A THEORETICAL AND EMPIRICAL INVESTIGATION OF DEBT MATURITY TIMING AND YIELD CURVE SLOPE ANALYSIS

Thesis for the Degree of Ph. D.

MICHIGAN STATE UNIVERSITY
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1973

This is to certify that the

thesis entitled

AN EMPIRICAL INVESTIGATION OF DEBT MATURITY
TIMING AND YIELD CURVE SLOPE ANALYSIS

presented by

William C. Handorf

has been accepted towards fulfillment
of the requirements for

Ph.D degree in Finance - Business

Major professor

Date 1/10/73

O-7639

ABSTRACT

A THEORETICAL AND EMPIRICAL
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By

William Charles Handorf

Economists have been attempting to explain the term structure of interest rates for more than a generation. In spite of considerable effort, the diversity of explanations remains large. Nevertheless, enough is now known to justify an effort to develop normative rules for debt management. Specifically, this study scrutinizes the information content of the term structure for elements that might aid in financial management. The research hypothesis tested in this study is that the slope of a yield curve contains information useful for debt management, both governmental and corporate. If the hypothesis cannot be rejected, debt interest cost benefits may be obtained by judicious timing of long-term issues. In order to do so, however, the traditional normative financial rule that debt maturity should

parallel asset life must be waived.

Selection of debt maturities and the timing of financing may be brought within a dynamic programming model. The framework minimizes present valued interest costs and requires that the issuer specify debt needs, maturity constraints and forecasted interest rates. The model concentrates on the relationship between a yield curve slope and forecasted interest rates. It does not explicitly consider flotation costs and financial risk of debt maturity decisions. Given that long-term rates are presumed to be equal to the serial sequence of future short-term rates and that the current yield curve is known, forward interest vectors may be forecast. If a yield curve slope contains statistically significant information, a rational basis for adjusting the forward interest vector facing the issuer exists.

The empirical test is designed to indicate whether the slope of a yield curve may predict deviations from projected forward interest rates. Funds needed for a given length of time may be supplied either by single-stage or by double-stage financings. Larger number of stages are possible but were not examined in this study. The slope of a yield curve is defined as the difference in yields between the yield on a single-stage issue and the yield on the first stage of a double-stage issue. By equating the known present interest cost of a single-stage issue to the unknown present interest cost of a double-stage issue, a breakeven yield for the second stage of the double stage may be computed. The real yield that later prevailed for the second-stage maturity is compared to the breakeven yield. If the later real yield is greater than the breakeven yield, the single issue would have been advantageous; if the later real yield is less than the breakeven yield, the double issue would have been advantageous.

An ordinary least squares regression tests the relationship between the realized yield minus the breakeven yield (dependent variable) and the yield curve slope as defined (independent variable). Desirability of stage financing is indicated by the combination of both the regression constant B_0 and the slope B_1. Regression estimators significantly different from zero based on the F test justify the hypothesis that information is contained by a yield curve slope. The two-stage least squares technique is used to reduce the effect of positive autoregression of residuals to acceptable limits. Various plans ranging in maturity from two to twenty years are tested on

quarterly yields over the 1952-71 period.

Interest cost minimization is often a stated goal of Treasury debt management. The purpose of this study is not advocacy of a procyclical approach; but, rather, illustration of the potential for interest cost reduction. The test of this hypothesis with the U.S. Treasury debt yield curve showed that the regression slope B_1 is not significantly different from zero at the ten per cent level. Knowledge of a yield curveslope is of no consequence for Treasury debt management with regard to present valued interest cost minimization. The expectations theory gains support because a yield curve of period t does reflect future interest rates in period t+1,...,t+n, regardless of the associated yield curve slope. For the plans tested the regression constant B_0 is not significantly different from zero at the ten per cent level. The existence of a liquidity premium is substantiated on the basis of an increasing B_0 as the maturity of the plans tested lengthens. Negative B_0's in the 1950's and positive B_0's in the 1960's are noted and weakly support a change from the money-substitute to the "normal" hypothesis of the liquidity preference theory and also reflect the cyclical increase in interest rates over the time period. Results do not support or reject the institutional theory since relative supply and demand is not measured.

A comprehensive yield curve for corporate debt does not exist. The substitute test was of a commercial paper alternative and a bank prime alternative where both plans are funded in year one by a nineteen year AA long-term Utility. The regression results of these tests are significant at the two per cent level and indicate that double-stage financing becomes more advantageous as the yield curve slopes more steeply upward. The information contained by a yield curve slope is ascribed to be a result of market participant overreaction. The bank prime plans is statistically more significant than the commercial paper plan. Market inefficiencies and administered rates increase the chance that information may be contained within the interest rate term structure. The astute corporate financial manager may lower the overall cost of creditor funds by recognition of the slope of a yield curve.

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A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Accounting and Financial Administration

1973

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ACKNOWLEDGMENTS

A dissertation does not stand by itself; but, rather, reflects the combined efforts of many involved and dedicated people. I wish to express my gratitude to these people and acknowledge the financial support provided by the Department of Accounting and Financial Administration, Michigan State University.

Professor Roland I. Robinson, Chairman, continuously provided very responsive and necessary guidance and structure. Professor Robinson's incredibly quick and insightful response to written work and open door for verbal communication greatly aided a successful completion. Professor Alden C. Olson skillfully edited the thesis and provided beneficial research structure. Professor Olson is also the only person, to my knowledge, that has volunteered to serve on a time consuming dissertation committee. Dr. Ronald M. Marshall enhanced the theoretical and statistical aspects while improving readability, especially for the "non-finance major."

Mr. Jerry St. Amand was extremely helpful in

untangling the administrative red tape of the computer facilities in addition to offering consistent and constructive computer programming advice. I appreciated the opportunity to repeatedly discuss dissertation related topics with Dr. Richard Walter, Mr. Larry Lang and fellow Doctoral Candidates. Appreciation is also extended to Jo McKenzie who typed the final draft and whom I had fullest confidence that the myriad administrative details for Michigan State University would be met.

Finally, I thank my wife, Linda, for being her wonderful self while remaining patient with our work and encouraging for our goals. It is her love that was and remains my greatest and most cherished asset.

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CHAPTER I

PURPOSE OF STUDY AND LITERATURE REVIEW

Introduction

The term structure of interest rates is the pattern of yields for a number of securities that differ only with respect to maturity. Usually this means the securities of a single issuer such as the United States government, but it might be corporate obligations of issuers of similar quality and character. Normally no issuer, or group of issuers emits an infinite number of securities so the pattern of yields to maturity is a series of point observations. However, if the number of points is large enough a continuous curve can be reasonably fitted to the patterns of points. This is a "yield curve" which is the graphic representation of the term structure of interest rates.

Irving Fisher was one of the first to note the difference between long-term and short-term yields.[1] Other

[1] Irving Fisher, The Theory of Interest (The MacMillan Company 1930), p. 210.

theorists, such as Hicks, Keynes, Kessel and Malkiel gave greater substance to the theory, but it was the work of an obscure Treasury Department economist, Henry Murphy, who first applied the idea to the practical problems of market financing.[2] Soon thereafter, Durand applied a similar set of ideas to the corporate bond market.[3] Except for Murphy's work, however, this set of ideas does not appear to have been used for decision making purposes in the capital markets.

Observation of the money and capital markets shows that not only do interest rates change, but the shape and slope of the yield curves that can be fitted to market observations also change. When such changes are material, there is a reasonable presumption that a shrewd manager of a market financing might profit from a flexible approach to the market; of picking the maturity for current financing that will, in the long run, reduce financing costs.

Economists have long attempted to explain the term structure of interest rates. In spite of repeated efforts, the subject is still far from settled, particularly in the

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[2]Henry C. Murphy, National Debt in War and Transition (McGraw-Hill, 1950), pp. 92-103.

[3]David Durand, Basic Yields of Corporate Bonds, 1900-1942, NBER Technical Paper #3 (1942).

area of application. This study attempts to develop a decision making model which will apply the term structure of interest rates to financial management. Specifically, the focus is on the term structure slope, the differentials between rates for various periods to maturity. Results of the research have practical implications for the issuer of debt, both governmental and corporate.

The hypothesis is that a combination of present and future maturities can reduce interest costs over that of a single maturity. The analysis departs from the strict assumption of traditional finance which has taught the financial manager to fit debt structure to approximate asset maturity. Interest cost reduction may be possible through more sophisticated debt timing. The decade of the 1960's witnessed many manhours expended to define an optimal debt/equity structure for corporations.[4] Numerous debt/equity studies have been made to support these propositions empirically. The results have not been conclusive.[5]

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[4]See for example, David Durand, "Cost of Debt and Equity Funds for Business: Trends and Problems of Measurement"; Franco Modigliani and Merton H. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," and numerous other replies and articles reprinted in <u>The Theory of Business Finance</u> (MacMillan Company, 1967) Stephen H. Archer and Charles A. D'Ambrosio, eds., pp. 92-253.

[5]The debate continues. Michael Davenport, "Leverage, Dividend Policy and the Cost of Capital: A Comment," <u>The Journal of Finance</u>, Vol. 25, No. 4 (September, 1970), pp. 893-7.

Attention is this paper is on the management of short-term and long-term debt.

Debt Maturity Management

Financial Return

Debt management, like any phase of financial management, does possess a greater potential for success with prior planning. Planning which requires the manager to time amount and maturity of debt should lead toward a more optimal debt policy. A model that incorrectly interprets future interest rates will be in the same sorry position as a 100 per cent variable ratio bond portfolio in a bull market. A financial manager who committed a large proportion of debt to a long-term maturity issue would indeed be upset to see long-term interest rates suddenly decline.

A debt structure plan may be analogous to a formula investment plan. In a constant ratio plan an investor keeps a fixed percentage of funds between stocks and bonds. When stock prices are rising the investor sells part of his stock investment and buys bonds to maintain the fixed ratio of stocks and bonds. By contrast, in a variable ratio plan the investor decreases the proportion of stocks held in a portfolio as stock prices rise and increases the proportion of equity funds as stock prices fall. The formula plans have the advantage of requiring the investor to engage in prior planning and rule against emotional judgements such as might be possible in a bear market. Formula plans, especially variable ratio plans, do not always give superior results.[6] A debt model might alter the proportion of debt maturity with regard to the slope of the yield curve. The same general advantages and disadvantages of formula plans are applicable to a debt allocation model. The financial environment modifies a normative model.

Corporate debt management mistakes are tempered by both income taxes and a smaller variability in long-term interest rates. Debt interest and flotation costs are legitimate business expenses. Additional expense is reduced by one minus the marginal corporate income tax rate. For example, if a firm did incur an extra interest cost of ½ per cent on debt, the effective after-tax cost amounts to ¼ per cent for a firm in the 50 per cent tax bracket. It is important to note that the income taxes also limit the potential gains from correct utilization of a normative structure.

Quarterly interest rates on twenty year to maturity AA Utility bonds have fluctuated between 2.95 per cent and

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[6]Jerome B. Cohen and Edward Z. Zinbarg, Investment Analysis and Portfolio Management (Richard D. Irwin, 1967), pp. 553-4.

9.04 per cent from 1952 to 1971. The recent variability in interest rates of this maturity have not been as dramatic. The lack of severe fluctuations minimizes the potential for both gains and losses in debt structuring. However, the difference of even ¼ per cent to the financial manager is great. A ¼ per cent after-tax reduction on debt amounts to a $15,000,000 a year saving for a firm with the debt capacity of General Motors Acceptance Corporation. Gains of this magnitude make the study both worthwhile and practical. The impact is very clear for the corporate manager. In addition to considering possible returns from debt management, we will seek to ascertain the contribution to a firm's risk.

Financial Risk

Financial risk increases as the proportion of funds provided by debt and fixed charges increase. All other things equal, an increase in fixed obligations increases the probability that future fixed charges may not be met. The chance of potential bankruptcy increases likewise and represents a very real risk to the creditors and shareholders of the firm. The maturity of debt within the capital structure represents a measure of risk as the following examples demonstrate.

A utility that funds its total debt with short-term maturities would subject itself to the possible whims of the money market. The possibility of being unable to refund short-term debt continually represents risk for the firm. Most financial managers would shun a continual short-term debt policy, especially with long life assets. Shorter debt maturity may be translated into greater economic risk exposure. A firm may magnify financial risk by using a very high proportion of debt funded by a very short-term maturity. On the other hand, a longer maturity allows the firm greater time to meet fixed obligations without the capriousness of a short-term market.

The next example demonstrates the opposite side of risk from debt usage. A sales finance company might fund its debt structure entirely with long-term debt. The assets of a finance company are short lived and demand is uncertain. The firm may find itself with unneeded long-term funds and uncancellable debt because of an economic downturn. Excess debt represents an unprofitable source of funds. This opportunity loss is a risk for the shareholder, although the consequences are not as disasterous as cash insolvency. These two examples represent the potential economic and opportunity losses that may result from improper debt maturity decisions.

Maturity of debt does represent risk. Shorter debt maturity carries a greater potential for an economic loss. Longer debt maturity carries a greater potential for an opportunity loss. The potential risk is easily recognized at the extremes; say average maturity of debt equal to one year or equal to twenty years.

Traditionalists have taught that debt should parallel asset maturity. With a given debt maturity that strictly reflects asset life, the manager may not make a decision with regard to return. A debt cost reduction may be possible with combinations of debt maturity equalling asset life. In particular, gains may be possible by changing the proportion of long-term debt with relation to the slope of the yield curve.

These techniques enable a manager to fine tune debt management. First, management must decide the total debt/equity relationship. Second, management must place bounds over which the debt maturity and debt composition may change and maintain the amount of financial risk the firm desires. Of course, the above two decisions are most important to debt management. Financial leverage affects the potential earnings and the risk attached to those earnings. Sophisticated financial managers are needed who are capable of determining proper debt decisions. Numerous texts and

financial literature exist to assist in determination of proper financial leverage and these broad topics are not contained in the decision structure of this paper.

The plan of this study is as follows. Chapter one continues with a presentation of the theories of the term structure of interest rates. Effort is made to indicate the implications for debt management that arise from each theory. Chapter two presents the theoretical debt decision structure and the research design. Chapter three presents and interprets results of the application of the debt decision model to governmental debt management. Chapter four presents applications of the model to corporate debt. Chapter five presents the summary conclusions.

Term Structure of Interest Rate Theory

Interest rates have differed among issues, dates of issue and maturities. Issuer interest rate differentials may be explained by risk of default probabilities assigned by investors and bond analysts.[7] Date of issue interest rate differentials may be explained by savings, rate of

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[7]Lawrence Fisher, "Determinants of Risk Premiums on Corporate Bonds," The Journal of Political Economy, No. 67 (June, 1959), pp. 217-237. Also Avery B. Cohhan discussed factors that affected yields in "Yields on Corporate Debt Directly Placed," NBER General Series #84 (1967).

real investment, money stock rate of growth, inflation expectations and returns desired by investors.[8] Considerable effort has been given to answering the question of how maturity of debt affects interest rate structure. Theories of the term structure of interest rates are diverse and no one theory has yet gained full acceptance.

The yield to maturity curve is the most widely used graphic device for showing the relationship between yield and remaining maturity for debt issues of similar credit quality. Years to maturity is measured along the abscissa and annual yield to maturity is measured along the ordinate. During World War II Secretary of the Treasury Morganthau ordered his Director of Research to make charts for members of the Federal Reserve comparing the pattern of interest rates between a base period of 1942 and later dates.[9] Each chart contained the yield curve for a particular point in time. This posting allowed the Federal Reserve and the Treasury to be aware of the interest position and provided a graphic history of governmental yields. The Federal Reserve felt short-term rates should be allowed to rise while the Treasury felt the rates should remain constant.

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[8]Reuben A. Kessel, The Cyclical Behavior of the Term Structure of Interest Rates, NBER Occasional Paper #91. (1965).

[9]Murphy, op. cit.

The yield curve provided the Treasury with graphical feedback to insure that short-term rates were not increasing. Thus, the Federal Reserve felt the rates were "impaled" since the short-term rate could not float upward. In any event, the graphical presentation of a yield curve has remained an integral part of debt management.

Many theories have been proposed to explain the interest rate term structure but these fall into three broad groups: expectations, liquidity preference and the institutional theory. Indeed, the theories may prove complementary.

To permit comparison of these theories a standard interest rate notation will be used. Forward yields may be calculated from market yields at a point in time. A capital case "R" stands for yields currently available from the market while a lower case "r" stands for forward rates anticipated for the future. A presubscript indicates the period in which the above yields are applicable. A postsubscript indicates the remaining time to maturity for the instrument. A second postsubscript, when used, indicates the base period from which a forward rate is calculated. Thus, ${}_{t+2}r_{1,t}$ indicates the forward one year to maturity yield for period t+2 as estimated in period t. Additional interest formulations are introduced as needed. Much of the analysis

observes the variation of a credit risk free rate of interest with the term to maturity of debt. The credit risk free rate of interest abstracts from financial and credit risks such as bankruptcy which face the corporate borrower. The rate for United States government issues may be used as a proxy for a credit risk free rate of interest. The basic term structure theories are now presented. Such empirical evidence as exists is cited and its usefulness for debt management is delineated.

Expectations Theory

Broadly interpreted, the expectations theory implies long-term interest rates are a geometric average of expected future short-term rates. The expectations theory assumes investors do not demand a premium for holding longer maturity issues. Calculated forward rates are equal to expected rates. Lutz, an early advocate of the expectations theory, portrayed the investor as making a choice between holding a bond to maturity or continually investing in a succession of short-term maturities at the expected forward short rates.[10] The theory abstracts from market imperfections and assumes investors pursue their goal of profit

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[10]Frederick A. Lutz, "The Structure of Interest Rates," Quarterly Journal of Economics (November, 1940), pp. 36-63.

maximization with a uniform set of expectations for future short-term interest rate movements. Given that future short-term interest rates are accurately forecast, long-term yields may be derived mathematically. For example, the current market rate for a one year to maturity instrument today is 4 per cent, ${}_tR_1 = .04$. The expected market rate for a one year to maturity instrument one year hence is 6 per cent, ${}_{t+1}r_{1,t} = .06$. Then, the current market rate for a two year to maturity instrument today is 5 per cent, ${}_tR_2 = .05$. If an investor had funds for two years he could receive a 5 per cent return from the two year maturity and approximately the same return by investing in the two one year maturities.[11] What return would an investor receive with funds to invest for only one year? The one year note would return 4 per cent. A two year note would offer a coupon of 5 per cent but be sold at a capital loss at the end of the first year. The original two year note must sell at a discount so as to offer new investors the current 6 per cent yield one year hence for a one year to maturity issue. The two year note sells at 99 per $100 par note and gives the original one year investor a return

[11]The mathematical approximations are as shown:

$(1+{}_tR_2)^2 = (1+{}_tR_1)(1+{}_{t+1}r_{1,t})$ is $(1.05)^2 = (1.04)(1.06)$ or $1.1025 \cong 1.1024$

of 4 per cent, the same as available from a one year alternative.[12] The example may be mathematically represented as:

$$(1 + {}_tR_2)^2 = (1 + {}_tR_{1,t})(1 + {}_{t+1}r_{1,t}) \qquad (1)$$

More generally the relationship among interest rates and maturities posited is:

$$(1 + {}_tR_n) = [(1 + {}_tR_1)(1 + {}_{t+1}r_1)\ldots(1 + {}_{t+n-1}r_1)]^{1/n} \qquad (2)$$

Long-term rates for period n are a function equal to the multiplication of n future short-term rates.

