DYNAMIC STRESS TRANSDUCERS AND THE USE OF CONTINUUM MECHANICS IN THE STUDY OF VARIOUS SOIL STRESS-STRAIN RELATIONSHIPS

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DYNAMIC STRESS TRANSDUCERS
AND THE USE OF CONTINUUM MECHANICS
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by

Wesley Lamar Harris

AN ABSTRACT

Submitted to the School for Advanced Graduate Studies of

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ABSTRACT

The changes in soil consolidation resulting from externally applied forces and the effect of these changes on the physical properties of the soil have been studied by many individuals. The results of one of the investigations revealed that the concept of continuum mechanics could be used as a mathematical model for studying the soil compaction problem. The development of soil stress-strain relationships which will permit the prediction of the changes in the state of compaction caused by various implements and power units will be a major contribution toward controlling soil compaction.

The concept of continuum mechanics was used to determine various stress-strain relationships. A Six Directional Stress Transducer capable of measuring sufficient data to determine the components of the general stress tensor was developed and compared with the method used by Vanden Berg (1958). A W Cell capable of measuring mean stress directly was developed and the values of mean stress calculated from the Type A and 6 DST data were compared.

The data from a series of 27 tests of 5 replications composed of three depths below the loading surface, three moisture contents and three rates of loading are presented.

The data was analyzed using MISTIC, an electronic digital computer, to determine the relationships between the invariants of the stress tensor and bulk density.

The hypothesis that changes in mean normal stress, an invariant of the stress tensor, are related to changes in volumetric strain was tested by measuring the stress tensor and bulk density in the soil while the soil was subjected to dynamic loads of various magnitudes. Based on the data presented, the hypothesis could not be accepted or rejected. The data indicated that of the four invariants of the stress tensor investigated the maximum shear stress related best to changes in bulk density.

The relationships between the invariants and bulk density were affected by the moisture content at the higher rate of loading and deeper depths. The rates of loading data was varied; therefore the effects on the relationships could not be determined.

The values of mean stress obtained directly from the W Cell compared best with the values calculated from the Type A data. Comparison of the two methods of measuring vertical stress with theoretical values determined with Froehlick's equation showed good agreement with the Type A values at the two deeper depths. For the 5-inch depth, both the 6 DST and Type A data were greater than the theoretical data for a given surface load.

The relationships between the invariants and bulk density are exponential for the soil studied. The relationship between mean stress and applied load appears to be linear for loads greater than 5 pounds per square inch.

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INTRODUCTION

A major factor in the advancement of civilization during the twentieth century has been the mechanization of agriculture. The extensive use of larger power units and associated equipment has benefited mankind, but it has been a source of problems too; i.e. inadequate soil air movement, reduced infiltration and percolation rates, mechanical impedance to roots and reduced crop yields are caused to some degree by excessive compaction. The complete solution of these problems will require the combined efforts of many branches of science. Soil compaction resulting from large externally applied forces has been studied by agricultural engineers and soil physicists. Unfortunately their results to date have not produced an adequate agricultural soil mechanics.

Although soil is one of the oldest materials used by man, accurate stress-strain relationships for all soil conditions and types of loading have not been developed. The main reason that this is true is that agricultural soils vary in density and texture and are non-homogeneous and inelastic. Vanden Berg(1960) stated that there is no analytical method for developing a rigorous stress-strain relationship. The stress and strain developed in a soil mass must be measured simultaneously and stress-strain relationships for soil determined empirically.

The study of soil compaction consists of two phases. The first phase involves determining the distribution of stresses in a soil mass caused by externally applied forces. The second phase involves determining the effect that these stresses or strains have on the soil mass. Since one of the effects of the stresses developed is to force a modification of the stress pattern, the two phases must be studied simultaneously.

In general the stress distribution developed by an externally applied load will depend upon several factors that include the following:

1. The magnitude and type of load
2. The size and shape of the contact area where the force is applied
3. The distribution of the pressure within the contact area
4. Moisture content of the soil
5. The initial bulk density of the soil mass.

The largest forces applied to the soil are due to tractor and implement traffic. While these forces are not the only causes of soil compaction, they are conceded to be the major cause. Since these forces are dynamic, to understand the effect of these forces, the volumetric strain produced by a dynamic load must be determined.

In order to conduct the above study, the model of a continuous medium for soils as proposed by Vanden Berg (1958) was used. He defined soil stress as a set of nine quantities in the form of a stress tensor instead of a single value. The stress tensor can be separated into two components, the mean normal stress tensor or spherical stress tensor and the stress deviator tensor. The spherical stress tensor is similiar to hydrostatic pressure and is determined by taking the algebraic mean of the normal stresses acting in three mutually perpendicular directions at a point. The stress deviator tensor differs from the stress tensor in that the mean normal stress is subtracted from each normal stress component.

Since any stress-strain relationship will be a complicated function depending on soil type, moisture content, rate at which the load is applied and others, many instrumentation problems are involved. Because of the great need for an instrument to measure the components of the stress tensor at a point in the soil, the major portion of the work presented in this thesis was directed toward the design, construction and development of a six directional stress transducer.

The primary objective of this study was to determine the relationship of dynamic forces, mean normal stress and volumetric strain. When this relationship is determined, it will be possible to predict soil compaction as caused by various implements and power units. This information will be a major contribution toward the development of means for controlling soil compaction.

REVIEW OF LITERATURE

Numerous studies have been conducted to evaluate the stress distribution in soils and the relationship between the stresses and changes in the soil mass. Obviously, if this relationship was known, the change in the state of compaction resulting from externally applied forces could be predicted. The soil stress-strain relationships were recently reviewed by Vanden Berg (1958).

The development of strain-gage pressure transducers led to the first real progress in accurately measuring stress in a soil mass. They have not only been used within the soil mass but at the loading surfaces such as at the soil-tire interface. The performance of the cells reported by Cooper (1956) makes the Type A Cell preferable to other types.

Soil-Tire Interface Pressures

The first recorded effort to measure the magnitude of the forces applied to the soil by a farm tractor tire was made by Lask (1958, 1959). Small strain-gage pressure transducers (column and diaphragm cells) were mounted in the tire so that the surfaces were flush with the tire surface. The lower inflation pressures gave a more even pressure distribution across the tire. The lugs of the tire carried a larger portion of the load than the undertread.

Additional studies were conducted by Trabbic (1959) using diaphragm-type pressure transducers in lugs as well as undertread. The results showed that as the drawbar load and tire inflation pressure were increased the soil-tire interface pressure generally increased on the undertread and leading lug side. The pressure decreased on the lug face and trailing lug side as the drawbar load was increased.

The soil-tire interface pressure was measured in a different manner on a smooth tire by Vanden Berg and Gill (1959). Larger diaphragm cells were placed flush with the surface in a densely packed sand and a tractor equipped with a smooth tire was towed across the instrument area. Peak pressures occured just as the tire made contact and broke contact with the soil. The highest pressures occured at the center of the tire and progressively decreased toward the outside edge.

Soehne (1958) theoretically calculated the soil-tire interface pressures and concluded from measurements made by Kraft for thin-walled tires on firm soil that the surface pressure over the entire contact area was approximately equal to the average pressure. In a study of the pressure distribution between a smooth tire and soil, however, Vanden Berg (1959) found that the pressure distribution within the contact area was not uniform. He concluded that Soehne's theory of uniform surface pressure can not be used without considerable error since the maximum pressures recorded were twice the average pressure for the contact area.

Measurement of Stresses in a Soil Mass

The stresses produced in a soil mass as a result of an externally applied force have been measured using various physical principles. A U.S. Waterways Experiment Station report, as reported by Cooper (1956), reviewed and described various types of soil pressure cells developed for soil mechanics studies prior to 1956.

Cooper *et al*. (1957) described a strain gage transducer for measuring normal stress pressures developed by the wheels of a tractor in the soil. Results obtained with the cell indicated that the stress distribution under a rolling wheel was similiar to that described by the empirical equation $\sigma_z = P_m (1-\cos^4\alpha)$ developed by Froehlich (1934).

Where:

σ_z = vertical normal stress

P_m = applied surface load

α = polar coordinate.

Reaves and Cooper (1959) studied the stress distribution under a 12-inch tractor track and a 13-38 tractor tire carrying the same total dynamic load and pulling the same drawbar load. They found that the stresses under the tire were in almost every case twice as large as those under the track for any position. Pressure measurements were recorded at 3-inch increments from the center line of tire

and track laterally 12 inches and downward to 42 inches in Congaree silt loam with the Type A Cells. Also in the same report, results of comparative stress curves at a depth of 9 inches in Hiwassee sandy loam under a 13-38 inch tire and a 12-inch track were presented. They found for the tire a smooth curve of higher magnitude and shorter duration then for the track. The curve for the track showed a vibrating stress which was correlated with stresses applied to the surface of the soil due to the action of the drive sprocket.

In experiments designed to determine the overall movement and compaction in a soil mass for the simplified case of piston sinkage, Soehne et al. (1959) found that "at some distance from the piston, lines of equal principal stress appeared to coincide fairly well with lines of equal compaction, but directly under the piston this was not the case". The movement of the soil was determined by placing small lead spheres in the soil and X-ray plates were made during each test. The method of determining the directions of the principal stresses from the deformation of a grid as developed by Haefeli and reported by Bekker (1957) was used.

Willits (1956) studied the stress produced in soils by traffic and the relationship between the stresses and compaction in undisturbed soils. He found that a maximum stress of over one hundred pounds per square inch near the surface of the soil was produced under the drive wheel of a Massey-Harris Clipper combine. The stresses developed by all traffic decreased rapidly with depth. The amount of compaction was determined by taking soil samples and determining the bulk density. The variables affecting the change in compaction were the vehicle, number of passes, original soil density and soil moisture content. Cores of undisturbed soil were subjected to various pressures in the laboratory to obtain the same change in bulk density as was produced by the passage of a tractor in the field. The pressures were similiar to those recorded by the pressure cells during field tests.

Soil Stress-Strain Relationships

A number of different theories have been applied to soils. One of the oldest, the Coulomb-Mohr formula (an empirical relationship) discussed by Terzaghi (1959), defines the stresses acting on a plane through the soil mass at the moment of failure.

In studies of agricultural implements 47 years ago, Berstein developed a sinkage equation that relates the ground pressure and sinkage of a given loading area. Bekker (1957) modified Berstein's equation and used it in his theory of land locomotion. Soil deformation was defined in terms of certain soil constants "practically independent" of the size and form of the loading area. Using the soil value system developed by Bekker, Stong (1960) found that the soil strength was decreased by plowing and disking. Vehicle traffic increased the soil strength by compacting the soil. Within the range of 10-24 percent moisture content, bulk density has a greater effect on soil strength than the moisture content. Vanden Berg (1960) stated that neither the Berstein equation or the Coulomb-Mohr formula is a logical basis for a general soil mechanics because they do not relate stress and strain.

Using the model of a continuous medium for soil, Vanden Berg (1958) defined soil stress in terms of a stress tensor. The stress tensor was divided into two tensors, the mean normal stress tensor and the deviatoric stress tensor. Applying theories of elasticity and plasticity, he proposed that volume strain is controlled by mean normal stress. Some of the observations made by Vanden Berg are:

1. The concept of continuum will apply to loose soils.
2. Of the four invariants of the stress tensors investigated the mean normal stress related best to bulk density.
3. It could not be concluded that soil compaction is independent of the deviatoric stress tensor.

Hovanesian (1958, 1959) found that the density of agricultural soils was related to mean normal stress by the following general formula:

$$\gamma = \gamma_0 + B \ln[(\sigma/\sigma_0) + K/(1+K)] \qquad (1)$$

Where:

γ_0 = initial bulk density of soil

σ_0 = initial mean stress

σ = mean stress

γ = bulk density

K and B are soil parameters, assumed constant for a given soil condition.

He also found that for a given value of mean stress, impact loads will cause less change in bulk density than that created by a gradually applied and released load.

In static compression tests, Soehne (1958) filled cylinders of outside diameter 11.2 inches, height 5.2 inches and volume 610 cubic inches, with undisturbed soil samples taken from the field or with loose soil. He found that the amount of compaction and the reduction of porosity was related to the pressure by a logarithmic law. The higher the moisture content the more the soil was compacted by a given pressure. When the kneading compaction test was compared with the static compaction of loose soil a steeper slope resulted from the kneading test.

The following equation was derived from an analysis of the compaction of arable soils.

$$n = -A \log p + c \qquad (2)$$

Where:

n = porosity

A = slope of the curve on a logarithmic scale

p = pressure

c = porosity at a pressure of 10 psi.

This relationship between porosity and pressure is similiar to the formula used in civil engineering soil mechanics discussed by Hough (1957).

From studies of the resistance to compression of confined fragmented soils, Reaves and Nichols (1955) found the relationship between pressure and amount of compression to be of the general form $y = a\,e^{bx}$ where:

y = amount of compression

x = pressure.

Hendrick (1960) found that the tensile strength of soil briquettes did not change for loading rates of 0.18 to 4.70 kg/cm^2/sec. Less strain energy was required to cause failure at the higher loading rates because the briquettes strained less.

The magnitude of volume strain may be less from a static load than from a dynamic load such as that produced by a track or a tractor tire. This latter action may cause an orientation of particles that will result in a greater volume strain. Terzaghi (1959) found that vibration of sand resulted in a greater compaction than could be caused by an equivalent static force. The effect of vibration on clay was much less because the cohesive bond between clay particles interferes with intergranular slippage.

THEORY

Using the model of a continuous medium proposed by Vanden Berg (1958) the forces acting on a volume element are completely specified by the stress tensor and volumetric strain by the change in bulk density by ignoring the shearing deformations and rigid body rotation. To define the state of stress at a point requires that six independent values be determined. The volumetric strain can be determined by measuring the change in bulk density.

The stress vector on any arbitrary plane can be determined by using matrix algebra (Murnaghan 1951. For example problems see Malvern 1957). For the following conditions,

1. a plane oriented so that its normal lies in the Y Z plane and bisects the angle formed by the positive Y and Z axes,
2. a plane oriented so that its normal lies in the Y X plane and bisects the angle formed by the positive Y and X axes,
3. and a plane oriented so that its normal lies in the X Z plane and bisects the angle formed by the positive X and Z axes;

the directions cosines of a normal vector to each of the planes are, respectively,

1. 0, $\sqrt{2}/2$, $\sqrt{2}/2$,

2. $\sqrt{2}/2$, $\sqrt{2}/2$, 0 ,

3. $\sqrt{2}/2$, 0, $\sqrt{2}/2$.

For the general stress state the stress tensor is

$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z . \end{pmatrix} \tag{3}$$

If $\vec{i}$, $\vec{j}$, $\vec{k}$ are unit vectors along the positive X, Y and Z axes respectively, then the components of the stress vector acting on the three planes described above can be obtained by matrix multiplication.

For plane one as defined by condition one the stress vector $\vec{T}_1$ is

$$\vec{T}_1 = (0, \sqrt{2}/2, \sqrt{2}/2) \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{pmatrix}$$

$$= \sqrt{2}/2(\tau_{xy}+\tau_{xz})\,\vec{i} + \sqrt{2}/2(\sigma_y+\tau_{yz})\,\vec{j} + \sqrt{2}/2(\tau_{yz}+\sigma_z)\,\vec{k}. \tag{4}$$

For plane two (condition two)

$$\vec{T}_2 = (\sqrt{2}/2, \sqrt{2}/2, 0) \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{pmatrix}$$

$$= \sqrt{2}/2(\sigma_x+\tau_{xy})\,\vec{i} + \sqrt{2}/2(\tau_{xy}+\sigma_y)\,\vec{j} + \sqrt{2}/2(\tau_{xz}+\tau_{yz})\,\vec{k}. \tag{5}$$

For plane three (condition three)

$$\vec{T}_3 = (\sqrt{2}/2,\ 0\ ,\sqrt{2}/2) \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{pmatrix}$$

$$= \sqrt{2}/2(\sigma_x+\tau_{xz})\ \vec{i} + \sqrt{2}/2(\tau_{xy} + \tau_{yz})\ \vec{j} + \sqrt{2}/2(\tau_{xz}+\sigma_z)\ \vec{k}. \qquad (6)$$

If the scalar product of a unit vector $\vec{n}$ in the direction of the normal to the plane and the stress vector $\vec{T}_i$ acting on the plane is determined this will be the magnitude of the normal stress acting on the plane. The normal stress σ_{n_i}, acting on the plane can be obtained as follows; for plane one the unit normal vector is,

$$\vec{n}_1 = 0\ \vec{i} + \sqrt{2}/2\ \vec{j} + \sqrt{2}/2\ \vec{k}$$

then

$$\sigma_{n_1} = \vec{n}_1 \cdot \vec{T}_1 \qquad (7a)$$

$$\sigma_{n_1} = \tfrac{1}{2}(\sigma_y+\sigma_z) + \tau_{yz} \qquad (7b)$$

for plane two the unit normal is,

$$\vec{n}_2 = \sqrt{2}/2\ \vec{i} + \sqrt{2}/2\ \vec{j} + 0\ \vec{k}$$

then

$$\sigma_{n_2} = \vec{n}_2 \cdot \vec{T}_2 \qquad (8a)$$

$$\sigma_{n_2} = \tfrac{1}{2}(\sigma_x+\sigma_y) + \tau_{xy} \qquad (8b)$$

for plane three the unit normal is,

$$\vec{n}_3 = \sqrt{2}/2\ \vec{i} + 0\ \vec{j} + \sqrt{2}/2\ \vec{k}$$

then,

$$\sigma_{n_3} = \vec{n}_3 \cdot \vec{T}_3 \tag{9a}$$

$$\sigma_{n_3} = \tfrac{1}{2}(\sigma_x + \sigma_z) + \tau_{xz} \; . \tag{9b}$$

The principal stresses can be determined from the stress tensor for the general stress state, however, it is easier to determine the principal values of the stress deviator tensor and then calculate the principal values for the stress tensor. The stress tensor can be separated into a spherical stress tensor and a stress deviator tensor as follows.

$$\underset{\text{Stress Tensor}}{\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{pmatrix}} = \underset{\substack{\text{Spherical} \\ \text{Stress Tensor}}}{\begin{pmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{pmatrix}} + \underset{\text{Deviator Tensor}}{\begin{pmatrix} \sigma_x - \sigma_m & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - \sigma_m & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma_m \end{pmatrix}} \tag{10}$$

Where:

$$\sigma_m = \tfrac{1}{3}\,(\sigma_x + \sigma_y + \sigma_z) \tag{11}$$

If S_i denotes the three principal deviator stresses, the following relationships are known:

$$S_1 = \sigma_1 - \sigma_m \tag{12}$$

$$S_2 = \sigma_2 - \sigma_m \tag{13}$$

$$S_3 = \sigma_3 - \sigma_m \tag{14}$$

σ_1, σ_2, and σ_3 are the principal stresses of the stress tensor.

It is possible to rotate the coordinate axes to such a position that all of the shear stresses will be zero and only these principal stresses will act on the plane. The problem is to determine the direction cosines n_x, n_y, and n_z so that this condition is present. If $\vec{n}$ is a unit vector in one of the principal directions and S the magnitude of the stress vector $\vec{T}_n$, the stress vector on this plane must be parallel to $\vec{n}$ since there are no shear stress component on the plane perpendicular to $\vec{n}$. Therefore,

$$\vec{T}_n = S\,\vec{n} \tag{15}$$

The three components of this vector equation can be determined by matrix algebra as follows

$$(n_x,\ n_y,\ n_z)\begin{pmatrix} S_x-S & \tau xy & \tau xz \\ \tau xy & S_y-S & \tau yz \\ \tau xz & \tau yz & S_z-S \end{pmatrix} =$$

$$(S_x-S)n_x + \tau yx\ n_y + \tau zx\ n_z = 0 \tag{16a}$$

$$\tau xy\ n_x + (S_y-S)n_y + \tau xy\ n_z = 0 \tag{16b}$$

$$\tau xz\ n_z + \tau yz\ n_y + (S_z-S)n_z = 0 \tag{16c}$$

Where:

$$S_x = \sigma_x-\sigma_m, \quad S_y = \sigma_y-\sigma_m, \text{ and } S_x = \sigma_z-\sigma_m$$

This is a set of three homogeneous linear algebraic equations for the three unknown direction cosines n_x, n_y, and n_z. The directions cosines must also satisfy the equation

$$n_x^2 + n_y^2 + n_z^2 = 1, \qquad (17)$$

and, therefore, all three cannot be zero. A system of linear homogeneous equations has a solution other than the trivial solution if and only if the determinant of the coefficients is equal to zero, that is, if

$$\begin{vmatrix} S_x-S & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & S_y-S & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & S_z-S \end{vmatrix} = 0 \qquad (18)$$

Expanding the determinant gives a cubic equation in terms of the unknown magnitude S;

$$S^3 - II_s\, S - III_s = 0 \qquad (19)$$

where II_s and III_s are algebraic invariants of the stress deviator tensor and are defined as follows;

$$II_s = \frac{1}{6}[(S_x-S_y)^2 + (S_z-S_x)^2 + (S_y-S_z)^2] + \tau_{yz}^2 + \tau_{zx}^2 + \tau_{xy}^2 \qquad (20a)$$

Substituting

$$S_x = \sigma_x-\sigma_m, \quad S_y = \sigma_y - \sigma_m, \quad S_z = \sigma_z - \sigma_m ;$$

$$II_s = \frac{1}{6}[(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2] + \tau_{xy}^2 + \tau_{zx}^2 + \tau_{xy}^2 \qquad (20b)$$

$$III_s = \begin{vmatrix} s_x & \tau_{xy} & \tau_{xz} \\ \tau_{yz} & s_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & s_z \end{vmatrix} = \begin{vmatrix} \sigma_x - \sigma_m & \tau_{xy} & \tau_{xz} \\ \tau_{yz} & \sigma_y - \sigma_m & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma_m \end{vmatrix} \qquad (21)$$

The roots of equation (19) are the three principal stresses. The solution may be obtained by making the substitution (Malvern 1957):

$$S = 2(\cos\alpha)\sqrt{II_s/3}. \qquad (22)$$

From this substitution the following relation is determined:

$$\text{Cos } 3\alpha_1 = \frac{2.6\ III_s}{(II_s)^{3/2}} \qquad (23)$$

Then $3\alpha_1$, $3\alpha_1 + 2\pi$, and $3\alpha_1 - 2\pi$ all have the same cosine given in terms of the invariants of the stress deviator. Thus the three roots of equation (19) are:

$$S_1 = 2(\cos\alpha_1)\sqrt{II_s/3}$$

$$S_2 = 2(\cos\alpha_2)\sqrt{II_s/3}$$

$$S_3 = 2(\cos\alpha_3)\sqrt{II_s/3}$$

Where:

$$\alpha_2 = \alpha_1 + \frac{2\pi}{3} \quad \text{and} \quad \alpha_3 = \alpha_1 - \frac{2\pi}{3}.$$

Now the principal stress values can be determined by using equations (12), (13) and (14). The values that are obtained are ordered algebraically from largest to smallest and designated by σ_I, σ_{II}, and σ_{III} respectively. The maximum shear stress, which is a function of the stress deviator, is given by

$$\tau_{max} = \tfrac{1}{2}(\sigma_I - \sigma_{III}) \tag{24}$$

In order to vertify the hypothesis that soil compaction developed under dynamic conditions is controlled by mean normal stress two things must be demonstrated:

1. That mean normal stress does correlate with bulk density and
2. That the deviator stress tensor does not correlate with bulk density.

The only measure of the spherical stress tensor is mean normal stress. Many expressions can be used as a measure of the deviator tensor. Since earlier investigations have indicated a relationship between maximum shear stress, maximum normal stress and bulk density, these relationships will be investigated.

APPARATUS AND INSTRUMENTATION

Design and Development of a Six Directional Stress Transducer

Two different models were designed during the development of the six directional stress transducer (6 DST). The first model designed consisted of a small octagonal brass box with eight sensing elements as shown in Figure 1. Each of the sensing elements was to be made of 0.025 inch thick stainless steel with two Type A-18, SR-4 electrical resistance strain gages cemented to the element. The diametrical pairs of sensing elements would form the four components of a Wheatstone bridge. With this arrangement maximum sensitivity and temperature compensation would be obtained.

Construction of several sensing elements revealed that pieces of stainless steel of this size and shape were difficult to work. In addition, since the element was designed to act as a simply-supported beam, the problem of protecting the gages mounted on the element without restricting the action of the beam was not satisfactorily accomplished.

Due to these difficulties the second model, the 6 DST, was designed and constructed. (Figure 2) A hollow brass sphere (3 inches outside diameter and 1 7/8 inches

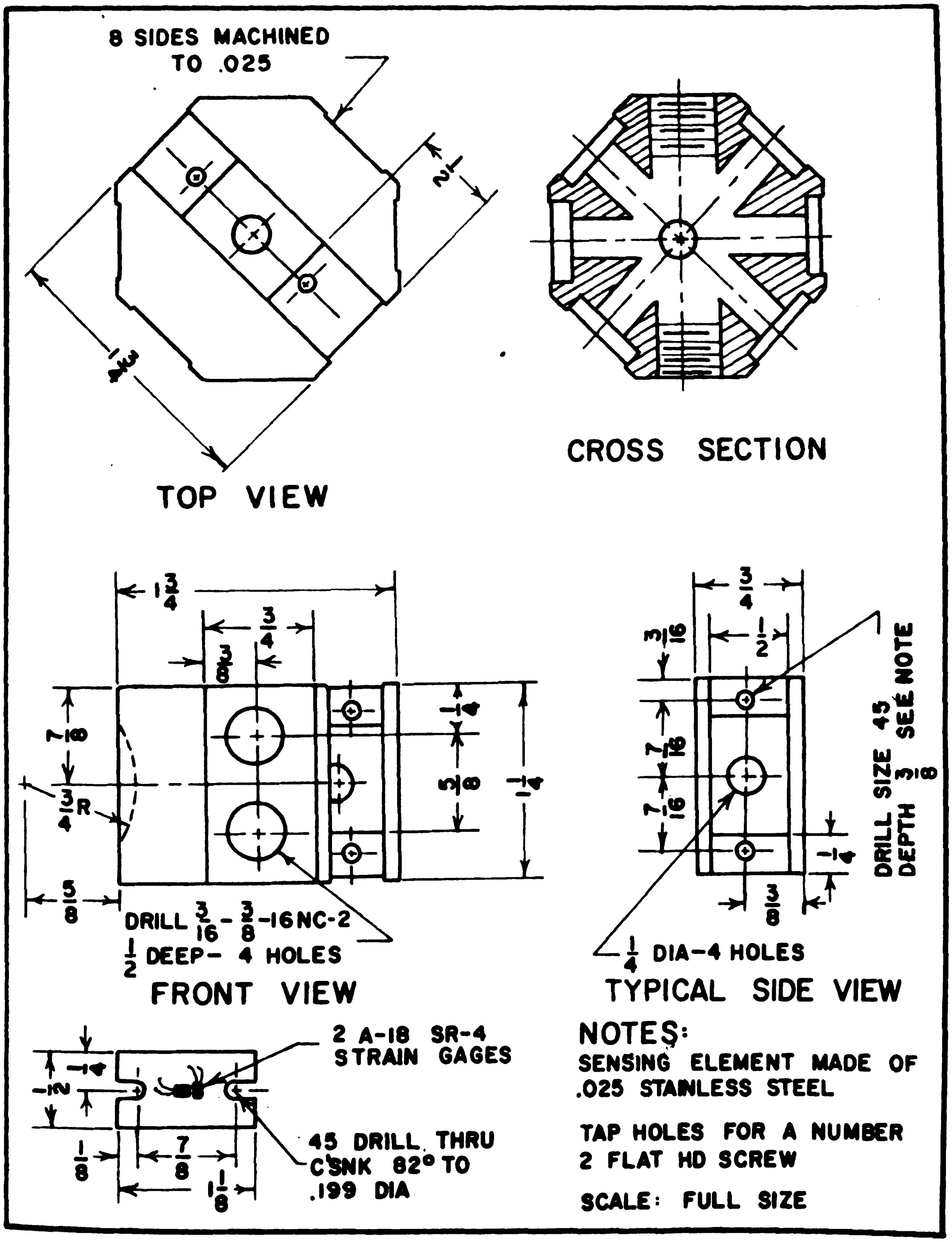

Figure 1. Detail drawing of the Four Directional Stress Transducer.

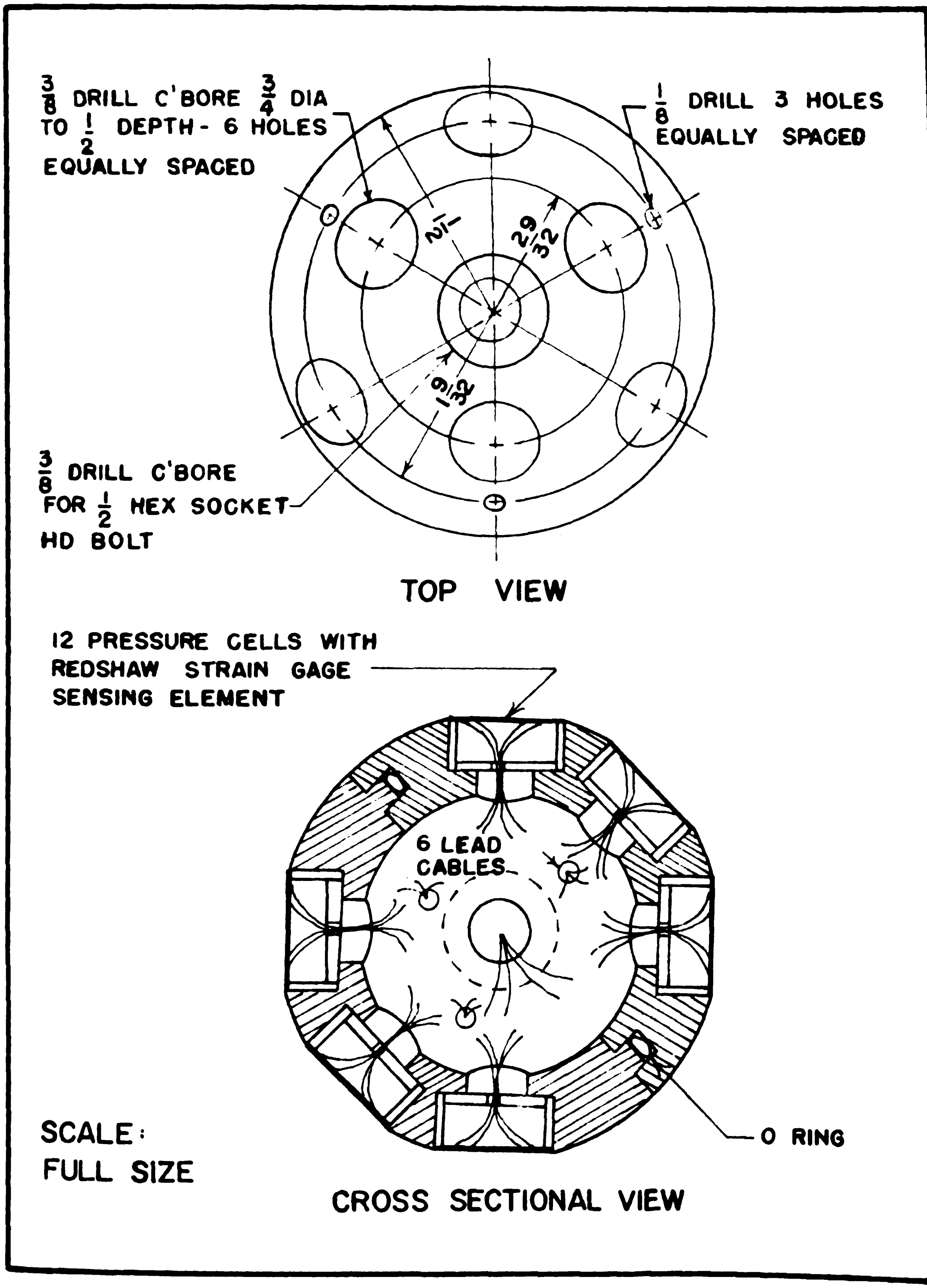

Figure 2. Drawing of the Six Directional Stress Transducer showing the Location of Diaphragm Pressure Cells.

inside diameter) was cast in two parts. The two halves were machined to permit the use of an "O" ring for a waterproof connection. A 3/8 inch hexagon socket head bolt was used to clamp the two hemispheres together. Six diaphragm pressure cells capable of measuring normal stress were located in each hemisphere. Three of the pressure cells are mutually perpendicular and the other three are oriented in the planes that bisect any two of the three mutually perpendicular directions. When the two half spheres were connected, the corresponding cells in each half sphere were oriented diametrically opposite each other. These pairs of cells were connected to form two legs of a Wheatstone bridge. Two 120-ohm wire resistors were used to complete the Wheatstone bridge.

The diaphragm cells which were used for the sensing elements were constructed in the following manner. A length of 3/4 inch diameter cold-rolled steel stock was chucked in a lathe and a 5/8 inch hole was drilled through the center of the piece. An 11/16 inch drill was used to enlarge the hole to a depth of 1/16 inch. A 3/8 inch long cylinder was then cut from the length.

Diaphragms made of 0.010-and 0.020-inch thick stainless steel were rough cut to a one inch diameter with a metal clipper and soldered with stainless steel solder to the cylinder wall at the end with the 11/16 inch inside hole. Finally, the cell was chucked in a lathe and the diaphragm was machined flush with the outside diameter of the cell wall.

After the cells were constructed, a Saunders-Roe foil strain gage (Redshaw 1/2-2 ED, 25 ohms, gage factor 2.1) was cemented to the inside surface of the stainless steel diaphragm. Since the Redshaw strain gage does not have lead wires attached as the SR-4 gages, a method for attaching lead wires to the tabs of the gage had to be devised. After several preliminary tests the best method found was to attach a piece of copper wire to the tip of a soldering gun. The tabs were tinned prior to being mounted on the diaphragm and after the curing process a 2-inch length of wire (Belden No. 8430) was soldered to each tab.

The gage was waterproofed with a thin layer of wax. To protect the strain gage from being damaged by a force applied to the lead wires, a rubber stopper was cut and placed in the open end of the pressure cell. The lead wires were conducted through a hole in the center that was sealed after the stopper was in place. A four-conductor shielded cable (Belden Strain Gage Cable No. 8434) was connected to the 2-inch wires to carry the signal to the amplifier.

The calibration device as reported by Lask (1958) was used to calibrate the individual cells. The maximum design value of 60 psi was selected since this was the maximum pressure recorded in the soil by present agricultural equipment. Calculations indicated that a 0.010 inch thick stainless steel diaphragm 3/4 inch in diameter could be used.

A problem was encountered since the Redshaw strain gage has only approximately 25 ohms resistance. The amplifiers and associated equipment available for conducting the experimental tests had a range of 50 to 500 ohms. Several calibration tests were conducted using different bridge arrangements to determine if the bridge could be balanced and the order of magnitude of the gage output.

A calibration curve for the 0.010 thick diaphragm using a 120 ohm wire resistor in series with the Redshaw gage is shown in Figure 3. The results of three tests show a linear relationship up to 25 psi. Within the range from 0 to 25 psi, one line deflection represents approximately 2 1/2 psi. As it was proposed to use two active gages in each bridge the sensitivity of the bridge would be doubled or one line deflection would represent 1 1/4 psi. Since the relationship was not linear up to 60 psi, it was concluded that a thicker diaphragm should be used.

A series of tests were made using 100 and 120 ohms resistors in series with the 25 ohm Redshaw gage mounted on a 0.020 inch thick diaphragm 3/4 inch in diameter. The results of three of these tests using a 100 ohm resistor are shown in Figure 4. A linear relation was obtained up to the maximum value of 80 psi used during the tests. One

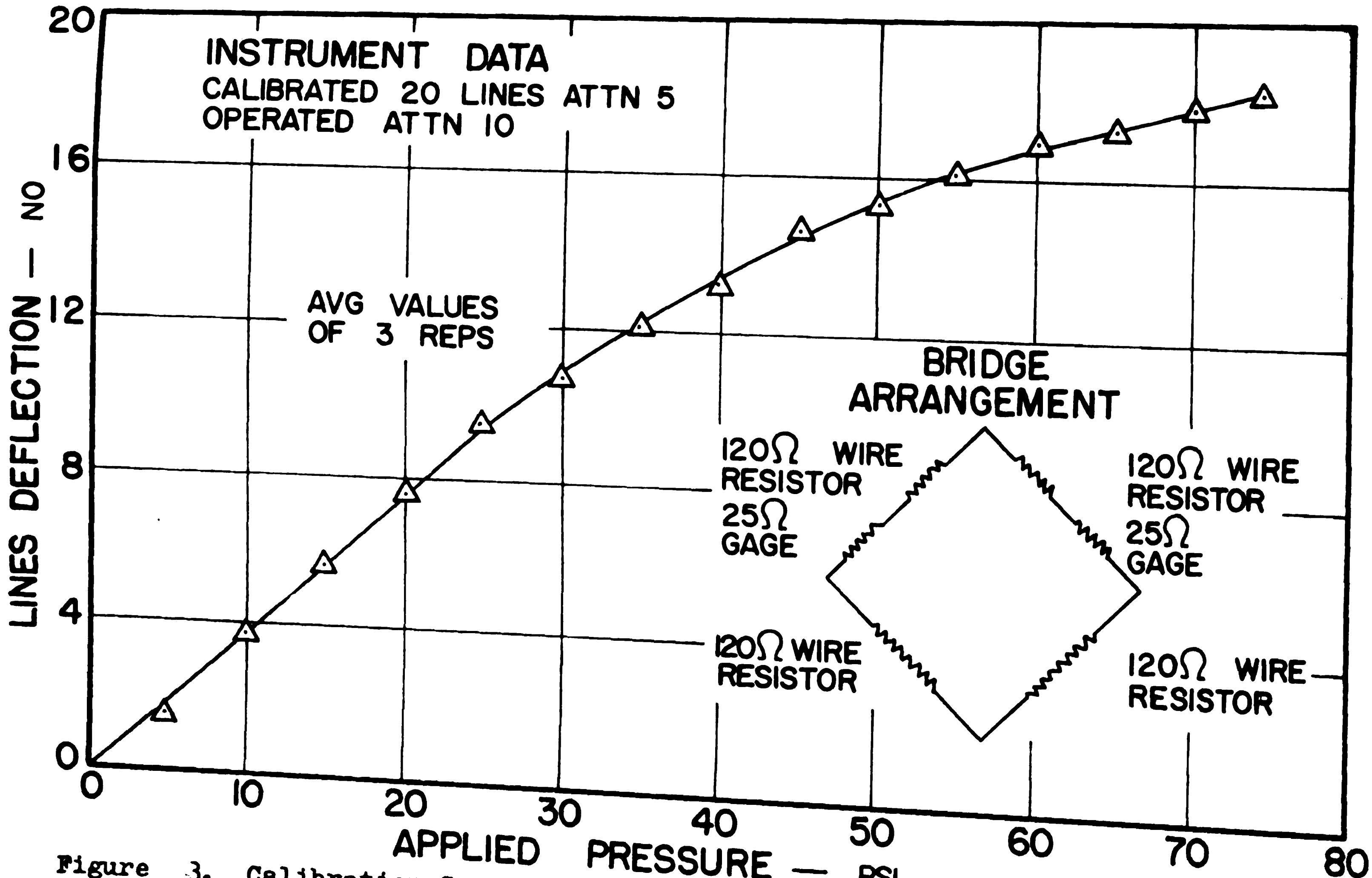

Figure 3. Calibration Curve for Redshaw Strain Gage on 0.010 inch Thick Diaphragm 3/4 inch in Diameter

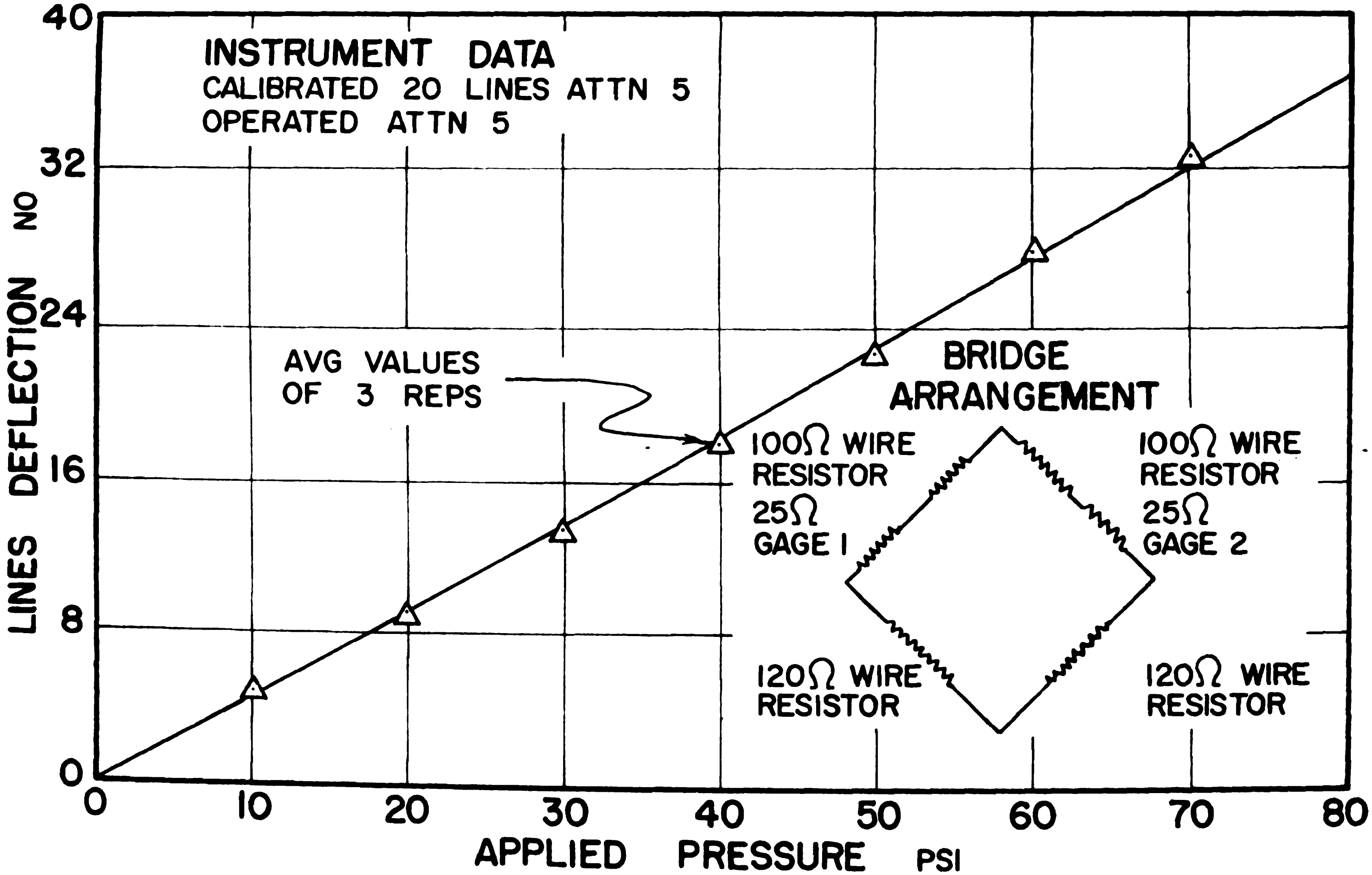

Figure 4. Calibration Curve for Redshaw Strain Gage Mounted on 0.020 inch Thick Diaphragm 3/4 inch in Diameter.

line deflection with the calibration data shown corresponds to approximately 2 1/4 psi. As a result of these tests it was concluded that the 0.020 inch thick stainless steel diaphragm 3/4 inch in diameter would be used for the sensing elements of the 6 DS transducer.

Since it was proposed to use two active gages in each bridge, tests were conducted using the arrangement in Figure 5. The calibration data were obtained from only one pressure cell being subjected to pressure. During the experimental tests both pressure cells in a bridge would be subjected to a load, therefore, a device for obtaining calibration curves with both cells being subjected to a load had to be designed and constructed. With 6 holes for lead wires, 12 for the sensing elements and two for the connecting bolt , the probability of maintaining a completely air sealed unit was very small. Therefore, a calibration device with two nozzles similiar to the one reported by Lask (1958) was constructed and is shown in Figure 7. Control valves were installed to permit separate control of each nozzle. Any combination of pressures could be obtained.

The calibration data shown in Table 1 proved that the bridge arrangement with two pressure cells would give an average of the two separate readings. This was expected, as it can be shown mathematically. Consider the bridge arrangement as shown in Figure 5. The output voltage of

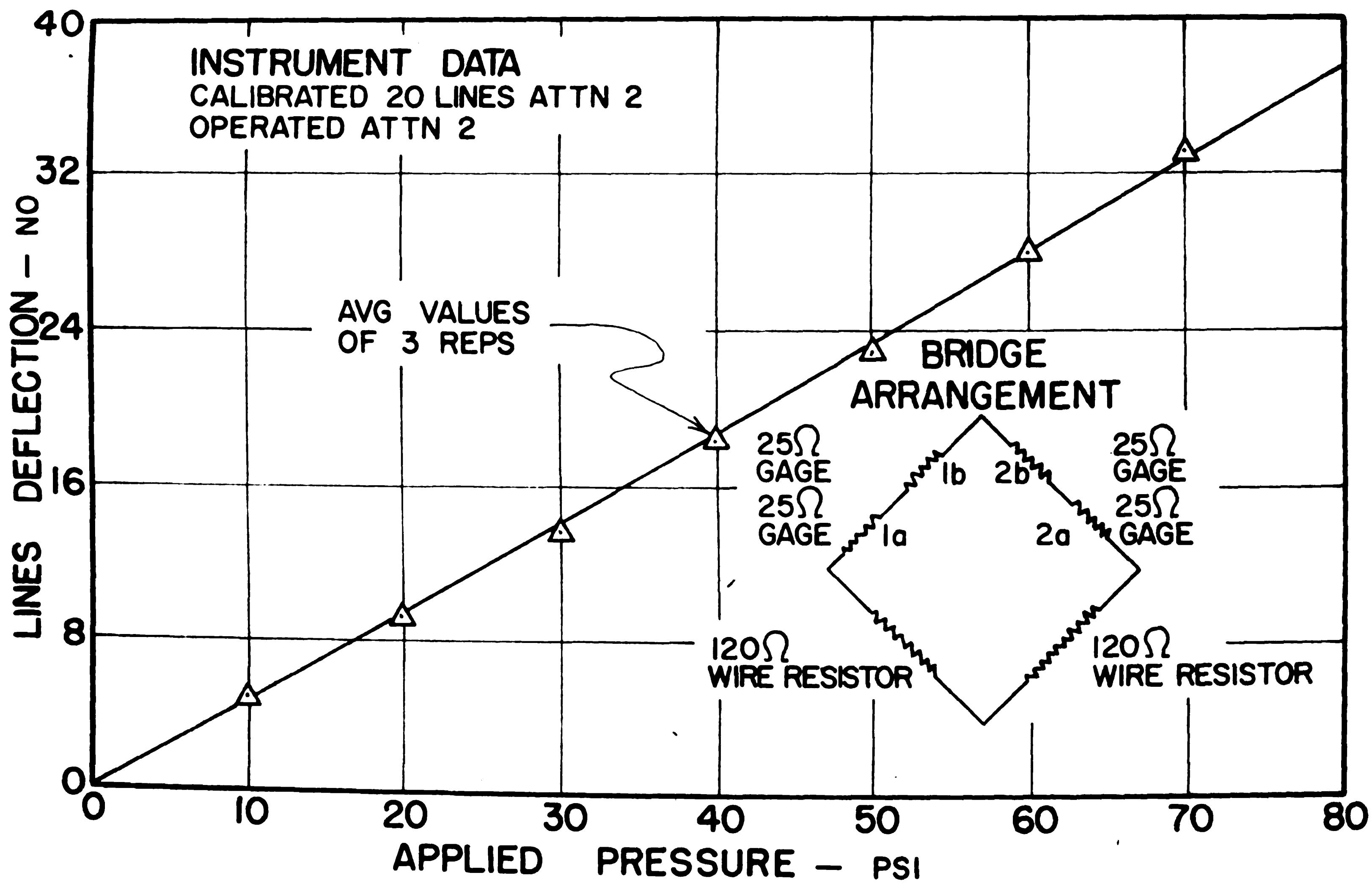

Figure 5. A Typical Calibration Curve Using Bridge Arrangement as Shown. One complete Redshaw 2-ED Type Strain Gage Used as Sensing Element and the other as Dummy Gages.

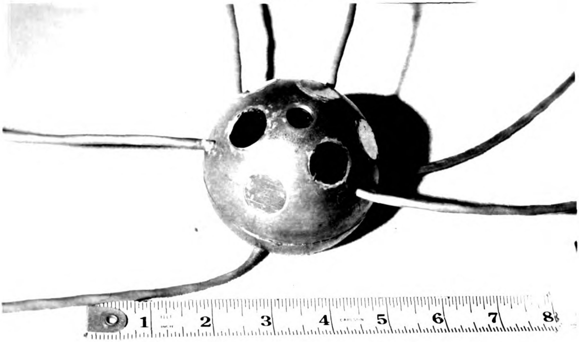

Figure 6. A View of the Six Directional Stress Transducer.

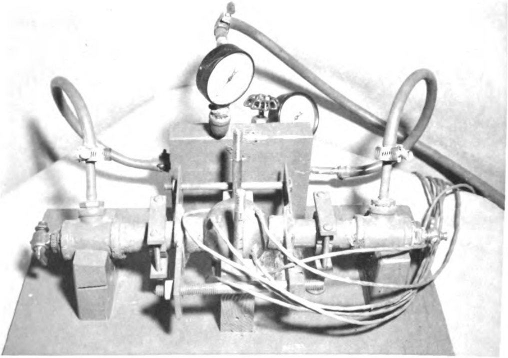

Figure 7. Calibration Device used to obtain Calibration Data for the 6 DST.

TABLE 1

AVERAGE OUTPUT OF NUMBER ONE SET OF GAGES IN 6 DST CALIBRATED FOR ONE LINE DEFLECTION EQUAL TO ONE PSI

Load (psi)			Lines Deflection
Cell 1	Cell 2	Average	Avg. of 3 reps.
20	0	10	9.8
20	10	15	14.8
30	10	20	20.0
40	20	30	30.0
40	40	40	40.1
40	0	20	19.1
40	10	25	24.3
40	20	30	29.0
40	30	35	34.7
40	40	40	40.0

the bridge is,

$$E_o = E\left[\frac{R_1}{R_1+R_4} - \frac{R_2}{R_2+R_3}\right] . \qquad (24)$$

Where:

E = supply voltage

$R_1 = R_{1a} + R_{1b}$ = resistance values of elements of Redshaw gages.

$R_2 = R_{2a} + R_{2b}$ = resistance values of elements of Redshaw gages.

$R_3 = R_4$ = resistance values of wire resistors

E_o = bridge output voltage

The bridge circuit is in balance and zero output voltage results when

$$\frac{R_1}{R_1 + R_4} = \frac{R_2}{R_2 + R_3} \,. \tag{25}$$

The change in output voltage for a change in resistances R_1 and R_2 can be determined by calculating the total differential for equation (24).

$$dE_o = \frac{\partial E_o}{\partial R_1} dR_1 + \frac{\partial E_o}{\partial R_2} dR_2 \tag{26a}$$

$$\frac{\partial E_o}{\partial R_1} = \frac{E(R_1+R_4-R_1)}{(R_1+R_4)^2} , \frac{\partial E_o}{\partial R_2} = \frac{E(R_2+R_3-R_2)}{(R_2+R_3)^2}$$

Substituting into equation (26a)

$$dE_o = E\left[\frac{R_4 dR_1}{(R_1+R_4)^2} - \frac{R_3 dR_2}{(R_2+R_3)^2}\right] \tag{26b}$$

For $R_{1a} = R_{1b} = R_{2a} = R_{2b} = R$ and all with gage factor F, and $R_3 = R_4 = XR$, then

$$dE_o = \frac{EX}{(2+X)^2} \left[\frac{dR_{1a}}{R} + \frac{dR_{1b}}{R} - \frac{dR_{2a}}{R} - \frac{dR_{2b}}{R}\right] \tag{26c}$$

Setting:

$$\frac{dR_{1a}}{R} = F\,\varepsilon_{1a}$$

$$\frac{dR_{1b}}{R} = F\,\varepsilon_{1b}$$

$$\frac{dR_{2a}}{R} = F\varepsilon_{2a}$$

$$\frac{dR_{2b}}{R} = F\varepsilon_{2b}$$

dE_o now becomes,

$$dE_o = \frac{EX}{(2+X)^2} F[\varepsilon_{1a} + \varepsilon_{1b} - \varepsilon_{2a} - \varepsilon_{2b}]. \qquad (27a)$$

Since the Redshaw had a resistance of 25 ohms, and the resistor R_3 and R_4 are 120 ohms, X is equal to 4.8.

$$dE_o = \frac{EF}{4.82}\left[\frac{\varepsilon_{1a} + \varepsilon_{1b}}{2} - \frac{\varepsilon_{2a} + \varepsilon_{2b}}{2}\right]. \qquad (27b)$$

Letting

$$\frac{\varepsilon_{1a} + \varepsilon_{1b}}{2} = \varepsilon_1$$

$$\frac{\varepsilon_{2a} + \varepsilon_{2b}}{2} = \varepsilon_2$$

$$dE_o = \frac{E\ F}{4.82}[\varepsilon_1 - \varepsilon_2]. \qquad (28)$$

Therefore, the proposed bridge gives the algebraic difference of the average strain in arm 1 and the average strain in arm 2. Since the gages in arm one will be measuring the change in strain due to tension and those in arm two due to compression, ε_1 will be a positive value and ε_2 will be negative. Our equation now becomes

$$dE_o = \frac{E\ F}{4.82}\ [|\varepsilon_1| + |\varepsilon_2|]\ . \qquad (29)$$

W Cell

Another instrument that was designed and constructed during the experimental tests is the W Cell. This instrument, Figure 8, consists of a non-collapsible plastic tubing connected to a spherical shaped rubber balloon approximately 3 cm in diameter. The other end of the tubing is connected to a cylinderical shaped housing containing one of the diaphragm pressure cells. Details of this housing are shown in Figure 10. The balloon, tubing and part A of the housing are completely filled with water. When the balloon is subjected to stress, its volume of water cannot decrease, therefore, the pressure within the balloon must change. This change in pressure is the change in mean normal stress as water cannot transmit shearing stresses.

Calibration tests were made with the device reported by Hovanesian (1958) and the results are shown in Figure 11.

Soil Handling Equipment

The need for controlling the soil parameters and providing an accurate means of reproducing the initial soil conditions to obtain replication results required the construction of special soil handling equipment. The equipment described in this thesis was built as a joint project between Mr. Jack Stong (1960), Graduate Assistant,

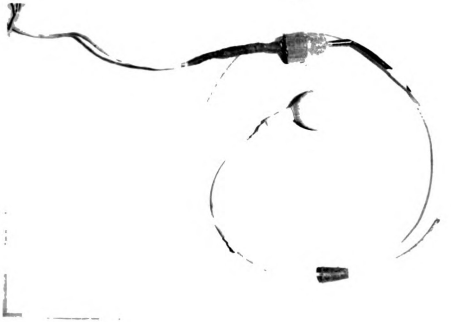

Figure 8. The W Cell used to measure Mean Stress directly.

