PRESTRESSED CONCRETE

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Jack Ellison Harney

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This is to certify that the

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Major professor

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PRESTRESSED CONCRETE

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INTRODUCTION

Disadvantages of Reinforced Concrete

Concrete has a compressive strength equal to more than ten times its tensile strength. Steel, on the other hand, has a compressive strength approximately equal to its tensile strength. This is the reason why concrete is reinforced with steel bars placed in the direction of the tensile forces.

In the combination of the two materials into reinforced concrete, many advantages are gained. However, at the same time, many disadvantages occur which are overcome only at great expense of either time or materials. Probably the greatest disadvantage of reinforced concrete is its complex, non-homogeneous nature, which results in uneconomical use of either the strength of steel or of concrete. In the parts of a structure where compression predominates, the steel is nearly always understressed. Where tension predominates, the designer always assumes that the steel resists it all which results in a rather large volume of concrete being "theoretically" unstressed. This volume does, however, add weight to the structure, but does no "work" in support of the structure.

Also, no account is taken of the tensile stresses developed in the concrete at or near the reinforcement, which results in the tensile stress in the concrete exceeding the ultimate value. When this occurs, cracks are formed as the concrete is unable to conform to the normal steel strain. These cracks are a particularly undesirable characteristic of reinforced concrete.

While the stress cracks are not immediately dangerous, they provide openings through which the concrete may be attacked by chemical action and the steel by corrosion. Further, stress due to shrinkage during hardening of the concrete may produce cracks even before the load has been applied.

Another disadvantage of reinforced concrete is that the dimensions of a beam or slab are often determined by diagonal tension. Thus, if the shearing force is large, a large beam is required, and for long spans the dead load becomes too large to be practical.

In ordinary reinforced concrete, it is not possible to make full use of high strength concrete. As Mr. Schorer (2)* points out in his paper, "It appears that the recent improvements in the compressive strength of concrete are largely offset by a reduced value of 'n' so that practically all concretes crack at the same low steel stress limits". In other words, it is the tensile stresses in the concrete at or near the reinforcement that determine the economical sizes of beams and slabs, rather than the compressive strength.

Development of Prestressed Concrete

As may be seen from the previous discussion, the principal disadvantage of concrete lies in its inability to resist tensile stress. In using reinforced concrete, the reinforcing steel may be utilized to confine the deformation of concrete within limits, or to produce stresses in the concrete which will help overcome the tensile stresses.

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* Refers to bibliography.

The idea of using one material to restrain the effects of stress in the other has been accepted as a possibility since the advent of reinforced concrete. One method of restraining the concrete deformations is illustrated by the case of a concrete column with spiral reinforcement. The steel acts as an "envelope" that prevents the column from bulging laterally under axial loads.

A general method of restraining the effects of stress by controlling the stresses in a desired manner may be called, or described by the term "prestressing".

Since concrete is much stronger in compression than in tension, it should be stressed in such a manner as to place it in compression. As the effects of tensile stress in concrete are highly undesirable, the compression in the concrete should be large enough to overcome any tensile stress caused by subsequent loading without exceeding the allowable fiber compression strength. This is the essence of prestressed concrete.

In 1888, P. H. Jackson of San Francisco used what may be called the first prestressed system. In order to strengthen a structure, he threaded the ends of the reinforcing steel and tightened it to produce compression in the concrete.

About the turn of this century, several men including M. Koenen of Berlin and G. Lund of Sweden did extensive work for the purpose of reducing tensile stresses and cracking in concrete beams. The method they proposed was to counteract the concrete tensile stress by stretching the tensile reinforcement.

Their work, for the most part, failed for two reasons. First, the initial prestress they used was too small and the losses in the initial prestress were only partly considered.

Then in 1927, a French engineer named E. Freyssinet concluded that mild steel was not a suitable material to use for maintaining a permanent compression in concrete. Through his experiements he discovered that a permanent compressive stress can be maintained in concrete only with steel having a very high yield point or yield stress. Because of his work, a truly controllable prestressed concrete is possible.

The chart on the following pages shows the development of various systems of prestressing concrete.

CHART I

Development of various systems of Prestressing

Year and Name of Proposer	Purpose of Prestress	Process Suggested	Way of Achieving Purpose
1888 P.H.Jackson San Francisco	Strengthening the structure	Tighten bars to a not determinate degree	Many methods for stretching the reinforcement
1896 J. Mandl Vienna	Reduction of concrete tensile stress and cracking	Counteraction due to stretching the tensile reinforcement	Stretching before pouring concrete
1907 M. Koenen Berlin	"	"	Stretching by Hydraulic jacks before concreting
1907 J. E. Lund Bjorn	"	"	Rods having threads tightened by nuts between prefabricated blocks
1908 C. R. Steiner California	"	"	Rods having threads tightened by nuts against green concrete and after hardening, stretched again.
1923-25 R.H. Dill Nebraska	Guaranteed crackless	Counteraction by full prestressing, the stretching force being of such magnitude that no tensile stress occurs	Destruction of bond by coating
1927 W.H. Hewett Minneapolis	Guaranteed crackless	Full Prestressing	Similar to R.H.Dills' proposal
1928 E. Freyssinet	"	"	High strength steel or wire stretched. Reinforcement substantially reduced.

Year and Name of Proposer	Purpose of Prestress	Process Suggested	Way of Achieving Purpose
1931 T.E.Nichols New York	Guaranteed crackless	Full Pre-stressing	Tensile reinforcement in excess of usual requirements
1934 F.O.Andregg Ohio	"	"	Tensioned tie rods extending thru per-forated ceramic blocks
1934 F.Dischinger Berlin	Extended applicability (Increase of span)	Tensioned ties in com-bination with normal re-inforced concrete	The ties, hanging in curved lines, engaged externally the reinforced concrete elements
1936 U.Fensterwalder Berlin	"	"	Similar to F.Dischinger's proposal
1939 F.Emperger Vienna	More effective reduction of cracking	Partial counteraction by combination of an effectively stretched and an unstretched reinforcement	Unstretched main rein-forcement in usual manner and bonded additional prestressed rods of superior strength
1940-42 P.W.Abeles London	Saving Steel	Partial Prestressing	Similar to F.Emperger's proposal, except the tensile reinforcement substantially reduced by the use of high strength steel wire for both stretched and unstretched reinforcement

By producing a net compression in a concrete member, nearly all of the disadvantages of reinforced concrete are eliminated.

Economical use of both materials is obtained since the total cross sectional area of concrete is effective in compression. In prestressed concrete, the steel is used to a maximum of its tensile properties to place the concrete in compression. The concrete in compression then is able to act in a homogeneous manner to resist bending and shear.

The effect of prestressing is opposite to that of final loading (see Fig. 1). As may be seen, it is possible to prestress a structural member in such a way that the working loads will never produce stresses great enough to cancel the internal stresses entirely.

Since there is no net tension, any cracks which may form due to shrinkage would be closed by the pressure of the steel reinforcement.

As the total area of concrete is effective in resisting loads, the size of a beam may be reduced by a considerable amount.

Since little tension exists, the dimensions of a beam do not depend upon diagonal tension.

Also, it is possible to use high strength concrete as its use is not dependent upon cracking strength (tensile stress).

Purpose of This Thesis

Prestressed concrete holds the promise of providing a new

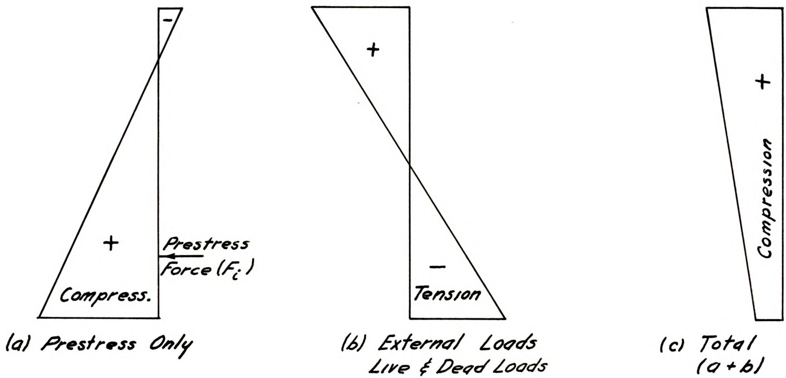


Fig. 1 Stresses in a Prestressed Beam

structural material by making more efficient use of the physical properties of steel and concrete in combination. The purpose of this thesis is to show the advantages and design principles of prestressed concrete and to thus help promote the use of this material.

The readers' attention is called to the fact that the notation of symbols on page 42 will be valuable in interpreting the various derivations and equations which follow.

DESIGN THEORY

Fundamental Conditions

Only simple beams, slabs, and girders are considered in the following derivations. The reason for this is that they are the structural elements which lend themselves most easily to prestressing. Also, after using them for a general analysis, the results may be expanded to include other structural elements.

As has been stated previously, the most important difference between a structural unit of reinforced concrete and one of prestressed concrete is that in the latter, all the concrete in any cross section must be kept in compression.

In general, a prestressed concrete beam subject to bending only must resist two moments. The first is produced while the prestress is being established and is the result of the moment due to the dead load plus the moment caused by the prestress force. These two moments are in opposition to each other and result in smaller total stresses (see Fig. 1).

The second is produced by loading the beam in actual use. This moment is generally called the live load moment and includes impact.

Next, consider a beam of cross-section A, a steel force of F_i, the eccentricity of the steel, e, and the distance from the neutral axis to the upper and lower fibers, y_1 and y_2, respectively as shown in Fig. 2.

Assume that the bending moments are such that the top fiber is in compression for a positive moment. Then the section of the beam at the point of maximum bending stress must satisfy the following conditions:

<u>In the Top Fiber</u>

A. As soon as the prestress has been established, the combination of the tensile stress in the concrete due to the beam weight, w_d, plus the stress due to the prestress force, F_i must not exceed the permissible concrete tensile stress, f_{cf}. Since the cross-section of a prestressed beam should always be in compression, f_{cf} is equal to zero as a minimum compression value.

B. After a period of time (for example 1 year), the beam is acting under w_d, the dead load and w_l, the live loads in combination. The compressive stress in the concrete must not exceed the permissible concrete compressive stress, f_c, if the steel is outside the "core" area or middle third of the beam. If the steel is within the middle third, this condition must be met at the time of prestressing.

<u>In the Bottom Fiber</u>

C. As soon as the prestress has been established the stress in the concrete due to the combined effects of the prestress moment and the dead load moment must not exceed the allowable compressive stress in the concrete, f_c.

D. After a period of time (as in condition B) under the combined live and dead loads plus the initial prestress, the tensile stresses in the concrete must not exceed f_{tc}. (f_{tc} usually equals zero.)

These four conditions described above will now be expressed mathematically:

To establish the mathematical relationships existing between the loads and the resulting stress, consider first the load of the beam, w_d, which will cause fiber stresses in the beam according to the general elastic stress formula.

$$S = \frac{M\ c}{I}$$

Since concrete is considered to be an elastic material within limits, and since the dead load moment is always in combination with the prestress moment, it is possible to conclude:

$$f_{du} = \frac{M_d\ y_1}{I}$$

and

$$f_{db} = -\frac{M_d\ y_2}{I} \quad \text{(- indicates tension)}$$

In a like manner, the live load w_l, produces a moment, M_l, which causes fiber stresses in the top and bottom of the beam of:

$$f_{ul} = \frac{M_l\ y_1}{I}$$

and

$$f_{lb} = \frac{M_l\ y_2}{I}$$

Using the general fiber stress equation, $s = \frac{P}{A} \pm \frac{Mc}{I}$ it will be possible to find the fiber stresses due to the prestress acting alone.

$$s = \frac{P}{A} \pm \frac{Mc}{I}$$

$$M = F_i\ e, \qquad P = F_i, \qquad \text{and} \qquad I = Ar^2$$

Then in the top fiber

$$f_{uc} = \frac{F_i}{A} - \frac{P_i\, y_2\, e}{I} = \frac{F_i}{A}\left(1 - \frac{ey_2}{r^2}\right)$$

And in the bottom fiber

$$f_{bc} = \frac{F_i}{A} + \frac{P_i\, y_2\, e}{I} = \frac{F_i}{A}\left(1 + \frac{ey_2}{r^2}\right)$$

Consider the quantity $\frac{ey}{r^2}$.

