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THE BANK LOAN DECISION UNDER THE

EXPECTED UTILITY HYPOTHESIS

presented by

John B. Carlson

has been accepted towards fulfillment
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THE BANK LOAN DECISION UNDER THE EXPECTED UTILITY HYPOTHESIS

By

John B. Carlson

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Economics

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ABSTRACT

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The supply of commercial loans is considered the primary function of the commercial bank. The bank's loan decision is complicated by the fact that the loan contract involves several dimensions, i.e. terms of the loan. Another aspect which presents an analytical problem is the risk inherent in the loan. Because of these factors, the theory of the banking firm has been slow to develop. The primary objective of this study is to offer a theoretical framework for the bank loan decision. From this framework, a loan offer function is derived which describes the relationship between five terms of the loan--size of loan, contract rate of interest, required percentages of collateral and compensating balances, and length of maturity. The theory is based on the Expected Utility Hypothesis and a theory of the interest rate. Utility is defined to be a function of profits. Profits are dependent on the random outcome of a loan. A fundamental basis is provided for the bank's perception of risk inherent in the loan.

A second purpose of the study is to provide a theory of the interest rate consistent with a weak version of the Efficient Markets

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Hypothesis. Several propositions about interest rate differentials (across markets) are offered and tested. The propositions describe time series which follow from efficient markets which allow for both transactions and information costs. These time series models are tested using Box-Jenkins techniques on money market rate differentials for various time periods from 1965-1977. The major finding of the tests is that the data do not contradict the hypothesis that interest rate differentials behave according to an IMA(1,q) stochastic process with 100% reversion after q periods. In economic terms, this means that one cannot deny the hypothesis of Efficient Markets when allowance is made for transactions and information costs. This theory of interest is invoked as part of the environment under which the loan offer function is derived.

The major theoretical result of the loan offer function is: Under certain specified conditions, the optimal size of loan increases with an increase in the contract rate of interest, the proportion of collateral, the percentage of compensating balances or a decrease in the length of maturity. This finding is consistent with the results of a widely acknowledged empirical model of the loan offer function.

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CHAPTER I

INTRODUCTION

The primary objective of this study is to develop a theoretical framework for the bank loan decision and to compare some implications of this theory with an empirical counterpart. The framework is based on the assumption that banks are expected utility maximizers.

A second purpose is to provide a theory of the interest rate based on a weak version of the Efficient Markets Hypothesis. It will be assumed that the 'bank' of this study adopts this theory in the determination of the maturity aspect of a loan.

The Loan Offer Function

One characteristic which distinguishes a bank from other financial intermediaries is the emphasis it places on its function as the primary supplier of commercial loans. This function in conjunction with the fact that the bank is the primary supplier of payment services is an essential factor in the determination of the money stock. Thus, a complete theory of money stock determination requires an adequate theory of the loan offer function--a theory which should derive from some fundamental principles of the banking firm. It is the intent of this thesis to take a step in this direction by offering a fundamental theory of the loan offer function.

The loan offer function to be derived is consistent with the notion defined by Hester (16). That is, the loan offer function is the relation that specifies the terms of a loan at which a given bank is willing to lend a borrower with a given set of characteristics. These characteristics determine the bank's perception of risk inherent in the loan prospect. The nature of this relationship is hypothesized for a general but realistic conception of a loan prospect to demonstrate its potential for practical application.

The 'Bank'

The 'bank' of this thesis is a risk-averse firm that maximizes expected utility from profits. It generates profits by selling two types of services: intermediary and payments.

The intermediary service has two faces. On one side, it produces income for saver-lenders. This income may be in money (explicit interest) or in kind (payments service). For this income and a claim against the bank,[1] the saver lends the bank use of its funds. On the other side, the bank lends the funds to a deficit spending unit, e.g. businessman seeking financing for an investment project. In return for the funds the bank receives a claim against the borrower (a loan contract) that includes payment of income to the bank.

The bank is able to sell this intermediary service because of its immunity to factors which preclude individual surplus income

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[1]This claim is often called an indirect or secondary claim since the bank is not considered a deficit spending unit.

units from lending directly to deficit spending units.[1] The value of this service will be proportional to the interest rate differential between the rate earned on assets and the rate paid on liabilities (including yield in kind).

Payment service was, until recently, a service which distinguished banks from other financial intermediaries. Although NOW (negotiable orders of withdrawal) accounts and share drafts provide means of payment produced by other financial intermediaries, banks still dominate in the production of this service. In a competitive banking market, a typical bank does not earn enough revenue from services charges (if there are indeed any at all) to pay for the resources it consumes in producing payment services. The bulk of the revenue comes from the employment of a portion of funds obtained from demand deposits in the creation of loans.

The model of this thesis will adopt the viewpoint that demand deposits are one source of funds for a bank. The bank will produce payment services in order to attract these funds. The yield to the deposit holder is the payment service less explicit charges. It will be assumed that as the bank employs more resources in this function it will be able to attract more deposits.

The Expected Utility Hypothesis

In the fields of Economics and Finance it is held by many theorists that the Expected Utility Hypothesis is the "correct" rule

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[1] These factors include: denomination size, transactions cost, liquidity needs of saver, default risk, etc. For a discussion of how banks obviate these barriers, the reader is referred to Hutchinson (19).

for choice among uncertain alternatives. It is deemed correct in the sense it does not impose any implausible restrictions on the nature of choice of the individual or on the distribution of the prospects considered. For example, it is well known that the Mean-Variance criterion (a competing rule) is valid only under one of two conditions: (1) individual choice is consistent with that characterized by a quadratic utility function, or (2) prospects are distributed multivariate normal.

Behavior characterized by a quadratic utility function implies that an individual becomes more averse to a risk situation involving a small fixed bet as his wealth increases. This contradicts the widely accepted axiom of decreasing absolute risk aversion. To consider only prospects which are distributed multivariate normal precludes the consideration of securities with limited liability, e.g. stocks and bonds. Clearly the fruitfulness of the Mean-Variance criterion comes at a cost. The more general hypothesis of expected utility maximization will be invoked by the model of this thesis.

The Organization of the Subject

The first stage of the bank model will be introduced in Chapter III. The initial model will abstract from time. From the first and second-order conditions of expected utility maximization, we shall derive implications about relationships between four major terms of a loan--size of loan, contract rate of interest, degree of collateralization and percent of compensating balances.

To introduce maturity as a term of a loan, we need a hypothesis about future interest rate movements. Chapter IV offers several propositions about interest rate behavior based on the Efficient Markets Hypotheses and two weakened versions of it.

Based on the theories of interest provided in Chapter IV, Chapter V deals with establishing a term structure of loan rates. This relationship between maturity and the interest rate is then linked in Chapter VI to the relationships of the other terms derived in the initial model. The integrated results of the theory are compared with an empirical examination of the loan offer function.

Chapter II offers a survey of the literature on the theory of the banking firm.

CHAPTER II

A SURVEY OF THE LITERATURE

The literature on the theory of the banking firm (including asset management) has been slow to develop. Perhaps the most significant reason for this is that a comprehensive theory must deal with the difficult problem of uncertainty in a multiperiod time horizon. Though progress is slow, much work has been done developing theories which abstract from either uncertainty and/or dynamics (in some way) in order to make the analysis tractable. Since there is abundant literature, this chapter will be organized in such a way as to focus on those studies which are most comprehensive in the approach they take.

We can identify four basic approaches found in the literature. The first will be termed the "rules of thumb" approach. It is best represented by the study of Robinson (29). Though it is not grounded in neoclassical economic theory, it probably describes quite accurately the manifest behavior of banks for sometime prior to its publication.

The second approach--standard neoclassical--utilizes the profit maximization framework. It is still being invoked as evidenced by the recent publication of Wood (34). This approach may utilize classical programming (as in the case of Wood) or linear programming. The linear program (LP) models were developed as

operational tools for banks. Though the practical solutions did not live up to expectations this approach provides many insights for understanding bank behavior.

Third, we find the approach that maximizes expected profits. The models that take this approach, including Hester and Pierce (17), at least incorporate uncertainty into the objective function. The weakness of such an approach is that it implicitly assumes the decision-maker is risk neutral. Regardless of this fact, these models produce some interesting and testable hypotheses.

Finally we will look at models which maximize the expected utility of profits (or wealth). This is the approach taken by this study. It allows the characteristic of risk aversion to be incorporated in the analysis.

The "Rules of Thumb" Approach

The "rules of thumb" approach to bank modeling derives from the actual past practice of banks. In a historical perspective, McKinney (25) relates that during the 1930s and 1940s banks experienced abundant liquidity relative to customer needs. He notes at that time, banks (ostensibly in the absence of competitive pressure) allocated their funds according to arbitrary percentages. A first step toward more efficient funds management comes with an identification of some basic objectives, e.g. profitability and safety. This step is provided in the study by Robinson (29).

To appreciate this seminal work by Robinson, one must consider that the first edition appeared in 1951 and the second in

1962. This was a period in which financial conditions had changed drastically. Prior to the "Accord of 1951" the Fed accommodated the Treasury by purchasing as much government debt as was necessary to peg the interest rate at low levels. Even after the Fed exercised its independence, the forced savings during the Second World War had left the U.S. economy quite liquid. This high level liquidity in conjunction with moderate to light demand for credit resulted in the relatively low interest rates throughout most of the fifties. However, by the early sixties, credit markets began to tighten and interest rates began to rise.

The relevance of all this for banks was that during this period of change, many of the "rules of thumb" for bank management became obsolete. The contribution of Robinson was to question the conventional wisdom of bank operating procedure. Rather than just describe the status quo technique of bank management, he asks why.

Robinson defines the banking problem as the resolution of the conflict between profitability and safety. This is still widely accepted as the general objective of bank management. As a solution to this problem, Robinson puts forth a schedule of priorities which serve as the "rules of thumb."[1]

The schedule of priorities are themselves established by two "rules of thumb:" (1) It is better to err on the side of safety than profitability, and (2) banks should exploit those credit

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[1]Robinson presages the obsolescence of this solution when he acknowledges that the rules must be reevaluated in environment which changes as rapidly as financial institutions do.

opportunities for which it enjoys a natural advantage and avoid those types for which it does not. Based on these guides, he establishes the following order of priorities for employment of funds; (1) primary reserves, (2) secondary reserves, (3) customer credit demands, and (4) open-market investments.

Given this schedule, banks make a series of decisions. First, before any funds are used the bank must determine how much cash is needed to provide adequate primary reserves. By "adequate" Robinson means the minimum cash position the law and ordinary standards of operation permit. The central provision for bank safety is met by the second decision, i.e. what level of secondary reserves should be maintained.

Secondary reserves are comprised of securities that can be converted to cash quickly without significant risk of loss of principle. Thus, secondary reserves permit the bank to obtain cash for even remote contingencies without needlessly foregoing income (necessary for long-run safety).

Once all contingencies are planned for, (i.e. bank is made safe) the bank concentrates on its third priority, that of making loans. According to Robinson, this is the banks true forte. This is a function in which they have a comparative advantage over other financial institutions and should thus be exploited as the principal source of income after safety has been provided.

The fourth priority, open-market investments is simply a residual function. If any funds are left over once customer credit

demands are met, they are put to work earning income from this source.

Robinson's model of bank funds management is useful in providing insights about the objectives of the bank. However, changing institutions manifested by bank liability management have nullified his implicit assumption of deposits being exogenous. Furthermore, theoretical advances in this method of solving the conflict between profitability and safety have supplanted his "rules of thumb" as a more defendable approach to such decisions.[1]

Another widely acknowledged work in the "rules of thumb" approach is the study by Hodgman (18). The bulk of the study is a systematic description of bank rules for loan and investment policy obtained from commercial banks via detailed interviews. Hodgman's major contribution comes in two Chapters (X and XI) where he develops the concept of the "customer relationship."

The customer relationship concept stresses the opportunity cost of not making a loan. By making a loan the bank establishes a relationship which enhances its opportunity to earn income via the provision of other bank services to the customer, e.g. payments services, trust services, etc. In particular, Hodgman emphasizes the potential for an intertemporal relation between current loans and future deposits. Thus, if a bank rejects a loan request it must consider the foregone benefits of the potential deposits.

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[1]It is interesting to note that Robinson's solution closely approximates the first 'bank' model discussed by Hester and Pierce. This model maximizes profit.

Like Robinson, Hodgman never defines a specific objective function and optimization procedure. However, his concept of the customer relationship does appear in some form in subsequent models which adopt this procedure.[1]

The Profit Maximization Approach

This approach has been used for two distinct purposes. First, it has been utilized to obtain theoretical results that will help explain certain phenomena related to banking. This is the primary purpose of the Wood (34) study. The second purpose is to provide an operational procedure for bank management. This is the intent of the Chambers and Charnes (7) and Cohen and Hammer (8) papers.

Wood is motivated by a desire to explain an apparent paradox in the cyclical variation of aggregate bank loans and security holdings vis à vis rates of return on loans and securities. Specifically, he notes that loan rates tend to increase substantially less than security yields during expansionary periods. This would seem to suggest that securities would become more attractive relative to loans. However, the data indicate that bank holdings of securities relative to loans, decline during expansion. The converse of this is also observed.

In an attempt to explain this empirical fact, Wood develops a theory in a multiperiod time horizon that accounts for the bank-customer relationship as defined by Hodgman. Wood's 'bank' maximizes

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[1] See Wood (34), Chapter 2, and Hester and Pierce (17), Chapter 3.

the discounted stream of current and future profits. Profits are derived from returns on loans and securities. This, the objective function, is maximized subject to five constraints, two of which embody the customer relationship. These are: the loan demand function that includes as a determinant past loans with customers and the bank's deposit function that also includes past loans as a determinant. The other constraints involve institutional factors. After substitutions the decision variables are the current loan level and loan rate.

Abstracting from uncertainty, Wood applies the classical programming method to obtain analytical results. These results relate the effects of changes in economic activity on loans and loan rates of the individual bank. The relationships are complicated functions of the parameters of the loan and deposit demand functions.[1]

Wood obtains time-series estimates of the parameters of these functions and concludes that estimates support the customer relationship hypothesis. Furthermore, he concludes that the estimates are consistent with a customer relationship that is capable of explaining the observed phenomena described above.

Finally Wood compares his theory with a paper on credit rationing by Jaffe and Modiglianni (discussed in the next section). The two theories offer alternatives for explaining the empirical regularities.

The second purpose identified above was that profit maximization be utilized to obtain practicable results for bank managers.

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[1]Because of space required we cannot specify the relationships here. They may be found in Wood (34), Chapter 4 and Appendix A.

The method invoked for this type of problem has been linear programming (LP).

In their path breaking article, Chambers and Charnes use LP in an attempt to find an asset mix which maximizes returns over a planning horizon. Returns are maximized subject to constraints that incorporate rules for maintaining safe and liquid distributions of assets. These rules impose restrictions on the bank balance sheet and have been determined by bank examiners of the Federal Reserve System. Also embodied in the constraints, are restrictions on acquisitions of assets. These restrictions reflect assumptions about sales of loans and securities before maturity.

The solution to this problem provides the optimal mix of specified bank assets including cash, government securities of various maturities, other securities and loans. Although the analysis is oversimplified for actual practice, it was one of the first models to explicitly define the problem in a multi-period time horizon.

Actual practical application of this approach did follow. Cohen and Hammer (8) generalized the Chamber-Charnes model in two ways: (1) they maximized the present value of the net income stream plus realized capital gains (losses) and stockholders equity at the end of the period; (2) they included more realistic formulations of restrictions about the market and intertemporal linkages.

According to McKinney (25) this model was used by a large New York City bank during much of the sixties. McKinney notes, however, that the model requires accurate forecasts of future interest

rates. It is no surprise that the anomalous events of the early seventies led to its demise in practice.

Expected Profits Maximization

Included in this section are perhaps two of the most interesting models. The first is that of Friemer and Gordon (14). This model is subsequently invoked and extended by Jaffe and Modigliani (20). Both papers utilize this model to provide a rationale for credit rationing. The second model is one developed by Hester and Pierce (17). Actually they offer a set of models from which they derive an empirically testable proposition about a bank's asset response to deposit shocks.[1] We shall discuss the Friemer and Gordon model first.

Friemer and Gordon derive a loan offer function for a bank faced with a number of borrowers, each seeking financing for an investment project. For each borrower the bank forms a perception of the risk dependent on the outcome of the project. This perception is in the form of a density function describing the end-of-period value of the firm. Assuming that the size of the project is fixed, the outcome of the loan is independent of the size of the loan. The loan is not the only source of funding, but the lending bank has sole claim to all income if proceeds from investment fall short of the contracted repayment.

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[1] We will concern ourselves with the development of the proposition but not the empirical test.

The outcome of the loan is divided into classes--full repayment and partial repayment. The expected revenues from full repayment equal the contracted repayment times the probability that the outcome of the project is of a size sufficient to make full repayment (including interest). The expected revenues under partial repayment equal expected repayment if proceeds are not sufficient for full repayment in which case the bank receives the entire amount of proceeds realized. Expected profits are simply the sum of these two components less the opportunity cost of the funds for the bank.

Expected profits are maximized with respect to the size of the loan. The first-order condition is interpreted to mean that the size of the loan is such that the probability of default is equal to the excess of the loan rate over the marginal opportunity cost discounted by the loan rate.

The loan offer function is obtained by solving the first-order condition for the size of the loan in terms of the interest rate. This gives the locus of optimal loan size, L^*, for different levels of interest, r.

