Accepted by the faculty of the Department of Communication, College of Communication Arts, Michigan State University, in partial fulfillment of the requirements for the Doctor of Philosophy degree.

John E. Hunter
Director of Thesis

Guidance Committee: Gerald R. Miller, Chairman

John E. Hunter, Co-Chairman

Richard V. Farace

[illegible]

[illegible]

ABSTRACT

DYNAMIC MATHEMATICAL MODELS OF DYADIC INTERACTION
BASED ON INFORMATION PROCESSING ASSUMPTIONS

By

Joseph N. Cappella

In order to treat the dynamic behavior of dyads as attitudes are being negotiated through the process of mutual influence, mathematical models are necessary. The reason is that even for dyadic influence the dynamics of change is too complex to be handled with purely verbal models.

The models developed for mutual influence in this thesis originate from Newcomb's structuring of dyads and, therefore, include variables for each person's attitude, each person's perception of the other's attitude, and each person's attraction to the other. In addition to these six variables, we consider two aspects of the communicative underchange: the rate of transmission of messages and the content of generated messages. In the case of content, two alternative models are considered: a veridical model in which the speaker's message always reflects his attitudes and a shift model in which the speaker's message is shifted a fraction of the distance toward the speaker's perception of the other's attitude.

Because Newcomb's paradigm for dyadic situations does not specify the form of the change equations for attitudes, perceptions, and attractions, two well-known theories of attitude change were invoked to

specify the form of the change equations for attraction (Social Judgment Theory) and for attitudes and perceptions (Information Processing or Reinforcement Theory). While Social Judgment Theory has not generally been applied to attraction change, there are sound reasons for doing so.

In order to solve the system of six non-linear differential equations for the veridical and shift message cases (that is, determine stability characteristics and "direction" of movement) generated from the assumptions of Information Processing Theory, several simplifications were made. First, with attraction and message transmission held constant, the shift and veridical message models always converged to a point of equilibrium although the points differed between the models. Second, with attraction constant but transmission varying, both message models tended to converge to a point at which one individual perceived no discrepancy from the other and was silent while the other perceived discrepancy and was transmitting. Third, with attraction varying but transmission constant, both message content models produced infinite dislike with actual and perceived attitudinal differences only when both persons' initial messages to the other were well outside his region of acceptance. In all other cases, infinite liking with no actual or perceived differences obtained. Finally, with both variable attraction and transmission results which were a combination of simplifications two and three above were obtained.

It was concluded that the method of analysis and framework for mutual influence had promise for future model building, theory construction, and research. However, that promise could be realized only by comparing other models with alternative psychological assumptions (for example, congruity and dissonance) and with alternative message content assumptions to one another.

DYNAMIC MATHEMATICAL MODELS OF DYADIC INTERACTION BASED ON INFORMATION PROCESSING ASSUMPTIONS

By

Joseph N. Cappella

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Communication

1974

DEDICATION

To Professor Robert O. Brennan, friend and mentor, who was first to confuse me with mathematics.

ACKNOWLEDGMENTS

I hope that before this is read each person that I wish to thank has been thanked in a more personal way. Nonetheless these intellectual and personal debts should be made public. Of course, my entire committee is deserving of my deepest gratitutde for all of their time, energy, and understanding in the completion of this work. John Hunter who directed and dissected this dissertation deserves as much credit for its completion as I do. His mark, both on this particular work and on the character of my future work, will be unmistakable. I must also express an admiration for his ability to combine a sharp critical attitude toward intellectual pursuits but tempered with enough personal warmth and support to keep one moving ahead. Gerald R. Miller in his role as adviser offered personal support and an intellectual openness without which this dissertation would never have been undertaken, let alone completed. Donald Cushman has been both friend and sounding board for the entirety of my graduate education. His influence on my orientation to theory and types of theory construction can be seen by any who have studied with him. Joseph Woelfel has offered by his example and personal discussions support for the aims of theory development. R. Vincent Farace's contacts with the Environmental Design and Management group headed by William Cooper and Herman Koenig made possible financial support for me in an environment free of time constraints and commitments. For his efforts in this regard,

I am deeply indebted.

The Department of Communication deserves mention since it has provided an open, flexible, and stimulating intellectual environment in which to pursue interests which would have been unacceptable in more constrained academic units. Elena, Jeffrey and Elise Cappella managed to maintain their own high spirits which often buoyed my own through the weeks and months of frustration, disappointment, and failure which preceded the completion of this work. Finally, the manuscript was professionally prepared and typed by Ruth Langenbacher.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure		Page

Figure | Page

CHAPTER I

INTRODUCTION

The purpose of this work is to discuss and develop mathematical models of small group interaction, particularly where the group is task-oriented and communicating about some issue of mutual importance and mutual relevance. Although the literature pertinent to small group processes is voluminous, there are at least two areas which have been sufficiently well-developed theoretically and researched empirically to warrant treatment in a mathematical framework. These include the balance-related theories of interpersonal situations (Heider, 1946; Newcomb, 1953, 1961, 1968; Osgood and Tannenbaum, 1955) and the attitude change theories of the passive communication situation (Festinger, 1957; Aronson, Turner, and Carlsmith, 1963; Hovland, Janis, and Kelly, 1953; Sherif, Sherif, and Nebergall, 1965; Hunter and Cohen, 1974).

Both of these points of view have their advantages and limitations. These can best be seen if we first adopt a structural and terminological framework for discussing the basic dyadic process. The framework is that supplied by Newcomb's ABX model (1953; 1961), which we shall refer to as the IJX model for reasons of uniformity of notation. Newcomb assumes that the components of the interpersonal situation consist of two persons, I and J, and an object of mutual concern, X.

The relations among the components are attraction between I and J and the orientations or attitudes of I to X and of J to X. In addition, each individual is presumed to have a perception of what the other's orientation toward X is. With the addition of these two perception relations, Newcomb can divide the IJX situation into parts on the basis of which of the six relations are relevant. These parts are the two individual (or intrapersonal systems) and the collective (or objective) system. The individual system is constructed from cognitions available to the focal individual. These include I's attraction to J, I's attitude toward X, and I's perception of J's attitude toward X. A similar set of relations constitute J's individual system. The collective system is constructed from two individual systems and, hence, is constructed from information which <u>at any point in time</u> is unavailable to either of the individuals in the interpersonal situation. The collective system consists of four relations: I's attraction to J, J's attraction to I, I's attitude toward X, and J's attitude toward X.

The beauty of Newcomb's structuring of IJX situations is found in the types of actions which an individual may undertake when he experiences individual strain. That is, when an individual acts to alter the IJX situation, his actions may be inner-directed toward changes in the individual system, or they may be outer-directed toward inducing changes in the collective system. Changes in the individual system would involve changes in attitude, attraction, or perception of the other. Actions directed toward the collective system would take the form of communicative acts, or persuasive efforts. Unfortunately, Newcomb's model does not go so far as to suggest the forms of the change equations for internal changes, or the processes by which attitudes, perceptions, and attraction are altered as a result of communication. We must turn to other models to provide answers to these questions. But Newcomb's model provides the framework for analysis which goes beyond either pure balance-related positions or pure attitude change theories.

But why build mathematical models rather than verbal models? The answer is two fold: complexity and precision. In accepting the Newcomb paradigm for IJX situations we have already focused upon three variables for each person: attitude, attraction, and perception of the other person. With two people this means six variables which are all changing as non-linear functions of one another. They are not merely structurally related to one another but dynamically related to one another. Even ignoring the type of messages that are generated or the rate at which they are transmitted, this is already a very complex system to analyze without some mathematically sophisticated tools. Secondly, verbal models of complex processes often have difficulty drawing correct and complete deductions from initial assumptions. In mathematical modeling, not only are assumptions quite explicit but the results of those assumptions constitute a complete and logically consistent set of conclusions. Thus, in terms of their ability to treat the dynamics of complex situations and their ability to completely and explicitly array the alternative assumptions and their implications, mathematical models are to be preferred to other forms of theory construction and model building.

Balance processes are presumed to be primarily cognitive and intrapersonal. The role of interpersonal processes of information transmission is minimized, obscured or left absent. In the most careful and thorough review of balance in small groups to date, Taylor (1970, p. 41) discusses the role of communication as follows: "Through communication with the other, the focal person discovers the other's attitude toward X. In this respect, communication allows the tension mechanism to operate in the balance process" (emphasis in the original). Thus, the only role that communication plays is that of informing the individual's perception of the other so that it becomes accurate. On the other hand, attitude change theories have been primarily interactive and interpersonal by involving explicitly the effects of messages. But this emphasis has thereby excluded processes whereby intrapersonal changes can occur in the absence of message input.

Secondly, changes in perception of the other have not been treated in detail by either balance or attitude change researchers. Its role in the passive communication context is minimal since there is usually no assumption of continued interaction with the speaker beyond a single message. However, in IJX situations where continued interaction is the focus, the role of perception of the other is crucial in understanding whether an individual's perceptions are accurate or inaccurate and, hence, whether individual and collective systems differ or are interchangeable.

Thirdly, balance-related theories usually begin with definitions of balance which are static. That is, at balance the system is presumed to be at rest. Deviations from the static balanced state define imbalance and point out the appropriate changes toward balance. However, --and this is our third criticism--the emphasis in balance research for IJX situations has ignored the dynamics of change (see Hunter, 1974 for an exception) and has focused primarily upon verifying the definition of balance. Attitude change theories, on the contrary, have been concerned in a fundamental way with the dynamics of attitude change (Hunter and Cohen, 1974) but have been less concerned with the dynamics of attraction change (this is not true of congruity theory or dissonance theory). As noted above, none of the passive communication models have been concerned with changes in perception of the other.

Finally, attitude change theories, because they have been cast in the passive paradigm, do not consider alternative processes for the generation of messages. In the passive paradigm, messages may be treated as a constant input from some source but in the interactive IJX situation messages are an output from an individual system whose content should change as the individual system changes.

With the above criticisms in mind, the present work seeks to develop alternate mathematical models of the dynamics of IJX situations based upon passive attitude change models (Hunter and Cohen, 1974) extended to fit more closely the Newcomb structuring of IJX situations. The extensions will include (1) processes of individual, intrapersonal change as well as interpersonal change due to message transmission, (2) processes of the generation of message content and of message transmission as a function of individual system states, and (3) a consideration of the process of change in the perception of the other as parallel to but independent of attitude change. The model to be considered is the information processing model (Hovland, Janis, and Kelly, 1953; Hunter and Cohen, 1974, pp. 28-39). This is developed in the following chapter.

CHAPTER II

DEVELOPING MATHEMATICAL MODELS OF CHANGE FOR IJX SITUATIONS

As indicated in the previous chapter, Newcomb's structuring of IJX situations offers a useful framework for the development of dynamic models of dyadic processes. Following Newcomb's discussion, any dynamic model must consider processes of internal as well as external change. That is, individuals would be expected to alter their cognitions in the direction of minimizing internal strain *independently of the messages that they receive from the other* and to alter their cognitions *as a function of the messages that they receive from the other*. Also following Newcomb, the cognitions which are altered are the relationships which constitute the individual system: I's attitude toward X, P_i, I's perception of J's attitude toward X, Q_{ij}, and I's attraction to J, A_{ij}. These constitute the three state variables of I's individual system. Together with P_j, Q_{ji}, and A_{ji} from J's individual system, the six variables are the state variables for the collective system. Because Newcomb's paradigm allows the individual to act on the collective system (that is, the other individual) through the process of communication, then any dynamic model of the IJX situation must also account for the generation of message content and the transmission of those messages.

As we also noted in the previous chapter, while Newcomb's IJX system is suggestive for structuring the dynamics of interpersonal processes, it does not go beyond suggestion to the specifics of change. To do so we shall be forced to invoke the assumptions of other, more developed, models and to extend them where necessary. In addition to presenting models of message transmission and the generation of message content, we shall invoke a model of attitude change which has been thoroughly researched and developed for the passive communication paradigm--the information processing model (Hovland, Janis, and Kelly, 1953; Hunter and Cohen, 1974, pp. 28-39). As we take up these models in turn, we shall first consider external changes in the six state variables as a function of the other's message, and then the comparable process of internal spontaneous change occurring independently of the other's message. Finally, we shall consider the message transmission and generation processes which link the output of each individual system to the input of the other individual system.

The key differences between the interactive models of this chapter and the passive models upon which they are based are found (1) in treating the perception of the other's attitude toward X, Q_{ij} and Q_{ji} as a relevant system variable, and (2) in considering the generation of message content and its transmission as the mathematical and substantive link between individual systems.

Information Processing Model

As Hunter and Cohen (1974, p. 30) point out, the fundamental tenets of the information processing models of attitude change are

> . . . that (1) the magnitude of change is proportional to the discrepancy between the receiver's attitude and the position advocated by the message and (2) the change is always in the direction advocated by the message.

These changes arise from the internal comparison processes which individuals undergo when the incoming message is compared to their own attitudinal position. The greater the difference between the incoming message and the individual's attitude, the greater the expected change in attitude. That is,

$$\begin{aligned} \text{change in } P_i &= P_i(t) - P_i(t\text{-}1) \\ &= \Delta P_i \\ &= a(M_{ji} - P_i) \end{aligned}$$

where M_{ji} is the message sent by J to I and a is a constant of proportionality which is greater than zero but less than one. As the discrepancy or distance between I's attitude and J's message increases, so does the expected amount of attitude change. That is, the amount of attitude change is a linear function of the amount of discrepancy between M_{ji} and P_i. Since this is the basic change characteristic of the information processing model, we shall also refer to it as the linear discrepancy (LD) model. The above change equation has a simple verbal interpretation. I's attitude toward X at time t is given by I's attitude toward X at the previous time (t-1) plus an increment in the direction of I's perception of J's attitude toward X. The fractional amount of that increment is given by a.

As Hunter and Cohen (1974, p. 34) point out, information processing theorists have given a great deal of attention to the effects of source credibility in inducing the desired amount of attitude change.

In purely interpersonal situations where the credibility of the source can be identified almost completely with his attractiveness, then the amount of attitude change depends upon the attractiveness of the source. That is, the more attractive the source, the greater the attitude change at least when the attraction is positive. However, when the source is thoroughly disliked so that his attractiveness is negative, then the attitude change can either (1) go to zero so that credibility is always positive, or (2) actually go negative thus causing the attitude to change <u>opposite</u> to the direction advocated by the message. We shall opt for the first alternative for two reasons: First, the research related to balance in IJX situations suggests that there exists a strong positive balance tendency in the amount of tension or strain produced in IJX situations Zajonc, 1968; Newcomb, 1968). That is, the change of perceptions in IJX situations does not arise when the other is negatively evaluated but only when he is positively evaluated and there is disagreement. Second, the experimental production of a boomerang effect is very difficult to achieve (Cohen, 1962; Cohen, 1964; Whittaker, 1967) and as a result should not be postulated as the primary mechanism of attitude change. Thus, the credibility factor should increase from zero for an infinitely incredible source to one for an infinitely credible source. A function which achieves this is $e^{A_{ij}} / (1 + e^{A_{ij}})$ so that

$$\Delta P_i = a \frac{e^{A_{ij}}}{1 + e^{A_{ij}}} (M_{ji} - P_i).$$

Our change model for attitudes will be complete when the factor of transmission from J, N_{ji}, is included. In the passive communication context the difference equation above can be applied again and again for each message that is generated. However, in the interactive mode, we desire to have the transmission process included explicitly so that once the process of interaction is begun, it will be terminated by the interactants with a termination of transmission. Also, as the rate of transmission increases, the more messages J sends to I and, hence, the faster I should change toward the message. When transmission is zero, then the change in attitudes should also be zero. This suggests that transmission, like credibility, is a multiplicative factor in the change equation for attitudes:

$$\Delta P_i = a \frac{e^{A_{ij}}}{1 + e^{A_{ij}}} (M_{ji} - P_i)\, N_{ji}. \tag{1}$$

Figure 1 shows the effects of discrepancy $(M_{ji} - P_i)$, attraction, A_{ij}, and transmission, N_{ji}, on the change in attitude. In Figure 1a, the more positive M_{ji} is than P_i, the greater will be the positive change in P_i (that is, in the direction of the message). For the same amount of discrepancy, the greater the transmission from J. the greater the amount of change in I's attitude. In frame b of the same figure, we see that for a fixed amount of discrepancy, the greater the attraction, the greater the attitude change. For a fixed discrepancy and fixed level of attraction to the source, the greater the transmission the greater the change in the direction advocated by the message.

Having laid the ground work for a change in I's attitudes as a function of J's messages according to information processing theory, developing the change equation for Q_{ij} is an easy matter. The change

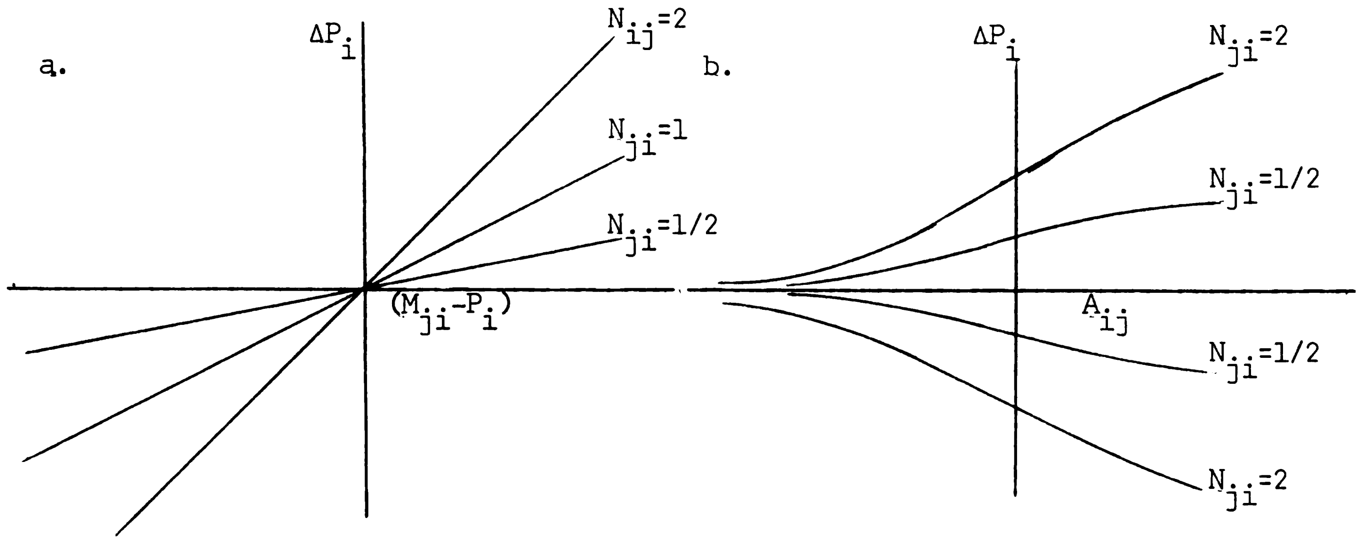


Figure 1. Changes in Attitude versus Discrepancy between Message and Attitude with Attraction = 0 (a) and versus Attraction with Discrepancy between Message and Attitude = +1 and -1 (b): Varying Levels of Transmission.

in I's perception of J's attitude should be exactly analogous to the change in I's attitude as a function of J's message. That is, the more discrepant J's message is from I's position, the more change in Q_{ij} in the direction of the message should be observed. Also the more attractive J is to I, the more change in Q_{ij} that should be realized. And as the transmission from J increases, the rate of change of Q_{ij} in the direction of the message should increase as well. Thus,

$$\Delta Q_{ij} \;=\; b \; \frac{e^{A_{ij}}}{1 + e^{A_{ij}}} \; (M_{ji} - Q_{ij})\, N_{ji}. \tag{2}$$

Figure 2 presents the graphical form of equation (2) for the cases of constant attraction and constant discrepancy with varying levels of transmission. Comparing Figures 1 and 2 for equations (1) and (2) shows in a striking manner the similarity of the two change equations. The only differences between equations (1) and (2) are found in the parameters a and b. Although they both have the same function, a is not necessarily equal to b and their ratio indicates whether a given message elicits more change in P_i ($a > b$) or in Q_{ij} ($b > a$). Based upon a study by Wackman and Beatty (1971), cited in Wackman (1973), we shall always assume in our examples that perceptions of the other are less resistant to change than are the attitudes that one holds. This means that for the same discrepancy, I's perception of J's attitude will change more in the direction of J's message than will I's own attitude, hence $b > a$. At this point, it is interesting to note that M_{ji} plays a dual role in equations (1) and (2). In equation (1) its effect is that of persuasion and in equation (2) its effect is that of informing I of J's position.

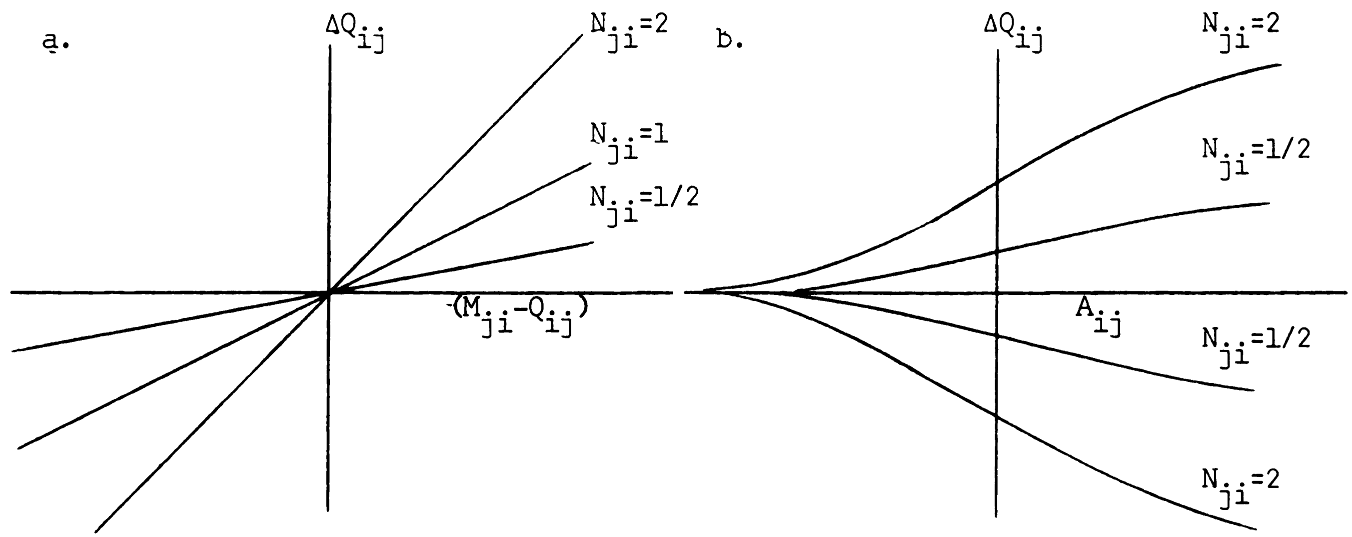


Figure 2. Changes in Perception of the Other versus Discrepancy between Message and Perception of the Other with Attraction = 0 (a) and versus Attraction with Discrepancy between Message and Perception = +1 and -1 (b): Varying Levels of Transmission.

This duality is not unreasonable since in attempting to persuade another of his position, I is simultaneously offering him information on the exact nature of his position.

Unfortunately, deriving a change equation for attraction from information processing assumptions is not as easy as it was for perceptions and attitudes. The reason, as Hunter and Cohen (1974, p. 38) note, is that the information processing theorists were not interested in change in the attraction of the source as a function of his message. In fact, attitude change theorists did not seriously begin to think about the effect that source change could have on the processes of attitude change until Aronson, Turner, and Carlsmith's famous attempt (1963) to explain nonlinear changes in attitude by invoking changes in the attractiveness of the source. Since the information processing research predated the Aronson, Turner, and Carlsmith piece, the information processing model takes no explicit stand on source change. However, certain requirements for the change in attraction can be stipulated: First, based upon the work of Byrne (1969) and his colleagues, we expect that changes in the attractiveness of the other will depend upon the degree of similarity that is perceived by the focal individual. Second, changes in attraction should be both positive and negative so that attraction is capable of either increasing or decreasing. Obviously, if attraction can <u>only</u> increase or <u>only</u> decrease, then the patterns of attraction which can emerge from such a model will be less than interesting. We believe that any model allowing <u>only</u> increases or <u>only</u> decreases in attraction lacks face validity and conflicts with everyday acquaintance processes. Newcomb's famous field study of the acquaintance process (1961) observed and measured both increases and decreases in attractiveness between members of a housing unit and related those changes to initial socioeconomic and religious similarities of the subjects at least at the early stages of acquaintance formation. The reason that this point is being emphasized is that dissonance theory of source change (as Hunter and Cohen, 1974, show) is one of pure source derogation. We feel that such a model of attraction change is too limited to apply to dyadic processes. Similarly, a straightforward extension of Byrne's so-called "law of attraction" would posit

$$\Delta A_{ij} = n\Delta \ |M_{ji} - P_i|.$$

However, this model has the peculiar characteristic that if I and J initially hate one another but upon interaction find that they agree (that is $M_{ji} = P_i$), then they will remain unattracted to one another despite being in agreement. We find this implausible and at odds with the evidence presented in Newcomb (1961).

Rather, we shall posit a model of attraction change which is basically social judgmental in character. That is, we assume that when $M_{ji} = P_i$, the change in attraction is positive and maximum. When M_{ji} and P_i are discrepant, then whether the change in attraction is positive or negative depends upon what amount of discrepancy the focal individual is willing to accept. That is, if person I is willing to accept a certain amount of disagreement but no more before his attraction to J begins to decrease, then that amount defines the boundaries of his acceptance region. The change equation which will describe the above process is (see Hunter and Cohen, 1974):

$$\Delta A_{ij} = c \frac{(t_{ij}^2 - (M_{ji} - P_i)^2)}{1 + t_{ij}^2} N_{ji} . \tag{3}$$

This equation is graphed in Figure 3. $2t_{ij}$ is the width of the

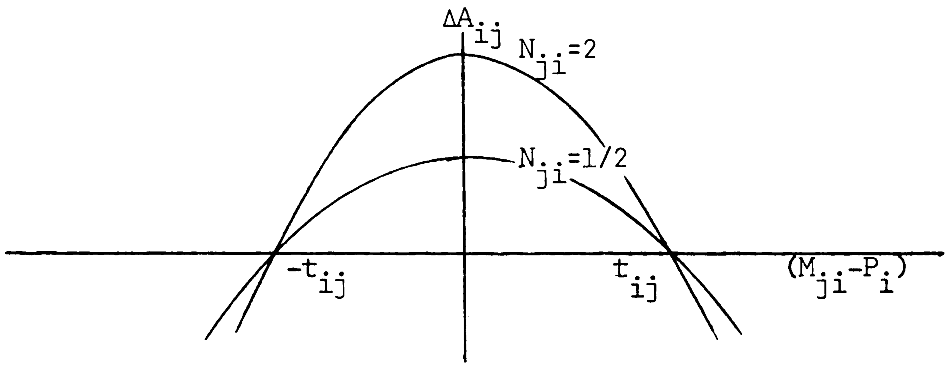


Figure 3. Changes in Attraction versus Discrepancy between Message and Attitude for Varying Levels of Transmission.

acceptance region centered at Pi. When $M_{ji} - P_i > t_{ij}$ or $M_{ji} - P_i <$ $-t_{ij}$, then the change in attraction decreases. When $-t_{ij} < M_{ji} - P_i$ $< t_{ij}$, then the change in attraction is positive and is a maximum for $M_{ji} - P_i = 0$ or perfect perceived agreement. When $M_{ji} - P_i$ falls exactly on the border between the acceptance and rejection regions, then the change in attraction is zero.

Obviously, the behavior of equation (3) depends upon the value of t_{ij}. We shall assume with Hunter and Cohen (1974, p. 49) and Sherif, Sherif, and Nebergall (1965, p. 189) that the width of the acceptance region depends at least in part upon the attractiveness of the other. The more attractive the other, the wider the acceptance region and the less attractive the other, the narrower the acceptance region. Specifically, it is assumed that $t_{ij} = e^{A_{ij}}$. In this way, the more positive the attraction, the more likely that discrepancies will fall within the acceptance region and produce even greater attraction. Also, the more negative the attraction, the more likely that discrepancies will fall outside the acceptance region and produce further decreases in attraction.

In sum, the information processing model for external changes is summarized by equations (1), (2), and (3). The key problem in the information processing model surrounds the choice of an attraction change equation which is consistent with the information processing position. Because the information processing point of view has failed to consider source change along with attitude change, we introduced independent criteria for attraction change and were led to a model which best fits the social judgment position. However, the change equations for P_i and Q_{ij} are <u>not</u> social judgment equations. We next turn to the development of models of the generation of message content and its transmission in order to complete the input and output characteristics for the information processing model of external changes.

<u>The Generation of Message Content</u>

We shall consider two models of the generation of message content. Each of these models will be highly speculative since the question of <u>what</u> is said in interaction has not been well researched.

First, suppose the subject always speaks his mind. That is, the message will just be his attitude or

$$M_{ij} = P_i \ . \tag{4}$$

This prediction constitutes the first model of message generation and has been the one most commonly adopted in interactive models of attitude change (Abelson, 1964; Taylor, 1968). It will be called the "veridical" model.

In the veridical model, the speaker says the same thing regardless of who the listener might be. But suppose that he seeks to ingratiate himself with J by shifting his message in the direction of his perception of J's attitude. That is,

$$M_{ij} = pP_i + (1-p)Q_{ij}$$

where p is a weighting factor between 0 and 1. If we presume that individuals are more ingratiating for more attractive others and less ingratiating for less attractive others, then the weighting factor p would be a function of attraction. Furthermore, the more attractive the other, the closer p should be to unity. This implies that p could be chosen to be $p = 1/(1 + e^{A_{ij}})$. Rewriting the above equation with this new expression for p, we have

$$M_{ij} = P_i + \frac{e^{A_{ij}}}{1 + e^{A_{ij}}} (Q_{ij} - P_i) \ . \qquad (5)$$

When I thoroughly dislikes J, then he speaks his mind (that is, is veridical) and does not seek interpersonal rewards from J by ingratiating him. When I likes J a great deal, then he seeks to further the favors and good graces from J by saying what he thinks J wishes to hear. We shall call this the "shift" model because of the cynicism associated with an "ingratiation" model.

The Transmission of Messages

Recall that Newcomb's discussion of message transmission was as a possible response to individual system strain. He presumes that the reaction to individual system strain through communication to the other actually took the form of attempts to influence the other's point of view concerning X. Consequently, influence attempts directed toward the other should arise from forces created by perceived discrepancies on X. That is, we assume that transmission is intended to alter the other's attitudes toward X and not to alter I's attraction to J. If N_{ij} is the number of influence attempts generated by I toward J, then we assume that

$$N_{ij} = \frac{d\,|Q_{ij}-P_i|}{\sqrt{1+(Q_{ij}-P_i)^2}} \left[1 + \frac{e^{A_{ij}}}{1. + e^{A_{ij}}} \right] \qquad (6)$$

where d is a positive constant.

Equation (6) is the product of a discrepancy term and an attraction term. It was chosen to yield the following specifications: (1) For constant attraction, the greater the discrepancy perceived by I, the greater the transmission from I. (2) For constant perceived discrepancy, the greater the attraction which I has for J, the greater the transmission from I (see Figure 4). (3) For large negative attraction, transmission is still positive and depends upon the amount of perceived disagreement (see $A_{ij} = -\infty$ in Figure 4).

The empirical evidence relevant to the evaluation of equation (6) is both limited and relatively old. In the early 1950's Festinger and his colleagues at the University of Michigan undertook a program of field and experimental search related to the question of communication and attitude change in the small group context. In summarizing the results of this research program Festinger (1951) states two propositions also predicted by our equation 6:

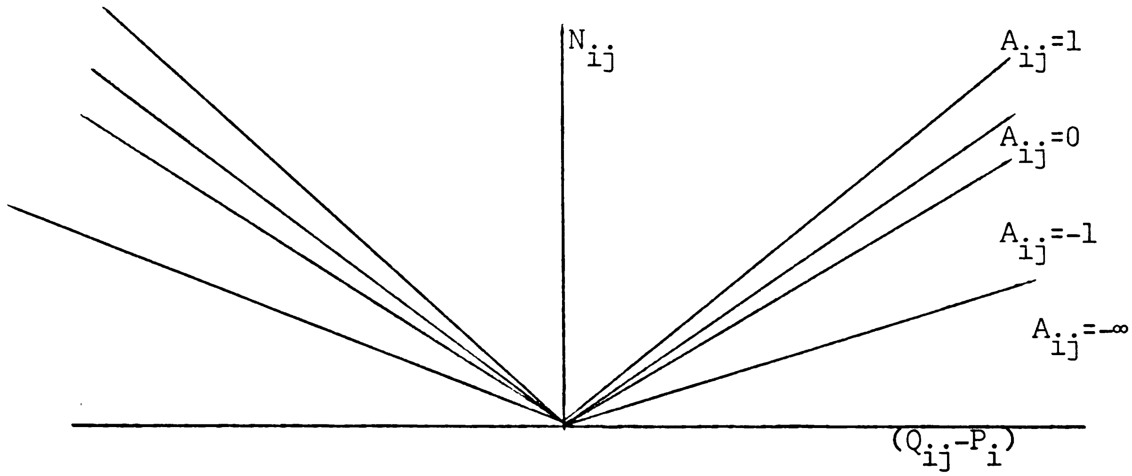


Figure 4. Transmission versus Perceived Discrepancy for Various Levels of Attraction.

