SPECTRUM CHARACTERIZATION OF AC/AC POWER CONVERSION SYSTEM

By

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ABSTRACT

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This Thesis proposes an analytical method based on three-dimensional Fourier integral to obtain accurate spectra of both the switching functions and the synthesized output voltages and input currents of matrix converter. The principle of this analytical is carried out in two steps. The first step is that the spectra of the switching functions are derived based on three-dimensional Fourier integral. The second step is that the spectra of the output voltages and input currents are evaluated using a convolution operation in the frequency domain.

The challenges associated with the spectral analysis of matrix converter waveforms are twofold. On one hand, the modulation signal contains both the input and output frequencies. Unlike the third harmonic injection in the modulation functions, the input frequency and the output frequency are typically independent from each other and will not form an integer ratio. On the other hand, it is very common that the switching frequency or the carrier frequency is not rational multiple of either the input frequency or the output frequency. These aforementioned challenges make it a very challenge task to accurately characterize the spectra of matrix converter waveforms through commonly resorted numerical methods such as fast Fourier transform (FFT). The proposed analytical approach, which is an extension of the double Fourier series expansion for the pulse width modulated waveforms of voltage source inverter will provide an accurate solution to spectral analysis of matrix converters.

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CHAPTER 1

Introduction

1.1 Background of AC/AC Power Conversion

The family of AC/AC power converters can be categorized into two types which are AC-to-AC converters without DC-link (direct matrix converter) and AC-to-AC converters with DC-link (indirect matrix converter).

a) AC-to- AC converter without DC-link:

This type of converter uses only one conversion stage. In other word, input AC voltages are connected directly into output AC voltages through proper operation of four-quadrant or voltage-and-current bidirectional switches.

b) AC-to- AC converter with DC-link:

This type of converter uses two conversion stages. First, the input AC voltages are converted into intermediate DC voltage and this stage is called rectifier stage. Then, the intermediate DC voltages are converted into output AC voltages and this stages is called inverter stage.

Each converter topology has its own advantages and disadvantages and the choice of the converter depends on the requirement of the application.

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1.2 Matrix Converter Concept

The matrix converter that was first introduced is a direct AC/AC power converter. It consists of nine four-quadrant or bidirectional switches which are used to connect directly the three-phase power supply to a three-phase load without using energy storage elements or DC-link as shown in Figure 1.1. Each four-quadrant switch employed in the matrix converter is typically implemented from two two-quadrant switches as illustrated in Figure 1.2.



Figure 1.1 Schematic of direct matrix converter.



Figure 1.2 Four-quadrant switch.

The nine bidirectional switches are modulated to generate the desired output waveform based on the input supply voltages and the demanded output voltages.

The matrix converter is usually fed on the input side by a three-phase voltage source. Each phase is connected with an input filter capacitor to ensure continuous input voltage waveforms. In addition, the matrix converter is connected to an inductive load which leads to continuous output current waveforms.

The matrix converter has received an increased amount of interest in the last three decades. Furthermore, it has been considered intensely as an alternative to conventional indirect power converter systems due to its several desirable features such as:

- Sinusoidal input and output current waveforms
- Generation of load voltage with variable magnitude and frequency
- Controllable input power factor for any load
- Simple and compact power circuit
- Inherent regeneration capability

Despite these advantages, there are some reasons for rejecting the matrix converter in industrial applications. The major disadvantages is the limitation of the voltage gain ratio up to 0.866. Consequently, electrical motors or any standard device connected as load to the matrix converter do not operate at their nominal rate voltage. Another drawback is that the number of the semiconductor switches is more than the number used in a dc-link converter. Therefore, the cost will be more expensive. In addition, with the absence of the dc-link, there is no decoupling between the input and output sides. Hence, and distortion in the input voltage is reflected in the output voltage at different frequencies. As a consequence, sub-harmonics can be generated. Furthermore, the control of the bidirectional switches of the matrix converter is very complicated.

1.3 Modulation Techniques

Pulse width modulation (PWM) strategies for matrix converters have received significant research effort recently [1]- [23]. The modulation research on matrix converter starts with the work of Venturini and Alesina [3]. The authors provided a mathematical approach to describe how the low frequency behaviors of the voltages and currents are generated at the load and the input. This strategy allows the full control of the output voltages and input power factor. This approach is also known as a direct matrix converter (DMC). Unfortunately, the drawback of this strategy is that the maximum voltage transfer ratio (q) which is known as the ratio of the magnitude of the output voltage to the magnitude of the input voltage, was limited to one half. Later, the same authors proposed an improved method in 1989 which increased the voltage transfer ratio to 0.866 by utilizing the third harmonic injection technique in the input and output

voltage waveforms. The third harmonic injection has been extended with the input power factor control which leading to a very powerful modulation strategy (optimum AV method) [4].

A well-known technique for controlling the fictitious rectifier and inverter was proposed in [8], [24]. The authors split the matrix converter into a two-stage system, namely a three-phase rectifier and a three-phase inverter connected together through a fictitious DC-link as shown in Figure 1.3. First, the input voltage rectified to create a fictitious DC-link. Then, inverted at the required output frequency. This technique was defined as indirect matrix converter (IMC).



Figure 1.3 Schematic of indirect matrix converter.

Space vector modulation (SVM) approach for matrix converters is another technique which was first proposed by Huber et al in 1989 [25], [26] to control only the output voltages. The space vector modulation technique was adapted to the matrix converter by employing a basic concept of indirect method. This technique was successively developed in order to achieve the

full control of the input power factor, to fully utilize the input voltages and to improve the modulation performance. Furthermore, it allows the maximum gain ratio of 0.866.

A general solution of the modulation problem for matrix converters based on the concept of *Duty-Cycle Space Vector* was presented in [**19**]. This method simplifies the study of the modulation strategy. In addition, it has been demonstrated that three degrees of freedom are available in defining the modulation strategy, allowing for the control of the instantaneous values of the output voltages and input power factor to be obtained. Thus, this technique has been considered as the best solution for achieving the highest voltage gain ratio.

In principle, the main task of PWM schemes is to create trains of switched pulses. The major problem with these trains of switched pulses is that they have unwanted harmonic components which should be reduced. Hence, for any PWM scheme, the main objective which can be identified is to determine the most effective way of arranging the switching processes to reduce unwanted harmonic distortion, switching losses, or any other performance criterion.

The analysis of PWM schemes has been the subject of intensive research for several decades. Some of this work has investigated the harmonic components which are produced from the modulation process. It is quite complex to accurately determine the spectrum of PWM waveforms that are typically present in any converter and it is often resorted to a fast Fourier transform (FFT) analysis. It is known that this approach does not yield accurate results if the ratio between the switching frequency to fundamental frequency is not integer. In contrast, a theoretical analysis which identifies the exact harmonic components when various PWM strategies are compared against each other was presented in [**33**]. This analysis is based on

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double Fourier integral analysis in two variables (high frequency carrier signal with low frequency sinusoidal signal) has been well developed in [16], [29]- [31].

The challenges associated with the spectral analysis of matrix converter waveforms are twofold. On one hand, the modulation signal contains input and output frequencies. Unlike the intentional third harmonic injection in the modulation functions of voltage source inverters (VSI), the input and output frequencies in the matrix converter are typically independent from each other and will not form an integer ratio. On the other hand, it is very common that the switching frequency or the carrier frequency is not rational multiple of either the input frequency or output frequency [**33**]. These aforementioned challenges make it a very difficult task to obtain an accurate spectrum of the matrix converter waveforms through commonly resorted numerical methods such as fast Fourier transform (FFT).

Thus, the research work here proposes a new analytical method based on three dimensional Fourier integral which exactly identifies accurate spectra of the switching functions and synthesized terminal quantities of the matrix converter. The proposed method is an extension of the double Fourier integral analysis of the output voltage waveforms of the voltage source inverter [**33**]. The scope of the thesis is organized as follows:

- Chapter 2, presents the concept of the existence function and summarizes the most important modulation strategies for matrix converter.
- In chapter 3, a general review of Fourier series analysis is given first. Then, the formulation of double Fourier integral analysis is presented. Finally, the triple Fourier integral analysis has been explained follow by its application to the pulse width modulation.

- In chapter 4, detailed spectral analysis of the matrix converter based on three dimensional Fourier integral to characterize the accurate spectra of the switching functions and synthesized output voltages and input currents of matrix converter which was proposed by Alesina and Venturini in 1981 [**3**] and verified with FFT.
- In chapter 5, the work presented in this thesis is summarized. Topics for further investigation are suggested as future work.

CHAPTER 2

Matrix Converter Modulation Strategies

In this chapter, a brief introduction to the existence function which used to provide a mathematical form for expression switching function is given first. Then, the main modulation strategies which used in matrix converters are considered (the Alesina and Venturini modulation methods are reviewed, SVM algorithm for matrix converter is presented and the concept of duty cycle space vector is considered).

