A NEW CRITERION FOR SATISFACTORY COMMUTATION AND A DIGITAL COMPUTER PROGRAM FOR DESIGN

Thesis for the Degree of Ph. D.

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Hiremaglur Krishnaswamy Kesavan

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This is to certify that the

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A NEW CRITERION FOR SATISFACTORY COMMUTATION AND A DIGITAL COMPUTER PROGRAM FOR DESIGN

By

Hiremaglur Krishnaswamy Kesavan

A THESIS

Submitted to the School for Advanced Graduate Studies
of Michigan State University of Agriculture and
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Department of Electrical Engineering

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An Abstract

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ABSTRACT

The manifold applications of commutator-type machines in control systems have placed exacting requirements on design. The chief difficulty encountered in this regard is the problem of commutation itself. The mechanical switch, for which there is no practical substitution at the present, places several limitations on the terminal characteristics that can be realized. The 'electrical' aspects of the mechanical switching problem have long proven formidable. It is generally recognized that new concepts must be developed before a satisfactory solution could be obtained for this old and difficult problem.

In recent years, the significant advances in the areas of network theory and the digital computer have had a direct impact on the study of commutation. It has not only resulted in the new formulation techniques of the problem but also has profoundly changed the concept of machine design.

In chapter I a brief survey of the several aspects of the problem is presented. The electrical aspects of the problem are separated from the rest and are considered under two categories. First, the inherent characteristics of the armature winding itself as they effect commutation. Second, the effect on commutation of the coupling between the fields and the armature circuits.

In chapter II a concise survey is made of the reference [6] , for purposes of laying the mathematical foundations for both aspects of the problem. The background is set for the consideration of the problem of the thesis.

The problem of the thesis is defined in chapter III. A criterion is

evolved which could serve as a basis for studying the relative commutating abilities of the various armature windings operating in the same frame and under the same terminal conditions. This criterion is referred to as the 'commutation factor'. By making use of the commutation factor, a reactance voltage is calculated which serves as a basis for comparing the commutating ability of various armature windings operating on different frames.

In chapter IV the basis for calculating the inductances which form the starting point for the calculation of the commutation factor is considered. New analytical expressions for the inductances are presented and some convenient terms defined. Also, a numerical method suited for the digital computer is given for the determination of the exact locations of the coils undergoing short-circuit.

Chapter V presents a complete digital computer program for the commutation design of the commutator-type machine.

In chapter VI the effects of varying several of the design parameters on the commutation factor are discussed. The changes in parameters necessary for realizing the desired commutation factor are discussed with reference to a flow-diagram of the computer program. The results of calculations on forty machines are presented in the form of a table wherein the commutation factor is tabulated against several pertinent design parameters. From this table, it is concluded that when two windings are compared on the same frame, a lower value of the commutation factor will result in better commutation.

The matrix notation is used extensively throughout the thesis.

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The author is grateful to Dr. H. E. Koenig, his thesis advisor, for his constant guidance in preparing this thesis. His fundamental work in the area of commutation study forms the background on which much of this thesis is based.

The author wishes to express his thanks to Dr. M. B. Reed and Dr. L. W. Von Tersch for their constant encouragement. Thanks are also due to the Reliance Electric and Engineering Company and to Mr. Lanier Greer, the chief D.C. Engineer in particular, for their cooperation and support.

CONTENTS

I. INTRODUCTION

The rapid growth of the control systems area has greatly enhanced the importance of the commutator type machine. This electro-mechanical component, by virtue of its unique characteristics, renders it a very versatile component, suitable for application in a wide area of control systems. However, the chief obstacle in its design is the problem of commutation itself. The mechanical switch used to realize the terminal characteristics presents several problems, many of which are not 'electrical' in nature. In a recent paper, Tur[1] points out, like many others on the subject, that although the differential equations of the problem can be obtained, these equations can not be solved analytically, as the voltage in a sliding contact depends upon a number of factors, eg., mechanical, thermal and chemical processes in the contact layer.

Wilhite[2] has studied in detail the effect of the quality of the carbon-brush on commutation. Korecki[3] has established criteria useful in selecting commutator bar material and design. In a recent paper, Hindmarsh[4] discusses the mechanical limitations of commutation.

Experimental investigations of commutation are difficult because of inadequate techniques of accurately measuring the factors effecting commutation. Thielers[5] points out that some machines which are found good commutation-wise in the first few days deteriorate with time while others improve with time.

In spite of all the many difficulties associated with commutator design, the compelling fact remains that there is no substitute at the present for the mechanical switch. With recent developments in transistors there is perhaps renewed hope in the possibility of replacing the mechanical switching system by one devoid of moving part. However, with the present over-voltage limitations on transistors, the problem of minimizing the terminal voltages of the armature circuits during the switching interval is expected to be as critical for a transistorized commutator as it is for the mechanical commutator. Moreover, the problem of replacing the commutating machine by some other device, however desirable, is truly a problem in invention and hence belongs to the realm of speculation.

Although it is generally recognized that the design and construction of mechanical commutators and the associated armature circuitry is a highly developed art, it is also generally recognized that a substantial improvement in the mathematical description of the switching system is essential to the design of commutator machines suitable for application to fast response control systems. In this thesis, attention is confined only to the electrical aspects of the problem.

The electrical problem can be broadly classified into two categories. First, the inherent characteristics of the armature winding itself as they effect commutation. Second, the effect on commutation of the coupling between the fields and the armature circuits. In this thesis, a mathematical foundation for the study of both aspects is presented but only the first is studied in detail, a criterion for the choice of the armature winding is presented, and a working digital computer program established for design based on this criterion.

II. EQUATIONS OF COMMUTATION

2.1 History

Reference is made first to the report entitled 'Commutation Study' (CS) by Koenig[6]. This work represents a departure from any other work found in the literature in at least two respects. First, the problem is viewed as a network switching problem with the exploitation of linear transformation theory. Second, new and different procedures are used to relate the inductance numbers to the machine geometry. These procedures, while too complex for calculation by hand, are ideally suited for digital computation. The theoretical development and results presented in that report constitute the starting point for this thesis. In view of its relationship to this thesis, the salient features of the report are reorganized and presented as they effect the development of this thesis.

2.2 Equations of Commutation

In the CS report, it is pointed out that for each interval of time during which a given number of commutator segments are contacted by the brush, a system of differential equations can be written which describe the interrelationship between the coil currents, the effects of armature and field currents, the mechanical position of the rotor and its velocity. These equations are valid for each interval of time during which a given set of commutator segments are short-circuited by the brushes. Fig. 2.1 shows a connection diagram for a 2-pole machine, for any interval in which there is no switching action. The mesh circuit equations for the network are of the form

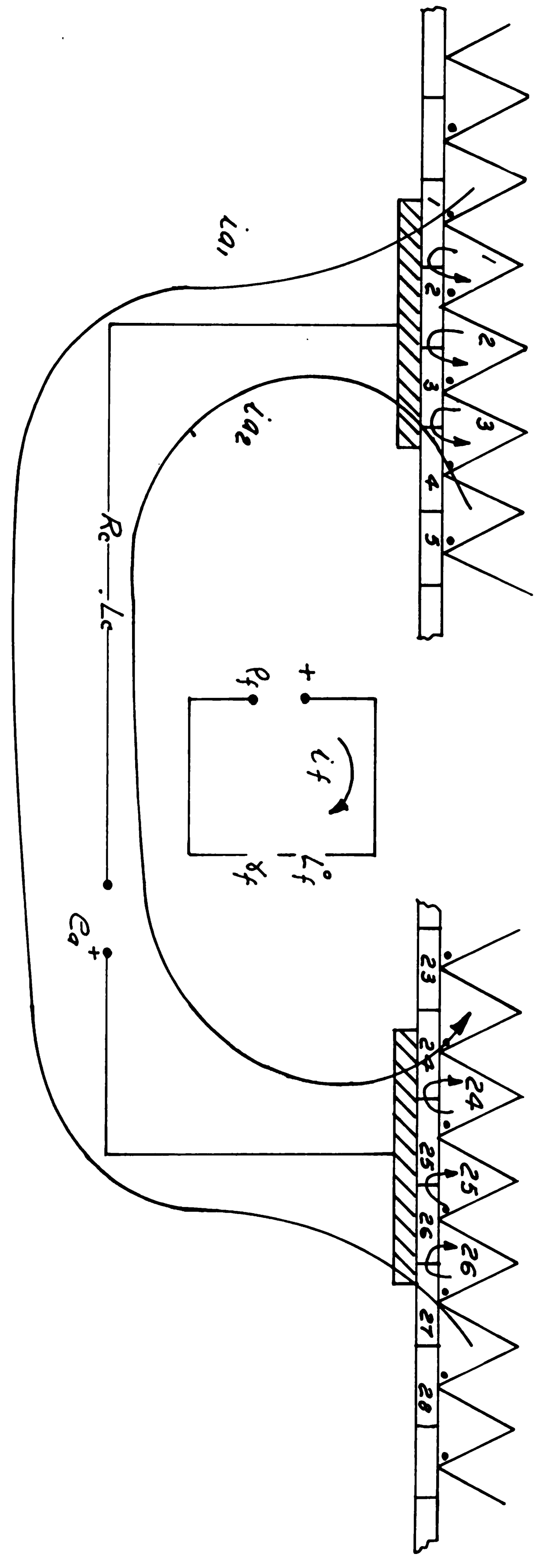


FIG. 2.1

$$\begin{bmatrix} \mathcal{V}_1(t) \\ \mathcal{V}_2(t) \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \mathcal{M}_{11}(t) & \mathcal{M}_{12}(t) \\ \mathcal{M}_{21}(t) & \mathcal{M}_{22}(t) \end{bmatrix}\begin{bmatrix} \mathcal{I}_1(t) \\ \mathcal{I}_2(t) \end{bmatrix}$$

$$= -\frac{d}{dt}\begin{bmatrix} \mathcal{M}_{13}(t) & \mathcal{M}_{14}(t) \\ \mathcal{M}_{23}(t) & \mathcal{M}_{24}(t) \end{bmatrix}\begin{bmatrix} \mathcal{I}_3(t) \\ \mathcal{I}_4(t) \end{bmatrix} \quad (2.2.1)$$

where the elements in the column matrices $\mathcal{I}_1(t)$ and $\mathcal{I}_2(t)$ represent respectively the currents in the circuits closed by the contact surfaces of brush one and two.

The elements in column matrices $\mathcal{I}_3(t)$ and $\mathcal{I}_4(t)$ represent currents in the main circuits of the armature and stator field windings respectively. In particular, for a 2-pole lap wound machine,

$$\mathcal{I}_3(t) = \begin{bmatrix} i_{a_1}(t) \\ i_{a_2}(t) \end{bmatrix}$$

$$\mathcal{I}_4(t) = i_f(t)$$

where $i_f(t)$ is the shunt field current.

For any given investigation of commutation the variables in $\mathcal{I}_3(t)$ and $\mathcal{I}_4(t)$ are considered as known functions of time consistent with terminal conditions for which the commutating characteristics are to be investigated.

The effect of eddy currents in the solid frame are included in the inductance numbers used for the various circuits. That is, these inductance numbers are evaluated at the frequencies of time variation involved in commutation.

The coefficients in submatrices $\mathcal{M}_{11}(t)$ and $\mathcal{M}_{22}(t)$ represent respectively the self and mutual inductance coefficients of the circuits closed by brush one and two, and $\mathcal{M}_{12}$ represents the mutual inductance coefficients between those circuits closed by brush one and those closed by brush two.

The elements of $\mathcal{I}_3(t)$ and $\mathcal{I}_4(t)$ are considered to be independent of the elements in $\mathcal{I}_1(t)$ and $\mathcal{I}_2(t)$ and depend only on the operating conditions of the machine.

The elements in $\mathcal{V}_1(t)$ and $\mathcal{V}_2(t)$ represent respectively, the contact voltages of brushes one and two. When the contact voltages are neglected and (2.2.1) is integrated between the limits 0 and t, there results

$$\begin{bmatrix} \Delta \lambda_1(t) \\ \Delta \lambda_2(t) \end{bmatrix} = \begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{bmatrix} \begin{bmatrix} \Delta \mathcal{I}_1(t) \\ \Delta \mathcal{I}_2(t) \end{bmatrix} \qquad (2.2.2)$$

where the variables in $\Delta \lambda(t)$ can be considered as changes in 'flux linkages' between commutating pole and armature circuits over the time interval 0-t. The variables in $\Delta \mathcal{I}(t)$ represent the change in the current variables associated with the circuits closed under the brushes over the same interval of time. No switching action takes place during the chosen interval.

The objective of the study presented in the CS report is to determine the values of $\Delta \lambda_1$ and $\Delta \lambda_2$ which would ensure the complete reversal of the commutating coil currents during the commutation period. It is proposed that this can be accomplished by solving for the changes in the magnitudes of the currents for an appropriate number of switching intervals. The criterion for satisfactory commutation as presented in the CS report is based on an actual time solution to the entire system of equations.

In calculating the inductance numbers of the commutating coils, it is not possible to detect a significant difference between the self and mutual

inductances of coils sharing the same slot. Consequently, when two or more sections of the armature winding sharing the same slot are undergoing commutation, the coefficient matrix of (2.2.2) is singular and the system of equations has no unique solution.

2.3 Linear Transformation of Variables and Definition of 'Slot Currents'

The difficulty encountered in the solution of (2.2.2) is overcome by choosing a set of new variables whose number is exactly equal to the rank of the coefficient matrix of (2.2.2). This transformation is best explained through the use of an example.

Example: Consider a 2-pole D.C. machine which has three coils closed by each brush. Let coils 1 and 2 under brush one share the same slots and coils 5 and 6 under the second brush share the same slots.