> The pressure on members to communicate to others in the group concerning "item x" increases monotonically with increase in the perceived discrepancy in opinion concerning "item x" among members of the group (p. 274).

and

> The pressure . . . to communicate . . . concerning "item x" increases montonically with increase in the cohesiveness of the group (p. 274).

We note that "cohesiveness" was generally operationalized as attractiveness: "Cohesiveness is the attraction of membership in a group for its members" (Back, 1951, p. 9). Specifically, research by Back (1951) found that increases in group cohesiveness resulted in greater total discussion as well as a greater number of influence attempts. This supports Festinger's second proposition. Research by Festinger and Thibaut (1951) found that the weighted number of communications to discrepant others decreased as the other's attitude moved from an extreme position toward a pre-established group norm. This supports Festinger's first proposition, and our model.

In the most explicit and well-known discussion of communication and rejection in small groups, Schachter (1951) presents and tests a model in which the effects of discrepancy and attraction on transmission were presumed independent and additive. This view agrees with that of Festinger (1951) but disagrees with our equation (6) where an interaction between attraction and perceived discrepancy is assumed. While Schachter's data are by no means unequivocal (see Berkowitz, 1971 for a discussion of interpretation difficulties), they do tend to support the attraction-transmission hypothesis. No interaction hypothesis is tested. However, certain of Schachter's data tend to support an interaction between discrepancy and attraction for predicting transmission.

In each experimental group, Schachter planted a "deviate" who took an opposite position to that of the group and maintained it, a "mode" who adopted the position most frequently chosen by the group members, and a "slider" who initially took an extreme position and over time converged toward the group norm. If the number of communications addressed to the slider over time is graphed, then an interpretation of the graph as the number of messages versus discrepancy is possible since the slider's position is systemically changed toward that of the group over time. This data is presented in Figure 5 for the high and low attraction conditions. Only the data for the groups discussing topics relevant to their purpose rather than irrelevant is presented. The reason for this omission is that Schachter did not measure the number of influence attempts or even the relevant messages but rather measured the gross number of communications. Berkowitz (1971, p. 238) has criticized this measure as a possible explanation of the equivocality of

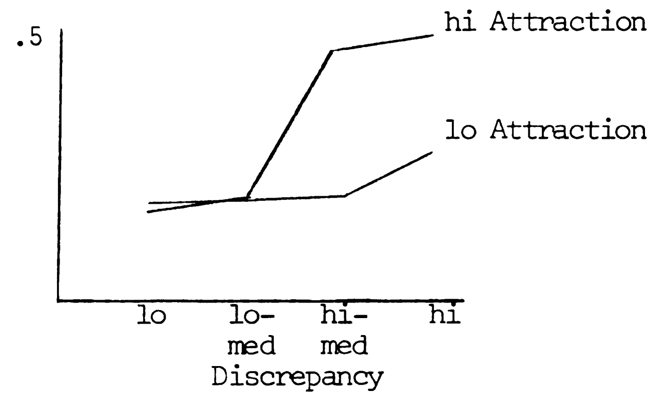


Figure 5. Transmission to the Slider versus Perceived Discrepancy for High and Low Attraction (Schachter, 1951, Table 8).

Schachter's results. In addition, a recent study by Rosenfeld and Sullwold (1969) found a large increase in irrelevant discussion as individuals who had little use for each other's information interacted over time. Thus, to increase the validity of Schachter's measure as an indicator of influence attempts, only the data from groups interacting over relevant topics is considered. This is especially true if the "lo attraction" condition of Figure 5 can be viewed as the high dislike state for the experiment. In this case, the high dislike case still produces transmission with low discrepancy rather than yielding no transmission.

Clearly, Figure 5 suggests that in predicting transmission, attraction and perceived discrepancy interact thus supporting the implications of equation (6).

With the completion of the message transmission model and the content generation models, we now have the input and output linkages between individual systems through the process of interaction. But before summarizing the external change model according to information processing theory, we consider an information processing model of <u>internal</u> changes.

<u>Internal Changes According to Information Processing Theory</u>

Before discussing the application of information processing theory to internal, spontaneous changes in the individual's cognitions in his intrapersonal system, it deserves mention that most information processing theorists would argue that spontaneous changes in perceptions and attitudes do not occur. Rather, changes in perceptions and attitudes occur as a function of the rational consideration of one's position relative to the available information and arguments to which one attends. However, there is one rationale which information processing theorists might find acceptable. Even when the other, J, is not present, I may think about him and may think what J would say about X. That is, I <u>imagines</u> J giving messages and those messages should have some impact. There are then two possibilities: I remembers J's actual messages in which case attitudes and perceptions change as some function of the actual message. This function would presumably reflect such effects as forgetting and selective retention. Or I can create messages for J based upon his perception of J in which case attitudes change as a function of his perception of J's position. We have opted for the latter in the models below.

In the individual system, it is not discrepancies between incoming messages and attitudes, or messages and perceptions which are evaluated relative to one another to determine change, but the discrepancies between the internal system variables P_i and Q_{ij}. That is, the change in P_i for internal information processing is exactly analogous to equation (1) for external information processing, except that the information being processed relative to the focal person's attitude is information about the other which is internal to one's own cognitive system, that is Q_{ij}. Thus, rather than evaluating his attitude relative to the "hard" information provided by the other's message, the focal individual evaluates his attitude relative to the "soft" information about the other which is already in storage. That is,

$$\Delta P_i = r \frac{e^{A_{ij}}}{1 + e^{A_{ij}}} (Q_{ij} - P_i) , \tag{7}$$

which is graphed in Figure 6. Notice that this equation, like equation (1), has a credibility multiplier so that more attractive others produce greater changes in attitudes in the direction of the perception of the other. Unlike equation (1), no transmission term is involved since transmission is relevant only in the interaction or external change processes.

Notice in Figure 6a that the greater the perceived discrepancy, the greater the change in P_i for constant attraction. For the same amount of perceived discrepancy, the greater the attraction the greater the change in P_i. Figure 6b shows that the change in P_i goes to zero as A_{ij} goes to negative infinity if $(Q_{ij}-P_i)$ stays finite. As A_{ij} goes to positive infinity the change in P_i becomes a simple linear function of the perceived discrepancy $(Q_{ij}-P_i)$ because the multiplier $e^{A_{ij}}/(1 + e^{A_{ij}})$ goes to 1. Thus, if I likes J a great deal and perceives that they differ on X, then his spontaneous changes in attitude should be large. And, the more different I perceives J to be, the more

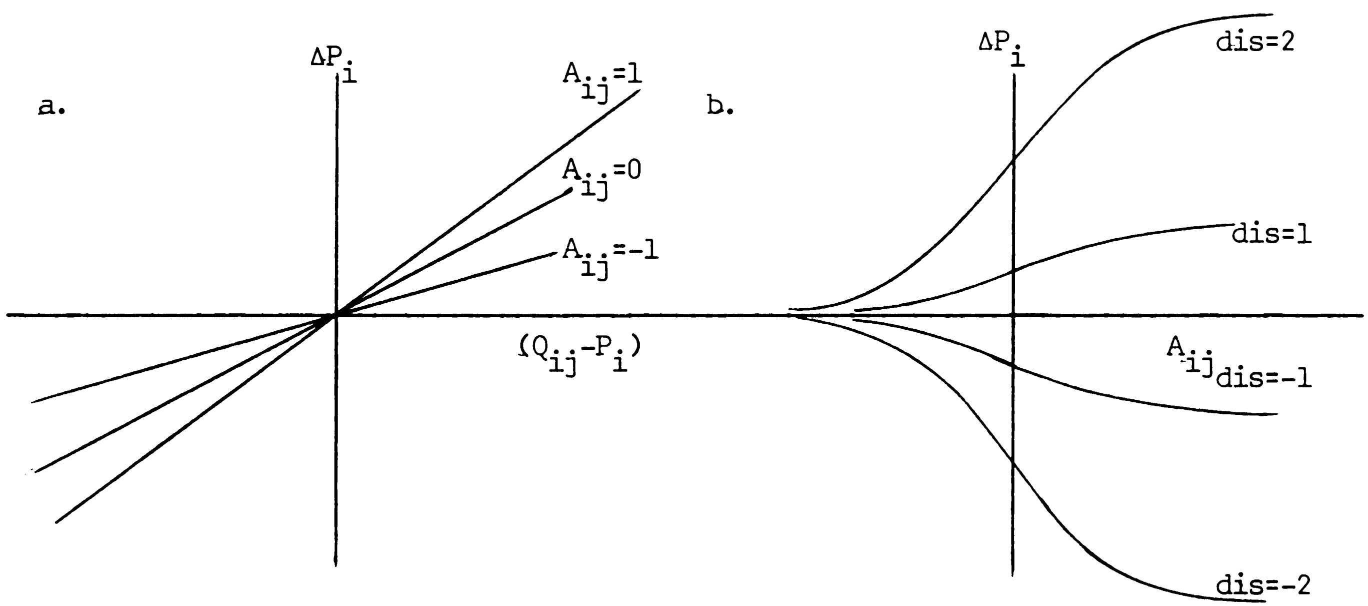


Figure 6. The Change in P_i as a Function of Perceived Discrepancy with Attraction held Constant (a) and as a Function of Attraction with Discrepancy held Constant (b) for Internal Forces.

he will change his attitude. However, if I dislikes J a great deal, then perceived discrepancies on X will produce little or no spontaneous changes in attitude.

An equation to represent internal changes in Q_{ij} is also simple to develop since it is exactly the same as equation (7) with the roles of P_i and Q_{ij} reversed. That is, in evaluating his perception of the other's position I compares it to his own position and changes his perception in proportion to the discrepancy between P_i and Q_{ij}. Thus, we may write an equation for Q_{ij} directly:

$$\Delta Q_{ij} = q \frac{e^{A_{ij}}}{1 + e^{A_{ij}}} (P_i - Q_{ij}) . \qquad (8)$$

The equation for the change in Q_{ij} due to internal forces has exactly the same qualitative description and the same graphical representation as those for P_i except that P_i and Q_{ij} have been interchanged. That is, Q_{ij} is changing in the direction of P_i by an amount which is a function of I's attraction to J and the constant Q. Like the constant r in equation (7), q is positive and less than or equal to 1. q is not necessarily equal to r and their ratio would indicate whether P_i is easier to change than Q_{ij} $(r > q)$ or Q_{ij} easier to change than P_i $(q > r)$ for internal change.

Finally, given the validity of equation (3) for changes in attraction due to messages from the other, the comparable equation for internal changes in attraction merely replaces the external information M_{ji} with the internal information Q_{ij}. Thus,

$$\Delta A_{ij} = s \frac{(t_{ij}^2 - (Q_{ij} - P_i)^2)}{1 + t_{ij}^2} , \qquad (9)$$

where we assume as before that $t_{ij} = e^{A_{ij}}$. Figure 7 shows the critical points for the change in attraction at differing amounts of perceived disagreement. When $Q_{ij}-P_i > t_{ij}$ or $Q_{ij}-P_i < -t_{ij}$, then ΔA_{ij} is less than zero. When $-t_{ij} < Q_{ij}-P_i < t_{ij}$, then ΔA_{ij} is greater than zero and should be maximum for $Q_{ij}-P_i = 0$ or perfect perceived agreement. When $Q_{ij}-P_i$ falls exactly on the border between the acceptance and rejection regions, then ΔA_{ij} is zero.

Figure 8 graphs the cases for $t_{ij} = \infty$ and $t_{ij} = 0$. When $t_{ij} = \infty$, the acceptance region is infinitely wide so that any finite discrepancy is perceived as "near zero" and the change in attraction is always positive. When $t_{ij} = 0$, the acceptance region is infinitesmal so that any non-zero discrepancy no matter how small produces decreases in attraction. This latter case is similar to the dissonance model of source derogation (Hunter and Cohen, 1974, p. 81).

Obviously, then, the behavior of equation (9) depends upon the value of t_{ij}. We shall assume, as we did in the case for external changes, that the width of the acceptance region depends at least in part upon the attractiveness of the other. The more attractive the other, the wider the acceptance region and the less attractive the other, the narrower the acceptance region. Specifically, it is assumed that $t_{ij} = e^{A_{ij}}$. In this way, the more positive the attraction, the more likely that discrepancies will fall within the acceptance region and produce even greater attraction. Also, the more negative the attraction, the more likely that discrepancies will fall outside the acceptance region and produce further decreases in attraction.

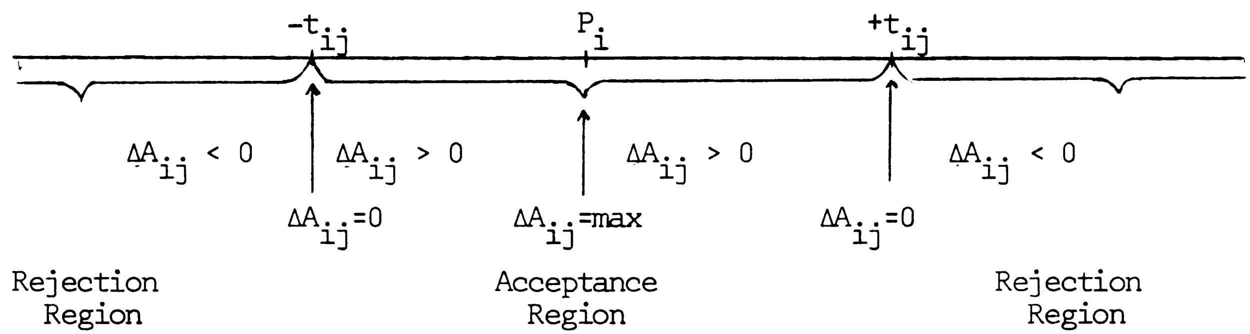


Figure 7. Regions of Acceptance and Rejection as Related to Changes in Attraction.

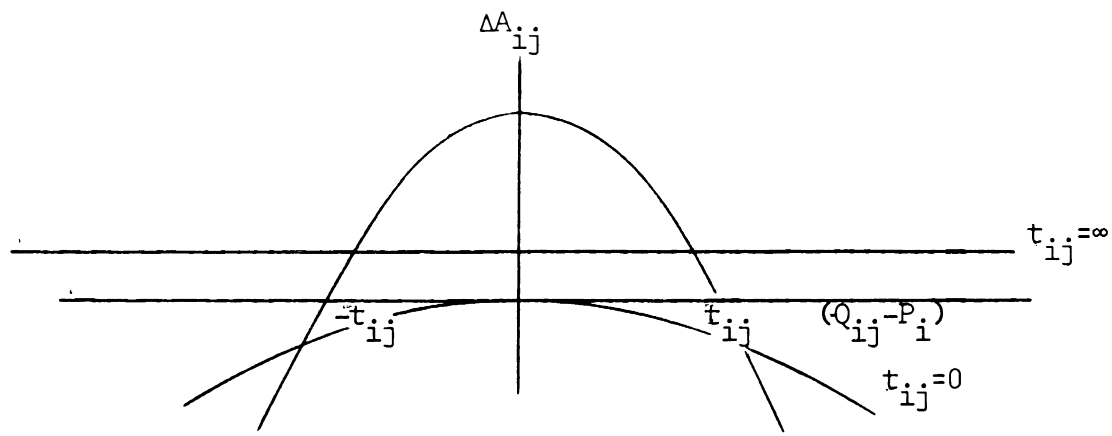


Figure 8. The Change in Attraction as a Function of Perceived Discrepancy According to a Social Judgment Model for Internal Forces.

It is important to recognize what we have done in equations (1) through (3) and (7)-(9) in the light of Newcomb's paradigm. Recall that Newcomb carefully distinguishes between an individual and a collective system and argues that changes in an individual's attitudes, perceptions, and attractions can arise both through internal, spontaneous changes and through the communicative and persuasive acts of the other. But as we also noted, Newcomb's model does not offer the specifics as to how internal and external changes occur nor how they may differ. What we have done thus far, is to bring the assumptions of information processing theory to describe external changes due to messages from the other and to extend the theory to describe internal changes as well. The change equations which have been developed are summarized in Table 1. Notice that in the external change equations, it is the information provided by the other--an outside source--which is compared to the internal reference points P_i and Q_{ij}. This gives rise to the difference terms in the external change equations. On the other hand, for the internal changes it is information which is directly available from the focal person's cognitive space which is compared. Thus, the difference terms in the internal change equations arise from comparisons of internal reference points to one another. In a sense, internal information processing is an "irrational" process which seeks to make compatible, information which is incompatible. By altering attitudes, perceptions, and attraction to the other on the basis of internal information alone, these alterations may or may not have a basis in fact. External information processing is "rational" at least in the sense that attitudes, perceptions, and attractions change on the

Table 1. Change Equations for Internal and External Changes Based upon Information Processing Theory.

	Internal	External
ΔP_i:	$r \dfrac{e^{A_{ij}}}{1 + e^{A_{ij}}} (Q_{ij}-P_i)$	$a \dfrac{e^{A_{ij}}}{1 + e^{A_{ij}}} (M_{ji}-P_i)N_{ji}$
ΔQ_{ij}:	$q \dfrac{e^{A_{ij}}}{1 + e^{A_{ij}}} (P_i-Q_{ij})$	$b \dfrac{e^{A_{ij}}}{1 + e^{A_{ij}}} (M_{ji}-Q_{ij})N_{ji}$
ΔA_{ij}:	$s \dfrac{t^2_{ij} - (Q_{ij}-P_i)^2}{1 + t^2_{ij}}$	$c \dfrac{t^2_{ij} - (M_{ji}-P_i)^2}{1 + t^2_{ij}} N_{ji}$

where $t_{ij} = e^{A_{ij}}$

Input-Output

Transmission	Content
$N_{ij} = d \dfrac{\lvert Q_{ij}-P_i \rvert}{\sqrt{1 + (Q_{ij}-P_i)^2}} \left[1 + \dfrac{e^{A_{ij}}}{1 + e^{A_{ij}}} \right]$	$M_{ij} = P_i$
	$M_{ij} = P_i + \dfrac{t_{ij}}{1 + t_{ij}} (Q_{ij}-P_i)$

basis of additional external information rather than through a "rationalizing" process of internal changes.

Simplifying the Dynamics of IJX Situations

Our general procedure for building an understanding of the complex dynamics embodied in the equations of this chapter will be to introduce simplifying assumptions and reductions of complexity initially and then to relax those assumptions. First, the equations of spontaneous change will be analyzed independently of the equations for induced change. This corresponds to the assumption that the internal and external processes do not operate at the same time. This would follow from our assumption that the autonomous forces arise from imagined messages from J to I which take place when I is thinking about J and J is not in fact present.

Second, models in which the rate of transmission is held constant will be considered separately. In fact some investigators have found equal transmission rates to be the norm if only two people are talking. Thus, our equal, fixed transmission rate models may ultimately prove to be more realistic for dyads than our fancy model which was derived from small group studies.

We will also consider separate models in which the attraction which I has for J will be assumed constant. These serve to build some understanding of the changes in P_i and Q_{ij} before considering the more complex case. The assumption of constant attraction will both reduce the number of equations to be considered and remove some pesky non-linearities. Substantively, constancy of attraction is associated with long-term friendships which are unlikely to change or with disagreement over relatively unimportant topics. Also we restrict our analysis to the interaction of two individual systems.

CHAPTER III

ANALYZING THE DYNAMICS OF CHANGE IN IJX SITUATIONS: INFORMATION PROCESSING THEORY

The previous chapter developed static descriptions of the change in state variables, P_i, Q_{ij}, A_{ij}, and M_{ij}. These change equations are summarized in Table 1. The task of this chapter is to develop an understanding of the behavior of each model over time so that comparisons between models can be made. In achieving this goal, we shall find it advantageous to move from a difference equation format to a differential equation format. In this way, the mathematics will be facilitated without any conceptual or substantive changes.

Understanding the dynamics of IJX situations will be developed through standard mathematical analysis for systems of differential equations when that is possible and through numerical analyses of the system when analytical procedures break down. Most of the mathematical analyses will be relegated to the two appendices A and B. The numerical results were generated on a CDC 6500 computer using a standard Runge-Kutta method. The program was developed at the Northwestern University Vogelback Computing Center by John Michelson and adapted for local use by the author. Most of the over time trajectories presented below were generated numerically.

Our general method of proceeding is as follows: The chapter is divided into two major sections. The first of these sections will take up the information processing (IP) model under the assumption of <u>constant</u> attraction. Within that section, we shall discuss the internal change model, the two message models with transmission constant, and then the two message models under conditions of varying transmission. The second major section will take up the IP model with <u>variable</u> attraction. Once again, within that section, we discuss the internal change model with varying attraction, the two message models with varying attraction but constant transmission, and then the two message models with varying attraction and variable transmission. Of primary interest in each model is the presence or absence of equilibria or critical points and the stability of those critical points. Since the critical point defines a point for which there are no changes in the state variables, those points (if there is more than one) define the balance points for the system. If there are no critical points, then we shall be interested in the direction that the state variables are moving as time t becomes infinite. That is, if the system is not going to a balance point where is it going?

Most mathematical discussions and derivations will be relegated to the appendices with the text reserved for graphical and verbal reports. We hope that such a format will facilitate comprehension without sacrificing mathematical rigor.

Information Processing with Constant Attraction

The information processing model with constant attraction consists of two parts: an internal change process due to spontaneous forces toward change and an external change process due to interaction between the individual systems. The reader is reminded that in keeping attraction and transmission constant, and separating internal and external change processes, certain strict substantive assumptions are presumed valid. Namely, attraction is deep-seated and long-term, the amount of discussion is rather evenly spaced throughout the period of interaction, and either internal or external processes dominate as a result of exogeneous factors enhancing or limiting discussion of the topic. With these strong assumptions the mathematical analysis of the IP case becomes somewhat simplified.

Internal Changes Only - The change equations governing spontaneous individual changes toward balance are the same for both individuals and are given by:

$$\frac{d\,P_i}{dt} = r\,k_{ij}\,(Q_{ij}-P_i) \tag{10a}$$

$$\frac{d\,Q_{ij}}{dt} = q\,k_{ij}\,(P_i-Q_{ij}) \tag{10b}$$

where k_{ij} replaces $e^{A_{ij}}/(1 + e^{A_{ij}})$ because A_{ij} is presumed constant. The analysis of this pair of linear equations is quite simple. From an intuitive point of view, since r, q, and k_{ij} are positive, equation (10a) says that the change in P_i is always toward Q_{ij} and (10b) says that Q_{ij} is always changing toward P_i. In a manner of speaking, Q_{ij} and P_i are "chasing each other" over time and, hence, converging toward one another. The bigger the value of k_{ij}, the faster that they will converge. If the ratio of r to q is large, then P_i will be changing more toward Q_{ij} than Q_{ij} will be changing toward P_i. Empirical studies by Kogan and Tagiuri (1958), Newcomb (1961), and Curry and Emerson (1970) support the view that it is the perception of the other that is changed while attitudes remain relatively stable (that is, the q to r ratio is large). Therefore, in our numerical examples we will take q/r to be 3 to 1.

The graphs of Figure 9 show the over time behavior of P_i and Q_{ij} when there is initial perceived agreement and initial perceived disagreement for two levels of attraction. Notice that in both graphs there is an asymmetry in that Q_{ij} changes more than P_i changes. This is due to setting the parameters r and q in a ratio of 1 to 3. An interesting case in Figure 9 is the over time behavior of the system when there is perceived agreement, $P_i = Q_{ij}$. In this case, the variables do not change over time at all. The system starts out "at rest" and remains there. Such a point is called an equilibrium point or critical point of the system. These points can always be found (assuming that they exist) by setting the equations for example (10a) and (10b) to zero and solving for those values of P_i and Q_{ij} which satisfy the equations. In the case at hand any pair in which the value of P_i equals Q_{ij} is a critical point. That is, we have an infinity of critical points which lie along the line $P_i = Q_{ij}$ in a plane of P_i, Q_{ij} points (see Figure 10a). As we shall see in this and upcoming sections, models which emphasize discrepancies often have such an infinity of critical points (which in non-linear cases produce certain other

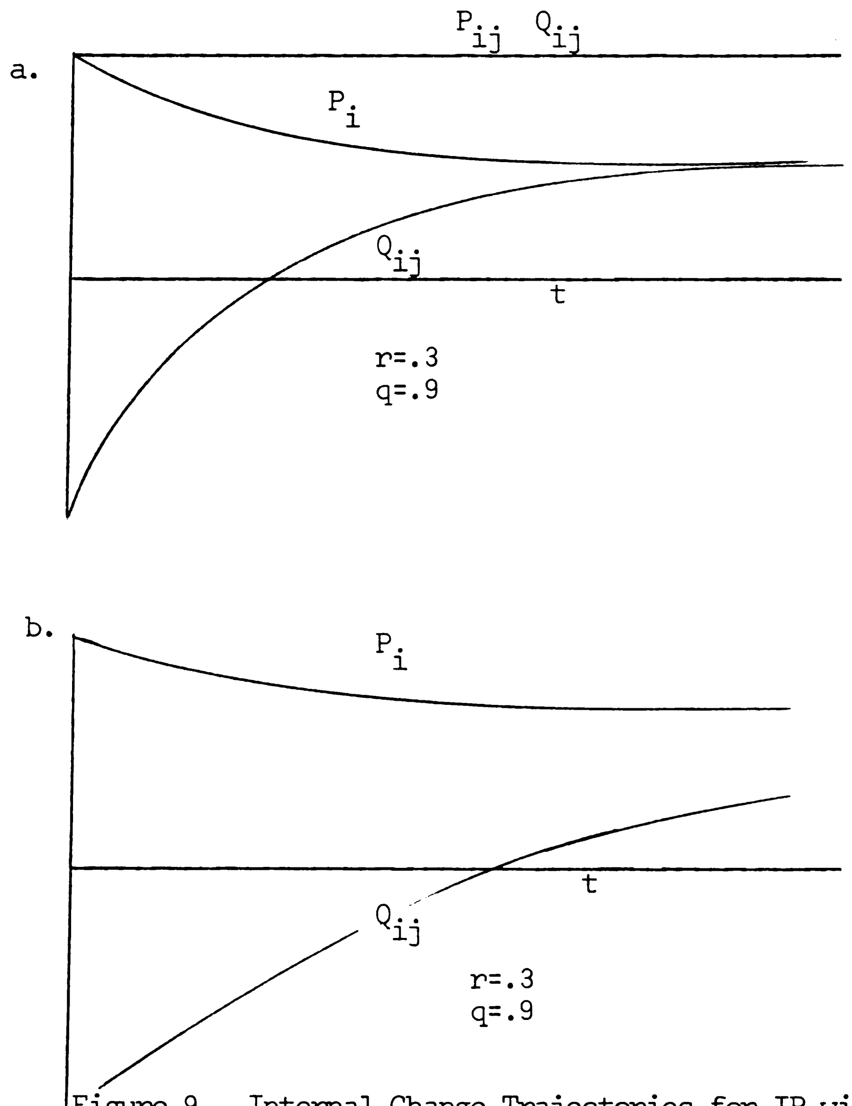


Figure 9. Internal Change Trajectories for IP with Positive Attraction (+3) (a) and Negative Attraction (=-3) (b) for Initial Perceived Disagreement.

mathematical difficulties).

Now that we have seen that the solutions to our equations converge toward one another and what the critical points of the system are, we are easily led to conclude that any set of initial values will converge toward a critical point. This result is shown mathematically in Appendix A. Figure 10 also shows this convergence for different values

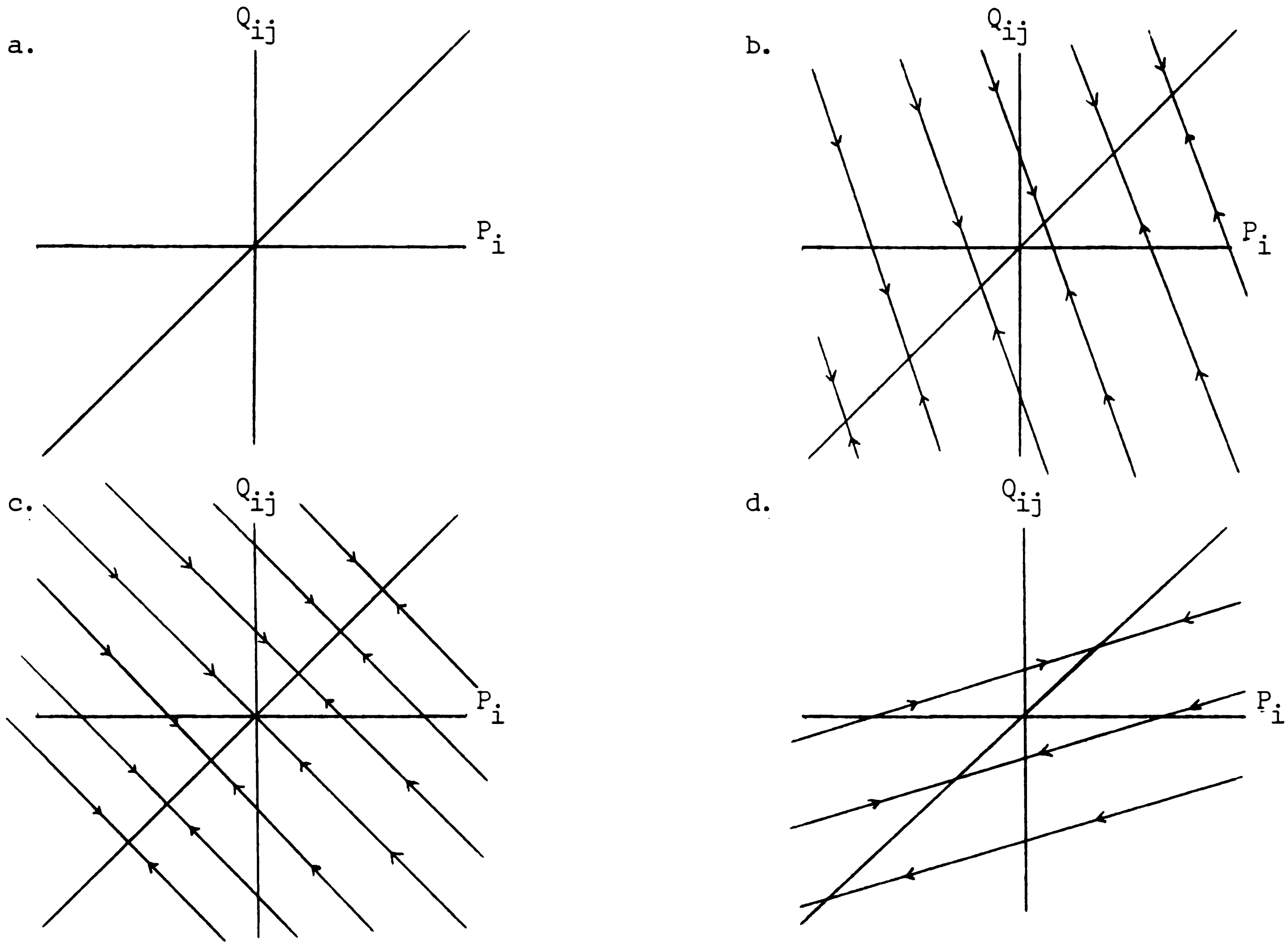


Figure 10. Phase Plane Trajectories for Internal Changes with Constant Attraction: (a) Line of Critical Points only, (b) Critical Points plus Trajectories for $q/r = 3$, (c) for $q/r = 1$, and (d) for $q/r = -.33$.

of the parameters. These graphs are called phase-plane trajectories and indicate possible movements of the pair of points (P_i, Q_{ij}) over time. The arrows on each line indicate the direction toward which the pair of values is moving. As can be seen, for all values of the parameters the trajectories terminate on the line $P_i = Q_{ij}$ which is the line of critical points. Thus, for any set of initial conditions and any set of parameter values (as long as they are positive) the system will converge toward a critical point.

What conclusions can be reached about individual system balance as a result of considering only the internal forces with attraction held constant? First, the system is unchanging when the perceived discrepancy is zero, $P_i = Q_{ij}$, <u>regardless of how I feels toward J</u>. This would mean, for example, that if I disliked J a great deal but perceived no differences between himself and J regarding X, then no spontaneous changes would ensue at all. Such a position is clearly predicted by Newcomb's positive balance model and by dissonance theory. In addition, the above description would be balanced in both Heider's and Festinger's descriptions of unchanging IJX situations. This principle also implies that if I likes J but perceives no differences between himself and J regarding X, then no spontaneous changes would result. This prediction agrees with Newcomb's, Festinger's, and Heider's models.

