INFLUENCE OF LOWER CHORD SIZE ON STRESSES IN TWO-HINGED TRUSSED ARCH

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Hedayat Behbehani

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Major professor

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INFLUENCE OF LOWER CHORD SIZE ON STRESSES IN TWO-HINGED TRUSSED ARCH

By

HEDAYAT BEHBEHANI

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9

TABLE OF CONTENTS

STATEMENT OF PROBLEM

The purpose of this analysis is;

1. to show the influence of lower chord size on stresses and

2. to determine the limiting cross-sectional area of the lower chord in a two-hinged trussed arch. In other words to investigate whether or not this structure will approach a three-hinged arch, by increasing the size of the lower chord.

INTRODUCTION

The analysis of statically indeterminate structures by classical methods has always been a very tedious job, since before the problem can be solved it is necessary to make some assumptions which will have to be corrrected later.

As an example, let it be required to find the stresses in a statically indeterminate truss for a given loading.

Before the problem can be solved it will be necessary to make an assumption regarding the cross-sectional area of the members. Having done this, then the stresses can be found by means of virtual work or any other known method. Generally speaking, these stresses are either too great or too small for the assumed areas and, it will be necessary to revise the original assumption. It is clear now that, when the cross-sectional area of the members are changed, the stresses obtained previously will no longer be true.

From the brief discussion presented, it can be concluded that in order to design an indeterminate truss for given loading, one would have to use a trial and error procedure. The number of trials required to get a close approximation depends greatly on the amount of experience that a person has with the particular kind of structure.

Time and energy will be saved if one could estimate in advnace the amount of variation in stress due to changes in area, because it would permit making a much closer and wiser assumption.

With this thought in mind, I have tried to show the variation in stress by changing the cross-sectional area of one group of members with respect to the others.

Although there are many different kinds of trusses for which this analysis could be used, the author is merely concerned with two-hinged trussed arch.

Since there are four reactions on this structure and all members are required to carry direct stress, it is statically indeterminate to the first degree.

In order to analyze this structure, it was necessary to assume some values for the cross-sectional area of the members. The assumption was made by setting a definite ratio between the size of the lower chord and the other members. By varying the size of lower chord and keeping the rest constant, some stresses were obtained which are plotted on the graph in dimensionless form.

The author sincerely hopes that the results of this analysis will be of some value to the designers of such structures.

ANALYSIS

The method of virtual work was used to obtain the stresses. I do not intend to discuss this method since it can be found in almost any textbook on Statically Indeterminate Structures. However, it might be well to point out the equations that were used and define the terms in them.

The equation of virtual work for a statically indeterminate truss to the first degree is;

$$\sum \frac{SuL}{AE} + R\sum \frac{u^2L}{AE} = 0$$

But since E is constant it can be omitted.

$$\sum \frac{SuL}{A} + R\sum \frac{u^2L}{A} = 0$$

or

$$R = -\frac{\sum \frac{SuL}{A}}{\sum \frac{u^2L}{A}}$$

Where;

S = Bar stress in the determinate truss.

R = Horizontal reaction which was assumed as the redundant.

L = Length of member.

A = Cross-sectional area of member.

E = Modulus of elasticity.

u = Bar stress due to a unit load applied at the point where the redundant was removed.

$\sum \frac{SuL}{AE}$ = Deflection due to the load.

$\sum \frac{u^2L}{AE}$ = Deflection due to a unit load.

$R\sum \frac{u^2L}{AE}$ = Deflection due to R.

Having R, the actual bar stresses can be found very easily since

Actual stress = $S + Ru$

For this analysis two trussed-arches were used, the dimensions of which will be shown in the next few pages. The following assumptions were made regarding the two structures;

1. The trusses are symmetrical.
2. The load on each panel point is equal to P.
3. The members are pin-connected.
4. The bottom chord panel points lie on a parabola.