Neglecting the contact voltages, the system of equations in detail are of the form

$$\begin{bmatrix} \Delta\lambda_1 \\ \Delta\lambda_2 \\ \Delta\lambda_3 \\ \hline \Delta\lambda_4 \\ \Delta\lambda_5 \\ \Delta\lambda_6 \end{bmatrix} = \left[\begin{array}{ccc|ccc} M_{11} & M_{11} & M_{13} & M_{14} & M_{15} & M_{15} \\ M_{11} & M_{11} & M_{13} & M_{14} & M_{15} & M_{15} \\ M_{13} & M_{13} & M_{33} & M_{34} & M_{35} & M_{35} \\ \hline M_{14} & M_{14} & M_{34} & M_{44} & M_{45} & M_{45} \\ M_{15} & M_{15} & M_{35} & M_{45} & M_{55} & M_{55} \\ M_{15} & M_{15} & M_{35} & M_{45} & M_{55} & M_{55} \end{array}\right] \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \hline i_4 \\ i_5 \\ i_6 \end{bmatrix} \qquad (2.3.1)$$

The first two and last two equations are identical because of the assumption that coils 1 and 2 and coils 5 and 6 have the same inductance numbers. Following the symbolic notation in (2.2.2), we write

$$\begin{bmatrix} \Delta\mathcal{T}_1 \\ \Delta\mathcal{T}_2 \end{bmatrix} = \begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{bmatrix} \begin{bmatrix} \Delta\mathcal{I}_1 \\ \Delta\mathcal{I}_2 \end{bmatrix} \qquad (2.3.2)$$

Define a new set of current variables as a linear combination of the old set as follows

$$\Delta \mathscr{I}_{S1} = S_1 \Delta \mathscr{I}_1 \quad \text{and} \quad \Delta \mathscr{I}_{S2} = S_2 \Delta \mathscr{I}_2$$

where

$$S_1 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad S_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

Similarly, let

$$\Delta \lambda_{S1} = S_1^{-1} \Delta \lambda_1 \quad \text{and} \quad \Delta \lambda_{S2} = S_2^{-1} \Delta \lambda_2$$

Clearly, S_1 and S_2 are non-singular. Substitution of the new variables into (2.3.1) yields

$$\begin{bmatrix} \Delta\lambda_1 \\ 0 \\ \Delta\lambda_3 \\ \hline \Delta\lambda_4 \\ \Delta\lambda_5 \\ 0 \end{bmatrix} = \left[\begin{array}{ccc|ccc} M_{11} & 0 & M_{13} & M_{14} & M_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ M_{13} & 0 & M_{33} & M_{34} & M_{35} & 0 \\ \hline M_{14} & 0 & M_{34} & M_{44} & M_{45} & 0 \\ M_{15} & 0 & M_{35} & M_{45} & M_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} \Delta i_1 + \Delta i_2 \\ \Delta i_1 - \Delta i_2 \\ \Delta i_3 \\ \hline \Delta i_4 \\ \Delta i_5 + \Delta i_6 \\ \Delta i_5 - \Delta i_6 \end{bmatrix} \qquad (2.3.3)$$

The system of equations in (2.3.3) has only four non-trivial equations, one for each slot. The variables $\Delta i_1 + \Delta i_2$ and $\Delta i_5 + \Delta i_6$ are defined as 'slot currents'.

Definition 2.3.1 Slot Current: The summation of all the individual coil currents sharing the same slot, is known as slot current.

To obtain the slot currents for a machine with η brushes, consider the following system of equations

$$\begin{bmatrix} \Delta \lambda_1 \\ \Delta \lambda_2 \\ \vdots \\ \Delta \lambda_n \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & m_{2n} \\ \vdots & & & \\ m_{n1} & \cdot & \cdots & m_{nn} \end{bmatrix} \begin{bmatrix} \Delta I_1 \\ \Delta I_2 \\ \vdots \\ \Delta I_n \end{bmatrix} \qquad (2.3.4)$$

Let p be the total number of slots short-circuited by the i^{th} brush ($i = 1,2, \ldots n$) and let j_k coils share the k^{th} slot ($k = 1,2, \ldots p$), where

$$\sum_{K=1}^{p} j_K$$ = total number of coils short-circuited under the i^{th} brush.

Then, the non-singular transformation matrix S_i is given by

$$S_i = \begin{bmatrix} J_1 & 0 & \cdots & 0 \\ 0 & J_2 & \cdots & 0 \\ \vdots & \vdots & & \\ 0 & 0 & \cdots & J_p \end{bmatrix} \qquad (2.3.5)$$

In (2.3.5),

a) each of the sub-matrices J_k is square,

b) each entry in the first row and first column of J_k is equal to +1 and finally,

c) the other main diagonal terms consists of -1 entries and all remaining entries are equal to zero.

The transformed equations are of the form

$$\begin{bmatrix} S_1^{-1}\Delta\lambda_1 \\ S_2^{-1}\Delta\lambda_2 \\ \vdots \\ S_n^{-1}\Delta\lambda_n \end{bmatrix} = \begin{bmatrix} \Delta\lambda s_1 \\ \Delta\lambda s_2 \\ \vdots \\ \Delta\lambda s_n \end{bmatrix} = \begin{bmatrix} (S_1^{-1} m_{11} S_1^{-1}) & (S_1^{-1} m_{12} S_2^{-1}) & \cdots & (S_1^{-1} m_{1n} S_n^{-1}) \\ (S_2^{-1} m_{12} S_1^{-1}) & (S_2^{-1} m_{22} S_2^{-1}) & \cdots & (S_2^{-1} m_{2n} S_n^{-1}) \\ \vdots & \vdots & & \vdots \\ (S_n^{-1} m_{n1} S_1^{-1}) & (S_n^{-1} m_{n2} S_2^{-1}) & \cdots & (S_n^{-1} m_{nn} S_n^{-1}) \end{bmatrix} \begin{bmatrix} \Delta J s_1 \\ \Delta J s_2 \\ \vdots \\ \Delta J s_n \end{bmatrix}$$

(2.3.6)

The coefficient matrix in (2.3.6) is symmetric since the original matrix of (2.3.4) is symmetric and S_1^{-1} is symmetric. Furthermore, (2.3.6) contains exactly p non-trivial equations, one corresponding to each slot current variable.

Having once defined the term 'slot current' and the form of the resulting equations, these results can be obtained directly by inspection.

2.4 Necessary Conditions for Satisfactory Commutation

In the CS report, the necessary conditions for satisfactory commutation are stated. In view of their importance for the further development of this thesis, these results are stated here in the form of a postulate and two theorems.

Postulate 2.4.1

Let S be the set of n coils sharing the same slot and τ_j and T_j represent respectively the time at which the j^{th} coil of S enters and leaves the commutating zone. Then, for satisfactory commutation,

$$\sum_{j=1}^{n} i_j(T_j) - i_j(\tau_j) = \sum_{j=1}^{n} I_{a2}(T_j) - I_{a1}(\tau_j) \qquad (2.4.1)$$

where i_j represents the j^{th} commutating coil current. The $I_{a1}(\tau_j)$ and $I_{a2}(T_j)$ terms represent respectively the currents in the armature circuits

from which and to which the commutating coils are switched.

The postulate (2.4.1) requires for its verification, the time-domain values of $I_{a1}(\tau'_j)$ and $I_{a2}(\tau_j)$ for the set of n coils during the successive switching intervals, which are encountered during the commutation period. Since these currents can not be determined either by direct measurement or by a solution to the system of differential equations of commutation, (2.4.1) can hardly be verified by direct experimentation. Rather, it is based on the observed properties of switching under similar conditions.

If the armature current per path is represented by I_a and is considered constant over the interval of time $(\tau_n - \tau_1)$, (2.4.1) reduces to

$$\sum_{j=1}^{n} i_j(\tau_j) - i_j(\tau_j') = 2 n I_a \tag{2.4.2}$$

Theorem 2.4.1

Let m be the number of switching intervals encountered as the armature rotates one slot and C_k be the total number of circuits closed by a brush during the k^{th} interval, then,

$$\sum_{k=1}^{m} \sum_{r=1}^{C_k} [i_r(t_k^-) - i_r(t_{k-1}^+)] = \sum_{j=1}^{n} [i_j(\tau_j) - i_j(\tau_j')] \tag{2.4.3}$$

Again, if I_a is constant over the commutation period, then

$$\sum_{k=1}^{m} \sum_{r=1}^{C_k} [i_r(t_k^-) - i_r(t_{k-1}^+)] = 2 n I_a \tag{2.4.4}$$

Theorem 2.4.2

Let δ and p represent respectively the slot pitch in degrees and number of poles and I_c represent the summation of all the currents of all the coils closed by all the brushes and ΔI_c the change in the variable as the armature rotates through an angle of $\frac{\delta}{p}$, then,

$$\Delta I_c = \sum_{j=1}^{n} I_{a_2}(\tau_j) - I_{a_1}(\tau_j) \qquad (2.4.5)$$

If the change in the magnitude of the armature terminal current is negligible over the commutation period of one slot, then

$$\Delta I_c = 2 n I_a \qquad (2.4.6)$$

The important consequence of the theorems (2.4.1) and (2.4.2) is that it is only necessary to consider the time-variation of only one current variable, I_c, which represents the summation of currents of all the coils and under all the brushes, and so, the study of commutation is finally reduced to the study of only a single differential equation instead of a system of differential equations representing all the coils undergoing commutation. Thereby, the rigid restriction of continuity on all the individual coil currents is relaxed and the study of the single differential equation mentioned above, for satisfactory commutation, marks a definite departure from all the study hitherto made on that subject.

III. THE COMMUTATION FACTOR

3.1 Definition of the Problem

The problem of this thesis can now be stated. Instead of focusing attention on realizing the commutating fields which would give satisfactory commutation, attention is directed towards developing a numerical measure of the relative commutating abilities of the various armature windings operating in the same frame and under the same terminal conditions. This numerical measure is called the 'commutation factor' (CF). In addition to establishing such a numerical measure of the relative commutating abilities of various armature winding configurations, a complete digital computer program is to be established for including the commutation factor in the design of commutating machines.

3.2 Commutation Factor

To define the proposed basis for comparing the commutating ability of various armature windings, consider the sketch of Fig. (3.2.1) which represents the commutating coil circuit in the process of being opened. Let this circuit (and possibly one under the second brush also) open at the end of the time interval under consideration. Then the equations representing the changes in commutating coil currents for this interval $t_n < t < t_{n+1}$ are of the form

$$\begin{bmatrix} \Delta \lambda_1 \\ \Delta \lambda_2 \end{bmatrix} + \int_{t_n}^{t_{n+1}} \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} dt = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} \Delta i_1 \\ \Delta i_2 \end{bmatrix} \tag{3.2.1}$$

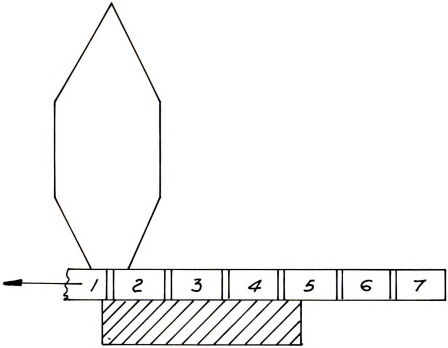


FIG. (3.2.1)

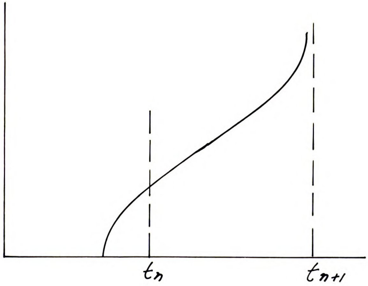


FIG. (3.2.2.)

While the contact surface closing the coil circuit of Fig. (3.2.1) decreases sharply toward the end of the period, the contact surfaces closing all other commutating coil circuits remain essentially constant and are relatively large. Consequently, it appears reasonable to neglect the contact voltages of all circuits except the one about to open. If this voltage between bars one and two of Fig. (3.2.1) is plotted as a function of time it is expected to be roughly of the form shown in Fig. (3.2.2) for undesirable commutating conditions. If the circuit to be opened at the end of the interval is included as the number one equation in (3.2.1) and the indicated integration carried out, the system of equations take on the form

$$\begin{bmatrix} \Delta \lambda_1 \\ \Delta \lambda_2 \end{bmatrix} + \begin{bmatrix} \Delta \Psi_1 \\ \Delta \Psi_2 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} \Delta \vartheta_1 \\ \Delta \vartheta_2 \end{bmatrix} \qquad (3.2.2)$$

where $\Delta \Psi_1 = \begin{bmatrix} \Delta \psi_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ and $\Delta \psi_1$ represents the area under the curve of Fig. (3.2.2) between t_n and t_{n+1}. Note that if there are an integral number of slots per pole, then a circuit will also be in the process of opening under the second brush. To include this possibility the $\Delta \Psi_2$ matrix, having the same form as $\Delta \Psi_1$ is included in the formulation. Commutation is expected to improve with reduced values of $\Delta \Psi$.

The properties of the system of equations in (3.2.2) are studied by the application of the theorem (2.4.2). The theorem is stated in terms of

a current variable representing the sum of the changes in all the currents closed by the brushes. To obtain such a current variable, a new set of equations are obtained by means of a non-singular transformation on the equations in (3.2.2)

Equations in (3.2.2) can be represented symbolically as,

$$\Delta \mathcal{H} + \Delta \Psi = \mathcal{M} \Delta \mathcal{I} \qquad (3.2.3)$$

Define a linear transformation as follows

$$\Delta \mathcal{I}^S = S \Delta \mathcal{I}$$

where the non-singular transformation matrix S has entries of +1 in the first row and along the main diagonal and with the rest of the entries equal to zero. The order of the S matrix is equal to the rows of $\Delta \mathcal{I}$. Also define,

$$\Delta \mathcal{H}^S = S \Delta \mathcal{H}$$

and

$$\Delta \Psi^S = S \Delta \Psi$$

Substituting the new variables into (3.2.3), we have,

$$\Delta \mathcal{H}^S + \Delta \Psi^S = (S \mathcal{M} S^{-1}) \Delta \mathcal{I}^S \qquad (3.2.4)$$

where S^{-1} is obtained by merely replacing all the elements of the first row of S, except the (1x1) element by -1.