Second, when there is initial perceived disagreement, then rates of change toward perceived agreement depend upon the attractiveness of the other. As Figure 9 illustrates, the less attractive the other the slower the rate of convergence toward perceived agreement. In the limit as attractiveness goes toward negative infinity, k_{ij} goes to zero and P_i and Q_{ij} remain constant. In other words, if I hates J and perceives that they disagree, he will not undergo any spontaneous changes toward perceived agreement. This is more in keeping with Newcomb's positive balance model since extreme dislike should not "engage" I in the IJX situation enough to result in spontaneous changes toward balance. On the other hand, in Heider's model only our predictions for positive attraction would yield "balanced" states. For finite negative attraction, the limiting states of this model are <u>im</u>balanced. For infinite negative attraction the final states are balanced only if the initial states happen to have opposite signs.

Thus, the key point of differentiation for the internal model is with regard to the level of attraction. When the attraction is in the vicinity of neutrality or is positive, the ultimate states are balanced according to Heider and Newcomb. When attraction is highly negative, the changes follow Newcomb's predictions from positive balance. However, for moderate negative attractions, the equilibrium of $P_i = Q_{ij}$ is reached. Such a point would be imbalanced according to Heider's view but non-balanced (or vacuously balanced) according to Newcomb's positive balance model. That is, Newcomb would not have predicted any change.

<u>External IP with Shift Message and Constant Attraction and Transmission</u>

When I and J are interacting so that the internal processes are not operative, then the change equations governing induced changes due to the messages transmitted are:

$$\frac{dP_i}{dt} = a\,k_{ij}\,(M_{ji} - P_i)\,N_{ji} \qquad (11a)$$

$$\frac{dQ_{ij}}{dt} = b\,k_{ij}\,(M_{ji} - Q_{ij})\,N_{ji} \qquad (11b)$$

$$\frac{dP_j}{dt} = a\, k_{ji}\, (M_{ij}-P_j)\, N_{ij} \tag{11c}$$

$$\frac{dQ_{ji}}{dt} = b\, k_{ji}\, (M_{ij}-Q_{ji})\, N_{ij} \tag{11d}$$

where N_{ij} and N_{ji} are assumed to be constant and k_{ij} and k_{ji} are the constant attraction parameters defined as before. Also, according to the shift model, $M_{ij} = P_i + k_{ij}(Q_{ij}-P_i)$ and $M_{ji} = P_j + k_{ji}(Q_{ji}-P_j)$. In this interactive case, both pairs of individual system variables are "chasing" the message values which the other is generating. But since the message values are themselves dependent upon individual system variables, then I's state variables P_i and Q_{ij} are chasing a weighted sum of J's state variables (as a message) and vice versa. Since a and b are positive constants less than one, and the attraction and transmission terms, k_{ij}, k_{ji}, and N_{ij} and N_{ji} respectively, are positive, then we might expect the system of equations (11) to converge upon one another so that eventually $P_i = Q_{ij} = P_j = Q_{ji}$. This is exactly what happens, as is proven in the second section of Appendix A. In fact, Appendix A presents the general solution for the common limit $P_i^* = P_j^* = Q_{ij}^* = Q_{ji}^*$ as a function of the initial values (for the present case of fixed attraction and fixed transmission). The conclusions found below by looking at examples are all borne out by examining that general solution.

The simplest way to begin analyzing the system of equations (11) is to consider the case in which I and J are equally attracted and transmit an equal number of messages. As we noted earlier, this model may best represent the interaction of natural dyads since there is some evidence to indicate both an equality in speaking time (Jaffe and

Feldstein, 1970) and in attraction (Willis and Burgess, 1974) for the dyadic case. Also, this case can be completely solved mathematically (Appendix A, section 2). Throughout our discussion of this model, it will be assumed that the parameter b > a and that the ratio is 3 to 1. A study by Wackman and Beatty (1971), reported also in Wackman (1973), supports the view that changes in perception of the other's position occur much more quickly in interaction than do changes in one's own position. Figure 11 presents four sets of trajectories for the equal attraction, equal transmission case. Obviously, there are an infinity of initial values and we may choose but a few substantively interesting ones to discuss. The four trajectories differ in their initial values: (1) In frame a, I and J initially disagree with J perceiving disagreement but I perceiving agreement; (2) in frame b, I and J agree but both perceive disagreement; (3) in frame c, I and J initially disagree but both perceive agreement; and (4) in frame d, I and J disagree and both perceive disagreement. The parameters, a and b, and the attraction and transmission terms all play important roles in determining the speed of convergence of each of the variables. First, we assume that b/a = 3. Therefore, with attraction and transmission equal we should observe more rapid changes in Q_{ij} and Q_{ji} toward the point of convergence than P_i and P_j. This is exactly what we find in all frames of Figure 11. Notice that in frames a and b, the attitude variable is changing <u>more</u> in <u>magnitude</u> than the perception variable. This agrees with the general result for final state derived in Appendix A, section 2. However, even in frames a and b, the perception variables reach the equilibrium point well before the attitude variables. This is more visible in frames c and d where the

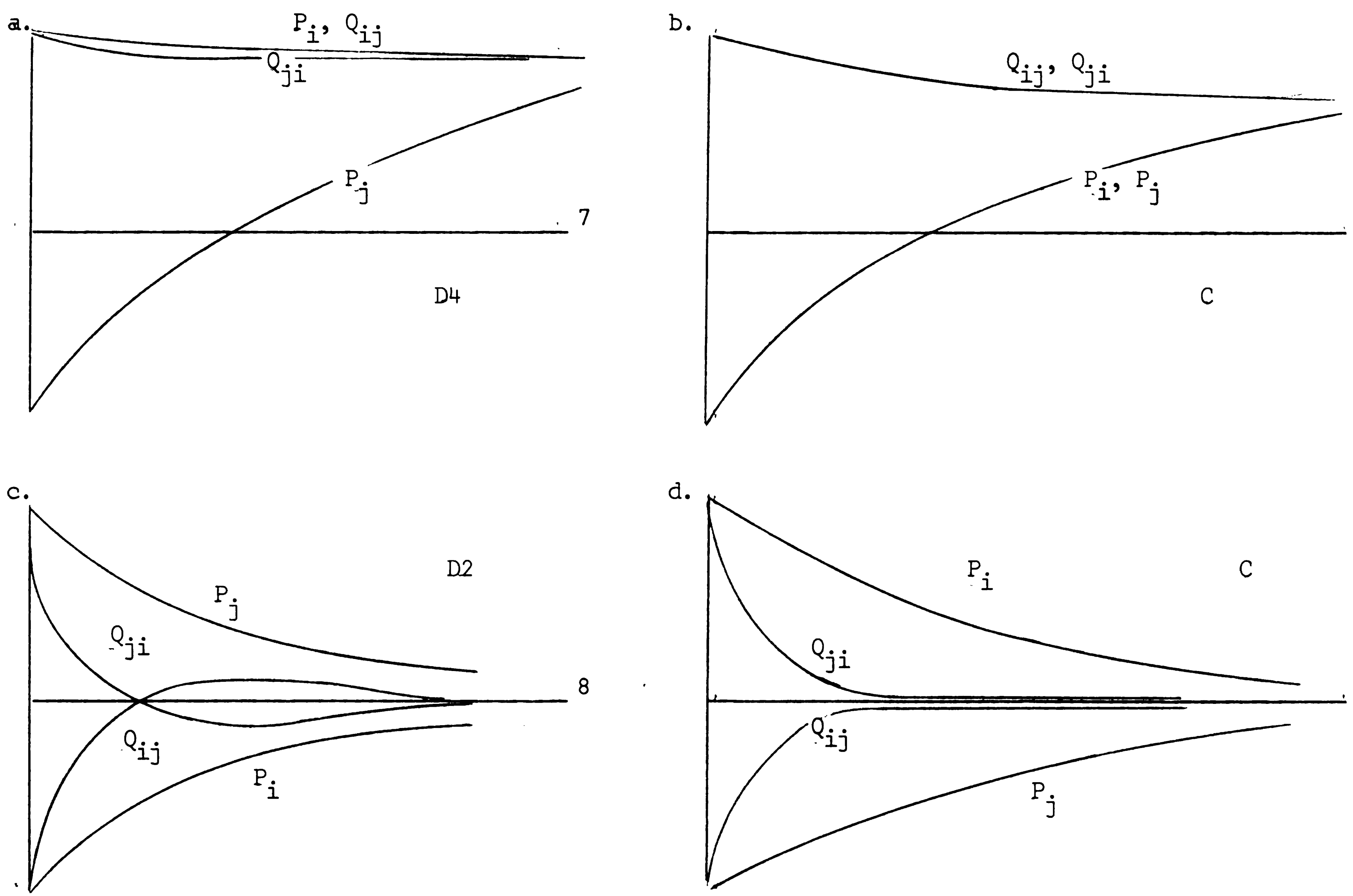


Figure 11. Trajectories for IP with Message Shift and Constant Attraction: Equal Attraction (3, 3) and Equal Transmission (1, 1).

equilibrium point (= 0) is equidistant from the initial values of all variables. Clearly, perceptions converge much more quickly than attitudes.

Thus far, we have been assuming that transmission is constant and, hence, can be assigned any values that we wish. However, some of these values are inconsistent with our model of varying transmission which stipulates that if I and J are equally attracted and both perceive disagreement, then transmission should be equal. If they are equally attracted but both perceive agreement, then transmission is equal and zero. When I and J are equally attracted, but I perceives agreement and J perceives disagreement, then J's transmission should be greater than I's which is zero. These cases and the eight other possible cases are summarized in Table 2. The table presents a comprehensive categorization

Table 2. Predictions of Relative Transmission between I and J as a Function of Relative Attraction and Initial Perceived Agreement (PA) and Perceived Disagreement (PD).

	PD I&J	PA I&J	PD(I) PA(J)	PD(J) PA(I)
$A_{ij} = A_{ji}$	$N_{ij} = N_{ji}$	0	$N_{ij} > 0 = N_{ji}$	$N_{ji} > 0 = N_{ij}$
$A_{ij} > A_{ji}$	$N_{ij} > N_{ji}$	0	$N_{ij} > 0 = N_{ji}$	$N_{ji} > 0 = N_{ij}$
$A_{ji} > A_{ij}$	$N_{ji} > N_{ij}$	0	$N_{ij} > 0 = N_{ji}$	$N_{ji} > 0 = N_{ij}$

of predictions about transmission based upon choice of attraction parameters and initial values on perceived agreement (PA) or perceived disagreement (PD). Each graph in Figure 11 is rated according to its consistency (C) with or degree of discrepancy (D1-D5) from the predictions of the transmission model. That is, if a graph receives a rating of D5, its initial values and attraction values are highly discrepant from the predictions that would be made based upon the transmission model. The rating scheme is summarized in Table 3 and can be used as a quick reference to ascertain the fit between attraction parameters, initial values and the predictions from the transmission model.

Table 3. Summary of Rating Scheme for the Degree of Fit between Attraction Parameters, Initial Values and Predictions from Transmission Model.

Symbol	Meaning
D1	If N_{ij} or N_{ji} is predicted to be zero, but one is greater than zero.
D2	If N_{ij} <u>and</u> N_{ji} are predicted to be zero, but both are set greater than zero.
D3	If $N_{ij} > N_{ji}$, or $N_{ji} > N_{ij}$, or $N_{ij} = N_{ji}$ is predicted, but the set values are different.
D4	If D1 and D3 hold.
D5	If D2 and D3 hold.
C	The predicted values of N_{ij}, N_{ji} are the ones chosen.

The ratings are presented for the graphs of Figure 11 in each frame. Frames b and d are consistent with the transmission assumptions while d is somewhat discrepant and a is very discrepant.

Consider the equal attraction but unequal transmission case. Figure 12 presents this case for the same set of initial conditions presented in the same order as for Figure 11. The most direct way of understanding Figure 12 is by comparison with Figure 11 since the two

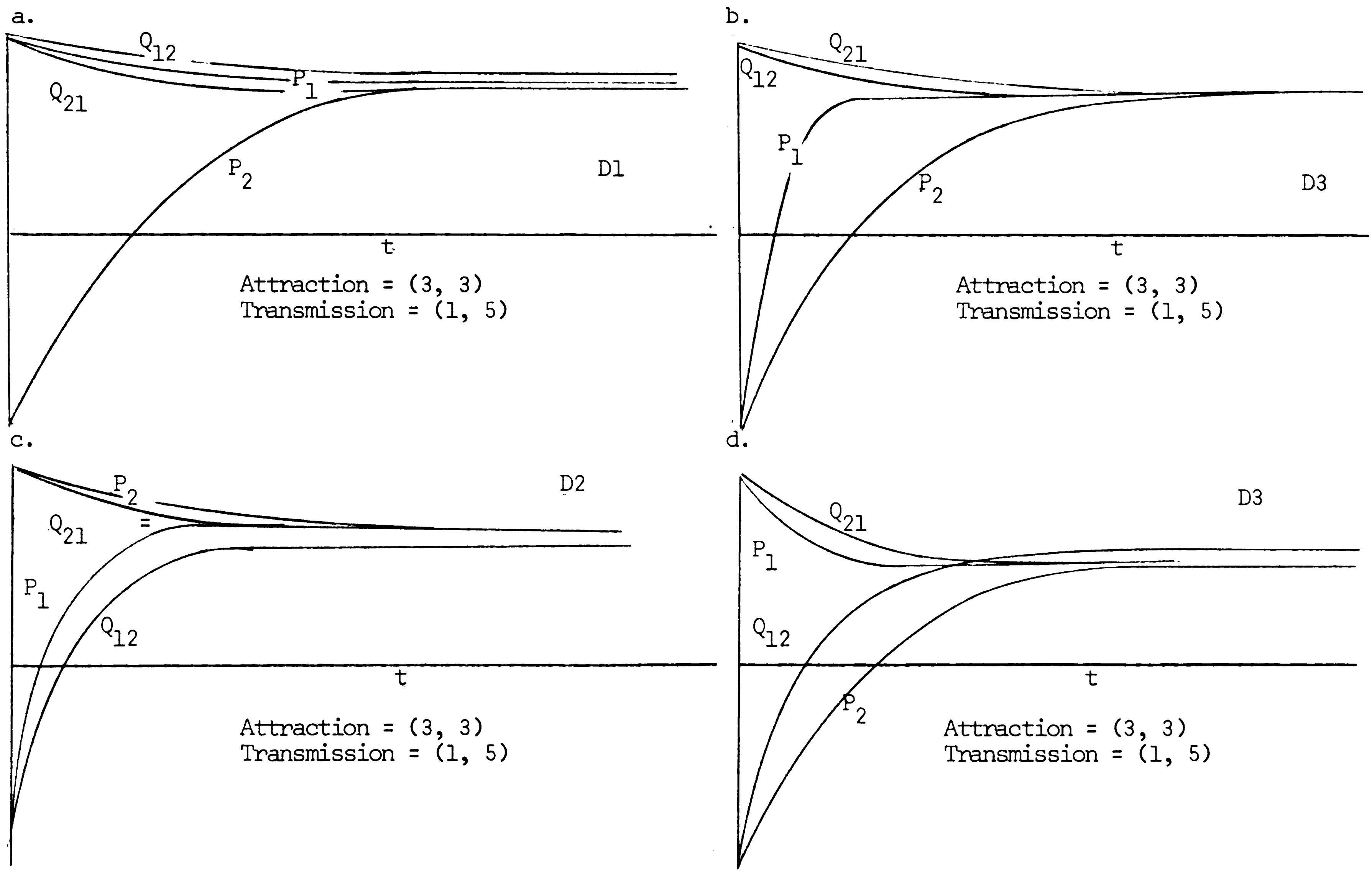


Figure 12. Trajectories for IP with Message Shift and Constant Attraction: Equal Attraction but Unequal Transmission.

differ only in the inequality of transmission. There are two observations to note: (1) the convergence points are not the same and (2) the rates of change of P_i and P_j can differ. The easiest case to see without performing a lot of calculation is in frame c. In Figure 11c the point of convergence was 0. But in Figure 12c, the convergence point is much closer to J's initial values. As we show with equation (A6) in Appendix A, for equal attraction, when J out-transmits I then I will do most of the changing toward a weighted sum of J's initial values. Frame c is such a case and the change in convergence point can literally be "seen" by comparison with Figure 11c. Secondly, in Figures 11b and 11c the convergence rates for P_i and P_j and for Q_{ij} and Q_{ji} were equal. In Figures 12b and 12c they are "distored" in the expected direction. That is, both P_i and Q_{ij} are changing more per unit time than their counterparts P_j and Q_{ji} precisely because J is transmitting more to I.

In Figure 13 we present the equal transmission, unequal attraction case with the same set of initial values as for Figures 11 and 12. The most striking aspect of the trajectories of this figure, compared to those of Figure 11, are the convergence points. In all cases, the fact that J dislikes I so much while I likes J yields the strong result that J's attitudes and perceptions change very little or very slowly while I's change a lot and rapidly. Furthermore, since J dislikes I so much, his message to I is for all intents and purposes just his attitude, which does not change much. As a result, all the other state variables converge to P_j which stays close to $P_j(0)$, just his attitude. That is, he does not shift his message so as to ingratiate I. Since his attitude does not change much, both I's perception of J and I's attitude converge

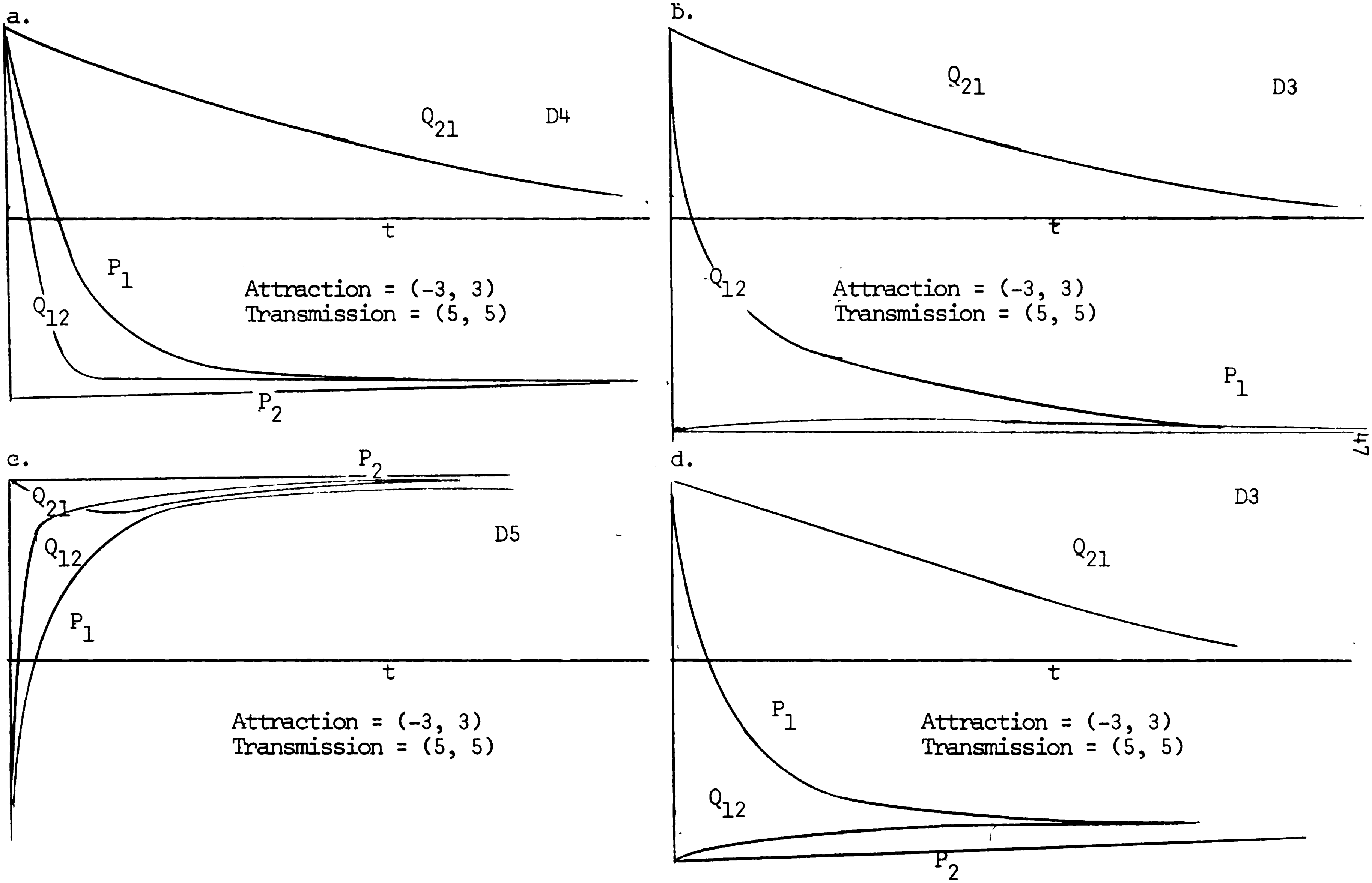


Figure 13. Trajectories for IP with Message Shift and Constant Attraction: Unequal Attraction and Equal Transmission.

to J's message value (which is still almost identical to his initial attitude). At this point I's message equals I's attitude which equals J's attitude. Hence, J's perception of I converges to I's message which is I's own attitude. As a result, all other state variables converge to P_j which stays close to $P_j(0)$.

Figures 14 and 15 can be discussed together since they represent the two forms of the unequal transmission and unequal attraction cases. Figure 14 offers the case which is at odds with our transmission predictions: namely, that the less attracted speaker will be the greater transmitter (see the ratings for Figure 14). Figure 15 presents the more plausible situation (Collins and Raven, 1968, p. 123) where attraction and transmission are positively correlated. As the ratings show, this figure has parameters and initial values which are more consistent with our own transmission assumptions.

Let us compare the convergence points in Figures 14 and 15. In both figures these points are very close to one another for the same set of initial conditions. That is, 14b and 15a have convergence points which are very similar, as do 14b and 15b, and so forth. The _only_ difference between Figure 14 and Figure 15 is that in the former J out-transmits I by a ratio of 5 to 1 while in the latter I out-transmits J by the same ratio. Thus, it is the attraction parameter which (3, -3) which almost completely determines the final state of the system given the same set of initial conditions. But the reason for this dominance has to do with the choice of parameters in this case. For attraction (3, -3), we have k_{ij} = .95 and k_{ji} = .05. With these values for attraction I would have to out-transmit J by a ratio of 19 to 1 to achieve an

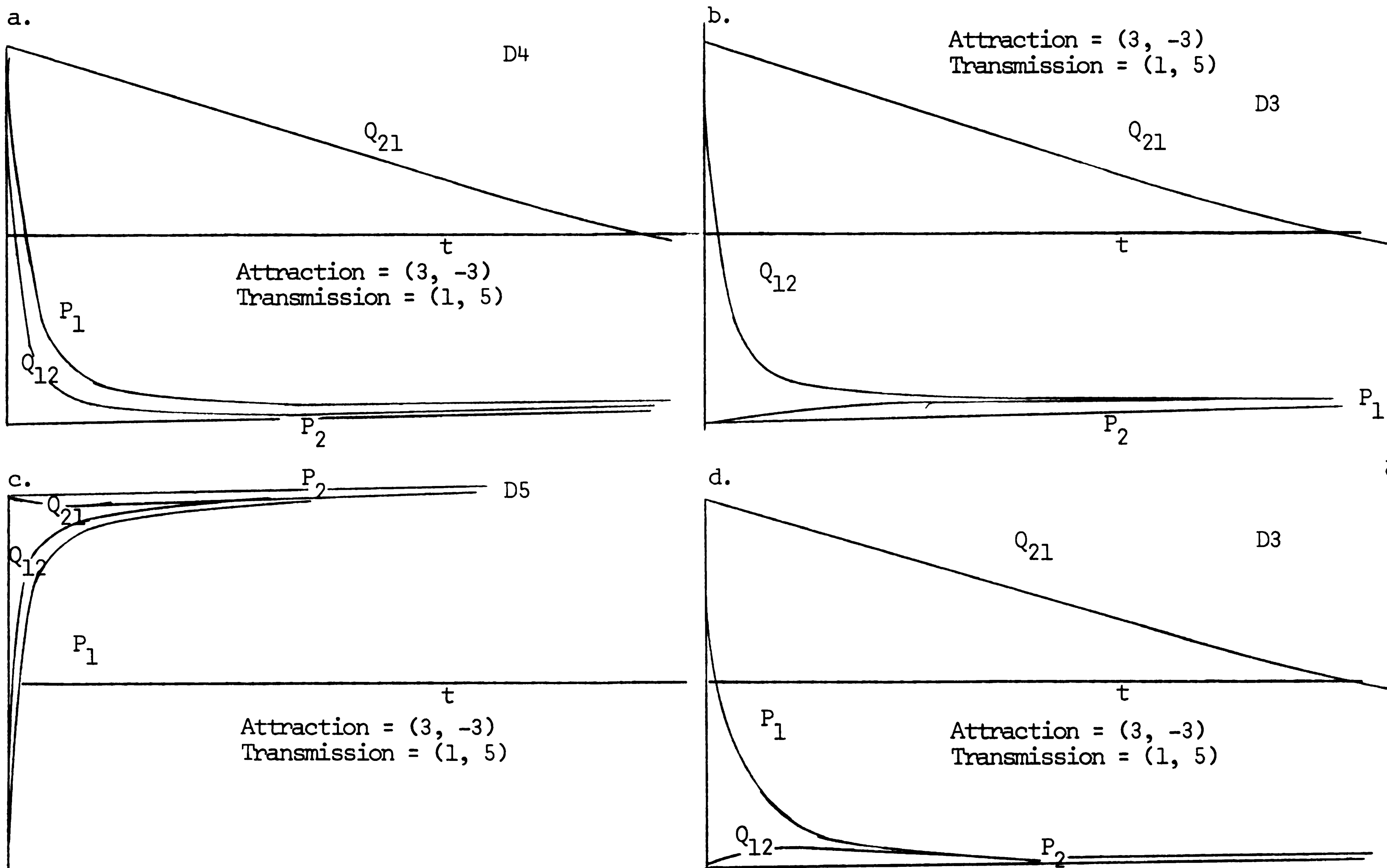
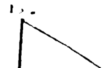


Figure 14. Trajectories for IP with Message Shift and Constant Attraction: Unequal Attraction and Unequal Transmission-Discrepant from Transmission Model.

b.



Attraction = (3, -3)

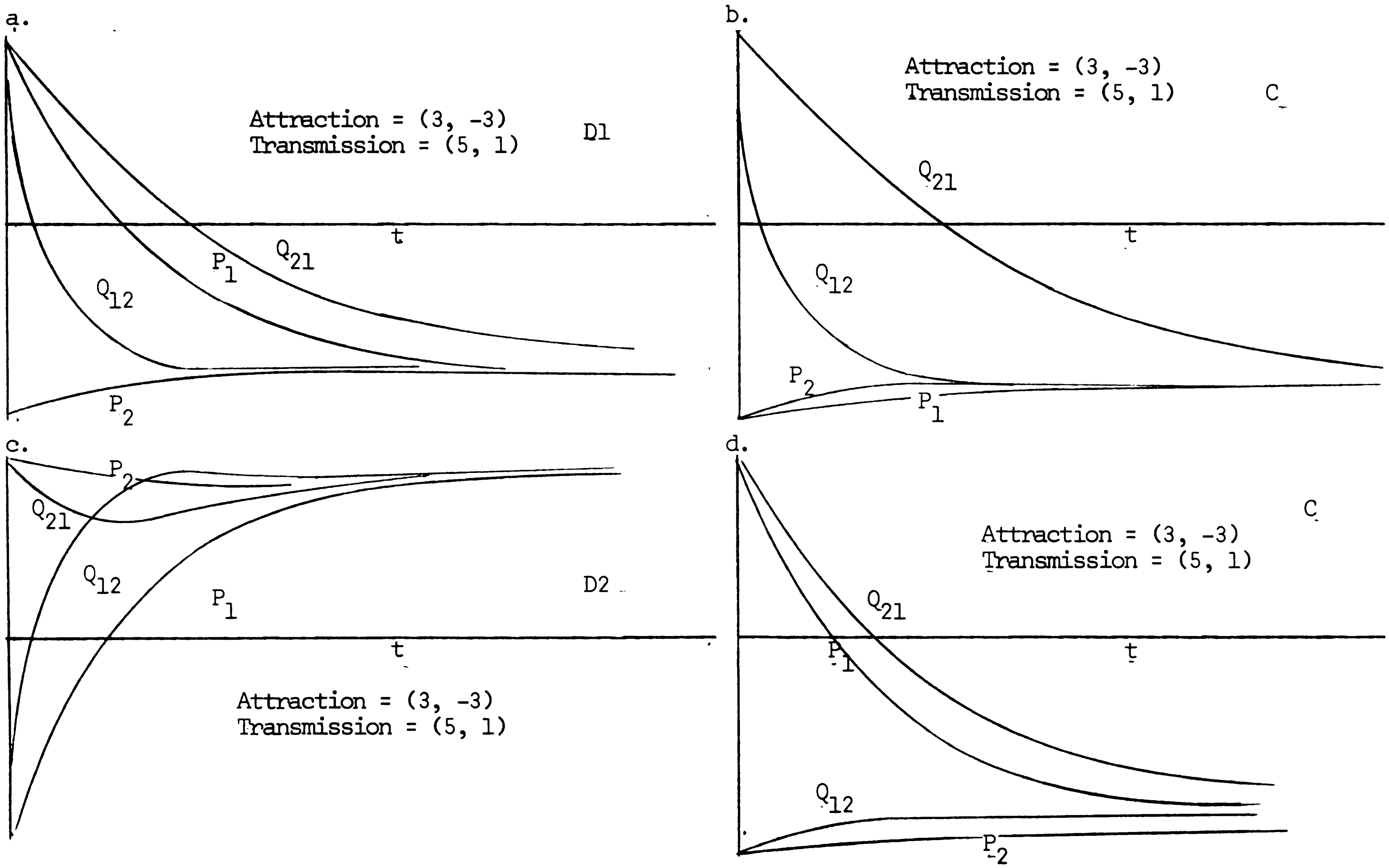


Figure 15. Trajectories for IP with Message Shift and Constant Attraction: Unequal Attraction and Unequal Transmission-Consistent with Transmission Model.

equal weighting for I's and J's initial values. When I out-transmits J by a ratio of 5 to 1 there will be somewhat more shifting toward I's initial values than if J out-transmits I by the same ratio. This is exactly what we see by comparing the two figures 14 and 15 frame by frame. For example, in frame a the point of convergence is somewhat less negative for Figure 15 than 14. There is also somewhat less change in P_i and Q_{ij} in Figure 15 than 14 and somewhat more change in P_j and Q_{ji}. Both of these characteristics are due to I's transmission being greater than J's in Figure 15 while the reverse is true in Figure 14.

Graphical representations of the over time behavior of a complex system such as that of equations (11) does not carry as much information about the overall pattern of trajectories as does a phase-plane graph (cf. Figure 10). However, when there are more than two variables to be considered, then a 3-, 4-, . . . , N-dimensional phase <u>space</u> is required. While there is no conceptual or mathematical limit to the dimensionality of a phase space, its heuristic value in portraying the set of possible trajectories for even three variables is essentially negligible. Thus, we are restricted primarily to representing only over time trajectories. The disadvantage of using the time trajectories is that the trajectories hold for only a single set of initial values. This is the reason that four different initial values were graphed in the Figures 11 through 15. In the IP case with constant attraction, the choice of initial conditions is not crucial since, as we show in Appendix A, all initial values ultimately converge to one of the critical points, $P_i = Q_{ij} = P_j = Q_{ji}$. In other words, the convergence or non-convergence of the state variables is independent of the initial conditions. Clearly, this does not mean that the initial conditions are not important since they do determine the point of convergence (along with the parameters) for the system with constant attraction.

Before considering some interesting special cases where convergence of the state variables to a common limit is not observed, a few remarks on the general character and properties of equations (11a) through (11d) will be made. As is shown in Appendix A, the equations (11) are a system of four linear differential equations with constant coefficients. It may also be written:

$$\frac{d\,\overline{S}}{dt} = \underline{W}\,\overline{S}\,, \qquad (12)$$

where $\overline{S}$ is a 4 x 1 vector of the state variables and $\underline{W}$ is a 4 x 4 matrix of coefficients. The interesting and useful result from linear systems theory is that the critical points of the system and their stability depend upon the matrix W. As is discussed in Appendix A, systems of equations like those of (11a) through (11d) yield matrices which allow one to conclude immediately that each solution converges to some critical point, and that there exist an infinity of critical points such that $P_i = Q_{ij} = P_j = Q_{ji}$. The interesting general observation is that <u>linear models built upon discrepancy assumptions often will have the characteristics necessary to conclude convergence to one of the critical points of the system</u>. Because linear discrepancy models have played an important role in social psychological theory, it is exciting to discover this linkage to mathematical conditions for stability.

