 50 to 25 the maximum number of entries I to be plotted in each cell. At each point (P2, S2), as many as eight entries I were placed inside the square cell; additional entries I were placed outside the plot and



TYPICAL DETAILS FOR THE PLOT

FIGURE 17

referenced with an alphabetic character inside the cell.

Finally then, each plot on transparent paper was to be for one of the five practical limitations, for one function and its range, and for one family, either RS(+) or RS(-). By stacking transparencies and peering through the stack with a strong light in back, the plots could be compared on the basis of function sensitivity or family sensitivity. This, in fact, was the way the Summaries of Plots were made.

With this plan in mind, the sorting program to read the data tape was written to tabulate the data, function-by-function, in sets of five columns, each column corresponding to a single practical limitation. The tape was read once for the RS(+) family, obtaining 25 such sets of five columns for each function; then the tape was read again for the RS(-) family, obtaining another 25 sets of five columns for each function.

THE RESULTS

Number of Design Failures by Kind

By subtracting the total number of design failures from 4050, the number of usable mechanisms obtained for each function is as shown in Fig. 18. In Fig. 19 are given numbers of failures attributed to the practical limitations of each kind, listed by function. Notice that not a single failure for any function was attributed to the change points limitation.

Plots

In all, seventy-two plots were made, as tabulated below:

Function No.	Branching		Closure		Transmission Angle		Link Length Ratio		Total
	RS(+)	RS(-)	RS(+)	RS(-)	RS(+)	RS(-)	RS(+)	RS(-)	
1	1	1	1	1	1	1	1	1	8
2	1	1	1	1	1	1	1	1	8
3	1	1	1	1	1	1	1	1	8
4	1	1	1	1	1	1	1	1	8
5	1	1	1	1	1	1	1	1	8
6	1	1	1	1	1	1	1	1	8
7	1	1	1	1	1	1	1	1	8
8	1	1	1	1	1	1	1	1	8
9	1	1	1	1	1	1	1	1	8
								Total	72

Several of these appear as Figs. 20 to 35, inclusive.

Summaries of Plots

By superimposing transparencies of the plots and comparing their patterns by groups, certain similarities of pattern were observed.

FUNCTION		NUMBER OF USABLE MECHANISMS OBTAINED FROM 4050 SYNTHESES
Nº	KIND AND RANGE	
1	$\frac{1}{X}$ $1 \leq X \leq 2$	333
2	e^X $-1 \leq X \leq 1$	377
3	e^X $0 \leq X \leq 10$	210
4	$\text{LOG}_{10} X$ $1 \leq X \leq 10$	369
5	SINH X $0 \leq X \leq 2$	407
6	COSH X $0 \leq X \leq 2$	301
7	TAN X $0 \leq X \leq 1.5$	331
8	ARCTAN X, $0 \leq X \leq 10$	216
9	X^2 $0 \leq X \leq 10$	340

FIGURE 18

NUMBERS OF FAILURES ATTRIBUTED TO THE PRACTICAL LIMITATIONS OF EACH KIND

FUNCTION Nº		BRANCHING	CLOSURE	TRANSMISSION ANGLE	LINK LENGTH RATIO	CHANGE POINTS
1	RS(+)	346	429	364	484	0
	RS(−)	435	441	344	475	0
	TOTAL	781	870	708	959	0
2	RS(+)	585	394	416	389	0
	RS(−)	530	437	582	340	0
	TOTAL	1115	831	998	729	0
3	RS(+)	939	252	339	379	0
	RS(−)	703	285	531	412	0
	TOTAL	1642	537	870	791	0
4	RS(+)	528	416	453	399	0
	RS(−)	475	407	621	382	0
	TOTAL	1003	823	1074	781	0
5	RS(+)	348	491	526	405	0
	RS(−)	438	443	634	358	0
	TOTAL	786	934	1160	763	0
6	RS(+)	807	280	367	369	0
	RS(−)	653	389	540	344	0
	TOTAL	1460	669	907	713	0
7	RS(+)	626	316	431	465	0
	RS(−)	397	313	670	501	0
	TOTAL	1023	629	1101	966	0
8	RS(+)	903	268	357	380	0
	RS(−)	677	333	569	347	0
	TOTAL	1580	601	926	727	0
9	RS(+)	656	354	397	389	0
	RS(−)	610	414	556	334	0
	TOTAL	1266	768	953	723	0

FIGURE 19

An attempt to represent these likenesses of pattern resulted in the Summaries of Plots, as listed below:

		Functions	
Branching	RS(+)	1 to 9, incl.	Fig. 36
Branching	RS(-)	1 to 9, incl.	Fig. 37
Closure	RS(+)	1 to 9, incl.	Fig. 38
Closure	RS(-)	1 to 9, incl.	Fig. 39
Transmission Angle. . . .	RS(+)	2,3,5,6,7,9	Fig. 40
Transmission Angle. . . .	RS(+)	1,4,8	Fig. 41
Transmission Angle. . . .	RS(-)	2,3,5,6,7,9	Fig. 42
Transmission Angle. . . .	RS(-)	1,4,8	Fig. 43
Link Length	RS(-)	1 to 9, incl.	Fig. 44

For all Summaries of Plots, the shaded area represents design failures due to that kind of limitation; the clear area represents freedom from failure due to that kind of limitation.

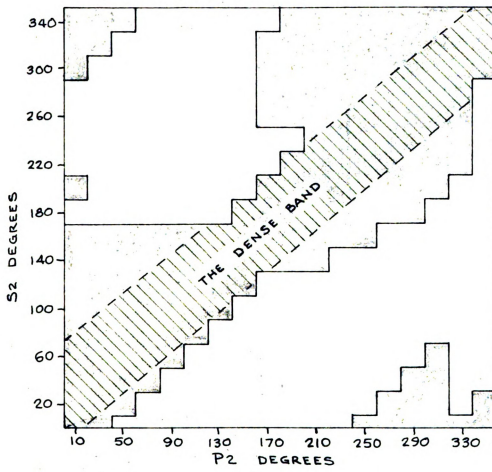
A certain judgement was employed in making the Summaries of Plots: where no more than two cells having one entry apiece coincided in a stack of six or more functions, this was regarded as clear area. Also, where only one cell having no more than three entries appeared in a stack of six or more functions, this was regarded as clear area.

A Summary of Plots was not made for Link Length Ratio, RS(+), because no pattern was discernible.

For the Summaries of Plots of the branching limitation, the dense regions caused by a concentration of entries in these regions were judged worthy of special notice.

Sample Output from Preliminary Study

Two pages of printed output and two pages of corresponding sketches of mechanisms are given as a sample of the output from the preliminary study.



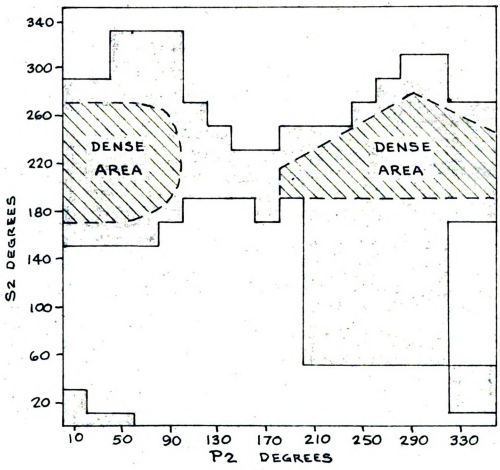
NOTE: SHADED AREA REPRESENTS DESIGN FAILURE.

SUMMARY OF PLOTS

FUNCTIONS: 1 TO 9, INCLUSIVE, RS(+)

BRANCHING

FIGURE 36



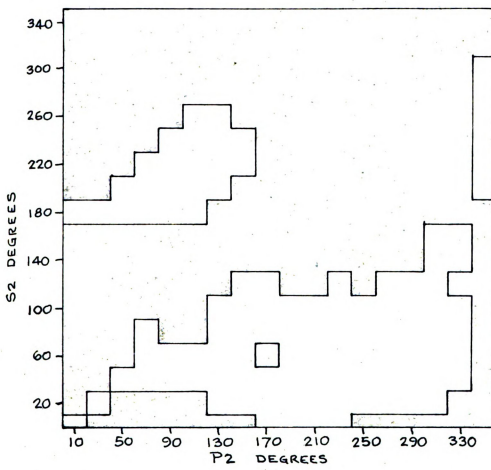
NOTE: SHADED AREA REPRESENTS DESIGN FAILURE.

SUMMARY OF PLOTS

FUNCTIONS: 1 TO 9, INCLUSIVE, RS(-)

BRANCHING

FIGURE 37



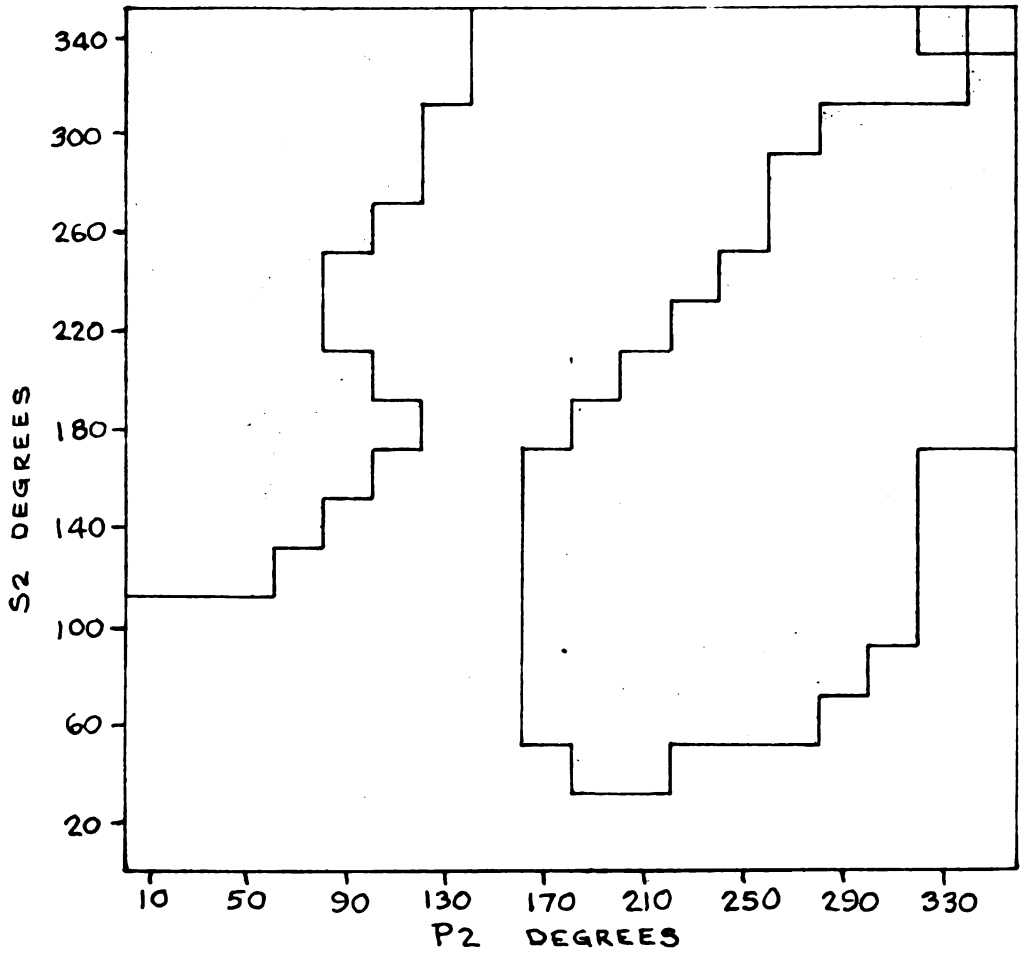
NOTE: SHADED AREA REPRESENTS DESIGN FAILURE.