For a two-pole machine or a machine with an even number of segments per pole pair, (3.2.4) can be written as

$$\begin{bmatrix} \Delta \mathcal{H}_{c1} \\ \Delta \mathcal{H}_{c2} \end{bmatrix} + \begin{bmatrix} \Delta \Psi_{c1} \\ \Delta \Psi_{c2} \end{bmatrix} = \begin{bmatrix} \mathcal{M}_{c11} & \mathcal{M}_{c12} \\ \mathcal{M}_{c21} & \mathcal{M}_{c22} \end{bmatrix} \begin{bmatrix} \Delta \mathcal{I}_{c1} \\ \Delta \mathcal{I}_{c2} \end{bmatrix} \qquad (3.2.5)$$

where $\Delta\Psi_{c_1}$ and $\Delta\Psi_{c_2}$ are column matrices with an entry $(\Delta\Psi_1 + \Delta\Psi_2)$ in the (1x1) position of $\Delta\Psi_{c_1}$ and all the other variables of the column matrices are the same as before the transformation. The first variable in $\Delta\mathcal{J}_{c_1}$ represents the sum of the changes in all the currents closed by both brushes. Theorem (2.4.2) is stated in terms of this variable. To separate this variable from the remaining, let (3.2.5) be repartitioned as follows

$$\begin{bmatrix} \Delta\lambda_{c1} \\ \Delta\lambda_{ca} \end{bmatrix} + \begin{bmatrix} \Delta\Psi_1 + \Delta\Psi_2 \\ \Delta\Psi_{ca} \end{bmatrix} = \begin{bmatrix} M_{c11} & m_{c1a} \\ m_{ca1} & m_{caa} \end{bmatrix} \begin{bmatrix} \Delta i_{c1} \\ \Delta\mathcal{J}_{ca} \end{bmatrix} \tag{3.2.6}$$

where $\Delta\mathcal{J}_{ca}$ includes all current variables in (3.2.6) except the first Δi_{c_1} variable.

The most efficient way to solve for (3.2.6) for Δi_{c_1} is to triangularize the coefficient matrix by multiplying both sides of the equation by

$$\begin{bmatrix} 1 & -m_{c1a} m_{caa}^{-1} \\ 0 & u \end{bmatrix} \tag{3.2.7}$$

with the result

$$\begin{bmatrix} \Delta\lambda_{c1} - m_{c1a} m_{caa}^{-1} \Delta\lambda_{ca} \\ \Delta\lambda_{ca} \end{bmatrix} + \begin{bmatrix} \Delta\Psi_1 + \Delta\Psi_2 - m_{c1a} m_{caa}^{-1} \Delta\Psi_{ca} \\ \Delta\Psi_{ca} \end{bmatrix}$$

$$= \begin{bmatrix} (M_{c11} - m_{c1a} m_{caa}^{-1} m_{ca1} & 0 \\ m_{ca1} & m_{caa} \end{bmatrix} \begin{bmatrix} \Delta i_{c1} \\ \Delta\mathcal{J}_{ca} \end{bmatrix} \tag{3.2.8}$$

From the top equation in (3.2.8) and with the assumption $\Delta\Psi_1 = \Delta\Psi$ and $\Delta\Psi_2 = 0$ the ratio $\frac{\Delta\Psi_1}{\Delta i_{c1}}$ takes on the form

$$\frac{\Delta\Psi_1}{\Delta i_{c1}} = \frac{(M_{c11} - m_{c1a}\, m_{caa}^{-1}\, m_{ca1}}{1 + K} - \frac{\Delta\lambda_{c1} - m_{c1a}\, m_{caa}^{-1}\, \Delta\lambda_{ca}}{\Delta i_{c1}\,(1 + K)}$$

where

$$K\,\Delta\Psi = -\left[m_{c1a}\, m_{caa}^{-1}\right] \Delta\Psi_{ca} \qquad (3.2.10)$$

all the non-zero elements of the column matrix $\Delta\Psi_{ca}$ are identically equal to $\Delta\psi$.

If only one circuit is opened at the end of the interval under commutation, then $\Delta\Psi_2 = 0$. Note that if the armature windings to be compared are assumed to have the same coupling to the external circuits or if the comparison is not to include the effects of the commutating field and external circuits, then the last term in (3.2.9) is not included and the ratio is a function of the inductance coefficient matrix of the winding only. Therefore, the commutation factor is defined as

$$\frac{\Delta\psi}{\Delta i_{c1}} = \frac{(M_{c11} - m_{c1a}\, m_{caa}^{-1}\, m_{ca1})}{(1 + K)} \qquad (3.2.11)$$

In evaluating the above commutation factor, the assumption was made that all the contact voltages of the column matrix of $\Delta\Psi$ are zero except for the coils which are about to leave the commutating zone. When these contact voltages are included they are multiplied by $m_{c1a}\, m_{caa}^{-1}$. This triple product is very small compared to other terms and the commutating factor defined above is not appreciably altered.

The commutating factor given in (3.2.11) is used as a basis of comparing the commutating ability of armature windings designed for the same

armature punchings and commutator and operating in the same frame. Under these conditions the switching sequences and length of intervals are identical and changes in the current variable Δi_{c1}, during the last interval of commutation, are essentially equal for various winding configurations such as lap, wave and frog-leg windings. Under these conditions the ratio gives a measure of the relative magnitudes of the average bar-to-bar voltage during the last interval of commutation.

The calculation of the commutation factor can be further simplified in windings like the simplex-lap where the winding repeats every pole or every pair of poles. The simplification is realized by means of 'equivalent representations' of the armature winding discussed in appendix B.

The commutation factor which gives the inherent characteristic of a given winding on a given frame, can not be used in its present form for comparing armature windings on different frames and of different current ratings. To achieve the latter purpose, the commutation factor is used to calculate the voltage between the brush and commutator bar at the time of opening. This voltage is known as the 'reactance voltage'.

3.3 Derivation of the Formula for the Reactance Voltage (B_r)

Definition 3.3.1 The ratio $\frac{\Delta \psi}{\Delta t}$ is known as the reactance voltage; thus,

$$E_R = \frac{\Delta \psi}{\Delta t} \tag{3.3.1}$$

Substituting (3.2.11) into (3.3.1), we have

$$E_R = (CF) \frac{\Delta i_c}{\Delta t} \tag{3.3.2}$$

In (3.3.2) Δt represents the time required for the current variable Δi_c to change from $+ I_a$ to $- I_a$. Thus, $\frac{\Delta i_c}{\Delta t}$ represents the average rate of change of the Δi_c variable during the commutation

period. Fig. (3.3.1) shows a diagrammatic representation of the time variation of the Δi_c variable during the time interval Δt.

Equation (3.3.2) can be stated in a form more convenient in design.

$$\Delta i_c = 2I_s$$

where I_s is the steady-state value of the current in the armature circuits.

$$= 2 m i_p$$

where m number of sections of the winding and

$$i_p = \frac{\text{Total amperes}}{\text{number of parallel paths of the winding}}$$

$$\Delta t = \delta / p n_s K$$

where δ is the slot pitch in commutator segments and is equal to m,

K is the total number of commutator segments,

p is the total number of poles,

n_s is the revolutions per second and finally,

$\frac{\delta}{pK}$ is the rotation of the armature in segments.

$$(CF)\frac{\Delta i_c}{\Delta t} = 2 i_p n_s K P (CF)$$

The commutation factor in (3.2.11) is calculated on the basis of only one turn/section. The total reactance voltage is obtained by multiplying this factor by $(\text{turns/section})^2$.

$$E_R = \frac{\Delta i_c}{\Delta t}(CF) = (2 i_p n_s KP)(CF)(\text{turns/section})^2 \tag{3.3.3}$$

Equation (3.3.3) can be expressed in terms of the ampere turns per pole (ATP).

$$ATP = \frac{i_p K (\text{turns/section})}{P} \tag{3.3.4}$$

A-B represents the commutating period.

Curve 1: represents the time variation of the ΔI_{C1} variable during the interval AB.

Curves 2, 3, 4 and 5: represent the time variation of the individual coil currents during the commutating period.

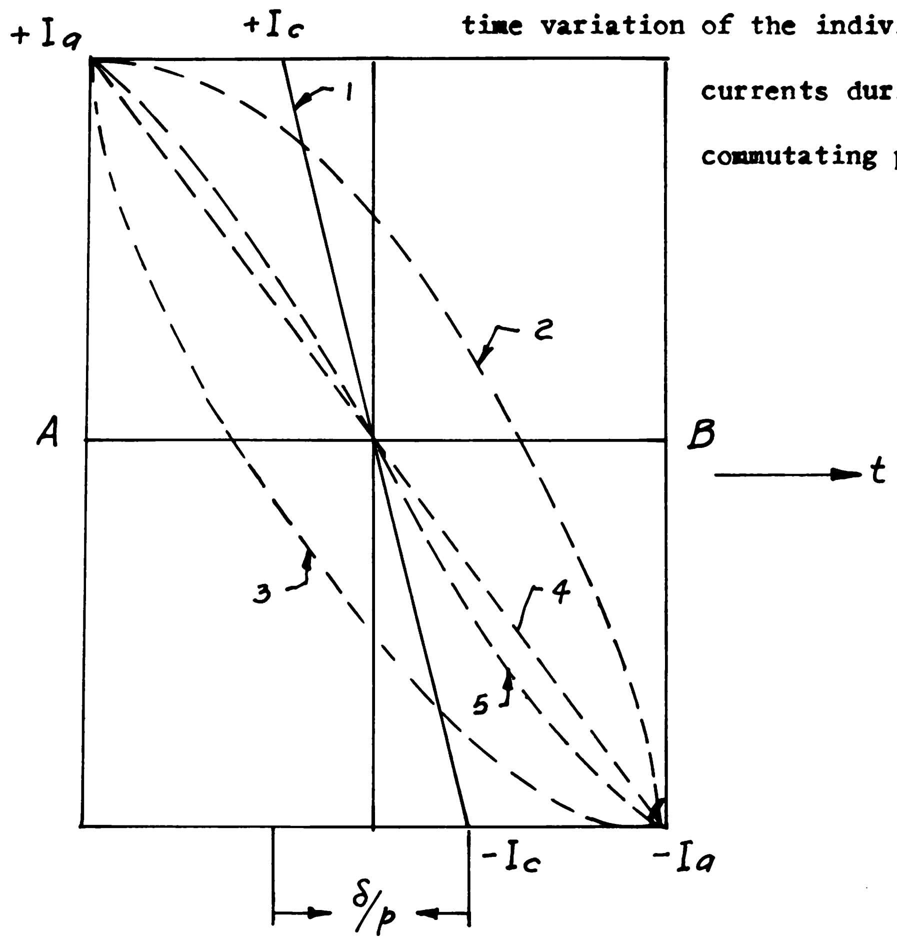


FIG. (3.3.1)

Coil currents as function of time during commutating period.

or,

$$i_p = \frac{\Delta TP \times P}{K \times \text{turns/section}} \qquad (3.3.5)$$

Substituting (3.3.5) into (3.3.3) we have,

$$E_R = 2\,(ATP) \times P^2 \times n_s \times (\text{turns/section})(CF) \qquad (3.3.6)$$

and substituting the relationship $n_s = \frac{RPM}{60}$ into (3.3.6), we have finally,

$$E_R = 2 \times ATP \times P^2 \times \frac{RPM}{60} \times (\text{turns/section}) \times (CF) \qquad (3.3.7)$$

IV. BASIS FOR INDUCTANCE CALCULATIONS

4.1 The commutation factor that is proposed is ultimately a numerical measure for the commutating ability of an armature winding on a given frame. Thus, for successful application of this criterion for design, the commutation factor should be calculated with utmost precision. The crucial step in its realization is the availability of accurate analytical expressions for the calculation of the inductance numbers of the commutating coils, which form the starting point of the derivation of the commutation factor. Dreyfus[7] and Thielers present methods of calculating some of the required inductance numbers for a two-pole machine under the assumption of a uniform airgap in the commutating pole region. However, these expressions do not warrant their application in the present study, because additional refinements have to be definitely added for more precise calculations of the inductance numbers. The analytical expressions presented in the CS report serve this need and are ideally suited to the digital computer.

A detailed discussion of the problem of determining the inductance numbers for rotating machines, by correlating them to the geometry of the machines, appears in reference (8). It also contains an extensive bibliography on the subject.

The mathematical expressions for calculating the commutating coil inductances are based on the results of a series of field maps of the commutating pole region appearing in R-29[9]. These maps were made for a relatively wide range of commutating pole airgaps and geometries under conditions corresponding to a constant excitation of the commutating pole. From these field maps the radial component of the airgap flux density per

ampere turn of m.m.f. so determined is plotted as a function of the position of the armature surface, a curve of the form shown in Fig. (4.1.1) results.

The basic assumption made in the calculation of the commutating coil inductances is that the radial component of the flux density in the commutating pole region when an armature coil is excited (instead of the commutating winding) is obtainable from the results of the field maps by simply changing the direction of the flux density as shown by curve in Fig. (4.1.2).

The airgap self and mutual inductances of the commutating coils are obtained by evaluating the surface integral of the flux density curve over appropriate areas.

To systematize this calculation of the inductance coefficients, the results of the field maps have been consolidated in the form of two curves. One curve, as in Fig. (4.1.3), shows the ratio of the third harmonic, C_3, to the peak value of flux density in a Fourier series representation of the airgap flux density curve of Fig. (4.1.1). In deriving the curves of Fig. (4.1.3) all harmonics above the third are neglected.

To obtain the value of C_1 (ratio of fundamental to the peak value of commutating pole flux density) simply subtract C_3 from unity. Thus, $C_1 = 1 - C_3$.

It should also be pointed out that the Fourier series expansion of the wave form in Fig. (4.1.1) is based on taking the distance $\tau/2$ as one half period. This quantity $\tau/2$ (referred to as extent of commutating field) is obtained from the curves shown in Fig. (4.1.4) by forming the product $a \times \omega = \tau/2$ where a appears as the ordinate of the curves in Fig. (4.1.4).

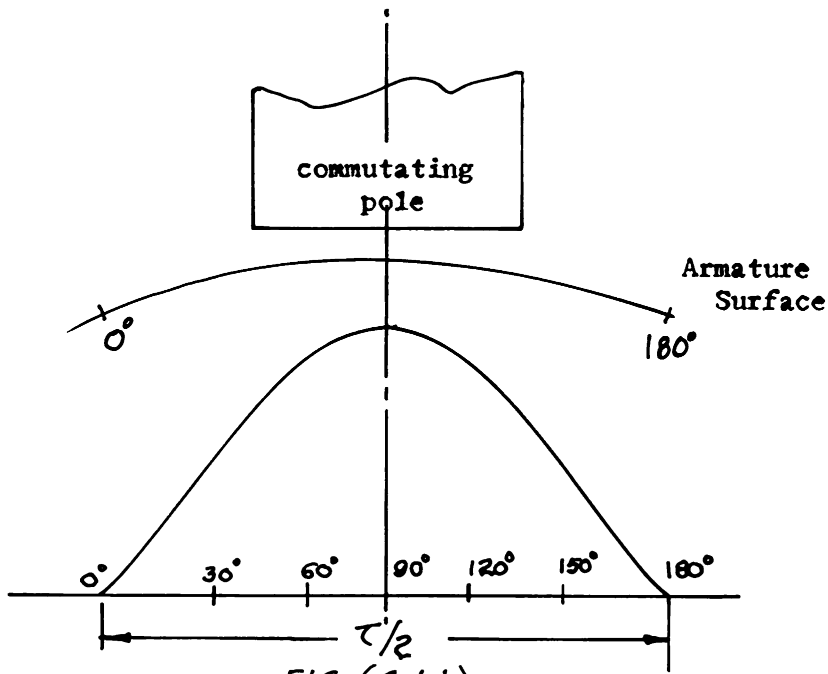


FIG. (4.1.1.)