Returning to the IP model of equations (11a) through (11d), thus far we know that for any fixed finite values for attraction, A_{ij} and A_{ji}, and for non-zero transmission, N_{ij} and N_{ji}, both individual systems and the collective system will converge over time toward perceived agreement and actual agreement. That is, one of the infinite set of critical points will be reached. Even for highly negative attractions the above result holds. Of course, the more negative the attraction, the slower the change toward convergence. Convergence is also guaranteed even for tiny transmission rates although the change is slower. Thus, the convergence point is the point where both individuals perceive no discrepancy between their own position and that of the other and their perceptions of the other are accurate.

If we do not insist that attraction be a finite parameter or that transmission be non-zero, then two interesting cases develop. If J likes I so that $A_{ji} > 0$ and $k_{ji} > 0$, and if I hates J so that A_{ij} is approaching negative infinity and k_{ij} is zero in the limit, then $d\,P_i/dt = 0$ and $d\,Q_{ij}/dt = 0$. This only leaves P_j and Q_{ji} as changeable. As we show in section 2 of Appendix A, the critical value for P_j and Q_{ji} is just the message which I initially sends and does not deviate from. That is, I does not change at all but J changes toward the position that I advocates. This means that in J's individual system there is perceived agreement and that in I's individual system there may or may not be perceived discrepancy depending upon the initial values of P_i and Q_{ij}. In the collective system there would be actual agreement. The reason is that since I hates J the message being sent by J is not shifted at all in the direction of J and, hence, is just I's constant attitude.

On the other hand if N_{ij} is zero while N_{ji} is non-zero, then J receives no messages from I and $dP_j/dt = 0$ and $dQ_{ji}\,/dt$. Only person I changes. Now if both individuals are moderately attracted to one another, then (see Appendix A), the critical point for I is just J's constant message. This message is shifted part of the distance toward $Q_{ji}(0)$. Since both P_i and Q_{ij} converge to M_{ji}, I's individual system ultimately has no perceived discrepancy. But there is actual discrepancy in the collective system: P_i will eventually be M_{ji}, not P_j. Since P_j remains constant at $P_j(0)$, the limit of P_i will always differ from P_j by the shift in J's message toward his initial perception of I. Thus, the absence of communication from I leads to a stable critical point which is also a point of collective system discrepancy in attitudes. Such a result could never have been anticipated by any of the balance theorists because the process of interaction is never explicitly included in their models. The reason for the discrepancy in attitudes in this case is twofold: (1) there is no communication from I thus leading J to retain his inaccurcies about I, and (2) the message content is shifted in the direction of the other. Were there no shift at all (that is if the veridical message model was operative), then $M_{ji} = P_j(0)$ and eventually $P_i(t) = P_j(0)$. That is, had J's message been veridical, then ultimately there would have been no discrepancy in attitude at the collective level.

To summarize, the IP model with constant attraction was shown to have an infinity of critical points satisfying $P_i = P_j = Q_{ji} = Q_{ji}$ when the parameter of attraction is finite and that of transmission is finite and non-zero. Furthermore, it was shown graphically and through some powerful theorems in Appendix A that the system always converges to a point of equality. That is, regardless of the initial values of the state variables, they will always converge toward a critical point of no discrepancy at either the individual level or the collective level. We also saw graphically that the role of attraction and transmission was to speed up or slow down the rates of convergence depending upon whether they were larger or smaller values. Finally, in taking up the special case of no transmission by one of the focal individuals, we saw a dramatic demonstration of the differences between collective system discrepancies and individual system discrepancies when the role of communication is explicitly included.

IP with Veridical Messages: Constant Attraction and Transmission

In the veridical message case, equations (11a) through (11d) still describe the behavior of the IJX system except that each person speaks his mind, or $M_{ij} = P_i$ and $M_{ji} = P_j$. In this way, P_i and Q_{ij} are "chasing" P_j and P_j and Q_{ji} are chasing P_i, and we might expect, as before, that the system will converge to the point of equality $P_i = P_j = Q_{ij} = Q_{ji}$. This is exactly the case (see Appendix B). Furthermore, the point of convergence is also shown to be a weighted sum of I's and J's initial messages which in this case means their attitudes. That is, the common limiting value $P_i^* = P_j^* = Q_{ij}^* = Q_{ji}^*$ is entirely independent of their perceptions of one another. The weights given each initial attitude in the equation for the limiting values are $k_{ij}N_{ji}$ for $P_j(0)$ and $k_{ji}N_{ij}$ for $P_i(0)$. That is, when the product of the credibility factor and transmission factor is greater for I than for J, $k_{ij}N_{ji} > k_{ji}N_{ij}$, then the final state is closer to $P_j(0)$ and I is doing most of the changing. When $k_{ij}N_{ji}$ and $k_{ji}N_{ij}$ are equal, then the final state merely "splits the difference" between I's and J's initial attitudes. Notice that the veridical message model differs from the shift model in that the final state does not depend upon Q_{ij} and Q_{ji} at all! The perception of the other is extraneous to the final state of the system. In fact the attitudes do not depend upon the perceptions at all. The equations for P_i and P_j in the veridical message model are

$$\frac{dP_i}{dt} = a\, k_{ij} N_{ji} (P_j - P_i)$$

$$\frac{dP_j}{dt} = a\, k_{ji} N_{ij} (P_i - P_j) \ .$$

And if the transmission rates are assumed constant, then Q_{ij} and Q_{ji} appear nowhere in the quations for P_i and P_j.

We should also note, that the IP shift and veridical models also undergo no change if N_{ij} and N_{ji} are zero, or if in the limit A_{ij} and A_{ji} go to negative infinity. If we are to be consistent with our model of transmission, then N_{ij} and N_{ji} should be zero only when $Q_{ij} - P_i = 0$ and $Q_{ji} - P_j = 0$ or when both individuals perceive no disagreement. Recall that transmission is <u>not</u> terminated by A_{ij} and A_{ji} going to negative infinity. But if they hate each other so much that $A_{ij} = A_{ji} = -\infty$, then both I and J view the other as infinitely incredible and stop changing in his direction. That is, if $A_{ij} = A_{ji} = -\infty$, there is no change in the system.

Like the shift model, the veridical model converges to an equilibrium point with all equal values for finite attraction and non-zero transmission. Unlike the shift model, the veridical model converges to a final state which depends only upon I's and J's initial attitudes and not upon a weighted sum of I's and J's initial attitudes and perceptions.

IP with Constant Attraction and Variable Transmission: Veridical and Shift Messages

General Discussion - In the next several sections we will consider the information processing models in which transmission is allowed to vary. In each case we use the basic equations

$$\frac{dP_i}{dt} = a\, k_{ij}\, N_{ji}\, (M_{ji} - P_i)$$

$$\frac{dQ_{ij}}{dt} = b\, k_{ij}\, N_{ji}\, (M_{ji} - Q_{ij})$$

in combination with the variable transmission equation

$$N_{ij} = \frac{\left| Q_{ij} - P_i \right|}{\sqrt{1 + (Q_{ij} - P_i)^2}}\, (1 + k_{ij})$$

with similar equations for P_j, Q_{ji}, and N_{ji}. Two distinct models of this type are defined by the two models of message generation.

In discussing the models in which transmission was assumed to be fixed, we noted that in each case the basic results fell into one of three categories: First there was the "typical" case in which neither attraction was negative infinity and in which neither transmission rate was taken to be 0. Second, there was the special case obtained if one or the other attraction was allowed to be negative infinity (and this will still be a special case below). Third, there was the special case in which one or the other of the transmission rates was taken to be zero.

In the "typical" case, both people kept transmitting messages to each other until all four system variables were driven to a common limit of

$$P_i^* = Q^*_{ij} = P_j^* = Q^*_{ji}\ .$$

The brunt of the analysis of each model then consisted of the determination of the relationship between the limiting value of the system and the four initial values.

In the "special" case in which one or the other of the transmission rates was assumed zero, the variables did not converge to a common limit. If for example N_{ij} were 0, then person I never transmitted a message to person J and hence person J never changed. Thus person J had his attitude and his perception fixed at whatever value they were to begin with. And since person J had a fixed attitude and perception of person I, he always sent the same message to person I. Person I then responded to these messages by having both of his values converge to that constant message value transmitted by J.

For the variable transmission models to be considered below, it turns out that the "typical" case is the case in which one or the other of the transmission rates is 0. To see this, we need merely look at the implications of the typical case for fixed transmission. If all four state variables were converging to the common asymptotic value P*, then in particular P_i and Q_{ij} would be converging to the same value P*. But if P_i and Q_{ij} are converging to the same value, then they are converging toward each other. That is, the discrepancy between P_i and Q_{ij} must be going to zero. But, if the discrepancy between P_i and Q_{ij} reaches 0, then so too does N_{ij}. That is, since $N_{ij} = 0$ whenever $|P_i - Q_{ij}| = 0$, the assumption of the typical case for fixed transmission implies the special case in which one or the other of the transmission rates becomes 0.

The critical question for the variable transmission models is this: Can one of the discrepancies $P_i - Q_{ij}$ or $P_j - Q_{ji}$ reach 0 before the other one does? If so, then the one that reaches 0 causes a cessation of messages to the other person and hence brings the other's discrepancy to a halt before it reaches 0. The answer to that question is: Yes. In most cases, one discrepancy will reach 0 first. To establish the plausibility of this fact we will first consider the two models under the admittedly unlikely assumption that the parameters a and b are equal. In this case, the mathematical results are simple and stark. After that we will give the only slightly more complicated conditions required to show that our claim is true if a and b are not equal.

<u>The Case of Equal Parameters: a = b</u> - Let us assign variable names to the two attitude-perception discrepancies. Let $x = P_i - Q_{ij}$ and let $y = P_j - Q_{ji}$. Then in Appendix A we show that regardless of the message generation model, we have the following differential equations for x and y wherever a = b:

$$\frac{dx}{dt} = -e \frac{|y|}{\sqrt{1 + y^2}} \quad x$$

$$\frac{dy}{dt} = f \frac{|x|}{\sqrt{1 + x^2}} \quad y$$

where e and f are complicated symmetric functions of the two constant attractions A_{ij} and A_{ji}. That is, if the parameters a and b are equal, then the two variables x and y obey a bivariate pair of differential equations that are each functions <u>only</u> of x and y. Thus x and y can be related to one another in a two dimensional phase plane. Two such phase planes are shown in Figure 16.

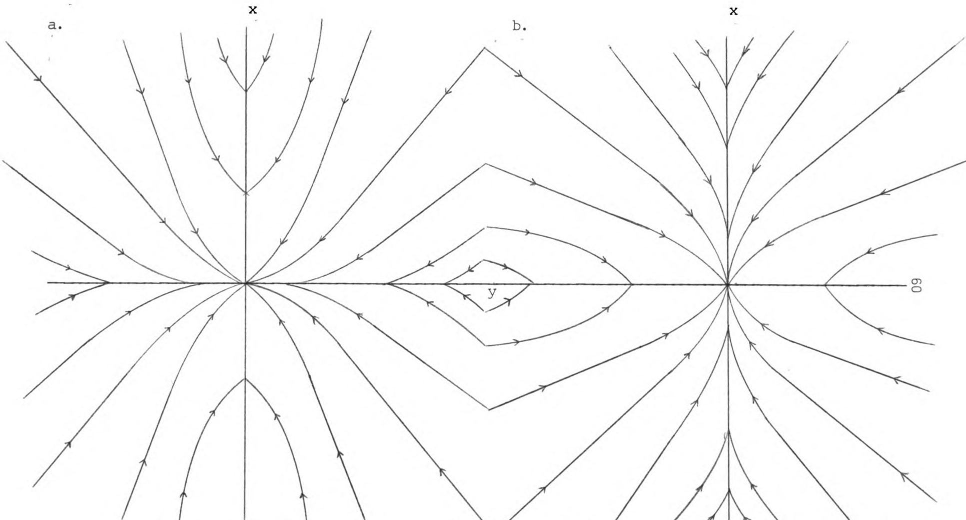


Figure 16. Phase Plane Plots of I's versus J's Perceived Discrepancy for IP with Message Shift and Varying Transmission: Attraction = (0, 1) for (a) and Attraction = (1, 0) for (b).

The important point about the phase planes in Figure 16 is that every trajectory converges to a point on either the x-axis or on the y-axis. Thus every trajectory converges to a point for which $y = 0$ or converges to a point for which $x = 0$. Thus it is always the case that at least one of the discrepancies converges to 0.

Where do we look in this phase plane for the possible common limit $P_i^* = Q_{ij}^* = P_j^* = Q_{ji}^*$? If all four were equal, then in particular $P_i = Q_{ij}$ or $x = 0$ and in particular $P_j = Q_{ji}$ or $y = 0$. Thus the case of the common limit is represented by the point $x = y = 0$ or the origin. That is, the only trajectories which represent all four system variables converging to a common limit are the trajectories in Figure 16 which converge to the origin. And that is clearly a very uncommon special case.

Thus if the parameters a and b are equal, we have proved that in all but unlikely special cases, either the discrepancy $P_i - Q_{ij}$ hits zero before the discrepancy for person J, or else the discrepancy $P_j - Q_{ji}$ hits 0 before the discrepancy for person I.

<u>The Case of Unequal Parameters</u> - If the parameters a and b are not equal, then the preceding development breaks down. Basically the problem is this: If $P_i - Q_{ij} = 0$, then at that moment in time $N_{ij} = 0$ and person J is not changing. However, the fact that $P_i - Q_{ij} = 0$ at that point in time need <u>not</u> imply that $P_i - Q_{ij}$ will <u>stay</u> equal to 0. This problem is illustrated by the trajectory in Figure 17.

The trajectory in Figure 17 starts out with $P_i = Q_{ij}$ and indeed at time 0, $N_{ij} = 0$. However since Q_{ij} changes three times as fast as does P_i, Q_{ij} decreases more rapidly toward M_{ji} than does P_i. Thus a

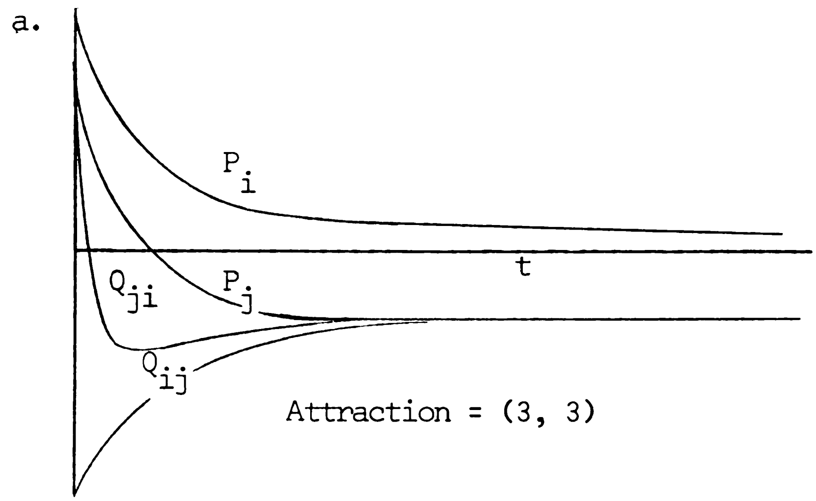


Figure 17. IP with Constant Attraction and Varying Transmission: Unequal Parameters.

gap opens between P_i and Q_{ij} and hence N_{ij} ceases to be zero.

Mathematically the corresponding problem in the case that $a \neq b$ is the fact that the differential equations for the discrepancies x and y are not functions of only x and y, but are functions of the other system variables as well. Thus if $a \neq b$, then there is no two dimensional phase plane for the two discrepancies.

If it is to be the case that one discrepancy hits 0 before the other does, then what is the additional condition that must be met beyond the condition $P_i - Q_{ij} = 0$? The problem is that although $P_i = Q_{ij}$ for one instant in time, they may not <u>stay</u> equal. In order that P_i stay equal to Q_{ij} it is necessary that in addition to equality of the variables we must have equality of the <u>derivatives</u>. If $P_i = Q_{ij}$, then we have

$$\frac{dP_i}{dt} = \frac{dQ_{ij}}{dt}$$

only if

$$a(M_{ji} - P_i) = b(M_{ji} - Q_{ij}) = b(M_{ji} - P_i) .$$

That is, only if

$$M_{ji} - P_i = M_{ji} - Q_{ij} = 0 .$$

Thus we are lead to consider the condition $P_i = Q_{ij} = M_{ji}$. And it requires only minimal checking to establish the fact that this determines a critical point.

How different is the condition $P_i = Q_{ij} = M_{ji}$ from the condition $P_i = Q_{ij}$? Not as different as might appear. If the parameters a = b, then $P_i = Q_{ij}$ implies that P_i stays equal to Q_{ij}. But that doesn't mean that either P_i or Q_{ij} remains constant. In point of fact it turns out that once $P_i = Q_{ij}$, then P_i and Q_{ij} converge together to M_{ji}. So even in the case of equal parameters, the critical point is obtained at $P_i = Q_{ij} = M_{ji}$, its just that the trajectory reaches $P_i = Q_{ij}$ first.

<u>The Unlikelihood of a Common Limit for the Unequal Parameters Case</u> - Thusfar, we have found out that the entire system stops changing when $P_i = Q_{ij} = M_{ji}$ for a not equal to b. In other words, $P_i = Q_{ij} = M_{ji}$ are a set of equilibria for the IP equations with constant attraction. However, these are not the only set of equilibria. If at the same time that P_i and Q_{ij} were converging to M_{ji}, P_j and Q_{ji} were converging to M_{ij}, then a possible outcome would be $P_i = Q_{ij} = P_j = Q_{ji}$. In this case, <u>both</u> I's perceived discrepancy and J's perceived discrepancy would be zero and $N_{ij} = N_{ji} = 0$. The system would stop changing because both I and J would stop transmitting. Also I and J would have reached agreement ($P_i = P_j$) and both would accurately perceive this agreement. This

situation constitutes a second set of equilibria for this model. What we wish to do for the remainder of this section is to make some educated guesses as to which equilibria is most likely to be reached: $P_i = Q_{ij} = M_{ji}$ with P_j and Q_{ji} arbitrary or $P_i = Q_{ij} = P_j = Q_{ji}$? Once again our remarks will be applicable to both the veridical and shift models although the trajectories to be presented are those of the shift model.

Figures 18 and 19 show the only trajectories which converge to a common limit. In each case, the initial values for J are all perfectly symmetric (in either a positive or negative sense, see Appendix B) to the initial values for I. Moreover, in each case both attraction values and transmission rays are equal. The least deviation from all of these highly unlikely conditions produces a trajectory for which one discrepancy hits 0 before the other one does. That is, the <u>set</u> of equilibria $P_i^* = Q_{ij}^* = P_j^* = Q_{ji}^*$ is <u>unstable</u>.

Figure 20 shows two trajectories for which attraction between I and J is equal but I's initial transmission is greater than J's. Although the same value of attraction is set for both I and J the system does not converge to a point of equality for all four state variables. Observe that the initial value of J's perceived discrepancy is 0 while I's is 2. Thus, I initially out-transmits J by .91 to 0. Of course, J's perceived discrepancy does not remain at zero since a message from I changes Q_{ji} more than P_j. But the damage has been done. That is, with I out-transmitting J initially, J converges to I's message considerably faster than I converges to J's message. The result is that $P_j = Q_{ji} = M_{ij}$ <u>before</u> P_i and Q_{ij} reach the same point. The transmission from J is shut off so that I stops changing. But I continues to transmit

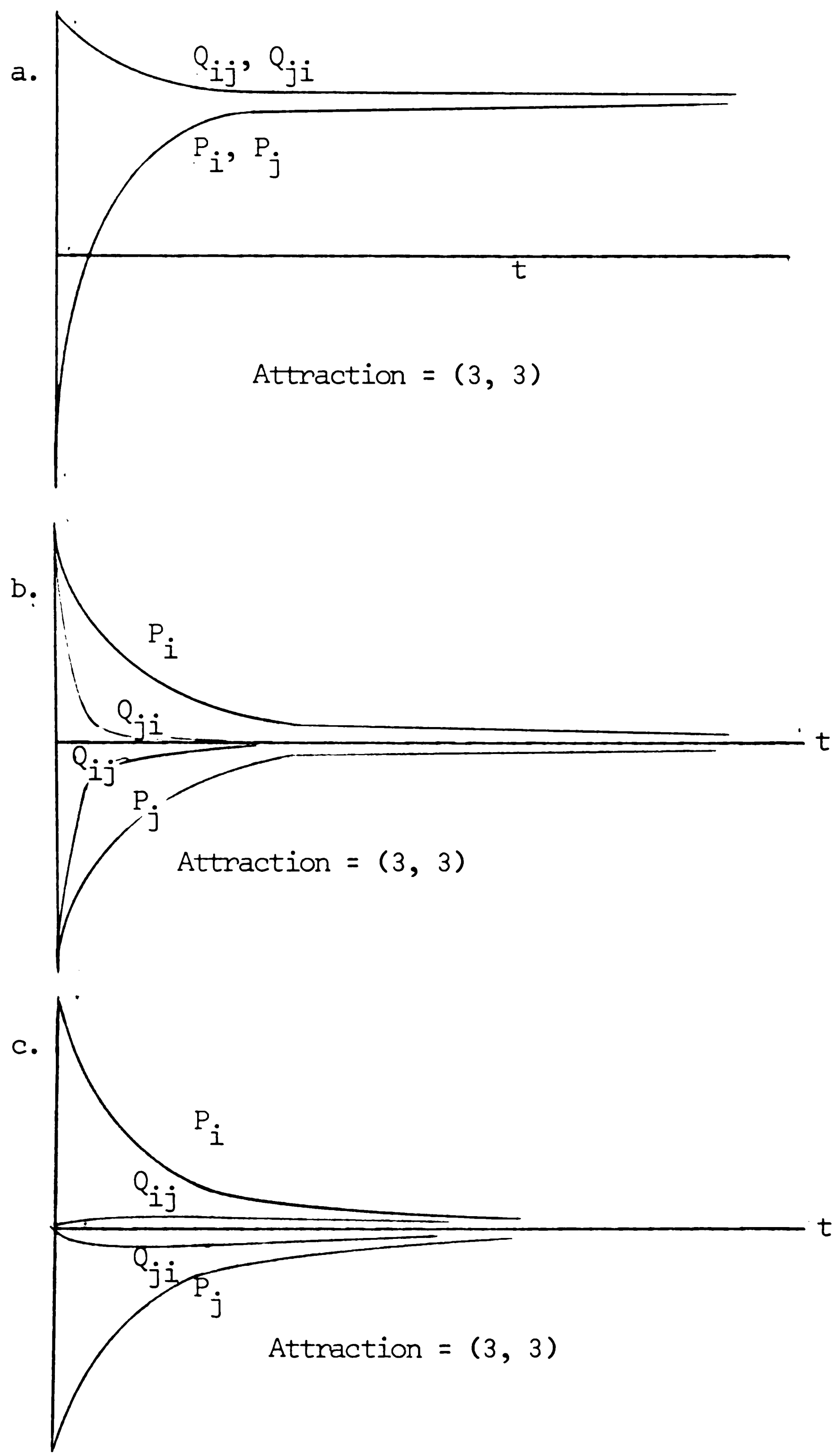


Figure 18. IP with Shift Message and Constant Attraction: Transmission Variable.

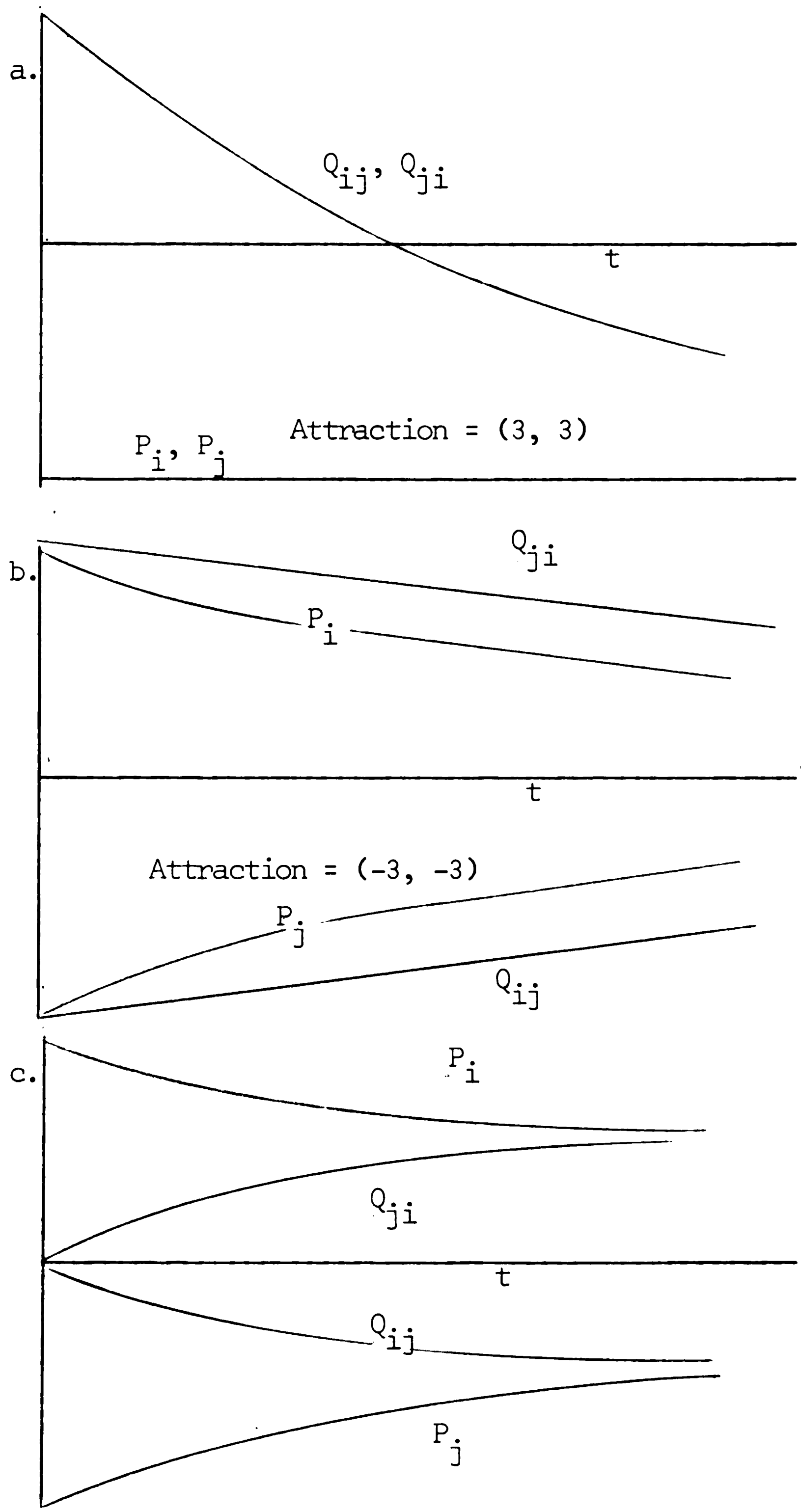


Figure 19. IP with Shift Message and Constant Attraction: Transmission Variable.

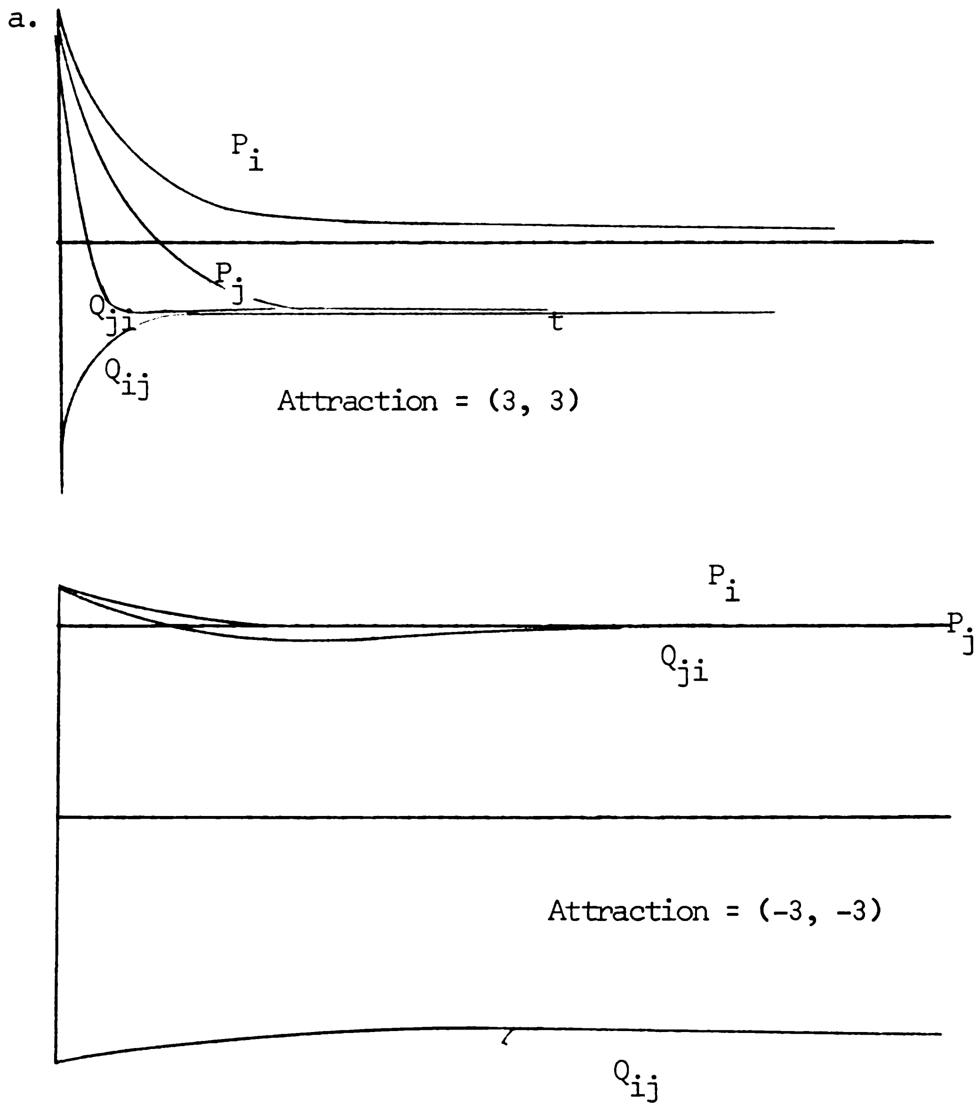


Figure 20. IP with Message Shift: Varying Transmission and Constant Attraction.

messages to J which are identical with J's attitudes and perceptions. Thus, even the slight asymmetry in the system due to differences in initial perceived agreement produce an equilibrium other than that of complete equality.

Figures 21 and 22 present trajectories which are symmetric in their initial conditions (that is, $|P_i - Q_{ij}| = |P_j - Q_{ji}|$, $|M_{ji} - P_i| = |M_{ij} - P_j|$, and $|M_{ji} - Q_{ij}| = |M_{ij} - Q_{ji}|$) but are asymmetric in attraction. In Figure 21 I is much more attracted to J than J is to I while in Figure 22, the reverse attraction pattern is present. In Figure 21, the system converges to $P_i = Q_{ij} = M_{ji}$ with P_j and Q_{ji} constant and in Figure 22 the system converges to $P_j = Q_{ji} = M_{ij}$ with P_i and Q_{ij} constant. This difference in which of the two individuals stops transmitting is due to the reversal in attraction patterns between the two figures. Let us focus upon Figure 21. Since I likes J much more than J likes I, then P_i and Q_{ij} will converge upon J's message very rapidly. As P_i and Q_{ij} both rush toward J's message, then they are also changing toward one another. Since N_{ij} depends upon the difference between P_i and Q_{ij} (which is getting smaller), then N_{ij} is getting smaller. As N_{ij} decreases then so does the rate of change of P_j and Q_{ji} since they depend directly upon how many messages are being received. Now, in the examples of Figures 21 and 22 we have deliberately chosen A_{ij} and A_{ji} very discrepant in order to represent in a dramatic way the failure of the system to converge to a common limit. However, there is no reason why much smaller asymmetries in attraction should not produce the same results, although the final differences among P_i, Q_{ij}, P_j, and Q_{ji} would be much smaller. Thus, even if the initial conditions on attitudes, perceptions, and messages <u>are</u> symmetrical, asymmetries in I's attraction to J and J's attraction to I will produce convergence to the more likely final state $P_i = Q_{ij} = M_{ji}$, $P_j = P_j^*$, $Q_{ji} = Q_{ji}^*$.