Investor anticipations of future short-term rates shape the yield curve. Assume the yield curve is currently flat, long-term rates are equal to short-term rates, and inflationary expectations do not exist. If investors now believe that interest rates are low with regard to "normal" rates or because inflation is expected, the resulting yield curve will slope upwards with increasing term to maturity. Given these expectations, the equilibrium level of short-term is lower than long-term rates so as to avoid a potential capital loss from a rising interest level. Investors would buy short-term debt and sell long-term debt. These buy and sell actions raise the price (lower yield) of short-

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[12]The price of the bond is found as follows:

$$\text{Price} = \frac{\text{coupon + par}}{1 + \text{yield}}, \quad \text{Price} = \frac{5 + 100}{1 + .06} = 99$$

term debt and decrease the price (increase yield) of long-term debt and give rise to the anticipated yield curve. Expectations may explain other yield curve shapes: flat, declining or humped.[13]

As previously noted, the empirical evidence does not explain completely nor accurately the term structure. In an effort to support an early and very simple form of the expectations theory, Macaulay found that time money rates anticipated a seasonal rise in call money rates.[14] This constituted evidence of successful forecasting for Macaulay. More recently, Sargent applied spectral analysis in an application of Macaulay's test and supported the earlier findings.[15] Hickman continued with the test of accurate forecasting as evidence for the expectations

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[13]Initial research of Professor Roland Robinson and this author indicates the humped yield curve may, in fact, be a myth. Long-term government maturities have been issued with an estate tax par redemption feature such that the yield curve points are not similar in all but remaining time to maturity. The impact of this possible "myth" is not incorporated into this research effort.

[14]Frederick D. Macaulay, The Movement of Interest Rates, Bonds and Stock Prices in the United States Since 1856, NBER (1938), p. 36.

[15]Thomas Sargent, "Expectations at the Short End of the Yield Curve: An Application of Macaulay's Test," NBER General Series # 93 (1971), pp. 391-412.

theory, but failed to find the relationship.[16] Hickman found mere inertia to be a better predictor, t's period yield curve will occur in t+l's period. Culbertson's empirical research was similar to Hickman's and also found lack of accurate predictions.[17] Culbertson found it difficult to believe that speculators would operate in the governments and predict as badly as his results indicated. Culbertson felt his findings supported the institutional theory which is discussed later. The underpinning of these original tests is one of accurate forecasting where realized rates are compared with calculated rates. Meiselman designed a test which did not require accurate forecasting and the sagging expectations theory was buoyed in the mid 1960's. Before turning to Meiselman's work, the implications for debt management from the pure expectations theory are examined.

Relative supply of securities has no relevance for interest rates in the expectations theory. Interest rates are tied together by the mathematical model of equation (2).

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[16]W. Braddock Hickman, "The Term Structure of Interest Rates: An Exploration Analysis," Unpublished manuscript, NBER (1942).

[17]John M. Culbertson, "The Term Structure of Interest Rates," Quarterly Journal of Economics, Vol. 71, No. 4 (November, 1957), p. 502.

The supply of securities only affects interest rates if the supply changes expectations for the investor. Buy and sell actions of investors shape the yield curve to conform with their general anticipations of future interest rates. Investors receive the same return, regardless of maturity held. Given these assumptions a governmental unit such as the Treasury or the Federal Reserve could not effect interest level changes by altering the composition of debt maturities. Unless the change in supply of a maturity changes the expectations, the government could finance its debt as cheaply with any maturity desired. DeLeeuew indicates:

> Debt management operations influence interest rates for a brief period, the average composition of the debt over longer periods does not have a perceptible influence.[18]

No interest cost advantages are possible through debt maturity decisions.

The corporate debt manager is faced with the same framework as the government under the expectations theory. If equal risk premiums are added to the credit risk free rate of interest of the United States government maturities, the corporate manager has no potential for interest cost

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[18]Frank DeLeeuew, "A Model of Financial Behavior," The Brookings Quarterly Economic Model of the United States, J. S. Duesenberry, G. Fromm, L. R. Klein and E. Kuh, eds. (1965), p. 503.

reduction in temporal placement of debt. The cost of two one-year loans is equal to the cost of one two-year loan. The logic is definitional, not one of economic behavior. The different maturities of debt may affect the financial risk of the firm, but is not considered within the context of expectations theory application. The expectations theory allows a financial manager to concentrate on matters other than debt maturity and debt timing since a gain is impossible. However, if unequal risk premiums are added to various maturities of debt, a possible cost advantage exists. In general, the pure expectations theory indicates government and corporations may not effect interest cost benefits via debt timing and yield curve analysis.

Error Learning Model

Meiselman devised a simple test for support of the expectations theory.[19] He assumes that market participants are able to derive forward rates from the existing term structure of interest rates equation (2). Long-term rates under the model are a function of future short-term rates and a change in these short-term estimates may affect the long-term rates. Next, Meiselman assumes that investors

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[19] David Meiselman, <u>The Term Structure of Interest Rates</u> (Prentice-Hall, 1962).

react positively to differences between present rates and forward rates corresponding to the present rates implied in the past.

> Thus, if actual rates are higher than had been anticipated, the market may systematically revise upward expectations of what short-term rates in the future are likely to be. Similarly, if actual rates are lower than had been anticipated, then the market may also systematically revise downward expectations of future short-term rates.[20]

Forward short-term rates change with regard to forecasting errors for the present short-term rate. This change may be shown notationally as:

$$_{t+n}r_{1,t} - {}_{t+n}r_{1,t-1} = f_n({}_tR_1 - {}_tr_{1,t-1}) \qquad (3)$$

where f_n is a function equal to the prediction error related to the difference in realized yield and forward yield for maturity n. The change may be represented as:

$$\Delta_{t+n}r_{1,t} = g_n(E_t) \qquad (4)$$

where g_n is again a function equal to the prediction error and E_t represents the difference of the realized yield and the forward yield. Assuming the relationship is linear, equation (4) may be solved by the regression:

$$\Delta_{t+n}r_{1,t} = a_n + b_nE_t \qquad (5)$$

This regression provides the basis for the test.

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[20] *Ibid.*, p. 20.

Meiselman found that changes in forward rates are highly correlated with the forecasting error. He used annual yield curve data on corporate bonds from 1901-54 developed by Durand. Forward rates behave as expected by the error-learning mechanism. Therefore, the expectations theory gains support. The existence of a near zero "a" term of the regression indicates no liquidity preference. The correlation coefficient of the regression varies inversely with the maturity of the dependent variable, the period in which forward interest rates are being forecast.[21] Investors are less likely to forecast effectively and seriously short-term rates far in the future. Although the error-learning model sparked new interest in the expectations theory and term structure research, several criticisms have been made.

Grant, using British data, found that the error-learning model does not provide good explanations of the yield changes.[22] However, Grant interpolated linearly between observed yields resulting in larger fluctuations than when yields are taken from a "best fit" yield curve as had

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[21] *Ibid.*, p. 22. The correlation coefficient, R, dropped from .952 to .590.

[22] J. A. G. Grant, "Meiselman on the Structure of Interest Rates: A British Test," *Economica* Vol. 31 (February, 1964), p. 61.

Meiselman. Buse suggested that the results obtained by Meiselman would be obtained by any set of smoothed yield curves and the test, therefore, had low discriminating power. Buse obtained the same results as Meiselman generated by both reversing the chronological order and random ordering of yield curves. The model does not discriminate between the behavior of investors acting on Meiselman postulates and alternative formulations. Thus, Buse reasoned:

> The Meiselman model is consistent with any set of smoothed yield curves in which the short rates show a greater variability.[23]

Malkiel and Kane provided support for the error-learning model for very near term (three month) forecasts based on questionnaires sent to financial institutions at different times.[24] This last test worked less well as the forward period increased. Modigliani and Sutch employed a technique similar to Meiselman's and were able to explain interest rate differentials between short and long rates for government securities.[25] The explanatory model of Modigliani and

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[23]A. Buse, "Interest Rates, The Meiselman Model and Random Numbers," Journal of Political Economy (February, 1967), p. 61.

[24]Edward J. Kane and Burton G. Malkiel, "The Term Structure of Interest Rates: An Analysis of a Survey of Interest-Rate Expectations," Review of Economics and Statistics (August, 1967), pp. 343-55.

[25]Franco Modigliani and Richard Sutch, "Debt Management and the Term Structure of Interest Rates: An Empirical Analysis," Journal of Political Economy (August, 1967 Supplement), pp. 569-89.

Sutch assumed two parts with regard to investors. First, future interest levels tend toward a "normal" level of rates based on past experience. Secondly, future interest rates move with regard to the most recent past (like Meiselman). By combining these two hypotheses the yield curve could be predicted. The results were offered as support of the expectations hypothesis. Criticism of the basis for the expectations theory and its assumptions are well phrased from this question raised by Weaver.

> One need only ask: Why it is not possible for changes in expectations about future short-term rates to have an influence directly upon the present supply and demand conditions which determine the current short rate to negate his [Meiselman's] analysis?[26]

The debate is by no means settled and controversy continues. Empirical tests and the logical framework for the support of those tests have not completely explained the expectations theory, nor the error-learning model variation.

Liquidity Preference Theory

Hicks argued that the expectations theory provides a good description of the term structure in a world of certainty but requires refinement for the real world

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[26]Alex R. H. Weaver, "The Uncertainty of the Expectations Theory of the Term Structure of Interest Rates," The Western Economic Journal, Vol. 4 (Spring, 1966), p. 133.

environment.[27] This followed the lead offered by the Keynesian theory of "normal backwardation" in the futures market. In particular, Hicks felt that a bond holder must be offered a risk premium for assuming the risk of greater price fluctuations for longer-term maturities. In a world of uncertainty shorter maturities are prefereable to longer maturities. Shorter maturities are more liquid and are able to be converted more quickly into cash, hence are more valuable. Longer maturities are subject to these risks over a longer period of time. Increased risk must be compensated by increased return; the essence of the liquidity preference theory.

The liquidation preference theorists do not disagree with the expectations theorists, but argue there is a natural increase in yield as maturity increases. Simple bond yield calculations show that for a given rise in the general interest rate level, long-term bond prices fall more than short-term prices. For example, observe the effect on bond prices for a 6 per cent coupon bond when the interest level changes from 6 per cent to 7 per cent. Market price drops to 99.05 for a one year to maturity note while the price drops to 89.32 for a twenty year to maturity

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[27] J. R. Hicks, Value and Capital (London, 1946), pp. 138-9.

bond. Clearly, the longer maturity leaves the investor open to a greater possible change in price. If increased variability in market price for a bond is risk to the investor, he should be compensated for this risk. A study of bonds from 1900-1957 indicates the mean return for bonds increases from 3.2 per cent for one year notes to 3.6 per cent for twenty year bonds; while the risk, as measured by the standard deviation, increases from 1.7 per cent to 3.6 per cent.[28] Longer maturity does leave the investor susceptible to a greater potential loss. These facts do lend support to the liquidity preference theory that risk increases as maturity increases.

A liquidity premium may be thought of as an amount that is added to the expected rate. Thus, the forward rate calculated is equal to the expected rate plus the anticipated liquidity premium. This may be defined notationally as:

$$(1 + {}_tR_2)^2 = (1 + {}_tR_1)(1 + {}_{t+1}r_1 + L_2) \tag{6}$$

where, $\frac{\partial L_2}{\partial t} > 0 \qquad \frac{\partial L_2{}^2}{\partial t^2} < 0$

The liquidity premium, L_t, is positive and increases with maturity but at a decreasing rate. Therefore, even if

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[28]William L. Wilbur, "A Theoretical and Empirical Investigation of Holding Period Yields on High Grade Corporate Bonds" (Ph.D. dissertation, University of North Carolina, 1967).

expectations assume no change in future interest rates, the yield curve slopes upward due to the existence of a liquidity premium. Malkiel has demonstrated logic for this effect through the mathematics of bond prices and interest rate movement.[29] First, for a given change in yield from the nominal yield, changes in bond prices are greater the longer the term to maturity. Second, the percentage price changes increase at a diminishing rate. The mathematics of bond prices also very neatly explain the "shoulder" observed in most yield curves because of the diminishing rate of price movement for increasing maturity. Conard has indicated the effect of the liquidity premium is most often felt by a maturity of three to five years.[30] The premium varies not only with maturity of an instrument but over the cyclical pattern of interest rates.

Two explanations exist for the cyclical movement of liquidity premiums. The first hypothesis indicates that the liquidity premium exists with respect to "normal" rates. One would expect liquidity premiums to be higher for long-term than short-term maturities when the interest level is

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[29]Burton G. Malkiel, The Term Structure of Interest Rates (Princeton University Press, 1966), pp. 50-9.

[30]Joseph W. Conard, The Behavior of Interest Rates (Columbia University Press, 1966), p. 80.

low. A low interest level subjects the long-term maturities to a greater potential of capital loss if yields increase. When the interest level is high the liquidity premium is low but always non-negative. The second hypothesis, the money-substitute hypothesis, operates inversely to the "normal" hypothesis. During a business expansion interest rates rise which makes money more expensive to hold. Money is exchanged for short-term securities which holds down short-term rates relative to long-term rates. The liquidity premium increases when the interest level rises and the opposite occurs when the interest level declines. Note this does not imply the spread between long and short securities increases as interest levels rise. The expectations theory indicates that if the interest level is high relative to normal rates, the yield curve will slope downwards. Therefore, the money-substitute hypothesis maintains that the downward slope of the yield curve is not as great as might be expected with high interest levels because of the increase of the liquidity premium.

Regardless of the hypothesis of the liquidity premium, investors are perceived to demand shorter maturities over longer maturities without a risk premium included. Borrowers, on the other side, desire to sell long-term debt to assure themselves of a constant source of funds. The desired supply of maturities does not equal the demand for maturities. Speculators are also considered to be risk-averters and must be paid a premium to accept longer maturities. The yield curve must possess a positive slope over time to equate the market investors and issuers. Before considering the empirical studies note Malkiel's comment:

> One must interpret the results of such studies [liquidity premium] very cautiously however. Since liquidity premiums can never be observed and only estimated, it is impossible to reach a completely definitive verdict regarding their behavior over time.[31]

The liquidity premium might exist implicitly in the minds of financiers but not explicitly. One may ask but not find an answer to the question: What is the liquidity premium for maturity n?

Kessel utilized a test similar to Meiselman and reasoned that forecasting errors did not invalidate the expectations theory.[32] Kessel felt anticipated and realized yields would only tend to be equal in a world of certainty. In the test Kessel found that the forward rates were consistently greater than realized rates and this positive

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[31]Burton G. Malkiel, "The Term Structure of Interest Rates: Theory Empirical Evidence, and Applications," (The McCaleb-Seiler Company, 1971). Footnote 28.

[32]Kessel, <u>op. cit</u>.

difference indicated the existence of a liquidity premium at the point in time the forward rate was calculated. Kessel also responded to Meiselman's charge that the existence of an "a" regression estimate from equation (5) close to zero invalidated the liquidity preference theory. Kessel found that the dependent variable, $\Delta_{t+n}r_{1,t}$, as defined would naturally find an "a" estimate close to zero because t-1's premium had been subtracted from t's premium. Cagan found that increasing the maturities of issues held for a set holding period led to increased returns.[33] Cagan reasoned that the returns were a result of the liquidity premium. Both Kessel and Cagan supported the money-substitute hypothesis of the liquidity premium; the liquidity premium varies directly with the interest level. Malkiel has offered a plausible explanation for the direct relationship between liquidity premiums and the interest level. Dealer risk aversion increases as the interest level increases, and widens their spread.[34] This increased dealer spread imparts a more positive bias to the slope of a yield

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[33]Phillip Cagan, "A Study of Liquidity Premiums on Federal and Municipal Government Securities," reprinted in Essays on Interest Rates, Vol. 1, NBER General Series #88 (1969).

[34]Malkiel, Term Structure of Interest Rates, p. 143.

curve when interest rates are high. Malkiel's recognition of transactions costs offers some practical explanation of Kessel's liquidity premium. Michaelsen in a test similar to Cagan's found that longer maturities led to both higher average holding period returns and standard deviation of those returns.[35] Theorists believe that the expectations theory and the liquidity preference theory are compatible and complementary. The liquidity preference theory allows the expectations theory to account for real world uncertainty.

The existence of liquidity premiums have implications for the management of debt. Financial risk and transactions cost aside, borrowers gain an interest cost reduction through continual finding by short-term debt. The borrower does not incur the liquidity premium and debt cost is reduced by that amount over time. The existence of the liquidity premium may enhance multi-stage financing as opposed to single-stage financing. If borrowers did issue only short-term debt issues, the liquidity premium would not exist since the demand by investors would equal the supply of borrowers. It is the imbalance of supply and

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[35]Jacob Michaelsen, "The Term Structure of Interest Rates and Holding Period Returns," Journal of Finance, Vol. 20 (September, 1965), pp. 444-63.

demand caused by risk aversion that leads toward the liquidity premium. The liquidity preference theory has not been subject to criticism to the extent of the pure expectations theory.

Institutional Theory

The institutional or hedging pressure theorists state that the interest rate differentials are a function of the relative demand and supply for given maturities. This theory holds that the debt market is segmented by investor and issuer preference for debt maturity. Commercial banks desire short-term maturities so as to be able to quickly liquidate debt for additional loans or reserve needs. Insurance companies are more interested in longer term debt because of their long-term liabilities. The theory states that investors are more interested in security of income over their holding period rather than potential capital gains. Institutions issue debt to parallel asset life. Implicit is the assumption that investors hold the debt to maturity. The institutionalists state that yield differentials are neither a function of expectations nor of liquidity preference but rather of supply and demand for a given maturity.

The institutional theory holds that the market for

maturities may be dis-continuous. Market participants are constrained by law and tradition in their choice of maturities. The rates of long-term debt do not affect the rates of short-term debt and vice-versa. The linking of yields of various maturities as implied by the mathematics of the expectations theory or the liquidity preference theory is not accepted by the institutionalists. Empirical evidence for this theory has been less substantive than the previous theories discussed.

Some market practitioners effectively argue for the institutional approach as reflected by their day to day working experience.