2.1 Existence Function

The existence functions which was proposed by Wood [32], provides a mathematical expression for describing the switching patterns. The existence function for a single switch when it is closed, the existence function has a value 1 and when it is opened, the existence function has a value 0.

For the matrix converter which is shown in Figure 1.1, the existence function $S_{uv}(t)$ for each of the switches can be expressed as follows:

$$S_{uv}(t) = \begin{cases} 1 & \text{when } S_{uv}(t) \text{ is closed} \\ 0 & \text{when } S_{uv}(t) \text{ is opened} \end{cases}$$
(2.1)

where $u \in \{1,2,3\}, v \in \{1,2,3\}$.

In order to prevent the short circuit in the capacitive input as well as the open circuit in the inductive output, only one switch on each output phase must be closed. Therefore,

$$\sum_{\nu=1}^{3} S_{1\nu} = \sum_{\nu=1}^{3} S_{2\nu} = \sum_{\nu=1}^{3} S_{3\nu} = 1$$
(2.2)

In Figure 1.1, the input voltages $V_{iu}(t)$ and output currents $I_{ov}(t)$ are assumed three-phase balanced with peak value V_i and I_o respectively as follows:

$$V_{iu}(t) = \begin{bmatrix} V_{i1}(t) \\ V_{i2}(t) \\ V_{i3(t)} \end{bmatrix} = V_i \begin{bmatrix} \cos(\omega_i t) \\ \cos(\omega_i t - 2\pi/3) \\ \cos(\omega_i t + 2\pi/3) \end{bmatrix}$$
(2.3)
$$I_{ov}(t) = \begin{bmatrix} I_{o1}(t) \\ I_{o2}(t) \\ I_{o3(t)} \end{bmatrix} = I_o \begin{bmatrix} \cos(\omega_o t) \\ \cos(\omega_o t - 2\pi/3) \\ \cos(\omega_o t + 2\pi/3) \end{bmatrix}$$
(2.4)

where ω_i is the angular frequency of the input voltages and ω_o is the angular frequency of the output currents.

The instantaneous voltage and current relationships are related to the state of the nine switches and can be written in a matrix form as:

$$\begin{bmatrix} V_{o1}(t) \\ V_{o2}(t) \\ V_{o3(t)} \end{bmatrix} = \begin{bmatrix} S_{11}(t) & S_{12}(t) & S_{13}(t) \\ S_{21}(t) & S_{22}(t) & S_{23}(t) \\ S_{31}(t) & S_{32}(t) & S_{33}(t) \end{bmatrix} \begin{bmatrix} V_{i1}(t) \\ V_{i2}(t) \\ V_{i3(t)} \end{bmatrix}$$

$$\begin{bmatrix} I_{i1}(t) \\ I_{i2}(t) \\ I_{i3(t)} \end{bmatrix} = \begin{bmatrix} S_{11}(t) & S_{21}(t) & S_{31}(t) \\ S_{12}(t) & S_{22}(t) & S_{32}(t) \\ S_{13}(t) & S_{23}(t) & S_{33}(t) \end{bmatrix} \begin{bmatrix} I_{o1}(t) \\ I_{o2}(t) \\ I_{o3(t)} \end{bmatrix}$$
(2.5)
$$(2.6)$$

Equations (2.5) and (2.6) are the basic of all modulation methods which consist in selecting appropriate combinations of switches to generate the desired output voltages.

2.2 Alesina and Venturini Method

Alesina and Venturini first proposed the high frequency synthesis technique for direct matrix converter [3], [4]. The high frequency technique allows the utilization of low frequency to calculate the existence functions for each switch in the matrix converter. Therefore, the aim of using Alesina and Venturini methods is to find the modulation matrix which satisfies the following equations:

$$\begin{bmatrix} V_{o1}(t) \\ V_{o2}(t) \\ V_{o3(t)} \end{bmatrix} = \begin{bmatrix} m_{11}(t) & m_{12}(t) & m_{13}(t) \\ m_{21}(t) & m_{22}(t) & m_{23}(t) \\ m_{31}(t) & m_{32}(t) & m_{33}(t) \end{bmatrix} \begin{bmatrix} V_{i1}(t) \\ V_{i2}(t) \\ V_{i3(t)} \end{bmatrix}$$
(2.7)

$$\begin{bmatrix} I_{i1}(t) \\ I_{i2}(t) \\ I_{i3(t)} \end{bmatrix} = \begin{bmatrix} m_{11}(t) & m_{21}(t) & m_{31}(t) \\ m_{12}(t) & m_{22}(t) & m_{32}(t) \\ m_{13}(t) & m_{23}(t) & m_{33}(t) \end{bmatrix} \begin{bmatrix} I_{o1}(t) \\ I_{o2}(t) \\ I_{o3(t)} \end{bmatrix}$$
(2.8)

where $m_{uv}(t)$ is the duty cycle of the nine switches $S_{uv}(t)$ with $0 \le m_{uv}(t) \le 1$.

In order to prevent a short circuit from the input side, the duty cycles matrix $m_{uv}(t)$ must satisfy the three following conditions:

$$m_{u1}(t) + m_{u2}(t) + m_{u3}(t) = 1$$
(2.9)

Alesina and Venturini proposed a control method for a nine-switch DMC using a mathematical approach to generate the desired output waveform. In [**3**], the first modulation strategy allows the control of the output voltages and input power factor. This strategy can be summarized in the following equation

$$m_{uv}(t) = \frac{1}{3} \left[1 + 2q \cos\left(\omega_0 t - (u-1)\frac{2\pi}{3}\right) \cos\left(\omega_i t - (v-1)\frac{2\pi}{3}\right) \right]$$
(2.10)

where q is the voltage transfer ratio. The solution given in (2.10) is valid for unity input power factor and the maximum output to input voltage ratio is limited to one half. This value represents the major drawback of this modulation strategy.

In later work by the same authors [4], it was shown that the maximum voltage transfer ratio increased up to 0.866 by the third harmonic injection techniques. This value represents an intrinsic limitation of three-phase to three-phase matrix converters balanced supply voltages. The

modulation strategy in this case is valid for unity input power factor and can be described in the following relationship:

$$m_{uv}(t) = \frac{1}{3} \left[1 + 2q \cos\left(\omega_i t - (v-1)\frac{2\pi}{3}\right) \cos\left[\left(\omega_o t - (u-1)\frac{2\pi}{3}\right) - \frac{1}{6}\cos(3\omega_o t) + \frac{1}{2\sqrt{3}}\cos(3\omega_i t)\right] \right] - \frac{1}{3} \left[\frac{2}{3\sqrt{3}}q \left[\cos\left(4\omega_i t - (v-1)\frac{2\pi}{3}\right) - \cos\left(2\omega_i t + (v-1)\frac{2\pi}{3}\right) \right] \right]$$
(2.11)

2.3 Space Vector Modulation Method

The first matrix converter using SVM technique was proposed by Huber et al in 1989 to control three-phase to three-phase matrix convert [25], [26]. These methods divided the matrix converter into two fictitious converters (rectifier and inverter) as shown in Figure 1.3. The rectifier section is controlled as a full-bridge diode rectifier and the space vector modulation is applied only to inverter section to control the output voltages. The application of space vector modulation is presented in the literature [27], [28]. The space vector modulation shows a better performance than the carried-based PWM control method in terms of output voltage harmonic distortion.

The instantaneous space vector \overline{x} representation of output voltage and input current can be expressed as follows:

$$\bar{x} = \frac{2}{3} \left[x_1 + x_2 \, e^{j2\pi/3} + x_3 \, e^{j4\pi/3} \right] \tag{2.12}$$

where x_1 , x_2 and x_3 are three time-varying variables which represent the output phase voltages and input line currents. The zero space vector x_o can be determined by the following relationship:

$$x_o = \frac{1}{3}(x_1 + x_2 + x_3) \tag{2.13}$$

The inverse transformation of (2.12) and (2.13) can be written as follows:

$$x_u = \frac{x_o}{3} + \bar{x}e^{j(u-1)2\pi/3}$$
(2.14)

where $u \in \{1, 2, 3\}$.

In three-phase matrix converters, there are twenty seven possible switching configurations and only twenty one can be applied in the space vector modulation algorithm as shown in Table 2.1 which can be categorized into three groups:

Group I. consists of eighteen switching combinations which determine the output voltage and input current vectors that have fixed directions. The magnitude of these vectors depend on the instantaneous values of input line-to-line voltages and output currents. These switching combinations are called active configurations.

Group II. consists of three switching combinations which determine zero output voltage and input current vectors. The three output phases are connected to the same input phase. These switching combinations are named zero configurations.

Croup III. Consists of six switching combinations which have each output phase is connected to a different input phase . The output voltage and input current vectors have variable directions, but cannot be used to synthesis the reference vectors.