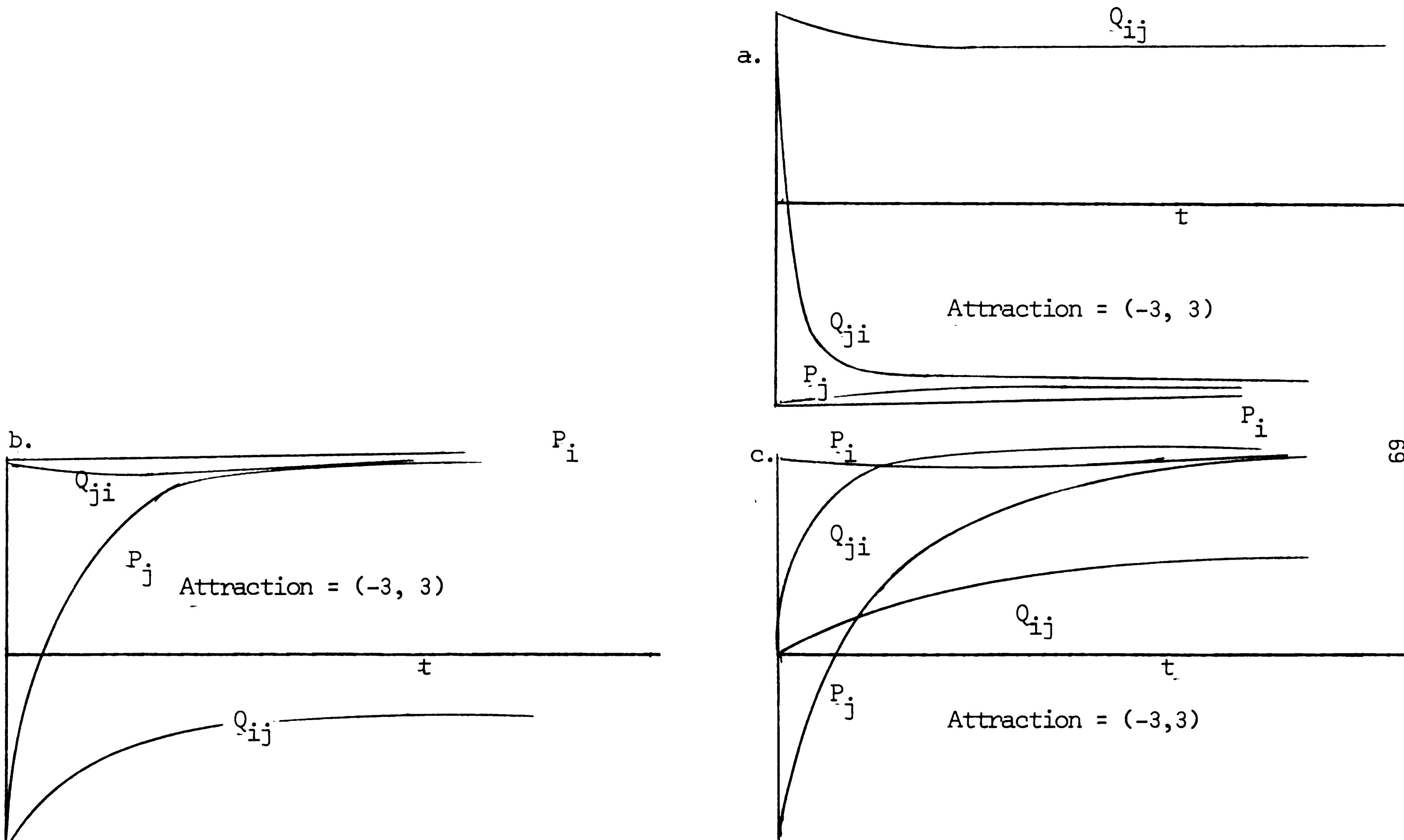


Figure 21. IP with Message Shift and Constant Attraction: Transmission Variable.

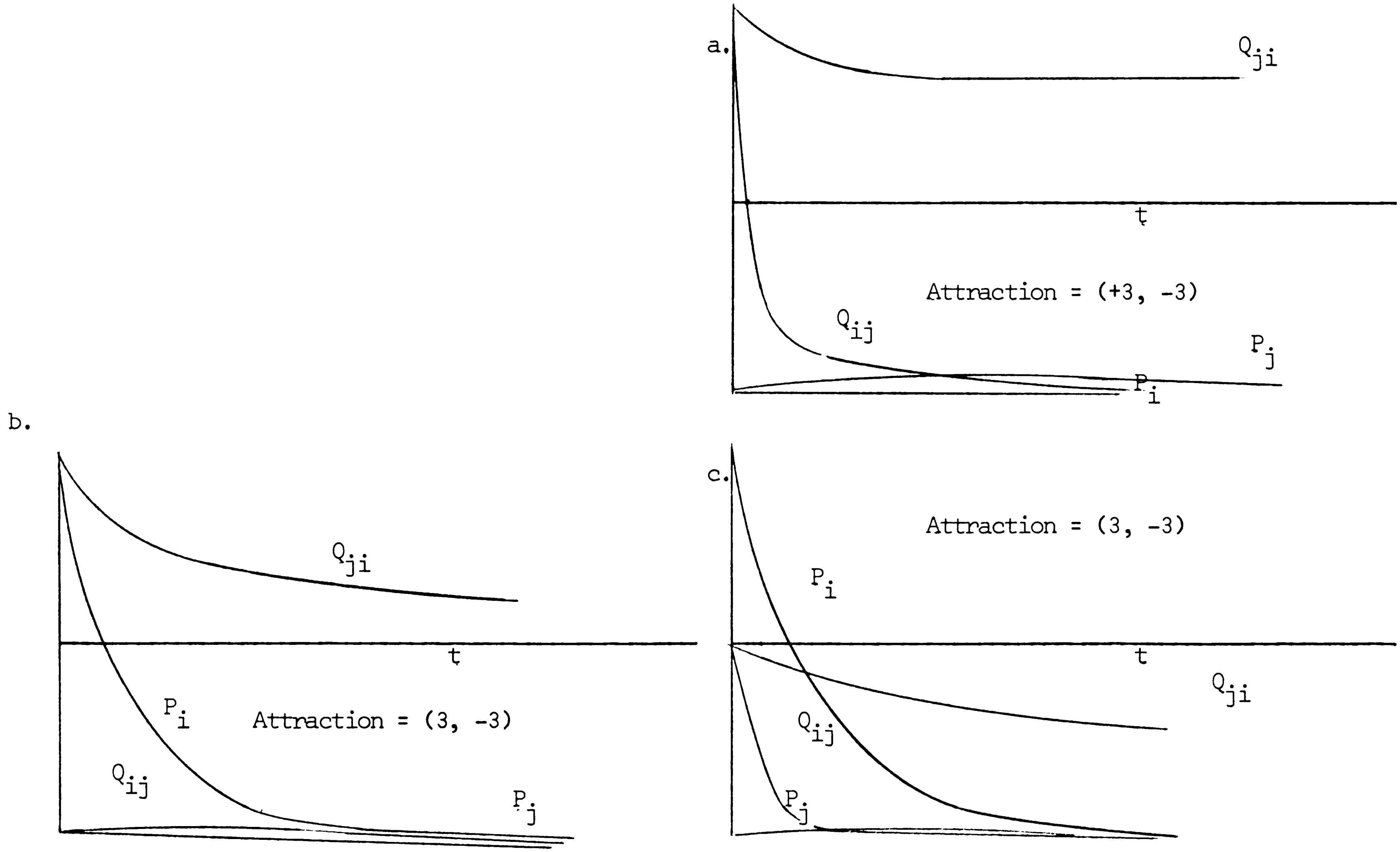


Figure 22. IP with Shift Message and Constant Attraction: Transmission Variable.

Finally, we consider the dual asymmetry in attraction and in initial transmission. Two example trajectories for this case are presented in Figure 23. Based upon our remarks above, we expect and find

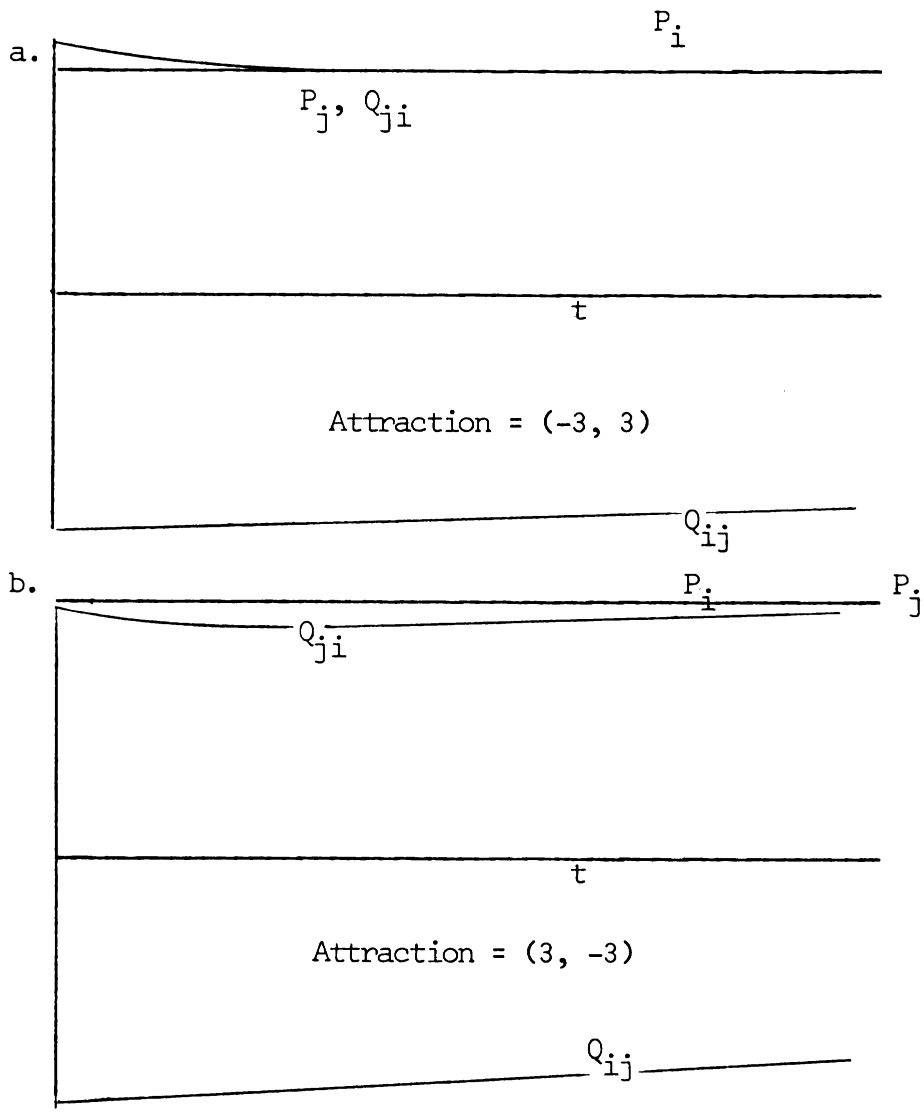


Figure 23. IP with Shift Messages: Constant Attraction and Variable Transmission.

that the system converges to $P_i = Q_{ij} = M_{ji}$ in frame a and $P_j = Q_{ji} = M_{ij}$ in frame b. That is, that there is convergence to the more likely equilibrium and failure to converge to the common limit.

What we find, then, for the IP model with message shift or with veridical messages is that when the initial conditions are symmetric the variable P_i, Q_{ij}, P_j, and Q_{ji} converge to a common limit $P_i = P_j = Q_{ij} = Q_{ji}$. If any of the symmetry conditions are violated, then it becomes possible for either P_i and Q_{ij} or P_j and Q_{ji} to converge to the other's message before the other converges to his message. In this case either the system will converge to $P_i = Q_{ij} = M_{ji}$ or to $P_j = Q_{ji} = M_{ij}$. If our analysis of the shift and veridical models for the case of variable transmission is correct, then convergence to a common limit is a very special and unstable result dependent upon some very unusual initial conditions. The more common result would be that one or the other of the persons stops transmitting while the other continues sending messages identical to his silent audience's attitudes and perceptions.

If trajectories for the veridical case had been presented, they would be qualitatively similar to those for the shift model. That is, the same conclusions about the final state of the system as a function of initial values could be supported. The chief difference between the two models would be found in the point at which the system reached equilibrium. Of course, this fact is traceable directly to the differences in message assumptions between the two models.

Information Processing with Varying Attraction

In this section, we consider the same set of models in the same order as the previous section but now permit attraction to vary both for the internal and external processes. Unfortunately, the powerful mathematical tools which were invoked in the previous section cannot be invoked with the IP models of varying attraction. First, all of the models in this section are nonlinear so that the mathematics of linear systems is precluded. Second, none of the models in this section have any critical points (except for the varying transmission case) thus precluding the usual techniques of analyzing the stability of critical points with linear approximations. As a result, we are forced to focus primarily upon the numerical solutions to the variable attraction models. On the positive side, we have a useful set of mathematical results to build upon from the previous section.

Internal Changes Only - The model for internal, spontaneous changes is summarized in the equations (7), (8), and (9) of Chapter II. The internal changes for I and J are independent of each other as we noted before. The key difference between the IP model with constant attraction of the previous section and the IP model with variable attraction is that attraction can now become infinite either in the positive or negative direction. As I's attraction to J becomes very positive, then (1) the changes of P_i and Q_{ij} in each other's direction becomes faster, and (2) the acceptance region for I becomes larger. As I's attraction to J becomes extremely negative, then (1) changes in P_i and Q_{ij} toward each slow down and eventually stop in the limit as attraction approaches negative infinity and (2) I's acceptance region approaches zero. Although there are no critical points in this model (or any of this section), we still can note that as attraction goes to positive infinity or as discrepancy goes to zero the experienced internal force would be zero. Although this is not a point in the sense that the number 6.34 is a point on the real line, it does indicate the direction that the IJX situation is moving and would be considered balanced by Newcomb and Heider.

That which determines whether the system will tend toward zero discrepancy with infinite attraction or finite discrepancy with negatively infinite attraction is where the individual system begins. First, suppose that I thinks that the discrepancy between himself and J is within his acceptance region. Then I's attraction will increase and his acceptance region will get wider. Hence, the discrepancy will be smaller and the acceptance region larger producing more positive increases in attraction and so forth. For the second case suppose that I thinks J is very discrepant so that he is <u>far</u> outside I's acceptance region. Then there will be little convergence of P_i and Q_{ij}, I will derogate J a great deal and I's acceptance region will shrink appreciably. Although the discrepancy will be slightly smaller, the acceptance region will be much smaller and the change in attraction will be very negative. This cycle will produce attraction going to negative infinity so fast that the discrepancy never reaches zero, but stops at a non-zero asymptote. Third, we consider the difficult case: when I perceives J to be only slightly outside the acceptance region. Now attraction will decrease a small amount and the convergence of P_i and Q_{ij} will be slowed. However, if P_i and Q_{ij} change enough to move back into the acceptance region, then attraction will increase again. The cycle toward positive infinity and zero discrepancy will have begun. If the convergence of P_i and Q_{ij} is too slow, then attraction will decrease again and the spiral toward negative attraction will have begun.

The intuitive arguments above can best be seen in the phase plane graph of attraction versus perceived discrepancy in Figure 24. The equations used to derive the integral curves are discussed in Appendix B. The arrows which indicate a flow to the right (toward positive attraction) represent the trajectories tending toward infinite, positive attraction and zero discrepancy. The arrows which indicate flows to the left represent trajectories tending toward infinite, negative attraction and non-zero discrepancies. The dotted line which separates trajectories flowing to the right from those flowing to the left is known as the separatrix. The separatrix is the dividing line between the cases whose initial values will yield a "right-flowing" trajectory from those whose initial values will yield a "left-flowing" trajectory. In other terms, the separatrix divides those individual systems with $A_{ij} = +\infty$, $P_i = Q_{ij}$ which are balanced and those with $A_{ij} = -\infty$, P_i, Q_{ij} arbitrary which are not balanced. In general, the separatrix must be numerically determined based upon the particular parameters of the equations being modeled.

To round out our discussion of internal changes, Figure 25 presents four over-time trajectories, two of which are balanced (a and b) and two which are not balanced (c and d). The four figures represent the same pair of initial conditions (discrepancy = 1 in all cases) but in the one case I is attracted to J initially and in the other I finds J unattractive enough initially so as to fail to converge back toward liking J.

Before taking up the external model, it will be useful to compare our results thus far with the results of the previous section. In the

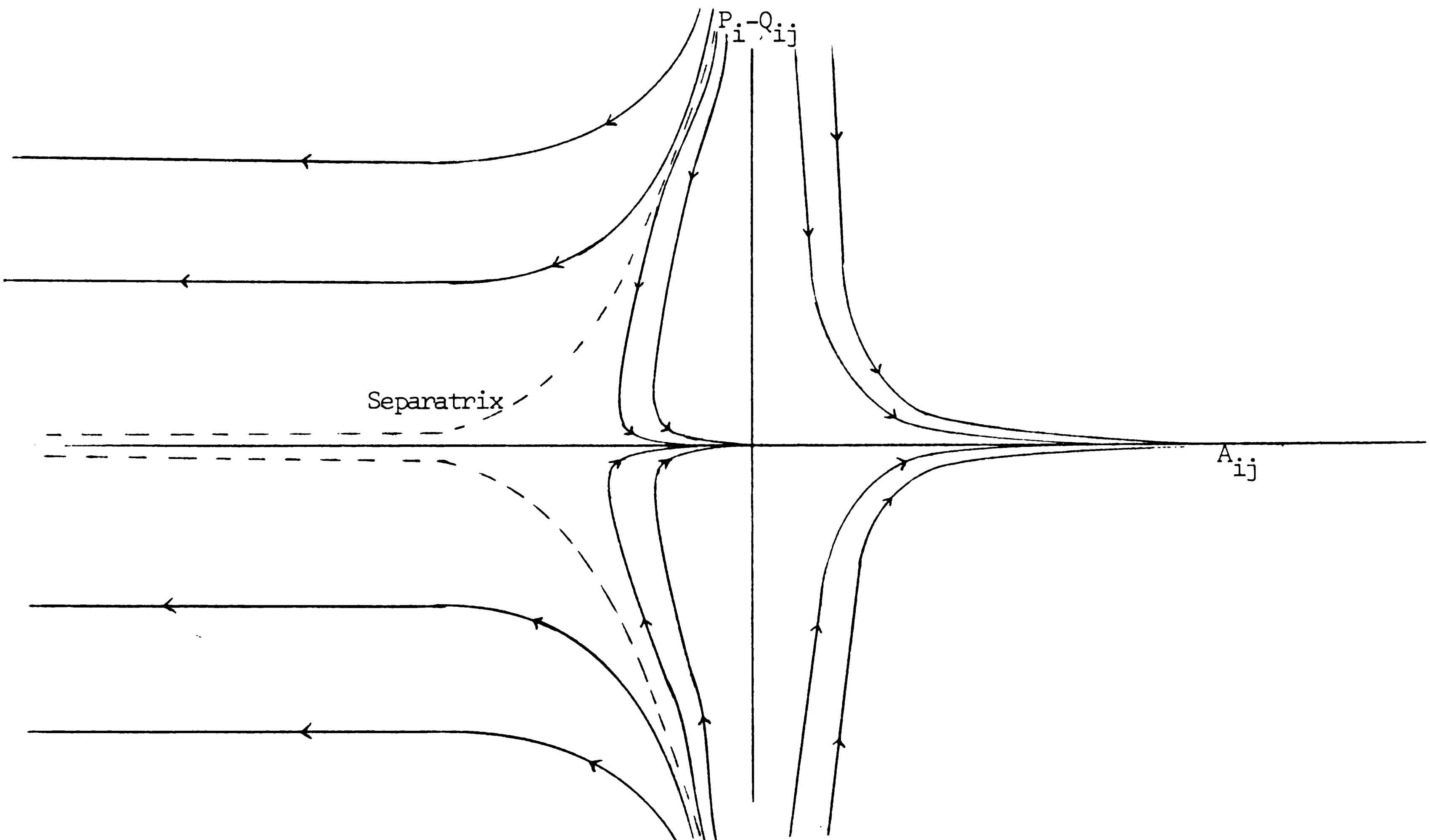


Figure 24. Phase Plane Trajectories for Internal Changes with Perceived Discrepancy versus Attraction.

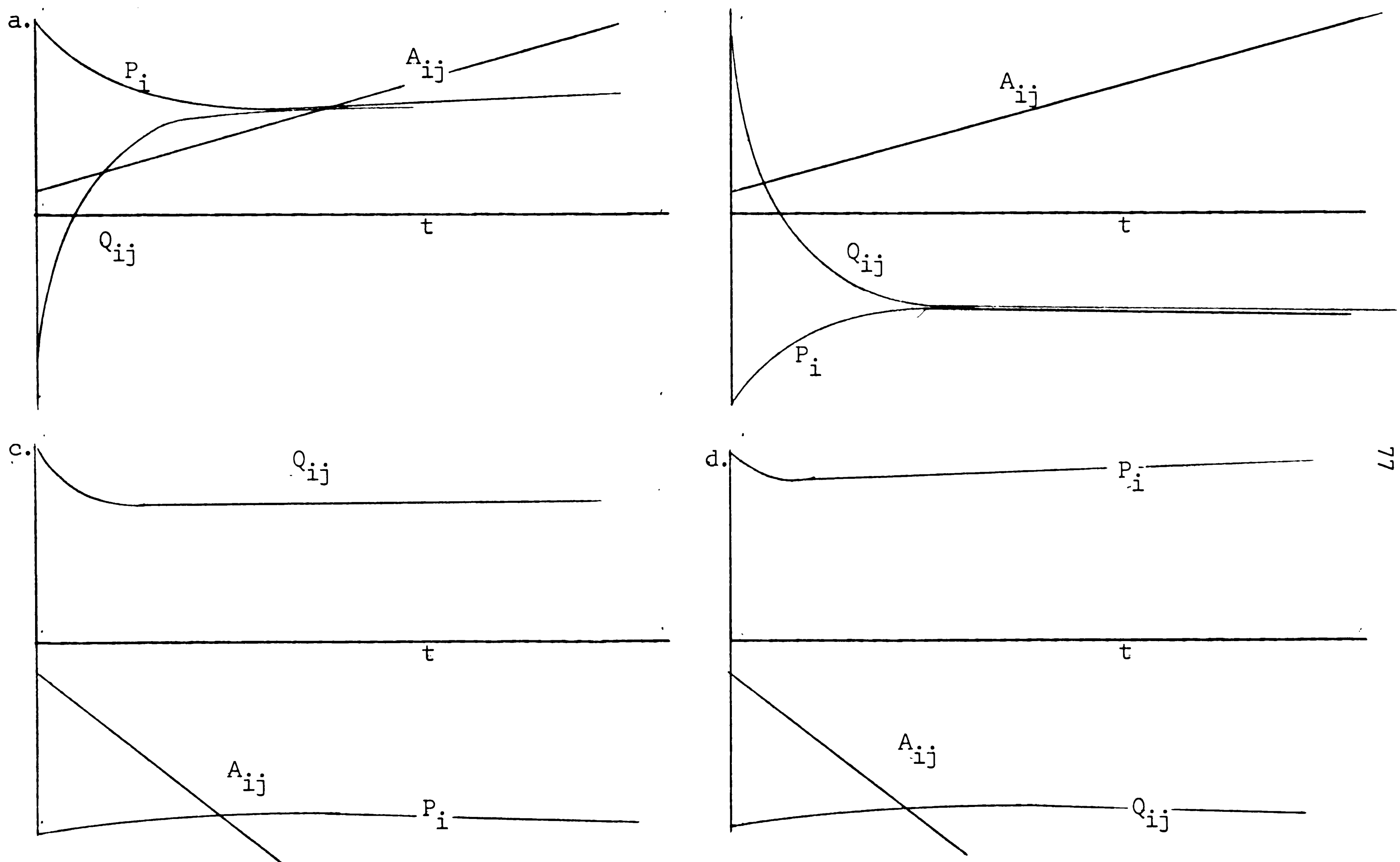


Figure 25. Time Trajectories for Internal Changes with IP and Varying Attraction: Initial Perceived Disagreement.

previous section, convergence toward zero discrepancy was always the case as long as attraction remained finite. This leads to zero discrepancy, negative attraction as well as zero discrepancy positive attraction final states. Such a result differs from the Heider approach to balance for the negative case. But when attraction is allowed to vary we find that positive infinite attraction accompanies zero discrepancy in support of positive balance and Heider's model. Also negative, infinite attraction accompanies finite discrepancy as Heider's view of IJX situations predicts.

IP with Message Shift and Variable Attraction: Transmission Constant

Equations (1), (2), and (3) and their counterparts for J constitute the mathematical system for IP with message shift and variable attraction. Based upon the results of the immediately preceding section we might expect that the behavior of the IJX system under equations (1) through (3) would depend upon the initial conditions for attraction and message values. That is, the direction which the system tends may depend upon whether the initially generated messages are within the other's latitude of acceptance or not. This is exactly what we shall conclude.

Let us discuss three cases: (1) both I's and J's initial messages within the other's acceptance region, (2) both I's and J's initial messages exterior to the other's acceptance region, and (3) I's initial message within J's acceptance region, but J's initial message outside I's acceptnace region. Since conclusions about I and J are symmetric, these above three cases cover all possible combinations of initial conditions. If I's initial message is within J's acceptance region and vice versa, then both I's attraction for J and J's attraction for I will increase. This mutual increase in attraction will simultaneously increase the width of both I's and J's acceptance regions and increase the rate of convergence of P_i, Q_{ij}, and P_j, Q_{ji}. Thus the "next round" of message interchanges will be even closer to the other person's attitude than the "first round" of message interchanges and will be within an even wider acceptance region. Hence, attraction will increase mutually once again and the cycle will continue. The result quite simply is that when both I and J extend messages which are initially within the latitude of acceptance of the other, then those IJX situations will produce infinite attraction for both individuals and zero discrepancy in the collective system and both individual systems. Figure 26 shows four numerically generated trajectories for the case where both I and J send initial messages within the other person's acceptance region. Notice that the attraction trajectories show positive changes across all values of time and that the variables P_i, P_j, Q_{ij} and Q_{ji} show a rapid convergence toward zero discrepancy.

If both I's and J's initial messages fall outside the other person's region of acceptance, then two situations need to be considered: (1) when the messages are both just outside the acceptance region of the other and (2) when the messages are both well outside the acceptance region. In the latter case, both I's attraction to J and J's to I will decrease by a large amount (the change in attraction recall is quadratic) which will shrink both regions of acceptance by a large amount (in fact at an exponential rate) and will slow down the convergence rate for P_i, Q_{ij} and P_j, Q_{ji}. With a large mutual decrease in attraction, the next

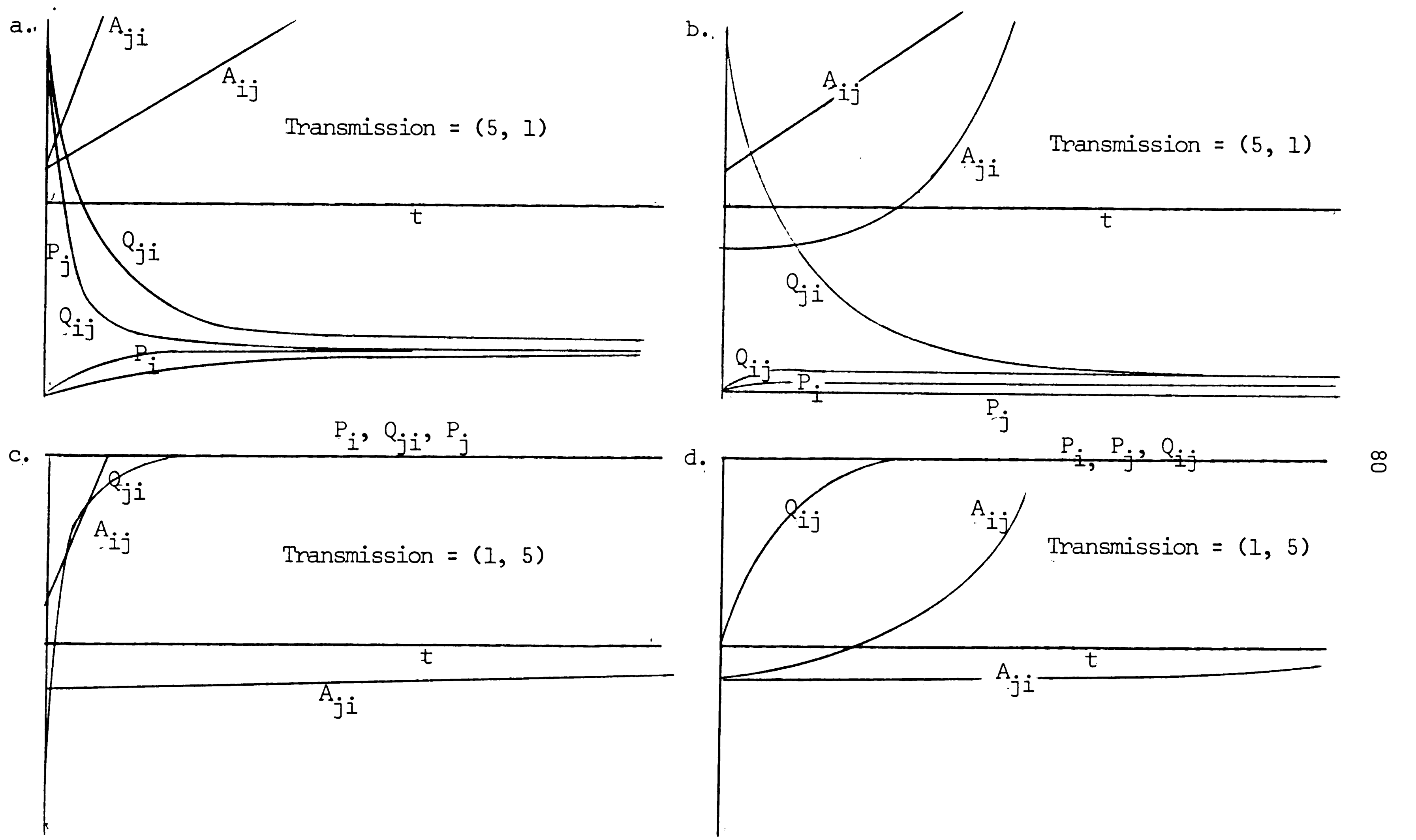


Figure 26. Time Trajectories for IP with Message Shift and Varying Attraction: Initial Messages both Inside the Other's Acceptance Region.

round of messages (which will not have changed much since P_i, Q_{ij}, P_j, and Q_{ji} will not have changed much) will be (relatively) even further outside the shrunken acceptance regions, thus producing greater decreases in attraction. This cycle will continue, giving rise to trajectories such as those in Figure 27. In both frames, the attraction goes toward negative infinity very rapidly and changes in P_i, Q_{ij}, P_j and Q_{ji} toward one another then cease. Note that it is only correct to say that changes in P_i, Q_{ij}, P_j and Q_{ji} crease in the limit as attraction becomes negatively infinite. Large negative (but finite) values of attraction may slow the process of change to a negligible level but only in the limit as attraction becomes _infinitely_ negative does convergence stop.

In the case where the initial messages generated by I and J are just outside each other's acceptance region, then attraction for both will decrease slightly. At the same time, both acceptance regions will shrink slightly and the convergence process will be slowed by a small amount. But convergence for P_i, Q_{ij}, P_j, and Q_{ji} is still taking place. If the rate of decrease in attraction is slow enough, then I's messages and J's messages can "catch" P_j and P_i respectively and produce changes in attraction which are positive. Once this change occurs for both I and J, then the spiral toward positive infinity and convergence of P_i, P_j, Q_{ij}, and Q_{ji} begins again. Figure 28 presents one of the few observed trajectories in which both messages were initially outside the other's acceptance region, and after slight initial decreases in attraction, the pattern of attraction change became positive. Substantively, the situation of Figure 28 represents the results of continuing interaction between two individuals whose differences are resolved by the a.

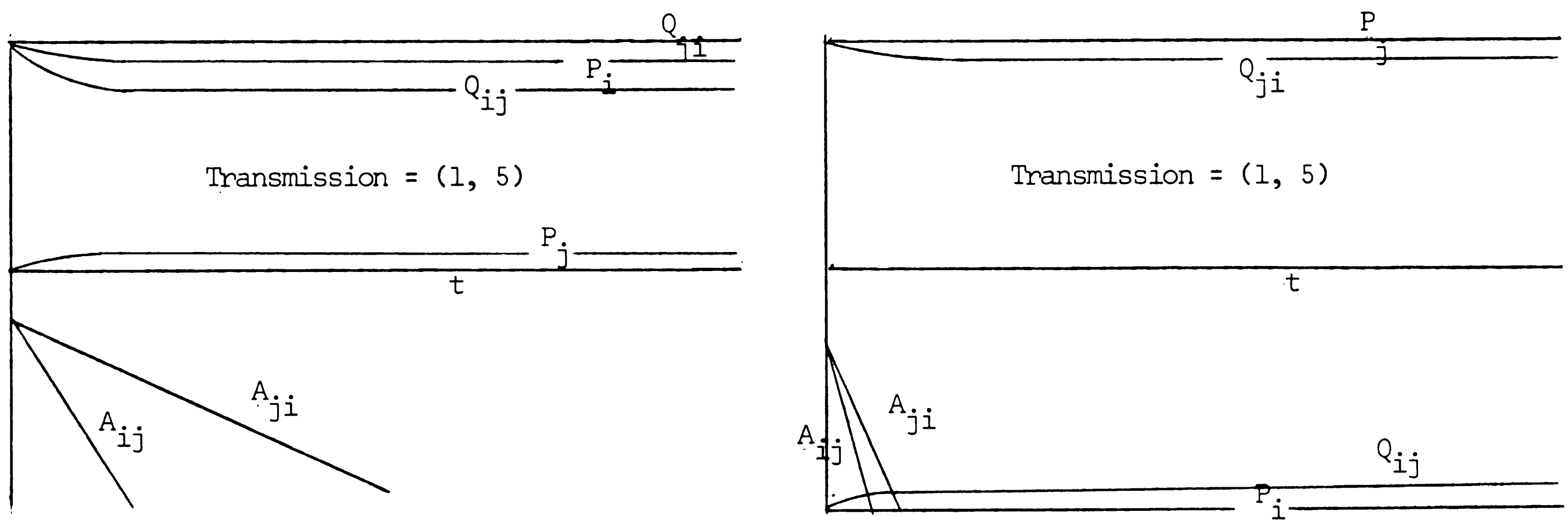


Figure 27. Time Trajectories for IP with Message Shift and Varying Attraction: Initial Messages well Outside Other's Acceptance Region.