SUMMARY OF PLOTS

FUNCTIONS: 1 TO 9, INCLUSIVE, RS(t)

CLOSURE

FIGURE 38



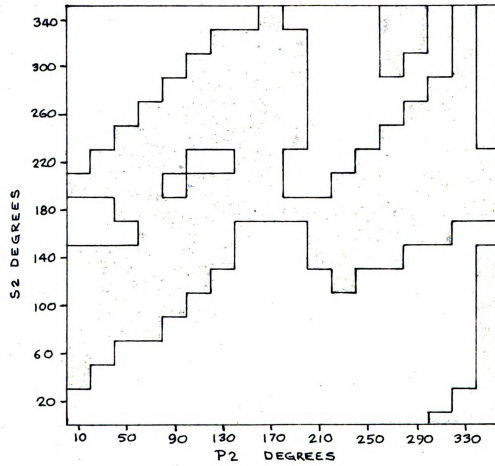
NOTE: SHADED AREA REPRESENTS DESIGN FAILURE.

SUMMARY OF PLOTS

FUNCTIONS: 1 TO 9, INCLUSIVE, RS(−)

CLOSURE

FIGURE 39

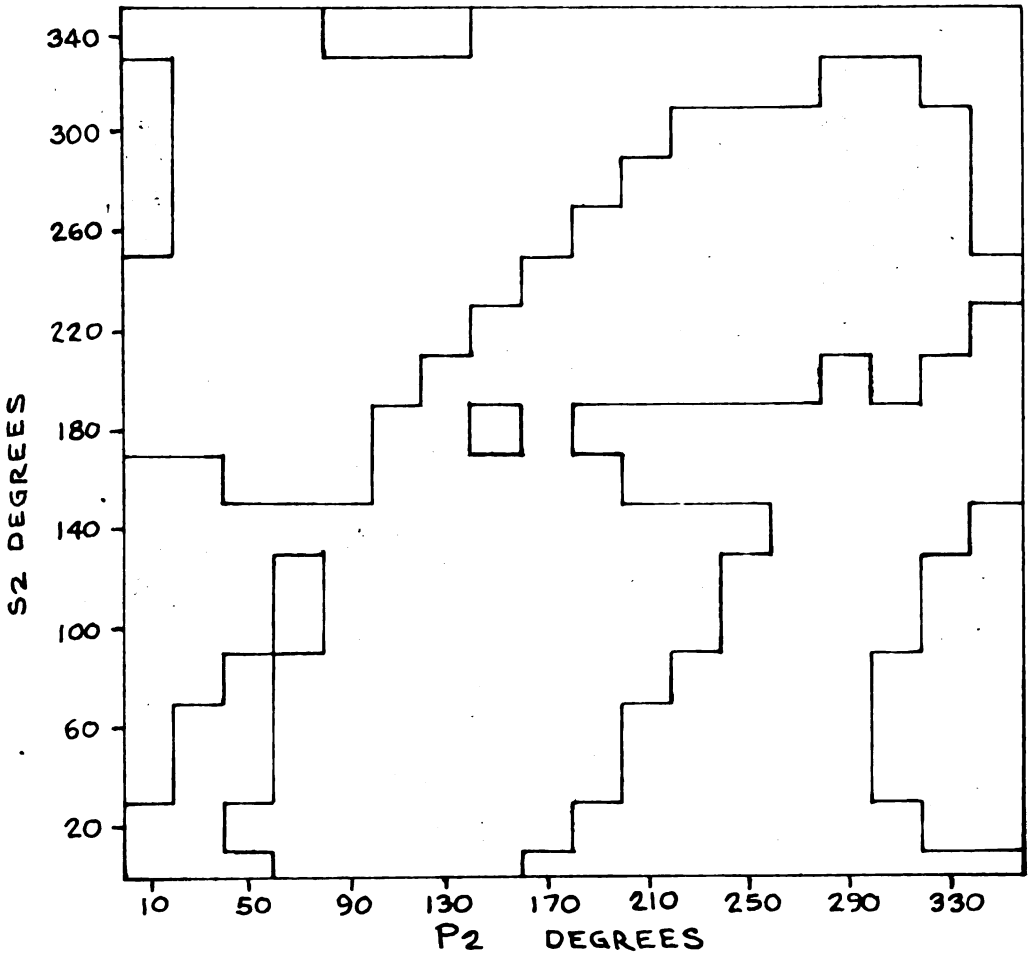


NOTE: SHADED AREA REPRESENTS DESIGN FAILURE.

SUMMARY OF PLOTS

FUNCTIONS: 2,3,5,6,7 AND 9, RS(+)
FIRST & SECOND DERIVATIVES WITH LIKE SIGNS.
TRANSMISSION ANGLE

FIGURE 40



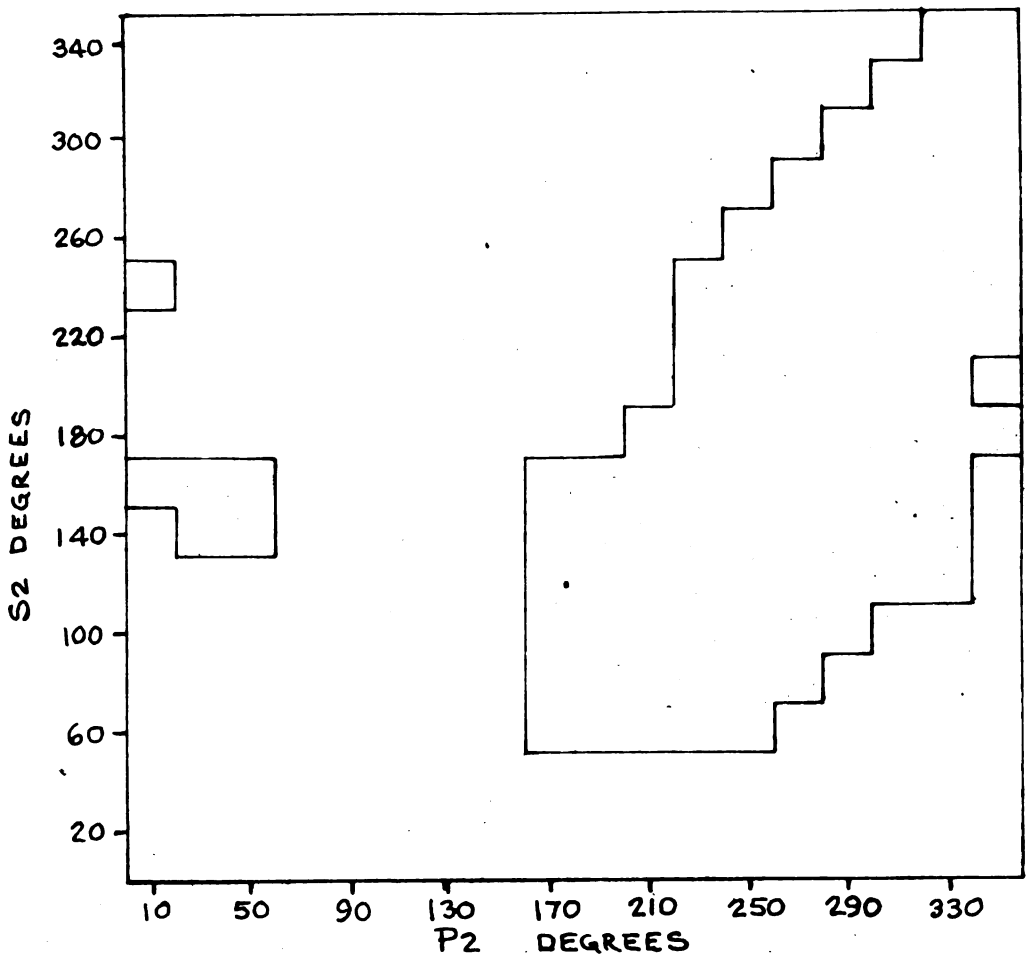
NOTE: SHADED AREA REPRESENTS DESIGN FAILURE.

SUMMARY OF PLOTS

FUNCTIONS: 1,4 AND 8, RS(+)
FIRST & SECOND DERIVATIVES WITH UNLIKE SIGNS.