EQUATION OF PARABOLA

The equation of the parabola is;

$$y = h - \frac{4h}{L^2} x^2$$

Where;

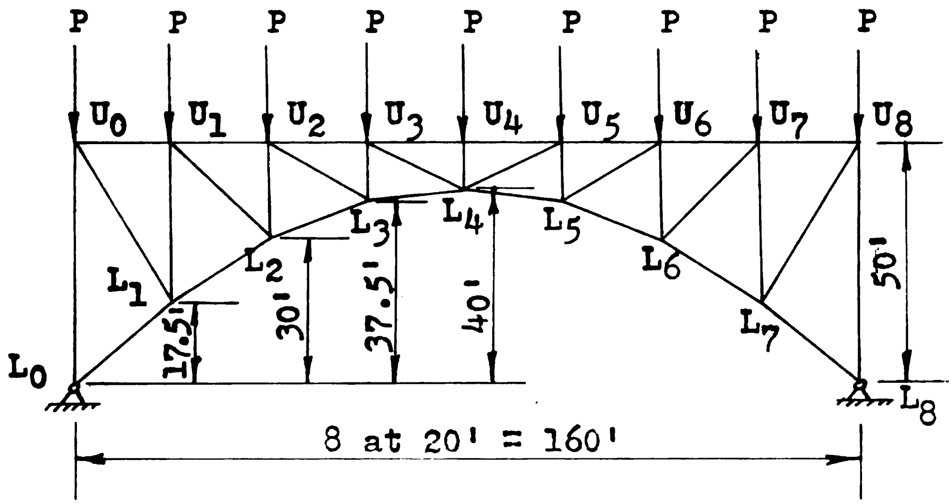
y = Distance from the x-axis to the parabola.

x = Distance from the y-axis.

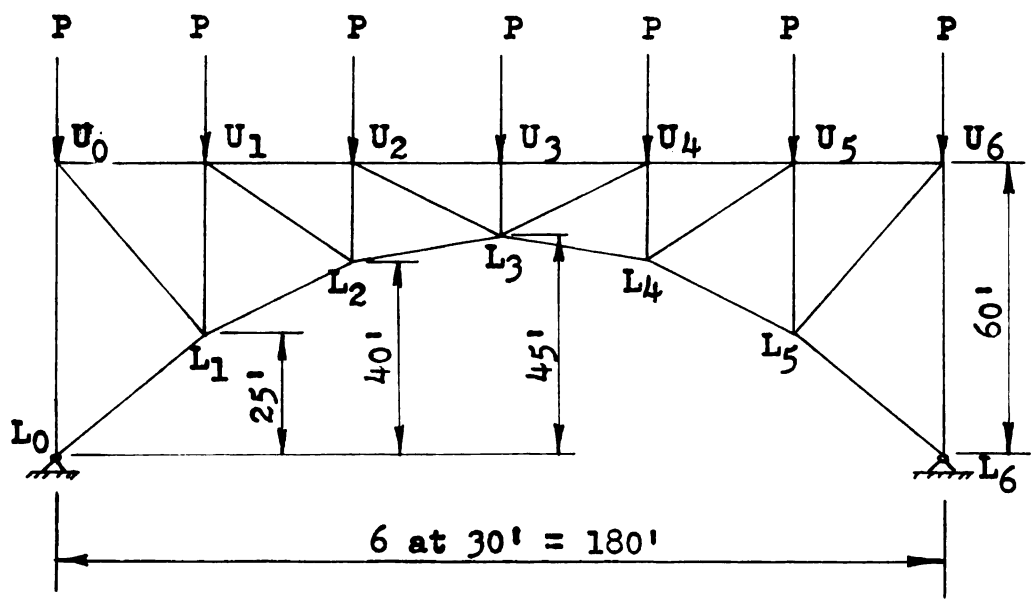
L = Length of the truss.

h = Maximum height of the parabola measured from the x-axis.

The following diagrams represent the trusses and the loading used in this analysis.

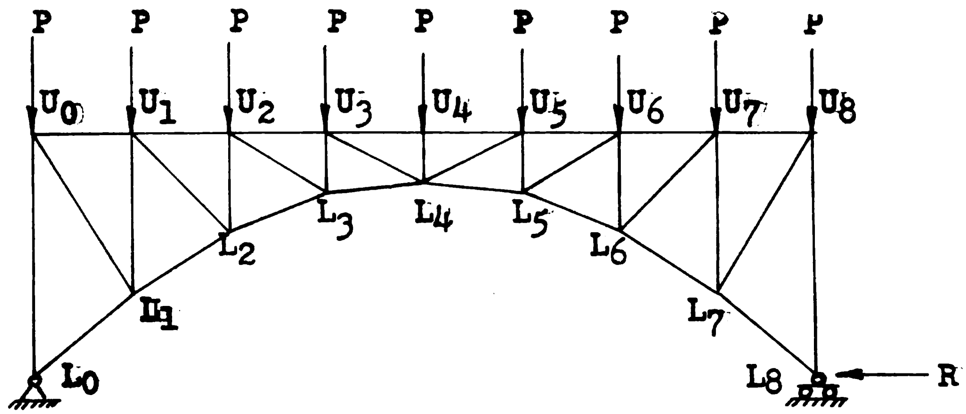


TRUSSED-ARCH "a"