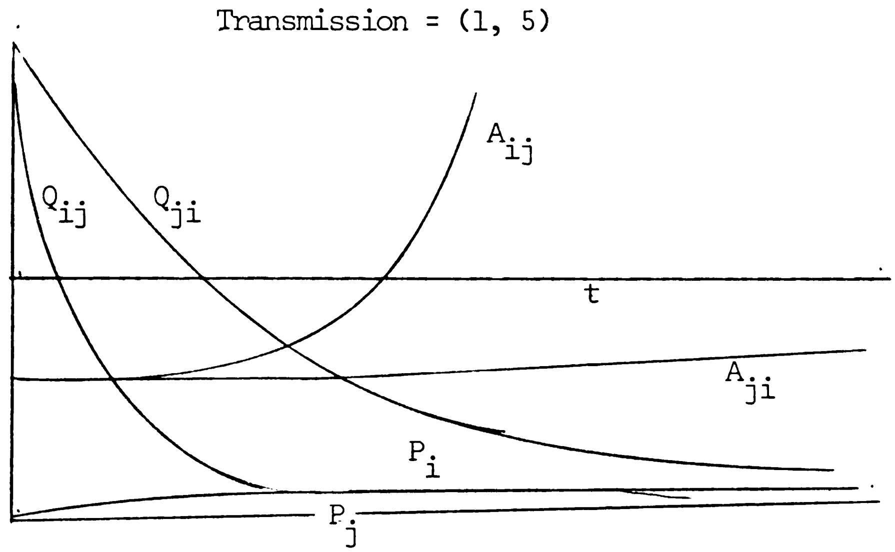


Figure 28. Time Trajectories for IP with Message Shift and Varying Attraction: Initial Messages both Outside the Other's Acceptance Region.

process of interaction.

Perhaps the most interesting set of results from the IP shift model with variable attraction is found for the case in which the initial message from I is outside J's acceptance region while the initial message from J is within I's acceptance region. With J's message inside I's acceptance region, I's attraction to J will increase and, hence, the width of the acceptance region will increase. More importantly, however, the rate of convergence of I's attitude and of I's perception of J toward J's message will increase. On the other hand, J's acceptance region will shrink and the rate of convergence of Q_{ji} and P_j toward J's message will decrease. The two key competing processes in this case are the shrinking of J's acceptance region and the convergence of I's attitude and perception of J toward J's message. As P_i and Q_{ij} converge toward J's message, then I's message will ultimately be identical with J's attitude. When this occurs $M_{ij} - P_j = 0$ and I's message will be within J's acceptance region even though J's acceptance region becomes very narrow. In other words, the ultimate result of an initial case where J's message is within I's acceptance region but I's message is outside J's region, is that I will change his position on X and his perception of J's position until they are coincident with J's actual position precisely because I likes J so much. When this occurs, J will begin to change his opinion of I and like him more and more. In a way this result represents the phenomenon of ingratiation on the part of I.

Figure 29 presents four of the approximately 30 trajectories for the ingratiation model that were generated. In each frame we see that almost all of the change in attitude and perception variables is done by the person whose attraction to the other is more positive. Most importantly, however, each frame shows that for the person receiving the "extreme" message, his attraction initially decreases but then reverses itself and changes in the positive direction. The most striking case of this is found in frame d where large initial decreases in attraction are followed by very small positive increments.

Despite the lack of the strong mathematical conclusions which characterized the IP model with constant attraction we still may offer some conclusions. First, whenever I's and J's initial messages are within the other's region of acceptance, or both are slightly outside the region, or when one message is within the other's region and one is

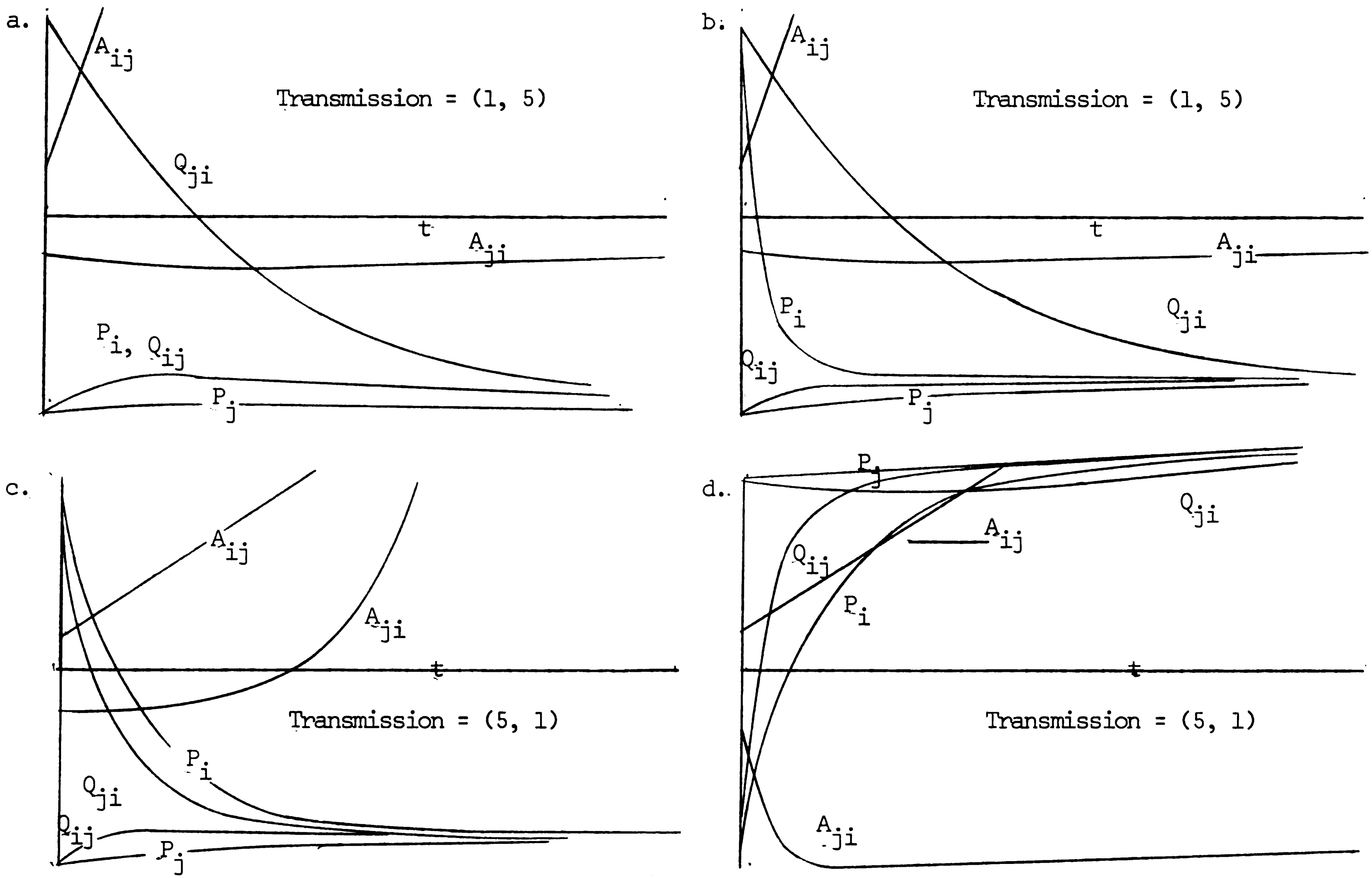


Figure 29. Time Trajectories for IP with Message Shift and Varying Attraction: Initial Message from J within I's Acceptance Region but from J Outside I's Acceptance Region.

outside, then the IJX situation will change toward increasing mutual positive attraction and toward the absence of discrepancy at the collective system level and for both individual systems. All these trajectories end in a state that is "balanced" according to positive balance theory as well as Heider's model. They are also consistent with dissonance theory.

Second, whenever both I's and J's initial messages are well outside the other's acceptance region, then the IJX situation will change toward increasing mutual dislike and toward non-zero discrepancies at the individual level, the collective level or both levels. These results would be consistent with dissonance theory or with Newcomb's model, but not with Heider.

The dissatisfying aspect of the IP shift model with varying attraction is our inability to obtain a more precise estimate of where the separatrix for the six variable model would lie in the six-dimensional space. Even reducing this space to four dimensions by treating discrepancies rather than the raw attitudes and perceptions, does not allow the clear graphic representation of the separatrix as did Figure 24 for the internal changes case. Rather we were forced to rely almost exclusively on the sample of trajectories which have been numerically generated.

IP with Veridical Messages and Varying Attraction: Transmission Constant

With the results from the shift model under our belt, analyzing the veridical model is quite easy although its implications are profoundly different. For the shift model, the final states of the system were dependent upon the location of I's and J's messages relative to the other's acceptance region. Since the form of the change equations has not altered by considering the new message model but only the messages have a new form, then the final states of the IP model with varying attraction should depend upon the location of each person's initial messages relative to the other's acceptance region. In the veridical case, the initial messages are just the initial attitudes of the person sending them. As a result, changes toward mutual positive attraction and zero discrepancy will depend upon either (1) I's and J's attitudes initially being within the other's acceptance region or (2) their initial attitudes being only slightly outside the other's region. On the other hand, change toward increasing mutual dislike results if I's and J's initial attitudes are well outside each other's acceptance region. In other words, the same relationship between initial messages and acceptance regions predicting the final states of the system are found in the shift and veridical models given our rough, qualitative results.

In the veridical case the messages would be unshifted from the speaker's position. In the shift case, the initial messages are shifted away from the speaker's attitude and toward the speaker's perception of his receiver's position. Let us explore the implications briefly. Suppose first that I and J disagree so that $P_i \neq P_j$ and both are well outside each other's acceptance region. If even one of the two persons in the IJX situation is accurate, let us say $Q_{ij} = P_j$, then under the shift and sequential models I's initial message will be shifted in the direction of J's <u>actual position</u> and the likelihood that I's initial message will fall into J's acceptance region is increased. If the shift message does fall into the acceptance region, then the IJX situation will tend toward a qualitatively different state (mutual positive attraction and zero discrepancy) than the veridical model would predict (mutual dislike with non-zero discrepancy). On the other hand, if I and J initially agreed but were both vastly <u>inaccurate</u>, then the veridical model would predict change toward mutual positive attraction and zero discrepancy and the shift and sequential models would predict mutual hostility and finite discrepancy. Of course, this does not cover all possibilities. But the two cases cited do serve to point out that the choice of message model is crucial in predicting where the IJX situation is tending under IP with varying attraction and constant transmission.

<u>IP with Varying Attraction and Transmission: Veridical and Shift Models</u>

Finally we take up the most complex of the IP models incorporating both variable attraction and variable transmission. Our discussion of this case will consider both the veridical and shift models simultaneously since the initial message values will be one of the crucial determinants of the system's behavior regardless of how those messages are determined.

With both transmission and attraction variable, there are two factors which can bring changes in P_i, Q_{ij}, P_j, and Q_{ji} to a halt: (1) zero transmission by both I and J and (2) mutual infinite dislike by both I and J. If the transmission from I and the transmission for J is zero, then all six state variables will stop changing. This <u>mutual cessation of transmission</u> can occur only if $P_i = Q_{ij}$ and $P_j = Q_{ji}$. If A_{ij} and A_{ji} go toward negative infinity, the transmission of information is <u>not</u> terminated but both I and J find one another totally without credibility and, hence, P_i, Q_{ij}, P_j, Q_{ji} will stop changing. A_{ij} and A_{ji} will continue to change. Thus, the two sets of crucial points are $P_i = Q_{ij} = P_j = Q_{ji}$ and $A_{ij} = A_{ji}$ equal to negative infinity. But there is another, more important, set of points determining the final states for the system. When we considered the varying transmission model with constant attraction, we found that $P_i = Q_{ij} = M_{ji}$ would shut the system down. This resulted since I stopped transmitting to J while J continued transmitting to I. In the variable attraction case, this same point is <u>not</u> an equilibrium point (see Appendix B) since if the system hits this point J will continue to transmit messages to I which are exactly equal to I's attitudes. Thus, I's attraction to J will be continually reinforced and will continue to increase toward positive infinity. Thus, we do not have an equilibrium point for $P_i = Q_{ij} = M_{ji}$ but only have an equilibrium point for $P_i = Q_{ij}$ and $P_j = Q_{ji}$. Notice that if the system converges to $P_i = Q_{ij} = P_j = Q_{ji}$, then both I and J stop transmitting and the system of equations shuts down. However, we will show that this latter equilibrium is both unstable and <u>very</u> unlikely to occur. Further, the more likely final states will be shown to be $P_i = Q_{ij} = M_{ji}$ with P_j and Q_{ji} arbitrary, A_{ji} going to positive infinity and A_{ij} constant <u>or</u> A_{ij} and A_{ji} going to negative infinity with P_i, P_j, Q_{ij}, and Q_{ji} approaching different asymptotic values. The best way of discussing the complexities of the variable attraction, variable transmission models with both veridical and shift messages is to review the results obtained previously for the constant transmission-variable attraction and variable attraction-constant transmission cases. The final states for both the veridical and shift models under constant transmission-variable attraction assumptions were seen to be dependent upon whether both I's and J's initial messages were (1) both within the other's acceptance region, (2) both well outside the other's acceptance region, (3) both <u>just</u> outside the other's acceptance region, or (4) whether one message was within while the other was outside the initial acceptance region. Only in condition 2 did the system move toward A_{ij} and A_{ji} at negative infinity with P_i, Q_{ij}, P_j and Q_{ji} discrepant from one another. More simply, for fixed (and non-zero) transmission the final state of the IP system for both veridical and shift messages depends upon the location of I's and J's initial messages relative to the other's acceptance region. Under the assumption of constant attraction and variable transmission, we concluded that the final states for both the veridical and shift models depended upon the symmetric or asymmetric configuration of the initial values. The initial values were called symmetric when $A_{ij} = A_{ji}$ ($\neq$ negative infinity), $|P_i(0) - Q_{ij}(0)| = |P_j(0) - Q_{ji}(0)|$, $|M_{ji}(0) - P_i(0)| = |M_{ij}(0) - P_j(0)|$ and $|M_{ji}(0) - Q_{ij}(0)| = |M_{ij}(0) - Q_{ji}(0)|$ and were called asymmetric otherwise. Under the symmetric conditions, the system would converge to the common limit $P_i = Q_{ij} = P_j = Q_{ji}$ and under the (much more likely) asymmetric conditions the system would converge to $P_i = Q_{ij} = M_{ji}$ with P_j and Q_{ji} arbitrary (if I converged on M_{ji} before J converged on M_{ij}). In simpler terms, the final states of the constant attraction-variable transmission IP model depends upon the symmetry or asymmetry of the initial conditions for both the veridical and shift models.

With both attraction and transmission varying the final states of the system should depend upon the location of the initial messages relative to the other's acceptance region and whether the initial conditions on attraction, attitudes, perceptions, and messages are symmetric or asymmetric. Table 4 summarizes all possible categories of initial conditions which could lead to different final states of the system. Cell IV-S in Table 4 is actually an empty cell since it is logically contradictory to require all initial conditions to be symmetric but to have one message outside and the other message inside the initial acceptance region. That is, with $|M_{ji}(0) - P_i(0)| = |M_{ij}(0) - P_j(0)|$ and $A_{ij}(0) = A_{ji}(0)$, either both messages are within or both are

Table 4. The Possible Combination of the Attraction Subsystem with Attitude-Perception Subsystem for IP with Message Shift.

		Attitude-Perception Subsystem	
		Symmetric	Asymmetric
Attraction Subsystem	Both Messages Within	I-S	I-A
	Both Messages Well Outside	II-S	II-A
	Both Messages Just Outside	III-S	III-A
	One Message Within and One Outside	IV-S	IV-A

outside the other's acceptance region. We also note that if both I's and J's initial messages are well outside the other's acceptance region, then A_{ij} and A_{ji} will tend toward negative infinity quickly whether the other initial values are symmetric or not. Thus, in both the symmetric and asymmetric cases with I's and J's initial messages well outside the other's acceptance region, A_{ij} and A_{ji} will go toward negative infinity and perceptions and attitudes will fail to converge to a common point. Figure 30 presents trajectories for both of these cases for shift messages. Frame a has both I's and J's well outside the other's acceptance region initially and also is asymmetric in that J's initial transmission is greater than I's. Frame b and c of the same figure are symmetric in every respect for I and J with both initial messages well outside the other's acceptance region. Clearly, all three cases result in A_{ij} and A_{ji} going to negative infinity with either I or J both perceiving some discrepancy.

Figure 31 presents three trajectories (also for shift messages) in which the initial messages are well within the other's acceptance regions and the initial values are completely symmetric in I and J. Clearly all three trajectories show P_i, Q_{ij}, P_j, and Q_{ji} converging to a common point and A_{ij} and A_{ji} going off to positive infinity. The rate of change of A_{ij} and A_{ji} will become constant in this case since both I's and J's transmission will cease. Figure 32, on the other hand, presents two trajectories (shift messages) in which both I's and J's initial messages are within the other's acceptance regions <u>but</u> the initial values on attitudes and perceptions are not symmetric. Notice that in both cases attitudes and attractions fail to converge to a common point despite the fact that attraction is always increasing or is at least constant and positive. In frame a, P_j and Q_{ji} converge to I's message (essentially equal to Q_{ji}), thus shutting off transmission to I. This causes A_{ij} to slow down and eventually just be a positive

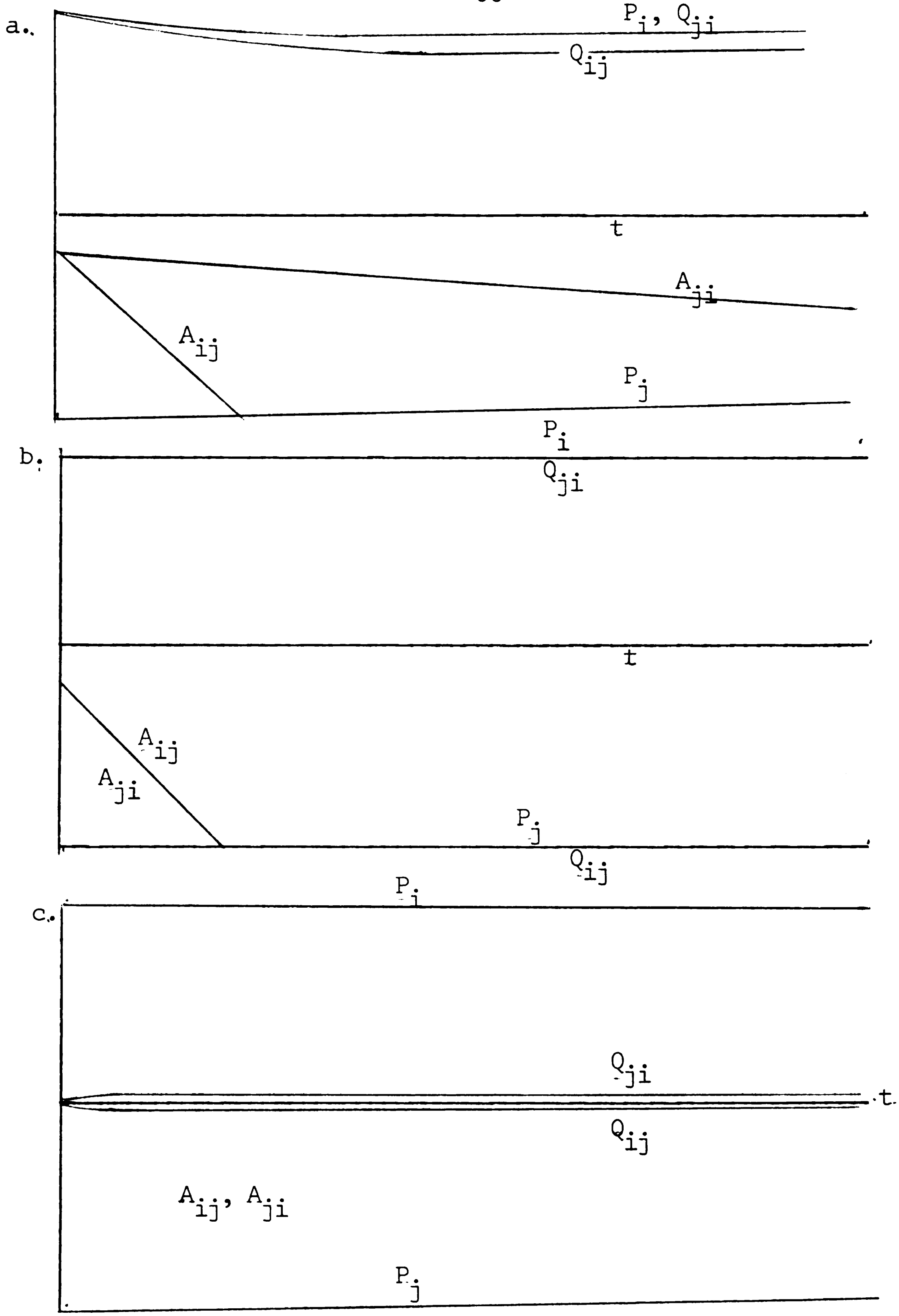


Figure 30. Time Trajectories for IP with Message Shift, Varying Attraction and Transmission: Both Initial Messages well Outside the Other's Acceptance Region.

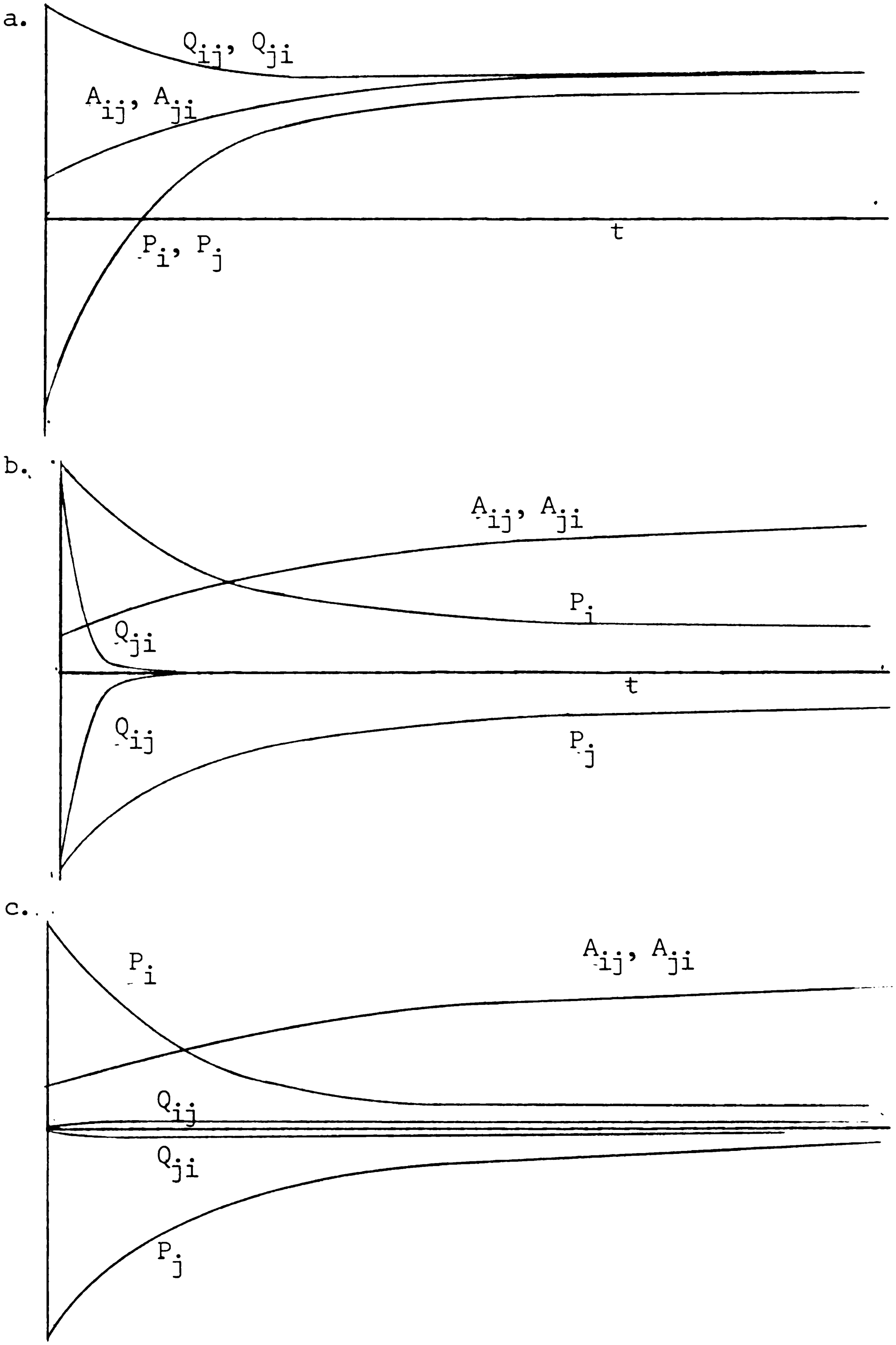


Figure 31. Time Trajectories for IP with Message Shift, Varying Attraction and Transmission: Both Initial Messages within the Other's Acceptance Regions.

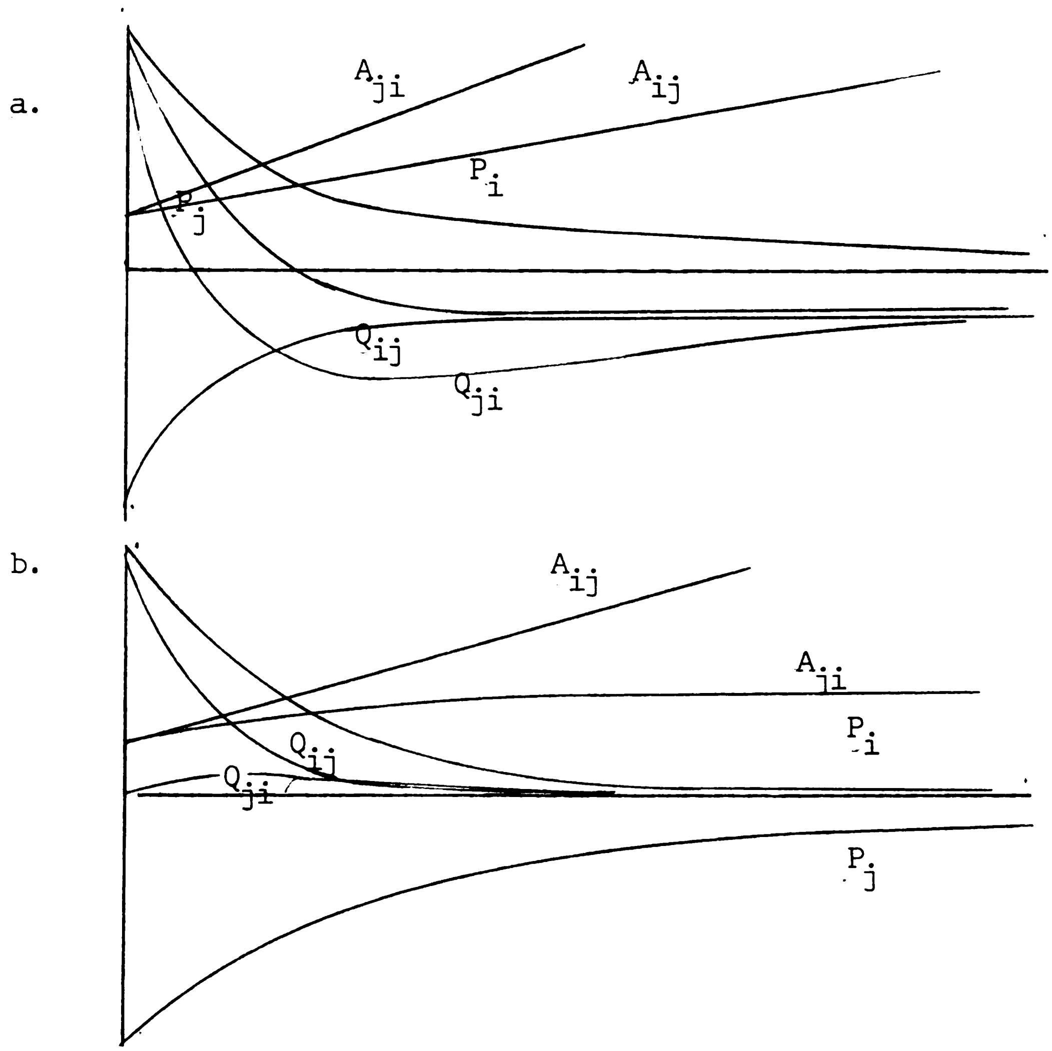


Figure 32. Time Trajectories for IP Message Shift, Varying Attraction and Transmission: Both Initial Messages within the Other's Acceptance Region.

constant. In frame b, it is I which converges to J's message so that A_{ji} becomes constant with the cessation of transmission by I. Notice that J does not stop transmitting and, hence, I's attraction to J continues to grow toward positive infinity. Thus, when both I's and J's initial conditions are symmetric the system tends toward constant positive attraction and $P_i = P_j = Q_{ij} = Q_{ji}$. When both messages are within the other's region but there is an asymmetry, the system tends toward positive attraction with perceived discrepancy by at least one of the persons. Of course it is the asymmetric case which would arise in fact; symmetric initial values are highly unlikely.

The symmetric and asymmetric cases for both messages initially just outside the other's acceptance region are presented in Figures 33 and 34 respectively. In Figure 33, with symmetric initial values, the attitudes and perceptions are clearly converging toward a common point and the attractions have reversed themselves to change in a positive direction. Since I's and J's attitudes are converging, then transmission from both I and J is decreasing. As a result, A_{ij} and A_{ji} will increase more and more slowly until, at the point of convergence of attitudes and perceptions, they become constant. For the asymmetric

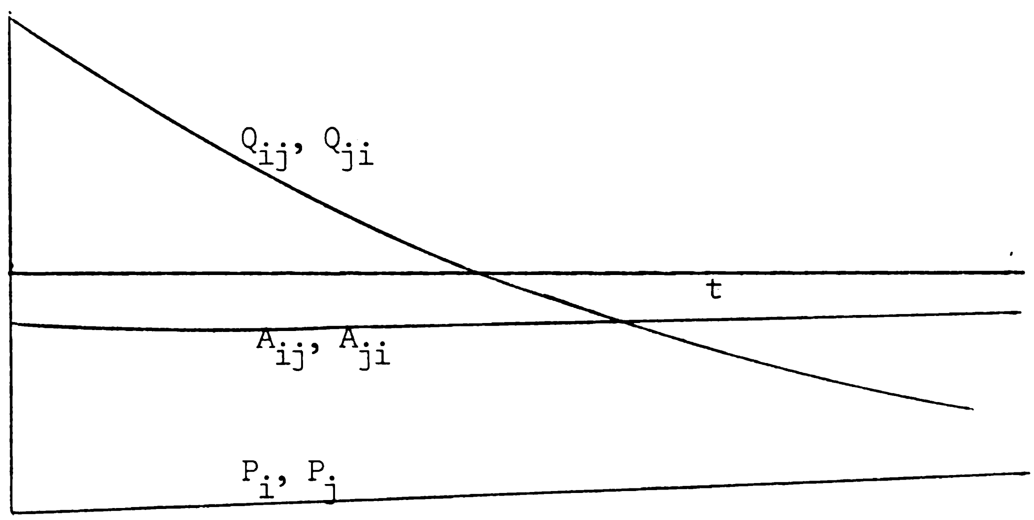


Figure 33. Time Trajectories for IP with Message Shift, Varying Attraction and Transmission: Both Initial Messages just Outside the Other's Acceptance Region.