TRANSMISSION ANGLE

FIGURE 41



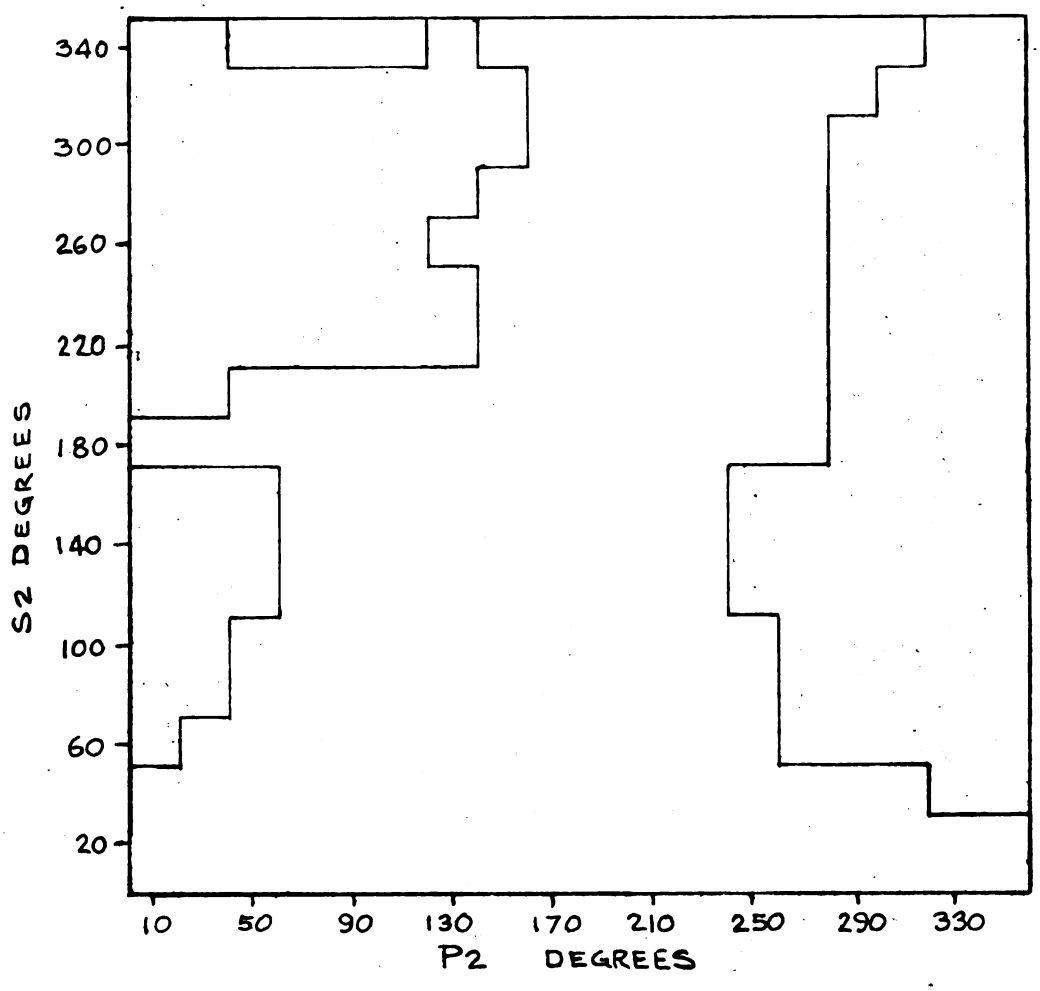
NOTE: SHADED AREA REPRESENTS DESIGN FAILURE.

SUMMARY OF PLOTS

FUNCTIONS: 2,3,5,6,7 AND 9 , RS(−)
FIRST & SECOND DERIVATIVES WITH LIKE SIGNS.

TRANSMISSION ANGLE

FIGURE 42



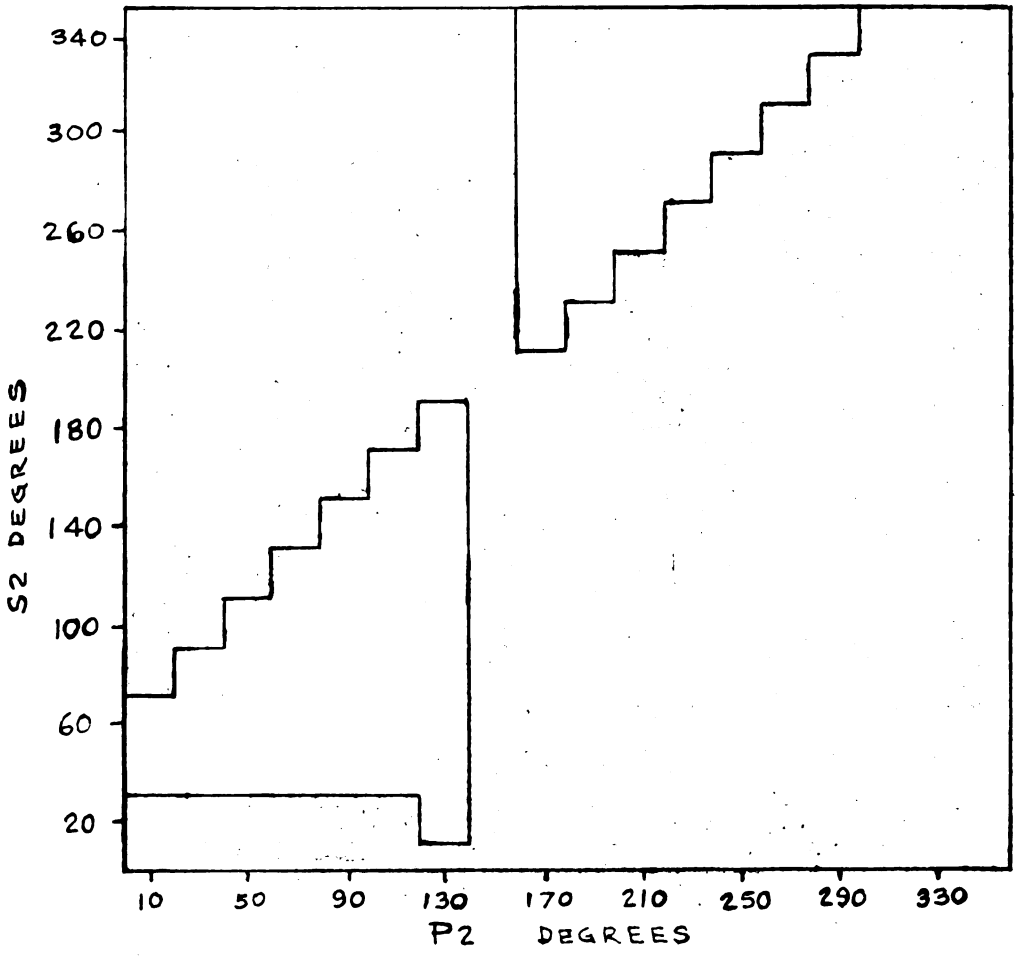
NOTE: SHADED AREA REPRESENTS DESIGN FAILURE.

SUMMARY OF PLOTS

FUNCTIONS: 1,4 AND 8 , RS (−)
FIRST & SECOND DERIVATIVES WITH UNLIKE SIGNS.

TRANSMISSION ANGLE

FIGURE 43



NOTE: SHADED AREA REPRESENTS DESIGN FAILURE.

SUMMARY OF PLOTS

FUNCTIONS: 1 TO 9, INCLUSIVE, RS(−)

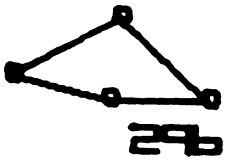
LINK LENGTH RATIO

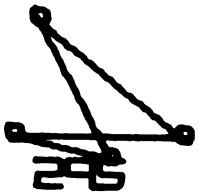
FIGURE 44

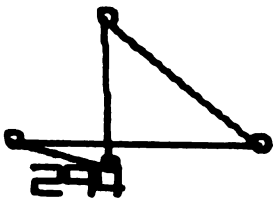
Among the 922 usable mechanisms obtained for generating the function $1/x$ in the range $1 \leq x \leq 2$, there are 190 mechanisms having less than one percent error and having the minimum transmission angle greater than 10 degrees. And among these 190 mechanisms, 46 have the transmission angle in the range of operation always greater than 40 degrees. For mechanisms with less than one percent error, the greatest value for minimum transmission angle in the range of operation was 68 degrees. See NPLOT = 296, p. 70. The smallest error for a mechanism with TAM $>$ 40 degrees was 0.1 percent. See NPLOT = 427, p. 71.

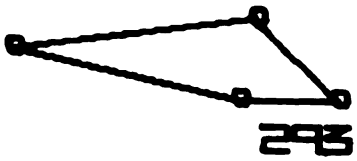
```
TAN LESS THAN 10-DEG. TAN = 8.12419628+000  PTM = 6.11549541-002
P2= 350.00 S2= 140.00 RP=  60.00 RS=  60.00

NPLOT =  295  NGRAS =    1  FP =  4.73738944+000
TAN= 2.08654581+001 PTM= 3.59915873+002 RATIO=1.90672849+000 NSPAC=  45
A=5.85496700-001 B=9.26094332-001 C=1.11638324+000 NEGP= 1 NEGS= 1
X2 =  1.05473545+000  X3 =  1.40392701+000  X4 =  1.89067644+000
P2= 350.00 S2= 140.00 RP=  60.00 RS= -60.00

GROSS ERROR.
P2= 350.00 S2= 320.00 RP=  60.00 RS=  90.00

RATIO EXCEEDS 15. RATIO = 2.27406935+001  LARGE =    1
P2= 350.00 S2= 320.00 RP=  60.00 RS= -60.00

UNABLE TO CLOSE LOOP.
A=1.47220967+000 B=3.9[illegible]-001 C=8.95[illegible]-001 NEGP= 1 NEGS= 2
P2= 350.00 S2= 320.00 RP=  60.00 RS= 120.00

NSPAC = 100.
P2= 350.00 S2= 320.00 RP=  60.00 RS=-120.00

BRANCHING.
A=1.02320520+000 B=3.57233[illegible]-001 C=4.524527[illegible]7-001 NEGP= 1 NEGS= 2
P2= 350.00 S2= 320.00 RP=  60.00 RS= 150.00

NSPAC = 100.
P2= 350.00 S2= 320.00 RP=  60.00 RS=-150.00

NPLOT =  296  NGRAS =    4  FP =  7.81106644-001
TAN= 6.77237327+001 PTM= 1.7[illegible]+002 RATIO=1.23237794+000 NSPAC=  43
A=9.95635440-001 B=1.22699965+000 C=1.17602223+000 NEGP= 2 NEGS= 1
X2 =  1.04159809+000  X3 =  1.36981563+000  X4 =  1.89075209+000
P2= 170.00 S2= 140.00 RP=  90.00 RS=  30.00
```