If $\frac{r^2}{y} = e$, the steel is just at the middle third point, and the top fiber stress is equal to zero

If $\frac{r^2}{y}$ is greater than e, its reciprocal is smaller than e, and the quantity $\frac{ey}{r^2}$ will be greater than 1. This would indicate a negative value for f_{tc} or that the top fiber is in tension. This is the usual condition in any beam at the point of maximum bending.

If the quantity $\frac{r^2}{y}$ is less than e, the steel is in the middle third, and the section will always be in compression.

There are two conditions to consider. These conditions will be when e is either greater or less than $\frac{r^2}{y}$.

Case 1

This case exists when the reinforcing steel is outside the middle third section of the beam.

Condition A

(Using a negative sign to indicate tension)

$$+\frac{F_i}{A}\left(1-\frac{ey_1}{r^2}\right)+f_{du} \leqq -f_{tc} \qquad (1)$$

$$-\frac{F_i}{A}\left(\frac{ey_1}{r^2}-1\right)+f_{du} \leqq -f_{tc}$$

Condition B

Let equal the amount of prestress left after a period of time. It will always be less than 1, since some initial prestress (F_i) is lost due to shrinkage, creep, etc.

Then:

$$+\eta\frac{F_i}{A}\left(1-\frac{ey_1}{r^2}\right)+f_{du}+f_{ul} \leqq f_c \qquad (2)$$

$$-\eta\frac{F_i}{A}\left(\frac{ey_1}{r^2}-1\right)+f_{du}+f_{ul} \leqq f_c$$

Condition C

Stress in bottom fiber due to prestress - stress due to dead load = f_c

$$+\frac{F_i}{A}\left(1-\frac{ey_2}{r^2}\right)-f_{cdb} \leqq f_c \qquad (3)$$

Condition D

$$-\eta\,\frac{F_i}{A}\left(1+\frac{ey_2}{r^2}\right)+f_{cdb}+f_{cdlb}\leqq o \text{ or } f_{ct} \qquad (4)$$

Case 2

Where e is less than $\frac{r^2}{y}$, or where the reinforcement is within the middle third.

Condition A

This condition will not apply in this case as the upper fiber will always be in compression when the reinforcement is within the middle third of the beam.

Condition B

$$-\frac{F_i}{A}\left(1-\frac{ey_1}{r^2}\right)+f_{cdu}+f_{cul}\leqq f_c$$

This condition must be met at time of prestressing, hence no "η" factor.

Condition C

Same as for Case I

Condition D

Same as for Case I

Value e, the eccentricity, is considered positive if it is below the neutral axis; that is, if it is measured in the same direction as value y_2.

Next, subtracting equations (1) and (2)

$$- \frac{F_i}{A}\left(\frac{ey_1}{r^2} - 1\right) + f_{du} \leqq -f_{tc}$$

$$-\eta \frac{F_i}{A}\left(\frac{ey_1}{r^2} - 1\right) + f_{du} + f_{ul} \leqq f_c$$

$$(-1 + \eta)\frac{F_i}{A}\left(\frac{ey_1}{r^2} - 1\right) \quad - f_{ul} \leqq -f_{tc} - f_c$$

or

$$(1-\eta)\frac{F_i}{A}\left(\frac{ey_1}{r^2} - 1\right) \quad + f_{ul} \leqq f_{tc} + f_c$$

Since $f_{ul} = \dfrac{M_1 \quad y_1}{I}$

Then $(1-\eta)\dfrac{F_i}{A}\left(\dfrac{ey_1}{r^2} - 1\right) + \dfrac{M_1 \quad y_1}{I} \leqq f_{tc} + f_c$

$$\frac{M_1 \quad y_1}{I} = f_{tc} + f_c - (1-\eta)\frac{F_i}{A}\left(\frac{ey_1}{r^2} - 1\right)$$

$$\frac{I}{M_1 \quad y_1} \geqq \frac{1}{f_{tc} + f_c - (1-\eta)\frac{F_i}{A}\left(\frac{ey_1}{r^2} - 1\right)}$$

or

$$\frac{I}{y_1} \geqq \frac{M_1}{f_{tc} + f_c - (1-\eta)\frac{Fi}{A}\left(\frac{ey_1}{r^2} - 1\right)} \tag{5}$$

Then adding equations (3) and (4)

$$+\frac{F_i}{A}\left(1 + \frac{ey_2}{r^2}\right) \quad - f_{db} \leqq f_c$$

$$-\eta\frac{F_i}{A}\left(1 + \frac{ey_2}{r^2}\right) \quad + f_{db} + f_{bl} \leqq f_{ct}$$

$$(1-\eta)\frac{F_i}{A}\left(1 + \frac{ey_2}{r^2}\right) \quad + f_{bl} \leqq f_c + f_{ct}$$

Substituting $f_{b1} = \dfrac{M_1 \; y_2}{I}$

and simplifying as in the previous derivation:

$$\frac{I}{y_2} \geqq \frac{M_1}{f_{tc} + f_c - (1-\eta)\left(\frac{F_i}{A}\right)\left(1 + \frac{ey_2}{r^2}\right)} \qquad (6)$$

Next, assume that there is no loss of prestress ($\eta = 1$) and substituting in (5) and (6)

$$\frac{I}{y_2} = \frac{I}{y_1} \geqq \frac{M_1}{f_{tc} + f_c}$$

Since f_{tc} is nearly always equal to zero,

$$\frac{I}{y_2} = \frac{I}{y_1} \geqq \frac{M_1}{f_c} \qquad (7)$$

This equation indicates that only the live load moment influences the design dimensions of a prestressed beam. From this it may be concluded that a prestressed beam will be smaller than an ordinary reinforced concrete beam.

Also, it will be noted that equation (7) satisfies only two of the four conditions deduced from equations 1 through 4. These two conditions are (1) and (3) which occur at the time of prestressing.

The other two condition equations (2) and (4) are used to determine the values of F_i and e.

<u>Equilibrium equations</u>

<u>Fiber Stresses in Bending</u>

Having considered some of the general theoretical conditions affecting the behavior of a prestressed beam, the next item will be the adaptation of these general conditions to practical situations.

As a step in this direction, an evaluation of the terms $(1-\eta)\frac{F_i}{A}\left(1+\frac{ey_2}{r^2}\right)$ and $(1-\eta)\frac{F_i}{A}\left(\frac{ey_1}{r^2}-1\right)$ from equations (5) and (6) will be considered.

Experimental data from various researches as well as mathematical demonstrations set a value of η generally at 0.85. The value of $\frac{F_i}{A}$ must not be much larger than .5 f_c to prevent tension in the top fibers due to prestressing. The quantities $\frac{ey_1}{r^2}$ and $\frac{ey_2}{r^2}$ are usually equal to 2.

Substituting these values in the two terms in question result in

$$(1-0.85)\ (.5\ f_c)\ (1+2) = 0.225\ f_c$$

$$(1-0.85)\ (0.5\ f_c)\ (2-1) = 0.075\ f_c$$

Equations (5) and (6) now become

$$\frac{I}{y_1} \geqq \frac{M_1}{0.925\ f_c + f_{tc}} \quad \ldots\ldots\ldots\ldots \quad (8)$$

$$\frac{I}{y_2} \geqq \frac{M_1}{f_{tc} + 0.775\ f_c} \quad \ldots\ldots\ldots\ldots \quad (9)$$

Using equations (8) and (9), the dimensions of a beam may be determined. These dimensions would be only temporary as they must be checked using all four condition equations, and revisions may be necessary.

Since equation (9) gives a larger section modulus than does

equation (8) this equation should be used when solving for values of a symmetrical section ($y_1 = y_2$).

Rectangular beams and Slabs

The section modulus for a rectangular beam is $\frac{I}{C} = \frac{bd^2}{6}$

Substituting this value in equation (8)

$$\frac{bd^2}{6} \geqq \frac{M_1}{0.775\ f_c} \qquad (10)$$

Solving for d

$$d = \sqrt{\frac{M_1\ 6}{b\ (0.775\ f_c)}} = 2.78\sqrt{\frac{M_1}{b\ \ f_c}} \qquad (11)$$

In this equation, M_1 is in # in., b is in inches, and f_c in #/in^2. The depth d, will then be in inches.

Graphical Analysis of Fundamental Conditions

The four condition equations have several characteristics in common. These are:

1. All contain the values F_i and e.
2. All values in each equation, except F_i and e, are constant for a given section of a given design.
3. All contain linear function expressions for F_i and e.

These characteristics indicate that a possible graphic solution may provide a rapid, easy method of determining desirable values for the concrete prestress, F_i, and the eccentricity, e.

A graphic analysis of each equation separately and collectively to determine the feasibility of the above assumption will follow.

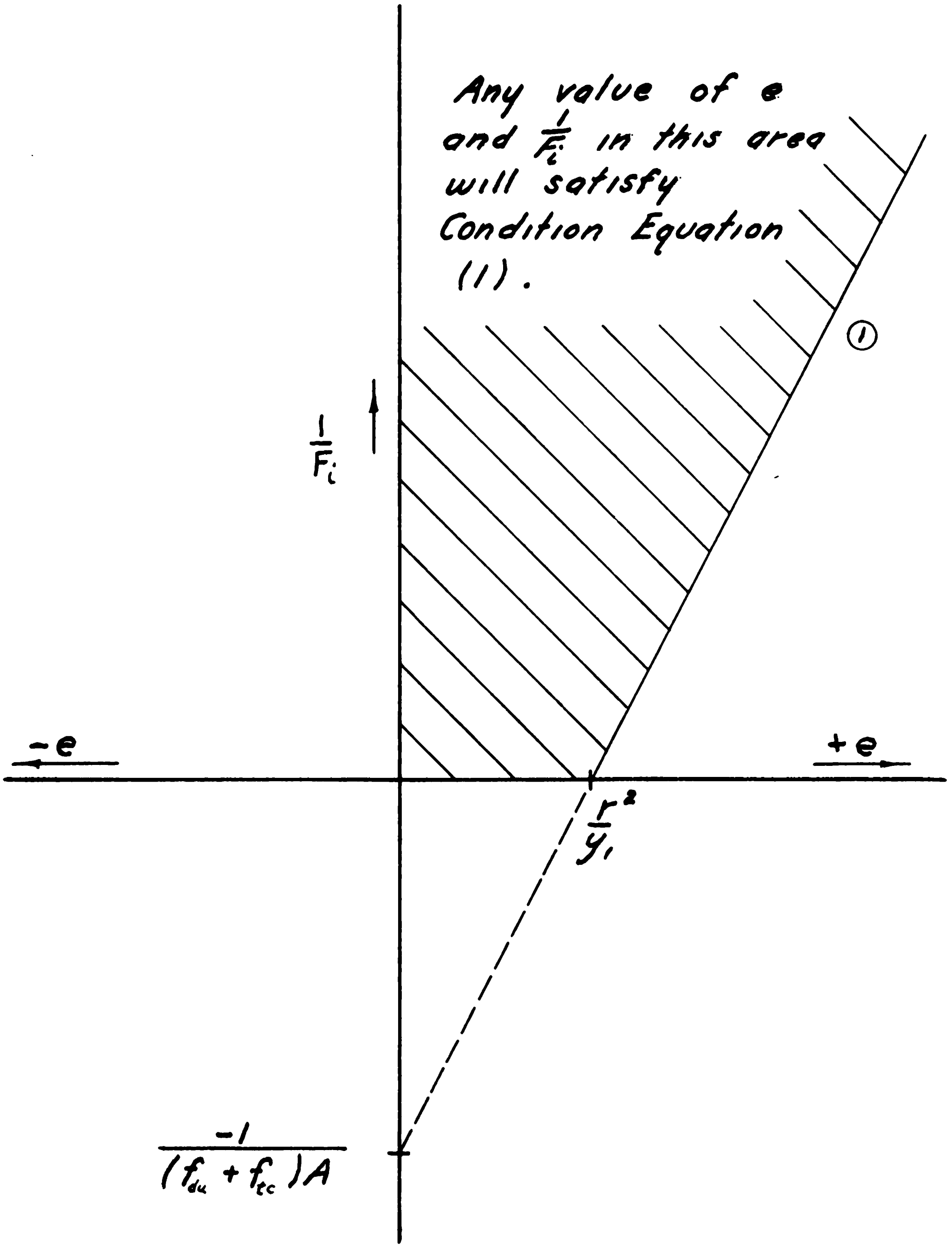

Any value of e and $\frac{1}{F_i}$ in this area will satisfy Condition Equation (1).
①
$\frac{1}{F_i}$
−e
+e
$\frac{r^2}{y_1}$
$\frac{-1}{(f_{du} + f_{tc})A}$

Fig. 3

Condition equation (1)

If e is less than $\frac{r^2}{y_1}$ any positive value of F_i and e will satisfy this condition.