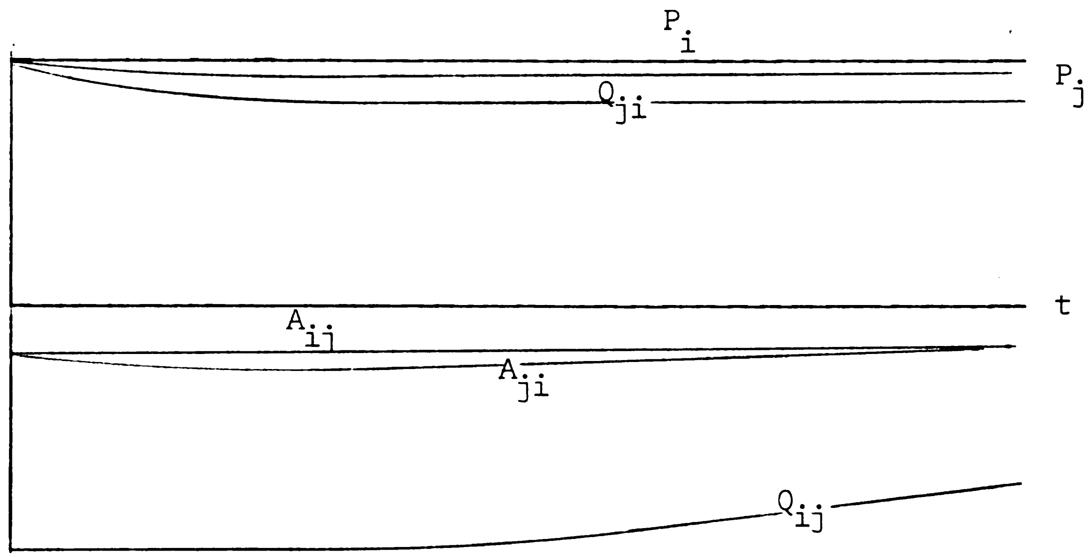


Figure 34. Trajectories for IP with Message Shift, Varying Attraction and Transmission: Both Initial Messages just Outside the Other's Acceptance Region.

case of Figure 34, the direction of movement of the trajectories is not clear. It is possible that P_j and Q_{ji} are close to convergence on I's message but the amount of change in Q_{ij} is puzzling. The likelihood is that Q_{ji} and P_j are converging on M_{ji} and the rate of change of Q_{ij} is decreasing. The problem here is that with A_{ij} and A_{ji} negative, the change in attitudes and perceptions is very slow. Therefore, after a much longer time interval than is graphed, $P_j = Q_{ji} = M_{ij}$ with P_i and Q_{ij} equal to their equilibrium values, A_{ij} constant and A_{ji} tending to positive infinity. Thus, the symmetric case once again produces convergence of attitudes and perceptions with A_{ij} and Q_{ji} constant. The asymmetric case produces perceived discrepancy for one of the individuals and perceived agreement for the other. However, we must caution the reader that the results for either case are only tentative.

Finally, in Figure 35 we consider three trajectories for the situation where J's message is within I's acceptance region but I's message is outside J's acceptance region. Since this situation is already asymmetric then we might expect that attitudes and perceptions

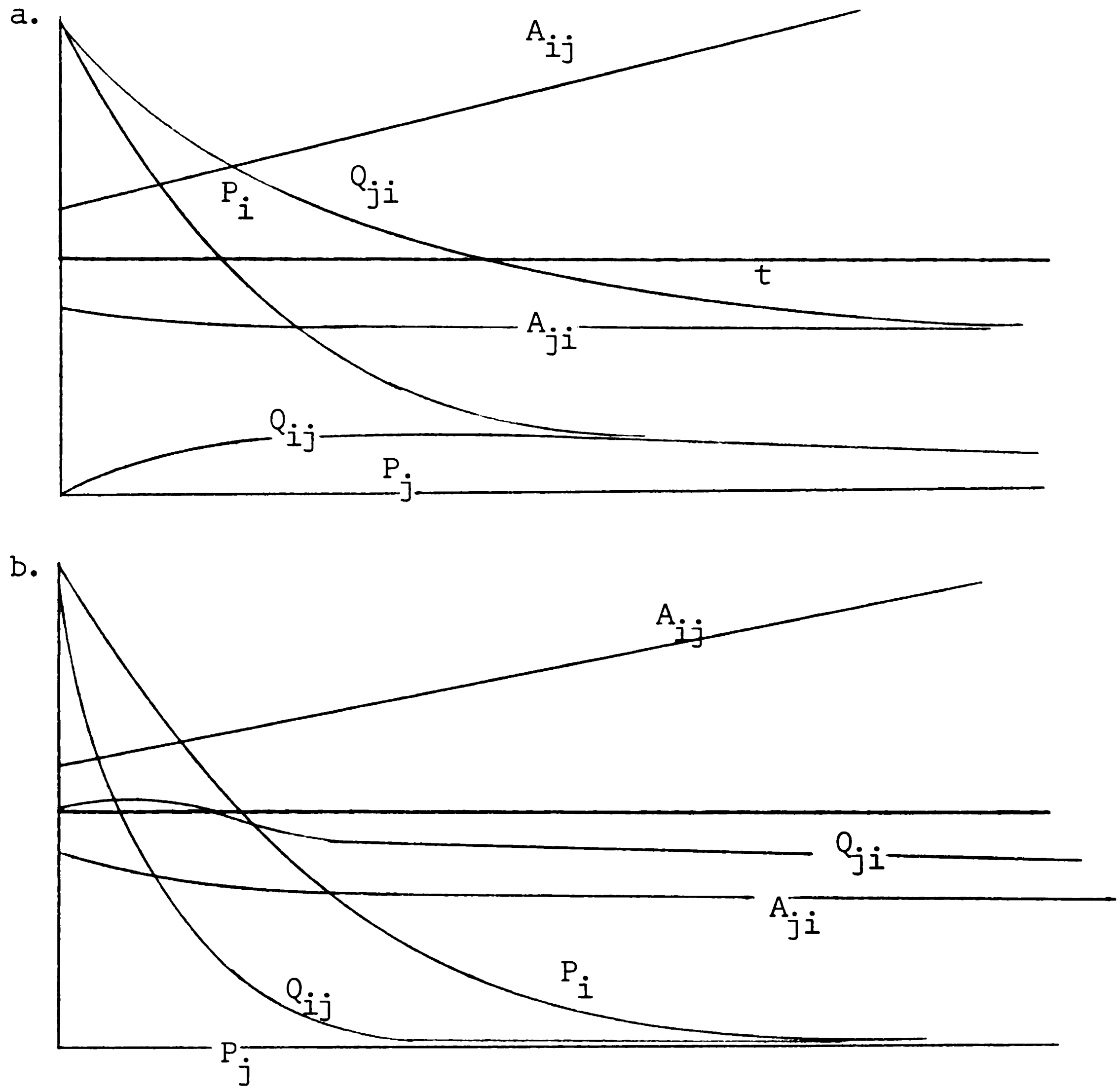


Figure 35. Time Trajectories for IP with Message Shift, Varying Attraction and Transmission: J's Initial Message within I's Acceptance Region while I's Message is Outside J's Region.

would fail to converge to a common point. This is exactly what happens in these numerical results. In each case I converges to J's message before J converges to I's message. I's transmission to J terminates and A_{ji} becomes a constant. Since J still perceives discrepancy, he continues to transmit to I causing A_{ij} to increase toward positive infinity.

While the varying transmission and varying attraction model is the most complex and least tractable of all the models considered thus far, it is also the most interesting. The symmetry of the initial values and the location of the initial messages relative to the other's acceptance region determine the final state of the model. These are summarized in Table 5. Of course these results are based upon a crude

Table 5. Tentative Final States for IP with Message Shift, Varying Attraction and Transmission, as a Function of the Initial Values.

	Symmetric	Asymmetric
Both Inside	A_{ij}, A_{ji} → +, constant $P_i = Q_{ij} = P_j = Q_{ji}$	$A_{ij} \rightarrow +\infty$, $A_{ji} \rightarrow$ + constant $P_i = Q_{ij} = M_{ji}$ $P_j = P_j^*$, $Q_{ji} = Q_{ji}^*$
Both Well Outside	A_{ij}, $A_{ji} \rightarrow -\infty$ $P_i^* \neq Q_{ij}^* \neq P_j^* \neq Q_{ji}^*$	A_{ij}, $A_{ji} \rightarrow -\infty$ $P_i^* \neq Q_{ij}^* \neq Q_{ji}^* \neq P_j^*$
Both Just Outside	A_{ij}, A_{ji} → +/-, constant $P_i = Q_{ij} = P_j = Q_{ji}$	$A_{ij} \rightarrow +\infty$, $A_{ji} \rightarrow$ constant $P_i = Q_{ij} = M_{ji}$ $P_j = P_j^*$, $Q_{ij} = Q_{ji}^*$
One-In, One-Out	A_{ij} → +, A_{ji} → +/-, constant $P_i = Q_{ij} = M_{ji}$ $P_j = P_j^*$, $Q_{ji} = Q_{ji}^*$	

N.B.: In this table "*" indicates a constant value.

analysis of a complex system. Nonetheless, even if the results are approximately correct (as they certainly are from our graphical results), the varying transmission and varying attraction case allows the IJX dyad to achieve final states which were unavailable in the varying attraction case alone. In particular, the "both inside-asymmetric" case shows that it is possible for I and J to be mutually attracted but for actual discrepancy to exist in the collective system. Thus a result obtains which would not be predicted by either Heider or Newcomb's analysis of IJX situations.

In IP with varying attraction and transmission we have discussed the veridical and shift models together. Although the trajectories presented were those for the shift message model only, qualitatively similar trajectories would be generated for the veridical model. The trajectories would not be identical however since in the veridical case both individuals are converging upon the other's attitude (his message in this case). Thus, Figure 31a would show convergence to a common limit which is the average of I's and J's initial attitudes rather than their initial shifted messages. The results of Table 5 are directly applicable to the veridical case with the minor observation that when initial conditions are asymmetric $M_{ji} = P_j$ and only Q_{ji} is arbitrary. The other major difference between the shift and veridical cases is to be found in the location of initial messages for the two cases relative to the other's acceptance region. As we noted in the constant transmission-variable attraction case, if I's and J's initial attitudes were within the other's acceptance region but both were very inaccurate, then veridical messages would be within and shift messages outside the acceptance regions. Each case would produce quite different final states. On the other hand, if I's and J's attitudes were well outside the other's acceptance region but both were quite accurate, then shift messages would be likely to be within the other's acceptance regions while veridical messages would not. Once again, we see that the veridical and shift message model give similar analyses but involve very different implications.

CHAPTER IV

CONCLUSIONS

The purpose of this chapter is to simultaneously summarize, evaluate, and extend (where necessary) the results obtained in our analysis of the information processing model. Some of the results obtained were <u>expected</u> and, in fact, the various models were built with assumptions which would produce these results. As a check and a summary, we will review some of these expected results first. On the other hand, certain results were <u>unexpected</u> in that the analysis showed that they flowed from our assumptions but the assumptions were not offered to yield these results. The unexpected results can be evaluated in terms of their plausibility and the assumptions retained, dismissed, or modified accordingly. A discussion of the unexpected results, their plausibility and the assumptions which produced them will constitute the second section of this chapter. Finally, we shall offer an experimental situation whose predictions would differentiate the veridical from the shift message models.

Some abbreviated notations will simplify our discussion: for constant attraction and constant transmission models write CA-CT, for constant attraction and varying transmission write CA-VT, for variable attraction and constant transmission write VA-CT, and for variable attraction and variable transmission write VA-VT. These four special cases together with the two message models, veridical and shift, constitute the eight models of information processing theory applied to interaction in IJX situations.

The Expected Results

The more foreseeable results are found in the CA-CT models and to a lesser extent in the VA-CT models. The predominant result for the CA-CT veridical and shift models was that attitudes and perceptions for both individuals converged to a common point for all initial conditions and for all values of attraction and transmission (assuming neither individual hated the other totally and assuming that both did some transmission). What this means quite simply is that under constant forced interaction, individuals will always come to some agreement and come to be accurate in their perceptions of the other even when there is a fair amount of negative sentiment between the individuals. However, this result is not as bad as it sounds. First, if the interacting individuls do dislike one another quite a bit, then convergence obtains in the long run but the "long run" may be so long as to be substantially greater than several normal lifetimes! Thus, for all intents and purposes, high negative attraction can sufficiently slow the convergence process to yield discrepancy among the individuals' perceptions and attitudes within "normal" time spans. Second, as long as we associate time with "real" time, then the assumption of constant transmission can be questioned since it assumes that individuals are in constant, mutual interaction for all time. However, if time means "the amount of time interacting about X" which may be interrupted by any number of factors unaccounted for by the model, then the constant transmission assumption becomes much more tenable. Thus, the strong assumptions which produce convergence in the CA-CT models are not as implausible in a different interpretive frame as they might at first seem.

Although convergence to a common value characterizes both the veridical and shift models under CA-CT assumptions, the two models predict convergence to quite different points. The person who does more changing should be the individual who is getting more information, hearing more arguments, and receiving unique restatements of position. This is exactly what we find in both message models. If both are equally attracted, then the person who is doing the most transmitting will induce greater change in the other toward his position. Similarly, under equal transmission, the greatest change is realized by the person who feels more positively toward the other. Notice, though, that these predictions merely verify that the CA-CT model operates as we have set it up. The assumptions on credibility and transmission built into the change equations of Chapter II directly predict the above results almost without a fancy mathematical treatment.

The results are also the expected implications of the assumptions in two cases of the VA-CT model. As we have reported, when initial messages are both inside or well outside the other's acceptance regions, then the final states for both the veridical and shift models are easy to see. In the former case attitudes and perceptions converge to a common point with both attractions becoming more and more positive. In the latter case, attitudes and perceptions stop changing at some finite discrepancy, and both attractions go off to increasing mutual dislike. These results are equivalent to statements which Byrne (1969) and his associates might make about interpersonal situations in which there is constant interaction. Namely, if two persons are in virtual agreement with one another, then their attraction to one another will be positively reinforced by the agreeing messages which are sent. If two persons severely disagree with one another, then their messages will be punishing and their attractions will decrease. In other words, the six simultaneous non-linear differential equations for VA-CT say not much more than Byrne's inelegant analysis at least for two of the four initial conditions.

The Unexpected Results

The "expected" results have the function of verifying for the model builder that what he thought would happen does indeed happen. The discovery, excitement and payoff of mathematical modeling arises in the evaluation of unexpected results in light of the assumptions made. Two cases for the VA-CT results were surprising: the case in which both initial messages were just outside the acceptance region of the other and the case in which one was inside and the other outside the acceptance region. In both cases, both persons' attitudes and perceptions converged to a common value and both attractions climbed toward positive infinity. As long as the interaction continues, convergence and increasing mutual attraction will result despite one person's initial disagreement with and derogation of the other. The plausibility of this prediction is difficult to assess since the author knows of no studies which have differentially manipulated attraction or agreement to produce the one-in-one-out case and knows no similarity-attraction model which predicts the above result. Consequently, a key test of the VA-CT model would be to set up the one-in-one-out initial situation and see where the attitudes, perceptions and attractions go. If the predictions of the model are borne out for this case, then the information processing model is to be preferred to Byrne's reinforcement model since it predicts his results and then some. On the other hand, failure of this hypothesis also suggests failure of the model as a whole.

There is an additional aspect of the VA-CT model which favors its viability. Of the two final states toward which this model tends, they both are states of mutual liking or mutual disliking. There are no final states in which one person likes the other but is in turn despised. Under assumptions of constant interaction, it seems reasonable that differences and similarities, affections, and disaffections would become more and more fully known by the interactants. That is, in accumulating information from the other person, it would seem plausible that an individual would come to know where he stood relative to the other's attitudes and thereby to assess his own favor in the eyes of the other.

Without question the most striking set of outcomes was obtained in the varying transmission models for both veridical and shift messages. Under conditions of mutual positive attraction both the CA-VT and VA-VT models produced a final state in which one individual stopped transmitting because he no longer perceived disagreement with the other while the other continued transmitting because he did perceive disagreement.

The only case in which equal transmission between the two persons was found occurred under the highly unlikely case of completely symmetric initial conditions. Thus these models predict that vast inequities in transmission should be common in dyadic interaction.

Let us turn to the literature for verification of what seems to be a highly unlikely result. We note first that most coding schemes for dyadic interaction presume that the interaction is <u>symmetric</u> between the individuals (Mark, 1970; Ericson, 1972). However, there is also direct evidence in favor of equal transmission. Jaffe and Feldstein (1970) who have authored what is perhaps the most comprehensive discussion of the dynamics of dialogue to date put it this way:

> There is a strong tendency for speakers in conversation to match the average duration of pauses that they alternately exhibit while speaking. This is not generally true of the durations of their respective vocalizations (as we measure them). This pause matching phenomenon is probably responsible for the positive correlations which several investigators have obtained between the average lengths of time that interacting speaker's 'hold the floor'. . . . The mutual pacing is referable to a bilateral adjustment of silence periods . . . . It is a conceivable mechanism for adjusting the linguistic information processing rates of the speakers to each other (p. 4).

This evidence suggests that the frequent unequal distribution of transmission between members of the dyad is highly unlikely and, further, suggests a modification of the transmission function to come closer to the "tit for a tat" norm which characterizes interaction.

Thus for dyadic interaction, the VA-CT model in which transmissions are equal and to a lesser extent the CA-CT model with equal transmissions seem to fare best. This does <u>not</u> contradict the data source on which our model was built. Schachter (1951) was always working with groups of more than two. In such a group, any one person _can_ stop transmitting without violating basic communication norms. So our model might still work in groups of three or more.

An Experimental Test

Consider the situation in which individuals are brought together for the purposes of interacting on two pre-selected topics. The topics and dyads would be selected in order to insure that there was essential agreement on one topic but severe disagreement on the other topic. The crucial aspect of the experiment in terms of its implications for our model is the _order_ in which the topics are discussed. There are four possible orderings: agree-agree, agree-disagree, disagree-agree and disagree-disagree. The revealing comparisons are to be found in pairing groups 1 and 3 (agree-agree with disagree-agree) and groups 2 and 4 (agree-disagree with disagree-disagree).

When the agree-topic is first, then, presuming that there is initial neutrality on attraction, the following is predicted by our information processing model: As a result of agreeing, each person should find the other more attractive and both should leave the first session with positive feelings about the other. In turn, this should produce a wider latitude of acceptance for the next session whether it be a disagree-topic or an agree-topic. When asked to interact on the disagree topic, the dyad should still reach a compromise position with further increases in attraction. Furthermore, if shift messages are the ones generated, then in the second session there should be a greater number of messages which exhibit tolerance for and even support of the other's position in the agree-disagree ordering than in the disagree-disagree ordering. On the other hand, if veridical messages are sent, then there should be little support of the other's position in the second session (disagree-topic) regardless of which topic preceded the disagree-topic.

The reason is that with the disagree-topic first, individuals who are neutral toward one another would discover that they disagree and mutually decrease their attraction to one another. With a relevant topic both acceptance regions would decrease and the second session would reinforce their dislike and disagreement. Hence, if messages were sent according to the shift model, they should be close to veridical.

With the agree-topic second, then dyads who disagree in the first session should decrease in attraction and shrink their acceptance regions for the second session. Although they essentially agree in the second session, their small acceptance regions should make complete agreement impossible and their attractions should not become more positive. In addition, if shift messages are being sent, then the beginning of the second session should exhibit more messages which are tolerant of and show support for the other's position in the agree-agree ordering than in the disagree-agree ordering. If messages are veridical then there should be no difference.

With careful selection of discussion topics and careful selection of dyad pairs on the agreement-disagreement dimension, a relatively simple experiment could shed light on the validity of the information processing model on the occurrence of message generation strategies, and on rates of transmission in live dyadic interaction. The predictions from information processing theory are striking and unique in the above experimental situation. We feel that this begins to show the potentialities of mathematical modeling for understanding of existing theories and their non-obvious implications.

Future Work

I think it is clear that the models treated in this paper only begin to scratch the surface of the possible models of IJX situations which can and should be carried out. In addition to modification of the social judgment model; reinforcement, congruity and dissonance models have not been treated. More important to the field of communication, however, is the inclusion of other message strategy models to couple with the models of attitudes, perception, and attraction change. A final intriguing possibility is to link persons in an IJX model who are employing <u>different</u> message strategies. In this way individual differences in styles of interaction would be incorporated directly into the mathematical model and perhaps produce qualitatively different results than the type of model treated in this thesis.

APPENDIX A

APPENDIX A

This appendix considers all the mathematical manipulations appropriate to the information processing models with attraction held constant. Its pertinence to the textual material may be found by references to the appendix within the text or it may be read separately.

Internal Changes Only

We first consider the derivation of the equations used to generate Figures 9 and 10 based upon the differential equations in matrix which allows us to determine the eigenvalues of the coefficient matrix almost

$$\frac{d}{dt}\begin{bmatrix} P_i \\ \\ Q_{ij} \end{bmatrix} = \begin{bmatrix} -k_{ij}\, r & k_{ij} r \\ \\ k_{ij}\, q & -k_{ij} q \end{bmatrix} \begin{bmatrix} P_i \\ \\ Q_{ij} \end{bmatrix}$$

by inspection. In this simple case, the eigenvalues are just zero and $-(r + q)k_{ij}$. If we denote the initial values of P_i and Q_{ij} by $P_i(0)$ and $Q_{ij}(0)$ respectively, then the complete solutions are

$$P_i = \frac{1}{1 + q/r}\left({}^{q}/r\ P_i(0) + Q_{ij}(0)\right) + \frac{(P_i(0) - Q_{ij}(0))\ e^{-(r+q)k_{ij}t}}{1 + q/r} \qquad Q_{ij} = \frac{1}{1 + q/r}\left(q/r\ P_i(0) + Q_{ij}(0)\right) - q\,\frac{(P_i(0) - Q_{ij}(0))^{-(r+q)k_{ij}t}}{(r + q)}$$

Note that the critical points of the system can be obtained by taking the limit as t tends to infinity or by setting the derivative to zero, and solving the homogeneous system of equations. The solution is just that $P_i = Q_{ij}$ are the critical points. In particular as t goes to infinity, both P_i and Q_{ij} converge to

$$P_i^* = Q_{ij}^* = \frac{q\,P_i(0) + r\,Q_{ij}(0)}{q + r}$$

To obtain the equations for the phase planes of Figure 10 we divide equation 10a by 10b yielding

$$\frac{d\,P_i}{d\,Q_{ij}} = \frac{-r}{q}$$

which upon integration gives the straight line integral curves

$$Q_{ij} = -q/r\,P_i + c$$

where c is an arbitrary constant of integration.

IP with Shift: Constant Attraction and Transmission

We first consider the special case where I's attraction to J and J's attraction to I are equal and their transmission to one another is also equal. We give special consideration to this case because (1) the mathematics is simple enough to allow a complete solution, (2) there is some evidence to indicate that interacting dyads share the speaking time between them (Jaffe and Feldstein, 1970) and (3) at least one study (Willis and Burgess, 1974) shows that persons both prefer and expect to receive the same amount of liking from an other as they give to that other person. For constant and equal attraction, let $k_{ij} = k_{ji} = k$, and for constant and equal transmission let $N_{ij} = N_{ji} = N$. Since we are dealing with the message shift model, $M_{ij} = (1 - k)\,P_i + kQ_{ij}$ with a similar equation for M_{ji}. Thus, the equations to be solved are:

$$\frac{dP_i}{dt} = akN\,((1-k)P_j + kQ_{ji} - P_i) \tag{A1}$$

$$\frac{dP_j}{dt} = akN\,((1-k)P_i + kQ_{ij} - P_j) \tag{A2}$$

$$\frac{dQ_{ij}}{dt} = bkN\,((1-k)P_j + kQ_{ji} - Q_{ij}) \tag{A3}$$

$$\frac{dQ_{ji}}{dt} = bkN\,((1-k)P_i + kQ_{ij} - Q_{ji}) \;. \tag{A4}$$

The most direct method of solution of the above system of equations is to first take sums and then differences of the P_i, P_j pair and Q_{ij} and Q_{ji} pair. For sums we have $P_s = P_i + P_j$, $Q_s = Q_{ij} + Q_{ji}$ and

$$\frac{dP_s}{dt} = ak^2N\,(Q_s - P_s)$$

$$\frac{dQ_s}{dt} = bkN\,(1-k)\,(P_s - Q_s)$$

which has the eigenvalues $r_1 = 0$, $r_2 = -kN(ak + b\,(1-k))$. Since $0 < a, b, k < 1$, notice that r_2 is negative. Given the eigenvalues we may write

$$P_s = a_1 + a_2 e^{+r_2 t}$$

$$Q_s = b_1 + b_2 e^{+r_2 t}$$

where the a's and b's immediately above are constants determined by the initial conditions. Now let us define the differences $P_d = P_i - P_j$ and $Q_d = Q_{ij} - Q_{ji}$ which yields the derivatives:

$$\frac{dP_d}{dt} = -akN\,((2-k)P_d + kQ_d)$$

$$\frac{dQ_d}{dt} = -bkN\,((1-k)P_d + (1+k)Q_d).$$

The eigenvalues for this system are

$$r_3, r_4 = \frac{-Nk}{2}\left[b(1+k) + a(2-k) \mp \sqrt{(b(1+k) + a(2-k))^2 - 8ab}\right]$$

which can be shown to be real and negative. Thus for differences we have

$$P_d = a_3 e^{r_3 t} + a_4 e^{r_4 t}$$

$$Q_d = b_3 e^{r_3 t} + b_4 e^{r_4 t} .$$

To get back to the original variables we note that

$$P_i = (P_s + P_d)/2$$

$$P_j = (P_s - P_d)/2$$

$$Q_{ij} = (Q_s + Q_d)/2$$

$$Q_{ji} = (Q_s - Q_d)/2 .$$

Thus, the state variables are just linear combinations of the solutions for P_s, P_d, Q_s, and Q_d which are themselves exponentially decreasing. That is,

$$P_i = 1/2\ (a_1 + a_2 e^{r_2 t} + a_3 e^{r_3 t} + a_4 e^{r_4 t})$$

$$P_j = 1/2\ (a_1 + a_2 e^{r_2 t} - a_3 e^{4_3 t} - a_4 e^{r_4 t})$$

$$Q_{ij} = 1/2\ (b_1 + b_2 e^{r_2 t} + b_3 e^{r_3 t} + b_4 e^{r_4 t})$$

$$Q_{ji} = 1/2\ (b_1 + b_2 e^{r_2 t} - b_3 e^{r_3 t} - b_4 e^{r_4 t}) .$$

It can be shown that $a_1 = b_1$ which equals

$$\frac{(r_2 + ak^2N)P_s(0) - ak^2N\ Q_s(0)}{r_2} .$$

Therefore, $1/2a_1$ is the value toward which P_i, P_j, Q_{ij}, and Q_{ji} converge in the limit as t goes to infinity. For the sake of completeness we also note that

$$a_2 = \frac{ak^2N}{r_2} (Q_s(0) - P_s(0)$$

$$a_3 = \frac{1}{r_3 - r_4} \left[(ak(k - 2)\ N - r_4)\ P_d(0) - kQ_d(0)\right]$$

$$a_4 = \frac{1}{r_3 - r_4} \left[(r_3 - ak(k - 2)N)\ P_d(0) + kQ_d(0)\right]$$

$$b_2 = \frac{r_2 + ak^2N}{r_2} \left[Q_s(0) - P_s(0)\right]$$

$$b_3 = \frac{ak(k - 2)N - r_3}{(r_3 - r_4)k} \left[(ak(k - 2)\ N - r_4)\ P_d(0) - kQ_d(0)\right]$$

$$b_4 = \frac{ak(k - 2)N - r_4}{r_3 - r_4)k} \left[(r_3 - ak(k - 2)\ N)\ P_d(0) + kQ_d(0)\right]$$

This gives us a complete solution for equations (A1) - (A4). Therefore, we may generate the trajectories of Figure 9 directly.

We note that if the parameters a and b were equal, then $r_2 = -akN$ and $a_1/2$ would be the average of the initial message values. That is, if a = b, then P_i, Q_{ij}, P_j, Q_{ji} would converge to the average of their initial messages.

Now, let us turn to the more general case for which both transmission and attraction are still constant but unequal.

If the expressions for M_{ij} and M_{ji} (that is, equation (5)) from the shift model are substituted into equations (11a) through (11d), then the matrix representation of this system can be denoted

$$\frac{d\bar{S}}{dt} = \underline{W}\bar{S} \tag{A5}$$

where S is the column vector:

$$S = (P_i, P_j, Q_{ij}, Q_{ji})$$

and W is the matrix:

$$\begin{bmatrix} -a\,k_{ij}\,N_{ji} & a\,k_{ij}(1 - k_{ji})\,N_{ji} & 0 & a\,k_{ij}\,k_{ji}\,N_{ji} \\ a\,k_{ji}(1 - k_{ij})\,N_{ij} & -a\,k_{ji}\,N_{ij} & a\,k_{ij}\,k_{ji}\,N_{ij} & 0 \\ 0 & b\,k_{ij}(1 - k_{ji})\,N_{ji} & -b\,k_{ij}\,N_{ji} & b\,k_{ij}\,k_{ji}\,N_{ji} \\ b\,k_{ji}(1 - k_{ij})\,N_{ij} & 0 & b\,k_{ji}\,k_{ij}\,N_{ij} & -b\,k_{ji}\,N_{ij} \end{bmatrix}$$

The matrix W has four important characteristics:

1. All diagonal elements are less than zero.
2. All off-diagonal elements are greater than zero.
3. The sum of the elements of each row is zero.
4. The matrix is compact (Abelson, 1964, p. 145) in that each variable is at least an indirect cause of every other variable.

Matrices with the above characteristics permit some direct and powerful inferences concerning the dynamic character of the system of linear equations. It is well-known in linear system theory that if the eigenvalues of the system (equation A5) can be determined, then the asymptotic characteristics are immediately known. If one or more of the eigenvalues is positive, the system is unstable. If all of the non-zero eigenvalues have negative real parts, then the trajectory for each set of initial values will converge to some critical point. A theorem which Abelson (1964, p. 145) invokes (see also, McKenzie, 1960) states that if the above four conditions are met, then all the non-zero eigenvalues will have negative real parts and at least one of the eigenvalues will be zero. For such a linear system, every trajectory converges to a critical point and every critical point is a right eigenvector of W for eigenvalue 0. If the matrix W is compact, then every right eigenvector is a scalar multiple of the column vector (1, 1, 1, 1). Thus, in our case we can use Abelson's theorem to conclude immediately that each of the variables converges to some common limit, i.e., P_i, Q_{ij}, P_j, Q_{ji} goes to L as t goes to ∞. However, this limit is not "stable": if S is randomly jarred from the critical point S* = (L, L, L, L), it will then converge to a new nearby critical point (L_1, L_1, L_1, L_1). The distance from L to L_1 is always less than or equal to the size of the random movement. That is, random events not accounted for by the model produce a random motion from one equilibrium point to another nearby. Thus, given any set of initial conditions the system (A5) will converge to a point at which there are no longer any changes. Such a point is by definition a critical point. The specific critical point toward which the system is converging is determined by the initial conditions.

We know that the system of equations converges to a point of equality but we do not know what that point is as a function of the initial conditions. However, this can be found in a straightforward but tedious way by determining the left eigenvector, V, for the system $\overline{V}\underline{W} = 0$ and then setting the initial dot product V.S equal to the asymptotic dot product V.S*. That is, we write

$$v_1x + v_2y + v_3z + v_4w = v_1x^* + v_2y^* + v_3z^* + v_4w^*.$$

But since all the limiting values are the same, we have

$$x^* = y^* = z^* = w^* = \frac{v_1 x + v_2 y + v_3 z + v_4 w}{v_1 + v_2 + v_3 + v_4}$$

Thus, we see that the final values all converge to a weighted average of the initial values.

The appropriate left eigenvector is

$$V = (bk_{ji}\ (1 - k_{ij})N_{ij},\ bk_{ij}(1 - k_{ji})N_{ji},\ ak_{ij}k_{ji}N_{ij},\ ak_{ij}k_{ji}N_{ji}).$$

The first and third components of this vector correspond to P_i and Q_{ij}, while the second and fourth correspond to P_j and Q_{ji}. Since these pairs each have a common factor, it is instructive to break up the final value by persons. If we let the sum of the weights be denoted s, i.e., $s = bk_{ji}(1 - k_{ij})\ N_{ij} + bk_{ij}$, then the contribution to the final value made by person I is

$$N_{ij}\ k_{ji}\ (b(1 - k_{ij})\ P_i + ak_{ij}\ Q_{ij})$$

divided by s, while the contribution made by person J is

$$N_{ji}\ k_{ij}\ (b(1 - k_{ji})\ P_j + ak_{ji}\ Q_{ji})$$

divided by s. We see then that the contribution to the final value made by person I is proportional to N_{ij} and k_{ji}. That is, the contribution made by person I is proportional to the rate at which I transmits to J and logistically proportional to how much I is liked by J. Similarly, the contribution to the final value made by person J is proportional to the rate at which J transmits to I and logistically proportional to how much J is liked by I.

