A COMPUTERIZED SENSITIVITY ANALYSIS OF SELECTED ASSUMPTIONS IN AN ANNUAL ECONOMETRIC MODEL OF THE U. S. ECONOMY: THE ELECTRIC KLEIN - GOLDBERGER

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ABSTRACT

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By

Arthur M. Havenner

The practicing macroeconometric modeler faces a host of problems competing for his attention. Among these are the econometric problems of efficiency, simultaneous equations inconsistency, autoregressive disturbances, lag specification, and nonlinearity. Often correcting any one of these problems precludes correcting others, and it is therefore important to assess the relative importance of each of these considerations. This thesis examines these five problems in the context of a revised Klein-Goldberger model of the U.S. economy, estimated over an extended data set encompassing the years 1929-41 and 1948-69. The results emphasize the importance of accurate lag specification and autoregression correction, with simultaneous equation inconsistency being relatively unimportant.

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I. Introduction

The practicing macroeconometric modeler faces a host of problems competing for his attention. Among these are the choices of model size, observation time period, and equation specification, as well as the econometric problems of errors of observation, autoregressive disturbances, efficiency, simultaneous equations inconsistency, lag specification, and nonlinearity. Often correcting any one of these problems makes it more difficult or impossible to account for some, or all, of the others, is therefore important to weigh the relative importance of these assorted complications.

This thesis examines the importance of five problems in the context of a revised Klein-Goldberger (henceforth K-G) model of the U.S. economy, estimated over an extended data set encompassing the years 1929-41 and 1948-69. The model equations and symbol definitions are given in Table 1.

As is apparent by inspection, the equations retain the basic structure of the K-G model -- particularly the tripartite income division conjoined in the consumption function -- although every stochastic equation except corporate profits has been respecified. In addition, the determination of investment has been disaggregated and the equations determining household and business liquidity have been dropped. The extended data set (two-thirds of which is post-war), gives a more modern version of the classic K-G model.

The five econometric problems examined in the context of the K-G model deal with (1) the gains from full information estimation procedures, (2) the practical importance of equation simultaneity, (3) the returns from generalized lag structures, (4) the importance of non-

Table 1

The Model Equations and Symbols

1. Consumption

$$C_t = \beta_{1,0} + \beta_{1,1}(1-\lambda_1)^{r_1} \sum_{i=o}^{\infty} \binom{r_1+i-1}{i} \lambda_1^i (Y_C)_{t-i} + U_{1,t}$$

2. Investment, plant and equipment

$$I_t^d = \beta_{2,0} + \beta_{2,1} 0_t + \beta_{2,2}(i_L) + \beta_{2,3} I_{t-1}^d + U_{2,t}$$

3. Investment, residential

$$I_t^r = \beta_{3,0} + \beta_{3,1} X_t + \beta_{3,2}(i_s)_t + U_{3,t}$$

4. Investment, inventories

$$I_t^i = \beta_{4,0} + \beta_{4,1} X_t^i + \beta_{4,2}(I_{STK}^i)_{t-1} + U_{4,t}$$

5. Imports

$$(F_I)_t = \beta_{5,0} + \beta_{5,1}(Y_d)_t + \beta_{5,2}(p_F)_t + \beta_{5,3}(F_I)_{t-1} + U_{5,t}$$

6. Production

$$X_t^p = \beta_{6,0} + \beta_{6,1}(h_m)_t + \beta_{6,2} K_t + U_{6,t}$$

7. Long term interest

$$(i_L)_t = \beta_{7,0} + \beta_{7,1}(1-\lambda_2)^{r_2} \sum_{i=o}^{\infty} \binom{r_2+i-1}{i} \lambda_2^i (i_s)_{t-i} + U_{7,t}$$

8. Short term interest

$$(i_s)_t = \beta_{8,0} + \beta_{8,1}(i_d)_t + \beta_{8,2} R_t + U_{8,t}$$

9. Corporate saving

$$(S_c)_t = \beta_{9,0} + \beta_{9,1}(P_c - T_c)_t + \beta_{9,2}(P_c - T_c - S_c)_{t-1} + U_{9,t}$$

10. Corporate profits

$$(P_c)_t = \beta_{10.0} + \beta_{10.1} P_t + U_{10,t}$$

Table 1 (continued)

11. Depreciation

$$D_t = \beta_{11,0} + \beta_{11,1} K_t + U_{11,t}$$

12. Agricultural income

$$(A_{P_A})_t = \beta_{12,0} + \beta_{12,1}(Y_{NF})_t + \beta_{12,2}(D_W)_t + U_{12,t}$$

13. Private wage bill

$$(W_1)_t = \beta_{13,0} + \beta_{13,1}(X^P)_t + \beta_{13,2} X^P_{t-1} + U_{13,t}$$

14. Nominal wage rate

$$\Delta w_t = \beta_{14,0} + \beta_{14,1} (\%U)_t^{-1} + \beta_{14,2} (\Delta p)_t + U_{14,t}$$

Identities

15. Gross national product

$$X_t = C_t + I_t^d + I_t^r + I_t^i + G_t + (F_X)_t - (F_I)_t$$

16. National income

$$Y_t = X_t - D_t - T_t$$

17. Profits

$$P_t = Y_t - (A_1)_t - (A_2)_t - (W_1)_t - (W_2)_t$$

18. Capital

$$K_t = I_t^d + I_t^r + I_t^i - D_t + K_{t-1}$$

19. Price level

$$p_t = \frac{w_t \cdot h_t \cdot (N_W)_t}{(W_1)_t + (W_2)_t}$$

Table 1 (continued)

Internal definitions

20. $X^i_t = X_t - I^i_t$

21. $X^p_t = X_t - (W_2)_t$

22. $(Y_c)_t = (W_1)_t + (W_2)_t - (T_W)_t + \left(\frac{\hat{\beta}_2}{\beta_1}\right) (A_1 + A_2 - T_A)_t + \left(\frac{\hat{\beta}_3}{\beta_1}\right) (P - S_c - T_p)_t$

23. $(Y_{NF})_t = (W_1)_t + (W_2)_t - (T_W)_t + P_t - (S_c)_t - (T_p)_t$

24. $0 = P_t + D_t + (A_1)_t + (A_2)_t - (T_A)_t - (T_p)_t$

25. $\Delta p_t = p_t - p_{t-1}$

26. $\Delta w_t = w_t - w_{t-1}$

27. $(Y_d)_t = Y_t - (T_W)_t - (T_A)_t - (T_p)_t$

28. $(p_F)_t = p_t / p_{M\,t}$

29. $(A_{p_A})_t = [p_t (A_1)_t] / (p_A)_t$

30. $(h_m)_t = h_t \, (N_W - N_G)_t$

31. $(P_c - T_c)_t = (P_c)_t - (T_c)_t$

Table 1 (continued)

Symbols

Symbol	Status*	Explanation (Value in billions of 1958 dollars unless otherwise noted).
C	*	Consumer expenditures
Y_c	*	Share weighted personal disposable income
I^d	*	Investment in plant and equipment
O	*	Disposable nonwage income
i_L	*	Average yield on corporate bonds (Moody's), percent
I^r	*	Residential construction
X	*	Gross national product
i_s	*	Yield on prime commercial paper, 4-6 months, percent
I^i	*	Change in business inventories
X^i	*	GNP less inventory investment
I^i_{STK}		Stock of inventories from arbitrary origin(1928≡0)
F_I	*	Imports of goods and services
Y_d	*	Disposable income
P_F	*	Ratio of domestic to import prices, 1958 ≡ 1.00
X^p	*	Private GNP
h_m	*	Index of manhours
K	*	End of year stock of private capital from arbitrary origin (1928≡0)
i_d		Average discount rate at all Federal Reserve Banks, percent

---

* means the variable is endogenous to the model.

Table 1 (continued)

Symbol	Status*	Explanation (Value in billions of 1958 dollars unless otherwise noted).
R		Year end ratio of member banks excess to required reserves, pure number
S_c	*	Corporate savings
P_c	*	Corporate profits
T_c		Corporate income taxes
p	*	Price index of gross national product, 1958≡1.00
w	*	Annual earnings, thousands of current dollars
h	*	Index of hours worked per year, 1954≡100
N_W	*	Number of wage and salary earners, millions of persons
W_1	*	Private employee compensation
W_2		Government employee compensation
A_1	*	Private farm income ($A_1 + A_2$ = total farm income)
A_2		Government payments to farmers
P	*	Nonwage nonfarm income
T_W		Personal and payroll taxes less transfers associated with wage and salary income
T_A		Taxes less transfers associated with farm income
T_P		Personal and corporate taxes less transfers associated with nonwage nonfarm income
Y_{NF}	*	Nonfarm income
D	*	Capital consumption allowances
Δp	*	Annual change in the GNP price index

Table 1 (continued)

Symbols	Status	Explanation (Value in billions of 1958 dollars unless otherwise noted).
Δw	*	Annual change in earnings, thousands of current dollars
A_{p_A}	*	Private farm income deflated by p_A
p_A	*	Index of agricultural prices, 1958≡1.00
N_G		Number of government employees, millions of persons
P_c-T_c	*	Corporate profits less corporate taxes
$\%U^{-1}$		Inverse of percent unemployed $[N/(N-N_W)]$
N		Number of persons in the labor force, millions of persons
p_M		Index of import prices, 1958≡1.00
F_X		Exports of goods and services
Y^P	*	$(1-\lambda_1)^{r_1} \sum_{i=o}^{\infty} \binom{r_1+i-1}{i} \lambda_1^i \left(Y_c\right)_{t-i}$, Pascal lag weighted Y_c
i^P	*	$(1-\lambda_2)^{r_2} \sum_{i=o}^{\infty} \binom{r_2+i-1}{i} \lambda_2^i \, (i_s)_{t-i}$, Pascal lag weighted i_s
Y_c^{-1}	*	Inverted Pascal lag on share weighted income } See Section III.A.4.c.
i_s^{-1}	*	Inverted Pascal lag on the short term interest rate } See Section III.A.4.c.
c_C		Restricted Pascal lag constant term, consumption } See Section III.A.1., Eqn. (6).
c_{i_L}		Restricted Pascal lag constant term, long term interest } See Section III.A.1., Eqn. (6).
T^*		Composite tax variable, T_w, T_A, T_p each at mean share.

linearities, and (5) the operational significance of the autoregression correction.

A. The Gains From Full Information Estimation

Unlike information contained in overidentifying restrictions, the information in across-equation disturbance variance-covariance matrices does not enter -- even asymptotically -- if not specifically allowed for in the estimation procedure. If the across-equations covariances are not large, however, adding the additional parameters may actually result in worse finite sample estimates due to the lost prior information (that the covariances are approximately zero). One of the objectives of this thesis is to examine alternative limited- and full-information estimators and make a judgment on which is superior under varying conditions.

B. Equation Simultaneity and Policy Endogeneity

The well-known difficulties resulting from simultaneous equations lead to elaborate estimators that are difficult to correct for the violation of other assumptions (e.g., disturbance autoregression and variance nonlinearity). Policy endogeneity is developed as a special case of equation simultaneity (as is the errors in variables problem). One of our objectives is to examine the importance of the resulting inconsistencies.

C. The Returns From Generalized Lags

The time-path of response of a macroeconometric model is of critical importance and hinges directly in the specification of the distributed lags. We shall estimate a more general lag formulation which more closely resembles the actual underlying processes. Generalized expectations and adjustment processes are developed and convoluted with each other, and then estimated by initial condition parameterization in order to make them more robust with respect to violation of other assumptions. The returns from this computationally expensive process are assessed.

D. Nonlinearities

One often has at least the suspicion that the world is significantly nonlinear. We shall work out the full conditions for generalizing the estimators and then to examine the practical effects of nonlinearities on various structural and reduced form coefficients.

E. Autoregression

Usually the most important single feature of time-series data is its autoregressive properties; and yet simultaneous macroeconomic models often are not corrected for autoregression because of the estimation difficulty. We derive an efficient, computationally feasible method for correcting autoregression in the presence of equation simultaneity, and analyze the importance of this problem.

After considering each of the above problems in turn and determining which were important and which were not important in the context of the specific model examined, we attempt to make estimation recommendations. In addition, forecasts and dynamic analysis of the model follow (in an appendix) to allow the reader to judge the model's correspondence to the world -- which necessarily conditions the results.

II. Econometric Problems

A. Three Stage Least Squares

1. Disturbance Properties

Given values for the parameters (r_1, r_2, λ_1, λ_2) of the two Pascal lags, the model equations[1] can be written in the form

$$(1)\quad \underset{(G\times G)}{(N+A)}\ \underset{(G\times T)}{Y} + \underset{(G\times G)}{B}\ \underset{(G\times T)}{Y_{-1}} + \underset{(G\times K)}{C}\ \underset{(K\times T)}{X} = \underset{(G\times T)}{U^*}$$

(the subscript -1 refers to a matrix whose value at time t is equal to the unsubscripted matrix's value at time t-1) where

G is the number of jointly dependent variables;

K is the number of predetermined variables;

T is the number of observations in the sample;

N is a normalizing matrix with $n_{ij}=1$ if the jth variable in Y is the normalizing variable in the ith equation, and $n_{ij}=0$ otherwise;

N,A,B, and C are parameters;

Y is the (GxT) matrix of sample values of the jointly dependent variables;

X is the (KxT) matrix of sample values of the predetermined variables, including the Pascal lag pseudo-variables;

and U* is the (GxT) matrix of stochastic error terms.

Since there are only fourteen stochastic equations in the model, all of the rows of U* after the first fourteen will be zeros. Defining U as the residual submatrix of U* after deleting the rows of zeros corresponding to the identities, and letting U(t) be the t^{th} column of U, the following assumptions complete the model:

(i) $E(U) = \emptyset$

(ii) $E[U(t)U(t)'] = \Sigma$, $t=1,2,\ldots, T$, Σ positive definite;

(iii) $E[U(t)U(s)']$ is diagonal, $t=1,2,\ldots, T$, $s=1,2,\ldots, T$, $t \neq s$;

(iv) $\text{plim } T^{-1} XU^{*\prime} = \text{plim } T^{-1} Y_{-1} U^{*\prime} = \emptyset$;

(v) The moment matrix of the jointly dependent and predetermined variables is well behaved in the limit;

(vi) N+A has an inverse.

[Assumptions (ii) and (iii) state that although there may be contemporaneous correlation of the disturbances across equations (measured by Σ), there is assumed to be no correlation of the disturbances of different equations at different time periods; under these assumptions, autoregression does not result in inconsistency unless the equation involves a lagged value of the normalizing variable.]

2. Parameter Estimates, Nonautoregression Corrected Model

Two sets of estimates of the parameters of the fourteen stochastic equations will be presented: (1) classical 2SLS and (3) 3SLS.

The format is

$$\begin{bmatrix} \text{2SLS} \\ \text{3SLS} \end{bmatrix}$$

with estimated coefficient standard errors in parentheses below the point estimates where appropriate. R^2, the standard error of estimate, and the Durbin-Watson statistic[1] are reported to the extreme right where applicable.

---

1 The Durbin-Watson statistic looses one additional observation due to the discontinuity at the war.

B. Simultaneous Equation Problems

1. Causality and instrument selection

a. Definitions

Many econometricians ignore the work of Simon[1], Koopmans[2], and Fisher[3] and lose some potential generality in defining endogenous, exogenous, jointly dependent, and predetermined variables.[4] The loss occurs when the system is decomposable or recursive so that different terms are necessary to describe the statistical properties of a variable, as opposed to its modeling relation to a complete subsystem.

Consider the model[5]

(1) $y_1 = \beta_o y_2 + \beta_1 Z + \beta_2 p_1 + \beta_3 (y_1)_{-1} + \epsilon_1$

(2) $y_2 = \beta_4 y_1 + \beta_5 Z + \beta_6 p_2 + \epsilon_2$

where y_1 and y_2 are to be determined, and ϵ_1 and ϵ_2 are stochastic disturbances. Z, p_1, and p_2 are exogenous to equations (1) and (2); that is "... they are variables which affect the economic system but are not in turn affected by it, or at least are only affected to a negligible degree by it."[6,7] The residual variables y_1 and y_2 -- which both affect the system and are affected by it -- are endogenous.

---

1 [54].

2 [33].

3 [21].

4 See Goldberger [24], pp. 294-295; Christ [5], pp. 179-180; Kmenta [42], pp. 532-534.

5 These equations are a modified version of Koopman's example [33], p. 202.

6 [52], p. 197.

7 This asymmetric relation has also be defined as "casuality"([54], pp. 6-7).

Statistically, the relevant question is whether any variables on the right of each equation are correlated with the disturbance of that equation; correlated right hand side variables are termed "jointly dependent."[1] If the system is fully integrated, all endogenous variables will be correlated, whether or not Σ, the across equations disturbance variance-covariance matrix, is diagonal (this is the well known simultaneous equations problem); thus the set of jointly dependent variables will include all current endogenous variables. Even in the case of diagonal Σ, however, although current endogeneity is sufficient cause for correlation with the disturbance and thus joint dependence, it is not necessary. For example, if anticipations are significant, then y_{1t} may depend on $y_{1,t+1}$, or lagging, $y_{1,t-1}$ may depend on $y_{1,t}$; then lagged endogenous variables can be jointly dependent, and not predetermined as they are usually classified.

If we add a third equation to the system (1) and (2),

(3) $Z = \beta_7 P_3 + \varepsilon_3$,

then Z becomes endogenous to the system composed of (1), (2), and (3), though still exogenous to the closed system (1) and (2): the expanded system is recursive. In this case, if ε_3 is uncorrelated with ε_1 and ε_2, Z may be treated as predetermined, when estimating the parameters of (1)

---

1 Errors in variables may be regarded as a special case of the simultaneous equations problem where the omitted equation is the one defining the error in the variable; not surprisingly, ordinary least squares estimates are inconsistent even when the variable error is uncorrelated with the equation error (Σ is diagonal).

and (2), although, if anticipations are important, $y_{1,t-1}$ may <u>not</u>. Then the set of jointly dependent variables is not equivalent to the set of endogenous variables, and the set of predetermined variables is not the union of the lagged endogenous[1] and exogenous variables. Endogenous/exogenous describes the relation of a variable to a particular complete sybsystem, while jointly dependent/predetermined indicates presence or absence of correlation of a right hand side variable with its equation error term.

b. Instruments

We shall consider two methods of computing the instrumental variables to be included in the second stage of 2SLS: (1) the classical method of including all predetermined variables in the first stage regression that appear with nonzero coefficients in the reduced form equation for the particular right hand side jointly dependent variable being considered; and (2) a tame version of Fisher's structurally ordered instrumental variables (SOIV) procedure.

---

1 Lagged endogenous variables have probably been automatically included in predetermined variables because of a confusion of temporal precedence with causality; causality is the basis of exogeneity, not temporal precedence, and it is exogeneity that guarantees no simultaneous equations problem. In our classification, "lagged endogenous" is just a special case of "endogenous."

Table 2
Nonautoregression Corrected Parameter Estimates
(Order: 2SLS, 3SLS)

Equation	R^2	S^2	DW
(1) Consumption	0.9991	16.12	1.15
(2) Investment, plant and equipment	0.9870	6.51	N/A
(3) Investment, housing	0.8363	12.15	0.53
(4) Investment, inventories	0.6570	8.46	1.55

(1) Consumption

$$C_t = \begin{bmatrix} -6.267 \\ (2.088) \\ \\ -13.607 \\ (0.012) \end{bmatrix} + \begin{bmatrix} 1.040 \\ (0.007) \\ \\ 1.019 \\ (0.003) \end{bmatrix} \underbrace{\left[(1-0.06)^4 \sum_{i=o}^{\infty} \binom{i+3}{i} (0.06)^i (Y_c)_{t-i}\right]_t}_{\equiv Y_t^P}$$

(2) Investment, plant and equipment

$$I_t^d = \begin{bmatrix} 0.315 \\ (0.051) \\ \\ 0.317 \\ (0.023) \end{bmatrix} + \begin{bmatrix} -1.173 \\ (0.413) \\ \\ -1.253 \\ (0.190) \end{bmatrix} O_t + \begin{bmatrix} -8.668 \\ (2.509) \\ \\ -8.189 \\ (1.430) \end{bmatrix} (i_L)_t + \begin{bmatrix} 0.505 \\ (0.095) \\ \\ 0.495 \\ (0.410) \end{bmatrix} I_{t-1}^d$$

(3) Investment, housing

$$I_t^r = \begin{bmatrix} 0.057 \\ (0.006) \\ \\ 0.056 \\ (0.004) \end{bmatrix} X_t + \begin{bmatrix} -2.099 \\ (0.629) \\ \\ -1.902 \\ (0.308) \end{bmatrix} (i_s)_t + \begin{bmatrix} -0.482 \\ (1.470) \\ \\ -0.782 \\ (1.336) \end{bmatrix}$$

(4) Investment, inventories

$$I_t^i = \begin{bmatrix} 0.118 \\ (0.020) \\ \\ 0.122 \\ (0.015) \end{bmatrix} X_t^i + \begin{bmatrix} -24.163 \\ (4.302) \\ \\ -24.773 \\ (3.237) \end{bmatrix} + \begin{bmatrix} -0.477 \\ (0.092) \\ \\ -0.502 \\ (0.067) \end{bmatrix} (I_{STK}^i)_{t-1}$$

(5) Imports

$$(F_I)_t = \begin{bmatrix} 0.028 \\ (0.010) \\ \\ 0.030 \\ (0.006) \end{bmatrix} (Y_d)_t + \begin{bmatrix} 5.672 \\ (2.556) \\ \\ 6.272 \\ (1.454) \end{bmatrix} (P_F)_t + \begin{bmatrix} -9.290 \\ (3.396) \\ \\ -10.037 \\ (1.954) \end{bmatrix} + \begin{bmatrix} 0.813 \\ (0.111) \\ \\ 0.779 \\ (0.064) \end{bmatrix} (F_I)_{t-1}$$

R^2 = 0.9891, S^2 = 1.52, DW = N/A

(6) Production

$$X^p_t = \begin{bmatrix} 6.231 \\ (0.509) \\ \\ 6.242 \\ (0.263) \end{bmatrix} (h_m)_t + \begin{bmatrix} 0.285 \\ (0.021) \\ \\ 0.281 \\ (0.011) \end{bmatrix} K_t + \begin{bmatrix} -17.685 \\ (15.350) \\ \\ -16.234 \\ (8.452) \end{bmatrix}$$

R^2 = 0.9961, S^2 = 104.44, DW = 0.38

(7) Long term interest

$$(i_L)_t = \begin{bmatrix} 0.746 \\ (0.261) \\ \\ -1.498 \\ (0.080) \end{bmatrix} + \begin{bmatrix} 1.600 \\ (0.113) \\ \\ 1.937 \\ (0.041) \end{bmatrix} \left[(1-0.91) \sum_{i=o}^{\infty} (0.91)^i (i_s)_{t-i}\right]_t$$

R^2 = 0.9350, S^2 = 0.17, DW = 0.77

(8) Short term interest

$$(i_s)_t = \begin{bmatrix} -0.228 \\ (0.232) \\ \\ -0.374 \\ (0.208) \end{bmatrix} + \begin{bmatrix} -1.435 \\ (0.284) \\ \\ -1.217 \\ (0.249) \end{bmatrix} R_t + \begin{bmatrix} 1.226 \\ (0.071) \\ \\ 1.263 \\ (0.065) \end{bmatrix} (i_d)_t$$

R^2 = 0.9435, S^2 = 0.43, DW = 1.34

(9) Corporate saving

$$(S_c)_t = \begin{bmatrix} 0.820 \\ (0.039) \\ \\ 0.779 \\ (0.026) \end{bmatrix} (P_c - T_c)_t + \begin{bmatrix} -2.606 \\ (0.951) \\ \\ -3.407 \\ (0.687) \end{bmatrix} + \begin{bmatrix} -0.466 \\ (0.121) \\ \\ -0.326 \\ (0.069) \end{bmatrix} (P_c - T_c - S_c)_{t-1}$$

$R^2 = 0.9662$, $S^2 = 2.96$, $DW = 1.94$

(10) Corporate profits

$$(P_c)_t = \begin{bmatrix} 0.570 \\ (0.015) \\ \\ 0.565 \\ (0.013) \end{bmatrix} P_t + \begin{bmatrix} -11.245 \\ (1.395) \\ \\ -10.977 \\ (1.229) \end{bmatrix}$$

$R^2 = 0.9797$, $S^2 = 10.91$, $DW = 0.30$

(11) Depreciation

$$D_t = \begin{bmatrix} 0.052 \\ (0.001) \\ \\ 0.051 \\ (0.001) \end{bmatrix} K_t + \begin{bmatrix} 14.475 \\ (0.448) \\ \\ 14.809 \\ (0.385) \end{bmatrix}$$

$R^2 = 0.9889$, $S^2 = 2.79$, $DW = 0.26$

(12) Agricultural income

$$(A_{P_A})_t = \begin{bmatrix} -0.002 \\ (0.003) \\ \vdots \\ -0.001 \\ (0.002) \end{bmatrix} (Y_{NF})_t + \begin{bmatrix} 9.983 \\ (0.554) \\ \vdots \\ 9.816 \\ (0.480) \end{bmatrix} + \begin{bmatrix} 3.393 \\ (0.707) \\ \vdots \\ 3.111 \\ (0.412) \end{bmatrix} (D_W)_t$$

R^2 = 0.6463, S^2 = 1.25, DW = 1.70

(13) Private wage bill

$$(V_1)_t = \begin{bmatrix} 0.414 \\ (0.033) \\ \vdots \\ 0.428 \\ (0.015) \end{bmatrix} X^P_t + \begin{bmatrix} 14.200 \\ (1.184) \\ \vdots \\ -14.456 \\ (1.130) \end{bmatrix} + \begin{bmatrix} 0.151 \\ (0.035) \\ \vdots \\ 0.137 \\ (0.015) \end{bmatrix} X^P_{t-1}$$

R^2 = 0.9991, S^2 = 7.53, DW = 1.09

(14) Nominal wage rate

$$\Delta w_t = \begin{bmatrix} 2.223 \\ (0.397) \\ \vdots \\ 2.284 \\ (0.156) \end{bmatrix} \Delta p_t + \begin{bmatrix} -0.082 \\ (0.017) \\ \vdots \\ -0.081 \\ (0.013) \end{bmatrix} + \begin{bmatrix} 0.030 \\ (0.003) \\ \vdots \\ 0.030 \\ (0.002) \end{bmatrix} U_t^{-1}$$

R^2 = 0.9314, S^2 = 0.001, DW = 1.94

(1) Classical instrument selection

Government expenditures and exports occur only in the GNP identity and thus can be combined into a composite variable $(G+F_X)$; then 29 variables[1] -- W_2, A_2, p_M, N_W, (N_W-N_G), p_A, $(G+F_X)$, R, i_d, T_c, T_W, T_A, T_p, T, U^{-1}, D_W, I^d_{-1}, $(F_I)_{-1}$, w_{-1}, $(I^i_{STK})_{-1}$, K_{-1}, X^P_{-1}, $(P_c-T_c-S_c)_{-1}$, p-1, (Y_c^{-1}), c_C, c_{i_L}, (i_s^{-1}), and a constant--appear with nonzero reduced from coefficients for those jointly dependent variables in the main block of model[1] (i_s and i_L in the recursive subblock will be computed only from the predetermined variables occuring in that block, as discussed above). All of the equations are overidentified and 2SLS is very close to OLS (since the number of instruments is very close to the number of observations).

(2) Structurally ordered instrument selection

Fisher has suggested that the selection of predetermined variables to be used in first stage regression be based on the casual structure of the model; since the efficiency of the estimate depends on the correlation of the instrumental variable and the variable being replaced, it is important to include those predetermined variables that are most directly related to the particular jointly dependent variable in the first stage regression for that variable.

---

1 (Y_c^{-1}) and (i_s^{-1}) are the predetermined variables originating in the (inverted) Pascal lags; c_C and c_{i_L} are the restricted intercept variables for the consumption and long-term interest rate equat.ons, respectively (see Section III.A.1. above).

2 There are 33 observations in the subsample from which the parameters are being estimated.

As Fisher explains[1]

> "Consider any particular endogenous variable in the equation to be estimated, other than the one explained by that equation. The right-hand endogenous variable will be termed of zero causal order. Consider the structural equation (either in its original for or with all variables lagged) that explains that variable. The variables other than the explained one appearing therein will be called of first causal order. Next, consider the structural equations explaining the first causal order endogenous variables. All variables appearing in those equations will be called of second causal order with the exception of the zero causal order variable and those endogenous variables of first causal order the equations for which have already been considered. Note that a given predetermined variable may be of more than one causal order. Take now those structural equations explaining endogenous variables of second casual order. All variables appearing in such equations will be called of third casual order except for the endogenous ones of lower causal order, and so forth. (Any predetermined variables never reached in this procedure are dropped from the eligible set while dealing with the given zero causal order variable).
>
> "The result of this procedure is to use the a priori structural information available to subdivide the set of predetermined variables according to closeness of causal relation to a given endogenous variable in the equation to be estimated."

Fisher then suggests that the predetermined variables be ordered lexicographically according to the lowest causal orders at which each occurs,[2] and then those that are highest (causally closest) in this ranking be retained if their contribution to the R^2 of the first stage equation is large enough according to some preselected criterion (the variables are deleted one by one, retaining those whose deletion causes R^2 to drop more than the criterion allows).

---

1 [21], p. 266; italics his.

2 If one predetermined variable occurs in the first and third (1,3) casual order, and another in the first and second (1,2), then the second predetermined variable ranks ahead of the first. (Alphabetizing is an instance of lexicographic ordering.)

This procedure has been modified in two ways in estimating the present model: (1) the criterion for inclusion of a predetermined variable has been that it appear by the second causal order (rather than using an R^2 test over the lexicographic ordering); and (2) all of the right-hand jointly dependent variables of a given equation use the same (pooled) set of predetermined variables in their first stage regressions (to guarantee orthogonality of all instrumental variables and the second stage disturbance), whereas Fisher's original[1] suggestion was to use a (possibly) different first stage regression for each right-hand jointly dependent variable (even within an equation).

We have also recognized the endogeneity of policy in the SOIV model. If there exist government reaction functions, then the control variables that are exogenous to the particular complete subsystem we are examining need not be predetermined variables. If we consider the meta-model consisting of the subsystem and the reaction functions, then the control variables are not uncorrelated with the equation disturbances (the familiar simultaneous equations problem again) and are not predetermined variables; the fact that they are potentially exogenous (the need not be affected by the subsystem) is irrelevant statistically if they are in fact affected through governmental reaction to the state of the economy.

---

[1] Fisher recognized the potential inconsistency of his original suggestion and noted the above procedure as a consistent alternative [21], p. 273.

Thus all taxes, the discount rate, the ratio of excess to required reserves, and the agricultural price level have been reclassified as jointly dependent.[1]

---

[1] Government expenditures remain predetermined, an inflexible policy instrument. The only predetermined variables obtained from the (otherwise unspecified) reaction functions are G and F_X from the GNP identity for the calculation if i_d and R (presumably some function of GNP).

Table 3
Nonautoregression Corrected Parameter Estimates
(Order: 2SLS, SOIV2SLS, 3SLS)

(1) Consumption

$$C_t = \begin{bmatrix} -6.267 \\ (2.088) \\ * \\ -13.607 \\ (0.012) \end{bmatrix} + \begin{bmatrix} 1.040 \\ (0.007) \\ * \\ 1.019 \\ (0.003) \end{bmatrix} \underbrace{\left[(1-0.06)^4 \sum_{i=o}^{\infty} \binom{i+3}{i} (0.06)^i (Y_c)_{t-i} \right]}_{\equiv Y_t^P} t$$

(2) Investment, plant and equipment

$$I_t^d = \begin{bmatrix} 0.315 \\ (0.051) \\ 0.313 \\ (0.056) \\ 0.317 \\ (0.023) \end{bmatrix} + \begin{bmatrix} -1.173 \\ (0.413) \\ -1.234 \\ (0.449) \\ -1.253 \\ (0.190) \end{bmatrix} O_t + \begin{bmatrix} -8.668 \\ (2.509) \\ -8.373 \\ (2.567) \\ -8.189 \\ (1.430) \end{bmatrix} (i_L)_t + \begin{bmatrix} 0.505 \\ (0.095) \\ 0.509 \\ (0.103) \\ 0.495 \\ (0.410) \end{bmatrix} I_{t-1}^d$$

(3) Investment, housing

$$I_t^r = \begin{bmatrix} 0.057 \\ (0.006) \\ 0.056 \\ (0.007) \\ 0.056 \\ (0.004) \end{bmatrix} X_t + \begin{bmatrix} -2.099 \\ (0.629) \\ -2.081 \\ (0.765) \\ -1.902 \\ (0.308) \end{bmatrix} (i_s)_t + \begin{bmatrix} -0.482 \\ (1.470) \\ -0.131 \\ (1.492) \\ -0.782 \\ (1.336) \end{bmatrix}$$

(4) Investment, inventories

$$I_t^i = \begin{bmatrix} 0.118 \\ (0.020) \\ 0.142 \\ (0.026) \\ 0.122 \\ (0.015) \end{bmatrix} X_t^i + \begin{bmatrix} -24.163 \\ (4.302) \\ 29.146 \\ (5.529) \\ -24.773 \\ (3.237) \end{bmatrix} + \begin{bmatrix} -0.477 \\ (0.092) \\ 0.584 \\ (0.119) \\ -0.502 \\ (0.067) \end{bmatrix} (I_{STK}^i)_{t-1}$$

Equation	R^2	S^2	DW
(1)	0.9991	16.12	1.15
(2)	0.9870	6.51	N/A
(2)	0.9870	6.51	N/A
(3)	0.8363	12.15	0.53
(3)	0.8359	12.17	0.53
(4)	0.6570	8.46	1.55
(4)	0.6414	8.85	1.27

* SOIV2SLS not estimated (see text)

(5) Imports

$$(F_I)_t = \begin{bmatrix} 0.028 \\ (0.010) \\ \\ 0.032 \\ (0.012 \\ \\ 0.030 \\ (0.006) \end{bmatrix} (Y_d)_t + \begin{bmatrix} 5.672 \\ (2.556) \\ \\ 7.205 \\ (3.048) \\ \\ 6.272 \\ (1.454) \end{bmatrix} (P_F)_t + \begin{bmatrix} -9.290 \\ (3.396) \\ \\ -11.275 \\ (4.093) \\ \\ -10.037 \\ (1.954) \end{bmatrix} + \begin{bmatrix} 0.81[illegible] \\ (0.111) \\ \\ 0.767 \\ (0.131) \\ \\ 0.779 \\ (0.064) \end{bmatrix} (F_I)_{t-1}$$

R^2	S^2	DW
0.9891	1.52	N/A
0.9890	1.54	N/A

(6) Production

$$X^P_t = \begin{bmatrix} 6.231 \\ (0.509) \\ \\ 6.293 \\ (0.511) \\ \\ 6.242 \\ (0.263) \end{bmatrix} (h_m)_t + \begin{bmatrix} 0.285 \\ (0.021) \\ \\ 0.283 \\ (0.021) \\ \\ 0.281 \\ (0.011) \end{bmatrix} K_t + \begin{bmatrix} -17.685 \\ (15.350) \\ \\ -19.431 \\ (15.396) \\ \\ -16.234 \\ (8.452) \end{bmatrix}$$

R^2	S^2	DW
0.9961	104.44	0.38
0.9961	104.50	0.38

(7) Long term interest

$$(i_L)_t = \begin{bmatrix} 0.746 \\ (0.261) \\ \\ * \\ -1.498 \\ (0.080) \end{bmatrix} + \begin{bmatrix} 1.600 \\ (0.113) \\ \\ * \\ 1.937 \\ (0.041) \end{bmatrix} \left[(1-0.91) \sum_{i=o}^{\infty} (0.91)^i (i_s)_{t-i}\right]_t$$

R^2	S^2	DW
0.9350	0.17	0.77

* SOIV2SLS not estimated (see text)

(8) Short term interest

$$(i_s)_t = \begin{bmatrix} -0.228 \\ (0.232) \\ \\ 0.241 \\ (0.499) \\ \\ -0.374 \\ (0.208) \end{bmatrix} + \begin{bmatrix} -1.435 \\ (0.284) \\ \\ -2.597 \\ (0.744) \\ \\ -1.217 \\ (0.249) \end{bmatrix} R_t + \begin{bmatrix} 1.226 \\ (0.071) \\ \\ 1.135 \\ (0.140) \\ \\ 1.263 \\ (0.065) \end{bmatrix} (i_d)_t$$

R^2	S^2	DW
0.9435	0.43	1.34
0.9112	0.29	1.48

(9) Corporate saving

$$(S_c)_t = \begin{bmatrix} 0.820 \\ (0.039) \\ \\ 0.793 \\ (0.056) \\ \\ 0.779 \\ (0.026) \end{bmatrix} (P_c - T_c)_t + \begin{bmatrix} -2.606 \\ (0.951) \\ \\ -2.787 \\ (0.997) \\ \\ -3.407 \\ (0.687) \end{bmatrix} + \begin{bmatrix} -0.466 \\ (0.121) \\ \\ -0.402 \\ 0.156 \\ \\ -0.326 \\ (0.069) \end{bmatrix} (P_c - T_c - S_c)_{t-1}$$

R^2	S^2	DW
0.9662	2.96	1.94
0.9657	3.00	1.71

(10) Corporate profits

$$(P_c)_t = \begin{bmatrix} 0.570 \\ (0.015) \\ \\ 0.562 \\ (0.015) \\ \\ 0.565 \\ (0.013) \end{bmatrix} P_t + \begin{bmatrix} -11.245 \\ (1.395) \\ \\ -10.576 \\ (1.417) \\ \\ -10.977 \\ (1.229) \end{bmatrix}$$

R^2	S^2	DW
0.9797	10.91	0.30
0.9795	11.01	0.31

(11) Depreciation

$$D_t = \begin{bmatrix} 0.052 \\ (0.001) \\ \\ 0.052 \\ (0.001) \\ \\ 0.051 \\ (0.001) \end{bmatrix} K_t + \begin{bmatrix} 14.475 \\ (0.448) \\ \\ 14.445 \\ (0.449) \\ \\ 14.809 \\ (0.385) \end{bmatrix}$$

R^2	S^2	DW
0.9889	2.79	0.26
0.9889	2.79	0.26

(12) Agricultural income

$$(A_{F_A})_t = \begin{bmatrix} -0.002 \\ (0.003) \\ \\ -0.002 \\ (0.003) \\ \\ -0.001 \\ (0.002) \end{bmatrix} (Y_{NF})_t + \begin{bmatrix} 9.983 \\ (0.554) \\ \\ 9.948 \\ (0.556) \\ \\ 9.816 \\ (0.480) \end{bmatrix} + \begin{bmatrix} 3.393 \\ (0.707) \\ \\ 3.348 \\ (0.710) \\ \\ 3.111 \\ (0.412) \end{bmatrix} (D_W)_t$$

(13) Private wage bill

$$(W_1)_t = \begin{bmatrix} 0.414 \\ (0.033) \\ \\ 0.549 \\ (0.088) \\ \\ 0.428 \\ (0.015) \end{bmatrix} X^P_t + \begin{bmatrix} 14.200 \\ (1.184) \\ \\ -13.770 \\ (1.504) \\ \\ -14.456 \\ (1.130) \end{bmatrix} + \begin{bmatrix} 0.151 \\ (0.035) \\ \\ 0.009 \\ (0.092) \\ \\ 0.137 \\ (0.015) \end{bmatrix} X^P_{t-1}$$

(14) Nominal wage rate

$$\Delta w_t = \begin{bmatrix} 2.223 \\ (0.397) \\ \\ 2.727 \\ (0.621) \\ \\ 2.284 \\ (0.156) \end{bmatrix} \Delta p_t + \begin{bmatrix} -0.082 \\ (0.017) \\ \\ -0.071 \\ (0.021) \\ \\ -0.081 \\ (0.013) \end{bmatrix} + \begin{bmatrix} 0.030 \\ (0.003) \\ \\ 0.027 \\ (0.004) \\ \\ 0.030 \\ (0.002) \end{bmatrix} U^{-1}_t$$

Equation	R^2	S^2	DW
(12)	0.6463	1.25	1.70
(12)	0.6462	1.25	1.69
(13)	0.9991	7.53	1.09
(13)	0.9986	11.80	0.72
(14)	0.9314	0.001	1.94
(14)	0.9276	0.001	2.09

C. The Pascal Lags

All variables are deflated by the implicit GNP deflator unless otherwise noted; the $U_{i,t}$ are stochastic error terms.