Switching	Switches On			V _O	α_o	i _i	β_i
	Sec	See	See	$2v_{res}/3$	0	2 ; $\sqrt{2}$	$-\pi/6$
T 1	511	J ₂₂	532	$2v_{12i}/3$	0	$2l_{o1}/\sqrt{3}$	$-\pi/0$
-1	<i>S</i> ₁₂	<i>S</i> ₂₁	<i>S</i> ₃₁	$-2v_{12i}/3$	0	$-2i_{01}/\sqrt{3}$	$-\pi/6$
+2	<i>S</i> ₁₂	<i>S</i> ₂₃	S ₃₃	$2v_{23i}/3$	0	$2i_{o1}/\sqrt{3}$	$\pi/2$
-2	<i>S</i> ₁₃	<i>S</i> ₂₂	<i>S</i> ₃₂	$-2v_{23i}/3$	0	$-2i_{01}/\sqrt{3}$	$\pi/2$
+ 3	<i>S</i> ₁₃	<i>S</i> ₂₁	<i>S</i> ₃₁	$2v_{31i}/3$	0	$2i_{o1}/\sqrt{3}$	$7\pi/6$
- 3	<i>S</i> ₁₁	<i>S</i> ₂₃	S ₃₃	$-2v_{31i}/3$	0	$-2i_{01}/\sqrt{3}$	$7\pi/6$
+ 4	<i>S</i> ₁₂	<i>S</i> ₂₁	<i>S</i> ₃₂	$2v_{12i}/3$	$2\pi/3$	$2i_{o2}/\sqrt{3}$	$-\pi/6$
- 4	<i>S</i> ₁₁	<i>S</i> ₂₂	<i>S</i> ₃₁	$-2v_{12i}/3$	$2\pi/3$	$-2i_{02}/\sqrt{3}$	$-\pi/6$
+ 5	<i>S</i> ₁₃	<i>S</i> ₂₂	<i>S</i> ₃₃	$2v_{23i}/3$	$2\pi/3$	$2i_{o2}/\sqrt{3}$	$\pi/2$
- 5	<i>S</i> ₁₂	<i>S</i> ₂₃	<i>S</i> ₃₂	$-2v_{23i}/3$	$2\pi/3$	$-2i_{02}/\sqrt{3}$	$\pi/2$
+ 6	<i>S</i> ₁₁	<i>S</i> ₂₃	<i>S</i> ₃₁	$2v_{31i}/3$	$2\pi/3$	$2i_{o2}/\sqrt{3}$	$7\pi/6$
- 6	<i>S</i> ₁₃	<i>S</i> ₂₁	S ₃₃	$-2v_{31i}/3$	$2\pi/3$	$-2i_{02}/\sqrt{3}$	$7\pi/6$
+ 7	<i>S</i> ₁₂	<i>S</i> ₂₂	<i>S</i> ₃₁	$2v_{12i}/3$	$4\pi/3$	$2i_{o3}/\sqrt{3}$	$-\pi/6$
-7	<i>S</i> ₁₁	<i>S</i> ₂₁	<i>S</i> ₃₂	$-2v_{12i}/3$	$4\pi/3$	$-2i_{03}/\sqrt{3}$	$-\pi/6$
+ 8	<i>S</i> ₁₃	<i>S</i> ₂₃	<i>S</i> ₃₂	$2v_{23i}/3$	$4\pi/3$	$2i_{o3}/\sqrt{3}$	$\pi/2$
- 8	<i>S</i> ₁₂	<i>S</i> ₂₂	S ₃₃	$-2v_{23i}/3$	$4\pi/3$	$-2i_{03}/\sqrt{3}$	$\pi/2$
+9	<i>S</i> ₁₁	<i>S</i> ₂₁	S ₃₃	$2v_{31i}/3$	$4\pi/3$	$2i_{o3}/\sqrt{3}$	$7\pi/6$
-9	<i>S</i> ₁₃	<i>S</i> ₂₃	<i>S</i> ₃₁	$-2v_{31i}/3$	$4\pi/3$	$-2i_{o3}/\sqrt{3}$	$7\pi/6$
01	<i>S</i> ₁₁	<i>S</i> ₂₁	<i>S</i> ₃₁	0	-	0	-
02	<i>S</i> ₁₂	<i>S</i> ₂₂	<i>S</i> ₂₃	0	-	0	-
03	<i>S</i> ₁₃	<i>S</i> ₂₃	S ₃₃	0	-	0	_

 Table 2.1 Switching configuration for three-phase matrix converters.

2.4 Duty-Cycle Space Vector Modulation Method

A general solution to the modulation problem for matrix converters based on the concept of Duty Cycle Space Vector, was presented in [19]. The duty-cycle space vector can be developed by utilizing space vector modulation. This method simplifies the study of the modulation strategy. In addition, it has been demonstrated that three degrees of freedom are available in defining the modulation strategy, allowing for the control of the instantaneous values of the output voltages and input power factor. Thus, this technique has been considered as the best solution for achieving the highest voltage gain ratio.

The nine duty cycle m_{uv} can be represented by the duty cycle space vector $\overline{m_u}$ by the following transformation:

$$\overline{m_u} = \frac{2}{3} \left[m_{u1} + m_{u2} \, e^{j2\pi/3} + m_{u3} \, e^{j4\pi/3} \right] \tag{2.15}$$

Taking into account conditions in (2.9), the inverse transformation of the equation (2.15) is:

$$m_{uv} = \frac{1}{3} + \overline{m_u} \, e^{j(v-1)2\pi/3} \tag{2.16}$$

By considering the constraints of the modulation function $0 \le m_{uv} \le 1$, the duty cycle space vector $\overline{m_u}$ is a geometrically illustrated in *d-q* plane and is always inside the equilateral triangle which represents in Figure 2.1.



Figure 2.1 Geometrical representation for the validity for $\overline{m_u}$.

The position of the duty cycle space vector $\overline{m_u}$ inside the triangle is determined by the switching commutations of S_{uv} in a cycle period.

It is noted that the output voltage and input current space vector can be written by using the equation (2.12) as follows:

$$\overline{v_o} = \frac{2}{3} \left[v_{o1} + v_{o2} \, e^{j2\pi/3} + v_{o3} \, e^{j4\pi/3} \right] \tag{2.17}$$

$$\overline{i_i} = \frac{2}{3} \Big[i_{i1} + i_{i2} e^{j2\pi/3} + i_{i3} e^{j4\pi/3} \Big]$$
(2.18)

By substituting the equations (2.7), (2.8) and (2.16) into (2.17) and (2.18), the output voltage and input current space vector can be rewritten as follows:

$$\overline{v_o} = \frac{\overline{v_i}}{2} \left[\overline{m}_1^* + \overline{m}_2^* e^{j2\pi/3} + \overline{m}_3^* e^{j4\pi/3} \right] + \frac{\overline{v_i}}{2} \left[\overline{m_1} + \overline{m_2} e^{j2\pi/3} + \overline{m_3} e^{j4\pi/3} \right]$$
(2.19)

$$\overline{i_i} = \frac{\overline{i_i}}{2} \left[\overline{m_1} + \overline{m_2} e^{j4\pi/3} + \overline{m_3} e^{j2\pi/3} \right] + \frac{\overline{i_i^*}}{2} \left[\overline{m_1^*} + \overline{m_2^*} e^{j4\pi/3} + \overline{m_3^*} e^{j2\pi/3} \right]$$
(2.20)

Equations (2.19) and (2.20) suggest three new variables $\overline{m_d}$, $\overline{m_i}$ and $\overline{m_0}$ which can be defined as:

$$\overline{m_d} = \frac{1}{3} \left[\overline{m_1} + \overline{m_2} \, e^{j2\pi/3} + \overline{m_3} \, e^{j4\pi/3} \right] \tag{2.21}$$

$$\overline{m_i} = \frac{1}{3} \left[\overline{m_1} + \overline{m_2} \, e^{j4\pi/3} + \overline{m_3} \, e^{j2\pi/3} \right] \tag{2.22}$$

$$\overline{m_0} = \frac{1}{3} \left[\overline{m_1} + \overline{m_2} + \overline{m_3} \right]$$
(2.23)

where $\overline{m_d}$, $\overline{m_i}$ and $\overline{m_0}$ are considered as direct, inverse and zero components of the duty cycle space vectors respectively.

The inverse transformation of (2.21), (2.22) and (2.23) can be expressed by:

$$\overline{m_u} = \overline{m_d} \, e^{j(1-u)2\pi/3} + \overline{m_i} \, e^{j(u-1)2\pi/3} + \overline{m_0} \tag{2.24}$$

By substituting (2.24) into (2.19) and (2.20), the output voltage and input current space vectors can be defined as:

$$\overline{v_o} = \frac{3}{2} \left[\overline{v_i} \times \overline{m}_i^* + \overline{v}_i^* \times \overline{m_d} \right]$$
(2.25)

$$\overline{i_i} = \frac{3}{2} \left[\overline{i_o} \times \overline{m_i} + \overline{i_o}^* \times \overline{m_d} \right]$$
(2.26)

Equations (2.25) and (2.26) represent the output voltage and input current space vector of threephase matrix converters in a compact form.