Radial component of flux density per ampere turn, commutating pole excited

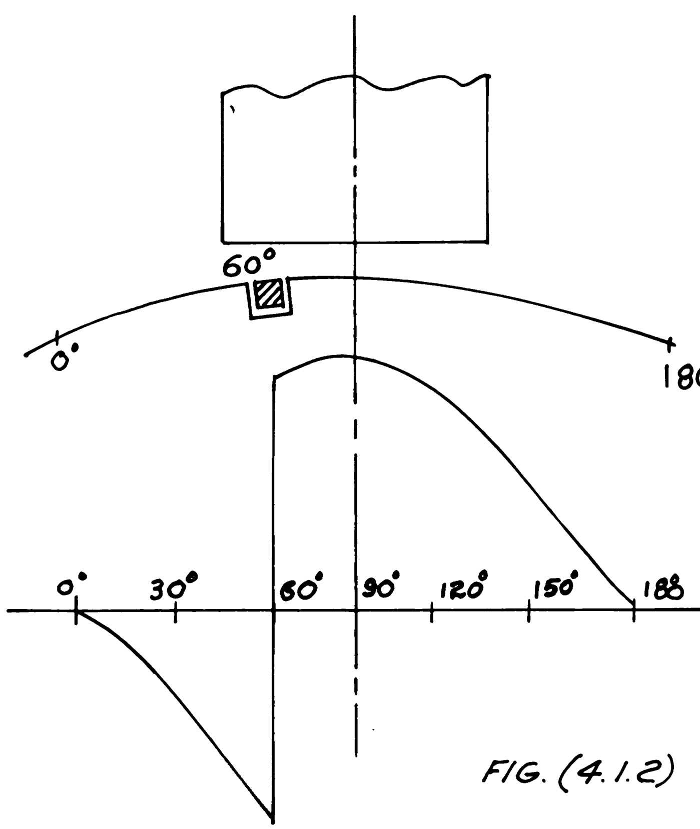


FIG. (4.1.2)

Assumed radial component of flux density per ampere turn, armature coil excited

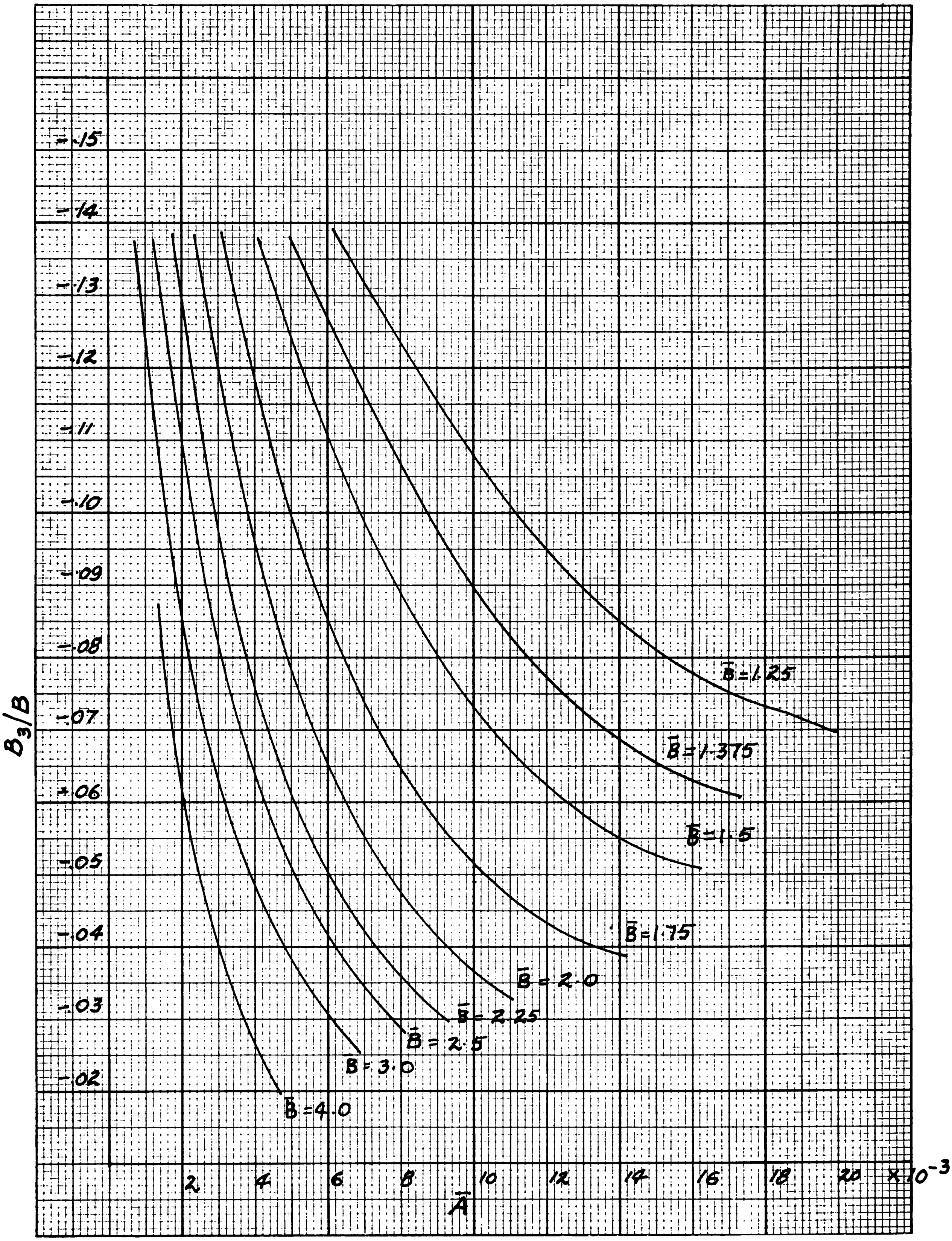
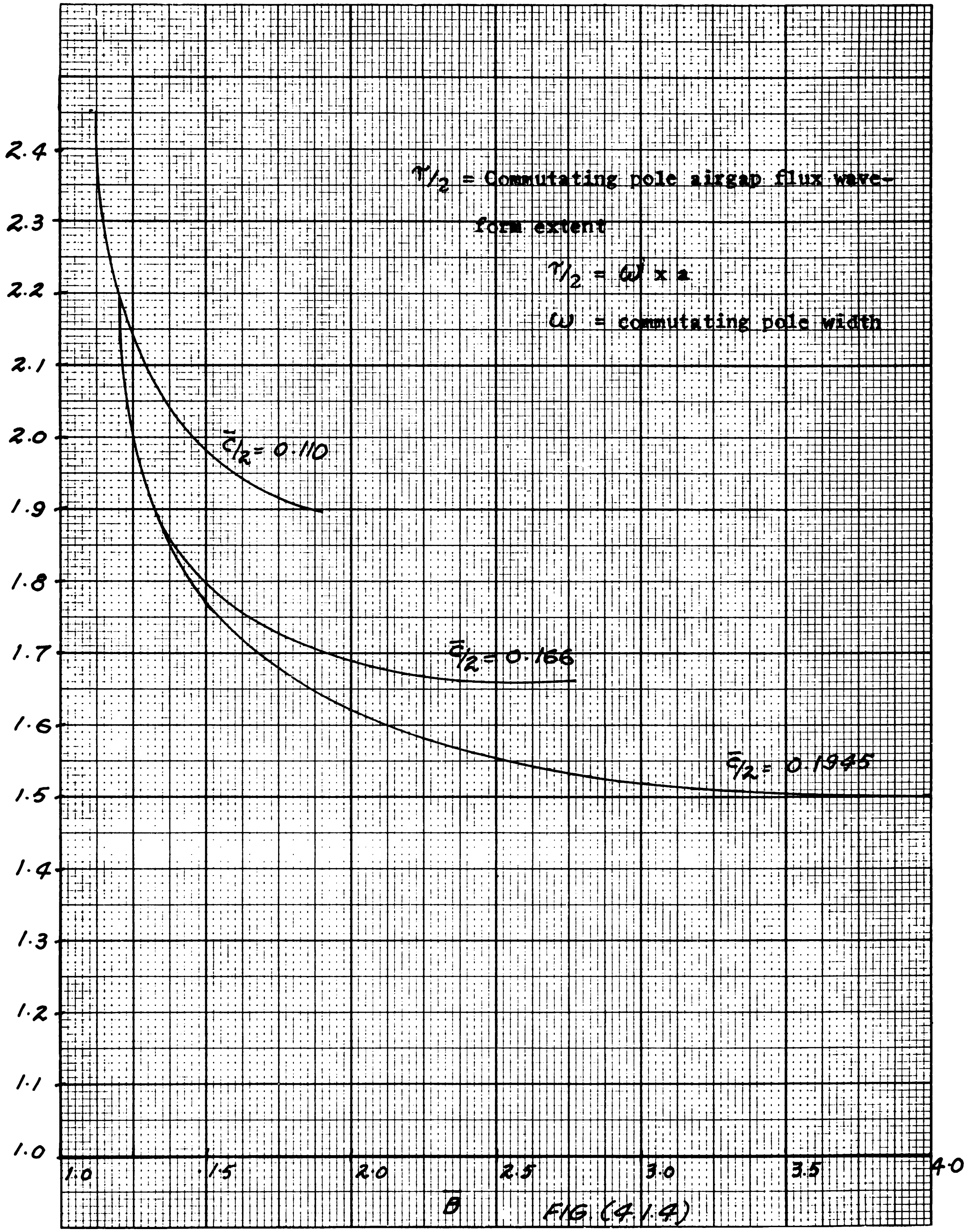

B_3/B
-.15
-.14
-.13
-.12
-.11
-.10
-.09
-.08
-.07
-.06
-.05
-.04
-.03
-.02
2
4
6
8
10
12
14
16
18
20
$\times 10^{-3}$
$\bar{A}$
$\bar{B} = 1.25$
$\bar{B} = 1.375$
$\bar{B} = 1.5$
$\bar{B} = 1.75$
$\bar{B} = 2.0$
$\bar{B} = 2.25$
$\bar{B} = 2.5$
$\bar{B} = 3.0$
$\bar{B} = 4.0$

FIG. (4.1.3.)



τ/2 = Commutating pole airgap flux wave-form extent
τ/2 = ω x a
ω = commutating pole width
c̄/2 = 0.110
c̄/2 = 0.166
c̄/2 = 0.1945
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
1.0
1.5
2.0
2.5
3.0
3.5
4.0
B̄

FIG (4.1.4)

4.2 Calculation of Commutating Coil Inductances for Salient Pole Machines

Fig. (4.2.1) gives a cross-sectional diagram of the iron boundary surface along with some details of the commutating pole and the armature. The details of Fig. (4.2.1) which enter into the mathematical expressions for the inductance calculations are presented below:

1.) Slot 1 is placed in a position wherein its top-section is about to break contact with the brush.

2.) γ is the angle between the center of slot 1 and the center of the nearest commutating pole.

3.) All angles are measured in the clockwise direction from the reference slot.

4.) If the top-section of slot j goes to the bottom-section of the slot k to form a coil, their distance α_{jK} is obtained by $\alpha_j - \alpha_K$ where α_j and α_K are the distances from the reference slot to the k^{th} and j^{th} slots respectively.

5.) σ is the chording angle and is equal to (π - coil pitch).

6.) CZ is the extent of the commutating field.

7.) The slot details of the armature of Fig. (4.2.1) are shown in Fig. (4.2.2).

In order to evaluate the inductances of an armature coil 3-3'., it is necessary to integrate the flux density over the surface a-b-c-d-e (Fig. 4.2.1) when the armature coil carries unit current. Let this surface of integration be divided into the following sub-surfaces:

a) integral over surface a-b and d-e define slot leakage inductance.

b) integral over surface b-d defines the airgap inductance.

