# A USE OF AUDIO-TUTORIAL TECHNIQUES IN ARITHMETIC FOR A REMEDIAL COLLEGE ALGEBRA CLASS

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This is to certify that the

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## ABSTRACT

# A USE OF AUDIO-TUTORIAL TECHNIQUES IN ARITHMETIC FOR A REMEDIAL COLLEGE ALGEBRA CLASS

By

Carl Gene Arendsen

The purposes of this study were:

1. To investigate the use of audio-tutorial techniques on remedial college algebra students' weaknesses in basic arithmetic.

2. To investigate the effect of students volunteering for additional aid on achievement in basic arithmetic in a remedial college algebra class.

3. To investigate the effect of the individual arithmetic audio-tutorial module on achievement of the objectives for that module.

4. To investigate the retention of material presented in the audio-tutorial modules.

To investigate the use of audio-tutorial techniques, ten modules, each consisting of slides, a tape, a list of objectives, a worksheet, a pretest, a posttest and answers to the pretest and worksheet, were prepared. These modules covered the basic operations of arithmetic using whole numbers, decimals and fractions, square root, per cent and the use of formulas to find areas and volumes. Each module was designed to provide instruction for from two to four objectives.

Based on the results of the Mathematics 082 Arithmetic Pretest, an individual list of at least four suggested modules was given to each of 77 of the 107 students enrolled in the non-credit course,

Mathematics 082 (Beginning Algebra 2), at Michigan State University during Spring Quarter, 1971. Using the 53 students who volunteered for additional aid through use of the modules, students were randomly assigned to a treatment group or to a control group. The remainder of the 77 students were placed in the non-volunteer group.

Three carrels to be used by the experimental group were placed in the MATHELP room in Wells Hall at MSU. MATHELP tutors and students from Methods in Teaching Secondary Mathematics (Education 327N) were used to supervise the carrels from 8:00 a.m. to 5:00 p.m. Monday through Friday. The experimental group was given approximately three weeks in which to complete the basic arithmetic material contained in the modules. This was done to provide all students with a chance to work in the carrels at some later time during the quarter and to encourage all students to erase arithmetic deficiencies as soon as possible.

At the end of the three-week period, the Mathematics 082 Arithmetic Posttest was given to all students still in the course. The test was repeated approximately 3½ weeks later, and results were obtained for the treatment and control groups. Statistical analysis of these test scores and scores on the module pretests and posttests were performed to aid in the investigation.

The first two purposes listed were examined using the Arithmetic Pretest score as co-variable and the Arithmetic Posttest score as the dependent variable in a one-way analysis of covariance. The test revealed significant differences in mean scores. Post Hoc testing (using Tukey's HSD test) showed that the mean score of the treatment

group was significantly higher than the mean scores of either the control group or the non-volunteer group. There was no significant difference between the mean scores of the control group and the non-volunteer group. Mean scores and standard deviations of the three groups on the Arithmetic Pretest also indicated no differences between groups in entering behavior.

The third purpose was pursued using individual module pretest and posttest mean scores and the number of students who completed each module at the 80 per cent and 90 per cent mastery levels. These statistics indicated that the experimental group students did learn the material in the modules they completed.

The retention level of the experimental group was determined using the scores of the Arithmetic Posttest and the Retention Test (the repeated Arithmetic Posttest) in a repeated measures design and analysis. The results indicated that the experimental group had maintained their achievement at approximately the same level above the control group, since significant differences were found between the two groups; but no significant interaction was present.

The results indicated that the audio-tutorial materials and techniques used in this college level remedial mathematics course were successful in achieving the objectives of the study.

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Carl Gene Arendsen

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DEDICATION

TO DEE

For Her Patience and Understanding

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# CHAPTER I

# INTRODUCTION

## Statement of the Problem

The purposes of this study were:

1. To investigate the use of audio-tutorial techniques on remedial college algebra students' weaknesses in basic arithmetic.

2. To investigate the effect of volunteering for additional aid on achievement in basic arithmetic in a remedial college algebra class.

3. To investigate the effect of the individual arithmetic module on achievement of the objectives for that module.

4. To investigate the retention of material contained in the modules.

For the investigation of the use of audio-tutorial techniques in arithmetic on the college level, materials consisting of audiotapes, slides (in PASS strips) and worksheets consistent with the objectives of the material were prepared. A room containing three carrels for student use of materials was also prepared. Treatment group scores on an arithmetic achievement test were compared to scores of a randomly equivalent control group.

Individual achievement on each module was examined through the use of a pretest and a posttest given immediately before and after use of each module.

The effect of volunteering on achievement level was also measured by use of the arithmetic achievement test. A comparison was made between the scores of a group of non-volunteers and a group of volunteers. Both groups received the same instruction in the six small sections of the course, Beginning Algebra 2 (Mathematics 082), at Michigan State University. These two groups were also compared for differences in achievement level on entry behavior. It should be pointed out here (and discussed more fully later in this chapter) that it was impossible to make these groups randomly equivalent.

To investigate the level of retention of material contained in the modules, the achievement test was repeated approximately 3½ weeks after the first test. The treatment group scores were compared to the scores of the control group.

## Importance of the Study

The instruction of remedial classes in college mathematics deserved attention for several reasons. Students beginning such a course have possible a great variety of entering behaviors. Their mathematics background may consist of four years of high school mathematics or of very little mathematics taken several years before enrollment in the course. Concerning the general variability among college students, McConnell and Heist, using American College Examination (ACE) scores, wrote:

> Most college instructors are probably well aware of differences in student ability, but is doubtful that most of them realize the degree of diversity that exists or adapt their teaching procedures accordingly. More than 85 per cent of the schools had an entering student body whose distribution of ACE scores extended over more than three standard deviations of the distribution of all entering students in the total sample. And 35 per cent of

> these schools had distributions extending over five standard deviations.[1]

Even students enrolled in a remedial mathematics class exhibited varying entry behavior. Jones gave an entering achievement level test to 126 remedial college algebra students. He stated:

> ... a significant number of students were unable to do even half of the basic arithmetic problems while an equally significant number of the students were quite competent on all but two or three problems of the test. It was painfully obvious to the writer that no conventional course could be offered which would meet the individual needs of these students.[2]

The study was also important because of the nature of remedial students. Experience in the instruction of remedial mathematics courses indicated that these students did not respond well to lecture classes. Indeed, Ryan wrote:

> Students with educational deficiencies, underachievers, do not do well in classes in which information is presented primarily or solely through lecture. They do better when methods are used which provide opportunity for immediate feedback, when material is presented in optimum sequence and when students are active rather than passive.[3]

Remedial non-credit mathematics classes often have poor attendance. When taught by the large lecture-discussion method at Michigan State University, attendance at the discussion sections usually fell off

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[1] T. P. McConnell and Paul Heist, "The Diverse Student Population," The American College, ed. by G. Nevitt (New York: John Wiley and Sons, 1961), p. 28.

[2] J. Howard Jones, "The Effects of Entering Achievement Level and Time Spent in Course Completion on Final Examination Performance in a Remedial Algebra Course for University Students" (unpublished Ph.D. dissertation, Michigan State University, 1971), p. 34.

[3] T. Antoinette Ryan, "Research: Guide for Teaching Improvement" (Address at Oregon State Conference on College Teaching, Willamette University, Jan., 1968).

sharply during the quarter in the remedial Mathematics 082. Attendance also declined in the large lecture. Of 143 students registered for the class during the Winter Quarter of 1971 a check revealed that less that 40 students attended each of the last two large lectures which were held each Thursday.

Another reason for conducting this study was an attempt to begin improving instruction and level of achievement in a remedial class. Examination of past records revealed failure rates (in a pass-fail system) of 40 to 60 per cent in Mathematics 082; with the usual rate close to 50 per cent.

Begle wrote of the need to continue researching mathematics education at the elementary and secondary levels:

> Our major mistake in mathematics education has been our failure to recognize that we have not possessed the tools needed to do a good job in improving mathematics education, and that in the course of carrying out our normal activities as teachers and as mathematicians we are not likely to be provided with these tools.
>
> Let me hasten to say that I do not believe that this mistake has had disastrous results. On the contrary, I am convinced that even though the guide-posts we followed and the tools we used in our attempts over the last decade to improve mathematics education were of dubious validity, we did move in the right direction and we have achieved positive results. ... The time and efforts we have devoted to reform during the last decade has not been wasted.
>
> Nevertheless, we cannot stop now. Further improvements are essential. Our children will live in an even more complicated and more quantified world than that of today. They need a better mathematics program than they are now getting. We still have many difficult problems to solve before we can make further improvements.[1]

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[1] E. G. Begle, "The Role of Research in the Improvement of Mathematics Education," Educational Studies in Mathematics, II, 237-38.

These comments apply equally to instruction at the college level in general.

Finally, Jones has commented on the need for further research and changes needed in the course with which this study was concerned:

> Approximately fifteen years of the lecture method has left the student in a position requiring him to take a remedial algebra class in college. We then offer the student another lecture course which is significantly different from courses in his prior experience only in the ratio of number of students to lecturer. The student has been trained to view mathematics as a spectator sport in which the number of spectators is ever increasing as the opportunity to perform is ever decreasing.
>
> A major change of method is due in the teaching of Mathematics 082 and such change should be in the direction of individualized instruction. Such methods as audio-visual-tutorial learning, programmed learning and computer assisted learning should be considered for immediate adoption to relieve the plight of the student in Mathematics 082.[1]

## Definitions

Terms used in the study with which the reader may not be familiar are defined below:

### Carrel

A carrel was a 46 inch by 22 inch by 22 inch booth designed for individual study. Each carrel was equipped with a cassette tape player, a PASS strip projector, a set of earphones, a screen suitable for the slides (frames) and a bulletin board upon which were mounted full directions for use of the carrel (Appendix A1).

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[1] Jones, "Effects of Entering Achievement Level and Time Spent on Examination Performance," pp. 17, 18.

### PASS Strip

A PASS strip was a cardboard strip 1 7/8 inches by 8 1/4 inches that contained eight half-frame (18 x 24 millimeter) pictures for use with the projector. "PASS" is an acronym for Plan A Show Strip.

### Module Packet

A module packet consisted of papers listing: materials needed for the module, behavioral objectives for the module, a pretest, a worksheet and answers to the pretest and worksheet (Appendix A2).

### Module

A module consisted of a set of 7 to 9 PASS strips, a 30-37 minute tape, a module packet, and a posttest. Each module was designed to be completed in approximately 75 minutes.

### Module Pretest

A test of 5 to 10 questions, given at the beginning of each module, covering the objectives for the module. A score of 80 per cent was the criterion level and indicated a student did not need the module.

### Module Posttest

A test parallel to the corresponding module pretest. This was given upon completion of each module to measure the extent to which the objectives for the module were attained (Appendix A3).

## Operating Procedure

During Fall Quarter, 1970, and Winter Quarter, 1971, arrangements were made for three carrels to be set up in the MATHELP room in

Wells Hall at Michigan State University. Also during this time ten modules were designed and prepared to cover basic arithmetic material. The modules contained instruction on addition, subtraction, multiplication and division of whole numbers and decimals, addition, multiplication and division of fractions, per cent, square root and the use of formulas in finding areas and volumes.

Since the sample used will be described in detail in Chapter III, only the operation of the carrel room is discussed below.

Using the Student Advisory Committee tutors for 15 hours per week and students from Education 327N, Methods in the Teaching of Secondary Mathematics,(each working 70 minutes per week) the carrel room was kept open from 8:00 a.m. to 5:00 p.m. Monday through Friday. During the three-week period in which the students in the experimental group were to complete their modules, sign up sheets for reserved carrel time were placed in the undergraduate mathematics office.

Before the study began, a meeting was held in order to acquaint all tutoring students with the procedure for running subjects through the carrels. In addition to this meeting, the writer was present at the beginning of the first of each student's working hours to recheck the procedure with him.

The carrel room contained, in addition to the three carrels, a five-drawer filing cabinet, three tables with two chairs each, and six armchair desks. The tables and desks were used for taking pretests and posttests when the carrels were all in use. Using this method no student had to wait for a carrel because of extra time needed by the previous student. The tables and chairs were also used by the tutors for other mathematics classes. The file contained all materials needed for completion of the modules. The top drawer held all the packets for each of the ten modules and the individual module reports (Appendix A4). Three complete sets of PASS strips for each module were kept in the second drawer. The third drawer contained four copies of each tape (one extra). The posttests and their keys were kept in the fourth drawer. Finally, the fifth drawer contained an extra tape player, an extra projector and an extra projector bulb. This drawer also contained a folder for each student in the study. After completion of each module, all papers upon which the student had written were paperclipped together and placed in his folder.

Each student was given a full list of the modules he should complete; however, if he could not remember which module he needed, a master list was kept in the fifth drawer of the file. An explanation of how these modules were determined is contained in Chapter III.

Upon entering the carrel room, the student went to the person in charge and told him which module he wanted to complete. The person in charge then went to the file and got an "Individual Module Report," a packet, the PASS strips and a tape for the particular module. The student was given the pretest, told to complete it, and to then return it to the person in charge for immediate grading. If the student reached the criterion score of 80 per cent,he was given the choice of leaving, completing the module or beginning another module. The students often chose to leave; however, some students still took the module. If the student did not reach the criterion score, he was given the materials for the module (including pretest and worksheet answers). The person in charge then checked to be certain the student knew that the tape, PASS strips and worksheet were to be used together, and that the student knew how to operate the carrel. The tape and PASS strips both contained instructions for the use of the worksheet. The student was also reminded that he could ask the person in charge questions at any time.

After completing the tape, PASS strips and worksheet, the student again went to the person in charge for the module posttest. After taking the posttest, the student took it back to the person in charge for immediate grading. If the criterion score was reached, the student recorded his score, straightened the carrel, returned the tape and PASS strips and then left the room. The person in charge then filed all the student's papers, on which there was writing, in that student's folder and returned the tape and PASS strips to their proper places. If the criterion score was not reached, the student could repeat the module or he could seek aid from the person in charge. Most students discussed their results with the person in charge and did not repeat the module.

During the three weeks in which the modules were to be completed, the writer maintained contact with the experimental group students. This was accomplished in part by attending each section of the course for approximately ten minutes on the first day of the second week of the study. The students were praised for their efforts and were encouraged to "keep up the good work." This visit was also made so that the students could connect a face with the voice on the tapes. On the first day of the third week each experimental group student was given a personal progress report. The student recieved a paper which listed his completed modules and the modules he had remaining (Appendix A5).