If e is greater than $\frac{r^2}{y_1}$, equation 1 may be expressed with F_i as a function of e as follows:

$$\frac{-F_i}{A}\left(\frac{ey_1}{r^2} - 1\right) + f_{du} \leqq -f_{tc} \qquad (1)$$

$$\frac{F_i}{A}\left(\frac{ey_1}{r^2} - 1\right) = f_{du} + f_{tc}$$

$$F_i\left(\frac{ey_1}{r^2} - 1\right) \geqq (f_{du} + f_{tc})\, A$$

$$\frac{1}{F_i\left(\frac{ey_1}{r^2} - 1\right)} \geqq \frac{1}{(f_{du} + f_{tc})\, A}$$

or

$$\frac{1}{F_i} \geqq \frac{\left(\frac{ey_1}{r^2} - 1\right)}{(f_{du} + f_{tc})\, A}$$

To graph this quantity, plot e vs. $\frac{1}{F_i}$, Then the ordinates will be

for $e = 0$ $\quad \frac{1}{F_i} = \frac{-1}{(f_{du} + f_{tc})\, A}$

and for $\frac{1}{F_i} = 0$

$$e = \frac{r^2}{y_1}$$

See Fig. 3 for the results.

Condition Equation (2)

If e is less than $\frac{r^2}{y_1}$, (case 2) equation 2 may be written as

$$\frac{1}{F_i} \geqq \frac{\left(1 - \frac{ey_1}{r^2}\right)}{(f_c - f_{du} - f_{ul})\, A}$$

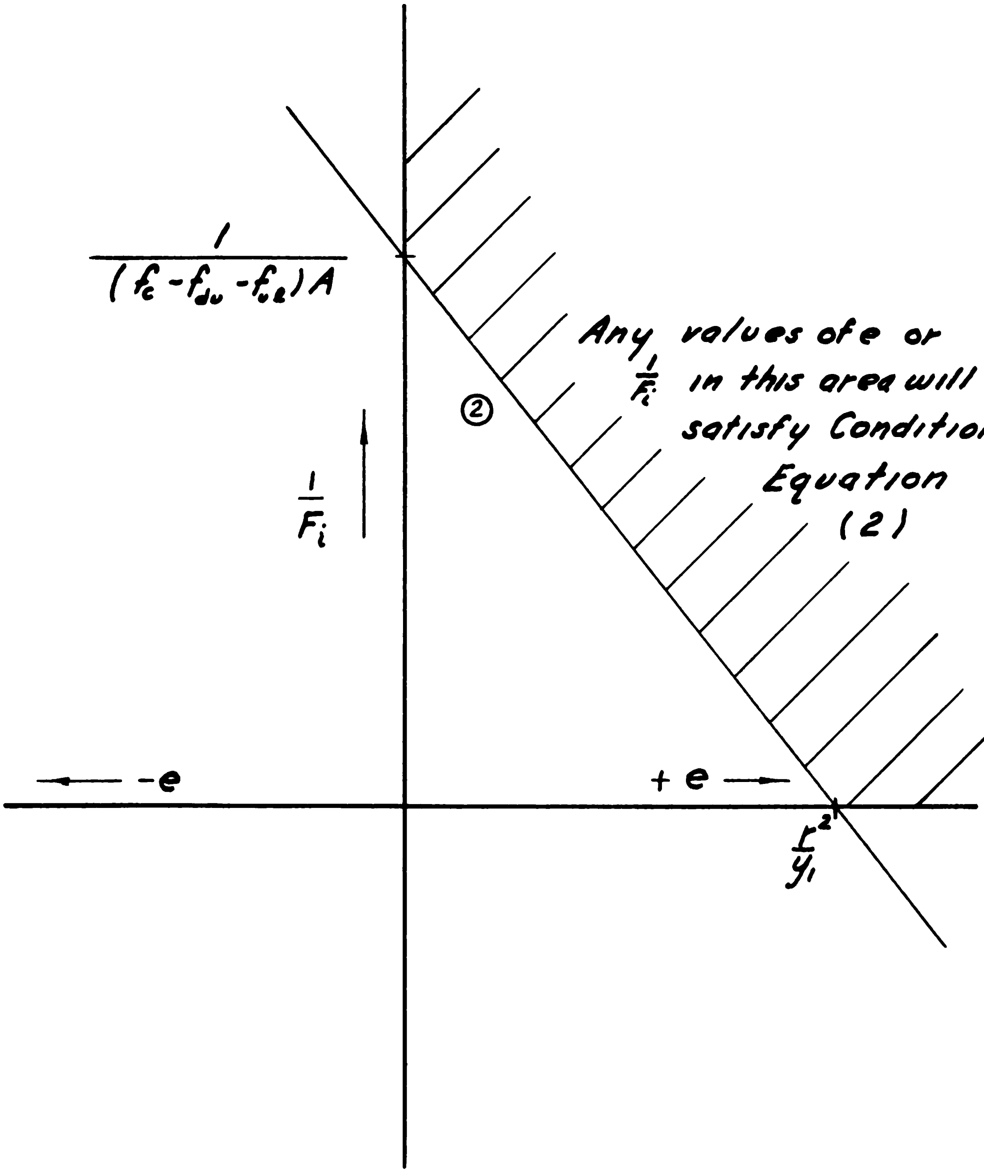
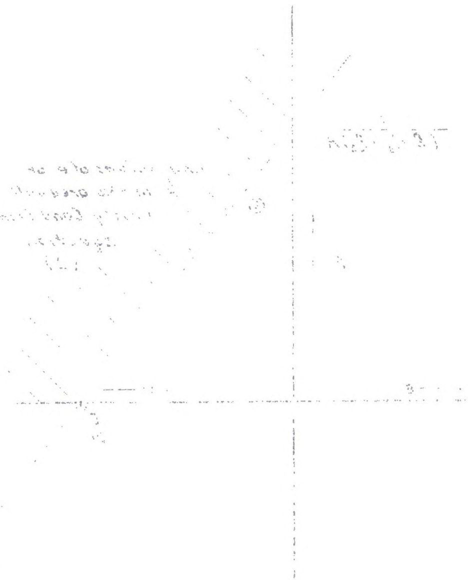


Fig. 4-A

For e less than $\frac{r^2}{y_1}$ (Case 2)
or where $f_c > f_{du} + f_{ul}$



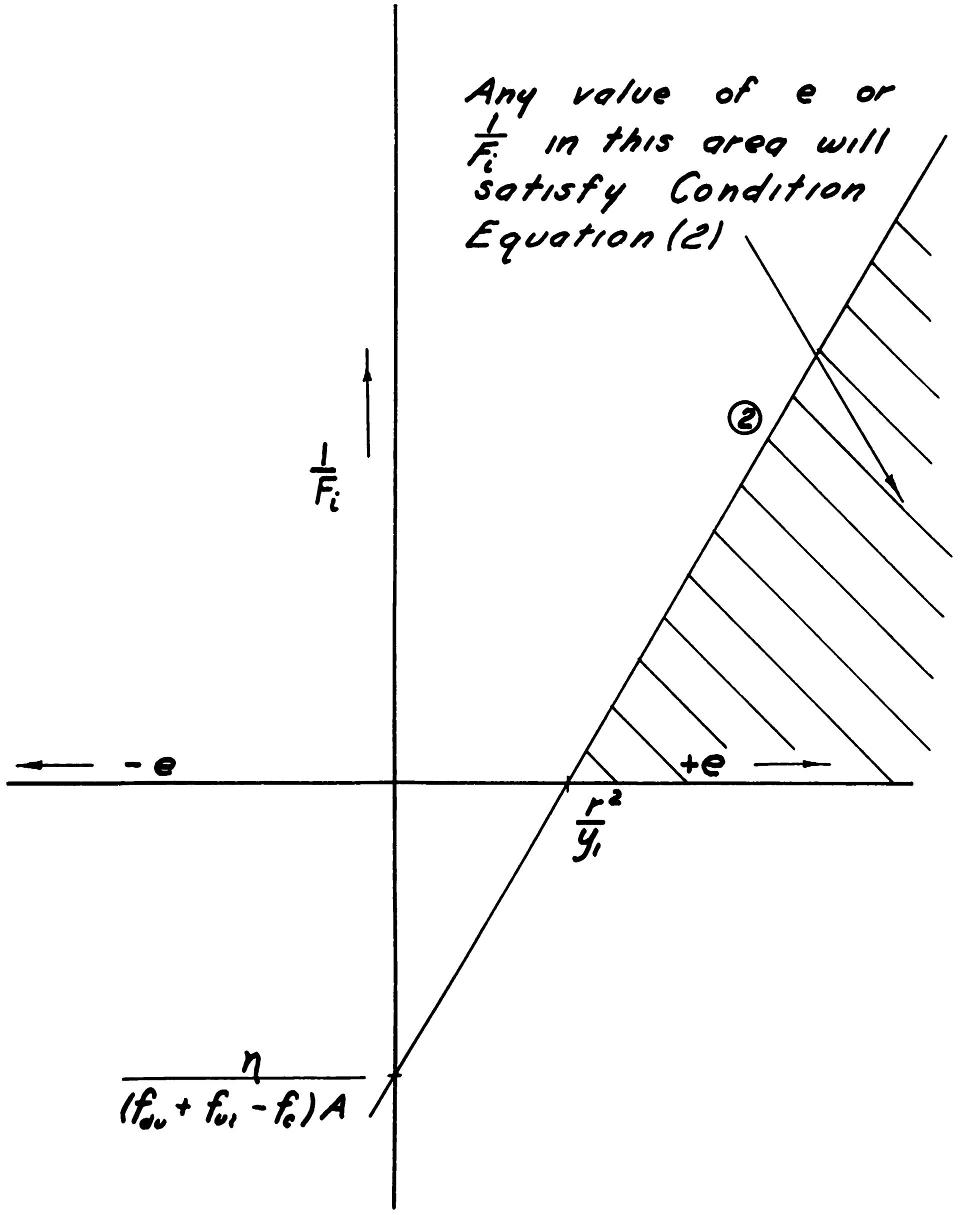


Fig. 4-B

For e greater than $\frac{r^2}{y_1}$ (Case 1) or where $f_c < f_{du} + f_{ul}$.

The ordinates will then be

for $e = 0$, $\frac{1}{F_i} \geqq \frac{1}{(f_c - f_{du} - f_{ul})\ A}$

for $\frac{1}{F_i} = 0$, $e = \frac{r^2}{y_1}$

and if e is greater than $\frac{r^2}{y_1}$ (case 1), equation 2 may be written as

$$\frac{1}{F_i} = \frac{-\eta\,(1 - \frac{ey_1}{r^2})}{(f_c - f_{du} - f_{ul})\ A} \quad \text{or} = \frac{\eta\,(1 - \frac{ey_1}{r^2})}{(f_{du} + f_{ul} - f_c)\ A}$$

The ordinates will then be:

for $e = 0$, $\frac{1}{F_i} = \frac{\eta}{(f_{du} + f_{ul} - f_c)\ A}$

and for $\frac{1}{F_i} = 0$, $e = \frac{r^2}{y_1}$

See Fig. 4 (A and B) for the results of these two conditions.

<u>Condition Equation</u> (3)

If e is greater than $-\frac{r^2}{y_1}$ (case 1), equation (3) may be written as:

$$\frac{1}{F_i} \geqq \frac{(1 - \frac{ey_2}{r^2})}{(f_c + f_{db})\ A}$$

The ordinates will be:

for $e = 0$, $\frac{1}{F_i} = \frac{1}{(f_c + f_{db})\ A}$

and for $\frac{1}{F_i} = 0$, $e = -\frac{r^2}{y_2}$

If e is less than $-\frac{r^2}{y_1}$, (case 2) condition (3) will be satisfied for any value of F_i and e.

See Fig. 5 for the results of these conditions.

Condition Equation (4)

If e is less than $-\frac{r^2}{y_2}$, (Case 2) this condition is never satisfied.

If e is greater than $-\frac{r^2}{y_2}$, this equation may be written:

$$\frac{1}{F_i} \geqq \frac{-\eta\left(1 - \frac{ey_2}{r^2}\right)}{(f_{tc} - f_{db} - f_{lb})\, A} \quad \text{or} \quad \frac{\eta\left(1 - \frac{ey_2}{r^2}\right)}{(f_{db} + f_{lb} - f_{tc})\, A}$$

for $e = 0$ $\qquad \frac{1}{F_i} = \frac{\eta}{(f_{db} + f_{lb} - f_{tc})\, A}$

for $\frac{1}{F_i} = 0$ $\qquad e = -\frac{r^2}{y_2}$

Fig. 6 shows the results.