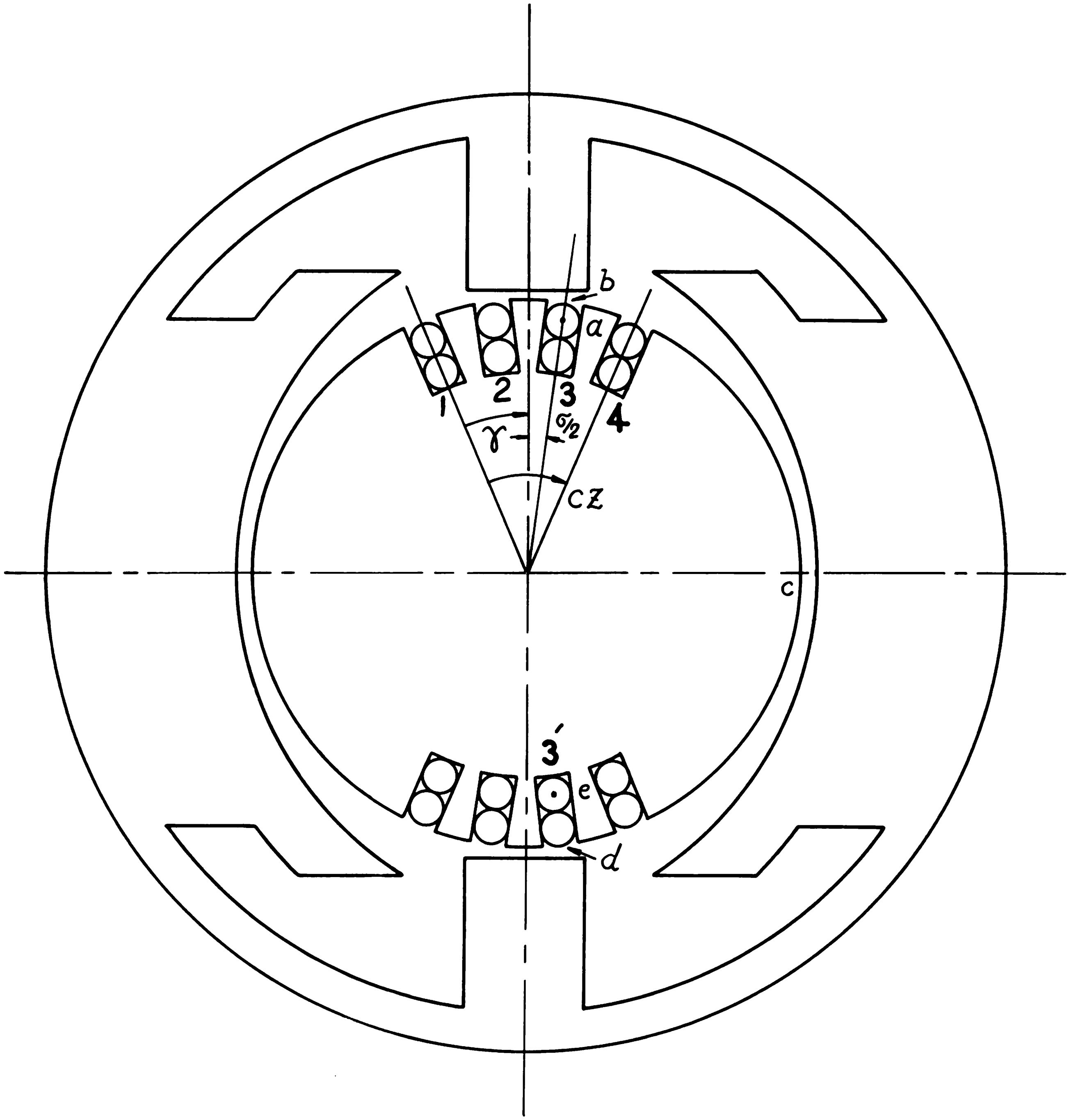


FIG (4.2.1)

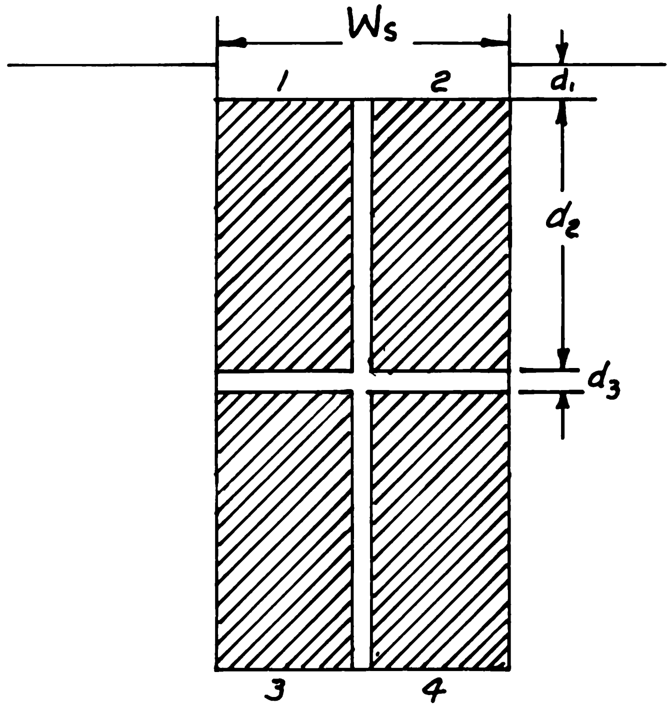


FIG (4.2.2.)

W_s = width of the slot

d_1 = distance between the wedge of the slot and the top coil of the slot

d_2 = height of the coil in the top section of the slot

d_3 = insulation between the coil sections in the slot

4.3 Slot Inductance

The field distribution in the slot is assumed to be normal to the sides of the slot.

It is easily shown that the self inductance

$$L_S = \frac{3.19 \times L}{W_S} (2d_1 + 5/3\, d_2 + d_3) \times 10^{-8} \text{ henry} \qquad (4.3.1)$$

and the mutual inductance between two conductors in the same slot is taken as

$$M_S = \frac{3.19 \times L}{W_S} (d_1 + d_2/2) \times 10^{-8} \text{ henry} \qquad (4.3.2)$$

4.4 Airgap Inductance

The assumption made in the calculation of the airgap inductance is that the excitation of the field poles will not alter the relative flux density distribution at the surface of the armature, due to the excitation of one armature coil. The airgap inductance is considered in two parts, that which is due to the commutating-pole flux and that due to the main-pole flux. Both the components of airgap inductance are obtained by surface integration over prcper areas. A detailed discussion appears in the CS report and will not be reported here. The flux density distribution curves for the commutating pole appear in the Engineering report R-29[9].

Only the final expressions for the airgap inductances are presented below. As they are obtained by the integration of different areas, depending upon the positions of the coils, their ranges of application are stated. The restrictions are stated in terms of the extent of the commutating pole field ($\tau/2$), the location of the slots in the commutator zones (α'S), and the switching angle(γ).

1) Expression valid for the region

$$-(\tau/2 + \gamma) \geqslant \alpha_j \leqslant \alpha_K \leqslant (\tau/2 - \gamma)$$

$$M_{jK} = \frac{\psi_{c1}}{4}\left\{\left[1 + \sin\Delta(\gamma + \alpha_j)\right]\left[1 - \sin\Delta(\gamma + \alpha_K)\right] + \left[1 + \sin\Delta(\gamma + \alpha_j - \sigma)\right]\left[1 - \sin\Delta(\gamma + \alpha_K - \sigma)\right]\right\} \times 10^{-8} \text{ henry} \qquad (4.4.1)$$

where,

ψ_{c1} = fundamental component of commutating pole flux per ampere turn of airgap m.m.f. (A detailed derivation of this quantity appears in the section (5.2)

$\Delta = \dfrac{\pi \times Ar\,OD}{P \times \tau/2}$ where P is the number of poles

σ is the chording angle

when j = k, (4.4.1) represents the self inductance

$$M_{jj} = \frac{\psi_{c1}}{4}\left\{\left[1 - (\sin\Delta\,\overline{\gamma + \alpha_j})^2\right] + \left[1 - (\sin\Delta\,\overline{\gamma + \alpha_j - \sigma})^2\right]\right\} \times 10^{-8} \qquad (4.4.2)$$

Equation (4.4.2) is applicable for calculating the self inductance terms of any coil and also for the mutual inductances of coils in slots under the same pole.

2.a) Expression valid for the region

$$-(\tau/2 + \gamma) \leqslant \alpha_K \leqslant \alpha_j + \sigma \leqslant (\tau/2 - \gamma)$$

$$M_{jK} = \frac{\psi_{c1}}{4}\left\{\left[1 - \sin\Delta(\gamma + \alpha_j - \sigma)\right]\right\}\left\{\left[1 + \sin\Delta(\gamma + \alpha_K)\right]\right\} \times 10^{-8} \qquad (4.4.3)$$

2.b) Expression valid for the region

$$-(\tau/2 + \gamma) \leqslant \alpha_j - \sigma \leqslant \alpha_K \leqslant (\tau/2 - \gamma)$$

$$M_{jK} = \frac{\psi_{c1}}{4}\left\{\left[1 + \sin\Delta(\gamma + \alpha_j - \sigma)\right]\left[1 - \sin\Delta(\gamma + \alpha_K)\right]\right\} \times 10^{-8} \qquad (4.4.4)$$

3.a) Expression valid for the region

$$(\tau/2 - \gamma) \geq \alpha_j \geq \alpha_K - \sigma \geq -(\tau/2 + \gamma)$$

$$M_{jK} = \frac{\psi_{c_1}}{4}\left\{\left[1 + \sin\Delta(\gamma + \alpha_K - \sigma)\right]\left[1 - \sin\Delta(\gamma + \alpha_j)\right]\right\} \times 10^{-8} \qquad (4.4.5)$$

3.b) Expression valid for the region

$$(\tau/2 - \gamma) \geq \alpha_K - \sigma \geq \alpha_j \geq (\tau/2 + \gamma)$$

$$M_{jK} = \frac{\psi_{c_1}}{4}\left\{\left[1 + \sin\Delta(\gamma + \alpha_j)\right]\left[1 - \sin\Delta(\gamma + \alpha_K - \sigma)\right]\right\} \times 10^{-8} \qquad (4.4.6)$$

Equations (4.4.2) to (4.4.6) involve the switching angle (γ) and the slot locations (α's). Hence, the next step is to establish mathematical expressions for their determination.

Out of the many possible switching intervals, the switching interval that is selected must meet the following conditions.

1.) It must serve as the criterion for comparing different machines,

2.) It must be such that it represents the worst possible condition of commutation,

3.) It must be valid for both directions of the armature rotation.

When the reference slot is in a state of leaving the commutating zone, the above three conditions are satisfied.

Definition 4.4.1 The zone of armature periphery in inches corresponding to the interval of rotation during which the first winding section in a given slot enters and the last winding section of the same slot leaves the commutating period is known as the commutating zone.

A detailed discussion of the commutator zone is found in reference (11). Only the result is given here.

$$\text{Commutator zone (CZ)} = \frac{\text{Ar O.D.}}{\text{com O.D.}}\left\{\frac{\text{Brush Thickness}}{\text{Cosine of Brush Angle}} - \text{mica thickness} + \frac{\pi \text{ com O.D.}}{\text{No. of bars}}\left[\text{Sections}\left(1 + \text{com. pitch deficiency}\right) - \frac{\text{Paths}}{\text{Poles}}\right]\right\} \qquad (4.4.7)$$

where the commutator pitch deficiency is given by

$$\left| \frac{\text{No. of com. bars}}{\text{Sections} \times \text{No. of poles}} - (\text{coil pitch}) \right| \tag{4.4.8}$$

Definition 4.4.2 One-half of the commutating zone when expressed in degrees is referred to as the switching angle (γ).

$$\gamma = \frac{1}{2}\left[\text{Com. Zone in degrees}\right] \tag{4.4.9}$$

4.5 Determinations of the Slot Locations (α's)

The positions of all the slots in the commutating zones defined by all the brushes can be determined by more than one method. Only the numerical method which is highly suited for the digital computer will be presented here. The method is best explained through the use of two examples. These examples also serve to point out the simplifications possible in calculating the airgap inductances when certain patterns of symmetries exist.

Example 4.5.1 The pertinent machine data used in the following calculations is:

a) Armature details:

1) Armature winding .... Simplex lap
2) Number of sections 4
3) Coil pitch .... 9 slots
4) Number of slots 38
5) Armature outer dia. .... 11.5 inches

b) Commutator details:

1) Number of com. segments 152

2) Commutator dia. .... 9.25 inches

3) Brush thickness .... 0.625 inches

4) Brush angle .... 30 degrees

5) Mica thickness .... 0.035 inches

c) Commutating pole data:

1) No. of commutating poles 4

Calculations:

1) Com. zone as calculated from (4.4.7) is 2.03 inches

2) Com. zone angle (CZ) = $\dfrac{\text{CZ in inches} \times 114.6}{\text{Ar O.D.}}$

3) Slot pitch (δ) in degrees = $\dfrac{360}{\text{No. of slots}}$

4) Tabulate the slot angles and commutator zone angles as in table (4.5.1)

5) From table (4.5.1), determine which slots are in the commutating zone.

From table (4.5.1), it is evident that slots 1, 2, and 3 are in the com. zone of brush one. Slots 11 and 12 are in the zone of brush two. Slots 21 and 22 are in the zone of brush three. And finally, slots 30 and 31 are in the zone of brush four. The angles locating the positions of the slots are:

$$\alpha_1 = 0$$
$$\alpha_2 = 1\delta$$
$$\alpha_3 = 2\delta$$
$$\alpha_4 = 10\delta$$
$$\alpha_5 = 11\delta$$
$$\alpha_6 = 20\delta$$
$$\alpha_7 = 21\delta$$
$$\alpha_8 = 29\delta$$
$$\alpha_9 = 30\delta$$

From table (4.5.1) slot 20 is in the same position with respect to the zone of the third brush as is slot 1 with respect to the zone of the first brush. This is always the case for simplex lap. Since the winding repeats every pair of poles, considerable simplification in the calculation of the inductance numbers is achieved by an equivalent two-pole representation. Such an equivalent two-pole representation is discussed in the appendix B.

Example 4.5.2 In this example, a machine with a simplex-wave winding is considered.

a) Armature details:

1) Armature winding .... Simplex-wave
2) No. of sections 3
3) Coil pitch .... 8 slots
4) No. of slots 33
5) Armature outer dia. .... 9.5 inches

b) Commutator details:

1) No. of Com. Segments 99
2) Com. dia. .... 7.223 inches
3) Brush thickness .... 0.5 inch
4) Brush angle .... 30 degrees
5) Mica thickness .... 0.035 inch

c) Commutator pole data:

1) No. of commutating poles 4

d) Calculations:

1) Com. zone from (4.4.7) will yield ...... 1.69 inches
2) Com. zone angle (CZ) = $\frac{1.69 \times 114.6}{9.5} = 20°.40$

3) Slot pitch (δ) in degrees .... = $\frac{360}{33} = 10°.9091$

4) Tabulate the slot angles and commutating zone angles as in Table (4.5.2).

5) From the table (4.5.2), the slot locations are:

$\alpha_1 = 0$
$\alpha_2 = 1\delta$
$\alpha_3 = 9\delta$
$\alpha_4 = 10\delta$
$\alpha_5 = 17\delta$
$\alpha_6 = 18\delta$
$\alpha_7 = 25\delta$
$\alpha_8 = 26\delta$

From table (4.5.2) reference slot 1, is about to leave its commutator zone. There are no other slots in a similar position. This is typical of simplex-wave windings. There is no repetition in the magnitude of the inductance numbers.

