RADIO FREQUENCY HEATING APPLIED TO MAGNETIC AND NON MAGNETIC CYLINDRICAL CONDUCTORS

THESIS FOR DEGREE OF M. S.

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This is to certify that the

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THESIS

RADIO FREQUENCY HEATING APPLIED TO MAGNETIC AND NON MAGNETIC CYLINDRICAL CONDUCTORS.

By
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A THESIS

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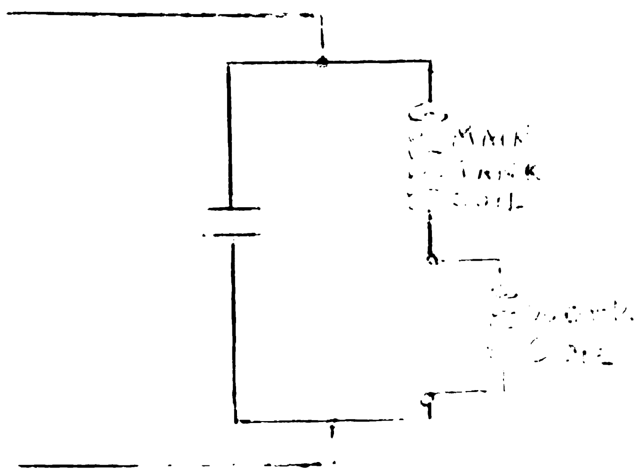
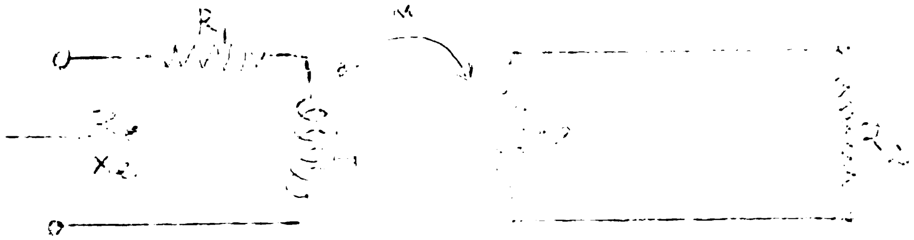
THESIS

The objective of this thesis is the study of the application of induction heating principles to the heating of several metallic pieces to obtain the desired results. By means of the formulas that will be presented later, the circuit requirements are calculated and suitable coils designed to accomplish a determined purpose, namely, to heat the material up to T degrees Farenheit in t seconds.

One of the sources of trouble in induction heating is the variation of the resistivity of metals with temperature. This effect causes the load power to vary and therefore the reflected impedance due to the load, and the Q of the work coil. As a consequence, the Q of the tank circuit, the tank voltage, and current change too. This can be best understood by inspection of the circuit diagram.



Considering the circuit shown as equivalent to the actual circuit, we can write



Work coil | Metal to be heated.

$$R_e = R_1 + \frac{(WM)^2 R_2}{R_2^2 + (WL_2)^2} \qquad (1)$$

$$X_e = X_1 - \frac{X_2 (WM)^2}{R_2^2 + (WL_2)^2}$$

Therefore, there is always an increase in resistance due to the secondary circuit and a decrease in reactance. It must be well understood that the original reactance, $\underline{X_1}$, to be considered, has the same magnetic circuit as the actual one with the only difference that the secondary circuit is open circuited.

Thus, in the case of heating a copper tubing, X_e is measured with the tubing inside the coil, and X_1 without the tubing. The experiment shows that X_e and R_e are 1.93 Ω and 0.161 Ω, X_1 and R_1 being 2.41 and 0.160 Ω.

The problems dealing with magnetic loads are somewhat more complicated, although the variation of R

follows the same behavior. In this case X_1 would be the coil with the magnetic core inside but not allowing any secondary current to flow. This is certainly not the same reactance as that measured with the coreless coil and can be thought of as the same coil with a laminated core of the same permeability . This way, starting from the coreless coil reactance X_o we can obtain X_e in the following steps:

1) - Coreless coil X_o

2) - Same coil with ideal laminated core $X_1 = X_o\mu$

3) - Same coil with actual core:

$$X_e = X_1 - \frac{X_2\ (WM)^2}{R_2^2 + X_2^2} = X_o\mu - \frac{X_2\ (WM)^2}{R_2^2 + X_2^2}$$

The following experiments show that $X_e > X_o$ and of course that $R_e > R_1$.

In all the problems on induction heating, the variation of the resistivity of the load plays an important part. While the computations on r - f - power deal with the resistivity that corresponds to the average temperature of the material to be heated, the design of the tank circuit must be made considering the minimum Q, that is the maximum resistance R_e of the circuit. Formula (1) says that this corresponds to the maximum resistivity of the core. During the operation, changes in R_2 due to the temperature, produce changes in R_e and therefore in the Q of the main tank. This, in turn, causes the tank voltage and current to change and as a consequence the D.C. plate voltage must be adjusted continuously to keep a constant tank current and so a constant magnetizing force. All this will be illustrated later with the aid of vector diagrams.

CALCULATION OF REQUIRED MAGNETIZING FORCE

"Radio Frequency Heating" by Brown, Hoyles and Blerwith, on page 29, gives the following general formula for heating cylindrical conductors.

$$P = \frac{3\pi H_o^2 \cdot \sigma}{\pi \cdot s} \cdot F$$

c.g.s. rationalized practical system of units.

When: P = Watts per unit length

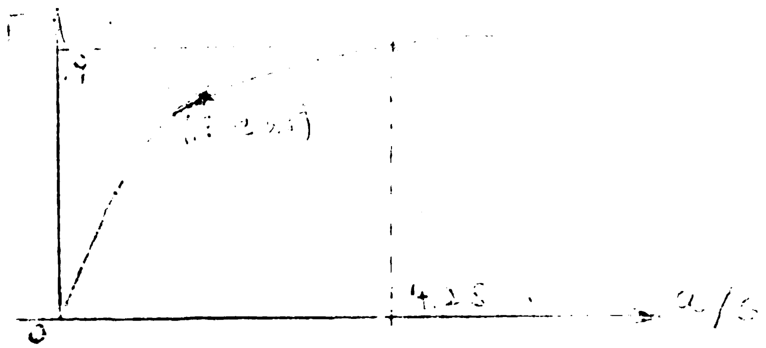
H_o = magnetizing force of the coil without the core.

σ = conductivity.

a = radius of cylinder

s = skin depth in the cylinder

F = a function of several Bessel's functions plotted in the following graph:



In all the experiments performed in connection with this thesis a/s was much higher than 5 and so we can set

F equal to one.

$$P = \frac{8 \pi H_o^2}{\sigma} \cdot \frac{a}{s} \quad \text{watts per cm length.}$$

The power density (watts per cm cube), becomes:

$$(1) \quad PD = \frac{8 \pi H_o^2}{[illegible]} \cdot \frac{a}{s} = \frac{8 H_o^2}{[illegible]\ a\ s} \quad \text{watts/cm}^3$$

(2) But

$$S = \frac{1}{2\pi\sqrt{10^{-9} [illegible]}} \quad \text{cm (Brown's page 17)}$$

Therefore;

$$(3) \quad P.D = [illegible]$$

From here on we will analyze separately the magnetic and non-magnetic materials.

MAGNETIC MATERIALS

By definition

[illegible]

Therefore:

[illegible]

But since induction heating is accomplished under sa turation conditions -

B = constant

and: [illegible] Where [illegible] $\frac{B}{M_o}$

Therefore: P.D = [illegible]

where ρ is the resistivity of the material

P.D = [illegible]

Where [illegible]

and [illegible]

The variation of P with the temperature causes the energy input to vary continuously so that the total energy input per cubic cm is:

[illegible]

Assuming now that the temperature increases linearly with time and neglecting the initial value, we can write:

$$t^{o} = K^{1}\ t\ \text{sec}.$$

Assuming also that the resistivity increases with temperature following a straight line through the origin, we can write:

$$\rho = K_{1}^{\ 1}\ t^{o} = K_{1}\ t_{\ \text{sec}}$$

Where $K_1 = K_1^{\ 1} \times K^1$

This assumption is not very far from the actual conditions as can be seen from the corresponding graph of $\rho = f\ (t^{o})$ Fig. (1). Besides, as far as power computations are concerned, it yields acceptable results since the main contributions of energy are effected at higher values of ρ when it approaches the straight line closely enough.

Under these conditions we can write:

[illegible]

But $\rho_{max} = K_1\ T_{sec}$.   Therefore:

$$E = G . \sqrt{\rho_{max}}\ \frac{2}{3}\ T_{sec}$$

If ρ is a linear function of temperature, as has been assumed, then:

$$\rho_{max} = 2\ \rho_{average}$$

Therefore:

$$E = G \times 0.945 \sqrt{\rho_{ave}}\ T_{sec.} = \frac{0.945 K\ H_o^{3/2}}{a} \sqrt{\rho_{ave}}\ T_{secs}$$

However:

$$E = (PD)_{ave} \times T_{secs.}$$

So:

$$(P\ D)_{ave} = \frac{0.945\ KH_o^{3/2}}{a} \sqrt{\rho_{ave}}$$

Hence:

$$H_o = \left(\frac{(P.D)_{ave}\ a}{0.945.K. \sqrt{\rho_{ave}}} \right)^{2/3}$$ cgs rationalized system of units

The value of $K = 159 \times 10^{-5} \frac{B}{\gamma_o}$ depends upon the magnetic induction of the material at saturation and, of course, it changes if we change the system of units.

For ferrous materials, "Electronic for Industry" by Bendz gives:

$$H_o = \left\{ \frac{PD \times d}{0.438 \sqrt{\rho_{ave}}} \right\}^{2/3}$$ oersteds

Where:

$P\ D$ = watts/cubic inch

d = diameter in inches

ρ = resistivity in Ω inches

H_o = peak magnetizing force (oersteds)

μ = relative permeability

B = magnetic induction (Gausses)

The condition

$\frac{a}{s} = \frac{d}{s} \geqslant 4.25$ (to make F $\doteqdot$ 1) expressed

in these units becomes

$$\frac{d(\text{inches})}{s(\text{inches})} = \frac{d}{3170\sqrt{\frac{\rho_{ave}}{\mu f}}} = \frac{d}{3170}\sqrt{\frac{\mu f}{\rho_{ave}}} \geqslant 4.25$$

$$d\sqrt{\frac{\mu f}{\rho_{ave}}} \geq 13400$$

The value of μ to be used in these formulas was found from experimentation (Bendz's, page 298) to be:

$$\mu = 1.8\,\frac{B\ \text{saturation}}{H_o} = 1.8 \times \frac{18000}{H_o} = \frac{32400}{H_o}$$

NON-MAGNETIC MATERIALS

Letting μ_r equal unity, equation (3) becomes:

$$P.D = \frac{159 \times 10^{-5} \times H_o^2}{a}\sqrt{\rho \times f}$$

Under the same assumptions done before, the following final formula can be obtained (Bendz's 304)

$$H_o = \sqrt{\frac{P\ D \times d}{2.59 \times 10^{-3}\sqrt{f\rho}}} \qquad \text{oersteds}$$

Where, as before:

d = diam. in inches

ρ = resistivity, Ω inches

f = frequency, cpo.

The condition $\frac{d}{s}$ [illegible] 4.25 becomes:

$$d \sqrt{\frac{[illegible]}{[illegible]}}$$ [illegible]

SELECTION OF FREQUENCY

The original formula

[illegible]

and the plot of F as a function of $\frac{a}{s}$ shown before, show that increasing $\frac{a}{s}$ in the range 0-2.25 (by increasing frequency) causes a high increment of F and therefore P.

When $\frac{a}{s}$ reaches 2.25, F reaches 0.8 and approaches saturation. Therefore, we can define a minimum frequency (for $\frac{a}{s}$ = 2.25) for which the coupling of energy is efficient.

In fact, let's analyze the variation of the ratio "power absorbed in the core to power lost in the exciting solenoid" as a function of F.

$$E = \frac{\text{Power core}}{I^2 \; R_{\text{solenoid}}}$$

But

$$R_{\text{solenoid}} = \frac{R_{dc}}{2} \; \frac{a_{sol}}{S_{sol}} \qquad \text{(Brown's, page 22)}$$

Besides from (2)

S = [illegible] Where the A's are constant for a given material

Therefore:

$$R_{sole} = A_2 \sqrt{f}$$

Similarly:

$$\frac{a\ core}{s\ core} = A_3 \sqrt{f}$$

So:

$$R_{sol.} = \frac{A_4 a\ core}{S\ core}$$

and

$$E = \frac{\text{Power core}}{I^2 A_4 \frac{a\ core}{s\ core}} = \frac{\text{Power core}}{A_5 \frac{a\ core}{s\ core} H_o^2}$$

substituting $\frac{P}{H_o{}^2}$

and from (4) gives:

$E = k_6 F$ If $\frac{a}{s} > 2.25$ F ≅ constant

Therefore ≅ constant.

With higher values of frequency (a/s > 2.25) we couple, of course, more energy into the core (for same H_o) but the ratio E is very nearly constant and so is, of course, the efficiency.

The minimum frequency for efficient heating is, therefore:

$$\frac{a}{s} = 2.25$$

So $\frac{a}{s} = a\, 2\pi \sqrt{\ \ }$ = 2.25 (cgs rationalized units)

$$f_{min} = \frac{128.5 \times 10^6}{\ \ }$$

In the specific case of the piece of iron that will be heated in the 1st experiment we have:

f_{min} = 26000 $\frac{1}{\ \ }$ (to obtain the higher f_{min})

$\mu_r = 938$

$d = 0.5'' = 1.25$ cm

Therefore:

$$f_{min} = \frac{128.5 \times 10^6}{26000 \times 938 \times 1.56} = 3.4 \text{ c/s}$$

The critical frequency to heat a 0.5" diam. copper cylinder is:

The equipment used to conduct the experiments has a frequency of 350 KC, thus obtaining the highest possible efficiency for a given material and work coil. The object of using so high a frequency is to couple more power into the core for a given circuit.

EQUIPMENT AND VECTORIAL DIAGRAMS

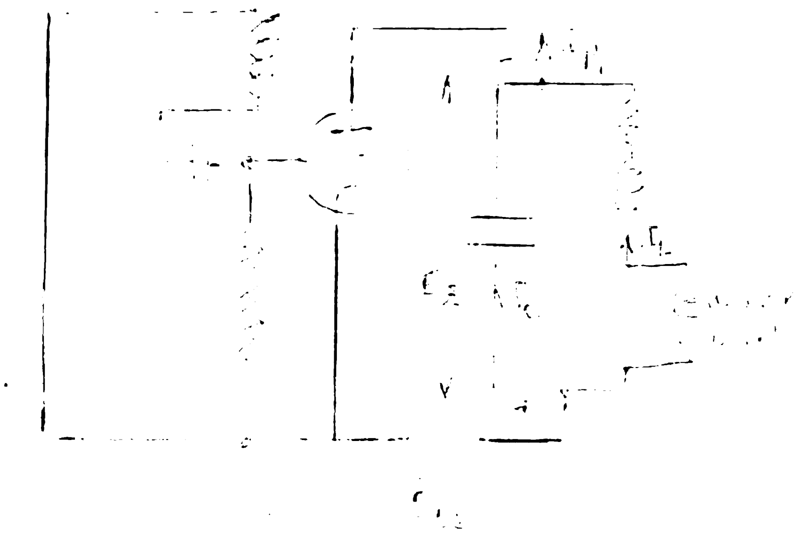
The equipment used during the experiments was a 1 KW, 20 amp. tank current, Westinghouse Generator. The equipment is so designed that we can obtain two different frequencies by changing the connections.

a) Hartly circuit - For dielectric heating with a frequency of 10 M c/s.