2.5 Summary

This chapter has reviewed the concept of the existence function and described the modulation strategies of three-phase matrix converters.

Alesina and Venturini approach presented a general waveform synthesis technique in [3]. The major drawback of this approach is the maximum output to input voltages ratio was only limited to one half. Later work by the same authors increased the ratio to 0.866 which is the maximum theoretical limit using high frequency synthesis [4]. In addition, the space vector modulation approach is based on the instantaneous output voltage and input current space vectors. By using this approach, the switching pattern will be defined by the switching configuration sequence. This approach was developed and introduced a new concept of duty cycle space vector, which simplifies the study of modulation strategies of three-phase matrix converters.

CHAPTER 3

Analytical Approach to Characterize Spectral Performance of Matrix Converters

PWM is one of the cornerstone of switched converter systems. In addition, it has been become the subject of intensive research for several decades. A vital part of this work has investigated the harmonics that are inevitably produced as part of the modulation process.

In general, the harmonic frequency components of the PWM waveforms is quite complex to characterize and it is often obtained by using FFT analysis to identify their magnitudes. However, it is known that this approach does not yield accurate results if the ratio between the fundamental frequency and carrier frequency is not integer. In contrast, to characterize the exact harmonic components of the PWM waveforms without relying on the numerical method, an analytical approach has been considered in [**33**]. This approach is termed double Fourier integral analysis, which exactly identifies the correct harmonics magnitude when various PWM strategies are compared against each other.

A new analytical approach for identifying the harmonic components of PWM waveforms for AC/AC converters has been proposed. This analytical approach is termed triple Fourier integral analysis, which is a natural extension of the double Fourier integral analysis that has been developed for VSI waveforms [**33**]. However, in this chapter, the Fourier series analysis is

reviewed first. Then, the formulation of the double Fourier analysis is presented. Finally, the triple Fourier analysis is proposed follow by its application to the pulse width modulation.

3.1 Fourier Series Analysis

The principle of Fourier series is that any periodic function f(t) can be expressed as an infinite summation of sinusoidal harmonic components as follows:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right]$$
(3.1)

where a_0 , a_n and b_n are called the Fourier coefficients and ω_o is the fundamental frequency of the periodic function.

The Fourier coefficients can be found as:

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos(n\omega_0 t) \, d\omega_0 t \quad \text{for} \quad n=0,1,2,3...$$
(3.2)

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin(n\omega_0 t) \, d\omega_0 t \quad \text{for} \quad n=1,2,3...$$
(3.3)

The cosine and sine functions in (3.2) and (3.3) can be expressed in Euler formula as follows:

$$\cos(n\omega_o t) = \frac{e^{jn\omega_o t} + e^{-jn\omega_o t}}{2}$$
(3.4)

$$\sin(n\omega_o t) = \frac{e^{jn\omega_o t} - e^{-jn\omega_o t}}{2j}$$
(3.5)

By substituting (3.4) and (3.5) into (3.2) and (3.3) respectively, the Fourier series can be expressed in a complex form as follows:

$$f(t) = \sum_{n = -\infty}^{\infty} c_n \times e^{jn\omega_0 t}$$
(3.6)

The complex coefficients are then given by:

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) \times e^{-jn\omega_0 t}$$
(3.7)

The complex coefficient c_n is related to the real-numbered a_n and b_n as the following:

$$c_n = \frac{a_n - jb_n}{2} \tag{3.8}$$

OR

$$a_n = a_n + a_{(-n)}$$

$$b_n = j(b_n - b_{(-n)}) \tag{3.9}$$

3.2 Double Fourier Integral Analysis

The challenge associated with the non-periodicity of PWM waveform was solved by the mathematical approach called double Fourier integral analysis. The double Fourier integral analysis is utilized as an analytical approach to identify the harmonic components of carrier-based PWM. This analytical approach was originally proposed for communication system by Bennet [**34**] and Black [**35**], later by Bowes and Bird [**36**] for power converters.

The analysis process assumes the existence of two time variables as follows:

$$x(t) = \omega_c t + \theta_c$$

$$y(t) = \omega_o t + \theta_o$$
 (3.10)

where x(t) and y(t) being defined as the time variation of the high frequency carrier waveform and low-frequency reference waveforms respectively. ω_c is the angular frequency of the carrier signal while ω_o is the angular frequency of the low-frequency modulation signal. θ_c and θ_o are the phase angles of the carrier and low-frequency signals respectively and are assumed to be zero.

Let us assume that the triangular signal c(x) is considered for this analysis. Then, the mathematical expression for the carrier signal c(x) is:

$$c(x) = \frac{1}{\pi}\arccos(\cos(x))$$
(3.11)

Furthermore, the modulation function m(y) is defined by

$$m(y) = \frac{1 + M\cos(y)}{2}$$
(3.12)

where *M* is the modulation index with range 0 < M < 1. Then, the switching function f(x, y) can be determined by the comparison of modulation function m(y) against carrier signal c(x) as follows:

$$f(x, y) = \Phi[m(y) - c(x)]$$
(3.13)

where $\Phi(\cdot)$ is defined by:

$$\Phi(u) = \begin{cases} 1 & \text{if } u > 0 \\ 0 & \text{if } u \le 0 \end{cases}$$
(3.14)

It can be seen from (3.14) that the switching function f(x, y) takes values of 1 or 0 for any combination of x and y.

Let a double variable function f(x, y) be periodic in both x and y directions. It is further assumed that x and y are phase-angle variables and the period in both directions is 2π .

$$f(x, y) = f(x + 2\pi, y) = f(x, y + 2\pi)$$
(3.15)

The Fourier expansion can be conducted in two steps. First, the function f(x, y) is expanded in *x* direction while *y* being kept a constant. In complex form, the Fourier series with coefficients being functions of *y* will read

$$f(x, y) = \sum_{m = -\infty}^{\infty} \left[F_m(y) \times e^{jmx} \right]$$
(3.16)

where the complex coefficients are determined by

$$F_m(y) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y) \times e^{-jmx} dx$$
(3.17)

It is obvious that $F_m(y)$ is periodic since f(x, y) is periodic in direction.

$$F_m(y+2\pi) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x, y+2\pi) \times e^{-jmx} dx$$

$$F_m(y+2\pi) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x,y) \times e^{-jmx} dx$$

$$F_m(y+2\pi) = F_m(y)$$
 (3.18)

Hence, the complex coefficient $F_m(y)$ can be expanded into

$$F_m(y) = \sum_{n=-\infty}^{\infty} \left[F_{mn} \times e^{j(mx+ny)} \right]$$
(3.19)

where

$$F_{mn} = \frac{1}{2\pi} \int_{-\pi}^{\pi} F_m(y) \times e^{-jny} dy$$
(3.20)

Substituting the equation (3.19) into (3.16), the double function variable f(x, y) can be rewritten as following:

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$$f(x,y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left[F_{mn} \times e^{j(mx+ny)} \right]$$
(3.21)

where

$$F_{mn} = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y) \times e^{-j(mx+ny)} dxdy$$
(3.22)

Consequently, the double Fourier can be expressed in real-numbered coefficients as following:

$$f(x, y) = \frac{A_{00}}{2} + \sum_{n=1}^{\infty} \left[A_{0n} \cos(ny) + B_{0n} \sin(ny) \right]$$

$$+\sum_{m=1}^{\infty} \left[A_{m0} \cos(mx) + B_{m0} \sin(mx) \right]$$

$$+\sum_{\substack{m=1\\m\neq 0}}^{\infty}\sum_{\substack{n=-\infty\\n\neq 0}}^{\infty} \left[A_{mn}\cos(mx+ny) + B_{mn}\sin(mx+ny)\right]$$
(3.22)

where the Fourier coefficients A_{mn} and B_{mn} are determined by the following double integral

$$A_{mn} = \frac{1}{2\pi^2} \int_{-\pi - \pi}^{\pi} \int_{-\pi}^{\pi} f(x, y) \cos(mx + ny) dx dy$$
(3.23)

$$B_{mn} = \frac{1}{2\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y) \sin(mx + ny) dx dy$$
(3.24)

The complex coefficient F_{mn} is related to the real-numbered A_{mn} and B_{mn} as the following

$$F_{mn} = \frac{A_{mn} - jB_{mn}}{2} \tag{3.25}$$

OR

$$A_{mn} = F_{mn} + F_{(-m)(-n)}$$

$$B_{mn} = j(F_{mn} - F_{(-m)(-n)})$$
(3.26)

Replacing x by $\omega_c t + \theta_c$ and y by $\omega_o t + \theta_o$, the equation (3.22) can be expressed in the timevarying form as:

$$f(t) = \frac{A_{00}}{2} + \sum_{n=1}^{\infty} \left[A_{0n} \cos(n(\omega_o t + \theta_o)) + B_{0n} \sin(n(\omega_o t + \theta_o)) \right]$$

$$+\sum_{m=1}^{\infty} \left[A_{m0} \cos(m(\omega_c t + \theta_c)) + B_{m0} \sin(m(\omega_c t + \theta_c)) \right]$$

$$+\sum_{\substack{m=1\\n\neq 0}}^{\infty}\sum_{\substack{n=-\infty\\n\neq 0}}^{\infty} \left[A_{mn}\cos(m(\omega_c t + \theta_c)) + n(\omega_o t + \theta_o)) + n(\omega_o t + \theta_o))\right]$$
(3.27)

where m and n are the carrier and baseband index variable respectively.