Table (4.5.1)

Slot No.	Degrees from Slot 1	Commutating Zone No.	Span in Degrees
1	0	1	0-20.2294
2	9.4736	2	90-110.2294
3	18.9472	3	180-200.2294
		4	270-290.2294
4	28.4208		
5	37.8944		
6	47.3680		
7	56.8416		
8	66.3152		
9	75.7888		
10	85.2624		
11	94.7360		
12	104.2096		
13	113.6832		
14	123.1568		
15	132.6304		
16	142.1040		
17	151.5776		
18	161.0512		
19	170.5248		
20	179.9984 (is actually 180.0; round off error of 0.0016)		
21	189.4720		
22	198.9456		
23	208.4192		
24	217.8928		
25	227.3664		
26	236.8400		
27	246.3136		
28	255.7872		
29	265.2608		
30	274.7344		
31	284.2080		
32	293.6816		
33	303.1552		
34	312.6288		
35	322.1024		
36	331.5760		
37	341.0496		
38	350.5232		

Table (4.5.1) shows that slots 1,2 and 3 are in commutating zone 1; slots 11 and 12 are in zone 2; slots 20,21 and 22 are in zone 3; and finally, slots 30 and 31 are in zone 4. The winding repeats every pair of poles.

Table (4.5.2)

Slot No.	Degrees from Slot 1	Commutating Zone No.	Span in Degrees
1	0	1	0-20.3867
2	10.9091	2	90-110.3867
		3	180-200.3867
3	21.8182	4	270-290.3867
4	32.7273		
5	43.6364		
6	54.5455		
7	65.4546		
8	76.3637		
9	87.2728		
10	98.1819		
11	109.0910		
12	120.0001		
13	130.9092		
14	141.8183		
15	152.7274		
16	163.6365		
17	174.5456		
18	185.4547		
19	196.3638		
20	207.2729		
21	218.1820		
22	229.0911		
23	240.0002		
24	250.9093		
25	261.8184		
26	272.7275		
27	283.6366		
28	294.5457		
29	305.4548		
30	316.3639		
31	327.2730		
32	338.1821		
33	349.0912		

Table (4.5.2) indicates that slots 1 and 2 are in commutating zone 1; slots 10 and 11 are in zone 2; 18 and 19 are in zone 3; and finally, slots 26 and 27 are in zone 4. The winding does not repeat.

V. DIGITAL COMPUTER DESIGN OF THE D.C. MACHINE ON THE BASIS OF THE COMMUTATION FACTOR

5.1 The subject of electrical machinery is at least 70 years old, and the first paper in the A.I.E.E. dates back to the year 1886. Since that date, the subject has grown enormously. However, many research and developmental problems of the commutator type of machine still exist. The various analyses of the commutation problem have not resulted in a practical design procedure, eliminating the necessity of adjustments on the test floor for achieving optimum commutation.

The design of the commutator type of machine, like all the other machinery, have hitherto depended on mathematical methods which are conducive for numerical analysis. Only numerical methods can be successfully applied to take account of, to mention only a few, the nonlinearities introduced by the saturation effects, the inclusion of the actual field maps of the region of the commutating pole, etc. Also, the numerical calculations in the design procedures were kept at a minimum, (within the range of the slide rule) by means of simplifying assumptions. Vienott[12], in a recent paper on the subject of induction machine design, speaks of this 'arithmetic barrier'.

The advent of the modern digital computer has seen tremendous changes in the area of machine design. These arithmetic barriers are definitely surmounted and consequently, the designer is no longer forced to make unnecessary simplifying assumptions so as to stay within simple mathematical expressions. This fact has definitely resulted in more precise calculations. Also, the speed of the computer has enabled the designer to look for several designs, and in effect, has raised the possibility of eliminating design changes on the test floor. The digital computer pro-

gram for commutation design presented in section (5.3) is established with this as the objective.

The main design is based on the commutation factor. Both the data and the calculations are presented in a manner best suited for digital computer application. The program details are then summarized in the form of a flow diagram, indicating the parameters which can be varied in a commutation factor investigation.

5.2 Begin Input Data

Rating:

1.		Run no.
2.		H.P./K.W. rating
3.		Line volts
4.	RPM	Revolutions per minute
5.		Amperes

Armature dimensions:

6.	Ar O.D.	Armature outer diameter
7.		Armature inner diameter
8.	L	Gross armature length
9.	N_s	No. of slots
10.		Slot depth
11.		Slot opening
12.	W_s	Widest slot width
13.		Slot width at bottom
14.	d_1	Distance between the wedge of the slot and the top coil of the slot
15.	d_2	Height of the coil in the top section of the slot

16.	d_3	Insulation between the coil sections in the slot

Armature winding details:

17.		Coil width less insulation
18.		Coil depth less insulation
19.	WPS	Conductors/slot
20.	m	No. of sections
21.		Type of winding
22.		Coil pitch in slots
23.		No. of parallel paths in the winding

Commutator and brush:

24.	K	No. of commutator bars
25.		Commutator outer dia.
26.		Mica thickness
27.		Brush width
28.		Brush thickness
29.		No. of brushes
30.		Brush angle

Main pole:

31.		Width of pole
32.		Length of pole
33.		Height of pole
34.		½ pole face chord
35.		No. of main poles
36.		Shunt field volts

Frame and commutating pole:

37. No. of commutating poles

38. Pole length

39. Pole width

40. Minimum inter-polar gap

41. Frame outer dia.

42. Frame inner dia.

43. Effective frame length

END OF INPUT DATA

CALCULATIONS BEGIN

5.3 Calculations

44. e' Embrasure $\dfrac{\text{No. of poles} \times \text{Sin}^{-1}}{180}\left[\dfrac{1/2 \text{ Pole face chord}}{0.5 \times \text{Ar O.D.}}\right]$

$(35) \times \text{Sin}^{-1}\left[\dfrac{\frac{1}{2} \times (34)}{0.5 \times (6)}\right]$

45. $\bar{A}$ $\dfrac{p \times IPG_{min}}{\pi \times Ar\ OD}$

46. $\bar{C}$ $\dfrac{IP_{arc}}{MP_{arc}}$ $\dfrac{\text{width of com. pole} \times \text{No. of poles}}{\pi \times \text{Ar OD} \times e'}$

$\dfrac{(39) \times (35)}{\pi \times (6) \times (44)}$

47. $\dfrac{p}{2\pi A}$ $\dfrac{(35)}{2\pi \times (45)}$

48. $\dfrac{\pi e' C}{p}$ $\dfrac{\pi \times (44) \times (46)}{(35)}$

49. $\bar{B}$ $1 + \dfrac{p}{2\pi \bar{A}}\left[\dfrac{1}{2}\left(\dfrac{\pi e' \bar{C}}{p}\right)^2 - \dfrac{1}{8}\left(\dfrac{\pi e' \bar{C}}{p}\right)^4 - \dfrac{1}{16}\left(\dfrac{\pi e' \bar{C}}{p}\right)^6\right]$

$1 + (47)\left[\dfrac{1}{2}(48)^2 - \dfrac{1}{8}(48)^4 - \dfrac{1}{16}(48)^6\right]$

50. IPG_{max} $\bar{B} \times IPG_{min}$

$(48) \times (39)$

51. Effective airgap of commutating pole

$$(40) + \frac{1}{3}\left[(50) - (40)\right]$$

52. Carter constant $= \dfrac{\text{widest slot width}}{\text{Effective airgap of com. pole}}$

$$= \frac{(12)}{(51)}$$

From (45), (46) and (49) and with the aid of a sub-routine, the numerical procedure of which is presented in appendix A, the values of C_1 and $\tau/2$ are calculated and located in (53) and (54).

53. C_1

54. $\tau/2$

55. Slot component of self-inductance of a coil (L_S) $\quad 3.19 \times 10^{-8} \times \frac{(8)}{(12)}\left[2(14) + \frac{5}{3}(15) + (16)\right]$

56. Slot component of mutual inductance between two coil-sides in the same slot. ($|M_S|$) $\quad 3.19 \times 10^{-8} \times \frac{(8)}{(12)}\left[(14) + \frac{(15)}{2}\right]$

57. $\Delta = \frac{\pi D}{P} \times \frac{1}{\tau/2}$ $\quad \dfrac{\pi \times (6)}{(35) \times (54)}$

58. B $\quad \dfrac{3.19 \times (40)}{(52)}$

59. Fundamental component of com. pole flux per amp. turn of airgap m.m.f. (Ψ_{c_1}) $\quad \dfrac{2(53)(58)(6)(8)}{(35)(57)}$

60. $\Psi_{c1}/4$ $\quad (59)/4$

61. Commutator pitch deficiency. $\quad \left|\dfrac{(24)}{(20) \times (35)} - (22)\right|$

62. Commutator zone in inches $\quad \dfrac{(6)}{(25)}\left\{\dfrac{(28)}{\cos(30)} - (26) + \dfrac{\pi(25)}{24}\left[(20)(1 + (61)) - \dfrac{(23)}{(35)}\right]\right\}.$

63. **Commutator zone angle in degrees** $\frac{(62)}{(6)} \times 114.6$

64. **Switching angle in electrical degrees** $\frac{(63)(35)}{4}$

65. **Pole-pitch in degrees** $\frac{360}{(35)}$

66. **Distances of the pole centers from the reference pole**

$0 \times (65) \ldots\ldots (66.0)$

$1 \times (65) \ldots\ldots (66.1)$

$2 \times (65) \ldots\ldots (66.2)$

$\vdots$

$(p-1) \times (65) \ldots\ldots (66.\overline{p-1})$

where $(p-1) = (35) - 1$

67. **Limits of the zones of commutation under each pole**

$(66.0) + (63) \ldots\ldots (67.0)$

$(66.1) + (63) \ldots\ldots (67.1)$

$(66.2) + (63) \ldots\ldots (67.2)$

$\vdots$

$(66.\overline{p-1}) + (63) \ldots (67.\overline{p-1})$

68. **Commutating zones under each pole**

$CZ_1 = (66.0)$ to (67.0)

$CZ_2 = (66.1)$ to (67.1)

$CZ_3 = (66.2)$ to (67.2)

$\vdots$

$CZ_p = (66.\overline{p-1})$ to $(67.\overline{p-1})$

69. **Slot pitch in degrees** $\frac{360}{(9)}$

70. Angles of the slots measured from the reference slot 1.

$0 \times (69) \ldots\ldots (70.0)$

$1 \times (69) \ldots\ldots (70.1)$

$2 \times (69) \ldots\ldots (70.2)$

$\vdots$

$(N_S - 1) \times (69) \ldots\ldots (70.N_S - 1)$

71. Search for the number of slots, 0,1,2..... $(N_S - 1)$ which lie in the regions of CZ_1 thru CZ_p. Renumber them in a sequence.

S_1

S_2

$\vdots$

S_n

72. Distance of the slots within all the com. zones from slot 1 measured in mechanical degrees. $(\alpha_1', \alpha_2', \ldots \alpha_n')$

$\alpha_1' = S_1 \times (69) \ldots\ldots (72.1)$

$\alpha_2' = S_2 \times (69) \ldots\ldots (72.2)$

$\vdots$

$\alpha_n' = S_n \times (69) \ldots\ldots (72.n)$

73. Test each α' for the following conditions and store them in a sequence after multiplying each angle by $\frac{(35)}{2}$ to convert them into electrical degrees.

If (i) $0 \leq \alpha' \leq (65)$, do not change the angle

(ii) $(65) \leq \alpha' \leq 2(65)$ subtract (65) from α'

(iii) $2(65) \leq \alpha' \leq 3(65)$ subtract 2(65) from α'

(iv) $\overline{p-1}(65) \leq \alpha' \leq p(65)$ subtract (p-1) (65) from α'

74. The distances of the slots from the reference slot 1 in electrical degrees

$\alpha_1 = (74.1)$

$\alpha_2 = (74.2)$

$\vdots$

$\alpha_n = (74.n)$

75. Coil-pitch in electrical degrees $\frac{(22) \times (65) \times (35)}{2}$

.76. Chording angle σ $\pi - (75)$

77. Airgap component of self-inductance $(60) \times 10^{-8} \left\{ 1 - \left(\sin\left[(57)\left\{(64) + \alpha_j\right\}\right]^2\right) \right\}$

78. $(64) + \alpha_j$

79. $(78) - (76)$

80. $\sin\left\{(57)(78)\right\}$

81. $\sin\left\{(57)(79)\right\}$

82. Airgap component of self-inductance $(60) \left\{ \left[1 - (80)^2\right] + \left[1 + (80)^2\right] \right\} \times 10^{-8}$

Equations in (82) are used to calculate all the self inductance of all coils in (74).

83. Total self inductance of the coil $(82) + (55)$

84. $(57) \left[(64) + \alpha_K\right]$

85. $(57) \left[(84) - (76)\right]$

86. $\sin (84)$

87. $\sin (85)$

The expressions for the airgap component of mutual inductances, M_{jk}, depend on the regions. Therefore a decision must be made to the region of application and the expression to be used.

88. If $-(\tau/2 + \gamma) \geq \alpha_j \leq \alpha_K \leq (\tau/2 - \gamma)$

$$M_{jK} = (60) \times 10^{-8} \left\{ \left[1 - (80)\right]\left[1 + (86)\right] + \left[1 + (81)\right]\left[1 + (87)\right] \right\}$$

89. If $-(\tau/2 + \gamma) \leq \alpha_K \leq \alpha_j + \sigma \leq \tau/2 - \gamma$

$$M_{jK} = (60) \times 10^{-8} \left\{ [1-(80)][1+(86)] \right\}$$

90. If $-(\tau/2 + \gamma) \leq \alpha_j - \sigma \leq \alpha_K \leq (\tau/2 - \gamma)$

$$M_{jK} = (60) \times 10^{-8} \left\{ [1+(80)][1-(86)] \right\}$$

91. If $(\tau/2 - \gamma) \geq \alpha_j \geq \alpha_K - \sigma \geq -(\tau/2 + \gamma)$

$$M_{jK} = (60) \times 10^{-8} \left\{ [1-(80)][1+(87)] \right\}$$

92. If $(\tau/2 - \gamma) \geq \alpha_K - \sigma \geq \alpha_j \geq (\tau/2 + \gamma)$

$$M_{jK} = (60) \times 10^{-8} \left\{ [1+(80)][1-(87)] \right\}$$

93. For the calculations of M_{jk}, from (88) thru (92), test whether there is the slot component of mutual inductance or not; if 'yes', add (56).

94. From the calculations (77) thru (92), form the inductance coefficient matrix $(\mathcal{M})$.

95. Form the transformation matrix S of the same order as the $\mathcal{M}$ matrix of (94).

96. Form the S^{-1} matrix. Since the form of the S^{-1} matrix is known, a routine for the inverse in going from (95) to (96) is not required.

