TRUCKLOAD MOTOR CARRIER RATES IN A NORMATIVE SPATIAL ENVIRONMENT

Thesis for the Degree of Ph. D.

MICHIGAN STATE UNIVERSITY

JAMES PATRICK HYNES

1971

This is to certify that the

thesis entitled

TRUCKLOAD MOTOR CARRIER RATES IN A
NORMATIVE SPATIAL ENVIRONMENT

presented by

James Patrick Hynes

has been accepted towards fulfillment
of the requirements for

Ph. D. degree in Marketing and
Transportation

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Date June 1, 1971

ABSTRACT

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Very little analytical attention has been focused on directional and spatial effects in truckload freight rate structures. Directional effects in freight rates have been widely discussed in the literature, but a general mathematical resolution of the problem has not evolved.

Directional effects are looked at here in terms of a profit maximizing motor carrier. Under normative assumptions, the characteristics of a rate structure which maximizes profit for the carrier are examined.

Rates on truckload freight traffic in an actual common motor carrier are examined for directional patterns which correspond with empty vehicle movement patterns prescribed by the normative model. The data indicate a positive statistically significant relationship.

It is concluded that directional effects are significant and measurable factors in a motor carrier's rate structure, and therefore should not be ignored in contemporary rate studies.

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By

James Patrick Hynes

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Marketing and Transportation

1971

ACKNOWLEDGMENTS

This dissertation was brought to fruition through the efforts of many people. The author wishes to extend his sincere appreciation for these contributions.

My gratitude is especially directed toward Dr. Frank H. Mossman, committee chairman, for providing me with the basic theoretical framework for implementing this research. His inspirational and material help made it possible.

I wish to thank Dr. Richard F. Gonzalez, committee member, for his invaluable critiques of the theoretical aspects of this thesis. A special thanks to Dr. Gardner M. Jones for providing insightful interpretations of the relations encountered in the research. Also, the aid of the motor carrier firm which provided the data is greatly appreciated.

My greatest thanks goes to my wife, Donna, whose willing sacrifices and encouragement contributed to the successful completion of this project.

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CHAPTER I

A FRAMEWORK FROM WHICH TO APPROACH TRUCKLOAD FREIGHT TRANSPORTATION RATES IN A MOTOR CARRIER SYSTEM

Introduction

This dissertation focuses on one facet of an issue in the field of transportation. The issue is truckload motor carrier freight rates. The intent here is to examine analytically and to explain spatial interrelationships among truckload freight rates in a multiple terminal motor carrier system.

It is recognized in transportation that freight rates are affected by the direction of freight movement.[1] For example, lower domestic air rates generally prevail from west to east and from south to north.[2] Nevertheless, no explicit analytical explanation to this phenomena has evolved in the literature. In some cases this phenomena

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[1] See D. P. Locklin, Economics of Transportation (5th ed.; Homewood, Ill.: R. D. Irwin, Inc., 1960), Chapters 20 and 23.

[2] *Ibid.*, p. 779.

is completely ignored. This dissertation presents an analytical resolution of the problem.

General Considerations

A volatile question for at least a half century has been the precise identification of the economic considerations faced by transportation firms and their customers, and the interaction processes affecting freight rates. The Pigou-Taussig controversy[1] of the early nineteen hundreds is evidence of the early differences in the interpretation of the transportation rate problem with regard to cost and demand factors. Since then the problem has been discussed by other economists, accountants, transportation experts, academicians, engineers, politicians, statisticians, business men, and operations researchers.[2] Progress has been realized through these interchanges, and with each advance it becomes more evident that current problems in transportation cannot be viewed solely from one reference point. Rather, these problems are multidisciplinary, and acceptable solutions must address and integrate a variety of contentions.

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[1]Principal documentary evidence of this controversy can be found in F. W. Taussig, "Railway Rates and Joint Costs Once More," Quarterly Journal of Economics, XXVII (February, 1913), 378-84. Also in A. C. Pigou, "Railway Rates and Joint Costs," Quarterly Journal of Economics, XXVII (May, 1913), 536-58.

[2]Specific citations to arguments put forth by these various groups are given in the literature review section.

Some day there may be a "transportation science"[1] which untangles and places arguments in integrated and structured perspectives. Unfortunately, this day is not at hand; work in the area must still proceed on a compartmentalized basis, hypothesizing, identifying, and measuring isolated interrelationships in transportation systems.

Viewpoints

What is the appropriate charge for moving a shipment of goods between two points? The answer to this depends on the vested interests of the individual faced with the question. The carrier entity providing the transportation service would be motivated to charge the most profitable price, or rate. The shipper entity utilizing the transportation service would be motivated to seek the lowest rate which would adequately fulfill his shipping needs. A community would be interested in enforcing rates which maintain and prepare transportation resources for national emergencies. An economist might be interested in setting rates so as to maximize the economic growth of the country. Another economist may be more concerned with rate stability as it relates to economic growth. All of these considerations must be taken into

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[1]See M. L. Manheim, "Principles of Transport Systems Analysis," Papers--Seventh Annual Meeting of the Transportation Research Forum (December, 1966), 9-21.

account when designing systems which adjust and control freight rates in a changing environment.

This dissertation proceeds on the assumption that although these and many other factors affect the logical development of freight rates, it is necessary from a practical standpoint to divide the problem into component parts, attack each part separately, and then synthesize results for further analysis. Of these many steps, one is undertaken here; the economic interactions among shippers and a motor carrier are examined with particular emphasis on the directional orientations of rates. The interactions are examined in terms of a normative economic model. Specific mathematical conditions are formulated which relate the supply of and the demand for transportation among multiple terminals.

The Problem

In a normative sense, freight rate levels are results of economic interactions between shippers and carriers. These interactions are often explained in terms of the cost of service and value of service concepts.[1] A rate is economically viable as long as it is greater than or equal to the carrier's cost of service and less than or

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[1]Elaborations of these concepts can be found in most any basic transportation text. See D. F. Pegrum, Transportation: Economics and Public Policy (revised ed.; Homewood, Ill.: R. D. Irwin, 1968), pp. 229-32.

equal to the shipper's value of the service.[1] Neither the cost of service nor value of service approaches clearly resolve rate problems. First, it is difficult to identify quantitatively the dollar value of the transportation service. Second, it is difficult to allocate costs in a motor carrier system. It is generally recognized that common and joint costs cannot be consistently and logically allocated to specific freight movements without taking into account some intangible factors.

> The distribution of the burden of these joint costs is a matter requiring an interpretation of the value-of-service or demand factor by kinds of freight, size of shipment, and directions of movement.[2]

Another author goes so far as to say:

> . . . that there is no cost curve in the economic sense for the total output of any transportation enterprise. There are a variety of costs co-existing at a moment of time, depending on specific conditions.[3]

Cost allocation problems arise in several different operational areas of a motor carrier.[4] The major areas are:

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[1]Hugh S. Norton, Modern Transportation Economics (2nd ed.; Columbus, Ohio: Charles E. Merrill Pub. Co., 1971), pp. 141-44.

[2]Charles A. Taff, Commercial Motor Transportation (Homewood, Ill.: R. D. Irwin, 1950), p. 122.

[3]George W. Wilson, "On the Output Unit in Transportation," Land Economics, XXXV (August, 1959), 276.

[4]Taff, op. cit., pp. 119-22.

1. The allocation of empty vehicle movement costs.
2. The allocation of pickup and delivery costs when multiple shipments and stops are involved.
3. The allocation of costs when excess equipment and facilities are utilized only during peak seasons.
4. The allocation of costs which relate to the entire operation of the firm; for example, executive salaries would fall in this category.

This dissertation is concerned with the total costs and revenues in the whole motor carrier system as opposed to being concerned with assigning costs to a particular freight shipment. In this context, the problem of identifying which specific freight shipment gave rise to some cost is, for the moment, a moot issue. Freight rates are approached from the point of view of a profit maximizing carrier. Of principal concern is the interaction among rates, freight flows, costs, and empty vehicle movements. An example illustrating basic interrelationships is presented below to indicate the nature of the problem.

Rate Adjustment and Profit Maximization

Consider a motor carrier firm which transports freight among five terminals. These terminals are labeled A, B, C, D, and E; their relative positions are shown in Figure 1.

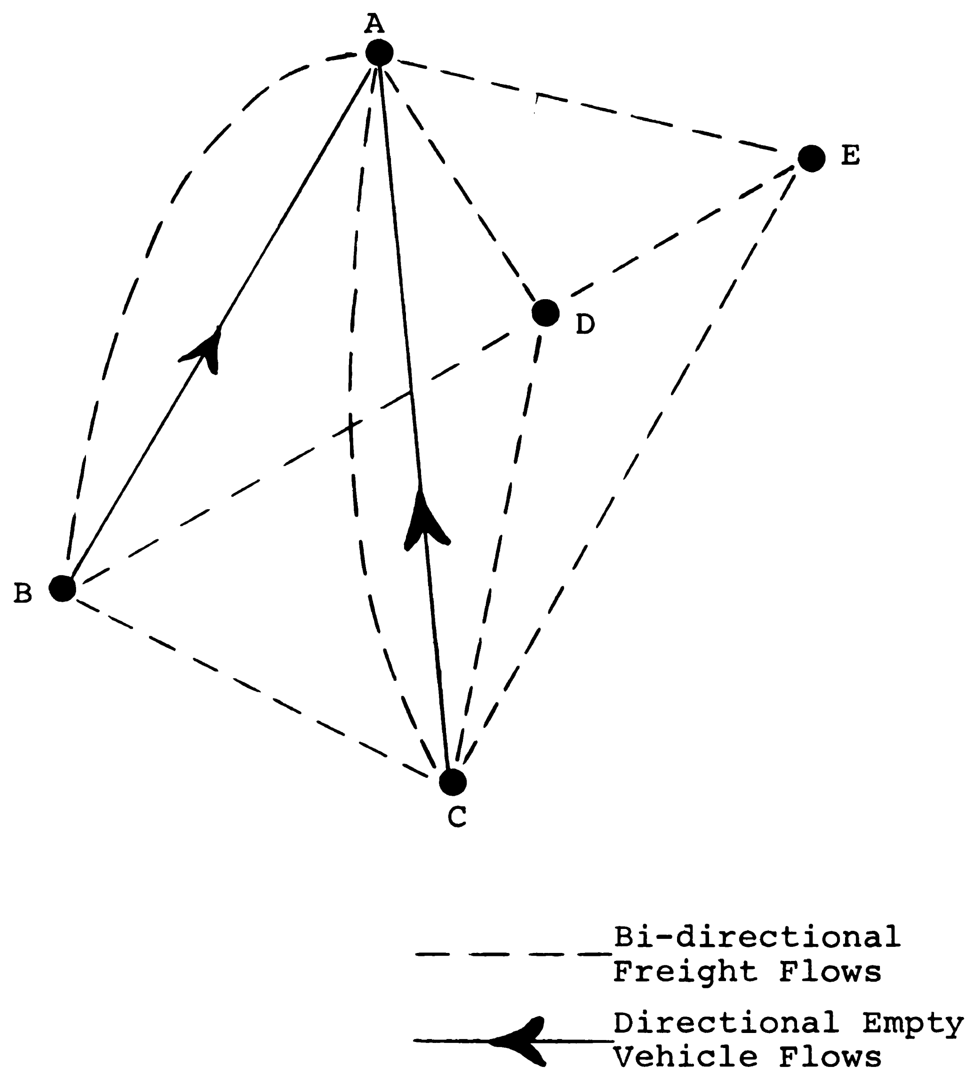

Figure 1.--Freight and vehicle flows in a hypothetical system.

Suppose that freight movements into and out of terminal D and E are relatively balanced; that is, through time total inbound freight movements are roughly equal to outbround freight movements and that there is neither a shortage nor surplus of vehicles at these terminals. Suppose, on the other hand, that freight traffic is unbalanced at terminals A, B, and C; terminals B and C have greater inbound freight movements than outbound freight movements, and terminal A has greater outbound freight movements than inbound movements. This imbalance is rectified in the short run by moving empty vehicles from B to A, and from C to A; this is commonly referred to as a backhaul situation. Is this the most profitable operating level, or should the carrier seek changes in certain freight rates?

Consider the following ranges of alternatives.

1. The carrier could increase the rates on shipments moving out of terminal A. This would most likely reduce outbound traffic and therefore would reduce the number of empty vehicles required by A. Presumably, an increase in outbound rates would induce shippers to employ other means of transport.
2. The carrier could reduce the rates on shipments moving to terminal A. This would most likely increase inbound traffic and would provide additional vehicles at terminal A. This would reduce the number and cost of empty vehicles moving to A.

3. The carrier could increase the rates on shipments moving to terminals B and C from A, D, and E. This would most likely reduce the freight traffic moving to B and C, and automatically provides a closer source of empty vehicles for A. For example, if shipments from D to B were curtailed, then the resulting availability of vehicles at D could move directly to A as opposed to the longer B to A route.
4. The carrier could leave freight rates as they are; this avenue would presume that the most profitable, or optimum, operating level was in effect.

These are only a few of the many possible rate adjustments that might take place. It is readily apparent that freight rates in one portion of the system are interrelated to freight rates and flows in other portions of the system. This complex problem is approached in this dissertation by analyzing the conditions which denote maximized profit in a normative system.

Literature Review

The topic of transportation rates has been widely discussed in the literature. Besides elementary transportation texts,[1] there are various publications which

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[1]For example: John B. Lansing, Transportation and Economic Policy (New York: The Free Press, 1966); Locklin, op. cit.; Norton, op. cit.; Pegrum, op. cit.; Charles A.

examine competition in transportation,[1] the feasibility of cost based rates,[2] value of service rates,[3] and supply and demand for transportation,[4] and problems in the regulation of transportation.[5] These documents recognize and

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Taff, Management of Traffic and Physical Distribution (3rd ed.; Homewood, Ill.: R. D. Irwin, 1964).

[1]John R. Meyer, Merton J. Peck, John Stenason, and Charles Zwick, The Economics of Competition in Transportation Industries (Cambridge, Mass.: Harvard University Press, 1959.

[2]Systems Analysis and Research Corporation, Cost-Based Freight Rates, Desirability and Feasibility (Washington, D.C.: U.S. Department of Commerce, Under Secretary for Transportation, 1966); John R. Meyer and Gerald Kraft, "Evaluation of Statistical Costing Techniques as Applied in the Transportation Industry," The American Economic Review, LI (May, 1961), 313-34; A. A. Walters, "The Allocation of Joint Costs with Demands as Probability Distributions," The American Economic Review, L, 3 (1960), 419-32.