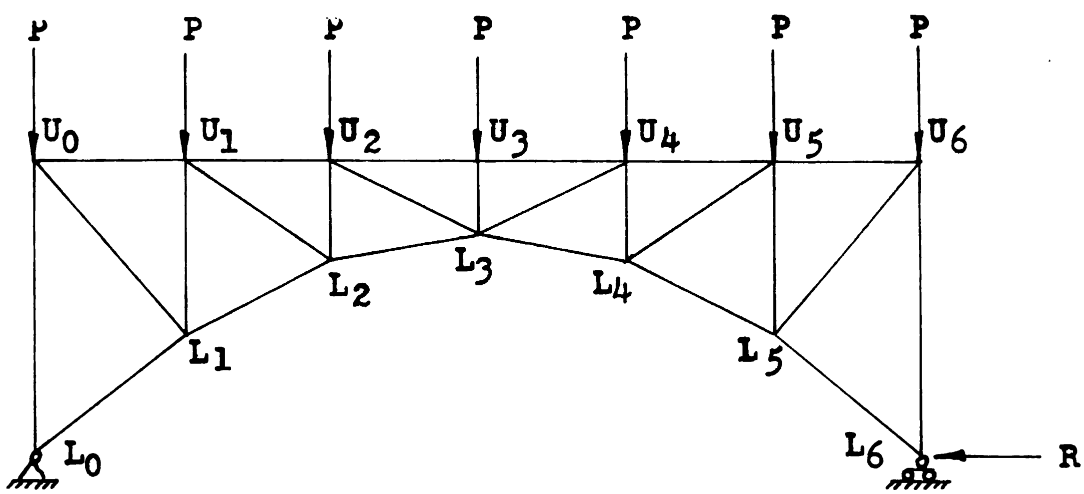


TRUSSED-ARCH "b"

As it can readily be seen both structures are statically indeterminate to the first degree. Following the method of virtual work, it is necessary to remove the redundant, which in this case was taken as the horizontal reaction at the right support. This redundant must be replaced with a force, namely R, as shown below.



TRUSSED-ARCH "a"



TRUSSED-ARCH "b"

Now from the previous discussion, reaction R and the stress in the members can be calculated by assuming a cross-sectional area for each member.

Let,

A_{bc} = cross-sectional area of each bottom chord member and

A = cross-sectional area of each other member.

In order to show the influence of bottom chord on stresses, it is necessary to keep A_{bc} constant and vary A. This is to say that;

$$A_{bc} = KA$$

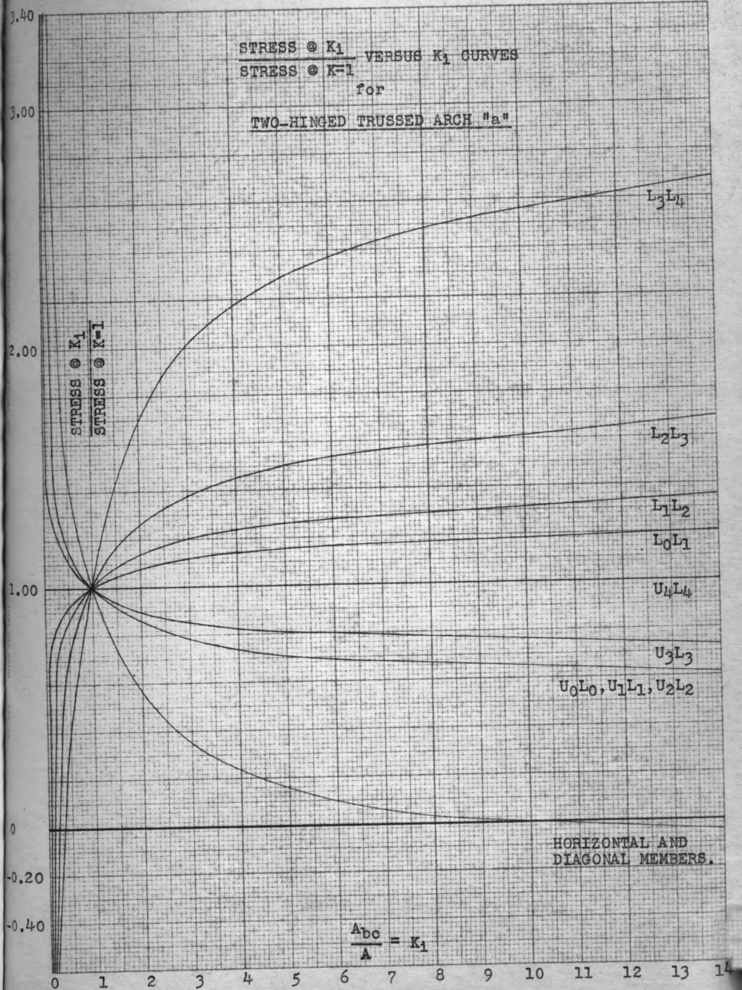
where K is a factor which can take on values from $0 \rightarrow \infty$ or $0 < K \leq \infty$. K can not reach a value of zero, because if $K = 0$ then