The simple Keynesian relation between current consumption and income may be unstable for several reasons: (1) variables other than income may be important in determining consumption; (2) the long run marginal propensity to consume (mpc) is systematically above its short run counterpart, suggesting a consumption "ratchet" or permanent income effect; (3) the separate components (durables, non-durables, and services) of consumption may depend on different variables; (4) aggregate consumption may depend on the distribution, as well as the level, of income; and (5) the decision variable should be disposable rather than total income.

Of the variables other than income that are often suggested, the most important are wealth variables, primarily liquid assets and occasionally unrealized capital gains. It can be shown, however, that the influence of wealth can be transformed into a distributed lag on income. Consider the household consumption function

(1) $C_t = \beta_0 + \beta_1 W_t + \beta_2 Y_t$

where C = household consumption

W = household wealth

Y = household income.

The change in wealth is defined to be household savings in year t,

(2) $\Delta W_t \equiv Y_t - C_t$,

while the total household wealth is cumulated savings plus the initial endowment, W_o:

(3) $$W_t = \sum_{i=1}^{T} (Y-C)_{t-i} + W_o$$

(assuming the household is in its $T^{\underline{th}}$ period of existence).

Substituting (3) into the household consumption function and rearranging:

(4) $$C_t + \beta_1 \sum_{i=1}^{T} C_{t-i} = \beta_o^* + \beta_2 Y_t + \beta_1 \sum_{i=1}^{T} Y_{t-i}$$

where $\beta_o^* = (\beta_o + \beta_1 W_o)$.

Rewriting (4) in terms of the lag operator L[1] we obtain

(5) $$(L^o + \beta_1 \sum_{i=1}^{T} L^i)C_t = \beta_o^* + (\beta_2 L^o + \beta_1 \sum_{i=1}^{T} L^i)Y_t \ .$$

It is then apparent that the consumption function including wealth has been transformed to one incorporating a rational lag[2] on income:

(6) $$C_t = \beta_o^* + \left[\frac{(\beta_2 L^o + \beta_1 \sum_{i=1}^{T} L^i)}{(L^o + \beta_1 \sum_{i=1}^{T} L^i)} \right] Y_t \ .$$

The divergence of the short run and long run mpc has been much discussed.[3] If consumers attempt to avoid rapid changes in their standard of living, they will adjust slowly to their expected future

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1 $L^i X_t \equiv X_{t-i}$

2 [31]. Since household wealth is not infinite, this series must be finite, the only condition necessary to transform it into a rational lag.

3 Esp. [23], [2].

consumption path. An approximation to this process will be developed by combining generalized[1] adaptive expectations and partial adjustment models.[2]

Consumers are assumed to adjust to permanent income (Y^P), hypothesized to be a stochastic function[3] of past income. If, for example, a second order rational lag describes this relation, then (for the particular case of two roots)

$$(7) \qquad (Y^P)_t = (1-\lambda_1)(1-\lambda_2)\sum_{h=0}^{\infty}\sum_{i=0}^{\infty} \lambda_1^h \lambda_2^i \; L^{h+i} \{Y_t+[(1-\lambda_1)(1-\lambda_2)]^{-1}\epsilon_{1t}\}$$

or, applying the Koyck transformation (inverting the lag),

$$(8) \qquad (Y^P)_t = (1-\lambda_1(1-\lambda_2)Y_t + (\lambda_1+\lambda_2)(Y^P)_{t-1} - \lambda_1\lambda_2(Y^P)_{t-2} + \epsilon_{1t},$$

where λ_1 and λ_2 are the roots of the two convoluted geometric lags that comprise the second order rational lag, and Y_t is current income. [Note that if $\lambda_2 = 0$, equation (8) reduces to the simple (geometric) adaptive expectations model.]

Desired consumption (C*) is a function of permanent income:

$$(9) \qquad C_t^* = \beta_0 + \beta_1(Y^P) + \epsilon_{2t} ,$$

---

1 The usual generalization allows interdependence between two first-order adjustment processes; the generalization considered here is the higher order adjustment of a single process.

2 [42], pp. 474-477.

3 This function can be very general, depending on the number of convolutions and/or the domain of the roots [31].

while actual consumption (C) depends on the (generalized) adjustment mechanism:

(10) $C_t - C_{t-1} = (1-\lambda_3)(1-\lambda_4)[C_t^* - C_{t-1}] + \lambda_3\lambda_4[C_{t-1} - C_{t-2}] + \epsilon_{3t}$

which, after recursive substitution, can be rewritten as

(11) $C_t = (1-\lambda_3)(1-\lambda_4) \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \lambda_3^j \lambda_4^k L^{j+k} \left[C_t^* + [(1-\lambda_3)(1-\lambda_4)]^{-1} \epsilon_{3t} \right]$.

Substituting (7) into (9) into (11) gives[1]

---

[1] $C_t = \underbrace{\beta_o (1-\lambda_3)(1-\lambda_4) \Sigma_j \Sigma_k \lambda_3^j \lambda_4^k}_{\equiv 1} + (1-\lambda_3)(1-\lambda_4) \Sigma_j \Sigma_k \lambda_3^j \lambda_4^k L^{j+k}$ (times)

(times) $\Big\{ \beta_1 (1-\lambda_i)(1-\lambda_2) \Sigma_h \Sigma_i \lambda_1^h \lambda_2^i L^{h+i} \left[Y_t + [(1-\lambda_i)(1-\lambda_2)]^{-1} \epsilon_{1t} \right]$

$+ \epsilon_{2t} + (1-\lambda_3)(1-\lambda_4)^{-1} \epsilon_{3t} \Big\}$

$= \beta_o + \beta_1 (1-\lambda_2)(1-\lambda_3)(1-\lambda_4) \Sigma_h \Sigma_i \Sigma_j \Sigma_k \lambda_i^h \lambda_2^i \lambda_3^j \lambda_4^k L^{h+i+j+k}$

$\left[Y_t + [1-\lambda_i)(1-\lambda_2)]^{-1} \epsilon_{3t} \right] + (1-\lambda_3)(1-\lambda_4) \Sigma_j \Sigma_k \lambda_3^j \lambda_4^k L^{j+k}$ (times)

$\left[(1-\lambda_3)(1-\lambda_4) \right]^{-1} \epsilon_{3t}$

Rewriting in inverted (Koyck transformed) notation:

$$C_t = \prod_{i=1}^{4} \left[\frac{(1-\lambda_i)}{(L^o - \lambda_i L)} \right] Y_t + \left[\frac{L^o}{\prod_{i=1}^{4} (L^o - \lambda_i L)} \right] \epsilon_{3t}$$

$$+ \prod_{i=3}^{4} \left[\frac{(1-\lambda_i)}{(L^o - \lambda_i L)} \right] \left[\epsilon_{2t} + [(1-\epsilon_3)(1-\epsilon_4)]^{-1} \epsilon_{3t} \right]$$

$$(12)\quad C_t = \prod_{i=1}^{4} \frac{(1-\lambda_i)}{L^o-\lambda_i L} Y_t + \prod_{i=1}^{4} \frac{L^o}{L^o-\lambda_i L} \epsilon_{3t} + \prod_{i=3}^{4} \frac{(1-\lambda_i)}{(L^o-\lambda_i L)} \left[\epsilon_{2t} + [(1-\epsilon_3)(1-\epsilon_4)]^{-1}\epsilon_{3t}\right] ,$$

a fourth order rational lag on income.

The lag as specified allows positive, negative, and complex roots; this is certainly extreme generality for estimation purposes, and perhaps even for specification -- it seems plausible that the adjustment roots should lie between 0 and 1, and not implausible that the expectations roots should also lie in this interval. The orders of the expectations and adjustment processes have not been specified (the second order example is only for expository purposes); given this freedom in the number of convolutions in each case, it may not be unduly restrictive to constrain the roots of each process to be equal. Combined with the interval restriction above, this means that the weights of each mechanism lie along a Pascal density function.[1] If we add the additional assumption that the roots of the adaptive expectations model are equal to the roots of the partial adjustment model,[2] then it is not necessary to determine separately the order of each process and the resulting lag is nonlinear in only two parameters: r, the combined number of

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[1] The Pascal (Solow) lag is a rational lag with equal roots λ that lie in the interval $0 < \lambda < 1$.

[2] In the simple geometric version of each model this implies that the adjustment coefficient is 1 minus the decay parameter of the expectations model. [This is strictly an assumption of convenience, since the Pascal lag is (nonspectrally) estimable while the rational lag is not (but see appendix III-A for later work to eliminate this assumption).]

convolutions; and λ, the value of all r roots. In addition, since the combined result is a Pascal lag, there is a procedure available for consistent parameter estimation.[1]

In specifying the consumption function, some care has been taken to allow inclusion of a partial adjustment mechanism; complete adjustment of nondurables and services consumption may or may not be immediate, but we would certainly not want to assume complete adjustment of durables "consumption." Because durables enter the national income accounts as they are purchased, rather than as they are consumed, their purchase decision is an investment decision. Assuming that the desired flow of services from durable goods depends on expected future income, but the consumer must purchase the entire service stream at once, in an uncertain world the prudent consumer will only partially adjust his actual purchases to the desired level. Since a separate equation for durables consumption has not been specified in the model,[2] it is important to allow this partial adjustment mechanism.

The three-way partition of income -- into wage income, farm income, and nonwage, nonfarm (profit) income -- is a basic feature of the K-G model, with equations being developed to determine these shares. Since changes in the distribution of income, ceteris paribus, result in changes in total consumption, this information should be incorporated into the consumption function. As Klein and Goldberger argue,[3] this functional distribution is an approximation to the size

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1 [47].

2 In order to retain the income shares consumption function discussed below.

3 [34], pp. 4-7.

distribution of income, and also includes the effects of: (1) the relative stability of wage income, allowing a higher mpc at the same level of risk, (2) business saving by individual proprietors due to better investment outlets and more liquidity preference, and (3) "... the influence of the rural way of life with comparatively fewer expenditure outlets for consumer items."[1] In estimating the three mpc's of the original K-G consumption function in order to avoid multicollinearity the ratios of the mpc's out of wage income, farm income, and nonwage, nonfarm income were constrained to values obtained from (extraneous) survey data. For example, instead of estimating the relation

(13) $C = \beta_0 + \beta_1$ (disposable wage income) $+ \beta_2$ (nonwage, nonfarm income $+ \beta_3$ (farm income) $+ \epsilon^*$,

first $\frac{\beta_2}{\beta_1}$ and $\frac{\beta_3}{\beta_1}$ would be estimated, and then time series data would be used to estimate β_1 in

(14) $C = \beta_0 + \beta_1$ [(wage income) $+ \widehat{\left(\frac{\beta_2}{\beta_1}\right)}$ (nonwage, nonfarm income) $+ \widehat{\left(\frac{\beta_3}{\beta_1}\right)}$ (farm income)] $+ \epsilon$

where $\widehat{\left(\frac{\beta_2}{\beta_1}\right)}$ and $\widehat{\left(\frac{\beta_3}{\beta_1}\right)}$ are survey estimates of the ratios of nonwage, nonfarm income and agricultural income mpc's, respectively, to the wage income mpc. [Note that only the ratios, not the levels, of the mpc's have been constrained.]

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[1] [3], p. 6.

Clearly personal disposable income is the appropriate variable for determining consumption; since disposable income by factor shares is not available, it will be specially constructed for the model following the method proposed by Frane and Klein.[1]

Putting all of these considerations together, the resulting consumption function is

$$(15)\quad C_t = \beta_{1,0} + \beta_{1,1}(1-\lambda_1)^{r_1} \sum_{i=o}^{\infty} \binom{r_1 + i - 1}{i} \lambda_1^i (Y_c)_{t-i} + U_{1,t}$$

with

$$(16)\quad (Y_c)_t = (W_1 + W_2 - T_W)_t + \widehat{\left(\frac{\beta_A}{\beta_W}\right)} (A_1 + A_2 - T_A)_t + \widehat{\left(\frac{\beta_p}{\beta_W}\right)} (P - S_c - T_p)_t$$

where r_1, λ_1, $\beta_{1,0}$, and $\beta_{1,1}$ are parameters, and

W_1 = private wages

W_2 = government wages

T_W = net taxes and transfers on wage income

A_1 = private agricultural income

A_2 = government payments to farmers

T_A = net taxes and transfers on agricultural income

P = nonwage, nonfarm (profit) income

S_C = corporate savings

T_p = net taxes and transfers on profit income.

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1 [22]. Since the National Income Accounts were revised in 1965, it was necessary to recalculate all of the years rather than accept the Klein and Frane estimates through 1952 (see Appendix II-B).

In estimating the Pascal (Solow[1]) lags, we have chosen a transformation that eliminates the infinite string of regressors and permits estimation that was originally suggested by Klein[2] for the geometric case and later generalized by Maddala[3] to the Pascal case. The basic procedure involves truncating the infinite sum and estimating the effect outside the sample as a parameter, since it is invariant over time. For the geometric case,

$$(2) \quad y_t = \beta_0 \, \beta_1(1-\lambda) \sum_{i=o}^{\infty} \lambda_1 X_{t-i} + \epsilon_t ,$$

Klein proposed that the equation be rewritten

$$(3) \quad y_t = \beta_0 + \beta_1 Z_{1t} + \theta \, Z_{2t} + \epsilon_t ,$$

$$\text{where } Z_{1t} = (1-\lambda)^r \sum_{i=o}^{t-1} \lambda^i X_{t-i}$$

$$Z_{2t} = \lambda^t$$

$$\theta \equiv E(y_o) - \beta_o$$

($\beta_1 Z_{1t}$ measures the effect on y_t of the regressors inside the sample period, while θ, the parameterized initial condition, contains the effect of the regressors outside the sample period discounted by λ^t).

Maddala has generalized this technique to include the Pascal family of lag distributions, lags with multiple convolutions but equal roots. In this case, equation (1) is written

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[1] [55].

[2] [41]; see also [7].

[3] [47].

$$(4) \quad y_t = \beta_0 + \beta_1 z_{1t} + \sum_{i=1}^{r} \theta_{i-1} Z_{i+1,t} + \epsilon_t ,$$

where $\theta_{i-1} = E(y_{i-1}) - \beta_0$

and

$$Z_{1t} = (1-\lambda)^r \sum_{i=o}^{t-r} \binom{r+i-1}{i} \lambda^i X_{t-i}$$

$$Z_{2t} = (-1)^{r-1} \begin{matrix} t-1 \\ r-1 \end{matrix} \lambda^t$$

$$Z_{3t} = - \frac{r-1}{1} \cdot \frac{t}{t-1} \cdot \frac{1}{\lambda} Z_t$$

$$Z_{4t} = - \frac{r-2}{2} \cdot \frac{t-1}{t-2} \cdot \frac{1}{\lambda} Z_{3t}$$

$$Z_{5t} = - \frac{r-3}{3} \cdot \frac{t-2}{t-3} \cdot \frac{1}{\lambda} Z_{4t}$$

$$\vdots$$

$$Z_{r+1,t} = \binom{t}{r-1} \lambda^{t-r+1}$$

except that for $t=1$, $Z_{3t}=1$, and the rest are zero, for $t=2$, $Z_{4t}=1$ and the rest are zero, and so on, until for $t=r-1$, $Z_{r+1,t}=1$, and the rest are zero. A generalized example of this derivation is given in Appendix III. For the particular case of three convolutions, (1) becomes

$$(5) \quad y_t = \beta_0 + \beta_1 (1-\lambda)^r \sum_{i=o}^{t-3} \frac{(i+2)(i+1)}{2} \lambda^i X_{t-i} + \frac{(t-1)(t-2)}{2} \lambda^t [E(y_o)-\beta_o]$$

$$- t(t-2)\lambda^{t-1}[E(y_1)-\beta_o] + \frac{t(t-1)}{2} \lambda^{t-2} [E(y_2)-\beta_o] + \epsilon_t .$$

Since $E(y_k) = y_1-\epsilon_i$, if we assume[1] that the disturbances on the para-

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[1] Instinct served me well; later work by Swamy and Rao ("Maximum Likelihood Estimation of a Distributed Lag Model with Autocorrelated Errors," unpublished paper, P.A.V.B. Swamy and J.N.K. Rao, Federal Reserve System and University of Manitoba) demonstrates that $\hat{\beta}_1$ is inconsistent when the initial conditions are estimated, but consistent if the initial conditions are set to any arbitrary constant.

meterized initial conditions are very small relative to y_i, we can replace $E(y_i)$ with y_i and reduce by r the number of parameters to be estimated. The restricted equation, for the case of r=3, obtained by substituting y_i for $E(y_i)$ and distributing the terms, is

$$(6) \quad y_t^* = \beta_o I_t^* + \beta_1 (1-\lambda)^3 \sum_{i=o}^{t-3} \frac{(i+2)(i+1)}{2} \lambda^i X_{t-i} + \epsilon_t$$

where

$$y_t^* = y_t - \frac{(t-1)(t-2)}{2} \lambda^t y_o + t(t-2)\lambda^{t-1} y_1 - \frac{t(t-1)}{2} \lambda^{t-2} y_2$$

$$I_t^* = 1 - \frac{(t-1)(t-2)}{2} \lambda^t + t(t-2)\lambda^{t-1} - \frac{t(t-1)}{2} \lambda^{t-2} .$$

y_t^* and I_t^* are similarly defined for other values of r so that there always remain only the two coefficients β_0 and β_1 to be estimated for each trial set of r and λ.

D. Nonlinearities

The model is nonlinear in both the parameters and the variables, but the four parameters of the two Pascal lags (r_1, λ_1, r_2, and λ_2) introduce the only parameter nonlinearity -- given these values the model is linear in the remaining parameters and is written so that the stochastic equations are strictly linear in both variables and parameters and the variable nonlinearity occurs only in the identities.

a. Parameter Nonlinearity

The model is to be estimated by two- and three-stage least squares (2SLS and 3SLS), techniques not customarily applied to systems nonlinear in parameters (although the generalization is not difficult). If the model is nonlinear only in the parameters, first stage estimation can be performed as usual, since the elements of the reduced form coefficient matrix are stable and thus the predicted values of the nonnormalizing jointly dependent variables are well defined.

Considering 2SLS as a special case of instrumental variable estimation that provides a particular method for obtaining the instruments, it is apparent that the second stage of 2SLS is simply an instance of the nonlinear least squares problem and can be solved in a number of ways [including differentiating the sum of squared errors with respect to the parameters and solving the resulting (nonlinear) differential equations, or, if the parameter domain is bounded (as in the present case[1]), by search techniques]. Models nonlinear only in the parameters are not conceptually difficult to estimate via 2SLS.

The extension to 3SLS is equally straightforward conceptually, although it imposes a large computational burden; in the 3SLS case the parameters would be chosen to minimize the generalized variance ($|\Sigma|$, the determinant of the across-equation disturbance variance-covariance matrix). While nonlinear 2SLS estimates will be calculated for the current model, 3SLS estimates will not--they would necessitate a four dimensional search (over r_1, λ_1, r_2, and λ_2), with a 3SLS estimate of the parameters of fourteen stochastic equations at each combination of trial values of the nonlinear parameters. The 3SLS estimates to be calculated will be conditional on the 2SLS estimates of the four Pascal lag parameters.

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[1] r_1 and r_2 are assumed to be integers between 1 and 5, and λ_1 and λ_2 are assumed to lie inside the interval from 0 to 1.

b. Variable Nonlinearity

Nonlinearity in the variables is more troublesome than parameter nonlinearity, since (1) the reduced form is often not in closed form, and (2), even when it is, it will generally be nonlinear in the variables such that the reduced form coefficients are unstable (the reduced form is nonlinear in its parameters).[1] To overcome these problems, Goldfeld and Quandt[2] proposed approximating the reduced form by a p^{th} degree polynominal in the predetermined variables, representing the first p terms of a Taylor series expansion. They assumed but did not prove consistency under this approximation; however, Kelejian[3] and Edgerton[4] have reestablished sufficient conditions for consistent 2SLS estimation using the Goldfeld-Quandt polynomial approximation (actually, Fisher published a much clearer discussion in an earlier article[5] in a slightly different context). Since there seems to be needless confusion on the requirements for consistent nonlinear 2SLS estimation, and since none of the articles summarize all of the conditions, we shall examine the problem in more depth.

Assume that a system of G structural equations is linear in the parameters, has additive error terms, and each equation can be noralized on one jointly dependent variable; the first equation can be written:

$$(1) \quad y_{1t} = X_{1t}\gamma_1 + F_{1t}\beta_1 + U_{1t} \qquad (t=1, \ldots, T)$$

where y_{1t} is the t^{th} observation on the normalizing variable in the

1 See especially [13], [26], [11], and [12].

2 [26], p. 116.

3 [32].

4 [12].

5 [21].

first equation, X_{1t} is the (1 x K*) vector of observations on the first equation predetermined variables at time t, γ_1 is the associated (K* x 1) vector of parameters, F_{1t} is a $[1 \times (G^{\Delta}-1)]$ vector of observations on $(G^{\Delta}-1)$ jointly dependent functions at a time t, β_1 is the corresponding $[(G^{\Delta}-1) \times 1]$ vector of coefficients, and U_{1t} is the t^{th} period stochastic error term. In addition, define X_t and Y_t, respectively, as the t^{th} observation on all predetermined and jointly dependent variables appearing in the G equation system and note that the elements of the vector of jointly dependent functions can be written

(2) $F_{1t} = [f_1(Y_t,X_t) \ \ldots \ f_{G^{\Delta}-1}(Y_t,X_t)]$.

To perform two stage least squares it is necessary to satisfy what we shall call the consistency conditions and obtain instruments that (1) in the probability limit are uncorrelated with the structural disturbance but (2) are correlated with the structural variables they replace. However, as Goldfeld and Quandt observe, there are two immediate candidates:[1] (1) the "hat of the function," e.g. $\widehat{f_j(Y_t,X_t)}$, and (2), the "function of the hat," e.g. $f_j(\hat{Y}_t,X_t)$. Goldfeld and Quandt guess that estimates based on the second method will be inconsistent; Kelejian proves it.[2] Each jointly dependent function, then, is to be considered as a random variable in order to calculate $\widehat{f_j(Y_t,X_t)}$, $j=1, \ldots, G^{\Delta}-1$. There is an unknown, perhaps nonanalytic function

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[1] For the instrument for the j^{th} jointly dependent function in the given equation. The predetermined variables are used as instruments for themselves.

[2] [32], p. 374.

relating each jointly dependent function to the predetermined variables; the nonstochastic part of this function is defined to be $h_j(X_t)$, so that

(3) $E[f_j(Y_t,X_t)|X_t] = h_j(X_t)$ $\quad$ (j=1, ..., GΔ-1; t=1, ..., T)

and

(4) $f_j(Y_t,X_t) = h_j(X_t) + v_{jt}$ $\quad$ (j=1, ..., GΔ-1, t-1, ..., T)

where v_{jt} is the actual reduced form disturbance, with $E[v_{jt}|X_t] = 0$. Since $h_j(X_t)$ is generally unknown (depending on the structural parameters), and may not be in closed form, the Goldfeld-Quandt polynomial approximation is introduced: let $P_j(X_t)$ be a $p^{\underline{th}}$ degree polynomial in X

(5) $P_j(x_t) = \pi_{0j} + \pi_{1j}\, x_{1t} + \ldots + \pi_{Kj}\, x_{Kt} + \ldots + \pi_{qj}\, x_{Kt}^p$ (j=1, ..., $G\Delta_{-1}$;

t=1, ..., T),

where $x_{1t}, \ldots, x_{Kt}$ are the elements of X_t and $(\pi_{0j}, \ldots, \pi_{qj}) \equiv \underset{T}{\text{plim}}$ $(\pi_{0j}, \ldots, \pi_{qj})$. Then (4) can be rewritten in terms of this approximation as

(6a) $f_j(Y_t,X_t) = P_j(X_t) + v_{jt}^*$

or $\quad$ (j=1, ..., GΔ-1; t=1, ..., T),

(6b) $\widehat{f_j(Y_t,X_t)} = \hat{P}_j(X_t)$

with v_{jt}^* defined as v_{jt} plus the error of approximation inherent in replacing $h_j(X_t)$ by $P_j(X_t)$. Equation (6b) forms the basis of the instrument calculation given the following two (sufficient)[1] subconditions for meeting consistency condition (1): (a) all of the predetermined variables that occur in the given structural equation occur as regressors

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[1] See Maddala, ([48], p. 2, footnote 2), for an explanation of how recursive models can provide counterexamples to the necessity of condition [1] for certain equations within the model.

in the first stage equation (i.e., X_{1t} is included in X_t), so that X_t is uncorrelated with the first stage residual; and (b) all instruments in a given structural equation are calculated from the same set of regressors (which implies that the approximating polynomials are all of the same degree). The reasoning behind these conditions is apparent from examination of the second stage equation after instrument substitution:

$$(7) \quad y_t = X_{1t} \gamma_1 + \hat{F}_{1t} \beta_1 + \underbrace{U_{1t} + (F_{1t} - \hat{F}_{1t}) \beta_1}_{\equiv U^{**}_{1t}} \;.$$

A linear combination of the first stage residual from each instrument equation is added to the structural disturbance every time period; thus, as Fisher remarked long before the present consistency discussions,

> "... consistency requires not only zero correlation in the probability limit between the original disturbance and all the variables used in the final regression but also zero correlation in the probability limit between the residuals from the earlier-stage regression equations and all such variables. If the same set of instruments is used when replacing every right-hand endogenous variable, and if that set includes the instruments explicitly in the equation, the latter requirement presents no problem since the normal equations of ordinary least squares imply that such correlations are zero even in the sample. When different instruments are used in the replacement of different variables, however, or when the instruments so used do not include those explicitly in the equation, the danger of inconsistency from this source does arise." [1]

Having established uncorrelatedness of the composite disturbance U^{**}_{1t} with the instruments in the second stage regression, it remains to satisfy the second consistency condition and show correlatedness of the

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1 [21], pp. 271-2.

instruments with the variables that they replace. Since X_{1t} is used as an instrument for itself the correlation is obvious, while the correlation of $\hat{F}_1$ [1] (calculated from the reduced form) with F_1 is implicit in the model assumptions: if the model is correctly specified, then $h_j(X_t)$ describes a causal[2] relationship and X_t includes all variables relevant to the determination of the jointly dependent functions. Since $P_j(X_t)$ is a p term approximation to $H_j(X_t)$, there exists[3] a p such that

$$\text{plim}\ [f_j(Y,X)'\ \widehat{f_j(Y,X)}] \neq 0,$$ [1,4]

which, with the previous two subconditions, establishes the consistency of 2SLS parameter estimates if they exist.

There are four nonlinear identities in the model, including the nontrivial price identity. Two of these identities--defining agricultural income and relative import prices--are sufficiently irregular as to be poorly approximated by a first degree Taylor expansion. In order to test model sensitivity to the linearization of these identities, reduced forms were calculated with (1) four Taylor expansions, (2) two Taylor expansions and two[5] identities fitted by ordinary least squares,

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[1] Vector symbols without t subscripts refer to matrices, each row of which is the original vector at a given t, with each matrix consisting of T rows.

[2] The predetermined variables, by definition, affect the jointly dependent variables but are not affected by them. This asymmetric relation may be defined as causality. See the immediately following sections.

[3] Only a pathological function would require $p > 1$.

[4] Regardless of the fact that $E(v^*_{jt}) \neq 0$ due to the error of approximation.

[5] The irregular identities (mentioned above) defining A_{p_A} and p_F.

and (3) four ordinary least squares approximations. Some of the results appear in Table 4, along with sample means of the exogenous variables and a measure of the importance of the change in the multiplier--the absolute value of the maximum difference between the 2SLS multipliers times the mean of the exogenous variable.

That the linearization chosen is important in determining the p_A^{-1} and p_M^{-1} multipliers is not surprising; none of the linearizations will provide accurate estimates of these multipliers.[1] Unfortunately, however, the government expenditures multipliers are also sensitive to the linearization chosen. Since we have used first degree polynomial approximations to the reduced form in the first stage of 2SLS, this impact multiplier sensitivity may be an indication that we have lost considerable first stage information in the linearization.[2] Of course we have yet to correct for autoregression, which will lower the multipliers and perhaps lessen the linearization sensitivity.

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[1] This disagrees with the Goldberger result ([25], pp. 136-138). The difference undoubtedly stems from the fact that the original K-G model was too heavily damped, while these estimates are not damped enough--thus the effect of the nonlinearity is understated in one case and overstated in the other.

Table 4

IMPACT MULTIPLIERS

(Nonautoregression Corrected Model)

Model	R	i_d	G	T	T_W	T_A	T_P	W_2	P_A^{-1}	P_m^{-1}
2SLS										
Four Taylor Expansions										
X	16.47	-14.07	4.96	-3.00	-3.81	-3.87	-4.37	1.10	-6.38	-22.49
p	0.038	-0.032	0.013	-0.007	-0.010	-0.010	-0.011	-0.008	-0.017	-0.060
h_m	2.25	-1.92	0.72	-0.42	-0.55	-0.55	-0.62	-0.028	-0.93	-3.27
Two OLS, Two Taylor										
X	17.68	-15.11	5.38	-3.23	-4.13	-4.18	4.72	0.85	-0.70	-10.65
p	0.041	-0.035	0.014	-0.008	-0.011	-0.011	0.012	-0.008	-0.002	-0.028
h_m	2.43	-2.07	0.78	-0.46	-0.60	-0.59	-0.67	-0.03	-0.010	-1.55
Four OLS										
X	18.12	-15.48	5.52	-3.31	-4.23	-4.29	-4.84	1.15	-0.72	-10.65
p	0.028	-0.024	0.010	-0.006	-0.008	-0.007	-0.018	-0.018	-0.001	-0.020
h_m	2.49	-2.13	0.80	-0.47	-0.62	-0.61	-0.69	0.01	-0.11	-1.59
Exogenous Variable Mean	0.197	2.648	75.376	36.581	14.605	1.010	24.307	40.010	1.571	1.607
Importance										
$\lvert$Maximum $\Delta X$$\rvert$	0.319	3.654	42.211	11.340	6.280	0.500	10.695	12.003	8.970	33.458
$\lvert$Maximum $\Delta p$$\rvert$	0.003	0.029	0.302	0.073	0.029	0.003	0.097	0.400	0.025	0.064
$\lvert$Maximum $\Delta h_m$$\rvert$	0.047	0.556	6.030	1.829	1.022	0.061	1.701	0.880	1.304	2.764
3SLS										
Two Taylor, Two OLS										
X	14.80	-15.36	5.76	-3.56	-4.41	-4.48	-5.06	0.86	-0.69	-12.63
p	0.034	-0.036	0.015	-0.009	-0.012	-0.011	-0.013	-0.009	-0.002	-0.033
h_m	2.04	-2.12	0.84	-0.50	-0.64	-0.64	-0.72	-0.03	-0.10	-1.84

E. Autoregression

1. Autoregression in simultaneous models[1]

The estimation of the parameters of simultaneous equation systems with both serial correlation and lagged endogenous variables is complicated by the inability to solve for the separate equation autoregression parameters piecewise even when the estimation technique is a single equation one such as two stage least squares (2SLS). The problem is further complicated by tradeoffs between large and small sample properties. A simple example (due to Ray Fair[2]) will make the exposition clearer.