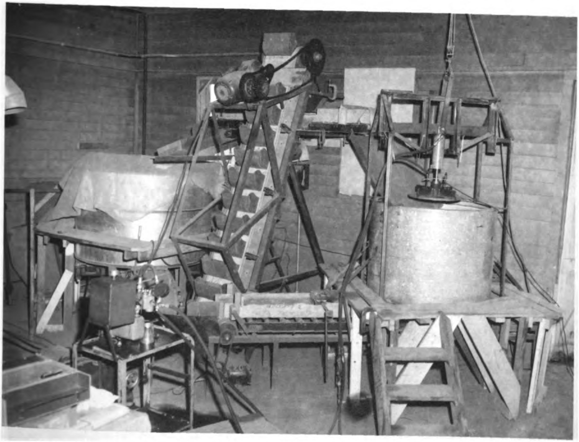

Figure 9. A View of the Soil Handling Equipment.

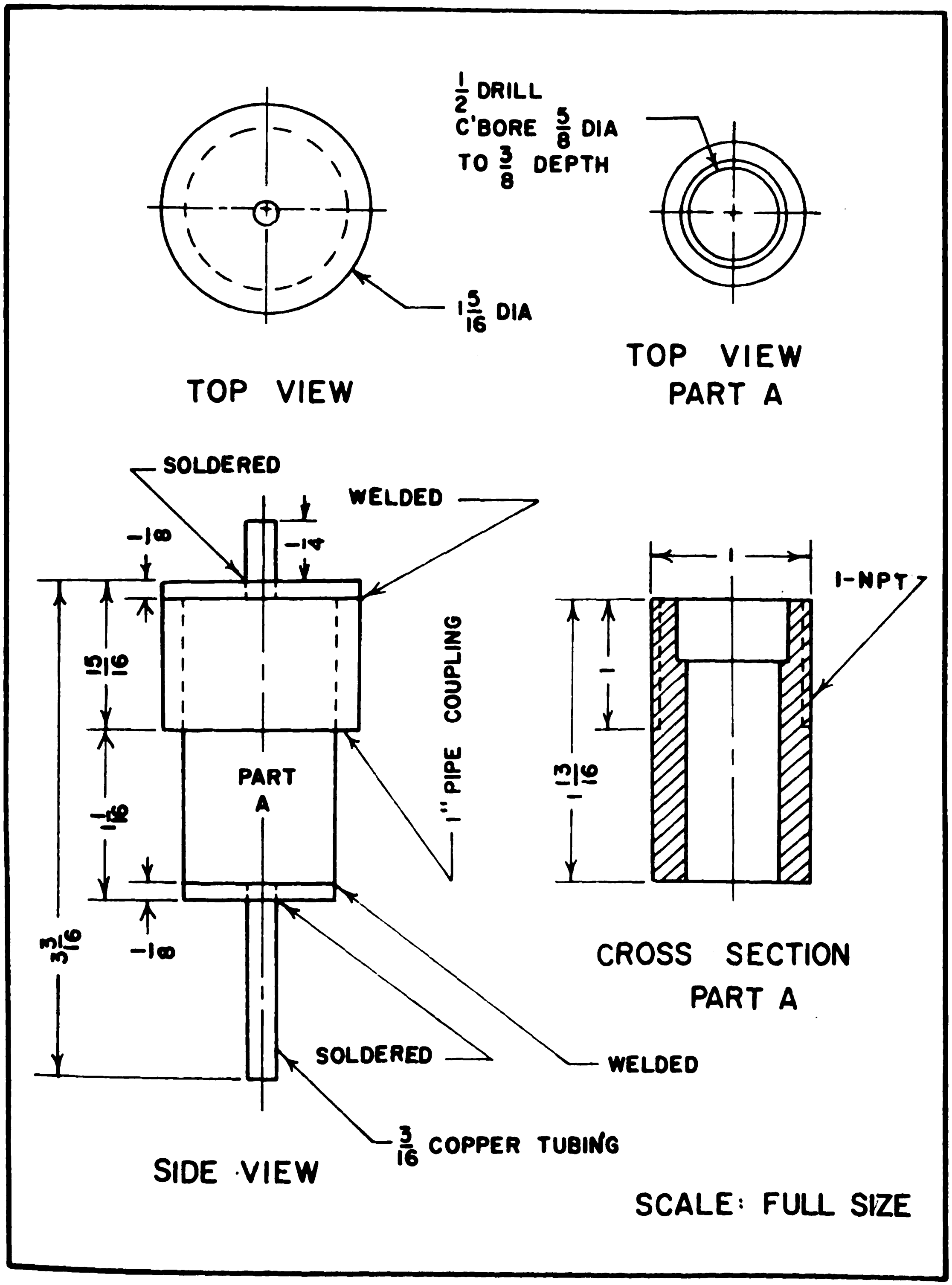

Figure 10. Detail Drawing of the W Cell used to Measure Mean Stress Directly.

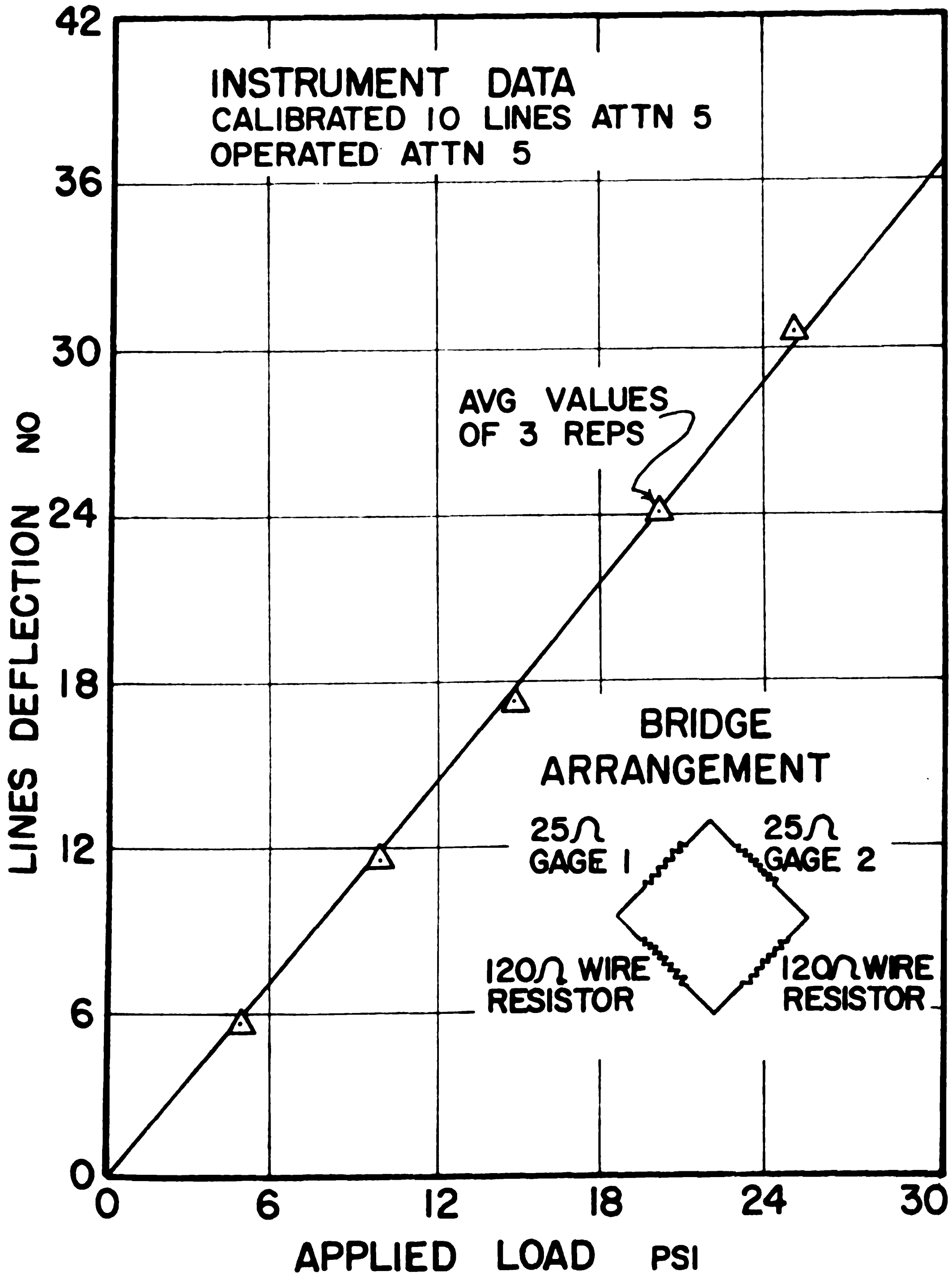

Figure 11. Calibration Curve for W Cell showing Number of Lines Deflection versus Applied Load.

Agricultural Engineering Department and the author.

The system as shown in Figure 9 page 35 was designed and constructed so that it was not limited to one particular experiment. There are two main soil tanks for experimental work and one storage tank. One tank is 5 feet in diameter and 42 inches high, the other is 4 feet in diameter and 3 feet high. Under each tank is located an 18-inch wide flat conveyor belt that transports the soil from the tank to the boot of a bucket elevator. The bucket elevator, which is 10 feet in height, was constructed of an 18-inch wide belting with 14-inch by 7-inch buckets placed 9 inches on center. At the head of the elevator is a 6-foot, reversible conveyor belt used for transporting the soil to a chute which delivers the soil into the tank to be filled.

A loading frame used in the application of dynamic vertical loads was constructed above the smaller tank. A 15-inch stroke, 4-inch bore hydraulic cylinder was suspended from the frame above the center of the tank. A 3/4-inch thick steel plate, 20 inches in diameter, was constructed for the loading plate (Figure 12). A strain-gage transducer designed to measure vertical forces only was connected between the end of the hydraulic cylinder piston and loading plate.

Force Transducer

A strain-gage force transducer designed and constructed

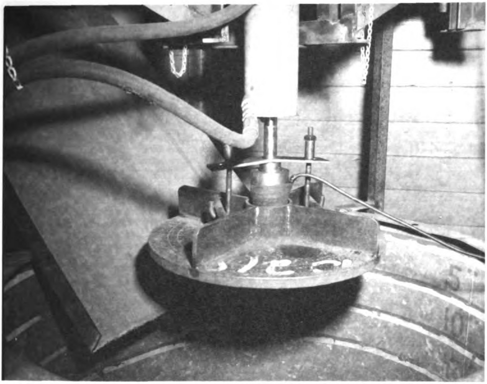

Figure 12. A View of the Loading Plate, Force Transducer and Hydraulic Cylinder.

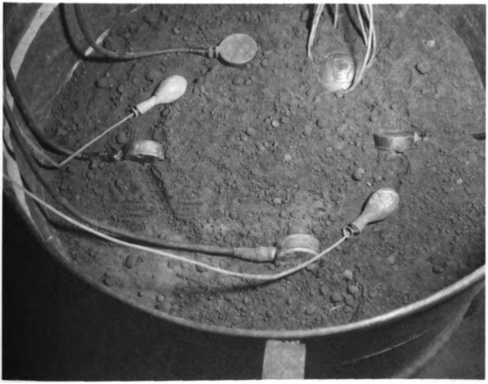

Figure 13. A View of the Pressure Transducers and Balloons used to obtain Data.

by Bellinger (1960) was modified to measure the total vertical force applied to the loading plate. The semi-ball as shown in Figure 14 was constructed to fit the external end of the hydraulic-cylinder piston. A seat to permit a flexible joint between the piston and force transducer was constructed and placed in the force transducer (Figure 14). Calibration test results made with a screw-type loading machine are shown in Figure 15.

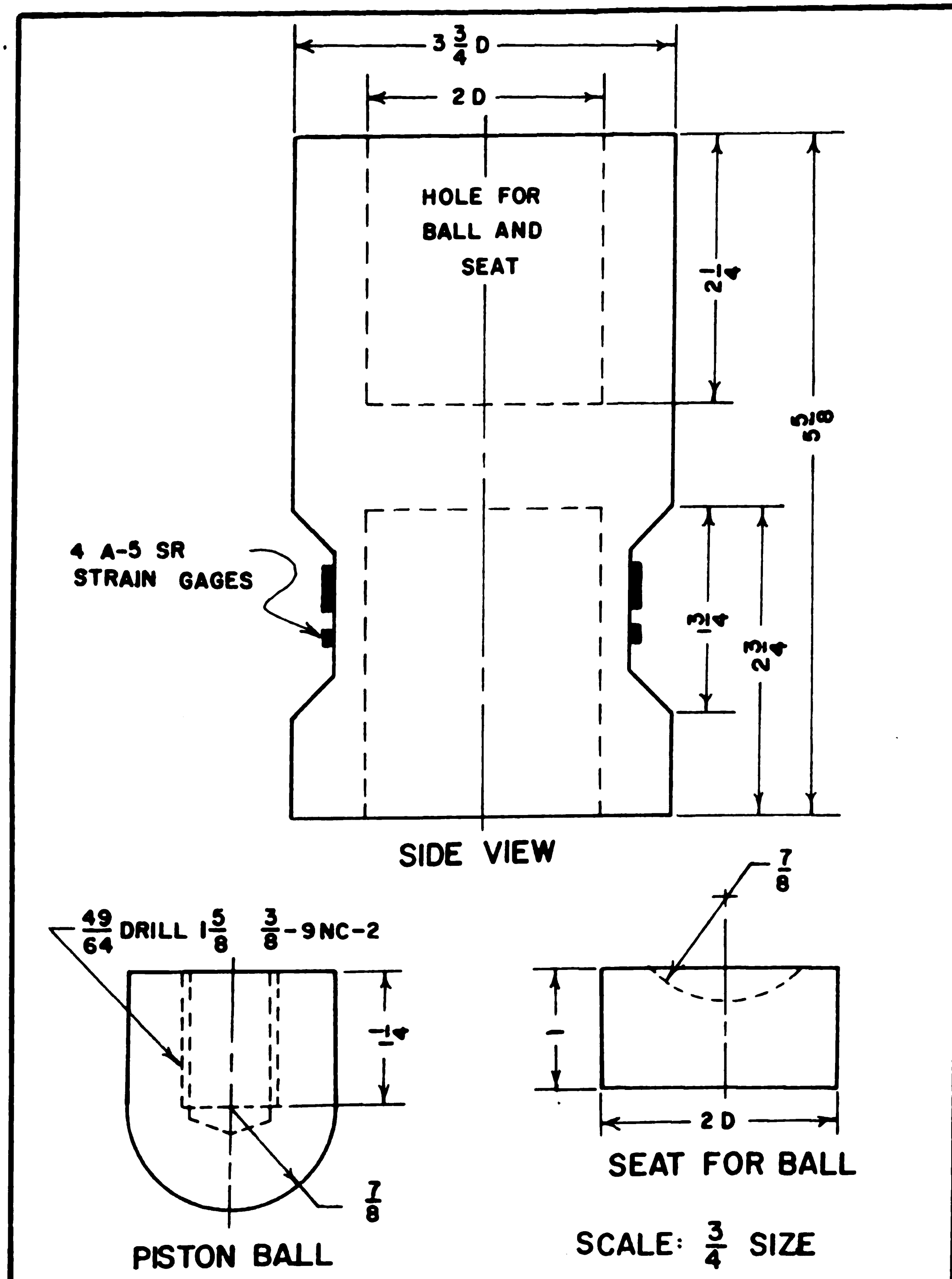

Figure 14. Detail drawing of Force Transducer showing location of Strain Gages and Ball and Socket joint.

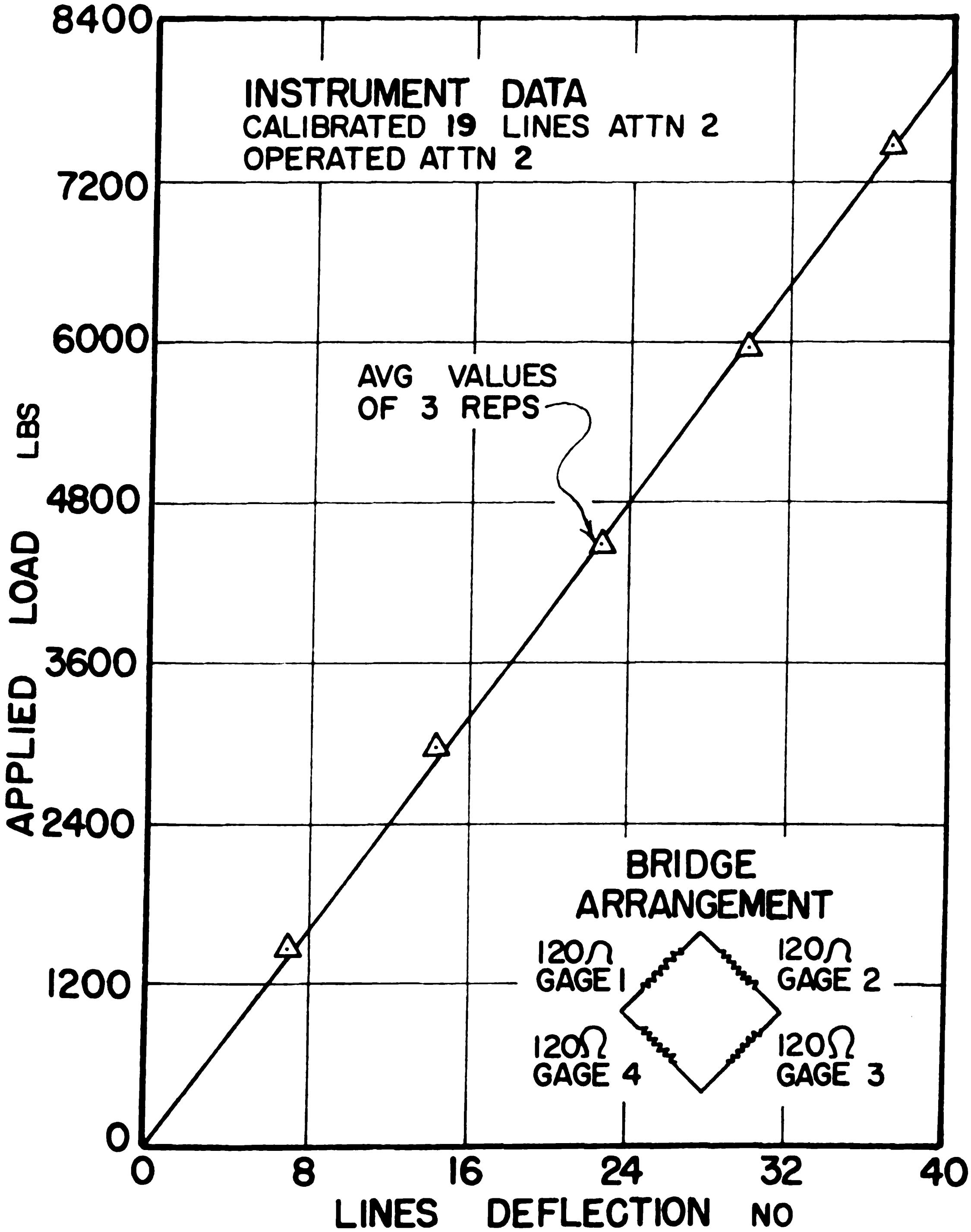

Figure 15 . Calibration Curve for Force Transducer showing Number of Lines Deflection versus Applied Load.

PROCEDURE

Original plans were to use a tank 4 feet in diameter. However, the results obtained by replication varied because the loading plate would not remain level during the loading process, and the contact surface therefore, could not be considered as a principal plane. This made it difficult to use the method of Vanden Berg's to check the results obtained with the six directional stress transducer. The soil flowed around the contact area and low values were obtained from the instruments in the soil and the load transducer.

It was decided to use a smaller tank to avoid these difficulties. Measurements showed that the loading plate would fit inside of a 55 gallon drum with approximately an inch of clearance between the plate and the inside wall of the drum. Tests were conducted to determine the distribution of vertical stress under the loading plate at the proposed test depth. Four of the Type A Cells were placed on the circumference of a 12-inch diameter circle. Results showed that the vertical stress pattern around the circumference of the circle was uniform.

The following proceduce was used while conducting a test. The soil was sprayed with water to obtain the desired moisture content and then stored in the larger tank one or two days. During this time a plastic sheet was placed over the soil to prevent moisture changes. Prior to conducting a series of tests at this level of moisture content a 3/4 by 2 inch expanded metal screen was placed over the top of the drum to remove large clods of soil that were formed during the wetting process. The soil was recirculated several times by the soil handling equipment to increase the homogeneity of the mass.

The small tank was filled to the desired level for the particular test being run and the surface leveled with a template. A circle 12 inches in diameter was drawn in the center of the tank and the Type A Cells, the six directional stress transducer and balloons for measuring changes in bulk density and in some tests the W Cell were positioned as shown in Figure 16. The position of the gages was checked with a hand level. To prevent movement of the instruments in the process of placing the desired depth of soil on top of them, loose soil was placed over all instruments by hand. After the tank was filled to the operating level, the surface was again leveled with the template. The loading plate was properly positioned and the recording instruments activated. The control lever of the portable hydraulic unit which operated the hydraulic cylinder was held open until the end of the piston stroke was reached or until a signal from the instrument operator was given. Upon completion of a test the soil and instruments were removed from the tank. The soil was passed through the 3/4 by 2 inch screen to remove large blocks of soil formed during the compaction process.

The rate of loading of the loading plate was changed by controlling the rate of fluid flow in the hydraulic lines. A needle valve was placed in the high pressure line and insured a constant rate of flow during the test run.

Moisture and bulk density samples were taken in the loose soil at the level of the instruments prior to the placement of the instruments. A standard core sampler was used to take the soil samples.

Since the six directional stress transducer was not completely airtight at high pressures, checks were made to determine the magnitude of the static pressure developed during a test. A piece of non-collapsible plastic tubing was connected to a static pressure gage and the open end placed at the same level of the instruments in the soil. Readings were taken during a test run and the maximum value obtained for all tests was 1.2 inches of water. This was so small that any effects on the pressure cells could be neglected.

The test soil was Brookston sandy loam, which was obtained from M.S.U. Farm Crops Farm located on Mt. Hope road. At the time of placing the soil in the testing tank the bulk density was approximately 0.95 to 1.05 (dry weight) depending upon the moisture content. The physical properties of the soil as reported by Stong (1960) are set forth in Table 2.

TABLE 2

PHYSICAL DESCRIPTION OF THE BROOKSTON SOIL USED IN TESTS

Mechanical Analysis	
Fine Gravel	1.2%
Coarse Sand	3.6%
Medium Sand	6.1%
Fine Sand	26.8%
Very Fine Sand	27.7%
50 Micron	13.4%
5 Micron	5.6%
2 Micron	15.6%
Hygroscopic Coefficient	1.6%
Moisture Equivalent	14.3%
Maximum Water Holding Capacity	63.8%
Soil Saturated	37.1%
60 cm Tension	25.4%
Permanent Wilting Point	8.7%
Lower Plastic Limit	21.0%
Upper Plastic Limit	25.5%
Plastic Range	4.5%
Density	2.6%

Since the recording instruments were set on zero reference after the soil tank had been filled and leveled, the effect of the weight of the soil on the pressure cells and balloons was not determined during the experimental tests. To determine the effect of the soil weight, a series of tests were run at 5, 10 and 15-inch depths with moisture contents of 12.21, 15.19 and 17.21 percents, and the results are presented in Table 3.

TABLE 3

CHANGES IN BULK DENSITY PRODUCED BY THE WEIGHT OF THE SOIL AS DETERMINED WITH THE VOLUMETRIC TRANSDUCER

	Percent Moisture Content					
	12.21		15.19		17.21	
Depth	B.D.	Stress	B.D.	Stress	B.D.	Stress
(in)	(gm/cc)	(psi)	(gm/cc)	(psi)	(gm/cc)	(psi)
0	1.064	0.0	0.962	0.0	0.932	0.0
5	1.068	0.9	0.967	1.0	0.947	1.1
10	1.072	1.2	0.969	1.2	0.952	1.3
15	1.075	1.4	0.970	1.7	0.959	1.8

These errors in initial bulk density cause errors in the measured values of bulk density obtained during a test. A 5.5 percent error for moisture contents less than 15.19 and an error of 8.5 percent for a moisture content of 17.21 are caused provided the bulk density change for the experimental test was as large as 0.2 gm/cc. These errors would change the intercept values of the relationships presented but would not change the values of the regression coefficients. The statistical analyses of the various relationships were based on the regression coefficients and therefore, are not affected by the error in neglecting the errors due to the weight of the soil.

The vertical stresses produced by the soil weight do not exceed 2.0 pounds per square inch. As mean stress is an average of three normal stresses the maximum error due to the weight of the soil mass would not exceed 1.0 pounds per square inch.

RESULTS AND DISCUSSION

To test the relationship between mean stress and bulk density under dynamic conditions, different stress states must be applied to a volume element. If different stress states are not used, all the stress tensor components will be linearly related to the applied load. In such cases all invariants of the stress tensor will be linearly related and will appear to be related to bulk density. Three different stress states were measured by varying the depth of instruments below the loading plates. Five replications of each stress state were taken and the calculated results are reported in Table A in the Appendix. The bulk density readings reported were an average of three measurements taken on the periphery of the circles.

The laboratory tests were conducted at three depths, three rates of loading and three moisture contents. A series of tests were run at a constant rate of loading and approximately the same moisture content at the three depths. Then the rate of loading was changed and the series repeated at the same moisture content. This procedure was followed until all possible combinations of the three rates, moisture contents and depths were used. For convenience a description of the tests is presented in Table 4. The values for initial bulk density were obtained

TABLE 4

DESCRIPTION OF THE LABORATORY TESTS

Test No.	Depth (in.)	Rate of Loading (in/sec)	Moisture Content (Per Cent)	Initial Bulk Density (gm/cc)
1	10	0.62	8.63	1.08
2	5	0.62	9.89	1.08
3	15	0.62	11.59	1.06
4	10	0.38	11.56	1.06
5	5	0.38	8.89	1.07
6	15	0.38	7.97	1.08
7	5	0.38	11.35	1.08
8	15	0.38	11.39	1.09
9	10	0.38	11.38	1.06
10	15	0.62	12.63	1.06
11	10	0.62	12.43	1.08
12	5	0.62	12.46	1.09
13	10	1.00	17.41	0.94
14	5	1.00	17.41	0.94
15	15	1.00	14.85	1.01
16	10	1.00	14.85	1.02
17	5	1.00	14.34	1.03
18	15	1.00	12.31	1.08
19	10	1.00	10.95	1.06
20	5	1.00	10.79	1.06
21	15	0.62	17.93	0.93
22	10	0.62	17.52	0.91
23	5	0.62	17.67	0.95
24	15	0.38	16.16	1.00
25	10	0.38	15.78	0.98
26	5	0.38	16.05	0.98
27	15	1.00	16.54	0.96

with a hand sampler at the instrument level prior to filling the tank.

The values of σ_m, II_s, σ_I, σ_{II}, σ_{III} and τ_{max} for the Type A Cells were computed from four measured stress values using the appropriate formulae as reported by Vanden Berg (1958). The values for the 6 DST were computed from 6 measured stress values using equations 7b, 8b, 9b, 12, 13, 14, 19 and 24. MISTIC, an electronic digital computer at Michigan State University was used to make the lengthy calculations involved in evaluating the above equations and the statistical analysis. The sum of least squares method was used to fit a straight line to the data plotted on semi-logarithmic paper. An estimate of standard error was determined for each relationship and a test of significance of the slope was made by using the "t" test. The confidence limits at the 95 per cent level for each relationship were also determine.

The results have been presented under the following headings:

1. The Relationship between Mean Normal Stress and Bulk Density
2. The Relationship between Second Invariant and Bulk Density
3. The relationship between Maximum Normal Stress and Bulk Density

Relationship between Mean Normal Stress and Bulk Density

The results obtained indicated an exponential relationship between mean normal stress and bulk density. This exponential relationship had been observed by other investigators (Soehne, 1953; Hovanesian, 1958; Vanden Berg, 1958). Instead of plotting the results using rectangular coordinates, semi-logarithmic paper was used to obtain a straight curve.

The variation which can be seen by examining the data in Table A of the Appendix required that statistical analysis be used. The sum of least squares method was used to determine the best predicting relationship. The natural logarithm of the values of mean stress were used instead of the quantity itself. In mathematical terms, bulk density, the dependent variable, would be described as a function of mean stress, the independent variable. In biological statistics the term regression is generally used and the relationship is defined by the regression equation.

The regression equations, estimates of standard error (S_{xy}) and confidence limits for both the Type A Cells and 6 DST data are given in Table 5. The Type A data is designated by an "A" following the test number and the 6 DST data by only the test number. The calculated values of "t" were compared with the distribution of "t" using the degrees of freedom (D.F.) shown. All calculated values

TABLE 5

STATISTICAL ANALYSIS FOR MEAN NORMAL STRESS VERSUS BULK DENSITY

Test No.	Regression Equation	Syx	D.F.	t	Confidence Limits
1	$\ln \sigma m=-36.82+30.50\gamma$	0.13	26	38.46	28.63-32.27
1A	$\ln \sigma m=-26.11+22.10\gamma$	0.12		25.68	20.33-23.87
2	$\ln \sigma m=-37.41+31.28\gamma$	0.20	33	27.93	24.60-29.04
2A	$\ln \sigma m=-25.41+21.65\gamma$	0.19		16.31	18.94-24.36
3	$\ln \sigma m=-23.08+20.26\gamma$	0.27	38	15.35	18.03-22.49
3A	$\ln \sigma m=-21.53+19.07\gamma$	0.20		18.93	17.37-20.77
4	$\ln \sigma m=-36.29+30.99\gamma$	0.24	18	12.01	25.57-36.41
4A	$\ln \sigma m=-29.90+25.72\gamma$	0.20		12.28	21.31-30.13
5	$\ln \sigma m=-34.47+28.76\gamma$	0.15	28	26.39	26.53-30.99
5A	$\ln \sigma m=-25.14+21.27\gamma$	0.12		24.00	19.45-23.09
6	$\ln \sigma m=-27.59+23.96\gamma$	0.19	38	21.59	22.09-25.83
6A	$\ln \sigma m=-25.98+22.44\gamma$	0.19		20.07	20.55-24.33
7	$\ln \sigma m=-34.21+29.83\gamma$	0.23	33	21.31	26.98-32.68
7A	$\ln \sigma m=-26.68+23.58\gamma$	0.18		21.60	21.36-25.88
8	$\ln \sigma m=-30.10+26.30\gamma$	0.50	38	10.08	21.90-30.70
8A	$\ln \sigma m=-23.97+21.14\gamma$	0.42		9.65	17.45-24.83
9	$\ln \sigma m=-26.84+24.41\gamma$	0.45	33	13.16	22.81-31.15
9A	$\ln \sigma m=-24.17+21.23\gamma$	0.25		14.26	18.20-24.26
10	$\ln \sigma m=-26.84+24.41\gamma$	0.45	38	11.05	20.68-28.14
10A	$\ln \sigma m=-20.19+18.77\gamma$	0.31		12.37	16.21-21.33
11	$\ln \sigma m=-34.02+29.84\gamma$	0.32	33	15.07	25.81-33.87
11A	$\ln \sigma m=-24.66+22.12\gamma$	0.21		17.16	19.50-24.74
12	$\ln \sigma m=-23.49+22.14\gamma$	0.26	33	16.77	19.46-24.82
12A	$\ln \sigma m=-20.02+19.06\gamma$	0.21		17.71	16.86-21.26
13	$\ln \sigma m=-38.41+37.96\gamma$	0.25	22	15.43	32.86-43.06
13A	$\ln \sigma m=-31.44+31.40\gamma$	0.25		12.95	26.38-36.42
14	$\ln \sigma m=-53.62+51.83\gamma$	0.25	18	14.44	44.29-59.37
14A	$\ln \sigma m=-41.76+40.72\gamma$	0.18		15.46	35.19-46.25

TABLE 5 (CONTINUED)

Test No.	Regression Equation	Syx	D.F.	t	Confidence Limits
15	$\ln \sigma_m = -51.75 + 49.72\gamma$	0.35	33	14.54	42.76-56.68
15A	$\ln \sigma_m = -38.15 + 37.13\gamma$	0.27		14.00	31.74-42.52
16	$\ln \sigma_m = -34.82 + 34.68\gamma$	0.17	33	26.05	32.11-37.53
16A	$\ln \sigma_m = -31.73 + 31.75\gamma$	0.17		22.19	28.84-34.66
17	$\ln \sigma_m = -32.79 + 32.83\gamma$	0.23	33	19.43	29.39-36.27
17A	$\ln \sigma_m = -26.36 + 26.65\gamma$	0.26		13.74	22.70-30.60
18	$\ln \sigma_m = -37.77 + 32.16\gamma$	0.18	38	31.84	30.46-33.86
18A	$\ln \sigma_m = -29.06 + 25.19\gamma$	0.17		26.41	23.59-26.79
19	$\ln \sigma_m = -43.71 + 37.50\gamma$	0.22	38	14.88	34.77-40.23
19A	$\ln \sigma_m = -33.44 + 29.05\gamma$	0.17		22.63	26.89-31.21
20	$\ln \sigma_m = -35.36 + 30.42\gamma$	0.22	38	22.53	28.14-32.70
20A	$\ln \sigma_m = -27.75 + 24.18\gamma$	0.21		18.31	21.95-26.41
21	$\ln \sigma_m = -40.48 + 39.81\gamma$	0.16	18	18.10	35.19-44.43
21A	$\ln \sigma_m = -33.23 + 32.88\gamma$	0.15		16.34	28.66-37.10
22	$\ln \sigma_m = -36.92 + 36.75\gamma$	0.22	13	9.10	28.02-45.48
22A	$\ln \sigma_m = -33.06 + 32.76\gamma$	0.14		12.10	26.91-38.61
23	$\ln \sigma_m = -27.91 + 27.87\gamma$	0.32	13	6.26	18.26-37.48
23A	$\ln \sigma_m = -21.16 + 21.37\gamma$	0.20		7.50	15.21-27.53
24	$\ln \sigma_m = -29.03 + 28.54\gamma$	0.29	23	13.27	24.09-32.99
24A	$\ln \sigma_m = -22.30 + 22.20\gamma$	0.20		14.68	19.08-25.32
25	$\ln \sigma_m = -23.63 + 23.73\gamma$	0.21	23	16.37	20.73-26.73
25A	$\ln \sigma_m = -19.24 + 19.56\gamma$	0.20		14.02	16.68-22.44
26	$\ln \sigma_m = -30.92 + 30.84\gamma$	0.29	23	11.34	25.21-36.47
26A	$\ln \sigma_m = -23.56 + 23.95\gamma$	0.24		10.51	19.23-28.67
27	$\ln \sigma_m = -26.19 + 25.23\gamma$	0.17	18	21.56	22.77-27.69
27A	$\ln \sigma_m = -19.32 + 18.70\gamma$	0.14		18.59	16.58-20.82

were highly significant which means that the regression coefficients or slopes are different than zero. The true regression coefficient is within the limits presented for each relationship. Assuming a normal distribution of error, one standard error (S_{xy}) would include 68.3% of the values used to determine the regression equation. The data obtained with the Six Directional Transducer are consistently more varied than that obtained with the Type A Cells as the standard errors are larger except for three tests.

If a quantity is related to bulk density in a general manner, the regression lines for each different stress state should not be significantly different for a given soil condition. The lines should be parallel or the difference between slopes should not be significant. The "t" test was used to test for differences among the lines for different stress states for each method used. The results of these tests for the Type A and 6 DST lines are presented in Table 6.

Differences between 1-3, 14-13, 17-16, 20-19, and 19-18 are significant at the 95 per cent level (*) and between 14-27, 13-27, 17-15, 23-22 and 23-21 are significant at the 99 per cent level (**) with data obtained with the Type A Cells.

The following tests for data obtained with the 6 DST show a significant difference; 2-3, 1-3, 5-6, 4-6, 7-9, 14-13, 14-27, 13-27, 17-15, 16-15, 20-19, 19-18, 23-21,

TABLE 6

COMPARISON OF REGRESSION COEFFICIENTS OF THE σ_m VERSUS BULK DENSITY RELATION FOR DIFFERENT STRESS STATES AND SAME STRESS STATE FOR THE TWO METHODS USED TO OBTAIN DATA

Tests Compared	Degrees of Freedom	t 6 DST	t Type A	Test No.	Depth In.	Degrees of Freedom	t
2-1	59	0.55	0.28	1	10	52	6.71**
2-3	71	6.44**	1.54	2	5	66	5.60**
1-3	64	6.39**	2.28*	3	15	76	0.72
5-4	46	0.80	1.95	4	10	36	1.58
5-6	66	3.09**	0.82	5	5	56	5.32**
4-6	56	2.50*	1.38	6	15	76	0.96
7-9	66	2.18*	1.27	7	5	66	3.52**
7-8	71	1.19	1.00	8	15	76	1.51
9-8	71	0.57	0.57	9	10	66	1.25
12-11	66	0.32	1.82	10	15	76	2.10*
12-10	71	0.88	0.16	11	10	66	3.27**
11-10	71	1.83	1.68	12	5	66	1.81
14-13	40	3.19**	2.61*	13	10	44	1.90
14-27	36	7.04**	7.82**	14	5	36	2.50*
13-27	40	4.67**	4.84**	15	15	66	2.91*
17-16	66	0.86	2.12*	16	10	66	1.50
17-15	66	4.43**	3.19**	17	5	66	2.40*
16-15	66	4.10**	1.79	18	15	76	5.03**
20-19	76	3.36**	2.65*	19	10	76	4.09**
20-18	76	1.03	0.62	20	5	76	3.30**
19-18	76	2.80**	2.42*	21	15	36	2.33*
23-22	26	1.48	2.90**	22	10	26	0.82
23-21	31	2.41*	3.30**	23	5	26	1.23
22-21	31	0.67	0.04	24	15	46	2.41*
26-25	46	2.31*	1.64	25	10	46	2.07*
26-24	46	0.66	0.64	26	5	46	1.94
25-24	46	1.86	1.29	27	15	36	4.22**

and 26-25.

The significant differences for the 6 DST and Type A data between 1-3 could be due to the difference in moisture content of 2.96%. The results vary between the two methods used except for the highest rate of loading and the higher moisture contents. On the basis of the data obtained the hypothesis that changes in bulk density are controlled by mean normal stress cannot be accepted or rejected.

The results of the comparison of the regression coefficients for the same stress state obtained by the two types of instruments are given in column four of Table 6. There are significant differences for tests 1, 2, 5, 7, 10, 11, 14, 15, 17, 18, 19, 20, 21, 24, 25 and 27.

Approximately fifty percent of the calculated values of "t" show a significant difference. A pattern appears between the 10- and 15-inch depths at high moisture contents and high rates of loading. At the lower moisture contents and lower rates of loading significant differences appear between the 5- and 10-inch depths and 5- and 15-inch depths.

A typical set of data showing the relationship between mean normal stress and bulk density are shown in Figures 16 and 17.

<u>Relationship between Second Invariant and Bulk Density</u>

The calculated results showing the relationships between the second invariant of the stress deviator tensor

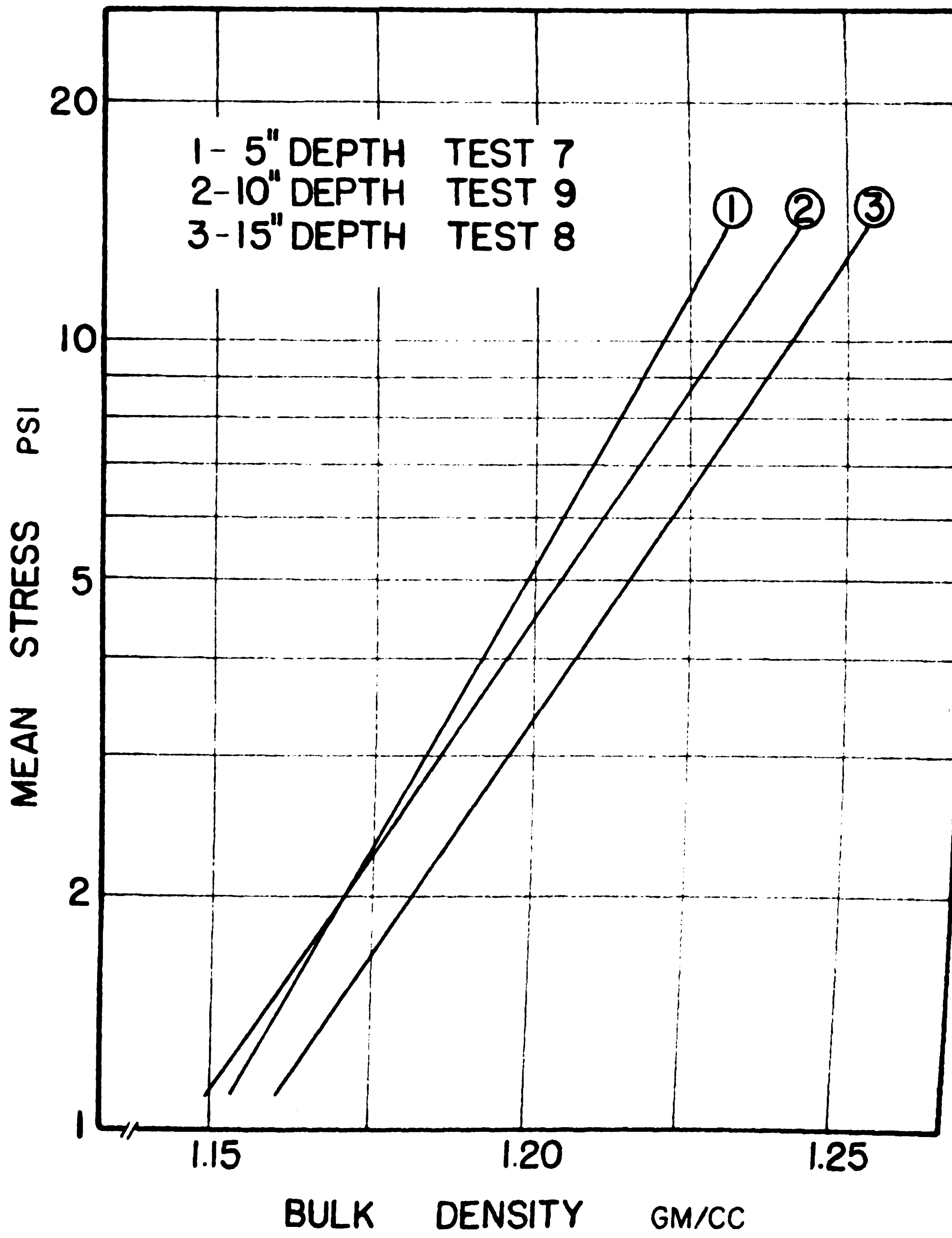

Figure 16. Mean Normal Stress versus Bulk Density for Data Obtained with Six Directional Stress Transducer.

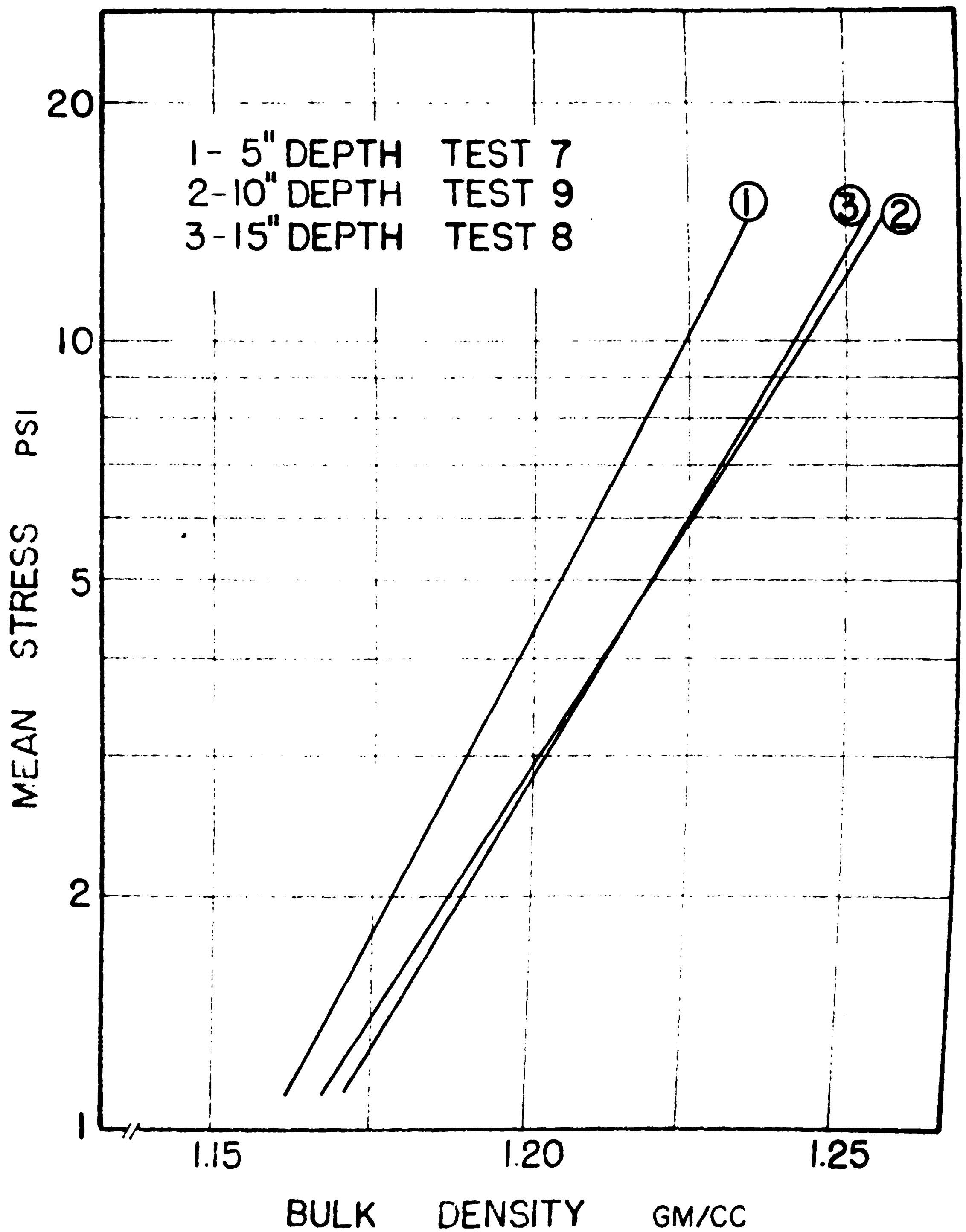

Figure 17. Mean Normal Stress versus Bulk Density for Data Obtained with Type A Pressure Cells.

and bulk density for both sets of data are presented in Table 7. The values of "t" are highly significant which means that the regression coefficients or slopes are different from zero. The estimates of the standard error for the 6 DST data are larger than the Type A values except for three tests.

If a quantity is not related to bulk density in a general manner, the regression line for each different stress state should be significantly different from one obtained for a different stress state. As the second invariant is one of the three functions of the stress deviator tensor the results of tests between different stress states are given in Table 8.

The results for the 6 DST show significant differences between tests 2-1, 2-3, 1-3, 5-6, 4-6, 12-11, 14-27, 13-27, 17-16, 17-15, 16-15, 20-19, 20-18, 23-21, and 26-25. For the Type A data significant differences are found between 1-3, 12-11, 14-13, 14-27, 13-27, 17-16, 17-15, 16-15, 20-19, 19-18, 23-22, 23-21, and 26-24.

Comparisons of the regression coefficients between the two methods used to obtain the data show significant differences for tests 1, 2, 5, 7, 10, 11, 15, 18, 21, 24, 25, 26, and 27.

These results are similiar to those for the relationship between mean normal stress and bulk density. The values for the 6 DST data show more differences than the

TABLE 7

STATISTICAL ANALYSIS FOR SECOND INVARIANT VERSUS BULK DENSITY

Test No.	Regression Equation	Syx	D.F.	t	Confidence Limits
1	$\ln II_s=-36.74+31.12\gamma$	0.13	26	34.20	29.25-32.99
1A	$\ln II_s=-25.14+21.80\gamma$	0.13		24.12	19.95-23.65
2	$\ln II_s=-28.38+25.18\gamma$	0.22	33	26.23	23.23-27.13
2A	$\ln II_s=-24.40+21.34\gamma$	0.20		15.69	18.57-24.11
3	$\ln II_s=-21.03+19.42\gamma$	0.27	38	14.71	17.19-21.65
3A	$\ln II_s=-20.92+18.99\gamma$	0.22		17.99	17.20-20.78
4	$\ln II_s=-34.73+30.38\gamma$	0.23	18	12.30	25.19-35.57
4A	$\ln II_s=-28.94+25.38\gamma$	0.17		13.66	21.47-29.29
5	$\ln II_s=-32.65+27.90\gamma$	0.15	28	25.60	25.67-30.13
5A	$\ln II_s=-24.79+21.52\gamma$	0.13		22.77	19.59-23.45
6	$\ln II_s=-27.19+24.31\gamma$	0.22	38	18.99	22.15-26.47
6A	$\ln II_s=-25.88+22.81\gamma$	0.19		20.47	20.94-24.68
7	$\ln II_s=-34.24+30.46\gamma$	0.24	33	20.86	27.49-33.43
7A	$\ln II_s=-25.91+23.43\gamma$	0.17		22.16	21.27-25.59
8	$\ln II_s=-28.45+25.65\gamma$	0.49	38	10.02	21.33-29.97
8A	$\ln II_s=-23.82+21.47\gamma$	0.40		10.37	17.98-24.96
9	$\ln II_s=-27.14+25.39\gamma$	0.49	33	13.85	24.94-33.52
9A	$\ln II_s=-23.99+21.60\gamma$	0.27		13.81	18.43-24.77
10	$\ln II_s=-27.14+25.39\gamma$	0.49	38	10.54	21.33-29.45
10A	$\ln II_s=-20.13+19.27\gamma$	0.31		12.72	16.71-21.93
11	$\ln II_s=-32.19+29.04\gamma$	0.35	33	13.38	24.63-33.45
11A	$\ln II_s=-24.78+22.68\gamma$	0.23		16.18	19.83-25.53
12	$\ln II_s=-20.58+20.32\gamma$	0.24	33	16.66	17.84-22.80
12A	$\ln II_s=-19.24+18.98\gamma$	0.21		17.89	16.82-21.14
13	$\ln II_s=-36.24+36.67\gamma$	0.26	22	14.38	31.38-41.96
13A	$\ln II_s=-30.32+30.89\gamma$	0.24		13.16	26.02-35.76
14	$\ln II_s=-41.87+41.47\gamma$	0.22	18	13.12	34.83-48.11
14A	$\ln II_s=-39.72+39.28\gamma$	0.18		15.44	33.94-44.62

TABLE 7 (CONTINUED)

Test No.	Regression Equation	Syx	D.F.	t	Confidence Limits
15	$\ln II_s=-50.39+49.24\gamma$	0.32	33	15.73	42.87-55.61
15A	$\ln II_s=-38.55+38.08\gamma$	0.38		13.72	32.43-43.73
16	$\ln II_s=-32.60+33.33\gamma$	0.17	33	25.06	30.62-36.04
16A	$\ln II_s=-30.59+31.26\gamma$	0.20		20.12	28.11-34.41
17	$\ln II_s=-27.84+28.88\gamma$	0.21	33	18.63	25.73-32.03
17A	$\ln II_s=-24.84+25.79\gamma$	0.25		14.03	22.05-29.53
18	$\ln II_s=-35.41+30.90\gamma$	0.17	38	32.53	29.30-32.50
18A	$\ln II_s=-28.45+25.15\gamma$	0.16		28.06	23.63-26.67
19	$\ln II_s=-37.41+33.00\gamma$	0.19	38	23.57	30.64-35.36
19A	$\ln II_s=-34.02+29.99\gamma$	0.18		22.14	27.71-32.27
20	$\ln II_s=-29.41+26.28\gamma$	0.21	38	20.37	24.11-28.45
20A	$\ln II_s=-27.23+24.16\gamma$	0.22		17.99	21.90-26.42
21	$\ln II_s=-42.99+43.00\gamma$	0.19	18	16.48	37.52-48.48
21A	$\ln II_s=-32.06+32.40\gamma$	0.16		15.16	27.90-36.90
22	$\ln II_s=-36.68+37.36\gamma$	0.20	13	10.15	29.41-45.31
22A	$\ln II_s=-32.01+32.32\gamma$	0.15		11.44	26.23-38.41
23	$\ln II_s=-25.42+26.25\gamma$	0.28	13	6.75	17.85-34.65
23A	$\ln II_s=-20.29+21.05\gamma$	0.21		7.10	14.66-27.44
24	$\ln II_s=-28.76+29.10\gamma$	0.29	23	13.53	24.65-33.55
24A	$\ln II_s=-21.65+22.24\gamma$	0.19		15.56	19.28-15.20
25	$\ln II_s=-23.30+24.27\gamma$	0.22	23	15.97	21.13-27.41
25A	$\ln II_s=-18.80+19.67\gamma$	0.21		13.47	16.65-22.69
26	$\ln II_s=-30.13+30.94\gamma$	0.30	23	11.01	25.13-36.75
26A	$\ln II_s=-22.22+23.25\gamma$	0.22		11.10	18.93-27.57
27	$\ln II_s=-24.88+24.94\gamma$	0.19	18	19.04	22.19-27.69
27A	$\ln II_s=-19.05+18.94\gamma$	0.13		21.69	17.11-20.77

TABLE 8

COMPARISON OF REGRESSION COEFFICIENTS OF THE II_s VERSUS BULK DENSITY RELATION FOR DIFFERENT STRESS STATES AND SAME STRESS STATE FOR THE TWO METHODS USED TO OBTAIN DATA

Tests Compared	Degrees of Freedom	t 6 DST	t Type A	Test No.	Depth In.	Degrees of Freedom	t
2-1	59	4.49**	0.28	1	10	52	7.28**
2-3	71	3.53**	1.36	2	5	66	2.31*
1-3	64	7.30**	2.02*	3	15	76	0.25
5-4	46	0.92	1.85	4	10	36	1.62
5-6	66	2.14*	0.89	5	5	56	4.43**
4-6	56	2.18*	1.19	6	15	76	0.89
7-9	66	1.98	0.97	7	5	66	3.90**
7-8	71	1.63	0.84	8	15	76	1.27
9-8	71	0.08	0.05	9	10	66	1.44
12-11	66	3.50**	2.11*	10	15	76	2.15*
12-10	71	1.88	0.16	11	10	66	2.46*
11-10	71	1.13	1.65	12	5	66	0.83
14-13	40	1.18	2.42*	13	10	44	1.67
14-27	36	4.83**	7.58**	14	5	36	0.54
13-27	40	4.09**	4.77**	15	15	66	2.67**
17-16	66	2.18*	2.27*	16	10	66	1.01
17-15	66	5.83**	3.69**	17	5	66	1.28
16-15	66	4.68**	2.14*	18	15	76	4.39**
20-19	76	3.53**	3.06**	19	10	76	1.55
20-18	76	2.88**	0.61	20	5	76	1.14
19-18	76	1.24	2.98**	21	15	36	3.14**
23-22	26	2.07	2.76**	22	10	26	1.09
23-21	31	3.58**	3.11**	23	5	26	1.06
22-21	31	1.25	0.02	24	15	46	2.66**
26-25	46	2.09*	1.40	25	10	46	2.18*
26-24	46	0.52	3.99**	26	5	46	2.20*
25-24	46	1.83	1.26	27	15	36	3.82**

Type A data, however, neither set of values support completely the part of the hypothesis that the second invariant is not related to changes in bulk density.