[3]Interstate Commerce Commission, Bureau of Transport Economics and Statistics, Value of Service in Rate-Making, Statement No. 5912 (Washington, D.C.: ICC, Bureau of Transport Economics and Statistics, 1959).

[4]For example: H. Benishay and G. E. Whitaker, Jr., "Demand and Supply in Freight Transportation," Journal of Industrial Economics, XIV (July, 1966), 243-62; Eugene D. Perle, The Demand for Transportation: Regional and Commodity Studies in the United States (Chicago: University of Chicago Press, 1966); Robert B. Adams, "An Input-Output Program for the Department of Transportation," Transportation Journal, VIII (Winter, 1968), 45-52; Z. S. Zannetos, The Theory of Oil Tankship Rates (Cambridge, Mass.: The MIT Press, 1966).

[5]Ann F. Friedlander, The Dilemma of Freight Transport Regulation (Washington, D.C.: The Brookings Institution, 1969); James Sloss, "Regulation of Motor Freight Transportation: A Quantitative Evaluation of Policy," The Bell Journal of Economics and Management Science, I (Autumn, 1970), 327-66; C. F. Phillips, The Economics of Regulation (rev. ed.; Homewood, Ill.: R. D. Irwin, Inc., 1969).

give lip service to the existence of unbalanced freight movements and loosely identify responses of freight rates to this backhaul condition. In some instances the directional rate and backhaul problem is sometimes assumed away.

> Obviously, the backhaul problem creates great instability in rate structures. . . . No doubt in the aggregate this effect balances out, but it may be a hardship on certain carriers.[1]

In other instances it is assumed to be an extremely complex problem which has no general solution.[2]

The approach taken in this dissertation is balanced between the above two extremes. The problem is formulated here so as to incorporate the basic factors which give rise to the problem and at the same time provide a precise explanation of the resulting directional interrelationships in freight rate structures.

A streamlined and tight examination of directional rates must logically proceed from an unambiguous foundation in the form of a mathematical model. The first notable analytical examination of the directional rate and backhaul problem was presented by T. C. Koopmans and S. Reiter.[3]

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[1]Norton, op. cit., p. 115.

[2]See Wilson, op. cit. Also M. L. Burstein, et al., The Cost of Trucking: Econometric Analysis (Dubuque, Iowa: Wm. C. Brown Company Publishers, 1965), pp. 112-13.

[3]T. C. Koopmans and S. Reiter, "A Model of Transportation," in Activity Analysis of Production and Allocation, ed. by T. C. Koopmans (New York: John Wiley & Sons, Inc., 1951), pp. 222-59.

The cited article identified the normative relationships between transportation rates and movements of empty ships, or ballast traffic.

All the major concepts in this dissertation were developed independently of Koopmans' article. The model presented in Chapter II is a generalization of this early transportation model. Specifically, Koopmans and Reiter assumed that cargo flows between terminals are fixed whereas here it is assumed that cargo flows between terminals are related to the transportation rate charged for the service. It is interesting to note that Koopmans was one of the first to identify the analogy between transportation systems and linear graph theory.

> The cultural lag of economic thought in the application of mathematical methods is strikingly illustrated by the fact that linear graphs are making their entrance into transportation theory just about a century after they were first studied in relation to electrical networks, although organized transportation systems are much older than the study of electricity.[1]

In following chapters, the linear graph analogy[2] itself is not emphasized because it tends to detract and focus attention away from issues relevant to transportation.

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[1]Koopmans and Reiter, op. cit., p. 258.

[2]Other linear graph analogies of economic and social systems are described in contemporary literature. For example, see F. Harary, R. Norman, and D. Cartwright, Structural Models: An Introduction to the Theory of Directed Graphs (New York: John Wiley & Sons, Inc., 1966); H. C. Koenig and T. J. Manetsch, System Analysis of the Social Sciences (East Lansing, Mich.: College of Engineering, Michigan State University, 1966). (Mimeographed.)

The concepts in Koopmans' and Reiter's basic model have been used mostly in transportation equipment scheduling. Indirect extensions have been in the areas of geography and spatial economics.[1] However, there have been no notable attempts to generalize the economic nature of transportation rates in the manner in which they are treated in this dissertation.

Transportation Equipment Scheduling

Transportation equipment scheduling problems[2] are generally posed in the following manner. Given the estimated requirements for the movement of freight between terminals in the system, and given the locations, capacities and costs of transportation equipment in the system,

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[1]There are numerous professional journals and articles concerned with these areas. Some reference articles are given here. W. L. Garrison, "Spatial Structure of the Economy: III," Annals of the Association of American Geographers, L (September, 1960), 357-73; F. R. Pitts, "A Graph Theoretic Interpretation of Historical Geography," The Professional Geographer, XVIII (May, 1956), 15-20; L. J. King, "The Analysis of Spatial Form and Its Relation to Geographic Theory," Annals of the Association of American Geographers, LIX (September, 1969), 573-95; Gunnar Olsson, "Central Place Systems, Spatial Interaction and Stochastic Processes," Papers and Proceedings, Regional Science Association, XVIII (1967), 13-45; R. E. Quandt and W. J. Baumol, "The Demand for Abstract Transport Modes: Some Hopes," Journal of Regional Science, IX, 1 (1969), 159-62; A. Stegman, "Accessibility Models and Residential Location," Journal of the American Institute of Planners, XXXV, 1 (1969), 22-29.

[2]For example see J. F. Pierce, "Direct Search Algorithms for Truck-Dispatching Problems," Transportation Research, III (1969), 1-42; Burstein, et al., op. cit., pp. 47-105; J. L. Saha, "An Algorithm for Bus Scheduling Problems," Operational Research Quarterly, XXI (December, 1970), 463-74.

the movements of equipment between terminals are scheduled in a manner which minimizes total costs while satisfying service requirements. Note again, as with the Koopmans' model, it is assumed that freight movement requirements are fixed; the normative models in this dissertation begin with the assumption that freight movement requirements are variable with the price, or rate, being charged for the service.

Objectives and Organization

The primary objective of this dissertation is to develop a normative model which will aid in the explanation and resolution of directional factors in truckload transportation rates. The secondary objective is to give empirical evidence showing that actual truckload motor carrier freight rates resemble the analytical patterns suggested in the model.

This dissertation is organized into four chapters. The first chapter introduces the general nature of the subject and problem area. It focuses on the literature and relevant issues which underscore the contemporary significance of the problem, and places in perspective the approach taken to solve the problem.

The second chapter develops a model which is used to examine the fundamental economics of a multiple terminal common motor carrier system. The model identifies the conditions which will occur when a motor carrier

facing known demand curves has adjusted truckload freight rates and empty vehicle flows so as to be at an operating level which maximizes profits. The conditions show that the rate structure which maximizes profit is not overly complex; rather, predictable directional patterns will result. According to the model, directional rates are ultimately due to the profit maximization motive of the motor carrier, and to the differences in demands for transportation among points in the transportation system. Differences in demand give rise to imbalanced freight movements which cause directional orientations in the incremental costs of freight movements. This induces the carrier to change freight rates, which, in turn, affect traffic flows and vehicle imbalances. This cycle is theoretically repeated until the carrier is at a point in which rates, costs, and traffic flows are such that profit can no longer be increased through additional changes.

From the relationships prescribed in the normative model it is hypothesized in Chapter III that patterns in the truckload freight rate structure are mathematically related to the patterns of empty vehicle flows and freight traffic imbalances in an actual system. This hypothesis is tested by examining the rates, costs, and freight imbalances in an actual motor carrier system. A motor common carrier provided the data for this empirical investigation. The empirical analysis measures the statistical significance of the system wide relationship between patterns in freight traffic imbalances and the freight rate structure in a manner suggested by the normative model. The analysis provides a basic foundation for examining the effects of unequal demands and vehicle balancing on truckload freight rates.

The fourth chapter synthesizes the conclusions drawn from the normative and empirical investigations, and shows how these conclusions can be used to resolve contemporary issues in transportation management, regulation, and planning.

CHAPTER II

CHARACTERISTICS OF PROFIT MAXIMIZATION IN A NORMATIVE MODEL OF A MULTIPLE TERMINAL MOTOR CARRIER

Introduction

A normative model of a motor carrier is developed in this chapter. The model specifies mathematical conditions for profit maximization. The development and analysis of this model is analogous to the economic analysis of a monopolistic firm,[1] modified, of course, to a motor carrier. The same general assumptions made by the economist for static equilibrium models are made here. The model focuses on equilibrium states of the system, where a carrier attains maximum profits. Time dependent adjustment processes are not explicitly examined here. In the latter part of the chapter, the model is interpreted with regard to its compatibility with contemporary economic views of transportation rates.

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[1]See C. E. Ferguson, Microeconomic Theory (Homewood, Ill.: R. D. Irwin, Inc., 1966), Chapter X and XI. Also, J. M. Henderson and R. E. Quandt, Microeconomic Theory (New York: McGraw-Hill Book Company, 1958), Chapter VI.

General Model Structure

This model is designed for examining the optimal economic characteristics of revenues and costs of general commodity truckload freight in a motor carrier system. The model represents a motor carrier facing known revenue, cost, and freight flow relationships; it is then assumed that the carrier maximizes profit with respect to these known conditions.

The crux of the profit maximization problem is the optimum adjustment of freight rates with respect to balanced vehicle movements in the system.

A fundamental operating constraint on a motor carrier is the equalization of inbound and outbound vehicle flow at each terminal. A surplus of vehicles at certain terminals and shortages of vehicles at other terminals are caused by unequal inbound and outbound freight flows at the terminals. This problem is rectified by adjusting freight rates and empty vehicle movements in manners which promote profitable balanced freight and vehicle flows in a system.

Segmentation of Carrier Activities

The model focuses on the economics of interterminal truckload freight, as opposed to freight activities such as less-than-truckload movements, intraterminal freight movements, and pickup and delivery. These other freight activities are not ignored in the model; rather, the effects of these other freight activities are incorporated in the model as known variables. This assumption simplifies the understanding of the problem, and provides a foundation for analyzing the economics of these other transport activities. For descriptive purposes, carrier activities are segmented into three mutually exclusive and exhaustive nominal categories. The first category is truckload freight movements, the second is empty vehicle movements, and the third is called other carrier activities.

Truckload freight movements refer to the movement of truckload freight between <u>separate</u> terminals in the system. Empty vehicle movements refer to the movements of empty vehicles between separate terminals in the system. Other carrier activities refer to activities which do not fall into the above two categories. These nominally segmented activities are synthesized in the model by assuming that the revenues, costs, and vehicle imbalances caused by <u>other</u> carrier activities have known effects in the system; the model then maximizes remaining profits on truckload freight with respect to vehicle balancing costs.

Freight and Vehicle Assumptions

The system under consideration is a motor carrier which transports general commodity freight among spatially separated stationary terminal facilities. The terminals in the system are numbered sequentially from (1) to (θ), where (θ) represents the number of terminals in the system.

It is assumed that truckload freight enters the system or originates, at some terminal, and is transported by a vehicle directly to its destination terminal without passing through other terminals. The freight then leaves the system. Freight and vehicle movements between terminals are designated by the origin and destination of the movement; they are identified by the terminal from which the movement originated, and the terminal for which the movement is destined. Given that there are (θ) terminals in the system, there are (θ^2) pairs of origins and destinations; this includes intraterminal movements. Intraterminal movements are movements in which both the origin and destination are in a region surrounding a terminal. In most cases they include pickup and delivery operations. As mentioned above, intraterminal movements are categorized as other carrier activities and are not explicitly examined. In all there are (θ) intraterminal movements; therefore, there are ($\theta^2 - \theta$) interterminal freight movements in the model.

Freight and vehicle movements in the system are measured here as flows; that is, quantity per unit of time. It is not absolutely necessary that specific units of freight and vehicle flow be assigned in this model because the variables and equations can be defined to handle any logical unit of measure. However, for descriptive purposes, hundredweight per month is used to specify freight flows, and hundredweight capacity per month is used to specify the flow of vehicles in the system. It is assumed the vehicle capacity flow from one terminal to another during a monthly period must equal or exceed the freight flow moving between those two terminals; the excess of vehicle capacity over freight flow is called empty vehicle flow. Empty vehicle flows arise because of the necessity to achieve balanced flows of vehicles in a system; these and other assumptions are detailed in following sections.

Mathematical Notation

The mathematical symbols used to represent freight and vehicle flows are identified in this section. Table 1 is a summary of these and other mathematical variables used in this chapter.

Number of Terminals

As mentioned previously, the Greek letter (θ) represents the number of terminals in the motor carrier system.

$$\text{Number of Terminals in System} = \theta \qquad (2\text{-}1)$$

The terminals are indexed by the letters i,j,k,m, and n in this and later chapters.

TABLE 1.--A summary of mathematical variables used in Chapter II.

(θ) is the number of terminals in the system.

q(i,j) is freight flow from terminal (i) to (j), (hundredweight per month).

(q) represents the freight flows in the entire system.

u(i,j) is empty vehicle flow from terminal (i) to (j), (hundredweight capacity per month).

(u) represents the empty vehicle flows in the entire system.

(q(i,j) + u(i,j)) is the vehicle capacity flow from terminal (i) to j), (hundredweight capacity per month).

s(i) is exogenous vehicle capacity available at terminal (i), (hundredweight capacity per month).

r(q_o,i,j) is the first derivative of the total revenue function with respect to q(i,j) evaluated at the solution (q_o). It is the marginal revenue on freight moving from terminal (i) to (j).

a(q_o,i,j) is the first derivative of the total direct cost function with respect to q(i,j) evaluated at the solution (q_o). It is the marginal direct cost on freight moving from terminal (i) to (j).

b(u_o,i,j) is the first derivative of the total cost function of moving empty vehicles with respect to u(i,j) evaluated at the solution (u_o). It is the marginal cost of moving empty vehicles from terminal (i) to (j).

p(q_o,u_o,i,j) is the marginal vehicle profit on vehicles moving from terminal (i) to (j). It is equal to (r(q_o,i,j) - a(q_o,i,j)) or (-b(u_o,i,j)), which ever is greater.