Jaffe and Modigliani (20) show that if we consider a uniform density function, the loan offer function has some general properties illustrated in Figure 2.1. That is, the loan offer function initially slopes upward but ultimately bends downwards. This downturn reflects two facts: the outcome of the investment is fixed with respect to the size of the loan and the bank will not extend a loan beyond the amount such that the contracted repayment exceeds the maximum possible outcome of the investment project. That is, $(1 + r)L^* \leq C$, where C

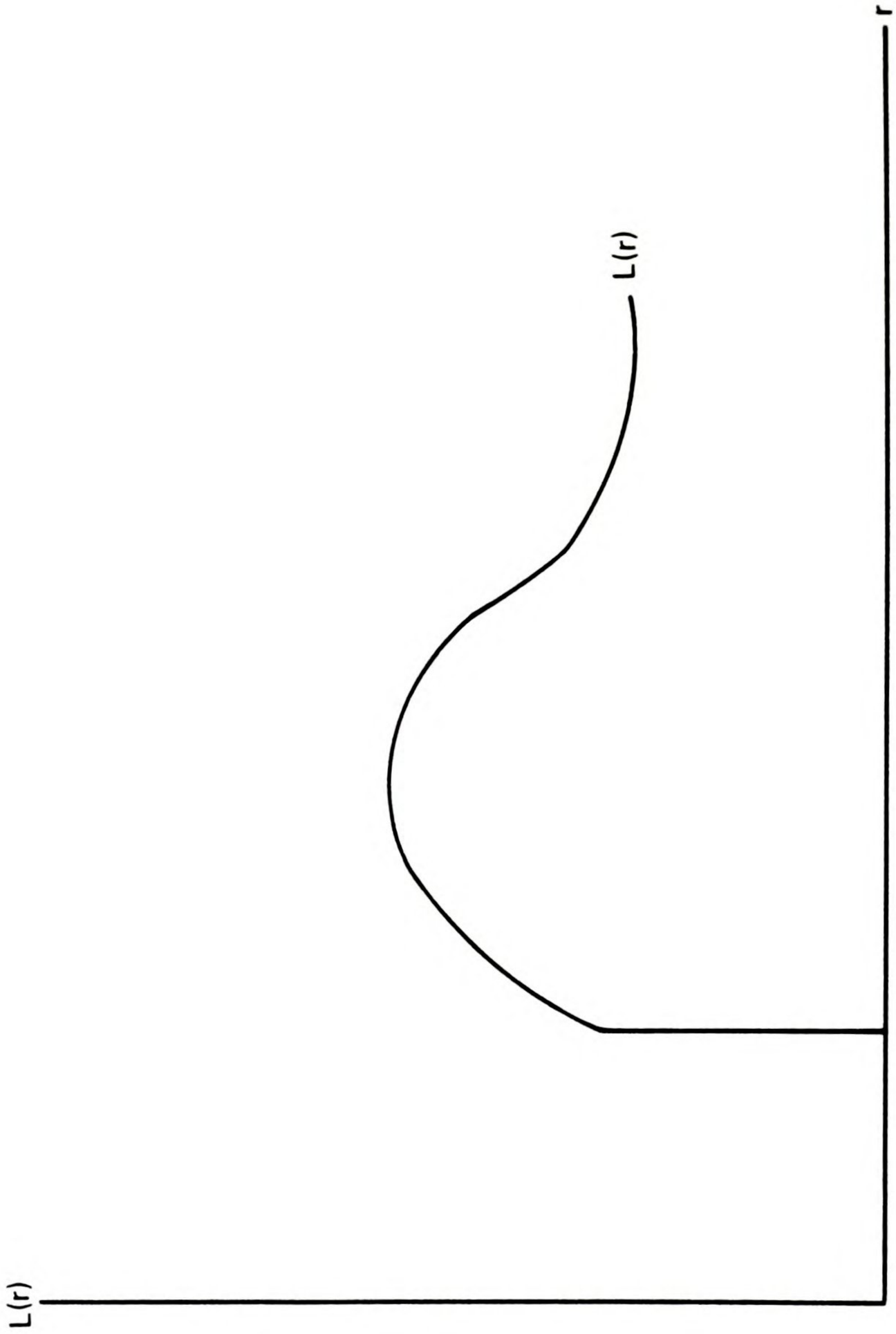


Figure 2.1--The Jaffe-Modigliani Loan Offer Function.

is the maximum possible outcome of the project in dollars. Thus the limit of L^* is zero as $r \to \infty$. Hence beyond some point the optimal size of the loan does not increase and eventually decreases.

Though this model explicitly incorporates uncertainty into the derivation of the loan offer function, it implicitly assumes banks are risk neutral. This is true of all models which maximize expected profits.

As noted above Hester and Pierce offer several bank models. The models are developed as a sequence of generalizations of a simple "archetypal" bank. For this first model, deposits are exogenous but vary within a known range. In the tradition of Robinson and Porter (28), the assets of the portfolio of such a bank are determined in the following manner. First, the cash level is determined by the reserve requirements on the initial level of deposits. Then the level of marketable securities is determined to ensure necessary liquidity for the deposit low. Finally, the most profitable but unmarketable asset, loans, are made with the balance of the funds. This solution follows from standard profit maximization under his given profits function and linear constraints.

The first step of generalization allows for deposit predictability. It adopts an expected profits maximization approach. Its major conclusion is that when deposits behave according to a stationary stochastic process, banks will place all their capital, but only a fraction of their deposits, in loans. Am empirical implication of this hypothesis is that banks with high capital-deposit ratios should have high loan-asset ratios.

Next, Hester and Pierce step back by assuming no deposit uncertainty, but introduce a model that accounts for growth of bank deposits, capacity ceilings on a non-portfolio factor of production and factor market imperfections. Though no analytical solution is obtained, it is clear from their sketch that this model is capable of exhibiting lagged portfolio adjustments.[1]

The authors then analyze bank behavior under uncertainty about deposits (that may grow over time) and with costs of portfolio adjustments (which are lumpy and derive from imperfect labor markets). This 'bank' seeks to maximize expected profits.

The major hypothesis of this model is that net rate of return (net cost of adjustment) for any given asset purchased with the funds from a deposit inflow is a function of the number of periods since the inflow occurred. Figure 2.2a illustrates the hypothesized relationships for loans, securities, and cash.[2] Loan response to deposit inflows is adumbrated by simulating six cases under selected regimes involving different degrees of foresight, loan maturity, and adjustment non-linearities. The major result of this model is that lending capacity, loan maturity, and deposit predictability are very important determinants of the time path of the bank's assets in

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[1] A set of simulation solutions is provided after this model is synthesized with the model of deposit uncertainty. A discussion of these results follows in a subsequent paragraph.

[2] Figure 2.2a should be interpreted as follows: Time zero is the point of unexpected deposit inflow. Because of adjustment cost cash yields the highest return until t_1 when net return on securities becomes higher. At t_1 profit maximizing banks dispose of cash and hold securities until t_2 when loans yield more. The bank then sells its securities and acquires loans.

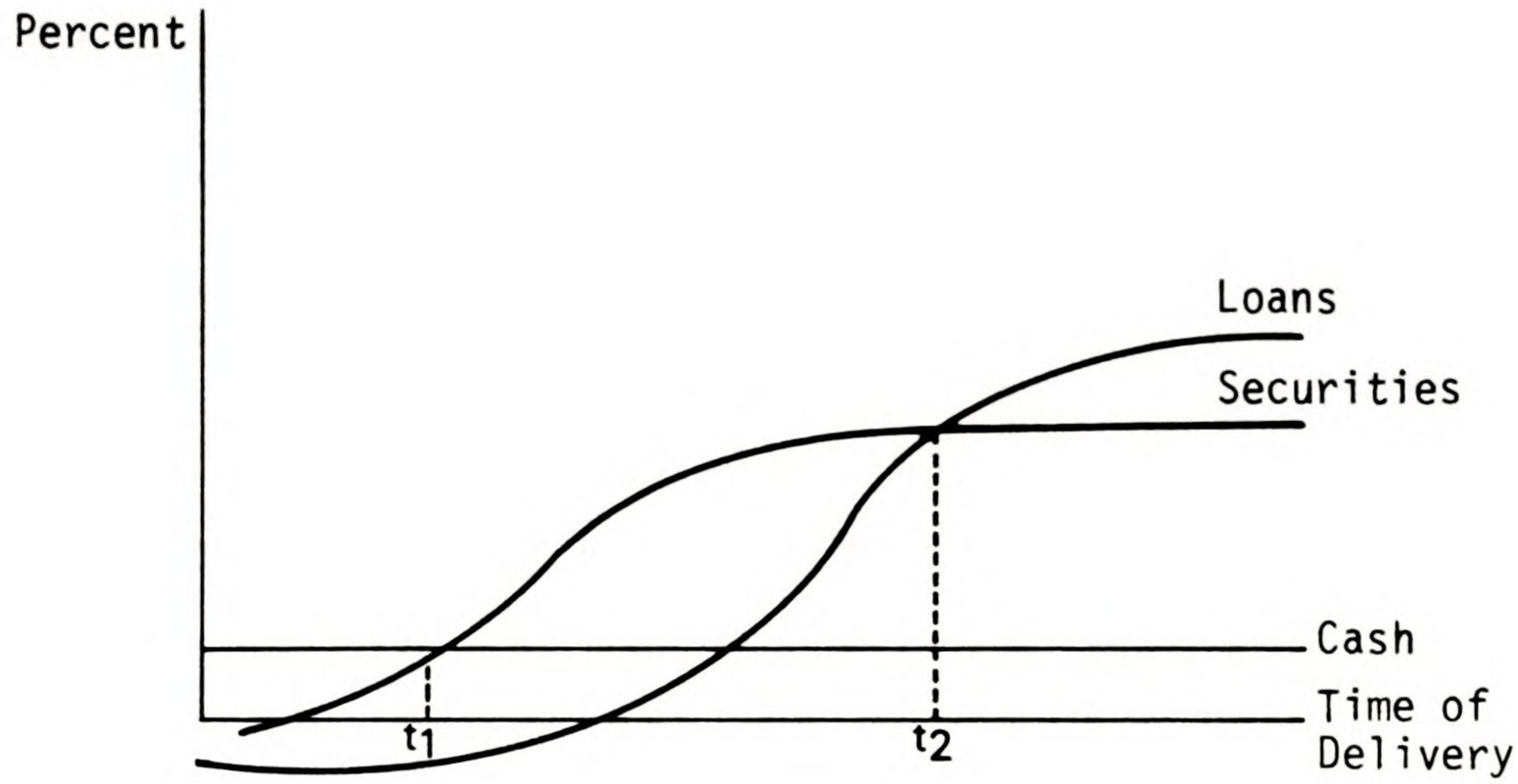


Figure 2.2a--Net Rates of Return for Different Short-Term Assets.

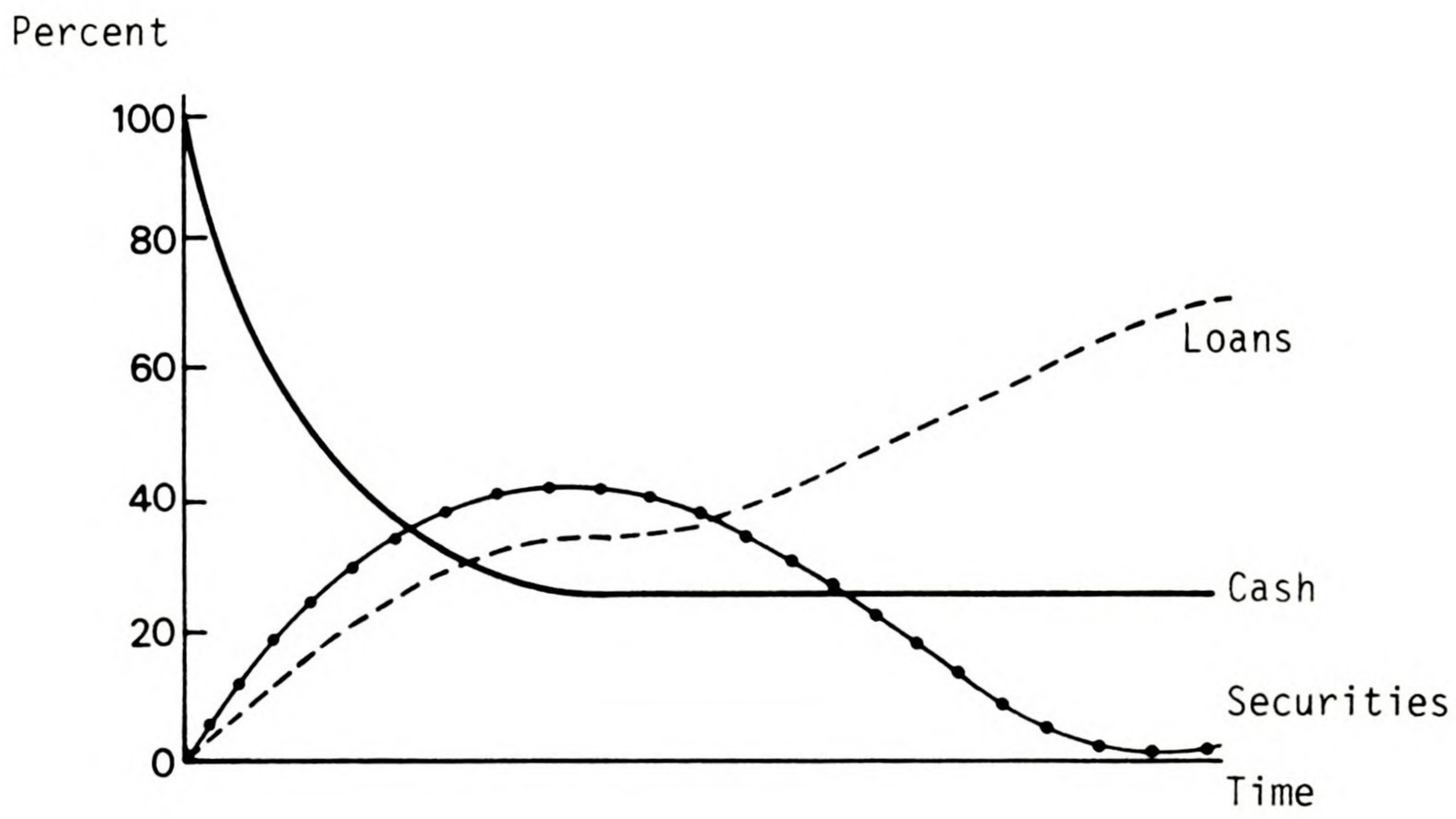


Figure 2.2b--Dynamic Portfolio Adjustment Paths.

response to deposit inflows. This model provides the basis for their subsequent empirical analysis.

The fifth model of the study deals with loan market imperfections. It is developed in a risk-return framework as an extension of the first model. Two results are: (1) deposit shocks should affect security levels instantaneously and the time sequence of deposits should accurately forecast changes in security levels. (2) Loan response to deposit shocks, on the other hand, should exhibit an irregular lag reflecting the fact that banks cannot instantaneously acquire loans at the highest available return.

From the collection of models summarized above, Hester and Pierce draw a priori predictions on the dynamics of bank portfolio adjustment to unanticipated deposit flows. In particular, they assume that deposit levels are described by an autoregressive process. Thus, the bank only expects to retain a small fraction of the deposit inflow. Because of the high transactions cost of making a loan the bank will hold the transitory component in cash or securities depending on its expected duration.

Considering factor market imperfections, banks will acknowledge lending costs of portfolio adjustment. These costs are subtracted from nominal rates to yield net rates of return. Hester and Pierce hypothesize that net rates follow a time path depicted by Figure 2.2a.

Finally, because of loan market imperfections, banks will hold the permanent component of deposit flows temporarily as securities until they are able to acquire the most profitable loans. This

result in conjunction with those discussed above, suggest that bank portfolios respond to deposit injections in a manner illustrated in Figure 2.2b.

Hester and Pierce empirically test the hypothesized lag structure. Though the data tend to confirm the hypothesis, we must note that the sample period ended in 1963. This is prior to the development of liability management. Thus, their assumption of deposit exogeneity is not so unrealistic.

The Expected Utility Maximization Approach

The models of this section are of two kinds. One is of the Tobin-Markowitz tradition implicitly assuming either a quadratic utility function or normality in the distribution of the prospect. The other is completely general. Both are attempts to answer questions about credit rationing.

Kane and Malkiel (22) modify the Tobin-Markowitz model to account for deposit variability and for considerations of long-run profits. The model maximizes expected utility. Utility is a function of expected profits and its risk as measured by the variance of profits. Expected profit is a function of the expected return from loans and from government securities, the only two assets of the model. The balance sheet constraint requires that the sum of these two equal the sum of deposits and capital. Deposits are assumed to be random.

Concern for long-run profit involves a variation of Hodgman's customer relationship concept. Kane and Malkiel introduce a variable

that accounts for the quality of relationship for each customer. This variable enters the model as a determinant of the probability distribution of deposits. Refusing loan accommodation to a customer may affect the banks aggregate risk exposure through its effect on this relation, i.e., alienation of customer caused by loan denial. Kane and Malkiel argue that such a class of customer exists. They call loan applications from this class L^* loans.

The main result of the paper follows from the condition at optimum: mere receipt of an L^* loan request disturbs a banks portfolio optimum. Accepting such a loan might leave the bank with a higher risk/profit ratio. But the consequences of rejecting the loan may be even greater.

The implication of this is that during expansion banks will tend to make more loans than traditional analysis would indicate. That is, many loans that would be turned down under traditional analysis will be accepted by the Kane-Malkiel bank.

The problem of credit rationing is also addressed in a paper by Azzi and Cox (3). The purpose of their paper is to establish a relationship between collateral, borrower equity and the interest rate a bank will charge for an offered loan. They conclude that existence of these relationships (and the consequent trade-offs at optimum) obviate the need for the assumption of Jaffe and Modigliani (that lenders are discriminating monopolists) to explain why borrowers with different demand functions are charged different rates of interest.

Their approach is similar to the one taken by this study. They consider a borrower who seeks funds for an investment project. The outcome of the project in relation to the investors equity determines the outcome of the loan.

Should the bank engage in this loan, its wealth at the end of the period becomes a function of the random outcome of the loan. The bank maximizes the expected utility of its terminal wealth. Collateral enters the relationship via its relation to the default rate of return. That is, the rate of return on the loan when the project fails and default ensues.

From optimum conditions, Azzi and Cox derive propositions relating collateral and equity levels to optimal size of loan under different market structures and assumptions about bank risk. Their major result is that optimal size of loan increases with an increase in collateral or equity. This result is consistent with one of the results of this study.

Azzi and Cox, however, do not consider other important terms of the loan--maturity and compensatory balances--which would also give results for their purpose.

CHAPTER III

THE LOAN OFFER FUNCTION--FIXED TIME PERIOD

In this chapter we shall investigate the relationship between four major terms of a loan: size of loan, contract rate of interest, collateral requirements and compensating balances.[1] The agent making the loan will be a bank. We shall assume that the bank maximizes utility in the von Newmann-Morgenstern sense. The utility function may be either (a) that of a single owner, (b) representative of the stockholders as a group, or (c) representative of the management. In any case, it will be assumed to exhibit risk aversion but not necessarily be independent of wealth.

In particular, we shall consider a loan which is sought to finance an investment proposal. It may be that the size of the project is small relative to the current market value of the firm, or it could be a totally new enterprise for a nascent firm. In the case of the latter, the entrepreneur would be seeking a loan as a supplement to the initial capital.

The Bank's Perception of Risk

It goes without saying that the bank will require information about the use of the funds. The bank will want to make its own

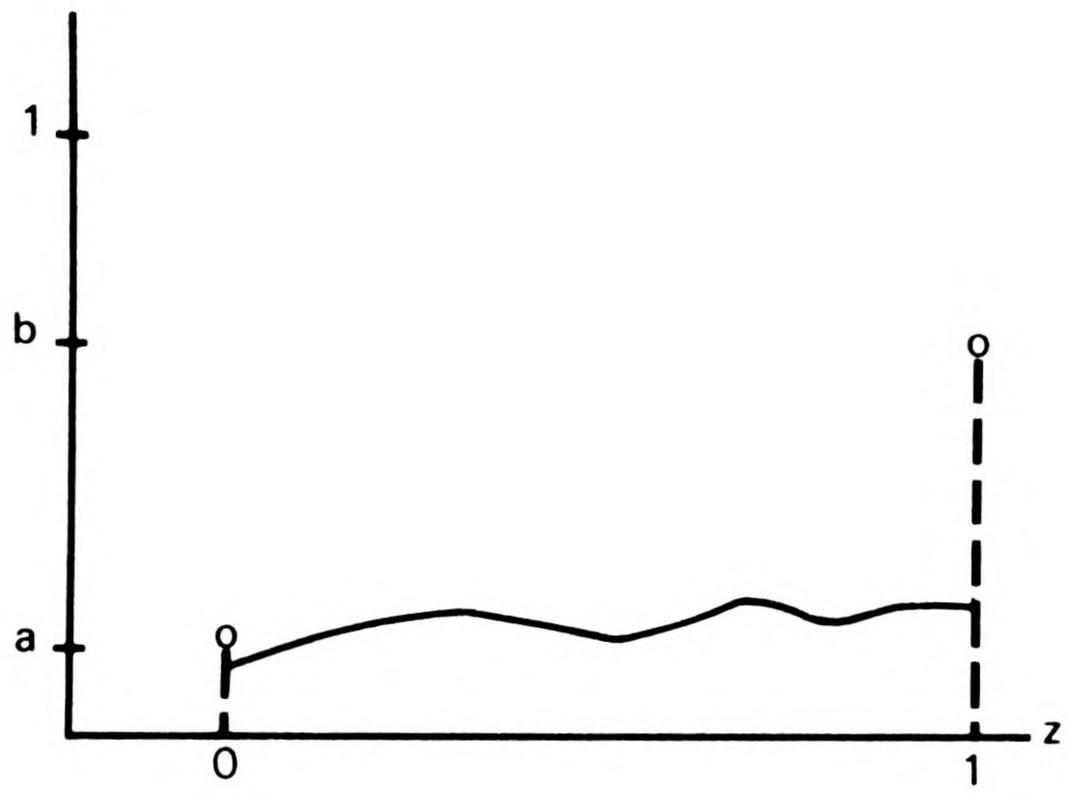
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[1] Because of the complexities which arise, we shall introduce maturity as a term in Chapter V.

judgment about the auspiciousness of the investment. It will seek information pertaining to the potential for success of the proposed investment. Success or failure will be measured by the ultimate impact of the project on the distribution of the future stream of the firm's profits. It will also seek information pertaining to the potential for success of the total enterprise during the period of the loan. This information is used by the bank to form some notion about the risks involved in making this loan.