If we look at the contribution made by each person separately, then we can assess the relative weight given to perceptions as compared to attitudes. For person I we have

$$\frac{\text{weight attitude}}{\text{weight perception}} = \frac{b(1 - k_{ij})}{ak_{ij}} = \frac{b}{ae^{A_{ij}}}$$

Thus, I's perception of J is weighted by parameter a, while I's attitude toward the issue is weighted by parameter b. Thus, according to Wackman's study (1973), this model would predict that the person's perception of the other is ultimately three times as important as his attitude toward the object or issue. If this is compared to the veridical model, then we see that the assumption of ingratiation is an extremely strong assumption.

Of course, there is a second term to the relative weight of perception to attitude, namely $e^{A_{ij}}$. If I likes J initially, then $e^{A_{ij}} > 1$, and perception will ultimately be weighted even more than three times as much as the person's attitude. On the other hand, if I initially dislikes J, then $e^{A_{ij}} < 1$, and his attitude has more weight in the final analysis. For $A_{ij} = -1.10$, the weights for perception and attitude become equal, while as A_{ij} goes to $-\infty$, the model approaches the assumption of veridicality.

Before leaving the shift model let us consider two special cases: (1) the limit as A_{ij} approaches negative infinity for A_{ji} finite and (2) the case of $N_{ij} = 0$ for $N_{ji} > 0$. Notice that both of these cases drastically change (A5) and, in particular, the vital characteristics of W. As A_{ij} goes to negative infinity k_{ij} goes to zero and $d\,P_i/dt = 0$ and $d\,Q_{ij}/dt = 0$. This implies that P_i and Q_{ij} are constant which we take to be their initial values $P_i(0)$ and $Q_{ij}(0)$. Since k_{ij} is zero, the message sent by I is not shifted so that $M_{ij} = P_i(0)$. Therefore,

$$\frac{d\,P_j}{dt} = a\,k_{ji}\,N_{ij}\,(P_i(0) - P_j)$$

$$\frac{d\,Q_{ji}}{dt} = b\,k_{ji}\,N_{ij}\,(P_i(0) - Q_{ji}).$$

The critical value for P_j and Q_{ji}, then, is just $P_i(0)$. This is stable critical point since the eigenvalue for each equation separately is negative and $P_i(0)$ is fixed. Thus, P_j and Q_{ji} will ultimately converge toward $P_i(0)$ and I will not change from the initial values, $P_i(0)$ and $Q_{ij}(0)$.

For $N_{ij} = 0$, we also have $d\,P_j/dt = 0$ and $d\,Q_{ji}/dt = 0$ which imply as above that $P_j = P_j(0)$ and $Q_{ji} = Q_{ji}(0)$ for all time. Now the message which J generates is shifted toward his perception of I

$M_{ji} = P_j(0) + k_{ji}\,(Q_{ji}(0) - P_j(0))$. But, $M_{ji}(0)$ is fixed over time and so,

$$\frac{d\,P_i}{dt} = a\,k_{ij}\,N_{ji}\,(M_{ji}(0) - P_i)$$

$$\frac{d\,Q_{ij}}{dt} = b\,k_{ij}\,N_{ji}\,(M_{ji}(0) - Q_{ij})$$

which has the same critical value: $P_i = M\ \ (0)$ and $Q_{ij} = M\ \ (0)$. That is, both converge to J's initial message. This critical point is stable as before. Thus P_i and Q_{ij} will ultimately converge toward $M_{ji}(0) = P_j(0) + k_{ji}\,(Q_{ji}(0) - P_j(0))$ while P_j and Q_{ji} remain constant at their initial values.

It is interesting to note at this juncture that if the model of equation (A5) had been extended to 3, 4, or more individuals, the expanded matrix would still satisfy the conditions dictated by Abelson.

As a result, the conclusions about critical points and their convergence reached for the dyadic case would hold identically for a larger group of persons. It is also interesting to note that the symmetries which give rise to the special case of W arise from the basic discrepancy principle which treats change as a function of the difference between a state variable (for example, I's attitude) and a target variable (for example, J's message). When presented in the language of target and state variables, the link to a general cybernetic format is implied.

IP with Veridical Messages: Constant Attraction and Transmission

The system of equations (11a) through (11d) for $M_{ij} = P_i$ and $M_{ji} = P_j$ also satisfies Abelson's Theorem as can be seen by inspection of the coefficient matrix:

$$\begin{bmatrix} -ak_{ij} N_{ji} & ak_{ij} N_{ji} & 0 & 0 \\ ak_{ji} N_{ij} & -ak_{ji} N_{ij} & 0 & 0 \\ bk_{ij} N_{ji} & 0 & -bk_{ij} N_{ji} & 0 \\ 0 & bk_{ji} N_{ij} & 0 & -bk_{ji} N_{ij} \end{bmatrix}$$

Therefore, in the veridical case I and J always converge to $P_i = P_j = Q_{ij} = Q_{ji}$. As before, the point of convergence can be related to the initial values. For the veridical case, I's and J's attitudes and perceptions converge to

$$\frac{k_{ji} N_{ij} P_i(0) + k_{ij} N_{ji} P_j(0)}{k_{ji} N_{ij} + k_{ij} N_{ji}}$$

This is the simpler final value of the two message models since it depends neither on Q_{ij}, Q_{ji} or the parameters a and b. Clearly, if $k_{ji} N_{ij} > k_{ij} N_{ji}$, then the final value will be closer to I's initial attitude than to J's which means that J will have done more changing than I. In the case where I and J transmit equally and $k_{ji} > k_{ij}$, then J's attitude changes k_{ji}/k_{ij} as much as I's attitude. Similarly, when I and J are equally attracted to each other, then J's attitude changes N_{ij}/N_{ji} as much as I's attitude. In general, the ratio of the weights $k_{ji} N_{ij}/k_{ij} N_{ji}$ indicates how much change J undergoes relative to I.

IP with Constant Attraction and Variable Transmission Shift and Veridical

If we consider the situation where N_{ij} and N_{ji} are variable but attraction is constant, then we can discuss both message models simultaneously. If we assume that a = b, then subtracting equation (11a) from (11c) and (11b) from (11d) <u>regardless of what M_{ij} and M_{ji} are</u>, we have

$$\frac{d\,(P_i - Q_{ij})}{dt} = \frac{a\,k_i \left| P_j - Q_{ji} \right|}{\sqrt{1 + (P_j - Q_{ji})^2}} \,(Q_{ij} - P_i)$$

$$\frac{d\,(P_j - Q_{ji})}{dt} = \frac{a\,k_j \left| P_i - Q_{ij} \right|}{\sqrt{1 + (P_i - Q_{ij})^2}} \,(Q_{ji} - P_j)$$

where $k_i = k_{ij}\,(1 + k_{ji})$ and $k_j = k_{ji}\,(1 + k_{ij})$. Let $P_i - Q_{ij} = S_1$ and $P_j - Q_{ji} = S_2$. This yields

$$\frac{dS_1}{dt} = \frac{a\,k_i \left| S_2 \right|}{\sqrt{1 + S_2^2}} \,(-S_1) \tag{A10}$$

$$\frac{dS_2}{dt} = \frac{a\,k_j \left| S_1 \right|}{\sqrt{1 + S_1^2}} \,(-S_2) \tag{A11}$$

We note immediately that $S_1 = 0$ or $S_2 = 0$ are equilibrium points for the variable transmission case. That is, <u>when</u> <u>a = b</u>, if either I or J or both perceives no disagreement, then $P_i - Q_{ij}$ and $P_j - Q_{ji}$ will remain constant from the point of zero perceived disagreement on. For $S_1, S_2 > 0$ or $S_1, S_2 < 0$, the equations (A10) and (A11) are separable yielding

$$\frac{dS_1}{dS_2} = \frac{k_i\sqrt{1 + S_1^2}}{k_j\sqrt{1 + S_2^2}}$$

which integrates to

$$k_j \ln (S_1 + \sqrt{1 + S_1^2}) + C = k_i \ln (S_2 + \sqrt{1 + S_2^2})$$

where C is an arbitrary integration constant. These constitute the integral curves of quadrants 1 and 3 of Figure 16. For $S_1 > 0$, $S_2 < 0$ or $S_1 < 0$, $S_2 > 0$ the integral curves are

$$k_j \ln (S_1 + \sqrt{1 + S_1^2}) + c = -k_i \ln (S_2 + \sqrt{1 + S_2^2})$$

thus yielding the integral curves for quadrants 2 and 4 of the same figure.

Note that when $S_1 = 0$, the slope $dS_2/dS_1 = (k_j/k_i)\sqrt{1 + S_2^2}$ which increases as S_2 increases in absolute value. Similarly when $S_2 = 0$ the slope dS_2/dS_1 is $(k_j/k_i)(1/\sqrt{1 + S_1^2})$. As S_1 increases in absolute value, then the slope of the phase plane trajectories approaches 0. These results suggest the nature of the trajectories in Figure 16 which are near to the S_1 and S_2 axes respectively. We remark that the above analysis holds for the veridical model and the shift model.

The unequal parameters case ($a \neq b$) behaves somewhat differently. First consider the situation in which A_{ij} is large and positive and A_{ji} is large and negative. For the shift message model, this implies that $M_{ij} \approx Q_{ij}$ and $M_{ji} \approx P_j$ and that k_{ij} is near unity while k_{ji} is near zero. The equations for this case have the form:

$$\frac{dP_i}{dt} = a\, N_{ji}\, (P_j - P_i) \; ; \; \frac{dQ_{ij}}{dt} = b\, N_{ji}\, (P_j - Q_{ij})$$

$$\frac{dP_j}{dt} = ak_{ji}\, N_{ij}\, (Q_{ij} - P_j) \; ; \; \frac{dQ_{ji}}{dt} = b\, k_{ji}\, N_{ij}\, (Q_{ij} - Q_{ji})$$

$$\approx 0 \qquad\qquad\qquad\qquad \approx 0$$

Since the variables for J are changing very slowly, P_j is almost constant. But the variables for I are moving rapidly toward P_j so that in a very short time $P_i = Q_{ij} = P_j$. In fact the greater the initial discrepancy, $P_j - Q_{ji}$, the more rapid the convergence of P_i, and Q_{ij} toward P_j. The convergence of P_i and Q_{ij} stops the transmission from I to J and, hence, "freezes" Q_{ji} at its current value. Although J continues to transmit to I, he is transmitting a message equal to P_j so that I does not deviate from $P_i = Q_{ij} = P_j$. These behaviors are depicted in the trajectories of Figures 21 and 22 in the text.

In the more general case, where messages are M_{ij} and M_{ji}, then if for any reason P_i and Q_{ij} converge to M_{ji} before P_j and Q_{ji} converge to M_{ij}, then $dP_i/dt = dQ_{ij}/dt = 0$ since $P_i = Q_{ij} = M_{ji}$ and $dP_j/dt = dQ_{ji}/dt = 0$ since N_{ij} will be zero due to I's perception of agreement. The key question, however, which we have been unable to answer with any precision is what conditions will necessarily produce such a convergence. Suppose $N_{ij}(0) = N_{ji}(0)$, $|M_{ji} - P_i| = |M_{ij} - P_j|$ initially, and

$|M_{ji} - Q_{ij}| = |M_{ij} - Q_{ji}|$ initially. Actually, we only need to assume two of the three of these conditions since the other follows immediately given any two. With these initial values and $k_{ij} > k_{ji}$, then P_i and Q_{ij} will always converge faster to M_{ji} than P_j and Q_{ji} converge to M_{ij}. With P_i and Q_{ij} converging faster, then N_{ij} will be decreasing faster than N_{ji} is decreasing. As a result, $P_i = Q_{ij} = M_{ji}$ with P_j and Q_{ji} arbitrary constants whenever the initial values of the state variables meet the conditions above and $k_{ij} > k_{ji}$.

This result also suggests sufficient conditions for convergence of the varying transmission system to $P_i = Q_{ij} = P_j = Q_{ji}$. Suppose $|P_i(0) - Q_{ij}(0)| = |P_j(0) - Q_{ji}(0)|$ or that I and J transmit equally at time t = 0. Also suppose that $|M_{ij}(0)| = |M_{ji}(0)|$ or that initial messages are of the same magnitude. If we further assume that $A_{ij} = A_{ji}$ and is not infinitely negative then we can show that the system of equations (11a) through (11d) converges to $P_i = Q_{ij} = P_j = Q_{ji}$ as follows: Consider the first increment in attitudes, ΔP_i and ΔP_j. We have

$$\Delta P_i = a\, N_{ji}(0)\; k_{ij}\; (M_{ji}(0) - P_i(0))$$

$$\Delta P_j = a\, N_{ij}(0)\; k_{ji}\; (M_{ij}(0) - P_j(0)).$$

But since $N_{ij}(0) = N_{ji}(0)$, and $k_{ij} = k_{ji}$, the difference between ΔP_i and ΔP_j depends only upon $M_{ji}(0) - P_i(0)$ and $M_{ij}(0) - P_j(0)$. But these two differences are equal in magnitude (but not necessarily in sign) since $|Q_{ij}(0) - P_i(0)| = |Q_{ji}(0) - P_j(0)|$ and $|P_i(0) + k_{ij}\; (Q_{ij}(0) - P_i(0))| = |P_j(0) + k_{ji}\; (Q_{ji}(0) - P_j(0))|$ according to the shift model. Therefore, $|\Delta P_i| = |\Delta P_j|$ in the first time increment. Similarly, we can show that $|\Delta Q_{ij}| = |\Delta Q_{ji}|$. However, it is <u>not</u> the case that $|\Delta Q_{ij}| = |\Delta P_i|$ since we assume that b > a. Since attitudes are changing by the same amount and perceptions are changing by the same amount, this implies that N_{ij} (1) = $N_{ji}(1)$, $|M_{ij}(1)| = |M_{ji}(1)|$ and further that the magnitude of the change in attitudes will be the same in the next time increment; similarly for the change in I's and J's perceptions. The overall implication of our "proof" is that P_i and P_j will converge on each other's messages by changing equal amounts and Q_{ij} and Q_{ji} will converge on each other's messages faster than P_i and P_j do but also by changing equal amounts. This means that $P_i = P_j = Q_{ij} = Q_{ji}$ when the restrictive symmetry assumptions detailed above hold. We note that the same proof can be made for the veridical model.

While the symmetric form of the model of equations (11a) through (11d) for varying transmission clearly will yield convergence to P_i = $P_j = Q_{ij} = Q_{ji}$ (that is, the symmetry conditions are sufficient), it is not clear that these restrictive conditions are also necessary. If they could be shown to be necessary, the result would be a strong one. If it is false, then we would be back to the issue that plagued our search for a separatrix in the other 3, 4, . . . variable models. Namely, how much asymmetry is possible before P_i, Q_{ij}, P_j, Q_{ji} fail to converge?

APPENDIX B

APPENDIX B

In this appendix the few mathematical results pertaining to the IP model with variable attraction are discussed. As noted in the text, this model consists of six first-order, nonlinear differential equations which have no critical points (the varying transmission case is an exception). The nonlinearities make the mathematics impossible to treat analytically while the absence of critical points makes the usual approximation procedures also inapplicable. The main procedures of this appendix are to discuss special cases of the models.

Internal Changes Only

If we let the parameters r and q be equal in equations (7) and (8) of Chapter II, then we may subtract to obtain

$$\frac{d(P_i - Q_{ij})}{dt} = 2r \frac{e^{A_{ij}}}{1 + e^{A_{ij}}} (Q_{ij} - P_i) \ .$$

Together with equation (9)

$$\frac{dA_{ij}}{dt} = s \frac{(e^{2A_{ij}} - (Q_{ij} - P_i)^2)}{1 + e^{2A_{ij}}}$$

we have the internal change model with varying attraction. Dividing the former equation into the latter does not yield a separable form and so we generate the trajectories of Figure 24 numerically. Notice that along the A_{ij} axis, $Q_{ij} - P_i = 0$ and

$$\frac{dA_{ij}}{dt} = s \frac{e^{2A_{ij}}}{1 + e^{2A_{ij}}}$$

which is always positive. This suggests that the separatrix (the dotted line in Figure 24) is asymptotic to the A_{ij} axis for large negative A_{ij}, $Q_{ij} - P_i = 0$.

IP with Message Shift and Varying Attraction: Transmission Constant

Let us consider some special cases of the IP message shift model with varying attraction so that the asymptotic behaviors can be understood. First, consider the case in which A_{ij} and A_{ji} are large and positive so that $e^{A_{ij}}/(1 + e^{A_{ij}})$ and $e^{A_{ji}}/(1 + e^{A_{ji}})$ are both approximately one, and $e^{A_{ij}}$ and $e^{A_{ji}}$ are extremely large. In this case the model reduces to

$$\frac{dP_i}{dt} = a\ N_{ji}\ (Q_{ji} - P_i) \tag{B1}$$

$$\frac{dQ_{ij}}{dt} = b\ N_{ji}\ (Q_{ji} - Q_{ij}) \tag{B2}$$

$$\frac{dA_{ij}}{dt} = c\ N_{ji} \tag{B3}$$

with equivalent equations for P_j, Q_{ji}, and A_{ji}. After attraction becomes large, it simply increases in a linear fashion with increasing time. Also, the change equations for attitudes and perceptions become linear discrepancy equations converging rapidly to $P_i = Q_{ij} = P_j = Q_{ji}$. Results which indicate such behavior may be found in Figure 26 especially frames a and b. Note that if $N_{ji} \neq N_{ij}$ such that $N_{ji} > N_{ij}$ then,

$dA_{ij}/dt > dA_{ji}/dt$ while both are increasing at a constant rate. In Figure 26a $N_{ij} > N_{ji}$ so that J's increase in attraction while constant is much greater than I's constant increase.

Next, consider the situation for which A_{ij} and A_{ji} are large negative values (note that $A_{ij} = -5$, yields $e^{-5} = .0067$) so that $e^{A_{ij}}/(1 + e^{A_{ij}})$ and $e^{A_{ji}}/(1 + e^{A_{ji}})$ are approximately zero, and so are $e^{A_{ij}}$ and $e^{A_{ji}}$. In this case the model reduces to

$$\frac{dP_i}{dt} = \frac{dP_j}{dt} = \frac{dQ_{ij}}{dt} = \frac{dQ_{ji}}{dt} = 0$$

$$\frac{dA_{ij}}{dt} = -c\, N_{ji}\, (P_j - P_i)^2 = -c\, N_{ji}\, m^2$$

$$\frac{dA_{ji}}{dt} = -c\, N_{ij}\, (P_i - P_j)^2 = -c\, N_{ij}\, m^2$$

where m is a constant since P_i and P_j are not changing. Note that for large negative values of A_{ij} and A_{ji}, attraction decreases in a linear fashion and the variables P_i, Q_{ij}, Q_{ji}, and P_j remain fixed. The trajectories of Figure 27 approximate such behavior. Once again, notice that $N_{ji} > N_{ij}$ will yield dA_{ij}/dt as the more negative. Figure 27a shows the effect of transmission asymmetry on the slopes of dA_{ij}/dt and dA_{ji}/dt.

Finally consider the situation in which the variables P_i, Q_{ij}, P_j, Q_{ji} have converged to a common limit. In this case we have that P_i, Q_{ij}, P_j, and Q_{ji} are unchanging as before but

$$\frac{dA_{ij}}{dt} = c\, N_{ji}\, \frac{e^{2A_{ij}}}{1 + e^{2A_{ij}}}$$

$$\frac{dA_{ji}}{dt} = c\, N_{ij}\, \frac{e^{2A_{ji}}}{1 + e^{2A_{ji}}}$$

both of these equations are directly integrable but this would be unnecessarily complicating. Rather, given the convergence of attitudes and perceptions, the rate at which attraction increases depends upon the value of attraction. For large positive values, the change is approximately constant, for large negative values it is very slow, and as attraction increases from negative to positive the rate of change of attraction also increases. The attraction trajectories in Figures 28 and 29 show each of these cases.

IP with Veridical Messages and Varying Attraction: Transmission Constant

The equations for the veridical case have a particularly simple form since the change in attitudes and attraction are independent of changes in perception. That is, for the veridical case, we have

$$\frac{dP_i}{dt} = a\, N_{ji} \frac{e^{A_{ij}}}{1 + e^{A_{ij}}} (P_j - P_i)$$

$$\frac{DP_j}{dt} = a\, N_{ij} \frac{e^{A_{ji}}}{1 + e^{A_{ji}}} (P_i - P_j)$$

$$\frac{dA_{ij}}{dt} = c\, \frac{e^{2A_{ij}} - (P_i - P_j)^2}{1 + e^{2A_{ij}}}$$

$$\frac{dA_{ji}}{dt} = c\, \frac{e^{2A_{ji}} - (P_j - P_i)^2}{1 + e^{2A_{ji}}}$$

which does not depend upon Q_{ij} or Q_{ji}. The change in Q_{ij} and Q_{ji} on the other hand, does depend upon the behavior of P_i, P_j, A_{ij}, and A_{ji}:

$$\frac{dQ_{ij}}{dt} = b\, N_{ji} \frac{e^{A_{ij}}}{1 + e^{A_{ij}}} (P_j - Q_{ij})$$

$$\frac{dQ_{ji}}{dt} = b\, N_{ij} \frac{e^{A_{ji}}}{1 + e^{A_{ji}}} (P_i - Q_{ji}).$$

If P_i and P_j are within the other's acceptance region, then P_i and P_j converge toward one another and A_{ij} and A_{ji} go toward negative infinity. As this happens Q_{ij} and Q_{ji} converge toward P_j and P_i respectively and, hence, toward one another. If P_i and P_j are well outside the other's acceptance region, then A_{ij} and A_{ji} will decrease toward negative infinity. As this occurs, Q_{ij} and Q_{ji} will stop changing. In any case, the behavior of the attitude-attraction subsystem can be studied independently of the perception subsystem. And the behavior of the attitude-attraction subsystem determines the behavior of the perception subsystem while the reverse is not true.

IP with Varying Attraction and Varying Transmission: Both Message Models

Perhaps the simplest way to come to some understanding of the varying attraction, varying transmission model is to consider the P_i, P_j, Q_{ij}, Q_{ji} subsystem and its interaction with the attraction subsystem. The equations for the attitude-perception subsystem are

$$\frac{dP_i}{dt} = a\, N_{ji} \frac{e^{A_{ij}}}{1 + e^{A_{ij}}} (M_{ji} - P_i)$$

$$\frac{dQ_{ij}}{dt} = b\, N_{ji} \frac{e^{A_{ij}}}{1 + e^{A_{ij}}} (M_{ji} - Q_{ij})$$

where

$$N_{ji} = \frac{\left|Q_{ij} - P_i\right|}{\sqrt{1 + (Q_{ij} - P_i)^2}} \left[1 + \frac{e^{A_{ij}}}{1 + e^{A_{ij}}} \right]$$

and M_{ji} depends upon the particular message model chosen. The comparable equations for J complete the attitude-perception subsystem. There are two characteristics of this subsystem that are crucial: (1) As long as both values for A_{ij} and A_{ji} are not large and negative, P_i, Q_{ij}, P_j, Q_{ji} will always change toward each other. (2) For fixed values of attraction, $P_i = Q_{ij} = M_{ji}$ is an equilibrium value for the subsystem. But note that for the attraction subsystem

$$\frac{dA_{ij}}{dt} = c\, N_{ji} \frac{e^{2A_{ij}} - (M_{ji} - P_i)^2}{1 + e^{2A_{ij}}}$$

$$\frac{dA_{ji}}{dt} = c\, N_{ij} \frac{e^{2A_{ji}} - (M_{ij} - P_j)^2}{1 + e^{2A_{ji}}}$$

that $P_i = Q_{ij}$ only insures that $N_{ij} = 0$. Thus $dA_{ji}/dt = 0$, $A_{ji} = A_{ji}*$. But $P_j = P_j*$, $Q_{ji} = Q_{ji}*$ when $P_i = Q_{ij} = M_{ji}$ and J's attitude and perception will not in general be equal. Therefore at $P_i = Q_{ij} = M_{ji}$

$$\frac{dA_{ij}}{dt} = (1 + k_{ji}*) \frac{\left|P_j* - Q_{ji}*\right|}{\sqrt{1 + (P_j* - Q_{ji}*)^2}} \quad \frac{e^{2A_{ij}}}{1 + e^{2A_{ij}}}$$

$$= N_{ji}* \quad \frac{e^{2A_{ij}}}{1 + e^{2A_{ij}}}$$

where $k_{ji}* = e^{A_{ji}*}/(1 + e^{A_{ji}*})$. In other words if the attitude-perception subsystem equilibrates, the attraction subsystem does not as A_{ij} will approach positive infinity at a rate equal to $N_{ji}*$. On the other hand, A_{ji} will be frozen at $A_{ji}*$ which can be positive or negative depending upon the rate of convergence to $P_i = Q_{ij} = M_{ji}$ and $A_{ji}(0)$, the initial attraction of J to I. The graphs of Figure 32 exemplify the above behaviors.

The infinite but unstable set of equilibria which shuts down both subsystems is $P_i = Q_{ij} = P_j = Q_{ji}$. If such a point is reached $N_{ij} = N_{ji} = 0$ and all changes are zero with $A_{ij} = A_{ij}^*$, $A_{ji} = A_{ji}^*$. As we showed in the constant attraction, varying transmission case, the equilibria $P_i = Q_{ij} = P_j = Q_{ji}$ will be reached only if the initial values are completely symmetric. In the varying attraction case, this means $A_{ij}(0) = A_{ji}(0)$, $|P_i(0) - Q_{ij}(0)| = |P_j(0) - Q_{ji}(0)|$, and the distance between the initial incoming messages and each person's attitudes are the same, and the distance between the initial incoming messages and each person's initial perception of the other is the same. When these restrictive initial conditions are met, we say that the system is symmetric. If the initial messages are in or not too far from the acceptance regions, then the system will converge to $P_i = Q_{ij} = P_j = Q_{ji}$ with $A_{ij} = A_{ji}$. As in the constant attraction, varying transmission case, it is <u>not</u> clear that the above conditions are necessary to have convergence to a common limit although they are clearly sufficient <u>with one qualification</u>: If I's and J's initial messages are well outside the other's acceptance regions, then A_{ij} and A_{ji} will go toward negative infinity very rapidly thus shutting off the attitude-perception subsystem at $P_i = P_i^*$, $Q_{ij} = Q_{ij}^*$, $P_j = P_j^*$, and $Q_{ji} = Q_{ji}^*$. In this case both A_{ij} and A_{ji} will go to negative infinity with the rate

$$\frac{-\left|P_i^* - Q_{ij}^*\right|}{\sqrt{1 + (P_i^* - Q_{ij}^*)^2}} \qquad (P_j^* - P_i^*)^2$$

for A_{ji} and a comparable rate for A_{ij}. Figures 30b and 30c exhibit trajectories with the above behavior since initial messages were well outside the other's acceptance region. Figure 31 shows three symmetric sets of initial conditions with initial messages within the other's acceptance region so that there is convergence of attitudes and perceptions to a common point with the change in A_{ij} and A_{ji} decreasing to a constant value.

Finally, we remark that the symmetric set of initial conditions described above is a very unlikely set to find. In fact if we consider a unit hypercube in the vicinity of the origin and assume that any initial conditions falling within that hypercube are equally likely to occur, then it is an easy matter to show that the set of initial conditions which we call symmetric have a probability of occurrence which for all intents and purposes is zero. Therefore, it is much more likely for the system to tend toward $P_i = Q_{ij} = M_{ji}$ with P_j, Q_{ji}, A_{ji} arbitrary and A_{ij} increasing toward positive infinity.

None of the results of this section of the appendix are specialized to either the veridical or shift models of message transmission. In part this reflects the crudity of our analysis and in part reflects the similarities between the two message models. However, this should not be taken to mean that there are no differences between the models since there will certainly be differences in the position of the convergence point at least.

BIBLIOGRAPHY

Aronson, E., J. A. Turner, and J. M. Carlsmith. "Communicator Credibility and Communicator Discrepancy as Determinants of Opinion Change," J. of Abnormal and Social Psychology, 1963, 67, 31-37.

Abelson, R. P. "Mathematical Models of the Distribution of Attitudes under Controversy." In N. Fredericksen and H. Gulliksen (editors), Contributions to Mathematical Psychology. New York: Holt, Rinehart, 1964. Pp. 141-160.

Back, K. W. "Influence through Social Communication," J. of Abnormal and Social Psychology, 1951, 46, 9-23.

Berkowitz, L. "Reporting an Experiment: A Case Study in Leveling, Sharpening, and Assimilation," J. of Experimental Social Psychology, 1971, 7, 237-243.

Byrne, D. "Attitudes and Attraction." In L. Berkowitz (editor), Advances in Experimental Social Psychology. New York: Academic Press, 1969. Pp. 35-90.

Cohen, A. R. Attitude Change and Social Influence. New York: Basic Books, 1964.

Cohen, A. R. "A Dissonance Analysis of the Boomerang Effect," J. of Personality, 1962, 30, 75-88.

Collins, B. E., and B. H. Raven. "Group Structure: Attraction, Coalitions, Communication and Power." In G. Lindzey and E. Aronson (editors), The Handbook of Social Psychology (2nd edition), Volume IV. Reading, Mass: Addison-Wesley, 1964. Pp. 102-204.

Curry, T. J., and R. M. Emerson. "Balance Theory: A Theory of Interpersonal Attraction," Sociometry, 1970, 33, 216-238.

Ericson, P. "Relational Communication: Complementarity and Symmetry and Their Relation to Dominance-Submission." Unpublished dissertation, Dept. of Communication, Michigan State University, 1972.

Festinger, L. "Informal Social Communication," Psychological Review, 1950, 57, 271-282.

Festinger, L. A Theory of Cognitive Dissonance. Evanston: Row, Peterson and Co., 1957.

Festinger, L., and J. Thibant. "Interpersonal Communication in Small Groups," J. of Abnormal Social Psychology, 1951, 46, 9-23.

Heider, F. "Attitudes and Cognitive Organization," J. of Psychology, 1946, 21, 107-112.

Hovland, C. I., I. L. Janis, and H. H. Kelley. Communication and Persuasion. New Haven: Yale University Press, 1953.

Hunter, J. E. "Dynamic Sociometry," unpublished manuscript. E. Lansing: Dept. of Psychology, Michigan State University, 1974.

Hunter, J. E., and S. H. Cohen. "Mathematical Models of Attitude Change in the Passive Communication Context," unpublished manuscript. E. Lansing: Dept. of Psychology, Michigan State University, 1974.

Jaffe, J., and S. A. Feldstein. Rhythms of Dialogue. New York: Academic Press, 1970.

Kogan, N., and R. Tagiuri. "Interpersonal Preference and Cognitive Organization," J. of Abnormal and Social Psychology, 1958, 56, 113-116.

Mark, R. A. "Parameters of Normal Family Communication in the Dyad," unpublished dissertation, Dept. of Communication, Michigan State University, 1970.

Newcomb, T. M. "An Approach to the Study of Communicative Acts," Psychological Review, 1953, 60, 393-404.

Newcomb, T. M. The Acquaintance Process. New York: Holt, Rinehart, and Winston, 1961.

Newcomb, T. M. "Interpersonal Balance." In R. P. Abelson, E. Aronson, W. T. McGuire, T. M. Newcomb, M. J. Rosenberg, and P. H. Tannenbaum (editors), Theories of Cognitive Consistency: A Sourcebook. Chicago: Rand-McNally, 1968.

Osgood, C. E., and P. H. Tannenbaum. "The Principle of Congruity in the Prediction of Attitude Change," Psychological Review, 1955, 62, 42-55.

Rosenfeld, H. M., and V. L. Sullwold. "Optimal Information Discrepancies for Persistent Communication," Behavioral Science, 1969, 14, 303-315.

Schachter, S. "Deviation, Rejection, and Communication," J. of Abnormal and Social Psychology, 1951, 46, 190-207.

Sherif, C. W., M. Sherif, and R. E. Nebergall. Attitude and Attitude Change; The Social Judgment-Involvement Approach. Philadelphia: W. B. Saunders Co., 1965.

Taylor, M. "Towards a Mathematical Theory of Influence and Attitude Change," Human Relations, 1968, 21, 121-140.

Taylor, H. F. Balance in Small Groups. New York: VanNostrand Reinhold Co, 1970.

Wackman, D. B. "Interpersonal Communication and Coorientation," American Behavioral Scientist, 1973, 16, 537-550.

Wackman, D. B., and D. J. F. Beatty. "A Comparison of Balance and Consensus Theories for Explaining Changes in ABX Systems." Presented to the International Communication Association, Phoenix, Arizona, 1971.

Whittaker, J. O. "Resolution of the Communication Discrepancy Issue in Attitude Change." In C. W. Sherif and M. Sherif (editors), Attitude, Ego-Involvement and Change. New York: John Wiley, 1967. Pp. 159-177.

Willis, R. H., and T. D. G. Burgess. "Cognitive and Affective Balance in Sociometric Dyads," J. of Personality and Social Psychology, 1974, 29, 145-152.

Zajonc, R. "Cognitive Theories in Social Psychology." In G. Lindzey and E. Aronson (editors), The Handbook of Social Psychology (2nd edition), Volume I. Reading, Mass: Addison-Wesley, 1968. Pp. 320-411.