> Homer and Johannesen (members of a large Wall Street firm specializing in bonds) do not regard short and long-term bonds as two ends of the same moustache, but rather... as different from each other as stocks are from bonds, or more so.[36]

Since the theories proposed to explain the term structure are attempting to predict that structure, the above comments from those close to the market are particularly revealing. The practitioners do not agree with the academicians.

In addition, Culbertson felt that since holding period returns increased as the maturities of that holding

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[36]Malkiel, "The Term Structure," p. 14.

period increased, the only logical explanation could be institutional. Wallace found that forward rates are influenced by the supply of maturities of loans greater than one year.[37] The effects proposed by Wallace were small but statistically significant. Modigliani and Sutch attempted to test the supply effect on term structure of governmental rates and found that changes in the supplies of government debt had little effect on interest rates. Empirical support for the institutional theory has been limited.

Sufficiently refined data has not existed for proper determination of a debt maturity effect. Data does not exist that properly reflects private and local government debt. In addition, debt maturity and cost are intertwined. If rates are high borrowers may refrain from issues until a more favorable interest level exists. It is difficult to assume that debt composition is a truly exogenous variable.

Malkiel tested the assumption that market participants are rigidly constrained by maturity preference from information of The Treasury Survey of Ownership of Government Securities.[38] The Survey provides information on the

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[37]Neil Wallace, "Buse on Meiselman -- A Comment," Journal of Political Economy, Vol. 77 (July, 1969), pp. 524-7.

[38]Malkiel, The Term Structure of Interest Rates, pp. 144-180.

maturity composition of securities by different financial institutions. The maturity composition by financial institutions is quite variable over time. The strict assumption of an extreme institutional theory is doubtful. The support or non-support of the general institutional theory will become clearer with the introduction of more refined debt composition data.

Implications for the institutional theory are very clear for the governmental debt manager. Given that supply affects the interest level, the government could shape a yield curve to its preference by debt maturity decisions.[39] The corporate manager might attempt to issue that maturity of debt that would least affect the interest level for that maturity. Thus, if many firms were issuing twenty year debt, the manager might find a cost advantage in issuing other than twenty year maturities.

Summary

Empirical evidence tends to support the expectations and the liquidity preference theories but not the

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[39]See Modigliani and Sutch for a discussion of the Governments Operation Twist where an attempt is made to change the yield curve shape through debt maturity decisions. Franco Modigliani and Richard Sutch, "Innovations in Interest Rate Policy," American Economic Review: Papers and Proceedings (May, 1966), pp. 178-97.

institutional theory. As more debt data becomes available all three theories might be viewed as being complementary. Disagreement exists with regard to any one theory or their combinations for explanatory and predictive value in the term structure of interest rates.

The theorists do agree that interest cost reductions may be possible with proper timing of debt maturities. Baxter notes:

> Although commercial paper and funded liabilities are not considered as good substitutes, most issuers indicate that they try to time their long-term issues to get the most attractive rates possible. The expectation of rising interest rates will generally speed up the long-term financing decision and that of falling rates will lead to its postponement. Borrowers utilize short-term debt, both bank loans and commercial paper, to provide funds until the long-term flotation.[40]

Malkiel indicates:

> Thus, there are reasons to suppose the existence of a strong a priori case in favor of funding which would tend to bias the distribution of corporate debt toward longer maturities. We hasten to point out, that this case for funding in no way rules out the possibility of anticipatory or delayed funding.[41]

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[40] Nevins D. Baxter, The Commercial Paper Market (Bankers Publishing Company, 1966), p. 75.

[41] Malkiel, The Term Structure of Interest Rates, p. 168.

> Historically, the most opportune times for long-term financing have occurred when the yield curve was steeply upward sloping.[42]

Limited empirical evidence has supported the idea that the slope of the known term structure of interest rates conveys useful knowledge for the management of debt. For such a policy to be feasible, the yield curve must reflect or potentially reflect information not fully explained by the term structure theories. This study places additional information in front of the financial manager for the timing of maturities of a desired amount of debt. Chapter two shows the theoretical decision structure and the research technique for the testing of the hypothesis: information is contained in the slope of a yield curve for debt management.

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[42]Malkiel, "The Term Structure," p. 18.

CHAPTER II

A THEORETICAL DEBT DECISION STRUCTURE AND RESEARCH DESIGN

Introduction

This chapter presents the research design for a test of the stated hypothesis. The hypothesis is that a debt issuer may reduce interest costs by issuing combinations of debt maturity based on the slope of a given yield curve. While the research technique tests the proposition within a macroeconomic framework, the results may be applied within the microeconomic environment. Attention is focused on the maturity composition of a given amount of total debt to be issued. The best method of minimizing debt cost could be determined with perfect foresight of the future interest rate structure. Since this foresight does not exist, we must do with the existing term structure and any historical perspective available with regard to interest rates. This research attempts to provide one such perspective within an interest minimizing debt decision structure. First, however, a debt maturity model is formalized so that the decision framework may be more clearly stated. The decision model represents only one of many such models that could be presented. Models may include alternative variables with an objective function other than interest cost minimization. The model selected concentrates on the relationship between the yield curve slope and the maturity of debt selected.

A Theoretical Debt Decision Structure

In this section the debt decision is formalized in a theoretical model.[1] In the model debt interest costs are minimized subject to certain constraints concerning the maturity of debt and financial risk which the maturity represents. These constraints may be imposed by management or creditors. Flotation costs associated with a debt issue are assumed to be zero[2] and call provisions are assumed to be absent.[3] Debt needs are predicted on the basis of cash

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[1]For this section's background see Charles R. Carr and Charles W. Howe, Quantitative Decision Procedures in Management and Economics (McGraw-Hill Co., 1964). The theoretical framework centers on the corporation, but is equally applicable to the government.

[2]Robert H. Litzenburger and David P. Rutenberg, "Size and Timing of Corporate Bond Flotations", Journal of Financial and Quantitative Analysis (January, 1972), pp. 1343-60.

[3]See Martin H. Weingartner, "Optimal Timing of Bond Refunding", Management Science, Vol. 13, No. 7 (March, 1967), pp. 511-524. Oswald D. Bowlin, "The Refunding Decision: Another Special Case in Capital Budgeting," The Journal of Finance (March, 1966), pp. 55-68.

flow for the firm. These limitations are particular to this decision framework and allow full concentration on the hypothesis. The limitations may be removed only with resulting increase in decision structure complexity.

Debt alternatives (X_i) are composed of varying debt maturities (x_{ij}) at an interest cost (c_{ij}) for each maturity. The subscript i represents the year of debt issue ($i=1,\ldots,p$) while the subscript j represents the remaining term to maturity for that issue ($j=0,\ldots,n$). Once a debt instrument, x_{ij}, is issued it continues to carry a cost of c_{ij} to retirement. The debt decision model for both a single-stage ($i=1$) and a multi-stage ($i=1,\ldots,p$) are shown. The model is formulated in terms of a decision structure consisting of a state space, a feasible decision set, a transformation set and an objective function; and an associated search process involving these decision structure components.

Multi-Stage Decision

The components of the decision structure identify the relevant aspects of the decision problem confronting the firm. In a multi-stage setting the financial manager lives in any period, say period t, with the consequence of decisions made previously (period $i=1,\ldots,t-1$).

Discretionary financing in the following period is limited to total debt requirements less the debt maturities existing from past periods. The decision structure components are now examined with greater detail.

State Space

The state space Θ is a set of elements θ representing those relevant aspects which are non-controllable insofar as the firm's debt decision. The elements may represent such given parameters as the current yield curve facing the firm and total debt requirements of the firm. Examples of possible θ's for period i are:

Θ = $\{\theta = (\theta_1, \ldots, \theta_k, \ldots, \theta_m)\}$ where

θ_1 = A n component row vestor of interest rates. This vector is simply the yield curve facing the firm in period i for maturities one through n. This vector for period i=1 is simply the current yield curve facing the firm. The manner in which future interest rates are forecast provide the crux of the empirical test of this research. The expectations theory projects future interest rates on the basis of mathematical extrapolation while the liquidity premium theory includes a risk premium for increasing maturities. Obviously, the yields applicable to future maturities may greatly affect the optimal decision.

θ_2 = The debt requirement for year i based on cash flow of the firm.

θ_3 = A n component row vector of dollar debt maturity constraints. The constraints may be imposed by management or creditors. The dollar constraint might not exist for all maturities. Since maturity of debt represents risk to the firm, the relative debt constraints may be imposed to maintain a desired amount of financial risk. However, once these constraints are fixed they become non-controllable in the decision model.

Feasible Decision Space

The decision space P refers to the set of feasible and alternative debt maturity decisions. Elements of the decision space are denoted as X which represents a vector of feasible debt dollar amounts within alternative maturities. The decision space is normally constrained so that debt maturity decisions are in fact feasible. The feasibility of a decision vector X is comprised of real numbers and is made feasible by meeting the constraints g_a. For example, the dollar amount of debt maturities selected should equal the total debt requirement. Examples of possible constraints based upon the examples of the state space previously indicated are:

$$P = \{X : g_a(\theta, X) = 0,\ a=1,\ldots,q\}$$ where:

$$g_1(\theta, X) = \sum_{j=1}^{n} x_j - \theta_2 = 0.$$ Hence, constraint one, g_1, indicates that total debt requirements are met for year 1. Maturities may proceed to a n year maximum.

$g_2(\theta, X) = X - \theta_3 = 0$. Constraint two, g_2, indicates the minimum and/or the maximum dollar amount that may be obtained by the different maturities of debt.

State Transformation Function

The transformation function T refers to the change in the state space from period i to period i+1 as a result of a decision X being made in a period i. In effect, the transformation function indicates how the state space changes over time. The most significant consequence is that annual debt maturity decisions involving a maturity greater than one year affect the discretionary debt requirements for subsequent periods. Decision X made in period i affects the state space in period i+1.[4] Illustrations for the sample elements of the state space are:

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[4]Note that for any $i \neq 1$, $g_a(\theta_i, X_i) = g_a(T(\theta_{i-1}, X_{i-1}), X_i) = 0$ Which in more general form reduces to:
$g_a(T_{i-1}(T_{i-2}(\ldots T_2(T_1(X_1, \theta_1), X_2), \ldots, X_{i-1})X_i)$ which is equivalent to:
$g_a(T(\theta_1, X_1, X_2, \ldots, X_{i-1}), X_i) = 0$ where
$(\theta_1, X_1, X_2, \ldots, X_{i-1}) = \theta_i$ and where
$T = T_{i-1} \cdot T_{i-2} \cdots T_i$, a composite function of previous transformations each of which is based on a particular decision and state. Thus, state θ_i for decision i depends on all previous decisions $X_{i-1}, X_{i-2}, \ldots, X_1$ and the initial state θ_1. Certainty exists only if one possible initial state, say $\theta_1 = \bar{\theta}_1$ and if all transformation functions are deterministic. A non-deterministic T_1 might be:
$\theta_{i+1} = T_i(\theta_i, X_i, Z_i)$ where Z_i is a random variate denoting uncertainty during period i.

$T(\theta, X) = \theta'\varepsilon\Theta$ where:

θ_1' -- A new yield curve will face the firm in period i+1. The yield curve will depend on how future interest rates are forecast from period i. It is on this area that the empirical research is focused. In particular, the test attempts to note information contained by a yield curve slope. Statistically significant results of the regression model applied in this study adjust the forecasted rates based on information contained by the slope.

θ_2' -- The discretionary debt need for the firm in period i+1 is equal to the total debt requirement for period i+1 minus the debt provided in the period i of maturity greater than two. Thus, decision X of period i affects the state of nature of period i+1. This relationship establishes the key to the debt maturity decision and the empirical test. Once a longer-term debt maturity is selected the interest cost is "locked-in" for the maturity of that debt and the feasible debt decision space in the future is limited.

θ_3' -- The firm faces new debt constraints as imposed in period zero. However, many managements will provide similar constraints for all periods of consideration and this element will not change from period i to i+1. Decision X in period i may affect whether constraint θ_3' becomes binding or not in period i+1.

Search Problem

The search process attempts to locate the maturities of debt and the timing of those maturities over the entire period of consideration so that the maturities are feasible and their cost is minimized. The objective function π_i for the firm in period i yields the present value of interest costs for maturity decisions taken in that period. The value of π_i is defined:

$$\pi_i(\theta_i, X_i) = \sum_{j=1}^{n} \sum_{k=1}^{j} \frac{c_{ij}x_j}{(1+c_{ij})^k}$$

The firm attempts to minimize the present value sum of these functions with respect to the alternative and feasible debt decisions. $\pi(\theta_1, X)$ is minimized.

$$\min_{X \varepsilon p} \pi(\theta_1, X) = \sum_{i=1}^{p} \frac{\pi i(\theta_i, X_i)}{(1+c_{ij})^i}$$

where $X = (X_1, \ldots, X_i, \ldots, X_p)$.

An optimal solution may be derived by dynamic programming. Figure 1 schematically indicates the relationship of the decision structure components.

FIGURE 1

MULTI-STAGE DECISION STRUCTURE

$$\begin{tikzcd} & X_i \varepsilon P_i & \\ \cdots \arrow[r, "\theta_i \varepsilon \Theta"] & \boxed{\text{Period } i} \arrow[u] \arrow[d] \arrow[r, "T(\theta_i, X_i) = \theta_{i+1} \varepsilon \Theta"] & {} \\ & \pi_i(\theta_i, X_i) = \sum_{j=1}^{n} \sum_{k=1}^{j} \frac{c_{ij}x_j}{(1+c_{ij})^k} & \end{tikzcd}$$

The impact of the multi-stage presentation is that todays decisions have an effect on future decisions and the present cost of those decisions. If the pure expectations theory holds, any one debt decision is as costly as any other. The hypothesis of this paper is that the slope of a yield curve may reflect useful information for the debt manager. If this hypothesis is not rejected, then there is basis for including this information in the transformed interest vector θ_1' of the state θ. Market imperfections which are contained within the yield curve slope would be introduced for future interest rate forecasts. The search process then selects maturities from the transformed data input.

Single-Stage Decision

In a single-stage debt model debt funds are provided for one period subject to existing debt, maturity constraints and cost minimization. When i=1 the multi-stage problem reduces to a single stage problem. For this special case the search problem may be represented as follows:

$$\min_{X_1} \{\pi_i(\theta_1, X_1)\}$$

In words the search problem states that management elects those maturities of debt which meet individual maturity constraints and total debt requirements while min

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minimizing interest cost. This is equivalent to selecting that maturity from the lowest point of a yield curve until meeting a constraint and proceeding to the next lowest cost and associated maturity. It may be desirable that integer constraints be imposed on debt which might alter the final solution from a strict implementation of linear programming.

The implications of a one-stage model is that a firm reacts only to the financial environment for the present year. Decisions made in period i are tolerated in period i+1. If the pure expectations theory best explained interest term structure, such a decision policy would not be economically undesirable since no cost advantage is possible through altering debt maturity. Few decisions of a firm should be made with regard to only one period and the multi-stage decision model is more realistic. The empirical test may be introduced now that the theoretical framework for debt maturity decisions has been explained.

Research Design

A Regression Model

The analysis assumes the issuer knows the time span needed for debt funds and is financing this constant amount over n years. Other costs are assumed not to exist. Transactions costs are assumed to be minimal and so do not enter the decision structure. An issuer may provide for debt funds through a single, double or multi-stage issue. As an example, if an issuer needs funds for a twenty year period, n=20, it may issue a single-stage twenty year bond in year zero, or might issue a double-stage issue by a one year note, j=1, in year zero to be funded in year one by a nineteen year bond. Other multi-stage issues of greater than two stages are possible, of course, but are not considered in this study. The issuer is assumed to follow that course of action which promises to minimize interest cost within the financial constraints.

The known present cost of a single-stage issue may be equated to the unknown present cost of a two-stage issue. The uncertainty of the two-stage issue is due to the unknown future interest cost of the second stage. We may solve this equation for a second stage breakeven interest rate. If the future realized rate for the second stage is above the breakeven rate, the single-stage issue is advantageous; for a future realized rate below the breakeven point, the two-stage issue is advantageous. Mathematically, the breakeven problem is as shown below where coupon rates are equal to current interest rates (no bond premium or discount).

Interest Cost Single-Stage = Interest Cost Double-Stage

$$\sum_{i=1}^{n} \frac{{}_mR_n}{(1+{}_mR_n)^i} = \sum_{i=1}^{j} \frac{{}_mR_j}{(1+{}_mR_n)^i} + \sum_{i=j+1}^{n} \frac{{}_{m+j}r_{n-j}}{(1+{}_mR_n)^i} \qquad (7)$$

where:

${}_mR_n$ = the known current long-term interest rate in period m

${}_mR_j$ = the known current short-term interest rate in period m and $j<n$

${}_{m+j}r_{n-j}$ = the unknown future long-term interest rate which is the breakeven rate for period m+j of maturity n-j

Financing by either alternative is for a period of n years. Single-stage n is equal to two-stage j + (n-j). The break-even rate applies to period m+j since the first stage funding goes j periods from the initial point in time, m. The breakeven rate is solved by rearranging terms in equation (7).

$$\sum_{i=j+1}^{n} \frac{{}_{m+j}r_{n-j}}{(1+{}_mR_n)^i} = \sum_{i=1}^{n} \frac{{}_mR_n}{(1+{}_mR_n)^i} - \sum_{i=1}^{j} \frac{{}_mR_j}{(1+{}_mR_n)^i} \qquad (8)$$

The determination of the calculated breakeven rate may be shown by annuity form. The notation $\$1a_{\overline{n|}}i$ indicates the present value of a stream of equal \$1 payments for n years discounted at the rate of i.[5] Using this notation

[5]See James C. T. Mao, Quantitative Analysis of Financial Decisions (MacMillan Company, 1969), pp. 184-5.

the breakeven rate is shown by:

Single-Stage Double-Stage

$$ {}_0R_n \, a_{\overline{n}|\,{}_0R_n} + \frac{1}{(1+{}_0R_n)^n} = ({}_0R_j - {}_jr_{n-j})\, a_{\overline{j}|\,{}_0R_n} + {}_jr_{n-j} \, a_{\overline{n}|\,{}_0R_n} + \frac{1}{(1+{}_0R_n)^n} \qquad (9) $$

The end period principal payments cancel. The two-stage substracts the present value of ${}_jr_{n-j} \, a_{\overline{j}|\,{}_0R_n}$ so that the interest payments are discounted only once for the applicable period, j+1 to n. Rearranging terms with ${}_jr_{n-j}$ on the right-hand side and simplifying results in:

$$ {}_jr_{n-j} = \frac{{}_0R_j \, a_{\overline{j}|\,{}_0R_n} - {}_0R_n \, a_{\overline{n}|\,{}_0R_n}}{a_{\overline{j}|\,{}_0R_n} - a_{\overline{n}|\,{}_0R_n}} \qquad (10) $$

This is the basis for the actual calculation of the rate used in the computer programs. The usefulness of the calculated breakeven interest rate becomes apparent in the context of the empirical test.