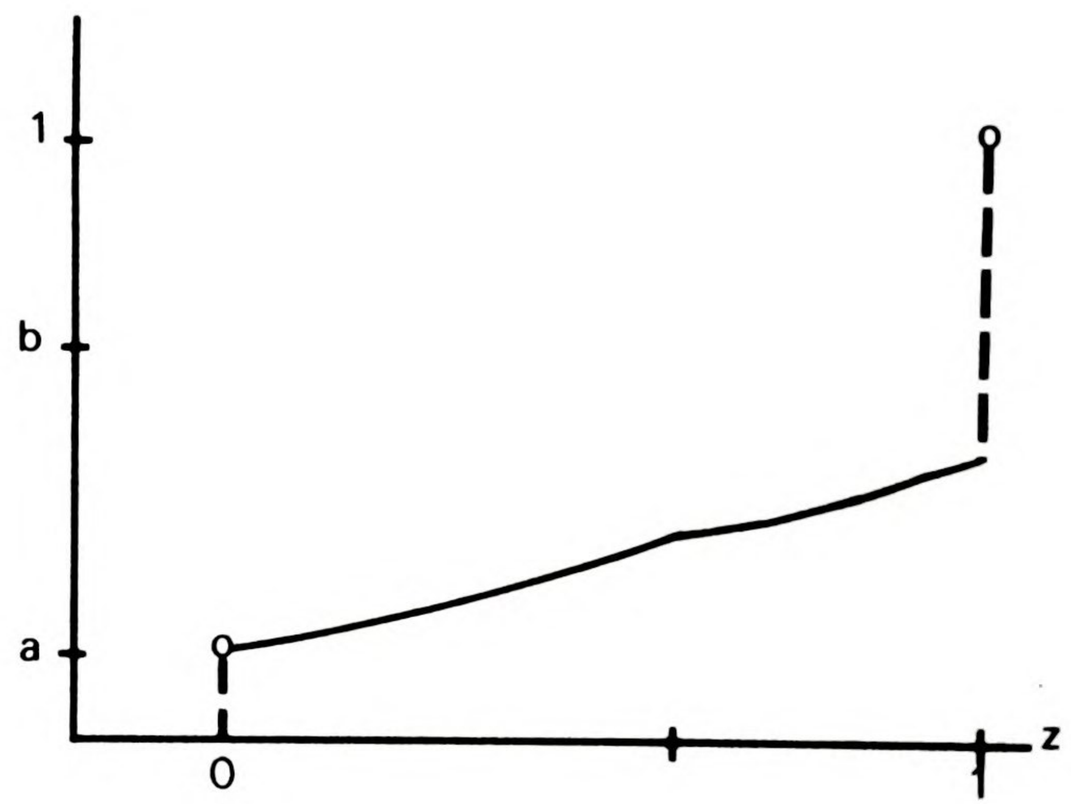
The bank will form some idea about the probability of complete success, i.e. full repayment. Graphically, this may be represented by point 'b' in Figure 3.1. For most bank loans which are ultimately made, 'b' will be large and near one, though in general it need not be. Also, the bank will form some idea about the probability of a complete failure, i.e. zero repayment. This may be represented by point 'a' in Figure 3.1 and will likely be small for loans which are made. The bank will also have some notion about the probability of partial repayment. This may be represented by the density over the open interval (0,1). The probability that Z, the proportion of repayment, will fall in this interval is the area under the density function.

A more fundamental basis for this perception of risk will now be provided. In forming its notion of risk, the bank will seek information concerning the future cash-flow of the firm as a whole and the financial obligations it has. Cash-flow is defined as total revenue less payments by the firm for goods and services employed. In the single period model, end-of-period cash-flow might be perceived



proportion of repayment

(a) probability density



proportion of repayment

(b) cumulative distribution

Figure 3.1*--A Bank's Perception of Risk.

*In general, points a and b may take any conceivable values and the density can assume any conceivable shape.

by the bank to have a probability density given by g(CF) in Figure 3.2.[1] Since we are dealing with a one-period model, all capital is assumed to be either consumed or sold for revenue at the end of the period. Thus, the value of capital consumed will be less than the total investment which is denoted by X in the figure. The investment is financed with funds obtained from the issuance of capital stock (repurchased at the end of the period) and funds obtained from general creditors.

The outcome of cash-flow determines the outcome of the loan. The outcome of the loan has been divided into three mutually exclusive classes: zero repayment, partial repayment and full repayment. Each of these classes corresponds to a range in the set of outcomes of CF.

The bank will not receive any repayment when the cash-flow is zero or less; thus, the probability of zero repayment will equal: $\int_{-\infty}^{0} g(CF)dCF$. In general, this will equal some number greater than or equal to zero but less than one; therefore the probability of zero repayment will be a point mass as depicted by 'a' in Figure 3.1.

Before we discuss the second class of outcomes of the loan, we can easily obtain the probability density of the third class which will also be a point mass. It is obtained by integrating the density g(CF) from L(1 + r), where L is the value of funds supplied by general creditors and r is the contract rate of interest on the loan, i.e. $\int_{L'}^{\infty} g(CF)dCF$, where L' = L(1 + r). This is simply the

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[1]Note: Figure 3.2 is only one possible form of g(CF). The function is completely general.

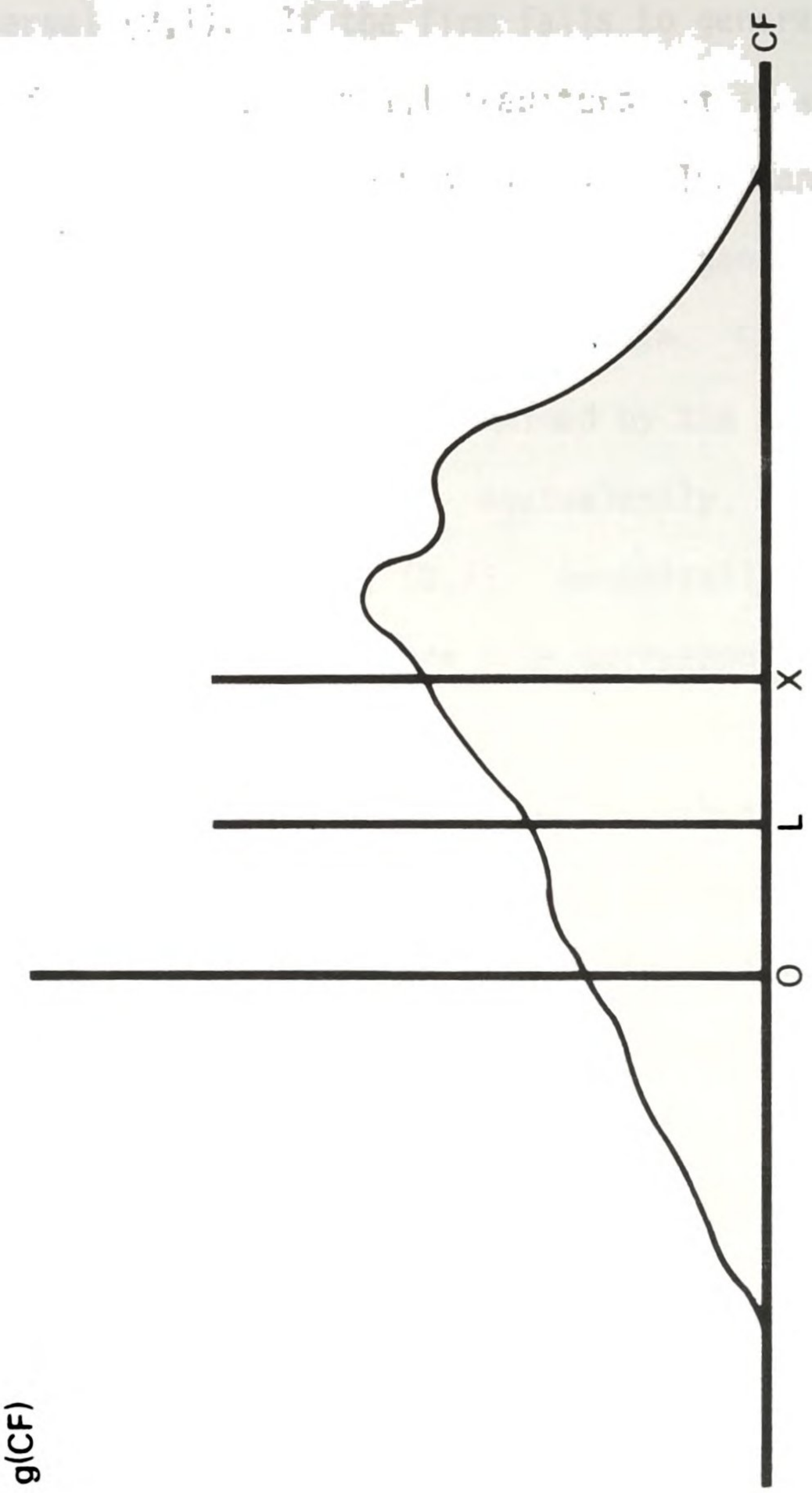


Figure 3.2--A Bank's Perception of Cash Flow.

probability that the cash-flow will be greater than that which is requisite for full repayment of all loans.

The second class of outcomes will be distributed along the open interval (0,1). If the firm fails to generate sufficient cash-flow for full repayment of all creditors, it is assumed that some creditor will force it into bankruptcy.[1] The Bankruptcy Act dictates that the funds available for such a group (general creditors) are to be distributed in proportion to each claim. Consequently the proportion of repayment, Z, is determined by the proportion of L' which is generated by the cash-flow: equivalently, Z is distributed the same as $\frac{CF}{L'}$ in the interval, (0,1). Graphically, the density function in the interval (0,1) in Figure 3.1a corresponds to the density in (0,L') in Figure 3.2.

In summary, we find that the distribution of Z will be, in general, both discrete and continuous, and may be conveniently expressed in the Stieltje's integral: $P(Z \leq x) = \int_0^x dF(z)$.

If we allow for collateral, we introduce the capacity to affect the lower bound of the distribution. For example, if the bank has a rule that a certain proportion of the loan is to be secured, this restricts the outcome of the loan. Specifically, suppose that λ is the proportion of the loan to be secured. The probability that Z will be less than λ is zero, but the probability that Z will be exactly λ is given by $P(Z \leq \lambda)$; i.e. $\int_0^\lambda dF(z)$. The distribution in the interval $(\lambda,1]$ will be identical to the distribution without

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[1] In the one-period world it is in the interest of some creditors to force bankruptcy. This may not be true in the multi-period milieu.

collateral. Graphically, this may be expressed in Figure 3.3. The point mass at λ will equal the point mass at zero (a) plus the area under the original density in the interval $(0,\lambda]$. This is represented by point 'a' in Figure 3.3.

A Model of the Bank's Loan Decision

Given the notion of risk developed above, the bank will make its decision whether or not to make the loan. If it chooses to make the loan, it must select the optimal size. As noted above, the bank does this by maximizing expected utility. It is assumed that utility is derived from profits which in turn is a function of the outcome of the loan, Z. The profit function is given by:

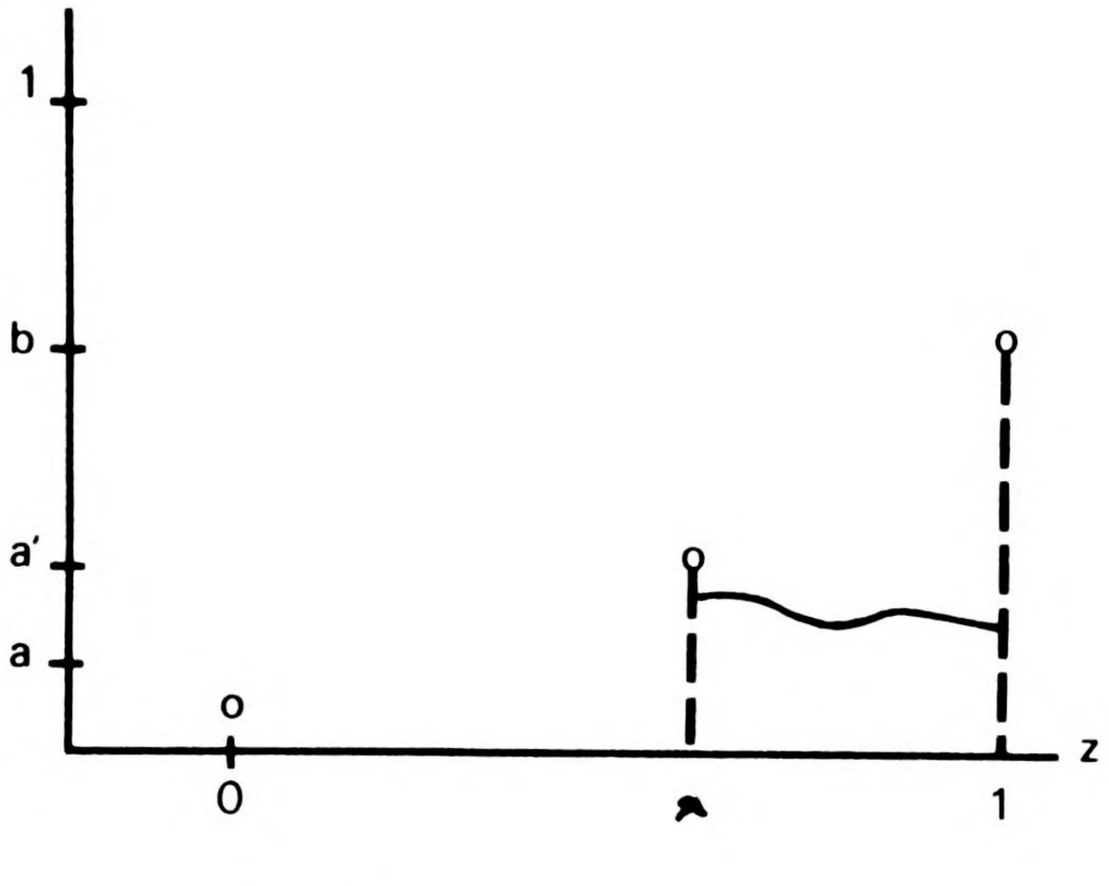
$$\pi_Z = \pi_O + A[Z(1 + r) - 1] - e(A)$$

where π_O equals profits which are independent of the loan decision (assumed to be known with certainty), A is the size of the loan, and e(A) is the cost function. Costs are assumed to increase with the size of the loan at a constant or increasing rate so that $e' > 0$ and $e'' > 0$. Since the bank has several sources to obtain its funds, it will utilize that source whose marginal cost is the lowest.[1]

In the model, A is the only decision variable; the other terms of the loan are assumed to be parameters and not under the direct

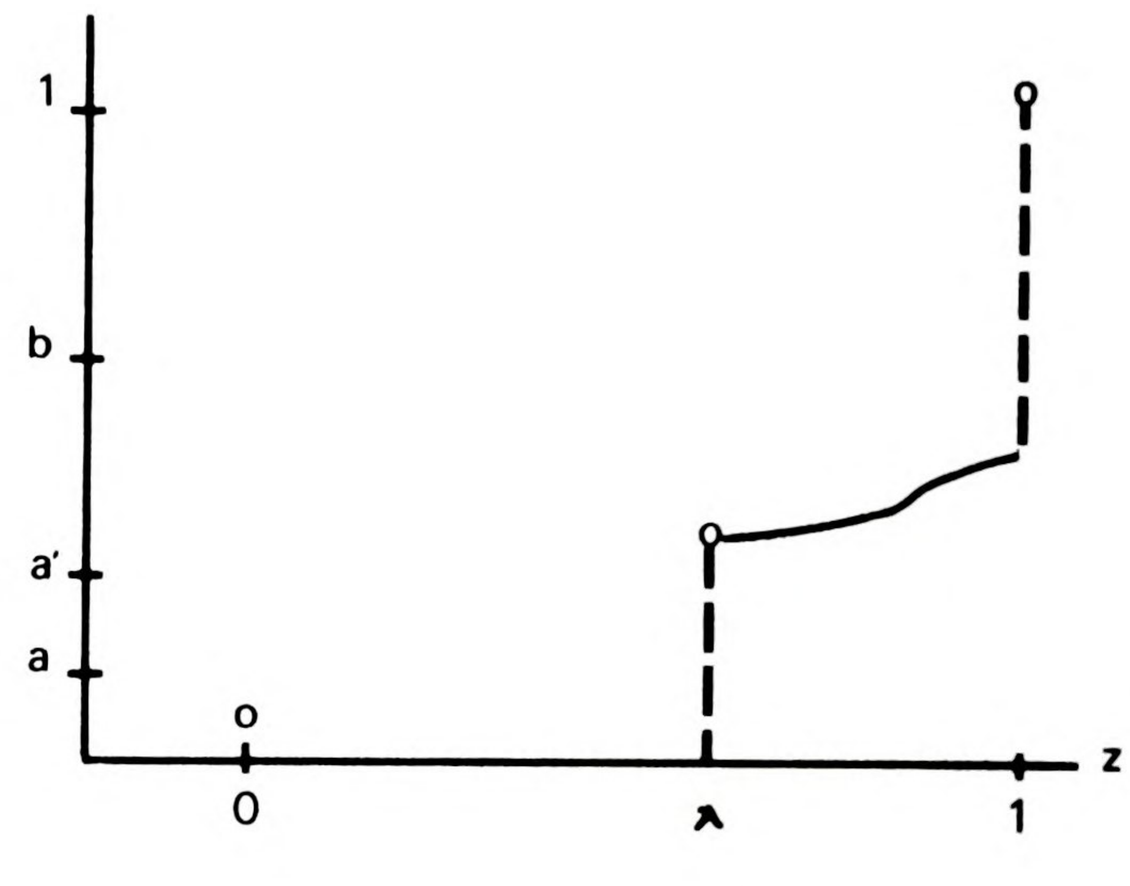
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[1]We assume here that the marginal cost of maintaining a dollar of funds for the current period is known with certainty. This is obviously true for funds acquired from the negotiable certificates of deposit.



proposition of repayment

(a) probability density f(z)



proposition of repayment

(b) cumulative distribution F(z)

Figure 3.3--Risk Perception with Collateral.

control of the decision maker.[1] Implicitly, then, utility is a function of random variable, Z, and decision variable, A, i.e. u(Z,A). We can now develop the model by starting with the simplest case.