## The Hypotheses

The hypotheses are given in testable form in Chapter III and are stated only briefly below. First, it is expected that members of the treatment group will achieve higher scores on the arithmetic achievement test (Appendix A6) than either the control group or the non-volunteer group. It is also expected that a large percentage of students taking each module will achieve the criterion score of 80 per cent after at most two trials. It is expected that no difference in achievement level between the control group and the non-volunteer group will be found. Finally, it is expected that there will be a significant difference in achievement test scores between the treatment group and the control group after 3 1/2 weeks.

## ASSUMPTIONS

1. The sample did not differ from the population at Michigan State University or from similar populations at other colleges.

2. It was assumed that no student had an opportunity to practice the questions on any examination.

3. Since each student operated independently in the carrels, it was assumed that there was no teacher effect. This assumption was further warranted by the fact that students were chosen for the study (by a random process) from each of the six small sections of the course. Each small section was taught by a different instructor. Specific arithmetic instruction was not part of the course.

## Limitations

The purposes of this study have been clearly stated. The study was limited by the number of students in the course, Beginning Algebra 2 (Mathematics 082). The effect of volunteering on achievement deserves short comment. The group using the modules and the control group were randomly equivalent. The group that consisted of non-volunteers could not be considered randomly equivalent to either of these other samples. Conclusions concerning comparisons between the non-volunteers and the volunteer group should be considered with this in mind. It was not a purpose of the study to examine reasons for volunteering. In order to justify the exclusion of students from the treatment group, opportunity had to be given all students for carrel time at a later date in the course. Therefore, the retention level test scores for the treatment group were compared to only the scores of members of the control group who later chose to complete no modules. The length of the quarter also limited the retention study.

## Succeeding Chapters

In Chapter II the background of the problem and a review of the related literature is given.

Chapter III contains a detailed description of the study. Included is a complete description of the sample and the statistical design of the study.

Chapter IV consists of the analysis of the collected data. Included is a description of the results and the interpretation of the data collected on the individual modules.

Chapter V lists the conclusions based on the results of the analysis and recommendations for further research.

# CHAPTER II

# BACKGROUND OF THE PROBLEM AND REVIEW OF THE LITERATURE

## Introduction

Curriculum modification is a continuing process. Over the last few years, many changes in content and methods of instruction have occurred. The search for better means of instruction has been and should continue to be one aim of education in general. The changes in the mathematics content and the amount of research in mathematics education at all age levels serves as one example of this continuous process.

The value of reviewing and examining past work to improve instruction is pointed out by Dewey:

> The way out of scholastic systems that make the past an end in itself is to make acquaintance with the past a means of understanding the present.[1]

Thus, we must not be content with the status quo. We must examine the past for clues to methods of improving mathematics education.

A brief history of the present mathematics curriculum will aid in understanding today's mathematics education. The next few pages present a view of some of the factors which have influenced present mathematics education. The literature is then reviewed in four areas which are closely connected with this study: 1) audio-tutorial instruction, 2) individualized instruction, 3) remedial instruction, and 4) mastery learning.

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[1] John Dewey, cited by Sidney Hook, Education for Modern Man - A New Perspective (New enlarged edition; New York: Alfred A. Knopf Publishing Co., 1966), p. i.

## Background of the Problem

Butler and Wren reported that the present American public secondary school had its beginning during the 1830's. The exact date of distinction between its two predecessors (the Latin grammar school and the academy) and our secondary school is not distinct. The change from one philosophy to the other was characterized by the effort to reform the secondary school along the lines of emerging social change.[1]

By 1860 the philosophy that was to shape the development of the public high school had begun to exert major influence over the Latin grammar school and the academy. Concerning the next 30-35 years Butler and Wren wrote:

> There was now a strong tendency toward expansion in both number and content of courses offered, as well as an attempt to blend intellectual and practical training in the same school. Curriculums were organized and expanded rapidly with no particular plan or definite educational objective in view. By 1890 (according to Kandel, 1930) this unrest had reached its highest point. This awkward and unsystematic expansion of the curriculum offered sufficient reason for a demand for reform, and mathematics received its share of the attack. Dissatisfaction arose from several sources relative to the results achieved in the teaching of secondary mathematics. Complaints came from the teachers of mathematics themselves that the subject was not being grasped by the pupils. A study of a large number of representative high schools (by the Committee of Ten on Secondary School Subjects) revealed that the largest percentages of failures were in Latin and mathematics. College faculties were not hesitant in letting it be known that students entered their freshman classes with poor mathematical training. Businessmen were doubtful of the opportunity for the application of high school mathematics, as taught, to problems of everyday life.[2]

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[1] Charles Butler and F. L. Wren, The Teaching of Secondary Mathematics (New York: McGraw-Hill Book Co., 1965), pp. 3-5.

[2] Ibid., pp. 8-9.

One committee which had a great influence was the Committee of Ten on Secondary School Subjects. It agreed that radical change in the secondary school curriculum was overdue. In fact, the Subcommittee on Mathematics suggested that a course in applied geometry be given in elementary school and the first year of algebra be given in the ninth grade. Other suggestions of the Committee included: 1) a formal course in geometry (including proofs) be given after the first year of algebra; 2) then, for the next two years, geometry should be taught along with algebra and some solid geometry could be included; 3) the Committee also suggested different programs for college-bound and noncollege-bound students. After the first year of algebra students expecting to go to college would spend time on trigonometry and more advanced topics in algebra, while noncollege-bound students would spend an extra year on the more "technical parts of arithmetic."[1]

Concerning the important developments and the reasons for these changes in the early 1900's, Wooton wrote:

> ... many people in the United States began to question the prevailing tenet that a high school education was reserved for the elite, and they began to view the high school as a possible vehicle for the education of the masses. This increased interest in the nature and function of secondary education derived from a number of factors. The assembly line seemed to be diminishing the need for some of the skills that demanded long apprenticeship, while at the same time white-collar jobs that demanded more formal education were growing in number. There seemed little purpose in a young man's foregoing additional formal education in favor of apprenticing to a trade that might cease to exist in a short time. Another factor was the large number of youths who were either immigrants or first-generation

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[1] Report of the Committee of Ten on Secondary School Subjects (New York: American Book Company, 1894), pp. 105-116.

> children of immigrants and whose assimilation into the American culture was a very real problem. A third factor was the wave of social conscience that swept across the country during the late 1800s and early 1900s. Overcrowded cities, low wages, monopolistic tendencies, and political corruption led to the enactment of such legislation as the Pure Food and Drug Act, the Hepburn Act, the Federal Reserve Act, and others. All these developments had implications for the secondary schools.[1]

Then in 1914 the National Education Association appointed a Commission on the Reorganization of the Secondary School. Its purpose was to reconsider the goals of America's secondary schools. The Committee issued a report in 1918 centering on what are known as The Seven Cardinal Principles of Secondary Education. These seven principles and their great influence on education were again pointed out by Wooton:

> The Commission asserted that secondary education should stress health, vocation, citizenship, ethical character, estimable use of leisure, worthy home life, and command of the fundamental processes, factors they deemed basic to the life of the citizenry of a twentieth-century democracy. This report had such a profound effect on the subsequent design of the secondary school curriculum of this country that it has been called the most influential single document in the history of American education.[2]

During the 20's and 30's major influence in the curriculum came from the report of the National Committee on Mathematical Requirements. The report titled The Reorganization of Mathematics in Secondary Education is now commonly called the "1923 Report." The significance of the report was partially due to the influence exerted by some of its writers, including E. H. Moore and J. W. Young.

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[1] William Wooton, SMSG the Making of a Curriculum (New Haven: Yale University Press, 1965), p. 1.

[2] Ibid., pp. 2-3.

Bidwell and Clason commented on the report and its influence on the curriculum:

> Of particular importance in this report was the solidification of the junior high school-senior high school curriculum. The foundations for the junior high school program were laid in the prior period. The result, as seen in this period, was the integrated or fused course we now frequently call "general mathematics." General mathematics, as originally conceived, is best represented by the mathematics course commonly taught today in grades 7 and 8. This course was conceived as a course for all students rather than as a separation point for college-bound and noncollege-bound students. Only later did the grade 9 course act as a separation point.
>
> The influence of the "1923 Report" continued until the report in 1959 of the Commission on Mathematics of the College Entrance Examination Board. That this influence did not create a corresponding change in practice was due mainly to the traditional inertia in educational practice and the depression of the 1930's.[1]

Commenting further on the substantial influence of the depression Bidwell and Clason stated:

> Because of the lack of employment more students stayed in school; at the same time, fewer students enrolled in college. Since mathematics was an elective subject, enrollments in traditional courses decreased. A new hybrid general mathematics course developed which is difficult to define and more difficult to justify. This course is still with us. Mathematics was in disfavor.[2]

The Progressive Education Association Committee on the Function of Mathematics in General Education issued its report in 1940. While this report had a stated philosophy, its effects were somewhat minimized by World War II.

With the advent of the war, the inadequacy of the secondary school

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[1] James Bidwell and Robert Clason, editors, Readings in the History of Mathematics Education (Washington, D.C.: National Council of Teachers of Mathematics, 1970), p. 361.

[2] Ibid., p. 531.

mathematics program became very obvious. The inductees into the war effort brought this deficiency to light.[1] The war effort needed trained men, and there was no need nor time to defend or to improve mathematics instruction.

As it became evident that the Allies were going to win the war, the National Council of Teachers of Mathematics (NCTM) created a Commission on Post-War Plans. The Commission issued two reports. The first was preliminary and asked for public aid in securing suggestions that would help provide adequate training in mathematics for <u>all</u> students.[2]

The second report of the Commission and its influence was summarized by Willoughby. Several recommendations were given by the Committee:

> The first was that the school should provide "functional competence" in mathematics for all pupils who are able to achieve it. The committee followed this nonspecific statement with a list of some 28 items (for example, use of per cents, measurements, vectors, axioms, trigonometry, and dollar-stretching) in which it thought pupils ought to be competent....Other recommendations of the committee included the following: consideration of arithmetic as a content subject as well as a tool subject; wiser use of drill in arithmetic, a two-track program for the ninth grade (with algebra the content of the faster track); a program to satisfy junior college students interested in mathematics for cultural reasons as well as those planning to continue in mathematics and those having prevocational needs; better preparation of mathematics teachers at all levels; and more effective use of multisensory aids in teaching mathematics. After this report, there was no significant committee report until the 1950's and the advent of so-called modern mathematics programs.[3]

As Willoughby stated, the 1950's saw the beginning of "modern

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[1] Butler and Wren, <u>Teaching Secondary Mathematics</u>, pp. 25-27.

[2] Commission on Post-War Plans First Report, <u>Mathematics Teacher</u>, XXXVII (1944), 226-232.

[3] Stephen Willoughby, <u>Contemporary Teaching of Secondary School Mathematics</u> (New York: John Wiley and Sons, Inc., 1967), p. 11.

mathematics." Krause also pointed to evidence of the modern mathematics movement prior to the launching of the USSR's Sputnik I.[1]

The past few years have been a flurry of activities. Indeed, Wooton has stated:

> The decade 1955-65 has witnessed a vast upheaval in parts of the secondary school curriculum. In particular, high school physics, chemistry, biology, and mathematics have all been subjected to intense scrutiny by subject-matter specialists, and far-reaching recommendations for changes in the content and teaching of courses in these subjects have resulted. It is difficult to find another decade in the history of education where so much attention has been devoted to such a narrow part of the curriculum by so many recognized authorities at the research level in the various sciences.[2]

In this light only two of the groups which had major influnece will be mentioned. The first of these is the School Mathematics Study Group (SMSG) which had its beginning in 1958. An Advisory Committee, consisting of teachers from all levels, mathematicians, educators and representatives of science and technology, was formed to work with the director, Professor E. G. Begle.[3]

The influence that SMSG has had on the modern curriculum is hard to overstate. Teachers and students alike today are aware of SMSG and its influence. SMSG emphasized, among other things, structure, the real number system, careful use of language, discovery and proof.[4]

Thompson and Poe provided the background and influence of the

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[1] Marina Krause, "The Modern Mathematics Movement: Evolution and Implications," Dissertation Abstracts XXIX A (Feb., 1969), 2539.

[2] Wooton, SMSG Curriculum, p. v.

[3] John Wagner, "The Objectives and Activities of the School Mathematics Study Group," The Mathematics Teacher, LIII (1960), 454-55.

[4] Willoughby, Contempory Teaching, p. 46.

second group to be mentioned, The Committee on the Undergraduate Program in Mathematics (CUPM):

> The Mathematical Association of America appointed its committee on the Undergraduate Programs in Mathematics (CUPM) in January, 1959. CUPM is charged with the responsibilities of recommending and influencing undergraduate mathematics curriculum changes. While carrying out its responsibilities, CUPM has exerted a dominant effect within a national mathematics curriculum reform movement. Many colleges and university mathematics departments have used CUPM recommendations as guidelines in structuring their present undergraduate mathematics course offerings to provide an efficient program of studies for the students of today.[1]

The reports of other committees such as the University of Illinois Committee on School Mathematics (UICSM) do not add any significant new information and have been omitted from the review. The purpose of this background was to present a brief outline of past influences on modern mathematics education. Some of the more important factors which have influenced our present-day student and his curriculum are now known.

The teaching model of Glaser has four basic components. These are: instructional objectives, student entering behavior, instructional procedures and performance assessment.[2] A brief look at present college instruction using this model will aid in understanding the methods of this study and the literature.

First, instructional objectives are rarely used in any classroom today. The text usually determines what will be taught in a course. The large number of students in many undergraduate courses are often

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[1] P. E. Thompson and R. L. Poe, "A Report of the CUPM Recommendations in the State of Texas," The American Mathematical Monthly, LXXIV, No. 10, (Dec., 1968), 1107.

[2] Robert Glaser, Psychology and Instructional Technology Training and Research Education (Pittsburg: University of Pittsburg Press, 1962), p. 6.

given departmental finals. To insure that all students cover the same material in preparation for the final, syllabi listing textbook sections to be covered are given to the instructors. Hence, the objective of the instructor is then to cover the material. Obviously, this is wrong. Objectives should be given which tell the student what is to be expected of him.

Especially in college mathematics classes, entering behavior is usually completely ignored. The only requirement for entering a course is often the completion of the previous course in the sequence. The "completion of the previous course" is obviously not a very good measure of entering behavior. It is quite likely that only a small number of students are actually finished with a course at quarter's end. Colleges must begin to pay greater attention to entering behavior. A much better system would be to first find the capabilities of each student; then, given a list of acceptable objectives for each course, determine where the student should begin on the basis of what he knows.

Instructional procedures have not changed much in many years. Students attend class, hear lectures, receive problem assignments, and possibly have some questions answered. This method is inadequate for many people. Failure rates of 40 to 60 per cent are not uncommon in remedial mathematics in other colleges. It is time to investigate more closely other instructional methods which may be applicable to large numbers of students.