It will be noted that in conditions (1), (3) and (4) the graphs will satisfy these conditions and may be plotted to be used to solve for values of F_i and e. However, condition (2) imposes two possibilities. The first of these two occur when e is greater than $\frac{r^2}{y_1}$ which is case 1. Note that this condition also occurs when $f_{du} + f_{ul}$ is greater than f_c. The second possibility occurs when e is less than $\frac{r^2}{y_1}$ which is case 2. Also, this possibility occurs when f_c is greater than $f_{du} + f_{ul}$. See Fig. 7 for results of these two possibilities.

Since the values of y_1, y_2, I, b, f_c and f_{tc} may be computed in a practical problem without the use of F_i or e, the graphic method may provide an easy, accurate means of solving for suitable values for F_i and e. It thus appears that a graphic method may be a plausable one for solution of a practical problem.

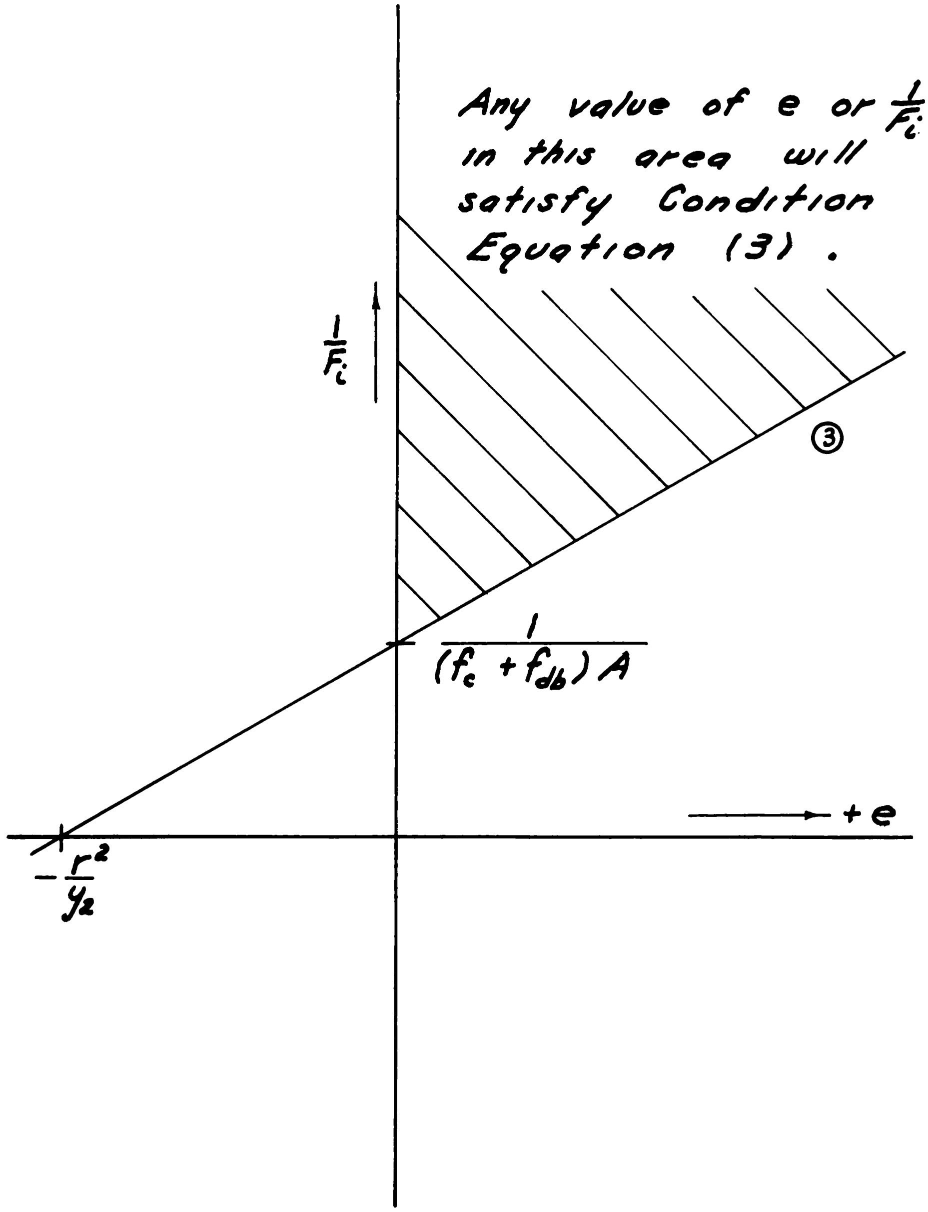
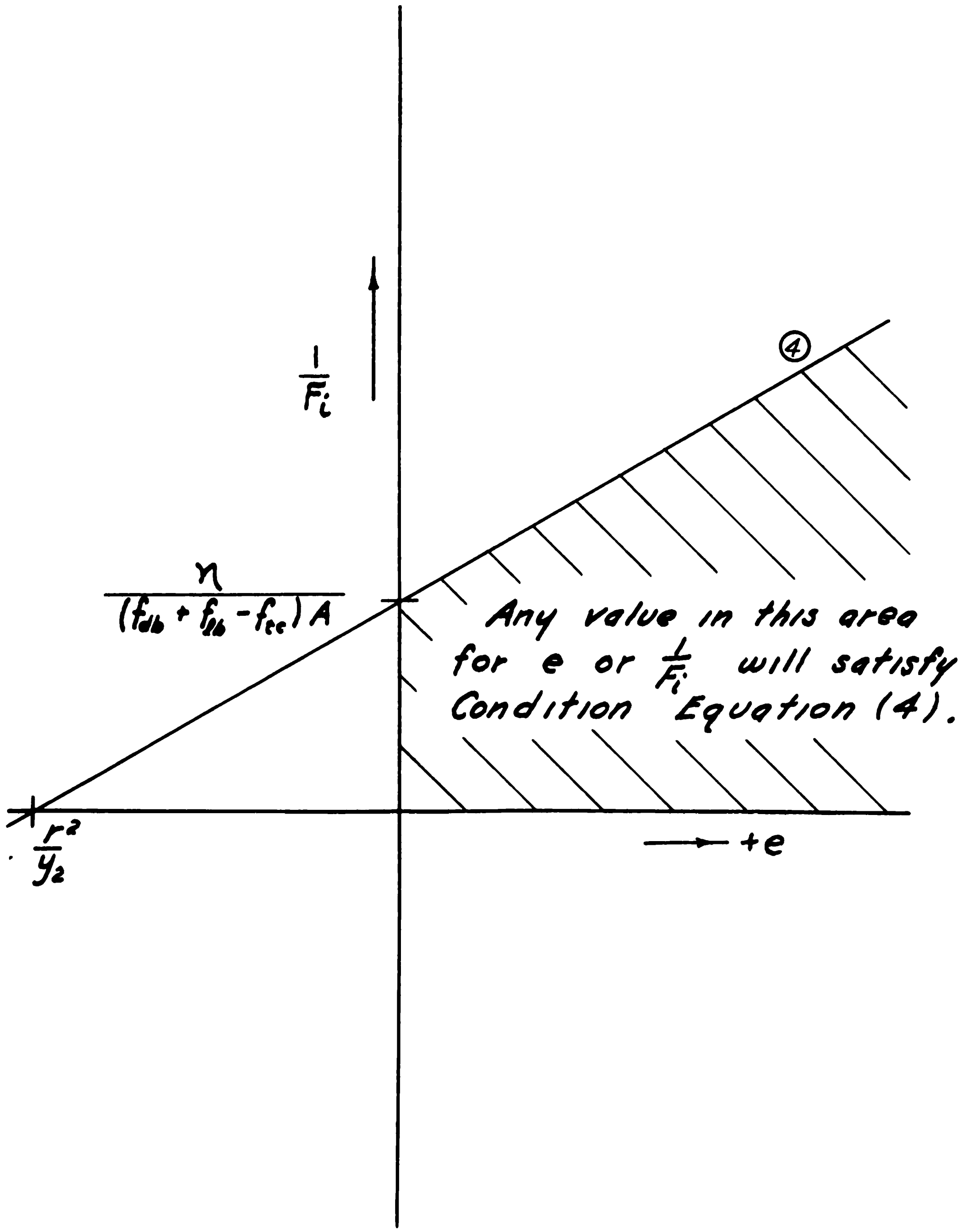

Any value of e or $\frac{1}{F_i}$ in this area will satisfy Condition Equation (3).
$\frac{1}{F_i}$
③
$\frac{1}{(f_c + f_{db})A}$
+e
$-\frac{r^2}{y_2}$

Fig. 5



$\frac{1}{F_i}$
④
$\frac{n}{(f_{db} + f_{gb} - f_{tc}) A}$
Any value in this area for e or $\frac{1}{F_i}$ will satisfy Condition Equation (4).
$\frac{r^2}{y_2}$
+e

Fig. 6

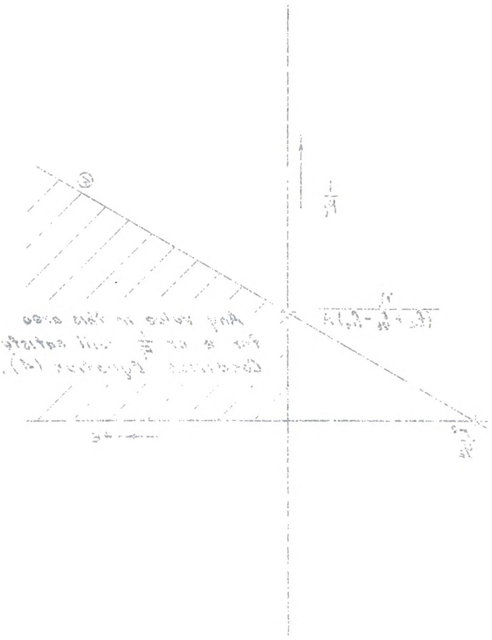
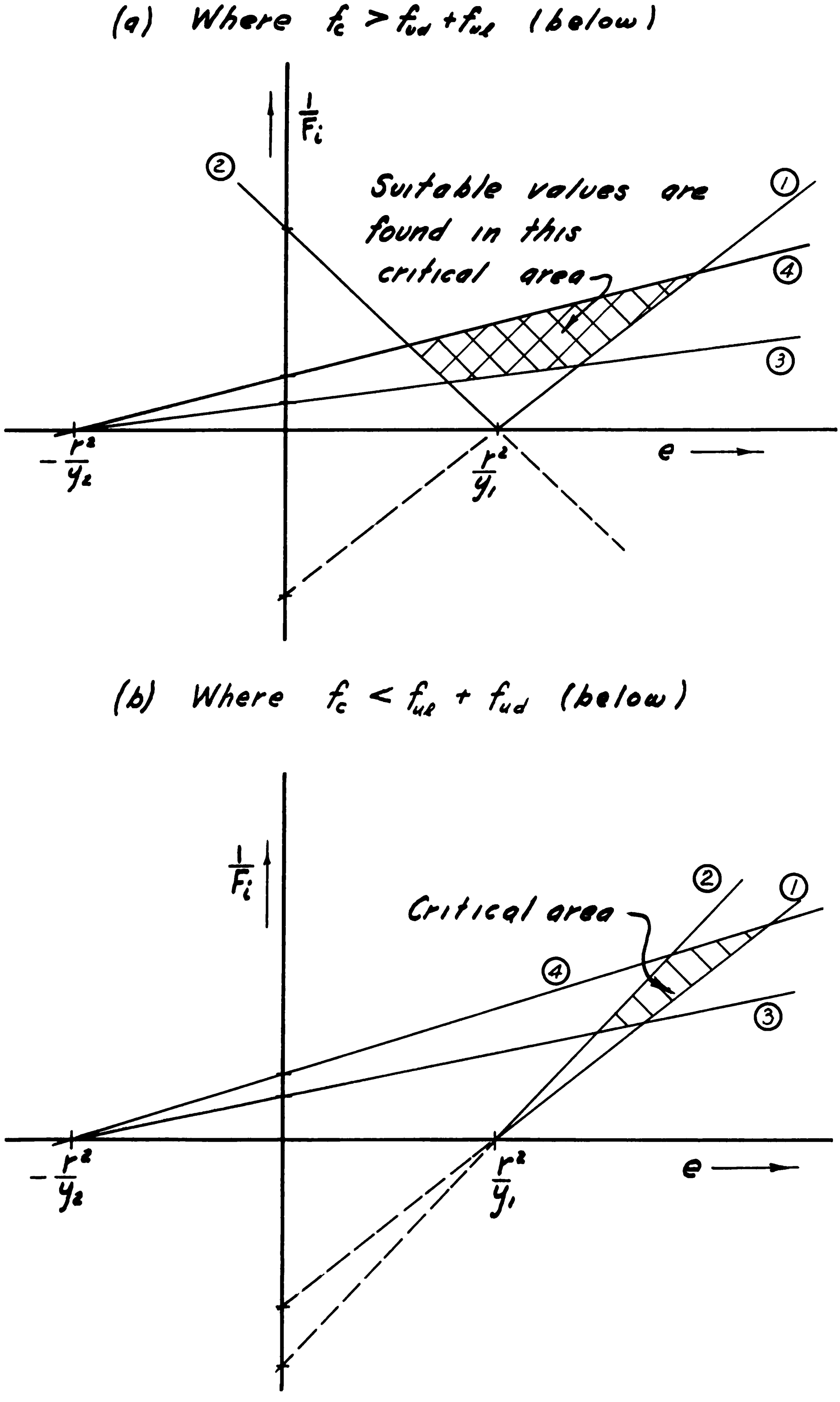
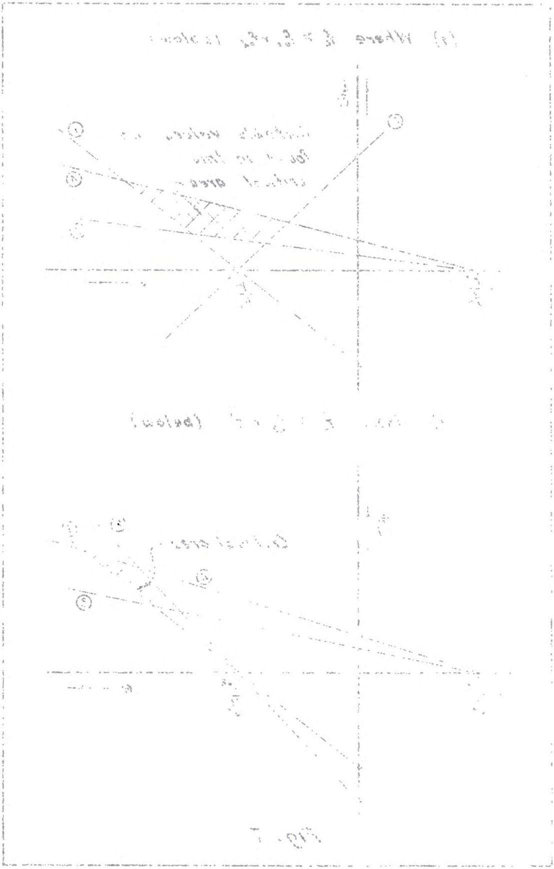