The hypothesis is tested by statistical least squares linear regression. More specifically, the null hypothesis is that the regression line estimates are not significantly different from zero. The independent variable, Z, is the slope of the yield curve. The slope is operationalized as the difference in yields between maturity n and maturity j at time period m. $Z = ({}_mR_n - {}_mR_j)$. The dependent variable, Y, is the difference between the realized rate and the calculated breakeven rate.

$Y = ({}_{m+j}R_{n-j} - {}_{m+j}r_{n-j})$. The regression may be stated in the familiar form of $Y = B_0 + B_1Z$. That is:

$$({}_{m+j}R_{n-j} - {}_{m+j}r_{n-j}) = B_0 + B_1({}_{m}R_{n} - {}_{m}R_{j}) \tag{11}$$

This basic regression is run on quarterly interest data for different combinations of holding periods n, and two-stage alternatives with variations of period j.

<u>The Regression Model and The Theoretical Debt Decision Structure</u>

This section explicitly shows the relationship between the regression model as specified and the theoretical debt decision structure. A decision maker may observe the current yield curve θ_1 from a market source. On the basis of θ_1, forward rates may be calculated using equation (2). The resulting yield curves computed by this equation contain observations for one year's less maturity as the time period for which the yield curve being estimated increases by one year. Hence, if θ_1 contains estimates for maturities of n years, the yield curve calculated for year n-1 may contain a forward rate for maturity equal to only one year. A calculated forward rate is equal to the breakeven rate specified within the dependent variable of the regression model. The regression compares the slope of the yield curve with the resulting relationship between the realized market yield from yield curve θ_1^* and the calculated break-even yield from yield curve θ_1' at time m+j of maturity n-j. Figure 2 schematically identifies the regression variables.

A regression may then be computed on the basis of succeeding i periods and recomputed for alternative single-stage and two-stage issues. Figure 3 schematically indicates the relevant aspects of the regression estimates. The diagonal lined portion of Figure 3 represents that area where given the yield curve slope and regression as specified, a single-stage issue would be superior. The non-lined area indicates that area where a two-stage issue is superior. The estimated position within the regression area depends upon both the constant B_0 and the slope B_1 of the regression line.

If the regression results are significant, evidence exists for correcting observations within yield curve θ_1' of the state space for the decision structure. In particular, the forward rate ${}_{m+j}r_{n-j}$ is corrected by adding the amount of $B_0 + B_1({}_mR_n - {}_mR_j)$. Therefore, the regression model specifies exactly how the forward rates should be changed on the basis of the existing yield curve slope. Non-significant regression results would indicate there exists no statistical basis for altering the forward rate from yield

FIGURE 2

OVERLAY OF REGRESSION VARIABLES

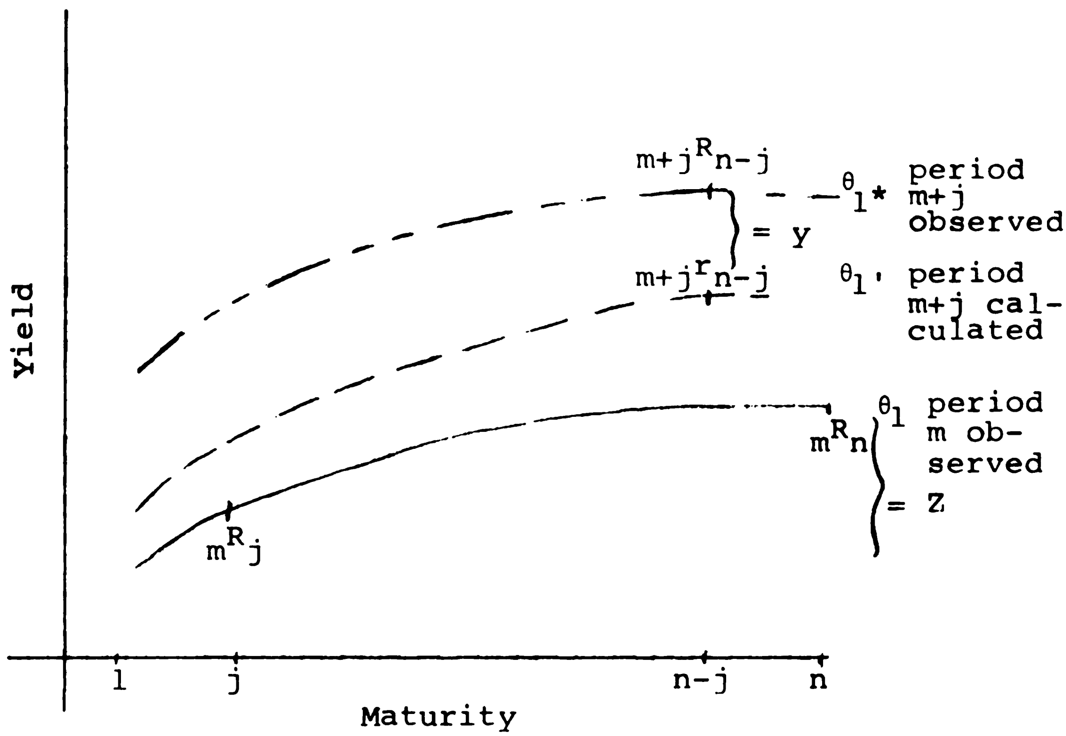
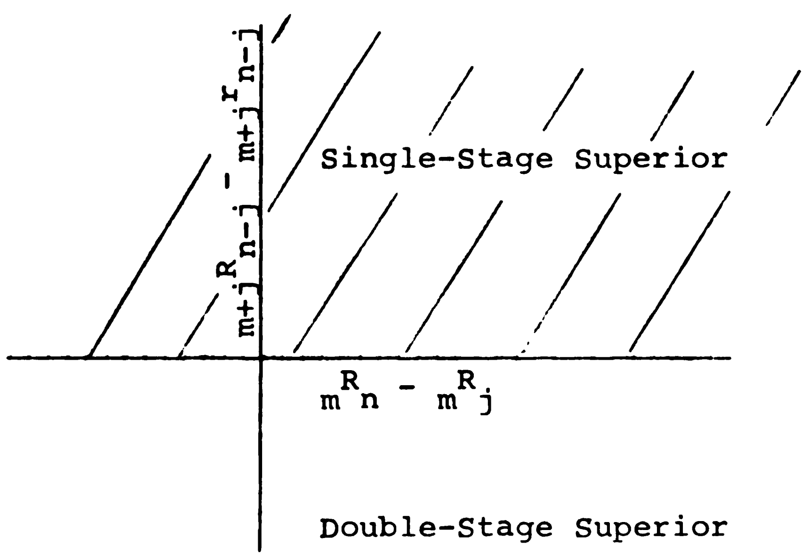


FIGURE 3

GRAPHICAL REGRESSION RESULTS AND FINANCING STAGE SUPERIORITY



curve slope knowledge. Results of the regression model directly affect the state space of the multi-state decision structure.

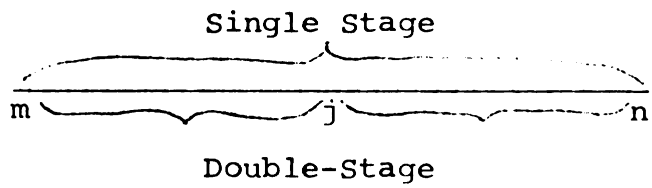
It may now be noted why the regression model indicated is used as opposed to an alternative formulation; say, ${}_{m+j}R_{n-j} = A_0 + A_1({}_mR_n - {}_mR_j)$. This later regression would provide a very easy and quick input for the θ_1' vector. In addition, the slope A_1 of the regression line would be expected to be significantly positive since the slope of the current yield curve θ_1 indicates the forward interest rate via expectations. However, the regression model used shows the deviation of observed rates from the breakeven rate that is based on the current yield curve. The significance of this regression model is not so obvious and provides a more reliable test for information contained by the yield curve slope. The element of expectations is removed so that the effect, if any, of the slope is directly observable.

A Rationale for the Model

Equation (8) discounted all dollar flows by the yield of the longest period under consideration, ${}_mR_n$. The yield of ${}_mR_n$ may be considered to be an opportunity cost for the issue of debt over period n. Schematically, the maturity decision is viewed as shown in Figure 4 where debt

FIGURE 4

MATURITY DECISION OF DEBT STAGES



funds are needed from period m to n.

The yield on ${}_mR_n$ provides a basis for comparison with alternative financing plans. The yield of ${}_mR_n$ is known at point m for the entire period of time from m to n. The rate for the two-stage process is not known. We reason the yield on the single-stage process is an opportunity cost for the period m to n and constitutes a proper discount rate for either financing plan. Support for the chosen rate is given by the following argument.

Equation (7) equated the present value of the single-stage financing plan to the two stage alternative. The following analysis assumes the issue of one bond with a par equal to $1,000. First the flows are determined for a single-stage debt issue. Period m, the initial period, is equal to zero. The borrower receives $1,000 in period zero for the payment of a stated dollar interest of ${}_0R_n$ ($1,000) for n years plus the repayment of $1,000 in period n.

Period	0	1	2	n	n
Cash Flow	\$1,000 =	\$1,000($_0R_n$) +	\$1,000($_0R_n$) +...	\$1,000($_0R_n$) +	\$1,000

factoring out \$1,000

$$1 = {}_0R_n + {}_0R_n + \ldots + {}_0R_n + 1$$

Combining terms the internal rate of return, k, may be calculated.

$$1 = \sum_{i=1}^{n} \frac{{}_0R_n}{(1+k)^i} + \frac{1}{(1+k)^n} \qquad (12)$$

The discount rate, k, is equal to the known interest cost over the period n. $k = {}_0R_n$.[6] Thus, the discounted present cost of a single-stage issue may be shown as:

$$1 = \sum_{i=1}^{n} \frac{{}_0R_n}{(1+{}_0R_n)^i} + \frac{1}{(1+{}_0R_n)^n} \qquad (13)$$

Similarly, the flows for a two-stage issue are shown.

Period	0	1	j	j+1	n
Cash Flow	\$1,000 =	\$1,000($_0R_j$) +...	\$1,000($_0R_j$) +	\$1,000($_jr_{n-j}$) +...	\$1,000

---

[6] For n=1, equation (12) is as follows:

$$1 = \frac{{}_0R_n}{(1+k)^1} + \frac{1}{(1+k)^1}$$

Multiplying both sides of the equation by (1+k) and combining terms results in $k = {}_0R_n$.

For n=2, equation (12) may be simplified to: $k^2 + 2k - {}_0R_nk - 2{}_0R_n = 0$ where $k = {}_0R_n$ is the only real root.

Therefore, we induce that $k = {}_0R_n$ will remain a real root as n increases beyond the special cases shown.

Again, factoring out \$1,000,

$$1 = {}_0R_j + \ldots + {}_0R_j + {}_jr_{n-j} + \ldots + {}_jr_{n-j} + 1$$

The flows may be discounted at a rate of k.

$$1 = \sum_{i=1}^{j} \frac{{}_0R_j}{(1+k)^i} + \sum_{i=j+1}^{n} \frac{{}_jr_{n-j}}{(1+k)^i} + \frac{1}{(1+k)^n} \qquad (14)$$

The original formulation (7) equated the single-stage cost to the two-stage cost. This point of indifference is found where $k = {}_0R_n$. We discount the flows at the rate of ${}_0R_n$ and solve for ${}_jr_{n-j}$. This, of course, was the original objective. The calculated breakeven rate may then be compared with the realized yield. It is well to note that even with the arguments cited for the rate, the choice does remain somewhat arbitrary. The use of other discount rates may be justified. The single-stage yield utilized lends itself to easier calculation of the breakeven rate and is able to be supported; hence its use.[7]

All n and j period combinations are multiples of twelve months. Diller introduced evidence that a repetitive seasonal variation in interest rates existed between 1955

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[7]Utility preference functions for debt maturity will be introduced in Chapter four whereby the discount rates for the two stages may not be equal.

and 1960 and again emerged after 1965.[8] Diller found short-term rates declined from a high in January through Spring to a trough in June, sharply increasing to September with a gradual rise to December. The seasonal variation declined as term to maturity increased but long-term rates went from a low of January through March rising to a plateau from June to October before subsequently declining. Therefore, any possible seasonal bias is eliminated from the study by the exclusion of periods not equal to multiples of twelve months.

Statistical Assumptions of the Model

Since the ordinary least squares regression technique is the statistical tool its assumptions are elaborated.[9] Violation of these assumptions do not, in all cases, invalidate results, but do reduce design efficiency. In particular we are most interested in the effect on the best linear unbiased estimates (BLUE) if the basic assumptions are violated.

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[8]Stanley Diller, "The Seasonal Variation in Interest Rates", reprinted in Essays on Interest Rates, Vol. II, NBER General Series #93 (1971), pp. 35-133.

[9]For a fuller treatment of the assumptions for least squares regression technique see Jan Kmenta, Elements of Econometrics (MacMillan Company, 1971), Chapter VIII.

1. Normalty. If normality is dropped the least squares regression is still BLUE although the regression is not the most efficient.

2. Zero mean of regression disturbance. In this case the estimate of B_1 is unaffected but B_0 becomes B_0^*. The relationship between the dependent and independent variables have been misspecified.

3. Heteroscedasticity. Residuals, e_1, may be calculated from a regression by subtracting a calculated dependent variable from a known dependent variable. $Y_i - \hat{Y}_i = e_i$. Least squares assumes $E(e_i^2) = \sigma^2$ for all i: the variance of disturbances is constant for all observations. Kmenta notes:

> This assumption may not be too troublesome for models involving observations over time.[10]

The regression for this analysis is over time.

4. Autoregression disturbance. Absence of serial correlation, autoregressive disturbance, implies that a disturbance at one point in time is not correlated with any other disturbance. $Cov(e_i, e_j) = 0$ for $i \neq j$. Kmenta notes the following:

> [autoregressive disturbance]...more frequently violated in case of relations estimated from time series than cross sectional data.[11]

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[10] *Ibid.*, p. 249. [11] *Ibid.*, p. 269.

> Thus, we have to conclude that the least squares estimates are not BLUE when disturbances are autoregressive. This implies that the least squares estimators are not efficient estimators.[12]
>
> To sum up, we have established that when the disturbances are autoregressive, the least squares estimates of the regression coefficients are unbiased and consistent, but they are not efficient or asymtotically efficient. Thus, if we use the least squares formulas when the disturbances are autoregressive, the resulting estimators will still have some desirable properties. However, if we want to use these estimators for the purpose of testing hypotheses or constructing confidence intervals we require unbiasedness not only of estimators themselves, but also of their estimated variances...when the disturbances are autoregressive, the conventional formulas for carrying out tests of significance lead to incorrect statements.[13]

The presence of autoregressive disturbance is extremely important for regressions over time. The Durbin-Watson d statistic tests for its presence.[14] The statistic is defined as:

$$d = \sum_{t=2}^{n} \frac{(e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2}$$

The d statistic equals the sum of the squared differences between residuals divided by the sum of the squared

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[12] Ibid., p. 275. [13] Ibid., pp. 278-9.

[14] J. Durbin and G. S. Watson, "Testing for Serial Correlation in Least Squares Regression II", Biometrica, Vol. 38 (June, 1951), pp. 159-178.

residuals. When the residuals exhibit a random distribution the d statistic is large. When d is large the hypothesis of autocorrelation may be rejected. Exact significance levels for d are not available. Bounds have been calculated for different numbers of observations and explanatory variables. If d is greater than the upper bound, d_u, the hypothesis of autocorrelated residuals is rejected. If d is less than the lower bound, d_l, the hypothesis of random residuals is rejected. The test is inconclusive for values between the two bounds. The Durbin-Watson d is calculated for each regression because of its importance for time series observations.

5. Stochastic explanatory model. The introduction of a stochastic explanatory variable may cause necessary complications. This problem does not exist since the variables in this test are deterministic.

Test Period and Financing Plans

When issuing debt, managers are most interested in the market rates for original issues. Unfortunately, sufficient information is not available on new issue market yields for a range of maturities. There is information on United States government issues and selected corporate issues. Much of the yield data is derived from the secondary issue market. We are forced to use this market. The yield information is obtained from Salomon Brothers' An Analytical Record of Yields and Yield Spreads. Salomon Brothers is a very large private dealer for both governmental and corporate debt issues. Both treasury bills and commercial paper are sold on a discount basis but are reported on a bond yield equivalent so that no further adjustments of basic data are necessary for this research. The Record provides historical information on 108 different yield series.

The time period 1952-71 provides the time span for study. Figure 5 and Figure 6 show the quarterly yields for selected governmental and corporate securities for this period. Both the initial and ending periods represent postwar situations for the United States economy. Four business contractions are contained within the time sample: 1953-54, 1957-58, 1960-61, 1970. Rising, humped, flat and declining yield curves exist for the period. It is essential that the different shapes be represented so that an effective test of various slopes may be made. The interest rates are studied in quarterly intervals: January, April, July and October. The periods and stage models tested are shown in Table 1.