A set of typical curves showing the relationship between the second invariant and bulk density are shown in Figures 18 and 19. A comparison of the second invariants for test 18 is shown in Figure 20. The large difference in the curves at high loads is due to the neglected values of the shearing stresses of equation (20b) in calculating the second invariant with the Type A data.

Relationship between Maximum Normal Stress and Bulk Density

The regression equations, standard errors and confidence limits for maximum normal stress versus bulk density are given in Table 9. Comparing the calculated values of "t" with those in the "t" table shows that all are highly significant. Therefore, all slopes are significantly different from zero.

Results of the "t" tests for the maximum normal stress-bulk density relationship are presented in Table 10. Differences between 2-1, 2-3, 1-3, 7-8, 12-11, 12-10, 14-27, 13-27, 17-16, 17-15, 16-15, 20-19, 18-20, 23-22, 23-21, 26-25, and 25-24 of the 6 DST data are significant. There are significant differences between 12-11, 14-27, 13-27, 17-16, 17-15, 16-15, 20-19, 19-18, 23-22, and 23-21 for the data obtained with the Type A Cells.

Tests 1, 2, 5, 7, 10, 15, 18, 21, 24, 26 and 27 show

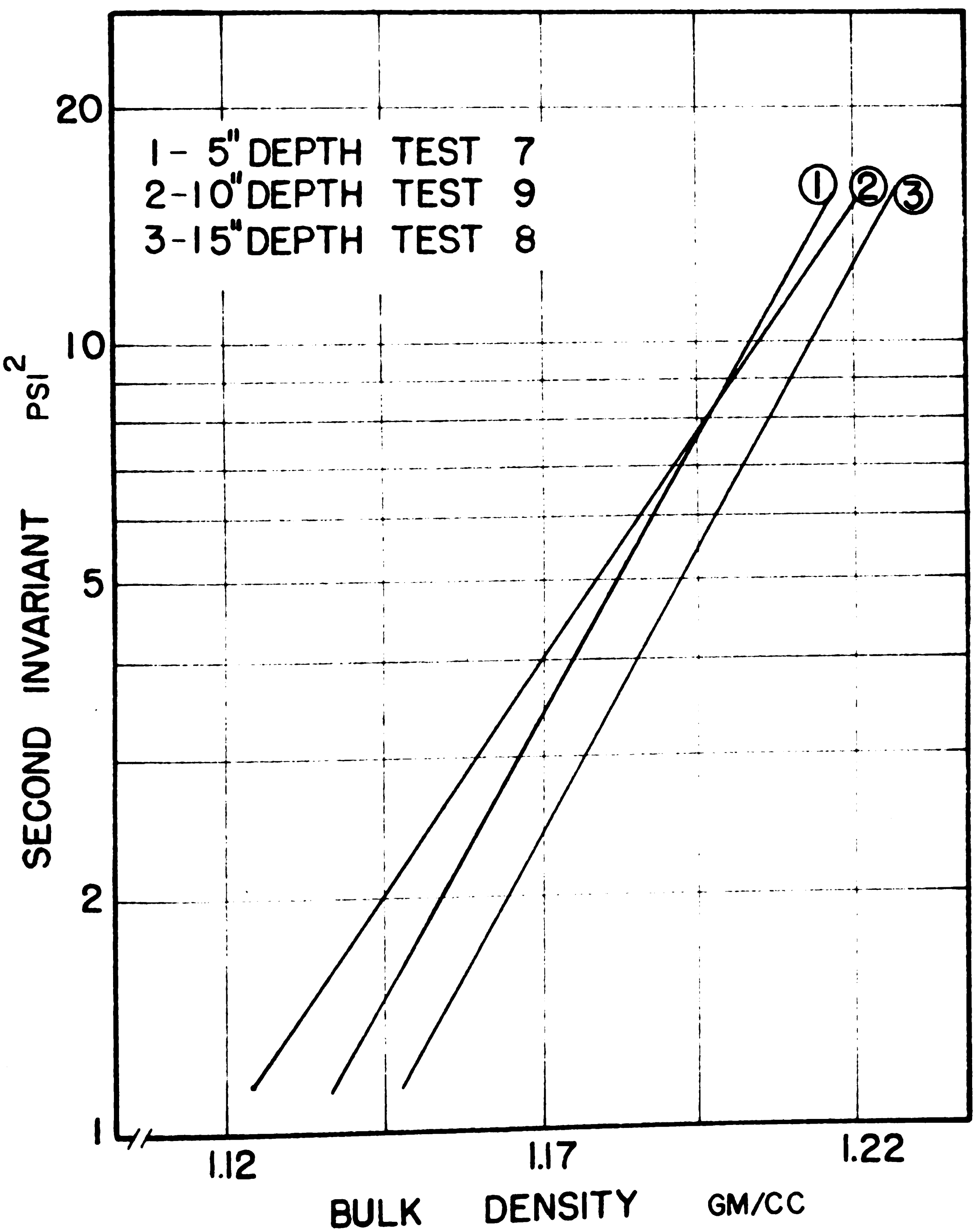

Figure 18. Second Invariant of Stress Deviator Tensor versus Bulk Density for Data Obtained with Six Directional Stress Transducer.

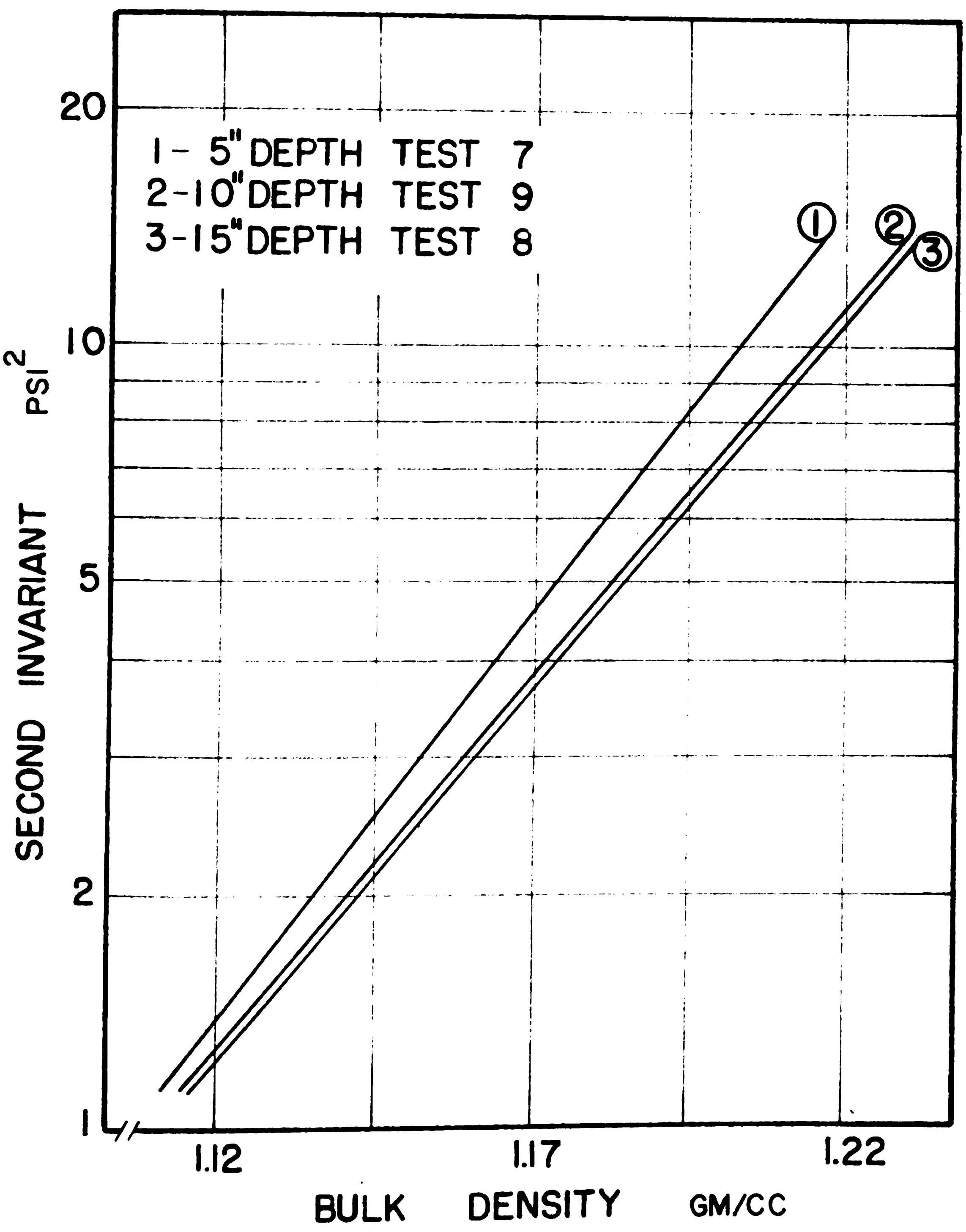

Figure 19. Second Invariant of Stress Deviator Tensor versus Bulk Density for Data Obtained with Type A Pressure Cells.

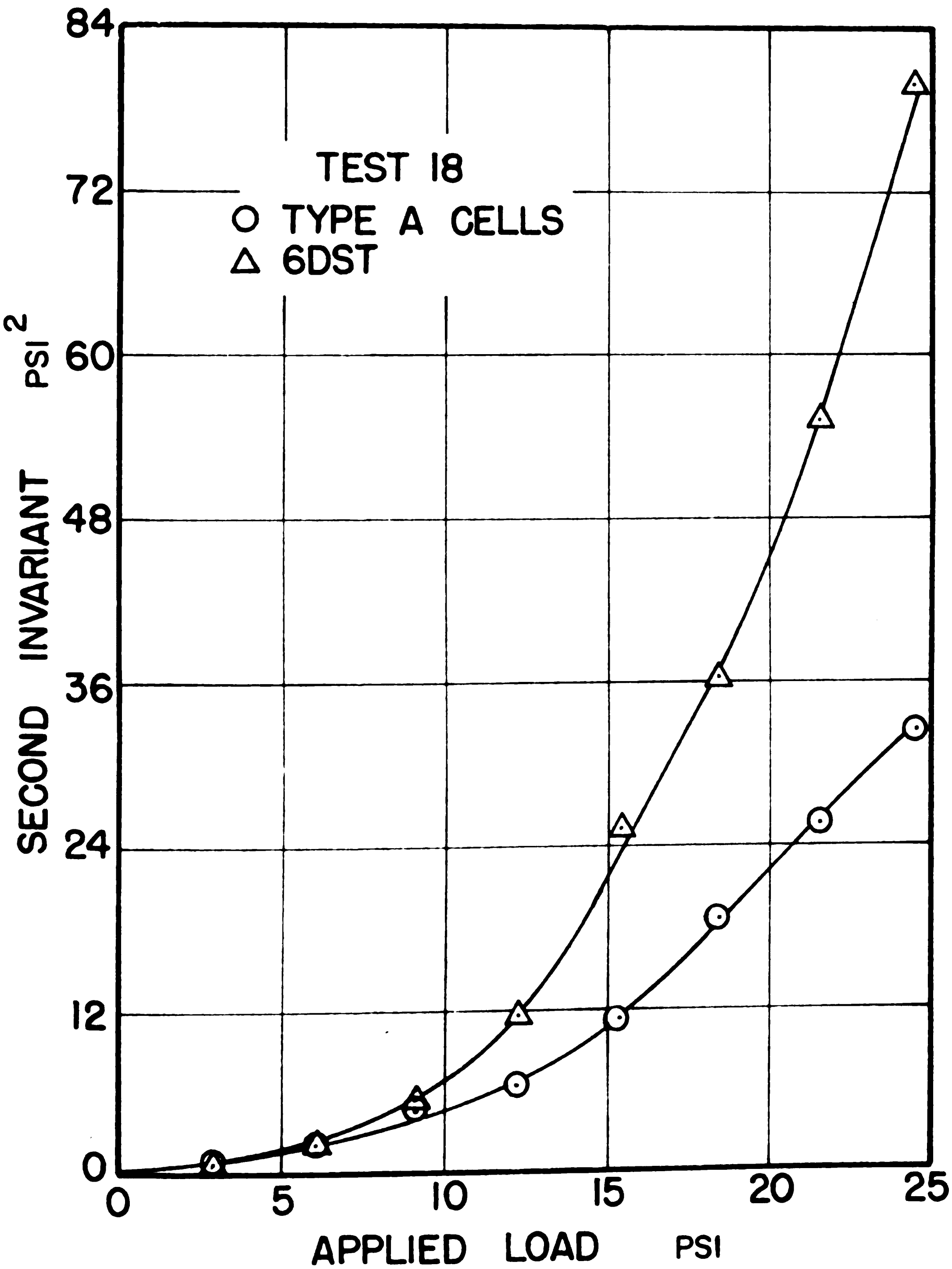

Figure 20. Comparison of the Values of the Second Invariant Calculated from the Type A and 6 DST Data.

TABLE 9

STATISTICAL ANALYSIS FOR MAXIMUM NORMAL STRESS VERSUS BULK DENSITY

Test No.	Regression Equation	Syx	D.F.	t	Confidence Limits
1	$\ln \sigma_I=-36.53+30.40\gamma$	0.16	26	33.41	28.53-32.27
1A	$\ln \sigma_I=-25.77+21.47\gamma$	0.26		39.51	19.25-23.69
2	$\ln \sigma_I=-27.30+24.01\gamma$	0.18	33	23.54	21.94-26.08
2A	$\ln \sigma_I=-23.97+20.18\gamma$	0.20		30.74	17.41-22.95
3	$\ln \sigma_I=-21.48+19.23\gamma$	0.27	38	14.64	16.55-21.91
3A	$\ln \sigma_I=-21.25+18.31\gamma$	0.24		32.56	16.35-20.27
4	$\ln \sigma_I=-34.39+29.45\gamma$	0.21	18	13.09	24.72-34.18
4A	$\ln \sigma_I=-30.09+25.46\gamma$	0.16		28.17	21.72-29.20
5	$\ln \sigma_I=-32.31+26.97\gamma$	0.16	28	23.25	24.60-29.34
5A	$\ln \sigma_I=-25.61+21.38\gamma$	0.15		39.45	19.13-23.63
6	$\ln \sigma_I=-27.76+24.13\gamma$	0.25	38	16.53	21.67-26.59
6A	$\ln \sigma_I=-26.97+22.93\gamma$	0.19		40.34	21.01-24.85
7	$\ln \sigma_I=-34.34+29.81\gamma$	0.25	33	19.48	26.70-32.92
7A	$\ln \sigma_I=-26.57+23.10\gamma$	0.17		44.09	20.96-25.24
8	$\ln \sigma_I=-26.97+23.79\gamma$	0.47	38	9.67	19.64-27.94
8A	$\ln \sigma_I=-25.00+21.56\gamma$	0.39		21.52	18.14-24.98
9	$\ln \sigma_I=-28.20+25.64\gamma$	0.49	33	14.73	25.27-33.35
9A	$\ln \sigma_I=-25.41+21.96\gamma$	0.30		25.33	18.24-25.50
10	$\ln \sigma_I=-28.20+25.64\gamma$	0.49	38	10.64	21.58-29.70
10A	$\ln \sigma_I=-21.37+19.47\gamma$	0.31		25.73	16.87-22.07
11	$\ln \sigma_I=-32.14+28.42\gamma$	0.35	33	10.49	22.91-33.93
11A	$\ln \sigma_I=-26.41+23.13\gamma$	0.27		28.00	19.73-26.53
12	$\ln \sigma_I=-19.62+18.80\gamma$	0.23	33	16.07	16.42-21.18
12A	$\ln \sigma_I=-19.74+18.59\gamma$	0.20		36.39	16.49-20.69
13	$\ln \sigma_I=-35.08+34.85\gamma$	0.31	22	11.42	28.52-41.18
13A	$\ln \sigma_I=-30.90+30.43\gamma$	0.26		23.80	25.14-35.72
14	$\ln \sigma_I=-35.94+35.23\gamma$	0.21	18	11.70	28.91-41.55
14A	$\ln \sigma_I=-35.59+34.39\gamma$	0.18		26.69	28.82-39.96

TABLE 9 (CONTINUED)

Test No.	Regression Equation	Syx	D.F.	t	Confidence Limit
15	$\ln \sigma I=-49.35+47.53\gamma$	0.29	33	16.80	41.77-53.29
15A	$\ln \sigma I=-40.33+38.77\gamma$	0.29		27.43	32.95-44.59
16	$\ln \sigma I=-31.48+31.52\gamma$	0.15	33	26.93	29.14-33.90
16A	$\ln \sigma I=-30.88+30.58\gamma$	0.22		36.20	26.12-34.04
17	$\ln \sigma I=-24.09+24.56\gamma$	0.21	33	15.85	21.41-27.71
17A	$\ln \sigma I=-24.42+24.38\gamma$	0.25		27.27	20.70-28.06
18	$\ln \sigma I=-33.49+28.67\gamma$	0.21	38	24.30	26.68-30.66
18A	$\ln \sigma I=-28.20+24.07\gamma$	0.17		50.94	22.42-25.72
19	$\ln \sigma I=-34.39+29.87\gamma$	0.18	38	22.46	27.63-32.11
19A	$\ln \sigma I=-35.88+30.16\gamma$	0.24		35.46	27.66-33.56
20	$\ln \sigma I=-25.74+22.66\gamma$	0.22	38	16.79	20.38-24.94
20A	$\ln \sigma I=-28.34+24.14\gamma$	0.24		32.92	21.66-26.62
21	$\ln \sigma I=-45.77+44.88\gamma$	0.25	18	13.08	37.68-52.08
21A	$\ln \sigma I=-32.68+32.06\gamma$	0.17		27.58	27.21-36.91
22	$\ln \sigma I=-37.10+37.07\gamma$	0.19	13	10.62	29.54-44.60
22A	$\ln \sigma I=-31.40+30.75\gamma$	0.19		17.63	23.08-38.42
23	$\ln \sigma I=-24.12+24.27\gamma$	0.24	13	7.29	17.08-31.46
23A	$\ln \sigma I=-21.16+20.81\gamma$	0.24		12.44	13.68-27.94
24	$\ln \sigma I=-29.04+28.66\gamma$	0.22	23	14.93	24.69-32.63
24A	$\ln \sigma I=-23.00+22.67\gamma$	0.19		31.90	19.77-25.57
25	$\ln \sigma I=-23.11+23.46\gamma$	0.22	23	15.43	20.32-26.60
25A	$\ln \sigma I=-19.66+19.45\gamma$	0.22		26.25	16.35-22.55
26	$\ln \sigma I=-30.39+30.48\gamma$	0.31	23	10.51	24.48-36.48
26A	$\ln \sigma I=-22.08+22.13\gamma$	0.22		21.87	17.87-26.39
27	$\ln \sigma I=-24.92+24.46\gamma$	0.21	18	16.87	21.41-27.51
27A	$\ln \sigma I=-20.16+19.01\gamma$	0.14		38.27	16.91-21.11

TABLE 10

COMPARISON OF REGRESSION COEFFICIENTS FOR σ_I VERSUS BULK DENSITY RELATION FOR DIFFERENT STRESS STATES AND SAME STRESS STATE FOR THE TWO METHODS USED TO OBTAIN DATA

Tests Compared	Degrees of Freedom	t 6 DST	t Type A	Test No.	Depth In.	Degrees of Freedom	t
2-1	59	4.67**	0.74	1	10	52	6.32**
2-3	71	2.87**	1.05	2	5	66	2.25*
1-3	64	6.97**	1.99	3	15	76	0.52
5-4	46	0.98	1.95	4	10	36	1.39
5-6	66	1.52	0.98	5	5	56	3.50**
4-6	56	1.98	1.20	6	15	76	0.65
7-9	66	1.66	0.56	7	5	66	3.62**
7-8	71	2.08*	0.67	8	15	76	0.70
9-8	71	0.58	0.15	9	10	66	1.39
12-11	66	3.26**	2.31*	10	15	76	2.16*
12-10	71	2.55*	0.47	11	10	66	1.66
11-10	71	0.77	1.61	12	5	66	0.13
14-13	40	0.09	1.08	13	10	44	1.11
14-27	36	3.22**	5.43**	14	5	36	0.21
13-27	40	3.08**	4.17**	15	15	66	2.18*
17-16	66	3.58**	2.50*	16	10	66	0.46
17-15	66	7.12**	4.25**	17	5	66	0.08
16-15	66	5.23**	2.46*	18	15	76	3.88**
20-19	76	3.80**	2.83**	19	10	76	0.34
20-18	76	3.35**	0.05	20	5	76	0.74
19-18	76	0.67	3.73**	21	15	36	3.10**
23-22	26	2.65*	2.05*	22	10	26	1.27
23-21	31	4.31**	2.79**	23	5	26	0.74
22-21	31	1.60	0.30	24	15	46	2.52*
26-25	46	2.14*	1.05	25	10	46	1.88
26-24	46	0.52	0.22	26	5	46	2.35*
25-24	46	2.12*	1.57	27	15	36	3.09**

a significant difference between the two sets of data obtained with the Type A Cells and 6 DST.

The significant differences again appear around the highest rate of loading and high moisture contents. The Type A values are not as varied as the 6 DST data but the conclusion must be drawn that the changes in bulk density are not independent of the maximum normal stress. This is particularly true at the lower moisture contents and lower rates of loading.

The set of typical curves for this relationship are shown in Figures 21 and 22.

Relationship between Maximum Shear Stress and Bulk Density

The third function used to represent the stress deviator tensor was the maximum shear stress. The regression equations, standard errors and confidence limits are listed in Table 11. Again all the calculated values of "t" are highly significant.

The results of comparisons of the stress states are given in Table 12. For the data obtained with the 6 DST, comparisons between 2-3, 1-3, 7-8, 12-11, 12-10, 14-27, 13-27, 17-16, 17-15, 16-15, 20-19, 20-18, 23-22, 23-21, 26-25 and 25-24 show significant differences. Results of the Type A data show the differences between 12-11, 17-16, 16-15, and 23-22 to be significant at the 95% level and 14-27, 13-27, 17-15, 20-19, 19-18, 23-22, and 23-21 to be significant at the 99% level. The differences

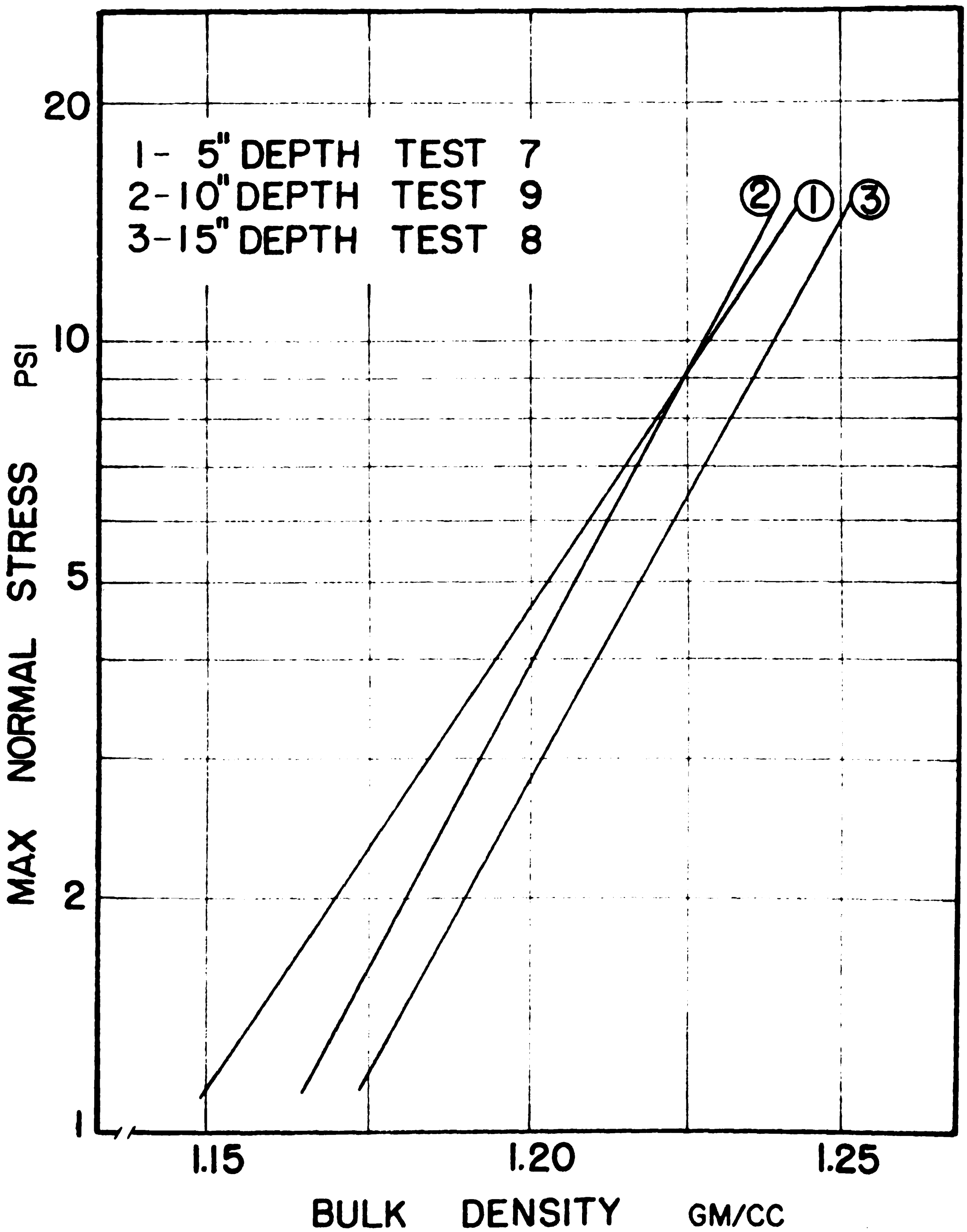

Figure 21. Maximum Normal Stress versus Bulk Density for Data Obtained with Six Directional Stress Transducer.

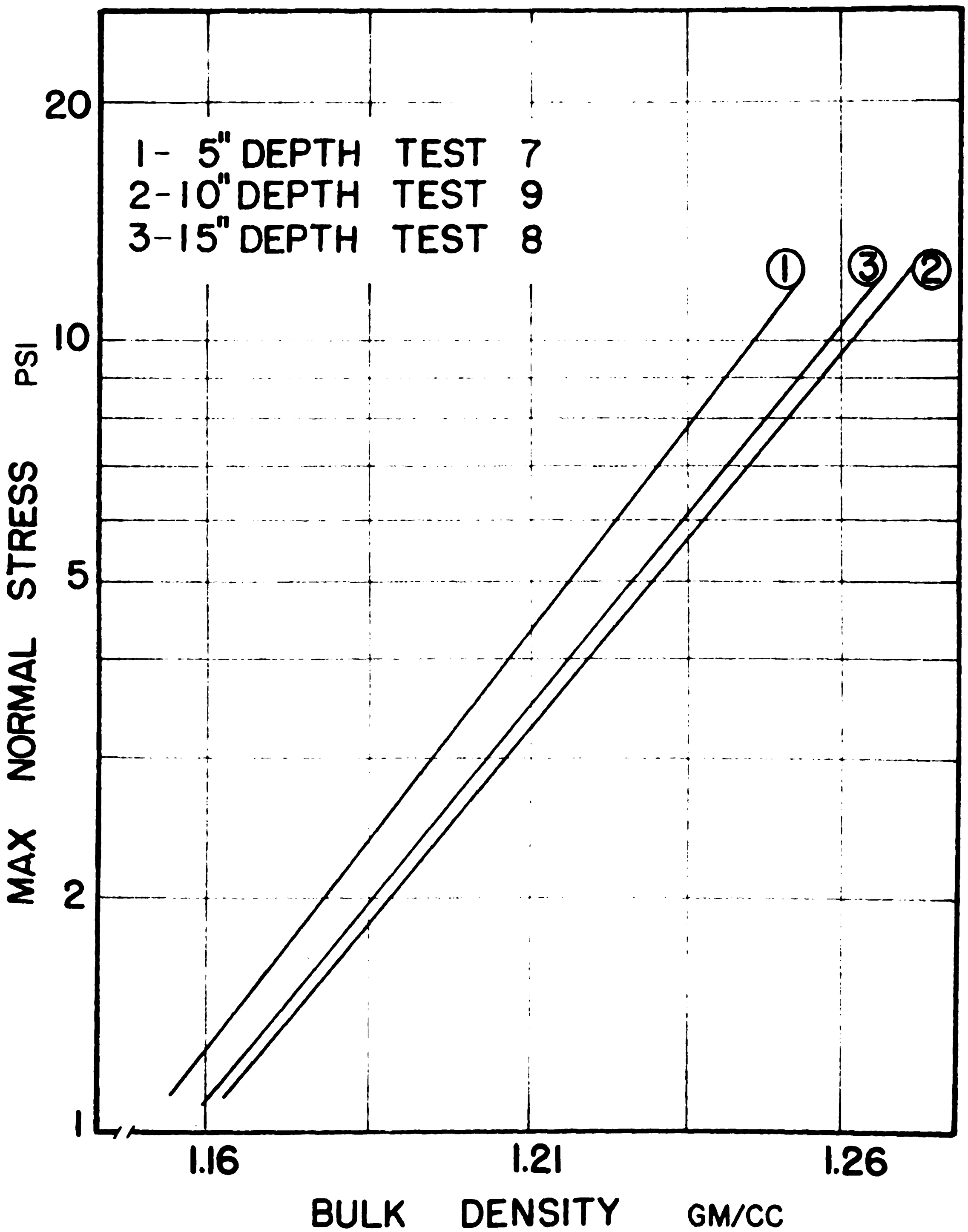

Figure 22 . Maximum Normal Stress versus Bulk Density for Data Obtained with Type A Pressure Cells.

TABLE 11

STATISTICAL ANALYSIS FOR MAXIMUM SHEAR STRESS VERSUS BULK DENSITY

Test No.	Regression Equation	Syx	D.F.	t	Confidence Limits
1	$\ln\tau_{max}=-74.29+61.85\gamma$	0.26	26	34.17	58.13-65.57
1A	$\ln\tau_{max}=-51.30+42.91\gamma$	0.29		21.13	38.74-47.08
2	$\ln\tau_{max}=-58.75+52.16\gamma$	0.39	33	19.78	47.30-57.02
2A	$\ln\tau_{max}=-49.37+41.65\gamma$	0.41		14.84	35.93-47.37
3	$\ln\tau_{max}=-74.29+61.85\gamma$	0.26	38	14.39	33.53-42.43
3A	$\ln\tau_{max}=-43.49+37.62\gamma$	0.48		15.86	33.62-41.62
4	$\ln\tau_{max}=-69.27+59.43\gamma$	0.44	18	12.59	49.51-69.35
4A	$\ln\tau_{max}=-59.17+50.20\gamma$	0.32		14.52	42.93-57.47
5	$\ln\tau_{max}=-64.71+54.11\gamma$	0.32	28	23.32	49.36-58.86
5A	$\ln\tau_{max}=-51.93+43.50\gamma$	0.30		19.77	37.99-49.01
6	$\ln\tau_{max}=-56.14+48.89\gamma$	0.50	38	16.74	43.97-53.81
6A	$\ln\tau_{max}=-53.98+45.92\gamma$	0.39		20.00	42.04-49.84
7	$\ln\tau_{max}=-70.84+61.58\gamma$	0.51	33	19.80	55.26-67.90
7A	$\ln\tau_{max}=-53.23+46.47\gamma$	0.35		21.51	42.08-50.86
8	$\ln\tau_{max}=-56.12+49.54\gamma$	0.97	38	9.77	40.99-58.09
8A	$\ln\tau_{max}=-50.43+43.62\gamma$	0.45		11.18	37.04-50.20
9	$\ln\tau_{max}=-57.07+51.99\gamma$	1.01	33	14.44	52.34-69.50
9A	$\ln\tau_{max}=-50.76+44.01\gamma$	0.59		12.77	36.99-51.03
10	$\ln\tau_{max}=-57.07+51.99\gamma$	1.01	38	10.48	43.63-60.35
10A	$\ln\tau_{max}=-43.21+39.56\gamma$	0.63		12.80	34.35-44.77
11	$\ln\tau_{max}=-64.40+57.02\gamma$	0.73	33	12.62	47.83-66.21
11A	$\ln\tau_{max}=-53.22+46.76\gamma$	0.53		14.23	40.07-53.45
12	$\ln\tau_{max}=-39.38+37.83\gamma$	0.46	33	16.17	33.07-42.59
12A	$\ln\tau_{max}=-39.76+37.61\gamma$	0.42		17.53	33.26-41.96
13	$\ln\tau_{max}=-70.98+70.58\gamma$	0.58	22	12.38	58.78-82.39
13A	$\ln\tau_{max}=-61.35+60.63\gamma$	0.51		12.16	50.28-70.98
14	$\ln\tau_{max}=-71.38+69.99\gamma$	0.41	18	11.34	57.03-82.95
14A	$\ln\tau_{max}=-73.05+70.63\gamma$	0.37		13.14	59.33-81.93

TABLE 11 (CONTINUED)

Test No.	Regression Equation	S_{yx}	D.F.	t	Confidence Limit
15	$\ln \tau_{max}=-100.09+96.53\gamma$	0.58	33	17.02	85.00-108.06
15A	$\ln \tau_{max}=-81.34+78.38\gamma$	0.61		13.17	66.28-80.48
16	$\ln \tau_{max}=-63.47+63.64\gamma$	0.32	33	25.46	58.56-68.72
16A	$\ln \tau_{max}=-61.91+61.49\gamma$	0.45		17.58	54.37-68.61
17	$\ln \tau_{max}=-49.85+50.84\gamma$	0.43	33	16.04	44.39-57.29
17A	$\ln \tau_{max}=-49.28+49.43\gamma$	0.49		13.71	42.09-56.77
18	$\ln \tau_{max}=-68.51+58.71\gamma$	0.41	38	25.53	54.83-62.59
18A	$\ln \tau_{max}=-58.12+49.71\gamma$	0.34		26.44	46.54-52.88
19	$\ln \tau_{max}=-69.11+60.15\gamma$	0.37	38	22.03	55.55-64.75
19A	$\ln \tau_{max}=-72.70+62.17\gamma$	0.48		17.69	56.25-68.09
20	$\ln \tau_{max}=-50.02+46.73\gamma$	0.43	38	18.25	42.26-51.20
20A	$\ln \tau_{max}=-56.49+48.26\gamma$	0.47		16.62	43.37-53.15
21	$\ln \tau_{max}=-92.95+91.28\gamma$	0.48	18	13.85	77.44-105.12
21A	$\ln \tau_{max}=-64.76+63.75\gamma$	0.33		13.86	54.09-73.41
22	$\ln \tau_{max}=-74.92+74.07\gamma$	0.38	13	10.73	59.87-90.07
22A	$\ln \tau_{max}=-63.84+62.64\gamma$	0.38		8.96	47.54-77.74
23	$\ln \tau_{max}=-49.00+49.38\gamma$	0.50	13	7.11	34.38-64.38
23A	$\ln \tau_{max}=-41.53+41.04\gamma$	0.47		6.28	26.91-55.17
24	$\ln \tau_{max}=-58.87+58.20\gamma$	0.55	23	14.30	49.78-66.62
24A	$\ln \tau_{max}=-45.16+44.68\gamma$	0.37		16.08	38.93-50.43
25	$\ln \tau_{max}=-47.21+47.92\gamma$	0.44	23	15.82	41.65-54.19
25A	$\ln \tau_{max}=-39.70+39.48\gamma$	0.46		12.57	32.98-45.98
26	$\ln \tau_{max}=-61.46+61.77\gamma$	0.63	23	10.47	49.57-73.97
26A	$\ln \tau_{max}=-44.70+44.96\gamma$	0.45		10.62	36.21-53.71
27	$\ln \tau_{max}=-50.05+49.14\gamma$	0.43	18	16.55	42.90-55.38
27A	$\ln \tau_{max}=-40.60+38.35\gamma$	0.29		19.29	34.17-42.53

TABLE 12

COMPARISON OF REGRESSION COEFFICIENTS FOR τ_{max} VERSUS BULK DENSITY RELATION FOR DIFFERENT STRESS STATES AND SAME STRESS STATE FOR THE TWO METHODS USED TO OBTAIN DATA

Tests Compared	Degrees of Freedom	t 6 DST	t Type A	Test No.	Depth In.	Degrees of Freedom	t
2-1	59	0.09	0.36	1	10	52	6.96**
2-3	71	6.63**	1.10	2	5	66	5.40**
1-3	64	7.46**	1.70	3	15	76	0.10
5-4	46	1.01	1.63	4	10	36	1.58
5-6	66	1.40	0.76	5	5	56	3.32**
4-6	56	1.90	1.03	6	15	76	0.80
7-9	66	1.83	0.60	7	5	66	3.99**
7-8	71	2.02*	0.64	8	15	76	0.93
9-8	71	0.37	0.07	9	10	66	1.46
12-11	66	3.77**	2.33*	10	15	76	2.13*
12-10	71	2.58*	0.52	11	10	66	1.84
11-10	71	0.75	1.60	12	5	66	0.07
14-13	40	0.07	1.36	13	10	44	1.31
14-27	36	3.04**	5.63**	14	5	36	0.08
13-27	40	3.34**	4.15**	15	15	66	2.21*
17-16	66	3.17**	2.40*	16	10	66	0.50
17-15	66	7.03**	4.16**	17	5	66	0.29
16-15	66	5.31**	2.45*	18	15	76	3.03**
20-19	76	3.53**	3.06**	19	10	76	0.45
20-18	76	3.41**	0.42	20	5	76	0.39
19-18	76	0.40	3.13**	21	15	36	3.43**
23-22	26	2.60*	2.26*	22	10	26	1.25
23-21	31	4.37**	2.84**	23	5	26	0.87
22-21	31	1.70	0.13	24	15	46	2.74**
26-25	46	2.09*	1.04	25	10	46	1.93
26-24	46	0.50	0.06	26	5	46	2.31
25-24	46	2.03*	1.24	27	15	36	3.02**

between tests 13-27 and 14-27 are probably partly due to the differences in moisture content, the moisture content of tests 13 and 14 is 17.41 percent and only 16.54 percent for test 27. To a smaller degree, this could also be true for the differences between test 15-17 and 16-17 because there is a 0.51% moisture content difference. The difference in moisture content for tests 18 and 19 is 1.36 percent and 1.52 percent for tests 18 and 20. The estimates of standard error for tests 2, 6, 9, 13, and 23 are larger than the other values of standard errors indicating more error in obtaining these test data. Based on this information and data, the conclusion that the maximum shear stress is best related to changes in bulk density might be made.

A typical set of curves showing the relationship between maximum shear stress and bulk density is shown in Figures 23 and 24.

<u>The Effect of Moisture Content on the Relationship between Mean Normal Stress and Bulk Density</u>

Since the moisture content of the soil is one of the variables in agricultural soils which has been found to be a factor in soil compaction, three different moisture contents were used during the experimental tests. The range used was from 7.97 percent to 17.93 percent (dry weight basis). Within the five replications of a test the moisture content remained constant. However, in tests

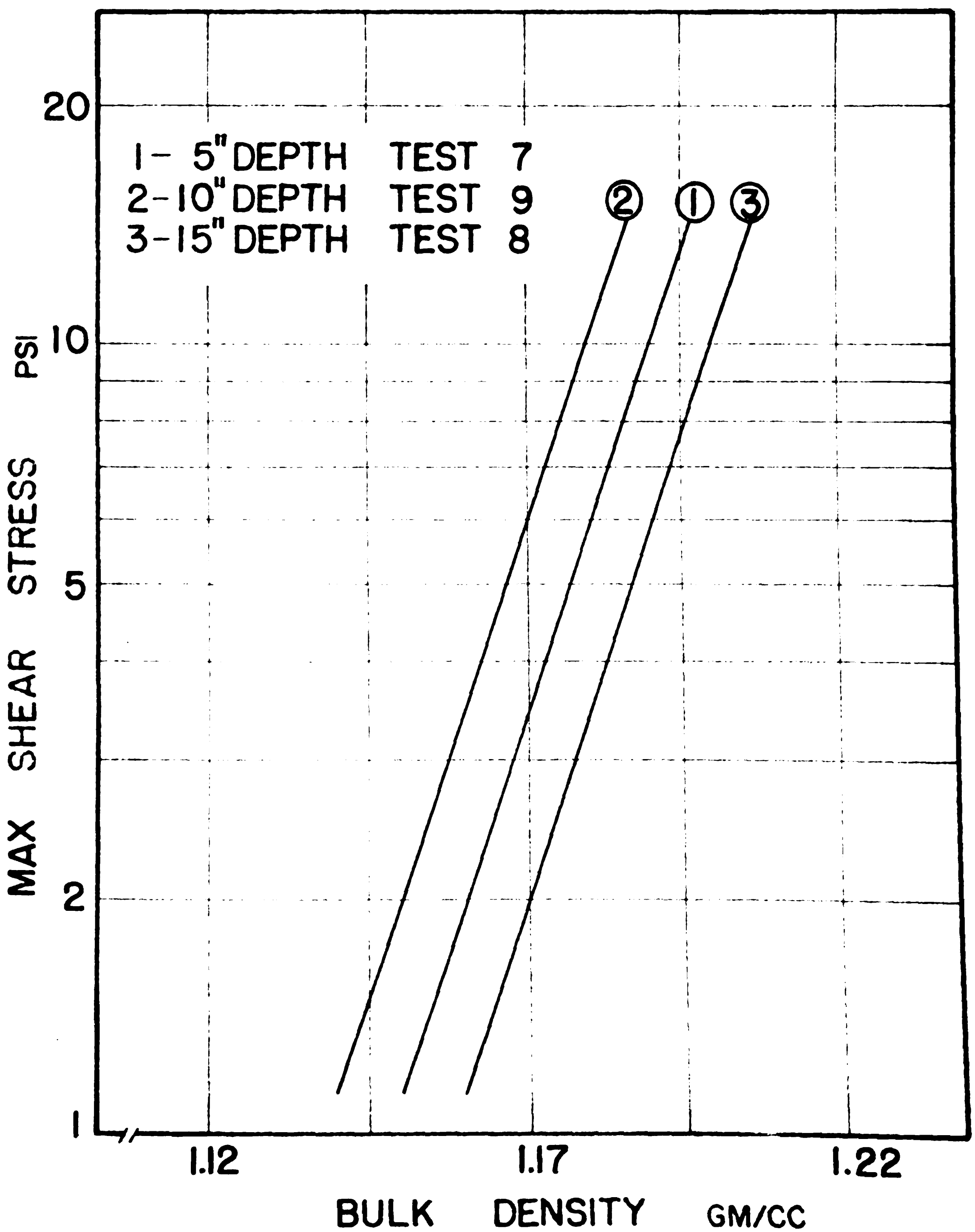

Figure 23. Maximum Shearing Stress versus Bulk Density for Data Obtained with Six Directional Stress Transducer.

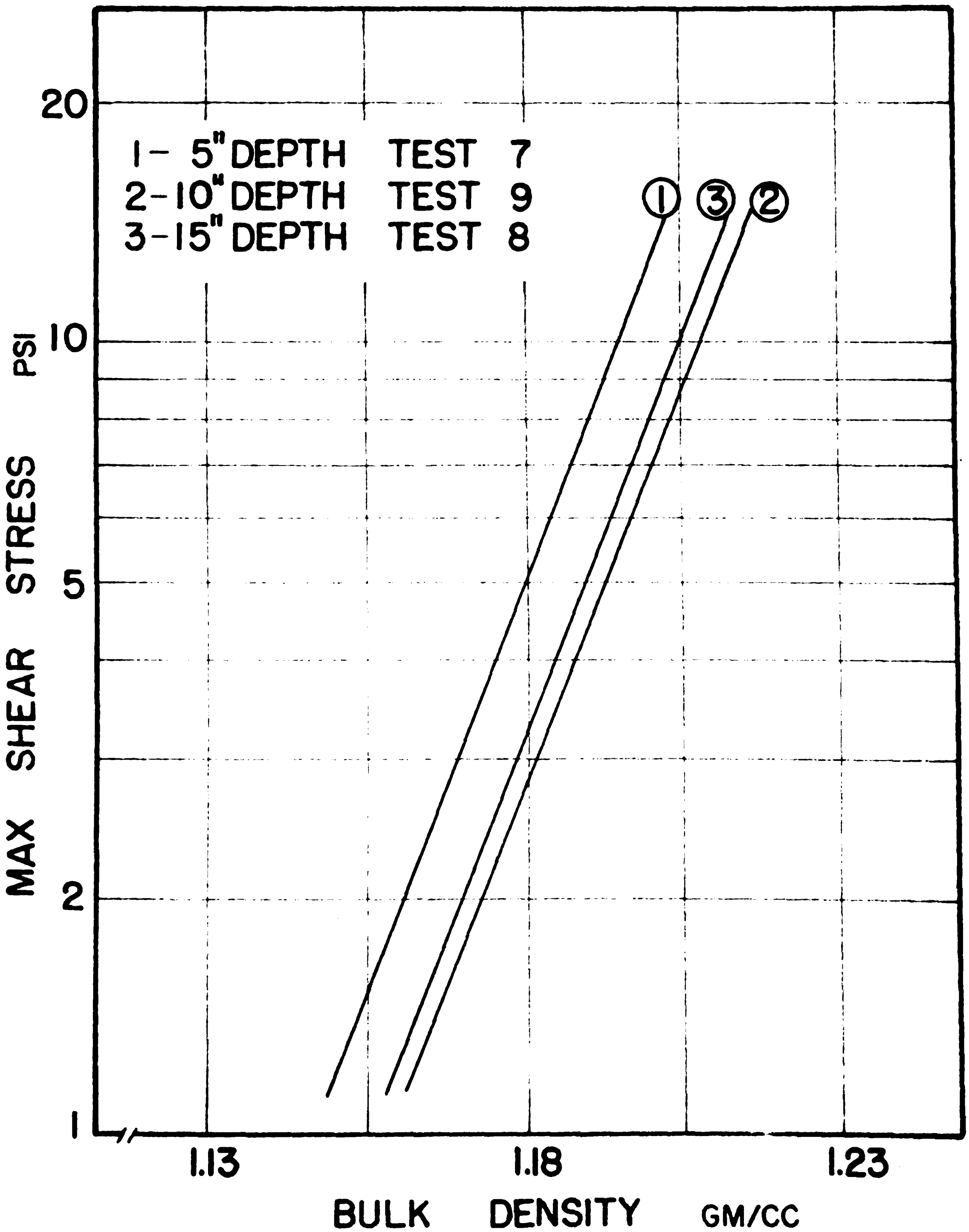

Figure 24 . Maximum Shearing Stress versus Bulk Density for Data Obtained with Type A Pressure Cells.

conducted a day or more apart the moisture content was not the same. The exact percent moisture was not known until after the tests were run, the soil samples dried, and the percentages calculated.

To determine the effect of moisture content on the relationship between mean normal stress and bulk density the "t" test for the regression coefficients was used. The coefficients from two relationships determined at the same depth and rate of loading but with different moisture contents were compared. The results of these comparisons are presented in Table 13.

For the 0.38 inch per second rate of loading significant differences were obtained only for the 6 DST data between tests 4-9, and 4-25. The moisture contents for tests 4 and 9 are 11.56- and 11.38-percent which indicates that an error must have been made in obtaining the data for test 4. The estimate of standard error, however, does not indicate a large variation in the data used to determine the regression coefficient. Under these conditions, the conclusion that changes in bulk density are independent of moisture content could be made.

Significant differences were found between tests 2-12, 3-21, and 10-21 for the 6 DST data and 1-22, 11-22, 3-21 and 10-21 for the Type A data under the 0.62 inch per second loading condition. The values for the Type A data were obtained for the 10- and 15-inch depths between

TABLE 13

COMPARISON OF REGRESSION COEFFICIENTS OF THE σ_m VERSUS BULK DENSITY RELATIONS AT DIFFERENT MOISTURE CONTENTS AND BETWEEN METHODS FOR A GIVEN TEST

Tests	D.F.	t Type A	t 6 DST	Test No.	M.C. %	D.F.	t
		Rate of Loading 0.38 Inch per Second					
5-7	61	1.64	0.60	5	8.89	56	5.32**
5-26	51	1.09	0.71	7	11.35	66	3.52**
7-26	56	0.15	0.33	26	16.05	46	1.94
4-9	51	1.74	3.20**	4	11.56	36	2.49*
4-25	41	2.45	5.01**	9	11.38	66	1.26
9-25	56	0.82	0.32	25	15.78	46	2.02*
6-8	76	0.53	0.83	6	7.97	76	0.96
6-24	61	0.13	1.89	8	11.39	76	1.51
8-24	61	0.40	0.66	24	16.16	46	2.41*
		Rate of Loading 0.62 Inch per Second					
2-12	66	1.51	5.34**	2	9.89	66	5.60**
2-23	46	0.09	0.87	12	12.46	66	1.80
12-23	46	0.76	1.10	23	17.67	26	1.12
1-11	59	0.01	0.30	1	8.63	52	6.71**
1-22	39	3.74**	1.50	11	12.43	66	3.27**
11-22	46	3.55**	1.54	22	17.52	26	0.82
3-10	76	0.16	1.61	3	11.59	76	0.72
3-21	56	6.14**	7.62**	10	12.63	76	2.10*
10-21	56	5.60**	4.94**	21	17.93	36	2.33*
		Rate of Loading 1.00 Inch per Second					
20-17	71	1.05	1.11	20	10.79	76	3.30**
20-14	56	5.62**	5.58**	17	14.34	66	2.40*
17-14	51	4.31**	4.79**	14	17.41	36	2.50*
19-16	71	1.41	1.34	19	10.95	76	4.09**
19-13	60	0.86	0.16	16	14.85	66	1.50
16-13	55	0.12	1.17	13	17.41	44	1.90
18-15	71	4.24**	4.92**	18	12.31	76	5.03**
18-27	56	4.68**	4.48**	15	14.85	66	2.91**
15-27	51	6.50**	6.78**	27	16.54	36	4.22**

moisture contents of 11.5- and 17.9-percent. This difference may not have been evident at the 5-inch depth due to instrument effects on the stress pattern so near the loading surface. It is not clear why the effect of moisture at the 10-inch depth was not found with the 6 DST unless the larger size of the instrument has a greater effect on the stress pattern for a deeper depth. The conclusion that moisture content does affect the relationship between mean normal stress and bulk density for 0.62 inch per second rate of loading and depths of 10 and 15 inches could be made.

For the 1.00 inch per second rate of loading condition, differences between tests 20-14, 17-14, 18-15, 18-27, and 15-27 were significant. These results are consistent except for the difference between tests at the 10-inch depth. Therefore, the conclusion could be made that the moisture content affects the relationship between mean stress and bulk density at the 5- and 15-inch depths at the 1.00 inch per second rate of loading.

The results from Tests 1,11, and 22 are shown in Figures 25 and 26.

<u>The Effect of Moisture Content on the Relationship between Second Invariant and Bulk Density</u>

Results of the comparison between regression coefficients for the relationship between the second invariant and bulk density are presented in Table 14. The significant

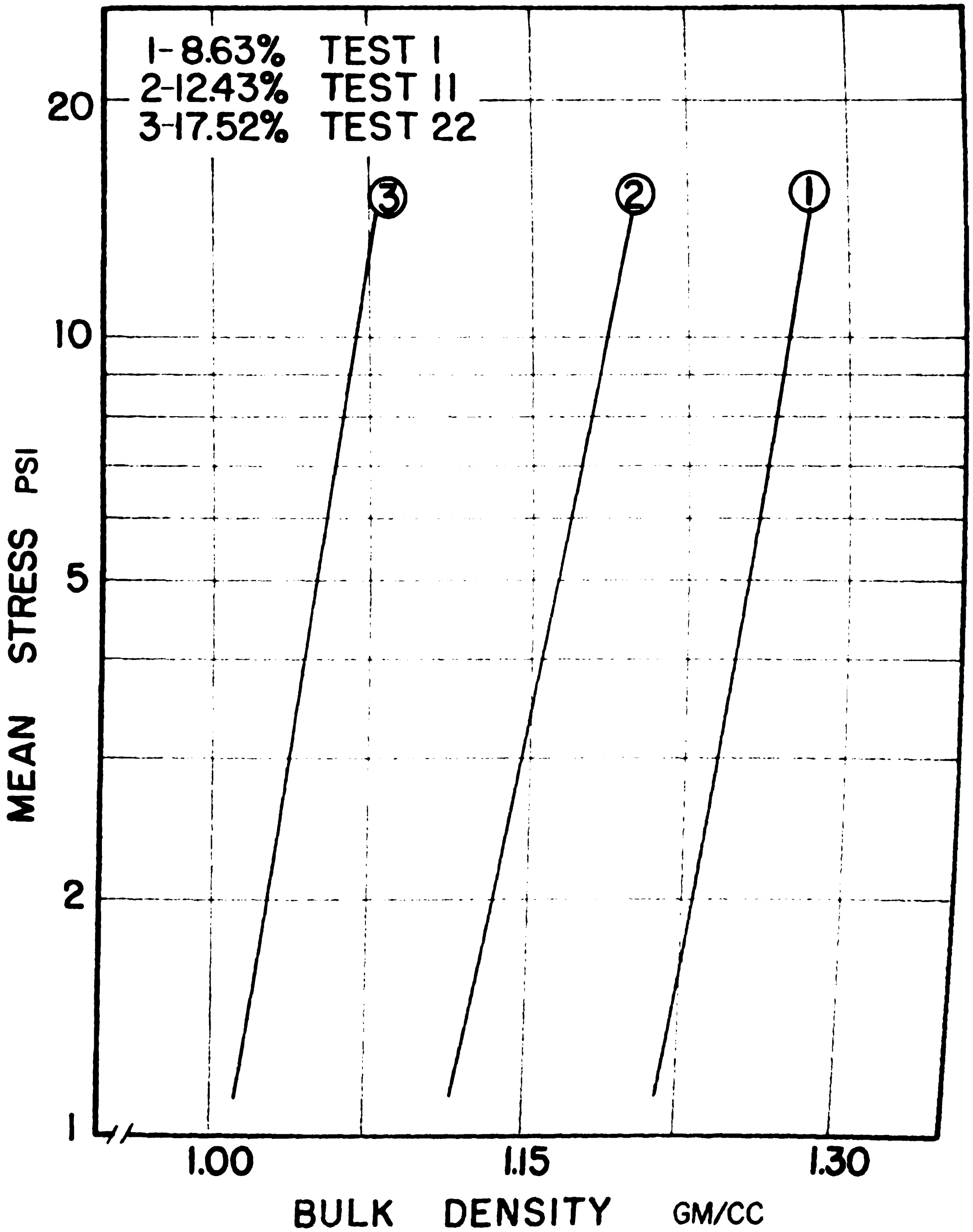

Figure 25. The Effect of Moisture Content on the Relationship between the Mean Stress and Bulk Density. Data obtained with 6" DST.