(scmvp) is an informal mathematical variable which represents the sum of marginal vehicle profits around an arbitrary circuitous route in the system.

tdc(q) is monthly total direct cost as a function of freight flow (q).

tec(u) is monthly total cost of empty vehicle movements as a function of empty vehicle flows (u).

TABLE 1--*Continued*

ttr(q) is monthly total truckload revenue as a function of freight flow (q).

(tpa) is monthly total profits on other carrier activities, a constant.

t(i) is the imputed marginal profitability of vehicle capacity at terminal (i). It is derived from the Kuhn-Tucker theorem, and is used to describe optimality conditions.

d,y(i,j) and z(i,j) are "dummy" variables used to apply the Kuhn-Tucker theorem to the motor carrier model. They are not given an explicit interpretation.

Freight Flows

The variable q(i,j) represents the hundredweight per month flow of truckload freight from origin (i) to destination (j). The flows of interterminal truckload freight for the entire system are designated as q(i,j), $(i=1,\ldots,\theta;\ j=1,\ldots,\theta;\ i\neq j)$. The letter q, without adjacent parentheses, is used to designate the freight flows in the entire system.

$$q \equiv q(i,j),\ (i=1,\ldots,\theta;\ j=1,\ldots,\theta;\ i\neq j) \qquad (2\text{-}2)$$

This equivalency is made for convenience in order to shorten the designation of system wide freight flows.

Empty Vehicle Flows

The variable u(i,j) represents the flow of empty vehicles from terminal (i) to terminal (j). This variable is measured in terms of hundredweight capacity per month. The letter u is used to represent system wide empty vehicle flows.

$$u \equiv u(i,j),\ (i=1,\ldots,\theta;\ j=1,\ldots,\theta;\ i\neq j) \qquad (2\text{-}3)$$

External Vehicle Capacity

Vehicle capacity which arises, or is required, at terminals as a result of other carrier activities in the system is represented by the variable s(i); it is the monthly surplus, or shortage, of vehicle capacity at terminal (i). The variable s(i) is measured in terms of hundredweight vehicle capacity per month. A positive value for this variable signifies that other carrier activities provide an excess of vehicles at the indexed terminal; a negative value signifies that other carrier activities require vehicles at the indexed terminal. System wide surpluses and shortages are represented by the letter s.

$$s \equiv s(i), \ (i=1,\ldots,\theta) \tag{2-4}$$

In the initial development of profit maximization characteristics, the values in s are assumed to be constant; during the interpretation of the profit maximization characteristics, this assumption is relaxed.

Flow Constraints

It is assumed that truckload freight is transported directly from the origin terminal to the destination terminal, and that the amount of vehicle capacity used to transport the truckload freight equals, or exceeds, the amount of freight transported.

The fundamental constraint on freight and vehicle flows is that the availability of vehicle capacity at any terminal must equal or exceed the amount of vehicle capacity required at the terminal through time. This constraint is informally expressed as follows. A terminal's inbound truckload freight, plus the terminal's inbound empty vehicles, minus the terminal's outbound truckload freight, minus the terminal's outbound empty vehicles, plus the vehicle capacity adjustments due to other carrier activities must be greater than or equal to zero.

A mathematical expression of the constraint is given below in (2-5) for terminal (m).

$$\sum_{\substack{i=1 \\ i \neq m}}^{\theta} q(i,m) + \sum_{\substack{i=1 \\ i \neq m}}^{\theta} u(i,m) - \sum_{\substack{i=1 \\ i \neq m}}^{\theta} q(m,i) - \sum_{\substack{i=1 \\ i \neq m}}^{\theta} u(m,i) + s(m) \geq 0 \qquad (2\text{-}5)$$

where

$$(\text{Inbound truckload freight flow}) = \sum_{\substack{i=1 \\ i \neq m}}^{\theta} q(i,m)$$

$$(\text{Inbound empty vehicles flow}) = \sum_{\substack{i=1 \\ i \neq m}}^{\theta} u(i,m)$$

$$(\text{Outbound truckload freight flow}) = \sum_{\substack{i=1 \\ i \neq m}}^{\theta} q(m,i)$$

$$(\text{Outbound empty vehicles flow}) = \sum_{\substack{i=1 \\ i \neq m}}^{\theta} u(m,i)$$

$$(\text{Vehicle capacity adjustments due to other carrier activities}) = s(m)$$

In all there are (θ) of these equations, each expressing the vehicle flow constraints at each of the (θ) terminals.

In adjusting rates and controlling the flow of freight and empty vehicles in the system, it is assumed that the carrier must operate within the confines of these constraints.

Cost and Revenue Assumptions

Four categories are identified here; the first category accounts for truckload transportation costs, the second accounts for empty vehicle movement costs, the third accounts for truckload freight revenues, and the fourth accounts for revenues and costs, or profit, on other carrier activities.

Mathematical relationships are identified so as to provide a basis to assess the profitability of feasible, or admissible, freight and vehicle flows.

The derivatives of the mathematical functions described in this section are used extensively in the model and, therefore, derivative notations are described along with the functions. It is assumed that all functions described below are continuously differentiable within the relevant ranges of the flow variables.

Direct Transportation Costs

Direct truckload freight transportation costs are defined here to include all costs related to the movement of transporting truckload freight among the terminals. Direct transportation costs do not include any costs related to the movement of empty vehicles. They are assumed to include the costs of pickup and delivery, loading and unloading, insuring, billing, linehaul costs, and all other costs directly related to the movement of freight.

A mathematical function is used to describe the relationship between truckload freight flows and total direct cost. The function describing the relationship is denoted as tdc(q).

$$\text{Monthly Total Direct Cost} = tdc(q) \qquad (2\text{-}6)$$

The symbol q represents freight flows in the system as defined in (2-2). For given values of q, the function tdc(q) gives monthly direct transport costs incurred by the system.

The partial derivative of tdc(q) with respect to q(i,j) evaluated at a point q_o is denoted as a (q_o,i,j).

$$a(q_o,i,j) = \left.\frac{\partial tdc(q)}{\partial q(i,j)}\right|_{q=q_o} \qquad (2\text{-}7)$$

In economic terms,[1] $a(q_o,i,j)$ is called the marginal direct cost of moving freight from terminal (i) to (j) at the point q_o.

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[1]Marginal cost is a standard economic term, see C. E. Ferguson, <u>op. cit.</u>, pp. 168-75.

Empty Vehicle Movement Costs

The cost related to the movement of empty vehicles in the system is expressed as a function of u.

Monthly Total Empty Vehicle Movement Cost = tec(u) (2-8)

The symbol u represents empty vehicle flows in the system as defined in (2-3). For given values of u, the function tec(u) gives monthly total empty vehicle movement cost. The partial derivative of tec(u) with respect to u(i,j) evaluated at a point u_o is denoted as $b(u_o,i,j)$.

$$b(u_o,i,j) = \left.\frac{\partial tec(u)}{\partial u(i,j)}\right|_{u=u_o} \tag{2-9}$$

In economic terms, $b(u_o,i,j)$ is called the marginal cost of moving empty vehicles from terminal (i) to (j) at the point u_o.

Truckload Freight Revenues

In this model it is assumed that the carrier is facing known demand curves for freight transportation service between terminals, and that the carrier can operate at any desirable point on the demand curve. It is assumed that the carrier can control the amount of truckload freight flowing between each pair of terminals by adjusting prices of the transportation service. Presumably, a high rate, or price will discourage the movement of freight between two terminals; likewise a low freight rate will increase the flow of freight between two terminals.

It is assumed that there are mathematical functions describing the relationships between freight rates and truckload freight flow for each pair of terminals; furthermore, it is assumed that these functions can be inverted to give the freight rate as a function of the freight flow. This makes it possible to express revenues as a mathematical function of freight flows in the system; the function ttr(q) gives the total truckload freight revenue as a function of q.

$$\text{Monthly Total Truckload Freight Revenue} = ttr(q) \qquad (2\text{-}10)$$

The symbol q represents freight flows in the system as defined in (2-2). For given values of q, the function ttr(q) gives monthly total revenue.

The assumptions regarding the above function are made in a mathematical sense in order to examine the profit maximizing characteristics of the model. Actual rate and marketing adjustments involved in controlling freight flows are not explicitly identified; rather it is assumed that these adjustment processes take place, and that the equation in (2-10) gives the relationship between truckload revenue and freight flows in the system.

The partial derivative of ttr(q) with respect to q(i,j) evaluated at a point q_o is denoted as $r(q_o,i,j)$.

$$r(q_o,i,j) = \left.\frac{\partial ttr(q)}{\partial q(i,j)}\right|_{q=q_o} \qquad (2\text{-}11)$$

In economic terms,[1] $r(q_o,i,j)$ is called the marginal revenue on freight moving from terminal (i) to (j) at the point q_o.

Profits on Other Carrier Activities

The variable (tpa) is used to represent monthly profits of other carrier activities.

Monthly Profits on Other Carrier Activities = tpa (2-12)

As previously defined, other carrier activities are those activities which are neither classified in the inter-terminal truckload freight category nor the empty vehicle movement category. For example, less-than-truckload freight movements and intraterminal freight movements fall in this category. Overhead items not included in truckload freight and balancing costs would also be included here. The variable (tpa) is assumed to be mathematically independent of q and u, and, therefore, is treated as a constant in this model.

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[1]Marginal revenue is a standard economic term, see Ferguson, op. cit., pp. 87-95.

Model Synthesis

The model is synthesized here in terms of the relevant costs, revenues, flow constraints, and profit maximization objectives. Profits are maximized by finding values for q and u which maximize equation (2-13) with respect to the constraints in equations (2-14), (2-15), and (2-16).

$$\text{Monthly Profit} = ttr(q) - tdc(q) - tec(u) + tpa \qquad (2\text{-}13)$$

$$\left(\left(\sum_{\substack{i=1 \\ i \neq m}}^{\theta} (q(m,i) + u(m,i) - q(i,m) - u(i,m))\right) - s(m)\right) \leq 0, \ (m=1,\ldots,\theta). \qquad (2\text{-}14)$$

$$q(i,j) \geq 0, \ (i=1,\ldots,\theta;\ j=1,\ldots,\theta;\ i \neq j) \qquad (2\text{-}15)$$

$$u(i,j) \geq 0, \ (i=1,\ldots,\theta;\ j=1,\ldots,\theta;\ i \neq j) \qquad (2\text{-}16)$$

The equation (2-13) was obtained by appropriately summing equations (2-6), (2-8), (2-10), and (2-12). The equations (2-14) were obtained by multiplying equation (2-5) by a minus one and consolidating the summation signs; this rearrangement facilitates the application of maximization conditions. The equations in (2-15) and (2-16) are nonnegative restrictions on freight and vehicle flow. The necessary conditions for maximization of (2-13) are outlined in the next section.

Necessary Profit Maximization Conditions

This section specifies the mathematical relationships which must occur when maximum profit is attained in the model presented in the preceding section. The necessary conditions stated here become necessary and sufficient conditions when (2-13) is concave.[1]

The application of the Kuhn-Tucker theorem on non-linear programming, as developed in Appendix B, to the problem stated in (2-13), (2-14), (2-15), and (2-16) gives the following necessary conditions. If q_o and u_o are solutions which maximize profits with respect to the constraints, then there must exist values for the variables $t(i)$, $(i=1,\ldots,\theta)$, and $y(i,j)$, $z(i,j)$, $(i=1,\ldots,\theta;\ j=1,\ldots,\theta;\ i\neq j)$, which satisfy (2-17), (2-18), (2-19), (2-20), (2-21), (2-22), (2-23), and (2-24).

$$t(i) \geq 0, \quad (i=1,\ldots,\theta) \tag{2-17}$$

$$t(i) \cdot \Big(\Big(\sum_{\substack{j=1 \\ j\neq i}}^{\theta} q_o(i,j) + u_o(i,j) - q_o(j,i) - u_o(j,i)\Big) - s(i)\Big) = 0, \quad (i=1,\ldots,\theta) \tag{2-18}$$

$$y(i,j) \geq 0, \quad (i=1,\ldots,\theta;\ j=1,\ldots,\theta;\ i\neq j) \tag{2-19}$$

$$y(i,j) \cdot q_o(i,j) = 0, \quad (i=1,\ldots,\theta;\ j=1,\ldots,\theta;\ i\neq j) \tag{2-20}$$

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[1] This aspect is expanded upon in a later section.

$$z(i,j) \geq 0, \quad (i=1,\ldots,\theta;\ j=1,\ldots,\theta;\ i\neq j) \tag{2-21}$$

$$z(i,j) \cdot u_o(i,j) = 0, \quad (i=1,\ldots,\theta;\ j=1,\ldots,\theta;\ i\neq j) \tag{2-22}$$

$$r(q_o,i,j) - a(q_o,i,j) - t(i) + t(j) + y(i,j) = 0, \quad (i=1,\ldots,\theta;\ j=1,\ldots,\theta;\ i\neq j). \tag{2-23}$$

$$-b(u_o,i,j) - t(i) + t(j) + z(i,j) = 0, \quad (i=1,\ldots,\theta;\ j=1,\ldots,\theta;\ i\neq j). \tag{2-24}$$

The above conditions are simplified in Table 2; the table was developed from the following observations.

Positive Freight and Empty Vehicle Flows

For (i,j) such that $q_o(i,j) > 0$ and $u_o(i,j) > 0$, then y(i,j) and z(i,j) must equal zero because of (2-20) and (2-22) respectively. Therefore, y(i,j) can be dropped from (2-23), and z(i,j) can be dropped from (2-24). The combined result is condition one in Table 2.

Positive Freight Flow and Zero Empty Vehicle Flow

For (i,j) such that $q_o(i,j) > 0$ and $u_o(i,j) = 0$, then y(i,j) = 0 because of (2-20) and can be dropped from (2-23). Because of (2-21), z(i,j) can be removed from (2-24) by replacing the equality sign with an inequality. The combined result is condition two in Table 2.

TABLE 2.--A summary of necessary conditions for maximized profit.

If q_o and u_o satisfy vehicle flow constraints, and maximize profit, then there must exist real values for t(i), (i=1,...,θ), which satisfy the following conditions.