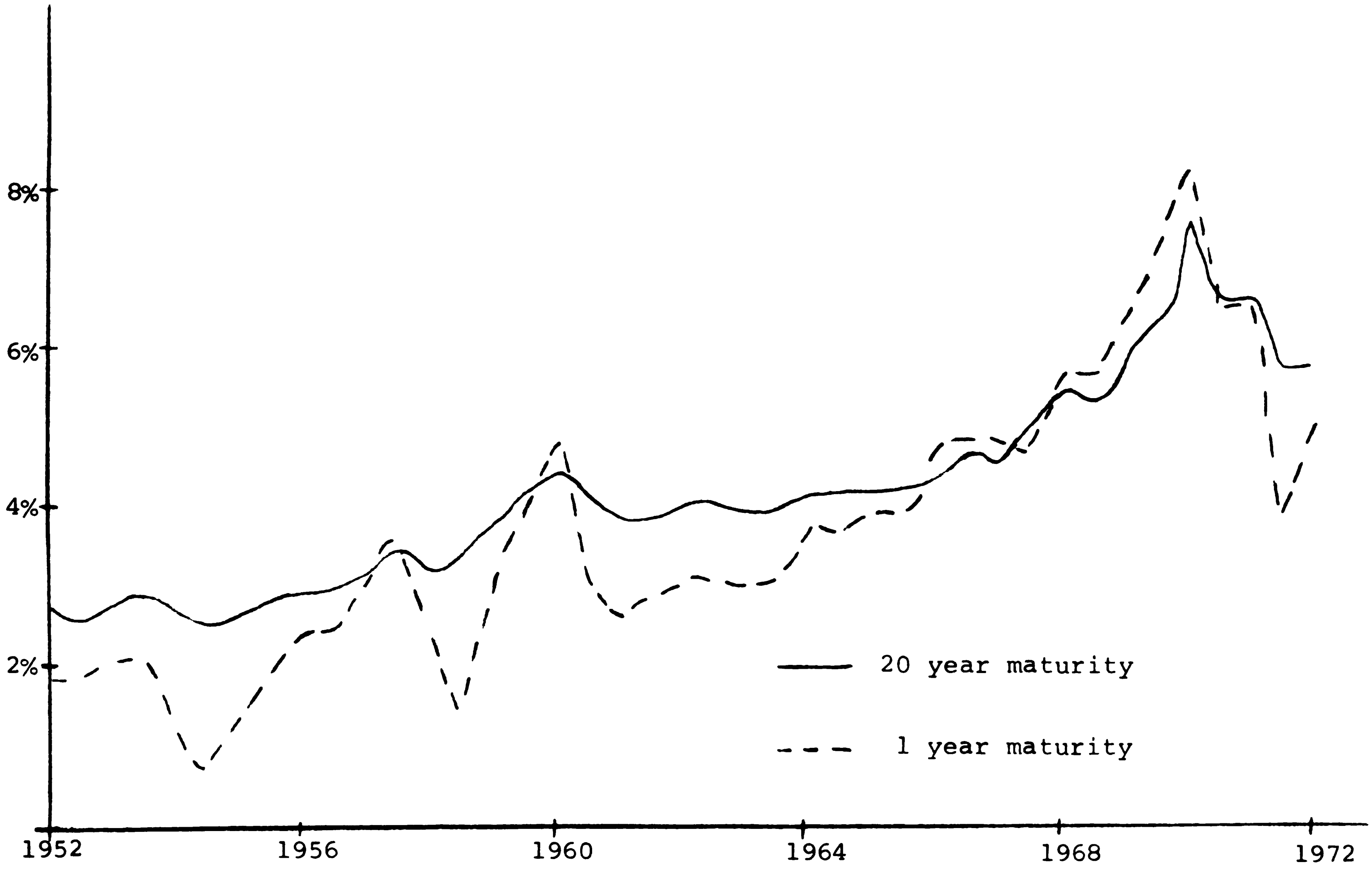

FIGURE 5

SELECTED GOVERNMENT YIELDS: 1952-71

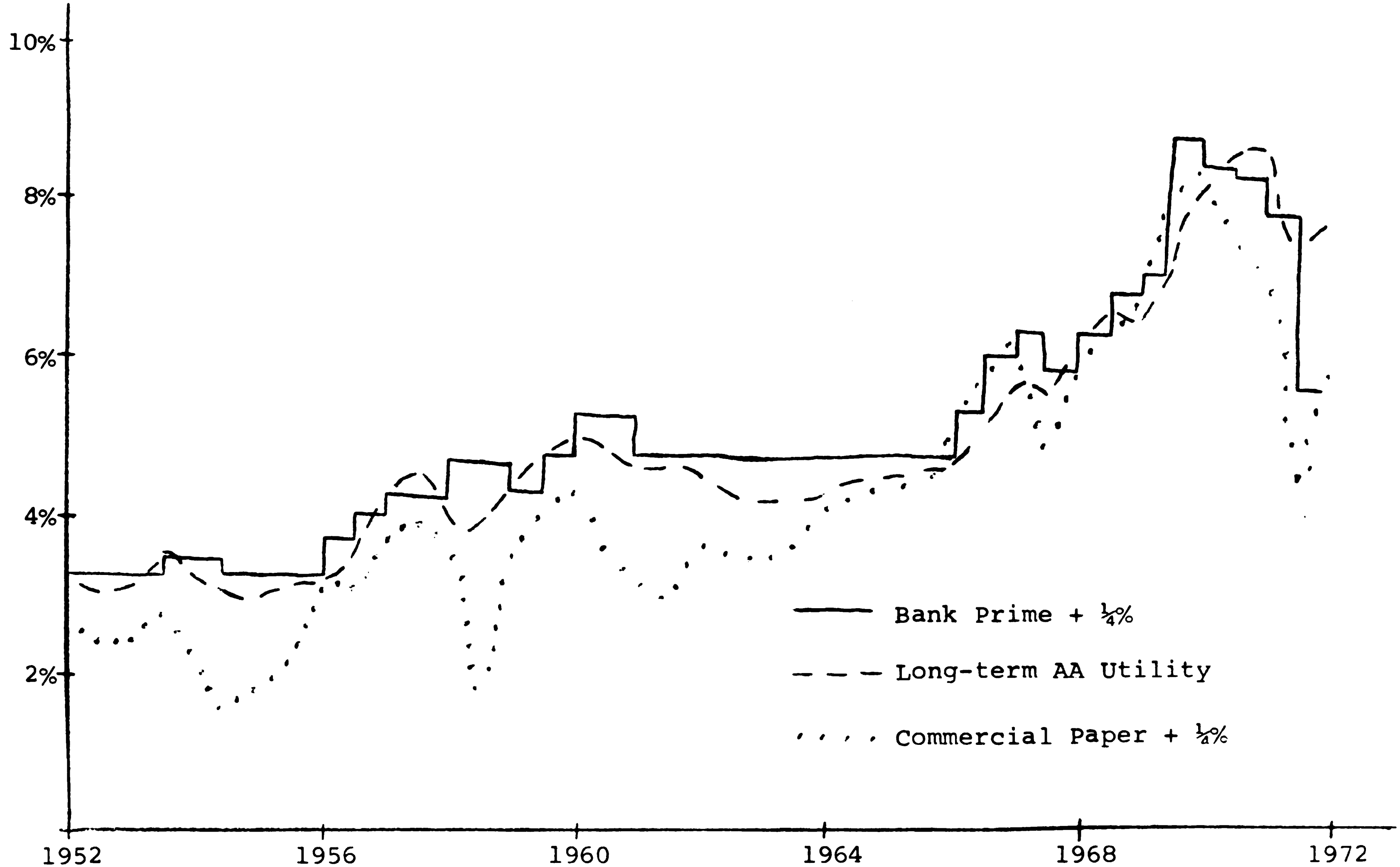

FIGURE 6

SELECTED CORPORATE YIELDS: 1952-71

TABLE 1

FINANCING PLANS TESTED

Time Period Debt Covered	Single-Stage	Two-Stage Alternatives 1st stage/ 2nd stage
Government		
2 years	${}_0R_2$	${}_0R_1/{}_1r_1$
5 years	${}_0R_5$	${}_0R_1/{}_1r_4$; ${}_0R_2/{}_2r_3$; ${}_0R_3/{}_3r_2$; ${}_0R_4/{}_4r_1$
10 years	${}_0R_{10}$	${}_0R_1/{}_1r_9$; ${}_0R_2/{}_2r_8$; ${}_0R_5/{}_5r_5$
20 years	${}_0R_{20}$	${}_0R_1/{}_1r_{19}$; ${}_0R_2/{}_2r_{18}$; ${}_0R_5/{}_5r_{15}$; ${}_0R_{10}/{}_{10}r_{10}$
Corporate		
20 years	${}_0R_{20}$	${}_0R_1/{}_1r_{19}$

where ${}_0R_{20}$ and ${}_0R_{19}$ are equal to the long-term AA utility bond yield and ${}_0R_1$ is equal to the commercial paper yield plus ¼ per cent.

20 years	${}_0R_{20}$	${}_0R_1/{}_1r_{19}$

where ${}_0R_{20}$ and ${}_0R_{19}$ are equal to the long-term AA utility bond yield and ${}_0R_1$ is equal to the bank prime rate plus ¼ per cent.

Interpretation of the Regression Model

We are attempting to see if the slope of the yield curve is useful in explaining the difference between a realized rate and a calculated breakeven rate. The significance of the sign of the dependent variable was previously noted for use in debt maturity decisions. The regression results indicate two items for the analysis. First, the slope of the regression line, B_1, indicates if the slope of the yield curve conveys useful information with regard to possible debt cost reduction. Second, the regression line, $B_0 + B_1 Z$, should be able to offer the debt manager more knowledge about specific debt issue's alternatives. The regression results may be used within the multi-period decision model framework postulated. Statistically significant findings could be incorporated into the interest vector of the state space. Specifically, the interest rates within the θ_1' vector would be adjusted by the addition of the regression constant and the regression slope estimate times the yield curve slope; $\theta_{1,n-j}' + (B_0 + B_1({}_mR_n - {}_mR_j))$. The process of dynamic programming would then elect the optimal maturities based on the transformed state. For those decisions not made within the rigorous decision model, useful information is available. The regression estimates, based on the period tested, will indicate whether information is contained by the yield curve slope. For example, a significantly negative B_1 estimate might cause a decision maker to question issuing a single-stage, long-term issue when the yield curve is sharply upward sloping. Of course, the final decision rests with the debt manager. The regression will merely offer additional information so that the manager may make that decision with more confidence.

Assuming linearity exists, the most important information is gathered from the sign of the B_1 regression coefficient. A B_1 coefficient of zero indicates that the slope of the yield curve is of no importance in explaining differences in the dependent variable. A positive B_1 coefficient indicates that as the yield curve slope increases, the realized minus the breakeven rate increases positively. Thus, a positive B_1 supports Malkiel's previously quoted statement that an issuer should increase long-term debt financing in periods of a sharply rising yield curve. Single-stage financing is advantageous with a sharply upward sloping yield curve when the realized rate becomes greater than a breakeven rate as the yield curve slope increases. A negative B_1 indicates the dependent variable increases negatively as the independent variable increases. With increasing slope of the yield curve, an issuer would tend to gain by financing in a two-stage process, financial risk and transaction costs aside. The statistical significance of a B_1 different from zero is tested by the "F" statistic.

$$H_0^1 : B_1 = 0$$

$$H_A^1 : B_1 \neq 0$$

The secular rise in the interest level for the time period may possibly bias results toward a positive B_1. However, the bias should be uniform with regard to the independent variable and therefore, affect the regression intercept, B_0.

On average the realized rate should equal the calculated breakeven rate, liquidity premium aside. The breakeven rate is dependent on the known yields of period n and period j which may be found from a yield curve of period m. As the n period increases the liquidity premium increases and causes the yield of ${}_mR_n$ to increase. By observation of equation (9), the breakeven rate increases as the yield of ${}_mR_n$ increases. However, the realized rate of ${}_{m+j}R_{n-j}$ has a shorter term to maturity than n and will include a smaller liquidity premium. Due to the above process B_0 is expected to be negative. The B_0 should become more negative as the period of refinancing, n-j, becomes smaller and approaches the smallest period considered, one year. The general rise in interest levels over the period of consideration may dominate the liquidity premium effect and show a less negative (more positive) B_0 than originally expected. The significance of the regression intercept is tested by the "F" statistic.

$$H_0^2 : B_0 = 0$$

$$H_A^2 : B_0 \neq 0$$

If the B_1 regression estimator points toward a financing plan based on the slope of the yield curve, an issuer would benefit by being able to predict the change in future yield curve slopes.

As noted in Chapter one, Macaulay found that time money rates anticipated a seasonal rise in call money rates.[15] Peaks and troughs in long-term rates should exist before those in short-term rates. Long-term rates are an average of future short-term rates. If the market can predict short-term turning points, long-term rates should anticipate these short-term movements. However, Kessel found that when the liquidity premium is considered, the market is unable to predict turning points.[16] Interest rates reach their peaks and troughs at the same point in time regardless of maturity. The prediction of interest rate turning points appear to be as difficult as the prediction of the interest rate level. Cagan noted that

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[15]Macaulay, op. cit., p. 36.

[16]Kessel, op. cit., p. 92.

turning points are hard to pinpoint--even with hindsight.[17] In addition, Cagan found that turning points of yields for different maturities have clustered closer to each other as time passes. Thus, a lead-lag relationship among rates is decreasing, if not already non-existent, and limits the ability to predict future yield curve slopes. All is not lost. The analysis is designed to assist the manager in debt decisions at a point in time and, as such, makes use of existing information. This analysis finds a historical perspective that may assist in proper debt maturity placement. Chapter three presents the results of the empirical test for governmental securities.

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[17]Phillip Cagan, "Changes in the Cyclical Behavior of Interest Rates", reprinted in Essays on Interest Rates, Vol. II, NBER General Series #93 (1971), pp. 3-34.

CHAPTER III

TEST OF HYPOTHESIS WITH YIELDS ON GOVERNMENT SECURITIES

Introduction

The Treasury is responsible for United States debt management and would be interested in potential for interest cost minimization through debt maturity placement.

> Public debt management takes as "given" the size of the debt and the general conditions prevailing in the money market. The function of public debt management is to establish the terms on which new issues are sold, and maturing public issues are refinanced. Public debt management means, then, making decisions concerning the types of public debt offered, the proportionate amounts of different debt forms to be used, the pattern of debt maturities, the pattern of debt ownership and determination of all other general characteristics of the public debt.[1]

Different goals, sometimes of a conflicting nature, have been ascribed to Treasury operations.[2] First, the Treasury should fund debt to longer-term maturities

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[1]Ansel M. Sharp and Bernard F. Sliger, Public Finance (The Dorsey Press, 1964), pp. 177-8.

[2]The Treasury goals mentioned are summarized from James M. Buchanan, The Public Finances (Richard D. Irwin, 1970), pp. 331-2.

whenever possible such that the mere physical burden of continually placing debt may be diminished. The maturity distribution of the debt has shortened almost steadily since the end of the Second World War.[3] In 1946 56 per cent of the government debt was of a maturity greater than five years while in 1971 about 18 per cent was greater than five years.[4] Thus, the Treasury must now go to the market more often to refinance retiring debt. Second, the Treasury should attempt to minimize the interest costs of governmental debt. While debt may be structured so as to minimize present interest costs, the Treasury must pay the "going rate" for its borrowing.[5] Third, the timing and maturities of debt should accommodate the needs of the various classes of investors. The third goal may be considered a subgoal of interest cost minimization since accommodation allows lower interest costs than would otherwise be available. This goal reflects a possible explanation for the shortening composition of public debt. Corporate Treasurers have become more sophisticated in short-term cash management and

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[3]Tilford C. Gaines, Techniques of Treasury Debt Management (The Free Press of Glencoe, 1962), p. 266.

[4]Board of Governors of The Federal Reserve System, Historical Chart Book 1971 (New York, 1972), pp. 40-1.

[5]Gaines, op. cit., p. 259.

have demanded safe, short-term investments. Fourth, and perhaps most important, the Treasury should secure an effective coordination between debt management and fiscal policies and the more general monetary policies of the Federal Reserve. Sharp and Sliger note the following:

> Public debt policy decisions, such as changing the pattern of maturities and ownership, determining the rate of interest,...may have economic effects which offset or foster the policy pursued by fiscal and monetary policies.[6]

Not all theorists feel the above mentioned policies should be followed by the Treasury with regard to public debt management. More specifically, the goal of interest cost minimization is dependent upon the type of Treasury policy followed: countercyclical, pro-cyclical or neutral.[7]

Simons and the Committee for Economic Development (CED) both indicated that public debt should be used as a countercyclical monetary device. Simons took as given the structure of debt but altered the absolute size of the debt. Consols were to be issued in times of inflation (high interest level) and to be purchased in times of deflation

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[6]Sharp and Sliger, op. cit., pp. 178-9.

[7]The alternative Treasury theories of debt management are summarized from William E. Laird, "The Changing Views of Debt Management," Quarterly Journal of Economics and Business, Vol. 3 (Autumn, 1963), pp. 7-17.

and economic stagnation (low interest level). These operations in conjunction with the Federal Reserve would tend to reduce the liquidity and money supply in inflationary periods and increase them otherwise. Meanwhile, the CED took as given the size of the debt, but altered the composition of that debt. The debt was to be lengthened during periods of high interest and shortened in periods of low interest. Either of the countercyclical approaches has the impact of increasing debt interest costs since long-term issues are increased when the interest level is highest.

The pro-cyclical approach has been the policy most often followed or at least mentioned as a normative objective by the Treasury. The debt should be "tailored to the market" and issued so as to minimize interest costs. Debt maturity would be lengthened during recessions and low interest rates and shortened during inflationary periods and high interest rates.

The third approach has been one of neutrality for debt management. Friedman and Gaines both argue for a dependable system of financing whereby debt operations would be "regular in timing, reasonably stable in amount and predictable in form."[8] Debt management would not be used for

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[8] Milton Friedman, A Program for Monetary Stability (Fordham University Press, 1959), p. 65.

economic stabilization. Interest costs are neither an objective nor a constraint for the neutral approach.

This brief background of Treasury debt management provides a framework from which the results of this research may be placed. Interest cost minimization is the goal of this study. However, this should not be construed as a bias to a pro-cyclical Treasury approach. Rather, the study should be viewed in the perspective of the potential for interest cost minimization with regard to knowledge of a current yield curve slope. These results may be useful to the Treasury if interest cost reduction continues to remain one of the several competing goals stated. Regardless of Treasury objectives advocated, the study will further illuminate term structure theory and is useful within that construct alone.

Ordinary Least Squares Empirical Results

The following sections present and interpret the regression results based on governmental securities. The ordinary least squares regression shows high positive autocorrelation as judged by the Durbin-Watson d statistic. The time series is positively autocorrelated since the d statistic is less than the lower bound allowed. Kmenta comments:

> However, if the test indicates autoregression then we have some reason to be concerned. One response is to re-estimate the equation, using one of the estimation methods designed for this situation (e.g. maximum likelihood or the two-stage procedure). Alternatively, we may take a second look at the specification of the regression model, since the autoregression of the disturbance may simply reflect the presence of some unexplained systematic influence on the dependent variable... Finally, if the result of the test is inconclusive we may or may not respond.[9]

Significant regression results could be used to alter the state space of the multi-period decision model. In particular, the forward yield curve θ_1, would be modified as indicated in Chapter two. Optimal debt maturity decisions might be changed as a result of this state transformation. Changes in debt maturity decisions would identify potentially cheaper debt financing alternatives. In addition, the regression model indicates whether the yield curve slope as defined predicts a realized yield minus calculated breakeven yield relationship. We identify cheaper debt financing plans with yield curve slopes for various periods and plans tested. It is important to remember that this identification assumes a position within the multi-period decision framework.

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[9]Kmenta, op. cit., p. 296-7.