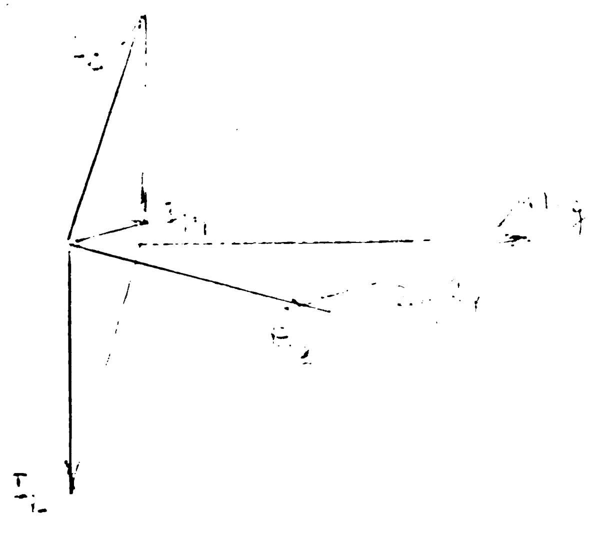
b) Feedback oscillator - For induction circuit with a rated frequency of 300 KC.

The experiments were run with the oscillator set in (b) and the frequency was checked to be 350 KC.

Essentially the circuit is as shown below:



The ectorial diagram can be drawn for the tank circuit as follows:



When the material to be heated is introduced in the work coil, it has been found that Q decreases due to the reflected resistance as has already been explained. This produces a slight shift in the vector diagram but the main effect is that the current i_L decreases.

In fact, the impedance of the tank circuit at resonance is:

$R_T =$ [illegible]

so, if R increases R_T decreases and, for the same plate current, E_T decreases too. This causes I_t and I_L to decrease.

When the material is being heated the reflected resistance changes continuously due to the variation of resistivity of the core with temperature which causes the above mentioned change to take place. As a result, if a constant magnetizing force (constant i_L) is desired, the anode supply voltage E_{bb} must be continuously adjusted.

The measurements of temperature were made with two iron constanton thermocouples whose calibration curves are attached, Fig.(2). The 1st couple was used during the 1st experiment and the 2nd one was used in the others.

HEATING SEVERAL SAMPLES OF LOW CARBON STEEL

The following specific heat and resistivity data was obtained from the "Handbook for Metallurgical Engineers".

SWEDISH IRON

°F	S HEAT
68	.1075
77	.108
104	.1097
140	.112
212	.114
302	.121
392	.128
482	.134
572	.140
752	.151
932	.163
1022	.172
1112	.188
1200	.208

DENSITY

70°F - 7.85
600 - 7.78
1200 - 7.63

RESISTIVITY

°C	°F	ρ
0	32	13.2
100	212	20.1
200	390	27.4
300	570	36.
400	750	46.2
500	930	58.3
600	1110	72.

So the following average specific heat values were obtained for different ranges:

70°F - .1075 BTU/pound °F = small col/

70 - 200 - .111

70 - 600 - .124

70 - 1200- .143

EXPERIMENT #I

Piece of iron (0.2% Carbon) 5.25" long, 0.5" diam. to be heated from $70^{o}F$ up to $600^{o}F$ in 1 minute.

1) Calculation of Thermal power required:

$TP = 17.6\ MC\ \Delta T$ where:

M = rate of material to be heated per minute (in pounds/minute).

C = average specific heat in BTU/pound ^{o}F

ΔT = temperature variation in ^{o}F

In this case it is:

Vol. = 1.03 cu. inch

Density at $70^{o}F = 7.85\ \frac{gr}{cm^3} = .284\ \frac{pound}{cub.inch}$

Hence:

$M = 1.03 \times 0.284 = 0.292\ \frac{pound}{minute}$

Besides

$C_{70\ -\ 600\ ^{o}F} = 0.124$

Therefore:

Thermal power = $17.6 \times 0.292 \times 0.124 \times 530 = 338$ watts.

2) Radiation, convection and conduction losses.

Conduction losses are very small because the piece is held by two 0.1" diam. pivots fixed one at each end.

The radiation and convection lossess are less than 10 watts (from Bendz's, page 301) and therefore are negligible.

3) Power density.

$$P.D = \frac{\text{Total input (watts)}}{\text{Volume of metal in coil}} = \frac{338}{1.03} = 328 \frac{\text{Watts}}{\text{Cub.inch}}$$

4) Peak magnetizing force:

The corresponding formula for a solid magnetic cylinder is:

$$H_{peak} = \left(\frac{PD \times d}{0.438 \;\cdots} \right)^{2/3} \text{ valid where d } \cdots$$

Where H = [illegible]
d = diameter (inches)
[illegible] = relative permeability
[illegible] = resistivity at average temp (335°F) =

[illegible]

[illegible] = frequency in C/S = 350,000 C/S

As already explained, the permeability to be used in these formulas is:

$$\mu = \frac{32400}{H}$$

The magnetizing force therefore becomes

$$H_{peak} = \left(\frac{328 \times 0.5}{0.438 \sqrt{35 \times 10^{\cdot} \times 10 \times 10^{-6}}} \right) = 200^{2/3} =$$

- 34.5

$$\mu = \frac{32400}{34.5} = 938$$

The condition

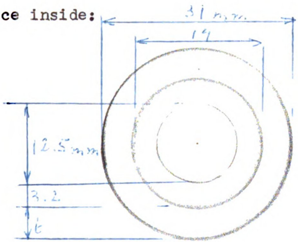
[illegible]

is satisfied.

5) Coil design:

The coil was designed to be 6.7" long so as to cover the material thoroughly. With a 19 min. interior diameter this allowed a clearance of 3.2 mm around the work piece. The coil was made of 19 turns of copper tubing 6 mm diameter. The outside diameter became 31

The sketch below shows the coil with the iron piece inside:



6) Coil current.

The values of H in oersteds and I in amperes are related by the following formula:

$$H = \frac{0.4\pi NI}{l\,(cm)} = \frac{0.4\pi NI}{2.5\, l\,(inch)} = 0.503\,\frac{NI}{l\,(inch)}$$

If H is to represent the peak magnetizing force and I is expressed in terms of (rms) of a sine wave, then:

$$H_{peak} = \sqrt{2} \times 0.503\,\frac{NI}{l} = 0.70\,\frac{N\,I_{rms}}{l\,(inch)}$$

Therefore:

$$I = \frac{H_{peak} \times l_{[illegible]}}{[illegible]} = 1.4 [illegible]$$

To take account of the coil-core spacing the factor K, is added:

$$I = \frac{[illegible]}{N} \cdot K_1 \qquad \text{(Bendz 312)}$$

K_1 being a function of $\frac{\text{coil length}}{\text{spacing}}$

In this example the current will be:

$$I = \frac{[illegible]}{[illegible]} \cdot K_1 = 17.[illegible] K_1$$

Where for a $\frac{\text{length}}{\text{spacing}}$ ratio equal to 45, K_1 is very nearly 1.02. Hence:

I = amp. 18.0

As the rating of the equipment to be used is 1 KW, 20 amp., this experiment can be run without further complications.

The table shows the experimental results where the values of temp, I and V were measured, thus allowing the calculation of the impedance to be made:

TABLE I

f	Secs.	I	Temp.(over room temp. Th.couple	°F	V	Z
350 KC	10		2.5	95	61	3.3
"	20		5	190	63	3.4
"	30		7.5	285	65	3.51
"	40		9.4	360	66	3.57
".	50		11	420	66	3.57
"	60		13.5	515	66.3	3.59
"	90		18.2	700	69.	3.73

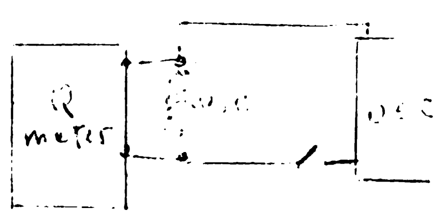
Temp. in 1 minute = 515 + 70 = 585°F

To separate both, the resistive and reactive components of Z , measmurements of the Q of the circuit were done by means of a Q meter. The material was heated up to a certain temperature and quickly connected to the Q meter and disconnected from the heating equipment. The same operation was repeated at different maximum temperatures.

The following data was obtained:

TABLE II

f	°F over room temp.	Q
350 KC	70	9
"	270	7
"	450	5
"	700	4
"	800	3.5
"	900	3.5



Plotting Z and Q as functions of temperatures in Fig.(4) provides a means of determining the values of R and X independently. The separation of R and X was made by means of:

$$\left\{\begin{array}{l} Z^2 = R^2 + X^2 \\ Q = \dfrac{X}{R} \end{array}\right.$$

Solving the above equations gives:

$$Z^2 = X^2 + \frac{X^2}{Q^2} = X^2 \left(1 + \frac{1}{Q^2}\right)$$

Hence

$$X^2 = \frac{Z^2}{1 + \frac{1}{Q^2}} \qquad \left\{\begin{array}{l} X = \dfrac{Z}{\sqrt{1 + \frac{1}{Q^2}}} = \dfrac{Z\,Q}{\sqrt{Q^2 + 1}} \\ R = \dfrac{X}{Q} \end{array}\right.$$

The following table gives the results obtained for the first sample tested:

TABLE III

^{o}F over room temp.	Z	Q	X	R
100	3.3	8.7	3.28	0.377
200	3.43	7.6	3.403	0.447
300	3.52	6.55	3.48	0.532
400	3.56	5.5	3.50	0.636
500	3.60	4.73	3.53	0.746
600	3.64	4.23	3.54	0.837
700	3.73	3.9	3.61	0.927

The same measurements were repeated for the coil alone over the range of temperature from room temperature up to the temperature that the coil attains when the material has been heated. It was found that Z and Q are constants, the values being:

TABLE IV

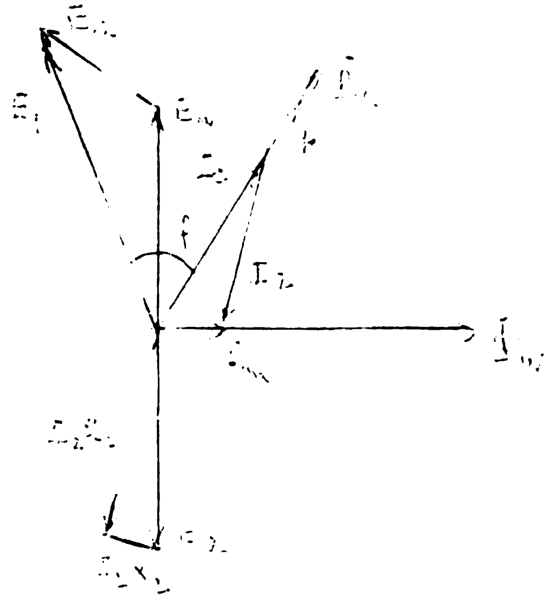
Coil Alone	Q	Z	X	R
The same range of variations of temperature	15	2.42	2.41	0.16

Where X and R have been calculated as before.

These curves are drawn on the same graph as the parometers of the cored coil (Fig. 4). The curves of Fig. 4 show to what extent the resistance and reactance of the coil increases when the core is set inside.

CALCULATION OF COIL VOLTAGE

The following vector diagram shows the core and coil currents and voltages and the distribution of fluxes through the core and air space:



Where:

Φ_W, E_W, flux through the work piece and corresponding emf in the coil -

Φ_A, E_A, flux through the air space and corresponding emf in the coil -

I_c - coil current

I_2 - core (secondary) current -

E_2 - secondary emf (core)

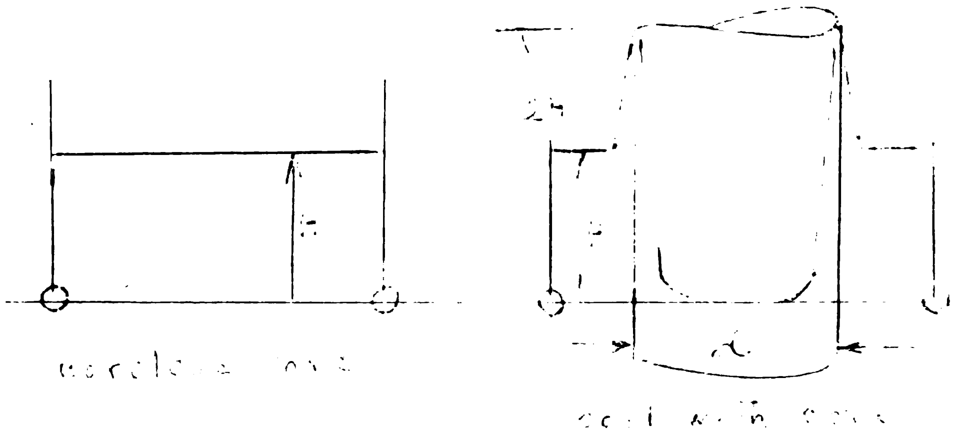
H - magnetizing force dueto the coil current.

The coil current produces the magnetizing force H and the flux Φ_A in phase with it. The flux in the core Φ_W is in phase with the magnetizing current I_m.

The emf induced in the core E_2 is 90° lagging Φ_W and the core current I_2 is lagging due to the reactive characteristic of the circuit.

The primary emfE_1 is made up of two parts: E_A due to the flux in the air space, and E_W due to the flux in the work piece. Both these vectors are leading the corresponding fluxes.

The variation of H (peak magnetizing force) vs. the diameter of the coil and work piece, is shown to be (Brown, p. 28) roughly:



Therefore, the flux through the air space is

$$\Phi = 6.45H\,(A_{coil} - A_w) \quad \begin{pmatrix} \Phi\text{, B and H} \\ \text{cgs units} \end{pmatrix}$$

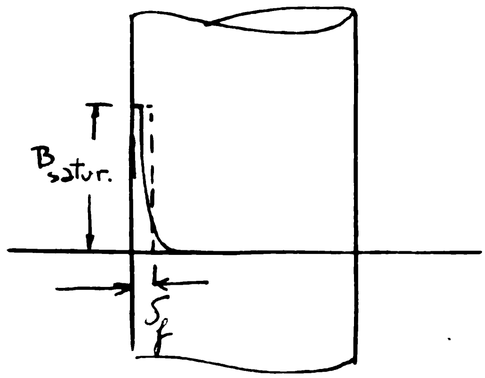
Where

A coil = cross sectional area coil in inches^2

A_w = cross sectional area work piece

The flux through the work piece is calculated assuming that it penetrates the material at a constant value B (phase and magnitud constants) to a depth equal to δ below the surface. It is further assumed that the material is saturated and B = 18000. (See transactions A1EE Vol. 63, June 1944, p. 273).

On this basis the total flux through the material is approximately:



$$\Phi_W = 18000\,\pi \,.\, d \,.\, \delta_f \times 6.45 \quad (\delta_f \text{ and } d \text{ in inches})$$

Where

$$\delta_f = 17.6\sqrt{\frac{H\,\rho_{max}}{f}} \doteq 17.6\sqrt{\frac{34.5 \times 14.6 \times 10^{-6}}{35 \times 10^{4}}} \text{ inches}$$

Where

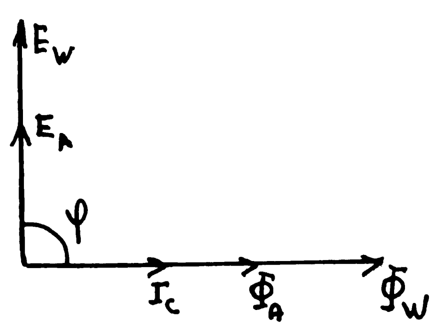
$$\rho_{max} = 37 \times 10^{-6}\,\Omega\,\text{cm} = 14.6 \times 10^{-6}\,\Omega\text{ inch}$$

(Corresponds to 600°F)

So:

$$\delta_f = \frac{17.6 \times 3.8}{10^5} = 67 \times 10^{-5} \text{ inch.}$$

Under the assumptions made Φ_W is in phase with B in the surface which is, in turn, in phase with I coil. Therefore, the diagram becomes:



In this ideal diagram we are neglecting the in phase component of E_W that is small compared with the reactive component.