Present performance assessment is also inadequate. The usual procedure now is to assign a grade to all students who have taken a test on the same day over the same material. Since students learn at such varying rates, this practice is unfortunate. Some students may be able to complete the objectives for a course in a much shorter time than given, while others need more time. Teachers could provide for different learning rates by altering their grading procedures.

College instruction has not changed much since Wood and Learned wrote in 1938:

> Each individual has some level peculiar to himself at which his education in any subject must begin. Instead of expecting the members of a college class to conform to an average, we might better arrange circumstances so that each student could make full use of what he has learned and could advance from the point where he really stands.[1]

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[1] W. S. Learned and Ben D. Wood, The Student and His Knowledge (New York: Carnege Foundation for the Advancement of Teaching, 1938), p. 44.

## Review of the Literature

This study examined audio-tutorial learning in a remedial college algebra class. The related literature was reviewed in the four areas: 1) audio-tutorial instruction, 2) individualized instruction, 3) remedial instruction and 4) mastery learning.

### Audio-tutorial Instruction

Audio-tutorial instruction in this study referred to the use of audiotapes and slides in conjunction with study assignments on worksheets. Studies in fields other than mathematics which used this technique were found and will be reported later. However, as Emery noted in 1970, studies that used the audio-tutorial technique of instruction in mathematics were very scarce.[1]

Emery used a similar method to teach a "modern mathematics" class to prospective elementary teachers. She designed and prepared multi-media materials consisting of slides, audiotapes, posters and problem worksheets appropriate to the junior college level and to the course for the prospective elementary teachers. Some commercial laboratory materials were also used. She reported that on units where no confounding variables beyond her control were introduced, the audio-tutorial method was successful above the control group.[2]

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[1] Harriett E. Emery, "Mathematics for Prospective Elementary Teachers in a Community College: A Comparison of Audio-tutorial and Conventional Teaching Materials and Modes" (unpublished Ph.D. dissertation, Michigan State University, 1970), p. 44.

[2] Ibid., pp. 1, 134-35.

A visit to the campus of Lansing Community College (LCC) in Lansing, Michigan, revealed other modified forms of the method in use. Students enroll in one of three remedial courses based on entrance examination scores. The courses are Basic Arithmetic, Beginning Algebra and Intermediate Algebra. For the Basic Arithmetic and the Beginning Algebra courses each student may choose between the usual lecture or a programmed text approach. For the Intermediate Algebra, the student may choose between the methods of traditional lecture or the use of audiotapes and workbook. The tapes and workbook used were prepared by the Merrill Publishing Company and were written by Robert Moon and Robert Davis of Fullerton Junior College in Fullerton, California. The faculty at LCC reported the method was successful, with students using the tape-workbook method achieving as much or more than the conventional classes.

A use of the audio-tutorial method similar to this study is being tried at the Oakridge Campus of Oakland Community College in Walled Lake, Michigan. Here the tapes and slides are used solely as support or supplementary material for first year algebra students.

The lack of research using audio-tutorial methods in mathematics was further noted by Emery when she wrote:

> A review of studies, which included approximately 70 reports submitted at the Audio-Tutorial System Conference at Purdue University on October 20-21, 1969, revealed that the method was used most frequently at the college level, two-year and four-year colleges (60 reports). The reports were in these areas: biology (20), botany (11), chemistry (1), physics (2), geography (2), earth or soil science (2), nursing (3), medicine (1), foods (2), social studies (5), electronics (2), and others; none were in mathematics or

mathematics education.[1]

A review of studies and papers presented at the Second Annual Audio-Tutorial Systems Conference at Purdue on November 9-10, 1970, showed the same results: no studies in mathematics or mathematics education. Thus, brief examinations of some successes of this approach in other areas are in order.

One comparative experiment using the audio-tutorial method was done in Biology at Wisconsin State University - LaCrosse. The study consisted of 190 students in an audio-tutorial section and a control group of 180 students in a conventional laboratory section. Both sections used the same test items. The study provided evidence that students enrolled in an audio-tutorial section of a general biology course achieved more than students enrolled in a conventional lecture-laboratory section.[2]

Another comparative experiment using the audio-tutorial approach was done at the Clearwater Campus of St. Petersburg Junior College in Florida. A randomly assigned audio-tutorial earth science class using one general meeting, one one-half hour quiz session and independent study hours as needed each week was compared to a control class using the usual lecture-laboratory. A 90 per cent confidence level using a Z-score test showed superiority for the audio-tutorial section.[3]

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[1] Ibid., p. 16.

[2] Phillip Sparks, Richard Nord and Loraine Unbehaun, "A Comparison of the Achievements of Audio-Tutorial and Conventional Biology Students - Wisconsin State University - LaCrosse" (Paper presented at Second Annual Audio-Tutorial Systems Conference at Purdue University, November 9-10, 1970).

[3] Joseph Gould and N. G. Langford, "A Comparison of Audio-Tutorial and Non-Audio-Tutorial Earth Science as Taught During Fall Session 69-70" (Paper presented at Purdue Conference, 1970).

No studies reviewed in any area reported the control group achieved more than an audio-tutorial class. Thus, it is time that mathematics educators begin to examine the use of audio-tutorial systems in their classes. This study was a small start.

No writing on audio-tutorial systems is complete without mentioning the work of Postlethwait at Purdue. He must certainly be regarded as one of the pioneers in the field. Introductory to his work is the statement:

> A fundamental guideline which must be given prime consideration is that "learning is an activity done by an individual and not something done to an individual." The structuring of an educational system should be done on the basis that the program must involve the learner. The teacher at best can only create a situation conducive to learning by providing the direction, facilities, and motivation to the individual learner. Immediately, it becomes apparent that the program must allow for individual differences in interests, capacity, and background.[1]

Postlethwait, Novak and Murry listed advantages of the audio-tutorial approach upon which there is general agreement:

> In the audio-tutorial approach:
>
> 1. Emphasis is placed on student learning rather than on teaching.
>
> 2. Students can adapt the study pace to their ability to assimilate the information. Exposure to difficult subjects are repeated as often as necessary for any particular student.
>
> 3. Better students are not a "captive audience" and can use their time most effectively. Their interests are not dulled by unnecessary repetition of information already learned but they are free to choose those activities which are more challenging and instructive.
>
> 4. The student can select a listening time adapted to his diurnal efficiency peak.

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[1] S. N. Postlethwait, J. Novak and H. T. Murry, Jr., The Audio-tutorial Approach to Learning (2nd ed.; Minneapolis: Burgess Publishing Co., 1969), p. 1.

5. Tapes demand the attention of the students. Students are not distracted by each other.

6. Students have more individual attention, if they desire it.

7. Scheduling problems are simplified. The four hours of scheduled time from which the students are relieved under the new system can now be distributed throughout the week as necessary to adjust to the student's activities.

8. More students can be accommodated in less laboratory space and with less staff.

9. Make-up labs and review sessions can be accommodated with a minimum of effort.

10. The student feels more keenly his responsibility for his own learning.

11. Each student is essentially "tutored" by a senior staff member.[1]

Other advantages listed for audio-tutorial instruction were: 1) repetition, 2) concentration, 3) association, 4) unit steps, 5) a communication vehicle appropriate to the objective, 6) a multiplicity of approaches and 7) integrated experiences. They also point out the extra advantage of free teacher time to aid those students who need extra help, and in many cases student-teacher contact was enhanced by the method.[2]

It is interesting to note that while others at Purdue have conducted studies on audio-tutorial teaching, Postlethwait participated in only one study. The study showed definite time savings for students but no significant difference in achievement.[3]

In view of the many successes in other disciplines and in view of

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[1] Ibid., p. 96.

[2] Ibid., pp. 3, 4, 20.

[3] D. D. Husband and S. N. Postlethwait, "The Concept of Audio-tutorial Teaching" (unpublished Ph.D. dissertation, Dept. of Biological Science, Purdue University), p. 25.

all its advantages, the audio-tutorial method of instruction in mathematics deserves close investigation.

One of the advantages claimed for the audio-tutorial method is that of individualizing instruction. A closer examination of this area is given below.

### Individualized Instruction

Recognition of the need to individualize instruction is not new. The Roman Quintillian of 35 to 100 A.D. argued that teachers should take into account individual differences in students,[1] and Plato in his Republic also mentioned the need for recognizing individual differences in learning.

The need to individualize instruction has been pointed out in more modern times by several people. White indicated that only through individualized instruction can we give lessons appropriate for each child and thus minimize the possibility of increasing students' problems through unrealistic demands and expectations.[2] Wood and Haefner wanted to use individualized instruction to provide "functional, stimulating school activities."[3] A conversation with Robert Davis, the director of the Educational Development Program at Michigan State University, revealed the concern for individualized instruction on the campus which led to

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[1] Phillip S. Jones, "The History of Mathematics Education," The American Mathematical Monthly, LXXIV, No. 1, Part 2, (Jan., 1967), p. 43.

[2] Verna White, Studying the Individual Pupil (New York: Harper and Bros., 1968), p. 12.

[3] Ben D. Wood and R. Haefner, Measuring and Guiding Individual Growth (New York: Silver Burdett, 1948, p. vii.

the development of Structured Learning and Training Environments (SLATES). SLATES are in part audio-tutorial learning situations in use on the campus. Foster, Kaufman and Fitzgerald indicated the need to individualize instruction with respect to content, level of presentation and characteristics of the learner.[1] Finally, the Comprehensive School Mathematics Program has been concerned with individualizing instruction since 1966; indeed, Kaufman and Haag said that:

> It has become clear to many people who think about mathematics education that the traditional process of fitting all children in a class to one instructional program must be replaced by a process by which programs are designed to fit the individual children.[2]

Although individualization is important, few courses do this at present. Wilhelms wanted:

> ... to point out again that it is not individual differences that have been aimed at but types of group differences (such as age) ... the common grouping systems will tend to hold two kinds of danger.
>
> First, there is the danger of stereotyping. With reference especially to the sectioning of classes into ability groups, it is notorious that administrators and teachers fall into thinking of each section as "homogeneous." The teacher often speaks of his "slow" group or his "fast" group as if all the members of each were the same. It is frequently obvious that his satisfaction with the system corresponds precisely with his relief at no longer having the bother of adapting his teaching to a range of differences.
>
> Second, specialized courses designed for particular groups introduce another danger. Being designed to do one job

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[1] G. Foster, B. Kaufman and W. Fitzgerald, "A First Step Towards the Implementation of the Cambridge Mathematics Curriculum in a K-12 Ungraded School" (Cooperative Research Project No. 5-405, Florida State U., 1966).

[2] B. Kaufman and V. Haag, The Comprehensive School Mathematics Program - An Experiment in Content-Orientation and Individualization of Instruction (Carbondale: Central Midwestern Regional Educational Laboratory, Inc.: 1971), p. 1.

> especially well, such courses are often narrow in scope and offer few internal choices.[1]

This is not to say that common goals for education are bad but rather than one must offer students different routes to these goals.

Goodlad also indicated that our schools hamper provision for individual differences by fixing grade level and course content,[2] and as indicated in a recent SMSG Newsletter:

> Unhappily for teachers, there is no book of any kind or description which has enough pages of varying difficulty to fit the range of needs in any given class.[3]

Fortunately, efforts have been made to individualize instruction at all levels of education and for the large number of students attending today's schools and colleges. Commercial companies are producing methods of instruction designed to individualize teaching. The Westinghouse Learning Corporation has produced individualized learning materials which are in use in the elementary grades at the McCulloch School in Jackson, Michigan. In this system each child has a yearly instructional plan (which may be changed by the teacher) designed for him by computer. The child proceeds through the system on his own, receiving help from the teacher when needed. A visit to the school showed that the system was well received. Students in all classes were busy learning, and it was reported that there were very few discipline problems. A child

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[1] Fred Wilhelms, "The Curriculum and Individual Differences," Individualizing Instruction, Sixty-first Yearbook of the National Society for the Study of Education, Part I (Chicago: University of Chicago Press, 1962), p. 102.

[2] R. Goodlad, New Approaches to Individualizing Instruction, Report of Conference on May 11, 1965 (Princeton: Educational Testing Service, 1965).

[3] SMSG Newsletter Number 33, Mathematics for the Disadvantaged and Low Achieving Students (Stanford: SMSG, 1970), p. 2.

working on his studies has no time to cause disturbances.

At the college level, all the previous studies mentioned in the audio-tutorial section were concerned with individualizing instruction. These studies also demonstrated that the audio-tutorial method can be used successfully with a large number of students. Educators can no longer use large classes as an excuse for not individualizing instruction

Other disciplines (see the audio-tutorial section) were successful in providing learning situations for individual students at the college level. Mathematics educators should search for methods to do the same. As Jones stated:

> We note that efforts to individualize learning are primarily made at the elementary school level. The greatest provision for individual difference is thus made when that difference is least. We should maintain our efforts at the elementary level and increase our committment to individualized instruction at the secondary and college levels.[1]

This study examined the audio-tutorial method in a remedial college algebra class. Realizing now that there is a need to individualize instruction and the the use of audio-tutorial methods is one way to accomplish this, consideration is next given to remedial instruction.

## Remedial Instruction

In 1965 the National Council of Teachers of Mathematics (NCTM) and the U.S. Office of Education cooperated in a conference on the low achiever in mathematics. Phillips gave some reasons for interest in the low achiever. Among these were: 1) lack of achievement has been

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[1] J. Howard Jones, "The Effects of Entering Achievement Level and Time Spent in Course Completion" (unpublished Ph.D. dissertation, Michigan State University, 1971), p. 28.

shown to be one principle reason for dislike of school; 2) training in basic mathematics is necessary for approximately two-thirds of the skilled and semiskilled job opportunities of today; and 3) success or measurable achievement in mathematics has a significant correlation with increased achievement in other disciplines.[1] Another reason for interest is that low achievement in mathematics does not necessarily mean low achievement in other areas. Remedial sections of mathematics at Michigan State often contain masters degree level students, and Ph.D. students are not uncommon. Fleishman agreed with this view.[2]

How does one get to be a remedial college mathematics student? One way is by low achievement in high school which may have as the cause low achievement in the elementary grades. Among the reasons for low achievement at this level are fear, lack of hope of achievement, lack of "motivation," and, of course, poor instruction. Then, repeated failure in lower grades often leads to the same in high school. Other ways to become a low achiever were given by Rosenbloom. Among these are: 1) Bad writing (or teaching) may make a bigger difference for slow learners than for average students, and 2) in most schools the teachers of remedial students are either lowest in the assignment order or are specialists on low achievers with little special knowledge of mathematics.[3]

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[1] Harry Phillips, "Why We are Concerned About Low Achievers in Mathematics," The Low Achiever in Mathematics, Report of a conference held in Washington, D.C. in March, 1964, Lauren G. Woodby, ed., U.S. Office of Education Bulletin No. 31 (Washington: U.S. Printing Office, 1965), pp. 2-3.