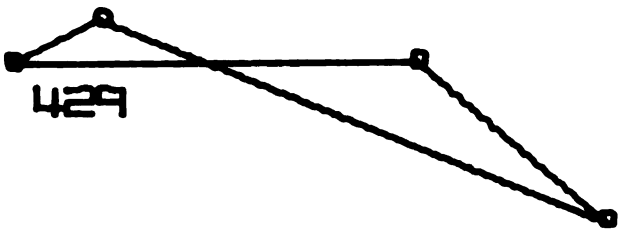
```
UNABLE TO CLOSE LOOP.
A=[illegible]-001 B=7.[illegible]-001 C=1.[illegible]-001 [illegible] 1 [illegible] 2
P2= 30.00 S2= 320.00 RF= 30.00 RS=-150.00

NPLOT = 427 NGRAS = 2 IP = 1.01[illegible]-001
TAN= 4.41100[illegible]+001 PIN= 2.6[illegible]+001 RATIO=2.0[illegible]+000 [illegible] 2
A=2.0[illegible]+000 B=1.[illegible]+000 C=1.[illegible]+000 [illegible] 1 [illegible] 1
X2 = 1.0[illegible]+000 X3 = 1.[illegible]+000 X4 = 1.9[illegible]+000
P2= 30.00 S2= 140.00 RF= 60.00 RS= 30.00

RATIO EXCEEDS 15. RATIO = 1.[illegible]+002 LARGE = 1
P2= 30.00 S2= 320.00 RF= 60.00 RS= -30.00

TAN LESS THAN 10-DEG. TAN = [illegible] PIN = 2.[illegible]
P2= 30.00 S2= 140.00 RF= 60.00 RS= [illegible]

TAN LESS THAN 10-DEG. TAN = [illegible] PIN = 2.[illegible]
P2= 30.00 S2= 320.00 RF= 60.00 RS= [illegible]

BRANCHING.
A=1.10[illegible]+000 B=2.[illegible]+000 C=2.[illegible]+000 [illegible] 1 [illegible] 2
P2= 30.00 S2= 320.00 RF= 60.00 RS= [illegible]

UNABLE TO CLOSE LOOP.
A=3.6570[illegible]-001 B=1.[illegible]+000 C=4.[illegible]-001 [illegible] 1 [illegible] 2
P2= 30.00 S2= 320.00 RF= 60.00 RS= -90.00

UNABLE TO CLOSE LOOP.
A=9.7[illegible]-001 B=1.[illegible]+000 C=[illegible]-001 [illegible] 1 [illegible] 2
P2= [illegible] S2= 320.00 RF= 60.00 RS= 1[illegible].00

UNABLE TO CLOSE LOOP.
A=4.[illegible]77794-001 B=[illegible]+001 C=3.[illegible]-001 [illegible] 1 [illegible] 2
P2= 30.00 S2= 320.00 RF= 60.00 RS= 1[illegible].00
```

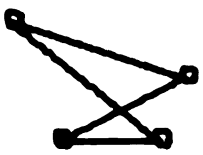


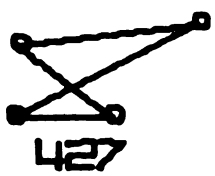


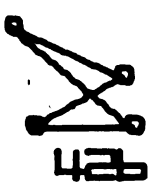


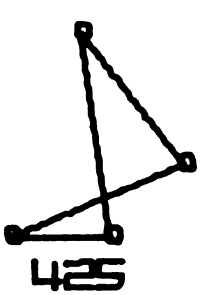



429


428


427


426


425

INTERPRETATION OF RESULTS

Only those results relatively independent of the function being generated were considered significant. Thus, in the Summary of Plots for branching, attention was focused on the dense areas.

However, the Summaries of Plots for transmission angle were clearly separable into those for Functions 2, 3, 5, 6, 7, and 9 and those for Functions 1, 4, and 8, according to similarities in pattern. Note that Functions 2, 3, 5, 6, 7, and 9 have like sign for their first and second derivatives while Functions 1, 4, and 8 have unlike signs for them.

Branching

For RS(+), the diagonal dense band in the Summary of Plots, Fig. 36, shows four-bar mechanisms having P2 = S2 to be quite susceptible to the branching limitation. The Summary of Plots for RS(-), Fig. 37, predicts frequent encounter with the branching limitation when S2 is in the third quadrant (180 to 270 degrees).

Closure

The Summaries of Plots for closure, Fig. 38 and Fig. 39, have enough resemblance to those for branching to suggest a correspondence. Branching is concerned with the two different configurations that are possible with four links after two of the links have been fixed relative to each other. Closure is concerned with the configurations that are impossible after two of the links have been fixed relative to each other. The nature of the correspondence, if there is any, is not clear. But it is clear that choosing P2 and S2 so as to avoid branching will also avoid closure problems.

Transmission Angle

FUNCTIONS HAVING LIKE SIGNS FOR FIRST AND SECOND DERIVATIVES

The Summary of Plots for RS(+), Fig. 40, indicates that S2 should be chosen in the first two quadrants and P2 anywhere but in the first quadrant, or S2 in the third quadrant and P2 in the first or third quadrants to have a good transmission angle.

For RS(-), the Summary of Plots, Fig. 42, condemns the third and fourth quadrants for P2 unless S2 is in the first or fourth quadrant.

FUNCTIONS HAVING UNLIKE SIGNS FOR FIRST AND SECOND DERIVATIVES

The Summary of Plots for RS(+), Fig. 41, shows by the shaded diagonal pattern that equal angles P2 and S2 give a poor transmission angle.

For RS(-), the Summary of Plots, Fig. 43, reveals the second and third quadrants for P2 to have relative freedom from the transmission angle limitation, whatever the value of S2.

Link Length Ratio

For RS(+), no pattern independent of the functions could be seen, so a Summary of Plots was not made. For RS(-), the Summary of Plots, Fig. 44, gives two clear areas and was impossible to ignore.

CONCLUSIONS

In designing a four-bar mechanism for a function generator, if a minimum transmission angle of 40 degrees in the range of operation and a maximum link length ratio of six are assumed, only about 8 percent of the solutions obtainable from a three precision point approximation are usable, exclusive of any consideration of error.

Choice of range angles from 30 degrees to 150 degrees has little influence on whether or not one of the practical limitations is encountered.

Branching is a practical limitation to such linkage design; making the starting angles for input and output links differ will reduce the probability of branching for input and output links rotating in the same direction if the function has positive slope or for input and output links rotating in opposite directions if the function has negative slope. Avoiding the third quadrant for the starting angle of the output link will reduce the probability of branching for input and output links rotating in opposite directions if the function has positive slope or for input and output links rotating in the same direction if the function has negative slope. These same measures will reduce the likelihood of problems with closure at the extremes of the range of operation.

To have good force transmission characteristics in four-bar mechanisms (i.e., to keep the transmission angle greater than 40 degrees in the range of operation), the starting angles for input and output links should be chosen from the clear areas of Fig. 40, 41, 42, or 43, pp. 64 to 67, inclusive, as appropriate.

As long as the starting angles of input and output links are not exactly the same, change points will not be a practical limitation.

No significant generalization can be formed from this work which will aid the linkage designer in avoiding the limitation of excessive link length ratio.

APPENDIX I

Derivation of Equations

THE SYNTHESIS EQUATION

This is the well-known Freudenstein Equation; its derivation (21) is given here primarily for the purpose of defining notation. Refer to Fig. 45, the four-bar mechanism.

$$x_2 = A \cos P \qquad y_2 = A \sin P$$

$$x_3 = C \cos S + F \qquad y_3 = C \sin S$$

By the Pythagorean Law:

$$B^2 = (x_3 - x_2)^2 + (y_3 - y_2)^2$$

$$= (C \cos S + F - A \cos P)^2 + (C \sin S - A \sin P)^2$$

$$= C^2(\sin^2 S + \cos^2 S) + A^2(\sin^2 P + \cos^2 P) + F^2$$

$$- 2AC \sin P \sin S - 2AF \cos P + 2(CF - AC \cos P)\cos S.$$

Rearrangement gives

$$2AC \sin P \sin S + 2(AC \cos P - CF) \cos S =$$

$$= A^2 - B^2 + C^2 + F^2 - 2AF \cos P.$$

And, after division by 2AC,

$$\sin P \sin S + (\cos P - \frac{F}{A})\cos S =$$

$$= \frac{A^2 - B^2 + C^2 + F^2}{2AC} - \frac{F}{C} \cos P \quad \ldots\ldots\ldots\ldots \text{(Eqn. 1)}$$

Using the trigonometric identity for the cosine of the difference of two angles,

$$\cos (P - S) = \cos P \cos S + \sin P \sin S,$$

in (Eqn. 1), it becomes

$$\cos (P-S) = \frac{A^2 - B^2 + C^2 + F^2}{2AC} - \frac{F}{C} \cos P + \frac{F}{A} \cos S, \text{ or}$$

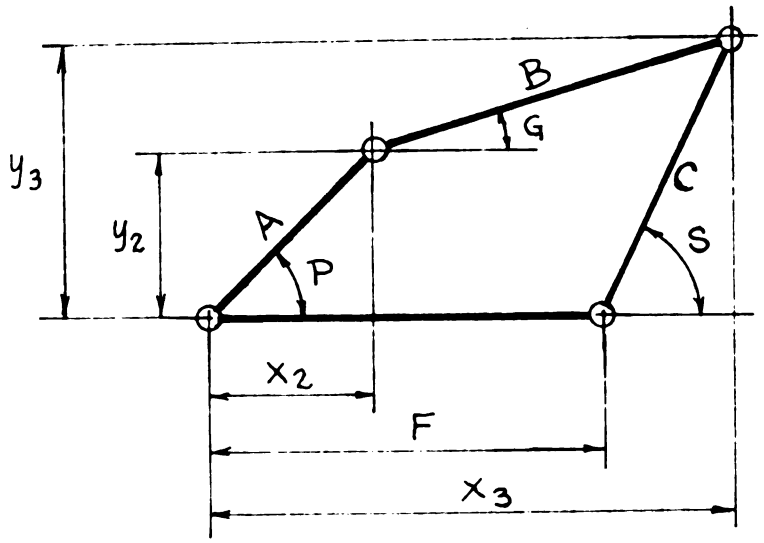


FIGURE 45

$$\frac{F}{A} \text{ Cos } S - \frac{F}{C} \text{ Cos } P + \frac{A^2 - B^2 + C^2 + F^2}{2AC} = \text{Cos } (P - S).$$

Let $H_1 = \frac{F}{A}$, $H_2 = \frac{F}{C}$ and

$$H_3 = \frac{A^2 - B^2 + C^2 + F^2}{2AC}.$$

Substitution in the above equation gives the Freudenstein Equation, or the Synthesis Equation:

$$H_1 \text{ Cos } S - H_2 \text{ Cos } P + H_3 = \text{Cos } (P - S) \ldots\ldots\ldots \quad \text{(Eqn. 2)}$$

This equation must hold whatever the values of P and S, since it was derived from the geometry of the four-bar mechanism.