Now we can compute E_W and E_A.

$$E_W = j28.6 \times f \times T \times 18000\,\pi\, d\, \delta_f \times 10^{-8} \qquad \text{volts.}$$
$$= j\,0.0162 \times f \times T \times d \times \delta_f = j\,36 \text{ volts}$$

Where T = coil turns = 19. .d = core diam = 0.5"

f = frequency.

The value of E_A must be calculated by means of the equivalent ideal coil. The same coil shall be discussed in connection with the heating of a non-magnetic material.

From there we get:

A coil - A_W = 0.67 - 0.196 = 0.474 sq. inch.

Therefore:

$$E_A = j\,28.6 \times f \times T \times H \times 0.474 \times 10^{-8} = j31 \text{ volts}$$

Neglecting the internal resistance and reactance voltage drops we have:

$$E_T = j31 + j36 = j\,67 \text{ volts.}$$

This value corresponds to $600^{\circ}F$ and coincides with the one measured in the experiment.

From there $Z = \frac{E}{I} = \frac{67}{18} = 3.73\,\Omega$ ($600^{\circ}F$)

When the core is taken out, all the voltage drop is due to the flux in the air and, assuming roughly the same current distribution in the coil, we get:

(A coil = 0.67) inch

$$\begin{cases} \Phi_W = 0 \\ \Phi_A = 6.45H \times A_{coil} = H \times 0.67 \times 6.45 \end{cases}$$

Therefore; if H is kept constant (I = 18.0 amp) we get:

$E_W = 0$

$E_A = j\ 28.6 \times f \times T \times H \times 0.67 \times 10^{-8} = j44$ volts

and the reactance of the coil would be:

$$X_{\substack{coil \\ alone}} = \frac{E}{I} = \frac{44}{18} = 2.44\ \Omega$$

This coincides with the value obtained from the experiment.

Therefore, the introduction of the iron core increases the reactance of the coil from 2.44 Ω to 3.73 Ω at 600°F and also reflects some resistive component that can be calculated by means of the power factor:

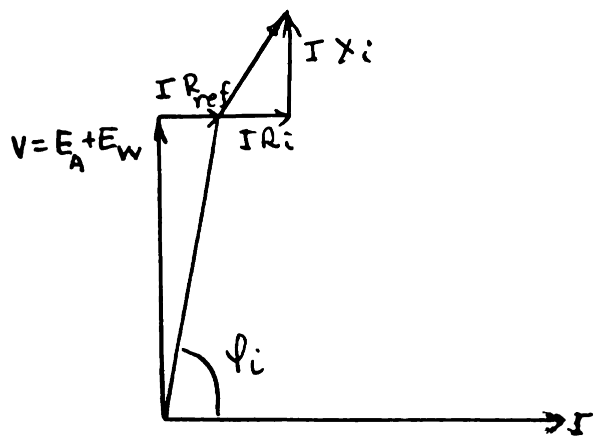
Internal Power factor is $= \dfrac{\text{Input to the core (Watts)}}{\text{VI (work coil)}} =$

Where V does not take account of the internal IR_1 and IX_1 drops.

$$\cos \varphi_i = \frac{338 \text{ watts}}{67 \times 18} = 0.28$$

$$\cos \varphi_i = 0.28$$

Where φ_i is the angle shown in the diagram



Therefore:

$R_{reflected} = 3.73 \times 0.28 = 1.045\ \Omega$ (at 600°F)

The total resistance of the coil with the core inside is the sum of this term plus the coil resistance. This was measured and was found to be:

$R_1 = 0.16\ \Omega$

Therefore:

$R_{Total} = 0.16 + 1.045 = 1.205\ \Omega$ (at 600°F)

This figure differs from 0.837 Ω found in the experiment by 30%. This is understandable if we consider how indirectly this experimental value was obtained.

The efficiency of the coil is:

$$\text{effy} = \frac{I^2\ 1.045}{I^2\ 1.205}\ 100 = 87\%.$$

and the total power factor

$$\cos \varphi_T = \frac{1.205}{3.73} = 0.323$$

The above results have been plotted in graphs 2, 3 and 4.

EXPERIMENT #2

Piece of iron (0.2% carbon) 6" long, 0.37" diameter to be heated from 70°F up to 1000°F in one minute.

1) Calculation of required thermal power:

   Again:

   TP = 17.6 x M x C x Δ T (units as explained before)

   In this case:

   Volume = 0.645 cub. inches

   M = 0.645 x .284 = 0.183 pounds/minute.

   C = average 70 - 1000°F = 0.137 BTU/pound x°F

   Therefore:

   TP = 17.6 x 0.183 x 0.137 x 930 = 410 watts.

2) Radiation, convection and conduction losses at maximum temperature are 17 watts (4%) and will be neglected during the calculations.

3) Power density

   $$PD = \frac{410}{0.645} = 637 \frac{\text{watts}}{\text{cub. inch}}$$

4) Peak magnetizing force:

The same formula used before applies in this problem:

$$H \text{ peak} = \left(\frac{PD \times d}{0.438\sqrt{fp}}\right)^{2/3} \quad \text{valid when } d\sqrt{\frac{\mu f}{\rho}} > 13{,}400$$

The resistivity is:

$$\rho = \rho_{535^{o}F} = 34\ \mu\Omega\,\text{cm} = 13.4 \times 10^{-6}\ \Omega\ \text{inch}$$

Therefore:

$$H_{peak} = \left(\frac{637 \times 0.37}{0.438\sqrt{35 \times 10^{4} \times 13.4 \times 10^{-6}}}\right)^{2/3}$$

$$= (249)^{2/3} = 39.6$$

The condition

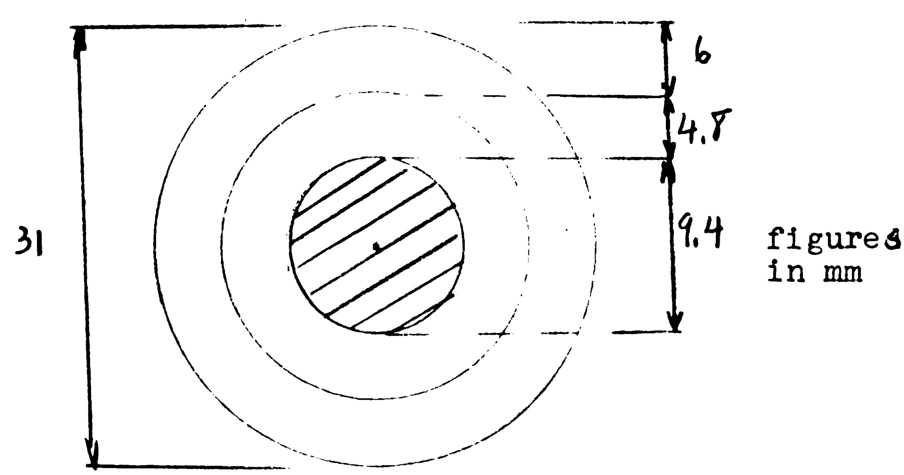
$$d\sqrt{\frac{\mu f}{\rho}} = 0.37\sqrt{\frac{818 \times 35 \times 10^{4}}{13.4 \times 10^{-6}}} = 0.37 \times 10^{5} \times 46$$

$$= 1{,}700{,}000 > 13{,}400$$

Where: $\mu = \frac{32400}{39.6} = 818$ is satisfied.

5) Coil design.

The same work coil employed in the preceding problem was also used for this experiment. A sketch of the coil and core is shown below.



Coil length = 6.7", made of 19 turns of copper tubing 6 mm in diameter.

6) Coil current:

As in the preceding example:

$$I = \frac{1.43 \times H_{peak} \times \ell \text{ (inch)}}{N} \; K_1$$

Where K_1 for a ratio:

$$\frac{\text{Coil length}}{\text{spacing}} = \frac{164}{4.8} = 35$$

is:

$K_1 = 1.03$

Therefore:

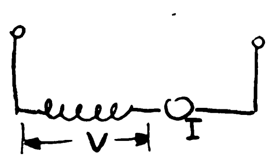
$$I = \frac{1.43 \times 39.6 \times 6.7}{19} \; 1.03 = 20.6 \text{ amp.}$$

The experiment was run at 20 amp, this being the maximum rating of the equipment used.

The following table gives the experimental results obtained:

TABLE V

f	secs	I	temp(over room temp) μa	°F	V	Z
350	10	20	3.5	150	63	3.15
350	20	20	6	250	66	3.3
350	30	20	9	375	68	3.4
350	40	20	12.2	520	70	3.5
350	50	20	16	660	75	3.75
350	60	20	19	790	78	3.9



Temperature in 1 minute = 790 + 70 = 860°F

Voltage data has also been obtained to calculate the impedance of the cored-coil.

Following the same procedure as in the first experiment, the Q of the cored coil was measured at different temperatures, the following results being obtained:

TABLE VI

f	°F over room temp.	Q
350	70°	9.5
350	300	7.
350	400	5.
350	500	5.
350	700	3.5

Both Q and Z were plotted on the same graph as a function of temperature (Fig. 5). Therefrom pairs of values at the same temperatures were obtained. As a next step X and R were calculated as before:

TABLE VII

°F over room temp.	Z	Q	X	R
100	3.05	9.3	3.035	0.327
200	3.25	8.1	3.22	0.397
300	3.35	6.8	3.32	0.488
400	3.4	5.6	3.35	0.6
500	3.5	4.7	3.43	0.73
600	3.63	4.2	3.53	0.84
700	3.8	3.7	3.68	0.995
800	3.95	3.4	3.79	1.11

As already stated the coil used in this experiment is the same as the one used before; its reactance and resistance being constant and of value:

$Z = 2.42\ \Omega$

$X = 2.41\ \Omega$

$R = 0.16\ \Omega$

These curves have been plotted on the same graph as the curves for the cored coil. The curves indicate to what extent the resistance and reactance of the coil increased when the core is set inside.

All the above results have been plotted graphically in Figs. No. 5 and 6.

EXPERIMENT #3

Piece of iron 1.48" long, 0.75" diameter to be heated from 70°F up to 600°F in one minute.

The procedure is the same followed before.

1) Calculation of required thermal power:

$T.P = 17.6 \times M \times C \times \Delta T$

In this case:

Volume = 0.655 cub. inch.

$M = 0.655 \times 0.284 = 0.186$ pounds/minute.

C = average 70-600°F = 0.124 BTU/pound °F.

Therefore:

$T.P = 17.6 \times 0.186 \times 0.124 \times 530 = 215$ watts.

2) Radiation, convection and conduction losses are negligible.

3) Power density.

$$P\ D = \frac{215}{0.655} = 329\ \frac{\text{Watt}}{\text{cub. inch}}$$

4) Peak magnetizing force:

The formula to be applied is the same as in the preceding problems:

$$H_{peak} = \left(\frac{PD \times d}{0.438\sqrt{f\rho}}\right)^{2/3} \quad \text{valid when}$$

$$d\sqrt{\frac{\mu f}{\rho}} \geqslant 13400$$

When:

$$\rho = \rho_{335^{o}F} = 25\ \mu\Omega\,cm = 10 \times 10^{-6}\ \Omega\ \text{inch.}$$

Therefore:

$$H_{peak} = \left\{\frac{329 \times 0.75}{0.438\sqrt{35 \times 10^{4} \times 10 \times 10^{-6}}}\right\}^{2/3} =$$

$300^{2/3} = 45.$

Hence:

$$\mu = \frac{32400}{45} = 720$$

The condition:

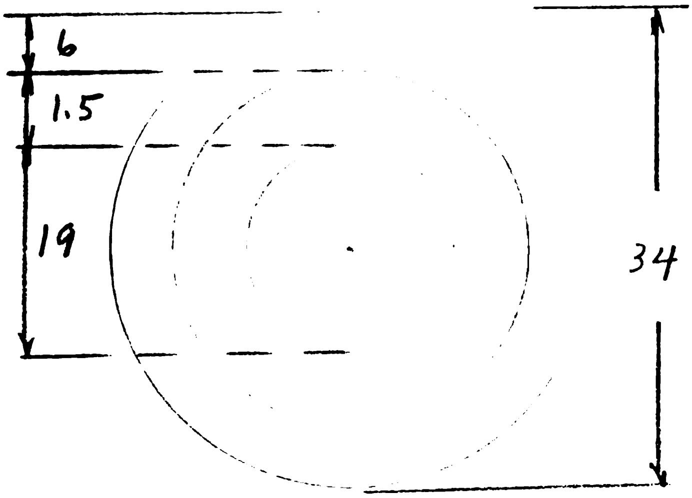
$$d\sqrt{\frac{\mu f}{\rho}} = 0.75\sqrt{\frac{720 \times 35 \times 10^{4}}{10 \times 10^{-6}}} = 75000 \times 50.3 =$$

$3{,}770{,}000 > 13400$

is satisfied.

5) Coil design:

The coil was made of 6 turns of 6 mm diameter copper tubing and was designed 5.7 cm (2.24 inches) in length and 34 mm outside diameter, thus allowing a clearance of 1.5 mm around the material.



Figures in mm.

6) Coil current:

$$I = \frac{1.43 \times H_{peak} \times l \text{ (inch)}}{N} K_1 =$$

$$\frac{1.43 \times 45 \times 2.24}{6} K_1$$

Where K_1 correspondends to a ratio:

$$\frac{\text{coil length}}{\text{spacing}} = \frac{57}{1.5} = 38 \qquad \text{So } K_1 = 1.03.$$

Therefore:

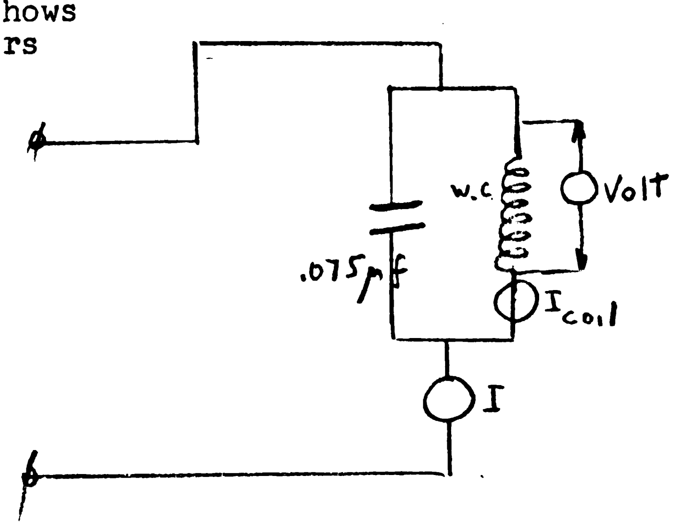
$I = 24.7$

To get this current through the work coil with the 20 amps rated equipment, the work coil was shunted with a high voltage .075 μf mica condenser. The following data was obtained:

TABLE VIII

f	secs	I_{coil}	μamp.	°F over room temp. °F	V	Z
350	10	25	2.5	105	22.3	0.893
350	20	25	4	170	23	0.92
350	30	25	5.5	230	23	0.92
350	40	25	7.5	310	23.8	0.953
350	50	25	9	375	23.8	0.953
350	60	25	10.2	430	24.1	0.964
350	80	25	14	580	25.	1.