$$A_{bc} = (0)A = 0$$

and if $A_{bc} = 0$, the structure will become unstable.

K, however, can be equal to infinity without endangering the stability of the structure, but it is neither practical nor economical to make A_{bc} so very much greater than A. Actually K never approaches zero or infinity in practice, but for the purpose of this analysis let us assume that it does.

If K is allowed to vary between the interval $0<K<\infty$ the stresses can be computed for each given K. The results are shown on graphs by plotting $\frac{\text{stress @ } K_1}{\text{stress @ } K = 1}$ against $\frac{A_{bc}}{A} = K_1$. Note that in either case the numerator and the denominator have the same units, which is to say that the quantity is in dimensionless form.


STRESS @ K1 / STRESS @ K=1 VERSUS K1 CURVES
for
TWO-HINGED TRUSSED ARCH "a"
STRESS @ K1 / STRESS @ K=1
L3L4
L2L3
L1L2
L0L1
U4L4
U3L3
U0L0, U1L1, U2L2
HORIZONTAL AND DIAGONAL MEMBERS.
Abc / A = K1
3.40
3.00
2.00
1.00
0
-0.20
-0.40
0
1
2
3
4
5
6
7
8
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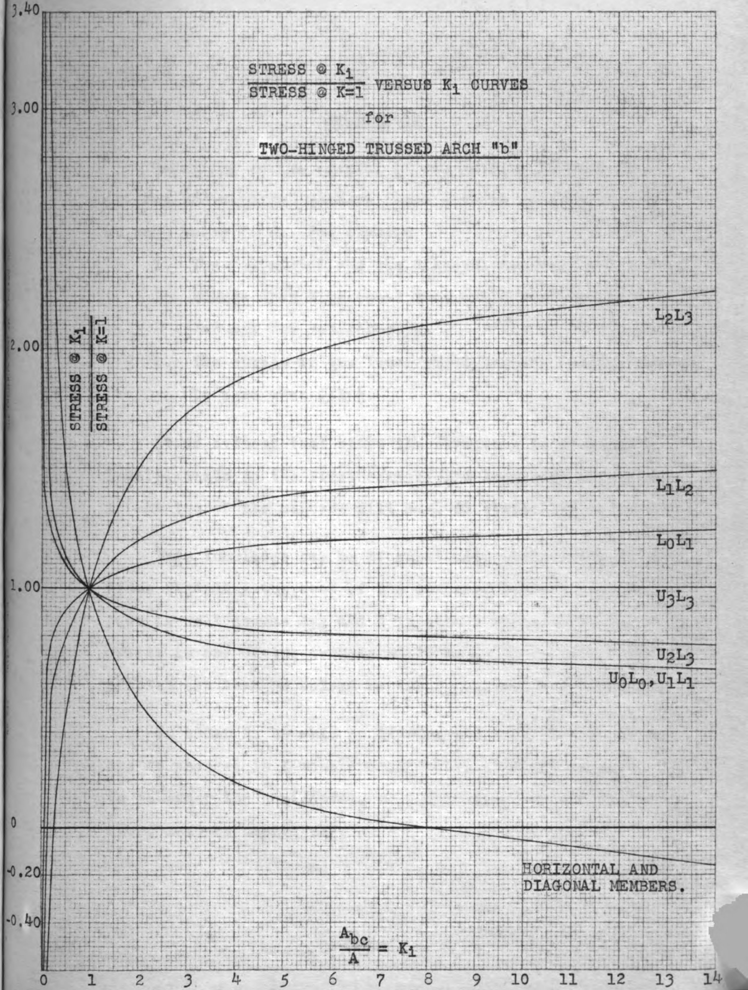
11
12
13
14