Conditions

1. For (i,j) such that
   $q_o(i,j) > 0$, and
   $u_o(i,j) > 0$, then

$$r(q_o,i,j) - a(q_o,i,j) = t(i) - t(j) = -b(u_o,i,j).$$

2. For (i,j) such that
   $q_o(i,j) > 0$, and
   $u_o(i,j) = 0$, then

$$r(q_o,i,j) - a(q_o,i,j) = t(i) - t(j) \geq -b(u_o,i,j).$$

3. For (i,j) such that
   $q_o(i,j) = 0$, and
   $u_o(i,j) > 0$, then

$$r(q_o,i,j) - a(q_o,i,j) \leq t(i) - t(j) = -b(u_o,i,j).$$

4. For (i,j) such that
   $q_o(i,j) = 0$, and
   $u_o(i,j) = 0$, then

$$r(q_o,i,j) - a(q_o,i,j) \leq t(i) - t(j) \geq -b(u_o,i,j).$$

5. For (i) such that

$$((\sum_{\substack{j=1 \\ j\neq i}}^{\theta} q_o(i,j) + u_o(i,j) - q_o(j,i) - u_o(j,i)) - s(i) = 0, \text{ then } t(i) \geq 0.$$

6. For (i) such that

$$((\sum_{\substack{j=1 \\ j\neq i}}^{\theta} q_o(i,j) + u_o(i,j) - q_o(j,i) - u_o(j,i)) - s(i) < 0, \text{ then } t(i) = 0.$$

Positive Empty Vehicle Flow and Zero Freight Flow

For (i,j) such that $q_o(i,j) = 0$ and $u_o(i,j) > 0$, then $y(i,j) \geq 0$ because of (2-19), and $z(i,j) = 0$ because of (2-22). Therefore, $y(i,j)$ can be removed from (2-23) by replacing the equality sign with an inequality, and $z(i,j)$ can be dropped from (2-24). The result is condition three in Table 2.

Zero Freight Flow and Zero Empty Vehicle Flow

For (i,j) such that $q_o(i,j) = 0$ and $u_o(i,j) = 0$, then $y(i,j) \geq 0$ and $z(i,j) \geq 0$ because of (2-19) and (2-21). Therefore, $y(i,j)$ and $z(i,j)$ can be removed from (2-23) and (2-24) by replacing the equality signs with inequalities. The result is condition four in Table 2.

Constrained Vehicle Flow at Terminals

From (2-17) the values for t(i) must in all cases be greater than or equal to zero for an optimal solution. Furthermore, for (i) such that

$$\Big(\sum_{\substack{j=1\\ j\neq i}}^{\theta} q_o(i,j) + u_o(i,j) - q_o(j,i) - u_o(j,i)\Big) - s(i) < 0, \qquad (2\text{-}25)$$

then t(i) must equal zero; this is concluded from (2-18). In words, this condition is saying that if there is excess vehicle capacity at a terminal, say (i), at an optimal solution then t(i) must equal zero. These are the fifth and sixth conditions in Table 2.

Interpretation of the Conditions for Maximized Profit

Motor vehicles, and hence vehicle capacities are an economic resource in a motor carrier system. This intuitively implies that there is an economic value attached to vehicle capacity and vehicle flows. It so happens that following this line of thought provides an interpretation of the equations in Table 2.

Terminal Multipliers

As pointed out in Appendix B, the variables t(i), y(i,j), and z(i,j) used in equations (2-17) through (2-24) are called Kuhn-Tucker multipliers and are analogous to Lagrange multipliers; each multiplier reflects the imputed effect on the objective function which would result from an incremental change on the associated constraint equation. From (2-18) it is apparent that t(i) is associated with the vehicle flow constraint on the (i)th terminal. Therefore, the value of t(i), for an optimal q_o and u_o, equals the imputed increase in profit that would result from an incremental unit increase in vehicle capacity at terminal (i).

Mathematically, (t(i) · d) equals the incremental increase in profit that would result if (d) replaced the zero in the right hand side of the (i)th equation in

(2-14), and the problem was resolved for optimum q_o and u_o. It is assumed that the absolute value of (d) is arbitrarily small.

Similarly, (-t(j) +t(i)) equals the imputed increase in profit that would result from removing a unit of vehicle capacity from terminal (j) and adding it to terminal (i). Mathematically ((-t(j) +t(i)) · d) equals the incremental increase in profit that would result if (-d) replaced the zero in the right hand side of the (j)th equation in (2-14), and (+d) replaced the zero in the right hand side of the (i)th equation in (2-14), and the problem was resolved for optimum q_o and u_o. The reverse would also hold; (-t(j) +t(i)) equals the imputed <u>decrease</u> in profit that would result from <u>adding</u> a unit of vehicle capacity to terminal (j) and removing it from terminal (i).

Marginal Vehicle Profit

The relationship of the t(i) to marginal revenues and costs of freight and vehicle movements can be better interpreted by defining a variable called marginal vehicle profit. Marginal vehicle profit is defined here to be the maximum and immediate incremental increase in profit that would accompany a unit increase in vehicle flow from one terminal to another. Marginal vehicle profit will be denoted as p(q,u,i,j), and is mathematically defined as follows.

$$\text{Terminal (i) to (j) Marginal Vehicle Profit} = p(q,u,i,j)$$

$$\left.\begin{array}{l} = r(q,i,j) - a(q,i,j) \\ \qquad\qquad \text{or} \\ = -b(u,i,j) \end{array}\right\} \begin{array}{l}\text{Whichever}\\ \text{is greater}\end{array} \qquad (2\text{-}26)$$

The value of $p(q,u,i,j)$ is found by finding the most profitable (least expensive) method of moving an incremental unit of vehicle flow from terminal (i) to (j). There are two alternative ways of moving an additional vehicle from, say, terminal (i) to (j). One would be to move an empty vehicle, and the other would be to induce additional freight flow; for the former, marginal profit (negative cost) would be $-b(u,i,j)$, for the latter it would be $(r(q,i,j) - a(q,i,j))$. Marginal vehicle profit is defined to be the most profitable of the two.

Optimal Relationships Among the Values of the Terminal Multipliers

By applying the definition of marginal vehicle profits to the conditions one through three in Table 2, the following relationship is derived. For (i,j) such that $q_o(i,j) > 0$, or $u_o(i,j) > 0$, or both, then

$$t(i) - t(j) = p(q_o,u_o,i,j). \qquad (2\text{-}27)$$

Stated in words, (2-27) is saying that at an optimal solution the imputed value of vehicle capacity at terminal (i), minus the imputed vehicle value at terminal (j), equals the marginal vehicle profit on vehicles which are flowing from (i) to (j). It is intuitively logical that this should be true because if (2-27) did not hold, then there is a direct implication that q_o and u_o are not optimal.

For example, suppose

$$t(i) - t(j) > p(q_o,u_o,i,j). \qquad (2\text{-}28)$$

Recall from the previous section that (t(i) - t(j)) equals the imputed decrease in profit that would result from adding a unit of vehicle capacity to terminal (j) and removing it from (i). Hence, (2-28) is saying that the imputed incremental loss in profit of shifting vehicle capacity from terminal (i) to (j) is greater than the marginal vehicle profit gained from the actual movement. Clearly, this is marginally unprofitable and vehicle flows from (i) to (j) should be reduced; the initial solution could not be optimal.

Consider the opposite situation; suppose

$$t(i) - t(j) < p(q_o,u_o,i,j). \qquad (2\text{-}29)$$

Here, the marginal vehicle profit from the actual movement is greater than the imputed decrease in profit resulting from the shift in vehicle capacity from terminal (i) to (j). This is marginally profitable, and therefore more vehicles should be sent from (i) to (j) to further increase profits; the initial solution could not be optimal. From the counter examples in (2-28) and (2-29), the relationship in (2-27) is intuitively consistent with logical analysis.

The relationships in (2-27), (2-28), and (2-29) were formulated under the assumption that vehicles were flowing between the relevant terminals. Condition four in Table 2 is now applied to specify relationships when there is no vehicle flow between two terminals. For (i,j) such that $q_o(i,j) = u_o(k,j) = 0$, then

$$t(i) - t(j) \geq p(q_o, u_o, i, j). \tag{2-30}$$

This is also intuitively logical; no vehicles are flowing from (i) to (j) because the imputed decrease in profit resulting from a shift in capacity from terminal (i) to (j) exceeds, or equals, the marginal vehicle profit which is to be gained.

In an abstract sense, the conditions stated in this section are implying that if economic returns (profits) are being maximized on a set of mobile resources (vehicles) then it is a necessary condition that the incremental values of the resources at various points in space be directly related to the returns on moving the resources between those points. In a loose sense, the model developed in this chapter provides a structure for identifying the spatial value of transportation resources.[1]

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[1]This also applies to the commodities being transported. See Paul Samuelson, "Spatial Price Equilibrium and

Marginal Vehicle Profits and Circuitous Routes

Special relationships occur on directed circuitous routes when profit is at a maximum. A directed circuitous route is defined here as a continuous path which begins at some terminal, and sequentially proceeds to other terminals, and eventually returns to the terminal from which it began. A directed circuitous route may contact any finite number of terminals; the only restriction is that it proceeds from one terminal to another, and eventually closes the loop by returning to its originating terminal.

At a profit maximizing solution, the sum of marginal vehicle profits on any directed circuitous route will always be less than or equal to zero; furthermore, if there are positive vehicle flows on each segment, then the sum of marginal vehicle profits must equal zero.

For example, consider the directed circuitous route which travels from, say, terminal (i) to terminal (j) to terminal (k) to terminal (m) to terminal (n), and back to terminal (i). The sum of marginal vehicle profits along this directed route is (scmvp), where

$$\text{scmvp} = p(q_o,u_o,i,j) + p(q_o,u_o,j,k) + p(q_o,u_o,k,m) + p(q_o,u_o,m,n) + p(q_o,u_o,n,i) \qquad (2\text{-}31)$$

Linear Programming," American Economic Review, XLII (June, 1952), 283-303.

If vehicle flows are positive along each segment of the route, then the relationship in (2-27) can be applied to give

$$\text{scmvp} = t(i) - t(j) + t(j) - t(k) + t(k) - t(m) + t(m) - t(n) + t(n) - t(i) = 0 \qquad (2\text{-}32)$$

This equals zero because for each positive terminal multiplier there is also a corresponding negative terminal multiplier.

For cases where there are some segments with zero vehicle flows in the circuit, the relationships in (2-30) are applied to find the value of (scmvp). In this case (scmvp) will be less than or equal to zero because each $p(q_o,u_o,i,j)$ will be less than or equal to $t(i) - t(j)$.

It is intuitively reasonable that such relationships occur in an optimal system because of the nature of the vehicle flow constraints. Notice that an increase, or decrease, in vehicle flow around a circuitous route will <u>not</u> cause vehicle imbalances because for each terminal along the route, changes in inbound flow will be offset by equal changes in outbound flow. Notice also, that the sum of marginal vehicle profits around a circuitous route equals the change in profit that would result from an incremental change in vehicle flow around the route. Therefore, a solution could not be optimal if there was some circuitous route in which (scmvp) was greater than zero because this would indicate that profits could be increased by increasing vehicle circulation[1] along the route. Similarly, a solution could not be optimal if there was some circuitous route in which vehicle flow is positive along all segments of the route and the value of (scmvp) was simultaneously not zero because an appropriate change in flow could increase profit.

Less formally, this necessary condition is saying that any feasible deviation from a profit maximizing solution cannot result in greater profit.

Changes in Terminal Vehicle Capacity Caused by Other Carrier Activities

At an optimal solution, the imputed effects on profits of changes in an s(i) are given by the value of t(i). An increase of (d) in s(i) has the same effect as increasing the right hand side of the (i)th equation by (d); from Appendix B, the imputed increase in profit would be (t(i) · d). It is assumed that the absolute value of (d) is relatively small.

This provides a valuable insight to the economics of other carrier activities, particularly less-than-truckload movements. The t(i) values prescribe the economic effects of vehicle inputs and outputs at each

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[1]The term circulation is used by Ford and Fulkerson to describe methods for maximizing flows and profits in constrained networks. See L. R. Ford, Jr., and D. R. Fulkerson, Flows in Networks (Santa Monica, Calif.: The RAND Corporation, 1962), pp. 50-53.

terminal on truckload freight profitability. With known values of t(i), a carrier could evaluate the effects of less-than-truckload freight on truckload freight profits and vice versa. For example, effects of <u>partially</u> loaded vehicles moving between terminals could be evaluated with respect to economic impacts on truckload freight.

The Relationship Between Marginal Revenue and Marginal Direct Cost

For $q_o(i,j) > 0$, it is not necessary that $r(q_o,i,j)$ equal $a(q_o,i,j)$ at an optimal solution q_o. In fact, according to the first condition in Table 2, it would only occur when $t(i) = t(j)$. This is a peculiar, but logical, result of the model. This relationship is pointed out here because in the economic analysis of production type firms it is repeatedly stated that "maximum profit is attained at that rate of output and sales for which marginal cost equals marginal revenue."[1] Clearly, this condition does not necessarily hold for the motor carrier model when the concepts of marginal costs and revenues are restricted to freight flows between two terminals. However, recall the relationship specified in (2-23); it was necessary that (scmvp) equal zero, or in other words, that the sum of marginal revenue equals the sum of marginal direct cost <u>around</u> a circuitous

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[1]Ferguson, <u>op. cit</u>., p. 232.

route. Therefore, the traditional economic notion that marginal revenue must equal marginal cost in order to maximize profit also applies here to circuitous routes.

If $r(q_o,i,j)$ equals $a(q_o,i,j)$ for all relevant (i) and (j) at an optimal solution, then it is implied, among other things, that the objective function in (2-13) is maximized without being actively constrained by (2-14). That is to say, it was possible to independently maximize profits on truckload freight moving between each pair of terminals without encountering vehicle balancing problems. This would be a very unique situation indeed. Therefore, it is presumed that for an optimal solution, $r(q_o,i,j)$ will generally not equal $a(q_o,i,j)$ even if all $u_o(i,j) = 0$ because of differences in the demands for transportation between terminals.

The Attainment of a Normative Optimum

So far, attention has been focused on the necessary conditions for maximized profit. This section briefly reviews the problems associated with actually maximizing (2-13) with respect to (2-14), (2-15), and (2-16).