Case I: A World Without Collateral or Compensating Balances

The objective function is given by the Stieltje's integral:

$$(1)\quad EU = \int_0^1 U(\pi_z)dF(z)$$

when (1) is maximized with respect to A we obtain:[2]

$$(2)\quad \text{FOC } \frac{\partial EU}{\partial A}: \int_0^1 U'(\pi_z)[z(1 + r) - 1 - e'_A]dF(z) = 0$$

$$(3)\quad \text{SOC } \frac{\partial E^2U}{(\partial A)^2}: \int_0^1 \{U''(\pi_z)[z(1 + r) - 1 - e'_A]^2 - U'(\pi_z)e''_A\}dF(z) < 0$$

The first question we ask is: when will the loan be made? That is, when will the optimal size loan, A^*, be greater than zero? A sufficient condition is given by $\frac{\partial EU}{\partial A}(A = 0) > 0$. The condition is satisfied when $E[Z(1 + r) - (1 + e'_0)] > 0$, i.e. when expected marginal profit from the first dollar loaned is positive.[3]

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[1]The effective rate of interest will be determined by the market. Since banks take as much collateral as they can reasonably get, λ is dictated by the circumstances. The determining factor for compensating balances will be discussed later.

[2]For the rule guiding differentiation under the Stieltje's integral sign see Apostol (1), p. 7.

[3]This follows from the fact that at A = 0 profits are independent of Z; hence so is utility which may then be factored out of the integral. This result should not be surprising since we know by

Alternatively, this means that the expected gain from the contracted repayment of the first dollar loaned is greater than the marginal loss resulting from costs of loaning that dollar, i.e. $E(Z)(1 + r) > (1 - e'_A)$.

Taken together, the conditions given by (2) and (3) are sufficient for an interior maximum. The SOC is satisfied under the original assumptions of the model. The FOC, however, may not be obtainable when marginal cost is constant. In this case, if $\frac{\partial EU}{\partial A}$ is positive for $A \geq 0$, then the bank will try to loan as much as the loanee will accept as long as it does not affect the distribution of repayment.[1] Ultimately, however, the cost of funds will increase or the distribution perceived will be modified to account for the increased risk.

An implication of the FOC is that the optimal size of the loan is less under uncertainty than it is under perfect certainty (i.e. if the distribution were collapsed on a point). Under the latter condition the FOC implies the standard result that marginal revenue, r, equal marginal cost, e'_A. This follows from the fact that under perfect certainty the FOC is:

Samuelson's Favorable Bet Theorem that any expected utility maximizer can be coaxed into a favorable bet if the scale is small enough; see Samuelson (32).

[1]Any enterprise which borrows money for investment is limited by its existing resources as to the way it can effectively use the funds. When these limits are approached the distribution of outcomes becomes conditional on A, $F_A(Z)$. As A increases the mass of the distribution shifts from right to left; that is, the probabilities of higher repayment decrease relative to probabilities of lower repayment.

$$U'(\pi_1)[(1 + r) - (1 + e'_A)] = 0$$

Since marginal utility is always positive, the condition holds only when $r = e'_A$.

For an interior solution under uncertainty, the expression $[Z(1 + r) - (1 + e'_A{}^*)]$ must change in sign at least once as Z increases from 0 to 1. Otherwise, the total expression on the left hand side of FOC will not net to zero. Since this expression monotonically increases with Z, the sign changes only once from negative to positive. Thus, when Z is at its maximum, 1, the term $[(1 + r) - (1 + e'_A{}^*)]$ must be positive. Thus: $r > e'_A{}^*$. Since e'_A is positive and non-decreasing with respect to A, the optimal A* must be less than the A^+ for which $r = e'_A$. That is, A* will be less than the optimal value of A under the certainty model. Further, to induce the bank to lend the same amount under uncertainty as it does under certainty the borrower must pay a higher rate of interest.

If the integrand of the FOC is strictly concave in Z, we may extend this result to apply increases in risk in the sense of a mean preserving spread.[1] That is, the greater the risk, the lower the optimal level of A. This follows from a Theorem of Rothchild and Stiglitz.[2] Stated in our notation it says:

> Theorem: Let $A^*(r)$ be the level of the control variable which maximizes $\int u(z,A)dF(z,r)$. If increases in r represent mean

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[1]The integrand is strictly concave for quadratic utility functions. For the definition of mean preserving spread see Rothchild and Stiglitz (30).

[2]See the sequel article by Rothchild and Stiglitz (31) Theorem III.

preserving increases in risk, then A* increases (decreases) with r if u_A is a strictly convex (concave) function of z, i.e. if $u_{Axx} > (<) 0$.

This result has implications for the marginal profit rate. As with traditional microtheory, we assume that we are operating on the rising portion of the marginal cost curve. Thus, a decrease in A^* implies a lower marginal cost at optimum. Since marginal revenue is constant with respect to A, the marginal profit rate increases with a decrease in A^*. Thus, an increase in risk results in higher optimum profit rate, i.e. a higher premium for risk.

We can now analyze the effect that a change in the parameter r will have on the optimal level of A. To do this we differentiate the FOC with respect to r and we find,

$$(4)\ \frac{dA}{dr} = -\frac{R}{SOC}$$

where

$$R = \int_0^1 \{U''(\pi_z)(Az)\ \frac{\partial \pi z}{\partial A} + U'(\pi_z)z\}dF(z)$$

and

$$\frac{\partial \pi_z}{\partial A} = Z(1 + r) - 1 \cdot e'_A$$

Since SOC < 0 under the assumptions above, $\frac{dA}{dr}$ will be the same sign as R.

Now, R may be rewritten so:[1]

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[1]Dividing by π_z and $U'(\pi_z)$ require that we assume that total profits are positive for all outcomes of the loan, i.e. profits

$$R = \int_0^1 \{U'(\pi_z) \cdot z \left[\frac{U''(\pi_z)}{U'(\pi_z)}(A)\frac{\partial \pi_z}{\partial A} + 1\right]\} dF(z)$$

$$= \int_0^1 \{U'(\pi_z) \cdot z \left[\frac{U''(\pi_z)\pi_z}{U'(\pi_z)} \cdot \frac{\partial \pi_z}{\partial A}\frac{A}{\pi_z} + 1\right]\} dF(z)$$

$$= \int_0^1 \{U'(\pi_z) \cdot z \left[-\rho(\pi_z) \cdot \varepsilon(\pi_z, A) + 1\right]\} dF(z)$$

where $\rho(\pi_z)$ is the relative risk aversion parameter, and $\varepsilon(\pi_z, A)$ is the elasticity of profits with respect to the size of the loan.

Careful inspection of $\rho(\pi_z)$ reveals that it is the elasticity of marginal utility with respect to profits. Hence the product of the two elasticities yields the elasticity of marginal utility with respect to the size of the loan, $\varepsilon(MU,A)$. If this elasticity is less than one, R will be positive; hence $\frac{dA}{dr}$ will also be positive. That is, $\varepsilon(MU,A) < 1$ is a sufficient condition for $\frac{dA}{dr}$ to be positive.

If we adopt Arrow's hypothesis of increasing relative risk aversion, we may obtain a weaker sufficient condition.[1] Under the hypothesis, $\rho(\pi_1) > \rho(\pi_z)$ for all $z < 1$. In addition, we will now show under what conditions $\varepsilon(\pi_z, A)$ is a monotonically increasing function of z so that it too is at its maximum when $z = 1$. Substituting for $\frac{\partial \pi_z}{\partial A}$ we get

$$\varepsilon(\pi_z, A) = \frac{[z(1 + r) - 1 - e'_A]A}{\pi_z}$$

and

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independent of this loan, π_o, are sufficiently large to cover all costs in event of total default. Marginal utility is positive by an earlier assumption.

[1] See Arrow (2), p. 98.

$$\frac{d\varepsilon}{dz} = \frac{A(1 + r)\pi_z - [z(1 + r) - 1 - e'_A]A \frac{\partial \pi_z}{\partial z}}{(\pi_z)^2}$$

Now recall

$$\pi_z = A[z(1 + r) - 1] - e(A)$$

so

$$\frac{\partial \pi_z}{\partial z} = A(1 + r)$$

hence

$$(5)\ \frac{d\varepsilon}{dz} = \frac{-e(A) + e'_A \cdot A}{\frac{(\pi_z)^2}{(1 + r)A}}$$

$$= \frac{MC - AC}{\frac{(\pi_z)^2}{(1 + r)A^2}}$$

where AC is average cost and MC is marginal cost. If we assume fixed costs are sufficiently small relative to the size of the loan so that MC > AC, then the numerator of (5) is positive. Since the denominator of (5) is positive, $\frac{d\varepsilon}{dA}$ is positive and

$$\varepsilon(\pi_1, A) > \varepsilon(\pi_z, A) \text{ for all } z < 1$$

Hence the condition

$$(6)\ \rho(\pi_1)\ \varepsilon(\pi_1, A) < 1$$

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is sufficient for

$$\frac{dA}{dr} > 0$$

In economic terms: if the bank exhibits increasing relative risk aversion, and if fixed costs are sufficiently small relative to the size of the loan, then the optimal size of the loan will increase with r if the elasticity of marginal utility with respect to the size of the loan under certainty is less than one.

There is some theoretical basis for condition (6). First, Arrow (2) defends the idea that relative risk aversion is numerically somewhere close to one:[1]

> The important theoretical point is that the variation of the relative risk aversion with changing wealth is intimately connected with the boundedness of the utility function. It can be shown as a mathematical proposition that if the utility function is to remain bounded as wealth becomes infinite, then the relative risk aversion cannot tend to a limit below one; similarly, for the utility function to be bounded (from below) as wealth approaches zero, the relative risk aversion cannot approach a limit above one as wealth tends to zero. . .
>
> Thus, broadly speaking, the relative risk aversion must hover around 1, being, if anything, somewhat less for low wealths and somewhat higher for high wealths. Two conclusions emerge: (1) it is broadly permissible to assume that relative risk aversion increases with wealth, though theory does not exclude some fluctuations; (2) if, for simplicity, we wish to assume a constant relative risk aversion, then the appropriate value is one. As can easily be seen, this implies that the utility of wealth equals its logarithm, a relation already suggested by Bernoulli.

Second, the leasticity of profits with respect to size of loan can be interpreted as the ratio of marginal profits over average

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[1] See Arrow (2), pp. 98-99. In the same chapter, he cites as empirical evidence for increasing relative risk aversion the fact that the wealth elasticity of money (a relatively safe asset) is greater than one.

profits. Under the assumptions of the model, the profits function for z = 1 will be positive at A = 0, increase with A at a decreasing rate until a maximum is reached at which point profits begin to decrease with A. Thus, marginal profits are everywhere less than average profits as illustrated in Figure 3.4.

Hence

$$\varepsilon(\pi_1, A) < 1$$

This result in conjunction with Arrow's theoretical argument provides some support for the conclusion that the product of the two elasticities is less than one.

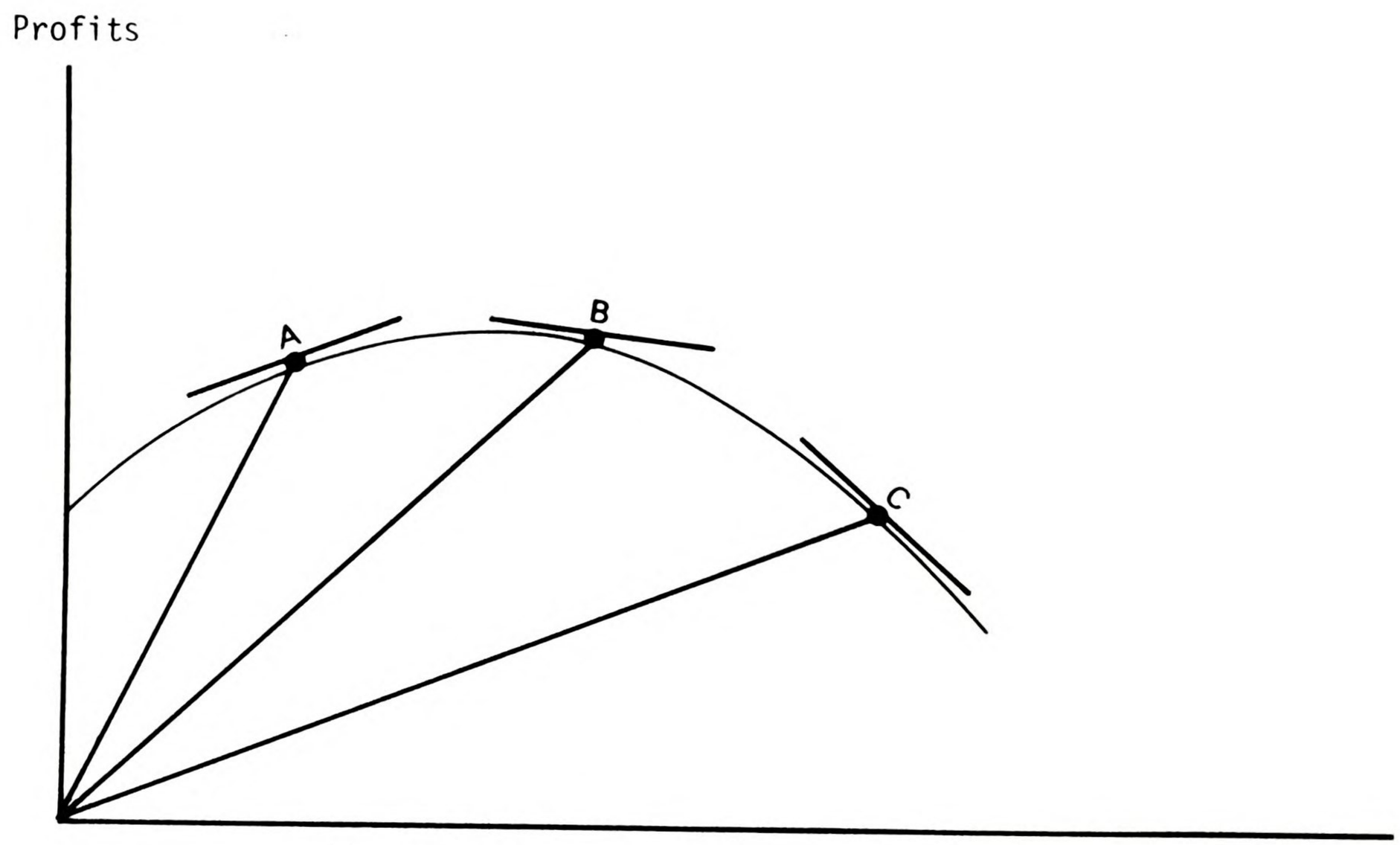
Case II. The Introduction of Collateral

The fact that collateral reduces risk to the lender is obvious. The exact nature by which the collateral relates to the other terms of the loan is not so obvious. We will now develop one plausible relationship.

Let λ be the proportion of the loan which is collateralized. The probability that the loan will default and that the bank will recover only the value of the collateral is the cumulative probability: $\int_0^{\lambda} dF(z) = p(\lambda)$.

The objective function in this case is given by the Stieltje's integral:

$$EU = p(\lambda)U(\pi_\lambda) + \int_\lambda^1 (\pi_z)dF(z)$$



Marginal Profits: Slope of function at point

Average Profits: Slope of ray to point

Figure 3.4--The Profits Function.

The first and second order conditions are:

$$\text{FOC}' \ \frac{\partial EU}{\partial A}: \quad p(\lambda)U'(\pi_\lambda)[\lambda(1 + r) - 1 - e'_A] + \int_\lambda^1 U'(\pi_z)[z(1 + r) - 1 - e'_A]dF(z) = 0$$

$$\text{SOC}' \ \frac{\partial E^2U}{(\partial A)^2}: \quad p(\lambda)\{U''(\pi_\lambda)[\lambda(1 + r) - 1 - e'_A]^2 - U'(\pi_\lambda)e''_A\} + \int_\lambda^1 \{U''(\pi_z)[z(1 + r) - 1 - e'_A]^2 - U'(\pi_z)e''_A\}dF(z) < 0$$

Again we find a loan will be made when $\frac{\partial EU}{\partial A}(A=0) > 0$. However, we can now make an additional statement for the case with collateral. A sufficient condition for making a loan is: $\lambda > \frac{1 + e'(A=0)}{1 + r}$. That is when a bank is guaranteed a profit (via security) on the first dollar loaned.

The second order condition, SOC', like SOC, is always negative, but like FOC, FOC' may not be attainable when marginal cost is constant. So, as in the case without collateral, a bank will be willing to lend as much as the borrower will take when $\frac{\partial EU}{\partial A} > 0$ for all $A \geq 0$.[1] A sufficient condition for this is: $\lambda > \frac{1 + e'_A}{1 + r}$ for all $A \geq 0$. An alternative way of interpreting this is that there must be some risk of negative profits for an interior solution to exist. That is: $z(1 + r) < 1 + e'_A$ for at least one outcome of the loan.

Since Rothchild and Stiglitz prove their theorem for a Stieltje's integral with arbitrary limits, the theorem also applies

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[1] Again we assumed that the amount lent does not affect the distribution of the outcome.

to the model with collateral. That is, as risk increases in the sense of a mean preserving spread the optimal size of loan decreases.

The effect of a change in the parameter r is the same result (in sign) as for the case without collateral. Again we have:

$$\frac{dA}{dr} = \frac{R'(\lambda,r)}{SOC'}$$

but where

$$R'(\lambda,r) = p(\lambda)\{U''(\pi_\lambda)A\lambda[\lambda(1+r) - 1 - e'_A] + U'(\pi_\lambda)\lambda\}$$

$$+ \int_\lambda^1 \{U''(\pi_z)(Az)[z(1+r) - 1 - e'_A] + U'(\pi_z)z\}dF(z)$$

This value of the term is dependent on the value of the relative risk aversion parameter, ρ, in the same way that R is. Hence condition (6) is also sufficient for the optimal size of loan to increase with r in the second case.

In addition to these results, we can now introduce a change in the parameter λ and analyze its effect on A*. To do this we differentiate FOC' with respect to λ and obtain:

$$M(\lambda,r) = U''(\pi_\lambda)A(1+r)[\lambda(1+r) - 1 - e'_A] + U'(\pi_\lambda)(1+r)$$

The sign of M is positive when $[\lambda(1+r) - 1 - e'_{A*}] < 0$. We shall show now that this condition holds for an interior solution. We know that $[z(1+r) - 1 - e'_{A*}] < 0$ for at least one $z\varepsilon[\lambda,1]$ when an interior solution exists. Now since the term is monotonic in z and since λ is the smallest value z takes, the term must be negative when evaluated at λ (again if interior solution exists). Furthermore,

$$\frac{dA}{d\lambda} = - \frac{M(\lambda,r)}{SOC} > 0$$

unambiguously. Thus the greater the proportion of the loan collateralized, the greater is the optimal size of the loan.

Case III: With Collateral and Compensating Balances

Compensating (or compensatory) balances as a term of the loan has been grossly neglected by economists. This term actually serves to obscure the effective rate of interest charged on the loan. Let δ denote the proportion of the loan held as compensating balances. The effective rate of interest will then equal $\frac{r}{1 - \delta}(=r_e(\delta))$ where r is the contract rate.