For a three precision point synthesis, (Eqn. 2) can be written three times, once with each pair of (P_2, S_2), (P_3, S_3) and (P_4, S_4). The result is three equations in three unknowns, H_1, H_2 and H_3. From H_1, H_2 and H_3, the ratios of the lengths of three of the links to the length of the fourth can be determined, e.g., $\frac{A}{F}$, $\frac{C}{F}$ and $\frac{B}{F}$. As used in subroutine FERD3, F = 1.

THE ANALYSIS EQUATION

Once link lengths have been determined, the Analysis Equation is needed to investigate the performance of the mechanism. That is, an equation is needed for S as a function of P. This is obtained from (Eqn. 1), above. To simplify the expression, let

$$Q_1 = \text{Sin } P, \qquad Q_2 = \text{Cos } P - \frac{F}{A} \text{ and}$$

$$Q_3 = \frac{A^2 - B^2 + C^2 + F^2}{2AC} - \frac{F}{C} \text{ Cos } P.$$

Then, (Eqn. 1) becomes

$$Q_1 \text{ Sin } S + Q_2 \text{ Cos } S = Q_3,$$

or, when sine and cosine are expressed in terms of the tangent of the half-angle,

$$Q_1 \frac{2 \operatorname{Tan} \frac{1}{2} S}{1 + \operatorname{Tan}^2 \frac{1}{2} S} + Q_2 \frac{1 - \operatorname{Tan}^2 \frac{1}{2} S}{1 + \operatorname{Tan}^2 \frac{1}{2} S} = Q_3 .$$

Now, this is a quadratic equation in $\operatorname{Tan} \frac{1}{2} S$ giving S as a function of P. After some rearrangement, the Analysis Equation appears:

$$S = 2 \operatorname{Arctan} \frac{Q_1 \pm (Q_1^2 + Q_2^2 - Q_3^2)^{\frac{1}{2}}}{Q_2 + Q_3} \quad \ldots\ldots\ldots \text{(Eqn. 3)}$$

The positive square root belongs to one branch of closure, the negative square root to the other branch.

APPENDIX II

Computer Programs in Fortran

MAIN PROGRAM FOR FINAL STUDY

```
$JOB,331707,FUNGEN,200,DOWELL,HAROLD,L.
$FTN,L,X
      PROGRAM FUNGEN
      DIMENSION X(5),Y(5),P(5),S(5),NA(3),XSA(5),E(105),NAME(10)
 5055 FORMAT(6I4)
 5056 FORMAT(10A8)
 5000 FORMAT(5F9.5)
 5005 FORMAT(23H  COMPLETED THROUGH LP=I3,5H LIS=I3,5H LRP=I3,5H LRS=I3,
     15H NRS=I3,7H NPLOT=I4)
      RADF(Z)=Z*3.14159265/180.
      TIM1=TIMEF(0)
      READ 5055,I1,I2,I3,I4,I5,NPLOT
      NEGP=1 $ NEGS = 1
      CALL COPY
      DO 200 NP=NPLOT,13
      READ 5056,(NAME(I),I=1,10)
      READ 5000,X(1),X(2),X(3),X(4),X(5)
      WRITE TAPE 5,(NAME(I),I=1,10)
 5060 PRINT 5055,I1,I2,I3,I4,I5,NPLOT
      F1=I1-1$F2=I2-1$F3=I3-1$F4=I4-1$F5=I5-1
      P(2)=10.+F1*40.
      DO 5001 J=1,5
 5001 XSA(J)=X(J)
      DO 5050 LP=I1,9
      P(2)=P(2)+40.
      S(2)=20.+F2*40.
      F2=0.
      DO 5050 LIS=I2,9
      I2=1
      S(2)=S(2)+40.
      RP=F3*30.
      F3=0.
      DO 5048 LRP=I3,5
      I3=1
      RP=RP+30.
      RS=F4*30.
      F4=0.
      DO 5048 LRS=I4,5
      I4=1
      RS=RS+30.
      DO 5048 NRS=1,2
      GO TO(5065,5066),I5
 5066 GO TO(5007,5009),NRS
 5065 I5=2 $ NRS=2
      GO TO 5066
 5009 RS=-1.*RS
 5007 NSPAC=0
 5008 CALL FERD3(X,Y,P,S,RP,RS,RX,RY,A,B,C,NEGP,NEGS,LARGE,NP)
      GO TO(5035,5012,5053),LARGE
 5053 IX=1
      WRITE TAPE 5, IX,P(2),S(2),RP,RS
      GO TO 5015
 5035 CALL MAMI(A,B,C,LS,LL,LE1,LE2,LE3,RATIO)
      IF(RATIO-15.)5011,5012,5012
 5012 IX=2
      WRITE TAPE 5, IX,P(2),S(2),RP,RS
      GO TO 5015
```

```
5011 CALL BRANCH(P,S,A,B,C,NS,NSD)
     GO TO (5014,5016,5053),NSD
5016 IX=3
     WRITE TAPE 5, IX,P(2),S(2),RP,RS
     GO TO 5015
5014 P1=P(1) $ P2=P(2) $ S2=S(2) $ X1=X(1) $ Y2=Y(2)
     CALL ERROR(P1,P2,S2,RP,RS,X1,Y2,RX,RY,NS,A,B,C,E,EMAX,XMAX,EP,
    1TAM,PTM,NE,NP)
     GO TO(5021,5018,5053),NE
5018 IX=4
     WRITE TAPE 5, IX,P(2),S(2),RP,RS
     GO TO 5015
5021 IF(TAM-40.)5024,5025,5025
5024 IX=7
     WRITE TAPE 5, IX,P(2),S(2),RP,RS
     GO TO 5015
5025 CALL MAMI(A,B,C,LS,LL,LE1,LE2,LE3,RATIO)
     IF(RATIO-6.)5027,5027,5028
5028 IX=8
     WRITE TAPE 5, IX,P(2),S(2),RP,RS
     GO TO 5015
5027 CALL GRAS(A,B,C,LS,LL,LE1,LE2,LE3,NGRAS)
     GO TO(5015,5015,5015,5015,8,9,10,5015),NGRAS
   8 IX=9
     WRITE TAPE 5, IX,P(2),S(2),RP,RS
     GO TO 5015
   9 IX=10
     WRITE TAPE 5, IX,P(2),S(2),RP,RS
     GO TO 5015
  10 IX=11
     WRITE TAPE 5, IX,P(2),S(2),RP,RS
5015 GO TO(5040,5041),NRS
5041 RS=-1.*RS
5040 GO TO(5042,5043),NEGP
5043 P(2)=P(2)-180.
     NEGP=1
     IF(P(2))5044,5042,5042
5044 P(2)=P(2)+360.
5042 GO TO(5045,5046),NEGS
5046 S(2)=S(2)-180.
     NEGS=1
     IF(S(2))5047,5045,5045
5047 S(2)=S(2)+360.
5045 DO 5049 J=1,5
5049 X(J)=XSA(J)
5048 CONTINUE
5050 CONTINUE
     END FILE 5
     TIM2=TIMEF(0)
     IF(TIM2.GT.TIM1)5062,5002
5062 IF((TIM2-TIM1).GT.5400000.0)5002,200
 200 CONTINUE
5002 PRINT 5005 ,LP,LIS,LRP,LRS,NRS,NP
     END FILE 5
     END FILE 5
     STOP
     END
```

SUBROUTINES

```
      SUBROUTINE FERD3(X,Y,P,S,RP,RS,RX,RY,A,B,C,NEGP,NEGS,LARGE)
      DIMENSION X(5),Y(5),P(5),S(5)
      RADF(Z)=Z*3.14159265/180.
      LARGE=1
      DO 501 J = 1,5
  501 X(J) = X(J) - X(1)
      RP = P(5) - P(1)
      RS = Y(5) - Y(1)
      P(1)=P(2)-RP*(X(2)-X(2))/RX
      S(1)=S(2)+RS*(Y(1)-Y(2))/RY
      P(5)=P(1)+RP
      S(5)=S(1)+RS
      DO 502 J=3,4
      P(J)=P(2)+RP*(X(J)-X(2))/RX
  502 S(J)=S(2)+RS*(Y(J)-Y(2))/RY
      DO 513 J=1,5
      IF (P(J))511,516,512
  511 P(J)=P(J)+360.
      GO TO 516
  512 IF(360.-P(J))513,513,516
  513 P(J)=P(J)-360.
  516 IF(S(J))514,518,515
  514 S(J)=360.+S(J)
      GO TO 515
  515 IF(360.-S(J))517,517,518
  517 S(J)=S(J)-360.
  518 CONTINUE
      W1 = COSF(RADF(P(2))) - COSF(RADF(P(3)))
      W2 = COSF(RADF(S(2))) - COSF(RADF(S(3)))
      W4 = COSF(RADF(P(2))) - COSF(RADF(P(4)))
      W5 = COSF(RADF(S(2))) - COSF(RADF(S(4)))
      W3 = COSF(RADF(P(2)-S(2))) - COSF(RADF(P(3)-S(3)))
      W6 = COSF(RADF(P(2)-S(2))) - COSF(RADF(P(4)-S(4)))
      A=(W1*W5-W2*W4)/(W1*W6-W3*W4)
      IF EXPONENT FAULT 519,520
  520 H1=1./A
      C=(W1*W5-W2*W4)/(W3*W6-W3*W5)
      IF EXPONENT FAULT 519,521
  519 LARGE=2
      RETURN
  521 H2=1./C
      H3=H2*COSF(RADF(P(3)))+COSF(RADF(P(3)-S(3)))-H1*COSF(RADF(S(3)))
      BF=A**2+C**2+1.-2.*A*C*H3
      IF(BF)522,523,523
  522 LARGE=3
      RETURN
  523 B=SQRTF(BF)
      IF (A) 503,504,504
  503 A = -1.* A
      DO 505 J = 1,5
      P(J) = P(J) + 180.
      IF (P(J) - 360.) 505,506,506
  506 P(J) = P(J) - 360.
  505 CONTINUE
      NEGP = 2
  504 IF (C) 507,508,508
```

```
  507 C = -1.*C
      DO 509 J = 1,3
      S(J) = S(J) + 180.
      IF (S(J) - 360.) 509,510,510
  510 S(J) = S(J) - 360.
  509 CONTINUE
      ARGS = 2
  508 RETURN
      END
```