The constraints (2-14), (2-15), and (2-16) are linear and, therefore, the admissible or feasible values of q and u are convex sets.[1] This simplifies the problem greatly, however, no general solution algorithm exists

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[1]See G. Hadley, Linear Programming (Reading, Mass.: Addison-Wesley Publishing Co., Inc., 1962), p. 60.

for the above problem unless (2-13) is concave, in which case a local maximum is necessarily a global maximum,[1] and the Kuhn-Tucker conditions become necessary and sufficient for a global maximum.[2]

The problem is simplified even more by assuming that (2-13) is a concave quadratic function of q and u; there are several algorithms available for solving this type of problem.[3] Simple functions which satisfy this restriction are given below.

Convenient Functional Relationships

Let truckload freight flow from, say, terminal (i) to (j) be a linear function of the rate;

$$q(i,j) = c(i,j) - d(i,j) \cdot \text{Rate}(i,j) \qquad (2\text{-}34)$$

where c(i,j) and d(i,j) are non-negative constants. Revenue on the (i) to (j) freight flow as a function of q(i,j) would, therefore, be

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[1] See G. Hadley, Nonlinear Programming (Reading, Mass.: Addison-Wesley Publishing Co., Inc., 1964), pp. 123-26.

[2] See J. Abadie, "On the Kuhn-Tucker Theorem," Nonlinear Programming, ed. by J. Abadie (Amsterdam: North-Holland Publishing Co., 1967), pp. 21-36.

[3] This is the standard quadratic programming problem. See L. Cooper and D. Steinberg, Introduction to Methods of Optimization (Philadelphia, Pa.: W. B. Saunders Co., 1970), pp. 295-301; F. S. Hillier and G. J. Lieberman, Introduction to Operations Research (San Francisco: Holden-Day, Inc., 1967), pp. 578-80.

$$\text{Revenue}(i,j) = (c(i,j)/d(i,j)) \cdot q(i,j) - (1/d(i,j)) \cdot q(i,j)^2 \qquad (2\text{-}35)$$

and marginal revenue would, therefore, be

$$r(q,i,j) = (c(i,j)/d(i,j)) - (2/d(i,j)) \cdot q(i,j) \qquad (2\text{-}36)$$

Let the direct cost be a linear function of q, and let empty movement costs be a linear function u; this means that a(q,i,j) and b(u,i,j) would be constants for all relevant values of (q) and (u). Given the values for these parameters, profit can be maximized in a finite number of steps using the cited algorithms.

Normative Optimality and the Behavior of an Actual Motor Carrier Firm

It would be convenient to say at this point that if differences appear between the model developed in this chapter and an actual motor carrier system, then the intent and purpose of the model has been misunderstood. With this statement all real world incompatibilities could be ignored for the sake of theoretical consistency. Needless to say, there are some very apparent and glaring problems which arise when one tries to compare the model with an actual system. These problems are highlighted below.

Management would have difficulty ascertaining cost functions, let alone revenue functions. These functions are often unknown or only estimates. This problem was avoided in the model by assuming that these functions were known.

Even if cost and revenue functions were known at a point in time, they would change rapidly through time.[1] This issue was avoided by implicitly assuming that a carrier makes instantaneous and optimal adjustments.

There are difficulties with regard to the continuity of cost and revenue functions. Truckload freight movements can only move in integer units; however, for the sake of making it possible to identify derivatives and profit maximization conditions, it was implicitly assumed that vehicles are continuously divisible.

Transportation is regulated by "a chaotic patchwork of inconsistent and often obsolete legislation and regulation."[2] The model assumes no regulation and that the carrier adjusts rates so as to maximize profits.

It is implicitly assumed that freight inputs are continuously flowing through the system; this avoids the issue of random fluctuations in freight and vehicle movements.

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[1]See Wilson, op. cit., pp. 266-76.

[2]A quote from President Kennedy's special transportation message to Congress in 1962. See the "Antitrust Head Urges an End to Regulation for Most Transport," Wall Street Journal, January 29, 1971, p. 6.

Even the assumption about profit maximization has dubious overtones; the inability to measure and estimate costs and revenues precisely, casts doubt on the carrier's ability to maximize profit.

These problems project the complex and transient nature of transportation rates. They are sobering, yet they reinforce a contention that if a model handled all of these aspects, then its complexity would be of such magnitude that few individuals, if any, would have the mental capacity to understand and interpret it.

These issues must be put in perspective with the original intent of the model developed here; its purpose is to provide an analytical framework from which to view and analyze the general nature of truckload freight rates. To do this it is necessary to step away from reality and reflect on the general nature of the problem (as opposed to focusing on specific intricacies), in order to make progress toward generalizations in behavior.

In this light, only general comparisons between the model and a real system can be sought; it would be naive to do otherwise. A motor carrier's behavior should, at most, loosely parallel the relationships found in the model; other factors intervene and disguise the effects put forth in the model.

Adjustment Process

The dominance of truck movement costs cannot be ignored by management.

> Inasmuch as transportation costs make up 50 per cent of total costs . . . it is savings from this source that appears to dominate the relation between output and total costs. It is interesting to note that this is where one would expect savings from scheduling and routing advantages to accumulate.[1]

Therefore, a motor carrier is strongly motivated to take into account marginal trucking costs when setting freight rates.

The concept concerning circuitous routes, and the summation of marginal vehicle profits around those routes is not only theoretically significant in the normative model, but is also operationally significant. If it can be assumed that marginal revenues and costs can be reasonably estimated by the carrier, and that they can be computed on circuitous routes, then the carrier has a workable method of identifying non-optimal conditions and can adjust operating levels accordingly.

For example, reconsider the situation described in (2-31), and also assume that vehicle flows are positive along all constituent segments in the circuit.

If the carrier recognizes that (scmvp) differs from zero, then it can go about adjusting rates and flows in the circuit to increase profit. Through time, (scmvp) will

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[1]Burstein, *et al.*, *op. cit.*, p. 33.

tend toward zero. This argument would apply to all circuits in the system; presumably, the circuits which showed the greatest profit potential would be adjusted first and the others would follow.

Obviously, it is not likely that all, or even one (scmvp) will reach zero because of errors in measurement, errors in judgment, regulatory restrictions, and changes in economic conditions; nevertheless, the adjustment toward zero would parallel the characteristics put forth in the normative model.

Conclusions

Three basic conclusions are drawn in this chapter. The first conclusion concerns the relationships between marginal revenues and costs with regard to circuitous routes. The second conclusion concerns the terminal multipliers and their interpretation. The third conclusion concerns the notion of normative optimality and the actual behavior of a motor carrier.

Marginal Revenues and Marginal Costs

It has been demonstrated in the model that marginal revenues on interterminal freight movements will generally not equal the marginal direct cost of transportation when a motor carrier is at a profit maximizing state. On the surface this appears to contradict microeconomic theory, but further analysis shows that vehicle

flow constraints differentiate motor carrier operations from the production type firm. In general, it is required in the motor carrier model that the sum of marginal revenues equals the sum of marginal costs around a circuitous route for profit to be maximized, as opposed to the equality of marginal revenue and marginal cost on a single trip.

In effect, transportation rates and costs in one portion of a system affect transportation rates in other portions of the system.

Terminal Multipliers

When a motor carrier maximizes profit, the value of vehicle capacity at each terminal can be imputed from the solution. These imputed values are associated with the terminals; they are the Kuhn-Tucker multipliers which embody the necessary, and sometimes sufficient, conditions for profit maximization in the model. The conceptual explanation of these multipliers revolves around the fact that when freight is transported from one terminal to another, vehicle capacity is required at the origin and is made available at the destination.

The value of a terminal multiplier at an optimal solution reflects the incremental increase in profit that would result if a unit of vehicle capacity was added to a terminal. In a logical manner they interrelate the

marginal vehicle profits on vehicles which are flowing between terminals.

In general, the differences between the marginal values of vehicles at two points in an optimal solution equals the marginal profit of moving a vehicle from the one point to the other.

Theoretical Behavior and Actual Behavior of a Motor Carrier Firm

The model developed in this chapter synthesizes some fundamental conditions encountered in the transportation of truckload freight; it is designed to formalize the relationships which occur when a motor carrier maximizes profits. In order to accomplish this goal, it is necessary to make simplifying assumptions; each assumption pushes the model further into an abstract realm. Nevertheless, the model incorporates basic features of a motor carrier system, and these same basic features are real forces and constraints in the management of a motor carrier. This gives reason to believe that the observed relationships among rates and costs of an actual motor carrier should parallel the relationships expressed in the normative model. At present this is only a hypothesis; in Chapter III this hypothesis is examined, and empirical evidence is presented to show that relationships which exist in the model also exist in an actual system.

CHAPTER III

AN EMPIRICAL INVESTIGATION

Introduction

In Chapter II a model identifies the normative characteristics of maximized profit on truckload freight in a multiple terminal motor carrier system. The purpose of this chapter is to provide empirical evidence showing that the rate and profit structure of an operating motor common carrier exhibits characteristics which are similar to those predicted in the normative model.

Motor carrier freight rates are a result of interactions among a variety of factors. Some of these factors are economical, some are legal, and others are political in nature. The rates examined here are a result of these factors. Some specific factors are analyzed and the others are assumed to affect the rate structure in a random manner.

Regression models are used to test the statistical significance of the relationships suggested in the normative model. The specific assumptions of these models are detailed in the analytical sections of the chapter.

A general hypothesis investigated in this chapter is addressed to the empirical relationships between truckload freight profitability patterns and freight flow imbalances in the system. The form of the hypothesis is related to the normative equations and relationships developed in Chapter II, and in this sense the normative model provides an explanation of the statistical results presented in this chapter. However, the statistical results presented here also stand independent of the normative model in that the results are significant in their own right, and depend on the normative model only for explanation, not justification.

Organization

This chapter is organized into five major sections. The first section introduces the general nature of the empirical analyses undertaken. The second section describes the source of the data and the mathematical notation used to denote the various observed variables. The third section is devoted to a verbal and informal mathematical description of the hypothesis being examined. The fourth section gives a formal mathematical statement of the statistical models and hypothesis being tested; statistical results are also presented. The last section presents conclusions and summarizes the empirical findings.

Data Source

A motor common carrier provided the data for this investigation. The reasons for using the data from this carrier are twofold. First, the carrier has significant freight traffic imbalances and has empty vehicle flows which should theoretically be reflected in the freight rate and profit structure; hence, the data provides an excellent setting for comparing actual profit interrelationships with normative relationships. Second, the carrier has detailed accounting records of freight traffic flow, revenues, and costs listed according to freight origin and destination; this, of course, greatly facilitated data collection.

The carrier operates from thirty-four terminals located in five states. Data from two nonconsecutive accounting periods were collected. The accounting periods were approximately a month in length, and were in the middle of the 1969 calendar year. Company officials stated that the data were representative of their operations during the respective periods. During these two periods, the carrier moved over four hundred million pounds of freight; approximately half this amount was moved in each period.

Rate and cost data of select truckload shipments were used for this investigation. Only shipment sizes between ten thousand and twenty thousand pounds, and traffic with both initial origin and final destination within the carrier's system were used in the rate and cost analyses. This shipment group is referred to here, and in the remainder of the chapter, as intraline group ten freight. Freight flow data on other freight groups were also used incidentally in certain parts of the analyses.

Intraline Group Ten Freight

Intraline group ten freight accounts for approximately 10 per cent of the total weight moved by the carrier. This particular group was selected for the following reasons. First, if a study is to analyze the rates and profitability of freight traffic between a set of terminals, then it is reasonable to restrict the analyses to similar shipment sizes because rates, costs, and profitability are related to the size of the shipment.[1] This homogeneous group of shipment sizes was used to avoid the possibility of attributing variations in profitability to traffic imbalances and demand factors when such variations might have been due to variations in shipment size.

Second, freight traffic which either originates or terminates with another carrier (interline traffic), is excluded because such traffic has different economic characteristics than intraline traffic. Revenue allocations between carriers on interline traffic involve certain regulatory "rules" and are not comparable with revenue on similar traffic which both originates and

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[1]This is commonly recognized by both carriers and regulatory agencies; see Locklin, op. cit., p. 649.

terminates within a single carrier's territory. In light of the need to analyze a homogeneous freight group, rate and cost data from interline traffic were not included.

Third, the normative model in Chapter II analyzed the economics of truckload shipments moving directly from an origin terminal to a destination terminal. Therefore, in order to compare actual data with the model it is necessary to consider freight traffic which moves directly from an origin terminal to a destination terminal and intraline group ten freight does this. Shipment sizes of less than ten thousand pounds are usually routed via intermediate terminals for linehaul consolidation with other shipments.

Fourth, intraline group ten freight shipments are of sufficient size to warrant point to point rate adjustments by the carrier. The rates are, of course, regulated by government agencies; however, the carrier retains some influence on rate levels. This influence is exercised in several ways. The carrier can actively solicit freight traffic which moves at favorable rates and de-emphasize unfavorable traffic. The carrier can also seek modifications of published rates which are disadvantageous.

Mathematical Notation of Data

To avoid ambiguity, data variables are defined and labeled in this portion of the chapter, and apply to the remaining portions of the chapter. The following list of variables represent the relevant data collected for analyses.

Unit Revenue

The variable $r_1(i,j)$ is the average unit revenue on intraline group ten freight moving from terminal (i) to (j) during the first of the two data collection periods. Similarly, $r_2(i,j)$ is the average unit revenue for the second data collection period. The $r_k(i,j)$, $(k=1,2)$, variables were computed by dividing the appropriate total revenue by the total weight; these variables are measured in dollars per hundredweight.

Direct Unit Cost

The variable $a_1(i,j)$ is the average direct unit cost of transporting intraline group ten freight from terminal (i) to (j) during the first of the two data collection periods. Similarly, $a_2(i,j)$ is the average direct cost for the second period. The $a_k(i,j)$, $(k=1,2)$, variables were computed by dividing the appropriate total direct cost by total weight; the measurement units are in dollars per hundredweight. Direct costs, as defined here, include the following basic cost centers: (1) pick-up and delivery costs, (2) dock and platform costs, (3) linehaul costs, (4) claim costs, and (5) office and billing costs. The costs in these centers are assigned to

freight movements according to standard motor carrier accounting techniques.[1]

With the exception of linehaul costs, the costs in these centers are also computed according to standard accounting techniques.[2] The exception arises because motor carrier accounting procedures treat the cost of moving empty vehicles between terminals as an overhead expense, and distribute it among linehaul costs. In order to maintain a comparison with the direct cost variable used in the normative model, it is necessary to treat empty vehicle movement costs separate from direct costs.