The scheme shows the parometers measured.



Final temperature in 1 minute = 430 + 70 = 500°F

The Q readings gave the following results:

TABLE IX

°F over room temp.	Q
70	7
200	7
500	5
700	4
1000	4

Finally, computations were made for the reactance and resistance of the coil as before.

TABLE X

°F over room temp.	Z	Q	X	R
100	0.89	7.35	0.885	0.12
200	0.92	6.7	0.911	0.136
300	0.945	6.2	0.934	0.151
400	0.96	5.6	0.946	0.169
500	0.975	5	0.956	0.191
600	1.	4.4	0.975	0.221

The corresponding readings for the coil alone are:

TABLE XI

t over room	Q	Z	X	R
70°	10	0.8	0.796	0.0796
600	10	0.8	0.796	0.0796

Measurements of the temperature were made at both the surface and the center of the piece under identical conditions and running the experiment with 20 amp. The following results were obtained:

TABLE XII

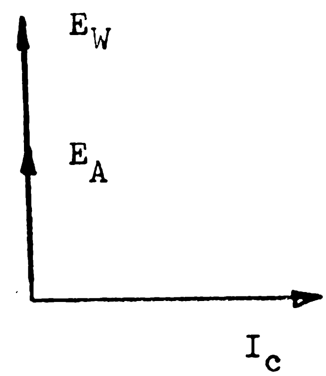
I_{coil}	°F over room temperature		Secs
	Center	Surface	
20 amp.	130	130	20
"	170	170	30
"	210	228	40
"	263	278	50
"	317	333	60
"	470	490	90

It can be seen that the temperature is fairly uniform due to the small size of the piece and the fact that the heating was rather slow.

All of the above results have been plotted graphically in Figs. 7, 8 and 9.

COIL VOLTAGE COMPUTATIONS

Following the same procedure as in the first experiment and neglecting the in phase component:



$E_W = j\ 0.016 \cdot d \times f \times T \times \delta_f$ volts
(δ and d in inches)

Where:

$$\delta_f = 17.6\sqrt{\frac{H \times \rho_{max.}}{f}} = 17.6\sqrt{\frac{45 \times 15 \times 10^{-6}}{35 \times 10^{4}}} \text{ inches}$$

Where

$$\rho_{max} = \rho_{600^{o}F} = 15 \times 10^{-6}\ \Omega \text{ inch}$$

Hence -

$$\delta_{f\,(600^{o})} = 77.4 \times 10^{-5} \text{ inches.}$$

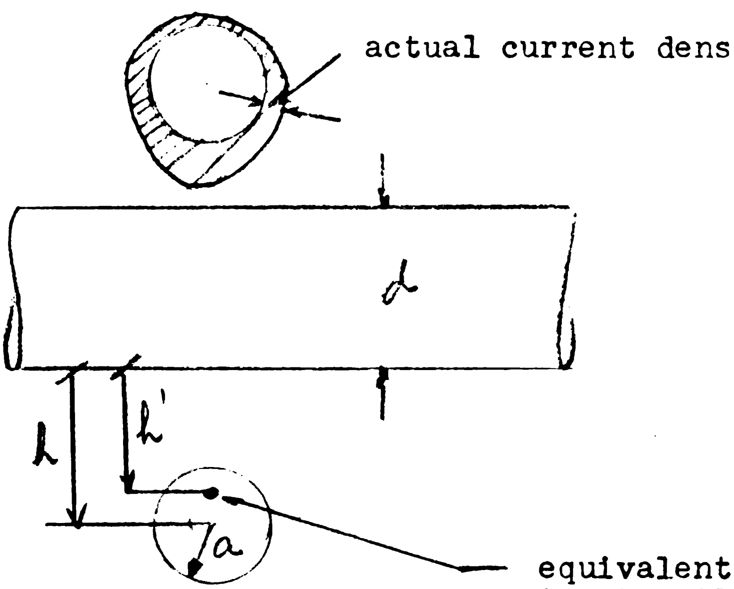
Therefore:

$E_{W\,(600^{o})} = j\ 0.0162 \times 0.75 \times 35 \times 10^{4} \times 6 \times 77.4 \times 10^{-5} = j\ 19.75$ volts

Besides:

$E_A = j\ 28.6 \times f \times T \times H\ (A_c - A_w) \times 10^{-8}$ volts.

$\underline{\underline{A_c}}$ is the cross sectional area of the equivalent ideal coil. As will be explained later in connection with the heating of a copper tubing, the spacing h' of this fictitious coil would be: (Brown's Chapter VIII)



$$h^1 = h \sqrt{1 - \left(\frac{a}{h}\right)^2} \qquad \text{Where } a = 3 \text{ mm}$$

$$h = 4.5 \text{ mm}$$

$$h^1 = 4.5 \sqrt{1 - 0.445} = 3.33 \text{ mm.}$$

Therefore the diameter of the ideal coil is:

d_c = 1.01" and the area: A_c = 0.8 sq. inch.

Hence:

$$E_A = j\ 28.6 \times 35 \times 10^4 \times 6 \times 45 \times 0.358 \times 10^{-8} =$$

$$= j\ 9.66 \text{ volts}$$

Therefore

$V = E_A + E_W = j\ 9.66 + j\ 19.75 = j\ 29.41$ volts(600°F)

And: $X = \frac{29.41}{24.7} = 1.19\ \Omega$

Actually E_W has an "in-phase component" that provides the power input to the load.

We can thus calculate the internal power factor (not including the IR_i, IX_i voltage drops in the coil):

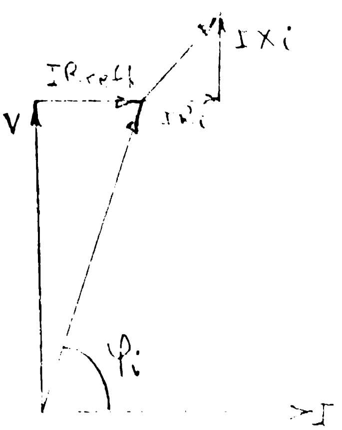
$$\text{internal power factor} \doteq \cos\varphi_i = \frac{\text{Input to the core (watts)}}{V\ I\ (\text{work coil})}$$

Where $V \doteq E_A + E_W$.

$$\cos\varphi_i \doteq \frac{215}{24.7 \times 29.41} = 0.296$$

Therefore:

$$R_{reflejada} = 1.19 \times 0.296 = 0.35352$$



The total resistance of the coil with the core inside is the sum of this plus the coil resistance. This was measured and was found to be:

$$R_1 = 0.0796$$

So:

$$R_{total} = 0.0796 + 0.353 \doteq 0.4326\ \Omega\ (600^{\circ}F)$$

This value is high compared with that found in the experiment (0.221). This disagreement is due partially

to the fact that the power was less than 215 watts as can be observed by looking at the heating curve. However the power could not be measured directly to check this.

Besides, the value of R from the experiment was found through a very indirect way basing our computations on the measurements of Q done with an inadequate instrument and for this reason it cannot be considered accurate.

The total power factor:

$$\cos \varphi_T = \frac{0.4326}{1.19} = 0.363$$

And the efficiency of the coil:

$$\text{effeciency} = \frac{I^2 \times 0.353}{I^2 \times 0.4326} \times 100 \doteq 87.8\%$$

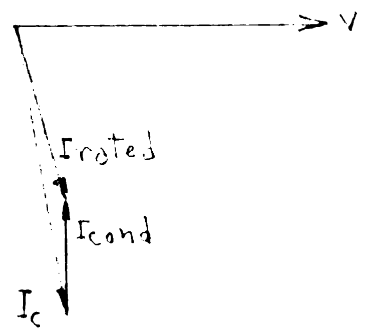
Condenser computations.

In the last experiment of this paper there is a complete discussion about the application of condensers in parallel with the work coil.

Starting from V = 29.41 volts and selecting I condenser so that:

$$I_{cond} = I_{coil} - I_{\substack{rated\\equip}} = 24.7 - 20 = 4.7 \text{ amp}$$

Even though this formula is not estrictly **correct** it can be applied successfully due to the fact that i_c is almost an entirely resistive current.



We can write

$$I_{cond} = V.\ C.\ W.$$

$$C = \frac{I_{cond}}{V\ W} = \frac{4.7 \times 10^6}{29.41 \times 2\pi \times 35 \times 10^4}\ \mu f.$$

$$C = 0.073\ \mu f$$

EXPERIMENT # 4

Iron tubing 3.22" long, 0.11" thick and 0.84" external diameter to be heated from 70°F up to 600°F in one minute.

The calculations follow the same procedure as before

1) Thermal power:

$$TP = 17.6 \times M \times C \times \Delta T$$

Where

$$\text{Volume} = \frac{\pi \times 0.84^2}{4} - \frac{0.62^2}{4} \times 3.22 =$$

0.805 Cub. inch

$$M = 0.805 \times 0.284 = 0.229 \text{ pounds/minute}$$

C = average 70 - 600°F = .124

= 530

Thence:

$$TP = 17.6 \times 0.229 \times 0.124 \times 530 = 265 \text{ watts.}$$

2) Radiation, correction, and conduction losses are negligible.

3) Power density.

$$PD = \frac{265}{\text{total volume}} = \frac{265}{1.78} = 149 \ \frac{\text{watts}}{\text{cub. inch}}$$

4) Peak magnetizing force. The same equation for solid magnetic cylinders applies for the hollow ones if the wall thickness is greater than the depth of current penetration or:

$$\text{Wall thickness} \geq 3170 \sqrt{\frac{\rho}{\mu f}}$$

Therefore:

$$H = \left(\frac{PD \times d}{0.438 \sqrt{f} \times \rho}\right)^{2/3} = \left(\frac{149 \times 0.84}{0.438 \sqrt{35 \times 10^4 \times 10 \times 10^{-6}}}\right)^{2/3}$$

$$\rho\,(335^\circ F) = 10 \times 10^{-6} \ \Omega \text{ inch}$$

$$H = \left(\frac{285}{1.87}\right)^{2/3} = 28.6$$

So:

$$\mu = \frac{32400}{28.6} = 1130$$

The maximum skin depth takes place at maximum temperature. It is:

$$\rho\,(600^\circ F) = 38 \mu\Omega\text{cm} = 15 \times 10^{-6} \ \Omega \text{ inch}$$

$$\text{Skin depth} = 3170 \sqrt{\frac{15 \times 10^{-6}}{1130 \times 34 \times 10^4}} = \frac{3170 \times 1.95}{10^7}$$

$$= 6.18 \times 10^{-4} \text{ inches}$$

The condidion

Wall thickness = 0.11"> 6.18×10^{-4} inch

is satisfied.

5) Coil design.

The coil was made of 10 turns of 6 mm diameter copper tubing and was designed 3.5" (8.9cm) long and 38 mms outside diameter thus allowing a clearance of 2 mms around the core.

6) Coil current

For $\frac{\text{coil length}}{\text{spacing}} = \frac{89}{2} = 45$ $K_1 = 1.02$ ([1]Bendz page 312)

Therefore:

$$I_{coil} = \frac{1.43 \times 28.6 \times 3.5}{10} \; 1.02 = 14.5 \text{ amp.}$$

The following data was obtained:

TABLE XIII

t secs.	I coil	°F over room Temp.		V	Z
10	15.5	1.3	60	36	2.32
20	"	3.2	140	38	2.45
30	"	5.4	230	42	2.7
40	"	7.	290	42	2.7
50	"	8.7	360	42	2.7
60	"	10.5	435	42	2.7
90	"	15	620	42	2.7

Final temp. in 1 minute = 435 + 70 = 505°F.

Following the same procedure as before the Q's were measured at different temperature.

TABLE XIV

Temp	Q
70	7.5
350	6
600	4
800	4

From the graph No. 11 the following table can be constructed:

TABLE XV

°F over room temp.	Z	Q	X	R
100	2.4	7.6	2.38	0.313
200	2.6	6.8	2.57	0.378
300	2.7	6.	2.665	0.444
400	2.7	5.2	2.65	0.51
500	2.7	4.5	2.64	0.587
600	2.7	4.2	2.63	0.603

The following readings were obtained taking the core out of the coil:

TABLE XVI

Coil alone	Temp. over room	Z	Q	X	R
	70	1.75	15	1.715	0.114
	450	"	"	"	"
	700	"	"	"	"

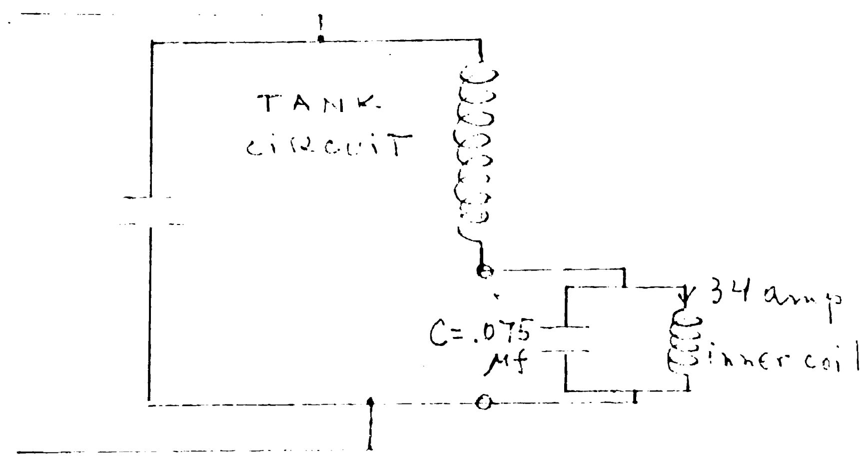
All these results are plotted in figs. 10 and 11. Conclusions are similar to those drawn from experiment No. 1.

EXPERIMENT # 5

The same iron tubing was heated with a coil inside, and a higher current using a condenser in parallel.

TABLE XVII

Coil current I	secs.	Temp over room μ.amp	Temp over room °F
34	10		
"	20	1	40
"	30	1.5	65
"	40	1.7	72
"	50	2.	85
"	60	2.7	115
"	90	4.5	190



The coil had the following characteristics:

external diameter = 13 mms
internal diameter = 5 mms
length = 8.5 = 3.35 inches
no. of turns = 15

The final ΔT in one minute was only 115°F. To get a temperature of 600°F in one minute ($\Delta T = 530$) the current should have been I_2, and the peak magnetizing force H_2.

But:

$$H = K_1(P.D)^{2/3} = K_2 \times (\text{Thermal power})^{2/3} = K_3 \times \Delta T^{2/3}.$$

The ratio between the actual H_1 and H_2 is:

$$\frac{H_2^{3/2}}{H_1^{3/2}} = \frac{530}{115} = 4.61 = \frac{I_2^{3/2}}{I_1^{3/2}}$$

Therefore:

$$I_2^{3/2} = I_1^{3/2} \times 4.61 =$$
$$I_2^{3/2} = (34.0)^{3/2} \times 4.61 = 198 \times 4.61 = 915$$

Hence:

$I_2 = 94.5$ amp and the value of H_2 peak magnetizing force for such a coil to **heat** the **material** up to 600°F in one minute, becomes:

$$H_2\text{peak} = \frac{0.70 \times 15 \times 94.5}{3.35} = 296.$$

Therefore in this case, the ratio of $\frac{H_{inner}}{H_{outer}}$ to accomplish the same purpose is:

$$\frac{H_{inner}}{H_{outer}} = \frac{296}{28.6} = 10.35$$

The results are shown in graph No. 10.