Any value in this area
for a or $\frac{1}{k}$ will satisfy
Condition Equation (45).

Fig. 5



(a) Where $f_c > f_{ud} + f_{ul}$ (below)
$\frac{1}{F_i}$
Suitable values are found in this critical area
① ② ③ ④
$-\frac{r^2}{y_2}$
$\frac{r^2}{y_1}$
e
(b) Where $f_c < f_{ul} + f_{ud}$ (below)
$\frac{1}{F_i}$
Critical area
① ② ③ ④
$-\frac{r^2}{y_2}$
$\frac{r^2}{y_1}$
e

Fig. 7



Bond:

In prestressed concrete, bond is of importance only in cases where the prestress is transferred from the steel to the concrete through the oond.

In cases where the prestress is established by contact or end bearing plates, there is in reality no bond present between the steel and concrete; therefore, no bond stresses are considered.

Where the reinforcement is embedded in the concrete under initial tensile stress, a very strong bond is produced as follows:

> Upon release of the force producing tension in the steel, the tension in the steel is reduced due to the elastic shortening of the concrete and subsequent volume changes, such as shrinkage. As this happens, the wire reinforcement develops a tendency to enlarge its diameter slightly, as determined by Poisson's Ratio. As may be readily seen, this action is similar in effect to that of a "forced fit", where considerable resistance to sliding is developed in spite of smooth contact surfaces.

Thus, the bond developed in prestressed concrete is greater than is developed in reinforced concrete due to the "force fit" action.

Rosov develops an equation which he suggests may be used to check the bond stresses. It is:

$$\mu = V \frac{pn\,(q-k)}{C \Sigma o\, d} \quad \text{or} \quad V \frac{pn\, e}{C \Sigma o\, d}$$

where μ = the unit bond stress in #/in^2.

V = the total shear at the section in question

p = percentage of steel in the entire cross section

n = ratio of $\frac{E_s}{E_c}$

$(q-b) = e$ = the eccentricity

C = a constant $\frac{1}{12} + (n-1)\ (pe^2)$

ΣO = Total perimeter of the steel

d = depth of a beam

He concludes with the statement "In fact, the most cases of prestressed beams, the bond to be provided is relatively weak and need not be considered".

This statement will be brought into sharper focus in the computation of bond for the bridge design problem.

Shear or Diagonal Tension

In conventional reinforced concrete design, diagonal tension is of great importance, and to offset it, an intricate system of special reinforcement must be designed.

The value of the diagonal tension may be expressed by the use of the equation for combined stresses which becomes:

$$T_d = \tfrac{1}{2}f \pm \sqrt{(\tfrac{1}{2}f)^2 + v^2} \qquad (12)$$

in which f is the fiber stress and v is the shear produced by the bending moment at a given section.

Consider next, the application of this equation to the prestressed beam or slab. Since the longitudinal stress is

always compressive for a prestressed beam, and since the maximum unit shear stress does not occur at the same place in the beam as the maximum fiber stress, the diagonal compression will never be much in excess of the allowable. However, if shear exists, there will always be a diagonal tensile stress existing.

Using a negative sign to denote compression, equation (12) may be written:

$$T_d = -\tfrac{1}{2}f + \sqrt{(\tfrac{1}{2}f)^2 + v^2}$$

$$T_d = -\frac{f}{2} + (\frac{f^2}{2} + v^2)^{\frac{1}{2}}$$

$$T_d = -\frac{f}{2} + \frac{f}{2}\left(1 + \frac{4v^2}{f^2}\right)^{\frac{1}{2}}$$

This may be expanded using the binomial theorem into:

$$T_d = -\frac{f}{2} + \frac{f}{2}\left[1 + \frac{2v^2}{f^2} - \frac{2v^4}{f^4} + \cdots\cdots\right]$$

This series is convergent only if $v < \tfrac{1}{2}f$. For values of v greater than $\frac{f}{2}$, the series does not define the radical.

For the first approximation,

$$T_d = \frac{v^2}{f} - \frac{v^4}{f^3} \qquad (13)$$

This equation may be used to check the diagonal tension at any point on a beam, computing the unit shear v from the equation:

$$v = \frac{VQ}{Ib}$$

and using the fiber stress determined from the combined prestress moment and the dead and live load moments at the given section.

From equation (12) it is seen that the value for diagonal tension is much smaller than for a conventional reinforced beam where the diagonal tension is computed from $v = \frac{8V}{7\,bd}$.

Limitations

Care must be taken in using the previously derived equations for two reasons:

(a) The equations may have been derived specifically for a certain structural element, that is a rectangular slab or beam.

(b) Many of the equations were developed on the assumption that there will be no tensile stresses taken by the concrete, that is, $f_{tc} = o$. Actually concrete may take some tensile stress up to 3% of the ultimate strength of the concrete in compression, (1941 J. C. specifications for flexure), and the beam may be designed for some tensile stresses in the concrete. This practice is not recommended, but circumstances may alter the recommendation.

APPLICATION OF DESIGN THEORY

Use of High Working Stresses

Freyssinet and other researchers (1,8,) have proven rather conclusively that mild steel is not suitable for use in prestressed concrete. Their studies show that high tensile strength steel and high strength concrete makes a good combination for prestressed work. The next question to be considered is one involving reasonable stress in both concrete and steel.

Concrete

The concrete used for prestressed work should have an early high strength as well as a high later strength. This would allow for the prestress force on the wires to be released sooner. In factory fabrication, this is particularly advantageous as it makes possible the more rapid release of space for further work.

For this reason, the concrete should have an ultimate strength of at least 5,000 #/in^2. Using a safety factor of 3 would make the allowable compressive stress equal to 1,667 #/in^2. Following the 1941 J.C. recommendations (Sec. 878)would make the allowable compressive stress equal to 0.45 (5,000), or 2,250 #/in^2.

Due to many variable factors in the processing of the concrete, a safety factor of at least 3 is recommended.

Steel

A general rule in the design of reinforced concrete beams

is not to permit steel stresses to be greater than about 50% of the yield stress. This is justified by many reasons. First, the bar may have a local defect. Also, the bars are subjected to severe variations in stress which may lead to fatigue.

In prestressed concrete, these factors do not exist. Actually, every wire is stretched separately to a stress higher than it will ever be subjected to afterward, and if a wire fails, it may be replaced before the concrete is poured. There is no danger of fatigue as the stress is nearly constant. The additional stress imposed by the live load on a prestressed beam rarely exceeds 10% of the stress in the steel due to prestressing. For these reasons, and backed by research investigations of several researchers [8,10], it is generally agreed that the stress in the wires should not exceed 0.8 of the equivalent yield stress (0.2 permanent strain) or 0.6 of the tensile strength, whichever is smaller.

It should be remembered that the stresses must not be exceeded as this allows for a much smaller factor of safety. This is allowable as the steel is made under more controlled conditions and is tested under an actual excess tensile stress before using it in the beam.

DESIGN OF A HIGHWAY BRIDGE

Given Data:

Span length 32' - 0"

Span width 46' - 0" (consisting of four 10 ft. lanes + 2,3'-0" sidewalks and rails)

Span loading - ASSHO H-20

Allowable stress in the concrete

compression, f_c, 1500 #/in^2

compression ultimate 4500 #/in^2

tension f_c 0.#/in^2

η = 0.85 (loss in prestress with age)

Allowable steel tension

120,000 #/in^2

This requires a steel tensile strength of

200,000 #/in^2

(0.6 of tensile strength or 0.8 yield strength)

Use a wire of 0.2 inch diameter for reinforcing.

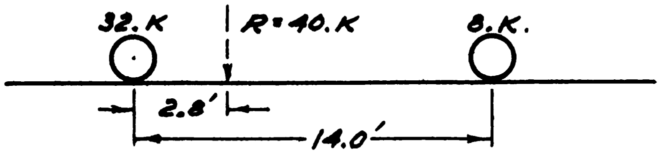
Live Loading

ASSHO H-20

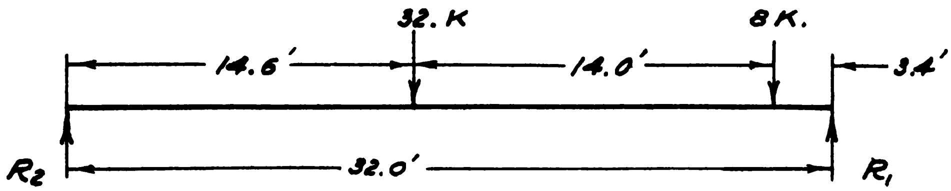
$$\text{Impact} = \frac{50}{L - 125} = \frac{50}{167} = 30\%$$

However, since the concrete stress was somewhat on the conservative side (0.33) a value of 25% for impact should be sufficient.