Assume the model consists of the two equations

(1a) $y_{1t} = -a_{12}\, y_{2t} - b_{11}\, x_{1t} - b_{12}\, x_{2t} - b_{14}\, y_{1,t-1} + u_{1t}$ $(t=1,2,...,T)$

(1b) $y_{2t} = -a_{21}\, y_{1t} - b_{22}\, x_{2t} - b_{23}\, x_{3t} - b_{24}\, y_{2,t-1} + u_{2t}$ $(t=1,2,...,T)$

where

(2a) $u_{1t} = r_{11}\, u_{1,t-1} + v_{1t}$ $(t=1,2,...,T)$

(2b) $u_{2t} = r_{22}\, u_{2,t-1} + v_{2t}$ $(t=1,2,...,T)$

Then the reduced form for y_{2t} [used in the first stage of 2SLS estimation of (1a)] is

---

1 This section borrows heavily from an article by Ray C. Fair, [16].

2 [16], p. 510.

$$(3)\quad y_{2y} = (1-a_{21}\ a_{12})^{-1}[(r_{22}-a_{21}\ a_{12}\ r_{11})y_{2,t-1}-b_{24}(Y_{2,t-1}-r_{22}Y_{2,t-2})$$

$$+ a_{21}(r_{22}-r_{11})y_{1,t-1} + a_{21}\ b_{14}(y_{1,t-1}-r_{11}\ y_{1,t-2})$$

$$+ a_{21}\ b_{11}(x_{1t}-r_{11}\ x_{1,t-1}) + (a_{21}\ b_{12}\ -b_{22})x_{2t}$$

$$- (a_{21}\ b_{12}\ r_{11}\ -b_{22}\ r_{22})x_{2,t-1}-b_{23}(x_{3t}-r_{22}\ x_{3,t-1})$$

$$- a_{21}\ v_{1t} + v_{2t}]\ ,$$

an expression involving both autoregressive parameters (r_{11} and r_{22}) of the system. Thus the (nonlinear) constraints implied by the autoregressive structure cannot be incorporated in the first stage unless either the autoregressive parameters are all simultaneously estimated or at least all but one are known. If the restrictions are forsaken, then an important part of the structure of the model is not incorporated into the choice of instruments ($\hat{y}$'s) for the second stage.[1] The objective is to include as much prior information as possible in the first stage without significantly damaging the second state properties. To this end an extremely rigid simplifying assumption--that the disturbances for each equation are not

---

[1] The asymptotic properties of this procedure (dropping the restrictions) are by far the easiest to prove; the small sample properties would be expected to be less than desirable, however. In this completely unstructured case, the autoregression coefficients in the first stage are not even constrained to lie between -1 and 1, while the number of first stage regressors is doubled (losing much prior information about the autoregressive process).

only uncorrelated with those of other equations, but in addition the autoregression parameter within each equation is the same for all equations--is imposed in the first stage of estimation and later disregarded in the second and third stages.

The model is

$$(4)\quad \underset{(GxG)}{(N+A)}\ \underset{(GxT)}{Y} + \underset{(GxG)}{B}\ \underset{(GxT)}{Y_{-1}} + \underset{(GxK)}{C}\ \underset{(KxT)}{X} = \underset{(GxT)}{U^*}$$

with

$$(5)\quad \underset{(GxT)}{U^*} = \underset{(GxG)}{R}\ \underset{(GxT)}{U^*_{-1}} + \underset{(GxT)}{V}$$

and the assumptions

(i) $E(V) = 0$

(ii) $E[v(t)v(t)'] = \begin{bmatrix} \Sigma & \phi \\ \phi & \phi \end{bmatrix}$, $t=1,2,\ldots,T$, Σ positive definite, where

$v(t) = t^{th}$ column of the matrix V;

(iii) $E[v(t)v(t')'] = \phi$, $t=1,2,\ldots,T$, $t \neq t'$;

(iv) $\text{plim } T^{-1} XV' = \text{plim } T^{-1} X_{-1}V' = \text{plim } T^{-1} Y_{-1}U' = \phi$;

(v) the moment matrix of the endogenous, lagged endogenous, predetermined, and lagged predetermined variables is well behaved in the limit;

(vi) R is a diagonal matrix of elements between zero and one;

(vii) N+A has an inverse.[1]

---

1 These assumptions have been taken from the Fair article [16] with only minor changes.

Solving (4) for the reduced form, we obtain

$$(6)\quad Y = -(N+A)^{-1} BY_{-1} - (N+A)^{-1}CX + (N+A)^{-1} U^{*}$$

which, after Koyck transformation with factor R, gives

$$(7)\quad Y-RY_{-1} = (N+A)^{-1}B(Y_{-1}-RY_{-2})-(N+A)^{-1}C(X-RX_{-1}) + (N+A)^{-1}(R-R)U^{*}_{-1} + (N+A)^{-1}V.$$

At this point we introduce the assumption that R is a diagonal matrix of identical elements, and thus can be written rI. We are now prepared to search[1] over r to minimize the first stage residuals $(Y-rY_{-1})-\widehat{(Y-rY_{-1})}$. Let r be the trial value for r; then the disturbance in (8) will be at a minimum for $\hat{r} = r$:

$$(8)\quad Y-\hat{r}Y_{-1} = -(N+A)^{-1}B(Y_{-1}-\hat{r}Y_{-2})-(N+A)^{-1}C(X-rX_{-1})+(N+A)^{-1}(r-\hat{r})U^{*}_{-1}+(N+A)^{-1}V.$$

The transformed structural form is

$$(9)\quad N(Y-\hat{r}Y_{-1}) = -A(Y-\hat{r}Y_{-1})-B(Y_{-1}-\hat{r}Y_{-2})-C(X-\hat{r}X_{-1}) + (r-\hat{r})U^{*}_{-1} + V;$$

if we carry out 2SLS replacing the nonnormalizing jointly dependent variables with predicted values calculated from (8) there is no guarantee that the minimum sum of squared errors (SSE) will occur at $\hat{r}=r$, since the combined disturbance on (9) after the instrument substitutions will be $(r-\hat{r})U^{*}_{-1} + V + A(Y-rY_{-1}) - A\widehat{(Y-rY_{-1})}$, and AY_{-1} and U^{*}_{-1} are not uncorrelated (and thus the combined disturbance cannot be minimized by parts). The r chosen would minimize the sum of the variances and the covariances, and thus would differ from r. The solution is to include enough variables in the instrument equation to guarantee the orthogonality of AY_{-1} and U^{*}_{-1}

---

[1] Or iterate.

by taking advantage of the fact that least squares residuals are orthogonal to the space spanned by the regressors.[1] Since

(10) $U^{*}_{-1} = (N+A)Y_{-1} + BY_{-2} + CX_{-1}$

we must include in the instrument equations for each structural equation lagged values of all variables that occur in that particular structural equation.[2] We call the resulting equations constrained augmented reduced form equations and write them

$$(11)\quad \underset{(GxT)}{(Y-rY_{-1})} = \underset{(Gx3G)}{\pi_g} \underset{(3GxT)}{\begin{bmatrix} Y_{-1} \\ Y_{-2} \\ X_{-1} \end{bmatrix}} \underbrace{-(N+A)^{-1}B(Y_{-1}-rY_{-2})}_{(GxT)} \underbrace{-(N+A)^{-1}C(X-rX_{-1})}_{(GxT)}$$

$$\underbrace{+(N+A)^{-1}(r-r)U_{-1}}_{(GxT)} + \underbrace{(N+A)^{-1}V}_{(GxT)}$$

where π_g is the augmenting matrix for the g^{th} equation:

$$\underset{(Gx3G)}{\pi_g} \equiv [\underset{(GxG)}{\pi_A} : \underset{(GxG)}{\pi_B} : \underset{(GxG)}{\pi_C}]_g$$

---

1 If the coefficients are restricted, then the least squares residuals are not necessarily orthogonal; hence the latter inclusion of separate, "extra" variables to eliminate the autoregression restrictions on the predetermined variables occurring in the structural equation.

2 The inclusion of these "extra" variables does not affect the disturbance term and may be regarded as a dropping of the nonlinear autoregression restriction relating current and lagged values of the variables occurring in the structural equation.

$$[\pi_A]_g = [(\pi_A)_{ij}] = 0 \text{ if the } g,j \text{ element of } A,\ [a_{gj}],\ = 0 \qquad (((g=1,2,\ldots G),\ i=1,2,\ldots G),\ j=1,2,\ldots G)$$

$$= [(\pi_A)_{ij}]^1 \text{ otherwise}$$

$$[\pi_B]_g = [(\pi_B)_{ij}] = 0 \text{ if the } g,j \text{ element of } B,[b_{gj}],\ = 0 \qquad (((g=1,2,\ldots G),\ i=1,2,\ldots G),\ j=1,1,\ldots G)$$

$$= [(\pi_B)_{ij}]^1 \text{ otherwise}$$

$$[\pi_C]_g = [(\pi_C)_{ij}] = 0 \text{ if the } g,j \text{ element of } C,[c_{gj}] = 0 \qquad (((g=1,2,\ldots G),\ i=1,2,\ldots G),\ j=1,2,\ldots G)$$

$$= [(\pi_C)_{ij}]^1 \text{ otherwise}$$

(the π_g matrix is simply a method of including lagged values of the predetermined variables that occur in the $g^{\underline{th}}$ structural equation in the constrained augmented reduced form used in obtaining instruments for that equation).

Two stage least squares can now be performed on each equation separately, both stages being calculated at some trial value $\hat{r}$ of the autoregression parameter, with the instruments being obtained from the constrained augmented reduced form equations (11) corresponding to the given structural equation. The second stage of this procedure, with the instruments substituted in, will be[2] (for the $g^{\underline{th}}$ equation)

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[1] The coefficient of the constrained augmented reduced form equation for the $g^{\underline{th}}$ equation.

[2] Note that $(\widehat{Y-rY_{-1}}) \equiv (\hat{Y}-\hat{r}Y_{-1})$.

$$(12)\quad y_{g,t}-\hat{r}y_{g,t-1} = -a_{11}(\widehat{y_{1,t}-ry_{1,t-1}}) -a_{12}(\widehat{y_{2,t}-ry_{2,t-1}})-\ldots-a_{1g}(\widehat{y_{g,t}-ry_{g,t-1}})$$

$$-b_{11}(y_{1,t-1}-\hat{r}y_{1,t-2})-b_{12}(y_{2,t-1}-\hat{r}y_{2,r-2})-\ldots-b_{1g}(y_{g,t-1}-\hat{r}y_{g,t-2})$$

$$-c_{11}(x_{1,t}-\hat{r}x_{1,t-1})-c_{12}(x_{2,t}-\hat{r}x_{2,t-1})-\ldots-c_{1g}(x_{g,t}-\hat{r}x_{g,t-1})$$

$$+\ (r-\hat{r})[(n+a)^{11}u_{1,t-1} + (n+a)^{12}u_{2,t-1}+\ldots+(n+a)^{1g}u_{g,t-1}]$$

$$+\quad (n+a)^{11}v_{1,t} + (n+a)^{12}v_{2,t} +\ldots+ (n+a)^{1g}v_{g,t}$$

$$+\ a_{11}[(y_{1,t}-\hat{r}y_{1,t-1})-(\widehat{y_{1,t}-ry_{1,t-1}})]+ a_{12}[(y_{2,t}-\hat{r}y_{2,t-1})$$

$$-\ (\widehat{y_{2,t}-ry_{2,t-1}})]+\ldots+ a_{1,g}[(y_{g,t}-\hat{r}y_{g,t-1})-(\widehat{y_{g,t}-ry_{g,t-1}})],$$

where the lower case subscripted letters refer to particular elements of the uppercase matrix, while superscripts denote elements of an inverted matrix.

The v_{ij}'s are independent of the rest of the disturbance by assumption; inclusion of the augmenting variables in the first stage guarantees orthogonality of the disturbance terms in u_{ij} and y_{ij}. Thus the minimum SSE will be at $\hat{r}=r$, as desired.

The estimation is carried out for each equation, choosing the estimate of the system autoregression parameter that minimizes SSE for the particular equation. (The across-equations restriction that the system autoregression parameter is assumed the same for all equations is purposely ignored everywhere except in the first stage for each equation).

Since V is hypothesized to have a cross-equation covariance [assumption (ii) above], if the model is overidentified there will be an efficiency gain from three stage least squares (3SLS) estimation. Considering 3SLS as Aitken's generalized estimation over 2SLS starting estimates (with instruments substituted for nonnormalizing jointly dependent variables), we can follow the usual procedure of replacing the unknown across-equation disturbance variance-covariance matrix, Σ, with a consistent estimate Σ obtained from the 2SLS residuals.[1]

Recalling the model

(4) $(N+A)Y + BY_{-1} + CX = U^*$

(5) $U^* = RU^*_{-1} + V$

we rewrite it after the autoregression transformation

(13) $(N+A)Y - R(N+A)Y_{-1} + BY_{-1} - RBY_{-2} + CX - RCX_{-1} = V$.

For general R (not necessarily diagonal), at time t the first equation can be written:

(14) $[a_{11}y_{1t} - (r_{11}a_{11} + r_{12}a_{21} + \ldots + r_{1g}a_{g1})y_{1,t-1}] + [a_{12}y_{2t} - (r_{11}a_{12} + \ldots + r_{1g}a_{g2})y_{2,t-1}]$

$+ \ldots + [a_{1g}y_{gt} - (r_{11}a_{1g} + r_{12}a_{2g} + \ldots + r_{1g}a_{gg})y_{g,t-1}]$

$+ [b_{11}y_{1,t-1} - (r_{11}b_{11} + r_{12}b_{21} + \ldots + r_{1g}b_{g1})y_{1,t-2}] + [b_{12}y_{2,t-1} - (r_{11}b_{12}$

$+ r_{12}b_{22} + \ldots + r_{1g}b_{g2})y_{2,t-2}]$

$+ \ldots + [b_{1g}y_{g,t-1} - (r_{11}b_{1g} + r_{12}b_{2g} + \ldots + r_{1g}b_{gg})y_{g,t-2}] \quad +$

---

[1] As Ruble mentions [53], Σ must be estimated from the 2SLS residuals calculated from the actual jointly dependent variables rather than their instruments in order for this procedure to give 3SLS estimates. The fact that there are as many separate estimates of the system autoregression parameter as there are stochastic equations in no way effects the consistency--given the assumptions--of Σ estimated from the residuals of these equations: the restriction across equations has simply not been incorporated into the estimation process .

$$+[c_{11}x_{1t}-(r_{11}c_{11}+r_{12}c_{21}+\ldots+r_{1g}c_{g1})x_{1,t-1}]+[c_{12}x_{2t}-(r_{11}c_{12}+r_{12}c_{22}$$

$$+\ldots+r_{1g}c_{g2})x_{2,t-1}]$$

$$+\ldots+[c_{1g}x_{gt}-(r_{11}c_{1g}+r_{12}c_{2g}+\ldots+r_{1g}c_{gg})x_{g,t-1}] = v_{11}.$$

The Aitken procedure does not directly include the knowledge that R is diagonal (as does the constrained first stage of 2SLS), and is itself consistent with a nondiagonal R (though the parameters are unidentified without further assumptions about R). With the assumption that R is diagonal all of the r_{ij} terms in (14) for which $i \neq j$ are zero and drop out and the equation will normally be overidentified,[1] giving rise to one or more nonlinear (linear given r_{ii}) restrictions on the coefficients within the equation. Two estimation possibilities present themselves: (1) estimate the equations via 3SLS ignoring the nonlinear restrictions,[2] since they will hold in the probability limit anyway; or (2) choose values for each of the r_{ii}, and perform 3SLS estimation subject to the now linear restrictions. Estimates from both methods will be obtained for the model, with the r_{ii} set at their 2SLS estimates in the latter estimation procedure.

---

1 So long as at least one exogenous variable occurs or not all of the jointly dependent variables occur with lags in the original specification.

2 The r_{ii} implied by the coefficient estimates for each equation can be calculated and used as an index of either sampling error or nondiagonality of R.

2. Structural parameters

Parameter estimates[1] based on the autoregression correction technique discussed above are presented in this section; Figure I gives the first stage regressors that appear in each constrained augmented reduced form. Orcutt's iterative technique has been employed in estimating the autoregression parameters for the Pascal lag equations, while a search method has been used for the remaining stochastic equations; the results of the search are tabled in Appendix III-B.

Three sets of estimates are given: 2SLS, 3SLS restricted, and 3SLS unrestricted. "3SLS restricted" refers to the estimation procedure mentioned at the end of the preceding section in which the autoregression parameter for each equation is restricted to its 2SLS estimate--thereby reducing the nonlinear relations between the coefficients of each current and lagged variable to linear relations conditional on the given value for the autoregression parameter. (The unrestricted 3SLS estimation drops these restrictions, allowing 3SLS to affect the values of the autoregression parameters but loosing the information contained in the between-coefficient restrictions.)

The format is

$$\begin{bmatrix} \text{2SLS} \\ \text{3SLS restricted} \\ \text{3SLS unrestricted} \end{bmatrix}$$

---

[1] All estimates are conditional on the nonautoregression corrected classical 2SLS estimates of the Pascal lag parameters. Five equations--normalized on I^d, I^1, F_I, i_s, and S_c--were assumed to be known nonautoregressive from their specification.

Table 5

Additional Instruments in the Constrained Augmented Reduced Form Equations[1]

Equation	Additional Variables $Y_{g,-1}$	$X_{g,-1}$	First Stage Degrees of Freedom
Consumption	$C_{-1}, (Y^P)_{-1}$	$(c_C)_{-1}$	1
Investment, durable	$[I^d_{-1}], O_{-1}, (i_d)_{-1}$	I^d_{-2}	1
Investment, residential	$I^r_{-1}, X_{-1}, (i_s)_{-1}$		1
Investment, inventories	I^i_{-1}, X^i_{-1}	$(I^i_{STK})_{-2}$	1
Imports	$[(F_I)_{-1}], (Y_d)_{-1}(P_F)_{-1}$	$(F_I)_{-2}$	1
Production	$[X^P_{-1}], (h_m)_{-1}, [K_{-1}]$		3
Long term interest	$(i_L)_{-1}, (i^P)_{-1}$	(c_{i_L})	27
Short term interest	$(i_s)_{-1}$	$R_{-1}, (i_d)_{-1}$	27(OLS)
Corporate savings	$(S_c)_{-1}, (P_c - T_c)_{-1}$	$(P_c - T_c - S_c)_{-2}$	1
Corporate profits	$(P_c)_{-1}, P_{-1}$		2
Depreciation	$D_{-1}, [K_{-1}]$		3
Agricultural income	$(A_{P_A})_{-1}, (Y_{NF})_{-1}$	$(D_W)_{-1}$	1
Wage bill	$(W_1)_{-1}, [X^P_{-1}]$	X^P_{-2}	2
Nominal wage rate	$(\Delta w)_{-1}, \Delta P_{-1}$	$(U^{-1})_{-1}$	1

[1] Brackets indicate that the variable already occurs in the equation. There are 33 observations and 29 predetermined variables:

$W_2, A_2, P_M, N_W, (N_W - N_G), P_A, (G+F_X), R, i_d, T_c, T_W, T_A, T_P, T, U^{-1}, D_W, I^d_{-1}, (F_I)_{-1},$
$w_{-1}, P_{-1}, (I^i_{STK})_{-1}, K_{-1}, X^P_{-1}, (P_c - T_c - S_c)_{-1}, (Y_c^{-1}), c_C, c_{i_L}, (i_s^{-1})$, constant.

with estimated standard errors in parentheses below the coefficients where appropriate. R^2, the standard error of estimates, and the Durbin-Watson statistic for the 2SLS estimates are reported to the extreme right, as before.

Table 6

Autoregression Corrected Parameter Estimates

(Order: 2SLS, 3SLS restricted, 3SLS unrestricted)

(1) Consumption ($\hat{\rho} = 0.54$)

$$C_t = \begin{bmatrix} -2.943 \\ -28.539 \\ -2.214 \\ (1.996) \end{bmatrix} + \begin{bmatrix} 1.039 \\ 1.210 \\ 0.765 \\ (0.107) \end{bmatrix} \underbrace{\left[(1-0.06)^4 \sum_{i=0}^{\infty} \binom{i+3}{i} \lambda^i (Y_c)_{t-i}\right]_t}_{\equiv Y_t^P} + \begin{bmatrix} -0.558 \\ -0.650 \\ -0.455 \\ (0.139) \end{bmatrix} Y^F_{t-1} + \begin{bmatrix} 0.537 \\ 0.537 \\ 0.713 \\ (0.139) \end{bmatrix} C_{t-1}$$

(2) Investment, plant and equipment ($\hat{\rho} = 0.0$)

$$I^d_t = \begin{bmatrix} 0.315 \\ (0.051) \\ 0.288 \\ 0.300 \\ (0.037) \end{bmatrix} O_t + \begin{bmatrix} -1.173 \\ (0.413) \\ -0.802 \\ -1.315 \\ (0.329) \end{bmatrix} (i_L)_t + \begin{bmatrix} -8.668 \\ -9.394 \\ -7.430 \\ (2.042) \end{bmatrix} + \begin{bmatrix} 0.505 \\ (0.095) \\ 0.555 \\ 0.530 \\ (0.067) \end{bmatrix} I^d_{t-1}$$

(3) Investment, residential ($\hat{\rho} = 0.8$)

$$I^r_t = \begin{bmatrix} 0.047 \\ 0.057 \\ 0.038 \\ (0.023) \end{bmatrix} X_t + \begin{bmatrix} -0.038 \\ -0.046 \\ -0.024 \\ (0.026) \end{bmatrix} X_{t-1} + \begin{bmatrix} -0.661 \\ -0.768 \\ -0.873 \\ (0.433) \end{bmatrix} (i_s)_t + \begin{bmatrix} 0.529 \\ 0.615 \\ 0.425 \\ (0.406) \end{bmatrix} (i_s)_{t-1} + \begin{bmatrix} -0.519 \\ -1.670 \\ -0.032 \\ (0.935) \end{bmatrix} + \begin{bmatrix} 0.800 \\ 0.800 \\ 0.730 \end{bmatrix} I^r_{t-1}$$

Equation	R^2	S^2	DW
(1)	0.9993	12.86	2.05
(2)	0.9870	6.51	N/A
(3)	0.9419	4.79	1.76

(4) Investment, inventories ($\hat{\rho} = 0.0$)

$$I^i_t = \begin{bmatrix} 0.118 \\ (0.020) \\ \\ 0.111 \\ \\ 0.114 \\ (0.015) \end{bmatrix} X^i_t + \begin{bmatrix} -24.163 \\ (4.302) \\ \\ -21.527 \\ \\ -23.220 \\ (3.318) \end{bmatrix} + \begin{bmatrix} -0.477 \\ (0.092) \\ \\ -0.468 \\ \\ -0.457 \\ (0.069) \end{bmatrix} (I^i_{STK})_{t-1}$$

$R^2 = 0.6570$, $S^2 = 8.46$, DW $= 1.55$

(5) Imports ($\hat{\rho} = 0.0$)

$$(F_I)_t = \begin{bmatrix} 0.028 \\ (0.010) \\ \\ 0.025 \\ \\ 0.029 \\ (0.007) \end{bmatrix} (Y_d)_t + \begin{bmatrix} 5.672 \\ (2.556) \\ \\ 5.673 \\ \\ 7.396 \\ (1.660) \end{bmatrix} (p_F)_t + \begin{bmatrix} -9.290 \\ (3.396) \\ \\ -8.492 \\ \\ -11.109 \\ (2.221) \end{bmatrix} + \begin{bmatrix} 0.813 \\ (0.111) \\ \\ 0.814 \\ \\ 0.785 \\ (0.073) \end{bmatrix} (F_I)_{t-1}$$

$R^2 = 0.9891$, $S^2 = 1.52$, DW $=$ N/A

(6) Production ($\hat{\rho} = 0.8$)

$$X^P_t = \begin{bmatrix} 6.524 \\ \\ 5.074 \\ \\ 6.734 \\ (0.783) \end{bmatrix} (h_m)_t + \begin{bmatrix} 0.271 \\ \\ 0.388 \\ \\ 0.126 \\ (0.192) \end{bmatrix} K_t + \begin{bmatrix} -4.710 \\ \\ -3.795 \\ \\ -4.798 \\ (14.078) \end{bmatrix} + \begin{bmatrix} -5.219 \\ \\ -4.059 \\ \\ -5.682 \\ (0.681) \end{bmatrix} (h_m)_{t-1} + \begin{bmatrix} -0.216 \\ \\ -0.311 \\ \\ -0.082 \\ (0.176) \end{bmatrix} K_{t-1} + \begin{bmatrix} 0.800 \\ \\ 0.800 \\ \\ 0.851 \\ (0.087) \end{bmatrix} X^P_{t-1}$$

$R^2 = 0.9986$, $S^2 = 42.01$, DW $= 1.77$

(7) Long term interest ($\hat{\rho} = 0.61$)

$$(i_L)_t = \begin{bmatrix} 0.208 \\ \\ 0.419 \\ \\ 0.396 \\ (0.217) \end{bmatrix} + \begin{bmatrix} 1.688 \\ \\ 1.493 \\ \\ 2.201 \\ (0.554) \end{bmatrix} \underbrace{\left[(1-0.91) \sum_{i=o}^{\infty} \lambda^i (i_s)_{t-i}\right]_t}_{\equiv\, i^P_t} + \begin{bmatrix} -1.037 \\ \\ -0.917 \\ \\ -1.546 \\ (0.628) \end{bmatrix} i^P_{t-1} + \begin{bmatrix} 0.614 \\ \\ 0.614 \\ \\ 0.543 \\ (0.104) \end{bmatrix} (i_L)_{t-1}$$

$R^2 = 0.9223$, $S^2 = 0.12$, DW $= 1.38$

(8) Short term interest ($\hat{\rho} = 0.0$) $\quad R^2 = 0.9435 \quad S^2 = 0.43 \quad DW = 1.34$

$$(i_s)_t = \begin{bmatrix} -0.228 \\ (0.232) \\ -0.256 \\ -0.334 \\ (0.209) \end{bmatrix} + \begin{bmatrix} -1.435 \\ (0.284) \\ -1.026 \\ -1.234 \\ (0.241) \end{bmatrix} R_t + \begin{bmatrix} 1.226 \\ (0.071) \\ 1.218 \\ 1.251 \\ (0.065) \end{bmatrix} (i_d)_t$$

(9) Corporate saving ($\hat{\rho} = 0.0$) $\quad R^2 = 0.9662 \quad S^2 = 2.96 \quad DW = 1.94$

$$(S_c)_t = \begin{bmatrix} 0.820 \\ (0.039) \\ 0.771 \\ 0.805 \\ (0.031) \end{bmatrix} (P_c - T_c)_t + \begin{bmatrix} -2.606 \\ (0.951) \\ -2.950 \\ -2.741 \\ (0.809) \end{bmatrix} + \begin{bmatrix} -0.466 \\ (0.121 \\ -0.345 \\ -0.427 \\ (0.091) \end{bmatrix} (P_c - T_c - S_c)_{-1}$$

(10) Corporate profits ($\hat{\rho} = 0.9$) $\quad R^2 = 0.9955 \quad S^2 = 2.58 \quad DW = 1.29$

$$(P_c)_t = \begin{bmatrix} 0.671 \\ 0.641 \\ 0.676 \\ (0.030) \end{bmatrix} P_t + \begin{bmatrix} -0.604 \\ -0.577 \\ -0.650 \\ (0.052) \end{bmatrix} P_{t-1} + \begin{bmatrix} -2.389 \\ -2.343 \\ -0.250 \\ (1.102) \end{bmatrix} + \begin{bmatrix} 0.900 \\ 0.900 \\ 0.934 \\ (0.071) \end{bmatrix} P_{c_{t-1}}$$

(11) Depreciation ($\hat{\rho} = 0.9$) $\quad R^2 = 0.9976 \quad S^2 = 0.66 \quad DW = 1.66$

$$D_t = \begin{bmatrix} 0.050 \\ 0.060 \\ 0.046 \\ (0.012) \end{bmatrix} K_t + \begin{bmatrix} -0.045 \\ -0.054 \\ -0.035 \\ (0.015) \end{bmatrix} K_{t-1} + \begin{bmatrix} 1.635 \\ 0.944 \\ 3.051 \\ (1.227) \end{bmatrix} + \begin{bmatrix} 0.900 \\ 0.900 \\ 0.797 \\ (0.080) \end{bmatrix} D_{t-1}$$

		R^2	S^2	DW
(12)	Agricultural income ($\hat{\rho} = 0.0$)			

$$(A_{P_A})_t = \begin{bmatrix} -0.002 \\ -0.008 \\ 0.000 \\ (0.003) \end{bmatrix} (Y_{NF})_t + \begin{bmatrix} 9.983 \\ 11.837 \\ 9.885 \\ (0.523) \end{bmatrix} + \begin{bmatrix} 3.393 \\ 3.265 \\ 2.481 \\ (0.515) \end{bmatrix} (D_W)_t \qquad R^2 = 0.6463 \quad S^2 = 1.25 \quad DW = 1.70$$

(13) Private wage bill ($\hat{\rho} = 0.7$)

$$(W_1)_t = \begin{bmatrix} 0.526 \\ 0.537 \\ 0.506 \\ (0.026) \end{bmatrix} X^P_t + \begin{bmatrix} -5.045 \\ -4.135 \\ -6.813 \\ (1.953) \end{bmatrix} + \begin{bmatrix} -0.327 \\ -0.351 \\ -0.255 \\ (0.083) \end{bmatrix} X^P_{t-1} + \begin{bmatrix} -0.029 \\ -0.017 \\ -0.014 \\ (0.009) \end{bmatrix} X^P_{t-2} + \begin{bmatrix} 0.700 \\ 0.700 \\ 0.583 \\ (0.132) \end{bmatrix} (W_1)_{t-1} \qquad R^2 = 0.9992 \quad S^2 = 6.88 \quad DW = 1.27$$

(14) Nominal wage rate ($\hat{\rho} = 0.0$)

$$\Delta w_t = \begin{bmatrix} 2.223 \\ (0.397) \\ 1.930 \\ 2.249 \\ (0.252) \end{bmatrix} \Delta p_t + \begin{bmatrix} -0.082 \\ (0.017) \\ -0.117 \\ -0.078 \\ (0.015) \end{bmatrix} + \begin{bmatrix} 0.030 \\ (0.003) \\ 0.035 \\ 0.029 \\ (0.003) \end{bmatrix} U_t^{-1} \qquad R^2 = 0.9314 \quad S^2 = 0.002 \quad DW = 1.94$$

3. Reduced form parameters

Selected reduced form parameters for all three auto-regression corrected models are recorded in Table 7.

Table 7

IMPACT MULTIPLIERS

(Autoregression Corrected Model)

Model	R	i_d	G	T	T_W	T_A	T_P	W_2
2SLS								
X	6.27	-5.35	5.16	-2.90	-4.05	-4.09	-4.62	1.17
P	0.0083	-0.0071	-0.0085	-0.0042	-0.0066	-0.0062	-0.0071	-0.0076
h_m	0.8383	-0.7163	-0.7299	-0.3974	-0.5719	-0.5651	-0.6401	0.0179
3SLS restricted								
X	9.08	-10.78	9.97	-6.21	-9.16	-8.49	-9.67	2.60
P	0.0211	-0.0250	0.0257	-0.0152	-0.0236	-0.0211	-0.0242	-0.0030
h_m	1.5337	-1.8213	1.7562	-1.0728	-1.6132	-1.4739	-1.6824	0.2707
3SLS unrestricted								
X	3.81	-3.86	2.72	-1.29	-1.54	-1.74	-1.95	0.51
P	0.0062	-0.0063	0.0051	-0.0022	-0.0029	-0.0031	-0.0035	-0.0017
h_m	0.5212	-0.5284	0.3895	-0.1795	-0.2213	-0.2443	-0.2739	-0.0730

Table 8

Autoregression Parameter Search Results[1]

Durable Investment

$$I_t^d = \hat{\beta}_{2,0}+\hat{\beta}_{2,1}0_t+\hat{\beta}_{2,2}(i_L)_t + \hat{\beta}_{2,3}I_{t-1}^d$$

$\hat{\rho}$	Model	SSE	0	i_L	Constant	$(I^d)_{-1}$
0.0	C	189	0.315	-1.17	-8.67	0.505
0.0	F	189	0.313	-1.23	-8.37	0.509
0.1	C	185	0.343	-1.17	-8.89	0.456
0.1	F	186	0.348	-1.29	-8.65	0.449
0.2	C	185	0.380	-1.15	-9.23	0.390
0.2	F	187	0.391	-1.36	-8.95	0.372
0.3	C	185	0.423	-1.08	-9.53	0.311
0.3	F	188	0.440	-1.41	-9.14	0.288
0.4	C	184	0.468	-0.95	-9.63	0.230
0.4	F	189	0.489	-1.46	-9.02	0.206
0.5	C	180	0.511	-0.75	-9.38	0.154
0.5	F	189	0.533	-1.51	-8.52	0.136
0.6	C	177	0.547	-0.49	-8.67	0.093
0.6	F	191	0.573	-1.57	-7.67	0.081
0.7	C	175	0.577	-0.20	-7.46	0.049
0.7	F	195	0.610	-1.61	-6.57	0.043
0.8	C	180	0.604	0.04	-5.78	0.025
0.8	F	201	0.649	-1.48	-5.33	0.022
0.9	C	194	0.640	0.20	-3.64	0.022
0.9	F	205	0.687	-0.88	-3.69	0.020

[1] C refers to the classical 2SLS model; F refers to Fisher's structurally ordered instrumental variables model.