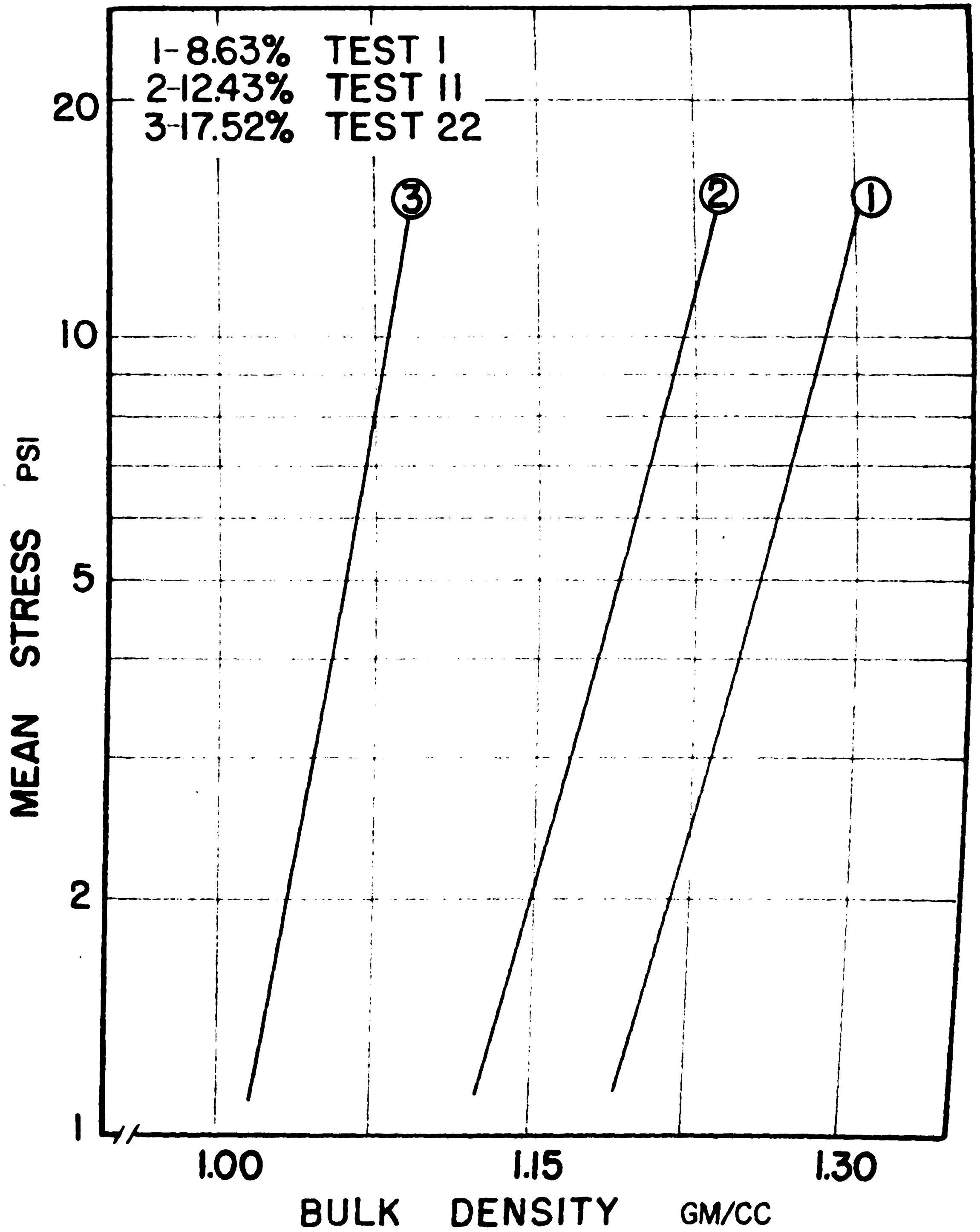

Figure 26 . The Effect of Moisture Content on the Relationship between Mean Stress and Bulk Density. Data obtained with Type A Pressure Cells.

TABLE 14

COMPARISON OF REGRESSION COEFFICIENTS OF THE II_s VERSUS BULK DENSITY RELATIONS AT CONSTANT RATE, CONSTANT DEPTH, AND DIFFERENT MOISTURE CONTENTS AND BETWEEN METHODS FOR A GIVEN TEST

Tests	D.F.	t Type A	t 6 DST	Test No.	M.C. %	D.F.	t
		Rate of Loading 0.38 Inch per Second					
5-7	61	1.35	1.41	5	8.89	56	4.43**
5-26	51	0.75	1.01	7	11.35	66	3.90**
7-26	56	0.08	0.15	26	16.05	46	2.20*
4-9	51	1.56	1.54	4	11.56	36	1.62
4-25	41	2.41*	2.10*	9	11.38	66	1.44
9-25	56	0.90	0.43	25	15.78	46	2.18
6-8	76	0.57	0.47	6	7.97	76	0.89
6-24	61	0.31	1.91	8	11.39	76	1.27
8-24	61	0.31	1.03	24	16.16	46	2.65**
		Rate of Loading 0.62 Inch per Second					
2-12	66	1.37	3.13**	2	9.89	66	2.31*
2-23	46	0.09	0.27	12	12.46	66	0.83
12-23	46	0.66	1.45	23	17.67	26	1.06
1-11	59	0.53	0.88	1	8.63	52	7.28**
1-22	39	3.55**	1.65	11	12.43	66	2.46*
11-22	46	3.06**	1.95	22	17.52	26	1.09
3-10	76	0.15	2.17*	3	11.59	76	0.25
3-21	56	5.62**	7.72**	10	12.63	76	2.14*
10-21	56	5.00**	4.68**	21	17.93	36	2.84*
		Rate of Loading 1.00 Inch per Second					
20-17	17	0.72	1.29	20	10.79	76	1.14
20-14	56	5.26**	4.45**	17	14.34	66	1.28
17-14	51	4.30**	3.58**	14	17.41	36	0.54
19-16	71	1.10	0.17	19	10.95	76	1.54
19-13	60	0.33	1.26	16	14.85	66	0.52
16-13	55	0.49	1.16	13	17.41	44	1.67
18-15	71	4.42**	5.61**	18	12.31	76	4.39**
18-27	56	4.96**	3.68**	15	14.85	66	2.66**
15-27	51	6.57**	7.16**	27	16.54	36	3.82**

differences follow the same pattern as found for the relationship between mean normal stress and bulk density. In general, at the higher rates of loading the moisture content of the soil does affect the relationship between the second invariant and bulk density. The relationships for Tests 1, 11 and 22 have been plotted and are shown in Figures 27 and 28. There are no significant differences for the 6 DST lines. However, for the Type A lines significant differences are found between Test 1-22 and 11-22, as can be seen in Figure 28. The slopes of Tests 1 and 11 are significantly different from the slope of Test 22.

<u>The Effect of Moisture Content on the Relationship between Maximum Normal Stress and Bulk Density</u>

The results of the "t" tests between slopes of the regression equations for the maximum normal stress-bulk density relationships are listed in Table 15. The difference between Tests 4 and 25 for the lowest rate of loading is significant for both the Type A and 6 DST data. For the 0.62 inch per second rate, significant differences are found between Tests 2-12, 3-10, 3-21, and 10-21 for the 6 DST lines and 1-22, 3-21, and 10-21 for the Type A. Significant differences are found between Tests 18-15, 18-27, and 15-27 for both sets of data under the 1.00 inch per second rate of loading. In addition differences between the Type A data for Tests 20-14, and 17-14 are significant. Based on this data the conclusion that the

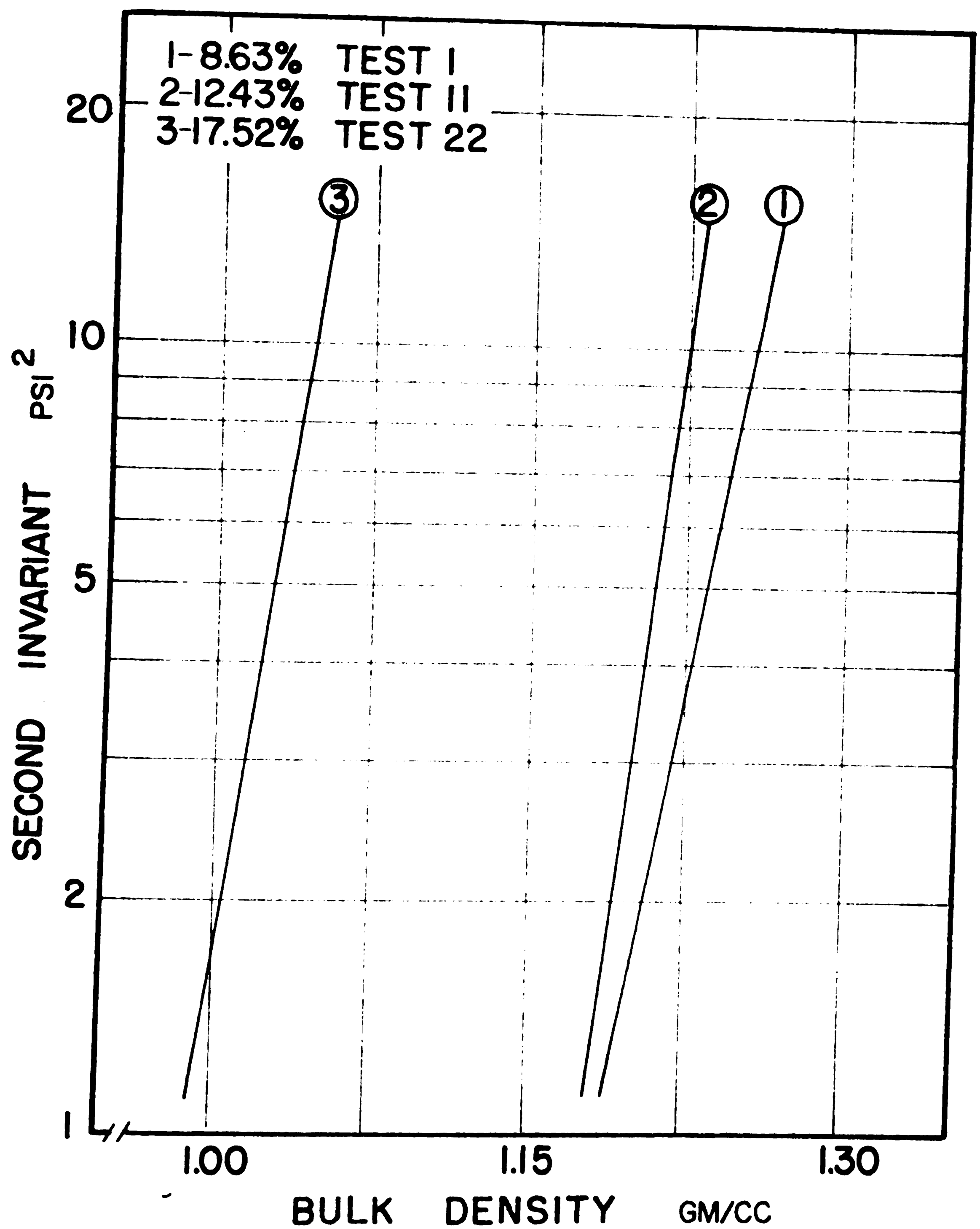

Figure 27. The Effect of Moisture Content on the Relationship between the Second Invariant and Bulk Density. Data obtained with 6 DST.

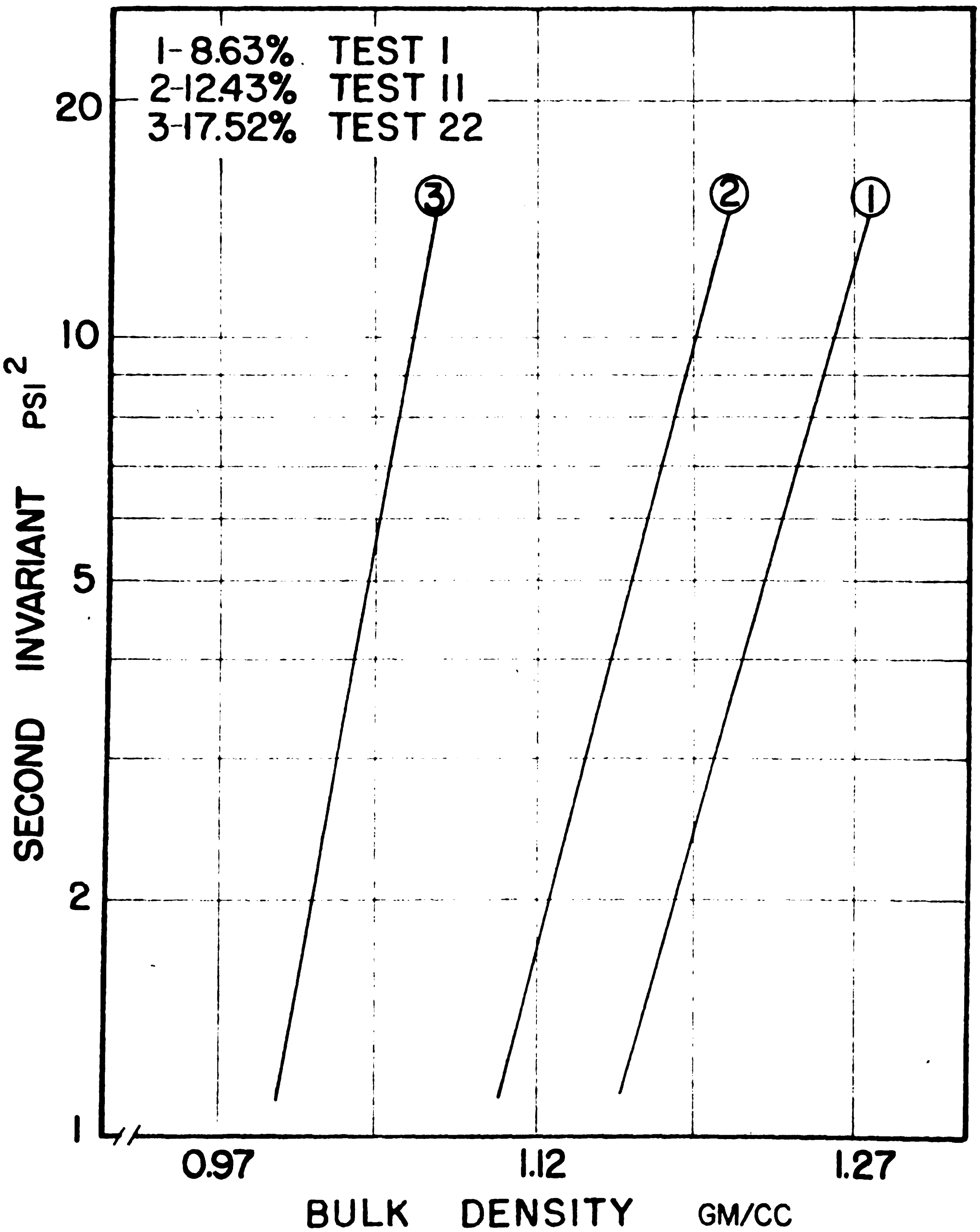

Figure 28. The Effect of Moisture Content on the Relationship between the Second Invariant and Bulk Density. Data obtained with Type A Pressure Cells.

TABLE 15

COMPARISON OF REGRESSION COEFFICIENTS OF THE σI VERSUS BULK DENSITY RELATIONS AT CONSTANT RATE, CONSTANT DEPTH AND DIFFERENT MOISTURE CONTENTS AND BETWEEN METHODS FOR A GIVEN TEST

Tests	D.F.	t Type A	t 6 DST	Test No.	M.C. %	D.F.	t
Rate of Loading 0.38 Inch per Second							
5-7	61	1.63	1.48	5	8.89	56	4.80**
5-26	51	0.36	1.12	7	11.35	66	3.62**
7-26	56	0.42	0.20	26	16.05	46	2.35*
4-9	51	1.41	1.27	4	11.56	36	1.39
4-25	41	2.58*	2.21*	9	11.38	66	1.39
9-25	56	1.09	0.87	25	15.78	46	1.88
6-8	76	0.59	0.14	6	7.97	76	1.04
6-24	61	0.14	2.35*	8	11.39	76	0.70
8-24	61	0.45	1.56	24	16.16	46	2.52*
Rate of Loading 0.62 Inch per Second							
2-12	66	0.93	3.36**	2	9.89	66	2.25*
2-23	46	0.18	0.07	12	12.46	66	0.13
12-23	46	0.64	1.55	23	17.67	26	0.74
1-11	59	0.83	0.69	1	8.63	52	6.32**
1-22	39	2.50*	1.85	11	12.43	66	1.66
11-22	46	1.94	1.96	22	17.52	26	1.27
3-10	76	0.60	2.33*	3	11.59	76	0.52
3-21	56	5.32**	6.98**	10	12.63	76	2.16*
10-21	56	4.53**	4.59**	21	17.93	36	3.10**
Rate of Loading 1.00 Inch per Second							
20-17	71	0.10	0.92	20	10.79	76	0.74
20-14	56	3.38**	0.78	17	14.34	66	0.08
17-14	51	3.12**	0.20	14	17.41	36	2.28*
19-16	71	0.01	0.93	19	10.95	76	0.34
19-13	60	0.06	1.50	16	14.85	66	0.46
16-13	55	0.05	1.02	13	17.41	44	1.11
18-15	71	4.86**	6.15**	18	12.31	76	3.00**
18-27	56	3.61**	2.25*	15	14.85	66	2.18*
15-27	51	6.52**	7.26**	27	16.54	36	3.09**

moisture content affects the relationship at the higher rates of loading and 15-inch depth could be made.

A set of typical curves showing the relationship between maximum normal stress and bulk density are presented in Figures 29 and 30.

The Effect of Moisture Content on the Relationship between Maximum Shear Stress and Bulk Density

The effect of moisture content on the relationship between maximum shear stress and bulk density follows the same pattern as the other relationships as shown in Table 16. The conclusion that moisture content affects the relationship at the higher rates of loading and the 15-inch depth could be made.

The values for Tests 1, 11 and 22 were used to plot the set of typical curves shown in Figure 31 and 32.

The Effect of Rate of Loading on the Relationship between Mean Normal Stress and Bulk Density

Three rates of loading were used as described in Table 4. In order to determine the effect of the rate of loading, the regression coefficients of two tests at the same depth and approximately the same moisture contents but with different rates were compared by means of the "t" test. The results of these comparisons are given in Table 17.

Difference between Tests 14-23 and 14-26, 5 inches below the loading surface are significant for both sets of

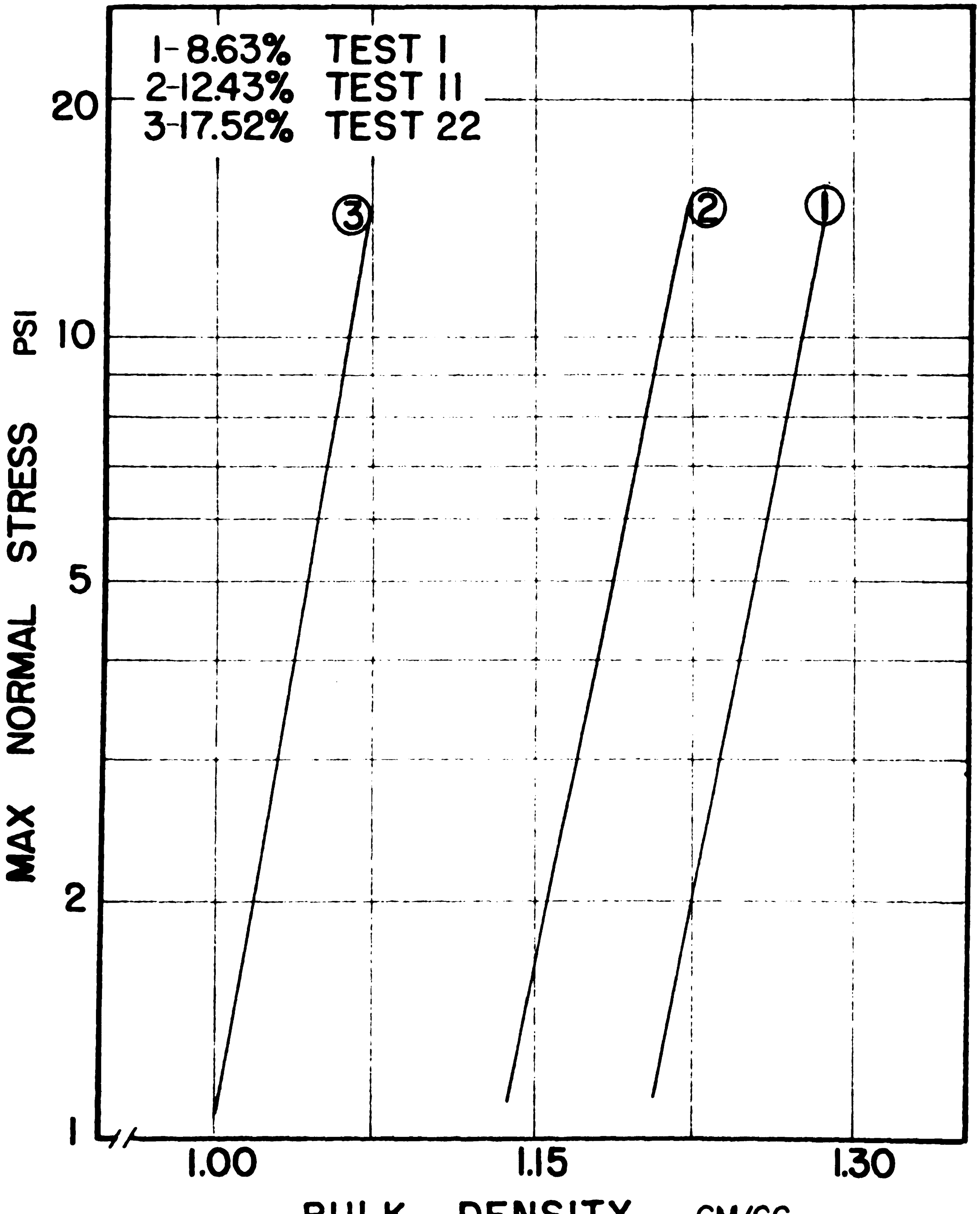

Figure 29. The Effect of Moisture Content on the Relationship between the Maximum Normal Stress and Bulk Density. Data Obtained with 6 DST.

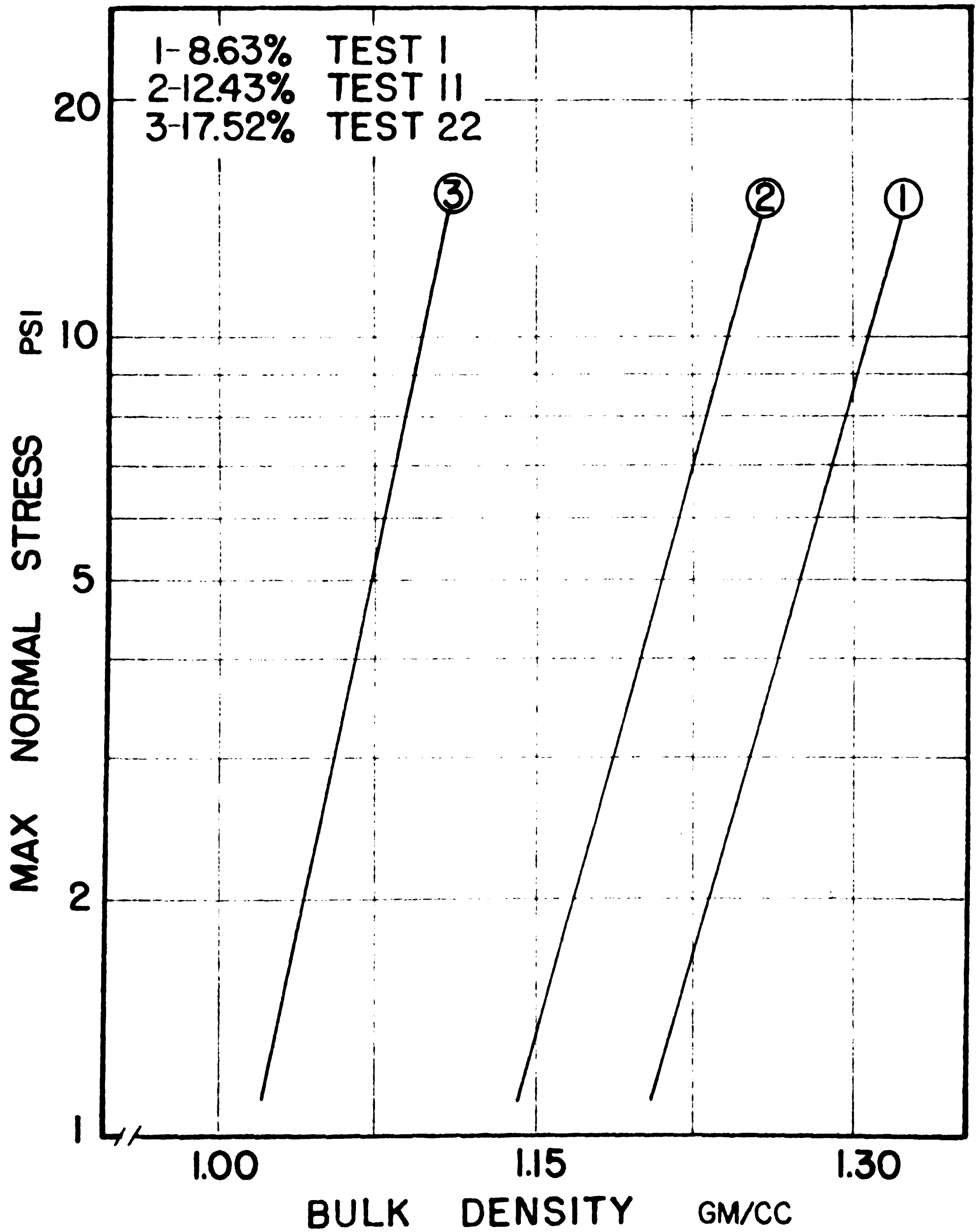

Figure 30. The Effect of Moisture Content on the Relationship between the Maximum Normal Stress and Bulk Density. Data obtained with Type A Pressure Cells.

TABLE 16

COMPARISON OF REGRESSION COEFFICIENTS OF THE τ_{max} VERSUS BULK DENSITY RELATIONS AT CONSTANT RATE, CONSTANT DEPTH AND DIFFERENT MOISTURE CONTENTS AND BETWEEN METHODS FOR A GIVEN TEST

Tests	D.F.	t Type 6	t 6 DST	Test No.	M.C. %	D.F.	t
		Rate of Loading 0.38 Inch per Second					
5-7	61	0.96	1.93	5	8.89	56	3.32**
5-26	51	0.31	1.21	7	11.35	66	3.99**
7-26	56	0.32	0.03	26	16.05	46	2.32*
4-9	51	1.27	1.19	4	11.56	36	1.60
4-25	41	2.29*	2.10*	9	11.38	66	1.46
9-25	56	0.97	0.80	25	15.78	46	1.91
6-8	76	0.51	0.11	6	7.97	76	0.80
6-24	61	0.34	1.86	8	11.39	76	0.93
8-24	61	0.22	1.33	24	16.16	46	2.74**
		Rate of Loading 0.62 Inch per Second					
2-12	66	1.14	7.10**	2	9.89	66	5.40**
2-23	46	0.09	1.66	12	12.46	66	0.07
12-23	46	0.50	1.57	23	17.67	26	0.87
1-11	59	1.00	0.99	1	8.63	52	6.96**
1-22	39	2.71**	1.82	11	12.43	66	1.84
11-22	46	2.06*	2.16*	22	17.52	26	1.25
3-10	76	0.50	2.49*	3	11.59	76	0.10
3-21	56	5.05**	7.51	10	12.63	76	2.13*
10-21	56	4.37**	4.76**	21	17.93	26	3.43**
		Rate of Loading 1.00 Inch per Second					
20-17	71	0.25	0.99	20	10.79	76	0.39
20-14	56	3.66**	3.46**	17	14.34	66	0.29
17-14	51	3.27**	2.76**	14	17.41	36	0.08
19-16	71	0.14	0.94	19	10.95	76	0.45
19-13	60	0.25	1.65	16	14.85	66	0.50
16-13	55	0.14	1.12	13	17.41	44	1.31
18-15	71	4.59**	6.18**	18	12.31	76	3.03**
18-27	56	4.15**	2.55*	15	14.85	66	2.21*
15-27	51	6.38**	7.40**	27	16.54	36	3.02**

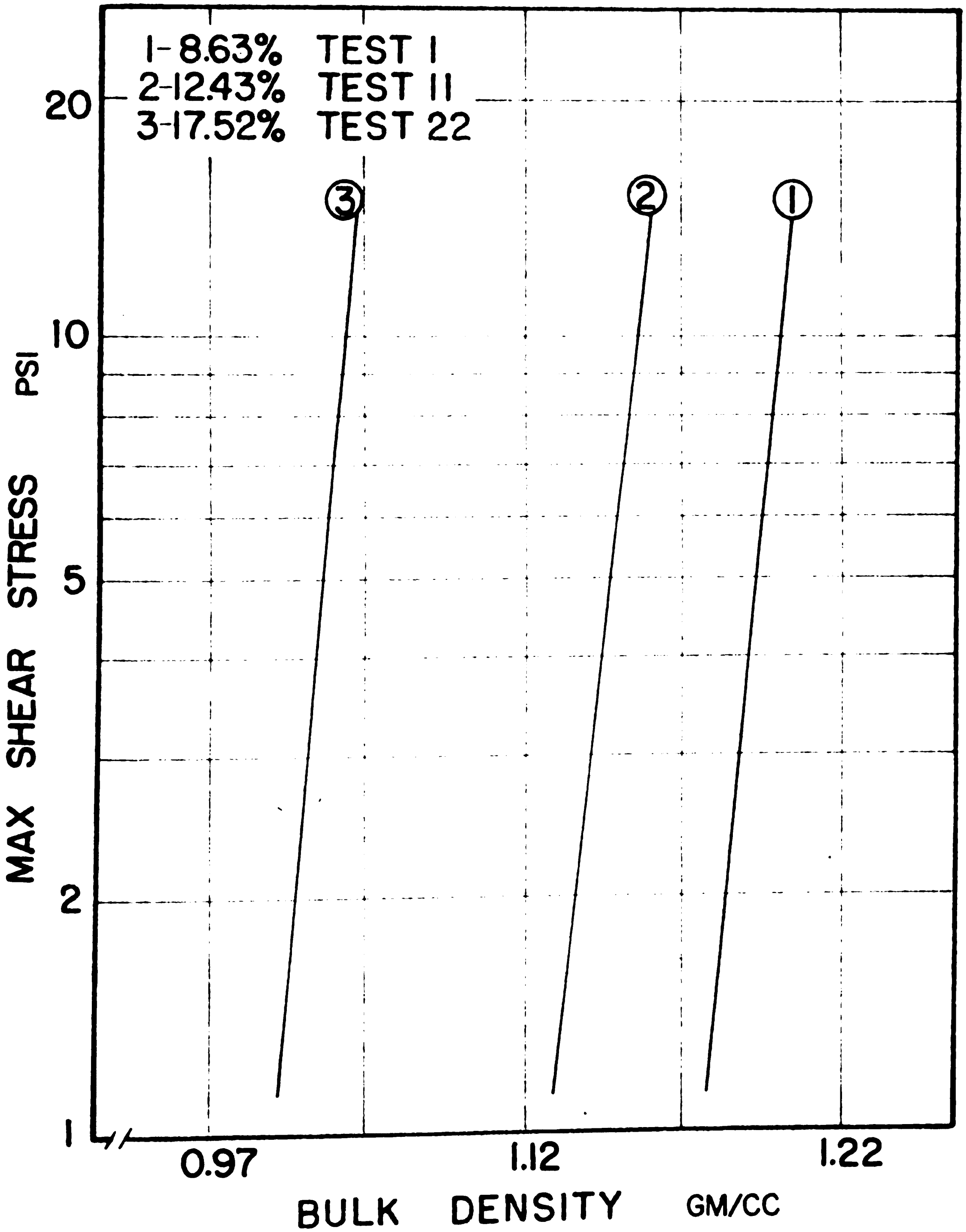

Figure 31. The Effect of Moisture Content on the Relationship between the Maximum Shear Stress and Bulk Density. Data obtained with 6 DST.

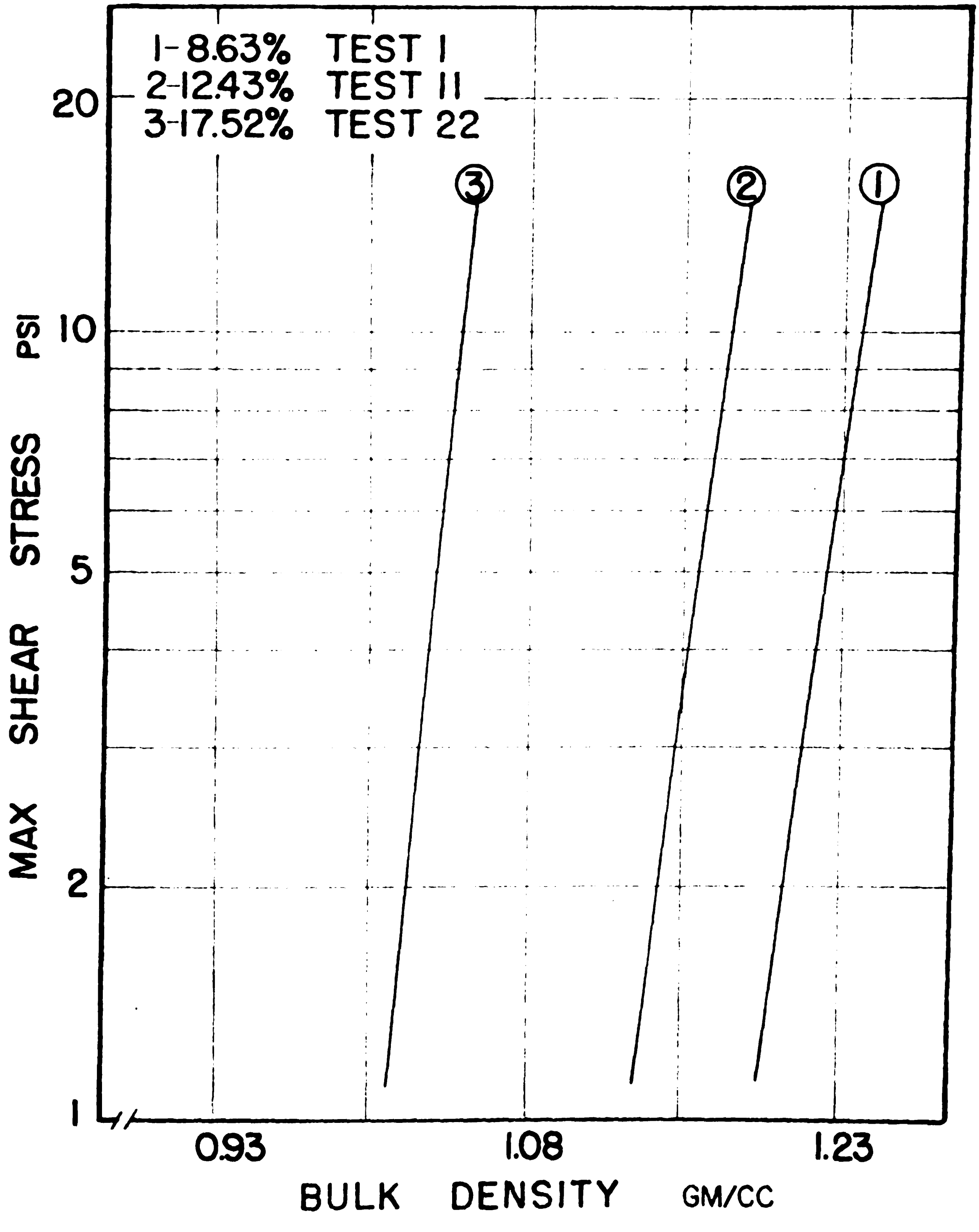

Figure 32. The Effect of Moisture Content on the Relationship between the Maximum Shear Stress and Bulk Density. Data obtained with Type A Pressure Cells.

TABLE 17

COMPARISON OF REGRESSION COEFFICIENTS OF THE σ_m VERSUS BULK DENSITY RELATIONS AT DIFFERENT RATES OF LOADING AND BETWEEN METHODS FOR A GIVEN TEST

Tests	D.F.	t Type A	t 6 DST	Test No.	Rate (in/sec)	D.F.	t
5 Inches below Loading Surface							
2-20	71	1.35	0.50	5	0.38	56	5.32**
2-5	61	0.24	1.63	2	0.62	66	5.60**
5-20	66	1.83	0.96	20	1.00	76	3.30**
14-23	31	4.99**	4.19**	14	1.00	36	2.50*
14-26	41	4.82**	4.66**	23	0.62	26	1.23
23-26	36	0.71	0.57	26	0.38	46	1.94
10 Inches below Loading Surface							
4-19	56	1.35	2.14*	19	1.00	76	4.09**
1-4	44	1.56	0.18	1	0.62	52	6.71**
1-19	64	4.51**	3.77**	4	0.38	36	1.58
13-22	35	0.37	0.26	13	1.00	44	1.90
13-25	45	4.24**	4.98**	22	0.62	26	0.82
22-25	36	4.33**	3.03**	25	0.38	46	2.08*
15 Inches below Loading Surface							
27-21	36	6.30**	5.85**	27	1.00	36	4.22**
27-24	41	1.93	1.35	21	0.62	36	2.33*
21-24	41	4.25**	3.66**	24	0.38	46	2.41*

data. The differences for the other tests compared could have been affected by the differences in moisture contents and therefore, were not significant. Based on the data of the three tests compared with approximately the same moisture contents the conclusion that rate of loading affects the relationship between mean normal stress and bulk density at the 5-inch depth and high moisture contents could be made.

For the relationships obtained from the 10-inch data, differences between Tests 1-19, 13-25, and 22-25 for both sets of data are significant. In addition, the difference between Tests 4 and 19 is significant for the 6 DST data. These differences are probably due to the difference in moisture contents of the tests compared. The conclusion that the rate of loading does not affect the relationship between mean stress and bulk density at the 10-inch depth could be made.

The results at the 15-inch depth show a significant difference between the 1.00 and 0.62 inch per second rates of loading and between the 0.62 and 0.38 rates. The difference between the 1.00 and 0.38 rates is not significant. The significant differences between the 1.00 - 0.62 and 0.62 - 0.38 rates could be due to the differences in moisture contents. Based on this data the conclusion that the rate of loading at the 15-inch depth affects the relationship between mean normal stress and bulk density could not be made.

A set of typical curves showing the results of Tests 5, 2, and 20 are shown in Figures 33 and 34.

<u>The Effect of Rate of Loading on the Relationship between Second Invariant and Bulk Density</u>

The results of the "t" tests between regression coefficients for the second invariant-bulk density relationships are presented in Table 18.

The comparisons for the 5- and 10-inch depths revealed results similiar to those for the mean stress relationships. The results for the 10-inch depth show significant differences between Tests 13-25 and 22-25 for both the Type A and 6 DST data. Differences between Test 4-19 and 1-19 are significant for the Type A data.

A set of typical curves are shown in Figures 35 and 36.

<u>The Effect of Rate of Loading on the Relationship between Maximum Normal Stress and Bulk Density</u>

Comparisons between the regression coefficients obtained from the regression equations of the maximum normal stress-bulk density relationships are shown in Table 19. The results for the 5-inch depth show significant differences between the Type A data for tests 14-23, and 14-26. Difference between Tests 5-20 and 14-23 are significant for the 6 DST data.

For the 10-inch depth, the Type A data show that the rate of loading affects the relationship between maximum

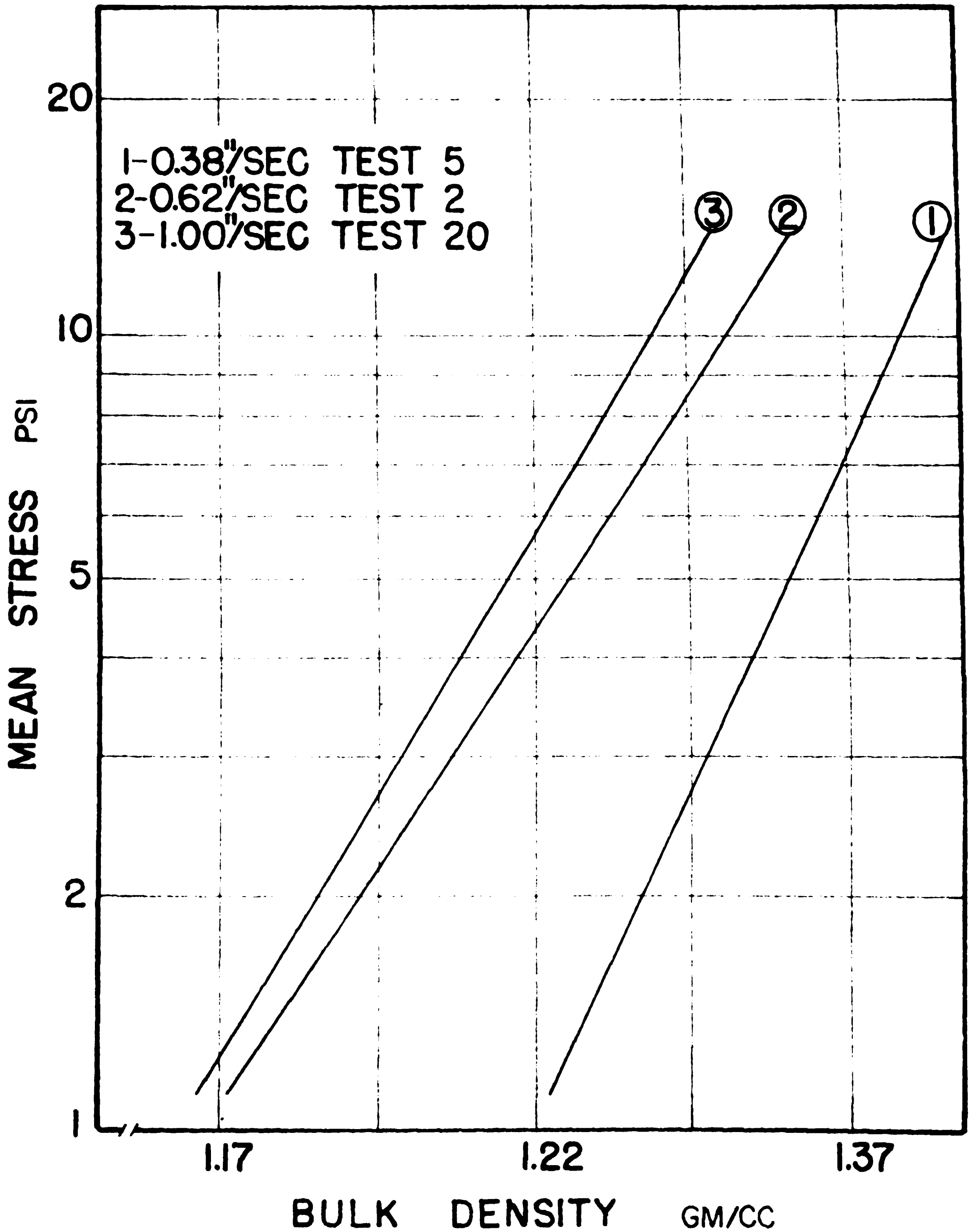

Figure 33. The Effect of Rate of Loading on the Relationship between Mean Stress and Bulk Density. Data obtained with 6 DST.

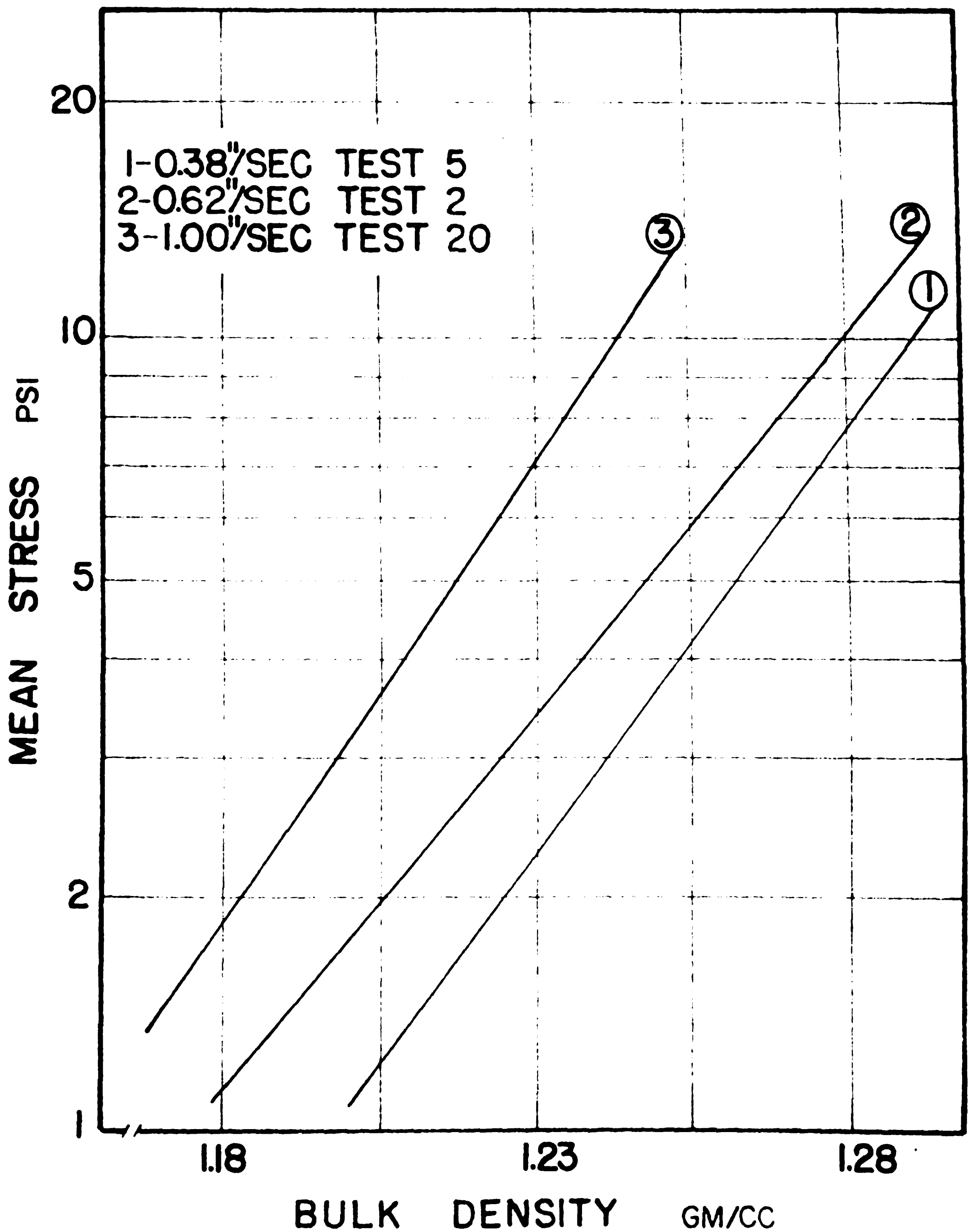

Figure 34 . The Effect of Rate of Loading on the Relationship between Mean Stress and Bulk Density. Data obtained with Type A Pressure Cells.

TABLE 18

COMPARISON OF REGRESSION COEFFICIENTS OF THE II_s VERSUS BULK DENSITY RELATIONS AT DIFFERENT RATES OF LOADING AND BETWEEN METHODS FOR A GIVEN TEST

Tests	D.F.	t Type A	t 6 DST	Test No.	Rate (in/sec)	D.F.	t
5 Inches below Loading Surface							
5-20	66	1.61	0.96	20	1.00	76	1.14
2-5	61	0.11	1.87	2	0.62	66	2.31*
2-20	71	1.48	0.68	5	0.38	56	4.43**
14-23	31	4.67**	3.04**	14	1.00	36	0.54
14-26	41	4.87**	2.49*	23	0.62	26	1.06
23-26	36	0.61	0.98	26	0.38	46	2.20*
10 Inches below Loading Surface							
4-19	56	2.00*	0.92	19	1.00	76	1.55
1-4	44	1.73	0.28	1	0.62	52	7.28**
1-19	64	5.05**	1.13	4	0.38	36	1.62
13-22	35	0.40	0.15	13	1.00	44	1.67
13-25	45	4.06**	4.18**	22	0.62	26	1.09
22-25	36	3.98**	3.29**	25	0.38	46	2.18*
15 Inches below Loading Surface							
27-21	36	5.83**	6.18**	27	1.00	36	3.82**
27-24	41	1.97	1.65	21	0.62	36	3.14**
21-24	41	3.95**	4.11**	24	0.38	46	2.66**

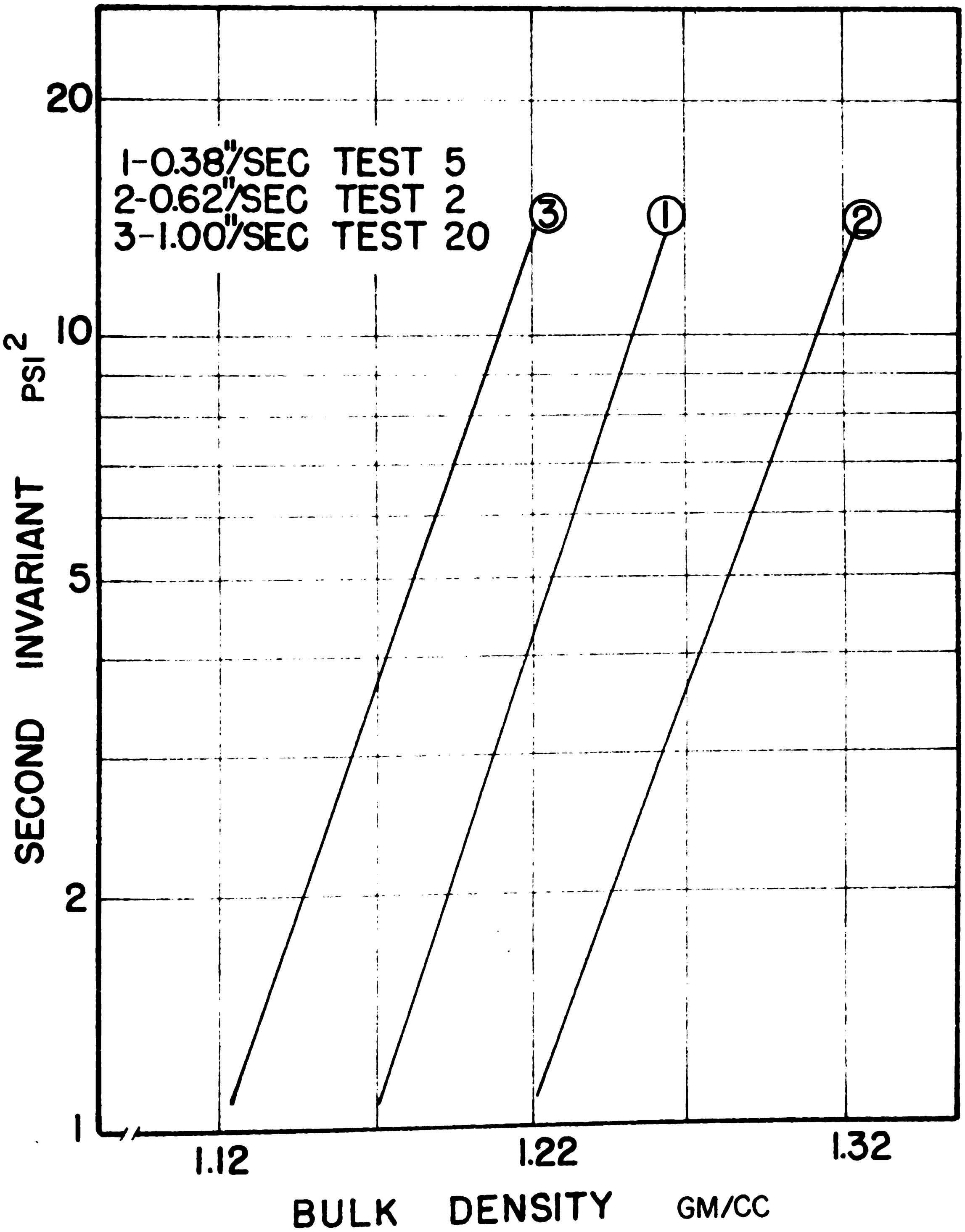

Figure 35. The Effect of Rate of Loading on the Relationship between the Second Invariant and Bulk Density. Data obtained with 6 DST.

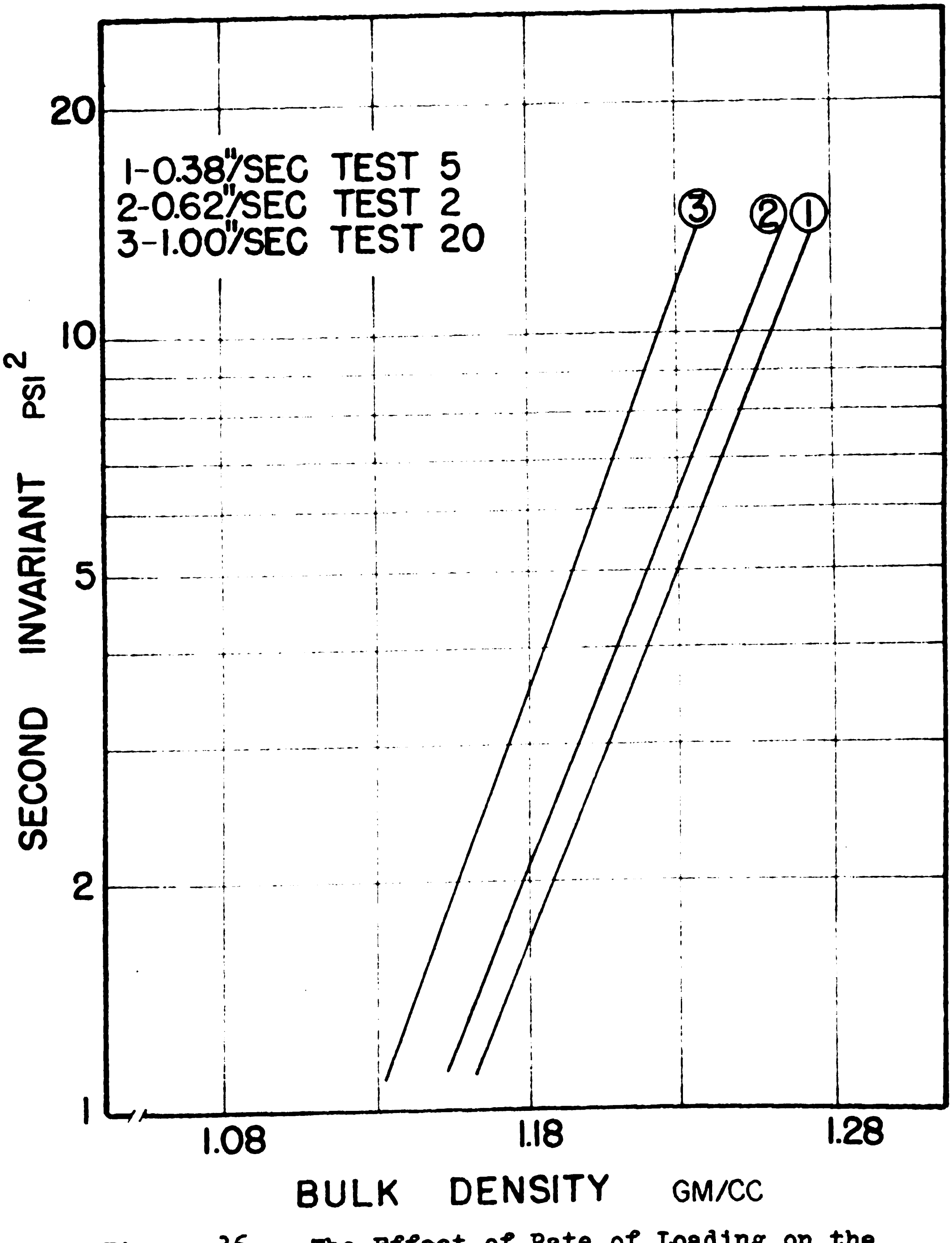

Figure 36 . The Effect of Rate of Loading on the Relationship between the Second Invariant and Bulk Density. Data obtained with Type A Pressure Cells.