FTN4,30 07/13/65

```
      SUBROUTINE MAMI(A,B,C,LS,LL,LE1,LE2,LE3,RATIO)
      LE1 = 1
      LE2 = 1
      LE3 = 1
      IF (A-B) 501,502,503
  503 IF (A-C) 504,505,506
  506 IF (A-1.)521,508,509
  509 RMAX = A
      LL = 1
      IF (B-C) 510,511,512
  512 IF (C-1.) 513,514,515
  515 RMIN = 1.
      LS = 4
      GO TO 600
  514 LE3 = 2
  513 RMIN = C
      LS = 3
      GO TO 600
  511 LE2 = 2
  510 IF (B-1.)516,517,515
  508 LE1 = 4
      GO TO 521
  517 LE2 = 3
  516 RMIN = B
      LS = 2
      GO TO 600
  502 LE1 = 2
  501 IF (B-C) 518,519,520
  520 IF (B-1.) 521,522,523
  523 RMAX = B
      LL = 2
      IF (A-C) 524,525,526
  526 IF (C-1.) 513,514,515
  525 LE1 = 3
  524 IF (A-1.) 527,528,515
  528 LE1 = 4
  527 RMIN = A
      LS = 1
      GO TO 600
  522 LE2 = 3
  521 RMAX = 1.
      LL = 4
      GO TO 534
  519 LE2 = 2
  518 IF (C-1.) 521,530,529
  529 RMAX = C
      LL = 3
      IF (A-B) 531,532,533
  533 IF (B-1.) 516,517,515
  532 LE1 = 2
  531 IF (A-1.) 527,528,515
  530 LE3 = 2
      GO TO 521
  534 IF (B-C) 535,536,537
  537 IF (C-A) 513,537,527
  535 LE1 = 3
```

```
      GO TO 513
  536 LE2 = 2
  535 IF (B-A) 516,538,527
  538 LE1 =2
      GO TO 516
  505 LE1 = 3
  504 IF (C-1.) 521,530,529
  600 RATIO = [illegible]
      GO TO(541,542,542,542),LE1
  541 DIF=A-B
      IF(DIF)550,551,552
  550 IF(.001+DIF)553,553,551
  551 LE1=2
      GO TO 553
  552 IF(.001-DIF)553,553,551
  553 DIF=A-C
      IF(DIF)554,555,556
  554 IF(.001+DIF)557,557,555
  555 LE1=3
      GO TO 557
  556 IF(.001-DIF)557,557,555
  557 DIF=A-1.
      IF(DIF)558,559,560
  558 IF(.001+DIF)542,542,559
  559 LE1=1
      GO TO 542
  560 IF(.001-DIF)542,542,559
  542 GO TO (561,543,543),LE2
  561 DIF=B-C
      IF(DIF)562,563,564
  562 IF(.001+DIF)565,565,563
  563 LE2=2
      GO TO 565
  564 IF(.001-DIF)565,565,563
  565 DIF=B-1.
      IF(DIF)566,567,568
  566 IF(.001+DIF)543,543,567
  567 LE2=3
      GO TO 543
  568 IF(.001-DIF)543,543,567
  543 GO TO (569,540),LE3
  569 DIF=C-1.
      IF(DIF)570,571,572
  570 IF(.001+DIF)540,540,571
  571 LE3=2
      GO TO 540
  572 IF(.001-DIF)540,540,571
  540 RETURN
      END
```

FTN4,30 07/13/65

```
      SUBROUTINE BRANCH(P,S,A,B,C,NS,NSD)
      DIMENSION P(5),S(5),SSP(4),SSM(4)
      RADF(Z)=Z*3.14159265/180.
      NS=0
      DO 4002 J=2,4
      Q1=SINF(RADF(P(J)))
      Q2=COSF(RADF(P(J)))-1./A
      Q3=(A**2+C**2+1.-B**2)/(2.*A*C)-COSF(RADF(P(J)))/C
      RF=Q1**2+Q2**2-Q3**2
      IF(RF)4001,4013,4013
 4001 NSD=3
      RETURN
 4013 R=SQRTF(RF)
      SNUMP=Q1*R
      SNUM=Q1-R
      SDEN=Q2+Q3
      CALL ATN (SNUMP,SDEN,HALFS)
      SSP(J)=2.*HALFS
      IF(SSP(J)-359.)4011,4012,4012
 4012 SSP(J)=SSP(J)-360.
 4011 CALL ATN(SNUM,SDEN,HALFS)
      SSM(J)=2.*HALFS
      IF(SSM(J)-359.)4002,4014,4014
 4014 SSM(J)=SSM(J)-360.
 4002 CONTINUE
      DO 4003 J=2,4
      SD=SSP(J)-S(J)
      IF(SD)4004,4003,4005
 4004 IF(3.+SD)4006,4003,4003
 4005 IF(3.-SD)4006,4003,4003
 4003 CONTINUE
      NS=1
      NSD=1
      RETURN
 4006 DO 4007 J=2,4
      SD=SSM(J)-S(J)
      IF(SD)4008,4007,4009
 4008 IF(3.+SD)4010,4007,4007
 4009 IF(3.-SD)4010,4007,4007
 4007 CONTINUE
      NS=2
      NSD=1
      RETURN
 4010 NSD=2
      RETURN
      END
```

```
      SUBROUTINE ERROR(P1,P2,S2,RP,RS,X1,Y2,RX,RY,NS,A,B,C,E,EMAX,XMAX,
     1EP,TV1,PTM,NE)
      DIMENSION E(105)
      RADF(Z)=Z*3.14159265/180.
      NE=1
      TAM = 90.
      EMAX = 0.
      DO 6001 J=1,101
      W=J-1
      PE=P1+W*RP/100.
      IF(PE)6030,6035,6031
 6030 IF(360.+PE)6032,6032,6034
 6032 PE=720.+PE
      GO TO 6035
 6031 IF(360.-PE)6033,6033,6035
 6033 PE=PE-360.
      GO TO 6035
 6034 PE=360.+PE
 6035 Q1=SIN(RADF(PE))
      Q2=COS(RADF(PE))-1./A
      QA3=(1./C)*COS(RADF(PE))
      Q3=(1.**2-B**2+Q**2+1.)/(2.*A*C)-QA3
      RF=Q1**2+Q2**2-Q3**2
      IF(RF)6012,6023,6023
 6012 NE=3
      RETURN
 6023 R=SQRTF(RF)
      GO TO(6002,6003),NS
 6002 SNUM = Q1+R
      GO TO 6004
 6003 SNUM = Q1-R
 6004 SDEN = Q2 + Q3
      CALL ATG (SNUM,SDEN,HALFS)
      SE=2.*HALFS
      IF(SE-360.)6040,6054,6054
 6054 SE=SE-360.
 6040 ST=SE-S2
      IF(ST)6055,6056,6056
 6055 IF(180.+ST)6057,6057,6053
 6056 IF(180.-ST)6057,6057,6053
 6057 IF(RP)6026,6026,6027
 6027 IF(RS)6029,6029,6028
 6026 IF(RS)6028,6028,6029
 6028 IF(PE-P2)6053,6037,6037
 6029 IF(PE-P2)6037,6036,6036
 6036 IF(ST)6053,6053,6039
 6037 IF(ST)6038,6053,6053
 6038 ST=ST+360.
      GO TO 6053
 6039 ST=ST-360.
 6053 YE=(RY/RS)*ST+Y2
      XA=X1+W*RX/100.
      YA=YFUN(XA)
      E(J)=YE-YA
      IF(E(J))6006,6007,6005
 6005 IF(EMAX-E(J))6008,6007,6007
```

```
      6008 EMAX=E(J)
           XMAX=XA
           GO TO 6007
      6006 IF(EMAX+E(J))6009,6007,6007
      6009 EMAX=-1.*E(J)
           XMAX=XA
      6007 IF(RY)6041,6041,6042
      6041 IF(EMAX+RY)6044,6010,6010
      6042 IF(EMAX-RY)6044,6010,6010
      6010 NE=2
           RETURN
      6044 GNUM=C*SINF(RADF(SE))-A*SINF(RADF(PE))
           GDEN=C*COSF(RADF(SE))+1.-A*COSF(RADF(PE))
           CALL ATQ (GNUM,GDEN,G)
           T=SE-G
           IF(T)6013,6014,6014
      6013 T=360.+T
      6014 IF(T-90.)6015,6015,6016
      6015 TA=T
           GO TO 6017
      6016 IF(T-180.)6018,6018,6019
      6018 TA=180.-T
           GO TO 6017
      6019 IF(T-270.)6020,6020,6021
      6020 TA=T-180.
           GO TO 6017
      6021 TA=360.-T
      6017 IF(TAM-TA)6001,6001,6022
      6022 TAM=TA
           PTM=PE
      6001 CONTINUE
           IF(RY)6043,6011,6011
      6043 EP=-100.*EMAX/RY
           RETURN
      6011 EP=100.*EMAX/RY
           RETURN
           END
```

FIN4.30 07/13/65

```
      SUBROUTINE GRAS(A,B,C,LS,LL,LE1,LE2,LE3,NGRAS)
      GO TO(8009,8002,8003,8004),LE1
 8003 GO TO(8009,8002,8005),LE2
 8002 GO TO(8009,8006),LE3
 8004 GO TO(8009,8006,8009),LE2
 8009 GO TO(8010,8011,8012,8013),LL
 8010 GO TO(8030,8014,8015,8016),LS
 8030 PUNCH 8031
 8031 FORMAT(16H   FAULT IN GRAS)
      NGRAS=8
      RETURN
 8011 GO TO(8014,8030,8017,8018),LS
 8012 GO TO(8015,8017,8030,8019),LS
 8013 GO TO(8016,8018,8019,8030),LS
 8014 XX=A+B
      YY=1.+C
      IF(XX-YY)8020,8025,8022
 8015 XX=A+C
      YY=1.+B
      IF(XX-YY)8020,8025,8022
 8016 XX=1.+A
      YY=B+C
      IF(XX-YY)8020,8025,8022
 8017 XX=A+C
      YY=1.+A
      IF(XX-YY)8020,8025,8022
 8018 XX=1.+B
      YY=A+C
      IF(XX-YY)8020,8025,8022
 8019 XX=1.+C
      YY=A+B
      IF(XX-YY)8020,8025,8022
 8020 GO TO(8021,8022,8023,8024),LS
 8021 NGRAS=1
      RETURN
 8024 NGRAS=2
      RETURN
 8023 NGRAS=3
      RETURN
 8022 NGRAS=4
      RETURN
 8025 NGRAS=5
      RETURN
 8006 NGRAS=6
      RETURN
 8005 NGRAS=7
      RETURN
      END
C AIO SUBROUTINE
```