Empty Vehicle Movement Cost

The variable b(i,j) is the average unit cost of moving empty vehicle capacity from terminal (i) to terminal (j). These variables are computed by taking the standard accounting cost of moving a tractor and a single bottom empty trailer from terminal (i) to (j), and dividing it by the standard hundredweight capacity of the trailer; resulting measurements are in terms of dollars per hundredweight capacity.

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[1]See John C. McWilliams, Motor Carrier Cost Techniques (Washington, D.C.: National Motor Freight Traffic Association, Inc., 1956). Also see Interstate Commerce Commission, op. cit.

[2]Ibid.

Direct Unit Profit

The variable $p_1(i,j)$ is the average direct unit profit on intraline group ten freight moving from terminal (i) to (j) during the first data collection period. This variable is measured in dollars per hundredweight, and equals $(r_1(i,j) - a_1(i,j))$. The variable $p_2(i,j)$ is similarly defined for the second data collection period.

Number of Shipments

The variables $N_1(i,j)$ and $N_2(i,j)$ are the number of intraline group ten shipments which moved from terminal (i) to terminal (j) during the first and second data collection periods respectively.

Total Freight Flow

The variable $w_1(i,j)$ equals the total freight flow from terminal (i) to (j) during the first data collection period. The variable $w_2(i,j)$ is similarly defined for the second period. These variables are measured in terms of total hundredweight per period. Note that these variables represent all the freight transported by the carrier; this includes all shipment sizes and interline freight.

Intraline Group Ten Freight Flow

The variable $q_k(i,j)$, $(k=1,2)$, equals the intraline group ten freight which flowed from terminal (i) to (j) during the (k)th data collection period. The flows are measured in terms of total hundredweight per period.

Distance Between Terminals

The variable d(i,j) is the over the road distance in statute miles between terminals (i) and (j). This variable is used in the analyses to reenforce certain statistical models and conclusions.

A Note on the Distinction Between Empirical Variables and Normative Variables

In this chapter it is necessary to refer to the data variables defined in the preceding section, and to the normative variables defined in the preceding chapter. Notation is similar but not identical. Table 3 is provided to summarize the notational and conceptual similarities and differences between the empirical and normative variables. Some of the empirical variables listed in the table are defined in the later parts of this chapter.

A Note on Weighted Observations

Due to the fact that the rate and cost variables defined above represent different numbers of freight shipments, it is necessary to give more importance or weight, to those which represent large numbers of shipments. In the statistical models used here, there are standard techniques for handling this situation; briefly, observed variables are weighted according to the number of shipments the variables represent. The procedures for weighting variables in linear statistical models are well

TABLE 3.--A list of normative and empirical variables.

Normative Variables Used in Chapter II		Empirical Variables Used in Chapter III	
$r(q,i,j)$	marginal revenue	$r_k(i,j)$	average revenue
$a(q,i,j)$	marginal direct cost	$a_k(i,j)$	average direct cost
$b(u,i,j)$	marginal empty movement cost	$b(i,j)$	average empty movement cost
$r(q,i,j) - a(q,i,j)$	marginal direct profit	$p_k(i,j)$	average direct profit
$p(q,u,i,j)$	marginal vehicle profitability	-	
$q(i,j)$	normative truckload freight flow	$q_k(i,j)$	intraline group ten freight flow
$u(i,j)$	normative empty vehicle flow	$u_*(i,j)$	imputed empty vehicle flow
$t(i)$	marginal value of vehicle capacity at a terminal	-	
$t(i) - t(j)$	marginal effect on profit of shifting vehicle capacity	$t_*(i) - t_*(j)$	imputed effect on balancing cost of shifting vehicle capacity
-		$w_k(i,j)$	total freight flow
-		$N_k(i,j)$	number of shipments
-		$d(i,j)$	distance
$d, z(i,j), y(i,j)$	dummy variables	-	

documented.[1] Assumptions are made about the variance of the error terms associated with an observation; the assumed variance of the error term becomes smaller as greater weight is assigned to the observation. The formal mathematical assumptions used for weighting the variables are described in the analytical section of this chapter.

A General Description of the Hypothesized Profit Patterns

The normative model in Chapter II prescribes the following pattern of marginal revenues and marginal costs in an optimal system. There will exist values for $t(i)$, $(i=1,\ldots,\theta)$, in which

$$r(q_o,i,j) - a(q_o,i,j) = t(i) - t(j) \text{ for } (i,j) \text{ such that } q_o(i,j) > 0, \quad (3\text{-}1)$$

and

$$-b(u_o,i,j) = t(i) - t(j) \text{ for } (i,j) \text{ such that } u_o(i,j) > 0. \quad (3\text{-}2)$$

These equations are a partial list of the conditions in Table 2; the variables are defined in Chapter II.

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[1]See Henry Scheffé, The Analysis of Variance (New York: John Wiley & Sons, Inc., 1959), pp. 19-22. For weighted regression see N. R. Draper and H. Smith, Applied Regression Analysis (New York: John Wiley & Sons, Inc., 1966), pp. 71-81.

It is implied from these equations that in an optimal normative system the marginal direct profitability patterns, $(r(q_o,i,j) - a(q_o,i,j))$, are related to the patterns of empty vehicle flows[1] through the differences among the t(i) variables. It can be generally assumed that $b(u_o,i,j)$ is always greater than zero, and therefore if there is some (i,j) such that $u_o(i,j) > 0$, then t(i) cannot equal t(j) because of (3-2). This difference between t(i) and t(j) will necessarily be reflected in the marginal direct profitability of positive freight flows which connect the two terminals because of (3-1). It is sufficient to say here that for a normative system which is interconnected by positive freight flows, the presence of empty vehicle flows at an optimal operating level implies that marginal revenues will differ from marginal direct costs on some, if not all, freight flows. With the above relationship in mind, it would certainly appear that perceptible relationships should exist between freight profitability and patterns of empty vehicle flows when significant empty vehicle flows are present in an actual system.

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[1]Notice that if there are no empty vehicle flows in an optimal normative system, then all that can be said is that $t(i) - t(j) \geq - b(u_o,i,j)$ for all (i,j). This does not imply that $\overline{t}(i)$ must equal t(j). As pointed out in Chapter II, the t(i), $(i=i,\ldots,\theta)$, will equal one another only in some very special circumstances.

The Carrier and Rate Adjustments

For the motor carrier firm which provided the data, approximately thirty-four thousand dollars was expensed each accounting period to empty vehicle movements. Over a year, this would represent nearly four hundred thousand dollars. The patterns of empty vehicle flows were similar in both periods. On a relative basis, twelve terminals consistently received empty vehicles, nine terminals consistently supplied empty vehicles, and the remaining thirteen terminals both supplied and received empty vehicles at various times during the two accounting periods.

The carrier's management stated that these vehicle imbalances stimulated rate adjustments in the interest of financial health. Government regulatory procedures, of course, monitored the adjustments which were implemented by the carrier.

In general, the carrier was very well aware of freight imbalances and empty vehicle flows. Without revealing specific policies, it can be said that special marketing adjustments were pursued on freight involving terminals that had major traffic imbalances. In this light, the data certainly provides a suitable setting for examining patterns in the direct unit profit structure.

Empty Vehicle Flows

The movements of empty vehicles in an actual motor carrier system are governed by a variety of factors, not all of which are compatible with the conceptual nature of the empty vehicle flow variable used in the normative model. The various complications which arise are discussed below.

For the motor carrier under consideration, empty vehicle routing decisions are made by a central dispatcher who is in direct contact with the terminals in the system. His decisions take into account minimum tractor and trailer requirements at terminals, vehicle availability, driver availability, and driver constraints. Consistent patterns of empty vehicle flows emerge in the system, but it is apparent from the data that empty vehicles are not routed directly from surplus terminals to deficit terminals. Indirect routings take place in many instances because of short term driver constraints, the company's policy on terminal trailer requirements, tractor shortages, and random circumstances.

For example, an empty vehicle might be routed out of surplus terminal A to a central terminal C, and then to a deficit terminal B. In terms of the data variables, this would appear as an empty vehicle movement from terminal A to terminal C and another empty vehicle movement from terminal C to terminal B, when in fact it is only an empty vehicle movement from terminal A to B.

The normative model presumes perfect information, and therefore, indirect routing of empty vehicles would never occur because it is clearly uneconomical to move a vehicle over a greater distance than necessary. Furthermore, if it is assumed that $b(u_o,i,j)$ is greater than zero for all (i,j), then the normative model precludes a circuitous route of positive empty vehicle flows; this follows from (3-2) in that no values could exist for the relevant t(i) in such a situation. In an actual system, empty vehicles may traverse all routes at one time or another.

These complications require a need for a generally applicable technique which identifies underlying empty vehicle flow patterns and which is compatible with the normative model. In addition, the technique should be somewhat independent of a carrier's transient empty vehicle routings and at the same time reflect actual patterns in the system.

One viable approach attacks the problem in terms of freight traffic imbalances. The approach imputes empty vehicle flows from actual freight flow imbalances; the imputed empty vehicle flows are determined by calculating an empty vehicle flow pattern which balances the system at the lowest total cost.[1] It is basically a linear

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[1]This approach to imputing empty vehicle flow is identical to the methodology set forth by Koopmans and Reiter, op. cit., pp. 222-59.

programming origin-destination problem, commonly called the transportation problem.[1] The terminals with greater freight inflow than outflow become the origins of empty vehicles, and the terminals with greater freight outflow than inflow become the destinations of empty vehicles. The resulting minimum total cost flows from origins to destinations are the imputed empty vehicle flows. Details of the procedure and its relationship to the normative model are presented later in the chapter.

General Assumptions

The following assumptions are made in order to make it possible to compare empirical data with the normative model. First, it is assumed that the carrier is motivated to adjust operating policies and freight rates so as to provide a favorable return on investment. This assumption connotes profit maximization with respect to the physical and economic constraints placed on the carrier. Second, it is assumed that average unit revenue is viewed by the carrier as a close approximation to marginal revenue. Third, it is assumed that the average unit costs of moving freight and empty vehicles between terminals are viewed by the carrier as close approximations to marginal costs. Fourth, it is assumed that the empty vehicle

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[1]There are a large number of references on the transportation problem; a broad and complete analysis of the subject is in Hadley, Linear Programming, Chapter 9.

flows imputed from the freight flow imbalances reflect the overall vehicle balancing problem facing the carrier. Fifth, it is assumed that factors not explicitly taken into account by the model affect the data in a normally random and independent manner. With these assumptions the empirical data variables can be applied to least squares regression models which incorporate the equations in (3-1) and (3-2).

A number of structural variations are developed in the regression models in order to provide results which are not dependent on a single set of assumptions. In a general sense, the regression models test for a statistically significant system wide relationship between average unit profit patterns and imputed empty vehicle flow patterns.

Statistical Models, Tests, and Results

The numerical differences among the t(i) values are calculated from the imputed empty vehicle flows, and then these differences are "fitted" to average direct unit profit in a least squares regression model. Statistical inferences are developed by making assumptions about the distributions of the error term associated with the empirical observations.

Imputed Empty Vehicle Flows and Differences Among (t) Values

Imputed empty vehicle flows are determined from the linear programming model shown in Table 4. The linear programming model determines the least cost method of routing empty vehicles in the system so as to provide each terminal with sufficient vehicle capacity to transport known freight flow requirements.

For notational purposes, the variables $u_*(i,j)$, $(i=1,\ldots,34;\ j=1,\ldots,34;\ i\neq j)$, are used to represent the minimum total cost solution of the linear programming model. With known values for $u_*(i,j)$ and $b(i,j)$, imputed differences among the $t(i)$ values can now be determined. For notational purposes, the variables $t_*(i)$, $(i=1,\ldots,34)$, are used to represent the $t(i)$ values determined from the imputed empty vehicle flows. From (3-2),

$$t_*(i) - t_*(j) = -b(i,j) \text{ for } (i,j) \text{ such that } u_*(i,j) > 0. \qquad (3\text{-}3)$$

In the minimum total cost solution there are thirty-three positive $u_*(i,j)$, and it so happens that these thirty-three non-zero values are sufficient to uniquely define the differences among <u>all</u> $t_*(i)$, $(i=1,\ldots,34)$. This is not a coincidence; in fact the differences among the $t_*(i)$ embody the necessary and sufficient conditions for the optimal solution to the linear programming model depicted

TABLE 4.--Linear programming model used for the determination of imputed empty vehicle flows.

Minimize Total Vehicle Balancing Cost:
$$\text{Total Vehicle Balancing Cost} = \sum_{i=1}^{34} \sum_{\substack{j=1 \\ j\neq i}}^{34} b(i,j) \cdot u(i,j)$$
Constraints:
$$\sum_{k=1}^{2} \left[\sum_{\substack{i=1 \\ i\neq m}}^{34} w_k(i,m) - \sum_{\substack{i=1 \\ i\neq m}}^{34} w_k(m,i) \right] \geq \sum_{\substack{i=1 \\ i\neq m}}^{34} u(m,i) - \sum_{\substack{i=1 \\ i\neq m}}^{34} u(i,m),$$
$(m=1,\ldots,34)$.
$u(i,j) \geq 0$ for all (i,j)
Definition of Variables:
$w_k(i,j)$ is the known total flow, of all freight groups, from terminal (i) to terminal (j) during period (k).
$b(i,j)$ is the known unit cost of moving a unit of empty vehicle capacity from terminal (i) to (j).
$u(i,j)$ is the unknown empty vehicle flow required to balance the system.

in Table 4.[1] It stands to reason that the t(i) values in the normative model are analogous to the $t_*(i)$ values computed from the linear programming model because a motor carrier system operating at an optimal level would necessarily employ a minimum total cost empty vehicle flow.