In (3.27), the first term represents the dc components of PWM waveform. The first summation term defines the output fundamental low-frequency waveform and it is called baseband harmonics. The second summation term corresponds to the carrier waveform harmonics and it is called carrier harmonics. The final double summation term defines the sideband harmonics around the carrier harmonic components.

3.3 Triple Fourier Integral Analysis

The triple Fourier integral analysis is a natural extension of the double Fourier integral analysis that can accurately characterize the spectrum of the PWM waveforms for the matrix converters. Without loss of generality, the sine-triangle naturally sampled modulation is considered for this analysis as well. The analysis process assumes the existence of three time variables:

$$x(t) = \omega_c t + \theta_c$$

$$y(t) = \omega_1 t + \theta_1$$

$$z(t) = \omega_2 t + \theta_2$$
(3.28)

where x(t), y(t) and z(t) being defined as the time variation of the high frequency carrier waveform and two low-frequency reference waveforms respectively. ω_c is the angular frequency of the carrier signal whereas ω_1 and ω_2 are two independent angular frequencies of the two low-frequency modulation signals. θ_c is the phase angle of the carrier frequency while θ_1 and θ_2 are the phase angles of the two low-frequency modulation signals and all the phase angles are assumed to be zero.

Let a triple variable function f(x, y, z) be periodic in x, y and z directions. It is further assumed that x, y and z are phase-angle variables and the period in all directions is 2π . Consequently,

$$f(x, y, z) = f(x + 2\pi, y, z) = f(x, y + 2\pi, z) = f(x, y, z + 2\pi)$$
(3.29)

With reference to the double Fourier expansion, the triple Fourier expansion is possible. Since the assumption of f(x, y, z) be periodic in x, y and z directions, the triple Fourier expansion can be written as the following:

$$f(x, y, z) = \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left[F_{kmn} \times e^{j(kx+my+nz)} \right]$$
(3.30)

where

$$F_{kmn} = \frac{1}{8\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y, z) \times e^{-j(kx + my + nz)} dx dy dz$$
(3.31)

Therefore, the triple Fourier can be expressed in real-numbered coefficients as following:

$$f(x, y, z) = \frac{A_{000}}{2} + \sum_{m=1}^{\infty} [A_{0m0} \cos(my) + B_{0m0} \sin(my)] + \sum_{n=1}^{\infty} [A_{00n} \cos(nz) + B_{00n} \sin(nz)] + \sum_{k=1}^{\infty} [A_{k00} \cos(kx) + B_{k00} \sin(kx)] + \sum_{k=1}^{\infty} \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ n \neq 0}}^{\infty} [A_{kmn} \cos(kx + my + nz) + B_{kmn} \sin(kx + my + nz)]$$
(3.32)

where the Fourier coefficients A_{kmn} and B_{kmn} are determined by the following triple integral

$$A_{kmn} = \frac{1}{8\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y, z) \cos(kx + my + nz) dx dy dz$$
(3.33)

$$B_{kmn} = \frac{1}{8\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y, z) \sin(kx + my + nz) dx dy dz$$
(3.34)

The real coefficients A_{kmn} and B_{kmn} are related to the complex coefficient F_{kmn} as the following:

$$A_{kmn} = F_{kmn} + F_{(-k)(-m)(-n)}$$

$$B_{kmn} = j(F_{kmn} - F_{(-k)(-m)(-n)})$$
(3.35)

$$F_{kmn} = \frac{A_{kmn} - jB_{kmn}}{2} \tag{3.36}$$
Replacing x by $\omega_c t + \theta_c$, y by $\omega_1 t + \theta_1$ and z by $\omega_2 t + \theta_2$ the equation (3.32) can be expressed in the time-varying form as:

$$f(t) = \frac{A_{000}}{2} + \sum_{m=1}^{\infty} [A_{0m0} \cos(m(\omega_1 t + \theta_1)) + B_{0m0} \sin(m(\omega_1 t + \theta_1))] + \sum_{n=1}^{\infty} [A_{00n} \cos(n(\omega_2 t + \theta_2)) + B_{00n} \sin(n(\omega_2 t + \theta_2))] + \sum_{k=1}^{\infty} [A_{k00} \cos(k(\omega_c t + \theta_c)) + B_{k00} \sin(k(\omega_c t + \theta_c))] + \sum_{k=1}^{\infty} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \left[A_{kmn} \cos(k(\omega_c t + \theta_c) + m(\omega_1 t + \theta_1) + n(\omega_2 t + \theta_2)) \\+ B_{kmn} \sin(k(\omega_c t + \theta_c) + m(\omega_1 t + \theta_1) + n(\omega_2 t + \theta_2))) \right]$$
(3.37)

where k is the carrier index variable while m and n are two baseband index variables.

The first term of the equation (3.37) represents the dc components of PWM waveform. The first and second summation terms define the two output fundamental low-frequency waveforms and they are called baseband harmonics. The third summation term corresponds to the carrier waveform harmonics and it is called carrier harmonics. The final triple summation term defines the sideband harmonics around the carrier harmonic components.

3.4 Application of Triple Fourier Integral Analysis to PWM

For the PWM process in matrix converters, there is one high-frequency signal called *carrier signal* and two low-frequency components are named *modulation signals*. The analysis process assumes the existence of three time variables:

$$x(t) = \omega_c t + \theta_c$$

$$y(t) = \omega_{m1} t + \theta_{m1}$$

$$z(t) = \omega_{m2} t + \theta_{m2}$$
(3.38)

where x(t), y(t) and z(t) being defined as the time variation of the high frequency carrier waveform and two low-frequency reference waveforms respectively. ω_c is the angular frequency of the carrier signal whereas ω_{m1} and ω_{m2} are two independent angular frequencies of the two low-frequency modulation signals. θ_c is the phase angle of the carrier frequency while θ_{m1} and θ_{m2} are the phase angles of the two low-frequency modulation signals and all the phase angles are assumed to be zero.

The comparison between the carrier signal c(x) and modulation function m(y, z) will give rise to the *switching function*. The switching function h(x, y, z) takes the value one when the modulation signal is greater than the carrier signal while it takes the value zero when the modulation signal is less than the carrier signal. The mathematical description of the modulation process can be written as follows:

$$h(x, y, z) = \Phi(m(y, z) - c(x))$$
(3.39)

$$\Phi(\cdot) = \begin{cases} 1 & \text{if } u > 0 \\ 0 & \text{if } u \le 0 \end{cases}$$
(3.40)

The triple Fourier series expansion of the nine switching functions can be given in the complex form by the following expression:

$$h(x, y, z) = \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left[H_{kmn} \times e^{j(kx+my+nz)} \right]$$
(3.41)

$$H_{kmn} = \frac{1}{8\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} h(x, y, z) \times e^{-j(kx + my + nz)} dx dy dz$$
(3.42)

If the modulation signal consists of two sinusoidal components of two independent frequencies and the carrier signal is triangular, then the three-dimensional integral in (3.42) will be conducted within the unit cube with each edge of the length of 2π as illustrated in Figure 3.1.



Figure 3.1 Illustration of the unit cube. For interpretation of the references to color in this and all other figures, the reader is refered to the electronic version of this thesis.

where x-axis represents the phase angle of the carrier signal whereas y-axis and z-axis represent the phase angles of the two components in the modulation function.

Within the unit cube, the switching function h(x, y, z) is non-zero only in the space between the two surfaces that are defined by (3.43).

$$m(y,z) - c(x) = 0 \tag{3.43}$$

In addition, the line which illustrated in Figure 3.1, is defined as follows:

$$y = \frac{\omega_{m1}}{\omega_c} x + \left[\theta_{m1} - \frac{\omega_{m1}}{\omega_c} \theta_c \right]$$
(3.44)

$$z = \frac{\omega_{m2}}{\omega_c} x + \left[\theta_{m2} - \frac{\omega_{m2}}{\omega_c} \theta_c \right]$$
(3.45)

The intersections of the line defined by (3.44), (3.45) and the surfaces defined by (3.43) determine the instants when the value of the switching function h(x, y, z) switches from 0 to 1 and vice versus.

Once the integration boundaries of the above equation are defined, the procedure of carrying out the integration will be straightforward process as defined in [**33**]. This will be clearly explained in the next chapter.