6



STRESS @ K_1 / STRESS @ K=1 VERSUS K_1 CURVES
for
TWO-HINGED TRUSSED ARCH "b"
STRESS @ K_1 / STRESS @ K=1
3.40
3.00
2.00
1.00
0
-0.20
-0.40
L_2L_3
L_1L_2
L_0L_1
U_3L_3
U_2L_3
U_0L_0, U_1L_1
HORIZONTAL AND
DIAGONAL MEMBERS.
$\frac{A_{bc}}{A} = K_1$
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14

DISCUSSION

Referring to the graphs, it can readily be seen that all but the vertical members reach a value of zero stress at some value of K. For example;

1. The horizontal and the diagonal members have zero stress when K is equal to 10 and 8 for trusses "a" and "b" respectively.
2. The stress in the lower chord members becomes equal to zero when $0<K<\frac{1}{2}$.

It is evident that since the structure is statically indeterminate to the first degree, the knowledge of stress in any two of the members will make it statically determinate.

This knowledge can be of great help when making assumptions regarding the area of the members, because one can just as well assume A_{bc} and A in such a manner that the stress in one of the members would be equal to zero. The problem can then be solved by the equations of equiblirum.

In order to be able to discuss the variation in stress due to changes in K more specifically, it might be well to consider the horizontal, vertical, diagonal and the lower chord members separately.

<u>HORIZONTAL and DIAGONAL MEMBERS</u>. These two sets of members are discussed together since the $\frac{\text{stress @ } K_1}{\text{stress @ } K = 1}$ versus K_1 curve is the same for both of them. This is caused by the fact that the horizontal compoent of a stress in a diagonal is equal to the resulting horizontal stress at the joint with an opposite sign.

The stress variation curve seems very abrupt, but it is not unreasonable, since the greater the area of a member the more stress it will take.

The stress in the horizontal and diagonal members is very small when K is greater than four. When K gets smaller than two, the stresses start to increase very rapidly since the lower chord members tend to take a smaller amount of the load. This stress variation is not unusual as the lower chord members seem to transfer their stress to the other members as K decreases.

<u>VERTICAL MEMBERS</u>. The stress variation in these members is more or less the same as in the diagonal and the horizontal members, with the exception that they do not decrease quite as much when K is increased.

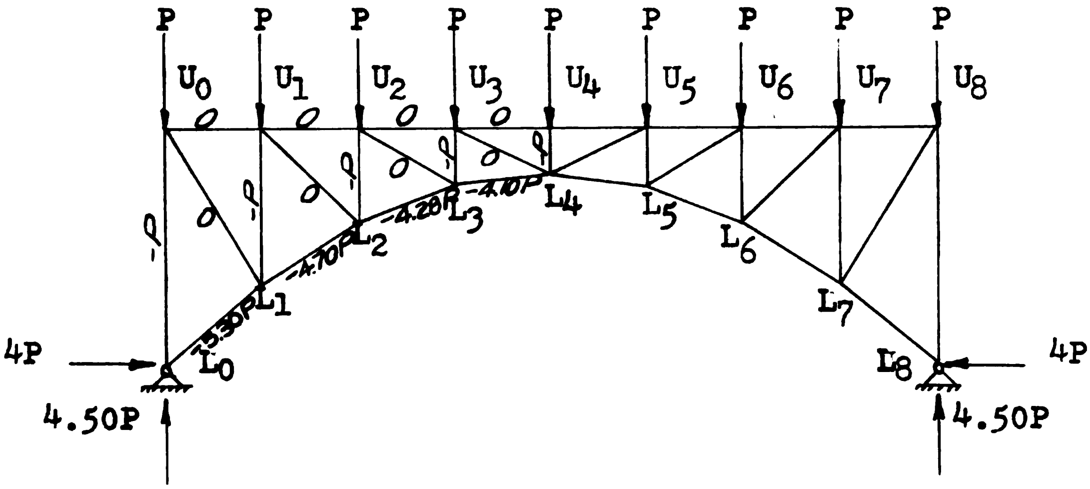
Note that the center member keeps the same stress regardless of K. The stress in this member is equal to the load applied directly above it.