EXPERIMENT # 6

Copper tubing to be heated from 70 to 320°F in two minutes.

The dimensions of the copper tubing are:

outer diameter = 0.5 inches

length = 6.2 inches

thickness = 0.018 inches

weight = 0.056 pounds

external volume = 1.215 cub. inches

The specific heat and resistivity are for copper:

TABLE XVIII

°F	S. heat	ρ $\mu\Omega$cm
	-	1.72
70	0.09	-
400	0.096	3.1
600	0.099	-
800	0.101	4.8
1200	0.106	6.6

<u>Hence</u>, average s. heat (70-320) = 0.093

1) Thermal power

$$TP = 17.6 \times 0.056 \times 0.093 \times \frac{250}{2} = 11.5 \text{ watts}$$

2) Radiation, convection and conduction losses are negligible.

3) Power density

$$PD = \frac{11.5}{\text{External volume}} = \frac{11.5}{1.215} = 9.46 \frac{\text{watts}}{\text{cub. inch}}$$

4) Peak magnetizing force:

For this hollow cylinder to be considered

equivalent to a thick-walled one, the following condition must be satisfied:

Wall thickness $\geqq$ skin depth

That is:

Wall thickness $\geqslant 3170\sqrt{\frac{\rho_{max}}{f}}=$ (Bendz's p.305)

When

$$\rho_{max} = \rho_{320} = 2.8 \times 10^{-6}\,\Omega\,cm = 1.1 \times 10^{-6}\,\Omega \text{ inch}$$

So:

$$\text{Wall thickness} \geqslant 3170\sqrt{\frac{1.1\times10^{-6}}{35\times10^4}} = \frac{3170}{10^6}1.77 = 5.6\times10^{-3}\text{in.}$$

The required condition is satisfied.

Under these conditions we can apply the following formula developed in this paper previously:

$$H = \sqrt{\frac{PD \times d}{2.59 \times 10^{-3}\sqrt{f\rho_{ave}}}} \quad \text{valid if } d\sqrt{\frac{f}{\rho_{ave}}} \geqslant 13400$$

Where $\rho_{ave} = \rho_{195} = 2.35 \times 10^{-6}\,\Omega\,cm = 0.93 \times 10^{-6}\,\Omega$ inch

The condition

$$d\sqrt{\frac{f}{\rho_{ave}}} = 0.5\sqrt{\frac{35\times10^4}{0.93\times10^{-6}}} = 0.5\times10^5\times6.14 = 3.07\times10^{+5} > 13400$$

is also satisfied.

Therefore:

$$H = \sqrt{\frac{9.46 \times 0.5}{2.59\times10^{-3}\sqrt{35\times10^4\times0.93\times10^{-6}}}} = \sqrt{3.2 \times 10^3} = 56.5$$

5) Coil design.

The same coil used in experiment No. 1, was used for this experiment, its characteristics being:

19 turns of copper tubing 6 ~~inches~~ in diameter

6.7" long

31 ~~inches~~ outside diameter

19 ~~inches~~ inside diameter

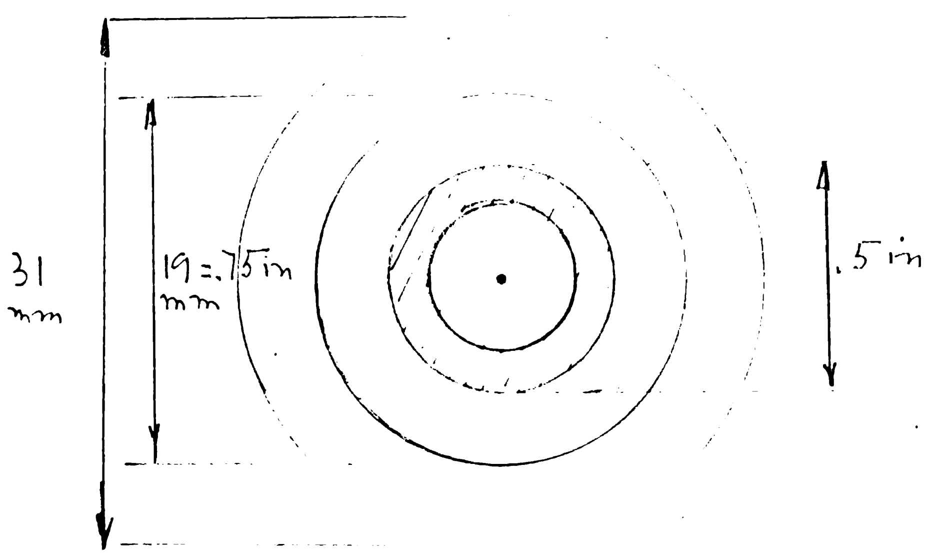
The clearance was 3.2 ~~inches~~ and the factor K_1 nearly unity.

6) Coil current

$$I = \frac{1.43 \times 56.5 \times 6.7}{19} = 28.5 \text{ amp.}$$

Coil voltage

The diagram shows the relative sizes of coil and core:

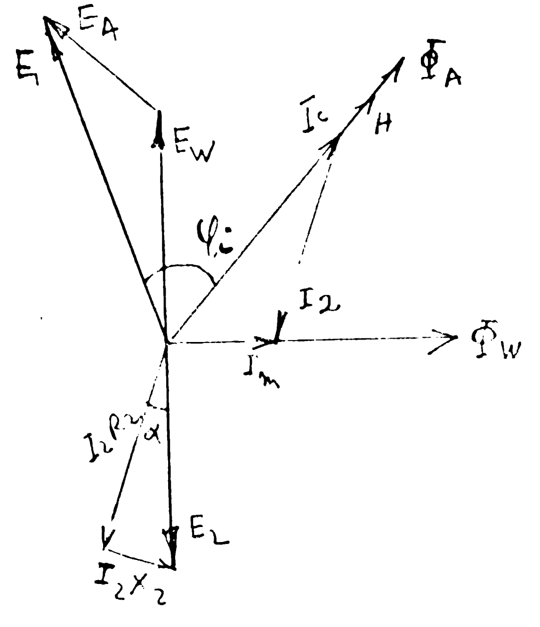


It is easily seen that we have to consider two different fluxes, the one through the work piece Φ_w and the one through the air space surrounding it, Φ_A.

Let a vector diagram be constructed. The coil current I_c produces the flux Φ_A in phase with it. The

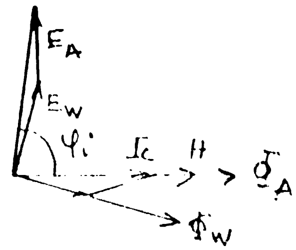
flux Φ_w is lagging due to the secondary current ($I_2 = I_{work}$), I_2 is yet to be found.

As a second step let's find the emf's. The emf induced in the core E_2 is 90^o lagging Φ_w and the core current is lagging α^o due to the resistive and inductive characteristic of it, considered as a secondary circuit short circuited. Thus, E_2 is made up of I_2R_2 and I_2X_2. Now adding up I_c and I_2 referred to the primary, we obtain the total core magnetizing current I_m. The primary emf E_1 is made up of 2 parts: E_a due to the flux in the air and EW due to the flux in the work piece. Each of these vectors is leading 90^o the corresponding flux.



$\cos \varphi_i$ is the internal power factor.

Shifting the diagram so as to make I_c be in the X axes we can write:



$$\Phi_A = H \times \text{Area air space}$$

$$\Phi_A = H\,(A_{coil} - A_w)$$

$$\Phi_W = A_w \cdot H(P - jQ)$$

Where A_{coil} and A_w represent the area of the coil and work piece and H the peak magnetizing force of the coil above; that is, the air space. P and Q therefore, take care of any change in the magnitude and phase of H while coming into the core.

The emf's core then, can be written:

$$E_a = j28.6 \times f \times T \times H\,(A_{coil} - A_w)\,10^{-8} \text{ volts.}$$

$$E_w = j28.6 \times f \times T \times A_w \cdot H\,(P - jQ) \times 10^{-8} \text{ volts.}$$

T represents the number of coil turns.

In the present experiment at a temperature of 320°F we have:

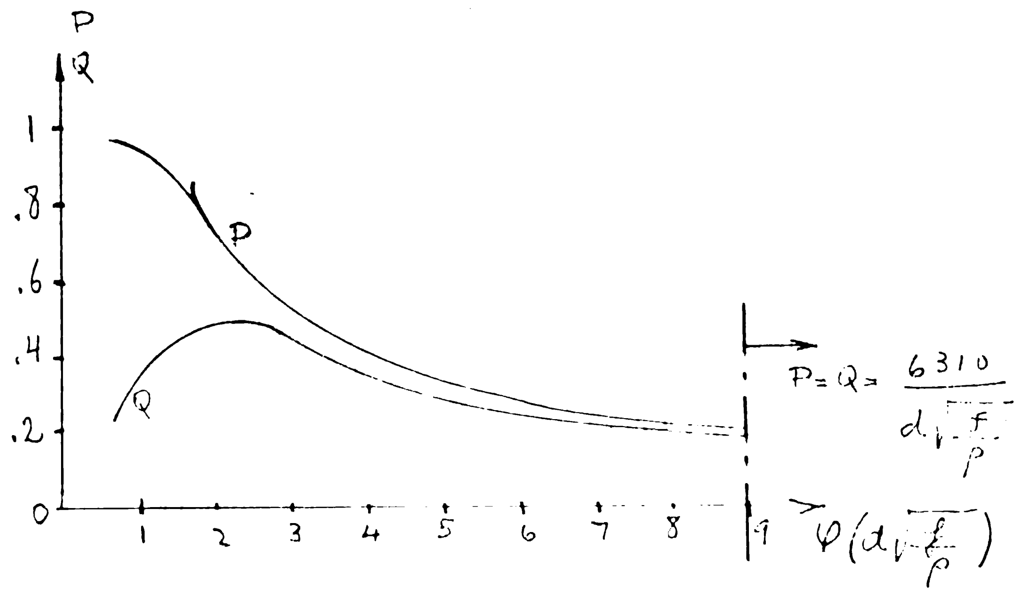
$$\rho_{max} = \rho_{320} = 1.1 \times 10^{-6} \text{ inch-}\Omega$$

$$d = \text{diameter} = 0.5 \text{ inch.}$$

Hence:

$$d\sqrt{\frac{f}{\rho}} = 0.5\sqrt{\frac{35 \times 10^4}{1.1\times10^{-6}}} = 0.5 \times 10^5 \times 5.63 = 282{,}000$$

The parameters $\underline{P}$ and $\underline{Q}$ vary as shown in the following graph. Taken from AIEE transactions Vol. 63, June 1944, p. 273.



The value $\varphi(d\sqrt{\frac{f}{\rho}}) = 9$ on the X axes corresponds to

$$d\sqrt{\frac{f}{\rho}} = 13400$$

Hence in the present case we are beyond that point and:

$$P = Q = \frac{6310}{282000} = 2.24 \times 10^{-2}$$

Therefore:

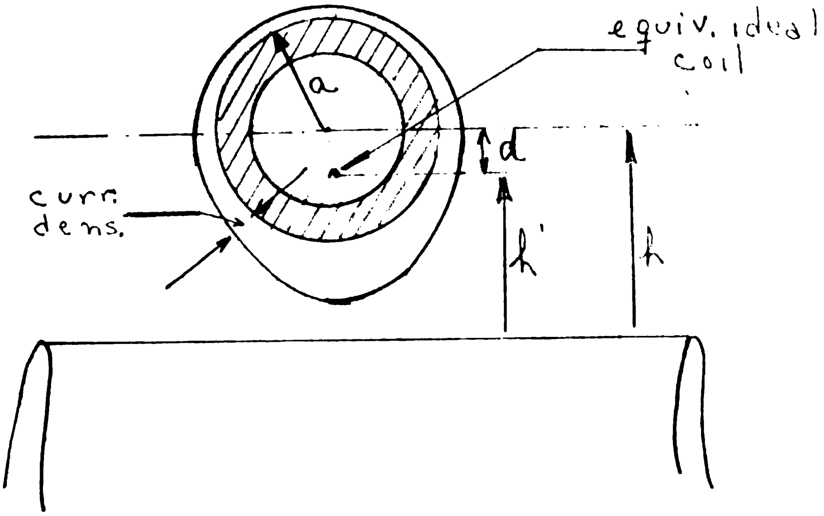
$$E_w = j\ 28.6 \times 35 \times 10^4 \times 19 \times 0.19 \times 56.5\ (2.24 - j2.24)10^{-10} \text{volts}$$

$$E_w = j\ (471 - j471)10^{-3} = 0.471 + j0.471 \text{ volts} = 0.665\ \underline{|45^\circ}$$

We can see that, in all the theoretical cases very small and the flux Φ_A can be neglected, the internal power factor angle is 45°.

The value of E_a is analyzed by considering an equivalent ideal coil.

Due to the currents flowing in the cylinder the current density in the coil increases at the inner boundary and decreases at the outer one as the diagram shows:



In Brown S. Hoiler's Chapter VIII there is a complete analysis of the matter where for a single turn coil they conclude that for the equivalent ideal coil the following equation applies:

$$h' = h\sqrt{1 - \left(\frac{a}{h}\right)^2}$$

In this case:

$h = 6$ inches

$a = 3$ inches

Therefore:

$h' = 6\sqrt{1-0.25} = 6 \times 0.865 = 5.2$ inches

and the diameter of this ficticious coil

$d_c = 23.4$ inches $= 0.923$ inches.

So:

$A_c = 0.67$ sq. inch.

$E_a = j28.6 \times 35 \times 10^4 \times 19 \times 565 \times (0.67-0.196) \times 10^{-8}$

$= j\,51$ volts $= 51\,\underline{|90^\circ}$ volts.

We must add to this internal voltage (emf) the voltage drops in the coil itself due to its internal impedance.