Residential Investment

$$I_t^r = \hat{\beta}_{3,0}+\hat{\beta}_{3,1}X_t+\hat{\beta}_{3,2}(i_s)_t$$

$\hat{\rho}$	Model	SSE	DW	Coefficients		
				X	i_s	Constant
0.0	C	364	0.53	0.0573	-2.10	-0.482
0.0	F	365	0.53	0.0562	-2.08	-0.131
0.1	C	305	0.62	0.0578	-2.13	-0.599
0.1	F	306	0.63	0.0575	-2.22	-0.292
0.2	C	255	0.74	0.0582	-2.13	-0.712
0.2	F	258	0.79	0.0592	-2.39	-0.479
0.3	C	215	0.89	0.0583	-2.09	-0.807
0.3	F	220	0.99	0.0611	-2.56	-0.677
0.4	C	184	1.06	0.0578	-1.97	-0.864
0.4	F	191	1.22	0.0622	-2.63	-0.823
0.5	C	161	1.23	0.0565	-1.76	-0.860
0.5	F	167	1.38	0.0601	-2.35	-0.752
0.6	C	143	1.41	0.0534	-1.36	0.738
0.6	F	148	1.38	0.0502	-1.39	-0.177
0.7	C	135	1.56	0.0508	-1.03	-0.641
0.7	F	158	1.43	0.0289	0.25	1.050
0.8	C	129	1.76	0.0474	-0.66	-0.519
0.8	F	215	1.73	-0.0035	1.91	2.540
0.9	C	130	1.92	0.0481	-0.47	-0.602
0.9	F	321	1.95	-0.0618	3.19	3.950

Inventory Investment

$$I^r_t = \hat{\beta}_{4,0} + \hat{\beta}_{4,1} X^i_t + \hat{\beta}_{4,2} (I^i_{STK})_{t-1}$$

$\hat{\rho}$	Model	SSE	DW	Coefficients		
				X^i	Constant	$(I^i_{STK})_{-1}$
0.0	C	254	1.55	0.118	-24.2	-0.477
0.0	F	265	1.27	0.142	-29.1	-0.584
0.1	C	238	1.64	0.124	-22.9	-0.504
0.1	F	249	1.34	0.148	-27.5	-0.613
0.2	C	226	1.72	0.132	-21.7	-0.538
0.2	F	235	1.41	0.156	-25.9	-0.650
0.3	C	216	1.78	0.141	-20.4	-0.581
0.3	F	224	1.46	0.167	-24.2	-0.696
0.4	C	207	1.84	0.153	-19.0	-0.631
0.4	F	216	1.51	0.179	-22.4	-0.749
0.5	C	200	1.88	0.166	-17.3	-0.687
0.5	F	208	1.56	0.193	-20.3	-0.808
0.6	C	195	1.93	0.180	-15.1	-0.744
0.6	F	203	1.63	0.208	-17.6	-0.864
0.7	C	192	1.99	0.194	-21.4	-0.800
0.7	F	199	1.73	0.221	-14.2	-0.910
0.8	C	192	2.06	0.208	- 9.0	-0.840
0.8	F	197	1.86	0.233	-10.3	-0.934
0.9	C	196	2.13	0.225	- 5.2	-0.861
0.9	F	199	2.00	0.245	- 5.8	-0.926

Imports

$$(F_I)_t = \hat{\beta}_{5,0} + \hat{\beta}_{5,1}(Y_d)_t + \hat{\beta}_{5,2}(P_f^{-1})_t + \hat{\beta}_{5,3}(F_I)_{t-1}$$

$\hat{\rho}$	Model	SSE	Coefficients			
			Y_d	P_f^{-1}	Constant	$(F_I)_{-1}$
0.0	C	44.0	0.0275	5.6	-9.2	0.813
0.0	F	44.5	0.0317	7.2	-11.3	0.767
0.1	C	44.5	0.0351	7.1	-10.4	0.731
0.1	F	45.2	0.0409	9.1	-12.8	0.668
0.2	C	45.5	0.0436	8.7	-11.2	0.639
0.2	F	46.6	0.0508	11.2	-13.8	0.562
0.3	C	46.7	0.0527	10.2	-11.5	0.543
0.3	F	48.2	0.0610	13.2	-14.3	0.456
0.4	C	47.7	0.0622	11.4	-11.2	0.446
0.4	F	49.5	0.0710	14.8	-13.9	0.356
0.5	C	48.2	0.0716	12.0	-10.2	0.353
0.5	F	50.3	0.0807	15.9	-12.7	0.265
0.6	C	47.8	0.0812	11.8	- 8.5	0.268
0.6	F	50.0	0.0901	16.0	-10.8	0.187
0.7	C	46.0	0.0911	10.7	- 6.5	0.192
0.7	F	47.9	0.0996	14.7	- 8.1	0.123
0.8	C	42.4	0.0102	8.8	- 4.3	0.132
0.8	F	44.0	0.1111	12.3	- 5.4	0.
0.9	C	38.3	0.1180	7.2	- 2.5	0.0 6
0.9	F	39.9	0.1310	10.5	- 3.2	0.049

Production

$$X_t^P=\hat{\beta}_{6,0}+\hat{\beta}_{6,1}(h_m)_t+\hat{\beta}_{6,2}K_t$$

$\hat{\rho}$	Model	SSE	DW	Coefficients		
				h_m	K	Constant
0.0	C	3133	0.38	6.23	0.285	-17.7
0.0	F	3135	0.38	6.29	0.282	-19.4
0.1	C	2670	0.45	6.24	0.285	-16.0
0.1	F	2672	0.45	6.31	0.282	-17.9
0.2	C	2269	0.55	6.25	9.285	-14.4
0.2	F	2271	0.55	6.33	0.281	-15.4
0.3	C	1928	0.68	6.26	0.284	-12.9
0.3	F	1931	0.68	6.37	0.280	-15.0
0.4	C	1648	0.86	6.29	0.283	-11.4
0.4	F	1652	0.86	6.42	0.277	-13.8
0.5	C	1429	1.07	6.32	0.281	- 9.9
0.5	F	1434	1.07	6.51	0.274	-12.5
0.6	C	1271	1.31	6.38	0.279	- 8.5
0.6	F	1277	1.31	6.61	0.269	-11.1
0.7	C	1172	1.56	6.45	0.275	- 6.8
0.7	F	1182	1.55	6.74	0.262	- 9.2
0.8	C	1134	1.77	6.52	0.271	- 4.7
0.8	F	1146	1.77	6.87	0.254	- 6.3
0.9	C	1155	1.92	6.57	0.263	- 1.9
0.9	F	1173	1.91	6.96	0.238	- 2.4

Short Term Interest Rate

$$(i_s)_t = \hat{\beta}_{8,0} + \hat{\beta}_{8,1} R_t + \hat{\beta}_{8,2} (i_d)_t$$

$\hat{\rho}$	Model	SSE	DW	Coefficients		
				Constant	R	i_d
0.0	C	5.55	1.34	-0.228	-1.43	1.23
0.0	F	8.72	1.48	-0.241	-2.60	1.14
0.1	C	5.30	1.43	-0.249	-1.35	1.24
0.1	F	8.18	1.59	-0.159	-2.53	1.16
0.2	C	5.11	1.52	-0.264	-1.24	1.25
0.2	F	7.77	1.72	-0.077	-2.45	1.18
0.3	C	4.97	1.61	-0.272	-1.10	1.26
0.3	F	7.46	1.84	-0.001	-2.34	1.21
0.4	C	4.87	1.69	-0.265	-0.94	1.27
0.4	F	7.24	1.96	-0.070	-2.21	1.25
0.5	C	4.79	1.76	-0.241	-0.76	1.28
0.5	F	7.12	2.07	-0.125	-2.05	1.29
0.6	C	4.74	1.81	-0.197	-0.57	1.27
0.6	F	7.12	2.17	-0.158	-1.88	1.33
0.7	C	4.71	1.85	-0.138	-0.39	1.24
0.7	F	7.28	2.25	-0.160	-1.71	1.37
0.8	C	4.71	1.87	-0.076	-0.25	1.20
0.8	F	7.72	2.31	-0.128	-1.58	1.41
0.9	C	4.80	1.88	-0.025	-0.15	1.16
0.9	F	8.66	2.33	-0.067	-1.56	1.45

Corporate Profits

$$(P_c)_t = \hat{\beta}_{10,0} + \hat{\beta}_{10,1} P_t$$

$\hat{\rho}$	Model	SSE	DW	Coefficients	
				P	Constant
0.0	C	338	0.30	0.570	-11.24
0.0	F	341	0.31	0.562	-10.58
0.1	C	285	0.36	0.569	-10.10
0.1	F	288	0.37	0.561	- 9.43
0.2	C	239	0.43	0.569	- 9.02
0.2	F	242	0.45	0.559	- 8.29
0.3	C	199	0.54	0.570	- 7.93
0.3	F	203	0.57	0.557	- 7.15
0.4	C	165	0.69	0.570	- 6.87
0.4	F	171	0.73	0.554	- 6.00
0.5	C	138	0.88	0.572	- 5.85
0.5	F	145	0.94	0.550	- 4.87
0.6	C	118	1.10	0.577	- 4.88
0.6	F	128	1.20	0.545	- 3.74
0.7	C	103	1.30	0.588	- 4.01
0.7	F	119	1.49	0.537	- 2.62
0.8	C	91	1.40	0.613	- 3.24
0.8	F	121	1.74	0.524	- 1.53
0.9	C	75	1.29	0.671	- 2.39
0.9	F	141	1.90	0.498	- 0.53

Corporate Savings

$$(S_c)_t = \hat{\beta}_{9,0}+\hat{\beta}_{9,1}(P_c-T_c)_t+\hat{\beta}_{9,2}(P_c-T_c-S_c)_{t-1}$$

$\hat{\rho}$	Model	SSE	DW	Coefficients		
				P_c-T_c	Constant	$(P_c-T_c-S_c)$
0.0	C	88.8	1.94	0.820	-2.61	-0.466
0.0	F	90.1	1.71	0.793	-2.79	-0.402
0.1	C	86.7	1.95	0.796	-2.68	-0.390
0.1	F	88.7	1.65	0.762	-2.86	-0.311
0.2	C	84.3	1.92	0.769	-2.73	-0.304
0.2	F	86.8	1.59	0.730	-2.86	-0.218
0.3	C	81.0	1.88	0.741	-2.71	-0.214
0.3	F	83.7	1.54	0.698	-2.76	-0.132
0.4	C	77.0	1.86	0.713	-2.59	-0.130
0.4	F	79.5	1.55	0.670	-2.53	-0.061
0.5	C	73.0	1.88	0.688	-2.34	-0.059
0.5	F	75.1	1.61	0.647	-2.20	-0.009
0.6	C	69.8	1.94	0.667	-1.98	-0.007
0.6	F	71.4	1.73	0.627	-1.79	0.025
0.7	C	68.3	2.03	0.649	-1.54	0.027
0.7	F	69.7	1.86	0.609	-1.34	0.042
0.8	C	69.4	2.11	0.636	-1.09	0.046
0.8	F	70.8	1.96	0.591	-0.90	0.048
0.9	C	73.6	2.16	0.632	-0.66	0.056
0.9	F	75.2	2.02	0.578	-0.52	0.047

Depreciation

$$D_t = \hat{\beta}_{11,0} + \hat{\beta}_{11,1} K_t$$

$\hat{\rho}$	Model	SSE	DW	Coefficients	
				K	Constant
0.0	C	86.5	0.25	0.0521	14.50
0.0	F	86.5	0.25	0.0522	14.40
0.1	C	71.1	0.29	0.0521	13.00
0.1	F	71.2	0.29	0.0522	13.00
0.2	C	57.8	0.34	0.0520	11.60
0.2	F	57.8	0.34	0.0521	11.60
0.3	C	46.4	0.42	0.0520	10.20
0.3	F	46.4	0.42	0.0521	10.20
0.4	C	36.9	0.54	0.0519	8.76
0.4	F	37.0	0.54	0.0520	8.73
0.5	C	29.5	0.72	0.0518	7.33
0.5	F	29.5	0.72	0.0520	7.30
0.6	C	24.0	0.96	0.0517	5.90
0.6	F	24.0	0.96	0.0519	5.87
0.7	C	20.4	1.24	0.0514	4.48
0.7	F	20.4	1.24	0.0517	4.44
0.8	C	18.8	1.51	0.0510	3.05
0.8	F	18.9	1.51	0.0516	3.00
0.9	C	19.1	1.66	0.0500	1.64
0.9	F	19.3	1.66	0.0515	1.55

Agricultural Income

$$(A_{P_A}) = \hat{\beta}_{12,0} + \hat{\beta}_{12,1}(Y_{NF})_t + \hat{\beta}_{12,2}(D_W)_t$$

$\hat{\rho}$	Model	SSE	DW	Coefficients		
				Y_{NF}	Constant	DW
0.0	C	37.4	1.70	-0.00224	9.98	3.39
0.0	F	37.5	1.69	-0.00199	9.95	3.35
0.1	C	37.1	1.96	-0.00187	8.95	3.29
0.1	F	37.1	1.95	-0.00168	8.93	3.26
0.2	C	37.3	2.22	-0.00137	7.92	3.16
0.2	F	37.3	2.22	-0.00130	7.91	3.15
0.3	C	38.2	2.46	-0.00073	6.88	2.99
0.3	F	38.2	2.46	-0.00084	6.89	3.02
0.4	C	39.7	2.67	0.00011	5.85	2.76
0.4	F	39.7	2.68	0.00026	5.88	2.84
0.5	C	42.0	2.86	0.00131	4.81	2.43
0.5	F	42.0	2.87	0.00047	4.87	2.60
0.6	C	44.8	3.00	0.00308	3.78	1.93
0.6	F	44.9	3.02	0.00145	3.87	2.25
0.7	C	48.3	3.11	0.00308	2.74	1.08
0.7	F	48.5	3.13	0.00276	2.88	1.72
0.8	C	52.3	3.20	0.01060	1.73	-0.58
0.8	F	52.8	3.21	0.00438	1.92	0.82
0.9	C	56.9	3.28	0.02020	0.77	-5.27
0.9	F	58.2	3.26	0.00457	1.02	-0.93

Private Wage Bill

$$(W_1)_t = \hat{\beta}_{13,0}+\hat{\beta}_{13,1}(X^p)_t+\hat{\beta}_{13,2}(X^p)_{t-1}$$

$\hat{\rho}$	Model	SSE	DW	Coefficients		
				X^p	Constant	$(X^p)_{-1}$
0.0	C	225	1.09	0.414	-14.2	0.1510
0.0	F	354	0.71	0.549	-13.8	0.0092
0.1	C	225	1.03	0.442	-13.0	0.1220
0.1	F	306	0.75	0.541	-12.6	0.0187
0.2	C	223	0.99	0.468	-11.6	0.0957
0.2	F	272	0.81	0.537	-11.4	0.0234
0.3	C	217	1.01	0.488	-10.3	0.0754
0.3	F	247	0.87	0.536	-10.2	0.0254
0.4	C	209	1.05	0.502	- 9.00	0.0612
0.4	F	229	0.95	0.537	- 8.97	0.0261
0.5	C	201	1.11	0.513	- 7.69	0.0517
0.5	F	215	1.04	0.538	- 7.77	0.0264
0.6	C	195	1.19	0.520	- 6.38	0.0454
0.6	F	206	1.13	0.540	- 6.56	0.0267
0.7	C	193	1.27	0.526	- 5.04	0.0414
0.7	F	201	1.22	0.543	- 5.37	0.0273
0.8	C	194	1.34	0.532	- 3.63	0.0387
0.8	F	203	1.30	0.549	- 4.17	0.0285
0.9	C	202	1.39	0.537	- 1.98	0.0364
0.9	F	217	1.34	0.564	- 2.92	0.0307

Change in the Nominal Wage Rate

$$\Delta w_t = \hat{\beta}_{14,0} + \hat{\beta}_{14,1}\Delta p_t + \hat{\beta}_{14,2}\%U_t^{-1}$$

$\hat{\rho}$	Model	SSE	DW	Coefficients		
				Δp_t	Constant	$\%U^{-1}$
0.0	C	0.0376	1.94	2.22	-0.0825	0.0298
0.0	F	0.0397	2.09	2.73	-0.0714	0.0268
0.1	C	0.0394	2.15	2.42	0.0007	0.0287
0.1	F	0.0410	2.24	2.80	-0.0549	0.0263
0.2	C	0.0415	2.33	2.57	-0.0007	0.0279
0.2	F	0.0423	2.36	2.77	-0.0549	0.0264
0.3	C	0.0441	2.46	2.64	-0.0516	0.0274
0.3	F	0.0445	2.47	2.74	-0.0482	0.0265
0.4	C	0.0473	2.56	2.67	-0.0433	0.0271
0.4	F	0.0475	2.56	2.73	-0.0412	0.0265
0.5	C	0.0513	2.63	2.68	-0.0355	0.0269
0.5	F	0.0514	2.64	2.72	-0.0341	0.0265
0.6	C	0.0561	2.69	2.69	-0.0278	0.0267
0.6	F	0.0562	2.69	2.71	-0.0267	0.0263
0.7	C	0.0617	2.73	2.69	-0.0203	0.0265
0.7	F	0.0617	2.73	2.71	-0.0194	0.0261
0.8	C	0.0680	2.76	2.69	-0.0129	0.0261
0.8	F	0.0681	2.76	2.71	-0.0121	0.0256
0.9	C	0.0751	2.77	2.70	-0.0058	0.0257
0.9	F	0.0752	2.78	2.71	-0.0051	0.0251

III. Conclusion

As is the case in classical hypothesis testing, model changes are notoriously difficult to assess unless they are "nested," that is, the change includes the prior specification as a special case; in this event the researcher at least knows that in the limit the change will not do damage, although with limited evidence information may still be lost if the generalization is unnecessary. Of the seven major types of changes that have been made to the model, five include the original K-G specifications as nested subcases, while two represent an alternate underlying set of assumptions. The two nonnested changes are the theoretical changes in equation specification and the SOIV treatment of the simultaneous equations problem. The nested changes were the improved treatment of variable nonlinearities in determining the multipliers, estimation of theoretically superior lags, the autoregression correction, use of an expanded data set, and the use of a "full-information" estimation procedure (3SLS). We shall examine each of the changes in turn.

A. Theoretical Changes in Equation Specification

Although we deviated from the original K-G specification in many places, the three most important were the disaggregation of investment, the specification of the two equations of the monetary sector, and the derivation of the consumption function. The consumption function will be disucssed later, when we evaluate the Pascal lags.

Although the disaggregation into durable, residential, and inventory investment (see sections II.A.2., 3., 4. above) is an obvious and much-needed improvement in the K-G model, the equation errors remain relatively large. The dominant variable in the durable investment equation, nonwage disposable income (retained from the original K-G investment specification) is unfortunately not one that the model is adept at tracking; the errors in I^d in the single period sample data solution correspond closely to the errors in the determination of nonwage disposable income.[1] In addition, the expense of estimating the Pascal lags restricted their application to only one equation in the main body of the model; had the consumption function not been the logical center of the K-G model, the durable investment equation would have been specified with the Pascal lag[2] -- where the convolution of expectations and adjustment mechanisms is likely to be very important and the simple first-order adjustment model less satisfactory. With the results of Appendix III it is feasible to estimate different expectations and adjustment roots; in order to simplify estimation each process may be specified a priori to have a known number of roots (based on the desired lag shape), and further, the roots may be specified to be equal within a process but different across processes: then the parameter search is only two dimensional. That is, the Pascal lag with unknown convolutions but equal roots would be exchanged for a

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1 See figures 13 and 24. Nonwage disposable income has not been plotted separately; since disposable wage income is relatively stable, the actual and solution values of nonwage disposable income are almost identical to the corresponding values of total disposable income.

2 The surprisingly good estimation residuals (which did not continue through the full model solution phase) further influenced the decision to use the Pascal lag on the consumption function.

rational lag of known convolutions with subsets of roots equal.[1] A durable investment equation of this type would likely have much improved the model.

Inspection of the single period residential investment solution (Appendix III) suggests that the shape of the curve is correct but the timing is wrong -- I_t^r corresponds more closely to $\hat{I}_{t+1}^r$ than to $\hat{I}_t^r$. Since the initial formulation of the equation contains no lagged variables, this effect must be a result either of the autoregression correction or of input[2] model solution values of the right-hand side endogenous variables. The Durbin-Watson statistic for the uncorrelated equation is an autoregressive 0.53 (Appendix III-B), and the sum of squared errors (SSE) drops from 364 to 129 when $\hat{\rho}$ is set to 0.8. Thus we conclude that the autoregression correction is necessary and the minimum is meaningful. Examining the input variables we note the slight lag in the solution value of GNP (Figure 11); this is evidently the source of the residential investment lag, and the equation itself performs adequately.

The inventory investment equation at least performs the function of confining the errors resulting from excessive time aggregation to the equation in which they originate; the poor tracking of this equation is primarily a result of the use of annual data.

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[1] Estimation algorithms are currently being developed for this case.

[2] We assume that the primary determinants of the endogenous variables are the variables in the equations normalized in the endogenous variables in question; in a fully integrated system there are clearly feedback effects from the other equations, but these are assumed small relative to the other effects.

B. Simultaneous Equations Problems

An annual model provides an opportunity to assess the importance of inconsistency resulting from simultaneous equations problems, since the longer the observation time period the more nearly simultaneous the equations become. First, however, a digression is in order.

There has been considerable confusion regarding the calculation of 2SLS parameter estimates when the number of first stage regressors equals or exceeds the number of observations; specifically, 2SLS is often said to be undefined when this condition prevails.[1] As Ruble demonstrates[2], however, the 2SLS estimates are perfectly well-defined in this case and are in fact equal to ordinary least squares (OLS) estimates. Two-stage least squares is an estimator in which the right-hand side jointly dependent variables (Y) are replaced by their estimated values $\hat{Y}$, the part of Y in the space spanned their first stage regressors (denoted X_I). If the number of observations equals the number of regressors, we can obtain coefficients relating (perfectly) Y and X_I: that is, $\hat{Y} \equiv Y$, or the entire Y vector is contained in the space spanned by X_I. If the number of first stage regressors exceeds the number of observations, unique coefficients cannot be obtained -- but the coefficients are not required, only the part of Y in the space of X_I, and <u>all</u> of Y is in this space again, so that $\hat{Y} \equiv Y$. But when $\hat{Y} \equiv Y$, 2SLS ≡ OLS. Asymptotically, of course, the two estimates will diverge, since the number of first-stage regressors will not increase as the number of observations does, but for the given sample the

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[1] See, for example, [37], p. 175.

[2] [53].

estimates are equivalent. Thus classical 2SLS is not a good way to take equations simultaneity into account.

In this particular model (estimated from 33 observations), there are 29 predetermined variables in the first-stage regressions for those right-hand jointly dependent variables in the main body of the model before the autoregression correction; after the required autoregression augmentations (see II.C.1 above), the median first-stage degree of freedom value is 1 (Table 5 above). Thus the $\hat{Y}$'s, although never quite equivalent to the Y's, are very close indeed[1], and our classical model is one that assigns little importance to simultaneous equations problems and results in estimates very close to ordinary least squares estimates.

The structurally ordered instrumental variable (SOIV) model is the opposite extreme,[2] placing heavy emphasis on equation simultaneity, since it is based not only on causal structure within the model, but also recognizes the existence of policy variable reaction functions outside the model. How different, then, were the parameter estimates resulting from these two models?

The unambiguous conclusion is that the estimates are not very different. The parameter estimates from the two models at each trial value of the autoregression parameter are tabled in Appendix III-B; in

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1 Subsequent calculations showed that the correlation between $\hat{Y}$ and Y was never less than 0.99 for any right-hand side jointly dependent variable.

2 The identification of the equations provides an index to the polarity of the two models: all of the SOIV model equations have between 0 and 3 overidentifying restrictions (Table 2 above), while the classical model equations have between 30 and 32 overidentifying restrictions.

only three cases are the differences worthy of comment, in the equations determining the short term interest rate, private wage bill, and imports.

In the short term interest rate equation the generalization to endogenous policy is likely to be important, since there may be strong feedbacks from the short term rate to the discount rate. The SOIV model tends to increase the coefficient on the reserve ratio and decrease the coefficient on the discount rate, although at the means (with $\hat{\rho} = 0$) the net change in the short rate is only 0.2%.

In the private wage bill equation the classical and SOIV estimates are in approximate agreement at $\hat{\rho} = 0.7$ (the minimum SSE), but diverge as drops; at $\hat{\rho} = 0$ there is considerable difference between the two models, with the SOIV model estimates remaining much closer to the previous estimates while the classical model begins to put more weight on the lagged private output term. It seems that the SOIV model is less likely to overstate the importance of lagged terms in the presence of autoregression, principally because many of the lagged endogenous first stage regressors were not included in calculation of the instruments -- thus lowering the correlation between the instruments and the second stage error and decreasing the inconsistency. Similarly, in the import equation the SOIV model consistently puts less weight on the lagged term, although in neither of these equations are the parameter differences important; when the SOIV model estimates are different at all, it is primarily due to better estimates in the presence of autoregression.

Thus the conclusion must be that the inconsistency introduced by equation simultaneity, whether derived from unstated policy reaction functions or from supposedly inappropriate estimation procedures, is negligably small; in fact, in large econometric models 2SLS is "sample equivalent" to OLS since (1) in the absence of significant expectations processes all of the lagged endogenous variables become predetermined (increasing the number of first stage regressors), (2) if the autoregression correction does not constrain the augmented reduced form equations as above the number of first stage regressors is vastly increased, and (3) if the model is nonlinear the reduced form Taylor expansion need not be arbitrarily stopped at first-order, and any number of first-stage regressors may be included. Any one of these, and certainly all together, virtually assumes that the number of first-stage regressors will exceed the number of observations, guaranteeing the sample equivalency of OLS and classical 2SLS. Further, if our results generalize, even the polar SOIV estimates will differ little from the OLS estimates, and estimation might just as well be carried out by the less expensive OLS method as far as consistency is concerned.

C. Variable Nonlinearities

Goldberger calculated the original K-G multipliers from a simplified, linearized reduced form;[1] we compared impact multipliers obtained from an unsimplified reduced form linearized in three separate ways and found them somewhat different (Table 4), although the comparison was distorted by overlarge multipliers calculated from the model as yet uncorrected for autoregression.

The full generalization of the Goldberger calculations is to numerically differentiate the model at the appropriate point; this is the technique underlying the multipliers in tables in Appendix IV.

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[1] [25], pp. 71-77.

Comparison with Table 7 (the autoregression corrected linearized impact multipliers) leads to the conclusion that the linearization loses significant information, even when the multipliers are smaller: the government expenditures-GNP multiplier differs by 14 per cent, the -manhours multiplier by 13 per cent, and the -price multiplier changes sign. In this particular model, at least, the multipliers are somewhat sensitive to variable nonlinearities. However, since the preferred technique (numerical differentiation) is usually the least expensive, all modern multipliers are calculated by this method (in 1959 Goldberger did not have available the nonlinear solution programs of today, and thus had to use the less-accurate labor intensive linearization); the linearization inaccuracy is of consequence only in its effects on the first-stage instrument calculation. Expansion of the approximating polynomial to a higher degree reduces 2SLS to OLS in most econometric models in the course of obtaining better first-stage estimates, and even then serious inefficiency may result if all of the variables (not just those suspected *a priori* of being important nonlinearly) are included with higher order terms, and inconsistency may result if they are not (see above). Thus the conclusion is that for most macroeconomic samples 2SLS has no advantages over OLS in the presence of variable nonlinearities.

D. Pascal Lags

Two equations, determining consumption and the long-term interest rate, have been estimated incorporating Pascal lags. The long term rate equation was discussed above (V.A.) with the conclusion that although the root is perhaps a little too large (due to sampling error) the equation still tracks very well. The consumption lag remains to be examined.

The original K-G model included, among other regressors, lagged consumption; if consumption decisions are based on an adaptive expectations/permanent income mechanism, the lag inversion will introduce serial correlation into the (transformed) disturbance. Except in the special case in which the autoregression parameter of the untransformed disturbance is equal to the root of the lag the resulting parameter estimates will be inconsistent, with the seriousness on the inconsistency depending on the difference vetween the lag root and the autoregression parameter. (This incidentally is not the case with the parameterized initial condition estimation we have undertaken since the lagged dependent variable does not appear). The K-G estimate[1] of the lagged consumption coefficient is 0.26, while our estimate of the untransformed disturbance autoregression parameter is 0.54; the difference (0.28) is large, resulting in an upward biased lagged consumption coefficient and too small an income coefficient. These effects are offset by the depression-dominated sample, which tends to raise the short run marginal propensities to consume (mpc's) -- although they remain low (see [14], p. 66). Our estimates of the short run mpc's from wage, agricultural, and profit income are 0.60, 0.37, and 0.45 respectively, compared to the K-G estimates[2] of 0.55, 0.34, and 0.41. While our short run estimates are considerably better than the K-G estimates, this is not entirely due to the Pascal lag; without the

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1 [34], p. 90.

2 [34], p. 90.

autoregression correction the estimates would be too high (the usual result in annual data including the depression).

In calculating the long run mpc's we have followed Evans' example[1] and assumed an equilibrium growing at 2 per cent per year,[2] giving long run mpc's of 0.94, 0.58, and 0.70. The aggregate long run mpc depends on the relative income shares and ranges from a sample

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1 [14], pp. 70-72.

2 Implying $Y_t = 1.02Y_{t-1}$, or $Y_{t-1} = (1.02^{-1}Y_t$. The inverted Pascal lag equation

$$(1)\ Y^P_t - 0.24\,Y^P_{t-1} + 0.0216\,Y^P_{t-2} - 0.000864\,Y^P_{t-3} + 0.00001296\,Y^P_{t-4} = 0.78(Y_c)_t$$

becomes

$$(2)\ Y^P_t - \frac{0.24}{1.02}\,Y^P_t + \frac{0.0216}{(1.02)^2}\,Y^P_t - \frac{0.000864}{(1.02)^3}\,Y^P_t + \frac{0.00001296}{(1.02)^4}\,Y^P_t = 0.78(Y_c)_t$$

or

$$(3)\ Y^P_t = 0.88(Y_c)_t.$$

The estimated consumption function

$$(4)\ C_t = -2.21 + 0.76\,Y^P_t = 0.45\,Y^P_{t-1} + 0.71\,C_{t-1}$$

Similarly discounted becomes

$$(5)\ (1 - \frac{0.72}{1.02})\,C_t = -2.21 + (0.76 - \frac{0.45}{1.02})\,Y^P_t.$$

Substitution of (3) for Y^P gives

$$(6)\ C_t = -2.21 + 0.94\,[W_1 + W_2 - T_W + 0.62\,(A_1 + A_2 - T_A) + 0.75\,(P - S_c - T_c)],$$

the discounted consumption function from which the long run mpc's are obtained.

high of 0.886 in 1969 to a low of 0.841 in 1933 (the corresponding short run aggregate mpc's were 0.565 and 0.537). These values are slightly higher than expected a priori [Evans' comparisons ([14], p.65), suggest 0.834 as the concensus on the aggregate long run marginal propensity to consume nondurables and services; the inclusion of durables would raise this estimate].

Altogether, the conclusion is that in the particular case of annual consumption data there are offsetting misspecifications in the simple inverted geometric lag framework that result in approximately the same estimares as are obtained from the more general Pascal formulation estimated by the statistically superior initial condition parameterization technique. This results from the offsetting biases discussed above combined with (1) the lessened importance of inverted V-lags in annual data, and (2) the absence of a partial adjustment mechanism, or the presence of a partial adjustment mechanism with a root radically different from the expectations root. Quarterly data will provide a better test of the Pascal lags, and investment data -- with the strong expectations/adjustment convolution -- will provide a better test of the generalized lags of Appendix III.

E. Autoregression

Serial correlation of the model disturbances may occur for several reasons; besides the economist's usual explanation of long cycles in unmeasurable omitted variables included in the disturbance, autoregression may also result from algebraic transformations (e.g., stock-flow translations[1] or lag inversion[2]), or from aggregation over time when there are lagged normalizing variables.[3]

Two of the equations, determining durable investment and imports, have lagged normalizing variables on the right; fortunately, aggregation is unlikely to be serious in these cases, although long cycles in the disturbance generation may be. As is well known, the Durbin-Watson test is not applicable in the presence of lagged normalizing variables; Durbin, however, has proposed an alternative test that also has the desirable characteristic of not requiring estimation of the autoregression parameter.[4] The test was originally designed for the ordinary least squares model; we shall temporarily reverse our assumptions and assume the simultaneous equation problem is negligible in our model and thus use the test as an approximation. The assumption that the disturbances are normally distributed is also added.

Consider the model

(1a) $$Y_t = \beta_o + \beta_1 X_t + \beta_2 Y_{t-1} + \epsilon_t \qquad (t-1,\dots,T)$$

(1b) $$\epsilon_t = \rho\epsilon_{t-1} + u_t \qquad u_t \sim N(0,\sigma^2)$$

and let β denote the set of parameters $(\beta_o,\beta_1,\beta_2,\sigma^2)$; the logarithm of the likelihood function $[L(\rho,\beta)]$ is

$$L(\rho,\beta) \equiv -\frac{T}{2}\log(2\pi\sigma^2) - \frac{1}{\sigma^2}\sum_{t=1}^{T}(\epsilon_t - \rho\epsilon_{t-1})^2.$$

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1 See [40] for an application of this often neglected fact.

2 See section II.A.1 above, for example.

3 See for example [49].

4 [9], p. 416 and p. 420.

If we set the derivatives of (2) with respect to ρ and β equal to zero and solve the resulting nonlinear differential system, the estimates obtained will be asymptotically normal with known variances and co-variznces. One way to solve such a system is to choose arbitrary starting estimates and use the Newton-Raphsen iterative technique. But we do not really need an estimate of ρ, we just want to know if it is significantly different from zero; we need a test on the change in $\hat{\rho}$ from its starting estimate of zero. If we expand the likelihood function (2) about ρ=0 and $\beta = \hat{\beta}$, where $\hat{\beta}$ is the ordinary least squares estimate of β, the result can be rearranged[1] as

$$(3)\ L(\rho,\beta) \equiv -\frac{T}{2}\log(2\pi\sigma)^2 - \frac{1}{2\sigma^2}\sum_{t=1}^{T}(e_t - d\rho\, e_{t-1} - d\beta_o - d\beta_1 X_t - d\beta_2 Y_{t-2})^2$$

where the e's are the ordinary least squares residuals from (1a). Setting the derivatives with respect to $d\rho$ and $d\beta$ -- not ρ and β -- equal to zero and solving the differential system obtained is equivalent to carrying out ordinary least squares on the equation

$$(4)\qquad \epsilon_t = (d\rho)e_{t-1} - (d\beta_1)X_t - (d\beta_2)Y_{t-1} + v_t.$$

Since the Taylor expansion implies multiplying each change in the parameters, $d\rho$ and $d\beta$, by the partial with respect to that parameter of the likelihood function (evaluated at $\rho = 0$ & $\beta = \hat{\beta}$), this is also equivalent to taking the first step in a Newton-Raphson iterative estimation of ρ and β. Thus we are testing whether the change in $\hat{\rho}$ resulting from the first Newton-Raphson iteration is significantly different from zero.

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[1] [9], p. 420.

Durbin shows that the estimate of $d\rho$ is asymptotically normal and the test of $\rho = 0$ can then be "carried out by testing the significance of the coefficient of $[e_{t-1}]$ by ordinary least-squares procedures."[1]

On the other hand, if the original structural disturbance (before lag inversion) was autoregressive, the models are reversed, with $\hat{\rho}=0.0$ the AE model and $\hat{\rho}=0.9$ the PA model. These models are hard to distinguish on the basis of the coefficient estimates alone. Theoretically we are included toward the PA hypothesis (see Section II.A.3. above), but important serial correlation would decrease our faith in this hypothesis. Applying the Durbin test gives the following results:

Imports

$$w_t = \underset{(0.148)}{0.218}\, e_{t-1} + \underset{(8.320)}{0.529} + \underset{(0.023)}{0.126}\, (Y_{d_t}) - \underset{(6.680)}{8.833}(p_F^{-1})_t + \underset{(0.291)}{0.197}\, (F_I)_{t-1}.$$

The appropriate t ratio is 1.470 so that we fail to reject the null hypothesis of no autoregression, choosing the estimates of $\hat{\rho}=0.0$ based on a partial adjustment hypothesis. In this case the autoregression decision is very important.

Autoregression decisions on the remaining stochastic equations were based on the Durbin-Watson (D.W.) statistic. Although all of the differenced equations (e.g., I^r, I^i, Δw) were candidates for negative autoregression[2], all of the D.W.'s were 2.0 or below, precluding

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1 [9], p. 420.

2 If, for example, the disturbance originates on the stock of houses (K^r), so that $K_t^r = \beta X_t + \epsilon_t$, then $I_t^r \equiv K_t^r - K_{t-1}^r = \beta(X_t - X_{t-1}) + \epsilon_t - \epsilon_{t-1}$ and $\text{Cov}(\epsilon_t - \epsilon_{t-1}, \epsilon_{t-1} - \epsilon_{t-2}) = -\sigma^2$ even if ϵ_t is serially independent.

negative serial correlation. Of the twelve equations, seven -- normalized on C, I^r, X^p, i_L, P_c D, and W_1 -- had D.W.'s below the lower bound; these were searched or iterated over $\hat{\rho}$, and the results obtained were reasonable (see Appendix III-B). Two more equations, determining Δw and A_{P_A}, had D.W. statistics above the positive autoregression upper bound, allowing us to reject the hypothesis of autoregressive disturbances. These equations were searched anyway and the minimum SSE was at $\hat{\rho}$=0.0, where the coefficients were plausible. Three other equations-normalized on I^i, i_s, and S_c -- resulted in D.W. statistics either above the upper bound for positive autoregression (giving strong evidence of no autoregression) or else in the inconclusive region. These equations were also searched over the positive $\hat{\rho}$ domain for completeness, although in the model $\hat{\rho}$ was constrained to zero in these cases (even though the minimum SSE was elsewhere) since the inventory investment and short-term interest rate equations were specifically hypothesized to be short run processes, while the corporate savings equation resulted in a negative adjustment coefficient at the minimum SSE (with $\hat{\rho}$=0.8).

Casual inspection of Appendix III-B shows that, as expected, the autoregression correction is of critical importance in almost all cases. In the short-term interest rate equation, for instance, the coefficient of R, the ratio of excess to required reserves, changes from -1.43 at $\hat{\rho}$=0.0 to -0.398 at $\hat{\rho}$=0.7, the minimum SSE over positive values of $\hat{\rho}$; the policy implications are enormous. Even with this particular $\hat{\rho}$ constrained to zero, the 2SLS monetary policy impact multipliers on GNP are reduced to less half of this previous values by the full model autoregression correction (line 1, Tables 4 and 7), primarily through changes in the residential investment coefficients. The fiscal policy multipliers, on the other hand, all increase slightly. Both of these results carry through the 3SLS estimation (also Tables 4 and 7). If these results are indicative the autoregression parameter is the single most important parameter of an equation.