Lane Loading - for 10' Lane



Span Loading for Maximum Moment



$\Sigma M_{r_2} = 0$

$+ 14.6\ (32) + 28.6\ (8) = 32\ R_1$

$58.4 + 28.6 = 4\ R_1$

$\frac{87}{4} = R_1 = 21.75$ K

$R_2 = 40 - 21.75$

$R_2 = 18.250$ K

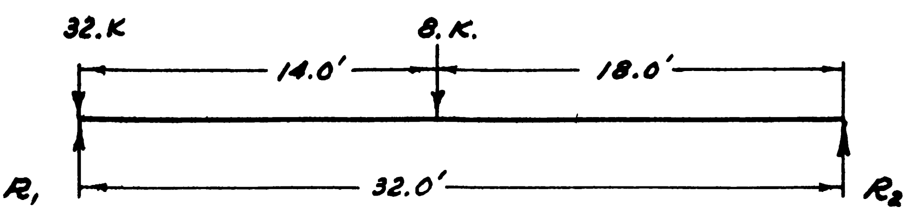
Maximum moment = 18,250 #(14.6)

= 226,450 ft. #   Lane Load

$$\text{Moment per foot} = \frac{226{,}450 \text{ ft. \#}}{10 \text{ ft. width}} = 26{,}645 \frac{\text{ft \#}}{\text{ft}} \text{width}$$

<u>Shear</u>

Span loading for maximum shear



$$R = V = 32{,}000\ \# + \frac{18}{32}(8) = 4.5\ K$$

$$V = 32{,}000\ \# + 4{,}500\ \#$$

$$V = 36{,}500\ \#\ \text{for lane loading}$$

$$\text{Shear per 1 foot width} = \frac{36{,}500\#}{10\ \text{ft.}} = 3650.\ \#/\text{ft.}$$

Maximum Live Load Moment

$$\text{Live Load} + 30\%\ \text{Impact} = 26{,}645\ \text{ft.}\ \# + .25\ (26{,}645)$$

$$= 33{,}306.\ \text{ft.}\ \#$$

COMPUTATIONS

(a) Compute the depth of the beam using (11)

$$d = 2.78\sqrt{\frac{M_1}{b\ f_c}} = 2.78\sqrt{\frac{33{,}306 \times 12}{12\ (1500)}} = 2.78\ \sqrt{22.2}$$

$$d = 2.78\ (4.71) = 13.1\ \text{inches}$$

(b) Assume a section to satisfy equations (8) and (9)

$$\frac{I}{y_2} \geqq \frac{M_1}{0.775\ f_c + f_{tc}} = \frac{33{,}306 \times 12}{1162.5} = 342.\ \text{in.}^3$$

Use a depth of 14" and assume section as shown.

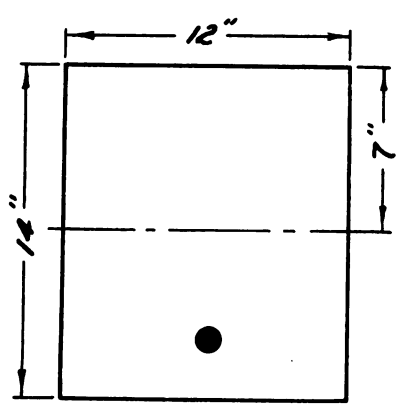
Checking

$$\frac{I}{C} = \frac{bd^2}{6}$$

$$= \frac{12 \times 14 \times 14}{6}$$

$$= 28 \times 14$$

$$= 392.\ \text{in.}^3$$



This section will work satisfactorily.

(c) Compute the dead load weight.

Using the weight of concrete = 150. #/ft.^{3}

$$w_d = 150 \times \frac{14}{12} = 175 \text{ \#/ft.}^2$$

$$M_d = \frac{1}{8} wl^2 = \frac{175\,(12)\,(32)\,(32)}{8} = 1536 \times 175$$

$$= 268{,}500 \text{ in. \#/ft. width}$$

(d) Compute the concrete stresses using $S = \frac{M\,c}{I}$

or $S = \frac{6\,M}{bd^2}$

$$f_{du} = f_{db} = \frac{268{,}500 \times 6}{12 \times 14 \times 14} = 686. \text{ \#/in}^2$$

$$f_{ul} = f_{lb} = \frac{33{,}306 \times 6 \times 12}{12 \times 14 \times 14} = 1020 \text{ \#/in}^2$$

(e) In this case, f_c is less than $f_{du} + f_{lb}$.

$$f_c = 1500 \text{ \#/in}^2$$

$$f_{du} + f_{lb} = 686 + 1020 = 1706$$

$$1500 < 1706$$

(f) Then draw a diagram similar to Fig. 7 (b) for a graphical solution to obtain F_i and e.

Thus $\frac{r^2}{y_2} = \frac{r^2}{y_1} = \frac{16.3}{7} = 2.33$ in.

Ordinates for e = 0 are as follows:

For line 1 $\frac{-1}{(f_{ul}+f_{tc})\,A} = \frac{-1}{(686 + 0)\,A} = -\frac{1.46}{1000\,A}$

For line 2 $\quad \dfrac{\eta}{(f_{du}+f_{ul}-f_c)A} = \dfrac{0.85}{(-1706+1500)\ A} = \dfrac{-0.85}{206A} = -\dfrac{4.13}{1000A}$

For line 3 $\quad \dfrac{1}{(f_c+f_{db})\ A} = \dfrac{1}{(1500+686)\ A} = \dfrac{1}{(2186)\ A} = \dfrac{0.457}{1000A}$

For line 4 $\quad \dfrac{\eta}{(f_{db}+f_{lb}-f_{tc})\ A} = \dfrac{0.85}{(1706-0)\ A} = \dfrac{0.498}{1000\ A}$

(See the next page for this diagram)

Then for a value of e = 3".

$$\frac{1}{F_i} \times 1000\ A = 1.12$$

$$F_i = \frac{1000\ (12)\ (14)}{1.12} = 150{,}000\ \#/\text{foot width}$$

(g) Compute steel area.

$$\frac{F_i}{\text{allowable steel stress}} = \frac{150{,}000}{120{,}000} = 1.25\ \text{in}^2/\text{ft.}$$

Then using wires 0.2" (#5 American wire size)

$$\frac{1.25\ \text{in}^2}{0.0314} = 39.9 \qquad \text{Use 40 wires}$$

Spacing of wires

3 rows of 14 wires each per foot

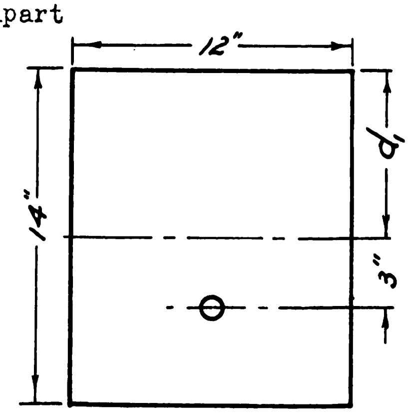
row spacing = .9" apart

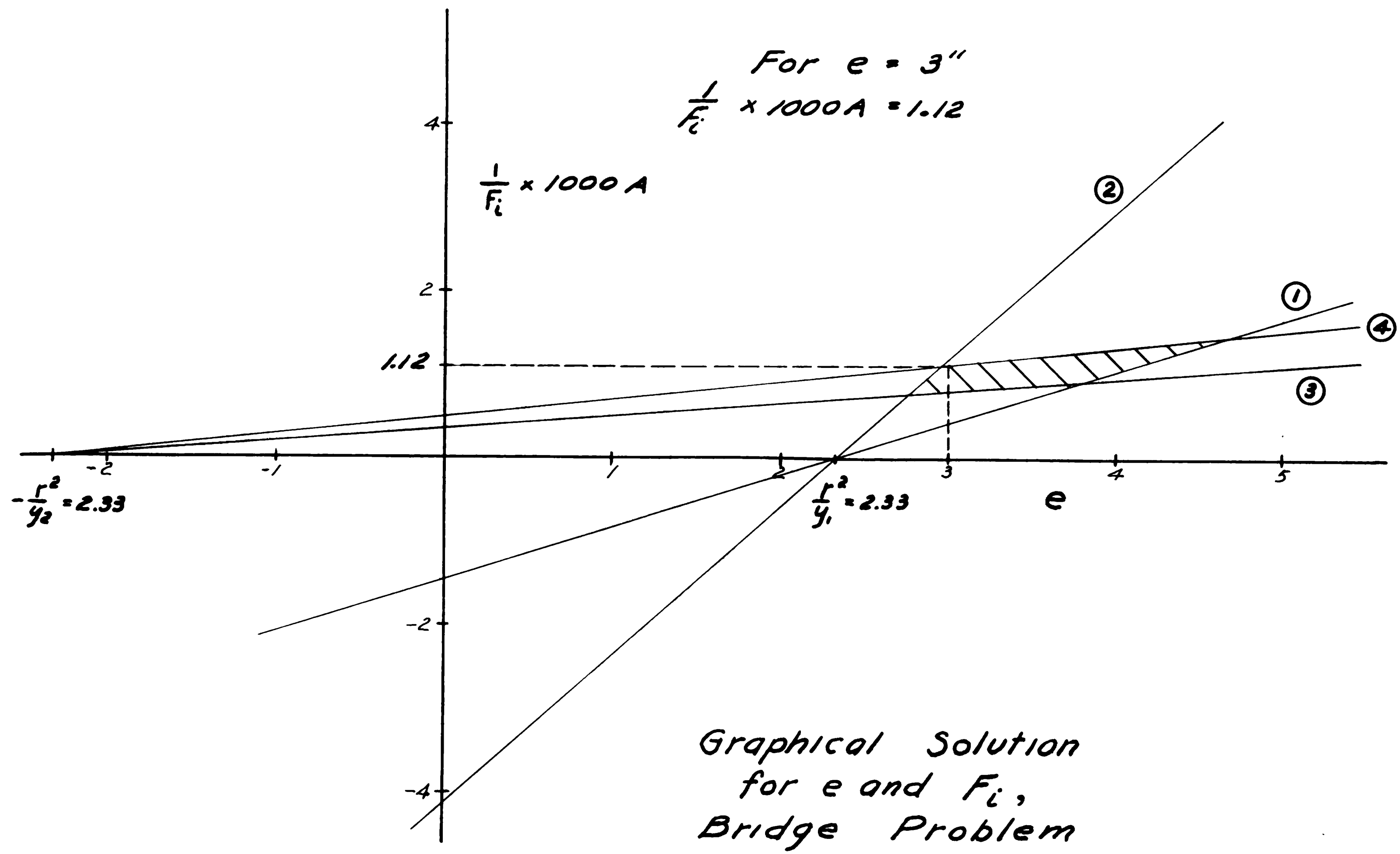
wire spacing = $\dfrac{12"}{14}$ = 0.85" apart

(h) Allow for reduced steel area

Find n. a. = d_1

a. $d_1 = bdx - a_s d$





Graphical Solution
for e and F_i,
Bridge Problem

$$d_1\,(166.75) = 7\,(14)(12) - 1.25\,(10)$$

$$= 1176 - 12.50$$

$$d = \frac{1163.5}{166.75} = 6.9775$$

$$d = 6.98$$

$y_1 = 6.98$

$y_2 = 7.02$

$I = 2732.75\ in^4$

$A = 166.75\ in^2$

$r^2 = 16.4$

$\frac{r^2}{y_1} = 2.342 \qquad \frac{r^2}{y_2} = 2.334$

$I = \frac{1}{12}bd^3 - Ad^2$

$I = \frac{1}{12}(12)(14)^3 - 9(1.25)$

$I = 2744 - 11.25$

$I = 2732.75\ in^4$

Stresses due to loads

$$f_{dt} = \frac{268,500\,(6.98)}{2732.7} = 689.\#/in^2$$

$$f_{db} = \frac{268,500\,(7.02)}{2732.7} = 693.\#/in^2$$

$$f_{at} = \frac{12(33,306)\,(6.98)}{2732.7} = 1021\ \#/in^2$$

$$f_{ab} = 1030\ \#/in^2$$

(i) Check concrete stresses

With $F_i = 150,000\ \#/ft.$ and $e = 3''$

Formula (1) $-\frac{F_i}{A}\left(\frac{ey_1}{r^2} - 1\right) + f_{dt} \leq f_t$

$$-\frac{150,000}{166.75}\left(3\,\frac{6.98}{16.4} - 1\right) + 689\#/in^2 = f_t$$

$$-900\,(1.273-1) + 689 = f_t$$
$$-900\,(0.273) + 689 =$$
$$-245 + 689 = f_t$$
$$+444 = f_t \qquad f_t = 0 \qquad + = \text{compression}$$

Formula (2)

$$-n \frac{F_i}{A} \left(\frac{ey_1}{r^2} -1 \right) + f_{dt} + f_{at} \leqq f_c$$

$$-0.85 \left(\frac{150,000}{166.75} \right) (0.273) + 689 + 1021 \leqq 1500$$

$$-0.85 \ (900) \ (0.273) + 689 + 1021 \leqq 1500 \ \#/in^2$$

$$-208 + 689 + 1021 \leqq 1500 \ \#/in^2$$

$$1502 \leqq 1500$$

Formula (3)

$$- \frac{F_i}{A} \left(1 + \frac{ey_2}{r^2} \right) + f_{db} \leqq f_c \quad 1500\#/in^2$$

$$-900 \left(1 + 3 \frac{7.02}{16.4} \right) + 693 \leqq 1500$$

$$-900 \left[1 + 3 \ (.428) \right] + 693 = 1500$$

$$-900 \ (1 + 1.282) + 693 < 1500$$

$$-2050 + 693 < 1500$$

$$-1357 < 1500$$

Formula (4)

$$-n \frac{F_i}{A} \left(1 + \frac{ey_2}{r^2} \right) + f_{db} + f_{ab} \leqq f_t = 0$$

$$-0.85 \ (900) \ (2.282) + 693 + 1030 \leqq 0$$

$$-1742.5 + 1723 \leqq 0$$

$$-19.5 = 0$$

The concrete stresses check within limits.