1952-71

Positive autoregression bias the variances of the regression estimators which invalidates testing of significance for those estimators. However, the B_0 and the B_1 estimates are non-biased and present useful information. Table 2 shows the results of the regression for governmental securities from 1952-71. To assist the interpretation of results the financing plan of n=2 and j=1 is used

TABLE 2

RESULTS OF LEAST SQUARES REGRESSION FOR FINANCING PLANS OF QUARTERLY YIELDS FOR GOVERNMENT SECURITIES: 1952-71

Financing n yrs	Plan j yrs	Observations	R	B_0	B_1	Durbin-Watson d
2	1	76	-.146	.0005	-1.3219	.4080
5	1	76	-.058	.0011	-0.1563	.4535
5	2	72	-.054	.0026	-0.2271	.4386
5	3	68	-.139	.0053	-1.0705	.4369
5	4	64	-.217	.0069	-3.8596	.3664
10	1	76	-.084	.0017	-0.0972	.4707
10	2	72	-.194	.0045	-0.3086	.4646
10	5	60	-.005	.0098	-0.0251	.2647
20	1	76	-.072	.0017	-0.0487	.4960
20	2	72	-.248	.0044	-0.2580	.4634
20	5	60	-.088	.0101	-0.2292	.1907
20	10	40	-.489	.0194	-2.7650	.4999

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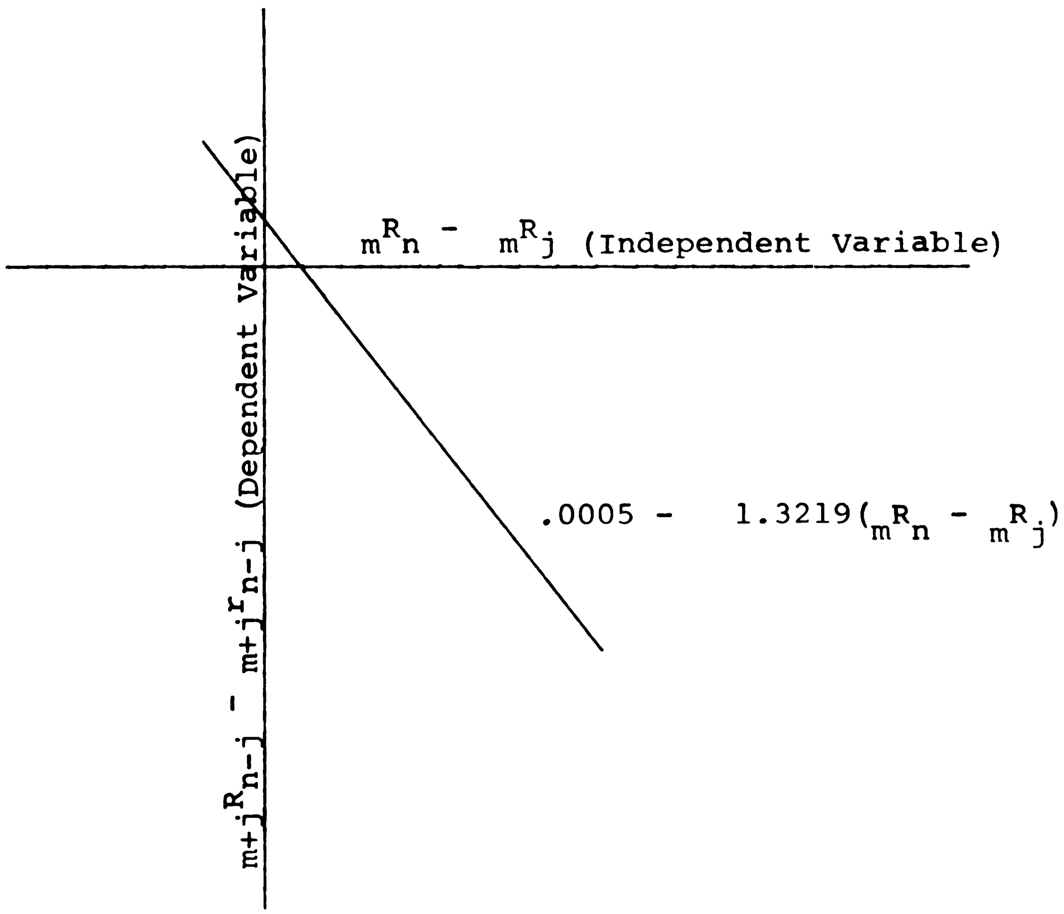
as an example. The B_0 = .0005 indicates that when the yield curve is flat, ${}_mR_n - {}_mR_j = .0000$, the realized rate lies above the breakeven rate by .05 or five basis points. The $B_1 = -1.3219$ indicates that the dependent variable becomes less positive (more negative) as the slope of the yield curve increases. In particular, for every basis point increase in the yield curve slope, the realized rate minus breakeven rate declines by 1.3 basis points. The yield curve slope may be increased by an increase in ${}_mR_n$ or a decrease in ${}_mR_j$ or a similar combination of the two yields. The breakeven yield is calculated at the time of the yield curve observation. Thus, the dependent variable becomes a function of the change in the realized yield compared to a stationary breakeven yield. The relationship for n=2 and j=1 is shown graphically in Figure 7. Based on the regression, two stage financing becomes more desirable whenever the yield for two years is approximately 4 basis points more than for one year. The desirability of stage financing is dependent upon the location in the regression graph; top half for single-state, and bottom half for two-stage.

From Table 2 note the regression constant B_0 is positive for all financing plans. The realized rate should possess a smaller liquidity premium since the time remaining

FIGURE 7

GRAPHICAL EXPRESSION OF REGRESSION MODEL RESULTS



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to maturity is n-j which is less than the entire n period. The general rise in interest rates over the 1952-71 period offsets the expected negative B_0 resulting from the liquidity premium effect. However, for a given j period the B_0 becomes greater as the n period increases. Equal number of observations are possible between financing plans when the same j period is used. The increasing B_0 estimates indicate the existence of an increasing liquidity premium for increasing maturity. The realized rate is greater as the n-j period increases for given j periods. In summary, the increasing trend of interest rates for the time period tested offsets the expected liquidity premium for any one financing plan but the liquidity premium may be observed when various plans are compared.

The regression slope B_1 is negative for all financing plans. As the slope of a yield curve increases, the difference between the realized rate and the calculated breakeven rate increases negatively. Thus, a two-stage financing plan becomes more desirable as the yield curve slope increases. The coefficient of correlation, R, is shown. The coefficient of correlation measures the linear relationship between the dependent variable and the independent variable. The negative R indicates the same inverse linear fit as does the negative B_1 estimator. Considering these statistics together note that as the B_0 becomes more positive, the B_1 estimate becomes more negative. This relationship holds for a given n period plan as the j period increases with the exception of j=5 years. When j=5 years for a n period of both 10 years and 20 years, the R statistic and the B_1 estimator "fall out of line."

A visual observation of yield curves indicates that more governmental yield curves are humped with peak equal to 5 years that any other period. This fact provides a possible clue to the difference in results with regard to humped and non-humped yield curves. A humped yield curve is defined as a curve where the highest yield occurs at a maturity other than the extremes of the maturities observed. When the basic regression is run on data where humped yield curve variables (30 of 76 data points) are removed, the results are inconclusive.

<u>1952-60 and 1961-71</u>

Dividing the 1952-71 time period into two subperiods of 1952-60 and 1961-71 uncovers interesting results. The Durbin-Watson d statistic of Table 3 shows the residuals remain autocorrelated and again usefulness of the estimators is diminished. Particularly noteworthy is the uniform alternating of signs for the coefficient of correlation

TABLE 3

RESULTS OF LEAST SQUARES REGRESSION FOR FINANCING PLANS
OF QUARTERLY YIELDS FOR GOVERNMENT
SECURITIES: 1952-60 AND 1961-71

Period	Financing n yrs	Plan j yrs	Observations	R	B_0	B_1	Durbin-Watson d
1952-60	2	1	36	.198	-.0064	1.7286	.5960
1961-71	2	1	40	-.411	.0049	-4.1956	.4879
1952-60	5	1	36	.406	-.0053	1.0497	.7415
1961-71	5	1	40	-.340	.0041	-1.0178	.4591
1952-60	5	2	36	.304	-.0038	1.1317	.5904
1961-71	5	2	36	-.208	.0064	-0.9324	.5156
1952-60	5	3	36	.291	-.0026	1.6643	.7405
1961-71	5	3	32	-.464	.0127	-4.1197	.6081
1952-60	5	4	36	.130	-.0014	1.5161	.8508
1961-71	5	4	28	-.504	.0155	-10.5226	.5441
1952-60	10	1	36	.362	-.0018	0.3824	.6702
1961-71	10	1	40	-.298	.0030	-0.3933	.4321
1952-60	10	2	36	.207	.0003	0.2599	.4757
1961-71	10	2	36	-.317	.0068	-0.5742	.5049
1952-60	10	5	36	.446	.0030	1.2485	.5855
1961-71	10	5	24	-.219	.0182	-1.1694	.4734
1952-60	20	1	36	.341	-.0007	0.2182	.5056
1961-71	20	1	40	-.223	.0024	-0.1731	.4976
1952-60	20	2	36	.079	.0014	0.0666	.3178
1961-71	20	2	36	-.316	.0060	-0.3754	.5659
1952-60	20	5	36	.395	.0046	0.5726	.3653
1961-71	20	5	24	-.371	.0166	-1.2403	.3496

between time periods. The correlation coefficient increased in absolute amounts from the aggregative period. In effect, single-stage financing becomes more desirable in the first decade as the yield curve slopes more sharply upward. Two-stage financing becomes more desirable during the second decade as the yield curve slopes more sharply upward. However, these "cost benefits" cited are tenuous since the significance of the estimators cannot be tested because of autoregression. The last financing plan of n=20 and j=10 is not subdivided since realized yields are not available past 1971.

The striking reversals of sign for the statistics between the two decades in Table 3 deserve additional comment. A different liquidity premium hypothesis may be applicable for each decade. The B_0 intercept of equation (11) identifies the point where the yield curve is flat with respect to maturity n and maturity j. The dependent variable is equal to the regression constant plus the regression slope times the yield curve slope which is zero where ${}_mR_n = {}_mR_j$. Hence, the B_0 intercept is the point identified with a flat yield curve. Generally, the yield curve is flat when the interest level is high with regard to a normal level. As such, the interest level may decline in the future as it tends toward normality. The money-substitute hypothesis indicates the liquidity premium varies directly with the interest rates. Therefore, when the yield curve is flat and the interest level high, the liquidity premium is greatest according to the money-substitute hypothesis. However, the premium will decline as the interest level falls. The "normal" liquidity premium hypothesis indicates the liquidity premium varies inversely with the interest level. When the yield curve is flat and the interest level high, the liquidity premium is lowest according to the "normal" hypothesis. However, the premium will increase as the interest level falls.

The B_0 estimates are negative for the decade of the 1950's. When the interest level does fall the liquidity premium declines according to the money-substitute hypothesis. There is a greater probability that the realized rate will include a smaller liquidity premium and cause the dependent variable to be negative. The negative B_0 estimates coincide with that expected for the money-substitute hypothesis for the 1950's. This hypothesis agrees with the cyclical liquidity preference research of Kessel and Cagan over similar time periods. The B_0 estimates become very much more positive for the decade of the 1960's. The liquidity premium increases as the interest level falls. There is a greater probability that the realized rate will include a larger liquidity premium and cause the dependent variable to be positive. The positive B_0 estimates may substantiate the "normal" hypothesis for the 1960's. The positive B_0 estimates may well be a result of the dramatic rise in interest rates during the 1960's. The rising interest level would, of course, increase the realized interest yields over that expected. In any event, the evidence offered is weak since the standard deviations of the estimates are biased and inference making is limited. The autoregression must be reduced to an acceptable level to gain more satisfactory statistical results.

Two-Stage Least Squares Empirical Results

Rationale

Several procedures exist to reduce the effect of autocorrelation on residuals such as maximum likelihood, first difference equations or two-stage least squares.[10] Serial correlation indicates a disturbance in period t is not independent but, rather, dependent on period t-1. All three methods attempt to remove period t-1's effect from period t. The convariance, p, among time periods measures the relationship from period t-1 to period t. The three

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[10] *Ibid.*, p. 282-92.

techniques differ with regard to determination of the co-variance. The first difference equations method assumes p is equal to one and as such the resulting B_0 approaches zero. The maximum likelihood method attempts to locate that p which minimizes the variance of a random variable u_t. The two-stage method uses a calculated p which makes it superior to the first difference equations since p may not, in fact, be equal to one. The two-stage method is nearly as efficient as the maximum likelihood method and facilitates ease of computation; hence, its use. The covariance may be calculated from residuals of a first-stage least squares regression as follows where e_t is the residual for period t:

$$p = \frac{\sum_{t=2}^{n} e_t e_{t-1}}{\sum_{t=2}^{n} e_{t-1}^2}$$

Normally, least squares' regressions assume that p is equal to zero. Once p is calculated, the value is used to reduce the serial disturbance. The logic follows. For period t the regression is:

$$Y_t = B_0 + B_1 Z_t + e_t \tag{15}$$

and for period t-1 the regression is:

$$Y_{t-1} = B_0 + B_1 Z_{t-1} + e_{t-1} \tag{16}$$

Multiplying (16) by p and subtracting from (15) results with:

$$Y_t - pY_{t-1} = B_0(1-p) + B_1(Z_t - pZ_{t-1}) + (e_t - e_{t-1}) \tag{17}$$

It may be shown that $e_t - e_{t-1} = u_t$ is normally and independently distributed.[11] The basic assumptions of the ordinary least squares model are satisfied. The second stage least squares is run on the following equation:

$$Y_t - pY_{t-1} = B_0(1-p) + B_1(Z_t - pZ_{t-1}) + u_t \quad t=2,\ldots,n \qquad (18)$$

This second stage may be run if p is known. The coefficient p is estimated from the residuals of the first stage regression. One observation, t=1, is lost from the second stage regression and the resulting estimators are nearly BLUE. Table 4 shows the covariance p for the following selected financing plans: short-term, n=2 and j=1; intermediate-term, n=5 and j=1, n=5 and j=2; long-term, n=20 and j=1, n=20 and j=10. The plans are representative of financing alternatives available to the government. However, it must be remembered that a majority of Treasury financing is confined to short-term maturities, especially bills that carry a maturity less than one year. The results are applicable to maturities greater than one year and the Treasury does finance with these maturities. Results of Table 4 indicate that the assumption of p=0 or p=1 is false and serial correlation does exist. With estimates of covariance known the second stage regression of the two-stage least squares technique may be performed.

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[11] *Ibid.*, p. 284.

TABLE 4

COVARIANCE AMONG RESIDUALS FROM FIRST-STAGE REGRESSION OF QUARTERLY GOVERNMENTAL YIELDS

Financing n yrs	Plan j yrs	Covariance, p
2	1	.7825
5	1	.7551
5	2	.7445
20	1	.7124
20	10	.7519

1952-71

Results for the transformed two-stage least squares for the time period 1952-71 are shown in Table 5. The estimates presented are for the transformed data as indicated by equation (18). Therefore, both B_0 and its standard deviation must be corrected by the amount $1/(1-p)$ for estimates of a non-transformed nature. Since p is approximately equal to .75 for the government securities, the values for B_0 and its standard deviation should be multiplied by a factor of approximately four. However, the F statistic and the significance remain the same because the numerator and the denominator are multiplied by a constant. The major objective of the regression model

TABLE 5

RESULTS OF TRANSFORMED TWO-STAGE LEAST SQUARES FOR FINANCING PLANS OF QUARTERLY YIELDS FOR GOVERNMENT SECURITIES: 1952-71

Financing n yrs	Plan j yrs	Observations	R	B_0	B_1	Durbin-Watson d
2	1	75	-.125	-.0003	-0.7740	1.0942
5	1	75	-.035	.0001	-0.0852	1.2537
5	2	71	-.134	.0007	-0.4848	1.6446
				(.0008)	(.4330)	
				(.384*)	(.267*)	
20	1	75	.045	.0003	0.0350	1.5291
				(.0004)	(.0918)	
				(.403*)	(.704*)	
20	10	39	-.238	.0047	-1.0073	1.8920
				(.0006)	(.6759)	
				(.001*)	(.145*)	

*Indicates the significance of the estimator based on the F test.

is a test of parameters significantly different from zero. This may be obtained from the transformed data as presented.[12] However, if the regression estimates were statistically significant, the e_1' vector must be corrected using the non-transformed data; the use of $B_0/(1-p)$ not B_0 as reported in the tables. The two-stage least squares does not eliminate autocorrelation with two financing plans; n=2 and j=1,

[12] The R statistic will also change when applied to the non-transformed data. Since the intent of the regression model is predictive, not explanatory, the R statistic conveys little information.

n=5 and j=1. For these plans the d statistic is less than the acceptable lower bound at the one per cent level. For these two plans we may only note the best regression estimators not the variances. The serial correlation is reduced to an acceptable level for the remaining plans. The standard deviations of non-biased estimates are noted within parentheses. Where the assumptions of least squares have been met, the familiar F statistic is employed for the determination of the significance of the B_0 intercept and the B_1 slope.

The slope	H_0^1:	$B_1 = 0$
	H_A^1:	$B_1 \neq 0$
The constant	H_0^2:	$B_0 = 0$
	H_A^2:	$B_0 \neq 0$

The significance found is reported with an asterisk so that investigators may attach their own interpretation of the results. However, this study regards significance at the .1 level for expository purposes. The alpha level of .1 indicates a 10 per cent chance of rejecting the null hypothesis H_0 when, in fact, the null hypothesis is true (type I error) is accepted.

In general, the results are non-significant at the .1 level. Significance is found only in the B_0 intercept for n=20 and j=10 plan with a positive constant. This last result leads the debt manager to prefer one long-term 20 year issue over two 10 year issues when the yield curve is flat or nearly so. The significance of the one B_0 intercept may be a result of the rising interest structure of the 1960's.

1952-60 and 1961-71

The total period is again subdivided into subperiods of 1952-60 and 1961-71.[13] Table 6 shows the results for the transformed data. Autocorrelation continues to exist for plan n=2 and j=1. From Table 6 note that only one estimator is significant; n=20 and j=1 during 1952-60 with a positive B_1 slope. This one statistic indicates that single-stage financing becomes more desirable as the slope of the yield curve increases. The forward interest vector could be corrected on the basis of the regression model using non-transformed variables. Immediate long-term financing via a single-stage would be more greatly favored. Since the autoregression has been reduced to an acceptable level confidence exists for the non-significance of regression estimators. Additional information is not uncovered

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[13]For a review of the effectiveness of Treasury debt management see Thomas R. Beard, "Debt Management: Its Relationship to Monetary Policy, 1951-63," National Banking Review, Vol. 2 (September, 1964), pp. 61-76.

TABLE 6

RESULTS OF TRANSFORMED TWO-STAGE LEAST SQUARES FOR FINANCING PLANS OF QUARTERLY YIELDS FOR GOVERNMENT SECURITIES: 1952-60 AND 1961-71

Period	Financing n yrs	Plan j yrs	Observa- tions	R	B_0	B_1	Durbin- Watson d
1952-60	2	1	35	-.098	-.0005	-0.5194	1.0721
1961-71	2	1	40	-.165	.0000	-1.3210	1.1894
1952-60	5	1	35	.138	-.0005	0.2879	1.4250
					(.0010)	(.3591)	
					(.648*)	(.428*)	
1961-71	5	1	40	-.212	.0005	-0.6201	1.2550
					(.0010)	(.4633)	
					(.631*)	(.189*)	
1952-60	5	2	35	-.043	-.0001	-0.1131	1.5024
					(.0010)	(.4494)	
					(.951*)	(.803*)	
1961-71	5	2	36	-.219	.0013	-1.1372	1.6668
					(.0012)	(.8699)	
					(.289*)	(.200*)	
1952-60	20	1	35	.379	-.0002	0.2461	1.6483
					(.0005)	(.1047)	
					(.591*)	(.025*)	
1961-71	20	1	40	-.169	.0005	-0.1564	1.5274
					(.0005)	(.1483)	
					(.365*)	(.298*)	

*Indicates the significance of the estimator based on the F test.

by eliminating the humped yield data points from the regression.