[2] E. A. Fleishman, "Human Abilities and Verbal Learning," Learning and Individual Differences, ed. by R. M. Gagne (Columbus: Charles E. Merrill, 1963), p. 59.

[3] Paul Rosenbloom, The Low Achiever in Mathematics, pp. 24-25.

The approach other than lecture most often applied for teaching remedial mathematics in college is the programmed text. Nagel used this method in a remedial college algebra class at Oregon State University. One of his results was that some remedial students covered more material, but the failure rate was unchanged.[1] White, at Utah State, used programmed material in a similar course. He showed a mean score difference in favor of programmed material for computation problems but no significant difference for problem solving questions.[2]

A comment on the use of programmed texts is in order. May cautions against their use for several reasons. First, he cautions that there is no conclusive evidence that students learn more by this method than by any other; and these remedial students are not learning very well by present methods. He also warns that programmed texts do not provide greater motivation for all students. He indicated that no fixed program can take care of all the objectives of mathematics education. Finally, he indicated that programmed materials should not replace teacher or human contact.[3]

With these cautions in mind, mathematics educators should examine other methods of remedial instruction at the college level.

This section closes with recommendations from various sources for the instruction of remedial classes. The SMSG indicated that it

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[1] T. S. Nagel, "Effects of Programmed Instruction in a Remedial College Algebra Class," The Mathematics Teacher, LX (Nov. 1967), 748-752.

[2] Charles White, "The Use of Programmed Texts for Remedial Mathematics Instruction in College" (unpublished Ph.D. dissertation, Utah State, 1970).

[3] Kenneth May, "Programming and Automation," The Mathematics Teacher, LIX (May, 1966), 444-452.

may be advisable for students to complete separate worksheets each day and not use a text. This allows students to consider what they have accomplished, not what they still must do.[1]

Meserve also presented some guidelines for planning a program for remedial students. His suggestions included 1) adjusting the reading level downward, 2) giving the material in short sections, and 3) using many examples.[2]

A number of recommendations resulted from the conference on the "Low Achiever in Mathematics" in Washington, D.C., in 1964. Some of these which are particularly related to the instruction of remedial classes are: 1) teams of specialists should select course content, 2) new materials should be devised since slowed-down mathematics seemed inappropriate, 3) opportunity for success should be a major aim of the instruction, 4) the classes should be taught by able and well-trained teachers, 5) special help should be available for both students and teachers, 6) laboratory settings may be especially effective for low achievers and 7) present material in language meaningful to the student.[3]

Nagel agrees with the statement that the divergence in background and abilities make college level remedial courses particularly difficult to teach.[4] Once again it is time to consider alternative

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[1] SMSG Newsletter Number 33, Mathematics for Disadvantaged and Low Achieving Students (Stanford, 1970), p. 4.

[2] Bruce Meserve, "The Teaching of Remedial Mathematics," The Mathematics Teacher, LIX (May, 1966), 437-443.

[3] Lauren Woodby, ed., The Low Achiever in Mathematics, pp. 85-93.

[4] Nagel, "Effects of Programmed Instruction," Mathematics Teacher (Nov., 1967), p. 752.

methods of teaching this type of class.

## Mastery Learning

In a usual classroom at the beginning of a new school session (be it semester, quarter, term or year) most teachers expect that approximately one-third of their students will fail or just "make it," another third will learn much of what is taught but not enough to be called a "good student" and another third will master the material.

Commenting on these expectations Bloom wrote:

> This set of expectations, which fixes the academic goals of teachers and students, is the most wasteful and destructive aspect of the present educational system. It reduces the aspirations of both teachers and students; it reduces motivation for learning in students; and it systematically destroys the ego and self-concept of a sizable group of students who are legally required to attend school for 10 to 12 years under conditions which are frustrating and humiliating year after year. The cost of this system in reducing opportunities for further learning and in alienating youth from both school and society is so great that no society can tolerate it for long.[1]

Some of these expectations have built up over years of conditioning in the present school system. The practice has been to classify students as "A, B, C, D or F students" in each class. Bloom also commented about the normal distribution and how it has influenced teaching.

> Having become "conditioned" to the normal distribution, we set grade policies in these terms and are horrified when some teacher attempts to recommend a very different distribution of grades. Administrators are constantly on the alert to control teachers who are "too easy" or "too hard" in their grading. A teacher whose grade distribution is normal will avoid difficulties with administrators. But even more important, we find ways of convincing students that they can only do C work or D work by our grading

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[1] Benjamin Bloom, "Learning for Mastery," in UCLA-Center for the Study of Evaluation of Instructional Programs, Evaluation Comment, Vol. I, No. 2 (May, 1968), p. 1.

> system and even by our system of quiz and progress testing. Finally, we proceed in our teaching as though only the minority of our students should be able to learn what we have to teach.[1]

Block gave the general approach used in teaching for mastery. First, mastery was defined in terms of attainment of educational objectives. Then, instruction was organized into well-defined units. Testing was done after the instruction to determine the level of mastery. Then, finally, on the basis of this testing, the original instruction was either supplemented so the student could learn to criterion, or upon mastery, the student went to the next unit.[2]

Studies have been done on mastery learning in mathematics. In 1941 Thompson reported on a matched experimental versus control group designed to investigate a series of studies carried out over a four year period (1936-40) with 7th-10th graders. Before beginning study of any section the student was given a test to see if he had mastered the material to be covered. If mastery was complete, a pretest was given for the next phase of instruction. The student was not allowed to proceed to unit N+1 until unit N was mastered. The material covered was arithmetic and algebra. The results indicated consistent gains in achievement. One study showed that in a 10-week period, the experimental group gained 1.41 years in arithmetic as measured by a standardized test while the control group using usual methods gained .40 years. The author claimed the method was effective because: 1) no pupil wasted time on topics he had already mastered; 2) no student

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[1] Ibid., p. 2.

[2] James Block, Mastery Learning Theory and Practice (New York: Holt, Rinehart and Winston, Inc., 1971), pp. 3-4.

had to wait for his whole class; and 3) no student left a topic until he had mastered it.[1]

Support for mastery learning in more recent times came from Airasian in 1969 who, in part of a study, used approximately sixty algebra students. Two curriculum specialists in algebra examined two specific chapters in an algebra test and separated the chapters into sections ranging from specific facts to relatively complex material requiring such processes as application of principles. The necessity of mastery learning was supported when it was found that 75 per cent of the students who missed lower level material also missed related material at the complex level.[2] This gave support to the usually accepted fact that failure in course N implies failure in course N+1.

Collins investigated mastery learning in 1969. The study involved an experimental and a control class in each of the areas of algebra for liberal arts majors and calculus for engineering and science majors. After using the general approach previously indicated (by Block) significant results were found. In the algebra classes, 75 per cent of the mastery students compared to 30 per cent of the control students achieved the same mastery criterion (of A or B grades). The calculus classes' results were similar with 65 per cent of the mastery class reaching the criterion while only 40 per cent of the control

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[1] R. Thompson, "Diagnosis and Remedial Instruction in Mathematics," <u>School Science and Mathematics</u>, IXLI (Feb., 1941), 125-128.

2. Peter Airasian, "Formative Evaluation Instruments: A Construction and Validation of Tests to Evaluate Learning Over Short Time Periods," <u>Mastery Learning Theory and Practice</u>, edited by J. Block (New York: Holt, Rinehart and Winston, Inc., 1971), p. 99.

group reached the criterion level. Collins also reported that, in the mastery classes, D and F grades were all but eliminated.[1]

Concerning mastery learning, Block wrote:

> The results from almost 40 major studies carried out under school conditions indicate that mastery learning has marked effects on student cognitive and affective development and their learning rate. In general, mastery strategies enable about three-fourths of students to learn to the same performance standards as the top fourth of students learning under conventional, group-based instructional approaches. The strategies seem to be especially effective for those students who typically have had problems learning under ordinary instructional conditions. For example, students with below average I.Q. scores seem to learn as well under mastery conditions as students with above average I.Q. scores under a traditional approach. For subjects where most of the students have achieved the prerequisite learnings, mastery procedures appear to be able to almost eliminate the effects of individual differences on level of achievement.
>
> Mastery methods also produce markedly greater interest in and better attitudes toward the material learned than more conventional approaches. They seem to help most students overcome feelings of defeatism and passivism brought to the learning. Their powerful affective consequences may be attributed to many factors, the most important of which seem to be the cooperative rather than competitive learning conditions, successful and rewarding learning experiences, personalized attention to each student's learning problems, and the use of certain correctives (e.g., student tutors and small group study sessions) which add a personal-social aspect to the learning not typical of group-based instruction.
>
> Finally, mastery approaches also make student learning increasingly efficient. Mastery of the earliest units in a school subject appears to facilitate the learning of the subsequent units, especially where the learning units are sequentially arranged. The instructional time spent to ensure adequate learning over the first units in the course seems to result in the need to spend less time than usual over the later units to maintain a high level of student performance.[2]

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[1] Kenneth Collins, "A Strategy for Mastery Learning in Freshmen Mathematics" (unpublished study, Purdue University, Division of Mathematical Sciences, 1969).

[2] Block, <u>Mastery Learning</u>, pp. 96-97.

## Summary

In Chapter 2 a brief history of the development of the present curriculum of mathematics education has been given. The literature has been reviewed in each of four areas: audio-tutorial learning, individualized instruction, remedial instruction and mastery learning.

Some conclusions from the literature which are important to the study are:

1. The reform movement begun in the early 1900's began to place emphasis on education for all students.

2. The curriculum during this period emphasized practical and social utility of the courses and their content.

3. The recent developments (since approximately 1950) have been mainly subject or content centered. The increasing demand for mathematically trained persons made it necessary to study more and better mathematics.

4. In general, present college instruction of remedial mathematics classes leaves much to be desired. Instruction could be improved by considering more closely the teaching model of Glaser.

5. The audio-tutorial method has been very successful in disciplines other than mathematics.

6. The audio-tutorial mode has seen only very limited use in the field of mathematics education. It is past time to examine the method more closely with regard to mathematics instruction.

7. Most educators agree that individualizing instruction is a goal worthy of more attention. Other disciplines have shown that it is

possible to individualize instruction with large groups of students.

8. Remedial instruction in mathematics is receiving some attention. Guidelines are available for this type of class, and they present a starting place for structuring a remedial course.

9. Requiring mastery learning in the classroom at the 80 per cent level is quite feasible. It provides not only greater achievement but may also decrease time spent studying later subjects.

# CHAPTER III

# DESIGN OF THE STUDY

## Introduction

The general operating procedure is given on pages 6 to 9 in Chapter I. The reader may wish to briefly review these pages before continuing.

This chapter contains: 1) a description of the sample, 2) a description of the population, 3) a discussion concerning the measures used, 4) a description of materials and operation of the audio-tutorial laboratory, 5) the statistical design of the study, 6) the testable hypotheses, and 7) a summary.

## The Sample

All students entering Michigan State University (MSU) are required to take a thirty-item mathematics placement exam (Appendix A7 contains a short discussion of this test). Based on the results of this exam the students are placed into either the first term of analytic geometry and calculus, a one-term course in algebra and trigonometry, the first of a two-term sequence in algebra and trigonometry or a one-term class titled Beginning Algebra 2. Exceptions to the placement criteria may be made by personal interview.

Beginning Algebra 2 (Mathematics 082) is a non-credit algebra class since it is regarded as remedial. At the completion of registration for Spring Quarter,1971, 107 students had registered for six sections of Mathematics 082. Each section had an enrollment of not more than 20 students and not less than 16 students.

The text used during Spring Quarter was Intermediate Algebra: Second Alternate Edition by William Wooton and Irving Drooyan. It was published by Wadsworth Publishign Company in 1968. The course content was based on a series of lectures prepared by Professor John G. Hocking of the Department of Mathematics during the summer of 1970. Major topics included were: properties of real numbers, polynomials, algebraic fractions, exponents, roots, radicals, and first and second degree equations and inequalities in one variable. No specific work on basic arithmetic was included in the text or the course, although such material was included in Professor Hocking's outline.

During the first week of classes in the Spring Quarter of 1971 each student was given a test to determine specific weaknesses in arithmetic including per cent, square root, use of formulas (areas and volumes) and operations with whole numbers, decimals and fractions (Appendix A8). The test questions were keyed to each of ten modules which had been prepared for instruction on the basic arithmetic material. Each module was designed to cover 2 to 4 (behavioral) objectives and the test was also keyed to these objectives. Appendix A9 (page 103) contains a complete list of module numbers and corresponding objectives.

A student was said to be deficient in the material contained in a module if he missed questions on the test over at least 50 per cent of the material contained in the module. Thus, a student was strongly urged to complete a module if he had obtained less than or equal to 50 per cent of the objectives for the particular area (or module).

Based on this criterion, 77 students enrolled in Mathematics 082 during Spring Quarter, 1971, were termed deficient in four or more of the ten modules which had been prepared. Each of these students was given a paper listing: 1) those modules in which he had a serious deficiency and 2) the complete list of objectives (Appendix A9). The modules in which the student was seriously deficient were indicated by putting a red square around the module number. If the student missed at least one but fewer than 50 per cent of the objectives for a module, a blue circle was put around the module number. No mark around a number meant that the student was probably competent in the material contained in the module. A red square and blue circle were used for those students who were color blind.

Fifty-three of the students who were deficient in four or more modules indicated that they would like to take the modules. This group of volunteers was randomly divided into two parts, called the treatment group and the control group. Due to the nature of the Mathematics 082 class and experience with attendance problems in the course, 30 students were randomly placed in the treatment group and 23 students were put into the control group. Of the 77 students termed deficient in the material contained in at least four modules, 24 did not indicate they wanted to take the modules suggested. This group was called the non-volunteer group.

Personal data collected on each student was used to describe the sample in detail. There were 45 males and 32 females in the study. The mathematics background of the students was quite varied. Five of the students had taken previous work in mathematics at the college or the junior college level. These courses were either arithmetic or remedial algebra. A large number (60 per cent) of students had one year of high school algebra and 8 per cent of the students had no mathematics beyond a ninth grade general mathematics course. Fifty-three per cent of the students also had one year of high school geometry and 4 per cent of the 77 students had a semester of trigonometry in high school. As these figures show, students could enter Mathematics 082 with anywhere from no high school mathematics to more than three years of high school mathematics. The previously mentioned arithmetic test given the first week of classes to all students also served to identify the variety of entering behavior. For a total possible score of 28, scores of 3 and 22 were attained, with a mean of 14.7.