3.5 Summary

A new analytical approach has been presented in this chapter for identifying the harmonic components of PWM waveforms for AC/AC converters. However, the analytical method based on three-dimensional Fourier integral will be explained in detail for obtaining accurate spectra

of the switching functions and synthesized terminal quantities of the matrix converter in the next chapter.

CHAPTER 4

Spectral Analysis of Matrix Converter Waveforms

This chapter presents an analytical approach based on three-dimensional Fourier integral to characterize the accurate spectra of the switching functions and synthesized output voltages and input currents of matrix converter which was proposed by Alesina and Venturini 1981[**3**]. The principle of this analysis is carried out in two steps. The first step is that the spectra of the switching functions are derived based on three-dimensional Fourier integral. The second step is that the spectra of the output voltages and input currents are evaluated using a convolution operation in the frequency domain. The analytical spectra have been compared with the numerical spectra that are obtained from FFT. In addition, the total harmonic distortion (THD) and weighted total harmonic distortion (WTHD) of the synthesized output voltages and input currents matrix converter are considered.

4.1 Modulation Process

Even though the modulation process of the matrix converter is well understood, the purpose of the section is just to introduce some notations which will be utilized in the subsequent sections.

The modulation function proposed in **[3**], which is defined in (2.10), can be rewritten in the following general form:

$$m_{uv}(t) = \frac{1 + q \cos\left[\omega_{m1}t - (u + v + 1)\frac{2\pi}{3}\right] + q \cos\left[\omega_{m2}t - (u - v)\frac{2\pi}{3}\right]}{3}$$
(4.1)

where $0 \le q \le 0.5$, ω_{m1} and ω_{m2} are related to the input frequency ω_i and output frequency ω_o as follows:

$$\omega_{m1} = \omega_i + \omega_o \tag{4.2}$$

$$\omega_{m2} = \omega_i - \omega_o \tag{4.3}$$

However, in order to prevent a short circuit on the input side and ensure uninterrupted load current flow, the switching function $h_{uv}(x, y, z)$ which is determined by (3.39), must satisfy the following constraint condition:

$$\sum_{\nu=1}^{3} [h_{u\nu}(x, y, z)] = 1 \qquad \text{for } u \in \{1, 2, 3\}$$
(4.4)

With $0 \le h_{uv}(x, y, z) \le 1$ for $u \in \{1, 2, 3\}, v \in \{1, 2, 3\}$.

The switching functions are generated in the following expression:

$$h_{uv}(x, y, z) = \begin{cases} \Phi[m_{u1}(y, z) - c(x)] & \text{if } v = 1 \\ \Phi[m_{u1}(y, z) + m_{u2}(y, z) - c(x)] - h_{u1}(x, y, z) & \text{if } v = 2 \\ 1 - h_{u1}(x, y, z) - h_{u2}(x, y, z) & \text{if } v = 3 \end{cases}$$
(4.5)

The synthesized output voltages (phase-to-neutral) and input currents can be determined by the following expression:

$$V_{ou} = \sum_{\nu=1}^{3} \left[h_{u\nu}(x, y, z) \times V_{iu} \right] \qquad \text{for } u \in \{1, 2, 3\}$$
(4.6)

$$I_{iv} = \sum_{u=1}^{3} \left[h_{uv}(x, y, z) \times I_{ov} \right] \qquad \text{for} \quad v \in \{1, 2, 3\}$$
(4.7)

4.2 Analytical Solution to the Spectra of Switching Functions

The triple Fourier expansion of the switching functions given by (4.5) can be written in complex form as follows:

$$h_{uv}(x, y, z) = \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left[H_{kmn}(u, v) \times e^{j(kx+my+nz)} \right]$$
(4.8)

where

$$H_{kmn}(u,v) = \frac{1}{8\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} h_{uv}(x,y,z) \times e^{-j(kx+my+nz)} dx dy dz$$
(4.9)

Equation (4.9) can be proceed with consideration of (3.43), (4.1) and (4.5). Hence, the spectrum of the nine switching functions can be determined in three different cases:

1) When v = 1, by substituting (4.5) into (4.9), this will lead to

$$H_{kmn}(u,1) = \frac{1}{8\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \Phi[m_{u1}(y,z) - c(x)] \times e^{-j(kx+my+nz)} dxdydz$$

$$=\frac{1}{8\pi^{3}}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\int_{-a}^{a}e^{-j(kx+my+nz)}dxdydz$$
(4.10)

where $a = \pi \times m_{u1}(y, z)$.

If k = 0, equation (4.10) will become as:

$$H_{0mn}(u,1) = \frac{1}{8\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{a} e^{-j(my+nz)} dx dy dz$$

$$H_{0mn}(u,1) = \begin{cases} \frac{1}{3} & \text{for } m = n = 0 \\ \frac{q}{3} e^{-j(u-2)2\pi/3} & \text{for } m = 1, n = 0 \\ \frac{q}{3} e^{-j(u-1)2\pi/3} & \text{for } m = 0, n = 1 \\ 0 & \text{for } m, n \neq 0 \end{cases}$$
(4.11)

If $k \neq 0$, the equation (4.10) will become as:

$$H_{kmn}(u,1) = \frac{1}{8\pi^3} \int_{-\pi - \pi - a}^{\pi} \int_{-\pi - \pi - a}^{a} e^{-j(kx + my + nz)} dx dy dz$$
$$= \frac{2\sin\left[\left(\frac{k}{3} + \frac{m + n}{2}\right)\pi\right]}{k\pi} \times J_m\left(\frac{k\pi \cdot q}{3}\right) \times J_n\left(\frac{k\pi \cdot q}{3}\right) \times e^{-j[m(u+2) + n(u-1)]2\pi/3}$$
(4.12)

where $J_m(\cdot)$ and $J_n(\cdot)$ are the *m*-th and *n*-th order Bessel functions of the first kind respectively.

- 2) When v = 2, by substituting (4.5) into (4.9), this will lead to two steps:
- The first step is;

$$H'_{kmn}(u,2) = \frac{1}{8\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \Phi[m_{u1}(y,z) + m_{u2}(y,z) - c(x)] \times e^{-j(kx+my+nz)} dxdydz$$

$$=\frac{1}{8\pi^{3}}\int_{-\pi}^{\pi}\int_{-\pi-b}^{-\pi}e^{-j(kx+my+nz)}dxdydz$$
(4.13)

where $b = \pi(m_{u1}(y, z) + m_{u2}(y, z))$.

If k = 0, the equation (4.13) will become as:

$$H_{0mn}'(u,2) = \frac{1}{8\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{b} e^{-j(my+nz)} dx dy dz$$

$$H_{0mn}'(u,2) = \begin{cases} \frac{2}{3} & \text{for } m = n = 0 \\ \frac{-q}{3} e^{-j(u+1)2\pi/3} & \text{for } m = 1, n = 0 \\ \frac{-q}{3} e^{-j(u)2\pi/3} & \text{for } m = 0, n = 1 \\ 0 & \text{for } m, n \neq 0 \end{cases}$$
(4.14)

If $k \neq 0$, the equation (4.13) will become as:

$$H'_{kmn}(u,2) = \frac{1}{8\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{b} e^{-j(kx+my+nz)} dx dy dz$$
$$= \frac{2\sin\left[\left(\frac{2k}{3} - \left(\frac{m+n}{2}\right)\right)\pi\right]}{k\pi} \times J_m\left(\frac{k\pi \cdot q}{3}\right) \times J_n\left(\frac{k\pi \cdot q}{3}\right) \times e^{-j[m(u+1)+n(u)]2\pi/3}$$
(4.15)

• The second step is;

$$H_{kmn}(u,2) = H'_{kmn}(u,2) - H_{kmn}(u,1)$$
(4.16)

The equation (4.16) is valid when $k \neq 0$. If k = 0, the equation (4.16) will become:

$$H_{0mn}(u,2) = \begin{cases} \frac{1}{3} & \text{for } m = n = 0 \\ \frac{q}{3}e^{-j(u)2\pi/3} & \text{for } m = 1, n = 0 \\ \frac{q}{3}e^{-j(u-1)2\pi/3} & \text{for } m = 0, n = 1 \\ 0 & \text{for } m, n \neq 0 \end{cases}$$
(4.17)

3) When v = 3, by substituting (4.5) into (4.9), this will lead to

$$H_{kmn}(u,3) = \frac{1}{8\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-c}^{c} e^{-j(kx+my+nz)} dx dy dz$$
(4.18)

where $c = \pi \times m_{u3}(y, z)$.