The rest of the vertical members reach a stress equal to the load, applied directly above them, if K is made large enough, namely 10 and 8 in trusses "a" and "b" respectively.

<u>LOWER CHORD MEMBERS</u>. These members act very much the same as expected. The stresses increase directly as K increases.

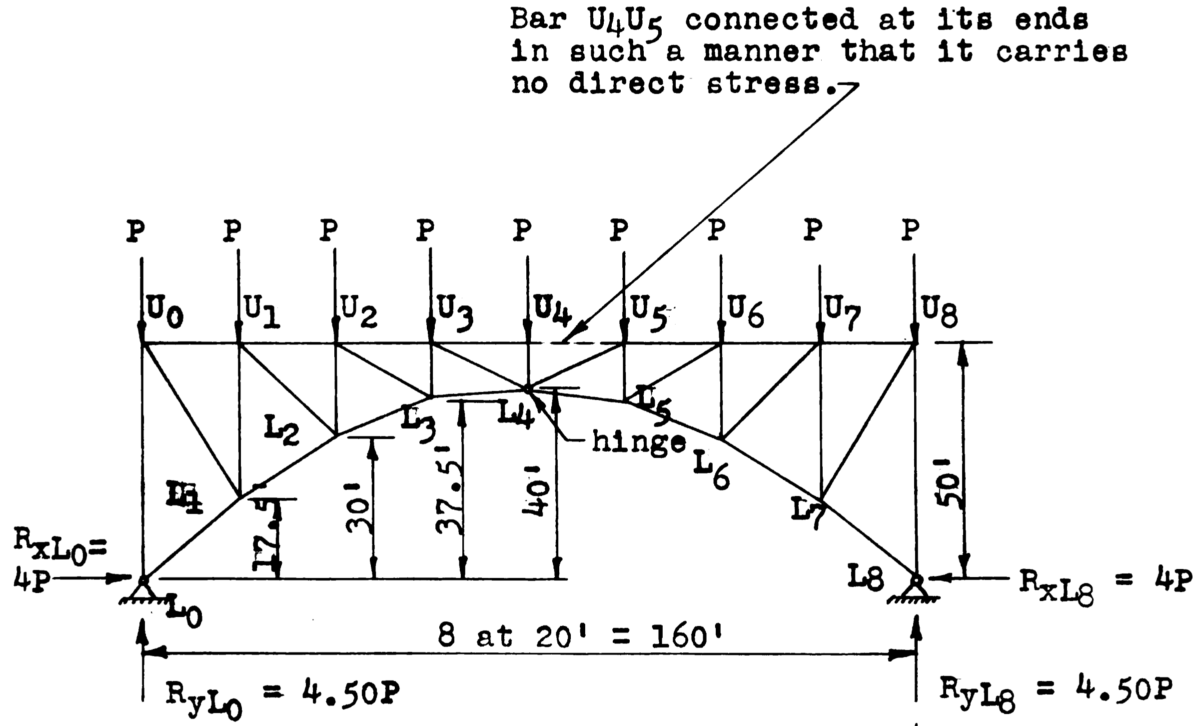
If trussed arch "a" is considered for K = 10 we can immediately see that the following stress condition exists;

1. The stress in each top chord is zero.
2. The stress in each diagonal is zero.
3. The stress in the vertical is equal to the top chord panel load.
4. The horizontal component of stress in each bottom chord is the same and equal to the horizontal reaction.



TRUSSED-ARCH "a"

It can therefore be said that the trussed-arch "a" is equivalent to the three-hinged trussed arch shown below.