(j) Check Bond

$$\mu = V \frac{pn \ (q-b)}{c \Sigma o \quad d}$$

V = Total Shear

p = steel percentage = $\frac{1.25}{168}$ = .00745 = 0.745%

$q = \frac{10}{14} = 0.714$

b = breadth = 12"

$C = \frac{1}{12} + (\frac{1}{2} - k)^2 + (n-1) \quad p_t(k-q_t)^2 + p_b(q_b - k)^2$

n = ratio, $\frac{E_s}{E_c}$ = 6 (from 1941 J.C.)

$\Sigma o = \pi d \times$ no. of bars = 3.14(0.2)(40) = 25.1 in.

d = 14."

$k = \frac{6.98}{14} = 0.499$

$$C = 0.083 + 0 + 5\ [.00745\ (0.714 - 0.499)]$$

$$C = 0.083 + 5\ [0.00745\ (.215)]$$

$$C = 0.083 + 5\ (0.0016)$$

$$C = 0.083 + 0.008$$

$$C = 0.091$$

$$\mu = 3650\ \#\ \frac{0.00745(6)\ (-11.286)}{0.091 \times 25.1\ (14)}$$

$$\mu = 3650\ \#\ (0.0158$$

$$\mu = 57.6\ \#/in^2$$

Allowable minimum bond = $0.04\ f_c'$

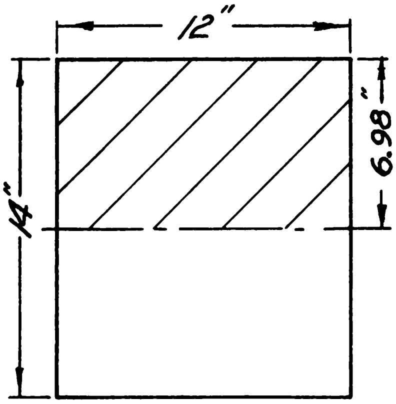
[using f_c' = 2000 (.04)] = 80#/in^2

(k) Diagonal Tension:

$f_d = \frac{v^2}{f_1}$ $\qquad f_1 = 1500\#/in^2$

$v = \frac{V}{bjd} = \frac{VQ}{Ib}$

$v = \frac{VQ}{I\ b}$



$$v = \frac{3650 \,\# \times 292 \text{ in}^3}{2732.75 \text{ in}^4 \times 12 \text{ in.}}$$

$$v = 32.6 \ \#/\text{in}^2$$

$$Q = 12 \times \frac{6.98 \times 6.98}{2}$$

$$Q = 6 \times 6.98 \times 6.98$$

$$Q = 292. \text{ in}^3$$

Using condition where shear is near end of beam, and only dead load stresses act-

$$f_1 = \frac{M\ c}{I} = \frac{Fe\ \ c}{I} = \frac{150{,}000(3)(7)}{2732} = \frac{1150}{2} = 575$$

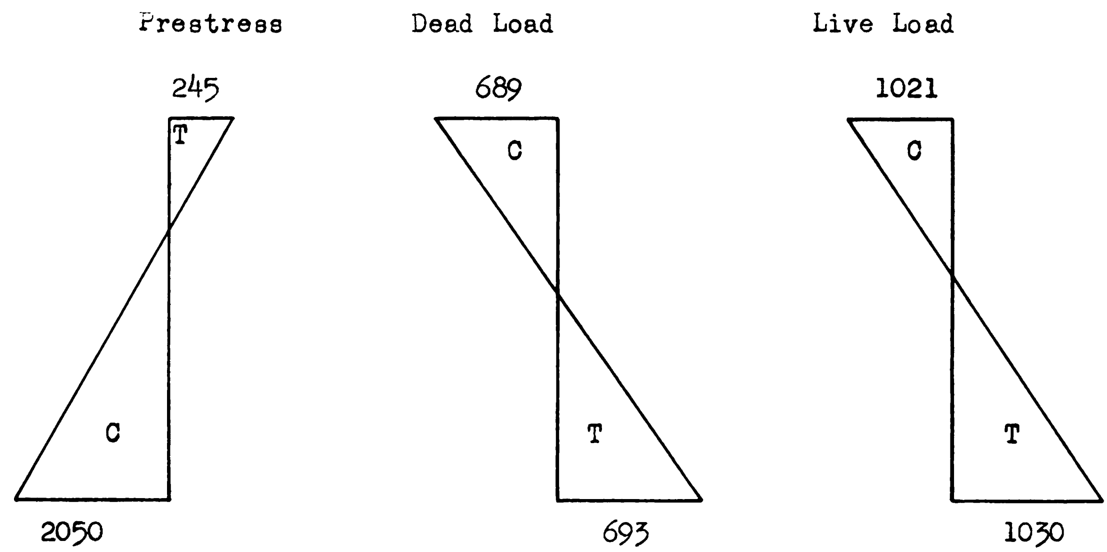
Using $\frac{1}{2}$ this value for a safety factor

$$f_d = \frac{(32.6)\ (32.6)}{575} = 1.85 \ \#/\text{in}^2$$

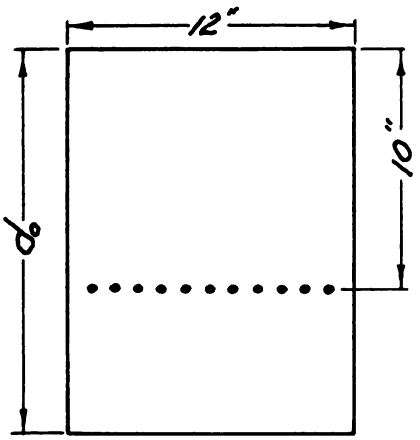
Allowable $.012\, f'_c$, (using as $f'_c = 2000$)= 24 $\#/\text{in}^2$

No condition of diagonal tension exists.

(1) Diagrams for Stresses in Fibers



(m) Section at Reaction Point or end of beam



Solve for d_o

$2/3\ d_o = 10"$

$d_o = \frac{10 \times 3}{2} = 15"$ minimum

$S = \frac{F_i}{A} \left(\frac{ey_1}{r^2} - 1 \right)$

$S = \frac{150{,}000}{180} \left[\frac{2.5}{18.8} \ (7.5) - 1 \right]$

$S = 834\ (0)$

$S = 0$

$I = \frac{1}{12} bd^3$

$I = (15)^3 = 3380\ in^4$

$r^2 = \frac{3380}{180} = 18.8$

Use $16" = d_o$

$A = 192$

$r^2 = 21.3$

$e = 2"$

$f_u = \frac{150{,}000}{192} \left[\frac{2\ (8)}{21.3} - 1 \right]$

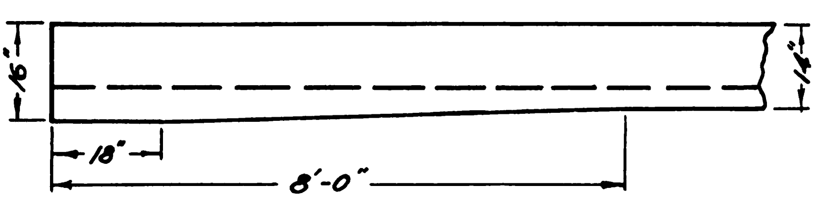
$f_u = 781\ (0.75 - 1)$

$f_u = + 195\ \#/in^2$ (compression)

$f_b = 781\ (1.75)$

$f_b = 1365.\#/in^2$

End section for slab shown below:



(o) Estimate of Steel used.

$\frac{40 \text{ wires}}{\text{ft.}} \times 48 \text{ ft.wide} \times 32 \text{ ft.long} = 61,440 \text{ ft.}$

weight of wire per ft. = 0.12 #

Cost of wire per # = 10¢

$61,440 \text{ ft.} \times \frac{.12\#}{\text{ft.}} \times \frac{\$0.10}{\#} = \$737.36$

The author has obtained from the Michigan State Highway Department a bridge similar to the design presented here, except it has been designed for conventional reinforced concrete.

Comparing the two designs on the basis of material costs only, the prestressed bridge is slightly cheaper. However, more savings may be obtained after considering the labor costs and savings in formwork. These factors will not be considered here.

Steel used in conventional design = 10 tons

Cost of steel at $140.00 per ton =	$1400.00
	- 737.36
Difference	$662.64

Savings in material-

10 yds. of concrete at $6.00 per yd. =	$ 60.00
Reinforcing steel	662.60
	$722.60

See drawing of this bridge in the inside back cover.

METHODS OF PRESTRESSING STEEL

One of the chief disadvantages in the use of prestressed concrete is the relative high cost and difficulty of applying the prestress force to the reinforcing steel. Many methods have been devised in the past to perform this operation and all have met with varying degrees of success. In general, these methods may be divided into mechanical and electrical prestressing.

Mechanical Prestressing

The first method to be described in this type is the Hoyer System. The system is usually adaptable only under factory conditions. The wire is stretched between two rigid butresses and the concrete is poured around them. After the beam or slab has hardened, the wires are cut next to the ends of the beam. The hardened beam is then ready for use. The prestress force in this system depends upon the bond between the steel and concrete.

Another method similar to the Hoyer System is called the Shorer System. The similarity is due to the fact that both depend upon the bond developed between steel and concrete for their proper operation. These units, along with test data are fully covered in an ACI Journal article by Shorer [2]. The units consist of a central tube of high strength steel covered with paper to prevent bonding to the concrete. Over the paper is wound the number of wires required. They are wound with a very wide pitch, half in one direction and half in the other. The wires are kept about 3/16" from the tubes by discs

placed along at regular intervals. A locking device, as shown in Fig. 8, is used at each end.

The manner of using this device is as follows:

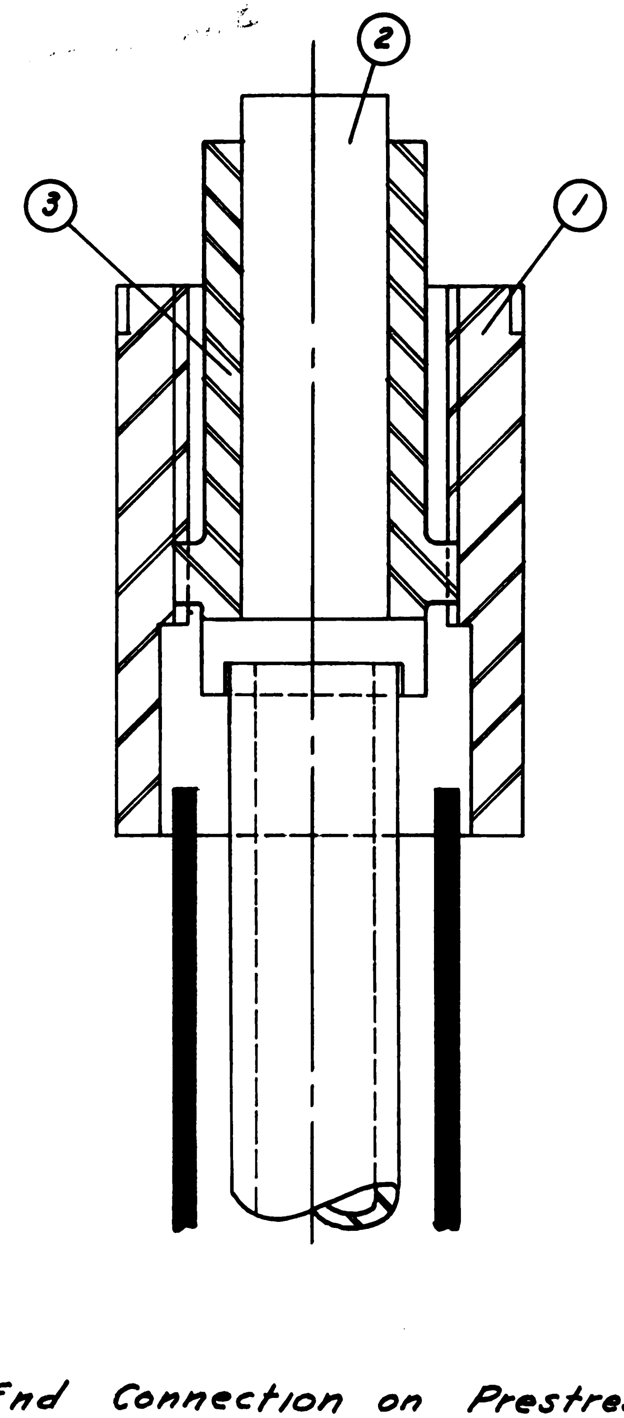
> The unit is placed in the conduit left open for it in the concrete. The locking device consists of two parts which slide one along the other and a locking nut (3). On one end, the locking nut is tightened.
>
> On the opposite end, the wires are clamped to part 1 by means of wedges. A hydraulic jack applies the required force between parts 1 and 2. When the proper value of the force is reached, the locking nut is tightened. After the concrete has hardened, the wires are unclamped, and the compression tube is removed and the holes are filled with grout. The compression is produced by bond.