If k = 0, equation (4.18) will become:

$$H_{0mn}(u,3) = \frac{1}{8\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{c} e^{-j(my+nz)} dx dy dz$$

$$H_{0mn}(u,3) = \begin{cases} \frac{1}{3} & \text{for } m = n = 0 \\ \frac{q}{3} e^{-j(u+1)2\pi/3} & \text{for } m = 1, n = 0 \\ \frac{q}{3} e^{-j(u)2\pi/3} & \text{for } m = 0, n = 1 \\ 0 & \text{for } m, n \neq 0 \end{cases}$$
(4.19)

If $k \neq 0$, equation (4.18) will become:

$$H_{kmn}(u,3) = \frac{1}{8\pi^3} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{c} e^{-j(kx+my+nz)} dx dy dz$$
$$= \frac{2\sin\left[\left(\frac{k}{3} + \frac{m+n}{2}\right)\pi\right]}{k\pi} \times J_m\left(\frac{k\pi \cdot q}{3}\right) \times J_n\left(\frac{k\pi \cdot q}{3}\right) \times e^{-j[m(u+1)+n(u)]2\pi/3}$$
(4.20)

Equations (4.11), (4.12), (4.16), (4.17), (4.19) and (4.20) can be rearranged to formulate the general form of the switching functions based on three-dimensional Fourier integral in (4.21), (4.22) and (4.23).

If k = 0,

$$H_{kmn}(u,v) = \begin{cases} \frac{1}{3} & \text{for } m = n = 0 \\ \frac{q}{3}e^{-j(u+v+1)2\pi/3} & \text{for } m = 1, n = 0 \\ \frac{q}{3}e^{-j(u-v)2\pi/3} & \text{for } m = 0, n = 1 \\ 0 & \text{for } m, n \neq 0 \end{cases}$$
(4.21)

If
$$k \neq 0$$
,

When v = 1,3

$$H_{kmn}(u,v) = \frac{2\sin\left[\left(\frac{(3v-1)k}{6} + \frac{m+n}{2}\right)\pi\right]}{k\pi} J_m\left(\frac{k\pi \cdot q}{3}\right) J_n\left(\frac{k\pi \cdot q}{3}\right) e^{-j[m(u+v+1)+n(u-v)]2\pi/3}$$
(4.22)

When v = 2

$$H_{kmn}(u,v) = H'_{kmn}(u,2) - H_{kmn}(u,1)$$
(4.23)

Hence, the Fourier harmonic component forms of (4.21), (4.22) and (4.23) can be developed for a triple variable controlled waveforms in time-varying $h_{uv}(t)$ as four different cases:

1) When v = 1, $u \in \{1, 2, 3\}$

$$h_{u1}(t) = \frac{A_{000}}{2} + \sum_{m=1}^{\infty} \left[A_{0m0} \cos\left(m\left(\omega_{m1}t - (u+2)\frac{2\pi}{3}\right)\right) \right]$$

$$+\sum_{n=1}^{\infty} \left[A_{00n} \cos\left(n \left(\omega_m t - (u+2)\frac{2\pi}{3} \right) \right) \right]$$
(4.24)

$$+\sum_{k=1}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty}\sum_{\substack{n=-\infty\\m\neq 0}}^{\infty}\left[A_{kmn}\cos\left(k\omega_{c}t+m\left(\omega_{m1}t-(u+2)\frac{2\pi}{3}\right)+n\left(\omega_{m2}t-(u+2)\frac{2\pi}{3}\right)\right)\right]$$

2) When $v = 2, u \in \{1,3\}$

$$h_{u2}(t) = \frac{A_{000}}{2} + \sum_{m=1}^{\infty} \left[A_{0m0} \cos\left(m \left(\omega_{m1}t - (u)\frac{2\pi}{3}\right)\right) \right] + \sum_{n=1}^{\infty} \left[A_{00n} \cos\left(n \left(\omega_{m}t - (u+1)\frac{2\pi}{3}\right)\right) \right]$$
(4.25)

$$+\sum_{k=1}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty}\sum_{\substack{n\neq 0\\m\neq 0}}^{\infty}\left[A_{kmn}^{'}\cos\left(k\omega_{c}t+m\left(\omega_{m1}t-(u)\frac{2\pi}{3}\right)+n\left(\omega_{m2}t-(u+1)\frac{2\pi}{3}\right)\right)\right]$$

3) When $v = 2, u \in \{2\}$

$$h_{u2}(t) = \frac{A_{000}}{2} + \sum_{m=1}^{\infty} \left[A_{0m0} \cos\left(m \left(\omega_{m1}t - (u)\frac{2\pi}{3}\right)\right) \right] + \sum_{n=1}^{\infty} \left[A_{00n} \cos\left(n \left(\omega_{m}t - (u+1)\frac{2\pi}{3}\right)\right) \right]$$
(4.26)

$$+\sum_{k=1}^{\infty}\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty}\sum_{\substack{n=-\infty\\m\neq 0}}^{\infty}\left[A_{kmn}^{''}\cos\left(k\omega_{c}t+m\left(\omega_{m1}t-(u)\frac{2\pi}{3}\right)+n\left(\omega_{m2}t-(u+1)\frac{2\pi}{3}\right)\right)\right] +B_{kmn}^{''}\sin\left(k\omega_{c}t+m\left(\omega_{m1}t-(u)\frac{2\pi}{3}\right)+n\left(\omega_{m2}t-(u+1)\frac{2\pi}{3}\right)\right)\right]$$

4) When $v = 3, u \in \{1, 2, 3\}$

$$h_{u3}(t) = \frac{A_{000}}{2} + \sum_{m=1}^{\infty} \left[A_{0m0} \cos\left(m \left(\omega_{m1}t - (u+1)\frac{2\pi}{3}\right)\right) \right] + \sum_{n=1}^{\infty} \left[A_{00n} \cos\left(n \left(\omega_{m}t - (u)\frac{2\pi}{3}\right)\right) \right] + \sum_{k=1}^{\infty} \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} \left[A_{kmn} \cos\left(k\omega_{c}t + m \left(\omega_{m1}t - (u+1)\frac{2\pi}{3}\right) + n \left(\omega_{m2}t - (u)\frac{2\pi}{3}\right) \right) \right]$$

$$(4.27)$$

Where the spectral coefficients are defined by a triple Fourier integral analysis as follows:

$$A_{000} = \frac{2}{3}, \ A_{010} = A_{001} = \frac{q}{3}$$

$$A_{kmn} = \frac{2 \times j_m \left(\frac{k\pi \cdot q}{3}\right) j_n \left(\frac{k\pi \cdot q}{3}\right)}{k\pi} \times \sin\left[\left(\frac{k}{3} + \frac{m+n}{2}\right)\pi\right]$$

$$A_{kmn}' = A_{kmn} \times \cos\left[(m+n)\frac{2\pi}{3}\right] \times \left[\sin\left(\left(\frac{2k}{3} - \frac{m}{2} - \frac{n}{2}\right)\pi\right) - \sin\left(\left(\frac{k}{3} + \frac{m+n}{2}\right)\pi\right)\right]$$

$$B_{kmn}' = A_{kmn} \times \sin\left[(-m-n)\frac{2\pi}{3}\right] \times \left[\sin\left(\left(\frac{2k}{3} - \frac{m}{2} - \frac{n}{2}\right)\pi\right) + \sin\left(\left(\frac{k}{3} + \frac{m+n}{2}\right)\pi\right)\right]$$

$$A_{kmn}^{"} = A_{kmn} \left[\sin\left(\left(\frac{2k}{3} - \frac{m}{2} - \frac{n}{2}\right)\pi\right) \times \cos\left((m+n)\frac{4\pi}{3}\right) - \sin\left(\left(\frac{k}{3} + \frac{m+n}{2}\right)\pi\right) \times \cos\left((m+n)\frac{2\pi}{3}\right) \right]$$
$$B_{kmn}^{"} = A_{kmn} \left[-\sin\left(\left(\frac{2k}{3} - \frac{m}{2} - \frac{n}{2}\right)\pi\right) \times \sin\left((-m-n)\frac{4\pi}{3}\right) + \sin\left(\left(\frac{k}{3} + \frac{m+n}{2}\right)\pi\right) \times \sin\left((-m-n)\frac{2\pi}{3}\right) \right]$$

4.3 Analytical Spectra of Output Voltages and Input Currents

Let $f_1(t)$ and $f_2(t)$ be two functions of t. The product of the two time-domain functions is denoted by

$$f(t) = f_1(t) \times f_2(t)$$
(4.28)

The convolution process can be illustrated by transforming the product defined in (4.28) into frequency domain $F(\omega)$, which can be obtained to a convolution integral of individual spectrum in the frequency domain as follows

$$F(\omega) = F_1(\omega) \otimes F_2(\omega)$$

$$= \int_{-\infty}^{\infty} F_1(\sigma) \times F_2(\omega - \sigma) d\sigma$$
(4.29)

where $F_1(\omega)$ and $F_2(\omega)$ are Fourier transforms of $f_1(t)$ and $f_2(t)$, respectively.

This principle can be applied to obtain the spectra of the synthesized output voltages due to the fact that they are the summation of the products of the time-domain switching functions and the input voltages as described in (4.6). In a similar manner, the spectra of the synthesized input currents as described in (4.7) can be determined.