Table 1 gives the age and class distribution of the sample:

TABLE 1: Age and Class Distribution of the Sample (in Per cent to the Nearest Tenth).

AGE

| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 or older |
|---|---|---|---|---|---|---|---|---|---|---|
| 3.9 | 13.0 | 22.1 | 15.6 | 7.8 | 6.5 | 5.2 | 7.8 | 3.9 | 5.2 | 9.1 |

CLASS

| Freshman | Sophomore | Junior | Senior | Graduate |
|---|---|---|---|---|
| 48.1 | 24.7 | 23.4 | 1.3 | 2.6 |

The information in Table 1 also indicated the wide variety of students in the class. An age span of 18 years (the oldest student was 35) was present with over 18 per cent of the students at least 25 years old. The aims, goals, interests and recall of past mathematics of the 17 and 18 year old students were probably different from those of the students who were 25 or older. These students all received the same course.

The fact that approximately 27 per cent of the students were at the junior level or above was somewhat surprising. Mathematics 082 is a freshman course and obviously many students are leaving their mathematics requirement until late in their college career.

A final note on the sample is due concerning the criteria for inclusion in the study. The inclusion of only those students needing four or more modules was not meant to imply the other students did not need additional aid. Similarly, students missing less than 50 per cent of the material on a given module also may have benefited from taking the module. Physical facilities, cost and time limited the number of students that could be included in the study.

## The Population

The sample was described in the preceeding section. The population was described using a Cornfield-Tukey inference:

> In almost any practical situation where analytical statistics is applied, the inference from the observations to the real conclusion has two parts, only the first of which is statistical. A genetic experiment on Drosophila will usually involve flies of a certain race of a certain species. The statistically based conclusions

> cannot extend beyond this race, yet the geneticist will usually, and often wisely, extend the conclusion to (a) the whole species, (b) all Drosophila, or (c) a larger group of insects. This wider extension may be implicit or explicit, but it is almost always present. If we take the simile of the bridge crossing a river by way of an island, there is a statistical span from the near bank to the island, and a subject-matter span from the island to the far bank. Both are important.[1]

Therefore, after carefully describing the sample and presenting the type of material taught in the course, Mathematics 082, at Michigan State University, the population was inferred: The population for the study was all students in similar classes now in existence or that will be in existence in colleges and universities. An outline of the algebraic material covered in the course is contained in Appendix All.

## The Measures

All tests used in this study were criterion-referenced tests. That is, the tests were used to measure a student's level of achievement with respect to some criterion or standard. Tests for this purpose are interpreted quite differently than norm-referenced tests which are used to measure the achievement of an individual in relation to the achievement of other students.

Popham and Husek wrote on criterion-referenced tests in 1969. Commenting on the use of the criterion-referenced tests they stated:

> Criterion-referenced tests are devised to make decisions both about individuals and treatments, e.g., instructional

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[1] J. Cornfield and J. Tukey, "Average Values of Mean Squares," Annals of Mathematical Statistics, XXVII (1956), 913.

> programs. In the case of decisions regarding individuals one might use a criterion-referenced test to determine whether a learner had mastered a criterion skill considered prerequisite to his commencing a new training program. In the case of decisions regarding treatments, one might design a criterion-referenced measure which reflected a set of instructional objectives supposedly achieved by a replicable instructional sequence. By administering the criterion-referenced measures to appropriate learners after they had completed the instructional sequence, one could reach a decision regarding the efficacy of the sequence (treatment).
>
> Although both norm-referenced and criterion-referenced tests are used to make decisions about individuals, there is usually a difference in the two contexts in which such decisions are made. Generally, a norm-referenced measure is employed where a degree of selectivity is required by the situation. For example, when there are only limited openings in a company's executive training program, the company is anxious to identify the best potential trainees. It is critical in such situations, therefore, that the measure permit relative comparisons among individuals. On the other hand, in situations where one is only interested in whether an individual possesses a particular competence, and there are no constraints regarding how many individuals can possess that skill, criterion-referenced measures are suitable.[1]

Realizing that norm-referenced tests are used to separate students, the scores on these tests should vary. With criterion-referenced tests the situation is quite different. This is the central difference between these two types of tests. Popham and Husek wrote:

> The subtle and not-so-subtle implications of this central difference in the relevance of variability must permeate any discussion of the two approaches to testing. For example, we all have been told that a test should be reliable and valid. We have all read about test construction and item analysis. The procedures may not always be simple, the formulas may not be trivial

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[1] W. J. Popham and T. R. Husek, "Implications of Criterion-referenced Measurement," Journal of Educational Measurement, VI, No. 1 (Spring, 1969), 2-3.

> but there are hundreds of books and thousands of articles to guide us. Unfortunately, most of what these "helpmates" outline as "good" things to do are not only irrelevant to criterion-referenced tests, but are actually injurious to their proper development and use. This is true because the treatments of validity, the suggestions about reliability, and the formulas for item analysis are all based on the desirability of variability among scores. The connection may not be obvious but it is always there.[1]

Later in the same article, writing on the use of reliability coefficients for criterion-referenced tests, Popham and Husek indicated that the reliability coefficient for this type of test may often be low and indeed will be near 0 if most students reach the criterion level. Hence, they stated that, "It is really unwise to apply such estimates."[2] when using a criterion referenced test. Therefore, reliability coefficients were not presented in the study.

## The Audio-tutorial Laboratory

Although partial descriptions of the laboratory were presented in previous sections of the study, a brief discussion concerning the materials used is in order. The laboratory was located in Room A138 Wells Hall at MSU. This room was used by the Student Advisory Committee MATHELP tutors who gave aid on the modules as needed to the experimental group and to other students (not in Mathematics 082). The MATHELP tutors and the Education 327N (Methods of Teaching Secondary Mathematics) students assisted the experimental group students in completion of the modules by: 1) administering and grading pretests, 2) giving the student the materials needed to complete the module,

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[1] Ibid., pp. 3-4.

[2] Ibid., p. 5

3) checking to be sure the student understood the directions for use of the module, 4) answering questions, as necessary, for students working on a specific module, 5) administering and grading the posttests with the keys that were provided for each module, and 6) filing the papers upon which the student had written.

The dimensions of the three carrels used were given in Chapter I. The carrels were designed for and borrowed from the Science and Mathematics Teaching Center at MSU. They were designed to fit table tops and thus were quite portable. Screens for the pictures were placed in each carrel so that the projector was always on the left side of the student. Thus, the PASS strips were always inserted into the projector the same way.

The projector used was made by the Taylor-Merchant Corporation (with offices in New York, N.Y.) using an adapter for the PASS strips. A Norelco tape recorder-player was used. The cassettes were altered to eliminate accidental erasing of the tapes. The headsets used were made by Handy Electronics, Inc. (in Rockford Ill.) and were wired for monaural sound.

The pictures for the study were taken by the writer on a half-frame camera using Kodak high-contrast black and white film. The negatives were used for the frames and color was added (as indicated on page 1 of Appendix A2) through the use of 1/8 inch transparent Chart-Pak tape. This resulted in a considerable reducation of cost.

The cost of each empty PASS strip was 10 cents, and the cost of each roll of film was $1.20. Since nine PASS strips were made from each role of film, the cost for each completed PASS strip was approximately 25 cents or about 3 cents per frame not including the labor. Special appreciation is given to Professor T. Wayne Taylor of MSU who originated and refined the system.

The recordings were made by the writer, in line with the recommendations of Postlethwait, so that students could identify the voice on the tapes with a known face. Multiple copies were made at the Instructional Media Center at MSU and were paid for in part by the Mathematics Department and in part by the Educational Development Program at MSU.

## The Statistical Design of the Study

The first two purposes of the study were:

1. To investigate the use of audio-tutorial techniques in a college-level remedial algebra class, and

2. To investigate the effect of volunteering for additional aid on achievement in basic arithmetic in a college level remedial algebra class.

The Mathematics 082 Arithmetic Pretest (Appendix A8) was given during the first class period to find the basic arithmetic weaknesses of the students. The experimental group (described previously) had been given three weeks to complete their modules. The three-week time limit was included since it was believed that students should erase their arithmetic weaknesses as soon as possible. This involved not more than three carrel sessions per week, each lasting approximately 75 minutes. At the end of this time a Mathematics 082 Arithmetic Posttest (Appendix A6) covering the same material as the arithmetic pretest was given. The score on the pretest was used as the co-variable in a one way analysis of covariance (ANCOVA). Data were assembled as in Table 2. The dependent variable (X) was the score on the Mathematics 082 Arithmetic Posttest.

TABLE 2: Data Matrix for the Analysis of Covariance

| GROUP | | |
|---|---|---|
| Experimental | Control | Non-volunteer |
| X | X | X |
| X | X | X |
| . | . | . |
| . | . | . |
| . | . | . |

The reader will recall that the sample had unequal numbers of subjects in each group. It was expected that some students would drop the class; and, as usual in Mathematics 082, some students would not come to classes. Thus, it was planned to run the ANCOVA with 20 subjects per cell and data were randomly eliminated to accomplish this. (Additional comment on those students who did not attend class is presented in Chapter IV.) An equal number of subjects per cell also facilitated the use of Post Hoc techniques and aided in meeting the assumptions necessary for ANCOVA.

There are five assumptions which must be met for proper use of ANCOVA. They are: 1) independence between and within cells, 2) a normal conditional distribution of the dependent variable scores, 3) homogeneity of variance, 4) parallel regression lines, and 5) a linear relationship between the co-variable and the dependent variable.

The general conduct of the study made the assumption of independence feasible. Concerning the assumptions of normality and equal variance (using a fixed effects model and an equal number of observations per cell as in the study) Kirk has written:

> In general, unless the departure from normality is so extreme that it can be readily detected by visual inspection of the data, the departure will have little effect on the probability associated with the test of significance.
>
> The F distribution is robust with respect to violation of the assumption of homogeneity of population-error variances provided that the number of observations in the samples is equal.[1]

Therefore, these two assumptions were made.

The assumption of parallel regression lines was more important. Winer presented a test for this assumption.[2] The within-class variation was subdivided into two parts and the test was performed. The specific formulas may be found on page 586 of Winer's book. The data are summarized in Table 3:

TABLE 3: Data for F-Test of Parallel Regression Lines

| Source | Sum of Squares | df | Mean Squares | F Ratio |
|---|---|---|---|---|
| Numerator | 60.6 | 2 | 30.3 | 4.5 |
| Denominator | 362.0 | 54 | 6.7 | |

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[1] R. Kirk, <u>Experimental Design: Procedures for the Behavioral Sciences</u> (Belmont: Wadsworth Publishing Company, 1968), p. 61.

[2] B. J. Winer, <u>Statistical Principles in Experimental Design</u> (New York, McGraw-Hill Book Company, 1962), pp. 586-87.

The hypothesis tested was: $H_o$: The regression lines were parallel; and, obviously, failure to reject is desired. The tabled F of 5.04 was found at the .01 level; and hence, the hypothesis was not rejected. Therefore, the assumption of parallel regression lines was met.

Without a linear relationship between the co-variable and the dependent variable, interpretation of the adjusted treatment means, as used in Post Hoc testing, is difficult. Therefore, this relationship was also investigated. Using material presented by Porter in the class, Experimental Design in Education (Ed. 969C) at MSU, a ratio was computed which gave the strength of the linear relationship in question. The ratio used values from the sums of squares found in the analysis. The values are given in Table 4.

TABLE 4. Data for Linear Relationship Ratio Between Co-variable and Dependent Variable.

| | SUMS OF SQUARES | | |
|---|---|---|---|
| Source | $R{:}T_{xy}$ | $R{:}T_x$ | $R{:}T_y$ |
| R:T | 491.2 | 846.5 | 647.5 |

R:T denotes the source: subjects nested in treatments

The subscripts denote sums of squares on the co-variable (x), the dependent variable (y) and the interrelation (xy).

The ratio computed uses the square root of the product (846.5)(647.5) which is 740.3 (to the nearest tenth). The ratio is 491.2/740.3 which is .7 to the nearest tenth. According to Porter, this indicated that a very strong linear relationship existed and that a good co-variable was used.

Thus, all assumptions necessary for ANCOVA were met, and the increase in precision given by ANCOVA was present.

The data were run on the Control Data Corporation 3600 computer in the MSU Computer Center. The Finn Multivariate Analysis of Variance program was employed. Use of the MSU computing facilities was made possible through support, in part, from the National Science Foundation and the Department of Education at MSU.

The null form of the hypothesis tested was: $H_o$: There will be no difference between the experimental group, the control group and the non-volunteer group mean scores on the Mathematics 082 Arithmetic Posttest.

Another purpose of the study was to investigate the retention of material studied in the modules. For this purpose, the Mathematics 082 Arithmetic Posttest was given to the experimental and the control groups approximately 3 1/2 weeks after the original testing. Operating within the limits of the quarter system and within the limits of normal testing in the small sections of the class, this was the longest time period between tests that was possible. The scores were collected and, again allowing for students dropping the course and for students not coming to class, data were randomly eliminated to leave an equal number of observations per cell. The data were listed as in Table 5, and were analyzed using a repeated measures design. As usual, X denoted the score on the proper test for the cell.

TABLE 5. Data Matrix for the Repeated Measures Design

| | | POSTTEST | RETENTION TEST |
|---|---|---|---|
| TREATMENT GROUP | $S_1$ | X | X |
| | $S_2$ | X | X |
| | ⋮ | ⋮ | ⋮ |
| | $S_N$ | X | X |
| CONTROL GROUP | $S_{N+1}$ | X | X |
| | ⋮ | ⋮ | ⋮ |
| | $S_{2N}$ | X | X |

The assumptions necessary for use of a repeated measures design are the three basic analysis of variance assumptions (independence, equal variance and normality) and a fourth assumption concerning correlations between the measures. The assumption is that all correlations between different measures are equal. Since the repeated measures design used in the study had only two measures, this assumption was met. The same arguments that applied to the three basic assumptions in the ANCOVA also applied here; and hence, all assumptions for the test were met.

The hypothesis tested within this design, given in null form was:

$H_o$: There will be no difference between the experimental group and the control group mean scores on the Mathematics 082 Arithmetic Posttest given approximately 3 1/2 weeks after the first use of the test.

Another purpose of the study was to investigate the effect of the individual arithmetic module on achievement over material contained in the module.

The reader will recall that the individual module pretests and posttests were criterion referenced tests. Therefore, since the students were expected to score above 80 per cent on the tests, and in fact were encouraged to seek aid and repeat the module if they did not, interpretation of these scores by conventional methods was inappropriate.