TABLE 19

COMPARISON OF REGRESSION COEFFICIENTS OF THE σ_I VERSUS BULK DENSITY RELATIONS AT DIFFERENT RATES OF LOADING AND BETWEEN METHODS FOR A GIVEN TEST

Tests	D.F.	t Type A	t 6 DST	Test No.	Rate (in/sec)	D.F.	t
5 Inches below Loading Surface							
5-20	66	1.50	2.42*	20	1.00	76	0.74
2-5	61	0.69	1.92	2	0.62	66	2.25*
2-20	71	1.98	0.80	5	0.38	56	3.50**
14-23	31	3.21**	2.44*	14	1.00	36	0.21
14-26	41	3.65**	1.14	23	0.62	26	0.74
23-26	36	0.34	1.41	26	0.38	46	2.35*
10 Inches below Loading Surface							
4-19	56	2.06*	0.16	19	1.00	76	0.34
1-4	44	1.92	0.39	1	0.62	52	6.32**
1-19	64	4.44**	0.33	4	0.38	36	1.39
13-22	35	0.07	0.48	13	1.00	44	1.11
13-22	45	3.71**	3.34**	22	0.62	26	1.27
22-25	36	2.93**	3.58**	25	0.38	46	1.88
15 Inches below Loading Surface							
27-21	36	5.18**	5.48**	27	1.00	36	3.09**
27-24	41	2.13*	1.75	21	0.62	36	3.10**
21-24	41	3.48**	4.13**	24	0.38	46	2.52*

normal stress and bulk density. The 6 DST data supports the Type A data for the higher moisture contents but not at the lower moisture contents.

The results for the 15-inch depth show agreement between the two methods except for Tests 27-24. The Type A data is significant, however, the 6 DST is not.

The results of Tests 1, 11, and 22 are shown in Figure 37 and 38.

<u>The Effect of Rate of Loading on the Relationship between Maximum Shear Stress and Bulk Density</u>

The results of the "t" tests are presented in Table 20. They are similiar to the results obtained for the relationship between maximum normal stress and bulk density. In general, the conclusion that the rate of loading affects the relationship between maximum shear stress and bulk density at the higher moisture contents and lower depths could be made.

A typical set of curves for this relation are shown in Figures 39 and 40.

<u>Comparison of Three Methods Used to Determine Mean Normal Stress</u>

The regression equations for the relationship between mean normal stress and applied surface loads for the calculated values of mean normal stress obtained from the Type A and 6 DST data and the values measured directly with the W-Cell were determined. The results are shown

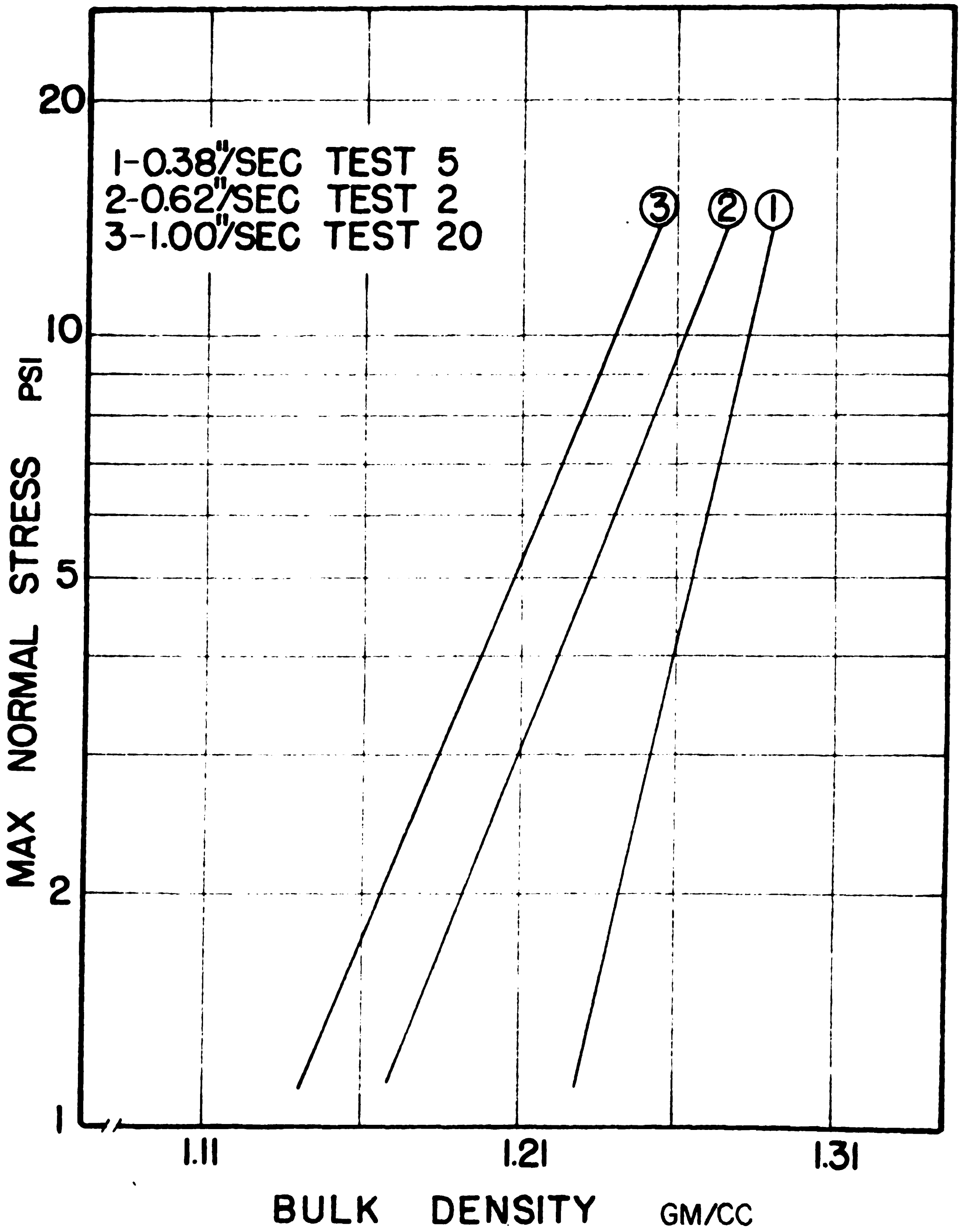

Figure 37 . The Effect of Rate of Loading on the Relationship between the Maximum Normal Stress and Bulk Density. Data obtained with 6 DST.

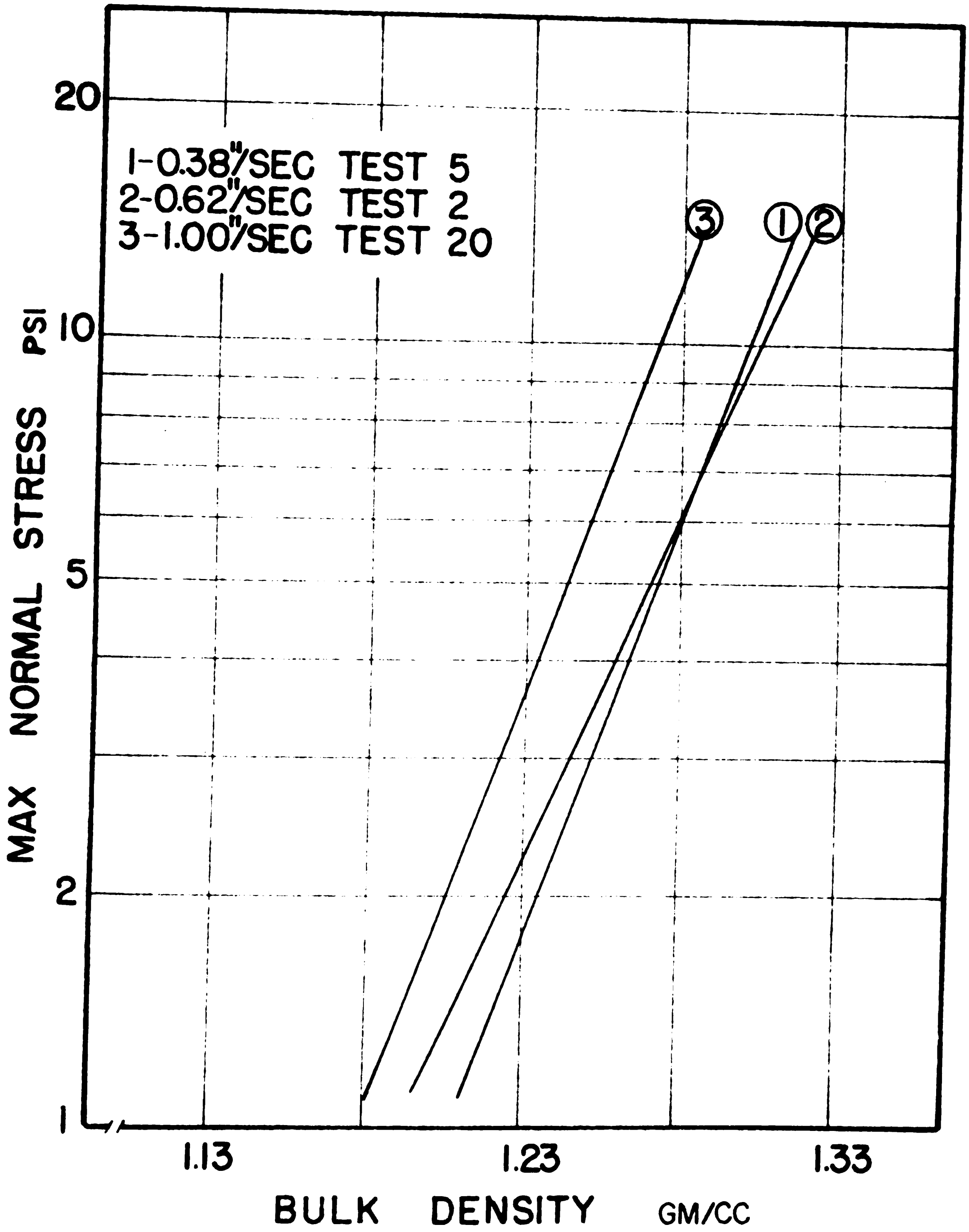

Figure 38. The Effect of Rate of Loading on the Relationship between the Maximum Normal Stress and Bulk Density. Data obtained with Type A Pressure Cells.

TABLE 20

COMPARISON OF REGRESSION COEFFICIENTS OF THE τ_{max} VERSUS BULK DENSITY RELATIONS AT DIFFERENT RATES OF LOADING AND BETWEEN METHODS FOR A GIVEN TEST

Tests	D.F.	t Type A	t 6 DST	Test No.	Rate (in/sec)	D.F.	t
			5 Inches below Loading Surface				
5-20	66	1.31	2.10*	20	1.00	76	0.39
2-5	61	0.52	2.24	2	0.62	66	5.40**
2-20	71	1.64	4.16**	5	0.38	56	3.32**
14-23	31	3.49**	2.22*	14	1.00	36	0.08
14-26	41	3.75**	0.96	23	0.62	26	0.87
23-26	36	0.50	1.36	36	0.38	46	2.32*
			10 Inches below Loading Surface				
4-19	56	2.43	0.13	19	1.00	76	0.45
1-4	44	1.82	0.48	1	0.62	52	6.96**
1-19	64	4.75**	0.52	4	0.38	36	1.58
13-22	35	0.23	0.49	13	1.00	44	1.31
13-25	45	3.59**	3.51**	22	0.62	26	1.25
22-25	36	3.02**	3.55**	25	0.38	46	1.93
			15 Inches below Loading Surface				
27-21	36	5.08**	5.83**	27	1.00	36	3.02**
27-24	41	1.85	1.80	21	0.62	36	3.42**
21-24	41	3.54**	4.27**	24	0.38	46	2.74**

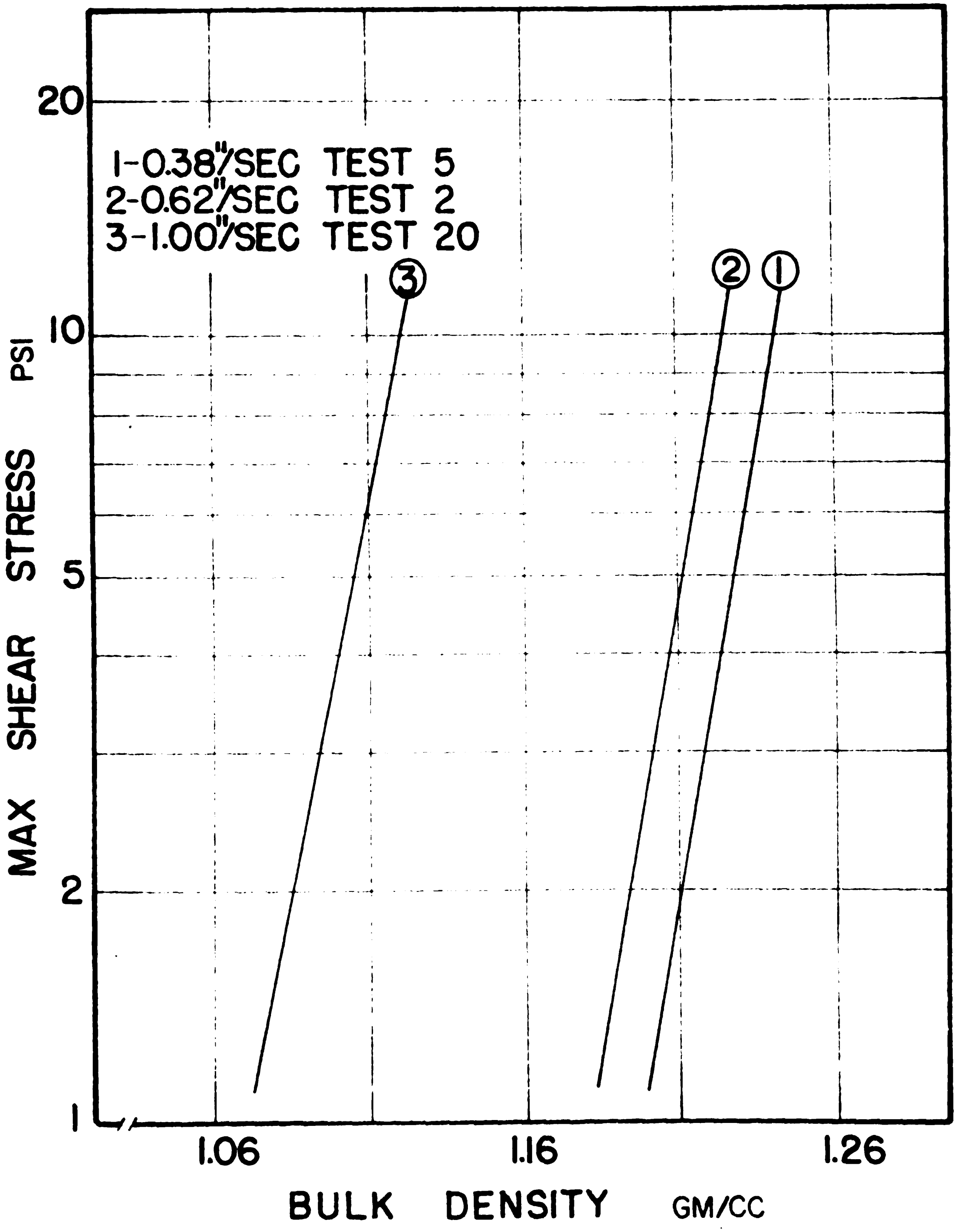

Figure 39. The Effect of Rate of Loading on the Relationship between the Maximum Shear Stress and Bulk Density. Data obtained with 6 DST.

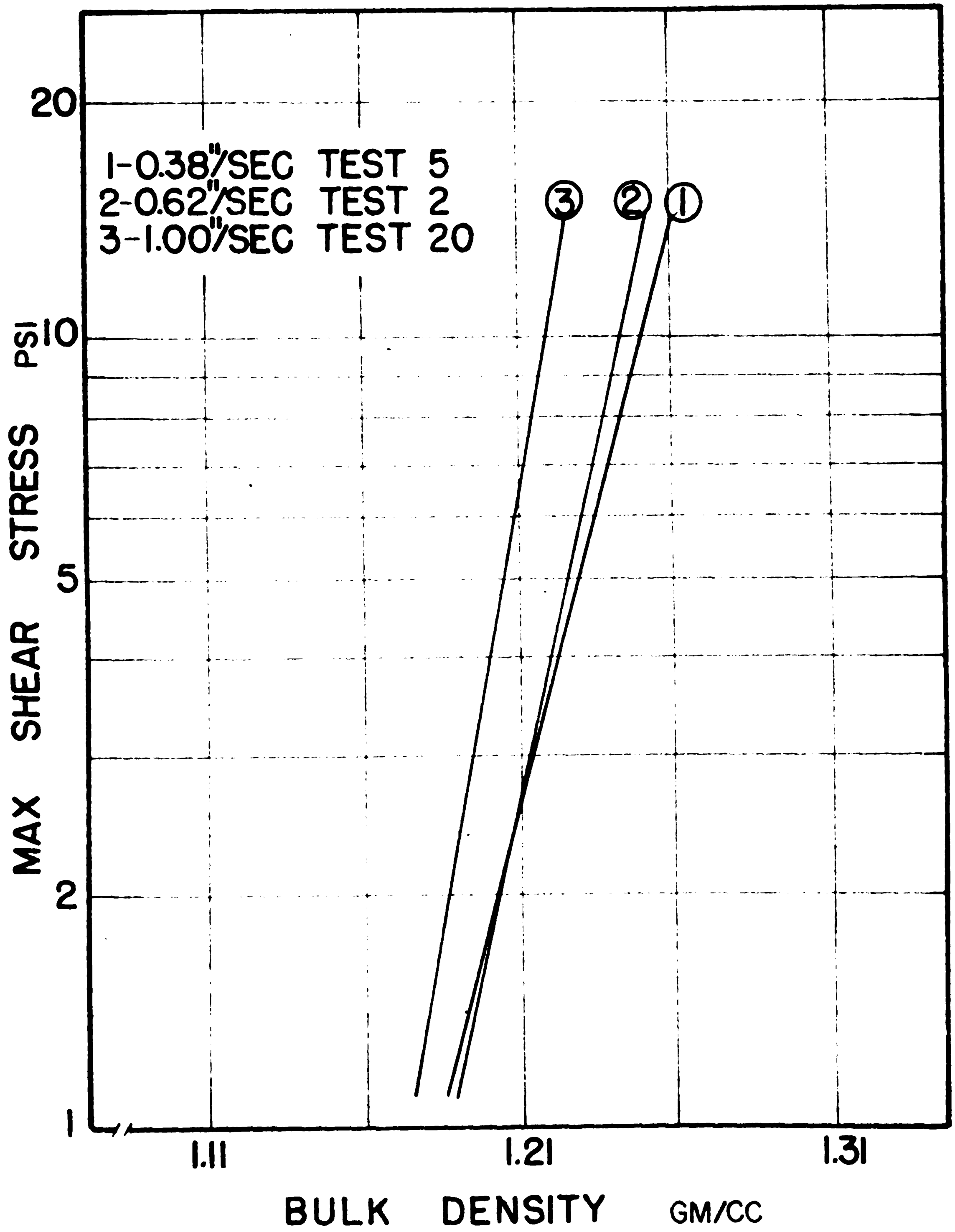

Figure 40. The Effect of Rate of Loading on the Relationship between the Maximum Shear Stress and Bulk Density. Data obtained with Type A Pressure Cells.

in Figures 41, 42, 43, 44, and 45. Theoretically, the relationships should go through the origin, however, as the curves show this was not true. The sum of least squares method of obtaining the best predicting straight line for the points was used to permit the use of a statistical method for comparing the three methods.

The "t" test was applied to the regression coefficients to check for significant differences and the results are presented in Table 21. The most consistent results were obtained between the Type A Cells and the W Cell. The differences between the Type A and 6 DST data were the most varied.

TABLE 21

COMPARISON OF THE REGRESSION COEFFICIENTS CALCULATED FOR THE RELATIONSHIP BETWEEN MEAN NORMAL STRESS AND APPLIED LOAD FOR A CONSTANT RATE OF LOADING OF 1.00 INCH PER SECOND.

Test No.	Depth In.	M.C. %	t Type A 6 DST	t Type A W Cell	t 6 DST W Cell	D.F.
14	5	17.41	4.07**	1.22	0.33	36
20	5	10.79	3.57**	1.36	1.26	76
13	10	17.41	0.06	0.65	2.75**	44
19	10	10.95	2.72**	2.82**	0.81	76
18	15	12.31	0.44	1.32	3.90**	76

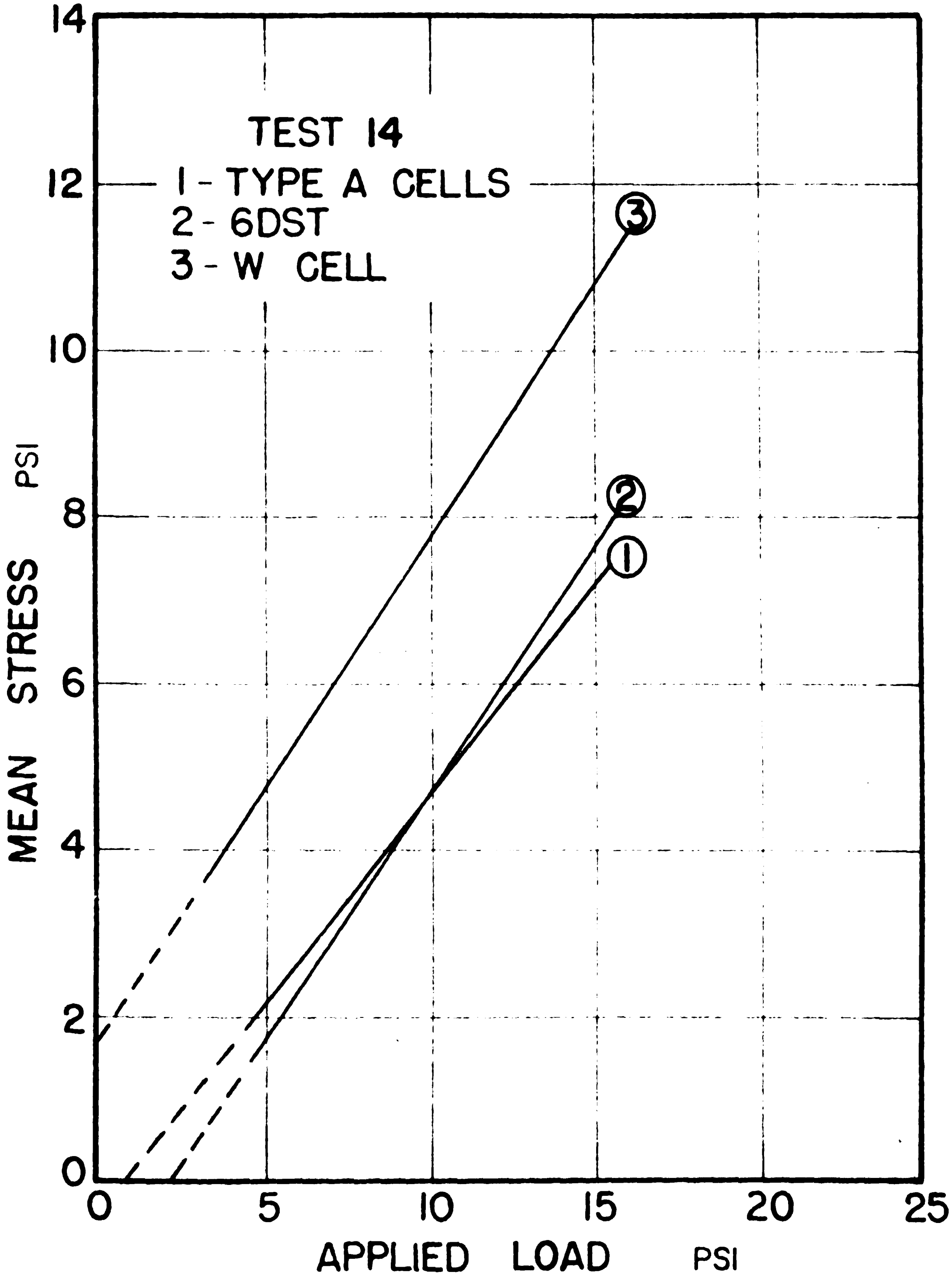

Figure 41. Comparison of the Regression Lines for the Three Methods used to Determine Mean Stress. Data obtained at the 5-inch Depth, 1.00 in/sec Rate of Loading and a Moisture Content of 17.41%.

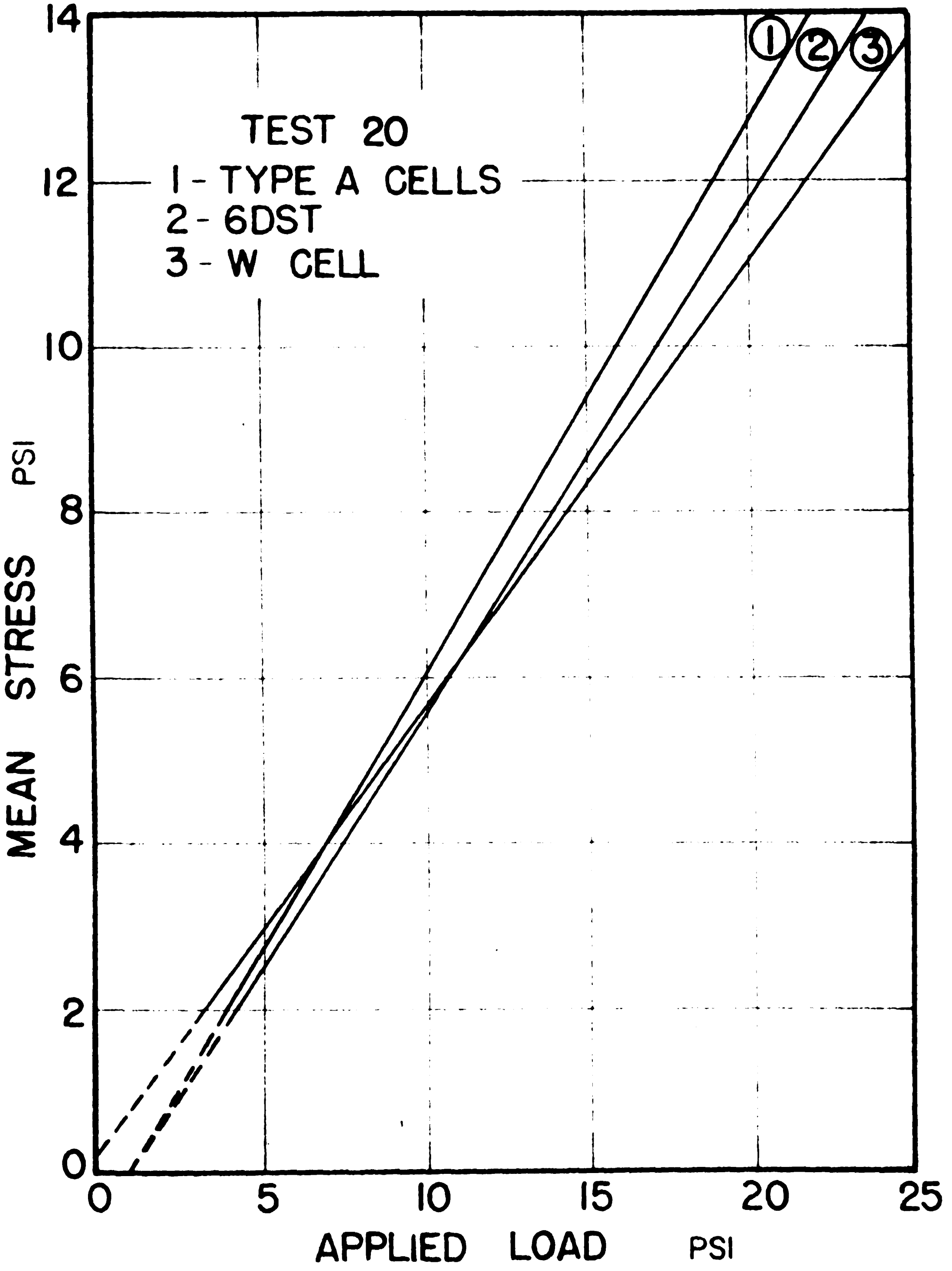

Figure 42 . Comparison of the Regression Lines for the Three Methods used to Determine Mean Stress. Data obtained at the 5-inch Depth, 1.00 in/sec Rate of Loading and a Moisture Content of 10.79%.

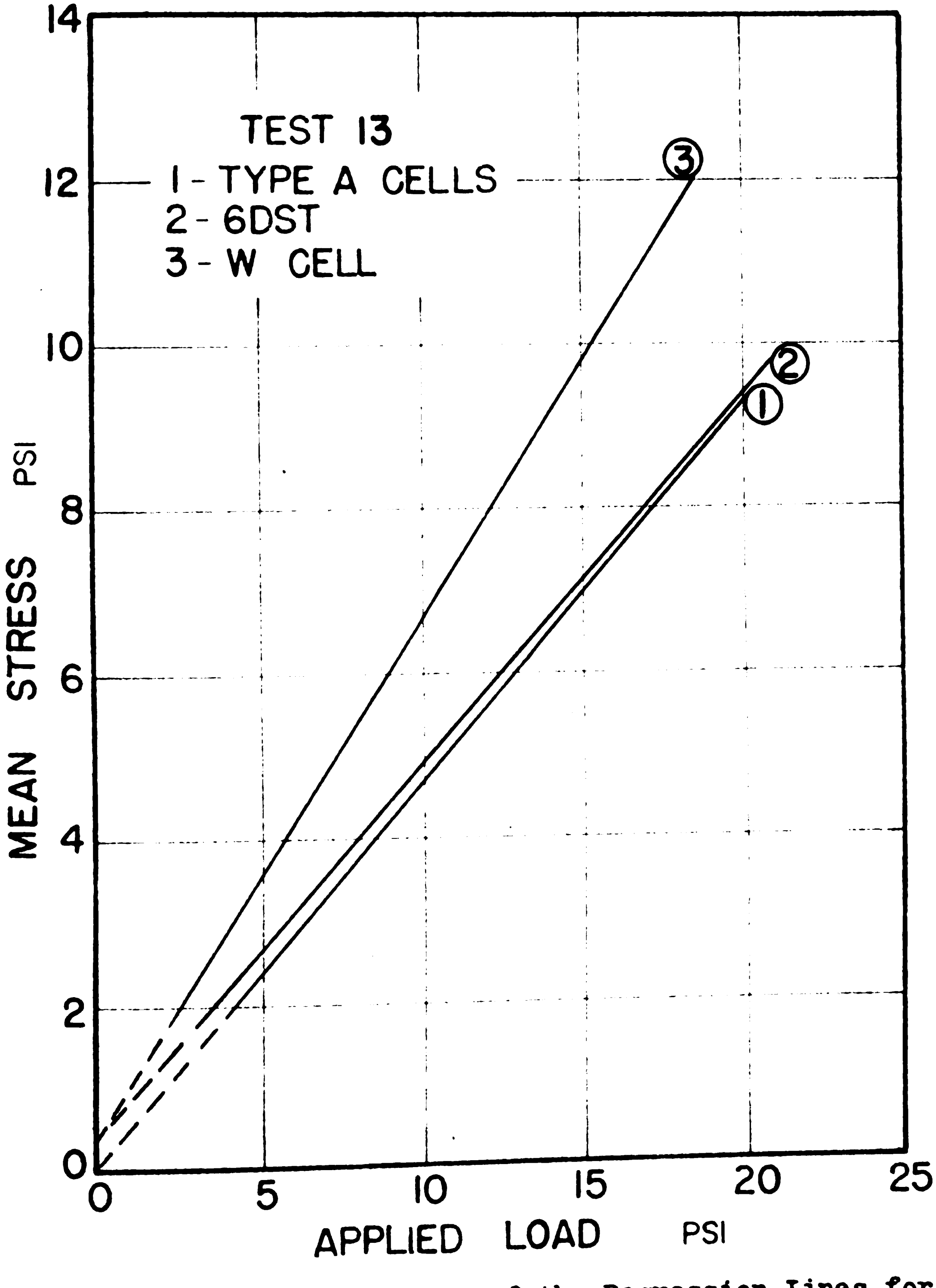

Figure 43 . Comparison of the Regression Lines for the Three Methods used to Determine Mean Stress. Data obtained at the 10-inch Depth, 1.00 in/sec Rate of Loading and a Moisture Content of 17.41%.

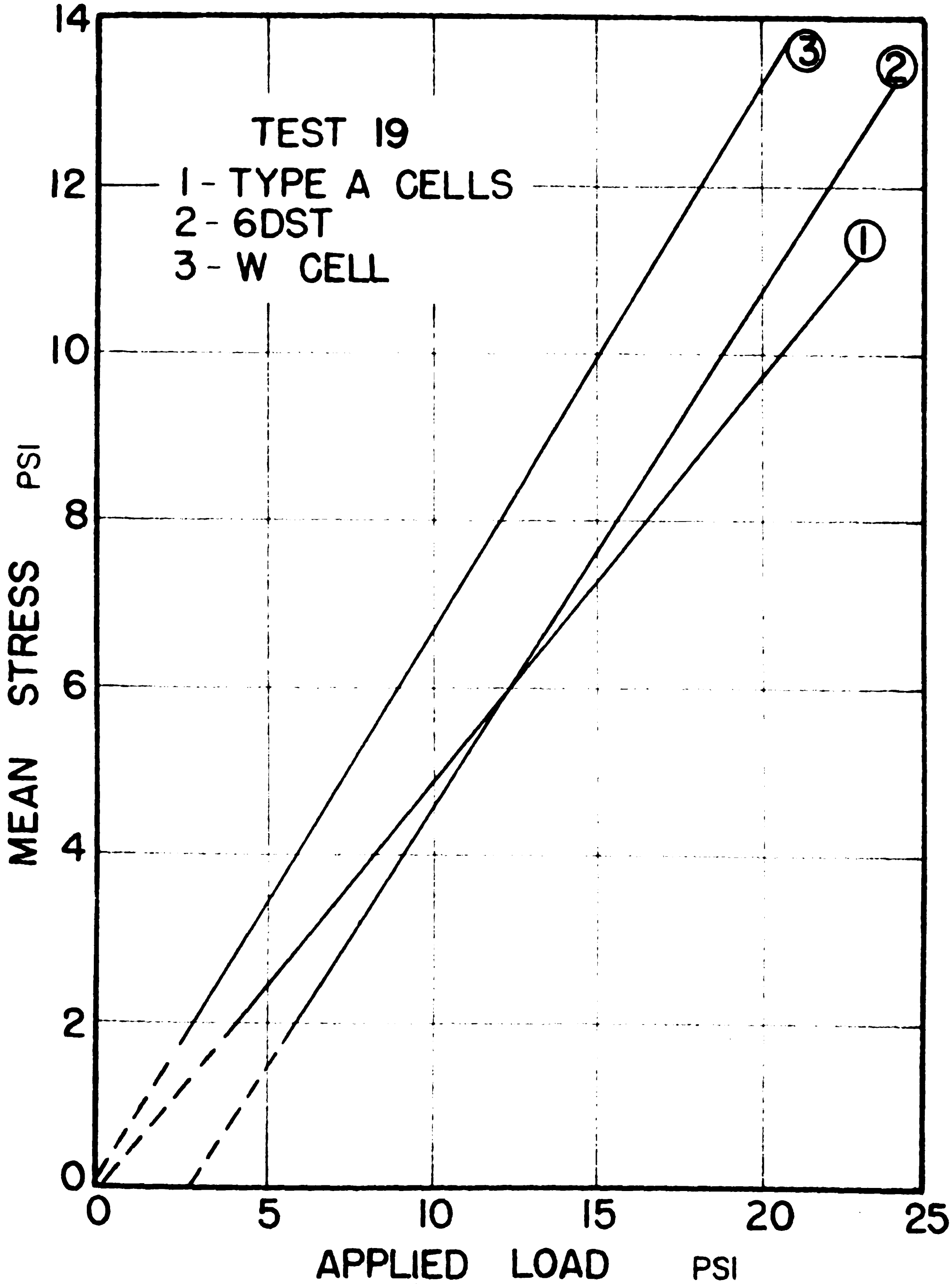

Figure 44. Comparison of the Regression Lines for the Three Methods Used to Determine Mean Stress. Data obtained at the 10-inch Depth, 1.00 in/sec Rate of Loading and a Moisture Content of 10.95%.

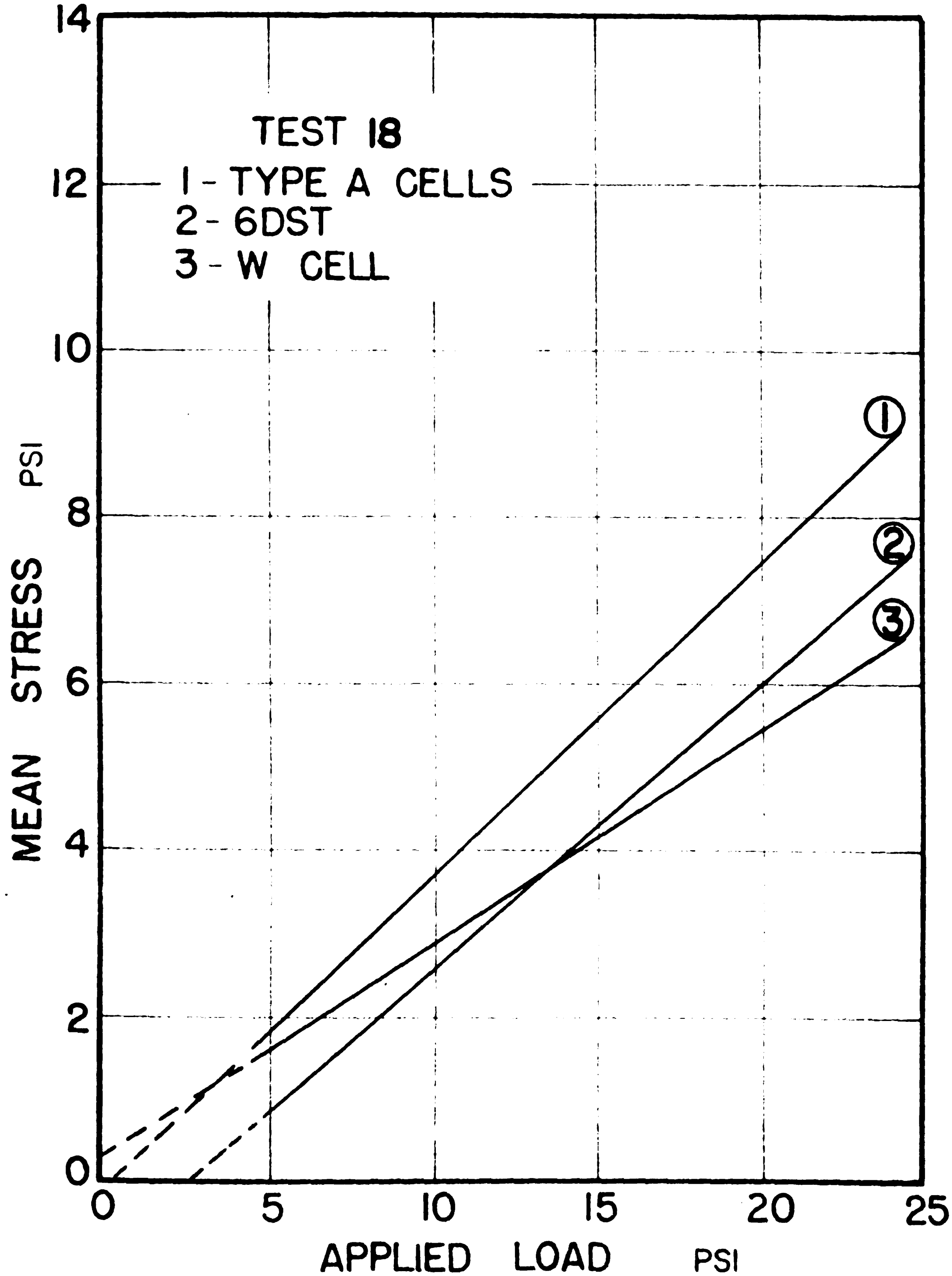

Figure 45. Comparison of the Regression Lines for the Three Methods used to Determine Mean Stress. Data obtained at the 15-inch Depth, 1.00 in/sec Rate of Loading and a Moisture Content of 12.31 %.

On the basis of this data the measured values of mean stress obtained with the W Cell will be similiar to those calculated with the Type A method.

The Effect of Rate of Loading on the Relationship between Mean Stress and Applied Load

To determine the effect of the rate of loading on the relationship between mean normal stress and applied surface load the average values of mean stress for the 5 replications were plotted versus applied load. The curves shown in Figures 46 through 49 show, in general the results obtained.

The curves persented in Figures 46 and 47 are for 5 inches below the surface and moisture content ranging from 8.89 to 10.79 percent. This difference in moisture content could contribute to part of the difference obtained. The differences between the 1.00 inch per second and 0.62 inch per second are similiar for the two sets of curves. There is only 0.90 percent difference in moisture content. The difference between the 0.38 inch per second rate and 1.00 inch per second rates is not as consistent, which could be due to the difference in moisture of 1.90 percent. In general, these results are the same at the 10- and 15-inch depths within the range of moisture content from 8.89 to 12.31 percent.

The results at a moisture content ranging from 16.05 to 17.67 percent and a depth of 5 inches are shown in Figures 48 and 49. The two sets of curves are in very

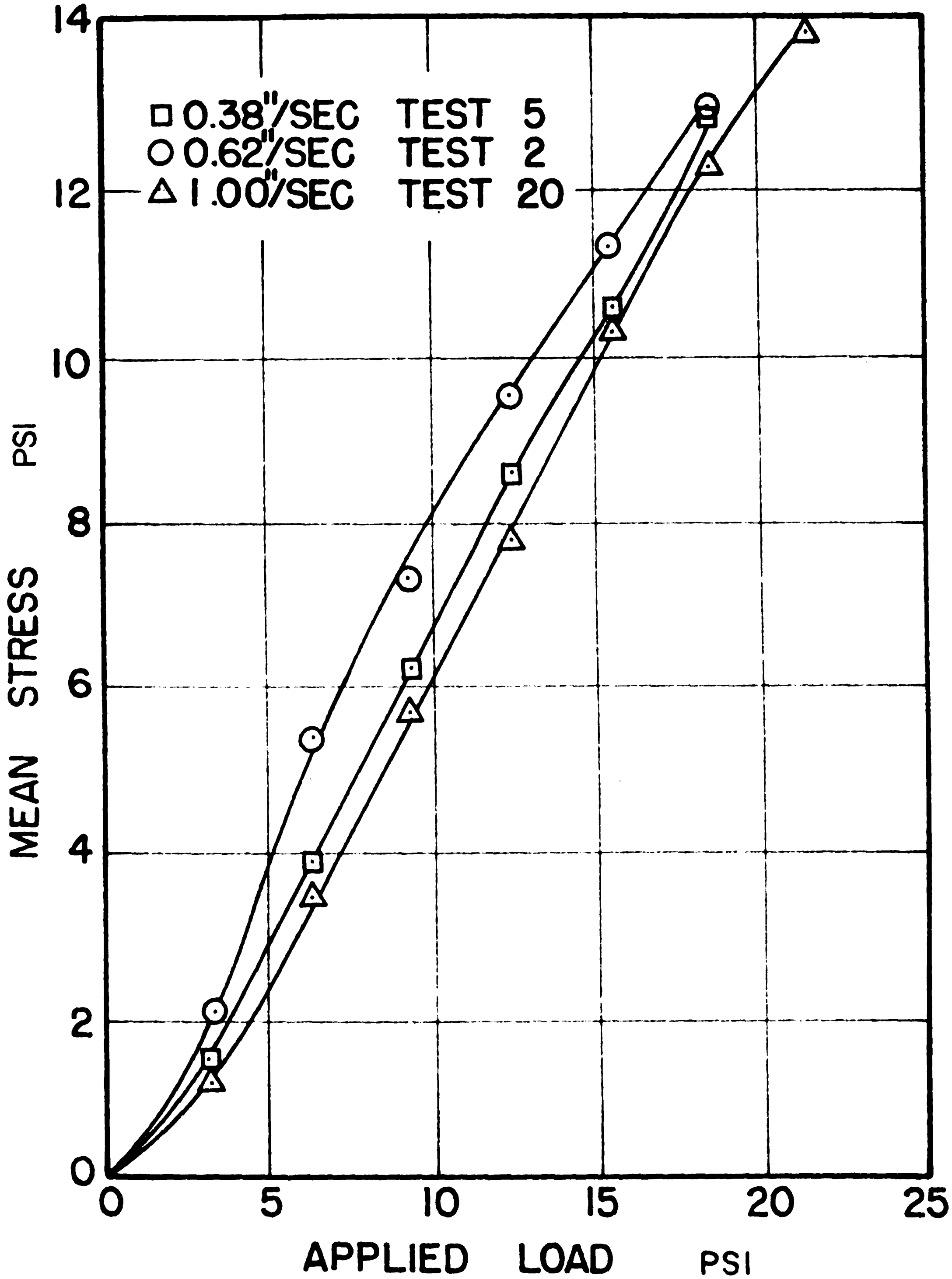

Figure 46. The Effect of Rate of Loading on the Relationship between Mean Stress and Applied Load at 5-inch Depth. Data obtained with 6 DST at a Moisture Content Ranging from 8.89 to 10.79%.

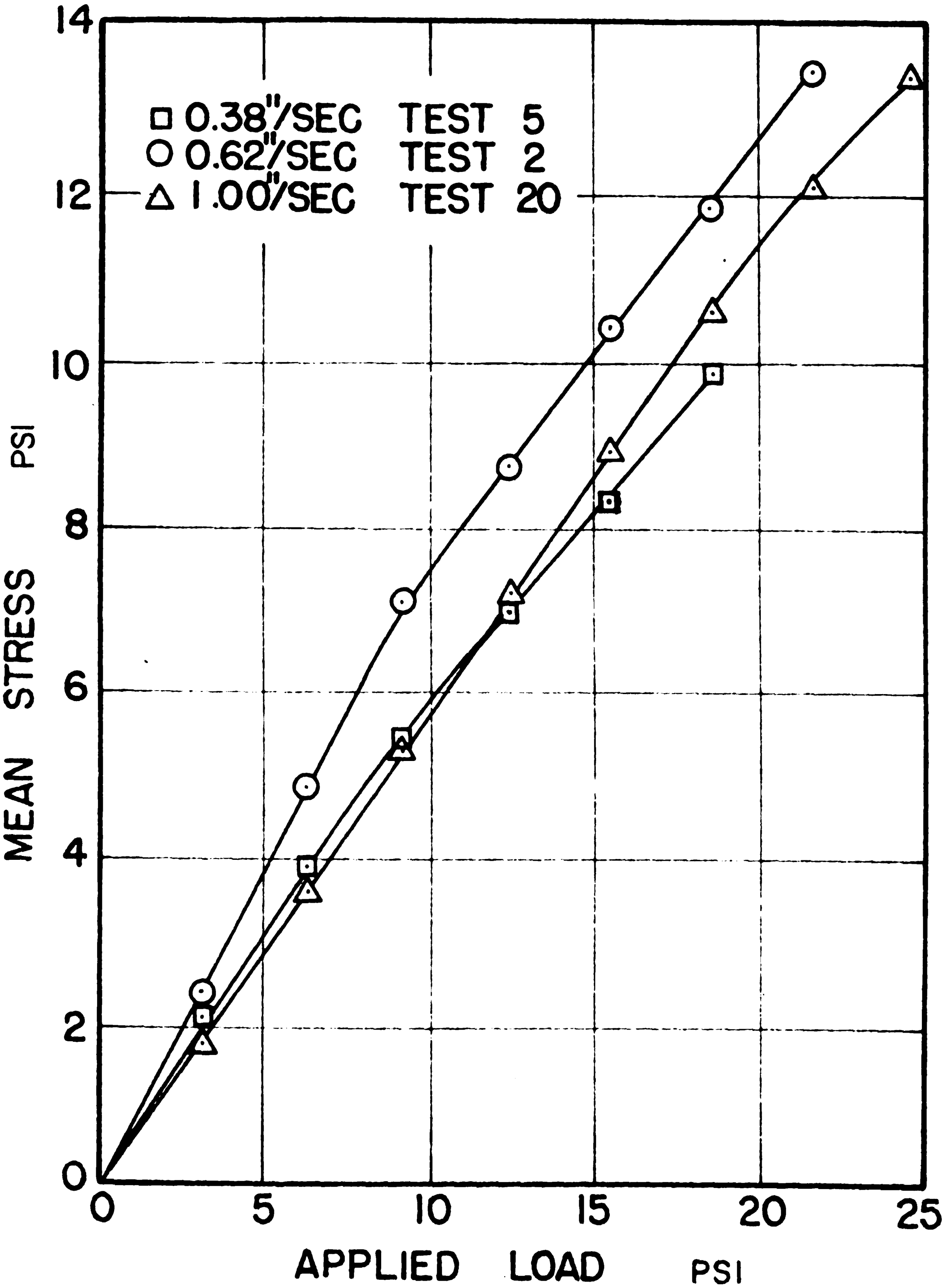

Figure 47. The Effect of Rate of Loading on the Relationship between Mean Stress and Applied Load at 5-inch Depth. Data obtained with Type A Cells at a Moisture Content Ranging from 8.89 to 10.79%.

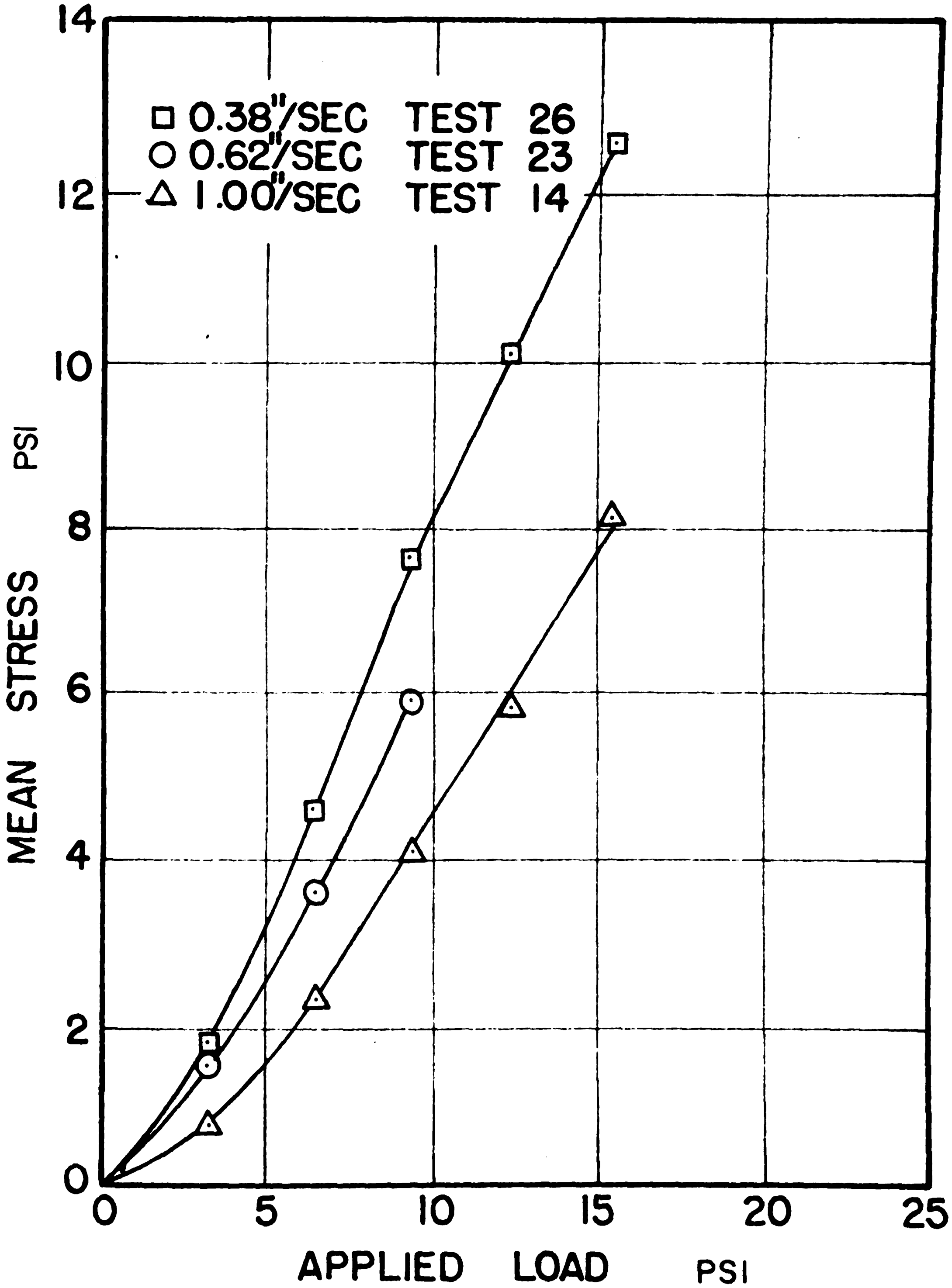

Figure 48. The Effect of Rate of Loading on the Relationship between Mean Stress and Applied Load at 5-inch Depth. Data obtained with 6 DST at a Moisture Content Ranging from 16.05 to 17.67%.