This method has the advantage of being adaptable for both field and factory use without the need for butress walls or other rigid units.

Other methods have been used to obtain the same results. One of these consists of stretching each wire individually and clamping it by means of wedges in a "sandwich plate". The difference in the various other methods consists of the manner by which the wires are held in place after being stretched.

Electrical Prestressing

Mr. Billner and Mr. Carlson (3) have devised a method



End Connection on Prestressed
Reinforcing Unit

Fig. 8

of establishing a prestress by using an electric current to heat the steel. This system consists of using steel bars which can be safely stressed to about 30,000 #/in^2. The bars are threaded at the ends and then coated with sulfur. These bars are then used in a manner similar to ordinary reinforcement, except they extend beyond the ends of the concrete.

After the concrete has hardened, the bars are heated for a period of about two minutes with electric current. The voltage applied is 5 volts per yard length of reinforcement. The current used is about 400 amps for every 0.15 sq. in. of cross sectional area of the bar. The resulting heat melts the sulfur, breaking the bond, and also elongates the bar. Nuts are then placed on the threaded ends of the bars and tightened. After the bar cools, it remains stretched and places the concrete in compression. Since the sulfur hardens upon cooling, the bond is re-established, and the nuts may be removed.

This method has several disadvantages. First a serious comparative loss of prestress results due to the small stress created by heating, and the low allowable stress for the steel. Also, it would be possible to produce a non-uniform prestress due to uneven heating. Finally, if moisture is present, there is every likelyhood that chemical action between the sulfur (as sulfurous acid) and concrete and steel would cause serious damage in time.

Extensive research is still being carried on in this field and in the future economical, satisfactory means will be developed for solving the problem of field prestressing.

NOTATION

A	Cross sectional area of a beam
A_s	Cross sectional area of reinforcing steel
b	Breadth or width of a rectangular beam
b	-as a subscript denotes the bottom fiber of a beam
C	as a subscript it denotes concrete
d	total depth of a beam or slab
d	as a subscript denotes "dead load" on the beam
e	eccentricity of the steel from the neutral axis or the eccentricity of the stretched wires from the centroid
E_c	Modulus of elasticity for concrete
E_s	Modulus of elasticity for steel
F_i	Initial prestress force supplied by the steel
f	denotes a fiber unit stress
f_c	allowable fiber unit stress for concrete in compression
f_{ct} or f_{tc}	- allowable fiber unit stress for concrete in tension
f_{du}	upper fiber stress due to the dead load
f_{ul}	upper fiber stress due to the live load
f_{db}	lower or bottom fiber stress due to the dead load
f_{dl} or f_{lb}	- lower or bottom fiber stress due to the live load
I	Moment of inertia of the effective area
I_1	Moment of inertia of the area resisting prestress
l	as a subscript denotes live load
M	Moment or total bending moment
M_l	Moment due to the live load on the beam

M_d	Moment due to the dead load on the beam
η	proportion of F_i that remains permanently, usually has a value of 0.85
n	ratio of $\frac{E_s}{E_c}$
p	percentage of steel in the entire cross section
r	radius of gyration of the concrete section
s	a subscript denoting steel
u	a subscript denoting upper or top fiber of a beam
V	total shear
v	unit shear stress
w	a uniformily distributed load in lbs/ft applied to a beam. It may be either live load or dead load.
y_1	distance from the neutral axis to the top fiber of a beam
y_2	distance from the neutral axis to the bottom fiber of a beam

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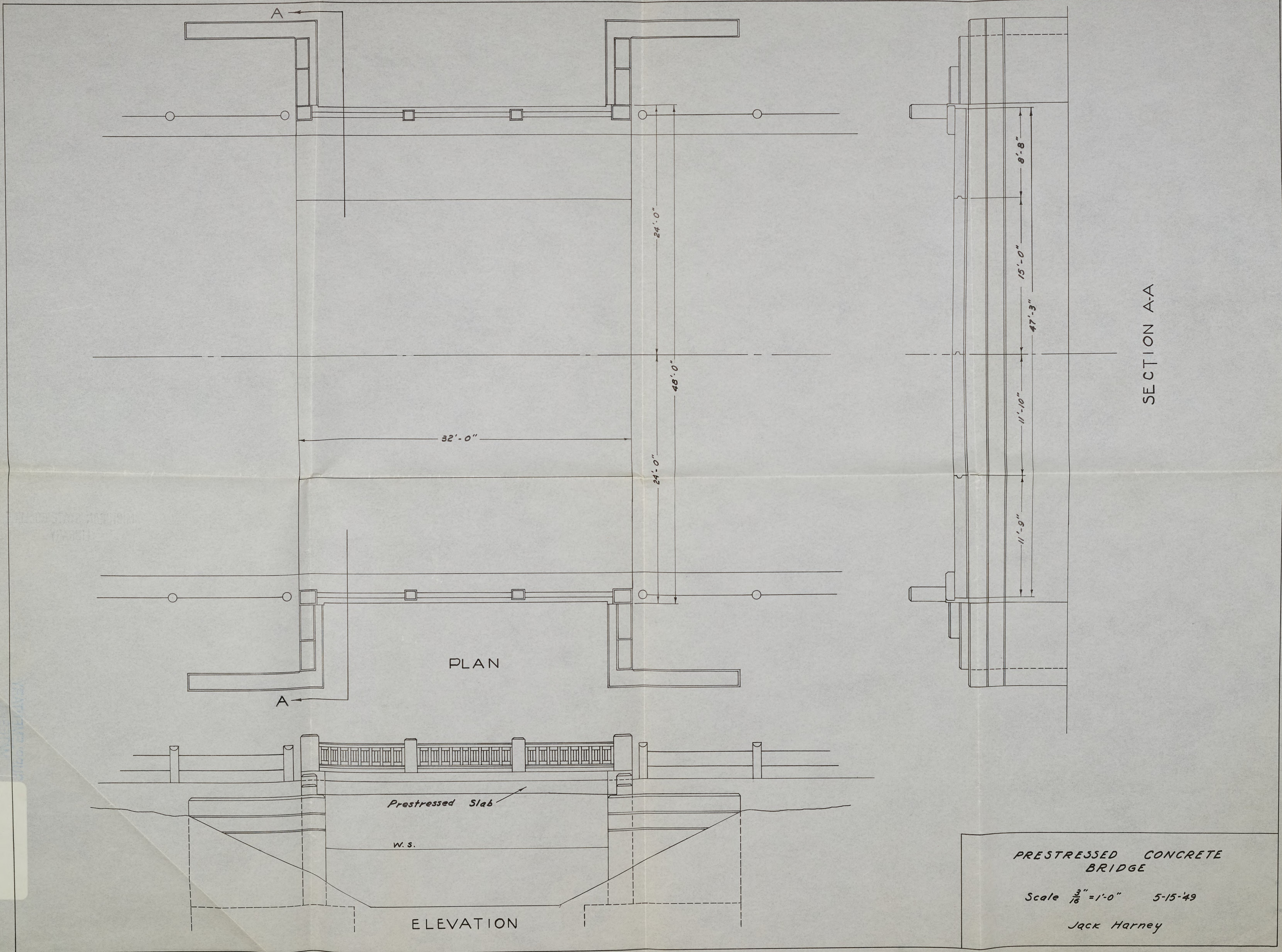
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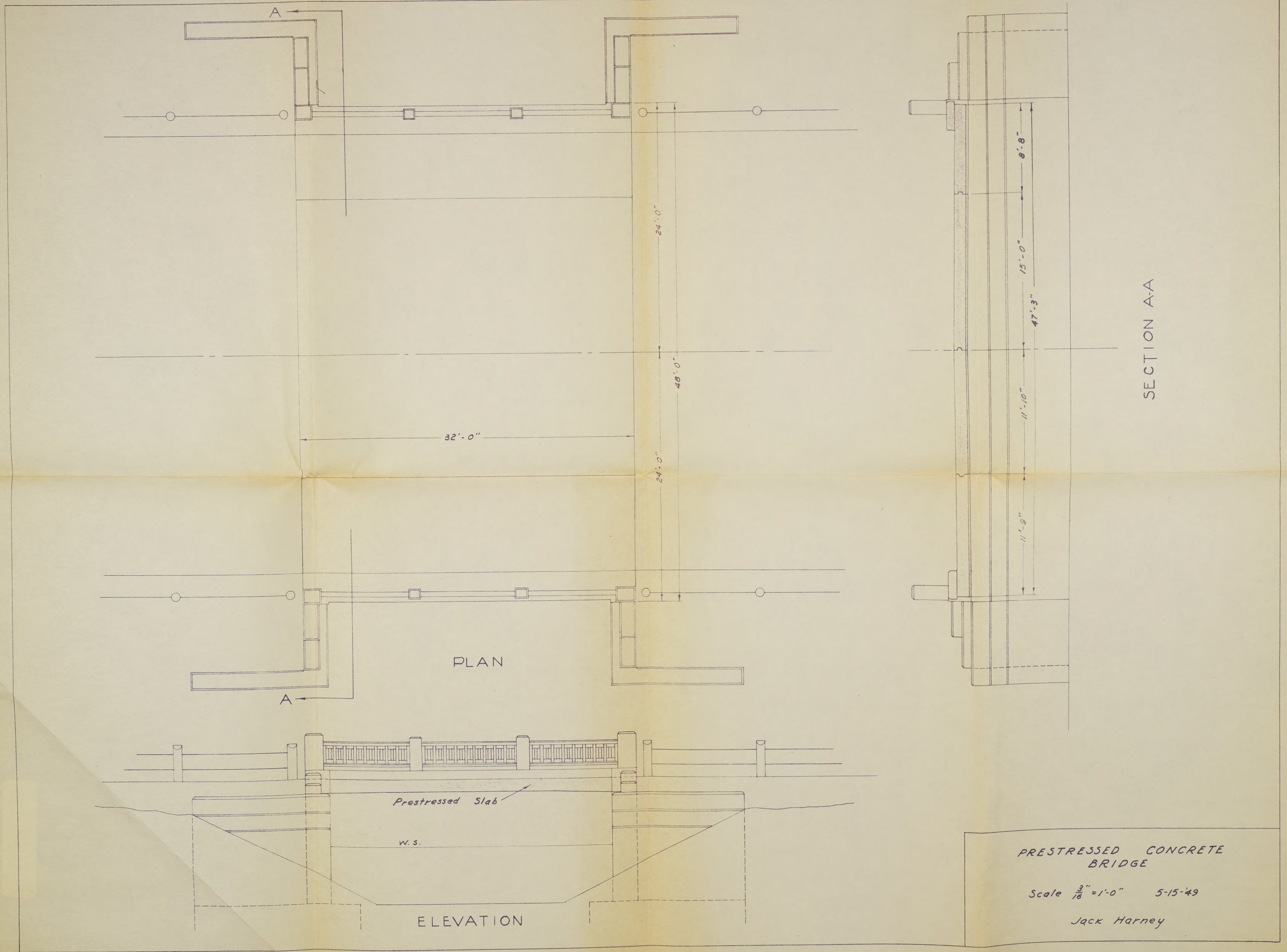
SECTION A-A
Prestressed Slab
W. S.
ELEVATION
PRESTRESSED CONCRETE
BRIDGE
Scale 3/16" = 1'-0"
5-15-'49
Jack Harney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