F. Data

The original K-G model was estimated from data for the 18 years 1929-1941, 1946-1950; less than a third of this data lies in the postwar period. The present revision has been estimated from 33 observations of which 21 lie in the postwar period, with the unrepresentative immediately postwar years dropped from the estimation and used as a forecasting test; it is hoped that this gives the model a much more current flavor, in addition to almost doubling the data set. The expanded data set also allowed us to perform the autoregression correction (autoregression was a serious problem in the original model)[1] and discontinue the arbitrary instrument set truncation (Klein and Goldberger used 15 predetermined variables in place of the 21 that actually occurred in the system in their calculations[2]), although this latter point is of small importance (see Section V.B. above).

G. Three-Stage Least Squares

Three-stage least squares can be proven to be asymptotically efficient relative to 2SLS. In the small sample, however, there is no guarantee that 3SLS is superior; indeed, if the number of stochastic equations exceeds the number of observations, or if the number of

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[1] [34], pp. 51-53.

[2] [34], pp. 50-51.

coefficients to be estimated is greater than the product of equations and observations, then conventional 3SLS is undefined, since -- the across-equations disturbance variance-covariance matrix -- is singular.[1] In our particular case this is not a problem (since there are 14 stochastic equations and 33 observations and the number of coefficients to be estimated is 59, against an equation-observation product of 462), but it will be interesting to examine the finite sample gain from 3SLS estimation.

Since at least three separate effects -- due to 3SLS estimation, autoregression correction, and lag specification -- are intermixed in the estimates, it is difficult to accurately assess the importance of 3SLS estimation alone. There are two comparisons that potentially might remove the effects of the autoregression corrections and allow us to concentrate on the changes due to 3SLS: (1) autoregression corrected 2SLS versus autoregression corrected but constrained[2] 3SLS (Table 6); and (2) nonautoregression corrected 2SLS versus uncorrected 3SLS (Table 3). In both cases we immediately observe that the standard errors of the coefficients are all much smaller than the 2SLS estimates; 3SLS is efficient relative to 2SLS, as expected. But how much did the point estimates of the coefficients themselves change?

In the first (autoregression corrected) case, the coefficients of twelve of the equations -- determining I^d, I^r, I^i, F_I, X^p, i_s, i_L, D, P_c, S_c, W_1, and 2 -- do not change much, usually less than one standard deviation. There is a larger change in the coefficients of the equation normalized on A_{P_A}, and an even larger change in the coefficients

1 See [37], p. 175, for example.

2 See Section III.c.2. above.

of the consumption function [the marginal propensity to consume wage income increases 14%, which results in impact multipliers that are far too large (see Table 7)]. The change in the agricultural income equation is probably due to misspecification, since the nonfarm income variable is not significant; the consumption function changes probably result from a bad 2SLS estimate of ρ, obtained by premature cessation of the Orcutt iterative technique (the later unrestricted 3SLS estimate of ρ is quite different). Thus over the equations that are comparable the three stage coefficients did not change much, even with noncomparable equations elsewhere in the system.

In the second and far more clear cut case, 2SLS and 3SLS are compared before any autoregression correction. Here the decrease in coefficient errors is even more striking, and almost[1] all of the 3SLS coefficients are withon one (3SLS) standard deviation of the 2SLS estimates. Not surprisingly, we find in Table 4 that the impact multipliers are also very similar, with the 3SLS monetary policy multipliers slightly lower while the fiscal policy multipliers are slightly higher.

The conclusion, then, is that 3SLS is very important for hypothesis testing, but has only marginal effects on the actual coefficient estimates obtained -- at least for this particular model.

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[1] Only the coefficients of the long-term interest rate and corporate savings equations were not within one standard deviation; the maximum i_L coefficient difference is three standard deviations, while the S_c coefficients are all within two.

H. An Interesting Implication

Much of the previous discussion has been leading inexorably toward the suggested use of an alternative estimator. Let us briefly review some of the major points.

First, variable nonlinearities are relatively important and at the same time most difficult to properly treat in the first stage regression, and sometimes (if nonlinear within an equation) very hard to handle in the second and third stages of estimation. At the same time, a crucially important nonlinearity relating GNP and the price level occurs in all meaningful macroeconomic models.

Second, some of the most important parameters of a model enter nonlinearly. This is true of the lag parameters--in this case causing limited use of the Pascal lag as a result--and also of the auto-regression parameters, which are extremely important in model estimation. Not only does this nonlinearity cause first-stage difficulties resulting in grossly inefficient estimates except under certain relatively restrictive sets of assumptions, but even more important it renders 3SLS inefficient in small samples-- either ρ is arbitrarily fixed at some value for each equation or else the non-linear restrictions between coefficients are dropped (assuming that the search over all possible, combinations of the $\hat{\rho}$'s is computationally impossible, even with modern electronic equipment).

Third, we have found that in this case at least the inconsistency that results from simultaneous equations is negligibly small. Two-stage least squares is a technique with the main purpose of eliminating this inconsistency, even at the price of efficiency. Three-stage least squares adds information from the disturbance variance-covariance matrix, which in our sample primarily affects the coefficient standard errors and only marginally changes the coefficients themselves; thus 3SLS coefficients are very similar to the 2SLS results. What is the price of using 2SLS with its emphasis on consistency above all?

Asymptotically we lose nothing, as the standard textbooks show; in the finite sample the cost is higher, however. Comparing unconstrained reduced forms to those derived from the estimated structure, it is apparent that they are quite different, as a result of the overidentifying restrictions not being included in the first stage. Thus the calculated instruments inputted into the second stage are very inefficient estimates, which in turn condition the second stage parameter estimates. Asymptotically the unconstrained and desired reduced forms converge, since each parameter estimate goes to the true parameter in the limit, and the restrictions hold between the true parameters. In a finite sample, however, these restrictions do not necessarily hold; to the extent that they do not, we are losing efficiency. In actual practice, the information included in these restrictions is likely to be far more important in the determination of coefficients than any minor inconsistency[1] as a result of simultaneity. It is critically important that simultaneous estimators

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[1] Wold's Proximity Theorem reassures us that statistically we are not in a Lipsey-Lancaster second best world where any deviation from the optimum results in completely lost properties; statistically, minor inconsistencies can be proven to be better than major inconsistencies so that the coefficients will be relatively unaffected if the inconsistency is small. See [21].

not sacrifice small sample efficiency in the quest for consistency.

Maddala has done some interesting work in attempting to incorporate these restrictions into first-stage estimation.[1] Essentially he proposes iterating until the estimated "instruments" obtained from the derived reduced form (derived RF) when substituted into the second stage calculation result in the same derived reduced form. (This is equivalent to stating that the structural $\hat{y}$'s, when substituted back into the second stage, result in the same structural $\hat{y}$'s.) Without reviewing the entire Maddala paper, we note that in this case 2SLS is not equivalent to instrumental variable estimation (IVE), and also that there are two different iterative techniques that can be followed. The first iterative technique Maddala terms the solved reduced form (SRF) method; it involves starting with the 2SLS derived RF and substituting the implied $\hat{y}$'s back into the second stage, reestimating the structural parameters, recalculating the derived RF, etc., until convergence is achieved. The second iterative technique is called the method of successive substitution (SS); it involves starting with an initial set of jointly dependent variable values and performing the second stage calculation over them to obtain structural coefficients, which are then solved for the implied (structural values of the jointly dependent variables, which are then substituted back into the second stage calculation, etc., until convergence is achieved. Maddala examined via Monte Carlo methods the

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1 [48]

properties of both of these iterative techniques for each of the two estimators, 2SLS and IVE, and concluded that the IVE-SRF method was by far the most likely to converge to unique values. The SS iterative method often failed to converge; although the inconsistency of the non-iterative procedure may be small (as the SOIV model demonstrated above), continued iteration causes it to accumulate. The 2SLS estimates often were not unique.

Maddala then reconciled the IVE-SRF estimator with full information maximum likelihood (FIML) estimation,[1] demonstrating that whereas IVE-SRF uses the estimated values from an unchanging derived RF as second stage instruments, FIML is equivalent to an instrumental variable estimator that adds a weighted structural disturbance to the above instruments. Thus FIML enforces a stochastic version of the overidentifying restrictions.

Two- and three-stage least squares have been touted for their computational simplicity relative to FIML; this simplicity entirely disappears in the face of parameter and variable nonlinearity however, both of which our results indicate are important. Further, 2SLS and 3SLS are inefficient to the extent that the derived reduced form coefficients differ from the unconstrained first-stage estimates; that these restrictions are important has been borne out by modeling experience -- a major problem in all large models is that each single equation tracks well given the actual values of the other endogenous variables, but the model as a whole performs poorly. It can be argued that loss of the across-equation information in the overidentifying restrictions

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[1] Assuming normality of the disturbances.

is the cause. Any attempt to include these restrictions *a la* Maddala is computationally impossible in the presence of even minor parameter nonlinearity, and 2SLS is not at all well adapted to handle variable nonlinearity. Thus for actual models FIML would seem to be the best estimator;[1] although computationally more difficult in the simple cases, it generalizes readily and makes better use of prior information in finite samples.

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[1] The "conventional wisdom" holds that FIML is undesirable because it is more sensitive to specification error. What is not considered is that the researcher *wants* to find the specification error; if the error does not appear in the estimated coefficients, it is likely that the model solution will inherit it. The present procedure of estimating and then solving for multipliers, elasticities, and long run properties in order to detect error is far more computationally demanding than would be direct FIML.

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APPENDIX I

Data Sources

There are 33 basic series from which the rest of the data are calculated. The primary 1929-65 data source (for 26 of the 33 series) is The National Income and Product Accounts of the United States, 1929-1965;[1] while the primary 1966-69 data source is the Survey of Current Business,[2] July, 1970; table and line numbers refer to these two sources (and are the same) unless noted.

The first eight entries are in constant (1958) dollars; an asterisk denotes a series based on current dollar sources but deflated for use in the model. All dollar magnitudes except the wage rate have been transformed to billions of dollars. The sources are:

- C Personal consumption expenditures; Table 1.2, line 2.
- I^d Gross private domestic investment, nonresidential, Table 1.2, line 8.
- I^r Gross private domestic investment, residential structures; Table 1.2, line 11
- I^i Gross private domestic investment, change in business inventories; Table 1.2, line 14.
- G Government purchases of goods and services; Table 1.2, line 20.
- F_X Net exports of goods and services, exports; Table 1.2, line 18.
- F_I Net exports of goods and services, imports; Table 1.2, line 19.

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1 The National Income and Product Accounts of the United States, 1929-1965: A Supplement to the Survey of Current Business, U.S. Department of Commerce, Office of Business Economics (August, 1966).

2 Survey of Current Business: July 1970, U.S. Department of Commerce, Office of Business Economics, Washington, D.C.

X Gross national product; Table 1.2, line 1.

N Labor force; 1929-38 from [37], p. 191; 1939-1968 from 1969 Business Statistics,[1] p. 67; 1969 from The Statistical Abstract of the U.S., 1970[2], p. 213, Table 316.

N_W Average number of full-time and part-time employees, all industries, total; Table 6.3, line 1.

N_G Average number of full-time and part-time employees, government and government enterprises; Table 6.3, line 73.

t Time, 1929=0, continues through the war.

w Annual earnings, thousands of dollars; 1929-63 from SCB, 7-70, Table 6.5, line 1; 1961-69 from assorted later SCBs.

p Implicit price deflator, gross national product; Table 8.1, line 1 (All times 10^{-2}).

p_M Implicit price deflator for gross national product, imports; Table 8.1, line 17 (All times 10^{-2}).

p_A Implicit price deflators for gross farm product, total value of farm output; Table 8.5, line 1 (All times 10^{-2}).

i_d Average discount rate at all Federal Reserve Banks; 1929-65 from [37], p. 191; 1966-69 from The Statistical Abstract of the U.S., 1970, weighted by days in effect.

i_L Bond and stock yields, corporate (Moodys) total; 1929-61 from [37], p. 191; 1962-69 from The Federal Reserve Bulletin,[3] January, 1970 (henceforth FRB, 1-70) p. A34.

R Year-end ratio of member banks' excess to required reserves; 1929-65 from [37], p. 191; 1966-69 from FRB, 1-70, p. A6.

i_s Yield on prime commercial paper, 4-6 months; 1929-61 from [37], p. 191; 1962-69 from FRB, 1-70, p. A33.

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1 1969 Business Statistics: A Supplement to the Survey of Current Business, U.S. Department of Commerce, Office of Business Economics, Washington, D.C.

2 The Statistical Abstract of the United States: 1970. (91st edition) U.S. Bureau of the Census, Washington, D.C. 1970.

3 Federal Reserve Bulletin: January 1970, Board of Governors of the Federal Reserve System, Washington, D.C.

D^* Capital consumption allowances; Table 1.9, line 2.

S^*_c Undistributed corporate profits after tax plus corporate inventory valuation adjustment; Table 1.10, lines 23 plus 24.

P^*_c Corporate profits and inventory valuation adjustment less corporate profits before tax, agriculture, forestries, and fisheries; Table 1.10, line 18 less Table 6.13, line 2.

W^*_1 Private employee compensation; Table 1.10, line 4 plus line 7 less Table 6.7, line 14.

W^*_2 Government employee compensation; Table 1.10, line 5 plus line 6 less Table 6.7, line 14.

A^*_1 Private farm income; Table 6.8, line 2 plus Table 6.13, line 2 less Table 1.17, line 13.

A^*_2 Government payments to farmers; Table 1.17, line 13.

P^* Nonwage, nonfarm income; Table 1.10 line 1 less $(W^*_1 + W^*_2 + A^*_1 + A^*_2)$.

T^* Indirect business tax and nontax liability, business transfer payments, statistical discrepancy, and subsidies less current surplus; Table 1.9, lines (4+5+6-7).

T^*_c Federal and State corporate profits tax liability, all industries less agriculture, forestry, and fisheries; Table 6.14, line 1, less line 2.

T^*_W, T^*_A, T^*_p — See Appendix II

APPENDIX II

Disposable Income by Distributive Shares

I. The Reconciliation

The U.S. Department of Commerce publishes data on National Income disaggregated into five distributive shares: (1) Compensation of employees, (2) Proprietors' income, (3) Rental income of persons, (4) Corporate profits and inventory valuation adjustment, and (5) Net Interest. The model supposes three distributive shares -- compensation of employees (W), agricultural income (A), and the residual (P) -- and, in the consumption function, requires data on personal disposable income by these three shares. To obtain these data, we must build the desired three shares from the National Income shares, and then distribute to our three shares the reconciling items added and subtracted to obtain personal disposable income. The reconciliation is as follows:

VARIABLE	USDC TABLES
pX	Gross National Product
pD	Less: Capital consumption allowances
	Equals: Net National Product
pT	Less: Indirect business tax and nontax liability
	Business transfer payments
	Statistical Discrepancy
	Plus: Subsidies less current surplus of government enterprises
	Equals: National Income

VARIABLE	USDC TABLES
pY	National Income
	Employees Compensation
pW_1	Private Employee Compensation
p^W_2	Government Employee Compensation
	Agricultural Income
pA_1	Private Agricultural income
pA_2	Government payments to farmers
pP	Residual
pT_A^*	Less: Corporate profits before tax, agricultural corporations
pT_P^*	Corporate profits before tax, nonagricultural corporations
pT_p^*	Inventory valuation adjustment
pT_W^*	Contributions for social insurance
pT_W^*	Wage accruals less disbursements
$-pT_W^*$	Plus: Government transfer payments to persons
$-pT_p^*$	Interest paid (net) by government & consumers
$-pT_p^*$	Dividends, nonagricultural corporations
$-pT_A^*$	Dividends, agricultural corporations
$-pT_W^*$	Business transfer payments
	Equals: Personal Income
	Less: Federal personal tax
	Federal personal income taxes less refunds

* Add all pT_p^**, PT_A^**, pT_W^** to obtain total pT_p, pT_A, pT_W.

VARIABLE	USDC TABLES
pT_W^*	Share from employees' compensation (See part II of this Appendix)
pT_A^*	Share from agricultural income (See part II of this Appendix)
pT_P^*	Residual (See part II of this Appendix)
pT_P^*	Federal estate and gift taxes
	State and local personal tax payments
pT_W^*	Share from employees' compensation (See part II of this Appendix)
pT_A^*	Share from agricultural income (See part II of this Appendix)
pT_P^*	Residual (See part II of this Appendix)
pT_P^*	State and local death and gift taxes
pT_P^*	State and local property taxes
	Nontax payments, licenses, permits, etc.
	(all other Federal, state and local payments)
	Equals: Personal Disposable Income
$pW-pT_W$	Disposable employees compensation
$pA-pT_A$	Disposable agricultural income
p^P-pT_P	Disposable residual income

II. Allocation of Federal, State, and Local Income Tax to Distributive Shares

A simplifying assumption has been made in the allocation of federal, state, and local income taxes: we assume that the relative tax shares vary directly with the relative incomes. Specifically, we assume the equations

$$\frac{\left(\frac{W_1 + W_2}{Y}\right)_t}{\left(\frac{W_1 + W_2}{Y}\right)_o} = \frac{\left(\frac{x_W}{x}\right)_t}{\left(\frac{x_W}{x}\right)}, \quad \frac{\left(\frac{A_1 + A_2}{Y}\right)_t}{\left(\frac{A_1 + A_2}{Y}\right)_o} = \frac{\left(\frac{x_A}{x}\right)_t}{\left(\frac{x_A}{x}\right)_o}, \text{ and } \frac{\left(\frac{P}{Y}\right)_t}{\left(\frac{P}{Y}\right)_o} = \frac{\left(\frac{x_P}{x}\right)_t}{\left(\frac{x_P}{x}\right)_o}$$

hold identically over the sample, providing us with definitions of x_W, x_A, and x_P from which we calculate their values (where x is the total federal income tax bill, x_W is the portion from pW, x_A is the portion from pA, and x_P is the residual; the zero subscript refers to an arbitrary base year):

$$(x_W)_t \equiv \left(\frac{x_W}{x}\right)_o \cdot \left(\frac{Y}{W_1 + W_2}\right)_o \cdot \left(\frac{W_1 + W_2}{Y}\right)_t \cdot x_t \equiv k_W X_t$$

$$(x_A)_t \equiv \left(\frac{x_A}{x}\right)_o \cdot \left(\frac{Y}{A_1 + A_2}\right)_o \cdot \left(\frac{A_1 + A_2}{Y}\right)_t \cdot x_t \equiv k_A X_t$$

$$(x_p)_t \equiv x_t - (x_w)_t - (x_A)_t .$$

We require x_W, x_A, and x_P for some (representative) base year, here 1959. The U.S. Internal Revenue Service provides data on sources of income and loss by twenty-six adjusted income classes, and the amount of income tax after credits paid by each adjusted income class.[1] Ten sources of income are tabled; they have been distributed to the three income shares (W, A, P) of the model as follows:

(1)	Salaries and Wages (net)	W
(2)	Dividends (after exclusions)	P
(3)	Interest received	P

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[1] <u>Statistics of Income, Individual Income Tax Returns for 1959</u>, U.S. Treasury Dept., Internal Revenue Service. Publication No. 79 (9-61), pp. 24-26.

(4) Business or professional profit: allocated to A,P on the basis of — P,A

$$\frac{\text{Agricultural net profit}^{1}}{\text{Total net profit}} = \frac{2{,}913{,}642}{21{,}516{,}876}$$

= 0.1354

(5) Partnership profit: allocated to A,P on the basis of — P,A

$$\frac{\text{Agricultural Partnerships with net profit}^{2}}{\text{Total Partnerships with net profit}}$$

$$= \frac{750{,}842}{9{,}720{,}805} = 0.0772$$

(6) Sales of capital assets: allocated to A,P on the basis of — P,A

$$\frac{\text{Sales of agricultural assets}^{3}}{\text{Total sales of assets}}$$

$$= \frac{1{,}579{,}384}{12{,}331{,}867} = 0.1281$$

(7) Sales of property other than capital assets — P

(8) Rents and royalties — P

(9) Estates and trusts — P

(10) Other income — P

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1 Statistics of Income, U.S. Business Tax Returns 1959-1960, I.R.S., p. 16.

2 Ibid., p. 44.

3 Statistics of Income, 1959, Supplemental Report: Sales of Capital Assets, I.R.S., p. 10.

From these data the 1959 relative shares were found to be:

$\frac{x_W}{x}$	k_W	$\frac{x_A}{x}$	k_A	$\frac{x_P}{x}$	x[1]	W_1+W_2[2]	A_a+A_2[2]	P[2]	Y[2]
0.7350	1.0535	0.0311	1.0472	0.2339	38.645299	274.69	11.69	107.33	393.58

The state and local income tax bill was distributed using the k_i's calculated above:

$$s_W = k_W \cdot \left(\frac{W_1 + W_2}{Y}\right)_t \cdot s_t$$

$$s_A = k_A \cdot \left(\frac{A_1 + A_2}{Y}\right)_t \cdot s_t$$

$$s_P = s - s_W - s_A$$

(where s is the total state and local income tax bill, and s_W, s_A, and s_P are the portions from W, A, and P income respectively).

This state and local allocation is somewhat suspect -- wage income undoubtedly bears a relatively heavier burden at this level than at the federal level -- but the figures themselves are small so the allocation error is assumed to be negligible.

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1 In billions of current dollars.

2 From the model data, not from IRS estimates.

APPENDIX III

A Generalized Example of Initial Condition Parameterization

A model based on a normalized rational lag of two convolutions can be written

$$(1) \quad y_t = \beta(1-\lambda_1)(1-\lambda_2) \sum_{i=o}^{\infty} \sum_{j=o}^{\infty} \lambda_1^i \lambda_2^j L^{k+j} X_t + \epsilon_t ,$$

which, upon expansion, is

$$(1a) \quad = \beta(1-\lambda_1)(1-\lambda_2) \left\{ \begin{array}{l} [\; X_t + \lambda_2 X_{t-1} + \lambda_2^2 X_{t-2} + \lambda_2^3 X_{t-3} + \lambda_2^4 X_{t-4} + \ldots] \\ + [\; \lambda_1 X_{t-1} + \lambda_1 \lambda_2 X_{t-2} + \lambda_1 \lambda_2^2 X_{t-3} + \lambda_1 \lambda_2^3 X_{t-4} + \ldots] \\ + [\; \lambda_1^2 X_{t-2} + \lambda_1^2 \lambda_2 X_{t-3} + \lambda_1^2 \lambda_2^2 X_{t-4} + \ldots] \\ \vdots \\ + [\lambda_1^i X_{t-i} + \lambda_1^i \lambda_2 X_{t-i-1} + \lambda_1^i \lambda_2^2 X_{t-i-2} + \ldots + \lambda_1^i \lambda_2^j X_{t-i-j} + \ldots] \\ \vdots \end{array} \right\} + \epsilon_t .$$

Regrouping coefficients by time period and partitioning the infinite sum into the part t-r (here t-2) periods back and the residual, we obtain

$$(2) \quad y_t = \beta(1-\lambda_1)(1-\lambda_2) \left(\sum_{i=o}^{t-2} \sum_{j=o}^{i} \lambda_1^j \lambda_2^{i-j} X_{t-i} + \sum_{i=t-1}^{\infty} \sum_{j=o}^{i} \lambda_2^j \lambda_2^{i-j} X_{t-i} \right) + \epsilon_t .$$

The objective is to write the second term -- the effect of the values that variable X assumed more than t-r periods back in time -- as a finite number of parameters (stable as t changes) so that they can be estimated. For notational ease, we name the second term of (2) Z_t:

$$(3) \quad Z_t \equiv \beta(1-\lambda_1)(1-\lambda_2) \sum_{t-1}^{\infty} \sum_{j=o}^{i} \lambda_1^i \lambda_2^{i-j} X_{t-i} .$$

Rebasing the summation over i at zero,

$$(3a) \quad Z_t \equiv \beta(1-\lambda_1)(1-\lambda_2) \sum_{i=o}^{\infty} \sum_{j=o}^{t-1+i} \left(\lambda_2^{t-1} \right) \lambda_1^j \lambda_2^{i-j} X_{1-i} .$$

and partitioning the inner summation such that the upper limit of one of the terms is i, we obtain

$$(4) \qquad Z_t \equiv \lambda_2^{t-1}\beta(1-\lambda_1)(1-\lambda_2)\sum_{i=o}^{\infty}\sum_{j=i+1}^{t-1+i}\lambda_1^j\lambda_2^{i-j}X_{1-i}$$

$$+\ \lambda_2^{t-1}\beta(1-\lambda_1)(1-\lambda_2)\sum_{i=o}^{\infty}\sum_{j=o}^{i}\lambda_1^j\lambda_2^{i-j}X_{1-i}\ .$$

The second term of (4) is simply $\lambda_2^{t-1}E(y_1)$, as is apparent by ignoring the partition in (2) and subsituting 1 for t. Adding and subtracting t times the last term of (4) and rebasing the inner summation of the first term at j=o, we obtain

$$(5) \qquad Z_t \equiv \lambda_2^{t-1}\beta(1-\lambda_1)(1-\lambda_2)\sum_{i=o}^{\infty}\lambda_1^i\sum_{j=o}^{t-2}\lambda_1^{j+1}\ \lambda_2^{-j-1}X_{1-i} + t\lambda_2^{t-1}E(y_1)$$

$$-\ (t-1)\lambda_2^{t-1}\underbrace{\beta(1-\lambda_1)(1-\lambda_2)\sum_{i=o}^{\infty}\sum_{j=o}^{i}\lambda_1^j\lambda_2^{i-j}X_{1-i}}_{E(y_1)}\ .$$

Combining the first and last terms of (5) and expanding

$$(6)\quad Z_t \equiv t\lambda_2^{t-1}E(y_1)-(t-1)\lambda_2^{t-1}\beta(1-\lambda_1)(1-\lambda_2)\Big\{\Big[\lambda_1^{o}-(t-1)^{-1}\lambda_1^{o}\Big(\frac{\lambda_1}{\lambda_2}+\frac{\lambda_1^2}{\lambda_2^2}+\dots+\frac{\lambda_1^{t-1}}{\lambda_2^{t-1}}\Big)\Big]X_1$$

$$+\lambda_2(\lambda_1^{o}\lambda_2^{o})X_o \quad + \quad \Big[\lambda_1^{1}-(t-1)^{-1}\lambda_1^{1}\Big(\frac{\lambda_1}{\lambda_2}+\frac{\lambda_1^2}{\lambda_2^2}+\dots+\frac{\lambda_1^{t-1}}{\lambda_2^{t-1}}\Big)\Big]X_o$$

$$+\lambda_2(\lambda_1^{o}\lambda_2^{1}+\lambda_1^{1}\lambda_2^{o})\,X_{-1} \quad + \quad \Big[\lambda_1^{2}-(t-1)^{-1}\lambda_1^{2}\Big(\frac{\lambda_1}{\lambda_2}+\frac{\lambda_1^2}{\lambda_2^2}+\dots+\frac{\lambda_1^{t-1}}{\lambda_2^{t-1}}\Big)\Big]X_{-1}$$

$$+\dots\Big\}.$$

But the terms enclosed in bold lines, when combined with the factor in front of the brace, can be written as $-(t-1)\,\lambda_2^t\,E(y_o)$:

$$(7)\quad Z_t \equiv t\,\lambda_2^{t-1}E(y_1) - (t-1)\lambda_2^t\,E(y_o)$$

$$+\,\lambda^{t-1}\beta(1-\lambda_1)(1-\lambda_2)\sum_{i=o}^{\infty}\lambda_1^i\left[\sum_{j=o}^{t-2}\left(\frac{\lambda_1^{j+1}}{\lambda_2{}^{j+1}}\right)-(t-1)\right]X_{1-i}$$

Note that the quantity inside the brackets depends on t but not on i, and can be brought outside the summation over i:

$$(8)\quad Z_t \equiv t\,\lambda_2^{t-1}E(y_1)-(t-1)\,\lambda_2^tE(y_o)$$

$$+\,\lambda_2^{t-1}\left[\sum_{j=o}^{t-2}\left(\frac{\lambda_1}{\lambda_2}\right)^{j+1}-(t-1)\right]\beta(1-\lambda_1)(1-\lambda_2)\sum_{i=o}^{\infty}\lambda_1^iX_{1-i}.$$

Substituting (8) for Z_t in (2) gives

$$(9) \quad y_t = \underbrace{\beta (1-\lambda_1)(1-\lambda_2) \sum_{i=o}^{t-2} \sum_{j=o}^{i} \lambda_1^j \lambda_2^{i-j} X_{t-i}}_{\text{data}} + \underbrace{t \lambda_2^{t-1}}_{\text{data}} E(y_1) - \underbrace{(t-1)\lambda_2^t}_{\text{data}} E(y_o)$$

$$+ \underbrace{\lambda_2^{t-1} \left[\sum_{j=o}^{t-2} \frac{\lambda_1}{\lambda_2}^{j+1} - (t-1) \right]}_{\text{data}} \beta(1-\lambda_1)(1-\lambda_2) \sum_{i=o}^{\infty} \lambda_1^i X_{1-i} + \epsilon_t,$$

where β, $E(y_1)$, $E(y_o)$, and $\beta(1-\lambda_1)(1-\lambda_2) \sum_{i=o}^{\infty} \lambda_1^i X_{1-i}$ are stable over t and therefore parameters, while their factors all vary with t and therefore can be treated as data; the rational lag is estimable in a manner similar to the Pascal lag except for an additional correction term. Note that if $\lambda_1 = \lambda_2$, the fourth term in (9) becomes zero since $\sum_{j=o}^{t-2} (1)^{j+1} = t-1$, and (9) can be written

$$(10) \quad y_t = \beta(1-\lambda_1)^2 \sum_{i=o}^{t-2} (i+1)\lambda^i X_{t-i} + t \lambda^{t-1} E(y_1) - (t-1)\lambda^t E(y_o) + \epsilon_t,$$

the Maddala result.

APPENDIX IV

Simulations

Unless otherwise noted, all parameter references are to the autoregression corrected three-stage least squares estimates.

A. Forecasts

The theoretical underpinnings of a model cannot and should not be independent of the data; consequently, subsampling and forecasting are necessary to validate the model. In our case the years 1945-1948 have not been used in the estimation process and have been saved for use as a forecasting test, a relatively stringent test since these immediately post war years are somewhat abnormal.

Three forecasts have been made, each using the actual values of the exogenous variables. Two of the forecasts using observed[1] values of the lagged endogenous variables. Over the twenty tabled variables,

---

1 Reliable (but war-distorted) data are available for I^d and X^P in 1945. Data for the Pascal lag variable (Y^P) in the consumption function was obtained by noting that since

$$(1) \quad (Y^P)_t = (1-\lambda)^4 \sum_{i=0}^{t=4} \binom{i+3}{i} \lambda^i (Y_c)_{t-i} ,$$

by inversion

$$(2) \quad (Y^P)_t \simeq (1-\lambda)^4 (Y_c)_t + 6\lambda Y^P_{t-1} - 10\lambda^2 Y^P_{t-2} + 6\lambda^3 Y^P_{t-3} - 1\lambda^4 Y^P_{t-4} .$$

(The relation would hold exactly if the limit on the summation in (1) was ∞ rather than t-4. For t=1947, t-4=14 which is close enough to ∞ when we remember that λ was estimated to be 0.06: $(0.06)^{14} \simeq 0$, and the approximation is good enough.)

The war values of Y^P were estimated by the simple autoregressive scheme

$$(3) \quad Y^P_t = 0.87 + 1.03\, Y^P_{t-1}$$

where the coefficients were obtained by ordinary least squares over the sample data; since we have the actual 1945 and 1946 values of Y^P and since λ is so small, the error is not excessive.

war[1] (the initial conditions of the 1947 forecast include data from the last war year, 1945, with government expenditures of $156.4 billion compared to $48.4 billion in 1946; the distortion continued well past the war, however, with, for example, government wage bill figures of $61.6 billion, $34.1 billion, and $25.1 billion in 1945, 1946, and 1947, respectively). If we were forecasting *ex ante*, extraneous information about specific residuals for the most affected equations would be included as add factors to those equations; *ex post*, the information is perfect and thus invalidates the comparison. Hence our forecast of two rather exceptional years must be made without utilizing *any* extraneous information about the residuals.

Ex ante forecasts of the Klein-Goldberger-Suits (henceforth K-G-S) model have been tabulated for the years 1953-1960.[2] A comparison of the forecast errors may be interesting even if not completely legitimate.[3] The GNP forecast errors listed in the Suits article range from a low of 0.1% in 1958 to a high of 4.0% in 1959. Our GNP forecast errors for 1947 and '48 were 6.6% and 4.8% respectively. The maximum Suits consumption error was 4.9%, while our '47 and '48 consumption errors were 4.1% and 2.7%. The Suits private wage bill errors ran from 0.1% to 8.1%; our 1947 error was 8.5%, reduced to 4.1% in 1948. Given the distortion of the initial conditions and the years to be forecast (combined with the inability to use add factors), our forecasts must be considered something of a success.

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1 Leaving these particular years for the forecast conserved data for estimation purposes, as previously explained.

2 [58], p. 602.

3 Problems (1) and (2) complicated the Suits forecasts, while (3) and (4) were not particularly important -- the opposite of our case.

B. Sample Data Simulations

While forecasts provide the only truly independent test of the model, there is still considerable information in the sample data concerning the model's performance. Various summary statistics based on the sample residuals have been presented with the parameter estimates. In addition, two other tests of the model fit to the sample data have been performed: (1) the sample data have been simulated by using actual exogenous and lagged endogenous values to solve the model for each single period, and (2) the postwar data have been simulated fully endogenously by using the model's own past solution values for the lagged endogenous variables. The plots[1] of selected variables are included as figures 11 through 29.