FTN4,30 07/13/65

```
      SUBROUTINE ATO (Y,X,V)
      PI = 3.14159265
      ZERO=.000000001
      IF (Y) 501,502,503
  501 IF(Y+ZERO)513,502,502
  502 IF (X) 511,512,512
  511 V = 180.
      GO TO 510
  503 IF(Y-ZERO)502,502,514
  513 IF(X)515,505,516
  505 V = 270.
      GO TO 510
  515 IF(X+ZERO)504,505,505
  516 IF(X-ZERO)505,505,506
  504 V = 180.+ ATANF(Y/X)*180./PI
      GO TO 510
  506 V = 360.-ATANF (-Y/X)*180./PI
      GO TO 510
  514 IF(X)517,508,518
  508 V = 90.
      GO TO 510
  517 IF(X+ZERO)507,508,508
  518 IF(X-ZERO)508,508,509
  507 V = 180.-ATANF(-Y/X)*180./PI
      GO TO 510
  509 V = ATANF (Y/X)* 180./PI
      GO TO 510
  512 V = 0.
  510 RETURN
      END
```

FTN4,30 07/13/65

```
      SUBROUTINE SPACE(X,E,XMAX,RX,RY,NOPT,NSPAC)
      DIMENSION X(5),E(105),EM(4),ZA(105),ZM(4),Z(5)
      NOPT=1
      M1=1 $ E(102)=0. $ E(103)=0. $ E(104)=0. $ E(105)=0.
      Z(1)=1. $ Z(5)=2. $ ZMAX=1.+(XMAX-X(1))/RX
      Z(2)=1.+(X(2)-X(1))/RX $ Z(3)=1.+(X(3)-X(1))/RX
      Z(4)=1.+(X(4)-X(1))/RX
      DO 7001 J=1,4
      RMX=Z(J+1)-Z(J)
      VA=100.*RMX
      M=VA+.5
      EM(J)=0.
      GO TO (7003,7003,7003,7019),J
 7019 M2=101
      GO TO 7020
 7003 M2=M1+M
 7020 DO 7008 K=M1,M2
      MA=K-M1
      ZA(K)=Z(J)+MA/100.
      IF(E(K))7005,7006,7006
 7006 IF(EM(J)-E(K))7007,7008,7008
 7007 EM(J)=E(K)
      ZM(J)=ZA(K)
      GO TO 7008
 7005 IF(EM(J)+E(K))7009,7008,7006
 7009 EM(J)=-1.*E(K)
      ZM(J)=ZA(K)
 7008 CONTINUE
 7001 M1=M2
      TOL=.001*RY
      IF(TOL)7022,7023,7023
 7022 TOL=-1.*TOL
 7023 DO 7004 J=1,3
      ETOL=EM(J)-EM(J+1)
      IF(ETOL)7010,7004,7011
 7010 IF(TOL+ETOL)7012,7012,7004
 7011 IF(TOL-ETOL)7012,7012,7004
 7004 CONTINUE
      NOPT=2
      RETURN
 7012 DO 7013 J=2,4
      IF(ZMAX-Z(J))7014,7015,7013
 7013 CONTINUE
      GO TO 7017
 7014 DOM=EM(1)+EM(2)+EM(3)+EM(4)
      GO TO (7016,7016,7018,7018),J
 7016 CHEK=EM(1)/DOM-.25
      IF(CHEK)7021,7024,7024
 7024 STEP=SQRT(CHEK)
      Z(2)=Z(2)-STEP*(Z(2)-Z(1))
      X(2)=X(1)+(Z(2)-1.)*RX
 7015 NSPAC = NSPAC+1
      RETURN
 7018 ZS=Z(J) $ ZS1=Z(J-1)
      STEP=EM(J-1)/DOM-.25
      PROP=ZMAX-ZS1
```

FTN4,30 07/13/65

```
      Z(J)=ZS-STEP*PROP
      X(J)=X(1)+(Z(J)-1.)*RX
      PROP=ZS-ZMAX
      Z(J-1)=ZS1+STEP*PROP
      X(J-1)=X(1)+(Z(J-1)-1.)*RX
      NSPAC=NSPAC+1
      RETURN
 7017 DOM=EM(1)+EM(2)+EM(3)+EM(4)
      CHEK=EM(4)/DOM-.25
      IF(CHEK)7021,7025,7025
 7025 STEP=SQRT(CHEK)
      Z(4)=Z(4)+STEP*(Z(5)-Z(4))
      X(4)=X(1)+(Z(4)-1.)*RX
      NSPAC=NSPAC+1
      RETURN
 7021 NOPT=3
      RETURN
      END
```

FTN4.30 07/13/65

```
      SUBROUTINE SCALE (A,B,C,LS,P,G,X2,X3,X4,Y2,Y3)
      RADF(X) = X*3.14159265/180.
      GO TO (501,502,503,504),LS
  501 AP = 1.
      BP = B/A
      CP = C/A
      X4 = 1./A
      GO TO 505
  502 BP = 1.
      AP = A/B
      CP = C/B
      X4 = 1./B
      GO TO 505
  503 CP = 1.
      AP = A/C
      BP = B/C
      X4 = 1./C
      GO TO 505
  504 AP = A
      BP = B
      CP = C
      X4 = 1.
  505 X2 = AP *COSF(RADF(P))
      X3 = X2 + BP *COSF(RADF(G))
      Y2 = AP *SINF(RADF(P))
      Y3 = Y2 + BP *SINF(RADF(G))
      RETURN
      END
```