Compensating balances may also serve as collateral for the loan. The implications of this are not difficult to incorporate into the model. By the way it is defined, λ is the proportion of the loan that is secured. Since a dollar of compensating balances serves as a dollar of collateral, λ increases by the same amount as δ, i.e. $\frac{d\lambda}{d\delta} = 1$. The profit function with compensating balances is

$$\pi_z = \pi_o + z\ A(1 + \frac{r}{1 - \delta}) - A - e(A).$$

And the objective function is given by:

$$EU = p(\delta)U(\pi_\delta) + \int_\delta^1 U(\pi_z)dF(z)$$

The first and second-order conditions are identical to Case II with δ substituted for λ and $r(\delta)$ for r.

$$\text{FOC}'' \quad \frac{\partial EU}{\partial A}: \quad p(\delta)(U'(\pi_\delta)[\delta(1 + \frac{r}{1-\delta}) - 1 - e_A'] + \int_\delta^1 U'(\pi_z)[z(1 + \frac{r}{1-\delta}) - 1 - e_A']dF(z) = 0$$

$$\text{SOC}'' \quad \frac{\partial E^2 U}{(\partial A)^2}: \quad p(\delta)\{U''(\pi_\delta)[\delta(1 + \frac{r}{1-\delta}) - 1 - e_A']^2 + U'(\pi_\delta)e_A''\} + \int_\delta^1 \{U''(\pi_z)[z(1 + \frac{r}{1-\delta}) - 1 - e_A']^2 + U'(\pi_z)e_A''\}dF(z) < 0$$

Since δ and $r(\delta)$ are independent of A and always positive, these conditions yield the same qualitative results as they did for Case II.

We can now analyze the effect of a change in δ on optimal A, i.e. obtain $\frac{dA}{d\delta}$ from FOC.

$$\frac{dA}{d\delta} = - \frac{\frac{\partial FOC}{\partial \delta}}{SOC}$$

Denote $\frac{\partial FOC}{\partial \delta}$ as $N(\delta, \frac{r}{1-\delta})$. The sign of $\frac{dA}{d\delta}$ will be the same sign as $N(\delta, \frac{r}{1-\delta})$.

Applying Leibnitz Rule:[1]

$$\begin{aligned} N(\delta, \tfrac{r}{1-\delta}) &= p'(\delta)U'(\pi_\delta)\frac{\partial \pi_\delta}{\partial A} \\ &+ p(\delta)U''(\pi_\delta)\frac{\partial \pi_\delta}{\partial \delta}\frac{\partial \pi_\delta}{\partial A} \\ &+ p(\delta)U'(\pi_\delta)\frac{\partial \pi_\delta}{\partial A \partial \delta} \\ &- U'(\pi_\delta)\frac{\partial \pi_\delta}{\partial A}p'(\delta) \\ &+ \int_\delta^1 \{U''(\pi_z)\frac{\partial \pi_z}{\partial A}\frac{\partial \pi_z}{\partial A} + U'(\pi_z)\frac{\partial \pi_z}{\partial A \partial \delta}\}\, dF(z) \end{aligned}$$

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[1] See Olmsted (27), p. 322. Also to conserve space, we will not write out the partial derivates of π_z.

The first and fourth terms cancel. Collecting the remaining terms, $N(\delta,\frac{r}{1-\delta})$ can be re-expressed as

$$N(\delta,\frac{r}{1-\delta}) = M(\delta,\frac{r}{1-\delta}) + \frac{r}{(1-\delta)^2} R'(\delta,\frac{r}{1-\delta})$$

Thus the sign of $\frac{dA}{d\delta}$ is the function of two effects, the collateral effect and the interest rate effect. The collateral effect as in Case II is unambiguously positive. The interest rate effect will also be positive under the sufficient condition given by inequality (6). Thus the total effect will be positive under this condition. That is, the optimal size of loan will increase with the percent of compensating balances.

Hence an increase in the proportion of compensating balances will result in an increase in optimal size of loan for banks whose elasticity of marginal utility with respect to size of loan is less than one.

Summary of Results

Sufficient conditions for making a loan are (1) that the expected gain in utility from the revenue of the first dollar is greater than the expected loss (in utility) resulting from the cost loaning that dollar, or (2) that collateral ensures a certain profit.

If marginal utility is strictly a concave function, we may conclude that the optimal size of loan decreases with an increase in risk, where increasing risk is defined as a mean preserving spread.

When an interior solution exists, we may make the following statements about the optimal size of loan. The optimal size of loan

increases unambiguously when the proportion of collateral is increased. When the elasticity of marginal utility with respect to size of loan is less than one, the optimal size of loan increases with an increase in either the contract rate of interest or proportion of compensating balances.

Up to this point, we have dealt with the relationship between four terms of a loan. In order to incorporate maturity into the loan decision, however, we need some theory for forecasting future values of the interest rate. This is the subject of the next chapter.

CHAPTER IV

MONEY RATES IN THE CONTEXT OF EFFICIENT MARKETS

It is well known that Irving Fisher believed that the nominal rate of interest determined in a well functioning market with perfect foresight is the sum of two components, the equilibrium real interest rate and the fully anticipated inflation rate. When one allows for uncertainty (the absence of perfect certainty), the hypothesis becomes: the nominal rate of interest equals the equilibrium expected real return plus the market's assessment of the expected rate of inflation.

Essentially the second hypothesis belongs to a class of models of price formation known as expected return models.[1] In this class, equilibrium conditions for the market can be stated in terms of expected return which is conditional on some relevant information set. When the model is in equilibrium, the interest rate "fully reflects" available information. Then by definition, the equilibrium interest rate is an efficient market rate.

It is the intent of this chapter to extend this model in its given context and to derive empirically testable propositions about money market interest rates. First, however, we shall review a recent paper by Fama on this subject.

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[1] For a description of this class of models see Fama (9).

Fama's Interpretation of Fisher's Theory of Interest

Eugeme Fama, a leading proponent of the theory of efficient capital markets, has brought attention to an apparent inconsistency in a claim made by Fisher.[1] Fisher found evidence that price changes affect interest rates in the proper direction but only after a lag. He claimed that the evidence of a relationship was support for his hypothesis. Fama points out that since the relationship occurs only after a lag, it is in fact inconsistent with a well functioning market in the efficient market sense.

Fama reformulates the fisherian concept in the following manner:

> If the inflation rate is to some extent predictable, and if the one-period equilibrium expected real return does not change in such a way as to exactly offset changes in the expected rate of inflation, then in an efficient market there will be a relationship between the one-period nominal interest rate observed at a point in time and the one-period rate of inflation subsequently observed. (p. 269)

In addition Fama notes:

> If the inflation rate is to some extent predictable and no such relationship exists, the market is inefficient: in setting the nominal interest rate, it overlooks relevant information about future inflation. (p. 269)

Fama then tests his proposition when applied to the treasury bill market, and for empirical expedience, he assumes that the equilibrium expected real return (EERR) is constant. Thus the empirical form of his proposition is given by (1).

$$(1)\quad \tilde{\Delta}_t = \alpha_o + \alpha_1 R_t + \tilde{e}_t \quad [2]$$

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[1] See Fama (10) and Fisher (13).

[2] The tilde denotes a random variable.

where $\tilde{\Delta}_t$ is the rate of change of purchasing power subsequently observed for the period, R_t is the t-bill rate at the beginning of the period, and $\tilde{e}_t$ is an error term. Under the hypothesis, α_1 should equal -1 and α_o should equal EERR. Using OLS on data taken from the 1953-71 period, Fama obtains an estimate of α_1 equal to -.98 which conforms quite closely to the hypothesis.

Fama's results have been challenged by Hess and Bicksler (15), Carlson (6), Joines (21), and Nelson and Schwert (26). These challenges raise serious doubt about two of the assumptions: (1) that the equilibrium expected real rate of interest is constant, (2) that the nominal rate of interest reflects all available information relevant for predicting inflation.

Despite these criticisms, the major proposition that expectations of inflation have accounted for the largest part of the variation in short-term interest rates seems to have survived. Both of the papers by Hess and Bicksler, and Nelson and Schwert tend to support the hypothesis of efficient markets but reject the assumption of a constant expected real return. Though the papers by Carlson and Joines raise some doubt about whether all relevant information for predicting inflation is reflected in the short-term rate, the evidence given in these papers supports the notion that the short-term rate is the best predictor of inflation of the predictors studied.

Fisherian Interest Rates Under A Fixed Risk Structure

As noted above, the model to be developed in this chapter will be an extension from the efficient markets context. The

hypothesis of efficient markets will be weakened to allow for the realities of information and transactions costs. In this context, the fisherian concept will be generalized in two ways. First the analysis will be extended from a univariate concept of an interest rate to a multivariate concept of money market rates.[1] Secondly, the model will account for the compensation of risk.[2] Unlike Fama we will not assume that the equilibrium expected real return is constant.

The nominal one-period interest rate R_{it} in the i^{th} money market for period t is hypothesized to equal the sum of three components: the equilibrium expected real risk-free return (EERRFR) for that period, $E(\tilde{r}_{ot})$; the expected rate of inflation during the period, $E(\tilde{\dot{p}}_t)$; and some premium for the risk as perceived for that market for the period, η_{it}.[3]

In equation form this reads:

$$(2) \quad R_{it} = E(\tilde{r}_{ot}) + E(\tilde{\dot{p}}_t) + \eta_{it}$$

Note that when differences between rates are obtained, the first two terms on the right hand side of (2) are netted out. Thus

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[1]Money markets are defined as funds markets in which funds are committed for less than a year. Included in such a category are markets for commercial paper, negotiable certificates of deposit, eurodollars, banker's acceptances, federal funds and bank loans (excluding term loans).

[2]Occasionally an event such as that witnessed by the bankruptcy of Penn Central will cause market participants to revise perceptions of risk, but for the most part these perceptions will be constant.

[3]It is noted that the equilibrium expected real risk-free rate is unobservable.

differences between interest rates are strictly a function of differences in risk premiums. This leads to the following proposition.

> Proposition IV.1: If all markets are efficient, and if investor's perceptions of risk for each market vis a vis each other market are fixed, then the differences between interest rates of one-period securities will be constant and stationary over time.

Thus under the conditions of the proposition, we find that:

$$(3) \quad R_{it} - R_{jt} = \alpha_{ij}$$

where α_{ij} is constant. Equivalently, variations in expectations about the equilibrium expected real risk-free return and/or expected inflation rate will result in a change in all nominal returns, each by the same amount. Thus the vector $\underline{R}_{t+k}$, nominal returns in period t+k, will be related to the vector $\underline{R}_t$, nominal returns in period t, in the following manner:

$$(4) \quad \underline{R}_{t+k} = \underline{R}_t + c\underline{L}$$

where c is some scalar reflecting changes in expectations about risk-free rate of return and inflation, and $\underline{L}$ is a unit vector of the same length of $\underline{R}_t$. Thus, such changes in expectations will effect each nominal rate uniformly.

Sufficient conditions for market efficiency in an expected return model are found in Fama's classic review of efficient markets (9). They are (i) there are no transactions costs in trading securities, (ii) all available information is costlessly available to the market, and (iii) all participants agree on the implications of

current information for current price and distribution of future prices of each security.

A heuristic but lucid interpretation of the proof of the sufficiency of these conditions for stock market prices is given by Baruch Lev (24). A parallel interpretation for money market interest rates is given in the following: Strong competition among investors is assured by the absence of both transactions and information costs. These forces in conjunction with agreement among investors as to the import of the information on money market rates, will cause interest rates to adjust instantaneously and in an unbiased manner.[1]

These are sufficient conditions and not necessary. In the real world foresight is less than perfect and information relevant to improving foresight is not costless. Even if such information were costless, variety in beliefs of market participants would result in a variety of interpretations of information. In addition, transactions involve costs. Thus, to induce net flows of funds into and out of money markets, expected rewards must be sufficient to cover costs. Obviously, a realistic characterization of the relationship between various money market rates cannot be deterministic. However, if relevant information is not wasted and if the relative risk structure is fixed, we might expect the levels of nominal rates of interest to be described by probability distributions which tend to

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[1]It can be argued that competition in turn will result in efficient utilization of information. Firms that consistently fail to correctly utilize freely available information bearing on future prices will be eliminated through competition.

be dispersed around the equilibrium values as described above. Implicit in such a model is the assumption that there are sufficient numbers of participants on both sides of the market to facilitate price adjustments.[1]

As a first step toward reality, let us weaken one of the conditions above. Let us consider a world where information is costly. At this stage, however, we continue to abstract from transactions cost.

Under the assumption of fixed risk structure we eliminate the event of new information relevant to relative risk structure.[2] All new information whether sufficiently disseminated or not will pertain to expectations about EERRFR and/or the inflation rate. As expectations of the equilibrium expected real interest rate and expected inflation rate change, the vector of equilibrium expected nominal interest rates will change by some amount which will be the same for each element. Changes in interest rates due to such new information (whether immediately incorporated or not) will be netted out when differences between interest rates are taken. Random discrepancies from α_{ij} may occur as a result of the cost in obtaining information about the actual level of interest determined in each market. Eventually this information becomes available to the public

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[1] In a mathematical sense this may mean: to rule out corner solutions which are due to a lack of resources.

[2] The assumption of fixed risk structure may not be as unreasonable as one may first think. In the absence of major events such as the bankruptcy of Penn Central, one might expect investors to share some fixed notion of the relative risk of each market. Thus, we would expect differences between risk premiums to be constant.

and market forces eliminate the discrepancy. Thus we find that the differences between money market rates are representable as a stationary stochastic process even though each interest rate may behave in a non-stationary manner. This type of model can be summarized in the following proposition:

> Proposition IV.2. If relevant information is not wasted, if transactions are costless, and if investors' perceptions of risk for each market vis a vis each other are fixed, then the vector of money market rates of the same maturity is characterized by a multivariate stochastic process in which each variable may behave in a non-stationary manner, but the stochastic process formed by the differences between any two rates will be stationary.

The proposition implies the existence of the following relationship:

$$(5)\quad \tilde{R}_{it} - \tilde{R}_{jt} = \alpha_{ij} + \tilde{u}_t$$

$$= \alpha_{ij}$$

where

$$E(\tilde{u}_t) = 0,\ \sigma^2(\tilde{u}_t) = \sigma^2$$

and

$$E(\tilde{u}_t, \tilde{u}_s) = 0 \text{ for all } s \neq t$$

Intertemporal independence of the error terms follows from the efficient markets hypothesis. It is perhaps more obvious when stated by the contrapositive: if the error terms are intertemporally dependent, all relevant information is not being used.

A time series generated by this stochastic process will tend to revert toward the mean. Proposition IV.2, however, allows for each interest rate to behave as though it were generated by a martingale process comprised of a sum of an independent series of random variables, each with zero mean. The time series of each interest rate will wander accordingly. That is, each series will lack any specific form of statistical equilibrium. The stochastic process of the differences in rates is stationary because all nonstationary elements are netted out. The error terms are generated by a "white noise" process.

In order to take a second step toward reality, let us now relax the condition of costless transactions. In this milieu, managers of portfolios of money market instruments are constrained and often cannot adjust portfolios to acquire those instruments they perceive as being underpriced. For example, it is not all that uncommon to find an institution (e.g. U.S. Treasury) come to the money market to borrow such large quantities of funds so as to affect price. Under our most general hypothesis we can explain this phenomenon by the fact that transaction costs inhibit the flow of funds necessary for an immediate return to equilibrium. In time, however, assets mature, portfolios turn over and the barrier imposed by the existence of transactions cost is obviated. This type of price adjustment is summarized in the following proposition:

> Proposition IV.3. If relevant information is not wasted and if the relative risk structure is fixed, then differences between money market rate of the same maturity will be characterized by an integrated moving-average stochastic process of order (0, 1, q) where q depends on transactions

costs and the sum of the moving-average coefficients will equal -1.[1]

Put more simply, this proposition states that under the hypotheses any shock leading to a deviation from equilibrium, will be reversed by 100 percent after q periods. The higher the transactions cost, the more inhibitive it will be. Consequently, the higher the transactions cost, the longer the period of adjustment (the larger the q).

Proposition IV.3 can also be interpreted to account for delays in price movements which are due to information costs. (Proposition IV.3 interpreted to account for both transactions costs and information costs will be denoted Proposition IV.3'.) Even if risk structure is fixed, market participants may have to expend resources periodically on financial analysis to determine this fact. In the interim they may look to market price for the information. That is, small discrepancies may not warrant the expenditure of resources required for one to determine for one's self whether there is a change in risk structure. Thus under the hypothesis of constant risk structure with information and transactions costs, we may find that displacement from equilibrium may occur for an extended period.

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[1] In Box-Jenkins notation an IMA (0,1,q) model is of the following form: $(1-B)z_t = (1-\Theta_1 B-\Theta_2 B^2 \ldots -\Theta_q B^q)a_t$, where B is a backward difference operator, z_t is the observed time series variable at time t and q_t is an element at time t in a series of independent shocks. (Denote $(1-\Theta_1 B-\Theta_2 B^2 \ldots -\Theta_q B^z)$ as $\Theta_q(b)$.) A more general model for describing times series is given by the autoregressive integrated moving-average (ARIMA) process of order (p,d,q) which is of the form $\Theta_p(B)(i-B)^d z_t = \Theta_q(B)q_t$ where $\Theta(B) = (1-B\Theta_1 - B^2\Theta_2 \ldots -B^p\Theta_p)$.

This period, if longer than time necessary for the portfolio to turnover, would indicate that transactions costs are not necessarily the inhibiting factor.

Empirical Tests for the Theories

A test for stationarity of (5) is an empirical test of Proposition IV.2. Should the test confirm the proposition, it would support the joint hypothesis of efficient markets (including the assumed behavior of investors toward risk) and fixed risk structure. Since factors common to all interest rates are netted out, we cannot identify inflationary expectations. However, the model is consistent with the notion that the expected inflation rate is a component of each individual rate. Confirmation of Proposition IV.2 fails to negate Fama's interpretation of Fisher's hypothesis. There are several ways in which to test for stationarity of (5). We shall confine ourselves to Box-Jenkins type tests.[1] The most obvious of these is to test whether $\tilde{u}_t$ is "white noise" directly. An indirect alternative, however, seems to be more informative. That is, we consider the time-series of the first differences of the original process:

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[1]Another general form of test is that of specification error. This test takes as data the observed α_{ijt}. It regresses this variable on all imaginable determinants. Under Proposition IV.2 no other variable should have a non-zero coefficient. For example, one likely regressor candidate is the inflation rate. Under the hypothesis, expected inflation rate is netted out. Since this is part of the premise, a proxy for expected inflation, e.g. inflation subsequently observed, is a reasonable candidate for the experiment. Likewise, a proxy for the expected real rate should be tested.

$$(6)\quad \tilde{\alpha}_{ijt} - \tilde{\alpha}_{ijt-1} = \tilde{u}_t - \tilde{u}_{t-1}$$

Under the hypothesis of Proposition IV.2 and the assumption that $\tilde{u}_t$ are intertemporally independent and identically distributed, the process generated by the first differences of the original process should follow a first-order moving-average with a coefficient of minus one.[1] Thus, if the data (observed time-series) confirms this statistical hypothesis, it confirms the hypothesis that $\tilde{u}_t$ is "white noise" and hence $\tilde{\alpha}_{ijt}$ is stationary.

Proposition IV.3 is explicit in its empirical implications. Under its conditions we expect to find that shocks to equilibrium are reversed by 100 percent. If transactions costs are the only inhibiting factor to efficient markets, the time required for complete reversion will be less than three periods reflecting the fact that we are using securities of approximately three month maturity. Thus, portfolios of securities of such length will turnover after three periods. Any evidence that reversion takes longer than three periods necessarily implies a need for weaker conditions, conceivably those allowing for information costs.

Results

Several variations of the integrated moving-average model were run for each differences between the two elements in each of the permutations (five money market rates taken two at a time).[2]

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[1] The first-order auto correlation coefficient of such a system equals $-\frac{1}{2}$.

[2] After experimenting with both weekly and monthly averages of interest rates, it became evident that the reaction time was

In addition, the hypothesis of taking first differences was tested for several of the series. The results are given in Table 4.1. All differences are of monthly averages of interest rates of approximately three month maturity.[1] The time periods for each series are given in the table.