Since the two-stage least squares reduced the effect of autocorrelation the testing of hypotheses is valid. However, based on the F statistic tests we cannot reject the hypothesis that B_0 or B_1 estimators are different from zero. The low coefficients of correlation ($-.17<R<.38$) indicate the linear fit is not a good one. The associated coefficients of determination, $R^2<.16$, indicate that a small amount of variation of the sum of squares is explained by the linear regression. Therefore, explanatory variables other than the simple slope differences are necessary to provide a more adequate description of the dependent variable for the debt manager. The null hypothesis of $B_1 = 0$ is not rejected. Thus, information is not contained within the slope of a yield curve for Treasury debt management. In essence, this result adds support for the expectations theory of interest rate term structure. A yield curve of period t reflects movements of future interest rates in periods t+1,...,t+n, regardless of the associated slope. The expectations theory assumes the slope of the yield curve indicates future interest level movements. No cost advantage exists for timing of debt maturities since long-term rates are presumed to be equal to the serial sequence of future short-term rates. Knowledge of a particular yield curve slope does not impart any other information than what is expected by that slope.

The B_0 estimates indicate a potential change in the liquidity premium hypothesis from the money-substitute in the 1950's to the "normal" hypothesis in the 1960's. However, none of the B_0 estimates are significantly different from zero. Again, the evidence for a change in the liquidity premium is weak based on the statistical technique of this research. The existence of a liquidity premium with regard to maturity is demonstrated by the B_0 estimates. For a given j period, j=1, increasing n-j periods include a larger constant which is ascribed to be the liquidity premium. The liquidity premium increases with increasing term to maturity.

Summary

An ordinary least squares regression run on quarterly governmental yield data shows positively autocorrelated results. Autocorrelation severely limits hypothesis testing of regression estimates since the variances of those estimates are biased. A review of the regression estimates for the 1952-71 period indicates that two-stage financing becomes cheaper as the yield curve slope increases. When the 1952-71 period is broken down into sub-periods, different results become apparent. For the 1952-60 period single-stage financing becomes more attractive as the yield curve slope increases. The 1961-71 period, one of sharply rising interest rates, shows that the two-stage financing plan becomes more attractive as the yield curve slopes sharply upward. However, since the significance of the regression estimates cannot be tested, the "cost benefits" available from the financing plans are tenuous.

The implementation of two-stage least squares reduces the positive autoregression to acceptable limits for some plans tested. The regression results are non-significant at the .1 level. The B_0 estimates again offer some support for the existence of a liquidity premium. A reversal of the sign of B_0 estimates between the decade of the 1950's and the 1960's offers weak support for a change from the money-substitute hypothesis to the "normal" liquidity premium hypothesis. The non-significance of the B_1 estimates indicate the yield curve slope is of no consequence in predicting the future realized yield. The yield curve does not impart any other information than expected. Support is generated for the expectations theory on the basis of the B_1 estimates while lesser support exists for the liquidity premium theory on the basis of the B_0 estimates.

In summary, the regression analysis suggests that knowledge of the yield curve slope does not provide information required for Treasury debt management. The results do not allow a confrontation of alternative Treasury debt management theories mentioned in this chapter's introduction. Interest costs may not be minimized from the knowledge of a current yield curve slope. Pro-cyclical advocates are not given a new technique by which interest costs may be minimized. However, the state of the art for debt management is subject to question as the following statement indicates.

> Nor do we even have clear scientific knowledge concerning the effects of debt management operations on the rate structure itself. Debt management is a much less important matter for economic stabilization than either monetary or fiscal policy and that there are probably many alternative ways of managing the federal debt that are entirely consistent with effective use of the other policy instruments.[14]

Further research may well be justified in an attempt to piece together federal debt management, interest rate term structure and monetary policy. Chapter four presents the regression results for corporate securities.

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[14] Warren L. Smith and Ronald L. Teigen, Readings in Money, National Income and Stabilization Policy (Richard D. Irwin, 1970), p. 398.

CHAPTER IV

TEST OF HYPOTHESIS WITH YIELDS ON CORPORATE SECURITIES

Corporate Plans Tested

Little empirical term structure research has been possible on corporate debt yields because of a dearth of information. In particular, new issue corporates with different term to maturities have been unavailable for study. More information is now being made available.[1] In this section we use the research technique described in Chapter two. Again, the same question is asked: Does knowledge of the yield curve slope convey useful information for corporate debt placement?

Two long-term financing plans are tested. Both utilize quarterly yield data from 1952-71 for AA long-term utilities as shown by Salomon Brothers' An Analytical Record of Yields and Yield Spreads. The two-stage financing plans are based on j periods of one year. The first plan

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[1]Since 1969 Salomon Brothers began publishing data on five and seven year corporate maturities.

includes a ${}_{m}R_{j}$ equal to the six month commercial paper rate + ¼ per cent. The second plan includes a ${}_{m}R_{j}$ equal to the existing commercial bank prime rate + ¼ per cent. The ¼ per cent is added to both basic yields to approximate the additional risk of going to a maturity of one year. The ¼ per cent added is slightly less than the average yield difference between the one month and six month commercial paper rate. The two-stage plans may be represented as ${}_{m}R_{CP+\frac{1}{4}\%}/{}_{m+1}r_{AA}$ and ${}_{m}R_{BP+\frac{1}{4}\%}/{}_{m+1}r_{AA}$. The AA long-term rate is used for both the single stage yield and the second stage of the two-stage plan.

Ordinary Least Squares Empirical Results

1952-71

Table 7 shows the results of the ordinary least squares regression: ${}_{m+j}R_{n-j} - {}_{m+j}r_{n-j,m} = B_0 + B_1({}_{m}R_{n} - {}_{m}R_{j})$. The residuals are autocorrelated as were the residuals of the governmental yields. Hence, we cannot test the significance of the regression estimates. Note the increased negative coefficient of correlation, R, for the corporate data. The fit between the dependent and the independent variable is better and more negative. Knowledge of a yield curve slope may be of greater consequence for corporates than for governments. The fact that the slope of a yield curve

TABLE 7

RESULTS OF LEAST SQUARES REGRESSION FOR FINANCING PLANS OF QUARTERLY YIELDS FOR CORPORATE SECURITIES: 1952-71

Financing n yrs	Plan j yrs	Observations	R	B_0	B_1	Durbin-Watson d
20	1					
AA	CP+¼%	76	-.403	.0045	-0.4305	.5136
AA	BP+¼%	76	-.443	.0010	-0.8018	.3809

TABLE 8

RESULTS OF LEAST SQUARES REGRESSION FOR FINANCING PLANS OF QUARTERLY YIELDS FOR CORPORATE SECURITIES: 1952-60 AND 1961-71

Period	Financing n yrs	Plan j yrs	Observations	R	B_0	B_1	Durbin-Watson d
	20	1					
1952-60	AA	CP+¼%	36	-.047	.0014	-0.0389	.6683
1961-71	AA	CP+¼%	40	-.609	.0066	-0.7595	.5267
1952-60	AA	BP+¼%	36	-.382	.0005	-0.7450	.5128
1961-71	AA	BP+¼%	40	-.459	.0016	-0.8019	.3289

proves beneficial for debt management may indicate market imperfections exist.

1952-60 and 1961-71

Table 8 shows the results for the subdivided period. Positive autoregression as shown by the Durbin-Watson d is still very much present. However, note the negative increase in R for the decade of the 1960's. There exists a greater potential for interest cost reduction by following a two-stage financing plan when the yield curve slopes steeply upward. However, since the regression estimators significance may not be tested because of the autoregression, the cost reduction potential can be accepted only tentatively. Again, there is a link between the regression model and the decision model. The regression model provides information for adjusting the forward interest vector θ_1' of the state space. Significant regression results provide the impetus for changing the vector as specified in Chapter two. The mechanics of changing the vector are elaborated later in this chapter's section, "Input for Multi-Stage Dynamic Programming Decision Model." The vector change may be translated into a possible different election of optimal debt maturity decisions.

There is a much more noticeable change in the regression coefficients for the commercial paper alternative than the bank alternative. The change in statistics for commercial paper between decades is more similar to those of governments than the bank prime. The yield for paper reflects market conditions much more rapidly than does the administered prime and as such is more like the government market. The B_0 estimates of both plans increase from the 1950's decade to the 1960's decade. The increase is a result of the sharply rising interest level of the 1960's. However, before too much is made of these estimates, the autoregression of residuals must be reduced to acceptable levels.

Two-Stage Least Squares Empirical Results

The two-stage least squares technique is employed to reduce the effect of positive autocorrelation. The covariance, p, for the residuals of the first stage regression is equal to .7928 for the commercial paper alternative and is equal to .8068 for the bank prime alternative. Again, the least squares assumption of covariance equal to zero was not met.

1952-71

Table 9 presents the results of the transformed two-stage least squares. The estimates are presented for

TABLE 9

RESULTS OF TRANSFORMED TWO-STAGE LEAST SQUARES FOR FINANCING PLANS OF QUARTERLY YIELDS FOR CORPORATE SECURITIES: 1952-71

Financing n yrs	Plan j yrs	Observations	R	B_0	B_1	Durbin-Watson d
20	1					
AA	CP+¼%	75	-.291	.0008 (.0005) (.073*)	-0.2441 (.1016) (.019*)	1.5137
AA	BP+¼%	75	-.548	.0001 (.0004) (.630*)	-0.8036 (.1435) (.001*)	5.5263

*Indicates the significance of the estimator based on the F test.

the transformed data as indicated by equation (18). The significance as judged by the transformed data is the same as non-transformed data. Both B_0 and its standard deviation must be corrected by the amount 1/(1+p) to be used within the non-transformed nature. The covariance is approximately equal to .8 for the corporate securities so the B_0 and its standard deviation must be multiplied by a factor of five for adjustment within the e_1' vector. The two-stage least squares reduces the positive autoregression of the residuals to acceptable limits. Testing of hypotheses for regression estimates are valid and both financing plans show significance at the .1 level. We can reject the null hypothesis of the B_1 estimate equal to zero for both corporate plans. The B_0 intercept of -.0008 for the commercial paper plan is significant at the 7½ per cent level while the null hypothesis of B_0 equal to zero for the bank prime plan is not rejected. What information do these statistics convey to the corporate debt manager?

Based on the data for the 1952-71 period and the regression as specified in Chapter two, the debt manager would benefit by studying the slope of the yield curve. The slope is measured by the difference between yields for maturity n and maturity j. The negative B_1 estimator of the two-stage least squares show that the realized rate minus the calculated breakeven rate increases negatively as the slope of the yield curve increases. A two-stage financing plan becomes more attractive as the yield curve slopes more steeply upward.

The significance, .001 and .019, of the negative B_1 estimates is especially noteworthy for both corporate plans. The slope conveys information other than predicted by the expectations theory. Thus, the capital market may have inefficiencies that the corporate manager may take advantage in debt maturity timing. When a yield curve is sharply upward sloping, say long-term rates 1½ per cent above short-term rates, the regression indicates a two-stage financing plan becomes desirable. The steeply upward

sloping yield curve might indicate current overreaction by investors with regard to market yields. Long-term rates may be inflated or short-term rates may be depressed. For either situation the calculated breakeven rate is higher which increases the probability that the realized rate might be less than the breakeven rate when the market eliminates the overreaction. The opposite situation exists when the yield curve is flat or declining. Long-term rates may be depressed and short-term rates inflated. The calculated breakeven rate is lower for either case and increases the likelihood that the realized rate will be higher than the breakeven rate. The possibility of market overreaction allows the financial manager a potential debt interest cost advantage. For example the sharply upward sloping yield curve mentioned above, indicates interest rates are expected to rise in the future. As a result of these expectations corporations may attempt to increase their long-term debt which would tend to further increase the yield curve slope. This overreaction will magnify the yield curve slope and helps to explain the information that is contained within the slope. The 1952-71 period is subdivided to analyze sub-period trends and see if market imperfections exist for each decade.

1952-60 and 1961-71

Table 10 shows the results for the 1952-60 and 1961-71 transformed corporate securities. Autoregression is successfully reduced to an acceptable level by the two-stage least squares technique for the corporate sub-periods. The commercial paper plan indicates the same trend as did the governmental statistics. For the 1952-60 period we cannot reject the null hypothesis of B_0 equal to zero or of B_1 equal to zero. In fact, note the "best fit"

TABLE 10

RESULTS OF TRANSFORMED TWO-STAGE LEAST SQUARES FOR FINANCING PLANS OF QUARTERLY YIELDS FOR CORPORATE SECURITIES: 1952-60 AND 1961-71

Period	Financing n yrs	Plan j yrs	Observations	R	B_0	B_1	Durbin-Watson d
	20	1					
1952-60	AA	CP+¼%	35	-.119	.0005 (.0007) (.473*)	-0.0881 (.1281) (.496*)	1.9384
1961-71	AA	CP+¼%	40	-.433	.0012 (.0006) (.059*)	-0.4775 (.1613) (.005*)	1.3630
1952-60	AA	BP+¼%	35	-.598	.0000 (.0005) (.936*)	-0.9331 (.2179) (.001*)	1.5336
1961-71	AA	BP+¼%	40	-.516	.0003 (.0006) (.609*)	-0.7240 (.1951) (.001*)	1.4414

*Indicates the significance of the estimator based on the F test.

regression line is nearly flat. The slope of the yield curve has little impact for a commercial paper alternative. The 1961-71 data indicates the slope of the regression is negative at a .005 significance. A two-stage financing plan becomes more attractive as the yield curve slopes more steeply. When the yield curve is upward sloping, the debt manager should look more closely to financing long-term debt needs with a first stage issue of commercial paper and funding the debt one year later.[2] Of course, all firms are not able to float commercial paper so the commercial bank alternative for obtaining short-term funds is examined.

Interestingly, the bank alternative indicates a decrease in the negative slope of the best fit regression line from the 1950's to the 1960's. However, the significance of the negative B_1 estimates again indicate the desirability of two-stage financing with an increasing slope of the yield curve. It is worth noting that the B_1 estimate for a bank alternative is more negative and statistically more significant than the commercial paper alternative. The commercial paper rate moves much more rapidly to market conditions than does the prime rate.

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[2]See Nevins D. Baxter, Commercial Paper Market (Bankers Publishing Company, 1966) for an excellent discussion of the commercial paper market.

The rapidity of moves is similar to that of the governments. Hence, the "stickiness" of the bank prime may allow a greater potential for debt cost reduction.[3] Before uniformly endorsing two-stage financing for periods of sharply upward sloping yield curves the total impact of a debt maturity decision must be considered. In particular, consideration is given to the effect of two-stage financing for a firm's risk.

Utilization of Corporate Empirical Results

The empirical evidence of the last sections indicate a two-stage financing plan becomes more beneficial to corporations as the yield curve slopes more steeply upward. The realized rate minus the breakeven rate increases negatively as the yield curve slope increases. A two-stage financing plan involves both increased financial risk and uncertainty of cost benefit. The average maturity of debt is shorter for a two-stage financing plan. For the 20 year n period considered for the corporate alternatives, the average debt maturity for a two-stage plan during the first year of issue is ½ year while it is 19½ years for the

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[3]This "stickiness" of bank prime rate movements may be a phenomena of the past. Several commercial banks now tie their prime rate to commercial paper yields. The adjustments to the prime are made weekly.

single-stage plan. Clearly, financial risk is involved for the firm; the risk of being unable to fund its debt at the end of year one.

Uncertainty of interest cost benefit arises from the unknown level of future interest rates. While the empirical evidence of this research indicates a two-stage financing plan <u>should</u> become more attractive as the yield curve slope increases, the realized rate <u>may</u> be greater than the breakeven rate. In retrospect, the correct decision is very easily made. While the decision is being made, uncertainty exists.

Input for Multi-Stage Dynamic Programming Decision Model

Returning to the theoretical decision framework of Chapter two, we introduce the effect of the empirical results. The research indicates a strict mathematical extrapolation of interest rates on the basis of the expectations theory is incorrect. An adjustment with regard to the given slope of the yield curve is appropriate. The e_1' vector of forward rates is changed. Such an adjustment is possible by "plugging in" the regression estimates. First it is necessary to adjust the transformed data by multiplication of the B_0 estimate by $1/(1-p)$ which for the corporate results is multiplication by an approximate factor of

five. For example, data is employed from the bank prime, AA long-term bond alternative. If the current AA yield is 2 per cent greater than the bank prime + ¼ per cent, the forward yield (calculated breakeven yield) is added to (5(.0001) + ((.0200)(-0.8036))) for input to the e_1' vector. The regression estimate of .0001 is B_0 and -0.8036 is B_1 from Table 9 while .0200 is the 2 per cent difference in yields of short- and long-term rates. The 5 adjusts the transformed results to a non-transformed nature. The regression model has corrected the forward rate for long-term rates for the yield curve one year hence. The forward yield has been reduced by .0156. This reduction would, of course, have the effect of favoring a two-stage issue. The remaining maturities of the forward yield curve may be similarly adjusted by the appropriate regression estimates. The limited results of this study only allow correction for a 19 year maturity of a forward yield curve one year hence. Additional time periods and financing plans must be tested for other forward rate adjustments. The element of financial risk is controlled by the estimates for the desired amount of debt maturity, e_3. Once these adjustments are made, the search process of dynamic programming problem "solves" for the optimal debt maturity. However, we still may not have correctly nor completely accounted for the uncertainty of the projected two-stage cost advantage.

Adjustment of the Discount Rate for Two-Stage Financing Plans

The uncertainty may be accounted for by a reexamination of the basic breakeven yield calculation (2).

$$\sum_{i=j+1}^{n} \frac{{}_{m+j}r_{n-j,m}}{(1+{}_{m}R_{n}^{*})^{i}} = \sum_{i=1}^{n} \frac{{}_{m}R_{n}}{(1+{}_{m}R_{n})^{i}} - \sum_{i=1}^{j} \frac{{}_{m}R_{n}}{(1+{}_{m}R_{n})^{i}} \qquad (19)$$

The manager may quantitatively indicate concern for financial risk by altering the discount rate, ${}_{m}R_{n}^{*}$, for the left-hand side of the equation (1). Increasing uncertainty decreases the discount rate, ${}_{m}R_{n}^{*}$. Computationally, the decrease in ${}_{m}R_{n}^{*}$ reduces the breakeven yield for the equating of the two financing plans. Hence, the resulting relationship of realized minus calculated breakeven yield becomes more positive (less negative) and the cost advantage of the two-stage issue diminishes. A two-stage least squares regression might then be run on the corporate yield data compared with the new breakeven yield. While the above analysis appears to be academically pleasing at this stage of development, the pragmatic usefulness for the corporate manager is less obvious. The treatment for uncertainty involves a process similar to utility preference theory.[4] How much

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[4]Ralph O. Swalm, "Utility Theory--Insights into Risk Raking," Harvard Business Review (November, 1966), pp. 123-36.

should the discount rate be decreased to account for what degree of uncertainty? The implementation of the process involves many problems. However, the framework does provide an area for additional study.