Popham and Husek gave suggestions for proper reporting and interpretation of scores on criterion-referenced tests:

> Some criterion-referenced tests yield scores which are essentially "on-off" in nature, that is, the individual has either mastered the criterion or he hasn't. For example, certain examinations in the chemistry laboratory may require a pupil to combine and treat chemical compounds in such a way that they produce hydrogen. In such tests it is sufficient to report whether or not the learner has displayed the desired criterion behavior.
>
> With respect to the evaluation of treatments, it has already been pointed out that norm-referenced measures are not the most suitable devices for such purposes since their emphasis on producing heterogeneous performance sometimes diverts them from adequately reflecting the treatment's intended objectives. In using criterion-referenced measures for purposes of treatment assessment, e.g., testing the merits of a new set of programmed mathematics materials, we have several alternatives. We could simply report the number of individuals who achieve the pre-established criterion. Although such a procedure seems to supply scant data, it has the advantage of making graphically clear the proportion of learners who did not achieve criterion level proficiency. Too often this result is masked through the use of statistical averages.
>
> We could also use traditional descriptive statistics such as means and standard deviations. Because one is often interested in the average performance produced by a treatment, as well as its variability, such statistics

> are useful. An average "percentage correct," however, is a helpful addition. Sometimes, if the criterion level for an individual has been set a particular level, it is useful to report the proportion of the group which reached that level. For instance, using 80 per cent as a criterion level, then one might describe a group's performance as 92 80, indicating that 92 per cent of the group had achieved 80 per cent or better on the test. Such reporting, however, overlooks the proportion and degree of the better-than-criterion performance. It would seem, then, that in using criterion-referenced measures to make decisions about treatments, the best course of action would be to employ a number of these schemes to report the group's performance in order to permit more enlightened interpretations.[1]

Heeding this advice, the individual module scores were interpreted using several statistics; also included was a discussion of these scores. Recalling that the experimental group taking the modules had 20 members, statistics reported were:

1. The number of students who took the module (20 maximum).

2. The percentage of people who achieved the criterion levels of 80 per cent and 90 per cent, respectively.

3. The mean pretest and posttest scores.

## Summary

In this chapter, the investigation of erasing specific arithmetic weaknesses of remedial college algebra students was described in detail. Ten audio-tutorial modules were prepared which contained instructional material on addition, subtraction, multiplication and division of whole numbers and decimals, addition, multiplication and division of positive rational numbers, square root, per cent, and the

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[1] Popham and Husek, "Implications of Criterion Referenced Measures," pp. 7-8.

use of formulas in computing areas and volumes of some specific geometric figures and solids.

A detailed description of the sample of Mathematics 082 students was given: After testing for weaknesses in these specific areas, 53 volunteers were randomly divided into an experimental group and a control group. The 24 students who did not volunteer for additional aid, but were deficient in their arithmetic skills, were denoted the non-volunteer group. To aid in the analysis students were randomly eliminated as necessary, leaving 20 students per group. It was found that approximately 73 per cent of the sample were freshman- and sophomore-level students and over 18 per cent of the students in the total sample were at least 25 years old. The mathematics background of the sample was quite varied. It ranged from no high school mathematics to more than three years of high school mathematics. There was also an age span of 18 years which could indicate a variety of aims and goals among the students.

The audio-tutorial laboratory operation and materials were described. The three carrels were equipped with slide projectors with adapters for PASS strips, a tape player, a screen and a bulletin board for announcements. The reader will recall from Chapter I that, using the paid MATHELP tutors and students from the course, Education 327N, the carrel room was kept open from 8:00 a.m. to 5:00 p.m., Monday through Friday, and that students could sign up for reserved carrel time in the undergraduate Mathematics Office.

The statistical design consisted of three parts. The analysis of covariance (ANCOVA), using the Mathematics 082 Arithmetic Pretest as co-variable, was used to investigate the overall effect of module use on achievement in material covered in the modules and the effect of volunteering for additional aid on achievement in the arithmetic material. The dependent variable was the student's score on the Mathematics 082 Arithmetic Posttest.

The retention of the material covered in the modules was investigated through the use of a repeated measures design. The Mathematics 082 Arithmetic Posttest was repeated approximately 3 1/2 weeks after the first test. Scores of the experimental and control groups were compared.

The effect of individual modules on achievement over material contained in the module was investigated by using several statistics: 1) the number of students who took the module, 2) the number of students who achieved the criterion levels of 80 per cent and 90 per cent respectively, and 3) the mean pretest and posttest scores for the module.

# CHAPTER IV

# ANALYSIS OF RESULTS

## Introduction

Data were collected for the statistical analysis of the study from results of the Mathematics 082 Arithmetic Pretest, the Mathematics 082 Arithmetic Posttest, the Individual Module Reports, the Retention Test, and the module pretests and posttests. Before considering the analysis of the data, special mention should be made of the data which, for various reasons, could not be collected.

The reader will recall that at the beginning of Spring Quarter, 1971, a total of 107 students had registered for Mathematics 082 at Michigan State University. One student added the class (through a late add procedure) after approximately four weeks and was not included in the study. By the end of the quarter, 17 students had withdrawn from the course, leaving a total enrollment of 91. Thus, approximately 16 per cent of the students originally enrolled in the course dropped it before the quarter ended.

Of the 30 students who were included in the experimental group, 7 completed fewer than two of the minimum of four modules which were suggested for them. These people were eliminated because this would influence their score on the arithmetic posttest used to evaluate the modules.

Lost test or carrel data arose for one or more of the following reasons: 1) student withdrawal from the course, 2) student nonattendance in class, 3) student studying credit courses and not the non-credit Mathematics 082, 4) student illness or death in the family, 5) student failure to take a test, 6) student changes in curriculum or major field, and 7) student withdrawal from school. Another reason given by one student was that since he worked late afternoons and evenings, he had difficulty getting up for his 8:00 a.m. section. He could not attend one of the 4:10 p.m. sections because of his job.

## The Statistical Analysis

The hypotheses tested involved mean test scores for each cell. For this reason the mean test scores for each cell were reported.

### The Analysis of Covariance

The background for the Analysis of Covariance was given on pages 50-54 in Chapter III. The reader may wish to review these pages before continuing.

Table 6 listed cell means and standard deviations for the test used in the Analysis of Covariance (ANCOVA). As previously stated, the arithmetic pretest score was used as the concomitant variable in the ANCOVA.

Further use of these scores will be made later in this chapter.

TABLE 6. Means and Standard Deviations of the Mathematics 082 Arithmetic Posttest.

| | GROUP | | |
|---|---|---|---|
| | Treatment | Control | Non-volunteer |
| Arithmetic Posttest Means | 22.00 | 16.60 | 16.55 |
| Standard Deviations | 2.4 | 3.9 | 3.6 |

The null hypothesis tested using the ANCOVA was: $H_0$: There will be no difference between the experimental group, the control group and the non-volunteer group mean scores on the Mathematics 082 Arithmetic Posttest.

The data were assembled and the analysis yielded the results presented in Table 7.

TABLE 7. One-way Analysis of Covariance Results.

| SOURCE | Adjusted Sum of Squares* | df | Adjusted Mean Squares | F Ratio | P Less Than |
|---|---|---|---|---|---|
| Between | 452.02 | 2 | 226.01 | 33.18 | .0001** |
| Within | 381.36 | 56 | 6.81 | | |

*The sums of squares were reported after adjustment for the co-variable.

**The decision was to reject the null hypothesis.

The null hypothesis was rejected. The rejection satisfied the necessary requirement for the introduction and use of Post Hoc testing. Kirk commented on the use of Post Hoc testing for pairwise comparisons:

> A multiple comparison test similar to the LSD test has been proposed by Tukey (1953). This test, which is called the HSD (honestly significant difference) test or the w procedure, sets the experimentwise error rate at $\alpha$. The HSD test was designed for making all pairwise comparisons among means. The basic assumptions of normality, homogeneity of variance, and so on, described in Section 2.1 in connection with a t ratio are also required for the HSD test. In addition, the n's in each treatment level must be equal or approximately equal.
>
> A comparison involving two means is declared to be significant if it exceeds HSD, which is given by
>
> $$HSD = q_{\alpha,\nu}\sqrt{\frac{MS_{error}}{n}}$$
>
> The value of q is obtained from the distribution of the studentized range statistic. This distribution is given in Table D.7. The sampling distribution of q is based on the fact that, for random samples, the range tends to be larger as the sample size is increased. In order to enter the table for q, two values are required - the degrees of freedom for $MS_{error}$ and k, the number (range) of treatment levels in the experiment. In a completely randomized design, an estimate of $MS_{error}$ is provided by $MS_{WG}$ with N-k degrees of freedom.[1]

The use of an equal number of students per cell made the HSD test more precise. The Tukey test for these pairwise comparisons was known to be among the more precise methods of Post Hoc Testing.

The comparisons of interest were:

1) (treatment group mean) - (control group mean),

2) (control group mean) - (non-volunteer group mean) and

3) (treatment group mean) - (non-volunteer group mean).

The value of HSD was calculated to be 2.50 to the nearest hundredth. The .01 level was used with 56 degrees of freedom, and comparisons were made using adjusted group means. Table 8 summarizes

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[1] Kirk, Experimental Design, p. 88.

the information.

TABLE 8. Post Hoc Comparisons

| Adjusted Group Mean Scores | | | Comparisons* |
|---|---|---|---|
| Treatment | Control | Non-volunteer | T - C = 5.74<br>C - N = 0.23<br>T - N = 5.97 |
| 22.28 | 16.54 | 16.31 | |

*A comparison was significant if the difference in means exceeded HSD = 2.50.

The comparisons between the experimental and control groups and between the experimental and non-volunteer groups were significant, while the comparison between the control and the non-volunteer groups was not significant.

Since the test of the null hypothesis used adjusted treatment means, the Post Hoc testing also used adjusted treatment means. In an effort to gain more information about differences between the groups on entering behavior, means and standard deviations for each group on the Mathematics 082 Arithmetic Pretest were given. Table 9 listed these scores. In particular, they are offered as additional evidence to show that the non-volunteer group did not differ from the other two groups in entering behavior.

TABLE 9. Mathematics 082 Arithmetic Pretest Means and Standard Deviations

| | GROUP | | |
|---|---|---|---|
| | Treatment | Control | Non-volunteer |
| Mean | 13.05 | 13.65 | 13.95 |
| Standard Deviation | 3.2 | 4.6 | 3.3 |

The group means on the pretest differed by at most approximately 1 point and the standard deviations differed by at most approximately 1.4 points.

This concluded the statistical analysis for the purposes of: 1) the investigation of the overall effect of the arithmetic modules and 2) the investigation of the influence of volunteering for additional aid on achievement in the arithmetic modules. A discussion of the conclusions appears in Chapter V.

### The Repeated Measures Analysis

The background for the repeated measures design was presented on pages 54-56 in Chapter III. The reader may wish to review these pages before continuing.

Since the only comparison of interest was the treatment group versus the control group, retention test scores for only these two groups were obtained. Table 10 presents the means and standard deviations for these groups on the retention test.

TABLE 10. Retention Analysis Means and Standard Deviations

| | RETENTION TEST | | POSTTEST | |
|---|---|---|---|---|
| | Treatment | Control | Treatment | Control |
| Means | 23.38 | 17.75 | 22.38 | 15.75 |
| Standard Deviations | 2.5 | 4.8 | 2.5 | 3.8 |

The Posttest mean scores were not the same as mean scores presented in Table 6 since additional data were lost because of student attrition over the time lapse.

The null hypothesis tested using the repeated measures design was: $H_0$: There will be no difference between the experimental group and the control group mean scores on the Mathematics 082 Arithmetic Posttest given approximately 3½ weeks after first use of the test.

The data were collected and the analysis yielded the results presented in Table 11.

TABLE 11. Repeated Measures Results

| Sources | Sums of Squares | df | Mean Squares | F |
|---|---|---|---|---|
| G* | 600.25 | 1 | 600.25 | 29.6** |
| S:G | 608.50 | 30 | 20.28 | |
| M | 36.00 | 1 | 36.00 | 8.1*** |
| GM | 4.00 | 1 | 4.00 | .9**** |
| SM:G | 133.00 | 30 | 4.43 | |
| Total | 1,381.75 | 63 | | |

*The symbols were:
G was the group.
S:G was students nested in a group.
M was measures.
GM was the group by measures interaction.
SM:G was the subject by measures interaction nested in the group.

**F = $MS_G/MS_{S:G}$ and the null hypothesis was rejected at the.001 level since $F_{1,30}(.001) = 13.29$.

***F = $MS_M/MS_{SM:G}$ and failure to reject at the.001 level.

****F = $MS_{GM}/MS_{SM:G}$ and failure to reject at the .001 level.

The null hypothesis of no overall difference between the two groups' mean scores on the measures was rejected at the .001 level of significance. The tests of differences in measures and group by measure interaction were not significant.

The repeated measures design concluded the statistical analysis for the purpose of the investigation of the retention of material learned in the modules. A discussion of the conclusions appears in Chapter V.

### The Individual Module Analysis

The background for the individual module analysis was given on pages 56-57 in Chapter III. The reader may wish to review these pages before continuing. Also, a complete list of individual module objectives is given in Appendix A9 (page 103).

As indicated in Chapter III, no hypothesis testing was done on the individual modules. Table 12 presented the statistics used in the individual module analysis. (See next page for Table 12.)

Note that out of the 90 modules completed, students did not reach the criterion level on eight of the posttests. Although students were encouraged to repeat the modules, they discussed their posttest scores (if they wished) and did not repeat any modules. One student failed to reach the criterion on modules 3 and 4 but the other six cases of failure to reach criterion levels were by six different students.

Also, note the considerable saving in time that resulted from the modules. During past quarters, all students were required to spend approximately two weeks completing the material contained in the modules. The students spent 3 hours per week in class and 4 to 6 hours per week on homework. Most students using the modules could cover the same material in less time by excluding parts they already knew.