97. $\mathcal{M}_c = S \mathcal{M} S^{-1}$

= (95) (94) (96)

98. Separate the (1x1) element of (97) — $M_{c_{11}}$

99. Form row matrix containing all the elements in the first row of (97) except (98) — $\mathcal{M}_{c_{1a}}$

100. Form column matrix containing all elements in the first column of (97) except (98) — $\mathcal{M}_{c_{a1}}$

101. Form the square obtained from (97) by deleting (98), (99) and (100) — $\mathcal{M}_{caa}$

102. Inverse of $\mathcal{M}_{caa}$ — $(101)^{-1}$ obtained by an inverse sub-routine.

103. Form triple matrix product $\mathcal{M}_{c1a}\,\mathcal{M}_{caa}^{-1}\,\mathcal{M}_{ca1}$ — (99) (102) (100) product obtained by a triple matrix product sub-routine.

104. Subtract matrices $M_{c11} - \mathcal{M}_{c1a}\,\mathcal{M}_{caa}^{-1}\,\mathcal{M}_{ca1}$ — (98)-(103)

105. Form the summing matrix of the same order as $\mathcal{M}_{caa}$ — $\mathcal{F}$

106. Form triple product $\mathcal{M}_{c1a}\,\mathcal{M}_{caa}^{-1}\,\mathcal{F}$ — (99) (102) (105) product obtained by a triple matrix product sub-routine.

107. Commutation factor — $\dfrac{104}{1-(106)}$

108. turns/section — $\dfrac{\text{conductors/slot}}{\text{2x no. of sections}}$

$$\frac{(19)}{(20)\text{x}2}$$

109. amperes/path = i_p — $\dfrac{(4)}{(23)}$

110. amp. turns/pole (ATP) — $\dfrac{\text{turns/sect. x amps x total no. of com. segments}}{\text{no. of parallel paths x poles}}$

$$\frac{(108)\ (109)\ (24)}{(35)}$$

111. Reactance voltage between bar and brush (E_R) — $\dfrac{\text{2xATP x } p^2 \text{ x turns/sect. x CF x RPM}}{10 \text{ x } 60}$

$$\frac{2\text{x}(110)\text{x}(35)^2\text{x}(108)\text{x}(4)\text{x}(107)}{10^7 \text{ x } (61)}$$

END OF CALCULATIONS.

VI. CONCLUSION

6.1 The digital computer program presented in section V is used for calculating the commutation factors of forty different machines. These machines have been built and their test performance is known. They are classified into 2-pole simplex-lap, 4-pole simplex-wave, 4-pole simplex-lap and 6-pole simplex-lap respectively. These results are presented in table (6.1.1). Table (6.1.1) includes a few pertinent details such as the length of the armature, the outer diameter of the armature, the volume (D^2L), the type of winding, the width of the slot, the commutating zone, the reactance voltage and the percentage buck and boost amperes for the "black-band" test along with the run numbers identifying the particular machine. The following conclusions are drawn from the table (6.1.1).

1.) For the same frame-size, a simplex-wave winding has a higher commutation factor than the simplex-lap winding.

It is borne out by experience that satisfactory commutation is difficult to achieve in large frame sizes using wave windings. As a matter of normal design practice, the wave-windings are avoided if possible in these machines.

2.) All other variables held constant, increase in the length of the armature results in an increase in the commutation factor.

The "Erie Sparking Factor" commonly used as a "rule of thumb" in commutation design is also directly proportional to the length of the armature.

3.) The variation of the commutation factor with volume is plotted in the form of two curves in Fig. (6.1.1) --- one for the simplex-lap and the other for the simplex-wave.

These curves show a direct correlation between the commutation factor and volume of the armature for the relative commutation pole region geometries, brush width and slot geometry used in the forty designs.

4.) All the other variables held constant, increase in slot width results in a decrease in the commutation factor.

5.) Decrease in the commutating zone results in an increase of the commutation factor, when all the other parameters are the same.

This conclusion is also related to experience in that a decrease in the width of the commutating period (with the number of coils shorted by the brushes remaining unchanged) increases the rate of change of commutating coil current.

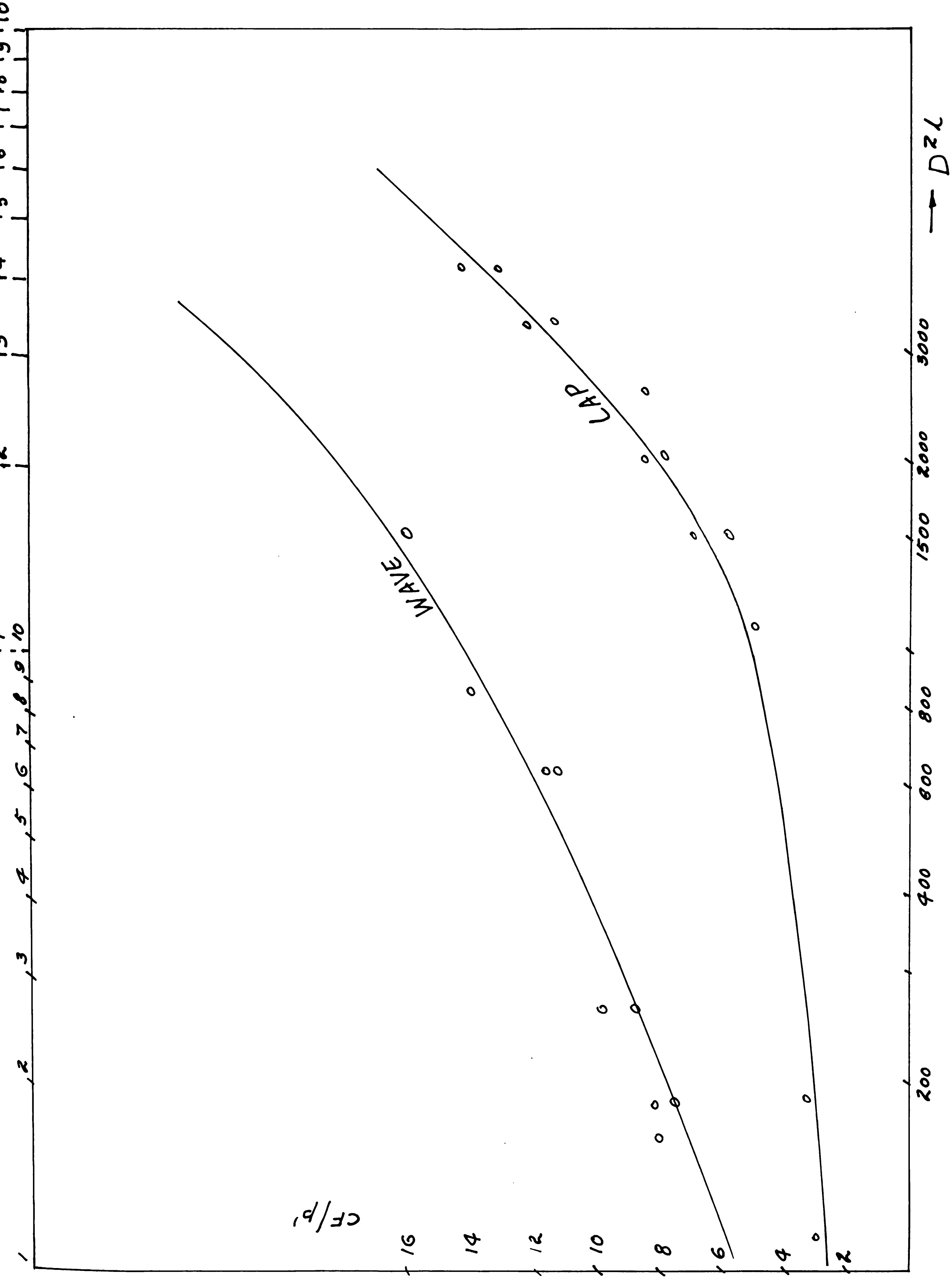
The switching angle chosen is exactly equal to half of the commutating zone. Hence, the variation of the commutation factor with switching angle is the same as that already discussed under 5).

6.) The changes in the width of the commutating pole also effect the commutation factor. However, since there is very little change in the width of the commutating pole for the samples chosen, this effect does not show in the tabulation.

7.) The total number of slots within the commutating zone also effect the commutation factor. However, again in the samples chosen, this factor remains unchanged within each group.

8.) The reactance voltage tabulated gives a numerical measure for comparing two windings in different frames designs. For the machine chosen, these numbers, in general, agree with experience.

9.) The commutation factors do not correlate with the percentage "buck and boost" obtained by the black-band tests. This is as expected,


WAVE
LAP
cF/p'
D²L
2 4 6 8 10 12 14 16
200 400 600 800 1500 2000 3000
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 110

since the commutation factor which as calculated does not include the effects of the commutating field.

The results in table (6.1.1) show that for the same frame, an armature winding with a lower commutation factor results in a better commutation. There are a number of design parameters that can be varied in the process of design. The digital computer program presented in section (5) is designed to include the possibility of varying these parameters. The exact stages in the program wherein these factors can be varied are explained by means of a 'flow diagram' of the computer program given in Fig. (6.2.1).

Box 1 represents the stage of calculations from 44 to 60 as presented in section (5.3). These calculations are used to establish the fundamental component of commutating pole flux per ampere turn of airgap m.m.f. (Ψ_{c_1}). The factors effecting the commutation factor at this stage are: (1) the minimum inter-polar gap and (2) the width of the commutating pole.

Box 2 represents the calculations involved in determining the slot inductances, ranging from 55 to 56. The factors effecting the commutation factor at this stage are: (1) the gross length of the armature and (2) the width of the slot.

The decision after box 2, whether to choose a lap winding or a wave winding, is crucial in the realization of satisfactory commutation characteristics.

Boxes 3, 4, and 5 represent the calculations ranging from 65 to 74. These calculations determine the number of slots in the commutating zones and their respective locations with respect to the reference slot. The number of slots in the commutating zones effect the commutation factor

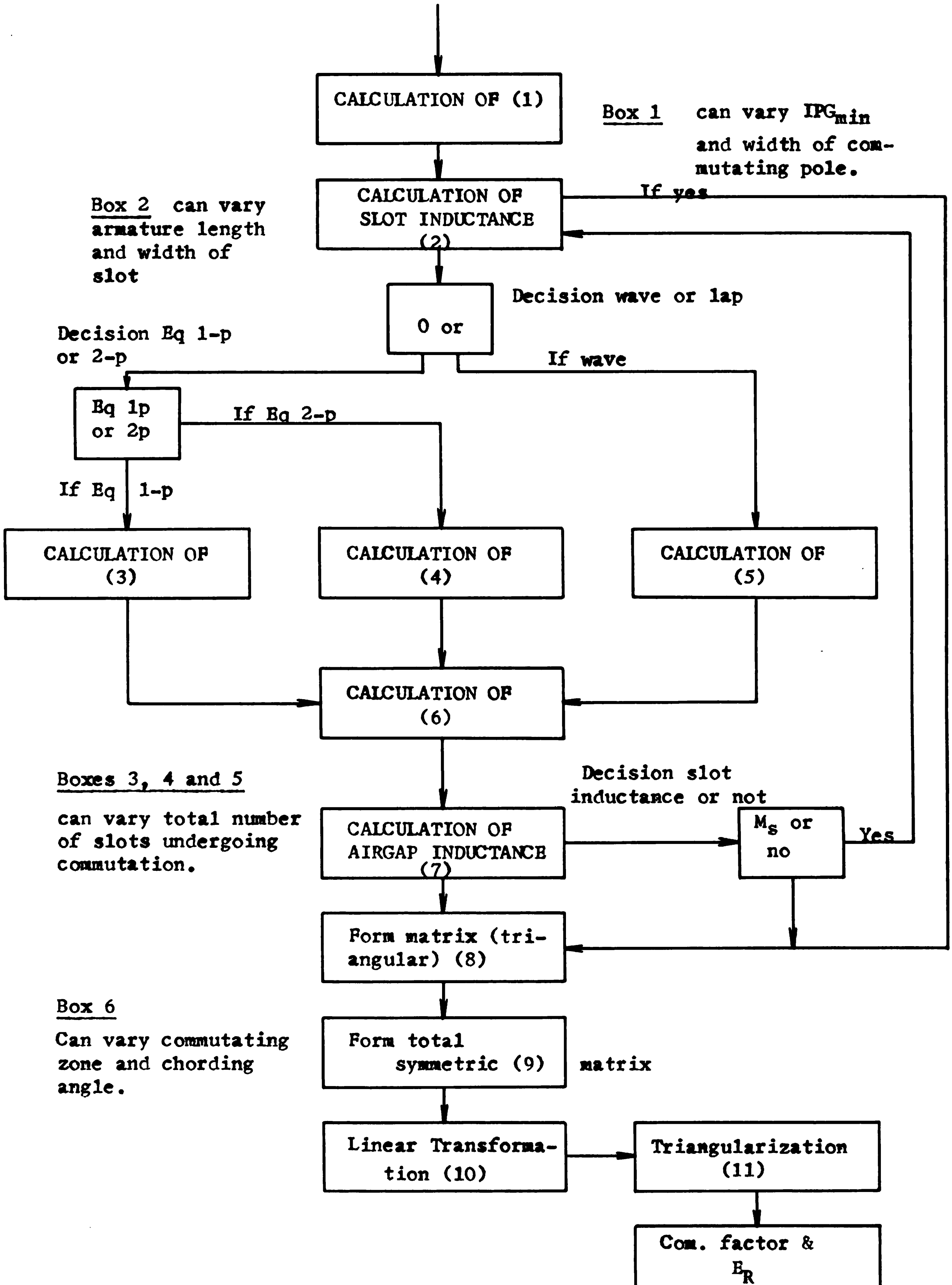
and can be varied to an advantage.

Boxes 6 and 7 represent the calculations ranging from 61 to 64 and 75 to 93 respectively. This is the stage where the airgap inductances are calculated. Here, the factors effecting the commutation factor are: (1) the commutating zone angle and (2) the coil pitch or the chording angle.

Boxes 8 through 12 represent the calculations from 94 to 107. There is no variable parameter in this stage of the program.

6.3 Unsolved Problem

Closely associated with the study of the armature windings, is a problem frequently referred to as the problem of 'field weakening'. When the main field is weakened beyond a particular value, very bad commutation results. The only method presently available is to make adjustments on the test floor to establish the number of turns and airgap of the commutating pole. These adjustments are made to give optimum commutation characteristics at rated conditions. This problem is included in the second phase of the commutation problem mentioned in the introduction. The work of Koenig brings out this problem in all its details. To achieve some concrete results from the mathematical foundations that have already been laid out, additional research of a long-term nature is needed. For the present it is sufficient to point out that the $\Delta\lambda$ term in (3.2.9) represents the effect of the commutating field and the external conditions on commutation. However, how this term is related to 'field weakening' is yet unknown.