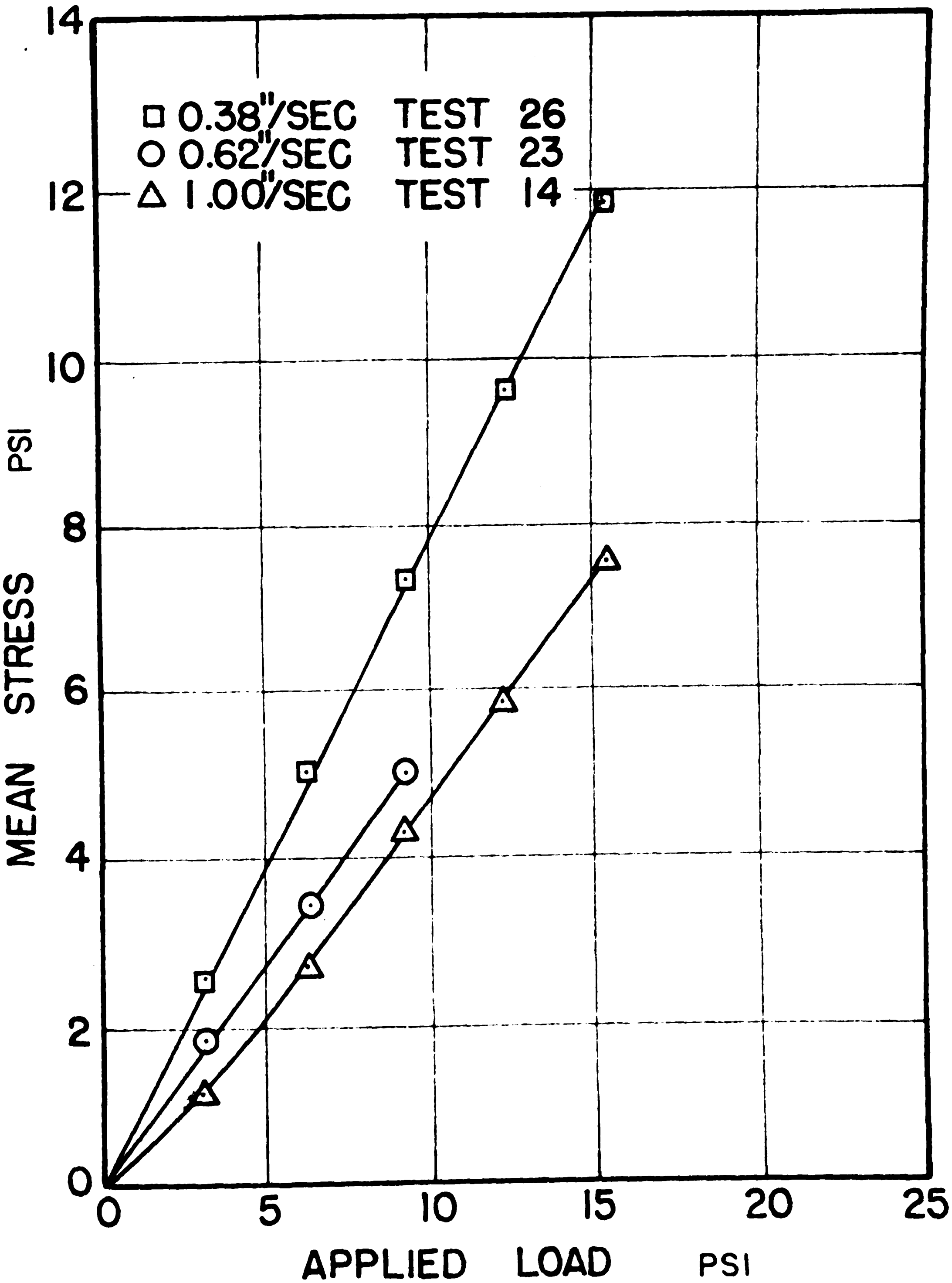

Figure 49. The Effect of Rate of Loading on the Relationship between Mean Stress and Applied Load at 5 inches Depth. Data obtained with Type A Cells. At a Moisture Content Ranging from 16.05 to 17.67%.

good agreement. In general, these results are similiar to those at the 10- and 15-inch depth for 15.78 to 17.93 percent moisture contents.

Based on this data, the conclusion is that for a given applied load, the lowest values of mean stress will be obtained for the 1.00 inch per second rate of loading.

The Effect of Moisture Content on Relationship between Mean Normal Stress and Applied Load

To determine the effect of moisture content on the relationship between mean normal stress and applied surface loads, the averages of 5 replications of data from the 6 DST and Type A cells were plotted. The results at the 5-inch depth for the 3 rates of loading are shown in Figures 50 through 53.

For the 0.38 inch per second rate (Figures 50 and 51) the results are approximately the same for the two methods. The curves show a difference between the highest and lowest moisture content. The "t" tests between the highest and lowest moisture contents for the relationship involving bulk density were also significant. These results are representative of results for the 10- and 15-inch depths. The conclusion that for a given load the highest values of mean stress would be obtained for the high moisture content could be made.

The results at the 0.62 inch per second rate (Figures 52 and 53) are similiar for both sets of curves. The

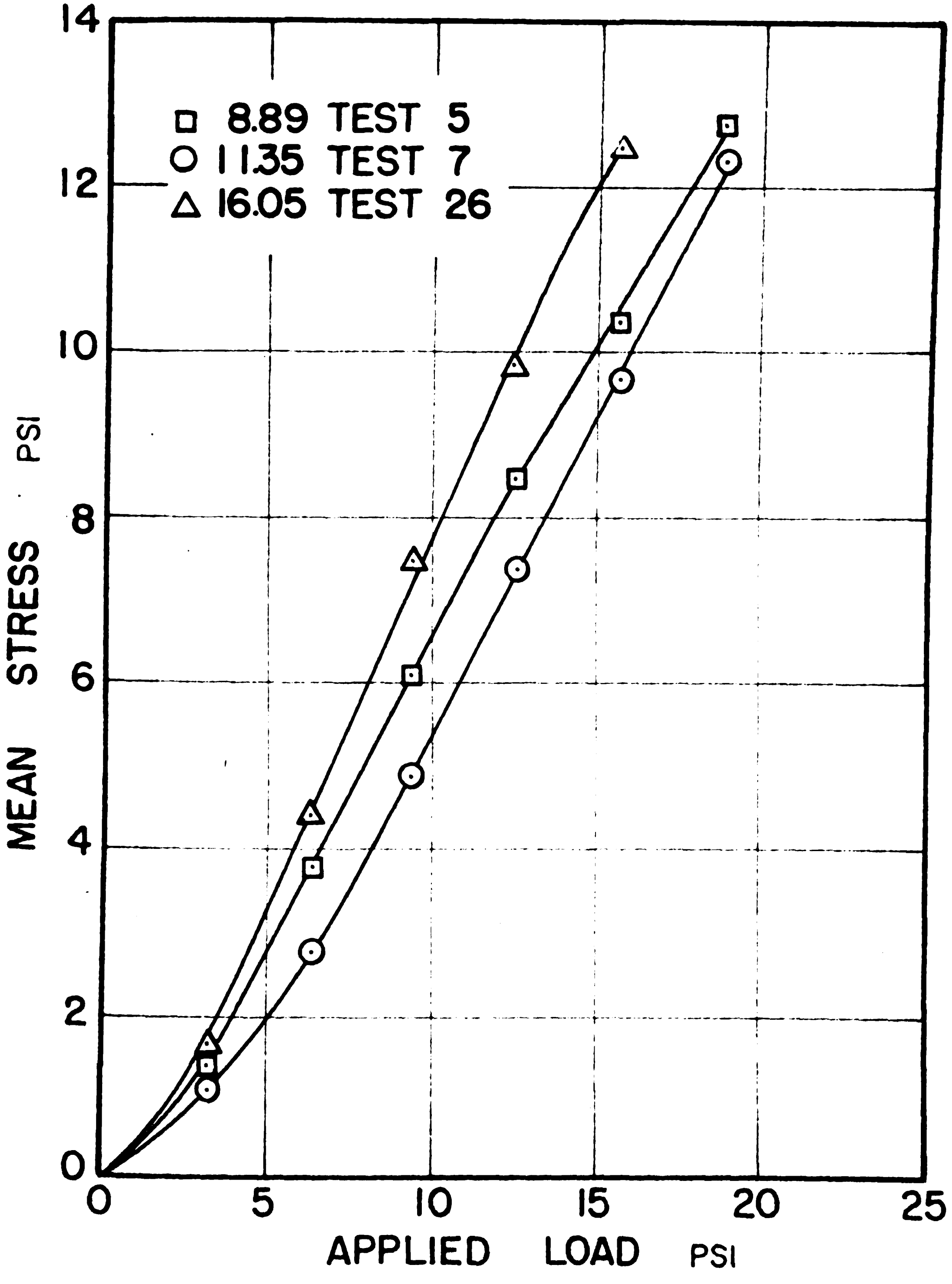

Figure 50. The Effect of Moisture Content on the Relationship between Mean Stress and Applied Load at 5-inch Depth and 0.38 in/sec Rate of Loading. Data obtained with 6 DST.

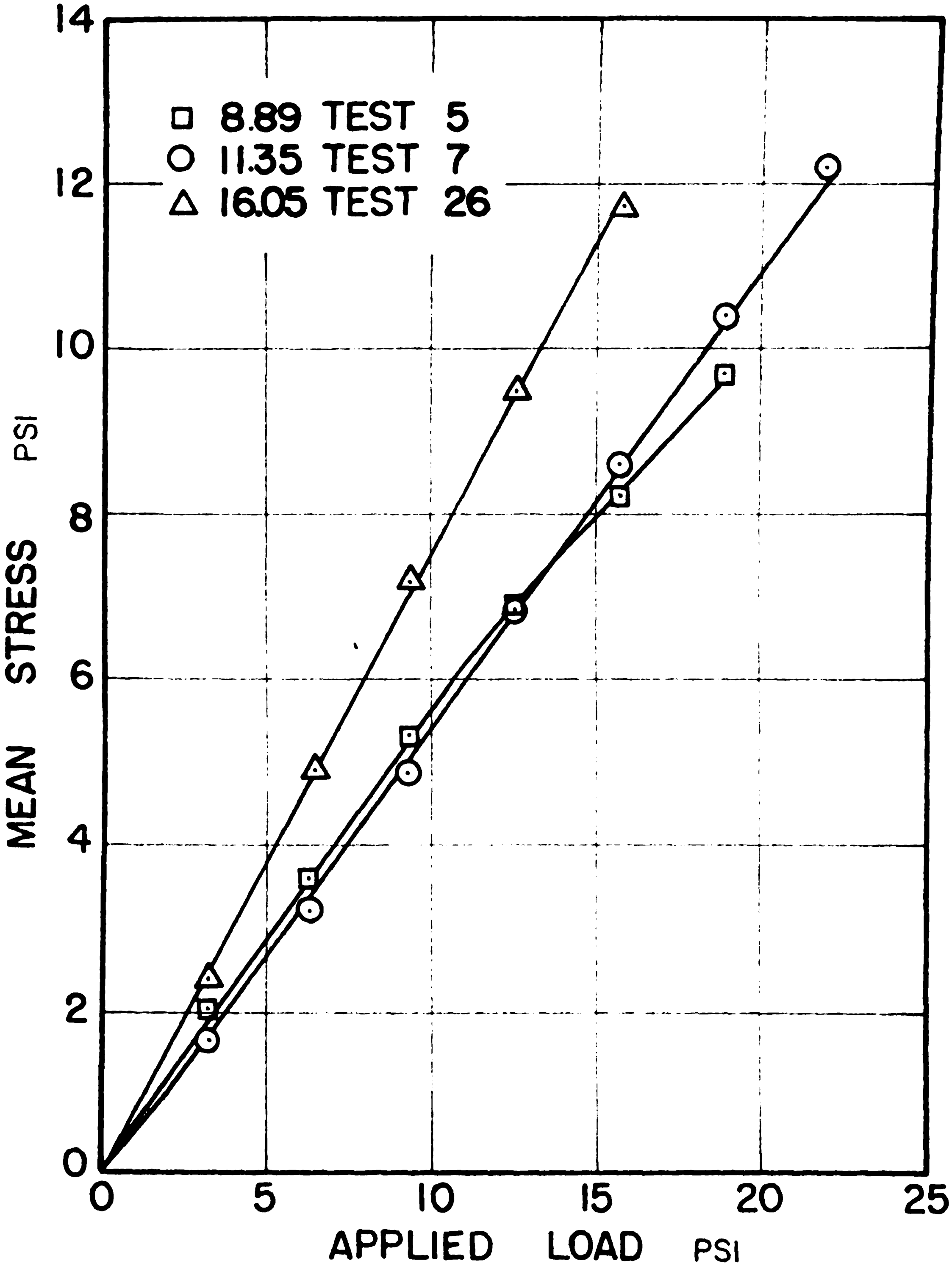

Figure 51 . The Effect of Moisture Content on the Relationship between Mean Stress and Applied Load at 5-inch Depth and 0.38 in/sec Rate of Loading. Data obtained with Type A Cells.

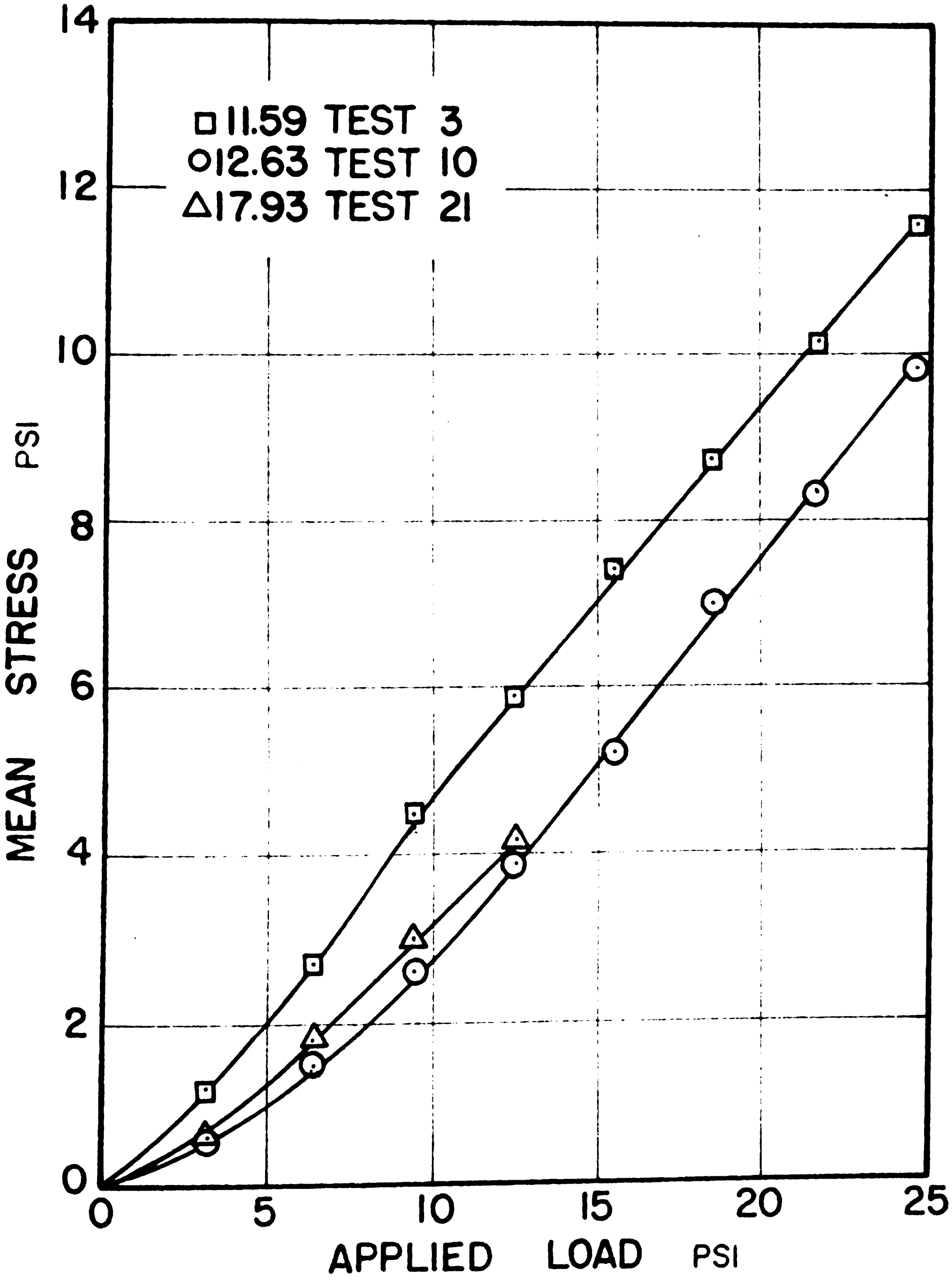

Figure 52 . The Effect of Moisture Content on the Relationship between Mean Stress and Applied Load at 15 Inches Depth and 0.62 in/sec Rate of Loading. Data obtained with 6 DST.

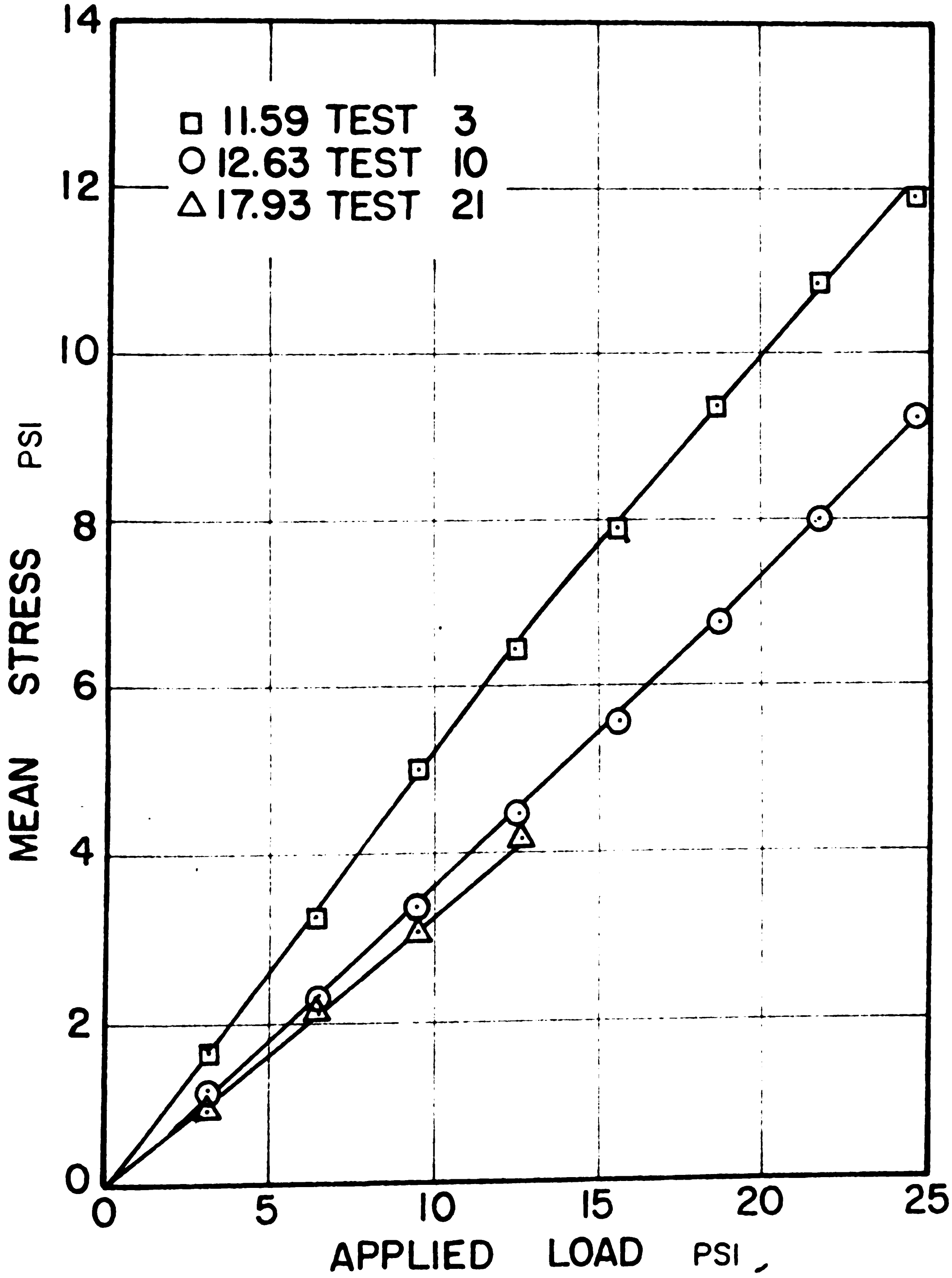

Figure 53 . The Effect of Moisture Content on the Relationship between Mean Stress and Applied Load at 15 Inches Depth and 0.62 in/sec Rate of Loading. Data obtained with Type A Pressure Cells.

results, in general, are the same for the 10- and 15-inch depths. The maximum value of mean stress that could be developed for the 17.93 percent moisture content was less than 5 pounds per square inch. The differences between the highest and lowest moisture contents appear to be significant.

Figures 54 and 55 show the curves for the 1.00 inch per second rate. The 6 DST and Type A curves are in good agreement. The results indicate little or no effect of moisture content on the mean stress-applied load relation. In addition, they represent the pattern at the 10- and 15-inch depths. The conclusion that moisture content within the range used for these tests has little or no effect on the relationship between mean stress and applied load at the 1.00 inch-per-second rate of loading could be made.

Comparison of Theoretical Values of Vertical Stress with Measured Values

The vertical normal stress produced in the soil by a loading plate can be determined with a semi-empirical equation developed by Froehlick as reviewed by Soehne (1957). The equation is as follows:

$$\sigma_Z = \frac{C \quad Q}{2 \pi Z^2} \cos^{(C+2)} \theta \qquad (30)$$

where:

C = concentration factor

Z = distance below loading plate

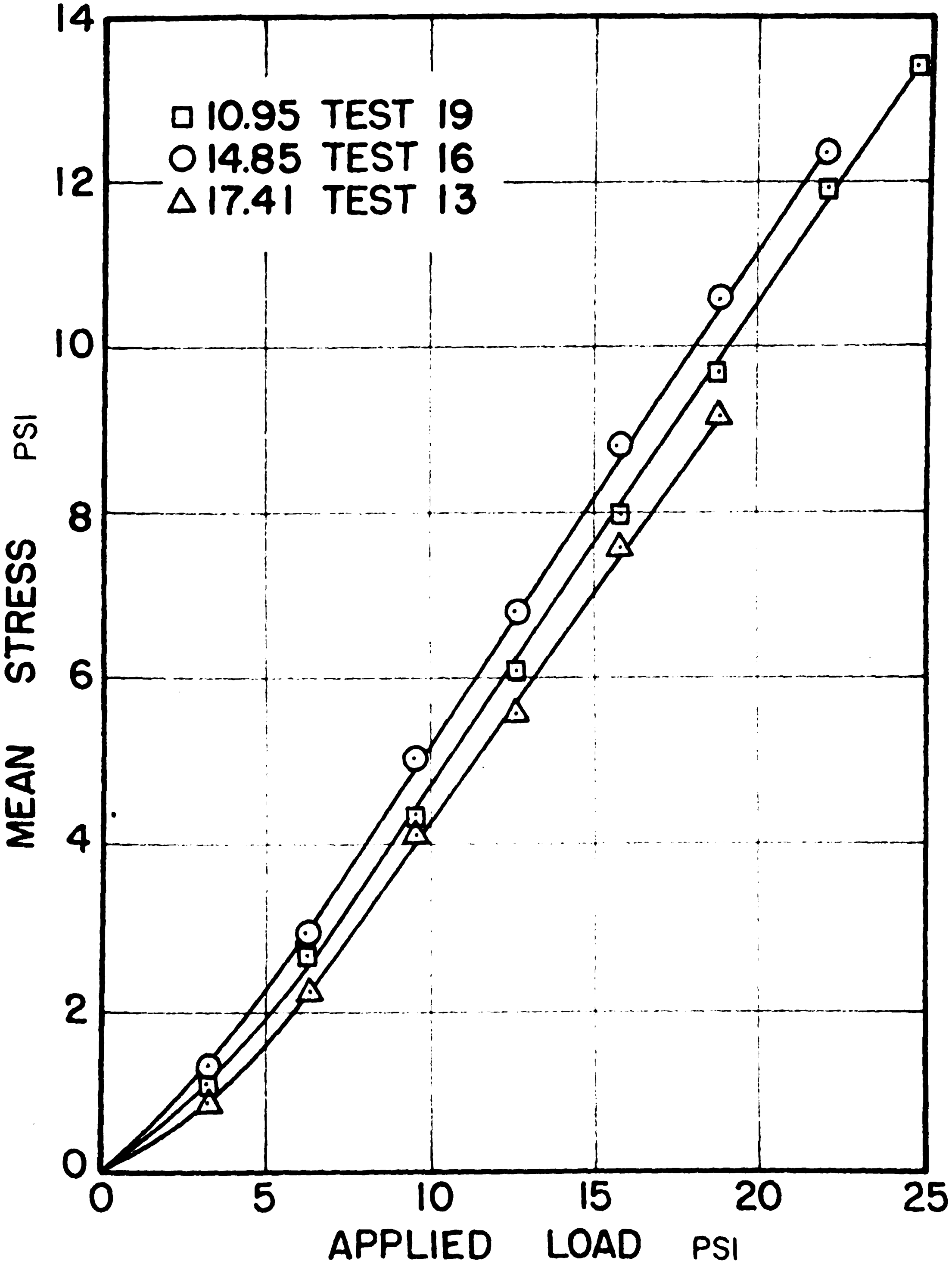

Figure 54 . The Effect of Moisture Content on the Relationship between Mean Stress and Applied Load at 10 Inches Depth and 1.00 in/sec Rate of Loading. Data obtained with 6 DST.

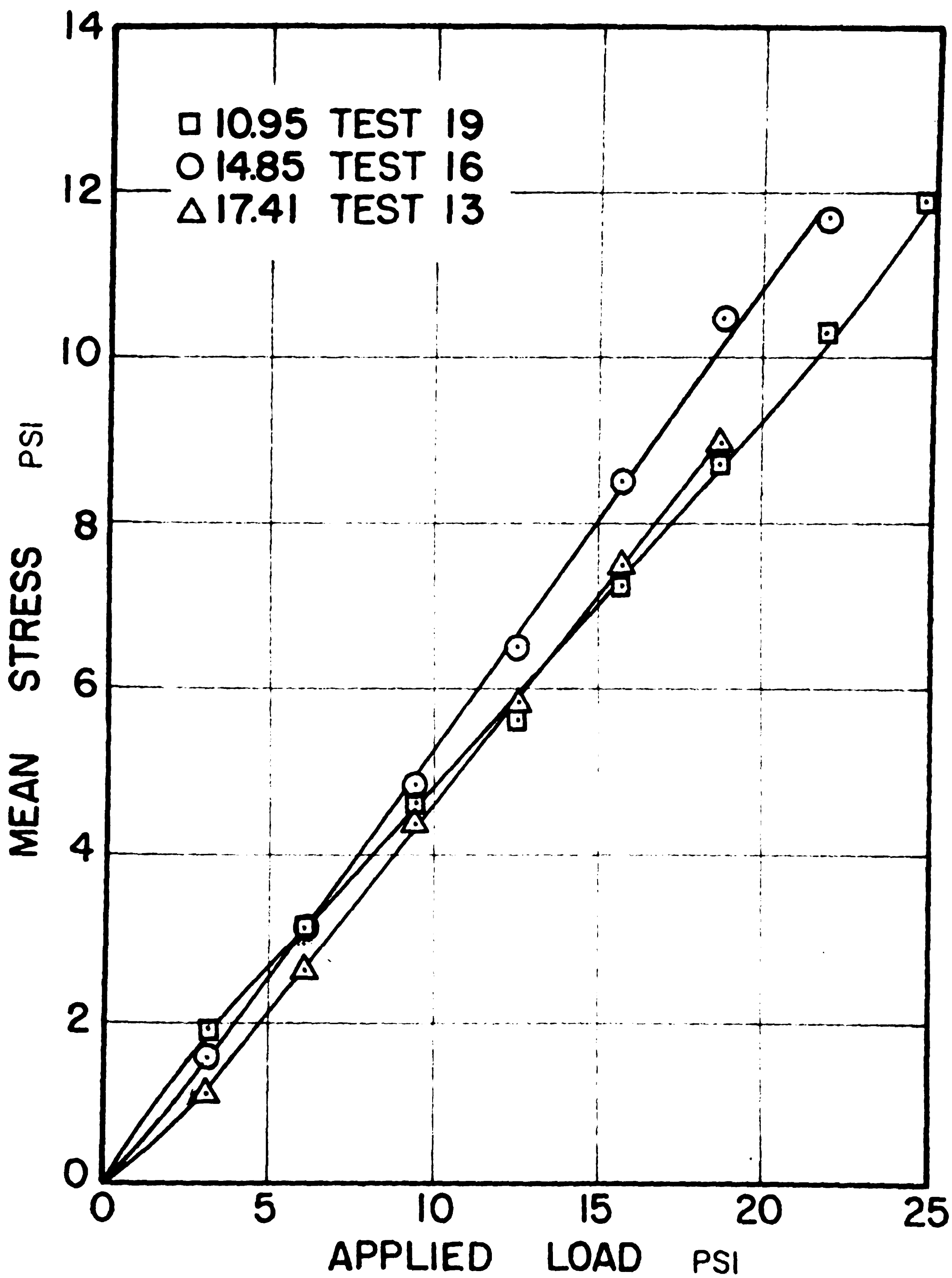

Figure 55. The Effect of Moisture Content on the Relationship between Mean Stress and Applied Load at 10 Inches Depth and 1.00 in/sec Rate of Loading. Data obtained with Type A Cells.

θ = polar coordinate

Q = force applied.

The average measured values of the vertical stress for the 5 replications at the 5, 10, and 15-inch depths at a moisture content of 14.34, 14.85 and 14.85 percent and the 1.00 inch per second rate of loading were plotted versus applied surface loads. Theoretical values were calculated from equation (30) using concentration factors to obtain curves corresponding to the measured values. The results are shown in Figures 56, 57 and 58.

The curves in Figure 56 are for the 5-inch depth. The theoretical curve was the maximum that could be obtained with any concentration factor. The value used was two. Both the measured values are greater than the theoretical. The differences may be due to the influence of the instruments on the stress pattern near the surface.

Results for the 10-inch depth are shown in Figure 57. With a concentration factor of six the theoretical and Type A values are in good agreement at the lower loads and only a 10.35 percent difference at the highest load. The 6 DST values are lower than the other two for loads less than 7.5 pounds per square inch and are increasingly greater with increasing load. The highest difference between the Type A and 6 DST values is 31.78 percent and 19.34 percent between the theoretical values and 6 DST values.

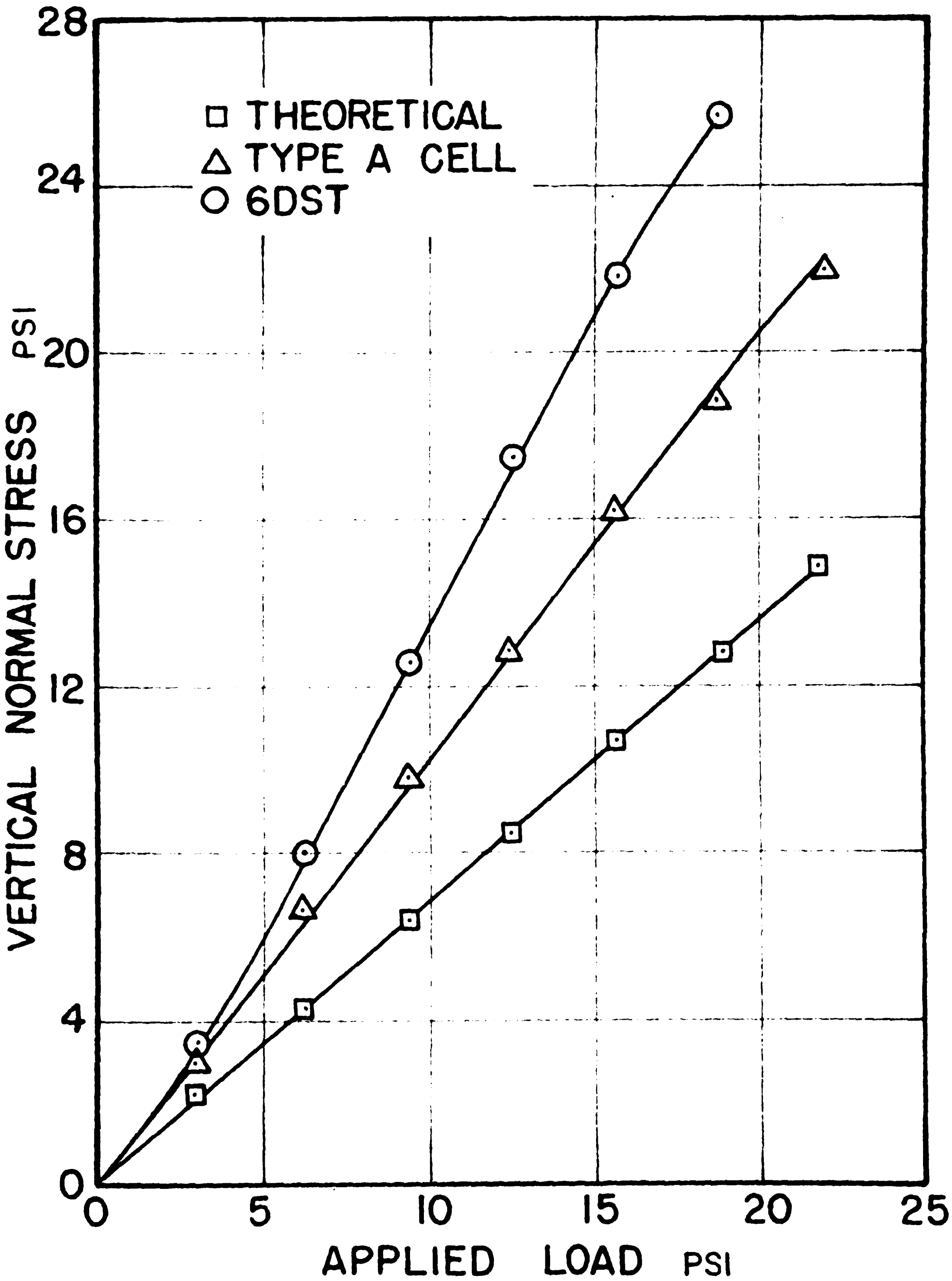

Figure 56. Comparison of the Two Instruments Used to Measure the Vertical Stress with Theoretical values at a Depth of 5 Inches below the Loading Surface. Measured Data obtained from Test 17.

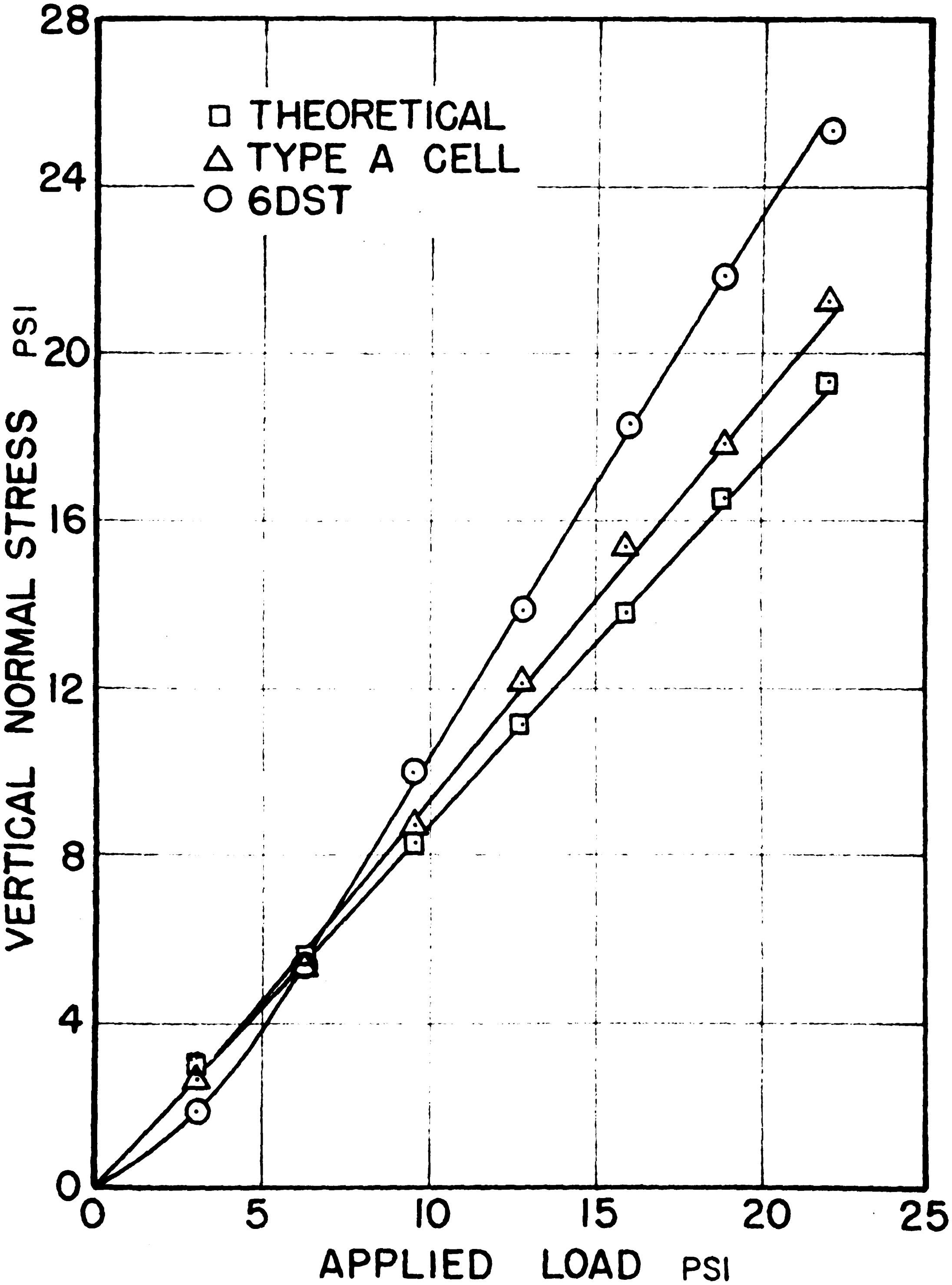

Figure 57 . Comparison of the Two Instruments Used to Measure the Vertical Stress with Theoretical Values at a Depth of 10 Inches below the Loading Surface. Measured Data obtained from Test 16.

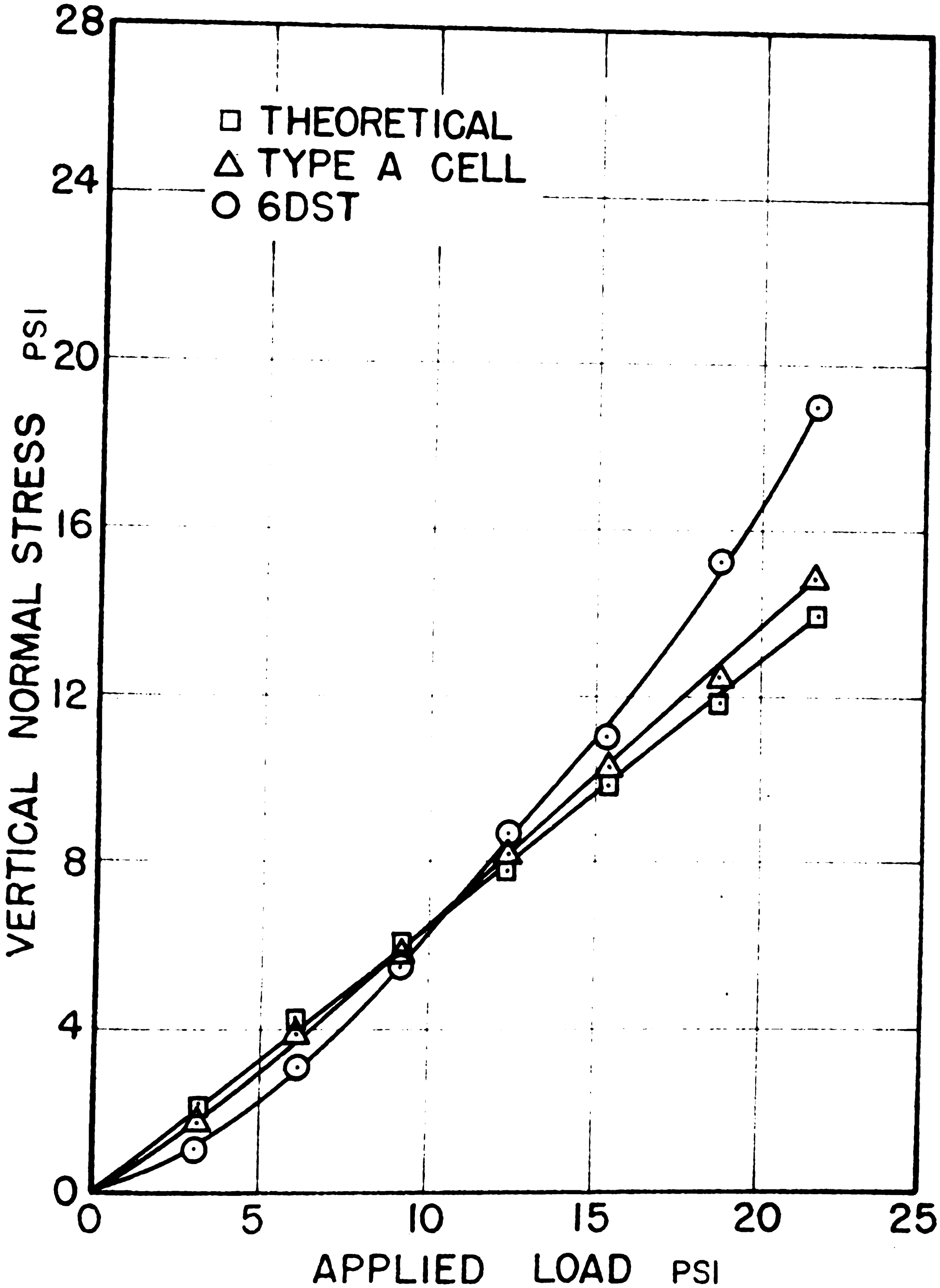

Figure 58. Comparison of the Two Instruments Used to Measure the Vertical Stress with Theoretical Values at a Depth of 15 Inches below the Loading Surface. Measured Data obtained from Test 15.

The curves for the 15-inch depth are shown in Figure 58. The theoretical and Type A curves again are in good agreement for a concentration factor of four. The maximum percent difference is only 7.75, which occured at the highest load of 21.6 pounds per square inch. The values obtained with the 6 DST are lower than the theoretical and Type A values for loads less than 12.0 pounds per square inch. Above 12.0 pounds per square inch the differences are greater with increased loads. The maximum difference between the Type A and 6 DST values is 27.55 percent as compared with 36.68 percent between the 6 DST and theoretical values.

On the basis of this data the conclusion that both instruments gave higher values than the theoretical values for loads greater than 12 pounds per square inch could be made. In addition the 6 DST gave measured values greater than the Type A values for loads greater than 12 pounds per square inch.

The differences between the theoretical values and measured values could be due to the differences in their volumes which may interfere with the stress distribution in the soil mass.

CONCLUSIONS

In the loose soil used for the experimental tests, data presented indicate the following.

1. The data obtained with the Six Directional Stress Transducer were more varied than that obtained with the Type A Cells.
2. The hypothesis that changes in bulk density are controlled by mean normal stress cannot be accepted or rejected.
3. The data obtained for both methods do not support completely the part of the hypothesis that the second invariant, maximum normal stress and maximum shear stress are not related to changes in bulk density.
4. The maximum shear stress was best related to changes in bulk density.
5. For the 0.38 inch per second rate of loading, a range of moisture content from 7.97 percent to 16.16 percent had no effect on the relationship between mean stress and bulk density.
6. The moisture content had an affect on the relationship between mean normal stress and bulk density for the 0.62 inch per second rate of loading at depths of 10 and 15 inches below the

loading surface and the 1.00 inch per second rate at the 15 inch depth.

7. The relationships between the invariants and bulk density at the higher rates of loading at the 15 inch depth were affected by the moisture content of the soil.
8. The mean stress-bulk density relationship was not affected by the rate of loading at the three depths below the loading surface.
9. The values of mean stress obtained directly with the W Cell compared best with the values calculated from the Type A data.
10. The lowest values of mean stress produced in the soil mass for a given applied load occured under the 1.00 inch per second rate of loading.
11. The vertical stresses measured with the Type A Cell were in good agreement with values calculated with Froehlick's equation at the 10- and 15-inch depths.
12. The relationships between the invariants and bulk density were exponential.

REFERENCES

Anonymous (1958). Soil compaction committee report. Joint American Society of Agricultural Engineers and Soil Science Society of American Committee Report. Agricultural Engineering 39:173-176.

Bekker, M. G.(1960). Mechanical properties of soil and compaction problems. Unpublished A.S.A.E. Paper No. 60-126. St. Joseph, Michigan.

Bekker, M. G.(1956). Theory of Land Locomotion, University of Michigan Press, Ann Arbor. 520 pp.

Cooper, Arthur W.(1956). Investigations of an instrumentation for measuring pressure distributions in soils. Thesis for degree of Ph.D., Michigan State Univ., East Lansing, (Unpublished).

Cooper, A. W., G. E. Vanden Berg and H. F. McColly (1957). Strain gage cell measure soil pressure. Agricultural Engineering 38:232-235, 246.

Gill, William R.(1959) Soil compaction by traffic. Agricultural Engineering 40:392-394, 400.

Hendrick, James G.(1960). Strain energy and tensile strength of a dynamically loaded clay soil. Thesis for degree of M.S., Auburn Univ., Auburn, Alabama (Unpublished).

Hough, B. K.(1957). Basic Soils Engineering. The Ronald Press Co., New York. 513 pp.

Hovanesian, J. F.(1958). Development and use of a volumetric transducer for studies of parameters upon soil compaction. Thesis for degree of Ph.D., Michigan State Univ., East Lansing (Unpublished).

Hovanesian, J. D., and W. F. Buchele (1959). Development of a recording volumetric transducer for studying effects of soil parameters on compaction. Transactions, A.S.A.E. 2:78-81.

Lask, Kay V.(1958). Instrumentation and measurement of soil-tire contact pressures. Thesis for degree of M.S., Michigan State Univ., East Lansing (Unpublished).

Lee, G. H.(1950). An Introduction to Experimental Stress Analysis. John Wiley and Sons, Inc., New York.

Malvern, L. E.(1957). Introduction to the mechanics of a continuous medium. Unpublished notes for graduate course in Applied Mechanics Dept., Michigan State Univ., East Lansing.

Murnaghan, J.(1951). Finite Deformation of an Elastic Solid. John Wiley and Sons, New York.

Perry, C. C. and H. R. Lissner (1955). The Strain Gage Primer. The Maple Press Co., York, Pa. 281 pp.

Popov, E. P.(1958). Mechanics of Materials. Prentice-Hall Civil Engineering and Engineering Mechanics Series, Englewood Cliffs, N. J.. 441 pp.

Reaves, C. A. and A. W. Cooper (1960). Stress distribution in soils under tractor loads. Agricultural Engineering 41:20-21, 31.

Reaves, C. A. and M. L. Nichols (1955). Surface soil reaction to pressure. Agricultural Engineering 36:813-816, 820.

Redshaw, S. C. (1954). A sensitive miniature pressure. Jour. of Sci. Inst. 31:467-469.

Reed, I. F., A. W. Cooper and C. A. Reaves (1959). Effects of two-wheel and tandem drives on traction and soil compacting stresses. Transactions, A.S.A.E. 2:22-25.

Rosenbach, Joseph B., Edwin A. Whitman and David Moskovitz (1943). Plane Trigonometry. Ginn and Company, New York. 144 pp.

Snedecor, George W. (1956). Statistical Methods. 5th ed. The Iowa State College Press, Ames. 534 pp.

Soehne, W.(1958). Fundamentals of pressure distribution and soil compaction under tractor tires. Agricultural Engineering 39:276-281, 290.

Soehne, W. H., W. J. Chancellor and R. H. Schmidt (1959). Soil deformation and compaction during piston sinkage. Unpublished A.S.A.E. Paper No. 59-100. St. Joseph, Michigan

Stong, Jack V.(1960). Basic factors affecting the strength and sinkage of tillable soils. Thesis for degree of M.S., Michigan State Univ., East Lansing (Unpublished).

Terzaghi, K.(1943). Theoretical Soil Mechanics. John Wiley and Sons, Inc., New York. 510 pp.

Timoshenko, S.(1936). Theory of Elastic Stability. McGraw Hill Book Co., New York. 518 pp.

Timoshenko, S. and J. N. Goodier (1951). Theory of Elasticity. McGraw Hill Book Co. 2nd ed. New York.

Trabbic, Gerald W.(1959). The effect of drawbar load and tire inflation on soil-tire interface pressure. Thesis for degree of M.S., Michigan State Univ., East Lansing (Unpublished).

Trabbic, G. W., K. V. Lask and W. R. Buchele (1959). Measurement of soil-tire interface pressures. Agricultural Engineering 40:678-681.

Vanden Berg, Glen E.(1958). Application of continuum mechanics to compaction in tillable soils. Thesis for degree of Ph.D., Michigan State Univ., East Lansing (Unpublished).

Vanden Berg, G. E.(1960). Requirements for a soil mechanics. Unpublished A.S.A.E. Paper No. 60-127. St. Joseph, Michigan.

Vanden Berg, G. E. and W. R. Gill (1959). Pressure distribution between a smooth tire and soil. Unpublished A.S.A.E. Paper No. 59-108. St. Joseph, Michigan.

Weaver, H. A. and V. C. Jamison (1951). Effects of moisture on tractor tire compaction of soil. Soil Science, 71:15-23.

Willits, Nathan A. (1956). Measurement of pressures in soils produced by traffic and the relationship between those pressure and compaction in undisturbed soils. Thesis for degree of Ph.D., Michigan State Univ., East Lansing (unpublished).

Wilson, S. D.(1952). Effect of compaction on soil properties. Proc. of the M.I.T. Conference on Soil Stabilization, 84-161 pp.