The calculated values of the $(t_*(i) - t_*(j))$, $(i=1,...,34;\ j=1,...,34;\ i \neq j)$, have an economic interpretation in the linear programming model which aids in understanding the nature of the regression models; therefore, this interpretation is elaborated upon here. The calculated value of $(t_*(i) - t_*(j))$ is the imputed increase in total empty vehicle movement cost that would result from one unit of vehicle capacity being removed from terminal (i) and added to terminal (j), and then having the problem resolved for a new optimum.[2] In effect, $(t_*(i) - t_*(j))$ reflects the change in total vehicle balancing, or backhaul cost that results when a unit of vehicle capacity is shifted from terminal (i) to (j); henceforth, $(t_*(i) - t_*(j))$ will be referred to as the imputed backhaul cost variable. In this light, it is reasonable to suspect that direct unit profits on freight $(r_k(i,j) - a_k(i,j))$, or simply $p_k(i,j)$, will have a

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[1]See Ibid., pp. 309-12. For additional coverage in a network context, see S. E. Elmaghraby, "The Theory of Networks and Management Science, Part I," Management Science, XVII (September, 1970), 24-25.

[2]Ibid.

positive relationship with the imputed backhaul cost $(t_*(i) - t_*(j))$ which is an indirect cost of freight movement. Stated in another way, the larger the value of $(t_*(i) - t_*(j))$, the larger would be the value of $p_k(i,j)$ because above average indirect backhaul costs must be compensated for by above average direct unit profits on the freight flow which gives rise to the backhaul cost.

Weighted Observations

The variance of a numerical average of independent homoscedastic random variables is equal to the variance of the individual random variable divided by the number of variables that comprises the average.[1] Hence, it is assumed that the variance of the error term associated with a dependent regression variable is σ^2 divided by the number of shipments which the dependent variable represents, where σ^2 is the unknown error variance of a single shipment. This is a standard technique in weighted regression analysis.[2]

Unit Profit Regression Models

These models measure the statistical relationship between the unit profit variable $p_k(i,j)$, and the imputed

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[1] See Emanuel Parzen, Modern Probability Theory and Its Applications (New York: John Wiley & Sons, Inc., 1960), p. 371.

[2] See Draper and Smith, *op. cit.*, pp. 71-81.

backhaul cost variable $(t_*(i) - t_*(j))$. Five least squares regression models are presented here; the models differ according to the number of concomitant variables which are fitted to the unit profit variable before the imputed backhaul cost variable. These different models are developed in order to preclude the possibility of attributing variations in unit profit to the imputed backhaul cost variable when the variations may have been due to known concomitant variables. It so happens that fitting these additional concomitant variables does not affect the relevant statistical inferences made here.

The forms of the five models are described in Table 5; they are labeled WC1LP2A, WC2LP2A, WC3LP2A, WC4LP2A, and WC5LP2A.

The statistical hypothesis examined here has the following form.

Null Hypothesis: $h=0$ (3-4)

Alternative Hypothesis: $h \neq 0$ (3-5)

The variable (h) is the regression coefficient associated with imputed backhaul cost variable; it is expected that the estimate of this variable will be positive.

The null hypothesis is rejected in all five models at the 10 per cent significance level using the analysis of variance F-test.[1] The analysis of variance tables for

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[1]See Scheffé, <u>op. cit.</u>, pp. 31-32.

TABLE 5.--Assumptions of the unit profit regression models.

WC1LP2A: $p_k(i,j) = c(1) + h \cdot (t_*(i) - t_*(j)) + e_k(i,j),$

WC2LP2A: $p_k(i,j) = c(1) + c(2) \cdot a_k(i,j) + h \cdot (t_*(i) - t_*(j)) + e_k(i,j),$

WC3LP2A: $p_k(i,j) = c(1) + c(2) \cdot a_k(i,j) + c(3) \cdot w_k(i,j) + h \cdot (t_*(i) - t_*(j)) + e_k(i,j),$

WC4LP2A: $p_k(i,j) = c(1) + c(2) \cdot a_k(i,j) + c(3) \cdot w_k(i,j) + c(4) \cdot \sqrt{a_k(i,j)} + h \cdot (t_*(i) - t_*(j)) + e_k(i,j),$

WC5LP2A: $p_k(i,j) = c(1) + c(2) \cdot a_k(i,j) + c(3) \cdot w_k(i,j) + c(4) \cdot \sqrt{a_k(i,j)} + c(5) \cdot \sqrt{w_k(i,j)} + h \cdot (t_*(i) - t_*(j)) + e_k(i,j),$

for all (i), (j), and (k) such that $N_k(i,j) > 0$.

$e_k(i,j)$ is a random error term. It is assumed to be independently normally distributed with a mean of zero, and a variance of $(\sigma^2/N_k(i,j))$.

c(i), (i=1,...,5), are the unknown regression coefficients; they are, of course, different for each model.

All other variables are empirical variables, and are defined in Table 3.

these models are contained in Appendix I. Specific significance levels, and the estimates of (h) are listed in Table 6; the estimates of (h) range approximately between (.05) and .07).

Concomitant Variables

The regression model WC1LP2A fits only the grand mean to the direct unit profit variables before the backhaul cost variables.

The regression model WC2LP2A fits the grand mean and the direct unit cost variables before the backhaul cost variables. It is reasoned here that some variations in direct unit profit are due to direct unit cost; specifically, it is reasonable to suspect that large direct unit profits will be accompanied by large direct unit costs.

In regression model WC3LP2A, the grand mean, direct unit cost, and the total freight flow variables are fitted to direct unit profits before the imputed backhaul cost variables. It is reasoned here that some variations in direct unit profits are also due to the total volume of traffic which moves with the intraline group ten freight. From an economic theory standpoint, a large volume of traffic from one terminal to another implies that the average direct cost variable is a more accurate approximation to marginal direct cost than in the case of small traffic volumes. In addition, large volumes of traffic imply a greater potential for competition; hence, unit

TABLE 6.--A summary of statistical tests.

Model	F-Statistic	Approximate Significance	Estimate of (h)
WC1LP2A	3.000	.08	.048
WC2LP2A	4.618	.04	.058
WC3LP2A	5.191	.025	.059
WC4LP2A	5.908	.02	.063
WC5LP2A	6.418	.015	.066

revenues may be a more accurate approximation of marginal revenue. In any case, the total traffic variables are fitted to the data without adversely affecting the statistical significance of the backhaul cost variables.

In the regression models WC4LP2A and WC5LP2A, curvilinear direct unit cost and total traffic variables are fitted in the models respectively, again without adverse affects on the backhaul cost variables.

The regression coefficients associated with the direct unit cost and total traffic flow variables are not revealed here for obvious proprietary reasons; however, it is sufficient to say that the coefficients were consistent with expectations.

<u>The Analysis of Residuals</u>

An analysis of the residual errors in the models did not reveal characteristics which would tend to discount the validity of the statistical inferences. The residuals were distributed in a bell shaped pattern, and this conforms with the normality assumption.

There was no significant correlation between the residuals of the two periods; this is accounted for by the fact that the data were from two non-consecutive periods, and that there were some variations in the traffic mix due to the yearly "model changes" in the plants which were served by the carrier. Although intertemporal correlation among the residuals was not a problem here, it could be a potential problem in other data.[1] Ways of handling this are to either combine the data as one period, or analyze each period separately. The latter was tried here, for the sake of curiosity, and the results were very similar to those already presented.

Alternative Regression Models

Several variations to the principal regression models were constructed to examine the stability of the statistical inferences under alternative formulations. In the interest of maintaining the continuity of the chapter, only the general aspects of these alternative regression models are presented. In general, the backhaul cost variable has an equal or stronger significance in these alternative models. Three general groups of variations to the regression models were implemented, and they are briefly described here.

Unit Revenues and Distances

Unit revenues, instead of direct unit profit, were treated as the independent variable, and the distance variables, d(i,j), were used, instead of direct unit cost, as concomitant variables. This variation was developed from a contention that the direct cost variables might have had hidden directional influences on unit profitability. By using unit revenue and the distance variables,

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[1] See J. Johnston, Econometric Methods (New York: McGraw-Hill Book Company, Inc., 1963), pp. 177-200.

direct unit cost variables were not present in the regression model. The inferences with regard to the significance of the backhaul variable were unchanged; the actual estimates of (h) in this alternative model were not appreciably different than the estimates in the unit profit models.

Deletion of Minor Terminals

For the data under consideration, each of the thirty-four terminals had at least six intraline group ten shipments moving in and out of the terminal during the combined accounting periods. However, to insure that small terminals did not have a disproportionate effect on the analysis, they were removed. An arbitrary cutoff of twenty-five group ten intraline shipments was established as a minimum inbound and outbound limit. This criterion eliminated nine of the original thirty-four terminals. All remaining terminals had at least twenty-five inbound intraline group ten shipments, and at least twenty-five outbound intraline group ten shipments. The data for these twenty-five terminals were applied to models identical to those previously described. In general, the significance of the backhaul cost variables were much stronger in these models than in the models where data from all terminals were included; this is a logical result in light of the erratic freight flows associated with the small terminals.

Alternative Computation of Imputed Backhaul Cost Variables

Instead of aggregating the freight imbalances over the two accounting periods and then computing the backhaul cost values from the linear programming model, backhaul cost values were computed for each period from the freight imbalances which occurred during the period. The resulting imputed backhaul cost values were not identical for each period, but did have a linear correlation coefficient greater than (.90). These backhaul costs were averaged from both periods, and then applied to the regression models in place of the backhaul costs which were imputed from aggregate freight imbalances. Here again, the results were not appreciably different than before.

Conclusions

The major conclusion of this empirical analysis is that the data exhibits a measurable statistically significant relationship between direct unit profit patterns on intraline group ten freight and the imputed empty vehicle flow patterns calculated from freight flow imbalances. Three implications result from this conclusion:

(1) the relationships prescribed in the normative model bear a significant resemblance to empirical data;

(2) freight imbalances are a perceptible concomitant factor in unit profit and rate patterns;

(3) backhaul effects are evident in the rate structure and do not necessarily "average out" for a carrier.

These implications are often ignored in contemporary transportation literature. For example, the transportation rate studies cited in the first chapter ignore the problem completely, or assume that it "averages out." This latter assumption is somewhat dubious when it can be shown that there are measurable patterns in the rate structure which are related to freight imbalances.

The empirical analysis indicates that the motor carrier under consideration is operating in a manner predicted by the normative model, and this supports the general hypothesis that the normative model provides a practical framework from which to view spatial effects in truckload freight rate structures.

CHAPTER IV

EVALUATION AND CONCLUSIONS

An Overview

This dissertation views motor common carrier truckload freight rates from the position of a motor carrier firm. It examines the normative and empirical interactions among three basic factors. The factors are:

(1) the physical necessity of equalizing vehicle flows into and out of each terminal;

(2) the dependency of freight flows on the freight rate being charged for the transportation service and the relative demand for the transportation service;

(3) the profit maximization motive which stimulates the motor carrier to adjust freight rates for economic gain.

A theoretical model consolidates these factors in order to characterize the behavior of a motor carrier firm. Principal attention is focused on the rate structure which maximizes profit for the carrier. In essence, the model describes how vehicle imbalances which arise from unequal demands affect the rate structure in a profit maximizing motor carrier firm. It is shown that marginal revenues and costs are interrelated by imputed variables associated with each terminal in the system; these variables reflect the marginal value of vehicle capacity at a terminal. General patterns result in the rate structure and they are directionally related to origins and destinations through the variables associated with the terminals.

It was also found that circuitous vehicle flows in the system are very significant from an economic theory standpoint. In general, it is theoretically incorrect to assume that marginal revenue will equal marginal cost on individual freight movements when profit is at a maximum. Rather, the correct relationship is that the sum of marginal revenues will equal the sum of marginal costs on circuitous vehicle flows in the system when profit is at a maximum.

Rate, cost, and freight flow data were collected from a motor common carrier, and applied to a statistical regression model in order to determine the extent to which the relationships prescribed by the theoretical models compare to the imperfections of the practical world. Imputed backhaul cost patterns were calculated from actual freight flow imbalances in a manner prescribed by the normative model, and then correlated with direct unit profits on truckload freight. A statistically significant relationship was found between the two. It is clearly indicated in the analysis that this cannot be interpreted to prove or disprove the normative model. However, the observed empirical relationships certainly enhance the credibility of the normative model, and strongly indicate that backhaul patterns have predictable effects in freight profitability patterns in a multiple terminal system.

Significance of the Normative Models

Actual motor carrier systems do not operate according to a simplified set of theoretical rules; rather, the real world tends to be a transient complex process. In order to make a complex problem manageable it is necessary to revert to simplified models which resemble the major elements of the problem. This is the situation here. The normative model encompasses only a portion of the total freight rate problem, and is designed to examine the interaction between the supply of and the demand for the motor carrier transportation service.

Shippers, Carriers, and Regulatory Agencies

The models are relevant to the problems of shippers, carriers, and regulatory agencies because they indicate how rate and cost structures respond to spatial differences in the demand for the transportation service. This is of concern to shippers in terms of plant location and physical distribution policies. It concerns the carrier in terms of its pursuit of profitable freight rate adjustments, equitable allocation of costs, and the identification of profitable and unprofitable freight traffic. It is of obvious concern to the regulatory agencies; they should be capable of recognizing the characteristics of an efficient transportation firm in order that they may better regulate the entire system.

Regulation and Economic Theory

The model identifies a problem with regulatory policies which treat empty vehicle movement costs as if they were an overhead item; they are typically allocated in this nondirectional manner in a motor carrier system.[1] A fundamental tenet of price theory is that prices must be commensurate with economic costs if there is to be an efficient allocation of resources under the pricing mechanism. The model clearly shows that empty vehicle movement costs have a significant directional effect in the rate structure, and, therefore, nondirectional freight rates cannot reflect the true economic costs of motor freight transportation. It is implied that a regulated freight rate structure which conforms to costs derived from the nondirectional allocation of empty vehicle

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[1]See McWilliams, op. cit. Also see Interstate Commerce Commission, op. cit.

movements costs will preclude proper economic adjustments in a system.

The Significance of the Empirical Findings

The empirical investigation indicates that freight rates and direct unit profits tend to polarize with respect to freight imbalances. These findings have definite implications with regard to motor carrier freight rate studies. The measured relationship between the rate structure and freight flow imbalances in a regulated carrier indicates that these factors should be recognized in rate studies.[1]

Potential Research Extensions

The framework presented in this dissertation can be extended in two general directions. One extension can be oriented toward the motor carrier firm, and the other directed toward regulation of the motor carrier industry.