From (Brown's page 22)

$$R_1 = X_1 = \frac{1}{\gamma \pi a^2} \times \frac{a}{25}$$

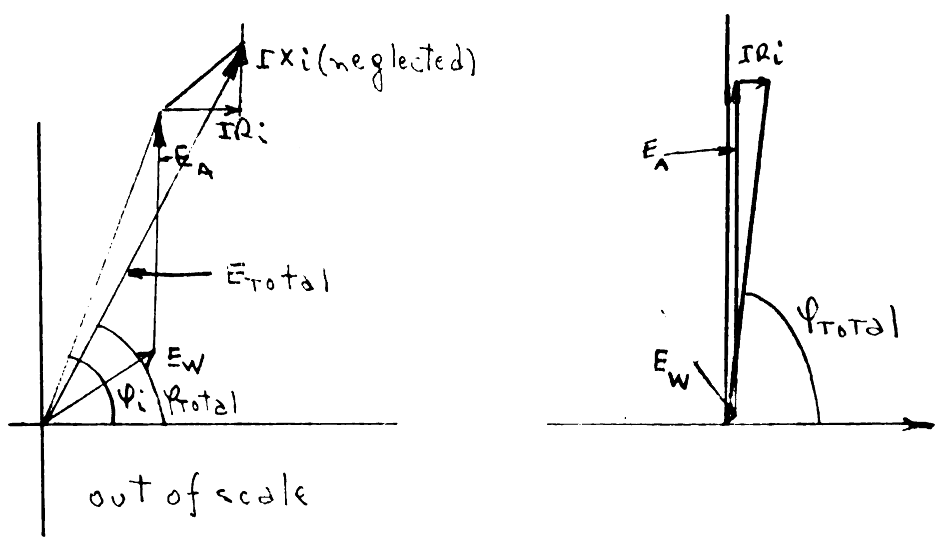
Theoretical calculations are not accurate, whereas a simple reading in the experiment gives (neglecting X_1)

$$R_1 = 0.16\ \Omega$$

Hence

$$IR_1 = 28.5 \times 0.16 = 4.5 \text{ volts}$$

We can see that in this case the total voltage isvery nearly the same as E_a and our φ_i becomes nearly 90°. The total coil voltage is shown in the following diagram:



Hence:

Total coil voltage: 52 volts $\lfloor 84^\circ$

Where 84° is the total φ since it comprises also the IR_1, IX_1, drops on the coil. The efficiency of the coil is:

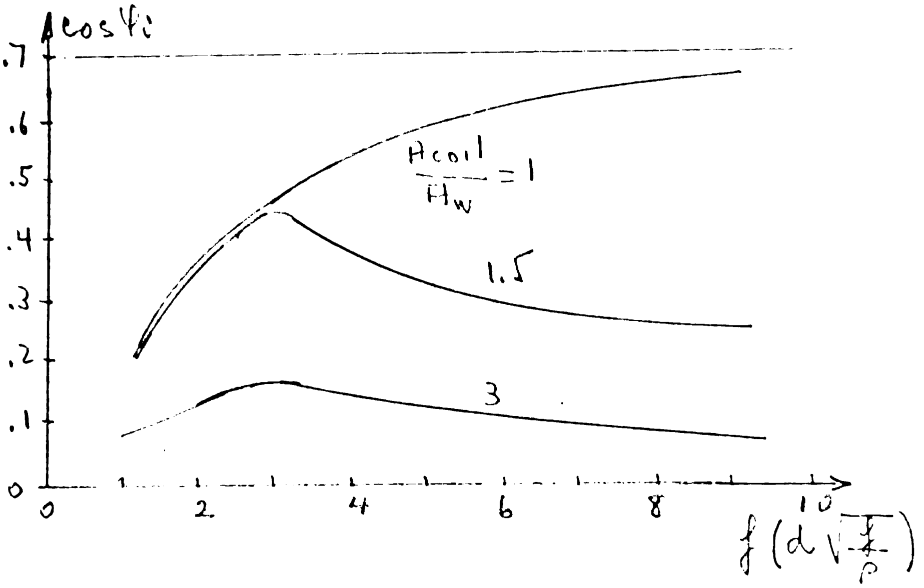
$$\text{Effy} = \frac{E_W' \text{ (in phase component) } I}{I^2R_1 + E_W'I} = \frac{E_W}{E_W' + IR_1}\,100$$

$$= \frac{0.471}{0.471 + 4.5}\,100 = \frac{0.471}{4.971}\,100 = 9.6\%.$$

Total power factor $= \cos \varphi_T = 0.1$

In the experiment the voltage through the coil was 7% higher than this value, due to the fact that a slight difference in the measure of the coil diameter increases greatly the area ($A_{coil} - A_W$) and as the coil was made by hand its diameter was not very uniform.

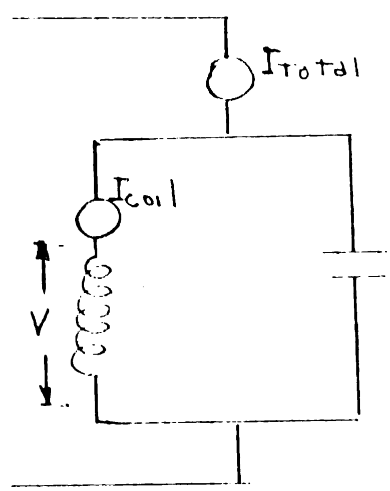
The variation of the internal power factor with the ratio $\frac{A_{coil}}{A_{work}}$ has been shown to be (AIEE transactions Vol. 63, June, 1944, page 273)



The experiment was run by connecting a condenser in parallel with the work coil as shown. The table gives the results obtained.

TABLE XIX

I_{total}	I_{coil}	secs.	°F over room temp		V	Z
20	29	10	1.8	75	56	1.935
"	"	20	2.5	105	"	"
"	"	30	3.	130	"	"
"	"	40	3.5	150	"	"
"	"	50	3.8	160	"	"
"	"	60	4.2	175	"	"
"	"	90	5.2	220	"	"
"	"	120	5.7	240	"	"



Final temperature in 2 minutes = 240 + 70 = 310°F

The value of Q was found to be constant through this temperature range and equal to Q = 11.

Therefore, R and X were calculated with the result listed below:

TABLE XX

Coil with hollow copper core.	temp. over room	Q	Z	X	R
	0-240	11	1.935	1.93	0.175

For the coil alone the corresponding parameters are also constants over the same range of temperature.

TABLE XXI

Coil alone same range temperature	Q	Z	X	R
	15	2.42	2.41	0.16

It is seen that the copper core acts as an inductive secondary circuit increasing the total resistance and decreasing the reactance.

The fact that the resistance of the secondary is very small compared with the reactance causes the total reactance and resistance to be fairly constant.

In fact, the total impedance is:

$$R_1 + \frac{(WM)^2 R_2}{R_2^2 + (WL_2)^2} + jw(l_1 - \frac{(WM)^2 L_2}{R_2^2 + (WL_2)^2}$$

That in this case reduces to:

$$R_1 + \frac{M^2 R_2}{L_2^2} + jw(L_1 - \frac{M^2}{L_2}) \quad \text{where } \frac{M^2 R^2}{L_2^2} \text{ is small.}$$

Condenser Computations

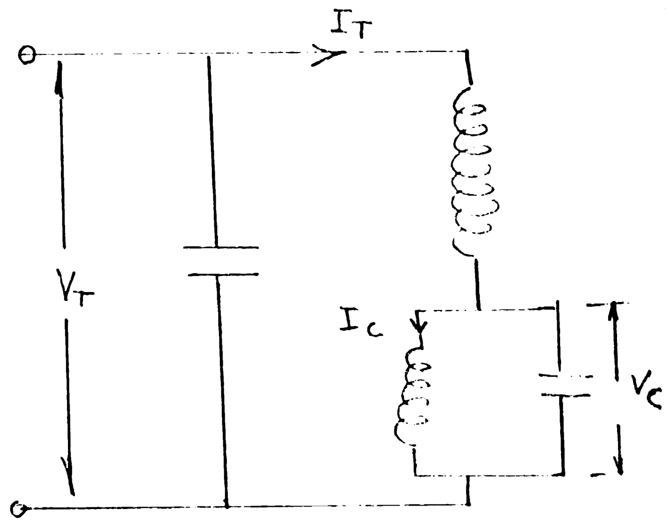
In applying a condenser in parallel with the work coil care must be taken to insure that this combination

will not form a tank circuit that can take control of the frequency of oscillation.

To help avoid this undesired possibility the following condition will be imposed.

$Q_{load} \leq 80\%$ Q main tank circuit with work coil

This can be written as: (Bendz's p.319)



$$\frac{\text{Coil voltage x coil current}}{\text{load watts}}$$

$$\leq 80\% \frac{\text{generator rated volt-amp}}{\text{load watts}}$$

or $V_c \times I_c \leq 80\% \ V_t I_t$

In addition, the Westinghouse Mfg. Co. recommends (1kw generator catalogue) that: $I_{coil} \leq 2I_T$ to keep the load tank circuit away from resonance.

In this range the condenser current is to be chosen

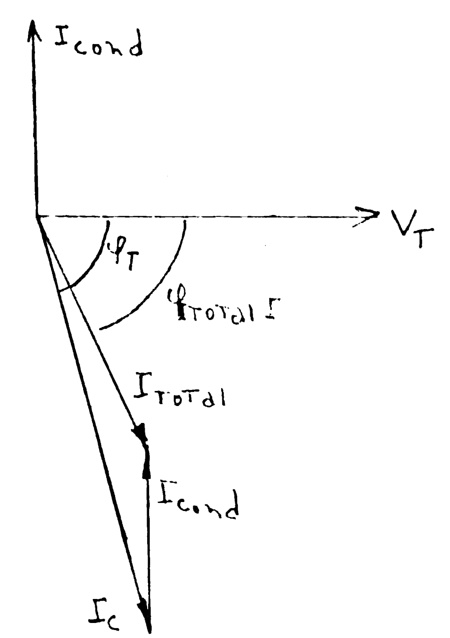
so that

$$I_{cond} = I_{coil} - I_{t_{rated}}$$

In this case:

$$I_{cond} = 29 - 20 = 9 \text{ amp}$$

This formula is only correct in the total absence of power in the circuit but in most practical problems it yields approximate results because φ_T is very nearly 90^{o} as analyzed before.



Finally:

$$C = \frac{I}{V_{cw}} = \frac{9 \times 10^{6}}{56 \times 2\pi \times 35 \times 10^{4}} = .073 \mu f$$

The results of this experiment are shown in graphs 12 and 13.

Conclusions

Developing the expression for 'E' found at the beginning of this paper we get:

For $F \gtrapprox 1$

$$E = \frac{P_{(\text{power core})}}{I^2 R_{\text{solen.}}} = \frac{8\pi H_o^2}{\sigma_{core}} \frac{A_{core}}{S_{core}} \frac{1}{I^2\ R_{dc}} \frac{2\ S_{sol}}{sol.\ A_{sol}}$$

For a given solenoid this becomes:

$$E = K \frac{A_{core}\ S_{sol}}{\sigma_{core}\ S_{core}} = K \frac{A_{core}}{\sigma_{core}} \sqrt{\frac{\mu_{core}\ \sigma_{core}}{\sigma_{sol}}}$$

$$E = K\ A_{core} \sqrt{\mu_{r\,core}\ \rho_{core}}$$ <u>independent of freq.</u>

This formula shows that for a given solenoid, E is much smaller when heating a piece of non magnetic material than heating a magnetic one, no matter what the frequency is, as long, of course, that we are over $\frac{a}{s} = 2.25$.

Assuming for instance:

$$A_{mag\ core} = A_{non\ mag\ core}$$

$$\mu_{r\,mag} = 800$$

$$\rho_{mag\ core} = 10\ \rho_{non\ mag\ core}$$

there results:

$$\frac{E_{mag}}{E_{non\ mag}} = \sqrt{800 \times 10} = 90$$

If we also assume an efficiency of 85% for the heating of the magnetic core we can write:

$$\text{effy}_{mag} = \frac{85}{85+15} = 85\% \text{ where } \begin{array}{l} \text{Power}_{core} = 85 \\ I^2R_{sol} = 15 \end{array}$$

For the same I^2R_{sol}, the power that gets to the load for non magnetic material would be:

$$\text{Power}_{\text{non mag}} = \frac{85}{90} = 0.95$$

and the effy:

$$\text{effy}_{\text{non mag}} = \frac{0.95}{0.95+15} = 6\%.$$

We can conclude, therefore, that to keep the efficiency of this method between reasonable values while heating non magnetic materials much larger coils must be used.

High frequency heating, then, is especially convenient to heat magnetic materials but in spite of its low efficiency is used many times to heat non magnetic ones due to its outstanding characteristics such as:

1) Heating can be accomplished faster than with other methods for most metal pieces.
2) Heating can be confined to the desired area which may be a small fraction of the total.
3) Heating starts instantly when radio frequency power is applied and stops instantly when radio frequency power is removed.

These features frequently outweigh the relatively high cost of the necessary equipment.

List of the equipment used in these experiments

-Westinghouse industrial radio-frequency generator; EE V 020, Style No. 867692, 1KW, 300 M c/s low frequency, and 10 M c/s high frequency.

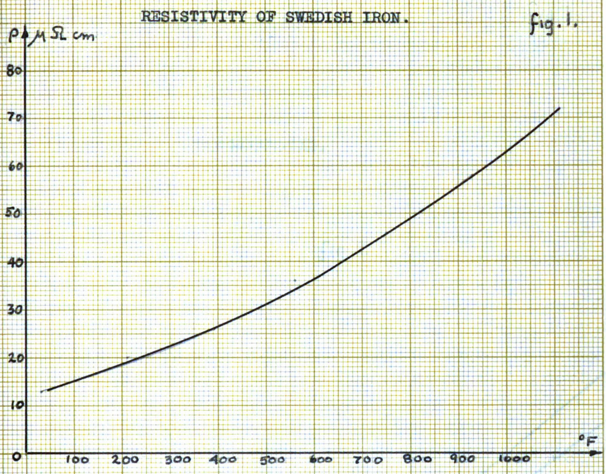
-Radio frequency 'Q' meter; Type 160 A, Serial No. 2632, Boonton Radio Corp.

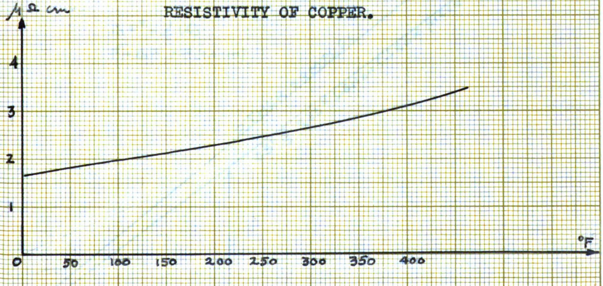
-Vacuum tube voltmeter; Type 726A, Serial No. 3091 General Radio Co.