MAIN PROGRAM FOR PRELIMINARY STUDY

```
      PROGRAM FUNGEN
      DIMENSION X(5),Y(5),P(5),S(5),NA(3),XSA(5),E(105),NAME(10)
 5056 FORMAT(10A8)
 5000 FORMAT(5F9.5)
 5051 FORMAT(25H  FUNCTION Y = 1/X, X1 = E15.8,7H  X5 = E15.8/)
 5054 FORMAT(18H  DF NEG IN FERDS.)
 5010 FORMAT(23H  UNABLE TO CLOSE LOOP.)
 5013 FORMAT(23H  RATIO EXCEEDS 15, RATIO = ,E14.8,10H  LARGE = ,I4)
 5017 FORMAT(12H  BRANCHING.)
 5019 FORMAT(14H  GROSS ERROR.)
 5052 FORMAT(20H  CHEK NEG IN SPACE.)
 5023 FORMAT(14H  NSPAC = 100.)
 5026 FORMAT(30H  TAM LESS THAN 10-NEG. TAM = ,E14.8,8H  PTM = ,E14.8)
 5029 FORMAT(27H  RATIO EXCEEDS 6, RATIO = ,E15.8)
 5030 FORMAT(15H  CHANGE POINTS.)
 5031 FORMAT(10H  DELTOID.)
 5032 FORMAT(15H  PARALLELOGRAM.)
 5034 FORMAT(10H  NPLOT = I4,10H  NGRAS = I4,7H  EP = E15.8)
 5036 FORMAT(6H  TAM=E15.8,5H PTM=E15.8,7H RATIO=E14.8,7H NSPAC=I4)
 5037 FORMAT(4H  A=E14.8,3H B=E14.8,3H C=E14.8,6H NEGP=I2,6H NEGS=I2)
 5038 FORMAT(7H  X2 = ,E15.8,7H  X3 = ,E15.8,7H  X4 = ,E15.8)
 5039 FORMAT(5H  P2=F7.2,4H S2=F7.2,4H RP=F7.2,4H RS=F7.2/)
 5005 FORMAT(25H  COMPLETED THROUGH LP=I3,5H LIS=I3,5H LRP=I3,5H LRS=I3,
     15H NRS=I3,7H NPLOT=I4)
      RADF(Z)=Z*3.14159265/180.
      TIM1=TIMEF(3)
      NEGP=1 $ NEGS = 1
      READ 5056,(NAME(I),I=1,10)
      READ 5000,X(1),X(2),X(3),X(4),X(5)
      PRINT 5000,X(1),X(2),X(3),X(4),X(5)
      READ 5055,I1,I2,I3,I4,I5,NPLOT
 5055 FORMAT(6I4)
      IF(I1.EQ.1.AND.I2.EQ.1.AND.I3.EQ.1.AND.I4.EQ.1.AND.I5.EQ.2)5057,
     15058
 5057 KS=2
      GO TO 5059
 5058 KS=1
 5059 KS=KLOKER(KS)
      GO TO (5060,5061),KS
 5061 WRITE TAPE 5,(NAME(I),I=1,10)
      WRITE TAPE 6,(NAME(I),I=1,10)
 5060 PRINT 5055,I1,I2,I3,I4,I5,NPLOT
      PRINT 5051,X(1),X(5)
      F1=I1-1$F2=I2-1$F3=I3-1$F4=I4-1$F5=I5-1
      P(2)=10.+F1*40.
      DO 5001 J=1,5
 5001 XSA(J)=X(J)
      DO 5050 LP=I1,9
      P(2)=P(2)+40.
      S(2)=20.+F2*40.
      F2=0.
      DO 5050 LIS=I2,9
      I2=1
      S(2)=S(2)+40.
      RP=F3*30..
      F3=0.
```

```
      DO 5048 LRP=I5,5
      I5=1
      RP=RP+30.
      RS=F4*30.
      F4=0.
      DO 5048 LRS=I4,5
      I4=1
      RS=RS+30.
      DO 5048 NRS=1,2
      GO TO(5065,5066),I5
 5066 GO TO(5007,5009),NRS
 5065 I5=2 $ NRS=2
      GO TO 5066
 5009 RS=-1.*RS
 5007 NSPAC=0
 5008 CALL FERD3(X,Y,P,S,RP,RS,RX,RY,A,B,C,NEGP,NEGS,LARGE)
      GO TO(5035,5012,5053),LARGE
 5053 PRINT 5054
 5006 PRINT 5010
      IX=1
      WRITE TAPE 5, IX,P(2),S(2),RP,RS
      PRINT 5037,A,B,C,NEGP,NEGS
      GO TO 5015
 5035 CALL MAMI(A,B,C,LS,LL,LE1,LE2,LE3,RATIO)
      IF(RATIO-15.)5011,5012,5012
 5012 PRINT 5013,RATIO,LARGE
      IX=2
      WRITE TAPE 5, IX,P(2),S(2),RP,RS
      GO TO 5015
 5011 CALL BRANCH(P,S,A,B,C,NS,NSD)
      GO TO (5014,5016,5006),NSD
 5016 PRINT 5017
      IX=3
      WRITE TAPE 5, IX,P(2),S(2),RP,RS
      PRINT 5037,A,B,C,NEGP,NEGS
      GO TO 5015
 5014 P1=P(1) $ P2=P(2) $ S2=S(2) $ X1=X(1) $ Y2=Y(2)
      CALL ERROR(P1,P2,S2,RP,RS,X1,Y2,RX,RY,NS,A,B,C,E,EMAX,XMAX,EP,
     1TAM,PTM,NE)
      GO TO(5003,5018,5006),NE
 5018 PRINT 5019
      IX=4
      WRITE TAPE 5, IX,P(2),S(2),RP,RS
      GO TO 5015
 5003 CALL SPACE(X,E,XMAX,RX,RY,NOPT,NSPAC)
      GO TO(5020,5021,5033),NOPT
 5033 PRINT 5052
      IX=5
      WRITE TAPE 5, IX,P(2),S(2),RP,RS
      GO TO 5015
 5020 IF(NSPAC-100)5008,5022,5022
 5022 PRINT 5023
      IX=6
      WRITE TAPE 5, IX,P(2),S(2),RP,RS
      GO TO 5015
 5021 IF(TAM-10.)5024,5025,5025
```

```
 5024 PRINT 5026,TAM,PTM
      IX=7
      WRITE TAPE 5, IX,P(2),S(2),RP,RS
      GO TO 5015
 5025 CALL MAMI(A,B,C,LS,LL,LE1,LE2,LE3,RATIO)
      IF(RATIO-6.)5027,5027,5028
 5028 PRINT 5029, RATIO
      IX=8
      WRITE TAPE 5, IX,P(2),S(2),RP,RS
      GO TO 5015
 5027 CALL GRAS(A,B,C,LS,LL,LE1,LE2,LE3,NGRAS)
      GO TO(7,7,7,7,8,9,10,5015),NGRAS
    8 PRINT 5030
      IX=9
      WRITE TAPE 5, IX,P(2),S(2),RP,RS
      GO TO 5015
    9 PRINT 5031
      IX=10
      WRITE TAPE 5, IX,P(2),S(2),RP,RS
      GO TO 5015
   10 PRINT 5032
      IX=11
      WRITE TAPE 5, IX,P(2),S(2),RP,RS
      GO TO 5015
    7 TY=C*SINF(RADF(S(2)))-A*SINF(RADF(P(2)))
      TX=C*COSF(RADF(S(2)))-A*COSF(RADF(P(2)))+1.
      CALL ATG(TY,TX,G)
      CALL SCALE(A,B,C,LS,P,G,X2,X3,X4,Y2,Y3)
      NPLOT=NPLOT+1
      CALL PLOT(0.,-11.,2,100.,100.)
      CALL PLOT(0.,0.,0,50.,50.)
      CALL PLOT(0.,10.,2,50.,50.)
      CALL PLOT(0.,0.,0,50.,50.)
      CALL CHAR(-.032,-.032,1HO,1,0.,.064,.064)
      CALL PLOT(0.,0.,2,50.,50.)
      CALL PLOT(Y2,X2,1,50.,50.)
      YP=Y2-.032 $ XP=X2-.032
      CALL CHAR(YP,XP,1HO,1,0.,.064,.064)
      CALL PLOT(Y2,X2,2,50.,50.)
      CALL PLOT(Y3,X3,1,50.,50.)
      YP=Y3-.032 $ XP=X3-.032
      CALL CHAR(YP,XP,1HO,1,0.,.064,.064)
      CALL PLOT(Y3,X3,2,50.,50.)
      CALL PLOT(0.,X4,1,50.,50.)
      YP=-.032 $ XP=X4-.032
      CALL CHAR(YP,XP,1HO,1,0.,.064,.064)
      CALL PLOT(0.,X4,2,50.,50.)
      CALL PLOT(0.,0.,1,50.,50.)
      NOP=NPLOT
      DO 100 J=1,3
      NA(J)=NOP-NOP/10*10
  100 NOP=NOP/10
      NPLO=((NA(3)*64+NA(2))*64+NA(1))*1073741824
      YP=-.5 $ XP=.25
      CALL CHAR(YP,XP,NPLO,3,0.,.125,.125)
      CALL PLOT(4.,0.,2,50.,50.)
```

```
      CALL PLOT (0.,0.,-1,100.,100.)
      WRITE TAPE 6,NPLOT,NGRAS,EP,TAM,PTM,RATIO,NSPAC,A,B,C,NEGP,NEGS,
     1X(2),X(3),X(4),P(2),S(2),RP,RS
      PRINT 5034,NPLOT,NGRAS,EP
      PRINT 5036,TAM,PTM,RATIO,NSPAC
      PRINT 5037,A,B,C,NEGP,NEGS
      PRINT 5038,X(2),X(3),X(4)
 5015 PRINT 5039,P(2),S(2),RP,RS
      TIM2=TIMEF(0)
      IF(TIM2.GT.TIM1)5062,5002
 5062 IF((TIM2-TIM1).GT.7200000.0)5002,5004
 5004 GO TO(5040,5041),NRS
 5041 RS=-1.*RS
 5040 GO TO(5042,5043),NEGP
 5043 P(2)=P(2)-180.
      NEGP=1
      IF(P(2))5044,5142,5042
 5044 P(2)=P(2)+360.
 5042 GO TO(5045,5046),NEGS
 5046 S(2)=S(2)-180.
      NEGS=1
      IF(S(2))5047,5145,5045
 5047 S(2)=S(2)+360.
 5045 DO 5049 J=1,5
 5049 X(J)=XSA(J)
 5048 CONTINUE
 5050 CONTINUE
 5002 PRINT 5005,LP,LIS,LRP,LRS,NRS,NPLOT
      END FILE 5
      END FILE 5
      END FILE 6
      END FILE 6
      STOP
      END
```

LIST OF REFERENCES

1. Svoboda, Antonín, Computing Mechanisms and Linkages, Dover Publications, New York, 1965. (First published by McGraw-Hill Book Co., Inc., 1948.)
2. Hrones, John A., and George L. Nelson, Analysis of the Four-bar Linkage, published jointly by Technology Press, M.I.T., Cambridge, Mass., and John Wiley & Sons, Inc., New York, 1951.
3. Hain, Kurt, Angewandte Getriebelehre, Hermann Schroedel Verlag K.G., Hannover, 1952.
4. Nickson, Philip T., "A Simplified Approach to Linkage Design", Trans. 1st Conf. on Mechanisms, Penton Pub. Co., Cleveland, 1953.
5. Schaffer, B. W. and I.Cochin, "Synthesis of Four Bar Mechanisms When the Position of Two Members is Prescribed", Trans. ASME, vol. 76, 1954.
6. Freudenstein, F., "An Analytical Approach to the Design of Four-Link Mechanisms", Trans. ASME, vol. 76, 1954.
7. Freudenstein, Ferdinand, "Approximate Synthesis of Four-Bar Linkages, ASME Paper No. 54-F-14, Trans. ASME, vol. 77, 1955.
8. Sieker, K. H. "Zur algebraischen Mass-Synthese ebener Kurbelgetriebe", Ingenieur-Archiv., vol. 24, Part I, pp. 188-215, 1956.
9. Allen,C. Wesley, "Point-Position-Reduction," Trans. 5th Conf. on Mechanisms, Penton Pub. Co., Cleveland, 1958.
10. Freudenstein, Ferdinand, "Four-Bar Function Generators", Trans. 5th Conf. on Mechanisms, Penton Pub. Co., Cleveland, 1958.
11. Michalec, George W., "Mechanical Analog Computer Components", Trans. 5th Conf. on Mechanisms, Penton Pub. Co., Cleveland, 1958.

12. Worthley, W. W. and R. T. Hinkle, "Four-Bar Linkages, Approximate Synthesis", ASME Paper No. 58-A-130, 1958.

13. Hahn, Edward E., "Series Design of Four-Bar Mechanisms", Trans. 6th Conf. on Mechanisms, Penton Pub. Co., Cleveland, 1960.

14. Vidosic, J. P. and H. L. Johnson, "Synthesis of Four-Bar Function Generators", Trans. 6th Conf. on Mechanisms, Penton Pub. Co., Cleveland, 1960.

15. Dunk, A. C. and C. L. Hanson, "Six-Bar Linkages", Trans. 6th Conf. on Mechanisms, Penton Pub. Co., Cleveland, 1960.

16. Hinkle, R. T., Kinematics of Machines, 2nd ed., Prentice-Hall, Inc., Englewood Cliffs, N. J., 1960.

17. McLarnan, C. W., "Design Equations for Four-Bar Function Generators", Trans. 7th Conf. on Mechanisms, Penton Pub. Co., Cleveland, 1962.

18. McLarnan, Charles W. and C. Tarver Hagan, "Synthesis of Four-Bar Function Generators with Five Constraints, ASME Paper No. 63-WA-200, New York, 1963.

19. McLarnan, Charles W., Equations for Four-Bar Linkage Function Generators, Engrg. Expt. Sta. Bull. 197, University Publications Sales, Ohio State University, Columbus, Ohio, 43210, (Not dated but apparently published in 1964).

20. Hirschhorn, Jeremy, Kinematics and Dynamics of Plane Mechanisms, McGraw-Hill Book Co., Inc. New York, 1962.

21. Hartenberg, Richard S. and Jecques Denavit, Kinematic Synthesis of Linkages, McGraw-Hill Book Co., New York, 1964.

22. Keller, Robert E., "Mechanism Design by Electronic Analog Computer Trans. 7th Conf. on Mechanisms, Penton Pub. Co., Cleveland, 1962.

MICHIGAN STATE UNIVERSITY LIBRARIES
3 1293 03071 3550