Perhaps surprisingly, the best results are found for the difference between the negotiable CD rate and the prime banker's acceptance rate (A24). The fact that at times in the past the prime banker's acceptance market has been "thin" in comparison to negotiable CD market makes one suspicious of the results. However, since the results are meaningful, the data appear to be more than adequate.[2] The data indicate for this series that it is quite reasonable to assume an integrated moving-average of the form (0, 1, q), i.e., IMA (1, q). This follows from the fact that the first-order autoregressive coefficient is not statistically different

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sufficiently extended, such that monthly averages were most appropriate. Thus, if any information was lost through aggregation, it appeared to be small relative to the benefit of eliminating some of the parameters.

[1]Data on interest rate series were taken from Board of Governors of the Federal Reserve System, Banking and Monetary Statistics and supplements to date.

[2]Outside of the fact banker's acceptances may be found in varying denominations, it is quite a close substitute for negotiable CDs in terms of its inherent qualities. That is, the risk inherent in holding either security is a function of commercial bank credit worthiness. The advent of money market mutuals which hold both securities assures that one institution will not allow the rates to drift too far apart without making some substitutions in its portfolio. This might explain why even in the most extreme of money market conditions the two rates are never more than fifty-nine basis points apart.

Table 4.1--Empirical Results.

Spread	Time Period	Form	$\underline{X}^2$	$\underline{\sigma}^2_a$
A12 (commercial paper rate--prime bankers' accept. rate)	(7/71-12/77)	a. $(1 - .93^{**}B)z_t = .04 + (1 - .69^{*}B)a_t$	22	.11
		b. $\Delta z_t = (1 - .75^{**}B)a_t$	25	.11
		c. $\Delta z_t = (1 - .74^{*}B - .21B^2 + .21B^3 + .07B^4 + .01B^5 - .18B^6)a_t \quad \sum_{i=1}^{6} \Theta i = .85$	18	.12
		d. $\Delta z_t = (1 - .89^{*}B - .20B^2 + .21B^3 + 0.0B^4 - .04B^5 - .10B^6 + .06B^7 - .18B^8 - .04B^9)a_t \quad \sum_{i=1}^{6} \Theta_i = 1.18$	14	.11
A13 (commercial paper rate--treasury bill rate)	(7/71-12/77)	a. $(1 - .88^{*}B)z_t = .78^{*} + a_t$	26	.39
		b. $\Delta z_t = (1 - .41^{*}B^4)a_t$	16	.37
		c. $\Delta z_t = (1 - .10B + .08B^2 - .14B^3 - .56^{*}B^4 - .04B^5 - .36^{*}B^6)a_t \quad \sum_{i=1}^{6} \Theta_i = 1.12$	14	.37
A14 (commercial paper rate--CD rate)	(7/71-(12/75)	a. $(1 - .98^{**}B)z_t = .36 + (1 - .66^{*}B - .57^{*}B^2)a_t$	12	.10
		b. $\Delta z_t = (1 - .41^{*}B - .32^{*}B^2)a_t$	12	.11

*statistically different from zero **not statistically different from one

Table 4.1--Continued.

Spread	Time Period	Form	$\underline{X}^2$	$\underline{\sigma}^2_a$
A14 (continued)		c. $\Delta z_t = (1 - .41*B - .32B^2 + .10B^3 + .03B^4 - .14B^5 + .01B^6)a_t \quad \sum_{i=1}^{6} \Theta_i = .73$	10	.11
A15 (commercial paper rate--eurodollar rate)	(7/71-12/75)	a. $z_t = (1 - .35*B - .30*B^2)a_t$	13	.43
		b. $\Delta z_t = (1 - .42*B - .37*B^2 + .30B^3 + .03B^4 - .05B^5 - .11B^6)a_t \quad \sum_{i=1}^{6} \Theta_i = .62$	11	.42
A23 (prime bankers' acc. rate--treasury bill rate)	(1/64-12/77)	a. $(1 - .88*B)z_t = .65* + a_t$	37	.29
		b. $\Delta z_t = (1 - .24*B^4 - .24*B^6)a_t$	21	.28
		c. $\Delta z_t = (1 + .01B + .03B^2 - .11B^3 - .24*B^4 + .02B^5 - .29*B^6)a_t \quad \sum_{i=1}^{6} \Theta_i = .58$	21	.28
		d. $\Delta z_t = (1 - 0.0B + .03B^2 - .14B^3 - .20*B^4 - .02B^5 - .26*B^6 - .18*B^{13} - .09B^{19})a_t \quad \sum_{i=1}^{19} \Theta_i = .86$	12	.27

*statistically different from zero **not statistically different from one

Table 4.1--Continued.

Spread	Time Period	Form	$\underline{X}^2$	$\underline{\sigma}^2_a$
A24 (prime bankers' acc. rate--CD rate)	(1/64-12/75)	a. $(1 - .79^{**}B)z_t = -.29^{*} + (1 - .29B - .16B^2)a_t$	19	.13
		b. $\Delta z_t = (1 - .46^{*}B - .30^{*}B^2)a_t$	24	.14
		c. $\Delta z_t = (1 - .57^{*}B - .26^{*}B^2 - .04B^3 + .27^{*}B^4 + .12B^5 - .52^{*}B^6)a_t \quad \sum_{i=1}^{6} \Theta_i = 1.00$	20	.13
A25 (prime bankers' acc. rate--eurodollar rate)	(1/64-12/75)	a. $\Delta z_t = (1 - .37^{*}(B - .23^{*}B^2 + .33^{*}B^3)a_t$	31	.36
		b. $\Delta z_t = (1 - .33^{*}B - .27^{*}B^2 + .33^{*}B^3 + .16B^4 + .05B^5 + .04B^6)a_t \quad \sum_{i=1}^{6} \Theta_i = .20$	30	.36
A34 (treasury bill rate--CD rate)	(1/64-12/75)	a. $\Delta z_t = (1 - .34^{*}B^6 - .20^{*}B^{13})a_t$	16	.31
		b. $\Delta z_t = (1 - .07B - .13B^2 - .07B^3 - .21^{*}B^4 - .04B^5 - .38^{*}B^6)a_t$	20	.32
A45 (CD rate--eurodollar rate)	(1/64-12/75)	a. $\Delta z_t = (1 - .41^{*}B - .20^{*}B^2 + .27B^3)a_t$	32	.38

*statistically different from zero **not statistically different from one

from one for the ARMA (1,2) model (A24a). We find that shocks to the original series are reversed by 76 percent after the second period and by exactly 100 percent after six periods. Though these results do not support Proposition II, they do provide strong support for Proposition IV.3'. This evidence will be discussed in the next section.

Series A12 lends support for both Proposition IV.2 and Proposition IV.3'. Again the data indicate IMA (1, q) as the correct form. Proposition IV.2 is supported by the fact that the first-order moving-average coefficient in equation A12b is not statistically different from one. The data also support Proposition IV.3', i.e. $q > 1$. Though 74 percent of the reversion occurs after one period it is not complete until sometime after the sixth period. This is indicated by the fact that the sum of the moving-average coefficients after six periods is not quite equal to one but is slightly greater than one when summed through the ninth period.

Differential A14 lends mild support for Proposition IV.3'. After two periods we find 73 percent reversion. However, even after nine periods we fail to get 100 percent reversion.

Differentials involving the treasury bill rate tend to have delayed but substantial reversion. The bulk of the reaction appears in the fourth and sixth period with negligible amounts coming before. In one case, that of A23, we find 18 percent of the reversion coming after one year. It appears that reaction on the part of investors to market conditions involving government debt come only after periods of sustained pressure.

Finally, series involving eurodollar rate over longer periods provide little, if indeed any, support for any of the propositions. We do, however, find a reversion of 65 percent for A15 in the period 5/71-12/75.

Conclusions

Though there is little support for Proposition IV.2, we find there is moderate support for Proposition IV.3' when applied to differentials involving the interest rates of commercial paper, prime banker's acceptances and negotiable certificates of deposit. For these differentials, most if not all of exogenous shocks to the original series are reversed after six months. Since this is longer than the three months necessary for a portfolio comprised solely of these instruments to turn over, it suggests that the inhibiting factor is information cost.

As noted in the last section, the differential between the banker's acceptance rate and the negotiable CD rate behaves in a manner most consistent with Proposition IV.3'. The sum of the coefficients over six months equals minus one. After two periods, there is a reversion of 83 percent. However, the fourth-order moving-average coefficient is positive. This may reflect a small degree of market segmentation. With market segmentation, we would find that there may be such periodic effects in the market.[1] When a

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[1]This periodic effect is to be distinguished from seasonality. The effect occurs in a time span after a shock to the system. If the shock is totally random and not due to seasonal factors, we can not associate the effect with a season. For differentials involving commercial paper, banker's acceptances or CDs, there did not appear

particular borrower or group of borrowers are in need of short-term funds they will seek funds in those markets which are available to them. Often they are limited to one or two markets. Likewise suppliers of funds may operate under legal restrictions or constraints which limit their participation across several markets or which inhibit their own sources of funds if they are intermediaries.[1]

Under these conditions, demand and supply conditions will not be neutralized over all money markets. Consequently, equilibrium expected real rates adjusted for risk may vary for each market. In particular, if excess demand exists in any one market, it may require some time before market forces accommodate with sufficient supply of funds. Thus reversion of the displacement from equilibrium may be inhibited by this factor.

In addition, one would expect that to find that part of the debt incurred in any given period will be to refinance after three periods (i.e. rolled over). This might explain why the fourth-order moving-average coefficient in model A24c is positive, reflecting the fact that anomolous market conditions are likely to recur in part at

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to be any significant seasonality. However differentials involving treasury bills did exhibit some signs--see A34a. For an explanation of these short-term yield spreads see Lawler (23).

[1]A classic case of the latter is found in the supply of the longer term mortgage market funds. The relationship between Regulation Q and disintermediation is well known. In periods of high interest rates alternatives to bank deposits become much more attractive. Hence, deposits tend to become scarce and since they are the ultimate (though not sole) source of funds available for mortgage, the mortgage market becomes "tight" with the mortgage rate rising provided of course that demand has remained unchanged.

the time of refinancing. After six months conditions appear to be fully anticipated and market forces complete their action.

The series A12 and A14 both involve the commercial paper rate. Because of the event of the Penn Central troubles, time was allowed for market perceptions of risk to crystallize. This limited the time period available as data. This might explain why these series did not offer stronger support for Proposition IV.3'.

CHAPTER V

THE MATURITY DECISION

In the previous chapter we developed several propositions about the behavior of interest rates in the money markets. One major result (Proposition IV.2) is that under a weak variant of Efficient Market Hypothesis and fixed risk perceptions for each market, the differences between various money market rates of like maturity behave according to a stationary stochastic process even when each individual rate is assumed to be generated by a non-stationary stochastic process. When the Efficient Markets Hypothesis is weakened to allow for both information and transactions costs Proposition IV.3' follows, which states that interest rate differentials will behave according to an IMA (1,q) stochastic process with 100 percent reversion.

As a demander of funds the bank depends quite heavily on the money markets. In fact the vast majority of funds the bank obtains are committed for the short-term (one period or less in our model). Long-term commitments generally exist in the form of savings certificates held by individual depositors.[1] Because long-term liabilities

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[1] We find today that many banks go to the capital markets for funds, i.e. sell bonds or issue new shares. Since this action is not a result of every day operations we shall ignore it.

make up such a small percentage of total liabilities, they will be ignored in our analysis. Thus, the model will only consider cases where the bank borrows short. On the other hand, the bank has many opportunities to lend long-term.

In Chapter I we defined a bank as a firm which provides two services: payment service and intermediary service. When it provides the payment service, the law permits the bank to use a percentage of the funds deposited for this purpose to purchase assets or create new assets in the form of loans.[1] In this capacity (as a producer of payment services) the bank acts as an intermediary. The converse of this statement is not true. That is, the provision of intermediary services does not imply the provision of payment services.

Time deposits and other liabilities are generated in the provision of purely intermediary services. Individuals who cannot purchase primary securities for various reasons[2] rely on banks and other financial intermediaries to provide access to ownership of income producing assets. Thus when the bank obtains funds via a creation of a liability on itself, it is generally providing a service of intermediation for the purchaser of the liability.

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[1]It is noted that in Illinois there is no reserve requirement. Thus, a bank not concerned with liquidity or safety could conceivably lend out 100 percent of its demand deposits.

[2]The reasons which are most often cited are related to wealth constraints. Individuals rarely have enough wealth to buy large primary securities which may be in $100,000 denominations. Wealth constraints also limit the volume of transactions which can take place and thus precludes quantity discounts which arise from economies in the size of transactions. Finally, wealth constraints limit opportunities for diversification of risk.

The Marginal Cost of Funds

Associated with each liability is a cost. For most liabilities the greater part of this cost is explicit in the form of an interest rate. For example, to obtain time deposits under competitive conditions, banks will offer an effective rate of return which is close to that which individual savers can get at other thrift institutions which provide time deposits of the same characteristics.[1] The provision of this opportunity for the saver also consumes resources in the form of management expertise, teller services and physical plant. The costs associated with these resources are largely fixed with respect to most decisions. This is assumed to be true for all short-term liabilities except demand deposits. Thus, marginal costs of funds obtained from time deposits and other short-term liabilities are approximately equal to the effective rates of return necessary to attract such funds.

Since banks are prohibited by Regulation Q from paying interest on their regular checking accounts, it appears as though their marginal cost is zero; but recall this account involves the production of payment services. The flow of this service is an implicit return to the depositor and it seems reasonable to assume that the costs will vary with the volume of transactions which in turn will depend on the size of this account. Under competitive conditions banks will have to provide a service which yields an

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[1] We assume here that interest rate ceilings are not effective. When interest rate ceilings are effective, however, banks may experience a loss of funds through disintermediation.

implicit return which will attract funds, i.e. some market rate. In competitive equilibrium banks will employ the amount of resources which are necessary to bring the marginal cost of an additional dollar from this account in line with the marginal costs of an additional dollar of other sources of funds.[1] Thus under competitive conditions the marginal cost of obtaining an additional dollar of funds from this account will approximately equal money market rates for other very short-term commitments adjusted for corresponding reserve requirements. In our model we shall assume for simplicity that the marginal cost of an additional dollar from any source is approximately equal to one of the short-term rates.[2]

The Loan Decision with Maturity Considerations

Case I: Under Proposition IV.2

As in Chapter III, we assume the bank maximizes expected utility derived from profits. The profit function now takes on an added dimension, time. In a simple one-period system, we assume the profit function to be:

$$(1) \quad \tilde{\pi}_{St} = \tilde{R}_{St}A_1 - \tilde{R}_{CDt}A_1 - a$$

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[1] Note that each dollar obtained from this amount must be adjusted for reserve requirements; i.e. for every dollar of funds acquired from this source only $\$(1 - r_{dd})$ is available for the bank to invest.

[2] We abstract from reserve requirements in order to facilitate correspondence with the empirical form of the hypotheses of Chapter IV.

where R_{St} is nominal rate for a one-period loan, $\tilde{R}_{CDt}$ is nominal rate for a one-period negotiable certificate of deposit, A_1 is the size of the loan (decision variable) and 'a' is some fixed cost associated with the loan. As in the previous chapter, we denote $\tilde{R}_{St} - \tilde{R}_{CDt}$ as $\tilde{\alpha}_{St}$. Since this is a difference between two money market rates, our theory of interest tells us that it is generated by a stationary stochastic process. Thus if a bank were to make a series of one-period loans of size A_1, the profit stream generated would equal:[1]

$$(2) \quad \underline{\tilde{\pi}}s = (\bar{\alpha}_{S1}A_1, \ldots, \tilde{\alpha}_{Sn}A_1)$$

The risk involved in such a commitment with n = 3 may be graphically represented as in Figure 5.1a. (In the first period both rates are known; hence so is their difference.) Under the assumptions of Proposition IV.2, the series is stationary. Consequently, forecasts of future rates will exhibit constant risk over time. This is inherent in the fact that the distributions of profits for periods 2 and 3 are identical.[2]

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[1]We assume throughout a payout ratio of 100 percent, i.e. no profits are reinvested. We also assume zero default risk so that we may focus our attention on market risk, i.e. risk associated with changes in interest rates.

[2]Strictly speaking, the bank is committed for only one period. If unfavorable circumstances should prevail at the beginning of the next period, the bank has the option of not making the loan. Thus, it avoids the risk of making negative profits. The actual distributions it faces will be truncated on the downside. The probability of zero profit is therefore a point density equal to the cumulative probability that the profit prospect will not be sufficient to make a loan at that time. We will assume this outcome is negligible.

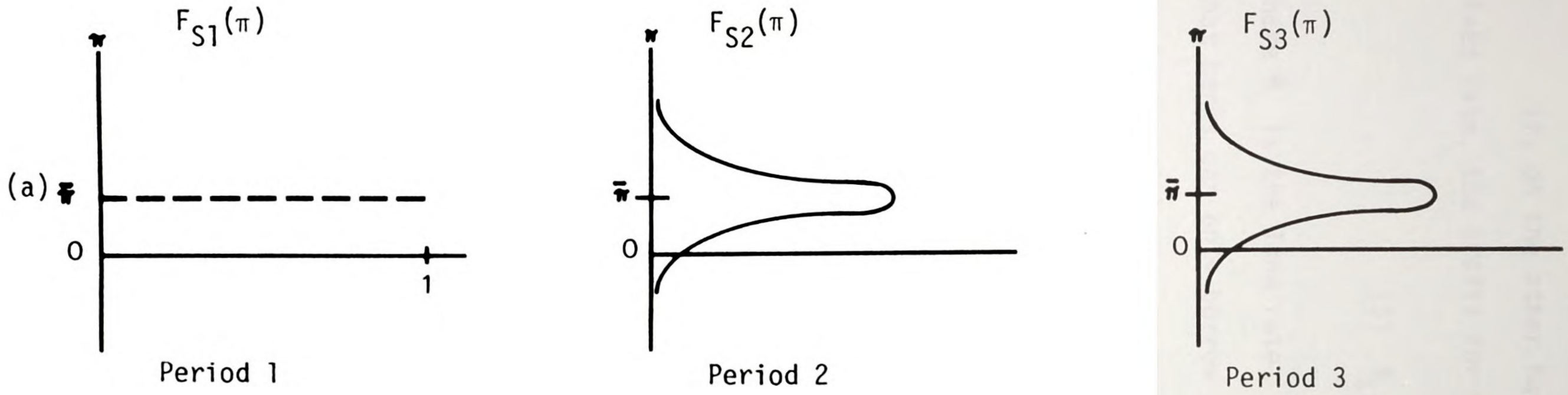


Figure 5.1--Risk in Making Loan.

If, on the other hand, a bank lends long (n periods) at a fixed rate, the profit for period t is given by:

$$(3) \quad \tilde{\pi}_{Lt} = \bar{R}_L A_1 - \tilde{R}_{CDt} A_1 - a$$

where $\bar{R}_L$ is the long rate. Costs are random because we have assumed that banks can only borrow short. Thus, at the end of each period the bank must borrow at the given one-period rate to pay back the loan borrowed at the beginning of that period.

The profit stream generated under such an arrangement is given by:

$$(4) \quad \underline{\tilde{\pi}}_L = (\bar{\alpha}_{L1} A_1, \tilde{\alpha}_{L2} A_1, \ldots, \tilde{\alpha}_{Ln} A_1)$$

where $\tilde{\alpha}_L$ equals $\bar{R}_L - \tilde{R}_{CDt}$.