-Iron constantan thermocouple with galvanometer Weston model 301, EE 2504, 30 micro amp

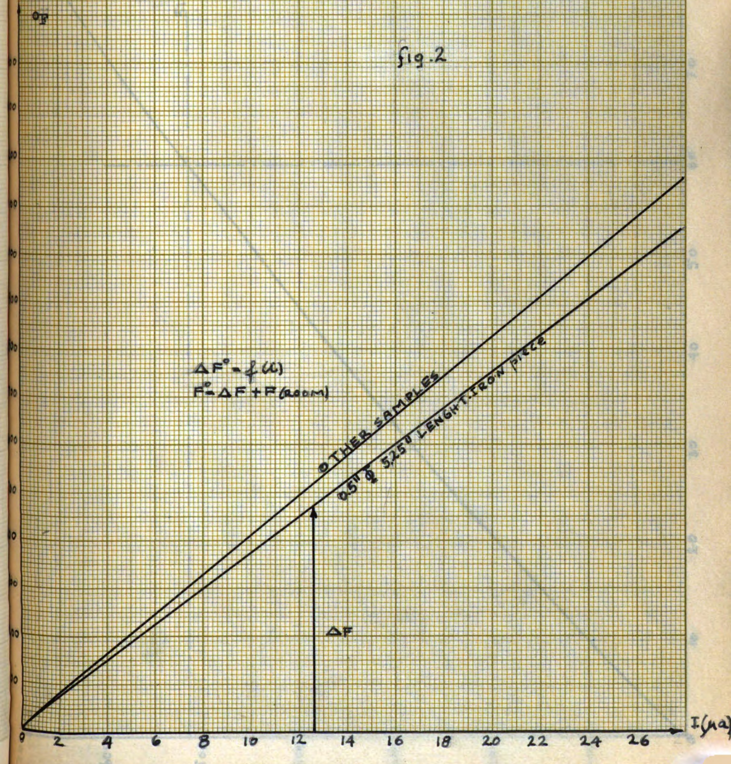
-Ammeters. GE Type CG 22037, 200 amps (rf)
EEAA124
GE Type CG 22041, 50 amps (rf)
EEAA125

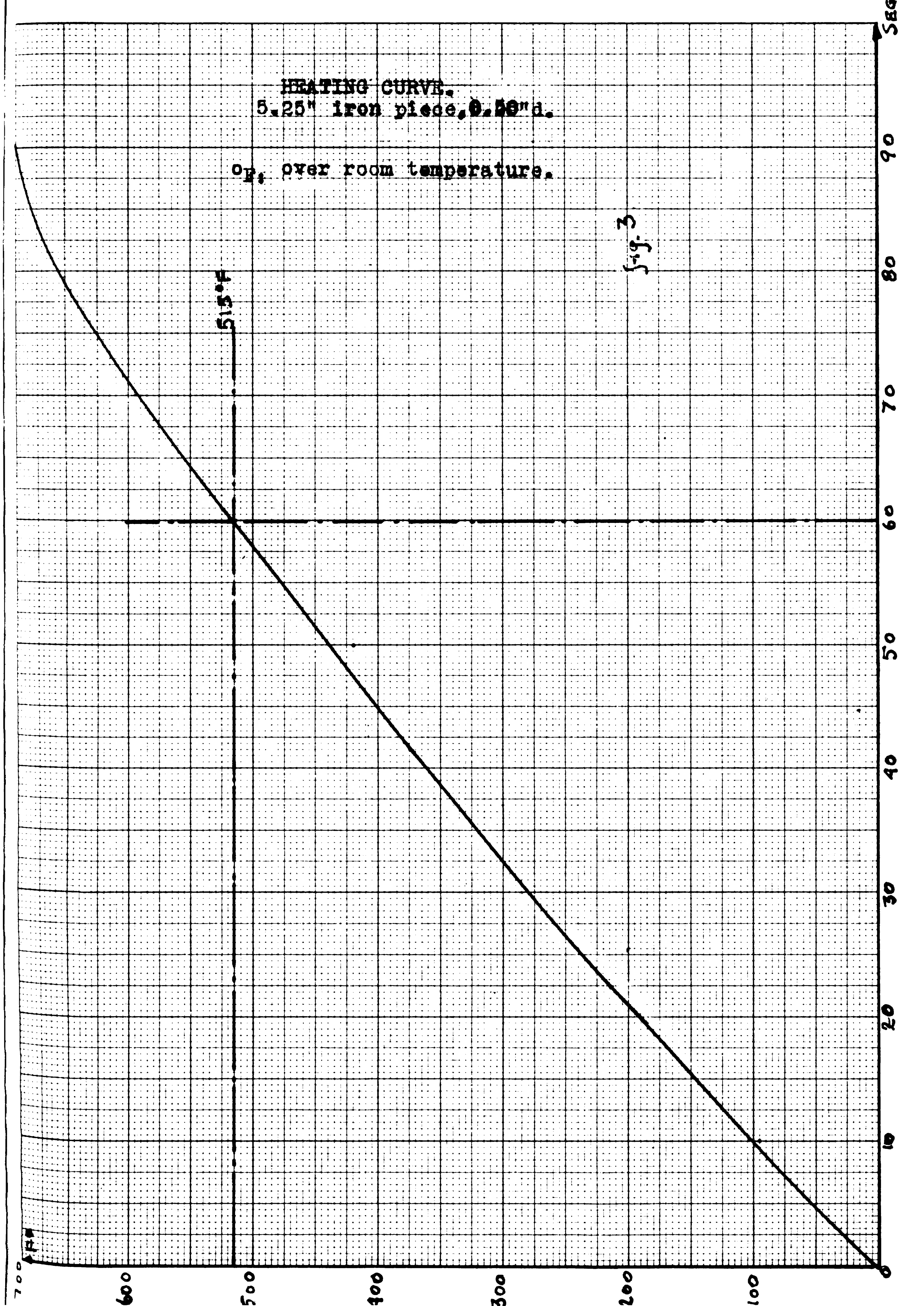
-High voltage condensers (rf)
Cornell - Dubilier GD 48449-5
EE 2819-0.03 mf
Cornell - Dubilier GD 48996-5
EE 2818 - 0.045 mf


RESISTIVITY OF SWEDISH IRON.
fig. 1.
ρ μΩ cm
80
70
60
50
40
30
20
10
0
100
200
300
400
500
600
700
800
900
1000
°F

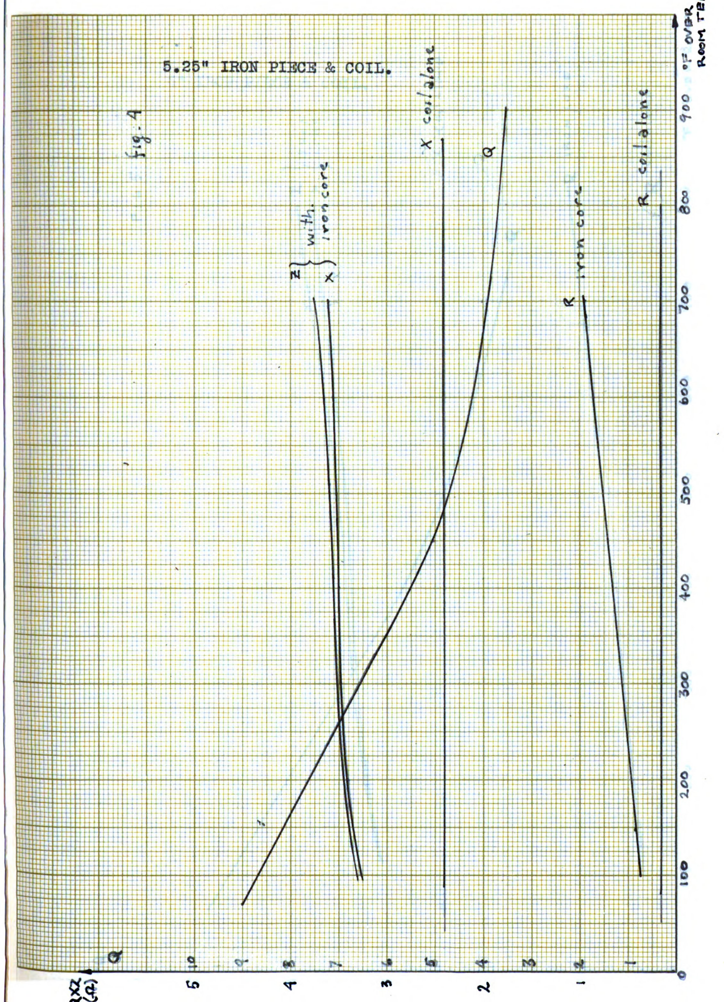

μΩ cm
RESISTIVITY OF COPPER.
4
3
2
1
0
50
100
150
200
250
300
350
400
°F

THERMOCOUPLES.

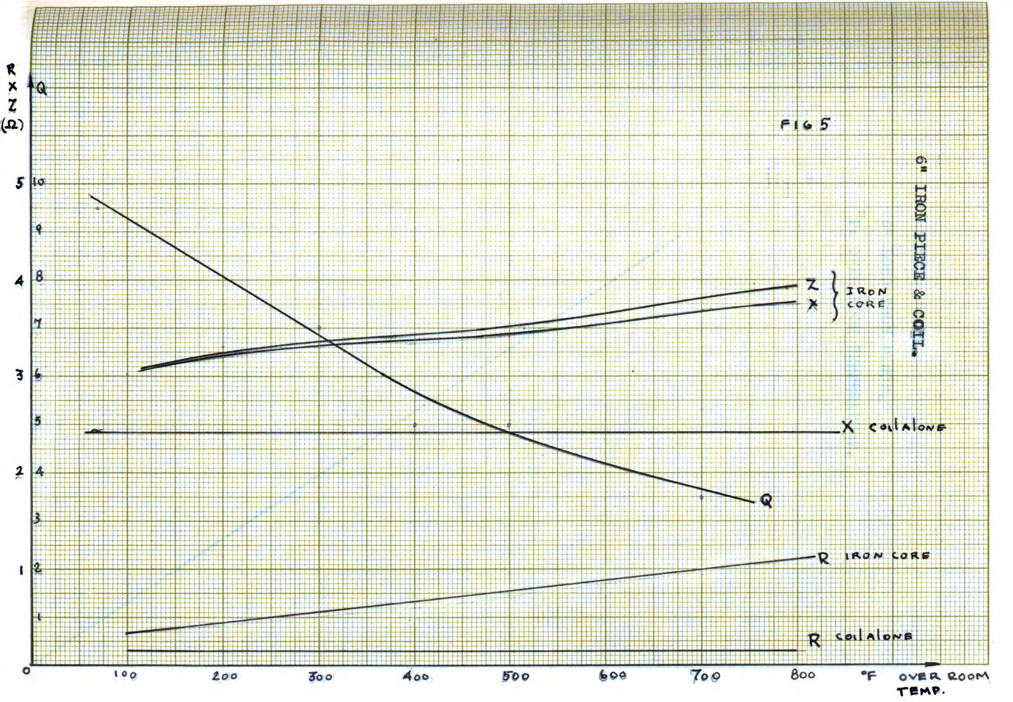


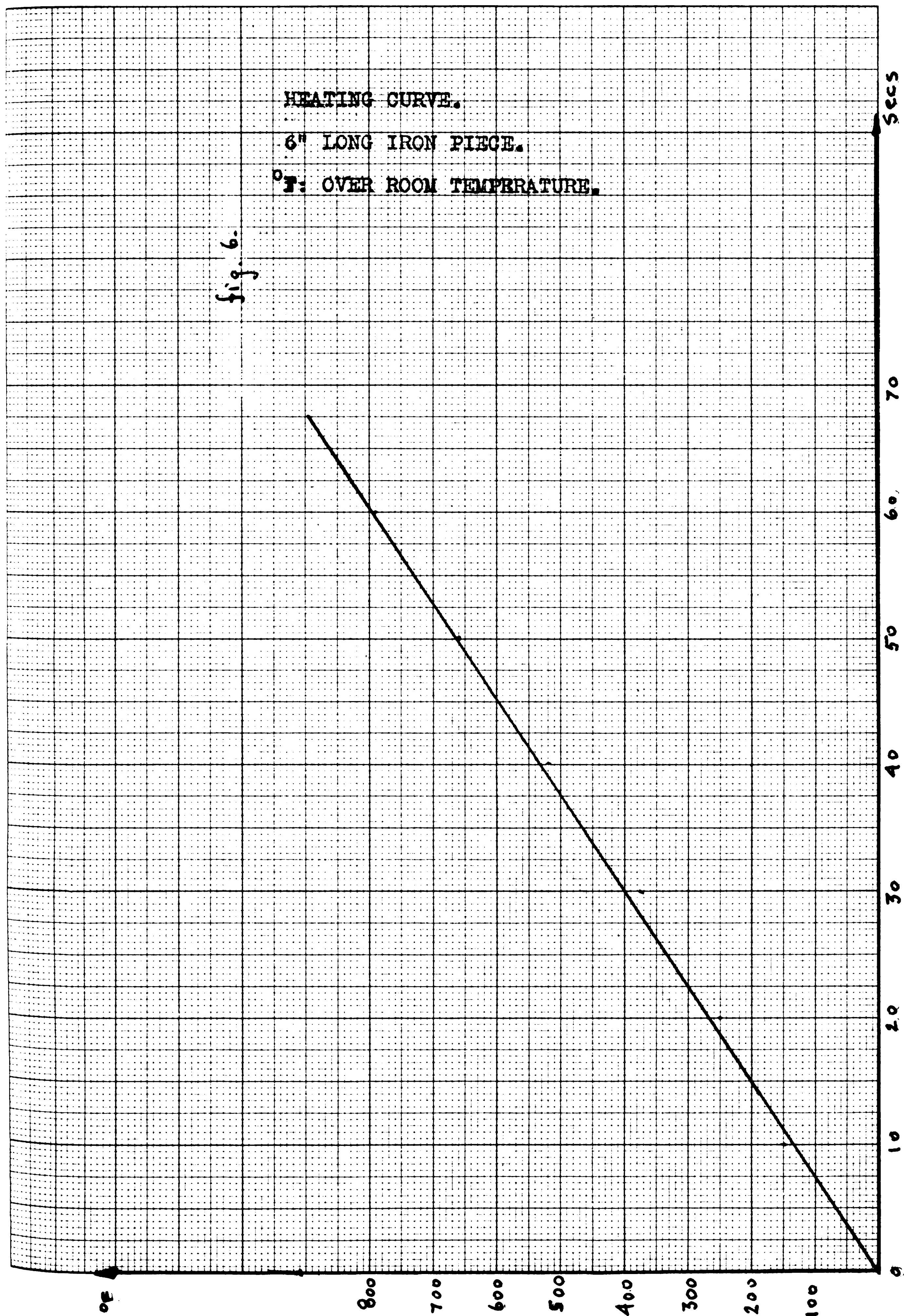


HEATING CURVE.
5.25" iron piece, 0.50"d.
°F, over room temperature.
fig. 3
515°F
°F
SEC
0
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700



5.25" IRON PIECE & COIL.
fig. 4
Z } with iron core
X
X coil alone
Q
R iron core
R coil alone
°F OVER ROOM TEMP.
100
200
300
400
500
600
700
800
900
Q
10
9
8
7
6
5
4
3
2
1
0
RXZ (Ω)
5
4
3
2
1


6" IRON PIECE & COIL.
FIG 5
R
X
Z
(Ω)
Q
5
4
3
2
1
10
9
8
7
6
5
4
3
2
1
0
100
200
300
400
500
600
700
800
°F OVER ROOM TEMP.
Z
X
IRON CORE
X COIL ALONE
Q
R IRON CORE
R COIL ALONE


HEATING CURVE.
6" LONG IRON PIECE.
°F: OVER ROOM TEMPRATURE.
fig. 6.
Secs
70
60
50
40
30
20
10
0
800
700
600
500
400
300
200
100
°F