TABLE 12. Individual Module Analysis

| Module Number | Number of Students Who Took the Module (20 Maximum)* | Number of Students Who Reached at Least 80% Criterion (Posttest) | Number of Students Who Reached at Least 90% Criterion (Posttest) | Total Possible on Pretest and Posttest | Mean Posttest Score* | Mean Pretest Score* |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 12 | 12 | 6 |
| 2 | 1 | 1 | 1 | 5 | 5 | 4 |
| 3 | 17 | 15** | 11 | 8 | 6.9 | 3.4 |
| 4 | 18 | 15 | 5 | 6 | 5.1 | 3.9 |
| 5 | 10 | 10 | 7 | 9 | 8.0 | 5.3 |
| 6 | 14 | 14 | 14 | 10 | 9.6 | 5.7 |
| 7 | 13 | 13 | 12 | 8 | 7.7 | 5.5 |
| 8 | 1 | 1 | 1 | 11 | 11 | 11 |
| 9 | 13 | 10 | 4 | 7 | 6.0 | 3.5 |
| 10 | 2 | 2 | 1 | 6 | 5.5 | 4.5 |

*Includes only those students who took the pretest, completed the module and took the posttest. Scores of 26 students who reached criterion levels on the pretest and did not complete the module were not included.

**The number of problems required to attain each criterion level was given to the nearest whole number of problems for modules 1, 2, 3, 5, 6, 7 and 8.

***If the number of problems required to attain each criterion level was the same (as in modules 4, 9 and 10) to the nearest whole number, a one problem split was enforced resulting in the 90 per cent level changing to the 100 per cent level.

Several students needed to cover only 4 of the 10 modules; since each module was designed to take approximately 75 minutes, these students could complete the material in less than half the time usually given. This time could be spent doing additional work on the algebra contained in the course.

A complete list of the individual module pretest and posttest scores used in the study is presented in Table 13. A blank space in the pretest column indicated that the student was not encouraged to complete the module. His arithmetic pretest score indicated that he was probably proficient in the material contained in the module. A "+" in the pretest column indicated that a student was encouraged to complete the module; however, for one or more of the reasons listed at the beginning of this chapter, he did not take the module. (See Table 13 on the following page.)

The number of blank pretest spaces on the modules indicated the time saved by the students in covering the material. Further discussion of the implications of the material presented in Tables 12 and 13 are given in Chapter V. This completed the analysis of the data for the purpose of investigating the effects of the individual module on achievement of the objectives for that module.

TABLE 13. Module Pretest and Posttest Scores

| Student Number | Module Number | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | | 9 | | 10 | |
| | Pre | Post | Pre | Post | Pre | Post | Pre | Post | Pre | Post | Pre | Post | Pre | Post | Pre | Post | Pre | Post | Pre | Post |
| 1 | | | | | 4 | 6 | 5** | 5 | 6 | 8 | 4 | 9 | 6** | 8 | | | 4 | 5 | | |
| 2 | | | | | 3 | 5 | 4 | 4 | | | 2 | 9 | 7 | * | 11 | * | 4 | 6 | 5 | * |
| 3 | | | | | 5 | 8 | 4 | 4 | | | + | | + | | | | + | | | |
| 4 | | | | | 4 | 7 | 5** | 6 | | | 6 | 10 | | | | | | | | |
| 5 | | | | | | | 4 | 5 | | | 7 | 10 | 8 | * | | | 6** | 7 | | |
| 6 | | | | | 2 | 6 | 4 | 5 | 7** | 9 | 5 | 10 | 8 | 7 | 11 | * | 5 | 6 | | |
| 7 | | | 5 | * | 3 | 8 | 4 | 6 | | | 10 | * | | | | | 5 | 6 | | |
| 8 | | | 5 | * | 5 | 7 | 4 | 5 | 7** | 9 | 10 | * | 6 | 7 | 11 | * | 2 | 7 | 6 | * |
| 9 | | | | | | | 3 | 5 | | | 7 | 10 | 7** | 8 | | | 4 | 5 | 6** | 5 |
| 10 | | | | | 5 | 8 | 4 | 5 | 5 | 7 | 8** | 9 | 7** | 8 | | | 6** | 7 | 6 | * |
| 11 | | | | | 4 | 7 | 3 | 5 | 6 | 9 | 6 | 10 | 3 | 8 | 11 | * | 2 | 6 | | |
| 12 | | | | | 2 | 6 | 3 | 4 | 6 | 8 | 7 | 9 | 1 | 8 | 9 | * | 1 | 6 | 3 | 6 |
| 13 | | | | | 5 | 8 | 3 | 6 | | | 5 | 10 | 6 | 8 | | | 2 | 6 | 6 | * |
| 14 | 6 | 12 | 4 | * | 3 | 7 | 4 | 5 | 5 | 7 | 4 | 9 | 7** | 8 | + | | + | | + | |
| 15 | | | 5 | * | 2 | 5 | 4 | 5 | | | 10 | * | 7** | * | 9 | * | 6 | * | | |
| 16 | | | 4 | 5 | | | + | | | | | | 6 | 6 | | | 7 | * | 6 | * |
| 17 | | | | | 3 | 8 | 4 | 6 | 3 | 8 | 3 | 10 | 4 | 8 | 11** | 11 | + | | | |
| 18 | | | | | 2 | 8 | | | 4 | 7 | | | 5 | 8 | 11 | * | + | | | |
| 19 | | | | | 5 | 8 | 4 | 6 | 4 | 8 | 6 | 10 | 6 | 8 | 11 | * | 3 | 4 | | |
| 20 | | | | | 1 | 6 | 4 | 5 | | | 10** | 10 | 8 | * | | | 2 | 7 | | |

*The criterion level (80 per cent) was reached on the pretest, and the student did not complete the module.

**The criterion level was reached on the pretest; however, the student wanted to complete the module.

## Summary

The statistical analysis of the study has been presented in Chapter IV. Table 14 listed the decisions made on the basis of the statistical analysis.

TABLE 14. Statistical Decisions

| For No Difference Between Groups on | Source of Comparison (Groups) | Decision | Level | Test |
|---|---|---|---|---|
| Achievement Level | All | Reject | .0001 | ANCOVA |
| Achievement Level | Treatment vs Control | Reject | .01 | Tukey Post Hoc |
| Achievement Level | Control vs Non-volunteer | Fail to Reject | .01 | Tukey Post Hoc |
| Retention Level | Treatment vs Control | Reject | .001 | Repeated Measures |
| Interaction (Retention) | Treatment vs Control | Fail to Reject | .001 | Repeated Measures |

# CHAPTER V

# SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

## Discussion

The analysis of covariance results indicated significant differences between groups on achievement level over material contained in the modules. Post Hoc testing showed that there was a difference between the treatment and control groups, and between the treatment and non-volunteer groups, while the difference between the control and non-volunteer groups was not significant. Table 9 (page 64) listed means and standard deviations of scores on the Arithmetic Pretest. These statistics indicated that the only difference between the non-volunteer group and the other two groups was the motivation factor. The Arithmetic Posttest analysis indicated that this factor had minimal influence on the test scores.

The significance of the repeated measures analysis for differences between groups on the retention test indicated that the treatment group achieved higher scores than the control group. The lack of significance on the test for interaction between the groups indicated that differences between the groups were approximately the same for both tests. Thus, the treatment group students maintained their higher level of achievement on both tests.

The fact that the means for both groups on the retention test were higher than the respective group means on the posttest (as indicated in Table 10, page 65) was probably due, in part to:

1) the instruction the students received in their classes and/or 2) the effect on scores of taking the same or similar tests on three different occasions.

The test for differences in the measures (see Table 11, page 66) indicated no rejection. Although not of major interest, the test was reported. Failure to reject was an indication of no differences between the mean score on the retention test (for both groups) and the mean score on the posttest.

Table 12 (page 68) listed the statistics used to investigate the effect of the individual module on achievement over material contained in that module. Differences in mean pretest and posttest scores gave an indication of the level of achievement of the material. When considering these scores, the reader should keep in mind the number of students who completed the module. Another indication of the success of each module was the number of students who reached the minimum criterion level (80 per cent) on each module. Again keeping in mind the number of students who took the module, indications were that students did learn from the modules. The small number of students taking modules 1 and 2 was not a surprise since it was expected that most students could work the whole number and decimal addition, subtraction and multiplication problems (the material covered in these two modules).

Table 13 (page 70) listed the module pretest and posttest scores. The discussion of the small number of students who completed modules 8 and 10 needed the scores in this table for completeness.

All nine of the students who took the pretest reached the criterion level for module eight and a similar result occurred for module ten. One possible explanation for this was that the students, upon completion of earlier modules, had mastered material which originally caused the deficiency in subject matter contained in these modules. The amount of time saved as a result was obvious.

The success rates in Mathematics 082 of each group of students included in the study were of some interest. The reader should view the results with caution for two reasons. First, the course is graded on a pass (P) or a no-grade (N) basis. Second, each instructor was responsible for the final grades of his section. This procedure was very different from the one used in past quarters. The usual method was to base the grade upon a departmental final examination. Such factors as attendance had an influence on the grades given during the quarter of this study, but this was not the usual procedure. Table 15 listed the percentage of students from each group who passed the course.

TABLE 15. Percentage of Students in Each Group that Passed Mathematics 082 During Spring Quarter, 1971.

| Treatment | Control | Non-volunteer |
|---|---|---|
| 100 | 86.7 | 87.5 |

Recall that the material covered in the modules was not specifically included for instruction in the course but was deemed essential for an understanding of material contained in the course.

Thus, the information presented above was not used to draw any conclusions. Note, however, the differences in percentages for the control group, the non-volunteer group, and the treatment group.

No statistical data were collected concerning student attitude toward the use of the audio-tutorial techniques used in the study; however, verbal comment of students and their small section instructors indicated favorable reaction toward the modules.

## Summary

The purposes of this study were:

1. To investigate the use of audio-tutorial techniques on remedial college algebra students weaknesses in basic arithmetic.

2. To investigate the effect of volunteering for additional aid on achievement in basic arithmetic in a remedial college algebra class.

3. To investigate the effect of the individual arithmetic module on achievement of the objectives for that module.

4. To investigate the retention of material presented in the modules.

To investigate the use of audio-tutorial techniques, materials consisting of audiotapes, slides and worksheets consistent with the objectives of the material were prepared during the Fall and Winter Quarters of the 1970-71 school year. Ten modules, each consisting of slides, a tape, a list of objectives, a worksheet, a pretest, a posttest and answers to the pretest and worksheet, were prepared. These modules covered the basic operations of arithmetic using whole numbers, decimals and fractions, square root, per cent and the use of formulas to find areas and volumes. Each module was designed to provide instruction for from two to four objectives.

Based on the results of the Mathematics 082 Arithmetic Pretest given during the first week of classes, an individual list of at least four suggested modules was given to each of 77 of the 107 students enrolled in the non-credit course, Mathematics 082, at Michigan State University (MSU) during Spring Quarter, 1971. Using the 53 students who volunteered for additional aid through use of the modules, students were randomly assigned to a treatment group or to a control group. The remainder of the 77 students were placed in the non-volunteer group.

Three carrels to be used by the experimental group were placed in the MATHELP room in Wells Hall at MSU. MATHELP tutors and Methods in Teaching Secondary Mathematics (Education 327N) students were used to keep the room open from 8:00 a.m. to 5:00 p.m. Monday through Friday. The experimental group was given approximately three weeks in which to complete the basic arithmetic material contained in the modules. This was done in order to provide all students with a chance to work in the carrels at some time during the quarter and to encourage all students to erase arithmetic deficiencies as soon as possible.

At the end of the 3 week period, the Mathematics 082 Arithmetic Posttest was given to all students still in the course. The test was repeated approximately 3½ weeks later, and results were obtained for the treatment and control groups. Using these test scores and scores on the module pretests and posttests, statistical tests were performed to aid in the investigation.

The first two purposes listed were examined using the Arithmetic Pretest score as co-variable and the Arithmetic Posttest score as the dependent variable in a one-way analysis of covariance. The test revealed significant differences in mean scores. Post Hoc testing (using Tukey's HSD test) showed that the treatment group mean score was significantly higher than the mean scores of the control group or the non-volunteer group. There was no significant difference between the mean scores of the control group and the non-volunteer group. Mean scores and standard deviations of the three groups on the Arithmetic Pretest also indicated no differences between groups in entering behavior.

The third purpose was investigated using individual module pretest and posttest mean scores and the number of students who completed each module at the 80 per cent and 90 per cent mastery levels. These statistics indicated that the experimental group students did learn the material in the modules they completed.

The retention level of the experimental group was investigated using the scores of the Arithmetic Posttest and the Retention Test (the repeated Arithmetic Posttest) in a repeated measures design and analysis. The results indicated that the experimental group had maintained their achievement at approximately the same level above the control group since significant differences were found between the two groups, but no significant interaction was present.

## Conclusions

The results of the analysis, the sample used and the conduct of the study led to the following conclusions:

1. Students in a non-credit remedial college algebra class did learn basic arithmetic material by using audio-tutorial techniques in a carrel setting.

2. By using audio-tutorial techniques, the student could concentrate on his particular module and could repeat sections of each module as he desired or needed. Thus, a student could progress through the material at his own rate.

3. The influence of volunteering for additional aid on student achievement in basic arithmetic in a non-credit remedial college algebra class was minimal.

4. The type of student used in this study can learn given material to a mastery level of 80 per cent. If one insists on this level of mastery, he should be certain that the student has already mastered the prerequisite material.

5. The audio-tutorial technique results in a considerable saving of student time involved in learning given material. In some cases time spent was reduced by a factor of one-half or more.

6. Material learned by students using audio-tutorial techniques in a non-credit remedial college algebra class was retained through the quarter. The students did recall, at least during the somewhat limited time of the study, material they had mastered over a period of at most eight and one-half weeks.

7. In the one non-credit remedial college algebra course studied the use of audio-tutorial techniques in basic arithmetic material was beneficial to the student.

## Recommendations

The following recommendations for further study of the use of audio-tutorial techniques in mathematics courses are made. Several possibilities are listed in the hope that further research will be completed:

1. A replication of the study to verify the results.

2. A replication of the study using all students in the course, Beginning Algebra II (Mathematics 082).

3. The use of audio-tutorial techniques should be examined in other courses at MSU. Specifically, an audio-tutorial laboratory could be set up and used for the lower level Mathematics 108 (College Algebra and Trigonometry I) and Mathematics 109 (College Algebra and Trigonometry II) courses.

4. An investigation of the use of audio-tutorial materials as supplementary aids for both voluntary and non-voluntary use by college level mathematics students.

5. An investigation of the use of mastery learning in beginning college level mathematics classes.

6. The use of a mathematics lecture-laboratory situation incorporating a lecture with laboratory experiences such as audio-tutorial techniques, programmed texts, slide rules, probability

materials, desk calculators and computers should be investigated at the college level.