Summary

The two-stage least squares reduces positive autocorrelation to acceptable limits and the null hypothesis of the regression slope equal to zero may be rejected. A corporation may effect potential interest cost benefits by financing via a multi-stage framework as a current yield curve slopes more steeply upward. The information provided by a yield curve slope is ascribed to be a result of participant overreaction and market inefficiency. Multi-stage financing increases financial risk of a firm since funding of debt must be accomplished more often. The increase in financial risk may be accounted by adjustment of the discount rate used for the breakeven yield computation. Additional empirical research may be attempted as corporate issues for various maturities become available.

CHAPTER V

SUMMARY AND IMPLICATIONS FOR FURTHER RESEARCH

Summary

Theories of Interest Rate Term Structure and Yield Curve Slope

This section summarizes the research technique, empirical results and interpretation of those results. The summary section may be omitted without loss of continuity by the faithful reader of the previous four chapters. Implications with regard to the theories of interest rate term structure and debt management applications are made. Several theorists have mentioned that debt interest cost benefits may be obtained by judicious timing of long-term issues based on the slope of a yield to maturity curve. This analysis represents a serious attempt to discover what information, if any, is contained within a yield curve slope. If information has historically been contained in the slope, use of that information may be profitably made in the future.

Various theories have been proposed to explain the interest rate term structure. The pure expectations theory implies long-term interest rates are a geometric average of expected future short-term rates. All interest rate movements are accounted for by the slope of the yield curve. Accordingly, the expectations theory indicates that a particular slope does not contain information applicable for debt management. The liquidity preference theory implies a natural increase in interest yields as maturity increases. Implications for information contained by the slope depend upon the liquidity preference hypothesis posited. The money-substitute hypothesis indicates the liquidity premium is greatest when the interest level is high and the premium is smallest when the interest level is low. When the interest level is high the slope of the yield curve is flat or declining while the curve is upward sloping when the interest level is low. Accordingly, future liquidity premiums increase as the future interest level increases as implied by a present upward sloping yield curve. The future liquidity premiums decrease as the future interest level declines as implied by a present downward sloping yield curve. Note the increase or decrease in the liquidity premium does not occur in the period the yield curve is observed. An upward sloping yield curve implies interest rates will increase as time passes. As the interest level increases which is implied by an upward sloping yield curve, the liquidity premium increases such that future realized yields are greater than expected. The converse is true for downward sloping yield curves. Alternatively, the "normal" liquidity premium hypothesis indicates the liquidity premium is greatest when the interest level is low. Thus, a downward sloping yield curve indicates the future liquidity premium will increase as the future interest level declines as implied by a downward sloping yield curve. The converse is true for upward sloping yield curves for the "normal" hypothesis. Either hypothesis of the liquidity preference theory implies information is contained by the slope since the premium changes with regard to the interest level. The institutional theory indicates that yield differentials are neither a function of expectations nor of liquidity preference but rather of supply and demand for a given maturity. The slope of the yield curve is inconsequential for potential interest cost benefits according to the institutional theory. In summary, if information is contained by the yield curve slope, market imperfections exist or one hypothesis of the liquidity preference theory is supported. If no information is contained by the slope, the expectations theory gains additional support.

Research Technique

The research hypothesis is that the slope of the yield curve contains information useful for debt management. Funds for a given amount of time may be financed through a single-stage issue or double-stage issue. Larger number of issues are possible but are not considered. A decision model minimizes debt interest costs by electing the optimal debt maturities based upon the debt needs, debt maturity constraints and forecasted interest structure. If information is contained by the yield curve slope, the interest vector of the issuer's state space may be profitably altered. The interest cost associated with a single-stage issue is known with certainty as is the first stage of a two-stage issue. Uncertainty and possible interest cost gain or loss exists with the yield realized for the second stage of the two-stage issue. By equating the present interest cost of the single-stage issue to the two-stage issue, a breakeven rate for the second stage may be calculated. The realized future rate is compared to the calculated breakeven rate. The slope of the yield curve is measured by the difference in yields between two different maturities at a point in time. Thus, the slope is the yield for a single-stage issue minus the slope of a first stage of a two-stage issue. An ordinary least squares regression is run between the realized yield minus the breakeven yield (dependent variable) and the slope (independent variable) over time.

The time period for analysis is 1952-71. Quarterly yields are analyzed for various governmental and corporate financing plans. Yields are obtained from Salomon Brother's An Analytical Record of Yields and Yield Spreads. Time series regressions are subject to autoregression which invalidates hypothesis testing. Therefore, two-stage least squares is employed in order to reduce the autoregression effect. Multicollinearity is not a problem since only one independent variable is indicated.

The regression estimate of the intercept B_0 is expected to be negative due to the existence of liquidity premiums. The realized rate for the second stage should contain a smaller liquidity premium since the time remaining to maturity is smaller. The liquidity premium changes with regard to maturity and over time--depending on the liquidity preference hypothesis. The general increase in the interest level over the period of 1952-71 does bias the results. The regression estimate of the slope B_1 indicates which type of financing plan may benefit, if any, as the slope of the yield curve changes. A B_1 of zero indicates no information is contained by the slope of the yield curve for prediction of the dependent variable. A negative B_1 indicates two-stage financing becomes more favorable with an upward sloping yield curve while a positive B_1 indicates single-stage financing becomes more favorable with an upward sloping yield curve. Significance of the regression estimates is tested by the F statistic. Significant results would justify a change in the interest vector facing the firm. A change in this state might allow a more optimal debt maturity selection by a dynamic programming process for the issuer.

Empirical Results

Governmental

Autoregression as shown by the Durbin-Watson d statistic exists for the ordinary least squares regression for the governmental plans tested. Although hypothesis testing is invalid, interesting points exist from the regression estimates. Generally, the negative B_1 regression estimates indicate that a two-stage financing plan becomes cheaper with an upward sloping yield curve. The existence of increasing liquidity premiums with increasing maturities is substantiated. For a given first stage financing period (j), the regression intercept B_0 increases as the time financing is needed (n) increases. The realized minus breakeven relationship increases as the length of time remaining to be financed increases. Thus, the realized yield increases as the maturity increases which is what the liquidity preference theory indicates.

More useful information is provided by subdividing the twenty year period into decades of 1952-60 and 1961-71. Autoregression continues to exist and inference making is again limited. Striking reversals of sign exist between the decade of the 1950's and the 1960's in the regression estimates and the correlation coefficients. A single-stage issue becomes cheaper in the 1950's while a two-stage issue becomes cheaper in the 1960's as the yield curve slopes more positively. The existence of an increasing liquidity premium with increasing maturity shows for both time periods. However, different liquidity preference hypotheses may be at work for the two sub-periods. The B_0 intercept is negative or less positive in the 1950's than it is in the 1960's. When the yield curve is flat at the B_0 intercept, interest rates are generally high and are expected to decline sometime in the future. As the interest rates do decline the realized rates include a smaller liquidity premium according to the money-substitute theory. Thus, the realized yield has a greater probability of being less than the breakeven rate and the fact is demonstrated by the results for the 1950's. This hypothesis agrees with the cyclical liquidity preference research of Kessel and Cagan over similar time periods. However, during the 1960's the B_0 regression estimate becomes much more positive. According to the "normal" liquidity premium hypothesis the premium is low when interest rates are high but increases as interest rates decline. This fact would cause the realized rate to increase more than the breakeven rate at the point of a flat yield curve and is evidenced by the positive B_0 regression estimates for the 1960's. However, standard deviations of the estimates are biased and the evidence offered is weak.

In most cases the two-stage least squares eliminates the effects of positive autoregression to acceptable limits and hypothesis testing is valid. Significance in regression estimates is lacking. This statistical evidence does not support the existence of information being contained by the slope of the yield curve. Again, as noted by the ordinary least squares results, a liquidity premium exists for the sub-periods tested. The money-substitute hypothesis gains weak support for the 1950's and the "normal" hypothesis for the 1960's. However, the rapid interest level increase of the 1960's may explain the positive B_0 regression estimates, not the "normal" hypothesis.

In summary, the governmental results indicate knowledge of a yield curve slope is immaterial for proper debt management. The null hypothesis of B_1 equal to zero cannot be rejected and the expectations theory gains additional support. The yield curve slope does not, by itself, exhibit any other information than expected. Weak evidence supports the existence of a liquidity premium and a change in the cyclical liquidity premium from the money-substitute hypothesis in the 1950's to the "normal" hypothesis in the 1960's. Results do not disprove the institutional theory since relative supply and demand is not measured.

Corporate

Limited data exists for corporate maturities and only two plans are tested; a commercial paper alternative and a bank prime alternative where both issues are funded in year one by a nineteen year AA long-term Utility issue. The regression and the time period are similar to that tested for the governments. Autoregression exists for the ordinary least squares regression. Implications are similar to those of the governments. However, with regard to either the entire time period of 1952-71 or the subperiod decades, two-stage issues become cheaper as the slope of the yield curve increases. This is particularly interesting since the period is one of generally increasing interest rates which bias the regressions toward single-stage financing.

The two-stage least squares eliminates positive autoregression to acceptable limits. The regression slopes of the corporates are negative and significantly negative in all periods except 1952-60 for the commercial paper alternative. Two-stage financing is indicated for steeply upward sloping yield curves. The regression intercepts are positive but not significantly different from zero. The regression intercept increases from the 1950's to the 1960's offering weak evidence for a change in the liquidity premium hypothesis. Knowledge of the yield curve slope offers potential interest cost reduction for the corporate financial manager.

The existence of statistically significant negative B_1 regression estimates have implications for the market perfection of corporate securities. The negative regression slope may be explained by market participant over-reaction. For example, a sharply upward sloping yield curve, long-term yields two per cent greater than short-term yields, indicates future interest rates are expected to increase. On the basis of this slope corporations may attempt to increase long-term debt issues so as to avoid a further yield increase. This increased demand for long-term maturities will generate additional pressure on the yield curve and increase the slope. The slope may be increased by an increase in long-term rates or a decrease in short-term rates. For either case the calculated breakeven rate necessary for comparison with the second stage realized yield will increase. Thus, once market overreaction has subsided there exists a greater probability that the realized rate will be less than the inflated breakeven rate. A similar circumstance exists when the yield curve slope is downward sloping. Corporations would issue short-term debt until long-term rates do, in fact, decline. These actions tend to accentuate the negative slope of the yield curve and decrease the calculated breakeven rate. The realized rate has a greater probability of being larger than the breakeven rate once market overreaction subsides. Therefore, a market overreaction phenomena may explain the corporate results and negative regression slope of this research.

The results for the commercial paper alternative are much closer to the governmental results than the bank prime alternative. The commercial paper rate reflects market conditions much more rapidly than the administered bank prime rate. However, now that some banks tie the prime to weekly movements in commercial paper yields, the effect may be pertinent only to historical debt needs. Market inefficiencies increase the chance that information may be contained by a yield curve slope.

Application of Corporate Results

Debt maturity decisions are not made on the sole basis of interest cost reduction. Maturity of debt represents risk to the issuer. Shorter debt maturity carries a greater potential for economic loss and cash insolvency as a result of being unable to continually fund debt. Longer debt maturity carries a greater potential for an opportunity loss from having unneeded long-term debt. Over the long run the maturity of debt should parallel that of its assets, although short run variations allow interest cost benefits. This analysis indicates a two-stage issue is most profitable when the yield curve is steeply upward sloping.

A two-stage issue offers the firm additional risk. First, the average maturity of debt is shorter and this increases potential for economic loss. Second, uncertainty remains with a realized yield of the second stage. Risk and uncertainty increase as the maturity of the first stage decreases. Average maturity of debt is smaller and any variations in the realized yield compared to expected yield affect the firm for a longer period of time. Risk is also present for the single-stage issue; the risk future interest rates may decline which would make the single-stage issue more expensive. Risk may be accounted for by a utility preference adjustment for debt maturity in the original breakeven rate calculation. Risk may also be judged by discounting the expected present cost benefit (loss) and its standard deviation over the debt need life relative to a single-stage issue. The expected present cost and standard deviation may be obtained from the regression estimates. The process allows the financial manager to make decisions on the basis of more information within a rational framework. An estimate may be made as to the probability of the two-stage interest cost being less than the single-stage cost based on a given yield curve slope.

Implications for Additional Research

Normative

This research indicates governmental interest rate term structure may be explained by the expectations theory in addition to weak evidence of a liquidity preference with regard to maturity. The cyclical liquidity premium is best explained by the money-substitute theory for the 1950's which substantiates other theorists' research. The liquidity

premium may be best explained by the "normal" hypothesis in the 1960's and offers a challenge to the previously supported hypothesis. As such, economists may wish to further explore this apparent shift for the liquidity premium and its implications for a normative Federal debt management model.

This research indicates the corporate debt manager may optimize debt costs with respect to the slope of the yield curve. First, the research technique of this analysis should be continued for other possible debt maturities as sufficient information becomes available. Second, the empirical results of this exercise represent a small portion of necessary inputs for an optimal debt policy. These results must be incorporated with existing research efforts for a more total corporate environment. This analysis abstracts from flotation costs and bond refunding which should be considered for a total representation of rational policy. A more optimal policy with respect to issuing debt maturity appears possible.

Given that interest costs are currently high we might expect two-stage financing to offer greater rewards in the 1970's if the interest level declines to a more normal range.[1] However, a seeming paradox exists. Future

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[1]Lindley H. Clark, "The Outlook". <u>The Wall Street Journal</u>, Vol. LII, No. 185, July 3, 1972, p. 1.

rates are expected to decrease with a declining or flat yield curve. Based on this research a single-stage not a two-stage issue would be advantageous when the yield curve is flat or declining. Two-stage financing becomes cheaper when the yield curve is sharply upward sloping which indicates interest rates are expected to increase. In general a firm would benefit by multi-stage financing when the interest level declines. In particular the existence of market overreaction offers the firm an additional area for potential interest cost reduction from debt maturity decisions and yield curve slope analysis.

Positive

The question of how corporations have actually placed debt maturity has not been addressed or answered. A positive study relating debt maturity and debt composition with regard to yield curve slope appears to be in order. Have corporate financial managers responded in a manner similar to that offered as more optimal by this empirical research? Debt policy may be measured by either average debt maturity or debt composition; percentage of total debt placed in short-term (0-1 year), intermediate-term (1-10 years) and long-term (10 years and longer). For either or both debt policy definitions it is important to note reliance on creditor funds so as to avoid spurious correlations of those firms with very small or large total debt usage. A particularly intriguing point of study is sales finance companies. These firms have the ability to quickly change debt policy in addition to possessing a large debt capacity. Utilities lie on the other side of the spectrum of firms with large long-term debt usage and, as such, provide a good balance with sales finance companies for a positive study of debt policy and yield curve analysis.

The entire area of debt/equity study may be enhanced by recognition of maturity of debt rather than total debt. Maturity of debt does affect the financial risk of the firm. Therefore, financial managers may be able to lower cost of capital by formally introducing debt maturity to the total debt component of a firm's capital structure. Recognition of debt maturity offers another dimension of risk to that of debt itself.

Additional research is necessary for analyzing basic data used in construction of yield curves. Are the points generated for a yield curve plot similar in all but remaining term to maturity? Obviously, if this condition is not met improper empirical conclusions may result. Certain yield curve shapes such as the humped curve may not trully exist. Further rigorous examination of the assumptions behind yield curve data points may prove especially useful.

Government Implications for Yield Curve Analysis

Government, in the most general sense of the word, may affect the basic interest rate term structure. The 1951 "accord" between the Treasury and the Federal Reserve marked the inception of modern public debt management. Under the accord, the Federal Reserve continued to ensure that the government would be able to finance its cash needs while maintaining the opportunity to promote economic stability and growth through judicious use of its monetary policies. Without the Federal Reserves support for pegging of prices for public debt, the Treasury began to pay the "going rate" for its issues. No longer was the government yield curve static in shape and level from period to period. Flexible interest rate policies allow financial managers an opportunity to time debt maturities so as to minimize interest costs. The Treasury has often included interest cost minimization as a goal; increasing long-term issues in recessions and short-term issues in inflationary periods. However, these pro-cyclical operations run counter to those monetary policies of the Federal Reserve; providing liquidity in recessions and restraint in inflationary periods. As a result, a neutral and more systematic debt management policy is emerging as the guideline for Treasury management. However, given that the Treasury is not fully certain of cash needs, an opportunity remains for partial interest cost minimization. Additional pursuit is deserving a policy that minimizes interest costs relevant to the uncertain Federal debt needs. Interest cost reduction, even in a partial sense, frees budget dollars for more worthwhile society demands. While this study does not statistically indicate the slope of a current yield curve is useful for present value interest cost minimization, other variables may. These variables appear worthy of pursuit. Interest cost minimization need not be a historical relic nor need it run counter to the intent of a general neutral debt management policy.

Meanwhile, private sectors of the financial community are being encouraged to artificially restrain interest rate movement. More specifically, commercial banks are being asked to maintain and isolate bank prime rates from those of the more volatile market.[2] The committee on interest and dividends feel that limiting bank prime increases will slow inflation. Artificial constraints by

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[2]Edward P. Foldessy, "Interest Panel Warning Bankers to Keep Loan Charges Down or Face Rate Controls". The Wall Street Journal, Vol. CLXXX, No. 88, November 6, 1972, p. 3.

government affect current interest rates and the perception of future interest rate term structure. The financial manager must be able to anticipate governmental interference with regard to calculating a forward interest rate vector. The corporate data of this study may be inappropriate for future use given potential governmental intervention. A definite need exists for a more general qualitative and quantitative analysis of government operations on interest rate term structure for corporate and governmental securities.

Conclusion

In conclusion, this study has attempted to see if information is contained within a yield curve's slope. The government market was more orderly and information contained by the slope is that anticipated by a combination of the expectations and the liquidity preference theory. Additional research may be warranted for a study of a possible shift of the cyclical liquidity premium of governments between the decade of the 1950's and the decade of the 1960's. Initial evidence supports the fact that information was present and may be usefully applied for a corporate financial policy. Additional research is warranted for both normative and positive implications of corporate debt maturity management and yield curve analysis. A continuing study of the financial return and risk for debt maturity is necessary. Finally, the impact of different government operations on the term structure must be ascertained. The existence of market imperfections resulting from participant overreaction allowed the astute financial manager a chance to lower the overall cost of creditor funds.

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