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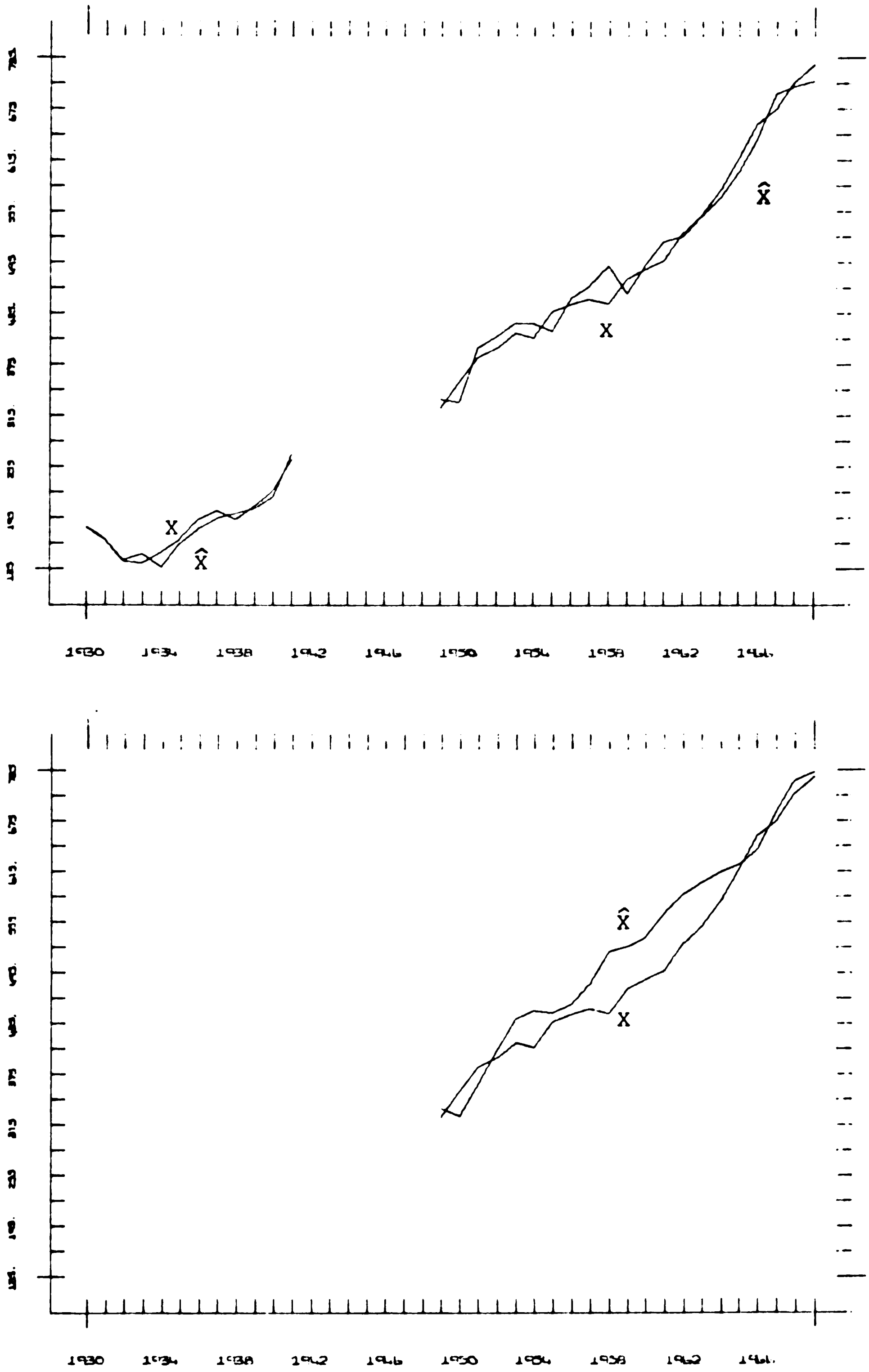
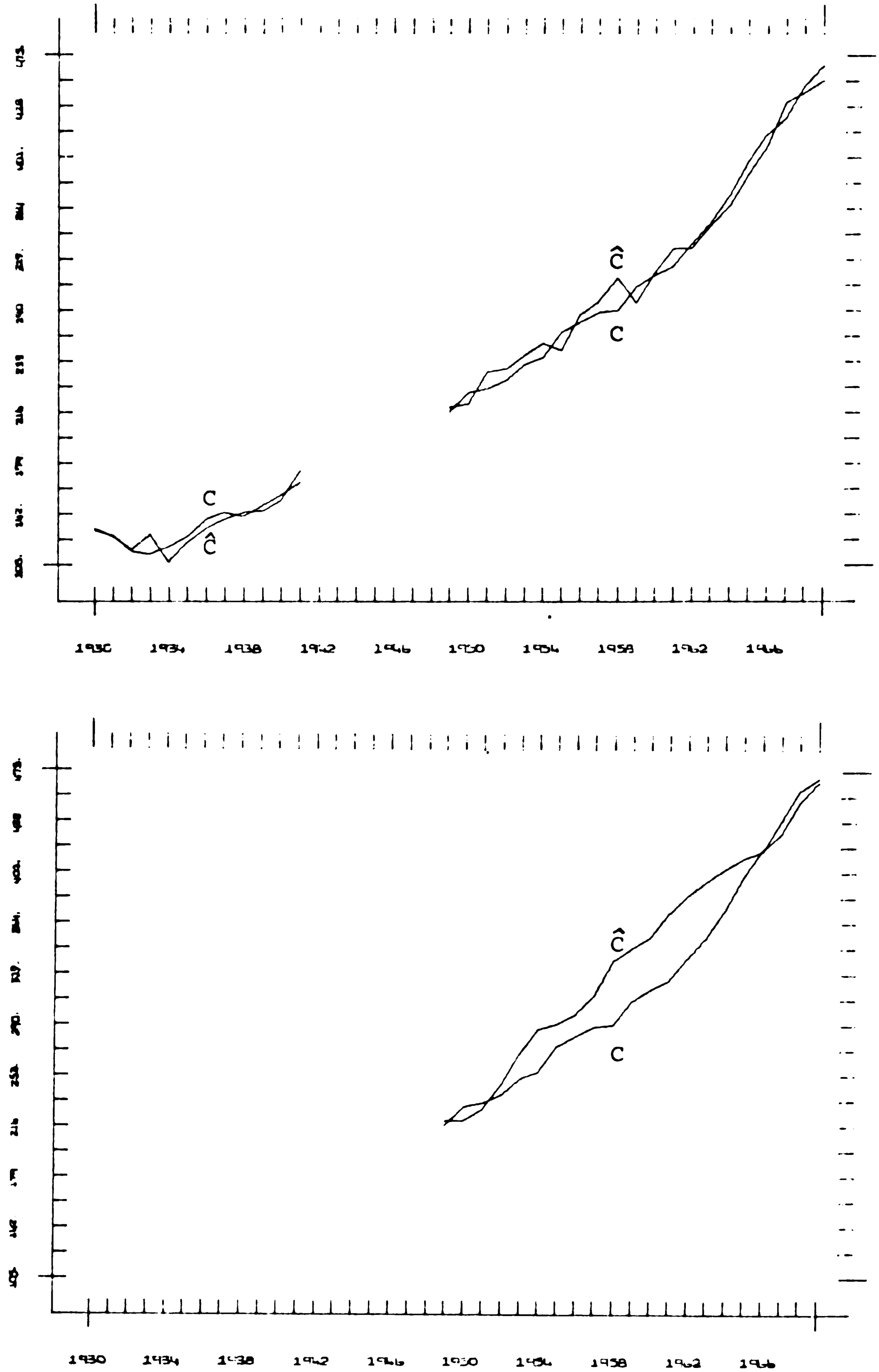
1 These plots can be misleading: the same percentage error on each variable is not represented by the same acutal distance on each plot.