Since we have assumed that individual interest rates are generated by a martingale process,[1] the profit stream, $\tilde{\pi}_{Lt}$, will also be a martingale.[2] This is illustrated in Figure 5.1b for n = 3. Note the nature of the change in the probability distribution of forecasts over time: it tends to spread out. This is a characteristic of the particular martingale process we assumed and is verified in the following.

In conventional notation, we assumed a martingale process defined by

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[1] For evidence of this hypothesis see Stiglitz (33).

[2] The martingale property is invariant to a linear transformation. For definition of martingale see Feller (12).

$$\tilde{x}_t = \tilde{x}_{t-1} + \tilde{\varepsilon}_t$$

where $E(\tilde{\varepsilon}_t) = 0$ and $E(\tilde{\varepsilon}_s, \tilde{\varepsilon}_t) = 0$ for all $s \neq t$.[1]

If we know the past values of x, x_1, x_2, . . ., x_T, and we desire to forecast future values, the one-period forecast is given by:

$$\begin{aligned}\hat{x}_T(1) &= E(\tilde{x}_{T+1} | x_1, \ldots, x_T) \\ &= x_T + E(\tilde{\varepsilon}_{T+1}) \\ &+ x_T\end{aligned}$$

Similarly the two-period forecast is:

$$\begin{aligned}\hat{x}_T(2) &= E(\tilde{x}_{T+2} \mid x_1, \ldots, x_T) \\ &= E(\tilde{x}_{T+1} + \tilde{\varepsilon}_{T+2}) \\ &= E(x_T + \tilde{\varepsilon}_{T+1} + \tilde{\varepsilon}_{T+2}) \\ &= x_T\end{aligned}$$

and the N period forecast is:

$$\begin{aligned}\hat{x}_T(N) &= E(\hat{x}_T(N-1) + \tilde{\varepsilon}_{T+N}) \\ &= x_T\end{aligned}$$

Thus, the forecast will be the same for all N, (i.e. the martingale property). This is represented in Figure 5.1b by the fact that $\bar{\pi}$, the mean of the forecast distribution, remains constant. We can now calculate the forecast errors to verify the spread in

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[1] If the error terms are identically distributed this process is known as a random walk. It is, in fact, a special case of the martingale (fair game) process and is often used in the financial literature as a model for movements in stock prices.

the distribution over time. The one-period forecast error is given by:

$$\begin{aligned}\tilde{e}_1 &= \tilde{x}_{T+1} - \hat{x}_T(1) \\ &= x_T + \tilde{\varepsilon}_{T+1} - x_T \\ &= \tilde{\varepsilon}_{T+1}\end{aligned}$$

The two-period forecast error is:

$$\begin{aligned}\tilde{e}_2 &= \tilde{x}_{T+2} - \hat{x}_T(2) \\ &= x_T + \tilde{\varepsilon}_{T+1} + \tilde{\varepsilon}_{T+2} - x_T \\ &= \tilde{\varepsilon}_{T+1} + \tilde{\varepsilon}_{T+2} \quad (\text{or } \tilde{e}_1 + \tilde{\varepsilon}_{T+2})\end{aligned}$$

Now since:

$$x_{T+N} = x_T + \sum_{i=1}^{N} \tilde{\varepsilon}_{T+i}$$

We have:

$$e = \sum_{i=1}^{N} \tilde{\varepsilon}_{T+i} \quad (\text{or } \tilde{e}_{N-1} + \tilde{\varepsilon}_{T+N})$$

Since the error terms are independent it is clear that we add variability to the forecast as the forecast horizon is extended.[1] That is, the forecast in period t+1 equals the forecast in period t plus white noise.

Rothchild and Stiglitz (30) have shown that a definition of increasing risk based on this type of increasing variability is equivalent to two other definitions.[2] One is based on a mean

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[1] If we assume the error terms are identically distributed with var $(\tilde{\varepsilon}_t) = \sigma_\varepsilon^2$, then the error variance will equal $N\sigma_\varepsilon^2$. That is, the standard error the forecast will increase with the square root of N.

[2] The proof of this equivalence is quite extensive and will not be reproduced here.

preserving spread, i.e. mass is shifted from the center of the distribution to the tails without changing the mean. The second, somewhat tautological,--is based on the preferences of risk averse expected utility maximizers. It states that if random variables X and Y have the same mean, but every risk averter prefers X to Y, i.e., if EU(X) > EU(Y) for all concave U, then X is less risky than Y.

In the context of our model, this means that when a bank commits itself to a long-term loan at a fixed rate, the riskiness of profit for each future period increases over time. Furthermore, the maturity term of the fixed rate loan will affect the level of expected utility of a loan prospect via its affect on the riskiness. In other words, since $\tilde{\pi}_{L,t+1}$ is riskier than $\tilde{\pi}_{Lt}$, (but has same expected value), $EU(\tilde{\pi}_{L,t+1}) < EU(\tilde{\pi}_{Lt})$.

Up to this point, we have shown that under existing assumptions, risk in each period is constant for a series of equal-valued one-period loans made at market rates at the beginning of the corresponding periods. We have also shown that inherent risk increases over time for a fixed-rate multiperiod loan. Thus, we have compared the distributions between time periods for each prospect independently. We have not yet compared distributions between prospects for given time periods.

Under existing assumptions, the random elements of the two prospects are generated by two different mechanisms. The randomness in the long prospect is due entirely to randomness of individual

interest rates.[1] In Chapter IV, we assumed that changes in interest rates are due to new information which is perceived to effect expectations about future inflation or the future real rate of interest. Randomness in the short prospect, on the other hand, is due to randomness in interest rate differentials, $\tilde{\alpha}_{st}$. Changes in these differentials reflect imperfect and unequal assimilation of the new information.

Under our theory in the preceding chapter, we concluded that assimilation of information into interest rates was inhibited by information and transactions costs. We also found evidence that this process of assimilation was ultimately completed and that there was at least partial adjustment in the periods immediately following the new information. On this basis we make the following assumption:

> Assumption V.1: Market forces reacting to disparate returns ensure that the districation of $\tilde{\alpha}_{s2}$ $(=\tilde{R}_{s2} - \tilde{R}_{CDt})$ is more "compact" than that of $\tilde{\alpha}_{L2}(\bar{R}_L - R_{CDt})$ in the sense that the distribution of $\tilde{\alpha}_{L2}$ can be obtained from the distribution of $\tilde{\alpha}_{s2}$ by a sequence of steps which shift mass from its center to its tails without changing its mean when $\bar{R}_L$ is adjusted to equal $E(\tilde{R}_{s2})$.

In order to illustrate the relationship between maturity and other terms of the loan we shall examine a special case in which the bank is confronted with a choice between the two prospects illustrated above. That is, the bank may lend short for three periods and borrow short, or it may lend long at fixed-rate and borrow short for three periods.

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[1] The profit rate is simply a linear transformation of the CD rate, i.e. $\tilde{\pi}_{Lt} = \tilde{\alpha}_{Lt}A_1$ where $\tilde{\alpha}_{Lt} = \bar{R}_L - \tilde{R}_{CDt}$.

As in Chapter III, we assume that banks are risk averse, i.e. the banks utility function is concave everywhere. In addition, we assume that the utility function is three times differentiable and additive in the time dimension with a pure rate of time preference equal to μ. Under the Expected Utility Hypothesis, the bank's choice between the two options will be indicated by the one with the greatest expected utility. Expected utility from the profits of each prospect, short and long, is given respectively by:

$$EU_S = U(\bar{\pi}_{S1}) + (1 - \mu)EU(\tilde{\pi}_{S2}) + (1 - \mu)^2 EU(\tilde{\pi}_{S3})$$
$$EU_L = U(\pi_{L1}) + (1 - \mu)EU(\tilde{\pi}_{L2}) + (1 - \mu)^2 EU(\tilde{\pi}_{L3}) \quad (5)$$

where $\underline{\tilde{\pi}}_S$ is distributed as $\underline{F}_S$ and $\underline{\tilde{\pi}}_L$ is distributed as $\underline{F}_L$.

We shall now derive propositions concerning the relative values of EU_S and EU_L based on the nature of the distributions of the two profit series. We first look at the case where interest rates are not expected to change, i.e. martingale property.

Proposition V.I: A bank will choose to lend long (i.e. choose F_L over F_S) only if R_L is greater than R_{S1} (i.e. there exists some liquidity premium).

Proof (by contradiction): Assume $\bar{R}_L = \bar{R}_{S1}$. Then: $EU(\bar{\pi}_{S1}) = EU(\bar{\pi}_{L1})$. $E(\tilde{R}_{S2}) = \bar{R}_{S1}$, by martingale property. Hence $\bar{R}_L = E(\tilde{R}_{S2})$. Therefore by assumption V.1, the distribution $\tilde{\alpha}_{L2}$ differs from that of $\tilde{\alpha}_{S2}$ by a mean preserving spread (MPS). Since $\tilde{\pi}_{L2}$ and $\tilde{\pi}_{S2}$ differ from $\tilde{\alpha}_{L2}$ and $\tilde{\alpha}_{S2}$ respectively by the same scale factor (A_1), the distribution of $\tilde{\pi}_{L2}$ will differ from that of $\tilde{\pi}_{S2}$ by a MPS.[1] Therefore,

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[1]The integral conditions which define MPS are invariant to equiscalar changes in both distributions.

by the Rothchild-Stiglitz equivalency theorem, $EU(\tilde{\pi}_{S2}) > EU(\tilde{\pi}_{L2})$. It is clear that since $\tilde{\alpha}_{S2}$ and $\tilde{\alpha}_{S3}$ are identically distributed (under Proposition IV.2), their scalar transforms, $\tilde{\pi}_{S2}$ $(=\tilde{\alpha}_{S2}A_1)$ and $\tilde{\pi}_{L2}$ $(=\tilde{\alpha}_{L2}A_1)$ will also be identically distributed. Hence $EU(\tilde{\pi}_{S2}) = EU(\tilde{\pi}_{S3})$. We have shown in the chapter that since $\tilde{\pi}_{L3}$ is equal in distribution to $\tilde{\pi}_{LS}$ plus some random noise, $EU(\tilde{\pi}_{L2}) > EU(\tilde{\pi}_{L3})$. Hence $EU(\tilde{\pi}_{S3}) > EU(\tilde{\pi}_{L2}) > EU(\tilde{\pi}_{L3})$. Therefore $\bar{R}_L = \bar{R}_{S1}$ implies $EU_S > EU_L$.

It is obvious that EU_L increases monotonically with $\bar{R}_L$. Hence $\bar{R}_L < \bar{R}_{S1}$ also implies $EU_S > EU_L$.

Therefore $EU_L > EU_S$ implies $\bar{R}_L > \bar{R}_{S1}$. Q.E.D.

As was noted, this proposition assumes that individual interest rates are generated by a martingale process. Implicit in this is the assumption that there is no discernable trend. That is, the expected level of the interest rate in period t+1 equals the actual level in period t.

Banks, however, are aware of factors which affect interest rates and employ resources to perceive hints of future interest rate movements. For example, there are several signals which forebode increases in interest rates--low inventory levels which signal inventory investment and its requisite financing, new financing of projected deficits by the US Treasury, increases in planned investment in capital goods, low stock of consumer durables requiring replacement with the consequent demand for consumer credit, and of course increases in inflationary expectations. Money managers use such knowledge to improve their own forecasts. That is, they are

able to reduce the dispersion of the forecasts of profitability of the prospects in their opportunity set.[1] In terms of our model, it is conceivable that bank management correctly perceives a downward trend in negotiable CD rates (and thus their marginal cost) which might enhance future profit prospects in the near term and it might appeal to some of the more risk averse decision-makers to borrow short in the near future. Thus, if a bank can solicit a loan at a fixed rate expected profits may increase for a time as illustrated in Figure 5.2.[2] Furthermore this increase in expected value may be sufficient to compensate for the fact that the dispersion of the forecast values will still increase with time.[3] Thus, there is no a priori rule for the existence of a liquidity premium. As in the theory of the term structure of interest rates, we find an expectations variant of loan rate theory which attempts to account for expected opportunity cost.

We may now state the second proposition:

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[1]Implicitly we are assuming a weak version of the Efficient Market Hypothesis which involves information costs. Sustained existence of financial intermediaries under such a regime requires that they expend resources to obtain information. Market participants cannot rely on market prices to always "fully reflect" all information. The irony of efficient markets is that in order for prices to reflect all available information some participants must be suspect of existing prices and employ resources to obtain additional information.

[2]This assumes ignorance on the part of the borrower, otherwise the borrower would not commit to a higher rate when lower rates are expected.

[3]This may be interpreted as meaning that increased dispersion does not imply increased risk if the mean has been increased.

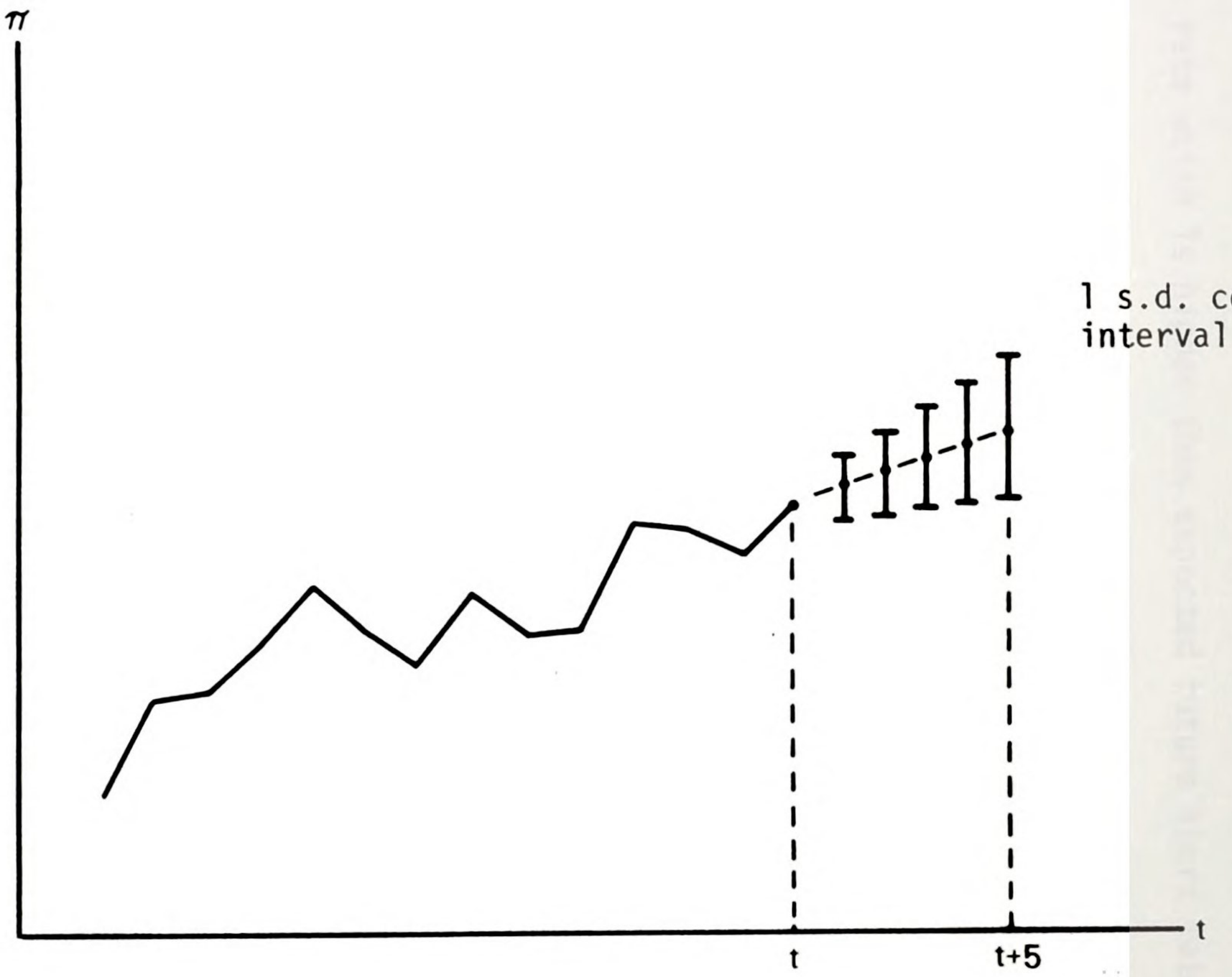


Figure 5.2--Forecasting a Random Walk with a Trend, e.g. A Submartingale.

> Proposition V.2: In a period of expected reductions of short-term interest rates, it is feasible that a bank lend long at a fixed rate equal to or less than the current short rate.

More specifically, what this says is: If the bank correctly forecasts future interest rate reductions that reduce its cost and is able to make a multiperiod loan at a fixed rate equal to the current short rate which is higher than expected future short rates, its expected profit rate would increase into the future by making such a loan. Thus, the subsequent expected profit rate increases compensate for the decrease in forecast ability (increase in dispersion). If this compensation is significant enough, expected utility maximizing banks will choose the multiperiod loan over the one-period loan. This corresponds to the situation of a humped yield curve in the theory of term structure of interest rates. That is, we find that long term assets are capable of being sold at higher prices (lower yields) because of expectations of future price movements.

There are two forces that work against each other. First when short-term interest rates are expected to decline, competition may force banks to offer fixed loan rates over two or more periods which are at lower levels than the current single period rate. As the maturity horizon is extended, however, forecasts become more fallible --resulting in increasing risk. Ultimately, increasing risk will dominate and be reflected in the interest rate. This can be summarized in the following:

> Proposition V.3: If a bank has a choice to make a loan for t periods at a fixed rate, $\overline{R}_S$, or a loan for t + k periods at another fixed rate, $\overline{R}_L$, it will, after some number of periods, c, choose to make the longer loan only if $\overline{R}_L \geq \overline{R}_S$ for all k where $k \geq c \geq 1$.

This proposition merely states that at some time period in the forecast horizon, the cost associated with an increase in dispersion will outweigh the benefit in compensation which comes from higher expected profits. That is, the yield curve will either flatten out or start increasing again.

Case II. Under Proposition IV.3'

As in the previous case, we have a proposition with implications for the forecastability of interest rate differentials and consequently of short-term loans. Again, this income prospect will be compared with the expected opportunity cost and risk of not lending long for the same period.

Recall that the import of Proposition IV.3' is that the interest rate differentials will behave in a manner characterized by an integrated moving-average process (IMA(1,q)) with 100 percent reversion after q periods. Forecasts of first-differences are thus stationary. In addition, since there is 100 percent reversion, the actual series itself is stationary. That is, the process has a long run (statistical) mean about which there exists a distribution that is independent of time. This is characterized by Figure 5.3. We see in this case (q = 3) that after three periods the confidence intervals become uniform. That is because the following distributions are identical. We consequently have a situation which is similar to the first one presented in Case I. Again, we invoke Assumption V.1 to facilitate another proposition.

Proposition V.4: A bank will choose to lend long only if $\overline{R}_L > \overline{R}_{S1}$.