For the case where the input voltage waveforms are sinusoid defined by (2.3), the corresponding spectrum can be calculated by

$$V_{iu}(\omega) = \frac{V_i}{2} \left[e^{-j(u-1)2\pi/3} \delta(\omega - \omega_i) + e^{j(u-1)2\pi/3} \delta(\omega + \omega_i) \right]$$
(4.30)

where
$$\delta(\omega) = \begin{cases} 1 & \text{if } \omega = 0 \\ 0 & \text{otherwise} \end{cases}$$
 (4.31)

Also, the spectrum of the switching functions $h_{uv}(x, y, z)$ that is defined by (4.8), can be represented as a function of ω .

$$H_{uv}(\omega) = \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left[H_{kmn}(u,v) \times \delta(\omega - (k\omega_c + m\omega_{m1} + n\omega_{m2})) \right]$$
(4.32)

where $H_{kmn}(u, v)$ is defined by (4.21), (4.22) and (4.23).

Therefore, the spectra of the synthesized output voltages $V_{ou}(\omega)$ can be easily determined since the spectrum of the input voltages switching functions and switching functions are known in (4.30) and (4.32) respectively.

$$V_{ou}(\omega) = \sum_{\nu=1}^{3} \left[\int_{-\infty}^{\infty} H_{u\nu}(\sigma) \times V_{iu}(\omega - \sigma) d\sigma \right]$$
$$= \frac{V_i}{2} \sum_{\nu=1}^{3} \left[e^{-j(u-1)2\pi/3} H_{u\nu}(\omega - \omega_i) + e^{j(u-1)2\pi/3} H_{u\nu}(\omega + \omega_i) \right]$$
(4.33)

Since the output current waveforms are sinusoid defined by (2.4), the spectra of the synthesized input currents as described in (4.7) can be determined as follows:

$$I_{iv}(\omega) = \sum_{u=1}^{3} \left[\int_{-\infty}^{\infty} H_{uv}(\sigma) \times I_{ov}(\omega - \sigma) d\sigma \right]$$

= $\frac{I_o}{2} \sum_{u=1}^{3} \left[e^{-j(v-1)2\pi/3} H_{uv}(\omega - \omega_o) + e^{j(v-1)2\pi/3} H_{uv}(\omega + \omega_o) \right]$ (4.34)

It can be seen that the equations (4.33) and (4.34) are very important results because they illustrate the spectral of the synthesized output voltages and input currents that are simply the superposition of the nine switching functions spectrum with frequency shifts of $\pm \omega_i$.

4.4 Analytical and Numerical Results

The analytical results have been verified against the numerical simulation based on FFT. Table 4.1 illustrates the list of parameters used in the simulation.

Input frequency	f_i	60 Hz
Output frequency	f_o	50 Hz
Input voltage amplitude	V _i	100 V
Output current amplitude	I _o	50 A
Carrier frequency	f_c	1260 Hz
Voltage gain ratio	q	0.5

Table 4.1 Illustration multiple unit-cube for one switching cycle.

The analytical spectra of the nine switching functions have been compared with the numerical spectra that obtained from FFT in Figure 4.1to Figure 4.3, which clearly show a very good agreement between the analytical and numerical results.



Figure 4.1 Analytical and numerical spectra of switching functions h_{11} , h_{12} and h_{13} .



Figure 4.2 Analytical and numerical spectra of switching functions h_{21} , h_{22} and h_{23} .



Figure 4.3 Analytical and numerical spectra of switching functions h_{31} , h_{32} and h_{33} .

Figure 4.4 illustrates the analytical and numerical spectra of phase-to- neutral output voltages whereas the analytical and numerical spectra of synthesized input currents shows in Figure 4.5. Again, a very good agreement between analytical and numerical spectra has been observed for both the synthesized output voltages and input currents.



Figure 4.4 Analytical and numerical spectra of switching functions V_{o1} , V_{o2} and V_{o3} .

Figure 4.4 (cont'd)





Figure 4.5 Analytical and numerical spectra of switching functions I_{i1} , I_{i2} and I_{i3} .

4.5 Total Harmonic Distortion (THD) and Weighted Total Harmonic Distortion (WTHD)

THD is the summation of all the harmonic components of the voltage or current waveform compared against the fundamental components of the voltage or current waveform which can be obtained as follows:

$$THD = \frac{1}{V_1} \sqrt{\sum_{n=2,3,\dots}^{\infty} \left[V_n^2 \right]}$$
(4.35)

On the other hand, *WTHD* is needed to compare harmonic distortions that caused mainly by lower order harmonics and it is defined as follows:

$$WTHD = \frac{1}{V_1} \sqrt{\sum_{n=2,3,...}^{\infty} \left[\frac{V_n^2}{n^2} \right]}$$
(4.36)

The two formula above show the calculation for THD and WTHD on a voltage waveform where V_1 is the amplitude of fundamental component n is the order of harmonic, and V_n is the amplitude of *nth* harmonic components. Equations (4.35) and (4.36) can be also utilized to calculate THD and WTHD for a input current waveform.

Figure 4.6 and Figure 4.7 show the *THD* versus the voltage gain ratio of output voltages and input currents respectively. It can be seen from both figures how the harmonic significance reduces with increasing the voltage ratio gain for both output voltages and input currents.



Figure 4.6 (a) THD versus voltage gain of output voltages (phase A-C), (b) (phase B).



Figure 4.7 (a) THD versus voltage gain of input currents (phase A-C), (b) (phase B).

Table 4.2 and Table 4.3 describe the THD magnitudes of Figure 4.6 and Figure 4.7 respectively. It can be noted from both tables that as the voltage gain increases, the of the output voltages and input currents will decrease.

Voltage gain ratio q	THD (Phases A - C)	THD (Phase B)
0.1	22.242	22.353
0.2	11.339	11.514
0.3	7.701	7.879
0.4	5.837	5.982
0.5	4.683	4.793

Table 4.2 THD magnitudes of three phase output voltages.

Table 4.3 THD magnitudes of three phase input currents.

Voltage gain ratio q	THD (Phases A - C)	THD (Phase B)
0.1	36.973	32.446
0.2	18.415	15.337
0.3	12.118	9.38
0.4	8.877	6.355
0.5	6.899	4.633

The WTHD versus the voltage ratio are summarized for the output voltages and input currents in Figure 4.8 and Figure 4.9, respectively. It can also be noted that the harmonic presented in Figure 4.8 and Figure 4.9 reduces as the voltage ratio (q) increases for both output voltages and input currents.



Figure 4.8 (a) WTHD versus voltage gain of output voltages (phase A-C), (b) (phase B).



Figure 4.9 (a) WTHD versus voltage gain of input currents (phase A-C), (b) (phase B).

Table 4.4 describes the WTHD magnitudes of Figure 4.8. It can be seen that the output voltages decrease as the voltage gain decreases. Whereas, Table 4.5 represents the WTHD magnitudes of Figure 4.9. The input currents also decrease with increasing in voltage gain.

Voltage gain ratio q	WTHD (Phases A - C)	WTHD (Phase B)
0.1	0.876	0.878
0.2	0.436	0.439
0.3	0.288	0.292
0.4	0.213	0.218
0.5	0.168	0.174

Table 4.4 WTHD magnitudes of three phase output voltages.

Table 4.5 WTHD magnitudes of three phase input currents.

Voltage gain ratio q	WTHD (Phases A - C)	WTHD (Phase B)
0.1	1.62	0.773
0.2	0.802	0.365
0.3	0.525	0.224
0.4	0.385	0.151
0.5	0.3	0.11

4.6 Summary

This chapter has presented an analytical approach based on three-dimensional Fourier integral for characterizing the spectra of the synthesized output voltages and input currents of the matrix converter which was proposed by Alesina and Venturini 1981 [**3**]. The analytical spectra of the synthesized output voltages and input currents has been observed a very good agreement

with the numerical spectra obtained from FFT. The analytical and numerical spectra of the nine switching functions has been shown a very good result as well.

CHAPTER 5

Conclusions and Future Work

5.1 Conclusions

This thesis has presented an analytical method based on three-dimensional Fourier integral to obtain accurate spectra of the switching functions and synthesized terminal qualities of matrix converters.

The high frequency modulation of the matrix converter which was proposed by Alesina and Venturini 1981 has been considered in this analysis [**3**]. The analytical spectra of the nine switching functions have been compared first with the numerical spectra obtained from FFT. The results shows very good agreement between analytical and numerical spectra for each switching functions. Then, the spectra of the synthesized output voltages and input currents have been verified against the numerical spectra based on FFT. Again, a very good agreement between the analytical and numerical spectra has been observed for both the synthesized output voltages and input currents.

5.2 Future Work

PWM strategies for matrix converters have been received increasing attention recently. The analytical approach of the three-dimensional Fourier integral which is presented in this thesis, is

demonstrated through the carrier based pulse width modulation of a conventional matrix converter.

It is worth nothing that the analytical approach is equally applicable to space vector based modulation once the equivalent modulation functions are determined. In addition, the modulation process of indirect matrix converters can also be analytically characterized using the proposed approach in this thesis.

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