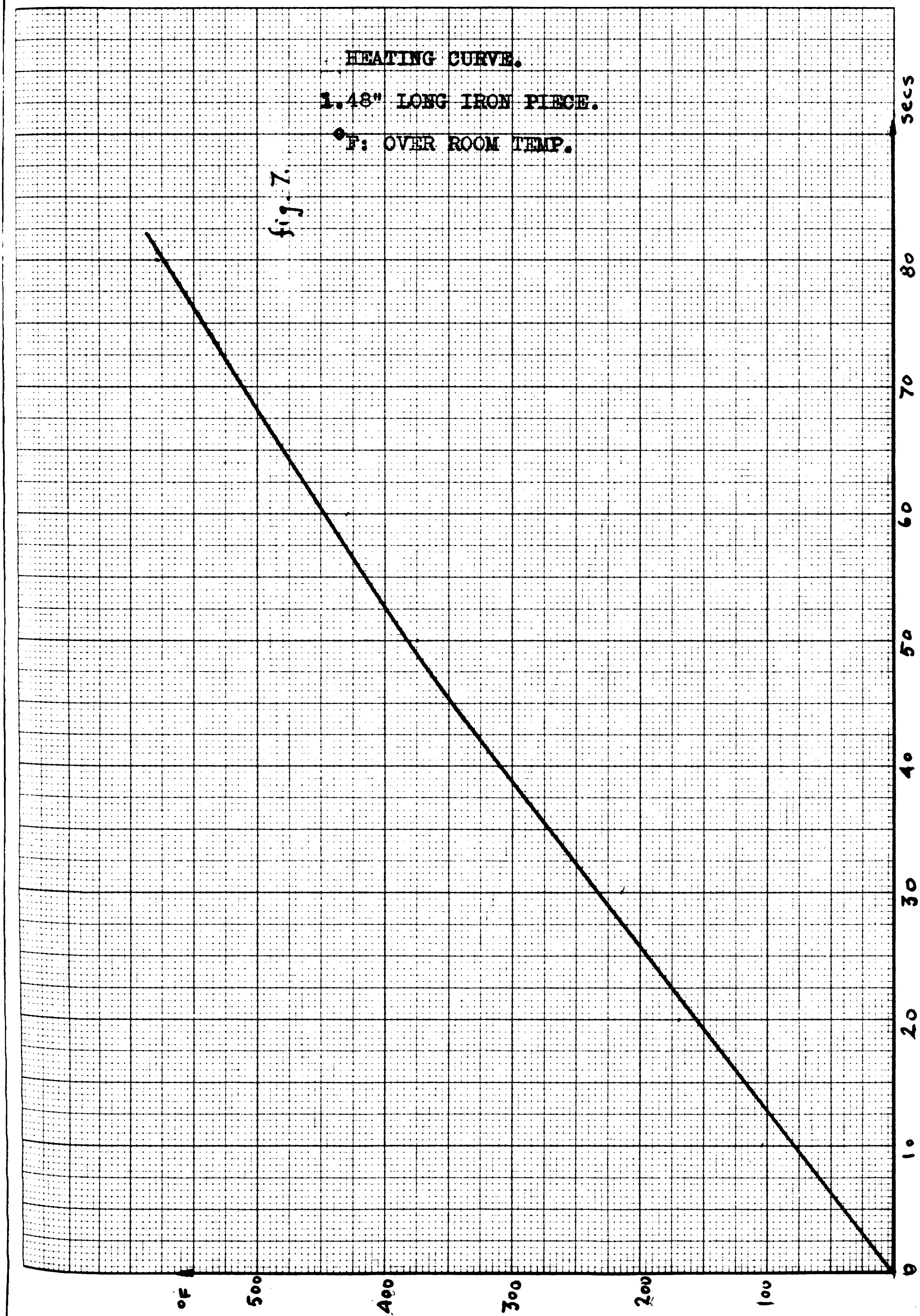
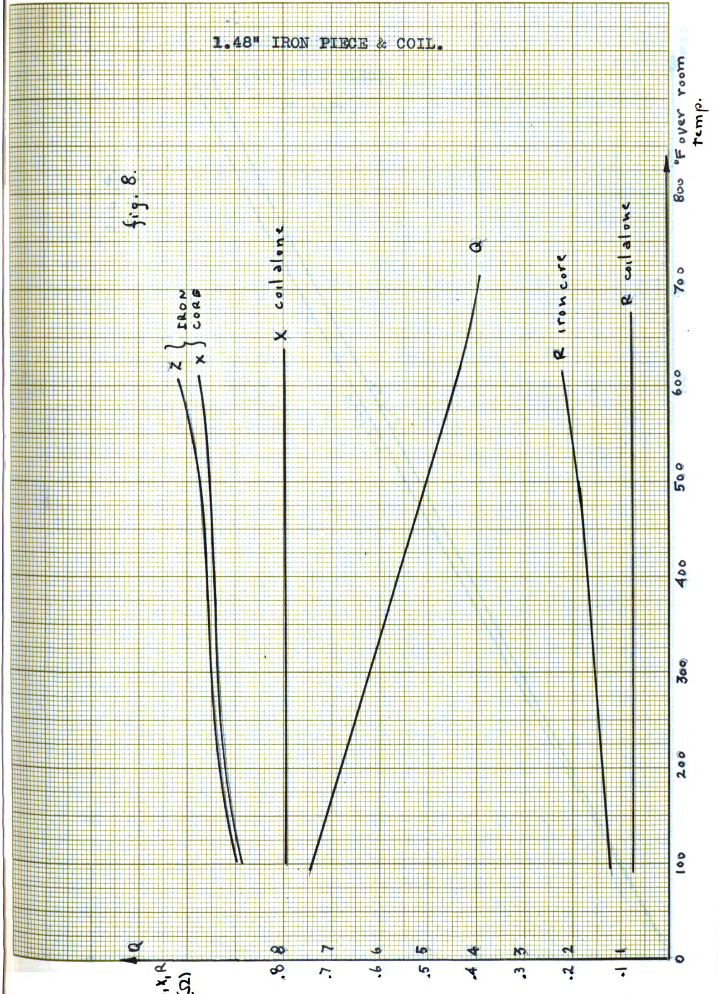

HEATING CURVE.
1.48" LONG IRON PIECE.
°F: OVER ROOM TEMP.
secs
80
70
60
50
40
30
20
10
0
°F
500
400
300
200
100

fig. 7.

1.48" IRON PIECE & COIL.



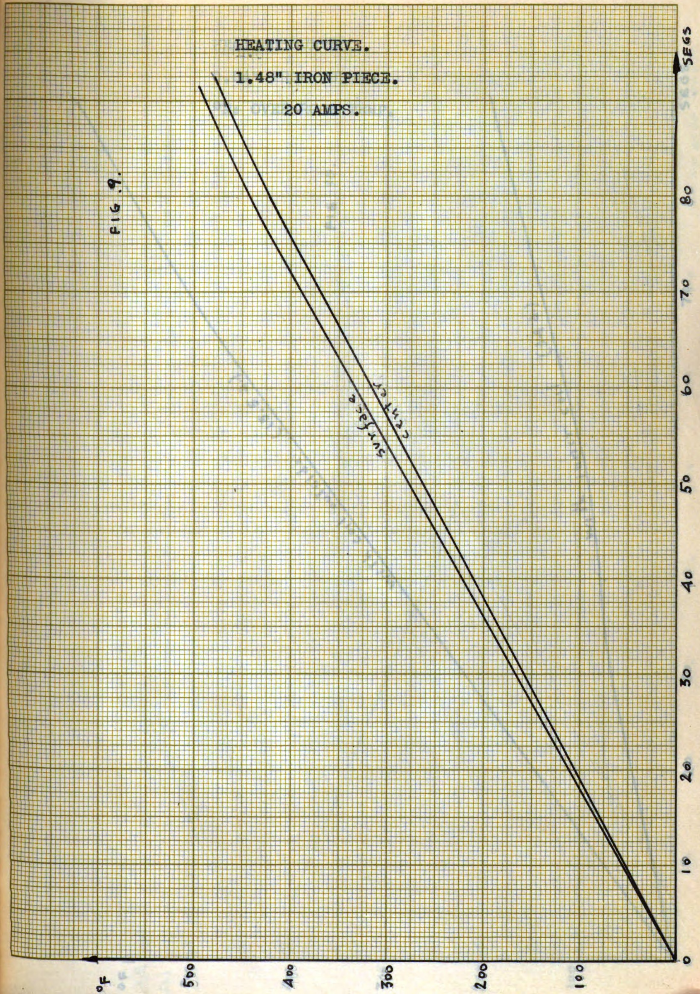
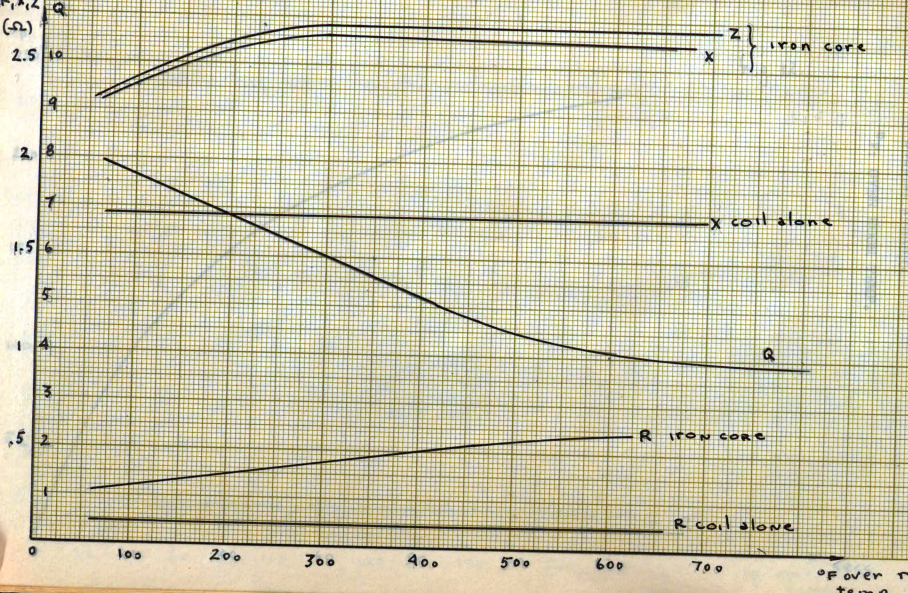
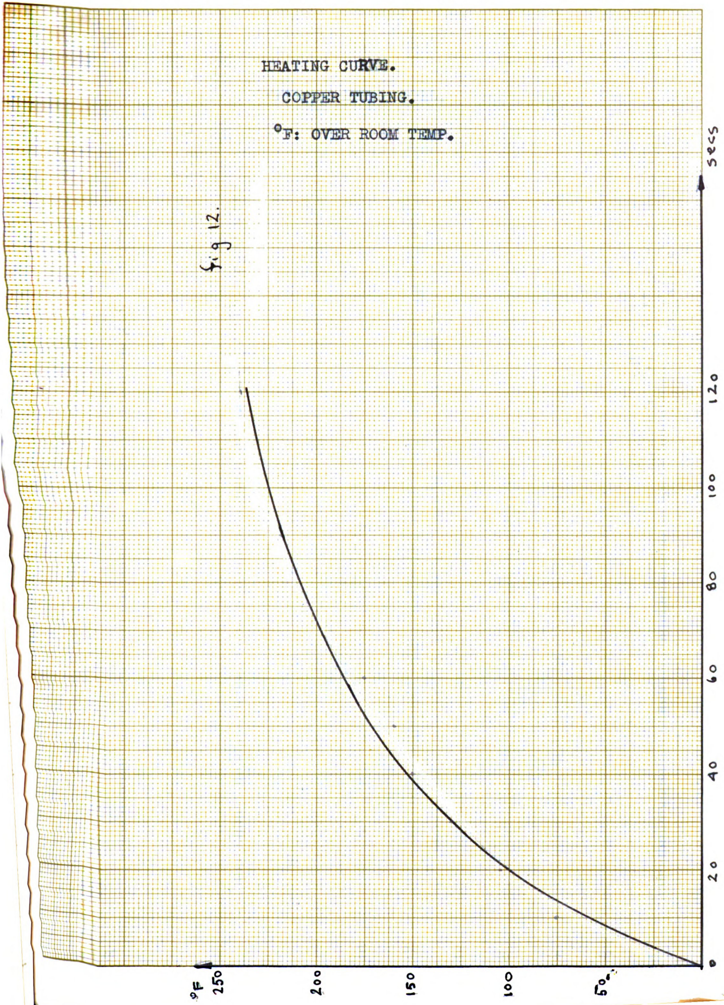

HEATING CURVE.
1.48" IRON PIECE.
20 AMPS.
SECS
80
70
60
50
40
30
20
10
0
°F
500
400
300
200
100
center
surface

FIG. 9.

fig 11.

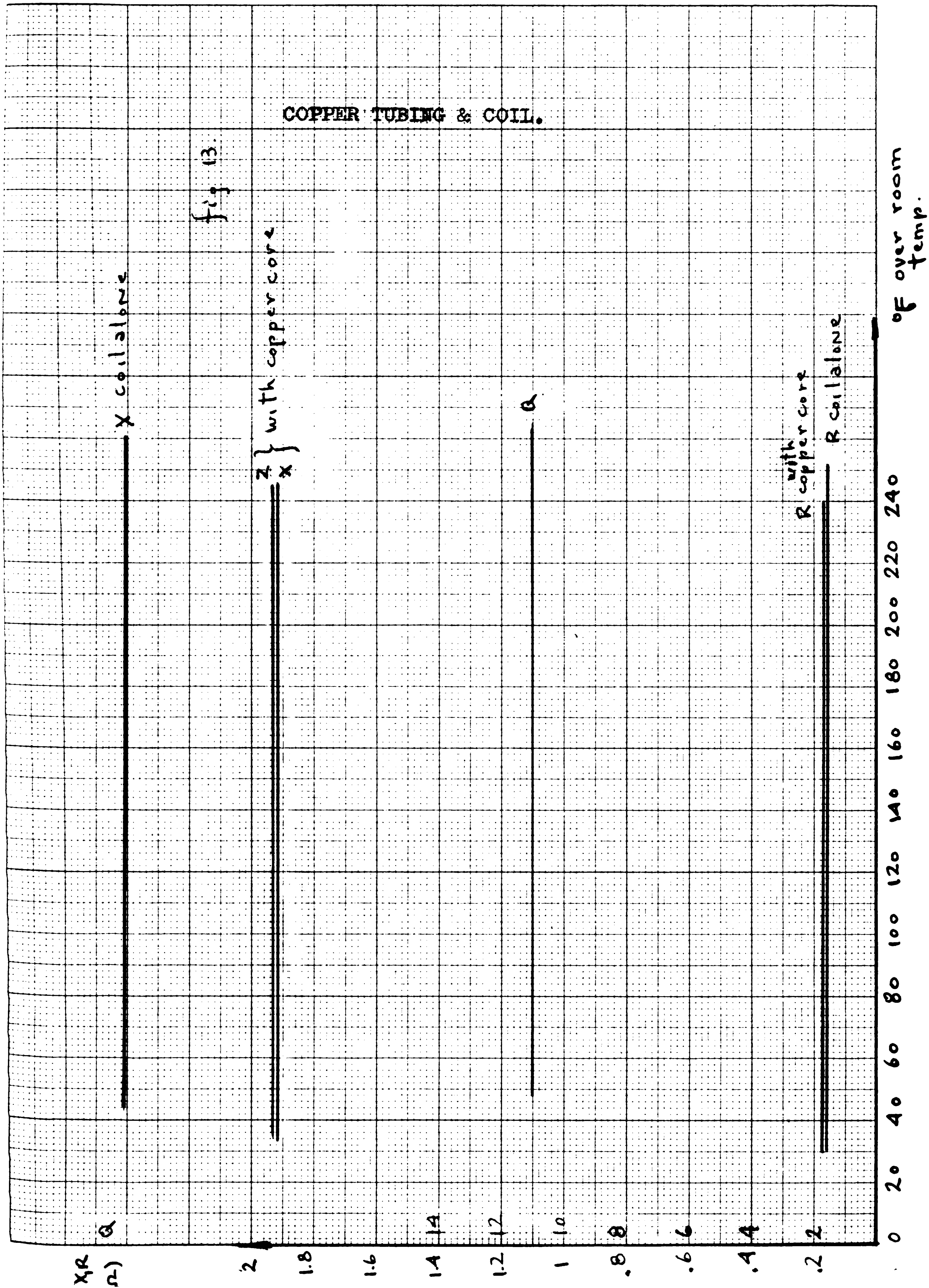


0.84" IRON TUBING & OUTER COIL.

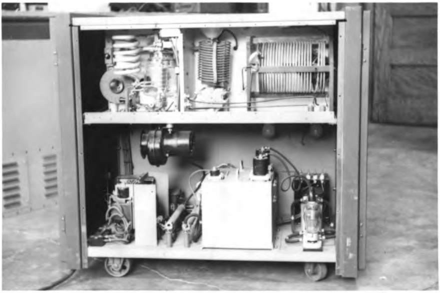


HEATING CURVE.
COPPER TUBING.
°F: OVER ROOM TEMP.
Fig 12.
secs
120
100
80
60
40
20
0
°F
250
200
150
100
50

COPPER TUBING & COIL.



FRONT VIEW OF THE EQUIPMENT USED



REAR VIEW

COILS AND CYLINDRICAL CONDUCTORS TO BE HEATED.

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