APPENDIX

TABLE A

CALCULATED VALUES OF MEAN NORMAL STRESS, SECOND INVARIANT OF DEVIATOR STRESS TENSOR, MAXIMUM AND MINIMUM PRINCIPAL STRESSES, AND MAXIMUM SHEAR STRESS ROR THE TYPE A CELLS AND THE SIX DIRECTIONAL STRESS TRANSDUCER

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)
							TEST 1						
Rep 1													
3.1	1.21	1.6	1.5	3.0	1.0	0.8	1.1	0.9	1.4	2.2	0.6	-0.1	1.1
6.2	1.24	2.9	3.6	5.1	1.9	1.7	1.7	2.4	11.5	6.0	2.2	-0.8	3.4
9.2	1.26	5.1	11.6	9.0	3.3	3.0	3.0	4.1	32.6	10.2	3.3	-1.1	5.7
12.3	1.27	6.9	18.5	11.9	4.5	4.4	3.7	6.8	83.7	16.8	4.7	-1.1	9.0
15.4	1.28	8.2	30.4	14.6	5.5	4.6	5.0	9.1	129.8	21.9	5.5	-1.1	11.0
18.5	1.28	9.2	39.4	16.4	6.0	5.2	5.6	10.0	157.7	24.0	6.2	-0.2	12.1
21.6	1.28	10.3	54.5	18.8	6.8	5.3	6.7	11.0	182.3	26.1	7.0	0.0	13.0
Rep 2													
3.1	1.20	1.8	1.8	3.4	1.2	0.8	1.3	0.8	1.0	1.9	.7	-0.1	1.0
6.2	1.24	3.2	5.3	5.9	2.2	1.6	2.1	2.4	7.0	5.2	2.0	0.0	2.6
9.2	1.26	5.4	14.5	9.7	3.6	2.8	3.5	4.1	22.3	9.4	2.4	0.4	4.5
12.3	1.27	6.8	23.0	12.2	4.8	3.3	4.5	6.4	65.1	15.6	3.2	0.5	7.6
15.4	1.28	8.5	42.3	15.8	6.2	3.5	6.2	8.7	109.3	20.7	4.1	1.3	9.7
18.5	1.28	9.5	50.8	17.6	6.8	4.2	6.7	9.6	125.6	23.4	5.0	1.4	10.5
21.6	1.28	10.6	61.4	19.6	7.3	5.0	7.3	10.8	162.9	25.2	6.1	1.0	12.1
Rep 3													
3.1	1.20	1.7	1.6	3.1	1.0	0.9	1.1	0.9	1.2	2.1	0.6	-0.1	1.1
6.2	1.23	3.1	5.1	5.7	2.0	1.6	2.0	2.3	7.0	5.0	2.0	-0.3	2.6
9.2	1.26	4.9	12.6	9.0	3.1	2.5	3.2	4.0	22.6	9.3	2.4	0.2	4.6
12.3	1.27	6.3	22.8	11.8	4.2	2.9	4.4	6.5	72.3	16.1	3.4	-0.1	8.1
15.4	1.28	8.4	37.9	15.5	5.5	4.2	5.6	8.9	118.5	21.3	4.9	0.7	10.3
18.5	1.28	9.3	47.4	17.3	5.9	4.8	6.2	10.0	136.2	23.2	5.8	1.0	11.1
21.6	1.28	10.6	55.2	19.2	6.8	6.0	6.7	11.1	167.6	25.7	6.4	1.2	12.3

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	6 DST σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 4													
3.1	1.20	1.8	1.6	3.3	1.2	1.0	1.2	0.9	0.9	1.9	0.7	0.1	0.9
6.2	1.24	3.3	4.7	5.8	2.2	1.9	2.0	2.4	7.6	5.4	1.8	0.0	2.7
9.2	1.26	5.4	13.1	9.5	3.5	3.1	3.2	4.5	33.1	10.9	2.8	-0.3	5.6
12.3	1.27	6.9	19.7	12.0	4.5	4.2	3.9	6.9	84.9	17.2	4.2	-0.7	8.9
15.4	1.28	8.6	28.8	14.8	5.6	5.3	4.7	8.7	114.7	20.8	5.0	0.3	10.2
18.5	1.28	9.4	36.6	16.4	5.9	5.8	5.3	10.1	160.0	24.4	5.6	0.4	12.0
21.6	1.28	10.6	48.0	18.6	6.9	6.4	6.1	11.4	187.2	26.9	6.3	1.0	12.9
TEST 2													
Rep 1													
3.1	1.23	2.5	3.1	4.5	1.8	1.2	1.7	2.0	6.2	4.7	1.3	-0.1	2.4
6.2	1.26	4.9	11.2	8.7	3.5	2.5	3.1	5.5	77.1	15.7	1.1	-0.1	7.9
9.2	1.27	7.0	24.0	12.7	4.6	3.8	4.4	6.9	83.5	17.4	2.7	0.6	8.4
12.3	1.28	8.7	38.6	15.8	5.7	4.5	5.7	9.1	139.4	22.5	4.1	0.5	11.0
15.4	1.28	10.6	54.3	19.1	6.7	6.0	6.5	11.1	171.8	26.0	6.0	1.3	12.3
18.5	1.28	12.1	66.0	21.5	7.7	7.2	7.2	12.7	178.3	27.7	8.0	2.2	12.8
21.6	1.29	13.8	84.0	24.4	8.6	8.5	8.0	14.0	175.3	30.0	9.1	3.9	12.5
Rep 2													
3.1	1.21	2.2	3.0	4.2	1.5	1.0	1.6	2.0	6.3	4.8	1.1	0.0	2.4
6.2	1.26	4.9	12.0	8.8	3.5	2.3	3.3	5.4	71.0	15.0	1.7	-0.5	7.8
9.2	1.27	7.2	26.1	13.0	5.0	3.5	4.7	7.4	101.8	19.0	2.9	0.3	9.3
12.3	1.28	8.6	43.1	16.1	5.6	4.0	6.0	9.5	158.4	24.0	3.2	1.3	11.3
15.4	1.28	10.0	60.4	18.9	6.7	4.5	7.2	11.4	189.6	27.2	4.9	2.1	12.6
18.5	1.28	11.6	72.5	21.4	7.3	6.2	7.6	12.9	185.5	28.4	7.3	3.0	12.7
21.6	1.29	13.4	87.3	24.2	8.2	7.8	8.2	14.1	178.0	29.3	8.7	4.3	12.5

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 3													
3.1	1.10	2.1	2.7	4.0	1.4	1.0	1.5	1.9	4.6	4.3	0.7	0.6	1.9
6.2	1.23	4.5	12.2	8.5	3.2	1.9	3.3	5.3	55.3	13.9	1.2	0.8	6.6
9.2	1.25	6.5	25.5	12.3	4.4	2.8	4.7	7.1	79.8	17.4	2.6	1.3	8.0
12.3	1.26	8.3	38.3	15.4	5.7	3.8	5.8	9.0	116.1	21.4	3.2	2.3	9.5
15.4	1.27	10.2	57.1	18.8	6.9	4.8	7.0	10.6	147.1	24.5	5.3	2.1	11.2
18.5	1.28	11.7	69.8	21.3	7.8	6.0	7.6	12.6	159.4	27.0	7.3	3.4	11.8
21.6	1.28	13.1	85.2	23.8	8.0	7.5	8.1	13.9	160.0	28.2	9.4	4.2	12.0
Rep 4													
3.1	1.22	2.5	3.2	4.5	1.7	1.2	1.7	2.0	5.5	4.6	1.3	0.1	2.3
6.2	1.26	4.9	11.6	8.8	3.5	2.5	3.2	5.6	50.4	13.8	1.9	1.0	6.4
9.2	1.27	7.2	27.1	13.1	4.9	3.5	4.8	7.6	74.5	17.4	4.1	1.2	8.1
12.3	1.28	8.8	42.1	16.2	5.6	4.5	5.9	10.1	122.0	22.6	5.8	1.8	10.4
15.4	1.28	10.3	62.0	19.4	6.0	5.5	6.9	11.8	142.4	25.3	7.6	2.6	11.4
18.5	1.29	12.0	76.4	22.1	7.1	6.8	7.6	13.5	276.3	28.5	8.6	3.4	12.6
21.6	1.29	13.5	95.0	24.7	8.0	7.7	8.5	14.4	176.9	29.3	10.1	3.8	12.8
Rep 5													
3.1	1.22	2.2	2.6	4.0	1.5	1.0	1.5	2.0	7.8	5.1	1.0	-0.2	2.7
6.2	1.25	4.7	13.2	8.8	3.2	2.0	3.4	4.9	46.7	12.7	1.8	0.2	6.3
9.2	1.27	6.9	27.8	12.5	4.8	3.5	4.5	7.1	87.8	17.9	1.9	0.6	8.6
12.3	1.28	8.4	36.2	15.3	5.8	4.2	5.6	9.3	141.4	22.9	4.2	0.8	11.0
14.4	1.28	10.2	48.4	18.2	6.8	5.5	6.3	11.3	176.6	26.5	5.8	1.7	12.4
18.5	1.28	11.4	57.5	20.1	7.2	6.8	6.7	12.8	203.0	28.9	7.8	1.8	13.6
21.6	1.28	13.1	83.0	23.6	7.9	7.8	7.9	13.9	179.4	29.1	9.2	3.6	12.7

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
TEST 3													
Rep 1													
3.1	1.15	1.5	0.6	2.3	1.2	0.9	0.7	1.3	4.4	3.7	0.1	0.0	1.8
6.2	1.19	3.3	4.6	5.8	2.2	2.0	1.9	2.8	2.1	8.1	0.3	0.0	4.0
9.2	1.21	5.3	9.8	8.9	3.5	3.4	2.7	4.4	42.9	12.0	1.1	0.2	5.9
12.3	1.23	6.7	17.8	11.6	4.3	4.2	3.7	5.7	67.6	15.2	1.5	0.4	7.3
15.4	1.24	8.0	22.6	13.5	5.3	5.3	4.1	6.8	78.8	17.0	2.5	0.9	8.0
18.5	1.25	9.5	32.0	16.0	6.5	6.0	5.0	8.4	128.6	21.5	3.1	0.7	10.4
21.6	1.25	10.8	41.1	18.2	7.2	7.0	5.6	10.0	193.2	25.9	3.6	0.4	12.8
24.7	1.26	11.7	42.0	19.2	8.0	8.0	5.6	11.5	268.2	30.2	4.2	0.0	15.1
Rep 2													
3.1	1.15	1.6	1.5	2.0	1.2	0.7	1.2	1.2	3.9	3.4	0.3	-0.2	1.8
6.2	1.22	3.4	5.1	6.0	2.2	2.1	2.0	2.5	15.2	6.9	1.0	-0.4	3.7
9.2	1.23	5.3	10.7	9.0	3.5	3.3	2.9	4.0	43.0	11.5	1.4	-0.9	6.2
12.3	1.24	6.9	18.3	11.8	4.6	4.2	3.8	5.3	68.8	14.8	1.8	-0.6	7.7
15.4	1.24	8.3	26.5	14.2	5.6	5.0	4.6	6.5	96.8	17.8	2.3	-0.5	9.1
18.5	1.25	9.8	37.3	16.8	6.5	6.0	5.4	8.5	156.1	22.9	2.6	0.1	11.4
21.6	1.25	11.2	49.8	19.3	7.3	6.8	6.3	10.0	220.0	27.0	3.0	-0.1	13.5
24.7	1.25	12.3	55.1	20.9	8.4	7.6	6.6	11.4	305.0	31.5	2.7	0.0	15.7
Rep 3													
3.1	1.14	1.5	1.0	2.6	1.0	0.8	0.9	1.4	5.5	4.1	0.1	-0.1	0.2
6.2	1.19	3.0	3.6	5.2	2.0	1.8	1.7	2.9	22.6	8.4	0.3	0.0	4.2
9.2	1.21	4.5	5.6	7.2	3.2	3.0	2.1	4.8	52.1	13.7	0.8	0.0	6.8
12.3	1.22	5.7	9.0	9.2	4.0	4.0	2.6	6.6	101.7	18.2	2.1	-0.4	9.3
15.4	1.23	7.2	14.3	11.5	5.2	4.8	3.4	7.4	112.3	20.0	2.2	0.4	9.6
18.5	1.24	8.8	24.5	14.5	6.5	5.5	4.5	9.1	176.3	24.4	2.4	0.6	11.9
21.6	1.24	10.3	37.4	17.3	7.3	6.2	5.5	10.8	243.2	28.7	3.4	0.3	14.2
24.7	1.25	11.6	50.8	19.8	8.0	7.0	6.4	12.2	298.8	32.0	4.3	0.2	15.9

TABLE 3 A (CONTINUED)

Load	Bulk Density	Type A Cell σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	6 DST σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 4													
3.1	1.19	1.6	0.5	2.3	1.4	1.0	0.7	1.3	4.4	3.7	0.2	0.0	1.9
6.2	1.21	3.2	4.4	5.6	2.1	1.9	1.9	2.8	22.9	8.4	0.3	-0.2	4.3
9.2	1.23	5.2	8.2	8.5	3.8	3.3	8.6	4.7	60.2	13.6	0.6	-0.2	6.9
12.3	1.23	6.6	13.5	10.8	4.5	4.4	3.2	6.3	106.1	18.2	1.1	-0.4	9.3
15.4	1.24	7.9	18.9	12.9	5.8	5.0	3.9	7.6	163.6	22.4	1.1	-0.5	11.4
18.5	1.24	9.2	19.4	14.3	7.0	6.4	4.0	9.3	238.4	27.0	1.6	-0.8	13.9
21.6	1.24	10.7	30.1	17.0	8.0	7.1	4.9	10.5	289.1	30.1	2.1	-0.6	15.3
24.7	1.25	12.2	46.0	20.0	8.6	8.0	6.0	11.5	315.2	31.9	2.5	-0.1	16.0
Rep 5													
3.1	1.14	1.6	0.9	2.5	1.3	1.0	0.8	1.1	3.4	3.2	0.1	-0.1	1.6
6.2	1.18	3.2	3.9	5.5	2.2	2.0	1.8	2.4	16.5	7.1	0.3	-0.1	3.6
9.2	1.20	4.8	8.8	8.2	3.1	3.0	2.6	4.4	56.6	13.1	0.5	-0.3	6.7
12.3	1.23	6.3	15.0	10.8	4.2	4.0	3.4	5.4	79.3	15.6	1.0	-0.4	8.0
15.4	1.24	7.9	20.4	13.5	5.4	5.2	4.0	6.9	130.4	20.1	1.2	-0.4	10.3
18.5	1.24	9.2	27.1	15.2	6.5	6.0	4.6	8.3	180.8	23.7	1.6	-0.5	12.1
21.6	1.25	10.9	43.6	18.5	7.5	6.7	5.9	9.4	233.3	27.0	2.0	-0.7	13.8
24.7	1.25	11.8	53.9	20.3	7.8	7.3	6.5	10.7	274.7	29.8	2.5	-0.2	15.0
TEST 4													
Rep 1													
3.1	1.20	1.6	1.8	3.2	1.0	0.7	1.2	1.4	2.9	3.3	0.8	0.1	1.6
6.2	1.22	3.4	5.0	6.0	2.4	1.9	2.1	3.4	15.8	8.0	1.6	0.7	3.6
9.2	1.23	5.0	10.9	8.8	3.5	2.8	3.0	5.3	34.5	12.1	2.8	1.2	5.4
12.3	1.24	6.6	17.8	11.4	4.8	3.6	3.9	7.6	68.0	17.0	3.9	1.8	7.6
15.4	1.24	8.2	27.5	14.2	6.0	4.4	4.9	10.0	108.8	21.7	5.1	2.5	9.6

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 2													
3.1	1.19	1.7	1.5	3.1	1.2	0.8	1.1	1.5	3.3	3.6	0.9	0.1	1.7
6.2	1.22	3.4	4.6	5.8	2.5	1.8	2.0	3.4	13.3	7.5	2.1	0.5	3.5
9.2	1.23	5.1	10.5	8.8	3.7	2.8	3.0	5.4	35.8	12.3	2.8	1.2	5.5
12.3	1.24	6.6	16.3	11.2	4.9	3.6	3.8	7.4	62.1	16.5	3.5	2.2	7.1
15.4	1.24	8.2	25.3	13.9	6.2	4.4	4.7	9.8	113.3	22.1	4.5	2.9	10.0
Rep 3													
3.1	1.17	1.5	0.7	2.5	1.1	1.0	0.8	1.3	2.6	3.2	0.5	0.3	1.4
6.2	1.20	3.4	4.0	5.7	2.5	2.0	1.9	3.5	13.2	7.7	1.9	1.0	3.3
9.2	1.22	5.0	9.1	8.5	3.5	3.1	2.7	5.5	33.1	12.1	3.0	1.4	5.3
12.3	1.23	6.6	12.8	10.7	5.0	4.1	3.3	7.4	60.3	16.3	3.9	2.0	7.2
15.4	1.24	8.3	23.3	13.8	6.5	4.6	4.6	9.4	113.1	22.1	5.0	2.5	9.8
Rep 4													
3.1	1.18	1.7	1.4	3.0	1.2	0.8	1.1	1.3	1.9	2.3	0.9	0.2	1.3
6.2	1.21	3.4	5.2	6.0	2.5	1.8	2.1	3.2	16.3	7.9	1.1	0.7	3.6
9.2	1.22	5.1	12.1	9.1	3.5	2.6	3.2	5.0	33.2	11.6	2.3	1.1	5.3
12.3	1.23	6.5	22.1	11.8	4.8	2.8	4.5	7.3	67.6	16.7	3.4	1.7	7.5
15.4	1.24	8.0	32.3	14.4	6.0	3.6	5.4	9.3	96.5	20.6	4.6	2.8	8.9
TEST 5													
Rep 1													
3.1	1.21	1.9	2.6	3.8	1.2	0.8	1.5	1.3	2.1	2.9	0.8	0.2	1.4
6.2	1.25	3.6	7.5	6.8	2.2	1.8	2.5	3.8	10.6	7.5	2.7	1.3	3.1
9.2	1.26	5.3	16.8	10.0	3.0	2.8	3.6	5.9	29.9	12.1	3.6	1.9	5.1
12.3	1.27	6.9	30.3	13.2	3.8	3.6	4.8	8.5	64.9	17.7	5.3	2.5	7.6
15.4	1.28	8.3	40.2	15.6	4.8	4.5	5.6	10.6	98.1	21.9	6.7	3.2	9.3
18.5	1.29	9.7	60.8	18.7	5.5	5.0	6.9	13.0	151.7	26.9	8.4	3.6	11.7

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 2													
3.1	1.22	2.0	2.3	3.7	1.2	1.0	1.4	1.5	2.7	3.4	0.7	0.4	1.5
6.2	1.26	3.6	7.8	6.8	2.2	1.8	2.5	3.9	11.9	7.8	2.7	1.3	3.3
9.2	1.27	5.3	18.4	10.2	3.2	2.5	3.9	6.3	32.8	12.8	3.7	2.3	5.3
12.3	1.28	6.9	33.9	13.6	3.9	3.2	5.2	8.7	70.8	18.3	5.2	2.6	7.8
15.4	1.28	8.3	42 0	15.8	4.6	4.5	5.6	10.3	95.0	21.4	6.8	2.9	9.2
18.5	1.29	9.8	64.1	19.0	5.2	5.2	7.0	12.6	150.0	26.4	8.1	3.3	11.6
Rep 3													
3.1	1.21	2.1	1.7	3.5	1.7	1.0	1.3	1.4	3.0	3.4	0.5	0.5	1.5
6.2	1.24	3.7	6.1	6.5	2.5	2.0	2.3	3.8	13.4	8.0	2.3	1.2	3.4
9.2	1.26	5.5	14.9	10.0	3.5	3.1	3.4	6.1	37.0	13.0	3.4	1.7	5.6
12.3	1.27	7.2	27.9	13.3	4.2	4.0	4.6	8.3	68.7	17.8	4.5	2.7	7.6
15.4	1.28	8.4	39.1	15.6	5.0	4.6	5.5	10.2	91.2	21.1	6.0	3.5	8.8
18.5	1.29	10.0	60.8	19.0	5.8	5.1	6.9	12.5	144.2	26.2	7.5	3.8	11.2
Rep 4													
3.1	1.21	1.9	2.0	3.5	1.1	1.0	1.3	1.5	2.0	3.1	1.0	0.4	1.3
6.2	1.24	3.5	8.5	6.8	2.0	1.6	2.6	3.7	12.9	7.8	2.1	1.2	3.3
9.2	1.26	5.1	15.6	9.7	3.0	2.6	3.5	5.9	31.1	12.3	3.4	2.0	5.1
12.3	1.27	6.7	31.6	13.2	3.8	3.2	5.0	8.3	64.8	17.5	4.8	2.7	7.4
15.4	1.27	8.0	44.9	15.7	4.9	3.5	6.1	10.7	89.9	21.6	6.6	4.1	8.8
18.5	1.28	9.7	65.3	19.0	5.7	4.5	7.3	12.9	139.1	26.4	7.9	4.5	11.0
Rep 5													
3.1	1.21	1.9	3.5	4.1	0.9	0.8	1.6	1.5	2.6	3.2	1.1	0.1	1.6
6.2	1.24	3.4	8.9	6.9	1.9	1.5	2.7	3.6	13.0	7.6	2.6	0.6	3.5
9.2	1.26	5.1	18.8	10.1	2.7	2.5	3.8	6.1	35.6	12.8	4.2	1.4	5.7
12.3	1.27	6.8	36.4	13.8	3.8	2.9	5.4	8.5	66.9	17.8	5.5	2.3	7.7
15.4	1.28	8.1	48.5	16.1	4.5	3.7	6.2	10.4	102.5	21.9	6.8	2.6	9.6
18.5	1.28	9.6	73.2	19.5	5.3	4.1	7.7	12.8	149.1	26.6	8.4	3.4	11.6

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
TEST 6													
Rep 1													
3.1	1.19	1.2	0.6	2.0	1.0	0.5	0.8	1.4	2.4	3.3	1.1	0.2	1.5
6.2	1.21	2.3	2.6	4.1	2.0	0.9	1.6	2.9	9.1	6.4	1.8	0.6	2.9
9.2	1.23	3.7	6.5	6.5	3.1	1.5	2.5	4.6	21.0	9.7	2.9	1.1	4.3
12.3	1.24	4.0	10.6	8.4	4.2	2.1	3.2	6.2	39.2	13.3	3.7	1.5	5.9
15.4	1.25	6.0	16.1	10.4	5.0	2.6	3.9	7.6	63.7	16.8	3.9	2.2	7.3
18.5	1.25	7.1	22.8	12.3	6.0	2.9	4.7	9.2	89.1	20.1	4.6	2.9	8.6
21.6	1.26	8.3	29.5	14.3	7.2	3.6	5.4	10.9	130.7	24.0	5.5	3.1	10.4
24.7	1.26	9.5	41.7	16.4	8.3	3.7	6.4	12.7	120.2	28.5	6.2	3.4	12.6
Rep 2													
3.1	1.17	1.3	0.6	2.1	1.2	0.6	0.8	1.4	1.9	3.0	0.8	0.4	1.3
6.2	1.20	2.7	2.1	4.3	2.5	1.3	1.5	2.9	10.2	6.5	1.5	0.6	3.0
9.2	1.21	4.0	5.1	6.4	3.7	2.0	2.2	4.5	25.4	10.3	2.1	1.1	4.6
12.3	1.22	5.3	9.5	8.7	4.8	2.6	3.1	6.0	43.7	13.5	2.9	1.4	6.1
15.4	1.23	6.4	15.1	10.6	5.7	2.9	3.8	7.6	69.4	17.2	3.9	1.8	7.7
18.5	1.24	7.6	23.4	12.8	6.5	3.4	4.7	9.4	111.7	21.6	4.5	2.2	9.7
21.6	1.25	8.7	32.3	14.8	7.5	3.7	5.6	11.1	157.6	25.5	5.3	2.5	11.5
24.7	1.25	9.6	40.2	16.5	8.5	3.9	6.3	12.6	206.6	29.1	5.8	2.8	13.1
Rep 3													
3.1	1.17	1.0	0.5	1.8	1.1	0.3	0.7	1.9	6.0	4.6	1.2	-0.2	2.4
6.2	1.20	2.2	2.5	3.8	2.1	0.7	1.6	3.0	14.1	7.4	1.1	0.6	3.4
9.2	1.21	3.4	6.3	6.0	3.1	1.0	2.5	4.5	26.6	10.4	1.8	1.2	4.6
12.3	1.23	4.5	12.4	8.2	4.0	1.3	3.5	6.1	51.9	14.4	2.3	1.6	6.4
15.4	1.24	5.2	18.8	10.5	5.2	2.0	4.3	7.8	90.7	18.7	3.0	1.6	8.6
18.5	1.25	6.9	27.9	12.6	6.0	2.2	5.2	9.3	118.6	21.9	4.0	2.1	10.0
21.6	1.25	8.0	36.7	14.4	7.1	2.4	6.0	10.4	133.2	23.7	4.5	3.0	10.4
24.7	1.26	9.1	43.4	16.2	8.0	3.1	6.5	12.0	181.8	27.5	5.0	3.4	12.1

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	6 DST σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 4													
3.1	1.17	1.6	1.3	2.9	1.2	0.8	1.1	1.5	2.5	3.4	0.7	0.5	1.4
6.2	1.20	2.9	3.9	5.1	2.2	1.3	1.9	3.3	13.7	7.5	1.9	0.6	3.5
9.2	1.21	4.2	8.9	7.6	3.1	2.0	2.8	5.5	45.2	13.2	2.5	0.8	6.2
12.3	1.23	5.4	14.3	9.7	4.0	2.6	3.6	7.1	79.5	17.3	3.2	0.9	8.2
15.4	1.24	6.8	22.6	12.2	5.0	3.2	4.5	8.9	123.0	21.6	3.8	1.3	10.2
18.5	1.25	7.7	27.7	13.6	5.9	3.5	5.0	10.2	151.7	24.2	4.9	1.4	11.4
21.6	1.25	8.7	36.1	15.5	6.8	3.9	5.7	11.8	201.9	28.0	5.4	1.9	13.1
24.7	1.26	9.4	42.6	16.7	7.7	4.0	6.4	13.2	251.7	31.3	6.3	1.9	14.7
Rep 5													
3.1	1.17	1.3	1.0	2.3	1.3	0.3	1.0	1.5	1.8	3.0	1.0	0.4	1.3
6.2	1.20	2.7	3.9	4.8	2.4	1.0	1.9	3.1	10.7	6.8	1.9	0.6	3.1
9.2	1.21	3.9	7.9	6.9	3.5	1.3	2.8	4.9	32.6	11.4	2.5	0.7	5.3
12.3	1.22	4.9	12.7	8.7	4.4	1.6	3.5	7.3	92.1	18.2	3.5	0.1	9.0
15.4	1.23	5.9	12.4	10.5	5.5	1.8	4.4	8.7	113.3	20.8	4.0	1.2	9.8
18.5	1.24	7.1	28.4	12.7	6.5	2.1	5.3	9.7	118.0	22.2	4.7	2.3	10.0
21.6	1.25	8.2	38.2	14.7	7.5	2.4	6.2	11.6	162.1	26.2	6.1	2.6	11.8
24.7	1.25	9.3	49.8	16.8	8.3	2.7	7.0	13.1	208.0	29.6	6.8	2.8	13.4
							TEST 7						
Rep 1													
3.1	1.15	1.6	1.1	2.8	1.0	0.9	1.0	1.1	0.8	2.1	0.9	0.4	0.8
6.2	1.19	3.3	4.6	5.8	2.2	2.0	1.9	2.8	3.8	5.0	1.8	1.5	1.7
9.2	1.21	5.3	12.0	9.3	3.6	3.1	3.1	5.1	20.6	10.3	2.9	2.1	4.1
12.3	1.22	7.2	21.5	12.6	4.9	4.2	4.2	7.5	53.6	15.9	3.7	2.9	6.5
15.4	1.22	9.1	29.9	15.4	6.4	5.5	4.9	9.9	88.0	20.7	5.1	3.8	8.4
18.5	1.23	10.9	49.3	19.0	7.0	6.8	6.1	12.4	138.0	25.9	6.3	5.0	10.5
21.6	1.23	12.3	73.0	22.2	7.5	7.3	7.5	14.2	171.7	29.4	7.1	6.3	11.5

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)
Rep 2													
3.1	1.14	1.4	1.2	2.6	0.8	0.8	0.9	1.0	1.2	2.3	0.5	0.3	1.0
6.2	1.16	2.9	4.6	5.4	1.9	1.5	1.9	2.9	4.0	5.1	2.1	1.3	1.9
9.2	1.19	4.7	12.5	8.7	3.0	2.3	3.2	5.0	23.2	10.5	2.6	1.9	4.3
12.3	1.21	6.8	21.6	12.1	4.5	3.7	4.2	7.2	52.7	15.6	3.5	2.5	6.5
15.4	1.22	8.9	34.2	15.6	6.1	4.9	5.3	9.7	96.7	21.1	4.4	3.7	8.7
18.5	1.23	10.2	45.5	18.0	6.3	6.2	5.9	12.2	151.6	26.4	5.7	4.5	11.0
21.6	1.23	12.0	70.5	21.7	7.2	7.0	7.3	14.0	184.5	29.7	6.7	5.7	12.0
Rep 3													
3.1	1.15	1.5	0.8	2.5	1.0	0.9	0.8	1.0	1.1	2.2	0.7	0.1	1.0
6.2	1.18	3.1	4.3	5.5	2.0	1.8	1.9	2.8	6.6	5.7	2.0	0.7	2.5
9.2	1.20	4.6	10.2	8.2	3.0	2.5	2.9	5.0	30.6	11.3	2.5	1.2	5.1
12.3	1.22	6.3	19.1	11.3	4.1	3.5	3.9	7.6	67.6	17.1	3.3	2.5	7.3
15.4	1.22	7.8	24.9	13.5	5.0	4.8	4.4	10.2	123.2	23.0	4.2	3.4	9.8
18.5	1.23	10.1	45.2	17.9	6.6	5.8	6.0	12.2	151.3	26.4	5.5	4.8	10.8
21.6	1.23	12.1	58.2	20.9	8.1	7.3	6.8	14.0	169.4	29.1	7.0	6.1	11.5
Rep 4													
3.1	1.15	1.8	1.6	3.2	1.1	1.0	1.1	1.0	1.0	2.1	0.8	0.2	0.9
6.2	1.19	3.6	7.7	6.8	2.0	2.0	2.4	2.6	3.7	4.8	1.8	1.2	1.8
9.2	1.21	5.3	16.7	10.0	3.1	2.8	3.6	4.9	24.1	10.6	2.4	1.8	4.4
12.3	1.22	7.2	34.1	13.9	4.0	3.7	5.1	7.2	54.0	15.6	3.0	2.1	6.8
15.4	1.23	8.8	46.5	16.6	5.2	4.5	6.1	9.5	107.4	21.4	4.1	2.9	9.3
18.5	1.23	10.3	65.3	19.7	6.0	5.3	7.2	12.3	172.5	27.4	4.8	4.6	11.4
21.6	1.23	12.3	78.1	22.5	7.3	7.0	7.7	14.1	187.3	30.0	6.5	5.8	12.0

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 5													
3.1	1.16	1.6	1.5	3.0	1.0	0.8	1.1	1.2	1.4	2.5	0.7	0.2	1.1
6.2	1.20	3.2	6.1	6.0	2.0	1.5	2.3	3.0	7.4	6.1	1.5	1.4	2.4
9.2	1.21	4.8	15.2	9.3	2.9	2.3	3.5	4.7	18.7	9.7	2.3	2.1	3.8
12.3	1.21	6.7	24.2	12.4	4.1	3.6	4.4	7.3	39.7	14.6	3.8	3.5	5.5
15.4	1.22	8.5	36.4	15.5	5.0	5.0	5.2	9.3	66.2	18.7	4.7	4.5	7.1
18.5	1.22	10.6	52.8	19.0	6.4	6.3	6.3	12.2	144.6	26.1	5.4	5.1	10.5
21.6	1.23	12.4	73.5	22.3	7.5	7.4	7.4	14.1	164.2	28.9	7.1	6.4	11.3
TEST 8													
Rep 1													
3.1	1.16	0.9	0.7	1.7	0.8	0.1	0.8	0.6	1.3	1.8	0.5	-0.4	1.1
6.2	1.18	1.9	1.8	3.4	1.5	0.8	1.3	1.7	4.3	3.8	1.5	-0.3	2.1
9.2	1.19	3.1	4.0	5.4	2.2	1.7	1.8	3.1	12.8	7.1	2.1	0.1	3.5
12.3	1.19	4.3	6.9	7.3	3.2	2.4	2.4	4.7	30.3	10.9	2.5	0.6	5.2
15.4	1.19	5.4	12.8	9.5	4.0	2.8	3.4	6.2	56.4	14.8	2.8	1.0	6.0
18.5	1.20	6.8	18.0	11.6	5.1	3.6	4.0	8.2	102.7	29.8	3.3	1.4	9.2
21.6	1.20	7.9	23.7	13.4	6.0	4.3	4.6	10.0	145.1	23.8	4.0	2.1	10.9
24.7	1.20	9.2	30.6	15.5	6.9	5.2	5.2	12.0	213.9	28.8	4.1	3.0	12.9
Rep 2													
3.1	1.13	0.7	0.2	1.2	0.6	0.3	0.5	0.7	0.7	1.6	0.4	0.0	0.8
6.2	1.15	2.0	1.2	3.2	1.5	1.2	1.0	1.6	2.3	3.3	1.2	0.4	1.5
9.2	1.18	3.0	3.2	5.0	2.3	1.7	1.7	3.3	13.5	7.5	1.4	0.9	3.3
12.3	1.20	4.2	6.3	7.0	3.0	2.5	2.3	4.7	30.5	11.1	1.7	1.3	4.9
15.4	1.20	5.4	11.6	9.3	4.0	3.0	3.2	6.2	56.9	14.9	2.1	1.6	6.7
18.5	1.21	6.6	15.9	11.1	4.8	3.8	3.7	8.2	111.1	20.4	2.7	1.6	9.4
21.6	1.21	8.7	28.0	14.7	6.5	4.8	5.0	10.2	146.0	24.1	4.0	2.5	10.8
24.7	1.21	9.8	36.1	16.7	7.2	5.6	5.6	12.1	215.8	29.0	4.3	2.9	13.1

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 3													
3.1	1.15	0.9	0.2	1.4	0.7	0.6	0.4	0.8	1.4	2.0	0.6	-0.3	1.2
6.2	1.19	2.0	2.0	3.7	1.3	1.1	1.3	1.9	6.5	4.7	1.1	-0.1	2.5
9.2	1.21	3.1	5.1	5.7	1.9	1.8	2.0	3.1	18.3	7.8	2.4	-0.7	4.2
12.3	1.22	4.5	10.0	8.1	2.8	2.5	2.8	4.9	37.1	11.8	2.5	0.3	5.7
15.4	1.22	5.7	14.3	10.1	3.9	3.2	3.4	6.3	62.1	15.3	3.0	0.6	7.3
18.5	1.22	7.1	21.8	12.5	4.7	4.1	4.2	8.5	113.0	20.7	3.3	1.4	9.7
21.6	1.22	8.6	29.5	14.9	6.2	4.8	5.0	10.1	140.8	23.7	4.1	2.3	10.7
24.7	1.22	9.8	33.0	16.4	7.0	6.0	5.2	12.0	199.3	28.3	4.2	3.4	12.4
Rep 4													
3.1	1.13	1.0	0.3	1.6	0.7	0.7	0.5	0.6	1.1	1.8	0.3	-0.2	1.0
6.2	1.20	2.0	1.7	3.5	1.3	1.2	1.2	1.4	2.1	3.0	1.0	0.2	1.4
9.2	1.22	3.0	4.6	5.5	1.8	1.8	1.9	3.3	15.0	7.8	1.2	1.0	3.4
12.3	1.22	3.8	7.1	6.9	2.3	2.2	2.3	4.9	32.1	11.5	1.9	1.4	5.0
15.4	1.23	4.9	10.9	8.7	3.4	2.6	3.0	6.6	63.9	15.8	2.2	1.7	7.1
18.5	1.23	6.1	18.1	11.0	4.2	3.1	3.9	8.3	102.8	20.0	2.7	2.1	8.9
21.6	1.23	7.6	24.9	13.2	5.8	3.8	4.7	10.2	161.4	24.8	3.1	2.4	11.2
24.7	1.23	9.0	33.7	15.7	6.5	4.9	5.4	12.2	231.5	29.7	4.1	2.7	13.5
Rep 5													
3.1	1.13	0.8	0.4	1.5	0.5	0.5	0.5	0.2	2.3	1.8	0.2	-0.1	1.5
6.2	1.15	2.2	1.3	3.5	1.7	1.4	1.1	1.6	3.3	3.6	1.2	0.1	1.7
9.2	1.21	3.2	4.0	5.5	2.1	2.0	1.8	3.4	14.4	7.8	1.6	0.9	3.4
12.3	1.22	4.6	8.8	8.0	3.0	3.0	2.6	4.9	30.8	11.3	2.1	1.3	5.0
15.4	1.23	5.7	15.5	10.2	3.6	3.2	3.5	6.2	48.7	14.2	2.9	1.5	6.4
18.5	1.23	7.1	24.8	12.8	4.5	4.0	4.4	8.2	95.8	19.4	3.1	1.9	8.8
21.6	1.23	8.7	30.8	15.0	6.3	4.7	5.2	10.0	136.8	23.5	3.5	2.9	10.3
24.7	1.24	10.0	35.3	16.8	7.1	6.0	5.4	12.2	201.6	28.6	4.8	3.3	12.7

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	6 DST σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
TEST 9													
Rep 1													
3.1	1.19	1.3	0.9	2.3	1.0	0.5	0.9	0.7	0.7	1.5	0.6	-0.1	0.8
6.2	1.21	2.6	3.7	4.8	1.8	1.3	1.8	1.9	2.6	3.7	1.7	0.4	1.6
9.2	1.22	4.0	8.0	7.2	2.5	2.2	2.5	3.5	10.6	7.0	2.8	0.6	3.2
12.3	1.22	5.4	14.1	9.7	3.2	3.2	3.3	5.7	33.6	12.4	3.2	1.6	5.4
15.4	1.23	7.0	23.0	12.5	4.2	4.2	4.2	7.8	75.0	17.8	3.6	2.1	7.8
18.5	1.23	8.4	32.4	15.0	5.3	5.0	5.0	10.1	133.6	23.5	4.0	3.0	10.3
21.6	1.23	9.9	43.9	17.6	6.4	5.8	5.9	11.5	134.0	25.4	5.5	3.6	10.9
Rep 2													
3.1	1.13	1.3	0.7	2.3	1.0	0.7	0.8	0.8	0.7	1.6	0.7	0.0	0.8
6.2	1.17	2.6	3.0	4.6	1.7	1.5	1.6	1.9	2.1	3.4	1.9	0.5	1.5
9.2	1.21	4.2	8.2	7.5	2.5	2.5	2.5	3.6	11.5	7.4	2.7	0.8	3.3
12.3	1.22	6.0	15.6	10.2	3.5	3.1	3.5	5.5	29.8	11.7	3.1	1.6	5.1
15.4	1.23	6.9	26.1	12.8	4.2	3.7	4.5	7.7	79.7	18.0	3.2	2.0	8.0
18.5	1.24	8.3	36.9	15.3	5.2	4.5	5.4	9.4	119.8	21.9	4.1	2.1	9.9
21.6	1.24	9.9	53.7	18.3	6.0	5.3	6.5	11.1	146.8	25.0	5.5	2.8	11.1
Rep 3													
3.1	1.15	1.4	1.1	2.6	0.8	0.7	0.9	0.9	0.4	1.6	0.9	0.3	0.6
6.2	1.18	2.6	6.2	5.5	1.5	0.9	2.3	2.3	3.0	4.3	1.7	1.0	1.7
9.2	1.20	4.0	12.1	8.0	2.3	1.7	3.2	3.9	12.6	7.9	2.5	1.3	3.3
12.3	1.21	5.7	23.7	11.3	3.2	2.6	4.4	5.6	31.3	12.0	2.9	1.8	5.1
15.4	1.22	7.1	41.3	14.5	4.0	2.7	5.9	7.5	57.4	16.2	4.2	2.2	7.0
18.5	1.23	8.7	54.8	17.2	5.0	3.8	6.7	9.4	103.1	21.0	4.6	2.4	9.3
21.6	1.24	10.4	100.8	21.8	6.1	3.2	9.3	11.6	150.0	25.6	5.8	3.3	11.2

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	6 DST σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 4													
3.1	1.15	1.2	0.8	2.2	0.8	0.6	0.8	1.2	0.6	2.0	1.0	0.6	0.7
6.2	1.19	2.5	4.0	4.8	1.8	0.9	1.9	2.4	4.3	4.7	2.0	0.6	2.0
9.2	1.21	3.8	10.0	7.4	2.5	1.4	3.0	4.2	15.1	8.7	2.4	1.5	3.6
12.3	1.22	5.2	21.3	10.4	3.5	1.6	4.4	6.1	40.8	13.4	3.2	1.7	5.9
15.4	1.23	6.4	35.0	13.1	4.2	2.0	5.6	8.0	86.6	18.7	3.8	1.6	8.5
18.5	1.23	7.8	49.8	15.7	5.3	2.3	6.7	9.8	124.3	22.7	4.3	2.5	10.1
21.6	1.24	9.3	101.1	21.0	6.5	5.5	10.2	11.7	158.2	26.2	4.9	3.9	11.1
Rep 5													
3.1	1.16	1.5	1.1	2.7	1.0	0.8	1.0	1.0	0.8	2.0	0.8	0.2	0.9
6.2	1.20	2.9	4.2	5.2	2.0	1.5	1.9	2.4	4.5	4.9	1.5	1.0	1.9
9.2	1.22	4.6	10.1	8.2	3.2	2.3	2.9	4.0	16.8	8.7	2.2	1.1	3.8
12.3	1.23	6.0	17.5	10.7	4.2	3.0	3.0	6.0	39.3	13.2	2.8	1.9	5.6
15.4	1.23	7.3	25.1	13.0	5.0	3.9	4.6	7.8	71.0	17.5	3.4	2.5	7.5
18.5	1.24	9.0	37.6	16.0	6.2	4.8	5.6	9.6	111.8	21.8	3.9	3.0	9.4
21.6	1.24	10.6	52.3	18.8	7.7	5.3	6.8	11.6	148.2	25.7	4.7	4.4	10.6
TEST 10													
Rep 1													
3.1	1.09	1.1	0.8	2.1	0.7	0.5	0.8	0.5	0.2	1.1	0.3	0.2	0.4
6.2	1.13	2.2	3.3	4.3	1.2	1.0	1.6	1.4	1.1	2.6	0.9	0.6	1.0
9.2	1.15	3.2	8.6	6.6	1.6	1.5	2.6	2.4	4.6	4.8	1.5	0.8	2.0
12.3	1.17	4.2	11.5	8.1	2.4	2.2	3.0	3.7	13.3	7.9	2.0	1.2	3.3
15.4	1.18	5.4	23.8	11.1	2.9	2.3	4.4	4.6	23.1	10.1	2.6	1.1	4.5
18.5	1.18	6.5	35.3	13.3	3.4	2.7	5.3	6.4	51.2	14.6	3.1	1.6	6.5
21.6	1.19	7.6	48.7	15.6	4.0	3.1	6.3	7.9	80.9	18.2	3.8	1.6	8.3
24.7	1.19	8.7	68.1	18.2	4.6	3.4	7.4	9.2	112.9	21.4	4.2	2.1	9.7

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 2													
3.1	1.09	1.0	0.5	1.8	0.7	0.5	0.6	0.5	0.3	1.2	0.3	0.1	0.5
6.2	1.13	2.0	2.5	3.8	1.2	1.0	1.4	1.5	2.8	3.3	1.1	0.0	1.7
9.2	1.16	3.2	5.0	5.8	1.9	1.8	2.0	2.6	7.4	5.6	1.6	0.5	2.6
12.3	1.17	4.1	9.2	7.6	2.5	2.3	2.7	3.7	18.2	8.6	1.7	0.8	3.9
15.4	1.18	5.2	14.3	9.6	3.2	2.8	3.4	5.0	33.9	11.7	2.3	1.0	5.3
18.5	1.18	6.3	23.7	11.9	3.7	3.3	4.3	6.5	62.3	15.6	2.8	1.2	7.2
21.6	1.19	7.5	33.2	14.2	4.3	4.1	5.0	7.8	88.9	18.7	3.4	1.4	8.6
24.7	1.19	8.7	46.2	16.5	4.8	4.7	5.9	9.3	117.3	21.8	4.0	2.1	9.8
Rep 3													
3.1	1.11	1.2	0.5	2.0	0.8	0.7	0.7	0.6	0.5	1.4	0.4	0.0	0.7
6.2	1.14	2.4	3.5	4.5	1.4	1.2	1.7	1.4	2.6	3.1	1.3	-0.1	1.6
9.2	1.16	3.5	6.6	6.5	2.1	2.0	2.2	2.5	8.3	5.7	1.8	0.1	2.8
12.3	1.17	4.6	10.1	8.3	2.8	2.7	2.8	3.7	20.0	8.7	2.2	0.2	4.2
15.4	1.17	5.6	19.0	10.6	3.2	2.9	3.8	5.3	42.6	12.8	2.5	0.6	6.1
18.5	1.18	7.1	29.6	13.4	4.1	3.9	4.8	7.0	83.9	17.5	3.1	0.4	8.5
21.6	1.18	8.3	42.0	15.8	4.9	4.2	5.8	8.2	110.7	20.2	3.6	0.7	9.7
24.7	1.19	9.5	55.2	18.1	5.5	4.9	6.6	9.9	156.2	24.2	4.1	1.3	11.5
Rep 4													
3.1	1.09	1.0	0.5	1.8	0.6	0.5	0.7	0.5	0.4	1.2	0.3	0.1	0.6
6.2	1.11	2.2	3.1	4.2	1.2	1.1	1.6	1.5	2.5	3.2	1.3	0.1	1.6
9.2	1.13	3.4	8.2	6.7	1.8	1.8	2.5	2.6	7.5	5.7	1.8	0.4	2.6
12.3	1.14	4.6	13.4	8.8	2.8	2.2	3.3	3.7	15.7	8.2	2.1	0.8	3.7
15.4	1.15	5.7	19.3	10.7	3.5	2.8	4.0	4.9	32.2	11.4	2.6	0.8	5.3
18.5	1.17	7.0	28.6	13.2	4.2	3.7	4.8	7.2	82.8	17.7	2.7	1.3	8.2
21.6	1.17	8.6	43.2	16.2	5.2	4.5	5.9	8.7	117.8	21.1	3.4	1.6	9.8
24.7	1.18	9.7	52.3	18.0	6.0	5.0	6.5	10.2	159.1	24.8	4.0	1.9	11.4

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	6 DST σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 5													
3.1	1.08	1.2	0.6	2.1	0.9	0.6	0.8	0.7	0.4	1.5	0.5	0.2	0.6
6.2	1.10	2.3	2.9	4.3	1.5	1.2	1.6	1.8	4.6	4.2	0.9	0.2	2.0
9.2	1.12	3.3	5.7	6.0	2.0	1.8	2.1	3.1	13.1	7.2	1.3	0.7	3.3
12.3	1.13	4.4	11.1	8.2	2.7	2.2	3.0	4.5	31.0	10.9	1.3	1.2	4.9
15.4	1.14	5.4	15.7	10.0	3.2	3.1	3.5	6.2	67.1	15.6	1.5	1.4	7.1
18.5	1.15	6.8	23.0	12.3	4.0	4.0	4.2	7.7	93.5	18.9	2.3	2.0	8.4
21.6	1.16	8.0	31.8	14.5	4.8	4.8	4.9	9.0	114.6	21.4	3.2	2.4	9.5
24.7	1.17	9.3	46.7	17.2	5.4	5.4	5.9	10.6	165.6	25.4	3.3	3.0	11.2
TEST 11													
Rep 1													
3.1	1.13	1.7	1.7	3.2	1.3	0.6	1.3	0.8	1.9	2.4	0.2	-0.2	1.3
6.2	1.20	3.5	6.3	6.3	2.8	1.4	2.4	2.4	10.4	6.0	1.3	-0.2	3.1
9.2	1.20	5.2	12.6	9.2	4.2	2.3	3.4	3.9	29.5	10.0	2.3	-0.5	5.2
12.3	1.21	6.7	23.1	12.2	4.8	3.1	4.5	5.7	62.6	14.4	3.5	-1.0	7.7
15.4	1.21	8.5	33.3	15.1	6.1	4.3	5.4	7.6	113.4	19.5	4.4	-1.0	10.3
18.5	1.22	10.2	54.0	18.6	7.2	4.9	6.9	9.4	167.0	23.9	5.5	-1.0	12.5
21.6	1.22	11.3	62.0	20.2	8.0	5.6	7.3	11.3	210.0	27.5	6.7	-0.3	13.9
Rep 2													
3.1	1.14	1.6	0.6	2.5	1.2	1.2	0.7	0.7	1.7	2.2	0.1	-0.2	1.2
6.2	1.16	3.3	3.0	5.3	2.3	2.3	1.5	2.7	11.4	6.5	1.5	0.1	3.2
9.2	1.18	4.7	8.7	8.1	3.0	3.0	2.6	4.3	30.4	10.6	2.3	0.1	5.2
12.3	1.20	6.4	15.4	10.9	4.3	3.9	3.5	6.8	73.7	16.6	2.8	0.9	7.9
15.4	1.21	8.1	30.6	14.5	5.4	4.5	5.0	9.0	136.2	22.3	3.9	0.7	10.8
18.5	1.21	9.6	40.1	16.9	6.5	5.5	5.7	10.5	187.9	26.1	5.0	0.3	12.9
21.6	1.22	11.2	56.3	19.8	7.3	6.4	6.7	11.9	218.4	28.7	6.2	0.8	13.9

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 3													
3.1	1.15	1.6	0.9	2.7	1.3	0.8	0.9	0.8	0.6	1.7	0.8	0.1	0.8
6.2	1.19	3.4	3.6	5.5	2.5	2.1	1.7	2.8	10.7	6.5	1.5	0.4	3.1
9.2	1.20	5.3	8.0	8.5	3.8	3.5	2.5	4.2	20.6	9.3	2.8	0.6	4.4
12.3	1.20	7.0	17.3	11.8	4.7	4.6	3.6	5.6	39.3	12.5	4.1	0.3	6.1
15.4	1.21	8.8	34.5	15.6	5.6	5.2	5.2	7.3	66.3	16.4	4.7	0.7	7.8
18.5	1.21	10.1	36.1	17.0	6.8	6.4	5.3	9.0	99.4	20.3	5.6	1.3	9.5
21.6	1.22	11.5	46.4	19.4	7.6	7.5	5.9	11.2	151.7	25.2	6.6	1.9	11.6
Rep 4													
3.1	1.14	1.5	1.2	2.7	1.3	0.6	1.1	0.8	0.6	1.7	0.6	0.2	0.8
6.2	1.15	3.3	5.3	5.9	2.3	1.7	2.1	2.8	15.6	7.3	1.4	-0.2	3.8
9.2	1.18	4.7	12.5	8.8	3.1	2.2	3.3	4.9	44.8	12.5	2.1	0.0	6.2
12.3	1.20	6.3	21.3	11.6	4.2	3.0	4.3	6.6	88.7	17.3	2.8	-0.3	8.8
15.4	1.20	7.9	35.1	14.6	5.3	3.7	5.5	8.2	119.8	20.6	4.0	0.0	10.3
18.5	1.21	9.6	60.0	18.4	6.2	4.1	7.2	10.2	186.1	25.7	4.8	0.0	12.8
21.6	1.21	10.9	60.9	19.9	7.0	5.9	7.0	11.9	229.3	29.2	5.5	1.1	14.1
Rep 5													
3.1	1.13	1.5	0.5	2.3	1.1	1.0	0.7	0.8	1.5	2.2	0.3	-0.2	1.2
6.2	1.17	3.2	2.9	5.2	2.3	2.2	1.5	2.2	6.9	5.0	1.8	-0.2	2.6
9.2	1.19	4.6	6.5	7.5	3.2	3.0	2.3	3.8	19.5	8.7	2.7	0.1	4.3
12.3	1.20	6.2	13.0	10.3	4.4	3.8	3.3	5.8	43.0	13.2	3.2	0.9	6.2
15.4	1.21	7.8	20.4	13.0	5.6	4.9	4.1	7.9	91.7	18.8	4.1	0.9	9.0
18.5	1.21	9.4	30.1	15.7	6.5	6.0	4.9	10.1	151.6	24.1	5.0	1.1	11.5
21.6	1.22	10.7	39.9	18.0	7.4	6.8	5.6	11.7	194.0	27.6	5.9	1.6	13.0

TABLE A (CONTINUED)

TEST 12

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 1													
3.1	1.09	1.6	1.7	3.1	0.9	0.9	1.2	1.3	3.2	3.4	0.6	0.1	1.7
6.2	1.14	3.6	12.3	7.6	1.8	1.3	3.2	3.4	15.2	7.8	1.8	0.5	3.7
9.2	1.16	5.4	25.0	11.1	2.9	2.1	4.5	6.3	46.3	14.1	3.1	1.7	6.2
12.3	1.17	6.9	44.0	14.5	3.6	2.5	6.0	8.6	89.4	19.5	4.2	2.1	8.7
15.4	1.17	8.9	64.6	18.1	4.7	3.8	7.2	11.5	175.2	26.7	5.3	2.5	12.1
18.5	1.18	10.5	87.5	21.3	5.5	4.6	8.3	13.2	189.8	29.0	6.9	3.6	12.7
21.6	1.18	12.3	105.6	24.2	6.8	5.9	9.1	14.4	199.6	30.5	8.3	4.3	13.1
Rep 2													
3.1	1.08	1.5	1.5	2.9	1.1	0.6	1.2	1.3	3.2	3.3	0.6	0.0	1.7
6.2	1.10	3.3	8.2	6.6	1.9	1.3	2.6	3.3	14.5	7.7	1.6	0.7	3.5
9.2	1.13	5.0	17.8	10.0	2.9	2.3	3.8	5.2	34.0	11.7	3.1	0.6	5.6
12.3	1.15	6.7	32.0	13.2	3.8	3.1	5.1	7.6	71.7	17.1	4.8	0.9	8.1
15.4	1.17	8.2	47.0	16.1	4.6	4.0	6.1	9.4	112.3	21.3	6.0	1.0	10.2
18.5	1.18	10.8	76.1	20.8	6.7	4.8	8.0	11.4	173.0	26.1	7.5	0.6	12.7
21.6	1.18	12.3	89.3	23.2	7.2	6.5	8.4	13.5	193.5	29.1	9.0	2.3	13.4
Rep 3													
3.1	1.08	1.5	2.0	3.2	0.7	0.7	1.2	1.4	5.0	3.9	0.8	-0.5	2.2
6.2	1.10	3.4	10.0	7.1	1.6	1.6	2.7	3.7	18.8	8.6	1.8	0.6	4.0
9.2	1.12	5.7	28.5	11.9	2.7	2.6	4.7	5.8	39.5	13.0	2.9	1.6	5.7
12.3	1.13	7.5	45.0	15.3	3.7	3.6	5.8	8.3	76.5	18.3	4.5	2.0	8.1
15.4	1.15	9.2	59.0	18.1	4.8	4.7	6.7	11.3	170.3	26.2	5.3	2.2	12.0
18.5	1.17	11.0	78.6	21.2	5.9	5.8	7.7	12.6	183.5	28.1	7.1	2.8	12.7
21.6	1.18	12.5	99.2	24.0	6.8	6.8	8.6	14.0	200.0	30.1	8.2	3.6	13.2

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 4													
3.1	1.09	1.8	1.8	3.4	1.0	1.0	1.2	1.1	3.1	3.1	0.5	-0.3	1.7
6.2	1.12	3.8	11.4	7.7	2.1	1.7	3.0	3.2	14.5	7.4	2.1	0.0	3.7
9.2	1.14	5.3	22.9	10.9	2.8	2.3	4.3	5.7	26.5	11.6	3.2	2.3	4.7
12.3	1.15	7.3	45.2	15.0	3.6	3.2	5.9	6.9	43.3	14.4	4.3	2.1	6.2
15.4	1.17	9.1	69.1	18.7	4.5	4.2	7.3	9.9	101.1	21.4	5.7	2.7	9.4
18.5	1.17	10.6	90.6	21.6	5.4	4.9	8.4	12.3	145.6	26.1	7.0	3.8	11.2
21.6	1.18	12.2	108.4	24.2	6.5	5.8	9.2	13.8	167.8	28.5	8.8	4.1	12.2
Rep 5													
3.1	1.08	1.6	1.7	3.0	1.2	0.5	1.3	1.3	4.0	3.6	0.7	-0.2	1.9
6.2	1.11	2.9	7.6	6.1	1.7	1.0	2.5	3.7	19.6	8.7	1.9	0.4	4.2
9.2	1.13	4.5	16.4	9.2	2.6	1.8	3.7	5.7	40.7	13.0	3.2	1.0	6.0
12.3	1.15	6.7	33.8	13.4	3.9	2.8	5.3	7.1	51.4	15.3	4.2	1.9	6.7
15.4	1.16	8.4	45.9	16.2	5.1	4.0	6.1	9.6	99.4	21.0	5.5	2.3	9.3
18.5	1.17	10.2	62.0	19.3	6.2	5.2	7.1	11.6	132.7	24.8	6.5	3.6	10.6
21.6	1.17	12.1	79.5	22.4	7.4	6.5	7.9	13.5	173.9	28.6	7.6	4.2	12.2
TEST 13													
Rep 1													
3.1	1.00	1.2	0.8	2.2	0.8	0.6	0.8	0.8	0.8	1.8	0.6	-0.1	0.9
6.2	1.02	2.6	4.4	5.0	1.5	1.3	1.9	2.0	7.0	5.0	1.0	0.0	2.5
9.2	1.04	4.0	10.1	7.6	2.8	1.5	3.0	3.1	13.8	7.3	1.8	0.3	3.5
12.3	1.06	5.5	17.3	10.2	4.0	2.3	4.0	4.6	27.1	10.6	2.2	1.1	4.6
15.4	1.06	7.1	21.0	12.4	5.1	3.9	4.2	6.8	50.2	15.0	3.4	2.1	6.4
18.5	1.06	8.8	22.7	14.3	6.2	6.0	4.2	8.8	70.7	18.5	4.4	3.4	7.5

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	6 DST σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 2													
3.1	1.02	1.1	0.3	1.8	0.8	0.8	0.5	0.9	0.5	1.7	0.5	0.5	0.6
6.2	1.03	2.5	2.9	4.5	1.8	1.3	1.6	2.1	8.5	5.3	1.5	-0.5	2.9
9.2	1.06	4.7	14.2	9.1	3.0	2.1	3.5	4.3	20.3	9.5	2.2	1.2	4.1
12.3	1.06	6.3	21.0	11.5	4.3	3.0	4.3	6.0	34.7	12.7	3.7	1.6	5.5
15.4	1.06	7.8	23.6	13.4	5.2	4.7	4.3	7.6	53.4	16.0	4.4	2.5	6.8
18.5	1.07	9.1	24.4	14.9	6.6	6.0	4.4	9.3	84.9	19.9	5.1	3.0	8.4
Rep 3													
3.1	1.01	1.3	0.7	2.3	0.9	0.8	0.8	0.7	0.8	1.7	0.5	-0.1	0.9
6.2	1.04	2.7	3.4	4.8	1.8	1.5	1.7	1.8	6.1	4.5	1.2	-0.3	2.4
9.2	1.05	4.3	8.3	7.6	2.7	2.5	2.5	3.4	17.7	8.0	2.3	-0.2	4.1
12.3	1.05	6.1	16.3	10.7	3.8	3.7	3.5	5.1	43.7	12.3	3.6	-0.6	6.5
15.4	1.06	7.7	25.5	13.6	4.9	4.7	4.4	6.9	58.6	15.5	4.4	0.8	7.3
18.5	1.06	9.3	29.9	15.6	6.4	5.8	4.9	7.9	67.0	17.0	5.4	1.2	7.9
Rep 4													
3.1	1.01	1.0	0.7	2.0	0.6	0.5	0.8	1.0	2.0	2.6	0.5	0.0	1.3
6.2	1.03	2.3	3.5	4.5	1.4	1.1	1.7	2.6	10.2	6.0	2.1	-0.3	3.2
9.2	1.06	4.2	11.7	8.1	2.5	2.0	3.1	5.0	35.5	11.6	3.5	0.0	5.8
12.3	1.06	5.3	16.6	9.9	3.6	2.4	3.8	6.4	60.3	15.0	4.5	-0.2	7.6
15.4	1.07	6.7	26.8	12.7	4.3	3.2	4.7	8.9	96.1	20.0	5.2	1.5	9.3
18.5	1.07	8.5	32.0	15.0	5.5	5.0	5.0	10.3	110.0	22.2	6.0	2.6	9.8
TEST 14													
Rep 1													
3.1	1.03	1.2	1.0	2.2	1.3	0.1	1.0	0.8	2.5	2.6	0.2	-0.4	1.5
6.2	1.06	2.8	2.9	4.5	2.8	1.1	1.7	2.1	7.3	5.1	1.5	-0.2	2.6
9.2	1.07	4.4	7.2	7.4	3.7	2.1	2.6	4.2	18.8	8.6	4.1	-0.1	4.3
12.3	1.07	5.9	14.0	10.1	4.8	2.9	3.6	5.7	34.4	11.9	5.0	0.2	5.8
15.4	1.07	7.4	22.8	12.8	5.6	3.7	4.5	8.0	58.2	16.0	7.3	0.8	7.6