The vertical reactions of this structure can be determined by taking moments about points L_0 and L_8. To obtain the horizontal reaction at L_0, the summation of moments of the left half of the structure about the hinge at L_4 can be set equal to zero. The horizontal and the vertical reactions are 4.00P and 4.50P respectively. Calculating the bar stresses the following is found to exist;

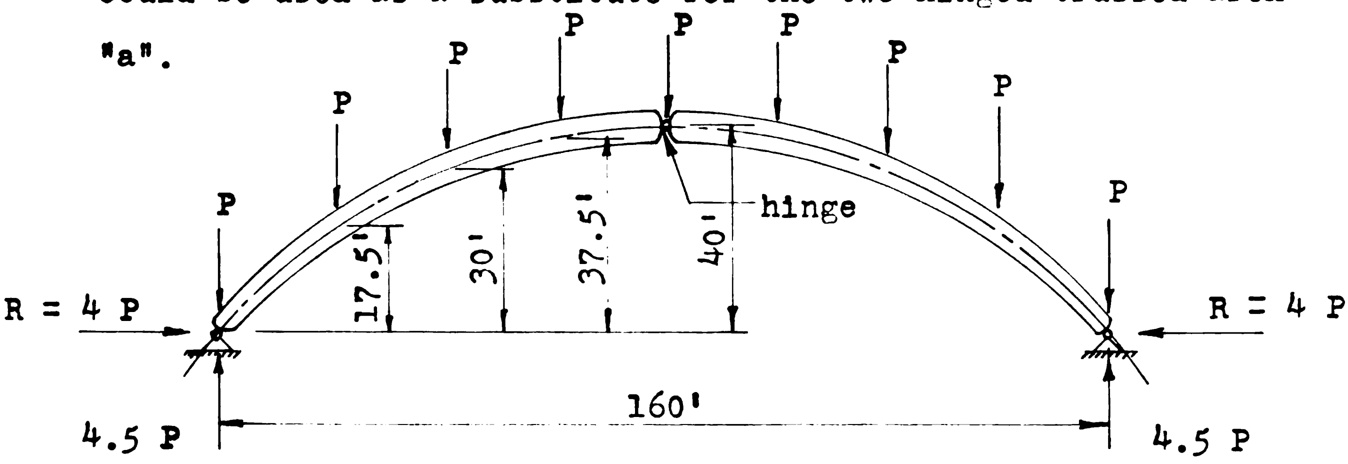
1. Zero stress in top chord members,
2. Zero stress in diagonal members,
3. Stress equal to top chord panel load in verticals, and
4. (-5.32P), (-4.70P), (-4.28P), (-4.10) in members L_0L_1, L_1L_2, L_2L_3 and L_3L_4 respectively.

Therefore, the two trusses mentioned are equivalent and it can be concluded that when $\frac{A_{bc}}{A} = 10$ in the trussed arch "a", it is possible to analyze it as a three-hinged arch and calculate the stresses in that manner. The reason for this is the fact that, when the stress in U_4U_5 becomes equal to zero, the hinge at L_4 becomes effective.

The above discussion also holds true for the trussed arch "b", except in this case K = 8 instead of 10.

Referring back to trussed arch "a", when K = 10, the stress in diagonal and horizontal members is zero. But these bars cannot be removed if the structure is to remain stable, because the lower chord members are pin-connected. The structure can and perhaps should be replaced by a three-hinged parabolic arch, which would be more economical.

The diagram below represents a three-hinged arch that could be used as a substitute for the two-hinged trussed arch "a".



THREE-HINGED PARABOLIC ARCH

The equation of parabola is;

$$y = 40 - \frac{x^2}{160}$$

The reactions can be found by the equations of equilibrium.

CONCLUSION

As a result of this analysis it can be concluded that in any two-hinged trussed arch there exists a definite relationship between the stress in members and the area of the lower chord.

A two-hinged trussed arch is effective as such only when the difference in the cross-sectional area of the bottom chord and the other members is relatively small. When this difference gets very large, the lower chord tends to take a much greater amount of the load, which in turn reduces the load on the remaining members. In other words, the lower chord carries the most load while the others help to keep the stability of the structure. Just how large this difference should be allowed to get for an economical design was not considered in this analysis, as that would involve many factors.

Two-hinged trussed arch can be treated as three-hinged trussed arch when K reaches a certain value, this value is different for each truss. When K does reach this value, however, the two structures become equivalent as the stress condition that was mentioned previously exists in the two.

At this point the structure may be replaced by a three-hinged arch with the same horizontal and vertical reactions resulting.

Perhaps the most important thing to be said, is that when K gets relatively large, the structure can be analyzed as a determinate, without an appreciable amount of error. This is caused by the fact that as K increases the stress in the horizontal members get closer and closer to zero, which in turn tends to make the center pin approach a hinge.