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# APPENDICES

# APPENDIX A1

## CARREL DIRECTIONS

1. Fill out all parts of the module report that apply, including starting time.
2. Take the pretest and bring it to the person in charge for grading.
3. Record your results on your sheet.
4. Insert the first pass strip into the open projector and focus by turning the front of the projector.
5. Plug in earphones, if necessary.
6. Insert cassette into tape player.
7. You may wish to adjust the volume (black) on the side of the recorder (start at 2).
8. If necessary, rewind the tape to start.
9. Complete the module.
10. Rewind the tape.
11. Get the posttest and take it.
12. Bring the posttest to the person in charge for grading.
13. Record your results on your sheet.
14. Record the ending time.
15. Clean and straighten the carrel, TURN OFF the projector.
16. Turn in the slides, the cassette, and the module report.

# APPENDIX A2

## MODULE PACKET

## MODULE 3

## DIVISION OF WHOLE NUMBERS AND DECIMALS

As Usual You Should Have:

1. The slides
2. The tape
3. The pretest
4. The objectives
5. The worksheet
6. The answers to the pretest and the worksheet.

As Usual:

1. Examples are in red
2. Definitions are in blue
3. Assignments on the worksheet are in green.

# MODULE 3 OBJECTIVES

Upon Completion of the Module you will:

1. Given a division problem, be able to label the quotient, divisor, dividend and remainder.
2. Be able to divide whole numbers.
3. Be able to divide decimal numbers.

Remember that objectives tell you what you will be able to do upon completion of the module. Do not worry if you cannot understand them now. Objectives tell you what will be on the posttest.

# MODULE 3 PRETEST

1. Label the dividend, divisor, quotient and remainder in:

a) $\frac{37}{5} = 7 + R\ 2$

b) $22 \div 5 = 4 + R\ 2$

2. Perform the following divisions

a) $19\overline{)372}$

b) $6\overline{)37.12}$

c) $746\overline{)78596}$

d) $231\overline{)12.16}$

e) $1.25\overline{)17.1}$

f) $17.3\overline{)1.6543}$

## WORKSHEET MODULE 3

1. Write in words:

a) $4\overline{)7}$ b) $\frac{11}{7}$ c) $15 \div 7$

2. Write the statement 7 divided by 5 using: $\div$ , ——— , $\overline{)\phantom{000}}$

Now return to slide 9

3. In each part label the dividend, the divisor and the quotient.

a) $\frac{9}{3} = 3$ b) $14 \div 2 = 7$ c) $\begin{array}{r}2\\18\overline{)36}\end{array}$

Now return to slide 21

4. Do the indicated divisions:

a) $\frac{344}{4}$ b) $10\overline{)74.57}$ c) $75\overline{)3760}$

d) $721\overline{)62.346}$ 3) $81\overline{)37.84}$

Now return to slide 49

M-3 WORKSHEET Page 2

5. Perform the indicated divisions

a) $7.3\overline{)\,84.1}$

b) $.27\overline{)\,326}$

c) $92\overline{)\,7.21}$

d) $8.43\overline{)\,2198.7}$

## MODULE 3 ANSWERS

### PRETEST

1. a) $\dfrac{\overset{\text{Dividend}}{37}}{\underset{\text{Divisor}}{5}} = \underset{\text{Quotient}}{7} + \overset{\text{Remainder}}{R\ 2}$     b) $\overset{\text{Dividend}}{22} \div \underset{\text{Divisor}}{5} = \overset{\text{Quotient}}{4} + R\ \underset{\text{Remainder}}{2}$

2. a) 19 R 11     b) 6.18 $\frac{4}{6}$ OR
6.18 R 0.04     c) 105 R 266

d) .05 R 0.61     e) 13 R 85     f) .095 R 0.108

### WORKSHEET

1. a) Seven divided by 4     b) Eleven divided by 3

c) Fifteen divided by seven

2. a) $7 \div 5$     b) $\frac{7}{5}$     c) $5\overline{)7}$

3. a) $\dfrac{\overset{\text{Dividend}}{9}}{\underset{\text{Divisor}}{3}} = \overset{\text{Quotient}}{3}$     b) $\underset{\text{Dividend}}{14} \div \overset{\text{Divisor}}{2} = \underset{\text{Quotient}}{7}$

c) $\begin{array}{r} \overset{\text{Quotient}}{2} \\ \underset{\text{Divisor}}{18}\overline{)\underset{\text{Dividend}}{36}} \end{array}$

4. a) 86     b) 7.45 R 0.07 OR
7.457     c) 50 R 10

d) .086 R 0.340     e) .46 R 0.58

5. a) 11 R 38     b) 1207 R 11     c) .07 R 0.77

d) 260 R 690

## APPENDIX A3

## MODULE POSTTEST

1. Label the dividend, quotient, divisor and remainder in:

a) $\begin{array}{r} 2\ R\ 6 \\ 15\overline{)\ 36} \end{array}$

b) $\frac{14}{5} = 2 + R\ 4$

2. Perform the indicated divisions:

a) $21\overline{)\ 521}$

b) $9\overline{)\ 83.5}$

c) $243\overline{)\ 25784}$

d) $78\overline{)\ 2.395}$

e) $2.41\overline{)123.4}$

f) $18.4\overline{)\ 1.459}$

# APPENDIX A4

## INDIVIDUAL MODULE REPORT

NAME ______________________________

MODULE TITLE AND NUMBER ____________________________________

Repeating the Module? YES NO (circle)

ENDING TIME ____________________

BEGINNING TIME _________________

PRETEST (count each part as one problem)

Number correct: _______________ Total possible: _____________

POSTTEST (count each part as one problem)

Number correct: _______________ Total possible: _____________

How do you feel about this Module?

## APPENDIX A5

## PROGRESS REPORT

NAME: ________________________________

SECTION: _______________

You are scheduled to complete modules ______________________________ by Friday, April 30.

Our records show that you have completed modules _____________________.

The carrel room has enough free time for you to complete these modules by this time. Please be certain you do, so that we will not hold up the next group of students.

# APPENDIX A6

## MATHEMATICS 082 ARITHMETIC POSTTEST

NAME ____________________

Accuracy is important. Questions will be graded right or wrong.

1. a) Find the sum: 701.24 + 26.2 + 638.72 + 977.83

b) Subtract: 274.6 - 87.43

2) Multiply: a) $\begin{array}{r} 834 \\ \underline{\times 603} \end{array}$ b) $\begin{array}{r} 3.75 \\ \underline{\times .0021} \end{array}$

3. Divide: a) $28\overline{)\,863}$ b) $26.3\overline{)5.617}$

4. a) Find the square of 26 b) Find the square root of 687 correct to 3 digits.

5. a) Reduce to lowest terms: $\frac{140}{210}$ b) Convert 2.74 to a common fraction and reduce.

c) Convert $\frac{47}{6}$ to a decimal (Use at most 2 places)

d) Convert $3\frac{7}{12}$ to a common fraction:

6. Multiply: a) $3\frac{1}{2} \times 8\frac{2}{3}$ b) $4\frac{3}{5} \times 2$

Divide: c) $6\frac{2}{7} \quad 3$ d) $\frac{5}{9} \quad \frac{2}{3}$

7. a) Circle the prime numbers:
2, 8, 7, 23, 42, 9

b) Write 140 as a product of prime numbers.

c) Find the lowest common denominator for the fractions:

$\frac{5}{12}, \frac{7}{45}, \frac{3}{50}$

8. a) Tell if each of the following pairs of numbers are equal fractions.

$\frac{4}{3}, \frac{12}{9}$ AND $\frac{2}{9}, \frac{3}{10}$

b) Convert $\frac{2}{9}$ to an equal fraction with 108 as denominator.

c) Add: $\frac{5}{12} + \frac{1}{8} + \frac{4}{15}$

9. a) Convert .8 to a per cent b) Convert 12% to a decimal

c) 68 is what per cent of 24? d) 6 is 31% of what number?

10. a) The area of a triangle is: $A = \frac{1}{2}bh$. Find the area of a triangle with base 32 and height 5.

b) The volume of a sphere is $V = \frac{4}{3}\pi r^3$. Find the volume of a sphere of radius 3. Use $\pi = 3.14$.

# APPENDIX A7

## MATHEMATICS PLACEMENT EXAMINATION

Inclusion of the Mathematics Placement Examination itself is, of course, impossible since that could terminate the examination's usefulness. The test is given to all new students at MSU regardless of class or transfer status.

A review of the examination shows that questions covering the following material appear on the test:

1. Operations with signed numbers, exponents (integral and rational), and rational algebraic experessions.
2. Absolute value.
3. Solution of linear, quadratic and two linear equations in two variables. A complex solution to a quadratic equation is present.
4. Factoring second degree expressions.
5. Simplification (including multiplication and division` of algebraic expressions of one and two terms.
6. Verbal problems.

Students with a "good" background in first (or second) year high school algebra should have little trouble with the test. There is no geometry on the examination.

## APPENDIX A8

## MATHEMATICS 082 ARITHMETIC PRETEST

NAME ________________________

Accuracy is important. Questions will be graded right or wrong.

1. a) Find the sum: 37.2 + 301.06 + 987.77 + 645.87

   b) Subtract: 236.5 - 78.73

2. Multiply: a) $\begin{array}{r} 761 \\ \underline{\text{x}802} \end{array}$ b) $\begin{array}{r} 2.35 \\ \underline{\text{x}.0013} \end{array}$

3. Divide: a) $42\overline{)\,875}$ b) $37.2\overline{)\,7.613}$

4. a) Find the square of 35 b) Find the square root of 1,240 correct to 3 digits.

5. a) Reduce to lowest terms: $\frac{150}{240}$ b) Convert 3.76 to a common fraction and reduce.

   c) Convert $\frac{39}{7}$ to a decimal fraction with at most 2 decimal places:

   d) Convert $4\frac{5}{7}$ to a common fraction:

6. Multiply: a) $4\frac{1}{2} \times 7\frac{2}{3}$ b) $7\frac{1}{8} \times 2$

Divide: c) $3\frac{7}{9} \div 6$ d) $\frac{3}{4} \div \frac{7}{8}$

7. a) Circle the prime numbers: 2, 4, 8, 9, 13, 27, 31

b) Write 130 as a product of prime numbers.

c) Find the lowest common denominator for the fractions:

$\frac{1}{12}, \frac{7}{15}, \frac{8}{25}$

8. a) Tell if the following pairs of numbers are equal fractions.

$\frac{2}{3}, \frac{4}{9}$ and $\frac{7}{5}, \frac{21}{15}$

b) Convert $\frac{7}{8}$ to an equal fraction with 96 as denominator.

c) Add: $\frac{2}{10} + \frac{7}{15} + \frac{5}{12}$

9. a) Convert .6 to a per cent b) Convert 12% to a decimal

c) 72 is what per cent of 14? d) 4 is 22% of what number?

10. a) The area of a triangle is: $A = \frac{1}{2} bh$. Find the area of a triangle with base 22 and height 4.

b) The volume of a right circular cylinder is $V = \pi r^2 h$. Find the volume of a right circular cylinder of radius 6 and height 4. Use $\pi = 3.14$.

# APPENDIX A9

## ASSIGNMENT SHEET AND OBJECTIVE LIST

NAME ______________________________

SECTION ___________________________

A Red Square means you should study this module. You have a serious deficiency in this area which may hinder your completion of Mathematics 082.

A Blue Circle means you are advised to study the module.

No mark means you are probably competent in this area but you may study the module if you wish.

The list of objectives tell what is contained in each module.

1 2 3 4 5 6 7 8 9 10

Each module consists of slides, a tape, a worksheet (for practice), a pretest and a posttest to indicate your success.

You may sign up for reserved carrel time in room A-212 WH. The modules will be available for study between 8 and 5 Monday through Friday. There will be a person in the carrel room (A-138 WH) to answer questions you have while you are working on a module.

## I

## OBJECTIVES

| Upon completion of module number: | You will be able to: |
|---|---|
| one | 1. add whole numbers.<br>2. add decimal numbers.<br>3. subtract whole numbers.<br>4. subtract decimal numbers |
| two | 1. label correctly the factors and the product in a multiplication problem.<br>2. find correct products in multiplication problems using whole numbers and decimals. |
| three | 1. given a division problem, label the quotient, divisor, dividend and remainder.<br>2. divide whole numbers.<br>3. divide decimal numbers. |
| four | 1. find the square of any given number.<br>2. find the square root of any number to a given number of digits. |
| five | 1. reduce a fraction to lowest terms.<br>2. convert a decimal fraction to a common fraction, and vice versa.<br>3. convert a mixed number to a common fraction. |
| six | 1. multiply fractions using whole numbers, mixed numbers and fractions.<br>2. divide fractions using whole numbers, mixed numbers and fractions. |
| seven | 1. pick out examples of prime numbers.<br>2. write any whole number greater than 1 as a product of primes.<br>3. find the LOWEST COMMON DENOMINATOR for two or more fractions. |

| | |
|---|---|
| eight | 1. pick out examples of equal fractions.<br>2. given any fraction, convert it to an equivalent fraction having a given denominator.<br>3. given two or more fractions, find the sum of the fractions. |
| nine | 1. convert decimals to per cents and vice versa.<br>2. find a given per cent of a number.<br>3. find what per cent one number is of another.<br>4. find a number when a per cent of it is known. |
| ten | 1. find the perimeter of triangles, rectangles and the circumference of a circle.<br>2. find the total surface area of triangles, rectangles, parallelepipeds and right prisms.<br>3. find the volume of a parallelepiped, a sphere and a right circular cylinder. |

## APPENDIX A10

## DATA SHEET

PLEASE PRINT

NAME ______________________________ AGE ______ SEX M F

CLASS: Fresh. Soph. Jr. Sr. Grad.

MAJOR(S): ______________________________

MINOR(S): ______________________________

NAME OF HIGH SCHOOL: ______________________________

GIVE THE NUMBER OF SEMESTERS COMPLETED IN HIGH SCHOOL

Semesters Completed

General Math: ____________________

Algebra: ____________________

Geometry: ____________________

Trigonometry: ____________________

COLLEGE OR JUNIOR COLLEGE MATHEMATICS CLASSES COMPLETED:

APPROXIMATE ATTENDANCE:

I usually missed 0-3 classes, 4-6 classes 7-10 classes,

more than 10 classes

# APPENDIX A11

## MATHEMATICS 082 MATERIAL FOR SPRING QUARTER, 1971

Following is a list of major topics listed for the course, Mathematics 082 during the time this study was in progress.

1. Sets and symbolism - real numbers
2. Equality and order axioms
3. Properties of real numbers
4. Sums, differences, products and quotients of signed numbers
5. Order of operations
6. Polynomials - sums, differences, products, quotients, and factoring (linear and quadratic)
7. Signs of algebraic fractions
8. Reduction of algebraic fractions
9. Sums, products and quotients of algebraic fractions
10. Integral, rational, zero and negative exponents
11. Radicals
12. Solution of first and second degree equations including the quadratic formula