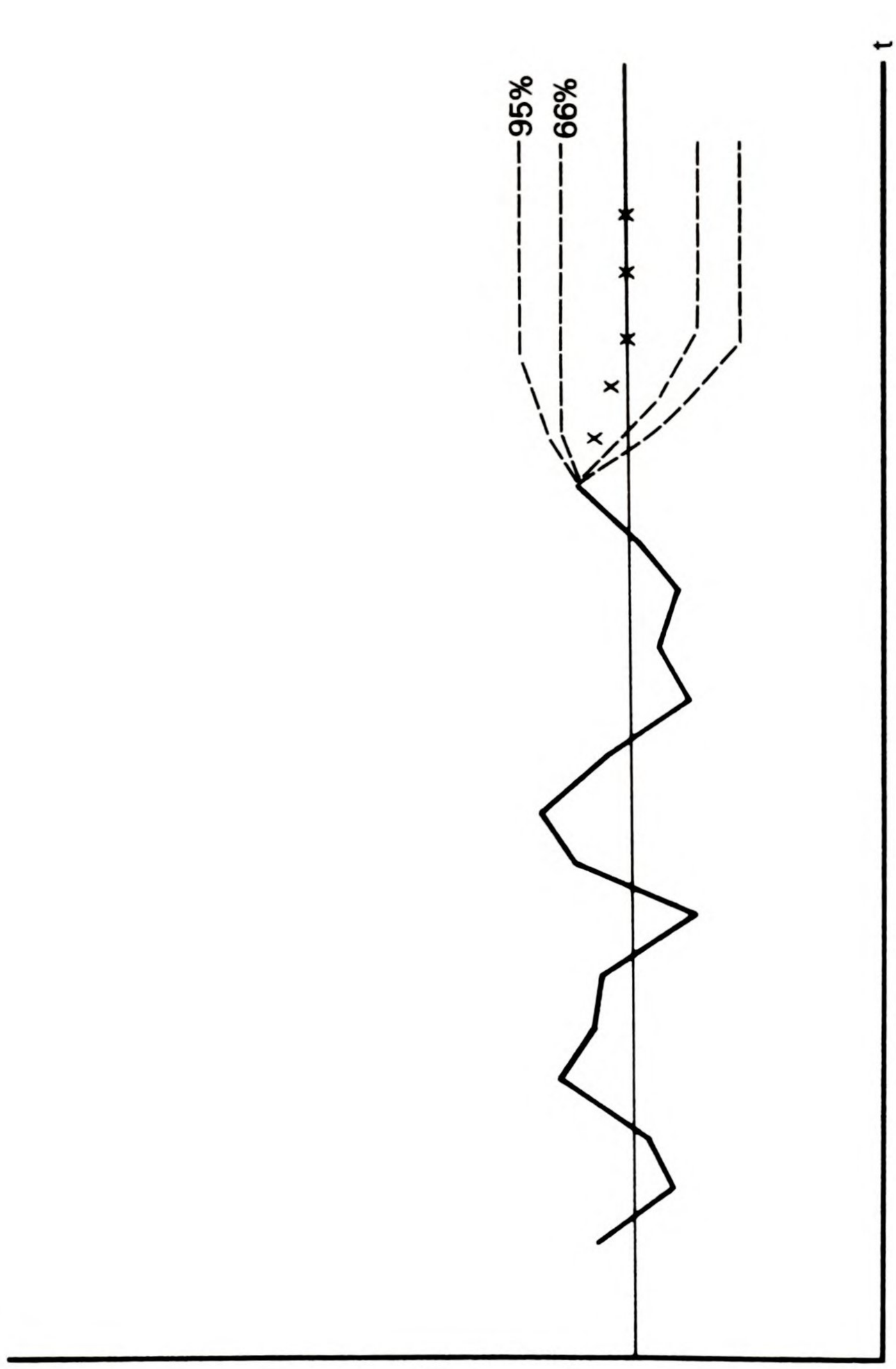


Figure 5.3--Forecasts and Confidence Intervals for an IMA(1,3) Process with 100 Percent Reversion.

We must deal with the fact that the distributions of the one-period differential spread out for q periods before they become uniform. The rate at which they spread out will be less than the rate at which a martingale will spread out. This follows for two reasons: First, by Assumption V.1, the distribution of the shock to the process is more compact for the one-period differential. Second, the process forecasts involve only a partial sum including only a fraction of each shock term whereas the forecasts of a martingale involve a total sum of 100 percent of the shock terms which are more disperse to begin with. Thus, for each period in the forecast horizon, the distribution of the short differential will be more "compact" than that of the long differential when assuming that $\tilde{R}_{CD}$ is formed by a martingale process.

Again, we may generalize and consider the possibility of detectable trends in interest rates. Banks that correctly forecast future reductions in short-term rates will engage in multiperiod commitments at a lower fixed-rate than they would with forecasts of no change in short-rates. If there are a sufficient number of efficient banks, and if expected reductions of short-rates are of sufficient size, then the efficient bank may engage in a multiperiod commitment at a fixed-rate equal to or less than the current short-term. This follows because under Proposition IV.3', the expected alternative cost of lending short is not reduced by the trend in short-rates. Thus, Proposition V.2 applies under Proposition IV.3'.

In addition, risks associated with forecasts of short-rate differentials (profit rate of short prospect) becomes time invariant

after q periods. On the other hand, risks associated with fixed-term multiperiod commitments will always increase with time. Ultimately increasing risk will dominate, and the yield curve will flatten out or begin to rise. Therefore, Proposition V.3 also applies under Proposition IV.3'.

Some Final Comments

In this chapter we have investigated the implication of various interest rate theories on the maturity decision of a bank loan. In particular we have focused on a choice betwen two alternatives--one involving a series of short-term loans and the other a long-term loan for the same amount and same number of periods, but at a fixed rate.

It seems apparent that a bank, in assessing its expected opportunity cost, will have to include more than just the two alternatives we looked at in this chapter. For example, it may consider the alternative of breaking down the long-term loan into two intermediate-term loans, or one such loan and a subsequent (or supersequent) series of one-period loans. It should be clear that the analysis presented in this chapter is capable of being extended to accommodate such alternatives.

CHAPTER VI

THE UNIFIED THEORY AND SOME EMPIRICAL OBSERVATIONS

The purpose of this chapter is two-fold. First, it is intended to integrate the theories developed in Chapters III and V into a unified theory of the banking firm. Secondly, the theoretical results will be juxtaposed with relationships observed in the real world. In particular, these results will be compared with the empirical results obtained in a paper by Hester (16).

The Theory as a Whole

In Chapter III, we analyzed the relationship between four terms of a loan--size of a loan, contract rate of interest, degree of collateralization, and percent of compensating balances--for a given perception of default risk. We abstracted from the maturity decision by considering only one-period loans. In Chapter V we introduced a perception of market risk by considering prospects of different maturity. In that chapter, however, we abstracted from default risk.

Under the assumption of efficient markets the two theories are both consistent and complementary. The first theory is based on the assumption that the effective rate of interest is dictated by the market when conditions are competitive or more precisely,

when the loan market is efficient. Under these conditions, we find fairly uniform interpretations about market risk. The term structure of loan rates dictated by the market will reflect the market risk. That is, if a bank is to solicit a loan, it cannot charge more than the going rate for the risk class and maturity considered. Thus, the relationship between maturity and the effective rate of interest is determined in the market.

The relationship between maturity and other terms of a loan is perhaps best viewed through its link with the effective rate of interest. In the last chapter, we discovered that when interest rates are expected to decline, there is no a priori rule for the sign of the relationship. When interest rates are expected to remain constant or increase, we concluded that expected utility maximizers will require a premium to be induced to make a long-term loan. Since expectations about future interest rate movements are not readily observed, it is difficult to control for this factor. However, some pertinent observations are in order.

In general, there is no reason to expect a trend of interest rates reductions the majority of the time. In fact, looking at the trend of inflation over the past ten years leads one to believe that if this trend was correctly anticipated, interest rates were expected to be increasing or constant the majority of the time.[1] During such periods, our theory tells us that we should expect to have observed a positive relationship between maturity of a loan and its contract

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[1]This assumes of course that there were no systematic trends in other determinants.

rate of interest, i.e. an upward sloping yield to maturity curve.

On the other hand, according to our theory: an expected trend of interest rate reductions does not imply that we should observe a negative relationship between maturity and the interest rate. If the slope of the trend is too slight the expected reduction in cost may be offset by the additional risk associated with increased forecast error.[1]

Another factor must be considered. Forecasts of potential for default will change over time. Relevant information concerning the solvency of the loan prospect may become available at any time in the future. If this information is ominous, a bank engaged in a loan of "short enough" maturity may be able to escape the additional risk. This ability to avoid exposure to additional risk will also be reflected in premiums for loans of longer maturity.

Thus, it is reasonable to conclude that, on balance, we might expect to see a premium for loans with longer lengths of maturity in the real world. In other terms: on balance the relationship between maturity and the contract rate of interest should be positive.

When a prospective borrower comes to a bank with a request for a loan, it is usually for a particular purpose. It may be to finance inventory replacement, seasonal cash-flow problems, or perhaps a capital expenditure. The time requirement for the funds will be

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[1]Even if expected rate reductions were great enough to be reflected in a decreasing yield to maturity curve, the trend would be expected to terminate more quickly. Thus, a downward sloping yield curve would be ephemeral and consequently even more rare.

determined by its purpose. From the borrower's point of view, it is not flexible as a term of a loan.[1]

Since we have assumed that market risk is already reflected in the term structure of loan rates for given risk classes, our model bank will be indifferent to the maturity term.[2] If the bank perceives more risk than is reflected in a given long-term market rate, it may offer a loan with a floating rate that is pegged to some other rate representing its cost of funds, or the bank may choose not to make the loan at the going rate.[3]

Once the maturity aspect is determined, the results of Chapter III follow. The bank faces an effective market rate for the given maturity. Assuming that the bank perceives the general form of the subjective probability distribution of default described in Chapter III, it chooses to make a loan of size A* so as to maximize the expected utility of the profits derived from the loan. The qualitative results derived from the comparative static analysis of such a hypothesis are summarized in the following: the bank may be induced to make a larger loan by being offered more collateral or a

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[1]It is recognized that when financing capital outlays, the borrower may use different sources at different stages, e.g. a firm may seek a term loan for the planning stage and float a bond for the construction stage. Under such an arrangement, maturity may be a negotiable term. We shall abstract from such cases in our analysis.

[2]In mathematical terms, maturity is no longer a control variable as it was in the analysis of Chapter V.

[3]The popularity of the floating-rate term loan seems to suggest that the long-term market rates involve too much risk for either suppliers or demanders of funds. Given the roller-coaster inflation experience in recent years, it is not difficult to understand why.

higher effective rate of interest--in the form of a higher contract rate of interest or a higher percent of compensating balances or both. When linking this to the relationship between the effective rate of interest and maturity, we may conclude: during periods when interest rates are expected to remain constant or rise, a bank will tend to lend a larger amount for a loan prospect with shorter maturity, other things being equal. Thus we have determined, theoretically, the trade-offs which exist between five terms of a loan. We will now compare these results with the results of a paper by Hester (16).

Hester's Empirical Examination of the Loan Offer Function

In his paper, Hester presents an econometric investigation of the existence of a loan offer function. For the model, he develops a cursory but practicable theory of the loan offer function. This theory is not based on fundamental principles as the model propounded in this thesis, but is developed on a level in which hypotheses are stated in empirical terms. Their theoretical justification is limited to pragmatic conjecture.

Hester defines a loan offer function in a manner which is analogous to the definition of a production-possibility frontier. Recall, a production-possibility frontier is defined, as the efficient set of outputs obtainable from a given set of inputs. The coordinates of the frontier are measured in terms of the outputs. The analogue to this is the loan offer function which is the efficient set of terms of a loan which are obtainable from a given set of characteristics of both loanee and bank. The coordinates of the

loan offer function are measured in units which are applicable to each term, e.g. maturity might be measured in months.

In its abstract form the loan offer function is defined by an implicit function:

$$F(\underline{t};\ \underline{u},\ \underline{v}) = 0$$

The terms of lending are represented by the vector $\underline{t}$. Vectors $\underline{u}$ and $\underline{v}$ represent sets of characteristics of the borrowers and lenders respectively. In this context, a loan offer function is the efficient set of terms of a loan, $\underline{t}^*$, obtainable from given sets of characteristics, $\underline{u}^o$ and $\underline{v}^o$.

Efficiency is analogous to efficiency in the Pareto sense in that the loan offer curve implies that a bank will not grant a more liberal loan from the point of view of one term or condition without worsening other terms. This concept of liberal terms of lending requires the pragmatic conjecture. For example, Hester says it is obvious that banks prefer loans which are made at higher rates of interest. Banks, he claims, also prefer relatively short maturity loans. His reasons for this coincide with some of the assumptions made in the previous chapter; but he fails to acknowledge the case where interest rates are expected to decline. In addition, he hypothesizes that banks prefer to make moderate size loans rather than very large or small ones. Small loans are less desirable because of fixed costs. These costs can be spread out as the size of the loan increases. Finally, he concludes that banks prefer to make secured loans because collateral reduces default risk.

To summarize: banks attempt to make secured loans of moderate amounts for short periods and with higher interest rates. If both banks and borrowers bargain effectively, banks will end up on their offer frontier, i.e. offer curve.

Hester then proceeds to analyze the extent to which the characteristics of both the borrower and the bank will affect the loan. The analysis of the characteristics of the borrower is consistent with the model of this thesis in that the characteristics may be embodied in the perceived distribution of the prospect. The analysis of bank characteristics may be somewhat outdated. Deposit variability, according to Hester, increases with bank size. Consequently, larger banks face more risk from deposit variability. The advent of negotiable CDs and the development of the Federal Funds Market, however, have helped to mitigate the effects of this risk.

The significant contribution of the Hester paper is his ingenious application of the method of canonical correlation to test an empirical form of his loan offer function. He defines two canonical variates:

$$q_1 = (\text{Log } R)' + k_1(\text{Log } M)' + k_2(\text{Log } A)' + k_3(\text{Log } S)' + u$$

$$q_2 = \sum_{i=1}^{I} a_i(\text{Log } W_i)' + \sum_{j=1}^{J} b_j(\text{Log } Z_j)' + v$$

where

R = the loan rate of interest (percent)

M = the maturity of the loan (months)

A = the amount of the loan (dollars)

S = "1" if the loan is secured, and "0" otherwise

W_i = the ith relevant characteristic of loan applicants, (i = 1, . . . , I)

Z_j = the jth relevant characteristic of lending banks, (j = 1, . . . , J)

u = stochastic variables independent of R, M, A and S

v = stochastic variable independent of all W_i and Z_j

Primes (') on variables indicate they are defined as deviations from the means.

Hester makes two assumptions about the world generating the data. First, he assumes that effective bargaining by borrowers forces bankers to lend at terms on the efficient frontier. Secondly, banks are assumed to be willing to substitute among terms of lending. Thus, real world data should yield canonical weights of q_1 on the efficiency frontier.

The computational procedure Hester uses estimates those parameters (a's, b's and k's) which maximize the correlation between q_1 and q_2 when q_1 and q_2 are normalized to have unit variance. Data for this model were obtained from three large commercial banks and a 1955 survey of business loans by the Board of Governors of the Federal Reserve System.[1] Actual estimates of the coefficients of the terms of lending and the canonical correlation coefficient are given in Table 6.1 for the four samples.[2]

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[1]For the three samples obtained from the individual banks, Hester could not obviously estimate coefficients regarding bank characteristics. On the other hand, the survey containing cross-sectional data on bank characteristics did not identify characteristics of the loanee.

[2]Since the estimates of the coefficients of the characteristics have little import for the hypothesis to be discussed they were omitted here to avoid extraneous explanation.

Table 6.1--Hester's Estimates of Canonical Correlations.

Variable	Bank 1	Bank 2	Bank 3
	A. From Samples of Term Loans (1955-1957)		
(Log R)'	1.0000	1.0000	1.0000
(Log M)'	-.053203	-.086959	-.070112
(Log A)'	-.012387	-.052804	-.032025
(Log S)'	-.015547	.000069	-.0013735
Canonical Correlation (Squared)	.799	.852	.796

Variable	Weight
B. From FRS Survey, 1955	
(Log R)'	1.0000
(Log M)'	-.036209
(Log A)'	-.38171
(Log S)'	.066075
Squared Canonical Correlation	.547

The estimates of the table tend to verify Hester's a priori predictions about trade-offs between terms of lending. For example, his theory predicts that for similar borrowers, the loan rate of interest should increase as the maturity of a loan increases, as the size of loan increases and should be lower if collateral is supplied. This prediction is evidenced by the signs of the a's (vis a bis the

loan rate). They are negative for all maturity terms and all size of loan terms. This indicates that an increase in either the maturity or size of the loan must be compensated by a higher interest rate to remain on the efficiency frontier, i.e. for q_1 to remain constant.

The weight of the dummy security variable failed to consistently yield the correct (positive) sign.[1] Hester claims this is not surprising because the variable has limited economic meaning. Since the theory of Chapter III brings to light the economic meaning of collateral, we must conclude that the inconsistency is due to other factors. It is more likely that banks do not substitute collateral for less liberal terms under many circumstances. When default risk is high, full security will be required regardless of the other terms. When default risk is low banks may be able to obtain security when it is virtually costless to the loanee and hence no sacrifice.

The important point to note is that the predictive hypotheses offered by Hester and supported by his empirical test are totally consistent with the theory developed in this thesis and summarized in Section 1 of this chapter.[2] Furthermore, since the model of this thesis is founded on more basic principles, it should supplant his conjecture as a fundamental basis for the loan offer function.

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[1] One expects the sign to be positive because zero (less) collateral is compensated only by a higher interest rate (for q_1 to remain constant).

[2] Recall that implications of the model are consistent with Hester's predictions only when interest rates are expected to remain constant or rise. Since the data was taken from the period of 1955 to 1957 (when interest rates were not expected to decline), it is not surprising that the data tend to confirm the hypotheses.

CHAPTER VII

SUMMARY AND CONCLUSIONS

This study has two main objectives. The first is to provide a theoretical framework for the bank loan decision. The development of this theory is in two parts. The first abstracts from time. It provides a basis for the relationship between four terms of a loan--size of loan, contract rate of interest, proportion of collateralization and percent of compensatory balances.

The second part presents a theory that relates the maturity of a loan to its rate of return. In order to make this decision the bank must have forecasts of future interest rates. The second objective of this study is to present a viable theory of the interest rate on which the bank may base its forecast.

The theory of the loan decision is based on the assumption that risk averse banks maximize expected utility. Utility derives from profits which in turn are a function of the random outcome of the loan. A fundamental basis for the bank's perception of the loan outcome is offered. It accounts for the existence of collateral and indirect affects of compensatory balances.

When an interior solution exists our analysis predicts that the optimal size of loan increases unambiguously with an increase in the proportion of collateral. The optimal size of loan increases

with an increase in either the contract rate of interest or the percent of compensatory balances when the bank's elasticity of marginal utility with respect to size of loan is less than one.

Under the hypothesis of efficient markets and with constant or increasing expectations about future interest rates our model concludes that the contract rate of interest will increase with an increase in maturity. When this result is linked to the static model, we conclude that the optimal size of loan decreases with an increase in maturity.

The theoretical results are compared with an empirical examination of the loan offer function by Hester (16). They are found to be consistent with the data used in that study.

The theory of the interest in this study is an extension of Fisherian notion. It generalizes from a univariate concept of the interest rate to a multi-variate one and accounts for a risk premium. Several propositions are drawn under progressively weakened versions of the Efficient Markets Hypothesis.

Under the strongest form of the hypothesis, we conclude that differentials of money market rates of the same maturity are constant when risk perceptions are fixed. When we allow for information costs (but abstract from transactions costs and differences in interpretations of information), the theory states that money rate differentials behave according to a stationary stochastic process if risk perceptions are fixed (even though individual rates may follow a random walk).

Under the weakest version of the hypothesis, a proposition is presented that states: under fixed risk perceptions any shock to interest rate differentials will be reversed by 100 percent after cost barriers are transcended. The higher the costs inhibiting the market, the longer it takes for complete reversion.

These propositions were tested using Box-Jenkins methods. There was moderate support for the proposition under the weakest version of Efficient Markets. There is evidence that information costs are a significant inhibiting factor.

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