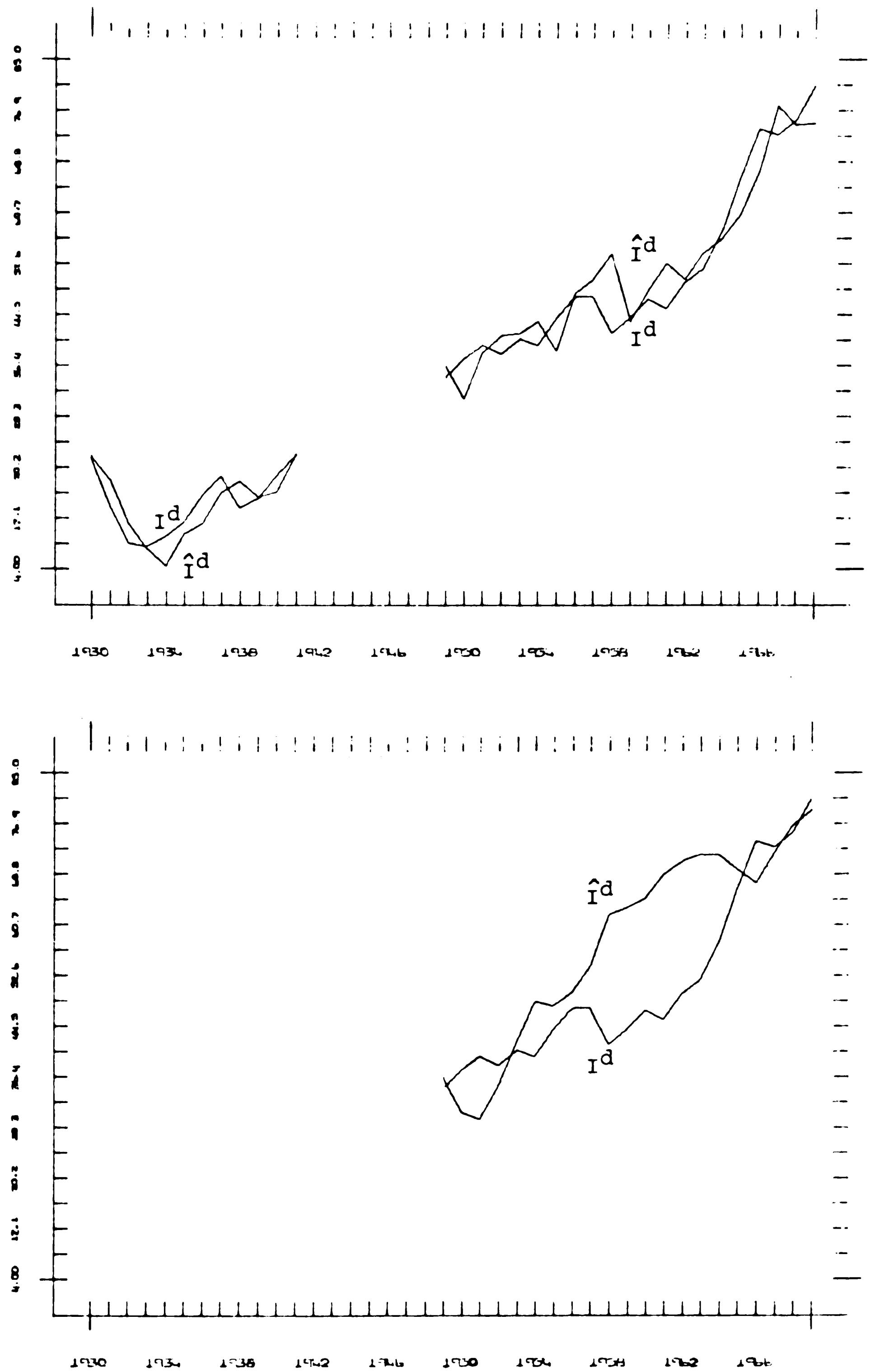
Gross National Product

Figure 11



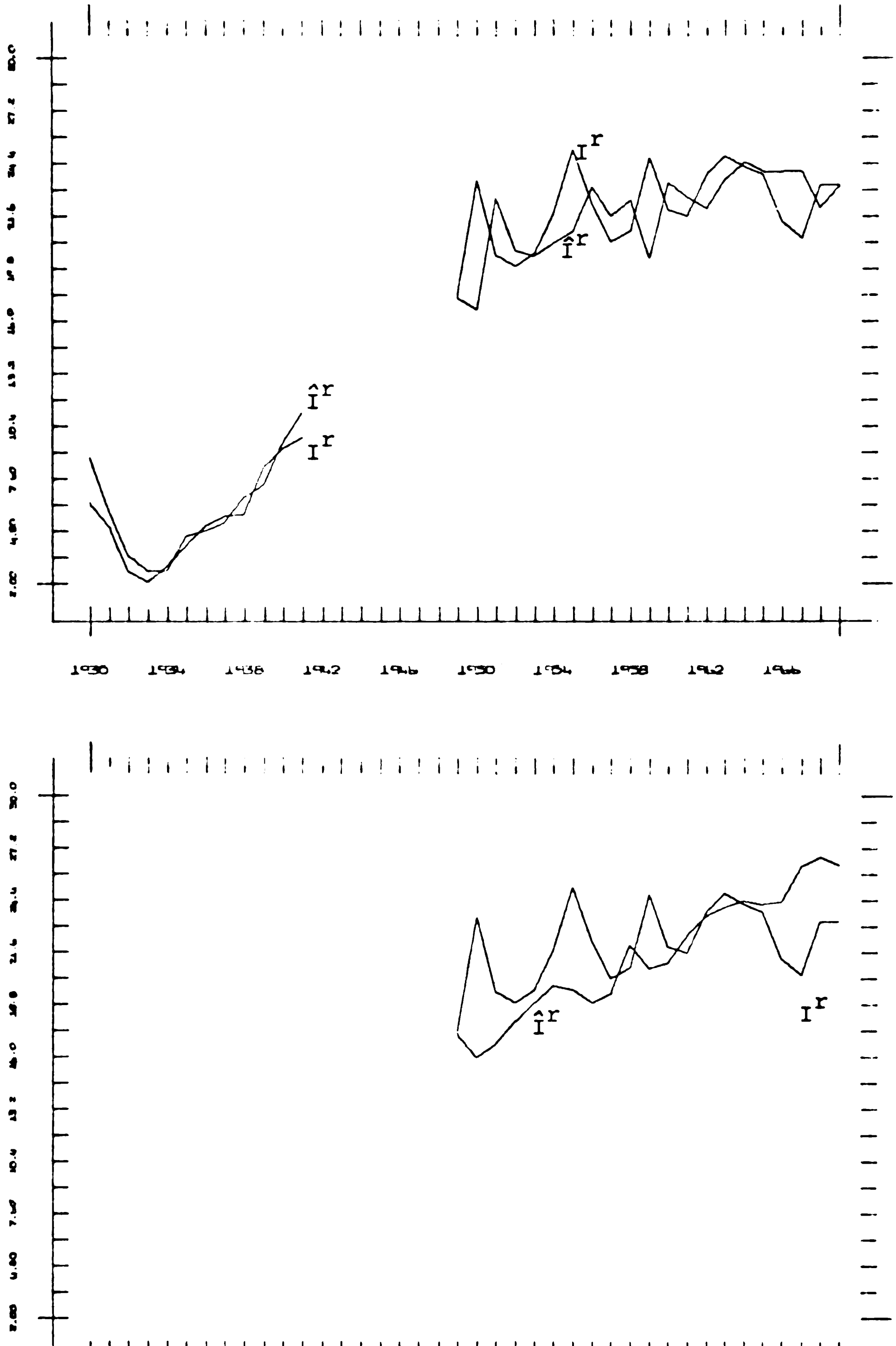
Consumption

Figure 12



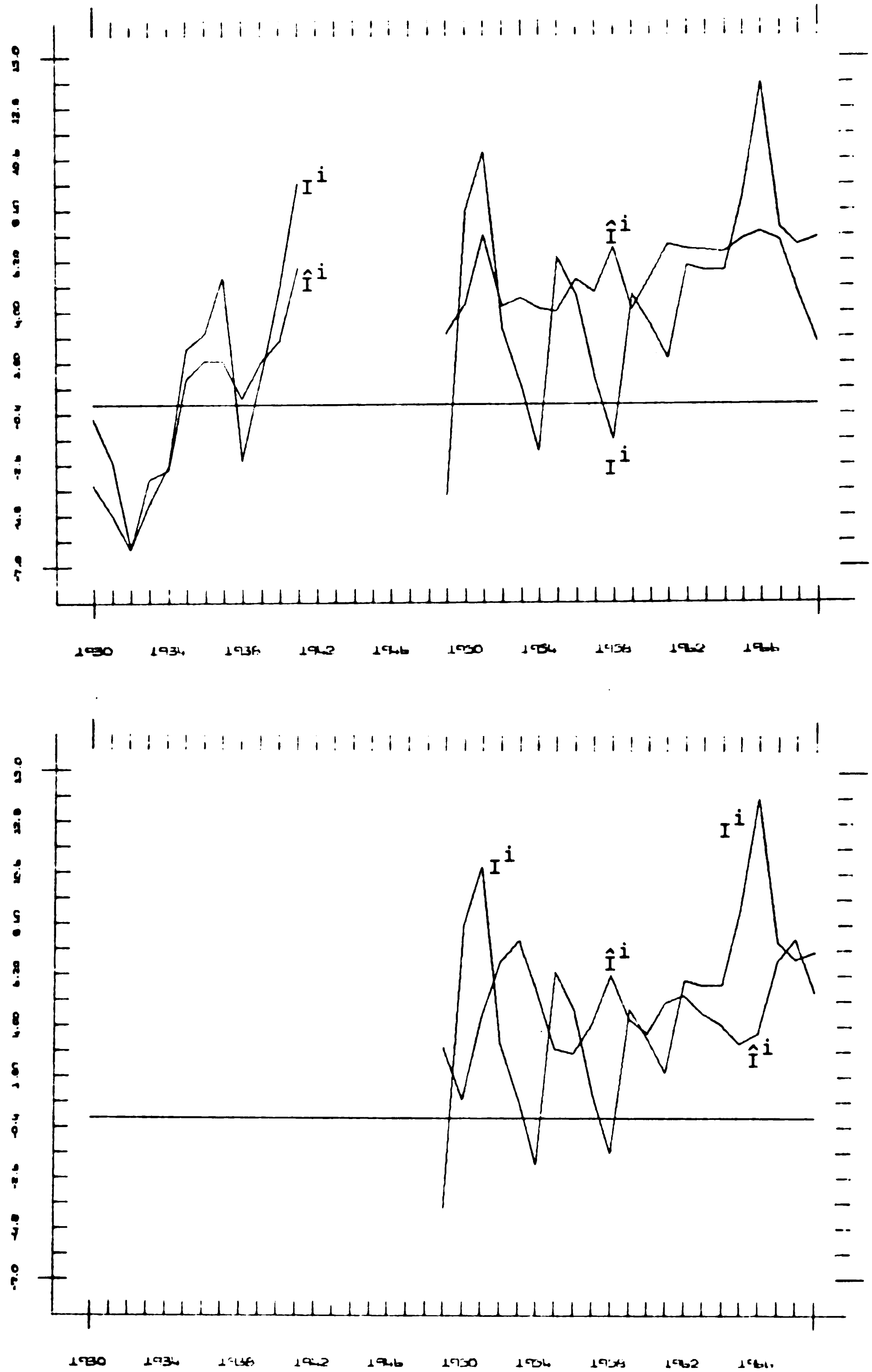
Durable Investment

Figure 13



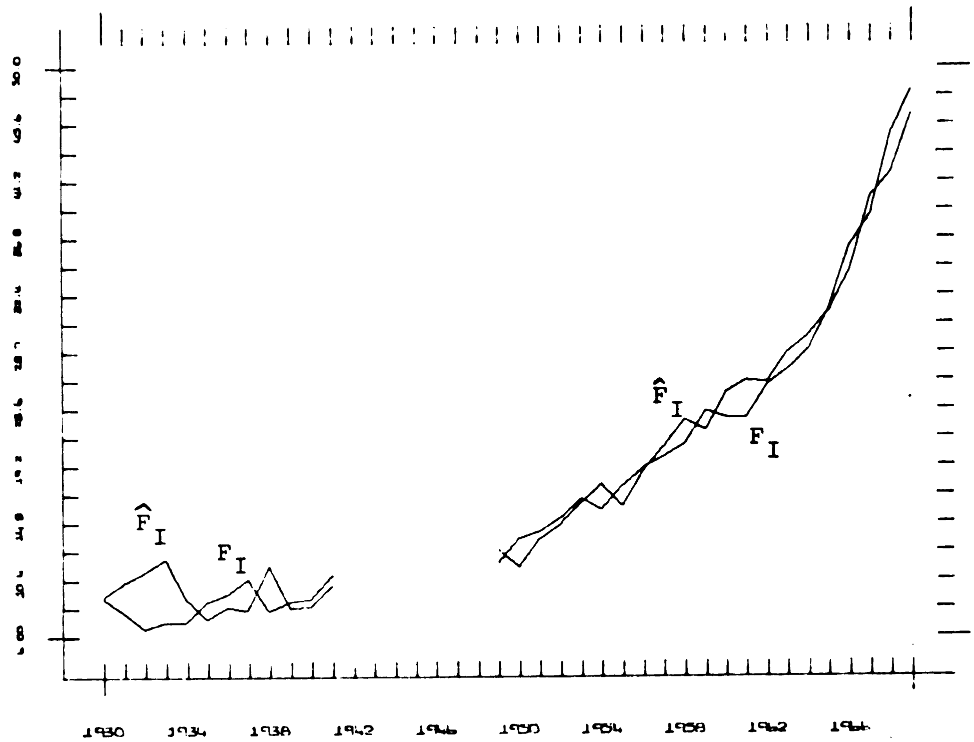
Residential Investment

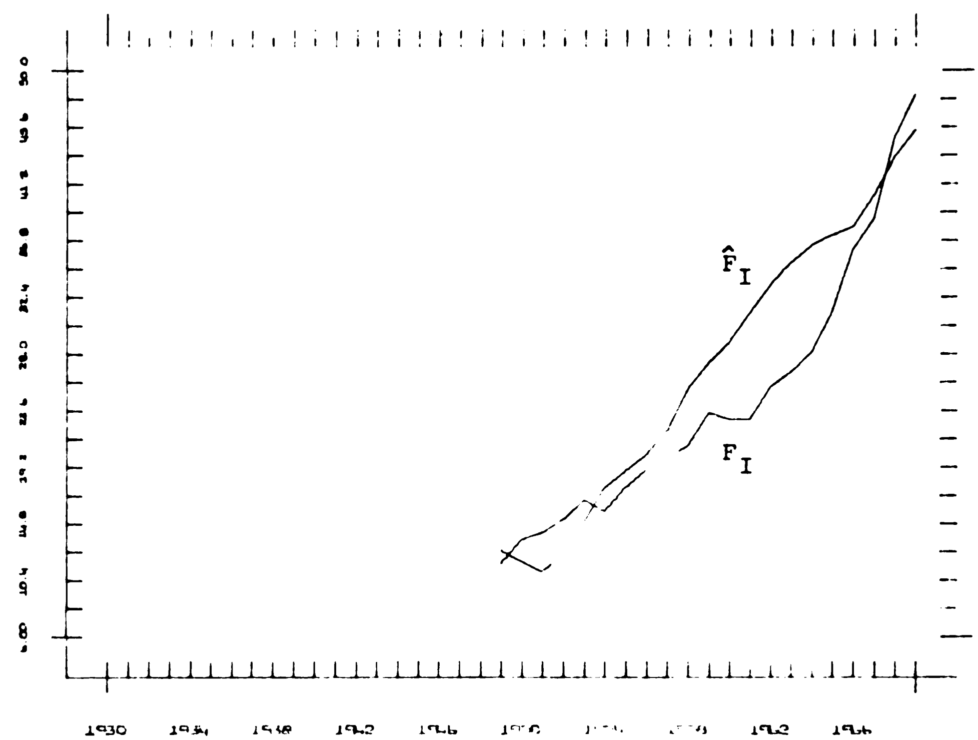
Figure 14



Inventory Investment

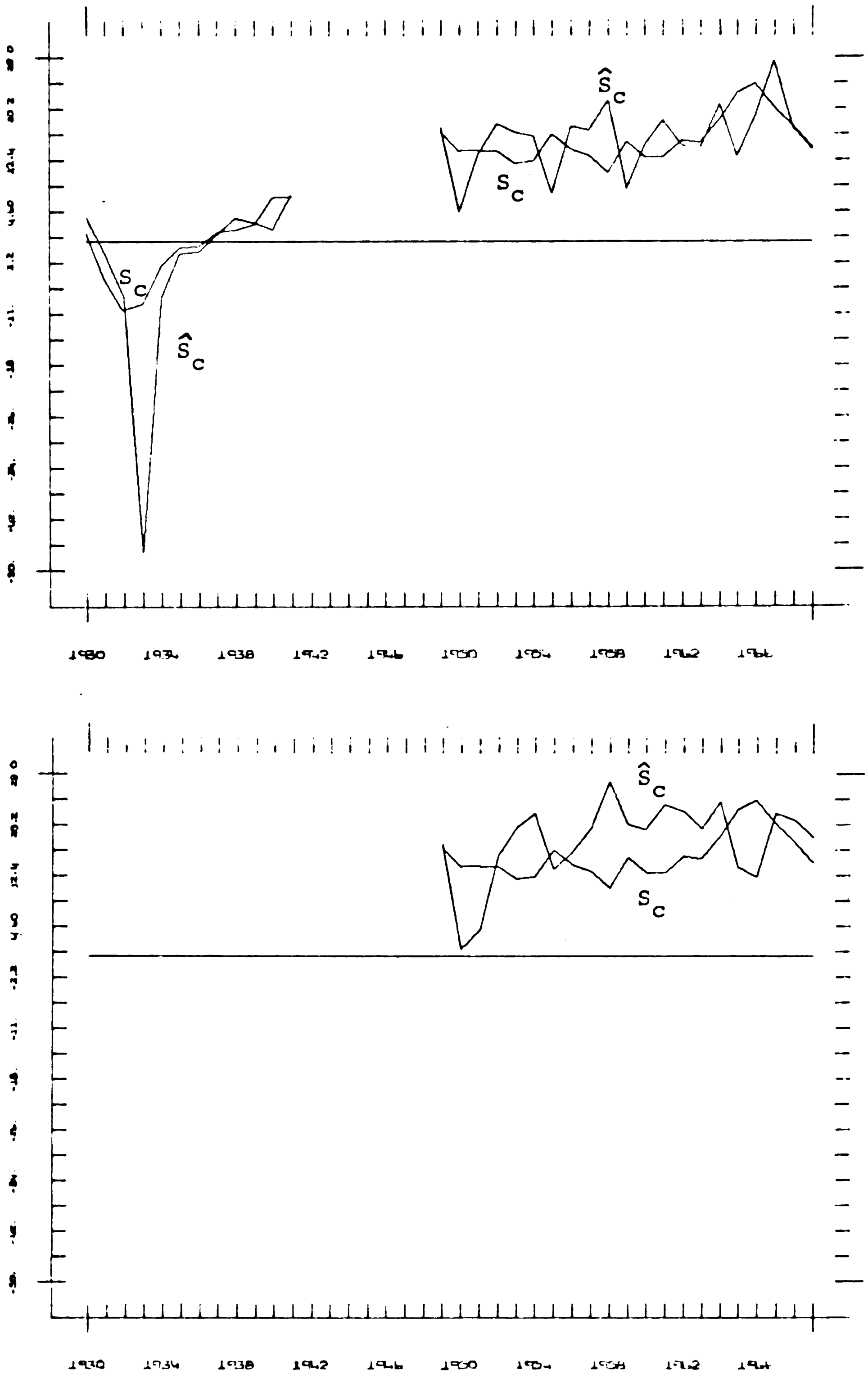
Figure 15





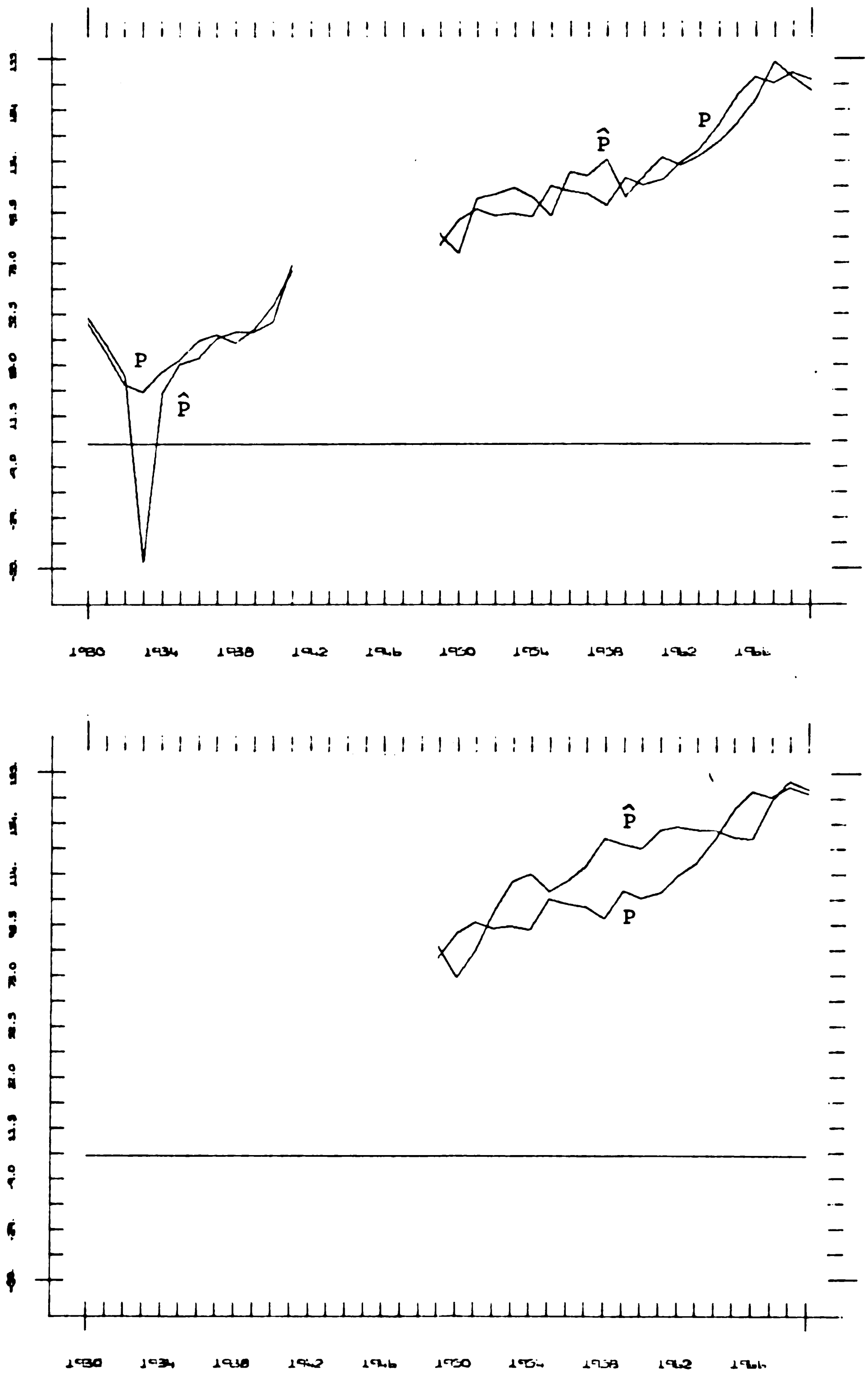
Imports

Figure 16



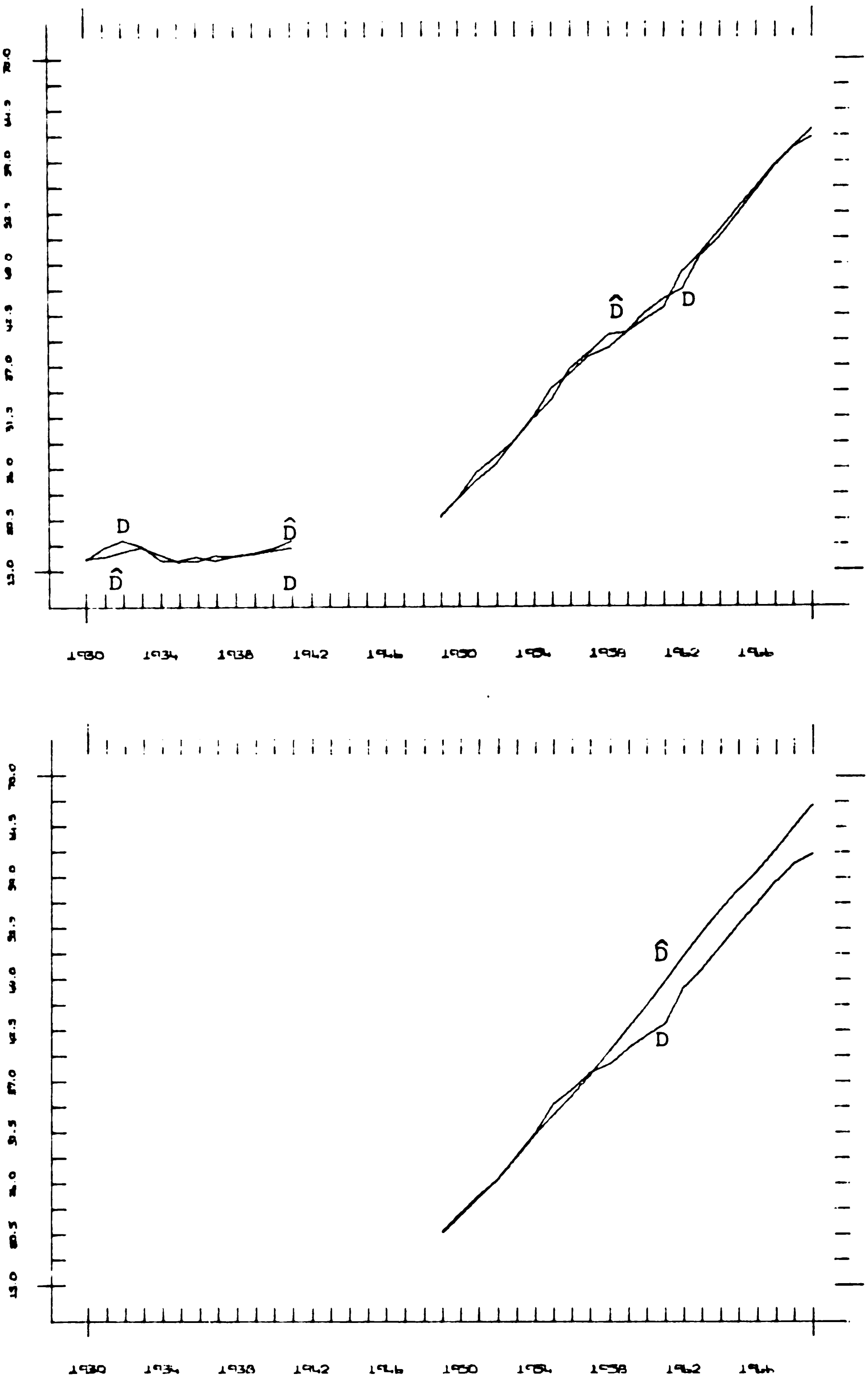
Corporate Saving

Figure 17



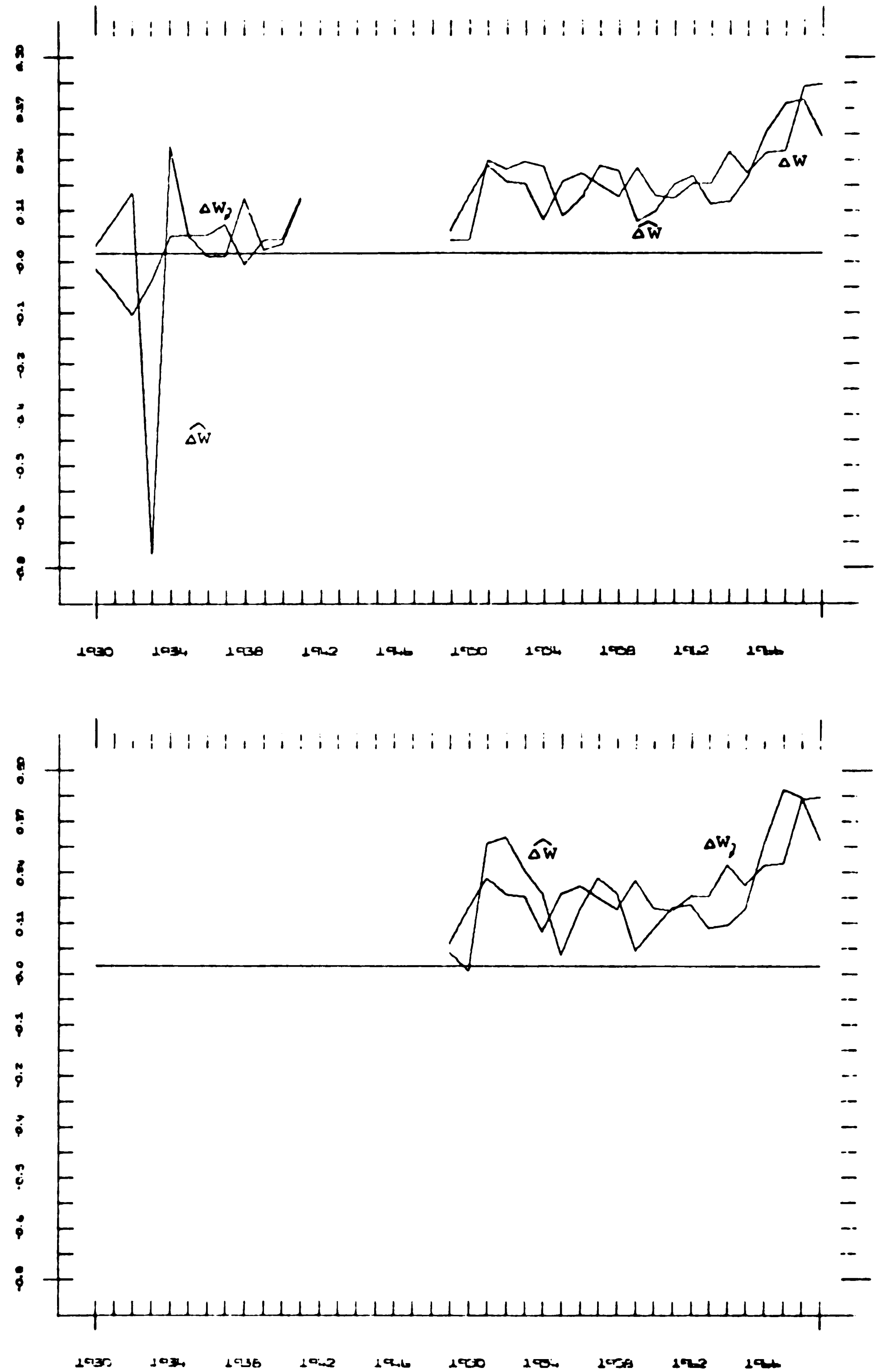
Profits

Figure 18



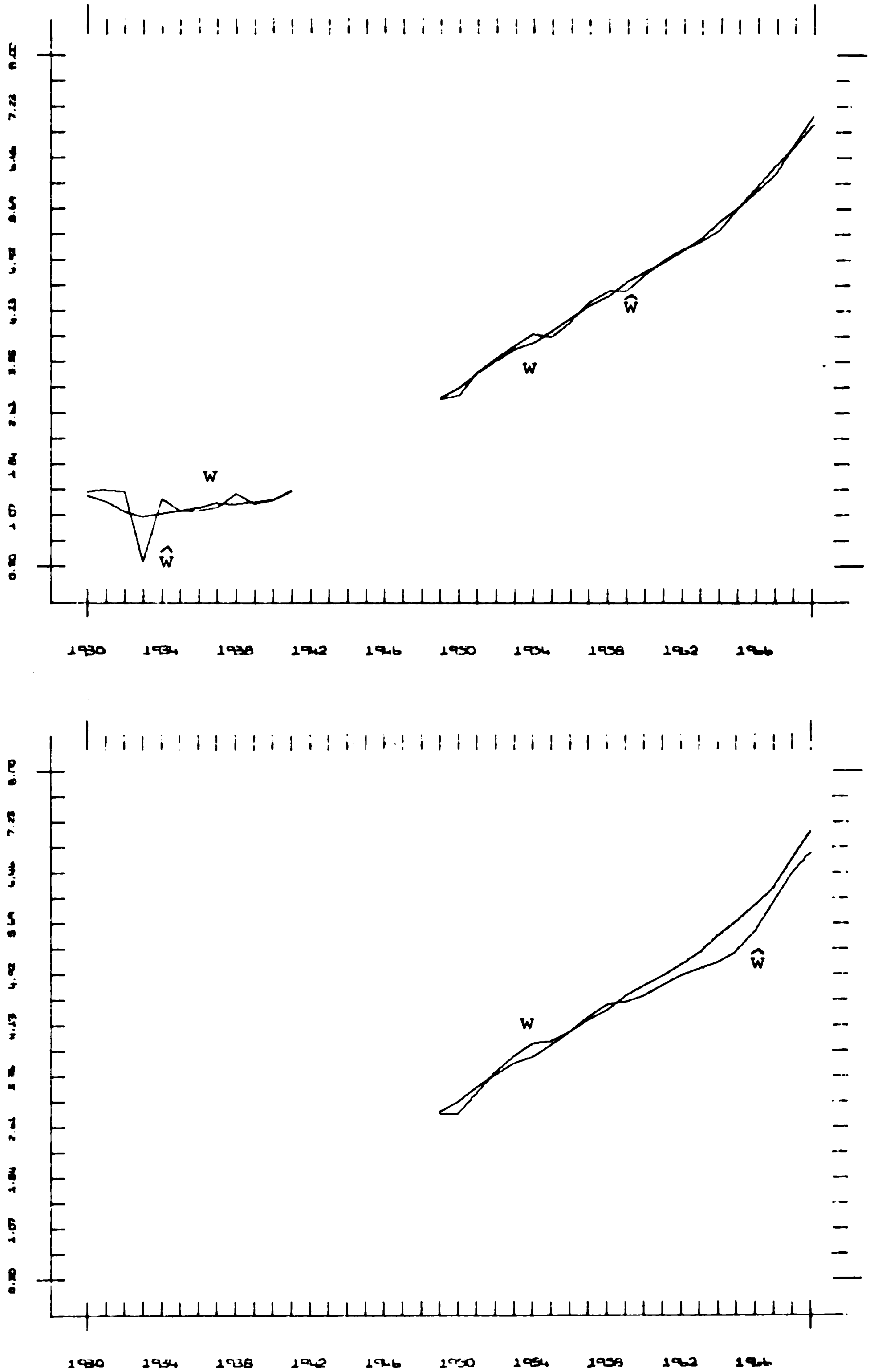
Depreciation

Figure 19



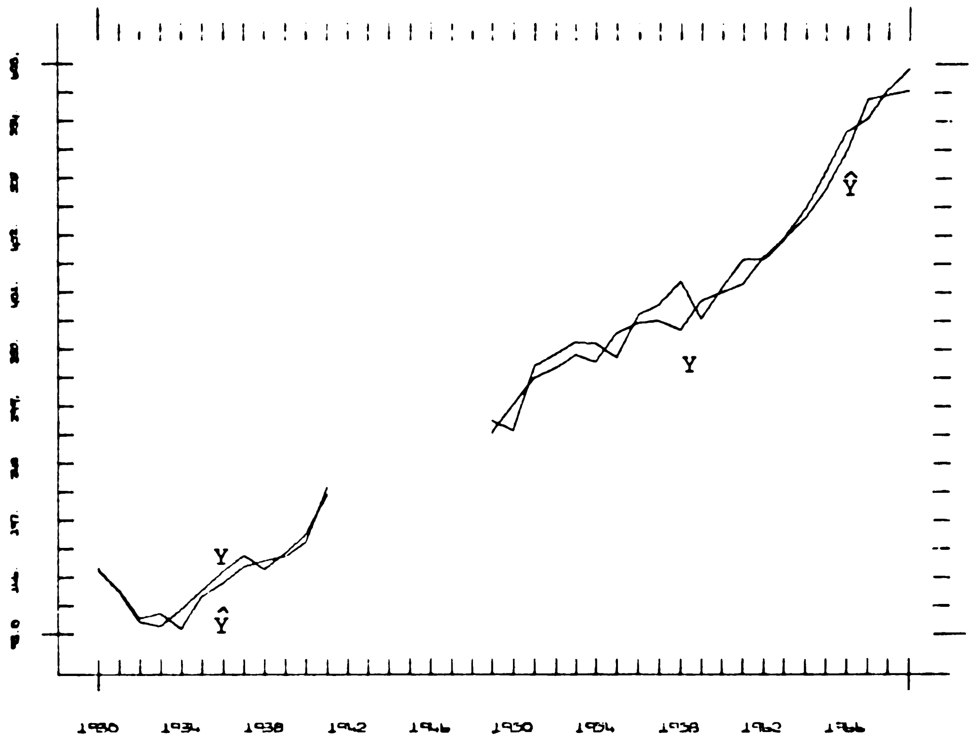
Change in the Nominal Wage Rate

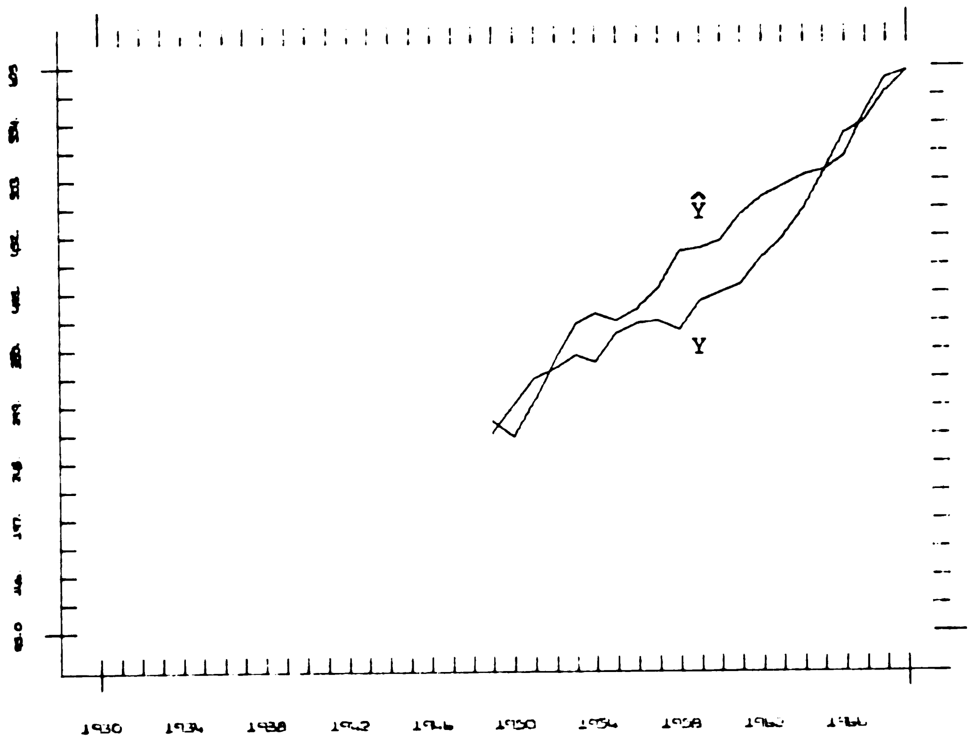
Figure 20



Nominal Wage Rate

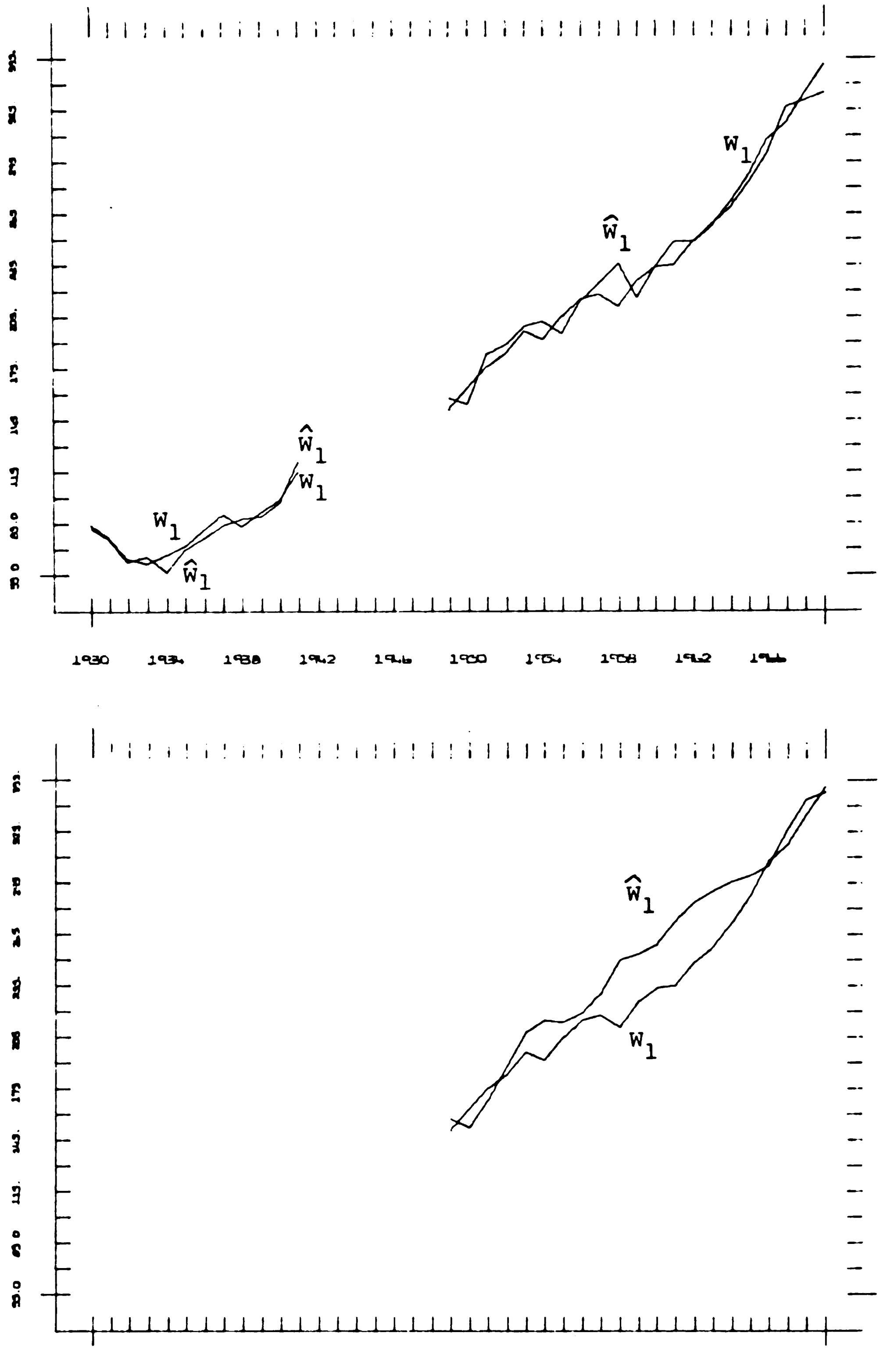
Figure 21





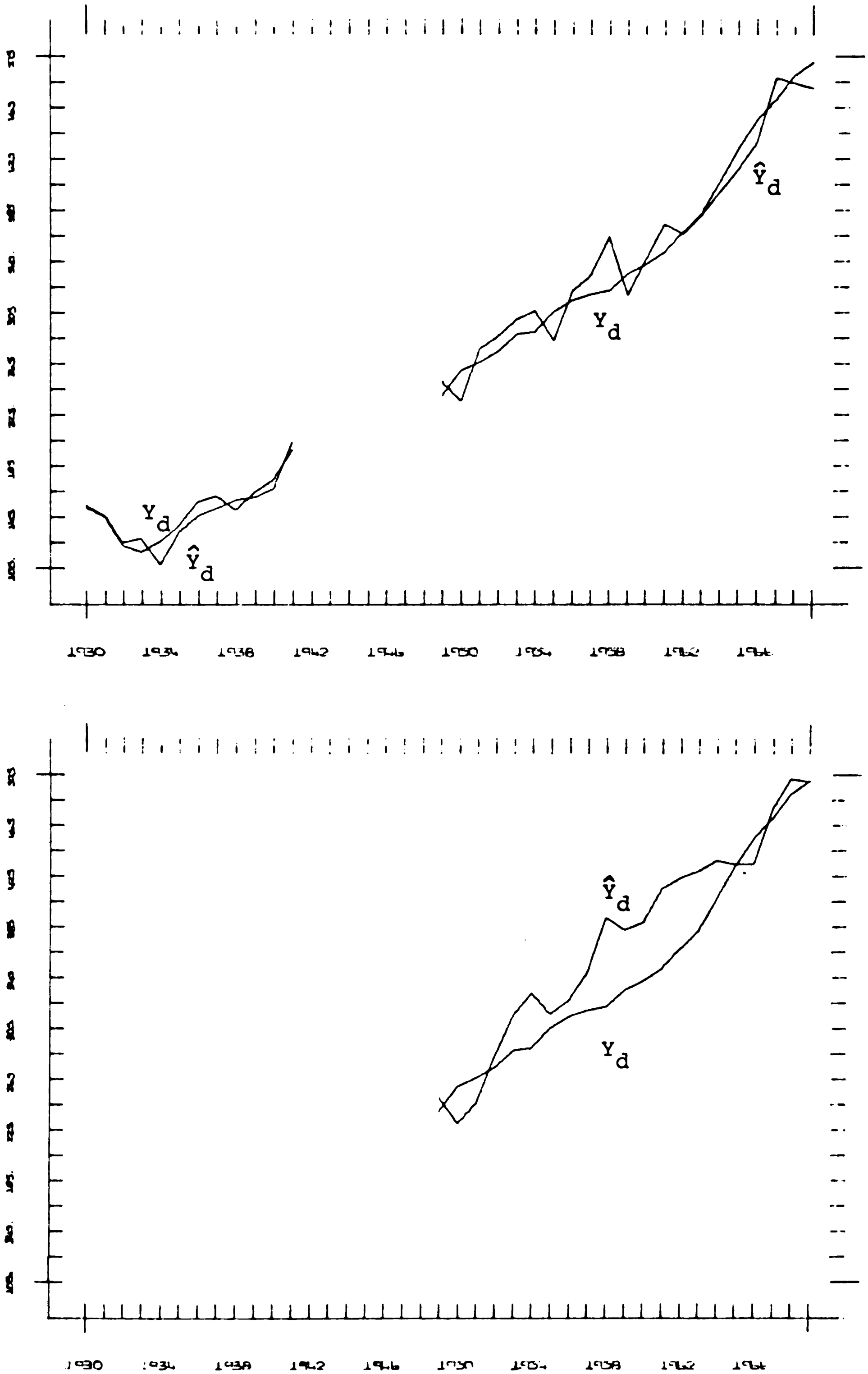
National Income

Figure 22



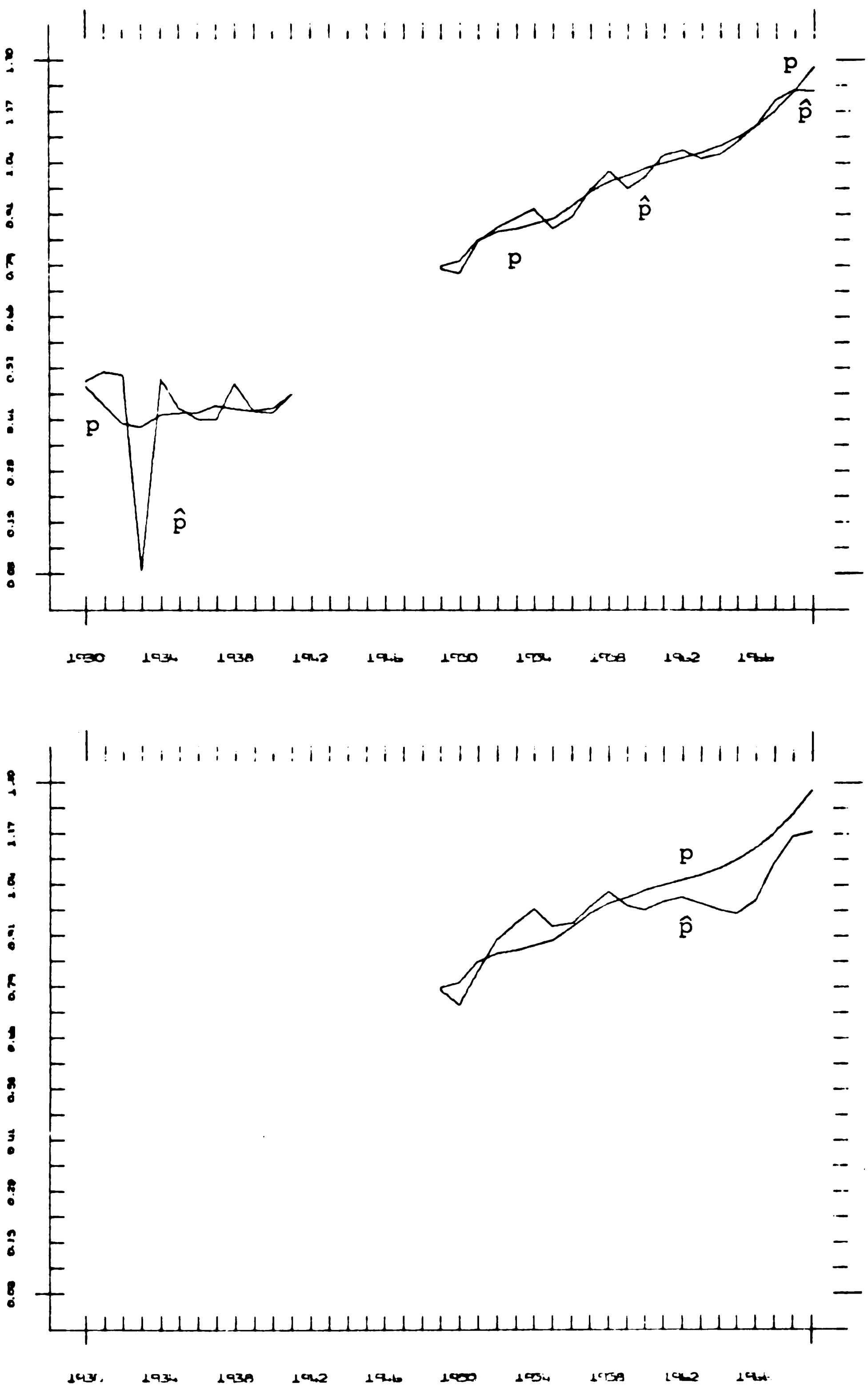
Private Wage Bill

Figure 23



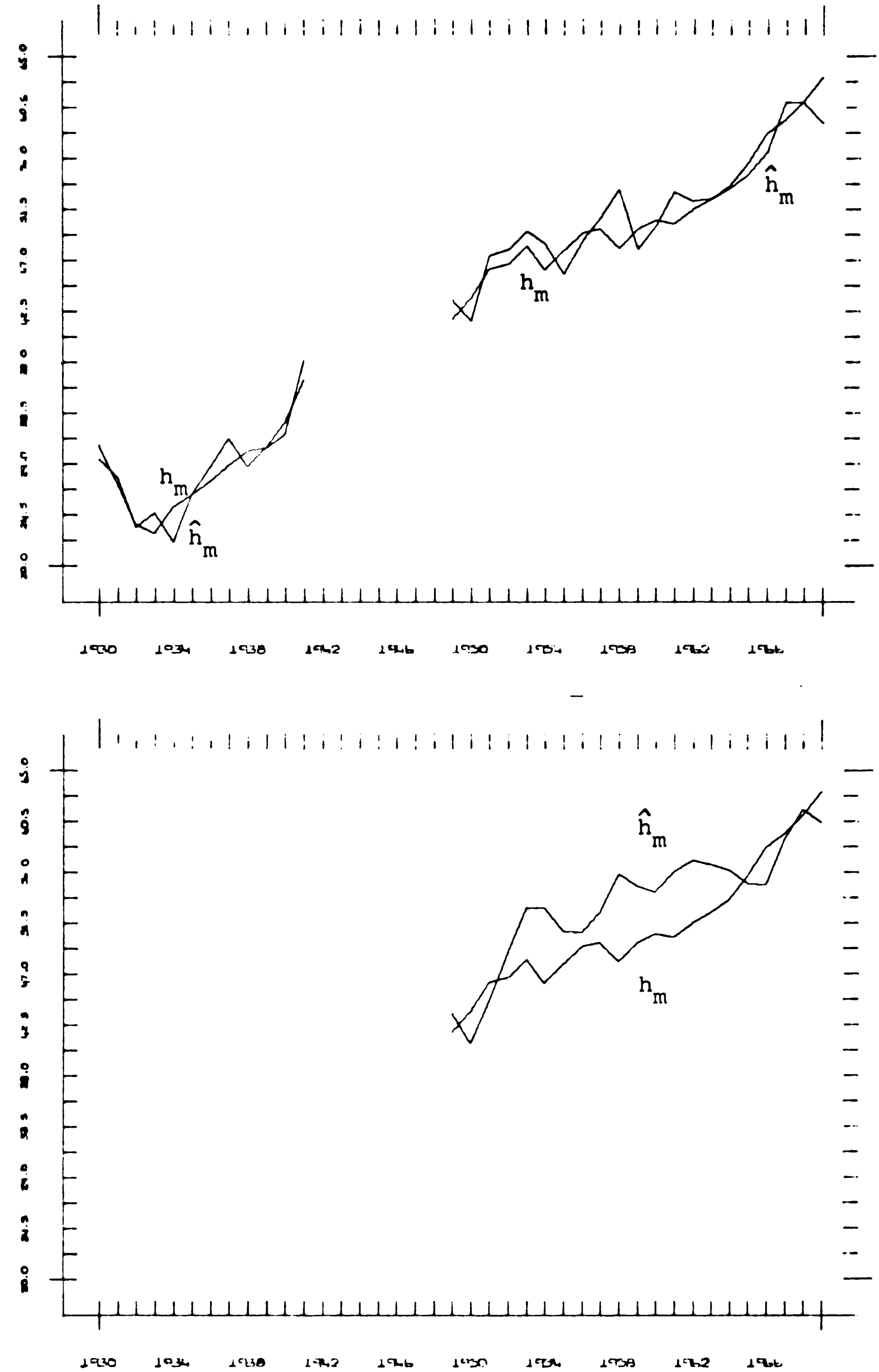
Disposable Income

Figure 24



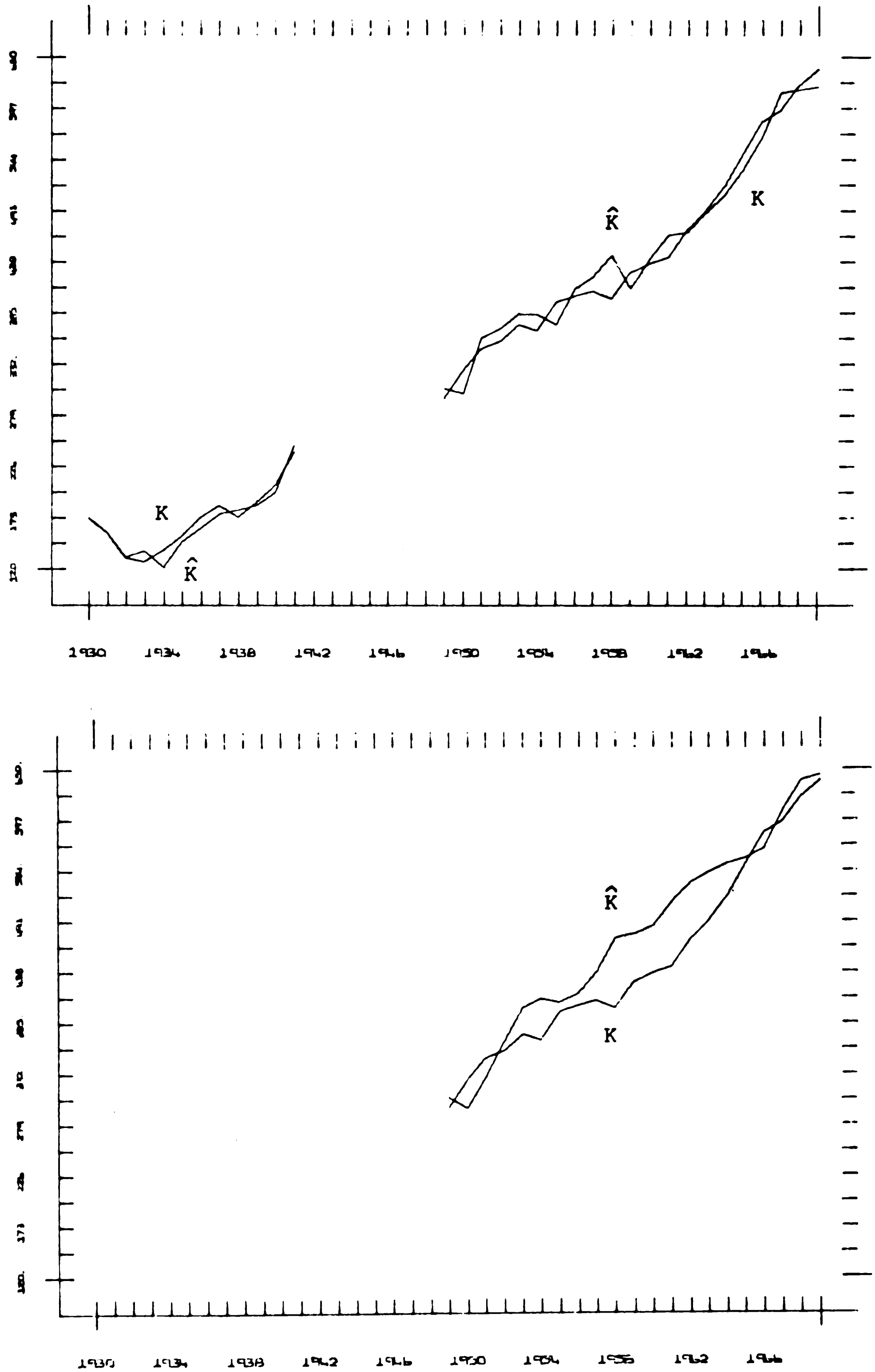
Price Level

Figure 25



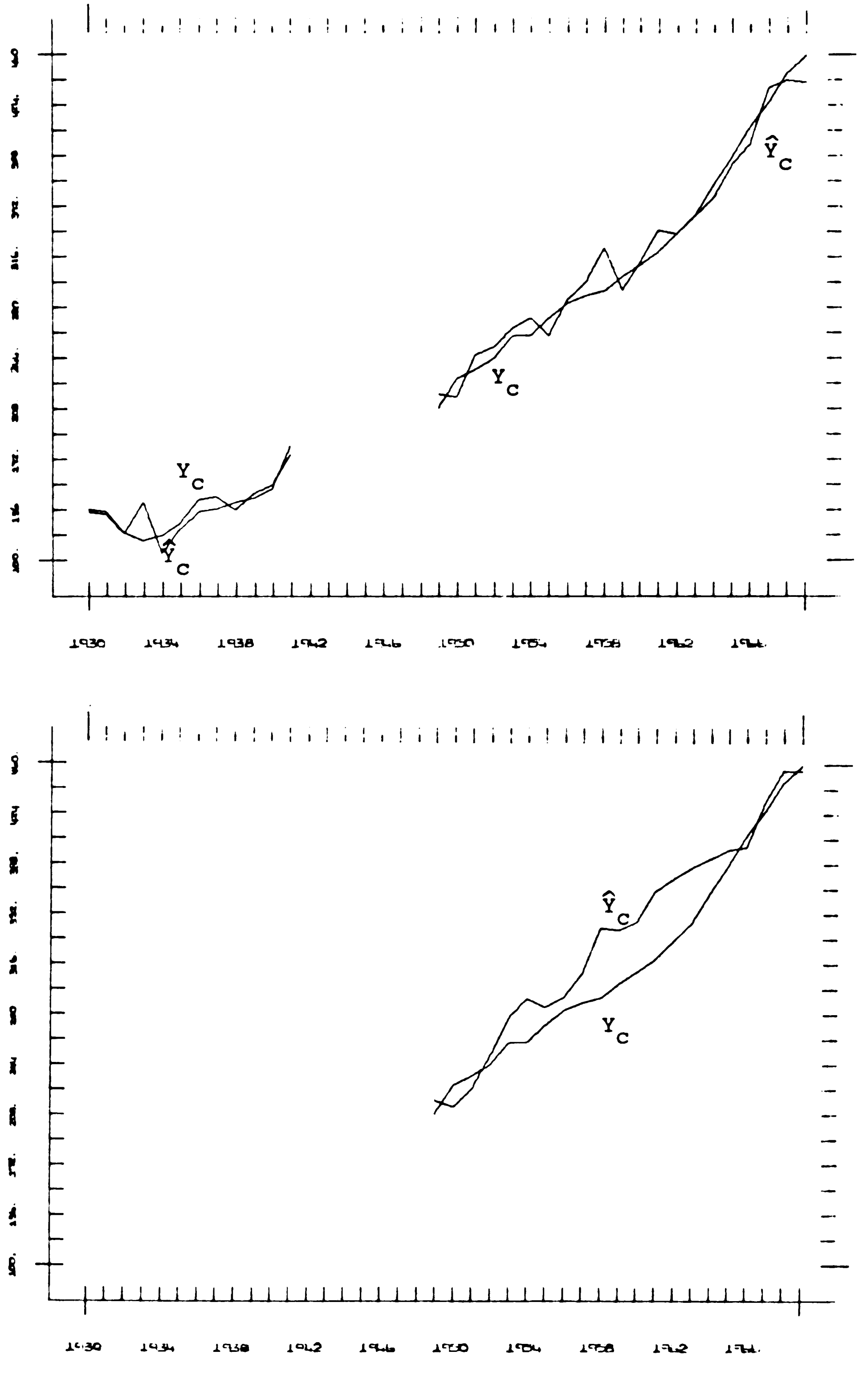
Manhours

Figure 26



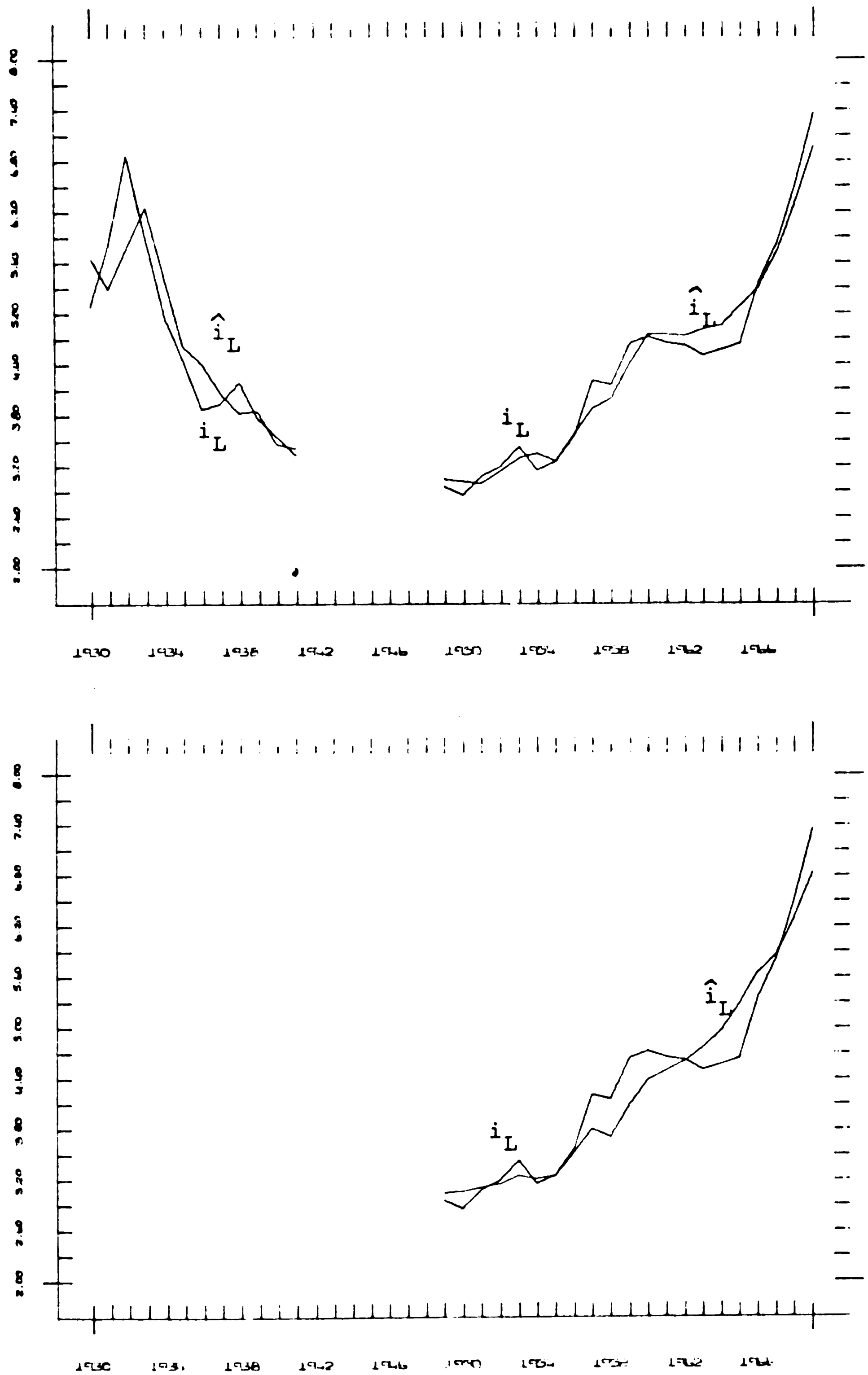
Capital

Figure 27



Share-Weighted Consumption

Figure 28



Long Term Interest Rate

Figure 29

1. Single Period Simulations

Examining the single period prewar simulation it is evident that the model malfunctioned in 1933 at the depth of the depression. The model was extremely difficult to solve[1] at this point; the solution that was finally obtained is economic nonsense, with wages, prices, and profits being far too severely depressed, and total income, manhours, and consumption not declining enough. This error can be traced to the particular nonlinearity incorporated into the Phillips curve, which, although satisfactory in the other sample years, is out of its range of adequate approximation in 1933. (Since the sample data include the depression they provide an excellent opportunity to allow the data to partially determine the nonlinearity by using a Box-Cox transformation.[2] _A priori_, the potential return did not appear to justify the cost of estimating an additional parameter nonlinearly; _a posteriori_, it seems the effort would have been well spent.)