CALCULATION OF (1)
Box 1 can vary IPG_{min} and width of commutating pole.
If yes
CALCULATION OF SLOT INDUCTANCE (2)
Box 2 can vary armature length and width of slot
0 or
Decision wave or lap
Decision Eq 1-p or 2-p
If wave
Eq 1p or 2p
If Eq 2-p
If Eq 1-p
CALCULATION OF (3)
CALCULATION OF (4)
CALCULATION OF (5)
CALCULATION OF (6)
Boxes 3, 4 and 5
can vary total number of slots undergoing commutation.
Decision slot inductance or not
CALCULATION OF AIRGAP INDUCTANCE (7)
M_s or no
Yes
Form matrix (triangular) (8)
Box 6
Can vary commutating zone and chording angle.
Form total symmetric (9) matrix
Linear Transformation (10)
Triangularization (11)
Com. factor & E_R

No.	Run No.	No. of Poles	Type of Winding	Dia.	Length of Ar.	D^2L	Width of slot	IPG	Com. zone	%BB	Com. fa	ER
1	1434	2	Lap	5.0	3.375	84.5	0.432	0.039	1.859	11	5.06	7.50
2	1429	2	Lap	5.0	4.50	112.5	0.432	0.048	1.921	30	6.00	5.50
3	856	2	Lap	5.0	4.50	112.5	0.432	0.059	1.859	14	6.02	7.64
4	1430	2	Lap	5.0	4.50	112.5	0.432	0.049	1.859	4.5	6.26	8.00
5	1433.1	2	Lap	5.0	4.50	112.5	0.432	0.048	1.859	18	6.28	6.60
6	1382	2	Lap	5.0	4.50	112.5	0.432	0.053	1.859	17	6.42	7.40
7	1398	4	Wave	6.0	3.625	130.5	0.216	0.057	1.190	6	5.70	13.2
8	645	4	Wave	6.0	4.50	162.0	0.190	0.053	1.10	17	6.32	5.4
9	1392	4	Wave	6.0	4.50	162.0	0.216	0.053	1.303	6.5	6.70	8.45
10	1258	4	Wave-Dead Coil	6.75	4.25	193.0	0.190	0.060	1.186	15.0	7.62	10.5
11	1393	4	Wave	6.0	4.50	162.0	0.190	0.053	1.107	9	8.02	19.3
12	1248	4	Wave	6.75	4.25	193.0	0.210	0.063	1.194	13	8.19	18.7
13	1301	4	Wave-Dead Coil	6.75	5.75	262.0	0.190	0.063	1.186	8	9.97	11.6
14	1256	4	Wave	6.75	5.75	262.0	0.210	0.062	1.194	8.5	10.4	16.9
15	428	4	Wave	6.75	5.75	262.0	0.210	0.058	1.186	26	10.6	9.2
16	1243	4	Wave	6.75	5.75	262.0	0.210	0.060	1.194	14	10.7	12.8
17	1543	4	Wave	9.50	7.125	645.0	0.260	0.112	1.691	5.5	11.3	9.15
18	1544	4	Wave	9.50	7.125	645.0	0.260	0.105	1.691	5.5	11.6	11.9
19	1545	4	Wave	9.50	9.50	860	0.260	0.172	1.583	18.5	14.0	8.5
20	1551	4	Wave	11.50	11.50	1520	0.329	0.140	2.036	15.5	16.1	9.3
21	1548	4	Wave	11.50	8.25	1090	0.329	0.165	1.907	14.5	32.9	28.8
22	1549	4	Lap	11.50	8.25	1090	0.329	0.180	2.040	16.0	9.82	14.8
23	1554	4	Lap	13.00	9.00	1520	0.337	0.159	2.242	21.0	11.40	12.8
24	1546	4	Lap	9.50	9.50	860	0.260	0.128	1.691	22.0	13.6	12.2
25	1552	4	Lap	11.50	11.50	1520	0.329	0.165	2.040	22.0	13.76	10.30

Table (6.6.1)

No.	Run No.	No. of Poles	Type of Winding	Dia.	Length of Ar.	D^2L	Width of slot	IPG	Com. Zone	%BB	Com. fa	BR
26	1556	4	Lap	13.0	12.00	2015	0.337	0.160	2.242	34.0	15.18	12.30
27	1557	4	Lap	13.0	12.0	2015	0.337	0.140	2.160	12.0	15.36	19.0
28	1561	4	Lap	15.5	11.0	3370	0.308	0.180	2.444	16.5	16.68	16.9
29	1558	4	Lap	13.0	12.0	2015	0.308	0.143	2.000	14.5	17.56	29.2
30	1564	4	Lap	15.50	14.0	3360	0.327	0.172	2.264	17.0	23.0	33.8
31	1563	4	Lap	15.50	14.0	3360	0.332	0.172	2.342	13.5	24.4	20.4
32	1598	6	Lap	30.099	19.25	17,450	0.510	0.432	2.794		22.32	41.1
33	1588	6	Lap	30.099	19.25	17,450	0.510	0.300	2.794		22.71	31.8
34	1599	6	Lap	30.099	19.25	17,450	0.562	0.302	2.769		26.25	60.3
35	1592	6	Lap	30.099	19.25	17,450	0.472	0.363	2.648		30.30	32.4
36	1590	6	Lap	30.099	19.25	17,450	0.472	0.370	2.640		30.30	38.1
37	1591	6	Lap	30.099	19.25	17,450	0.432	0.328	2.346		36.6	56.1
38	1589	6	Lap	30.099	19.25	17,450	0.432	0.330	2.346		37.8	52.5
39	1593	6	Lap	30.099	19.25	17,450	0.357	0.333	2.497		42.3	156
40	1594	6	Lap	30.099	31.50	28,500	0.370	0.328	2.292		69.0	81.6

Table (6.1.1) Continued

Appendix A

Numerical Procedure for the Calculation of C_1 and $\tau/2$

The numerical procedure for calculating items (53) and (54) in the computer program of section (5.3) is given below in nine discrete steps.

1.) Input the family of curves in Fig. (4.1.3), by the method of polynomial approximation using the method of 'least squares'. Curve-fitting as the basis of this numerical method forms an auxiliary subroutine. All the curves within the family are approximated by a polynomial of the same degree.

2.) Form another polynomial of the same degree whose whose coefficients are a function of $\bar{B}$.

For a value of $\bar{B}$, for which a curve is already drawn such as in Fig. (4.1.3) and for which the polynomial approximation is already made in step 1, the new polynomial of step 2) coincides with it.

3.) For a value of $\bar{B}$ different from the $\bar{B}$'s of Fig. (4.1.3), the coefficients of the polynomial are calculated by interpolation formulas.

4.) Given the value of $\bar{A}$, with the aid of the polynomial in 3), calculate C_3.

5.) Value of C_1 is given by the relationship, $C_1 = 1-C_3$.

6.) Steps 1) and 2) are applied to the family of curves of Fig. (4.1.4).

7.) For a given value of $c/2$, the coefficients of the polynomial relating $a = \tau/2w$ and $\bar{B}$ are calculated by similar interpolation formulas as in step 3).

8.) Given the value of $\bar{B}$ from the expression resulting from step 7), calculate a.

9.) Calculate $\tau/2$ by multiplying a by ω = width of the commutating pole.

9.) Calculate $\tau/2$ by multiplying a by ω = width of the commutating pole.

Appendix B

Two-pole Equivalent Representations

For a machine with more than two poles, an 'equivalent two-pole' representation is possible for certain windings such as the simplex-lap. Such 'equivalent two-pole' representations are not possible in simplex-wave windings since the winding does not repeat every pair of poles. The properties of the coefficient matrix and the transformations necessary to realize the equivalent two-pole representation are discussed for a six-pole machine with a simplex-lap winding in which three slots are short-circuited by each brush. The system of equations are of the form:

$$\begin{bmatrix} \Delta \psi_1 \\ \Delta \psi_2 \\ \hline \Delta \psi_3 \\ \Delta \psi_4 \\ \hline \Delta \psi_5 \\ \Delta \psi_6 \end{bmatrix} = \left[\begin{array}{cc|cc|cc} m_{11} & m_{12} & m_{13} & m_{14} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} & m_{23} & m_{24} \\ \hline m_{13} & m_{14} & m_{11} & m_{12} & m_{13} & m_{14} \\ m_{23} & m_{24} & m_{21} & m_{22} & m_{23} & m_{24} \\ \hline m_{13} & m_{14} & m_{13} & m_{14} & m_{11} & m_{12} \\ m_{23} & m_{24} & m_{23} & m_{24} & m_{21} & m_{22} \end{array}\right] \begin{bmatrix} \Delta i_1 \\ \Delta i_2 \\ \hline \Delta i_3 \\ \Delta i_4 \\ \hline \Delta i_5 \\ \Delta i_6 \end{bmatrix} \tag{B.1}$$

where $\Delta \psi$ and Δi represent the same quantities as in eq. (3.2.5). The subscripts 1 through 6 refer to the respective brushes and the order of each m matrix is equal to the number of slots closed by the corresponding brush. Following the indicated lines of partitioning equation (B.1) can be written as:

$$\begin{bmatrix} \Delta \psi_A \\ \Delta \psi_B \\ \Delta \psi_C \end{bmatrix} = \begin{bmatrix} m_{aa} & m_{ab} & m_{ab} \\ m_{ab} & m_{aa} & m_{ab} \\ m_{ab} & m_{ab} & m_{aa} \end{bmatrix} \begin{bmatrix} \Delta i_A \\ \Delta i_B \\ \Delta i_C \end{bmatrix} \tag{B.2}$$

For windings that repeat every pair of poles, equation (B.2) has the following properties. First, the entire matrix is symmetric. Secondly, the submatrix in the (1,3) position is the same as that in the (1,2) position and the submatrix in the (2,3) position is the same as that in the (3,1) position. In other words, the entire coefficient matrix is cyclic with respect to the six submatrices. It is to be emphasized here, that it is this cyclic character of the coefficient matrix that makes an 'equivalent two-pole' representation possible.

The coefficient matrix is diagonalized with respect to the submatrices by the following linear transformation of variables. Let

$$\begin{bmatrix} \Delta \Psi_A^s \\ \Delta \Psi_B^s \\ \Delta \Psi_C^s \end{bmatrix} = S \begin{bmatrix} \Delta \Psi_A \\ \Delta \Psi_B \\ \Delta \Psi_C \end{bmatrix} \tag{B.3}$$

and

$$\begin{bmatrix} \Delta \mathcal{I}_A^s \\ \Delta \mathcal{I}_B^s \\ \Delta \mathcal{I}_C^s \end{bmatrix} = S \begin{bmatrix} \Delta \mathcal{I}_A \\ \Delta \mathcal{I}_B \\ \Delta \mathcal{I}_C \end{bmatrix} \tag{B.4}$$

where the symmetrical component transformation matrix

$$S = \begin{bmatrix} \mathcal{U} & \mathcal{U} & \mathcal{U} \\ \mathcal{U} & a\mathcal{U} & a^2\mathcal{U} \\ \mathcal{U} & a^2\mathcal{U} & a\mathcal{U} \end{bmatrix} \tag{B.5}$$

where $\mathcal{U}$ is the unit matrix and $a = e^{j2\pi/3}$. The order of the unit matrix $\mathcal{U}$ is the same as the order of the square submatrices in (B.2). In this particular example, it is (6x6) since 3 slots are short-circuited by each

brush. The equations resulting from the change of variables are:

$$\begin{bmatrix} \Delta \psi_A^s \\ \Delta \psi_B^s \\ \Delta \psi_C^s \end{bmatrix} = \begin{bmatrix} (\mathcal{M}_{aa} + 2\mathcal{M}_{ab}) & 0 & 0 \\ 0 & (\mathcal{M}_{aa} - \mathcal{M}_{ab}) & 0 \\ 0 & 0 & (\mathcal{M}_{aa} - \mathcal{M}_{ab}) \end{bmatrix} \begin{bmatrix} \Delta \mathcal{I}_A^s \\ \Delta \mathcal{I}_B^s \\ \Delta \mathcal{I}_C^s \end{bmatrix} \tag{B.6}$$

when $\Delta \mathcal{I}_A = \Delta \mathcal{I}_B = \Delta \mathcal{I}_C$ and $\Delta \psi_A = \Delta \psi_B = \Delta \psi_C$, we have $\Delta \psi_A^s = 3\,\Delta \psi_A$, $\Delta \mathcal{I}_A^S = 3\Delta \mathcal{I}_A$ and the top equation in (B.6) reads

$$\Delta \psi_A^S = (\mathcal{M}_{aa} + 2\mathcal{M}_{ab})\,\Delta \mathcal{I}_A^S \tag{B.7}$$

Equation (B.7) represents the equations for the two pole equivalent of the machine. In terms of the submatrix notation in (B.1) the equivalent two-pole representation appears as

$$\begin{bmatrix} \Delta \psi_{P_1} \\ \Delta \psi_{P_2} \end{bmatrix} = \begin{bmatrix} \mathcal{M}_{P_{11}} & \mathcal{M}_{P_{12}} \\ \mathcal{M}_{P_{21}} & \mathcal{M}_{P_{22}} \end{bmatrix} \begin{bmatrix} \mathcal{I}_{P_1} \\ \mathcal{I}_{P_2} \end{bmatrix} \tag{B.8}$$

where

$$\mathcal{M}_{P_{11}} = (\mathcal{M}_{11} + 2\mathcal{M}_{13}) \quad \text{and} \quad \mathcal{M}_{P_{21}} = (\mathcal{M}_{21} + 2\mathcal{M}_{23})$$

$$\mathcal{M}_{P_{12}} = (\mathcal{M}_{12} + 2\mathcal{M}_{14}) \quad \text{and} \quad \mathcal{M}_{P_{22}} = (\mathcal{M}_{22} + 2\mathcal{M}_{24})$$

The subscripts P_1 and P_2 refer to the equivalent two poles. Equation (B.8) has the same form as equation (3.2.5). Note that the coefficients in the two-pole equivalent are a simple linear sum of the coefficients appearing in the first two rows of (B.1).

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