Using the basic structure of the normative model, operational accounting methods can be developed for imputing interterminal backhaul costs. They could be imputed from freight and vehicle flows. Implementation of such a procedure would demand, of course, certain assumptions regarding overall carrier service policies, the stability, randomness, seasonal factors, and the forecasted trends of freight inputs. Once a procedure

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[1]See Walter Y. Oi and Arthur P. Hurter, Economics of Private Truck Transportation (Dubuque, Iowa: Wm. C. Brown, 1965), p. 187.

is established for imputing backhaul movement costs on a system wide basis, the profitability of specific freight movements can be ascertained.

A cost assignment system which reflects the inherent directional cost characteristics of motor carriers would provide a logical foundation for rate regulation policies. This aspect deserves special attention in view of the recent and strong criticisms aimed at the motor carrier regulatory policies of the Interstate Commerce Commission.[1] Some critics have called for complete deregulation of the industry. This dissertation provides a framework for examining the general economic implications of such proposals.

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[1]Friedlaender, op. cit., Chapter 7. Also "Anti-trust Head Urges An End to Regulation for Most Transport," Wall Street Journal, January 29, 1971, p. 6; "Council of Economic Advisers Join Ranks of Those Assailing Transport Regulation," Wall Street Journal, February 2, 1971, p. 2.

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APPENDICES

APPENDIX A

ANALYSIS OF VARIANCE TABLES

TABLE 7.--Analysis of variance table for regression model WC1LP2A.

Independent Variable	Estimated Coefficient	Coefficient Estimate	Reduction in Sum of Squares	Degrees of Freedom	Mean Square	F-Statistic
Grand Mean	c(1)	*	781.301	1		
$(t_*(i) - t_*(j))$	h	.048	0.708	1	0.708	3.000
Residual			201.654	855	0.2358	
Totals			983.663	857		

TABLE 8.--Analysis of variance table for regression model WC2LP2A.

Independent Variable	Estimated Coefficient	Coefficient Estimate	Reduction in Sum of Squares	Degrees of Freedom	Mean Square	F-Statistic
Grand Mean	c(1)	*	781.301	1		
$a_k(i,j)$	c(2)	*	6.8727	1		
$(t_*(i) - t_*(j))$	h	0.0581	1.0525	1	1.0525	4.618
Residual			194.4370	854	0.227	
Totals			983.6630	857		

TABLE 9.--Analysis of variance table for regression model WC3LP2A.

Independent Variable	Estimated Coefficient	Coefficient Estimate	Reduction in Sum of Squares	Degrees of Freedom	Mean Square	F-Statistic
Grand Mean	c(1)	*	781.301	1		
$a_k(i,j)$	c(2)	*	6.873	1		
$w_k(i,j)$	c(3)	*	13.140	1		
$(t_*(i) - t_*(j))$	h	0.59	1.103	1	1.103	5.191
Residual			181.246	853	.212	
Totals			983.663	857		

TABLE 10.--Analysis of variance table for regression model WC4LP2A.

Independent Variable	Estimated Coefficient	Coefficient Estimate	Reduction in Sum of Squares	Degrees of Freedom	Mean Square	F-Statistic
Grand Mean	c(1)	*	781.301	1		
$a_k(i,j)$	c(2)	*	6.873	1		
$w_k(i,j)$	c(3)	*	13.140	1		
$\sqrt{a_k(i,j)}$	c(4)	*	4.839	1		
$(t_*(i) - t_*(j))$	h	.063	1.223	1	1.223	5.908
Residual			176.288	852	.203	
Totals			983.663	857		

TABLE 11.--Analysis of variance table for regression model WC5LP2A.

Independent Variable	Estimated Coefficient	Coefficient Estimate	Reduction in Sum of Squares	Degrees of Freedom	Mean Square	F-Statistic
Grand Mean	c(1)	*	781.301	1		
$a_k(i,j)$	c(2)	*	6.873	1		
$w_k(i,j)$	c(3)	*	13.140	1		
$\sqrt{a_k(i,j)}$	c(4)	*	4.839	1		
$\sqrt{w_k(i,j)}$	c(5)	*	.320	1		
$(t_*(i) - t_*(j))$	h	.066	1.326	1	1.326	6.418
Residual			175.864	851	.207	
Totals			983.663	857		

APPENDIX B

THE KUHN-TUCKER THEOREM

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THE KUHN-TUCKER THEOREM

Introduction

The Kuhn-Tucker theorem[1] is a generalization of the well-known theorem on Lagrange multipliers. The Lagrange multiplier theorem specifies certain necessary conditions for the optimization of a mathematical function subject to equality constraints; the Kuhn-Tucker theorem specifies certain necessary conditions for the optimization of a function subject to inequality constraints. It should be noted that the Lagrange theorem can be derived from the Kuhn-Tucker theorem.

Under general assumptions, the Kuhn-Tucker theorem is a necessary condition for an optimal solution; that is to say, an optimal solution will always satisfy the conditions of the theorem, along with possibly other certain solutions. In special cases, the theorem becomes both a necessary and sufficient condition for optimality; that is to say, the only feasible solution which can satisfy the

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[1]H. W. Kuhn and A. W. Tucker, "Nonlinear Programming," in Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, ed. by J. Neyman (Berkeley: University of California Press, 1951).

theorem is the optimal solution. This special case occurs when the mathematical functions in the problem satisfy certain convex and concave requirements.

The development of the Kuhn-Tucker conditions here begins with the general form of the theorem, and then focuses on the special case of linear constraints.

General Form of the Kuhn-Tucker Theorem

Consider the following mathematical program.

Maximize $f = f(x)$ (1)

Subject to $g_i(x) \leq 0$, $(i=1,\ldots,m)$ (2)

where

(x) is a vector composed of the elements x(j), (j=1,...,n).

x(j), (j=1,...,n), are the (n) controllable variables in the mathematical program; the problem is to find values for these (n) variables which maximize (f) with respect to the given constraints.

f(x) is a known mathematical function of the vector (x). It is assumed that this function is continuously differentiable.

$g_i(x)$,(i=1,...,m), are (m) known functional inequality constraints on (x). It is assumed that each function is continuously differentiable, and that any set of nonlinear $g_i(x)$ functions are not tangent to one another at the optimal solution.[1]

The Kuhn-Tucker theorem says that if x_o is a feasible optimal solution to the mathematical program specified

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[1]The "tangent" condition is dealt with by what is called the constraint qualification property, and is discussed in later references.

above in (1) and (2), then there exists a set of variables $t(i)$, $(i=1,\ldots,m)$, which satisfy (3), (4), and (5).

$$t(i) \geq 0, \quad (i=1,\ldots,m) \qquad (3)$$

$$t(i) \cdot g_i(x_o) = 0, \quad (i=1,\ldots,m) \qquad (4)$$

$$\left.\frac{\partial f(x)}{\partial x(j)}\right|_{x_o=x} - \sum_{i=1}^{m} \left(t(i) \cdot \left.\frac{\partial g_i(x)}{\partial x(j)}\right|_{x_o=x}\right) = 0, \quad (j=1,\ldots,n) \qquad (5)$$

where

$x_o(j)$, $(j=1,\ldots,n)$, is an optimal solution to the mathematical program. Of course, this optimal solution must satisfy the (m) constraint equations specified in (2).

$t(i)$, $(i=1,\ldots,m)$, are called the Kuhn-Tucker multipliers in the same context as are Lagrange multipliers. The (i)th multiplier is associated with the (i)th constraint.

$\left.\frac{\partial f(x)}{\partial x(j)}\right|_{x_o=x}$ is the partial derivative of the objective function (1) with respect to the (j)th controllable variable evaluated at x_o.

$\left.\frac{\partial g_i(x)}{\partial x(j)}\right|_{x_o=x}$ is the partial derivative of the (i)th functional constraint with respect to the (j)th controllable variable evaluated at x_o.

The equations in (4) show that if the value of the (i)th functional constraint evaluated at x_o is strictly less than zero, then the (i)th multiplier must equal zero; on the other hand, if the value of the (i)th functional constraint evaluated at x_o is equal to zero, then the (i)th multiplier need not be equal to zero. The equation in (3) shows that each t(i) must be greater than or equal to zero. In summary, the (i)th multiplier may be greater than or equal to zero if the (i)th constraint is at its upper bound; however, if the (i)th constraint is not at its upper bound, then the (i)th multiplier must equal zero.

The equations in (5) show that at the optimal solution the partial derivative of the objective function with respect to the (j)th variable must equal the summation of the products of the multipliers times the partial derivatives of the constraints with respect to the (j)th variable.

It is assumed in (1) and (2) that the functions are continuously differentiable, and that the constraint functions satisfy the constraint qualification property.[1] However, there are variations to these assumptions, for example, it could be assumed that the functions f(x) and $g_i(x)$, need only be differentiable in the neighborhood of the optimal solution.[2]

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[1]See C. Almons, Matrix Methods in Economics (Reading, Mass.: Addison-Wesley Pub. Co., 1967), pp. 101-04. Also, G. Hadley, Nonlinear and Dynamic Programming (Reading, Mass.: Addison-Wesley Pub. Co., 1964), p. 195.

[2]J. Abadie, "On the Kuhn-Tucker Theorem," in Nonlinear Programming, ed. by J. Abadie (Amsterdam: North-Holland Pub. Co., 1967), p. 22.

The Values of the Multipliers

It so happens that a value of t(i) can be used to reflect the effect of incremental changes in the (i)th constraint on the optimal value of the objective function. This characteristic of the multipliers is demonstrated below.[1]

Suppose the program in (1) and (2) was revised as follows.

$$\text{Maximize } f_* = f(x) \qquad (6)$$

$$\text{Subject to} \quad g_i(x) \leq e(i), \ (i=1,\ldots,m) \qquad (7)$$

where

e(i), (i=1,...,m) have values which are in a sufficiently small neighborhood of zero.

f(x) and $g_i(x)$, (i=1,...,m), are functions identical to those defined in (1) and (2).

After solving this revised program, the value of f_* will differ from the value of f by approximately

$$\left(\sum_{i=1}^{m} e(i) \cdot t(i) \right). \qquad (8)$$

where

f is the optimal value of the objective function of the original problem in (1) and (2).

f_* is the optimal value of the objective function of the revised program in (6) and (7).

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[1]For greater detail, see J. Abadie, <u>op. cit</u>., p. xiii.

t(i), (i=1,...,m), are a suitable set of multipliers in the optimal conditions of the original problem.

The Kuhn-Tucker Theorem with Linear Constraints and Non-negativity Restrictions on the Controllable Variables

In the case where all the constraint functions in (2) are linear, and the controllable variables (x) are restricted to being non-negative, then the Kuhn-Tucker theorem is applied as follows.

Consider the following mathematical program.

$$\text{Maximize } f = f(x) \tag{9}$$

$$\text{Subject to } \quad Gx - s \leq 0 \tag{10}$$

$$x \geq 0 \tag{11}$$

where

f(x) and (x) are as defined in (1) and (2).

(G) is a matrix which has (m) rows and (n) columns. The elements in (G) are known constants.

(s) is a column vector which has (m) rows. The elements of (s) are known constants.

The constraints in (10) and (11) can be written in algebraic form as follows.

$$\left(\sum_{j=1}^{n} G(i,j) \cdot x(j)\right) - s(i) \leq 0, \quad (i=1,\ldots,m) \tag{12}$$

$$x(j) \geq 0, \quad (j=1,\ldots,n) \tag{13}$$

where

G(i,j) is the element in the (i)th row and (j)th column of the matrix (G).

s(i) is the element in the (i)th row of the vector (s).

x(j) is the element in the (j)th row of the column vector (x).

The Kuhn-Tucker theorem says that if x_o is a feasible optimal solution to the problem stated in (9), (10), and (11), then there must exist a (m) by (1) column vector (t), and a (n) by (1) column vector (z) which satisfy (14), (15), (16), (17), and (18).

$$t \geq 0 \tag{14}$$

$$t' \cdot (G\, x_o - s) = 0 \tag{15}$$

$$z \geq 0 \tag{16}$$

$$z' \cdot x_o = 0 \tag{17}$$

$$\left.\frac{\partial f(x)}{\partial x}\right|_{x_o = x} - G' \cdot t + z = 0 \tag{18}$$

where

t',z', and G' are the transposes of the vector t, the vector z, and the matrix G respectively.

$\left.\frac{\partial f(x)}{\partial x}\right|_{x_o = x}$ is a column vector representing the derivatives of (f(x)) with respect to x(j), (j=1,...,n); the (j)th element is $(\partial f/\partial x(j))$.

The above matrix expressions are expressed in algebraic form as follows in equations (19) through (23) respectively.

$$t(i) \geq 0, \quad (i=1,\ldots,m) \tag{19}$$

$$t(i) \cdot \left(\left(\sum_{j=1}^{n} G(i,j) \cdot x_o(j)\right) - s(i)\right) = 0, \quad (i=1,\ldots,m) \tag{20}$$

$$z(j) \geq 0, \quad (j=1,\ldots,n) \tag{21}$$

$$z(j) \cdot x_o(j) = 0, \quad (j=1,\ldots,n) \tag{22}$$

$$\left.\frac{\partial f(x)}{\partial x(j)}\right|_{x_o=x} - \left(\sum_{i=1}^{m} t(i) \cdot G(i,j)\right) + z(j) = 0, \quad (j=1,\ldots,n) \tag{23}$$

The equations in (19) and (20) show that if the (i)th constraint of (G) is at its upper bound, then t(i) can be greater than or equal to zero; however, if the (i)th constraint is not at its upper bound, then t(i) must equal zero. The equations in (21) and (22) show that if $x_o(j)$ is strictly greater than zero, then z(j) equals zero; on the other hand, if $x_o(j)$ is equal to zero then z(j) may be either equal to or greater than zero.

Note here that if the objective function in (9) is linear, then the program becomes a linear programming problem, and the z(j) and t(i) are the elements in the "Z-C" row of the simplex method;[1] each z(j) is associated

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[1]See Hadley, Linear Programming, pp. 108-48.

with an x(j), and each t(i) is associated with the slack variable in the (i)th constraint.

The constraint qualifications, which were mentioned previously, are always satisfied when the constraints are linear.[1]

The Kuhn-Tucker conditions become necessary and sufficient conditions for global optimality when the objective function is concave and the constraint functions are convex. Linear constraints always define a convex feasible solution space; therefore, if the objective function in (9) is concave, then a solution which satisfies the Kuhn-Tucker conditions is a global optimum to the program.[2]

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[1]See Hadley, Nonlinear and Dynamic Programming, p. 201.

[2]*Ibid*., p. 212.