The postwar performance is considerably better, although there is evidence that the model overstates the lags somewhat. There are four postwar recessions in the period covered by the sample, with troughs in 1949, 1954, 1958, and 1960;[3] in all four cases the model

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[1] Both Newton-Raphson and short-step Newton-Raphson algorithms were unsuccessful in solving the model for this year (although the model normally solved in four N-R iterations); the 1933 solution was ultimately obtained after 30 relaxation iterations.

[2] [4].

[3] Due to the timing of the '49 and '60 recessions, there is no actual decline in annual real GNP figures for these years.

solution was low in the immediately following year. The 1949 solution is surprisingly accurate; the 1954 solution is higher than the actual data, but the downturn from '53 is generally evident. It is in 1958 that the model does most poorly -- the variables continue steadily upward, only to crash in 1959. By the 1960 recession the model is back on course, and minor errors continue through the period of steady growth in the sixties.

What caused the model to miss the '58 recession and still track the others, including the relatively severe 1954 downturn? The answer lies in the causes of the 1958 recession, which appears to have been the result of restrictive monetary policy from late 1956 until mid 1958. While it is true that both of the model's monetary variables (R and i_d) fell during this period, these do not adequately reflect the much larger change in the growth rate of M1. In addition, it is likely that the importance of money is underestimated while the impact of the sizeable (5%) change in government expenditures in 1958 is overstated. Similarly, the model missed the large 1959 residential investment figure--primarily a result of much looser monetary policy in late '58 and early '59, after the extended through in residential investment due to the preceeding tight monetary policy. Thus the model, with its two equation monetary sector, did not track the 1958 recession, even though its performance in the other three post-war recessions was adequate.[1]

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[1] It should perhaps be reemphasized that the plots are of full model solution values; the individual equation (estimation) residuals are of course much smaller and less systematic.

2. Fully Endogenous Simulations

The fully endogenous simulation is a very severe test; surprisingly, the turning point behavior of the broad aggregates--GNP, national income, and consumption--is better here than in the single period simulation. The model begins to run high in the '54 recession, which it projects as a major slowdown but not an absolute decline; the '58 recession increases the upward error, which diminishes to a reasonable size by the early '60's. The unexpected result is that, with 21 years of compunded errors, the general shape of the actual and simulated curves is not dissimilar (figures 11, 22, and 12), and the ending GNP error is only 0.7%. The turning points of the volatile components of GNP--I^d, I^r, I^i, and F_I-- and the income shares--P, A_1, S_c--are not nearly as accurate, however.

C. Dynamic Multipliers

Since the model is nonlinear in the variables the dynamic multipliers depend on the levels of the exogenous and lagged endogenous variables and are difficult to obtain analytically; consequently, these multipliers have been calculated by numerical differentiation at the sample means.[1] Dynamic multipliers for ten exogenous variables are

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[1] The method used is simple ans well known: first a control solution is calculated with the exogenous variables set at their sample means over the entire simulation, and then a disturbed solution is calculated with one exogenous variable set at its mean plus one for the first time period only. The differences between the second and first solutions are the dynamic multipliers for the sidturbed exogenous variable.

presented in Tables 9 through 18. The ten period sum is given as an approximation to the long run multiplier.

The multipliers of the original K-G model were far too low, a fact not unrecognized by the authors.[1] Two important demand leakages--endogenous taxes and exports[2]--were not included in the original K-G model nor in the current revision, implying that the calculated multipliers should be higher than those obtained from models including these functions. The original multipliers were considerably lower than those computed from models with some or all of these additional endogenous functions[3]; the multipliers from the current revision are higher, as expected.

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1 See discussion in [25], p. 68-71.

2 Endogenous due to price level effects.

3 See [15], p. 355, and [14], pp. 567-568 for multiplier comparisons.

Table IV-2

DYNAMIC MULTIPLIERS

DISCOUNT RATE

Variable	t	t+1	t+2	t+3	t+4	t+5	Ten Period Sum
C	-1.804	-2.139	-2.106	-1.863	-1.479	-1.021	-10.117
I^d	-0.984	-1.300	-1.382	-1.309	-1.143	-0.926	-8.460
I^r	-1.259	-0.430	-0.362	-0.296	-0.233	-0.175	-2.973
I^i	-0.452	-0.218	-0.109	-0.010	0.068	0.123	0.040
F_I	-0.072	-0.140	-0.202	-0.253	-0.291	-0.315	-2.538
i_L	0.248	0.186	0.148	0.123	0.106	0.093	1.186
i_s	1.251	0.0	0.0	0.0	0.0	0.0	1.251
S_c	-0.972	-0.655	-0.556	-0.411	-0.255	-0.105	-2.275
P_c	-1.208	-0.939	-0.641	-0.662	-0.450	-0.234	-3.561
P	-1.786	-1.435	-1.326	-1.091	-0.600	-0.492	-6.334
D	-0.118	-0.202	-0.280	-0.345	-0.395	-0.428	-3.511
A_1	-0.286	-0.138	-0.050	0.056	0.164	0.262	1.695
W_1	-2.243	-2.178	-2.106	-1.851	-1.470	-1.031	-10.900
w	0.010	0.005	0.002	-0.002	-0.006	-0.009	-0.061
p	0.005	0.002	0.001	-0.001	-0.003	-0.004	-0.027
X	-4.425	-3.944	-3.754	-3.223	-2.494	-1.681	-18.973
K	-2.577	-4.323	-5.896	-7.166	-8.079	-8.629	-70.583
h_m	-0.609	-0.490	-0.415	-0.291	-0.144	0.008	-0.669
Y	-4.310	-3.745	-3.477	-2.881	-2.102	-1.256	-15.468
Y_d	-4.312	-3.748	-3.479	-2.883	-2.104	-1.258	-15.510

Table IV-2 (cont.)

DYNAMIC MULTIPLIERS

RATIO OF EXCESS TO REQUIRED RESERVES

Variable	t	t+1	t+2	t+3	t+4	t+5	Ten Period Sum
C	1.777	2.105	2.072	1.834	1.457	1.009	9.997
I^d	0.966	1.276	1.357	1.286	1.123	0.911	8.341
I^r	1.241	0.423	0.355	0.291	0.229	0.171	2.924
I^i	0.444	0.213	0.107	0.010	-0.067	-0.121	-0.038
F_I	0.072	0.139	0.200	0.250	0.287	0.309	2.492
i_L	-0.245	-0.184	-0.146	-0.121	-0.104	-0.091	-1.171
i_s	-1.234	0.0	0.0	0.0	0.0	0.0	-1.234
S_c	0.959	0.643	0.545	0.403	0.250	0.104	2.260
P_c	1.191	0.922	0.825	0.649	0.441	0.231	3.532
P	1.761	1.410	1.301	1.070	0.764	0.485	6.276
D	0.116	0.198	0.275	0.339	0.368	0.421	3.452
A_1	0.269	0.129	0.047	-0.056	-0.160	-0.258	-1.724
W_1	2.204	2.135	2.062	1.811	1.438	1.008	10.670
w	-0.010	-0.005	-0.002	0.002	0.006	0.009	0.060
p	-0.004	-0.002	-0.001	0.001	0.003	0.004	0.027
X	4.358	3.880	3.692	3.172	2.458	1.663	18.750
K	2.536	4.250	5.794	7.041	7.938	8.478	69.382
h_m	0.600	0.482	0.407	0.286	0.142	-0.007	0.667
Y	4.240	3.680	3.415	2.830	2.067	1.240	15.273
Y_d	4.238	3.677	3.412	2.828	2.065	1.237	15.250

Table IV-2 (cont.)

DYNAMIC MULTIPLIERS

GOVERNMENT EXPENDITURES

Variable	t	t+1	t+2	t+3	t+4	t+5	Ten Period Sum
C	1.305	0.958	0.593	0.320	0.112	−0.047	2.264
I^d	0.468	0.413	0.314	0.206	0.106	0.027	1.167
I^r	0.119	0.063	0.044	0.028	0.013	0.002	0.209
I^i	0.323	0.007	−0.052	−0.074	−0.076	−0.069	−0.096
F_I	0.049	0.070	0.080	0.081	0.077	0.070	0.617
i_L	0.0	0.0	0.0	0.0	0.0	0.0	0.0
i_s	0.0	0.0	0.0	0.0	0.0	0.0	0.0
S_c	0.707	0.155	0.061	−0.006	−0.050	−0.077	0.415
P_c	0.878	0.284	0.144	0.036	−0.039	−0.090	0.681
P	1.298	0.454	0.257	0.102	−0.010	−0.088	1.344
D	0.040	0.061	0.073	0.079	0.079	0.075	0.621
A_1	0.221	0.033	−0.013	−0.049	−0.073	−0.087	−0.333
W_1	1.602	0.817	0.497	0.262	0.076	−0.062	2.257
w	−0.008	−0.001	0.000	0.002	0.003	0.003	0.012
p	−0.004	−0.001	0.000	0.001	0.001	0.001	0.005
X	3.168	1.372	0.821	0.401	0.082	−0.155	3.966
K	0.870	1.292	1.526	1.607	1.574	1.460	12.279
h_m	0.454	0.173	0.081	0.013	−0.038	−0.074	0.179
Y	3.126	1.309	0.746	0.320	0.000	−0.232	3.319
Y_d	3.123	1.306	0.743	0.318	−0.002	−0.235	3.297

Table IV-2 (cont.)

DYNAMIC MULTIPLIERS

WAGE TAXES

Variable	t	t+1	t+2	t+3	t+4	t+5	Ten Period Sum
C	-1.338	-0.853	-0.469	-0.254	-0.087	0.035	-2.256
I^d	-0.269	-0.296	-0.237	-0.158	-0.064	-0.022	-0.809
I^r	-0.068	-0.051	-0.034	-0.021	-0.011	-0.002	-0.146
I^i	-0.164	-0.045	0.026	0.051	0.056	0.051	0.069
F_I	-0.057	-0.069	-0.072	-0.070	-0.064	-0.057	-0.534
i_L	0.0	0.0	0.0	0.0	0.0	0.0	0.0
i_s	0.0	0.0	0.0	0.0	0.0	0.0	0.0
S_c	-0.404	-0.178	-0.060	0.000	0.036	0.057	-0.274
P_c	-0.502	-0.273	-0.126	-0.034	0.026	0.066	-0.452
P	-0.742	-0.424	-0.215	-0.084	0.005	0.066	-0.897
D	-0.023	-0.040	-0.050	-0.054	-0.055	-0.052	-0.422
A_1	-0.129	-0.048	0.001	0.030	0.049	0.060	0.215
W_1	-0.914	-0.670	-0.403	-0.211	-0.067	0.039	-1.562
w	0.005	0.002	0.000	-0.001	-0.002	-0.002	-0.007
p	0.002	0.001	0.000	-0.000	-0.001	-0.001	-0.003
X	-1.801	-1.174	-0.660	-0.311	-0.059	0.122	-2.590
K	-0.498	-0.851	-1.046	-1.120	-1.104	-1.024	-8.382
h_m	-0.258	-0.155	-0.071	-0.014	0.025	0.053	-0.110
Y	-1.781	-1.137	-0.613	-0.259	-0.007	0.171	-2.193
Y_d	-2.783	-1.139	-0.615	-0.261	-0.009	0.169	-3.215

Table IV-2 (cont.)

DYNAMIC MULTIPLIERS

AGRICULTURAL TAXES

Variable	t	t+1	t+2	t+3	t+4	t+5	Ten Period Sum
C	-1.195	-0.977	-0.681	-0.413	-0.180	0.012	-2.306
I^d	-0.601	-0.528	-0.410	-0.285	-0.167	-0.066	-1.698
I^r	-0.076	-0.067	-0.051	-0.034	-0.019	-0.005	-0.188
I^i	-0.206	-0.076	0.009	0.052	0.069	0.072	0.109
F_I	-0.061	-0.080	-0.091	-0.094	-0.092	-0.085	-0.743
i_L	0.0	0.0	0.0	0.0	0.0	0.0	0.0
i_s	0.0	0.0	0.0	0.0	0.0	0.0	0.0
S_c	-0.448	-0.251	-0.120	-0.031	0.031	0.072	-0.315
P_c	-0.557	-0.370	-0.211	-0.087	0.008	0.078	-0.537
P	-0.823	-0.569	-0.348	-0.171	-0.032	0.073	-1.090
D	-0.039	-0.067	-0.086	-0.096	-0.099	-0.096	-0.779
A_1	-0.138	-0.059	0.000	0.045	0.078	0.099	0.470
W_1	-1.023	-0.876	-0.613	-0.369	-0.157	0.016	-2.000
w	0.005	0.002	0.000	-0.002	-0.003	-0.004	-0.017
p	0.002	0.001	0.000	-0.001	-0.001	-0.002	-0.007
X	-2.015	-1.564	-1.040	-0.584	-0.203	0.099	-3.322
K	-0.845	-1.447	-1.813	-1.964	-2.001	-1.905	-15.478
h_m	-0.284	-0.200	-0.109	-0.032	0.030	0.077	0.001
Y	-1.979	-1.499	-0.956	-0.490	-0.106	0.193	-2.566
Y_d	-2.982	-1.502	-0.958	-0.492	-0.109	0.191	-3.591

Table IV-2 (cont.)

DYNAMIC MULTIPLIERS

PROFIT TAXES

Variable	t	t+1	t+2	t+3	t+4	t+5	Ten Period Sum
C	-1.370	-1.092	-0.751	-0.453	-0.197	0.011	-2.637
I^d	-0.637	-0.568	-0.444	-0.308	-0.181	-0.071	-1.821
I^r	-0.065	-0.074	-0.056	-0.038	-0.021	-0.006	-0.209
I^i	-0.231	-0.082	0.012	0.058	0.077	0.079	0.118
F_I	-0.065	-0.087	-0.099	-0.102	-0.100	-0.092	-0.803
i_L	0.0	0.0	0.0	0.0	0.0	0.0	0.0
i_s	0.0	0.0	0.0	0.0	0.0	0.0	0.0
S_c	-0.502	-0.276	-0.130	-0.033	0.034	0.079	-0.359
P_c	-0.624	-0.408	-0.231	-0.094	0.010	0.085	-0.609
P	-0.922	-0.628	-0.381	-0.186	-0.034	0.079	-1.229
D	-0.042	-0.073	-0.093	-0.104	-0.107	-0.104	-0.839
A_1	-0.155	-0.066	-0.000	0.049	0.084	0.107	0.500
W_1	-1.145	-0.969	-0.672	-0.402	-0.171	0.017	-2.235
w	0.006	0.002	0.000	-0.002	-0.003	-0.004	-0.018
p	0.002	0.001	0.000	-0.001	-0.001	-0.002	-0.008
X	-2.256	-1.728	-1.138	-0.636	-0.221	0.107	-3.727
K	-0.911	-1.562	-1.957	-2.140	-2.156	-2.052	-16.681
h_m	-0.318	-0.222	-0.120	-0.035	0.032	0.083	-0.020
Y	-2.217	-1.658	-1.047	-0.535	-0.116	0.209	-2.914
Y_d	-3.219	-1.660	-1.050	-0.537	-0.119	0.206	-3.936

Table IV-2 (cont.)

DYNAMIC MULTIPLIERS

INDIRECT TAXES

Variable	t	t+1	t+2	t+3	t+4	t+5	Ten Period Sum
C	-0.811	-0.795	-0.612	-0.401	-0.202	-0.032	-1.980
I^d	-0.522	-0.454	-0.362	-0.260	-0.162	-0.075	-1.573
I^r	-0.056	-0.055	-0.045	-0.032	-0.019	-0.007	-0.166
I^i	-0.152	-0.071	-0.005	0.035	0.054	0.059	0.087
F_I	-0.053	-0.068	-0.078	-0.082	-0.081	-0.076	-0.660
i_L	0.0	0.0	0.0	0.0	0.0	0.0	0.0
i_s	0.0	0.0	0.0	0.0	0.0	0.0	0.0
S_c	-0.875	-0.148	-0.084	-0.022	0.027	0.063	-0.650
P_c	-1.087	-0.296	-0.183	-0.080	0.003	0.065	-1.039
P	-1.607	-0.480	-0.323	-0.176	-0.054	0.040	-1.971
D	-0.032	-0.057	-0.074	-0.084	-0.088	-0.086	-0.692
A_1	-0.100	-0.051	-0.005	0.033	0.063	0.083	0.413
W_1	-0.755	-0.724	-0.549	-0.355	-0.174	-0.022	-1.779
w	0.004	0.002	0.000	-0.001	-0.002	-0.003	-0.015
p	0.002	0.001	0.000	-0.001	-0.001	-0.001	-0.006
X	-1.487	-1.305	-0.943	-0.574	-0.246	0.023	-2.953
K	-0.699	-1.222	-1.559	-1.733	-1.772	-1.709	-13.790
h_m	-0.208	-0.167	-0.101	-0.038	0.016	0.058	-0.003
Y	-2.457	-1.251	-0.872	-0.493	-0.161	0.107	-3.286
Y_d	-2.459	-1.253	-0.874	-0.495	-0.163	0.104	-3.306

Table IV-2 (cont.)

DYNAMIC MULTIPLIERS

GOVERNMENT WAGE BILL

Variable	t	t+1	t+2	t+3	t+4	t+5	Ten Period Sum
C	0.058	-0.164	-0.212	-0.188	-0.139	-0.083	-0.613
I^d	-0.181	-0.139	-0.124	-0.106	-0.082	-0.057	-0.698
I^r	-0.008	-0.015	-0.017	-0.015	-0.012	-0.008	-0.076
I^i	-0.022	-0.031	-0.019	-0.004	0.008	0.014	0.013
F_I	0.057	0.035	0.018	0.005	-0.004	-0.010	0.042
i_L	0.0	0.0	0.0	0.0	0.0	0.0	0.0
i_s	0.0	0.0	0.0	0.0	0.0	0.0	0.0
S_c	-0.139	-0.048	-0.057	-0.039	-0.019	-0.003	-0.214
P_c	-0.172	-0.077	-0.086	-0.064	-0.037	-0.013	-0.338
P	-0.255	-0.120	-0.137	-0.107	-0.070	-0.034	-0.612
D	-0.009	-0.017	-0.024	-0.029	-0.032	-0.033	-0.266
A_1	-0.337	-0.007	-0.007	0.004	0.014	0.023	-0.172
W_1	-0.614	-0.246	-0.228	-0.191	-0.139	-0.085	-1.425
w	0.012	0.000	0.000	-0.000	-0.000	-0.001	0.006
p	0.005	0.000	0.000	-0.000	-0.000	-0.000	0.003
X	-0.208	-0.383	-0.388	-0.315	-0.219	-0.122	-1.399
K	-0.201	-0.369	-0.505	-0.601	-0.656	-0.673	-5.345
h_m	-0.176	-0.048	-0.045	-0.030	-0.013	0.003	-0.188
Y	-0.201	-0.368	-0.367	-0.289	-0.190	-0.091	-1.158
Y_d	-0.203	-0.370	-0.369	-0.291	-0.192	-0.094	-1.180

Table IV-2 (cont.)

DYNAMIC MULTIPLIERS

GOVERNMENT PAYMENTS TO FARMERS

Variable	t	t+1	t+2	t+3	t+4	t+5	Ten Period Sum
C	0.385	0.181	0.066	0.011	-0.022	-0.044	0.329
I^d	0.077	0.071	0.046	0.022	0.003	-0.010	0.110
I^r	0.020	0.011	0.006	0.002	-0.001	-0.002	0.017
I^i	0.053	0.004	-0.014	-0.017	-0.016	-0.012	-0.024
F_I	0.008	0.012	0.013	0.012	0.011	0.009	0.084
i_L	0.0	0.0	0.0	0.0	0.0	0.0	0.0
i_s	0.0	0.0	0.0	0.0	0.0	0.0	0.0
S_c	-0.429	0.102	0.035	0.006	-0.004	-0.010	-0.341
P_c	-0.533	0.072	0.027	0.005	-0.007	-0.013	-0.512
P	-0.788	0.085	0.023	-0.008	-0.024	-0.034	-0.897
D	0.007	0.010	0.012	0.012	0.011	0.010	0.081
A_1	0.037	0.007	-0.006	-0.012	-0.015	-0.016	-0.062
W_1	0.265	0.148	0.058	0.006	-0.024	-0.044	0.170
w	-0.001	-0.000	0.000	0.000	0.001	0.001	0.002
p	-0.001	-0.000	0.000	0.000	0.000	0.000	0.001
X	0.528	0.257	0.094	0.008	-0.045	-0.077	0.366
K	0.143	0.219	0.245	0.240	0.216	0.181	1.566
h_m	0.076	0.033	0.007	-0.006	-0.014	-0.019	-0.002
Y	0.519	0.245	0.080	-0.007	-0.058	-0.089	0.261
Y_d	0.517	0.242	0.078	-0.009	-0.060	-0.091	0.238

Table IV-2 (cont.)

DYNAMIC MULTIPLIERS

INVERSE OF THE IMPORT PRICE LEVEL

Variable	t	t+1	t+2	t+3	t+4	t+5	Ten Period Sum
C	0.344	0.522	0.566	0.526	0.445	0.337	3.028
I^d	0.122	0.204	0.243	0.246	0.222	0.182	1.470
I^r	0.031	0.041	0.044	0.042	0.036	0.029	0.260
I^i	0.084	0.068	0.040	0.012	-0.010	-0.026	0.017
F_I	-0.250	-0.178	-0.119	-0.072	-0.036	-0.010	-0.579
i_L	0.0	0.0	0.0	0.0	0.0	0.0	0.0
i_s	0.0	0.0	0.0	0.0	0.0	0.0	0.0
S_c	0.184	0.186	0.163	0.126	0.086	0.048	0.703
P_c	0.229	0.255	0.238	0.197	0.145	0.090	1.082
P	0.338	0.366	0.371	0.319	0.248	0.172	1.883
D	0.010	0.024	0.038	0.050	0.060	0.067	0.525
A_1	0.058	0.054	0.039	0.018	-0.005	-0.026	-0.117
W_1	0.420	0.544	0.557	0.506	0.419	0.313	3.007
w	-0.002	-0.002	-0.001	-0.001	0.000	0.001	0.004
p	-0.001	-0.001	-0.001	-0.000	0.000	0.000	0.002
X	0.834	1.015	1.013	0.902	0.730	0.533	5.374
K	0.228	0.517	0.806	1.055	1.242	1.359	10.560
h_m	0.120	0.139	0.131	0.106	0.073	0.038	0.470
Y	0.821	0.989	0.973	0.849	0.667	0.464	4.824
Y_d	0.819	0.987	0.970	0.846	0.665	0.462	4.802

Multiplier comparisons are difficult since superficially similar models often have quite different sets of endogenous and exogenous variables; thus even though we have the original K-G model and the Suits revision to compare to the present model (denoted K-G-H), the only[1] multipliers that are available and approximately comparable are the government expenditures multipliers on GNP, consumption, and durable investment (reproduced in table 19).

Examination of table 19 shows that there is considerable disagreement between the models about the timing of the impact of government expenditure changes; while the long run multipliers of the K-G-H and K-G models are in approximate agreement, the K-G-H model concentrates the effects more heavily in the beginning years. We take this to be a marked improvement over the original K-G model which, as Goldberger states[3] ". . . must be judged deficient on the basis of its sample period record. It is reasonable to infer that the structural lags specified in the model were, in general, too long." In table 19, all of the K-G-H multipliers peak in the first year, while the K-G multipliers peak in the second year (with large third year effects), and the K-G-S model exhibits a peak consumption multiplier

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1 The three models use completely different monetary instruments for instance.

2 Even these are not directly comparable: The Suits multipliers include the effects of endogenous tax functions, while the K-G investment multipliers apply to total--not just durable--investment.

3 [25], p. 68.

in the fourth year--a clearly unreasonable result. Both the K-G and K-G-S models constrain all impact multipliers on durable and residential investment[1] to zero--a questionable practice in an annual model--which further lowers the GNP impact multiplier. The K-G-S investment multipliers are questionably low, while the long run GNP multiplier (admittedly only a five year approximation) seems excessive. Thus, of the three sets of multipliers tabled, the timing of the K-G-H appears to be most reasonable.

Since a one unit change in each of the instruments brings about vastly differing changes in the target variables, to examine the policy implications of the estimated multipliers we shall first normalize the exogenous variable changes to those magnitudes necessary to effect a 10% increase in mean GNP (or a 5.34% increase in 1969 GNP) in the long run.[2] Three instruments will be normalized: (1) the discount rate, (2) government expenditures, and (3) a composite personal tax variable (T*) that maintains wage, farm, and profit taxes at their mean proportions while the combined changes result in the required 10% increase in GNP.[3]

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1 The K-G model also constrains the impact multiplier on inventory investment to zero, since investment is not disaggregated.

2 The sum of the first ten dynamic multipliers is taken as the long run multiplier.

3 Let t_W, t_A, t_P be the solution tax changes. Then

$$t_W/(t_W+t_A+t_P) = \text{the mean wage tax proportion} = 0.366$$

and

$$t_P/(t_W+t_A+t_P) = \text{the mean profit tax proportion} = 0.609,$$

while from the long run multipliers

$$-2.590t_W - 3.322t_A - 3.727t_P = X = 38.853.$$

Solving the three equations gives the values of t_W, t_A, and t_P that make up the composite personal tax change.

Table 21 gives the policy effects on the income side of the accounts. The expected relative stability of private wages and volatility of profits appears; less expected, perhaps, is the long run negative effect on private agricultural income brought about through price level changes--all of the policies exhibit important percentage changes in farm income in the long run.[1]

[1] The result should be tempered with misgivings about the lag structure of price changes and the specification of the farm income equation; farm income is almost certain to be price level sensitive, however, even if the exact timing and magnitude of the changes are unknown.

Table IV-3

GOVERNMENT EXPENDITURES MULTIPLIERS[1]

Variable	Model	YEAR t	t+1	t+2	t+3	t+4	Long run[2]
X	K-G-H	3.168	1.372	0.821	0.401	0.082	3.966
	K-G	1.386	1.421	1.078	0.680	0.323	4.745[3]
	K-G-S	1.304	1.619	1.582	1.545	1.335	7.385
C	K-G-H	1.305	0.958	0.593	0.320	0.112	2.264
	K-G	0.398	0.619	0.574	0.418	0.242	2.302
	K-G-S	0.295	0.426	0.460	0.500	0.465	2.146
I^d	K-G-H	0.468	0.413	0.314	0.206	0.108	1.187
	K-G	0.0	0.826	0.533	0.294	0.112	1.624
	K-G-S	0.0	0.186	0.173	0.133	0.082	0.574

1 The Klein-Goldberger multipliers are from [25], p. 87; the Suits multipliers are from [58], p. 610 (the aggregated consumption multiplier is the sum of the four separate Suits consumption multipliers).

2 The long run multipliers are ten, seven, and five period sums for the K-G-H, K-G, and K-G-S models, respectively.

3 The long run K-G GNP multiplier is elsewhere estimated at 2.93 ([25], p. 87 ff.).

Table IV-4

POLICY NORMALIZED MULTIPLIER EFFECTS[1]

ENDOGENOUS VARIABLE	EXOGENOUS VARIABLE	t		t+1		t+2		t+3		t+4		t+5		Ten Period Sum	
		$	%	$	%	$	%	$	%	$	%	$	%	$	%
X	i_d	9.06	2.33	6.08	2.08	7.69	1.96	6.60	1.70	5.11	1.31	3.44	0.69	38.85	10.00
	G	31.04	7.99	13.44	3.46	8.05	2.07	3.93	1.01	0.80	0.21	-1.52	-0.39	38.86	10.00
	T*	24.52	6.31	17.91	4.61	11.31	2.91	6.07	1.56	1.90	0.49	-1.32	-0.34	38.65	10.00
C	i_d	3.69	1.46	4.38	1.73	4.31	1.70	3.81	1.51	3.03	1.20	2.09	0.63	20.71	8.18
	G	12.79	5.05	9.38	3.71	5.81	2.29	3.14	1.24	1.09	0.43	-0.46	-0.18	22.18	8.76
	T*	15.94	6.29	11.79	4.66	7.69	3.04	4.46	1.76	1.84	0.73	-0.24	-0.09	29.30	11.57
I^d	i_d	2.01	5.30	2.66	7.00	2.83	7.44	2.68	7.06	2.34	6.16	1.90	4.99	17.36	45.69
	G	4.59	12.08	4.05	10.65	3.08	8.10	2.02	5.32	1.06	2.79	0.26	0.70	11.63	30.60
	T*	5.90	15.52	5.50	14.49	4.32	11.37	2.97	7.82	1.70	4.48	0.62	1.64	17.04	44.84
I^r	i_d	2.58	16.11	0.88	5.51	0.74	4.63	0.61	3.79	0.48	2.99	0.36	2.23	6.09	38.05
	G	1.17	7.28	0.62	3.85	0.43	2.69	0.27	1.66	0.13	0.82	0.02	0.11	2.05	12.78
	T*	0.92	5.78	0.77	4.79	0.56	3.51	0.37	2.32	0.20	1.25	0.05	0.33	2.18	13.65
I^i	i_d	0.93	28.06	0.45	13.51	0.22	6.77	0.02	0.63	-0.14	-4.24	-0.25	-7.65	-0.08	-2.49
	G	3.16	95.79	0.07	2.10	-0.51	-15.44	-0.72	-21.83	-0.74	-22.57	-0.68	-20.49	-0.94	-28.40
	T*	2.51	76.06	0.81	24.42	-0.20	-5.99	-0.65	-19.73	-0.81	-24.54	-0.81	-24.44	-1.18	-35.62
F_I	i_d	0.15	0.09	0.29	1.54	0.41	2.21	0.52	2.77	0.60	3.19	0.65	3.45	5.20	27.79
	G	0.48	2.57	0.69	3.69	0.78	4.17	0.79	4.25	0.76	4.05	0.69	3.69	6.05	32.34
	T*	0.73	3.89	0.94	5.03	1.04	5.58	1.06	5.67	1.02	5.44	0.93	4.99	8.27	44.24

[1] Each instrument is set at the value required to obtain a change in GNP of $38.85 b, 10% of mean GNP or 5.34% of 1969 GNP.

Table IV-5

POLICY NORMALIZED MULTIPLIER EFFECTS[1]

ENDOGENOUS VARIABLE	EXOGENOUS VARIABLE	t		t+1		t+2		t+3		t+4		t+5		Ten Period Sum	
		$	%	$	%	$	%	$	%	$	%	$	%	$	%
Y	i_d	8.82	2.76	7.67	2.40	7.12	2.23	5.90	1.85	4.30	1.35	2.57	0.80	31.71	9.93
	G	30.63	9.59	12.82	4.01	7.31	2.29	3.14	0.98	0.00	0.00	-2.28	-0.71	32.52	10.18
	T^*	24.15	7.56	17.22	5.39	10.43	3.26	5.09	1.59	0.90	0.28	-2.29	-0.72	31.09	9.73
Y_d	i_d	8.62	3.16	7.67	2.74	7.12	2.55	5.90	2.11	4.31	1.54	2.58	0.92	31.71	11.36
	G	30.63	10.95	12.82	4.58	7.31	2.61	3.14	1.11	0.00	-0.01	-2.28	-0.82	32.52	11.55
	T^*	35.94	12.85	17.22	6.17	10.43	3.74	5.09	1.83	0.90	0.33	-2.29	-0.81	43.12	15.42
W_1	i_d	4.59	2.54	4.46	2.47	4.31	2.38	3.79	2.10	3.01	1.67	2.11	1.17	22.32	12.34
	G	15.69	8.66	8.01	4.43	4.87	2.69	2.57	1.42	0.77	0.42	-0.60	-0.33	22.11	12.23
	T^*	12.44	6.88	10.09	5.58	6.73	3.72	3.90	2.16	1.56	0.86	-0.30	-0.16	23.34	12.91
A_1	i_d	0.59	5.10	0.28	2.45	0.10	0.90	-0.12	-1.00	-0.33	-2.91	-0.54	-4.66	-3.47	-30.19
	G	2.16	18.81	0.32	2.78	-0.13	-1.11	-0.48	-4.17	-0.71	-6.22	-0.86	-7.45	-3.26	-28.36
	T^*	1.71	14.86	0.70	6.09	-0.00	-0.02	-0.49	-4.30	-0.84	-7.28	-1.06	-9.19	-4.65	-40.40
P	i_d	3.66	4.24	2.94	3.41	2.71	3.15	2.23	2.59	1.64	1.90	1.01	1.17	12.97	15.05
	G	12.72	14.76	4.45	5.16	2.52	2.92	1.00	1.16	-0.10	-0.11	-0.87	-1.00	13.17	15.28
	T^*	10.05	11.66	6.49	7.53	3.76	4.36	1.75	2.03	0.23	0.27	-0.87	-1.01	12.99	15.07

[1] Each instrument is set at the value required to obtain a change in GNP of $38.85 b, 10% of mean GNP or 5.34% of 1969 GNP.

Table IV-6

POLICY NORMALIZED MULTIPLIER EFFECTS[1]

ENDOGENOUS VARIABLE	EXOGENOUS VARIABLE	t		t+1		t+2		t+3		t+4		t+5		Ten Period Sum	
		$	%	$	%	$	%	$	%	$	%	$	%	$	%
p	i_d	-0.01	-1.16	-0.00	-0.57	-0.00	-0.21	0.00	0.22	0.01	0.66	0.01	1.07	0.06	6.99
	G^*	-0.03	-4.35	-0.01	-0.64	0.00	0.26	0.01	0.96	0.01	1.42	0.01	1.70	0.05	6.44
	T^*	-0.03	-3.42	-0.01	-1.42	-0.00	-0.02	0.01	0.96	0.01	1.65	0.02	2.09	0.07	9.11
w	i_d	1.25	2.90	1.00	2.34	0.85	1.98	0.60	1.39	0.30	0.69	-0.02	-0.04	1.41	3.28
	G^*	4.45	10.35	1.69	3.94	0.80	1.85	0.12	0.29	-0.37	-0.87	-0.72	-1.68	1.75	4.08
	T^*	3.48	8.09	2.31	5.38	1.20	2.78	0.32	0.75	-0.35	-0.81	-0.85	-1.97	0.62	1.44
h_m	i_d	-0.02	-0.61	-0.01	-0.30	-0.00	-0.11	0.00	0.12	0.01	0.35	0.02	0.56	0.13	3.68
	G^*	-0.08	-2.29	-0.01	-0.34	0.00	0.13	0.02	0.50	0.03	0.75	0.03	0.90	0.12	3.40
	T^*	-0.06	-1.80	-0.03	-0.75	-0.00	-0.01	0.02	0.51	0.03	0.87	0.04	1.10	0.16	4.60

[1] Each instrument is set at the value required to obtain a change in GNP of $38.85 b, 10% of mean GNP or 5.34% of 1969 GNP.

Table 22 shows the effects of the normalized policies on prices, wages, and manhours; both prices and manhours move in the opposite direction from the expected, and remain negative for three years (although the long run effects are plausible). This unlikely result becomes even more doubtful when we remember that the impact multipliers calculated from the Taylor expanded identities were positive for both price and manhours (table 7 above). Thus we conclude that these multipliers are very sensitive to the nonlinearities while the other multipliers are only moderately sensitive. Since the price level and manhours are of such critical importance to policy makers they should be determined as normalizing variables in stochastic equations if possible in order to avoid the accumulated multiplier errors that result from determining them residually.[1]

To be honest, we must accept all of the conclusions based on the dynamic multpliers with reservation. Examination of figures 11-29 shows that although the fully endogenous solution tracks better than expected a priori, there remain large inaccuracies in the model solution, especially in the less aggregative variables. In addition, the dynamic multipliers are only point estimates: they may have very large variances, or variances that increase rapidly as the time from the impact multiplier increases.[2] Consequently, the results should be interpreted as suggestive, pending an elaborate sensitivity analysis and variance estimation.

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1 Unfortunately, solving the production function for manhours (transforming it to a labor demand function) results in multi-collinearity among the regressors (capital and output). See [14], pp. 253-255.

2 Only the acute shortage of computer time kept these variances from being calculated (see [28] for the method).