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 2													
3.1	1.03	1.2	0.6	2.1	0.9	0.6	0.7	0.7	2.7	2.4	0.5	-0.9	1.6
6.2	1.05	2.5	2.0	4.1	2.1	1.3	1.4	2.2	4.4	4.4	2.0	0.2	2.1
9.2	1.06	3.7	4.5	6.1	3.0	1.9	2.1	3.6	13.9	7.5	3.2	0.1	3.7
12.3	1.07	5.5	12.1	9.4	4.3	2.7	3.3	5.4	31.5	11.4	4.5	0.2	5.6
15.4	1.07	7.1	17.0	11.7	5.5	4.0	3.9	7.7	49.4	15.0	7.2	1.0	7.0
Rep 3													
3.1	1.03	1.2	0.8	2.2	1.0	0.4	0.9	0.7	1.7	2.1	0.5	-0.5	1.3
6.2	1.05	2.7	2.4	4.5	2.3	1.4	1.5	2.1	4.8	4.3	2.0	0.0	2.2
9.2	1.06	4.4	4.9	7.0	3.5	2.8	2.1	4.0	15.0	8.0	3.8	0.3	3.9
12.3	1.07	6.0	11.1	9.8	4.6	3.6	3.1	5.5	27.6	10.8	5.4	0.3	5.3
15.4	1.07	7.7	14.8	12.1	5.9	5.0	3.5	7.7	51.6	15.3	6.8	1.0	7.1
Rep 4													
3.1	1.03	1.0	0.9	1.9	1.0	0.1	0.9	0.7	2.3	2.4	4.5	-0.6	1.5
6.2	1.04	2.6	2.4	4.2	2.6	1.1	1.6	2.3	7.2	5.0	2.1	-0.3	2.7
9.2	1.06	4.1	5.7	6.7	3.5	2.1	2.3	4.2	19.9	8.8	3.9	-0.1	4.4
12.3	1.07	5.7	15.2	10.1	4.3	2.7	3.7	6.1	35.4	12.3	5.4	0.5	5.9
15.4	1.07	7.3	20.8	12.5	5.4	4.0	4.3	8.5	57.5	16.9	6.4	2.2	7.4
TEST 15													
Rep 1													
3.1	1.04	1.0	0.3	1.6	0.8	0.6	0.5	0.4	0.4	1.1	0.4	-0.1	0.6
6.2	1.05	2.3	1.7	3.5	1.3	1.3	1.1	1.5	3.3	3.5	0.8	0.1	1.7
9.2	1.07	3.1	4.3	5.5	2.0	1.8	1.9	2.6	15.8	7.0	1.4	-0.7	3.8
12.3	1.07	4.3	9.3	7.8	2.8	2.3	2.8	3.9	34.0	10.4	2.2	-0.8	5.6
15.4	1.08	5.4	16.2	10.0	3.5	2.7	3.7	5.4	62.6	14.0	3.6	-1.5	7.8
18.5	1.08	6.6	23.7	12.2	4.5	3.2	4.5	7.0	80.9	17.1	4.1	-0.2	8.6
21.6	1.08	7.9	30.7	14.3	5.2	4.3	5.0	8.7	105.4	20.4	4.5	1.2	9.6

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 2													
3.1	1.04	0.9	0.4	1.7	0.6	0.4	0.6	0.5	0.4	1.2	0.3	0.0	0.6
6.2	1.06	2.3	3.3	4.4	1.4	1.1	1.6	1.5	3.1	3.5	0.6	0.4	1.5
9.2	1.07	3.4	7.4	6.5	1.9	1.8	2.4	2.8	10.3	6.5	1.1	0.8	2.8
12.3	1.07	5.1	17.7	9.9	2.7	2.6	3.7	4.2	22.4	10.0	1.7	1.2	4.2
15.4	1.07	6.0	22.3	11.4	3.3	3.2	4.1	5.4	33.5	12.1	2.6	1.6	5.2
18.5	1.08	7.0	27.9	13.1	4.1	3.8	4.6	7.1	60.8	16.1	3.1	2.1	7.0
21.6	1.08	8.3	37.2	15.3	5.0	4.5	5.4	8.9	102.7	20.6	3.8	2.4	9.1
Rep 3													
3.1	1.04	1.0	0.5	1.8	0.8	0.4	0.7	0.5	0.7	1.4	0.5	-0.3	0.8
6.2	1.05	2.1	3.0	4.1	1.2	1.0	1.5	1.7	3.8	3.9	0.8	0.3	1.8
9.2	1.07	3.3	4.6	5.7	2.3	1.8	2.0	3.0	10.5	6.7	1.5	0.7	3.0
12.3	1.07	4.5	10.0	8.1	2.8	2.5	2.8	4.3	23.2	9.9	1.8	1.3	4.3
15.4	1.08	5.9	18.0	10.8	3.5	3.3	3.7	5.4	34.5	12.1	2.4	1.6	5.3
18.5	1.08	7.1	25.8	13.0	4.6	3.8	4.6	7.9	73.9	17.8	3.7	2.2	7.8
21.6	1.08	8.6	35.7	15.5	5.7	4.7	5.4	9.3	104.1	21.1	4.2	2.7	9.2
Rep 4													
3.1	1.02	1.1	0.5	1.9	0.8	0.5	0.7	0.5	0.3	1.1	0.3	0.0	0.6
6.2	1.04	2.0	2.9	3.9	1.1	9.5	1.5	1.2	1.8	2.8	0.5	0.4	1.2
9.2	1.05	2.8	3.2	4.9	1.8	1.8	1.6	2.3	6.0	5.1	0.9	0.8	2.2
12.3	1.06	4.0	9.2	7.5	2.3	2.3	2.6	3.3	11.8	7.2	1.4	1.2	3.0
15.4	1.07	5.3	16.6	10.0	3.0	3.0	3.5	4.5	23.9	10.2	1.8	1.7	4.3
18.5	1.07	6.7	26.1	12.6	3.9	3.5	4.5	6.4	47.5	14.3	2.7	2.1	6.1
21.6	1.08	8.0	38.2	15.2	4.6	4.3	5.4	8.0	73.8	18.0	3.5	2.7	7.6

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)
Rep 5													
3.1	1.02	0.9	0.3	1.5	0.6	0.6	0.5	0.5	0.4	1.2	0.3	-0.1	0.6
6.2	1.04	2.1	1.9	3.7	1.3	1.2	1.2	1.4	2.8	3.3	0.5	0.4	1.5
9.2	1.05	3.1	5.9	5.9	1.8	1.6	2.2	2.4	7.2	5.5	1.0	0.8	2.4
12.3	1.06	4.4	12.1	8.4	2.5	2.2	3.1	3.7	16.9	8.4	1.5	1.1	3.7
15.4	1.07	5.6	17.8	10.4	3.2	3.1	3.7	4.6	28.9	10.8	1.9	1.2	4.8
18.5	1.07	6.6	24.0	12.3	3.8	3.8	4.2	5.8	41.8	13.3	2.6	1.7	5.8
21.6	1.08	8.2	36.9	15.2	4.7	4.6	5.3	7.9	69.2	17.5	3.3	2.9	7.3
TEST 16													
Rep 1													
3.1	1.01	1.4	1.6	2.9	0.9	0.5	1.2	1.1	2.1	2.7	0.9	-0.2	1.4
6.2	1.04	3.0	6.1	5.8	1.8	1.3	2.3	2.9	10.5	6.5	2.1	0.2	3.2
9.2	1.06	4.6	12.1	8.6	2.9	2.4	3.1	5.3	23.8	10.6	4.2	1.0	4.8
12.3	1.06	6.4	25.8	12.3	3.6	3.3	4.5	7.3	42.4	14.5	5.8	1.7	6.4
15.4	1.07	8.5	41.5	15.9	4.8	4.7	5.6	9.9	76.1	19.4	8.1	2.3	8.6
18.5	1.07	10.2	56.2	18.8	5.9	5.8	6.5	11.5	107.9	23.0	8.9	3.0	10.1
21.6	1.07	11.8	70.5	21.5	7.0	7.0	7.3	13.0	132.2	25.7	10.0	3.3	11.2
Rep 2													
3.1	1.01	1.4	1.5	2.8	0.8	0.6	1.1	1.2	2.3	2.9	0.7	-0.1	1.5
6.2	1.03	3.0	6.6	6.0	1.6	1.5	2.3	2.8	10.8	6.5	1.8	0.2	3.2
9.2	1.05	4.8	13.5	9.0	2.8	2.5	3.3	5.0	33.5	11.6	3.0	0.5	5.6
12.3	1.06	6.7	30.5	13.0	3.8	3.2	4.9	6.7	58.2	15.3	3.9	0.8	7.3
15.4	1.06	8.6	47.5	16.5	5.0	4.3	6.1	8.9	91.3	19.8	5.0	1.0	8.9
18.5	1.07	10.4	66.8	19.8	6.2	5.2	7.3	10.7	133.6	23.9	5.9	2.4	10.8
21.6	1.07	12.0	83.6	22.6	7.2	6.3	8.1	12.3	170.0	27.2	6.7	3.0	12.1

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 3													
3.1	1.01	1.6	1.6	3.1	1.0	0.7	1.2	1.2	2.5	3.0	0.4	0.2	1.4
6.2	1.03	3.2	6.4	6.1	2.0	1.4	2.3	2.8	10.5	6.6	1.2	0.7	2.9
9.2	1.04	5.2	18.1	10.1	3.0	2.4	3.8	5.2	30.4	11.6	2.3	1.8	4.9
12.3	1.06	6.8	31.1	13.3	3.8	3.4	4.9	6.9	51.1	15.2	2.9	2.7	6.2
15.4	1.06	9.1	53.8	17.6	5.0	4.7	6.4	8.8	68.4	18.4	4.4	3.7	7.3
18.5	1.07	11.0	68.2	20.5	6.8	5.8	7.4	11.1	106.7	23.1	5.5	4.9	9.1
21.6	1.07	12.0	79.1	22.3	7.2	6.6	7.9	12.7	129.2	25.8	6.6	5.6	10.1
Rep 4													
3.1	1.01	1.2	0.5	2.0	0.8	0.8	0.6	1.1	1.4	2.5	0.5	0.4	1.0
6.2	1.03	3.0	4.8	5.5	2.0	1.5	2.0	2.9	11.4	6.7	1.6	0.3	3.2
9.2	1.05	4.4	10.1	8.0	3.0	2.1	3.0	4.4	25.3	10.1	2.5	0.6	4.8
12.3	1.06	6.0	21.1	11.3	3.8	3.0	4.2	6.1	47.3	14.0	3.0	1.4	6.3
15.4	1.07	7.8	34.5	14.5	4.8	4.0	5.3	8.1	91.3	19.0	3.7	1.5	8.8
18.5	1.07	9.6	53.0	18.0	5.9	5.0	6.5	9.8	126.9	22.8	4.3	2.3	10.2
21.6	1.08	11.3	70.3	21.0	7.0	6.0	7.5	12.2	154.4	26.6	5.4	4.8	10.9
Rep 5													
3.1	1.01	1.2	0.8	2.2	0.8	0.6	0.8	1.2	1.7	2.5	2.1	-0.1	1.3
6.2	1.04	2.6	4.4	5.0	1.6	1.2	1.9	2.6	9.2	5.9	2.1	-0.1	3.0
9.2	1.05	4.4	9.8	8.0	2.8	2.4	2.8	4.7	28.5	10.6	3.2	0.2	5.2
12.3	1.06	6.1	18.2	11.0	4.2	3.2	3.9	6.4	54.8	14.8	4.0	0.6	7.1
15.4	1.07	7.8	26.2	13.6	5.8	3.9	4.8	7.7	80.5	17.8	4.8	0.5	8.6
18.5	1.07	9.2	36.7	16.1	6.8	4.7	5.7	9.4	100.4	20.8	5.5	1.9	9.4
21.6	1.08	11.1	53.3	19.4	8.0	5.9	6.8	11.4	138.9	24.8	6.9	2.6	1.1

Load	Bulk Density	Type A Cell σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	6 DST σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
TEST 17													
Rep 1													
3.1	1.02	1.6	1.6	3.1	1.0	0.8	1.1	1.5	5.0	4.0	0.8	-0.4	2.2
6.2	1.06	3.5	7.2	6.6	2.0	1.9	2.3	4.3	19.5	9.4	2.0	1.5	4.0
9.2	1.07	5.0	15.4	9.6	2.8	2.7	3.4	6.3	49.8	14.4	2.7	1.8	6.3
12.3	1.07	6.6	25.9	12.5	3.7	3.7	4.4	8.6	93.9	19.7	4.2	1.9	8.9
15.4	1.07	8.8	38.9	16.0	5.2	5.2	5.4	11.0	131.2	24.2	5.9	3.1	10.5
18.5	1.08	10.4	46.4	18.3	6.5	6.5	5.9	12.5	160.6	27.0	6.5	3.9	11.6
21.6	1.08	12.3	60.1	21.2	7.8	7.8	6.7	14.8	201.7	31.1	8.2	5.1	13.0
Rep 2													
3.1	1.01	1.7	1.9	3.2	1.0	0.8	1.2	1.5	4.2	3.8	0.9	-0.2	2.0
6.2	1.05	3.9	10.8	7.7	2.1	2.0	2.9	4.4	21.8	9.7	2.1	1.3	4.2
9.2	1.06	5.8	21.8	11.2	3.2	3.0	4.1	6.5	45.0	14.2	2.7	2.5	5.9
12.3	1.07	7.6	36.7	14.6	4.2	4.1	5.3	8.7	68.5	18.3	4.1	3.8	7.3
15.4	1.07	9.3	48.8	17.3	5.3	5.2	6.1	10.8	100.2	22.3	5.7	4.3	9.0
18.5	1.07	11.6	67.1	21.1	7.0	6.7	7.2	12.7	135.0	26.1	6.7	5.3	10.4
21.6	1.08	13.2	69.7	22.8	8.5	8.2	7.3	14.9	193.2	31.0	7.7	6.1	12.4
Rep 3													
3.1	1.01	1.5	1.4	2.8	0.8	0.8	1.0	1.3	4.4	3.6	0.9	-0.5	2.1
6.2	1.05	3.5	7.2	6.6	2.1	1.9	2.4	4.5	18.5	9.3	3.1	1.1	4.1
9.2	1.06	5.0	12.6	9.1	3.0	2.9	3.1	6.4	31.1	12.6	4.8	1.8	5.4
12.3	1.07	6.5	18.6	11.5	4.1	4.0	3.8	8.7	53.9	16.9	6.4	2.8	7.1
15.4	1.07	8.8	29.4	15.0	5.8	5.5	4.8	11.3	78.0	21.0	8.9	3.8	8.6
18.5	1.08	10.8	43.0	18.3	7.0	7.0	5.7	12.5	106.7	24.1	9.5	4.1	10.0
21.6	1.08	12.5	52.6	20.8	8.3	8.3	6.3	14.7	174.1	29.5	10.2	4.3	12.6

Load	Bulk Density	Type A Cell σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	6 DST σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)
Rep 4													
3.1	1.00	1.7	1.3	3.0	1.1	1.0	1.0	1.1	3.5	3.1	0.8	-0.6	1.9
6.2	1.02	3.6	7.8	6.8	2.0	1.9	2.4	3.2	10.9	6.9	2.0	0.7	3.1
9.2	1.05	5.3	30.8	11.7	2.8	1.5	5.1	5.7	28.9	11.9	2.9	2.2	4.8
12.3	1.06	7.3	31.6	13.8	4.1	4.0	4.9	7.9	53.4	16.4	4.0	3.4	6.5
15.4	1.07	9.6	50.3	17.8	5.5	5.5	6.1	10.5	101.1	22.0	5.8	3.6	9.2
18.5	1.07	10.8	60.9	19.8	6.3	6.2	6.8	12.9	132.1	26.1	7.0	5.4	10.3
21.6	1.07	12.9	65.2	22.2	8.4	8.0	7.1	15.1	206.4	31.6	8.3	5.3	13.1
Rep 5													
3.1	1.01	1.8	1.4	3.1	1.2	1.0	1.1	1.2	4.8	3.6	0.6	-0.6	2.1
6.2	1.03	3.7	7.2	6.8	2.1	2.1	2.3	3.3	19.4	8.3	1.3	0.1	4.1
9.2	1.05	5.4	16.4	10.0	3.2	2.9	3.6	5.5	38.6	12.7	2.4	1.5	5.6
12.3	1.06	7.2	24.7	12.9	4.7	4.0	4.5	7.9	68.8	17.5	3.7	2.5	7.5
15.4	1.06	9.1	37.5	16.2	5.7	5.4	5.4	10.4	102.0	22.0	5.4	3.7	9.2
18.5	1.07	10.7	49.4	18.9	6.7	6.6	6.1	12.5	129.0	25.6	6.5	5.4	10.1
21.6	1.07	12.8	62.3	21.9	8.4	8.1	6.9	14.7	216.9	31.6	7.5	5.0	13.3
TEST 18													
Rep 1													
3.1	1.15	0.9	0.3	1.5	0.8	0.4	0.6	0.4	0.6	1.3	0.1	-0.2	0.7
6.2	1.17	1.8	1.5	3.2	1.5	0.8	1.2	1.1	0.8	2.1	0.8	0.4	0.9
9.2	1.19	2.9	4.3	5.2	2.4	1.2	2.0	1.8	4.4	4.2	1.1	0.2	2.0
12.3	1.21	4.6	9.3	8.0	3.5	2.2	2.9	3.0	12.7	7.0	1.5	0.3	3.3
15.4	1.22	5.5	10.8	9.2	4.0	3.2	3.0	3.8	30.6	10.0	2.0	-0.6	5.3
18.5	1.23	7.3	19.4	12.3	5.3	4.2	4.1	5.2	42.8	12.6	2.7	0.2	6.2
21.6	1.23	8.7	25.8	14.5	6.0	5.5	4.5	6.4	68.4	15.7	3.7	-0.2	7.9
24.7	1.24	9.6	30.9	16.0	6.5	6.3	4.9	7.7	101.1	18.9	5.0	-0.7	9.8

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	6 DST σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)
Rep 2													
3.1	1.16	0.9	0.5	1.7	0.6	0.4	0.7	0.5	0.3	1.1	0.4	0.0	0.6
6.2	1.18	2.0	1.8	3.5	1.5	1.0	1.3	1.1	1.3	2.4	0.9	0.1	1.1
9.2	1.20	3.1	4.7	5.5	2.2	1.5	2.0	2.1	4.8	4.6	1.2	0.5	2.1
12.3	1.21	4.3	8.3	7.5	3.3	2.1	2.7	3.3	13.2	7.5	1.9	0.6	3.4
15.4	1.22	5.6	14.9	10.0	4.0	2.8	3.6	4.5	25.2	10.3	2.3	1.0	4.6
18.5	1.22	6.6	21.3	11.8	4.8	3.2	4.3	5.7	32.6	12.3	3.2	1.7	5.3
21.6	1.23	7.8	26.0	13.6	5.8	3.9	4.8	6.9	43.1	14.5	3.7	2.6	5.9
24.7	1.23	9.2	34.1	15.9	6.8	4.9	5.5	8.0	57.9	16.7	4.5	2.8	7.0
Rep 3													
3.1	1.15	0.9	0.4	1.6	0.6	0.4	0.6	0.5	0.7	1.4	0.5	-0.3	0.8
6.2	1.18	1.7	1.3	3.0	1.0	1.0	1.0	1.1	1.7	2.6	0.8	-0.4	1.3
9.2	1.19	3.0	3.5	5.1	2.1	1.7	1.7	2.4	6.5	5.2	1.5	0.3	2.4
12.3	1.21	4.1	6.1	6.9	2.9	2.4	2.2	3.3	16.3	7.8	2.0	0.0	3.9
15.4	1.22	5.1	9.1	8.6	3.6	3.1	2.7	4.3	30.5	10.5	2.5	0.0	5.3
18.5	1.23	6.5	14.3	10.8	4.3	4.3	3.3	5.7	44.1	13.1	3.5	0.4	6.4
21.6	1.24	7.6	20.4	12.8	5.1	5.0	3.9	6.9	58.2	15.5	4.3	1.0	7.3
24.7	1.24	8.9	25.8	14.7	6.1	5.8	4.5	8.4	79.6	18.5	5.2	1.6	8.5
Rep 4													
3.1	1.15	0.9	0.3	1.5	0.8	0.5	0.5	0.5	0.4	1.2	0.5	-0.1	0.6
6.2	1.18	1.7	1.7	3.2	1.2	0.8	1.2	1.2	2.3	2.9	0.6	0.0	1.4
9.2	1.21	2.8	4.9	5.3	2.0	1.1	2.1	2.2	6.5	5.1	1.0	0.5	2.3
12.3	1.22	3.8	8.0	7.0	2.5	1.8	2.6	3.3	10.1	7.0	1.7	1.3	2.8
15.4	1.23	4.6	10.0	8.3	2.9	2.7	2.8	4.8	21.8	10.2	2.4	1.8	4.2
18.5	1.23	5.8	15.8	10.4	3.6	3.3	3.5	5.8	30.7	12.1	2.9	2.2	5.0
21.6	1.24	6.9	24.5	12.6	4.2	4.0	4.3	7.0	42.4	14.5	3.6	2.9	5.8
24.7	1.24	8.5	35.3	15.3	5.1	5.0	5.2	8.2	70.2	17.8	4.1	2.7	7.6

Load	Bulk Density	Type A Cell σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	6 DST σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 5													
3.1	1.15	1.0	0.3	1.6	0.8	0.6	0.5	0.4	0.5	1.1	0.4	-0.2	0.7
6.2	1.18	1.9	2.2	3.6	1.1	1.0	1.3	0.9	1.9	2.4	0.6	-0.3	1.4
9.2	1.19	2.9	4.7	5.4	1.7	1.5	1.9	1.9	3.7	4.1	0.9	0.6	1.8
12.3	1.21	4.1	9.7	7.7	2.3	2.2	2.8	3.2	6.5	6.0	2.5	1.1	2.5
15.4	1.22	5.0	12.7	9.1	3.0	3.0	3.1	4.6	17.2	9.3	2.8	1.6	3.9
18.5	1.22	6.2	20.8	11.4	3.6	3.5	4.0	5.6	30.8	12.0	3.1	1.8	5.1
21.6	1.23	7.3	30.3	13.7	4.3	3.9	4.9	7.2	63.0	16.3	3.7	1.6	7.3
24.7	1.23	8.9	35.2	15.7	5.5	5.5	5.1	8.3	89.6	19.1	4.0	1.7	8.7
TEST 19													
Rep 1													
3.1	1.17	1.6	0.8	2.6	1.2	1.0	0.8	0.9	3.5	3.0	0.0	-0.4	1.7
6.2	1.20	3.1	4.9	5.7	1.8	1.8	1.9	2.0	10.6	5.8	0.5	-0.2	3.0
9.2	1.21	4.5	11.3	8.4	2.6	2.5	2.9	3.7	20.2	8.9	1.9	0.4	4.2
12.3	1.22	6.0	18.6	11.0	3.6	3.5	3.8	5.3	35.8	12.1	2.8	0.9	5.6
15.4	1.22	7.7	28.6	13.8	4.7	4.5	4.7	7.0	57.0	15.6	4.2	1.3	7.2
18.5	1.23	9.2	37.1	16.2	5.8	5.6	5.3	9.1	89.2	20.0	5.1	2.3	8.8
21.6	1.23	10.7	50.2	18.9	7.0	6.2	6.3	12.0	151.4	26.1	6.3	3.6	11.3
24.7	1.23	12.2	60.3	21.1	8.0	7.4	6.9	13.2	160.2	27.7	6.9	4.8	11.4
Rep 2													
3.1	1.16	1.6	0.4	2.3	1.3	1.2	0.6	1.0	3.7	3.2	-0.1	-0.2	1.7
6.2	1.19	3.0	3.1	5.0	2.0	2.0	1.5	2.1	11.3	5.9	1.1	-0.6	3.3
9.2	1.20	5.1	12.4	9.1	3.1	3.0	3.1	4.0	20.4	8.9	2.9	0.1	4.4
12.3	1.21	5.9	14.8	10.4	4.0	3.4	3.5	5.8	40.2	12.8	4.2	0.4	6.2
15.4	1.22	7.7	23.8	13.3	5.0	4.7	4.3	8.1	71.8	17.7	4.9	1.7	8.0
18.5	1.22	9.2	32.0	15.7	6.2	5.6	5.0	9.7	98.4	20.9	5.8	2.3	9.3
21.6	1.22	10.8	45.6	18.6	7.5	6.4	6.1	12.3	156.0	26.6	6.4	3.8	11.4
24.7	1.23	12.2	61.2	21.1	8.7	6.7	7.2	13.4	181.9	28.9	7.4	4.0	12.4

Load	Bulk Density	Type A Cell σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	6 DST σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 3													
3.1	1.16	1.8	0.7	2.7	1.5	1.1	0.8	1.1	2.2	2.8	0.4	0.0	1.4
6.2	1.19	3.3	3.7	5.5	2.3	2.0	1.7	2.6	10.0	6.2	1.4	0.2	3.0
9.2	1.21	4.7	9.3	8.2	3.2	2.6	2.8	4.2	23.9	9.7	2.6	0.4	4.7
12.3	1.22	6.2	14.7	10.6	4.5	3.5	3.5	5.8	41.5	13.2	3.2	1.1	6.0
15.4	1.22	7.9	24.1	13.5	5.8	4.5	4.5	8.0	90.8	18.8	4.5	0.8	9.0
18.5	1.23	9.5	34.6	16.3	6.8	5.5	5.4	9.4	116.8	21.6	5.2	1.3	10.2
21.6	1.23	11.2	47.0	19.1	8.0	6.5	6.3	12.0	182.3	27.4	6.0	2.4	12.5
24.7	1.23	12.7	59.0	21.4	9.5	7.1	7.2	13.2	194.4	29.1	7.2	3.2	13.0
Rep 4													
3.1	1.17	1.7	0.6	2.5	1.5	1.0	0.8	1.0	3.4	3.1	0.4	-0.4	1.8
6.2	1.20	3.2	2.7	5.1	2.5	2.0	1.5	3.5	19.4	8.6	1.5	0.5	4.0
9.2	1.21	4.4	4.1	6.7	3.4	3.1	1.8	5.1	36.9	12.1	2.1	1.2	5.5
12.3	1.22	5.7	9.6	9.3	4.3	3.6	2.8	6.9	70.0	16.6	2.7	1.5	7.6
15.4	1.22	7.1	15.9	11.6	5.5	4.2	3.7	8.0	113.7	21.1	3.5	1.9	9.6
18.5	1.23	8.8	27.1	14.8	6.5	5.1	4.8	10.3	141.2	24.0	4.2	2.6	10.7
21.6	1.23	10.2	33.1	16.8	7.5	6.3	5.3	11.6	164.7	26.4	5.1	3.3	11.5
24.7	1.24	11.5	47.1	19.4	8.4	6.8	6.3	13.3	210.5	30.0	5.9	3.9	13.0
Rep 5													
3.1	1.16	1.3	1.5	2.5	1.3	0.1	1.2	1.0	3.1	3.0	0.1	-0.2	1.6
6.2	1.19	2.5	3.5	4.7	1.6	1.3	1.7	2.7	7.0	5.7	1.6	0.7	2.5
9.2	1.20	3.9	9.6	7.5	2.2	2.0	2.7	4.0	15.2	8.4	2.5	1.0	3.7
12.3	1.21	5.1	17.3	9.9	2.8	2.5	3.7	5.6	31.0	12.0	3.5	1.5	5.2
15.4	1.22	6.8	30.6	13.2	3.6	3.5	4.8	7.6	54.6	16.1	4.3	2.4	6.8
18.5	1.22	7.8	41.0	15.2	4.5	3.7	5.7	9.6	91.4	20.6	5.0	3.3	8.7
21.6	1.22	9.4	58.5	18.2	5.5	4.5	6.9	11.4	125.7	24.3	5.5	4.3	10.0
24.7	1.23	11.1	74.3	21.0	6.8	5.4	7.8	13.3	154.4	27.6	6.4	5.8	10.9

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)
TEST 20													
Rep 1													
3.1	1.18	1.6	0.7	2.5	1.3	1.0	0.8	1.3	7.3	4.2	0.6	-0.1	2.6
6.2	1.21	3.2	3.5	5.3	2.2	2.0	1.7	3.7	21.1	8.9	1.5	0.5	4.2
9.2	1.22	4.8	8.3	8.2	3.3	3.0	2.6	5.8	38.4	13.0	2.4	2.1	5.5
12.3	1.23	6.9	14.9	11.3	4.8	4.5	3.4	8.1	91.1	19.1	2.9	2.3	8.4
15.4	1.24	8.1	23.6	13.7	5.5	5.0	4.3	10.8	130.3	23.9	4.6	3.8	10.1
18.5	1.24	10.2	32.5	16.7	7.0	6.8	5.0	12.9	209.3	29.6	5.1	4.1	12.8
21.6	1.25	11.2	37.2	18.2	8.3	7.2	5.5	14.5	277.8	33.6	6.4	3.4	15.1
24.7	1.25	13.1	57.9	21.8	9.5	8.0	6.9	15.5	299.4	35.4	7.2	3.9	15.7
Rep 2													
3.1	1.19	1.9	1.1	3.1	1.3	1.2	0.9	1.3	6.7	4.1	0.7	-0.1	2.5
6.2	1.21	3.8	4.9	6.3	2.7	2.3	2.0	3.9	21.1	9.1	2.0	0.5	4.3
9.2	1.22	5.1	8.7	8.5	3.5	3.3	2.6	6.0	40.2	13.3	2.7	1.9	5.7
12.3	1.23	7.8	19.3	12.9	5.5	5.0	3.9	8.1	76.2	18.2	3.3	2.8	7.7
15.4	1.23	8.8	22.1	14.2	6.1	6.0	4.1	10.6	125.7	23.6	4.8	3.5	10.0
18.5	1.24	10.5	27.0	16.5	7.5	7.5	4.5	13.2	205.1	29.8	5.4	4.6	12.6
21.6	1.24	12.4	40.2	19.7	8.8	8.7	5.5	14.5	256.0	33.0	5.4	5.2	13.9
24.7	1.25	13.5	54.0	22.0	9.5	9.1	6.5	15.7	274.8	34.8	7.1	5.2	14.8
Rep 3													
3.1	1.17	1.8	0.9	2.8	1.5	1.0	0.9	1.3	5.3	3.8	0.9	-0.7	2.3
6.2	1.21	3.8	3.7	6.0	3.2	2.3	1.9	3.6	17.0	8.3	2.1	0.4	3.9
9.2	1.22	5.7	10.1	9.3	4.5	3.2	3.0	6.0	37.3	13.0	3.1	1.9	5.6
12.3	1.23	7.2	14.6	11.5	5.5	4.5	3.5	7.8	52.7	16.1	4.5	2.7	6.7
15.4	1.24	9.4	22.8	14.7	8.0	5.5	4.6	10.4	94.7	21.6	5.7	4.0	8.8
18.5	1.24	10.8	28.6	16.8	9.2	6.5	5.2	12.2	122.3	24.9	6.9	4.8	10.1
21.6	1.24	12.1	38.7	19.1	10.0	7.1	6.0	13.8	140.9	27.4	7.9	5.9	10.7
24.7	1.25	13.7	51.1	21.8	10.9	8.3	6.7	15.2	215.2	32.0	8.5	5.1	13.5

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 4													
3.1	1.17	1.8	0.9	2.8	1.5	1.0	0.9	1.3	5.3	3.8	0.9	-0.7	2.3
6.2	1.21	3.8	3.7	6.0	3.2	2.3	1.9	3.6	17.0	8.3	2.1	0.4	3.9
9.2	1.22	5.7	10.1	9.3	4.5	3.2	3.0	6.0	37.3	13.0	3.1	1.9	5.6
12.3	1.23	7.2	14.6	11.5	5.5	4.5	3.5	7.8	52.7	16.1	4.5	2.7	6.7
15.4	1.24	9.4	22.8	14.7	8.0	5.5	4.6	10.4	94.7	21.6	5.7	4.0	8.8
18.5	1.24	10.8	28.6	16.8	9.2	6.5	5.2	12.2	122.3	24.9	6.9	4.8	10.1
21.6	1.24	12.1	38.7	19.1	10.0	7.1	6.0	13.8	140.9	27.4	7.9	5.9	10.7
24.7	1.25	13.7	51.1	21.8	10.9	8.3	6.7	15.2	215.2	32.0	8.5	5.1	13.5
Rep 5													
3.1	1.16	1.7	0.9	2.8	1.3	1.1	0.9	1.0	4.2	3.4	0.2	-0.5	1.9
6.2	1.18	3.4	4.5	5.8	2.7	1.7	2.0	2.4	14.5	6.8	0.9	-0.4	3.6
9.2	1.21	5.1	11.0	9.0	3.5	2.9	3.0	4.7	52.8	13.0	1.6	-0.5	6.8
12.3	1.22	6.7	21.0	11.9	5.0	3.3	4.3	6.4	99.4	17.7	2.8	-1.2	9.5
15.4	1.23	8.5	32.5	15.0	6.1	4.5	5.3	8.9	160.2	23.3	3.8	-0.5	11.9
18.5	1.24	10.1	41.1	17.5	7.3	5.6	5.9	10.4	205.5	26.8	4.6	-0.1	13.4
21.6	1.24	11.8	58.2	20.5	9.0	6.0	7.2	12.8	264.0	31.4	5.7	1.3	15.0
24.7	1.25	13.0	74.0	22.7	10.0	6.3	8.2	14.2	301.0	34.1	6.4	2.1	16.0

TEST 21

Load	Bulk Density	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
Rep 1													
3.1	1.01	0.9	0.6	1.8	0.5	0.5	0.7	0.7	0.3	1.3	0.5	0.2	0.5
6.2	1.03	1.9	2.7	3.8	1.0	9.0	1.5	1.6	2.9	3.5	1.0	0.3	1.6
9.2	1.04	3.0	6.8	6.0	1.6	1.4	2.3	2.9	9.6	6.4	1.6	0.6	2.9
12.3	1.05	4.0	11.1	7.8	2.3	1.8	3.0	4.1	22.9	9.6	2.1	0.7	4.5

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)
Rep 2													
3.1	1.01	1.0	0.7	2.0	0.6	0.4	0.8	0.7	0.7	1.6	0.3	0.2	0.7
6.2	1.04	2.1	2.9	4.1	1.3	0.9	1.6	1.7	3.3	3.8	1.0	0.4	1.7
9.2	1.05	3.1	5.8	5.8	2.0	1.4	2.2	2.9	10.4	6.6	1.3	0.8	2.9
12.3	1.05	4.1	10.3	7.8	2.7	1.9	3.0	4.1	23.3	10.0	1.7	0.9	4.4
Rep 3													
3.1	1.01	0.9	0.5	1.7	0.5	0.5	0.6	0.7	0.2	1.2	0.5	0.5	0.4
6.2	1.03	1.9	2.3	3.6	1.0	1.0	1.3	1.9	3.5	4.1	1.1	0.6	1.7
9.2	1.05	2.8	5.3	5.5	1.5	1.5	2.0	3.3	11.2	7.1	1.6	1.0	3.0
12.3	1.06	3.9	9.2	7.4	2.3	2.0	2.7	4.4	23.3	10.0	1.8	1.5	4.2
Rep 4													
3.1	1.01	1.0	0.7	1.9	0.6	0.4	0.7	0.8	0.6	1.6	0.6	0.1	0.8
6.2	1.03	2.0	3.1	4.1	1.1	0.9	1.6	1.9	3.8	4.2	0.9	0.7	1.7
9.2	1.04	3.2	7.8	6.5	1.7	1.5	2.5	3.1	12.5	7.1	1.5	0.6	3.3
12.3	1.05	4.3	11.2	8.2	2.5	2.3	3.0	4.3	21.1	9.5	2.2	1.1	4.2
Rep 5													
3.1	1.01	0.9	0.6	1.8	0.5	0.5	0.7	0.7	0.5	1.5	0.5	0.1	0.7
6.2	1.03	1.9	2.9	3.9	1.0	0.9	1.5	1.8	4.1	4.1	1.2	0.2	1.9
9.2	1.04	3.1	7.3	6.2	1.6	1.4	2.4	3.0	13.4	7.1	1.4	0.3	3.4
12.3	1.05	4.2	10.2	7.8	2.7	2.0	2.9	4.2	23.7	9.7	2.0	0.8	4.5
TEST 22													
Rep 1													
3.1	1.02	1.2	0.7	2.1	0.8	0.6	0.8	1.5	3.3	3.5	0.9	0.1	1.7
6.2	1.04	2.5	4.3	4.9	1.8	0.9	2.0	3.0	15.7	7.5	1.5	0.0	3.7
9.2	1.05	3.7	8.4	7.0	2.7	1.5	2.8	5.6	48.4	13.5	2.9	0.3	6.6

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 2													
3.1	1.02	1.2	0.6	2.1	1.0	0.6	0.8	1.4	2.8	3.2	1.1	-0.1	1.7
6.2	1.04	2.6	2.9	4.5	1.9	1.3	1.6	3.0	14.8	7.3	1.4	0.1	3.6
9.2	1.05	3.9	6.3	6.8	2.9	2.1	2.4	5.5	45.0	13.1	3.3	0.3	6.4
Rep 3													
3.1	1.01	1.1	0.8	2.1	0.9	0.4	0.9	1.5	3.3	3.6	0.9	0.1	1.8
6.2	1.03	2.3	3.1	4.3	1.4	1.2	1.6	3.0	15.4	7.5	1.3	0.2	3.6
9.2	1.04	3.8	6.1	6.7	2.8	2.0	2.3	5.7	44.5	13.3	3.1	0.8	6.3
Rep 4													
3.1	1.01	1.2	0.9	2.3	0.9	0.5	0.9	1.5	3.1	3.5	0.8	0.2	1.6
6.2	1.04	2.4	4.4	4.8	1.4	1.0	1.9	3.3	15.2	7.8	1.4	0.8	3.5
9.2	1.05	3.9	7.4	7.0	2.8	2.0	2.5	5.7	46.3	13.4	2.8	0.7	6.3
Rep 5													
3.1	1.02	1.3	0.6	2.2	1.0	0.8	0.7	1.3	3.3	3.4	0.6	0.0	1.7
6.2	1.04	2.5	2.9	4.5	1.6	1.4	1.5	3.1	22.0	8.4	1.4	-0.5	4.5
9.2	1.05	4.1	5.2	6.7	3.1	2.5	2.1	5.5	48.7	13.5	2.6	0.5	6.5
TEST 23													
Rep 1													
3.1	1.01	1.7	1.3	3.0	1.1	1.0	1.0	1.5	2.8	3.3	0.9	0.2	1.6
6.2	1.03	3.3	4.7	5.8	2.3	1.8	2.0	3.4	14.2	7.6	2.3	0.3	3.6
9.2	1.05	5.0	10.8	8.8	3.5	2.8	3.0	5.7	38.5	12.7	3.6	0.8	5.9
Rep 2													
3.1	1.02	1.6	1.1	2.8	1.0	1.0	0.9	1.2	3.1	3.1	0.8	-0.4	1.7
6.2	1.05	3.2	4.0	5.5	2.1	2.0	1.8	3.3	11.5	7.1	2.3	0.5	3.3
9.2	1.06	4.8	8.2	8.1	3.3	3.1	2.5	5.7	33.3	12.2	3.4	1.4	5.4

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 3													
3.1	1.02	1.8	1.4	3.1	1.3	1.0	1.1	1.3	2.7	3.2	0.7	0.1	1.6
6.2	1.06	3.3	5.4	6.0	2.2	1.8	2.1	3.1	12.9	7.1	1.9	0.2	3.5
9.2	1.06	5.1	11.7	9.1	3.6	2.7	3.2	5.5	33.9	12.1	3.1	1.2	5.5
Rep 4													
3.1	1.02	1.6	0.9	2.6	1.1	1.0	0.8	1.7	2.8	3.6	1.1	0.5	1.6
6.2	1.04	3.2	2.5	5.0	2.3	2.3	1.4	4.4	16.3	9.0	2.4	1.7	3.7
9.2	1.06	4.7	6.2	7.5	3.3	3.2	2.2	6.5	38.6	13.6	3.7	2.2	5.7
Rep 5													
3.1	1.02	1.9	1.0	2.9	1.8	0.9	1.0	1.4	3.2	3.3	0.9	-0.2	1.7
6.2	1.06	3.3	3.5	5.4	2.8	1.7	1.8	3.3	14.2	7.6	1.8	0.4	3.6
9.2	1.06	4.9	7.9	8.0	4.3	2.5	2.8	5.7	30.8	12.0	3.5	1.6	5.2
TEST 24													
Rep 1													
3.1	1.00	1.3	1.2	2.5	0.9	0.5	1.0	0.8	0.8	1.8	0.5	-0.1	0.9
6.2	1.05	2.9	7.0	5.8	2.0	0.8	2.5	2.0	6.5	4.9	0.7	0.4	2.3
9.2	1.07	4.1	13.0	8.1	2.8	1.3	3.4	4.0	25.5	9.8	1.3	0.8	4.5
12.3	1.08	5.5	22.0	10.8	3.6	2.0	4.4	6.1	50.4	14.2	2.9	1.1	6.6
15.4	1.08	6.9	33.3	13.5	4.5	2.8	5.4	7.9	80.1	18.1	4.0	1.6	8.3
Rep 2													
3.1	1.00	1.0	0.8	2.0	0.5	0.5	0.8	0.8	0.9	1.8	0.5	0.1	0.9
6.2	1.06	2.3	0.4	4.6	1.3	1.0	1.8	2.0	7.5	5.0	1.0	-0.2	2.6
9.2	1.07	3.7	11.0	7.5	2.1	1.5	3.0	3.8	24.9	9.4	1.8	-0.1	4.7
12.3	1.07	4.8	14.2	9.1	2.9	2.3	3.4	5.8	54.1	14.2	2.7	0.6	6.8
15.4	1.08	6.4	27.4	12.4	4.1	2.8	4.8	8.0	76.5	18.0	3.6	2.3	7.9

Load	Bulk Density	Type A Cell σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	6 DST σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 3													
3.1	1.02	1.1	0.7	2.0	0.8	0.4	0.8	0.8	0.5	1.5	0.6	0.2	0.7
6.2	1.06	2.4	4.4	4.8	1.6	0.9	2.0	2.0	6.5	4.9	1.2	0.0	2.5
9.2	1.07	4.0	11.7	7.9	2.7	1.5	3.2	4.3	22.5	9.7	2.6	0.7	4.5
12.3	1.08	5.4	20.9	10.6	3.6	2.0	4.3	6.1	44.5	13.7	3.2	1.3	6.2
15.4	1.08	6.7	30.9	13.0	4.6	2.5	5.3	8.0	74.0	17.8	4.0	2.1	7.9
Rep 4													
3.1	1.02	1.2	1.3	2.5	0.6	0.4	1.1	0.8	1.1	1.8	0.8	-0.2	1.0
6.2	1.06	2.6	6.3	5.4	1.5	0.8	2.3	2.1	7.8	5.2	1.2	-0.1	2.7
9.2	1.07	3.9	1.6	8.5	2.3	1.0	3.8	3.9	26.7	9.7	2.2	-0.2	5.0
12.3	1.08	5.2	2.2	10.5	3.2	1.8	4.4	6.1	53.0	14.4	3.1	0.8	6.8
15.4	1.08	6.6	3.4	13.2	4.3	2.3	5.5	8.0	81.1	18.3	4.0	1.7	8.3
Rep 5													
3.1	1.00	1.0	0.7	2.0	0.6	0.5	0.8	1.9	1.0	2.0	0.4	0.2	0.9
6.2	1.04	2.5	4.8	5.0	1.5	1.0	2.0	2.2	8.2	5.5	1.0	0.1	2.7
9.2	1.07	3.9	13.3	8.1	2.2	1.5	3.3	3.9	30.8	10.2	1.6	-0.1	5.2
12.3	1.07	5.0	17.5	9.8	3.3	2.0	3.9	5.9	60.9	14.9	2.6	0.4	7.2
15.4	1.08	6.4	29.2	12.5	4.1	2.5	5.0	7.9	90.6	18.8	3.3	1.5	8.7
TEST 25													
Rep 1													
3.1	1.01	1.7	1.5	3.1	1.3	0.8	1.2	1.3	3.0	3.2	0.9	-0.1	1.6
6.2	1.05	3.7	6.0	6.5	2.3	2.2	2.2	3.9	25.3	9.5	2.4	-0.2	4.9
9.2	1.07	5.7	12.9	9.9	3.9	3.4	3.2	6.6	68.2	15.9	3.7	0.2	7.9
12.3	1.08	7.4	24.5	13.2	4.7	4.5	4.3	9.1	136.3	22.3	5.0	0.0	11.1
15.4	1.09	9.5	42.5	17.1	6.0	5.5	5.8	11.2	206.7	27.4	6.0	0.1	13.7

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 2													
3.1	1.02	2.0	1.9	3.6	1.2	1.2	1.2	1.4	2.8	3.2	1.2	-0.2	1.7
6.2	1.05	3.9	7.1	7.0	2.4	2.4	2.3	3.7	19.9	8.6	2.8	-0.2	4.4
9.2	1.07	5.8	15.5	10.3	3.5	3.5	3.4	6.3	61.7	15.1	3.8	0.0	7.6
12.3	1.08	8.0	30.3	14.4	4.9	4.7	4.8	9.2	138.5	22.5	5.2	0.0	11.2
15.4	1.09	9.9	45.4	17.7	6.2	5.8	5.9	11.1	207.3	27.4	5.8	0.1	13.6
Rep 3													
3.1	1.01	1.6	1.2	2.9	1.0	1.0	1.0	1.5	4.1	3.8	0.9	-0.2	2.0
6.2	1.07	3.5	5.9	6.3	2.3	2.0	2.2	3.8	25.0	9.3	2.3	-0.3	4.8
9.2	1.08	5.4	12.8	9.5	3.4	3.3	3.1	6.3	81.9	16.6	2.7	-0.5	8.5
12.3	1.09	7.5	24.5	13.2	4.8	4.5	4.4	8.8	155.7	22.8	4.6	-0.1	11.9
15.4	1.09	9.5	37.7	16.6	6.0	5.8	5.4	11.0	224.3	27.9	5.6	-0.6	14.2
Rep 4													
3.1	1.01	1.9	1.3	3.2	1.3	1.1	1.1	1.7	4.6	4.0	1.3	-0.2	2.1
6.2	1.07	4.1	7.7	7.3	2.8	2.3	2.5	4.2	28.0	10.1	2.6	-0.2	5.1
9.2	1.08	5.4	8.5	8.7	4.0	3.4	2.7	6.6	78.9	16.7	3.3	-0.1	8.4
12 3	1.09	8.4	32.9	15.0	5.2	5.0	5.0	9.3	153.7	23.4	4.4	0.2	11.6
15.4	1.09	10.4	51.5	18.7	6.6	5.9	6.4	11.4	231.9	28.7	5.7	0.1	14.4
Rep 5													
3.1	1.01	1.8	1.1	3.0	1.4	1.0	1.0	1.7	5.5	4.0	1.6	-0.6	2.3
6.2	1.08	3.9	5.6	6.6	2.8	2.2	2.2	4.3	30.8	10.6	2.5	0.0	5.3
9.2	1.08	5.8	14.5	10.2	4.0	3.2	3.5	6.8	77.5	16.7	3.5	0.1	8.3
12.3	1.09	7.9	27.6	14.0	5.1	4.6	4.7	9.5	145.9	23.2	5.0	0.3	11.4
15.4	1.09	9.8	45.2	17.6	6.2	5.7	5.9	11.5	228.1	28.6	6.0	-0.1	14.3

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
TEST 26													
Rep 1													
3.1	1.02	2.8	5.4	5.5	1.5	1.4	2.0	1.5	4.1	3.7	1.0	-0.3	2.0
6.2	1.06	5.5	20.6	10.7	3.0	2.8	4.0	4.6	33.7	11.2	2.3	0.3	5.5
9.2	1.07	7.8	40.1	15.1	4.4	4.0	5.6	7.5	92.5	18.5	3.4	0.6	8.9
12.3	1.08	10.4	62.6	19.6	6.2	5.5	7.0	10.0	162.1	24.5	4.9	0.6	11.9
15.4	1.08	12.8	80.7	23.2	8.0	7.3	8.0	12.7	236.5	30.2	6.6	1.3	14.4
Rep 2													
3.1	1.02	2.3	2.1	4.0	1.6	1.4	1.3	1.8	5.6	4.6	0.7	0.2	2.2
6.2	1.04	4.7	8.2	8.0	3.2	2.9	2.6	4.4	33.8	11.1	1.6	0.6	5.3
9.2	1.05	7.4	15.5	11.8	5.8	4.5	3.7	7.3	89.5	18.2	2.4	1.3	8.5
12.3	1.06	9.4	32.1	15.9	6.2	6.0	5.0	10.0	158.0	24.4	3.7	1.8	11.3
15.4	1.07	11.5	49.1	19.6	7.5	7.5	6.1	12.9	253.4	31.2	5.7	1.9	14.7
Rep 3													
3.1	1.02	2.2	2.0	3.8	1.8	1.1	1.4	1.7	6.7	4.6	5.8	-0.2	2.4
6.2	1.06	4.7	10.0	8.3	3.5	2.2	3.0	4.6	34.1	11.3	2.0	0.5	5.4
9.2	1.07	6.6	18.4	11.4	5.3	3.1	4.1	7.5	101.9	19.1	2.1	1.2	8.9
12.3	1.07	8.8	35.8	15.4	7.0	3.9	5.8	9.5	166.5	24.4	2.7	1.4	11.5
15.4	1.08	11.2	56.8	19.8	8.5	5.4	7.2	11.7	227.2	29.0	3.7	2.2	13.4
Rep 4													
3.1	1.02	2.4	3.8	4.6	1.3	1.2	1.7	1.7	3.4	3.8	1.0	0.3	1.8
6.2	1.07	5.0	15.2	9.5	2.8	2.6	3.4	4.9	31.2	11.2	2.7	0.7	5.3
9.2	1.07	7.2	27.6	13.2	4.3	4.0	4.6	8.2	99.3	19.6	3.5	1.4	9.1
12.3	1.08	9.7	49.5	17.8	5.8	5.5	6.2	10.4	164.1	25.1	5.0	1.2	11.9
15.4	1.08	11.9	68.6	21.5	7.2	7.1	7.2	12.9	238.2	30.6	5.8	2.2	14.2

TABLE A (CONTINUED)

		Type A Cell						6 DST					
Load	Bulk Density	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)	(psi)	(psi)^2	(psi)	(psi)	(psi)	(psi)
Rep 5													
3.1	1.03	2.3	7.9	5.4	1.7	-0.1	2.8	1.7	2.8	3.6	1.2	0.4	1.6
6.2	1.07	4.8	10.2	8.5	3.2	2.8	2.9	4.2	30.9	10.6	1.3	0.7	5.0
9.2	1.07	7.1	20.0	12.2	5.0	4.1	4.1	6.8	92.5	17.8	2.3	0.2	8.8
12.3	1.08	9.3	37.8	16.3	6.2	5.3	5.5	9.8	162.7	24.4	4.5	0.6	11.9
15.4	1.08	11.3	57.4	20.0	7.4	6.5	6.8	12.5	245.1	30.4	5.9	1.3	14.6
TEST 27													
Rep 1													
3.1	1.02	0.8	0.2	1.3	0.7	0.4	0.5	0.7	1.0	1.8	0.6	-0.2	1.0
6.2	1.07	1.7	1.5	3.1	1.4	0.6	1.2	1.9	6.4	4.7	1.3	-0.2	2.5
9.2	1.09	2.6	3.2	4.6	2.0	1.1	1.7	3.5	27.0	9.1	2.3	-1.1	5.1
12.3	1.10	3.6	4.0	5.9	2.8	2.1	1.9	5.0	63.3	13.7	3.3	-1.9	7.8
15.4	1.12	4.7	9.5	8.1	3.9	2.1	3.0	6.9	108.5	18.3	4.3	-2.0	10.2
Rep 2													
3.1	1.02	0.8	0.3	1.4	0.6	0.3	0.6	0.7	0.9	1.7	0.5	-0.2	1.0
6.2	1.07	1.6	1.3	2.9	1.3	0.7	1.1	1.9	6.3	4.6	1.5	-0.3	2.5
9.2	1.09	2.4	3.7	4.5	1.8	0.8	1.9	3.4	27.2	9.1	2.5	-1.2	5.2
12.3	1.10	3.4	4.4	5.8	2.5	2.0	1.9	4.8	68.2	13.7	3.4	-2.6	8.2
15.4	1.11	4.6	10.7	8.2	3.8	1.9	3.2	6.6	118.3	18.4	4.3	-3.0	10.7
Rep 3													
3.1	1.02	0.9	0.2	1.3	0.8	0.5	0.4	0.6	1.4	1.8	0.4	-0.5	1.2
6.2	1.06	1.8	0.9	2.9	1.5	1.0	0.9	1.8	6.9	4.8	0.9	-0.3	2.5
9.2	1.09	2.6	2.3	4.3	2.1	1.5	1.4	3.3	27.8	9.0	2.4	-1.5	5.2
12.3	1.10	3.7	4.0	6.0	3.0	2.1	1.9	5.0	56.6	13.2	3.1	-1.5	7.4
15.4	1.12	4.8	8.1	8.0	4.0	2.5	2.8	7.0	95.5	17.8	4.5	-1.3	9.5

TABLE A (CONTINUED)

Load	Bulk Density	Type A Cell						6 DST					
		σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}	σ_m	II_s	σ_I	σ_{II}	σ_{III}	τ_{max}
(psi)	(gm/cc)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)	(psi)	$(psi)^2$	(psi)	(psi)	(psi)	(psi)
Rep 4													
3.1	1.02	0.8	0.2	1.3	0.6	0.5	0.4	0.6	1.6	1.9	0.5	-0.6	1.2
6.2	1.07	1.8	1.3	3.1	1.4	0.8	1.1	1.8	8.5	4.9	1.2	-0.8	2.9
9.2	1.08	2.7	4.0	5.0	2.0	1.1	1.9	3.4	31.0	9.5	2.3	-1.5	5.5
12.3	1.09	3.9	3.6	6.0	2.8	2.8	1.6	5.3	74.0	14.7	3.2	-2.1	8.4
15.4	1.10	4.8	8.8	8.2	3.6	2.6	2.8	7.1	123.7	19.5	4.0	-2.1	10.8

ROOM USE ONLY